PN-ABE-323
64789

A Dual Approach to the Theory of Trade and Growth

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August 1988

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1. Introduction

The neoclassical theory of trade and growth combines the theory of growth developed for the closed economy and the theory of trade based on the Heckscher-Ohlin framework. The factor proportion theory of the Heckscher-Ohlin model requires a two-sector, two-factor framework in which one sector produces the exportable and the other the importable. Because of the similarity in the production structure, most of the dynamic Heckscher-Ohlin models represent the extension of the two-sector models of growth developed by Uzawa (1961, 1963) and others into an open economy. These two-sector growth models are an outgrowth of the one-sector models originally developed by Solow (1956) and Swan (1956).[1]

A dynamic Heckscher-Ohlin model was first introduced by Oniki and Uzawa (1965). They examined a two-country model of trade and growth in which both countries have identical natural rates of growth and identical technologies (production functions) within each sector between the two countries. By assuming that each country saves a constant proportion of its national income and that the consumption good sector is more capital intensive than the capital good sector (the so-called capital intensity condition), they characterized the regions of specialization in the world economy and showed that the world steady-state equilibrium is unique and globally stable. Since there is a possibility of complete specialization for certain ranges of commodity prices and capital-labor ratios, they overcame the inherent complexity by making extensive use of the offer functions for the two growing

countries. Their technique of analysis has since been widely used in the literature.

Kemp (1970) made an important extension of the Oniki and Uzawa model by allowing unequal natural rates of growth between the two countries.[2] He showed that the analysis of the dynamic growth path becomes much more difficult since the steady-state loci of the phase diagrams shift over time. He also showed that with the capital intensity condition, the country with a higher rate of growth will be producing both goods in the long run, and that the world steady-state equilibrium is impossible although each country asymptotically approaches that equilibrium. In addition, he showed that for both countries to be incompletely specialized, it is necessary that $n/(\alpha/\theta_K) = n^*/(\alpha^*/\theta_K^*)$, where n and n^* are the home and foreign country's rates of population growth, α and α^* are the home and foreign country's average savings rates out of national income, and θ_K and θ_K^* are the home and foreign country's capital shares in national income, respectively.

Bardhan (1965) introduced a different model of trade and growth. By employing a classical savings function in which a country's saving consists of a constant fraction out of profit income but none out of wage income, he examined a model where the rates of growth are identical, but the technologies may be different between the two countries. He proved that if both countries are completely specialized, the world steady-state equilibrium is globally stable. On the other hand, if both countries are incompletely specialized, equilibrium may not be unique and stable. Bardhan's a priori specification of the patterns of specialization was later criticized by Takayama (1972), who analyzed Bardhan's model under the assumption that the consumption good sector is less capital intensive than the capital good sector. He completely characterized the patterns of specialization and concluded that, in general, one country will lead to complete specialization as time goes to infinity, and

that global stability of the world steady-state equilibrium is assured.

The generalization of Bardhan's model to the case of unequal natural rates of growth was attempted by Khang (1971). By employing the loci-shifting phase diagram technique similar to the one used by Kemp (1970), Khang obtained many interesting conclusions. For example, with the capital intensity condition and assuming identical technologies between the two countries, the country that has a higher rate of growth cannot become completely specialized in the long run. Both countries' capital-labor endowment ratios asymptotically approach a world steady-state equilibrium, and for both countries to be incompletely specialized in the long run, it is necessary that $n/s = n^*/s^*$, where n and n^* are the home and foreign country's natural rates of growth, and s and s^* are the home and foreign country's saving propensities out of profit income.

There have been many other contributions to the theory of trade and growth. For example, by using the geometric method, Johnson (1971) examined a two-country model of growth and trade; Vanek (1971) proved that an increase in the price of the consumption good which is more capital intensive will raise the steady-state capital-labor ratio of a small open economy; and Deardorff (1974) analyzed the small country case by using an aggregated per-worker production function in the two-sector economy. In addition, there have been studies examining the changes in the steady-state levels of per capita consumption between equilibria in autarky and free trade.[3]

Although the neoclassical models of trade and growth have been extensively researched in the literature, most of the models are quite complicated in nature. The purpose of this paper is to provide a dual approach to the theory of trade and growth. It will be shown that some of the analysis can be greatly simplified and new insight can be gained by the use of such an approach. Recent contributions by Woodland (1977) and Mussa (1979)

have shown that the structure of the two-sector general equilibrium model can be conveniently analyzed in terms of its dual. Their analyses, however, have been confined to the static framework. The present paper extends the dual approach to the context of growth and trade by examining the popular model with a classical savings function, which has been previously examined by Bardhan (1965), Khang (1971), Tukayama (1972), and Smith (1984), among others. In this paper, the distinction between the short-run and long-run impacts of trade and growth will be made clear. The time paths of the real wage rate and the interest rate can be conveniently illustrated, and the process of capital accumulation can be readily determined by the way the specialization regions are characterized in this paper. The dual approach makes it easier to compare the autarkic and free trade steady-state capital-labor ratios as well as per capita consumptions. In particular, when neither country is small in the sense of being a price taker, the present paper makes it easier to compare the long-run patterns of specialization and the long-run determinants of trade.

Section 2 of the paper presents the basic model and derives the structural relationships among the various variables. The autarkic steady-state equilibrium is described in this section. Section 3 analyzes the patterns of specialization, the dynamic adjustments of the real wage rate and the interest rate, and the time path of the country's capital-labor endowment ratio. The steady-state equilibrium for the case of a small open economy is established. Section 4 examines a two-country world equilibrium and analyzes the determinants of long-run comparative advantage and patterns of specialization. Section 5 examines the effect of trade and growth on a country's steady-state levels of per capita consumption. Finally, some concluding remarks are given in Section 6.

