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This manual contains the lecture notes and course materials, including a microcomputer disk of supporting data dn routines for classroom exercises from the 15-day shortcourse on linear programming applications to economic development and policy analysis offered by Oklahoma State University under the Agricultural Policy Analysis Project (APAP). Subjects covered include: Introduction to Linear Programming and the Microcomputer; Introduction to the farm Model; Extensions to the Farm Model, including input Levels; Resource Quality, Home Consumptions, Crop/livestock Models, Cash flows, and Risk Analysis; the Sector Model, including Introduction, Demand Equations, Risk, Constructing, Validating, Maintaining, and Policy Analysis.

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APAP SHORTCOURSE:
LINEAR PROGRAMMING
APPLICATIONS TO
ECONOMIC DEVELOPMENT
AND POLICY ANALYSIS:
LECTURE NOTES AND COURSE
MATERIALS

By: Arthur Stoecker
Elton Li

SEPTEMBER, 1988

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LINEAR PROGRAMMING APPLICATIONS TO ECONOMIC DEVELOPMENT AND POLICY ANALYSIS: LECTURE NOTES AND COURSE MATERIALS

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MATHEMATICAL PROGRAMMING APPLICATIONS TO POLICY ANALYSIS
(May 16 - June 3, 1988)

Reference Text: Hazell, B.R. and R.D. Norton, *Mathematical Programming for Economic Analysis in Agriculture*, Macmillan Inc., 1986.

Course Schedule and Topics Covered

Day 1: Introduction to Linear Programming and the Microcomputer, ch. 1-2.

Day 2: Introduction to the Farm Model, ch. 2-3.

Day 3: Extensions to the Farm Model- Input levels, Resource Quality, Home Consumption, Crop/Livestock Models, ch. 4.

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Day 5: Investment analysis with Multiperiod Models, ch. 4.

Day 6: Risk analysis with the Farm Model, ch. 5.

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Day 8: Introduction to the Sector Model, ch. 7.

Day 9: Incorporation of Demand Equations in the Sector Model, ch. 8-9.

Day 10: Risk in the Sector Model, ch. 10.

Day 11: Introduction to the Domingo Model.

Day 12: Techniques for Constructing, Validating, and Maintaining Sector Models, ch. 11.

Day 13: Policy Analysis with the Sector Model, ch. 12-13.

Day 14: Policy Analysis with the Sector Model, ch. 12-13.

Day 15: Policy Analysis with the Sector Model, Course Evaluation.

PART I: LINEAR PROGRAMMING AND THE FARM MODEL

INTRODUCTION TO LINEAR PROGRAMMING AND THE MICROCOMPUTER

References: Hazell and Norton, *Mathematical Programming for Economic Analysis in Agriculture*, Chapters 1-2.

O'Leary, *Lotus 1-2-3 Student Edition*, Chapters 1-2.

Introduction

Background and Course Objectives

Linear programming, which deals with the optimization of a linear objective function subject to a set of linear constraints, is an important economic planning and policy analysis tool. It is a natural tool in the sense that a major objective of planning and policy analysis is to make the best possible improvements within constraints imposed by limited resources.

The Simplex method of solving linear programs was developed by George Dantzig during the World War II period. The first applications of linear programming were for military purposes but linear programming has since been widely applied within agriculture. The development of the microcomputer and spreadsheet software has made linear programming a tool which is readily available to any analyst with a microcomputer. This has increased the need for training in both linear programming and in microcomputer skills.

The objectives of this course are to develop the necessary skills for using linear programming models in policy analysis at both the farm and at the agricultural sector levels; and the ability to implement L.P. models on microcomputers. The specific objectives are:

a. Develop the understanding and ability to apply the theory and procedures needed for building linear programming models for farm and sector applications.

b. Develop the ability to modify, manipulate, and interpret linear programming models for the analysis of policy questions.

c. Develop an understanding of linear programming models used for policy analysis in developing countries.

d. Develop the necessary microcomputer skill to effectively manage data input, obtain solutions, summarize and present results, and to maintain large scale programming models.

e. Develop the skills necessary to solve problems in constructing, solving, validating, and maintaining large scale sector models.

Characteristics of Problems Which Can be Solved by Linear Programming

1. Activities or decision variables which are non-negative.

2. The criterion for selecting the "best" values of the decision variables is a linear function of the variables.

3. The factors or resources which restrict the model can be expressed as a set of linear equations or inequalities.

Types of Models to be Considered

The Linear Programming models discussed in this course will be:

a) the farm model
b) the agricultural sector model.

LINEAR PROGRAMMING ON THE MICROCOMPUTER: SOME PRELIMINARIES

The objective is to provide some background on microcomputer techniques and software packages suitable for obtaining linear programming (LP) solutions with microcomputers.

What Hardware Do You Need?

Discussion will be limited to the IBM PC series of machines and their compatibles. The smaller examples in H&N (Hazell and Norton's book) can be solved on a "minimal" machine. By a minimal machine, we mean a IBM PC computer with 256K of RAM (Random Access Memory), and two floppy drives. Of course a printer would be essential if you want to print answers.

With a minimal machine and the right software, we can realistically solve problems of up to, say, about 50 equations. From this minimal machines, hardware options can be added on to make it more useful for linear programming.

A useful option is a math co-processor. The heart of any computer is the Central Processing Unit (CPU). Most microcomputer CPU's are not designed to perform arithmetic on floating point numbers *naturally*. Roughly speaking, a floating point number is one that has a decimal point in it, versus an integer which does not have a decimal point. Thus 234.156 is a floating point number whereas 2 is an integer. In the IBM PC series of computers, a floating point number can generally range from with about (+ or -) 4.19×10^{-307} to (+ or -) $1.67 \times 10^{+308}$ with 16 significant digits of accuracy. On the contrary, an integer generally can only range from -32,768 to 32,767 with no decimal point. As you can see, integers are fairly useless as far as numerical computation goes.

Whereas floating point arithmetic cannot be done naturally in the IBM PC, it can be emulated. In fact, many things that cannot be done naturally by computers can be emulated by software. That is precisely the reason that makes computers so useful. But emulation has a price — namely speed and, to a lesser extent, program size. With a math co-processor, floating point arithmetic can be done naturally, i.e., directly by hardware and not emulated by software. Thus floating point arithmetic is about 30 times faster with a math co-processor than without one. That does not mean that your LP will run 30 times faster — LP algorithms involve more than just floating point arithmetic alone. Moreover, the speed of floating point arithmetic depends on how efficiently the particular software package utilizes the math-coprocessor. Nonetheless, you would expect a healthy improvement in speed in most cases when a math co-processor is present.

In fact, many LP packages will not even run if you don't have a math co-processor. I guess the authors of these programs think that nobody in his/her right mind would actually run LP on an IBM PC without a math coprocessor; since it usually drastically speeds up the calculations but yet only costs less than 10 percent of a typical system.

More random access memory (RAM) would also be useful. In general, IBM PC running under MS DOS can use up to 640K bytes of RAM. (MS stands for MicroSoft which is the name of the company which produces the most popular DOS for the IBM PC. MS DOS is also known as PC DOS.) One K usually means 1000, but in computer terms it means 1024. Thus 640K bytes of RAM actually means 640x1024=655360 bytes. One byte of information is roughly equivalent to one character of information. It usually takes eight bytes to represent a floating point number. Although there are ways to extend the memory on the PC beyond 640K of memory, very few LP programs are capable of taking advantage of expanded or extended memory at this writing.

When a program is running, both the program and data reside in RAM. The disk operating system (DOS), which is a program in itself, also takes up some of that space. The larger the program, the more RAM it will occupy. But let's say DOS and the program together take up 300K of RAM, which is fairly typical for an LP program. On a 640K machine, this would leave about 340K for data. How many floating point numbers can be stored in 340K of RAM?

As mentioned before, 8 bytes are required to store a floating point number. If there are 340K bytes of RAM left for data after the program is loaded into memory, that would mean about 340x1024/8 = 43,520 floating point numbers can be stored — about a 208 by 208 matrix. Moreover, many matrix operations needed in LP, e.g. matrix inversions, require additional space for intermediate results. Thus while 640K seems to be an immense amount of space at first sight, it is not much when you try to fit an LP tableau into it.

Of course, most good LP packages use a sparse matrix storage technique to store the tableau. In most LP problems, less than 10 percent of the elements are non-zero. Thus one can develop some clever storage scheme whereby only the non-zero elements are recorded — together with the necessary information that keeps track of where the

non-zero elements are. Depending on the storage scheme and the solution algorithm employed, a very large LP tableau can be represented with 340K of RAM.

Another useful option is a hard disk. There are two main reasons to get a hard disk. First it stores a lot of information and second, it is faster. Some LP programs utilize the disk to store temporarily results when it runs out of RAM. In this case the speed of the solution depends on the speed to read from and write to the disk. Moreover, when working with large LP you are usually working with a lot of data, so a hard disk will always come in handy. Using a hard disk also eliminates the inconveniences of swapping diskettes when changing from one software package to another.

The final hardware option is to get a faster machine. The standard IBM uses the Intel 8088 chip running at a clock speed of 4.77 mHz. The higher the clock speed, the faster the machine if other factors remain constant. Many IBM compatibles run at a clock speed of 8 mHz and thus are about 60 percent faster than the standard PC.

Another factor which affects speed is the capability of the chip itself. The original IBM PC-AT runs at 6 mhz but uses a 80286 chip. It however runs about 3 times faster than the standard IBM-PC with the 8088 running at 4.77 mHz.

Some machines on the market now uses the 80386 chip. These machines can be 3 times faster than the original AT, or an astonishing 9 times faster than the original PC.

But note that with a math co-processor, the speed of LP solution depends not only on the main processor, but also on the math co-processor. Unfortunately, the math-coprocessor in the IBM PC-AT is not that much faster than the math-coprocessor in the IBM PC. Thus you would not really notice a 3 times increase in speed of the LP solution as advertised above since in LP, most of the work would be done by the co-processor and not the main processor. Experience has shown that the AT is only slightly faster than the PC in these cases.

What Software Do You Need?

LP packages are among the most expensive category of software in the market. Many of these software packages have descended from mainframe or mini computer packages. Comparative studies on microcomputer LP packages done by Sharda concluded that while mainframe packages are still better, the micro packages are not bad.

But having an LP package and knowing how to run it is just one link in the whole process of "running LP on the microcomputer." According to Rice et al.:

> ...when the linear model is valid, present limitations on its usefulness do not lie in its computational algorithms but rather in the amount of work required to prepare a large problem and use its answers.

Made in the early '80s, the above quotation was referring to solving LP with mainframe computers but applies to the modern microcomputer environment as well. Behind every realistic-sized LP problem is a data management problem. With mainframes, solving these data management problems often requires expertise in computer programming. On the microcomputer, many of these data management problems can be solved with easy-to-use tools like spreadsheets or editors.

The software packages that will be referred to subsequently in this shortcourse are:

Lotus 1-2-3 release 2 spreadsheet package — This will be used as an easy and intuitive vehicle for designing the LP tableau.

Musah86 — An easy-to-use LP program with interface to Lotus 1-2-3 intended for small educational LP problems.

ToMpsx — A program which translates an LP tableau coded in Lotus 1-2-3 to MPSX format.

Santiago — A matrix generator for use with Lotus 1-2-3.

b

Exercises

1. If you have not had any experience with computers, or if you have not had any experience with the IBM PC microcomputer in particular, go through the tutorial diskette "Exploring the IBM PC."
2. If you have not had any experience with spreadsheet software. Or if you have some experience with spreadsheet software but no experience with Lotus 1-2-3 in particular. Go through the Lotus Tutorial. You do not need to know all of Lotus but only the part 1 and 2. Make sure you know:
   a. Entering numbers in the worksheet.
   b. Entering labels in the worksheet.
   c. Formatting numbers.
   d. Justifying labels.
   e. Printing the worksheet.
   f. Saving and retrieving worksheet files.
3. Make a backup copy of the IBM DOS system disk.
4. Backup the diskette labelled 'Musah86' with the diskcopy command.
5. Format two diskettes for data storage.

INTRODUCTION TO THE FARM MODEL AND THE SIMPLEX METHOD

The example from the text is for the Mayaland farmer. The source of the information shown in the text is not given. However we assume that enterprise budgets have been constructed (possibly from survey information) for each of the crops (Corn, Beans, Sorghum, and Peanuts).

The production choices open to the Mayaland farmer include:

- **Production of Corn**. One hectare of corn requires 1.42 person and 1.45 mule months of labor. The net returns (total value of production less all variable costs except labor and mules) are 1372 pesos per year.

- **Production of Beans.** One hectare of beans requires 1.87 person and 1.27 mule months of labor. The net returns from one hectare are 1219 pesos.

- **Production of Sorghum.** One hectare of sorghum requires 1.92 person and 1.16 mule months of labor. The net return from one hectare of Sorghum is 1523 pesos.

- **Production of Peanuts.** One hectare of peanuts requires 2.64 person and 1.45 mule months of labor. The net returns from one hectare of peanuts is 4874 pesos. However the farmer faces a limited market for peanuts and can sell no more than 0.5 tons per year.

We also determine that each year the farmer is able to use resource services which consist of 5 hectares of land, 16.5 person months of labor, and 10 months of mule labor.

Objectives of the Model

Construction of the model begins by asking three questions.

1. What do you want to determine from the model? What are the activities, variables or unknowns?

2. What are the constraints, factors or resources which limit the variables or activities of the model?

3. What is the goal or objective to be achieved?

The three basic questions and the answers in the case of the Mayaland example are:

a) What are the activities or decision variables?

Answer:
The number of hectares of corn, beans, sorghum and peanuts to be grown each year.

b) What are the factors or resources which limit the amount which can be produced.

Answer:
Supply of land- only 5 hectares per year available.
Supply of labor- only 16.5 months per year available.
Supply of mule labor- only 10 months per year available.
Supply of market capacity for peanuts, can only market 0.5 tons per year.

c) What is the goal or objective of the Mayaland farmer?

Answer:
To maximize net farm income.

Programming Tableau

The tableau shown below (Table 2.2 from the text) just lists the resources required and the net returns for each enterprise or activity underneath the activity name. The supply of each resource has been entered in the column labeled RHS (right hand side).

	Corn (ha)	Beans (ha)	Sorghum (ha)	Peanuts (ha)	Type const.	Rhs
Objective function	1372	1219	1523	4874		
Land (ha)	1	1	1	1	<=	5.0
Labor (months)	1.42	1.87	1.92	2.64	<=	16.5
Mules (months)	1.45	1.27	1.16	1.45	<=	10.0
Market constraint (tons)				0.983	<=	0.5

Algebraic Form. If we let

X1 = hectares of Corn,
X2 = hectares of Beans,
X3 = hectares of Sorghum, and
X4 = hectares of Peanuts,

then the problem can be expressed in form of linear equations. The problem is Maximize Z where

$$Z = 1372 * X1 + 1219 * X2 + 1523 * X3 + 4874 * X4$$

Subject to

$$\begin{aligned}
1 * X1 + 1 * X2 + 1 * X3 + 1 * X4 &<= 5.0 \\
1.42 * X1 + 1.87 * X2 + 1.92 * X3 + 2.64 * X4 &<= 16.5 \\
1.45 * X1 + 1.27 * X2 + 1.16 * X3 + 1.45 * X4 &<= 10.0 \\
0 * X1 + 0 * X2 + 0 * X3 + 0.98 * X4 &<= 0.5
\end{aligned}$$

$$X1, X2, X3, X4 >= 0.$$

Preparing the Problem for Solution

Before the problem can be solved, it is necessary to convert all inequalities to equalities by adding slack variables. A slack variable measures the amount of unused resources. This is usually done for the user by the linear programming software but the process is reviewed here because we will be referring the values of the slack vectors in the output.

For the first constraint, land use,

Corn(ha) + Beans(ha) + Sorghum(ha) + Peanuts(ha) <= 5

or

X1 + X2 + X3 + X4 <= 5.

Define a new variable, X5 which is

X5 = 5 - X1 -X2 - X3 - X4

which can be written as

X1 + X2 + X3 + X4 + X5 = 5.

The new variable X5 is a slack variable which measures the amount of unused land. The variable X5 may be referred to as a land slack.

In a similar manner, define slack variables for each of the remaining resources in the problem. Let X6 (labor slack) represent the unused months of human labor. Then the second constraint can be written as an equality

1.42*Corn +1.87*Beans + 1.92*Sorghum + 2.64*Peanuts + X6 = 16.5.

Let X7 (mule slack) represent unused months of mule labor. Then the third constraint can be written as an equality

1.45*Corn + 1.27*Beans + 1.16*Sorghum + 1.45*Peanuts + X7 = 10.

Let X8 (market slack) represent the unused peanut marketing capacity. Then the fourth constraint becomes

.983*Peanuts + X8 = .5.

The problem in Tableau form becomes:

	Corn (ha) X1	Beans (ha) X2	Sorghum (ha) X3	Peanuts (ha) X4	Lnd S (ha) X5	Lab S (mon) X6	Mul S. (mon) X7	Mk S. (ton) X8	Rhs
Obj.	1372	1219	1523	4874					
Land	1	1	1	1	1				5.0
Lab.	1.42	1.87	1.92	2.64		1			16.5
Mule	1.45	1.27	1.16	1.45			1		10.0
Mark.				0.983				1	0.5.

In equation form the problem can be stated as:

Maximize Z where

$$Z = 1372 \ *X1 + 1219 \ *X2 + 1523 \ *X3 + 4874 \ *X4$$

Subject to

$$\begin{aligned}
1 \ *X1 + 1 \ *X2 + 1 \ *X3 + 1 \ *X4 + X5 &= 5.0 \\
1.42 \ *X1 + 1.87 \ *X2 + 1.92 \ *X3 + 2.64 \ *X4 + X6 &= 16.5 \\
1.45 \ *X1 + 1.27 \ *X2 + 1.16 \ *X3 + 1.45 \ *X4 + X7 &= 10.0 \\
0 \ *X1 + 0 \ *X2 + 0 \ *X3 + 0.98 \ *X4 + X8 &= 0.5 \\
X1, X2, X3, X4, X5, X6, X7, X8 &>= 0.
\end{aligned}$$

Outline of the Simplex Method

The steps of the simplex method for the Mayaland example are described on pages 23-29 of the text. The simplex calculations are reviewed here by spreadsheet formulas to make the actual calculations. The completed calculations are shown in Table 1.

Table 1. Solution of Mayaland Example by Simplex Calculation Using a Lotus Worksheet

	A	B	C	D	E	F	G	H	I	J	K	L
1			"Rhs	Corn	Beans	Sorg	Pean	Lnd.S	Lab.S	Mul.S	Mk.S	Ratio
2				ha	ha	ha	ha	ha	mon	mon	ton	
3				X1	X2	X3	X4	X5	X6	X7	X8	
4	Ci	Act	Level	1372	1219	1523	4874	0	0	0	0	
5	0	X5	5	1	1	1	1	1	0	0	0	5
6	0	X6	16.5	1.42	1.87	1.92	2.64	0	1	0	0	6.25
7	0	X7	10	1.45	1.27	1.16	1.45	0	0	1	0	6.896
8	0	X8	0.5	0	0	0	0.983	0	0	0	1	0.508
9		Z	0	0	0	0	0	0	0	0	0	
10		Z-C		-1372	-1219	-1523	-4874	0	0	0	0	
11							PC					
12												
13	0	X5	4.491	1	1	1	0	1	0	0	-1.01	4.491
14	0	X6	15.15	1.42	1.87	1.92	0	0	1	0	-2.68	7.894
15	0	X7	9.262	1.45	1.27	1.16	0	0	0	1	-1.47	7.984
16	4874	Pean	0.508	0	0	0	1	0	0	0	1.017	NA
17		Z	2479.	0	0	0	4874	0	0	0	4958.	
18		Z-C		-1372	-1219	-1523	0	0	0	0	4958.	
19						PC						
20												
21	1523	Sorg	4.491	1	1	1	0	1	0	0	-1.01	
22	0	X6	6.533	-0.5	-0.05	0	0	-1.92	1	0	-0.73	
23	0	X7	4.052	0.29	0.11	0	0	-1.16	0	1	-0.29	
24	4874	Pean	0.508	0	0	0	1	0	0	0	1.017	
25		Z	9319.	1523	1523	1523	4874	1523	0	0	3408.	
26		Z-C		151	304	0	0	1523	0	0	3408.	

There are three main steps to the Simplex method for finding the combination of activities which yield the maximum value of the objective function. These steps are:

Initialization Step: Identify an initial basic feasible solution.

Iterative step: Move to an adjacent basic feasible solution where the value of the objective function is greater or at least as great as the current solution.

Stopping rule: Stop when no adjacent basic feasible solution will improve the value of the objective function.

Initialization Step

The initial basic feasible solution involves setting the variables X1 - X4 equal to zero and setting the slack variables X5-X8, equal to the RHS values, (X5=5, X6=16.5, X7=10, X8=.5).

Starting Lotus. Insert the Lotus system disk in drive A, a formatted data disk in drive B and turn on the Computer.

Enter the date `5/16/88` **and press the** `enter` **key.**

Enter the current time. Eg. for 10:30 am enter `10:30` **and press the** `enter` **key.**

You should see the system prompt `A:>`

Type `LOTUS` **(press** `enter`**)**

Move the cursor to 1-2-3 and press `enter`**.**

Enter the the Initial Simplex Tableau for the Mayaland problem in the form shown below **for rows 1-8 only.**

	A	B	C	D	E	F	G	H	I	J	K
1			Rhs	Corn	Beans	Sorghum	Peanuts	Lnd.S	Lab.S	Mul.S	Mk.S
2				(ha)	(ha)	(ha)	(ha)	(ha)	(mon)	(mon)	(ton)
3				X1	X2	X3	X4	X5	X6	X7	X8
4	Ci	Act	level	1372	1219	1523	4874	0	0	0	0
5	0	X5	5.0	1	1	1	1	1	0	0	0
6	0	X6	16.5	1.42	1.87	1.92	2.64	0	1	0	0
7	0	X7	10.0	1.45	1.27	1.16	1.45	0	0	1	0
8	0	x8	0.5	0	0	0	0.983	0	0	0	1
9		Z	0	0	0	0	0	0	0	0	0
10		Z-C		-1372	-1219	-1523	-4874	0	0	0	0

The initial feasible solution for X1, X2, X3, X4, X5, X6, X7 and X8 is 0, 0, 0, 0, 5, 16.5, 10, and .5, respectively. The values in the rows labeled Z and Z-C are calculated.

We will use Lotus formulas to make these calculations. First calculate the values in the row labeled Z. **Move the cursor to cell C9, enter the following formula**

```
+$A5*C5+$A6*C6+$A7*C7+$A8*C8
```

This formula multiplies the values of the objective function in column A by the RHS values in column C and sums the result. Be sure and include the $ signs as shown above so the formulas can be copied. **Use the /Copy command to copy the formula in C9 to the cells D9-K9.**

Next subtract the values in the Objective function (C row) from the values in the row labeled Z. **Move the cursor to D10 and enter the formula**

```
+D9-D$4
```

Use the /Copy command to copy the formula in cell D10 to K10.

The tableau should look like the one shown above.

Check to see if the solution is optimal. The row labeled Z-C contains four negative values so the solution is not optimal.

Iterative Step

— Part 1. Choose the variable with the most negative value in the Z-C row to enter the solution.

The activity with the most negative entry in the Z-C row is Peanuts or X4. The column of the entering basic variable (X4 in this case) will be called the PIVOT COLUMN. **Move the cursor under the column X4 and mark this column (PC).**

— Part 2. Determine the leaving basic variable. Construct a ratio column by dividing the RHS elements (Column C) by the elements in the PIVOT COLUMN, which is Column C (see Table 1). The leaving basic variable is the basic variable associated with the row having the smallest non-negative ratio. This row with the smallest non-negative

ratio will be called the PIVOT ROW. The element or entry in the intersection of the PIVOT ROW and PIVOT COLUMN is called the PIVOT NUMBER.

In Lotus to set up the ratio column, **move the cursor to cell L5, enter the formula**

```
@IF(G5>0,C5/G5,@na)
```

Use the /Copy command to copy the formula in Cell L5 to cells L6-L8.

The IF statement in the formula causes Lotus to check whether or not the entry in column G is positive. If the entry is positive, the current RHS value in Column C is divided by the element in Column G. If the entry in Column G is not positive, `@na` (meaning not available) is entered. `@na` is used here to signify that this variable will never be considered as the leaving basic variable. In our case here, Peanuts will be the incoming Basic Variable and X8 (marketing slack) will be the leaving basic variable.

— **Part 3. Update the Tableau.** Determine the new basic feasible solution by constructing a new Simplex Tableau below the current one. (The operations are very similar to the Gauss-Jordan method for solution of simultaneous equations.)

The new pivot row is constructed by dividing every element in the OLD PIVOT ROW by the PIVOT NUMBER. The word **PEAN** will replace the name X8 in the list of basic variables and the C value, **4874** will replace the C value for X8 in the Column labeled Ci.

Use the /Copy command to copy the labels in cells A5-B10 to cell A13.

Move the cursor to A16. Type **Peanuts** (the name for X4)
Move the cursor to B16. Type **4874** (obj value for X4).

The Lotus formulas to calculate the new pivot row are entered. **Move the cursor to cell C16. Type +c8/g8.**

Copy the formula in Cell C16 to Cells D16 thru K16. The new pivot row has been completed.

The remaining rows (all rows except the PIVOT ROW) are updated by the relation New element = Old element - (Incoming element * Pivot element), i.e., N = O - (I * P).

Move the cursor to Cell C13. Type +C5 - ($C16*$G5).

The old element is in cell C5, the incoming element in the pivot row is in Cell C16 and the Pivot element is in cell G5. Be sure and include the $ signs as shown. **Now copy the formula in C13 to cells C13 thru K16.** The values in the Z and Z-C rows are updated by **copying the formulas in Cells C9 through K10 to Cell C17.** The formulas which should be entered in the block of cells from C13 to K16 are shown in Figure 1. The updated tableau is should look like the tableau in block 2 of Table 1.

	A	B	C	D	E	F	G	H	I	J	K
13			*Rhs	Corn	Beans	Sorg	Peas	Ind.S	Lab.S	Mul.S	Mx.S
14	0	X5	+C5-(C$16*$G5)	+D5-(D$16*$G5)	+E5-(E$16*$G5)	+F5-(F$16*$G5)	+G5-(G$16*$G5)	+H5-(H$16*$G5)	+I5-(I$16*$G5)	+J5-(J$16*$G5)	+K5-(K$16*$G5)
15	0	X6	+C6-(C$16*$G6)	+D6-(D$16*$G6)	+E6-(E$16*$G6)	+F6-(F$16*$G6)	+G6-(G$16*$G6)	+H6-(H$16*$G6)	+I6-(I$16*$G6)	+J6-(J$16*$G6)	+K6-(K$16*$G6)
16	0	X7	+C7-(C$16*$G7)	+D7-(D$16*$G7)	+E7-(E$16*$G7)	+F7-(F$16*$G7)	+G7-(G$16*$G7)	+H7-(H$16*$G7)	+I7-(I$16*$G7)	+J7-(J$16*$G7)	+K7-(K$16*$G7)
74	Peas		+C8/$G$8	+D8/$G$8	+E8/$G$8	+F8/$G$8	+G8/$G$8	+H8/$G$8	+I8/$G$8	+J8/$G$8	+K8/$G$87

Figure 1. Worksheet Formulas to Update the LP Tableau for Iteration 1.

The value of each basic variable (X5, X6, X7, and X4) is equal to value on the right side of the equation. The new feasible solution for the respective variables X5-X7 and X4 is 4.49, 15.15, 9.26, and .51. The values of the remaining variables X1, X2, X3, and X8 are 0. The value of the objective function is 2479 pesos. This completes the first iteration.

Check the Stopping or Optimality rule. There are three negative entries in the Z row. The solution is not optimal since the value can be improved by bringing X1, X2 or X3 into the solution.

Iteration 2. X3 (Sorghum) is chosen to be the incoming basic variable since it has the largest negative Z-C value. This will be the PIVOT COLUMN. The leaving basic variable in chosen by forming a ratio column by dividing the RHS elements by the non-zero entries in column F (rows 13-16). **Move the cursor to Cell L13. Type the formula**

```
@if(f13>0,c13/f13,0).
```

Copy the formula in L13 to L14 thru L16.

The smallest non-negative entry occurs in row 13 so X5 (land slack) will be the leaving basic variable. The PIVOT NUMBER is the element in the intersection of the PIVOT COLUMN and the PIVOT ROW (1).

The new tableau (block 3) is created by the following steps.

Step 1. Copy the information in cells **A13 thru B18 to Cell A21.**

Step 2. Enter 1523 in cell A21 and Sorg in Cell B21.

Step 3. Divide each element in row 13 by the PIVOT NUMBER in cell F13. This is the NEW PIVOT ROW.

Step 4. Update the remaining technical coefficients by using the relationship New = Old - Incoming * Pivot.

Step 5. Update the rest of the Tableau (the objective function and rows 1 and 2).

Step 6. Use the /Copy command to copy the formulas for the new Z and Z-C rows from block two.

The updated tableau should look like that in block 3 of Table 1. The solution is optimal since all the entries in the Z-C row are non negative.

Meaning of the Optimal Solution

The net farm income is 9319.47 pesos per year.

The values of the real production activities or variables which give highest income are 4.49 hectares of Sorghum and .51 hectares of peanuts. The producer has 6.53 months of unused human labor and 4.05 months of unused mule labor.

The value of the marginal product VMP for an additional hectare of Land is 1523 pesos per year and the VMP or shadow price for an additional ton of marketing capacity for peanuts is 3408 pesos per year. The VMP of additional human and mule labor is zero since these resources are already in excess.

Concepts and Terms Used

Basic and non-basic variables: A linear programming problem has more variables than equations. If it has m primary variables or activities and n constraints, then when all slack variables are included, there will be n+m variables and only n equations.

The extra m variables will be set equal to zero and called non-basic variables.

The remaining n variables will be allowed to take on non-negative values and will be referred to as basic variables.

Pivot operation: A sequence of mathematical operations which reduces a system of equations to a form in which a specified variable has a unit coefficient in one equation and zeros in all other equations (a simplex iteration).

Feasible solution: A solution for which all constraints are satisfied.

Basic solution: A solution in a system of m equations which consists of exactly m of a possible m+n total variables. The remaining n non-basic or non-active variables are set equal to zero.

Basic feasible solution: A basic solution where all basic variables are also non-negative. The theory of linear programming tell us that the optimal solution (if one exists) will always be a basic feasible solution. Therefore we will examine only basic feasible solutions. Adjacent-basic feasible solutions: Two basic feasible solutions which differ from each other by only one basic variable.

Optimal solution: A basic feasible solution that has the most favorable value of the objective function.

Signs of a coefficient in the initial tableau (in general): If a process or activity 'uses' a resource, (creates a demand for a resource) the coefficient pertaining to that resource has a positive sign. If a process or activity adds to a resource (increments the supply) the coefficient pertaining to that resource has a negative sign.

Summary of the Simplex Method (for maximization)

1. Choose the Incoming and Outgoing Basic Variables.

   a. The entering basic variable is the column with the most negative coefficient in the Z-C row. This will be the PIVOT COLUMN.

   b. Choose the leaving basic variables by constructing a ratio column and choosing the row having the smallest non-negative entry in the ratio column. Construct the ratio column by dividing each RHS element by the corresponding element in the outgoing column. (Note: the smallest ratio indicates how much the chosen activity can be increased.)

2. Construct a Revised Tableau

   a. Create a new incoming row by dividing every element in the old pivot row by the pivot number. This is the entry in the intersection of the outgoing column and old pivot row.

b. Compute all other entries by the formula New=O - (I* P)
O is the element in the old tableau
I is the value in the incoming row
P is the element in the column of the incoming basic variable and the row which is being updated.

5. Revise the entries in the C column and recalculate the entries in the Z-C row.

6. Check the entries in the Z-C row, if all entries are non-negative stop, otherwise repeat steps 1-6 until all entries in the Z-C row are non-negative.

Assumptions of Linear Programming

(See page 13 of Hazell and Norton for a more complete discussion.)

Proportionality. For any value of an activity Xj (lower bound <= Xj <= upper bound) we assume:

1. The total net return from any activity is measured by Cj*Xj (the total return from any activity is given by the product of the objective function coefficient and the activity level).

2. The resources required per unit of output are constant regardless of the level of the activity. Total use of each resource by an activity is proportional to the level of the activity, aij Xj. (assumes no startup cost).

Additivity or independence. Additivity assumes there are no interactions between any of the activities. The real production function of one activity is not affected by the output of another commodity or activity.

Divisibility. Activity units can be divided into fractional parts. (In reality, some variables are meaningful only if they are integer in nature).

Certainty. This means that the parameters of the model; the coefficients of the objective function (c_j), the technical coefficients (a_{ij}), and the resource values on the RHS (b_i) are known with certainty. Various extensions of linear programming are available to deal with risk analysis and sensitivity analysis can be used to play what-if games and determine those variables which are most important to the stability of the solution.

USING THE MUSAH86 LP WITH LOTUS

An Example From Hazell & Norton

We will use the example from page 12 of Hazell & Norton (H&N) to illustrate how to enter an LP tableau using Lotus 1-2-3 for solution with the Musah86 program. Familiarity with the Musah86 format is important. Later on you will see how the Musah86 format can be translated automatically into the de facto standard MPS (Mathematical Programming System) format -- A format compatible with most mainframe, mini, or micro-computer LP packages.

What Do You Need to Know Before You Begin?

Before we begin, it is assumed that you already know how to enter numbers and labels into a Lotus spreadsheet. We also assume that you know the difference between a number and a label. Moreover, you should know how to right justify or center a label in a cell using respectively the " and the ^ prefix. You must of course know how to save and retrieve Lotus worksheet files using the /File Save, /File Xtract, and /File Retrieve commands. When working through the following, you should have H&N, this tutorial, and the data available in front of you.

Objective

Enter in the example from page 12 of H&N and obtain a solution with the Musah86 program.

Procedure

1. Start up the computer in the usual manner. With the Lotus system disk in drive A, invoke Lotus. Drive B should contain the diskette labelled "Musah86". This diskette contains the Musah86 program and will also be used for storing your work.

Building the Lotus Worksheet

You should now have a blank worksheet staring at you with the cursor at cell A1. Since we are going to store this worksheet and use it as input to an LP program later on, some conventions must be followed for proper "handshaking". Simply put, there are rules to follow. But these are very simple and intuitive rules. We will not explicate these rules for now. It will become obvious later on what these rules are.

We are to arrive at a worksheet as displayed in Figure 2. Contrast Figure 2 here with Table 2.2 in page 12 of H&N. They are really very much similar except that the "right hand side" or the constraint levels are actually put on the left. Moreover, instead of using < to express inequality, we used L to stand for less than or equal to. Zero entries in the tableau are left blank although actually putting in zeros is also acceptable. The names of the constraints are also shortened.

2. Examine Figure 2 again. Cell A1 is the name of the problem, in this case MAYALAND. This name is used to identified the problem. *Legal names are 1 to eight characters composed of the alphabets (A to Z) or the numerals (0 to 9). The first character be one of the 26 letters of the alphabet. No embedded space is allowed.* Lower case letters may be used but will be converted to upper case.

Examples of legal names are:

```
L
L1
LABOR
LABORJAN
```

Examples of illegal names are:

```
JANUARYLABOR - Name is too long
JAN&FEB      - Contain character other than (A to Z) or (1 to 9)
JUN LAB      - Embedded space
1L           - First character not an alphabet.
```

Those who are familiar with the MPS format will notice that these requirements are more restrictive than the official MPS requirements. But many commercial package

	A	B	C	D	E	F	G
1	MAYALAND	MAXIMIZE	RHS	CORN	BEANS	SORGHUM	PEANUTS
2	OBJ			1372	1219	1523	4874
3	LAND	L	5	1	1	1	1
4	LABOR	L	16.5	1.42	1.87	1.92	2.64
5	MULES	L	10	1.45	1.27	1.16	1.45
6	MARKET	L	0.5				0.983

Figure 2.

advertised to be compatible with the MPS formats impose these additional restrictions as well.

Type MAYALAND in cell A1.

2. Cell B1 can contain one of two things: MAXIMIZE or MINIMIZE. It, as you would have guessed, specifies whether the problem is a minimization problem or maximization problem.

Type MAXIMIZE in cell B1.

3. Cell C1 contains the name of the right hand side or the constraint levels. It may not be obvious why we should have to give the right hand side a name, but it is done to maintain compatibility with MPSX. We have named the right hand side B here, but again any other legal names can be used instead.

Type RHS in cell C1.

4. **Cell D1 contains** the name of the first activity, namely **CORN**. The name of the second activity should be coded in cell E1, the third in cell F1 and so on.

Type in the names of the activities.

5. **Move the cursor to cell A2 and type OBJ.** Cell A2 contains the name of the objective function. **Move the cursor to cell D2 and type 1372.** Cell D2 is the coefficient of the objective function associated with the CORN

activity. In this case the value is 1372. Cell E2 is the coefficient of the objective function associated with BEANS, and so on. Thus one way to look at row 2 is:

C = 1372 x CORN + 1279 x BEANS + 1532 x SORGHUM + 4874 x PEANUTS

Type in the remaining coefficients of the objective function.

6. The constraints begin in row 3. Begin with row 3. First we should type the name of the constraint, LAND, in cell A3.

Type the names of the remaining constraints in cells A4 through A6.

7. In cell B3, you should have the type of the constraint. There are only three allowable types, namely Less than or equal to, Greater than or equal to, or strict Equality. These are, as hinted by the underlines, denoted by L, G, and E respectively. As can be seen, all the constraints in this problem are L type constraints. (Note to MPSX fans: N type constraints are not allowed here).

Type in the constraint types in cells B3 through B6.

8. Type in the nonzero input-output coefficients or A_{ij}'s for the model. Cell C3 contains the constraint level for LAND, 5 in this case. Cell D3, E3, F3, and G3 contains respectively the A_{ij} values for CORN, BEANS, SORGHUM and PEANUTS associated with the LAND constraint. Rows 4, 5, and 6 represent entries for the constraint LABOR, MULES, and MARKET respectively. They are constructed similar to row 3. Enter in the constraint levels.

Complete the tableau by filling in the A_{ij}'s in cells D3 through G6.

Double check all the entries, if everything is OK, you are then ready to save the tableau.

Saving the Tableau

9. Usually, saving a Lotus 1-2-3 worksheet is done by invoking the /File Save command. However, we will instead use the /File Xtract command (Xtract stands for "extract"). The reason is that the save command can sometimes save areas of the worksheet beyond the area of the tableau. This can confuse Musah86. Using Xtract, we can control precisely the area of the worksheet we intend to save.

Type /FXV for the commands /File Xtract Values. You will then be asked the name of the file to extract by the prompt **Enter Xtract File Name:**. This is the name of the file wherein the extracted information will be saved. **Type ML** (for Mayaland). If there is a file already named ML on the diskette, you will be asked whether you would like to replace the original content of the file ML with the new information, or to cancel the command, in which case, nothing is saved and the original content of ML would remain intact. Usually, you would want to choose Replace.

You are then asked to choose the range to extract. We want this range to be precisely the area of the tableau, *no more and no less*. In this case the range is A1.G6. **You can either specify the range by actually entering A1.G6** or use cursor movements to cover the appropriate area and then press enter.

Drive B should now spin as the saving operation takes place. After the saving is done, quit Lotus. We are now ready to use Musah86 to solve the LP problem.

Using Musah86 to Solve the LP

10. After exiting Lotus, control is returned to DOS. Now remember that both the Musah86 program and the ML worksheet is in drive B. We will leave the A drive as the default drive and invoke Musah86 by entering

B:MUSAH86 at the A> prompt.

11. As Musah86 comes up, you will first be asked for the name of the input file. In this case the name of the input file is ML. But since the default drive is A, if we just specify

ML, Musah86 will looked for it in drive A which of course is not there since drive A still contains the Lotus system disk. Thus we should explicitly **specify B:ML** to tell Musah86 to look for ML at drive B.

After the name of the input file is specified, Musah86 begins to read in and decode the information buried in the Lotus 1-2-3 worksheet ML. When the input file is completely read in and decoded, Musah86 then prompts for the name of an output file. The output from Musah86 is another Lotus worksheet file. The contents of this file include the initial simplex tableau and the final simplex tableau. The value of the objective function, shadow prices and other relevant information are also included. In short, Musah86 does not generate any solution information directly but rather creates a Lotus worksheet file. The detail solution is buried in that worksheet file. You must then invoke Lotus later on to inspect that file.

