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"Optimal Sampling for the Estimation
of Age Composition With an Age-Length Key:
Analysis and the Program, 'AGECOMP'"
by
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PREFACE

Even under the best of biological circumstances, estimating age composition and related parameters is costly, time consuming, and subject to the variability from reading errors and natural causes. To a large extent, these factors can all be treated as a complicated statistical problem involving a multistage sampling design and a cost function that relates the variance of estimates and the cost of processing samples for ageing. The starting point is the well-known age-length key which relates the number of fish assigned different ages that fall into the same length catagory.

The paper by Dr. Lai published in the Fishery Bulletin presents an analysis which optimizes the balance between the dollars spent on ageing and the variance of the estimates of age composition. Naturally, as dollars (or time) spent increase we expect variance to decrease. The methods are currently in use in several Northeast Pacific fisheries and are also part of the ageing project carried out on the corvina fishery in the Gulf of Nicoya in Costa Rica. This work is funded by the USAID CRSP Stock Assessment Project.

Dr. Lai's published article does not contain the computer program AGECOMP and the User's Guide which are both included in this technical report. The program is available for either the IBM-PC or VAX from Dr. Lai and/or from

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OPTIMUM ALLOCATION FOR ESTIMATING AGE COMPOSITION USING AGE-LENGTH KEY

HAN-LIN LAI[1]

ABSTRACT

A new optimum allocation method for age-length keys (ALK) was developed by applying Kimura's Vartot, an error index of estimated age composition. The method is applied to Pacific cod, sablefish, and walleye pollock. At the present working capacity, the total of 10,000 minutes (about 70 working days) will approximate the most effective cost to estimate age composition for the three species. Increasing costs beyond this level will show no more gain.

Age-length keys (ALK) are widely used for estimating age compositions in fisheries. The theory of ALK is based on a double sampling technique with stratification (Tanaka 1953). The first stage involves a simple random sampling for a relatively large size, less costly length sample. The second stage involves a stratified random sampling for a smaller size, more costly age subsample from each length stratum. Following the approximation of Kutkuhn (1963) and Southward (1963), the proportion of fish at the ith age class (p_i) and variance of p_i are estimated as

$$p_i = \sum_{j=1}^{L} l_j q_{ij} \tag{1}$$

$$\mathrm{Var}(p_i) = \sum_{j=1}^{L} \left[\frac{l_j^2 q_{ij}(1 - q_{ij})}{n_j} + \frac{l_j (q_{ij} - p_i)^2}{N} \right] \tag{2}$$

where l_j is the proportion of fish that fall into the jth length stratum,
N is total length sample size,
n_j is the size of age subsample in the jth length stratum,
q_{ij} is the proportion of n_j fish classified into the ith age class,
A is the number of age classes, and
L is the number of length strata.

Kimura (1977) defined Vartot as the sum of all variances of the p_i:

$$\mathrm{Vartot} = \sum_{i=1}^{A} \mathrm{Var}(p_i) = E\left[\sum_{i=1}^{A} (\hat{p}_i - p_i)^2 \right] \tag{3}$$

which is an error index for assessing precision of the ALK. Furthermore, Vartot is the expectation of the squared distance between the estimated age composition $\hat{P}' = (\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_A)$ and the true age composition of the population $P' = (p_1, p_2, \ldots, p_A)$. Then, $D = \sqrt{\mathrm{Vartot}}$ can be interpreted as a kind of average Euclidean distance between $\hat{P}$ and P in an A-dimensional space. Kimura (1983) indicated that D can be viewed as the percent error of the estimated accumulated age proportion (i.e., the percent error of $\sum p_i = 1$).

This paper derives a new method for the optimum allocation of ALK, applying the properties of Vartot and Cauchy-Schwarz inequality (Kendall and Stuart 1977). The optimum sizes of length sample and age subsample are determined so that either Vartot is minimized subject to a fixed total cost or the total cost is minimized subject to a desired level of Vartot. Although this method is basically derived for the problem that all age classes are of equal interest, it can be modified by adding weighting factors to the ages which are important to population dynamics. This method was applied to Pacific cod, *Gadus macrocephalus*, from the Washington coast, sablefish, *Anoplopoma fimbria*, from the Gulf of Alaska; and walleye pollock, *Theragra chalcogramma*, from the eastern Bering Sea.