2. The Model and Autarkic Steady-State Equilibrium

Consider a neoclassical model of trade and growth in which there are two

sectors, one producing the investment good (sector 1) and the other the consumption good (sector 2). Each sector employs capital and labor with a constant-returns-to-scale technology. Assume that the population grows at a constant rate, n, and that a constant fraction, s, of capitalists' income is saved and none is saved out of wage income. Let p_j be the nominal price of the jth good, W the nominal wage rate, and R the nominal rental rate to capital. For convenience, the investment good is chosen as the numeraire. Let w $(=W/p_1)$ be the real wage rate in terms of good 1, and p $(= p_2/p_1)$ be the relative price of consumption good. Without loss of generality, assume that capital does not depreciate. Then the rate of interest, i, is equal to R/p_1. Under perfect competition, the zero-profit conditions are:

(1) $$Wa_{L1} + Ra_{K1} = p_1$$

(2) $$Wa_{L2} + Ra_{K2} = p_2,$$

where a_{ij} is the amount of the ith factor needed to produce one unit of the jth good. Full employment of both factors of production implies

(3) $$a_{K1}X_1 + a_{K2}X_2 = K$$

(4) $$a_{L1}X_1 + a_{L2}X_2 = L,$$

where X_j is the output produced in the jth sector.

Let dK/dt be the time rate of change of capital stock. Then the equality between saving and investment requires

(5) $$dK/dt = siK.$$

Cost minimization implies that each a_{ij} is a function of W and R, namely, $a_{ij} = a_{ij}(W, R)$. Since a_{ij} is homogeneous of degree zero in W and R, we have $a_{ij}(W, R) = a_{ij}(W/p_1, R/p_1) = a_{ij}(w, i)$. Therefore, (1) and (2) can be rewritten as

(1') $$wa_{L1}(w, i) + ia_{K1}(w, i) \equiv c_1(i, w) = 1$$

(2') $$wa_{L2}(w, i) + ia_{K2}(w, i) \equiv c_2(i, w) = p.$$

At a constant p, (1') and (2') each defines a factor-price curve in the (i, w)

space. By using the first-order conditions $Wda_{Lj} + Rda_{Kj} = p_1(wda_{Lj} + ida_{Kj}) = 0$, we have $\partial c_j/\partial i = a_{Kj}$ and $\partial c_j/\partial w = a_{Lj}$, $j = 1, 2$. Therefore, the slope of each factor-price curve is the negative of that sector's capital-labor ratio, $k_j (\equiv a_{Kj}/a_{Lj})$:

(6) $$dw/di = -k_j, \qquad j = 1, 2.$$

Moreover, each factor-price curve is convex since[4]

(7) $$d^2w/di^2 = k_j\sigma_j/(i\theta_{Lj}) \geq 0,$$

where σ_j is the elasticity of factor substitution in the jth sector, and θ_{Lj} is the labor share in sector j. In the fixed coefficient case, $\sigma_j = 0$ and the factor-price curve becomes a straight line.

Fig. 1 depicts the two factor-price curves. The $c_1 = 1$ curve is independent of p but the $c_2 = p$ curve will shift outwards when p is increased. The shifting is a radial expansion since at constant w/i, each a_{ij} is a constant. For both sectors to make zero profit, the equilibrium point must be at A when the autarkic price is p^a. The corresponding i^a and w^a are the autarkic equilibrium i and w associated with the equilibrium p^a.[5] Sector 2's factor-price curve is drawn to be steeper than that of sector 1. This implies that sector 2 is more capital intensive than sector 1. The normals to the two curves at A are labeled v_1 and v_2, respectively. They are the input-output coefficient vectors, $v_j = (a_{Kj}, a_{Lj})$.

(Fig. 1 may be inserted around here)

The autarkic short-run equilibrium is characterized by the five equations (3), (4), (1'), (2') and $X_1 = siK$. They determine the five variables w, i, p, X_1, and X_2, given K and L. In view of (5), the dynamic adjustment of the economy's capital-labor ratio, k, is determined by

(5') $$dk/dt = (si - n)k.$$

The autarkic steady-state equilibrium in which capital and labor grow at the same rate is therefore characterized by

(8) $i = n/s$.

Once i is determined, w^a and p^a are determined by (1') and (2').

Denoting the per capita output in the jth sector by x_j ($\equiv X_j/L$), we can rewrite (3) and (4) as

(3') $a_{K1}x_1 + a_{K2}x_2 = k$

(4') $a_{L1}x_1 + a_{L2}x_2 = 1$.

Once i^a and w^a are determined, a_{ij} are also determined. Therefore, (3') and (4') together with $x_1 = sik$ jointly determine x_1^a, x_2^a, and k^a. By viewing (3') and (4') as a system of $x_1v_1 + x_2v_2 = v$, where $v = (k, 1)$, we see that v is a linear combination of the two sectoral input-output coefficient vectors v_1 and v_2. Since $x_j^a > 0$, v must lie in the interior of the cone formed by v_1 and v_2. x_j can be measured along the extended rays of the v_j vectors. This is done by using point A and the end point of vector v to form a parallelogram with the extended rays of v_1 and v_2 as its two sides. The scales on the extended v_j rays can be set in such a way that the end point of v_j measures one unit of x_j.

The comparative dynamics of autarkic steady-state equilibrium can be easily analyzed. Fig. 1 shows that the autarkic steady-state equilibrium is determined by n/s. If n/s is increased, i^a must also increase. Since sector 2 is capital intensive, its relative price must go up. This implies that the $c_2 = p^a$ curve must shift outwards as a result of an increase in p^a. Point A therefore slides down along the $c_1 = 1$ curve. At the new point A (not shown in Fig. 1) with a higher i^a and a lower w^a, both sectoral capital-labor ratios, k_j, are decreased. The cone v_1v_2, having slid down along the $c_1 = 1$ curve, has rotated in the counter clockwise direction. Moreover, x_2^a is increased and x_1^a decreased as a result of an increase in p^a. It should be noted that in the present case where the consumption good sector is more capital intensive than the capital good sector, the steady-state capital-labor

ratio is known to be positively related to the nominal wage/rental, and hence w/i, ratio. Thus in the present case, a nation with a lower savings rate or a higher natural rate of growth will have a lower autarkic steady-state capital-labor ratio.