13. **Respond to the prompt for output file with the name B:MLX.** Musah86 then writes the initial tableau to MLX in drive B and begins the solution process. Since this is a very small problem, the solution should finish almost instantaneously. When the solution is finished, **Musah86 asks whether you would like to store the solution.** You should normally **reply Y (for Yes)**. After the saving operation is done, Musah86 asks whether solution of another LP problem is desired. **Reply N for No.** Musah86 then exits and control is returned to DOS.

Viewing Answers with Lotus

14. Musah86 generates very little useful information about the solution on the screen nor does Musah86 generate any printed output. Instead, the detailed information is stored in the file we called MLX. MLX is a Lotus worksheet file that can be only be viewed or printed from within Lotus. **To look at MLX, invoke Lotus and retrieve the worksheet MLX.** The worksheet should look like Figure 3.

Refer to Figure 3. The first part of the contents of MLX, the area A1.G6, is merely a duplication of the input tableau, e.g. the contents of ML. The area immediately below contains the solution and the final tableau.

Compare Figure 3 with the final tableau in Table 2.4 on page 24 of H&N. The optimal value of the objective function, 9319.476, is contained in cell D14. The worksheet area A15.H27 contains the final tableau.

	A	B	C	D	E	F	G
1	MAYALAND	MAXIMIZE	RHS	CORN	BEANS	SORGHUM	PEANUTS
2	OBJ			1372	1219	1523	4874
3	LAND	L	5	1	1	1	1
4	LABOR	L	16.5	1.42	1.87	1.92	2.64
5	MULES	L	10	1.45	1.27	1.16	1.45
6	MARKET	L	0.5				0.983
7							
8							
9							
10							
11							
12		S o l u t i o n					
13		OPTIMAL					
14		function Value:	9319.476				
15							
16					CORN	BEANS	LAND	MARKET
17					1372	1219	0	0
18	Returns	Name	Type	Level	real	real	slack	slack
19								
20	1523	SORGHUM	real	4.491353	1	1	1	-1.01729
21	4874	PEANUTS	real	0.508646				1.017293
22	0	LABOR	slack	6.533774	-0.5	-0.05	-1.92	-0.73245
23	0	MULES	slack	4.052492	0.29	0.11	-1.16	-0.29501
24								
25	Z				1523	1523	1523	3408.952
26	Shadow Price				151	304	1523	3408.952
27								
28								
29								
30								
31								

Figure 3.

The final tableau presented in MLX is similar to the one presented in H&N except that the basis columns are not included. Row 17, from E17 to H17, represents the Cj row. Row 18 from F18 to H18 indicates whether the corresponding activity is a real activity or a slack activity. A real activity is one which started out as a column in the initial tableau. A slack activity is one which started out as a row in the initial tableau.

The range from B20 to B23 contains the names of the activities or resources which are in the solution. The corresponding entries in column D are the activity levels. In our solution, 4.49 ha of SORGHUM is produced with each ha produced yielding a gross margin of 1523 (content of A20). Similarly, about 0.5 hectare of PEANUTS is produced. There are 6.53 of the LABOR resource left over or remained idle. Likewise, 4.05 units of MULES are also not used in the production process.

Activities not in the solution are CORN, BEANS, LAND, and MARKET. CORN and BEANS are production activities. Their absences from the solution imply that no CORN and BEANS are being produced. The absence of LAND, a resource activity, from the solution means that no LAND is left over: all 5 acres of LAND have been used. The MARKET constraint is also binding since it is not in the solution. The shadow price in cell H26 indicates that it the farmer were allowed to market one addition ton of peanuts over the 0.5 ton already allowed, he should increase his total gross margins by 3409 pesos (at the margin).

Printing the Answers

15. Since Musah86 does not generate a hardcopy of the solution, a hardcopy must be obtained from within Lotus. We will go through the process of obtaining a printout similar to Figure 2 (but without the row and column designations.)

Invoke the Lotus Print command by typing **/P P**. Then get into the print option menu by press O for Option. The print option menu allows options to be specified for controlling the appearance the printout.

Notice that the printing in Figure 2 is done in compressed print. Compressed printing allows more characters on a line and, in this case, allows the printing of the entire tableau on one page.

Two things you must do to print compressed characters. First, you need to tell Lotus that you are expanding your right margin from the usual 76 to the wider 132. This is done by pressing M for Margin at the print option menu. Set the Left margin to 4, Right margin to 132, both Top and Bottom margins at 4. This tells Lotus to leave 4 blank spaces before printing a line (left margin), and to leave 4 blank lines both at the top and at the bottom of the page (Top and Bottom Margins). The Right margin setting of 132 informs Lotus to use the full width of the page, or alternatively stated, not to leave any right margin. With compressed print, a maximum of 132 characters can be printed on a line (the regular print allows 80).

After specifying all four margin settings as above. Press the Escape key to return to the print option menu.

Specifying a wider margin does not in itself switch the printer into compressed computer mode. To tell the printer to print in compressed mode, a 'setup string' must be send to inform the printer hardware. To do so, choose Setup from the print option menu and **enter in \015** (note that \ is a back-slash and not the ordinary slash /).

Now we have specified our option and are ready to print. First of all, check to see if the printer is turned on and connected properly to your computer. Then line up the paper in the printer to the top of the page. **Press Q (for Quit)** from the print option menu.

We must now tell Lotus which part of the worksheet we would like to print. To do so, choose Range. When asked for the range, **specify A1.H27 (i.e. the whole thing)**. After the range is specified, **press A (for Aligned)** to tell Lotus that the paper in the printer is lined up to the top. Then **press G (for Go)**.

Exercises

1. Bring up the tableau ML with Lotus. Change the coefficient of the objective function for CORN from 1372 to 1523. Save this tableau under the name ML1. Solve ML1 with Musah86 and name the output file ML1X. Make a printout similar to Figure 2 for ML1X. The farmer is now indifferent between growing CORN and growing SORGHUM. Why?
2. Bring up the tableau ML with Lotus. Change the coefficient of the objective function for CORN from 1372 to 1523.1. Save this tableau under the name ML2. Solve ML2 with Musah86 and name the output file ML2X. Printout ML2X with Lotus. Comment on the change in the farmer's product mix.
3. Bring up the tableau ML with Lotus. Change the LAND, LABOR and MULES requirements for the BEANS activity to zero. Save this tableau under the name ML3. Solve ML2 with Musah86 and name the output file ML3X. Printout ML3X with Lotus. Explain why the solution is now unbounded.

DUALITY IN LINEAR PROGRAMMING

Reference: Chapter 2, "Mathematical Programming and the Farm Model" in *Mathmatical Programming for Economic Analysis in Agriculture.*

Every (Primal) linear programming problem has an associated or Dual programming problem. The solution to the primal problem gives the solution to the dual problem and/or the solution to the dual gives a solution to the primal.

The following relationships hold between the original (Primal) problem and the Dual.

1. The objective function coefficients of the primal become the right hand side (RHS) constants of the Dual. The RHS constants of the Primal become the objective function coefficients of the Dual.
2. The inequalities are reversed between the Primal and Dual.
3. If the objective of the primal is to *maximize*, the objective of the Dual is to *minimize*.
4. Each column of the primal corresponds to a constraint in the Dual. The number of dual constraints = number primal variables.
5. Each constraint in the primal becomes a column in the dual. The number of variables in the dual is equal to the number of constraints in the Primal.
6. The Dual of the Dual is the Primal.

The points listed above imply there is a symmetric relationship between the primal and the dual LP problem. If the Primal problem is to maximize subject to less than or equal to constraints, the Dual is to minimize subject to greater than or equal to constraints.

Properties of Dual Linear Programs

Assume you have a feasible solution to the dual problem and also another solution which is feasible for the primal problem. Then the value of the dual objective function must be greater than or equal to the value of the primal objective function.

Call the solution to the dual y^0. y^0 must be non-negative. Call the solution to the Primal X^0. X^0 must be non-negative.

The primal problem is

$$\max\ c'x$$
$$\text{subject to } Ax \le b$$
$$x \ge 0$$

the dual problem is

$$\min\ y'b$$
$$\text{subject to } A'y \ge c$$
$$y \ge 0$$

where

A is m x n, b is m x 1, c is n x 1, y is m x 1, x is n x 1.

It can be shown the value of the dual objective function ($y^{0'}$ b) is greater than the value of the Primal objective function (c' x^0). We want to show that

$$y^{0'} b \ \ge c' x^0.$$

Since X^0 is feasible, we know that

$$Ax^0 \le b \text{ and } x^0 \ge 0. \qquad (1)$$

Since y^0 is feasible for the dual, we know that

$$A' y^0 \ge c \text{ and } y^0 \ge 0. \qquad (2)$$

If both sides of an inequality are multiplied by a non-negative number, the inequality is unchanged. Therefore multiply relation 1 by $y^{0'}$ to obtain,

$$y^{0'} A x^0 \le y^{0'} b.$$

Therefore multiply relation 2 by x^0 to obtain,

$$y^{0'} A x^0 \ge c^{0'} x.$$

This implies that

$$c'x \le y^{0'}Ax \le y^{0'} b$$

Implications

1. The value primal (maximization) objective function is a lower bound on the dual objective funct.

2. The dual objective function of a minimization problem with a feasible solution is an upper bound to the value of a primal max problem.

3. If the primal is feasible but its objective function is unbounded, then there is no feasible solution to the dual.

4. If the dual has a feasible solution but unbounded objective function, then the primal has no feasible solution.

Optimal Criterion

If you have a feasible solution for both the primal and the dual ($x^0 y^0$) and the value of the primal and dual objective functions are equal, ic $c'x^0 = y^0'b$ then x^0 and y^0 are optimal solutions.

x^0 must be optimal because for any other solution $\overline{x}$ we must have $c'\overline{x} \leq y^0 b = c'x^0$, hence $c'\overline{x} \leq c'x^0$.

Main theorem. If the primal and the dual problems have feasible solutions then

a. both have optimal solutions

b. the optimal value of the dual is equal to the optimal value of the primal.

Primal and Dual Forms of the Mayaland Problem

The Mayaland problem is used to illustrate the principles discussed above. The initial problem is formed as a maximization. The LP tableau and the resulting solution are presented in Table 1. The Solution values of the primal and dual variables are highlighted.

The dual of the Mayaland problem is formed by transposing the coefficient matrix, interchanging the objective function and the right hand side, changing the less than or equal constraints to greater than or equal to, and changing from maximization to minimization. The LP tableau and solution output are shown in Table 2. The solution values of the primal and dual variables are highlighted. Notice that the values of the variables which were "in the solution" for the primal are found in the Z-C row of the dual.

Table 1. The Tableau and Solution of the Primal Problem of the Mayaland Example

MAYALAN	MAX	OBJ	CORN	BEANS	SORGHUM	PEANUTS
Obj			1372	1219	1523	4874
LAND	L	5	1	1	1	1
LABOR	L	16.5	1.42	1.87	1.92	2.64
MULES	L	10	1.45	1.27	1.16	1.45
MARKET	L	0.5				0.983

OPTIMAL Solution
function Value: 9319.4

Returns	Name	Type	Level	CORN 1372 real	BEANS 1219 real	LAND 0 slack	MARKET 0 slack
1523	SORGHUM	real	4.4913	1	1	1	-1.017
4874	PEANUTS	real	0.5086				1.0172
0	LABOR	slack	6.5337	-0.5	-0.05	-1.92	-0.732
0	MULES	slack	4.0524	0.29	0.11	-1.16	-0.295
Z				1523	1523	1523	3408.9
Shadow Price				151	304	1523	3408.9

Table 2. The Tableau and Solution of the Mayaland Dual Problem

MAYALAN	MIN	RHS	LAND	LABOR	MULES	MARKET
OBJ			5	16.5	10	0.5
CORN	G	1372	1	1.42	1.45	
BEANS	G	1219	1	1.87	1.27	
SORGHUM	G	1523	1	1.92	1.16	
PEANUTS	G	4874	1	2.64	1.45	0.983

OPTIMAL Solution
function Value: 9319.4

Cost	Name	Type	Level	LABOR 16.5 real	MULES 10 real	SORGHUM 0 slack	PEANUTS 0 slack
5	LAND	real	1523	1.92	1.16	-1	
0.5	MARKET	real	3408.9	0.7324	0.2950	1.0172	-1.017
0	CORN	slack	151	0.5	-0.29	-1	
0	BEANS	slack	304	0.05	-0.11	-1	
Z				9.9662	5.9475	-4.491	-0.508
Reduced Cost				-6.533	-4.052	-4.491	-0.508

EXERCISES TO EXPAND THE FARM MODEL

The following are Musah86 solutions of the Mayland model in Hazell and Norton. Variations as discussed in the text are also incorporated into the model and solved. Enter in these model and obtain solutions with Musah86. Try to understand why solutions change from one variation to another.

I. Mayaland Example

MAYALAND	MAXIMIZE	RHS	CORN	BEANS	SORGHUM	PEANUTS
OBJ			1372	1219	1523	4874
LAND	L	5	1	1	1	1
LABOR	L	16.5	1.42	1.87	1.92	2.64
MULES	L	10	1.45	1.27	1.16	1.45
MARKET	L	0.5				0.983

S o l u t i o n
OPTIMAL
function Value: 9319.476

Returns	Name	Type	Level	CORN 1372 real	BEANS 1219 real	LAND 0 slack	MARKET 0 slack
1523	SORGHUM	real	4.491353	1	1	1	-1.01729
4874	PEANUTS	real	0.508646				1.017293
0	LABOR	slack	6.533774	-0.5	-0.05	-1.92	-0.73245
0	MULES	slack	4.052492	0.29	0.11	-1.16	-0.29501
Z				1523	1523	1523	3408.952
Shadow Price				151	304	1523	3408.952

II. Mayaland Example with Labor

Reduced to 8 units

MAYALAND MAXIMIZE		RHS	CORN	BEANS	SORGHUM	PEANUTS
OBJ			1372	1219	1523	4874
LAND	L	5	1	1	1	1
LABOR	L	8	1.42	1.87	1.92	2.64
MULES	L	10	1.45	1.27	1.16	1.45
MARKET	L	0.5				0.983

S o l u t i o n
OPTIMAL
function Value: 8725.675

Returns	Name	Type	Level	BEANS 1219 real	LAND 0 slack	LABOR 0 slack	MARKET 0 slack
1372	CORN	real	3.932451	0.1	3.84	-2	1.464903
1523	SORGHUM	real	0.558901	0.9	-2.84	2	-2.48219
4874	PEANUTS	real	0.508646				1.017293
0	MULES	slack	2.912081	0.081	-2.2736	0.58	-0.71983
Z				1507.9	943.16	302	3187.751
Shadow Price				288.9	943.16	302	3187.751

III. Factor Substitution

MAYALAND MAXIMIZE		RHS	CORN MAN	CORN	BEANS	SORGHUM	PEANUTS
OBJ			1372	1372	1219	1523	4874
LAND	L	5	1	1	1	1	1
LABOR	L	8	1.42	1	1.87	1.92	2.64
MULES	L	10	1.45	2	1.27	1.16	1.45
MARKET	L	0.5					0.983

S o l u t i o n
OPTIMAL
function Value: 8996.758

Returns	Name	Type	Level	CORN-MAN 1372 real	BEANS 1219 real	LAND 0 slack	LABOR 0 slack	MARKET 0 slack
1372	CORN	real	2.137201	0.543478	0.054347	2.086956	-1.08695	0.796143
1523	SORGHUM	real	2.354151	0.456521	0.945652	-1.08695	1.086956	-1.81343
4874	PEANUTS	real	0.508646					1.017293
0	MULES	slack	2.257242	-0.16652	0.064347	-2.91304	0.913043	-0.96377
Z				1440.934	1514.793	1207.869	164.1304	3288.734
Shadow Price				68.93478	295.7934	1207.869	164.1304	3288.734

IV. Quality Differences in Resources

MAYALAND	MAXIMIZE	RHS	CORN	CORNIRR	BEANS	SORGHUM	PEANUTS
OBJ			1372	1509.2	1219	1523	4874
DRYLAND	L	4	1		1	1	1
IRRLAND	L	1		1			
LABOR	L	8	1.42	1.562	1.87	1.92	2.64
MULES	L	10	1.45	1.595	1.27	1.16	1.45
MARKET	L	0.5					0.983

S o l u t i o n
OPTIMAL
function Value: 8819.991

Returns	Name	Type	Level	BEANS 1219 real	DRYLAND 0 slack	IRRLAND 0 slack	LABOR 0 slack	MARKET 0 slack
1372	CORN	real	3.216451	0.1	3.84	3.124	-2	1.464903
1509.2	CORNIRR	real	1			1		
1523	SORGHUM	real	0.274901	0.9	-2.84	-3.124	2	-2.48219
4874	PEANUTS	real	0.508646					1.017293
0	MULES	slack	2.684721	0.081	-2.2736	-2.50096	0.58	-0.71983
Z				1507.9	943.16	1037.476	302	3187.751
Shadow Price				288.9	943.16	1037.476	302	3187.751

V. Quality Differences in Resources with Transfer

MAYALAND	MAXIMIZE	RHS	CORN	BEANS	SORGHUM	PEANUTS	LABTRAN
OBJ			1372	1219	1523	4874	0
LAND	L	5	1	1	1	1	
FEMLABO	L	1	0.3	0.2			1
MIXLABO	L	7	1.12	1.67	1.92	2.64	-1
MULES	L	10	1.45	1.27	1.16	1.45	
MARKET	L	0.5				0.983	

S o l u t i o n
OPTIMAL
function Value: 8578.523

Returns	Name	Type	Level	BEANS 1219 real	LABTRAN 0 real	FEMLABO 0 slack	MIXLABO 0 slack	MARKET 0 slack
1372	CORN	real	3.333333	0.666666	3.333333	3.333333		
1523	SORGHUM	real	1.001999	0.480902	-2.46527	-1.94444	0.520833	-1.39877
4874	PEANUTS	real	0.508646					1.017293
0	LAND	slack	0.156020	-0.14756	-0.86805	-1.38888	-0.52083	0.381485
0	MULES	slack	3.266809	-0.25451	-1.97361	-2.57777	-0.60416	0.147507
Z				1647.081	818.7152	1611.944	793.2291	2827.950
Shadow Price				428.0815	818.7152	1611.944	793.2291	2827.950

VI. Seasonal Labor Requirements

MAYALAND	MAXIMIZE	RHS	CORN	BEANS	SORGHUM	PEANUTS
OBJ			1372	1219	1523	4874
LAND	L	5	1	1	1	1
LABOR1	L	0.666666		0.36	0.16	0.22
LABOR2	L	0.666666		0.34		0.32
LABOR3	L	0.666666	0.27	0.37		0.22
LABOR4	L	0.666666	0.36		0.06	0.27
LABOR5	L	0.666666	0.36			0.3
LABOR6	L	0.666666	0.09		0.36	0.08
LABOR7	L	0.666666		0.36	0.32	
LABOR8	L	0.666666	0.08	0.34		0.2
LABOR9	L	0.666666	0.26		0.32	
LABOR10	L	0.666666		0.1	0.09	0.45
LABOR11	L	0.666666			0.34	0.33
LABOR12	L	0.666666			0.27	0.25
MULES	L	10	1.45	1.27	1.16	1.45
MARKET	L	0.5				0.983

S o l u t i o n
OPTIMAL
function Value: 6700.155

Returns	Name	Type	Level	LABOR3 0 slack	LABOR7 0 slack	LABOR9 0 slack	MARKET 0 slack
1372	CORN	real	1.032653	1.861427	-1.91313	1.913133	-0.41659
1219	BEANS	real	0.745805	1.344364	1.396070	-1.39607	-0.30087
1523	SORGHUM	real	1.244302	-1.51240	1.554420	1.570579	0.338484
4874	PEANUTS	real	0.508646				1.017293
0	LAND	slack	1.468591	-1.69338	-1.03735	-2.08764	-0.63830
0	LABOR1	slack	0.087186	-0.24198	-0.75129	0.251292	-0.16964
0	LABOR2	slack	0.250325	-0.45708	-0.47466	0.474663	-0.22323
0	LABOR4	slack	0.082918	-0.57936	0.595462	-0.78296	-0.14500
0	LABOR5	slack	0.142317	-0.67011	0.688728	-0.68872	-0.15521
0	LABOR6	slack	0.085087	0.376938	-0.38740	-0.73759	-0.16574
0	LABOR8	slack	0.228751	-0.60599	-0.32161	0.321613	-0.06783
0	LABOR10	slack	0.251207	0.001680	-0.27950	-0.00174	-0.45815
0	LABOR11	slack	0.075750	0.514219	-0.52850	-0.53399	-0.45079
0	LABOR12	slack	0.203543	0.408350	-0.41969	-0.42405	-0.34571
0	MULES	slack	5.374551	-2.65201	-0.80209	-2.82290	-0.88154
Z				1889.258	1444.373	3315.001	4535.466
Shadow Price				1889.258	1444.373	3315.001	4535.466

VII. Buying Options (a)

MAYALAND MAXIMIZE		RHS	CORN-MAN	CORN	BEANS	SORGHUM	PEANUTS	REN_LAND	HIRE_LAB
OBJ			1372	1372	1219	1523	4874	-500	-50
LAND	L	5	1	1	1	1	1	-1	
LABOR	L	8	1	1.42	1.87	1.92	2.64		-1
MULES	L	10	2	1.45	1.27	1.16	1.45		
MARKET	L	0.5					0.983		

S o l u t i o n

OPTIMAL

function Value: 12459.66

Returns	Name	Type	Level	CORN-MAN 1372 real	CORN 1372 real	BEANS 1219 real	LAND 0 slack	LABOR 0 slack	MULES 0 slack	MARKET 0 slack
1523	SORGHUM	real	7.984880	1.724137	1.25	1.094827			0.862068	-1.27161
4874	PEANUTS	real	0.508646							1.017293
-500	REN_LAND	real	3.493527	0.724137	0.25	0.094827	-1		0.862068	-0.25432
-50	HIRE_LAB	real	8.673799	2.310344	0.98	0.232068		-1	1.655172	0.244150
Z				2148.275	1729.75	1608.405	500	50	799.1379	3136.571
Shadow Price				776.2758	357.75	389.4051	500	50	799.1379	3136.571

VIII. Buying Options (b)

MAYALAND MAXIMIZE		RHS	CORN	BEANS	SORGHUM	PEANUTS	BUY_UREA	BUY_COMP	BUY_MURI
OBJ			1700	1219	1523	4874	-0.6	-0.7	-0.5
LAND	L	5	1	1	1	1			
LABOR	L	16.5	1.42	1.87	1.92	2.64			
MULES	L	10	1.45	1.27	1.16	1.45			
MARKET	L	0.5				0.983			
NITROGEN	L	0	25				-0.46	-0.16	
POTASH	L	0	10					-0.1	-0.3

S o l u t i o n
OPTIMAL
function Value: 9893.132

Returns	Name	Type	Level	BEANS 1219 real	SORGHUM 1523 real	BUY_COMP -0.7 real	LAND 0 slack	MARKET 0 slack	NITROGEN 0 slack	POTASH 0 slack
1700	CORN	real	4.491353	1	1		1	-1.01729		
4874	PEANUTS	real	0.508646					1.017293		
-0.6	BUY_UREA	real	244.0952	54.34782	54.34782	0.347826	54.34782	-55.2877	-2.17391	
-0.5	BUY_MURI	real	149.7117	33.33333	33.33333	0.333333	33.33333	-33.9097		-3.33333
0	LABOR	slack	8.779450	0.45	0.5		-1.42	-1.24109		
0	MULES	slack	2.75	-0.18	-0.29		-1.45	-5.9E-12		
Z				1650.724	1650.724	-0.37536	1650.724	3279.018	1.304347	1.666666
Shadow Price				431.7246	127.7246	0.324637	1650.724	3279.018	1.304347	1.666666

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IX. Corn-Bean Intercropping

MAYALAND	MAXIMIZE	RHS	CORN	CORNBEAN	BEANS	SORGHUM	PEANUTS
OBJ			1372	1739	1219	1523	4874
LAND	L	5	1	1	1	1	1
LABOR	L	10	1.42	2	1.87	1.92	2.64
MULES	L	10	1.45	1.5	1.27	1.16	1.45
MARKET	L	0.5					0.983

S o l u t i o n
OPTIMAL
function Value: 10083.62

Returns	Name	Type	Level	BEANS 1219 real	SORGHUM 1523 real	LAND 0 slack	LABOR 0 slack	MARKET 0 slack
1372	CORN	real	0.561265	0.224137	0.137931	3.448275	-1.72413	1.122531
1739	CORNBEAN	real	3.930087	0.775862	0.862068	-2.44827	1.724137	-2.13982
4874	PEANUTS	real	0.508646					1.017293
0	MULES	slack	2.553495	-0.21879	-0.33310	-1.32758	-0.08620	0.106991
Z				1656.741	1688.379	473.4827	632.7586	2777.247
Shadow Price				437.7413	165.3793	473.4827	632.7586	2777.247

RESOURCE CONSTRAINTS BY QUALITY AND TIME OF USE

References: Hazell and Norton, *Mathematical Programming for Economic Analysis in Agriculture*, Chapter 3.

Beneke and Winterboer, *Linear Programming Applications to Agriculture*, Chapter 3.

Objective

The objective of this section is to demonstrate techniqes for constructing linear programming models where resources vary by quality and by time of use.

Variation in Resource Quality

It is well known that agricultural resources vary greatly within a relative small area and even within the same farm. Land classification schemes rate classes of land according to suitability for various kind of crops by considering such factors such as slope, depth of topsoil, natural fertility, drainage, and erosion potential. The suitability of the labor force may vary by age, skill or sex.

In general, when there are significant differences in resource quality, the technique is to treat resources of different qualities as distinctly different resources. Thus a farmer with two different grades of land may be modeled as having two or more land resources. Each resource class would have its own set of production activities. The case of multiple grades of land is covered by Hazell and Norton and by Beneke and Winterboer.

Resource Variation by Time Period

There are many circumstances in which it is necessary to distinguish differences in resource use by time period. Producers are well aware of the need to complete crop seeding operations, irrigation applications, weed control, and insecticide applications within a very narrow time limits. In these cases labor in June is a very different resource than labor in December and it is not realistic to treat one as a substitute for the other. The

general remedy is to provide seasonal, monthly or even bi-weekly labor constraints in the model.

In countries where multiple cropping is possible, the producer may produce more than one crop per year on the same land. In such cases in is possible to define land use by season or on a monthly basis. The following example provides an illustration of the use land by two different qualities and by time periods within each quality.

An Example with Seasonal and Quality Differences in Land

The example here deals with setting up a farm model where not only are there different classes of land but also where multiple cropping on the same land is possible. The data are drawn from a farm in northern Thailand but would be applicable in many of the more temperate climates of the world.

The farmer has 20 rai (one rai is about 1/6 of a hectare) of land. He has 8 rai which which is irrigated lowland and 10 rai of croppable upland.

The following crop choices are possible on the irrigated lowland.

Crop	Growing period	yield kg/rai	labor hrs/rai	capital b/rai	net return/rai
rice, native	June-December	336	104	34	272
rice, new var.	July-December	350	110	50	300
rice, new var.	Jan-May	360	115	60	310

The crops which may be grown on the upland area are:

corn	June-Sept	280	102	33	263
mungbean	Sept-Dec	75	43	41	170
mungbean	Nov-Feb	87	52	43	185
soybean	June-Sept	120	68	49	218
soybean	Oct-Jan	115	65	45	207
groundnut	Aug-Nov	357	183	100	400

Set up a linear programming model for crop selection. Assume the wet-season labor supply (June-Dec) is 1800 hours and that the dry-season labor supply is 1600 hours. Assume the supply of production capital for the entire year is 1500 baht.

Structure of the Thai Multiple Cropping Model

A listing of the tableau for the Thai multiple cropping model is shown in the upper portion of Table 1. Two levels of season detail were used with the land constraints. Seasonal constraints were used with the lowland rice area while monthly constraints were used in the case of the upland area.

Table 1. Listing of Tableau and Solution for Thai Multiple Cropping Example

Thai	MAX	RHS	RicNat	RicNV	RicNV2	Corn	MbSD	MbNF	SyJS	SyOJa	GnAN	CapTWD
obj			272	300	310	263	170	185	218	207	400	0
LowJnD	L	8	1	1								
LowJaM	L	8			1							
UpJan	L	10						1		1		
UpFeb	L	10						1				
UpMar	L	10										
UpApr	L	10										
UpMay	L	10										
UpJun	L	10				1			1			
UpJly	L	10				1			1			
UpAug	L	10				1			1		1	
UpSep	L	10				1	1		1		1	
Upoct	L	10					1			1	1	
UpNov	L	10					1	1		1	1	
UpDec	L	10					1	1		1		
Labws	L	1800	104	110		102	43	26	68	53	400	
Labds	L	1600			115			26		12		
CapWs	L	1500	34	50		33	41	22	49	36	100	1
CapDs	L	0			60			21		9		-1

OPTIMAL S o l u t i o n

function Value: 8472.0 baht

Activities in Solution

Obj.Va	Name	Type	Level
272	RicNat	real	8.0
310	RicNV2	real	8.0
263	Corn	real	4.7
170	MbSD	real	0.0
185	MbNF	real	10.0
218	SyJS	real	3.3
0	CapTWD	real	690.0
0	UpFeb	slack	0.0
0	UpMar	slack	10.0
0	UpApr	slack	10.0
0	UpMay	slack	10.0
0	UpJun	slack	2.0
0	UpJly	slack	2.0
0	UpAug	slack	2.0
0	UpSep	slack	2.0
0	Upoct	slack	10.0
0	UpDec	slack	0.0
0	Labds	slack	420.0

Activities Not in Solution

Name	Obj.Va	Type	Shad.Price
RicNV	300	real	309.69
SyOJa	207	real	243.97
GnAN	400	real	1001.22
LowJnDc	0	slack	3.29
LowJaMy	0	slack	215.19
UpJan	0	slack	46.98
UpNov	0	slack	16.32
Labws	0	slack	2.07
CapWs	0	slack	1.58
CapDs	0	slack	1.'8

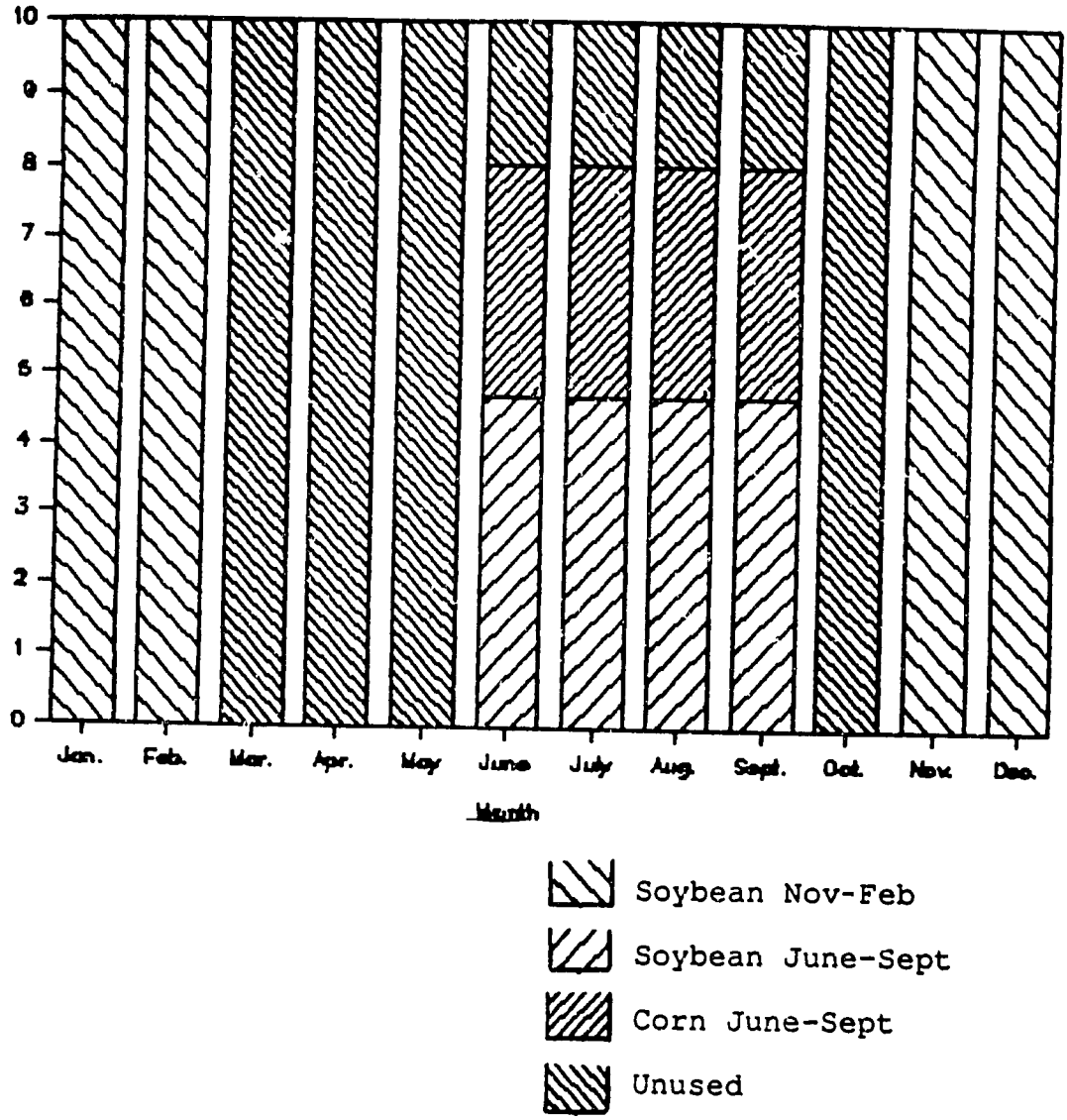

Figure 1. Monthly Use of Upland Crop Area for Thai Multiple Crop Example.

In the case of the lowland area there were fewer crop choices available than for the upland area. In the wet season, if the native variety was planted, the new variety was precluded and vice versa. Hence a single wet season constraint was sufficient. There was only one crop choice in the dry season for the lowland area. A single dry season constraint was adequate.

The upland area contains several overlapping possibilities. Here monthly constraints were defined. The tableau actually contains three empty rows (March, April, and May) which could be eliminated in the current model. The land use constraints for June and July were identical and one of these could be eliminated. Even if some of the empty rows were eliminated the analyst may still find evidences of degeneracy such as activities at a zero level in the final solution .

Results Thai Multiple Cropping Model

The results in the lower part of Table 1 indicate the lowland rice choices were native rice in the wet season followed by an improved variety in the dry season. Note the native variety had slightly lower returns on a per rai basis than the improved variety. However both wet season labor and capital were limiting and the native variety used less of the scarce inputs per baht of net return. The upland crops selected were corn mungbeans (Sept-Dec), soybeans (July-Sept), and soybeans (Oct.-Jan.). Labor and capital limitations during the wet season prevent all of the upland crop from being planted.

It may be necessary to specify the seasonal constraints on labor and capital on a monthly basis if the model were to be used for policy analysis at the farm level.

Lotus graphics were used to display the results. The cropping pattern shown in Figure 1. Table 2 was generated by multiplying the land area of each crop by the monthly cropping coefficients. The stacked bar chart option was used.

WORKING WITH INTERDEPENDENCE BETWEEN ACTIVITIES: COMPOSITE ACTIVITIES (Rotations and Intercropping)

References: Hazell and Norton, *Mathematical Programming for Economic Analysis in Agriculture*, Chapter 3.

Beneke and Winterboer, *Linear Programming Applications to Agriculture*, Chapter 3.

Objective

The objectives of this section are to illustrate a technique for modeling situations where there is interdependence among activities within the model. That is when the input-output coefficients for one activity would be affected by the level on another activity in the solution. The examples which follow deal with crop rotations and with intercropping.

Independence between activities. One assumption of L.P. is that the coefficients of one activity are independent of the level of any other activity. However, legumes are often included in rotations with the direct purpose of increasing the yield of the following crop by increasing the amount of nitrogen in the soil. That is the yield of a crop such as corn or sorghum with the same amount of applied nutrients is expected to be higher if it follows a legume than if it follows itself. But in an L.P. the yield of a sorghum activity must be independent of the level of any legume activities.

A way around the problem is to define a composite activity. Assume our objective is to find the most profitable rotation. Following Beneke, one rotation (CCOM) might be a four year sequence consisting of two years of corn, a year of oats and a year of a meadow crop.

Define a 4 acre activity:

Activity	Acre	Yield	Variable Cost
First Year Corn	1	100 bus	$180
Second Year Corn	1	90 bus	180
Oats	1	60 bus	100
Meadow/legume	1	2 tons	75
total	4		$535
Continuous Corn	1	85	180

The resulting LP model might appear as:

	RHS	CCom	CCorn	CornSel	OatSel	HaySel
Obj	0	-535	-180	2.40	1.80	40
Land	160>=	4	1			
Corntra	0>=	-190	-85	1		
Oatstra	0>=	-60			1	
Haytra	0>=	-2				1

OPTIMAL S o l u t i o n
function Value 4360

Returns	Name	Type	Level	CCorn -180 real	Land 0 slack	corntra 0 slack	Oatstra 0 slack	Haytra 0 slack
-535	CCom	real	40	0.25	0.25			
2.4	CornSel	real	7600	-37.5	47.5	1		
1.8	OatSel	real	2400	15	15		1	
40	HaySel	real	80	0.5	0.5			1
Z				-176.7	27.25	2.4	1.8	40
Shadow Price				3.25	27.25	2.4	1.8	40

AN INTERCROPPING EXAMPLE

(Data from Niger) The farm has 6 hectares. The household contains 6 people. The objective is to maximize earnings with the stipulation that enough sorghum or millet is produced to provide 250 kg of grain per member of the household. There are 3 workers who can each work 23 days per month during the 3 month cropping season. The family has 100 CFM in cash to meet production expenses. There are no off farm wage opportunities. The crop budgets are given in Table 3.

Table 3. Budget Information for the Niger Farm Example

item	units/ha	CFM/unit	Amount
	Sorghum		
Yield	434 kg	65.	28210
Seed	16 kg	65.	1040.
Fungicide	1.	15.	15.
Labor	51 days	500.	25,500.
	Groundnuts (Peanuts)		
Yield	585 kg	90	52,650
Seed	60 kg	90	5,400
Fungicide	1.	15	15
Labor	61. days	500	30,500
	Millet - Cow pea intercrop		
Yield(millet)	385 kg	70	26,950
Yield(cowpeas)	322 kg	90	28,980
Seed(millet)	8 kg	70	560
Seed(cowpeas)	24 kg	90	2,376
Fungicide	1	15	15
Labor days	106.3 days	500	53,150

Labor Schedule for Niger Farm

Sorghum

operation	month	days/ha
field prep	Apr	13
Planting	June	2
Weed 1	July	15
Weed 2	Aug	11
Fung/fert	July	1
Harvest	Nov	8
Total		50

Groundnuts

operation	month	days/ha
field prep	May	13
Planting	July	2
Weed 1	July	15
Weed 2	Aug	11
Harvest	Oct	20
Total		61

Cow Peas - Millet Intercrop

operation	month	days/ha
field prep	May	13
Planting Millet	June	2
Plant CowPea	July	9
Weed 1	July	15
Weed 2	Aug	11
Fung/fert	July	1
Harvest Millet	Oct	8
Harvest CowPea	Nov	17
Total		76

The Niger Model

The tableau shown in Table 4 contains three production activities; sorghum, groundnuts and a millet-cowpea intercropping activity. The millet-cow pea intercrop was treated as a joint production activity.

One class of land was specified and monthly labor constraints were used. The labor coefficients from the budgets were entered in the respective rows. Note that with the present activities, no labor was used in September so this constraint could have been deleted.

The capital coefficients represent the amount of cash required for actual cash purchases. For this reason, only the purchases of fertilizer or fungicide were assumed to require cash. The producer was assumed to be saving the seed (seed requirements have been deducted from the yields), and the family labor received no actual wage.

The last constraint insured that 1500 kg of sorghum and millet be retained for household use. The value of the objective function measured the value of cash sales less the cost of purchased inputs.

Results and Suggestions for Further Refinement

The information in the Niger budgets represents a first pass. The results in the lower part of Table 4 indicated the solution was feasible and optimal but that cash sales were limited to the cowpeas which were a joint product with millet. The other crop produced was sorghum. The land slack was in the final basis as 2.26 hectares of land were unused. The shadow price on the home consumption requirement indicated the cost of being self sufficient (HcMil= 107.81/kg), was greater than the market price for either sorghum or millet. The market prices of the commodities were 65 and 70 CFM/kg. The reason the cost of meeting the home consumption requirement exceeds the market price of sorghum or millet was that production of the more profitable groundnuts was precluded by the home consumption requirement.