METHODS

Two subsampling schemes related to ALK are frequently used by fisheries biologists: 1) fixed age sub-

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sampling, in which the size of age subsample in all length strata is constant (i.e., $n_j = n/L$, where $n = \sum n_j$ is total age subsample size), and 2) random age subsampling, in which the size of the age subsample in each length stratum is proportional to the length sample size for all length strata (i.e., $n_j = nl_j$). Thus applying Equations (2) and (3), Vartot for a fixed age subsample (Appendix A) is

$$\text{Vartot} = \frac{a_1}{n} + \frac{a_2}{N} \tag{4}$$

and Vartot of a random age subsample is

$$\text{Vartot} = \frac{b_1}{n} + \frac{b_2}{N} \tag{5}$$

where $a_1 = \sum_{i=1}^{A} \sum_{j=1}^{L} [L\, l_j^2\, q_{ij}\,(1 - q_{ij})]$

$$a_2 = b_2 = \sum_{i=1}^{A} \sum_{j=1}^{L} [l_j\,(q_{ij} - p_i)^2]$$

$$b_1 = \sum_{i=1}^{A} \sum_{j=1}^{L} [l_j\, q_{ij}\,(1 - q_{ij})].$$

It should be noted that a_1, a_2, b_1, and b_2 are all positive values.

The total cost of a survey can be expressed as some function of the costs of the two sampling stages. The commonly used form of the cost function is a simple linear equation relating the unit cost of observing the length and the age of a fish to N and n (Tanaka 1953; Kutkuhn 1963; Southward 1963):

$$C = c_1 N + c_2 n \tag{6}$$

where C is the total cost, c_1 is the unit cost of observing the length of a fish, and c_2 is the unit cost of determining the age of a fish. For optimum allocation, the problem becomes one of determining the values of N and n, which will provide an estimated age composition with a minimum Vartot subject to a given total cost C.

Cauchy-Schwarz inequality, which is frequently used in sampling theory (Cochran 1977; Kendall and Stuart 1977; Schweigert and Sibert 1983), is applied to find optimum set of N^* and n^* for an ALK. The set of N^* and n^* is obtained when they minimize the product of D^2 (= Vartot) and C.

For a fixed age subsample, using Equations (4) and (6) and the Cauchy-Schwarz inequality (Appendix B), we obtain

$$r^* = (n/N)^* = \sqrt{a_1 c_1 / a_2 c_2}. \tag{7}$$

This quantity is the optimum subsampling ratio (r^*) required to reach the minimum (min.) Vartot subject to the cost function given in Equation (6). Therefore, N^* and n^* are dependent on Equation (6):

$$N^* = C/(c_1 + c_2 r^*) \tag{8}$$

$$n^* = r^* N^* \tag{9}$$

$$\text{min. Vartot} = \frac{a_1}{n^*} + \frac{a_2}{N^*}. \tag{10}$$

Similarly, the optimum allocation of a random age subsample can be obtained using Equations (5) and (6) and the Cauchy-Schwarz inequality. The solutions of r^*, N^*, n^*, and min. Vartot are

$$r^* = (n/N)^* = \sqrt{b_1 c_1 / b_2 c_2} \tag{11}$$

$$N^* = C/(c_1 + c_2 r^*) \tag{12}$$

$$n^* = r^* N^* \tag{13}$$

$$\text{min. Vartot} = \frac{b_1}{n^*} + \frac{b_2}{N^*}. \tag{14}$$

Generally, survey designs are based on two constraints (Schweigert and Sibert 1983). The first, as derived previously, relates to obtaining the precision of p_i, viz., to minimize Vartot at a fixed total cost. The second determines the total cost required to achieve a given precision, that is, to minimize total cost (min. C) at a desired level of Vartot. In the latter problem, r^* in Equations (7) and (11) will also minimize the product of D^2C irrespective of the value of C and D^2, i.e., it will minimize D^2 for fixed C or C for fixed D^2 (Kendall and Stuart 1977, Section 39.20). The solutions of n^* and N^* are now dependent on the desired level of Vartot.

