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LONGEVITY, INTRAFAMILY TRANSFERS AND ECONOMIC GROWTH

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INTRODUCTION

A primary objective of recent contributions to the "new economic development" literature has been to explain the wide diversity in levels and rates of economic growth that has been observed across countries and over time--what Lucas (1988) and others have termed "the problem of development" (see Ehrlich, Hariharan and Lutter, 1989). A corollary issue to be explored is "the process of development": accounting for the historical pattern of economic growth and development, especially its accompanying "demographic transition", which has recurred with remarkable regularity in the now-developed economies of the West, as well as in the still-developing countries of the Third World. In this paper we address both of these issues by focusing on the role of longevity in economic development. Our thesis is that health-related technological developments that contribute to a significant increase in the life expectancies of specific population groups may influence the onset of a growth and development process and account for differences in growth rates among developing and developed economies.

The potential empirical relevance of our thesis is easily demonstrable using both World Bank data and recently assembled statistics by Summers and Heston (1988). Examining World Bank data pertaining to 105 countries over the period 1965-1986, we find that the annual rate of per-capita real GNP growth in different countries over that period is negatively and significantly correlated with the crude death rates in the respective countries in 1965. The coefficient of correlation between the growth rate and the overall death rate is -0.3471, which is significant at the 0.01% level for a 1-tail test, and the one corresponding to the growth rate and infant mortality is -0.2933, significant at the 0.1% level. Using the Summers and Heston data for 112 countries over the same period, we find the correlation coefficients

pertaining to identically defined variables over the same period to be -0.3935 and -0.3385, respectively, both significant at the 0.1% level. This data source also allows the computation of simple correlations between the growth rate and alternative measures of longevity in 70 to 90 countries over the period 1960-1985. The results, exhibiting a similar pattern, are summarized in Table 1.

It is arguable, of course, that the zero-order correlations mask a two-way causality between population longevity and economic growth. The former can "cause" latter and vice versa. Indeed, the idea that life expectancy is to a degree an "economic" choice variable which is influenced by the economy's growth prospects and initial human capital per capita is defensible on general economic principles (see Ehrlich and Chuma, 1989). But evidence assembled by authors of the demographic transition hypothesis (see section 2 below) indicates that historically the decline in the rate of mortality in the population has tended to <u>precede</u> the onset of the process of development and demographic transition. Could it be that the increase in longevity is in some way responsible for this process?

The approach we pursue to analyze this question links longevity, fertility and economic growth through their interaction with human capital accumulation, which is the engine of growth in our model. Like Lucas (1988), and Becker and Murphy (1989) we postulate that per-capita income and consumption grow over time as a result of the accumulation of general and specific knowledge (including knowledge that influences technological discoveries, as modeled in Romer (1988)), and that such human capital can grow over time without bound because past stocks of knowledge possessed by an older generation augment the productivity of resources they devote to educating the younger one. Like Becker and Murphy we assume, more specifically, that growth

TABLE 1: Correlation Coefficients of The Growth Rate and Alternative Measures of Longevity.

	DR55-59	DR60	LE20	LE40
Correlation with YG60-85	-0.2471	-0.2763	0.3893	0.2135
1-tail significance level	0.9%	0.7%	0.0%	3.8%
Number of countries	90	80	70	70

DR55-59 - average crude death rate from 1955 to 1959. (The crude death rate of 1960 is used if the average from 1955 to 1959 is not available.)

DR60 - crude death rate in 1960.

LE20 - life expectancy at age 20. (Latest available data before 1960 are used.)

LE40 - life expectancy at age 40. (Latest available data before 1960 are used).

YG60-85 - annual growth rate of real per capita GNP from 1960 to 1985.

Sources: Summers and Heston (1988) and Demographic Yearbook, various issues.

in human capital depends on the willingness of parents to invest sufficient resources to produce for their children a stock of human capital that is superior to their own.

A distinct feature of our approach, however, concerns the specification of the objective function underlying parental choices. Becker and Murphy adopt a dynastic utility maximization rule in which altruistic parents maximize the aggregate utility of their entire dynasty (as they set it up) but not necessarily the individual utility of each of their children. We adopt, instead, an overlapping-generations model in which parents consider the preferences of each of their children concerning his own welfare. We do not resort to a bargaining model in order to bring into balance the separate interests of parents and their children. We assume, instead, that parents are induced to consider the preferences of their children at least partly because of self-interest: If they reach old age, parents become dependent on their adult children for a combination of financial, physical, and moral support, including direct utility from the presence of surviving children which we term "companionship". It follows that parents must "live with" the consequences of their behavior towards their children, especially their investments in the children's productive capacity. Accumulation of human capital thus occurs in our model through implicit contracts involving intrafamily transfers between parents and children. In sections 1 and 2 we analyze the conditions under which such contracts are self-enforceable and time consistent.

This approach provides a natural framework for analyzing the role of longevity. We show that a higher probability of survival of children into adulthood and of adults into old age increases the intertemporal rate of return on investment in children, and thus indirectly the intergenerational rate of growth of real consumption per-capita. Our major results include:

1) Optimal intrafamily transfers may lead not just to a self-sustaining growth equilibrium, but also to a maximization of the attainable rate of growth in per-capita consumption.

2) A steady state of persistent growth will ultimately lead parents to specialize in investment in the quality rather than the quantity of children, i.e., the rate of fertility will decline to its lowest permissible level.

3) A sufficiently large increase in the probability of survival of children and young adults can set in motion a process of development from a steady-state development equilibrium into a growth equilibrium that is accompanied by a "demographic transition".

4) Exogenous changes in longevity also increase the rate of economic growth in countries that have already attained a growth equilibrium. Variations in longevity can thus explain both the economy's stage of development as well as differences in the rates of economic growth across developed economies.

5) Social Security and related social insurance programs can generally be expected to lower the growth rate of developed economies.

6) Population control policies that are designed to precipitate the onset of growth may in principle result in opposite outcomes.

Due to the complex "mechanics" of the dynamic systems we analyze, we present these results in two related models. In section 1 we assume that parents invest in children as a means of self-insurance of old-age needs. In section 2 we extend this model to consider "companionship" as an independent and critical motive for having children. In section 3 we introduce social insurance as an alternative to families' self-insurance through investments in children. We conclude with a summary of issues to be explored in future work.

1. BASIC MODEL: LONGEVITY, SELF-INSURANCE, AND GROWTH

In what we call the "basic model", the relationship between longevity, human capital formation and economic growth is derived on the assumption that intrafamily transfers take place as a result of implicit contracts between overlapping generations. Specifically, parents invest in the quantity and quality (i.e., the human capital) of their children in order to secure old-age insurance for themselves. Financial support during the period of old age dependency is here identified as the sole motive for having children. We later extend the analysis to consider alternative motives and alternative sources of old-age income, such as social security. For simplicity, we assume that human capital and "raw labor" are the only factors of production in the economy.

Formally, we view the economy as comprised of three overlapping generations of representative agents: children, young parents, and old parents. Children and old parents are dependent on the financial support of young parents. The latter are endowed with one unit of labor time, a fixed amount of "raw labor", $\bar{H}$, and an acquired stock of human capital, H_t, due to parental investments in the preceding period of childhood.

To raise each child, a parent must devote v units of labor time, where $0 < v < 1$. The technology of imparting human capital to children exhibits constant returns to scale in the amount of time a parent devotes to educating each child, h_t, as well as in the level of human capital attained by the parent. Specifically,

(1) $H_{t+1} = A(H_t + \bar{H})h_t$,

where A represents a positive technological parameter. As in Lucas (1988) and Becker and Murphy (1989), this technology identifies human capital as the economy's engine of growth.

A young parent forms an implicit contract with each child. By this contract, the parent receives from a grown-up child an amount of old-age financial support that is proportional to the stock of human capital produced in the latter, $w_{t+1}H_{t+1}$. The compensation ratio, w_{t+1}, is to be endogenously determined in this model although not necessarily through any direct negotiations between parents and their children. Parents can unilaterally set an amount of compensation that is optimal from the point of view of children.[1] A "bad" grown-up child (young parent) can choose to ignore the contract and refuse to pay the old-age insurance obligation to the old parent. We assume, however, that such behavior will entail an automatic penalty to the young parent, since his children will learn from his behavior and will not support him during his own old age. Such parent will therefore forego the return on any investment he makes in his own children. A "good" parent, in contrast, both honors the contract and sets an optimal compensation rate for his child. We shall first assume that contracts will be honored in every generation. We shall later demonstrate why under the assumed automatic sanction and related conditions the implicit contracts are indeed self-enforcing and time-consistent.

A representative agent may not survive through all three stages of the life-cycle. Let the probabilities of an agent surviving through young and old adulthoods be π_1 and π_2 , respectively. The expected consumption of a young adult at time t is then given by

$$c_1(t) = (H_t+\bar{H})(1-vn_t-h_tn_t)-\pi_2w_tH_t \ ,$$

or alternately, after substituting equation (1) into this expression, by

$$(2) \quad c_1(t)=(H_t+\bar{H})(1-vn_t)-(n_tH_{t+1}/A)-\pi_2w_tH_t \ .$$

In these equations, expected consumption or, equivalently, the actual consumption of the representative young adult equals gross earnings net of the

expected financial support committed to the old parent, as modified by the latter's probability of survival. Gross earnings, in turn, are a product of the amount of time allocated to market activity and the productive capacity (the sum of human capital and the human capital equivalent of raw labor) of the young adult.

The expected consumption level of the representative agent once reaching the period of old-age dependency at time t + 1 is the product of the surviving number of children, $\pi_1 n_t$, and the amount of financial support provided by each

(3) $c_2(t+1) = \pi_1 n_t w_{t+1} H_{t+1}$.

While the conditional levels of consumption during young adulthood and old age are uncertain prospects, for analytical simplicity we henceforth consider their expected values as stated in equations (2) and (3) to be certain magnitudes. Accordingly, and using the now-conventional iso-elastic specification of the one-period utility function, the expected utility of the representative young parent (or parents)[2] is given by

(4) $u_t = [1/(1-\sigma)][c_1(t)^{1-\sigma}-1] + \delta\pi_2[1/(1-\sigma)][c_2(t+1)^{1-\sigma}-1]$

where σ represents the inverse value of the (constant) intertemporal elasticity of substitution in consumption. The utility of old-age consumption is discounted here for both the associated probability of survival, π_2 , and the rate of time preference, ρ , where $\delta = 1/(1+\rho)$.