Table 4. Linear Programming Tableau and Solution for First Solution to Niger Farm Example

Niger	MAX	RHS	Sorg	GNut	Mil-C.	HcSg	HsMl	Ssor	Sgn	Smil	Scp
Obj			-15	-15	-15	0	0	65	90	70	90
Land	L	6	1	1	1						
LbApr	L	69	13								
LbMay	L	69		13	13						
LbJun	L	69	2	2	2						
LbJul	L	69	16	17	24						
LbAug	L	69	11	11	11						
LbSep	L	69									
LbOct	L	69		20	8						
LvNov	L	69	8		17						
Cap	L	100	15	15	15						
Sortr	L	0	-418			1		1			
Gntra	L	0		-525					1		
Miltr	L	0			-377		1			1	
CPtr	L	0			-298						1
HCmil	G	1500				1	1				

OPTIMAL S o l u t i o n
function Value: 32410

Activities in Solution

Obj Va	Name	Type	Level
-15	Sorg	real	2.5
-15	Mil-C.	real	1.2
0	HcSg	real	1043.6
0	HsMl	real	456.4
90	Sgn	real	0.0
90	Scp	real	360.7
0	Land	slack	2.3
0	LbApr	slack	36.5
0	LbMay	slack	53.3
0	LbJun	slack	61.6
0	LbAug	slack	28.2
0	LbSep	slack	69.0
0	LbOct	slack	59.3
0	LvNov	slack	28.4
0	Cap	slack	44.4

Activities Not in Solution

Name	Obj.Va	Type	Shad.Price
GNut	-15	real	408.12
Ssor	65	real	42.31
Smil	70	real	37.31
LbJul	0	slack	2802.54
Sortr	0	slack	107.31
Gntra	0	slack	90.00
Miltr	0	slack	107.31
CPtr	0	slack	90.00
HCmil	0	slack	107.31

The most limiting resource was July labor. The labor use was summarized in Table 5 and shown in Figure 2. (The data in Table 5 which is shown in Figure 2 were calculated in a spreadsheet by multiplying the solution levels of the activities by the coefficients in the LP tableau.)

Examination of the budgets shows that weeding was the main task to be done in July. There may be need for further refinement of the model at this point. Budgets are often constructed assuming that tasks will carried out at the optimal or recommended time. However, more than two hectares were left unplanted because of the limitation on labor for weeding. In this case the analyst should first recheck the information used to construct the budgets. Figure 1 indicates that very little labor was used in June. Sometimes the assigning of specific task such as weeding to a specific time period can be somewhat arbitrary. If the budget information is correct, then the analyst may want to consider estimating additional activities with weeding done at suboptimal times. That is, it may be more reasonable to assume that the family would plant as much as possible and later weed only a portion of the total area at the optimal time and complete the weeding at a later date. Such activities would have lower yields but use weeding labor at different time periods.

Table 5. Supply and Utilization of Labor by Crop in Initial Solution to Niger Model

Month	Supply	Use by Crop		
		Sorghum	Mill.-CP.	Unused
	Person Days			
Apr.	69.0	32.5	0.0	36.5
May	69.0	0.0	15.7	53.3
June	69.0	5.0	2.4	61.6
July	69.0	39.9	29.1	0.0
Aug.	69.0	27.5	13.3	28.2
Sept.	69.0	0.0	0.0	69.0
Oct.	69.0	0.0	9.7	59.3
Nov.	69.0	20.0	20.6	28.4

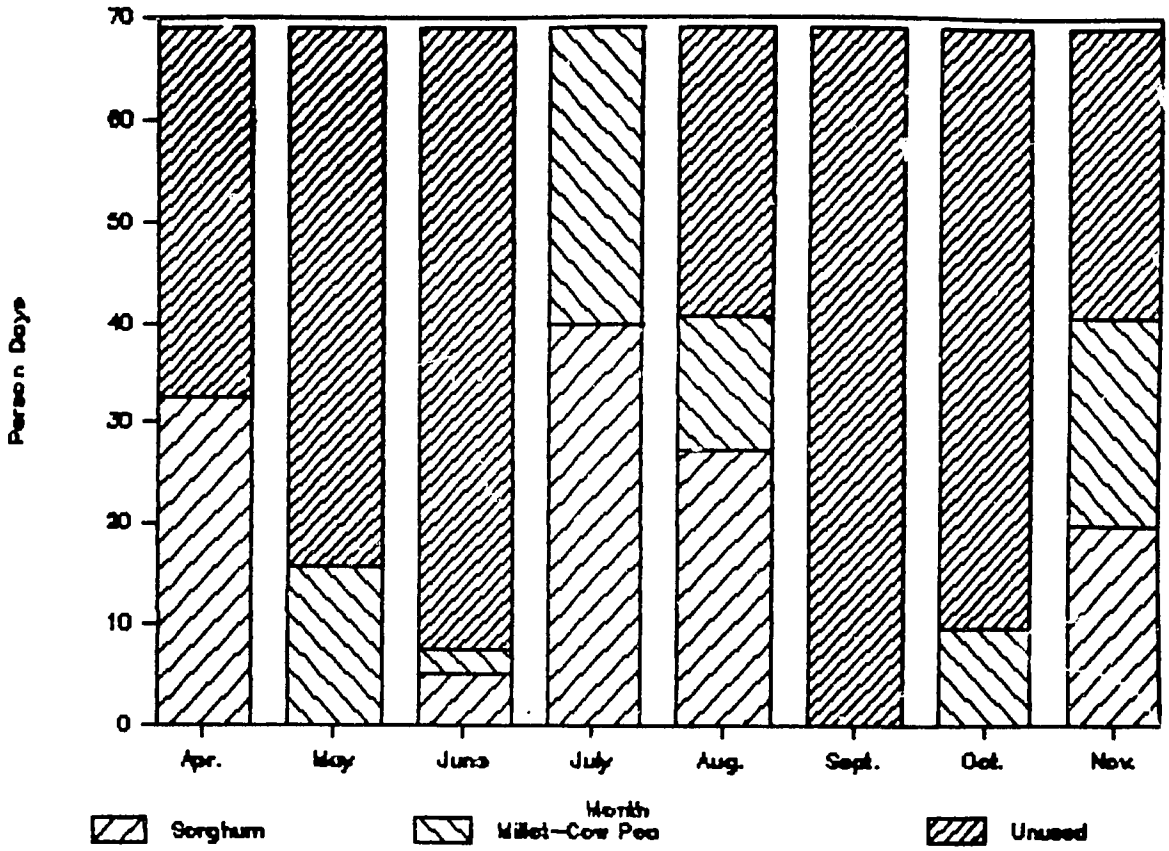

Figure 2. Monthly Use of Labor for Niger Intercrop Example.

Set up an L.P. model for the Niger farm.

Niger	MAXIMI	RHS	Sorg	GNut	Mil-C.	Ssor	Sgn	Smil	Scp
Obj			-15	-15	-15	65	90	70	90
Land	L	6	1	1	1				
LbApr	L	69	13						
LbMay	L	69		13	13				
LbJun	L	69	2	2	2				
LbJul	L	69	16	15	22				
LbAug	L	69	11	11	11				
LbSep	L	69							
LbOct	L	69		20	8				
LvNov	L	69	8		17				
Cap	L	100	15	15	15				
Sortr	L	0	-418			1			
Gntra	L	0		-525			1		
Miltr	L	0			-377			1	
CPtr	L	0			-298				1
HCmil	G	1500	418		377				

OPTIMAL S o l u t i o n
function Val 141372.3

Return	Name	Type	Level	GNut -15 real	LbJul 0 slack	Sortr 0 slack	Gntra 0 slack	Miltr 0 slack	CPtr 0 slack	HCmil 0 slack
-15	Sorg	real	2.21	-1.78	-0.11					-0.00
-15	Mil-C.	real	1.53	1.981	0.132					0.005
65	Ssor	real	923.06	-747.	-49.8	1				-2.90
90	Sgn	real	0.00	-525			1			
70	Smil	real	576.94	747.0	49.80			1		1.906
90	Scp	real	456.04	590.5	39.36				1	1.506
0	Land	slack	2.26	0.805	-0.01					0.001
0	LbApr	slack	40.29	23.23	1.548					0.090
0	LbMay	slack	49.11	-12.7	-1.71					-0.06
0	LbJun	slack	61.52	1.611	-0.02					0.003
0	LbAug	slack	27.88	8.861	-0.14					0.020
0	LbSep	slack	69.00							
0	LbOct	slack	56.76	4.146	-1.05					-0.04
0	LvNov	slack	25.32	-19.3	-1.29					-0.03
0	Cap	slack	43.92	12.08	-0.19					0.028
Z				9630.	3792.	65	90	70	90	80.18
Shadow Price				9645.	3792.	65	90	70	90	80.18

CASH FLOW MODELING

References: Hazell and Norton, *Mathematical Programming for Economic Analysis in Agriculture*, Chapter 3.

Beneke and Winterboer, *Linear Programming Applications to Agriculture*, Chapter 3.

Objective

To insure the producer can meet financial obligations through out the planning period. The specific objectives of this exercise are

a. To review the techniques for including interest rate calculations in linear programming models.

b. To review the use of comparative static modeling techniques in analyzing the potential complementary effects of policies at the farm level.

c. To illustrate how spreadsheet formulas can be used to update and modify LP tableaus.

Requirements: To model the producer's cash flow, it is necessary not only to know amount of a producers' purchases and sales but also the time at which the transactions occur. For the purposes of modeling, the planning period may be divided into quarterly, monthly, or biweekly periods. However for instructional purposes, the Mayaland farm model, the year has been divided into three periods. These periods are:

Period 1: Jan. 1-Apr. 30.
Period 2: May 1-Aug. 31.
Period 3: Sep. 1-Dec. 31.

Credit restrictions: Two cases will be considered. In the first case the producer is assumed to be able to borrow all of the short term capital needed at the prevailing rate of interest. For the second case the amount of capital which can be borrowed is limited to 2000 pesos.

It would be more realistic to have monthly or biweekly constraints but three periods are used here to reduce the size of the model.

Timing of Input Purchases and Crop Sales

We assume the budget indicated that the dates for the purchase of inputs and the sale of products for the crops in the Mayaland farmer were:

1. Corn: Fertilizer is purchased in period 1 and receipts from the sale are received in period 2.
2. Beans: Receipts are received in period 2.
3. Sorghum: Fertilizer is purchased in period 2 and any cash is received at the end of the year.
4. Groundnuts: Fertilizer is purchased in period 1 while receipts are received in period 3.

Other Assumptions

The annual rate of interest for production credit is 15% so the interest rate for each four month period is 5%. The price of fertilizer is 4 pesos per kg. In addition there are family living expenses of 3000 pesos for each four month period. The farmer has 4000 pesos on January 1.

The assumption about the crop yields, the crop prices, the interest rato, and the fertilizer price are shown at the top of the LP tableau. Note that formulas have been entered into the cells of the LP tableau so that if the values at the top of the tableau are changed during the research process, the tableau will be revised automatically.

Constructing the Model

The model contains one cash flow balance for each period plus one row to calculate the balance at the end of the year. We will assume all transactions take place on the first day of each period.

The cash balance equation for each period is of the form

Beg. Bal. + Receipts + Cash Borrowed ≥

Living Exp. + Fert. Cost + Loan Repayment + End Balance.

For period 1, an equation which requires the supply of money to be greater than or equal to the demand or use of money can be written

$$BB1 + CR1 + CB1 \geq LE1 + FC1 + LR1 + EB1.$$

Where BB1 is the beginning balance for period 1.

CR1 is the amount of all cash received in period 1. (This will be calculated as the sum of the total sales of all crop receipts in period 1.)

CB1 is the amount of cash borrowed in period 1.

LE1 is the amount of living expense in period 1.

FC1 is the amount of Fertilizer cost in period 1.

LR1 is the amount of Loan repaid in period 1.

EB1 is the amount of ending balance in period 1.

Without solving the model, we do know the beginning balance and the living expense. However we do not know the amount of cash received, the fertilizer expense, the amount of money borrowed, the amount of money repaid or the ending balance. We can rewrite the equation as

$$BB1 \geq FC1 + LE1 + LR1 - CR1 - CB1 + EB1.$$

This allows the variables for cash receipts and borrowing of capital to be determined by the model. The variables CR1 and CB1 represent additions to the supply of money in the first period. (In this model the amount of living expense is a known constant but we will leave this in the tableau as a variable because it would be negative when moved to the constant column. Some LP routines do not allow negative entries in the constant column).

For period 2, the equation is similar

$$BB2 + CR2 + CB2 \leq LE2 + FC2 + LR2 + EB2.$$

However we do not know the beginning balance for period two because we will not know how much money is left from period one until after the model has been solved. The ending balance for period one is the same as the beginning balance for period two so we

substitute EB1 for BB2. However EB1 is a variable so it must be moved to the tableau side of the inequality. The amount of loan repayment due in period 2 is equal to one plus the period rate of interest times the amount borrowed at the beginning of period 1. For example, if one peso was borrowed in period one and the rate of interest for the period (4 months) is 5% the amount due in period two is 1.05 pesos. Tnat is

$$LR2 = CR1*(1 + r),$$

where r is the per period rate of interest. After the substitutions

$$0 \geq - CR2 + LE2 + FC2 + [CB1*(1+r)] - EB1 - CB2 + EB2.$$

For period 3 there is a similar equation. Again we do not know the beginning balance for period 3. We do know that the beginning balance for period 3 is the ending balance from period 2 and that the debt to be repaid in period 3 is equal to (1+r) times the amount borrowed in period 2. (Again we assume the producer will either repay all previous loans or refinance all outstanding debt in each period). The constraint for period 3 can be written as

$$0 \geq -CR3 + FC3 + LE3 + [CB2*(1+r)] - EB2 - CB3 + EB3.$$

A final ending balance accounting constraint can be added to calculate the cash available at the end of the year.

We can now incorporate parts of the above example into the tableau. We shall use a mini tableau or portion of the larger tableau to discuss each part. The first mini tableau is shown in Table 1. First we can begin with the fertilizer purchase and production activities.

Timing of Cash Purchases and Sales

The cash flow or short term capital constraints are indicated at the lower portion of the tableau.

Cap1. The row labeled cap1 is the cash constraint for the first period. The cash on hand at the beginning of the year (4000 pesos) is entered in the RHS column in the

Table 1. Mini Tableau Showing Production, Selling, and Purchase Activities for Cash-Flow Model

Mland	Max	Rhs	corn	beans	sorghum	peanut	Scorn	Sbean	SSorg	SPnut	BuyFer
obj							1.853	3.208	2.2	5.004	- 4.0
land	L	5	1	1	1	1					
labor	L	16.5	1.42	1.87	1.92	2.64					
mules	L	10	1.45	1.27	1.16	1.45					
market	L	0.5				0.983					
cornba	L	0	-750				1				
beanba	L	0		-380				1			
sorgba	L	0			-700				1		
pnutba	L	0				-983				1	
fertba	L	0	20		20	50					-1
cap1	L	4000	80			200					
cap2	L	0		-1219	80						
cap3	L	0				-4918					
capend	L	0	-1389		-1540						

Table 2. Mini Tableau With Borrowing and Ending Balance Activities for Cash-Flow Problem

Mland	Max	Rhs			CB1	CB2	CB3	EB1	EB2	EB3	LvExp
obj					-0.05	-0.05	-0.05				
sorgba	L	0									
pnutba	L	0	P	A							
fertba	L	0	R	C							
cap1	L	4000	O	T	-1			1			3000
cap2	L	0	D	I	1.05	-1		-1	1		3000
cap3	L	0	U	V		1.05	-1		-1	1	3000
capend	L	0	C	I			1.05			-1	
LivExp	E	1	T	E							1
mcb1	L	2000	I	S	1						
mcb2	L	2000	O			1					
mcb3	L	2000	N				1				

Cap1 row. The value in the corn column is the cost of fertilizer purchased for corn in period 1 (20 x 4). The entry in the Peanut column and Cap1 row is also the value of the fertilizer purchased for peanuts in period 1 (50kg x ' pesos).

Cap2. If beans are grown, they can be sold in the second period. The revenue from the sale of beans (380kg x3.208 = 1219) is entered as a negative value because it represents an addition to the supply of cash. The entry in the Sorghum column represents the cost of fertilizer for sorghum (20kg x 4).

Cap3. The only entry is the revenue from the sale of peanuts (983kg x5.004 = 4918) which is received in period 3.

Capend. The entry in the corn column results from the sale of the corn which is received on the last day of the year (750kg x1.853 = 1389). The entry in the sorghum column represents receipts from the sale of grain sorghum which are also received on the last day of the year.

Capital Borrowing and Transfer

The capital borrowing and capital transfer activities are shown in the Table 2. In addition a family living constraint and a family living expense activity is also shown. The last three constraints would be used in cases where the access to capital is limited and the producer is unable to borrow all the money he or she wants at the stated rate of interest.

The borrowing activities (CBt), are represented by a -1 in the t'th period and a positive value (1 + r) in period t + 1, where r is the rate of interest. The one period loan adds to the cash supply in period t and results in a balance owed of 1 + r in the next period. The loan taken out in one period must be repaid from cash receipts or by additional borrowing in the next period. All debts must be settled by the last day of the year.

The Ending Balance Activities (EBt) transfer any unused cash from one period to the next. The entry in period t is positive (a use of money) and there is a negative entry in the period t + 1 as the money adds to the supply in period t + 1.

Limited access to the capital market. Producers may not be able to borrow all of the production credit they desire at the market rate of interest. The last three restrictions in the tableau limit the amount which can be borrowed in each time period or the maximum amount of short term debt a farmer can accumulate in each of the time periods.

Family living expense. An activity for family living expense is shown in the right most column of the Tableau. This activity which uses 3000 pesos in each time period is forced to enter the solution at one unit. In this example the family living expense is a known constant and could have been entered in the RHS column. However the right hand side entry would have been negative unless the inequality were reversed by multiplying through the constraint by a -1. As stated above, some LP routines will not allow negative entries to be entered in the RHS column.

Preparing the Model for Farm Level Policy Analysis

The basic Mayaland model is presented in Table 3. However before attempting to modify your Mayaland model, consider how Lotus formulas might be used to make further modifications of the model even easier.

First, what kinds of modifications might we be interested in making? Assume our supervisor has just returned from a visit to the Mayaland farming region. Producers there have complained that each year their financial position has become more difficult. After family living expenses have been deducted, there is less money for production expenses the next year than there was the previous year.

Table 3. Mayaland Model With Cash Flow Constraints and Limits on Short Term Capital Borrowing

Mland	Maximi	Rhs	corn	beans	sorgh	peanut	Scorn	Sbean	SSorg	SPnut	BuyFer	CB1	CB2	CB3	EB1	EB2	EB3	LivExp
obj							1.853	3.208	2.2	5.04	-4	-0.05	-0.05	-0.05				
land	L	5	1	1	1	1												
labor	L	16.5	1.42	1.87	1.92	2.64												
mules	L	10	1.45	1.27	1.16	1.45												
market	L	0.5				0.983												
cornba	L	0	-750				1											
beanba	L	0		-380				1										
sorgba	L	0			-700				1									
pnutba	L	0				-983				1								
fertba	L	0	20		20	50					-1							
cap1	L	4000	80			200						-1			1			3000
cap2	L	0		-1219	80							1.05	-1		-1	1		3000
cap3	L	0				-4954							1.05	-1		-1	1	3000
capend	L	0	-1389		-1540									1.05			-1	
LivExp	E	1																1
mcb1	L	2000										1						
mcb2	L	2000											1					
mcb3	L	2000												1				

Our supervisor instructs us to use the model to see if the producer's complaint is valid. Furthermore the supervisor wants to know if the farmers' income would be improved if:

a) Farmers used more fertilizer as a result of being provided more information about trials showing showing fertilizer response to sorghum. To do this the extension service would have to be enhanced and fertilizer prices would be reduced from 4 pesos to 2 pesos per kilogram through a subsidy.

b) Just having better access to lower cost credit alone would relieve the problem. Producers would have unlimited access to credit at 12% per year.

c) Both approaches were followed. That is farmers used more fertilizer (at the lower price) and had better access to credit at a lower interest rate (12% per year).

The instructions imply the need for at least four solutions from our model.

1. A base line solution using current credit restrictions and current fertilizer prices.
2. A solution in which the producer has access to unlimited credit at 12% per year.
3. A solution in which the producer considers the use of more fertilizer on sorghum and enjoys reduced fertilizer prices.
4. A solution in which the producer has access to unlimited credit at 12% and may consider the uses of more fertilizer at a lower cost.

The modifications require that we need to be able to quickly change the the values in the model which are affected by the interest rate and the fertilizer price. While we are at it, we may also change the calculations which are affected when crop prices change because that may be our next assignment.

A possible modification of the tableau is shown in Table 4. Note that at the top of the table we have a list of the prices and the interest rate used in the model. The difference is that some of the values in the cells of the tableau are calculated by the formulas. The advantage is that if one or more of the prices at the top of the table are modified, all entries in the tableau which are affected by that price are recalculated.

Note that you should save the tableau which contains the equations in a Master file. Then Xtract (under another name) only the Values in tableau portion of the Master file for use by the MUSAH86 program. When you revise the coefficients in the tableau, retrieve the master file, change the data in the formulas, then Xtract a new tableau from the master file for MUSAH86.

Solution 1. The baseline model shown in Table 3 or Table 4 is solved first, the results of the base line solution are shown below in Table 5.

It may be instructive to review the results of the solution in terms of the cash flow constraints and credit restrictions.

Table 4. Mayaland Cash Flow Model with Lotus Formulas to Permit Rapid Updating of the Tableau when the Parameters Are Changed

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1		interest=			0.05		Price	Sorghum=		2.2									
2		price corn=			1.853		Price	Peanuts=		5.04									
3		price beans=			3.208		Price	fert=		4									
4	Mland	Maximi	Rhs	corn	beans	sorgh	peanut	Scorn	Sbean	SSorg	SPnut	BuyFer	CB1	CB2	CB3	EB1	EB2	EB3	LvExp
5	obj							1.853	3.208	2.2	5.04	-4	-0.05	-0.05	-0.05				
6	land	L	5	1	1	1	1												
7	labor	L	16.5	1.42	1.87	1.92	2.64	+E2	+E3	+J1	+J2	-J3							
8	mules	L	10	1.45	1.27	1.16	1.45							-E1					
9	market	L	0.5				0.983												
10	cornba	L	0	-750				1											
11	beanba	L	0		-380				1										
12	sorgba	L	0			-700				1									
13	pnutba	L	0				-983				1								
14	fertba	L	0	20		20	50					-1							
15	cap1	L	4000	80	+D14*J3		200	+G14*J3					-1			1			3000
16	cap2	L	0		-1219	80	+F14*J3						1.05	-1		-1	1		3000
17	cap3	L	0		+E11*E3		-4954							1.05	-1		-1	1	3000
18	capend	L	0	-1389		-1540	+G13*J2								1.05			-1	
19	LivExp	E	1	+D10*E2		+F12*J1						(1+E1)							1
20	mcb1	L	2000										1						
21	mcb2	L	2000											1					
22	mcb3	L	2000												1				

Table 5. Base Line Solution with Credit Restrictions and High Fertilizer Prices

	Variables in the Solution			Variables Not in the Solution			
Name	type	ObjVal	Act. Level	Name	type	ObjVal	Shad.Price
corn	real	0	0.00	Scorn	real	1.853	0.20
beans	real	0	0.78	CB1	real	-0.05	0.06
sorgh	real	0	3.71	EB2	real	0	0.06
peanuts	real	0	0.51	EB3	real	0	0.13
Sbean	real	3.208	296.45	land	slack	0	1445.16
SSorg	real	2.2	2597.86	market	slack	0	3979.00
SPnut	real	5.04	500.00	cornbal	slack	0	2.05
BuyFert	real	-4	99.66	beanbal	slack	0	3.21
CB2	real	-0.05	1447.62	sorgbal	slack	0	2.20
CB3	real	-0.05	2000.00	pnutbal	slack	0	5.04
EB1	real	0	898.27	fertbal	slack	0	4.00
LvExp	real	0	1.00	cap1	slack	0	0.19
labor	slack	0	6.57	cap2	slack	0	0.19
mules	slack	0	3.97	cap3	slack	0	0.13
capend	slack	0	3615.29	LivExp	slack	-1.0E+14	1.0E+14
mcb1	slack	0	2000.00	mcb3	slack	0	0.08
mcb2	slack	0	552.38				

In the first period, the producer begins with 4000 pesos. There are no cash receipts or capital borrowing in period 1 (CB1 = 0). Expenses in the first period are

Fertilizer for peanuts .51 * 200 =	102.00	
Family living expense	3000.00	
Total expense for period 1	3102.00	
Balance = 4000 - 3102	898.00	(note EB1 = 898.27)
Period 2. Beginning balance =	898.27	
Sale of beans (.78 * 1219)=	951.00	
Amount borrowed (CB2)	1447.62	
Total cash available	3296.80	
Fertilizer for Sorghum 3.71 * 80 =	296.80	
Family living expense	3000.00	
Total expense for period 2	3296.80	
Balance at end of period 2	0.00	
Period 3.		
Balance at beginning of period	0.00	
Sale of Peanuts 500 * 5.04=	2520.00	
Amount borrowed(CB3)	2000.00	
Total money available	4520.00	
Expenses		
Repay/refin old loan (1.05*1447.62)=	1520.00	
Family living expense	3000.00	
Total period 3 expense	4520.00	
End of Year		
Transferred from period 3	0.00	
Sale of Sorghum 2.2 * 2597.86=	5715.29	
Repay Per 3 loan 2000*1.05	2100.00	
balance	3615.29	
capend= 3615.29		

The value of capend indicates the ending cash position is less than the beginning cash position. Note that beans have entered the solution. Beans were not competitive in the previous solutions. Bean production is a substitute for borrowing which is limited. Beans entered the solution because the receipts are received in period 2 and the money can be used to pay living expense and buy fertilizer for sorghum.

Solution 2. Retrieve the Master File. Revise the interest rate. Xtract the portion of the tableau without the capital restrictions. Resolve. Summarize the solution. What effect will increased amounts of lower cost credit have on income and the ending cash balance?

Solution 3. Retrieve the Master File. Set the interest rate at 5%. Modify the model to include the following information about sorghum response to fertilizer.

Fertilizer used	20kg (current)	80kg	140kg
Yield	700	900	1000
labor	1.92	2.0	2.1
mules	1.16	1.16	1.16

All other information remains the same. Be sure and include the credit restriction in the model since it is assumed there is no change in credit policy. Xtract the model, obtain and summarize the solution. Does the solution indicate that an extension service program in which farmers were provided information about fertilizer use and crop response would be successful in increasing farm income?

Solution 4. Retrieve the Master file. Set the interest rate at 4% and reduce the price of fertilizer to 2 pesos. Include the fertilizer response information in the model. Remove the capital borrowing restrictions. Xtract the relevant portion of the tableau and obtain the solution. What is the combined effect of the fertilizer and the credit policy on income? What is the effect on the choice of crops to produce?

MULTIPERIOD MODELS

References: Hazell, B.R. and R.D. Norton, Chapter 4 in *Mathematical Programming for Economic Analysis in Agriculture.*

Beneke, R.R. and R. Winterboer, Chapter 9 in *Linear Programming Applications to Agriculture.*

Objectives

1. Review the use of discounting in investment analysis.
2. Discuss the use of Stationary Equilibrium Programming Models in Investment analysis.
3. Discuss the use of Dynamic or Multiperiod Programming Models in Investment analysis.

Review of Discounting, Present Value, and Future Value

The Future Value at time t of an amount of a sum of money invested now at a rate of interest r is

$$FV_t = PV_o(1+r)^t.$$

The Present value of an amount to be received in a future time t is

$$PV_o = FV_t \ / \ (1+r)^t$$

The commonly used Net Present Value concept is the discounted sum of the differences between costs and returns received over a planning period of T periods.

$$NPV = \frac{R_o-C_o}{(1+r)^0} + \frac{R_1-C_1}{(1+r)^1} + \frac{R_2-C_2}{(1+r)^2} + \qquad \frac{R_T-C_T}{(1+r)^T}$$

or $$NPV = \Sigma_t \frac{R_t-C_t}{(1+r)^t}$$

An important special case is when the Net Returns are constant over the the T period Planning horizon. When the net returns for each period are the same, the formula can be written as

$$NPV = (R-C) * \Sigma_t \frac{1}{(1+r)^t} = (R-C) \frac{1 - (1+r)^{-T}}{r}$$

When the length of the planning period is very long, the term, $(1+r)^{-T}$ approaches zero and the formula becomes

$$NPV = \frac{R-C}{r}$$

which is the standard capitalization formula.

Dynamic Vs Static Approaches to Investment Analysis.

Analysis of Investments can be made with Static-Stationary or Dynamic-Multiperiod approaches. A major reason for using linear programming in investment analysis is to determine the optimal size and timing of investments subject to the constraints faced by the investor. Most investment analysis is dynamic or involves changes in the firm over a period of time. Multiperiod linear programming models can be used for this type of analysis. However the models expand in size very rapidly and the analyst may be forced to choose between the amount of annual detail or constraints to be concluded and the number of investment periods to be considered.

Static or Stationary Equilibrium Models

Static or Stationary Equilibrium models can be used when it is not unreasonable to assume the producer makes a one time adjustment in the production unit. This requires constant relative prices over the planning period. A one time adjustment would imply the producer is not growing in size and that crop yields are stable. It is also required that the producer be able to borrow all of the money needed for the investment at the market rate of interest. The possibility of using a single period linear programming model for investment analysis follows from the above case where the difference between the costs and returns were constant over the T period planning horizon.

The use can be illustrated with the example from Hazell and Norton (page 55). The Tableau produced below Table 1 is taken from Table 4.1 in the text. One modification has

been made. The problem in the text is to determine the optimum number of breeding sows to utilize the resources of the Mayaland farm. However the cost of constructing the facilities for the swine operation are not considered. It is possible to determine the optimal investment in swine facilities within the same model. Assume the cost of constructing the facilities for the swine herd is 20,000 pesos per sow and that the facilities have an expected 10 year life. (We also assume the optimum size of swine here will be reached the first year and remain constant over the 10 year period).

If the facilities are constructed at a cost of 20,000 pesos per sow and last for 10 years (with no salvage value), and if the annual rate of interest is 10%, then the annualized cost of borrowing the 20,000 pesos per sow is given by the formula

$$\text{Annual cost/sow} = 20{,}000 * \frac{i}{1-(1.1)^{-10}}$$

$$= 20{,}000 * .16274 = 3255 \text{ pesos.}$$

The annualized marginal cost of adding one additional sow to the farm is estimated to be 3255 pesos. This is equivalent to an annual mortgage payment on the cost of facilities for adding an additional sow unit.

A corn buying activity has been added to the model so the producer can either sell his own corn, feed his own corn to the livestock, or buy additional corn for the livestock.

Results. The results in Table 1 indicate the optimal investment in swine capacity was 1.95 units and that 15 pigs would be sold each year. The corn produced on the farm would be marketed through the swine herd and an additional 4933 kg of corn would be purchased each year. The limiting resources are labor (651 pesos/mo) and land (522 pesos /ha).

Table 1. Stationary Equilibrium Investment Model to Determine the Optimal Size of Swine Facility for the Mayaland Producer

PigInv	MAX	RHS	BredS	MrktP	RaiSw	BySow	Corn	SlCrn	ByCrn	HCap
Obj			-150	2000	-180	-8000	-52	1.9	-2	-3255
Land	L	5	0.2		0.25		1			
Labor	L	16.5	1.1	0.5	0.9		1.42			
Mules	L	10					1.45			
HerdRe	L	0	0.25		-1	-1				
LitCon	L	0	-8	1	1					
CornTr	L	0	300	500	350		-750	1	-1	
Sowmax	L	0	1							-1

S o l u t i o n
OPTIMAL
function Value: 13363

Return	Name	Type	Level	BySow -8000 real	SlCrn 1.9 real	Land 0 slack	Labor 0 slack	HerdRe 0 slack	LitCon 0 slack	CornTr 0 slack	Sowmax 0 slack
-150	BreedS	real	1.947	-0.00		-0.29	0.207	0.009	-0.10		
2000	Market	real	15.09	-1.07		-2.27	1.605	1.072	0.197		
-180	RearSw	real	0.486	0.997		-0.07	0.051	-0.99	-0.02		
-52	Corn	real	4.488	-0.24		1.077	-0.05	0.247	0.027		
-2	BuyCrn	real	4933.	-4.07	-1	-2061	923.7	4.070	38.10	-1	
-3255	HCap	real	1.947	-0.00		-0.29	0.207	0.009	-0.10		-1
0	Mules	slack	3.491	0.358		-1.56	0.078	-0.35	-0.03		
Z				-2271	2	522.8	651.4	2271.	674.2	2	3255
Shadow Price				5728.	0.1	522.8	651.4	2271.	674.2	2	3255

Multiperiod Models

These models are more general and allow an analysis of the change in producer decisions over time. A multiperiod model

a) contains two or more periods in which decisions must be made,

b) allows linkage between periods through the objective function and through the constraints,

c) should include a sufficient number of periods so that the decisions at the beginning of the planning period are independent of the ending conditions of the model.

The issues to be determined in establishing a multiperiod model include

a) The number of periods to be included.

b) Assignment of terminal values to investments which extend beyond the planning horizon.

c) Selection of a discount rate.

d) Initialization of the model to reflect the actual starting conditions for the producer.

Method of forming the objective function. The operator can choose to maximize

a) Net Present Value. The entries in the objective function are discounted back to the start of the planning period.

b) Ending Wealth or Terminal Value. Cash flows from the current period are transferred forward to the next or the last period. The only objective function value occurs in the last period.

Tree planting example. Many countries face problems in encouraging producers to plant rubber, coffee, tea, and cocoa trees because there is a long period between the date of planting and the time when the trees are in full production. The problem can be illustrated with the following example.

Assumptions a producer has

1. A 10 year planning horizon.
2. A choice between a tree crop and an annual crop.
3. 10 units of land
4. 600 dollars for production expense year 1.
5. Ability to borrow at 10% per year.
6. Ability to save at 5% per year.

The annual crop has a return of 10 dollars per hectare every year. The tree crop requires an initial investment of 100 dollars. The life of the tree is 7 years. The Returns from the tree crop are:

Years	return
2-3	0
4	50
5	70
6	80
7	60

Since the length of the planning horizon is only 10 years, the tree planting activities which are initiated after the first 4 years of the planning horizon must have terminal values calculated. The terminal values for tree planting activities in year 5 and 6 (T5 and T6) were determined by discounting the expected future returns at the rate of borrowing. This is a process which seems innocent but can have some unexpected effects on the solution.

Before any linear programming process is undertaken, it would be useful to carry out the normal Net Present Value and Internal Rate of Return calculations which are normally done. Lotus has some built in formulas which make this process easy. A sample spreadsheet the formulas and calculations are shown below.

Table 2. Calculation of the Net Present Value and the Internal Rate of Return from the Tree Cop Investment

	A	B	C	D	E
1					
2					
3			End of year		return-cost
4			1		-100
5			2		
6			3		
7			4		50
8			5		70
9			6		80
10			7		60
11					---
12			Total		160
13					
14			NPV @ 10%=		62.65347 <----@NPV(0.1,E4..E10)
15					
16			IRR=		0.238931 <----@IRR(0.1,E4..E10)

The @NPV formula requires the user to specify the discount rate (.1 is used) and the location of the values to be discounted. Note that the first value in the series (-100 in this case) will be discounted by 1/(1+r). The NPV is positive when a 10% discount rate is used so we know the internal rate of return exceeds 10%.

The @IRR formula requires an initial guess for the IRR and the location of the data. An initial estimate of 10% was used. The actual IRR was 23.8 %. This indicates the tree crop should be quite profitable.

If the analysis were to be done with the Static or Stationary approach, we would define a composite land unit of 7 hectares (like in the previous rotation example). One hectare would have trees between 0-1 years, one hectare would have trees aged 1-2, one hectare with trees 2-3, and so forth. The return return from all 7 hectares would be $160. The LP model would appear as

		Annual Crop	Tree Crop
OBj		10	160
Land	10 >=	1	7

The solution would be to immediately put all 10 hectares into the tree crop since the average annual income would be higher.

The purpose of the Multiperiod model is to indicate how the producer should move from a position of having no trees to the desired position of having the entire farm in the tree crop. The year by year adjustment in the planted area is one of the results to be obtained from the solution.

A first solution is listed in Table 4. First note that the producer does not have sufficient capital on hand to plant the entire area to tree crops. The producer plants 6.34 hectares of land to trees. This money is obtained by using the 600 dollars on hand plus the money from the 3.66 hectares of corn. Note that the producer does not borrow any money although the opportunity is available.

The way in which the returns from beyond the end of the planning period were credited to the terminal year for activities T5 and T6 account for the fact that no trees were planted in years 3, 4, and 5. The problem is that the IRR of 21% is greater than the 10% discount rate used. A quick analysis of activities T1, T5, and T6 indicates that the IRR of activities T5 and T6 were increased because the value future returns was brought forward at 10%.

	T1	T5	T6
1	-100	-100	-100
2			
3			
4	50	50	50
5	70	70	192.3140
6	80	134	
7	60		
NPV	62.653	62.346	62.653
IRR	0.239	0.245	0.264

A second solution was obtained by discounting the returns received after the end of the period by 0.23. The values of 134 and 192 were replaced by values of 128 and 174 respectively. A second solution was obtained which is shown in Table 5.

Table 3. Master Tableau for Multiperiod Tree Crop Example for Maximization of Ending Worth

annual crop return= 10 IRR= 0.23

borrowing rate= 0.1

Saving rate = 0.05

Initial cost 100

TreeMAXI		RHS	T1	CH1	T2	CH2	T3	CH3	T4	CH4	T5	CH5	T6	CH6	CH7	CH8	CH9	CH10	SV1	SV2	SV3	SV4	SV5	SV6	SV7	SV8	SV9	B1	B2	B3	B4	B5	B6	B7	B8	E9	ENDV
Obj																																					
L1	L	10	1	1																																	1
L2	L	10	1		1	1																															
L3	L	10	1		1		1	1																													
L4	L	10	1		1		1		1	1																											
L5	L	10	1		1		1		1		1	1																									
L6	L	10	1		1		1		1		1		1	1																							
L7	L	10	1		1		1		1		1		1		1																						
L8	L	10			1		1		1		1		1			1																					
L9	L	10					1		1		1		1				1																				
L10	L	10							1		1		1					1																			
C1	L	600	100	-10															1									-1									
C2	L	0			100	-10													-1.05	1								1.1	-1								
C3	L	0					100	-10												-1.05	1								1.1	-1							
C4	L	0	-50						100	-10											-1.05	1								1.1	-1						
C5	L	0	-70		-50						100	-10										-1.05	1								1.1	-1					
C6	L	0	-80		-70		-50						100	-10									-1.05	1								1.1	-1				
C7	L	0	-60		-80		-70		-50						-10									-1.05	1								1.1	-1			
C8	L	0			-60		-80		-70		-50					-10									-1.05	1								1.1	-1		
C9	L	0					-60		-80		-70		-50				-10									-1.05	1								1.1	-1	
C10	L	0							-60		-134		-192					-10									-1.05									1.1	1

Table 4. Solution to Tree Crop Model With Distortions Caused by Treatment of Returns Beyond the Planning Period

function Value: 2905.31

Activities in Solution				Activities Not in Soluti			
ObjVal	Name	Type	Level	Name	Obj Val	Type	Shad.Pri
0	T1	real	6.36	T3	0	real	1.199
0	CN1	real	3.64	T4	0	real	4.381
0	T2	real	0.33	T5	0	real	0.484
0	CN2	real	3.31	CN7	0	real	68.007
0	CN3	real	3.31	SV1	0	real	0.029
0	CN4	real	3.31	SV2	0	real	0.019
0	CN5	real	3.31	B1	0	real	0.046
0	T6	real	3.31	B2	0	real	0.051
0	CN6	real	0.00	B3	0	real	0.067
0	CN8	real	6.36	B4	0	real	0.064
0	CN9	real	6.69	B5	0	real	0.061
0	CN10	real	6.69	B6	0	real	0.058
0	SV3	real	33.06	B7	0	real	0.055
0	SV4	real	385.95	B8	0	real	0.053
0	SV5	real	900.29	B9	0	real	0.050
0	SV6	real	1146.96	L1	0	slack	16.009
0	SV7	real	1612.57	L2	0	slack	14.969
0	SV8	real	1776.67	L3	0	slack	14.071
0	SV9	real	2097.73	L4	0	slack	13.401
1	ENDV	real	2905.31	L5	0	slack	12.763
				L6	0	slack	12.155
				L7	0	slack	79.583
				L8	0	slack	11.025
				L9	0	slack	10.500
				L10	0	slack	10.000
				C1	0	slack	1.601
				C2	0	slack	1.497
				C3	0	slack	1.407
				C4	0	slack	1.340
				C5	0	slack	1.276
				C6	0	slack	1.216
				C7	0	slack	1.158
				C8	0	slack	1.102
				C9	0	slack	1.050
				C10	0	slack	1.000

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The results in Table 5 indicate the producer would increase his planting of the tree crop each year through year 4. The amount of planting each year is determined by the cash on hand and from the sale of the annual crop. There is a big increase in planting in year 4 because that is when the first returns from the trees planted in year 1 (T1) is received.