3. Patterns of Specialization, Dynamic Adjustment, and Steady-State Growth

This section examines the short-run and long-run impacts of trade on a small open economy. The economy is said to undergo short-run adjustments when its factor endowments remain unchanged while its factor prices, sectoral capital-labor ratios, and outputs respond to a change in the commodity price ratio. On the other hand, the economy is said to undergo long-run adjustments when its factor endowments also change over time. Assume that the economy is initially in an autarkic steady-state equilibrium with i^a, w^a, and p^a shown in Fig. 2, which maintains the assumption that the consumption good sector is more capital intensive than the capital good sector. Consider now the state in which the *economy*'s factor endowments are unchanged and that the commodity price p is increased as a result of trade. The c_2 curve shifts up and point A slides down along the c_1 curve. If the economy is still producing both goods, the new i and w are determined by the new intersection point of the c_1 and c_2 curves. Thus a positive relation between p and i can be traced out in the lower half of Fig. 2. As i is increased and w decreased, both sectoral capital-labor ratios are decreased. When p is sufficiently high at p^a_{max}, the economy begins to specialize in good 2. The factor prices i and w are then determined by point S_2 where k_2 is equal to the given k and therefore $v = x_2 v_2$. For $p > p^a_{max}$, the economy continues to specialize in good 2. But i becomes related to p by a straight line such as $S_2'B'$, whose extension will go through the origin. This is because $i = pf_2'(k_2)$, where $f'_2(k_2)$ is the marginal product of capital in sector 2, with k_2 equal to the constant k. Moreover, since $w = p[f_2(k_2) - k_2 f'_2(k_2)]$, w, i, and p will all change by the

same proportion whenever p is changed in this specialized range. Therefore, w and i are also linked by a straight line such as S_2B, whose extension should also go through the origin. On the other hand, if p is sufficiently low, reaching the value p^a_{min}, the economy moves to S_1 and becomes completely specialized in good 1. At S_1, $k_1 = k$ and $v = x_1v_1$. As p is further decreased, the $c_2 = p$ curve shifts further downwards and the economy will continue specializing in good 1. Thus i and w are fixed at the point S_1 for any $p \leq p^a_{min}$. The relation between i and p is therefore a vertical line labeled as $i_{min}S'_1$ in Fig. 2. The characterization of the regions of specialization for a given k is now complete. For any value of p, the associated i and w can now be readily found from the S_1AS_2B curve.

(Figure 2 may be inserted around here)

It is easy to see that points S_1 and S_2 are determined by the particular value of the autarkic steady-state capital-labor endowment ratio, k^a. A higher value of k^a will shift both S_1 and S_2 up along the c_1 curve, and the corresponding values of p^a_{min} and p^a_{max} will both decrease. Hence the points S'_1 and S'_2 in the *lower* half of the diagram will move with S_1 and S_2, respectively. p^a_{min} and p^a_{max} therefore signify the particular values of p_{min} and p_{max} determined by $k = k^a$.

The case where $k_2 < k_1$ is shown in Fig. 3. For the incomplete specialization region ($p^a_{min} < p < p^a_{max}$), p and i have a negative relationship. This is because as p is increased, the $c_2 = p$ curve moves upwards and point A also moves up along the $c_1 = 1$ curve. When $p = p^a_{max}$, the economy becomes completely specialized in good 2. A further increase in p beyond p^a_{max} will cause both i and w to grow at the same rate as p. We therefore have the two line segments S_2B and S'_2B', whose extensions will both go through the origin. On the other hand, when $p \leq p^a_{min}$, the economy specializes in good 1 and remains at point S_1. Again for any value of p, the short-run i and w can be

obtained from the corresponding point on the S_1AS_2B curve. If k^a is increased, both S_1 and S_2 will move up along the c_1 curve. The corresponding values of p^a_{min} and p^a_{max} will be higher.

(Figure 3 may be inserted around here)

For a small open economy, the short-run impact of trade on its factor prices and on its patterns of specialization can be readily inferred from Figs. 2 and 3. The dynamic adjustment to a long-run equilibrium can now be conveniently analyzed also. Consider first the case where $k_2 > k_1$ and assume that the small open economy faces a free trade price p^t which is greater than p^a. Fig. 4 illustrates a short-run equilibrium at point E, with associated $i = \tilde{i}$. The economy at E specializes in good 2. Since $\tilde{i} > n/s$, $dk/dt > 0$ by (5'). Thus the increase in k will induce an increase in w and a decrease in i, resulting in an increase in k_2. The economy therefore moves along the $c_2 = p^t$ curve from E to the free-trade equilibrium point F where the steady-state rate of interest, i^t, is equal to n/s, and k has converged to the new steady-state value k^t. An increase in the price of the capital-intensive good as a result of trade thus results in a temporary jump of i to $\tilde{i}$ and then a gradual decrease of i to the autarkic steady-state value. On the other hand, the short-run impact effect of trade on w is ambiguous. As one can easily verify from Fig. 4, the short-run value of w may or may not increase depending on whether or not p is sufficiently increased. However, as k is increased over time in the long run, w will increase until w^t is reached, which is greater than w^a. Since $w^t/i^t > w^a/i^a$, we have $k^t = k^t_2 > k^a_2 > k^a$.

(Figure 4 may be inserted around here)

If the opening of trade results instead in a decrease in p, the economy may instantly move, for example, from A to E'. At E', the associated $\tilde{i}' < n/s$ and k will decrease. Eventually, k will decrease to the value of k'_1 which is determined by the values of $\tilde{i}'$ and $\tilde{w}'$. The economy will then become

completely specialized in the investment good. A continued decrease in k will result in further increases in i and decreases in w, and k_1 will therefore decrease. Thus the economy will move down from E' towards A along the S_1S_2 curve which is a part of the $c_1 = 1$ curve. In this case, $i^t = i^a$ and $w^t = w^a$, but $k^t < k^a$ since $k^t = k_1^a < k^a$.