The objective of the representative young parent is to choose the optimal values of quantity (gross fertility) and quality of children, $n_t \geq 0$ and $H_{t+1} \geq 0$ respectively, by maximizing the expected utility function (4) subject to equations (2) and (3), taking w_t, w_{t+1}, and H_t as given. The Lagrangian is

(5) $L = u_t - \mu_1(t)[c_1(t) - (H_t+\bar{H})(1-vn_t) + (n_t H_{t+1}/A) + \pi_2 w_2 H_t]$

$- \mu_2(t)[c_2(t+1) - \pi_1 n_t w_{t+1} H_{t+1}]$,

where μ_1 and μ_2 are Langrangian multipliers. Since the optimal solution requires that n_t and H_{t+1} be positive in value [3], the first order optimality conditions can be stated as

(6) $\mu_1/\mu_2 = \delta[c_2(t+1)/c_1(t)]^{\sigma}(1/\pi_2)$

(7) $(c_2/c_1)^{\sigma} \geq \delta[A\pi_1\pi_2 w_{t+1}][h_t/(v+h_t)] \equiv \delta R_n$

(8) $(c_2/c_1)^{\sigma} \geq \delta[A\pi_1\pi_2 w_{t+1}] \equiv \delta R_h$,

where δR_n and δR_h can be interpreted as the discounted expected rates of return to parental investments in the quantity and quality of children, respectively.

An immediate implication of this analysis is that the rate of return to investment in quality exceeds that to quantity at all levels of investment. The reason is that while an equal proportional increase in the quantity and quality of children yields the same marginal benefits in terms of old-age consumption, the marginal cost of quantity incorporates both the cost of rearing and educating children (given the desired investment in the human capital of each child), whereas the marginal cost of quality is comprised of only the latter cost component and is thus necessarily lower. It is thus more economical to produce old-age insurance by emphasizing quality relative to quantity of children. Indeed, the optimal solution requires that equation (7) be stated as strict inequality and equation (8) as strict equality, with a resulting "corner solution" in n_t at its lowest permissible value. And although the latter is, in principle, an arbitrarily small positive value (see fn.2), empirically, the lowest possible value of gross births for the representative agent must be compatible with the survival of at least one child, or $n_t = n = 1/\pi_1$. We shall henceforth take this to be the equilibrium number of births.

By combining equations (2), (3) and (8), the discounted rate of return to investment in the quality of children can also be stated as

(9) $\{[\pi_1 n w_{t+1} H_{t+1}]/[(H_t+\bar{H})(1-vn)-(H_{t+1}/A)n-\pi_2 w_t H_t]\}^{\sigma} = \delta A \pi_1 \pi_2 w_{t+1}$,

and this equation can be manipulated to yield the following law of motion for human capital accumulation across generations

(10) $H_{t+1} = a_{t+1} H_t + b_{t+1}$,

where

(11) $a_{t+1} = [AJ_{t+1}(1-vn-\pi_2 w_t)]/[n(A+J_{t+1})]$

(12) $b_{t+1} = [AJ_{t+1}(1-vn)\bar{H}/[n(A+J_{t+1})]$, and

(13) $J_{t+1} = [\delta A \pi_1^{1-\sigma} \pi_2 w_{t+1}^{1-\sigma}]^{1/\sigma}$.

Since the coefficients a_{t+1} and b_{t+1} depend on the compensation ratio w_{t+1} , we now turn to determine its equilibrium value.

A good parent will choose a value of w_{t+1} that will be optimal for each child. The representative child's expected utility is given by

(14) $u_{t+1} = [(1/(1-\sigma)][c_1(t+1)^{1-\sigma}-1]+\delta\pi_2[(1/(1-\sigma)][c_2(t+2)^{1-\sigma}-1]$,

where $c_1(t+1)$ and $c_2(t+2)$ denote the child's expected consumption levels in periods (t+1) and (t+2) as specified in equations (2) and (3), but with the number of children set at its optimal value $n_{t+1} = n = 1/\pi_1$. By the envelope theorem, or the dynamic optimization condition requiring that $\partial u_{t+1}/\partial H_{t+2} = 0$, the first-order necessary condition for the optimal compensation ratio w_{t+1}^* is found from

$dU_{t+1}/dw_{t+1} = (\partial U_{t+1}/\partial H_{t+1})(\partial H_{t+1}/\partial w_{t+1}) + \partial U_{t+1}/\partial w_{t+1} = 0$,

or, using equation (10)

(15) $(1-vn)A(1-\sigma) = \pi_2 w_{t+1}[A+J_{t+1}\sigma]$.

Since the right-hand side of equation (15) is a monotonically increasing function of w_{t+1}, this equation provides a unique solution for w_{t+1} that applies in all time periods. And since the second order conditions for an interior solution in w require that $0 < \sigma < 1$, that value is necessarily positive, $w_{t+1} = w > 0$.[4]

Equations (10), (13) and (15) fully describe the dynamic system of human capital accumulation and growth. Since a constant optimal compensation ratio $w > 0$ implies that the coefficients a_{t+1} and b_{t+1} are also positive constants over time, equation (10) becomes

(10a) $H_{t+1} = aH_t + b$,

and the economy can easily be shown to have a unique solution.

Proposition 1. For any given set of parameters, there can be only one steady state equilibrium for H_t .

The proof is easily inferred from Figure 1 since H_{t+1} is a simple linear function of H_t with b representing a positive coefficient. If $a \geq 1$, the economy's steady state is a growth equilibrium (see phase line II in Figure 1) with the growth rate of human capital asymptotically approaching the value of the coefficient a (from above). Moreover, equations (2) and (3) imply that for a sufficiently large value of H_t, this would be the growth rate of per-capita consumption as well. If $a < 1$, in contrast, the economy converges to a steady state development equilibrium with $H_t = b/(1-a)$ for all t (see point X in Figure 1). The multiple equilibria result of Becker and Murphy (1988) cannot occur in this specific model.

Two numerical examples illustrate this result. Let $\sigma=0.5$, $A=5$, $\bar{H}=1$, $\pi_1=0.99$, $\pi_2=0.95$ and $v=0.2$. Then $w=0.2641$, $a=1.4657$ and $b=2.1381$. Since $a > 1$ this is a growth equilibrium. If we retain all the parametric values but change A to 3, we obtain $w=0.2997$, $a=0.6791$, and $b=1.0557$. This is a development equilibrium with $H_t = 3.2898$ for all t.

Proposition 2. The optimal compensation ratio as solved from (15) maximizes the constant marginal growth rate of human capital, a, and hence the economy's long-run rate of growth in a growth equilibrium.

Proof: Since $w_{t+1} = w_t = w$ and $a = AJ(1-vn-w\pi_2)/(A+J)n$, simple differentiation