For a fixed age subsample, substituting r^* of Equation (7) into Equation (4), we obtain

$$N^* = (a_1/r^* + a_2)/D^2 \tag{15}$$

$$n^* = r^* N^* \tag{16}$$

$$\text{min. } C = c_1 N^* + c_2 n^* \tag{17}$$

For a random age subsample, N^* is obtained by substituting Equation (11) into Equation (5)

$$N^* = (b_1 r^* + b_2)/D^2 \quad (18)$$

$$n^* = r^* N^* \quad (19)$$

$$\min. C = c_1 N^* + c_2 n^*. \quad (20)$$

Because $0 < r \leq 1$, Equations (7) and (11) indicate that

$c_1/c_2 \leq a_2/a_1$ for fixed age subsample and

$c_1/c_2 \leq b_2/b_1$ for random age subsample

must be held for the optimum allocation. When equality holds, $r = 1$ and then ALK degenerates to a simple random sampling for age samples.

EXAMPLES

Three sets of ALK data are used for the example: Pacific cod, aged by the scale method, from the Washington coast (Kimura 1977); sablefish, aged by the otolith method, from the Gulf of Alaska; and walleye pollock, aged by the otolith method, from the eastern Bering Sea (Lai 1985). The parameters of Vartot, a's and b's, are calculated and summarized in Table 1. Accurate cost estimates are difficult to determine; therefore, time measurements required for observing a length and determining an age of fish are used for c_1 and c_2. The total cost C is thus the total time required to build an ALK. The measurements of c_1 and c_2 for the three species (Table 1) are primarily based on the author's experience.

Given a total of 120 working days and 6 working hours per day to a fisheries scientist, the optimum allocation is summarized in Table 2. Under this budget, a random age subsample can provide higher precision than a fixed age subsample (improved 10%, 15%, and 18% respectively for Pacific cod, sablefish, and walleye pollock). Also, for this budget, the error of estimated cumulated age proportion is less than 2.5% for the three species using either fixed or random age subsamples.

Using Equations (7), (15), (16), and (17) for a fixed age subsample and Equations (11), (18), (19), and (20) for a random age subsample, costs under various desired D are minimized for the three species (Table 3). At the same level of D, a fixed age subsample requires much larger sample sizes of length and age than a random age subsample does for the three species. The greatest benefit of using random age subsample is that it drastically reduces the total cost required to obtain the same level of D. The total cost is reduced by 35%, 45%, and 55% respectively for Pacific cod, sablefish, and walleye pollock for any given D when a random instead of a fixed age subsample is used.

Figure 1 shows the relationships of D and total cost. Whether a fixed or a random age subsample is used for the three species, it is obvious that D decreases rapidly until $C = 10{,}000$ minutes, which is nearly 70 working days. A point of diminishing

TABLE 1.—Parameters of Vartot and costs for optimum allocation.

	Pacific cod	Sablefish	Walleye pollock
a_1	0.4495	1.2222	1.5501
a_2	0.2600	0.1717	0.1217
b_1	0.2768	0.6235	0.6834
b_2	0.2660	0.1717	0.1217
[1]C	43,200	43,200	43,200
[1]c_1	0.5	0.5	0.5
[1]c_2	15	15	15

[1]In minutes, assuming 120 working days for each species and 6 working hours per day.

TABLE 2.—Optimum allocation of minimizing Vartot for Pacific cod, sablefish and walleye pollock. ($D = \sqrt{\text{Vartot}}$.)

	Pacific cod		Sablefish		Walleye pollock	
	fixed	random	fixed	random	fixed	random
N^*	10,641	13,121	5,534	7,555	5,093	7,551
n^*	2,525	2,443	2,695	2,629	4,064	3,943
D^*	0.014	0.013	0.022	0.019	0.020	0.017

TABLE 3.—Optimum allocation minimizing total cost for various desired precision level ($D = \sqrt{\text{Vartot}}$). Parameters and c_1 and c_2 are listed in Table 1.

		Pacific cod		Sablefish		Walleye pollock	
D		fixed	random	fixed	random	fixed	random
0.05	N^*	865	701	1,073	788	828	557
	n^*	206	130	523	273	659	291
	C^*	3,508	2,307	8,372	4,492	7,002	3,188
0.04	N^*	1,351	1,095	1,678	1,228	1,290	870
	n^*	321	204	817	427	1,030	454
	C^*	5,482	3,605	13,080	7,019	11,940	4,978
0.03	N^*	2,401	1,947	2,979	2,182	2,310	1,547
	n^*	570	362	1,451	759	1,831	808
	C^*	9,745	6,410	23,254	12,478	19,450	8,850
0.02	N^*	5,401	4,360	6,702	4,910	5,160	3,480
	n^*	1,282	815	3,265	1,708	4,118	1,817
	C^*	21,924	14,422	52,320	28,076	43,781	19,912
0.01	N^*	21,604	17,520	26,809	19,640	20,640	13,922
	n^*	5,121	3,262	13,059	6,832	16,473	7,289
	C^*	87,699	57,687	209,279	112,305	175,043	79,650

C^* in minutes.