The other case where $k_2 < k_1$ can be similarly analyzed. As in the preceding case where $k_2 > k_1$, the free trade steady-state equilibrium will be characterized by a specialization in the consumption or investment good depending upon whether p^t is greater or less than p^a. There is, however, one important difference. If $k_2 > k_1$, i is a nondecreasing function of p; therefore, k increases or decreases depending upon p^t is greater or less than p^a. In the case where $k_2 < k_1$, i is a nonincreasing function of p when $p < p^a_{max}$, but is an increasing function of p when $p > p^a_{max}$. As shown in Fig. 3, if $p^t < p^a$ or $p^t > \bar{p}$, the associated short-run i will be greater than n/s. This will lead to increases in k, resulting in $k^t > k^a$. On the other hand, if $p^a < p^t < \bar{p}$, k will decrease and $k^t < k^a$. The steady-state value of i, i^t, is equal to i^a no matter what is the value of p^t. However, w^t is higher than w^a if $p^t > p^a$ and is equal to w^a if $p^t < p^a$.

4. The Two-Country Case

Consider now the long-run impact of trade and growth in a two-country world. Assume that the technologies are identical between the two countries and that each country is initially at its autarkic steady-state equilibrium. Let the superscript * denote the foreign country. Then n^* is the foreign country's natural rate of growth, s^* is its savings rate out of profit income, and p^{*a} is its autarkic steady-state price ratio. Its autarkic steady-state interest rate, i^{*a}, is equal to n^*/s^*.

Consider the case where $n = n^*$ and $s > s^*$. This implies that $i^a = n/s < n^*/s^* = i^{*a}$. In the case where $k_2 > k_1$, the autarkic steady-state equilibria

are illustrated in Fig. 5. The home and foreign country's equilibrium points are at A and A^*, respectively. Therefore, we have $p^a < p^{*a}$.[6] Thus the high saving country is seen to have a comparative advantage in the capital-intensive good. Since $w^a/i^a > w^{*a}/i^{*a}$, we have $k^a > k^{*a}$ in the present case. This follows from the well-known result that under the classical savings function and the capital intensity condition, the equilibrium endowment ratio, k, is an increasing function of w/i.

(Figure 5 may be inserted around here)

Once the world economy is open to free trade, it undergoes long-run adjustments in both countries. Let p^t (= p^{*t}) be the final free trade price facing the two countries.[7] p^t must be bounded by p^a and p^{*a}. Suppose that $p^a < p^t < p^{*a}$ as illustrated in Fig. 5. By using the results obtained in Section 3, we know that the home country's free trade equilibrium point is at F and that of foreign country at F^* which is the same point as A^*. It follows that $w^t > w^a$, $i^t = i^a$, $w^{*t} = w^{*a}$, and $i^{*t} = i^{*a}$. Moreover, it can be verified that $k^t > k^a$ and $k^{*t} < k^{*a}$. The home country specializes in the production of the consumption good and the foreign country in the other good.

The other case where $k_2 < k_1$ can be similarly analyzed. In this case, the home country will have a long-run comparative advantage in good 1 which is capital intensive, given the assumption that $n/s < n^*/s^*$. Combining the two cases, we conclude that if both countries have the same natural rates of growth, the country with a higher savings rate will have a long-run comparative advantage in the capital-intensive good, while the other country will have a long-run comparative advantage in the labor-intensive good.

If the natural rates of growth are different between the two countries, the crucial determinants of the long-run comparative advantage are again n/s and n^*/s^*. The country having the lower ratio of its natural rate of growth to its savings rate out of profit income will have a long-run comparative

advantage in the capital-intensive good.

Finally, the long-run pattern of specialization is affected by the natural rates of growth. The country with a higher natural rate of growth will eventually become a "large" one. As pointed out by Khang (1971), the large country cannot be completely specialized in the long run. Moreover, as long as $n/s \neq n^*/s^*$, at least one country must be completely specialized in the long run.

5. Steady-State per Capita Consumption

This section compares the steady-state levels of per capita consumption between free trade and autarky. Let z be the per capita consumption:

(9) $$z = [(1 - s)ik + w]/p.$$

Differentiating (9), we have

(10) $$pdz = (1 - s)kdi + dw - zdp + (1 - s)idk.$$

Also differentiating (1') and (2'), and using the first-order conditions $wda_{Lj} + ida_{Kj} = 0$, we obtain

(11) $$a_{K1}di + a_{L1}dw = 0$$

(12) $$a_{K2}di + a_{L2}dw = dp.$$

Multiplying (11) by x_1 and (12) by x_2, adding the resulting two equations, and using the full employment conditions (3') and (4'), we have

(13) $$dw = x_2dp - kdi.$$

Substituting (13) into (10), and noting that $di = 0$ since $i^t = i^a = n/s$, we obtain the fundamental equation:

(14) $$pdz = (x_2 - z)dp + (1 - s)idk.$$

dz represents the comparative steady states $z^t - z^a$. It is seen to be influenced by the terms-of-trade effect (dp) and the accumulation effect (dk).

We can now apply the results obtained in Section 3 to determine the sign of dz. As shown in Section 3, if $dp > 0$, the economy will specialize in good 2; hence $x_2 - z$ is positive. On the other hand, if $dp < 0$, it will specialize

in good 1; therefore, $x_2 = 0$ and $x_2 - z < 0$. It follows that $(x_2 - z)dp$ is always positive. Section 3 also shows that $dk > 0$ if either (i) $k_2 > k_1$ and $dp > 0$, or (ii) $k_2 < k_1$ and $dp < 0$. It follows that $z^t > z^a$ if either (i) or (ii) holds. In the case where $k_2 > k_1$ and $dp < 0$, we have $dk < 0$; therefore, the sign of dz is ambiguous. Finally, in the case where $k_2 < k_1$ and $dp > 0$, the sign of dk depends upon the magnitude of dp. From Fig. 3, we see that if $dp > \bar{p} - p^a$, then $dk > 0$ and we have $dz > 0$. On the other hand, if $0 < dp < \bar{p} - p^a$, then $dk < 0$, and dz can be of any sign.

