PN-AAC-374

CRITERIA USED IN SCREENING TECHNOLOGY PRIOR TO FIELD RECOMMENDATION

RAPHEL OUSEPH THATTIL

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
UNIVERSITY OF THE PHILIPPINES AT LOS BAÑOS
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE
DEGREE OF

MASTER OF SCIENCE
(Statistics)

March, 1980

The thesis hereto attached, entitled "CRITERIA USED IN SCREENING TECHNOLOGY PRIOR TO FIELD RECOMMENDATION", prepared and submitted by RAPHEL O. THATTIL, in partial fulfillment of the requirements for the degree of Master of Science (Statistics) is hereby accepted.

ANN INEZ N. GIRONELLA
Member, Guidance Committee

Date Signed

TIRSO B. PARIS, JR.
Member, Guidance Committee

Date Signed

JOHN C. FLINN
Member, Guidance Committee

Date Signed

MARIANO B. DE RAMOS
Adviser and Chairman
Guidance Committee

Date Signed

Accepted as partial fulfillment of the requirements for the degree of Master of Science (Statistics).

DOLORES A. RAMIREZ
Dean, Graduate School
University of the Philippines
at Los Baños

Date Signed

BIOGRAPHICAL SKETCH

RAPHEL O. THATTIL, son of Poulose and Mariam Thattil, was born on January 26, 1948. He is the 2nd child in a family of thirteen. He received his primary education at St. John's Dematagoda and completed his pre-University course at St. Benedict's, Kotehena.

In 1969 he entered the University of Peradeniya Sri Lanka to do a course in Agricultural Science leading to the BSc (Agric) degree. He was awarded the turf Club Scholarship in 1970. During his 4 years at the campus he represented the University in soccer and hockey. In 1974 he completed the BSc (Agric) degree, passing with second class honours.

Immediately after receiving the degree he was employed by the Department of Agricultural Economics as a research assistant. During the latter part of the same year he was appointed temporary assistant lecturer in Agric. Economics. In 1975 October he was selected to the newly created Div. of Biometry as a probationary assistant lecturer.

In November 1977 he was awarded a scholarship by the ADC (Agricultural Development Council) to pursue a M.S. degree in Statistics in the University of the Philippines at Los Baños.

He is married to Manorie Dharmawansa and has a child, Asanji born on July 2nd in the Philippines.

RAFHEL O. THATTIL

ACKNOWLEDGEMENT

I am thankful to the Agricultural Development Council for awarding a M.S. Scholarship that enabled me to undertake this study at the Department of Statistics, University of the Philippines at Los Baños, from 1977 November to 1980 March. I am also indebted to the International Rice Research Institute (IRRI) for granting me a research followship, which enabled me to complete my thesis work.

My grateful thanks go to Dr. Edwin Price who was instrumental in getting me to work at IRRI. I benifitted greatly from his help and valuable suggestions. To Dr. Sisira Jayasuriya I owe perhaps the greatest intellectual debt. He provided me with expert guidance throughout the preparation of this thesis and in the absence of Dr. Flinn, acted as member of my committee.

Among the others who assisted me, I wish in particular to thank Dr. Sid David and Dr. Ann Gironella. They gave me valuable ideas at crucial stages of the study. I also wish to express my sincere appreciation of my Adviser Dr. Mariano B. de Ramos and committee members Dr. Tirso Paris and Dr. John Flinn for their help and assistance. To Dr. James Roumasset who stimulated my interest in curve fitting I owe a great deal. He also provided me with many articles on the subject.

I am indebted to Yollie and Norma (Dept. of Economics, IRRI) for their help in converting illegible handwriting into presentable drafts and to Jovy Manguiat (Dept. of Statistics, UPLB) in doing a fast and neat job on the final form of the thesis.

I owe a big 'Thank you' to Oyie Umali for help in programming, Sreeni Jayasuriya for helping me out with the graphs and Danny for the expert work he performed on the final forms of the graph.

However this thesis would never have been completed if not for the inspiration and loving devotion of my wife Manorie.

TABLE OF CONTENTS

	PAGE
INTRODUCTION	1
Objectives	2
Area of Study and Data Source	3
REVIEW OF LITERATURE	5
Estimation of Yield Distribution	5
Criteria Used in Screening Technology	9
METHODOLOGY	12
Tests of Normality	13
Pearson System	16
Method of Moments and Type I Function	19
Maximum Likelihood Estimation	27
Screening of Technologies	34
RESULTS AND DISCUSSION	41
Preliminary Tests	41
Method of Moment Estimates	43
Screening of Technologies	59
SUMMARY AND CONCLUSIONS	68
LITERATURE CITED	71
APPENDIX	74

LIST OF TABLES

TABLE		PAGE
1	Stratification of data	12
2	Pearson system of curves	18
3	Tests of normality and values of criterion	42
4	Type I parameters, α_1 and α_2 unspecified	44
5	Type I parameters, $\alpha_1 = 0$	46
6	Type I parameters, $\alpha_1 = 0$, $\alpha_2 = \alpha$	47
7	Maximum likelihood estimates	49
8	Probabilities for yield $\leq$ 2 tons/ha.	57
9	Probabilities of loss and means for the different technologies	65

LIST OF FIGURES

FIGURE		PAGE
1	Pearson system and the criterion K	17
2	Flowchart of Fortran program to compute maximum likelihood estimates	35
3a-3j	Fitted curves superimposed on histograms for the different strata	50
4	Empirical cumulative distribution functions for high labour, low and high cash inputs, second crop.	56
5a	Cumulative density functions of profit, first crop	60
5b	Cumulative density functions of profit, second crop	61
6	Illustration of SSD	62
7	SSD cumulative curves	64

ABSTRACT

THATTIL, RAPHEL OUSEPH, University of the Philippines at Los Baños, March 1980. Criteria Used in Screening Technology Prior to Field Recommendation. Major Professor: Dr. Mariano B. de Ramos.

The yield data were categorized into 6 strata, using 3 levels of labour (high, Medium and Low) and 2 levels of cash input costs (High and Low), for the 1st and 2nd rice crops.

The distribution of the data in each strata was satisfactorily explained by the Pearson type I function. The method of moments was initially used in estimating the parameters of the distribution and later improved by using the maximum likelihood method. The concept of stochastic dominance was then used to screen the different technologies.

The conclusions that follow from the study are briefly:

1. Yield distribution of rice for a given technology are generally non-normal.
2. The Pearson type I function can be used to adquately represent the yield distributions.
3. The distributions become progressively more negatively skewed as levels of labour and cash inputs are increased.
4. The type I moment estimates with the extremities of the curve specified, may be acceptable substitutes for the maximum likelihood estimates.
5. Medium labour technologies form the risk efficient set.

INTRODUCTION

Agricultural research is primarily geared towards increasing productivity of crops. In the field of rice research, it is centered around evolving early maturing varieties, with desirable characteristics such as high yields, pest and disease resistance and good grain quality, and methods of crop establishment which enable cropping intensity to be increased. These varieties however tend to require high levels of inputs and management to realize their potential. Successful introduction of this new technology to farmers, used to late maturing varieties and traditional methods of farming, is possible, only if it can pass the test of economic efficiency.

Economic efficiency is usually measured in terms of profitability of new technologies. However in the past, most of the analysis of new technologies for profitability have ignored the inherent variability in crop yields. Profitability has been based on the mean value of yields. But crop yields are stochastic; and more and more researchers are discovering that adoption of new technology is influenced by the variability of yields and related profits. Various studies on technology adoption have pinpointed the fact that many new technologies which have higher average yields but also high variance under field conditions are rejected by farmers for more stable but lower yielding technologies. This has made researchers aware of the need to screen new technologies not only for higher mean levels of performance but also for higher stability in the performance.

The last few years have witnessed the incorporation of risk[1] considerations both in partial as well as whole farm type analysis of the performance of new technologies. However, these studies have tended to suffer from the limitation that they have explicitly (or more frequently implicitly) assumed normality in the underlying yield (or gross margin) distributions. Such an assumption cannot of course be justified on 'a priori' grounds. In fact, since yields are always non-negative and have a biologic maximum, there are 'a priori' grounds for rejecting the normality assumption. Despite this very few empirical studies have attempted to look at the actual form of the yield functions.

Objectives

This study will be concerned only with rice farming. The specific objectives can be classified into two. (a) A sample of rice yields generated from IRRI cropping systems research will be used to estimate the probability distribution of yield for different levels of cash input and labour costs. (b) The profit distribution obtained by transformation of the yield distribution will be used to screen the technologies for 'risk efficiency'.[2]

---

[1] According to Frank Knight risk refers to a situation where alternative outcomes exist with known probabilities and uncertainty to the case where probabilities are unknown. However in modern decision theory uncertainty is usually subsumed under risk, and we shall adopt the latter approach in this study.

[2] The concept of 'risk efficiency' will be discussed in the next two chapters using different performance criteria.

Area of Study and Data Source

The data used in this study come from the cropping pattern trials conducted by the International Rice Research Institute (IRRI) in collaboration with the Philippine Bureau of Plant Industries (BPI) in Pangasinan during 1975-1978. Pangasinan was chosen as a research site, for the following reasons:

(i) The represents land areas parts of which are fully irrigated, while some others are partly irrigated and still others rainfed; which are typical conditions of major agricultural lands of Asian countries.

(ii) In depth and reliable data of the project area are available at IRRI.

(iii) The area is easily accessible to the researchers.

Within the site, research is performed under varied micro-economic and physical conditions, so that after such broadly determining factors such as rainfall pattern are considered, the appropriateness of new cropping patterns to micro-variants as soil texture or farm size can also be learned.

There are 2 groups of farmer-cooperators in the site, namely 'agronomic' and 'economic' cooperators. The agronomic cooperators make portions of their land available for agronomic investigations of cropping patterns. They perform experiments (cropping pattern trials) directed by IRRI researchers and use inputs donated by IRRI. A research team, composed of members from four IRRI departments and

the BPI are assigned to the research site to implement cropping systems research on farmer's fields.

The climate in the area on the average has, six consecutive dry months with less than 100 mm of rain and 4 consecutive wet months with more than 200 mm of rainfall, of which at least one month average greater than 500 mm of rainfall. The yearly average of rainfall in 1980 mm. Taxonomically most soils at the site fall within the Eutropept great group. Clay loam is the modal surface soil texture. The land system is composed of flat plains of the river terrace sub-system.

REVIEW OF LITERATURE

This chapter comprises 2 sections. The first section discusses the estimation of yield distributions and the second reviews the various criteria used in screening technologies.

Estimation of Yield Distribution

Karl Pearson in a series of papers beginning in 1895, (1895, 1902) was one of the first to study a family of curves which would represent empirically useful distributions satisfactorily. He looked at a class of frequency functions which (a) have a single mode and (b) have smooth contact with the X-axis at the extremities. These conditions are obeyed by density functions that satisfy the differential equation

$$\frac{d \log y}{dx} = \frac{x + a}{b_o + b_1 x + b_2 x^2} \qquad (1)$$

where a, b_o, b_1 and b_2 are constants and $y = f(x)$ is the frequency function of the random variable x.

The family of curves given by the above equation is known as the Pearson system of curves. This system covers a wide range of curves which include the Normal, Chi-square, Beta and Gamma distributions.

For all members of the system it can be shown that

$$nb_o\mu'_{n-1} + \{(n+1)\ b_1 + a\}\mu'_n + \{(n+2)\ b_2 + 1\}\ \mu'_{n+1} = 0 \qquad (2)$$

where μ_n' is the nth moment about zero given by

$$\int_{-\infty}^{\infty} x^n f(x)\, dx$$

Equation (2) permits the determination of any higher order moment from those of lower orders. In fact the four constants can be expressed in terms of the first four moments about zero. Putting $n = 0, 1, 2, 3$, successively in equation (2), a. b_o, b_1 and b_2 can be found in terms of μ_1 to μ_4. In general it follows that the Pearson system of distributions is completely determined by its first 4 moments.

For explicit expression of the frequency function $y = f(x)$, the right hand side of equation (1) has to be integrated. According to the nature of the roots of the quadratic $b_o + b_1x + b_2x^2$, integration yields the main types of the Pearson system.

Elderton and Johnson (1969) give a systematic account of the technique of fitting Pearson distributions. Day (1965) used the Pearson system to fit distributions to field crop yields. He found that yield distributions of crops were in general non-normal and non-log normal.

Fisher (1922) showed that the method of moments was statistically inefficient and proposed the method of maximum likelihood. The likelihood function of n random variables $x_1, x_2, \ldots, x_n$ is the joint density of random variables $f_{x_1, x_2, \ldots, x_n}(x_1, x_2, \ldots, x_n; \theta)$ which is considered a function of the parameter θ. The likelihood for a random sample $x_1, x_2, \ldots, x_n$ is $L(\theta) = f(x_1, \theta)\, f(x_2, \theta) \ldots f(x_n, \theta)$

The maximum likelihood estimate is a solution to the equation,

$$\frac{d\ L\ (\theta)}{d\theta} = 0$$

For p parameters there will be p equations to solve.

The maximum likelihood method has several optimal properties. However, one difficulty of using the method in general curve fitting is that the equations cannot be solved directly and iterative techniques have to be used to get explicit solutions to the parameters.

Koshal (1933) carried out Fisher's suggestion of using an iterative scheme for improving the efficiency of coefficients estimated by the method of moments. Koshal's method of approximation was based on maximizing

$$L = \sum_{k=1}^{c} n_k \log p_k$$

where n_k $k = 1, 2, \ldots c$ denotes the observed frequency of the kth class and p_k is the probability of an observation falling in this class, as determined from the curve (and is thus a function of the parameters). He obtained values of L by varying one parameter at a time and then by varying 2 parameters at the same time. Increments for the parameters were chosen at random. The method is very laborious and is obsolete with the availability of good computer facilities.

Myers (1934) followed up Koshal's work using the same data, as he found that Koshal had made some computational errors.

Serquiña (1977) and Paris (1978) working with rainfall data showed that the maximum likelihood estimates gave very good fits, as judged by the chi-square values.

There are also methods other than use of the Pearson system to describe a family of distributions. One method due to Bruns, Gram, Charlier and Edgeworth (see Kendall and Stuart, 1958) seek to represent a given frequency function as a series of derivatives of the normal frequency function. But the method does not satisfactorily represent frequency curves nor is it an improvement over the method of moments (see Elderton and Johnsom (1969) for worked examples).

The third approach is due to Edgeworth (1898) and later writers like Johnson (1949). In this method the variables are transformed, whereby the transformed variables may be considered to have a normal distribution. Based on the type of transformation used Johnson developed 3 systems which he called the S_L (lognormal curve), S_B (range of variation bounded) and S_U (range of variation unbounded) systems. For the S_U system he provides tables to facilitate the fitting.

Other methods, such as the least squares method, cannot be used as general methods for fitting curves. Least squares for example suffers from the weakness that it often leads to equations incapable of solution.

In practical applications many researchers have used hand smoothing in fitting frequency distributions {Schlaifer (1969), Roumasset (1976)}. Roumasset actually combines the method of

moments with hand smoothing. Anderson (1973) confronted with sparse data used the jth order statistic to estimate the $\frac{j}{n+1}$ fractile of the distribution. He advocates drawing a hand smoothed curve through these points to then obtain the cumulative distribution function.

The dearth of applications of the maximum likelihood method in the past have been due to the problems encountered in seeking solutions to the likelihood equations. But now with greater computer facilities this problem is less servere. Kale (1962) investigated the iterative solution of the likelihood equation for a multiparametric case. He gives various methods of solution which include the Newton-Raphson process and the method of scoring for parameters due to Fisher. Barnett (1966) went further and reviewed the methods of solution where the likelihood equation has multiple roots. If good computer facilities are available the iterative techniques like the Newton-Raphson process and the method of false positions (Reguli falsi - see Methodology) present no insurmountable problems. Barnett shows that the method of false positions is the most appropriate when the likelihood equation has multiple roots.