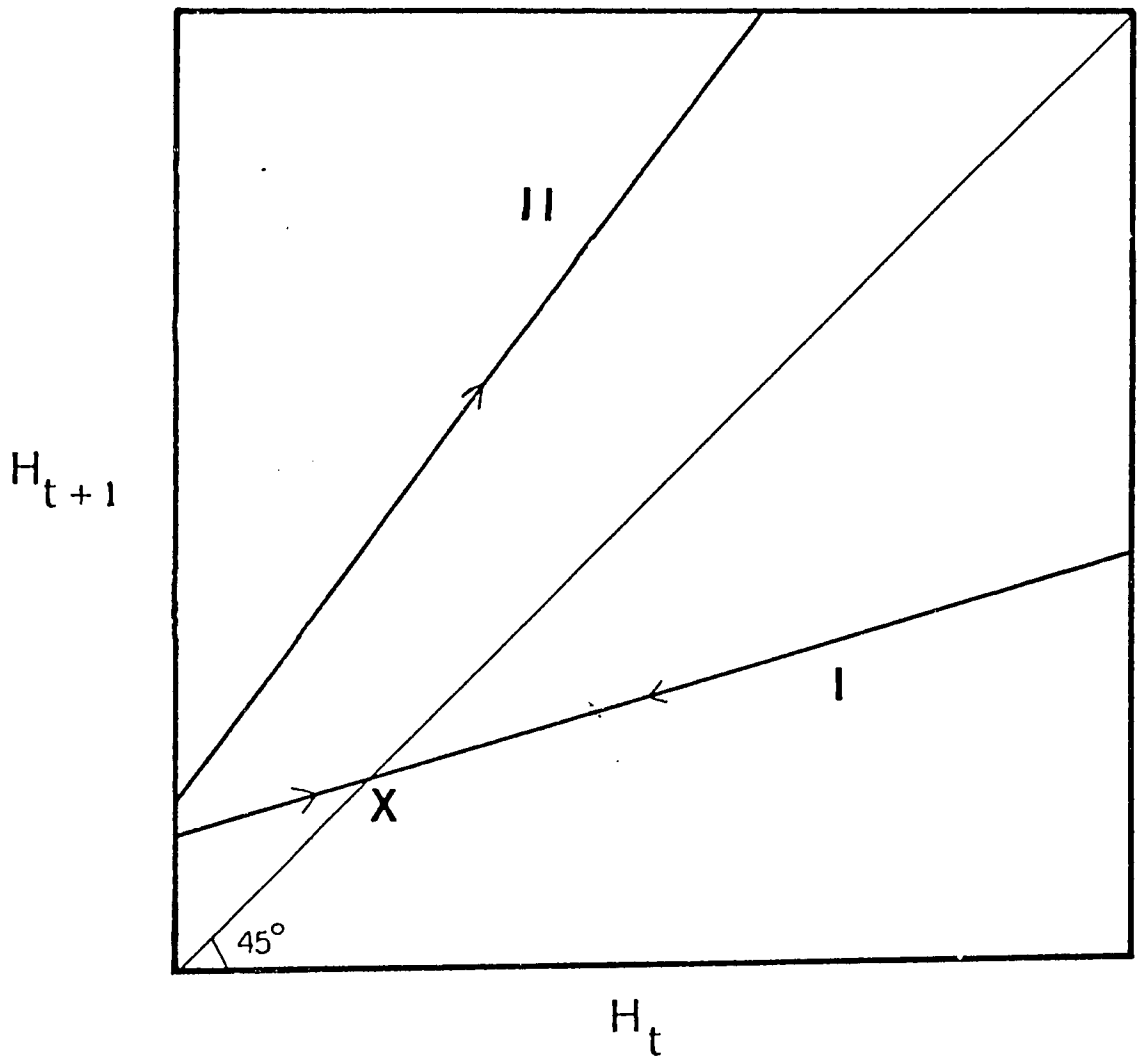

FIGURE 1

shows that $da/dw=0$, if and only if, $(1-vn)A(1-\sigma)=\pi_2 w[(A+J\sigma)]$, which also satisfied $da^2/dw^2 < 0$. The value of w that satisfies this condition also satisfies equation (15) when $w_{t+1}= w$. Equation (15), in turn, implies that $c_1(t+1)$ is maximized for any given level of $c_1(t)$. Put differently, the compensation ratio that maximizes the young adults' rate of growth of consumption over time also maximizes the constant marginal rate of growth of human capital across generations--a in equation (10a). If the economy were in a steady-state growth equilibrium, the latter rate would also equal the economy's long-run rate of growth of income $(H_t+\bar{H})$ and consumption per capita $g = c_1(t+1)/c_1(t)=c_2(t+1)/c_2(t)$. This result is a manifestation of the Coase Theorem in the context of economic growth.

We now turn to the behavioral implications of the model concerning the effects of longevity and related parameters on the equilibrium growth rate.

Proposition 3. Increases in longevity through either π_1 or π_2 will raise the economy's long-run growth rate and level, as indicated by a and b in equation (10a). Likewise, an increase in the efficiency of investment, A, a reduction in the rate of time preference (an increase in δ), or a reduction in the cost of rearing children (v) will also raise the economy's level and rate of economic growth.[5]

Proof: Since $da/dw=0$ and n is constant, by the envelope theorem, and using equations (11) and (15), we obtain

$da/d\pi_1= \partial a/\partial\pi_1= AJ/(A+J) > 0$, and

$da/d\pi_2=\partial a/\partial\pi_2=AJw\sigma/[n(A+J)(1-\sigma)] > 0$.

The remaining propositions can be similarly proved.

The main contribution of this analysis to understanding the "problem of development" is the prediction that the growth rate will be an increasing function of longevity. The simple rationale is that a rise in longevity of

either old parents or young adults increases the intertemporal rate of return to parents from any investment in the human capital of their children and thus the optimal rate of investment children's education and the rate of growth of human assets across generations. A similar effect is associated with the decrease in the cost of raising a child or in the rate of time preference, since both raise the incentive to defer consumption from adulthood to old age. More generally, the argument is that an increase in the discounted intertemporal rate of return to parental investment in children from any source would increase both the intertemporal and the intergenerational rate of growth of consumption per person through the increased accumulation of human capital across generations.

Proposition 4. An exogenous increase in fertility will reduce the economy's growth rate or level depending upon whether it is initially in a growth or a development equilibrium. An exogenous increase in the rate of population growth will thus have mixed effects on these variables.

Proof: By the envelope theorem and equations (11) and (15)

$da/dn = [-AJ(1-\pi_2 w)]/[n^2(A+J)] < 0$, and $db/dn = -AJ/[n^2(A+J)] < 0$.

While an exogenous increase in fertility is shown to reduce the economy's growth rate if it is already in a steady state of growth equilibrium, an exogenous increase in longevity was just shown to increase it. The net effect of an exogenous increase in population on the rate of economic growth therefore depends on the source of such increase and its magnitude.

The Time Consistency Problem

We have so far assumed that the implicit contract concerning intrafamily transfers between parent and child will be honored. To show that the contract is self-enforcing we assume: a) Parents and children have identical preferences; b) Violating the contract always entails a penalty in the form of

a similar behavior by the violator's own children; c) Consumption during young adulthood and old age are sufficiently nonsubstitutable; d) The economy has an infinite horizon.

Define, for convenience, $M_1 = H_t + \bar{H}$; $M_2 = (H_t + \bar{H})vn_t + (H_{t+1}n_t/A) + \pi_2 w_t H_t$; $M_3 = \pi_1 n_t w_t \bar{H}_{t+1}$. If the young adult is a "good parent" and his children support him in his old age, his utility is given by

$U^1 = [(M_1 - M_2)^{1-\sigma} - 1]/(1-\sigma) + \delta\pi_2[M_3^{1-\sigma} - 1]/(1-\sigma)$.

If he violates his commitment to his parent who has been a good parent, he knows his children will behave in the same manner towards him and thus he has no incentive to invest in his own children. His expected utility in this case is

$U^2 = [M_1^{1-\sigma} - 1]/(1-\sigma) + \delta\pi_2[-1/(1-\sigma)]$.

Clearly, $U^1 > U^2$ if and only if $(M_1 - M_2)^{1-\sigma} + \delta\pi_2 M_3^{1-\sigma} > M_1^{1-\sigma}$. If σ approaches unity, so that consumption is sufficiently nonsubstitutable intertemporally, the latter inequality becomes $(1+\pi_2) > 1$. Hence $U^1 > U^2$.

The fundamental force leading to compliance with the implicit contract is the threat of being penalized by the loss of old-age benefits. Such threat may itself be the result of a "norm" (such as the fifth commandment requiring that one "honor thy father and thy mother... so that thy days may be prolonged") as well as the interaction between overlapping generations: if children are aware of their parents' treatment of their grandparents they will behave similarly when they reach adulthood since they are assumed to have identical preferences and motivation. A parent will honor the contract and invest in his children's human capital if he knows that his children will behave likewise. Hence, provided that the economy does not end, this behavior will be time-consistent. In contrast, violating a contract will not be time-consistent since it would lead to no investment in children. And since the

compensation rate is set to maximize a child's as well as a parent's expected utility, there would be no incentive for the child to deviate from the optimal compensation rate.

This conclusion presupposes that there is no saving or social insurance in the economy. The time consistency problem in the presence of such alternatives will be considered in the following section.

2. EXTENDED MODEL: DEMOGRAPHIC TRANSITION AND THE DEVELOPMENT PROCESS

Our growth model has so far relied on parents' need for old-age financial support as the main force underlying intrafamily transfers and accumulation of human capital. This formulation of the problem leads to optimal fertility decisions involving only the lowest permissible number of surviving children per parent or family and thus to a prediction of a stable population level or trend over time. One of the most striking features of the process of development in the now-developed countries of Europe and North America over the last 200 years, as well as in the newly developing countries of Asia and Africa in more recent decades, is the phenomena known as demographic transition. The transition refers to a shift in reproductive behavior in countries experiencing growth from a state of high birth and death rates to a state of low birth and death rates consummating the development process. More specifically, authors of the demographic transition hypothesis characterize the process of development as consisting roughly of the following stages:[6]

1. The onset of growth and development is preceded by a marked decline in the rate of mortality in the population due to significant technological innovations in medical science and related urban developments bearing on public health.

2. In the following "first phase" of development, population growth is rising

due to the decline in mortality rates coupled with either a more modest decline, or frequently an actual increase in fertility rates.

3. In the more advanced phase of development, in contrast, there is a continuous and pronounced decline in fertility rates apparently to the point where the latter approach a minimum feasible level consistent with a stable population. This is evident not just in the now-developed Western economies, but also in the economies of many developing countries, especially in Latin America and East and Southeast Asia. In this section we develop an extended version of our basic model that can account for these basic features of the development process.

The extension centers around identifying a more general motive for investment in children. We assume that parents invest in the quantity and quality of children in order to achieve direct utility in addition to financial security. The added utility is due to a variety of services good parents can expect from good adult children. These may include a sense of family strength and influence, which is especially important in less civilized societies where the legal system and law enforcement are weak, companionship and moral support, or the sheer pleasure from intrafamily interactions (a form of altruism). And even though these nonpecuniary benefits, lumped together as "companionship", may be enjoyed by parents throughout all phases of their children's lives, we focus on the importance of companionship during the period of old-age dependency.

Formally, we specify the companionship function as

$$(16) \quad c_3(t+1) = B(\pi_1 n_t)^{\beta} H_{t+1}^{\alpha} ,$$

where $0 < \alpha < 1$, $0 < \beta < 1$, $B=1$ if young adults honor their old parents, and $B=0$ if they do not. By this specification, companionship is a function of the number of surviving children, $\pi_1 n_t$, and their quality, H_{t+1} , with its

intensity determined by the coefficient B. The terms of the implicit contract between parents and children now include companionship services as well as financial support.[7] As in section 1 we shall first make, and then defend, the assumption that contracts will be honored by all generations. To simplify the mechanics of the following analysis, however, we shall take the financial compensation rate w_{t+1} and the intensity of companionship B to be exogenously determined constants.

The utility function of a representative young adult is thus given by

$$(17)\quad u_t^* = [1/(1-\sigma)][c_1(t)^{1-\sigma}-1]+\delta\pi_2\{[1/(1-\sigma)][c_2(t+1)^{1-\sigma}-1]+ [1/(1-\sigma)][c_3(t+1)^{1-\sigma}-1]\},$$

and the young parent chooses the optimal quantity and quality of children by maximizing equation (17) subject to equations (2), (3), and (16), the boundary restrictions $H_{t+1} \geq 0$ and $n_t \geq 1/\pi_1$, and the initial value of human capital H_t.