Savings begin in year 4 and increase through the planning period. The ending value 2891 is the amount of cash at the end of the planning period. The present value of that amount would be determined by discounting.

The producer begins planting annual crops again in years 8, 9, and 10. This is a response to the approaching end of the planning period. If the model had contained more periods, this effect would have been postponed. The usual practice is to build models with planning periods which are longer than planning period desired by the decision maker to minimize the effect of the ending conditions.

The producer did not take advantage of borrowing even though the internal rate of return (20%) on the investment was greater than the cost of borrowing. Apparently it was even more profitable to invest the money from the annual crops and raise the investment capital internally than it was to borrow the money. If the returns from the annual crops are set to zero, the model will borrow the money and plant all of the farm to tree crops in the first year.

Table 5. Solution to Multiperiod Tree Crop Model When Returns Received Beyone the Planning Period are Discounted at the Internal Rate of Return

function Value: 2891.786

Activities in Solution				Activities Not in Solution			
Returns	Name	Type	Level	Name	Obj val	Type	Shad.Pric
0	T1	real	6.36	T5	0	real	1.868
0	CN1	real	3.64	T6	0	real	13.234
0	T2	real	0.33	CN7	0	real	63.627
0	CN2	real	3.31	SV1	0	real	0.024
0	T3	real	0.30	SV2	0	real	0.026
0	CN3	real	3.01	SV3	0	real	0.029
0	T4	real	3.01	B1	0	real	0.053
0	CN4	real	0.00	B2	0	real	0.046
0	CN5	real	0.00	B3	0	real	0.038
0	CN6	real	0.00	B4	0	real	0.064
0	CN8	real	6.36	B5	0	real	0.061
0	CN9	real	6.69	B6	0	real	0.058
0	CN10	real	6.99	B7	0	real	0.055
0	SV4	real	17.66	B8	0	real	0.053
0	SV5	real	480.52	B9	0	real	0.050
0	SV6	real	1051.81	L1	0	slack	16.347
0	SV7	real	1683.96	L2	0	slack	15.341
0	SV8	real	2086.04	L3	0	slack	14.360
0	SV9	real	2515.74	L4	0	slack	13.401
1	ENDV	real	2891.79	L5	0	slack	12.763
				L6	0	slack	12.155
				L7	0	slack	75.203
				L8	0	slack	11.025
				L9	0	slack	10.500
				L10	0	slack	10.000
				C1	0	slack	1.635
				C2	0	slack	1.534
				C3	0	slack	1.436
				C4	0	slack	1.340
				C5	0	slack	1.276
				C6	0	slack	1.216
				C7	0	slack	1.158
				C8	0	slack	1.103
				C9	0	slack	1.050
				C10	0	slack	1.000

CASE STUDY: LIBERIAN FARM MODEL

Summarizing Results of a Representative Farm Model using Lotus 1-2-3

Reference: Epplin, F.M. and J.G. Jusah. "A Representative Farm Planning Model for Liberia." In Proceedings of the Liberian Agricultural Policy Seminar, 1985. Oklahoma State University, APAP Project Resource Report B-23, February 1987.

The enclosed initial and solution tableau is from a model of a Liberian representative farm by Epplin and Musah. The variable definitions are as shown in Table 1.

You are to complete the table in Figure 1 using the information from the enclosed initial and final tableau using Lotus. The table dressing are already typed in for you, all you need to do is to supply the numbers. The worksheet is stored under the name SUM.

You may have to do some hand calculations, but use Lotus formulas as much as possible. For example, total palm oil production in cell C53 should be computed by the formula @sum(C41.C52), which contains the monthly production figures. We will discuss more about Lotus formulas in class as we go along. When typing in numbers into the tables, use as many significant figures as possible, but the numbers in the table should be formatted consistently using the Lotus /Range Format Command.

If you need any immediate tables in order to facilitate calculations with Lotus, please feel free to create your own.

Design 1 or 2 graphs to summarize the labor utilization information (stack bars would be useful here).

Table 1. Variables

Columns

RICEOK	Rice/Okra acreage (Intercrop)
RICEPEP	Rice/Pepper acreage
RICEBBLS	Rice/Bitter-balls acreage
RICECAS	Rice/Cassava acreage
SWRICE	Rice acreage
CASSAVA	Cassava acreage
COCOA	Cocoa acreage
COFFEE	Coffee acreage
SCANE	Sugur Cane acreage
OKRA	Okra acreage
PEPPER	Pepper acreage
BBALLS	Bitter balls acreage
RUBBER	Rubber acreage
RICSSC	Rice/Cassava/Sugar Cane acreage (2 year rotation with rice and cassava in year 1 and sugar cane in year 2)
PALM	(Cultivated) palm acreage
POJAN	Palm oil collection, January (Gallons)
POFEB	Palm oil collection, February (Gallons)
POMAR	Palm oil collection, March (Gallons)
POAPR	Palm oil collection, April (Gallons)
POMAY	Palm oil collection, May (Gallons)
POJUN	Palm oil collection, June (Gallons)
POJUL	Palm oil collection, July (Gallons)
POAUG	Palm oil collection, August (Gallons)
POSEP	Palm oil collection, September (Gallons)
POOCT	Palm oil collection, October (Gallons)
PONOV	Palm oil collection, November (Gallons)
PODEC	Palm oil collection, December (Gallons)
RISELL	Sell rice (lbs)
OKSELL	Sell Okra (lbs)
PEPSELL	Sell Pepper (lbs)
BBSELL	Sell Bitter-balls (lbs)
CASSELL	Sell Cassava (lbs)
COCSELL	Sell Cocoa (lbs)
COFSELL	Sell Coffee (lbs)
CANESELL	Sell Cane Juice (Gallons)
RUBSELL	Sell Rubber (Gallons)
PALMSELL	Sell Palm (Tons)
POSELL	Sell Palm oil (Gallons)
JMLBY	Male Labor Hired (Mandays) January
FMLBY	Male Labor Hired (Mandays) February
MRMLBY	Male Labor Hired (Mandays) March
APMLBY	Male Labor Hired (Mandays) April
MYMLBY	Male Labor Hired (Mandays) May
JNMLBY	Male Labor Hired (Mandays) June
JLMLBY	Male Labor Hired (Mandays) July
AGMLBY	Male Labor Hired (Mandays) August
SPMLBY	Male Labor Hired (Mandays) September
OCMLBY	Male Labor Hired (Mandays) October

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NOMLBY	Male Labor Hired (Mandays) November
DEMLBY	Male Labor Hired (Mandays) December
GVTCAP	Borrow capital from government
BORROW	Borrow capital from other sources
RICTR	Rice transfer
OKCTR	Okra transfer
PECTR	Pepper transfer
BBCTR	Bitter ball transfer
CVCTR	Cassava transfer
SCTR	Sugar Cane transfer
POCTR	Palm oil transfer

Rows

C	Objective function
MLJAN	Male labor availability: January (person-days)
MLFEB	Male labor availability: February (person-days)
MLMAR	Male labor availability: March (person-days)
MLAPR	Male labor availability: April (person-days)
MLMAY	Male labor availability: May (person-days)
MLJUN	Male labor availability: June (person-days)
MLJUL	Male labor availability: July (person-days)
MLAUG	Male labor availability: August (person-days)
MLSEP	Male labor availability: September (person-days)
MLOCT	Male labor availability: October (person-days)
MLNOV	Male labor availability: November (person-days)
MLDEC	Male labor availability: December (person-days)
FLJAN	Female labor availability: January (person-days)
FLFEB	Female labor availability: February (person-days)
FLMAR	Female labor availability: March (person-days)
FLAPR	Female labor availability: April (person-days)
FLMAY	Female labor availability: May (person-days)
FLJUN	Female labor availability: June (person-days)
FLJUL	Female labor availability: July (person-days)
FLAUG	Female labor availability: August (person-days)
FLSEP	Female labor availability: September (person-days)
FLOCT	Female labor availability: October (person-days)
FLNOV	Female labor availability: November (person-days)
FLDEC	Female labor availability: December (person-days)
LAND	Land Constraint (acres)
CAPITAL	Own capital availability ($)
RICETRS	Rice transfer
OKATRS	Okra transfer
PEPTRAS	Pepper transfer
BBLSTRS	Bitter balls transfer
CASSTRS	Cassava transfer
COCOATRS	Cocoa transfer
COFFETRS	Coffee transfer
SCANETRS	Sugar cane transfer
PALMTR	Cultivated palm transfer
POTR	Indigenous palm (oil) transfer
POMAX	Maximum palm oil production
RUBTRS	Rubber transfer

GVTBMAX	Maximum borrowing from government ($)
CASMAX	Maximum cassava (lbs)
RICONS	Rice consumption requirement (lbs)
OKCONS	Okra consumption requirement (lbs)
PECONS	Pepper consumption requirement (lbs)
BBCONS	Bitter balls consumption requirement (lbs)
CVCONS	Cassava consumption requirement (lbs)
SCCONS	Sugar Cane Juice consumption requirement (gallons)
POCONS	Palm-oil consumption requirement (gallons)

	A	B	C	D	E	F	G	H
1				Labor Summary of Optimal Farm Plan				
2								
3				Male Labor			Female Labor	
4		Avail	Used	Idle	Hired	Avail	Used	Idle
5								
6	January							
7	February							
8	March							
9	April							
10	May							
11	June							
12	July							
13	August							
14	September							
15	October							
16	November							
17	December							
18								
19	Total							
20								
21								
22								
23								
24				Production Levels for Optimal Farm Plan				
25								
26					Family			
27			Quantity		Con-			Gross
28			Produced	Unit	sumption	Sales	$/Unit	Receipts
29								
30	Rice			Pounds				
31	Pepper			Pounds				
32	Cassava							
33	Cocoa							
34	Coffee							
35	Sugar cane							
36	Okra							
37	Bitter-balls							
38	Rubber							
39	Palm							
40	Palm Oil							
41	Jan							
42	Feb							
43	Mar							
44	Apr							
45	May							
46	Jun							
47	Jul							
48	Aug							
49	Sep							
50	Oct							
51	Nov							
52	Dec							
53	Total Palm Oil							
54								
55								

Table 1. Tableau of the Liberian Farm Linear Programming Model

NIMBA	MAX	B	RICEOX	RICEPEP	RICEBBLS	RICECAS	SMRICE	CASSAVA	COCOA	COFFEE	SCANE	OKRA	PEPPER	BBALLS	RUBBER	RICESSC	PALM	POJAN	POFEB	POMAR	POAPR	POMAY	POJUN	POJUL	POAUG	POSEP	POOCT
C			-57.5	-57.5	-57.5	-57.5	-45.61	-8.65	-21.11	-22.1	-144.58	-3.85	-4.01	-3.41	-62	-202.08	-27	-0.65	-0.65	-0.65	65	-0.65	-0.65	-0.65	-0.65	-0.65	-0.65
NLJAN	L	50	6	6	6	6	3		1.36	1.36					5.05	6		1									
NLFEB	L	50	8	8	8	8		8	0.8	1.2	10				6.18	8			1								
NLMAR	L	50	5	5	5	5	10	12	0.44	0.44	19				5.24	21	3.6			1							
NLAPR	L	50	11	11	11	11	4	5	2	2.04	20				5.16	24	0.3				1						
NLMAY	L	50	10	10	10	10	7		0.8	0.6	20				6.38	25	5.4					1					
NLJUN	L	50	10	10	10	10	3		1.24	1.24	4				7.29	10	1.5						1				
NLJUL	L	50					3.5		0.56	0.4					5.19		2.4							1			
NLAUG	L	50					2		0.68	0.28					4.91										1		
NLSEP	L	50	1	1	1	1			1	1.2					6.16	1	1.5									1	
NLOCT	L	50	2	2	2	2			0.6	0.52					4.91	2	1										1
NLNOV	L	50	2	2	2	2	2		1.04	1.6					8.41	2	5.5										
NLDEC	L	50					5	2	0.4	0.28	3				5.19	3											1
FLJAN	L	50	2	2	2	2	7					7.5	7.5	7.5		2		1									
FLFEB	L	50					5												1								
FLMAR	L	50																		1							
FLAPR	L	50	5	5	5	5						10	10	10		5					1						
FLMAY	L	50	4	4	4	4						10	10	10		4						1					
FLJUN	L	50	3	3	3	3	3	10								3							1				
FLJUL	L	50	7	7	7	7	3	6								7								1			
FLAUG	L	50	6	6	6	6	4	4								6									1		
FLSEP	L	50	2	2	2	2	8		0.6			8	8	8		2										1	
FLOCT	L	50	8	8	8	8	4		0.8	0.2		20	20	20		8											1
FLNOV	L	50	6	6	6	6	6		0.8	0.8						6											
FLDEC	L	50	6	6	6	6	10		0.48	0.2						6											1
LAND	L	10	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1										
CAPITAL	L	170	28.75	28.75	28.75	28.75	26.6	6.49	10.55	11.05	10.31	0.96	2	0.85	22	39.06											
RICEYRS	L	0	-960	-960	-960	-960	-1100									-960											
OKRAYRS	L	0	-100									-800															
PEPYRS	L	0		-150									-800														
BBLSYRS	L	0			-100									-800													
CASSYRS	L	0				-3000		-6000								-3000											
COCOAYRS	L	0							-400																		
COFFEYRS	L	0								-450																	
SCANEYRS	L	0									-80					-90											
PALMYR	L	0															-5.5										
POYR	L	0																-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5
POMAX	L	31																2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
RUBYRS	L	0													-985												
GVTRMAX	L	1000																									
CASMAX	L	351				3000		6000								3000											
RICCONS	G	1000																									
OKCONS	G	96																									
PECONS	G	128																									
BBCONS	G	96																									
CVCONS	G	270																									
SCCONS	G	20																									
PCCONS	G	24																									

Table 1. Tableau of the Liberian Farm Linear Programming Model (Continued)

BIMBA	POBOV	PODEC	RISELL	OESELL	PEPSELL	BESELL	CASSELL	COCSELL	COFSELL	CAMESELL	RUBSELL	PALMSL	POSELL	JMLBY	FMLBY	MMLBY	APMLBY	MYMLBY	JMMLBY	JLMLBY	AGMLBY	SPMLBY	OCMLBY	NOMLBY	DEMLBY	GVTCAP
C	-0.65	-0.65	6.12	0.11	0.1	0.1	0.05	0.45	0.55	5	0.32	30	4	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-0.15
HLJAN														-1												
HLFEB															-1											
HLMAR																-1										
HLAPR																	-1									
HLMAY																		-1								
HLJUN																			-1							
HLJUL																				-1						
HLAUG																					-1					
HLSEP																						-1				
HLOCT																							-1			
HLNOV	1																							-1		
HLDEC		1																							-1	
FLJAN																										-1
FLFEB																										
FLMAR																										
FLAPR																										
FLMAY																										
FLJUN																										
FLJUL																										
FLAUG																										
FLSEP																										
FLOCT																										
FLNOV	1																									
FLDEC		1																								
LAND																										
CAPITAL																										
RICETRS			1																							-1
OBATRS				1																						
PEPTRAS					1																					
BELTRS						1																				
CASTRS							1																			
COCONTRS								1																		
COFFETRS									1																	
SCAMETRS										1																
PALMTR												1														
POTR	-2.5	-2.5											1													
POMAX	2.5	2.5												1												
BUYTRS																										
GVTMAX											1															
CAMMAX																										
RICURS																										1
OBCURS																										
PECURS																										
BECURS																										
CUCURS																										
SCCURS																										
POCURS																										

Table 1. Tableau of the Liberian Farm Linear Programming Model (Continued)

RHSA	BORROW	RICEBY	RICTR	ORCTR	PECTR	BBCTR	CWCTR	SCCTR	PCCTR
C	-0.3	-0.3	0	0	0	0	0	0	0
MLJAN									
MLFEB									
MLMAR									
MLAPR									
MLMAY									
MLJUN									
MLJUL									
MLAUG									
MLSEP									
MLOCT									
MLNOV									
MLDEC									
FLJAN									
FLFEB									
FLMAR									
FLAPR									
FLMAY									
FLJUN									
FLJUL									
FLAUG									
FLSEP									
FLOCT									
FLNOV									
FLDEC									
LAND									
CAPITAL	-1								
RICETRS		-1	1						
OKRATRS				1					
PEPTRAS					1				
BBLPTRS						1			
CASSTRS							1		
COCONS.S									
COFFETRS									
SCABETRS								1	
PALMTR									
POUR									1
POUR									
BBETRS									
OPTRMIX									
CASMIX									
RICONS			1						
OKCONS				1					
PECONS					1				
BBCONS						1			
CWCONS							1		
SCCONS								1	
PCCONS									1

Table 2. Solution to the Liberian Farm Linear Programming Model

OPTIMAL S o l u t i o n
function Value:1761.221

Activities in the Final Solution

Returns	Name	Type	Level
-45.61	SWRICE	real	1.61
-22.1	COFFEE	real	2.58
-144.58	SCANE	real	0.13
-3.85	OKRA	real	0.12
-4.01	PEPPER	real	0.16
-3.41	BBALLS	real	0.12
-62	RUBBER	real	5.04
-202.08	RICSSC	real	0.12
-0.65	POJAN	real	5.30
-0.65	POAPR	real	6.81
-0.65	POJUN	real	0.29
0.05	CASSELL	real	81.00
0.45	COCSELL	real	0.00
0.55	COFSELL	real	1161.86
0.32	RUBSELL	real	4968.45
30	PALMSL	real	0.00
4	POSELL	real	7.00
-2.5	MYMLBY	real	0.56
-0.15	GVTCAP	real	18.72
0	RICTR	real	1880.00
0	OKCTR	real	96.00
0	PECTR	real	128.00
0	BBCTR	real	96.00
0	CVCTR	real	270.00
0	SCCTR	real	20.00
0	POCTR	real	24.00
0	MLJAN	slack	10.19
0	MLFEB	slack	13.46
0	MLMAL	slack	1.51
0	MLJUL	slack	17.16
0	MLAUG	slack	21.30
0	MLSEP	slack	15.71
0	MLOCT	slack	23.66
0	MLDEC	slack	14.31
0	FLJAN	slack	30.22
0	FLFEB	slack	41.97
0	FLMAR	slack	50.00
0	FLAPR	slack	38.61
0	FLMAY	slack	45.53
0	FLJUN	slack	44.54
0	FLJUL	slack	44.36
0	FLAUG	slack	42.87
0	FLSEP	slack	33.71
0	FLOCT	slack	34.12
0	FLNOV	slack	37.59
0	FLDEC	slack	32.71
0	GVTBMAX	slack	981.28

Activities Not in the Solution

Name	Obj Val	Type	Shad.Price
RICEOK	-57.5	real	28.690
RICEPEP	-57.5	real	14.663
RICEBBLS	-57.5	real	28.747
RICECAS	-57.5	real	12.500
CASSAVA	-8.65	real	140.864
COCOA	-21.11	real	65.972
PALM	-27	real	104.447
POFEB	-0.65	real	0.000
POMAR	-0.65	real	0.000
POMAY	-0.65	real	2.500
POJUL	-0.65	real	0.000
POAUG	-0.65	real	0.000
POSEP	-0.65	real	0.000
POOCT	-0.65	real	0.000
PONOV	-0.65	real	1.719
PODEC	-0.65	real	0.000
RISELL	0.12	real	0.144
OKSELL	0.11	real	0.169
PEPSELL	0.1	real	0.180
BBSELL	0.1	real	0.179
CANESELL	5	real	0.195
JMLBY	-2.5	real	2.500
FMLBY	-2.5	real	2.500
MRMLBY	-2.5	real	2.500
APMLBY	-2.5	real	2.500
JNMLBY	-2.5	real	2.500
JLMLBY	-2.5	real	2.500
AGMLBY	-2.5	real	2.500
SPMLBY	-2.5	real	2.500
OCMLBY	-2.5	real	2.500
NOMLBY	-2.5	real	0.781
DEMLBY	-2.5	real	2.500
BORROW	-0.3	real	0.150
RICEBY	-0.3	real	0.036
MLAPR	0	slack	0.000
MLMAY	0	slack	2.500
MLJUN	0	slack	0.000
MLNOV	0	slack	1.719
LAND	0	slack	219.492
CAPITAL	0	slack	0.150
RICETRS	0	slack	0.264
OKRATRS	0	slack	0.279
PEPTRAS	0	slack	0.280
BBLSTRS	0	slack	0.279
CASSTRS	0	slack	0.050
COCOATRS	0	slack	0.450
COFFETRS	0	slack	0.550
SCANETRS	0	slack	5.195
PALMTR	0	slack	30.000
POTR	0	slack	4.000
POMAX	0	slack	3.740
RUBTRS	0	slack	0.320
CASMAX	0	slack	0.035
RICONS	0	slack	0.264
OKCONS	0	slack	0.279
PECONS	0	slack	0.280
BBCONS	0	slack	0.279
CVCONS	0	slack	0.050
SCCONS	0	slack	5.195
POCONS	0	slack	4.000

ANALYSIS OF RISK IN THE FARM MODEL

References: Hazell, B.R. and R.D. Norton. "Risk in the Farm Model." Chapter 5 in *Mathematical Programming for Economic Analysis*.

Anderson, J.R., J.L. Dillon and B. Hardaker. "Whole Farm Planning Under Risk." Chapter 7 in *Agricultural Decision Analysis.*

Objectives

The objectives of this section are:

a) To review measures of risk and the principles of diversification for risk reduction
b) To present selected criteria for using linear programming to make efficient decisions in risky situations
c) To examine how the optimal mix of production activities varies with the degree of risk aversion of the decision maker.

Introduction

The suggested outline for this section includes a brief review of approaches to risk analysis but the main concentration will be on the use of the MOTAD Linear Programming model which was introduced by Hazell in 1971.

The following linear programming approaches to risk analysis in the farm model will be reviewed

1) Maximin Criterion,
2) MOTAD (Minimization of Total Absolute Deviations), and
3) Target MOTAD.

Risk and uncertainty—Frank Knight. An early classical work on risk and uncertainty was done by Frank Knight. Knight defined risk as the case where outcomes were variable but where the parameters of the distribution (mean, variance, skewness) could be empirically determined. Uncertainty referred to outcomes where probabilities of an event occurring could not be established empirically. However, since very little practical work could be done with uncertain situations, modern analysts use subjective probabilities to convert uncertain situations to risky situations and go ahead. Of course such assumptions should be fully documented.

Measures of variability. The most common measures of variability are the variance or standard error and the sum or mean of the absolute deviations.. With a set of yields observed over a 5 year period, the estimates of the above parameters are

Year	Yield	Yield Squared		Absolute Deviation $\lvert Y_i - \bar{Y} \rvert$
Year 1	25	625		2.8
Year 2	39	1521		11.2
Year 3	10	100		17.8
Year 4	45	2025		17.2
Year 5	20	400		7.8
Sum	139	4671	Tot.Abs.Dev=	56.8
Mean	27.8		MAD=	11.36

σ^2= 201.7

σ= 14.20211

$$\sigma^2 = \frac{(\Sigma\, Y_i^2 - (\Sigma\, Y_i)^2/T)}{(T-1)}$$

$$\text{Tot.Abs.Dev.} = \Sigma \lvert Y_i - \bar{Y} \rvert$$

$$\sigma = \text{SQRT}(\sigma^2)$$

$$\text{MAD} = \frac{\text{Tot.Abs.Dev}}{T}$$

The MAD (mean absolute deviation) estimator of the variability is similar in magnitude to the standard error. MAD is a simple mean of deviations while the standard error is a geometric mean. The MAD estimator is readily used in linear programming models, while the variance approach requires the use of more complicated routines like quadratic programming. When the distribution is normal, the variance and MAD estimators are related as

$$\text{Var} = F * \text{MAD}^2 = \frac{T * \pi}{2(T-1)} (\text{MAD}^2).$$

Measuring Income Deviations in the Linear Programming Model

The linear programming techniques which follow involve including income for various time periods as rows in the linear programming model.

Calculating annual farm income in the model. The year by year variability of the net returns (objective function coefficients) of the activities from the Hazell and Norton example are shown below. In a model in which expected income is maximized, the average values would be entered in the objective function.

Year	Activity			
	X1	X2	X3	X4
YEAR1	292	-128	420	579
YEAR2	179	560	187	639
YEAR3	114	648	366	379
YEAR4	247	544	249	924
YEAR5	426	182	322	5
YEAR6	259	850	159	569
	---	---	---	---
AveInc	252.83	442.67	283.83	515.83

The expected total farm income in year one is obtained by multiplying the earnings of each activity by the number of units of that activity

Farm Income Year 1 = 292*X1 - 128*X2 + 420*X3 + 579*X4.

And,

Farm Income Year 2 = 179*X1 + 560*X2 + 187*X3 + 639*X4.

If the annual levels of farm income are entered into the model as separate constraints or rows, then the model is capable of calculating the income generated each year by a particular farm plan and is capable of measuring the difference between income levels.

Deviations in income levels. Some of the models discussed below require deviations in income be entered in the tableau. The amount by which farm income in each year is above or below average income is determined by first calculating the amount by which each activity has income above or below average and multiplying by the number of units of that activity. The sum over all activities gives the deviation in total farm income for that year.

Deviations From Average Income For Each Activity

	X1	X2	X3	X4
YEAR1	39.17	-570.67	136.17	63.17
YEAR2	-73.83	117.33	-96.83	123.17
YEAR3	-138.83	205.33	82.17	-136.83
YEAR4	-5.83	101.33	-34.83	408.17
YEAR5	173.17	-260.67	38.17	-510.83
YEAR6	6.17	407.33	-124.83	53.17
AveDev	0.00	0.00	0.00	0.00

For year one, the amount by which total farm income deviates from mean farm income for a given value of X1 through X4 is

Dev Yr1 = 39.17*X1 - 570.67*X2 + 136.17*X3 + 63.17*X4.

A similar calculation for years 2-6 would measure the deviations in farm income over the six year historical period.

A Game Theory Model: Maximin Criterion

Objective

Select the farm plan that provides the highest expected income under the worst circumstance or state of nature. The model is to find the highest income under the worst circumstances.

The model can be stated as Max M

subject to $\sum_j c_{jt} X_j \geq M$, for all t

$\sum_j c_j X_j = \lambda$ (a parametric income constraint)

$\sum_j a_{ij} X_j \leq b_i$ for all i

$X_j, M \geq 0.$

The model is shown below in Tableau form. The modeling approach differs from the standard LP modeling approach in that

1) A series of solutions will be obtained. The utility function of the decision maker is affected a) positively on the level of expected income and b) negatively by an increase in the variability of income. Thus we obtain a series of solutions which trace out an M-Expected income frontier.

2) The objective function contains a single coefficient for the variable M. M is constrained to be no greater than the lowest income for a specified expected level of income.

Table 1. Linear Programming Tableau for the MAXIMIN Problem

Maximin	MAX	RHS	X1	X2	X3	X4	M
OBJ			0	0	0	0	1
EXPINC	E	65000	253	443	284	516	
LAND	L	200	1	1	1	1	
LABOR	L	10000	25	36	27	87	
ROTREQ	L	0	-1	1	-1	1	
YEAR1	G	0	292	-128	420	579	-1
YEAR2	G	0	179	560	187	639	-1
YEAR3	G	0	114	648	366	379	-1
YEAR4	G	0	247	544	249	924	-1
YEAR5	G	0	426	182	322	5	-1
YEAR6	G	0	259	850	159	569	-1

In the above tableau, the expected income is constrained to be equal to some feasible value. The maximization of M leads to the farm plan which has the maximum worst income for a given level of expected income. For example an average income of 65,000 pesos can be obtained with the lowest income in a single year being 60,163 pesos. However an average income of 77,996 pesos cannot be obtained without an annual income one year in six being as low as 37,559 pesos. The alternative solutions which trace out the E-M curve are determined by repeatedly changing the constraint on expected income and resolving the problem. The solution with the highest income is the normal profit maximizing solution. The results with series of 5 solutions (which are similar to the text) are

Farm Plan	I	II	III	IV	V
Exp Inc	65000	70000	73000	75000	77996
M(pesos)	60163	56203	52274	47707	37559
X1 (ha)	85.91	112.18	101.30	100.00	0.00
X2 (ha)	30.55	37.85	48.73	26.03	27.45
X3 (ha)	55.50	4.03	0.00	0.00	100.00
X4 (ha)	27.08	45.94	49.97	73.97	72.55

The MOTAD Approach

The MOTAD approach was proposed by Hazell in 1971. MOTAD stands for Minimization Of Total Absolute Deviations. This model is a linear programming approximation of the Mean-Variance programming model. Instead of tracing out an E-V

frontier, the MOTAD approach will derive an E-A frontier. (As pointed out above, when the distributions are normal, the the MAD estimator is equivalent to the Variance estimator.)

Again the problem requires a series of solutions. One approach is to specify the level of income and then minimize the annual deviations associated with the income. The level of income is revised, and the problem is resolved. The solution with the maximum income and maximum level of deviations is the standard profit maximizing solution. The E-M frontier can also be traced out by setting the amount of annual income fluctuation, and maximizing income. The variability constraint is revised and another solution is obtained.

The name MOTAD implies model is designed to minimize Total Deviations from a mean income. However it is more efficient to minimize only the negative income deviations (since the sum of the deviations below the mean are equal to the sum of the deviations above the mean). Accordingly the model can be specified as

$$\text{Min} \quad .5\sqrt{W} = \sum_t Z_t$$

$$\text{subject to} \quad \sum_j (\bar{C}_{jt} - C_j)X_j + Z_{\bar{t}} \geq 0 \quad \text{all } t$$

$$\sum_j \bar{C}_j X_j = \lambda \quad \text{(an income parameter)}$$

$$\sum_j A_{ij} X_j \leq b_i$$

The E-M curve for the example in Hazell and Norton is derived and discussed below. First the LP solutions to trace out the E-M curve were obtained through the use of a series of solutions in which the income constraint was varied from 62000, 66000, 70000, 74000 and 77996 pesos. Note these values differ from the income levels used in the text. Ideally parametric programming would be used for this purpose but the MUSAH86 routine does not have this option. The tableau for the MOTAD model is shown below.

MOTAD	MIN	RHS	X1	X2	X3	X4	Z1	Z2	Z3	Z4	Z5	Z6
OBJ			0	0	0	0	1	1	1	1	1	1
ExpGM	E	62000	253	443	284	516						
LAND	L	200	1	1	1	1						
LABOR	L	10000	25	36	27	87						
ROTATE	L	0	-1	1	-1	1						
YEAR1	G	0	39	-570	136	63	1					
YEAR2	G	0	-74	117	-97	123		1				
YEAR3	G	0	-138	205	82	-137			1			
YEAR4	G	0	-6	101	-35	408				1		
YEAR5	G	0	173	-261	38	-510					1	
YEAR6	G	0	6	408	-124	53						1

The LP problem in the above tableau is to minimize the sum of the variables Z1 through Z6 subject to meeting the other constraints in the model. One of the constraints in the model is that farm income be equal to some specified value. Each Z variable measures the negative deviations in total farm income for a single year. The solution of the problem will determine the farm plan which will give the minimum amount of variability (estimated by deviations) for the required level of farm income. For year 6 for example, if we omit the variable Z6, the last constraint requires,

$$6*X1 + 408*X2 - 124*X3 + 53*X4 \geq 0.$$

If farm income in year 6 is above average the constraint holds with the inequality and the variable Z6 would not enter the solution. However if income is below average, then Z6 enters the solution and the constraint holds as an equality,

$$6*X1 + 408*X2 - 124*X3 + 53*X4 + Z6 = 0,$$

as the variable Z6 measures the amount by which income falls below average. The objective is to find a farm plan which provides the required level of income, meets the other resource constraints and which has the minimum amount of variability.

The results are summarized in Table 2 below.

Table 2. Results of the LP Solution Used to Trace Out the E-M Curve from the Hazell and Norton MOTAD Example

	Basic Solution				
	I	II	III	IV	V
Exp.Inc	62000	66000	70000	74000	77996
Dev(objval)	8132	14076	21338	28934	40364
Ave Ann. T.Dev	2711	4692	7113	9645	13455
X1	71.59	60.72	46.08	31.44	0.00
X2	26.46	27.17	27.63	28.10	27.45
X3	82.56	82.96	82.17	81.38	100.00
X4	16.89	29.15	44.11	59.08	72.55

Note that the treatment of the deviations in Table 1 differs from that in the text. If desired, the deviations of income in the objective function can be converted to a Variance of income.

$$\text{Var} = \frac{F\ (2 * \text{Obj value})^2}{T^2} \qquad \text{Where } F = \frac{T\ \pi}{2\,(T-1)}$$

The curve showing the maximum expected income for each level of income variability is plotted in the top half of Figure 1 using the risk-return data in Table 2. The changes in the cropping pattern which result from a systematic variation of the level of acceptable risk are plotted in the lower portion of Figure 1. Note that crop X1 enters the solution under extreme risk averse circumstances but steadily declines in less risk averse solutions. Conversely activity X4 steadily increases in solutions as both the level of income and risk are increased. This should raise some questions as to why the cropping pattern changes in the manner shown in Figure 1.

The effect of relaxing the risk constraint (becoming more risk neutral), on the mix of activities can be understood by examining

a) The amount of risk activity per activity unit and the amount of risk per dollar of expected income from the alternative enterprises.

b) The correlation between net returns from the various activities. (A principle of diversification is that total risk is less when enterprises whose returns are negatively correlated are combined).

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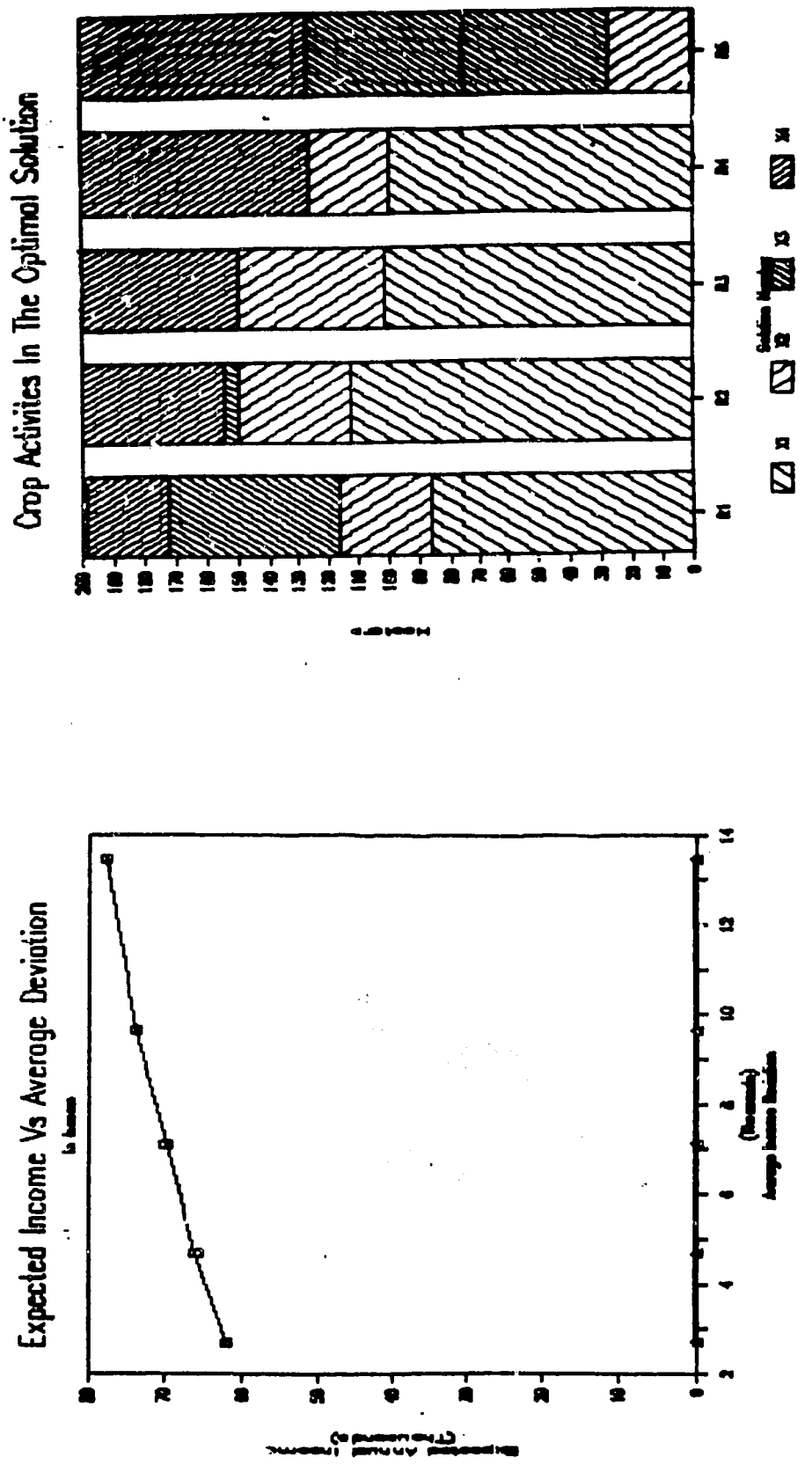

Figure 1. Expected Income vs. Annual Income Deviations and Activity Levels from the Solutions to the MOTAD Problem.

The first concept can be seen by examining the coefficient of variation of income. Its counterpart (the sum of the negative deviations divided by the expected income) is the concept of risk used in the model. These values are shown in Table 3

Table 3. Measures of Risk Per Dollar of Expected Income from the Alternative Activities in the Model

	Activity			
	X1	X2	X3	X4
Mean income/ha	253	443	284	516
Stand Dev	15.91	21.05	16.85	22.72
Coefficient of Variation	6.29	4.75	5.93	4.40
		Correlation		
X1	1.000	-0.547	0.133	-0.483
X2		1.000	-0.750	0.271
X3			1.000	-0.349
X4				1.000

Coefficient of Variation is the st. error / mean.

The measures shown in top of Table 3 reflect both the amount of total variability and the amount of variability per dollar of expected income. The results indicate that the income from activities X2 and X4 is more stable (has a lower coefficient of variation) than the income from activities X1 and X3. In general, the activity which provides the greatest returns to the most limiting constraint tends to enter the solution. When risk aversion is great, the most profitable activity X4 is less competitive because it has the greatest amount of variability (note the standard error) per activity unit. As successively higher levels of income are required, X4 increases in the solution. Additional insights can be gained by examining the correlation matrix between the net returns from the activities. The correlation matrix (Table 3.) shows that the net profits from activities X1 and X4, from X2 and X3 and from X4 and X3 are negatively correlated. Thus if the producer is very risk averse, these correlations are very important and he would tend to diversify by planting those crops which are negatively correlated. As the producer moves toward a risk neutral profit maximizing position these correlations become less important. Note that labor

limitations and rotation restrictions also affect the solution. In fact rotations which are specified for the sole purpose of risk diversification become redundant in a MOTAD model.

Target MOTAD

The target MOTAD belongs to the class of safety first models. Safety first models are designed to insure that the producer first meets goals related to survival such as food for the family, sufficient income to meet mortgage and other fixed cost obligations.

Target MOTAD has also received considerable attention in the literature because of its relationship to stochastic dominance techniques.

The Target MOTAD was developed by Tauer in 1983. The model calls for maximization of expected income subject to the requirement that income deviations below the target income not exceed some specified level. The normal farm model programming constraints must also be met. The formal model can be stated as

$$\text{Maximize } E = \sum_j \bar{C}_j X_j$$

subject to

$$Y_0 - \sum_j C_{jt} X_j - Z_t \leq 0 \quad \text{all } t$$

$$\sum_j p_t Z_t \leq \lambda$$

$$\sum_j a_{ij} X_j \leq b_i \quad \text{all } i$$

$$X_j, Z_t \geq 0 \quad \text{all } j, t.$$

Where

Y_0 is target income

Z_t are negative deviations

$\bar{C}_j$ is average income per activity unit

p_t is a probability

Pt is the probability of the events recorded in year t reoccurring at a future time. The LP model used in the previous examples which has been revised for the Target MOTAD approach is shown below in Table 4.

