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ECONOMIC DEVELOPMENT

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By: LUTHER G. TWEETEN, Editor,
et. al.

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Dr. William Goodwin
U.S. Agency for International Development
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Washington, D.C. 20523

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Abel, Daft & Earley, 1339 Wisconsin Avenue, N.W., Washington, D.C. 20007 • (202) 342-7620
Oklahoma State University, Department of Agricultural Economics, Stillwater, Oklahoma 74078 • (405) 624-6157

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"Tools" provides approaches to and techniques useful in analyzing very real world problems in agricultural policy analysis. In addition to an Introduction, the 14 chapters cover regression and statistical analysis, use of microcomputers, demand theory, supply theory, elasticity estimation, classical welfare analysis, simulation and systems modeling, enterprise budgets, mathematical programming, investment analysis, natural resources, trade macroeconomic linkages to agriculture and index numbers. The book originated out of the APAP microcomputer workshop held annually at Oklahoma State University and thus it serves nicely as a text on agricultural policy analysis to accompany practical, "hands-on" training in policy analysis.

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WORK SHEET

AGRICULTURAL POLICY ANALYSIS TOOLS FOR ECONOMIC DEVELOPMENT

By

Daniel D. Badger
Francis M. Epplin
David Henneberry
Shida Henneberry
Darrel Kletke
Elton Li
David Pyles
Daryll E. Ray
Dean F. Schreiner
Arthur L. Stoecker
James N. Trapp
Luther Tweeten
Michael P. Welsh

Edited by Luther Tweeten

PREFACE

Agricultural Policy Analysis Tools for Economic Development was written to make modern policy analysis methods accessible to policy analysts. All too frequently wrong policy decisions have exacted massive human and material cost in lost opportunities. *Tools* can improve policy decisions by combining the best analytical methods with the power of analysts' and decision makers' good judgment and with microcomputer hardware and software.

This book originated out of a microcomputer workshop held annually at Oklahoma State University for policy analysts from developing countries. Students in the workshop had at least a Bachelors Degree and many had a graduate degree. Hands-on experience with policy analysis tools on microcomputers in the afternoon was preceded in the morning by lectures explicating the conceptual foundation and past applications. *Agricultural Policy Analysis Tools for Economic Development* provided background and elaboration for the lectures.

Tools can be used as a text, reference, handbook, or book of readings. *Tools* can be used as a text in a course in policy analysis methods. It can be used as a major source-book in courses in economic development, agricultural development, research methodology, agricultural policy, and a host of other courses teaching methods and applications.

Chapters are written to be as easily understood as the material allows. The book is written to serve the needs of a wide variety of agricultural policy analysts including the experienced and inexperienced, the young and the old, those from developed or developing countries, and by persons with wide differences in backgrounds. Some parts can be understood by those with little background; other parts require calculus. Chapters of *Tools* are designed to "stand alone" to meet unique needs. Difficult parts can be omitted by those with limited backgrounds. In cookbook sequences the methods can be applied to very real policy problems by those who prefer to omit the conceptual background. However, understanding of the concepts underlying the tools is highly desirable to avoid unrealistic and inappropriate applications and interpretations.

The authors are deeply grateful to the many who made this book possible. Support and encouragement from the U.S. Department of Agriculture and the U.S. Agency for International Development was indispensable. Oklahoma State University and The Ohio State University provided a facilitative environment. Cloye Brower was a patient and competent typist.

Luther Tweeten
The Ohio State University

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1. INTRODUCTION

by Luther Tweeten

Policy matters. In recent decades countries with similar resource endowments and culture have experienced sharply different economic progress because of different economic policies. Examples are Kenya versus Tanzania, North Korea versus South Korea, and Taiwan and Japan versus China. Many countries are deeply committed if not desperate to improve economic performance but do not know how.

Policy analysis is the process of providing in-depth information to improve policy decisions. The ultimate objective of policy analysis is to improve the well-being of society but the immediate objective is to help administrators, managers, and policy makers make better decisions in allocating scarce resources to meet ends or goals.

OBJECTIVES

The objectives of this chapter are to:

1. Introduce the role of policy analysis.
2. Outline basic steps in policy analysis.
3. Briefly describe contents of remaining chapters.

KEY POINTS

1. Microcomputers have placed powerful capabilities at the disposal of policy analysts and decision makers even in less developed countries with limited budgets. The methods and tools presented in this book are often most easily applied using computer analysis.
2. A sound conceptual framework and adequate data are essential for sound policy analysis. Ability to process data by computers complements rather than competes with use of quality data and theory in policy analysis.
3. The policy analyst and decision maker have different roles. The former is to inform, the latter is to make decisions. Public advocacy of policy positions

by analysts confuses the two roles and can compromise objectivity and usefulness of the analyst in the policy process.

CONCEPTS

Policy analysis is a process of interpreting and analyzing data and of communicating results to decision makers. It is typically performed by staff analysts located in various ministries, divisions, and bureaus of government, by public or private research institutions such as institutes and universities, and with contributions from consultants and other sources of expertise.

Policy analysis need not be highly sophisticated and formal. Skilled, experienced analysts relying on judgment and a minimum of data and computational devices can make significant contributions to the policy process. However, more sophisticated analysis can help to identify interactions, indirect effects, long-term effects, and the inconsistency in policies that help selected groups but hurt society. The human brain alone cannot integrate the mass of data and concepts required to analyze many policy problems.

The microcomputer has made powerful analytical capabilities accessible to policy analysts even in governments of developing countries with tight budgets. However, mechanistic application of undigested numbers generated by computers can miss the major, oftentimes nonquantifiable, policy issues and can result in more misguided policies than forthcoming from experienced analysts working without sophisticated analytical tools. Computational capacity alone does not make sound policy analysis. Such analysis requires careful planning, a conceptual framework, adequate data, and other components discussed below. In short, the best policy analysis comes from combining experience and sound judgment of analysts with more or less sophisticated policy analysis tools such as outlined in subsequent chapters.

Steps in the Decision Cycle

Seven stages of the decision cycle are:

1. Defining the problem.
2. Listing for analysis of options (hypotheses) to diagnose or alleviate the problem.
3. Assembling data.
4. Analyzing of options (or testing of hypotheses).
5. Making of a decision.
6. Implementing the decision.
7. Evaluating results.

A *problem* is a situation of confusion, doubt, uncertainty; a lack of knowledge of the most advantageous means to close the gap between the current situation and the desired outcome or goal. Defining the problem often is the most difficult stage in analysis. It has been said that a problem well-defined is a problem half-solved. Inability to write down or state the problem usually means the problem is not clear in the mind of the analyst or

decision maker. It is difficult to proceed with analyzing options or testing hypotheses and with making and implementing decisions without a clear definition of the problem.

A problem with only one solution requires no decision. There is only one choice. Most problems have many possible options to diagnose and resolve them. In more formal analysis, these options are listed as hypotheses. Hypotheses are tentative answers to questions or solutions to problems. Hypotheses focus analysis. The decision maker is likely to play a major role in choosing which options the analyst will examine. Spreadsheet analysis described later is especially well suited to examine the implications of various alternatives including sensitivity tests of the impact on results of changes in values of basic parameters not known with precision.

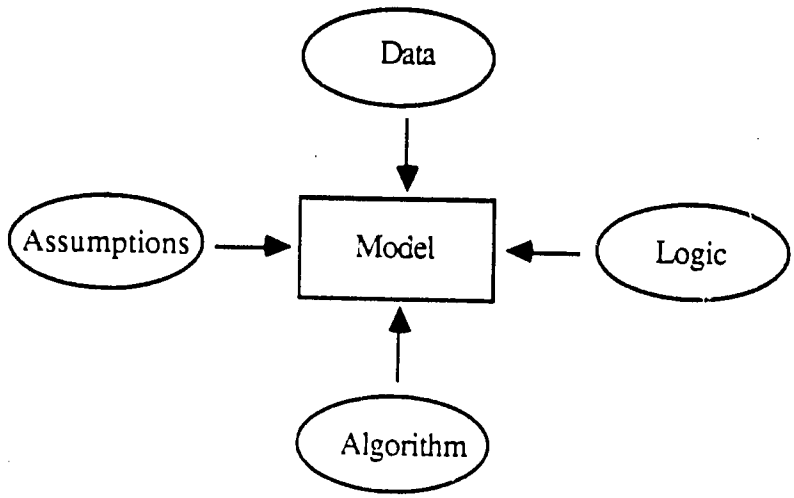
Data gathering (step 3) can be expensive, hence it is well to rely on secondary (already gathered and processed) data where possible. More will be said of data in the later discussion of analytical models. Step 4 is to analyze options. Raw data may be of little value. A key role for the analyst is to process data into supply and demand parameters, budgets, optional farm plans, or into whatever form is suitable to analyze options and predict outcomes. Analysis of options also means sorting out those alternatives that are most relevant for policy, and presenting them in oral, written, graphic, and tabular form for convenient examination and appraisal by decision makers.

It is wise for the analyst to involve the decision maker in each stage of the decision cycle but the major role of the decision maker is in step 5, the making of the decision, and step 6, implementing that decision. Bearing the consequences of that decision is also a role of the decision maker that might have been listed above. Feedback is critical to the analyst -- if the option implemented does not resolve the problem as predicted, the analyst needs to know why so that mistakes are not repeated. That is why step 7, evaluating results, is so important.

Models

Formal models are sometimes useful approaches for policy analysis. Models, although abstractions from reality, retain essential elements of reality necessary to explain and predict. A microcomputer is of the most help in policy analysis if it is used with an appropriate conceptual and data system. The real world is far too complex to analyze in full detail.

Models range all the way from qualitative informal mental models each of us use to process information for life's everyday decisions to large, highly complex quantitative models requiring analysis by sizable computers.



A formal model used by policy analysts ordinarily will include the four components above. The *logic* and *assumptions* together constitute the conceptual or theoretical foundation of the model. A well-conceived conceptual framework is useful but the model may have little value for policy analysis without proper *data*. Data may be *primary*, gathered from say farmers or marketing agents especially for the study, or *secondary*, previously collected and processed data available from published or unpublished sources. The phrase "garbage is, garbage out" refers to inability to obtain useful results from even a well-specified, sophisticated model using inadequate data.

Assumptions are important. It is often said that an analyst "assumes away the problem" when assumptions ignore critical components of a system. Good judgment is required to decide which simplifying assumptions facilitate diagnosing and solving problems and which represent serious and damaging departures from reality. Assumptions and other components of models need to be kept as simple as possible consistent with providing essential information. The world is littered with abandoned sophisticated, complex models which developing countries did not have data, personnel, and funding to maintain.

Models need a logical (coherent) structure along with defensible assumptions to form a sound theoretical (conceptual) framework. Nothing is more practical than good theory. It indicates how components fit together. For example, it tells how demand and supply response estimates fit together is classical welfare analysis and in simulation analysis presented in subsequent chapters.

An *algorithm* refers to a special mathematical procedure for solving a numerical problem. Most quantitative models will utilize some algorithm. It may be to maximize net income or to minimize cost as in linear programming, to minimize prediction error as in an econometric or simulation model, or to solve for equilibrium pricing and output in a prediction model. The computational burden of an algorithm can be overcome by modern computer routines.

Attributes of Worthwhile Analysis

Worthwhile analysis has *internal validity* and *external worth*. An analysis is internally valid if it is *clear, logical,* and *reliable* (see Ghebremedhin and Tweeten, pp. 7, 11).

1. Internal Validity. Analysis does not have to be simple, but it needs to have clarity in the sense of being no more complex nor demanding of assumptions than necessary.

Internal validity requires a logical or coherent structure. Valid analysis must make sense. It may be necessary to make a case that A is caused by systematic structural forces B, C, and D. However, A also may be influenced by random elements called unexplained shocks or disturbances.

Finally, internal validity of analysis depends on reliability as measured by ability of its propositions to predict. That predictive ability is apparent by how closely the propositions correspond to past experience and, more importantly, how accurately future or previously unknown phenomena are predicted.

Analysis is *objective* if researchers are willing to have their work reviewed for clarity, logic, and predictiveness and are willing to abide by the results (Johnson). Objectivity is apparent if two or more researchers independently studying the same phenomenon consistently reach the same conclusions.

Intellectual honesty or *integrity* in research means that analysts rise above conventional wisdom and above political and social pressures to report findings well grounded in fact and logic. Integrity means that analysts do not discard correct information which may displease the decision maker and do not distort outcomes by selecting only information to please the decision maker. Such practices ultimately result in uninformed decisions and wrong policies.

2. External Worth. Analysis with only internal validity may have value as a curiosity or arcane diversion, but its value to society derives from external validity (Ghebremedhin and Tweeten). Such validity is in turn a function of (1) the ability to turn resources devoted to the analysis into output (information), an attribute closely related to internal validity, and (2) the value of that output. Tradeoffs are apparent. Meteorology is unable to turn its resources into information with much reliability. But weather forecasts are so highly valued that imprecision is tolerated. Economics is similar. On the other hand, with minimal resources some fields of astronomy are able to predict precisely the course of objects in space. Even if predicting the course of a distant star is of little value, the science of astronomy is considered valuable. Thus, two sciences can be equally worthwhile: one's predictions (outputs) are highly valued even though resources devoted to it seldom predict correctly whereas another science consistently predicts well but each precise prediction is not of much value.

The Boundaries of Analysis

Analysis is as varied as the imagination of the analyst and the demands for decisions on wide-ranging problems. Nonetheless, analysis can be broadly classified into two categories: normative and positive. *Normative* policy analysis specifies "what ought to be." In extreme form, normative analysis entails advocacy by the analyst of means and ends of the analyst's own choosing rather than the means and ends specified by decision makers or society. Such normative analysis is unscientific and unobjective and is best avoided by analysts.

Positive policy analysis studies options to serve ends or goals set by decision makers and society rather than by the analyst. It is well to note, however, that the role of the analyst will differ depending on his position. A policy analyst working for a private firm or the ministry of agriculture will serve the goals of his or her employer. A policy analyst in a public (tax supported) university will wish to analyze policy options to improve the well-being of society rather than only serve a special interest group such as a private firm or the ministry of agriculture. The publicly employed analyst serves the public interest rather than private special interests. Determining what is the public interest demands great care and judgment regarding the social welfare function. In general, policies that improve the level, distribution, and stability of real income of society are preferred to policies which do not.

Positive analysis may be *predictive* (e.g. forecasting commodity prices for next year), *descriptive* (e.g. estimating current marketing margins for staples), or *prescriptive* (e.g. indicating what would be the advantages and disadvantages for producers, consumers, taxpayers, and the nation of a tax on irrigation water versus on farm exports to raise revenue).

The analyst informs; the political process makes decisions. Analysts who fail to heed this advice and depart from objective analysis to subjective advocacy are forsaking the role of analyst for the role of politician. However, especially for publicly employed analysts testing and analyzing policy options, it is often well to look at options not necessarily requested by the decision maker but which will improve the level, distribution, and stability of income for society as a whole. The outcomes from this option can supplement information from requested options. The cost in foregone benefits to society can be shown for a policy serving special interests. The decision makers can decide which option to follow. However, the analyst interacts with the decision maker in determining what is economically feasible, socially desirable, and politically acceptable. Good economics is not necessarily good politics. However, a political process consistently ignoring sound economics is likely to fail.

The world of policy analysis is sometimes divided into *institutional* and *quantitative* approaches. The institutionalist is recognized for knowing well the way the people and institutions work in his field of inquiry. History, laws, traditions, and modes of operation are important to the institutionalist. The quantitative analyst is recognized for knowledge of mathematical and other quantitative or sophisticated nonquantitative tools to diagnose and solve problems. Quantitative techniques, data sources, and computer access are important

to the quantitative analyst. Pressures of specialization often force the institutional-quantitative division. That is, the institutionalist provides shallow analysis ignoring powerful tools of diagnosis and prediction whereas the quantitative analyst provides sophisticated results of no relevance to the real world. Problems are best diagnosed and solved by combining the best features of institutional and quantitative approaches. This can be done either by finding both of these qualities in one analyst or by finding two or more individuals with the respective specialties to work together on problems. Although this book emphasizes quantitative approaches to policy analysis, it in no way intends to minimize the importance of the knowledge of institutions.

Outline of Chapters

We turn now from the above brief introduction to the philosophy of policy analysis to the tools presented in subsequent chapters.

Chapter 2. Regression and Statistical Methods. Statistical tools are widely used in policy analysis. This chapter provides descriptive statistics such as means to measure central tendencies and standard deviations to measure variation around the mean. It also shows how to use regression analysis to quantify the impact of a unit increase in X (the cause) on variable Y (the effect) while controlling for other variables. It shows how to use inferential statistics in testing hypotheses. It explains, for example, that a t test shows the probability of obtaining a sample statistic as large or larger when sampling from a population in which the null hypothesis is true. It shows procedures allowing the analyst to reject or fail to reject a null hypothesis based on probability.

Chapter 3. Microcomputers in Agricultural Policy Analysis in Developing Countries. This chapter explains the place of microcomputers in policy analysis and, more specifically, the place of spreadsheet analysis. Spreadsheet software programs are flexible and adaptable to a wide range of policy analysis, including that described in later chapters.

Chapter 4. Demand Theory and Elasticity Matrix Construction. Demand functions relating demand quantity to prices, incomes, and other variables are of value for predicting consequences for consumers of changes in price or the impact on price of shifts in supply. The chapter shows how to generate a complete set of own- and cross-price elasticities of demand useful for prediction, classical welfare analysis, simulation, and other purposes.

Chapter 5. Supply and Elasticity Estimation. Supply along with demand determines pricing and output. Supply responses or elasticities are essential building blocks for a wide range of policy analysis. Supply response estimates help to predict how much output will change if price supports are raised or lowered. Elasticities (or their inverse) indicate how much prices will change if the demand shifts due to a change in exports, income, or population. Statistical methods described in Chapter 2 can be used to

estimate a few supply elasticities but rarely a complete set or matrix of farm commodity own- and cross-price elasticities for a country. A procedure for using theory to fill in missing elasticities and to form a complete matrix is included in the chapter.

Chapter 6. Classical Welfare Analysis. Classical welfare analysis uses demand and supply relationships to determine the level and distribution of gains and losses among consumers, producers, taxpayers, and society from changes in economic policy. The technique is useful for policy analysts determining costs and benefits of intervening in markets. Classical welfare analysis is especially well suited to determine whether policies add to or subtract from national income (efficiency). Appropriately modified, welfare analysis is also suited for analyzing equity in distribution of benefits and costs.

Chapter 7. Simulation and Systems Modeling. Simulation utilizes mathematical relationships to depict on a small scale the essential elements of an economic or other system. Social experiments trying new policies on the actual system are prohibitively expensive and intrusive; the same experiment can be simulated on a model of the system at low cost and with no intrusion into personal lives. Modern computers have the capacity to bring more realism into models including random shocks from weather and other factors. Simulation models have wide application to economic problems facing developing countries.

Chapter 8. Enterprise Budgets. Budgets show costs and returns from farming enterprises. The farm budget is of interest in itself and also is a basic building block for analysis of comparative advantage, economies of farm size, farm firm impacts of policies, and for other tools such as linear programming. Many countries will desire to have a set of budgets for typical farms covering major resource situations for the country. Microcomputer programs can be used conveniently to update prices, yields, and other elements in these budgets as conditions change.

Chapter 9. Mathematical Programming. Mathematical programming can be used for a wide range of problems including maximizing net income from a large number of enterprises facing a myriad of resource constraints on a farm or for a region. It is well suited for analyzing the impact on and likely reaction of producers to a change in policy. Implications can be traced at the farm, regional, or national level. Programming of optimal outcomes using border prices and omitting taxes and subsidies provides insights into comparative advantage for a region or country. Results can indicate how much income is sacrificed by forcing uneconomic production to pursue goals such as import substitution and self-sufficiency.

Chapter 10. Agricultural Project and Investment Analysis. In attempting to use resources wisely, countries face difficult decision regarding whether to invest in a road, school, irrigation system, or agricultural research and extension. Project analysis helps decision makers by showing whether benefits are likely to exceed costs of such projects and which projects provide the greatest net payoff. Project analysis is used

especially where large investments are made in durable resources so that costs or benefits accrue over a period of years.

Chapter 11. Policy Applications in Natural Resource Projects. Natural resources along with culture and institutions determine economic development. An abundance of natural resources (high "land-man" ratio) does not assure a high living standard but facilitates growth, other things equal. The well-being of a nation depends on wise use or maintenance of its environmental and natural resource base. Economic analysis presented in chapter 11 and other chapters can help analysts assemble the information needed by decision makers to decide what is "wise use."

Chapter 12. International Trade Policies. Many countries intervene in international trade with taxes, subsidies, quotas, or other tools. Sometimes the country intervening pays the largest price in foregone income. This chapter provides an analytical framework for quantifying the level and distribution of gains and losses to consumers, producers, taxpayers, and society from intervention in trade markets.

Chapter 13. Macroeconomic Linkages to Agriculture. Monetary and fiscal policies often create more uncertainty and loss of farm income than do natural elements such as weather and pests. Many developing countries are plagued by macroeconomic policy failures apparent in high inflation, overvalued currency, high unemployment (or underemployment), excessive bureaucracy, government budget deficits, trade deficits, delinquent or burdensome international debt, and economic and related political instability. This chapter discusses elements of sound macroeconomic policies as well as consequences for agriculture of unsound policies. Information provided can be helpful in constructing a macroeconomic model tying agriculture to the general economy.

Chapter 14. Index Numbers and Aggregation. Whether for information or analysis, aggregate measure of farm prices and output are useful. Examples are aggregate farm output, imports, multifactor productivity, indices of prices paid and received by farmers, the ratio of prices paid to received by farmers, or the consumer price index. The chapter shows the conceptual basis and computational procedure for constructing indices.

CONCLUSIONS

The chapters of this book describe tools for policy analysis. Combined with the power and convenience of microcomputers and with capable analysts, the result can be information needed by administrators and policy makers to make decisions improving well-being of people. The tools are not a substitute for sound judgment or reliable data, but they do allow the analyst to extract more information from data and other available resources.

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2. REGRESSION AND STATISTICAL METHODS

by Daryll E. Ray

Policy analysis often requires using two kinds of statistical techniques: Descriptive statistics and inferential statistics. The term *descriptive statistics* includes a wide range of activities. The most common descriptive statistics measure central tendency using sample means, medians, and modes. Descriptive statistics also measure dispersion using ranges, mean absolute differences, variances, standard deviations, and coefficients of variation. Also, classifying data, constructing frequency distributions and histograms, and drawing other kinds of graphs such as line, bar, or pie graphs are in the domain of descriptive statistics[1].

Inferential statistics uses information from selected observation units (say, 1,000 farms) to reach conclusions about the totality of the unit base (all farms in a given region). The selected observation units form the sample, and "the totality of the unit base" is referred to as the population. Thus, statistical inference deals with inferring about the population based on a (usually random) sample. Statistical inference allows researchers to make predictions based on sample information and to test hypotheses.

OBJECTIVES

The focus of this chapter is the subset of inferential statistics encompassing linear regression and its applications. Specifically, the objectives are to discuss the underlying assumptions of the classical regression model, the least squares estimation procedure, and the most common hypothesis tests. Extensions of the classical regression model are also presented including the use of alternative functional forms, and dummy or binary variables.

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[1]For a comprehensive treatment of descriptive statistics and graphical representation of data see Hayslett (pp. 6-34), Huntsberger and Billingsley (pp. 1-56), and Moore (pp. 129-233).

Since the underlying assumptions of the classical model are often not fulfilled, some of the most common econometric techniques that can be used to overcome violation of assumptions are briefly summarized[2]. Some statistical theory and notation are used but the exposition favors intuitive explanation over mathematical sophistication.

KEY POINTS

Many economic relationships postulate that changes in one variable, say the supply quantity of a commodity, can be explained by changes in several other variables such as commodity price expectations, input prices, technology, and weather. Regression analysis provides a means to quantify this cause-and-effect behavioral or technical relationship. Regression and its econometric extensions are the primary statistical analysis tools for giving empirical content to economic relationships.

When using regression, the variable to be explained is identified as the dependent variable while the variables that do the explaining are called independent variables. The independent variables are the causes and the dependent variable the effect. An analyst may use a regression-estimated economic relationship in several ways: To quantify an economic parameter, such as an elasticity of supply or the marginal propensity to save, or to test a hypothesis concerning a parameter value or set of values. Often the estimated relationship is used to forecast the value of the dependent variable for given values of the independent variables. The same regression would be run in all cases but the results would be used or emphasized differently depending on the purpose of the analysis.

The first step in applying regression analysis includes specifying the explanatory variables and functional form of the model. But it also includes a set of assumptions regarding the randomness of disturbances and the dependent variable, linearity of parameters, and other properties of the disturbance term and independent variables.

Ordinary least squares provides estimates of the model coefficients that minimize the sum of squared deviations of the actual observations and the model predictions of the dependent variable. With the classical assumptions fulfilled, model coefficients estimated with ordinary least squares have desirable statistical properties including unbiasedness and efficiency. The total variation of the dependent variable can be split into two parts: the part explained by the regression and the part unexplained or due to error. This decomposition is summarized in an analysis of variance table and the coefficient of determination of R^2 value. R^2 is the proportion of the variation of the dependent variable explained by the regression.

Significance testing provides a procedure for judging if there is any statistical difference between estimated parameters and hypothesized values for population parameters. The t distribution is used to test hypotheses that involve only one parameter or linear combinations of parameters. The most common use of the t distribution is to test a single coefficient against a null hypothesis that the parameter is equal to zero. The t is also

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[2]Books by Johnston, Kmenta, Maddala, and Pindyck and Rubinfeld are excellent references for those wanting additional information on the regression and econometric topics discussed in this chapter.

used to develop confidence intervals around individual model parameters and around forecasts of the dependent variable. The F distribution must be used to test hypotheses that involve setting more than one model parameter to specified values or involve more than one linear combination of the model parameters. Regardless of the specific hypothesis being tested, a standard set of procedural steps are followed.

Various functional forms are consistent with the statistical assumptions of the classical regression model. However, the estimation of elasticities and interpretations of coefficients vary from one functional form to another. Dummy or binary variables can be used whenever a set of observations can be separated into classes such as male and female or the various regions of a country. One of the most common applications of dummy variables is to account for seasonality such as allowing shifts in a function by quarter or season.

Violation of one or more of the assumptions of the classical model can cause difficulties in the use of the regression. Specification error means the wrong model was estimated. For example, the wrong set of independent variables and/or functional form may have been used. The model estimates are no longer unbiased under such circumstances. Furthermore, model estimates are biased if (1) an independent variable is measured incorrectly, (2) the disturbances are correlated with one another, (3) the disturbances have different variances, or (4) an explanatory variable is correlated with the disturbances. Econometric procedures are available to address each of these difficulties.

CONCEPTS AND APPLICATIONS

Model Specification

The first step in regression analysis is to specify the general form of the model. In economic applications, the variable to be explained (dependent variable) y is specified as a function of K independent variables -- $x_1, x_2, x_3, \ldots x_K$. The independent variables are selected based on economic theory, knowledge about the economic phenomenon being investigated, and previous research. Ideally, during the specification stage, model construction should not be influenced by data availability or other constraints. The functional (mathematical) form must also be specified. Economic theory often provides little insight regarding functional form. Previous research and especially the researcher's knowledge about economic activity being investigated usually must be relied upon.

1. Meaning of Linear. The classical regression model requires the function to be linear in the parameters. That is, the true regression coefficients, referred to as ß's later on, must have an exponent of one and must not be multiplied or divided by another. However, the independent variables or x's need not be linear. Thus, the researcher must decide, based on his conception of the *true* function, whether the x's should be used as they are measured or transformed, say, by taking logarithms of the independent variables or logarithms of both dependent and independent variables. Or, in an addition to original x's, the researcher may decide to include the square (and cube) of certain variables to allow curvature, or the product of two variables to allow interaction. Again, the specification

represents the researcher's belief about the *true* set of independent variables and the functional form of the relationship explaining variation in the dependent variable.

2. ***Random Nature of the Dependent Variable.*** In contrast to the physical and engineering sciences where relationships tend to be precise, a regression relationship in the social sciences is usually less exact. The value of a dependent variable (say, family milk consumption) varies from one observation unit to another even though the x or independent variable *values are identical* (say, income, family size, family age and sex composition, cultural heritage, occupation(s), time of year, and price). Thus, there is a *distribution* of y values for *each set* of values for the K independent variables. Even if the x variables and functional form were known with certainty (and all other rules are met), the regression relationship is an estimate of the dependent variable's expected or average value. Put another way, the regression relationship estimates the mean of a distribution of y values that would be generated from multiple samples of the dependent variable *for identical values of x.*

More formally, the regression model assumes that for each set of values for the K independent variables, there is a distribution of y_t values such that the mean of the distribution is given by

$$E(y_t) = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + \ldots \beta_k x_{tk} + \ldots + \beta_K x_{tK} \tag{2.1}$$

where t represents one (say the 4th) of the T total number of observations in the sample (say, 153); t = 1, . . ., T;

k represents one of the K total number of explanatory variables (including the intercept):

y_t denotes the tth observation of the dependent variable;

x_t's denote the tth observation of the K explanatory variables;

β's are the *true* but *unknown* population parameters; β_k (k = 2, . . ., K) is the slope of the relationship between the independent variable x_k and the dependent variable holding all other independent variables constant. β_1 is the intercept which represents the expected value of y when the all the remaining (nonintercept) variables are zero.

The inclusion of the intercept can be made explicit by defining $x_{t1} \equiv 1$ for t = 1,..., T; then

$$E(y_t) = \beta_1 x_{t1} + \beta_2 x_{t2} \ldots + \beta_K x_{tK} \tag{2.2}$$

The x_{t1} variable will be defined to be 1 for all t in subsequent discussion unless otherwise specified.

3. Inclusion of Random Disturbances. Equations 2.1 and 2.2 represent the theoretical relationship to explain the average value of y_t from repeated sampling with the x variables held constant. As an average, the equation is postulated to be accurate, but for a *single* observation of y_t the model must allow for imperfect prediction. This is done by including an error term

$$y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + \ldots \beta_k x_{tk} + \ldots + \beta_K x_{tK} + u_t. \tag{2.3}$$

The error term u_t is the deviation of the value of y from its distribution mean. The error term summarizes the collective influence on y of many variables, each of which is assumed to be of little importance and may be difficult or impossible to quantify. The error term may be the result of unknowable disturbances, sampling imperfection, or variables left out of the equation. The latter source of *specification error* should be minimized by including all variables if possible.

Representations of the Model

The regression model can be represented in various ways. Three of the more common representations are presented in this section. The matrix notation version of the model is often used since it includes all the observations of the variables and yet is convenient to manipulate.

1. Summation Notation. Using only the tth observations on y and the x's the model can be represented using summation notation as:

$$y_t = \sum_{k=1}^{K} \beta_k x_{tk} + u_t. \tag{2.4}$$

2. Vector Notation. Because the summation part of equation 2.4 defines the vector multiplication of a row vector of the x values and a column vector containing the ß's, the model can also be written as

$$y_t = \underbrace{[x_{t1}, x_{t2}, \ldots, x_{tK}]}_{\underset{\sim}{x}} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_K \end{bmatrix}}_{\underset{\sim}{\beta}} + u_t \quad . \tag{2.5}$$

Using the $\underset{\sim}{x}_t$ and $\underset{\sim}{\beta}$ definitions suggested in equation 2.5, the vector representation for the tth observations of the y and the x's is

$$y_t = \underset{\sim}{x}_t \underset{\sim}{\beta} + u_t. \tag{2.6}$$

3. Matrix Notation. Suppose we write the relationship given by equation 2.5 for the first observation. Assigning t equal to 1 we have

$$t = 1 \qquad y_1 = [x_{11}, x_{12}, \ldots, x_{1K}] \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_K \end{bmatrix} + [u_1] \tag{2.7}$$

Continuing on, let t = 2 and add a second "row" to equation 2.7 to represent the second observations on y, the x's, and u. Doing similarly for t = 3, . . ., T results in a set of vectors and matrices that contain all the observations for all the variables in the model

$$\underset{\sim}{y} \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_T \end{bmatrix} = \underset{X}{\begin{bmatrix} x_{11} \cdots x_{1K} \\ x_{21} \cdots x_{2K} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{T1} \cdots x_{TK} \end{bmatrix}} \underset{\sim}{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_K \end{bmatrix}} + \underset{\sim}{\begin{bmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_T \end{bmatrix}} \tag{2.8}$$

or more compactly:

$$\underset{\sim}{y} = X \underset{\sim}{\beta} + \underset{\sim}{u} \quad \text{or simply} \quad y = X\beta + u \tag{2.9}$$

where

y is the Tx1 vector of observations on the dependent variable;
X is a TxK matrix of observations on the independent variable;
ß is a Kx1 vector of *true* but *unknown* parameters;
u is a Tx1 vector of *unknown* disturbances.

Assumptions

Certain assumptions must be met to estimate appropriately the population parameters and to conduct tests of hypotheses under the OLS framework. As noted previously, the correct independent variables and functional form must be used and the function must be linear in the parameters. Of course, the data on the y and x variables must be measured accurately. Other assumptions are listed below. Implications of violating

each of the critical assumptions are discussed later but are briefly noted here within parentheses.

1. For each set of observations, the mean value of the error term is 0. That is, for x_{t1} x_{t2}, . . ., x_{tK}, $Eu_t = 0$ for all t. (The regression estimate for the intercept β_1 is adversely affected if this assumption is invalid; estimates for the remaining β's are unaffected.)
2. For each set of observations, the error or disturbance term has a constant variance of σ^2, that is, $VAR(u_t) = \sigma^2$ for all t. (Disturbances are said to be *homoskedastic* when this assumption is valid; *heteroskedastic* when it is not. Regression parameter *estimates* do not have least variance when the assumption is invalid).
3. For any two sets of observations, the error or disturbance terms are uncorrelated, that is $COV(u_t, u_s) = 0$. (Disturbances are said to be nonautocorrelated when this assumption is fulfilled and autocorrelated when it is not. Regression parameter *estimates* do not have least variance when the assumption is invalid).
4. Each independent variable is uncorrelated with the disturbance term. Thus, for each x_k, $COV(x_k, u) = 0$. (The *averages* of regression parameter estimates *over many samples* are accurate or unbiased when the assumption is valid but biased when the assumption is invalid.)
5. No exact linear relationships exist among the independent variables. (Estimation is impossible if, for example, variable C is exactly equal to twice variable D minus one-fifth variable F, etc. "Perfect multicollinearity" is said to exist if this assumption is invalid.)
6. The number of observations must exceed the number of explanatory variables, including the intercept. That is, T must be greater than K. (Estimation is impossible if we have fewer years of observations than parameters to estimate.)
7. In repeated sampling of y and u for a given set of the K independent variables, the disturbance u_t has a normal distribution. This assumption is not necessary to compute regression parameter estimates, but it is necessary if tests of hypothesis based on t and F distributions are to be conducted.

Notation for Estimated Parameters and Variables

So far the equations and most of the discussion have focused on the *true* relationship between the dependent variable and the K independent variables in the *population*. The regression analysis technique is used to estimate the values of the true population parameters using data from a finite sample of size T taken from the population. To distinguish the population model from the estimated model, the estimated empirical model is written as:

$$\hat{y}_t = \hat{\beta}_1 + \hat{\beta}_2 x_{t2} + \hat{\beta}_3 x_{t3} + \ldots \hat{\beta}_k x_{tk} + \ldots + \hat{\beta}_K x_{tK} \tag{2.10}$$

where the *estimates* of the true values are designated by "hats". The right hand side of the equation provides the prediction of y given the x values and the ß estimates.

The estimated residual $\hat{u}$'s are the estimates of the population disturbance but also pick up the empirical slack between the actual observations on y and their predicted values. Thus

$$y_t = \hat{y}_t + \hat{u}_t \tag{2.11}$$

where the predicted part, $\hat{y}_t$, is defined as

$$\hat{y}_t = \hat{\beta}_1 x_{t1} + \hat{\beta}_2 x_{t2} + \ldots + \hat{\beta}_K x_{tK} \tag{2.12}$$

or using matrix notation the Tx1 vector $\hat{y}$ is computed as

$$\hat{y} = X\hat{\beta} \tag{2.13}$$

By rearranging equation 2.11, the estimated population disturbance or residual appears as

$$\hat{u}_t = y_t - \hat{y}_t \tag{2.14}$$

or using matrix notation, the T observations on $\hat{u}$ are

$$\hat{u} = y - \hat{y} = y - X\hat{\beta} \tag{2.15}$$

Ordinary Least Squares

The $\hat{\beta}$'s are estimated using the least squares criterion, commonly called ordinary least squares (OLS) regression. The procedure is called least squares because it finds the set of ß estimates that causes the sum of the square of the $\hat{u}$'s ($\hat{u}_t$, t=1 . . . T) to be the smallest. Any *other* set of estimates for ß would cause the corresponding sum of squared $\hat{u}$'s to have a larger value. Thus, ordinary least squares finds the values for $\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_K$ that minimize the sum of the squared residuals:

$$\sum_{t=1}^{T} \hat{u}_t^2 = \hat{u}'\hat{u}\,. \tag{2.16}$$

Using the definition of $\hat{u}$, an equivalent way to express ordinary least squares is to say it finds those estimates for $\beta_1, \beta_2, \ldots, \beta_K$ that minimize the sum of the squared deviations of the actual and predicted observations on y, or

$$\sum_{t=1}^{T} (y_t - \hat{y}_t)^2 = \sum_{t=1}^{T} \hat{u}_t^2 . \tag{2.17}$$

Finding the $\hat{\beta}$s that minimize the sum of squared residuals does not require hit and miss guessing. Luckily, application of differential calculus provides exact formulas, which will be presented without derivation.

Simple Regression Estimation

In the case of a simple regression model containing only an intercept and one additional independent variable, $y_t = \beta_1 + \beta_2 x_{t2} + u_t$, the slope for x_2 is computed as

$$\hat{\beta}_2 = \frac{\Sigma(x_{t2} - \bar{x}_2)(y_t - \bar{y})}{\Sigma(x_{t2} - \bar{x}_2)^2} \tag{2.18}$$

where the bar over a variable represents the variable's sample mean. The intercept is computed as $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}_2$.

Multiple Regression Estimation

In general, using matrix notation, the K x 1 vector $\hat{\beta}$ (with the first element being the intercept) is computed as (Johnston, p. 171; Kmenta, pp. 350-353)

$$\hat{\beta} = (X'X)^{-1} X'y . \tag{2.19}$$

$(X'X)^{-1}$ corresponds to the denominator of equation 2.18 for the simple regression case and X'y corresponds to the numerator of equation 2.18. By including x_1 as a column of one's in the X matrix, the intercept is computed along with the slopes. Also, the column of one's allows the y and x variable values to be used directly without adjusting for means.

Each β Estimate Is From a Distribution

The $\hat{\beta}$ values are estimates of the population β parameters in the true model based on a *sampling of the population*. That is, given the observations for the x variables, the calculation of the $\hat{\beta}$'s are based on the specific sampling of the (T number of) y values. Due to the random component, the y observations from another sampling of size T would be different *even though the values for the x variables are the same*. A change in the y values changes the estimates for the $\hat{\beta}$'s. Thus, we can envision a distribution of estimates for each $\hat{\beta}_k$. For example the estimates for $\hat{\beta}_2$ from repeated regressions on fresh random samples on y of size T from the population will not be identical, but will be distributed around the overall mean value for $\hat{\beta}_2$.

The degree of variability of a $\hat{\beta}_k$ estimate about its mean can be measured as the estimated standard deviation of the sampling distribution. This estimate is known as the standard error and is denoted by a small "s" subscripted with the symbol for the estimated coefficient of interest $s_{\hat{\beta}_k}$. The true but unknown ß's, of course, have no distribution because they are fixed parameters or constants.

The estimated standard error for the simple regression case $\hat{\beta}_2$ is

$$s_{\hat{\beta}_2} = \sqrt{\frac{\sum_{t=1}^{T} (y_t - \hat{y}_t)^2 / (T-2)}{\sum_{t=1}^{T} (x_{t2} - \bar{x}_2)^2}} \tag{2.20}$$

or

$$s_{\hat{\beta}_2} = \sqrt{\frac{s^2}{\sum_{t=1}^{T} (x_{t2} - \bar{x}_2)^2}} \; . \tag{2.21}$$

The s^2 in equation 2.21, defined as the numerator in equation 2.20, is the estimated residual variance or variance of $\hat{u}$. This can be seen by noting that $\hat{u}_t = y_t - \hat{y}_t$, and that the mean of the estimated residual, $\hat{u}_t$, is zero. Hence, the numerator of equation 2.20 corresponds to the usual formula for computing a variable's variance except the degrees of freedom (d.f.) and divisor is T - 2 rather than T - 1. The T - 2 divisor is required in case of simple regression because the d.f. or number of independent pieces of information to compute the variance is the number of observations minus 1 for the mean (intercept in the regression model) and 1 for the computing the coefficient on the independent variable x_2. For future reference note that, in general, the degrees of freedom and hence the divisor for computing the residual s^2 in a multiple regression model is T - K.

Thus, equation 2.21 indicates that the variability or dispersion of the $\hat{\beta}$'s is related to the estimated variance of the residuals and to the inverse of the variability of the x variable(s). This causal relationship is even more evident in the formula for estimating the standard errors for the *vector* of estimated coefficients in the general multiple regression case using matrix representation

$$s_{\hat{\beta}} = \sqrt{s^2 \, (X'X)^{-1}} \; . \tag{2.22}$$

The smaller the residual variance (or, in less precise terms, the smaller the errors in predicting the dependent variable) the smaller will be the standard errors of the $\hat{\beta}$. Similarly, everything else equal, the greater the variation in the independent variables (individually), the smaller the $\hat{\beta}$ standard errors. Also, although less obvious, the larger the sample size and the more *unrelated* the independent variables are to one another, the lower the standard errors.

Desirable Properties of Least Squares

One reason for the popularity of OLS is the desirable properties of the estimators. Specifically, the estimators for the population β's are said to be unbiased and efficient. Unbiased means that *with repeated sampling* , the *average* of the parameter estimates for a given β_k (that is, the mean of the distribution for the estimated parameter) *equals* the *true* population *value* for the parameter. Unbiasedness does *not* ensure that any particular regression estimate is equal to its population counterpart. Coefficient estimates may be individually overestimated or underestimated in the application of least squares to a single sample of size T. Unbiasedness only requires averages of parameter estimates over many samples of size T be equal to their respective true values.

Being efficient and unbiased means that the β estimates have the smallest standard errors (or variances) of any other unbiased estimators. The Gauss-Markov theorem proves that OLS estimators for the β's are unbiased and efficient as long as the model is specified correctly and assumptions 1 through 6 are satisfied (Johnston, pp. 173-174). While unbiasedness and efficiency are "good properties" for estimators, it is important to remember that *any single estimate* could be in the tail of "a repeated sample distribution of the estimated parameter" and be far afield from the parameter's true value.

Decomposition of Total Sum of Squares

When using regression, a set of independent variables is used to explain variation in the dependent variable. Even under the best modeling conditions, only part of the variation is "explained" by the regression. Full explanation would require $y_t - \hat{y}_t = 0$ for all T observations. Thus, the unexplained portion occurs because the regression does not perfectly predict the dependent variable. The most common way to decompose the variability of the dependent variable into the portions explained and unexplained by the regression is to take the relationship $y_t = \hat{y}_t + \hat{u}_t$ and rewrite it as the sum of squared deviations about the respective means

$$\sum_{t=1}^{T} (y_t - \bar{y})^2 = \sum_{t=1}^{T} (\hat{y}_t - \bar{\hat{y}})^2 + \sum_{t=1}^{T} (\hat{u}_t - \bar{\hat{u}})^2 . \qquad (2.23)$$

Two of the properties of ordinary least squares estimates can be used to simplify this equation somewhat. One property is that the mean of the estimated residuals is zero.

The other property of interest is that ordinary least squares forces the mean of the $\hat{y}$'s to be equal to the mean of the actual observations on y. Thus, by replacing $\bar{\hat{y}}$ with $\bar{y}$ in the middle term and omitting the $\bar{\hat{u}}$ and parentheses in the last term, equation 2.23 can be written as:

$$\sum_{t=1}^{T}(y_t - \bar{y})^2 = \sum_{t=1}^{T}(\hat{y}_t - \bar{y})^2 + \sum_{t=1}^{T}\hat{u}_t^2. \tag{2.24}$$

Using TSS to represent the total sum of squares of the dependent variable about its mean, RSS to represent the sum of squares due to regression, and ESS to represent the sum of squares due to error, the decomposition of the total sum of squares of the dependent variable can be written as

$$TSS = RSS + ESS \tag{2.25}$$

Analysis of Variance

The Analysis of Variance (ANOV) table provides a convenient vehicle for summarizing regression results (Table 2.1). In addition to the sum of squares, the ANOV lists the degrees of freedom (the number of independent pieces of information used to compute the sum of squares) for each of the three sum of squares. The last column of the table contains the mean square for the regression and mean square error. These mean squares are computed by dividing the sum of squares by corresponding degrees of freedom. Note that the mean square error (in the "Residual" row of the ANOV) is the definition of s^2 or estimated residual variance.

Goodness of Fit

The most common measure of goodness-of-fit for regression is R^2 or the coefficient of determination. It measures the *proportion* of the total sum of squares explained by the regression. Because RSS is defined as the sum of squares accounted for by the regression and TSS is the total sum of squares of the dependent variable, R^2 is simply the ratio of RSS to TSS. Alternatively, the ratio of ESS to TSS is the proportion of the variation *unexplained* by the regression, hence, 1.0 minus this ratio is another way to compute R^2:

$$R^2 = \frac{RSS}{TSS} = 1 - \frac{ESS}{TSS}. \tag{2.26}$$

Table 2.1. A Generalized Analysis of Variance Table.

Source	Sum of Squares	Degrees of Freedom	Mean Squares
Due to Regression	RSS or $\Sigma(\hat{y}_t - \bar{y})^2$	K-1	$\Sigma(\hat{y}_t - \bar{y})^2/(K-1)$
Residual	ESS or $\Sigma \hat{u}_t^2$ or TSS - RSS	T-K	$\Sigma \hat{u}_t^2 / (T-K)$
Total	TSS or $\Sigma(y_t - \bar{y}_t)^2$	T-1	

Because R^2 is defined as a proportion, its value can only range from 0 to 1. The R^2 computation assumes that the model includes an intercept term. (The intercept is required to compute the sum of squares about the means, i.e., TSS and RSS). R^2 is affected by the number of observations and independent variables used in the regression as well as the "explanatory power " of the independent variables. For example, R^2 increases as the number of independent variables approaches the number of observations, reaching a perfect value of 1 when they are equal. An adjusted R^2 is sometimes computed to correct for the effect of different combinations of K and T. It is defined as

$$\bar{R}^2 = 1 - \left[(1-R^2) \cdot \frac{T-1}{T-K}\right]. \tag{2.27}$$

Tests of Significance

In addition to goodness-of-fit, researchers are often interested in judging if there is any statistical difference between estimated parameters and hypothesized values for population parameters. Significance testing provides a procedure for making such judgments. The basic approach is to specify a hypothesized value for a *population parameter*, say β_k, (or a collection of β's) and then determine if the estimated parameter(s) provides evidence to *refute* the hypothesis. Thus, a statistical hypothesis is a statement about a population *parameter*, or *parameters* -- NOT about the estimator(s) of the parameter(s).

1. The Concept of Null Hypothesis. Typically, in regression analysis, the hypothesis to be tested, called *null hypothesis*, is framed in what is thought to be a false statement. Hence, rejecting the null hypothesis is most often the expected outcome. For example, it is common to test null hypotheses in which a regression population parameter

or a set of parameters is specified to be zero. Rejecting these "zero" hypotheses, although not validating the estimated value itself, provides evidence that the independent variable of interest aids in the explanation of the dependent variable. Rejecting the zero null hypothesis does not suggest that the estimated coefficient "is correct" but that there is statistical evidence that the difference between the sample statistic and hypothesized parameter, zero in this case, is too great to be attributed to chance. We either *reject* or *fail to reject* a null hypothesis. We do not *accept* the null or any other hypothesis.

Unfortunately, inability to reject the null hypothesis such as $\beta = 0$ has led some to conclude that β in fact equals zero. Statistical inference biased towards zero by small samples and sampling error is avoided by using the approach suggested by Tweeten (p. 550) of using economic theory, the best previous empirical estimate, or an average of previous empirical estimates as the null hypothesis. Thus the null hypothesis is likely to be a true statement. Failure to reject it because our sample is too small will not tempt us to conclude that $\beta = 0$, and hence does not lead to biased statistical inference.

2. *Level of Significance.* We reject the null hypothesis if the chance of obtaining a larger value of the test statistic t or F is very small when sampling from a population in which the null hypothesis is true. That is, the difference between the sample statistic and hypothesized parameter is too great to be attributed to chance. The definition of what is too great to be attributed to chance is stated in terms of probability level attached to a test statistic such as a t or F. This probability level is called the *level of significance.* The significance or probability level establishes the threshold value from a known distribution that the computed test statistic must equal or exceed to reject the null hypothesis. The probability idea reflects recognition that even though a computed value of a test statistic, say the t, may have a large value (which suggests rejecting the null hypothesis), there will be times, despite the large t or F, that the null hypothesis is actually true.

The significance level reflects the average number of times out of 100 that a *fluke* will occur in the form of rejecting the null hypothesis when in fact it is true. Or viewed differently, one minus the significance level reflects the percentage probability that such a fluke will *not* occur. The *greater the significance level,* as loosely used in everyday language (but actually 1.0 minus the significance level), the larger is the tabled critical value of t or F that the computed test statistic must exceed for us to reject the null hypothesis. Hence, a conclusion to reject the null hypothesis will be correct 95 times out of 100 when the computed test statistic is greater than the tabled critical value representing the .05 or 5 percent significance level. Five times out of 100 the conclusion will be incorrect. If the significance level were .01, the critical value would be greater and the risk of rejecting the null hypothesis when it is true would decline to 1 out of 100.

The incorrect rejection of a true null hypothesis is called a Type I error. Hence, the level of significance and probability of a Type I error are the same. It is also possible, of course, to err by failing to reject a false null hypothesis. This error of failing to reject the null hypothesis when the alternative hypothesis is true is called a Type II error. The

probability of a Type II error is usually impossible to calculate because a specific value must be used in the alternative hypothesis, a procedure rare in social science research.

Use of t and F Distributions in Regression Analysis

In regression analysis, the t distribution can be shown to be an appropriate test statistic for hypotheses concerning individual ß's or a single linear combination of ß's. The t distribution is also used to construct confidence intervals around individual ß's and forecasts of the dependent variable. More complex hypotheses that involve setting several ß's to specified values, e.g., setting all ß's to zero, or that involve more than one linear combination of ß's require the F distribution. Only the most common hypothesis tests used in regression analysis are summarized.

1. Hypothesis Testing of an Individual ß With the t. Suppose we want to test the following generalized hypothesis for one of the ß's, say $ß_k$

$$\text{Null Hyp: } ß_k = ß_k^o \tag{2.28}$$

$$\text{Alt. Hyp: } ß_k \neq ß_k^o \ . \tag{2.29}$$

The null hypothesis is that the population parameter $ß_k$ is equal to a specific value, represented in equation 2.28 as $ß_k^o$. The task is to determine if there is a difference between this specific hypothesized value for the population parameter $ß_k$ and its regression estimated counterpart $\hat{ß}_k$ that is too great to be due to chance factors. If the answer is yes, the null hypothesis is rejected; if the answer is no, we fail to reject the null hypothesis. The alternative hypothesis summarizes what can be said if the null hypothesis is rejected. Equation 2.29 is a *nondirectional alternative hypothesis* which means that a rejection of the null hypothesis would lead us to conclude that $ß_k$ does not equal the specified value $ß_k^o$ but we have no a priori basis to expect $ß_k$ to be smaller or greater than the $ß_k^o$. An alternative hypothesis which is directional will be discussed later.

In general the t test statistic is computed as

$$t = \frac{\text{Estimated Parameter - Hypothesized Value}}{\text{Standard Error of the Estimated Parameter}}$$

Hence, the t statistic for the current application is

$$t = \frac{\hat{ß}_k - ß_k^o}{s_{\hat{ß}_k}} \tag{2.30}$$

with T - K degrees of freedom which is the degrees of freedom of s^2.

The next step is to look up the critical value of the t distribution for T-K degrees of freedom with a *two-tailed*, say .05, significance level. A two-tailed test is used because the

alternative hypothesis is nondirectional. With a two-tailed t test, the area representing the level of significance is placed in each tail of the t distribution. The t distribution is similar to the bell–shaped normal distribution, especially as degrees of freedom become large. If the (absolute value of the) computed t statistic using equation 2.30 exceeds the tabled value, the null hypothesis is rejected in favor of the alternative hypothesis. If the tabled t value is larger, we fail to reject the null hypothesis.

2. Testing β Against Zero. Although β probably should be the most likely true value based on theory or previous empirical estimates, as indicated earlier the most common null hypothesis for a single regression coefficient is

$$\text{Null Hyp: } \beta_k = 0 \tag{2.31}$$

This hypothesis states that the individual coefficient in the population is zero. Alternatively stated, the test is to determine if the estimated coefficient is significantly different from zero. If the statistical testing procedure suggests rejection of this null hypothesis, the coefficient is said to be statistically different from zero (at a specified significance level). If the procedure gives the contrary result, we fail to reject the null hypothesis. (Technical reasons involving statistical theory and rules of logic preclude using the term "accept" in place of "fail to reject." (See Henkel, pp. 40-41; Tweeten, p. 548.)

The alternative hypothesis could be nondirectional as in equation 2.29 or it may be directional as in equation 2.32. With a directional alternative hypothesis, a rejection of the null hypothesis is in favor of the alternative hypothesis in the indicated direction. For example, if the alternative hypothesis were as follows:

$$\text{Alt. Hyp: } \beta_k < 0 \tag{2.32}$$

then rejecting the null hypothesis suggests that $\hat{\beta}_k$ is not only significantly different from zero, it also has a negative sign. This test situation, or with the inequality reversed in the alternative hypothesis, is referred to as a one-tailed test.

The alternative hypothesis chosen for the test is usually determined by the amount of information the researcher has about the relationship being modeled. For example, if a supply equation is being estimated, the researcher will, based on theory and review of literature, be in a position to specify the signs of the coefficients. The parameter attached to the commodity's own price is expected to be positive; coefficients on prices of other commodities that compete for production resources are expected to be negative. In this case, with a null hypothesis equal to zero, the alternative hypothesis is for the coefficient on own price to be greater than zero and for each of the alternative hypotheses relating to the other prices to be negative.

On the other hand, if information about the relationship is minimal or either coefficient sign is conceivable, a nondirectional alternative hypothesis is appropriate. Researchers estimating parameters in economic and policy models usually have information on which to base sign expectations. Hence, most tests on individual coefficients from

economic relationships can be framed as one-tailed tests, even though two-tailed tests are commonly reported in the literature.

Having specified the zero null hypothesis and a one-tailed alternative hypothesis, the procedure is the same as discussed earlier but now the rejection region is all in one tail of the t distribution. After selecting the level of significance, the critical value is determined from the table for the "one-tailed" t statistic for T - K degrees of freedom. The critical value is compared with the computed t statistic. The computed t statistic is computed as before but because the hypothesized value is zero, the computation simplifies to:

$$t = \frac{\hat{\beta}_k}{s_{\hat{\beta}_k}}. \tag{2.33}$$

If the absolute value of the computed t exceeds the tabled t, the null hypothesis is rejected. That is, the difference between $\hat{\beta}_k$ and the hypothesized value, 0, is so great that we cannot attribute the difference to chance factors in the sampling process. Hence, we reject the null hypothesis and conclude that $\hat{\beta}_k$ is significantly different from zero. If the computed t is less than the tabled t, we fail to reject the null hypothesis. In this case, the sample statistic, $\hat{\beta}_k$, is close enough to the hypothesized value, 0, that the difference between the two is considered to be due to chance and, hence, there is no reason to doubt the null hypothesis. The probability is small that the null hypothesis is false.

When reporting regression analyses in a table or other format, researchers will often include a computed t statistic in parentheses below each $\hat{\beta}$. This reported t is computed as shown in equation 2.33. That is, it is the t for testing the zero null hypothesis. Occasionally, rather than the computed t's, the standard errors of the estimated coefficients are inserted in parentheses below the coefficient estimates. Either way, each coefficient's "t" and standard error can be read directly or be easily computed. Also, values identified as "t statistics or values" in computer regression or econometric programs are computed using the ratio in equation 2.33. A common practice with computerized statistical packages is to report the probability of a greater t value if the null hypothesis is true. An advantage of this approach is that it avoids chosing an arbitrary critical level for t, allowing the reader to select the critical level.

It is well to remember that statistical tests provide a useful tool for inferences about population parameters but they should not be used dogmatically. For example, if a price variable in a demand equation has the correct sign but is statistically insignificant, the researcher may leave the variable in the equation because theory and previous estimates strongly support including the variable. Omitting the variable under these circumstances would be a specification error which will be discussed later.

On the other end of the spectrum, finding a variable to be significantly different from zero does not always mean that it is important. Statistical significance does not necessarily imply economic, political, or social significance (Schroeder, Sjoquist, and Stephan, p. 53). The strength of the relationship may be so small, even though statistically significant, that the variable is of little consequence. A one unit change in an x variable

which causes a dollar change in a dependent variable measured in billions of dollars may not be economically significant even if it is statistically significant. Computations of elasticities or standardized coefficients from regression results, which are described in a later section, may more nearly identify economic importance by striping away units of measurement through using proportional changes.

3. Use of t in Confidence Intervals. Although not often reported in applied regression work, computation of a *confidence interval about the true ß* illustrates the important fact that parameter estimation is subject to a margin of error, sometimes wide. The confidence interval (Lower Bound, Upper Bound) for $ß_k$ is computed as:

$$\text{Lower Bound} = \hat{ß}_k - s_{\hat{ß}_k} t_{\text{two-tail,T-K d.f.}}$$

$$\text{Upper Bound} = \hat{ß}_k + s_{\hat{ß}_k} t_{\text{two-tail,T-K d.f.}}$$

where $\hat{ß}_k$ is the estimated regression coefficient, $s_{\hat{ß}_k}$ is standard error of $\hat{ß}_k$ and $t_{\text{two-tail,T-K}}$ is the critical value of the t-statistic found in the two-tailed t-table for the selected significance level with T - K degrees of freedom.

A 95 percent confidence interval for, say, $ß_3$ means that if a large number of T-size samples from the population were drawn and a regression and confidence interval for $ß_3$ were computed for each sample, then about 95 percent of the intervals would contain the true population value for $ß_3$.

In computing the confidence interval for forecasts, a forecast value for y, y_F, is found by substituting forecast values for the regressors into equation 2.12, xk, k = 2, ... ,K (x1 always equals 1):

$$\hat{y}_F = \hat{ß}_1 + \hat{ß}_2 x_{F2} + \ldots + \hat{ß}_K x_{tFK}. \tag{2.34}$$

A confidence interval for y_F is given by:

$$\text{Lower Bound} = \hat{y}_F - s_{\hat{F}} t_{\text{two-tail,T-K d.f.}}$$

$$\text{Upper Bound} = \hat{y}_F + s_{\hat{F}} t_{\text{two-tail,T-K d.f.}}$$

where the t is defined as before and $s_{\hat{F}}$ is the estimated standard error of the forecast. $s_{\hat{F}}$ is defined as the *square root* of:

$$s^2 + s^2 X_F'(X'X)^{-1}X_F \tag{2.35}$$

where X_F is the K x 1 vector of forecast values for the regressors. The first term in this expression results from estimating (assuming) the error term as zero; the second term results from the use of OLS estimates rather than the true values (Kennedy, p. 211). Note

that the second term contains the estimated variance-covariance matrix of the estimated regressors, $s^2 (X'X)^{-1}$.

The interpretation is similar to that for confidence intervals for a ß: If a large number of T-size samples from the population were drawn, regressions were run for each sample and a 95 percent confidence interval for y_F were computed for each sample, then about 95 percent of the intervals would contain the actual value being forecast. The interval is smallest at the set of average x values used to estimate the regression parameters. The intervals become larger and larger as the values for the x variables are further and further from their average values used in the regression.

4. Use of F to Test Significance of all ß's Excluding Intercept. Probably the second most popular test of significance in regression analysis -- the t test to test a single population coefficient against zero is likely first -- is to determine if the joint effect of all nonintercept independent variables on the dependent variable is significantly different from zero. The null and alternative hypotheses are

$$\begin{aligned} &\text{Null Hyp.: } \beta_2 = \beta_3 = \ldots = \beta_K = 0 \\ &\text{Alt. Hyp.: Not the Null} \end{aligned} \tag{2.36}$$

As mentioned earlier, the t test statistic can only be used when the null hypothesis concerns only one population parameter or only one linear combination of population parameters. Hence, another test statistic must be used to complete the test in equation 2.36 because the null hypothesis includes statements about K - 1 population parameters: β_2 is hypothesized to be zero, β_3 is hypothesized to be zero and so on. It can be shown that the F distribution provides the appropriate test statistic when multiple coefficients are involved in the null hypothesis.

The test is based on unrestricted versus restricted models. The number of independent parts of the null hypothesis, such as K-1 in the current case, can be expressed as the number of restrictions imposed on the model by the null hypothesis. Viewed this way, the original or full model becomes the *unrestricted model.* The original model is unrestricted because coefficients are to be estimated by least squares with no constraints of any kind. A *restricted model* is the model that results when the null hypothesis is imposed on the original model.

For example, in (2.36) the hypothesis constrains all the coefficients, except the intercept, to be zero leaving the dependent variable as a function of only the intercept or for the tth observation $y_t = \beta_1 + u_t$. This regression can actually be run by simply regressing y on x_1 where x_1 is, as previously defined, a column of "1's." The estimate for β_1 in this situation turns out to be the mean of the dependent variable. Thus, each of the predicted values for y or $\hat{y}$'s is equal to the mean of original y variable. Other statistics for this restricted model can also be computed. Having the $\hat{y}$'s, the $\hat{u}$'s can be computed; then the residual or error sum of squares (ESS) can be calculated. Now suppose we identify this error sum of squares, from what we have called the *restricted model*, as $ESS_{Restricted}$. The

error sum of squares of the original or *unrestricted model*, can be identified as $ESS_{Unrestricted}$. This is the ESS discussed earlier and that appears in the ANOV table.

In computation of the F statistic, the general form is

$$F = \frac{(ESS_{Restricted} - ESS_{Unrestricted}) / \text{No. of restrictions in null hyp.}}{ESS_{Unrestricted} / \text{d.f. of the } ESS_{Unrestricted}} \qquad (2.37)$$

Basically the F test as used in regression analysis compares the relative magnitude of the two error sum of squares — the ESS for the restricted model against the ESS for the unrestricted model.

This comparison of ESS's makes intuitive sense. For example, suppose the null hypothesis -- that all the nonintercept population coefficients are jointly zero in this particular application -- is true. Further suppose, for argument's sake, that the regression estimating the unrestricted or full model results in exactly zero coefficient estimates for the nonintercept variables. Hence, with this situation, the $\hat{y}$'s, the residuals errors or $\hat{u}$'s, and the error sum of squares of this unrestricted model reflect the situation in which the nonintercept variables provide no explanation of the dependent variable.

Now consider the restricted model. The dependent variable is regressed only on the intercept term which, by omitting the other independent variables, reflects the restrictions imposed by the null hypothesis. Of course, the regression statistics including the error sum of squares reflect no explanation of the dependent variable by the nonintercept regressors. But in this case, because the unrestricted model also resulted in no explanation by these regressors, there was no penalty for using the restricted model. The estimated coefficients are the same, the error sum of squares are the same and hence the computed F calculated using equation 2.37 is zero. Thus, the degree of correspondence between the set of hypothesized ß's and the estimated values from the regression is gauged by the difference in the error sum of squares of the restricted and unrestricted models. Ever increasing differences in the hypothesized values for the parameters and the estimated parameters result in ever larger differences in the error sum of squares of the restricted and unrestricted models.

The F test provides a means to determine if the error sum of squares of the restricted model, which represents the statements of the null hypothesis, is significantly different from the sum of squares of the unrestricted model -- the model in which the *error sum of squares is minimized* without constraints of any kind. The *difference* in the restricted and unrestricted error sum of squares is measured *relative to* the error sum of squares of the unrestricted model. Both the difference in the numerator and the unrestricted error sum of squares in the denominator are divided by their respective degrees of freedom. The d.f. in the numerator takes into account the number of restrictions imposed on the model by the null hypothesis and the d.f. in the denominator takes into account the number of observations and independent variables used to estimate the unrestricted error sum of squares.

All F tests in regression analysis can be posed using unrestricted and restricted models and their corresponding error sum of squares to compute the F value with equation

2.37. However, it is possible to use the TSS = RSS + ESS equation and the definition of R^2 to derive other equivalent representations of the F test statistic for specific applications. Also, note that the denominator of equation 2.37 is s^2 for the unrestricted model. It can be shown, for example, that for this particular null hypothesis, equation 2.37 is equivalent to the following two formulations:

$$F = \frac{RSS/(K-1)}{s^2} = \frac{R^2/(K-1)}{(1-R^2)\ /\ (T-K)} . \tag{2.38}$$

As with the t test, the critical value for the test depends on the selected significance level and the degrees of freedom. Unlike the t, the distribution of the F is a function of two sets of degrees of freedom, one from the numerator and one from the denominator. Thus, the critical value is determined by finding the intersection of the two sets of degrees of freedom -- working with the numerator's d.f. along the top and the denominator's d.f. along the left side -- in a F table for the selected significance level. The conclusion from the test follows the same pattern as with the t test: The null hypothesis is rejected if the computed F is greater than the tabled F, and we fail to reject if the computed F is less than the tabled F.

Thus, if $ESS_{Restricted}$ is sufficiently close to $ESS_{Unrestricted}$ that the difference between the two can be considered to be due to chance, we have no basis for doubting the null hypothesis that the nonintercept ß's are jointly equal to zero and so we fail to reject. By failing to reject the null hypothesis, we are saying that there is *no statistical evidence* that independent variables taken together (exclusive of the intercept) aid in explaining the variation of the dependent variable. If the difference between $ESS_{Restricted}$ and $ESS_{Unrestricted}$ is so large that we cannot attribute the difference to randomness of sampling, we conclude that the difference is because the null hypothesis is false. That is, the coefficients are jointly significantly different from zero. By rejecting the null hypothesis, we are saying that the nonintercept independent regressors, taken together, do significantly aid in the explanation of the dependent variable.

5. ***Using F to Test a Subset of Coefficients.*** Another common procedure in regression analysis is to test if a subset of the K ß's is significantly different from zero. For notational convenience, suppose the last K-H ß's are to be jointly tested against zero. The null and alternative hypotheses are

$$\begin{aligned} &\text{Null Hyp.: } \beta_{H+1} = \beta_{H+2} = \ldots \beta_{K-1} = \beta_K = 0 \\ &\text{Alt. Hyp.: Not the Null.} \end{aligned} \tag{2.39}$$

The keys to using the F to test this hypothesis (and most hypotheses) are to determine (1) the unrestricted model, (2) the restricted model, and (3) the d.f. used as divisors in equation 2.37. After working through these preliminaries, completing the test is routine.

As always, the *unrestricted model* is the specification of the model that is not constrained by statements in the null hypothesis. Hence, the unrestricted model is

$$y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + \ldots + \beta_K x_{tK} + u_t. \tag{2.40}$$

The *restricted model* is the model that results when the constraints given by the null hypothesis are incorporated into the original model. In this case the constraints are that the last K - H ß's are zero. Those restrictions amount to omitting the last K - H independent variables. Hence, the resulting restricted model does not have K independent variables but H (K - H fewer) regressors

$$y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + \ldots + \beta_H x_{tH} + u_t . \tag{2.41}$$

For example, suppose the original model had five independent variables and the null hypothesis is that β_2 and β_4 are each and simultaneously equal to zero. The restricted model would contain the intercept, x_3, and x_5 while the unrestricted model would include all five independent variables.

The numerator *degrees of freedom* is the number of restrictions imposed on the original model by the null hypothesis. In the current application, K - H coefficients are restricted to zero in the null hypothesis. Hence, the degrees for freedom of the numerator of equation 2.37 is K - H. In our five regressor example, two coefficients were set to zero in the null hypothesis so the numerator d.f. would be 2.

The degrees of freedom of the denominator of the F is the degrees of freedom of the error sum of squares of the *unrestricted* model. Hence, the degrees of freedom for the denominator is T - K. If 30 observations were used to estimate the unrestricted model in the five regressor example, the d.f. for the denominator would be 25.

After using OLS to obtain the ESS estimates for the unrestricted and restricted models and completing calculations of equation 2.37, the standard procedure is followed: Find the critical value for the F with K - H and T- K degrees of freedom and the selected significance level, compare the critical value with the computed F, then reject the null if the computed F exceeds the tabled F and fail to reject otherwise.

Significance Test Summary

Examples have been given of the most common types of significance tests used in regression analysis. Many other significance tests can be performed but they all follow the same general format and the logic of the procedure is the same. Hence, it might be useful to briefly summarize the general procedure for completing tests of significance.

The central idea with significance testing is to compare the hypothesized parameter value with the estimate for the parameter in the case of the t, or compare the two error sum of squares in the case of the F -- in light of the amount of variation which can be expected on the basis of chance factors. The sampling distribution, t or F, gives the amount and "shape" of the variation expected on the basis of chance. The steps to properly use a test of significance are:

1. meeting the assumptions of the statistical technique being applied, in our case, meeting the list of seven assumptions discussed earlier required to use OLS;

2. knowing or deriving the appropriate test statistic formula and knowing its distribution (t or F);
3. calculating the test statistic; and
4. comparing the computed test statistic with the sampling distribution to determine if it is reasonable to attribute differences to sampling error:
   (a) *If the absolute value of the computed test statistic is less than the critical value* given in the table for the sampling distribution (with the appropriate d.f. and selected significance level), we fail to reject the null hypothesis. That is, the estimate of the parameter is close enough to the hypothesized parameter value (or the two ESS's are sufficiently close in case of F) that *the difference is considered to be due to chance factors* in the sampling process. Hence, there are no grounds for doubting the null hypothesis;
   (b) *If the absolute value of the computed test statistic is larger than the critical value*, the difference in the hypothesized value and estimated parameter or ESS's is so great that *the difference is unlikely to be accounted for by chance factors* from sampling. Because the discrepancy is too large to be attributed to chance, we conclude that the null hypothesis is false.

Interpreting Regression Results

Although numerous formats are used for presenting regression results, a common format is

$$\hat{y}_t = \underset{}{511.20} + \underset{(5.32)}{0.0435x_{2t}} + \underset{(2.65)}{305.64x_{3t}} \qquad (2.42)$$

$$R^2 = 0.85 \quad \bar{y} = 4140.67 \quad ESS = 45.94 \quad T = 73 \quad DW = 1.75 .$$

The intercept value estimates the expected or average value of y when x_2 through x_K are zero. In this case, the predicted value for y is 511.20 *when x_2 and x_3 are zero.* The estimates for the remaining ß's are usually of more interest. Each coefficient estimate (say $\hat{ß}_2$) measures the expected change on y from a *one* unit change in the corresponding independent variable (x_2) *while holding the other independent variables constant.*

Suppose the dependent variable in equation 2.42 is dollars of family expenditures on food and x_2 is dollars of family income and x_3 is family size. Thus, *holding income constant*, average food expenditures are expected to increase by $305.64 when family size increases by one. A one dollar change in income with no change in family size is associated with a 4.35 cent change in food expenditures.

The numbers in parentheses are computed t statistics to test the null hypothesis that the corresponding ß equals 0. The t value of 5.32 in parentheses under the estimated coefficient for x_2 in equation 2.42 can be used to test $ß_2 = 0$. The DW, or Durbin-Watson statistic, is used to gauge autocorrelation of the disturbances and is discussed in a later

section. Note that by giving the R^2 and error sum of squares values, the total sum of squares (TSS) and regression sum of squares (RSS) can be computed using the formulas for R^2.

Regression results can be used to compute *elasticities* for any nonintercept independent variable. Using regression terminology an elasticity is the percentage change in the dependent variable associated with a one percent change in the independent variable holding the values of all other independent variables constant. Because percentage changes are used, elasticities are unitless. The elasticity of y with respect to x_2 measured at the tth observation is

$$\text{Elast } y, x_2 = \frac{\% \Delta y_t}{\% \Delta x_{2t}} = \frac{\dfrac{\Delta y_t}{y_t}}{\dfrac{\Delta x_{2t}}{x_{2t}}} = \frac{\Delta y_t}{\Delta x_{2t}} \frac{x_{2t}}{y_t} \tag{2.43}$$

where Δ represents change. By letting Δ in x_2 be one, note that $\frac{\Delta y_t}{\Delta x_{2t}} = \hat{\beta}_2$. Thus

$$\text{Elast}_{y,x_2} = \hat{\beta}_2 \frac{x_{2t}}{y_t} . \tag{2.44}$$

Computed at the means of the dependent and the independent variable the elasticity is:

$$\text{Elast}_{y,x_2} = \hat{\beta}_2 \frac{\bar{x}_2}{\bar{y}} \tag{2.45}$$

Hence, the estimated coefficient for income (x_2), 0.0435, from equation 2.42 and the mean of the food expenditures (y), 4,140.67, would be used to compute the elasticity of food expenditures with respect to income but the mean of the income variable would also be needed. If the mean of the income variable were 11,422.54, the elasticity would be

$$\text{Elast}_{y,x_2} = .0435 \frac{11,422.54}{4,140.62} = 0.12 . \tag{2.46}$$

Each 1 percent increase in income is associated with a .12 percent increase in food expenditures, everything else is held the same.

Occasionally, elasticities will be reported with the regression results. More often, the elasticities are discussed in the text portion of the article or displayed in a separate table.

Standardized regression coefficients frequently are used like elasticities to indicate the relative impact of an independent variable on the dependent variable. The procedure is the same as for computing the elasticity except the standard deviation for x is substituted for the mean of x and the standard deviation of y is substituted for the mean of y in equation

2.45. If s_{x2} / s_y is 3.2, each standard deviation change in income is associated with a .14 (.0435 x 3.2) standard deviation increase in food expenditures.

Alternative Specifications of the Regression Model

Thus far, the regression model has been assumed to be linear in the variables as well as in the parameters and the data on the variables have been assumed to be continuous. As already pointed out, regression analysis requires linearity of parameters but variables may be nonlinear. The next sections review a few of the most common nonlinear functions and briefly discuss the use of variables that only take on values of 1 or 0.

1. Functional Forms. The usual regression model in which y is regressed directly on x_1 through x_K results in constant slope values for all observations but the elasticities vary across observations. As noted, means of the y and x variables are often used to compute "representative" elasticities. Other functional forms imply other assumptions about regression parameters. Some of the most often used functional forms are reviewed. Goldberger (pp. 213 - 216) and Berry and Feldman (pp. 57-67) provide more detail.

The double-log form, a common alternative formulation, specifies that the elasticities rather than the slopes are constant. The exponential or "Cobb-Douglas" functional form provides this property:

$$y = e^{\beta 1} x_2^{\beta 2} x_3^{\beta 3} \ldots x_K^{\beta K} e^u \tag{2.47}$$

where e is the base of the natural logarithms. By taking logarithms of the entire equation, the relationship can be transformed to an equation that is linear in the *parameters*:

$$\ln y = \beta_1 + \beta_2 \ln x_2 + \beta_3 \ln x_3 + \ldots + \beta_K \ln x_K + u \,. \tag{2.48}$$

Because in this case $\beta_k = \Delta(\ln y)/\Delta(\ln x_k)$ and since $\Delta(\ln y) \approx \Delta y/y$ and $\Delta(\ln x_k) \approx \Delta x_k/x_k$, the β's directly turn out to be the elasticities and those elasticities remain constant over all observations. Hence, the estimate for β_3 indicates the percentage change in y for a 1 percent change in x_3.

Researchers will sometimes specify equation 2.47 as

$$y = \beta_0 x_2^{\beta 2} x_3^{\beta 2} \ldots x_K^{\beta K} e^u. \tag{2.49}$$

This equation would be estimated as

$$\ln y = \ln \beta_0 + \beta_2 \ln x_2 + \ldots + \beta_K \ln x_K + u \,. \tag{2.50}$$

The interpretation for the estimated relationship is the same except for the intercept which must be transformed by taking its antilog, i.e. $\beta_1 = \ln \beta_0$.

In a *semi-log form* the regressors may include the logarithmic transformation of one or more of the independent variables. This nonlinear function may be appropriate when the elasticity is thought to decrease in absolute value as y increases. In this case neither the slope nor the elasticity remains constant for changes in the independent variable. For example given

$$y = \beta_1 + \beta_2 \ln x_2 + \beta_3 x_3 + \ldots + \beta_K x_K + u \tag{2.51}$$

β_3 through β_K are interpreted as in the usual linear form. For the second independent variable, the slope ($\Delta y/\Delta x_2$) is computed as β_2/x_2 and the elasticity of y with respect to x_2 is computed as β_2/y.

Another semi-log function allows estimating a relationship in which the researcher believes that a one unit change in an independent variable causes y to change by a constant percentage. The equation is in the form

$$y = e^{\beta_1 + \beta_2 x_2 + \ldots + \beta_K x_K + u} \tag{2.52}$$

and the estimation equation is

$$\ln y = \beta_1 + \beta_2 x_2 + \ldots + \beta_K x_K + u\,. \tag{2.53}$$

The slope for the kth independent variable is $\beta_k y$ and the elasticity is $\beta_k x_k$. Hence, the slope increases with y and if x_k changes by 1 unit, y will change by β_k percent.

Another method of handling nonlinear relationships is to allow the slopes to depend on the level of the independent variable itself. For example, a second-degree *polynomial form* in x_3 would be

$$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_3^2 + u \tag{2.54}$$

The slope for x_2 is unchanged from the strictly linear model but the slope of x_3 is $\beta_3 + 2\beta_4 x_3$. U-shaped curves are often fit with second-degree polynomials.

Including *interaction terms* for independent variables allows the influence of say x_2 on y to be influenced by the level of x_3. For example, the influence of nitrogen fertilizer on wheat yields may be different depending on the amount of phosphorus that is applied. Thus, a model might appear as

$$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_2 x_3 + u\,. \tag{2.55}$$

The slope for x_2 becomes $\beta_2 + \beta_4 x_3$ and for x_3 becomes $\beta_3 + \beta_4 x_2$. In working with crop production functions, it is not uncommon to have interactions among the major fertilizer nutrients (as well as the squares of the nutrient variables) as part of the regressor set.

2. Dummy Variables. The independent variables have been implicitly assumed to be continuous and thus each can take on an infinite number of values. However, some

variables defy such quantification and can only be represented by binary or dummy variables.

Dummy variables can be used any time the set of observations can be separated into classes. For example, those who smoke and those who do not; men and women; under 30 and over 30, Eastern, Western, Southern, and Northern regions of the country; and B.S., M.S., and Ph.D. are all ways of classifying observations. A dummy variable has only two possible values, 0 and 1. It is assigned to be 1 if the attribute is present in the observation case and zero if it is not.

For example, an observation on income from a female would cause the researcher to insert a 1 for the corresponding observation for the female dummy variable and a zero for the male dummy variable and vice versa for an observation on a male. Since the intercept is always available to represent one of the categories in the classification system, the number of dummy variables actually included in the regression is *always* one less than the number of classes of data. Thus, if female and male dummy variables were initially constructed, only one, say the female dummy variable, would be included in the regression. The intercept includes the relationship for the male. The coefficient on the female dummy variable represents the difference between male and female (or *net affect* of being female on the average value of the dependent variable). Thus, the intercept for the female case is found by *adding* the coefficient on the female dummy variable to the intercept (which represents the constant for the male).

If more than one set of classifications is used in a regression problem, one dummy variable from each classification must be dropped. For example, if smokers and nonsmokers as well as female and male classifications are to be used, then, in addition to dropping one dummy from the male-female classification, one of the (two) dummy variables from the smoker-nonsmoker classification must also be dropped:

$$\hat{y} = \hat{\beta}_1 + \hat{\beta}_2 \text{female} + \hat{\beta}_3 \text{nosmoke} + \text{continuous variables, etc.} \qquad (2.56)$$

By choosing to drop the smoker dummy, the intercept applies to a male who smokes. The coefficient on the non-smoker variable measures the net effect of not smoking. To find the intercept value for a women who does not smoke, the coefficients for the female and non-smoker dummy variables, $\hat{\beta}_2$ and $\hat{\beta}_3$, (the net effects of being female and not smoking) would be added to the intercept as originally estimated, $\hat{\beta}_1$.

The usual significance tests can be applied -- t tests for a single coefficient (is β_2 significantly different from zero?) and an F for a test involving more than one parameter (are β_2 and β_3, taken together, significantly different from zero?).

One of the most common applications of dummy variables in economic analysis is to account for seasonality. If in addition to the usual causal variables such as price and income, demand for a product, say turkey, is thought to vary by quarter of the year, a set of (three or would it be four?) quarterly dummies could be included (that's right, three). The interpretation is the same as before: Each estimated quarterly dummy coefficient adjusts the *intercept* up or down from its "standardized" level (the level appropriate for the

quarter with no dummy variable). The significance level will vary depending on which quarter has no dummy variable, and hence is part of the intercept.

So far only adjustments in intercepts have been discussed. It is also possible to account for differences in the slope of, say, the price variable in a demand equation. The slope for a continuous variable is allowed to change by class of data by creating interaction terms between the dummy variables and the continuous variable. Thus, each of the three quarterly dummies would be multiplied times the price variable to allow differing price slopes by quarter.

If the winter dummy was initially dropped, the coefficient on the price variable would represent the slope for the winter quarter. The slope for the spring quarter would be the slope on the price variable (winter quarter slope) *plus* the coefficient on the spring-quarter-dummy-*times*-price regressor. A single regression equation could include dummy variables to allow intercept changes and interactions of the dummy variables with one *or more* continuous variables to pick up slope changes from one class of data to another.

Violation of Assumptions and Other Pitfalls in Regression

The computations, desirable properties, and tests associated with regression analysis all presuppose that (1) the correct model equation is being used, (2) the variables are measured accurately, and (3) the seven assumptions presented earlier are satisfied. Violation of any of the preconditions can cause problems. This section reviews some of the most common difficulties that can arise while using regression. In each case the violation will be *described*, the *consequences* of ignoring it on regression results will be identified, and, as appropriate, procedures on *how to deal with the problem* will be mentioned.

1. Specification Error. Specification error means the "wrong" model was estimated. The model may not contain the correct variables or have the correct functional form. The regression technique cannot discern a well specified model from a misspecified model. The analyst must rely on theory combined with previous research and his/her knowledge about the phenomenon being investigated.

Omitting a relevant independent variable causes the OLS estimates of the remaining coefficients to be biased. The only exception is if the observations of the omitted variable(s) are uncorrelated in the sample with the observations of the other independent variables. This so called *orthogonal* relationship between the omitted and other variables, which is rare in the social sciences, allows the slope coefficients to be unbiased but the intercept will generally be biased. In any case the coefficient standard errors are biased upward.

The inclusion of an irrelevant independent variable does not affect the unbiasedness properties of the OLS ß estimates or their standard errors. If the irrelevant variable is orthogonal to the other variables, the ß estimates and standard errors are unaffected. The assumption of orthogonality rarely holds, however.

The consequences of using the wrong functional form, i.e., omitting the square of a variable or neglecting to take logarithms when the true function requires it, are identical to omitting a relevant variable.

Economic theory, knowledge of the literature, introspection, and experience are the classical ways to guard against misspecification. Kennedy (pp. 69-72) reviews some techniques that have been suggested for selecting/testing for omitted independent variables and erroneous functional forms. None is very robust or much used.

2. *Measurement Error.* Measurement error, sometimes referred to as errors in variables, refers to inaccurate observation data for one or more variables. The cause could be inaccurate recording of observations , improper data collection techniques, or inaccurate responses to surveys or to the use of proxy variables to represent variables that have not or cannot be measured.

Errors in measuring the dependent variable are lumped into the disturbance term and cause no problems if randomly distributed. Measurement errors in the independent variable(s) cause the regression coefficients to be biased because the independent variables are no longer uncorrelated with the disturbance term. As specified with the correctly measured regressors, the model has a disturbance term that is uncorrelated with the independent variables. Replacing one of these variables with an improperly measured replacement creates a new disturbance term which includes the error component of the regressor. Thus, the error shows up in both the measured independent variable and the disturbance. Correlation of the two is now unavoidable, resulting in biased ß estimates.

Instrumental variables can be used in situations such as this where an independent variable is contemporaneously correlated with the disturbance. It involves replacing the independent variable with another variable (its instrument) which is highly correlated with the original independent variable and is uncorrelated with the disturbance term. An alternative formula which uses the original variables and instruments, called the instrumental estimator, is used to estimate the regression coefficients (Kmenta, pp. 308-315; Maddala, 296- 300). Because the procedure includes a type of proxy variable, the estimators are still biased. However, the estimates do converge to the true population parameters as sample size becomes very large, i.e., they are consistent.

Numerous problems attend using instrumental estimators. It is difficult to find a variable with the desired qualities -- correlated with the independent variable and uncorrelated with the disturbance. If one is found, it is not known if other more efficient variables could be found. It is difficult to determine if the instrument is uncorrelated with the disturbance. Because the instrument may have a low correlation with the independent variable, the coefficient standard errors may be excessively high. This forces the analyst to choose between consistency of the instrumental variable technique and lower coefficient variability of OLS. Typically, in the case of errors in variables, researchers will use OLS as "the least of two evils," but good research methodology requires informing the reader about suspected measurement errors and the implications.

3. Autocorrelated Disturbances. Autocorrelated disturbances mean that the disturbance for the current time period (observation number) is correlated with disturbances for one or more previous time periods. This is a direct violation of assumption three. Autocorrelated disturbances, which are more prevalent with time series data than cross-section data, can occur due to shocks that linger beyond one measurement period. A strike, export embargo, weather cycle, and other "random" events may not only affect the disturbance term of the current period but may also affect future disturbances. The shorter the period of observation the more likely disturbances are autocorrelated. Sometimes due to inertia, past actions influence current actions so that a disturbance in one period might be acted upon in another period. Some natural causes of autocorrelated disturbances can be addressed with procedures discussed later. The appearance of "autocorrelated disturbances" can be caused by omitting relevant variables, failure to use distributed lag forms, use of an inappropriate functional form, and data manipulation. But the culprit here is misspecification and should be addressed as such.

A plot of the residuals against observation number is often helpful in identifying possible autocorrelation of disturbances. A string of positive residuals followed by a string of negative residuals and so on is a strong indication of positive autocorrelation. A pattern characterized by individual residuals that alternate between positive and negative suggests negative autocorrelation. The most common formal test for detecting autocorrelated disturbances is the Durbin-Watson d statistic (Johnston, pp. 313-321; Kmenta pp. 294–297). It is calculated from the residual of the OLS regression and tests for first-order autocorrelation. The d statistic is 2.0 in the absence of autocorrelation. As the statistic moves closer to 0 below 2 and closer to 4 above 2, the less confident we can be that autocorrelation does not exist. Tabled values for the d statistic are available but unfortunately the exact distribution is unknown. That causes complications in its use that are beyond our discussion here. One approach is to consider remedies if the Durbin-Watson d is below 1.25 and above 2.75. Most computer packages routinely print the d statistic.

With autocorrelation the OLS estimates remain unbiased but are no longer the minimum variance estimators (Goldberger pp. 238-243; Johnston, pp. 310-313). If the parameter representing the degree of autocorrelation is known, then an alternative procedure called generalized least squares provides minimum variance unbiased estimates for the ß's. Because the OLS estimated variances and thus standard errors are biased when disturbances are autocorrelated, t and F tests are also invalidated. With positive autocorrelation, which is the most prevalent form, the *standard errors* of the estimated coefficients are often *underestimated.* Hence, *t tests will be too large* which could cause coefficients to be found significantly different from zero when in fact they are not.

The first thing to do when observing a d statistic below, say, 1.5 or above 2.5, is to convince yourself that the model is specified correctly. Any misspecification problems should be remedied before looking for a technique to address autocorrelated disturbances. If the disturbances are autocorrelated due to lingering affects, procedures are available which transform the dependent and independent variables using an estimate of the autocorrelation parameter (Johnston, pp. 323-325; Kmenta, pp 287-289). The Cochrane-

Orcutt procedure is the best known and is available with most econometric regression packages. Because the autocorrelation parameter is not known but must be estimated, the ß estimators turn out not to be unbiased but only consistent.

4. Heteroskedastic Disturbances. Assumption number two says the disturbances must have uniform variance which is also known as *homoskedastic* disturbances. If the disturbance variances are not constant, the disturbances are said to be *heteroskedastic*. The problem is more frequent in cross-sectional data than in time series data. Frequently disturbance variance varies with one of the independent variables. For example, if consumption is a function of income, the larger the income the larger may be the associated disturbance. Having satisfied the basic needs of life, consumers with large incomes may have very disparate spending habits. Some may spend it all, others may save a large percentage. Those with small incomes have no choice but to spend most of their income. Hence, disturbance variance may increase with income in such a relationship.
As with autocorrelated disturbances, visual inspection of the OLS residuals can be very helpful. In the case of heteroskedastic disturbances, the residuals are plotted against the variable that may be related to the disturbance variance. If the dispersion of points is comparable from small to large values for the independent variable, probably no heteroskedasticity exits. If the dispersion spreads out or narrows with changes in independent variables, heteroskedasticity is likely.

The consequences of using OLS in the presence of heteroskedastic disturbances are the same as using OLS in the presence of autocorrelated disturbances. The regression estimates are unbiased but inefficient. Heteroskedasticity affects the size of the standard errors and thus also affects hypothesis test results. The direction of bias of the standard errors depends on the cause and pattern of the heteroskedastic disturbances.

The first approach to heteroskedasticity is to make sure the specification is appropriate. For example, a double-logarithm function with proportional error may pose less difficulty than a linear equation with additive error. A form of generalized least squares can be used to overcome heteroskedasticity. A function of the independent variable that varies with disturbance variance is used to give differential weights to the observations which equalizes the disturbance variances. Here again, the ß estimates remain biased but converge to the true population parameters as sample size increases.

5. Multicollinearity. Multicollinearity occurs when two or more independent variables are highly correlated with one another. Multicollinearity is quite common in estimation of economic relationships, especially when using time series data, because many economic variables tend to move together. Multicollinearity is a data problem and is usually a matter of degree. Most regression models in the social sciences exhibit multicollinearity because it is extremely unlikely that the independent variables would be perfectly uncorrelated. Whether or not multicollinearity is a problem depends on its severity. In the extreme case where two regressor variables are perfectly correlated, it is impossible to estimate two coefficients from what is essentially one independent variable.

With multicollinearity, the OLS estimates remain unbiased. In fact, all desirable properties of the OLS estimates are retained even though in practice multicollinearity is one of the most pressing problems associated with using regression. High multicollinearity causes imprecise and unstable coefficient estimates and large coefficient standard errors. Adding another variable may cause coefficients to change markedly. When two or more variables are highly correlated, OLS has difficulty separating out the independent effects of each regressor on the dependent variable. Taken together the independent variables may explain a large proportion of the variation in the dependent variable, but individually the coefficients could have large standard errors and extremely low t statistics. Thus, an estimated relationship with a high R^2 *and* no significant regression coefficient probably is plagued with high multicollinearity. Simple correlations among pairs of independent variables and regressions of each independent variable on the other independent variables will often uncover the sources of the multicollinearity.

If the major use of the estimated relationship is forecasting and the same linear relationships among independent variables are likely to continue into the future, the estimated equation may be used to predict even though it was estimated in the presence of extreme multicollinearity. On the other hand, if the purpose of the analysis is to estimate parameters, alternative remedies should be considered. The problem is that most suggestions for reducing multicollinearity may not be feasible for the problem at hand or cause misspecification of the model. Suggestions include increasing sample size, obtaining another sample, explicitly specifying a linear relationship among regressors (or parameters), such as a weighted average of past prices in a supply equation, incorporating information from another study, or dropping one or more variables (Kmenta, pp. 380-391; Maddala, pp. 183-201).

*6. **Simultaneous Equations.*** Single equation multiple regression models assume a one-way causation -- the x's explain y. In many situations the dependency runs both ways. One of the x's may be dependent on y. To remedy this situation, a second equation is added to the original and the two equations become a simultaneous system of equations.

Using OLS to estimate equations from a system of simultaneous equations results in biased estimates and the bias remains as sample size increases. The difficulty is called *simultaneous equation bias.*

Numerous techniques are available for estimating coefficients of a simultaneous model. The most popular are two-stage least squares and three-stage least squares, both of which are readily available in many computer regression packages. Prior to applying an estimation technique, it must be determined if it is mathematically possible to calculate unique values for each of the model's parameters (Kmenta, pp. 531-585).

ACTIVITIES

1. Write the regression model using a) matrix notation, b) vector notation, and c) summation notation.

2. Write out the elements of the y, X, β, and u of the matrix representation of the regression model. Include the dimensions of each vector or matrix {let T = number of observations and K = number of independent variables (including a column of ones for the intercept)}.

3. What assumptions are usually associated with OLS?

4. OLS estimators for the population β's are said to be unbiased and efficient. Comment on the following statement: "Since OLS estimates are unbiased the estimates for a single regression are equal to the true population estimates."

5. Using the data below:

   Identify a) the y vector, b) the X matrix, what must be added if x_1 is to represent the constant term, c) what is done when the X matrix is transposed, and d) using matrix notation, give the formulas for computing $\hat{\beta}$, $\hat{y}$, and $\hat{u}$.

Time Period	Milk Consumption Per Capita	Retail Price of Milk	Medium Family Income	Population Density
	(lbs.)	($/cwt)	(0005)	(People/Sq. km)
1	270	21.52	11.10	77
2	269	22.63	12.40	29
3	228	25.93	10.54	130
4	233	26.33	9.68	115
5	286	21.22	12.00	178
6	296	25.16	11.22	65
7	264	25.31	12.40	41
8	286	23.70	14.10	226
9	236	26.40	11.05	13
10	240	22.08	9.44	182
11	241	25.45	10.72	320
12	286	20.95	12.79	68
13	292	21.84	15.66	284
14	228	26.76	9.13	234
15	259	25.25	11.15	12
16	219	26.86	9.96	77
17	326	22.95	13.89	20
18	271	25.78	10.31	26
19	228	27.24	8.71	55
20	300	23.24	13.11	499

6. a) Enter the y vector and X matrix (including the extra column of "1's") into a spreadsheet program, b) using commands of the spreadsheet program compute X', $X'X$, $(X'X)^{-1}$, $X'y$ $\hat{\beta}$, $\hat{y}$, and $\hat{u}$.

7. Now perform this same regression using the regression command of the spreadsheet program. (What are the two ways to handle the computation of the constant term? Which did you use?)

8. Based on information from the regression output, test the hypothesis that coefficient on the price variable is equal to zero. Fully explain the result of the test.

9. Test the hypothesis that all the nonintercept coefficients are equal to zero.

10. a) What does R^2 mean; how is it computed?; b) interpret the meaning of the estimated coefficient in the retail milk price variable; c) compute the elasticity of per capita milk consumption with respect to the retail price of milk with all variables measured at their means; d) explain the purpose and applications of the Durbin-Watson d statistic.

11. Use micro TSP or another regression package to redo the above regression.

12. Give an example of how you might apply dummy variables in a regression model.

13. What is meant by autocorrelated disturbances? Is an OLS assumption violated? If yes, which one? How can autocorrelated disturbances be detected? What remedies are available?

14. Answer the questions in activity 13 for a) heteroskedastic disturbances, b) multicollinearity, and c) simultaneous equations.

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3. Using Microcomputers for Policy Analysis

by Elton Li and Arthur L. Stoecker

The microcomputer revolution which is transforming the professional work-place on a global scale grew out of efforts by hobbyists and amateurs to utilize the microprocessors developed during the early 1970's. Policy analysts are among the segments of society who can greatly benefit from microcomputer technology, a technology which is becoming commonly available in both the developing and developed world. The modern microcomputer can place powerful analytical capacities within the hands of analysts and technicians in both developing and developed countries. However, like other technological innovations it has its costs as well as benefits. Some of these costs are associated with training needs of the user population which has exploded to include novices and the non-technical users.

OBJECTIVES

The general objective of this chapter is to discuss the potential for using microcomputers in agricultural policy analysis. Specific objectives are to discuss the role of microcomputers in the professional work-place for policy analysts, define some of the commonly used terminologies, and discuss the major types of software useful for agricultural policy analysis in both developed and developing countries. Since the microcomputer field is undergoing rapid evolution, the chapter will focus on what are believed to long term concepts rather than the current technology.

KEY POINTS

1. Policy analysis involves problem identification, analyses of policy alternatives, and presentation of results to policy makers. The use of a microcomputer system can facilitate this process.

2. Modern microcomputer hardware is comparable in capacity to many mainframe computers in the past but at greatly reduced cost. This means that lack of computing power is no longer the constraint which limits more through analysis of alternative policies.

3. Modern microcomputer software is designed to be used by non-computer experts. A novice can achieve meaningful results with an electronic spreadsheet only after a few hours of instruction.

4. The microcomputer can facilitate the building of more complex policy models and reduce the effort required to maintain and update these models.

5. Useful microcomputer software for policy analysis includes spreadsheets, database management systems, computer languages, statistical packages, project management software, word processing programs, and business graphics.

CONCEPTS AND APPLICATIONS

The Policy Analysis Process

Inadequate capacity to perform an adequate analysis before a policy has been adopted and implemented has often resulted in agricultural policies falling short of serving the needs of decision makers and/or society at large. The policy analysis process requires the identification of a problem or problems, the analysis of alternative solutions, and the presentation of these results to policy makers. The mechanics of the policy analysis process are not unlike those of simple report writing. Specific tasks include collection of information, the organization and distillation of key points from the data, and report preparation. The last step involves producing a narrative text with accompanying tables and graphs which is edited and redrafted to produce an acceptable final copy. A final executive summary presented to a busy policy maker may not exceed two pages in length.