Trade may result in a lower steady-state per capita consumption in free trade than in autarky. As Fig. 5 shows, if the natural rates of growth are equal in the two countries, the "impatient" one (the foreign country since $s^* < s$) saves less and has a higher rate of interest in autarkic steady-state equilibrium. Through trade, it experiences a sudden reduction in the rate of interest, and eventually a specialization in the labor-intensive good. This entices the "impatient" country to consume more, resulting in a decline in its capital-labor ratio. However, the phenomenon of a lower steady-state per capita consumption in free trade than in autarky does not vitiate the notion of the gains from trade. The intertemporal gains from trade must still obtain, with the "impatient" country having the tendency to take the gains from trade in the present rather than in the future. This point has been elaborated by Smith (1984).

6. Concluding Remarks

The neoclassical models of trade and growth are noted for their complexity of analysis. This seems to be inevitable in the context of a two-country, two-factor, two-good world. This paper uses the dual approach to analyze one of the most popular models of trade and growth. Such an approach greatly simplifies some aspects of the analysis and provides new insight into the processes of growth and trade.

This paper has made a clear distinction between the short-run impact of trade and long-run impacts of trade and growth. It has examined the short-run and long-run dynamic adjustment paths of an economy as a result of opening to trade. The global characterization of the regions of specialization readily helps to determine the direction of changes in a country's capital-labor ratio. One result from this characterization is that in the case where the consumption good sector is less capital intensive than the capital good sector, an increase in the price of the consumption good as a result of trade may not lead to a decrease in the rate of interest in the short run. This has an important implication in comparing the capital-labor ratios as well as per capita consumptions across steady states.

It is well known that for a small open economy, the two-sector model can be aggregated to become isomorphic to a one-sector Solow model with the aggregate per-worker production function exhibiting a flat segment. The use of such an aggregate production function is better suited for models with a fixed ratio of saving to national income. The present model, with its classical savings function, is better analyzed by the dual approach. In particular, the dual approach renders the comparison of steady states extremely easy. However, the full dynamic process of long-run growth and trade and the characterization of the regions of specialization in the two-country case cannot be advantageously analyzed by the dual approach.

Footnotes

1. See Tsiang (1964) for applying the one-sector model to analyze growth in Rostovian stages. See also Wan (1971), Findlay (1984), and Smith (1984) for more references.

2. Bardhan (1966) attempted to generalize Oniki and Uzawa's model (1965) by considering a Cambridge savings function in which the capitalists' propensity to save is different from that of workers. He provided some new conditions for the offer function to shift in a determinable way and added a finer characterization of the regions of specialization. However, Bardhan did not provide a dynamic analysis in his model. For the use of this savings function in a one-sector model with a full dynamic analysis, see Chang (1969).

3. See, for example, Bertrand (1975), Chang and Chiang (1986), Deardorff (1973), Smith (1976, 1977, 1984), etc.

4. Let a "^" over a variable *denote* the proportional rate of change of that variable (e.g., $\hat{w} \equiv dw/w$), and θ_{ij} be the ith factor share in the jth sector ($\theta_{L1} = wa_{L1}$, $\theta_{K1} = ia_{K1}$, $\theta_{L2} = wa_{L2}/p$, and $\theta_{K2} = ia_{K2}/p$). By definition, $\sigma_j = (\hat{a}_{Kj} - \hat{a}_{Lj})/(\hat{W} - \hat{R}) = \hat{k}_j/(\hat{w} - \hat{i})$. Using (6), we have $d^2w/di^2 = -k_j\hat{k}_j/(i\hat{i}) = -k_j\sigma_j(\hat{w} - \hat{i})/(i\hat{i}) = (k_j\sigma_j/i)(1 + \theta_{Kj}/\theta_{Lj}) = k_j\sigma_j/(i\theta_{Lj})$, since $\theta_{Kj} + \theta_{Lj} = 1$ and, from (6), $\hat{w}/\hat{i} = -\theta_{Kj}/\theta_{Lj}$.

5. Throughout this paper, a variable with a superscript "a" signifies the autarkic state.

6. To avoid cluttering the diagram, the complete specialization ranges are not illustrated in the lower half of Fig. 5.

7. In this section, only some comparative dynamics are analyzed. For the full dynamic equilibrium in which the terms of trade, p, depend upon k and k^* so that the patterns of specialization are endogenously determined by k and k^*, see Khang (1971) and Takayama (1972).

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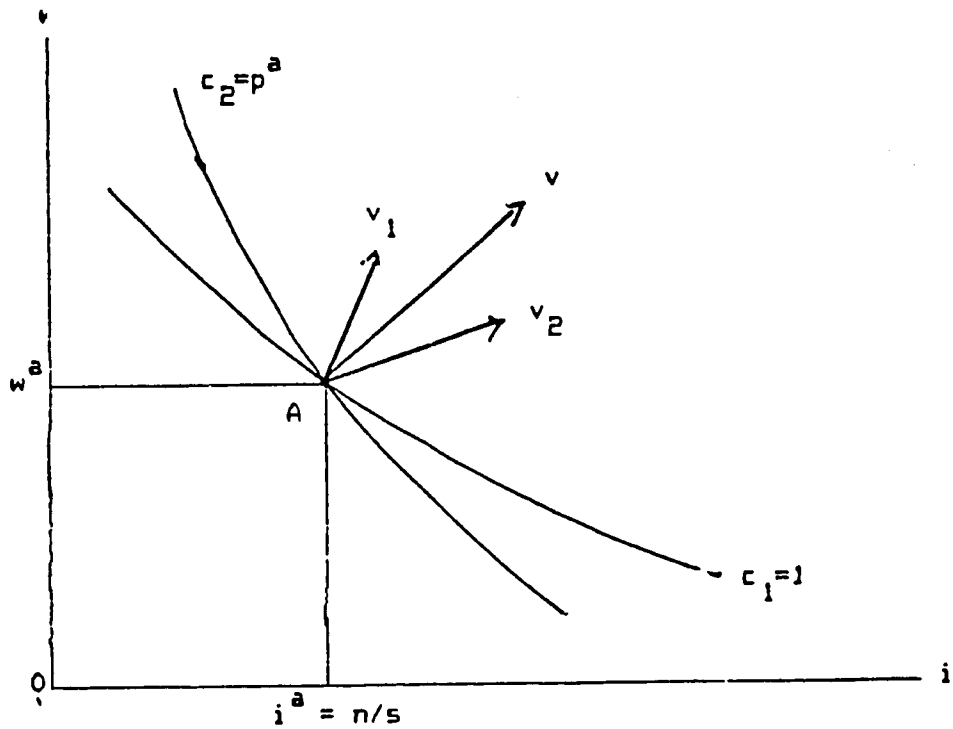