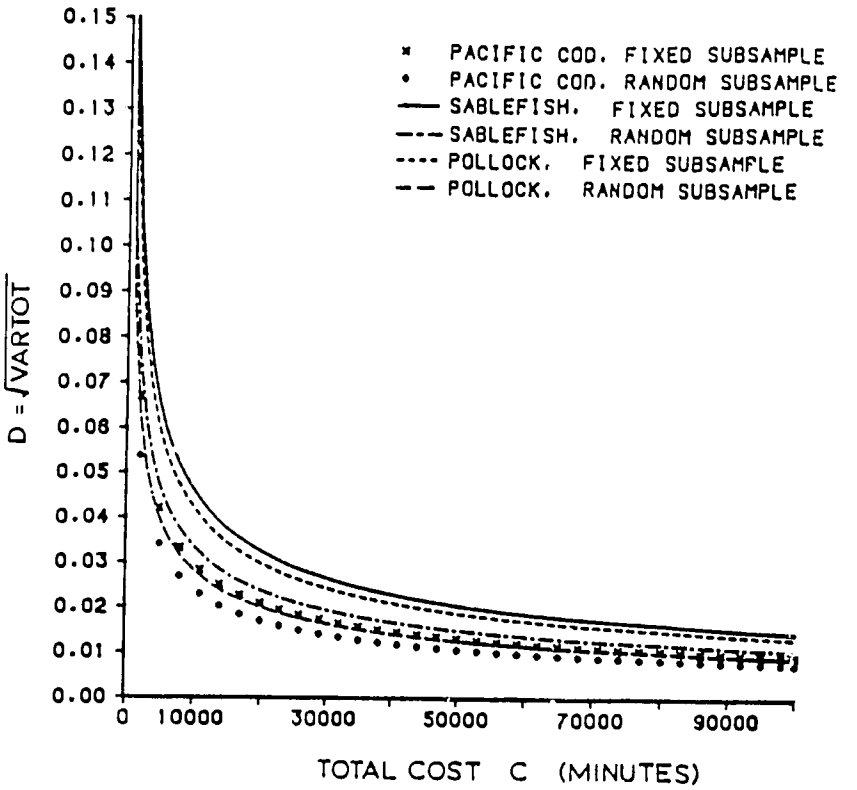

FIGURE 1.—The relationship of D ($= \sqrt{\text{Vartot}}$) and total cost for Pacific cod, sablefish, and walleye pollock, using fixed or random age subsample.

returns is reached beyond this total cost and the curves become flatter for C greater than 10,000. These results indicate that setting a precision at $D = 0.02$, 0.025, and 0.03 respectively for Pacific cod, walleye pollock, and sablefish using random age subsamples would represent a reasonable compromise between cost and precision. Increasing total cost beyond this level will show no more gains from the ALK.

DISCUSSION

It is obvious that the random subsampling scheme is superior to the fixed subsampling scheme. However, it is more important to realize that there is a cap on total cost for ALK. This cap represents the most effective budget for ALK. Vartot of estimated age composition will not decrease significantly for a greater budget. For the three species, total cost of 10,000 minutes (about 70 working days) is the upper limit. This indicates that approximately 2,000 length observations and 800 random age subsamples for sablefish, 2,500 and 1,200 for walleye pollock, and 3,000 and 600 for Pacific cod represent the best compromise between cost and precision of estimates ($\sqrt{\text{Vartot}} = 0.03$, 0.025, and 0.02 for the three species respectively).

Although it can be argued that minimizing Vartot may not be sufficient for optimum sampling design for all age classes, it is necessary to consider that some age classes are rare in the commercial catch and are therefore difficult to sample precisely. However, these age classes do not generally represent significant contributions to biomass, and it therefore seems reasonable to concentrate on the major age classes. If these rare age classes are important to population dynamics, the optimum allocation can be addressed as a multiple minima. The objective function can be rewritten as

$$M(N,n) = \sum w_i \operatorname{Var}(p_i), \qquad \text{for } i = 1, 2, \ldots, A$$

where w_i is weighting factor. A larger w_i must be given to those age classes which are of interest, whereas the mathematical expressions of optimum allocation are the same as Equations (7) to (14), and subject to the same cost function (Equation (6)), ex-

cept that a's and b's are weighted by w_i. In fact, minimizing Vartot is a special case of minimizing $M(N,n)$ = Vartot for all $w_i = 1$.