The major developments in microcomputer hardware and software are in the areas of integrating the collection of information, the analysis of the data, and the publishing of reports. The microcomputer allows the analysts to consider more policy alternatives because the process of data tabulation and report writing can be quickly and readily revised. The microcomputer is a relatively low cost means to store and promptly retrieve data, perform numerical computations, and enhance the process of transforming conceptual ideas to functional procedures, and deliver serviceable laser printed results to decision makers.

Preconditions for Policy Analysis. An agency's capacity to perform analysis requires at least three preconditions: availability of supporting databases, conceptual capacity, and logistical capability.

Supporting Database. A well organized database can provide a numerical history of past events in the agricultural and non-agricultural economy. The organization of the data must be such that data are readily available, reliable, and can be easily updated and

expanded. Once the database has been defined, the data can be transferred to the database by typing from printed or handcoded information, by diskette, by electronic network and/or by telecommunication linkages. The process of entering and checking the data for accuracy is time consuming and expensive but is necessary if any confidence is to be placed on the data. If the microcomputer system is to be used as an electronic filing cabinet, then steps must be taken to insure the data are organized, consistent, non-redundant, accurate and up-to-date and are capable of being shared for different applications in an agency and by different agencies. Electronic storage of information is low in cost and requires little physical space. It allows quick retrieval of the relevant facts and easy preparation of necessary data in a format suitable for further analysis. The difference between raw information and information in a well organized data base is not unlike the relation between a lowgrade ore and the refined metal: much processing is required to convert the former into the latter.

The data base entry and maintenance process is largely clerical and thus requires one of the less technically trained components of the work force to interact with the computer. Yet Danziger and Kraemer (1986) in a long term study of computer use in government reported that most data entry personnel had little trouble but that success was higher where adequate support was provided. The faster operating speeds, larger hard disk units, newer operating systems, and the emergence of optical disks will remove many of the present limitations of using microcomputers to handle larger data sets.

Conceptual Capacity. Conceptual capacity is the capacity to develop the logical framework used to explain why past economic events occurred, predict future events and estimate the benefits and costs of alternative policy actions. In agricultural policy analysis, this training usually includes such subject matter areas as agricultural economics, economics, political science, sociology, statistics, and operations research. Microcomputer training is often included in course work or available through special courses in the current graduate curriculums. However continuing education through special shortcourses (Li and Stoecker) or on-the-job training may be necessary to reduce the amount of time required for previously trained professionals to reach the level where they can fully exploit the potential of the microcomputer.

Although microcomputer skills are no replacement for conceptual capacity, microcomputers are invaluable in providing a laboratory surrounding where concepts can be implemented, experimented with, and learned. A staff that has ready computer access and receives quick turnaround is encouraged to experiment. Experimentation can increase the reliability of an analysis by solidifying theoretical concepts and inviting further learning. As an example, the economics of a representative farm can be investigated with linear programming or simulation models. Modern microcomputers can provide solutions to most realistic sized linear programming and simulation models in less time than was required by small mainframe computers a few years ago. Modern mainframe computers are of course much faster. However the real measure of time is the amount of time elapsed between the submission of the job and the return of the results. A somewhat slower interactive microcomputer system may still provide more rapid feedback than fast

mainframe which is shared by many users and is located away from the policy analysis group. The system with shortest total turnaround time will provide the greatest opportunity to gain insights through additional experimentation with model parameter changes.

Logistical Capability

Logistical capacity is concerned with the process of converting conceptual ideas to operational procedures and with the subsequential execution of these operational procedures. In a modeling situation, this encompasses construction of the model by the model builder, followed by utilization, updating, and maintenance of the model by the user or operator. The gap from conceptualizing a model to the derivation of results which are usable by decision makers is substantial. There are many logistical issues including assembling and/or reducing data, obtaining solutions, summarizing and finally reporting results which must be resolved. These logistical problems are not usually difficult in a conceptual sense but rather they tend to be voluminous, cumbersome, time-consuming, and error-prone when performed by hand. The continued operation, updating, and maintenance of a model often involves similar logistical problems. Models or analytical tools which are designed without regard to their usability, operability and maintainability may be unused or underutilized. Alternatively model builders resort to more simplistic models only because more complex models cannot logistically be operated by an agency. Over time microcomputers can be used to alleviate this logistical impairment to model development and usage in two ways. First, continuous and effective use of microcomputer systems increases an agency's confidence and dexterity in assembling and operating models. Second, with additional effort, the model builders can develop more automated procedures for updating and maintaining a model. For example, an agricultural sector model can be partially maintained and updated by developing programs or procedures which copy specified parameters or values from various data files or individual spreadsheets which are routinely updated and insert the values into the main tableau. An agency may routinely update cost of production budgets for various agricultural enterprises. Spreadsheets with linkages to these budget files or special computer programs can be used to copy information into the main model whenever budgets or population sensitive parameters have been changed.

Concepts of a Microcomputer System

One major difference between using a mainframe computer and microcomputer is that microcomputer user must also operate the computer. As mentioned above the microcomputer explosion has brought the computer novice and the non-technically trained operator (Otte; Danzieger and Kraemer) into the role of being a computer operator. Although in principle, operation of a microcomputer does not require a through understanding of it, some basic understanding is desirable. This section provides basic concepts and terminologies needed for understanding a modern microcomputer system.

The Central Processing Unit. A computer's heart is the central processing unit (CPU). CPU is central in the same sense as an engine is central to a car. It is where the computational and data processing power of a computer is generated. Also like an engine, the task performed by the CPU is relatively simple-minded. A CPU processes information, coded as 1's and 0's, needed by the rest of a computer system.

Computer Memory: ROM and RAM. A CPU processes information but cannot retain it. It must operate with a scratch pad or place where programs and data and data can be stored. This scratch pad is the Random Access Memory or RAM. Random access is a fancy way to say that the CPU can both read from and write to it in any order. This is in contrast to ROM or Read Only Memory, whose contents can be read but not be altered (written to) by CPU. ROM generally contains special programs provided by the computer manufacturer for enhancing the CPU.

Bits and Bytes. The unit of information in the computer is a byte. A byte contains 8 binary digits or bits. A binary digit can either be one or zero . Thus one bit can contain only information that has two states, 1 or 0. In Morse Code characters were represented by combinations of dots and dashes. Two bits can coded as 00, 01, 10, or 11 and hence represent 4 unique characters. Thus, a byte, which contains 8 bits can represent 2 to the eighth power or 256 different characters. This is sufficient to represent the English character set, which with 26 upper case characters, 26 lower case characters, 10 Arabic numerical digits, punctuation, and other special characters, consists of about 256 different symbols. For example, the letter "A" is represented as 01000001, the symbol "." is represented as 00101110. This particular coding scheme is internationally recognized and is referred to as ASCII code, where ASCII stands for American Standard Code for Information Interchange. This code was originally developed as a standard for sending Teletype messages. Other coding schemes are used on certain mini and mainframe computer systems. Since one byte holds one English character, the words byte and character are used interchangeably. A modern microcomputer usually contains at least 512K bytes of RAM, where 1K stands for 1024 or roughly 1000. Some advanced microcomputers contain 8M bytes of RAM, where 1M is 1024K. One page of a double-spaced document in English would occupy about 1.5K of storage. Wolf (1983) provides a good discussion of the microcomputer components the buyer encounters in purchasing microcomputer hardware.

Secondary Storage and Disk Operating System

The information stored in RAM is lost whenever electricity is turned off. It is also a relatively expensive storage device. Secondary storage is used for permanent storage of data and programs. Secondary storage usually refers to diskettes, fixed disk and magnetic tape. In the near future optical disks may be used for low cost mass storage. A diskette is removeable from the computer and allows relatively compact data storage. One diskette depending on type can store about 360K to 1.4M bytes of information. A fixed disk is not removable. It usually has a much higher capacity than diskettes, ranging from 10M bytes

to 80M bytes or more. It also has the additional advantage over diskettes in that information stored on it can be accessed by the computer much faster. Diskettes and fixed disks are secondary in the sense that the information must first be transferred to RAM before it can be processed by the CPU. Then the processed data must be rewritten to secondary device for permanent storage. The Disk Operating System (DOS) is the computer program which controls this traffic between RAM and secondary storage.

Because DOS is a computer program, it occupies part of the RAM when the computer is running. You then request another program to be loaded into RAM by issuing the appropriate DOS command. Newer Disk Operating Systems are appearing which will allow the CPU to address larger memory size, allow multiple programs to run simultaneously in RAM and support multiple users.

COMPUTER SOFTWARE

The impact of the microcomputer on the modern workplace would have been much smaller without the concurrent progress in the design of computer software. Many of the advances in human factors studies since the 1970's have been incorporated in modern microcomputer software. Ideally the user can interact with the software in a natural way and have easy recovery from errors. This allows the user to concentrate on the problem at hand rather than worrying about the computer program. This concentration on the interface with the end user has allowed microcomputer software to advance to where it is easier to acquire the necessary skills to operate microcomputer programs than it is to operate mainframe programs. Kelly et al. (1983) have published a directory of software available for socioeconomic analysis.

The modern spreadsheet may be one piece of software which is capable of doing the calculations common to most types of policy analysis. These calculations include tabulations involving row and columns of numbers, financial analysis, solution of equations, development of graphs and charts and statistical analysis. Indeed the first electronic spreadsheet was invented by a business school student to escape the boredom and exhaustion of the many repetitive calculations required for case studies in business and finance. This program called Visicalc[tm] was an instant success and a significant factor in the success of the Apple computer[tm].

SPREADSHEET APPLICATIONS

Fundamentally an electronic spreadsheet is an interactive electronic blackboard which appears to the user in a row and column orientation or as a two dimensional matrix of cells. An individual cell may be empty, contain a data entry, a label, or a formula which relates two or more cells. If the cell contains a formula, the user sees the result of the calculation rather than the formula itself. Each cell has a row and column coordinate. Thus C2 refers to the contents of the cell in column c and row 2. If the formula 2 + a1 + b1 is inserted in cell C2, the user would see the result (2 plus the value in cell a1 plus the value in cell b1).

The spreadsheet in Figure 1 illustrates the calculations for ending and beginning inventories in a commodity balance sheet. The user enters data from production and consumption estimates along with imports and exports and the initial beginning stock value. The values for the ending stock are calculated as the beginning inventory plus production and imports less consumption and exports. The beginning stock each year is taken to be the ending stock from the previous year. The formulas which would be entered into column F calculate the ending inventory and the formulas which would be entered into Column A transfer the ending inventory from the previous year to be the beginning inventory for the next year. If any of the data entries are revised, the calculated values for the ending and beginning inventories are automatically recalculated.

Most spreadsheets have built-in functions for financial calculations like net present value and the internal rate of return. Other features may include database functions (sorting and query), graphics, matrix and statistical operations. The main attraction of the spreadsheet is its flexibility and versatility in the sense that data and formulas are easily entered and revised

Spreadsheet Applications. The electronic spreadsheet will handle nearly all of tasks related to data tabulation, preparation of tables and graphic presentations. There are many spreadsheet applications in this text. These include applications to welfare analysis in Chapter 6, enterprise budgeting in Chapter 8, construction of linear programming tableaus in Chapter 9, simulation analysis in Chapter 7 and project analysis in chapter 10. Templates for many agricultural applications have been developed by many of the Land Grant Universities are available at nominal costs. Other researchers have developed templates for many kinds of analysis required for government planning.

COMPUTER LANGUAGES

Traditionally most use of the computer has involved writing or using a computer language. Even early microcomputers were equipped with BASIC: a simple computer language designed for beginners. Every computer comes with its own native language, expressed in 1's and 0's, called machine language. Few programmers write programs in machine language. Instead, some programmers use assembler language, which allows the use of nemonic and symbolic names. More often, programmers prefer writing in a higher-level language such as FORTRAN, BASIC, COBAL, or PASCAL. However, a computer cannot execute a program until it is translated in to machine language form. This translation is done with computer programs called compilers or interpreters. A wide variety of language compilers are available on the microcomputer today.

With microcomputer capacity approaching that of mainframe computers of just a few years ago, computer programs and models developed previously on mainframe computers can now be compiled to operate on microcomputers. Many mainframe models and programs were written in FORTRAN: a language designed for scientists and engineers. FORTRAN is a relatively standardized language, thus most FORTRAN programs written for the mainframe would not require major modifications when ported to

Formulas		A	B	C	D	E	F	formulas for
for	|1|	Beg.In.	Prod.	Imp	Xprt	Cons	End I.	column F
Column A	|2|	80	100	0	20	60	100	<- +A2+B2-C2-D2
+F2-->	|3|	100	90	0	30	70	90	<- +A3+B3-C3-D3
+F3 ->	|4|	90	85	0	30	75	70	<- +A4+B4-C4-D4
+F4 ->	|5|	70	80	0	25	75	50	<- +A5+B5-C5-D5
+F5 ->	|6|	50						

Figure 1. Spreadsheet Calculation for Commodity Balance Sheet.

the microcomputer. It is not uncommon for FORTRAN programs of over 20 thousand lines to compile successfully on the microcomputer. Other popular mainframe languages, such as COBOL and PL/1, are also available on the microcomputer.

FORTRAN, COBOL, PL/1, and BASIC originated before 1970 and do not contain features for implementing modern programming concepts such as abstract data types, modularity and object originated programming. Many programmers today prefer the more modern languages such as PASCAL, C, MODULA-2, and ADA.

Computer languages have a steep learning curve and considerable practice is required to gain proficiency. Boussard (1986) suggested that programmers should devote a large fraction -- perhaps 1/3 or 1/2 of the time for qualification upgrading, and this must be accepted as an additional cost inherent to the use of computers. Much of the popularity of electronic spreadsheet is that the novice can achieve meaningful results after a few minutes of instruction. However, there are applications which cannot be achieved with spreadsheets and which require special programs. One example might be a report writer for a linear programming model. Another example might be transforming a survey from past year's format into current format.

Database Management Systems

A database is a set of data stored in some special way on secondary computer storage. A database management system (DBMS) is the software that handles the storage and retrieval of the records in this database. Most microcomputer DBMS are self-contained in the sense that they use their own storage format, and provide their own query and retrieval procedures. DBMS packages typically contain facilities for designing data entry forms and producing standard reports. The more advanced DBMS contain their own programming language.

Many useful applications can be easily done with DBMS without using their programming language. This includes developing mailing lists and processing payrolls. More advanced applications such as survey processing would require some form of programming, usually in the language provided by the DBMS. In this case, as Boussard indicates, the difficulty of learning about the basic concepts in the DBMS is equivalent to the difficulty of learning a general programming language.

The size of a data base which can be managed on the microcomputer is limited. The first limitation is the size of the secondary storage, the second is the time needed to locate the relevant information as the size of the database increases. The emerging technology in greater processor speed, larger and faster fixed disk and optical disks should reduce this difficulty. However, very large database applications may remain with large central computers.

Other Software

Many other microcomputer software useful for policy analysts include statistical packages, project management packages, and business graphics.

Conclusions

We have tried in this chapter to provide some guidelines on potential and limitations of microcomputers for policy analysis. Advances in microcomputer technology have reduced the capital cost of quantitative analysis and have opened up opportunities for more sophisticated and timely analysis. Taking advantage of this opportunity requires, however, an increase in analytical capability: the subject matter of the other chapters of this book.

Advances in microcomputer technology have reduced the capital cost of quantitative analysis which creates opportunities for more timely and complete analysis of alternative policies. To take advantage of these opportunities however, an agency must increase both the computer and the analytical capabilities of its staff members.

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4. DEMAND THEORY AND ELASTICITY MATRIX CONSTRUCTION

by David Pyles

The demand for a particular commodity is generally defined as a function relating the quantity of the commodity that buyers would be willing and able to purchase, over a given interval of time, to commodity prices, income levels, and possibly other variables. Knowledge of demand functions and of their parameters is essential to any analysis concerned with prices and quantities of traded goods. For this reason, estimation of demand functions and demand function parameters is basic to policy analysis. For example, an analyst equipped with estimates of the demand functions for food items in a particular country could examine and predict the likely consequences of government intervention designed to influence nutrition using programs involving subsidies to producers, retail price ceilings, or targeted food assistance.

The primary focus of this chapter is the estimation of demand functions. A brief summary of the classical theory of the consumer demand function is presented. It is then shown how the theory may be empirically incorporated so as to facilitate the estimation.

OBJECTIVES

The objectives of this chapter are to:

1. Present a brief summary of consumer demand theory with emphasis given to the properties that such theory implies for the elasticities of demand
2. Introduce the concept of strongly separable utility functions, and explain the peculiarities of the demand elasticities under such utility functions.
3. Demonstrate how certain of the price elasticities may be estimated from estimates of the income elasticities when the underlying utility function is strongly separable.

KEY POINTS

1. Data limitations and statistical considerations generally forbid the direct estimation of all elements in demand matrices.
2. If the utility function is strongly separable, then certain of the price elasticities may be indirectly estimated with use of estimates for the income elasticities.

CONCEPTS AND APPLICATIONS

A typical *demand function* for a commodity, say commodity a, is of the general form:

$$q_a = q_a(p_a, p_b, p_c, y)$$

where p_a is the price of commodity a; p_b and p_c are prices of commodities that are related to commodity a, and y is the income of the potential purchaser. Given that all other factors influencing willingness and ability to purchase commodity a are held constant, $q_a(p_a, p_b, p_c, y)$ indicates the quantity demanded of commodity a at alternative levels of p_a, p_b, p_c, and y.

With any demand function, there is an underlying assumption that factors influencing demand other than those listed among the function's arguments are held fixed as one moves along the surface of the function. These assumptions are most commonly called the *ceteris paribus* assumptions. An analyst using a demand function needs to be aware of the underlying ceteris paribus assumptions.

The factors affecting willingness and ability to purchase a typical commodity include the price of the commodity, the prices of related commodities, consumer income, consumer wealth, interest rates, liquidity of consumer assets, government redistributions, expectations with regard to future movements in all of the preceding, the degree of certainty with which such expectations are held, and consumer tastes and preferences. In a typical demand function, some of these factors would be assumed variable, while others would be assumed fixed. The choice as to which factors are variable and which are fixed is quite arbitrary; however, most observed demand functions have the commodity's own price included among the variable factors.

A case of extreme ceteris paribus assumptions occurs with the popular demand curve, where quantity demanded is plotted against commodity price. Here it is assumed that all factors other than commodity price are held constant as one moves along the curve. A second extreme case occurs with the *Engel Curve*. Here, quantity demanded is plotted against consumer income; accordingly, all factors other than income are assumed fixed as one moves along the curve. The remainder of this chapter utilizes demand functions for which the variable factors are commodity price, prices of all other commodities, and consumer income.

As in supply analysis, the elasticity serves a useful role in demand analysis. Suppose a consumer purchases a total of n commodities. Let the prices of these commodities be denoted by $p = (p_1, p_2, \ldots, p_n)'$; let the consumer's income be denoted

by y, and let the n demand functions be q_1 (p, y) through q_n (p, y). Then the elasticities of demand are defined as:

$$\varepsilon_{ii} = \partial q_i / \partial p_i \, (p_i / q_i) = \partial \ln q_i / \partial \ln p_i$$
$$\varepsilon_{ij} = \partial q_i / \partial p_j \, (p_j / q_i) = \partial \ln q_i / \partial \ln p_j$$
$$\varepsilon_{iy} = \partial q_i / \partial y \, (y / q_i) = \partial \ln q_i / \partial \ln y.$$

Recall that an elasticity measures the percentage change in the dependent variable induced by a 1 percent change in the independent variable. In many contexts the elasticity is a more workable measure than the corresponding partial derivative because the elasticity is a unitless measure whereas the partial derivative is not.

Demand functions are typically estimated as regression equations. Although this is a convenient and direct approach, it poses several problems. One is that economic theory, while generally specifying the relevant variables affecting demand, seldomly prescribes the general form of the demand function; e.g. linear, polynomial, log-linear, etc. It is popular to assume that demand functions are either linear or log-linear, because these functional forms are quite flexible and are easily handled. One then estimates the demand functions with regression equations of the assumed form (see Tweeten). The log-log form is particularly popular because the parameters of this function are elasticities. Log-log regression equations corresponding to the demand system mentioned above are:

$$\ln q_1 = \beta_{10} + \beta_{11} \ln p_1 + \ldots + \beta_{1n} \ln p_n + \beta_{1y} \ln y + u_1$$
$$\ln q_2 = \beta_{20} + \beta_{21} \ln p_1 + \ldots + \beta_{2n} \ln p_n + \beta_{2y} \ln y + u_2$$
$$\vdots$$
$$\ln q_n = \beta_{n0} + \beta_{n1} \ln p_1 + \ldots + \beta_{nn} \ln p_n + \beta_{ny} \ln y + u_n$$

where the u_1 are disturbance terms. The β_{ij} are estimates of the elasticities.

A difficulty with the above approach is that the elasticities are assumed to be constant when in fact an elasticity can vary as one moves along the demand surface. For example, empirical studies have found income elasticities to be quite variable for some commodities. Knudsen and Scandizzo have estimated the income elasticity for food in India and Indonesia to be .8 for the poorest of the population, and .3 for the richest, thus indicating that the elasticity varies with the level of income. For this reason, various functional forms should generally be tested when attempting to estimate demand functions, including forms allowing elasticities to vary. The linear form admits variable elasticities but assumes constant partial derivatives. Of course, any functional form entails some degree of restrictiveness.

Many analyses can be accomplished with but one or two of the demand functions. However, comprehensive studies may require the entire demand system. In this context, the *demand matrix* often becomes a useful instrument. The demand matrix is the matrix of

all price and income elasticities corresponding to a complete set of consumer demand functions. For a consumer who purchases a total of n commodities the demand matrix is:

$$\begin{matrix} \varepsilon_{11} & \varepsilon_{12} & \dots & \varepsilon_{1n} & \varepsilon_{1y} \\ \varepsilon_{21} & \varepsilon_{22} & \dots & \varepsilon_{2n} & \varepsilon_{2y} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \varepsilon_{n1} & \varepsilon_{n2} & \dots & \varepsilon_{nn} & \varepsilon_{ny} \end{matrix}$$

Data and statistical limitations generally prohibit the estimation of complete demand matrices with regression equations. Hundreds of commodities may be purchased by a typical consumer. Data limitations alone preclude the estimation of such a large number of elasticities. Also, price data tend to be highly collinear; consequently, the estimation problem is compounded by the presence of multicollinearity. In the remainder of this chapter, we present the basic classical theory underlying the consumer demand function. We then utilize this theory in the development of a scheme wherein many of these difficulties can be reduced or eliminated to obtain a generally practical procedure for the estimation of demand matrices.

Utility Maximization and Demand Function Derivation

Consumer demand functions are generally derived from the utility maximization problem. The consumer is assumed to allocate income to the purchase of the various commodities such that utility is maximized. In classical consumer demand theory, the utility maximization problem is mathematically stated as follows:

$$\begin{aligned} &\text{maximize } (x)\text{: } U(x) \\ &\text{subject to: } p'x = y \end{aligned}$$

where $U(x)$ is the consumer's utility function and is assumed to be strictly quasiconcave and twice continuously differentiable; $x = (x_1, \dots, x_n)'$ denotes the vector of commodities, and $p = (p_1, \dots, p_n)'$ denotes the vector of commodity prices. $U(x)$ has the property that if x_1 is preferred to x_2, then $U(x_1) > U(x_2)$, and if the consumer is indifferent between x_1 and x_2, then $U(x_1) = U(x_2)$. Hence, the objective of the utility maximization problem is to find the most preferred of commodity combinations attainable at the given set of commodity prices and the given level of income. It is generally assumed that the consumer always prefers more to less; subsequently, the most preferred commodity combination must exhaust the consumer's income, that is $p'x = y$. The assumption that all income is spent may be somewhat relaxed by designating one of the n commodities as savings and assigning the commodity a price equal to one.

As mentioned above, the utility function has the properties: (1) if x_1 is preferred to x_2, then $U(x_1) > U(x_2)$, and (2) if the consumer is indifferent between x_1 and x_2, then $U(x_1) = U(x_2)$. However, in some contexts these are taken to be more than mere

properties; they are frequently taken to be the very criteria defining a utility function. That is, any function satisfying these two properties is, by this definition, a utility function. Utility so defined is called *ordinal* utility. When the utility function is limited to ordinality, the actual value of the function is of significance only insofar as it reflects the ranking of commodity bundles with respect to preference. Subsequently, if $U(x_1) = 2$ and $U(x_2) = 1$, it could be said that the consumer prefers x_1 to x_2, but it could not be said that the consumer prefers x_1 twice as much as x_2, even though the utility function doubles upon moving from x_2 to x_1. Any utility function having significance in the latter sense is said to be *cardinal.*

Existence of one ordinal utility function implies existence of infinite other ordinal utility functions. For example, if $F(x)$ were an ordinal utility function, then $U(x) = a + bF(x)$ for $b > 0$ would be an ordinal utility function also, because $U(x)$ does indeed rank commodity bundles according to preference. More generally, if $F(x)$ is an ordinal utility function and if $T(.)$ is any monotonic-increasing transformation, then $U(x) = T[F(x)]$ is also an ordinal utility function.[1]

Though there will be an infinite number of ordinal utility functions, each is optimized by the same commodity bundle when incorporated in the utility maximization problem. Subsequently, if $\bar{x}$ is a solution to the utility maximization problem when the utility function is $F(x)$, then $\bar{x}$ will also solve the problem when the utility function is $T[F(x)]$, where T is any monotonic-increasing transformation. Hence, the optimal solution to the utility maximization problem is invariant with respect to monotonic-increasing transformations of the utility function. This follows from the fact that a monotonic-increasing transformation preserves the extrema of a function.

If the maximization problem above is intended to characterize an aggregate market consisting of numerous consumers, then the existence of a representative consumer must be assumed. The representative consumer is defined as one who will purchase average per-capita quantities when provided with average per-capita income. Accordingly, x is taken to be the vector of average per-capita quantities, and $U(x)$ is taken to be the utility function of the representative consumer.

The Lagrangian to the utility maximization problem is

$$L(x, \lambda) = U(x) + \lambda(y - p'x)$$

Because $U(x)$ is differentiable, an optimal solution must satisfy the first-order conditions

$$\partial L / \partial x_i = \partial U / \partial x_i - \lambda p_i = 0; \qquad i = 1, 2, \ldots, n$$
$$\partial L / \partial \lambda = y - \Sigma x_i p_i = 0.$$

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[1]A monotonic-increasing transformation is defined as any function having positive slope.

The second condition is simply the requirement that all income be spent. As for the first set of conditions, upon taking a ratio of any two of these, one obtains the *proportionality rules*

$$U_i / U_j = p_i / p_j; \qquad i, j = 1, 2, \ldots, n.$$

The price ratio on the right represents the rate at which one unit of x_i can be exchanged for x_j on the market. On the other hand, the ratio of marginal utilities represents the marginal rate at which the consumer would be willing to exchange one unit of x_i for x_j. For example, if the marginal utility of x_i were twice that of x_j, then the consumer would be willing to exchange no more than two units of x_j for one unit of x_i. This ratio is known as the *marginal rate of substitution*. If the consumer and the market are not in agreement as to the rate of exchange between commodities, then utility gained may be had by substituting some commodities for others. As utility is assumed to be maximized here, no such gains should be possible.

Under general regularity conditions, $x_i^*(p, y)$ and $\lambda^*(p, y)$ will exist such that $x_i = x_i^*(p, y)$ and $\lambda = \lambda^*(p, y)$ will solve the maximization problem and the first-order conditions. The $x_i^*(p, y)$ are by definition the demand functions for a utility maximizing consumer. Also, one can define

$$U^*(p, y) = U[x_i^*(p, y), x_2^*(p, y), \ldots, x_n^*(p, y)].$$

$U^*(p, y)$ is known as the *indirect utility function*, and is equal to the utility of a utility-maximizing consumer with income y and when commodity prices are p. It can be shown that

$$\partial U^* / \partial y = \lambda.$$

Hence, λ is the marginal utility of income when such income is disposed in a utility maximizing fashion.

We now complete this section with an example in which a utility function is maximized and in which demand functions and an indirect utility function are derived. Suppose that the utility function is of the form

$$F(x_1, x_2) = a x_1^{\alpha_1} x_2^{\alpha_2}$$

where a, α_1, and α_2 are all positive. We take the utility function to be of ordinal significance only, hence any monotonic-increasing transformation of the above function is an equally acceptable measure of utility. For example, consider the transformation

$$T(z) = \log(z / a); \qquad a > 0.$$

Because the derivative $T'(z) = 1 / z > 0$ for all $z > 0$, the transformation is needed monotonic-increasing over the relevant domain. Upon applying this transformation to $F(x_1, x_2)$, we obtain the second ordinal utility function

$$U(x_1, x_2) = T[F(x_1, x_2)] = \alpha_1 \log(x_1) + \alpha_2 \log(x_2)$$

We are at liberty to use either of these utility functions. Because $U(x_1, x_2)$ is more convenient for analysis, we shall henceforth use it.

The utility maximization problem becomes

$$\text{maximize } (x_1, x_2): \ \alpha_1 \log(x_1) + \alpha_2 \log(x_2)$$
$$\text{subject to: } p_1 x_1 + p_2 x_2 = y.$$

The Lagrangian to this problem is

$$L(x_1, x_2, \lambda) = \alpha_1 \log(x_1) + \alpha_2 \log(x_2) + \lambda(y - p_1 x_1 + p_2 x_2).$$

The first-order conditions require that all partial derivatives of the Lagrangian be equal to zero; hence

$$\partial L / \partial x_1 = \alpha_1 / x_1 - \lambda p_1 = 0$$
$$\partial L / \partial x_2 = \alpha_2 / x_2 - \lambda p_2 = 0$$
$$\partial L / \partial \lambda = y - p_1 x_1 - p_2 x_2 = 0.$$

We can solve the first two equations for x_1 and x_2 and substitute the results into the third equation to obtain

$$y - \alpha_1 / \lambda - \alpha_2 / \lambda = 0$$

which implies

$$\lambda = \lambda^* (p_1, p_2, y) = (\alpha_1 + \alpha_2) / y = \alpha / y$$

where we have defined $\alpha = \alpha_1 + \alpha_2$. Substituting the expression for λ back into the first-order conditions, we obtain the demand functions

$$x_1^* (p_1, p_2, y) = \alpha_1 y / \alpha p_1$$

$$x_2^* (p_1, p_2, y) = \alpha_2 y / \alpha p_2.$$

Because the demand for both quantities is proportional to income, the income elasticities are equal to one. Likewise, the inverse proportionality between quantity demanded and own-price indicates that own-price elasticities are equal to minus one. The

cross-price elasticities are clearly equal to zero. Hence, the demand matrix for this system is

$$\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{1y} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{2y} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}.$$

Finally, we can substitute the demand functions back into the utility function to obtain the indirect utility function:

$$U^*(p_1, p_2, y) = \alpha_1 \log(\alpha_1 / p_1) + \alpha_2 \log(\alpha_2 / p_2) + \alpha \log(y / \alpha).$$

By differentiating $U^*(p_1, p_2, y)$ with respect to y, we confirm our interpretation of λ

$$\partial u^* / \partial y = \alpha / y = \lambda.$$

It may be confirmed that the same demand functions would be found by optimizing $F(x_1, x_2)$ rather than $U(x_1, x_2)$. However, a different expression for λ would be obtained. As noted earlier, the solutions to an optimization problem are invariant with respect to monotonic-increasing transformations of the objective function; however, the Lagrangian multipliers will generally be changed by such transformations. Therefore, if the utility function is taken to be only of ordinal significance, then no significance can be attached to the value of λ.

Properties of Demand Functions

If one has a set of demand functions deriving from the utility maximization problem of the previous section, then several properties for such functions are implied. We will present some of these properties here. First, the following notation is needful:

$$\begin{aligned}
\gamma &= \partial\lambda^* / \partial y\,(y / \lambda) \\
w_i &= p_i x_i / y \\
U_i &= \partial U / \partial x_i, \quad U_{ij} = \partial^2 U / \partial x_i \partial x_j \\
\phi_{ij} &= \partial U_i / \partial x_j\,(x_j / U_i) = U_{ij}\,(x_j / U_i) \\
\Phi &= \{\phi_{ij}\} \\
\{\phi^{ij}\} &= \Phi^{-1}.
\end{aligned}$$

Hence, γ is the elasticity of λ with respect to income. γ is sometimes called the *flexibility of money*. w_i is the expenditure proportion on the ith commodity. ϕ_{ij} is the elasticity of the ith marginal utility with respect to the jth commodity, and Φ is the nxn matrix consisting of such terms. The ϕ_{ij} are sometimes called *utility accelerators*. ϕ^{ij} is the (i, j) term in Φ^{-1}. The ϕ^{ij} are sometimes called *want elasticities*.

Although not proven here, utility maximization implies the following properties in the demand functions:

1. $\sum_j \varepsilon_{ij} + \varepsilon_{iy} = 0$
2. $\sum w_i \varepsilon_{iy} = 1$
3. $\sum_i w_i \varepsilon_{ij} = -w_j$
4. $w_i \varepsilon_{ij} = w_j \varepsilon_{ji} + w_i w_j (\varepsilon_{jy} - \varepsilon_{iy})$
5. $\varepsilon_{ij} = \phi^{ij} - w_j \varepsilon_{iy} \varepsilon_{jy} / \gamma - w_j \varepsilon_{iy}$
6. $\varepsilon_{iy} = \gamma \sum_j \phi^{ij}$
7. $\gamma = 1 / (\sum_i \sum_j w_i \phi^{ij})$.

There is some degree of redundancy among these properties. That is, some properties are implied by others. For example, property four is implied by property five.

The first property is known as the *homogeneity condition.* The property implies and is implied by homogeneity of degree zero in the demand functions with respect to prices and income. The second property is known as the *Engel aggregation condition.* The third property is the *Cournot aggregation condition*, and the fourth property is the *Slutsky symmetry relation.* The fifth, sixth, and seventh properties have no generally accepted names.

Because of the above relationships between the elasticities, certain of the elasticities may be determined if other of the elasticities are known. For example, consider the following incomplete demand matrix corresponding to a system of three demand equations

-1	?	?	1
?	-2	1	-1
?	?	?	?

where "?" has been used to denote missing elasticities. Of the 12 elasticities in the demand matrix, seven are presently unknown. Let the expenditure proportions for the three goods be: $w_1 = 1/4$, $w_2 = 1/4$, and $w_3 = 1/2$. The properties of the elasticities may be used to find the missing elements as follows

1. Using the homogeneity condition, ε_{21} is found to be equal to 2.
2. The Slutsky symmetry relation may then be used to determine $\varepsilon_{12} = 1$.
3. The homogeneity condition may then be used to determine $\varepsilon_{13} = -1$.
4. Engel aggregation is used to determine $\varepsilon_{3y} = 2$.
5. Cournot aggregation may then be used to determine: $\varepsilon_{31} = -1$, $\varepsilon_{32} = 0$, and $\varepsilon_{33} = -1$.

Hence, the complete demand matrix is found to be

$$\begin{matrix} -1 & 1 & -1 & 1 \\ 2 & -2 & 1 & -1 \\ -1 & 0 & -1 & 2. \end{matrix}$$

A system of n demand equations contains $n^2 + n$ elasticities, including both price elasticities and income elasticities. Of these, n are determined by the homogeneity condition; n are determined by the Cournot aggregation condition and one is determined by the Engel aggregation condition. Therefore, if all but 2n + 1 of the elasticities are known, the remaining elasticities may be found. It can be shown that the Slutsky symmetry condition is implied by the homogeneity, Cournot aggregation, and Engel aggregation conditions; consequently, the dimensionality of the problem is not further reduced by the inclusion of this restriction. However, in the next section, we show that the dimensionality of the problem can be further reduced using certain information concerning the structure of the utility function.

Separability

Recall that of the various elasticity relations implied by the utility maximization problem, we had the condition

$$\varepsilon_{ij} = \phi^{ij} - w_j \varepsilon_{iy} \varepsilon_{jy} / \gamma - w_j \varepsilon_{iy}.$$

Observe that if $\phi^{ij} = 0$, then ε_{ij} may be calculated from income elasticities and the flexibility of money γ. Indeed, there will be a transformation of utility under which some of the ϕ^{ij} are equal to zero if the utility function is of the following form

$$U(x) = T[U^1(x^1) + U^2(x^2) + \ldots + U^g(x^g)]$$

where $x = (x^{1'}, \ldots, x^{g'})'$ and where T(.) is an arbitrary monotonic increasing transformation. Here, the commodity bundle is divided into g groups, where x^i is the vector of commodities belonging to the ith group. If x_i belongs to a different commodity group than x_j, then x_i is said to be *strongly separable* from x_j. Accordingly, such utility functions are said to be *strongly separable.*

Let I be the set of all indices for commodities belonging to the ith group and define J accordingly, then it is easy to verify that the strongly separable function satisfies

$$\partial(U_i / U_j) / \partial x_k = 0; \qquad i \in I, j \in J, k \notin I, J$$

where $\in$ denotes *element of.* The above implies that if x_k belongs to a different group than those containing x_i and x_j, then the marginal rate of substitution between x_i and x_j is independent of x_k.

An important special case of strong separability occurs when each of the commodity groups contains only one commodity. In this case, the function takes the general form

$$U(x) = T[U^1(x_1) + U^2(x_2) + \ldots + U^n(x_n)].$$

Such functions are said to be *pointwise separable*. The pointwise separable function satisfies

$$\partial(U_i / U_j) / \partial x_k = 0; \qquad k \neq i, j.$$

That is, the marginal rate of substitution between any two commodities is independent of all other commodities.

Because the demand functions and resulting elasticities are invariant with respect to monotonic-increasing transformations of the utility function, it becomes both proper and convenient to choose the particular transformation of utility that is most amenable to analysis. If the utility function is strongly separable, then there is a transformation under which the utility function takes the form

$$U(x) = U^1(x^1) + U^2(x^2) + \ldots + U^g(x^g)$$

This form is referred to as the *block-additive* representation of utility. In the case of pointwise separability, we have the *additive* representation, which is

$$U(x) = U^1(x_1) + U^2(x_2) + \ldots + U^n(x_n).$$

Henceforth, the analysis will be conducted using these forms.

While it is true that the elasticities are variant to the various transformations of utility, invariance does not hold for the want elasticities or the flexibility of money. Consequently, if these measures are to be used in actual calculations, it becomes necessary to specify the particular transformation to which the measures correspond. Henceforth, it will be understood that all want elasticities and the flexibility of money correspond to the block-additive or additive representations of utility.

Demand Matrix Construction Under Strong Separability

If the utility function is strongly separable, then under the block-additive representation, the matrix of utility accelerators Φ is block-diagonal where there are g blocks, each corresponding to a commodity group. Specifically, it can be said

$$\phi_{ij} = 0; \qquad i \in I, j \notin I.$$

Because the inverse of a block-diagonal matrix is obtained by inverting each block, Φ^{-1} is also block diagonal with elements satisfying

$$\phi^{ij} = 0; \qquad i \in I, j \notin I.$$

As a consequence of the block-diagonality of Φ^{-1}, the fifth property may be rewritten as

$$\varepsilon_{ij} = \phi^{ij} - w_j \varepsilon_{iy} \varepsilon_{jy} / \gamma - w_j \varepsilon_{iy}; \qquad i, j \in I$$
$$\varepsilon_{ij} = - w_j \varepsilon_{iy} \varepsilon_{jy} / \gamma - w_j \varepsilon_{iy}; \qquad i \in I, j \notin I.$$

The second equation indicates that if one has (1) a full set of income elasticities, (2) γ, and (3) a full set of expenditure proportions, then all of the cross-price elasticities between commodities in different groups can be calculated.

The second of the latter equations may also be used to calculate γ if one cross-group, cross-price elasticity is known. Specifically, if ε_{ij} is known, then the second equation implies

$$\gamma = -(w_j \varepsilon_{iy} \varepsilon_{jy}) / (\varepsilon_{ij} + w_j \varepsilon_{iy}). \tag{4.1}$$

An alternative specification for γ can be derived using properties two, five, and seven, and the fact that Φ^{-1} is block diagonal. Let

$$w^I = \Sigma_{i \in I} w_i; \qquad I = 1, 2, \ldots, g$$
$$\varepsilon^I = \Sigma_{i \in I} w_i \varepsilon_{iy}; \qquad I = 1, 2, \ldots, g$$
$$\sigma^I = \Sigma_{i \in I} \Sigma_{j \in I} w_i \varepsilon_{ij}; \qquad I = 1, 2, \ldots, g$$

then:

$$\gamma = \frac{[1 - \Sigma_I (\varepsilon^I)^2]}{\Sigma_I \sigma^I + \Sigma_I w^I \varepsilon^I}. \tag{4.2}$$

As a consequence of the above, demand matrix estimation under strong separability can be accomplished with estimates available for the expenditure proportions, within-group price elasticities, and income elasticities. These estimates permit completion of the remainder of the matrix.

The above procedure poses difficulties in obtaining a set of estimates satisfying the homogeneity and Cournot aggregation properties. It is easy to confirm that the cross-group, cross-price elasticities automatically satisfy Slutsky symmetry. Consequently, if the within-group elasticities satisfy the same, then the resulting estimate for the demand matrix will be consistent with this property. Because Slutsky symmetry is a linear condition, it is not difficult to impose on least squares estimators for the within-group elasticities. Likewise, the Engel aggregation property is linear, and consequently, is easily imposed as a restriction. However, if the homogeneity or Cournot aggregation properties are to be satisfied, then complicated nonlinear restrictions must be imposed upon the estimators of the within-group price elasticities.

If compatibility with the homogeneity and Cournot aggregation properties is deemed necessary, then a possible alternative is to use a different estimate of γ for each row of the demand matrix. In particular, the estimate for γ could be chosen such that the homogeneity property is satisfied. It can be shown that if the Slutsky symmetry, Engel

aggregation, and homogeneity properties hold, then the Cournot aggregation property must hold also. Consequently, an estimate of the demand matrix satisfying all properties is found by: (1) imposing the Slutsky symmetry property on the estimators of the within-group, cross-price elasticities, (2) imposing the Engel aggregation property on the estimators for the income elasticities, and (3) choosing a different estimate of γ for each row, where in each case, the estimate is chosen such that the homogeneity property is satisfied. If this course is taken, then the appropriate estimate of γ for the ith row is:

$$\gamma_i = \frac{\varepsilon_{iy}\,(1 - \varepsilon^I)}{\sum_{j \in I} \varepsilon_{ij} + w^I\,\varepsilon_{iy}}; \qquad i \in I.$$

Demand Matrix Construction Under Pointwise Separability

If the utility function is pointwise separable, then under the additive representation, Φ^{-1} is simply diagonal. Consequently, the fifth property becomes:

$$\begin{aligned} \varepsilon_{ii} &= \phi^{ii} - w_i\,\varepsilon_{iy}\,\varepsilon_{iy} / \gamma - w_i\,\varepsilon_{iy}; \\ \varepsilon_{ij} &= - w_j\,\varepsilon_{iy}\,\varepsilon_{jy} / \gamma - w_j\,\varepsilon_{iy}; \qquad i \neq j. \end{aligned} \tag{4.3}$$

Also, the sixth property becomes:

$$\varepsilon_{iy} = \gamma\,\phi^{ii}$$

which implies $\phi^{ii} = \varepsilon_{iy} / \gamma$. Upon substituting this expression into the equation for ε_{ii}, one obtains:

$$\varepsilon_{ii} = \varepsilon_{iy} / \gamma - w_j\,\varepsilon_{iy}\,\varepsilon_{jy} / \gamma - w_j\,\varepsilon_{iy} \tag{4.4}$$

which involves only income elasticities, the flexibility of money, and expenditure proportions. The same may be said of the ε_{ij}; consequently, the entire demand matrix can be constructed if one has: (1) a full set of income elasticities, (2) γ, and (3) a full set of expenditure proportions. Using the same procedure as before, γ may be calculated from (4.1) if a single cross-price elasticity is known, or from (4.4) if a single own-price elasticity is known. It is not difficult to verify that if the estimates of the income elasticities satisfy the Engel aggregation property, then the resulting estimate of the demand matrix satisfies the Slutsky symmetry, homogeneity, and Cournot aggregation properties.

We now illustrate the procedure with an example. Later, we will imagine that the foregoing data and elasticities pertain to consumption in Pakistan; however, the data used here are purely hypothetical.

Suppose that the commodity bundle consists of the following goods, and is characterized by the indicated expenditure proportions and income elasticities:

Commodity	Expenditure Proportion	Income Elasticity
cotton	.07	1.25
sugarcane	.03	1.00
rice	.05	.25
wheat	.12	.25
misc crops	.09	.25
livestock	.14	1.20
nonagricultural	.50	1.30.

Suppose that the utility function is pointwise separable. Moreover, suppose that the own-price elasticity for cotton is known to be -1, then with use of (4.4), the flexibility of money is found to be $\gamma = -1.25$. Using both (4.3) and (4.4) and the data above, the demand matrix is found to be

	cotton	sugarcane	rice	wheat	misc crop	livestock	nonag	inc
cotton	-1	-.0075	-.050	-.1200	-.0900	-.00700	.025	1.25
sugarcane	0	-.8060	-.040	-.0960	-.0720	-.00560	.020	1.00
rice	0	-.0015	-.210	-.0240	-.0180	-.00140	.005	.25
wheat	0	-.0015	-.010	-.2240	-.0180	-.00140	.005	.25
misc crop	0	-.0015	-.010	-.0240	-.2180	-.00140	.005	.25
livestock	0	-.0072	-.048	-.1152	-.0864	-.96672	.024	1.20
nonagric	0	-.0078	-.052	-.1248	-.0936	-.00728	-1.014	1.30.

It is not difficult to confirm that the above matrix satisfies the homogeneity, Cournot aggregation, Engel aggregation, and Slutsky symmetry properties.

Summary

The classical theory of demand is based upon the assumption that consumers allocate income to the purchase of commodities such that the utility deriving from the allocation is maximized. Utility is generally understood to have only ordinal significance; consequently, the actual value assigned by the utility function to a particular commodity bundle is meaningful only insofar as it serves to rank commodity bundles according to preference. Utility so defined is said to be *ordinal*." Although there will be an infinite number of ordinal utility functions corresponding to any one consumer, all functions render the same optimal commodity combination. The demand functions are mathematically derived from the utility maximization problem, and are functions rendering the optimizing commodity combinations at various commodity prices and various levels of income when all other factors are held constant.

Utility maximization implies that certain relationships must hold between the demand elasticities. As a consequence of these relationships, certain of the elasticities may be calculated if other of the elasticities are known. The number of elements requiring direct estimation is further reduced if the utility function is known or assumed to be strongly separable or pointwise separable.

If the utility function is strongly separable, then the cross-group, cross-price elasticities can be calculated using estimates for the expenditure proportions, income elasticities, and the flexibility of money. The flexibility of money may be estimated with (4.1) if a single cross-group, cross-price elasticity is known. The parameter may also be estimated with (4.2). If the Slutsky symmetry property is imposed upon the estimators of the within-group elasticities, then the resulting estimate for the demand matrix will be fully consistent with the Slutsky symmetry property. An estimate of the demand matrix satisfying the Slutsky symmetry, homogeneity, and Cournot aggregation properties may be had by (1) imposing Slutsky symmetry upon the estimators for the within-group price elasticities, (2) imposing Engel aggregation upon the estimators of the income elasticities, and (3) choosing a different estimate of the flexibility of money for the various rows of the demand matrix, where for each row the estimate is chosen so as to produce the homogeneity property.

If the utility function is pointwise separable, then an estimate for the demand matrix may be fully constructed using estimates for the income elasticities and expenditure proportions, and an estimate of the flexibility of money. If the income elasticity estimates satisfy the Engel aggregation property, then the resulting estimate for the demand matrix will satisfy the Slutsky symmetry, homogeneity, and Cournot aggregation properties.

ACTIVITIES

1. Suppose a commodity bundle consists of two goods. The expenditure proportions are $w_1 = w_2 = 1/2$. Find the missing elasticities in the following demand matrix:

$$\begin{matrix} -2 & ? & 1 \\ ? & ? & ? \end{matrix}$$

2. Suppose a commodity bundle consists of n goods. Let the following data pertain to the ith and jth goods: $w_i = 1/10$, $w_j = 1/4$, $\varepsilon_{iy} = 1/2$, $\varepsilon_{jy} = 1$, and $\varepsilon_{ij} = -1$. Find ε_{ji}.

3. Consider a commodity bundle consisting of three goods. Assume that the utility function is pointwise separable. ε_{11} has been estimated to be -1. The income elasticities and expenditure proportions have been estimated to be as follows:

commodity	expenditure proportion	income elasticity
1	.20	.5
2	.50	1.2
3	.30	1.0.

a. Are the income elasticities consistent with the Engel aggregation property?
b. Construct the demand matrix.
c. Confirm that the matrix obtained in part b satisfies homogeneity, Cournot aggregation, and Slutsky symmetry.

4. Consider a commodity bundle consisting of four commodities. The utility function is strongly separable with the first two commodities comprising one group and the last two commodities comprising the other. Each commodity represents one-fourth of total expenditures. Find the missing elasticities in the following demand matrix:

-1.0	?	.2	?	1
.5	-.5	?	?	?
?	?	-2.0	1.0	1
?	?	?	-.2	2.

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5. SUPPLY AND ELASTICITY ESTIMATION

by Luther Tweeten, David Pyles, and Shida Henneberry

The *supply curve* is a schedule of relationships between real prices of a commodity and quantities supplied by producers at each price for a given market per unit of time, other things equal. Many variables such as technology, weather, investment in infrastructure, and prices of related commodities and of inputs shift the supply curve. A *supply function* contains all of these variables and can be used to estimate the supply curve and sources of shifts in the supply curve.

Supply curves and functions are extremely useful to answer many questions often confronted in agricultural policy analysis. If supply price is raised or lowered, how will the quantity supplied of commodities respond? Is the response different for a short run or 1 - 2 years versus a long run of 10 years or more? Is it more effective and efficient to increase output by raising commodity prices to move along the supply curve or to shift the supply curve forward by investment in agricultural education, research, and extension? If price is raised for one commodity, how is the production of other commodities affected? Is a subsidy to fertilizer more effective than a commodity price increase in generating output?

OBJECTIVES

The objectives of this chapter are to:

1. Present briefly the conceptual framework for agricultural commodity supply curves and functions.
2. Present empirical models for estimating agricultural supply parameters, noting advantages and disadvantages of each.
3. Illustrate a new method for computing a complete matrix of supply parameters useful in classical welfare analysis and simulation presented in later chapters.
4. Show selected applications of supply response estimates.

KEY POINTS

1. Supply response both influences and is influenced by public policy, hence knowledge of supply response to price and other variables influenced by public policy is essential in a wide variety of policy analyses.
2. Measuring supply response parameters is often difficult, especially because data are inadequate. The sophisticated econometric engines of parameter estimation function poorly on the crude fuel of available data, hence there is a high premium on using simple econometric models least demanding of precise data.
3. Given estimates of a few key parameters from statistical models, remaining supply parameters in a complete matrix of supply parameters can be filled in using economic theory.
4. Supply response may differ for the short run versus long run, for rising prices and falling prices, and for a crop that occupies a large share of land versus a small share. Any one parameter estimation is unlikely to utilize all information and is subject to considerable error; the most reliable estimates of supply response will utilize not only direct econometric estimates at hand, but also results of previous estimates along with good judgment based on knowledge of circumstances.

CONCEPTS AND APPLICATIONS

Assuming producers act to increase profit, the *supply function* expresses supply quantity as a function of several variables:

$$O_{it} = f\left(\frac{P_i}{W}, \frac{P_j}{W}, I, T, G, X, \varepsilon\right) \tag{5.1}$$

where O_i is production of commodity i; P_i is own-price of i; P_j is the price of competing or complementary commodities (represented by an index of prices received by producers or by separate price variables for each commodity); W is prices paid for inputs (represented by an index of prices paid for all inputs or by separate prices of fertilizer, machinery, labor, pesticides, and other inputs); I is fixed capital inputs or infrastructure such as public irrigation capacity; T is technology such as high-yielding varieties or productivity which is often represented by a time trend; G is government programs such as extension education or supply controls; X is weather and other factors such as pests over which the producer has little or no control in the short run; and ε is unaccounted for error. Other variables such as the *variation* in past prices may also influence the supply quantity.

Elasticities are frequently used for convenience to express the supply response to price. The elasticity E shows the percentage change in one variable associated with the percentage change in another variable and hence is independent of the units of measurement. For example, the own-price elasticity of supply response is

$$E_{ii} = \frac{\Delta 0_i}{0_i} / \frac{\Delta P_i}{P_i} = \frac{d0_i}{dP_i} \frac{P_i}{0_i} = \frac{d \ln 0_i}{d \ln P_i},$$

the cross-price elasticity for related commodities is

$$E_{ij} = \frac{\Delta 0_i}{0_i} / \frac{\Delta P_j}{P_j} = \frac{d0_i}{dP_j} \frac{P_j}{0_i} = \frac{d \ln 0_i}{d \ln P_j},$$

and the elasticity of response to input prices is

$$E_{iw} = \frac{\Delta 0_i}{0_i} / \frac{\Delta W}{W} = \frac{d0_i}{dW} \frac{W}{0_i} = \frac{d \ln 0_i}{d \ln W},$$

where Δ is the change in price or quantity and d refers to a very small change. Elasticities are frequently measured at the mean of price and quantity. In theory, the sum of these elasticities is zero. That is, a proportional increase in all prices does not change output if farmers react to real (relative) prices as shown in equation 5.1 rather than to absolute (nominal) prices.

Expectation Models

Supply response elasticities can be estimated empirically by multiple regression statistical analysis utilizing data over time on the variables in equation 5.1. In doing so, it is important to recognize that when producers plant they ordinarily do not know what actual price will be when they sell their commodity. They plan based on expected prices. Numerous functional forms have been used to represent producers' expectations which cannot be observed directly. One widely used *adaptive expectations* form is from Nerlove and assumes that the change in price expectations in the current year (left side of equation 5.2) is some proportion β if the error made in formulating expectations last year (right side of equation 5.2) or

$$(P/W)_t^* - (P/W)_{t-1}^* = \beta[(P/W)_{t-1} - (P/W)_{t-1}^*] \tag{5.2}$$

where the asterisk refers to expected price in year t or t-1. Suppose in a highly simplified explicit form of equation 5.1 that current output 0_t is a linear function of expected price with a long-term response γ_t plus a constant α and error μ_t or

$$0_t = \alpha + \gamma\ (P/W)_t^* + \mu_t \tag{5.3}$$

Substituting equation 5.2 into equation 5.3 yields

$$0_t = \alpha\beta + \gamma\beta(P/W)_{t-1} + (1-\beta)\ 0_{t-1} + (\mu_t - \mu_{t-1}) \tag{5.4}$$

The latter equation contains only observed variables 0_t, $(P/W)_{t-1}$, and 0_{t-1} and can be estimated by ordinary least squares or other regression procedure. The short-run response

of output to price is $\gamma\beta$ from statistically estimated equation 5.4. The long-run response γ is the short-run response divided by 1.0 minus the coefficient of 0_{t-1} in equation 5.4.

Other forms of expectation models are sometimes used. If the coefficient of expectations β is 1.0, it is apparent from equation 5.2 that expected price is equal to price of the previous period. Such naive behavior gives rise to an adaptive expectation pattern of supply response or commodity cycle called the "cobweb" model. A high price begets a high quantity in the next period which, given a downward sloping demand curve, brings a low price which in turn brings a low quantity and high price in the next period.

Adaptive models form price expectations from weighting past prices such as

$$(Pi/W)_t^* = w_1 (P_i/W)_{t-1} + w_2 (Pi/W)_{t-2} + \ldots + w_n (Pi/W)_{t-n} \qquad (5.5)$$

where in the Nerlove or Koyck expectation models the weights are geometrically declining and are $1-\beta$ for t-1, $(1-\beta)^2$ for t-2 to $(1-\beta)^n$ for t-n where n approaches infinity. Another alternative is to insert a large number of past values for Pi/W for $(P/W)_t^*$ in equation 5.3 and let the regression equation determine weights w_i. Such methods break down because of multicollinearity defined as high correlation among two or more so called independent variables.

A useful approach is to assume a pattern of weights. Some trial and error can be used, but too much trial and error invalidates statistical tests of significance. Producers' expectations of price in year t may be formed from a weighted average of prices in previous years. For example, the expected real own-price $(P_i/W)^*$ for commodity i may be represented by

$$(P_i/W)_t^* = .50(P_i/W)_{t-1} + .33(P_i/W)_{t-2} + .17(P_i/W)_{t-3} \qquad (5.6)$$

which can be constructed from time series of past prices.

Finally, *rational expectations* presume producers make no systematic error such as production cycles noted in the adaptive expectations models above. All worthwhile information is accounted for in forming rational expectations. Use of future market quotes for the time of harvest as the expected price in equation 5.3 comes close to a rational expectations model, but such futures market data are often not available for developing countries.

Adjustment Models

Once producers are subjectively certain of price, they may adjust output slowly because of caution, inertia, or costs of making adjustments. The actual adjustment of supply quantity in one time period may be some proportion g of intended full adjustment to O_i^* as expressed below (Nerlove):

$$O_{it} - O_{it-1} = g(O_{it}^* - O_{it-1}). \qquad O < g < 1 \qquad (5.7)$$

A typical linear equation of long-run supply is of the form

$$O_{it}^* = a + b(P_i/W)_t^* + \varepsilon_t \tag{5.8}$$

where E_t is error in year t. If the expected price is from equation 5.2, then ignoring the error term and substituting equation 5.8 into equation 5.7 yields the Nerlovian short-run supply equation

$$O_{it} = \beta ag + \beta bg\,(P/W)_{t-1} + (1-\beta)\,O_{it-1} - (1-\beta)\,(1-g)\,O_{it-2}. \tag{5.9}$$

An equation such as (5.9) ordinarily cannot be estimated with reliability from time series data because of multicollinearity especially caused by two lagged values of the dependent variable. More reliable estimates are obtained using an expectation model such as equation 5.6. Then if the long-term model of supply is

$$O_{it} = b\,(P_i/W)_t^* + c\,I_t + e\,T \tag{5.10}$$

where I is infrastructure in year t and T is technology as represented by a time variable, the equation to be estimated is

$$O_{it} = ag + bg(P_i/W)_t^* + cgH_t + geT + (1-g)\,O_{it-1} \tag{5.11}$$

where the error terms are omitted to save space. This equation can be estimated by the ordinary least squares multiple regression from time series data. The adjustment rate g is calculated from the coefficient 1-g on the lagged variable O_{it-1}. The short-run supply response to P_{it-1} is .5 times the short-run coefficient of bg from equation 5.11. The long-run response b to price is the short-run coefficient bg in equation 5.11 divided by the adjustment rate g. If the equations are in original values, the marginal response to price can be converted into elasticities by multiplying them by the appropriate ratios of price to quantity. If the variables are in logarithms, the coefficients in equation 5.11 will be elasticities. A typical empirical supply elasticity is .1 in the short run and 1.0 in many years if the price increase is maintained. Supply elasticities have been estimated and reported for a number of commodities and countries (Nerlove; Askari and Cummings; Henneberry).

Figure 5.1 illustrates the use of a supply curve for policy analysis. Output initially is q_s and price is p. An increase in the price to p_1 from an increase in demand, an increase in support price, termination of a tax, or from other sources raises the supply quantity to q_{s1}. If the increase in price is 10 percent and the supply elasticity is .1 in the short run and 1.0 in the long run, supply quantity is predicted to increase by 1.0 percent in the short run and by 10 percent in the long run if the higher price is maintained. If the original quantity q_s is 150 kilos and the original price p is 20 rupees, the long-run elasticity implies that a price increase of 2 rupees increases quantity 1.5 kilos in the short run and by 15 kilos if the price increase is maintained for several years.

If the price is decreased to p_2 from the initial price p, the quantity supplied falls to q_{s2}. If the real price decrease from p is 20 percent and price is held at that lower level with the supply elasticity as above, the quantity supplied is expected to fall 2.0 percent in one year and 20 percent in many years.

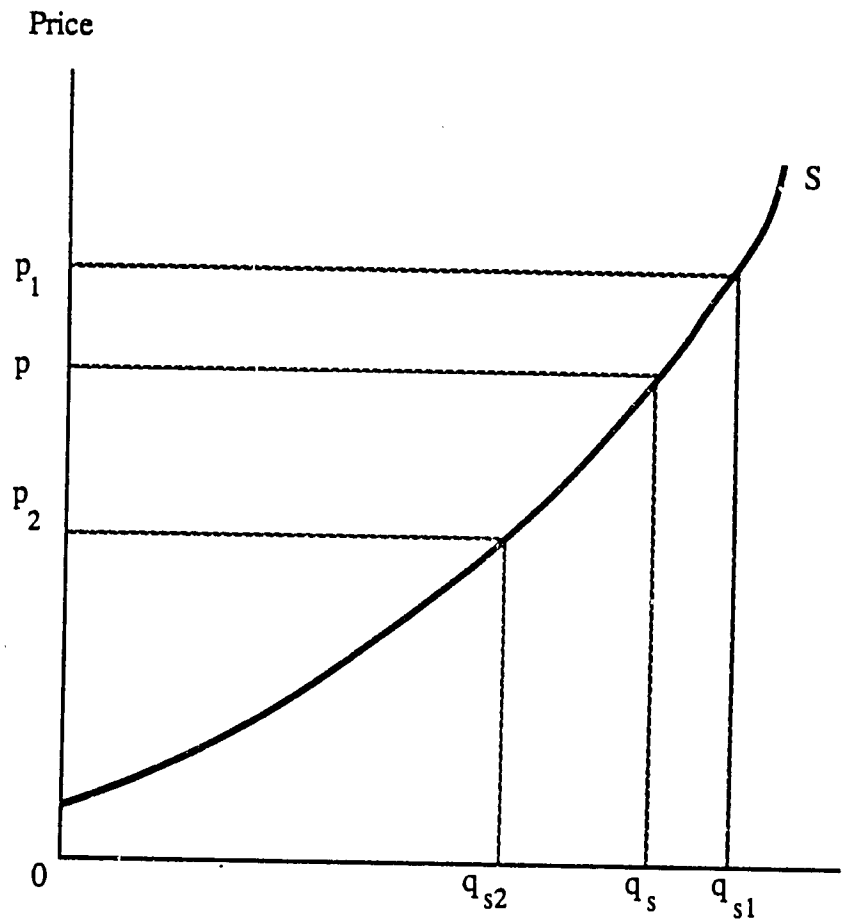

Figure 5.1. Illustration of Supply Curve S

Frequently, the analyst desires more information than just the supply response to price. Suppose it is useful to know the contributions of specific resources and of yield and area components to supply response. Supply response to price is a function of the physical response of output to use of inputs and the behavioral response of input use to price, the latter represented by the input elasticity of demand. The physical response of output to inputs is expressed by the production function

$$O_i = f(X_1, X_2, \ldots, X_n) \tag{5.12}$$

where $X_1, X_2, \ldots, X_n$ refer to inputs such as fertilizer, pesticides, irrigation water, land, labor, and other inputs. The production elasticity of output O_i with respect to any input X_k is

$$E_{ik} = \frac{\Delta O_i}{O_i} / \frac{\Delta X_k}{X_k} = \frac{dO_i}{dX_k} \frac{X_k}{O_i} = \frac{d \ln O_i}{d \ln X_k}$$

and the elasticity of input demand with respect to product price P_i is

$$E_{ki} = \frac{\Delta X_k}{X_k} / \frac{\Delta P_i}{P_i} = \frac{dX_k}{dP_i} \frac{P_i}{X_k} = \frac{d \ln X_k}{d \ln P_i}.$$

It has been shown (Tweeten and Quance) that under specified assumptions:

$$E_{ii} = \sum_k E_{ik} E_{ki}.$$

The elasticity of output supply is the input demand elasticity multiplied by production elasticity summed over all inputs. The contribution of input k to the elasticity of supply is $E_{ik} E_{ki}$.

Various methods can be used to estimate the production function. One method is ordinary least squares. A typical formulation is

$$\ln O_i = \ln b_0 + b_1 \ln X_1 + b_2 \ln X_2 + \ldots + b_n \ln X_n \tag{5.13}$$

where coefficients $b_1, b_2, \ldots, b_n$ are elasticities of production. Taking anti-logarithms, equation 5.13 is the Cobb-Douglas production function

$$O_i = b_0 X_1^{b_1} X_2^{b_2} \ldots X_n^{b_n}. \tag{5.14}$$

Because input quantities tend to move together through time, the high correlation among inputs often causes statistical problems in estimating equation 5.14 directly from time series. Indirect methods such as the "dual" discussed later are used.

In theory, the marginal product of input k is equal to the ratio of input price to product price

$$\frac{dO_i}{dX_k} = \frac{P_k}{P_i}.$$

Multiplying both sides by the ratio of input quantity X_k to output, it is apparent that in competitive equilibrium

$$\frac{dO_i}{dX_k}\frac{X_k}{O_i} = \frac{P_k X_k}{P_i O_i}$$

and the elasticity of production can be measured by the factor share, the right hand side, which can be computed directly from secondary data sources. Because the elasticity of production equals the factor share only in competitive equilibrium, methods have been devised to estimate the elasticity of production from factor shares in an economy in disequilibrium (Tyner and Tweeten).

Input demand equations may be specified in a form similar to that for the supply equation 5.1 but with input quantity the dependent variable. Technology may be a less prominent variable in the input demand equation. Suppose that the input elasticity of demand has been calculated and that the price elasticity of demand for input with respect to input price P_k is equal (sign reversed) to the elasticity of demand with respect to output price P_i. The contribution to supply elasticity of O_i is shown as:

Input	Elasticity of production	Short Run		Long Run	
		Elas.of input dem.	Cont. to sup.elas.	Elas.of input dem.	Cont. to sup. elas.
Fertilizer	.1	.5	.05	2.0	.20
Land	.3	.0	.00	.3	.09
Labor	.3	.1	.03	.5	.15
Machinery	.1	.0	.00	3.0	.30
Irrigation	.2	.1	.02	1.5	.30
	1.0		.10		1.04

A 1 percent increase in the price of commodity i increases fertilizer use by .5 percent in the short run and by 2.0 percent in the long run as indicated by the elasticity of input demand. Because fertilizer is a small proportion of all inputs, its elasticity of production is not large. But because demand for fertilizer is responsive to price, the contribution to supply elasticity of O_i is greater for fertilizer than for labor which has a larger elasticity of production.

Analysts may also find it useful to disaggregate supply response into area and yield components. It can be shown (Tweeten and Quance, p. 349) that the total supply elasticity E_{ii} can be expressed as

$$E_{ii} = E_{yi} + E_{ai}(1 + E_{ya}) \tag{5.15}$$

where E_{yi} is the elasticity of yield with respect to product price P_i, E_{ai} is the elasticity of crop area with respect to P_i, and E_{ya} is the elasticity of yield with respect to area. If crop area is expanded on marginal lands, E_{ya} is negative; if area is expanded on superior lands (say recently irrigated), E_{ya} is positive. If yield and area are independent, then the total supply elasticity of O_i is a simple sum of the yield and area elasticities.

The tie between input demand and product supply is apparent from the following input demand equation

$$X_t = \alpha (P/W)^{\beta_{t-1}} S^{\gamma_{t-1}} \mu_t \qquad \text{(Input demand)}$$

where X_t is aggregate input, P/W is the ratio of output price to input price, S is the stock of relatively fixed assets such as public irrigation facilities and μ is error. Suppose the technology is measured by a productivity index $T = O_t / X_t$, or output per unit of farm inputs and is independent of price. Multiplying by sides of the input demand equation by productivity, the result is

$$X_t (O_t / X_t) = O_t = \alpha (P/W)^{\beta_{t-1}} S^{\gamma_{t-1}} T^{\delta} \mu_t \qquad \text{(Output supply)}$$

where δ estimated in a log-log ordinary least squares equation should be nearly 1.0.

Some problems with estimating the supply equation are apparent. The dependent variable O is also the numerator of the independent variable T, hence simultaneous equation or least squares bias is present. This is avoided by replacing the productivity variable T with a simple time trend, composed of say the last two digits of the current year in time series. But because productivity is the correct variable measured very imperfectly by the time trend, specification error is introduced and is a much more serious shortcoming than is simultaneous equation bias. The obvious way out of the dilemma is to estimate the input demand equation directly. By taking logarithms of the input demand equation

$$\ln X_t = \ln \alpha + \beta \ln P_{t-1} - \beta \ln W_{t-1} + \gamma \ln S_{t-1} + \ln \mu_t$$

and of the output supply equation

$$\ln O_t = \ln \alpha + \beta \ln P_{t-1} - \beta \ln W_{t-1} + \gamma \ln S_{t-1} + \delta \ln T + \ln \mu_t$$

it is apparent that the price elasticity of output supply is equal to the price elasticity of input demand with respect to product price or

$$\frac{\partial O_t}{\partial P_{t-1}} \frac{P_{t-1}}{O_t} = \frac{\partial \ln O_t}{\partial \ln P_{t-1}} = \frac{\partial X_t}{\partial P_{t-1}} \frac{P_{t-1}}{X_t} = \frac{\partial \ln X_t}{\partial \ln P_{t-1}} = \beta$$

which in turn is equal to the elasticity of input demand

$$\frac{\partial X_t}{\partial P_{t-1}} \frac{P_{t-1}}{X_t} = \frac{\partial \ln X_t}{\partial \ln P_{t-1}} = \frac{\partial X_t}{\partial P_{t-1}} \frac{P_{t-1}}{X_t} = \frac{\partial \ln X_t}{\partial \ln P_{t-1}} = -\beta .$$

Unfortunately, measures of input demand frequently are not available and supply response must be estimated from output -- whatever the pitfalls.

Finally, the identification problem must be noted. In supply, quantity is a function of price. Quantity is also a function of price in the product demand equation. If price and quantity in the current year are jointly determined by demand and supply functions in the current year, a simultaneous equation system must be used to estimate parameters. However, if supply quantity is predetermined by expected prices based on previous years' prices or other variables not related to current output, then the single equation approach is appropriate.

The above output supply function is a short-run equation. S is fixed in the short run, but is variable and perhaps responsive to price in the long run. The response of S to price can be estimated separately in an adjustment model. The result can be substituted for S into the short-run supply function above. The long-run response of O to P through S then can be calculated.

The Duality Approach

The "Dual" method is another approach for estimating supply responses. This approach has become popular during the last ten years.

> A reason for the increasing popularity of the use of duality in applied economic analysis is that it allows greater flexibility in the specification of factor demand and output supply response equations and permits a very close relationship between economics and practice. The use of duality allows us to side-step the problems of solving first order conditions by directly specifying suitable minimum cost function or maximum profit function rather than production or transformation functions. An advantage of starting by specifying a cost or profit function rather than the underlying transformation function is that in order to derive the estimating factor demand and output supply responses there is no need to solve any complex system of first order conditions. The behavioral response equations are obtained by simple differentiation of the dual function with respect to input and/or output prices. The major advantage of this is that it implies less algebraic manipulations and, more importantly, it allows us to specify more complex functional forms which impose much less restriction on the estimating equations (i.e., we do not need to impose restrictions on the values of the elasticities of substitution, separability, homotheticity, etc.) [Lopez, p. 353].

The foundations for the dual approach are the indirect profit and cost functions which are obtained from profit maximization and constrained cost minimization (the primal problems).[1]

1. The Profit Function Approach. From the primal solution, factor demand and product supply equations can be derived from profit maximization. Equation (5.16) represents the profit function for a single product (the direct profit function).

$$\P = P \cdot Y - \Sigma_{i=1}^{n} r_i x_i \tag{5.16}$$

where

P = price of output
Y = the quantity of output
r_i = the price of input i
x_i = the quantity of input i, and

the production function is

$$Y = f(x_1, \ldots, x_n). \tag{5.17}$$

Duality theory uses an indirect profit function which is defined as the maximum profit associated with given output and input prices. One way to derive the indirect profit function is to obtain factor demand and product supply functions from primal solution, that is, from equation 5.16 (assuming one output and two inputs)

$$\frac{\partial \P}{\partial x_1} = 0 \Rightarrow x_1^* = x_1^* \ (P, r_1, r_2) \tag{5.18}$$

$$\frac{\partial \P}{\partial x_2} = 0 \Rightarrow x_2^* = x_2^* \ (P, r_1, r_2) \tag{5.19}$$

where x_1^* and x_2^* are profit maximizing levels of inputs. Substituting for x_1 and x_2 from (5.18) and (5.19) into (5.17) we obtain the profit maximizing level of output (Y^*)

$$Y^* = f(P, r_1, r_2) \tag{5.20}$$

and substituting (5.18), (5.19), and (5.20) into (5.16) gives

$$\tilde{\P} = P \cdot Y^* (P, r_1, r_2) - r_1 x_1^* (P, r_1, r_2) - r_2 x_2^* (P, r_1, r_2)$$

or

[1]Subsequent discussion of the dual is based on Beattie and Taylor, pp. 227 - 236.

$$\tilde{\pi} = \tilde{\pi}\ (P, r_1, r_2). \tag{5.21}$$

Equation (5.21) is an indirect profit function. Note that this profit function shows profit as a function of prices only and not quantities. There is an advantage in estimating an indirect profit function that is a function of prices (and quantities of fixed factors of production) compared to estimating a production function (or revenue function) that expresses quantity of output as a function of quantities of all inputs. The advantage is that no endogenous variable (output or input levels) is included in the indirect profit function as an explanatory variable and therefore simultaneous equation problems are avoided in the econometric estimates (Lopez).

This becomes especially important when estimating output elasticities with respect to the fixed factors of production. The reduced form elasticities obtained from an indirect profit function (the duality approach) reflect the output supply response of a profit-maximizing, price-taking firm assuming prices of variable factors and quantities of other fixed factors remaining constant. However, unlike the production function elasticities, they do allow for the adjustment of the quantities of variable factors to an increase in the "fixed" factor. As the quantity of a fixed factor (such as capital) increases, the marginal productivity of all variable factors is expected to rise. This increase in the marginal productivity will shift the factor demand curves to the right, and as a result the profit maximizing firm will employ more of the variable factors. Under these conditions the *mutatis mutandis* elasticities obtained from the duality approach seem to be more appropriate in policy analysis than the *ceteris paribus* elasticities obtained from a direct product function (Lau and Yotopoulos, p. 17).

An important concept in duality is the Envelope Theorem. A notable result of the Envelope Theorem is that by taking partial derivatives of the indirect profit functions with respect to the price of output and the prices of inputs we obtain the output supply and input demand equations, that is

$$\frac{\partial \tilde{\pi}}{\partial P} = Y^*\ (P, r_1, r_2) \tag{5.22}$$

$$\frac{\partial \tilde{\pi}}{\partial r_i} = -x_i^*\ (P, r_1, r_2). \tag{5.23}$$

Equation (5.22) represents an output supply equation while (5.23) represents input demand equations.

> The convenience of duality should be apparent. If we know the indirect profit function we can get the unconditional factor demand and product supply functions by simple partial differentiation -- quite an analytical advantage, indeed [Beattie and Taylor, p. 226].

The following example from Lau and Yotopoulos is given for further clarification of the duality approach. Assume a firm's production function as

$$Y = f(x_1, \ldots, x_n; z_1, \ldots, z_m) \tag{5.24}$$

where

Y = quantity of output
x = quantity of variable inputs, and
z = quantity of fixed inputs.

Substituting (5.24) into (5.16) we will have

$$\P = P \cdot f(x_1, \ldots, x_n; z_1, \ldots, z_m) - \Sigma_{i=1}^{n} r_i x_i \tag{5.25}$$

where:

$\P$ = profit defined as current revenues less current total variable costs
P = unit price of output
r_i = unit price of the ith variable input.

Dividing both sides of equation 5.25 by P we have

$$\P' = \frac{\P}{P} = f(x_1, \ldots, x_n; z_1, \ldots, z_m) - \Sigma_{i=1}^{n} r_i' x_i \tag{5.26}$$

where

$\frac{\P}{P}$ = $\P'$ is defined as the unit-output-price (UOP) profit , and

r_i' = $\frac{r_i}{P}$ is the normalized price of the ith input.

Let us rewrite equations 5.18 and 5.19 as

$$x_i^* = f_i(r', z), \qquad i = 1, \ldots, n \tag{5.27}$$

where

x_i^* = the optimal quantities of variable input i
r' = the vector of normalized price of variable inputs, and
z = the vector of fixed inputs.

By substituting (5.27) into (5.25) we have

$$\tilde{\P} = P \cdot [f(x_i^*, \ldots, x_n^*; z_1, \ldots, z_m) - \Sigma_{i=1}^{n} r_i' x_i^*] \tag{5.28}$$

Equation 5.28 gives the maximized value of the profit for each set of values for P, r_i', and z.

Rewriting (5.28) as a function of r' and z (from 5.27)

$$\tilde{\P} = P \cdot G^* (r_1', \ldots, r_n'; z_1, \ldots, z_m) \tag{5.29}$$

the UOP profit function is therefore given by

$$\frac{\tilde{\P}}{P} = \tilde{\P}' = G^* (r_1', \ldots, r_n'; z_1, \ldots, z_m). \tag{5.30}$$

From $\tilde{\P}'$ (the UOP profit function) one can always find $\tilde{\P}$ and visa versa. It is known that the UOP production function is decreasing and convex in the normalized prices of variable inputs and increasing in quantities of fixed inputs and in the money price of the output.

From the Envelope Theorem

$$x_i^* = -\frac{\partial \P'(r', z)}{\partial r_i'} \qquad i = 1, \ldots, n \tag{5.31}$$

$$Y^* = \tilde{\P}'(r', z) - \Sigma_{i=1}^{n} \frac{\partial \P'(r', z)}{\partial r_i} \cdot r_i' \tag{5.32}$$

where (5.32) is the supply function.

Derived input demand functions (equation 5.31) and the UOP (sometimes referred to as normalized) profit function (equation 5.30), or alternatively the supply function (equation 5.32) must be estimated jointly because of the existence of parameters that are common to both equations. In most empirical studies a method developed by Zellner is used which imposes the restriction of equality of parameters in these equations.

The advantages of working with the UOP profit function instead of the traditional production function are summarized as follows (Lau and Yotopoulos, pp. 12-13):

1. The input demand and output supply functions (equations 5.31 and 5.32) can be directly derived from an *arbitrary* normalized indirect profit function (UOP profit function) which is decreasing and convex in the normalized variable input prices and increasing in the fixed factors of production. Hence, explicit specification of the production function (equation 5.24) is not needed. And therefore, without solving for first order conditions for a profit-maximizing firm ($P \cdot \frac{\partial f}{\partial x_i} = r_i$ or $MP_i = r_i'$) the input demand functions can be derived. This provides flexibility in empirical analysis.

2. By the duality approach, because the derived factor demand and output supply functions are obtained from a profit function, the assumptions of profit maximization and competitive markets are assured.

3. As was mentioned before, the simultaneous equation bias is avoided because the profit, output supply, and input demand functions are explicitly written as functions of exogenous variables (output and variable input prices and quantities of fixed factors) which are considered to be determined independently of the firm's behavior.

The indirect profit functions associated with various single-output, multi-input production functions (Generalized Cobb-Douglas, Quadratic, CES) are given in Henneberry (1986).

2. The Cost Function Approach. This approach has been used to measure factor demand elasticities, elasticities of substitution and technical change in agriculture (Lopez, p. 354). In this method an indirect cost function is defined as the minimum cost required to produce a given level of output at given factor prices. Equation 5.33 represents an indirect cost function:

$$\tilde{c} = \tilde{c}\ (r_1, \ldots, r_n, y) \tag{5.33}$$

Another important result of the Envelope Theorem (usually referred to as Shephard's Lemma) is that by partial differentiating of the indirect cost function (equation 5.33) with respect to input prices we obtain the input demand functions, that is:

$$\frac{\partial\ \tilde{c}}{\partial\ r_i} = X_i^c\ (r_1, \ldots, r_n, y) \tag{5.34}$$

where X_i^c is the conditional factor demand equation (conditional on the level of output), also:

$$\frac{\partial\ \tilde{c}}{\partial\ y} = MC(r_1, \ldots, r_n, y) \tag{5.35}$$

which is the marginal cost function. By equating 5.35 to price we obtain the product supply equation:

$$Y^* = Y^*\ (P, r_1, \ldots, r_n) \tag{5.36}$$

The indirect cost functions associated with various single-output, multi-input production functions (Cobb-Douglas, Quadratic, CES) are given by Henneberry.

3. Limitations. An important limitation of the cost function approach of the dual method is that:

> ... it assumes that output levels are not affected by factor price changes and, thus, the indirect effect of factor price changes (via output levels) on factor demands are ignored. Moreover, the inclusion of output

> levels as explanatory variables may lead to simultaneous equation biases if output levels are not indeed exogenous. The profit function approach allows one to overcome most of these problems although at the cost of requiring a stronger behavioral assumption. The profit maximization assumption may be substantially more difficult to support in agriculture than simple cost minimization because of risk related problems which are mainly related to the variability of output yields and price rather than to costs of production [Lopez, pp. 356-357].

Limitations of the application of the duality approach (indirect profit or cost functions) are:

1. The duality approach is best suited for the micro-firm level and it should be applied to data from a profit-maximizing individual firm. However, many of the empirical estimations in agricultural economics have applied the duality approach to aggregate data in order to derive output supply and input demand functions for a country or region. The appropriateness of the results is therefore questionable.

2. In most cases, it is difficult to determine whether the estimated elasticities from the dual method are short-run or long-run elasticities. One approach used to distinguish short- and long-run elasticities has been to estimate the indirect profit or cost functions with and without fixed factors of production. Including fixed factor presumably estimates short-run elasticities. If the indirect profit or cost functions do not include any fixed inputs, then the estimated elasticities are assumed to be long-run. The dilemma is that designation by the analyst of fixed and variable factors is arbitrary.

3. The dual approach assumes a profit maximizing or cost minimizing production unit. As a result, this approach may not be the appropriate to depict behavior of subsistence farmers in developing countries . These farmers may be risk minimizers rather than profit maximizers or cost minimizers

4. Price expectations, lagged adjustment, and numerous supply shifter variables have not yet been incorporated into the dual approach. It is appropriate to include expected prices rather than current prices in the indirect profit function.

The dual method is conceptually elegant and has wide appeal to those analyzing supply. However, the dual must be used with caution for reasons listed above.

A Matrix of Own- and Cross-Price Elasticities of Supply for Pakistan

A wise strategy in estimating supply parameters is to keep econometric models as simple as possible but to use economic theory to complete a matrix of needed coefficients. Tables 5.1 and 5.2, showing own-price and cross-price elasticities of supply for specific commodities for Pakistan, are based on supply responses calculated econometrically but

adjusted as judged appropriate by results of other studies. Where cross-elasticity of supply coefficients were unavailable, economic theory was used to solve for missing coefficients based on own-price elasticities shown in the text and revenue shares. Revenue share weights are 24 percent for wheat, 10 percent for rice, 13 percent for cotton, 5 percent for sugarcane, 28 percent for livestock and livestock products, and 20 percent for miscellaneous crops for a total of 100 percent.

1. Own-Price Elasticity. Own-price elasticities are diagonal elements and cross-price elasticities are off-diagonal elements in Tables 5.1 and 5.2. Cotton is used as the example for interpretation of some supply elasticities.

The own-price elasticity of cotton supply is .30 in the short run (1-2 years) and 1.00 in the long run (5 years or more). A 10 percent increase in cotton price PC with all other commodity and input prices held constant is predicted to increase cotton production OC by 3.0 percent in the short run and 10.0 percent in the long run.

2. Cross-Price Elasticity. The cross-price elasticity of sugar supply with respect to the price of cotton is -.043 in the short run and -.114 in the long run. A 10 percent increase in the price of cotton PC is predicted to reduce sugar output OS by .43 percent in the short run and by 1.14 percent in the long run. Alternatively, a 10 percent increase in the price of sugar PS is predicted to decrease cotton output OC by .15 percent in the short run and by .33 percent in the long run as measured by elasticities of cotton output with respect to sugar price.

3. Aggregate Cotton Response. The sum of own-price and cross-price elasticities is zero for cotton as apparent in the total of all elasticities in row OC. That is, a proportional increase in all prices paid and received by farmers does not increase cotton output. A 10 percent increase in cotton price increases output 10.0 percent in the long run, and a simultaneous increase in other commodity prices increases cotton output by 10 times the sum of the cross-price elasticities, 2.0 percent. The total increase in cotton output, 10.0 + 2.0 = 12.0 percent, is exactly offset by the increase in prices paid by farmers W, 12.0 percent, as indicated by the elasticity - 1.20 in the W column and OC row.

4. Aggregate Output. The lower right hand coefficients in Tables 5.1 and 5.2 are aggregate supply response to the index of prices paid by farmers (negative sign) and to the index of prices received by farmers (positive sign). The aggregate supply elasticity is .15 percent in the short run and .60 percent in the long run. If the index of all prices received by farmers increases by 10 percent and remains at that higher level with all input prices

Table 5.1. Short Run (1-2 years) Own-Price and Cross-Price Elasticities of Supply in Pakistan[a]

Quantity	Price						
	PC	PS	PR	PW	PM	PL	W
OC	.300	-.015	-.010	.000	.000	.000	-.275
OS	-.043	.300	-.009	.000	.000	.000	-.248
OR	-.028	-.009	.200	.000	.000	.000	-.119
OW	-.020	-.007	-.004	.150	.000	.000	-.119
OM	-.020	-.007	-.004	.000	.000	.000	-.119
OL	-.020	-.007	-.004	.000	.000	.150	-.119
OT	.019	.008	.015	.036	.030	.042	-.150

[a]Definitions of variables:

C = Cotton
S = Sugarcane
R = Rice
W = Wheat
M = Miscellaneous (other) crops
L = Livestock and livestock products
T = Total agricultural production
O = Output (production)
P = Price
W = Index of prices paid by farmers. Because the elasticities sum to zero for each commodity (row) in the above tables, it follows that the supply elasticity with respect to W is equal to the sum of the elasticities (sign reversed) with respect to commodity prices received by producers PC, PS, PR, PW, PM, and PL. Thus a 1 percent decrease in W has the same impact on output as a 1 percent increase in all prices received by producers.

Table 5.2. Long Run (5 years or more) Own-Price and Cross-Price Elasticities of Supply in Pakistan[a]

Quantity	Price						
	PC	PS	PR	PW	PM	PL	W
OC	1.000	-.033	.000	.114	.048	.073	-1.202
OS	-.114	.900	.000	.092	.039	.059	-.976
OR	-.074	-.017	.600	.060	.025	.038	-.632
OW	-.045	-.010	.000	.400	.015	.023	-.383
OM	-.059	-.014	.000	.048	.500	.030	-.505
OL	-.058	-.013	.000	.046	.020	.500	-.495
OT	.078	.030	.060	.144	.120	.168	-.600

[a]Variables defined in Table 5.1.

unchanged, total output is predicted to increase 1.5 percent in one to two years and 6.0 percent in five years or more.

The increase of total farm output by .15 percent in the short run from an increase of 1 percent in the index of prices received by farmers is the weighted average of the sum of a .275 percent increase in cotton output OC, .248 percent increase in sugar output OS, .163 percent increase in rice output OR, and so forth reading down the right hand column of Table 5.1 for a total of .150. In the long run, the total elasticity .60 is the weighted sum of a 1.202 percent increase from OC, .976 from OS, .632 from OR, and so forth reading down the right hand column of Table 5.2. The interpretation is the same for an increase in the index of prices paid by farmers W but with signs reversed to negative values.

The last row in Tables 5.1 and 5.2 shows the percentage increase in aggregate output in Pakistan resulting from a 1 percent increase in the respective commodity prices. Again, all elasticities sum to zero; a proportional increase in all prices paid and received by farmers does not influence aggregate output. Although own-price elasticities for cotton, sugarcane, and rice are greater than for wheat, miscellaneous crops, and livestock, the latter commodities have a greater impact on total output from a 1 percent increase in their respective prices because they account for a greater share of total output. For example, a 1 percent increase in cotton price increases aggregate output of all commodities by .019 percent in the short run (Table 5.1) and .078 percent in the long run (Table 5.2). The same percentage increase in the price of livestock and livestock products raises aggregate output of agriculture in Pakistan by .042 percent in the short run and by .168 percent in the long run.

5. ***Use of Matrix for Prediction.*** Tables 5.1 and 5.2 are especially useful for econometric analysis of market and policy impacts from changes in prices. All price and quantity series may be viewed as logarithms or as normalized to the same initial index of 100. Coefficients are elasticities. The elasticities may be converted into coefficients suitable for other price and quantity units by multiplying elasticities by the appropriate price and quantity units. For example, the cross elasticities can be converted into coefficients of price response by multiplying them by the ratio of output to price. The marginal response of commodity output to a change in price is

$$\frac{\partial O_i}{\partial P_j} = E_{ij} \frac{O_i}{P_j}$$

where O_i and P_j are in units chosen by the analyst to fit the data at hand.

For prediction, it is frequently desirable to account for non-price elements. A prediction equation can be formed by adding to the price coefficients from Table 5.1 or 5.2 a time, technology, and/or fixed investment impact. For the latter, this analysis suggests that irrigation capabilities enhanced by public investment projects are important. For technology, introduction of high-yielding varieties are important. Analysis performed but

not shown indicates that adoption of high-yielding varieties is price-sensitive, hence the impact of price needs to be accounted for if improved varieties are made available.

Price elasticities of supply are not shown for the intermediate run of two to five years. These may be approximated by a linear interpolation between elasticities in Tables 5.1 and 5.2.

Economists have long posited that the supply curve for agricultural output is not reversible. That is, farmers expand output in response to a higher price along a supply curve that is relatively flat. When prices fall, farmers do not abandon the durable capital inputs they purchased when prices increased, hence they reduce output along a relatively steeply sloped (inelastic) supply curve. The choice method to segment price is to include a dummy independent variable D = 1 for years of rising prices and D= 0 for years of falling prices. The dummy independent variable D can be included in a least squares equation as is on interacting with price , i.e. DP_i. A significant coefficient on D alone indicates the intercept is significantly different for rising prices than for falling prices. A significant coefficient on DP_i indicates a slope significantly different for rising prices than for failing prices. Using the above and several other approaches, Tweeten et al. (1981) in extensive analysis for the United States found no evidence that the supply curve is irreversible. Until evidence is found to the contrary, it may be assumed that the elasticities in Tables 5.1 and 5.2 apply to either falling or rising prices.

The procedure for computing cross-price elasticities is given below where parameters are defined as

$$E_{ij} = \frac{\partial O_i}{\partial P_j} \frac{P_j}{O_i}. \tag{5.37}$$

$$E = \frac{\partial O}{\partial P} \frac{P}{O} \tag{5.38}$$

$$r_i = \frac{P_i}{P} \frac{O_i}{O} = \frac{\partial P}{\partial P_i} \frac{P_i}{P} = \alpha_i \tag{5.39}$$

E_{ij} is the elasticity of output of commodity i with respect to price of commodity j, O is aggregate farm output, and P is the index of prices received by farmers for commodities. Hence, (5.37) is the own- (i = j) or cross- (i ≠ j) price elasticity of commodity supply, (5.38) is aggregate supply elasticity, and (5.39) is revenue share of output i with respect to the total value of output.

For Pakistan, econometric estimates were first made of own-price elasticities E_{ii}, the revenue share r_i, and aggregate elasticity E (.15 in short run and .10 in long run) because these were the only parameters that could be estimated directly with acceptable reliability. These values were then substituted into equation 5.40 which was solved by the Newton-Raphson method for γ_w but can also be solved by trial and error.

$$-2E = \Sigma\{-(\alpha_i E \gamma_w + E) + [(\alpha_i E \gamma_w + E)^2 - 4 r_i E \gamma_w E_{ii}]^{1/2}\}. \tag{5.40}$$

$$E_{iw} = \frac{-(\alpha_i E \gamma_w + E) + [(\alpha_i E \gamma_w + E)^2 - 4 r_i E \gamma_w E_{ii}]^{1/2}}{2r_i} \tag{5.41}$$

$$E_{ii} = -r_i E_{iw}^2 / E \gamma_w - \alpha_i E_{iw} - E_{iw} / \gamma_w \tag{5.42}$$

$$E_{ij} = -r_j E_{iw} / E_{jw} / E \gamma_w - \alpha_j E_{iw}; \qquad i \neq j. \tag{5.43}$$

The value of γ_w calculated from equation 5.40 is used to estimate the elasticities E_{iw} of output i with respect to prices paid by farmers W from equation 5.41. These values in turn can be inserted into equation 5.43 to determine the cross-price elasticities of supply.

The supply functions are homogeneous of degree zero in prices, hence the sum of the elasticities in any supply equation sum to zero as noted in equation 5.44.

$$E_{iw} = \Sigma_{j=1}^{n} E_{ij} \qquad (i = 1, 2, \ldots, n) \tag{5.44}$$

$$E_{wj} = \Sigma_{i=1}^{n} r_i \; E_{ij} \qquad (j = 1, 2, \ldots, n) \tag{5.45}$$

Also, the sum of the elasticities weighted by revenue shares is equal to the elasticity of aggregate supply (or input demand) with respect to commodity prices. This is the basis for calculation of the last, the aggregate, row in a matrix of supply elasticities.

Given the system of coefficients derived, the following system of equations 5.46 can be solved for the constants $K_0, K_1, \ldots K_{n+1}$ which predict output in a historic period

$$\begin{array}{lllll} E_{11} \ln P_1 + E_{12} \ln P_2 + \ldots + E_{1w} \ln W + \ln K_1 & = \ln O_1 \\ E_{21} \ln P_1 + E_{22} \ln P_2 + \ldots + E_{2w} \ln W + \ln K_2 & = \ln O_2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots & = \vdots \\ E_{n1} \ln P_1 + E_{n2} \ln P_2 + \ldots + E_{nw} \ln W + \ln K_n & = \ln O_n \\ E_{w1} \ln P_1 + E_{w2} \ln P_2 + \ldots + E \ln W + \ln K_{n+1} & = \ln O \end{array} \tag{5.46}$$

and where O is aggregate output. Time trend variables could be added to depict supply functions over time.

The system of equations was estimated empirically for Pakistan. Own-price elasticities were derived from simple least squares or autoregressive least squares equations placing minimal demands on available data. As an example, the aggregate supply response used (along with estimates from other sources) to estimate E was estimated from annual national data in logarithms with results as follows:

Dependent Variable: OTA

Variable	Parameter Estimate	Standard Error	Prob > | t |
Intercept	-3.3484	0.9238	0.0019
IRP	0.9704	0.0851	0.0001
HYV	0.0197	0.0062	0.0054
WR19	0.3522	0.2049	0.1028

n = 22	R^2 = .976	DW = 1.646	ESR = .176
(1961/62 to 1982/83)	$\bar{R}^2$ = .972	rho = .146	ELR = .352

where

OTA = Production of agriculture, contribution to GDP, Pakistan, billion 1959/60 rupees
IRP = Irrigation water, farmgate, Pakistan, million acre feet
HYV = Area in high-yielding varieties of rice and wheat, Pakistan, percent
PIM = Prices received by farmers index for marketed output, Pakistan, 1959/60 = 100
PGD = Price index, GDP deflator, Pakistan, 1959/60 = 1.0.
R19 = PIM/PGD

All variables in statistical equations are in natural logarithms for year t unless preceded by L for year t-1. WR19 is a weighted average of past prices PIM/PGD. Weights are .5 for year t - 1, .33 for year t - 2, and .17 for year t - 3. Because the Durbin-Watson (DW) statistic indicated autocorrelation in residuals, the equation was re-estimated with autoregressive least squares but without any improvement in results.

The R^2s indicate that 97.6 percent of the variation in OTA was accounted for by the independent variables without adjustment for degrees of freedom and 97.2 percent with adjustment for degrees of freedom. ELR, the long-run elasticity of supply can be read directly from the coefficient of WR19 because the equation is in logarithms. The short-run elasticity of supply ESR is half of the long-run elasticity and is .176 as noted above.

ACTIVITIES

1. Using revenue shares, own-price elasticities, and formulas as shown in Chapter 5, reproduce the cross-price elasticities and the entire matrix of Tables 5.1 and 5.2 on a spreadsheet.

2. Calculate the values of the constants ln K_i and K_i assuming that each of the prices is 100 and OC = 50, OS = 75, OR = 25, OW = 60, OM = 100, OL = 30 and OT = 90 in Tables 5.1 and 5.2.

3. Estimate a supply function for some commodity using data for a country of your choice.

   (a) Would estimation in original values or logarithms be most appropriate?

   (b) Is a distributed lag model appropriate? Why?

   (c) What are advantages and disadvantages of using more years of time series data to estimate the supply equation?

   (d) What criteria are appropriate for determining whether the estimated supply equation is a good representation of the true supply function? On what basis do you decide which empirical equation is "best"?

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6. CLASSICAL WELFARE ANALYSIS

by Luther Tweeten

Classical welfare analysis refers to use of supply and demand relationships to determine the level and distribution of gains and losses among consumers, producers, taxpayers, and society from changes in economic policy. The technique is useful for policy analysts determining an economically efficient allocation of resources and determining whether it is appropriate or not to intervene in markets. The technique is useful for analysts to estimate who gains and who loses from market failure and from government distortions of markets. With appropriate modification, classical welfare analysis is suited to analyze equity (distribution) issues as well as economic efficiency.

In contrast to conventional marginal analysis which shows economic efficiency only at the margin, classical welfare analysis shows whether inefficiency entails loss of $1 or $1 billion. The strong assumptions (some listed later) required for classical welfare analysis caused it to fall into disuse for decades. It has been revived because it is so well suited to empirical applications. The technique applies not just to market economies. Diminishing returns, scarce resources, and foregone opportunities expressed by the supply curve and diminishing incremental satisfaction from greater consumption of a commodity as expressed by the demand curve characterize centrally planned economies and market economies alike.