Criteria Used in Screening Technologies

Technologies generated at research institutes need to be screened for economic efficiency and farmer acceptability before widescale recommendation. Since farming decisions are essentially investment decisions under conditions of risk and uncertainty,

screening for 'risk efficiency' of new technologies can lead to better recommendations.

There is an extensive body of literature dealing with investment decisions under risk. Roumasset (1976), Anderson et al (1977) give good reviews of decision theory and models of choice under uncertainty, for farmers.

This study deals with criteria that can be used in screening for 'risk efficient' technologies. The expected utility maximization rule provides an appealing theoretical framework for normative decisions analysis under uncertainty. It takes into account the risk attitudes and beliefs of the decision maker. It states that provided there exists a unique 'cardinal' preference scaling function for risky prospects, generally called a 'utility function' in the literature, then the 'utility' of any risky prospect can be calculated as a mathematical expectation of the utility function. According to the rule the decision maker should act in such a manner as to maximize the (subjective) expected utility, if he is to be consistent with his preferences.

The utility function of a decision maker can be constructed, based on a set of axioms of rational choice {von Neuman and Morgenstern, 1947}. However in real life applications of this rule, many problems arise, including the need for the elicitation of risk attitudes of individuals.

Markowitz (1952, 1959) and Tobin (1958) pioneered the "Mean-variance' approach to portfolio selection. This approach has been

widely used because it does not need the elicitation of preferences. It considers a portfolio as efficient if no other portfolio having the same or smaller variance has a larger mean or if no other portfolio with the same or larger mean has a smaller variance. But it has been shown by Feldstein (1969) and Borch (1969) that this approach is consistent with the expected utility maximization rule only if the individual's utility function is quadratic or if the expected income is normally distributed. However many researchers {Eg. Levy and Markowitz, 1979} have defended the mean-variance approach by arguing that the expected utility can be approximated well be a function of mean and variance, under most real world situations.

The weaknesses of the theoretical underpinings of the mean-variance approach have led to the search for more practical approaches which nevertheless are more in conformity with the rules of expected utility maximization. The use of the concept of 'stochastic dominance' has been put forward as one such approach. Anderson et al (1977) describe the concepts and its empirical application in some detail.

There are various other alternative approaches to decision making under uncertainty, for example, Shakle's focus loss and safety first rules of thumb (Roumasset, 1976, Anderson et al, 1977). Fishburn (1977) looks at a measure of risk which assumes that 'decision makers in investment contexts frequently associate risk with failure to attain a target return'. Webster's New World Dictionary defines risk as the chance of loss and it is commonly used in this sense. In this study the probability of loss is used as alternative risk criterion.

METHODOLOGY

Rice yield data are first stratified according to 3 levels of labour costs and 2 levels of material input costs (Table 1). The 3 levels of labour costs are designated high (H), low (L) and medium (M). Labor costs here include all costs incurred in land preparation. The levels were chosen according to the intensity of labour used in the various operations. Cash input costs, greater than 1,100 pesos were termed high (H) while costs lower than 1,100 pesos were termed low (L). The same levels were used in stratification of the first as well as the second crop of rice. More strata were not possible as the frequency per stratum would then have been very small (also see appendix).

Table 1. Stratification of Data (Costs in ₱/ha./season.)

LABOUR COST	CASH INPUT COSTS	
	High (> ₱1,100)	Low (< ₱1,100)
High (> ₱1,900)	HH	HL
Medium (₱1,500-1,900)	MH	ML
Low (< ₱1,500)	LH	LL

For the first and second crops separate stratifications were done. Preliminary testing for normality was done for data in each stratum. The method of moments was then used to obtain estimates to the parameters of the distribution function. In the next stage the maximum likelihood method was used to improve the parameters. Finally the concept of stochastic dominance was used in the screening of the 'risk efficient' technologies.

Tests of Normality

Chi-square Goodness of Fit Test

The data are grouped into classes to form a frequency distribution. The sample mean and standard deviation are then calculated. Using these quantities a normal distribution is fitted, which enables the calculations of the expected value within a class. The test criterion is

$$X^2 = \sum_{i=1}^{k} \frac{(o_i - E_i)^2}{E_i}$$

where i = 1, 2, ..., k is the number of classes

o_i = observed frequency in the ith class

E_i = expected frequency in the ith class

If the data are normally distributed, the above quantity follows the theoretical X^2 distribution with (k-3) degrees of freedom. (Snedecor and Cochran, 1967). If the data are non-normal computed

values of X^2 are large, and cause the rejection of the null hypothesis of normality.

The X^2 test can be used in general to test the goodness of fit of a theoretical probability distribution to the observed data.

The test is however a non-specific test. It does not tell one whether non-normality is due to skewness or kurtosis. Therefore other tests have been developed to test specifically for skewness or kurtosis.

Test for Skewness

A measure of skewness is afforded by the third moment about the mean (μ_3). If high values extend far above the mean, while low values are close to the mean, μ_3 will be positive and conversely if higher values are closer to the mean than the lower values μ_3 will be negative. The ratio μ_3/σ^3 (where σ is the standard deviation) is called the coefficient of skewness. It is unitless and is denoted by $\beta_1^{1/2}$. The sample estimate of this coefficient is denoted by $b_1^{1/2}$

$$b_1^{1/2} = \frac{m_3}{m_2^{3/2}}$$

where

$$m_3 = \frac{\Sigma(x - \bar{x})^3}{n}$$

$$m_2 = \frac{\Sigma(x - \bar{x})^2}{n-1}$$

If the sample comes from a normal population $b_1^{1/2}$ is approximately normally distributed with mean zero and variance 6/n and this can be used as the basis to test whether the sample data is skewed or not. For sample sizes less than 150 the assumption of normality is less accurate. For this case, the one tailed 5% and 1% significant levels of $\beta_1^{1/2}$ are available for sample sizes from 25 to 200. (Pearson and Hartley (Ed.), 1970).

Test for Kurtosis

The fourth moment about the mean, μ_4, can be used as a measure of kurtosis, which is the degree of 'flatness' of a density near its center. Values of μ_4/σ^4 (denoted by β_2) greater than 3 are sometimes used to indicate that a density is more 'peaked' around its centre than the density of a normal curve for which β_2 is three. (Mood et al 1963).

The sample estimate of β_2 is denoted by b_2.

$$b_2 = \frac{m_4}{m_2^2}$$

where $m_4 = \frac{\Sigma(x - \bar{x})^4}{n}$

The amount of kurtosis is also measured by

$$g_2 = b_2 - 3$$

In this case g_2 will be zero for a normal density, and positive values indicate a 'peaked' distribution. The distribution of b_2

does not approach the normal closely until the sample size is over 1000 (Snedecor and Cochran 1967). For sample sizes between 50 and 1000 tables are available. (Pearson and Hartley (Ed) 1970). However, for sample sizes less than 50 no tables of the significant levels of g_2 are available. Geary (1936) developed an alternative test criterion

$$a = \frac{\text{mean deviation}}{\text{standard deviation}} = \frac{\Sigma|x - \bar{x}|}{n\, m_2^{1/2}}$$

These tables are also available for sample sizes down to eleven.

Pearson System

The system of density functions developed by Karl Pearson is derived from the differential equation

$$\frac{d \log y}{dx} = \frac{x + a}{b_o + b_1 x + b_2 x^2} \qquad (3)$$

To express the above equation in the form $y = f(x)$, the right hand side of the equation has to be integrated. The form of the integral will depend on the roots of the quadratic $b_o + b_1 x + b_2 x^2$ which can be expressed as

$$b_2\{x - \frac{-b_1 + (b_1^2 - 4b_o b_2)^{1/2}}{2b_2}\}\{x - \frac{-b_1 - (b_1^2 - 4b_o b_2)^{1/2}}{2b_2}\}$$

and the criterion for fixing the form of the equation in a particular case is the same as that for the nature of the roots of the equation

$$b_o + b_1 x + b_2 x^2 = 0 \text{ viz. } b_1^2/4b_o b_2.$$

This can be shown to be identical to (see Elderton and Johnson (1969)

$$\frac{\beta_1 \ (\beta_2 + 3)^2}{4 \ (2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)} \qquad (4)$$

The above quantity is denoted by K and referred to as 'the criterion'.

Three "main types" are distinguished according to the nature of the roots (Table 2). In the limiting cases when one type changes to another simpler forms of transition curves are obtained.

The criterion K can take any value from $-\infty$ to $+\infty$. Figure 1 shows how the types cover all the possible values of K without overlapping.

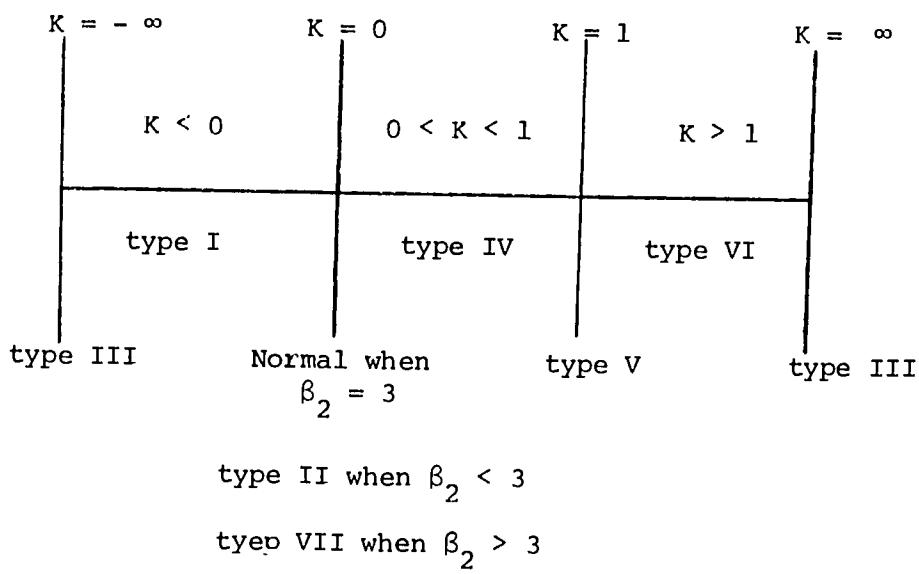


Figure 1. Pearson system and the criterion K

Source: Elderton and Johnson 1969, Systems of Frequency Curves p. 49.

Table 2. Pearson System of Curves

PEARSON TYPE NUMBER	VALUE OF CRITERION (K)	NATURE OF ROOTS
Main types		
I	$K < 0$	real, different signs
IV	$0 < K < 1$	complex
VI	$K > 1$	real, same sign
Transition types		
Normal curve	$K = 0$ $\beta_1 = 0$ $\beta_2 = 3$	$b_1 = b_2 = 0$
II	$K = 0$ $\beta_1 = 0$ $\beta_2 < 3$	roots equal, opposite sign
VII	$K = 0$ $\beta_1 = 0$ $\beta_2 > 3$	special case of Type IV
III	K very large	
	$2\beta_2 = 6 + 3\beta_1$	one root is ∞
V	$K = 1$	roots equal
VIII	$K < 0$, $5\beta_2 - 6\beta_1 - 9 < 0$	special case of Type I
IX	$K < 0$, $5\beta_2 - 6\beta_1 - 9 > 0$	special case of Type I
X	$\beta_1 = 4$, $\beta_2 = 9$	special case ot Type III
XI	$K < 1$ $2\beta_2 - 3\beta_1 - 6 > 0$	special case ot Type VI
XII	$5\beta_2 - 6\beta_1 - 9 = 0$	special case of Type I

Pearson Type I

Elderton and Johnson (1969) give a good general account of all the types and the process of fitting using the method of moments. Unfortunately they do not deal with cases where certain restrictions can be placed on the parameters.

As mentioned earlier the range of crop yields is limited. The Pearson type I function seems to be a appropriate functional form on a priori grounds for crop yield distributions, as the function is of limited range and can give rise to any kind of skewness. And, it is this function that will be discussed in detail, together with cases where 'a priori' information pertaining to the lower and upper limits of the curve can be accomodated.

Method of Moments and the Type I Function

The Different Forms of the Type I Function

Assuming that the roots α_2 and α_1 of the quadratic $b_1 + b_1x + b_2x^2$ are real and $\alpha_2 > \alpha_1$, the differential equation (3) can be written as

$$\frac{d \log y}{dx} = \frac{(x - a)}{b_2 (x - \alpha_1)(x - \alpha_2)} \tag{5}$$

$$= \{\frac{m_1}{x - \alpha_1} - \frac{m_2}{\alpha_2 - x}\} \tag{6}$$

where

$$m_1 = \frac{a - \alpha_1}{b_2(\alpha_2 - \alpha_1)} \quad \text{and} \quad m_2 = \frac{\alpha_2 - a}{b_2(\alpha_2 - \alpha_1)}$$

Expression (4) on integration yields

$$\log y = m_1 \log (x - \alpha_1) + m_2 \log (\alpha_2 - x) + C \qquad (7)$$

where C is the constant of integration.

Taking antilogs of equation (5) one obtains

$$y = C(x - \alpha_1)^{m_1} (\alpha_2 - x)^{m_2} \qquad (8)$$

which is the Pearson's type I density function for

$$\alpha_1 \leq x \leq \alpha_2$$

The Pearson's type I function can take a variety of forms depending on the values of m_1 and M_2: (a) m_1 and m_2 both positive, results in the bell, cocked hat and bowl shaped forms. (b) m_1 and m_2 having opposite signs give the J and twisted-J forms; while (c) m_1 and m_2 both negative yield the U shaped forms. When μ_3 is positive m_2 is greater than m_1 and vice versa.

The parameters α_1 and α_2 give the left and right extremeties of the density function. When 'a priori' information can be accomodated α_1 or α_2 or both can be fixed. In the case where both α_1 and α_2 values can be fixed the method of estimation is the simplest, since only m_1 and m_2 have to be estimated.

Method of Moments Estimation: Case I

If no prior hypothesis is made about the magnitude and signs of the parameters, there are 4 parameters to be estimated namely α_1, α_2, m_1 and m_2.