Table 4. Target MOTAD Linear Programming Model

MOTAD	MIN	RHS	X1	X2	X3	X4	Z1	Z2	Z3	Z4	Z5	Z6
OBJ			253	443	284	516						
LAND	L	200	1	1	1	1						
LABOR	L	10000	25	36	27	87						
ROTATE	L	0	-1	1	-1	1						
ExpSho	E	0					0.16	0.16	0.16	0.16	0.16	0.16
YEAR1	G	55000	292	-128	420	579	1					
YEAR2	G	55000	179	560	187	639		1				
YEAR3	G	55000	114	648	366	379			1			
YEAR4	G	55000	247	544	249	924				1		
YEAR5	G	55000	426	182	322	5					1	
YEAR6	G	55000	259	850	159	569						1

The MOTAD problem shown in Table 4 has a target income of 55000 pesos which has been entered in the constraint rows for each of the years. The constraints shown at the bottom of Table 4 are the constraints which require the difference between the target income (Y_0) and the estimated farm income for year to be less than the deficit Zt.

$$Y_0 - \sum_j c_{jt} x_j \leq z_t$$

or which can be rewritten as

$$Y_0 \leq \sum_j c_{jt} x_j + z_t .$$

In the latter form the constraint requires that estimated farm income plus the deficit be greater than or equal to the target income. The constraint becomes an equality if estimated income fall below the target income by an amount Z_t.

The constraint

$$\sum_t p_t z_t = \lambda$$

places an upper limit on the amount of total deficit which may be accumulated over the historical period. Another feature of the above constraint is that a probability pt is used which relates to the frequency with which an observed state is expected to occur. In the example in table 4, the events in the 6 years are expected to occur with equal frequency

so a probability of 1/6 is assigned to each year. (Note this concept can be extended to the previous models as well).

The objective of the model then is to maximize expected income subject to the constraint that actual income does not deviate below the target income by more than λ. The usual LP constraints on land, labor and capital must also be satisfied.

The results of four target MOTAD solutions where expected average total deviations from the target income were constrained to be less than or equal to 0, 1000, 2000, and 3000 pesos are shown below in Table 5.

Table 5. Target MOTAD Solutions To Hazell and Norton Problem

	Solution			
	I	II	III	IV
Exp.Inc.	71003	74467	76419	77996
Max.Def.	0	1000	2000	
X1	111.07	100.00	56.06	0.00
X2	40.91	33.33	25.25	27.45
X3	0.00	0.00	43.94	100.00
X4	48.02	66.67	74.75	72.55

Exercise

Complete the MOTAD worksheet as shown in Figure 2 using the following procedures.

1. Type in the worksheet area A1..E13. The averages should be computed with the @avg function in Lotus. The appropriate entry in Cell B12 is:

   @AVG(B5.B10)

   After cell B12 is entered, use the /Copy command to copy B12 to C12.E12.

2. Now type in the LP tableau from A15..M26. Use formula reference to the gross margins tables as much as possible. For example, cell D17 should be defined as +B12. The /Copy command can then be used to copy this formula to E12..G12.

3. The area D21..G26 should again be computed by formula reference to the gross margins section. Cell D21, for example, should be computed by the formula +B5-B$12. This can then be copied to the area D21..G26 using the /Copy command. (Make sure you understand why there should be a $ sign in front of 12 in B$12. What would happen if we don't have the $ sign there?)

4. Save this worksheet. This will be a Musah master tableau.

5. Solve this tableau for expected incomes of 60000, 64000, 68000, 72000 and 76000. Compare the changes in average annual total deviations and cropping patterns.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Gross Margins												
2													
3		X1	X2	X3	X4								
4													
5	YEAR 1	292	-127	420	579								
6	YEAR 2	179	560	187	639								
7	YEAR 3	115	648	366	379								
8	YEAR 4	247	544	249	924								
9	YEAR 5	426	182	322	6								
10	YEAR 6	259	851	160	569								
11													
12	Average	253	443	284	516								
13													
14													
15	MOTAD	MIN	RHS	X1	X2	X3	X4	Z1	Z2	Z3	Z4	Z5	Z6
16	OBJ							1	1	1	1	1	1
17	EXPGM	E	60000	253	443	284	516						
18	LAND	L	200	1	1	1	1						
19	LABOR	L	10000	25	36	27	87						
20	ROTATE	L	0	-1	1	-1	1						
21	YEAR1	G	0	39	-570	136	63	1					
22	YEAR2	G	0	-74	117	-97	123		1				
23	YEAR3	G	0	-138	205	82	-137			1			
24	YEAR4	G	0	-6	101	-35	408				1		
25	YEAR4	G	0	173	-261	38	-510					1	
26	YEAR6	G	0	6	408	-124	53						1

Figure 2. Master Tableau for Risk Analysis in a Lotus Spreadsheet.

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PART II: THE SECTOR MODEL

INTRODUCTION TO SECTOR PROGRAMMING MODELS

References: Hazell, B.R. and R.D. Norton, *Mathematical Programming For Economic Analysis in Agriculture*, Chapter 7.

Heady & Srivastava *Spatial Sector Programming Models in Agriculture* Chapter 1. Review other chapters for examples of applied studies.

Mathematical programming has been used to simulate market behavior since Samuelson noted that the maximization of consumer plus producer surplus lead to competitive equilibrium solution. The initial mathematical programming studies were limited but models have expanded in size and complexity as researchers gained experience, as computer size increased, and as computer software has become more reliable.

Elements of the Sector Model

Hazell and Norton indicate the sector model contains five components. These are:

1. Description of types of economic behavior (profit maximization, risk aversion, goals such as home consumption).
2. Description of production technology available to producers.
3. Description of Resources
4. Specification of the market environment (perfect competition, monopoly, monopolistic competition, access to interregional or international trade)
5. Specification of the policy environment for the sector (taxes, subsidies, price supports, import quotas tariffs).

Structure of the Sector Model

Objective Function. The objective function of the programming model which is maximized or minimized by setting the levels of the variables in the model. In the case of the regional model, the objective function is driven by factors which influence the supply equations (costs of production) and consumer demand. Most commonly the objective is

to find the prices and quantities of the various commodities which are consistent with a competitive equilibrium.

Algebraic Form of the Sector Model

Objective function.

$$Z = [\Sigma_j f(D_j) \text{ or } \Sigma_j f(D_{jr})] - \Sigma_j \Sigma_t \Sigma_r \Sigma_h c_{kjtrh} X_{jtrh}$$

Resource Constraints

$$\Sigma_j \Sigma_t a_{kjtrh} X_{jtrh} \leq b_{krh}$$

where
- j = commodity index
- t = technology index
- h = farm type (small, medium, etc)
- r = region
- k = resource type (land class, labor type, capital)

Coefficients and Variables

X_{jtrh} is one unit of an enterprise which produces commodity j with technology t, in region r by household type h.

C_{kjtrh} is the variable cost of production of commodity j using technology t, in region r by household h.

a_{jtrhk} is the amount of resource k required the production of one unit of commodity j using technology t, in region r, by household type h.

Y_{jtrh} is the quantity j resulting from one unit of (X_{jtrh}) commodity j, technology t, in region r by household h.

b_{krh} is the amount of resource k in region r held by household h.

D_j or (D_{jr}) is the demand function for commodity j (in region r)

Commodity Balances

$$- \Sigma_t \Sigma_r \Sigma_h Y_{jtrh} X_{jtrh} + D_j <= 0$$

Matrix Form of the Sector Model

In matrix tableau form, a two region model with no imports or exports would appear as

Activities	X1 Region 1	X2 Region 2	D Demand	
OBJ	-C1	-C2	f(Dj)	
Resource1	A1			≤ B1
Resource2		A2		≤ B2
Com.Bal	-Y1	-Y2	I	≤ 0

Sector Programming Models with Fixed Demands

We will begin with a review of programming models with fixed demand quantities. We will then continue with techniques to incorporate demand equations in the programming models.

Objectives

The objectives of the models with fixed point demand equations vary but usually the objectives are related to finding the simultaneous location of production and transportation that will minimize the total cost of production and transportation necessary to meet fixed or predetermined level of final demand. More advanced models use non-linear programming or linear programming approximations to problems involving price variable consumer demands.

More specific objectives would include:

1. To determine the cost of meeting final demands of various commodities.
2. To determine the subregions with the greatest comparative advantage in producing various commodities.
3. To determine the pattern of transportation consistent with meeting final demands of all commodities of least cost.
4. To determine competitive rents for fixed resources (land) and competitive prices for commodities in the model.

Historical Review

The first substantial Agricultural Sector Model was formulated and solved by Egbert and Heady. The model was rather simple. It dealt with finding the pattern of production (location and amount of production) of wheat for food, wheat for feed and feed grains that would meet final consumption needs for the United States plus export demands. Accordingly the crop producing areas of the U.S. were divided into 104 homogeneous producing areas. The producing areas are shown in Figure 1. The objective is to define a region area so that it is as homogeneous as possible with respect to major soil type and climate. It is also desirable that producing areas do not cross political boundaries.

The basic regional linear programming model developed by Egbert and Heady can be stated as

$$\text{Min } Z = \sum_{k=1}^{104} \sum_{q=1}^{3} C_{kq} \; A_{kq}$$

where

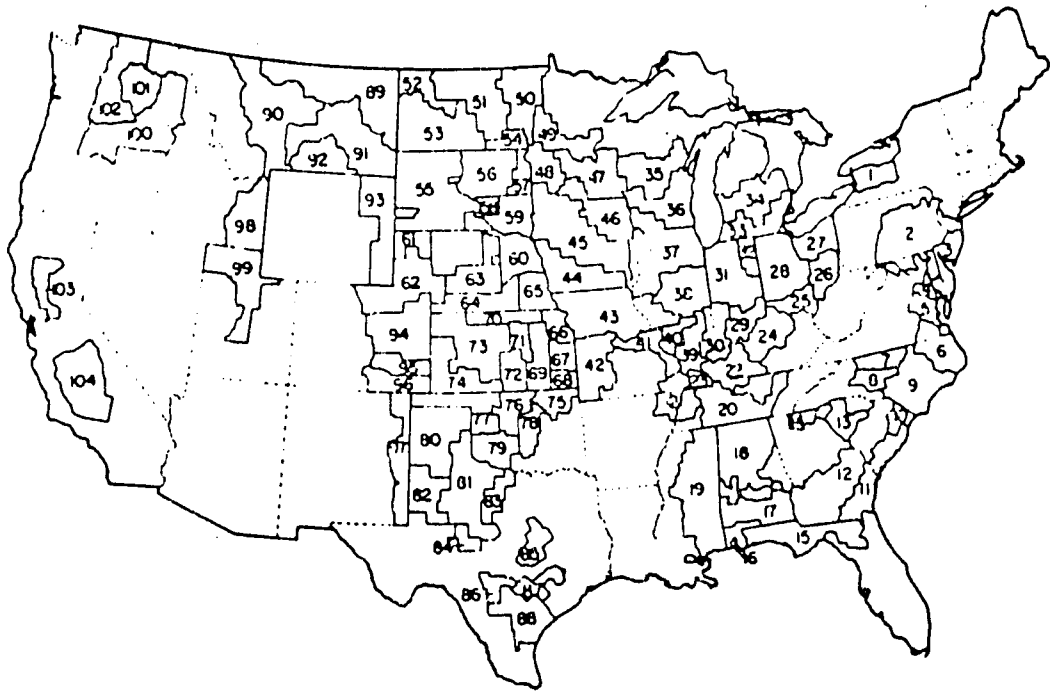
q=1 wheat for food
q=2 wheat for feed
q=3 feed grains

subject to $\sum_{q=1}^{104} A_{kq} <= L_k$ Land restraints

$$\sum_{k=1}^{104} b_{k1} \; A_{k1} = R_1 \text{ Wheat for food}$$

$$\sum_{k=1}^{104} b_{k2} \; A_{k2} = R_2 \text{ Feed grains}$$

The Tableau for this model is shown below in Figure 2. The model is too simplistic in the consumption sector to be of current policy value but it represented a major effort (people said it couldn't be done) in the mid 50's. The limitation with the consumption equations is that they are national and do not represent the final commodities being delivered to any particular point and that demands are perfectly inealstic.



Source: Heady and Egbert. Chapter 1 in *Spatial Sector Programming Models in Agriculture.*

Figure 1. Producing Areas in Egbert Heady Study.

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	X11	X12	X13	X21	X22	X23	...	Xm1	Xm2	Xm3		
Obj	C11	C12	C13	C21	C22	C23	...	Cm1	Cm2	Cm3		
Rg1	1	1	1	0	0	0	...	0	0	0	≤	L1
Rg2	0	0	0	1	1	1	...	0	0	0	≤	L2
Rg3	0	0	0	0	0	0	...	0	0	0	≤	L3
	.	.	.	.	.	.	...	.	.	.	. .	.
	.	.	.	.	.	.	...	.	.	.	. .	.
	.	.	.	.	.	.	...	.	.	.	. .	.
Rg104	0	0	0	0	0	0	...	1	1	1	≤	L104
Food	Y11	0	0	Y21	0	0	...	Ym1	0	0	=	R1
Feed	0	Y12	Y13	0	Y22	Y23	...	0	Ym2	Ym3	=	R2

Figure 2. Design of the LP Tableau Egbert-Heady Feed Grain Study.

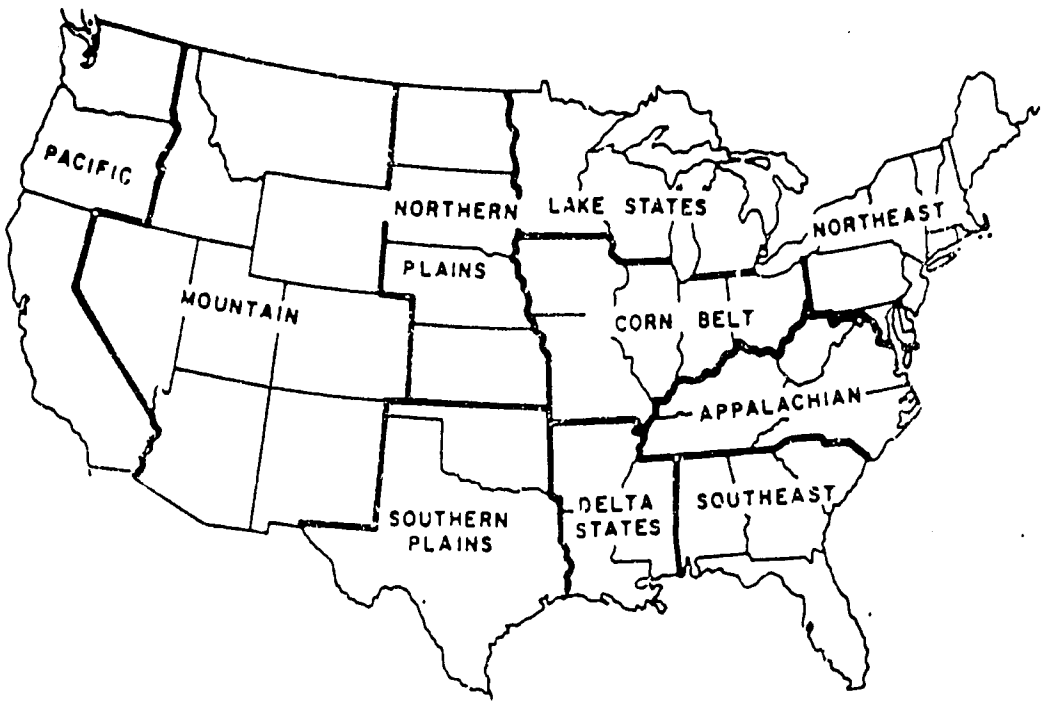
Consuming Regions

Consuming regions are defined when there are spatial differences between prices in various parts of a country because of transportation costs. This problem is modeled by defining separate geographic points or sub areas to represent points of consumption. A commodity balance equation for each commodity (which requires that the total supply of each commodity be greater than or equal to the demand for that commodity) is specified at each consumption point. We require production plus imports from other areas to be greater than or equal to consumption plus exports to other areas for each commodity.

In a fixed demand model, consumption of each commodity is usually calculated by multiplying per capita consumption by the number of people in a given region. As the number of both producing areas and the number of consuming regions is increased, the solutions become more realistic. Figure 3 shows a set of consuming regions.

Consumption is usually represented as accruing at a specified point (a large city) in each consuming region. Exports and imports take place at specified parts of entry. Defining specific consumption points allows the estimation of the cost of shipping each commodity from one point to another.

Figure 4 shows the pattern of shipment of food wheat to minimize the cost of production and transportation necessary to meet final demands for wheat for one solution.

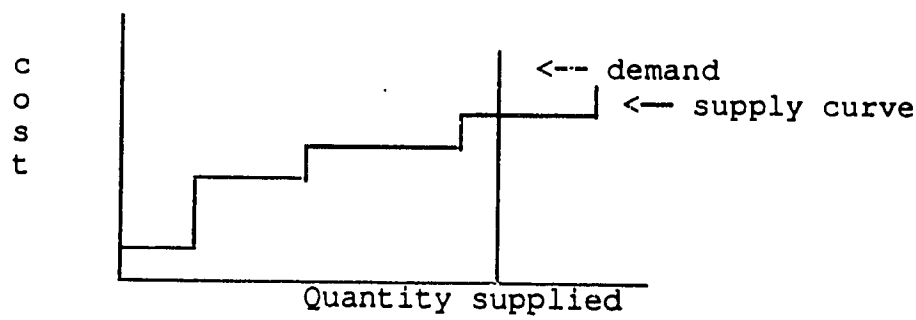


Source: Heady and Egbert. Chapter 1 in *Spatial Sector Programming Models in Agriculture.*

Figure 3. Typical Consuming Regions for Specifications of Regional Commodity Balance Equations.

The effects of changes in government policy, transportation costs, or producer technology are found by revising the relevant part of the model and obtaining a new solution.

The nature of competitive equilibrium prices determined by a model with fixed demands is illustrated below. Fixed demands are perfectly inelastic. The supply curve for a given commodity from a model with several producing areas tends to be stair stepped. Each step represents the cost of production in a given producing area. Production first occurs in the lowest cost producing region and proceeds to the highest cost region. The competitive equilibrium price necessary to induce producers to exactly meet the final demand is the shadow price from the demand row for that commodity.



Construction of a Spatial Problem
Comparative Advantage With Fixed Demands

Assume there are two regions which can each produce one or both of commodity A one or commodity B. The objective is to determine which region has the comparative advantage in the production of commodity A and which region has the comparative advantage in the production of commodity B. More specifically solution should give:

1. The amount of commodity A produced in region 1 and in region 2.
2. The amount of commodity B produced in region 1 and in region 2.
3. The supply price of commodity A in region 1 and in region 2.
4. The supply price of commodity B in region 1 and in region 2.
5. The annual rental value of land in region 1 and in region 2.
6. The Amount of Commodity A and Commodity B which is transported from region 1 to region 2 or from region 2 to region 1. Let

   DEa1 be the demand in Region 1 for commodity A
   DEb1 be the demand in Region 1 for commodity B
   DEa2 be the demand in Region 2 for commodity A
   DEb2 be the demand in Region 2 for commodity B.

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Production Activities, Production Costs, and Yields

Xa1 is the production of one acre of commodity A in region 1. The variable cost for producing one acre commodity A in region 1 is $10 and the Yield is 2 tons per acre.

Xb1 is the production of one acre of commodity B in region 1. The variable cost of production for commodity B in region 1 is is $15/acre. The yield is 1 ton/acre.

Xa2 is production of one acre of commodity 1 in region 2. The variable cost of producing commodity A in region 2 is $20/acre. The yield is 1.5 ton per acre.

Xb2 is production of one acre of commodity in region 2. The variable cost of producing commodity B in region 2 is $40/acre. The yield is 2 tons.

Resource Supplies

Land Region 1 = 40
Land Region 2 = 50

Transportation Activities and Transportation Costs

Let Ta12 be transportation of 1 ton of commodity A from region 1 to region 2. Let Sa12 be the cost of shipping one ton of commodity A from region 1 to region 2. Assume that Sa12 is $2/ton.

Let Ta21 be the transportation of one ton of commodity A from region 2 to region 1. Let Sa21 be the cost of shipping one ton of commodity A from region 2 to region 1. Assume that Sa21 is $2/ton.

Let Tb12 be the transportation of one ton of commodity B from region 1 to region 2. Let Sb12 be the cost of shipping one ton of commodity B from region 1 to region 2. Assume that Sb12 is $5/ton.

Let Tb21 be the transportation of one ton of commodity B from region 2 to region 1. Let Sb21 be the cost of shipping one ton of commodity B from region 2 to region 1. Assume Sb21 is $5/ton.

Final Demands

The final consumption requirements for products A and B in regions 1 and 2 are:

	region 1	region 2
commodity A	40	38
commodity B	20	50.

Albegric Statement of the Model

The spatial equilibrium problem with fixed demands can be stated as

Minimize total production and transportation cost subject to

1. Use of available resource supplies
2. Meeting final demands

Minimize $z = \Sigma_r \Sigma_j \; X_{rj} \; C_{rj} \quad + \Sigma_r \Sigma_j \Sigma_{k \atop j<>k} T_{rjk} \; S_{rjk}$

Subject to Land constraints

$$\Sigma_j \; X_{rj} \leq R_r$$

final demands

$$\Sigma_j \; X_{rj} \; Y_{rji} + \Sigma_k \; T_{irk} - \Sigma_k \; T_{ikr} \geq Dem_{ri}$$

Where

Dem_{ri} is the amount of commodity i required in region r.

R_r is the amount of land in region r,

X_{rj} is the number of acres used in production of the j'th activity in region r.

C_{rj} is the cost of one unit of X_{rj}

Y_{rji} is the production from one unit of X_{rj} (commodity i from activity j in region r)

T_{irk} is the amount of commidity i exported from region r to region k.

T_{jkr} is the amount of commidity i imported from region k into region r.

S_{irk} is the cost of shipping one unit of commidity i from region r to region k.

S_{ikr} is the cost of shipping one unit of commidity i from region k to region r.

The summary of the Minimization Tableau and the solution are shown in Table 1. the six points outlined above can be drawn from the solution which is summarized in Table 2.

Table 2. Summary of Results of the Regional Programming Example

Regional Production, Consumption, and Shipment

	Region 1			Region 2		
Commodity	Demand	Prod.	Net Xport	Demand	Prod.	Net Xport
A	40	78	38	38	0	(38)
B	20	1	(19)	19	6	19

Land Use

Supply	Used	Unused	Supply	Used	Unused
40	40	0	50	34.5	15.5

Resource and Product Prices (Calculated from Shadow Prices)

	Region 1	Region 2
	Product Prices	
Commodity A	10	12
Commodity B	25	20
	Resource Rental Values	
Land	10	0

Additional Questions About the Solution

1. What accounts for the difference in the price of commodity A between region 1 and region 2? What accounts for the difference in the price of commodity B in between regions 1 and 2?

2. What yield would producers in region 2 require before they could produce commodity A on a competitive basis?

USING DEMAND EQUATIONS IN A LINEAR PROGRAMMING MODEL

References: Duloy, J.H. and R. D. Norton. "Prices and Incomes in Linear Programming Models." *American Journal of Agricultural Economics*, pp 591-600, November 1975.

Objectives

One of the improvements to a regional model is to replace the fixed demand requirements by segmented demand equations for individual commodities.

The method by which commodity demand equations can be included into linear programming models is explained by Duloy and Norton (1975). Examples are provided below which describe the procedure and logic of incorporating both separable and interdependent demand equations into linear programming models. The case of separable demand equations will be discussed first.

Separable Demand or Price Equations

A separable demand equation is one in which the quantity demanded of one product can be given as a function of the price of that product,

$$Q = Ao + A1*P. \tag{1}$$

The inverse of equation 1 or a average price equation can be thought of as,

$$P = ao + a1*Q, \text{ where } ao = -Ao/A1 \text{ and } a1 = 1/A1. \tag{2}$$

Type of market. Before proceeding, the researcher must determine whether the market is best described by many small producers or whether it is a market which is dominated by one or only a few producers. That is, the researcher must determine whether the market is best described by assumptions of competitive or monopolistic behavior. If the objective is to find a competitive equilibrium solution, the desired solution must result in an equation of the price of each product with the marginal cost of producing that product. On the other hand, a monopoly solution is one in which the marginal revenue from the sale of the product is equated to the marginal cost of producing the product. The

respective monopoly and competitive equilibrium solutions represented by Qm and Qc are shown in Figure 1.

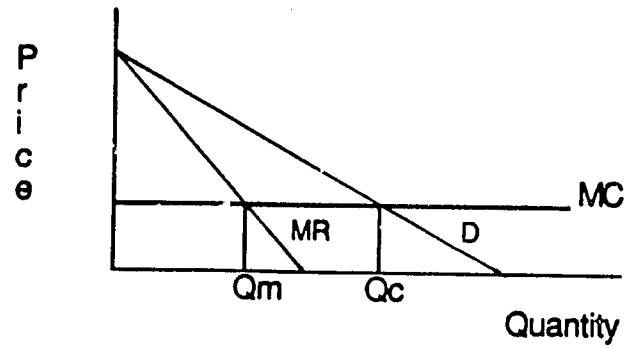

Figure 1. Review of Monopoly and Competitive Equilibrium Solutions.

A straight forward maximization of profits at the market level will result in a monopoly solution. This is shown by using a definition of profits as the difference between total revenue and total cost and using equation 2 for the price of the product,

$$\begin{aligned} \text{Profit} &= \text{Total Revenue} - \text{Total cost} \\ \text{Profit} &= (P * Q) - \text{Total cost} \\ \text{Profit} &= [\alpha o - (\alpha 1 * Q)] * Q - \text{Total cost} \\ \text{Profit} &= (\alpha o * Q) - (\alpha 1 * Q^2) - \text{Total cost} \end{aligned} \tag{3}$$

so that after taking the derivative with respect to Q,

$$\frac{d\,(\text{profit})}{d\,Q} = \alpha o - 2\,\alpha 1\,Q - \frac{d(\text{Total cost})}{d\,Q} = 0 \tag{4}$$

or,

$$\alpha o - 2\,\alpha 1\,Q = \frac{d(\text{total cost})}{d\,Q}$$

which is of the form, marginal revenue = marginal cost.

The above result occurs because the first derivative of the total revenue function is the marginal revenue function. The monopoly solution is the same as maximizing the area under the marginal revenue curve and above the marginal cost curve. However, if the user is seeking a competitive equilibrium solution, then the user must seek to maximize a different function which is constructed in such a way that the maximization or differentiation process results in a price or average revenue function being equated to the

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marginal cost function. This is accomplished by maximizing the area under the price, or average revenue function, and above the marginal cost curve. This area is referred to as the sum of producer plus consumer surplus. Recall that the total revenue function can be obtained by integrating over the marginal revenue function. If we integrate over the demand function, we obtain, not a revenue function, but a welfare function which measures the amount of consumer plus producer surplus. This process of finding a competitive market equilibrium was originally developed by Paul Samuelson. The important point to keep in mind with this maximization is that it is not the mathematical or numerical value of the welfare function that is of central importance, but rather, that the taking of the first derivative in the maximization process results in a price equation which is equated with a marginal cost equation.

Consider the following numerical example where the price equation is given as,

$$p = 10 - .1\ Q. \tag{5}$$

If the marginal and average costs are constant at \$6 per unit, then the objective (profit) function for the monopoly solution is, profit = $10\ Q - .1\ Q^2 - 6\ Q$.

The maximum profit point is determined by finding the level of Q that equates the first derivative equal to zero,

$$\frac{d(\text{profit})}{dQ} = 10 - .2\ Q - 6 = 0$$

so that,

$$Q = \frac{-4}{-.2} = 20.$$

If 20 units are produced, then the marginal revenue is 10-(.2)(4)= \$6. The price is 10 - (.1)(40)= \$8, which is greater than the marginal cost.

Competitive equilibrium. Duloy and Norton (1975) show the expression for consumers' plus producers' surplus as the function which results from the integration over the price or inverse demand equation and the subtraction of total production cost. If we

first integrate over the price equation in the above example, we obtain the following expression for the total area under the price equation as,

$$W' = (10-.1\,Q)\,dQ = 10\,Q - .05\,Q. \tag{6}$$

The total cost of production (the area under the supply function) is then subtracted to arrive at the area under the price equation and above the supply curve. In the example, the total cost function is simply 6Q. The level of Q that maximizes the sum of producer and consumer surplus is obtained by differentiating the welfare function with respect to Q and solving the resulting equation for Q,

$$dW/dQ = 10 - .1Q - 6 = 0$$

or,

$$10-.1Q = 6.$$

which is of the form, price = marginal cost. The solution to the above equation is 40 units. If 40 units are produced, the price will be \$6 (10-.1*40), which is equal to marginal cost, and a competitive market solution is obtained.

Inclusion into a programming model. The above relationships relating to total revenue or welfare derived from a linear price equation are quadratic and cannot directly be included in the objective function of a linear programming model. The equation (6), for the area under the demand equation is evaluated at selected intervals (say 5 units each) of Q as shown in Figure 2 and Table 1. The individual segments of the W' function can be included as individual selling activities in the programming model with one additional constraint which restricts the sum of such activities to be less than one. It should be noted that the marginal change in column 4 of Table 1 divided by the quantity change of 5 units corresponds to the average price over the interval. Notice also that the W' function reaches it's maximum point where the average price equation goes to zero. The total revenue function reaches its maximum point where the marginal revenue goes to zero.

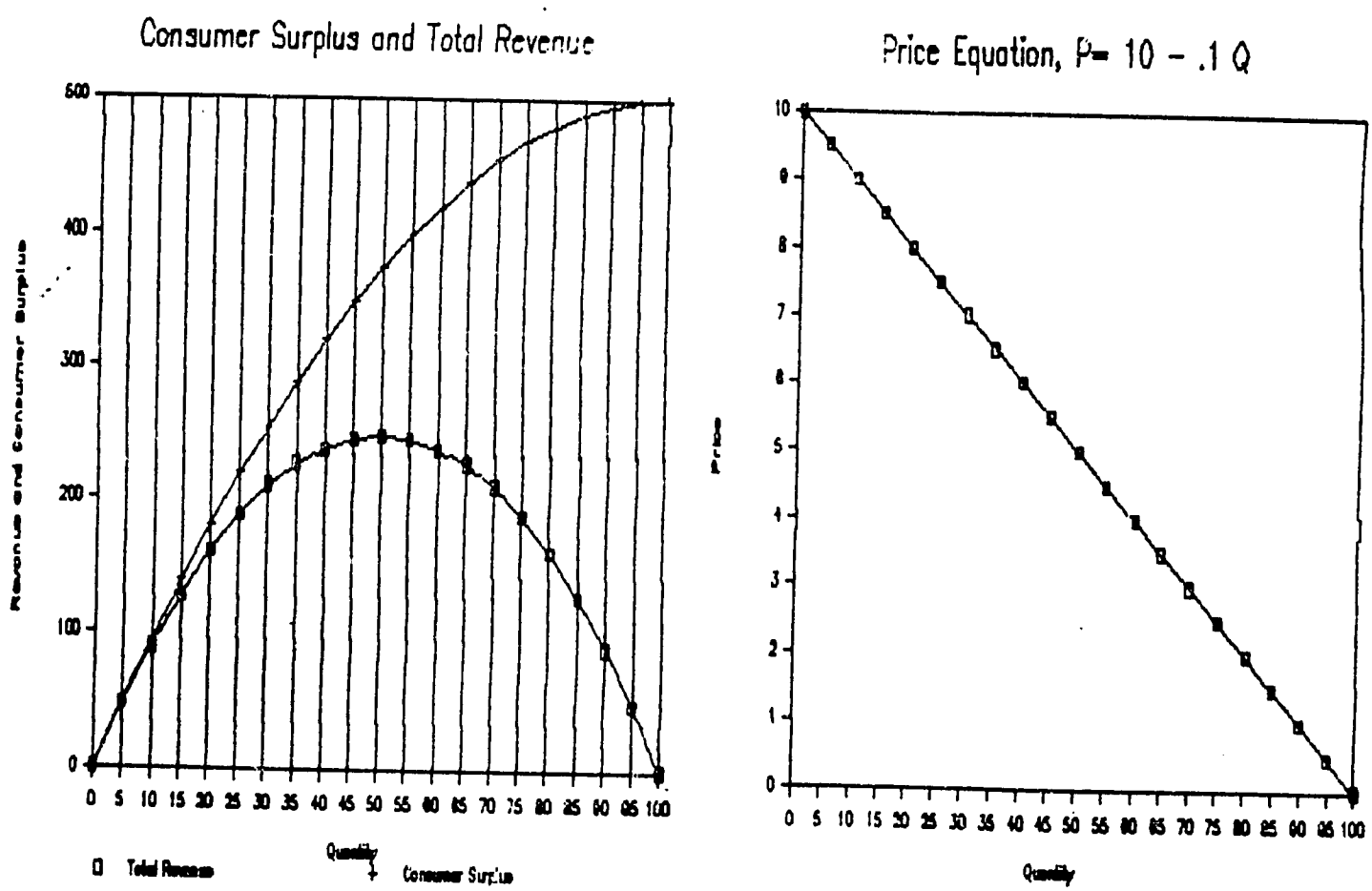

Figure 2. Relation Between Demand or Price Equation, the Total Revenue Function and the Consumer Surplus Function.

Table 1. Calculation of Area Under the Price Equation and Total Revenue at Selected Points Along the Welfare Equation

Seg.	Quant	Price	T.Rev	Welfare
D1	5	9.50	47.50	48.75
D2	10	9.00	90.00	95.00
D3	15	8.50	127.50	138.75
D4	20	8.00	160.00	180.00
D5	25	7.50	187.50	218.75
D6	30	7.00	210.00	255.00
D7	35	6.50	227.50	288.75
D8	40	6.00	240.00	320.00
D9	45	5.50	247.50	348.75
D10	50	5.00	250.00	375.00
D11	55	4.50	247.50	398.75
D12	60	4.00	240.00	420.00
D13	65	3.50	227.50	438.75
D14	70	3.00	210.00	455.00
D15	75	2.50	187.50	468.75
D16	80	2.00	160.00	480.00
D17	85	1.50	127.50	488.75
D18	90	1.00	90.00	495.00
D19	95	.50	47.50	498.75
D20	100	-.00	-.00	500.00

The method of putting the equations into a programming model requires the addition of selling activities corresponding to the segments on the price or welfare function. When any of the selling activities enters the solution at level one, it corresponds to the maximum amount that will clear the market at the indicated price. It is necessary to add a convex constraint which restricts the sum of the selling activities to be less than or equal to one.

In the following linear programming tableau (Table 2), the price equation discussed above (p=10-.1Q) is included as twenty separate selling activities. The objective function for each activity is the total amount of consumer surplus generated from the sale of the number of units shown in column 1 of Table 1 or in the first constraint row of the tableau in Table 2.

Two numerical solutions to the sample problem are provided in Table 3. The first solution represents the case described above where the supply price of the product is constant (perfectly elastic) at $6.00 per unit. The value of the objective function for the

Table 2. Example Linear Programming Tableau With One Segmented Price Equation and a Perfectly Elastic Supply Equation

Dem	MAX	Rhs	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Obj				95	138.7	180	218.7	255	288.7	320	348.7	375
Dq1	L	0	5	10	15	20	25	30	35	40	45	50
Convex	L	1	1	1	1	1	1	1	1	1	1	1

	D11	D12	D13	D14	D15	D16	D17	D18	D19	D20	SUPP
Obj	398.7	420	438.7	455	468.7	480	488.7	495	498.7	500	-1
Dq1	55	60	65	70	75	80	85	90	95	100	-1
Convex	1	1	1	1	1	1	1	1	1	1	

Table 3. Solutions to the Example Linear Programming Problem With One Segmented Price Equation When the Supply Cost is Constant at $1.00 and $6.00 per Unit

OPTIMAL Solution

function Value: 80 function Value: 405

Name	Type	Obj	Level	Name	Type	Obj	Level
D8	real	320	1	D18	real	495	1
SUPP	real	-6	40	SUPP	real	-1	90

Activities not in Solution

Name	Obj.v	Type	Shad.Pr.	Name	Obj.v	Type	Shad.Pr.
D1	48.8	real	110.0	D1	48.8	real	361.2
D2	95.0	real	140.0	D2	95.0	real	320.0
D3	138.8	real	170.0	D3	138.8	real	281.2
D4	180.0	real	200.0	D4	180.0	real	245.0
D5	218.8	real	230.0	D5	218.8	real	211.2
D6	255.0	real	260.0	D6	255.0	real	180.0
D7	288.8	real	290.0	D7	288.8	real	151.3
D9	348.8	real	350.0	D8	320.0	real	125.0
D10	375.0	real	380.0	D9	348.8	real	101.2
D11	398.8	real	410.0	D10	375.0	real	80.0
D12	420.0	real	440.0	D11	398.8	real	61.3
D13	438.8	real	470.0	D12	420.0	real	45.0
D14	455.0	real	500.0	D13	438.8	real	31.2
D15	468.8	real	530.0	D14	455.0	real	20.0
D16	480.0	real	560.0	D15	468.8	real	11.3
D17	488.8	real	590.0	D16	480.0	real	5.0
D18	495.0	real	620.0	D17	488.8	real	1.3
D19	498.8	real	650.0	D19	498.8	real	1.3
D20	500.0	real	680.0	D20	500.0	real	5.0
DQ1	0.0	slack	6.0	Dq1	0.0	slack	1.0
CONVEX	0.0	slack	80.0	Convex	0.0	slack	405.0

optimal solution is 80 which is the value of consumer plus producer surplus. Activity P6.00 (which measures the total area under the price line when the market price is $6.00) enters the solution at one unit. The activity CP enters the solution at 40 units which corresponds to 40 units of the product being produced. The equilibrium price of the product is $6.00 and this is indicated by the shadow price on the commodity balance row. The sum of producer plus consumer surplus is given by the value of the objective function. The shadow price on the convex constraint equation measures the amount of consumer surplus. The producer surplus is zero because the supply curve is perfectly horizontal.

The second solution is identical to the first except that the cost of production or supply price of the commodity is assumed constant at $1.00. In the second solution, the value of the objective solution increases to 405. The activity P1.00 enters the solution at level 1. The activity CP enters the solution at 90 units which is the amount of the commodity demanded when the price is $1.00. Again the commodity price is indicated by the shadow price on the commodity balance row which is $1.00.

The tableau of the model is expanded to include two producing areas in Table 4. It is assumed that both areas produce the same product. Producing area one can produce up to 30 units at a cost of $0.50 per unit while producing area two can produce up to 40 units at a cost $1.00. Again the objective is to find a competitive equilibrium. When there are resource limitations, then economic rents accrue to scarce resources. The definition of the competitive equilibrium is generalized to require the price of the product to be less than or equal to marginal cost plus rent accruing to fixed resources. A quick check of the above program indicates that the maximum production cannot exceed 70 units. However the price would not decline to $1.00 until 90 units are produced. The solution to the problem is presented in Table 5. As anticipated, producing areas one and two each produce to full capacity with 30 and 40 units respectively. The activity P3.00 enters the solution at one unit. The equilibrium market price is calculated to be $2.75. The shadow prices, or economic rents, for resource limitation in producing areas 1 and 2 are $2.25

Table 4. Example Competitive Equilibrium Linear Programming Tableau With Segmented Demand Equation and Two Producing Regions

DEM	MAXIMI	RHS	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
OBJ			48.75	95	138.75	180	218.75	255	288.7	320	348.7	375
DQ1	L	0	5	10	15	20	25	30	35	40	45	50
CONVEX	L	1	1	1	1	1	1	1	1	1	1	1
LPA1	L	3										
LPA2	L	4										

Tableau Continued.