OBJECTIVES

The objectives of this chapter are to:

1. Illustrate conceptually how demand and supply can be used to measure economic efficiency and the distribution of costs and benefits of economic interventions among consumers, producers, and taxpayers.
2. Provide specific examples of where government interventions in markets are appropriate and where inappropriate.

3. Show actual empirical applications of classical welfare analysis to issues in developing countries.
4. Provide empirical examples of classical welfare analysis including equity as well as efficiency by employing estimates of the marginal utility of income.

KEY POINTS

1. Classical welfare analysis is a powerful conceptual and empirical tool for determining who gains and who loses from changes in public policy.
2. Government interventions in markets are often appropriate for (a) providing public goods such as agricultural research and infrastructure, (b) bringing private benefits (or costs) in line with social benefits (or costs), (c) reducing monopoly power and for dealing with natural monopolies, and (d) providing for those who are unable to be supported by themselves, their families, or by others.
3. Government interventions reduce national income and are ordinarily inappropriate when used to (a) arbitrarily raise or lower farm and food prices from normal, border prices, (b) establish quotas or other restrictions on agricultural exports or imports, (c) cause a departure of exchange rates from market clearing levels, and (d) subsidize inputs, exports, or other goods or services except as noted in (2) above.
4. Government interventions such as land reform or funding of investment projects with conventional benefit-cost ratios below 1.0 may be appropriate under some circumstances where redistributions are from the wealthy to the poor, but the most efficient redistribution policy ordinarily is a direct income transfer.
5. The foundation for public policies promoting equity in distribution of income and resources is universal human resource investments in schooling and basic health care -- measures which increase access to opportunity for all including capability to participate in making the rules that guide the economic system.
6. In many situations, the issue of whether it is appropriate or inappropriate for government to intervene in the market is unclear. In such instances, historical experience indicates the wisest decision is often to rely on the private market because government interventions frequently have generated greater social losses than the market failures those interventions were designed to correct.

CONCEPTS AND APPLICATIONS

In this section we use classical welfare analysis to illustrate where public interventions may be appropriate; then examine where public interventions in markets tend to reduce gross domestic product. But first it is necessary to understand the concepts of classical welfare analysis.

Understanding the Concepts of Classical Welfare Analysis

Publicly employed economists who lay out options for those who make policy decisions need to include in their options those which have a high probability of increasing well-being of people (society), along with other options to serve narrower objectives such as increased sector income, employment, or stability. An option to increase well-being requires a workable concept to specify the impact on national well-being of changes in policy. Well-being is influenced by income level, distribution, and variability. These variables in turn are respectively related to economic efficiency, equity, and security. The first two are discussed below.

1. Measuring Economic Efficiency and the Distribution of Economic Gains and Losses. Classical welfare analysis with industry supply S and demand D curves in Figure 6.1 shows the efficiency of competitive market output. If a perfectly discriminatory monopolist charges consumers the maximum demand price d_1 for the first unit(s) q_1 of a commodity, the revenue from the first unit(s) is d_1q_1. The maximum price for the second unit is d_2, hence revenue from it is d_2q_2. Revenue from the third unit is d_3q_3. Continuing this procedure until price is zero (demand curve D intersects the horizontal axis at q_{14}), it is apparent that the demand curve is a marginal revenue curve. The area beneath it is total or gross revenue, and total revenue is greatest with quantity q_{14}.

The marginal cost of producing the first unit(s) q_1 is s_1, hence the total variable cost for the first unit(s) is s_1q_1. The marginal cost of the second unit is s_2 and of the third unit is s_3, hence total variable costs for these units are s_2q_2 and s_3q_3, respectively. The sum of these marginal variable costs, the total variable cost, is the area beneath S in Figure 6.1.

Net incremental revenue, the difference between the demand price d and the marginal cost s for each quantity, is positive proceeding from 0 to q_8. Net revenues for incremental quantities to the right of q_8 are negative because the marginal cost exceeds marginal revenue. It follows that for a perfectly discriminating monopolist, maximum net revenue is triangle $d_1d_8s_1$ forthcoming by producing and consuming q_8. The demand curve shows the maximum price consumers would pay for various quantities of a commodity. The demand curve is a measure of the incremental benefit derived by consumers from another unit of the commodity. Moving to the right along the demand curve, price falls because less and less satisfaction is derived from consuming more units of q.

If producers are rational and supply additional output when the supply price covers marginal cost, then the incremental costs $s_1, s_2, \ldots, s_{14}$ trace out the supply curve. The supply price may be viewed as the opportunity cost or incremental value to society foregone by use of resources for producing and consuming commodity q rather than other goods and services. The supply price (marginal cost) rises because the first units of a farm commodity, for example, can be produced at low cost on very productive land. As more units are produced, less productive land and other resources are brought into production. As additional variable resources are shifted from producing other goods and services to

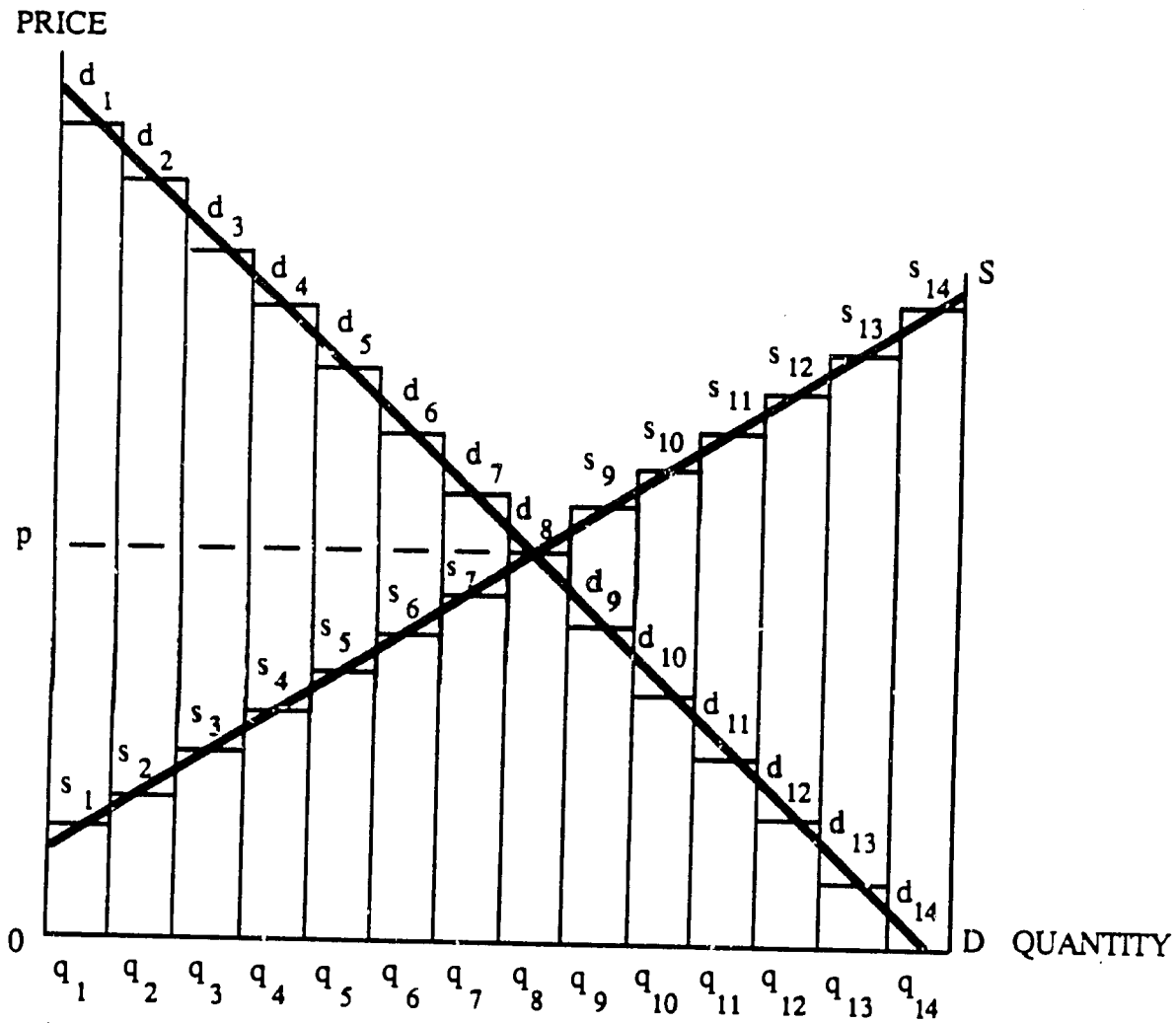

Figure 6.1. Economic Costs and Benefits Illustrated by Supply and Demand Curves.

q, incremental costs rise and less goods and services other than q are available. Fixed costs are irrelevant and are not included because they are committed to q and cannot be used to produce other commodities; they do not influence the decision to produce q.

Because a competitive market results in output q_8 where D and S intersect, it follows that such a competitive allocation maximizes net revenue or net social benefit which is real national income. If the marginal utility of income is constant, each dollar of net revenue can be interpreted as utility. It follows that the competitive output also maximizes net utility under the assumption that the marginal utility of income is constant and equal among people.

Whereas a perfectly discriminating monopolist would receive all net revenue, competitive markets divide net social benefit among many consumers and producers. The area above the price p and below the demand curve D, the amount consumers would be willing to pay for q in excess of what they are required to pay, is called *consumer surplus* and is triangle d_1d_8p in Figure 6.1. The area below equilibrium price p and above the supply curve S, the amount producers are paid in excess of variable costs of production is called *producer surplus* and is the area of triangle ps_8s_1 in Figure 6.1. Producer surplus is gross revenue to producers less total variable cost.

Consumer surplus plus producer surplus equal net social benefit. Net social benefit is maximized at the equilibrium quantity under workable competition. It is important to note that the equilibrium price and quantity under classical welfare analysis depends on the initial distribution of resources. The allocation at q_8 is the efficiency of a *Pareto optimum*, an allocation from which it is not possible to make someone better off without making someone else worse off. There are many Pareto optimums, depending on the initial distribution of resources before markets work. The global maximum net social gain which maximizes the common good or utility of society is also a Pareto optimum. Although the allocation in Figure 6.1 is described as a *market* equilibrium, the same principles apply for a socialist, barter, or other economy equilibrium. The final distribution of net social benefits will be most equitable if the initial distribution of resources and access to opportunity are most equitable -- a subject to be examined later.

Given time all resources become variable and values are bid up to remove pure profit. With producer surplus zero in the long run, it follows that consumers receive all net social benefits. This outcome is called consumer sovereignty in economics.

Classical welfare analysis is a powerful tool of economic policy analysis. It is sufficiently flexible to encompass not only social costs from an inefficient output (foregone net social benefit or deadweight loss) as can be analysed with Figure 6.1 but also from an inefficient resource mix (x-inefficiency), from excessive spoilage and administrative cost, and from lost output in resources "wasted" in political-economic seeking of transfers

(PEST) activities.[1] This cardinal approach requires stronger assumptions than Pareto-partial analysis which reveals inefficiency only as disequilibrium at the margin. Pareto-partial marginal analysis is capable of indicating presence of inefficiency but not its magnitude. Classical welfare analysis also shows disequilibrium at the margin but adds marginal units to indicate impacts on national income.

Classical welfare analysis is sufficiently flexible to estimate the net social benefits from *workable competition*, a term which recognizes that perfect competition is an abstraction and that economic efficiency must be measured in an environment of imperfect information, varying degrees of mobility of resources, and sometimes of an industry able to achieve economies of size with only one or a few firms. Workable competition does not require resources to be perfectly mobile but requires resources to adjust to other uses if incremental benefits exceed costs of doing so. Workable competition does not require perfect knowledge but requires information to be collected and utilized if marginal benefits exceed costs of doing so. If costs are sufficiently lower with monopoly than with atomistic (many buyers and sellers) competition, then a well-performing monopoly may be preferred over inefficient atomistic competition — based on classical welfare analysis. In short, classical welfare analysis can be used to measure efficiency even when the assumptions of perfect competition are not met.

2. ***Limitations***. The Marshallian concepts of producer surplus and consumer surplus have been controversial as well as useful. Major criticisms are listed below

1. Consumer surplus must be estimated from a demand curve which ordinarily is estimated from time series data. The historic price-quantity data used to estimate the demand curve are likely to be narrow in range. The shape of the demand curve outside the range of historic price-quantity data is unknown. Consumer surplus outside the historic price-quantity data range can be measured imperfectly at best. The appropriate means to alleviate the shortcoming is to estimate consumer surplus only within the historic price-quantity data range and only as deviations from equilibrium as shown in the illustrations to follow.
2. A second major criticism of consumer surplus is that price as a measure of marginal utility and the area beneath the demand curve as a measure of total utility requires constant marginal utility of income. Ideally, consumer surplus would be estimated from an income-compensated demand curve. Such a curve

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[1]The term "x-inefficiency" is often used to refer to inexplicable cost increases characteristic of absence of a competitive environment but here may also be interpreted as deadweight losses in input markets which shift the commodity supply price upward.

PEST activities include lobbying and other means to obtain favors from government. Such activities require resources which have value in other uses, hence entail real costs and are not merely transfers.

is not easily estimated. If the change in income is small from changes in consumer and producer surpluses being considered, error introduced by failure to adjust demand for changing marginal utility is small. Willig (p. 594) shows how conventional measures of consumer surplus can be adjusted for the changing marginal utility of income.

3. Marginal utilities differ among individuals, creating problems in using aggregate demand to measure utility. Although differences within groups such as consumers or producers may average out, systematic differences are apparent between groups by income level. Classical welfare analysis applies to group rather than to individual behavior and hence is well suited for policy analysis.
4. Producer surplus is a return to fixed resources — a net return after subtracting variable costs from gross returns. Supply curves are behavioral relationships. They are imprecise measures of economic rent in the long and short run. Often the best solution to this problem is to estimate producer surplus as gross receipts less all operating costs defined as costs which are variable in the time period under consideration. Estimates of operating costs may be available for commodities from farm management surveys or other sources.
5. Classical welfare analysis often is but need not be confined to a partial, single commodity analysis. Equilibrium prices and quantities ideally would account for interactions among commodities in a simultaneous system of demand and supply equations.
6. Long-run as well as short-run elasticities often are not but can be recognized in analysis.
7. Finally, classical welfare economics primarily relates to static economic efficiency and current income rather than to long-term efficiency and economic growth which requires savings and investment in high-return durable capital. Net social benefit may be consumed today or invested to produce future social benefits. If net social benefit is consumed today, it provides satisfaction today. If it is saved and invested, it provides a flow of future output benefiting later generations. The later will be apparent in dynamic efficiency. Dynamic efficiency can best be measured by trends in resource productivity, national income, or national income per capita. Historical data indicate that countries with the least amount of government interventions distorting prices (lowest net social cost) have had the fastest rates of economic growth (Agarwala).

Net Benefits and Redistributions from Public Interventions Improving the Economy

Individuals pursuing self-interests without a coordinating framework would not necessarily increase well-being of society. Some coordinating system is essential, either the private market sector or the public sector. The latter includes the family and all other non-market allocators but here refers mainly to allocations by government. Classical welfare analysis can help to identify situations of market failure or public sector failure

which reduce net social benefits. Figure 6.2 is used to illustrate market failure — where markets alone do not minimize social cost and where the public sector potentially can intervene to increase well-being.

1. Externalities. Panel A in Figure 6.2 shows the impact of disassociation of private and social costs — an *externality* such as costs incurred by downstream farmers when an upstream farmer allows high rates of soil erosion. The private supply curve S_p reflects costs incurred by and entering the accounts only of the upstream producer but does not reflect costs incurred by downstream producers when sediment destroys their crops. S_s is the social supply curve which includes costs borne by all producers. The social demand curve for the good being produced is D_s. Net social costs are appropriately measured from social rather than private supply and demand curves.

The firm responds to the private rather than the social supply curve, hence output is q_p and price is p_p. Efficient pricing and output determined by the social supply and demand curves would be q_s and p_s. The net social gain and redistribution of income from efficient output at q_s rather than at the inefficient level q_p are summarized as follows:

Area	
Loss to consumers	2 + 3
Gain to producers	2 + 3 + 4
Net gain to society	4.

In the absence of market intervention, the full producer surplus (calculated from curve S_s), including the cost to downstream producers, at price p_p is 5 - 3 - 4. After market intervention producer surplus is 2 + 5, hence the net gain to producers from market intervention is 2 + 5 - (5 - 3 - 4) or 2 + 3 + 4. The loss to consumers is more than offset by the gain to producers so society is better off with intervention.

In other instances, private costs exceed social costs, reversing the position of the supply curves in Figure 6.2A. An example is costs of storing buffer stocks of farm commodities. The private storage trade requires a high return on capital to compensate for high risk. For society as a whole, risks are less because many unfavorable and favorable outcomes average out for a large storage operation. The private trade alone may provide too little storage to bring optimal stability to farm and food prices.

Another externality occurs when private benefits exceed social benefits so that the private demand curve is D_p and the social demand curve is D_s in Figure 6.2A. An example is health problems caused consumers by residues in food from pesticides used by producers on crops or livestock. Consumers are unaware of the source of problems and purchase q_p when the optimal quantity would have been q_s. Benefits of pesticides are greater to producers than to consumers when residues remain in foods. The net social cost is area 4 in Figure 6.2A.

In other instances social benefits exceed private benefits. D_s and D_p then trade positions in Figure 6.2A. An example of such externalities is general schooling. The

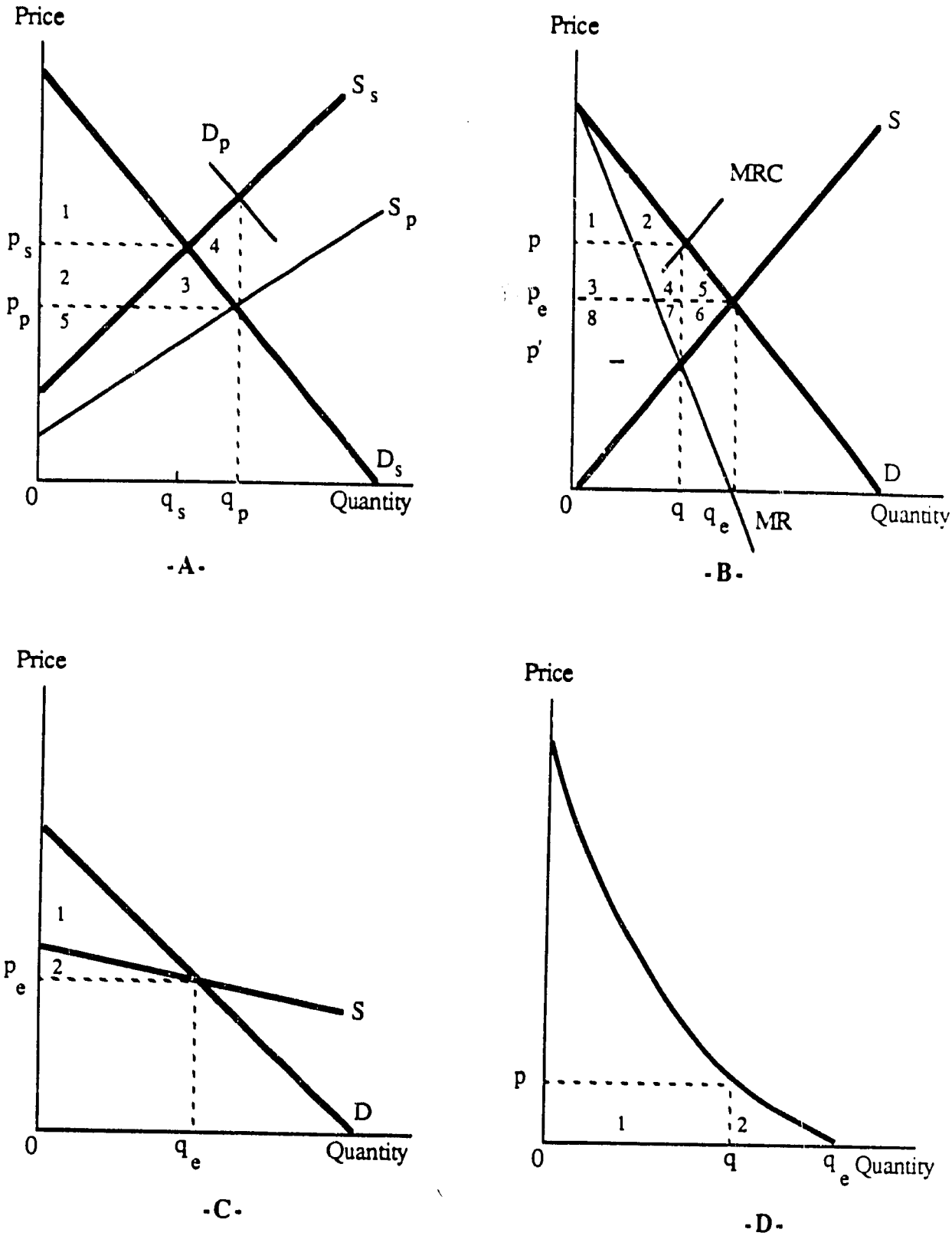


Figure 6.2. Redistributions and Costs of Market Failure.

benefits of a literate, informed citizenry able to vote intelligently and provide military skills may exceed private benefits of schooling to the individual. Hence, reliance on the market alone results in an inefficient, low quantity of schooling and a net social cost or lost real income measured by the value in area 4 if D_s and D_p are parallel.

Considerable evidence indicates that producers and consumers are rational in responding to private incentives. Thus a system of subsidies and taxes correcting externalities so that private costs (benefits) align with social costs (benefits) give signals to producers and consumers causing them to act in the public interest. In other instances regulations are required to bring socially efficient output q_s. These corrections to improve the economy will raise *real* national product but will reduce the *money value* of national income. The former rather than the latter is the appropriate measure of well-being and the appropriate target for expansion.

2. ***Imperfect Competition.*** Imperfect competition is another source of inefficiency in markets. The social gain from intervention to bring about a competitive market is shown in Figure 6.2B. A monopolist produces where marginal revenue MR equals marginal cost S at output q and price p. Such pricing and output maximizes net revenue to the firm but not to society. Competitive pricing at p_e and output at q_e increases consumer surplus by area 3 + 4 + 5 and reduces producer surplus by area 3 + 4 - 6. The net gain or addition to real national income is 5 + 6 from competitive markets. On the other hand if economies of size are substantial so that an atomized industry of many sellers would have a supply curve intersecting D above price p in Figure 6.2B, then monopoly pricing and output would have less social cost than would competitive pricing and output.

Now turning from the case of monopoly (single seller) to monopsony (single buyer), the monopsonist utilizes a resource q to the point where marginal revenue D equals marginal resource cost MRC. Quantity is q and the price paid suppliers is p'. Net social cost is area 5 + 6.

Market efficiency is relatively robust with respect to number of firms. A market with only one domestic firm may perform efficiently if the firm is not protected from foreign competition or entry into the industry by other firms. Antitrust action or termination of government regulations that shield imperfect competition may be appropriate responses.

A characteristic which gives rise to imperfect competition is a decreasing cost industry depicted in Figure 6.2C. Competitive pricing and output result in price p_e and quantity q_e. Consumer surplus is area 1 + 2 and producer surplus is area 2, a loss. Because producers lose money, they will choose not to produce. The benefits from producing and consuming at competitive price p_e and output q_e rather than not producing are summarized as follows:

Area	
Gain to consumers	1 + 2
Loss to producers	2
Net gain to society	1.

Intervention to bring output q_e increases national income by area 1.

Sometimes only one firm can operate in an industry at lowest cost per unit. Frequently, natural monopolies also are characterized by decreasing marginal cost illustrated by the curve S in Figure 6.2C. If one private firm operates, however, it is likely to practice monopoly pricing such as in Figure 6.2B to avoid losses.

Given a situation of natural monopoly, several options are available to society. A private firm may be regulated and subsidized out of taxes or the industry may be served by a public utility owned by government. In many rural areas, cooperatives such as grain elevators operate to avoid the onerous consequences of natural monopolies. Atomization of an industry (breaking up a single dominant firm into so many firms that no one can influence price) may raise production costs so high that the social loss from higher costs are larger than from monopoly pricing and output. In such instances where only one firm can serve the industry at low cost, then cooperatives, public utilities, or regulated private firms may be the answer. In reality, social costs of inefficiently administered regulations or of poorly managed cooperatives or public firms frequently exceed the social cost of private monopoly the interventions were attempting to eliminate.

3. ***Public Goods.*** Markets function best where goods are rival and exclusionary. In the case of a pure public good, consumption by one person does not reduce consumption available to another, hence the good is not rival. For example the marginal cost is zero as depicted in Figure 6.2D for providing another person national defense or another car access to an underutilized highway or bridge. Because marginal cost is zero, the social supply curve lies flat on the horizontal axis. To arbitrarily charge a price p to each individual who uses a highway or bridge reduces quantity from optimal amount q_e to q. Termination of the toll p increases use at no cost to society. The net gain to society from marginal cost pricing in an efficient market has components in Figure 6.2D as follows:

Area	
Gain to consumers	1 + 2
Loss to producers	1
Net gain to society	2.

Benefits of some goods cannot be appropriated by private firms. For the market to function efficiently, goods must be exclusionary; that is, if the good is made available to one person X it must be possible to exclude it from other persons Y and Z. If the latter can obtain the good without charge if X purchases it, the supplier will be unable to appropriate benefits. Everyone will want to be a free rider. No one will pay to produce it and the good will be under-produced by the market.

In some cases of public goods such as a sidewalk or lighthouse, it is possible to collect for use but not practical because the cost of collection is large relative to benefits. For other goods such as the technology in a seed variety easily reproduced by farmers, it is not possible for a private firm to appropriate enough benefits from users to cover costs of developing the technology however large the social benefits.

Some goods such as agricultural research, extension, and statistical (data) information have some but not necessarily all properties of public goods. Extension of patent rights can in some instances make nonrival biological technology appropriable and hence provide incentives to private firms but private firms are likely to price higher than the social marginal cost specified above as economically efficient. Marginal cost pricing requires public sector funding for developing public goods technology. Such funding is frequently not forthcoming especially in developing countries despite high social rates of return on investment. The resulting economic inefficiency of under-investment implies that use of the private sector to develop applied technology may be more efficient even if the sector prices in excess of marginal cost. Basic research, however, is likely to be under-funded if left solely to private firms because even with patent rights the firm may not be able to appropriate receipts to pay for research or may view outcomes as too risky. Because reliance on the public sector alone or the private sector alone for public goods such as agricultural technology has advantages and disadvantages, mixed public-private systems are often preferable and are used.

Net Costs and Redistributions from Public Interventions Distorting and Depressing the Economy

We now turn to public interventions in markets which reduce economic efficiency and national income as illustrated in Figure 6.3. Removing these interventions will raise national income but not necessarily income of agriculture or consumers. The private supply and demand curves are assumed to be the social curves.

1. Commodity Program Interventions. Panel A is used to show distributions and costs of mandatory supply control and of price supports without supply control. Deviations are examined from open market equilibrium at price p_e and quantity q_e. Suppose mandatory supply control reduces output to q_1 and raises price to p_1. Loss of consumer surplus compared to open market equilibrium is area 2 + 3. The increase in producer surplus is area 2 - 8, a value likely to be positive if demand is inelastic. Results for mandatory production controls are summarized as follows:

Area	
Loss to consumers	2 + 3
Gain to producers	2 - 8
Net loss to society	3 + 8.

The net loss to society, the excess of consumer losses over producer gains, is area 3 + 8. Transfer inefficiency as measured by net social loss divided by the gain to producers is (3 + 8)/(2 - 8). Inefficiency is zero for an efficient transfer.

Next consider costs and redistributions of income with price supports without production controls. Price supported at p_1 brings production q_2. At least two options are available to clear the market which will not clear at price p_2. One is to dispose of $q_2 - q_1$

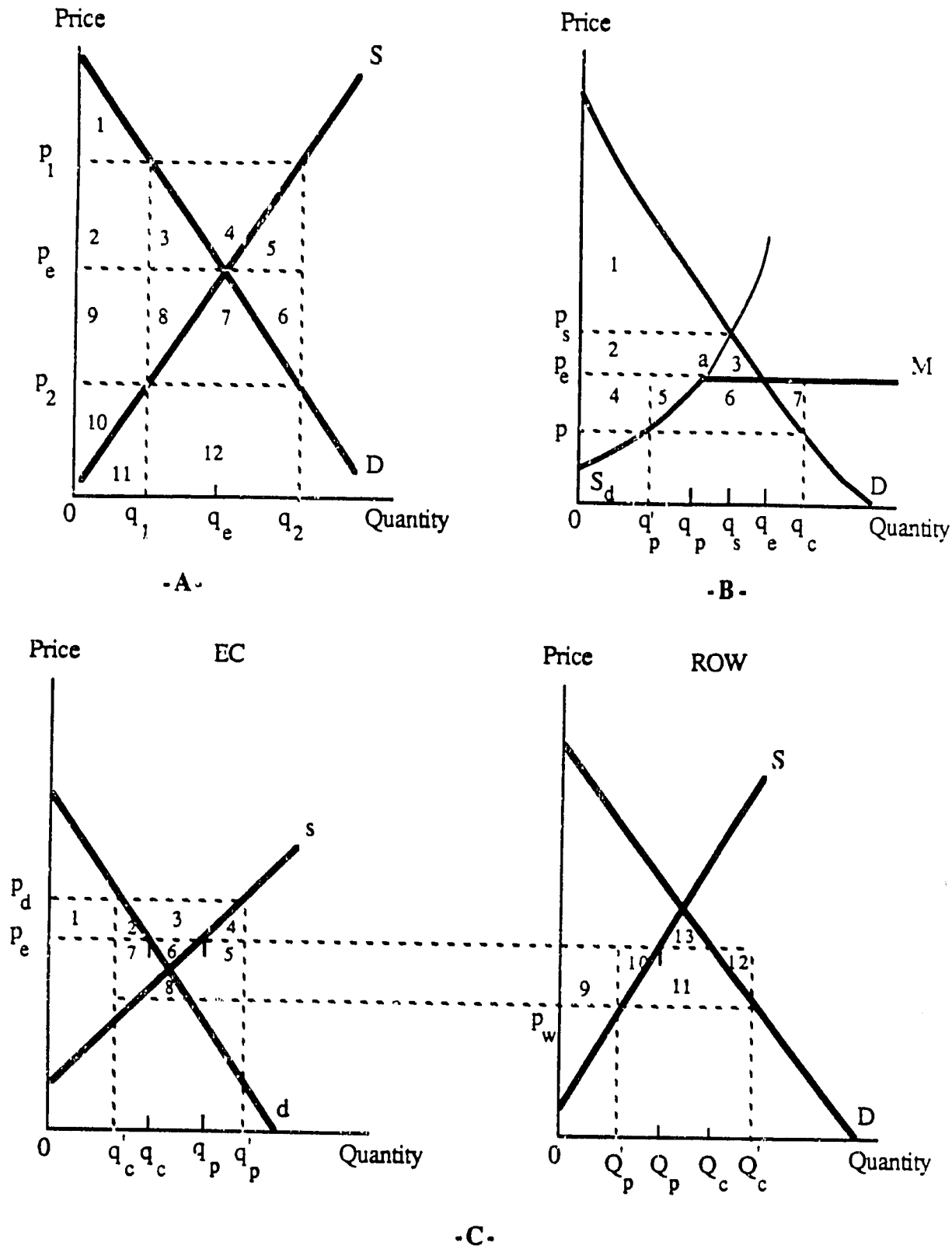


Figure 6.3 Redistributions and Costs of Government Interventions in Markets

outside the market. The second is to subsidize the difference between supply price p_1 and the demand price p_2 which clears the market.

Consider the latter first. In the case of transfer (subsidies) from taxpayers to hold producers' price at p_1 and consumers' price at p_2, the price p_1 requires a transfer from taxpayers of 2 + 3 + 4 + 5 to producers, increasing their surplus by 2 + 3 + 4, hence net social cost is 5 and transfer inefficiency is 5/(2 + 3 + 4). For consumers, holding price at p_2 instead of p_e requires a tax transfer equal to area 6 + 7 + 8 + 9 which increases consumer surplus by area 7 + 8 + 9 Taxpayers' cost exceeds consumers' gain by area 6, the net social cost. Transfer inefficiency is 6/(7 + 8 + 9) and transfer efficiency is (7 + 8 + 9)/(6 + 7 + 8 + 9). Compared to a well-functioning market equilibrium, the resulting combined welfare implications are summarized as follows:

Area	
Gain to consumers	7 + 8 + 9
Gain to producers	2 + 3 + 4
Loss to taxpayers	2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
Net loss to society	5 + 6.

The overall transfer inefficiency is (5 + 6)/(2 + 3 + 4 + 7 + 8 + 9).

All countries experience a *food price dilemma* which may be defined as a free market equilibrium price deemed too low to avoid hardship and reduce poverty among producers and too high to avoid hardship and reduce poverty among consumers. Subsidies to producers and consumers to address the food price dilemma tend to leave one or both groups worse off. That is because taxpayers are either producers or consumers. The subsidies are a negative sum game; that is, consumers and producers lose more as taxpayers than they gain as consumers and producers. Despite the shortcomings of this game, many countries play it.

An alternative to subsidies is to dispose of surpluses by spoilage or by allocating them to uses with little or no value. Disposing of quantity $q_2 - q_1$ in such manner provides the following welfare implications from Figure 6.3A:

Area	
Loss to consumers	2 + 3
Gain to producers	2 + 3 + 4
Loss to taxpayers	3 + 4 + 5 + 6 + 7 + 8 + 12
Net loss to society	3 + 5 + 6 + 7 + 8 + 12.

The welfare loss is large relative to transfer gain to the beneficiary, producers, from the policy. Transfer inefficiency is (3 + 5 + 6 + 7 + 8 + 12)/(2 + 3 + 4).

Figure 6.3B illustrates redistributions and social costs from government interventions in markets to provide self-sufficiency and to reduce food cost through a ceiling on food price. Domestic demand is D and domestic supply is S_d. The horizontal, perfectly elastic import supply curve M is for a small country which can import all it wants without

changing world price from p_e. Total supply is the curve of lowest cost sources of q, or $S_d aM$. Open market equilibrium quantity is q_e which is equal to consumption. Supply quantity is q_p from the domestic market and $q_e - q_p$ from imports.

Raising price to p_s attains self sufficiency by increasing domestic quantity supplied by $q_s - q_p$ and decreasing quantity demanded by $q_e - q_s$. The welfare impact is as follows:

Area	
Loss to consumers	2 + 3
Gain to producers	2
Loss to society	3.

It is apparent that the loss to consumers offsets the gain to producers, hence society is worse off from self-sufficiency. As with other welfare measures in this section, only economic efficiency (national income) is considered and other objectives such as equity and stability are ignored. Instability increases danger of food shortages if self-sufficiency is achieved at the expense of lost economic means to purchase supplies from abroad when domestic production is short, if self-sufficiency is achieved by use of fertilizers and pesticides purchased from abroad more subject than food imports to interruption, and if domestic production is variable from year to year. Equity may not be served by self-sufficiency because high food prices especially oppress the poor, who spend a high proportion of income for food.

Food prices may be held down by a price ceiling at p in Figure 6.3B. This reduces domestic production to q_p' and increases consumption to q_c. An import subsidy of 5 + 6 + 7 is required to avoid a massive food shortage. Welfare impacts are summarized as follows:

Area	
Gain to consumers	4 + 5+ 6
Loss to producers	4 + 5
Loss to taxpayers	5 + 6 + 7
Loss to society	5 + 7.

Transfer inefficiency (5 + 7)/(4 + 5 + 6) per dollar of gain to consumers tends to be low if the p_e - p is small and domestic supply and demand are highly inelastic.

2. ***Interventions Influencing World Markets.*** Many less developed countries "tax" producers by holding down food prices as shown above; developed countries often subsidize agriculture. The example in Figure 6.3C is hypothetical but broadly resembles the European Community which generates excess production with high price supports in the absence of supply controls and subsidizes export of surpluses. With free trade, welfare is maximized at equilibrium world price (no transport costs assumed) p_e, domestic quantity supplied q_p, and quantity consumed q_c in the European Community (EC), and Q_p and Q_c respectively in the rest of world (ROW). Small initial exports from

the EC, $q_p - q_c$, equal small imports $Q_c - Q_p$ by ROW. Price supports in EC bring prices to p_d, raising production to q_p' and lowering consumption to q_c'. In the absence of supply control, the EC elects to export the surplus $q_p' - q_c'$ to ROW which lowers world price to p_w. The subsidy per unit is $p_d - p_w$ and the total subsidy is area 2 + 3 + 4 + 5 + 6 + 7 + 8. The area 2 + 3 + 4 of this subsidy raising price from p_e to p_d is a transfer to producers, but area 5 + 6 + 7 + 8 is a subsidy to ROW, equal to area 10 + 11 + 12.

The welfare impact compared to open market equilibrium is summarized as follows:

EC	*Area*
Loss to consumers	1 + 2
Gain to producers	1 + 2 + 3
Loss to taxpayers	2 + 3 + 4 + 5 + 6 + 7 + 8
Loss to society	2 + 4 + 5 + 6 + 7 + 8.

The loss to society of area 2 + 4 can be charged to holding domestic price above p_e; the loss of 5 + 6 + 7 + 8 can be traced to forcing world price below p_e. The impact on ROW is:

ROW	*Area*
Gain to consumers	9 + 10 + 11
Loss to producers	9 + 10
Gain to society	11.

Consumers gain more than producers lose in ROW, leaving a net gain to ROW of area 11 from the policies of the EC. Of the net loss 2 + 4 + 5 + 6 + 7 + 8 to the EC, 2 + 4 was deadweight loss and 5 + 6 + 7 + 8 = 10 + 11 + 12 was a transfer to ROW. Of the transfer, only 11 was a real gain to ROW; 10 + 12 was deadweight loss. Hence the world deadweight loss from the EC support programs without supply control is 2 + 4 + 10 + 12. It is apparent that losses to consumers and taxpayers in the EC fall well short of gains to producers, hence the EC would appear to benefit from ending the Common Agricultural Policy (CAP). On the other hand, consumers gain more than producers lose in the ROW, hence the ROW would appear to benefit from the EC continuing the CAP. That the EC in fact favors continuation of the CAP while the ROW opposes it is probably not a testimony to the inadequacy of the analysis in Figure 6.3C but rather testimony to the political power of producers in the EC.

In concluding this section it is well to note that governments can go too far or not far enough to bring about efficient allocations as noted in Figures 6.2 and 6.3. Frequent government policy errors include an overvalued exchange rate, price controls holding traded goods prices below world price levels, excessive reliance on parastatal corporations or government agencies to produce and market, neglect of infrastructure and social overhead in general, provision of credit at excessively subsidized rates, too rapid increase in money supply and inflation, federal budget deficits, import substitution, excessive central planning, and finally but not least important, unstable policies that thwart sound long-term decision-making.

In some cases, the decision whether the government should intervene or let markets work for efficiency is clear-cut. However, in many instances the case for government interventions in markets is not clear-cut. Experience with problems actually encountered in developing countries suggests that when in doubt the appropriate policy is to rely on the private market. That is because government interventions often entailed higher social costs than did the private market failure the policies were attempting to correct.

Equity

Economic efficiency entails allocating resources to uses contributing the most to output. Economic growth results from economic efficiency over time, including savings and investment allocated to earn high returns in building human and material capital. As indicated earlier, the supply price is a schedule of incremental cost to firms and society of production whereas the demand price is the incremental benefit to society from consuming another unit of the commodity. But an efficient allocation is not necessarily an equitable allocation. Markets can be in equilibrium while some persons are starving and others chronically obese.

The food price dilemma with producers wanting higher prices than p_e and consumers wanting lower prices than p_e to promote equity is illustrated in Figure 6.4. Suppose taxpayers provide a subsidy $p_p - p_e$ to producers, bringing the supply price to p_p and inducing output to expand to q_1. Suppose also that a subsidy of $p_e - p_c$ is provided to bring the price of food to p_c per unit to consumers. The lower price of food raises consumption to q_1.

Consumers would have been willing to pay area 1 + 2 + 6 + 7 + 8 + 9 for q_1, but only have to pay area 8 + 9, hence consumer surplus is area 1 + 2 + 6 + 7 — a gain of area 6 + 7 compared to the initial efficient output q_e. Subsidies plus market receipts less variable costs of area 4 + 5 + 6 + 9 for q_1 leave producer surplus of area 2 + 3 + 7 + 8 — a gain of 2 + 3 compared to the efficient output at q_e. The results for output q_1, featuring subsidies to raise producer prices to p_p and subsidies to lower consumer prices to p_c compared to efficient price p_e and output q_e, are summarized as follows:

Area	
Gain to producers	2 + 3
Gain to consumers	6 + 7
Loss to taxpayers	2 + 3 + 4 + 5 + 6 + 7
Net loss to society	4 + 5.

National income is reduced by the value of area 4 + 5 in Figure 6.4 from an attempt to serve equity by lowering prices to consumers and raising prices to producers. The effort reduced economic efficiency in this example. Economic efficiency and equity do not always conflict, however. Investments in schooling of disadvantaged youth can be both economically efficient and equitable.

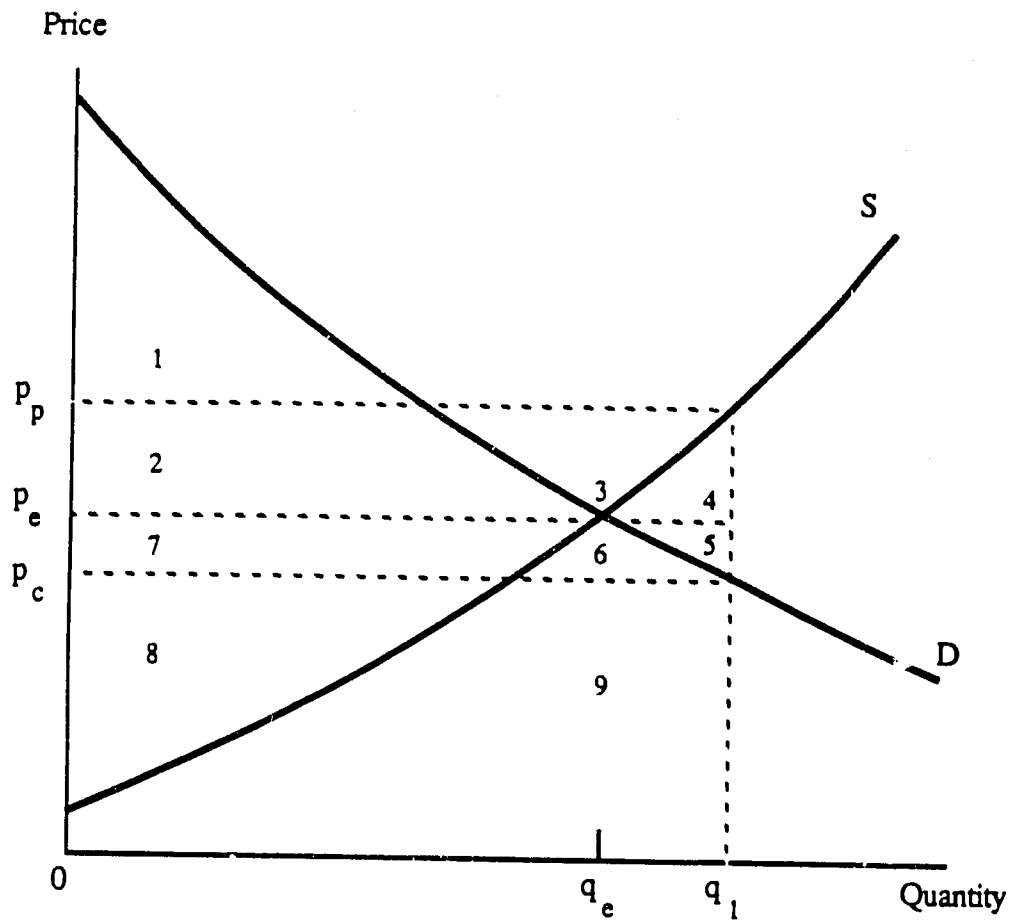


Figure 6.4 Economic Efficiency Illustrated with Demand and Supply Curves.

The market may be efficient in allocating resources to bring outcomes shown as p_e and q_e in Figure 6.4. If some are destitute while others are opulent at market equilibrium, the efficient market outcome is unlikely to be judged a fair outcome -- hence the public sector may intervene to promote equity. Society may be more concerned about providing equality of opportunity to succeed than equality of outcomes. And equitable opportunities or outcomes do not necessarily mean equal opportunities or outcomes.

Traditionally, determining an equitable distribution of income and resources has been viewed as a value judgment to be made by the political process with little or no input from socio-economic analysts. Modern socio-economic science and modern statistical techniques are reducing subjectivity and increasing objectivity in analysis of equity issues.

If an objective of an economic system is to raise the well-being of people, then resources and goods and services need to be allocated to uses where incremental (marginal) utility is greatest. Marginal utility (satisfaction derived from an additional unit of income) differs among individuals and is subjective, creating problems in measuring utility. Attitudinal scales developed and validated by psychologists and sociologists to measure well-being of individuals through personal interview surveys are as yet unreliable to predict marginal utility for any one individual but show promise to predict marginal utility for groups such as income classes because individual errors in measurement tend to average out.

The marginal (incremental) utility of income was estimated by Tweeten and Mlay for the United States using socio-psychological attitudinal scales in personal interviews. The marginal utility of income was estimated to be:

$$MU = 1.3582 - .3582Y \quad (MU = 0 \text{ for } Y > 3.8) \tag{6.1}$$

where MU is utility as a proportion of that derived by a family with median income and Y is family income as a proportion of median family income. Marginal utility differed systematically by age, education, and income, but no major differences in marginal utility were observed over time, sectors (rural-urban), or regions. Results were from a comprehensive set of socio-psychological scales relating to various domains of well-being and based on a large, random sample of one adult from each of approximately 1,500 U.S. families in 1976 and another sample of like size in 1980. Alternatively, marginal utility for various alternative income levels might be specified by the government through the political process.

Estimates of marginal utility can be used to adjust policy analysis results such as those from Figure 6.4. Suppose that weighted median income of taxpayers is 2.15 times the national median, of consumers is 1.19 times the national median, and of producers is .89 times the national median income. The marginal utility for each of these groups computed from equation 6.1 is shown in column "MU" in Table 6.1. The unadjusted policy analysis results show a net loss from the program or policy. (Marginal utilities were simply multiplied by gains and losses under the assumption that changes in income from the project were too small to change MU.) Because the income redistribution was from

higher income taxpayers to lower income producers, the utility-adjusted results show a net social gain rather than a loss.

Estimates in Table 6.1 are crude; additional refinement can be introduced by assigning utility weights *by income class* to the results from Figure 6.4 to express economic gains and losses as social gains and losses. Gains and losses from subsidies to consumers and producers are distributed among income classes as shown in Table 6.2. The upper left panel shows economic gains and losses (negatives) and the upper right panel shows social gains and losses (economic outcome multiplied by marginal utility) for price interventions from Figure 6.4.

Because the interventions especially benefit lower income producers and consumers with high MU at a cost especially to high income taxpayers who have a low MU, the net economic loss of $20 million is a net social gain of 102 million utility adjusted dollars.

This hypothetical sample combining the equity and efficiency dimensions to measure social gains and losses does not necessarily justify the market intervention policy. The lower panel in Table 6.2 demonstrates an important principle: a pure cash transfer payment is in principle socially more beneficial than an economically inefficient intervention such as a price ceiling or price support to redistribute income to the disadvantaged. The net social gain is $114 million with a direct transfer payment compared to $102 million by intervening in the market to promote equity.

Administrative costs are not included but are likely to be less with cash transfer payments than with price interventions. Cash transfers are not fully "pure" — they in fact will distort allocations and hence create deadweight losses. Such losses are likely to be small, however, compared to those of most market interventions designed to foster equity.

In short, the "efficient" equilibrium such as p_e and q_e in Figure 6.4 is a function of the initial distribution of resources. Allowing prices to bring about an economically efficient allocation and providing pure cash transfers to producers would have provided a larger adjusted net social gain (without loss to anyone) than the market interventions depicted in Figure 6.4. The overall most equitable and efficient system maximizing social gain adjusted for utility must provide for equitable and efficient distribution of resources. That is why human resource development programs and access to public services and resources are of high priority in developing countries. Efforts of developing countries to reduce farm poverty by nationwide support of farm commodity prices and to reduce consumer poverty by nationwide ceilings on food prices have sometimes placed burdens on taxpayers and the economy, ultimately slowing the economy and hurting the poor. Targeted food assistance programs such as food stamps or sale of staples consumed by the poor at subsidized prices in fair price shops have placed less burden on taxpayers and have been successful in some countries.

Estimating the level and distribution of economic gains and losses from government interventions requires considerable effort. Calculations of marginal utilities and weighting of gains and losses among income classes is often impossible. A sound approach, in many instances, is to provide data on distribution of gains and losses by income class, then leave to policy makers the decision regarding what is an appropriate redistribution.

Table 6.1 Illustration of Redistribution and Net Social Loss (Gain) from Figure 6.4 Adjusted for Marginal Utility of Income.

Item	Unadjusted Policy Results	Adjusted for Marginal Utility	
	($ Mil.)	(MU/$)	($ Mil.)
Gain to producers	160	1.041	166.6
Gain to consumers	120	.933	111.9
Loss to taxpayers	300	.588	176.4
Loss to society	20		(gain) -102.1

Table 6.2. Illustration of Redistribution and Net Loss (Gain) to Society from Subsidies in Figure 6.4 Adjusted by Marginal Utility of Money Among Income Classes.

Family	Economic				Social		
Income ($)	Producers	Consumers	Taxpayers	MU	Producers	Consumers	Taxpayers
			(Price Intervention; Million Dollars)				
0-10,000	65	40	-40	1.2729	82.7	50.9	-50.9
10,000-20,000	50	30	-45	1.1023	55.1	33.1	-49.6
20,000-30,000	30	25	-60	.8465	25.4	21.2	-50.8
30,000-40,000	10	20	-75	.3349	3.4	6.7	-25.1
40,000-80,000	5	5	-80	.0000	0.0	0.0	0.0
	160	120	-300		166.6	111.9	-176.4
Net Loss to Society			20				(gain)-102.1
			(Pure Transfer; Million Dollars)				
0-10,000	65	40	-37	1.2729	82.7	50.9	-47.1
10,000-20,000	50	30	-42	1.1023	55.1	33.1	-46.3
20,000-30,000	30	25	-56	.8465	25.4	21.2	-47.4
30,000-40,000	10	20	-70	.3349	3.4	6.7	-23.4
40,000-80,000	5	5	-75	.0000	0.0	0.0	0.0
	160	120	-280		166.6	111.9	-164.2
Net Loss to Society							(gain)-114.3

ACTIVITIES

Applications of classical welfare analysis to developing countries are shown below. In many instances the information initially available or more readily attainable for situations such as Figure 6.4 include prices to producers p_p, to consumers p_c, and at the border p_e along with an initial quantity q_e or q_1. Values represented by areas such as 4 and 5 in Figure 6.4 can be approximated by recalling that the area of a triangle is one-half of the base (here Δq) times the height (here Δp). It is apparent that Δq is approximately

$$\Delta q = q_1 - q_e \approx \frac{p_p - p_e}{p_e} \left(\frac{dq}{dp}\frac{p}{q}\right)_s q_1 \tag{6.2}$$

where $(p_p - p_e)/p_e$ is the proportional increase in price, and the term in parenthesis is the price elasticity of supply. The net social cost (value of goods and services or national income foregone) by the producer subsidy is $.5\ \Delta q\ \Delta p$ where $\Delta p = p_p - p_e$.

Similarly, the net social cost (value of goods and services or national income foregone) by the consumer subsidy is computed as $.5\ \Delta q \Delta p$, where

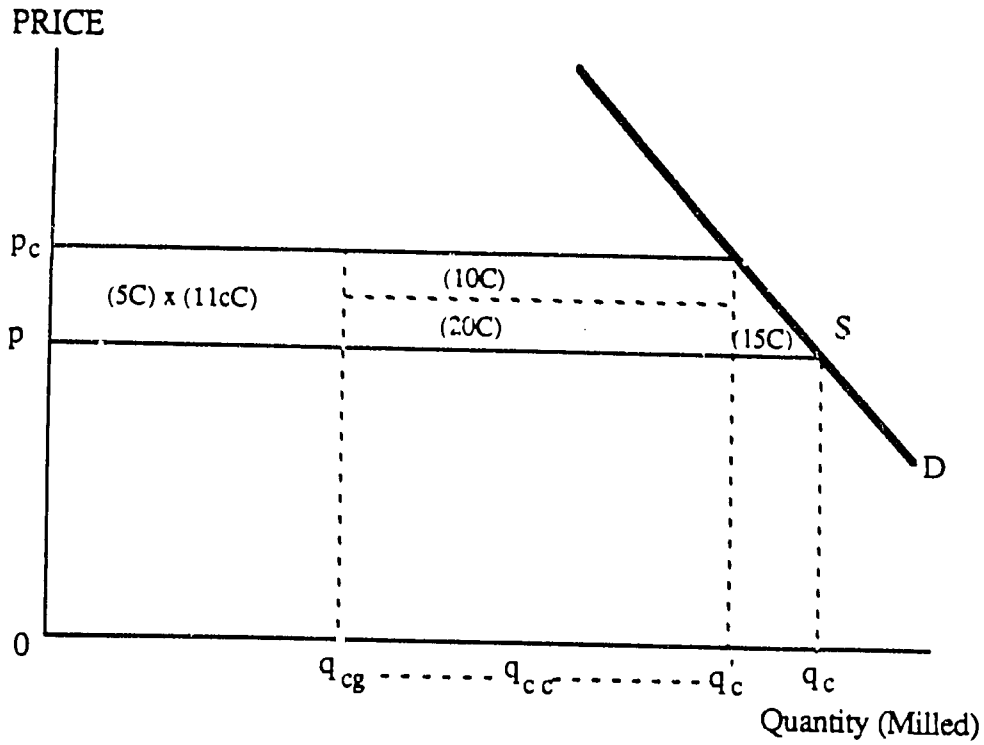
$$\Delta q = q_1 - q_e \approx \frac{p_e - p_c}{p_e} \left(\frac{dq}{dp}\frac{p}{q}\right)_d q_1 . \tag{6.3}$$

The term in parenthesis is the price elasticity of demand and Δp is $p_e - p_c$. Equations 6.2 and 6.3 are unlikely to give exactly the same measure of Δq, in which case an average of the two estimates might be used to calculate areas 4 and 5 in Figure 6.4. Other measures in Figure 6.4 are rather easily calculated given the above estimates. For example, producer surplus is the addition to revenue of producers $0q_1\ (p_p - p_e)$ less area 4.

Liberian Example

Table 6.3 and figure 6.5 show estimated gains and losses from government market interventions in the Liberian rice economy in 1982, 1983, and 1984. Analysis is at the producer level (P) expressed in paddy rice and at the consumer level (C) in milled clean rice.

First consider impacts on producers as estimated in the first panel (P) in Table 6.3 and the lower panel of Figure 6.5. The official support price at the county receiving stations was 18 cents per pound paddy but at the farm level was approximately 12 cents per pound p_p. The effective support price was lower in 1984 because the official support price could not be sustained for lack of funds. In the absence of supports, farm price (border price, cif) was estimated to be p_f = 8 cents per pound, hence the effective proportional subsidy was 50 percent in 1982 and 1983 as shown in row (8P). The support price generated a market surplus quantity $q_p' - q_p$. The subsidy of $(p_p - p_f)\ (q_p' - q_p)$ to producers was partially offset by additional production costs as shown in row (11P).



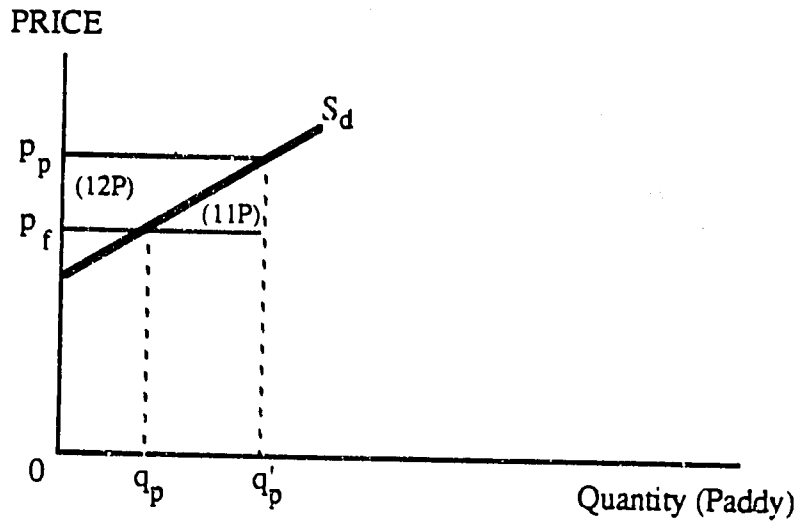


Figure 6.5. Graphic Illustration of Liberian Rice Policy Intervention

Table 6.3. Gains and Losses from Government Policies, Liberian Rice Economy.

Item	Notation	Producers (Farm Level) P Units	1982	1983	1984
(1) Domestic production sold to LPMC	q_p'	1,000 mt paddy	10.00	17.19	9.25
(2) Guaranteed producer price	p_g	$/mt (18¢/lb.)	396.90	396.90	396.90
(3) Effective producer price	p_p	$/mt	264.60	264.60	198.36
(4) Producer receipts (1) x (3)	$q_p' p_p$	$1,000	2,646.00	4,548.47	1,834.83
(5) Normal market price, farm level	p_f	$/mt (8¢/lb.)	176.40	176.40	176.40
(6) Producer subsidy (3) - (5)	$p_p - p_f$	$/mt	88.20	88.20	21.96
(7) Policy transfer to producers (1) x (6)	$q_p'(p_p - p_f)$	$1,000	882.00	1,516.16	203.13
(8) Proportional subsidy (6/5) x 100	$(p_p - p_f) / p_f$	Percent	50.00	50.00	12.45
(9) Direct price elasticity of market surplus		Percent	1.30	1.30	1.30
(10) Quantity generated by production subsidy (1) x (8) x (9) / 100	$q_p' - q_p$	1,000 mt	6.50	11.17	1.50
(11) Production value loss .5 (6) x (10)	$.5(p_p - p_f)(q_p' - q_p)$	$1,000	286.65	492.75	16.44
(12) Gain to producers (addition to producers surplus) (7) - (11)		$1,000	595.35	1,023.41	186.69
(13) Production value loss per unit of gain to producers (11/12)		$	.48	.48	.09
(14) LPMC quantity sold from local production		1,000 mt	4.00	5.35	5.15
(15) LPMC marketing cost of local production sold		$/mt	205.06	205.06	205.06
(16) Normal marketing cost for competitive sector		$/mt	132.30	132.30	132.30
(17) Excess resource cost of marketing (15 - 16) x (14)		$1,000	291.04	389.27	374.71
(18) LPMC purchases less sales (1) - (14)		1,000 mt	6.00	11.84	4.10
(19) Value lost from waste .5 (5) x (18) (assume half loss)		$1,000	529.20	1,044.29	361.62
(20) Sum of social costs (11) + (17) + (19)		$1,000	1,106.89	1,926.31	752.77
(21) Social cost per unit gain to producers (20) / (12)		$	1.86	1.88	4.03

Table 6.3. (Continued).

Item	Notation	Consumption (Wholesale Level) C Units	1982	1983	1984
(1) Total quantity marketed and consumed	q_c'	1,000 mt	95.40	102.40	102.40
(2) Support price, wholesale	p_c	$/mt	465.00	440.00	474.00
(3) Consumption cost (1) x (2)	$q_c' p_c$	$1,000	44,361.00	45,056.00	48,537.60
(4) Computed cif world wholesale price	p	$/mt	419.50	391.10	403.00
(5) Consumption tax (2) - (4)	$p_c - p$	$/mt	45.50	48.90	71.00
(6) Policy tax on consumers (1) x (5)	$q_c' (p_c - p)$	$1,000	4,340.70	5,007.36	7,270.40
(7) Proportional tax (5/4) x (100)	$(p_c - p)/p$	Percent	10.85	12.50	17.62
(8) Commercial imports	q_{cc}	1,000 mt	50.00	55.00	55.00
(9) Prescribed import margin .03 (4)		$/mt	12.59	11.73	12.09
(10) Planned commercial tax revenue (5-9) x (8)		$1,000	1,645.75	2,044.18	3,240.05
(11) LPMC a) PL 480		1,000 mt	43.00	45.00	46.40
b) In-country purchases		1,000 mt	2.40	2.94	2.68
c) Total	q_{cg}	1,000 mt	45.40	47.94	49.08
(12) Policy tax transfer to GOL (5) x (11c) + (10)		$1,000	3,711.45	4,388.45	6,724.73
(13) Direct price elasticity of demand		Percent	-.60	-.60	-.60
(14) Consumption lost by tax (1 x 7 x 13) / - 100	$q_c - q_c'$	1,000 mt	6.21	7.68	10.82
(15) Consumption value loss .5 x (5) x (14)	$.5(p_c - p)(q_c - q_c')$	$1,000	141.24	187.82	384.27
(16) Loss to consumers (6) + (15)		$1,000	4,481.84	5,195.18	7,654.67
(17) Consumption value lost per unit of tax (15) / (12)		$	.04	.04	.06
(18) Spoilage and waste above normal		1,000 mt	1.50	1.50	1.50
(19) Cost of spoilage (4) x (18)		$1,000	629.25	586.65	604.50
(20) Gain to commercial importers (6) - (12)		$1,000	629.25	618.19	545.67

Table 6.3. (Continued).

Society Gains and Losses from Market Intervention[a]

Private Sector	Source	Units	1982	1983	1984
+ Gain to producers	(12P)	$1,000	595.35	1,023.41	186.69
- Loss to consumers	(16C)	$1,000	4,481.94	5,195.18	7,654.67
+ Gain to commercial importers	(20C)	$1,000	629.25	618.91	545.67
Net		$1,000	-3,257 34	-3,552.87	-6,922.30
Public Sector					
- Policy transfer to producers	(7P)	$1,000	882.00	1,516.16	203.13
- Excess cost of country marketing	(17P)	$1,000	291.04	389.27	374.71
- Value lost from country waste	(19P)	$1,000	529.20	1,044.29	361.62
+ Policy transfer from consumers to GOL	(12C)	$1,000	3,711.45	4,388.45	6,724.73
- Spoilage and waste	(19C)	$1,000	629.25	586.65	604.50
Net		$1,000	1,379.96	852.09	5,180.77
Net cost of public intervention to society (Loss to private sector less gain to public sector)		$1,000	1,877.38	2,700.78	1,741.54

[a]Omitted from analysis:
(a) Net costs and benefits of PL 480 imports -- could be established as separate account.
(b) The subsistence rice production-consumption sector.

Only the market surplus is assumed to be effected by rice price support in the above calculations. Approximately the same additional quantity is produced in row (10P) if it is assumed that total rice supply elasticity is .1 and all Liberian rice production, even in remote areas for subsistence use, is effectively raised in price by 25 percent.

Mainly because of limited milling capacity, the Liberian Produce Marketing Corporation (LPMC) was only able to market a portion of paddy acquisitions as noted in row (14P). At least half of the unmarketed quantity was lost as waste at a value shown in row (19P). In addition, marketing costs were estimated to exceed competitive marketing costs. The excess resource cost for marketing is shown in row (17P). The sum of the lost value from three sources (production value lost, excess marketing cost, and spoilage) is shown in row (20P). The loss of well over $1 in goods and services to transfer $1 of income to producers as shown in row (21P) indicates very low efficiency in transferring income to producers.

Effects of government rice policy on market consumers (C) are shown in the second panel in Table 6.3 and the upper panel of Figure 6.5. The Liberian price p_c was supported above the cif world price level p as shown in row (5C), reducing consumption from q_c to q_c' as shown in Figure 6.5. Of this consumption, $q_{cc} = q_c' - q_{cg}$ was imported commercially and q_{cg} was from LPMC in-country and PL 480 acquisitions as shown in row (11C) of Table 6.3. The loss to consumers from the consumption tax was (16C). The government received part of the tax directly and (10C) of the tax indirectly from a duty on commercial imports. Commercial importers gained (20C) of the tax as economic rent, hence consumers lost (15C) not gained by government or commercial importers. In addition, social costs roughly estimated in (19C) were incurred due to above normal spoilage of LPMC stocks.

The distribution of gains and losses from market interventions is summarized in the final panel of Table 6.3. Gains to producers and commercial importers were offset by losses to consumers so that the private sector incurred a net loss of over $3 million each year. The public sector gained because transfers from consumers more than offset losses from price supporters to producers, excessive marketing costs, and spoilage and waste. The net loss to society was over $2 million in 1983 because losses to the private sector exceeded gains to the public sector. The social cost must be balanced against unaccounted for benefits of rice policies such as stability of rice prices, public employment, and political support. Net costs and benefits of PL 480 counterpart funds are not included in the calculations.

Exercise 1. Recompute results using spreadsheet analysis with a market surplus supply elasticity of .6 in (9P).

Recompute results with a price elasticity of demand in (13C) of -1.00.

Jordanian Example

We now turn to the impact on consumers, producers, and society of market interventions raising domestic price to p' from border price p for potatoes in Jordan.

Figure 6.6 shows domestic demand D_d, domestic supply S_d, and import supply pS (small country assumption) for a total supply curve S_daS giving market equilibrium price p. Importing at price p along import supply curve pS gives domestic production q_p, consumption q_c, and imports $q_c - q_p$ without intervention. The parastatal (AMPCO) intervenes to import only quantity $q_c' - q_p'$ holding domestic price at p', and giving results depicted below assuming the parastatal retains the import tax:

Compared to no intervention:	Line in Table 6.4	Area in Figure 6.6
Loss to consumers	(18)	1 + 2 + 3 + 4 + 5
Gain to producers	(12)	1
Gain to parastatal	(19)	3 + 4
Net social cost	(21)	2 + 5

Figure 6.6 also can show the impact on consumers, producers, and society of market intervention to bring self-sufficiency at price p_e and quantity q_e. Given supply and demand depicted in Figure 6.6, self-sufficiency is attained by raising price to p_e. The import gap of $q_c - q_p$ is closed by greater domestic supply quantity $q_e - q_p$ and reduced domestic demand $q_c - q_e$ or

$$q_c - q_p = (q_e - q_p) + (q_c - q_e).$$

The implications for producer, consumers, and the nation compared to market equilibrium at p are summarized as:

	Line in Table 6.4	Area in Figure 6.6
Loss to consumers	(44)	1 thru 8
Gain to producers	(42)	1 + 8
Net social cost	(45)	2 thru 7

Six key prior estimates are production and net imports, prices, and elasticities for potatoes in 1986:

Amman wholesale price	138.70	Jordanian Dinar (JD)/ton
Amman "border" price (Aqaba price plus transport and other charges to Amman)	117.90	JD/t
Jordan production	39,000	tons
Jordan net imports	15,178	tons
Supply elasticity (domestic)	.50	
Demand elasticity	-2.00	

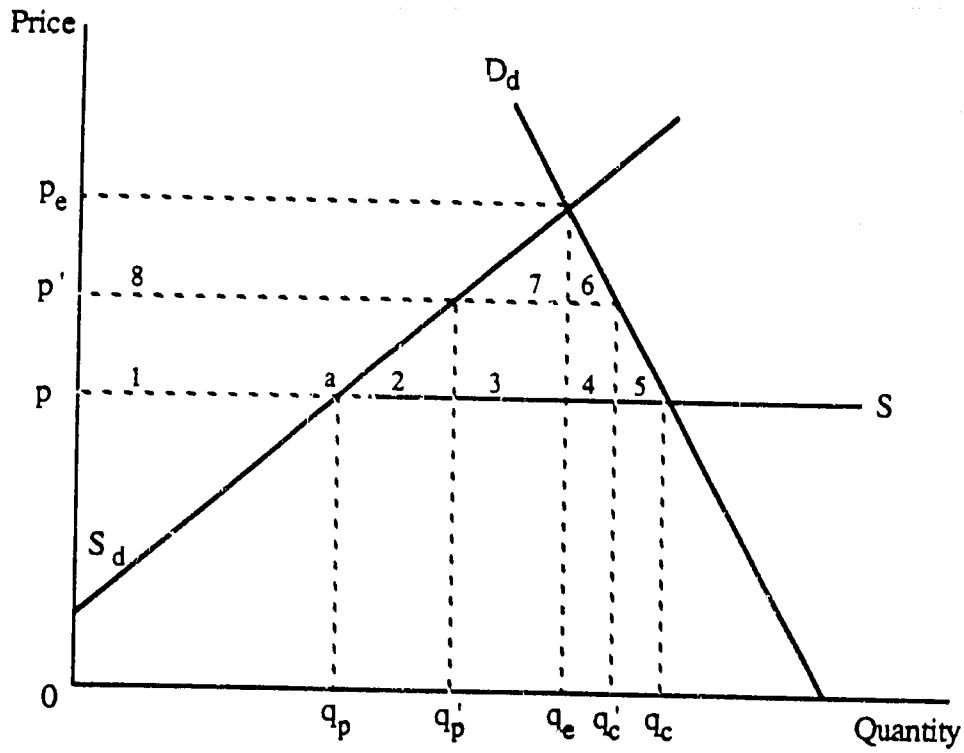


Figure 6.6. Supply and Demand Curves for a Fruit or Vegetable in Jordan

Table 6.4. Distribution of Benefits and Costs Among Producers, Consumers, AMPCO Parastatal, and the Nation from Potato Import Policy, 1986, with Demand Elasticity - 2.0.

Item	(See Figure 6.6)		
(1) Domestic Production (tons)	Qp'		39,000.000
(2) Net Imports (tons)	Qc' - Qp'		15,178.000
(3) Wholesale Price, AMMAN (JD/ton)	P'		138.700
(4) Border Price, CIF, AMMAN (JD/ton)	P		117.900
(5) Elasticity of Supply	Es		0.500
(6) Elasticity of Demand	|Ed|		2.000
(7) Price Gain from Intervention (JD/ton)	P' - P		20.800
(8) Proportional Price Gain	(P' - P)/P		0.176
(9) Addition of Price to Receipts (JD)	Qp' (P' - P)	1 + 2	811,200.000
(10) Addition to Production (tons)	(Qp' - Qp) = Qp' Es (P' - P)/P		3440.204
(11) Net Social Cost from Production (JD)	.5(P' - P) (Qp' - Qp)	2	35,778.117
(12) Producers Surplus Gain (JD)	(9) - (11)	1	775,421.883
(13) Other Producer Transfer (JD)	--	-	--
(14) Consumption (tons)	Qc' = Qp' + (Qc' - Qp')		54,178.000
(15) Addition of Price to Consumer Cost (JD)	(P' - P)Qc'	1 + 2 + 3 + 4	1,126,902.400
(16) Loss of Consumption (tons)	Qc - Qc' = Qc' Ed (P' - P)/P		19,116.241
(17) Net Social Cost from Consumption (JD)	.5 (P' - P) (Qc - Qc')	5	198,808.905
(18) Consumers Surplus Loss (JD)		1 thru 5	1,325,711.305
(19) Transfer to Parastatal (JD)	(P' - P) (Qc' - Qp')	3 + 4	315,702.400
(20) Other Social Cost (JD)	--	--	--
(21) Total Net Social Cost		2 + 5	234,587.022
(22) Transfer Inefficiency to Producers		(2 + 5)/1	0.303
(23) Social Cost as Percent of Producers Receipts		(2 + 5)/Qp'P'	4.337
(24) Ln of Supply Intercept	ln a = ln Qp' - Es ln P'		8.105
(25) Ln of Demand Intercept	ln b = ln Qc' + Ed ln P'		20.765
(26) Ln of Pe	(ln b - ln a)/(Es + Ed)		5.064
(27) Pe (JD)	exp (ln Pe)		158.190
(28) Ln Qe	ln a + Es ln Pe		10.637
(29) Ln Qe	ln b - Ed ln Pe		10.637
(30) Qe (tons)	exp (ln Qe)		41,650.122
(31) Revenue (JD)	Pe Qe		6588,644.652
(32) Market Equil. Production (tons)	Qp = Qp' - (Qp' - Qp)		35,559.796
(33) Market Equil. Consumption (tons)	Qc = Qc' + (Qc - Qc')		73,294.241
(34) Market Equil. Imports (tons)	Qc - Qp		37,734.444
(35) Isolation Price Gain (JD)	dP = Pe - P		40.290
(36) Proportional Price Gain	(Pe - P)/.5 (Pe + P)		0.292
(37) Production Gain in Isolation (tons)	dQs = QpEs (Pe - P)/P		5,189.296
(38) Consumption Loss in Isolation (tons)	dQd = QcEd (Pe - P)/P		42,783.770
(39) Change in Net Imports (tons)	Qc - Qp = dQs + dQd		47,973.066
(40) Policy Transfer to Producers (JD)	Qe (Pe - P)	1 + 2 + 3 + 7 + 8	1,678,095.251
(41) Net Social Cost - Supply (JD)	.5 (dP) (dQs)	2 + 3 + 7	104,539.112
(42) Net Gain in Producers Surplus (JD)		1 + 8	1,573,556.139
(43) Net Social Costs - Consumers (JD)	.5 (dP) (dQd)	4 + 5 + 6	861,885.116
(44) Net Loss in Consumers Surplus (JD)		1 thru 8	2,539,980.367
(45) Total Net Social Cost - Isolation (JD)		2 thru 7	966,424.228
(46) Transfer Inefficiency to Producers (JD)		(2 thru 7)/1 + 8	0.614
(47) Social Cost as Percent of Producers Receipts		(2 thru 7)/Pe Qe	14.668

The loss in national income per JD transferred to producers (assuming the latter is the objective of the policy) is JD .30 for AMPCO intervention to raise domestic prices but would be JD .61 for a policy of self-sufficiency. Administrative and PEST costs are not included.

Direct income transfers to producers would be more efficient. Administrative costs typically are 5 to 10 percent of transfers so only JD .05 to .10 of national income would be lost to add each JD to income of producers.

Exercise 2. Recompute results using spreadsheet analysis for a price elasticity of supply of 1.00 in (5) of Table 6.4.

Recompute results for a price elasticity of supply 1.00 in (6) of Table 6.4.

Exercise 3. Return to Figure 6.3C. Show that the net gain from competitive trade versus isolation is area 6 for the EC and area 13 for ROW.

Exercise 4. The partial equilibrium analysis herein assumes that prices of other commodities are fixed. Chapter 4 presents a demand elasticity matrix and Chapter 5 presents a supply elasticity matrix for agricultural products in Pakistan. By making certain assumptions regarding market interventions, advanced students with understanding of matrix manipulation using computers can employ the elasticity matrices to calculate a general equilibrium classical welfare analysis allowing prices and quantities of several commodities to vary.

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7. SIMULATION AND SYSTEMS MODELING

by James N. Trapp

This chapter provides the reader with an overview of systems analysis and the process of constructing a computerized simulation model. In addition, several specific modeling techniques found to be useful in modeling and analyzing a broad array of price control policies in developing nations are presented. One such tool is the Generalized Econometric Spreadsheet Simulation (GESS) Model. This model provides a framework for conceptualizing and implementing an econometric model. A second major tool dealt with in detail is the modeling of random variables in economic systems. The use of computer generated random number series to represent price and production uncertainties is illustrated.

OBJECTIVES

The objectives of this chapter are to:

1. Provide a general overview of the systems approach to modeling.
2. Give a working knowledge of the concept of own- and cross-price elasticities and their use in econometric modeling.
3. Illustrate a case development of a Generalized Econometric Spreadsheet Simulation Model (GESS Model) with detail allowing readers with LOTUS 1-2-3 programming experience to construct their own GESS model and modify it to address their unique policy analysis needs.
4. Present key procedures for modeling stochastic events with detail allowing readers with LOTUS 1-2-3 programming experience to use the techniques in spreadsheet modeling applications.

KEY POINTS

1. Simulation models need to be as simple as possible consistent with availability of analytical resources and the needs for analysis. New microcomputer software and modeling techniques have made it feasible to develop a functional econometric simulation model which can be used for applied policy analysis with knowledge of only a few key elasticities of supply and demand. These key elasticity parameters may be estimated, obtained from previous studies, or specified by judgment.
2. Use of spreadsheet programming methods allows functional econometric policy analysis models to be implemented on a microcomputer with a minimum amount of computer training (1 to 2 weeks) and used by individuals with virtually no computer training.
3. Econometric models provide a useful way to analyze the direct influences of commodity price control policies upon supply and demand of the commodity whose price is being controlled. Perhaps more importantly, econometric models also provide a way to analyze the indirect effect of price control policies for one commodity upon the supply and demand for related commodities. They also provide a method through which the overall affect of policies on such variables as trade balances, farm sector gross revenue, and nutrition levels can be analyzed.
4. In many policy making decisions, including "food security policy", it is just as important to be able to model the stochastic characteristics of the sector/market in question as it is to model the expected and/or optimal conditions for the sector/model.
5. Useable stochastic econometric simulation policy analysis models can be developed with the techniques presented here in a matter of weeks, and need not involve large staffs of professionals.

CONCEPTS AND APPLICATIONS

An Overview of Simulation Analysis

Simulation analysis involves the study of a "system" where a system is broadly defined as a set of interconnected elements (components) organized toward a goal or set of goals. Analysis of a system involves defining the interconnections (structure) of the system in question and the goal(s) of the system. In identifying the goals of the system one must necessarily also consider the participants in the system, i.e. producers, consumers, and policy makers.

Simulation is defined as the process of numerically solving a computerized mathematical model in an attempt to reproduce the actual essential elements of a operating system. Operating systems of interest to economics include firms, markets, and government agencies. However, the same general procedures used for economic modeling

have been found useful in modeling physical and biological processes, such as plant and animal growth or engine performance.

Simulation attempts to reproduce on a small scale the actual essential elements of an operating market, socio-economic system, or other unit of inquiry. By allowing the analyst to observe and experiment with the working of the system in the "laboratory" of a computer model, simulation is a powerful tool of policy analysis. Because of computational requirements of simulation models, it is not coincidental that their popularity has been closely correlated with the use of computers.

Simulation may be simple or complex. Tradeoffs are inevitable: large models provide greater realism, but are costly to build and maintain; small models are less costly, but may lack essential realism. There is merit in keeping simulation models as simple as possible consistent with meeting the needs of the policy analysis at hand.

1. System Simulation Models. System simulation emphasizes accounting for all components in the system that bear significantly on the analysis and solutions to problems. System simulation models offer a broad and flexible means of dealing with the complexities of monitoring and evaluating food and agricultural policies, programs, and projects. Their flexibility rests in their building-block organization, allowing for the development and incorporation of new components and the substitution of complex components for simpler ones as necessary to suit the particular analysis. They also allow for the use of a wide range of modeling techniques from different disciplines, depending on the needs at hand. Simulation is adaptive in that improved knowledge about data and structural relations can be progressively incorporated into the models.

A simulation model can incorporate one or more of several models discussed in this book including supply, demand, and balance sheet components; linear programming and other optimizing procedures; and behavioral equations. It can predict real world behavior or show the impact of policy experiments including alternative tax rates, quotas, price controls, investments in land and water development projects, land reform, and population planning.

Because any model is only an approximation of reality, its absolute predictions will always contain errors. Relative consequences, however, while still subject to error, tend to be more reliable in that data and structural inaccuracies usually affect the results of different experiments in the same ways. For example, if simulation of Policy A results in $100 million of additional farm income by 1995 while Policy B yields $150 million, the fact that Policy B gave 50 percent more income relative to A may be a more accurate piece of information than the absolute numbers of $100 million and $150 million.

2. Selection of Model Type and Modeling Approach. Simulation models can have numerous forms including (Manetsch):

1. a microscopic (individual or firm) versus macroscopic view (sector or national);

2. static versus dynamic behavior, where dynamic models are generally characterized by feedback loops and time lags;
3. deterministic versus stochastic behavior, where stochastic models explicitly include probability distributions to treat data series and parameters as random variables;
4. linear versus nonlinear behavior, where an example of a linear model is a doubling of fertilizer application causing a doubling of rice yield, and where a nonlinear model allows for diminishing returns, thresholds, saturation points, and other types of constraints;
5. optimizing versus nonoptimizing behavior, where an optimizing model attempts to identify decisions which maximize profits, security, happiness, or efficiency or which minimize costs, poverty, malnourishment, or trade deficits; and
6. structural versus predictive, where predictive models emphasize ability to forecast future outcomes whereas structural approaches emphasize causal relationships among important variables of the system. These structural relationships are *deduced* using appropriate theory from disciplines such as economics, sociology, biology, and physics. The structure of the model, once specified, determines what data and statistical analyses will be needed to estimate parameter values and test the model's validity.

Emphasis in structural models is on the theoretical and logical validity of the causal relationships in order to understand why certain behavior occurs in response to past and proposed policy conditions. For this reason, structural models are typically better at predicting and explaining qualitative behavior and responses than at making forecasts of absolute numbers.

By contrast, behavioral predictive models are developed through an inductive process. Beginning with empirical data on the behavior of variables of interest, relationships are chosen according to some criterion such as a least-squares fit. These statistical relationships will be a satisfactory predictor of what behavior may occur under conditions similar to those used to estimate the relationships. However, an understanding of how or why that behavior occurs (i.e., causal relationships) is important in making decisions about the future under conditions which never existed before -- a common situation in development planning. Because structural and predictive modeling have their own respective strengths and weaknesses, it is not surprising that practical models for policy analysis are hybrids typically falling somewhere between the two extremes.

Some general statements can be made about the requirements of developing and operating various types of models (Manetsch):

1. Dynamic models are usually more costly to develop than static models. However, they usually provide decision makers with significantly more useful information.

2. Stochastic models usually are not much more expensive to build than deterministic models, but they are much more expensive to operate on computers.
3. Optimizing models are usually more expensive to operate than nonoptimizing models.

3. The Modeling Process. In the sections that follow, an overview will be given of the steps involved in specifying and implementing a systems simulation model. Space does not permit a detailed elaboration of each of the steps, but only an overview.

•***Model Specification.*** Specification of a systems model begins with a *Feasibility Evaluation.* This initial step is perhaps the most important in the modeling process. The basic components of a Feasibility Evaluation include: (1) Needs Analysis; (2) Systems Identification; (3) Problem Statement; (4) Generation of Systems Alternatives, and (5) Assessment of Physical, Social, and Political Realizeability.

The first step of the Feasibility Evaluation is to determine the basic need to be addressed. An example is a need to estimate the cost of a given price support program, or a need to determine a least-cost production method. In determining the nature of the need(s) the participants of the system who have this need should be identified and questioned so that the nature of their need is clearly understood.

In the process of understanding the needs of the system participants, Systems Identification is initiated. This is a critical step of the Feasibility Evaluation, identifying the structure of the system with respect to the need(s). Figure 7.1 depicts the key components to be determined in the Systems Identification process. From the needs analysis it should be evident what outputs of the system are desirable and undesirable. All outputs should be listed and quantifiable measures for them identified. Secondly, the inputs that influence the system should be identified. Inputs are basically of three types: those exogenous to the system and a reflection of the environment, those controllable by the system manager, and those which must be overtly provided by the manager, but over which management has no control.

Separation of the system and environment defines the scope of the model. A basic rule to be observed here is that the output of the system should not essentially change the environment, otherwise the system should be expanded to include these critical interconnections. Variables chosen to describe the environment should be restricted to key variables required to reflect major uncontrollable influences on the system.

The most important input variables to define are those controllable by the managers of the systems. These variables will be the focus of the analysis process. However, one must also establish what other inputs management must allow for but may not control. For example, a tax rate may be a control variable but the cost of staffing a tax collection agency is a necessary input that cannot be avoided. Tax agency staffing cost is likely to be independent of the actual tax rate but, nevertheless, a required expense to collect a tax.

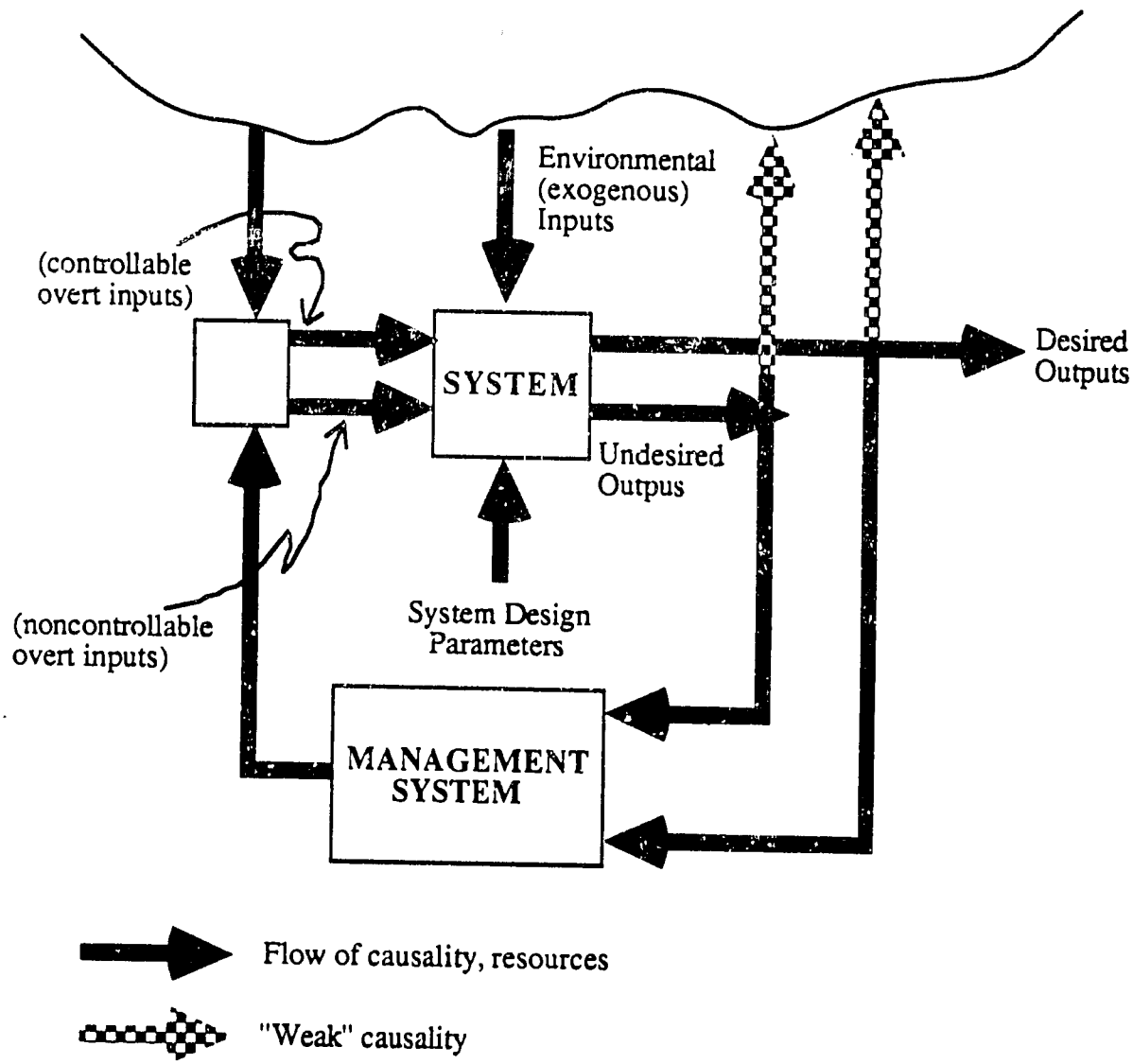


Figure 7.1 System Identification as Part of the Systems Approach (Feasibility Evaluation Phase)

Two linkages exist between system inputs and system outputs. One is the system itself and the other is management. It is important to consider what management system has previously existed and may exist in the future. Understanding of management's past practices and policies is critical to understanding total system behavior. The objective of the systems analysis in many cases will be to revise the management system, i.e., to devise new policies for setting controllable variables in response to various system outputs. The other link between system inputs and system outputs is the system itself. The structure of that system must be defined to a degree such that changes in system outputs in response to changes in system inputs can be determined. In economic analysis this traditionally has been done using statistical estimation procedures. In its simplest form, a system may be a production function. In a more complex form, it may be a system of supply and demand equations for a multicommodity market. Within the feasibility study and the System Identification step, estimation of the structure is not undertaken. What is undertaken is identification of the cause and effect linkages of the system. To this end, causal loop and block diagramming procedures are often used. Space does not permit a discussion of diagraming techniques here. The interested reader is referred to Manetsch.

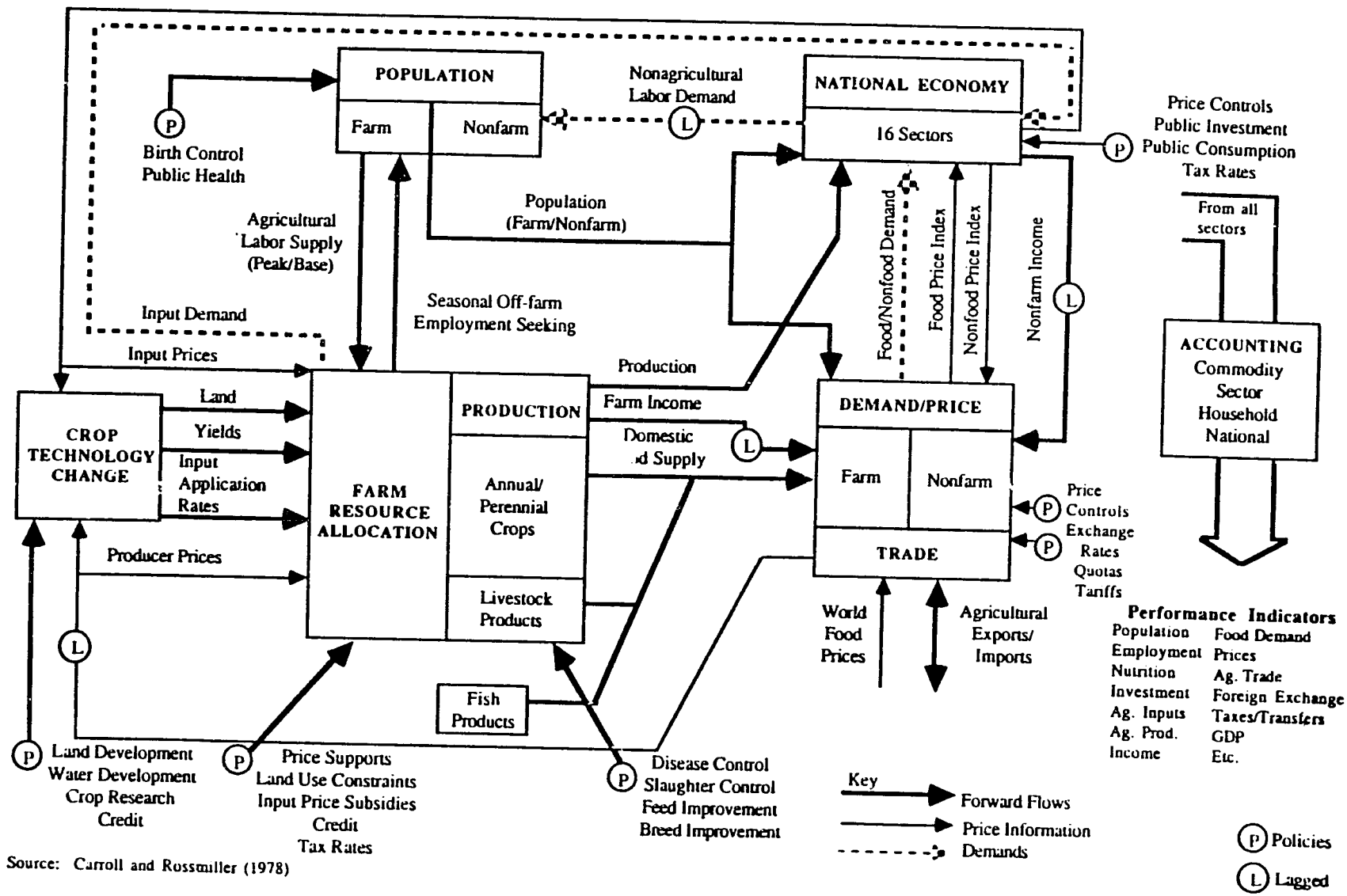
The building-block approach is often most suitable for construction of system simulation models. For example, Figure 7.2 is a diagram of a complex simulation model developed for agricultural sector policy analysis which uses a variety of modeling approaches: the national economy component uses a 16-sector input-output model; the population component utilizes a cohort-survival model; the crop technology change component follows a production function model; the resource allocation component is based on a recursive linear programming model; and the demand/price/trade component operates as a simultaneous-equation, market-clearing model. The interested reader is referred to Chapters 5-10 in Rossmiller for further background and descriptions of the overall system model and its components.

The Needs Analysis and System Identification phases of the Feasibility Evaluation allows the analyst to produce a Problem Statement. The Problem Statement should be concise with regard to what performance is desired, what controls influence the performance, what management system is needed to implement these controls, and what environmental factors must be dealt with in achieving the desired performance.

The final two steps of the Feasibility Evaluation involve first a search for all conceivable management systems, policy controls, and modifications to the system itself that may be feasible. This is followed by a screening of these alternatives for physical, political, and social acceptability.

•Data Sources and Requirements. Once the system has been identified and system alternatives specified in the Feasibility Study, the model must be implemented. In most cases, implementation begins with data collection.

The aggregation levels chosen in the model specification stage for commodities, population groups, or geographic regions heavily influence specific data requirements, data sources, and procedures for estimation. For example, depending on the country and the purposes for the model, it might be important to treat rice, corn, wheat, beef, and milk as



Source: Carroll and Rossmiller (1978)

Figure 7.2. A Five-component Agricultural Sector Simulation Model

separate commodities (although even these products are aggregates of components with different characteristics) while grouping other commodities such as fruits, vegetables, and industrial crops.

As an illustration, Table 7.1 shows a few of the data requirements of the agricultural sector simulation model described above and in Figure 7.2. For each of the five component models, selected data requirements are listed in three major categories:

1. initial conditions of internally computed variables;
2. values of parameters measuring the strengths of technical, institutional, and behavioral relationships; and
3. values, for each time period to be simulated, of exogenous variables such as government decisions.

Sensitivity testing is an important method for determining how much the outputs of the model are influenced by possible errors in the model's data series and parameter values. It involves making systematic changes in appropriate data and parameters, based on the degree of uncertainty regarding their accuracy, and then examining the effects of those changes in one or more model outputs. Sensitivity testing helps set priorities on efforts to improve the parameter estimates and data in the model, thereby allocating scarce resources for data collection and analysis in a cost-effective manner.

•***Abstract Model Implementation.*** Given data and a detailed systems identification diagram, a mathematical representation of the system can be undertaken. Economist generally relate variables within the system to each other by means of regression analysis. Time series data (or cross-section data) of the variables in question are collected and used together with regression analysis to estimate parameters. However, the general art of systems modeling does not restrict itself to defining the parameters of the system with regression analysis. In some cases, such as physical transformations, a single observation or experiment is adequate to define a parameter. In other cases, theory may be specific enough to provide the parameter(s) in question. For example, cross-elasticities of demand systems can be theoretically deduced from own-price and income elasticities, and market share information. Likewise some parameters of the system may be defined from knowledge of the decision rules used by managers and policy makers. If none of the above methods succeed in providing the necessary parameters for the system, the modeler is faced with either eliminating that portion of the model structure and ignoring it, or providing judgment estimates of the parameters in question. Depending on the purpose of the model and the nature of the parameter in question, it may be preferable, at least initially, to provide subjective parameter estimates rather than to ignore the structural feature at issue.

•***Computer Implementation.*** Simulating a model refers to the process of determining the behavior of the endogenous variables over time, given particular initial conditions of those variables and values of input variables (e.g., policies, weather conditions) over time. In the case of simple models, it may be possible to apply

Table 7.1. Selected Data Requirements and Sources for an Agricultural Sector Simulation Model

Component	Initial Conditions		Technical, Institutional, Behavioural Parameters		Exogenous Variables					
					Policy Variables		Component Inputs		System Inputs	
	Requirement	Source	Requirement	Source	Requirement	Source	Requirement	Source	Requirement	Source
Population and Migration	population by age, sex, and sector	population and housing census, agricultural census, analysis and adjustments	birthrates death rates migration rates	demographic and special surveys, population and housing census, model life tables, population and housing census analysis	military manpower family planning	assumption assumed birth-rates	nonagricultural employment	nonagricultural censuses and surveys	urban unemployment	nonagricultural censuses and surveys
Crop Technology Change	crop yields input use land classes	agricultural production survey agricultural production survey analysis agricultural production survey, soil survey	diffusion rates input elasticities productivity growth on new land	analysis analysis technical research experiments	land and water development investment crop improvement extension services	assumption assumption and analysis assumption	producer prices crop areas farm income	price survey agricultural production survey farm household survey	land development costs maximum potential land improvement	engineering data assumption and analysis
Farm Resource Allocation and Production	crop areas herd sizes farm savings	agricultural production survey livestock survey farm household survey	unit resource requirements depreciation rates	farm management survey assumption and technical data	agricultural finance policies maximum feed imports	assumption assumption and analysis	producer prices input prices land available farm labor available	price survey price survey and analysis agricultural census and analysis, rural labor survey and analysis	livestock yields	livestock survey special studies
Demand/Price/Trade	per capita consumption producer prices	food consumption survey food balance analysis and adjustments price survey	price and income elasticities	analyses based on data from price surveys and rural and urban household surveys	price bounds exchange rates	assumption assumption	population food supply income	population census and analysis, analysis, household surveys and analysis	per capita consumption targets world prices	nutrition and trend analysis trade statistics and analysis
National Economy	capital stock gross investment	industrial surveys special studies	input-output coefficients capital-output ratios	input-output tables/analysis special studies	tax rates, import substitution	assumption, assumption and analysis	agricultural input demand, nonfood expenditures	farm management surveys rural and urban household surveys	labor productivity world prices	labor surveys and analysis trade statistics and analysis

Source: Adapted from Thodey.

mathematical techniques such as calculus and algebra to arrive at a general solution to equations of the endogenous variables as functions of the input variables. However, such solutions are generally impossible for large, complex, nonlinear, socio-economic models. In such cases, it is necessary to use numerical analysis techniques, the basis of simulation, in which solutions are found by stepping the model through time in small increments to generate a particular time series of numerical values for the endogenous variables.