Another argument may relate to the possibility that the cost function may be more complicated so that traveling and overhead costs can be taken into account. In such cases, cost function may become a nonlinear form, and the explicit expressions of n^* and N^* cannot be obtained. However, the technique of nonlinear programming can be applied to find numerical solutions of n^* and N^*. In general, it is to

$$\text{minimize } M(N,n) = \sum w_i \operatorname{Var}(p_i)$$

$$\text{subject to } C = c(N,n)$$

where C is total cost and $c(N,n)$ is cost function. Many optimization programs can be employed from popular computer software packages for main frame computers. Bunday (1984) provided several BASIC programs for constrained optimization, which may be useful in personal computers. It should be noted, however, that the sufficient and necessary conditions of this constrained minimum must be proved. Theoretically, there is a unique minimum if objective function is convex and constraint function is concave (Bunday 1984).

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APPENDIX A. Derivation of Vartot

For a fixed age subsampling, substituting $n_j = n/L$ into Equation (2), the $\mathrm{Var}(p_i)$ is

$$\mathrm{Var}(p_i) = \sum_{j=1}^{L} \left[\frac{L\, l_j^2\, q_{ij}\,(1 - q_{ij})}{n} + \frac{l_j\,(q_{ij} - p_i)^2}{N} \right]. \qquad \text{(A.1)}$$

Applying Equation (3),

$$\begin{aligned}
\mathrm{Vartot} &= \sum_{i=1}^{A} \mathrm{Var}(p_i) \\
&= \sum_{i=1}^{A} \sum_{j=1}^{L} \left[\frac{L\, l_j^2\, q_{ij}\,(1 - q_{ij})}{n} + \frac{l_j\,(q_{ij} - p_i)^2}{N} \right] \\
&= \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [L\, l_j^2\, q_{ij}\,(1 - q_{ij})]}{n} + \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [l_j\,(q_{ij} - p_{ij})^2]}{N} \\
&= \frac{a_1}{n} + \frac{a_2}{N}
\end{aligned}$$

which is Equation (4).

For a random age subsampling, substituting $n_j = n\, l_j$ into Equation (2) and applying Equation (3), the Vartot is

$$\begin{aligned}
\mathrm{Vartot} &= \sum_{i=1}^{A} \sum_{j=1}^{L} \left[\frac{l_j\, q_{ij}\,(1 - q_{ij})}{n} + \frac{l_j\,(q_{ij} - p_i)^2}{N} \right] \\
&= \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [l_j\, q_{ij}\,(1 - q_{ij})]}{n} + \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [l_j\,(q_{ij} - p_i)^2]}{N} \\
&= \frac{b_1}{n} + \frac{b_2}{N}
\end{aligned}$$

which is Equation (5).

APPENDIX B. Derivation of Equation (7)

The Cauchy-Schwarz inequality (Cochran 1977, p. 97) is

$$(\sum_h A_h^2)(\sum_h B_h^2) - (\sum_h A_h B_h)^2 = \sum_i \sum_{i>j} (A_i B_j - A_j B_i)^2 \geq 0. \quad \text{(B.1)}$$

Therefore,

$$(\sum_h A_h^2)(\sum_h B_h^2) \geq (\sum_h A_h B_h)^2.$$

For a fixed age subsample, let

$$A_1 = \sqrt{a_1/n};\ A_2 = \sqrt{a_2/N};\ B_1 = \sqrt{c_2 n};\ \text{and}\ B_2 = \sqrt{c_1 N}.$$

Applying Equation (B.1), the product of D^2 and C is

$$D^2 C = \left(\frac{a_1}{n} + \frac{a_2}{N}\right)(c_2 n + c_1 N) \geq \left(\sqrt{a_1 c_2} + \sqrt{a_2 c_1}\right)^2. \quad \text{(B.2)}$$

The product D^2C will be minimized, provided that the equality of Equation (B.2) holds. Setting equality of Equation (B.2) and expanding both sides, we find the solution is

$$r^* = (n/N)^* = \sqrt{a_1 c_1 / a_2 c_2} \quad \text{(B.3)}$$

which is Equation (7).