Fig. 1 $k_2 > k_1$

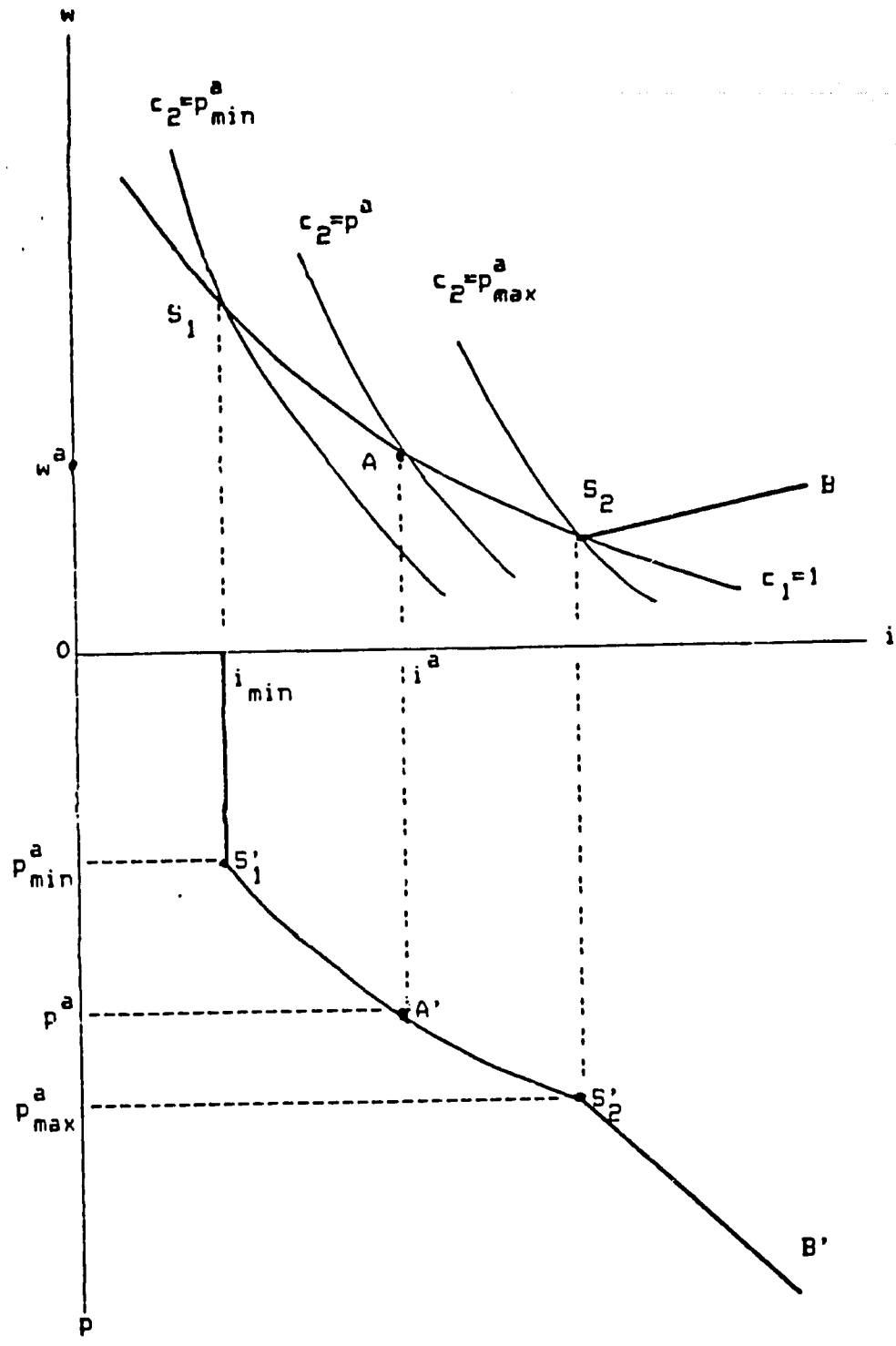
w
$c_2=p^a_{min}$
$c_2=p^a$
$c_2=p^a_{max}$
S_1
w^a
A
S_2
B
$c_1=1$
0
i
i_{min}
i^a
p^a_{min}
S'_1
p^a
A'
p^a_{max}
S'_2
B'
p