The kth moment about α_1 is given by

$$E(x - \alpha_1)^k = \mu_k' = C \int_{\alpha_1}^{\alpha_2} (x - \alpha_1)^{m_1 + k} (\alpha_2 - x)^{m_2} dx \qquad (9)$$

using the transformation

$$x = \alpha_1 + (\alpha_2 - \alpha_1) \; y \text{ then } y = \frac{x - \alpha_1}{\alpha_2 - \alpha_1}$$

$$dx = (\alpha_2 - \alpha_1) \; dy$$

substituting in equation (9)

$$\mu_k' = C \int_0^1 \{(\alpha_2 - \alpha_1)y\}^{m_1 + k} \{(\alpha_2 - \alpha_1)(1 - y)\}^{m_2} (\alpha_2 - \alpha_1) dy$$

$$= C \int_0^1 (\alpha_2 - \alpha_1)^{m_1+m_2+k+1} y^{m_1+k} (1 - y)^{m_2} dy$$

$$= C (\alpha_2 - \alpha_1)^{m_1+m_2+k+1} B(m_1 + k + 1, m_2 + 1) \qquad (10)$$

where $B(m_1 + k + 1, m_2 + 1) = \int_0^1 y^{m_1+k} (1 - y)^{m_2} dy$

For equation (8) to be a density function

$$\int_{\alpha_1}^{\alpha_2} C(x - \alpha_1)^{m_1} (\alpha_2 - x)^{m_2} dx = 1$$

$$\text{ie} \quad \mu_0' = 1$$

Using this Eq. (8) gives $C = \dfrac{1}{(\alpha_2 - \alpha_1)^{m_1+m_2+1} B(m_1 + 1, m_2 + 1)}$ (11)

Using the relationships

$$B(a, b) = \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(a+b)} \tag{12}$$

and

$$\Gamma(a) = (a-1)\ \Gamma(a-1) \tag{13}$$

in equation (10) results in

$$\mu_1' = \frac{(\alpha_2 - \alpha_1)(m_1 + 1)}{(m_1 + m_2 + 2)} \tag{14}$$

$$\mu_2' = \frac{(\alpha_2 - \alpha_1)^2\ (m_1 + 2)(m_1 + 1)}{(m_1 + m_2 + 3)(m_1 + m_2 + 2)} \tag{15}$$

$$\mu_3' = \frac{(\alpha_2 - \alpha_1)^3\ (m_1 + 3)(m_1 + 2)(m_1 + 1)}{(m_1 + m_2 + 4)(m_1 + m_2 + 3)(m_1 + m_2 + 2)} \tag{16}$$

$$\mu_4' = \frac{(\alpha_2 - \alpha_1)^4\ (m_1 + 4)\ \ldots\ldots\ldots\ (m_1 + 1)}{(m_1 + m_2 + 5)\ \ldots\ldots\ (m_1 + m_2 + 2)} \tag{17}$$

To express the parameters in terms of the moments about the mean μ_k, the following relationship can be used.

$$\mu_k = E(x - \mu)^k = \sum_{j=0}^{k} \binom{k}{j} \mu'_{k-j} (-\mu_1')^j$$

Then, equations 15-17 can be written in terms of the moments about the means as:

$$\mu_2 = \frac{(\alpha_2 - \alpha_1)^2\ (m_1 + 1)(m_2 + 1)}{(m_1 + m_2 + 2)^2\ (m_1 + m_2 + 3)} \tag{18}$$

$$\mu_3 = \frac{2(\alpha_2 - \alpha_1)^3\ (m_1 + 1)(m_2 + 1)(m_2 - m_1)}{(m_1 + m_2 + 2)^3\ (m_1 + m_2 + 4)(m_1 + m_2 + 3)} \tag{19}$$

$$\mu_4 = \frac{3(\alpha_2 - \alpha_1)^4 (m_1 + 1)(m_2 + 1) \{(m_1 + 1)(m_1 + m_2 - 4) + 2(m_1 + m_2 + 2)^2\}}{(m_1 + m_2 + 2)^4 (m_1 + m_2 + 5)(m_1 + m_2 + 4)(m_1 + m_2 + 3)} \quad (20)$$

Let $\alpha = \alpha_2 - \alpha_1$, $r = m_1 + m_2 + 2$ and $e = (m_1 + 1)(m_2 + 1)$

then,

$$\mu_2 = \frac{\alpha^2 e}{r^2 (r + 1)} \quad (21)$$

$$\mu_3 = \frac{2\alpha^3 e(m_2 - m_1)}{r^3 (r+2)(r+1)} \quad (22)$$

$$\mu_4 = \frac{3\alpha^4}{r^4} \frac{e \{e(r-6) + 2 r^2\}}{(r+3)(r+2)(r+1)} \quad (23)$$

$$\mu = E(x) = E(x - \alpha_1) + \alpha_1 = \frac{\alpha_1 (m_2 + 1) + \alpha_2 (m_1 + 1)}{r} \quad (24)$$

Equations (21) to (24) can be now used to solve for the parameters α_1, α_2, m_1 and m_2 giving

$$m_1 = \frac{r - 2 \div (t) (r^2 - 4e)^{1/2}}{2} \quad (25)$$

where

($t = 1$ when $\mu_3 < 0$)

($t = -1$ when $\mu_3 < 0$)

$$m_2 = r - m_1 - 2 \quad (26)$$

$$\alpha_1 = \mu_1 - \frac{\alpha(m_1 + 1)}{r} \quad (27)$$

$$\alpha_2 = \alpha + \alpha_1 \quad (28)$$

The procedure of using the method of moments is as follows:

(a) compute the sample moments μ_1^* to μ_4^*

(b) using this calculate the sample coefficients β_1^* and β_2^*

(c) using β_1^* and β_2^* and substituting in the following equations obtain values for r, α and e(see Elderton and Johnson, 1969 for derivation).

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{3\beta_1 - 2\beta_2 + 6} \tag{29}$$

$$\alpha = \frac{\sigma}{2}\{\beta_1 (r+2)^2 + 16 (r+1)\}^{1/2} \text{ where } \sigma = \mu_2^{1/2} \tag{30}$$

$$e = \frac{4r^2 (r+1)}{16 (r+1) + \beta_1 (r+2)^2} \tag{31}$$

(d) using the values of r, α and e from (c) and the estimate of the mean, solve equations (25) to (28) to obtain estimates of the parameters, whence the fitted distribution is obtained.

Method of Moments Estimation: Case II

Yields of field crops are always non-negative quantities and therefore it is reasonable to assume 'a priori' that the frequency function must start from zero. This means that a restriction is placed on one of the parameters namely $\alpha_1 = 0$. The frequency function will then be of the form

$$y = Cx^{m_1}(\alpha_2 - x)^{m_2}$$

Taking moments about zero, one obtains

$$E(x) = \mu_1' = \frac{\alpha_2(m_1 + 1)}{r} \tag{32}$$

$$E(x^2) = \mu_2' = \frac{\alpha_2^2 (m_1 + 2)(m_1 + 1)}{r(r + 1)} \tag{33}$$

$$E(x^3) = \mu_3' = \frac{\alpha_2^3 (m_1 + 3)(m_1 + 2)(m_1 + 1)}{r(r + 2)(r + 1)} \tag{34}$$

Let $g_1 = \mu_2'/\mu_1'^2$ and $g_2 = \mu_3'/\mu_1' \mu_2'$.

Then,

$$g_1 = \frac{(r)}{(r + 1)} \frac{(m_1 + 2)}{(m_1 + 1)} \tag{35}$$

$$g_2 = \frac{(r)}{(r + 2)} \frac{(m_1 + 3)}{(m_1 + 1)} \tag{36}$$

From (32), $\alpha_2 = \dfrac{r\mu_1}{m_1 + 1}$ (37)

From (35) and (36)

$$r = \frac{g_1}{\left(\dfrac{m_1 + 2}{m_1 + 1} - g_1\right)} \tag{38}$$

$$m_1 = \frac{2(g_1 - g_2)}{2g_2 - g_1 - g_1 g_2} - 1 \tag{38}$$

$$m_2 = r - m_1 - 2 \tag{40}$$

The method of moments procedure to obtain estimates of the parameters α_2, m_1 and m_2 is similar to Case I. However, only 3 moments are required in contrast to the 4 moments needed for Case I, as there are only 3 estimates to be obtained.

The efficiency of the moment estimates drops as the order of the moment increases (Elderton and Johnson, 1969). Therefore using

only the first 3 moments about zero must give rise to better estimates of the parameters α_2, m_1 and m_2 than for Case I.

Method of Moment Estimates - Case III

Field crop yields, in any given environment, have an upper limit imposed by the innate genetic potential of the crop under consideration. Thus, in estimating crop yield distributions, we are also able to place an upper limit. The value of the upper limit is a function of the crop and its environment. This means that one can decide on the values of α_1, and α_2 before estimation. In this case there will be only 2 parameters m_1 and m_2 to be estimated.
Using equations (32) and (33) and setting

$$h_1 = \frac{\mu_1'}{\alpha_2} = \frac{m_1 + 1}{r}$$

$$h_2 = \frac{\mu_2'}{\mu_1 \alpha_2} = \frac{m_1 + 2}{r + 1}$$

one obtains estimates of m_1 and m_2 given by

$$m_1 = \frac{h_1 \ (h_2 - 1)}{h_1 - h_2} - 1$$

$$m_2 = \frac{(h_1 - 1)(h_2 - 1)}{h_2 - h_1} - 1$$

The efficiency of the estimates of the parameters m_1 and m_2 will be improved compared to Cases I and II, since only the first 2 moments are involved in their computation.

Maximum Likelihood Method of Estimation of Parameters

Fisher (1922) showed that the method of moments was statistically inefficient. The moment estimates in general lack good small sample properties such as unbiasedness and uniformly minimum variance and (perhaps what is worse) also large sample properties such as consistency and asymptotic normality.

In contrast the maximum likelihood method of estimation possesses several optimal large sample properties. A maximum likelihood estimator is asymptotically normally distributed, while a sequence of maximum likelihood estimators is best asymptotically normal (BAN). Also the variance of the asymptotically normal distribution of the maximum likelihood estimator is the Cramer-Rao lower bound (see Mood et at 1974). In the case of a K-dimensional parameter too, maximum likelihood estimators possess similar optimum large-sample properties.

In addition maximum likelihood estimators posses the properties of invariance and minimal sufficiency statistics.

Unbiasedness is not a general property of maximum likelihood estimators. However, in many cases the unbiased estimator is a simple function of the maximum likelihood estimator. In this case this function is also the minimum variance unbiased estimator (UMVUE), if the UMVUE exists.

Likelihood and Support Functions for Type I Curve

The likelihood function for the type I density function (equations 6) is

$$L = \prod_{i=1}^{n} C(x_i - \alpha_i)^{m_1} (\alpha_2 - x_i)^{m_2} \tag{41}$$

The support function, defined as the natural logarithm of the likelihood function (Edwards 1972), is given by

$$S = m_1 \Sigma\log(x_i - \alpha_1) + m_2 \Sigma\log(\alpha_2 - x_i)$$

$$-n(m_1 + m_2 + 1)\log(\alpha_2 - \alpha_1) - n \log B(m_1 + 1, m_2 + 1)$$

where $\log B(m_1 + 1, m_2 + 1) = \log \Gamma(m_1 + 1) + \log \Gamma(m_2 + 1)$ (42)

$$- \log \Gamma(m_1 + m_2 + 2)$$

The system of likelihood equation is

$$\frac{\partial S}{\partial \theta_i} = 0 \text{ for } i = 1, 2, 3, 4.$$

where

$$\theta = \begin{bmatrix} m_1 \\ m_2 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}$$

The system of equations can be expressed as a vectorial equation

$$G(\theta) = 0 \tag{43}$$

where $G(\theta) = \left[\frac{\partial S}{\partial m_1}, \frac{\partial S}{\partial m_2}, \frac{\partial S}{\partial \alpha_1}, \frac{\partial S}{\partial \alpha_2} \right]$ (44)

The solution to equation (43) yields the maximum likelihood estimates, provided the matrix

$$\left[\frac{\partial^2 S}{\partial \theta_r \partial \theta_t} \right] \qquad r = 1, 2, 3, 4; \quad t = 1, 2, 3, 4$$

is negative definite.

System of Equation for Type I Function: Case III

Where α_1 and α_2 assume 'a priori' values, the system of likelihood equations to be solved will be

$$\frac{\partial S}{\partial m_1} = \Sigma \log x_i - N \log \alpha_2 - \frac{N \, \partial \log B(m_1 + 1, m_2 + 1)}{\partial m_1} = 0 \quad (45)$$

and $$\frac{\partial S}{\partial m_2} = \Sigma \log (\alpha_2 - x_i) - N \log \alpha_2 - \frac{N \partial \log B(m_1 + 1, m_2 + 1)}{\partial m_2} = 0 \quad (46)$$

where N is the sample size.

Equation (45) and (46) do not admit explicit solutions and iterative techniques have to be used.

Brief Note on Iterative Techniques to Solve for the Likelihood Equations

The problem of finding solutions to equations 5b and 6b can be viewed as the problem of finding roots for these simultaneous non-linear equations. Some of the algorithms used are briefly outlined below.

(a) Direct search method, is mainly developed for single variable functions. An interval known to include the optimum is identified. The size of the interval is then systematically reduced in a way that guarantees that the optimum is not missed. The length of the interval containing the optimum can be made as small as desired (Taha 1976).

(b) Gradient method, is used for maximizing the value of functions which are twice continuously differentiable. The idea is to generate successive points starting from a given initial point in the direction of the fastest increase (maximization) of the function (Taha 1976).

Among the gradient methods are the Newton-Raphon process and the method of steepest ascent. In the Newton-Raphson process for a single parameter θ an initial starting point t (which may be the moment estimate) is used to obtain an approximate value for $\hat{\theta}$ say θ''

$$\theta'' = t - \frac{(\partial S)}{\partial \theta} \Big/ \frac{\partial^2 S}{\partial \theta^2} \qquad (47)$$

where the differentials are obtained at $\theta = t$. Repeating the process with the new $t = \theta''$ will, under suitable conditions, lead to the evaluate θ. (Edwards, 1922). In another method due to Fisher (1925) known as 'the method of scoring for parameters' instead of equation 47 we use

$$\theta'' = t - \frac{\partial S}{\partial \theta} \Big/ E \left[\frac{\partial^2 S}{\partial \theta^2} \right] \qquad (48)$$

For the case of several parameters Edwards (1972) gives the Newton-Raphson iteration formula as

$$\theta'' = \theta' + B^{-1}\, T(\theta')$$

where θ' is the vector of trial values, $T(\theta')$ is the vector of the partial derivatives $\frac{\partial S}{\partial \theta_i}$ evaluated at θ'. B is known as the information matrix and is the matrix of the second partial derivatives of the support function, in which the element in the ith row and j^{th} column is $\frac{\theta^2 S}{\partial\theta_i \partial\theta_j}$ evaluated at θ'.

(c) In cases where the likelihood equation has multiple roots Barnett (1966) advocates the use of the method of false positions (Reguli falsi). This method is used for the case of a single parameter. Initially 2 values of θ , a_0 and b_0 are chosen such that

$$a_0 < b_o,\ \frac{\partial L}{\partial\theta}\Big|_{a_0} > 0,\ \frac{\partial L}{\partial\theta}\Big|_{b_0} < 0$$

writing $\frac{\partial L}{\partial\theta}\Big|_x$ as $f(x)$ and putting

$$x_i = \frac{a_{i-1}\, f(b_{i-1}) - b_{i-1}\, f(a_{i-1})}{f\,(b_{i-1}) - f\,(a_{i-1})}$$

successive a_i, b_i are obtained as

$$\left.\begin{aligned} a_i &= x_i \\ b_i &= b_{i-1} \end{aligned}\right\} \quad , f(x_i) > 0$$

or

$$\left.\begin{array}{l} a_i = a_{i-1} \\ b_i = x_i \end{array}\right\} , \; f(x_i) < 0$$

The process results in a_i and b_i continuing to enclose and converge on a relative maximum of L, and is terminated when $b_i - a_i$ is sufficiently small and yields the maximum as $1/2\ (a_i + b_i)$.

For quick convergence of the iterative processes, initial values chosen for the parameters must be very close to the true values. One way of choosing is to consider the moment estimates as initial values. However moment estimates obtained may not be close to the true values. If the trial values are for apart the Newton-Raphson process will not converge.

Bernett (1966) and Kale (1961) surveys the commonly used iterative techniques. Conte and De Boor (1972) give various iteration techniques for the solution of non-linear equations, which include an improvement on the method of false positions, known as modified reguli falsi.

Iterative Technique Used in the Study

The computer available at the University of the Philippines at Los Baños, did not carry the necessary packages to obtain derivatives of the Beta and Gamma functions. Therefore none of the iterative techniques mentioned above could be used. The method employed in the present study was carried out in 2 stages.

(a) Using the method of moment estimates from Case III as initial values, a series of values of the likelihood function was evaluated directly for values of the parameters ranging from $m_i^* - 5$ to $m_i^* + 5$ (where m_i^*, i=1, 2 are the moment estimates). The parameter values were incremented by 0.1 so that for each value of m_1 there were 50 values of m_2 in .1 increments and there were 50 x 50 = 2500 values of the likelihood function. The likelihood values were observed and the values o. m_1 and m_2 corresponding to the maximum value of the likelihood function were chosen to be used in the next stage.