DEM	D11	D12	D13	D14	D15	D16	D17	D18	D19	D20	PA1	PA2
OBJ	398.7	420	438.7	455	468.75	480	488.75	495	498.7	500	-5	-10
DQ1	55	60	65	70	75	80	85	90	95	100	-10	-10
CONVEX	1	1	1	1	1	1	1	1	1	1		
LPA1											1	
LPA2												1

Table 5. Solution to Competitive Equilibrium Problem With Two Producing Areas

OPTIMAL Solution
function Value: 400

Activities in Solution				Activities Not in Solution			
Return	Name	Type	Level	Name	Obj.Va	Type	Shad.Price
455	D14	real	1	D1	48.75	real	276.25
468.7	D15	real	0	D2	95.00	real	290.00
-5	PA1	real	3	D3	138.75	real	303.75
-10	PA2	real	4	D4	180.00	real	317.50
				D5	218.75	real	331.25
				D6	255.00	real	345.00
				D7	288.75	real	358.75
				D8	320.00	real	372.50
				D9	348.75	real	386.25
				D10	375.00	real	400.00
				D11	398.75	real	413.75
				D12	420.00	real	427.50
				D13	438.75	real	441.25
				D16	480.00	real	482.50
				D17	488.75	real	496.25
				D18	495.00	real	510.00
				D19	498.75	real	523.75
				D20	500.00	real	537.50
				DQ1	0.00	slack	2.75
				CONVEX	0.00	slack	262.50
				LPA1	0.00	slack	22.50
				LPA2	0.00	slack	17.50

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and $1.75 respectively. The solution satisfies the condition for a competitive equilibrium since for region one the cost of production plus rent is equal to the price of the product, ($2.75 = .5 + 2.25). The same condition is satisfied for region two, (2.75 = 1.00 + 1.75). The total quantity supplied by both regions at a market price of $2.75 is equal to the quantity demanded at $2.75 per unit

Interdependent Commodity Demand and Price Relationships

The demand for a given product is usually taken to be interdependent if the quantity demanded of that product depends not only on the price of that product, but also on the price of competing products. For example, Q1 = f1(P1,P2) and Q2 = f2(P1,P2), or in the inverse demand equation form used above, P1 = g1(Q1,Q2) and P2 = g2(Q1,Q2). The objective function for the linear programming model in the interdependent multiproduct case is derived in a manner similar to that for the single product case. Assume there are n final commodities and that the price of each commodity is contained in the price vector P which is calculated from the market quantities as,

$$P = A + BQ$$

where:

P is an nx1 vector of commodity prices
Q is an nx1 vector of final commodities
B is a symmetric nxn negative semidefinate matrix (or inverse demand matrix) of coefficients bij which relate the level of the i'th commodity price to the amount of the j'th commodity.
A is an nx1 vector of intercepts for the average price equations.

If B is a symmetric matrix, the welfare function W' is given as the area underneath the set of price equations:

$$W' = A'Q + .5\, Q'BQ. \tag{7}$$

The objective function is obtained by subtracting the total cost of production from the area under the price equations:

$$W = W' - TC = A'Q - .5Q'BQ - \Sigma_k \Sigma_j C_{kj} X_{kj} \tag{8}$$

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where:

C_{kj} is the cost of producing the j'th product in the k'th producing area

X_{kj} is the level of the j'th production activity in the k'th producing area.

The "welfare" selling activities for the interdependent case are derived in the same way described above for the separable demand case, that is, the "welfare" selling activities represent the area underneath the price equation which results from the consumption of specified amounts of each of the n final commodities ties. For example, we may calculate the welfare derived from the consumption of say 40, 60, and 80 units of commodity one in combination with 10, 20, and 30 units of a second commodity, requiring 9 selling activities. A single convex constraint is added to the model to insure that the sum of the "welfare" selling activities is less than or equal to one. To see this more clearly, suppose we have the following two

price equations for commodities one and two:

$$p_1 = 12 - .1\, q_1 - .1\, q_2$$

$$p_2 = 20 - .1\, q_1 - .4\, q_2. \tag{9}$$

The function for the total area under the price equation after integration, in matrix form, is,

$$W' = [12,20]\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + .5\,[q_1,q_2]\begin{bmatrix} -.1 & -.1 \\ -.1 & -.4 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{10}$$

or in equation form is,

$$W' = 12q_1 + 20q_2 - .05q_1^2 - .1q_1q_2 - .2q_2^2. \tag{10'}$$

The total revenue function for the same price equations is obtained by multiplying the equation for the price of each commodity by the amount of that commodity. The total revenue equation for the above set of price equations is,

$$TR = 12q_1 + 20q_2 - .1q_1^2 - .2q_1q_2 - .4q_2^2. \tag{11}$$

The difference between equation 10' and equation 11 is in the coefficients for the quadratic and interaction terms. If equation 10 is evaluated at nine points corresponding

the 40, 60, and 80 units of commodity 1 in combination with 10, 20 and 30 units of commodity 2, the welfare levels shown in Table 6 are obtained.

Table 6. Calculation of Consumer Surplus Values for A Two Commodity Model With Interdependent Price Flexibility Equations

Segment	Q1	Q2	P1	P2	T.Rev.	Con.Surp	
1	40	10	7	12	400	540	540
2	40	20	6	8	400	640	640
3	40	30	5	4	320	700	700
4	60	10	5	10	400	660	660
5	60	20	4	6	360	740	740
6	60	30	3	2	240	780	780
7	80	10	3	8	320	740	740
8	80	20	2	4	240	800	800
9	80	30	1	0	80	820	820

A linear programming example. An example linear programming tableau which contains the 9 points generated in Table 6 along with three producing regions is presented in Table 7. The 9 points on the W' function are included as activities W1-W9. Note that the objective function value for each Wi activity is the area under the price equation associated with the consumption of the amounts of q1 and q2. The amount of q1 and q2 associated with each of the welfare values appear in the respective commodity balance rows (labeled DQ1 and DQ2) and represent consumption. The production sector simply consists of three producing areas. Each area can produce either of the two products. Producing area 1 has the lowest costs of production while area 3 is more marginal in terms of having higher costs of production.

Table 7. Competitive Equilibrium Linear Programming Model for Two Interdependent Commodities and Three Producing Regions

IDEM	Max	RHS	P11	P12	P21	P22	P31	P32	D1	D2	D3	D4	D5	D6	D7	D8	D9
Obj			-1.5	-4	-2	-3.5	-5	-7.5	540	640	700	660	740	780	740	800	820
DQ1	L	0	-3		-2		-2.5		40	40	40	60	60	60	80	80	80
DQ2	L	0		-2.5		-2		-2	10	20	30	10	20	30	10	20	30
Convx	L	1							1	1	1	1	1	1	1	1	1
LPa1	L	10	1	1													
LPa2	L	20			1	1											
LPa3	L	10					1	1									

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Table 8. Solution to Example Linear Programming Problem With Interdependent Commodities and Three Producing Areas

OPTIMAL Solution
function Value: 666.25

Obj.Va.	Name	Type	Level	NAME	Obj.Va	Type	Shad.Price
-1.5	P11	real	10.00	P12	-4	real	2.00
-2	P21	real	7.50	P32	-7.5	real	2.38
-3.5	P22	real	12.50	D1	540	real	115.00
-5	P31	real	10.00	D2	640	real	50.00
780	D6	real	0.50	D3	700	real	25.00
800	D8	real	0.50	D4	660	real	50.00
				D5	740	real	5.00
				D7	740	real	25.00
				D9	820	real	15.00
				DQ1	0	slack	2.75
				DQ2	0	slack	3.50
				Convx	0	slack	510.00
				LPa1	0	slack	6.75
				LPa2	0	slack	3.50
				LPa3	0	slack	1.88

The solution to the problem is shown in Table 8. The calculations for Supply of each product are:

Product 1			
Activity	Level	Yield	Amount
P11	10	3	30
P21	7.5	2	15
P31	10.	2.5	25
Total			70
Product 2			
P22	12.5	2	25

The solution indicates that producing area one should produce 30 units of product 1. Producing area 2 should produce 15 units of product 1 and 25 units of product 2. Producing area 3 should produce 25 units of product 1. The total production of products one and two is respectively 70 and 25 units. The respective shadow prices on the rows labeled DQ1 and DQ2 indicate that the market prices of products one and two are $2.75

and $3.50. The selling activities in the solution are .5 units of W6 and .5 units of W8. The quantities demanded are calculated as,

		W6			W8		Total
Q1=	.5*	60	+	.5*	80	=	70
Q2=	.5*	30	+	.5*	20	=	25.

This corresponds to commodity levels of 70 units of Q1 and 25 units of Q2. The respective quantities can be inserted into the price equations to verify that the shadow prices correspond to the equilibrium product prices. The calculated price for commodity 1 is p1 = 12 - (.1)(70) - (.1)(25)= 2.50. The calculated price for the second commodity is p2 = 20 - (.1)(70) - (.4)(30) = 1. The price approximation in this example could be improved by defining additional segments in the area of the solution and resolving the program.

Each production activity in the final solution should satisfy the criterion that the price of the product should be equal to the cost of production plus the amount of economic rent on the resources used in production. Using the respective shadow prices or land rents for the rows PA1, PA2, and PA3 ($2.00, $1.50, and $ .50), it can be seen that the final solution satisfies the condition;

P11 = 30 units, with market price, $.5 + $2.0 = 2.5
P21 = 15 units, with market price, $1.0 +$1.5 = 2.5
P22 = 25 units, with market price, $1.5 +$1.5 = 3.0
P31 = 25 units, with market price, $2.0 +$ .5 = 2.5.

Exercise

Part I

Objective

To perform calculation needed for demand segmentation with Lotus. The completed worksheet is shown in Table 9.

Given:

1. Number of segments (N segs) of the demand curve, in cell B1.
2. Price Elasticities of Demand. (Ada) in cell B2.
3. Initial Price (P0) in cell B5.
4. Initial Quantity (Q0) in cell B6.

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Table 9.

	A	B	C	D	E	F	G	H	I	J	K
1	N Segs	11									
2	Ada	-0.8									
3	Alpha	225									
4	Beta	0.625									
5	P0	100									
6	Q0	200									
7	P-lower	50									
8	P-upper	200									
9											
10	1	2	3	4	5	6	7	8	9	10	11
11											
12	200	185	170	155	140	125	110	95	80	65	50
13	40	64	88	112	136	160	184	208	232	256	280
14	8500	13120	17380	21280	24820	28000	30820	33280	35380	37120	38500
15	8000	11840	14960	17360	19040	20000	20240	19760	18560	16640	14000
16											

Procedure

1. Enter in B1, B2, B5 and B6. These should just be numbers.
2. Calculate Alpha and Beta.

   a. In cell B4. enter in the formula for Beta:

   (-1/B2)*(B5/B6)

   b. In cell B3, enter in the formula for Alpha:

   +B5+(B4*B6)

3. Set price range of variation to (0.5*P0, 2*P0). i.e. Set P-lower and P-upper.

   a. In cell B7, enter in the formula for P-lower:

   0.5*B5

   b. In cell B8, enter in the formula for P-upper:

   2*B5

4. Window dressing.

   a. Draw lines from A9 to K9. Do this by first entering

   \-

   in cell A9 and then copy cell A9 to the range B9.K9 with the /Copy command.

   b. Fill row 10 starting from A10 to K10 with the /Data Fill command. Press /DF. Then specify the fill range as A10.K10. Start should be 1. Step should also be 1. End should be 11.

   c. Copy line from row 9 to row 11 using the /Copy command.

d. Copy line from row 11 to row 16 using the /Copy command.

5. Enter in the formulas for prices in row 12. The entry in A12 should be:

```
+$B7+($B8-$B7)/($B1-1)*($B1-A10)
```

copy the formula from A12 to B12.K12

6. Enter in the formulas for theta's (entries in the supply-demand balance row) in row 13. The entry in cell A13 should be:

```
(($B3-$B8)/$B4)+(($B8-$B7)/$B4)*(A10-1)/($B1-1)
```

copy the formula from A13 to B13.K13

7. Enter in the formulas for Omega's (entries in the OBJ) in row 14. The entry in cell A14 should be:

```
+$B3*A3-0.5*$B4*A13^2
```

copy the formula from A14 to B14.K14

8. Enter in the formulas for Rho's (entries in the farm income row) in row 15. The entry in cell A15 should be:

```
+$B3*A3-$B4*A13^2
```

copy the formula from A15 to B15.K15

9. Save this file under the name DEM.

Part II

Objective

Enter in a small sector model with exogenous demand. The model is as shown in Figure 1.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	MAYALAND	MAXIMIZE	B	MA11	MA21	BE1	MA12	MA22	BE2	DMAZ	DBE	LH1	LH2
2	OBJ			0	0	0	0	0	0	150	250	-5	-5
3	CBMA	L	0	-1.5	-2.5		-1.2	-2		1			
4	CBBE	L	0			-0.5			-0.6		1		
5	LAND1	L	100	1	1	1							
6	LAND2	L	300				1	1	1				
7	LABOR1	L	9000	30	40	35						-1	
8	LABOR2	L	6750				30	38	36				-1

Solution
OPTIMAL
function Value: 104250

Returns	Name	Type	Level	MA11	BE1	MA12	BE2	LH1	CBMA	CBBE	LAND1	LAND2	LABOR2
				0	0	0	0	-5	0	0	0	0	0
				real	real	real	real	real	slack	slack	slack	slack	slack
0	MA21	real	100	1	1								
0	MA22	real	300			1	1				1		
150	DMAZ	real	850	1	2.5	0.8	2		1			1	
250	DBE	real	0		-0.5		-0.6			1	2.5	2	
-5	LH2	real	4650			1	2						
0	LABOR1	slack	5000	-10	-5			-1			-40	38	-1
Z				150	250	80	140	0	150	250	375	110	5
Shadow Price				150	250	80	140	5	150	250	375	110	5

Figure 1.

Columns

MA11 Maize production technology 1, region 1
MA21 Maize production technology 2, region 2
BE1 Beans production region 1
MA12 Maize production technology 1, region 2
MA22 Maize production technology 2, region 2
BE2 Beans production region 2
DMAZ Demand for maize
DBE Demand for bean
LH1 Hire labor, region 1
LH2 Hire Labor, region 2

Rows

CBMA Commodity balance, maize
CBBE Commodity balance, beans
LAND1 Land constraint, region 1
LAND2 Land constraint, region 2
LABOR1 Labor constraint, region 1
LABOR2 Labor constraint, region 2

Save this tableau under the name DEM2. Use Musah86 to obtain solution.

Part III

Objective

Construct a small sector model with endogenous demand.

Procedure

We will arrive at the tableau in Figure 2 by modifying the tableau in Part I and then combining it with the segmentation calculations that was already done in Part II.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1											N Segs	11												
2											Ada	-0.8												
3											Alpha	337.5												
4											Beta	0.2205												
5											P0	150												
6											Q0	950												
7											P-lower	75												
8											P-upper	300												
9																								
10											1	2	3	4	5	6	7	8	9	10	11			
11																								
12											300	277.5	255	232.5	210	187.5	165	142.5	120	97.5	75			
13											170	272	374	476	578	680	782	884	986	1088	1190			
14											54187.	83640	110797	135660	158227	178500	196477	212160	225547	236640	245437			
15											51000	75480	95370	110670	121380	127500	129030	125970	118320	106080	89250			
16																								
17																								
18	TINY	MAX	B	MA11	MA21	BE1	MA12	MA22	BE2	DBE	DMA21	DMA22	DMA23	DMA24	DMA25	DMA26	DMA27	DMA28	DMA29	DMA210	DMA211	LB1	LB2	Y
19	OBJ		0	0	0	0	0	0	0	250	54187.	83640	110797	135660	158227	178500	196477	212160	225547	236640	245437	-5	-5	
20	CMGA	L	0	-1.5	-2.5		-1.2	-2			170	272	374	476	578	680	782	884	986	1088	1190			
21	CVGMA	L	1								1	1	1	1	1	1	1	1	1	1	1			
22	CBBE	L	0			-0.5			-0.6	1														
23	LAND1	L	100	1	1	1																		
24	LAND2	L	200				1	1	1															
25	LABOR1	L	9000	30	40	35																		
26	LABOR2	L	6750				30	38	36													-1		
27	INCOME	E	0																				-1	
28										250	51000	75480	95370	110670	121380	127500	129030	125970	118320	106080	89250	-5	-5	-1

Figure 2.

1. Compare carefully Figure 2 with Figure 1 and Figure 3 and think how you might arrive at the worksheet in Figure 2 from Figure 1 and 3.

	A	B	C	D	E	F	G	H	I	J	K
1	N Segs	11									
2	Eta	-0.8									
3	Alpha	225									
4	Beta	0.625									
5	P0	100									
6	Q0	200									
7	P-lower	50									
8	P-upper	200									
9											
10	1	2	3	4	5	6	7	8	9	10	11
11											
12	200	185	170	155	140	125	110	95	80	65	50
13	40	64	88	112	136	160	184	208	232	256	280
14	8500	13120	17380	21280	24820	28000	30820	33280	35380	37120	38500
15	8000	11840	14960	17360	19040	20000	20240	19760	18560	16640	14000
16											
17											

Figure 3.

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2. First, insert 16 rows starting from row 1 to accommodate the segmentation calculations.

3. Insert 10 columns starting from column L to accommodate the 10 extra demand activities.

4. Type in the names of the demand activities, but don't type in any numbers yet.

5. We will now put in the demand segmentations calculations. But since we already have done this in Part I (stored in DEM), we will *combine* DEM with this worksheet. We will do this with the /File Combine command:

To do so, first position the cursor in cell K1. Then invoke:

```
/File Combine Copy Entire-File
```

and supply DEM as the file name when prompted.

The segmentation information should then be merged into the worksheet.

6. Now change P0 in cell L5 to 150 and Q0 in cell L6 to 850.

7. Now complete the rest of the tableau, be sure to use formula reference to the assumption section as much as possible.

9. Save this tableau under the name DEM3. Note that this is a "Master tableau" which Musah cannot solve directly. The /File Xtract command must be used to extract the tableau part. DO NOT EXTRACT TO A FILE USING WITH THE SAME NAME OR YOU WILL LOSE YOUR MASTER TABLEAU.

10. Compare the solution you obtained in Part II and III.

    a. What is the price of Maize?
    b. What is the quantity demand of maize?
    c. What is farm income?
    d. What is consumer surplus?
    e. What is producer surplus?

11. Change the demand elasticity to -1.6 and obtain another solution. Interpret the result.

INTRODUCTION TO THE DOMINGO MODEL

Reference: Benito, Carlos. "Agricultural Accounting Framework for the Dominican Republic: A Guide for the Development and Use of Domingo." APAP Report A-40, Oklahoma State University, May 1987.

Objective

The objective this section is to introduce is a small sector programming model being developed for the Dominican Republic. The model is called Domingo.

The Domingo model is designed as a agricultural sector accounting model. The purpose is to develop a model which can be quickly solved and readily maintained on a microcomputer. Thus the model is smaller and more aggregated than the typical sector models which have been solved on mainframe computers. The model depicts the production of annual and perennial crops in the Dominican Republic.

Production Components of the Domingo Model

The agricultural production activities of the Domingo model can be divided into three groups. These are Small Farms (8-79 tarea), Medium and Large Farms (80 tarea or more) and producers of perennial crops.

Crops and Crop Production Technology for Small Farmers

The production activities for the small producers and the constraints they face are shown in Table 1. The number 1 is used in the Domingo model as part of the coding to refer to crops produced and resources used by small farms. The crops produced are rice, maize, sorghum, red beans, cassava, peanuts, pigeon peas, and tobacco. Most crops have two technologies. The yields differ according to the levels of inputs. Irrigation is a possibility for some crops.

Table 1. Listing of Activities and Constraints for the Small Farmers in the Domingo Model

MODEL: DOMINGO

			Rice			Maize			Sorghum		Beans		Cassava	Peanut	Pigeon Pea		Tobacco								
PPeal2	MAX		Ric11	Ric12	Ric13	Corn11	Corn12	CrnBn1	Sorg11	Sorg12	Bean11	Bean12	Cas11	Pnut11	PPea11	PPea11	Tob11	Tob12	Tob13	Tob14	Tob15	Tob16	Tob17	Tob18
Obj			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PrRice	L	0	-3.5	-6	-7																			
PrMaiz	L	0				-2	-2.25	-1.2																
PrSorg	L	0							-3	-3.9														
PrBean	L	0						-0.3			-0.55	-0.8												
PrTobac	L	0															-1.6	-2	-2.25	-1.25	-1.65	-1.92	-1.35	-1.9	-2.2
Cassa	L	0											-8.4												
PrSPot	L	0														-8.8									
PrPnut	L	0												-1.1											
PrPPea	L	0													-1.6	-1.1									
LnfSfrl	L	2685	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Irrigl	L	0		1	1														1		1				1
Chwml	L	0	10.82	23.34	30.3					8.43		9.56						9.7	16.19		5.7	15.65		8.89	17.81
OthInpl	L	0	55.31	63.52	93.22	0.81	12.7	4.86	28.03	12.57	16.25	31	8.2	17.79	2	17	2.14	7.64	12.39	2.14	6.54	11.97	2.14	12.14	18.42
Laborl	L	0	3	6.25	5.5	3	2	4.75	0.84	1.14	2.85	2.95	4.09	3.04	4	6	11	12	13	9	10	11.25	8.5	11.5	12.5

The crops which can be grown by small farmers (size 1) are;

Rice	Ric11	nonirrigated rice tech 1
	Ric12	irrigated rice low yield
	Ric13	irrigated rice high yield
Maize	Corn11	Maize, low input
	Corn12	Maize, high input
Sorghum	Sorg11	Sorghum, technology 1
	Sorg12	Sorghum, technology 2
Red Bean	Bean11	Beans, Technology 1
	Bean12	Beans, technology 2
Cassava	Cas11	Cassava, technology 1
Peanut	Pnut11	Peanut, technology 1
Pigeon pea	PPea11	Pigeon Pea, technology 1
Pea/S.Pot	PPSP11	Pigeon Pea-Sweet Potato intercrop
Tobacco	Tob11	Tobacco, amarillo, technology 1
	Tob12	Tobacco, amarillo, technology 2
	Tob13	Tobacco, amarillo, technology 3
	Tob14	Tobacco, diaz, technology 1
	Tob15	Tobacco, diaz, technology 2
	Tob16	Tobacco, diaz, technology 3
	Tob17	Tobacco, Cuban, technology 1
	Tob18	Tobacco, Cuban, technology 2
	Tob19	Tobacco, Cuban, technology 3

The land area and the amount of irrigation available (not shown) are the only physical constraints expressed in the model. The cost of the chemical, other inputs, labor days, and capital are constrained by purchasing activities which are not shown in the model. The specific rows related to agricultural inputs for the small farm are:

LndSfr1. The constraint requires the use of land by all small farmers cannot exceed the area of 2685 thousand tarea.

Irrig1. This constraint accounts for the use of irrigation resources.

Chem1. This constraint is used to account for the use of chemical inputs (measured in pesos) which are used in production.

OthInp1. This constraint is used to account for the use of all other variable inputs except chemical, land, and labor. The input is measured in pesos.

Labor1. This constraint is to account for the quantity of labor used in production by small farmers.

The remaining rows shown in Table 1 are commodity rows which are used to transfer the production for each crop from the farm the the processor or the market.

Crops and Crop Production Technology for Medium and Large Farms

The activities with their respective coefficients and constraints which represent production by medium and large farms are shown in Table 2. The list of crops and their definition is the same as those for the small farms. The number 2 is used to distinguish

Table 2. Annual Crop Production Activities and Resource Restraints for Medium and Large Farms in the Domingo Model

MODEL: DOMINGO

			Rice		Maize		Maize/ Bean	Sorghum		Red Bean		Tobacco									Cassava	Peanut		Pigeon Pea
Doming	MAX		Ric22	Ric23	Corn21	Corn22	CrBn21	Sorg21	Sorg22	Bean21	Bean22	Tob21	Tob22	Tob23	Tob24	Tob25	Tob26	Tob2	Tob28	Tob29	Casv21	Pnut21	Pnut22	PPea21	PPea21
Obj			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0
PrRice	L	0	-6	-7																					
PrMaiz	L	0			-2.25	-4.75	-1.2																		
PrSorg	L	0						-3.9	-4																
PrBean	L	0					-0.25			-0.8	-1														
PrTobac	L	0										-1.6	-2	-2.25	-1.25	-1.65	-1.92	-1.	-1.9	-2.2					
Casaa	L	0																			-8.4				
PrSPot	L	0																							
PrPnut	L	0																				-1.1	-1.4		-8.8
PrPPea	L	0																						-1.1	-1.1
LnfSfr2	L	2750	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1
Irrig2	L	0	1	1		1					1			1			1			1			1		
Chem2	L	0	23.34	30.3		19.92				9.56	21.5		9.7	16.19		9.7	15.65		8.89	17.81			7.67		15
OthInp2	L	0	63.52	93.22	12.7	31.8	4.86	8.43	10.85	31	41.75	2.14	7.64	12.39	2.14	6.64	11.97	2.	12.14	18.42	27.2	21.97	23.52	2	17
Labor2	L	0	6.25	5.5	2.5	2.5	4.75	1.14	0.99	2.95	2.95	11	12	13	9	10	11.25	8	11.5	12.5	3.8	2.72	2.83	4	6
CpRice2	L	0	141.07	171.89																					
CpMaiz2	L	0			33.45	73.58	31.46																		
CpSorg2	L	0						27.39	43.95																
CpBean2	L	0								57.08	79.57														
CpToba2	L	0										63.74	84.54	101.38	52.54	72.34	90.62	49.	85.43	108.73					
CpCasa2	L	0																			36.32				
CpOth2	L	0																				37.63	49.68	25	64

the activity names and the resources used by the medium and large farms from those used by small farms.

The resource constraints defined for the medium and large farms have the same definition as those defined for the small farms. Again the digit 2 is included in the resource name to distinguish resources used by larger farms from the resources used by the small farms.

Perennial Crops and Production Technology

The production activities for the coffee, cocao, plantain, and guineo perennial crops are shown in Table 3. (The sugar cane has not yet been incorporated into the model.) Each perennial crop is represented with two levels of technology. The coffee activities from Table 3 are isolated below for discussion.

	Type	RHS	Coff11	Coff12	CoffRp	
PrCoff	L	0	-2.5	-.3	0	to market-->
Chem1	L	0	27	0	113	
Othinp	L	0	20	0	81	
Labor	L	0	7	2	25	
Ldcoff	L	2250	1	1	4	
Cpcoff	L	0	56	5	255	
Rpcoff	L	0	.1	.01	-1	

The activities coff11 and coff12 represent the production of coffee at two rates of intensity. Activity Coff11 requires more inputs and has a much higher level of production than activity Coff12. The activity coffRp is a replanted tara of coffee which has not yet reached production. The activity Coff11 requires .1 tara of replanted coffee per tarea of producing coffee. This implies that 10% of the area is replanted each year. The activity Coff12 requires only .05 tarea of replanted coffee each year which implies that 5% of the area is replanted. Activities Coff11 and Coff12 represent average inputs and output from a grove of trees of mixed ages. Since coff11 has a higher rate of replanting each year, the average age of trees in activity coff11 would be less than those represented by activity coff12.

Table 3. Activities and Constraints in the Perennial Crop Section of the Domingo Model

MODEL: DOMINGO

			Coffee			Cocao			Plantain			Guineo-Govt.			Guineo-Private		
Doming	MAX		Coff11	Coff21	Coff12	Coco11	Coco12	CocoRP	Plant1	Plant2	PlanRP	GuinG1	GuinG2	GuiGRP	GuinP1	GuinP2	GuiPRP
Obj			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PrCoff	L	0	-2.5	-0.3													
PrCocoa	L	0				-0.06	-0.04										
PrPlant	L	0							-2.6	-1.5							
PrGuin	L	0										-20	-84		-20	-84	
LnfSfr1	L	2685															
Irriq1	L	0															
Chem1	L	0	27		113	20	5	58	18	6	28		2	15		2	15
OthInp1	L	0	20		81	3.55	3	17	4	3	14	2	4	12	2	4	12
Labor1	L	0	7	2	25	4.25	3.25	12	6.55	5	6.85	4	8	7	4	8	7
LdCoff	L	2250	1	1													
CpCoff	L	0	56	5	255												
RpCoff	L	0	0.1	0.01	-1												
LdCaco	L	1870				1	1	3									
CpCaco	L	0				37	16	104									
RpCaco	L	0				0.1	0.05	-1									
LdPlat	L	460							1	1	1						
IrPlat	L	225							1		1						
CpPlat	L	0							38	21	58						
RpPlat	L	0							0.25	0.1	-1						
LdGuinG	L	80										1	1	1			
LdGuinP	L	80													1	1	1
IrGuinG	L	10											1				
IrGuinP	L	10														1	
CpGuinG	L	0										8	26	48			
CpGuinP	L	0													8	26	48
RpGuinG	L	0										0.1	0.2	-1			
RpGuinP	L	0													0.1	0.2	-1

The remaining perennial crops shown in 3 are treated in a similar way. Each crop has its own land area. Each crops has two level of production and a replanting activity. The capital restraints are separate for each crop so that crop specific credit programs can be modeled.

The inclusion of perennial crops in an annual model always presents some problems. The best kind of a model for perennial crops is a dynamic or multiperiod model which we examined previously, but these become very large. The compromise is to include activities which represent production from a group of trees of various ages. The problem with this approach however, is that producers cannot change from production with a grove of trees which have an average age of 20 years to a more intensive level of production from a grove with an average age of 10 years in the span of a year.

Production, Marketng, Consumption of a Single Crop

Another view of the model is gained be collecting the activities which relate to the production, marketing, processing, and consumption of a single crop. These activities are shown in Table 4 for rice.

Table 4. Rice Production, Milling, Consumption and Trade Activities in the Domingo Model

[illegible]			Small Farm			Medium-Large Farm			Rice Mill	Mkt. Rice Bran	Rice Bran	Consumption of Rice					Cost				Costing Activities												
Domingo MRC			Rice11	Rice12	Rice13	Rice21	Rice22	Rice23	Qrice	QBrice	QBbra	BBbra	Brice1	Brice2	Brice3	Brice4	Brice5	Clnd	Cmble	Crcnl	Cpovt	Cirig1	Cchnl	Cotbl	Clnd1	Cblcl1	Cblcl2	Cblcl3	Cirig2	Cchnl2	Cotbl2	Clnd2	Cblcl1	Cblcl2	Cblcl3
Obj			0	0	0	0	0	0	0	0	-0.060	0	769	762	775	786	797	-0.001	-0.001	-0.002	0.001	-0.000	-0.001	-0.001	-0.000	-0.000	-0.000	-0.000	-0.000	-0.001	-0.701	-0.000	-0.000	-0.000	-0.000
PrRice	L	0	-3.5	-6	-7	-3.5	-6	-7	1																								
LnRbfr1	L	2000	1	1	1																												
Irrig1	L	0		1	1																												
Chem1	L	0	10.02	23.34	26.3																	-1											
OthInp1	L	0	35.31	63.02	93.22																		-1										
Labor1	L	0	3	6.25	8.5																			-1									
CpHired1	L	0	73	162	127																				-1								
LnRbfr2	L	2750				1	1	1																		-1	-1	-1					
Irrig2	L	0					1	1																									
Chem2	L	0				10.02	23.34	26.3																					-1				
OthInp2	L	0				35.31	63.02	93.22																						-1			
Labor2	L	0				3	6.25	8.5																							-1		
CpHired2	L	0				93.66	141.87	171.99																								-1	
TrRice	L	637							-0.61	1	-1																						-1	-1	-1
TrBbra	L	0							-0.1			1000																							
Indust	L	0							0.66																										
WhLoc	L	0																-1																	
Retail	L	0								1.8									-1																
Govt	L	0								6.1										-1															
WhRice	L	0				-			-13.67		-80.3										1														
ConsHr	L	1								-1			7500	7750	8000	8250	8500																		
ConsRic	L	1										1																							
Irr1	L	943											1	1	1	1	1																		
Irr2	L	1170																																	
LnRicHr	L	400									1																		1						
QCRice11	L	35000																								1									
QRice12	L	45000																									1								
QCRice13	L	30000																										1							
QRice21	L	60000																											1						
QRice22	L	60000																															1		
QRice23	L	30000																																1	
RiceBrn	E	0	1	-0.1	-0.1	1	-0.1	-0.1																											1

Production of rice. Three are six activities for the production of rice in the current form of the Domingo model, three activities for small farms and three activities for large farms. The total land supply for all crops is shown in the RHS column. However the rice activities must compete with the other crops in the model for land. The input-output coefficients for the are the same for the two farms with the exception of the requirements for short-term capital. The capital requirements for the large farm are larger than those for the small farm. Apparently this reflects the greater use of hired labor by medium and large farms. (In a model such as this, the objective function coefficient in the objective function may represent both actual wages page and reservation wage).

Processing of rice. The Activity Qric serves as a variable to measure the farm production of rice (paddy) and also as a rice milling activity. Each unit of paddy rice is milled into .61 units of white rice and .1 units of rice bran. The cost of milling the rice is 5.66 pesos. There is a government subsidy of 13.67 pesos per unit of paddy rice milled. Rice may also be imported. The cost of the imported rice is .06 per unit. There is also a government subsidy of 20.7 pesos per ton. There is an upper limit placed on the quantity of rice which may be imported (see row IMRicMx- imported rice maximum).

Marketing rice. The Qwric represents the movement of rice from the mill or from the outside world through the wholesale and retail channels to the domestic market. The marketing margins for which rice at the wholesale and retail levels are 1.2 and 4.1 per unit white rice. The row "WhRice" requires that the supply of rice moving through the retail channels is greater than or equal to the domestic demand for rice.

Consumption of rice. The domestic consumption of rice at the retail level is accounted for by the demand activities Dric1-5. These represent 5 segments on the equation which relates the price of rice to the quantity of rice consumed. There is a convex constraint for

the 5 demand activities for rice. There is also one activity for the consumption of rice bran. The demand for rice bran is a derived demand since the main use is for livestock feed.

The remaining activities shown in Table 4 are the activities used to set the prices for inputs. Note that separate costing or charging activities are provided for small and for large farms. This is to facilitate policies designed to affect input costs differently between small and large farms.

Production of maize. The maize (corn) production, consumption, and marketing activities from the Domingo model are presented are in Table 5. The method of modeling the maize production activities is similar to the method of modeling the rice producing activities (The other annual crop activities are handled in a similar manner). There are three levels of technology for each size of farm. The technologies for the smaller farms place greater emphasis on the use of labor while the activities for the larger farms use more purchased inputs. Two capital borrowing activities are included for each farm. The purpose was to model two sources of credit at different rates of interest for each size of farm. For the small farms, the first activity CCrn11 would have the lower rate. This source would be used first. If it was profitable to borrow additional funds at a higher rate, then unlimited borrowing would be possible through activity CCrn12. The capital borrowing activities for the large farms (CCrn21 and CCrn22 operate in a similar manner.

The is no processing of corn. The Qcorn activity transfers the corn from the market to the wholesale level. The wholesale margin is assumed to be .83 pesos per hundredweight. Corn can also be imported. There is a constraint on the maximum amount of corn which can be imported. Imported corn also moves into the wholesale market

Consumption of corn. The consumption of corn is modeled at the wholesale level. The segments of the demand equation are represented by activities Dcrn1-5 (Activity Dcrn1 was left out). There is a convex constraint for the corn consumption activities.

Table 5. Maize Production, Marketing, Import, and Consumption Activities

MODEL: DOMINGO

			Small Farm			Medium-Large Farm			Market	Import	Domestic Consumption				Credit Activities Small Farm		Credit Activities Large Farm	
Doming	MAX		Corn11	Corn12	Corn13	Corn21	Corn22	CrBn21	QCorn	ImCrn	Dcrn2	Dcrn3	Dcrn4	Dcrn5	CCrn11	CCrn12	CCrn21	CCrn22
Obj			0	0	0	0	0	0	0	-0.017	186	199	211	223	-0.000	-0.000	-0.000	-0.000
PrMaiz	L	0	-2	-2.25	-1.2	-2.25	-4.75	-1.2	1									
LnfSfr1	L	2685	1	1	1													
Irrig1	L	0																
Chem1	L	0																
OthInp1	L	0	0.81	12.7	4.86													
Labor1	L	0	3	2	4.75													
CpMaiz1	L	0	8.01	18.71	16.26													
LnfSfr2	L	2750				1	1	1							-1	-1		
Irrig2	L	0					1											
Chem2	L	0					19.92											
OthInp2	L	0				12.7	31.8	4.86										
Labor2	L	0				2.5	2.5	4.75										
CpMaiz2	L	0				33.45	73.58	31.46										
Indust	L	0															-1	-1
Whols	L	0							0.83	2.38								
Retail	L	0																
govert	L	0																
Whcorn	L	0							-1	-1	5350	5850	6350	6850				
ImxCrn	L	1									1	1	1	1				
ImxCrn	L	4000								1								
CpSFCrn	L	2000																
CpLfCrn	L	2350													1			
CrnBanR	L	0	0.35	0.35	-1	0.35	0.35	-1									1	

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PROCESSING ACTIVITIES AND MARKETING MARGINS

Reference: Hazell, B.R. and R.D. Norton, Chapter 8.

Objectives

The objectives of this section are to illustrate how processing activities and costs of marketing at the wholesale and retail levels can be formulated in agricultural sector models. Consideration is also given to Goverenment taxes and subsidies.

Background

The path or marketing channels by which a raw agricultural commodity moves from the agricultural producer through the processor (if any) to final consumer varies by product and by country.

Processing activities are a form of intermediate production activities. A raw agricultural product like paddy or hulled rice is transformed into final products like white rice and rice bran. The rice bran may be sold by a wholesaler to producers for animal feed. The wholesaler may deliver the milled rice to an exporter or to a retailer who in turn sells it to the final consumer.

A numerical example is used to show the process described above both as a diagram and in linear programming format.

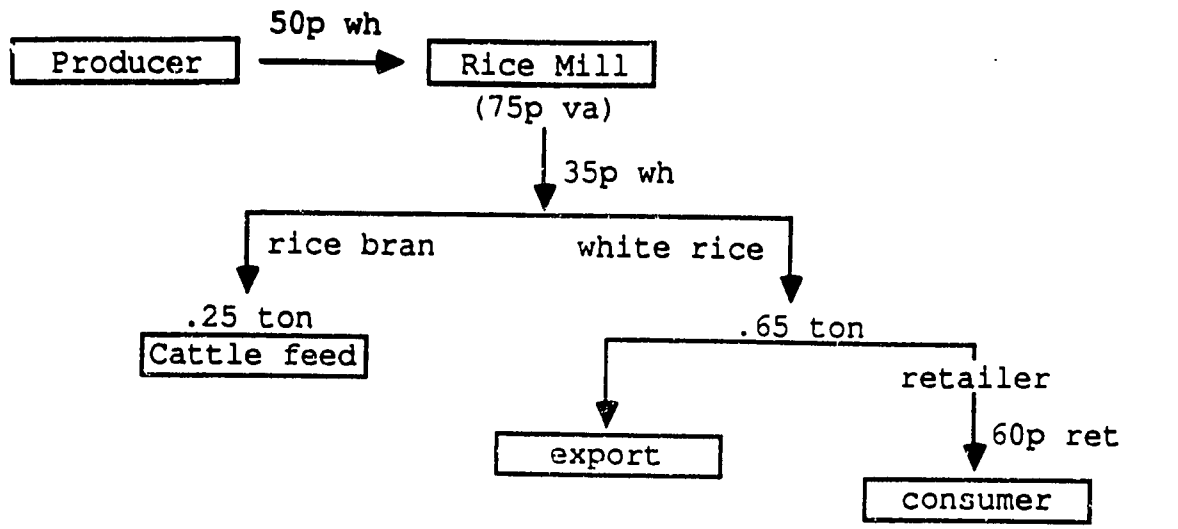

Figure 1. Marketing and Processing Channel for Rice.

In the above diagram, one ton of paddy or hulled rice is assumed to be milled into .65 tons of white rice and .25 tons of rice bran. The cost of moving the rice from the farm to the mill is 50 pesos per ton. The cost of milling (value added in agricualtual manfacturing) is 75 pesos per ton of paddy rice. The wholesale cost of moving the rice from the mill to either the exporter or the retail distributer is is 35 pesos per ton of paddy. If the rice is exported, the price is P_e. If the rice is delivered to the retailer, the retailer adds 60 pesos per ton of white rice.

The information in the diagram in Figure 1 is shown in a linear programming tableau in Figure 2.