The first order optimality conditions include, in addition to equation (6)

$$(18)\quad (c_2/c_1)^{\sigma} \geq \delta A\pi_1\pi_2 w[1+\beta(c_3/c_2)^{1-\sigma}][h_t/(v+h_t)] \equiv \delta R_n^* \text{ , and}$$

$$(19)\quad (c_2/c_1)^{\sigma} \geq \delta A\pi_1\pi_2 w[1+\alpha(c_3/c_2)^{1-\sigma}] \equiv \delta R_h^* \text{ ,}$$

where the δR^*s represent, as before, the discounted expected rates of return on investment in children. In conformity with our analysis in section 1 we maintain the assumption that $0 < \sigma < 1$. This guarantees in the context of our extended model that real rates of return will ultimately converge to a constant level $A\pi_1\pi_2 w$ in a long-run , steady-state growth equilibrium, since $(c_3/c_2)^{1-\sigma}$ in equations (18) and (19) will be an increasing function of H_t if $\sigma > 1$, but will ultimately approach zero as H_t rises without bound if $\sigma < 1$.

Interior solutions in both quantity and quality of children imply that equations (18) and (19) become strict equalities, in which case the rates of return R_n^* and R_h^* are equal. Note that if the quantity of surviving children

has as much weight in determining companionship as their quality, or $\alpha=\beta$, then the rate of return to quality will continue to dominate that to quantity, as in our basic model, so that optimal fertility once again would become a corner solution at all times. In order for interior solutions in n_t and H_{t+1} to exist, we must restrict $\beta > \alpha$, i.e., companionship must be influenced more strongly by the number of surviving children relative to the human capital attained by each. More generally, equations (18) and (19) imply that

(20) $R_h^* \gtrless R_n^*$ as $v \gtrless (\beta-\alpha)(c_3/c_2)^{1-\sigma} h_t$,

where $(c_3/c_2)^{1-\sigma} = [(\pi_1 n_t)^{\beta-1} H_{t+1}^{\alpha-1} w_{t+1}^{-1}]^{1-\sigma}$.

Equation (20) leads to two important auxiliary results:

Lemma 1. If H_t is sufficiently large, $R_h^* > R_n^*$.

Proof: For consumption of young adults to be positive, the time spent raising children must have an upper bound $vn_t < 1$, as does the time spent educating each child. Consequently, optimal fertility and investment per child are bounded from above and below: $n_t \in [1/\pi_1, 1/v)$, and $h_t \in [0, \pi_1 - v)$. These constraints combined with the fact that the ratio $(c_3/c_2)^{1-\sigma}$ is a decreasing function of H_t, implies that at a sufficiently high level of H_t, the R.H.S. of equation (20) must become lower in magnitude relative to its L.H.S. Hence $R_h^* > R_n^*$.

Technically, this result is due to the fact that an increase in human capital reduces the ratio $(c_3/c_2)^{1-\sigma}$ which represents the share of companionship relative to financial support as a component of the rates of return on investment in children. Since this ratio has a higher weight as a determinant of the return to quantity relative to quality of children ($\beta > \alpha$), and since n_t and h_t are bounded, a large enough increase in H_{t+1} will, cause a reduction in R_n^*/R_h^*. Looked at from an alternative perspective, the result is due, partly, to the underlying technology of producing human capital whereby a higher level of human capital increases the marginal cost of rearing

children relative to the marginal cost of educating them. And since growth of human capital must ultimately lower the marginal benefits from quantity relative to quality of children, given the parental objective function (17), a high enough H_t must ultimately reduce R_n^* relative to R_h^* .

Lemma 2. It is not possible that $R_h^* < R_n^*$.

Proof: Assume that $R_h^* < R_n^*$. Equation (20) then becomes $v < (\beta-\alpha)(c_3/c_2)^{1-\sigma} h_t$. The dominance of the rate of return on quantity over quality of children implies a corner solution for h_t at its minimum feasible level. As h_t approaches zero, however, the preceding inequality cannot be satisfied. Hence R_n^* cannot fall below R_h^*.

Steady State Equilibria and Growth Regimes.

Lemmas 1 and 2 provide important insights for understanding properties of alternative steady state equilibria. By Lemma 2 investment in human capital must have an interior solution. Lemma 1 suggests, in contrast, that an interior solution in both fertility and investment in human capital is incompatable with a steady state of persistent growth which requires that human capital will grow without bound across generations. A steady state of growth equilibrium must therefore entail a corner solution for fertility at its minimum value $n=1/\pi_1$.

To simplify the technical derivation of steady state equilibria we shall assume, without loss of generality, that $\alpha=0$. This assumption satisfies the condition $\beta > \alpha$ which is necessary to assure the existence of interior solutions in both n_t and h_t. Since H_{t+1} must have an interior solution, equation (19) becomes

(19a) $(c_2/c_1)^{\sigma} = \delta A \pi_1 \pi_2 w$,

which is identical to equation (8) in our basic model. Consequently, equation (10) applies in this case as well. The system of equations governing the dynamic equilibria becomes

(21) $H_{t+1} = a_{t+1}H_t + b_{t+1}$,

where $a_{t+1} = [AJ(1-vn_t-\pi_2 w)]/[n_t(A+J)]$, $b_{t+1} = [AJ(1-vn_t)\bar{H}]/[n_t(A+J)]$, and $J = [A\pi_1^{1-\sigma}\pi_2 w^{1-\sigma}]^{1/\sigma}$; and

(22) $A(\bar{H}+H_t)v \geq \beta(c_3/c_2)^{1-\sigma}H_{t+1}$,

where $(c_3/c_2) = (\pi_1 n_t)^{\beta-1}w^{-1}H_t^{-1}$, which is a modification of equation (20) for $\alpha=0$. Equations (21) and (22) represent the equations of motion in human capital accumulation and fertility rates across generations.

The system represents two possible stable steady states. One is a stable development equilibrium of zero growth rate with an interior solution in $H_t = H^*$ and an interior or a corner solution in $n_t = n^*$. The other is a stable and persistent growth equilibrium with a perpetual growth in H_t which, by Lemma 2 must lead to a corner solution in $n^* = 1/\pi_1$. An illustration of these two steady states is given by phase lines I and II in Figure 2. These curves, linking H_{t+1} to H_t, are necessarily upward sloping (see appendix). Their nonlinearity is due to the interaction between human capital and fertility choices over the period of transition to a steady state. The linear segments of both curves represent alternative growth regimes within both the development and the growth equilibrium states in which fertility reaches its minimum value $n^* = 1/\pi_1$. These growth regimes are analogous to the one derived in our basic model.[8]

Comparative and Transitional Dynamics

We now turn to some of the model's key results concerning the process of development. Assume that the economy, which is initially in a stable development equilibrium, experiences a technological shock raising the

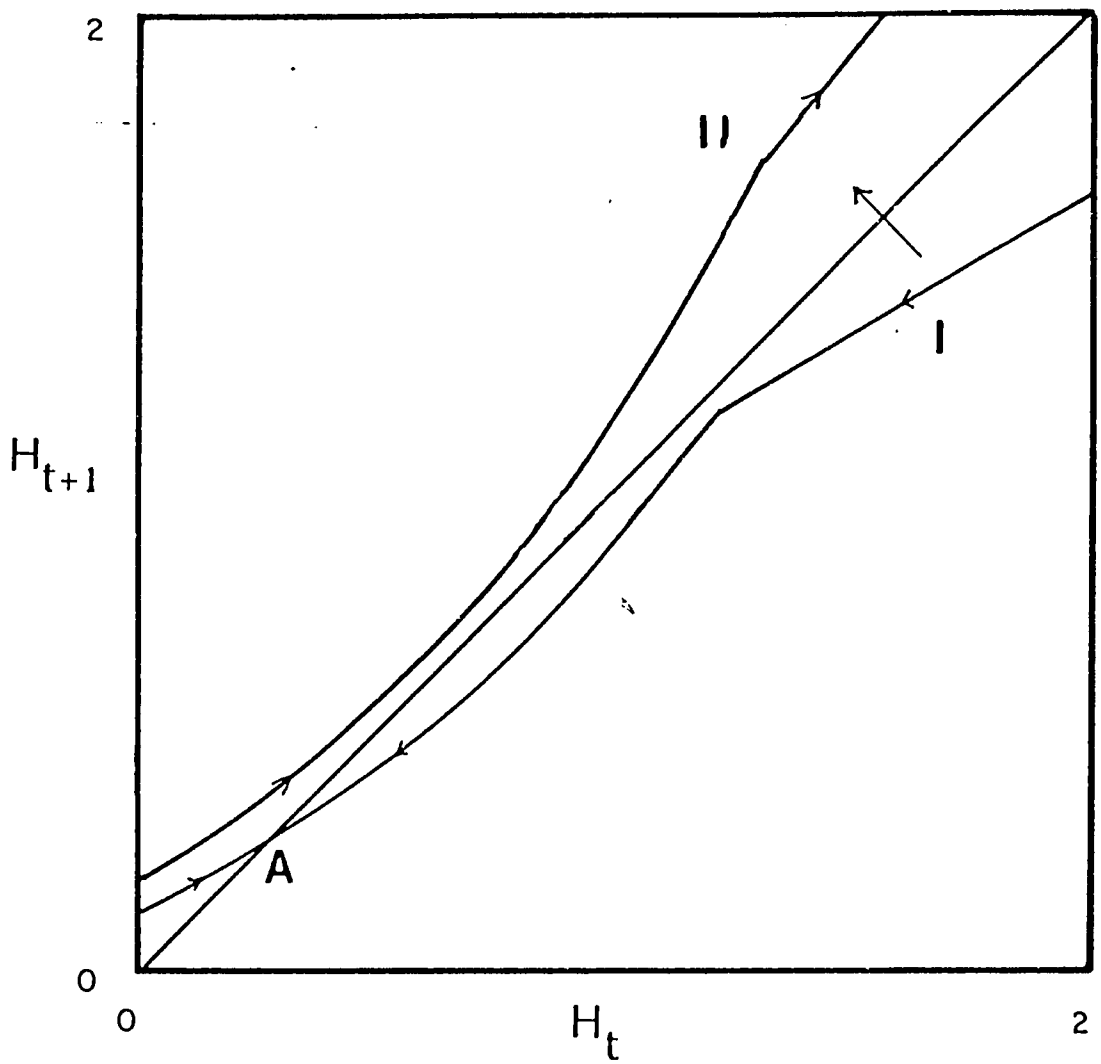

FIGURE 2

survival probabilities of children into adulthood, π_1, or adults into old age, π_2. There are then two possible outcomes depending on the extent of increase in longevity.