(b) In the second stage the value of $m_1^0 = (m_1 - .2)$ and $m_2^0 = m_2 - .2$ were used as starting points in the iteration. The corresponding value of the likelihood function $L^{0,0}$ was then calculated;[3] m_1^0 was then incremented by 0.001 and $L^{1,0}$ was calculated. If $L^{1,0} > L^{0,0}$ the incrementing process was carried out further unit $L^{S,0} < L^{S-1,0}$ where S denotes the Sth increment to m_1. At this point the value of m_1^{S-1} was kept fixed and $m_2^0 = m_1 - .2$ was incremented by 0.001 until $L^{S-1,r} < L^{S-1,r-1}$ where r denotes the rth increment to m_2. The value of m_2 was now kept fixed and m_1^{S-1} was incremented again until the value

---

[3] $L^{i,j}$ refers to the likelihood value computed for values of m_1^i (i^{th} increment to m_1) and m_2^j (j^{th} increment for m_2).

of L stopped increasing. Then m_2 was incremented again from m_2^{r-1} till the same thing happened. Iteration was stopped when increases in m_1 or m_2 did not further increase the value of L. The value of m_1 and m_2 obtained at this stage are the maximum likelihood estimates of the parameters. Figure 2 given below shows the flow-chart for the program used in the second stage of iteration.

Screening of Techniques

Different management techniques give rise to different yield distributions for the same crop in a given environment. Once the yield distributions are estimated, they can be used to generate the profit distributions associated with the different management techniques. Such profit distributions can then be screened using appropriate criteria to eliminate the 'non-risk efficient' subset of the management techniques.

In the present study the yield distributions will be transformed to generate profit distributions for each management technique, to be used in screening for economic viability which will explicitly incorporate risk considerations.

In screening of new technologies intended for adoption by large groups of farmers, the criteria that is used should not require information about risk attitudes of each farmer; clearly gathering such information cannot be a realistic proposition for agricultural researchers. Therefore in the present study we propose

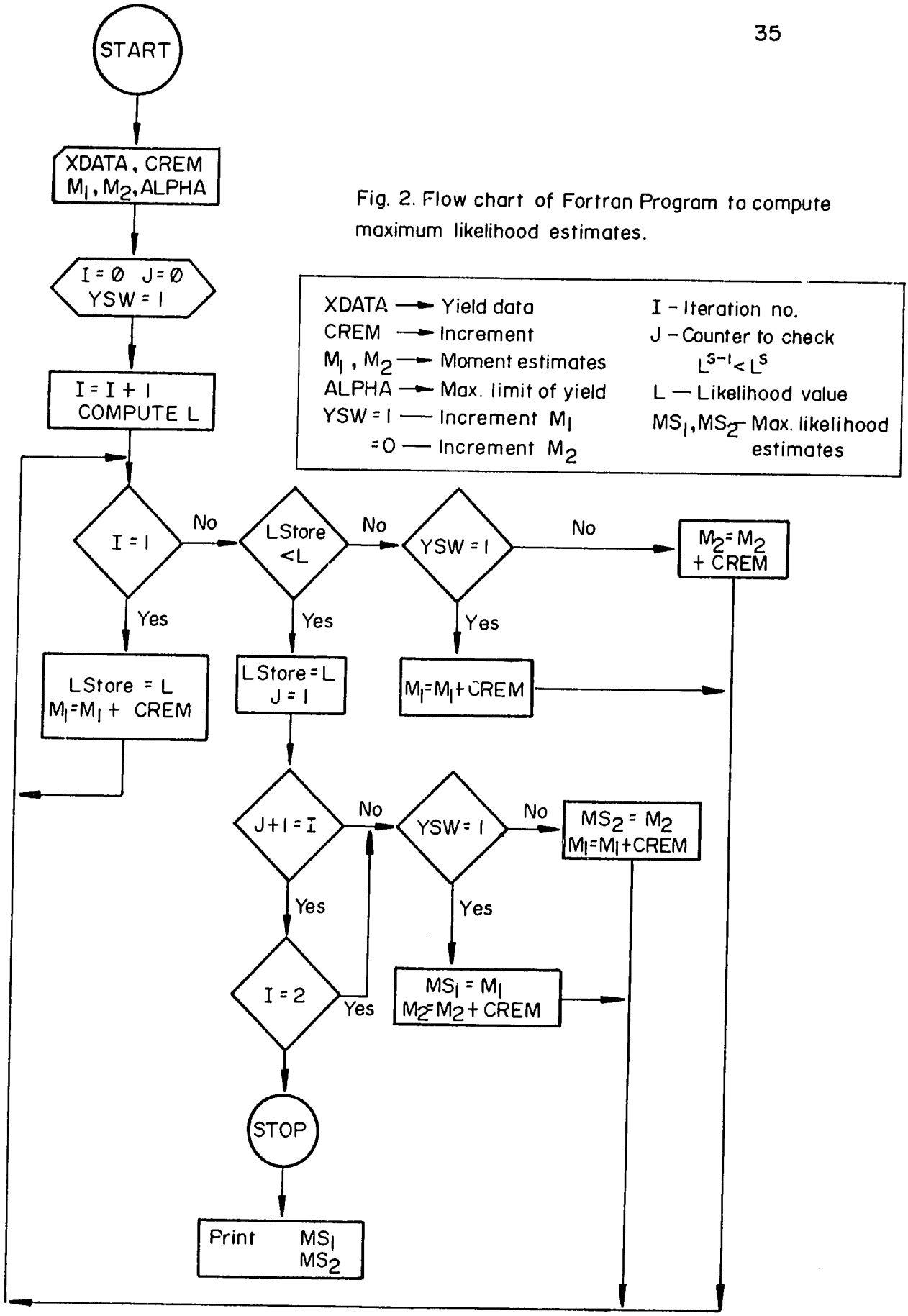


Fig. 2. Flow chart of Fortran Program to compute maximum likelihood estimates.

to use two methods for screening which do not require such information.

Probability of Loss

The first method that we will use is based on the concept of the probability of loss. The basis for screening (and ranking) of technologies using this method will be the probability of obtaining net returns less than zero, for each management package. Here we make the following assumptions:

(a) Small farmers are risk averse.

(b) Technologies with lower probabilities of loss are preferred.

The first assumption, of course, is widely made and is backed by substantial empirical evidence.

The second assumption, however can be more restrictive, since it assumes that only the probability of loss is important and ignores other characteristics of the profit distribution such as the mean level of profit.

Concepts of Stochastic Efficiency

The second more theoritically appealing approach is that based on the concept of stochastic dominance. Here, we proceed from the very simple assumption of risk aversion through progressively stronger assumption regarding the nature of risk attitudes.

(a) First-degree stochastic dominance (FSD) rests on a very reasonable behavioural assumption that decision makers prefer

more to less of x. More formally the assumption is that the utility function of an individual is monotonically increasing.

The first degree rule is generally stated in terms of the cumulative distribution function (CDF). If F_1 and G_1 are a pair of continuous CDFs associated with 2 risky prospects F and G, within the interval a, b.

where $$F_1(y) = \int_a^y f(x)\, dx$$

and $$G_1(y) = \int_a^y g(x)\, dx,$$

Then, F is said to dominate G in the sense of first degree stochastic dominance if $F_1(y) \leq G_1(y)$ for all possible y in the range {a, b} with at least one strong inequality.

An undominated set is called an 'efficient set'. The efficiency criteria is transitive, i.e. if F dominate G and G dominates H then F dominates H.

(b) Second degree stochastic efficiency (SSE) concept requires an additional behavioural assumption namely, that the decision makers are risk averse. This implies that the utility function is not only monotonically increasing but also strictly concave.

$$\text{ie } \frac{d\,U(x)}{dx} > 0$$

and $$\frac{d^2\,U(x)}{dx^2} < 0$$

where U(x) is the utility function.

The 2nd degree stochastic efficiency rule is defined in terms of the area under the CDF.

if $\quad F_2(y) = \int_a^y F_1(x)\,dx$

and $\quad G_2(y) = \int_a^y G_1(x)\,dx$

Then F is said to dominate G in the sense of second degree stochastic dominance (SSD) if $F_2(y) \leq G_2(y)$ for all possible y with at least one strong inequality.

Using the FSD and SSD rules we end with the SSE set. If an additional assumption about U (x) is made we can go on and isolate the third degree stochastic efficient (TSE) set.

The 3rd degree stochastic dominance (TSD) concept requires a further assumption that as people become richer they become decreasingly averse to risk. This is equivalent to saying that

$$\frac{d^3 U(x)}{dx^3} > 0$$

To state the TSD rule we consider

$$F_3(y) = \int_a^y F_2(x)\,dx$$

and

$$G_3(y) = \int_a^y G_2(x)\,dx$$

which are the areas under the SSD cumulative functions. F is now said to dominate G in the sense of TSD if $F_3(y) \leq G_3(y)$ for all y, with the strict inequality holding for at one value of y, and if $F_2(b) \leq G_2(b)$ where b is the upper range.

With the use of this rule we end up with third degree stochastically efficient (TSE) set. This is as far as we go in the present

study. For higher degrees of stochastic dominance further assumptions regarding risk attitudes have to be made, which may be unduly restrictive. To use the stochastic dominance approach it is necessary to obtain the definitive integrals of the type I function. This makes it necessary that the estimated function be transformed in an appropriate manner. This is carried out as follows:

Consider the type I function derived using the assumptions that $\alpha_1 = 0$ and $\alpha_2 = \alpha$ where α is the hypothesized upper limit of the curve.

$$y = C\, x^{m_1} (\alpha - x)^{m_2}, \quad 0 \leq x \leq \alpha$$

where

$$C = \frac{1}{\alpha^{m_1 + m_2 + 1} B(m_1 + 1, m_2 + 1)}$$

If the transformation

$$z = x/\alpha$$

is used, ie x measured in units of α.

then $$\frac{dx}{dz} = \alpha$$

and we obtain

$$y = \frac{1}{B(m_1 + 1, m_2 + 1)} z^{m_1} (1-z)^{m_2} I_{(0,1)}(z) \qquad (49)$$

Equation (49) gives function which belong to the family of beta distributions.

The cumulative distribution function $F_Z(z; m_1 + 1, m_2 + 1) = I_{(0,1)}(z) \int_0^z \frac{1}{B(m_1+1, m_2+1)} u^{m_1} (1-u)^{m_2} du + I_{(1, \infty)}(z)$ is called

the incomplete beta and has been tabulated extensively.

However, the integrals that can be obtained from the tables are only for discrete value of m_1 and m_2. Therefore the method employed in obtaining the integrals in the present case is by the use of Simpson's rule of approximation for integrals.

RESULTS AND DISCUSSION

When the yield data for each season is stratified as mentioned earlier, it is observed that for the HL and HH strata of the second crop the number of observations are very few (five or less). These 2 strata are therefore omitted in the analysis except to find the probability of loss, associated with these technologies, using the empirical cumulative distribution function to obtain the cumulative distribution function of profit.

Preliminary Tests

Test of Normality

The yield data in each stratum was tested for normality using the test of skewness and Geary's test of kurtosis for small samples. Table 3 shows the values of the test statistics $b_1^{1/2}$ and a. The table also gives their level of significance and the value of the criterion K used to distinguish between the different types of Pearson curves.

It is seen from the table 3 that yield distributions in seven out of the ten strata deviate significantly from the normal in terms of the skewness or kurtosis or both. The strata MH and HH for the first crop and ML and MH for the second crop shows departure from the normal with respect to their skewness, while the strata ML, HL and HH for the first crop and LL for the second crop differ from the normal with respect to kurtosis. It is

Table 3. Tests of Normality and Value of Criterion.

STRATA		TEST FOR SKEWNESS ($b_1^{1/2}$)	TEST FOR KURTOSIS (a)	CRITERION (K)	NUMBER OF OBSERVATION
First crop:	LL	.1846	.7947	-.0252	54
	ML	-.1513	.8557*	-.0085	51
	HL	.0488	.7319*	.00267	36
	LH	.2783	.7733	-.0498	21
	MH	-1.2289**	.7749	-3.3336	35
	HH	-.6395*	.7531*	2.7098	53
Second crop:					
	LL	.1317	.8561*	-.00575	53
	ML	-.6744*	.7575	-.2553	19
	LH	-.3642	.83	-.05254	63
	MH	-.8327*	.8256	-.2129	29

* 5% level of significance
** 1% level of significance
a = .7979 for normal distribution
$b_1^{1/2}$ = 0 for normal distribution

interesting to note that for the strata which show significant departure in skewness, the coefficient of skewness is less than zero, i.e. the observations in these strata viz. MH and HH for first crop and ML and MH for second crop are negatively skewed. It is also observed that LL for both crops show positive skewness (though not significant at the 5% level). Also, in general there exists a tendency towards greater negative skewness as the levels of inputs and labour (especially labour) are increased.

Type of Pearson Curve

The value of the criterion K for the different strata as shown in Table 3 is seen to be generally negative except in the case of HL and HH for the first crop. This indicates that generally conditional yield distributions follow the type I Pearson distribution.

Positive values of K for HL and HH for the first crop may be due to the inefficiency of moment estimation. The above results lend support to the "a priori" hypothesis that the conditional yield distribution of rice can be represented by the Pearson type I distribution.

Method of Moments Estimates of Parameters For Type I Function

Case I: α_1 and α_2 Unspecified

The type I function was used in fitting distributions to the yield data of each stratum. The parameters α_1 and α_2 which deter-

mine the extremities of the curve were unspecified. Thus the values of the 4 parameters m_1, m_2, α_1 and α_2 are determined only for the strata which showed negative values of K. Table 4 presents the results of fitting the type I function to the data in these strata.

Table 4. Type I Parameters with α_1 and α_2 Unspecified.

STRATA		TYPE I PARAMETERS m_1	m_2	α_1	MINIMUM VALUE OF OBSERVED YIELD	α_2	MAXIMUM OBSERVED YIELD VALUE
First crop	LL	4.053	2.683	-1.7087	0.1	5.4455	4.952
	ML	0.4395	0.198	2.248	2.10	5.5075	5.375
	LH	3.367	1.805	-2.2668	0.1	5.3414	5.2
	MH	29.302	0.9845	-16.3805	1.3	5.8791	5.727
Second crop	LL	0.054	-0.0941	-0.224	0.272	4.4948	4.588
	ML	3.2325	0.6016	-0.0433	1.8	5.6509	5.4
	LH	0.8933	0.2018	-0.47	0.253	4.6811	4.665
	MH	0.0759	-0.5595	2.7429	2.7	4.67	4.769

In nearly all cases the parameters m_1 and m_2 are positive which means that the type I functions are of the cocked hat, bell or bowl shaped forms. The two exceptions to this are in LL and MH second crops, where m_2 is negative. For these cases the form of the function is of the J and twisted J shapes.

A look at the values of α_1 reveal that in 2 cases, namely ML (1st crop) and MH (2nd crop), the left extreme points of the curve are greater than the minimum yield values observed for those strata. This is a serious shortcoming of the method of moments, because it would mean that the probability of observing values less than α_1

is zero. Similarly the probability of observing values greater than α_2 is zero. But values greater than α_2 is observed in LL (2nd crop) and MH (2nd crop). In the MH strata (1st crop) the value of α_1 (-16.3805) is much lower than zero.

Crop yields being positive, the probability of obtaining a negative yield is zero. Incorporating this restriction, namely that $\alpha_1 = 0$, would improve the estimates of the parameters. In the next stage of estimation this is done.

Case II α_2 Unspecified, $\alpha_1 = 0$

When one of the parameters (in this case α_1) is specified, the calculation of the remaining parameters involve only the first 3 moments. In the calculation of the moments about zero, the x values are taken to the power of the order of the moment. The higher the order of the moment, higher the power of x. Thus, in the calculation of the fourth moment x is raised to the power of four. The precision of estimation of moments is therefore more affected as the order of the moment increases. The first 3 moments are estimated with more precision than the fourth moment. In the present case as only 3 moments are involved, the parameters m_1, m_2 and α_2 would be estimated with more precision.

Table 5 presents the results of fitting the type I function with the restriction $\alpha_1 = 0$, to the yield data in the different strata.

Table 5. Type I Parameters with α_2 Unspecified, $\alpha_1 = 0$.