•***Model Testing and Credibility.*** The credibility of a model is tested every time it is used for policy analysis. Credibility testing involves checking:

1. the *logical structure*, defined as the conceptual validity and internal consistency of the model's structure;
2. the *predictive capability*, defined as the model's ability to predict the real world. Failure of the model to track historical values within some acceptable range indicates problems in the structure of the model, in estimated parameter values, or in the time-series data supporting it;
3. the *clarity* of the model, i.e., that it be unambiguous and easily communicated to analysts and decision makers; and
4. the *workability* both of the model, in terms of its utility in operation, and of the policy recommendations resulting from its use in policy analysis.

Finally, decision makers themselves make judgments about the utility of a model in terms of (1) the value of the information received as compared to the cost of acquiring it and (2) the workability of the policies which have been implemented partly on the basis of the results of the modeling exercise. Ultimately, this may be the most important test, for it concerns the usefulness of the information for decision making -- the overall objective of the information system.

With regard to achieving creditability of a model with decision makers, a good practice is to involve the decision maker in the modeling process as soon as possible, preferable beginning with the Needs Analysis phase. The more the decision makers understand the modeling process and provide input, the more credibility they will give to the model.

Simulation models formulated for various developing countries are described elsewhere (see Thorbecke and Hall; Rossmiller). Such models are highly diverse and are capable of addressing issues of efficiency, equity, and variability. Some stochastic models are suited to analyze commodity stock operations and other programs to reduce variation in food supplies and prices.

Agricultural Sector Econometric Modeling Using Spreadsheets

A particular application of simulation that has been found to be very adaptable and useful for modeling of agricultural policy decisions is the Generalized Econometric Spreadsheet Simulation Model (GESS Model). In the past most econometric simulation

models have been developed case by case. Individually unique models using FORTRAN, PL-1, or other basic computer languages were built for each modeling effort. No generalized modeling framework existed to aid in the model's computer implementation. The GESS Model provides a generalized framework for computer implementation of an econometric model of a developing country's agricultural sector.

The following section of this chapter will present the general methodology of the GESS Model. First, the general structure assumed for the model will be presented. Second, the theoretical underpinnings of this structure will be discussed. Lastly the procedure developed for implementing the model in a spreadsheet format will be presented.

1. Model Structure. A large majority of small developing countries have agricultural sectors with an economic market structure similar to that depicted in Figure 7.3. Prices are essentially determined by two exogenous forces: the government and/or the world market. These prices in turn determine the demand and supply of agricultural products. To the extent that these resulting supply and demand quantities do not match, incentives exist for imports or exports.

Two key assumptions and simplifications are evident in the structure depicted in Figure 7.3. The first is that all prices are exogenously determined. In small countries, or countries with significant price controls, this is a realistic assumption. A small country cannot influence world market prices through either its changes in consumption or production. Such a country, therefore, becomes a price taker in the world market. Because of this, many developing countries have a policy of controlling their agricultural prices to partially protect their producers and/or consumers from the instability of the world market. Thus the assumption of exogenous prices appears reasonable for many developing country cases.

A second simplifying assumption is that any imbalance between supply and demand, given the exogenous prices, will result in trade. In actuality, only *incentives* for trade are created which may or may not be fulfilled. These incentives may be constrained by government intervention in the form of quotas, tariffs, and/or lack of foreign currency due the economy-wide general trade balance. Thus, the supply/demand imbalance generated by the exogenous price situation should realistically be viewed as the resulting balance of incentives for trade. To determine actual trade the policies directly affecting trade also would need to be considered.

Figure 7.4 develops in more detail the structure diagramed in Figure 7.3. The retail, wholesale, and farm level market, as well as government policies influencing these three market levels and their link to the world market are considered. World prices are directly linked to the wholesale price. Depending on the size, efficiency, and distortions in the market in question, a country's wholesale price may be a few percentage points above the world market price for imports and a few percentage points below the world market price for exports. The primary cause of the spread is the marketing cost and profit involved in importing and exporting. A government tax or subsidy on imports or exports can drastically change the wholesale/world market price spread. An export tax in essence adds a marketing expense. As a result, the exporting company will reduce its domestic

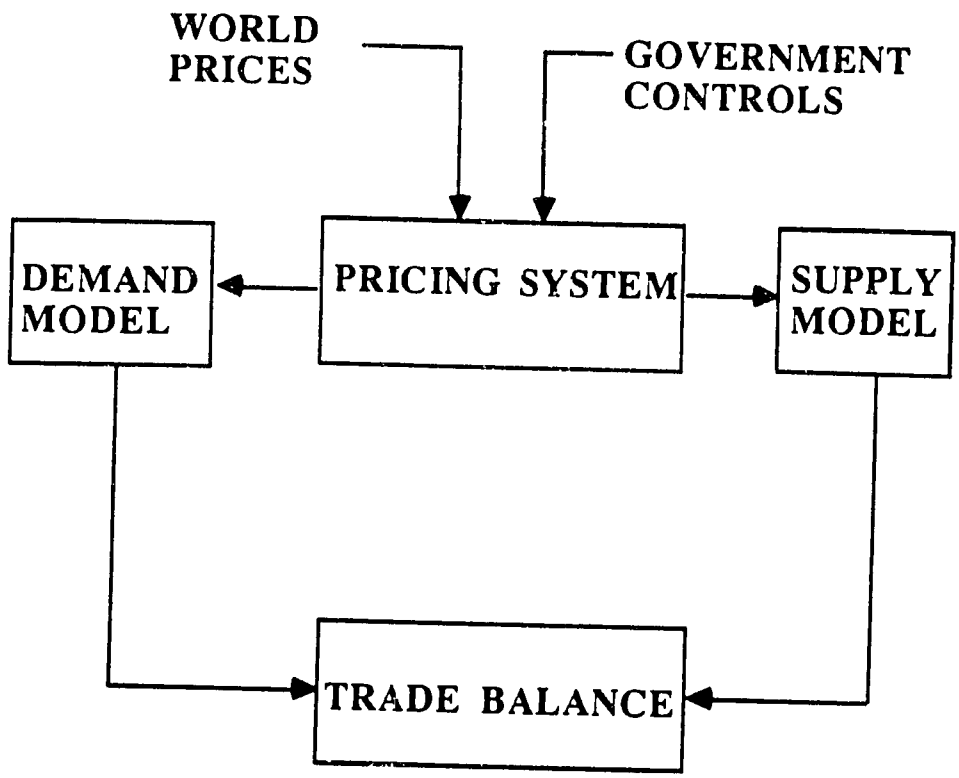


Figure 7.3. General Model Structure

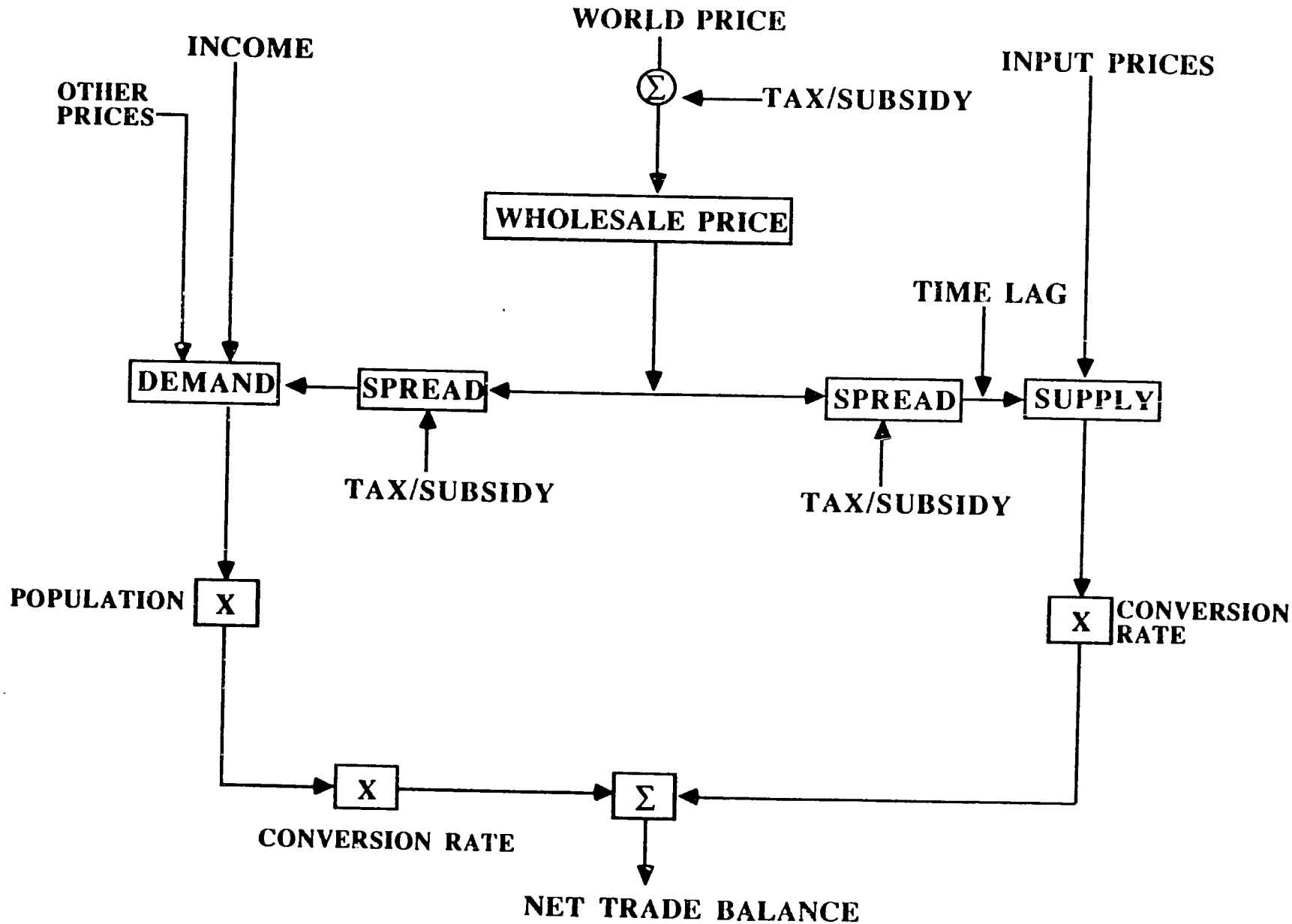


FIGURE 7.4. Detailed Model Structure

wholesale purchasing price and widen the wholesale/world price spread to cover this cost. Likewise, a subsidy on an export would allow the wholesale exporter to pay more for the domestic product and still cover his costs. In fact a large subsidy could result in the domestic price of an export product exceeding the world price. Similar logic can be used to deduce the effect of taxes and subsidies on imports.

Wholesale prices are linked to retail and farm level prices by marketing/processing spreads. This spread is essentially equal to the marketing and processing cost for the product, plus a profit margin. However, if the government intervenes to tax or subsidize the commodity in question, the spread between these markets will be affected in the same manner as discussed for the import/export market.

In modeling the market spread between market levels one must also consider that the physical units may be changed by processing. Rice may be milled and animals slaughtered. Physical changes in the units involved are denoted in Figure 7.4 by the presence of "conversion rate" variables.

Modeling of supply response, as noted in Chapter 5, needs to consider in may cases that supply does not adjust immediately to price changes because of the biological time involved in production. Today's changes in supply may be the result of past price changes. Supply may be influenced by input prices as well as output prices. Input prices more often than product prices are exogenous to the agricultural sector market. If input prices are not set by government, they are generally exogenous to the agricultural sector because they are determined by markets outside of the agricultural sector. Because of their exogenous nature, input prices can be stable as agricultural demand for inputs varies.

Demand is generally modeled as per capita demand and is specified to be a function of its own price, the price of other substitute and complimentary goods, and income as noted in Chapter 4. Multiplication of per capita demand by population will give total retail demand. Use of a physical conversion factor, if necessary, will convert retail quantity to wholesale and/or farm level quantities.

The existence of other competing and complementary food prices in the demand model, and input prices in the supply model (which may also be prices of other agricultural products) raises an issue with regard to modeling the total agricultural sector. Figure 7.4 only depicts the model structure for one commodity. For a sector model, a structural model needs to be specified for all major agricultural commodities in the sector. These individual models in turn need to be linked together through common prices. As long as all prices are assumed to be exogenously determined this linkage is relatively straightforward. Methods for modeling this linkage will be discussed in detail later.

The general econometric model structure described above has several advantages and disadvantages. The assumption of exogenous prices if reasonably realistic is a major advantage because it avoids complexity in the computerization and solution process. Endogenous price determination can be modeled with a spreadsheet program but much of the simplicity and power of the spreadsheet approach is lost.

A second advantage of the described model structure is the large number of policy variables it allows in a relatively simple model. For example, the impact can be considered

of import and export taxes and subsidies, producer price supports, input subsidies, consumer price subsidies and taxes, self-sufficiency, and even per capita nutritional level.

A third advantage of the described structure and use of a spreadsheet program is the ease with which the model can be developed and implemented, providing either reliable demand and supply elasticities are available on time series data (ten or more years preferably) and/or cross section data exist with which to estimate them. The need for aggregate data or large amounts of cross-sectional survey data, as opposed to limited quantities of firm level data, is one of the distinct disadvantages of this modeling approach and the econometric modeling approach in general. However, several compromises discussed later in the chapter help to alleviate the problem of inadequate data.

Another advantage or disadvantage, depending on the purpose of the analysis, is the focus of the model on short-term (annual) responses to price and policy changes. In general, the model is valid for looking at marginal changes in policy for one to perhaps five years into the future. Long-run impacts of drastic changes in price and policies which change structure cannot be adequately analyzed by the model.

2. ***Theoretical Underpinnings.*** At the heart of any econometric model is a set of supply and demand responses or elasticities (see Chapters 4 and 5). The modeling approach presented uses elasticities to relate price and quantity changes.

Elasticities are formally defined as shown in equations 7.1 and 7.2.

$$E_{own} = \frac{\Delta Qa}{\Delta Pa} \times \frac{Pa}{Qa} \tag{7.1}$$

$$E_{cross} = \frac{\Delta Qa}{\Delta Pb} \times \frac{Pb}{Qa} \tag{7.2}$$

The above basic definition of an elasticity can readily be related to the parameters of supply and/or demand equations of the general form Q = f(Pa, Pb). If a linear functional form is chosen, the following relation exists:

$$Qa = a + bPa + cPb \tag{7.3}$$

Equations 7.4 and 7.5 can be derived from equations 7.1 and 7.2.

$$E_{own} \times \frac{Qa}{Pa} = \frac{\Delta Qa}{\Delta Pa} \tag{7.4}$$

$$E_{cross} \times \frac{Qa}{Pb} = \frac{\Delta Qa}{\Delta Pb} \tag{7.5}$$

$\partial Q/\partial P$ is the continuous equivalent of $\Delta Q/\Delta P$. Knowing this, equations 7.6 and 7.7 follow:

$$E_{own} \times \frac{Qa}{Pa} = \frac{\partial Qa}{\partial Pa} = b \tag{7.6}$$

$$E_{cross} \times \frac{Qa}{Pb} = \frac{\partial Qa}{\partial Pb} = c \tag{7.7}$$

or alternatively equations 7.8 and 7.9 follow:

$$E_{own} = b \frac{Pa}{Qa} \tag{7.8}$$

$$E_{cross} = c \frac{Pb}{Qa}. \tag{7.9}$$

Likewise, if a double log functional form is chosen and specified as in equation 7.10, it can be shown that the parameters of the equation are elasticities.

$$Qa = a P_a^{b_1} P_b^{b_2} \tag{7.10}$$

$$\frac{\partial Qa}{\partial Pa} = b_1 a P_a^{b_1 - 1} P_b^{b_2} \tag{7.11}$$

Substituting equation 7.11 into equation 7.1 gives equation 7.12:

$$E_{own} = \frac{b_1 a P_a^{b_1} P_b^{b_2} Pa}{Pa\ Qa}. \tag{7.12}$$

Note that the numerator of 7.12 contains Qa as shown in equation 7.10. By substituting Q_a into equation 7.12 to form equation 7.13 and then cancelling terms, it is apparent that the own-price elasticity is equal to b_1:

$$E_{own} = \frac{b_1\ Qa\ Pa}{Pa\ Qa} = b_1. \tag{7.13}$$

Likewise, it can be shown by a similar argument that the cross-price elasticity Qa with respect to Pb is equal to the parameter b_2 in equation 7.10.

$$E_{cross} = \frac{b_2\ Qa\ Pb}{Pb\ Qa} = b_2 \tag{7.14}$$

In summary, elasticities and slopes of supply (or demand) equations are directly related. Given one, the other can be deduced. Equations 7.6 and 7.7 for the linear case, and 7.13 and 7.14 for the log linear case, show how all but the intercept/constant parameter of an equation can be deduced given own- and cross-price elasticities and base prices and quantities when computerization is done in the GESS modeling approach. Once the parameters of a linear or log linear function are known the intercept/constant parameter of the respective equation can be readily determined by equation 7.15 in the linear case and 7.16 in the log linear case.

$$a = Qa - bPa - cPc \tag{7.15}$$

$$a = Qa/\left(P_a^{b1} P_b^{b2}\right) \tag{7.16}$$

The prices and quantities used can be either the average prices and quantities over a period of time or from a specific point in time. It should be noted that income elasticities, partial adjustment coefficients for lagged dependent variables, and price flexibilities (for price dependent functions) can be dealt with using the same methodology as above.

The above procedure for deducing supply or demand equations from own and cross elasticities allows the heart of the GESS Model to be summarized in a matrix of supply elasticities and a matrix of demand elasticities. Using the equations listed above, the entire set of relationships required for an econometric model can be generated in a spreadsheet from two matrices of elasticities and a set of base year prices and quantities.

Chapters 4 and 5 of this book showed how supply and demand theory can be used to determine consistent relationships among own- and cross-price elasticities. The use of supply and demand elasticity matrices as the core of the GESS modeling approach can facilitate the incorporation of previous research results and/or general knowledge about elasticities into the model. If previous studies are deemed valid, the elasticity estimates from them may be directly incorporated into the GESS Model. In some cases, a good literature review can substitute for costly data collection and parameter estimation.

Sometimes it is necessary to rely heavily upon subjective knowledge and judgment to obtain the required elasticities. Done correctly, the GESS Modeling approach together with subjectively applied elasticities can provide much useful information. A review of the literature reveals some similarities or patterns among elasticities for commodities across countries. Thus, the literature from other countries coupled with expert opinion may provide a credible set of own-price elasticities for a given country. Then the procedure presented in Chapters 4 and 5 can be used along with basic market share information to generate a complete set of supply and demand elasticities.

The results of a model with only judgment-based elasticities can provide two types of valuable information. First, the model can provide estimates of the general order of magnitude of various pricing policy effects. It can draw attention to multicommodity effects not intuitively obvious even to the trained economist. Second, sensitivity testing indicates which elasticities are critical to the model results. Sensitivity testing essentially involves changing one or more of the assumed elasticities by a given amount and observing the changes in the model's results. Attention then can be focused upon obtaining accurate specification of the parameters found to be critical in evaluating the policy in question. The computerized spreadsheet format of the GESS Model facilitates sensitivity testing.

3. Computer Implementation of the GESS Model. Computer implementation of the GESS Model with a Microcomputer Spreadsheet program provides several distinct advantages. The first advantage is that learning to "program" or design models with a spreadsheet is much easier than learning to program with a computer language such as BASIC or FORTRAN. Indeed, the level of expertise required to implement the GESS Model using a LOTUS 1-2-3 spreadsheet program can be obtained in a one to two week workshop or self-taught over a somewhat longer period. Actual

implementation of the model (assuming all elasticities and base period prices and quantities are known) can be achieved in as little as two days, but certainly within two weeks in most cases.

A second advantage is that operation of the model is intuitive and easily learned. It is reasonable to expect that operational familiarity with the model can be obtained in less than a day by someone with no prior computer experience. This is the case because operation of the model becomes essentially a process of changing policy variables, and perhaps elasticity parameters, within what appears on the microcomputer screen as a set of well documented tables. Reading the output of the model involves simply observing computer generated tables. To the model user the process of simulating the impact of various policy changes is one of "modeling by table editing." Tables of policy variables are edited on the computer screen and the computer then automatically makes the necessary calculations to derive new tables of output projecting the impact of the specified policies.

In the presentation that follows a programer/modeler's overview will be given of how to develop a LOTUS 1-2-3 spreadsheet template for a GESS Model. Familiarity with the LOTUS 1-2-3 spreadsheet command and capabilities as discussed in Chapter 3 will be assumed. Those not familiar with LOTUS 1-2-3 commands will, however, be able to understand the essence of the process.

If the reader is interested in constructing an operating demonstration version of the GESS model, a tutorial assisting in this process has been provided in Annex 7.1 of this chapter. The tutorial does not provide key stroke by key stroke guide to completing a demonstration GESS model. Basic knowledge of LOTUS 1-2-3 is assumed. Key formulas are presented followed by a discussion of the procedure used to copy and replicate these formulas to complete the remainder of the model. The purpose of the tutorial is to provide the reader/programmer with enough experience and knowledge of the model that they can proceed to develop modified versions of the model suited to their own cases.

The task at hand for the LOTUS 1-2-3 spreadsheet modeler is essentially to link tables of own- and cross-price elasticities with tables of base period prices, quantities, and pricing policy variables so that tables of resulting supplies and demands can be generated. The methodology discussed which links elasticities to equation parameters provides the fundamental procedure for achieving this. These generated tables may then be augmented and reformulated to summarize the results in more meaningful ways, such as in terms of net trade balances, total government costs, and/or revenues.

For the purpose of this illustration a three-commodity model will be assumed. Figure 7.5 lays out the spreadsheet design for the heart of the model. It contains six basic tables: a Demand Table, Supply Table, Retail Price Table, Farm Price Table, Demand Elasticity Table, and Supply Elasticity Table. The Demand, Supply, Retail Price, and Farm

	A	B	C	D	E	F	G	H	I	J	K
1		RETAIL DEMAND (MILLION LBS.)									
2		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE			DEMAND ELASTICITIES			EQUATION CONSTANT TERM
3	Q#1	QD_1	S_1d_1	S_2d_1	$(S_2d_1 - S_1d_1)/S_1d_1$	Q#1	Q#1 E_{11}	Q#2 E_{12}	Q#3 E_{13}	INCOME I_1	a_1
4	Q#2	QD_2	S_1d_2	S_2d_2	$(S_2d_2 - S_1d_2)/S_1d_2$	Q#2	E_{21}	E_{22}	E_{23}	I_2	a_2
5	Q#3	QD_3	S_1d_3	S_2d_3	$(S_2d_3 - S_1d_3)/S_1d_3$	Q#3	E_{31}	E_{32}	E_{33}	I_3	a_3
6		FARM LEVEL SUPPLY (MILLION LBS.)						SUPPLY ELASTICITIES			
7		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE						EQUATION CONSTANT TERM
8	Q#1	QS_1	S_1a_1	S_2a_1	$(S_2a_1 - S_1a_1)/S_1d_1$	Q#1	Q#1 S_{11}	Q#2 S_{12}	Q#3 S_{13}	COST C_1	b_1
9	Q#2	QS_2	S_1a_2	S_2a_2	$(S_2a_2 - S_1a_2)/S_1d_2$	Q#2	S_{21}	S_{22}	S_{23}	C_2	b_2
10	Q#3	QS_3	S_1a_3	S_2a_3	$(S_2a_3 - S_1a_3)/S_1d_3$	Q#3	S_{31}	S_{32}	S_{33}	C_3	b_3
11		RETAIL PRICE ($/CWT)									
12		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE						
13	Q#1	RP_1	S_1rp_1	S_2rp_1	$(S_2rp_1 - S_1rp_1)/S_1rp_1$						
14	Q#2	RP_2	S_1rp_2	S_2rp_2	$(S_2rp_2 - S_1rp_2)/S_1rp_2$						
15	Q#3	RP_3	S_1rp_3	S_2rp_3	$(S_2rp_3 - S_1rp_3)/S_1rp_3$						
16		FARM PRICE ($/CWT)									
17		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE						
18	Q#1	FP_1	S_1fp_1	S_2fp_1	$(S_2fp_1 - S_1fp_1)/S_1fp_1$						
19	Q#2	FP_2	S_1fp_2	S_2fp_2	$(S_2fp_2 - S_1fp_2)/S_1fp_2$						
20	Q#3	FP_3	S_1fp_3	S_2fp_3	$(S_2fp_3 - S_1fp_3)/S_1fp_3$						

Figure 7.5. Core Table Structure of the GESS Model

Price Tables each contain five columns of information, (columns A through E), and three rows (one row for each of the three commodities considered). The entries in the tables in capital letters must be user supplied; those in small letters will be calculated by the program. Column A contains row names for the tables. In reality instead of Q#1, Q#2, etc., these names would be Rice, Corn, Chicken, etc. In Column B base period values are provided by the user or calculated from base period values provided elsewhere in the model. In this case, base period farm and retail prices are calculated by the program from world prices which are user specified elsewhere in the spreadsheet. Provision of world prices and other variables elsewhere in the spreadsheet will be discussed presently. Columns C and D report the outcomes of alternative scenarios of different policies and/or world price situations. Column E calculates and compares the percentage differences between Scenarios #1 and #2 as reported in columns C and D, respectively. Columns F through J contain the required supply and demand elasticities.

The size of the GESS Model precludes displaying at one time all of the tables of information required. In fact, for a model with many commodities it is usually best to design the program so only one table shows per screen and then use the "page down", "page up", or "page across" keys to move from table to table. In this illustration, actual "paging" breaks Figure 7.5 into two screens or pages, with the elasticity matrices appearing on a separate page. Figure 7.6 gives an overview or "screen map" of the tables in this three-commodity GESS Model. For the moment, concentrate only on the tables shown in Figure 7.5.

The model's operation is centered on the calculation of the values in Column C (or D) and in particular on the calculation of the quantities demanded and supplied, i.e., cells C3, C4, C5, C8, C9, and C10. These cells are calculated using double log functions of the form reported in equation 7.10. Using LOTUS 1-2-3 notation, cell C3 would be calculated by equation 7.17.

$$C3 = (K4 * (C13\text{^}G4) * (C14\text{^}H4) * (C15\text{^}I4) * (C68\text{^}J4)) * C71 \qquad (7.17)$$

For the reader not familiar with Lotus notation the "*" symbol denotes multiplication, and the "^" symbol denotes raising a variable to a power. The letter and number combination variables denote cell locations in the spreadsheet from which values are obtained, i.e. G4 denotes the intersection of Column G and row 4, otherwise denoted in Figure 7.5 as E_{11} or the own-price demand elasticity for Q#1. In essence, equation 7.17 calculates the projected demand for Q#1 by using the top row of values in the demand elasticity matrix together with the retail prices given in Column C of the Retail Price Table, i.e. in cells C13, C14, and C15. Each of the prices is raised to the power of the elasticity reported in the top row of the demand elasticity table. K4 is the constant term of the equation. The demand derived (per capita demand) is multiplied by population, as given in the cell C71, to determine total demand.

Cells C4 and C5 can be calculated in a similar fashion to project the demand for Q#2 and Q#3. Use of the supply elasticities in conjunction with farm prices in a similar

A . . . . . E F . . . . . K

1 Screen 1A (A1 ... E20)* Screen 1B (F1 ... K20)*

Retail Demand Demand Elasticities

Farm Supply Supply Elasticities

Retail Price

Farm Price

20

21 Screen 2A (A21 ... E40)*

Wholesale Price

Net Trade Balance

Wholesale/World Price Margin

Farm/Wholesale Price Margin

40

41 Screen 3A (A41 ... E60)*

Wholesale/Retail Price Margin

World Price

Import/Export Taxes and Subsidies

Farm Level Taxes and Subsidies

60

61 Screen 4A (A61 ... E80)*

Retail Level Taxes and Subsidies

Per Capita Income

Population

Production Cost Index

80

*Values in parenthesis denote the coordinates of the upper left hand and lower right hand corners of the screen in question.

Figure 7.6. GESS Model "Screen Map."

manner will generate projections of total supply. For example, cell C8 is calculated as follows:

$$C8 = K11 * (C18\text{^}G11) * (C19\text{^}H11) * (C20\text{^}I11) * (C74\text{^}J11). \quad (7.18)$$

In viewing Figure 7.5 and considering the variables and parameters used in equation 7.17 and 7.18, it is noted that only the elasticity parameters of equation 7.17 and 7.18 must be user provided (recall that capitalized variables in Figure 7.5 denote user supplied values and the small letters denote program calculated variables). Calculation of the non-user supplied variables/parameters of these equation must be done before the equations can be solved. Calculation of these variables warrants some explanation.

K4 in equation 7.17 denotes the constant term of the demand function for Q#1. It can be calculated within the program by using equation 7.16 as derived in the theoretical underpinnings section. In Lotus notation for the spreadsheet depicted in Figure 7.5, the calculation of K4 is as follows:

$$K4 = (B3/B71) / ((B13\text{^}G4) * (B14\text{^}H4) * (B15\text{^}14) * (B68\text{^}J4)) \quad (7.19)$$

Basically equation 7.19 is calculated using base period values as reported or calculated in column B, and elasticities. B3 / B71 calculates per capita demand in the base period. By dividing it by the products of the variables and parameters on the right-hand side of the demand equation, less the constant term, the constant term is solved for as explained in equation 7.16 in the theoretical discussion. Once K4 is calculated, it can be used in equation 7.17. Sequencing the calculation of K4 before equation 7.17 is not a problem with LOTUS 1-2-3. It's "natural order" sequencing of calculation will automatically cause K4 to be calculated before the value is needed in equation 7.17.

Returning again to Figure 7.5 and equation 7.17 we see that the values C13, C14, and C15 are internally calculated. These values are the retail prices for the commodities in question. At this point, it should be recalled that all prices are assumed to be exogenous and determined by either the world market or government controls. The internal calculation of C13, C14, and C15 is designed to reflect this assumption. Additional tables must be used to provide information about world prices, price margins, and government taxation/subsidization activities. These tables are provided on screens 2A, 3A, and 4A (see the Screen Map in Figure 7.6). The structure of these tables is summarized in Figures 7.7, 7.8, and 7.9. Calculation of the farm level and retail prices appearing in rows 13, 14, 15, 18, 19, and 20 respectively requires linking all the data in Figures 7.7, 7.8, and 7.9 together, i.e., screens 2A, 3A, and 4A. The starting point of this linkage is with the World Price data entered by the user in rows 48 to 50 as shown in Figure 7.8. The Wholesale Price can be calculated from the World Price, given the Wholesale to World Price Margin (found in rows 33-35), and any Import/Export Taxes and Subsidies (found in rows 53-55). Equation 7.20 shows the calculation of the wholesale price for Q#1 under Scenario #1, i.e. cell C23:

$$C23 = (C48 * (C33 / 100)) * ((100 + (C53 * (B28 / @ABS(B28)))) / 100). \quad (7.20)$$

	A	B	C	D	E
21		WHOLESALE PRICE ($/CWT)			
22		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
23	Q#1	WHP_1	S_1whp_1	S_2whp_1	(d - c)/c
24	Q#2	WHP_2	S_1whp_2	S_2whp_2	(d - c)/c
25	Q#3	WHP_3	S_1whp_3	S_2whp_3	(d - c)/c
26		NET TRADE BALANCE (MILLION LBS.)			
27		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
28	Q#1	NTB_1	S_1ntb_1	S_2ntb_1	(d - c)/c
29	Q#2	NTB_2	S_1ntb_2	S_2ntb_2	(d - c)/c
30	Q#3	NTB_3	S_1ntb_3	S_2ntb_3	(d - c)/c
31	WHOLESALE/WORLD PRICE MARGIN (WHOLESALE PRICE AS A PERCENT OF WORLD)				
32		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
33	Q#1	WWM_1	S_1WWM_1	S_2WWM_1	(d - c)/c
34	Q#2	WWM_2	S_1WWM_2	S_2WWM_2	(d - c)/c
35	Q#3	WWM_3	S_1WWM_3	S_2WWM_3	(d - c)/c
36	FARM/WHOLESALE PRICE MARGIN (FARM PRICE AS A PERCENT OF WHOLESALE)				
37		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
38	Q#1	FWM_1	S_1FWM_1	S_2FWM_1	(d - c)/c
39	Q#2	FWM_2	S_1FWM_2	S_2FWM_2	(d - c)/c
40	Q#3	FWM_3	S_1FWM_3	S_2FWM_3	(d - c)/c

Figure 7.7. Screen 2A of the GESS Model

	A	B	C	D	E
41	WHOLESALE/RETAIL MARGIN PRICE (RETAIL PRICE AS A PERCENT OF WHOLESALE)				
42		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
43	Q#1	WRM_1	S_1WRM_1	S_2WRM_1	(d - c)/c
44	Q#2	WRM_2	S_1WRM_2	S_2WRM_2	(d - c)/c
45	Q#3	WRM_3	S_1WRM_3	S_2WRM_3	(d - c)/c
46			WORLD PRICE		
47		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
48	Q#1	WP_1	S_1WP_1	S_2WP_1	(d - c)/c
49	Q#2	WP_2	S_1WP_2	S_2WP_2	(d - c)/c
50	Q#3	WP_3	S_1WP_3	S_2WP_3	(d - c)/c
51	IMPORT/EXPORT TAXES (-) AND SUBSIDIES (+) (AS A PERCENT OF FARM LEVEL				
52		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
53	Q#1	$IETS_1$	S_1IETS_1	S_2IETS_1	(d - c)/c
54	Q#2	$IETS_2$	S_1IETS_2	S_2IETS_2	(d - c)/c
55	Q#3	$IETS_3$	S_1IETS_3	S_2IETS_3	(d - c)/c
56	FARM/LEVEL TAXES (-) AND SUBSIDIES (+) (AS A PERCENT OF FARM LEVEL PR)				
57		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
53	Q#1				
54	Q#2				
60	Q#3				

Figure 7.8. Screen 3A of the GESS Model

	A	B	C	D	E
61	RETAIL LEVEL TAXES (-) AND SUBSIDIES (+) (AS A PERCENT OF RETAIL LEVEL)				
62		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
63	Q#1	RT_1	S_1RT_1	S_2RT_1	(d - c)/c
64	Q#2	RT_2	S_1RT_2	S_2RT_2	(d - c)/c
65	Q#3	RT_3	S_1RT_3	S_2RT_3	(d - c)/c
66			PER CAPITA INCOME		
67		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
68		PCI	S_1PCI	S_2PCI	(d - c)/c
69			POPULATION (THOUSANDS)		
70		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
71		POP	S_1POP	S_2POP	(d - c)/c
72			PRODUCTION COST INDEX		
73		BASE YEAR	SCENARIO#1	SCENARIO#2	PERCENT CHANGE
74	Q#1	PC_1	S_1PC_2	S_2PC_2	(d - c)/c
75	Q#2	PC_2	S_1PC_2	S_2PC_2	(d - c)/c
76	Q#3	PC_3	S_1PC_3	S_2PC_3	(d - c)/c

Figure 7.9. Screen 4A of the GESS Model

It is assumed that the Wholesale to World price spread is expressed by the World Price as a percentage of the Wholesale Price. Likewise, it is assumed any taxes or subsidies on imports or exports are reported as a percentage of the Wholesale Price, with taxes reported as a negative percent and subsidies as a positive percent.

Briefly, the logic of equation 7.20 is as follows. In the first term of the equation the World Price C48 is multiplied by the Wholesale Price expressed as a percentage of World Price as reported by the user in cell C33. Because C33 is reported in percentage terms, it must be converted to a decimal by dividing by 100. Thus, the first term of the equation calculates the Wholesale Price from the World Price.

The second term of the equation modifies the Wholesale Price calculated by the first term based on the effect of any tax or subsidy imposed on international trade. This modification is complicated by the fact that the impact of a tax or subsidy upon the domestic wholesale price is different depending on whether the commodity in question is an import or export. For example, a tax on an export will act like an increase in the cost of exporting and will cause the domestic Wholesale Price to be lowered relative to the World Price. On the other hand, a tax on an import will be an increase in import cost that will be passed on by raising the domestic Wholesale Price. Because taxes are reported as negative values in the Import/Export Tax/Subsidy table, the basic assumption is that all commodities listed are export commodities. If this is not the case, it is necessary to reverse the sign of the tax rate variable. A similar argument exists in the case of a subsidy. The net trade balance for the commodity in question is contained in cell B28. Positive values denote positive trade balances, or net exportation, and negative values denote net importation. The sign of the trade balance can be determined by dividing it's basic value by it's absolute value, i.e. B28/@ABS(B28). If this yields a negative value, i.e., a-1, the result can be taken times the basic tax/subsidy rate variable in cell C53 and the sign reversed to reflect the case of an import commodity. If the division yields a+1, multiplication of the tax/subsidy rate by the result will produce the desired effect of not changing the tax/subsidy rate specified. Thus, the operation of the second term of equation 7.20 ascertains the sign of the net trade variable, modifies the sign of the tax/subsidy rate appropriately, adds the appropriate percentage tax/subsidy to 100, and then converts the value obtained to a decimal by dividing by 100. Thus, the second term becomes the appropriate value for multiplying times the Wholesale Price to reflect the impact of a tax or subsidy on international trade.

Given the Wholesale Price, the Farm Level and Retail Level Prices can be calculated. The procedure is analogous to that used for the Wholesale Price calculation, except it is not complicated by the balance of trade issue. Equation 7.21 show the calculation of Farm Price for Q#1 while equation 7.22 shows the calculation of the Retail Price for Q#1:

$$C18 = (C23 * (C38 / 100)) * ((100 + C58) / 100) \tag{7.21}$$

$$C13 = (C23 * (C43 / 100)) * ((100 - C63) / 100). \tag{7.22}$$

With the above system of interlinked prices, the effect of any world price change or price policy change will be passed through the entire system, i.e., any World Price or policy variable change made in rows 48 through 65 will change the Retail and/or Farm Prices. These changes in turn will change consumption and production.

To complete the discussion of variables that must be user-provided, several more tables illustrated in Figure 7.9 must be mentioned. These tables provide values for variables that are exogenous to the agricultural sector. They include tables of values for per capita income, population, and production costs. The specific nature of the production cost variables provided will depend upon the input supply elasticities available.

The tables discussed so far provide for the basic operation of the model. Numerous other tables of information can be derived from them. One that has been derived and reported in rows 28-29 is a net physical trade balance table. The calculation of this table follows directly from the supply and demand tables with account being taken that the physical units at the demand level may be different than the physical units at the farm level. Given the net trade balance table and world price table, it is a simple step to calculate the country's monetary net trade balance for the commodities considered. Likewise, given the various tax rates and subsidy rates and volumes of production, consumption, and trade, the direct government costs and revenues of the various subsidies and taxes can be computed for a tabular summary. Total consumption can be converted to per capita consumption. Given per capita physical consumption and caloric data for the foods in question, per capita caloric consumption can be calculated. Thus, the flexibility of the spreadsheet system allows many additional output summary tables to be added with little or no additional input information being required.

In summary, the concepts presented here for the GESS Model are meant to be illustrative of the basic structure of the GESS Model and its implementation. From this base, many modifications can be developed. For example, additional commodities can be added. The supply response portion of the model can be broken into acreage and yield components. By providing two matrices of demand elasticities, rural and urban consumption subcomponents can be modeled. Rather than considering two scenarios one year into the future the model can be made to consider a sequence of policies for several years. This can be done by changing the scenario columns to yearly columns and then making the base year for each column the column immediately to its left. The model's basic structure can even be modified to endogenize the price determination process. This could be done with moderate ease for a recursive model structure and is feasible for a simultaneous model structure.

Modeling of Stochastic Conditions

A powerful capability of a simulation model is the ability to consider the effect of random events upon a system. Typical random events of concern in agricultural models include weather, pests, exogenous price conditions, and even producer and consumer reactions to price. With the use of random number generators that are available with most microcomputer languages, the measurement of the effects of randomness upon a system

can be undertaken with a simulation model. In the following sections several key procedures for doing this will be developed. In the concluding section randomness will be injected into the exogenous world prices specified in the three-commodity GESS model. The resulting impact upon the net monetary balance of trade will be analyzed.

1. Generating Random Variables. Modeling of random variables begins with the need to generate random values with the desired distribution. This presumes that the modeler has been able to observe actual events and obtain a description of the distributions involved. The computer is capable of generating only one random distribution. That distribution is a zero to one uniform distribution. The task of the modeler is to convert that distribution to the desired distribution. Fortunately, numerous methods are available for generating desired distributions. Three of the most common methods used will be presented here. The interested reader is referred to Naylor et al. ("Computer Simulation Techniques," Chapter 4), for discussion of additional techniques.

Annex 7.2 provides a hands-on exercise in applying each of the methods to be discussed. An experienced LOTUS programmer can likely read the following section and learn to program the stochastic methods discussed. Annex 7.2 provides the less experienced programmer with additional guidance. The exercise provides step-by-step instruction to generate various random distributions and then uses the random variables generated in small sample models to demonstrate their application. The final application demonstrated in the exercise is to add stochastic world prices to the GESS model presented in the previous section.

•Direct Transformation Techniques. The simplest method of generating a desired random number distribution is to use a formula to directly transform a series of uniform 0-1 random variants into another distribution. A simple example is the case where a uniform distribution between one and ten is desired instead of a 0-1 uniform distribution. The basic formula for making this transformation, given R represents a 0-1 uniform random number series, is shown in equation 7.23:

$$X = 1 + (10 - 1) * R. \tag{7.23}$$

Figure 7.10 presents a generalized spreadsheet design for producing random numbers of any desired uniform distribution. Cells D3 and D4 are to be specified by the user. Cell D5 is defined by the Lotus equivalent of equation 7.23 and makes use of the Lotus @RAND function. The @RAND function will generate a 0-1 uniform random variable every time the spreadsheet is recalculated. Equation 7.24 below gives the formula for cell D5:

$$D3 + (D4 - D3) * @RAND. \tag{7.24}$$

Thus, each time the spreadsheet is recalculated (by pressing the F9 key) a new 1-10 uniformly distributed random value will be generated in cell D5. For modeling purposes, this value could be transferred to any location in the model desired, i.e. it could be used as

	A	B	C	D	E
1		UNIFORM DISTRIBUTION GENERATION			
2					
3			MINIMUM	1	
4			MAXIMUM	10	
5			RANDOM VALUE	3.00	
6					

Figure 7.10. Spreadsheet Structure for Generating a Uniform Distribution

a variable in an equation. By repeatedly solving the model, each time with a new random variable, and keeping a table of the resulting solutions, the mean and variance can be calculated of key system output variables in response to the random variable. For example, a simple model into which to inject random variation might be a crop model where yield is specified to be random, acreage is constant, and the price received for the crop is a linearly declining function of total production. The variable of interest is the stochastic total revenue generated. Calculating the mean and variance of the total revenue given the mean and variance of the yield would not be a simple analytical task but it is a rather simple simulation model task.

The exponential distribution can also be generated with a formula transformation. Given the expected value of the distribution as EX and R is a 0-1 uniform random variable, an exponential distribution series X can be generated by the following formula:

$$X = -EX * LOG(R). \tag{7.25}$$

A normal (0,1) distribution can also be generated by the formula method. Given R1 and R2 are two independent 0-1 uniformally distributed variables, two normal (0,1) variables can be generated by the following formulas:

$$X1 = (-2 * @\ LN(R1))\text{^}0.5 * (@\ COS(2 * @\ PI * R2)) \tag{7.26}$$

$$X2 = (-2 * @\ LN(R1))\text{^}0.5 * (@\ SIN(2 * @\ PI * R2)) \tag{7.27}$$

where @ LN is defined as the log base e, and @PI is the Lotus function for generating the value π. Given a normal (0,1) variable it can be further transformed into any other desired normal distribution, XD, by the following formula where EX is the expected mean, STDX is the expected standard deviation, and X is a normal (0,1) random variable:

$$XD = STDX * X + EX. \tag{7.28}$$

Figure 7.11 illustrates the use of equations 7.26 and 7.27 to generate two normal (0,1) random variables. Cells E4 and E5 contain the command @RAND and generate 0-1 uniform random variables while cells E6 and E7 contain equations 7.26 and 7.27 respectively and generate standard normal (0,1) random variants. The stochastic crop revenue model previously discussed has also been added to this spreadsheet example to show the ease of using random numbers in a simple but useful model. Cells D11, D12, and D13 are user-supplied parameters. Cell D14 is the key cell of the model and follows the form of equation 7.28. Specifically, the Lotus formula for cell D14 is as follows:

$$D11 * (D13\text{^}0.5 * E6 + D12) \tag{7.29}$$

where D11 is acres of land, D13 is the yield variance, E6 is a normal (0,1) random variant, and D12 is the expected yield. Price is calculated as a function of the random total

	A	B	C	D	E	F	G
1		GENERATION OF A STANDARD NORMAL DISTRIBUTION					
2							
3							
4		0-1 UNIFORM VARIABLE #1			0.384465		
5		0-1 UNIFORM VARIABLE #2			0.416864		
6		NORMAL (0.1) VARIABLE #1			-1.19829		
7		NORMAL (0,1) VARIABLE #2			0.689848		
8							
9		STOCHASTIC CROP REVENUE MODEL					
10							
11			ACRES	50.00			
12			MEAN YIELD	40.00			
13			YIELD VARIANCE	256.00			
14			TOTAL PRODUCTION	833.09			
15			CALCULATED PRICE	4.75			
16			TOTAL REVENUE	3957.49			

Figure 7.11. Spreadsheet Structure for Generating a Uniform (0,1) Random Distribution and Applying the Results to a Crop Revenue Model

production by a simple price dependent demand function specified in cell C15. Specifically for this example, that formula is:

$$6-(0.0015 * D14) \tag{7.30}$$

Total revenue is calculated in cell D16 as the product of cells D15 and D14. This revenue value will also be a random variant. Its mean and variance can be calculated by iteratively recalculating the entire spreadsheet (by pressing the F9 key) and keeping a table of the resulting revenue values.

•***Sampling Method.*** Some distributions cannot be generated by direct formula transformation of a 0-1 uniform distribution. The binomial, a rather commonly used distribution, cannot be generated this way. A binomial distribution essentially gives the probability that a given event occurs X times out of N trials, where the probability of the event occurring is P. A relevant situation for which the binomial distribution is useful in agriculture is to simulate the random death of animals over a given period. For example, suppose one has 50 head of cattle being fed and transported, and the expected death rate is 2 percent. A binomial distribution could be used to simulate a random distribution of deaths.

A simple method for generating a binomial variant if N is of moderate size is illustrated for the above livestock death loss case. To randomly determine how many animals out of fifty would die, one would generate fifty 0-1 uniform random variates and then count how many of these variants were equal to or less than .02, the expected death rate. A LOTUS 1-2-3 Spreadsheet designed to generate a binomial distribution for up to ten events with any probability of success is given in Figure 7.12. This spreadsheet design could be expanded to consider any number of events.

The two key parameters of the binomial distribution are supplied by the user in cells D6 and D7. Respectively, they are the number of events and the probability of the specified event occurring. These two variables are in turn used to generate the desired random variable which is reported in cell D9. Cells F5 ... F14 contain the function @RAND and generate 0-1 uniform variants. Cell G5 contains the formula @IF(F5<=$D7,1,0). This formula generates a value of 1 if the random value in F5 is deemed an "event" of the nature in question, i.e. F5 has a random number equal to or less than the probability specified in cell D7. Otherwise a value of zero is returned in cell G5. Cells G6 ... G14 contain replications of G5. Column H contains a series of @SUM functions that sum all the values in column G to the row in which the @SUM appears, i.e., the command in cell H9 is @SUM($G$5 ... G9). The "Number of Events" variable supplied by the user is then used in a Lotus @VLOOKUP function to pick out the appropriate sum in column H to enter in cell D9 as the binomial random variant. Specifically, the formula in cell D9 is @VLOOKUP(D6, E5 ... H14,3). Briefly, this command says consider the table of values whose upper left hand corner is cell E5 and whose lower right hand corner is cell H14, pick the row of the table equal to the value in

	A	B	C	D	E	F	G	H
1		BINOMIAL DISTRIBUTION GENERATOR FOR UP TO 10 EVENTS						
2								
3								
4						RANDOM	EVENT	
5						VALUE	INDICATOR	SUM
6		NUMBER OF EVENTS		5	1	0.885	0	0
7		PROBABILITY OF SUCCESS		0.6	2	0.912	0	0
8					3	0.145	1	1
9		NUMBER OF SUCCESSES		3	4	0.202	1	2
10					5	0.545	1	3
11					6	0.906	0	3
12					7	0.484	1	4
13					8	0.128	1	5
14					9	0.707	0	5
15					10	0.338	1	6

Figure 7.12. Spreadsheet Structure for Generating a Binomial Random Distribution

cell D6, pick the column in the table that is as may columns to the right of the first column as the last number in the command (3 in this case), and return the table value at the intersection of this row and column.

A normal (0,1) distribution can also be generated by sampling. In essence, the process involves generating twelve 0-1 uniform random variates and subtracting the expected value of their sum, i.e., six, from this randomly generated sum. The result is a normal (0,1) random variant. This approach is based on the central limit theorem.

•***Inverse Transformation Method.*** An approximate set of random variants can be generated from a distribution's cumulative distribution function. The computerization of this method of generating random variables is somewhat more difficult than the Direct Transformation or Sampling methods. Thus, the Sampling or Direct Transformation methods are preferable if adequate data are available to undertake them and the distribution can be generated by one of the two methods. The advantage of the Inverse Transformation method is that it allows users to generate random numbers from subjectively developed distributions. In many cases data are not available to estimate the exact properties of a distribution, but a good subjective knowledge of the distribution is available. For example, farmers in an area can report the average yield, and the best and worst yield they would expect over a 10 year period.

This kind of information can readily be sketched into a cumulative distribution function such as appears in Figure 7.13. In viewing Figure 7.13 it is evident that every yield is uniquely associated with a probability value between zero and one. Thus, random yield values reflecting the distribution represented by the cumulative distribution function in the graph can be generated by reading the graph "inversely". The procedure would be to first generate a 0-1 uniform random number and then inversely read the graph, or cumulative distribution function if it is available, for the associated yield value.

Computer implementation of this process could proceed in two ways. One way would be to use regression analysis to estimate an Inverse Cumulative Distribution function for the subjectively graphed function. However, because the function is subjectively specified in the first place, a numerical approximation would seem adequate and is much more readily implemented. This method involves constructing a table of the subjectively graphed function and then reading from the table the yields associated with any 0-1 uniform random variant generated. Such a table must, of course, have a limited number of values. The 0-1 uniform random distribution generates a continuous array of numbers which rarely will exactly match any one number in the table. The solution to this problem is to interpolate between values in the table or between points in the graph. For simplicity, interpolation is linear. Given the subjectiveness of the general procedure, this is probably as much accuracy as is warranted if a reasonable number of points is used. Figure 7.14 displays a Lotus spreadsheet program to generate random yields from the cumulative distribution function graphed in Figure 7.13. Values in Columns F and G are provided by the user and represent a table of 10 evenly spaced points of the cumulative distribution graphed in Figure 7.13. Cell D5 is generated by the formula @RAND*10. Multiplication

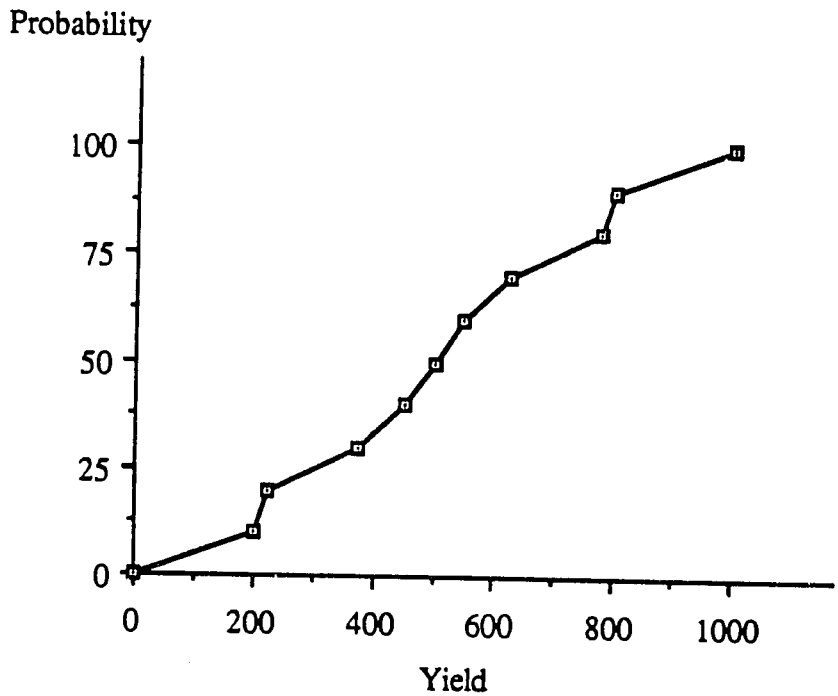


Figure 7.13. Subjective Cumulative Yield Distribution Function

	A	B	C	D	E	F	G
1	INVERSE CUMULATIVE DISTRIBUTION METHOD OF RANDOM NUMBER GENERATION						
2							
3					CUMULATIVE DISTRIBUTION		
4						PROB.	VALUE
5		CUMULATIVE PROBABILITY		26.83		0	0
6						10	200
7						20	225
8			RANDOM VALUE	327.43		30	375
9						40	450
10						50	500
11						60	550
12						70	625
13						80	775
14						90	800
15						100	1000

Figure 7.14. Spreadsheet Structure to Generate Random Variants From an Inverse Cumulative Distribution Function

by 10 is for the convenience of working in percentage terms. The key to the spreadsheet program is the formula in cell D8, given in the equation below.

@VLOOKUP(D5, F5 ... G15, 1) + (@VLOOKUP(D5 + 10, F5 ... G15, 1) - @VLOOKUP(D5, F5 ... G15, 1)) * (D5/10 - @INT(D5/10)). (7.31)

This seemingly complex function is not difficult given an analyst's working knowledge of the @VLOOKUP function. The interested reader with knowledge of Lotus is encouraged to analyze equation 7.31. In essence, the function provides a linearly interpolated inverse cumulative distribution function.

2. ***Modeling Correlated Random Events.*** Thus far the presentation of modeling of random events has focused on generating random values for a single variable. Oftentimes, random events in a system are correlated. For example, when the yield in one field is above average it is likely that yields in nearby fields will also be high. Because of the strong substitutability of many grains traded on the world market, when one grain price is high in the world market, all other grain prices will be above the average of previous years.

Correlation among random variables, be it negative or positive, can be modeled if the variance-covariance matrix for the variables in question is known. Naylor et al. in *Computer Simulation Techniques*, Chapter 4 and Clements et al. in "A Procedure for Correlating Events in a Farm Firm Simulation Model" both describe the methodology for modeling correlated random events. That procedure will be followed here and adapted to work with a LOTUS 1-2-3 spreadsheet program.

As an example a three-correlated-random-variable case will be considered. For reference and illustration these three variables will be identified as the three World Prices specified for Q#1, Q#2, and Q#3 in the GESS Model. Using matrix notation, three random correlated prices can be generated as follows:

$$P = \bar{P} + A^*R \tag{7.32}$$

where P is a 3 x 1 vector of random price, $\bar{P}$ is a 3 x 1 vector of expected prices, A* is a 3 x 3 matrix of coefficients, and R is a 3 x 1 vector of independent random normal (0,1) deviates. Given the discussion in the previous section on generating independent random normal distributions, implementation of equation 7.32 is straightforward except for the A* matrix. A* is an upper triangular matrix of coefficients that is derived from the variance-covariance matrix of the variables (World Prices) in question.

•***Development of the A* Matrix.*** Given the variance-covariance matrix of P, which will be referred to as V, two 3x3 triangular matrices A and A', which respectively are upper and lower triangular matrices, can be defined such that their product equals V.

$$V = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}. \tag{7.33}$$

Because the covariances are the same irrespective of order, the variance-covariance matrix is symmetric, i.e. $\sigma_{12} = \sigma_{21}$, $\sigma_{13} = \sigma_{31}$, and $\sigma_{23} = \sigma_{32}$. Likewise the upper and lower triangular matrices will be symmetric such that $A_{12} = A_{21}$, or $A_{ij} = A_{ji}$. Hence, the product of the two triangular matrices can be expressed as follows:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ & a_{22} & a_{23} \\ & & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & & \\ a_{12} & a_{22} & \\ a_{13} & a_{23} & a_{33} \end{bmatrix} = \begin{bmatrix} a_{11}^2+a_{12}^2+a_{13}^2 & a_{12}a_{22}+a_{13}a_{23} & a_{13}a_{33} \\ & a_{22}^2+a_{23}^2 & a_{23}a_{33} \\ & & a_{33}^2 \end{bmatrix}. \tag{7.34}$$

Let this new product matrix, which is also an upper triangular matrix, be called A*. Assume that the V matrix has the following numerical values and is equal, as previously stated, to the A* matrix.

$$V = \begin{bmatrix} 40 & -10 & -10 \\ -10 & 30 & 10 \\ -10 & 10 & 20 \end{bmatrix} \begin{bmatrix} a_{11}^2+a_{12}^2+a_{13}^2 & a_{12}a_{22}+a_{13}a_{23} & a_{13}a_{33} \\ & a_{22}^2+a_{23}^2 & a_{23}a_{33} \\ & & a_{33}^2 \end{bmatrix} \tag{7.35}$$

The values for the upper triangular matrix A* can be solved from the above relationship in a sequential process. Starting at the lower right hand corner of the A* matrix, it follows that:

$$\begin{aligned} a_{33}^2 &= 20 \\ a_{33} &= 4.472 \end{aligned} \tag{7.36}$$

given $a_{33} = 4.472$ it follows that:

$$
\begin{aligned}
a_{23}\, a_{33} &= 10 \\
a_{23} &= 10/a_{33} \\
a_{23} &= 10/4.472 \\
a_{23} &= 2.236
\end{aligned}
\tag{7.37}
$$

likewise, given $a_{33} = 4.472$ it follows that:

$$
\begin{aligned}
a_{13}\, a_{33} &= -10 \\
a_{13} &= -10/a_{33} \\
a_{13} &= -10/4.472 \\
a_{13} &= -2.236.
\end{aligned}
\tag{7.38}
$$

Thus, the third column of the upper triangular matrix A* is defined. In a similar fashion, the second column can be defined:

$$
\begin{aligned}
a_{22}^2 + a_{23}^2 &= 30 \\
a_{22}^2 &= 30 - a_{23}^2 \\
a_{22}^2 &= 30 - (-2.236)\text{^}2 \\
a_{22}^2 &= 25 \\
a_{22} &= 5.0
\end{aligned}
\tag{7.39}
$$

$$
\begin{aligned}
a_{12}\, a_{22} + a_{13}\, a_{23} &= -10 \\
a_{12}\, a_{22} &= -10 - a_{13}\, a_{23} \\
a_{22} &= (-10 - a_{13}\, a_{23})/a_{12} \\
a_{22} &= (-10 + 4.999)/5 \\
a_{22} &= -1.0.
\end{aligned}
\tag{7.40}
$$

And finally a_{11} equals:

$$
\begin{aligned}
a_{11}^2 + a_{12}^2 + a_{13}^2 &= 40 \\
a_{11}^2 &= 40 - a_{12}^2 - a_{13}^2 \\
a_{11}^2 &= 40 - (-1)\text{^}2 - (-2.236)\text{^}2 \\
a_{11}^2 &= 34 \\
a_{11} &= 5.831.
\end{aligned}
\tag{7.41}
$$

The upper triangular A* matrix for the variance-covariance given in equation 7.35 is:

$$
A^* = \begin{bmatrix} 5.831 & -1.000 & -2.236 \\ & 5.000 & 2.236 \\ & & 4.472 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ & a_{22} & a_{23} \\ & & a_{33} \end{bmatrix}. \tag{7.42}
$$

The same general procedure as described above can be used to develop an A* matrix of any dimension desired. When coupled with an equation of the form specified by equation 7.32, any desired number of random correlated events can be simulated.

•***Application of Random Correlated Variables.*** Figure 7.15 displays the Lotus spreadsheet structure for generating random World Prices for the GESS Model using the above described approach and the values of the illustrated variance-covariance matrix. According to the Screen Map given for the GESS Model, the entries displayed in Figure 7.15 would be displayed on Screen 2B.

The key formulas in the spreadsheet depicted in Figure 7.15 are contained in cells K32, K33, and K34. They execute equation 7.32 as given above. In Lotus format they are as follows:

L32 + 5.83 * M37 - 1.0 * N37 - 2.24 * 037 (7.40)

L33 + 5.0 * N37 + 2.24 * 037 (7.41)

L34 + 4.47 * 037 (7.42)

The first variable, L32, L33, and L34 in each equation, is the expected mean World Price of Q#1, Q#2, and Q#3 respectively. In each equation, this variable is followed by multiplication of the appropriate values in the A* matrix times three random independent normal (0,1) variants. These three variants are calculated off the screen in cells M37, N37, and 037. After being calculated the randomly generated prices reported in cells K32 ... K34 are transferred to the World Price Table in the GESS Model under the Scenario #2 column. Every recalculation of the spreadsheet results in a new set of random World Prices and a new model solution.

To analyze the impact of these random prices upon the net monetary trade balance, the mean, standard deviation, minimum and maximum of the simulated stochastic trade balance series are calculated. This is done in cells K23 ... K26 respectively. The actual trade balance on each individual iteration, or random draw, is calculated in cell K30. The trade balance is calculated by multiplying the net trade balances, as calculated in the GESS Model, by the random World Prices generated. Cells K36 and K37 keep count of the iterations run. Cell 36 is user entered. Entering a zero will start a new series of iterations and zero out all of the statistics being kept in cells K23 ... K26. The formula in cell K37 keeps the iteration count and is defined as @IF(K36>0,K37 + 1,0).

Calculating the statistics reported in K23 ... K26 requires keeping a table of the net monetary trade balances generated in cell K30. The spreadsheet structure for doing this is shown in Figure 7.16. In looking at Figure 7.16, it should be noted that columns N, O, and P are not part of the stochastic net trade balance table; rather they are used in generating the three normal (0,1) random variants. The sampling method of generating random variables is used here, i.e. twelve 0-1 uniform variants are generated in cells N24 ... N35. Cell C37 then generates a normal (0,1) variant by the formula 6-@SUM(N24 ... N35).

	G	H	I	J	K	L
21			STOCHASTIC TRADE BALANCE ANALYSIS			
22						
23				ESTIMATED MEAN	491529	
24				ESTIMATED STD.	93279	
25				MINIMUM BALANCE	224365	
26				MAXIMUM BALANCE	623324	
27						
28						
29					ACTUAL	EXPECTED
30		NET MONETARY TRADE BALANCE			470014	470270
31						
32				WORLD PRICE FOR Q#1	90.93862	100.00
33				WORLD PRICE FOR Q#1	81.50355	80.00
34				WORLD PRICE FOR Q#1	54.33669	50.00
35						
36				RESTART=0 RUN>=1	1	
37				ITERATION NUMBER	24	
38						
39						

Figure 7.15. Spreadsheet Structure For Modeling Correlated Random World Price Variation

	M	N	O	P	Q	R	S
21			NORMAL (0,1)		0	RANDOM VALUE	VARIANCE
22		RANDOM NUMBER GENERATION			1	458199	1624855840
23		#1	#2	#3	2	416461	6731746534
24		0.5830	0.4710	0.4574	3	465956	1059675960
25		0.3897	0.2176	0.4612	4	504486	35727442
26		0.1389	0.6317	0.8346	5	538156	1571933905
27		0.2179	0.7457	0.7157	6	504623	37388970
28		0.0790	0.7471	0.7872	7	560201	3805946744
29		0.9133	0.7282	0.1083	8	485433	170970042
30		0.8061	0.8738	0.6954	9	402198	9275728981
31		0.8137	0.8101	0.7428	10	480175	336095350
32		0.7771	0.4258	0.9559	11	587969	8003174941
33		0.1531	0.3526	0.7918	12	504980	41878268
34		0.2264	0.7624	0.5193	13	615295	13639205666
35		0.6458	0.5684	0.2389	14	552922	2960862859
36					15	424032	5546727595
37		0.2558	-1.3344	-1.1084	16	475593	525135626
38					17	512453	194466303
39					18	457212	705422481

Figure 7.16. Spreadsheet Structure For Constructing a Table for The Stochastic Trade Balance Series

Columns O and P repeat this process to generate two additional random normal (0,1) variants.

Column R contains the random trade balances generated and column S contains the variance of each trade balances, i.e., the squared value of the mean trade balance less the value of the specific balance in question. Getting each iteratively generated trade balance value to appear at the proper place in the table requires innovative Lotus programming. The iteration count value calculated in cell K24 must be used to direct the value reported for the trade balance each iteration, as it appears in cell K30, to the proper table location. The formula for doing that is listed in equation 7.43. The formula for calculating the variance of each individual trade balance is reported in equation 7.44.

R22 = @IF($K$37 = Q22 - 1, $K$30, @IF($K$37<Q22,0,R22)) (7.43)

S22 = @IF($K$37 = Q22 - 1, $K$30, @IF($K$73 - R22)^2,0) (7.44)

The two equations written in 7.43 and 7.44 are for the top cells of columns R and S, but replication of them down their respective columns with the COPY command will give the rest of the formulas for completing the table. Once the two table columns are completed, i.e., the random value column and variance column, they can be summed and divided by the number of iterations and the resulting value reported in cells K23 and K24 as the estimated mean and standard deviation. Reporting the standard deviation would, of course, necessitate taking the square root of the calculated variance.

The minimum and maximum trade balances reported in K25 and K26 are found more easily than the mean and standard deviation. Formulas for their determination are given in equations 7.45 and 7.46.

@IF(K36 = 0, K30, @IF(K25 < K30, K25, K30)) (7.45)

@IF(K36 = 0, K30, @IF(K26 > K30, K26, K30)) (7.46)

3. Summary. Use of random variables within the GESS Model, or any simulation model can be a powerful tool for analyzing a large number of critical questions involving risk. For example, with some further extension of the above GESS modeling process the question of food security for countries relying upon the world market for part of their food and having a tight trade balance could readily be examined. Estimates of the buffer stocks or foreign currency reserves necessary to assure desired food supplies in the face of volatile world prices could be made. Indeed, this was the topic of a study by Trapp, Rogers, and Wilkens for Liberia.

The examples provided here do not begin to exhaust the possible uses of simulation modeling and Monte Carlo Technique. However, they give the reader a chance to develop fundamental skills that can serve them well in future creative endeavors to describe and model real world systems and problems.

ACTIVITIES

1. Define in your own words the meaning of the terms "system," "simulation," and "model."

2. Calculate the parameters of the equation below given the following information:

   a. a 10 percent increase in the price of A will cause a 5 percent decrease in the quantity of A consumed.
   b. a 10 percent increase in the price of B will cause a 2.5 percent increase in the quantity of A consumed.
   c. base values for P_a, p_b, and Q_a are 60, 30, and 80 respectively.

   The equation is $Q_a = a\, P_a^{b_1}\, P_b^{b_2}$

3. Write a Lotus 1-2-3 program to generate normally distributed random variables with a mean of 25 and variance of 16.

4. Determine the transformed A* matrix for the following 2 x 2 Variance/Covariance matrix:

$$\begin{bmatrix} 16 & 4 \\ 4 & 25 \end{bmatrix}$$

5. See Annex 7.1 for other activities.

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ANNEX 7.1

This tutorial is designed to guide the reader through the process of developing a Generalized Econometric Spreadsheet Simulation (GESS) Model. To use it, you will need a Lotus template named GESSA, and, of course, a Lotus Program. The GESSA file contains the table headings and initialization data for the 3 x 3 example GESS model discussed in this chapter.

This tutorial is not designed to provide a key stroke by key stroke guide to completing the sample GESS model contained in the Lotus template referred to above. Rather, it discusses the general nature of the formulas for various tables of the Model and indicates the Lotus commands to use in completing the model table by table. Therefore, a basic knowledge of Lotus 1-2-3 is assumed. It is also assumed that the tutorial user has read and has access to the sections of this chapter which discuss the GESS Model and its theoretical underpinnings.

The GESSA File

The GESSA template file contains all the table headings and initial data the user must supply to build a 3 x 3 GESS Model. Load the file and use the "PgUp," "PgDn," and horizontal page key (just above the Ctrl key) to look through the spreadsheet and see what data are supplied. The overall spreadsheet design and tables within it are described in Figure 7.6 of this chapter. A mental note should be made because this is the data the modeler would have to provide if he or she were starting a new model for a different country or set of commodities. Briefly, requirements are the base year data for retail demand, farm supply, price margins, world price, population, income, production cost, conversion factors, and demand and supply elasticities. The demand and supply elasticities are probably the most difficult of the required initial data to provide. They must be determined either through literature review, estimation by regression analysis, or as a last resort, by subjective judgment. Exogenous data and policy control variable values are also provided for Scenarios #1 and #2 so that all equations developed will have test data available. However, all policy variables dealing with taxes and subsidies have arbitrarily been set to zero.

Step #1

First , a formula is provided for the trade balance cell located at B28. This cell basically is determined by the difference between the supply and demand balance. However, since demand is expressed in the model at the retail level and supply at the farm level, they must be transformed to wholesale level physical units before they are added together. The conversion factors for doing this are located in cells B84 ... B86. The equation for B28 is as follows:

(B8*($B84/100)) - (B3*(100/$C84)) (1)

Equations for B29 and B30 can be generated by the COPY command. Answers for these three cells should be -433, 1955, and 7144. Cells C28 ... D30 can also be completed with the COPY command. They will generate zero values until more of the model is completed.

Cells E28 ... E30 are simple percent change formulas. The formulas for E28 is given below:

((D28 - C28) / C28)*100 (2)

Cells E29 and E30 can be generated by copying cell E28 to them. In fact, the whole E column can be filled by copying cell E28 to the appropriate cells in the column. Do this now. In many cases, you will get a response of ERR when you copy the formula. This error is due to division by zero. This error will disappear when the spreadsheet is completed and the zeros in columns C and D are replaced.

Step #2

Enter the Wholesale Price formula in cell B23 next. Wholesale prices are calculated from World prices according to the provided Wholesale Market Spreads and any taxes or subsidies in existence. The Wholesale Price formula is specified as follows:

(B48*(B33/100))*((100 + (B53*($B28/@ABS($B28))))/100) (3)

This formula is explained in the chapter. Cells B24 and B25 can be filled with the COPY command. The resulting values should be 110, 88, and 55. It should be noted that all cells that contain prices have been formatted for two decimal points using the RANGE FORMAT command. This is indicated by the fact that an (F2) appears in front of the formula for any cell that has been formatted for two decimal places. Cells C23 ... D25 can also be filled by the COPY command. Answers for them should be identical to those in Column B.

Step #3

Enter the formula for Retail Price in cell B13. It is calculated from the Wholesale Price by considering the margin and any taxes or subsidies charged at the retail level. The Retail Price formula is explained in the text regarding equation 7.22. It appears as follows:

(B23*(B43/100))*((100 - B63)/100) (4)

Cells B14 and B15 can be generated by copying cell B13. The prices generated should be 231.00, 158.40, and 88.00 respectively. Cells C13 ... D15 can be filled by copying cell B13 also. They should have identical values to column B.

Step #4

Enter the formula for Farm Price in cell B18. The Farm Price formula is similar to the Retail Price formula and is explained in the text surrounding equation 7.22. It appears as follows:

(B23*(B38/100))*((100 + B58)/100) (5)

Copy the formula to fill the rest of the table. The prices generated should be 66.00, 52.80, and 33.00.

Step #5

Fill in the formula for Retail Demand in cell C3. Cells C3 ... C5 and D3 ... D5 contain demand functions that use the elasticities provided on Screen 1B, i.e., cells F1 ... K20. Cell C3 is described by the following equations. The equation is explained in the text surrounding equation 7.17.

($K4*(C$13^$G4)*(C$14^$H4)*(C$15^$I4)*(C$68^$J4))*C$71 (6)

After you have typed in the formula for C3 the cell should contain the value zero. This is because the variable K4 in the equation is not defined yet. K4 is the equation constant and can be defined from the base period data and elasticities provided. The equation for K4 is explained with equation 7.19 of this chapter. It appears as follows:

(B3/$B$71)/(($B$13^G4)*($B$14^H4)*($B$15^I4)*($B$68^J4)) (7)

Cells K5 and K6 can be generated by copying K4 into them. Finally the rest or the Retail Demand Table can be completed by copying cell C3 into the rest of the table.

Step #6

Completion of the Farm Supply Table is done in the same manner as the Retail Demand Table. The formula for cell C8 is as follows:

+$K11*(C$18^$G11)*(C$19^$H11)*(C$20^$I11)*(C74^$J11) (8)

The above equation is essentially a supply equation. Its derivation and logic are explained in the text surrounding equation 7.18 of this chapter. When this equation is entered it will generate a zero because the constant value in cell K11 is undefined. Proceed to cell K11 and enter the formula given below:

+B8/(($B$18^G11)*($B$19^H11)*($B$20^I11)*($B74^J11)) (9)

Copy the above formula into cells K12 and K13. The calculated intercepts should be 36946, 44938, and 26885. The value in cell C8 should now be 34000. Copy cell C8 into

the appropriate cells in the rest of the Farm Supply Table. The values which are generated in Columns C and D should match those in Column B.

Applications

The model is now complete. You are ready to run policy scenarios with the model and/or conduct sensitivity tests of the elasticity values. The suggested exercises that follow are provided to start thinking in terms of how the model can be used. You are invited to do further experimentation with the use and expansion of the model on your own.

World Price Response

Change the price of Q#1 in Scenario #1 to 110. As a result you should see production of Q#1 rise, consumption fall, and the trade balance switch to that of a net exporter. Note also that the consumption levels and trade levels of Q#2 and Q#3 are changed. It is this type of interaction to which the GESS Model is good at drawing attention to a providing estimates of. However, supply levels of Q#2 and Q#3 are not changed in this case because all supply cross elasticities are assumed to be zero. This case example has drawn approximate parameters and base period data from the American commodity market for beef, pork, and poultry. Very little supply interaction is deemed to exist between these commodities but demand interaction is strong. In most crop commodity cases supply interaction would be expected because crops often compete for the same land.

Elasticity Sensitivity

Leave the Price of Q#1 in Scenario #1 at 110 and note the magnitude of several key values such as net trade of all three commodities and the demand for Q#1. Having done this proceed to change the own-price elasticity for Q#1 in G4 to -.8. Observe the difference this change makes in the responses of the variables observed before.

Taxes and Subsidies

On your own try a number of taxes and subsidies at different market levels. You may want to tax under one Scenario and subsidize under another. Question the direction of changes with regard to whether they are the expected responses to a tax and/or subsidy. One exercise to try is to calculate a tax/subsidy scheme that would make the country "self sufficient" in commodity Q#1, i.e., cause it to have at least a zero balance of trade for Q#1.

Calculating a Monetary Trade Balance

Go to rows 93 to 100 and construct a Monetary Trade Balance Table for Scenario #1 and #2 and a percentage change column for the Monetary Trade Balances. The basic process for doing this is to multiply the Net Trade Balance figures in rows 28 to 30 together with the World Prices.

Another table you may want to construct is the "Net Direct Government Cost" table for all taxes and subsidies considered. This would involve multiplying the appropriate tax and subsidy rates times the appropriate quantities of retail demand, farm supply, and international trade, and then adding the results together to obtain a net government cost.

Other Applications

If you have further interest in the GESS Modeling approach you may wish to obtain and review a copy of the Liberian GESS Model and Dominican Republic Livestock Sector GESS Model. Both of these models contain several structural features and table derivations not shown here. A review of them will provide further illustration of the potential uses of the GESS Modeling approach. The Liberian GESS Model focuses upon rice policy. It describes supply in terms of both an acreage component and yield component. Additionally the Liberian model has both an urban and rural demand model. The Dominican Republic Model is structured as a recursive model and uses lagged price responses in the supply model. Templates of both models are available from the author.

Annex 7.2

An Exercise in Modeling Stochastic Values

This exercise is designed to give the student experience in developing Lotus Spreadsheet programs which generate stochastic variables that can be used to model stochastic systems. The exercise will closely follow the material presented in this chapter. Specifically, the material from equation 7.32 to the end of the chapter. The student is expected to have read and have access to this material.

The @RAND Function

The basis for generating stochastic variables is the @RAND Function. When typed as a Lotus command it generates a uniform random number between zero and one each time the spreadsheet is recalculated. Spreadsheet recalculation is done every time an entry is made and the enter key is depressed. Recalculation is also accomplished by pressing the F9 key. To familiarize yourself with the @RAND function enter it as a formula in cell A1 and then press the F9 key several times and watch the results. The random values generated can be used like any other spreadsheet value. For example, enter a 2 in cell B1 and multiply cell A1 and B1 together to form cell C1. Another experiment to try is to multiply two random numbers together to get a random product. What is the expected value and variance of the product of two 0-1 uniform random variants? Later is this exercise, we will learn how to simulate and answer this question. After familiarizing yourself with the @RAND command, clear your spreadsheet with the command/WEY and proceed with the next section of this exercise.

Uniform Distribution

Figure 1 below shows the spreadsheet design for generating a generalized uniform distribution. This program, when completed, will generate uniform random numbers between any Minimum and Maximum values specified. The procedure to create the program is given in the following steps:

	A	B	C	D	E
1		UNIFORM DISTRIBUTION GENERATION			
2					
3			MINIMUM	1	
4			MAXIMUM	10	
5			RANDOM VALUE	3.00	
6					

Figure 1. Spreadsheet Structure for Generating a Uniform Distribution.

Step #1

Enter the required titles in the cells indicated.

B1 Uniform Distribution Generation
B3 Minimum
B4 Maximum
B5 Random Value

After entering the titles you will have to experiment with the number of blank spaces to insert in front of the words Minimum, Maximum, etc., to get them to align properly. Use the F2 or edit key together with the arrow cursor keys on the numeric key pad and the DEL key to do this. If you are unfamiliar with these keys ask the lab instructor for help.

Step #2

Enter a 1 in cell D3 and a 10 in cell D4. These two values will serve as initial examples of a range over which to simulate uniformly distributed random numbers. Later they can be replaced by any two number desired.

Step #3

Enter the following formula in cell D5:

+D3 + (D4 - D3)*@RAND. (1)

The logic of this equation is explained on page 148 of this chapter.

Step #4

Press the F9 key several times to test the program. Observe the random values generated and see if they uniformly fall between 1 and 10. Change the Minimum and Maximum value several times and observe the change in the random numbers generated.

Step #5

Save the program created under the file name UNIFORM. To do this enter the command /FS and then press the ESC key to allow a new file name to be typed in. Type in the name UNIFORM. Clear the spreadsheet for the next exercise by entering /WEY.

Generating a Normal Distribution

Figure 2 displays the spreadsheet for generating a set of Normal (0,1) random numbers. Statements have also been added to the spreadsheet to transform the Normal (0,1) distribution into a second normal distribution having the mean and variance of the yield of a selected crop. This has been done to show how a standard Normal (0,1)

distribution can be changed to a normal distribution with any desired mean and variance. It also illustrates the use of random numbers in a simple model, i.e., a crop gross revenue model. Figure 2 and the steps for creating it are given below:

	A	B	C	D	E	F	G
1		GENERATION OF A STANDARD NORMAL DISTRIBUTION					
2							
3							
4		0-1 UNIFORM VARIABLE #1			0.384465		
5		0-1 UNIFORM VARIABLE #2			0.416864		
6		NORMAL (0.1) VARIABLE #1			-1.19829		
7		NORMAL (0,1) VARIABLE #2			0.689848		
8							
9		STOCHASTIC CROP REVENUE MODEL					
10							
11			ACRES	50.00			
12			MEAN YIELD	40.00			
13			YIELD VARIANCE	256.00			
14			TOTAL PRODUCTION	833.09			
15			CALCULATED PRICE	4.75			
16			TOTAL REVENUE	3957.49			

Figure 2. Spreadsheet Structure for Generating a Normal (0,1) Random Distribution and Stochastic Crop Revenue Model.

Step #1

Enter the following titles in the cells listed. Once again a number of the titles will have to be aligned using the F2 edit key.

B1 Generation of A Standard Normal Distribution
B4 '0-1 Uniform Variable #1
B5 '0-1 Uniform Variable #2
B6 Normal (0,1) Variable #1
B7 Normal (0,1) Variable #2
B9 Stochastic Crop Revenue Model
B11 Acres
B12 Mean Yield
B13 Yield Variance
B14 Total Production
B15 Calculated Price
B16 Total Revenue

Step #2

Enter the @RAND function in cells E4 and E5.

Step #3

Enter the following formulas in cell E6 and E7 to generate two Normal (0,1) random variants.

(-2*@LN(E4))^0.5*(@COS(2*@PI*E5)) (2)

(-2*@LN(E4))^0.5*(@SIN(2*@PI*E5)). (3)

Step #4

Enter the values 50, 40, and 256 in cells D11, D12, and D13, respectively.

Step #5

Enter the following formula in cell D14:

+D11*(D13^0.5*E6+D12). (4)

The rationale behind this equation is explained in the text explaining equation 7.29 of this chapter.

Step #6

Enter the following price dependent demand equation formula in cell D15:

6-(0.0015*D14). (5)

Step #7

Enter the formula for calculating total revenue as the product of production times price, i.e., +D14*D15. Press the F9 key several times and observe the performance of the program. You may want to convert cells D11 ... D16 to 2 decimal place values with the command /RFF2.

Step #8

Save the program under the name NORMAL.

Application Problem

As an added challenge attempt to expand the simple crop model program developed above by making price a random variable. Use the calculated price in D15 as the expected

price. Assume a variance of 0.8 and use the Normal (0,1) random variable in cell E7 to generate a random price. Use this random price in the calculation of total revenue.

Binomial Distribution

Figure 3 shows the spreadsheet structure for generating a Binomial Distribution for up to ten events. A Binomial Distribution random number generator randomly determines the number of occurrences of a specified type in N events given the expected probability of that event occurring. The steps for creating the Binomial Distribution program displayed in Figure 3 are given below:

	A	B	C	D	E	F	G	H
1		BINOMIAL DISTRIBUTION GENERATOR FOR UP TO 10 EVENTS						
2								
3						RANDOM	EVENT	
4						VALUE	INDICATOR	SUM
5					1	0.885	0	0
6		NUMBER OR EVENTS		5	2	0.912	0	0
7	PROBABILITY OF SUCCESS			0.6	3	0.145	1	1
8					4	0.202	1	2
9	NUMBER OF SUCCESSES			3	5	0.545	1	3
10					6	0.906	0	3
11					7	0.484	1	4
12					8	0.128	1	5
13					9	0.707	0	5
14					10	0.338	1	6
15								

Figure 3. Spreadsheet Structure for Generating a Binomial Random Distribution.

Step #1

Enter the following titles in the cells listed. Some editing of the titles will be necessary to achieve the desired alignment.

B1 Binomial Distribution Generator For Up to 10 Events
A6 Number of Events
A7 Probability of Success
A9 Number of Successes
F3 Random
G3 Event
F4 Value
G4 Indicator Sum

Step #2

Enter test values for "Number of Events" and "Probability of Success" in cells D6 and D7 -- for this example, enter 10 in cell D6 and 0.5 in cell D7.

Step #3

Enter the numbers 1 through 10 in cells E5 through E14. Then enter the @RAND command in cells F5 through F14. The copy command can be used to help in this process. You may want to format the random numbers in cells F5 ... F14 to 3 decimal places with the command /RFF3.

Step #4

Enter the Event Indicator Statement. This statement's purpose is to consider the random value in the Random Value Column and compare it to the probability value in cell D7. If the random value is less than or equal to the probability, a "1" should be generated to indicate the event in question has occurred. If the random value is greater than the probability specified, a zero should be generated. A Lotus @IF statement is used to accomplish this choice. The statement for cell G5, the first Event Indicator cell, is listed below.

@IF(F5<=$D$7,1,0). (6)

An @IF statement functions as follows: Within the parenthesis of the statement are three arguments/terms. The first argument is a comparison statement. If it is true the second argument/term is displayed for the cell. If the comparison is not true the third argument/term is displayed.

The formula for cell G5 can be copied to cells G6 to G14. Note this is possible because D7 is entered as a fixed location, while F5 is not. Hence, as the cell is copied F5 will change to F6, F7, and so on, so that the comparison being made is always between the random number to the left of the cell in question and the specified probability.

Step #5

The number of successes in N events is determined by summing the Event Indicator column over N values starting from the top of the column. The "Sum" column performs this function. The formula to do this for cell H5 is listed below:

@SUM($G$5 ... G5). (7)

This formula may seem redundant. It says sum from G5 to G5 which is just one number. For a one-event case no summing is needed to find the random number of events. But copying this command into cells G6 to G14 yields a series of summations where each summation sums one additional number, i.e., since the first term $G$5 is a fixed location

but the second term changes relatively as it is copied, the series of sums made will be $G$5 ...G6, $G$5 ... G7, and so on. Try it and see for yourself.

Step #6

Fill in the formula for cell D9. It is as follows:

@VLOOKUP(D6, E5 ... H14,3). (8)

The purpose of this command is to pick the appropriate summation value out of the "Sum" column. For example, if 5 events are specified in cell D6, then cell H9 should be read and entered in cell D9. The @VLOOKUP command is designed to perform such a task. The command has three arguments/values. The middle argument/value is the key argument. It defines a table range over which numbers can be retrieved. In this case, columns E and F and rows 5 through 14 are selected to be in the table. The first term of the command indicates what row to look at. The row selected is determined by matching the first argument of the command with a value in the left hand column of the designated table. In this case the value to be matched is given by D6, the number of events variable. The third argument/value of the command designates a column to look in. The number in the third argument tells how many columns to the right of the first column to look. In this case a 3 is designated. Thus, the resulting value is the value stored in the cell at the intersection of the row determined by the D6 value and the "Sum" column.

Step #7

Test the program by changing the number of events and probability. Then save the program under the file name BINOMIAL. Clear the spreadsheet with the command /WEY.

Inverse Cumulative Distribution Function

Randomly distributed numbers can be generated by reading a distribution's Cumulative Distribution function inversely (see text). Figure 4 below presents a program for reporting a cumulative distribution function in tabular form and then reading that table to generate random values distributed according to the distribution represented by the cumulative distribution function. Steps for programming the procedure are given below:

	A	B	C	D	E	F	G
1	INVERSE CUMMULATIVE DISTRIBUTION METHOD OF RANDOM NUMBER GENERATION						
2							
3						CUMMULATIVE DISTRIBUTION	
4						PROB.	VALUE
5	CUMMULATIVE PROBABILITY			26.83		0	0
6						10	200
7						20	225
8		RANDOM VALUE		327.43		30	375
9						40	450
10						50	500
11						60	550
12						70	625
13						80	775
14						90	800
15						100	1000

Figure 4. Spreadsheet Structure to Generate Random Variants From an Inverse Cumulative Distribution Function.

Step #1

Enter the listed titles in the designated cells.

A1 Inverse Cumulative Distribution Method of Random Number Generation
A5 Cumulative Probability
B8 Random Value
F3 Cumulative Distribution
F4 Probability Value

Step #2

Fill in the table portion of the spreadsheet. Cells F5 to F15 should be filled with the values 0, 10, 20, etc., up to 100 in cell F15. These values represent the cumulative probabilities of the values in the column to the right having occurred. Next fill in the values in column G. These values are the distribution function values, i.e., the two columns together form ten sets of points on a Cumulative Distribution function. As test values for the Cumulative Distribution function enter the values shown in Figure 4 in cells G5 to G15.

Step #3

Enter the formula @RAND*100 in cell D5. Then use the command /RFF2 to specify the number as having 2 decimals. This command will generate a probability value between 0 and 100.

Step #4

Convert cell D8 to a 2 decimal place format with the command /RFF2. Enter the following formula in cell D8.

```
@VLOOKUP(D5, F5 ... G15, 1) + (@VLOOKUP(D5 + 10, F5 ...
G15, 1) - @VLOOKUP(D5, F5 ... G15, 1))*(D5/10-@INT(D5/10))        (9)
```

This formula is the key to the entire program. It looks complex but it is not. The formula consist basically of three @VLOOKUP functions which you learned to use in the last exercise on Binomial Distributions. To understand the above formula one more aspect of the @VLOOKUP command must be noted. When the first argument of the @VLOOKUP is being matched with a value in the left column of the table to designate a row, if an exact match is not found the row with the largest value not exceeding the argument is selected, i.e., in this case argument values between 20 and 29.99 will result in the same row being selected. Row selection in this case is keyed by the Cumulative Distribution value drawn.

The basic purpose of the above formula is to linearly interpolate between values in the table. To see what happens in the formula consider the case where it is assumed the cumulative probability drawn is 45. The formula in cell D8 would be evaluated as follows:

```
450 + (500 - 450) * (45/10 - @INT(45/10))
450 + 50 * (4.5 - 4)
450 + (50 * 0.5)
450 + 25
475                                                              (10)
```

The three @VLOOKUP commands retrieve the values 450, 500, and 450. The last term determines the percent of the distance of 45 between 40 and 50. To do this use is made of the @INT command. This command truncates the value in parenthesis to form an integer value. Thus, the last term of the formula in this case ascertains that the value 45 lies 50 percent of the way from 40 to 50. Thus, 50 percent of the difference between the two distribution values, 500 and 450, should be added to the base value 450 to arrive at a linearly interpolated value for the probability 45.

Step #5

Test the program several times by pressing the F9 key. You may also wish to change the values in the Cumulative Distribution value column. The table need not be restricted to ten points. It could be expanded to hold 20 or even 100 points. The formula in cell D8 would have to be adjusted as the table size is adjusted.

Step #6

Save the program under the name TABLEDIS.

Evaluating a Subjective Cumulative Distribution Function

In most cases the table values for the Inverse Cumulative Distribution function generated will have been subjectively derived. Hence, the distribution's mean and variance are likely not known. The Inverse Cumulative Distribution program can be augmented to calculate the mean and variance of the values simulated. The steps for doing this are listed below.

Step #1

Load the TABLEDIS file into your Lotus program if it is not currently loaded. Add the following titles to the program in the indicated cell locations.

A3 Restart=0 Run=1
A4 Iteration Number
A10 Estimated Mean
A11 Estimated Variance
A12 Estimated Std. Dev.

Step #2

Enter a 0 in cell D3. Enter the following formula in cell D4 to keep an iteration count each time a random number is generated:

@IF(D3>0, D4 + 1,0). (11)

Step #3

Enter a 0 in cell I1. In cell I2 enter the formula +I1 + 1. Copy the formula down the column over the range I3 ... I101. This process should generate the numbers from 1 to 100.

Step #4

Enter the following formula in cell J2:

@IF($D$4 = I2, $D$8, @IF($D$4>I2, J2, 0)). (12)

The purpose of this formula is to enter each random value generated into the table as it is generated. From iteration to iteration some way must be found to enter the random value generated in the correct cell. If the cell is not being dealt with during the current iteration it should hold its old value of zero or a previously generated random number. In addition, from run to run all the random values previously stored must be zeroed out to start over. The above statement does all this. By copying it down over the range J3 ... J101 this function will be performed correctly for 100 random number generations. At least 50

random observations or more should be drawn in order to estimate the mean and variance of any distribution.

Briefly the logic of equation 12 is as follows. If the iteration number in D4 equals the observation number in column I, enter the random number generated during the iteration into the cell to the right of the matching number's cell. If the iteration number does not match, check the condition of a second @IF statement. This statement asks if the iteration count is greater than the observation number in the adjacent cell. If the count is greater than the observation number the cell has already been filled with a random number and should be refilled with the same number again, otherwise a 0 should be entered.

Enter the following formula in cell J103.

@SUM(J2 ... J101)/D4. (13)

This formula will keep an iteration by iteration running average of the mean of the random numbers generated thus far. To transfer this value back into the cell next to the title "Estimated Mean" enter a +J103 in cell D10.

Step #5

Enter the following formula in cell K2:

@IF($D$4>=I2, ($D$10 - J2)^2, 0). (14)

The logic of this formula is similar to that in column J, but its purpose is to calculate the variance of the random values generated. Copy the formula down into cell K3 ... K101.

Enter the following formula in cell K103:

@SUM(K2 ... K101)/(D4 - 1). (15)

This formula keeps an iteration by iteration calculation of the variance. Transfer its value to cell D11 by entering a +K103 in D11. Find the standard deviation of the variance and enter it in cell D12 by entering the formula +K103^0.5 in cell D12.

You may wish to convert the values in cells D10 ... D12 to two decimal point values with the command /RFF2.

Step #6

Enter a 0 in cell D3 to make sure all values are zeroed out. Then enter a 1 in cell D3. Proceed to press the F9 key several times, up to 100 if you wish, but it will take some time. As you press the F9 key watch the estimated values change. Also, observe the table in columns J and K fill up. For the example Cumulative Distribution function given, the mean should be about 500 and the standard deviation about 260. You may wish to experiment with other cumulative distribution functions to see if you can specify one with a smaller or larger variance, etc.

Step #7

Save the program under the name TABLEMU for Table Distribution with Mean and Variance calculated. It should be noted that this same basic procedure can be added to the Uniform, Normal, and Binomial distribution programs previously generated in order to see if they generate the expected means and variances, i.e., their expected means and variances are known and can be compared to the simulated one. Clear the spreadsheet for the next exercise by entering /WEY.

Randomizing the GESS Model

Figures 5 and 6 below display the spreadsheet structure for injecting correlated random World Prices into the GESS Model you previously developed. The purpose and motivation for this activity were discussed in Chapter 7.

	G	H	I	J	K	L
21			STOCHASTIC TRADE BALANCE ANALYSIS			
22						
23			ESTIMATED MEAN		491529	
24			ESTIMATED STD.		93279	
25			MINIMUM BALANCE		224365	
26			MAXIMUM BALANCE		623324	
27						
28						
29				ACTUAL	EXPECTED	
30	NET MONETARY TRADE BALANCE			470014	470270	
31						
32	WORLD PRICE FOR Q#1			90.93862	100.00	
33	WORLD PRICE FOR Q#1			81.50355	80.00	
34	WORLD PRICE FOR Q#1			54.33669	50.00	
35						
36	RESTART = 0 RUN> = 1			1		
37	ITERATION NUMBER			24		
38						
39						

Figure 5. Spreadsheet Structure For Modeling Correlated Random World Price Variables.

	M	N	O	P	Q	R	S
21			NORMAL (0,1)		0	RANDOM VALUE	VARIANCE
22		RANDOM NUMBER GENERATION			1	458199	1624855840
23		#1	#2	#3	2	416461	6731746534
24		0.5830	0.4710	0.4574	3	465956	1059675960
25		0.3897	0.2176	0.4612	4	504485	35727442
26		0.1389	0.6317	0.8346	5	538156	1571933905
27		0.2179	0.7457	0.7157	6	504623	37388970
28		0.0790	0.7471	0.7872	7	560201	3805946744
29		0.9133	0.7282	0.1083	8	485433	170970042
30		0.8061	0.8738	0.6954	9	402198	9275728981
31		0.8137	0.8101	0.7428	10	480175	336095350
32		0.7771	0.4258	0.9559	11	587969	8003174941
33		0.1531	0.3526	0.7918	12	504980	41878268
34		0.2264	0.7624	0.3193	13	615295	13639205666
35		0.6458	0.5684	0.2389	14	552922	2960862859
36					15	424032	5546727595
37		0.2558	-1.3344	-1.1084	16	475593	525135626
38					17	512453	194466303
39					18	457212	1705422481

Figure 6. Spreadsheet Structure for Constructing a Table of Random Stochastic Trade Balances.

Step #1

Load the GESSB Model you developed in a previous exercise. Enter the following titles in the cells indicated. Edit them for placement as needed.

I21 Stochastic Trade Balance Analysis
I23 Estimated Mean
I24 Estimated Std.
I25 Minimum Balance
I26 Maximum Balance
K29 Actual Expected
H30 Net Monetary Trade Balance
H32 World Price for Q#1
H33 World Price for Q#2
H34 World Price for Q#3
I36 Restart = 0 Run = 1
I37 Iteration Number
O21 Normal (0, 1)
N22 Random Number Generation

N23 '#1 #2 #3
R21 Random Value Variance

Step #2

Scenario #1 in Column C will be used as a non-stochastic base model column. Scenario #2 in column D will be made stochastic. Enter the following formula in cell L30 to calculate the expected non-stochastic trade balance:

+C28*C48 + C29*C49 + C30*C50. (16)

Enter the following formula in K30 to calculate the current iteration's stochastic trade balance:

+D28*D48 + D29*D49 + D30*D50. (17)

Make cells K31 and L31 display integer values by changing their format to zero decimal points with command /RFF0.

Step #3

Transfer the base scenario World Prices into cells L32 ... L34 by entering +C48, +C49, and +C50 in cells L32 ... L34.

Next use these expected prices together with the A* matrix values developed in Chapter 7 to generate random World Prices. Enter the following formulas in the indicated cells:

Cell K32	+L32 + 5.83*N37 - 1.0*O37 - 2.24*P37	(18)
Cell K33	+L33 + 5.0*O37 + 2.24*P37	(19)
Cell K34	+L34 + 4.47*P37.	(20)

Change the format in cells K32 ... L34 to a currency format with the command /RFC2.

Transfer the values calculated here for random prices to the Scenario #2 column by entering the terms +K32, +K33, and +K34 in cells D48, D49, and D50 respectively. Change these cells to display 2 decimal points with the command /RFF2.