	Rice Producer	Rice Mill	Export	Retailer	Consumption D1	D2	D3	Act ..Dn
Obj	-Cp		Pe		W1	W2	W3	..Wn
Paddy rice	-Yld	1						
White Rice (wh)		-.65	1	1				
Rice Bran (wh)		-.25						
White Rice (ret)				-1	q1	q2	q2	..qn
Val.Add Manf		75						
Whs.Margin		85						
Ret.Margin				60				
Conv.Const.					1	1	1	1

The rice milling activity in the above tableau actually combines the operations of wholesale marketing between the farm and the mill and between the mill and either the exporter or retailer. The margins for the value added by the rice miller (as agricultural manufacturer) and by the wholesale and retail activities are entered as coefficients in the tableau. "Purchasing" activities for value added by wholesalers, retailers, and manufacturers with a "-1" in the respective rows will complete the model. The coefficients for wholesale and retail margins and manufacturing could have been subtracted from the objective function but having the coefficients in the tableau allows an accounting of each item.

The Sugar Processing Subsector from the Domingo Model

Many agricultural commodities are consumed in processed form. Sugar cane is converted to sugar, molasses, bagassee. Also as the product moves through the wholesale and retail channels. There is a markup involving transportation and the services of the various agents who handle the product. In addition, there may be government taxes and subsidizes imposed at various points in the marketing chain.

The modeling of the commodity as it moves through the marketing chain usually requires the use of several transfer rows and columns.

The linkage between the Agricultural Production Sector and the Processing Sector will be illustrated using a Sugar model from the Dominican Republic. The Sub model was formulated by Dr. Carlos Benito of the Berkeley Research Institute, in Berkeley, California. The basic data on Sugar production, processing, exports and prices is presented in Appendix A.

Structure of the Model

The model contains farm level production activities, A single processing sector which produces white sugar, molasses, honey, and furfural (an oily liquid used in making

lacquers and dyes). Each of the separate by-products can be consumed domestically or exported.

Farm Production of Sugar Cane. The production of sugar cane is represented by three major types of producers. (Colonial or private plantations, government sugar plantations CEA, and other producers) The three production activities are shown below in Table 1.

The total sugar land available for sugar production for each group and in total is

Private Plantation	1330
Government Plantation (cea)	1290
Other Private	995
Total Sugar Cane	3615

Sugar Cane Production Activities. Sugar Cane is a perennial crop. Sugar cane is typically replanted every 5 years. A description of the sugar cane activities for the Private Plantations, Government Plantation, and other private producers is shown in Table 1. The sugar cane activity is a composite activity which consists of 1.2 units of land. One land unit is the ratoon sugar cane which is harvested. Two tenths of a land unit is the sugar cane which is being replanted. In Table 1, the respective activities were defined by adding one unit of a ratoon crop to .2 units of a replanting activity. The sugar cane activities in the model are shown in the right hand column of Table 2.

The resource requirements except for labor (which is in person days) are given in terms of pesos. The major inputs have been grouped into labor, chemical, other variable costs and short term capital. The inputs coefficients for the three activities are the same however the yields (in metric tons) vary.

The Mini Tableau Section of the model which contains the sugar cane production activities and the Sugar Cane Milling or processing activity is shown above in Table 2. The respective production activities are grown in different regions or locations. However they are assumed to be affected equally by changes in wage rates, chemical costs, other variable costs, and changes in interest rates.

Table 1. Combining Ratoon Sugar Cane Production Activities with Replanting Activities to Give Combined Production-Replant Activities for Sugar Cane Production

Private Plantation Activities

S.Cane	Ratoon AAZUCR1		Replant		SCANEPP
Obj	0				0
PrdSCane	-2.15		0		-2.15
Chempur	20.01		29.48		25.906
Othcst	0.72		30.18		6.756
Labcost	0.65	+.2*	1	=	0.85
LndPPlan	1		1		1.0
LndGPlan	0		1		0
LndOth	0		1		0
St-Cap	24.37		65.26		37.422

Government Plantation Activities

	Ratoon		Replant		SCANEGP
Obj	0		0		0
PrdSCane	-3		0		-3
Chempur	20.01		29.48		25.906
Othcst	0.72		30.18		6.756
Labcost	0.65	+.2*	1	=	0.85
LndPPlan	0		0		0
LndGPlan	1		1		1.0
LndOth	0		0		0
St-Cap	24.37		65.26		37.422

Other Sugar Cane Production Activities

	Ratoon		Replant		SCANEOTH
Obj	0		0		0
PrdSCane	-2.75		29.48		-2.75
Chempur	20.01		30.18		25.906
Othcst	0.72	+.2*	1	=	6.756
Labcost	0.65		1		0.85
LndPPlan	0		0		0
LndGPlan	0		0		0
LndOth	1		1		1.0
St-Cap	24.37		65.26		37.422

Table 2. Sugar Cane and Sugar Production Activities from the Sugar from the Sugar Model

S.Cane	Maxim	RHs	SCANEPP	SCANEGP	SCANEOTH	SuMill	MarWS	XPWSUS
Obj			0	0	0	0	0	298
PrdSCane	L	0	-2.15	-3	-2.75	1		
Chempur	L	0	25.906	25.906	25.906			
Othcst	L	0	6.756	6.756	6.756			
Labcost	L	0	0.85	0.85	0.85			
LndPPlan	L	1330	1	0	0			
LndGPlan	L	1290	0	1	0			
LndOth	L	995	0	0	1			
St-Cap	L	0	37.422	37.422	37.422			
PrdWSug	L	0				-0.11	1	1
PrdMola	L	0				-6.63		
PrdHon	L	0				-0.3		
PrdFurfu	L	0				-0.0036		
Ind-VA	L	0				58.4		
WhsMarg	L	0					99	
RetMarg	L	0					72	
GTaxSub	E	0				59.2		
WhSug	L	0					-1	
WhMolas	L	0						

Activities

SCANEPP is one tarea producing of sugar cane grown on private plantations. The yield is 2.15 metric ton of sugar cane per tarea.

SCANEGP is one tarea producing of sugar cane grown on government plantations. The yield is 3 metric tons of sugar cane per tarea.

SCANEOTH is one tarea producing of sugar cane grown on other private holdings. The yield is 2.75 metric tons of sugar cane per tarea.

SumIII represents the processing on one metric tone of sugar cane. One metric tone of sugar cane yields .11 tons of white sugar, 6.63 gallons of molasses, .3 gallons of honey, and .0036 tons of furfural which is used for making lacquer. The value added by the surgar processor (the cost of processing) is 58.4. The entry in the Gtaxsub row represents a government subsidy because the the objective function value of the CoGovt activity in Table 5 is positive.

MarWS is an activity which transfers refined (white) sugar to the domestic market. The sugar which moves to the domestic market goes through the wholes and retail channels. The cost of marketing at the wholesale is 99 pesos per ton and the cost of marketing at the retail level is 72 pesos per ton.

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Rows

PrdSCane is a row for transferring surgar cane from the farm to the mill. The unit is in metric tons.

Chempur. This row measures the purchase of chemical items used in the production of agricultural commodities. The units are in pesos rather the quantities of inputs.

Othcst. This row measures the purchase of other items excluding chemicals and wages used in the production of agricultural commodities. The units are pesos.

Labday. This row measures the quantity of labor (in person days) used in the production of agricultural commodities.

LandPPlan. This row accounts for the quantity of land on private plantations which may be used for producing sugar cane.

LndGPlan. This row accounts for the quantity of land on government plantations which may be used for producing sugar cane.

LndOth. This row accounts for the quantity of land on other private holdings which may be used for producing sugar cane.

St-Cap. This row accounts for the use of short term capital used in agricultural production.

PrdWsug. Accounting row for the production of white or refined sugar. Measured in metric tons.

PrdMola. Accounting row for the production of molasses. Measured in gallons.

PrdHon. Accounting row for the production and utilization of honey (Mieles) which is produced in sugar refining. Units are in gallons.

PrdFurfu. Accounting row for the production and utilization of furfural which is produced in sugar refining. Units are unknown.

Ind-VA. Accounting row for the value added or the processors cost in refining sugar. The units are in pesos.

WhsMarg. Accounting row for measuring the wholesale markup for commodities that move through the wholesale marketing channel (in pesos).

RetMarg. Accounting row for measuring the the wholesale markup for commodities that move through the retail channel (in pesos).

GTtaxSub. Accounting row for measuring Government subsidies or taxes, (in pesos).

Table 3. Marketing and Consumption activities for Refined or White Sugar

S.Cane	Max	SuMill	MarWS	XPWSUS	DeWS1	DeWS2	DeWs3	DeWS4	DeWs5
Obj		0	0	298	277556	292089	304839	315806	324989
PrdSCan	L	1							
PrdWSug	L	-0.11	1	1					
PrdMoll	L	-6.63							
PrdHon	L	-0.3							
PrdFurf	L	-0.003							
Ind-VA	L	58.4							
WhsMarg	L		99						
RetMarg	L		72						
GTaxSub	E	59.2							
WhSug	L		-1		220	240	260	280	300
WhMolas	L								
ConxSug	L				1	1	1	1	1
Xpmxsug	L			1					

WhSug. Accounting row for white or refined sugar for domestic consumption (measured in metric tons).

WhMolas. Accounting row for molasses which is consumed domestically.

The marketing section of the tableau traces the movement of the sugar products after they leave the sugar mill. Only the relevant rows are shown. The sugar cane milling activity is repeated for a reference.

As each ton of refined sugar leaves the mill (row PRDWSug) it can be exported through activity SPWSUS (export white sugar to the US) or it can be transferred to the domestic market.

SPWSUS. This is a sugar export activity. If sugar is exported, the returns are 298 pesos per ton up to a limit of 780 (000) metric tons. The activity MarWS (market white sugar) represents the transfer of sugar to domestic consumption. The sugar is assumed to move through commercial channels and the respective wholesale and retail costs are 99 and 72 pesos per ton.

WhSug row. This is an accounting row for sugar in the domestic market.

DeWS1-5. These are segments from the price flexibility equation for the domestic consumption of sugar. The values in the objective function represent the area underneath the price equation up to the quantities shown in the row WhSug.

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ConxSug. This is the convex constraint equation for the segmented price flexibility equation.

Marketing and Consumption of other Sugar Products

The section of the tableau for the marketing, consumption of the remaining sugar byproducts is shown below in Table 4.

The byproducts produced in the manufacture of refined sugar are molasses, honey, and furfural. The molasses (row Prdmol) can either be exported or transferred to the domestic market.

XpMol is the activity which exports one gallon of molasses. The export price is .3 pesos per gallon. There is an upper limit of 27000 gallons which can be exported. The upper limit is provided by the constraint row Xpmxmol.

Table 4. Marketing, Domestic Consumption and Export Activities for Sugar Byproducts

S.Cane	Max	H	RHS	SuMill	MarMol	XPMol	XPHon	XPFurf	DEMMOL
Obj				0		0.3	0.95	627	8856
PrdSCan	L			1					
PrdWSug	L			-0.11					
PrdMol	L			-6.63	1	1			
PrdHon	L			-0.3			1		
PrdFurf	L			-0.003				1	
Ind-VA	L			58.4					
WhsMarg	L								
RetMarg	L								
GTaxSub	E			59.2					
WhMolas	L		0		-1				25305
ConvMol	L		1						1
Xpmxmol	L		27000			1			
Xpmxhon	L		3060				1		
XpmxFur	L		35					1	

MarMol. This activity transfers molasses to the wholesale market. Note that there is no wholesale charge. This is likely because of the lack of information on the wholesale margin.

DEMMOL. One unit of this activity represents the consumption of 25,305 gallons of molasses. The objective function value represents the revenue from the 25,305 gallons.

XPHon represents the export of 1 gallon of Honey at a price of .95 pesos. The export market is the only outlet provided for honey.

Xpmxhox. This restriction places an upper limit of 2060 gallons on the quantity of honey which can be exported.

XPFurf. This activity represents the export of one unit of furfural at a price of 627 pesos per gallon.

XpmpFur. This constraint places a limit of 35 on the number of units of furfural which can be exported.

The final section of the tableau shown in Table 5 shows the purchasing or costing activities for chemical, other inputs, labor, the marketing margins, value added in manufacturing, and government subsidies.

Table 5. Purchasing or Charging Activities for Inputs used in the Sugar Cane Model

S.Cane	Max	RHs	CIndus	COWhsl	CORetl	COGovt	COchem	COoth	COLab
Obj			-1	-1	-1	1	-1	-1	-8
PrdSCan	L	0							
Chempur	L	0					-1		
Othcst	L	0						-1	
Labcost	L	0							-1
Ind-VA	L	0	-1						
WhsMarg	L	0		-1					
RetMarg	L	0			-1				
GTaxSub	E	0				-1			

The activities shown in Table 5 are used to provide the appropriate cost for the input services used in the model. The effects of inflation could be accounted for by replacing the objective function value with a cost index. Note that the value on the government activity is positive. This is a subsidy which has the effect of reducing the cost of sugar processing and makes the product more competitive in both the export and domestic markets.

The completed tableau is shown in Table 6. The solution to the problem shown in Table 6 is presented in Table 7.

Table 6. Tableau of Sugarcane Production, Processing and Marketing Submodel

S.Cane	Basis	Rhs	SCANEPP	SCANEGP	SCANEOTH	SuMill	MarSU	XPMSUS	MarMol	XPMol	XPBcs	XPFurf	DEMMS1	DEMMS2	DEMMS3	DEMMS4	DEMMS5	DEMMOL	Smol-15	COINDUS	COMhsl	CMMol	COGovt	COchem	COcth	COLab	CapBor
Obj			0	0	0	0	0	290		0.3	0.95	627	277556	292092	306839	315806	324989	8856	0.15	-1	-1	-1	0	-1	-1	-8	-0.10
PrdSCane	L	0	-2.15	-3	-2.75	1																					
Changsr	L	0	25.906	25.906	25.906																						
Ckhsst	L	0	6.756	6.756	6.756																			-1			
Labcost	L	0	0.85	0.85	0.85																				-1		
LndPPlan	L	1330	1	0	0																					-1	
LndGPlan	L	1290	0	1	0																						
LndOth	L	995	0	0	1																						
St-Cap	L	0	37.422	37.422	37.422																						
PrdMSug	L	0				-0.11	1	1																			-1
PrdMolla	L	0				-6.63			1	1																	
PrdBcs	L	0				-0.3					1																
PrdFurfu	L	0				-0.0036						1															
Ind-VA	L	0				58.4																					
MhsMarg	L	0					99													-1							
RetMarg	L	0					72														-1						
GTanSub	E	0				59.2																-1					
MhSug	L	0					-1						220	240	260	280	300						-1				
MhMolas	L	0							-1																		
CanSug	L	1																25305	1								
CanMol	L	1											1	1	1	1	1										
XpmSug	L	700						1										1									
XpmMol	L	37000								1																	
XpmBcs	L	3060									1																
XpmFur	L	25										1															

Table 7. Solution to the Sugar Sector Production, Processing and Marketing Model

OPTIMAL Solution
function Value: 379360.0

Activities in Solution

Obj	Name	Type	Level
0	SCANEPP	real	1330.00
0	SCANEGP	real	1290.00
0	SCANEOTH	real	995.00
0	SuMill	real	9465.75
0	MarWS	real	280.00
298	XPWSUS	real	761.23
0	MarMol	real	25305.00
0.3	XPMol	real	37000.00
0.95	XPHon	real	2839.72
627	XPFurf	real	34.08
315806	DEMWS4	real	1.00
8856	DEMMOL	real	1.00
-1	COINDUS	real	552799.8
-1	COWhsl	real	27720.00
-1	CORetl	real	20160.00
1	COGovt	real	560372.4
-1	COchem	real	93650.19
-1	COoth	real	24422.94
-8	COLab	real	3072.75
-0.18	CapBor	real	135280.5
0	Prdmola	slack	452.92
0	Xpmxsug	slack	18.77
0	Xpmxhon	slack	220.28
0	XpmxFur	slack	0.92

Activities not in Solution

Name	Obj	Type	Shad.Price
DEMWS1	277556	real	10110.00
DEMWS2	292089	real	4957.00
DEMWS3	304839	real	1587.00
DEMWS5	324989	real	197.00
PrdSCane	0	slack	36.12
Chempur	0	slack	1.00
Othcst	0	slack	1.00
Labcost	0	slack	8.00
LndPPlan	0	slack	31.46
LndGPlan	0	slack	62.17
LndOth	0	slack	53.14
St-Cap	0	slack	0.18
PrdWSug	0	slack	298.00
PrdHon	0	slack	0.95
PrdFurfu	0	slack	627.00
Ind-VA	0	slack	1.00
WhsMarg	0	slack	1.00
RetMarg	0	slack	1.00
GTaxSub	-1.0E+14	slack	1.00E+14
WhSug	0	slack	469.00
WhMolas	0	slack	0.00
ConxSug	0	slack	184486.00
ConxMol	0	slack	8856.00
Xpmxmol	0	slack	0.30

Appendix A. Data for the Sugar Sector of the DOMINGO MODEL

SUGAR CANE: PRODUCTION AND UTILIZATION
(UNITS IN OOO)

Year	Area Planted Tareas	Cane Harvested Ton.	Sugar Tn.	Molasses Gall.	Honey Gall	Furfural
1980						
1981						
1982		11805	1285	77992	2501	39
1983		11520	1209	67900	2996	34
1984		10271	1130	65904	3047	31
1985	2166	8420	921	49520	3062	27
1986		5455	894	52832	2678	38
Average	2166	9494.2	1087.8	62829.6	2856.8	33.8

REFINED SUGAR: SUPPLY AND UTLIZATION
(000 TON)

Year	BegStock Jan. 1 TON.	Produc-tion Ton.	Total Supply Tn.	Consump tion	Export tion	EndStock Dec 31
1980						
1981						
1982	250	1285	1534	222	850	462
1983	462	1209	1670	239	956	475
1984	477	1130	1607	258	885	464
1985	464	921	1399	304	722	374
1986	374	894	1235	294	480	461
Average	405.4	1087.8	1489	263.4	778.6	447.2

EXPORTS OF SUGAR

Year	Quantity TM	Value	Price /TM
1980			
1981			
1982	860	264	310.59
1983	959	267	311.66
1984	885	288	325.92
1985	722	177	245.88
1986			
Average	856.5	249	298.5125

MOLASSESS: SUPPLY AND UTILIZATION
Gallones (000)

Year	BegStock Jan. 1	Production	Consumption Direct	Consumption Industry	Export	EndStock Dec 31
1980						
1981						
1982	9354	77992	18363	8828	53170	6986
1983	6616	67900	18822	3914	35834	9578
1984	9578	65904	24742	11113	35771	3856
1985	3856	49520	19699	9746	27260	1694
1986	1694	49961	4277	10051	32714	4612
Average	6219.6	62255.4	17180.6	9930.4	36949.8	5345.2

HONEY: SUPPLY AND UTILIZATION

Year	BegStock Jan. 1 Tas.	Production Gal	Export	BegStock Dec.31	Value Exports	Price Gallon /Gall A
1980						
1981						
1982	2501	2479		50		0.99
1983	2996	2993		53		0.9
1984	3045	3010		37		0.99
1985	3062	3054		45		0.88
1986	3678	2668		55		0.99
Average	3056.4	2840.8		48		0.95

FURFURAL: SUPPLY AND UTILIZATION

Year	BegStock Jan. 1	Production	Export	BegStock Dec.31	Exportac	Valor Tn	Price
1980							
1981	34	34		3		773.2	
1982	39	37		5		683.7	
1983	34	36		4		628.4	
1984	31	33		2		604.5	
1985	29	28		3		589.84	
1986	38	35		4			
Average	34.2	33.8		3.6		626.61	

SUGAR REFINED PRICES

Year	Price at Mill	Price at Wholesale	Price at Retail	Consumer Price
1982	11.45	12.54	13.04	15
1983	11.45	12.54	13.04	15
1984	11.45	12.54	13.04	15
1985	13.95	15.05	15.65	18
1986	25	25	26.69	30
Average	17.76720	22.11420	23.23494	26.51634

RISK IN THE SECTOR MODEL

Reference: Hazell, B.R. and R.D. Norton, "Risk in the Sector Model." Chapter 10 in *Mathematical Programming for Economic Analysis in Agriculture.*

Hazell, B.R. and P.L. Scondizzo. "Market Intervention Policies when Production is Risky," *Amer. J. Ag. Econ.* 57:641-49, 1977.

Objectives

The objective of this section is to present a method of incorporating risk analysis in the sector model and illustrate the process with the Mayaland model.

Current method for studying risk in sector models was developed by Hazell and Scandizzo. The method requires 3 main assumptions.

1. The source of risk lies *only* in yield variations. Any price variability is a result of quantity variations in the market.

2. Farmers operate in a competitive market and maximize utility according to either an income variance approach (EV) or income standard deviation approach (Eσ).

3. Production depends on lagged prices because farmers must commit their resources each year before yields and prices are known (farmers base productions on past prices).

Single Product Market

Equations	Definitions
$D_t = a - bP_t$	Quantity demanded depends on price
$S_t = \lambda Y_t P_t^*$	Quantity supplied depends on realized yields and expected prices
$D_t = S_t$	The market clears each year

Variables

Y_t is a random yield with mean yield $\overline{Y}$, variance r_y^2

a and b are the intercept and slope of the demand equation.

λ is a constant in the supply function.

P_t^* is a forecast price. This could be a function of lagged prices (Nerlove), last year's price (Cobweb Model) or any other unbiased forecast.

The supply function (anticipated) is

$$S_t = \lambda \overline{Y} P_t^*$$

dependent on average yields and forecast prices. The actual supply function rotates as the actual yield varies from the mean yield.

The market clearing price in year t is obtained by substituting the demand and supply equations into the market clearing equation.

$$D_t = S_t$$

$$a - bP_t = \lambda Y_t P_t^*$$

so $$P_t = \frac{a - \lambda Y_t P_t^*}{b}.$$

For a given year t, the actual or realized price P_t is not necessarily equal to the anticipated price P_t^*. However a sufficiently long period the should converge.

If the market does converge so that

$$\lim E[P_t] = \lim E[P_{t-1}]$$

then the expected equilibrium price is

$$P_e = \frac{a}{b + \lambda \overline{Y}}.$$

The equilibrium solution is then modeled in a mathematical programming model by maximizing the shaded area in Figure 1 or see Figure 10.2 in the text.

$$w = \int_0^{\overline{P}} (a - bP)\, dP - \int_0^{\overline{P}} \lambda \overline{Y} P dP$$

$$= a\overline{P} - 1/2\,(b + \lambda \overline{Y})\overline{P}^2.$$

which is maximized by

$$\overline{P} = \frac{a}{b + \lambda \overline{Y}}$$

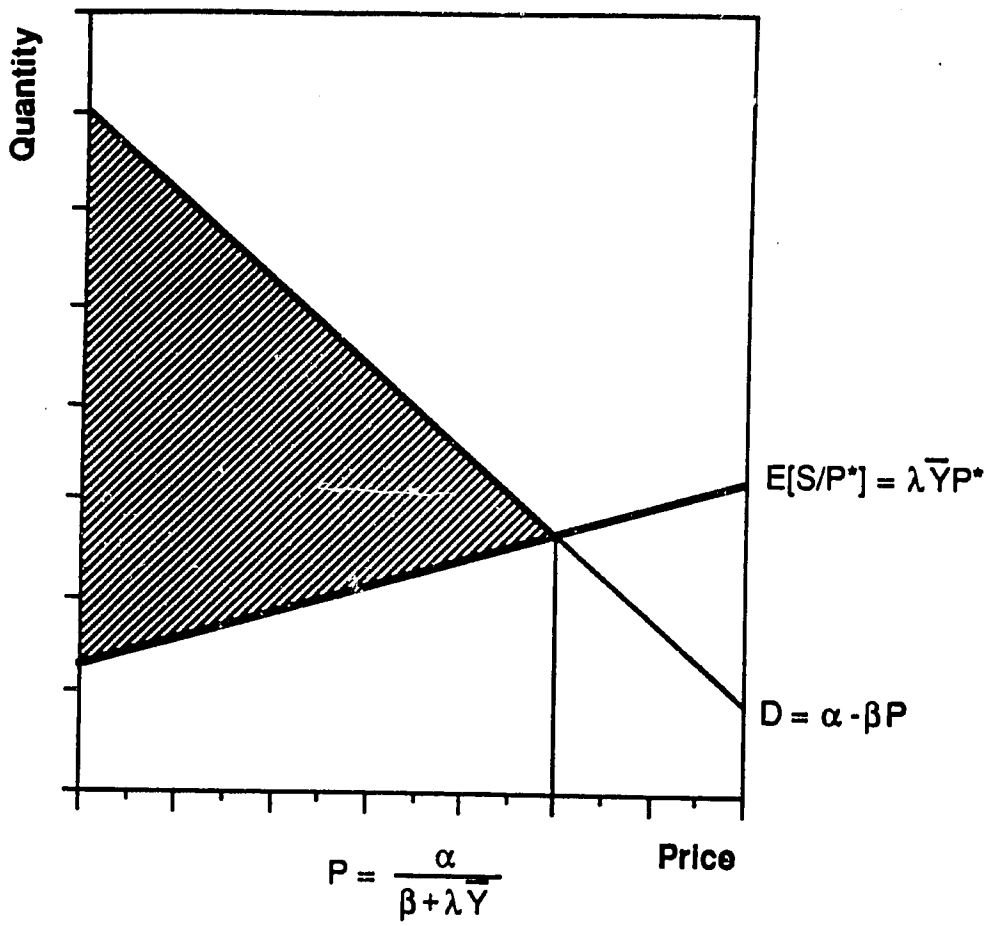

Figure 1. The Expected Market Equilibrium Price with Yield Risk.

The individual farm in the sector model. Models can be set up to max utility - by

a) Expected Income — Variance Model

b) Expected Income — Standard Deviation Model (we will work with b)

and by using

a) a forecast of expected price

b) a forecast of revenue per unit.

Hazell shows the revenue forecast is superior to the use of a price forecast. However we will develop a model in which a price forecast is used.

The basic farm model is

$$\text{max utility} = P^* E(Y) X - CX - \phi (X \Omega X)^{.5}$$
$$\text{subject to } AX \leq b$$

Hazell forms a Lagrangian

$$L = P^* E(Y) X - C'X - \phi (X \Omega X)^{.5} + v(b - Ax)$$

which is maximized with respect to X

$$\frac{\partial L}{\partial X} = \underbrace{P^* E(Y)}_{\text{revenue}} - \underbrace{C}_{\text{cost}} - \underbrace{\phi (X \Omega X)^{-.5} \sum W_{ij} X_i}_{\text{risk premium}} - A'V = 0.$$

where ϕ is a risk aversion coefficient

$X \Omega X$ is the variance of income

w_{ij} is a covariance term in Ω.

The maximization of utility for a risk averse producer requires that at the activity level,

marginal activity revenue = marginal cost + rent + risk discount or premium.

(A more risk averse producer will apply less input than will a risk neutral producer.)

In the sector model the aggregation requires that all producers have the same risk aversion coefficients.

There are two terms in the objective function which must be linearized, the term which measures the area under the demand equations and the term for the square root of the variance. Under the assumption that producers form price expectations the relevant objective function for the sector model is

$$\max X'NA - .5X'N'BNX - C'X - \phi(X'\Omega X)^{.5}$$

where

N is an n x n diagonal matrix with expected yields ($\overline{Y}$) on the diagonal

B is a diagonal matrix of slopes of demand equations. The demand structures are assumed independent so the off diagonal elements are zero

X is a vector of activity levels

C is a vector of costs for activity units

Ω is a variance-covariance matrix of activity revenues.

The first two terms are linearized by using the approach for segmenting demand equations which was used earlier.

The risk term is linearized using the deviations approach. The MAD estimator of

$$\phi(X'\Omega X)^{.5} \text{ is } \frac{\sum_t \sum_j |r_{jt} - \overline{r}_j| \cdot S}{T}$$

where $S = \sqrt{\frac{T\ \pi}{2(T-1)}}$

T = number of years.

Thus the measures of risk to include in the programming tableau are

$\Sigma (r_{jt} - \overline{r}_j) X_j + Z_t^- \geq 0$, where Z_t^- measures negative deviation in year t.

This form measures half of the total deviations. The estimate of the standard error of income is

$$\sigma = K \Sigma Z_t^- = \frac{2S}{T} \sum_t Z_t^-,$$

where $K = \frac{2S}{T}$

Risk Constraints in the Domingo Feedgrain Subsector

The corn and sorghum related activities of the DOMINGO model are shown as submodel in Table 1 to illustrate how risk coefficients are included in a sector model. Note that the model has the same structure as a regular sector model except for the nine risk related rows at the bottom and the Z_t and Sigma colums on the right of the model.

The Z_t^- activities at the right of the tableau work the same in the sector model as in the farm model. They measure negative deviations from average income.

The last equation in the model calculates the standard error of income which is measured by the variable sigma.

$$\sigma = \frac{2S}{T} \sum_t Z_t^- = .3349 \sum_t Z_t^-.$$

In this example, the relative variabilities in the yield of all corn activities are the same for a given farm size only because there was insufficient information about how inputs such as fertilizer and insecticides affect the variability of yields to make other assumptions. Similar assumptions were made for sorghum. In each case the deviations in net returns were calculated as

$$R_{jt} = P_j (Y_{jt} - Y_j)$$

where

- R_{jt} are the amount by which net returns in year t for activity j are above or below average returns
- P_j is the estimated equilibrium price
- Y_{jt} is the expected yield for activity j in year t
- Y_j is the mean yield for activity j.

The relative year to year variability in yields between the large and the small farms are the same for each crop. This is somewhat reasonable since there no reason to believe that a yield deficit by a small farm would not be matched by a similar deficit on a larger farm in the same region.

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Table 1. Domingo Corn and Sorghum Subsector Model With Risk Constraints

Doming	Max	Rhs	Corn11	Corn12	Corn13	Sorg11	Sorg12	Rnt1	Corn21	Corn22	Sorg21	Sorg22	Rnt2	QCorn	Qsorg	ImCrn	Dcrn1	Dcrn2	Dcrn3	Dcrn4
Obj			-0.81	-12.7	-4.86	-28.03	-21	30	-12.7	-51.72	-8.429	-10.85	30	0	0	-30	243504	274476	302400	327276
PrMaiz	L	0	-2	-2.25	-1.2				-2.25	-4.75				1						
PrSorg	L	0				-3	-3.9				-3.9	-4			1					
LnfSfr1	L	1500	1	1	1	1	1	1												
Labor1	L	0	3	2	4.75	3	3													
StCap1	L	0	8.01	18.71	16.26	30.05	23.74													
LnfSfr2	L	1200							1	1	1	1	1							
Labor2	L	0							2.5	2.5	2.2	2								
Stcap2	L	0							33.45	73.58	27.39	43.95								
Wholsal	L	0																		
WhsCorn	L	0												0.83		2.38				
WhsSor	L	0												-1		-1	4700	5500	6300	7100
CxCorn	L	1													-1					
CxSorg	L	1															1	1	1	1
ImxCrn	L	4000														1				
Year1	G	0	-6.420	-7.222	-3.852	6.6700	8.6710		-7.222	-15.24	8.6710	8.8934								
Year2	G	0	5.0690	5.7027	3.0414	7.2773	9.4605		5.7027	12.039	9.4605	9.7031								
Year3	G	0	-8.608	-9.684	-5.164	1.2057	1.5674		-9.684	-20.44	1.5674	1.6076								
Year4	G	0	-2.009	-2.260	-1.205	-20.70	-26.91		-2.260	-4.772	-26.91	-27.60								
Year5	G	0	13.430	15.108	8.0580	-11.30	-14.69		15.108	31.896	-14.69	-15.06								
Year6	G	0	13.645	15.350	8.1871	3.0986	4.0282		15.350	32.407	4.0282	4.1315								
Year7	G	0	-6.972	-7.844	-4.183	14.872	19.334		-7.844	-16.56	19.334	19.830								
Year8	G	0	-8.251	-9.283	-4.951	-1.119	-1.455		-9.283	-19.59	-1.455	-1.492								
StERR	E	0																		

Table 1. Domingo Corn and Sorghum Subsector Model With Risk Constraints (Continued)

Doming	Dcrn5	Dsor1	Dsor2	Dsor3	Dsor4	Dsor5	Bcap1	Bcap2	Hlab1	Hlab2	Cwhsl	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Sigm
Obj	351619	23687	25460	26909	28353	29687	-0.7	-0.7	-8	-8	-1	0	0	0	0	0	0	0	0	-1
PrMaiz																				
PrSorg																				
LnfSfr1																				
Labor1									-1											
StCap1							-1													
LnfSfr2																				
Labor2										-1										
Stcap2								-1												
Wholsal											-1									
WhsCorn	8000																			
WhsSor		775	855	925	1000	1075														
CxCorn	1																			
CxSorg		1	1	1	1	1														
ImxCrn																				
Year1												1								
Year2													1							
Year3														1						
Year4															1					
Year5																1				
Year6																	1			
Year7																		1		
Year8																			1	
StERR												0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	-1

The rental activities represent the opportunity to use land to produce crops not included in the model. The rental returns is also treated as a low but certain return. The model also allows for up to 4000 hundred weight (Cwt) of corn to be imported at a final cost of 32.38 pesos (30 import + 2.38 market cost) per cwt.

The results of the risk averse (Theta = 1) solution and the risk neutral solution are shown in Table 2. The results that production of the more risky crops (e.g., corn and sorghum) is reduced, product prices are increased, imports are increased and safer though lower paying activities (renting land for production of other crops) are increased when it is assumed producers exhibit risk averse rather than risk neutral behavior.

The general conclusion is that if producers are quite risk averse, the omission of risk from the sector model would lead to an over estimate of supply and an underestimate of prices.

Exercise

Complete the worksheet as shown in Figure 2 by following the procedures below:

1. As you can see, the worksheet is very similar to the sector model we had been working with, with the exception of slightly different treatment of labor and the addition of risk rows. In particular, the demand generator is the same one that we have been working with. Thus the best thing to do is to start with that demand generator. So, first load the demand generator into the worksheet, and use the **/Move** command to move the demand generator to the are beginning in K1.

2. Now complete the Musah tableau, making formula reference to the demand generator. For example, cell K19 should be defined as +K14. Then the **/Copy** command can be used to copy cell K19 to the range L19..U19.

3. The range Y35 to AD35 contains the formula:

$$\frac{2}{T}\left(\frac{T\ \pi}{2(T-1)}\right)^{\frac{1}{2}}$$

Table 2. Comparison of DOMINGO Feed Grain Submodel With Risk Averse and Risk Neutral Behavior

Item	Unit	Risk Averse	Risk Neutral
		Small Farms	
Corn Area	tt	750	1355
Corn Prod.	tcwt	1500	4064
Sorg.Area	tt	0	0
Sorg. Prod	tcwt	0	0
Area Rented	tt	750	145
		Medium/Large Farms	
Corn Area	tt	0	924
Corn Prod.	tcwt	0	2310
Sorg.Area	tt	256	276
Sorg. Prod	tcwt	1000	1075
Area Rented	tt	944	0
		Total Corn	
Total Prod	tcwt	1500	7100
Import	tcwt	4000	0
Consumed	tcwt	5500	7100
Retail Price	p/qq	36.44	31.03
		Total Sorghum	
Total Prod	tcwt	1000	1075
Consumed	tcwt	1000	1075
Retail Price	p/qq	19.18	16.78

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1											N Segs	11																			
2											Ada	-0.8																			
3											Alpha	337.5																			
4											Beta	0.2205																			
5											P0	150																			
6											Q0	850																			
7											P-lower	75																			
8											P-upper	300																			
9																															
10											1	2	3	4	5	6	7	8	9	10	11										
11											30.	277.5	255	232.5	210	187.5	165	142.5	120	97.5	75										
12											170	272	374	476	578	680	782	884	986	1088	1190										
13											54187.	83640	110797	135660	158227	178500	196477	212160	225547	236640	245437										
14											51000	75480	95370	110670	121380	127500	129030	125970	118320	106080	89250										
15																															
16																															
17																															
18	TINY	MAX	B	MA11	MA21	BE1	MA12	MA22	BE2	DBE	DMAZ1	DMAZ2	DMAZ3	DMAZ4	DMAZ5	DMAZ6	DMAZ7	DMAZ8	DMAZ9	DMAZ10	DMAZ11	LH1	LH2	Y	Z1	Z2	Z3	Z4	Z5	Z6	SIGMA
19	OBJ			0	0	0	0	0	0	250	54187.	83640	110797	135660	158227	178500	196477	212160	225547	236640	245437	-5	-5									-1
20	CBMA	L	0	-1.5	-2.5		-1.2	-2			170	272	374	476	578	680	782	884	986	1088	1190										
21	CVBMA	L	1								1	1	1	1	1	1	1	1	1	1	1										
22	CBME	L	0			-0.5			-0.6	1																					
23	LAND1	L	100	1	1	1																									
24	LAND2	L	300				1	1	1																						
25	LABOR1	L	0	30	40	35																-1									
26	LABOR2	L	0				30	38	36														-1								
27	BLH1	L	9000																			1									
28	BLH2	L	6750																				1								
29	INCOME	E	0							250	51000	75480	95370	110670	121380	127500	129030	125970	118320	106080	89250	-5	-5	-1							
30	RISK1	G	0	39	31.5	4	77	-285	5																1						
31	RISK2	G	0	-74	130	120	-68	58.5	-3																	1					
32	RISK3	G	0	-138	-67.5	-4	41	102.5	4																		1				
33	RISK4	G	0	-6	300	-110	-17.5	50.5	-7																			1			
34	RISK5	G	0	173	-255	-50	34	-130.5	5																				1		
35	RISK6	G	0	6	-139	40	-66.5	204	-4																					1	
36	SIGMAID	E	0																						1	1	1	1	1	1	-3

Figure 2. Small Sector Model With Risk Constraints.

where T is the number of risk rows. See if you can represent this formula in Lotus (hint: simplify the formula first).

4. The cell AE19 in the objective function is the NEGATIVE of the risk aversion coefficient. Risk aversion is 0.5 in the tableau (thus a -0.5 should be entered). A zero here represents no risk aversion.

5. After completion, this would be a Musah master tableau. Solve the model as given, then with no risk aversion, risk aversion = -1, and risk aversion = -0.5. Remember that the NEGATIVE of risk aversion should be entered into the objective function.

6. For the different solutions, compare the cropping patterns and income. The solutions for risk aversion = -1 and no risk aversion are shown in Table 3.

Table 3. Solution to Example Problem With No Risk Aversion and With the Risk Aversion Coefficient Equal to -1

	Theta = 0	Theta = -1
function Value:	109896.0	54809.7

Activities in the Final Solution

Returns	Name	Type	Level
0	MA21	real	100.00
0	MA22	real	177.63
0	BE2	real	0.00
158227.5	DMAZ5	real	0.73
178500	DMAZ6	real	0.27
-5	LH1	real	4000.00
-5	LH2	real	6750.00
0	Y	real	69265.79
0	Z1	real	47475.00
0	Z5	real	48680.92
0	SIGMA	real	362499.2
0	LAND2	slack	122.37
0	BLH1	slack	5000.00
0	RISK2	slack	23391.45
0	RISK3	slack	11457.24
0	RISK4	slack	38970.39
0	RISK6	slack	22336.84

Activities in the Final Solution

Returns	Name	Type	Level
0	MA11	real	43.53
0	MA21	real	15.95
0	BE1	real	40.52
0	MA12	real	143.39
0	MA22	real	48.38
250	DBE	real	20.26
110797.5	DMAZ3	real	1.00
-5	LH1	real	3362.07
-5	LH2	real	6140.18
0	Y	real	52923.61
0	Z1	real	385.93
0	Z2	real	3206.04
-1	SIGMA	real	13541.40
0	LAND2	slack	108.23
0	BLH1	slack	5637.93
0	BLH2	slack	609.82
0	RISK3	slack	3591.97

Activities Not in Solution

Name	Obj	Type	Shad.Pric
MA11	0	real	148.75
BE1	0	real	158.06
MA12	0	real	75.32
DBE	250	real	377.63
DMAZ1	54187.5	real	22950.00
DMAZ2	83640	real	13770.00
DMAZ3	110797.5	real	6885.00
DMAZ4	135660	real	2295.00
DMAZ7	196477.5	real	2295.00
DMAZ8	212160	real	6885.00
DMAZ9	225547.5	real	13770.00
DMAZ10	236640	real	22950.00
DMAZ11	245437.5	real	34425.00
Z2	0	real	0.00
Z3	0	real	0.00
Z4	0	real	0.00
Z6	0	real	0.00
CBMA	0	slack	198.75
CVXMA	0	slack	43350.00
CBBE	0	slack	627.63
LAND1	0	slack	296.87
LABOR1	0	slack	5.00
LABOR2	0	slack	10.46
BLH2	0	slack	5.46
INCOME	-1.0E+14	slack	1.0E+14
RISK1	0	slack	0.00
RISK5	0	slack	0.00
SIGMAID	-1.0E+14	slack	1.0E+14

Activities Not in Solution

Name	Obj	Type	Shad.Pric
BE2	0	real	33.73
DMAZ1	54187.5	real	4754.62
DMAZ2	83640	real	1229.81
DMAZ4	135660	real	1065.19
DMAZ5	158227.5	real	4425.38
DMAZ6	178500	real	10080.57
DMAZ7	196477.5	real	18030.76
DMAZ8	212160	real	28275.95
DMAZ9	225547.5	real	40816.15
DMAZ10	236640	real	55651.34
DMAZ11	245437.5	real	72781.53
Z3	0	real	3.77
Z4	0	real	3.00
Z5	0	real	2.18
Z6	0	real	0.32
CBMA	0	slack	254.19
CVXMA	0	slack	15729.30
CBBE	0	slack	250.00
LAND1	0	slack	390.80
LABOR1	0	slack	5.00
LABOR2	0	slack	5.00
INCOME	-1.0E+14	slack	1.0E+14
RISK1	0	slack	3.77
RISK2	0	slack	3.77
RISK4	0	slack	0.77
RISK5	0	slack	1.59
RISK6	0	slack	3.45
SIGMAID	-1.0E+14	slack	-1.0E+14

INTRODUCTION TO POLICY ANALYSIS

References: Hazell, B.R. and R.D. Norton. Chapter 12 in *Mathematical Programming for Economic Analysis in Agriculture.*

Tweeten, L. "Introduction to Agricultural Policy Analysis: The Distribution of Economic Costs and Benefits from Market Intervention." Oklahoma State University APAP Report B-5, June 1985.