STRATA		TYPE I PARAMETERS WITH $\alpha_1 = 0$			
		m_1	m_2	α_2	Maximum Observed Yield
First crop	LL	1.168	1.82	5.59	4.952
	ML	7.833	5.4	6.945	5.375
	HL	16.016	18.85	9.651	6.4
	LH	0.55	1.235	5.777	5.2
	MH	2.435	-.248	5.498	5.727
	HH	2.527	.457	6.77	6.984
Second crop	LL	.082	.265	5.019	4.588
	ML	3.159	.578	5.643	5.4
	LH	0.249	-.179	4.443	4.665
	MH	8.576	1.204	5.056	4.769

Here again as in Case I the range conditions are violated. MH and HH (first crop) and LH (2nd crop) have the estimates of α_2 less than the maximum observed yield values in these strata. The parameter estimates of the yield function in these strata cannot therefore be used as starting values in the maximum likelihood iteration process.

Case III: $\alpha_1 = 0$, $\alpha_2 = \alpha$

As noted earlier the yield of a rice crop is limited by the level of management, and the inherent genetic potential. Therefore based on the level of management used, the following maximum limits to rice yields that could be obtained in farmers fields were specified.

Strata	Specified value of $\alpha_2 = \alpha$
LL	5.0
ML	5.5
HL	6.5
LH	5.5
MH	6.0
HH	7.0

When α_1 and α_2 are both specified the only remaining parameters to be estimated are the shape parameters m_1 and m_2. These parameters can be expressed in terms of the first two moments only, and by using the same argument as in the last section, we can say that the precision of the estimates will be further improved. Table 6 shows the estimates of m_1 and m_2 for the different strata.

Table 6. Type I parameters, $\alpha_1 = 0$, $\alpha_2 = \alpha$

STRATA		TYPE I PARAMETERS α_1 AND α_2 SPECIFIED	
		m_1	m_2
First crop	LL	0.8812	0.9904
	ML	4.267	0.926
	HL	8.53	3.377
	LH	0.461	0.935
	MH	4.129	0.693
	HH	2.887	0.792
Second crop	LL	0.0753	0.249
	ML	2.798	0.312
	LH	0.906	1.004
	MH	15.807	6.728

It is noticed that m_1 and m_2 are both positive. This means that the distributions of yields in all the strata are of the cocked hat, bowl or bell-shaped forms. For both crops m_2 is greater than m_1 in the strata LL and LH which means that the distribution in these strata show positive skewness. When the level of labour is increased the distributions become negatively skewed. This is in accordance with the preliminary analysis of the data which also showed a tendency towards greater negative skewness as labour and input levels were increased.

The moment estimates from Case III were next used as initial values in the maximum likelihood iteration process.

Maximum Likelihood Estimates of Type I
Function With α_1 and α_2 Specified

The maximum likelihood estimates obtained from the iteration process are shown in Table 7, together with the moment estimates from Case III for easy comparison.

Comparison between the parameter estimates obtained from the moment method and the maximum likelihood method shows that the estimates from the moment method are very close to the maximum likelihood estimates. It is also obserced from the above table that whenever m_1 is greater than m_2 (negative skewness) in the maximum likelihood estimated, the same is true of the moment estimates, and vice versa. Another interesting feature is that when the moment estimate of m_1 is greater than its corresponding maximum likelihood

Table 7. Maximum Likelihood and Moment Estimates with α_1 and α_2 Specified.

STRATA		SAMPLE SIZE (n)	MAXIMUM LIKELIHOOD ESTIMATES		MOMENT ESTIMATES	
			m_1	m_2	m_1	m_2
1st crop	LL	54	0.587	0.65	0.8812	0.9904
	ML	51	3.976	0.776	4.267	0.926
	HL	36	6.327	2.302	8.53	3.377
	LH	21	0.318	0.677	0.461	0.935
	MH	35	4.608	0.895	4.129	0.693
	HH	53	2.081	0.381	2.887	0.792
2nd crop	LL	53	0.216	0.381	0.0753	0.249
	ML	19	2.473	0.173	2.798	0.312
	LH	63	0.836	0.992	0.906	1.004
	MH	29	17.253	7.422	15.807	6.728

estimate, the same is true of the estimate of m_2. Similarly when the moment estimate of m_1 is less than its maximum likelihood estimate the same holds true for the estimate of m_2. It is therefore apparent that the moment estimates of m_1 and m_2 are both either slightly underestimated or slightly overestimated compared to the maximum likelihood estimates.

An important observation is that when the end points of the distribution can be specified (as done in Case III), the moment estimates of m_1 and m_2 tend to be very close to the maximum likelihood estimates. The slight under (over)- estimation involved is compensated by the great reduction in computations that result from the use of the methods of moments.

Figure 3a to 3j shows the fitted curves superimposed on the histograms of the observed data, for the different strata.

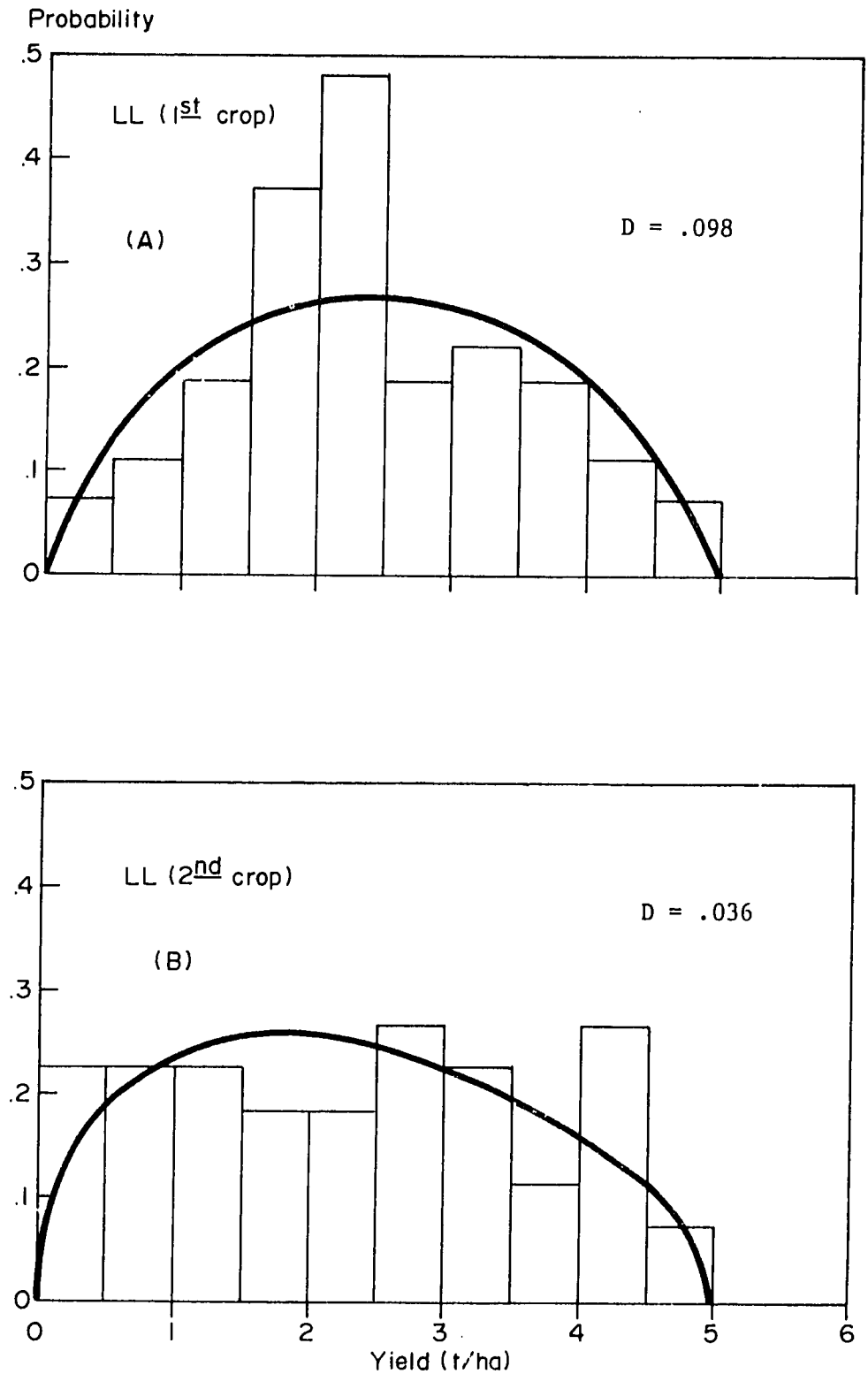


Fig. 3. Fitted curves supperimposed on histograms for the different strata.

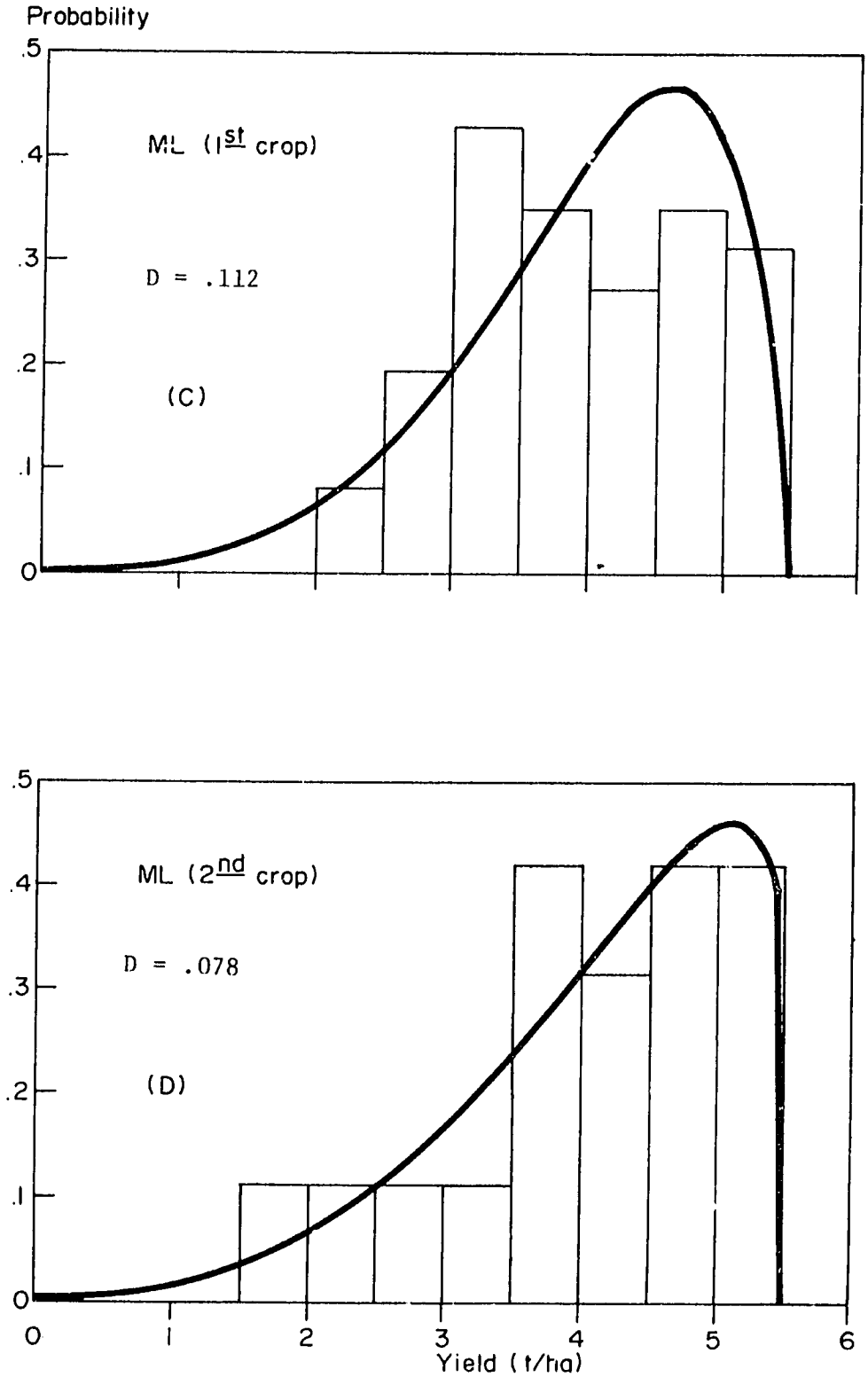


Fig. 3. Fitted curves supperimposed on histograms for the different strata.

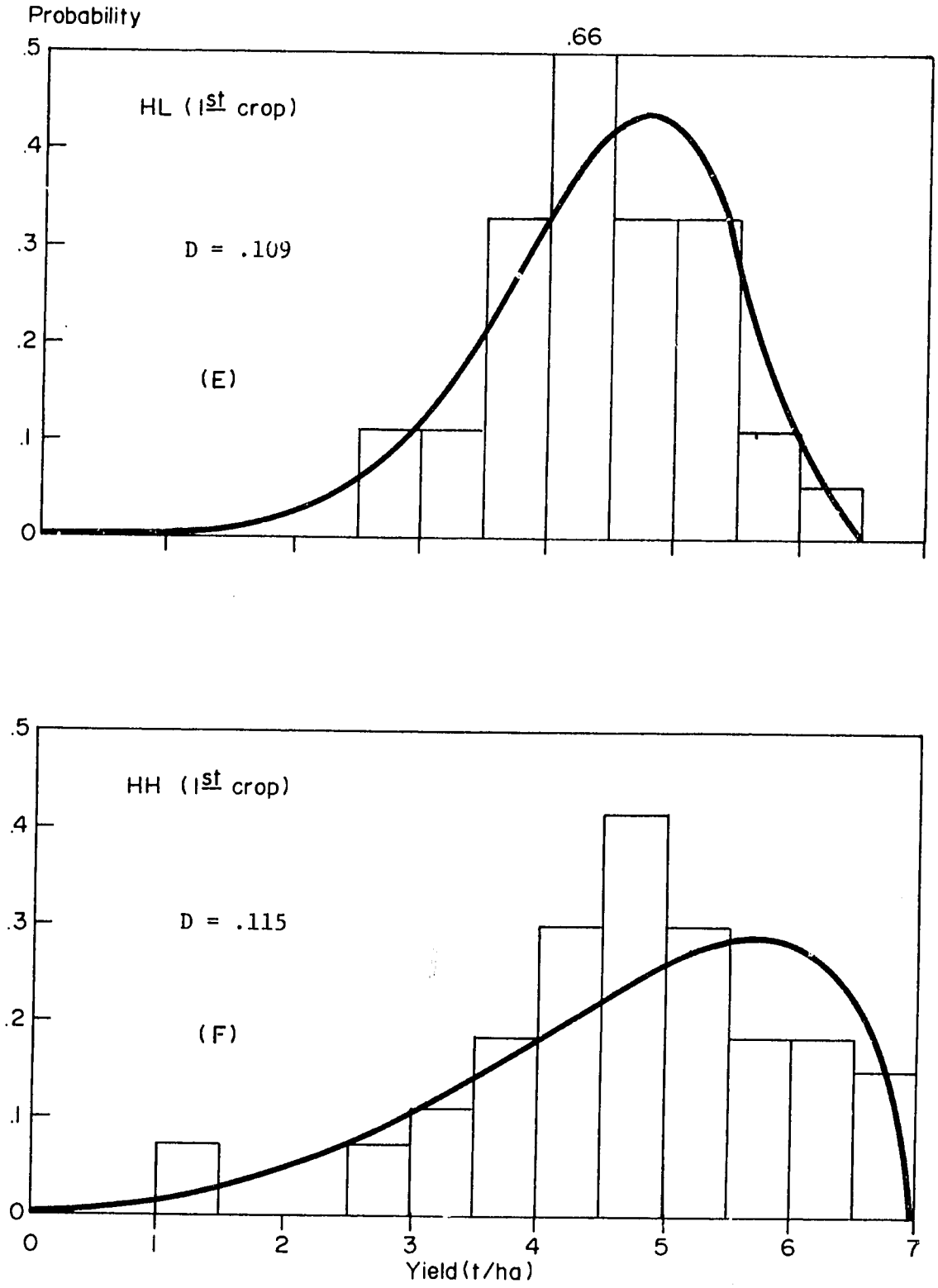


Fig. 3. Fitted curves supperimposed on histograms for the different strata.