Fig. 2 $k_2 > k_1$

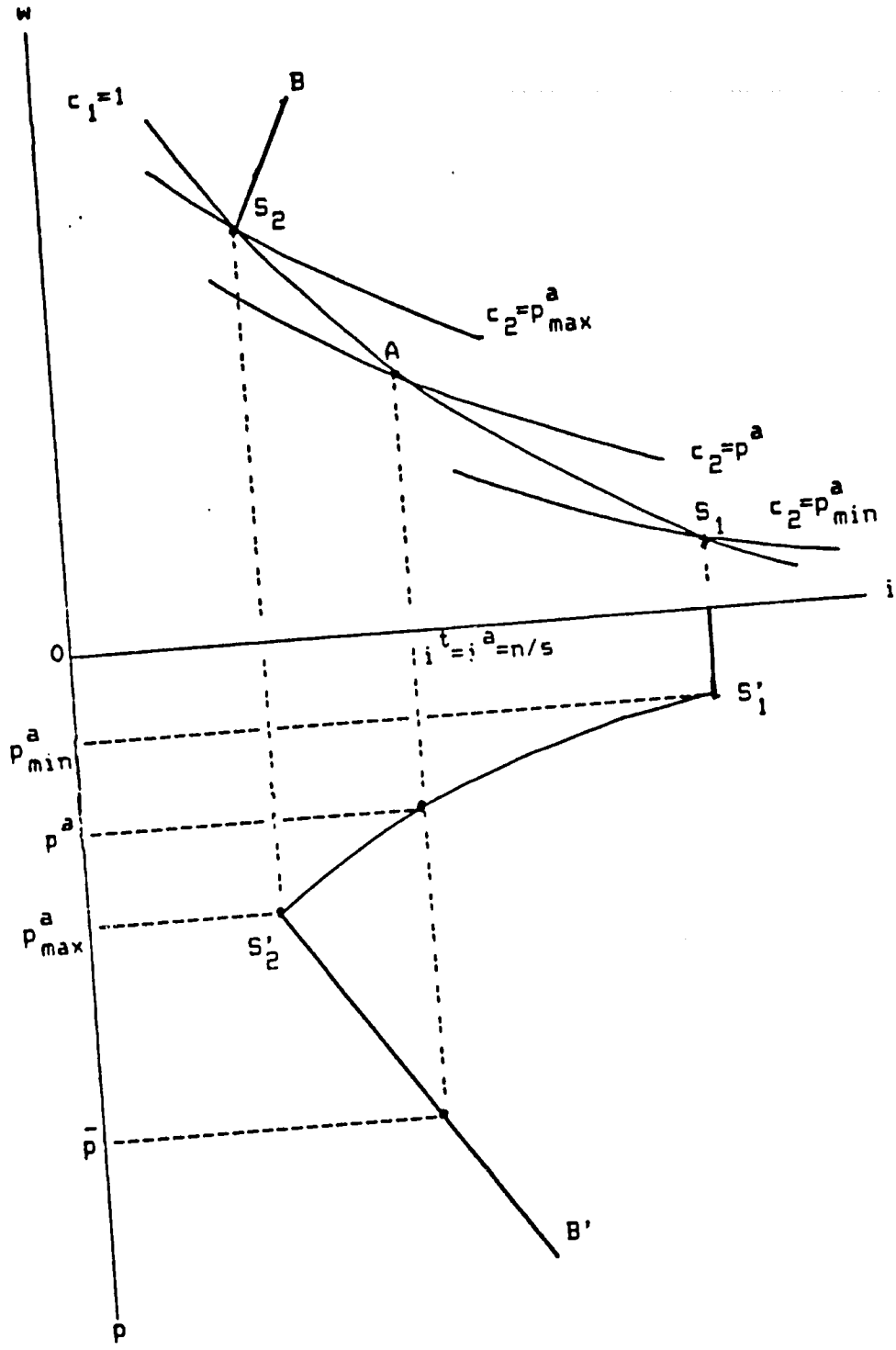
w
B
$c_1=1$
S_2
$c_2=p^a_{max}$
A
$c_2=p^a$
S_1
$c_2=p^a_{min}$
i
0
$i^t=i^a=n/s$
S'_1
p^a_{min}
p^a
p^a_{max}
S'_2
$\bar{p}$
B'
p

Fig. 3 $k_2 < k_1$

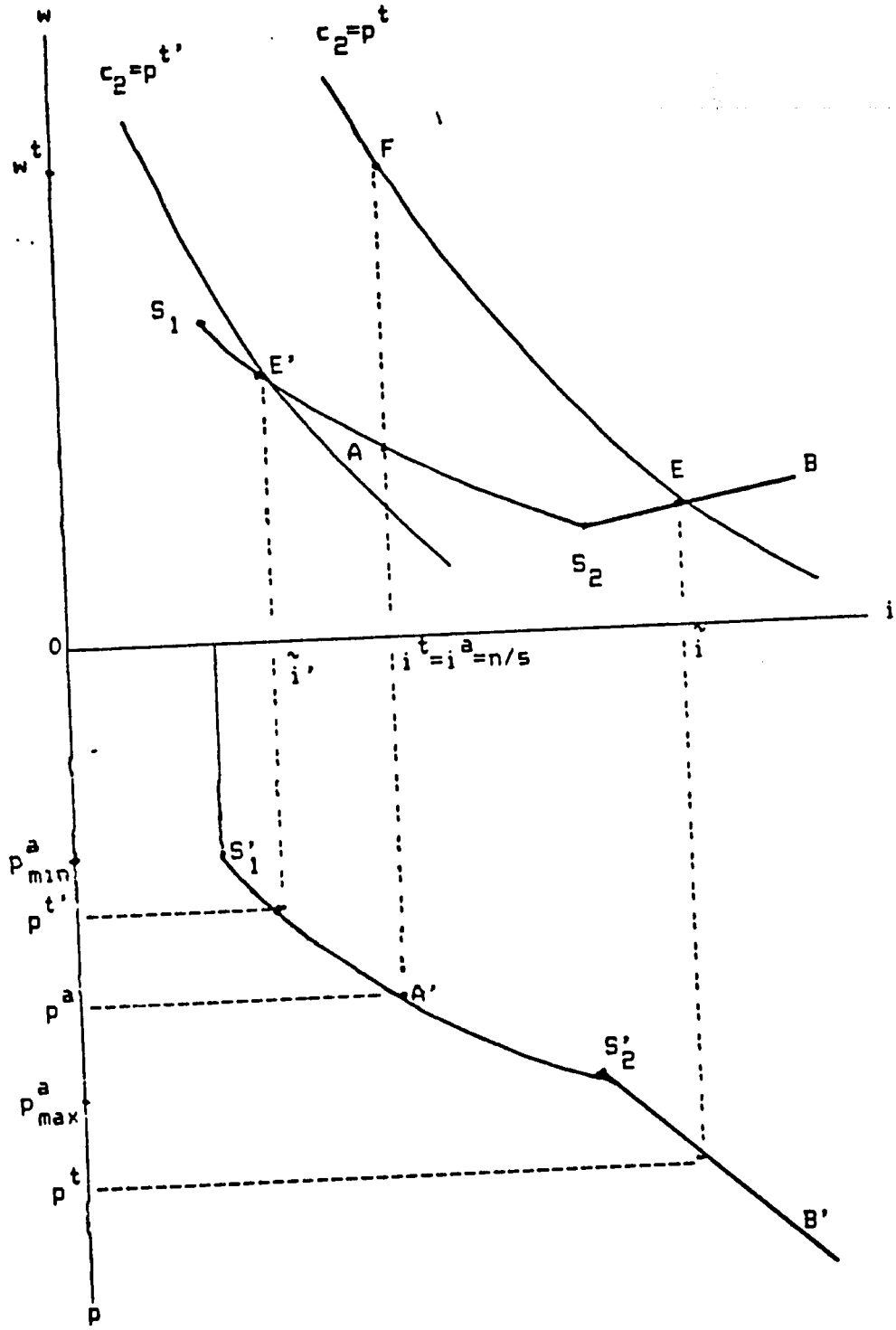
w
$c_2=p^{t'}$
$c_2=p^t$
w^t
F
S_1
E'
A
E
B
S_2
i
0
$\tilde{i}'$
$i^t=i^a=n/s$
$\tilde{i}$
p^a_{min}
S'_1
$p^{t'}$
p^a
A'
S'_2
p^a_{max}
p^t
B'
p