Objectives

The objective is to indicate how changes in producer and Consumer surplus can be derived from linear programing solutions and used to measure changes in producer benefits, consumer benefits and government costs.

Base Free Market Solution

Step 1 — Incorporating supply curves in LP models. Recall the supply curve is the marginal cost curve. The total variable cost curve is the integral over the marginal cost curve.

If $MC = b_0 + b_1 Q_s$, then $TVC = \int_0^{Q_s} (b_0 + b_1 Q_s) dQ_s$

$TVC = b_0 Q_s + \frac{b_1}{2} Q_s^2$.

The supply curve an be incorporated into the model using linear approximations as was done with the demand equations.

A Semi-Real Supply and Demand Model

The supply or aggregate marginal cost curve in the Liberian model referenced by Tweeten can be approximated as

$$MC = 128.9 + 13.56\ Q_s,$$

where Q_s = thousand metric tons (all quantities are in thousand metric tons)

MC = dollars per metric ton.

The demand or price flexibility equation used in the examples is

$$P = 705.6 - 44.1\ Q_d,$$

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which was derived from the assumptions that $E = -.6$, $\overline{P} = 264.6$, $\overline{Q} = 10$.

The base LP model which incorporates the simple supply and demand equation is shown in Table 1. The problem is shown graphically in Figure 1. The specific results in Table 1 which are discussed below are in bold type.

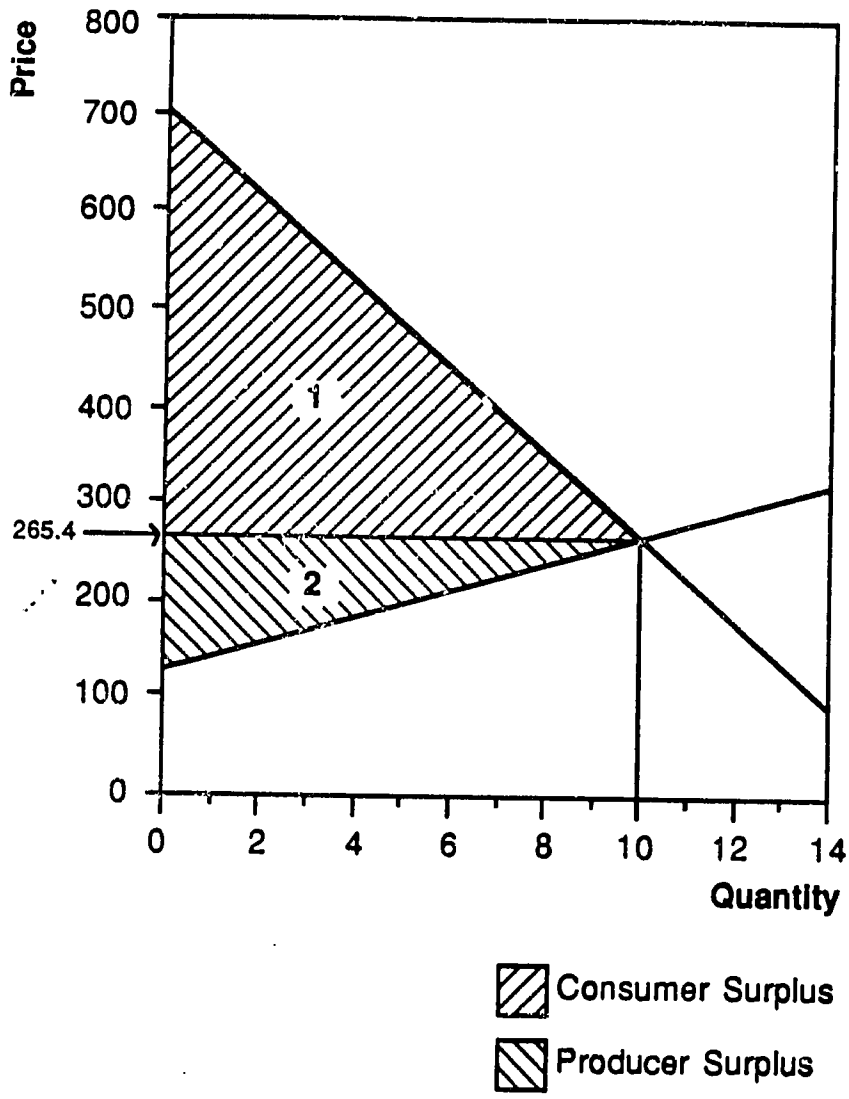

Figure 1. Competitive Market Equilibrium with Producer and Consumer Surplus.

Table 1. Linear Programming Tableau With Market Supply Curve and Consumer Surplus And Competitive Equilibrium Solution

SupplyMAXIMI		RHS	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	Market	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13
OBJ			-624.	-814.	-1017	-1234	-1465	-1709	-1967	-2238	-2523	-2822	-3134	0	2469.	2976.	3439.	3858.	4233.	4564.	4851	5093.	5292	5446.
QSUP	L	0	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	1										
CxSup	L	1	1	1	1	1	1	1	1	1	1	1	1											
QDEM	L	0												-1	4	5	6	7	8	9	10	11	12	13
CxDem	L	1													1	1	1	1	1	1	1	1	1	1

S o l u t i o n
OPTIMAL
Function Value 2883.

Return	Name	Type	Level	S4	S5	S6	S7	S8	S11	S12	S13	S14	D4	D5	D6	D7	D8	D9	D11	D12	D13	QSUP	CxSup	QDEM	CxDem
				-624.	-814.	-1017	-1234	-1465	-2238	-2523	-2822	-3134	2469.	2976.	3439.	3858.	4233.	4564.	5093.	5292	5446.	0	0	0	0
				real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	slack	slack	slack	slack
-1709	S9	real	0	6	5	4	3	2	-1	-2	-3	-4	-6	-5	-4	-3	-2	-1	1	2	3	1	10	1	-10
-1967	S10	real	1	-5	-4	-3	-2	-1	2	3	4	5	6	5	4	3	2	1	-1	-2	-3	-1	-9	-1	10
0	Market	real	10										6	5	4	3	2	1	-1	-2	-3			-1	10
4851	D10	real	1										1	1	1	1	1	1	1	1	1				1
Z				-420.	-678.	-936.	-1194	-1451	-2225	-2483	-2740	-2998	3304.	3561.	3819.	4077.	4335.	4593.	5108.	5366.	5624.	257.8	610.6	257.8	2272.
Shadow Price				203.5	135.6	81.41	40.70	13.56	13.56	40.70	81.41	135.6	834.5	585.1	379.9	218.8	101.7	28.83	15.26	74.62	178.0	257.8	610.6	257.8	2272.

The initial basic LP solution for the competitive market equilibrium is

$CS + PS = 2883$

$Q_s = Q_d = 10$

$Q_{sup} = Q_{dem}$ (shadow prices) = 257.8

C x Sup (shadow price) = 610.6 = producers surplus

C x Dem (shadow price) = 2272. = consumers surplus.

The above LP solution is an approximation. The exact solution can be found by the calculus approach of maximizing producer plus consumer surplus. A Lagrangian equation is formed in which producer plus consumer surplus is maximized subject to the constraint that the quantity supplied equal the quantity demanded.

$$\max L = 705.6\, Q_d - \frac{44.1}{2} Q_d^2 - 128.9\, Q_s - \frac{13.56}{2} Q_s^2 + \lambda(Q_s - Q_d)$$

The Lagrangian is maximized with respect to the quantity demanded (Q_d), the quantity supplied (Q_s), and the Lagrangian multiplier (λ).

$$\frac{\partial L}{\partial Q_d} = 705.6 - 44.1\, Q_d - \lambda = 0$$

$$\frac{\partial L}{\partial Q_s} = -128.9 - 13.56\, Q_s + \lambda = 0$$

$$\frac{\partial L}{\partial \lambda_1} = Q_d - Q_s = 0$$

or in matrix notation,

$$\begin{bmatrix} -44.1 & 0 & -1 \\ 0 & -13.56 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Q_s \\ Q_d \\ \lambda \end{bmatrix} = \begin{bmatrix} -705.6 \\ 128.9 \\ 0 \end{bmatrix}, \quad \begin{matrix} Q_s \\ Q_d \\ \lambda \end{matrix} = \begin{bmatrix} 10 \\ 10 \\ 264.5 \end{bmatrix}$$

The Lagrangian multiplier is the equilibrium price.

Lotus Solution of Linear Equations

The above solution is readily found by using the Lotus matrix commands.

a. enter the coefficients (put cursor in the upper left cell of the coefficient matrix)

b. Type **/DMI** for the commands **/Data Matrix Invert.**

— indicate the range of the spreadsheet containing the matrix

— indicate where you want the inverse stored.

c. Multiply the inverse of the coefficient matrix by the RHS vector.

Type **/DMM** for the commands **/Data Matrix Multiply**

— Indicate the range of the inverse for matrix 1

— Indicate the range of the RHS array for matrix 2

— Indicate where the results are to be stored.

The solution is also shown in Figure 1. A comparison between the exact calculation of the producer and consumer surplus from Figure 1 and the LP solution indicates

Calculus Solution			LP solution
Area 1,	Consumer surplus $(705.6 - 264.5) * \frac{10}{2}$	$= 2205.5$	2272
Area 1,	Producer surplus $(264.5 - 128.9) * \frac{10}{2}$	$= 678$	610.6
	Equilibrium price	$= 264.5$	257.8

This check provides an indication of the accuracy we might expect from the LP approximation.

Policy 1: Market Price Support with Government Purchase of Crop — No Resale

Objective — determine benefits to producers, consumers, government cost and total net social change from a price support by Government Purchase and Storage program.

Assume the price is to be supported at $292 per ton. From Figure 2 about 12 thousand ton would be supplied.

The Calculus Solution

The exact solution can be determined by solving the following problem

$$\max L = 705.6Q_d - \frac{44.1}{2}Q_d^2 - 128.9Q_s - \frac{13.56}{2}Q_s^2 - 292GP + \lambda(Q_s - Q_d - GP)$$

The Lagrangiam problem contains one additional variable, Government Purchases. The government purchases represent an additional demand. The constraint requires the total quantity demanded (Q_d + GP), to be equal to the total quantity supplied.

GP = Government purchase

$$\frac{\partial L}{\partial Q_d} = 705.6 - 44.1\, Q_d \qquad - \lambda = 0$$

$$\frac{\partial L}{\partial Q_s} = -128.9 \qquad -13.56\, Q_s \qquad + \lambda = 0$$

$$\frac{\partial L}{\partial GP} = -292 \qquad - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda_1} = \qquad - Q_d \qquad + Q_s - GP \qquad = 0$$

or

$$\begin{bmatrix} -44.1 & 0 & 0 & -1 \\ 0 & -13.5 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} Q_d \\ Q_s \\ GP \\ \lambda \end{bmatrix} = \begin{bmatrix} -705.6 \\ 128.9 \\ -292 \\ 0 \end{bmatrix}, \quad \begin{matrix} Q_d \\ Q_s \\ GP \\ \lambda \end{matrix} = \begin{bmatrix} 9.378\text{mt} \\ 12.01\text{mt} \\ 2.64\text{mt} \\ \$292/\text{ton} \end{bmatrix}$$

The solution indicated the quantity demand 9.38 thousand mt at a price of $292 per ton. The quantity supplied 12.01 thousand mt. To support the price at $292 per ton, the government must purchase 2.64 thousand mt. The graphical solution is shown in Figure 2. The gain to producers is Area 1 plus Area 2 plus Area 3 or

$$(292 - 264.5)\,(10 + \frac{2}{2}) = 302.5.$$

The loss to consumers is Area 1 plus Area 2 or

$$(292 - 264.5)\left[9.378 + \frac{(10 - 9.378)}{2}\right] = 266.45.$$

The cost to the government is the quantity purchased (2.64 thousand tons) times the price $292 or 770.88.

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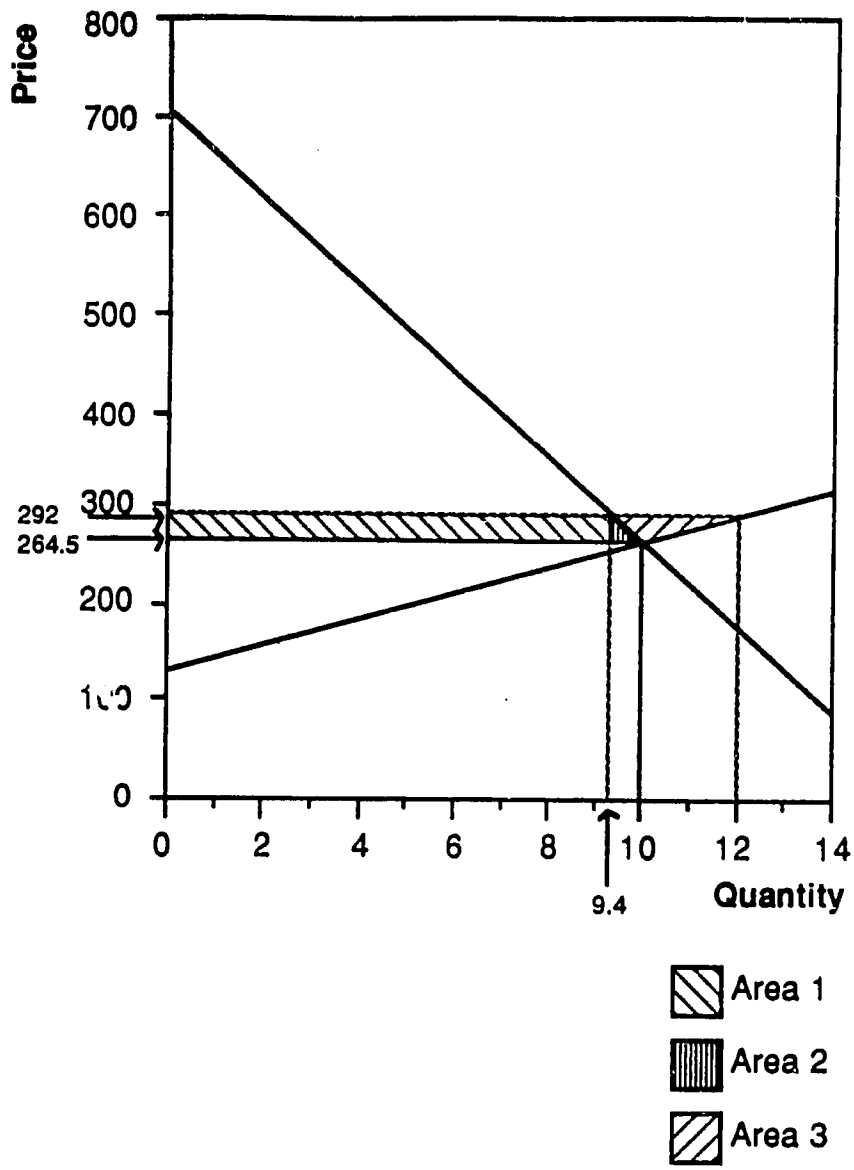

Figure 2. Calculation of Gains and Losses with Government Purchase to Support Prices when There Is No Resale.

The net gain to society is

Area 1 + 2 + 3	Gain to Producers	302.50
Area 1 + 2	Gain to Consumers	-266.45
	Gain to Taxpayers	-770.80
	Net Gain	-734.83

The linear programming approach. The linear programming approach to the same policy problem is shown in Table 2. A government buying activity Gbuy is added which

Table 2. Linear Programming Tableau for Calculation of Consumer and Producer Surplus when the Farm Price is Supported by Direct Government Purchase and No Resale

Supply	MAXIMI	RHS	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	Market	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	Gbuy
OBJ			-624.	-814.14	-1817	-1234.7	-1465	-1709.7	-1967	-2238	-2523	-2822	-3134	0	2469.	2976.	3439.	3858.	4233.	4564.	4851	5093.	5292	5446.	292
CSUP	L	0	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	1											
CxSup	L	1	1	1	1	1	1	1	1	1	1	1	1												
QDEM	L	0																							
CxDem	L	1												-1	4	5	6	7	8	9	10	11	12	13	1
															1	1	1	1	1	1	1	1	1	1	

S o l u t i o n
OPTIMAL
Function Value: 2916.49

Return	Name	Type	Level	S4	S5	S6	S7	S8	S9	S10	S11	S13	S14	D4	D5	D6	D7	D8	D10	D11	D12	D13	QSUP	CxSup	QDEM	CxDem
				-624.18	-814.	-1017.6	-1234	-1465.4	-1709	-1967	-2238	-2822	-3134	2469.	2976.	3439.	3858.	4233.	4851	5093.	5292	5446.	0	0	0	0
				real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	real	slack	slack	slack	slack
-2523	S12	real	1	1	1	1	1	1	1	1	1	1	1													
0	Market	real	12	8	7	6	5	4	3	2	1	-1	-2											1		
4564.	D9	real	1																				1	12		
292	Gbuy	real	3	8	7	6	5	4	3	2	1	-1	-2	1 -5	1 -4	1 -3	1 -2	1 -1	1 1	1 2	1 3	1 4	1	12	1	1 -9
Z				-187.85	-479.	-771.85	-1063	-1355.8	-1647	-1939	-2231	-2815	-3107	3104.	3396.	3688.	3980.	4272.	4856.	5148.	5440.	5732.	292	980.1	292	1936.
Shadow Price				436.328	334.2	245.832	170.9	109.612	61.85	27.66	7.049	6.519	26.60	634.7	419.6	248.5	121.5	38.75	5.349	54.79	148.3	286	292	980.1	292	1936.

	Govt.Pur. Price Sup.	Base Sol	Change
PS+CS=	2916.50	2883.48	33.02
Marketed	12.00	10.00	2.00
Consumed	9.00	10.00	-1.00
Govt pur(thos ton)	3.00		3.00
Cxsup	980.15	610.60	369.54
Mark(sp)	292.00	257.81	34.19
Cxdem	1936.35	2272.88	-336.53
Govt cost (-292*3)=	-876		-876.00
Net Society Benefits (369.54-336.53 - 876)=			-842.98

"consumes" the products of $292 per mt. The changes in the consumer and producer surplus are calculated by subtracting the similar values in the base solution.

A comparison with the LP and the simultaneous equation solution indicates there are differences because the LP solution is an approximation. These differences can be minimized by increasing the number of demand and supply segments in the area of the solution.

The estimate of changes in consumer surplus differ greatly between the two solutions because of differences in the equilibrium prices and because of differences in the equilibrium quantities.

Policy 2: Analysis of Price Support Where Product Is Sold Back on the Market

Assume the policy maker wants to support the farm price at $292 per metric ton but the government will resell all purchases on the domestic market at whatever price it can get. The support price will be above the free market equilibrium. The objective is to estimate the amount of subsidy required per ton the total government cost and changes in producer and consumer surplus.

Calculus Approach

The non-linear model for this problem can be stated as a constrained maximization where we require the supply price to be $292 per metric ton and require the quantity supplied to be equal to the quantity demanded. In the second constraint the supply price P_S is $P_S = b_0 + b_1 Q_S$. The constraint requires the supply price to be equal to $292 per ton.

$$\text{Max } L = A_0 Q_d - \frac{A_1}{2} Q_d^2 - b_0 Q_s - \frac{b_1}{2} Q_s^2 + \lambda_1 (Q_S - Q_D) + \lambda_2 (b_0 + b_1 Q_s - 292).$$

Take derivatives WRT Q_S, Q_d, λ_1, λ_2

$$\frac{\partial L}{\partial Q_d} = A_0 - A_1 Q_d - \lambda_1 = 0$$

$$\frac{\partial L}{\partial Q_s} = b_0 - b_1 Q_s + \lambda_1 - b_1 \lambda_2 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = \quad - Q_d \quad + Q_s \quad = 0$$

$$\frac{\partial L}{\partial \lambda_2} = b_0 - 292 \quad + b_1 Q_s \quad = 0$$

λ_1 will be the marked clearing price

λ_2 will be a subsidy.

Inserting the coefficients into the matrix we have

Q_d	Q_s	λ_1	λ_2	
-44.1	0	-1	0	= -705
0	-13.56	1	13.56	= 128.9
1	-1	0	0	= 0
0	13.56	0	13.56	= 163

Using the Lotus matrix invert and multiply commands the solution is

Q_d = 12.02

Q_s = 12.02

λ_1 = 175.1 = market clearing price

λ_2 = 8.616

The subsidy per ton is 8.616 * 13.56 = 116.83.

A graphical view of the solution is shown in Figure 3.

Linear programming solution. The linear programming analogy is less obvious. However trial and error can be used to enter a subsidy in an activity which transfers between the producer and the consumer. A trial solution is shown in Table 3 where previously estimated subsidy of $116 per ton was entered in the objective function of the market activity (G.Mark). Note that the $116 ton subsidy is counted as a "benefit" or added to the objective function value in the solution.

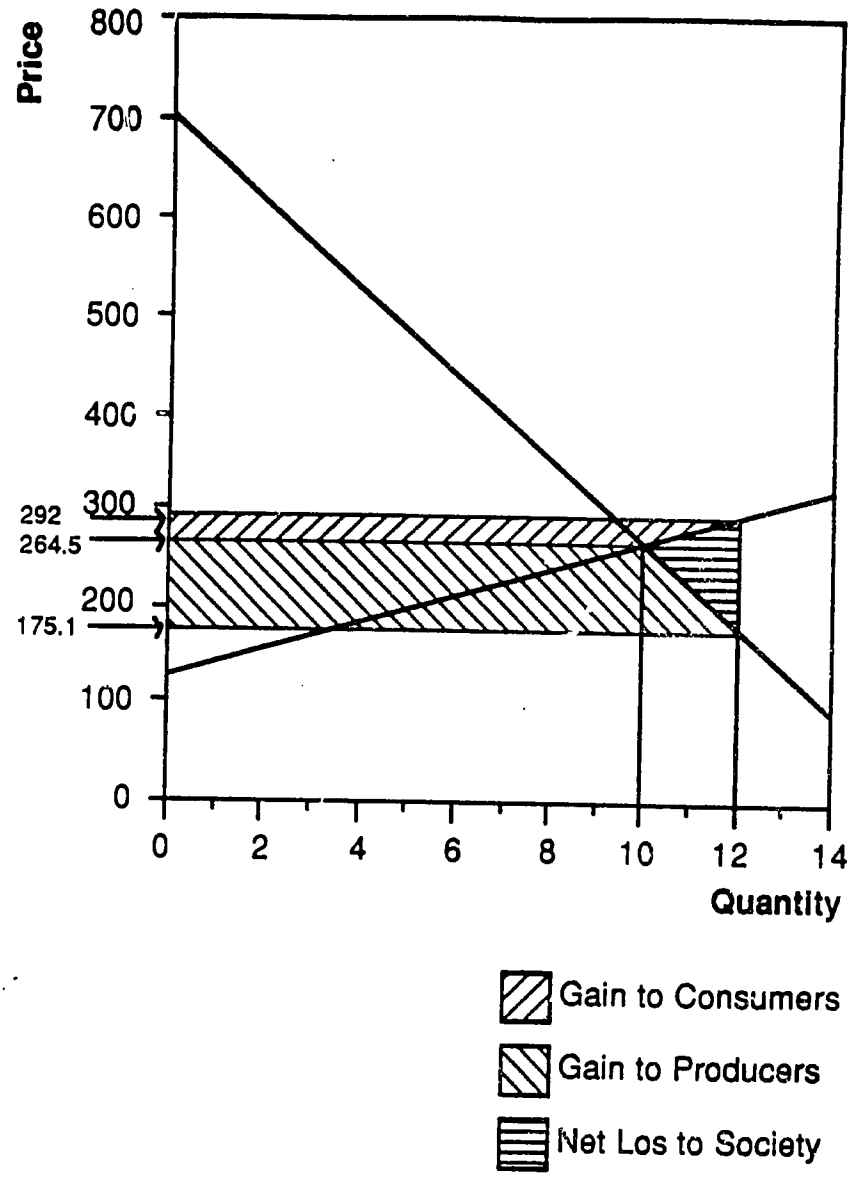

Figure 3. Market Equilibrium with Government Price Support and Subsidized Resale to Consumers.

Table 3. Linear Programming Tableau to Calculated Producer and Consumer Benefits From a Price Support Policy When Government Purchases are Resold on the Market

Supply	MAXIMI	RHS	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	G.Mark	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13
OBJ			-624.	-814.	-1017	-1234	-1465	-1709	-1967	-2238	-2523	-2822	-3134	116	2469.	2976.	3439.	3858.	4233.	4564.	4851	5093.	5292	5446.
QSUP	L	0	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	1										
CxSup	L	1	1	1	1	1	1	1	1	1	1	1	1											
QDEM	L	0																						
CxDem	L	1												-1	4	5	6	7	8	9	10	11	12	13
															1	1	1	1	1	1	1	1	1	1

S o l u t i o n
OPTIMAL
function Value: 4160.

Return	Name	Type	Level	S4 -624. real	S5 -814. real	S6 -1017 real	S7 -1234 real	S8 -1465 real	S9 -1709 real	S10 -1967 real	S11 -2238 real	S14 -3134 real	D4 2469. real	D5 2976. real	D6 3439. real	D7 3858. real	D8 4233. real	D9 4564. real	D10 4851 real	D11 5093. real	D13 5446. real	QSUP 0 slack	CxSup 0 slack	QDEM 0 slack	CxDem 0 slack
-2523	S12	real	1	9	8	7	6	5	4	3	2	-1	-8	-7	-6	-5	-4	-3	-2	-1	1	1	13	1	-12
-2822	S13	real	0	-8	-7	-6	-5	-4	-3	-2	-1	2	8	7	6	5	4	3	2	1	-1	-1	-12	-1	12
116	Market	real	12										8	7	6	5	4	3	2	1	-1			-1	12
5292	D12	real	1										1	1	1	1	1	1	1	1	1				1
Z				-135.	-434.	-732.	-1031	-1329	-1628	-1926	-2225	-3120	3831.	4014.	4196.	4379.	4561.	4744.	4926.	5109.	5474.	298.5	1058.	182.5	3101.
Shadow Price				488.4	379.9	284.9	203.5	135.6	81.41	40.70	13.56	13.56	1362.	1037.	757.0	520.6	328.3	180.0	75.96	15.93	28.16	298.5	1058.	182.5	3101.

	Govt. Pur&Resale	Base Solution	Difference Cur Sol - Base Sol
PS+CS=	4160.	2883.	1276.
QS=QD=	12	10	2
Cxsup	1058.	610.6	447.7
Mark(sp)	298.5	257.8	40.70
Consumer Price	182.5	257.8	-75.2
Cxdem	3101.	2272.	828.8
Govt Sub	116		116
Govt cost (-116*12)	-1392		-1392
Net Society cost= (447.7 +828.8 - 3504)=			-115.

Policy 3: Price Support When There Are International Trade Possibilities

Assume that imports are available on the international market at a price of $210.3 per mt. The graphical solution is shown in Figure 4. The open market solution can be found by calculus as:

$$\text{Max } L = A_0Q_d - A_1Q_d^2 - b_0Q_s - \frac{b_1}{2}Q_s^2 - cIm + \lambda(Q_S + QImp - Q_D)$$

$$\begin{array}{lcrrrrcl} & & Q_d & Q_s & Im & \lambda & & \\ \frac{\partial L}{\partial Q_d} & = & A_0 & -A_1 & 0 & 0 & -1 & = 0 \\ \frac{\partial L}{\partial Q_s} & = & b_0 & 0 & -b_1 & 0 & 1 & = 0 \\ \frac{\partial L}{\partial Im} & = & -c & 0 & 0 & 0 & 1 & = 0 \\ \frac{\partial L}{\partial \lambda} & = & 0 & -1 & 1 & 1 & 0 & = 0 \end{array}$$

where cIm is the cost of the imported commodity and QImp is the amount of the import. The appropriate coefficients can be inserted into a Lotus worksheet and the system of equations solved to obtain the exact solution.

The Model with Import is of the form Ax = b, so the sollution is of the form $x = A^{-1}B$.

$$\begin{array}{c} \\ Qdem \\ Qsup \\ Imp \\ psup \end{array} \begin{array}{c} \begin{array}{cccc} Qd & Qs & QImp & \lambda \end{array} \\ \begin{bmatrix} -44.1 & 0 & 0 & -1 \\ 0 & -13.5 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \end{array} = \begin{array}{r} -705 \\ 13.56 \\ 210.3 \\ 0 \end{array}$$

$$A^{-1} = \begin{bmatrix} -0.02 & 0 & -0.02 & 0 \\ 0 & -0.07 & 0.073 & 0 \\ -0.02 & 0.073 & -0.09 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{array}{lcr} Qd & = & 11.23 \\ Qs & = & 5.498 \\ QImp & = & 5.23 \\ \lambda(price) & = & 210.3 \end{array}$$

The Linear Programming model with imports is shown in Table 4. The imports are simply a purchasing activity. The results are fairly close because the import price corresponded with a segment on the supply curve.

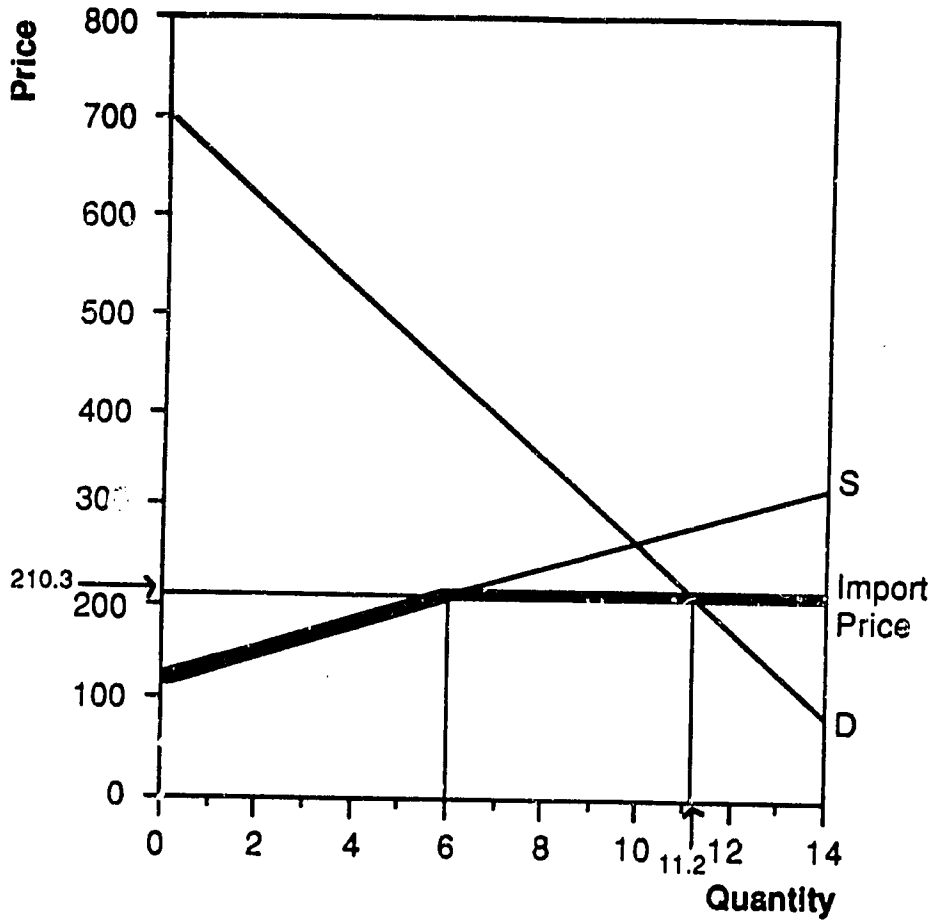

Figure 4. Market Equilibrium Solution with Commodity Imports.

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Table 4. Linear Programming Model and Base Solution For Analysis of Export-Import Policies

Supply	MAXINIZ	RHS	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	Market	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	QImp
OBJ			-624.1	-814.1	-1017.	-1234.	-1465.	-1709.	-1967.	-2238.	-2523.	-2822.	-3134.	0	2469.6	2976.7	3439.8	3858.7	4233.6	4554.3	4851	5093.5	5292	5446.3	-210.3
QSUP	L	0	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	1											
CaSup	L	1	1	1	1	1	1	1	1	1	1	1	1												
QDEM	L	0												-1	4	5	6	7	8	9	10	11	12	13	-1
CaDem	L	1													1	1	1	1	1	1	1	1	1	1	

Solution
Optimal
Function Value: 3024.3

Returns	Name	Type	Level	S4	S5	S7	S8	S9	S10	S11	S12	S13	S14	D4	D5	D6	D7	D8	D9	D10	D12	D13	QSUP	CaSup	QDEM	CaDem
				-624.1	-814.1	-1234.	-1465.	-1709.	-1967.	-2238.	-2523.	-2822.	-3134.	2469.6	2976.7	3439.8	3858.7	4233.6	4554.3	4851	5292	5446.3	0	0	0	0
-1017.	S6	real	1	1	1	1	1	1	1	1	1	1	1										slack	slack	slack	slack
0	Market	real	6	2	1	-1	-2	-3	-4	-5	-6	-7	-8											1		
5093.5	S11	real	1																				1	6		
-210.3	Imp	real	5	-2	-1	1	2	3	4	5	6	7	8	1	1	1	1	1	1	1	1	1				1
														7	6	5	4	3	2	1	-1	-2	-1	-6	-1	11
Z				-597.0	-807.3	-1227.	-1438.	-1648.	-1858.	-2069.	-2279.	-2489.	-2700.	3621.4	3831.7	4042.0	4252.3	4462.6	4672.9	4883.2	5303.3	5514.1	210.3	244.11	210.3	2780.2
Shadow Price				27.096	6.7635	6.8055	27.180	61.123	108.63	169.71	244.36	332.58	434.37	1151.8	855	602.25	393.6	229.05	108.60	32.25	11.849	67.759	210.3	244.11	210.3	2780.2

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	LP	Exact
CS+PS	3024.70	3025.30
PS	2780.25	2781.10
CS	244.16	244.20
DomP	6.00	6.00
Import	5.00	5.23
Consump	11.00	11.23
Price	210.30	210.30

Policy 4: Analysis of an Import Tariff

The quantity of "cheap" imports may cause producers to request protection.

Assume that a tariff of $30 is imposed which raises the import price to 240.3 per ton. The revised exact solution is

```
Qd    =  10.55
Qs    =   8.21
Imp   =   2.34
Price = 240.30.
```

The calculations on changes in consumer surplus, producer surplus and government revenue shown in Figure 5 are

Change in Consumer Surplus (Areas 1 + 2 + 3 + 4)

$$-30\left[10.55 + \frac{(11.23 - 10.55)}{2}\right] = -326.40$$

Change in Producer Surplus (Area 1)

$$30\left[6 + \frac{(8.21 - 6)}{2}\right] = 213.15$$

Change in Government Revenue (Area 3)

$$30\ (2.34) = 70.20$$

Net Social Change $\quad -25.05.$

The LP model shown in the upper part of Table 4 was revised by increasing the import cost to 240.3. The changes caused by a change in the import tariff are shown in Table 5. The LP solution would be more exact if there were more segments in the demand and supply equations in the area of the solution.

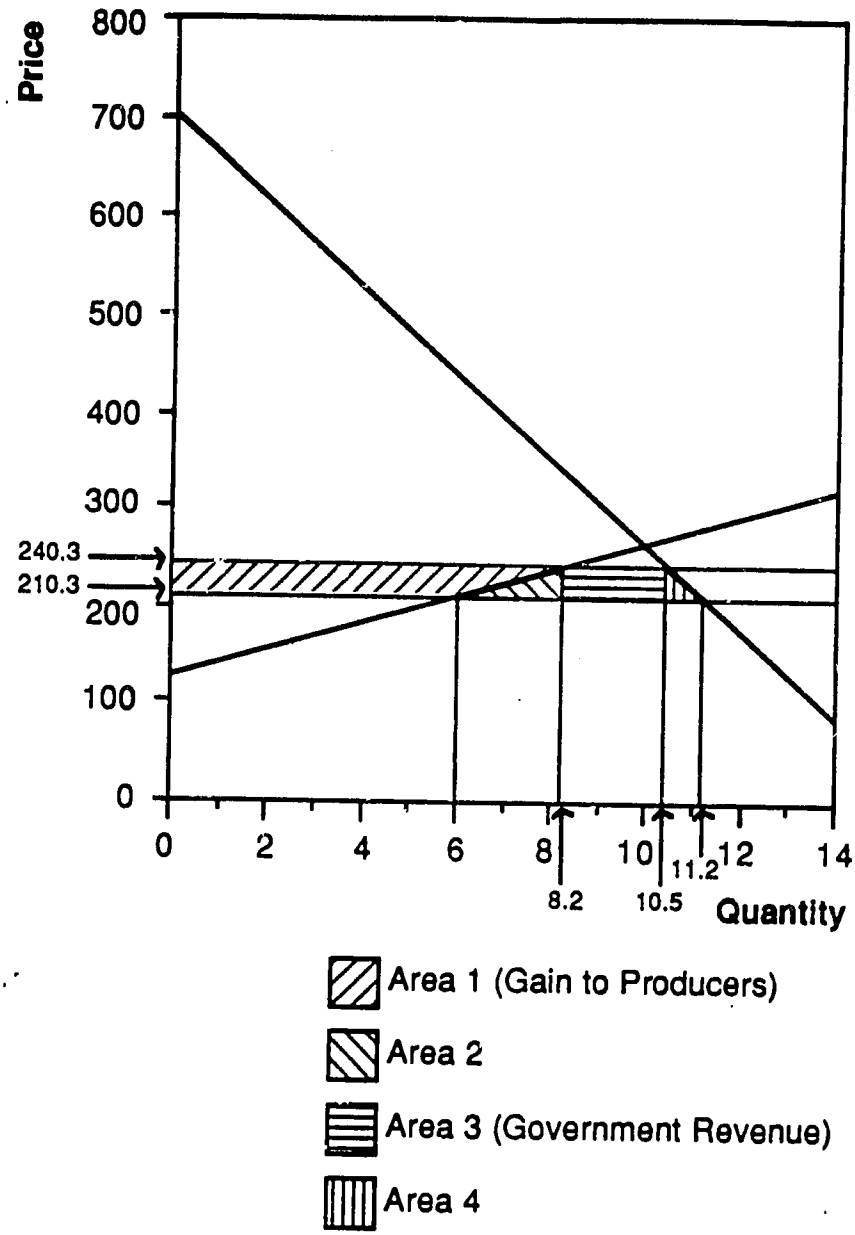

Figure 5. Changes in Producer and Consumer Surplus Resulting from an Import Tariff.

Table 5. Linear Programming Estimates of Changes in Producer and Consumer Surplus from the Implementation of a Tariff

	After Tariff of $30/ton	Base Sol With Imports	Diff
PS+CS=	2907.1	3024.3	-117.1
Dom Prod	8	6	2
Dom Con	11	11	0
Import	3	5	-2
Prod.Price	240.3	210.3	30
Cxsup	456.93	244.11	212.82
ComsumerP	240.3	210.3	30
Cxdem	2450.2	2780.2	-330
Govt.Rev.(3*30	90		
Net to Society(212.82 - 330 + 90)			-27.17

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