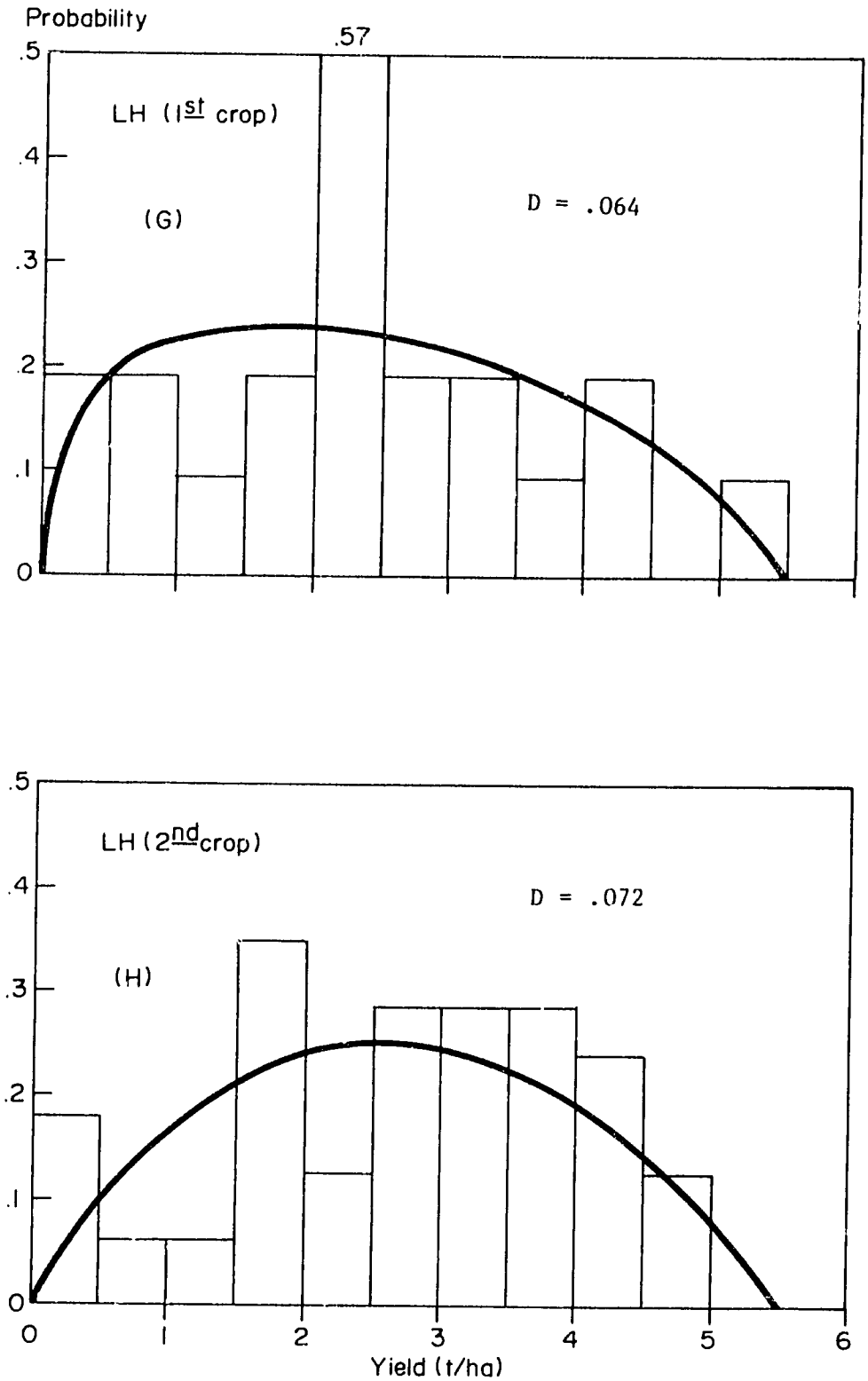
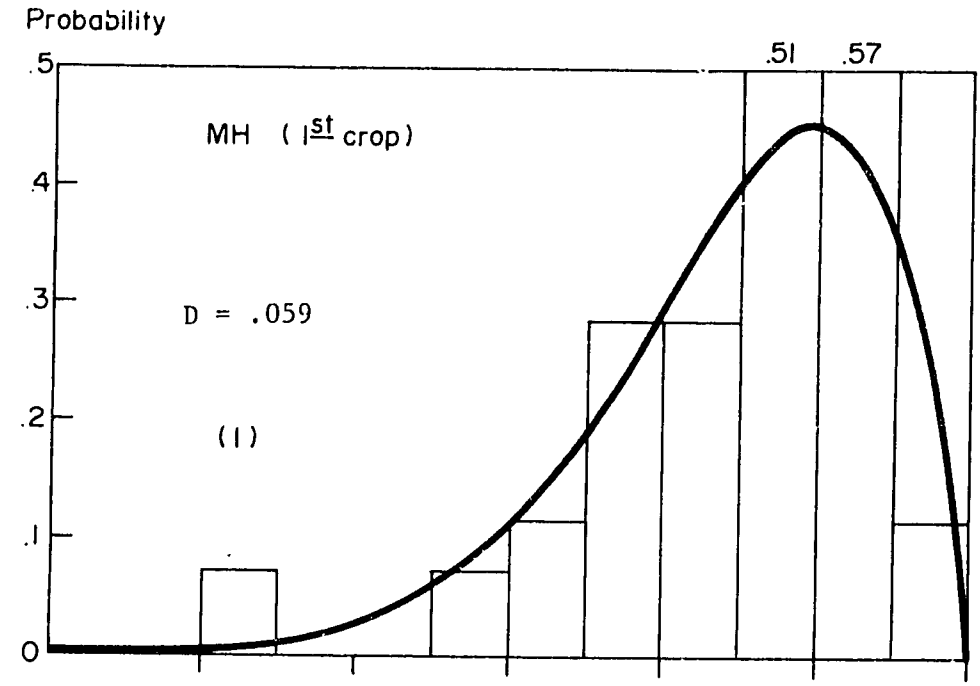


Fig. 3. Fitted curves supperimposed on histograms for the different strata.



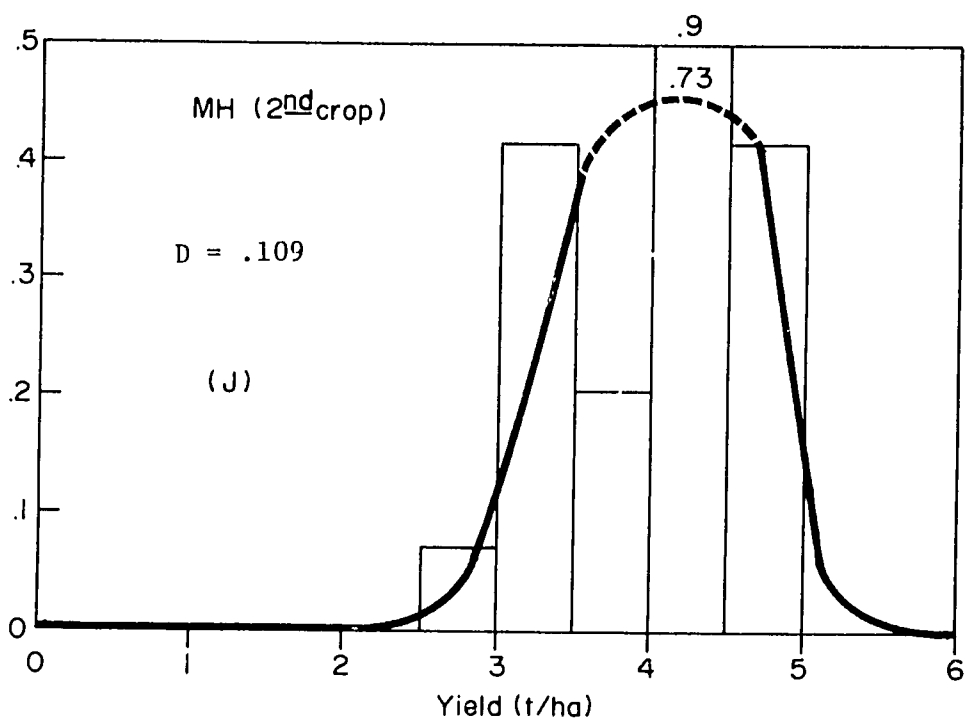


Fig. 3. Fitted curves supperimposed on histograms for the different strata.

The Histograms are drawn using the relative frequencies rather than the actual frequencies, so that the area under the histograms add up to unity. The class interval chosen in this case was 0.5 ton per hectare. The Kolmogorov-Smirnov test is used to test the goodness of fit, as this test is more powerful than the more widely used X^2 test (Siegel, 1956). The value of the Kolmogorov-Smirnov test statistic D is also given with the different curves. The null hypothesis stated as "The fitted distribution is not significantly different from the observed distribution (histograms)' cannot be rejected even at the 20% level of significance. The fit as judged from the calculated D values seem to be very close, though unfortunately it cannot be tested at a significant level greater than 20%, since the tables of D go only from significant levels of 1 to 20 percent.

Empirical Cumulative Distribution Function

In a situation where the number of observations is very small, the jth order statistic can be used to estimate the $j_{/n+1}$ fractile of the empirical cumulative distribution function (Mood and Graybill, 1963). For HL and HH of the 2nd crop this method was used to obtain the CDFs of yield and profit.

The CDFs of yields are given in figure 4. From the CDFs the probability of obtaining yields less than 2 tons/ha was calculated.

Table 8 gives the probabilities of obtaining a yield of tons or less per hectare for the different technologies.

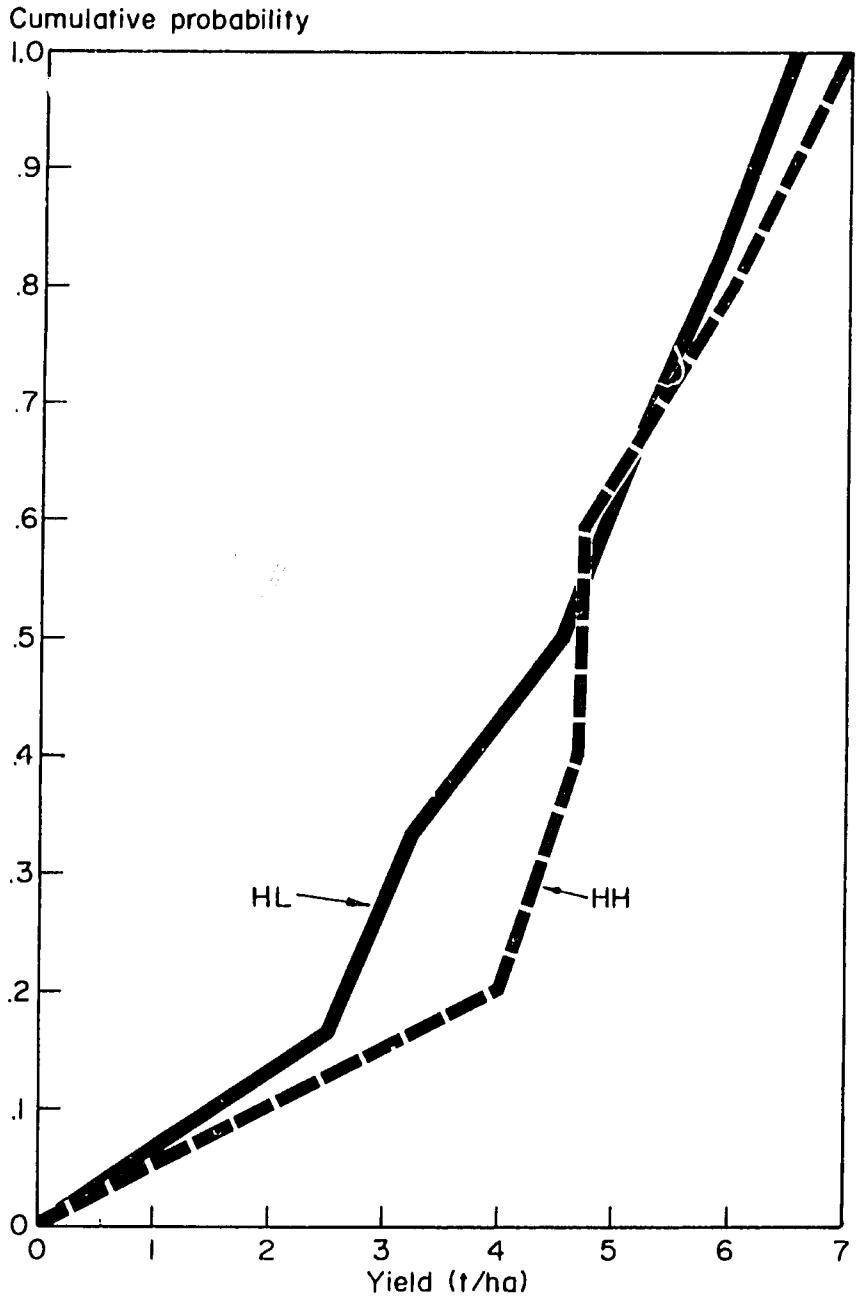


Fig. 4. Empirical cumulative distribution functions for high labor, low and high cash inputs, second crop.

Table 8. Probabilities for yield $\leq$ 2 tons/ha.

	STRATA	P(Y $\leq$ 2)	MEAN YIELDS (tons/ha)
First crop	LL	.381	2.43
	ML	.021	4.03
	HL	.005	4.45
	LH	.42	2.37
	MH	.009	4.51
	HH	.035	4.79
Second crop	LL	.43	2.31
	ML	.039	4.09
	HL	.132	4.07
	LH	.337	2.68
	MH	.0001	4.11
	HH	.098	4.86

Results show that the probability of obtaining a small yield (2 tons or less) is low in the case of the medium labour high input technology. The probability for the MH technology is 0.009 for the first crop and 0.0001 for the second crop. The LL and LH technologies give the highest probability of a low yield for both crops. This means that when labour level is low, high input levels do not decrease the probability of a low yield. On the other hand for a medium labour technology increase of the input level reduced the probability of loss for both crops. The same tendency is seen for the high labour technology for the second crop. For the first crop however, increase in material inputs in the high labour strata has not reduced the probability of a low yield, but the probabilities are yet very small, .005 for HL and .035 for HH.

It is clearly seen that increasing the labour component in the low input technologies lead to the progressive reduction of the probability of a low yield. For the high input case the reduction in probabilities occurs only when one goes from low to medium labour. When one goes from MH to HH the probabilities of low yields actually increase. This may be due to sampling variation or due to the limitations of the stratification method used (see limitations).

The general indication is that medium levels of labour with high input levels are associated with reduction of the probability of obtaining low yields.

The Profit Function

From the estimated yield functions, the profit functions for the various technologies can be derived.

The general form of the yield function is

$$y = C\, x^{m_1} (\alpha - x)^{m_2} I_{\{0,\alpha\}}(x)$$

where x is yield measured in tons/ha.

Using the transformation $\pi = p.x - \bar{L} - \bar{I}$

where p = price/ton of rice

$\bar{L}$ = mean labour cost for a given stratum

$\bar{I}$ = mean input cost for a given stratum

$$x = \frac{\pi + \bar{L} + \bar{I}}{p}$$

$$dx = 1/p \; d\pi$$

The profit function is

$$y = \frac{c}{p} \left(\frac{\pi + \bar{L} + \bar{I}}{p}\right)^{m_1} \left(\alpha - \frac{(\pi + \bar{L} + \bar{I})}{p}\right)^{m_2}$$

$$- (\bar{L} + \bar{I}) \leq \pi \leq \alpha . p - \bar{L} - \bar{I}$$

The integrals of the profit function are calculated using numerical integration (Simpson's rule).