Fig. 4 $k_2 > k_1$

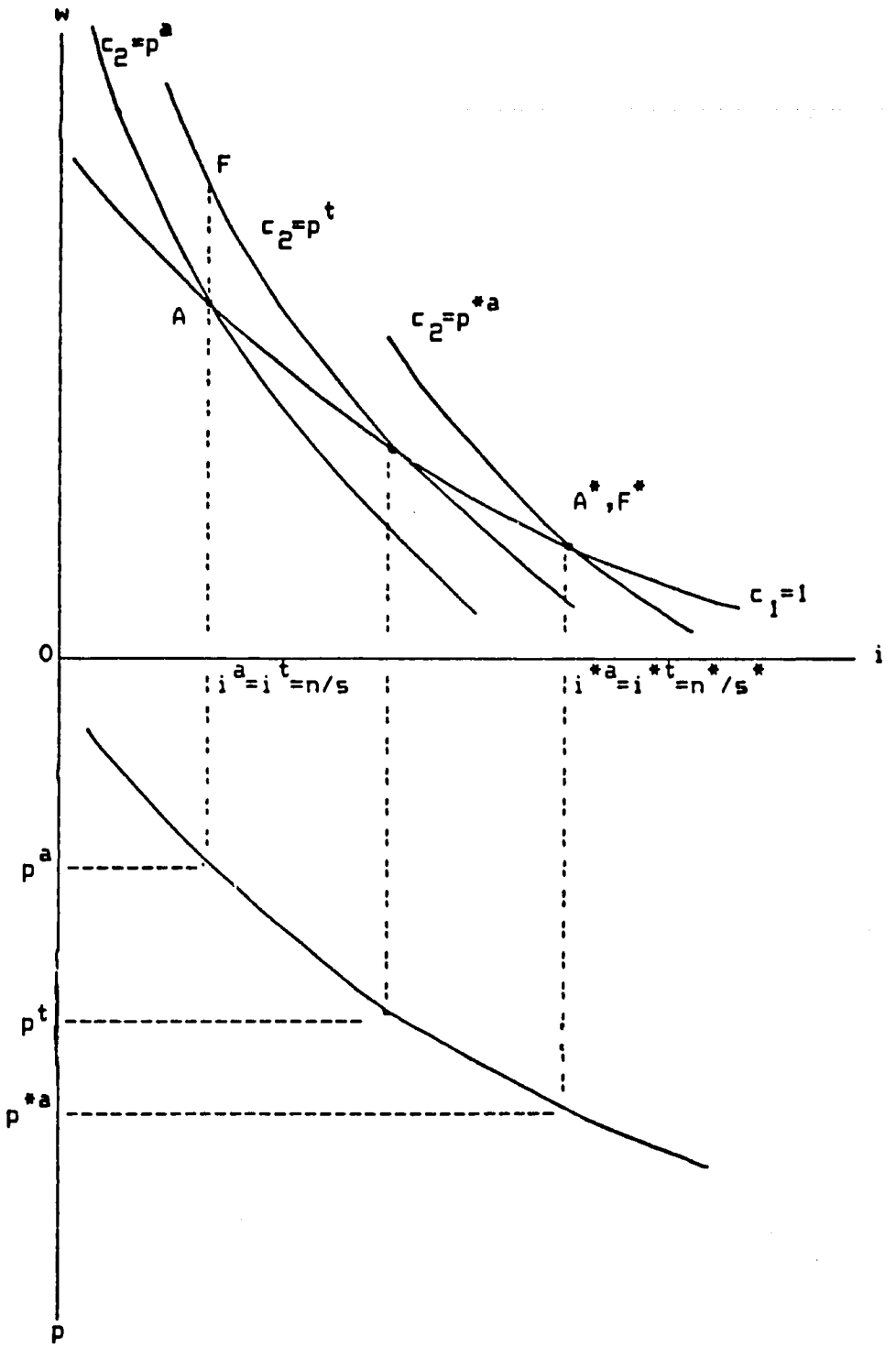

Fig. 5 $k_2 > k_1$