Step #4

Set up the iteration counter. Enter a 1 in cell K36. Enter the following formula in cell K37:

@IF(K36>0, K37 + 1, 0). (21)

Step #5

Set up the commands to generate three Normal (0, 1) random numbers. The sampling method, as opposed to the formula method, is used here to generate the three desired Normal (0, 1) random variables. Enter an @RAND value in cell N24. Proceed to copy it into cells N25 ... N35 and cells O24 ... P35. Change the number of decimals displayed to 4 with the command /RFF4. Then enter the following summation command in cell N37:

6 - @SUM(N24 ... N35). (22)

Copy the command to cells O37 and P37. Change these three cells to also display only 4 decimal places.

Step #6

Build a three column table to store the randomly calculated trade balances and calculate their mean and variance. This task is basically the same task that was performed in augmenting the Inverse Cumulative Distribution program to calculate a mean and variance of the generated random values.

Enter a 0 in cell Q21. Then enter the formula +Q21 + 1 in cell Q22. Copy the formula down the column to cells Q23 to Q71. This should result in the numbers 1 to 50 being printed down the Q column.

Enter the following formula in cell R22:

@IF($K$37 = Q22, $K$30, @IF($K$37< Q22, 0, R22)). (23)

Copy the formula down the R column to cells R23 ... R71.

Enter the following formula in cell S22:

@IF($K$37> = Q22, ($R$73 - R22)^ 2,0). (24)

The formula calculates the variance of each random number. Copy it down the S column to cells S23 ... S71.

Enter the following summation formula in cell R73:

@SUM(R22 ... R71)/K37 (25)

and the following summation formula in cell S73:

@SUM(S22 ... S71)/(K37 - 1). (26)

Step #7

Complete the statistical reporting portion of the spreadsheet. Enter the value +R73 in cell K23. Enter +S73 ^ 0.5 in cell K24. Enter the following formula in cell K25.

@IF($K$36 = 0, $K$30, @IF(K25 < K30, K25, K30)). (27)

Finally, enter the following formula in cell K26:

@IF($K$36 = 0, $K$30, @IF(K26 > K30, K26, K30)). (28)

Step #8

The model should be ready to operate at this point. Enter a 0 in cell K36 to initialize the model. Then enter a 1 in cell K36 to start the iterative simulation process. Press the F9 key several times and watch the action take place. There is quite a bit going on. Before you get too involved in simulating the model, be sure to save it under the name GESSR for GESS Random.

The main purpose of the exercise was to find the degree of variation in the trade balance. In this case, a positive trade balance is a certainty. However, if the supply and demand situation were shifted a bit it would not be. A possible further expansion of a stochastic trade balance model such as this is to calculate the buffer stocks or cash reserves needed to deal with a stochastic trade balance situation, i.e., if the trade balance was rather tight but expected to average positive, how much buffer stock reserves would have to be held to cover any short-term deficits? Programming to augment the above GESS Model so that estimates of the required buffer stocks for different supply and demand conditions and stochastic World Price conditions could be made is not all that difficult, but can not be taken up here. The study by Trapp, Rogers, and Wilkens, as referenced in this chapter, discusses the methodology for estimating buffer stocks and makes estimates of required Liberian rice buffer stocks.

8. ENTERPRISE BUDGETS

by Darrel Kletke

Enterprise budgets are estimates of costs and returns expected from crop and livestock enterprises. Those making decisions about farm operations need to know the profitability of alternatives. Enterprise budgets have value to many users other than farm level decision makers. Lenders can use enterprise budgets to evaluate the viability of proposed farm investments. Farm policy analysts can use the information to determine expected impacts of policy alternatives. And those interested in long-range planning for a country can use the estimates to determine comparative advantage. Budgets provide critical inputs for other analytical tools such as linear programming.

The diverse uses of enterprise budgets prompt several different approaches to their construction. When constructed for a particular farm, it is desirable to use expected production and costs for that farm. When working on national or regional problems, the analyst wants budgets to reflect average or typical conditions rather than those of a unique farm. Before beginning the preparation of a budget, it is necessary to have clear intentions for its use.

OBJECTIVES

The objectives of this chapter are to:

1. Present alternative uses for enterprise budgets.
2. Illustrate the steps required to develop enterprise budgets.
3. Work though a hands-on example of budget preparation for use on a micro-computer spreadsheet.

Computationally, enterprise budgets are not complicated. For most normal situations good estimates can be prepared quickly using nothing more than a calculator. However, accurate determination of costs, particularly fixed costs, may require more complex calculations. For budgets used in farm level analysis, an error of several cents in the annual cost of machinery may not make much difference in the mix of crops chosen for

the farm. If the budgets are being used to determine national estimates of production costs, a small error in the annual machinery depreciation charge can make a substantial difference in estimated aggregate farm income. The budget preparer must take into account the intended audience for the budget.

KEY POINTS

1. Enterprise budgets contain estimates of costs and returns for producing an acre (hectare) of a crop or one unit of a livestock enterprise.
2. Enterprise budgets have many alternative uses. The detail included in a particular budget may depend on the use intended.
3. Enterprise budgets properly prepared contain much of the information needed to efficiently allocate the farm's scarce resources among the alternative crop and livestock enterprises.
4. Both variable and fixed costs should be included in enterprise budgets.
   - Variable costs vary with the quantity of the input being used.
   - Fixed costs are associated with ownership. These costs occur with or without production.
5. Net returns can be either returns over cash costs or returns over all costs. The choice depends on the problem being analyzed.
6. Spreadsheets are a useful microcomputer tool for preparing enterprise budgets.

CONCEPTS AND APPLICATIONS

An enterprise budget is an estimate of the costs and returns for producing a unit of an enterprise. For crops the typical unit is an acre or hectare. Most livestock enterprise budgets are constructed for a specified number of animals. However, the costs and returns are usually stated on a per head basis. For example, a budget may be prepared assuming the herd would contain 100 cows, but the enterprise budget would give estimates of costs and returns for one cow. This is done primarily so that the enterprise budgets are divisible and can easily be compared with other budgets computed similarly. Having the budgets on a per unit basis also makes them easier to use in farm planning.

Enterprise budgets must be prepared for a given output and a corresponding set of inputs. For example, in the production function illustrated in Figure 8.1 the set of inputs specified by point a on the input axis result in the output Y_a. The input b results in Y_b, etc. Each of these combinations of inputs and output would result in a unique enterprise budget. Because preparers of enterprise budgets are looking only at one point on the production function, it is necessary that care be taken to ensure that the inputs and output are appropriately matched.

Development of accurate enterprise budgets requires that the enterprise size be considered. This is particularly important for the correct allocation of fixed costs. If, for example, a farm of 160 cropland acres were compared with a farm having 640 cropland acres, it would be possible to size the machinery being used so that the fixed machinery costs (taxes, housing, insurance, and depreciation) would be nearly the same per acre for

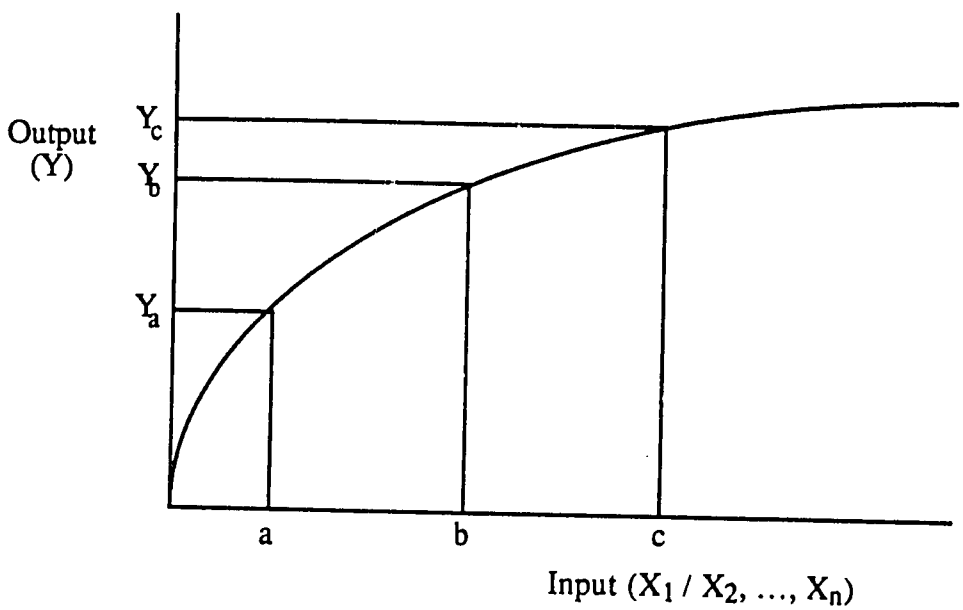

Figure 8.1. Production Function

each farm. However, the labor cost per acre for the small farm would be substantially greater as more labor would be required per acre because of small machine size. If the same set of machines were used on the two farm sizes, the labor per acre would be the same, but the fixed machinery cost per acre would be much greater for the small farm. Economies of size have a profound affect on cost and returns, hence the use to be made of the budget must be known when it is being prepared.

Budgets prepared for farm planning normally contain *expected* costs and returns. They constitute the preparer's best estimates of what is likely to occur. Depending upon the situation and data available, they may not represent what has occurred in the previous year or an average of previous years adjusted for trend.

When budgets are being prepared for aggregate policy analysis, fixed costs must be accurately specified, but the intended use of the budget may require that the allocation of fixed costs be based on a historical average. An enterprise budget being used for aggregate analysis may be more of an enterprise accounting than an estimated expected cost and return. Again it is cautioned that users of enterprise budgets ascertain that the budgets are appropriate for their intended use.

Enterprise budgets prepared for a particular farm must have fixed costs allocated appropriately for that farm. In farm planning, accurate specification of fixed costs for an enterprise may determine whether or not that crop or livestock alternative will be chosen for the farm. The planning horizon within which decisions are being made determines what costs are fixed and what costs are variable. For a short period, one year, all machinery ownership costs would be considered fixed and therefore would not be used in determining the optimum farm plan. If a farm plan were being considered over a longer period, say five or more years, the ownership costs for machinery would be considered variable because the owner would have the option of having or not having machinery over that period. Land may be considered a fixed cost over a five year period but variable over a longer period. Where land ownership decisions are being analyzed, a budget will consider all costs to be variable.

Budget Development

The main components of an enterprise budget are expected returns, costs, and net returns. Returns are computationally the most simple and will be discussed first although in the budget they may not be listed first. Under costs, variable costs will be covered first followed by fixed costs. Because estimating fixed costs can be more complex, separate sections dealing with machinery cost computation and land charge estimation are included as annexes to this chapter. Finally, a discussion of net returns computation and interpretation will be given. A typical crop enterprise budget is illustrated in the first table and a typical livestock enterprise budget is illustrated in the second table.

The wheat budget in Table 8.1 is for one acre and assumes that the machinery used is properly matched to the farm in question. If this assumption does not hold, the fixed machinery costs may be incorrectly allocated. The livestock budget in Table 8.2 is for one cow assuming the herd is 100 cows. The costs and returns estimates per cow are

Table 8.1. Wheat for Grain
Class I and II Clay and Loam Soils
Large equipment

	Unit	Price or Cost/Unit	Quantity	Value
Production:				
Wheat	bu.	$2.70	32.00	$86.40
Wheat Pasture	AUM	0.00	0.75	0.00
Total Receipts				$86.40
Operating Inputs:				
Wheat seed	bu.	$ 4.10	1	$ 4.10
18-46-0 fertilizer	cwt.	11.30	1	11.30
Nitrogen	lbs.	0.22	40	8.80
Insecticide	acre	4.50	1	4.50
Custom combine	acre	16.00	1	16.00
Custom hauling	bu.	0.14	32	4.48
Rent for fertilizer spreader	acre	1.25	2	2.50
Annual operating capital	dol.	0.12	29.93	3.59
Labor charges	hour	4.83	1.52	7.33
Machinery fuel, lube, repair	acre			20.16
Total Operating Costs				$82.76
Fixed Costs:				
Machinery				
Interest at 12.0%	dol.	0.12	$104.25	$12.51
Deprec., taxes, insurance	dol.			14.54
Land				
Interest	dol.	0.0	500.00	0.0
Taxes	dol.			0.0
Total Fixed Costs				27.05
Returns above total operating costs				$3.64
Returns above all costs except overhead, risk, and management				($23.41)

100 lbs. 18-46-0 fertilizer in the Fall
40 lbs. nitrogen in the Spring

Table 8.2. Cow-Calf Costs and Returns Per Cow
100 Cow Herd - Spring Calving
Native Pasture

	Unit	Price or Cost/Unit	Quantity	Value
Production:				
Steer Calves	cwt.	$74.00	2.024	$149.78
Heifer Calves	cwt.	64.00	1.392	89.09
Cull Cow	cwt.	35.00	0.950	33.25
Aged Bull	cwt.	45.00	0.160	7.20
Total Receipts				$279.32
Operating Inputs:				
41-45% supplement	lbs.	$0.09	$338.24	$30.44
Prairie hay	tons	42.00	0.179	7.53
Salt & minerals	lbs.	0.09	26.88	2.42
Veterinary & medicine	head	4.00	1.12	4.48
Hauling & marketing	head	5.00	1.00	5.00
Personal taxes	head	3.00	1.00	3.00
Annual operating capital	dol.	0.12	26.13	3.14
Machinery labor	hour	4.00	2.40	9.60
Equipment labor	hour	4.00	0.75	3.00
Livestock labor	hour	4.00	5.92	23.68
Machinery fuel, lube, repair	dol.			12.12
Equipment fuel, lube, repair	dol.			4.02
Total Operating Costs				$108.43
Fixed Costs:				
Machinery				
Interest	dol.	0.12	$26.22	$ 3.15
Deprec., taxes, insurance	dol.			5.13
Equipment				
Interest	dol.	0.12	170.79	20.49
Deprec., taxes, insurance	dol.			18.22
Livestock				
Beef cow	dol.	624.00		
Beef bull	dol.	32.40		
Beef heifer	dol.	78.00		
Horse	dol.	6.80		
Interest	dol.	0.12	741.20	88.94
Deprec., taxes, insurance	dol.			9.53
Land				
Pasture	AUM	0.0	13.44	0.0
Interest	dol.	0.0	175.00	0.0
Taxes	dol.			0.0
Total Fixed Costs				145.46
Returns above total operating costs				$170.89
Returns above all costs except overhead, risk, and management				$25.43
Protein supplement and hay in bad weather				
88% calf crop				

reasonably accurate for herd sizes near 100 cows. However, as herd sizes increase or decrease from the selected size, the fixed cost estimates will not be accurate.

*1. **Production**.* The intended use of the budget will influence the production quantities and prices used to estimate the total receipts expected for the enterprise. If the budget is to represent a particular farm, the production quantities and prices should indicate what would occur on that farm. For crops, the production quantity may be the established long-term average yield adjusted for trend. The price may be an average seasonal price adjusted for expected transportation costs. For budgets being used for aggregate analysis, the yields and prices should be long-term averages appropriate for a typical or representative farm type or region.

The budget in Table 8.1 is for the wheat enterprise. The yield is 32 bushels per acre and the price is $2.70 per bushel. Multiplying these gives the total receipts from wheat grain, $86.40. A byproduct of wheat production in Oklahoma is small grain grazing during the winter months. For this budget, .75 AUMs (animal unit months) are produced per acre. In this budget no price is specified for the small grain grazing because the grazing, if used, is an intermediate product use as an input for beef production which can be priced separately. Not including a price for grazing prevents double counting of incomes for the whole farm. The grazing production line is included, however, to fully reflect what is being produced by the enterprise.

A livestock budget may allow for multiple items produced. For example, a cow-calf budget may produce part of a cull cow, part of a steer, and part of a heifer. The budget developer must have a clear idea of the production plan represented by the budget to accurately specify the coefficients. In Table 8.2, the items produced times the prices determine the value of each item produced. The sum of the individual values determines the total receipts for the enterprise. Receipts from livestock production in Table 8.2 total $279.32 per head.

*2. **Operating Inputs**.* The operating or variable inputs section should include all inputs used just because the crop or livestock enterprise is produced. Not included are fixed costs which occur whether or not the enterprise is produced. For the operating inputs Tables 8.1 and 8.2 show the price of the input and the quantity used for the budget unit. If the unit of production is an acre, the quantity of input per acre should be included in the quantity column. If the unit of production is one head of livestock, the quantity of the input used per head should be included.

For many inputs, quantities are relatively easy to estimate. In Table 8.2, wheat seed, fertilizer, insecticide, harvesting, and spreader rental are usually priced on (or are easily converted to) a per acre basis. The last three items -- annual operating capital, labor, and machinery operating expense -- are more difficult to estimate. Annual operating capital charges depend on the timing of other expenditures; labor and machinery operating expense depend on the sizes of machinery. Estimation of annual operating capital and machinery labor and machinery operating expenses are illustrated in Annexes 8.1 and 8.2.

As with other budget components, the prices used for inputs depend on the

intended use of the budgets. If they are to be used in aggregate policy analysis, appropriate prices may be long-term regional averages (or border prices if the purpose is to estimate comparative advantage.) If the budgets are to be used for planning on a particular farm, the appropriate prices are those expected for that farm when production takes place or is sold..

The value column is computed by multiplying price times quantity. The interest, labor, and machinery operating costs are computed as illustrated in Annexes 8.1 and 8.2. In the crop budget, Table 8.1, the Total Operating Cost, $82.76, is the sum of the individual operating costs.

In livestock budgets, costs are computed much like they are for crop budgets. It is necessary to estimate the amount of labor used for livestock and the fuel, lubricants, and repair requirements. These computations are illustrated in Annex 8.3. The Total Operating Cost for the cow-calf budget in Table 8.2 is $108.43.

3. Fixed Costs. Fixed costs accrue whether or not production occurs on the farm. Normally, fixed costs are included for machinery and land. Procedures for estimating fixed machinery costs are illustrated in Annex 8.2. Cost categories included are interest on investment, depreciation, taxes, and insurance. In Table 8.1, depreciation, taxes, and insurance are aggregated.

The average machinery investment per acre for the wheat budget in Table 8.1 is $104.25. If the interest rate is 12%, the charge for machinery investment is $12.51 per acre. Depreciation, taxes, and insurance total $14.54 per acre.

Land charges may depend on the intended use of the budget. When working with a farmer for planning purposes, it may not be necessary to include a land charge because the cost will not affect decisions concerning farm organization. However, studies comparing efficiency from the cost of production among farm sizes would include a land charge.

The land charge may be the cash or share annual rental charge to reflect a rental situation. Or it may consist of an interest charge (less capital gain if economic rather than cash cost is being considered) and a tax payment to reflect costs for an owner with or without (opportunity cost) debt. Whatever charges are included should be appropriate for the intended use of the budget.

In Table 8.1, no land charges are included. However, the $500 in the quantity column indicates that the value of land is $500 per acre. The total fixed cost is $27.05 per acre. Including a land charge would considerably raise this total.

In the livestock budget, Table 8.2, the fixed cost items include livestock buildings, feeders, water tanks, other capital equipment, and the fixed costs of the livestock inventory. Cost estimation procedures are illustrated in Annex 8.3.

4. Enterprise Returns. Returns can be stated in alternative ways, but two are used in the budget examples. The first, returns above total operating cost, is good for short-term planning. In the short term the objective is to choose enterprises with greatest returns above variable (operating) costs. It is not necessary to cover all fixed or ownership expenses and thus they are not relevant to the decision process. Returns above total operating costs are found by subtracting total operating costs from total revenue. In Table

8.1, the total operating cost, $82.76, is subtracted from total revenue, $86.40, to give a return above total operating cost of $3.64 per acre.

The second returns figure (returns above all costs except overhead, risk, and management) deducts selected fixed costs and is negative. Note, as the title implies, that no charges are made for overhead, risk, or management. Overhead includes charges for bookkeeping, building expenses, and other items difficult to allocate to a particular enterprise. The enterprise is assumed to be risk neutral. That is, most enterprise budgets do not adjust for alternative levels of risk. Some enterprises may average a $20 net return per year but vary from a loss of $30 to a gain of $100. Another enterprise may average $20 with a range from $10 to $30 in net returns.

The title also indicates that no charge is made for management. It is expected that the farm manager will take some time, skill, and effort to make decisions about what crops are to be grown, when field work will be done, when and if to apply pesticides, when to harvest, and the like. Such management would require an expense to hire. However, no charge is made in the enterprise budget for these activities.

Although not stated in the title for the net returns figure, family living expenses are not allocated to the individual enterprises. All of the above mentioned expense items, (overhead, risk, management, and family living) must be covered from the "Returns above all costs except overhead, risk, and management." In Table 8.1 and 8.2, this return is found by subtracting operating costs and fixed costs from total revenue.

ACTIVITIES

1. "Annual capital" is an estimate of capital required on an annual basis. For example, $1,000 of capital used six months is $500 of annual capital (see Annex 8.1). It is normally assumed that capital is borrowed until the harvest month. If the harvest month is June, what is the annual capital for an acre of an enterprise when expenses consist of $20 in August, $25 in October, $12 in February, and $18 in June?

2. How many hectares per hour can be covered by a machine 5 meters wide, traveling 6 kilometers per hour if the machine efficiency is 80 percent?

3. Determine the expected repairs during the ownership period for a machine having an initial list price of $10,000. Assume the machine is purchased new and will be used 200 hours per year for 4 years. Typical machines are used 2,400 hours during their useful life. Assume also that during the life of the machine, repairs are expected to total 0.75 times the initial list price. This is RC1. The repair shaping coefficients are RC2 = .000251 and RC3 = 1.8 (see Annex 8.2).

4. What is the expected repair cost per hour for the above machine?

5. If the machine covered the number of hectares per year (hectares per hour times 200) determined in activity 2 above, what is the expected repair charge per hectare?

6. Determine the salvage value for a $10,000 machine being used 4 years using the salvage value equation: $RV\% = 56 \times (9.885)^n$, where RV percent = remaining value as a percent of list price and n = number of years of ownership.

7. What is the average investment over the four years of ownership?

8. What is the average depreciation per year over the four-year period?

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ANNEX 8.1

Estimation of Annual Operating Capital

Annual operating capital is an estimate of capital required on an annual basis. For example, $30 borrowed at 6% interest for six months is equivalent to $15 borrowed for one year at 12%. The annual operating capital equivalent of the $30 borrowed for six months is $15. There are several methods for estimating annual capital. Two will be discussed here. The first, rather crude, assumes that all expenses other than for harvest occur in the month the crop is planted. The second method is more precise but requires additional information -- the month each expense occurs.

1. Simple Method. For many budgets it is not necessary to precisely estimate annual capital. In these situations, operating expenses other than harvest expenses and interest are totaled. For the budget in Table 8.1, this total is $58.69 ($82.76 - $16.00 - $4.48 - $3.59). The crop is planted in October and harvested in June, which is eight months. Thus, from October through June $58.69 is invested in each acre. The $58.69 for eight months is equivalent to $39.13 ($58.69 * 8 / 12) for twelve months. For this example, the crude estimate is about $10 greater than the more precise estimate given in Table 8.1, $29.93. Although not precise, this method is much easier to use and can provide satisfactory estimates for most problems. This method works much better for budgets where the crop is harvested in only one period than it does for crops harvested several times per year. It does not work well for livestock budgets where expenses occur throughout the ownership period.

2. Complex Method. Accurately computing annual operating capital requires calculation of the net amount by which expense exceeds revenue for each month. This is the net capital required for each month. The monthly cumulative of the net capital is the net capital used each month. The procedure illustrated in the following table assumes no interest charge in the month the interest is repaid in full. For crop budgets this is usually the harvest month. In the illustration, month five is chosen as the month during which interest charges terminate.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Monthly expenses	10	12	15	37	50	20	0	20	15	12	0	10
Monthly income	0	0	0	0	100	0	15	0	0	80	0	0
Net capital required	10	12	15	37	-50	20	-15	20	14	-68	0	10
Cumulative capital	20	32	47	84	34	20	5	25	40	0	0	10
Monthly operating capital	20	32	47	84	0	20	5	25	40	0	0	10

Annual operating capital	$283	÷	12	=	$23.58
Operating interest	$23.58	x	.12	=	$2.83

The above example can be used to compute annual operating capital for an enterprise with multiple sale periods during the year. The example shows expenses in ten months and receipts in three months. Month five contains no monthly operating capital because in this month interest charges are terminated for the year. In the tenth month, revenues exceed expenditures by an amount greater than the capital borrowed in the preceding month. Thus, there is no monthly operating capital in the tenth month.

Operating interest for each month is computed for the above table by multiplying the monthly operating capital row by the interest rate and dividing by 12. Normally, only an annual interest charge is needed which can be computed by summing the monthly operating capital row, dividing the result by 12, and multiplying by the annual interest rate. For the example in the above table the annual operating capital is $23.58. If the annual interest rate were 12%, the annual operating interest would be $2.83.

Computing an annual operating capital equivalent, $23.58, makes it easy to adjust enterprise budgets for changing interest rates. If the rate were to change, the new interest charge could be computed by multiplying the new rate by the annual capital amount.

ANNEX 8.2

Estimation of Machinery Costs

Machinery costs consist of both variable and fixed components. The variable costs are fuel, lubricants, and repairs. Fixed costs consist of depreciation, interest, insurance, and taxes. Also, important for allocating machinery costs to a per acre basis is the amount of time required for each machine to cover an acre. Along with the discussion in each of the following sections, an example using a disk 28 feet wide being pulled with a 285 horsepower four-wheel drive tractor will be used.

1. Machine Capacity. Estimation of machine capacity depends on the speed traveled with the machine, the width of the machine, and the efficiency with which work is accomplished. Fortunately, estimates of the needed parameters are available for most machines.[1]

The formula for determining the number of acres covered by a machine in one hour is:

Acres/hour = (Speed X Width X Efficiency) / 8.25.

When working with hectares and kilometers per hour, the only change necessary to convert the above formula to the metric system is to replace the 8 25 with 10.

If an offset disk is 28 feet wide, is pulled at four miles per hour, and has an efficiency of 83%, the number of acres covered per hour is:

Acres/hour = (4.8 X 28 X .83) / 8.25 = 13.52 acres.

If 13.52 acres can be covered in one hour, then the number of hours required per acre is:

Hours/acre = 1 / Acres per hour = 1/13.52 = .074.

This means that one acre is covered every .074 hours or one acre every 4.44 minutes.

2. Tractor Hours. Tractors hours are normally considered to somewhat exceed machine hours estimated above. This is expected because it takes tractor time to get to and from the field. A convenient estimate is that tractor hours are 10 percent greater than machine hours.

If the above machine were used to cultivate 320 acres, the machine hours required would be:

Machine hours = 320 X.074 = 23.68 hours.

In 23.68 hours of actual work time, the 320 acres could be cultivated with the offset disk.

Tractor hours = 1.1 X machine hours

Tractor hours = 1.1 X 23.68 = 26.05 hours

The tractor time involved in cultivating with the offset disk is 26.05 hours for the 320 acres or .0814 hours per acre.

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[1]American Society of Agricultural Engineers. Data ASAS D230.2, Agricultural Machinery Management Data, revised September 1977, *1981-82 Agricultural Engineers Yearbook*, St. Joseph, Michigan, Section 5, p. 236.

3. Labor Hours. The hours of labor required per acre depend on the number of hours the tractor is used. Normally, labor hours will be greater than tractor hours by the amount of time it takes to service the equipment, make minor repairs, and adjust the equipment. A rule of thumb is that labor hours are 10 percent greater than tractor hours when a tractor is used. For self-propelled implements the rule of thumb is that labor hours are 20 percent greater than machine hours.

For the offset disk the estimate of labor hours is:

Labor hours = 1.1 X tractor hours

Labor hours = 1.1 X 26.05 = 28.66 hours.

The estimate of labor time for cultivating the 320 acres with the offset disk is 28.66 hours. This is about 0.09 hours per acre. Similar procedures can be used to estimate machine, tractor, and labor hours for all field operations.

4. Machinery Variable Costs. The variable costs associated with operation of machinery are those which vary directly with usage. Costs can be estimated for repairs, fuel, and lubricants.

•***Repairs.*** Repairs are usually the most variable component of machinery costs. They are influenced by variables such as management, maintenance level, variability in local repair costs, and the effects of climate and soils. Besides varying considerably from machine to machine, repair costs vary substantially from year to year. A good source of equations available for estimating repair costs is the Agricultural Engineers Yearbook.[2] In the handbook as well as other locations, estimates of factors for estimating repair costs can be found. The first step is estimating total accumulated repairs for the period of time the machine is owned. A part of this equation is the percent of machine life that the current owner is expected to own the machine. The percent life is computed from age in years, annual usage in hours, and the total expected life in hours.

Percent life = (Years owned X Hours used annually)/Hours of life

Using the offset disk as an example, assume the expected ownership interval is 10 years, the usage is 150 hours per year, and the total expected machine life is 2,000 hours. The percent life is:

Percent life = (10 X 150) / 2,000 = .75 or 75%.

In this example the current owner expects to utilize the offset disk about 75% of the total lifetime use expected from the disk.

In the *Agricultural Engineers Yearbook*, the equation for estimating total accumulated repairs for the machine is a function of machine list price, several repair cost factors, and the percent life estimated above.

Total accumulated repairs = Initial List X RC1 X RC2 X $(\%\ \text{Life})^{RC3}$

where RC1 = Repairs expected during the life of the machine as proportion of list price.

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[2]Ibid, p. 325.

RC2 and RC3 are a pair of curve shaping coefficients
% Life is a whole number (Use 75 for 75%).

For the offset disk, if total repairs over the full life of a machine with a list price of $17,000 are expected to be 65% of the list price and the curve shaping coefficients RC2 and RC3 are .000251 and 1.8, respectively, the expected total repairs during the first 75% of the machine's life are:

Total accumulated repairs = $17000 \times .65 \times .000251 \times 75^{1.8}$
Total accumulated repairs = $6,578.79.

The $6,578.79 is the total repair cost expected over the 10 years of ownership. Thus, expected average annual repairs for the disk are $657.88.

The expected annual usage is 150 hours per year. Dividing the $657.88 per year by 150 gives a repair charge of $4.39 per hour. In the machine capacity section above, it was determined that it takes .074 hours to cultivate one acre. The per acre repair estimate for the offset disk is:

Repairs per acre for offset disk = .074 X 4.39 = $0.32.

The expected repair cost per acre is 32 cents. Reliable estimates of the repair cost coefficients are essential to estimating repair costs using the above procedure.

•***Fuel Costs.*** Fuel costs depend on the amount of fuel consumed which depends on the size of tractor being used and the load being pulled. The size of tractor is generally more important than the load factor if implements are properly sized to the tractor. If available, records can be used to estimate annual fuel consumption per acre. However, it is difficult to ascertain from records the amount of fuel used per acre for (say) pulling the disc harrow. Thus most developers of enterprise budgets estimate fuel consumption for tractors using one of the following relationships.

Gasoline consumption per hour = .068 X Drawbar horsepower
LP gas consumption per hour = .08 X Drawbar horsepower
Diesel consumption per hour = .048 X Drawbar horsepower

Using the results of the appropriate above equation the fuel cost per acre can be estimated by multiplying the consumption per hour times the fuel price per gallon times the hours the tractor is used per acre.

Fuel cost per acre = Consumption per hour X fuel price X hours per acre

For the disking example above, the fuel consumption for a 285 horsepower four-wheel drive tractor is estimated as:

Diesel consumption per hour = .048 X 285 = 13.68 gallons.

If the diesel price is $0.95 per gallon and the tractors hours per acre is .0814, the estimated fuel cost per acre is:

Fuel cost per acre = 13.68 X .95 X .0814 = $1.06 per acre.

•***Lubricant Cost.*** Surveys[3] indicate that the cost of lubricants is approximately 15 percent of fuel costs. The equation for lubricant cost is:

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[3]Ibid, p. 237.

Lubricant cost = 0.15 X Fuel cost per acre

For the example offset disk where fuel cost per acre was estimated to be $1.06 per acre, the estimated lubricant cost is:

Lubricant cost per acre = 0.15 X 1.06 = $0.16

Lubricant costs are estimated only for machines having an engine. The lubricants used for machines without engines are assumed to be part of repair cost.

•***Labor Costs.*** For the example offset disk, the labor hours per acre was estimated above to be 0.09 hours. The labor cost is determined by multiplying the hours of labor by the wage rate. For a wage rate of $4.75 per hour, the labor cost for the disk is:

Labor cost = Hours per acre X Wage rate

Labor cost = 0.09 X 4.75 = $0.43 per acre

5. Machinery Fixed Costs. Fixed costs for machinery include charges for depreciation, interest, taxes, and insurance. These are classified as fixed because the owner will incur the expense even if the machines are not used. These may also be called ownership costs Most of these costs depend on the average investment in the machine. The average investment depends on the salvage value where the salvage value is defined as the remaining farm value at the time the current owner sells or trades the machine.

•***Salvage Value.*** There are several ways of estimating salvage value. A method which allows generalization to a number of machines uses one of the following equations:

Wheel tractors, Stationary Power Units

Remaining value as percent of list = 68 X $(0.92)^n$

Combines, Cotton Pickers, SP Windrowers

Remaining value as percent of list = 64 X $(0.885)^n$

Balers, Blowers, Forage Harvesters, SP (self-propelled) Sprayers

Remaining value as percent of list = 56 X $(0.885)^n$

All Other Field Machines

Remaining value as percent of list = 60 X $(0.885)^n$

where n is the age of the implement when sold or traded.

For the offset disk we will use the last equation. If the list price of the disk is $17,000 and machine will be used 12 years the expected salvage value as a percent of list price is:

Remaining value as percent of list = 60 X $(.885)^{12}$ = 13.85%.

The dollar amount of salvage is:

Remaining value = Remaining value as percent of list X List price

Remaining value = 13.85% X $17,000 = $2,346.

The tractor being used is a 285 horsepower four-wheel drive unit, used ten years, with a list price of $107,000. The first of the four equations above will be used to determine remaining value.

Remaining value as percent of list = 68 X $(.92)^{10}$ = 29.54%

The dollar amount of salvage is:

Remaining value = Remaining value as Percent of list X List price
Remaining value = 29.54% X $107,000 = $31,608.

•***Average Investment.***. The average investment in a machine of the ownership period is:

$$\text{Average investment} = \frac{\text{Purchase price} + \text{Salvage value}}{2}.$$

For the $17,000 offset disk with a salvage value of $2,346 used for a period of 12 years, the average investment is:

$$\text{Average investment} = \frac{\$17{,}000 + \$2{,}346}{2} = \$9{,}673.$$

For the $107,000 tractor with a salvage value of $31,608 used for a period of 10 years, the average investment is:

$$\text{Average investment} = \frac{\$107{,}000 + \$31608}{2} = \$69{,}304.$$

•***Depreciation Costs.*** The loss in value during the ownership period is the difference between the purchase price and the salvage value.

Total Depreciation = Purchase price - Salvage value

The annual average depreciation is:

$$\text{Depreciation per year} = \frac{\text{Purchase price} - \text{Salvage value}}{\text{Years owned}}.$$

Depreciation cost per hour depends on the number of hours the machine is used during the year. The annual use can best be estimated by summing the hours of use over all acres. If this is not feasible, a normal annual use can be used.

$$\text{Depreciation per hour} = \frac{\text{Depreciation per year}}{\text{Hours of annual use}}.$$

The depreciation cost per acre can be estimated by multiplying the depreciation per hour by the number of hours the implement is used on an acre.

Depreciation per acre = Depreciation per hour X Hours per acre

For the offset disk the depreciation cost per acre is:

Total depreciation = $17,000 - $2,346 = $14,654
Depreciation per year = $14,654 / 12 years = $1,221 per year
Depreciation per hour = $1,221 / 150 hours per year = $8.14 per hour.
Depreciation per acre = $8.14 X 0.074 = $0.60

For the tractor the depreciation cost per acre is:

Total depreciation = $107,000 - $31,608 = $75,392
Depreciation per year = $75,392 / 10 years = $7,539 per year

Depreciation per hour = $7,539 / 600 hours per year = $12.57 per hour
Depreciation per acre = $12.57 X 0.0814 = $1.02.

•***Interest Cost.*** Interest charges should be made whether or not funds are actually borrowed. If not an actual expense, the charge should be made as an opportunity cost. Interest costs may be estimated as:

Interest cost per year = Interest rate X Average investment

$$\text{Interest cost per hour} = \frac{\text{Interest cost per year}}{\text{Hours of annual use}}$$

Interest cost per acre = Interest cost per hour X Hours per acre
If for the offset disk a 12 percent interest rate is assumed, the interest cost is:
Annual interest = 12% X $9,673 = $1,161
Interest cost per hour = $1,161 / 150 = $7.74 per hour
Interest cost per acre = $7.74 X .074 = $0.57 per acre.
If for the tractor a 12 percent interest rate is assumed the interest cost is:
Annual interest = 12% X $69,304 = $8,316
Interest cost per hour = $8,316 / 600 = $13.86 per hour
Interest cost per acre = $13.86 X .0814 = $1.13 per acre.

•***Insurance Costs.*** Insurance charges may or may not be charged depending on actual practice. As with interest, insurance costs are most easily estimated using average investment:

Insurance cost per year = Insurance rate X Average investment

$$\text{Insurance cost per hour} = \frac{\text{Insurance cost per year}}{\text{Hours of annual use}}$$

Insurance cost per acre = Insurance cost per hour X Hours per acre.
If for the offset disk an insurance charge of .5% of average investment is assessed, the insurance cost is:
Annual insurance = 0.5% X $9,673 = $48.37
Insurance cost per hour = $48.37 / 150 = $0.32 per hour
Insurance cost per acre = $0.32 X .074 = $0.02 per acre.
If for the tractor an insurance charge of .5% of average investment is assessed, the insurance cost is:
Annual insurance = 0.5% X $69,304 = $346.52
Insurance cost per hour = $346.52 / 600 = $0.58 per hour
Insurance cost per acre = $0.58 X .0814 = $0.05 per acre.

•***Tax Cost.*** Taxes are normally computed based on the actual purchase price of the machine.

Annual taxes = Purchase price X Tax rate per dollar
Taxes per hour = Annual taxes / Annual hours of use
Taxes per acre = Taxes per hour X Hours per acre

If the tax rate is 1 percent of purchase price, the tax cost estimate for the offset disk is:

Annual taxes = $17,000 X .01 = $170

Taxes per hour = $170 / 150 hours = $1.13 per hour

Taxes per acre = $1.13 X .074 = $0.08 per acre.

If the tax rate is 1 percent of purchase price, the tax cost estimate for the tractor is:

Annual taxes = $107,000 X .01 = $1,070

Taxes per hour = $1070 / 600 hours = $1.78 per hour

Taxes per acre = $1.78 X .0814 = $0.14 per acre

•***Cost Summary***. Variable Costs per Acre of Use for the Offset Disk

Repairs	$0.32
Fuel	1.06
Lubricants	0.16
Labor	0.43
Total Variable	$1.97

Fixed Costs per Acre of Use for the Offset Disk

Depreciation	$0.60
Interest	0.57
Insurance	0.02
Taxes	0.08
Total Fixed	$1.27

Fixed Costs per Acre of Use for the Tractor

Depreciation	$1.02
Interest	1.13
Insurance	0.05
Taxes	0.14
Total Fixed	$2.34

Total Fixed and Variable Cost for Discing One Acre $5.58

A similar procedure as outlined above could be used to estimate the costs for other machines used to produce the acre of wheat. Because the computations are repetitious, computers are useful for estimating machinery costs.

ANNEX 8.3

Estimation of Livestock and Equipment Costs

Livestock and equipment costs have about the same components as machinery costs. The variable costs associated with livestock ownership are normally part of the operating inputs. The fixed costs associated with livestock ownership are interest; depreciation on cows, bulls, etc.; taxes; and insurance. Livestock equipment includes shelter, feeding equipment, watering equipment, and other items necessary for maintenance of a livestock enterprise. The variable costs associated with equipment include repair and labor for operation and maintenance. The fixed costs associated with equipment include interest, depreciation, taxes, and insurance.

1. Variable Costs. Fixed costs are discussed before examining variable costs.

•***Livestock.*** Labor is the primary variable cost that must be estimated independent of the individual operating costs. Livestock requires labor for feeding, observation, medication, and so forth. The budget developer must be careful to include all labor and yet not double count. Estimates of the amount of labor associated with livestock can be estimated directly from records or can be summed over the various operations associated with livestock production. If labor is computed for farm machinery, pickup, trailer, truck, and other items; then that labor should not be included in the direct estimate of labor involved for livestock production. For the example livestock budget in Table 8.2, livestock labor is estimated to be 5.92 hours per head.

•***Livestock Equipment.*** The variable costs associated with livestock equipment are included for labor, fuel, lubricants, and repairs. Some equipment may not include all cost categories. The budget preparer may choose to include equipment labor with the livestock labor. Or, the labor may be estimated for each equipment item. One approach is to include in the equipment labor estimate only that labor which is associated with maintaining and repairing the equipment. All labor associated with usage would be included in livestock labor. In Table 8.2, equipment labor is estimated to be .75 hours.

Equipment repairs are difficult to estimate. On a fence, for example, it may be easiest to estimate what repairs will total over the life of the fence. It might be determined that the fence would cost $2,500 per mile to construct and that over the expected life of 25 years fence repair will total about 25% of the construction cost. With this information, an annual fence repair cost can be estimated.

$$\text{Annual repair cost} = \frac{\text{Construction cost X Expected repair percentage}}{\text{Years of expected life}}$$

For one mile of fence:

$$\text{Annual repair cost} = \frac{\$2{,}500 \times .25}{25} = \$25$$

After determining the annual repair cost, it is necessary to allocate the cost to the budget. Complicating the procedure is that there may be more than one unit, one mile of fence, on the farm. In addition, the budget unit may not be a whole farm. In our case, we are building a budget for one cow but are assuming that there will be 100 cows on the farm.

Repairs for Budget Unit = Annual repair cost X Number of units X Budget proportion

For our fence the annual repair cost is $25. The number of units (miles of fence) is three, and the budget proportion (one cow of 100 cow unit) is .01.

Repairs for budget unit = $25 X 3 X (1 / 100) = $.75 per cow per year

If the equipment has an engine, it will be necessary to estimate a fuel and lubricant charge. In the rare cases when an engine is included, the fuel and lubricant equations in Annex 8.2 may be used to estimate costs.

2. ***Fixed Costs.*** The fixed costs for livestock normally include depreciation, interest, insurance, and taxes. For the most part, computation of fixed costs can be the same for both livestock and livestock equipment. As with machinery, it is necessary to know the salvage value and average investment to compute the other fixed costs.

•***Salvage Value.*** The expected residual or salvage value is computed by multiplying the purchase price by the expected residual percentage.

Salvage value = Purchase price X Expected residual percentage

For the cow in our sample, the residual value is set at 92% of the purchase price. For a cow purchased for $650, the salvage value is:

Salvage value = $650 X .92 = $598

The fence has no remaining value at the end of 25 years, so the salvage value is zero.

•***Average Investment.*** The average investment for both livestock and equipment is:

$$\text{Average investment} = \frac{\text{Purchase price} + \text{Salvage value}}{2}.$$

For the cow:

$$\text{Average investment} = \frac{\$650 + 598}{2} = \$624 \text{ per cow.}$$

For the fence:

$$\text{Average investment} = \frac{\$2{,}500 + 0}{2} = \$1{,}250.$$

•***Depreciation.*** The straight line estimate of depreciation is:

$$\text{Depreciation per year} = \frac{\text{Purchase price - Salvage value}}{\text{Years of life}}.$$

The depreciation for the cow assuming the cow is kept seven years is:

$$\text{Depreciation per year} = \frac{\$650 - \$598}{7} = \$7.43 \text{ per cow per year.}$$

The depreciation for the fence assuming the fence is used for 25 years:

$$\text{Depreciation per year} = \frac{\$1250 - \$0}{25} = \$50 \text{ per year per mile of fence.}$$

The fence depreciation must be allocated to the individual budget. For the example, there are three miles of fence and 100 cows on the farm.

Depreciation for budget = Depreciation per year X Number of units X Budget proportion

For the fence:

Depreciation for budget = $50 X 3 miles X (1 / 100) = $1.50 per cow per year

•***Interest.*** Annual interest is computed by multiplying the average investment by the interest rate.

Annual interest charge = Interest rate X Average investment

Cow annual interest charge = 12% X $624 = $74.88

Fence annual interest charge = 12% X $1,250 = $150

Allocated to the budget:

Interest charge per cow = Annual Interest X Number units X Budget Proportion

Cow interest charge per cow = $74.88 X 1 X 1 = $74.88 per cow per year

Fence interest charge per cow = $150 X 3 X (1/100) = $4.50 per cow per year.

•***Insurance.*** Annual insurance is computed by multiplying the average investment by the insurance rate.

Annual insurance charge = Insurance rate X Average investment

Assuming an insurance rate of .5%:

Cow annual insurance charge = .5% X $624 = $3.12

Fence annual insurance charge = .5% X $1,250 = $6.25.

Allocated to the budget the insurance for the fence is:

Insurance charge per cow = Annual Insurance X Number units X Budget Proportion

Cow insurance charge per cow = $3.12 X 1 X 1 = $3.12 per cow per year

Fence insurance charge per cow = $6.25 X 3 X (1/100) = $0.19 per cow per year.

•***Taxes.*** Taxes are usually charged on a specified formula. One way of estimating the taxes is to estimate the tax rate as a percent of the purchase price. For the following examples, assume the tax rate is 1 percent of the purchase price. It will be necessary to decide for each item whether or not taxes are appropriate. This will vary by political jurisdiction.

Annual tax charge = Tax rate X Purchase price

For the cow:

Annual tax charge = 1% X $650 = $6.50 per cow per year.

For the fence:

Annual tax charge = 1% X $1,250 = $12.50 per mile of fence per year

Tax charge per cow = Annual tax charge X Number units X Budget proportion

Tax charge per cow = $12.50 X 3 X (1/100) = $0.38 per cow per year.

3. *Fixed Cost Summary*

For the cow:

Depreciation	$ 7.43
Interest	74.88
Insurance	3.12
Taxes	6.50
Total	$91.93

For the fence on a per budget basis:

Depreciation	$ 1.50
Interest	4.50
Insurance	0.19
Taxes	0.38
Total	$ 6.57

In preparing the budget, costs must be computed using the above formulas for all livestock used in the budget. In the example budget, it takes a cow, part of a bull, part of a heifer, and part of a horse. Costs for all livestock items must be included to reflect accurately the production costs. Also, besides the fence used to illustrate equipment costs, a water tank and lot fences are charged as equipment used in the production of livestock.

When estimating enterprise cost and returns budgets it is necessary to be consistent in how costs are estimated for all enterprises. Also, other methods of estimating the costs may be feasible. If the taxes and insurance costs are known for a particular operation, those figures should be used rather than the formulas presented herein.

9. MATHEMATICAL PROGRAMMING

by Francis M. Epplin and Arthur Stoecker

Mathematical programming is a formal procedure for optimizing an objective function subject to numerous constraints such as those imposed by scarce resources. The technique is a logically consistent method for evaluating alternative economic policies. Mathematical programming can be applied to compute the "optimal" allocation of available resources among competing uses for any level of economic unit. For example, the technique can be used to evaluate the impacts of alternative economic policies on selected variables for a country, sector, region, firm, or family farm.

Models can be constructed to quantify the economic consequences of alternative policies on resource use, production, trade, and on the relative well-being of producers as well as consumers. The impact of policies such as price intervention, market quotas, input and capital subsidies, input and capital rationing, technology enhancement, and trade restrictions can be evaluated with the technique. By using representative or typical farm resource situations and by replacing market prices with border prices (international prices adjusted for transportation costs, etc.), mathematical programming is well suited to measure comparative advantage.

OBJECTIVES

The objectives of the material included in this chapter are to:

1. Present the standard mathematical programming model.
2. Illustrate the relationship between activity (enterprise) budgets and production activities for a mathematical programming model.
3. Demonstrate how to construct models which can be used to evaluate policies which involve alternative levels of input use and input substitution, as well as policies with various impacts across enterprises.
4. Provide examples of problems solved and results interpreted.

It is not possible to include all extensions of mathematical programming which have potential for application to policy analysis. (For construction of farm planning models see Heady and Candler; Beneke and Winterborer; Hazell and Norton). Numerous examples here illustrate the process of model construction and how to interpret results of solved models.

KEY POINTS

1. Mathematical programming is a formal procedure for optimizing an objective function subject to constraints. It is a logically consistent method for evaluating alternative economic policies.
2. Most comprehensive farm planning models simultaneously need to address a multitude of issues regarding optimal factor use and optimal product mix.
3. Application of mathematical programming for agricultural policy analysis requires consistent estimates of resource requirements for alternative crop and livestock production activities.
4. Crop and livestock enterprise budgets, when prepared in a consistent manner, can provide much of the data essential for farm planning and agricultural sector models.
5. Solved models provide estimates of optimal farm plans, including level of alternative activities, resource use, and opportunity cost of nonoptimal activities.
6. Through the process of optimization, the values of resources are revealed as computed shadow prices. Accurate computed shadow prices require that all relevant constraints and activities are included in the model.

CONCEPTS AND APPLICATIONS

The Mathematical Model

The standard form of the mathematical programming model is as follows:

Maximize $Z(x_1, x_2, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$

Subject to:

$$
\begin{array}{l}
a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \leq b_1 \\
a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n \leq b_2 \\
\quad \vdots \\
a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \leq b_m \\
x_1, x_2, \ldots, x_n \geq 0
\end{array}
$$

where:

Z = value to be optimized, that is, either maximized or minimized.

x_j = a choice variable which represents the level of activity j; the solution of the model provides the "optimal" level of activity j, for j = 1, 2, ..., n

c_j = the marginal change in the value of the objective function Z resulting from a one unit change in activity j

a_{ij} = the input-output coefficient which indicates the amount of resource i required to produce a unit of activity j

b_i = the initial quantity of resource or other constraint b available for allocation to the alternative activities, for i = 1, 2, ..., m.

1. Constraints. The model is provided with m potentially scarce resources. Each row in the model represents a unique resource which may constrain production. Hence, rows are often referred to as constraints. The quantity of each scarce resource, which is to be allocated in an optimal fashion by the model, is represented as b. When a constraint equation is written in standard algebraic form, the quantity b of the scarce resource i is the value on the right hand side of the equal sign (or weak inequality sign). Hence, it is often referred to as the "right hand side", or RHS, value of resource i.

For empirical modelling, it is convenient to assign short but descriptive and unique names to rows. For example, LABJAN might be used to name the row which represents a constraint imposed upon production by scarce January labor. Each constraint must be assigned a unit of measure which must be consistent across the row. For example, labor constraints may be defined in terms of hours, days, months, or some other unit. The practitioner must ensure that the unit of measure is consistent for each row and each column of the problem.

2. Activities. Each of the "j" columns in the model represents a unique production process. These columns are called activities. The symbol x_j represents the quantity of activity j. It is convenient to assign short but descriptive and unique alphanumeric names to activities. For example, the 27th column of a model may represent a corn production activity using technology 1 on land class 1. CORNT1L1 is more descriptive than x_{27}. Descriptive names will, in general, reduce the amount of time necessary to interpret the results of a solution.

3. Input-Output Coefficients. The a_{ij}'s are also called input-output coefficients. They indicate the quantity of resource i required to produce one unit of activity j. A production activity represents a unique point on a classical production surface. All a_{ij}'s for an activity describe the input levels of each designated scarce resource associated with that particular point on the production surface. Alternative points on the production surface can be represented by other production activities (columns) in the model.

4. Objective Function Values. The c_j's indicate the amount of change in the objective function value, Z, for a one unit change in the level of activity j. Thus, the objective of the model is to allocate the available b quantity of the m scarce resources to the

n activities in such a manner so the value of Z is optimized. In the process of optimization, solution values for all x_j's are obtained. An optimal value of Z is obtained when no alternative allocation of the scarce resources to the potential activities will result in a greater value for Z.

5. The Optimization Process. It is not the objective of this chapter to derive and present optimization routines. Numerous sources explain common procedures for solving mathematical programming problems. Many computer algorithms use a revised simplex procedure (for example, see Hazell and Norton). The rows (constraints) of a properly formulated model form a convex set (an n dimensional box in n dimensional space) with a finite number of vertices, or corners. The values of the objective function are used in conjunction with a logical iterative procedure to determine the optimal vertex (corner) at which Z is maximized and the level of each activity is determined. Solutions can be obtained with relative ease using available computer hardware and software. Hence, the crucial element for policy analysis is model construction and interpretation of results.

Examples of Application

Many of the agricultural production issues addressed by economists and farm management specialists can be classified into three types: product-product, factor-product, and factor-factor. The term "factor" refers to a factor (input) of production. "Product" refers to the output of the production process or activity. The product-product model is used to determine the optimal combination of products to produce, given limited resources. The factor-product model is used to evaluate the optimal factor-product combination given prices and other constraints. In some cases alternative combinations of factors or alternative factors can be used to produce a product. The factor-factor model can be used to select the optimal combination of factors. Applications which involve each of the three general classifications of production issues are included in the examples which follow.

1. Enterprise Budget Data. Consistent enterprise budgets provide a good data foundation for farm planning models. Four corn grain production budgets are included in Table 9.1. The budgets are very similar. Each of the four reflects costs and returns estimates for alternative levels of nitrogen fertilization applied to a fixed (one acre) unit of land. Alternative levels of input use, in this case fertilizer, are represented by the alternative budgets. The number of budgets needed to construct a policy model depends upon the type of policies under consideration and the level of aggregation and precision desired. If policy makers are interested in the optimal quantity of fertilizer use, activities which reflect alternative levels of fertilization should be included in the model.

Table 9.1. Estimated Returns and Costs Per Acre for Irrigated Corn Production for Four Alternative Ammonium Nitrate Fertilization Levels.

			Alternative Nitrogen Fertilization Levels (lbs./acre of Actual Nitrogen)							
			49		81		133		160	
	Unit	Price or Cost / Unit	Quantity	Value	Quantity	Value	Quantity	Value	Quantity	Value
Gross Returns										
Corn Grain	bu.	$2.00	130	260.00	150	300.00	170	340.00	170	340.00
Cash Costs										
Nitrogen (Ammonium Nitrate)	lbs.	$.095	148	14.11	245	23.32	403	38.29	485	46.06
Dry Fertilizer Application	acre	$2.00	1	2.00	1	2.00	1	2.00	1	2.00
Phosphate	lbs.	$.27	65	17.55	65	17.55	65	17.55	65	17.55
Potash	lbs.	$.14	20	2.80	20	2.80	20	2.80	20	2.80
Herbicide	acre	$6.88	1	6.88	1	6.88	1	6.88	1	6.88
Pesticide	acre	$7.50	1	7.50	1	7.50	1	7.50	1	7.50
Seed	lbs.	$1.00	22	22.00	22	22.00	22	22.00	22	22.00
Fuel and Lubricants										
Machinery	acre	$2.89	1	2.89	1	2.89	1	2.89	1	2.89
Irrigation	acre	$91.00	1	91.00	1	91.00	1	91.00	1	91.00
Repairs										
Machinery	acre	$9.27	1	9.27	1	9.27	1	9.27	1	9.27
Irrigation	acre	$7.00	1	7.00	1	7.00	1	7.00	1	7.00
Custom Harvesting										
Base Charge	acre	$12.00	1	12.00	1	12.00	1	12.00	1	12.00
Excess for > 20	bu.	$.12	110	13.20	130	15.60	150	18.00	150	18.00
Custom Hauling	bu.	$.15	130	19.50	150	22.50	170	25.50	170	25.50
Annual Operating Capital	$	$.1200	141	16.87	146	17.52	155	18.56	159	19.11
Total Cash Costs per acre	$			244.57		259.83		281.24		289.56
Total Cash Costs per bu.	$			1.88		1.73		1.65		1.70
Returns to Machinery and Equipment Depreciation, Insurance, Taxes, and Interest and Labor, Water, Land, Management, Overhead, and Risk ($/acre)				15.43		40.17		58.76		50.44

Labor	hours	Water	acre inches
Jan - Mar	.30	January	6
Apr - Jun	.74	May	5
Jul - Sep	.52	June	5
Oct - Dec	1.68	July	5
		August	5

The four corn budgets represent four points along a continuous production function. Inputs other than nitrogen, and those factors which vary when the level of nitrogen use is changed, are held constant. The names CORN49, CORN 81, CORN133, and CORN160 refer to the alternative budgets. CORN49 refers to the initial budget which reflects the use of 49 units of actual nitrogen per acre. The relationship is graphed in Figure 1. The yield of CORN160 is identical to that of CORN133. If corn grain is the only desirable output, the CORN160 activity would never be optimal and need not be included in a model. Under the budgeted prices, returns above the specified costs of production are greatest for the CORN133 activity. However, if factor prices change, CORN81 or CORN49 might be preferred.

To illustrate the process of model construction, consider a farm in a fictitious area of Panland. The farm is composed of 160 acres of cropland. All of the land can be irrigated. The farm has sufficient machinery and equipment for producing either sorghum or corn. A sorghum budget is included in Table 9.2. The only constraints on production are land (LAND) and the total labor available during the three fall months of October, November, and December (LABOD). Other resources, such as available labor in other periods, are assumed to be available in sufficient quantities.

The objective of example 1 is to determine whether sorghum (SORGHUM) or corn (CORN133) should be produced on the 160 acres of land. The information contained in each budget is represented on an acre basis. The per acre returns to the fixed resources can be computed from the budgets. CORN133 returns $58.76 per acre and SORGHUM returns $45.48. LABOD requirements are 1.68 hours per acre for CORN133 and 0.66 hours per acre for SORGHUM. The farm family can provide 600 hours of labor over the three month period. A mathematical representation of the model follows:

Maximize Z (CORN133, SORGHUM) = (58.76 * CORN133) + (45.48 * SORGHUM)
Subject to:

$$(1 * \text{CORN133}) + (1 * \text{SORGHUM}) \leq 160$$
$$(1.68 * \text{CORN133}) + (0.66 * \text{SORGHUM}) \leq 600$$
$$\text{CORN133} \geq 0 \qquad \text{SORGHUM} \geq 0.$$

For the sake of convenience, practitioners prepare models in tabular form rather than in explicit written equations. The information contained in the equations of example 1 is implicitly represented in the following tableau which was prepared with a microcomputer spreadsheet program. This particular format is used by the MUSAH microcomputer software package (Li, Ray, and Stoecker; Epplin and Li).

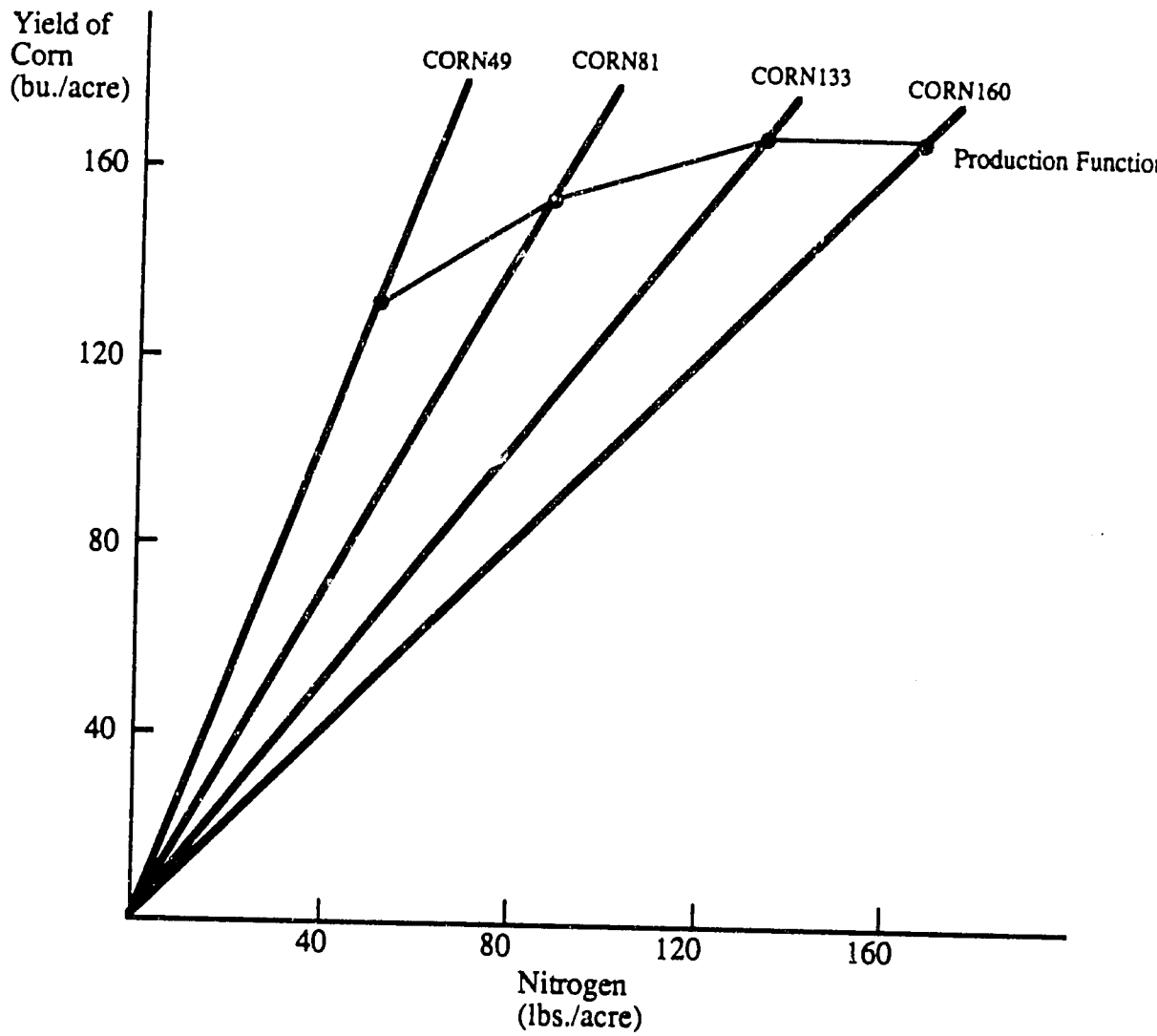

Figure 9.1 Corn Response to Nitrogen Function.
(The Factor-Product Example)

Table 9.2. Estimated Returns and Costs Per Acre for Irrigated Grain Sorghum.

	Unit	Price or Cost / Unit	Quantity	Value
Gross Returns				
Sorghum Grain	cwt	$3.00	80	240.00
Cash Costs				
Nitrogen (Ammonium Nitrate)	lbs.	$.095	400	38.00
Dry Fertilizer Application	acre	$2.00	1	2.00
Phosphate	lbs.	$.27	15	4.05
Potash	lbs.	$.14	10	1.40
Seed	lbs.	$.75	6	4.50
Milogard Herbicide	lbs.	$3.40	2	6.80
Pesticide	acre	$5.00	1	5.00
Fuel and Lubricants				
Machinery	acre	$2.89	1	2.89
Irrigation	acre	$63.00	1	63.00
Repairs				
Machinery	acre	$9.27	1	9.27
Irrigation	acre	$4.85	1	4.85
Custom Harvesting				
Base Charge	acre	$12.00	1	12.00
Excess for > 12	cwt.	$.12	68	8.16
Custom Hauling	cwt.	$.25	80	20.00
Annual Operating Capital	$	$.1200	105	12.60
Total Cash Costs per Acre	$			194.52
Total Cash Costs per Cwt.	$			2.43
Returns to Machinery and Equipment Depreciation Insurance, Taxes, and Interest and Labor, Water, Land, Management, Overhead, and Risk ($/acre)				45.48

Labor	hours	Water	acre inches
Jan - Mar	.56	April	3
Apr - Jun	.62	June	5
Jul - Sep	.30	July	5
Oct - Dec	.66	August	5

	A	B	C	D	E
1	EX1	MAXIMIZE B		CORN133	SORGHUM
2	C			58.76	45.48
3	LAND	L	160	1	1
4	LABOD	L	600	1.68	.66

Row 1 is used for naming. Values for the objective function are included in row 2. The constraints of the model are included in rows 3 and 4. Additional constraints could be added in the rows which follow. Column A is reserved for row names. Cell A1 can be used to provide a name for the problem. The name EX1 is used to identify example 1. Cell B1 indicates that the objective function is to be maximized. For models in which the objective function is to be minimized, cell B1 should contain the word MINIMIZE. Cell B2 is blank. The remaining cells in column B indicate the mathematical sign of the constraint equations. In the example, both are "less than or equal to" constraints, designated by the letter L to indicate ≤. The letter E is used for equality constraints and G for "greater than or equal to" constraints.

Column C contains the "right hand side" values beginning with column C3. Columns to the right of column C contain the objective function and input-output values for all activities of the model. The tableau implicitly contains all information included in the original equations.

The solution follows:

Solution
OPTIMAL
Function Value: 9401.6

				SORGHUM	LAND
				45.48	0
Returns	Name	Type	Level	Real	Slack
58.76	CORN133	real	160	1	1
0	LABOD	slack	331.2	-1.02	-1.68
Z				58.76	58.76
Shadow Price				13.28	58.76

The final tableau contains the same number of rows and columns as the initial tableau. In the iterative process of optimization, activities included in the initial tableau as columns may be included in the solution tableau as either rows or columns. If it is optimal to "conduct" the activity, it will "become" a row. If it is not optimal to "conduct" the

activity, it may remain as a column, or in some instances become a row and be included at a zero level. Similarly, rows for resources that do not limit output will remain as rows, whereas those that do limit output will become columns.

The optimal value of the objective function for EX1 is $9,401.60. The column labelled "Returns" contains the c_j value for rows that began as columns and 0 for rows that began as rows. Row or column names are included in the "Name" column. The "Type" column will indicate "real" for rows that began as columns and "slack" for rows that were rows in the original tableau. The optimal x_j values are printed in the column labelled "Level". For EX1, the optimal solution is to produce 160 acres of CORN133. Only 268.8 hours of LABOD will be used with the remaining 331.2 hours in "slack". All 160 acres of LAND is used.

The bottom row of the final tableau indicates that the "shadow price" of SORGHUM is $13.28. If one acre of SORGHUM were grown, one acre less of CORN133 could be planted. The budgets and the c_j's for the activities show that the estimated return to the fixed resources for CORN133 is $58.76 per acre which is $13.28 more than the estimate of $45.48 for SORGHUM. Thus, the activity shadow prices are computed for activities not included in the optimal farm plan. They indicate the opportunity cost or "income penalty" associated with nonoptimal activities. For this particular example, it would be easy to obtain the shadow price of SORGHUM by evaluating the differences in returns on the enterprise budgets. However, as problems become more complicated, computation of shadow prices without the use of mathematical programming becomes increasingly difficult.

The shadow price of LAND is estimated to be $58.76 per acre. This indicates that the last (marginal) unit of land contributed an additional $58.76 to the value of Z. It is the value of marginal product of land. The results indicate that the value of marginal product of labor, the increase in Z associated with another hour of LABOD, is zero.

The total value of the objective function is a return to the limited resources of the farm. Because LAND is the only restricting resource, the entire value of $9,401.60, is attributed to land. LABOD remains as a row, and its shadow price is 0. This is a remarkable characteristic of mathematical programming models. The value of the objective function can be computed by $c_1 x_1 + c_2 x_2 = (58.76 * 160) + (45.48 * 0) = 9{,}401.60$. The shadow prices computed by the model allocate or impute this entire amount to the scarce resources. $(\text{Shadow Price}_1 * b_1 + \text{Shadow Price}_2 * b_2) = (58.76 * 160) + (0 * 600) = 9{,}401.60$. The latter is the *dual* of the former objective function.

The results for EX1 may seem trivial. However, it is useful to associate the construction of models and the results with the underlying assumptions included in the enterprise budgets. The validity of all results generated by the model depends upon the estimates obtained from the budgets.

2. A Product-Product Example. Activities can be included in a model to reflect a complementary relationship that may exist among crops grown within a crop rotation. For example, the two crops of Panland could be grown in a two-year crop rotation with CORN133 followed by SORGHUM and SORGHUM followed by

CORN133. Rotations may be beneficial for controlling weeds and other pests, reducing risk, and improving the efficiency of labor and machinery use.

Assume that the yield of CORN133 grown in a field in which sorghum was grown in the previous year is enhanced or "complemented", by three bushels per acre. Even though the amount of inputs applied is held constant, the yield is greater than if the crops were grown continuously. Also assume that the rotation does not influence the yield of SORGHUM.

If the entire crop acreage were included in the rotation, half of the farm could be planted to CORN133 and half to SORGHUM each year. The three bushels of corn could be sold for $2 per bushel. Harvesting costs are $0.27 per bushel. If no other costs or returns change, the gross margin for two acres of land allocated to the crop rotation would be $58.76 + {3 bu. * ($2 - $0.27)} + $45.48 = $109.43. Labor requirements for two acres of the rotation would be 1.68 + 0.66 = 2.34 hours. The crop rotation is represented as the CRNSORRT activity in the EX2 tableau.

	A	B	C	D	E	F
1 | EX2	MAXIMIZE B			CORN133	SORGHUM	CRNSORRT
2 | C				58.76	45.48	109.43
3 | LAND		L	160	1	1	2
4 | LABOD		L	600	1.68	.66	2.34

The model is constructed by adding the CRNSORRT activity to EX1. The activity was arbitrarily defined in terms of two units of land. The solution tableau for model EX2 follows.

Solution
OPTIMAL
Function Value: 9401.60

				SORGHUM	CRNSORRT	LAND
				45.48	109.43	0
Returns	Name	Type	Level	Real	Real	Slack
58.76	CORN133	real	160	1	2	1
0	LABOD	slack	331.2	-1.02	-1.02	-1.68
Z				58.76	117.52	58.76
Shadow Price				13.28	8.09	58.76

Because a unit of the CRNSORRT activity does not generate more returns to the fixed resources per acre than CORN133, the results for EX2 are very similar to those obtained for EX1. The shadow price of CRNSORRT is $8.09. An acre of the activity would generate $4.045 less returns than an acre of CORN133. Since a unit of CRNSORRT was defined to require two acres of LAND, the shadow price for a unit (two acres) of the activity is $8.09.

This model provides for three alternative uses of the two fixed resources. The three activities define three points on a production possibilities curve. The objective is to select the optimal product combination (output) subject to the available resources. Hence, it is a product-product problem. Figure 9.2 graphically depicts the problem.

3. A Factor-Product Example. Only one of the four corn budgets included in Table 9.1 was used in examples 1 and 2. When the prices and factor requirements are fixed at the level indicated in the budgets, the CORN133 activity generates more return to the fixed resources and is the preferred corn production alternative. However, if factor prices or requirements change, CORN49 or CORN81 may be preferred. For models designed to evaluate the impact of policies involving factor prices and restrictions on factor use, such as fertilizer allocation per farm, multiple activities can be included to reflect the impact of alternative levels of factor use. The tableau which follows includes the three relevant corn production activities.

	A	B	C	D	E	F
1 | EX3	MAXIMIZE B			CORN49	CORN81	CORN133
2 | C				15.43	40.17	58.76
3 | LAND		L	160	1	1	1
4 | LABOD		L	600	1.68	1.68	1.68

CORN133 requires the same amount of LAND and LABOD as the other two activities, but generates more return to the fixed resources. Hence, it is the preferred activity and is said to "dominate" the other two activities. To evaluate the impacts of alternative nitrogen prices, the objective function values must be recomputed. Alternatively, it is possible and often desirable to separate factor purchase activities from crop production activities. For example, in the EX4 tableau which follows, an ammonium nitrate purchase activity is included. The cost of the ammonium nitrate fertilizer will be computed by the BYAMMNIT activity.

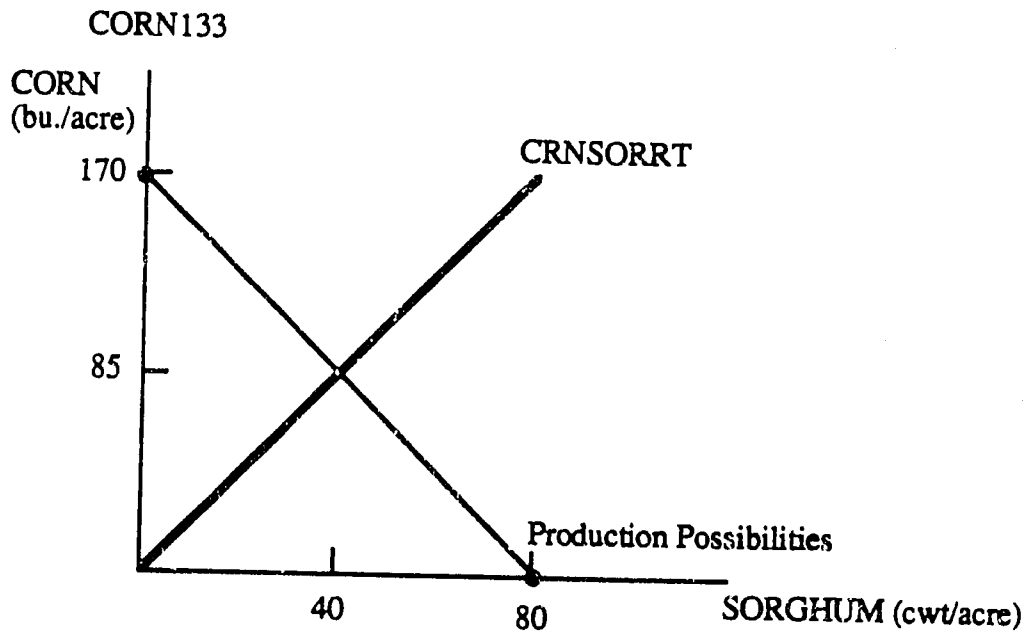

Figure 9.2. Production Possibilities From One Acre of Land. (The Product - Product Example)

=====A==========B==========C==========D==========E==========F==========G=====

1 | EX4	MAXIMIZE B		CORN49	CORN81	CORN133	BYAMMNIT
2 | C			29.54	63.49	97.05	-.095
3 | LAND	L	160	1	1	1	
4 | LABOD	L	~~200~~ 600	1.68	1.68	1.68	
5 | AMNTBL	L		49	81	133	-.33

==========

Ammonium nitrate contains 33 percent of actual nitrogen per unit and costs $0.095 per pound. The BYAMMNIT activity reflects the cost $0.095 per unit in the objective function. The objective function values for the other three activities are adjusted by adding the cost of ammonium nitrate to the previous gross margins. An ammonium nitrate balance (transfer) row (AMNTBL) is added to the model. The algebraic equation represented by the AMNTBL row is:

$$\{49 * (CORN49)\} + \{81 * (CORN81)\} + \{133 * (CORN133)\} - \{0.33 * (BYAMMNIT)\} \leq 0.$$

The purchase of one unit of ammonium nitrate reduces the value of the objective function by $0.095 and provides 0.33 units of actual nitrogen which can be used by any of the three corn production activities. The equation which follows is an alternative algebraic representation of the AMNTBL row. A negative input-output coefficient indicates that increasing the level of the activity, in this case BYAMMNIT, increases the amount available for use. It has the effect of "adding" to the RHS.

$$\{49 * (CORN49)\} + \{81 * (CORN81)\} + \{133 * (CORN133)\} \leq \{0.33 * (BYAMMNIT)\}$$

The equation restricts nitrogen use to be less than or equal to the amount purchased. Because nitrogen purchase will reduce the value of the objective function by $0.095 per unit, all purchased nitrogen will be used. Hence, although the sign of the equation is not a strict equality, purchases will equal use.

With this model configuration, the price of BYAMMNIT can be changed easily without recomputing costs for each of the production activities. The sensitivity of the results to alternative prices can be evaluated without explicitly computing new enterprise budgets for each system. However, the quantity of operating capital will vary with the price of nitrogen. If operating capital is a limiting resource, a row should be included in the model and the quantity of capital required for purchasing the fertilizer should be reflected in the BYAMMNIT activity.

The solution tableau for EX4 is as follows:

Solution
OPTIMAL
Function Value: 9401.939

Returns	Name	Type	Level	CORN49 29.54 Real	CORN81 63.49 Real	LAND 0 Slack	AMNTBL 0 Slack
97.05	CORN133	real	160	1	1	1	
-0.095	BYAMMNIT	real	64484.84	254.5454	157.5757	403.0303	-3.03030
0	LABOD	slack	331.2			-1.68	
Z				72.86818	82.08030	58.76212	0.287878
Shadow Price				43.32818	18.59030	58.76212	0.287878

The addition of the fertilizer purchase activity (BYAMMNIT) and balance row (AMNTBL) does not influence the optimal solution. The results are essentially identical to those of EX1. CORN133 is the preferred activity, LAND is the limiting resource, and its shadow price is still $58.76 per acre. The solution also indicates that it is optimal to purchase 64,484.8 pounds of ammonium nitrate. From the enterprise budget we know that each acre of CORN133 requires 133 pounds of actual nitrogen. One pound of the fertilizer contains 0.33 pounds of actual nitrogen. Further, 133/0.33 = 403.03, and 160 * 403.03 = 64,484.8.

4. The Factor-Factor Example. In many instances alternative inputs or alternative combinations of inputs can be used to produce a particular product. For example, there are numerous sources of nitrogen fertilizer. The model which follows includes activities for producing corn with either ammonium nitrate or an alternative nitrogen source, anhydrous ammonia.

=====A=========B=========C======D========E=========F=========G=======H======I======

1 | EX5	MAXIMIZE B		CORN81	CORN133	BYAMMNIT	CN81AH	CN133AH	BYAH
2 | C			63.49	97.05	-.095	59.81	93.87	-.12
3 | LAND	L	160	1	1		1	1	
4 | LABOD	L	~~200~~ 600	1.68	1.68		1.68	1.68	
5 | AMNTBL	L		81	133	-.33			
6 | ANHYBL	L					81	133	-.82

CN81AH and CN133AH are corn production activities requiring anhydrous ammonia rather than ammonium nitrate. The field operations and associated costs such as fuel and operating capital are different for the two sources of nitrogen. Hence, the objective function value for CORN81 is different than that for CN81AH. A graphical representation of this problem is included in Figure 9.3.

The final tableau for EX5 indicates that, given the budgeted information, anhydrous ammonia is a more economical source of nitrogen than ammonium nitrate. The 160 acres of CN133AH return $11,905.05 to the limiting resources. LABOD is in slack; its shadow price remains at zero. Hence, LAND is the only resource which restricts production in the model. Since the price of corn is held constant, the more economical production activity augments the value of the limiting resource. The shadow price of LAND increases to $74.41 per acre. The increase in returns (value) resulting from the more economical production process is added by the program into the value of the scarce resource LAND.

Solution
OPTIMAL
Function Value: 11905.05

				CORN81	BYAMMNIT	CN81AH	LAND	AMNTBL	ANHYBL
				63.49	-0.095	59.81	0	0	0
Returns	Name	Type	Level	Real	Real	Slack	Slack	Slack	Slack
97.05	CORN133	real	0	0.609022	-0.00248			0.007518	
93.87	CN133AH	real	160	0.390977	0.002481	1	1	-0.00751	
-0.12	BYAH	real	25951.21	63.41463	0.402439	63.41463	162.1951	-1.21951	-1.21951
0	LABOD	slack	331.2				-1.68		
z				88.19693	-0.05618	86.26024	74.40658	0.170251	0.146341
Shadow Price				24.70693	0.038817	26.45024	74.40658	0.170251	0.146341

5. A Factor-Product, Product-Product, Factor-Factor Example. Most comprehensive farm planning models address a multitude of issues regarding optimal factor use and optimal product mix. The example problem modelled in EX6 contains elements of each of the three types of classical production problems. Solution of the model will answer questions regarding (1) the optimal amount of input use (factor-product) -- 81 versus 133 pounds of actual nitrogen per acre of corn, (2) the optimal combination of nitrogen source (factor-factor) -- ammonium nitrate versus anhydrous ammonia, as well as (3) the optimal enterprise combination (product-product) -- corn versus a corn-sorghum rotation.

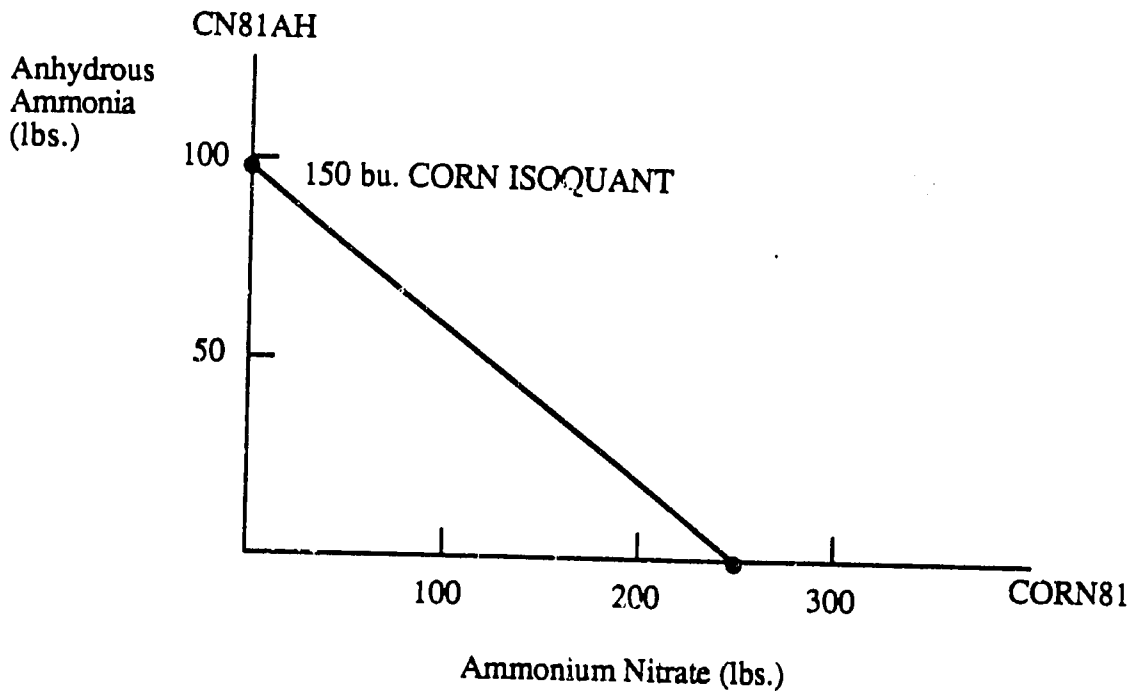

Figure 9.3. Corn Isoquant Reflecting Production of 150 bu. From Alternative Combinations of Anhydrous Ammonia And Ammonium Nitrate (The Factor - Factor Example)

1 | EX6	MAXIMIZE B		CORN81	CORN133	BYAMMNIT	CN81AH	CN133AH	BYAH	CRNSORRT
2 | C			63.49	97.05	-.095	59.81	93.87	-.12	109.43
3 | LAND	L	160	1	1		1	1		2
4 | LABOD	L	600 ~~200~~	1.68	1.68		1.68	1.68		2.34
5 | AMNTBL	L		81	133	-.33				
6 | ANHYBL	L					81	133	-.82	

6. Marketing Activities. Policy analysts are often interested in the impact of alternative product prices on crop production and resource use. For this reason models are usually constructed with separate activities to reflect the marketing and production components for a particular crop. Corn and sorghum selling activities are included in the EX7 tableau.

1 | EX7	MAXIMIZE B		CORN133	SORGHUM	CRNSORRT	SLCORN	SLSOR
2 | C			-281.24	-194.52	-476.57	2	3
3 | LAND	L	160	1	1	2		
4 | LABOD	L	600	1.68	.66	2.34		
5 | CORNBL	L		-170		-173	1	
6 | SORBL	L			-80	-80		1

The objective function values for the production activities of EX7 include only production costs. The negative signs for the objective function values indicate that production (without marketing) reduces the value of the objective function. Crop yields, 170 bushels for CORN133 and 80 cwt for SORGHUM, are "transferred" to the corn (SLCORN) and sorghum (SLSOR) marketing activities via the commodity balance or transfer rows (equations), CORNBL and SORBL. Positive returns are generated when the products are sold. With this model specification, product prices can easily be changed. The explicit algebraic equation represented by the SORBL row is:

$$\{-80 * (SORGHUM)\} + \{-80 * (CRNSORRT)\} + \{1 * (SLSOR)\} \leq 0.$$

Rewriting, the equation becomes:

$$\{1 * (SLSOR)\} \leq \{80 * (SORGHUM)\} + \{80 * (CRNSORRT)\}.$$

The amount of sorghum sold must be less than or equal to the amount of sorghum produced. Because sales will increase the value of the objective function, all production will be sold. Thus, sales will equal production.

The EX8 tableau includes activities for selling corn and sorghum as well as an activity for purchasing ammonium nitrate. Activities could be added to the model to enable additional land acquisition either by renting or purchasing. Alternatively, an activity could be added to permit the rental of a unit of land to another farmer.

1 | EX8	MAXIMIZE B		CORN133	SORGHUM	SLCORN	SLSOR	BYAMMNIT
2 | C			-242.95	-156.52	2	3	-.095
3 | LAND	L	160	1	1			
4 | LABOD	L	600	1.68	.66			
5 | CORNBL	L		-170		1		
6 | SORBL	L			-80		1	
7 | AMNTBL	L		403	400			-1

*7. **Labor Use.*** Policy analysts are often concerned with the impact of alternative crop and livestock production activities on labor requirements. Often a major objective of a policy model is to evaluate labor use. Successful crop production depends upon proper timing of the various field operations required to seed or plant, cultivate, and harvest. In some cases, expected weather patterns -- rainfall and time required for drying saturated soils -- may influence the expected number of field work days. Hence, special care is necessary to properly model labor requirements and use.

The number of labor constraints necessary to include in a model depends upon the (1) length of the planting and harvesting "windows" for the major crops and (2) the flexibility of the prevailing labor market. In some cases it may not be practical to hire labor for less than one month or one week. In other cases, it may be possible to add to family income by off-farm employment of family labor during slack times on the farm. In other instances, it may not be possible for family labor to locate part-time off-farm work.

*8. **Reservation Wage.*** When a quantity of labor is included in the RHS of a labor constraint, it is assumed to be available for allocation at no explicit charge to the objective function. In periods of excess or slack family labor, if no other uses are permitted, it will have an implied opportunity cost of zero. In some cases this may not properly characterize family labor. For example, if the return from work is very low, family members may derive more utility from leisure than from the goods purchased with the income generated by working on the farm. In other words, the opportunity cost of labor is equivalent to the marginal value of leisure, which could be expected to exceed zero.

The FMODLAB activity included in the EX9 tableau is constructed to place a minimum wage on family labor of $2 per hour. Family labor is restricted to 600 hours by the MXFMOD constraint. If a unit of labor contributes less than $2 to the value of the objective function, it will not be used. The price of $2 per hour indicates that the family considers an hour of leisure to be as valuable as $2 of market goods.

An unlimited quantity of additional (nonfamily) labor may be hired for $4.50 per hour as reflected in the HRODLAB activity. In some cases hired labor may be less efficient than family labor. In the example, an hour of hired labor is assumed to provide only 0.9 hours of labor in terms of family labor equivalent. Because family labor is less expensive, no labor will be hired until the 600 hours of family labor are used.

=====A==========B==========C======D========E=========F=========G=====

1 | EX9	MAXIMIZE B		CORN133	SORGHUM	FMODLAB	HRODLAB
2 | C			58.76	45.48	-2	-4.5
3 | LAND	L	160	1	1		
4 | LABOD	L		1.68	.66	-1	-.9
5 | MXFMOD	L	600			1	
6 | INCOME	G		58.76	45.48		-4.5

As constructed, the value of the objective function will not include the value of family labor. To the contrary, the objective function value will be reduced by $2 for each hour of family labor used. Including an INCOME row in the model is a convenient procedure for computing the returns to the fixed resources and family labor. The only purpose of the INCOME row is to account, or add the income, including the implied income from family labor. The algebraic sign of the equation is G rather than L. A G will ensure that the constraint does not interfere with the solution. Note that the input-output coefficient in the INCOME row for the FMODLAB activity is zero rather than -$2. Thus, the "cost" of the family labor will not be subtracted from the RHS of the INCOME row.

Solution
OPTIMAL
Function Value: 8864

				SORGHUM	HRODLAB	LAND	LABOR
				45.48	-4.5	0	0
Returns	Name	Type	Level	Real	Real	Slack	Slack
58.76	CORN133	real	160	1		1	
-2	FMODLAB	real	268.8	1.02	0.9	1.68	-1
0	MXFMOD	slack	331.2	-1.02	-0.9	-1.68	1
0	INCOME	slack	9401.6	13.28	4.5	58.76	
Z				56.72	-1.8	55.4	2
Shadow Price				11.24	2.7	55.4	2

The final tableau for EX9 indicates that the value of the objective function will decline from $9,401.60 to $8,864 when a charge of $2 per hour is explicitly included for family labor. Algebraically, {$9,401.60 - ($2/hour * 268.8 hours)} = $8,864. The "Level" of the INCOME row does contain the original value of $9,401.60. When the value of the family labor resource is fixed to be at least $2 per hour, the shadow price of the other limiting resource, LAND, is reduced. In the example, it declines by $3.36 -- from $58.76 to $55.40. Labor in the amount of 1.68 hours is required to produce one acre of CORN133. The reservation wage "cost" of labor required per land unit (acre) is ($2/hour * 1.68 hours/acre) = $3.36.

If the impact on labor use resulting from a policy is of major concern, it might be appropriate to include reservation wages for all relevant labor periods. Shadow prices of other limiting resources such as land will decline as more restrictions are placed on the model. However, realistic estimates of shadow prices are forthcoming only if the model approximates reality in terms of specifying labor and other critical timing restrictions such as available field work days. In the EX10 tableau, labor constraints are included for April - June (LABAJ) as well as October - December (LABOD). Activities are added to include the reservation wage of $2 per hour for family labor (FMAJLAB) and unrestricted hired labor (HRAJLAB) for $4.50 per hour.

	A	B	C	D	E	F	G	H	I
1	EX10	MAXIMIZE B		CORN133	SORGHUM	FMODLAB	HRODLAB	FMAJLAB	HRAJLAB
2	C			58.76	45.48	-2	-4.5	-2	-.4.5
3	LAND	L	160	1	1				
4	LABAJ	L		.74	.62			-1	-.9
5	LABOD	L		1.68	.66	-1	-.9		
6	MXFMAJ	L	600					1	
7	MXFMOD	L	600			1			
8	INCOME	G		58.76	45.48		-4.5		-4.5

9. Family Consumption. Rows may be included in a model to reflect the family's consumption requirement for items produced on the farm. The EX11 tableau contains a row, SORCONS, which requires the production of 12 cwt of sorghum. (In addition, a sorghum purchase activity could be included to permit the model to purchase rather than produce the sorghum if it is more economical to do so.) The TSOR activity transfers sorghum from the sorghum balance (SORBL) row to the sorghum consumption row.

	A	B	C	D	E	F	G	H
1	EX11	MAXIMIZE B		CORN133	SORGHUM	SLCORN	SLSOR	TSOR
2	C			-281.24	-194.52	2	3	
3	LAND	L	160	1	1			
4	LABOD	L	600	1.68	.66			
5	CORNBL	L		-170		1		
6	SORBL	L			-80		1	1
7	SORCONS	G	12					1

10. Consumption Function. As modelled in EX11, sorghum consumption is assumed to be perfectly inelastic with respect to family income. In model EX 12, family sorghum consumption requirements increase by 0.001 cwt for each dollar increase in the objective function value. The consumption function is SORCONS (INCTR) = 12 + (0.001 * INCTR).

	A	B	C	D	E	F	G	H	I
1	EX12	MAXIMIZE B		CORN133	SORGHUM	SLCORN	SLSOR	TSOR	INCTR
2	C								1
3	LAND	L	160	1	1				
4	LABOD	L	600	1.68	.66				
5	CORNBL	L		-170		1			
6	SORBL	L			-80		1	1	
7	SORCONS	G	12					1	-.001
8	NETINC	E		-281.24	-194.52	2	3		-1

The net income row (NETINC) is used to count the income. The INCTR activity transfers income to the objective function. For each dollar transferred to the objective function, sorghum consumption requirements are increased by 0.001 cwt. The equation implied by the NETINC row is as follows:

INCTR = (2 * SLCORN) + (3 * SLSOR) - (281.24 * CORN133) - (194.52 * SORGHUM).

The net income to be optimized is equal to sales minus production expenses. Fixed production costs associated with fixed farm resources not included in rows of the model could be included in the B column for the NETINC row.

Sorghum production for consumption must be transferred from the sorghum balance row by the TSOR activity and be greater than or equal to 12 plus 0.001 times the value of the objective function. The equation implied by the SORCONS row is (1 * TSOR) ≥ (12) + (0.001 * INCTR).

Solution
OPTIMAL
Function Value: 9334.056

Returns	Name	Type	Level	SLSOR	LAND	CORNBL	SORBL	SORCONS	NETINC
				0	0	0	0	0	-1.0E+14
				Real	Slack	Slack	Slack	Slack	Slack
0	CORN133	real	159.7333	0.012497	0.999267	-0.00002	0.012460	0.012460	0.000012
0	SORGHUM	real	0.266675	-0.01249	0.000732	0.000024	-0.01246	-0.01246	-0.00001
0	SLCORN	real	27154.66	2.124648	169.8755	0.995763	2.118293	2.118293	0.002118
0	TSOR	real	21.3340	0.000165	0.058574	0.001993	0.003156	-0.99684	-0.00099
1	INCTR	real	9334.056	0.165476	58.57455	1.993687	3.156008	3.156008	-0.99684
0	LABOD	slack	331.4720	-0.01274	-1.67925	0.000025	-0.01270	-0.01270	-0.00001
Z				0.165476	58.57455	1.993687	3.156008	3.156008	-0.99684
Shadow Price				0.165476	58.57455	1.993687	3.156008	3.156008	1.0E+14

The solution tableau indicates that 0.267 acres of sorghum are required to produce the required amount of 21.334 cwt of sorghum. The value of the objective function declines to $9,334.056. The "cost" of the sorghum is the $3 per cwt market price plus the "opportunity cost" associated with not producing the more valuable crop (CORN133) on the 0.267 acres. The total cost of the sorghum is approximately {($3/cwt * 21.334 cwt) + ($13.28/acre * 0.267 acres)} = $67.55. The net value of the objective function then is ($9,401.60 - $67.55) = $9,334.05. Because $67.55/21.334 = $3.16, this is the "shadow price" of sorghum. The shadow price for sorghum consumption (SORCONS) in the EX12 final tableau is $3.156. If sorghum could be purchased for less than $3.16 per cwt, the farm family would be better off by purchasing sorghum and producing corn for sale.

11. The Two-Region Model. The example contained in tableau EX13 is of a multiple-region model. Two production regions and one consumption region are included. The objective function of the model is to minimize the cost of meeting the demands of 30,000 bushels of corn and 3,000 cwt of sorghum, either by production in regions 1 or 2 or by importing. Production in region 2 is constrained by 40 acres of land and 120 hours of October - December labor. The model includes four transportation activities. Transportation costs of moving corn and sorghum from the production regions to the consumption region are included as objective function values for the transfer activities. Input requirements, production, and costs vary between the two regions. The CIMPT and SIMPT activities reflect the cost of imported corn and sorghum, respectively.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	EX13	MINIMIZE B		CORN1	SOR1	CORN2	SOR2	C1TR	S1TR	C2TR	S2TR	CIMPT	SIMPT
2	C			281.24	194.52	256.27	195.62	0.05	0.09	0.12	0.22	5	9
3	LAND1	L	160	1	1								
4	LABOD1	L	600	1.68	0.66								
5	CORNBL1	L	0	-170				1					
6	SORBL1	L	0		-80				1				
7	LAND2	L	40			1	1						
8	LABOD2	L	120			2.11	0.76						
9	CORNBL2	L	0			-150				1			
10	SORBL2	L	0				-82				1		
11	CDEMAND	G	30000					1		1		1	
12	SDEMAND	G	3000						1		1		1

The optimal plan for the model is to produce 160 acres of CORN1 and 0 acres of SOR1 on LAND1, and 3.41 acres of CORN2 and 36.59 acres of SOR2 on LAND2. Corn demand is met by producing and transporting 27,200 bushels from region 1 and 512 bushels from region 2, and importing 2,288 bushels. All 3,000 cwt of sorghum is produced in region 2 and transported to the demand location. The total cost to meet the fixed demands is $66,550.78.

Solution
OPTIMAL
Function Value: 66550.78

Cost	Name	Type	Level	SOR1 194.52 Real	SIMPT 9 Real	LAND1 0 Slack	CORNBL1 0 Slack	SORBL1 0 Slack	LAND2 0 Slack	CORNBL2 0 Slack	SORBL2 0 Slack	CDEMAND 0 Slack	SDEMAND 0 Slack
281.24	CORN1	real	160	1		1							
256.27	CORN2	real	3.414634	-0.97560	-0.01219			0.0121295	1		0.012195		0.012195
195.62	SOR2	real	36.58536	0.975609	0.012195			-0.01219			-0.01219		-0.01219
0.05	C1TR	real	27200	170		170	1						
0.09	S1TR	real	0.00	-80				1					
0.12	C2TR	real	512.1951	-146.341	-1.82926			1.829268	150	1	1.829268		1.829268
0.22	S2TR	real	3000	80	1			-1					-1
5	CIMPT	real	2287.804	-23.6585	1.829268	-170	-1	-1.82926	-150	-1	-1.82926	-1	-1.82926
0	LABOD1	slack	531.2	-1.02		-1.68							
0	LABOD2	slack	84.99024	1.317073	0.016463			-0.01646	-2.11		-0.01646		-0.01646
Z				105.1156	8.407195	-560.26	-4.95	-8.31719	-475.73	-4.88	-8.18719	-5	-8.40719
Reduced Cost				-89.4043	-0.59280	-560.26	-4.95	-8.31719	-475.73	-4.88	-8.18719	-5	-8.40719

The shadow price of LAND1 is $560.26 per acre. If an additional acre of LAND1 were available, corn imports could be reduced by 170 bushels. Corn could be produced and transported to the demand location. The cost of importing 170 bushels is $5 * 170 = $850. The cost of producing ($281.24) and transporting 170 bushels ($0.05 * 170 = $8.50) is $289.74. Thus, if an additional acre of LAND1 were available, the cost of meeting the fixed demands could be reduced by $850 - $289.74 = $560.26. An acre of LAND2 is less valuable with a shadow price of $475.73.