For a random age subsample, let

$$A_1 = \sqrt{b_1/n};\ A_2 = \sqrt{b_2/N};\ B_1 = \sqrt{c_2 n};\ \text{and}\ B_2 = \sqrt{c_1 N}.$$

The reader can derive Equation (11) by the procedures identical to Equations (B.2) and (B.3).

USER'S GUIDE FOR PROGRAM "AGECOMP": ESTIMATION OF AGE COMPOSITION AND OPTIMUM SAMPLING FOR AN AGE-LENGTH KEY

Introduction

This document is a guide to the use of the program AGECOMP, a computer program that estimates the age composition in a catch from age-length data. The reader should refer to the papers by Lai (1987) and Quinn et al (1983) for details. Appendix I summarizes the major equations for the computation. The program is written in FORTRAN-77 and runs in an interactive mode on the VAX-VMS the MS-DOS operating systems. To obtain a copy of the source program in ASCII code, please send a floppy disk ($5\frac{1}{4}$" MS-DOS) to Vincent F. Gallucci or Han-Lin Lai, Management Assistance for Artisanal Fisheries (MAAF), Center for Quantitative Science, HR-20, University of Washington, Seattle, WA 98195.

This program is organized into a main program with seven subroutines:

1) SUBROUTINE INPUT: input age-length key and length frequency data.
2) SUBROUTINE REGROUP: combines length groups into larger length intervals.
3) SUBROUTINE AGECOMP: estimates proportion of age and its variance.
4) SUBROUTINE CAGE: estimates proportion of catch at age and associated variance.
5) SUBROUTINE SAVE: saves catch at age and age composition in permanent file.
6) SUBROUTINE OPTA: optimum allocation for age-length key.
7) SUBROUTINE OPT1: called by OPTA, optimum allocation minimizing Vartot subject to given total cost.
8) SUBROUTINE OPT2: called by OPTA, optimum allocation minimizing total cost subject to desired Vartot

The common statements of the program are saved in a separate file called 'COMMON.FOR'. Should the user wish to change the dimension size of the variables in the common statements (i.e., the number of age classes and/or the number of length intervals), it is only necessary to change the values in the 'PARAMETER' statements. The

stores the age-length key (see next section). Thus, the size of length interval is not explicitly entered.

Field II is the frequency (number of individuals) in that length interval. An empty length group (frequency = 0) must be entered as 0. Field III is the average weight of fish in that length interval. The number of length groups is automatically counted as the file is read. Each column should be separated by a "," or a blank.

B. File for age-length key

The second file (named 'ALK.DAT' in our example) contains the age-length key (Table 2). Each record (line) is arranged in the following format:

Field I II III

<numerical value> , <number of fish aged in corresponding age group> , ...

Field I is the numerical value of the length interval that corresponds to the interval described in Section A. Fields II, III, ...,etc., are the number of fish aged that fall into each age group but also fall into that length interval. Enter "0" (zero) for empty cells (mandatory). The program will ask you for the range (lowest and highest ages) of the ages in the age-length key.

RUNNING THE PROGRAM

The following VMS commands are required:

1) $ FOR AGECOMP: compile the program
2) $ LINK AGECOMP: link object files
3) $ RUN AGECOMP: execution

When execution begins, you will be prompted to respond with an "Y" if you wish to see the List of Options displayed, or an "N" if you wish to proceed directly to the prompt for a choice of options. Should you type "Y", the following will appear on the screen:

'INCLUDE' statement will copy the COMMON statements stored in the file 'COMMON.FOR' into the program directly. For details of using the 'INCLUDE' and 'PARAMETER' statements refer to a FORTRAN-77 user manual for the VAX II.

How to Use Program AGE

PREPARATION OF INPUT FILES:

Prior to use the program AGECOMP, user must set up two files: 1) for the length frequency data and mean weight at each length group, and 2) for the age-length key. These two files must be saved as permanent files in your account. Your data will be entered in these files, which can be called by the program. Tables 1 and 2 are examples of these two files. All input values are read in **free format.** These sample data have been used in Professors Gallucci and Quinn's class, "Fishery Computation (CQSI 545)", University of Washington, in Spring Quarter 1985.

A. File for length frequency data

The first file (named 'LFREQ.DAT' in our example, users can choose another name) contains the length frequency and mean weight data (Table 1). The first record in the file is the total catch in weight, if this is not available enter 0.