Proposition 5. (Demographic Transition). If the economy is initially in a development equilibrium, an increase in π_1 will cause the human capital stock to increase over time. H_{t+1} will either converge to a higher constant level at a new development equilibrium, or it may continue to grow without bound. At the initial phase following the increase in π_1 fertility n_t may increase or fall. If human capital continues to grow without bound, n_t must eventually start declining towards its minimum level, $1/\pi_1$, at which point the economy would reach a stable steady state of growth equilibrium. An increase in π_2 is expected to result in the same qualitative effects on the time paths of H_{t+1} and n_t with the exception that fertility must consequently increase in the first phase of the economy's transition from a development to a growth equilibrium.[9]

Proof: See appendix.

The basic rationale behind proposition 5 lies in the effect of longevity on the rates of return to quantity and quality of children: An increase in π_1 increases both R_h^* and R_n^*, but the direct impact on the latter is lower because a higher π_1 reduces the benefits from companionship relative to old age consumption, c_3/c_2, which is a more important component of R_n^*. This explains why the increase in π_1 triggers a definite increase in the human capital of succeeding generations while fertility may rise or fall in the first phase of transition. If the increase in longevity is modest the full impact of increased longevity on the system would not be sufficient to cause a permanent reduction in R_n^* relative to R_h^*, and the economy would then resettle at a new development equilibrium of a higher H^* and a higher or lower n^*. A

large increase in longevity, however, may destabilize the development steady state if it brings about a sufficiently high accumulation of human capital in subsequent generations. By Lemma 1, then, R_n^* must fall below R_h^*. In this case an increase in π_1 sets off a transition from the development equilibrium into a growth equilibrium, during which fertility declines continuously, following a possible initial surge, until the economy attains a steady state of self-sustaining growth equilibrium. The dynamic effects of an increase in π_2 on the economy are similar, except that since a higher π_2 causes R_n^* and R_h^* to rise by the same proportion during the first phase of transition to a new steady state, fertility must then rise along with human capital.

Proposition 5 can be illustrated by means of numerical simulations. Let the model's parameter values be $A=5.5$, $v=0.13$, $\beta=0.75$, $\sigma=0.5$, $\pi_1=0.6$, $\pi_2=0.6$, $w=0.45$ and $\bar{H}=0.5$. The economy then reaches a globally stable development equilibrium at $H^*=0.292$ and $n^*=2.793$. Path I in Figure 2 depicts this simulation.[10] If we now let π_1 increase from 0.6 to 1, the development equilibrium will be destabilized. A transition toward a growth equilibrium will set in as the curve relating H_{t+1} to H_t shifts above the 45° line in Figure 2. The corresponding time paths of H_{t+1} and n_t over the transition period are traced in Figures 3 and 4.

Figure 2 thus illustrates graphically the comparative dynamics of the effect of exogenous shifts in longevity on the economy's level and rate of economic growth. If the initial values of human capital or longevity are low, an increase in π_1 would not move curve I upwards sufficiently, and the higher curve would simply intersect the 45° line at a higher level of H^*. A more significant increase in π_1 would shift both the level and slope of curve I so as to make it identical to curve II. By the same reasoning, a large enough reduction in longevity due to a long period of famine, disease, or war can

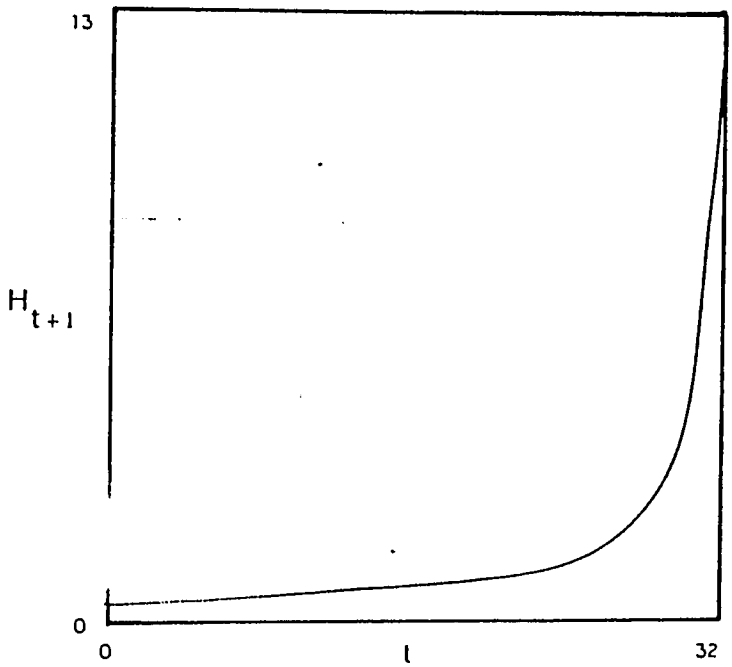

FIGURE 3

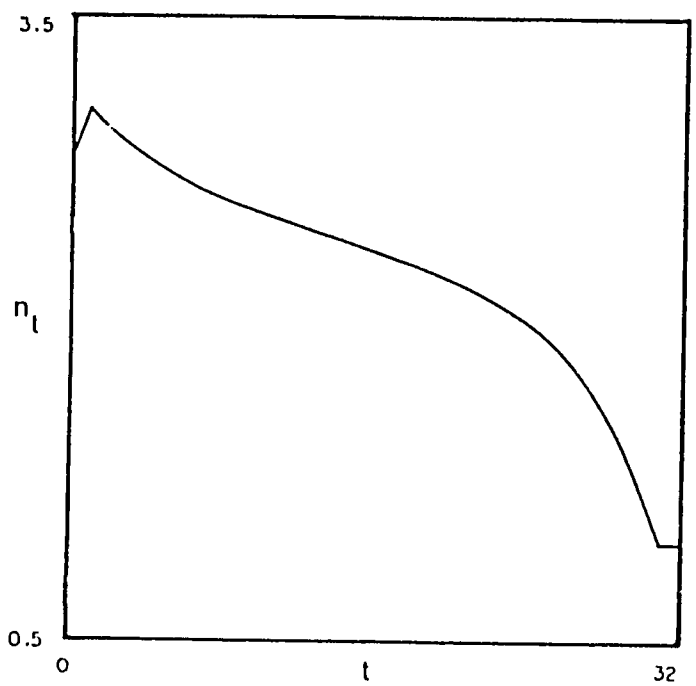

FIGURE 4

move curve II backwards in the direction of curve I. In such a case an economy can sink back from a growth equilibrium to a development equilibrium.

Note that the onset of a development process can be effected similarly by a technology shock that increases the efficiency of investment in human capita A, rather than longevity. Proposition 5 focuses on the role of the latter in conformity with the stylized facts of the demographic transition process.

Proposition 6. (Efficacy of Population Control Policies). An increase in the cost of having children will increase the level of human capital and per-capita income if the economy shifts from an initial state of development equilibrium involving an interior solution in fertility to a new development equilibrium. The opposite results would obtain, however, if the initial development equilibrium involved a corner solution in fertility. Moreover, if the economy were initially in transition toward a growth equilibrium steady state, an increase in v could either lower the steady-state growth rate or cause the economy to sink back into a development steady state.

Proof: See appendix.

The ambiguity of the net effect of the cost of having children on growth in our model is due to its potentially conflicting effects on the total costs of rearing children on the one hand, and the desired level of fertility on the other (as can be seen from equation (21)). If the equilibrium value of the elasticity of desired fertility with respect to its direct costs v is greater than or equal to unity, then an increase in v will reduce the total cost of children and thus the opportunity costs (in terms of foregone consumption in period 1) of investing in children's education. This is the case if the economy is initially at a development equilibrium involving interior solutions in both n_t and H_{t+1}. If the equilibrium value of the elasticity is

sufficiently less than unity, or zero, as is the situation in the other examples included in proposition 6, then the incentive to invest in children will necessarily fall.

A natural application of proposition 6 concerns the efficacy of governmental population control policies as a means of promoting economic growth and development. Since population control methods of the kind instituted in China (PRC) increase the marginal cost of children through the imposition of various sanctions, such methods may in fact achieve results that are opposite to those intended. Of course, even if population control policies can induce a higher level of economic growth, they are necessarily welfare-reducing since by the envelope theorem it can easily be shown that an increase in the cost of children must reduce the expected utility to young parents.

Time Consistency

The introduction of companionship as a motive for investment in children improves the prospect that the implicit contracts between parents and children will be self-enforcing and time consistent. This is because the threat of losing companionship in addition to old-age insurance benefits amounts to a more severe sanction than that considered in our basic model. Moreover, loss of companionship remains an effective sanction even if we allow for savings and physical capital formation. The existence of savings reduces the need to rely on financial support from children during old age dependency, but to the extent that companionship and old-age consumption as defined in equations (16) and (3) are not perfect substitutes, savings cannot eliminate the demand for companionship. The set of conditions imposed in section 1 to assure the self-enforceability and time consistency of the implicit contracts between parents and children will equally well assure these outcomes in this extended model.

3. SOCIAL INSURANCE, SELF-INSURANCE AND GROWTH

Both our basic and extended models ascribe investments in children, and thus economic growth, to parental incentives to self-insure old-age needs. It is thus natural to consider in this context what possible impact social insurance programs, such as social security, are expected to have on the economy's growth prospects. As we show in this section, the answer depends on both the form of the social insurance program and its interaction with private self-insurance within a given growth regime.

In a "pay as you go" social security system, the government pays insurance benefits to old adults and collects taxes that are related in some linear fashion to the income of young adults so as to maintain the social security fund in balance. More specifically, the typical tax rule can be thought of in a dynamic context as involving a combination of some lump sum tax and an amount proportional to human capital. Since the dynamic implications of each of these tax components is somewhat different, we shall consider them separately.