Comprehensive Farm Models

Policy models may be constructed to evaluate any number of crop and livestock production alternatives produced with various levels of input combinations which represent various levels of technology. It is essential that estimates regarding factor requirements and production are consistent across activities included in a model. If resource requirements or production are overstated for one crop relative to competing crops, all results may be distorted. Policy recommendations based upon distorted results may not contribute to the welfare of consumers and producers.

Available land, labor, and capital resources can be differentiated by time, quality, and uses. All relevant constraints and activities should be included. However, prior planning is essential to determine which policy issues are to be addressed. Knowledge of which policy issues and hypotheses are to be evaluated can be used to focus model construction in an appropriate manner.

In some instances it may be necessary to include hundreds or even thousands of activities and constraints in a mathematical programming model designed to evaluate economic policies. However, the mathematics of larger models is no more sophisticated than that required for the greatly simplified examples included in this chapter.

ACTIVITIES

1. Complete the tutorial for the MUSAH (or some other) linear programming system.
2. Thirteen example problems are included in the chapter. Solve each and interpret the solutions.
3. Use budgets from Chapter 8 (or from other sources) to build a farm planning model. Include marketing activities, a reservation wage, and a consumption function.
4. Determine what information would be required to evaluate the comparative advantage status of a country with a mathematical programming model.

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10. AGRICULTURAL PROJECT INVESTMENT ANALYSIS

by Dean F. Schreiner

Project analysis and the comparing of present value of incremental net benefits of alternative development projects is the most widely used analytical tool in development planning for the Third World. Methods of project analysis in comparing alternative projects for achieving policy objectives have advanced significantly during the last two decades. Policy objectives no longer are limited to choosing development projects which maximize economic growth but objectives may also include income distribution and the consumption benefits of projects. Although project analysis is still a partial equilibrium solution, holding many other things constant, the methods of analysis have been extended to include indirect or downstream effects, shadow pricing of resource use, and determining foreign exchange costs. International development agencies have formulated extensive procedures and training programs for project analysis. The advent of the microcomputer has now put the use of these procedures and analytical methodologies at the disposal of field personnel in their day to day operations of gathering and managing project data, designing project alternatives, and evaluating projects for achieving stated policy objectives.

OBJECTIVES

The objectives of this chapter are to:

1. Briefly relate project analysis to economic development planning.
2. Review methods for putting development projects on a comparative basis.
3. Relate project analysis to methods for achieving policy objectives.
4. Provide a set of activities for using the microcomputer in evaluation of agricultural projects employing various methods of analysis. Several of these activities come directly from examples and data given in J. Price

Gittinger (1982). The activities relate analytical procedures to objectives of development planning.

KEY POINTS

Agricultural project analysis is a methodology highly useful to planners in Third World countries. The methodology imputes a return to all resources used in a development project. When all resources but capital are imputed their opportunity cost, investment criteria are used to select projects which meet policy objectives of maximizing income growth and social well-being.

The methodology can be reduced to a few key steps that can be applied by practitioners in the field. These steps are categorized by the following:

Cash Flow - The contribution of the project must be specified in terms of what would be the state of the enterprise or economy with and without the project. Cash flow is a measurement of the contribution of the project.

Discounting - Projections of cash flow must be compared for alternative development projects. Choice of the discount rate and application of the discount rate to the projections of cash flow allow comparison in present worth of alternative projects.

Project Selection - Application of investment criteria such as net present worth, benefit-cost ratio, and internal rate of return allow selection of alternative projects which maximizes returns to capital.

Project analysis is most useful in the areas of investment planning--the allocating of limited funds to competing uses. However, project analysis increasingly is used for policy objectives not only of income growth but also of improved distribution of income. Therefore, project investment is viewed from several perspectives requiring the following analyses:

Financial Analysis - This analysis views projects from the perspective of those contributing equity capital to the project. Market prices and values are used in measuring project inputs and outputs. At issue is whether investors will earn more from the project than they invested in it.

Economic Analysis - Efficiency of resource use is the perspective of this analysis. Project inputs and outputs are valued according to the opportunity costs for the whole of society with no institutional constraints and no ownership rights. At issue is if society will benefit more from the project than they paid in cost for it.

Social Analysis - Economic analysis is combined with information on the distribution of project benefits and costs from the financial analysis to form the basis of social analysis. However, social analysis accepts policy makers weights attached to who receives the benefits (increased income and consumption) from the project.

CONCEPTS AND APPLICATIONS

Project Analysis in Economic Development

Leading authorities of the World Bank have stated that the project approach--combining sound investments with project-related policy and institutional reforms--has been the Bank's most important single contribution to the development process around the world (Baum and Tolbert, p. 9). How do projects contribute to the development process? The beginning point is always what would society be like "without" the project and what would it be like "with" the project. The difference measures the contribution of the project.

Two major issues arise in measuring the contribution of the project. The first is knowing what to measure in assessing social well-being or how society is better off with the project than without the project. The second issue relates to valuing what goes into the project by means of resources and what comes out from the project by means of products. And the two major issues are related because we frequently refer to the "benefits" of the project (i.e. society's valuing of what comes out from the project) and the "costs" of the project (i.e. society's valuing of what goes into the project). A rational society would thus choose only those projects whose benefits are greater than the costs.

The first issue in knowing what to measure depends on the collective preferences of society; on this subject the "jury is still out." What is generally assumed for societies is that more income is preferred to less, a more equal income distribution is preferred to a less equal distribution, social stability is preferred to social instability, and more personal freedom is preferred to less personal freedom once a basic set of individual rights are guaranteed. The appropriate combination of these and other preferences for a society determines the choice of policy objectives. Project analysis thus far has emphasized the policy objectives of income growth and project benefit (income) distribution. Furthermore, income growth is generally based upon the caveat of efficiency of resource use under the freedom of individual market choice.

The second issue in measuring the contribution of a project is valuing the inputs and outputs of the project. Most resources going into a project have several potential uses. Thus committing a resource to a specific project necessarily implies forgoing its use in other projects. The *opportunity cost* of a resource used in a particular project is defined as the value it could command if it were used for the best available alternative (Baum and Tolbert, p. 425). Similarly, the output resulting from a project may lead to higher consumption or exports and thus its value is determined by the *willingness-to-pay* of consumers (both domestic and international) for project output under freedom of market choice.

The basic principle needed to make the most efficient use of an economy's resources is that the price of any product (the willingness-to-pay by consumers) should be equal to the marginal cost (the opportunity cost or foregone output from resource use) of producing the last unit sold. A free competitive market would achieve the desired result of bringing prices into line with marginal costs, thereby allocating resources efficiently. The existing price system of a country provides the initial basis for valuing a project's inputs

and outputs. Whether the existing price system is allocating resources efficiently to the project will depend on whether those prices reflect opportunity cost of resource use and the willingness-to-pay by consumers for project output. Even when there is a high degree of competition, the results produced by the price system are efficient only for a given distribution of income. Obtaining other policy objectives, such as social stability or income distribution, require further criteria for evaluating project benefits and costs.

A major difficulty to evaluating investments in many Third World development projects is the lack of an appropriate market test for determining project benefits. Using the criterion of willingness-to-pay will undoubtedly lead to under-investing in such projects as education, health care, family planning, agricultural and other scientific research, and local and national security. In some cases the market test would reserve benefits only for the affluent, such as with education. In other cases, the "free-rider" problem results because providing the service to one automatically provides it to all, such as with forms of local and national security. And in still other cases, the costs of administering the collection of fees is greater than the costs of providing the service, such as with tolls for road use.

Competitive markets are not always possible or even desirable. Many infrastructure projects such as community water services, port facilities, telephone, and electricity distribution are best handled as a public utility or a not-for-profit firm because the economic structure of such projects is a natural monopoly. If left to the market, consumers may pay more than the marginal cost of providing the good or service either through duplication of investments in a decreasing cost industry or through controlling output and increasing price.

In the absence of an appropriate market test, project appraisers have become ingenious in ways to measure directly and indirectly development project benefits and costs and thus assist policy makers in making decisions on investments. This chapter does not present all of these alternative ways and methodologies but it presents the more common techniques of project analysis and then leaves it to the reader to seek out applications of specific interest.

Putting Development Projects on a Comparative Basis

The list of potential development projects is long. This is only natural because development projects arise out of the insatiable desires expressed by individuals and governments. The problem is to assemble information on development projects so that valid comparisons can be made among projects on the use of scarce resources in attaining the insatiable desires. The procedures for accomplishing this are to (1) estimate the *cash flow* of the project, (2) *discount* all future benefits and costs of the project to present values, and (3) apply a *criterion* for project selection. These procedures are discussed in order.

1. Cash Flow Analysis. Cash flow analysis measures what the economic results would be with and without the project. These economic results will differ between the perspective of a small private landholder (smallholder) and the perspective of the entire

society. The various viewpoints of project analysis are discussed in the next section but here we note that *cash flow* is not always limited to cash transactions. In fact, when measuring the contribution of a development project using the resources of an agricultural smallholder, many transactions are not in cash but are measured through increases in home-consumed production and foregone earnings from using the smallholder's resources in this project instead of an alternative project (including using his labor for off-farm employment). Similarly, when measuring *cash flow* for society, resource use of the project may be valued not by cash transactions (market prices) but by opportunity costs (efficiency prices) of those resources.[1] Cash flow also abstracts from inflation or an increase in the general level of prices. That is, output and input prices are assumed constant and take on the general price level for the base year (year one value). If relative (real) prices for project inputs and outputs are expected to change over the life of the project, these relative changes should be incorporated in the cash flow of the project. But such changes in relative prices should be based on empirical analysis of long-term trends in market conditions for specific project inputs and outputs.

To illustrate cash flow, data for a smallholder cashewnut project in Karnataka, India are adapted from Gittinger and presented in Table 10.1. Value of production, investment, and operating expenditures are shown for a 1-hectare planting model. Production is expected to begin in year three from plantings in year one with yields increasing as trees mature until maximum yield is attained beginning in year ten. Market price (rupee) is based on projected f.o.b. price adjusted to give the export parity value at the farm gate.

Land rent is assumed to represent the *without project* or opportunity cost of the land unit. In this case smallholders are presumed to be using land with no other economic use and thus the opportunity cost is zero. Labor and materials cost (plant materials, fertilizer, etc.) are the only project costs. Investment costs are arbitrarily considered to be all labor and materials costs for the first three years or until production starts. Operating expenditures are assumed to start in year four and continue throughout the project period. The original project period and data given in Gittinger extend through 40 years. However, because one of the objectives of this analysis is to provide a set of activities for introducing the use of microcomputers in analysis, certain liberties were taken in modifying and reducing volume of data and thus time spent on data entry.

A unit activity budget is presented in Table 10.2 to show project inflow, outflow, incremental net benefit before financing, financing, incremental net benefit after financing,

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[1]Thus the use of the term *cash flow* in project analysis is not in the conventional farm management useage which describes actual cash flow requirements for the enterprise or farm. In that case, the difference between gross revenue and cash flow is generally the net returns to fixed resources, management, and farm family labor. In project analysis, cash flow is the difference between gross output of the project and total economic (resource) cost of the project.

Table 10.1. Value of Production, Investment, and Operating Expenditure, 1-Hectare Planting Model, Cashewnut Project, Small Landholder Component, Karnataka, India.

Item	Project Year										
	1	2	3	4	5	6	7	8	9	10	11-20
	Value of Production										
Yield (kg/ha)	0	0	0	50	150	400	600	700	800	900	900
Price (Rs/kg)	-	-	-	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1
Gross Value of Production (Rs)	0	0	0	255	765	2,040	3,060	3,570	4,080	4,590	4,590
	Investment and Operating Expenditure										
Land Rent (opportunity cost)	0	0	0	0	0	0	0	0	0	0	0
Labor											
Work Days	110	56	62	40	49	53	59	63	67	71	71
Wage (Rs/day)	8	8	8	8	8	8	8	8	8	8	8
Labor Cost (Rs)	880	448	496	320	392	424	472	504	536	568	568
Materials Cost (Rs)	260	360	490	570	595	595	595	595	595	595	595
Investment Cost (Rs)[a]	1,140	808	986	0	0	0	0	0	0	0	0
Total Operating Expenditures[b]	0	0	0	890	987	1,019	1,067	1,099	1,131	1,163	1,163

Source: Adapted from Gittinger (pp. 144-145).

[a] Investment cost equal to labor and materials cost for years 1-3.
[b] Total operating expenditures equal labor and materials cost for years 4-20.

Table 10.2. Unit Activity Budget, 1-Hectare Planting, India Cashewnut Project (Rs).

Item	Project Year 1	2	3	4	5	6	7	8	9	10	11-20
Inflow											
Gross value of production	0	0	0	255	765	2,040	3,060	3,570	4,080	4,590	4,590
Subsidy	300	300	300	0	0	0	0	0	0	0	0
Total	300	300	300	255	765	2,040	3,060	3,570	4,080	4,590	4,590
Outflow											
Investment	1,140	808	986	0	0	0	0	0	0	0	0
Operat. Expend.	0	0	0	890	987	1,019	1,067	1,099	1,131	1,163	1,163
Total	1,140	808	986	890	987	1,019	1,067	1,099	1,131	1,163	1,163
Incremental net benefit before financing	(840)	(508)	(686)	(635)	(222)	1,021	1,993	2,471	2,949	3,427	3,427
Financing											
Loan receipts[a]	620	396	562	555	124	0	0	0	0	0	0
Debt service[b]	0	0	0	0	0	927	927	927	927	927	0
Net financing	620	396	562	555	124	(927)	(927)	(927)	(927)	(927)	0
Incremental net benefit after financing	(220)	(112)	(124)	(80)	(98)	94	1,066	1,544	2,022	2,500	3,427
Farm income[c]	660	336	372	240	294	518	1,538	2,048	2,558	3,068	3,995

Source: Adapted from Gittinger (pp. 148-149).

a. Borrowing is equal to 75 percent of labor costs, plus materials cost, minus inflow including subsidy.

b Debt service is on loan receipts compounded at 12 percent interest.

c Farm income is defined as income available to the farm family and equal to labor cost plus incremental net benefit after financing.

and farm income. Items in the project data (Table 10.1) and unit activity budget (Table 10.2) are identified such that the project analyst may easily use a microcomputer spreadsheet to analyze variations in yield, price of cashewnuts, opportunity cost of land, wage rates, materials cost, subsidy, and financing. For instance, financing is assumed to cover 75 percent of labor and materials costs during the period of a negative incremental net benefit before financing and at an interest cost of 12 percent. Because income available to the farm family depends on financing arrangements, it may be important to consider alternative financial plans to assure sufficient resources for family living. Similarly, the effect of a higher or lower subsidy on financing costs, farm income, project profitability, and, eventually, government cost can be analyzed.

Cash flow analysis of the India Cashewnut Project is summarized in Table 10.3 in terms of incremental benefit, incremental cost, and incremental net benefit or cash flow. The incremental net benefit or cash flow in this instance is a measurement of the returns of the Cashewnut Project to the smallholder. Incremental costs are an accounting of the costs to the smallholder of all resources put into the project. This includes the cost of labor, either wage payments made by the small landholder to day workers or the opportunity cost of the landholder himself if he supplies the labor, and the cost of financing investment. The incremental (gross) benefit is the value of cashewnuts produced and the subsidy received by the smallholder. The incremental net benefit is the return to the capital resources (equity) put into the Cashewnut Project by the smallholder. It is this return (cash flow) that the smallholder must compare with the alternatives available to him. These alternatives may include expansion of some other enterprise such as use of an improved technology in production of domestic food commodities.

2. ***Discounting.*** The cash flow of the Cashewnut Project is typical of most development projects in that there is a period of investment (negative cash flow) followed by a period of positive cash flow. But how do we compare the Cashewnut Project with the alternative development project? Seldom do development projects have the same annual rate of cash flow and thus provide a basis for direct comparison of the time streams of net incremental benefits. Furthermore, how do we know the smallholder's preference for less consumption in years one through five so that there is greater consumption in years six through 20?

To answer these questions we must be able to reduce the time stream of future net incremental benefits to a present value or to determine if the increase in future consumption compensates for a decrease in present consumption. This is the process of discounting. Once an appropriate discount rate is determined, the cash flow of the Cashewnut Project can be reduced to a present value and compared with the present value of the cash flow for an alternative development project.

Determining the appropriate discount rate is important. However, not knowing the exact discount rate should not deter the practical analyst from evaluating development projects. Frequently, choices can be made by comparing projects assuming a discount rate close to what is felt to be realistic and then varying the rate up and down, and again

Table 10.3. Cash Flow Analysis of the India Cashewnut Project (Rs).

Year	Incremental Benefit			Incremental Cost				Incremental Net Benefit (Cash Flow)
	Gross Value of Production	Subsidy	Total	Investment	Operat. Expend.	Net Financing	Total	
1	0	300	300	1,140	0	(620)	520	(220)
2	0	300	300	808	0	(396)	412	(112)
3	0	300	300	986	0	(562)	424	(124)
4	255	0	255	0	890	(555)	335	(80)
5	765	0	765	0	987	(124)	863	(98)
6	2,040	0	2,040	0	1,019	927	1,946	94
7	3,060	0	3,060	0	1,067	927	1,994	1,066
8	3,570	0	3,570	0	1,099	927	2,026	1,544
9	4,080	0	4,080	0	1,131	927	2,058	2,022
10	4,590	0	4,590	0	1,163	927	2,090	2,500
11	4,590	0	4,590	0	1,163	0	1,163	3,427
12	4,590	0	4,590	0	1,163	0	1,163	3,427
13	4,590	0	4,590	0	1,163	0	1,163	3,427
14	4,590	0	4,590	0	1,163	0	1,163	3,427
15	4,590	0	4,590	0	1,163	0	1,163	3,427
16	4,590	0	4,590	0	1,163	0	1,163	3,427
17	4,590	0	4,590	0	1,163	0	1,163	3,427
18	4,590	0	4,590	0	1,163	0	1,163	3,427
19	4,590	0	4,590	0	1,163	0	1,163	3,427
20	4,590	0	4,590	0	1,163	0	1,163	3,427
Total	64,260	900	65,160	2,934	19,986	2,378	24,298	40,862

comparing the projects. This sensitivity analysis on the discount rate can be accomplished quite rapidly using a microcomputer spreadsheet.

The smallholder bases his decision whether to invest in the Cashewnut Project on his *marginal time preference rate* (MTPR) and on the *alternative* to the project. If the smallholder is indifferent between the prospect of Rs 1 extra consumption in year one and the prospect of Rs 1.10 extra consumption in the following year he is said to have a marginal time preference rate of 0.10 or 10 percent per year (Sugden and Williams). *Marginal* is used because a MTPR measures an individual's preferences between small increments of consumption in different periods. An individual's MTPR likely varies according to the relative size of his claims on present and future consumption. As an individual gives up more and more present consumption, the more dear to him becomes his remaining claims to present consumption. Thus it takes more future consumption gain to compensate for this current consumption loss.

The problem is how to discover the smallholder's MTPR. This is where the market interest rate becomes important. If the smallholder is willing to borrow at a market interest rate of i percent to invest in the project, his MTPR is as great or greater than the market rate of interest. He is willing to give up claims on future consumption equal to the investment costs of the Cashewnut Project plus the interest costs so he doesn't have to finance the project out of his current consumption.

Market interest rates, however, are usually expressed in *nominal* or current price terms. Thus the MTPR inferred from the market interest rate is in nominal terms but the cash flow is in constant prices or *real* terms. If the smallholder expects the rate of inflation in future years to be the same as the current (or recent past) rate of inflation, then his real MTPR can be inferred from the deflated market interest rate. The real rate is the market or nominal rate less the inflation rate. This becomes important as we select discount rates for developing project criteria in the next section.

Farmers usually have a MTPR greater than a government subsidized interest rate. Hence, the subsidized interest rate should not be used as the individual's discount rate. For one thing, the farmer generally is not able to borrow all that he would like at the subsidized rate. So the interest rate is no longer the constraint. The constraint is the amount the government is willing to loan. The same may be true with a market rate of interest if the lending institution requires certain amounts of collateral from the farmer in exchange for the loan amounts. The starting point in determining the individual's MTPR is the marginal cost of capital (interest rate) for the marginal amount of capital the individual wishes to borrow. If the individual is willing to borrow at this interest rate in return for a future repayment of capital plus interest cost, his MTPR is said to be greater than the interest rate.

But only the question on the minimum possible MTPR has been answered.[2] What about the earning capacity of the Cashewnut Project and the earning capacity of alternative projects? To determine these we must develop further criteria for project selection.

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[2]The issue of a social time preference rate (STPR) is taken up in the section on viewpoints of project analysis.

3. Criteria for Project Selection. The smallholder is considering investment in the Cashewnut Project. The minimum earning capacity of the project must be at least 12 percent since this is the marginal cost of capital to the smallholder. Even after borrowing Rs 2,257 at a 12 percent interest rate, the landholder must still forego some consumption in years one through five to have increased consumption in years six through 20. One discount rate that can be applied to the incremental net benefit or cash flow of the project is 12 percent or the marginal cost of capital to the smallholder. If the present worth of the incremental net benefit is positive, the smallholder may continue to consider the Cashewnut Project. But does this project represent the best return possible or is there an alternative development project he should consider? To answer these questions we must first compute the profitability of the Cashewnut Project and then compare this expected return with any other alternative available to the smallholder.

Several measures of project profitability may be considered including the net present worth (NPW), benefit-cost ratio (B/C), and internal rate of return (IRR). Each measure uses a discounting procedure and provides a valid means for computing profitability of the project. The NPW criterion is simply the present worth of the incremental net benefit or incremental cash flow stream:

$$NPW = \frac{B_1 - C_1}{(1+i)^1} + \frac{B_2 - C_2}{(1+i)^2} + \dots + \frac{B_n - C_n}{(1+i)^n} = \sum_{t=1}^{n} \frac{B_t - C_t}{(1+i)^t} \qquad (10.1)$$

where

B_t = real (constant prices) incremental benefit in each year
C_t = real incremental cost in each year
t = 1, 2, ..., n
n = number of years in the project
i = real interest (discount) rate

Computing the net present worth requires a discount rate. In Table 10.4, the net present worth of the Cashewnut Project to the smallholder at a 12 percent discount rate is Rs 8,443. This means that the project will compensate the smallholder 12 percent annually for deferred consumption and return an additional Rs 8,443 in present value. The decision to go ahead with the project is up to him. The smallholder would have to have a very high real MTPR not to consider this project. In fact, one might consider the government subsidy to establish the cashewnut trees very generous to compensate for current consumption the smallholder must forego in early years.

Is this the best alternative available to the smallholder or is another project more profitable? If the smallholder has an alternative, the NPW of the Cashewnut Project can be compared with the alternative. If the Cashewnut Project exhausts all of the capital resources available to the smallholder (including the amount he can borrow) and if the alternative to the project also exhausts all resources, a direct comparison of the two NPWs can be made. The project with the highest NPW is the one to recommend to the

Table 10.4. Financial Analysis of the India Cashewnut Project

Year	Incremental Benefit (Rs)	Incremental Cost (Rs)	Incremental Net Benefit (Cash Flow) (Rs)	Discount Factor 12%	Present Worth of Cash Flow 12% (Rs)
1	300	520	(220)	0.893	(196)
2	300	412	(112)	0.797	(89)
3	300	424	(124)	0.712	(88)
4	255	335	(80)	0.636	(51)
5	765	863	(98)	0.567	(56)
6	2,040	1,946	94	0.507	48
7	3,060	1,994	1,066	0.452	482
8	3,570	2,026	1,544	0.404	624
9	4,080	2,058	2,022	0.361	730
10	4,590	2,090	2,500	0.322	805
11-20	4,590	1,163	3,427	1.819	6,234
Total	65,160	24,298	40,862		8,443

Net present worth at 12 percent = Rs 8,443

B/C ratio at 12 percent = Rs 16,476/Rs 8,034 = 2.1

IRR > 50%

smallholder. Seldom can such direct comparisons be made, however. The smallholder may have the alternative of improving the technology on another crop (i.e. supplemental irrigation) which will utilize only part of his limited capital resources but leave some for an additional project such as a reduced version of the Cashewnut Project. Then it becomes a case of combining the appropriate amounts of each alternative which gives the greatest total net present worth.

The B/C ratio is computed as the following:

$$B/C = \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \Big/ \sum_{t=1}^{n} \frac{C_t}{(1+i)^t} \tag{10.2}$$

where all variables are defined as before. The computed ratio is a comparison of the discounted project benefits with the discounted project costs. If the ratio is greater than one the project is returning more benefits than it costs. The computed B/C ratio for the Cashewnut Project is a very favorable 2.1. Assuming the discount (interest) rate used is a true measure of the small landholder's MTPR, he again would be very happy to go forward with the project.

Some would like to use B/C ratios to rank project alternatives when there is a limited capital budget. In the case of the smallholder this may be appropriate because we would probably use the same procedures in computing the incremental benefit and cost streams for all alternatives. However, one can readily see from Table 10.3 that the size of the B/C ratio may be influenced by different accounting procedures. Suppose operating expenditures were subtracted from gross value of production so that the incremental benefit stream were reduced. This also would lower the incremental cost stream. But because the operating costs come later relative to the total costs (investments come early), they have a greater impact on discounted incremental costs than they do on discounted incremental benefits. In the case of the Cashewnut Project the B/C ratio is increased to 4.3. An additional change in accounting procedure to influence the size of the B/C ratio would be to move loan receipts from incremental cost to incremental benefit, thus increasing marginally incremental benefit. By significantly increasing total incremental costs which come early in the project, this change would reduce the B/C ratio to 1.9.

Another inappropriate way to use B/C ratios is ranking of mutually exclusive projects. As we will see in a later section, the appropriate criteria to use in such cases is NPW or measures that utilize the cash flow difference.

The IRR is the discount rate i that makes the present worth of the incremental net benefit (cash flow) equal to zero

$$\sum_{t=1}^{n} \frac{B_t - C_t}{(1+i)^t} = 0 . \tag{10.3}$$

It can also be expressed as the maximum real interest rate a project can pay if it is to recover its investment and operating costs and still break even. It is the earning capacity of the

project or the yield on investment in the project. This is the measure of project profitability most frequently used by the World Bank.

The IRR of the Cashewnut Project is greater than 50 percent, indicating a very profitable project to the smallholder. The rational smallholder would need to possess a very high real MTPR to reject the project. Perhaps because of the high expected yield on investment, the government was willing to encourage cashewnut plantings by subsidizing the first three years of the project.

Calculating the IRR is straightforward with a microcomputer spreadsheet requiring only one command once the income stream of the incremental net benefit is computed. When using discounting tables an iteractive procedure may be used by selecting discount rates which bracket the IRR. The lower discount rate gives a positive NPW of the cash flow whereas the higher discount rate gives a negative NPW of the same cash flow. The true IRR lies between the two discount rates and can be found by applying interpolation procedures to the two rates (see Gittinger, pp. 332-339).

Viewpoints of Project Analysis

The group supplying the equity capital is especially interested in the results from analyzing an agricultural project investment. If the project fails, the group looses its equity. If the project succeeds, the group is entitled to the net incremental benefits and may distribute those benefits to members of the group as it wishes. Traditionally, projects have been viewed from two perspectives: (1) the perspective of the individuals having private ownership rights to the equity capital, and (2) the perspective of society as a whole with no ownership rights on project capital but with an interest in maximizing total income to society. The first perspective is generally referred to as financial analysis and the second as economic analysis. Results may differ from applying the selection criteria to the same project from the two different perspectives.

A third perspective is receiving increased interest for Third World countries whose distribution of resource ownership is highly skewed. Project selection criteria in this third perspective utilizes income or consumption weights and applies those weights to project generated income. This perspective recognizes that, collectively, society may utilize policy to influence project selection and the distribution of net incremental benefits.

1. Financial Analysis. The Cashewnut Project as presented in Tables 10.3 and 10.4 represents financial analysis. The individual smallholders are providing the equity capital and receiving the incremental net benefits. Project inputs are valued at market prices or opportunity costs to the smallholder. As an example, materials cost (Table 10.1) are the actual costs paid by the smallholder for nursery stock, chemicals, and other inputs. Labor cost is the number of work days times wage rate and represents either the cost to the smallholder if he hires the labor or his own opportunity cost of spending this time in cashewnut production versus spending the time in an alternative such as leisure or off-farm employment.

Financial analysis is important because it shows how a policy will affect the smallholder. The policy may influence the magnitude of the cash flow on which the owners of equity capital base their decision whether to invest or not to invest. In the case of the Cashewnut Project, policy was used to grant a subsidy for establishment of the cashewnut trees. This reduced the deficit in cash flow for the first three years and increased the private IRR. Alternatively, policy could have been used to decrease the incremental net benefit by taxing cashewnut production.

2. ***Economic Analysis.*** Society is concerned about efficient resource use--extracting the greatest total product (income) from available resources. Economic analysis ignores all institutional constraints and visualizes an economy with no ownership rights to resources, no underutilization of resources, no constraints to free trade of resources and products, no interest in restricting output and increasing price because of a natural monopoly, and no government interference except to ensure a competitive economy with freedom of choice and a complete internalizing of all external project costs and benefits. A truly fictitious economy! No project has been evaluated under these conditions.

The usual approach to economic analysis of agricultural projects is to start from where the economy is currently and to conceptualize what the conditions would be if the major restrictions to efficient resource use were removed. What would farmers receive if all taxes, subsidies, and price supports were removed? What would import prices be if all import controls were removed? If there are underutilized resources, what are the opportunity costs of employing additional units of those unused resources? What would be the incremental net benefit if external costs and benefits of the project were included in the analysis?

Conceptualization is not enough, however. Results of the removal of these restrictions must be estimated and interpreted in terms of changes in project input and output prices and changes in the incremental net benefit. Project analysis literature is becoming rich in methodologies and applications for addressing the issues of imperfect markets (Little and Mirrlees; Squire and van der Tak; Pearce and Nash). However, to reduce the level of potential frustration in removing these restrictions and completing an economic analysis, one must accept that project analysis at most is a partial equilibrium analysis. To take it any further, one must analyze the project within a general equilibrium model to evaluate the ultimate full effects of removing such institutional constraints as price supports and price subsidies not only on this project but all other projects and activities in the economy (see Hazell and Norton, Chapters 9 and 12).

An economic analysis of the Cashewnut Project is shown in Table 10.5. Incremental benefit was reduced from the financial analysis by the amount of the subsidy. The rationale for this is that the subsidy is merely a transfer from one group in society to another group. The assumption is that removing the subsidy does not affect resource use or resource prices. Incremental cost was changed from the financial analysis to exclude the net financing results. The rationale for this is that in economic analysis there is no concern

Table 10.5. Economic Analysis of the India Cashewnut Project

Year	Incremental Benefit (Rs)	Incremental Cost (Rs)	Incremental Net Benefit (Cash Flow) (Rs)	Discount Factor 12%	Present Worth of Cash Flow 12% (Rs)
1	0	1,140	(1,140)	0.893	(1,018)
2	0	808	(808)	0.797	(644)
3	0	986	(986)	0.712	(702)
4	255	890	(635)	0.636	(404)
5	765	987	(222)	0.567	(126)
6	2,040	1,019	1,021	0.507	518
7	3,060	1,067	1,993	0.452	901
8	3,570	1,099	2,471	0.404	998
9	4,080	1,131	2,949	0.361	1,065
10	4,590	1,163	3,427	0.322	1,103
11-20	4,590	1,163	3,427	1.819	6,234
Total	64,260	21,920	42,340		7,925

Net present worth at 12 percent = Rs 7,925

B/C ratio at 12 percent = Rs 15,755/Rs 7,830 = 2.0

IRR = 30.4 percent

about resource ownership and hence no interest in who contributes to the project and who benefits from the project. We are concerned about costs and benefits to society irrespective of who gains and loses. Returns from all capital used in the projecrt are now in the incremental net benefit (cash flow).

Economic analysis of the project now shows a net present worth at 12 percent discount rate of Rs 7,925 and a B/C ratio of 2.0, slightly less than the financial analysis. The calculated IRR is 30.4 percent.

The discount rate used for the economic analysis of the Cashewnut Project in India is 12 percent. Is this the appropriate rate? Does it reflect the economic opportunity cost of capital? Are there sufficient funds available to finance all investment projects with a 12 percent or greater rate of return? Is this the optimum amount of funds that society should invest? If the answer is yes to all of these questions, then we may conclude that for economic efficiency the Cashewnut Project should be encouraged for implementation because its return exceeds the discount rate.

The discount rate used in economic analysis should reflect the *social opportunity cost of capital (SOC)* and the *social time preference rate (STPR)* (Pearce and Nash). The SOC measures the rate of return that could be achieved by society from sacrificing a unit of consumption now to invest for the future. The rate of return would again be under the conditions of our fictitious economy--no institutional constraints and competitive markets. However, the starting point would be the estimated return to an additional dollar of aggregate investment in the economy as it exists. This return would then need to be adjusted for market imperfections and institutional constraints.

Knowing the rate of return to a particular level of aggregate investment is not sufficient. Because we generally expect diminishing marginal productivity of capital, the marginal rate of return will depend upon the aggregate level of investment. The amount of foregone consumption for a given period of time will determine the amount available for investment and thus the marginal rate of return on investment in alternatives. This rate represents the SOC of capital. It is this rate to which individual project returns are compared. If the project does not show a comparable rate of return, it is rejected. That is, when the incremental net benefit is discounted at the SOC of capital, it must show a zero or positive NPW or a B/C ratio equal to or greater than one. Similarly, the computed IRR must be equal to or greater than this SOC of capital.

The analysis above assumed an amount of foregone consumption by society, an amount available for investment. The amount of foregone consumption is determined by the social time preference rate (STPR) or the rate at which society is prepared to substitute future for present consumption. As we saw earlier, a high MTPR indicates a premium on consuming now and unwillingness to save and invest to obtain future output and consumption. The willingness of individuals to borrow or lend at the market interest rate is an indication that their individual marginal time preference rates (MTPR) are above the market interest rate. The STPR probably is lower than the MTPR based on (1) externalities from looking across generations, (2) collective agreement on investments that spread their effects broadly across society, and (3) projects that are risky for individuals and require a high MRPR but may be relatively secure when averaged over many projects for society,

and hence require a low STPR. Because the preference rate is indeterminate, some would argue that the choice of discount rate should be left to policy makers. Perhaps this is the reason Gittinger suggests letting the National Planning Institute determine the social discount rate and each individual planning unit assume this is the SOC of capital. As a minimum this assures that development projects across all sectors will be evaluated at the same discount rate.

Returning to the economic analysis of the Cashewnut Project (Table 10.5), it is obvious that society would benefit from investment in the project if the social discount rate is 12 percent. The expected IRR of the project is more than double the social discount rate. Furthermore, if we consider labor to be undertutilized in the region so that the wage rate and labor cost as given in Table 10.1 are twice the opportunity cost, the economic feasibility of the project would be even more favorable.

In viewing the incremental net benefit of the economic analysis (Table 10.5), policy makers were doubtful that smallholders would be able to sustain five years of substantial decreases in current consumption to finance establishment of the cashewnut trees. They suspected the MTPR of the small landholders was at least 30 percent or greater and thus they might be indifferent to the Cashewnut Project. Therefore, it was a policy decision to combine a subsidy program along with a finance program to reduce the size of the negative cash flow during the first five years (Table 10.4) and thus increase the private IRR to the smallholders but not change the IRR to society.

*3. **Social Analysis.*** The discussion on viewpoints of project analysis stated that the group putting up the equity capital is the most concerned about analyzing an agricultural project investment. As we saw in financial analysis, those putting up the equity capital are easily identified and thus easily compensated for project equity. But who puts up the equity capital in the economic analysis? The answer is, *society*. Then isn't it logical for society to have a say in the distribution of income growth due to the project or due to the equity capital put into the project?

Most societies have institutional constraints including ownership rights to resources which determine the distribution of benefits. For a society with strong property rights somewhat evenly distributed this should not be viewed as a disadvantage because it forms the cornerstone to the free enterprise system and incentive to maximizing income growth. In a competitive system, resources are compensated for their marginal value product. The more sizable and productive the resources owned by given individuals and groups, the greater the returns (income) they receive. But some in society have few resources to bring to the market place and hence return with little income. The question to be asked is should the returns (income) received from a project by those with few resources be valued the same as the returns (income) received from a project by those with many resources? In social analysis, society has a choice.

Little and Mirrlees (pp. 235-236) state,

> It is widely accepted that (the social value of an extra rupee) should be smaller, the more consumption a man has in total: that an extra rupee is better given to a poor man than to a rich man. It

> may be claimed, in support of this view, that happiness is increased less (or misery reduced by less) with each successive rupee given to a man. Not everyone would accept the truth--or even the meaningfulness--of this claim. Perhaps one should simply say that it seems fair to give the poorer man some priority, since the rich man has already been able to enjoy a rupee equivalent to the poor man's extra one.

The problem is knowing by how much to value an extra unit of income (consumption) by income group and whether project analysis should even be used to bring about a socially improved income distribution. Other instruments of policy, such as progressive taxation and targeted food assistance, might achieve as much equality as desirable, and could achieve it more efficiently than by choice of investment project. These decisions, however, are best left to policy makers.

One method of devising income weights is to assume that all additional returns to a project are consumed and that comparisons can be made between the marginal utilities of consumption at different consumption levels (Squire and van der Tak). For instance, if marginal utility is expressed as Uc, a function that expresses diminishing marginal utility of consumption is the following

$$Uc = c^{-n} \tag{10.4}$$

where c is the level of consumption and n is a parameter of the utility function. Although this formula is only one of many that could be used to show diminishing marginal utility of consumption, it has the advantage that the higher the value of n, the higher is the rate of diminishing marginal utility. For a value of n=1 (2), marginal utility is two (four) times higher for a person with a given level of consumption than for a person with a consumption level twice as high. For most policy makers, n would probably center around 1 with values close to zero or 2 considered extreme. In fact, for n=0, the marginal utility of consumption is independent of the level of consumption and thus our previous economic analysis of projects would suffice.

One possible set of weights would be to compare the marginal utility of consumption for a group at their consumption (income) level, Uc_i, with the marginal utility of consumption at the overall society's level of consumption, $U\bar{c}$

$$d_i = \frac{Uc_i}{U\bar{c}} = \left(\frac{\bar{c}}{c}\right)^n \tag{10.5}$$

where d_i is the consumption distribution weight. In our example of the Cashewnut Project let us assume three income groups with income levels as given in Table 10.6. Suppose agricultural laborers on the project in the region realize incomes of Rs 4,500. The smallholder has an average income level of Rs 10,000. The national average income level is Rs 7,500. If we assume policy makers choose a value of n=1, then the income weights for marginal additions to income (consumption) would be 1.67 for agricultural labor, 0.75 for farm operators, and 1.0 for the national average.

Table 10.6. Income Weighting, India Cashewnut Project

Income Group	Income Level (Rs)	Income Weight d_i (n=1)
Agriculture Labor	4,500	1.67
Farm Operator	10,000	0.75
National Average	7,500	1.00

We can now do a social analysis of the Cashewnut Project. The net social benefits of the project are defined as the following (see Squire and van der Tak, for more detail)

$$S = E - \sum_i C_i (B_i - d_i) \tag{10.6}$$

where

S	=	net social benefits
E	=	net efficiency benefits (economic cash flow)
C_i	=	increase in income (consumption) for income group i measured in financial (market) terms
B_i	=	adjustment factor for estimating the real (economic) resource costs of consumption C_i and is assumed equal to 1 for the Cashewnut Project
d_i	=	income (consumption) weight for group i
$\sum_i C_i (B_i - d_i)$	=	net social cost of increased consumption

Notice two things about equation 10.6. First, net social benefit begins with the estimation of net economic benefit. Second, the financial analysis is necessary because we need to have estimates of the increase in net income accruing to the various income groups.

Using income weights from Table 10.6 and results of the financial and economic analyses from Tables 10.4 and 10.5, results for a social analysis are given in Table 10.7. Incremental net economic benefits (E) are reproduced from Table 10.5. Net social cost of increased consumption $[\sum_i C_i (B_i - d_i)]$ must be calculated for the two income groups.

Table 10.7. Social Analysis Using Income Weights by Income Group, India Cashewnut Project (Rs).

Year	Incremental Net Economic Benefit (Cash Flow) (E)	Agricultural Labor		Smallholder		Incremental Net Social Benefit (Cash Flow) (S)
		Increased Consumption	Net Social Cost	Increased Consumption	Net Social Cost	
		$[\sum_i C_i (B_i - d_i)]$				
1	(1,140)	0	0	(220)	(55.0)	(1,085.0)
2	(808)	0	0	(112)	(28.0)	(780.0)
3	(986)	0	0	(124)	(31.0)	(955.0)
4	(635)	0	0	(80)	(20.0)	(615.0)
5	(222)	0	0	(98)	(24.5)	(197.5)
6	1,021	0	0	94	23.5	997.5
7	1,993	0	0	1,066	265.5	1,727.5
8	2,471	0	0	1,544	386.0	2,085.0
9	2,949	0	0	2,022	505.5	2,443.5
10	3,427	0	0	2,500	625.0	2,802.0
11-20	3,427	0	0	3,427	856.8	2,570.2
Total	42,340	0	0	40,862	10,215.0	32,125.0

Net present worth at 12 percent = Rs 5,815

IRR = 28.1 percent

This means we must know (calculate) the increase in net income accruing to the income groups due to the project. Under the assumptions of the economic analysis, agricultural labor has no increase in net income due to the project. The assumption was that the opportunity cost for labor is equal to the wage rate which is Rs 8 per day. If labor were not used in the Cashewnut Project, it could be employed elsewhere at this wage rate. Hence, there is no net change in the social cost of increased consumption by agricultural labor.

The Cashewnut Project does increase net income for the smallholders and the change is equal to the financial cash flow. However, because farm operators are at a higher income level than the national average, policy makers are valuing their increased consumption at 75 percent. Because $B_i = 1.0$ the assumption is that increased consumption is at full economic cost. Thus there is a net social cost of increased consumption by smallholders of 25 percent [i.e. C_i (1.0 - 0.75)]. The increased consumption of smallholders of Rs 40,862 has a net social cost of Rs 10,215.

The undiscounted incremental net social benefits (S) of Rs 32,125 are significantly lower than the undiscounted incremental net economic benefits of Rs 42,340. Present worth of the incremental net social benefits at 12 percent discount rate is Rs 5,815 and the IRR is 28.1 percent. The social value of the Cashewnut Project is still very favorable but much less so than the economic value which assumes each rupee is valued the same no matter who receives it.

Table 10.8 shows results for a different set of assumptions on the value of labor inputs to the project. In the previous analysis we assumed that the smallholder was either hiring labor or using his own labor and that it had an economic (opportunity) cost of Rs 8 per day. Now let us assume the smallholder is employing agricultural day workers and the market wage rate is still Rs 8 but there are many more laborers willing to work at that rate than there are jobs available. In fact, let us assume the opportunity cost of agricultural labor is one-half the market wage rate. This changes the incremental net economic benefit (E) as shown in Table 10.8. It also changes the net social cost of the project.

The project now increases income (consumption) for agricultural labor. Without the project, the opportunity (economic) cost of agricultural labor is only one-half the market wage rate. If we assume the Cashewnut Project will have only minimal effect on the total agricultural labor market, labor employed in the project will also have an opportunity cost of one-half the market wage rate. But agricultural labor is actually paid the full market wage rate. Hence, the increase in income for agricultural labor is the difference between the market wage rate of Rs 8 and the opportunity cost of Rs 4 or the equivalent of Rs 4 per day. This is reflected as increased consumption by agricultural labor and is shown in Table 10.8. The net social cost of the increased consumption, however, is weighted by the real resource costs of consumption ($B_i = 1$) minus the income (consumption) weight ($d_i = 1.67$) for agricultural labor. The net social cost of increased consumption is negative and thus adds to the net economic benefit in computing the net social benefit.

Increased consumption and the net social cost of increased consumption remains the same for the smallholder. The incremental net social benefit (S) increases to Rs 41,076.4 in Table 10.8 over the value of Rs 32,125 in Table 10.7. Present worth of the incremental net social benefit at 12 percent discount rate is Rs 9,124 and the IRR is 48.1 percent. The

Table 10.8. Social Analysis Using Income Weights by Income Group and Opportunity Cost for Agricultural Labor, India Cashewnut Project (Rs).

Year	Incremental Net Economic Benefit (Cash Flow) (E)	Agricultural Labor: Increased Consumption	Agricultural Labor: Net Social Cost	Small Landholder: Increased Consumption	Small Landholder: Net Social Cost	Incremental Net Social Benefit (Cash Flow) (S)
		$[\sum_i C_i (B_i - d_i)]$				
1	(700)	440	(294.8)	(220)	(55.0)	(350.2)
2	(584)	224	(150.1)	(112)	(28.0)	(405.9)
3	(738)	248	(166.2)	(124)	(31.0)	(540.8)
4	(475)	160	(107.2)	(80)	(20.0)	(347.8)
5	(26)	196	(131.3)	(98)	(24.5)	129.8
6	1,233	212	(142.0)	94	23.5	1,351.5
7	2,229	236	(158.1)	1,066	265.5	2,121.6
8	2,723	252	(168.8)	1,544	386.0	2,505.8
9	3,217	268	(179.6)	2,022	505.5	2,891.1
10	3,711	284	(190.3)	2,500	625.0	3,276.3
11-20	3,711	284	(190.3)	3,427	856.8	3,044.5
Total	47,700	5,360	(3,591.4)	40,862	10,215.0	41,076.4

Net present worth at 12 percent = Rs 9,124

IRR = 48.1 percent

changes in assumptions regarding who benefits from the Cashewnut Project caused substantial increases in net social benefit in Table 10.8 over the results of Table 10.7.

To carry out a social analysis, data are needed on B_i, d_i, and C_i. The National Planning Office should supply the parameter data for B_i and d_i. Results on C_i should come from the agricultural project analysis group. Other methods to compute the marginal utility of income were presented in Chapter 6.

Choosing Among Mutually Exclusive Alternatives

Choice of one development project alternative frequently precludes the choice of another, irrespective of a limited capital budget. Constructing a dam at a specific site on a river for a water use project generally precludes construction of a similar dam on the same river at a different site or constructing a dam of a different size (changing water holding capacity) at the same site or a different site. Which technology to propose for a government-financed agricultural development project is a frequent concern. Choice of one technology generally means investment in fixed assets not easily readapted to a different technology. Thus choice of one technology precludes the use of an alternative technology. Multi-purpose projects can generally be designed to include or exclude some specific purpose(s). The contribution of each purpose should be evaluated whether the returns to resources put into the project for that purpose equal or exceed their opportunity cost. Because of a limited market and economies of scale in operation, location of an agricultural processing facility generally precludes locating a similar facility at an alternative location. A choice needs to be made on which location is most profitable. These examples all show the need to choose among mutually exclusive project alternatives. More emphasis needs to be placed during the design stage of project preparation on testing alternatives. There is generally more than one way to design a project and project analysis should include evaluations of these alternatives.

The preferred criterion for selecting among mutually exclusive projects is the net present worth (NPW). For project alternatives where additional resources are adding to the basic project, such as scale of operation or additional purposes in a multi-purpose project, the internal rate of return (IRR) on the cash flow difference between the alternatives is an appropriate criterion to use for project selection because it gives a measure of the marginal contribution to the additional or marginal resources used. In this case, using the criteria of B/C ratio or IRR for each complete alternative may give wrong results for purposes of ranking alternatives since the criteria are giving an average return over all resources.

1. Alternative Technologies. A reforestation project in Tunisia has a choice of technology between manual clearing and mechanical clearing (Gittinger, pp. 383-388). Outputs or benefits under the two alternatives are the same so attention can be focused on choosing the least-cost alternative. Project data are given in Table 10.9. Total undiscounted costs for the five-year project are greater under manual clearing (US$239,250) than for mechanical clearing (US$215,008). However, because equipment costs come early in the mechanical clearing alternative and then are followed by lower

annual costs, the time streams of cost are very different for the two methods. Under these conditions the discount rate or opportunity cost of capital becomes important in choice of technology. Mechanical clearing will have a cost advantage at lower discount rates because costs come early whereas manual clearing will have a cost advantage at higher discount rates because costs are deferred to later periods.

Table 10.9 shows results of discounting project costs for the two alternatives. Mechanical clearing is preferred at a 10 percent discount rate because it has a lower net present cost than manual clearing. However, at a 15 percent discount rate the opposite is true --manual clearing has a lower net present cost than mechanical clearing.

The discount rate where the changeover occurs between mechanical and manual clearing can be determined by computing the cost stream difference between the two alternatives and solving for the IRR of this difference. This rate is called the *crossover discount rate*. For the Tunisia reforestation project the crossover discount rate is 14 percent. At an opportunity cost of capital less than 14 percent mechanical clearing is preferred whereas at an opportunity cost greater than 14 percent manual clearing is preferred. At exactly 14 percent, one could be indifferent to choice of technology.

The results of the Tunisia reforestation project are presented graphically in Figure 10.1. Discount rate is plotted on the horizontal axis and present worth of cost is plotted on the vertical axis. Straight lines connect present worth of cost at discount rates of 10 and 15 percent for each alternative. The crossover discount rate (CDR) is the rate where the lines intersect indicating equal costs for both technologies (point a). At rates of opportunity cost for capital below the CDR, mechanical clearing is less costly and thus the preferred technology and at rates above the CDR manual clearing is preferred.

Assume the opportunity cost of labor is slightly less than the market wage rate and thus for economic analysis labor costs are reduced. This will have the effect of decreasing costs for both technologies but decreasing costs under manual clearing more than under mechanical clearing. The CDR will move to the left (point b) indicating a lower opportunity cost for capital is required before shifting from manual clearing to mechanical clearing. Similarly, if we assume for economic analysis there is a foreign exchange premium increasing the costs of machinery imports, the costs of mechanical clearing will increase more than the costs of manual clearing. The CDR would shift from point a to point c, indicating an increase in the advantage of manual clearing over mechanical clearing. If the opportunity cost of labor decreases wage rates and, simultaneously, a foreign exchange premium increases costs of imports, a substantial decrease in the CDR is apparent with a shift to point d.

2. ***Alternative Size Project.*** Data for alternative size irrigation projects are presented in Table 10.10. The projects are mutually exclusive and, as the data show, for a 623 percent increase in cost the increase in benefits is 323 percent. Using the net present worth (NPW) criterion, we see that at a 12 percent discount rate (opportunity cost of capital) the large-scale alternative is preferred--it returns an additional US$ 241,900 over the small-scale alternative. However, at a 15 percent discount rate, the small-scale alternative is preferred. If we choose to implement the large-scale alternative, we would be

Table 10.9. Choice Between Manual and Mechanical Land Clearing for a Reforestation Project in Tunisia (US$).

	Alternative A Manual Clearing			Alternative B Mechanical Clearing			
Project Year	Wages	Other Costs	Total Costs	Equipment Cost	Operation & Maintenance	Total Cost	Cost Stream Difference A-B
1	44,050	3,800	47,850	90,700	21,586	112,286	(64,436)
2	44,050	3,800	47,850	0	25,134	25,134	22,716
3	44,050	3,800	47,850	0	25,134	25,134	22,716
4	44,050	3,800	47,850	0	26,227	26,227	21,623
5	44,050	3,800	47,850	0	26,227	26,227	21,623
TOTAL	220,250	19,000	239,250	90,700	124,308	215,008	24,242

Net Present Cost at 10 Percent

Alternative A: $181,399 Alternative B: **$175,905**

Net Present Cost at 15 Percent

Alternative A: **$160,393** Alternative B: $161,265

Crossover Discount Rate

IRR of the cost stream difference: **14 percent**

Source: Adapted from Gittinger (pp. 383-388).

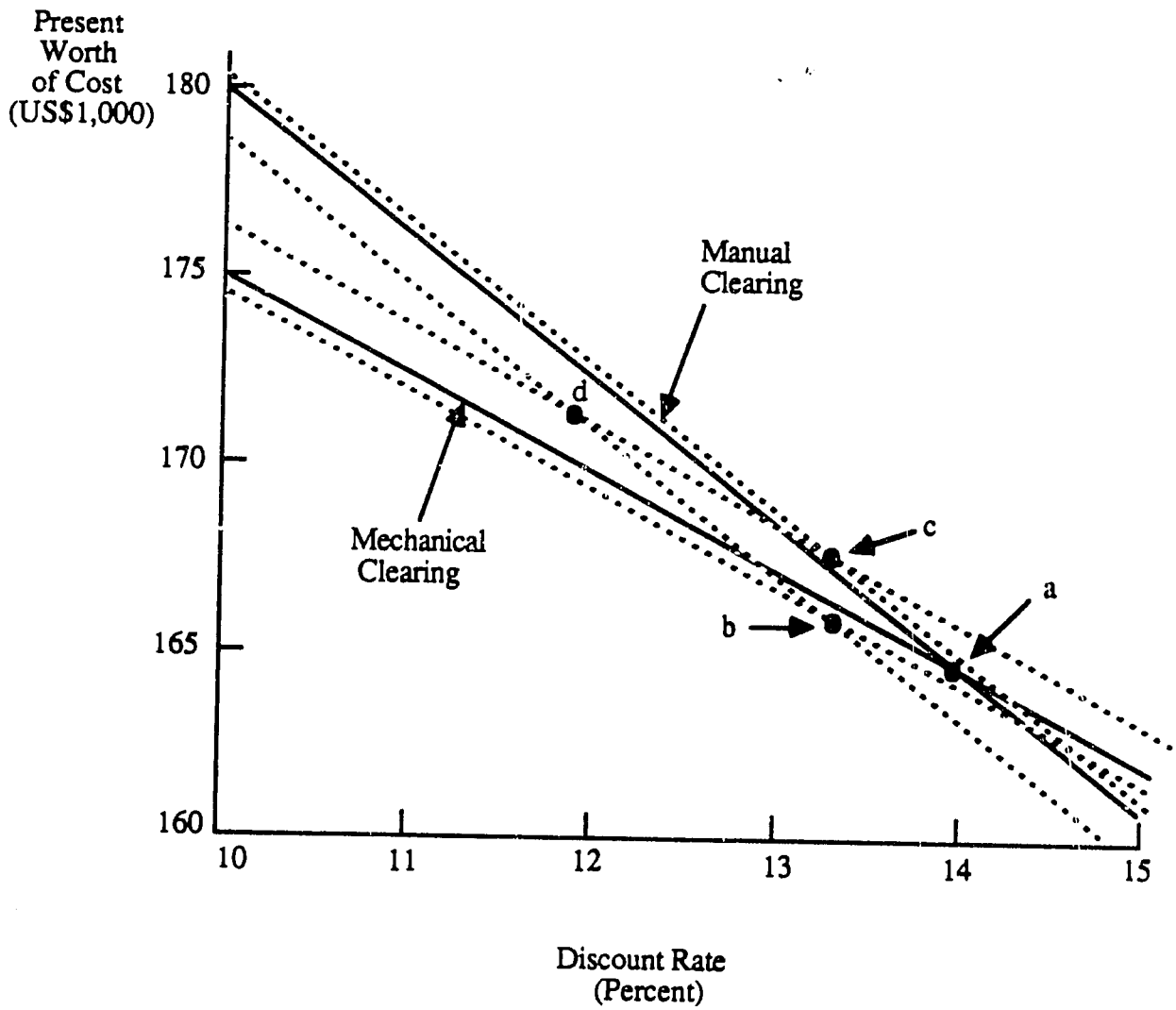

Figure 10.1. Graphic Description of Crossover Discount Rate, Manual Versus Mechanical Land Clearing for Reforestation, Tunisia

Table 10.10. Choice Between Small Scale and Large Scale Irrigation Project (US$1,000).

Project Year	Large Scale Alternative (A) Incremental Cost	Incremental Benefit	Incremental Net Benefit	Small Scale Alternative (B) Incremental Cost	Incremental Benefit	Incremental Net Benefit	Cash Flow Difference (A)-(B)
1	1,500	-	(1,500)	500	-	(500)	(1,000)
2	1,000	-	(1,000)	5	140	135	(1,135)
3	100	350	250	5	140	135	115
4	100	450	350	5	140	135	215
5	100	550	450	5	140	135	315
6 - 20	100	660	560	5	140	135	425
TOTAL	4,300	11,250	6,950	595	2,660	2,065	4,885

	Large Scale Alternative (A)	Small Scale Alternative (B)	Cash Flow Difference
NPW (12% DR)	**683.1**	441.2	**241.9**
NPW (15% DR)	155.2	**292.0**	**(136.8)**
B/C (12% DR)	1.25	1.92	-
B/C (15% DR)	0.76	1.63	-
IRR	16 percent	27 percent	14 percent

Source: Adapted from Gittinger (pp. 376-381).

giving up US$136,800 that could be more profitably invested in some other development project. The correct decisions are made by using the NPW criterion.

Notice that the B/C ratios at 12 percent discount rate would have led to the wrong decision. The small-scale alternative has a higher B/C ratio than the large-scale alternative. Similarly, the internal rate of return (IRR) criterion would lead us to select the small-scale alternative at both a 12 percent and 15 percent discount rate even though we see that at the 12 percent rate we would be giving up US$241,900 by not selecting the large-scale alternative.

Results of the cash flow difference are the same as results of the NPW and thus are correct. The NPW of the cash flow difference is positive at the 12 percent discount rate. Similarly the IRR of the cash flow difference is 14 percent and thus tells us that the large scale alternative is profitable at a 12 percent discount rate but not profitable at a 15 percent discount rate. One way to interpret these results is to say that the resources put into the large-scale alternative equivalent to those put into the small-scale alternative are giving a 27 percent return; the additional resources for the large-scale alternative are giving a 14 percent return for an overall average rate of return of 16 percent to all resources put into the large-scale project. The cash flow difference allows testing of the marginal contribution to the larger project.

Separable and Alternative Costs

Multi-purpose projects can frequently be designed to include or exclude a specific purpose. In the case of water resource projects, the project design perhaps can include or exclude irrigation or recreation. In an integrated rural development project, the project design perhaps can include or exclude processing of raw materials produced in the project or provide for direct marketing of project output versus disposing through a wholesale market. At issue is whether to include or exclude a specific (marginal) purpose in project design.

Data on a multi-purpose river basin project are presented in Table 10.11. The purposes of the project are flood control, irrigation, and power. Separable costs are those costs identified with a specific purpose. If irrigation is excluded, project costs are reduced by US$12,265,000. Alternative costs are the costs associated with providing the same level of benefits (see Table 10.11) in the best (least cost) alternative to the multi-purpose project. For instance, to provide the same level of benefits through irrigation would cost US$20,055,000 in the next best alternative.

No purpose should be included in the multi-purpose project if the alternative cost is less than the separable cost. In the case of the multi-purpose river basin project, discounting at 2.5 percent and 5 percent shows that the separable costs for each purpose are less than the alternative costs. So now we can proceed to test for a marginal purpose.

Table 10.11. Separable and Alternative Costs, Multi-Purpose River Basin Project, United States (US$1,000).

Project Year	Flood Control	Irrigation	Power
		Separable Costs	
1	84	1,132	600
2	1,244	1,970	1,530
3	907	2,865	2,085
4-50	5	134	61
Total	2,470	12,265	6,082
		Alternative Costs	
1	2,380	1,880	715
2	4,740	5,270	2,100
3	1,370	2,565	1,495
4-50	22	220	226
Total	9,524	20,055	14,932
	Present Worth at 2.5 Percent Discount Rate		
Separable Cost:	2,237	9,086	5,547
Alternative Cost:	8,672	14,890	9,897
	Present Worth at 5 Percent Discount Rate		
Separable Cost:	2,070	7,421	4,708
Alternative Cost:	8,091	12,203	7,388

Source: Adapted from Gittinger (pp. 388-393).

Benefit and cost data are presented in Table 10.12 to test whether irrigation should be included in the multi-purpose river basin project. Undiscounted cost increases by 87.4 percent by including irrigation and undiscounted incremental net benefit increases by 37.3 percent. Present worth at 2.5 percent discount rate of the incremental net benefit with irrigation is US$4,574,000 versus US$3,887,000 without irrigation. If the opportunity cost of capital is 2.5 percent we would forego US$687,000 by not including the marginal purpose of irrigation. This latter result is also the present value of the difference between cash flow with irrigation and without irrigation.

If the opportunity cost of capital increases to 5 percent we would not only forego irrigation but we would forego the entire project. Present worth of the incremental net benefit for the cash flow difference is negative. The present worth of the incremental net benefit of the project without irrigation also is negative. The internal rate of return of the cash flow difference tells us at what opportunity cost of capital irrigation just pays for itself. For our example this is at about 3.3 percent. For an opportunity cost of capital greater than 3.3 percent, irrigation should be excluded.

The computed internal rate of return with irrigation and without irrigation again points up the danger of using IRR's for purposes of ranking of mutually exclusive projects or using the IRR's for determining cut-off points. Suppose the opportunity cost of capital is 4 percent. By looking at the IRR's we would accept the with-irrigation alternative because it has a 4.1 percent IRR which is greater than the 4.0 percent opportunity cost of capital. But according to our previous analysis, the IRR of the marginal contribution of irrigation is only 3.3 percent. We would have made an error in accepting the project with irrigation. The correct interpretation is that the IRR on resources put into the without-irrigation project is equal to 4.4 percent, the IRR of the marginal resources put into the project for irrigation is 3.3 percent, and the average IRR for all resources including those for irrigation is 4.1 percent. But our decision should be made on the marginal contribution of irrigation.

Optimum Size and Timing of Project

Investment projects frequently must allow for growth in the demand for the project's services. For instance, agricultural processing plants may need to consider growth in the supply of raw materials available in a region for processing. An agricultural input industry, such as nitrogen fertilizer, may need to consider growth in demand for the input as farmers respond to new technologies or perhaps as farmers respond to subsidies by governments. Similarly, road improvement projects may need to consider growth in demand for transport services. Community water projects should anticipate growth in demand for water as population and income grows.

Frequently, these types of investment projects show decreasing unit costs for investment and operation and maintenance. So the larger the project, the lower will be the

Table 10.12. Measures of Project Worth With and Without Irrigation, U.S. Multi-Purpose River Basin Project (US$1,000).

	With Irrigation					Without Irrigation				
		Benefit					Benefit			
Project Year	Cost	Flood Control	Irrigation	Power	Incremental Net Benefit	Cost	Flood Control	Power	Incremental Net Benefit	Cash Flow Difference
1	3,588	-	-	-	(3,588)	2,456	-	-	(2,456)	(1,132)
2	8,188	-	-	-	(8,188)	6,218	-	-	(6,218)	(1,970)
3	4,848	-	-	-	(4,848)	1,983	-	-	(1,983)	(2,865)
4-50	206	271	380	347	792	72	271	347	546	246
Total	26,306	12,737	17,860	16,309	20,600	14,041	12,737	16,309	15,005	5,595

Net Present Worth at 2.5 Percent

With Irrigation:	4,574
Without Irrigation:	3,887
Cash Flow Difference:	687

Net Present Worth at 5 Percent

With Irrigation:	(2,730)
Without Irrigation:	(1,211)
Cash Flow Difference:	(1,519)

Internal Rate of Return

With Irrigation:	4.1 percent
Without Irrigation:	4.4 percent
Cash Flow Difference:	3.3 percent

Source: Adapted from Gittinger (pp. 388-393)

unit cost of supplying the project services. We would like to have the projects as large as possible in anticipation of growth in demand for project services and thus benefitting from the reduced unit costs. However, it is also true that the sooner we commit resources to a project for benefits to be received at a later point in time, the more costly becomes the project. It pays to defer investment as long as possible since future costs are more heavily discounted than present costs. Thus there are cost advantages and disadvantages of building excess capacity into a project in anticipation of growth in demand and reduced unit costs from increased capacity. The question becomes one of determining the optimum size and timing of a project.

Data are taken from a community water project study to illustrate alternative size and timing of project investments (Myoung and Schreiner). The original study used mathematical programming to determine the optimal size and timing of community water project investments. However, for discussion purposes specific alternatives will be compared in determining which alternative is the least cost means to provide capacity for meeting projected water demand over a 20 year planning period. Analysis is limited to planning time units of 5 years. That is, decisions to increase water services capacity are limited to once each 5 years and that capacity must serve for at least the following 5 year planning period. Water demand is assumed to be growing at an 8 percent annual rate. For this analysis, water demand is assumed to be price inelastic so the problem of capacity is reduced to one of determining the least-cost alternative to providing fixed water requirements.

The following cost functions were taken from the previous study:

Capital Investment Cost

$$TI = 203,456.4 + 7,973.8\ S \tag{10.7}$$

where

TI = total investment cost ($)

S = water system capacity in million gallons per year (mgy)

Operation and Maintenance (O&M)

$$TC = 24,278.7 + 353.7\ Q \tag{10.8}$$

where

TC = total annual O&M cost ($)

Q = quantity of water supplied in million gallons per year (mgy)

Investment is assumed to have a 20 year life. Both cost functions show decreasing unit costs as capacity and water supplied increases.

Quantity of water demanded is shown for the initial period and for the last year in each 5 year planning period with an 8 percent annual growth:

Project Year	Compounding Factor (8%)	Quantity of Water Demanded (mgy)
0	1.000	46.2
5	1.469	67.9
10	2.159	99.7
15	3.172	146.5
20	4.661	215.3

Investment planning is used to assure that sufficient capacity is installed at the end of each planning period to cover water supply requirements for at least the next planning period. Current water demand is 46.2 mgy and the assumption is that the community is initiating a community water supply; there is no existing capacity. To provide sufficient capacity for existing water demand and the projected growth through the first five-year planning period, the system should begin with capacity for at least 67.9 mgy.

Three alternative timings and size of water capacity additions for our community water project are presented in Table 10.13. Alternative 1 builds sufficient capacity at the beginning to serve community water demands for the entire 20 year planning period.That is, capacity is built in year 0 to deliver 215.3 mgy even though that capacity will not be used fully until the 20th year. Total investment cost is $1,920,216 and is based upon equation 10.7. At the end of 20 years, investment is used up (depreciated out) so replacement of all capacity is required in the 21st year.

Alternative 2 is the opposite extreme in that it builds only sufficient capacity to get through the next planning period. In the last year of each 5-year planning period capacity is added for the next planning period. Total investment cost increases to $2,530,584 since the fixed component of total investment ($203,456) must be paid for each addition to capacity. However, at the end of the 20-year planning period $966,611 investment in capacity remains to carry over into the next long-term planning period. Total investment costs used up in the current project period are $1,563,973.

Alternative 3 builds sufficient capacity in year 0 for the first two 5-year planning periods and then follows Alternative 2 for the remainder. Total investment costs lie between Alternative 1 and 2 at $2,327,128 and investment costs used up are $1,474,772.

Discounted investment costs for the three alternatives are compared in Table 10.14. O&M costs can be ignored since they will be the same for all alternatives. Alternative 3 has the lowest investment cost at zero discount rate. Constructing a larger system decreases unit cost and this savings more than offsets the depreciation costs of having excess capacity for one 5-year planning period. In fact, Alternative 3 is the least-cost alternative at discount

Table 10.13. Alternative Timing and Size of Water Capacity Additions, Community Water Project

Alternative 1

Project Year	Quantity Demanded (mgy)	Capacity Additions (mgy) Year 0	5	10	15	Investment Cost ($)
0	46.2	215.3				1,920,216
5	67.9					
10	99.7					
15	146.5					
20	215.3					
21						
					Total	1,920,216

Alternative 2

Project Year	Quantity Demanded (mgy)	Capacity Additions (mgy) Year 0	5	10	15	Investment Cost ($)
0	46.2	67.9				744,877
5	67.9		31.8			457,023
10	99.7			46.8		576,630
15	146.5				68.8	752,054
20	215.3					
21						(966,611)
					Total	1,563,973

Alternative 3

Project Year	Quantity Demanded (mgy)	Capacity Additions (mgy) Year 0	5	10	15	Investment Cost ($)
0	46.2	99.7				998,444
5	67.9					
10	99.7			46.8		576,630
15	146.5				68.8	752,054
20	215.3					
21						(852,356)
					Total	1,474,772

Table 10.14. Determining Optimum Size and Timing of Community Water Project Investments (US$).

Project Year	O&M Cost (5 yr. period)	Alternative 1		Alternative 2		Alternative 3	
		Investment Cost	Total Cost	Investment Cost	Total Cost	Investment Cost	Total Cost
0	-	1,920,216	1,920,216	744,877	744,877	998,444	998,444
1-5	225,411		225,411	457,023	682,434	-	225,411
5-10	274,270		274,270	576,630	850,900	576,630	850,900
11-15	346,020		346,020	752,054	1,098,074	752,054	1,098,074
16-20	451,454		451,454	-	451,454	-	451,454
21	-		-	(966,611)	(966,611)	(852,356)	(852,356)
TOTAL	1,297,155	1,920,216	3,217,371	1,563,973	2,861,128	1,474,772	2,771,927

Net Present Worth of Investment Costs

Discount Rate	Alternative 1	Alternative 2	Alternative 3
0 percent	1,920,216	1,563,973	1,474,772
5 percent	1,920,216	1,471,958	1,408,237
10 percent	1,920,216	1,302,014	1,285,696
15 percent	1,920,216	1,155,717	1,188,199

rates of 5 and 10 percent. This indicates the savings from decreased unit costs more than compensates for depreciation and interest costs at least up to 10 percent. At a 15 percent discount rate Alternative 2 becomes the least-cost alternative. This means that at high discount rates it is preferable to delay committing resources to a project for as long as possible. At the 15 percent discount rate, the savings from constructing a larger system and achieving reduced unit costs are not sufficient to cover the opportunity cost of capital for maintaining excess project capacity. This is the reason Alternative 1 is always more costly than either of the other two alternatives even though investment costs per mgy capacity are $8,919 versus $11,754 for Alternative 2 and $10,809 for Alternative 3.

Domestic Resource Cost

A partial analysis of the foreign exchange earnings capacity of a development project can be computed if the following information is known (Gittinger, p. 400): (1) the foreign exchange value of the product to be produced; (2) the foreign exchange cost incurred to produce the product; (3) the domestic currency cost of producing the output; and (4) the opportunity cost of capital. Development projects are frequently promoted on the basis of their foreign exchange savings from import substitution or foreign exchange earnings from exports. Computing the domestic resource cost to earn a unit of foreign exchange is a measure of the project's foreign exchange earning capacity.

Table 10.15 presents data for a fertilizer project in India for purposes of substituting domestic production for imports. Foreign exchange is measured in U.S. dollars and output of the fertilizer project is measured in terms of dollar savings from not importing. Value of the fertilizer produced during the project life is US$597,000,000. Value of foreign exchange costs is US$313,000,000 of which US$37,000,000 is investment costs and US$276,000,000 is production costs. The incremental saving of foreign exchange is US$284,000,000.

The domestic currency costs of the fertilizer project output is Rs 1,543,000,000 which is composed of both investment cost and production cost. The domestic resource cost is then computed as the ratio of the present worth of domestic cost of realizing foreign exchange saving (measured in domestic currency) to the present worth of net foreign exchange saving (measured in dollars or other foreign currency). This ratio is compared with the exchange rate for the foreign currency. If the exchange rate is less than the computed ratio, it is costing the country more in terms of domestic resource cost to produce a unit of project output than if the country imported the unit of output. If the exchange rate is more than the computed ratio, the opposite is true.

In the case of the India fertilizer project the domestic resource cost at a 12 percent discount rate (opportunity cost of capital) is 8.92 meaning it costs Rs 8.92 to save US$1 of foreign exchange by domestically producing fertilizer. The official exchange rate was Rs 7.50 = US$1, thus it cost more to produce fertilizer domestically than to import fertilizer. However, as Gittinger mentions, at the time of the analysis it was widely considered in India that a foreign exchange premium of at least 25 percent better reflected the true value of the rupee. This would make the shadow exchange rate Rs 9.375 = US$1

Table 10.15. Domestic Resource Cost, India Fertilizer Project.

Project Year	Foreign Exchange Savings (Million US$)				Domestic Currency Costs (Million Rs)		
	Value of Production	Foreign Exchange Costs		Incremental Saving	Investment	Production	Incremental Cost
		Investment	Production				
1970	-	7	-	(7)	107	-	107
1971	-	15	-	(15)	172	-	172
1972	-	15	-	(15)	56	-	56
1973	12	-	12	0	24	38	62
1974	38	-	18	20	-	81	81
1975	43	-	20	23	-	90	90
1976	56	-	26	30	-	109	109
1977	56	-	25	31	-	109	109
1978	56	-	25	31	-	109	109
1979-84	56	-	25	31	-	108	108
Total	597	37	276	284	359	1,184	1,543

$$\text{Domestic Resource Cost at 12 percent discount rate} = \frac{\text{Present worth of domestic currency cost of realizing foreign exchange saving (Rs)}}{\text{Present worth of net foreign exchange saving (\$)}} = \text{Rs } 696 \div \text{US\$}78 = 8.92$$

Source: Adapted from Gittinger (pp. 398-400).

(7.50 x 1.25 = 9.375). At this rate of exchange the domestic resource cost of producing fertilizer is less than the rate of exchange and the country would gain by producing fertilizer domestically.

The above procedure should be recognized as only a partial analysis of foreign exchange savings. The analysis is generally limited to accounting only for direct foreign exchange costs. Domestic currency costs may also have indirect elements of foreign exchange costs that are not captured in this measurement. Capturing all direct and indirect foreign exchange requirements would call for an input-output table for the country.

ACTIVITIES

The fourth objective of this chapter is to provide a set of activities for using the microcomputer in evaluation of agricultural projects. A set of Lessons have been prepared in detail on *Evaluation of Agricultural Projects using LOTUS 1-2-3* (see E. Li and A. Stoecker). The lessons demonstrate how LOTUS 1-2-3 can be used for performing computations and tabulations often required in the evaluation of agricultural projects. A template for the first four activities is available with the *Lessons* that reduces amount of data entry. A sensitivity analysis for the Cashewnut Project is carried out on yield, price, land rent, wage rate, investment cost, subsidy, interest (discount) rate, and financing to determine effect on project evaluation results such as farm income, NPW, B/C ratio, and IRR. Alternatively, the COMRAN project evaluation software package is designed specifically for project analysis (McCullough et al.).

1. *Formatting data for project evaluation.* Using LOTUS 1-2-3 set up the project data given in Tables 10.1 and 10.2. For those using the *Lessons* from *Evaluation of Agricultural Projects Using LOTUS 1-2-3*, start with the template and follow the *Lesson* procedures.

   a. Compare the results when changing yield and price individually by +25 percent and -25 percent.

   b. Compare the results when assuming land rent increases to Rs 100/ha and then to Rs 200/ha.

   c. Compare farm income when (1) subsidy is removed, (2) interest rate increases to 15 percent, and (3) borrowing is reduced to 50 percent of labor costs, plus materials cost, minus inflow.

2. *Computing financial and economic analysis of agricultural projects.* Using LOTUS 1-2-3 set up a financial and economic cash flow similar to the format of Table 10.3.

a. Assuming a 15 percent discount rate compute the net present values, benefit-cost ratios, and internal rates of return and compare with the results of Tables 10.4 and 10.5.

b. Compare the same project criteria when making the changes in project data given in activity one above.

c. Assume agricultural labor has an opportunity cost equal to 50 percent of the market wage rate. Compute the economic cash flow and compare it with column one of Table 10.8.

3. *Choosing between alternative technologies.* Using LOTUS 1-2-3 set up the land clearing project data given in Table 10.9.

a. Calculate the net present cost for each alternative at discount rates of 10, 12, 14, and 16 percent.

b. Using LOTUS graph the results of *a* above for each alternative with net present cost on the vertical axis and discount rate on the horizontal axis.

c. Calculate the crossover discount rate for the two alternatives and compare with the graphic results from *b* above.

4. *Choosing between alternative size project.* Using LOTUS 1-2-3 set up the irrigation project data given in Table 10.10.

a. Compute the NPW of the incremental net benefit for each project size and the cash flow difference at discount rates of 12 and 15 percent. If the opportunity cost of capital is correctly specified at 12 percent, what would we be giving up by building the small scale alternative?

b. Compute the B/C ratio for each alternative at discount rates of 12 and 15 percent. Why can't we use benefit-cost ratios for mutually exclusive projects?

c. Compute the IRR for the cash flow difference between the two alternative size projects. What does the IRR of the cash flow difference mean? If the opportunity cost of capital is 16 percent which alternative should be selected?

5. *Optimum size and timing of community water project.* Review the data and analysis given for the community water project.

a. What are at least two additional alternative capacity additions that could be evaluated?

b. Why can't this community water project be compared with an alternative development project? What additional information is needed?

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11. POLICY APPLICATIONS IN NATURAL RESOURCE PROJECTS

by Dean F. Schreiner, Daniel D. Badger, and Michael P. Welsh

A large number of development projects in Third World countries involve the natural resource base either in terms of developing the natural resource as an input into the project, such as land and water, or in determining the effects of a project on natural resources, such as the effects of a crop intensification program on soil depletion or the effects of agricultural chemical use on surface and ground water quality. Markets for many natural resources are imperfect (Randall, 1981, p. 83). Thus the results of investments in natural resource projects may differ from the results of the analysis of perfect markets as discussed in Chapter 6. Markets may be imperfect because of externalities, natural monopoly, indivisibility of project inputs, or government policy intervention. A general result of imperfect markets, such as project externalities or government policies of subsidized inputs and output price supports, is that natural resource projects are designed too large and too many resources are used for the benefit of all of society. This chapter shows how the analysis of natural resource projects can deal with market imperfections.

OBJECTIVES

The objectives of this chapter are to:

1. Briefly present methods of natural resource project valuation.
2. Analyze effects of selected government policies on size of natural resource projects.
3. Present examples of the analysis of imperfect markets on natural resource projects.

KEY POINTS

Natural resource projects require special analysis because they are often unique in methods of calculating project benefits, frequently must incorporate externalities in decision processes, and may be easily influenced by government interventions. Unless these factors are considered in project investment decisions, projects may be designed and constructed too large or too small and may give unanticipated results in the distribution of project benefits and costs. Emphasis in this chapter is on natural resource valuation techniques and the effects of imperfect markets and government interventions on analysis of natural resource projects.

Valuation Techniques. What is a natural resource project worth? If we are able to measure only the resource costs of the project we are limited in judgements we can make about projects. Without knowing the benefits of natural resource projects we are unable to compare those projects with other development projects and we are unable to determine exactly how many resources to put into those projects. Valuation techniques of natural resource projects include *market price*, *derived demand*, and other *nonmarket techniques*.

Market Price is a measure of what consumers or producers are willing to pay for the output(s) of a natural resource project. But frequently there is no direct competitive market for natural resource project outputs such as water, colonization services, or soil conservation

Derived Demand is an indirect measure of the value of project output through the demand for commodities produced by the project resources. Farmers and others participating in the project combine varying amounts of project resources with their own fixed resources in producing goods and services. The aggregate value of all project participants' marginal value product curves from using project resources is a measure of derived demand.

Imperfect Markets. The social value of a natural resource project may be quite different than the value determined from applying market prices because such prices were determined in an imperfect market. Markets for outputs of natural resource projects are imperfect most often because of *decreasing costs* and *external effects*.

Natural Monopoly is the result of decreasing costs in expanding the size of natural resource projects. One project serves the entire market and thus eliminates competitively determining a market price. Special techniques of pricing and valuing project outputs are required to compare such projects with other development projects.

External Effects are the results of benefits and/or costs accruing to members of society outside the project and outside the decision framework for project investments. The

decision to include those external effects (externalities) in natural resource project investment decisions frequently affect the size of project and the pricing of project resources.

Government Policy Interventions. Government *subsidies* and *price supports* influence the *size* of natural resource projects and the *distribution* of project benefits and costs among groups in society. Without careful analysis of policy interventions, natural resource projects are frequently designed too large, society suffers significant welfare losses, commodity and factor markets are distorted, and unanticipated results on distributions of benefits and costs occur.

CONCEPTS AND APPLICATIONS

Natural Resource Project Valuation

It is important to value both outputs and inputs of development projects. As an example, consider the community water project presented in Chapter 10 (Table 10.14). As a result of that analysis, we know that for a community of a given size and for a given growth rate in *water requirements*, the least cost investment alternative identified for supplying water over a twenty year time period and at a 10 percent discount rate is $1,285,696. When including the net present cost of the project operation and maintenance (O & M), the total cost of supplying water to the community over the planning period is $1,773,976. But how can we compare that development project to another development project, such as the Cashewnut Project which is giving us an economic rate of return of 30.4 percent (Table 10.5). Until we value the consumption and other utilization benefits of the community water project we will not be able to directly compare the two development projects. Valuation of *both* outputs and inputs is important if we want to compare development projects for investing limited funds in Third World countries.

1. Market Price. The usual procedure for valuing project outputs is by the market price which is a measure of what consumers are willing to pay. Using this line of reasoning we were able to construct a demand schedule in Chapter 6 showing how much consumers are willing to take of a good at different prices. This also led to estimates of consumers surplus which is an aggregate measure of consumer benefits. In the same chapter it was shown how much producers are willing to supply of a good at different prices. The difference between what producers are willing to supply at a given price and the price the producers actually receive is a measure of producers surplus and an estimate of gross returns. The efficient price is where the sum of consumers and producers surplus is a maximum and it is also the price which clears the market or where demand is equal to supply.

The market price in a competitive economy with many buyers and sellers of a good was shown in Chapter 6 to be an efficient price. These market determined efficient prices

are relevant when the development project would use all inputs and would produce all outputs in very small quantities relative to the total markets for those inputs and outputs (Randall, 1981, Chapters 15 and 16). But many natural resource development projects do not meet the conditions for competitive markets. The assumption of marginality may be invalid for at least some output or some input of a natural resource project. Large irrigation projects frequently can influence crop prices, especially if the project is located such that it produces for a limited domestic urban market. For example, to dispose of more vegetables on the domestic market, the project may have to lower prices. Similarly, the size of the project may be such that to obtain sufficient labor the project will have to draw from more distant labor pools and at higher wage rates.

2. ***Natural Monopoly***. A frequent case of an imperfect market for a natural resource project is that of the natural monopoly where the minimum average cost of producing a product occurs at a rate of output more than sufficient to supply the entire market at a price covering full costs. The product produced must also have limited substitutes. This is the general case for rural community water systems (Schreiner and Kang, 1988). In this case there is no competitive market price since to have two or more private community water systems would mean duplicate water lines, excess water storage capacity, and increased unit water cost because each private system would be higher on the declining average cost curve.

A graphic presentation of the community water supply project and household demand for water is presented in Figure 11.1. The typical household monthly water demand (D_H) in Figure 11.1 (b) is shown downward sloping indicating households will use less water if the unit cost of water increases. This is different than the assumption used in the analysis of the community water project in Chapter 10 where an average water consumption rate (in annual terms for that analysis) was used irrespective of the unit cost of water. It is not uncommon to have price elasticities for water demand from -.4 to -.8 or higher depending on household characteristics, environmental characteristics, and water use (Dellenbarger, Schreiner, and Kang, 1986). Similarly, income elasticities for water demand are reported at .4 and higher. Thus, communities should plan their water supply capacity not only for expected growth in population but also for expected growth in income.

The community water supply project is characterized with long-run average cost (LAC), long-run marginal cost (LMC), and aggregate community water demand (D_A) as shown in Figure 11.1(a). The community water project is a natural monopoly since the aggregate water demand curve is to the left of the minimum long-run average cost. The most efficient price is P_m since it is at this price that the long-run marginal cost of resources used in this project are equal to the marginal benefits of consuming the last unit of water by the community. The typical household water demand at P_m is q_h in Figure 11.1(b). The aggregate quantity of water supplied by the project is Q_a in Figure 11.1(a). To supply less, the community benefits from consuming marginal additions of water are greater than the marginal costs of supplying more water through the project. To supply more water than

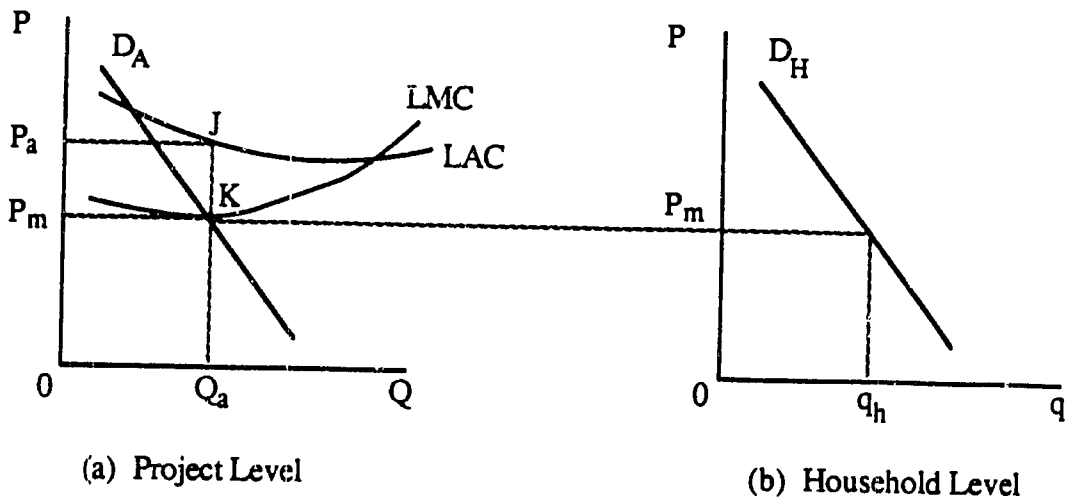


Figure 11.1. Community Water Supply Project

Q_a, the community benefits derived from consuming the last units are less than the marginal costs of supplying those units. Clearly, society is at a welfare equilibrium by building the community water project at a size to deliver Q_a quantity of water and at a price of P_m for the marginal quantity consumed by households.

Because the LAC curve is declining at Q_a quantity of water, and thus LMC is less than LAC, total revenue for the project is less than total cost at P_m. The project will experience a loss or negative cash flow equal to the area P_a JK P_m. The project is not financially feasible to the community under these conditions. Two approaches may be considered to meet the revenue requirement. The first approach is to make up all revenue loss, P_a JK P_m, through a public subsidy to the community. This meets the revenue requirement at the efficient marginal price of water but it is at the cost of redistributing income within society from those outside the community to those inside the community. As we will see later with benefit weighting, this may be socially efficient if members of the community are at lower income than the national average.

A second approach is to use block rate schedules which means charging households higher prices for initial quantities of water consumed but charging the price P_m for the last marginal quantities of water consumed. The project remains at the size giving efficient use of society resources and with no redistribution of income. This approach is frequently used even with public subsidy when the subsidy is not large enough to cover all revenue loss.

*3. **Derived Demand.*** Many natural resource projects produce a good or service which has no market determined transaction price. Irrigation projects provide water to farmers and colonization projects provide resources and services to potential colonizers. Neither type of project has a direct output easily negotiated in a competitive market so there is little basis to consider a market determined transaction price for water or for colonization services. However, each project is expected to use the project resources (outputs) and produce commodities easily negotiated in competitive markets. Thus it is possible to estimate the *derived demand* for project resources indirectly through the demand for commodities produced with the project resources.

The derived demand for outputs of a natural resource project is shown in Figure 11.2. Part (b) shows how *farmers* would combine their own resources with resources (outputs) from the *project* to produce a (several) commodity(ies). The marginal value product (MVP) curve shows the marginal return to the farmer from using additional units of the project resources in producing a commodity. The MVP curve is downward sloping indicating diminishing returns as the farmer uses additional project resources combined with his own fixed resources such as land, labor, or capital. The farmer would be willing to use project resources up to the point where the marginal value product of those resources is equal to the price the farmer must pay for those resources. This will give the farmer maximum net returns to fixed resources. To use less than this amount of project resources will reduce the returns to fixed resources and to use more than this amount will add more to total costs than received in total returns. Algebraically, we can express the equilibrium point of maximum net returns as follows:

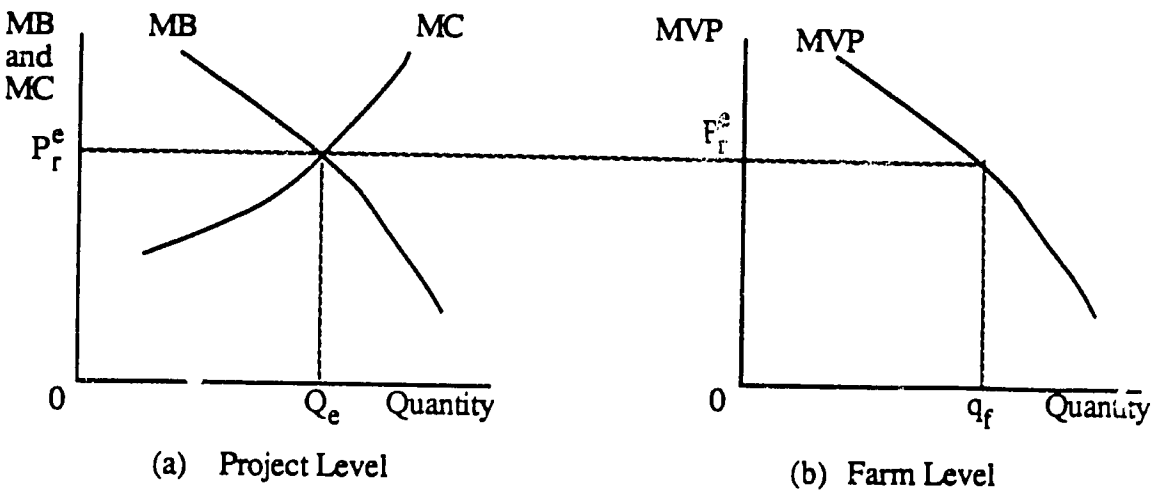


Figure 11.2. Derived Demand for Project Resources

$$MVP = MPP \cdot P_c = P_r \tag{11.1}$$

where

MPP = marginal physical product of using project resources
P_c = price of the commodity produced
P_r = price of the project resource

Notice what influences the amount of project resources the farmer uses. First, if the price of the resource (P_r) decreases, the farmer will use more of the resource by moving down on the MVP curve. Secondly, if the price of the commodity (P_c) increases, the MVP curve shifts out and for the same resource price the farmer will use more of the resource. Finally, if the farmer becomes more efficient in combining the project resource with other resources, perhaps through adopting an improved technology, less of the resource will be used for each unit of commodity output, the MPP increases, and the MVP shifts up for the same P_c. If P_r remains constant the farmer will actually use more of the project resource.

We are not restricted in our analysis of the use of project resources to the production of one commodity. We can use the techniques and methodologies discussed in Chapters 8 and 9 to combine project resources with the farmer's other resources in producing the optimum mix of enterprises and commodities. The MVP is then a measurement of the return to the project resources from producing a bundle of commodities. The MVP will then change for a change in any commodity price and for a change in the price of the project resource.

To answer the question of how many resources the *project* should produce or what *size* the project should be we need to look at Part (a) of Figure 11.2. The marginal benefit (MB) curve is the summation of the MVP curves for all farmers using the project resources. It is a measure of the marginal benefits derived by farmers in the aggregate from additional units used of the project resources (outputs). Thus it is a measure of *derived demand* since as the price of the resource varies the farmer will vary the use of the resource in equating price with MVP. The marginal benefit (derived demand) curve will shift as commodity prices and the marginal physical product vary, just as in the case of shifts in farmers' MVP curves.

The location of the MB curve is determined by the number of farmers served by the project. This number itself may be determined by such things as actual physical connection to the project such as connection through a water distribution system; distance from the project where at some distance the cost of delivering the project services is such that it is greater than the highest point on the farmers' MVP curve; or institutional limits such as a prespecified number of colonizers to be included in a colonization project. At times the number of farmers may be a variable and become part of the project solution. The trade-off may be between serving more farmers or distributing the same quantity of project resources to fewer farmers.

The marginal cost of supplying more project resources is shown in Figure 11.2 (a) by the MC curve. It is shown upward sloping indicating increased costs to expand the

quantity of resources offered by the project. This is frequently the case for irrigation projects where the unit cost of reservoir capacity (water holding capacity) increases as capacity increases. This may be due to increased construction costs as the heighth of the reservoir dam increases or the increased opportunity cost of larger surface areas needed for each additional unit of water holding capacity. Similarly, water projects using ground water will first exhaust water supplies nearer the surface and then subsequently tap water supplies at greater depths with the consequent higher pumping costs. Colonization projects will increase the number of land units only at an increasing cost as more remote areas are included. The result is increasing marginal cost as the *size* of the project increases.

Now we can answer the question, what is the optimum size of the project? The optimum size of the project is where the benefit of the last unit of project resource produced is equal to the cost of producing that resource. In Figure 11.2 (a) this is at Q_e where the marginal benefit (MB) is equal to the marginal cost (MC). This is also the point where the difference between total benefits and total costs is the greatest. But this is the marginal benefit and marginal cost in the aggregate. By dividing the MB and MC by the *number* (n) of participansts (farmers) in the aggregate we obtain the equilibrium *price* P_r^e of the project resources.

To assure the project is exactly of the size to produce Q_e (output) resources, we must charge the farmers P_r^e for each unit of project resources they use. To maximize their net returns they will equate P_r^e with their MVP and use q_f quantity of resources. At this price the MVP will be equal for all farmers, the sum of project resources used by all farmers will be equal to Q_e, and MB will equal MC. The project is of optimum size. If the farmers were charged less than P_r^e they would use more than q_f of project resources, the sum of the resources used by all farmers in the project would be greater than Q_e, and the project would be constructed too large since at that size the marginal benefits from the project would be less than the marginal costs of the project. If the farmers were charged more than P_r^e, the farmers would use fewer than q_f resources, the project would be constructed too small, and society would forego some benefits.

As seen in Chapter 10, most projects are constructed with a time stream of costs and a time stream of benefits. The analysis as presented in Figure 11.2 abstracts from time but could be modified to include time by merely incorporating a discount rate and expressing all values on the vertical axis as present value benefits, present value costs, and present value MVP. The optimum size project would still be at Q_e where present value marginal benefits (PVMB) is equal to present value marginal costs (PVMC).

Project Externalities

The analysis so far assumes that the project participants (farmers) are the only ones effected by the project. They appropriate the project benefits to themselves and they pay the project costs unless, as in the case of the natural monopoly, the government provides

the project a subsidy to cover the cash flow deficit where marginal cost pricing for all project resources is followed.

The project participants may or may not be the managers or decision makers for the project. If they are the decision makers and if the project must cover total costs, then project resources will be priced to the participants such that the total revenue equals total costs. Furthermore, if the participants as managers do not play favoritism (discrimination) in distributing project resources, and if the cost of supplying the project resources to all participants is the same, then one price per unit of resource will be used in assessing charges to participants and this will assure that all participants will obtain the same MVP for the last unit of the resource used. Thus, to meet the revenue requirements, the project participants must be charged a price equal to or greater than P_r^e in Figure 11.2. However, project participants as managers will want to maximize returns to their own fixed resources and thus obtain the project resources at the lowest possible price. This price will be P_r^e. We will thus have the efficient size project Q_e, marginal benefits will equal marginal costs, and all project participants will have the same MVP and equal to P_r^e.

What would be the results if the project managers are not the project participants but rather, they are government officials? If project management can be assumed equally efficient under the two alternatives, and if government officials are charged with the objective of overall efficiency of resource use by society, the results should be the same. The government would build the project of the size to deliver Q_e project resources, marginal benefits would equal marginal costs, and project participants would be charged P_r^e per unit of resource.

What would be the result if some of the project benefits or costs are external to the project decision makers or to the project participants? That is, there are some project benefits or costs which accrue to or are imposed upon individuals who play no part in the decisions of the project (see Randall, 1981, Chapter 8). For example, an irrigation project may be able to better control water flow and thus reduce flood damage to producers downstream who are not part of the irrigation system. Or the farmers in an irrigation project may use surface water to leach their fields, and these salts combined with chemical runoff from the project reduces water quality to users downstream and either increases their costs of restoring water quality or reduces the value of water in their production and consumption processes. A colonization project in the Amazon Basin that reduces the amount of natural vegetation may increase the amount of siltation downstream, reduce the amount of wildlife habitatant, and displace indigenous populations. A sanitary community water supply may reduce infectious diseases and benefit not only that community but surrounding communities by reducing their medical costs.