Family labour was costed at a constant rate of ₱7.50/man day. Price of rice was taken as ₱1.00/kilo. For mean labour and cash input costs see appendix.

Screening of Technologies

First Degree Stochastic Dominance (FSD)

The FSD rule, outlined in the earlier Chapter can be restated as follows. A FSD curve (CDF) to be called efficient (undominated) must lie nowhere to the left of any other FSD curve. Figure 5a and 5b show the CDFs of the 6 different technologies for the first and second crop respectively.

It is clear from figure 5a that the CDFs of ML and MH for the first crop lie to the right of all other curves, except at the uppermost region. Thus if the FSD rule is applied strictly, no single technology can be said to dominate another in the FSD sense. The same is true for the second crop as seen in Figure 5b.

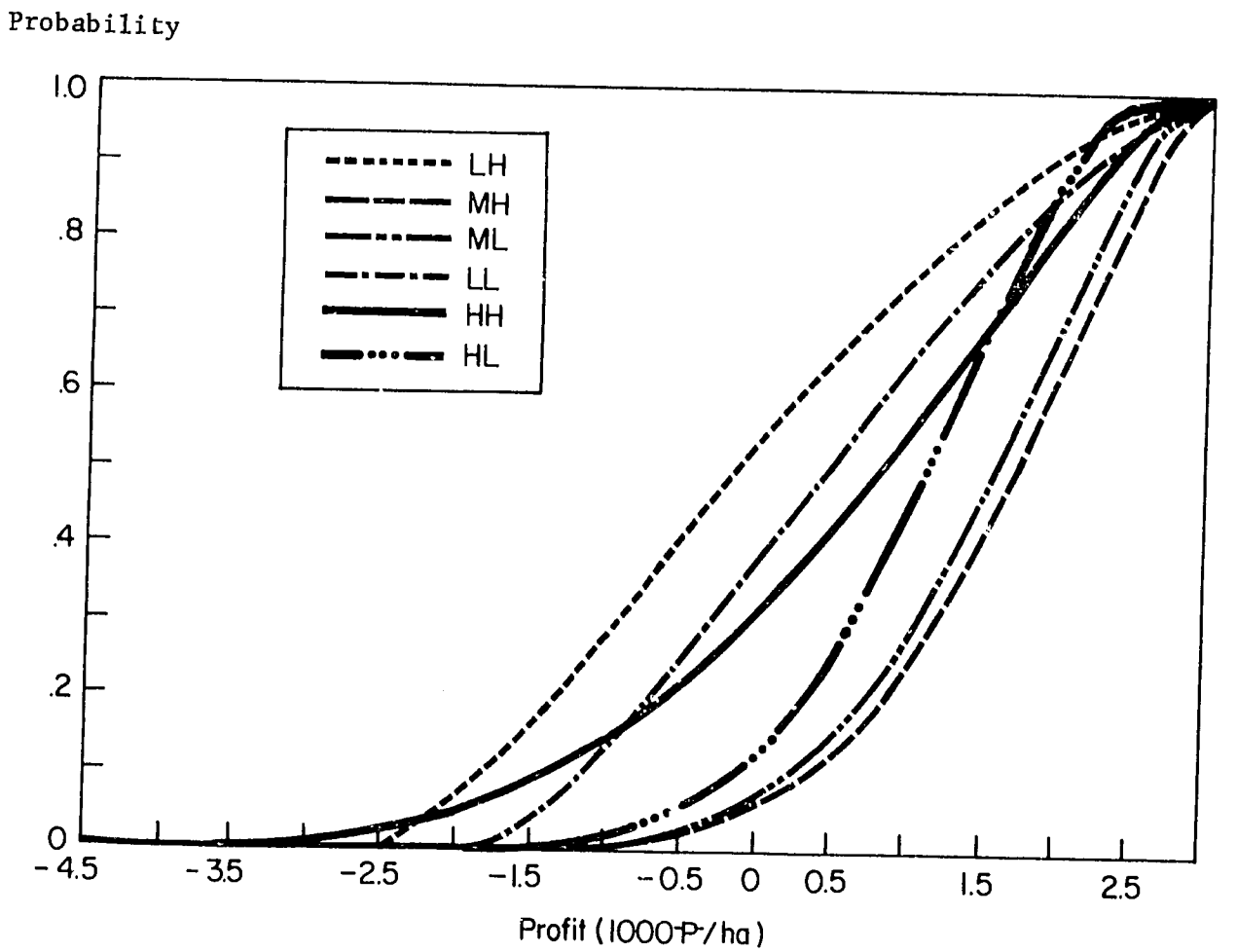


Fig. 5a. Cummulative density functions of profit, first crop. (H= High, M= Medium, L= Low).

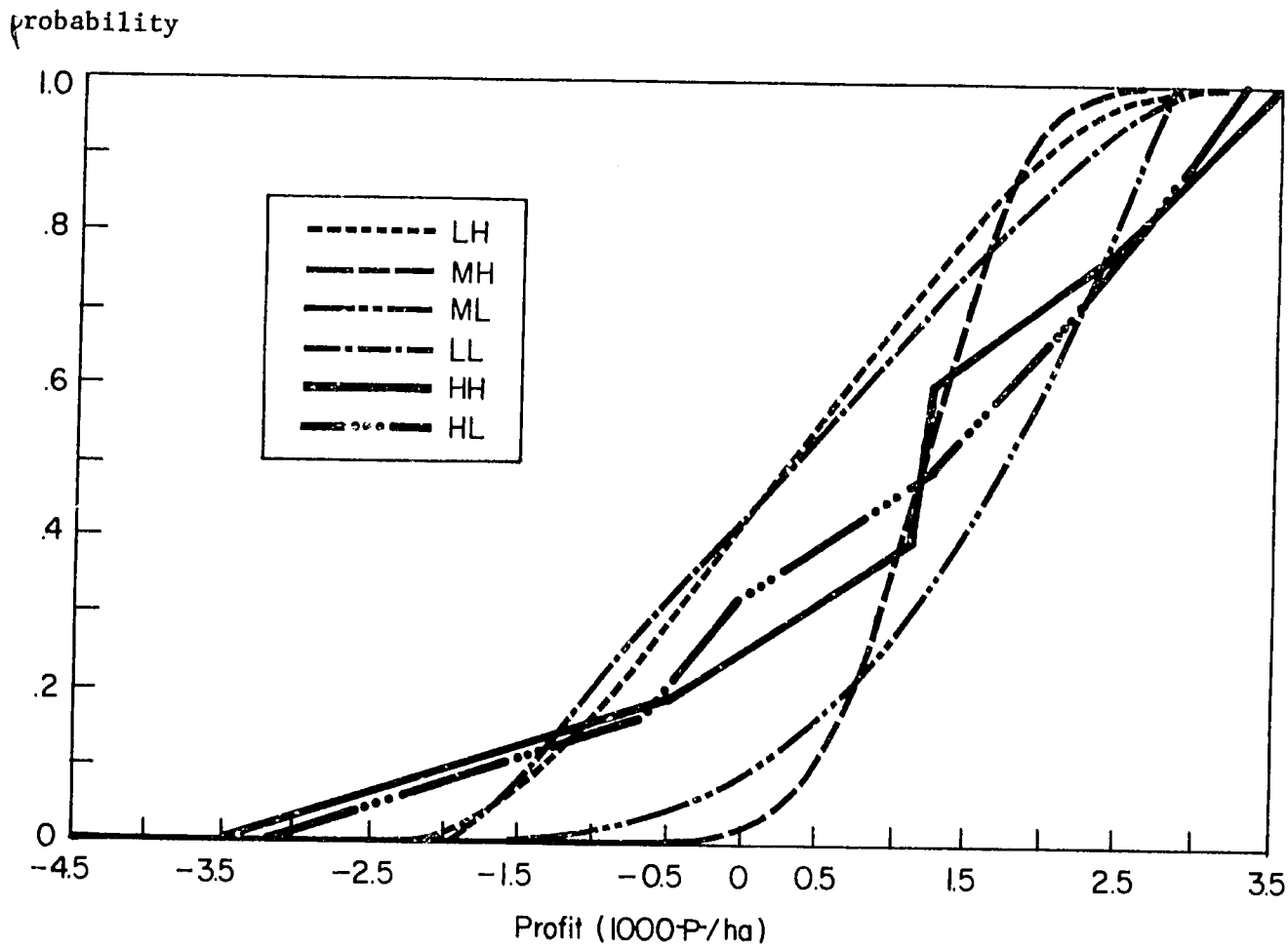


Fig. 5b. Cummulative density functions of profits, second crop. (H = High , M = Medium, L = Low) [illegible]

Second Degree Stochastic Dominance (SSD)

To eliminate distributions from the first degree stochastic efficient set, we need to use the SSD rule. The SSD rule given in Chapter 3 can be restrated, so that it is easy to view it graphically.

Consider the distributions F_1 and G_1 given in Figure 6.

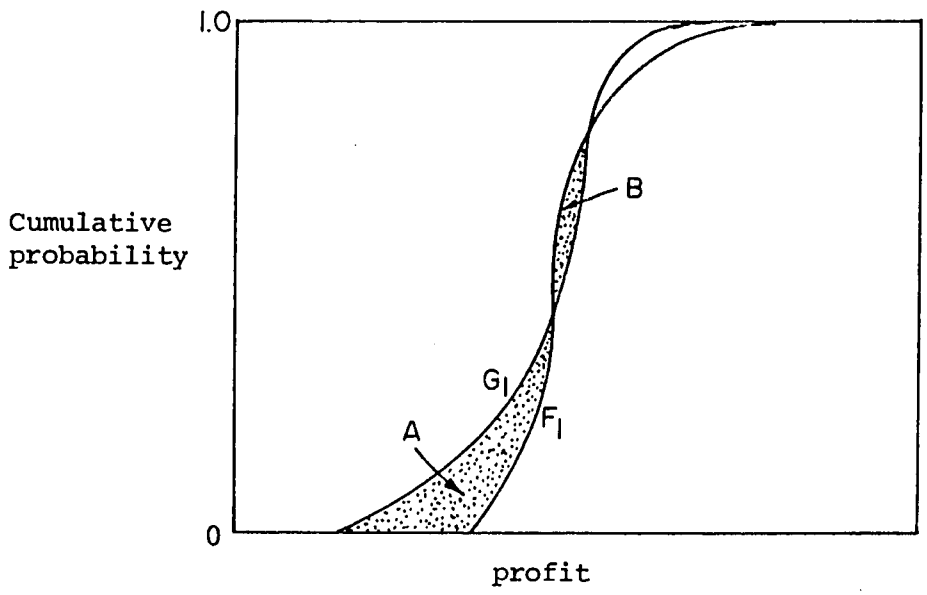


Figure 6. Illustration of SSD (area A > area B)

Source: Anderson, Dillon and Hardaker (1977). Agricultural Decision Analysis, Iowa State University Press, p. 285.

The SSD rule can be viewed graphically as follows. F_1 is said to dominate G_1 if it lies more to the right in terms of differences in area between CDF curves cumulative from the lower values of profit. This means that the area marked A must exceed area marked B for F_1 to dominate G_1.

Applying this rule to the CDFs it can be seen from Figure 5a that MH dominates all other technologies in the case of the first crop. In fact a ranking can be made as follows,

$$MH > ML > HL > HH = LL > LH$$

For the second crop the technologies ML and MH from the second degree stochastically efficient set (SSE set). While LL and LH are the most inefficient technologies. Thus for both crops, it is observed that in general the medium labour technologies are the most efficient.

Third Degree Stochastic Dominance (TSD)

In the case of the first crop there is no need to go into third degree stochastic dominance, as the SSD rule has eliminated LL, ML, HL, LH and HH leaving only MH in the SSE set. But for the 2nd crop the SSE set is made up of MH and ML. Therefore if technologies are to be screened further the TSD rules have to be used.

Similar to the restatement of the SSD rule, the TSD rule can be stated in terms of the second degree CDFs. Given 2 technologies F and G, F is said to dominate G if F_2 (2nd degree CDF of F) lies more to the right of G_2 (2nd degree CDF of G) in terms of differences in area between the 2nd degree CDF curves, cumulative from the lower values of profit.

The second degree CDFs of ML and MH for the 2nd crop is shown in Figure 7.

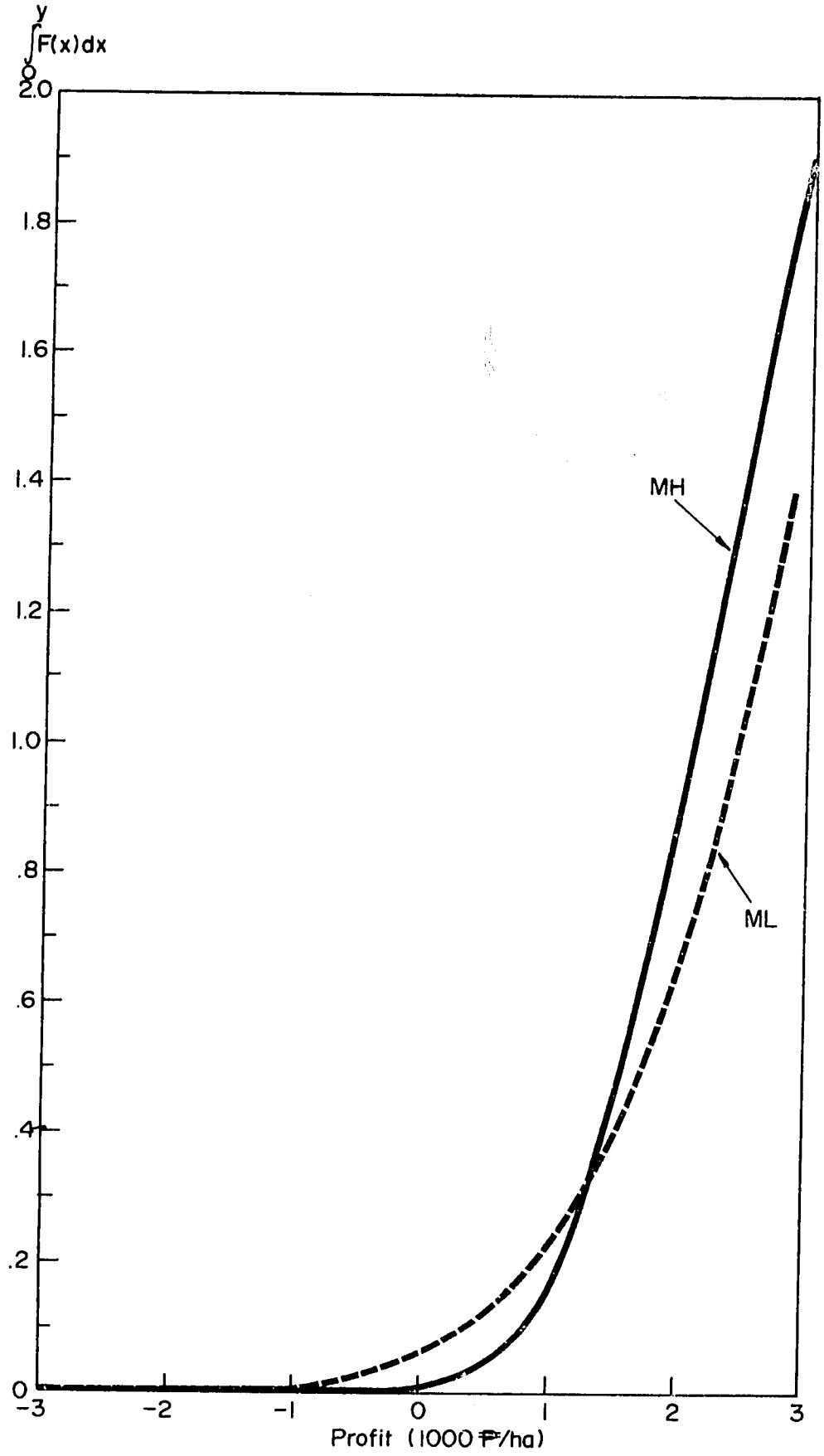


Fig. 7. SSD cumulative curves for MH and ML.