A. A Proportional Social Security Tax

The representative young parent pays a social security tax that is proportional to his human capital stock, i.e., an amount θH_t, where $\theta > 0$. The parent will receive an insurance benefit in the amount of S_{t+1} if he survives to old age. Since the fund must be balanced, the expected benefits must equal the expected tax receipts from children. The benefit per recipient of social security payments is thus

(23) $S_{t+1} = (\pi_1/\pi_2) n_t \theta H_{t+1}$,

and the young parent's consumption levels at young and old age are given by

(24) $c_1(t) = (H_1 + \bar{H})(1 - vn_t) - (n_t H_{t+1}/A) - \pi_2 w_t H_t - \theta H_t$, and

(25) $c_2(t+1) = \pi_1 n_t w_{t+1} H_{t+1} + S_{t+1}$.

The following analysis applies equally well in the growth regimes of both our basic and extended models, i.e., regardless of whether investments in children are motivated partly by "companionship", provided that there is a corner solution in fertility, i.e., $n^* = 1/\pi_1$. The optimization problem can thus be described as maximizing the utility functions (4) or (17), subject to equations (23), (24) and (25), $H_{t+1} \geq 0$ and $n_t = 1/\pi_1$. Since the social security benefit is taken to be exogenously given, we obtain the same first order optimality conditions as in equations (8) or (19a). Specifically $(c_2/c_1)^\sigma = \delta A \pi_1 \pi_2 w_{t+1}$. Substituting equations (23), (24) and (25) into this expression we obtain after some manipulation

(26) $\{[\pi_1 W_{t+1} n_t H_{t+1}]/[(H_t + \bar{H})(1 - vn_t) - (n_t H_{t+1}/A) - \pi_2 W_{t+1} H_t]\}^\sigma = \delta A \pi_1 \pi_2 w_{t+1}$,

where $W_{t+1} \equiv w_{t+1} + \theta/\pi_2$.

It can easily be shown that if the social security benefit S_{t+1} were not taken as exogenously fixed, but treated as an endogenous outcome of the optimal choice of n_t and H_{t+1}, given θ, then the R.H.S. of equation (26) would become $\delta A \pi_1 \pi_2 W_{t+1}$, in which case equation (26) would be identical to equation (9). The solutions pertaining to capital accumulation in the growth regimes where $n^* = 1/\pi_1$ in sections 1 and 2 would thus apply in this case as well. In particular, the introduction of social security would then leave human capital accumulation and economic growth unchanged. The reason is that parents and children are free to determine the overall optimal rate of intertemporal transfers, as indicated by $W_{t+1} = w_{t+1} + \theta/\pi_2$. Parents can thus choose to use the insurance benefits to reduce the monetary compensations owed them by children, or pay the latter's social security taxes. Indeed, if the tax rate θ is raised, the optimal compensation ratio, w_{t+1}, will be lowered to maintain the overall (voluntary) rate of taxation W_{t+1} unchanged. The independence of self-insurance through investment in children from social insurance in this

case is a manifestation of the so-called "Ricardian Equivalence Theorem". The government can simulate this solution only by making the insurance benefits received by parents proportional to the human capital of their respective children.

Absent such solution, S_{t+1} is an exogenously determined constant and equation (26) yields the following law of motion in human capital accumulation:

(27) $H_{t+1} = a^*_{t+1} + b^*_{t+1}$, where

$a^*_{t+1} = \{Aj_{t+1}(1-vn_t-\pi_2 w_t)\}/[n_t(AW_{t+1}+ j_{t+1})]$;

$b^*_{t+1} = [Aj_{t+1}(1-vn_t)\bar{H}]/[n_t(AW_{j+1}+ j_{t+1})]$ and $j_{t+1} = (A\pi_1^{1-\sigma}\pi_2 w_{t+1})^{1/\sigma}$.

The optimal compensation ratio w_{t+1} can now be determined by applying the methodology we used in section 1. The optimality condition is given by

(28) $A(1-vn_{t+1}- \pi_2 W_{t+1})\{(1-\sigma)+[\theta/(\pi_2 w_{t+1})]\} = \pi_2\sigma(j_{t+1}+AW_{t+1})$,

and the dynamic system is then fully described by equations (27), (28), and the optimal solution for the fertility rate $n_t = 1/\pi_1$. Note that when $\theta=0$, $W_{t+1} = w_{t+1}$ and equation (28) becomes identical to the condition for the optimal compensation ratio used in our development of the basic model (equation (15)).

Equation (28) provides a unique solution for the optimal compensation ratio $w_{t+1} = w^*$, which is conditional only on the model's basic parameters.[10] Therefore, the coefficients of the equation of motion in human capital (27) are also constant over time, $a_{t+1} = a^*$ and $b_{t+1} = b^*$. Since the latter equation becomes a first-order linear difference equation such as (10a), we obtain here the counterparts of Propositions 1 and 2. To wit,

<u>Proposition 7</u>. Under a social security system with a proportional tax rate θ and a given set of parameters, there can be only one steady state equilibrium for H_t provided that fertility is at a corner solution $n^* = 1/\pi_1$. The optimal compensation rate w^* then maximizes the attainable steady state growth rate

(a^* in equation (27)).

The major effect of the social security system under a fixed tax rate is

Proposition 8. The introduction of a social security system in which the tax rate is proportional to the human capital stock of the representative tax payer will lower the steady-state growth rate the economy can attain without taxes.

Proof: Since w^* maximizes the growth rate a^*, the envelope theorem applied here. Consequently,

$$(da^*/d\theta)=(\partial a^*/\partial\theta)= -(Aj/n)\{1/(j+AW)+[(1-vn-\pi_2W)A]/[\pi_2(1+AW)^2]< 0 .$$

The basic reason a proportional social security tax rate affects adversely human capital accumulation and growth is that the representative individual has no incentive to consider the marginal benefits he confers on the rest of society through the marginal investment in children in the form of higher social security benefits for all. This "free rider's problem" causes a negative effect of social insurance on self-insurance and economic growth.

B. A Lump-Sum Social Security Tax.

The tax levy in this case is a constant γ and the insurance benefit per recipient is $(\pi_2/\pi_1)n_t\gamma$. Equations (27) and (28) thus change to

(29) $H_{t+1}= a^o_{t+1} H_t + b^o_{t+1}$, where

$a^o_{t+1} = [AJ_{t+1}(1-vn_t-\pi_2w_t)]/[n_t(A+J_{t+1})]$,

$b^o_{t+1} = \{AJ_{t+1}[(1-vn_t)\bar{H}-\gamma]-[n_t\gamma A/(\pi_2w_{t+1})]\}/[n_t(A+J_{t+1})]$;

and

(30) $A(1-vn_t-\pi_2w_{t+1})\{(1-\sigma)H_{t+1}+ [\gamma/(\pi_2w_{t+1})]\} = \pi_2\sigma\ w_{t+1}(A+J_{t+1})H_{t+1}$,

where J_{t+1} is defined in equation (13).

Equation (30) implies that the optimal compensation rate w_{t+1} will vary with the per-capita level of human capital. The equation of motion (29) thus becomes a nonlinear difference equation since a^o_{t+1} and b^o_{t+1} depend on w_{t+1}

and will vary with the level of human capital.

Proposition 9. A social security system subject to a lump sum social security tax can generate several steady state equilibria for the economy under a given set of parameters.

The following simulation illustrates this case. Let $A=5$, $\sigma=0.5$, $w=0.2$, $\pi_1=0.99$, $\pi_2=0.95$, $\bar{H}=1$ and $\gamma=0.7$. If we plot H_{t+1} against H_t, the curve cuts the 45° line twice at $H_t=0.2017$ and $H_t=1.0058$. If the initial value of H_t is less than 1.0058, the economy will converge to the low level solution for H at point A in Figure 5. Otherwise it will attain a growth equilibrium at point B in Figure 5 where the growth rate is $a^o=1.4657$.

The significance of this result from a policy perspective is that the fixed social security tax rate introduces potential nonconvexities in the phase line III linking H_{t+1} with H_t. If the economy is at an early stage of a growth equilibrium, a small change in the tax rate γ or other relevant parameters which lower the phase line III in Figure 5 may result in a big change in the growth equilibrium, with the economy sinking back to a development equilibrium. This potential outcome is due to the distortive consequences of a fixed social security tax, the severity of which depends on the economy's growth level.

Proposition 10. The steady-state growth rate attained under a lump sum social security tax, a^o cannot be higher than that attained without such tax, a. In a development equilibrium $a^o < a$. In a steady-state growth equilibrium $a = a$.

Proof: In a steady-state development equilibrium, where $H_{t+1}=H_t= H^o$ and $w_{t+1}=w_t= w^o$ by equation (30), a_{t+1} in equation (29) becomes a constant, a^o, which is identical in form to the growth rate in an economy free of taxes, a. However, the value of w^o which determines a^o is different from the one