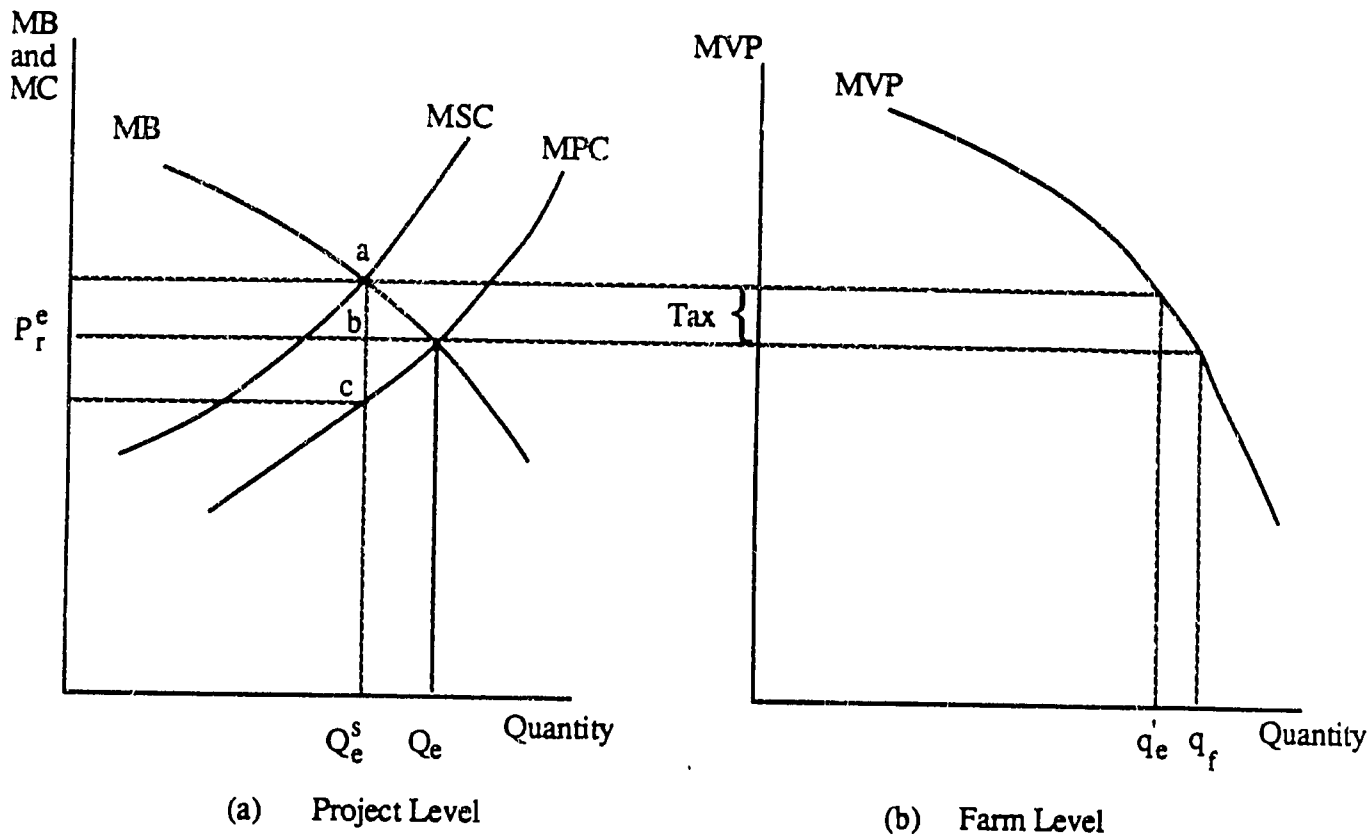


Figure 11.3. Effects of Externalities on Quantity of Project Resources

In the *economic* analysis of projects as discussed in Chapter 10, the distribution of project benefits and costs was ignored because ownership rights to resources were ignored and the *efficiency* of resource use to society as a whole was emphasized. Consider the project presented in Figure 11.2 but with an external cost to society proportional to the quantity of project output (resources). These results are shown in Figure 11.3 (a) where the marginal project (private) costs are shown as MPC and the marginal social costs including the external cost are shown as MSC. For economic efficiency and the economic analysis of Chapter 10 the project should be of size Q_e^s which is smaller than the project size of Q_e of Figure 11.2 (a). At this project size marginal benefit is equal to the marginal social cost (MSC) and we have an efficient allocation of society's resources to the project.

But with a smaller size project we have fewer project resources to distribute to the farmers at the price P_r^e. How do we limit project resources to the farmers? One way is to ration resources to farmers so that just the amount of Q_e^s project resources are produced and distributed to farmers. Farmers who received q_f resources previously, or who expected to receive q_f resources at price P_r^e, now receive a smaller proportion. This proportion could be the same as the proportional reduction in size of project from Q_e to Q_e^s. But rationing project resources could be a cumbersome administrative procedure.

A second procedure is to assess a tax to farmers equal to the amount *ab* in Figure 11.3 (a). Farmers would now pay the price P_r^e plus the tax *ab* and would demand q_e' quantity of the project resource. In the aggregate farmers would demand Q_e^s amount of project resources and this is the efficient size for the project or where MB = MSC.

Either of the two procedures above will give us economic efficiency and the economic analysis results of Chapter 10. But what has happened to the distribution of project costs and benefits? Remember, in our economic analysis of Chapter 10 distribution of project costs and benefits was ignored.

Clearly, project externalities could create a distribution problem. Rationing the amount of project resources received by farmers creates a project (producers) surplus equal to $ac \cdot Q_e^s$. This is equal to the external cost to society from implementing the project. By recognizing these costs we have reduced the size of the project to meet the economic efficiency criteria of Chapter 10. But nothing so far has been stated about compensating those who have suffered a loss. The producers surplus accrues to the project and, if left to the decision makers for the project, the surplus may be distributed as they wish. It could be distributed as lump sum payments to the project participants. If participants are low-resource, low-income producers, policy makers may deem this a beneficial use of the producers surplus.

However, if society wishes to compensate those who suffer a loss due to the project and those individuals can be identified and the amount of their loss estimated, policy makers may wish to transfer the producers surplus to them. For instance, project participants (farmers) may be assessed a tax on use of project resources equal to *ab*. As before, this gives the optimum size project. The project itself may be assessed a tax equal

to *bc* times Q_e^s. Now the full producers surplus is available to policy makers to compensate those in society who have suffered a loss due to the project. If it is difficult to identify and compensate those who have suffered a loss, policy makers may still wish to extract the full producers surplus from the project and add it to the general government revenue.

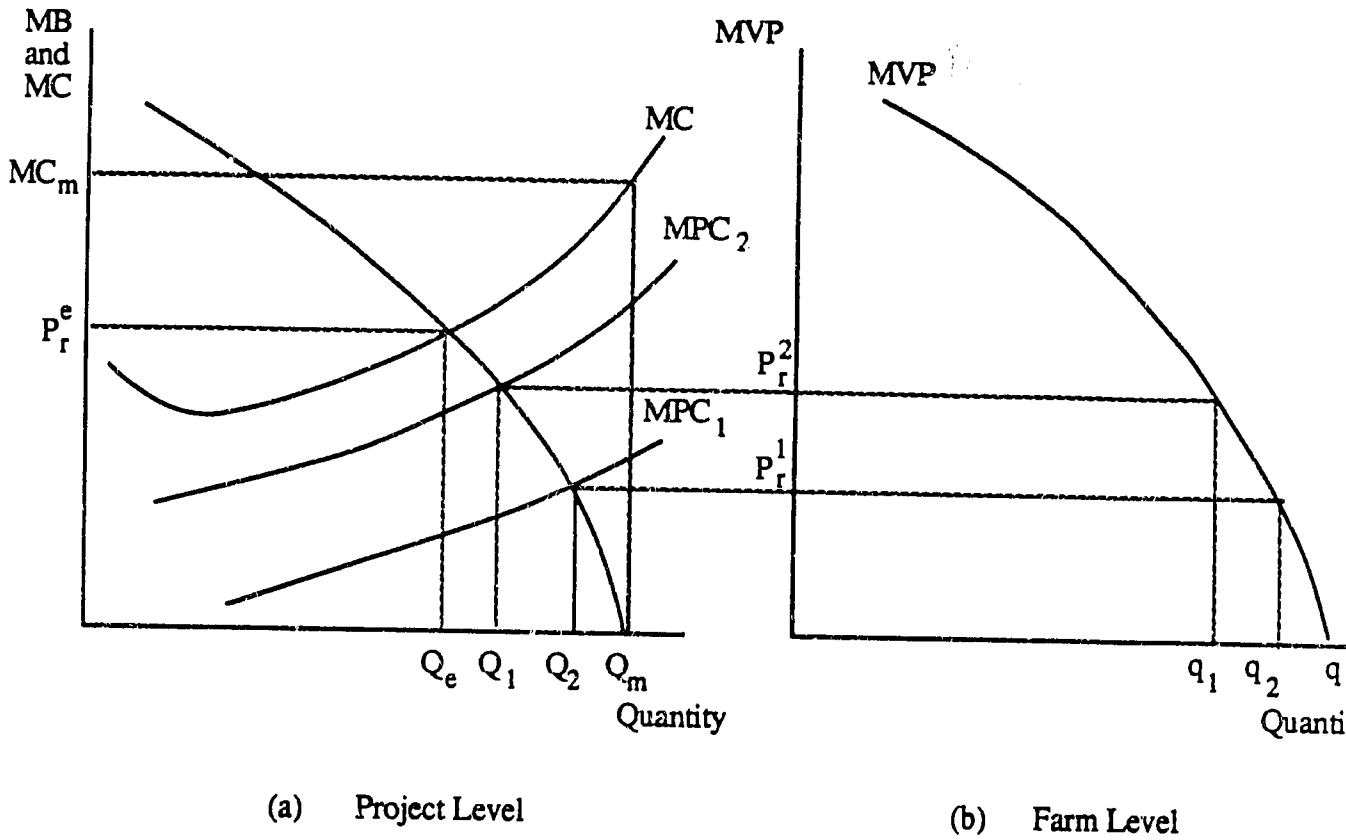
Government Policy Interventions

Governments frequently intervene directly in natural resource projects. Governments may decide to subsidize project participants by not charging them the full costs of providing them the project services. Another form of direct project intervention is to limit the number of project resources any one participant may receive. Governments also intervene indirectly by supporting prices of products produced or assessing tariffs on commodities produced or inputs used by the projects. Government intervention in natural resource projects generally changes the size of the project and either puts too many or too few resources into a project than is optimum from society's efficient level. A few examples of government intervention are analyzed below.

1. Subsidies. Most natural resource projects in Third World countries are subsidized. The full costs of irrigation projects are seldom assessed to those who are using the water. Colonization projects have frequently been justified on the basis of assisting low income groups or those with significant underemployment of family labor resources. Hence, many of the costs of colonization are absorbed by governments and are viewed as transfer payments to the colonizers. However, if participants are not assessed full costs of the project, they will tend to use too many of the project resources, projects will be built too large, and society could suffer large economic losses.

Consider the example in Figure 11.4 The project is producing resources used at the farm level. The MVP curve is driven down to zero at q_m quantity of project resources indicating the maximum amount of output a farmer can gain from combining unlimited project resources with his other fixed resources. If the farmer was charged nothing for project resources it would be rational to use q_m quantity. If all participants obtained project reosources at zero price they would all use resources to the point of MVP = 0 and the MB curve would be zero at Q_m.

The cost of the project, however, is not zero. The marginal cost of supplying project resources is shown by the curve MC. The efficient size of the project is in supplying Q_e resources to project participants. However, if farmers are not charged for project resources it could be difficult to control their demand for resources and the project could be grossly overbuilt to the extent of supplying Q_m quantity of resources at a marginal project cost of MC_m. Some irrigation projects in fact do not charge farmers for water or charge a very minimal annual water fee.



(a) Project Level

(b) Farm Level

Figure 11.4. Effects of Subsidies on Quantity of Project Resources

A set of marginal cost curves are shown in Figure 11.4 (a). The curve MPC_1 shows the marginal project (private) cost when accounting for only one-half of the actual operation and maintenance (0 & M) cost of the project. In the aggregate, if project policy is to charge participants one-half of the 0 & M costs, and if participants could have all of the project resources they wanted at that price, then the aggregate demand for project resources would be Q_2, the price would be P_r^1, and each participant would demand q_2 resources. MPC_2 represents marginal project costs when accounting for all of the 0 & M costs but not the annualized investment costs. If project policy is to charge participants full 0 & M costs then project size would be Q_1, the price would be P_r^2, and each participant would demand q_1 resources.

Many natural resource projects in Third World countries that do not charge participants even the 0 & M costs of the project often fail to adequately maintain the project because of inadequate funding from the general government budget. The project suffers, project participants cannot count on timely delivery of project resources, and social costs of project neglect could be substantial. Often times project participants themselves would be better off if they were charged all or a significant part of the 0 & M costs to be used at the project level for purposes of appropriately operating and maintaining the project.

Let's look in detail at the distribution of benefits and costs from alternative policies for a natural resource project. In Figure 11.5, MB represents the benefits derived by all farmers or participants using the project resources. No other benefits accrue to society. MPC_2 represents the marginal 0 & M costs of the project and MPC_1 represents exactly one-half of those costs. MC represents total annual marginal costs for the paroject including 0 & M and annualized investment costs. The results of the analysis would be the same if the curves represented present value of costs for the planning period which would be more consistent with how project analysis is actually carried out. MPC_1 and MPC_2 are shown constant and thus MPC_2 is equal to average 0 & M cost. MC is decreasing and then increasing indicating that unit investment costs decrease with size of project over some range of output and then increase.

The project policies are in two categories: (1) prices are determined at the project level and project participants can obtain as many project resources as they want at that price. Size of the project will be determined as a result of the pricing decision rule. (2) Project size is determined, a pricing decision rule is set, and the amount of rationing of project resources is the end result. The pricing decisions are critical in determining the distribution of project benefits and costs. Gross benefits are defined as the area under the MB curve. Total cost of project output (resources) is the area under the MC curve up to that quantity of output. Net benefits to project participants are equal to gross benefits minus the costs they are charged. If project participants are not charged full costs of the project the assumption is that the remaining costs must be paid out of the public treasury. Net social benefits are thus the difference between gross benefits and all costs, both participant and public. For any project size greater than Q_e, net social benefits must be reduced by the welfare loss from committing too many resources to this project. Since MC is a measure of the opportunity costs of resources used in this project, the difference

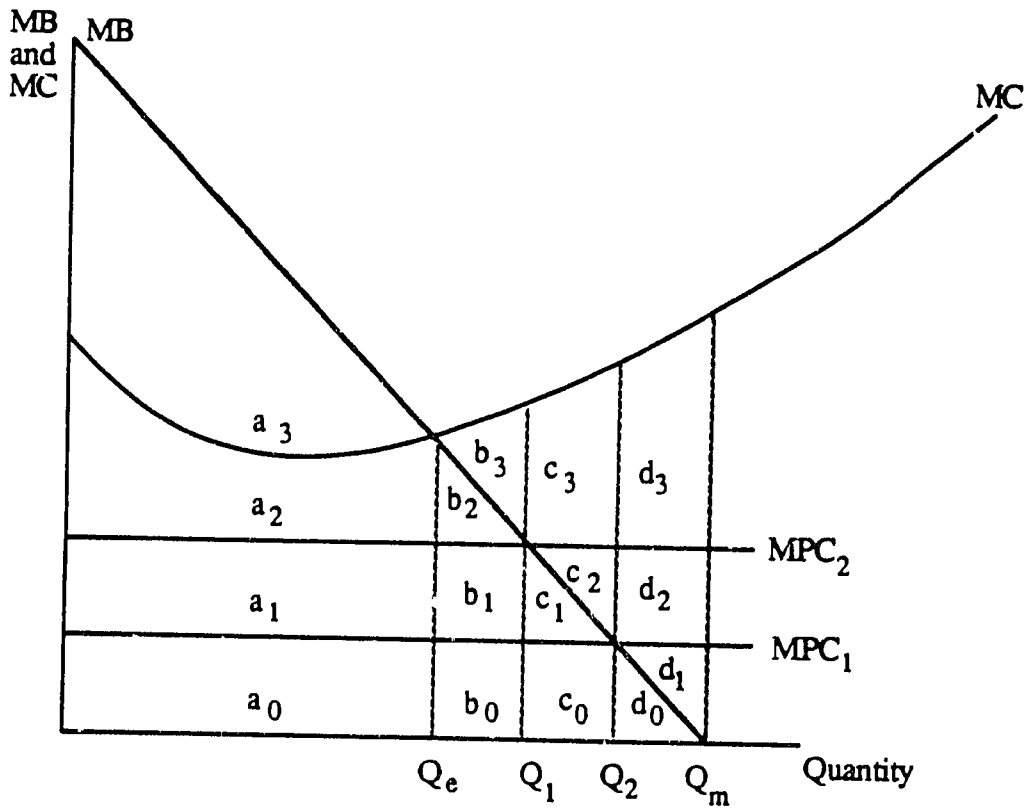


Figure 11.5. Distribution of Benefits and Costs Under Alternative Pricing Rules for Project Resources

Table 11.1. Distribution of Benefits and Costs Under Alternative Project Policies.

Size of Project, Resource Rationing, and Distribution of Benefits and Costs	Price Policy Decision Rules			
	$P_r = 0$ (1)	$P_r = MPC_1$ (2)	$P_r = MPC_2$ (3)	$P_r = MC$ (4)
1. Project Size = Q_0				
Resource Rationing	$Q_m - Q_0$	$Q_2 - Q_0$	$Q_1 - Q_0$	None
Participant Gross Benefits	$a_0 + a_1 + a_2 + a_3$	$a_0 + a_1 + a_2 + a_3$	$a_0 + a_1 + a_2 + a_3$	$a_0 + a_1 + a_2 + a_3$
Participant Costs	None	a_0	$a_0 + a_1$	$a_0 + a_1 + a_2$
Net Participant Benefits	$a_0 + a_1 + a_2 + a_3$	$a_1 + a_2 + a_3$	$a_2 + a_3$	a_3
Public Costs	$a_0 + a_1 + a_2$	$a_1 + a_2$	a_1	None
Welfare Loss	None	None	None	None
Net Social Benefits	a_3	a_3	a_3	a_3
2. Project Size = Q_1				
Resource Rationing	$Q_m - Q_1$	$Q_2 - Q_1$	None	
Participant Gross Benefits	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2$	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2$	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2$	
Participant Costs	None	$a_0 + b_0$	$a_0 + a_1 + b_0 + b_1$	
Net Participant Benefits	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2$	$a_1 + a_2 + a_3 + b_1 + b_2$	$a_2 + a_3 + b_2$	
Public Costs	$a_0 + a_1 + a_2 + b_0 + b_1 + b_2 + b_3$	$a_1 + a_2 + b_1 + b_2 + b_3$	$a_2 + b_2 + b_3$	
Welfare Loss	b_3	b_3	b_3	
Net Social Benefits	$a_3 - b_3$	$a_3 - b_3$	$a_3 - b_3$	
3. Project Size = Q_2				
Resource Rationing	$Q_m - Q_2$	None		
Participant Gross Benefits	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2 + c_0 + c_1$	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2 + c_0 + c_1$		
Participant Costs	None	$a_0 + b_0 + c_0$		
Net Participant Benefits	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2 + c_0 + c_1$	$a_1 + a_2 + a_3 + b_1 + b_2 + c_1$		
Public Costs	$a_0 + a_1 + a_2 + b_0 + b_1 + b_2 + b_3 + c_0 + c_1 + c_2 + c_3$	$a_1 + a_2 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3$		
Welfare Loss	$b_3 + c_2 + c_3$	$b_3 + c_2 + c_3$		
Net Social Benefits	$a_3 - b_3 - c_2 - c_3$	$a_3 - b_3 - c_2 - c_3$		
4. Project Size = Q_m				
Resource Rationing	None			
Participant Gross Benefits	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2 + c_0 + c_1 + d_0$			
Participant Costs	None			
Net Participant Benefits	$a_0 + a_1 + a_2 + a_3 + b_0 + b_1 + b_2 + c_0 + c_1 + d_0$			
Public Costs	$a_0 + a_1 + a_2 + b_0 + b_1 + b_2 + b_3 + c_0 + c_1 + c_2 + c_3 + d_0 + d_1 + d_2 + d_3$			
Welfare Loss	$b_3 + c_2 + c_3 + d_1 + d_2 + d_3$			
Net Social Benefits	$a_3 - b_3 - c_2 - c_3 - d_1 - d_2 - d_3$			

between MC and MB beyond Q_e is a measure of the welfare loss or what society could gain by using these resources in producing alternative goods and services.

Distribution of benefits and costs from alternative project policies are shown in Table 11.1. Across the columns of the table are the price policy decision rules and include: (1) price of project resources to participants is zero, $P_r = 0$; (2) price of project resources is one-half of 0 & M costs, $P_r = MPC_1$; (3) price of porject resources is equal to 0 & M costs, $P_r = MPC_2$; and (4) price of project resources is at full cost, $P_r = MC$. The left hand margin shows the size of project, amount of resource rationing, and distribution of project benefits and costs. The distribution of benefits is only to project participants since it is assumed there are no externalities. Project costs are distributed between project participants and the public and depend upon the size of project and price policy decision rule. Welfare losses to the public are shown for projects of size larger than Q_e. The quantity of project resources that have to be rationed are shown and depend on the size of the project and the price policy.

When project participants are charged the full cost of the project (Column 4) the efficient size project is built (Q_e); there are no public costs, no welfare loss, and no project resource rationing; and net social benefits are a maximum and equal net participant benefits. For the same size project, as participants are charged less for project resources (going from Column 4 to Column 1), net participant benefits increase, public costs increase, and resource rationing increases. However, there is no welfare loss and net social benefits remain a maximum.

Viewing Column (1) for zero project costs to participants, as project size increases, net participant benefits increase, public costs increase, welfare loss increases, net social benefits decrease, and project resource rationing decreases. Clearly, the project participants benefit from increasing project size under this pricing policy. However, these benefits can come at extremely high social costs. It is not uncommon to have natural resource projects where the public costs exceed the project participant benefits. This may become a very costly program where public costs of transfering a unit value of project benefits to target groups significantly exceeds that unit value. It would be less costly to make a direct transfer to the target group.

The lower right triangle of Table 11.1 does not exist. For projects larger than Q_e, marginal costs are such that the participants will utilize project resources only at subsidized rates. For instance, at project size Q_1, participants would be willing to take Q_e quantity of resources at full marginal cost but would be willing to take more resources only at lower costs. They would be willing to take the last units of project resources only at a price equal to MPC_2.

Frequently, project managers feel a project is unsuccessful if it is not always operating at full capacity and thus they push for subsidies to reduce prices of project resources to participants. Assume a project was overbuilt and of size Q_2. The long-run marginal cost curves of Figure 11.5 are no longer applicable for determing efficient level of project operation. Furthermore, the MB curve may have shifted up or down since the project was built. The efficient level of operation is now where the short-run marginal cost

is equal to marginal benefit. This level of project output may require a subsidy if the short-run marginal cost is less than average cost including any fixed annual payments.

2. Price Supports. Governments frequently support the prices of selected agricultural commodities. The policy objectives may be to (1) increase incomes to farmers or producers of those commodities, (2) increase commodity supplies and reduce prices to consumers, and/or (3) stabilize markets by reducing variability in prices and quantities.

Increasing farm income through price supports tends to increase the size of natural resource projects thus diverting resources away from uses with potentially higher opportunity costs. Consider the example in Figure 11.6. The MB and MVP curves are, respectively, the marginal project benefits and the marginal value product to farmers from using project resources in the absence of any price supports. The project is of size Q_e and farmers use q_f quantity of project resources.

Now assume the government has chosen to support the price(s) of the commodity(ies) produced with the project resources so that the MVP curve shifts up to MVP'. This is the same as increasing P_c in equation (11.1) to P_o' The project marginal benefit curve in Figure 11.6(a) also shifts up to MB'. If the project is still in the design stage and if project management expects the price supports to continue for some time into the future, they may decide to increase the size of the project from Q_e to Q' and price the project resource to farmers such that they use q' quantity. Thus the size of the project increases and total benefits at the project level increase. However, the returns on farmers fixed resources may increase or decrease as we will see in the following analysis. But what has happened to society as a whole from the government price supports?

Consider the example in Figure 11.7. The MB and MB' curves are, respectively, the marginal benefit curves without and with price supports on commodities produced with the project resources. Two cases are considered after the decision has been made to support commodity prices: (1) allow the project size to increase to Q' and (2) keep the project at size Q_e and ration project resources to farmers. The distribution of benefits and costs for the two cases are shown in Table 11.2. The social costs of the price support program itself are not included in Table 11.2 because the price supports pertain to commodities produced by farmers not only in this project but for all farmers. Also, the asumption is that the policy objective is to increase farmers income rather than increase commodity supplies and reduce prices to consumers.

Column (A) shows the distribtuion of benefits and costs under conditions of no price supports and marginal cost pricing of project resources to farmers. It is the most efficient use of resources by society and results in the maximum unweighted social welfare. However, the decision is made to increase farm income through price supports. Column (B) shows the distribution of benefits and costs under the assumption that price supports will continue into the future, project resources are priced to farmers at the higher marginal cost (MB' = MC), and the size of the project is allowed to increase to Q'. Under these conditions, the changes in the distribution of benefits and costs are shown as the

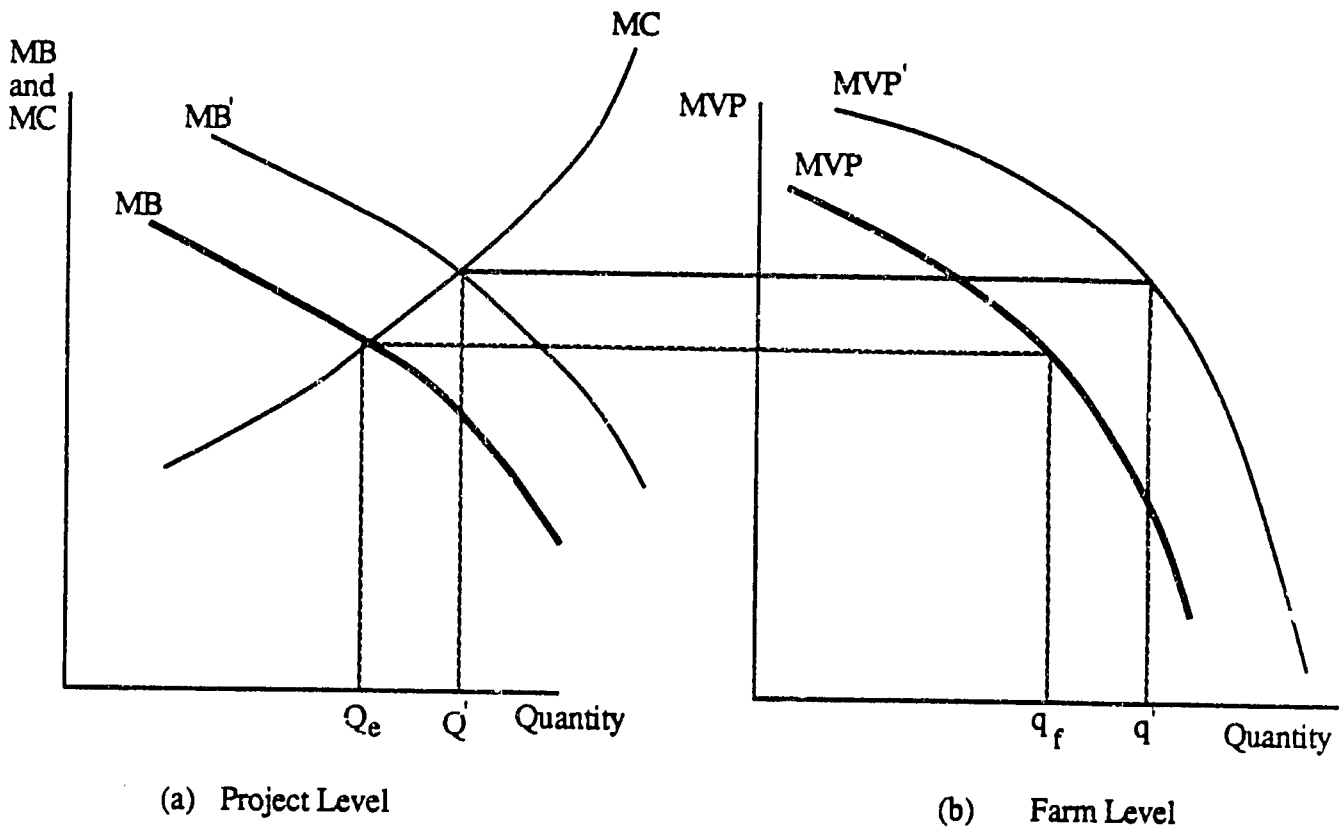


Figure 11.6. Effects of Price Supports on Quantity of Project Resources

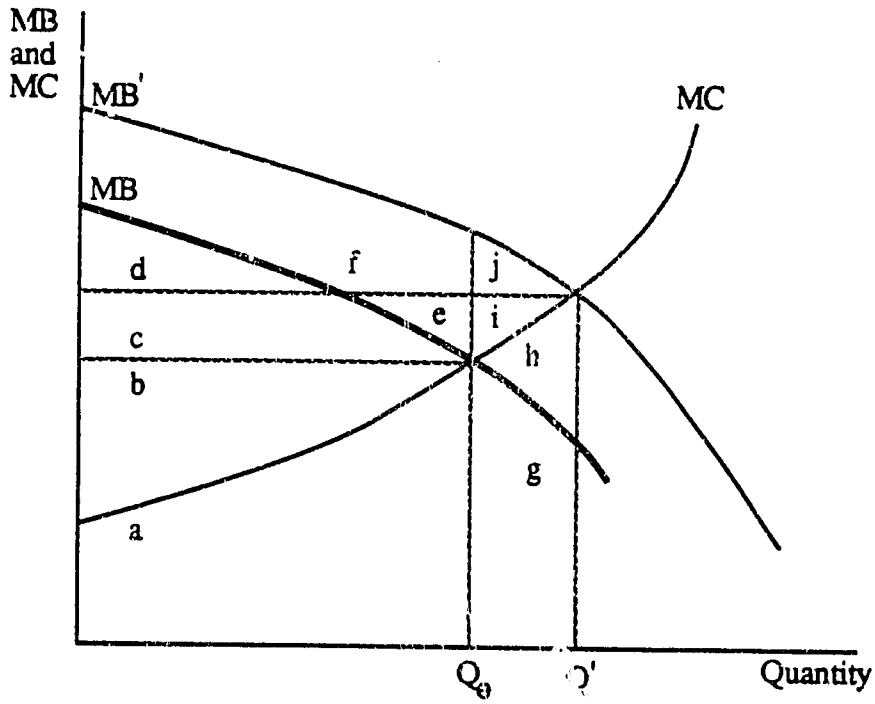


Figure 11.7. Effects on Quantity of Project Resources from Government Price Supports on Commodities Produced by the Project

Table 11.2. Distribution of Project Benefits and Costs Under Commodity Price Supports.

Benefits and Costs	No Price Supports (A)	Price Supports and No Constraint in Size of Project (B)	Difference (B) - (A) = (C)	Price Supports and Constrained to Q_e Project Size (D)	Difference (D) - (A) = (E)
Project Size	Q_e	Q'	$Q' - Q_e$	Q_e	None
Project Participants					
Gross Benefits	a + b + c + d	a + b + c + d + e + f + g + h + i + j	e + f + g + h + i + j	a + b + c + d + e + f	e + f
Costs	a + b	a + b + c + e + g + h + i	c + e + g + h + i	a + b	None
Net Benefits	c + d	d + f + j	f + j - c	c + d + e + f	e + f
Project					
Gross Benefits	a + b	a + b + c + e + g + h + i	c + e + g + h + i	a + b	None
Costs	a	a + g + h	g + h	a	None
Net Benefits	b	b + c + e + i	c + e + i	b	None
Society					
Gross Benefits	a + b + c + d	a + b + c + d + e + f + g + h + i + j	e + f + g + h + i + j	a + b + c + d + e + f	e + f
Costs	a	a + g	g	a	None
Welfare Loss	None	h	h	None	None
Net Benefits	b + c + d	b + c + d + e + f + i + j	e + f + i + j	b + c + d + e + f	e + f

differences in Column (C). Project participants (farmers) are better off by the area f + j - c. If this net difference is small (or negative) , the project participants could be compensated through a transfer of all or part of the gain received at the project level which is equal to c + e + i. This would be possible since the producers' surplus accruing at the project level (area b + c + e + i) is available to distribute back to project participants or to the government, depending upon ownership. Even if project ownership is by the government, a policy decision can be made to distribute all or part of this producers' surplus back to project participants.

The real loss from the larger size project is to society. Society suffers a welfare loss equal to the area h which shows the opportunity cost of resources used in building the project larger than Q_e. The marginal social benefit of these resources is the area g + i + j and the marginal social cost is g + h. Therefore, it costs society a welfare loss equal to area h so that project participants may have additional benefits equal to the area i + j. Transferring these marginal benefits to farmers through project resources may turn out to be very costly for society.

A more interesting result, however, is the second case where the size of the project is held at Q_e, price supports shift the marginal benefit curve to MB', project resources are priced to farmers where MB = MC, and rationing of resources are necessary. The distribtuion of benefits and costs are shown in Column (D) with the difference between this case and the efficient result of no price supports shown in Column (E). Project participants are better off by the area e + f, there is no change in project costs, and there is no welfare loss. Of course, society must pay the support price to project participants (and all other farmers) but soceity does not suffer the welfare loss for putting too many resources into this project.

The efficiency of the price support program in attaining the policy objective of increasing farm incomes is not addressed in the above analysis. Our concern here is only the result price supports have on the use of project resources and, in turn, the distribution of benefits and costs of building the natural resource project. In addition, the analysis is limited to a partial equilibrium analysis assuming all other prices remain constant and all other resources at the farm level fixed with only the amount of project resources varying in the production processes. However, other things may and do change. If the price of project resources increase (i.e., price of water or land), farmers may substitute other resources for project resources. Prices of commodities and other inputs (resources) may change because farm incomes change and factor substitutions occur. A later section shows the general equilibrium results from a change in government policy.

Empirical Applications

Many concepts were presented in the preceeding sections to show how project valuation, project externalities, and government policy interventions impact upon natural

resource project evaluations. This section presents some empirical analyses to show specifically how the concepts can be applied for more effective decision making.

*1. **Estimating Community Water Project Benefits.*** In Chapter 10 elements of a community water project were presented. Quantity of annual water demand was given by five-year planning periods. *Water demand* was assumed a *water requirement* and the project alternatives were specified as alternative timing and size of water capacity additions. The decision was to choose the least cost alternative given the cost relationships and a discount rate. But the question was asked earlier how can we compare a community water project with the Cashewnut Project for limited development resources in a Third World country? What are the benefits of a community water project?

We must turn to the valuation techniques discussed above. But is there a market price for water services delivered by a community water project? Probably not because such projects generally typify a natural monopoly and hence have no competitor. However, costs of community water projects can generally be estimated from engineering data or from analyzing a cross section of water projects. Hence, the long-run average cost and the long-run marginal cost of a unit of water can usually be estimated as shown in Figure 11.1(a). But as discussed in that section, the social welfare equilibrium may be at a quantity where the marginal price (equal to LMC) is less than the long-run average cost of supplying the water. If we valued water at this marginal price, the B/C ratio would be less than one and the project would be considered infeasible.

One alternative is to design the project for the social welfare quilibrium quantity, calculate the average cost of supplying that quantity of water, and use a block rate schedule to price different quantities of water taken such that total costs are covered and then making sure that the last rate is equal to the equilibrium marginal price. Now total revenue is equal to total cost, marginal benefit is equal to marginal cost, and the B/C ratio is equal to one.

How does the community water project compare with the Cashewnut Project now? If the discount rate is chosen such that there are sufficient investment funds for all projects with a rate of return equal to or greater than the discount rate, there would be sufficient funds to implement both the community water project and the Cashewnut Project. Suppose we are not sure about the discount rate and we would like to know the array of projects if the rate were slightly higher. In this case, the community water project would drop out of the array since its B/C ratio would be less than one, but the Cashewnut Project would remain because it's B/C ratio remains high. If the Cashewnut Project were in the same community as the water project, the community may question this decision of dropping the community water project and implementing the Cashewnut Project.

A second alternative is to estimate the total benefits the community appears to place on water services. This can be done by estimating the area under the marginal benefit curve. These total benefits can be compared with the total costs and a B/C ratio estimated. The demand curve for community water services is an estimate of the marginal benefit curve. That all or part of these benefits actually exist can be verified if households are willing to pay higher prices for initial quantities of water taken such as with a declining block rate water pricing schedule. However, the community water project in Chapter 10

assumed an infinitely price inelastic demand curve for water. If this is the case, the benefits derived from the community water services are also infinite and the water project would always be superior to the Cashewnut Project.

Now suppose the policy makers made this assumption, ignored the high engineering cost estimates for a specific community water project design, built the project, computed the average and marginal costs, designed a block rate schedule to cover total costs, and found that the community consumed much less water than the previously assumed *water requirements*. At the lower quantities of water consumed, total revenue would be less than total costs and an *ex post* evaluation would show a B/C ratio less than one. The assumption of an infinitely price inelastic demand for water was wrong.

Now let us assume a reasonable marginal price elasticity of water demand of -.6 based on other empirical studies (Kang and Schreiner, 1987).[2] Evidence from studies of other systems may also indicate that communities of the same population size, same income level, and similar characteristics for water use consume 46.2 mgy as in the example of Chapter 10 at a marginal price of $2.12 per thousand gallons. With this information, we can estimate a water demand curve for our community water project. The price elasticity of water demand is -.6 and is defined as the following:

$$\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -.6 \tag{11.2}$$

where

Q = quantity of water in million gallons per year (mgy)
P = marginal price of water in dollars per 1,000 gallons
$\frac{dQ}{dP}$ = the inverse of the slope of the demand curve or the change in quantity consumed for a change in price

If empirical results of other studies would indicate that for our community's characteristics and a consumption of 46.2 mgy of water at $2.12 per 1,000 gallons the price elasticity is -.6 then the slope of the demand curve is the following:

$$\frac{1}{\frac{dQ}{dP}} = -\frac{1}{.6} \cdot \frac{2.12}{46.2} = -0.07648 \tag{11.3}$$

Assuming the following linear demand function:

$$Q = a - bP \tag{11.4}$$

---

[2]Evidence exists that households may respond to marginal water price, monthly water bill, or a combination of the two (Schreiner and Kang, 1988). For purposes here it is assumed households respond to marginal price.

then $\frac{1}{b}$ is the absolute value of the slope of the demand curve and equal to 0.07648 and a is the intercept term and equal to 73.92. The inverse demand function is:

$$P = \alpha - \beta Q \tag{11.5}$$

where

$$\alpha = \frac{a}{b} = 5.654$$

and

$$\beta = \frac{1}{b} = 0.07648$$

The inverse demand function is shown graphically in Figure 11.8. The area under the demand curve (marginal benefit curve) at Q_e is total (gross) benefits and equal to

$$GB = \frac{1}{2}(\alpha - P_e)\, Q_e + P_e\, Q_e \tag{11.6}$$

or for this community water project at $Q_e = 46.2$

$$GB = 0.5\,(5.654 - 2.12)\,46.2 + 2.12\,(46.2) = \$179{,}579$$

Equation (11.6) can be rewritten for any quantity of water services by substituting equation (11.5) for price so that

$$GB = \frac{1}{2}(\alpha - \alpha + \beta Q)\, Q + (\alpha - \beta Q)\, Q = \alpha Q - 0.5\, \beta Q^2 \tag{11.7a}$$

or for this community water project

$$GB = 5.654\, Q - 0.03824\, Q^2 \tag{11.7b}$$

The marginal benefit function is equal to

$$MB = \frac{dGB}{dQ} = 5.654 - 0.07648Q \tag{11.8}$$

which is the same as the inverse demand function of equation (11.5).

We now have a basis for computing the benefits of the various alternatives for the community water project of Chapter 10. Furthermore, we have a basis for comparing the benefits and costs of the community water project with the Cashewnut Project or any other development project.

Before computing the benefits of the community water project there is one more calculation that must be done with the marginal benefit (demand) function. The marginal

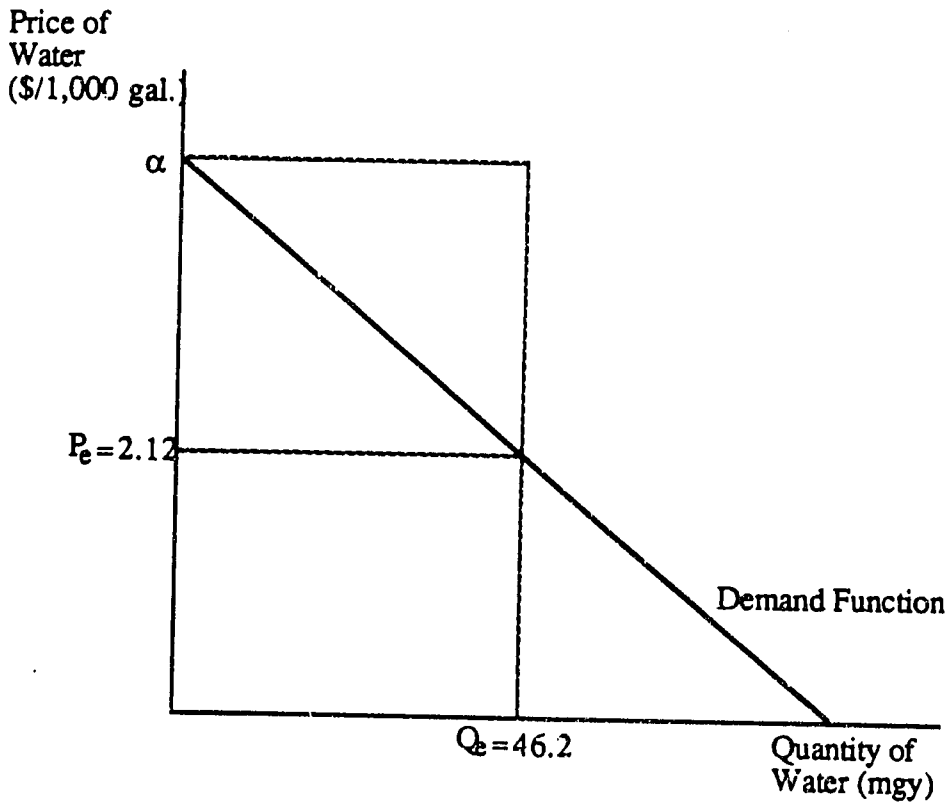


Figure 11.8. Demand Function for Community Water

benefit function represented by equation (11.8) is for the base or initial period of the water project. As assumed in Chapter 10, the demand for water is increasing by eight percent per year over the 20-year planning period. Growth in demand comes about through growth in community population and income. The assumption is that additional households will respond in the same way to price changes as do existing households. Therefore, the growth in water demand during the planning period merely means a rotation of the demand curve with demand in the next period at the same price equal to the demand in the current period plus growth. This is shown in Figure 11.9 with the actual data given in Table 11.3.

The price-quantity relationship shows that if the price level is P_0 = \$2.12 per 1,000 gallons, Q_0 = 46.2 mgy of water services is purchased by households at time unit zero. With an annual growth in water demand of eight percent due to growth in community population and income, at a price of P_0 = \$2.12 demand for water will be Q_1 = 56.0 mgy at the midpoint of the first five-year time unit. The demand curve becomes flatter with the slope equal to the slope of the initial demand curve divided by the growth factor 1.2122. The process for computing the annual quantity of water demand and the slope of the demand curve is the same for other years in the planning period.

Annual gross benefits are computed from equation (11.7) and shown in Table 11.3. These results are combined with the cost information from Table 10.14 to compute the benefits and costs of the community water project as shown in Table 11.4. Annual gross benefits from Table 11.3 are multiplied by five to estimate total benefits for the five-year planning period. Similarly, annual O & M costs are multiplied by five to estimate costs for the five-year planning period. The 0 & M costs are slightly different in Table 11.4 than shown in Table 10.15 but only because O & M costs were calculated for the midpoint of the planning period in Table 11.4 to be consistent with the midpoint evaluation of benefits.

The present worth of net benefits (NPW) and the benefit-cost ratio (B/C) of the community water project were calculated at discount rates of 0, 5, and 10 percent with results reported in Table 11.4. Results indicate very favorable B/C ratios. The computed internal rate of return (IRR) for the net benefit stream is a very favorable 20.4 percent.

2. Subsidized Colonization Project. Using mathematical programming techniques discussed in Chapter 9 the benefits of a colonization project in Peru can be estimated.[3] The gross benefits vary with the size of the project and are measured in the Peruvian currency of Intis (I/.). Costs of the colonization project are estimated and based on information from a project design team. It is assumed there is a fixed number of colonizers and each colonizing family brings with them a set of fixed resources such as labor and financing to purchase variable inputs and small capital items. The colonizers combine their own fixed resources with colonization services from the project to maximize net returns. The government subsidizes the colonization services provided by the project.

[3]The functions and data used for this analysis were adapted from the study by Schreiner, Chantaworn, and Badger, 1987.

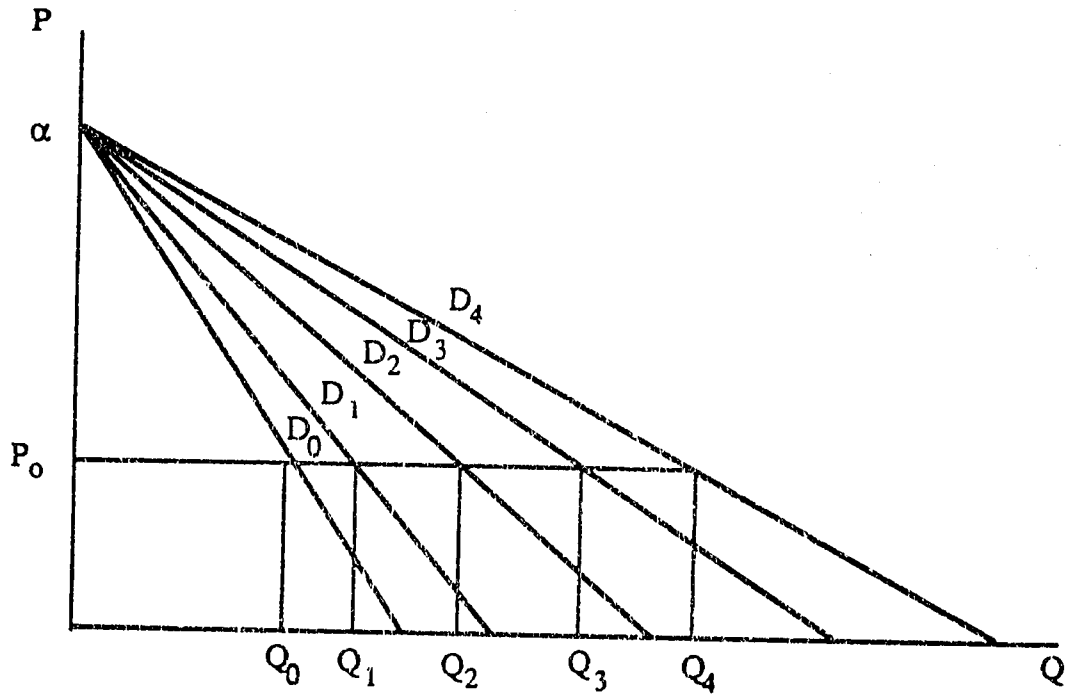


Figure 11.9. Growth in Community Water Demand

Table 11.3. Growth in Annual Water Demand and Annual Gross Benefit

Project Years	Compounding Growth Factor Midpoint [a] (8%)	Annual Quantity (Q) of Water Demand Midpoint (mgy)	Slope of Demand Curve β	Annual Gross Benefit[b] (US$)
0	1.0000	46.2	0.07648	179,594
1-5	1.2122	56.0	0.06309	217,699
6-10	1.7811	82.3	0.04294	319,901
11-15	2.6170	120.9	0.02922	470,017
16-20	3.8452	177.6	0.01989	690,467

[a]$(1+i)^n$ where i = 0.08 and n = 0, 2.5, 7.5, 12.5, and 17.5, respectively.
[b]$GB = 5.654\ Q - 0.5\ \beta\ Q^2$

Table 11.4. Benefits and Costs of the Community Water Project.

Project Years	Gross Benefits	Costs[a] 0 & M	Investment Alternative 3	Total	Net Benefit	Discount Factor[b] Midpoint (5%)
0	--	--	998,444	998,444	(998,444)	1.0000
1-5	1,088,495	220.430	--	220,430	868,065	0.8852
6-10	1,599,505	266,941	576,630	843,571	755,934	0.6936
11-15	2,350,085	335,205	752,054	1,087,259	1,262,826	0.5434
16-20	3,452,335	435,479	--	435,479	3,016,856	0.4258
21	--	--	(852,356)	(852,356)	852,356	0.3589
Total	8,490,420	1,258,055	1,474,772	2,732,827	5,757,593	

Discount Rate	NPW	B/C
0 Percent	5,757,593	3.1
5 Percent	2,570,991	2.1
10 Percent	1,123,249	1.6

IRR = 20.4 Percent

[a]See Chapter 10 for data.

[b]$\frac{1}{(1+i)^n}$ where i = 0.05 and n = 0, 2.5, 7.5, 12.5, 17.5, and 21, respectively.

The following analysis shows the results of government subsidy on size of project, distribution of project benefits and costs, and a potential welfare loss.

The total benefit function is

$$TB = 1.1888Q - 0.00006915\ Q^2 \tag{11.9}$$

where

TB = gross benefits (I/.million)
Q = quantity of colonization services (1,000 units).

Correspondingly, the marginal benefit (derived demand) function is

$$MB = \frac{dTB}{dQ} = 1.1888 - 0.0001383\ Q \tag{11.10}$$

Colonization services are basically units of virgin land laid out in a colonization scheme but may also include other services such as land clearing, roads and other basic infrastructure, extension and technical production assistance, marketing services, and other community and social amenities (Nelson, 1973). The emphasis here is not to test for the economic feasibility of the additional or any particular service as with the example in Chapter 10. But interest is in showing diminishing marginal returns to additional colonization services, not just with additional land units but also with additional other services provided. Thus it is assumed there is a continuum of services and decreasing returns as you add additional units.

For simplicity, costs are assumed linear and marginal costs are equal to average unit costs. By fixing the number of colonizers and assuming static resource availability to the colonizers the derived demand for colonization services is known and we are able to determine the optimum size of the project from the beginning initial conditions. These are all simplifying assumptions and can be relaxed for more realistic problem analysis.

The government investment cost is the following:

$$TC_{g\ inv} = 1.45\ Q \tag{11.11}$$

where

$TC_{g\ inv}$ = total government investment cost (I/.million).

This investment is amortized over 25 years at a five percent discount rate. The amortized annual investment cost is

$$TC_{g\ a\ inv} = (0.070952)\ 1.45Q = 0.1029Q \tag{11.12}$$

where 0.070952 is the capital recovery factor for 25 years at five percent interest. The amortized annual marginal investment cost is

$$MC_{g\ a\ inv} = \frac{dTC_{g\ a\ inv}}{dQ} = 0.1029 \tag{11.13}$$

The operation and maintenance (O & M) cost of the colonization project itself is

$$TC_{O\&M} = 0.52Q \tag{11.14}$$

where

$TC_{O\&M}$ = total O & M cost (I/.million).

The O & M marginal cost is

$$MC_{O\&M} = \frac{dTC_{O\&M}}{dQ} = 0.52 \tag{11.15}$$

The total long-run marginal cost is the sum of $MC_{g\ a\ inv}$ and $MC_{O\&M}$ and equal to

$$MC = 0.6229 \tag{11.16}$$

The optimum size colonization project for economic efficiency is where the long-run marginal cost is equal to marginal benefit or

$$MC = MB = 0.6229 = 1.1888 - 0.0001383Q \tag{11.17}$$

Solving for Q in equation (11.17) shows that the efficient size colonization project is 4,092 thousand units.

The Peruvian government, however, may choose to subsidize the colonizers because of policy objectives other than economic efficiency such as reducing urban unemployment, reducing population pressure in nearby highland areas, increasing the total cultivated area for high value crops, creating a more extensive economic base for the eastern Andean marginal highway, or a further opening-up of the Amazon basin for future development. Most will argue that the economists are not to question the legitimacy of these other policy objectives but to assist the policy makers in evaluation of the expected results of the policy decisions. The policy options identified for the colonization project are the same as those discussed in a previous section on *subsidies* and shown in Figure 11.5 and Table 11.1 The subsidy policy options are: (1) full subsidy of all project costs, Pr = 0; (2) subsidy of investment plus one-half of O & M costs, Pr = MPC_1; (3) subsidy of only investment costs, Pr = MPC_2; and (4) zero subsidy, Pr = MC. The policy choices also exist to limit the size of the project under each subsidy option and ration colonization services accordingly. The form of the benefit and cost functions are shown in Figure 11.10.

Empirical estimates for the colonization project of the policy options on size of the project and the distribution of benefits and costs are given in Table 11.5. These empirical results are consistent with the graphic results of Figure 11.10 and Table 11.1. The policy options for subsidizing colonization services are shown across the columns of Table 11.5

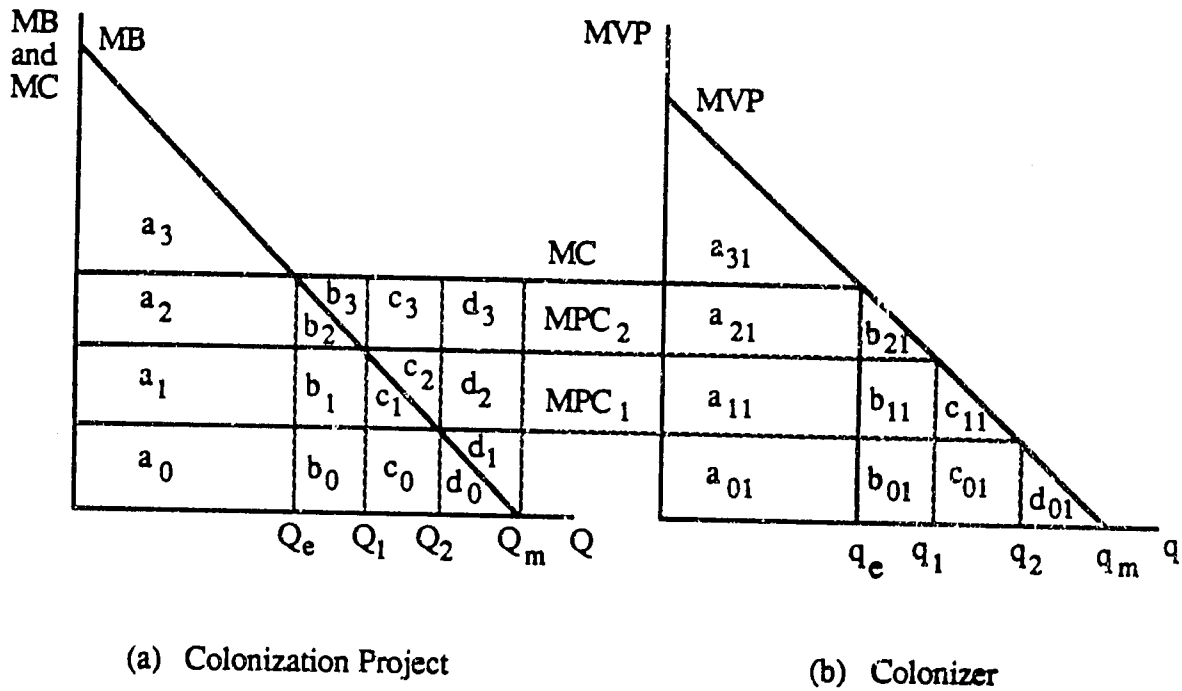


Figure 11.10. Effects of Colonization Project Subsidies on Size of Project and Distribution of Benefits and Costs

Table 11.5. Results of Policy Options on Size of Project, Rationing of Project Services, and Distribution of Benefits and Costs.

Size of Project, Units Rationed, and Distribution of Benefits and Costs	Policy Options (Subsidy per Unit)			
	I/.622.90 Full Cost Subsidy (1)	I/.362.90 Investment Plus One-half O & M Cost (2)	I/.102.90 Investment Cost (3)	I/.0.00 Zero Subsidy (4)
1. Project Size = Q_e = 4,091,829 units				
Units Rationed (1,000)	4,504	2,624	744	None
Benefits (I/.million)				
Colonizers	3,706.1	3,706.1	3,706.1	3,706.1
Costs (I/.million)				
Colonizers	0.0	1,063.9	2,127.8	2,548.8
Public	2,548.8	1,484.9	421.0	0.0
Net Benefits (I/.million)				
Colonizers	3,706.1	2,642.2	1,578.3	1,157.3
Society	1,157.3	1,157.3	1,157.3	1,157.3
Welfare Loss (I/.million)	0.0	0.0	0.0	0.0
2. Project Size = Q_1 = 4,835,864 units				
Units Rationed (1,000)	3,760	1,880	None	
Benefits (I/.million)				
Colonizers	4,131.2	4,131.2	4,131.2	
Costs (I/.million)				
Colonizers	0.0	1,257.3	2,514.6	
Public	3,012.2	1,754.9	497.6	
Net Benefits (I/.million)				
Colonizers	4,131.2	2,873.9	1,616.6	
Society	1,119.0	1,119.0	1,119.0	
Welfare Loss (I/.million)	38.3	38.3	38.3	
3. Project Size = Q_2 = 6,715,835 units				
Units Rationed (1,000)	1,880	None		
Benefits (I/.million)				
Colonizers	4,864.4	4,864.4		
Costs (I/.million)				
Colonizers	0.0	1,746.1		
Public	4,183.2	2,437.1		
Net Benefits (I/.million)				
Colonizers	4,864.4	3,118.3		
Society	681.2	681.2		
Welfare Loss (I/.million)	476.1	476.1		
4. Project Size = Q_m = 8,595,806 units				
Units Rationed (1,000)	None			
Benefits (I/.million)				
Colonizers	5,108.8			
Costs (I/.million)				
Colonizers	0.0			
Public	5,354.2			
Net Benefits (I/.million)				
Colonizers	5,108.8			
Society	(245.4)			
Welfare Loss	1,402.7			

and go from a full cost subsidy in column (1) to zero subsidy in column (4). In addition, policy makers may choose the options of limiting the size of the colonization project and maintaining the full array of policy options on subsidizing costs of the services knowing that under some subsidy options rationing of services will be required. The project size options are consistent with equating marginal benefit with the policy options of marginal costs charged to colonizers. Therefore, four alternative project sizes are considered and are listed down the rows of Table 11.5.

The results of analysis of the policy options are in the distribution of benefits and costs, potential welfare loss, and amount of colonization services rationed. Distribution of net benefits of the project are to colonizers and to society with each unit of net benefit equally weighted. Costs are borne by the colonizers or the public if subsidies are used. For each Inti (I/.) of subsidy the cost is transferred from the colonizer to society thus increasing the net benefit to the colonizer. Only if the size of the project increases beyond the point where the marginal benefit is equal to the full marginal cost does the Inti of subsidy decrease the net benefits to society. This occurs because the public is required to pay the difference between the marginal benefit to the colonizer and the higher marginal cost of supplying the additional colonization service. In fact society (the public) looses twice because it must pay the higher cost of providing the additional colonization services beyond the efficient size (Q_e) and then it looses what could have been produced with those resources or what is termed the welfare loss.

Let us look at the results in Table 11.5 in more detail. The efficient size project (Q_e) supplies 4,091,829 units of colonization services. The colonizers receive gross benefits equal to I/.3,706.1 (million). The costs of supplying this number of colonization units is I/.2,548.8 (million). The net benefits to colonizers depends upon how much the colonization services are subsidized. If the colonizers pay nothing for the services their net benefits equal their gross benefits and they have a return of I/.3,706.1 (million) on their fixed resources. If the colonizers pay the full marginal cost of supplying the colonization services their costs (in the aggregate) increase by I/.2,548.8 (million), net benefits are reduced to I/.1,157.3 (million), and their return to fixed resources are significantly reduced. The opposite occurs with public costs. If the colonizers pay the full cost of the colonization services public costs are zero. As the amount of subsidy increases, the public costs increase as the amount of the colonizers' costs decrease until the public costs reach the maximum of I/.2,548.8 (million).

However, with Q_e size project society is still efficient. Net benefits to society remain at I/.157.3 (million) no matter the size of the subsidy. There is merely a transfer within society and the net effect remains the same. This assumes equal utility for the unit gained by one group and the unit lost by the other group. If the colonizers represent a low-resource, low-income group, there may be a social gain from providing some level of subsidy. Welfare loss, like net benefits to society, remains the same for all subsidy levels and equal to zero. That is, resources are used in this colonization project just up to the point where marginal benefits are equal to full marginal costs. No other uses of these resources would give a greater return.

As project size increases, gross benefits to colonizers increase. But so do public costs and at an increasing rate. Comparing the efficient size project (Q_e) with the largest size project (Q_m), benefits to colonizers increase by about 38 percent and public costs increase by 110 percent. In fact, net benefits to society at the largest size project are negative meaning that the public is paying out more for supplying colonization services than the colonizers are receiving in benefits. In addition, society is suffering a welfare loss equal to I/.1,402.7 million that could be used to produce other goods and services. This would be a very costly way of transferring income to colonizers even if they were extremely low-resource, low-income people.

Clearly, public costs are less and society is better off if the colonization project can remain at or near the efficient size. But the returns to the colonizers fixed resources may not be adequate to interest them in colonization without subsidizing part of the project costs. And if there are policy objectives other than, or in addition to, economic efficiency it may be rational for policy makers to subsidize part of the colonization costs. As the analysis shows, however, if colonization services are subsidized and priced to the colonizers at less than the full marginal cost, the colonizers will want more services, the size of the project will expand, and net social benefits will decrease. The alternative is to limit the size of the project to Q_e and ration services. The problem here is that the greater the subsidy the greater is the amount of rationing that must occur. At the extreme of full cost subsidy at Q_e size project, colonizers would be limited to less than half of the services they would like.

Annualized investment costs account for 16.5 percent of the annual project costs. At Q_e project size, subsidizing the annualized investment costs increase net benefits to colonizers by 36.4 percent and at a public cost of I/.421 (million). This means only 744,000 units would need to be rationed. Even if the project would be allowed to expand to size Q_1 which is consistent with allowing colonizers to expand their use of services to equate their marginal benefit to MFC_2, public costs remain modest, net benefits to society decrease only slightly, and welfare loss is only I/.38.3 (million). It is only when O & M costs are subsidized do we have significant changes in public costs, net social benefits, and welfare loss.

4. General Equilibrium Results. The analyses and applications so far have been in the tradition of partial equilibrium analysis. A number of potential effects of policy were assumed away so we could study and measure the primary result of a policy or policy change on natural resource project investment decisions. For example, a subsidy to colonizers in the colonization project means a resource transfer from others in society to the colonizers. What are some of the results of this policy? (1) As we saw in the previous analysis, the primary result is an increase in net benefits (and thus incomes) to the colonizers. But this is the policy result we desired. (2) The resource transfers from others in society reduced their potential incomes. Since there are generally many more people not associated with the project than there are colonizers, the effect of resource transfers on any one person in society may be minimal but it does exist. (3) If the project is allowed to increase in size beyond the Q_e level, the resource transfers are larger and the effects more

significant. In addition, aggregate welfare (GDP) is (slightly) reduced since the additional resources put into the project are returning less (income) than if those resources were used elsewhere in the economy. (4) Higher incomes for the colonizers and marginally lower incomes for the rest of society from the initial resource transfer marginally changes the demand for all goods and services. The results will be a shifting out in the demand curve for some goods and services and a shifting in of demand for others. (5) Changes in the demand for all goods and services will result in new market equilibriums and changes in all commodity and factor prices. In fact, the price(s) of the commodity(ies) produced by the colonization project will change thus shifting the marginal value product (MVP) curve for the colonizers, shifting of the project marginal benefit (MB) curve, and, potentially, changing the equilibrium price (P^e) for the project resources supplied to the colonizers. (6) Changes in the price of the project resources and changes in all factor prices from the new commodity market equilibriums identified above, changes net returns to all resources and thus to incomes of those in the project and of all in society. Potentially, we have come full circle and could expect some second round effects. Thus the general equilibrium result is a change in all commodity and factor prices, a change in the quantity of all goods and services produced and consumed, and a change in income for all population groups.

General equilibrium results usually show a relative change in incomes for different population groups. This is because the changes in commodity and factor prices will change the net returns to resources held by individuals and groups of individuals and thus change their relative incomes. Some individuals and groups will have higher incomes and others will have lower incomes. It is important to know the ultimate result of a policy change on the incomes of various population groups. If incomes of low-resource groups decrease significantly, they may have insufficient funds to purchase basic food commodities and thus put them at greater nutritional risk.

Results of a general equilibrium analysis of fertilizer subsidy in rice production for Indonesia are shown in Table 11.6 (Suprapto). The three columns across the top of the table correspond to three different policy alternatives. Column (1) represents the general equilibrium results for the initial state of the economy. Resources are assumed fully employed or at least in a state of equilibrium for the alternatives available to resource owners. Markets are assumed in equilibrium so that there is a balance between demand and supply at the current commodity prices. Household and institutional incomes are formed and are the results of resource ownership, compensation (rents) for use of resources through factor input prices, and transfers among households and institutions. The initial state of the economy includes a 37.5 percent price subsidy for fertilizer in the production of paddy rice for Bimas producers. Bimas is a government program providing inputs and services to rice producers in Indonesia.

Column (2) represents general equilibrium results under the policy alternative of complete removal of the fertilizer subsidy. All other conditions of the economy are assumed to remain the same. However, with the removal of fertilizer subsidies, Bimas rice producers are expected to use less fertilizer, rice production decreases, all commodity and input prices change, production of all other commodities change, and household and institutional incomes change.

Table 11.6. General Equilibrium Results (Indices) for Three Policy Options in Rice Production for Indonesia.

Impact Variables	Policy Options		
	Fertilizer Subsidy to Bimas Rice Producers (1)	Removal of Fertilizer Subsidy to Bimas Producers (2)	Government Transfers to Households Equal to Subsidy (3)
Input Prices			
Agricultural Laborers	1.00000	1.02532	1.02383
Production Workers	1.00000	1.02698	1.02487
Sales and Services	1.00000	1.01992	1.01894
Professional and Management	1.00000	1.02785	1.02001
Capital	1.00000	1.15340	1.14518
Household Real Income[a]			
Agricultural Laborers	1.00000	0.92462	0.96637
Agricultural Operators	1.00000	0.97382	0.99166
Non-agricultural Rural Low-income	1.00000	0.96379	0.97780
Non-agricultural Rural High-income	1.00000	0.93820	0.97419
Urban Low-income	1.00000	0.95426	0.97539
Urban High-income	1.00000	0.94218	0.97754
Total Household Income	1.00000	0.95864	0.97862
Institutional Real Income[a]			
Private Companies	1.00000	1.02826	1.01324
Government	1.00000	1.01455	1.00831
Production			
Paddy (Bimas)	1.00000	0.96386	0.96386
Paddy (Non-Bimas)	1.00000	1.04170	1.20865
Food Non-paddy	1.00000	0.88303	0.92679
Agricultural Non-food Crops	1.00000	0.96868	1.00993
Mining, Industry, Gas, etc.	1.00000	1.03857	1.03940
Chemical and Fertilizer	1.00000	1.01105	1.01238
Trade, Transport and Communication	1.00000	0.97882	0.95015
Services	1.00000	0.99147	0.99662
Total Gross Output	1.00000	0.98539	0.98844
Commodity Prices			
Rice	1.00000	1.07062	1.06586
Food Non-rice	1.00000	1.15830	1.13220
Agricultural Non-food	1.00000	1.11742	1.11210
Mining, Industry, Gas, etc.	1.00000	1.12993	1.11580
Chemical and Fertilizer	1.00000	1.12783	1.11355
Trade, Transport and Communication	1.00000	1.09696	1.04385
Services	1.00000	1.07[illegible]2	1.06972
Aggregate Price Index[b]	1.00000	1.1210[illegible]	1.09853
Rice Consumption			
Agricultural Laborers	1.00000	0.93992	0.98695
Agricultural Operators	1.00000	0.97809	1.00000
Non-agricultural Rural Low-income	1.00000	0.97302	0.99374
Non-agricultural Rural High-income	1.00000	0.95449	0.99320
Urban Low-income	1.00000	0.96792	0.9[illegible]945
Urban High-income	1.00000	0.92766	0.99923

[a]Household and institutional income are deflated by the aggregate price index.

[b]Computed as the sum of nominal price indices weighted by the base year average budget share of the commodity in national consumption.

Column (3) shows the results of removing the fertilizer subsidy in Bimas rice production but including resource transfers from the government to household groups equal to the government savings from removing the subsidy and in proportion to the income loss for each household group after the subsidy removal. This policy represents a more market-oriented alternative to maintaining incomes of low-resource groups. Commodity exports were assumed constant for all policy alternatives. Commodity imports were assumed noncompetitive and proportional to sector output. Investment and government final demand expenditures were assumed constant. Therefore, equilibrium between government revenue and government expenditure, savings and investment, and rest-of-world inflow and outflow (balance of payments) was not assumed. The results are indicative of changes in commodity and factor markets for a given policy change but not adjusted for changes in foreign exchange rates, interest rates, or government budget deficits or excesses.

The rows of Table 11.6 represent some of the variables affected by the general equilibrium results and include input prices, household and institutional real incomes, commodity production, commodity prices, and rice consumption. Column (1) is a column of ones indicating equilibrium results of the initial conditions. These are the results of normalizing the data from an available Social Accounting Matrix (SAM) for Indonesia. The actual level of the variables may be inferred from the SAM. The General Equilibrium Model (GEM) itself is available in Suprapto and was validated by using the SAM data to generate the initial equilibrium results. Column (1) is referred to as the base results and includes the effects of the 37.5 percent fertilizer price subsidy for paddy (Bimas) production.

The objective of the policy simulation leading to the results in Column (2) is to analyze the impact of reducing fertilizer subsidy to paddy (Bimas) producers on overall rice production, sector output, factor and commodity prices, household and institutional income, and basic nutrition. The initial effect of a decrease in fertilizer subsidy is an increase in private cost of paddy production by Bimas producers. Since fertilizer costs are about 7 percent of total costs of paddy production, total elimination of fertilizer subsidy increases cost of paddy by about 9 percent.

The reduction in fertilizer subsidy causes all factor prices to increase. In general, input prices are expected to increase with fixed resources and a reduction in fertilizer subsidy. The price of capital, for example, increases from 1.00 in the base solution to 1.1534 (a 15.3 percent increase). Factor prices for labor increase from 2.0 percent to 2.8 percent, depending on the labor category. The significant impact on capital is due in part to land rents which are classified as returns to capital. As a result of the reduction in fertilizer subsidy, the demand for capital (including land) and labor shift to the right, thus increasing all factor prices.

Increases in all factor prices means a shift to the left in the commodity supply functions. Thus, given constant commodity demand functions, reducing the fertilizer subsidy increases the total private costs of producing all commodities, shifting the supply functions to the left and causing all commodity prices to increase. As shown in Column (2) of Table 11.6, removing all the fertilizer subsidy causes the commodity prices of rice, food

non-rice, agricultural non-food, mining and industry, chemical and fertilizer, transport and communication, and services to increase by 7.1 percent, 15.8 percent, 11.7 percent, 12.9 percent, 12.8 percent, 9.7 percent, and 8.0 percent, respectively. The aggregate price index increases by 12.1 percent. Interestingly, the price of rice increases less than the aggregate price index, while the price of food non-rice and agricultural non-food increases more than the price of rice and the price of food non-rice increases more than the aggregate price index.

Removing the fertilizer subsidy to Bimas paddy producers causes output of food non-rice to decrease by 12 percent and the output of agricultural non-food to decrease by 3 percent. This implies a reduction in total supply for both commodities. Holding the total commodity exports constant at the base SAM level and reducing total supply causes a significant reduction in quantity sold in the domestic market. Consequently, with world price remaining at 1.00, domestic price increases. This price increase is 15.8 percent for food non-rice and 11.7 percent for agricultural non-food. Adoption of this policy hurts domestic consumers since they must pay higher prices for those commodities.

The impact of removing the fertilizer subsidy to Bimas paddy producers on household real income is found to vary across the household classes. All household classes experienced a reduction in income. Impact on incomes can largely be explained by the relative factor income shares. Agricultural laborers had the largest income reduction at about 8 percent and it is the group with the highest aggregate labor income share. Agricultural operators had the lowest income reduction at 3 percent and it had the lowest aggregate labor income share and the highest capital income share. This also would be the group with the highest land rent share. Non-agricultural rural high-income and urban high-income groups had lower capital income shares than non-agricultural rural low-income and urban low-income groups and hence, the former two groups had higher income reductions.

To state the reverse of this -- the Bimas fertilizer subsidy program has benefitted agricultural laborers and consumers the most. It has benefitted agricultural laborers since it has reduced factor prices less for labor, which they have in abundance, relative to capital, which they have very little. It has benefitted consumers by holding down the relative price for rice, freeing up resources for the production of food non-rice (an increase in production of 12 percent), and reducing the aggregate price index by about 12 percent. The urban and non-agricultural rural groups have particularly benefitted from the reduced aggregate price index.

Agricultural producers (operators), on the other hand, have not benefitted as much from the Bimas fertilizer subsidy program. Their incomes have increased by only 3 percent versus an overall income increase of 4 percent. Fertilizer is a direct substitute for land and since their capital income share is 53 percent versus an aggregate labor income share of 43 percent, they have benefitted less from the relative changes in factor prices and the reduced aggregate price level.

Removing the fertilizer subsidy has the effect of reducing resources available to the economy equal to the cost of the subsidy program. These resources are taken out of the economy and not put back by means of a tax reduction or a government transfer to households. The overall effect is a reduction in real incomes to households by 4 percent.

This is offset in part by a 2.8 percent increase in real income to private companies. The policy alternative to be discussed next returns these resources to the economy by means of government transfers to households equal to the reduction in the cost of the fertilizer subsidy program.

Reducing the fertilizer subsidy has a smaller than expected change on rice production. The reason, in part, is that even though Bimas farmers decrease their rice production in response to a reduction in fertilizer subsidy, non-Bimas farmers increase their production in response to an increase in the price of rice. The effect of removing the fertilizer subsidy to Bimas producers is a reduction in total rice production of 2.6 percent. Therefore, adoption of this policy might not harm "food self-sufficiency" or "basic nutrition" to the extent previously thought.

The production indices show the dramatic effect fertilizer subsidy to paddy (Bimas) producers has on the food non-rice and agricultural non-food sectors. Removing the fertilizer subsidy reduces output of the food non-rice sector by about 12 percent and the agricultural non-food sector by about 3 percent. Or stated in the reverse, the fertilizer subsidy program of Bimas has freed up sufficient resources to expand the food non-rice sector by 12 percent and the agricultural non-food sector by 3 percent. With a strong export market for commodities of the agricultural non-food sector, the effects of the Bimas fertilizer subsidy program may be important in generating foreign exchange and increasing farm incomes.

As shown above, removing the fertilizer subsidy to Bimas paddy producers causes household real incomes to decrease. This implies that household purchasing power also decreases. A basic concern is the effect of this decrease on basic nutrition. Rice is the basic food commodity in the Indonesian diet accounting for a 25 percent budget share in low income groups. Removing the fertilizer subsidy decreases rice consumption by 6.0 percent for agricultural laborers, 2.2 percent by agricultural operators, and 2.7 percent by non-agricultural rural low-income households. Since these are the three lowest income groups, the Indonesian government may wish to consider income transfers to these groups equal to the reduction caused by removing the fertilizer subsidy. Results of this policy alternative are shown in Column (3) of Table 11.6.

Removing the fertilizer subsidy decreases household real incomes for all household classes. This is expected because government is taking resources out of the system equal to the subsidy. The objective of the policy simulation leading to the results of Column (3) is to analyze the effect of giving the resources back to the system by means of government transfers to households. An alternative means of giving the resources back to the system would be to reduce government taxes. However, uniformly reducing taxation rates would undoubtedly increase income inequalities among household classes. Thus, results of the previous policy simulation were used to weight government transfer payments to households. Households experiencing a greater reduction in real income received higher weights in income transfers, and vice versa. The purpose is to restore household real incomes as close as possible to the base SAM. By so doing, household purchasing power remains close to the same level as before removal of the fertilizer subsidy.

Results of this policy simulation show that government transfers to households were not able to restore incomes completely to the base SAM levels but came within 2 percentage points in most cases. Removing the fertilizer subsidy causes all factor input prices to increase just as with the previous policy simulation but at a slightly reduced rate. The reasons for higher factor prices are as before -- with fertilizer subsidy removed, farmers tend to use more land (capital) and labor in paddy production, less is available for production in other sectors, and this results in a rise in all factor input prices. Production changes are most significant in paddy (non-Bimas), food non-paddy, and agricultural non-food crops. In comparing Column (3) with Column (2), production of paddy (non-Bimas) increases from an index of 1.042 to 1.209, food non-paddy increases from 0.883 to 0.927, and agricultural non-food crops increases from 0.969 to 1.010. The aggregate price index is slightly lower for the policy alternative in Column (3) compared to the policy alternative in Column (2). Restoring income levels restores purchasing power and brings rice consumption levels back closer to the base SAM for all household classes. Agricultural laborers is the household group that has the highest reduction in rice consumption and it is only 1.3 percent less than the base SAM level.

The previous analysis required three main information and analytical components: (1) a Social Accounting Matrix (SAM) for Indonesia, (2) specification of a General Equilibrium Model (GEM), and (3) modeling of rice producers response to fertilizer subsidies. These three components for the Indonesian example are presented in (Suprapto). Social Accounting Methodology is available in (Pyatt and Round). Specification of a GEM is contained in (Hazel and Norton) and (Dervis, de Melo, and Robinson).

ACTIVITIES

1. A farmer participates in a natural resource project. He is able to combine resources from the project with his own fixed resources to produce maize. He can obtain project resources in increments of 20 units. The following represents production response from the resources (farm-gate price of maize is M1.10/kg):

Project Resources (units)	Maize Production (kg.)	Marginal Physical Product (MPP) (kg/unit)	Value of Production (M)	Marginal Value Product (MVP) (M/unit)
0	5,200	--	5,720	--
20	5,660	23	6,226	25.30
40	6,080	21	6,688	23.10
60	6,460	19	7,106	20.90
80	6,800	17	7,480	18.70
100	7,100	15	7,810	16.50
120	7,360	13	8,096	14.30
140	7,580	11	8,338	12.10
160	7,760	9	8,536	9.90
180	7,900	7	8,690	7.70
200	8,000	5	8,800	5.50
220	8,060	3	8,866	3.30
240	8,080	1	8,888	1.10
260	8,060	(1)	8,866	(1.10)

a. Graph the marginal value product (MVP) against the quantity of project resources.

b. Show how may project resources the farmer would use if he paid M4.00 per unit. How much maize would the farmer produce? What would be the return on the farmer's fixed resources?

c. Show that the MVP has the following functional form:

$$MVP = 26.4 - 0.11\,q$$

where q is the quantity of project resources used by the farmer. Show what the functional forms are for MPP, production, total revenue, and total cost.

2. Assume there are 100 farmers participating in the natural resource project and that each farmer has the same MVP shown in activity 1 above. Show that the marginal benefit curve for the project is the following:

$$MB = \sum_{i=1}^{n} MVP_i = 2{,}640 - 0.11Q$$

where $Q = 100q$

n = number of farmers = 100

a. If the following represents the marginal cost of expanding the project size

$$MC = 400 + 0.06Q$$

what is the optimum size of the project?

b. If the policy makers wish to price the project resource (P_r) where MC = MB show that this price to the farmer is equal to

$$\frac{\sum_{i=1}^{n} MVP_i}{n} = \frac{MB}{n} = \frac{MC}{n} = P_r$$

c. How many project resources will the farmer use?

d. In the aggregate, how many project resources will the farmers demand?

3. Compare the distribution of benefits and costs under the pricing policy of marginal cost pricing versus average cost pricing for the project in activity 2.

4. How would the results of activity 3 differ if there was a price support for maize equal to M1.50/kg?

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12. INTERNATIONAL TRADE POLICIES

*by David Henneberry and Shida Henneberry**

OBJECTIVES

The general objective of this chapter is to analyze the classical welfare effects of commonly employed price policies and quantitative restrictions in the international marketplace. The price policies analyzed are import taxes (more commonly referred to as duties or tariffs), export taxes, import subsidies, and export subsidies. The quantitative restrictions analyzed are import quotas and export quotas. Each policy will be presented from the perspective of a small country and a large country in international markets. A partial equilibrium analysis will identify the impact of the policy on producers, consumers, government revenues, and net social welfare within the society. A shortcoming of the analysis is that tertiary impacts on consumers, producers, and governments located outside of the country adopting the policy are ignored.

KEY POINTS

Small Country Versus Large Country

1. In the analysis of international trade policy impacts, it is crucial to differentiate between a "small" and "large" country. In this context, small and large do not refer to the population, geographic size, or gross national product of the country in question. Rather, small and large refer to the relative size of the country in the market for the good or commodity analyzed.
2. A small country's imports and/or exports are so small relative to the volume

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of world trade that it does affect the world price of the commodity through the policies it adopts. Conversely, a large country's policies do have an impact on the world price. It is the induced change in the world price level that makes it so important to differentiate between large and small country policy impacts. It may be the case that a policy which decreases net social welfare in a small country will increase net social welfare in a large country.

3. Because the concept of "large" country and "small" country refers only to a particular commodity, it is possible for a country to be large with respect to one commodity (e.g. corn) and small with respect to another (e.g. honey). As the level of production varies over time and across regions, large country status may be relevant for a given country in some years but not in others. Policy makers should consider which scenario to employ based upon the specific circumstances in each case.

Net Social Welfare

In the classical welfare analysis of trade policies which follows, a two-stage process will be employed to derive the net social welfare impact on society. First, the impact on consumers and producers will be identified as changes in consumer and producer surplus (see Chapter 6). Government revenues and expenditures will be identified. Second, the gains and losses accruing to these groups will be balanced against one another to deduce the net impact on societal welfare.

The implicit assumption is that a one dollar gain to consumers exactly offsets a one dollar loss to producers or the government (and vice-versa). In other words, the marginal utility of money is held constant across all groups. In this context, a "net social welfare gain" should be interpreted to mean that the net value of the gain exceeds the net value of the losses so that the gainers could fully compensate the losers and have positive gains remaining. Presumably, these redistributions would occur through government taxation and expenditures. This analysis is not concerned with whether or not such redistributions do occur, nor with how they would occur, but only with whether or not they could occur. Thus, in order to simplify and clarify the presentation of the classical welfare analysis, some important issues will not be addressed.

Government Revenues and Expenditures

The calculation of government revenues and expenditures is fairly straightforward for the price policies of taxes and subsidies. For the quantitative restrictions (quotas), it is not intuitively clear how government revenues occur. Import quotas and export quotas have an economic value equal to the difference between the world price and the domestic price multiplied by the quantity of trade approved under the quota system. If the government does not charge for the quota, these gains will accrue to the traders. In this analysis, it is assumed that the government sells the right to import or export under the quota in a perfectly competitive auction market. The government revenues thus equal the full economic value of the quota.

The Optimal Policy Problem

For the case of a large country in international markets, the net welfare effect of a specific policy depends upon the net change in world and domestic prices that result from the policy. Three outcomes are possible: a net social welfare gain, loss, or stalemate. The optimal policy is one for which the size of the tax or quota is calculated to maximize the difference between net gains and losses (subsidies always result in a net social welfare loss in this analysis). The mathematical formulation of the optimal policy problem is not addressed in this chapter. Rather, the analysis will simply compare the size of the counter balancing areas as identified on supply and demand graphs. Using procedures outlined in Chapter 6, the designated areas could be measured and used by decision makers to compare costs and benefits of alternative policies.

The World Price and Domestic Price

In this analysis, it is always assumed that initially world prices are directly translated into domestic prices. Only after the adoption of a policy does a difference between world and domestic prices emerge. Furthermore, it is assumed that domestic prices apply equally to consumers and producers. The differential price policies such as consumer price ceilings and producer price floors that are adopted in many countries are not incorporated into this analysis. While the student may wish to consider the many interesting policy combinations that are possible, their presence here would confuse the exposition of the basic international trade policies that are the focus of this chapter.

CONCEPTS AND APPLICATIONS

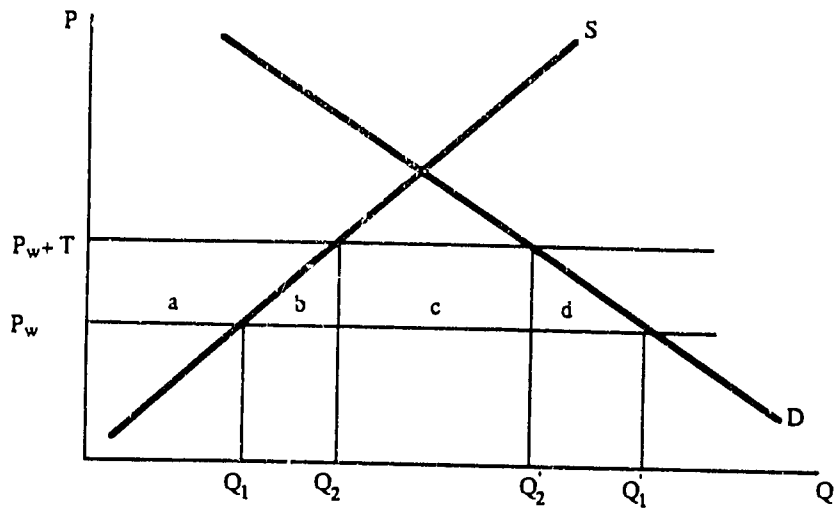
The following section of this chapter provides the graphical analysis of the following policies:

Figure 12.1. Import Tax - Small Country
Figure 12.2. Import Tax - Large Country
Figure 12.3. Export Tax - Small Country
Figure 12.4. Export Tax - Large Country
Figure 12.5. Import Subsidy - Small Country
Figure 12.6. Import Subsidy - Large Country
Figure 12.7. Export Subsidy - Small Country
Figure 12.8. Export Subsidy - Large Country
Figure 12.9. Export Quota - Small Country
Figure 12.10. Export Quota - Large Country
Figure 12.11. Import Quota - Small Country
Figure 12.12. Import Quota - Large Country

For each graph, a key to the symbols used is provided. A "sequence of steps" will briefly lead the reader through the policy implementation process and attempt to amplify the

temporal process concealed by comparative statics. A "welfare analysis" section illustrates the changes in consumer surplus, producer surplus, government revenue, and net social welfare in the society. In all cases, a small letter is used to indicate an area and a capital letter a point on the graphs. A brief summary is provided after the large country graph for each policy.

Figure 12.1. Welfare Analysis of an Import Tax (Tariff) for the Small Country Case



P_W	=	world price (price faced by producers and consumers in the country before the tariff)
$P_W + T$	=	price faced by consumers and producers in the country with the tariff
D	=	domestic demand
S	=	domestic supply
$Q'_1 - Q_1$	=	imports before tariff
$Q'_2 - Q_2$	=	imports after tariff

Sequence of Events

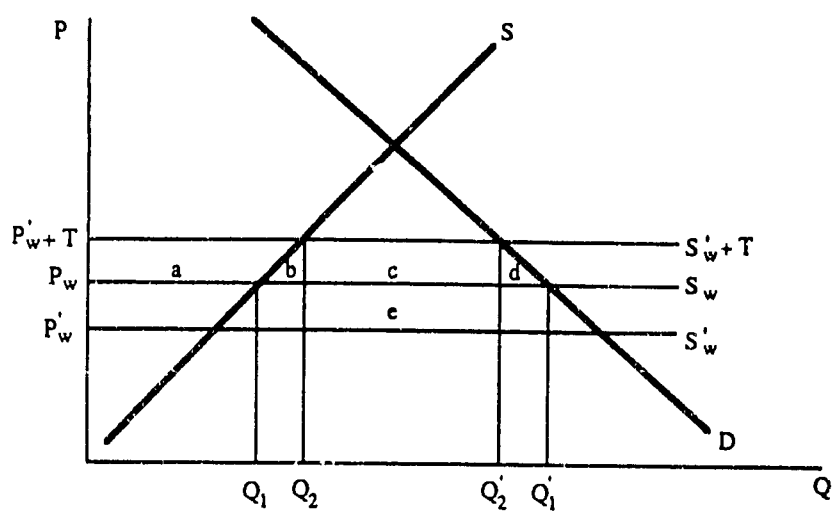
1. Country imposes an import tax (tariff) of T on the good
2. As a result the price faced by producers and consumers in the country rises from P_W to $P_W + T$
3. Imports fall from $Q'_1 - Q_1$ to $Q'_2 - Q_2$.

Welfare Analysis

Consumer surplus loss	=	- a - b - c - d
Producer surplus gain	=	+ a
Government revenue gain	=	+ c
Net social welfare loss	=	- b - d

Note: Government revenue gain is calculated by multiplying the amount of imports after the tariff (in this case $Q'_2 - Q_2$) by the amount of the tariff $[(P_w + T) - P_w = T]$ which gives the area c.

Figure 12.2. Welfare Analysis of an Import Tax (Tariff) for the Large Country Case



P_w	=	world price (price faced by producers and consumers in the country before the tariff)
P'_w	=	reduced world price as a result of decreased world demand (resulting from tariff)
$P'_w + T$	=	price faced by consumers and producers in the country with the tariff
D	=	domestic demand
S	=	domestic supply
$Q'_1 - Q_1$	=	imports before tariff
$Q'_2 - Q_2$	=	imports after tariff

Sequence of Events

1. Country imposes tariff
2. Due to the tariff the country reduces imports. Because this is a large country, the world

demand decreases as a result of the reduction in imports. This leads to a decline in world price from P_w to P'_w.

3. As a result the price faced by consumers and producers in the country goes from P_w to $P'_w + T$.
4. Imports fall from $Q'_1 - Q_1$ to $Q'_2 - Q_2$.

Welfare Analysis

Consumer surplus loss	=	- a - b - c - d
Producer surplus gain	=	+ a
Government revenue gain	=	+ c + e
Net social welfare loss/gain	=	+ e - b - d

Note: Government revenue gain equals the amount of the tariff $[(P'_w + T) - P'_w = T]$ times the quantity imported after the tariff $(Q'_2 - Q_2)$ which equals the area c + e.

The optimum tariff argument:

The country gains from the tariff when e > b + d

The country looses from the tariff when e < b + d

The optimum tariff would be that tariff which maximizes the area e - (b + d).

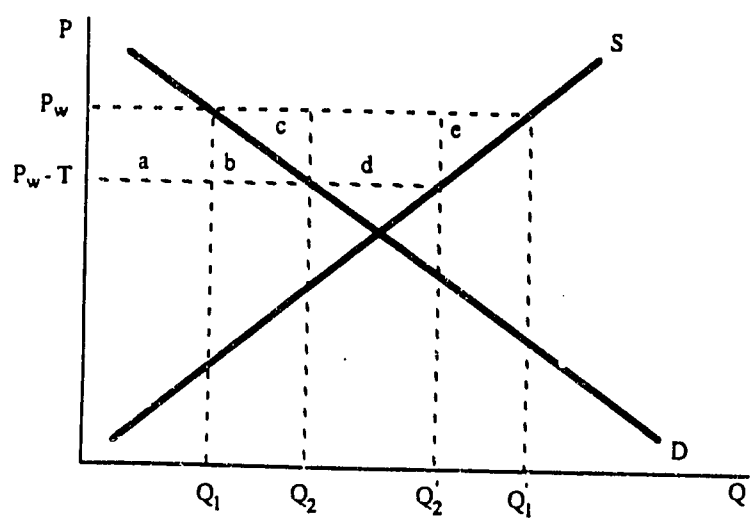
Tariff Summary (for the large and small country cases):

A tariff is designed to reduce the quantity of imports by increasing domestic production and decreasing and domestic consumption.

In the small country case, a tariff always results in a net social welfare loss.

In the large country case a tariff may result in a net social welfare gain or a net social welfare loss.

Figure 12.3. Welfare Analysis of an Import Tax for the Small Country Case



P_w = price before the export tax
$P_w - T$ = price with the export tax
$Q_1' - Q_1$ = exports before the export tax
$Q_2' - Q_2$ = exports with the export tax

Sequence of Events:

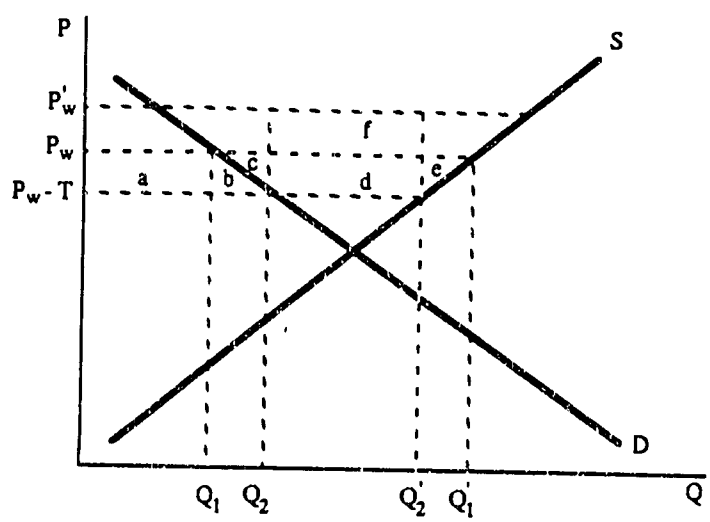
1. A tax is imposed on exports resulting in a price reduction from P_w to $P_w - T$.
2. This results in a reduction of exports from $Q_1' - Q_1$ to $Q_2' - Q_2$.

Welfare Analysis:

Consumer surplus gain	=	+ a + b
Producer surplus loss	=	- a - b - c - d - e
Government revenue gain	=	+ d
Net social welfare loss	=	- c - e

Note: Government revenue gain $= P_w - (P_w - T) \times$ exports after the export tax
$= T \times (Q_2' - Q_2)$
= area d

Figure 12.4. Welfare Analysis of an Export Tax for the Large Country Case



P_w = world price before the export tax
P_w' = world price resulting from a reduction in world supply which occurs as a result of reduced exports by the large country

$P'_w - T$ = price faced by domestic producers and consumers as a result of the export tax
$Q'_1 - Q_1$ = exports before the export tax
$Q'_2 - Q_2$ = exports with the export tax

Sequence of Events:

1. Country imposes a tax on exports. This results in a decline in exports by that country. Because the country is large, the decline in exports by the country will result in a reduction in world supply. This in turn will cause world price to rise from P_w to P'_w.
2. The final price faced by consumers and producers in the country will be $P'_w - T$.
3. Exports fall from $Q'_1 - Q_1$ to $Q'_2 - Q_2$.

Welfare Analysis:

Consumer surplus gain	=	+ a + b
Producer surplus loss	=	- a - b - c - d - e
Government revenue gain	=	+ f + d
Net social welfare loss/gain	=	+ f - (c + e)

Country gains when f > c + e
Country loses when f < c + e
Optimal export tax is one that maximizes the area f - (c + e)

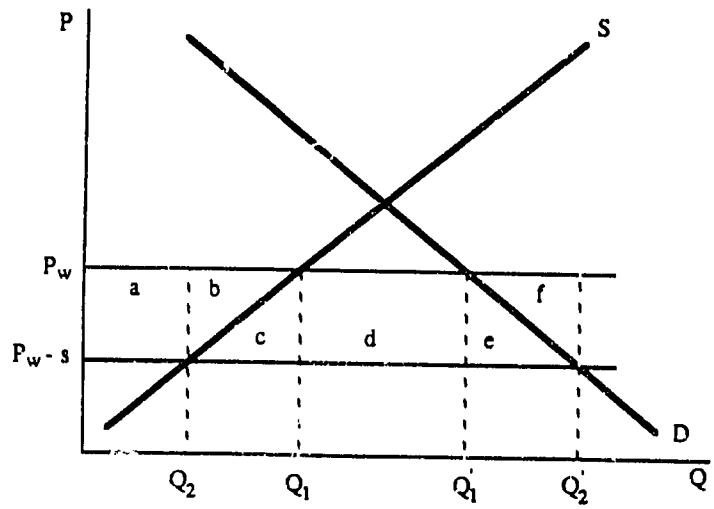
Summary of Export Tax (for the large and small country cases)

The export tax is a policy that improves consumer welfare in the country. This is accomplished by reducing the price and increasing domestic consumption.

For a small country an export tax always results in a net social welfare loss.

For a large country an export tax may result in a net social welfare gain or loss. It should be noted, however, that a net welfare gain to the country in question is more than offset by welfare losses in other countries so the world is worse off from an export tax.

Figure 12.5. Welfare Analysis of an Import Subsidy for the Small Country Case



P_w	=	world price (price faced by domestic producers and consumers before the subsidy)
P_w - s	=	price faced by domestic producers and consumers with the subsidy
S	=	domestic supply
$Q_1' - Q_1$	=	imports before the subsidy
$Q_2' - Q_2$	=	imports with the subsidy

Sequence of Events:

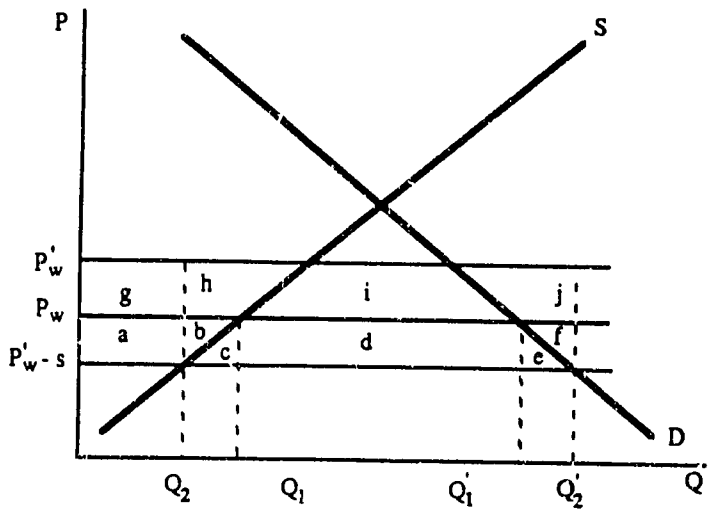
1. Country places a subsidy on the good resulting in a downward shift in price from P_w to P_w - s.
2. This results in an increase in imports from $Q_1' - Q_1$ to $Q_2' - Q_2$.

Welfare Analysis:

Consumer surplus gain	=	+ a + b + c + d + e
Producer surplus loss	=	- a - b
Government revenue loss	=	- b - c - d - e - f
Net social welfare loss	=	- b - f

Note: Government revenue loss is calculated by multiplying the amount of the subsidy, which is equal to S, by the quantity imported after the subsidy, which is equal to $Q_2' - Q_2$. This results in the area b + c + d + e + f.

Figure 12.6. Welfare Analysis of an Import Subsidy for the Large Country Case



P_w	=	world price before the subsidy
P'_w	=	world price resulting from an increase in demand which occurs as a result of the import subsidy
$P'_w - s$	=	price faced by domestic producers and consumers with the subsidy
S	=	domestic supply
$Q'_1 - Q_1$	=	imports before the subsidy
$Q'_2 - Q_2$	=	imports with the subsidy

Sequence of Events:

1. Country places an import subsidy on the good
2. This results in an increase in imports by the country. Since the country is large, the increase in imports causes world demand to increase. This results in a price increase from P_w to P'_w.
3. The final price faced by domestic producers and consumers equals the new world price P'_w minus the subsidy $S = P'_w - s$.