It is observed that even the use of the TSD rule cannot separate the ML and MH technologies. Thus the third degree stochastically efficient set is the same as the SSE set.

In conclusion we can state that by the S. D. criteria MH is the most efficient technology in the case of the first crop, while ML and MH form the efficient set in the case of the second crop.

Probability of Loss

As discussed earlier, a simple approach to screening technologies for 'riskiness' is to rank them by the probability of loss. While this criterion by itself ignores the mean performance, in this section we present the corresponding mean values. Table 9 gives the estimated probabilities of loss and the means for the different technologies.

Table 9. Probability of Loss and Means for the Different Technologies.

TECHNOLOGY (STRATA)	PROBABILITY OF LOSS		MEAN	
	1st crop	2nd crop	1st crop	2nd crop
LL	.3945	.4293	444.09	344.9
ML	.0739	.0941	1417.07	1512.44
HL	.1423	.332	988.55	1079.0
LH	.5334	.4322	62.06	275.58
MH	.0701	.0181	1476.78	1186.84
HH	.3181	.261	574.69	1351.25

In terms of this criterion, the low labour high input (LH) technology has the worst performance for both crops. It has the highest probability of loss (.5334 for the 1st crop and .4322 for the 2nd crop) and the lowest mean profit (62.00 for the first crop and 275.58 for the second crop). The technologies associated with a low level of labour have a high probability of loss and low mean profit. The medium labour technologies have the lowest probability of loss while the high labour technologies seem to be intermediate. This is generally true for both crops.

The MH technology has the lowest probability of loss for both crops and also the highest mean profit in the case of the first crop. While the ML technology has the highest mean profit for the 2nd crop (₱1512.44). However the MH technology also yields a high level of profit for the 2nd crop (₱1186.84).

The indication is that medium labour technologies, especially the MH technology, perform very well in terms of low probability of loss and high profit levels. The results here, agree closely with that obtained using stochastic dominance rules.

Limitations of the Study

The limitations of the study are maily due to the limitations imposed by the data that was available. Variations in planting dates and topography were not incorporated in the stratification as these would have made the number of observations per stratum very small. For the same reason more categories of labour and inputs costs could not be made.

In the stratification, we used technology in a broad sense and associated different technologies with different levels of inputs and the strictly technical differences between different technologies were not explicitly taken into account. The discrepancy observed in some of the results obtained may be due to this.

The results obtained and conclusions reached cannot be indiscrimately extrapolated to any region. The magnitudes and the type of variability in yields obtained would be different in regions dissimilar in the environmental conditions.

SUMMARY AND CONCLUSION

The yield data were categorized into 6 strata, using 3 levels of labour costs (low (L), medium (M) and high (H) and 2 levels of input costs (low (L) and high (H)) for both first and second rice crops. The high labour (HL and HH) technologies for the second crop had very few observations and were therefore not used in the main part of the analysis.

Preliminary analysis of the data in the other 10 strata (6 for 1st crop, 4 for 2nd crop) involved testing for normality. Data in seven out of the ten strata deviated significantly from the normal distribution in terms of skewness or kurtosis or both. A tendency towards greater negative skewness as the levels of labour and inputs (especially labour) were increased, was seen.

The 'criterion' K used to distinguish between the different types in the Pearson system gave negative values for all but 2 strata. Therefore the Pearson type I curve was used in the general curve fitting. Type I estimation with no specification and with the left extreme of the curve specified lead to observations that fell outside the ranges of the curves.

It was therefore decided to specify both the left extreme and the right extreme of the curve, in the moment estimation. This was possible as yields are non-negative and have a biological maximum. The estimation now involved only 2 parameters (m_1 and m_2). Results showed that the estimated values of m_1 and m_2 were both positive,

whereby the cocked hat, bowl or bell shaped forms of the type I function were implied.

The moment estimates (with α_1 and α_2 specified) were used as initial values in the maximum likelihood iteration procedure. The iteration procedure involved maximizing the likelihood functions directly, as packages and functions needed for the usual procedures (Newton-Raphson procedure, method of false positions, method of scoring, etc) where not available in the computer centre of UPLB. It was found that the moment estimates were very close to the maximum likelihood estimates. The slight under (over) estimation involved is negligible compared to the simplicity of calculating the moment estimates.

The empirical cumulative distribution was used for the high labour technologies (HL, HH) of the 2nd crop, due to the small sample sizes for these technologies. The CDF was derived by using the jth order statistic to estimate the $j/_{n+1}$ fractile of the distribution.

The various technologies were screened using the concept of stochastic dominance. For this the yield distributions were transformed to give CDFs for profit. It was found that the medium labour technologies were in the efficient set. Risk measured as the probability of loss also showed that the medium labour technologies (especially with high input) gave the lowest probabilities of loss.

The conclusions that follow from the study can be briefly enumerated as follows:

1. Yield distribution of rice (for a given technology) are generally significantly non-normal.

2. Yield distributions of rice can be adequately represented by the Pearson type I function.
3. The distributions become progressively more negatively skewed as labour and input levels (especially labour) are increased.
4. The type I moment estimates with the extremes of the curve specified, may be acceptable substitutes for the maximum likelihood estimates.
5. Medium labour high cash input technologies form the risk efficient set for the first crop; this result is obtained when both stochastic dominance criteria and the probability of loss criteria is used. For the second crop the medium labour technologies (MH and ML) were in the efficient set when stochastic dominance criteria are used. However the medium labour high cash input technology gave the lowest probability of loss.

LITERATURE CITED

ANDERSON, J. R. 1974. Sparse data, estimational variability and risk efficient decisions. American Journal of Ag. Econ. 56 (3): 564-72.

ANDERSON, J. R.; J. L. DILLON and J. B. HARDAKER. 1977. Agricultural decision analysis. Iowa State University Press. Amer. 321 p.

BARNETT, V. D. 1966. Evaluation of maximum likelihood estimator where the likelihood equation has multiple roots. Biometrika 53 (1 and 2): 151-164.

BORCH, K. 1969. A note on uncertainity and indifference curves. Review of Econ. Studies 36: 1-4.

CONTE, S. D. and D. DE BOOR. 1972. Elementary numerical analysis. McGraw-Hill, Kogakusha Ltd., Tokyo 396 p.

DAY, R. H. 1965. Probability distributions of fied crop yields. Journal of Farm Econ. 47: 713-714.

EDGEWORTH, F. Y. 1898. Journal of Royal Stat. Soc. 61: 670.

EDWARDS, A. W. F. 1972. Likelihood, Cambridge University Press, London, 235 p.

ELDERTON, W. P. and N. L. JOHNSON. 1969. Systems of frequency curves. Cambridge University Press, London, 216 p.

FELDSTEIN, M. S. 1969. Mean variance analysis in the theory of liquidity preference and portfolio selection. Review of Econ. Studies. 36: 5-12.

FISHBURN, P. C. 1977. Mean-rish analysis with risk associated with below-target returns. American Econ. Review, Vol. 6 (1).

FISHER, R. A. 1922. On the mathematical foundations of theoretical statistics. Phil. Tran. of the Royal Soc. of London. Series A 222: 309-368.

FISHER, R. A. 1925. Theory of statistical estimation. Proc. Cambridge Phil. Soc. 22: 700-25.

FISHER, R. A. 1937. Professor Karl Pearson and the method of moments, Ann. of Eugenics. Vol. 7, Pt. 4: 303-318.

GEARY, R. C. 1947. Testing for normality. Biometrika. 34: 209-242.

JAYASURIYA, S. K. W. 1977. The long term investment decision. Unpub. Ph. D. Thesis. Australian National University.

JOHNSON, N. L. 1949. Systems of frequency curves generated by the method of translation. Biometrika, 36: 149-176.

JOHNSON, N. L. 1965. Tables to facilities fitting S_u frequency curves. Biometrika 52(3 and 4): 547.

KALE, B. K. 1962. On the solution of likelihood equations by iteration processes. The multiparametric case. Biometrika 49(3 and 4): 479-486.

KENDALL, M. G. and A. STUART. 1958. The advanced theory of statistics Vol. 1(1958) and Vol. 11(1961). London: Charles Griffen and Co. 690 p.

KOSHAL, R. S. 1933. Application of the method of maximum likelihood to the improvement of curves fitted by the method of moments. Jour. of Royal Stat. Soc. 22: 700-725.

LEVY, H.; MARKOWITZ, H. M. 1979. Approximating expected utility by a function of mean and variance. American Economic review. 69 (3): 308-317.

MARKOWITZ, H. 1952. Portfolio selection. Journ. of Finance, 7 (1).

MARKOWITZ, H. 1959. Portfolio selection - Efficient Deversification of investments; Wiley, N. Y.

MOOD, A. M. and F. A. GRAYBILL, 1963. Introduction to the theory of statistics. Kogakusha Co. Ltd. Tokyo. 443 p.

MOOD, A. M.; F. A. GRAYBILL and D. C. BOES. 1974. Introduction to the theory of statistics, International Student Edition. McGraw-Hill, Kogakusha, LTD. Tokyo. 564 p.

MYERS, R. J. 1934. Note on Koshal's method of improving the parameters of curves by the use of the method of maximum likelihood. Ann. Math. Stat. 5:320-23.

PARIS, Jr. T. B. 1978. Systems analysis and simulation of upland rice based multiple cropping systems in the Philippines. Unpub. Ph. D. Thesis. Michigan State University.

PEARSON, E. S. 1963. Some problems arising in approximating to probability distributions, using moments. Biometrika L, 95: 284-300.

PEARSON, E. S. and H. O. HARTLEY. 1970. Biometrika tables for statisticians. Vol. 1. 3rd edition, Cambridge, The University Press. pp. 207-208.

PEARSON, K. 1895. Contributions to the mathematical theory of evolution. 11 Skew variation in homogeneous material. Phil. tran. of the Royal Soc. of London, Series A, 186: 343-414.

PEARSON, K. 1902. Mathematical contributions to the theory of evolution. X. Supplement to a Memoir on Skew variation. Phil. tran. of the Royal Soc. of London. Series A, 198: 235-299.

PEARSON, K. 1916. Mathematical contributions to the theory of evolution. XIS. Second supplement to a Memoir on Skew variation. Phil. trans. of the Royal Soc. of London, Series A, 216: 429-457.

PEARSON, K. 1924. On the moments of the hypogeometric series. Biometrika 16, 157.

PRICE, E. C. 1976. Economic criteria for cropping pattern design. In Proceedings, Symposium on cropping systems and development for the Asian farmer. IRRI, Philippine, pp. 167-185.

ROUMASSET, J. A. 1976. Rice and risk, North Holland Pub. Co. N. Y. 251 p.

SERQUIÑA, C. de los S. 1977. Statistical analysis of Philippine rainfall data. Unpublished M. S. thesis. UPLB, Philippines, June 1977.

SIEGEL, S. 1956. Nonparametric statistics for the behavioral sciences. International Student Edition, McGraw-Hill, Kogahusha Ltd. Tokyo, 312 p.

SNEDECOR, G. W. and W. G. COCHPAN, 1967. Statistical methods. 6th edition. Iowa State Univ. Press. Ames, U.S.A. 593 p.

TAHA, H. 1971. Operations Research. McGraw-Hill Co. 648 p.

TOBIN, J. 1958. Liquidity preference as behavior towards risk. The Review of Econ. Studies 25(2).

VON NEUMAN, J. and O. MORGENSTERN, 1947. Theory of games and economic behaviour, 2nd ed. Princeton, Princeton University Press.

APPENDIX

A.1. Mean Labour and Cash Input Costs for the Different Strata (₱/ha).

STRATA	MEAN LABOUR COSTS		MEAN CASH INPUT COSTS	
	1st crop	2nd crop	1st crop	2nd crop
LL	1126.77	1084.98	888.33	911.28
ML	1704.49	1675.95	931.76	923.00
HL	2552.22	2222.00	939.94	948.40
LH	1073.19	1115.35	1409.24	1247.00
MH	1746.66	1621.14	1261.17	1297.66
HH	2828.36	2170.00	1403.43	1336.50

A.2. Frequency Distributions for Labour and Material Input Costs.

LABOUR COSTS (₱/ha)	<1100	1100-	1200-	1300-	1400-	1500-	1600-	1700-	1800-	1900-	2000	>2100
Frequency (1st crop)	26	10	15	13	11	23	13	21	17	11	12	78
Frequency (2nd crop)	54	9	12	20	21	20	16	7	3	2	4	5
Material costs (₱/ha)	<700	700-	800-	900-	1000-	1100-	1200-	1300	>1400			
Frequency (1st crop)	8	18	26	39	50	44	23	15	27			
Frequency (2nd crop)	15	5	17	14	26	46	18	15	17			

A.3. Source Program for Calculating Maximum Likelihood Estimates.

```
      DIMENSION X(70)
      REAL*8 M1,M2,MS1,MS2,L,LS,LX,ALPHA,Z,R1,R2,R3,GA,Y,YY,DA,GB,
     1G1,G2,G3,CREM
      DATA x/70*0./
      WRITE(6,6)
      READ(5,7)CREM
      DO 10 I=1,10
      READ(5,1,END=90)      M,M1,M2,APLHA,(X(II), II=1,M)
      DA=DLOG(ALPHA)
      Y=0
      YY=0
   90 DO 20 J=1,M
      LX=X(J)
      IF(LX.LE.0)GO TO 11
      Y=Y+DLOG(LX)
      Z=ALPHA-LX
      IF(Z.LE.0)GO TO 12
   20 YY=YY+DLOG(Z)
      NS=0
      NSY=0
      NSW=1
      N=0
   99 IF(NSY.EQ,1)NS=N
      N=N+1
      R1+M1+1
      R2=M2+1
      R3=R1+R2
      G1=DGAMMA(R1)
      G2=DGAMMA(R2)
      G3=DGAMMA(R3)
      GA=G1*G2/G3
      IF(GA.LE.0)GO TO 13
      GB=DLOG(GA)
      L=M1*Y+M2*YY-M*GB-M*(R3-1.)*DA
      IF(N.EQ.1)GO TO 98
      IF(L.LT.LS)GO TO 97
      NSY=0
      IF(NSW.EQ.1)GO TO 98
      LS=L
      MS2=M2
      M2=M2+CREM
   98 LS=L
      MS1=M1
      M1=M1+CREM
      GO TO 99
   97 NSY=1
      NS1=NS+1
      IF(NS1.QE.N)GO TO 96
```

Appendix A.3. (Continued)

```
0048          IF(NSW.EQ.1)GO TO 95
0049          NSW=1
0050          M1=M1+CREM
0051          GO TO 99
0052       96 IF(N.EQ.2)GO TO 95
0053          GO TO 10
0054       95 NSW=2
0055          M2=M1+CREM
0056          GO TO 99
0057       11 WRITE(6,3)LX
0058          GO TO 999
0059       12 WRITE(6,4)A,ALPHA,LX
0060          GO TO 999
0061       13 WRITE(6,5)GA,G1,G2,G3
0062          GO TO 999
0063       10 WRITE(6,2)L,N,MS1,MS2,I,M
0064      999 STOP
0065        1 FORMAT(I2,3F6.1    /4(20F4.3/))
0066        2 FORMAT(1X,D16.8,5X,I4,7X,D16.8,D16.8,T110,I2,T126,I2)
0067        3 FORMAT(1X,D16.8)
0068        4 FORMAT(1X,3D16.8)
0069        5 FORMAT(1X,4D16.8)
0070        6 FORMAT(1H1,T4,'LIKELIHOOD',T20,'ITERATION',T108,'DATA SET',
           T122,"N lUMBER OF'/T6,'VALUE',T22,'NUMBER',T40,'M1'.T56,
           'M2',T109,'NUMBER', 1T121,'OBSERVATIONS'/)
0071        7 FORMAT(F6.3)
0072          END
```