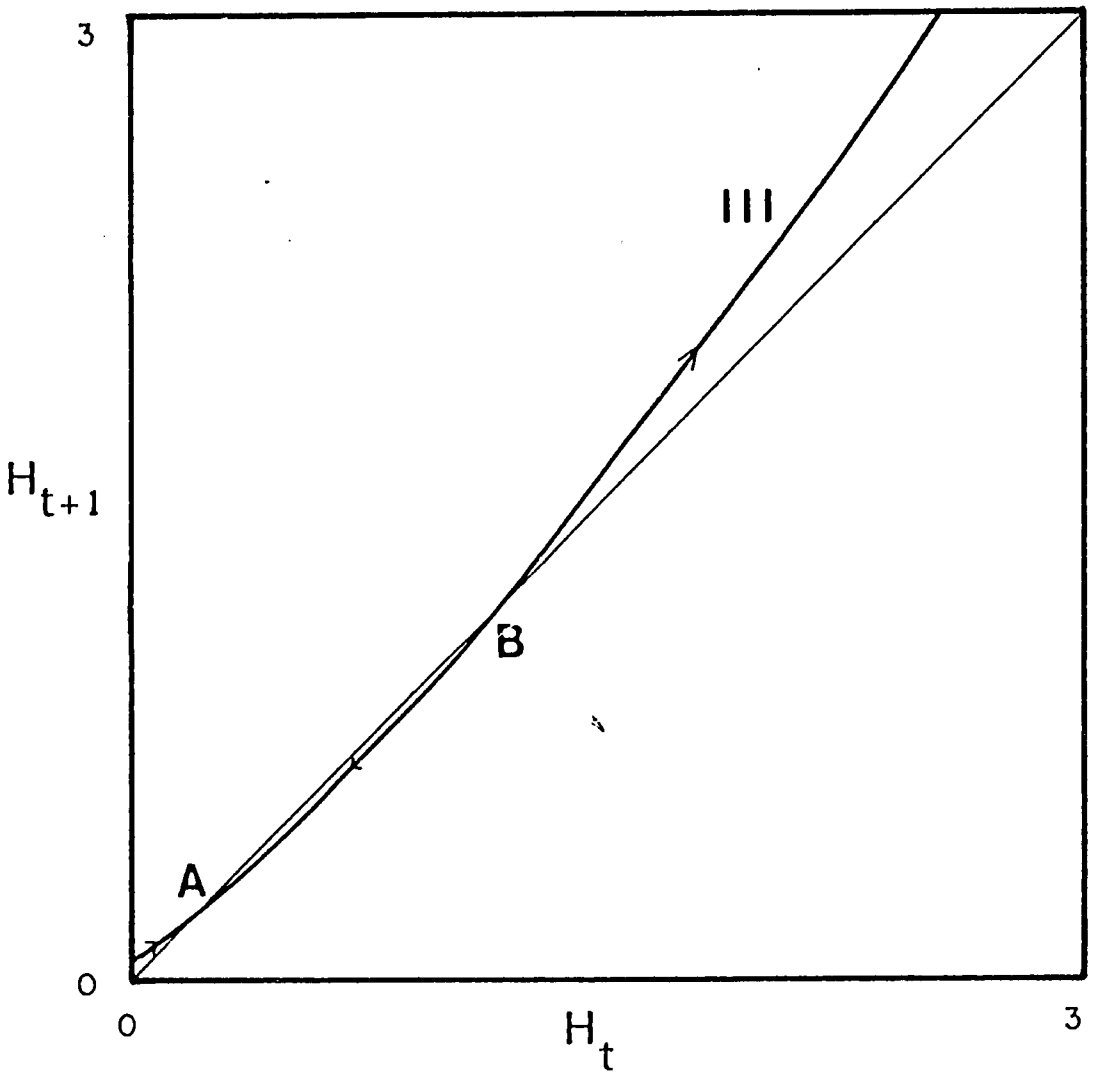

FIGURE 5

determining the maximum value of a. Hence w^o cannot be a maximizer, and $a^o < a$.

In a steady-state growth equilibrium, in contrast, the term γ/H_{t+1} must approach zero as H_t increases without bound. Thus equation (30) becomes asymptotically the same as equation (15) and $a^o = a$. The intuition is simple: while a fixed tax rate distorts the growth rate, the distortive effect becomes lower as the economy grows larger relative to the tax level.[11]

The conclusions of this analysis can be summed up as follows: a social security system based on taxes that are proportional to income $(\bar{H} + H_t)$ will lower the growth rate attainable by an economy free of such taxes, and such distortion increases with the magnitude of the tax rate. This result is proved in the preceding analyses at least for the case where the economy has attained a growth equilibrium with a low fertility rate $n^*=1/\pi_1$, but may apply in other growth regimes as well. The essential reason is that the introduction of an exogenously determined social insurance lowers the incentives parents have to self-insure their old-age needs through investments in children.

More important, we expect this analysis to hold even when we generalize our extended model to allow for physical capital accumulation and savings as an alternative instrument of self-insurance of old-age needs. We plan to address this generalization in future work.

CONCLUSION

We have argued in this paper that a period of significant improvements in the life expectancies of different age groups in the population can trigger the onset of an economy's takeoff from a low-level, stagnant "development equilibrium" into a state of self-sustaining, persistent "growth equilibrium" and that, furthermore, variations in the population's level of longevity may

help explain the observed diversity in the rates of per-capita income growth across developed economies. We trace the association between longevity and growth to the former's interaction with human capital accumulation across generations. Specifically, parents who benefit both directly and indirectly from a higher productive capacity of their children, and are concerned about the welfare level of each child, find it optimal to invest in their children's education an amount of resources that is enhanced by the respective generations' probabilities of survival. By this view, the long-term rate of growth of developed economies is not destined to slow down or fall relative to that in developing economies, as the neo-classical "convergence hypothesis" might imply, but may actually be enhanced by the secular upward trend in the life expectancies of the populations of these economies. Our analysis also underscores the role of the family unit in general, and optimal intrafamily transfers specifically, in assuring an endogenous growth equilibrium.

These conclusions are derived within a simplified dynamic setting in which changes in longevity are taken to be exogenous to the economy and where "raw labor" and human capital are the only factors of production. Longevity, however, is a symptom of greater personal "health capital", which is a distinct component of human capital. Changes in longevity, even those due to technological improvements, should therefore be viewed within a more complete model as an endogenous outcome of the system's basic parameters and initial conditions, including the initial level of human capital in the population. Also, a more complete model should incorporate savings as an alternative means of securing old-age income for parents. By incorporating these "missing elements" into our dynamic system in future work, we expect to obtain additional behavioral implications that may be relevant for explaining what we called in this paper the "problem" and the "process" development.

Our present analysis leads, however, to a number of policy implications that should remain valid under a more general framework of analysis as well:

1. Investments in health and related means of extending life expectancy can have a high payoff in terms of the economy's growth prospects. It seems appropriate to direct economic assistance efforts in less developed countries towards investments in this and other sources of human capital growth which may exert lasting effects on the economy by encouraging a transition from a state of dependency toward a state of self-sustaining and persistent growth.

2. Governmental efforts that attempt to achieve higher rates of economic growth through population control policies may be misguided. Since these policies can only raise the cost of having children but cannot raise the rate of return to investment in the human capital of children, they may have adverse effects on the prospect of an economy's transition from a development equilibrium into a growth equilibrium.

3. Externally imposed social insurance plans that operate as a "pay as you go" system are likely to reduce families' efforts at self-insurance through investments in children. They can thus have deleterious effects on the economy's long-run rate of growth, unless the social insurance tax is independent of the economy's level of human capital. While the present literature on the burden of social security taxation has emphasized its static effects on economic efficiency, our analysis shows that such taxation can exert important dynamic consequences as well.

APPENDIX

In this appendix, we prove propositions 5 and 6. We also establish that in the extended model, the curve linking H_{t+1} and H_t is upward sloping.

Equations (21) and (22) define, respectively, the following implicit n_t functions:

(A1) $n_t = [(1-\pi_2 w)H_t+\bar{H}]/[\{(A+J)H_t/AJ\} + v(H_t+\bar{H})]$

$\equiv x(H_{t+1},H_t,\pi_1,\pi_2,v)$

(A2) $n_t = \{H_{t+1}^{\sigma}\ \beta/[(H_t+\bar{H})\ Avw^{1-\sigma}\pi_1^{(1-\sigma)(1-\beta)}]\}^{1/(1-\beta)(1-\sigma)}$

$\equiv y(H_{t+1},H_t,\pi_t,v)$.

Define x_i and y_i to be the derivatives of $x(\cdot)$ and $y(\cdot)$ with respect to their ith arguments. It is possible to show that $x_1 < 0$, $x_2 > 0$, $x_3 > 0$, $x_4 > 0$, $x_5 < 0$, $y_1 > 0$, $y_2 < 0$, $y_3 < 0$, and $y_4 < 0$. In establishing $x_4 > 0$, we have made use of the condition that the value of w in the growth regime where n has an interior solution (regime 2) is smaller than its value in the growth regime where $n = 1/\pi_1$ (regime 1), which can be easily verified. By setting $x(\cdot) = y(\cdot)$, we can prove the following:

(A3) $dH_{t+1}/d\pi_1 = (x_3-y_3)/(y_1-x_1) > 0$.

(A4) $dH_{t+1}/d\pi_2 = x_4/(y_1-x_1) > 0$.

(A5) $dH_{t+1}/dH_t = (x_2-y_2)/(y_1-x_1) > 0$.

An increase in π_1 or π_2 will cause H_{t+1} to go up. This increase will further raise H_{t+2}. The process repeats itself. The human capital stock will either converge to a higher development equilibrium or grow without bound. (A5) also proves that the curve linking H_{t+1} with H_t is upward sloping.

Let $f(n_t,H_t,\pi_1,\pi_2,v)$ and $g(n_t,H_t,\pi_1,v)$ be the implicit H_{t+1} functions defined by equation (21) and (22), respectively. Again, let f_i and g_i represent their derivatives with respect to their ith arguments. The signs of

these derivatives are as follows: $f_1 < 0$, $f_2 > 0$, $f_3 > 0$, $f_4 > 0$, $f_5 < 0$, $g_1 > 0$, $g_2 > 0$, $g_3 > 0$, and $g_4 > 0$. In obtaining $f_4 > 0$, we have again made use of the condition that the value of w in regime 2 is smaller than that in regime 1. We equate $f(\cdot)$ and $g(\cdot)$ to derive the following:

(A6) $dn_t/d\pi_1 = (f_3-g_3)/(g_1-f_1) \gtrless 0$

(A7) $dn_t/d\pi_2 = f_4/(g_1-f_1) > 0$

(A8) $dn_t/dH_t = (f_2-g_2)/(g_1-f_1) < 0$ if and only if $f_2 < g_2$.

Differentiation of $f(\cdot)$ and $g(\cdot)$ shows that g_2 must be larger than f_2 when H_t is sufficiently large. Hence, if an improvement in longevity causes H_t to go up over time without bound, n_t must also eventually decline continuously until it reaches its minimal value. This proves proposition 5.

To establish proposition 6, we first note that

(A9) $dH_{t+1}/dv = (x_5-y_4)/(y_1-x_1) > 0$,

since the numerator can be shown to be strictly positive. This implies that when the economy is still in regime 2, an increase in v must shift the curve relating H_{t+1} and H_t upwards. In contrast, if the economy is in regime 1 where n_t is a constant, $da_{t+1}/dv < 0$ and $db_{t+1}/dv < 0$. An increase in v in this case must lower both the growth rate and level of H_{t+1}. It can also be shown that $dn_t/dv < 0$. Hence when v is larger, the corner solution for n_t is attained at a lower value of H_t.