The second and subsequent records are the length frequency and average weight for each length group. The numerical values in each of these records are arranged in the following format:

Field:	I	II	III
	<numerical value of the length interval>,	<frequency>,	<mean weight>

Field I is the numerical value associated with each length interval. It is not important whether you use the lower end, upper end, or midpoint of the range of the interval---but one of these must be entered and kept consistent throughout this file and in the file that

Table 1. Length frequancy data stored in the file 'LFREQ.DAT'. See text for a detailed explanation.

```
108663.7
13,1,0.
14,0,0
15,0,0
16,0,0
17,0,0
18,0,0
19,0,0
20,0,0
21,0,0
22,0,0
23,0,0
24,0,0
25,0,0
26,2,0.3
27,0,0
28,1,0.2
29,0,0
30,3,0.8
31,5,1.4
32,4,1.3
33,6,2.1
34,10,3.8
35,18,7.6
36,33,15.2
37,43,21.6
38,51,27.9
39,63,37.6
40,74,48.0
41,52,36.5
42,80,60.7
43,65,53.2
44,74,65.3
45,55,52.2
46,36,36.7
47,28,30.6
48,34,39.7
49,25,31.2
50,23,30.7
51,22,31.3
52,13,19.7
53,20,32.2
54,11,18.8
55,15,27.3
56,13,25.1
57,9,18.4
58,10,21.6
59,8,18.2
60,9,21.7
61,8,20.3
62,6,16.1
63,6,16.9
64,8,23.8
```

Table 2. Age-length key stored in the file 'ALK.DAT'. See the text for a detailed explanation.

```
13,1,0,0,0,0,0,0,0
14,0,0,0,0,0,0,0,0
15,0,0,0,0,0,0,0,0
16,0,0,0,0,0,0,0,0
17,0,0,0,0,0,0,0,0
18,0,0,0,0,0,0,0,0
19,0,0,0,0,0,0,0,0
20,0,0,0,0,0,0,0,0
21,0,0,0,0,0,0,0,0
22,0,0,0,0,0,0,0,0
23,0,0,0,0,0,0,0,0
24,0,0,0,0,0,0,0,0
25,0,0,0,0,0,0,0,0
26,0,1,0,0,0,0,0,0
27,0,0,0,0,0,0,0,0
28,0,0,0,0,0,0,0,1,0
29,0,0,0,0,0,0,0,0
30,0,2,0,0,0,0,0,0
31,0,1,1,0,0,0,0,0
32,0,0,2,0,0,0,0,0
33,0,0,2,1,0,0,0,0
34,0,0,3,0,0,0,0,0
35,0,0,1,2,0,0,0,0
36,0,0,2,1,0,0,0,0
37,0,0,2,1,0,0,0,0
38,0,0,2,1,0,0,0,0
39,0,0,0,2,0,1,0,0
40,0,0,0,3,0,0,0,0
41,0,0,0,3,0,0,0,0
42,0,0,1,1,1,0,0,0
43,0,0,0,1,2,0,0,0
44,0,0,0,2,1,0,0,0
45,0,0,0,2,1,0,0,0
46,0,0,0,2,1,0,0,0
47,0,0,1,2,0,0,0,0
48,0,0,0,1,2,0,0,0
49,0,0,0,2,1,0,0,0
50,0,0,0,0,2,1,0,0
51,0,0,0,0,3,0,0,0
52,0,0,0,0,1,1,1,0
53,0,0,0,0,1,2,0,0
54,0,0,0,0,2,1,0,0
55,0,0,0,0,1,2,0,0
56,0,0,0,0,2,0,0,1
57,0,0,0,0,0,3,0,0
58,0,0,0,0,0,2,1,0
59,0,0,0,0,0,2,1,0
60,0,0,0,0,1,1,1,0
61,0,0,0,0,0,2,0,1
62,0,0,0,0,0,2,1,0
63,0,0,0,0,0,1,0,2
64,0,0,0,0,0,1,1,1
```

```
      LIST OF OPTIONS
 0 : LIST THE OPTIONS
 1 : INPUT AGE-LENGTH KEY AND LENGTH FREQUENCY
 2 : REGROUP LENGTH INTERVAL
 3 : ESTIMATING PROPORTION OF AGE
 4 : ESTIMATING CATCH AT AGE
 5 : SAVE CATCH AT AGE AND AGE PROPORTION
 6 : OPTIMUM ALLOCATION FOR AGE-LENGTH KEY
-1 : STOP.
     WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
```