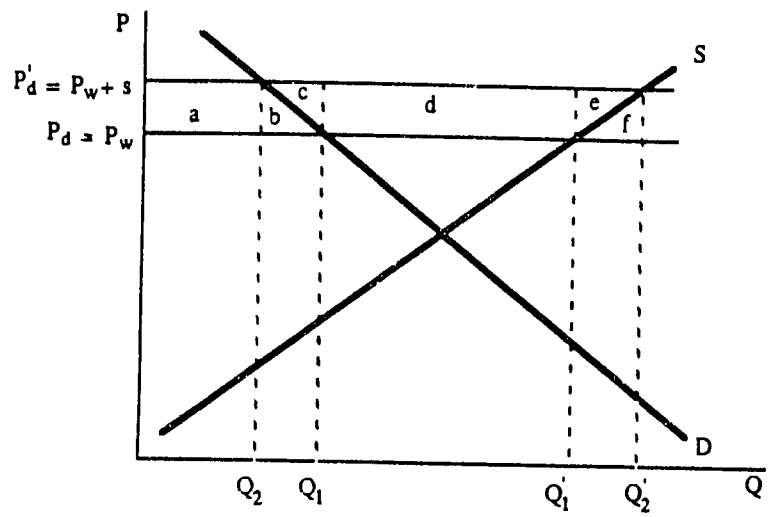
Welfare Analysis:

Consumer surplus gain	=	+ a + b + c + d + e
Producer surplus loss	=	- a - b
Government revenue loss	=	- b - c - d - e - f - h - i - j
Net social welfare loss	=	- b - f - h - i - j

Summary of Import Subsidy (for the large and small country cases):

An import subsidy is a policy designed to increase consumer welfare. The result of an import subsidy is reduced prices and increased imports. However, in both the small and large country cases a net social welfare loss results from this policy.

Figure 12.7. Welfare Analysis of an Export Subsidy for the Small Country Case



$P_d = P_w$	=	world price and domestic price before the subsidy
$P_d' = P_w + s$	=	domestic price after the subsidy has been imposed
$Q_1' - Q_1$	=	exports before the subsidy
$Q_2' - Q_2$	=	exports with the subsidy

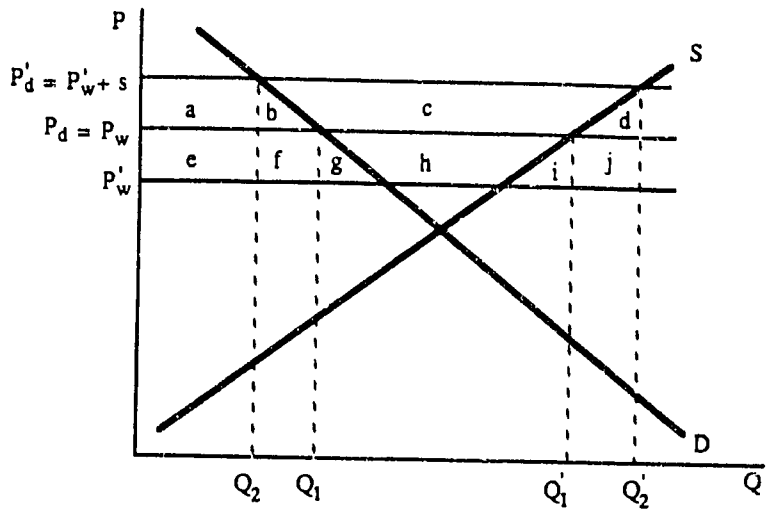
Sequence of Events:

1. A subsidy of s is placed on exports causing domestic price to rise from P_d to P_d'.
2. This results in an increase in exports from $Q_1' - Q_1$ to $Q_2' - Q_2$ from an increase in the amount supplied by domestic producers (Q_2') and a reduction in the amount purchased by domestic consumers (Q_2).

Welfare Analysis:

Consumer surplus loss	=	- a - b
Producer surplus gain	=	+ a + b + c + d + e
Government revenue loss	=	- b - c - d - e - f
Net social welfare loss	=	- b - f

Figure 12.8. Welfare Analysis of an Export Subsidy for the Large Country Case



$P_d = P_w$	=	world price and domestic price before the subsidy
P'_w	=	world price after the subsidy is imposed which results from an increase in world supply due to increased exports
$P'_d = P'_w + s$	=	final price faced by domestic producers and consumers after the world price has adjusted to the increase in supply

Sequence of Events:

1. Country imposes an export subsidy on the good.
2. In response to the subsidy the country increases exports causing world supply to increase which in turn results in a world price decrease from P_w to P'_w.
3. The final price faced by domestic producers and consumers equals the new world price P'_w + the subsidy $s = P'_w + s = P'_d$.
4. Exports have been increased from $Q'_1 - Q_1$ to $Q'_2 - Q_2$.

Welfare Analysis:

Consumer surplus loss	=	- a - b
Producer surplus gain	=	+ a + b + c
Government revenue loss	=	- b - c - d - f - g - h - i - j
Net social welfare loss	=	- b - d - f - g - h - i - j

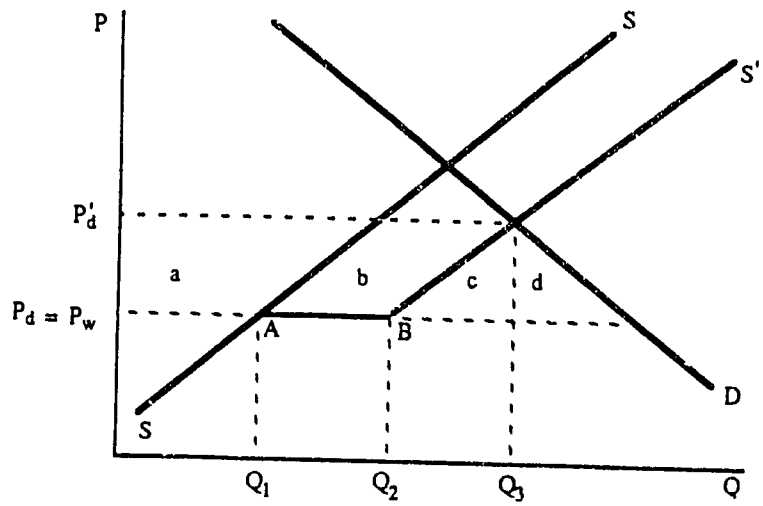
Government revenue loss	=	amount of the subsidy x exports after the subsidy
	=	$s \times (Q'_2 - Q_2)$

Summary of Export Subsidies (for the large and small country cases)

An export subsidy is designed to aid producers in the country in two ways: (1) by increasing prices paid to the producer and (2) by increasing the amount exported.

In both the small and large country cases export subsidies result in a net social welfare loss.

Figure 12.9. Welfare Analysis of an Import Quota for the Small Country Case



$P_d = P_w$	=	domestic price (which is equal to world price) before the quota
P_d'	=	domestic price after the quota has been imposed
SS	=	supply curve faced by domestic producers and consumers without the quota
SABS'	=	supply curve faced by domestic producers and consumers with the quota
$Q_2 - Q_1$	=	import quota

Sequence of Events:

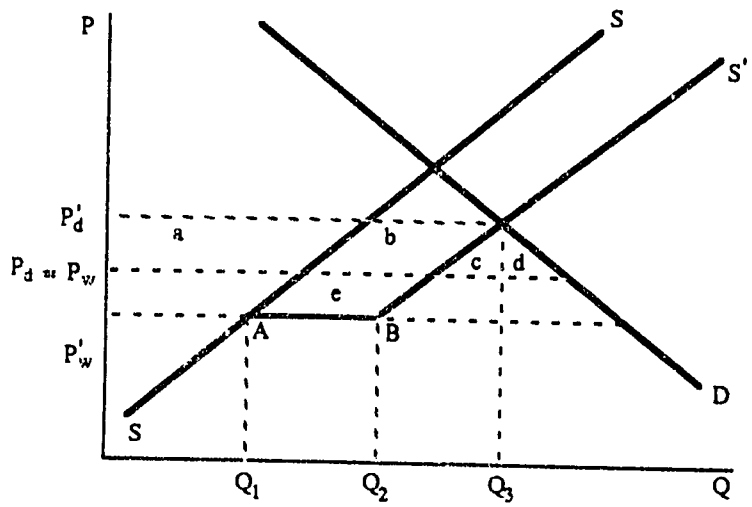
1. A quota of $Q_2 - Q_1$ is placed on imports.
2. This results in a supply decrease causing the domestic price to rise from P_d to P_d' where a new equilibrium is attained.
3. Domestic production increases from Q_1 to $Q_1 + Q_3 - Q_2$ with the quota.

Welfare Analysis:

Consumer surplus loss	=	- a - b - c - d
Producer surplus gain	=	+ a
Government revenue gain	=	+ b
Net social welfare loss	=	- c - d

Note: In the case of an import quota the government earns revenue if it sells the right to import. However, if the government allows traders to import without charge the welfare benefits accrue to the traders themselves.

Figure 12.10. Welfare Analysis of an Import Quota for the Large Country Case



$P_d = P_w$	=	domestic price (which is equal to world price) before the quota
P'_w	=	world price resulting from reduced demand due to the quota
P'_d	=	domestic price after the quota is placed in effect
$Q_2 - Q_1$	=	quota
SS	=	domestic supply before the quota
SABS'	=	Supply curve faced by domestic producers and consumers with the quota

Sequence of Events:

1. A quota of $Q_2 - Q_1$ is placed on imports. This causes world demand to decline (since this is a large country) resulting in a decline in world price from P_w to P'_w.
2. The result of the quota is a supply decrease in the country which causes the domestic price to rise from P_d to P'_d where a new equilibrium is attained.

Welfare Analysis:

Consumer surplus loss	=	- a - b - c - d
Producer surplus gain	=	+ a
Government revenue gain	=	+ b + e
Net social welfare loss/gain	=	+ e - (c + d)

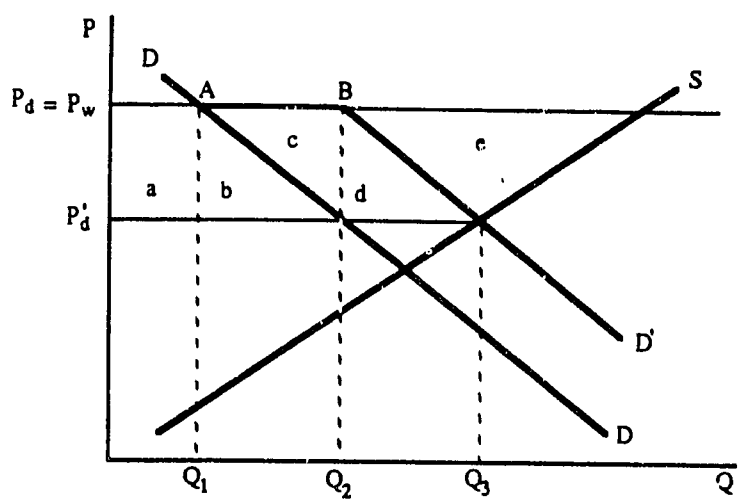
There is a net social welfare gain if e > c + d
There is a net social welfare loss if e < c + d
The optimal import quota is that quota that maximizes the area e - (c + d).

Summary of Import Quotas (for the large and small country cases)

Import quotas are designed to increase the welfare of domestic producers.

In the small country case import quotas result in a net social welfare loss. In the large country case import quotas may result in a net social gain or loss.

Figure 12.11. Welfare Analysis of an Export Quota for the Small Country Case



$P_d = P_w$	=	domestic price (which is equal to the world price) before the quota is imposed
P'_d	=	domestic price after the quota has been placed in effect
$Q_2 - Q_1$	=	export quota
DD	=	original demand curve
DABD'	=	demand curve after the quota has been placed in effect

Sequence of Events:

1. A quota of $Q_2 - Q_1$ is placed on exports. This results in an excess supply in the country which in turn causes domestic price to fall from P_d to P_d' where a new equilibrium is reached.
2. Domestic consumption is increased from Q_1 to $Q_1 + Q_3 - Q_2$ as a result of the new equilibrium attained from the intersection of the new demand curve DABD' and the supply curve S.

Welfare Analysis:

Consumer surplus gain	=	+ a + b
Producer surplus loss	=	- a - b - c - d - e
Government revenue gain	=	+ b + c
Net social welfare loss	=	- e (since b = d)

Proof that b = d:

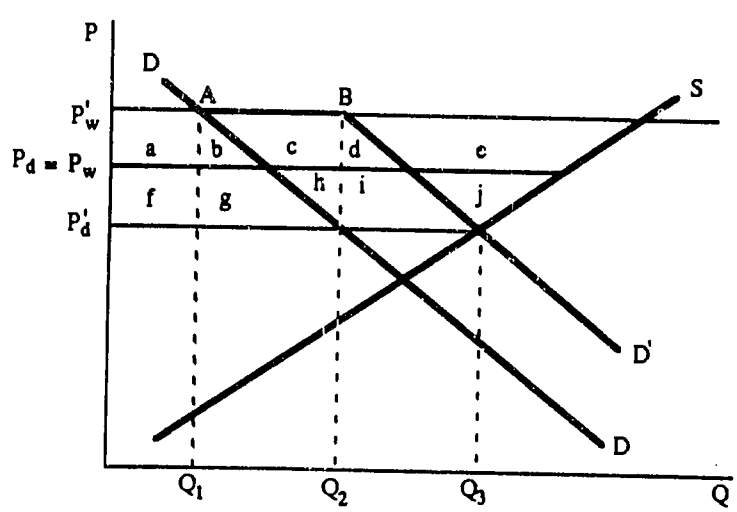
Both b and d are right triangles.

It is clear that the vertical sides of both b and d are equal to the distance $P_d - P_d'$.

It is also obvious that the angles of the hypotenuses for both b and d are equal because they are formed from lines that are parallel.

Because both triangles have 2 angles in common (the right angle and the upper angle) then they must also share a common third angle. And because the distance of the left side for both triangles is obviously the same, the other sides must be the same length also. As a result, both triangles possess the same area.

Figure 12.12. Welfare Analysis of an Export Quota for the Large Country Case



$P_d = P_w$	=	domestic price (which is equal to world price) before the quota
P_w'	=	world price resulting from reduced exports
P_d'	=	domestic price after the export quota has been imposed
$Q_2 - Q_1$	=	export quota
DD	=	original demand curve
DABD'	=	demand curve resulting from the quota

Sequence of Events:

1. A quota of $Q_2 - Q_1$ is placed on exports. As a result exports decrease causing a decrease in world supply. This causes world price to increase from P_w to P_w'.
2. The reduction in exports results in an excess domestic supply which causes price to fall from P_d to P_d' where a new equilibrium is attained

Welfare Analysis:

Consumer surplus gain	=	+ f + g
Producer surplus loss	=	- f - g - h - i - j
Government revenue gain	=	+ b + c + g + h
Net social welfare loss/gain	=	+ b + c + g - i - j

Because g = i, the net social welfare gain (loss) = (b + c) - j
There is a gain if b + c > j
There is a loss if b + c < j
The welfare maximizing export quota for the country maximizes the area (b + c) - j.

Note: b + g = d + i can be shown in the same way as described under Figure 12.11. It is clear that b = d, therefore g must equal i.

Summary of Export Quotas (for the large and small country cases)

Export quotas are designed to increase consumer welfare. In the small country case net social welfare losses occur with an export quota. In the large country case gains or losses may occur.

Summary

The international trade policies discussed above have clear, identifiable impacts on consumers, producers, government revenues, and net social welfare. An impact summary of international trade policies is presented in Table 12.1. Impacts can be quantified using procedures outlined in Chapter 6.

Small countries have less advantageous policy options than large countries. For all

policies analyzed, the small country case resulted in an unambiguous decrease in net social welfare. In the large country case, only import subsidies and export subsidies resulted in an unambiguous decrease in net social welfare. When large countries adopt import taxes, export taxes, import quotas, or export quotas, the opportunity exists to use their large country market power to manipulate world prices so as to attain an increase in net social welfare from the policy. However, a poorly constructed policy could potentially decrease net social welfare in large countries. And a policy that increases welfare in a large country may reduce welfare in other countries and the world as a whole.

Consumers benefit from export taxes, import subsidies, and export quotas in both the large and small country cases. Consumer welfare decreases from import taxes, export subsidies, and import quotas.

Producers benefit from import taxes, export subsidies and import quotas. Producer surplus decreases from export taxes, import subsidies, and export quotas in both the small country and large country cases.

Government revenues increase from taxes and quotas (whether applied to exports or imports), and decrease from subsidies. The magnitude of the change in government revenues is influenced by whether or not the country is large or small in the market for the commodity analyzed.

Table 12.1. Impact Summary of International Trade Policies.

	Consumer Welfare		Producer Welfare		Government Revenue		Net Social Welfare	
Policy	Increase	Decrease	Increase	Decrease	Increase	Decrease	Increase	Decrease
Small Country								
Import Tax		-	+		+			-
Export Tax	+			-	+			-
Import Subsidy	+			-		-		-
Export Subsidy		-	+			-		-
Import Quota		-	+		+			-
Export Quota	+			-	+			-
Large Country								
Import Tax		-	+		+		+	-
Export Tax	+			-	+		+	-
Import Subsidy	+			-		-		-
Export Subsidy		-	+			-		-
Import Quota		-	+		+		+	-
Export Quota	+			-	+		+	-

ACTIVITIES

1. In international trade, combinations of policies are often important. Students should attempt to combine the various policies introduced in this chapter. For example, an import quota in a small country may be employed in conjunction with an import tax.

2. Students can extend the analysis presented in this chapter by introducing the two-country, single-commodity model of international trade. How does each policy affect the excess supply of exporting countries and the excess demand of importing countries? What is the change in the world price?

3. For each policy, attempt to identify the difference between the large country and small country cases. When a significant difference in the impact of a given policy is evident, can a supplemental policy be developed to equalize the impact of the original policy across large and small country cases?

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13. MACROECONOMIC LINKAGES TO AGRICULTURE

by Luther Tweeten

Macroeconomic policies refer to monetary, fiscal, and structural policies of government to influence aggregate income, employment, general price level, wages, and foreign exchange. Rates are of special concern: the inflation rate, unemployment rate, wage rate, exchange rate, and interest rate. Macroeconomic policies favorable to meeting the needs of agriculture and the nation are not easy to get right and few developing or developed countries get them right. Many countries are plagued by macroeconomic policy failures as is apparent in government budget deficits, high inflation, overvalued currency, high unemployment (or under-employment), excessive bureaucracy, trade deficits, shortage of foreign exchange, delinquent international debt, and economic and related political instability.

Macroeconomic variables mightily influence agriculture. For example, an overvalued currency is an implicit tax on agriculture and a subsidy to importers, many of them wealthy urban consumers. Despite strong macroeconomic linkages to agriculture and despite the fact that macroeconomic policies often dwarf the influence of agricultural policies administered by the Ministry of Agriculture, usually neither the Minister of Agriculture nor agricultural interests in general have much voice in shaping macroeconomic policy. Given the importance of the agricultural economy in developing countries, macroeconomic policy is far too important to leave solely to analysts and decision makers outside of agriculture.

OBJECTIVES

The objectives of this chapter are to:

1. Explain how macroeconomic policies influence agriculture and the economy as a whole.

2. Present various theories of macroeconomic (mainly monetary and fiscal) policies as they relate to income and employment and to inflation, interest, and exchange rates.
3. Discuss the various markets in which inflation, interest, and exchange rates are formed.
4. Outline elements of sound macroeconomic policy consistent with national goals such as sustained economic growth with full employment at a reasonably stable general price level.

KEY POINTS

1. Macroeconomic policies critically determine success or failure in reaching development objectives such as (a) national income growth with full employment at stable prices and (b) an efficient agriculture realizing lower poverty and higher income per capita while providing abundant supplies of quality food and fiber at reasonable prices.
2. Powerful incentives pull governments of developing countries toward unsound, unsustainable macroeconomic policies that slow growth and create economic instability and inequality.
3. Microeconomic decisions at the individual, family, and firm level that are the proximate sources of a nation's economic progress are heavily influenced by that nation's macroeconomic policies.
4. Much more is known of what constitutes sound macroeconomic policies than is being applied. The macroeconomic prescription for economic progress is relatively simple and straightforward. The challenge facing developing countries is to muster the discipline and political environment to implement sound policies.

CONCEPTS AND APPLICATIONS

The sequence in this section is to first define macroeconomic policies, then show how they relate to national variables of interest including inflation, interest, wage, unemployment, and exchange rates. Linkages to agriculture of macroeconomic policies and the variables they generate then are discussed.

Macroeconomic Policies

Fiscal, monetary, and structural macroeconomic policies are discussed below. Related international trade policies were discussed in the previous chapter, natural resource policies in Chapter 11, and agricultural policies in Chapter 6.

1. Fiscal Policy. Fiscal policy refers to aggregate government revenues and expenditures, the balance between them as apparent in budget deficits or surpluses, and impacts of the variables on national income, employment, and other macro variables. For

decades governments paid little attention to aggregate supply and demand balance, relying instead on *Say's Law* that supply generated its own demand. That Law says people bring the products of their resources to the market only to purchase something in return so that the flow of output (total value of goods and services produced) equals the flow of factor payments (wages, salaries, rents, profits, royalties). Planned savings (foregone consumption) are assumed to equal planned investment in durable inputs to provide a future flow of output. But economic depressions occurred periodically when the flow of aggregate demand (for consumption and investment) fell short of aggregate supply of goods and services produced. The most severe was the Great Depression of the 1930s. The British economist John Maynard Keynes noted that government could provide liquidity when, because of unfavorable expectations for the economy and for other reasons people saved more than investors were willing to invest. That is, government spending could fill the void to create full employment when aggregate demand (consumption plus investment) fell short of aggregate supply of goods and services.

Aggregate demand can be expressed as Gross Domestic Product Y_d which is the sum of domestic consumption C plus domestic investment I plus government spending G plus exports X less imports M or

$$Y_d = C + I + G + X - M. \tag{13.1}$$

Suppose that consumption is a function of income, that is

$$C = C_0 + cY_d \qquad 0 < c < 1 \tag{13.2}$$

where C_0 is consumption when income is zero. Suppose also that suppliers of goods and services are willing to provide whatever quantity Y_s that is demanded so aggregate demand generates its own supply in this Keynesian model. Hence, in equilibrium, national income Y is

$$Y_d = Y_s = Y. \tag{13.3}$$

Given that c is the marginal propensity to consume, assume that s = 1 - c is the marginal propensity to save. Substituting (13.2) and (13.3) into (13.1) yields

$$Y = A + G/s \qquad 0 < s < 1 \tag{13.4}$$

where A is a constant of all variables not directly influenced by fiscal policy. Two notable implications of simplified equation 13.4 are that (1) where aggregate demand falls short of full-employment aggregate demand the government can fill the gap to increase income Y, (2) and government fiscal policy to stimulate the economy will be most effective in raising income and bringing full employment if the propensity to save approaches zero. An increase of one dollar of G will raise national income by 1/s dollars, the Keynesian multiplier. So a policy of government spending will be most effective in raising income as the propensity to consume approaches 1.0 and all that is produced is consumed. That

spending should be a virtue and savings a vice seemed happily to contradict the virtue of thrift.

Keynesian economics outlined above is a useful conceptual framework for lifting an economy out of recession or depression characterized by high unemployment and idle industrial capacity. Unfortunately, Keynesian economics has been used to justify chronic government spending for consumption rather than investment. An example is national retirement programs based not on savings and investment in an actuarially sound program but instead retirement programs of taxing from the young for immediate payment of retirement for the elderly. Such policies lower the propensity to save and raise the Keynesian multiplier. But as s approaches zero and the income multiplier 1/s from a unit of government spending approaches infinity, all that is produced is consumed and the infinite resulting nominal income is only inflation! It is not real goods and services. Without savings there is no investment, no real capital formation, and no real growth. *Neo-Keynesian* economics which features large government deficits in a full-employment economy is to be avoided.

Supply-side economics emerged in reaction to the absurdity of the Keynesian model applied to a full-employment economy. This point can be illustrated with the Harrod-Domar growth model. In equilibrium, investment I equals savings S in (13.5). Investment is defined as net capital formation ΔK in (13.6). The average and marginal output-capital ratio g is defined in (13.7) and is a measure of the efficiency of the system in converting human and material capital into output. The average and marginal propensity to save s is defined in (13.8).

$$I = S \tag{13.5}$$

$$I = \Delta K \tag{13.6}$$

$$\frac{Y}{K} = \frac{\Delta Y}{I} = g \tag{13.7}$$

$$S = sY; s = \frac{S}{Y} \tag{13.8}$$

Defining r as the growth in product $\Delta Y/Y$ and rearranging terms in (13.7), it is apparent that

$$r = \frac{\Delta Y}{Y} = \frac{I}{K} = \frac{S}{K} = \frac{Y}{K}\frac{S}{Y} = gs. \tag{13.9}$$

National product or real income grows at a rate gs. If g is .2 and s is .3, then output grows at a rate .2(.3) = .06 or 6 per cent per year. A high propensity to save and invest coupled with efficient use of resources (high g) lead to high income, a conclusion quite opposite that of the Keynesian model. The government sector can be introduced as forced savings and investment if planned savings are unsynchronized with investment so that national

growth is too high, low, or unstable. But forced savings and subsequent investment by government must pay attention to the lessons of equation 13.9 to contribute most effectively to real growth -- the savings must be invested efficiently where payoffs are high. Keynesian deficit spending is appropriate in recession or depression but continuing "pump priming" for consumption in a full-employment economy creates inflationary pressures and economic stagnation.

The suggested general rule of fiscal policy is for the government budget to be in deficit in recession but to be balanced in a full employment economy. The government budget deficit can grow at the same rate as national income without becoming an increasing burden but ordinarily the budget should be balanced or in surplus in a full-employment economy. Borrowing is justified from external sources for capital improvements only if expected returns exceed the cost of money; otherwise borrowing is a burden on future generations.

In short, savings and investment of those savings in high payoff areas such as infrastructure, education, and science to create improved technology is important if growth is important. Because the private sector is often better at foregoing consumption and finding worthy investments than is government, government can largely confine its spending to public goods investments such as providing security, stability, and more tangible undertakings such as roads, education, agricultural research and extension, and targeted food assistance. Project analysis as outlined in Chapter 10 is useful to ensure that investments by the public sector are in fact in high payoff activities.

2. ***Monetary Policy.*** Monetary policy refers to control of the money supply. By issuing currency or controlling bank loan-deposit ratios, by buying or selling government bonds, by regulations on credit and foreign investment, and by other means the money supply can be increased or decreased. Increasing the money supply faster then real output causes inflation, defined as an increase in the general price level.

Keynesian analysis gives little or no insight into inflation. Presumably, additional investment and government spending do not cause inflation as long as excess capacity exists so that additional income will come forth in real goods and services rather than in inflated values of the former real volume of goods and services.

Disenchantment with the Keynesian model led to revival of the quantity theory of money to explain national income and inflation. In equation 13.10, M is money supply, V is the velocity or rate of circulation of money, P is the national price level, and Q is real national output of goods and services. Y is nominal output as defined earlier.

$$MV = PQ = Y \quad \text{or } P = MV/Q \tag{13.10}$$

Although (13.10) is an identity, it provides very valuable insights into inflation, defined as an increase in P. (P is actually an average of all prices in the economy, weighted by quantities of goods and services; thus an increase in P does not necessarily mean that all prices have increased, only that a weighted average of prices has increased.) It is apparent from equation 13.10 that the general price level is directly proportional to the money supply

and inversely proportional to output, given the velocity of money V. Thus, input or output prices increased by higher import costs of oil, by a generous union wage settlement, or by a crop failure are *inflationary*, but in theory do not directly cause a rise in the general price level unless validated by an increase in money supply. Without an increase in money supply, other prices adjust downward to leave the price level unchanged. The theory notwithstanding, inflation does result from inflationary forces because (1) a rise in wages or oil prices tends to reduce real output and/or increase the velocity of money, and (2) governments tend to increase money supply to accommodate inflationary forces because the alternative is unemployment, idle industry capacity, and recession as the economy slowly adjusts to lower prices for some goods with M constant. The velocity of money V in equation 13.10 varies widely and is not easily predicted but tends to increase when inflationary pressures and expectations are strong.

Monetarism, defined as economics emphasizing money supply to explain aggregate pricing, output, and employment, alone is an inadequate macroeconomic theory. Much can be learned from the quantity theory of money despite its imprecision and the inexplicable variations in the velocity of money. Money does matter although the adage that "inflation is always and everywhere a monetary phenomenon" holds only for the long run. The money supply can be increased at the rate of growth of output without causing inflation. Modest inflation may be tolerable because it allows cuts in real wages or prices without reducing nominal wages and prices, hence enhances wage and price flexibility. The suggested rule of thumb for monetary policy is that if the tolerable or desired inflation rate is 3 percent and output is expanding 5 percent per year, money supply on average can be increased 8 percent per year with a slightly faster rate in recession and a slower rate in a full employment economy. Fine-tuning is not possible, however.

Some years ago, the *Phillips Curve* was published showing a strong negative association between the inflation rate and the unemployment rate. The implication was that a country willing to tolerate inflation could have low unemployment rates. This finding plus Keynesian economics coupled with strong pressures to reduce unemployment encouraged governments to use monetary-fiscal policies in search of low unemployment rates.

Yet it is not possible to use such macroeconomic policies to persistently hold the unemployment rate below the natural (long-term equilibrium) rate. Every economy at any point in time has a "natural" rate of unemployment, say 6 percent which is irreducible because people are between jobs or are "unemployable". The theory of *rational expectations* explains why governments create stagflation (high unemployment and inflation rates simultaneously) when using only macroeconomic policies to reduce unemployment below the natural rate. In an expansion phase of the inflation cycle a monetary-fiscal stimulus increases cash balances and aggregate demand relative to supply. People with bigger cash balances turn them into purchases of real goods and services. This lifts product prices, firms raise nominal wages to hire more workers who initially do not realize items they purchase have increased in price so their real wages have not increased. When firms and workers realize real wages and prices are no higher than before, the economy eventually returns to the former equilibrium and employment but only after

overshooting in the form of recession and unemployment in the stabilization phase of the inflation cycle. Firms and workers may be fooled initially by monetary-fiscal stimulus, but according to rational expectations theory they do not make systematic errors. They learn not to respond to the transitory impacts of monetary-fiscal stimulus.

Persistence of commodity cycles caused by naive expectations that current prices will continue indicates that expectations are not always rational. But people do learn and rational expectations theory is correct in noting that monetary-fiscal stimulus repeated often enough will lose impact. Monetary-fiscal stimulus sets in motion the so-called *inflation cycle* of an economic expansion phase followed by a stabilization or contraction phase. Averaged over the inflation cycle, income and employment are not greater (and might be lower) than with sound (steady) monetary-fiscal policy. The lesson is that excessive monetary-fiscal stimulus does not on the average add to national income or employment but instead creates severe *cost-price*, *cash-flow*, *real wealth*, and *instability* problems for agriculture -- problems to be discussed later. Monetary-fiscal stimulus needs to be avoided that attempts to reduce unemployment below the natural rate. Rather, reducing the natural unemployment rate needs to be addressed by structural policies.

*3. **Structural Policies.*** Structural policies refer to a host of public policies other than monetary, fiscal, and trade measures which influence macro variables. Structural policies include the choice of the public sector versus the private sector to allocate resources, use of parastatals to market output or supply inputs, rules and regulations that foster or discourage labor or industry monopolies, minimum wage laws, and other measures that influence income and employment given the monetary-fiscal stimulus. The political process frequently is shortsighted (political decision makers have relatively short planning horizons), serves not the public interest but special interests (political elites such as civil servants, the military, students, urban consumers, or large landowners are favored), and operates with highly imperfect information. One result is that governments attempt to short-circuit the development process. Developed countries pay high wages; why not use minimum wage laws to require industrialists to pay high wages in developing countries? Developed countries use tractors; why not subsidize tractors and other labor-saving inputs to quickly improve output per worker and living standards in developing countries? The problem is that these and related structural policies or measures are often counterproductive. For example, artificially forcing high wages increases unemployment, idles industry capacity, and slows economic growth in the absence of human and material capital investments raising productivity of workers to the value of the output they receive in compensation. National income is reduced by introducing capital and technology for which costs exceed benefits. Thus macroeconomic policy cannot be separated from sound microeconomic policy.

Favorable microeconomic policy is a necessary but not a sufficient condition for favorable macroeconomic policy. Favorable microeconomic policy emphasizes technical efficiency -- firms operate on the highest production function, isoquant, or production possibility curve given available resources. It also emphasizes allocating private savings, business earnings, and other resources to uses providing highest returns. It emphasizes

correction of market distortions and supplementing the market in provision of public goods and services as discussed earlier in Chapter 6. A useful rule is governments should follow structure policies that increase national income rather than reduce national income.

Finally, some policies not ordinarily viewed as macroeconomic policies can have a strong influence on national economic performance. An example is a free press -- a potent force to expose and reduce corruption, inefficiency, and inequities in government and business operations. Favorable macroeconomic outcomes depend not just on favorable policies, but also on a minimum of inefficiency and corruption in carrying them out. Culture, the work ethic, moral standards, and entrepreneurial zeal are important but are not dealt with here.

Linking Macroeconomic Policy to Agriculture

Principal elements of a favorable macroeconomic policy were discussed earlier. Increase money supply at the rate of growth of real output plus (say) 3 percentage points; the government budget in deficit in recession but balanced or surplus in a full-employment economy but with the average deficit increasing over time at a rate no faster than the rate of gain in real GDP; pursue structural policies such as elimination of barriers to competition and commerce to reduce the natural unemployment rate; let the value of foreign exchange and interest rates be determined by the market or at least not deviate for long from market rates.

These principles of sound policy are rarely followed. Two key elements, fiscal policy and monetary policy, instead of being moderate, usually are overly expansionary or overly restrained as follows:

	Monetary Policy	
Fiscal Policy	Expansionary	Restrained
Expansionary	A	B
Restrained	C	D

giving rise to four combinations A through D of unsound macroeconomic policy. Option D, excessive restraint on monetary and fiscal policy results in recession and stagnation, hence can be only temporary. Option A, highly stimulative monetary and fiscal policy, trends toward hyperinflation and also is temporary. Options B and C though not indefinitely sustainable tend to last for some time and are common modes of operation in developing countries. Hence, implications of each are presented in Figure 13.1 and discussed below with ramifications. The objective is two-fold: (1) to describe how macroeconomic policies influence agriculture for good or ill, and (2) to lay out rudiments of policies, markets, variables, and their consequences that can be used in quantitative modeling of macroeconomic linkages. Outcomes and formulations are unique to individual

countries; Figure 13.1 is a generalization which will not hold in all situations. The figure identifies agricultural problems of *cash-flow*, *real wealth loss*, *cost-price stress*, and *instability* caused by unsound macroeconomic policies. Problems are listed in capital letters in the *policy* columns. Key linkage variables are in capital letters in the *market* column.

1. Aggregate Demand and Supply. Monetary and fiscal policy work through aggregate demand and supply to determine national income and the general price level. The income growth rate and inflation rate may be used to monitor performance. Policies B and C stimulate aggregate demand relative to supply, especially if they are unanticipated. Under policy B demand is stimulated directly and under policy C it is stimulated indirectly as expanded money supply expands cash balances in the hands of individuals and firms. Faced with disequilibrium in the form of too much cash relative to goods and services, they enter the market and expand aggregate demand by using cash balances to purchase goods and services. Expectations for a favorable economic outcome initially are improved, stimulating the economy during the expansionary phase. When expected favorable real factor returns and product prices do not materialize, however, losses in the stabilization phase will offset gains in the expansionary phase as noted earlier. Unlike the short-term, the *long-term growth rate* is not very sensitive to monetary-fiscal policies. In developing countries, demand for farm output is heavily influenced by growth in consumers' incomes. Policies B or C tend to increase farm income in the expansion phase and to retard farm income in the stabilization phase, hence, the policies contribute to farm economic instability but do little to raise farm income or employment over a longer period of time.

2. Labor. Not much will be said of the labor market determining wage and unemployment rates. It is well to note, however, the pitfalls for developing countries of attempting to shorten the development processes by forcing high wages through legislation or allowing high concentration of economic power in organized labor. This precludes poor countries from using their one major source of comparative advantage, low-cost labor, to attract international investment and generate domestic employment. Because progress in agriculture depends heavily on availability of jobs for those released from agriculture as development proceeds, it is critical that developing countries allow markets to work in a climate conducive to industrial growth. In some instances, that growth is in small, light industries producing inputs which will improve agricultural productivity. The gradually increasing income can be invested in human and material capital to make labor more productive, thereby providing the foundation for more capital intensive, high-wage industry. A productive agriculture provides low-cost food for industrial workers (wage goods), raw materials (for example, fibers), and markets for industry. Thus, balanced

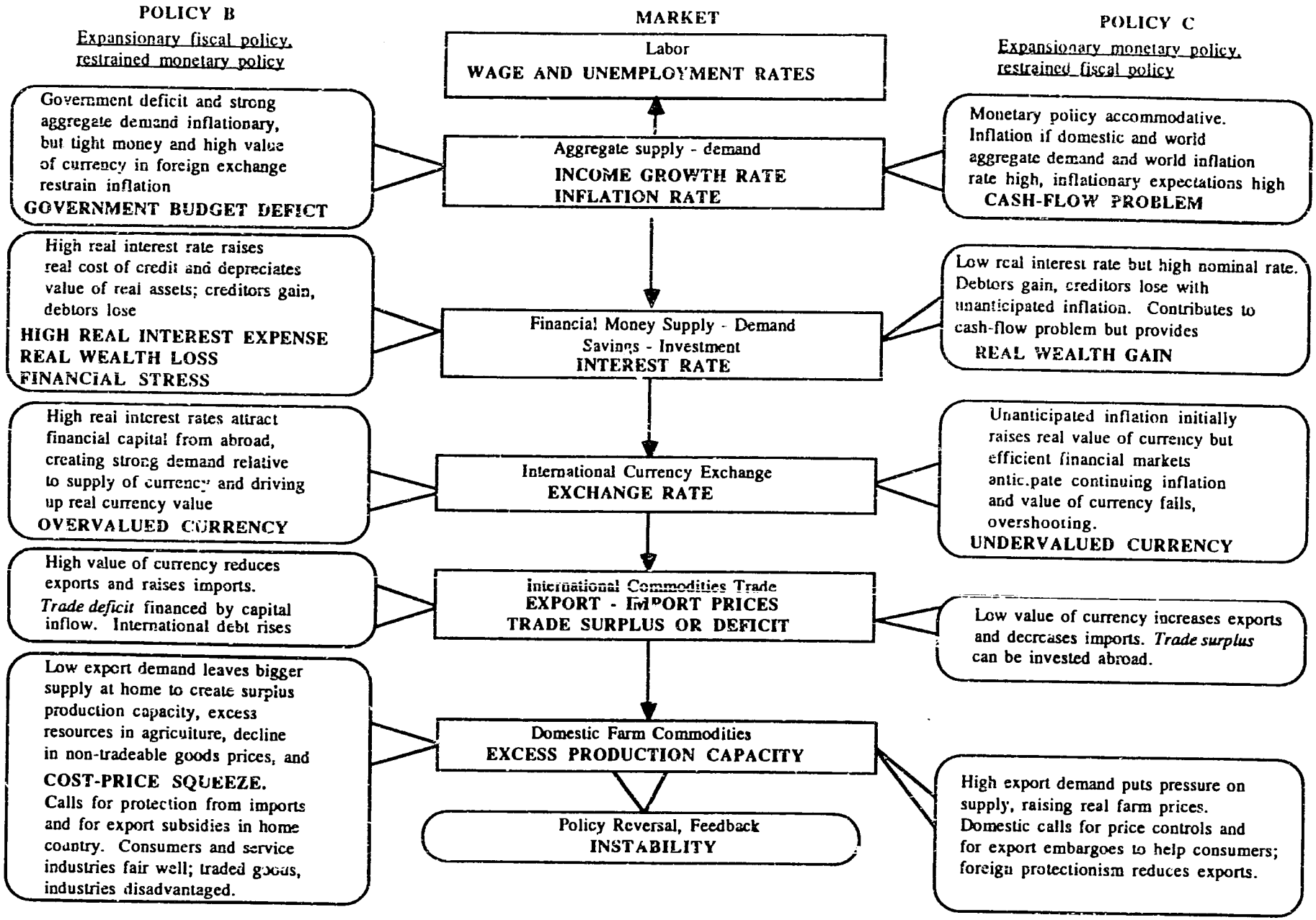

Figure 13.1. Flow Chart Showing Behavior of Markets and Farm Problems Under Alternative Macroeconomic Policies.

agriculture-industry growth is advantageous in promoting overall growth in per capita national income.

*3. **Domestic Financial Market and the Interest Rate.*** In the long run, interest rates are determined by two principal forces: the premium or discount rate which consumers place on present versus future consumption and the expected rate of return on investment. Most people prefer to consume now rather than later, hence, the time discount rate is positive.

Firms and individuals are doing a service by foregoing consumption to save and it is appropriate to compensate for that service. The interest rate is the result of forces of demand and supply working in financial markets. A high interest rate encourages savings and increases the supply of financial capital which speeds economic growth but discourages investment and reduces the demand for capital which retards growth. In actual markets the short-run supply of loanable funds is determined not only by savings but also by money supply and other factors. And the demand for loanable funds is influenced not only by expected rates of return on investment but also by factors such as the need for liquidity. Rates vary by risk involved. Risk-aversion dominates so rates of return are higher in risky investments than in secure investments. High real interest rates can increase price instability as they lower reserves by raising costs of holding reserve buffer stocks of commodities.

Two types of interest rates are important: (1) the nominal or market rate, and (2) the real rate which is the market rate less the inflation rate. If the market interest rate is 30 percent and the inflation rate is 20 percent, then the real interest rate is 10 percent. This implies that at the margin people will forego consumption and save and invest to obtain future income streams (hence, contribute to long-term national income) if they can receive a rate of return of 10 percent or more.

The financial market has a major influence on the farming economy. Under expansionary fiscal and tight money policy B, the demand for savings to finance government deficits combined with a strong private demand for funds relative to a limited supply of funds (due in part to monetary restraint) causes real interest rates to be high. The immediate impact is high real interest expenses to firms and individuals.

Other impacts are more subtle. One is to create *real-wealth* and *cash-flow* problems for farmers. In a well-functioning market a durable asset such as land receives a price equal to the expected future earnings discounted at a rate α -- the desired real rate of return. If land earnings or rents R_t are expected to increase or decrease at a constant percentage rate, the rent in year t is

$$R_t = R_0 e^{(i + i')t} \tag{13.11}$$

where subscript zero refers to the initial year, e is the base of natural logarithms, i is the expected inflation rate and i' is the expected annual future increase in real rents. Integrating over all future rents, the discounted value is the present land price P_0, i.e.

$$P_0 = R_0 / (\alpha - i') \tag{13.12}$$

or

$$R_0 / P_0 = \alpha - i'. \qquad \text{(current rate of return).}$$

The ratio of rent to land price is the current rate of return on land investment.

Because land rent and land prices are related to each other by a constant $\alpha - i'$ each year, it follows that land price is equal to

$$P_t = P_0 e^{(i + i')t}, \tag{13.13}$$

hence, the rate of increase in land price or rate of capital gain is $i + i'$, the same as the rate of increase in rent. Total nominal return on land is current return plus capital gain or summed from (13.12) and (13.13) is

$$(\alpha - i') + (i + i') = \alpha + i \tag{13.14}$$

Adjusting for inflation by subtracting i from the nominal rate of return in (13.14), the real rate of return is α.

Bonds are frequently used to finance farmland mortgages and pay a fixed nominal return each year. That nominal return declines in real terms by the inflation rate, hence $i' = -i$ in equation 13.12. It follows that the current return or market interest rate on a bond is $R_0 / P_0 = \alpha + i$ if α is the desired real rate of return.

The cash-flow problem can now be identified. The cash-flow shortfall on land as a percent of land price may be defined as the current interest rate $(\alpha + i)$ less the current rate of return on land $(\alpha - i')$ or $i + i'$. Assume i' is zero for simplicity. Then if the inflation rate is 15 percent, earnings from land will fall short of the interest on a fully indebted hectare by 15 percent of the land price. The shortfall as a proportion of land price on a fully indebted hectare is the inflation rate, hence will be even higher for a higher inflation rate. If the real rate of return $\alpha = 10$ percent and inflation is 20 percent, then the interest rate is 30 percent and it is normal for the current earnings from three hectares to be required to pay the interest on one hectare. Yet the real return is 10 percent when capital gain is included, thus markets are in equilibrium. Inflation raises immediate costs to farmers but defers returns -- the cash-flow problem with policy C in Figure 13.1.

Under an expansionary monetary policy that brings unanticipated inflation, farmers benefit from real wealth gains. When interest rates on intermediate and long-term loans are fixed, unanticipated inflation reduces the real rate of interest and hence real interest expenses to debtors. Because farmers on the whole tend to be net debtors, they benefit at the expense of creditors by lower real interest costs and by asset appreciation. As noted in equation 13.12, the value of current assets is inversely proportional to the real interest rate. A lower real interest rate raises present value and, along with lower interest costs, brings real wealth gains under policy C.

The converse is *real wealth loss* and *financial stress* problems of expansionary fiscal policy B. With the present value of farmland inversely proportional to the real interest rate, higher real interest rates under policy B reduce farm asset values and bring real capital losses. Along with higher real interest expenses and a transitory *cost-price* squeeze (discussed later), the result is financial stress.

4. International Finance and Currency Markets. High real interest rates associated with a strong demand for currency to invest in a country relative to the international supply of the currency raises the value of the currency in foreign exchange markets under expansionary fiscal policy B. Low real interest rates under expansionary monetary policy C work in the opposite direction to reduce the value of currency. Initially, inflation tends to overvalue currency. Exchange market participants quickly perceive the impact of inflation and adjust the value of currency accordingly, often overreacting and causing unduly low real rates. The real exchange rate is the nominal rate adjusted for differences in inflation rates between countries. For example, if the exchange rate in India changes from 10 rupees per dollar to 20 rupees per dollar while inflation in India is 100 percent more than in the United States, the real rupee exchange rate remains unchanged with respect to the dollar.

Rather than valuing their currency in the market, many developing countries peg their currency to the dollar, pound, or franc. Because inflation tends to be higher in developing countries than elsewhere, the eventual result is an overvalued currency. Because developing countries face strong internal pressure for government spending in excess of tax revenue or borrowing; they often chose to finance expenditures by expanding the money supply which brings inflation. Well-to-do, politically influential domestic consumers of imported goods have a vested interest in an overvalued currency, hence, overvalued currencies are more the rule than the exception in developing countries. Agricultural producers usually are worse off from an overvalued currency because they lose more from low exports and stiff competition from imported farm products than they gain from cheaper imported inputs and consumed goods. For urban consumers, the situation typically is the reverse.

An overvalued currency causes numerous distortions. Imported tractors are implicitly subsidized, causing uneconomic substitution of tractors for labor and other inputs. Exchange controls are likely to be imposed on imports and the politically influential elites rather than sound economics tend to determine what imports to allow. A market-determined flexible exchange rate may be optimal but is highly unstable. A second-best solution is a pegged rate allowed to float periodically and adjusted between floats for relative inflation rates compared to trading partners or a world currency such as the U. S. dollar.

5. International Trade Markets. An overvalued currency makes imports cheap and exports dear, shifting a country towards a negative balance of trade. If the agricultural exporting country goes from equilibrium to an overvalued currency, export demand will fall and reduce the agricultural trade surplus. A balance of payments deficit

will need to be offset by foreign grants or loans. An expansionary fiscal policy B is likely to accelerate tendencies for a developed country to be a net debtor. But again, a distortion is apparent -- loans and transfers ordinarily should be used to finance high payoff investments in infrastructure and other durable public goods rather than to subsidize consumption in a country living beyond its means as apparent in a trade deficit in excess of justifiable foreign investment inflow.

An exchange rate equating the demand with the supply of exchange is preferred, but an error in undervaluing exchange has certain advantages to overvaluing exchange. An undervalued exchange rate raises exports and employment but may reduce living standards because imports are dear. An overvalued rate reduces exports and employment but living standards of the employed temporarily are improved by lower import prices. The former policy can be sustained longer than the latter because financial crisis comes more quickly with overvalued exchange and trade deficits.

6. Domestic Commodity Markets. With expansionary fiscal policy B, the overvalued currency and attendant low export demand reduce overall demand for farm products, creating excess production capacity and excess resources at current prices. In the absence of commodity price supports which few developing countries can afford, the result is a *cost-price squeeze*, defined as falling product prices relative to prices paid by producers. Producers resist falling prices and resource adjustments, instead calling for export subsidies, commodity import quotas, or other market interventions.

With expansionary monetary policy C and a functioning exchange market, currency is likely to become undervalued and exports are likely to expand. Domestic producers may be hard-pressed to meet foreign and domestic demand for agricultural products. The rise in real domestic prices reduces real income of consumers and brings calls for food price controls, export restrictions, and other market interventions. Inflation or disinflation is not neutral with respect to commodity prices; a change in the general price level has real price effects at the individual input and commodity level.

An important point is that although policies B and C can last longer than policies A and D, the former also are not sustainable. Expansionary fiscal policy B allows a nation to live beyond its means for a time until money and credit run short. Mounting federal debt and trade deficits cause loss of confidence by the domestic and foreign lenders that may bring a financial crisis. The second phase, austerity, under policy B may be brought on by the International Monetary Fund or other sources called in to rescue a country from financial collapse. Expansionary monetary policy C also is two-phased and unsustainable. High inflation rates cause loss of both political support and economic stimulus after the expansionary phase. The stabilization phase follows, offsetting gains made in the expansionary phase.

In summary, *instability* may be the single greatest problem caused by unsound macroeconomic policies. Price signals are distorted, long-term plans for investment in durables are thwarted; capital flight is encouraged. A second problem caused by unsound macroeconomic policies is microeconomic price distortions and market interventions as noted below.

7. Interaction and Feedback. In Figure 13.1, markets are treated as a single causal chain emanating from monetary-fiscal policy. In fact, interaction and feedback characterize the political and economic relationships. Economic instability begets political instability and vice versa. Financial stress caused by high real interest rates and falling asset values is reinforced by the cost-price squeeze induced by reduced export demand. Government interventions to protect groups from macroeconomic policy distortions in turn distort markets and weaken the ability of the price system to bring needed allocations for economic growth. Distortions, though detrimental to society as a whole, benefit some groups. Thus, once in place, interventions are difficult to dismantle even after the original purpose for them has long disappeared.

A typical scenario begins with expansion of government spending for military or other purposes. Increasing taxes to balance the budget meet resistance. Obtaining financial capital from abroad to finance the deficit may be impossible and borrowing locally would drive up interest rates to slow investment and economic growth. An attractive alternative, especially for politicians with a short planning horizon, is to expand the money supply to finance the deficit and keep interest rates low. The result is inflation which "taxes" those holding cash balances and those unable to increase prices of what they sell to pay for inflated prices of what they buy. With a fixed exchange rate the currency becomes overvalued, causing imports to rise and exports to fall, hurting the traded goods industries. Price gains in nontraded goods and services are restrained by imports. The trade deficit becomes unmanageable so import quotas are imposed. Exchange controls may be imposed, with allocations of limited import licenses determined by bureaucrats. A parallel market develops for currency and for other items in short supply. If import quotas raise input prices for the agricultural sector more than for other sectors, the result is reduced agricultural *terms of trade* defined as ratios of prices between agricultural and nonagricultural sectors.

The tangled web of reactions and counter-reactions could be traced further, but it is apparent that one unfortunate macroeconomic policy such as deficit spending can lead to a complex set of distortions, not only causing misallocation of resources but also shifting the distribution of income and wealth. The initial shock could have come from a short harvest at home or rising import prices for oil. In any case, attempts to shield an economy from major nontransitory shocks eventually build economic pressures for change to crisis proportions. The time to stop a hangover is "the night before." Accepting market realities with a series of small adjustments avoids the large crisis of "the morning after."

Many macroeconomic distortions can be explained by the political influence of those who gain:

1. *Minimum wage laws* and antitrust exemption for organized labor are supported by the industrial elite workers who have little regard for disadvantaged workers denied employment by wages arbitrarily raised above the value of marginal worker's contribution to output.

2. *Overvalued exchange rates* and import subsidies are supported by industrialists producing nontraded goods from imported inputs and by middle and upper class consumers who purchase imports.
3. *Low institutional interest rates* are supported by borrowers -- but not the poor and underprivileged borrowers who are denied access to institutional credit and instead rely on informal credit sources.
4. *Government deficit spending* is favored by those drawing funds from government -- bureaucrats, military, publicly supported students, and by industrialists selling to government. Taxpayers' understandable opposition to higher taxes also leads to deficits.
5. *Low food prices* through controls are supported especially by urban consumers. These consumers on the average are much more wealthy than rural consumers and producers, hence it is not possible to justify price controls in the name of equity. (Targeted food assistance to the poorest of the poor in urban or rural areas can be justified, however.)
6. *Taxing of exports* is supported by beneficiaries of taxes listed above when faced with lack of alternative sources of funds. Income, estate, and property taxes would be more equitable and efficient (interfere less with optimal resource allocation) but such taxes are difficult to collect in developing countries.
7. *Inflation* is favored or at least tolerated by holders of real property and of hard currencies (often held in foreign financial institutions), and by debtors. (Those with least influence in society often cannot increase their income to keep up with inflation and hence pay the "tax" of a nation living beyond its means.)

Benefits from well-functioning markets are widely dispersed whereas costs focus on a few. For example, the benefits of free trade to large numbers of consumers far exceed costs. But a few determined if inefficient producers who lose big from free trade are more than a match in the political arena for millions of indifferent consumers, no one of whom is motivated to political activity. Also, many workers and consumers who would gain from sound macroeconomic and other policies do not work politically for such policies because "they never miss what they never had." Meanwhile, losers from terminating unsound policies tend to be informed and work to maintain such policies even at great loss to society. The political elites -- urban industrial workers and consumers, bureaucrats, students, large landowners, and the military -- account for many of the seemingly irrational macroeconomic economic policies that bring losses to society. Rural people, poor people, and marginal and underemployed workers are most frequently disadvantaged by such policies. The solution is not to replace a few buyers or a few monopolist sellers with one buyer or seller (the government), or to replace a few large landholders with one (the government) but to diffuse economic and political power (check and balance system of government) and rely to the extent possible on market systems that turn private greed into public good.

ACTIVITIES

1. Economists say there is no "free lunch." With this in mind, trace out the economic implications of a country living beyond its means as apparent in large government budget deficits.

2. Construct a macroeconomic model for a selected developing country (see Nainggolan and other references at end for help). Consider equations for:
   a. Real exchange rate
   b. Real interest rate
   c. Inflation rate
   d. Unemployment (or employment) rate
   e. Income growth rate
   f. Exports
   g. Imports

3. What variables belong in each equation in 2 above? Where can you find these variables?

4. Discuss advantages and disadvantages for agriculture and the general economy of having:
   a. An overvalued currency in foreign exchange
   b. An undervalued currency in foreign exchange

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14. INDEX NUMBERS AND AGGREGATION

by Luther Tweeten

Aggregate measures of economic performances are extremely useful for public policy analysis. These include measures over time of real gross domestic product to gauge national economic progress, of real crop and livestock output to gauge agricultural progress, or the ratio of real agricultural output to real agricultural production inputs to gauge productivity advances. Aggregate real import or export quantity are of interest. Nominal revenue or cost figures are inadequate because changes over time may be the result of price rather than quantity changes.

Useful aggregate price measures include the consumer price index, the index of producer or wholesale prices, index of prices received by farmers for crops and livestock, index of prices paid by farmers for production inputs, and the ratio of prices received by farmers to prices paid by farmers also called the parity ratio or terms of trade for agriculture. Trends in composite variables such as real wages and foreign exchange rates are also important. Again nominal values defined as current prices times quantities are not sufficient -- the change may be due to changes in quantity rather than price.

OBJECTIVES

The objectives of this chapter are to:

1. Illustrate a conceptual framework to measure composite prices and composite quantities.
2. Provide empirical examples of how to use actual disaggregated price and quantity series to measure a composite price index and a composite quantity index.
3. Compare results, including advantages and disadvantages, of alternative formula (indexes) to measure composite variables.

KEY POINTS

1. Meaningful aggregation to form composite indices requires strong assumptions, some of which will not be met in reality.
2. The so called index number problem, the dilemma of whether to weight by current or base period prices (or quantities), has no exact solution, only compromises of which some are useful despite imperfections.
3. The choice of aggregation procedure depends heavily on availability of data as well as on conceptual validity.

CONCEPTS AND APPLICATIONS

Measuring the Contributions of Inputs to Output

To understand the conceptual basis for aggregation, it is useful to begin with *Euler's Theorem* which states that with constant returns to scale (production function homogeneous of degree 1.0), output Q is equal to the marginal product of input i ($\partial Q/\partial q_i$) times the level of input q_i summed over all inputs. Suppose only two inputs a and b are used to produce Q, then

$$Q = \frac{\partial Q}{\partial q_a} q_a + \frac{\partial Q}{\partial q_b} q_b . \tag{14.1}$$

Multiplying both sides by product price gives

$$QP = \frac{\partial Q}{\partial q_a} Pq_a + \frac{\partial Q}{\partial q_b} Pq_b . \tag{14.2}$$

The value of output is the sum of value of marginal products multiplied by input quantities. In competitive equilibrium, value of marginal product, $(\partial Q / \partial q_i)P$, is equal to input price p_i so (14.2) becomes

$$QP = p_a q_a + p_b q_b . \tag{14.3}$$

The value of output or revenue is equal to the value of inputs or cost in equation 14.3.

Designating the cost in a base period as C_0 and in the current period as C_1, then

$$C_0 = p_{a0}q_{a0} + p_{b0}q_{b0} = \Sigma\, p_0 q_0 \tag{14.4}$$
$$C_1 = p_{a1}q_{a1} + p_{b1}q_{b1} = \Sigma\, p_1 q_1 . \tag{14.5}$$

Index Numbers are Based on Costs and Revenues

It is apparent that aggregations are formed from component prices and quantities. The procedure in measuring *composite* input price or *composite* input quantity is to compare aggregations called real cost. Similarly composite product price or composite product quantity is measured by real revenue changes as illustrated later.

Equations 14.4 and 14.5 do not compare real cost or constant currency unit costs, but only compare nominal cost. Total current cost changes will measure neither composite price change (quantities could be changing C) nor composite quantity change (prices could be changing C). It is necessary to hold quantities constant when measuring changes in composite price and to hold prices constant when measuring changes in composite quantity.

To measure the price change, we can weight by base period quantity.

$$p_{a0}q_{a0} + p_{b0}q_{b0} = \Sigma\, p_0 q_0 \tag{14.6}$$

and

$$p_{a1}q_{a0} + p_{b1}q_{b0} = \Sigma\, p_1 q_0 . \tag{14.7}$$

The ratio of composite cost for any period 1 compared to the base period 0 from the ratio of equation 14.7 to equation 14.6 is

$$L(p) = \frac{\Sigma\, p_1\, q_0}{\Sigma\, p_0\, q_0} \tag{14.8}$$

and is called the *Laspeyres price index*. Alternatively, we could have calculated constant currency or real change in cost by weighting price with current quantity. The result would be

$$p_{a0}q_{a1} + p_{b0}q_{a1} = \Sigma\, p_0 q_1 \tag{14.9}$$

for the base period, and for any current period

$$p_{a1}q_{a1} + p_{b1}q_{a1} = \Sigma\, p_1 q_1 . \tag{14.10}$$

The proportional change in real cost with quantity held constant from period 0 to period 1 is the ratio of (14.10) to (14.9) or

$$P(p) = \frac{\Sigma\, p_1\, q_1}{\Sigma\, p_0\, q_1} \tag{14.11}$$

which is called the *Paasche price index.*

Similarly, composite quantity changes can be measured from the change in real cost by weighting quantities with constant prices. Weighting with prices of the base period, the *Laspeyres quantity index* is

$$L(q) = \frac{\Sigma\, p_0\, q_1}{\Sigma\, p_0\, q_0} \tag{14.12}$$

and the *Paasche quantity index* is

$$P(q) = \frac{\Sigma\, p_1\, q_1}{\Sigma\, p_1\, q_0}. \tag{14.13}$$

Real cost is used to measure composite *input* price or quantity over time; real revenue is used to measure composite *product* price or quantity over time.[1] If p is redefined as product price (instead of input price) and q is redefined as product quantity, then the above formulas L(p), P(p), L(q), and P(q) can be used to gauge composite product price and quantity over time by measuring changes in real revenue.

Graphic Illustration of Concepts

The Laspeyres and Paasche indices obviously are two quite different ways of measuring composite variables over time. The difference between base and current period weighting is illustrated conceptually by Figures 14.1 and 14.2. In (14.1), suppose that technology changes so that a *given* quantity of output $Q = Q_0 = Q_1$ could be produced by combinations of input q_a and q_b along Q_0 in period 0 and along Q_1 in period 1. At issue is whether productivity has advanced as measured by real cost (constant input price) of producing a given output $Q_0 = Q_1$.

The dotted isocost line C_0^L tangent to Q_0 at d is the highest attainable and hence an equilibrium given base period technology and prices. The equation for that isocost line is

$$C_0^L = p_{a0}q_{a0} + p_{b0}q_{b0} = \Sigma\, p_0 q_0 \tag{14.14}$$

If $q_a = 0$, it is apparent from equation 14.14 that the intercept of the isocost line on the horizontal axis in terms of q_b is $q_{b0} = C_0^L/p_{b0}$. If $q_b = 0$, the intercept of the isocost line on the vertical axis is $q_{a0} = C_0^L/p_{a0}$. The Laspeyres isocost equation 14.15 is

$$C_1^L = p_{a0}\, q_{a1} + p_{b0}\, q_{b1} = \Sigma\, p_0\, q_1 \tag{14.15}$$

and utilizes the same price weights p_{a0} and p_{b0} as in equation 14.14 but utilizes quantities q_{a1} and q_{b1} where the isocost line is tangent to Q_1 at e in Figure 14.1. The intercepts for the isocost line are C_1/p_{ao} on the vertical axis and C_1/p_{bo} on the horizontal axis. The Laspeyres quantity index is formed by the ratio of (14.15) to (14.14) or

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[1]The product price may be viewed as marginal utility, hence total revenue is an index of gross utility (see Chapter 6). Input price may also be viewed as marginal utility of the input. The value of marginal product of input q_i is $\frac{\partial Q}{\partial q_i} P = p_i$ where Q is output, P is output price, and p_i is input price. If $P = \frac{\partial U}{\partial Q}$ as noted above, then $\frac{\partial Q}{\partial q_i}\frac{\partial U}{\partial Q} = \frac{\partial U}{\partial q_i} = p_i$.

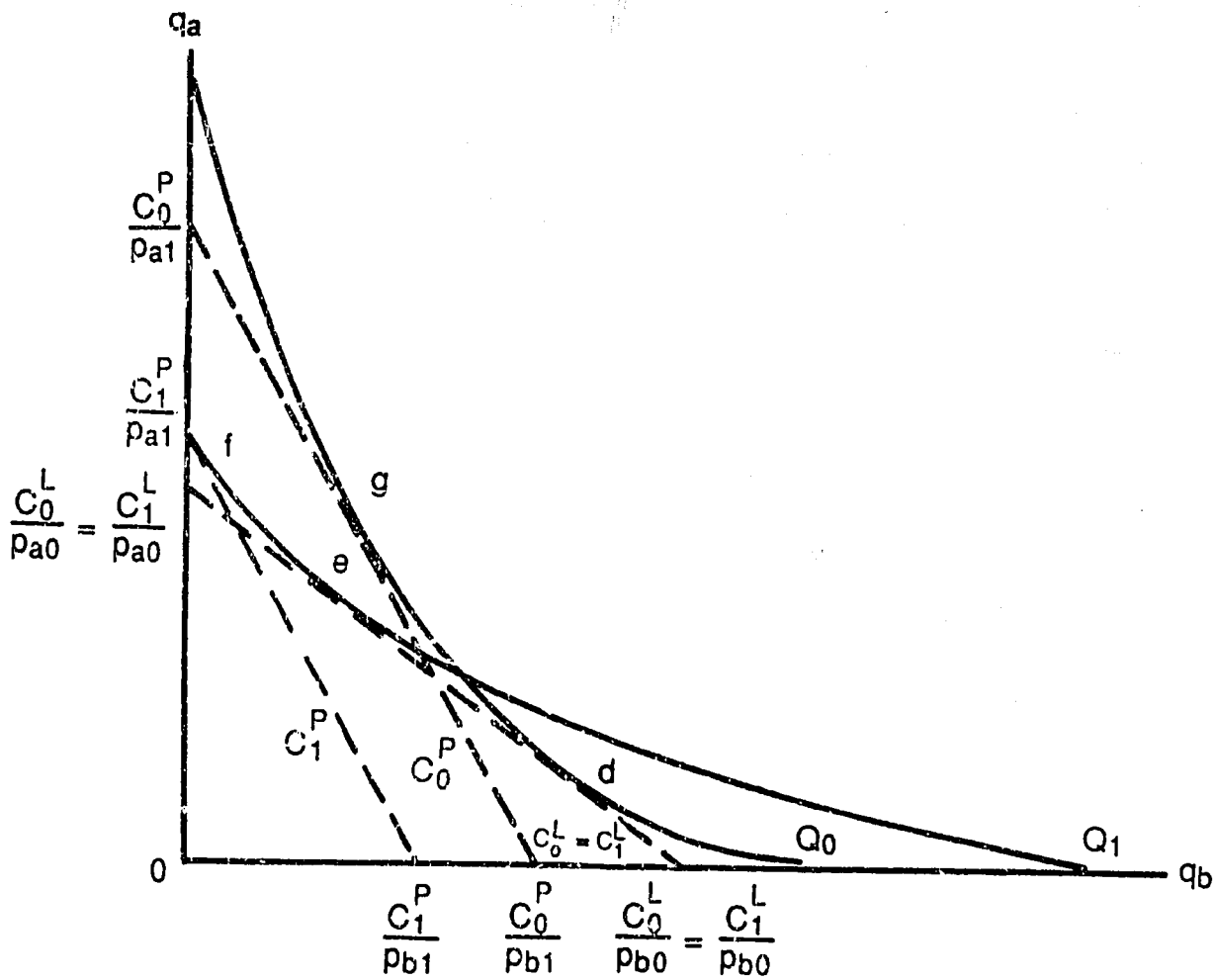

Figure 14.1 Isoquant Curves and Isocost Lines Used to Illustrate Index Number Dilemma for Resources q_a and q_b.

$$L(q) = \frac{\Sigma\, p_0\, q_1}{\Sigma\, p_0\, q_0} = \frac{C_1^L/p_{ao}}{C_0^L/p_{ao}} = \frac{C_1^L/p_{bo}}{C_0^L/\, p_{bo}} = \frac{C_1^L}{C_0^L}\ .$$

The ratio is 1.0 or in conventional index number terminology (percent of base period in current period) is 100. It is apparent that the isocost lines for technology Q_0 and Q_1 are the same, $C_1^L = C_0^L$, and the Laspeyres formula indicates no change in productivity from period 0 to period 1.

The Paasche index provides a quite different estimate of the change in constant-price real cost, and hence of productivity gains. Based on prices of period 1 and technology of period 1, the isocost line C_1^P as defined by

$$C_1^P = p_{a1}\, q_{ao} + p_{b1}\, q_{b0} = \Sigma\, p_1\, q_1 \tag{14.16}$$

is tangent to Q_1 at f. If $q_b = 0$, the intercept of (14.15) defined in terms of q_a on the vertical axis is $C_1^P\ /p_{a1}$. If $q_a = 0$, the intercept of the isocost line or the least cost given Q_1 technology, defined in terms of q_b is $C_1^P\ /p_{b1}$.

Real cost C_0^P at period 1 prices and period 0 technology is tangent to Q_0 at g, giving intercepts C_0^P/p_{a1} on the vertical axis and C_0^P/p_{b1} on the horizontal axis. The isocost line equation is

$$C_0^P = p_{a1}\, q_{a0} + p_{b1}\, q_{b0} = \Sigma\, p_1\, q_0 \tag{14.17}$$

The Paasche formula is

$$P(q) = \frac{\Sigma\, p_1\, q_1}{\Sigma\, p_1\, q_0} = \frac{C_1^P\ /\ p_{a1}}{C_0^P\ /\ p_{a1}} = \frac{C_1^P\ /\ p_{b1}}{C_0^P\ /\ p_{b1}} = \frac{C_1^P}{C_0^P}$$

P(q) obviously provides a quite different measure of productivity change than does L(q).

Figure 14.2 further illustrates the index number problem with production possibility curves q_0 for a given quantity of inputs using technology of period 0 and q_1 for the *same* quantity of inputs using technology of period 1. The quantity of product x is q_x and of product y is q_y. Given base period technology and prices, the formula for the maximum isorevenue line (dotted) tangent to q_0 at h is

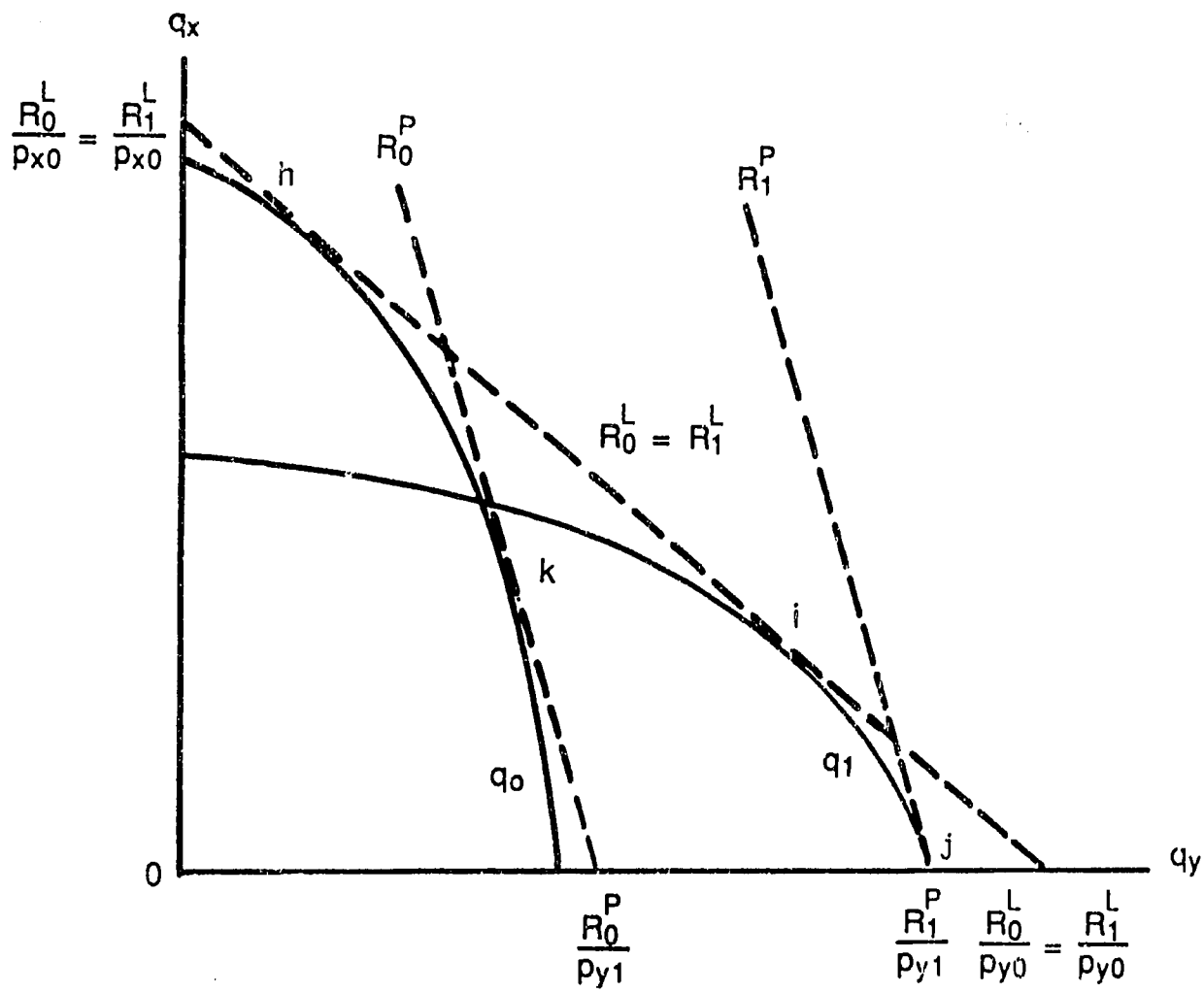

Figure 14.2. Production Possibility Curves and Isorevenue Lines Used to Illustrate Index Number Dilemma for Products q_x and q_y.

$$R_0^L = p_{x0}\, q_{x0} + p_{y0}\, q_{y0} = \Sigma\, p_0\, q_0. \tag{14.18}$$

The intercept on the q_x axis is R_0^L / p_{x0} and on the q_y axis is R_0^L / p_{y0}.

The highest attainable real revenue line using prices of base period 0 for technology q_1 is

$$R_1^L = p_{x0}\, q_{x1} + p_{y0}\, q_{y1} = \Sigma\, p_0\, q_1 \tag{14.19}$$

and the intercept on the q_x axis is R_1^L / p_{x0} and on the q_y axis is R_0^L / p_{y0}. Thus the Laspeyres formula for an increase in productivity is

$$L(q) = \frac{\Sigma\, p_0\, q_1}{\Sigma\, p_0\, q_0} = \frac{R_1^L / p_{x0}}{R_0^L / p_{x0}} = \frac{R_1^L / p_{y0}}{R_0^L / p_{x0}} = R_0^L / R_1^L .$$

In this case $R_0^L = R_1^L$ and the change in technology does not change productivity.

Now weighting by prices of period 1 to form the Paasche index, we begin with the highest isorevenue line attainable tangent to q_1 at j given prices of period 1:

$$R_1^P = p_{x1}\, q_{x1} + p_{y1}\, q_{y1} = \Sigma\, p_1\, q_1 . \tag{14.20}$$

The intercept on the vertical axis and expressed in terms of q_x is not shown but the intercept on the horizontal axis is R_1^P / p_{y1}. The maximum real (constant price) revenue available with q_0 technology and prices of period 1 is represented by tangency on the dotted line at k and is

$$R_0^P = p_{x1}\, q_{x0} + p_{y1}\, q_{y0} = \Sigma\, p_1\, q_0 \tag{14.21}$$

and the horizontal axis intercept is R_0^P / p_{y1} with $q_x = 0$. The Paasche quantity index is therefore

$$P(q) = \frac{\Sigma\, p_1\, q_1}{\Sigma\, p_1\, q_0} = \frac{R_0^P / p_{x1}}{R_1^P / p_{x0}} = \frac{R_1^P / p_{y1}}{R_0^P / p_{y1}} = \frac{R_1^P}{R_0^P} .$$

Thus the Laspeyres index showed no increase in productivity and the Paasche index shows a considerable increase in productivity in Figure 14.2. That the Laspeyres index showed no change was purely coincidental with the design of the illustration. Other combinations of prices and technologies could have produced different results but the examples indicate clearly that the choice of price (or quantity) weights can have a major impact on results.

Although technology has changed from isoquant Q_0 to Q_1 in Figure 14.1 to produce a given amount of output, the Laspeyres index shows no gain in productivity while the Paasche index shows less real cost C_1^P is required under the new than under the old technology C_0^P. And although technology has changed from production possibility line q_0 to q_1 in Figure 14.2 to produce products with a given volume of resources, the Laspeyres formula shows no increase in productivity whereas the Paasche formula shows an increase in productivity. These are special cases, but the two forms of measurement ordinarily will not agree. Which formulation and answer is correct? We do not know. The index number problem cannot be resolved, although there may be reasons to prefer one formula over another as will be noted later.

An Example of Index Number Formulation

As noted above, physical quantities or prices for individual commodities such as cotton and oranges cannot merely be added together to measure composite output, but must be weighted properly to form meaningful indices. Indices in any given year are expressed as a percentage of the index in some base year.

1. Laspeyres Index. The most widely used aggregation procedure is to weight prices (quantities) by base period quantities (prices) to form a Laspeyres Index. The procedure is illustrated with a simple example where the commodities (inputs or outputs) are Z and Y and p is price and q is quantity (Table 14.1). The conventional notation is to designate price or quantity in the base year by the subscript zero (0) and in any given year other than the base by the subscript one (1). The Laspeyres quantity index L(q) for the data in Table 14.1 is

$$L(q) = \frac{p_{0Z}\, q_{1Z} + p_{0Y}\, q_{1Y}}{p_{0Z}\, q_{0Z} + p_{0Y}\, q_{0Y}} = \frac{\Sigma\, p_0\, q_1}{\Sigma\, p_0\, q_0}$$

or

					L(q)
1965	(.60 x 1,161)	+	(60 x 60)	= 4,296.60	100
1975	(.60 x 725)	+	(60 x 85)	= 5,535.00	129
1985	(.60 x 249)	+	(60 x 110)	= 6,749.40	157.

These values expressed as percent of the base year (1965) constitute the Laspeyres Index and indicate that the aggregate quantity of X and Z increased to 129 percent of the 1965 level in 1975 and to 157 percent of the 1965 level in 1985.

Table 14.1. Data to be Aggregated to Illustrate Index Number Formulas

Year	Commodity Z		Commodity Y		Gross Value ($)
	Price p($/unit)	Quantity q(units)	Price p($/unit)	Quantity q(units)	
1965	.60	1,161	60	60	4,296.60
1975	1.20	725	80	85	7,670.00
1985	2.50	249	120	110	13,822.50

The Laspeyres price index L(p) is defined as

$$L(p) = \frac{p_{1Z}\, q_{0Z} + p_{1Y}\, q_{0Y}}{p_{0Z}\, q_{0Z} + p_{0Y}\, q_{0Z}} = \frac{\Sigma\, p_1\, q_0}{\Sigma\, p_0\, q_0}$$

or

				L(p)
1965	(.60 x 1,161) +	(60 x 60) =	4,296.60	100
1975	(1.20 x 1,161) +	(80 x 60) =	6,193.20	144
1985	(2.50 x 1,161) +	(120 x 60) =	10,102.50	235.

The Laspeyres Index of price increased from 100 in the base year (1965) to 144 in 1975 and to 235 in 1985.

2. ***Paasche Index.*** In contrast to the Laspeyres Index which uses base year weights, the Paasche Index uses weights of the given or "current" year. The Paasche formula for a quantity index P(q) is

$$P(q) = \frac{p_{1Z}\, q_{1Z} + p_{1Y}\, q_{1Y}}{p_{1Z}\, q_{0Z} + p_{1Y}\, q_{0Y}} = \frac{\Sigma\, p_1\, q_1}{\Sigma\, p_1\, q_0}$$

or from Table 14.1 is

		P(q)
1965	$\frac{(.60 \times 1{,}161) + (60 \times 60)}{(.60 \times 1{,}161) + (60 \times 60)} = \frac{4{,}296.60}{4{,}296.60}$	100
1975	$\frac{(1.20 \times 725) + (80 \times 85)}{(1.20 \times 1{,}161) + (80 \times 60)} = \frac{7{,}670.00}{6{,}193.20}$	124
1985	$\frac{(2.50 \times 249) + (120 \times 110)}{(2.50 \times 1{,}161) + (120 \times 60)} = \frac{13{,}822.50}{10{,}102.50}$	137.

The Paasche Index for price is

$$P(p) = \frac{P_{1Z}\, q_{1Z} + P_{1Y}\, q_{1Y}}{P_{0Z}\, q_{1Z} + P_{0Y}\, q_{1Y}} = \frac{\Sigma\, P_1\, q_1}{\Sigma\, P_0\, q_1}$$

or

		P(p)
1965	$\frac{(.60 \times 1161) + (60 \times 60)}{(.60 \times 1161) + (60 \times 60)} = \frac{4{,}296.60}{4{,}296.60}$	100
1975	$\frac{(1.20 \times 725) + (80 \times 85)}{(.60 \times 725) + (60 \times 85)} = \frac{7{,}670.00}{5{,}335.00}$	139

$$1985 \quad \frac{(2.50 \times 249) + (120 \times 110)}{(.60 \times 249) + (60 \times 110)} = \frac{13{,}822.50}{6{,}749.40} \qquad 205.$$

*3. **Fisher Ideal Index.*** An advantage of the Laspeyres Index and a reason it is most widely used is its convenience. Only base period prices are needed to compute the quantity index and only base period quantities are needed to compute the price index. This saves expenses of gathering data. In this example, the Laspeyres Index gave the higher estimate of the composite price and quantity changes. The Paasche Index gave lower estimates of the composite price and quantity changes for 1965 to 1985.

Although the index number problem has no solution, the Fisher Ideal Index has appeal as a compromise between the extreme results from the base period weighting under the Laspeyres Index and given period weighting under the Paasche Index. The Fisher Ideal is the geometric mean of the Laspeyres and Paasche Indices: $F(p) = \sqrt{L(p)\,P(p)}$ for price and $F(q) = \sqrt{L(q)\,P(q)}$ for quantity. The Fisher Ideal not only gives results between the Laspeyres Index and the Paasche Index, it also meets the so called *factor reversibility test.* That is, the Fisher Index for price multiplied by the Fisher Index for quantity gives the actual exact index for value computed directly from the price and quantity data in Table 14.1.

*4. **Divisia (Chain Link) Index.*** The Divisia Index has certain properties of the Laspeyres and Paasche Indexes and tends to lie between their values. Ordinarily, the Divisia Index would be formed year by year sequentially but data for decades will be used from Table 14.1.

The Divisia (chain link Laspeyres approximation) price index is defined as

$$D(p) = \frac{\Sigma\, p_1 q_0}{\Sigma\, p_0 q_0} \times \frac{\Sigma\, p_2 q_1}{\Sigma\, p_1 q_1} \times \ldots \times \frac{\Sigma\, p_t q_{t-1}}{\Sigma\, p_{t-1} q_{t-1}}.$$

Given

$$\begin{aligned}
\Sigma\, p(65)\, q(65) &= (.60 \times 1{,}161) + (60 \times 60) = 4{,}296.60 \\
\Sigma\, p(75)\, q(65) &= (1.20 \times 1{,}161) + (80 \times 60) = 6{,}193.20 \\
\Sigma\, p(75)\, q(75) &= (1.20 \times 725) + (80 \times 85) = 7{,}670.00 \\
\Sigma\, p(85)\, q(75) &= (2.50 \times 725) + (120 \times 85) = 12{,}012.50
\end{aligned}$$

and the quantity index is

$$D(q) = \frac{\Sigma\, p_0 q_1}{\Sigma\, p_0 q_0} \times \frac{\Sigma\, q_1 q_2}{\Sigma\, p_1 q_1} \times \ldots \times \frac{\Sigma\, p_{t-1} q_t}{\Sigma\, p_{t-1} q_{t-1}}.$$

Given

$$\Sigma\, p\,(65)\, q\,(65) = (\ .60 \times 1{,}161) + (60 \times 60) = 4{,}296.60$$
$$\Sigma\, p\,(65)\, q\,(75) = (\ .60 \times 725) + (60 \times 85) = 5{,}535.00$$
$$\Sigma\, p\,(75)\, q\,(75) = (1.20 \times 725) + (80 \times 85) = 7{,}670.00$$
$$\Sigma\, p\,(75)\, q\,(85) = (1.20 \times 249) + (80 \times 110) = 9{,}098.80$$

then the quantity index is

			D(p)
1965	$\Sigma\, p\,(65)\, q\,(65)$	$= 4{,}296.60$	100
1975	$\dfrac{\Sigma\, p(65)\, q\,(75)}{\Sigma\, p(65)\, q\,(65)}$	$= \dfrac{5{,}535.00}{4{,}296.60}$	129
1985	$\dfrac{\Sigma\, p(65)\, q\,(75)}{\Sigma\, p(65)\, q\,(65)} \times \dfrac{\Sigma\, p(75)\, q(85)}{\Sigma\, p(75)\, q(75)}$	$= \dfrac{5{,}535.00}{4{,}296.60} \times \dfrac{9{,}098.80}{7{,}670.00}$.	153.

The Divisia price index is

			D(p)
1965	$\Sigma\, p\,(65)\, q\,(65)$	$= 4{,}296.60$	100
1975	$\dfrac{\Sigma\, p(75)\, q\,(65)}{\Sigma\, p(65)\, q\,(65)}$	$= \dfrac{6{,}193.20}{4{,}296.60}$	144
1985	$\dfrac{\Sigma\, p(75)\, q(65)}{\Sigma\, p(65)\, q(65)} \times \dfrac{\Sigma\, p(85)\, q(75)}{\Sigma\, p(75)\, q(75)}$	$= \dfrac{6{,}193.20}{4{,}296.60} \times \dfrac{12{,}012.50}{7{,}670.00}$.	226.

Results are summarized for the example in Table 14.2. The Fisher Ideal Index is preferred conceptually but requires more data than the Laspeyres Index. The above index number formulas do not exhaust the possibilities -- other index number formulas have been proposed and utilized (see Tomek and Robinson, pp. 204-269; Yamane, ch. 11, for further discussion of formulations). The actual choice of formulas will depend on the data and computational resources available. A common approach is a combination of the Paasche and Laspeyres formulations by using base period weights but updating them periodically to better represent current price or quantity weights.

Splicing Indices

The Laspeyres Index requires price or quantity weights only for a base year, hence requires less data and is compiled at lower cost than other indexes. Weights eventually become outdated and error becomes large. Therefore, a frequent procedure is to use a Laspeyres formula for 5 or 10 years, then change the weights. For example, suppose that

Table 14.2. Summary of Quantity, Price, and Value Indices Computed from Laspeyres, Paasche, Fisher Ideal, and Divisia Formulas, 1965 = 100.

Year	Quantity	Price	Value	Actual Value
		Laspeyres		
	L(q)	L(p)	L(q) L(p)	
1965	100	100	100	100
1975	129	144	186	179
1985	157	235	369	322
		Paasche		
	P(q)	P(p)	P(q) P(p)	
1965	100	100	100	100
1975	124	139	172	179
1985	137	205	281	322
		Fisher Ideal		
	$\sqrt{L(q)\,P(q)}$	$\sqrt{L(p)\,P(p)}$	F(q) F(p)	
1965	100	100	100	100
1975	126	142	179	179
1985	147	219	322	322
		Divisia		
	D(q)	D(p)	D(q) D(p)	
1965	100	100	100	100
1975	129	144	186	179
1985	153	226	346	322

a Laspeyres price index is formed using average 1974-76 quantities to weight prices in the 1970s and average 1984-86 quantities to weight to weight prices in the 1980s with results in Table 14.3.

The price indexes for the overlapping year, 1980, are used to form a consistent series. The constant 86/115 = .75 formed from overlapping observations is multiplied by each of the 1975 series prices to convert them to the merged series so that 1985 = 100. (An equally valid alternative would be to multiply each of the 1985 based series by 115/86 = 1.34 to express all years as a percent of 1975 = 100.) The Laspeyres and Paasche indexes will usually represent the upper and lower bounds of the true index. The above procedure combines the advantage of the low cost of the Laspeyres fixed weights for a few years with the advantage of current "Paasche" weights in updating the weights periodically. It is useful to note that the "base" in a Laspeyres formula may be any one year, an average of all years, or an average of selected years. It need not be the first year or the year of which the index is expressed as a percentage.

Illustrating Uses of Index Numbers

Examples abound of composite series useful in policy analysis. Two widely used series are indices of prices received and paid by producers and indices of factor or multifactor productivity.

1. Price Indices. An index of prices received by producers measures trends in a composite of commodities produced by farmers. The composite may be for commodity groups such as coarse grains, food grains, all crops, all livestock and livestock products, or for crops and livestock combined. Trends in such series may be of interest directly or indirectly for inclusion in supply analysis discussed earlier or in other analysis.

An index of prices paid by producers measures trends in prices of a composite of inputs. The composite may be for an input group such as purchased inputs, production inputs, purchases for consumption, farm produced inputs, or for all inputs. An input price index may be used to adjust commodity support prices or to deflate prices in supply analysis.

The ratio of the index of prices received by producers to the index of prices paid by producers measures the terms of trade for agriculture. Changes in terms of trade can measure changes in farm economic well-being or incentives to produce. It is important to note, however, that the well-being of persons in agriculture depends on productivity, off-farm income, and other factors as well as on terms of trade.

The composite price in Table 14.4 is assumed to be a consumer price index and is used to deflate nominal income to express real income (purchasing power) and real prices paid in Table 14.4. Price indexes are expressed in percentages, hence it is necessary to move the decimal point over two places to the left before dividing (deflating) nominal income and prices paid to express them in real terms.

Table 14.3. Example of Splicing Times Series to Convert to a Percentage of 1985, i.e. 1985 = 100.

Year	Price Indexes		
	1974-76 Quantity Weights	1984 - 86 Quantity Weights	Merged Series
	(1975 = 100)	(1985 = 100)	(1985 = 100)
1975	100	--	75
1976	105	--	79
1977	108	--	81
1978	110	--	82
1979	112	--	84
1980	115	86	86
1981	--	89	89
1982	--	90	90
1983	--	94	94
1984	--	93	93
1985	--	100	100

2. *Productivity*. Productivity generally refers to ratios of output to input (U.S. Department of Agriculture 1980). If only one input and one output are involved, productivity can be measured by *technical efficiency* With more than one input or output, *economic efficiency* is involved because aggregation is necessary using price-weight procedures described above.

Productivity does not necessarily measure technological change as noted earlier, in Figures 14.1 and 14.2 or economic efficiency (Tweeten, ch.5). The divergence between productivity and economic efficiency is greatest for partial productivity measures. Common partial productivity measures include gross farm output per unit of farm labor, persons supplied per farm worker, crop yield per hectare, or crop yield per unit of fossil fuel. Improving economic efficiency entails substitution of profitable and productive improved capital inputs for less profitable and productive conventional inputs such as farm labor and land. In the process, output per unit of labor increases as output expands or as less efficient labor resources are released to other uses. Thus increased labor productivity as measured by gross output per unit of labor may be a sign of labor inefficiency -- economically inefficient labor in a sector is replaced by improved capital inputs. If oxen are replaced by more profitable tractors in a newly industrialized economy, we do not say that oxen are efficient because gross output per oxen has increased. It makes sense to measure economic efficiency by output per unit of fossil fuel energy only if fossil fuel is the sole input with any value (opportunity cost).

Measuring the economic efficiency of agriculture requires both static and dynamic measures. Static measures of efficiency were listed in earlier chapters and include rates of return on resources, benefit-cost ratios, and net social loss or gain. A traditional farming economy characterized by a closed set of technologies for decades is likely to have made resource adjustments eliminating economic disequilibrium. That is, resources are allocated to their highest and best use. Lack of profitable alternatives that would reward producers for foregoing consumption to invest for the future retards economic progress. Dynamic efficiency is unlikely to be evident despite presence of static efficiency in a traditional economy.

Multifactor productivity as measured by total input relative to total output over time is a useful measure of dynamic efficiency (Braha and Tweeten). As productive and profitable improved inputs are made available from agricultural research, extension, and education in a traditional economy, economic disequilibrium will become apparent in higher rates of return on new inputs such as high-yielding varieties. As producers adopt these new inputs, total inputs will be reduced or output expanded to increase multifactor productivity. Thus, a dynamic agriculture *may* show static inefficiency but will show dynamic efficiency. Multifactor productivity gains indicate dynamic efficiency but do not necessarily indicate static efficiency -- static measures of efficiency such as rates of return are likely to show disequilibrium over a period of rapid multifactor productivity growth.

Table 14.4. Example of Nominal and Real (Deflated) Income

Year	Consumer Price Index	Nominal (Current) Income	Real Constant Dollar Income[a]	Index of Prices Paid by Farmers[b]	Index of Prices Received by Farmers[b]
	(1985 = 100)	($ Million)	(Million 1985 dollars)	(1985 = 100)	(1985 = 100)
1975	75	525	700	80	107
1976	79	560	709	82	104
1977	81	720	889	86	106
1978	82	732	893	84	102
1979	84	740	881	87	104
1980	86	725	843	87	101
1981	89	802	901	88	99
1982	90	908	1,009	89	99
1983	94	941	1,001	90	96
1984	93	960	1,032	94	101
1985	100	990	990	100	100

[a]Nominal income deflated by the consumer price index. For example for 1975, 525/.75 = 700, for 1979 560/.79 = 709, etc.

[b]The Parity Ratio or terms of trade is the ratio of the index of prices received to the index of prices paid by farmers. For example the index for 1975 is 107/.80 = 134, for 1976 is 104/.82 = 127, etc.

Static measures of efficiency are useful to show opportunities for allocating resources and products in ways that reduce real costs or increase real output, and to show the social costs of not seizing such opportunities. An economy that creates static disequilibrium through improved inputs from agricultural research and extension and has producers who quickly allocate resources to reduce static disequilibrium will display much dynamic efficiency apparent in productivity gains and national income growth. Human resource investments can be decisive in increasing capabilities of producers to respond appropriately to disequilibrium. If producers respond to reduce static disequilibrium, the result will be apparent in increased multifactor productivity and national income. But in a truly dynamic economy, agricultural disequilibrium will constantly be generated from a continuing flow of improved capital inputs, technology, management, marketing, and product mixes. Disequilibrium is unlikely to be fully eliminated even if producers are responsive to continuing incentives for change in a growing economy.

Multifactor productivity ideally is measured by total output divided by total inputs including conventional farm labor, management, and capital (including land) as well as nonconventional inputs such as general and vocational education and agricultural research and extension. In the case of durable resources the input is measured by the service flow cost -- real interest, depreciation, repairs, and maintenance. The real interest rate is the market rate less inflation. In agriculture, the real cost of assets needs to account for market interest rates less capital gains in an inflationary economy.

Nonconventional inputs are not easily measured and a common procedure is to estimate productivity as real farm output per unit of conventional farm production inputs. This series has the advantage of showing shifts in the aggregate farm supply curve over time and hence for predicting future supply-demand balance and real price trends. But it has the disadvantage of any partial productivity measure. That is, the way to increase efficiency of conventional resources is to use nonconventional resources to the point where their marginal productivity is zero. Because these nonconventional resources have an opportunity cost (resources used for research, education, and extension have value elsewhere), it is inefficient to drive their marginal product to zero. It follows that an analytical program must consider the efficient use of both conventional and nonconventional resources. Rates of return on conventional inputs or nonconventional inputs (see Ruttan) above or below rates of return elsewhere (adjusted for risk) imply static inefficiency which eventually will be apparent in dynamic inefficiency.

Improvising with Inadequate Data

Inadequate data more often than inadequate conceptual models constrain analysts assembling an index. To illustrate some difficulties and means to deal with them, consider constructing an index of prices received by farmers.

First, we take a survey of sales at the farmgate to determine shares of the commodities in the market basket. (Sales from one farmer to another are excluded to avoid double counting.) Sales at the farmgate for the base period are found to be distributed half to grains, two-fifths to vegetables and fruits, and one-tenth to livestock as follows:

Period	Grains		Vegetables and Fruits		Livestock		Index of Prices Received
	Weight (%)	Price	Weight (%)	Price	Weight (%)	Price	
0	50	100	40	100	10	100	100.0
1		120		105		103	112.3
2		125		110		102	116.7

In reality, more detail by commodity would be obtained. The price of each commodity is arbitrarily normalized to a percent of the base period for convenience. The value proportions in the base period are multiplied by prices to form the composite index of price received by farmers.

The grains component may be made up of millet, grain sorghum, maize, wheat, and rice. Suppose it is not possible to obtain prices for each of these grains because of limited funds or other reasons. If wheat is the principal staple, if it is traded regularly in the market, and if its price is known to be highly correlated with prices of other grains, then regular enumeration of the wheat price alone may be sufficient for grains. Suppose that the enumeration band on a market survey indicates wheat prices (and presumably all grain prices in period) are 120 percent of those in the base period 1 and 125 percent of the base in period 2. (If the quality of grain changes from one year to the next it is important to obtain prices for comparable quality each year.)

Suppose that no market survey data of any kind are available for vegetable and fruit prices in periods 1 and 2. An alternative may be to contact the agricultural extension service or home economics advisors in various locations to obtain their best estimates of prices. These are then summarized and averaged, giving an estimated price for fruits and vegetables of 105 and 110 percent of that in the base period in periods 1 and 2, respectively.

Determining livestock prices might pose other problems. Suppose that livestock buyers refuse to reveal what they paid farmers for animals to be slaughtered. Prices from the few auction markets in the country might be used or farmers themselves queried. Perhaps neither of these approaches is feasible. Suppose it is found that past trends in wholesale meat prices are highly correlated with past trends in farmgate livestock prices. Then wholesale meat prices might be used to update livestock prices. Suppose the best estimate is that livestock prices were 103 percent of those in the base period in period 1 and were 102 percent of the base period in period 2. Multiplying each weight by price gives the index of prices received for each year:

Period 0	.50(100) + .40(100) + .10(100) = 100.0
Period 1	.50(120) + .40(105) + .10(103) = 112.3
Period 2	.50(125) + .40(110) + .10(102) = 116.7

It is apparent that the price weighing most heavily and varying most warrants special attention when improving data. In this case, that price is for grains. Accordingly, grains might be given more detail with an enlarged special effort to estimate the weight and price trend for, say, maize if it was important and its price diverged considerably from the wheat price movement. The Laspeyres weights need to be updated periodically also. For additional examples, see Collins and Hill.

ACTIVITIES

1. Work through illustrations in the previous section to improve understanding and to check results.
2. Find data on prices and quantities of crops or livestock for a country and construct one or more indexes, but beginning with a Laspeyres.
3. Find time series data on current fertilizer or other input prices and change to real prices by deflating according to an appropriate measure of purchasing power.

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