These results are consistent with the three possibilities depicted in figures A1, A2, and A3. In each case, curve I links H_{t+1} and H_t before the increase in v, and curve II links them following the increase. In figure A1, an increase in v causes the development equilibrium to move up from A to B. In figure A2, the development equilibrium declines, instead, from C to D. In figure A3, an economy which is initially growing may sink to a no-growth equilibrium at E. These alternative possibilities establish proposition 6.

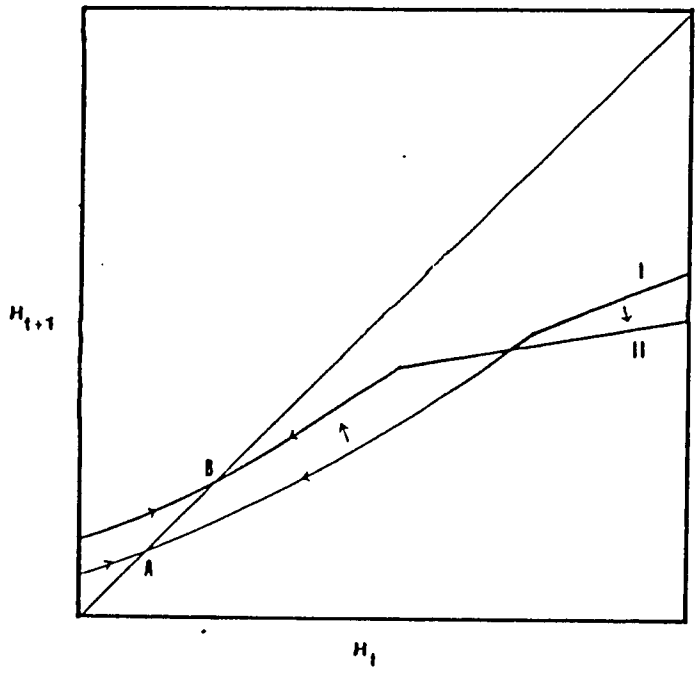

FIGURE A1

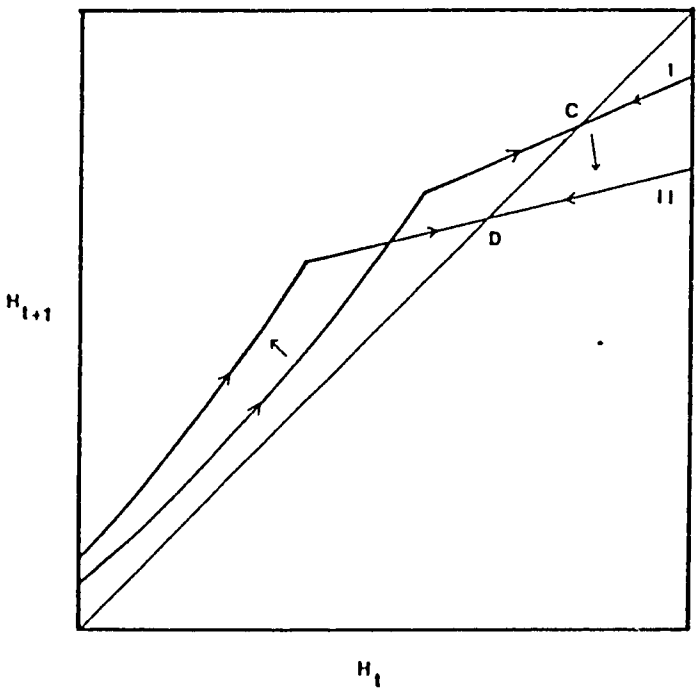

FIGURE A2

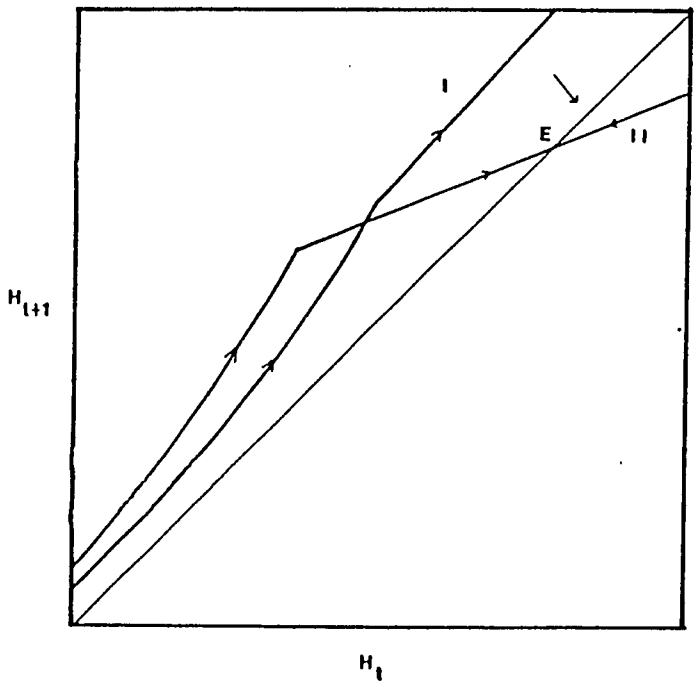

FIGURE A3

FOOTNOTES

1 This avoids the difficulty discussed in Becker and Murphy (1988) that children of young age cannot enter explicit contracts with their parents. The possibility of explicit contracts between parents and older children need not be entirely ruled out, since older dependent children often do borrow family funds in pursuit of investment opportunities. Such contracts can therefore be anticipated by parents.

2 The decision making unit may be a single parent or a pair of parents. In the latter case, the opportunity set represents the outcomes to both parents from investments in children, and n_t may be interpreted as the number of births per family rather than per parent.

3 The solution $n_t = 0$ or $H_{t+1} = 0$ would be incompatible with an optimum since it implies that $c_2(t+1)=0$, and thus that $\mu_2/\mu_1 = \infty$ in equation (6). Even a very small investment in children involving a tradeoff of present for future consumption would be welfare-improving. In addition, of course, time consistency requires that $n_t > 0$.

4 If $\sigma \geq 1$, the solution for equation (15) is nonpositive but it then represents a minimum solution for the expected utility u_{t+1}. Consequently, the optimal value of w, w^*, will be a very small but <u>positive</u> magnitude as well. The influence of σ on w^* is due to the role of the intertemporal elasticity of substitution in consumption: the lower the latter (the higher is σ), the smaller is one's incentive to trade consumption in the period of adulthood (c_1) for an increment in consumption in the period of old age (c_2). Since we wish to develop behavioral implications relating to interior optima in w and h, we shall henceforth assume that $0 < \sigma < 1$.

5 An increase in the intertemporal elasticity of substitution in consumption (a decrease in σ) may or may not raise the economy's growth rate. Formally, $da/d\sigma = -(\ln\delta A\pi_1\pi_2 w)[Aa/(A+J)\sigma^2]$. It follows that $da/d\sigma$ will be positive or negative depending upon whether the discounted expected rate of return $\delta R_h = \delta A\pi_1\pi_2 w$ is smaller or greater than one. If σ is close enough to a unitary value, equation (15) indicates that w will approach zero. In this case $da/d\sigma > 0$.

6 See, for example, Coale (1987), Easterlin (1987) and references therein. The article by Dyson and Murphy (1985) focuses on empirical evidence from many developing countries showing that fertility rates tend to surge before the onset of their decline during the demographic transition.

7 An alternative interpretation of the two types of benefits old parents derive from their children are 1) services involving a direct use of children's time which influence the consumption level they "produce" for old parents, $c_2(t+1)$; and 2) direct utility parents derive from good children, $c_3(t+1)$. Thus, the resources transferred from young to old adults need not involve direct monetary payments. This alternative interpretation of the assumed objective function involves only minor modifications of the relevant opportunity set and results in the same basic implications we derive in this section.

8 The nonlinearity of the curves linking H_{t+1} and H_t suggests, in fact, the possibility of multiple equilibria, since curves I and II in Figure 2 can intersect the 45° line more than once. Some development equilibria may therefore be unstable in this case. Note, however, that in this analysis the compensation ratio, w, is taken to be a constant.

9 Apparently similar results concerning the behavior of fertility following a decline in mortality risk have been derived by Becker and Barro (1988) and Srinivasan (1988). In the former paper, however, an exogenous decline in mortality has no permanent effect on the desired number of children. The percentage decrease in the birth rate must be equal to the percentage increase in the probability of survival. In our model, a once-and-for-all increase in longevity may in principle initiate a development process and cause both the birth rate and the desired number of children to decline over time. We can thus explain both the demographic transition and systematic differences in population growth rates across developing and developed countries at a point in time. Srinivasan's paper, like ours, relies on parents' demand for old-age support as a motive for having children. He shows that the fertility rate may increase or decrease following a decline in the mortality rate. However, the model does not offer a fully dynamic structure that links trends in fertility with the process of development and the demographic transition.

10 A development equilibrium can also be compatible with a corner solution at $n^* = 1/\pi_1$ if the phase line relating H_{t+1} to H_t intersects the 45° line along its linear segments. An example of such equilibrium is provided through a simulation involving the same parameter values considered in the previous illustration with the exception of a lower value of $\pi_1 = 0.33$.

11 Because in this version of our extended model the rate of return to human capital $R_h^* = A\pi_1\pi_2 w^*$ is independent of the level of H_{t+1}, a large enough externally imposed social security tax could "crowd out" entirely self-insurance through investment in children ($h_t = 0$). The effects on growth would then be extreme since the economy would sink into what Becker and Murphy (1989) call an "underdevelopment equilibrium". This result would

not be forthcoming--an interior solution in h_t would be guaranteed--if we invoke the more general version of our extended model where R_h^* is a decreasing function of H_{t+1} (see equation (19)). In the following analysis we assume that the social security tax is sufficiently low so that $h_t > 0$.

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