The user must type the option number, once each time. Usually, a user will start with Option 1 and select options in numerical order until Option 5, or 6 is reached. In general, the user **must** call a lower option number before calling a higher number. Each of the options does the following:

Option	
0	Lists the options shown above.
1	Accepts the different sets of length frequency and age-length key data. The program will ask the file names.
2	Combines length frequency data into larger interval sizes. **Note**: this is a required option even if you use the length interval which is the same as in the input file.
3	Estimates age composition by proportion and the variance (Lai 1987 and Quinn et al. 1983a, b).
4	Estimates catch at age (in number) and the variance (see Appendix I).
5	Saves the results from Options 3 and 4 in one or more different files for later use.
6	An optimum allocation scheme for sampling effort (Appendix I).
-1	Terminates the job.

Options 1 and 2 are required for every session with a new data set even if the size of the length interval is the same as in the input files. You can request a list of options any time during the session by selecting Option 0.

AN EXAMPLE

The following is a sample input/output from an interactive session using the data set shown in Tables 1 and 2. Explanatory remarks have been added in italics in the right column.

Appears on Screen	Explanation
$ FOR AGECOMP $ LINK AGECOMP $ RUN AGECOMP	*VAX/VMS commands*
	PROGRAM START
DO YOU WANT TO SEE THE OPTIONS? YES(Y)/NO(N) Y	*WANT TO SEE THE OPTIONS*
LIST OF OPTIONS 0 : LIST THE OPTIONS 1 : INPUT AGE-LENGTH KEY AND LENGTH FREQUENCY 2 : REGROUP LENGTH INTERVAL 3 : ESTIMATING PROPORTION OF AGE 4 : ESTIMATING CATCH AT AGE 5 : SAVE CATCH AT AGE AND AGE PROPORTION -1 : STOP	*LIST OF OPTIONS*
WHAT IS YOUR CHOICE OF AN OPTION NUMBER? 1	*INPUT LENGTH FREQUENCY & AGE-LENGTH KEY*
NEW FILES FOR AGE-LENGTH KEY AND LENGTH FREQUENCY? YES(Y)/NO(N) Y	*START WITH NEW FILES*
ENTER FILE NAME FOR LENGTH FREQUENCY FILE NAME --->LFREQ	*LFREQ. DAT STORES LENGTH FREQUENCY* *.DAT ia not necessary.*
ENTER FILE NAME FOR AGE-LENGTH KEY FILE NAME --->ALK	*ALK. DAT STORES AGE-LENGTH KEY* *.DAT is not necessary.*

DATA SET #1

OUTPUT IDENTIFICATION OF DATA SET

ENTER LOWEST AND HIGHEST AGES OF THE AGE-LENGTH KEY
LOWEST AGE --->
1
HIGHEST AGES --->
8

LOWEST AGE IN TABLE 2

HIGHEST AGES IN TABLE 2

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
2

REGROUP THE LENGTH INTERVAL

SIZE OF LENGTH INTERVAL TO REGROUP AGE-LENGTH KEY
1

REGROUPED BY 1cm INTERVAL

DO YOU WANT TO SEE THE REGROUPED AGE-LENGTH KEY?
YES(Y)/NO(N).
N

SUPPRESS DISPLAY OF AGE-LENGTH KEY

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
3

CALCULATE AGE PROPORTION & STANDARD DEVIATION

GIVE THE TITLE OF THIS ESTIMATION (UP TO 60 CHARACTERS)
HALIBUT, 1 STRATUM, LENGTH INTERVAL = 1cm

TITLE OF TABLE

ESTIMATION FOR DATA SET # 1
HALIBUT, 1 STRATUM, LENGTH INTERVAL = 1 CM

AGE	PROPORTION	VARIANCE	STDEV
1	.1059322E-02	.1120974E-05	.1058761E-02
2	.7944915E-02	.1185612E-04	.3443271E-02
3	.1558969E+00	.1230407E-02	.3507716E-01
4	.4548023E+00	.2993679E-02	.5471453E-01
5	.2418786E+00	.2180646E-02	.4669739E-01
6	.1038136E+00	.5854911E-03	.2419692E-01
7	.1906780E-01	.6253911E-04	.7908167E-02
8	.1447740E-01	.4279423E-04	.6541730E-02

OUTPUT AGE COMPOSITION

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
4

CALCULATE CATCH AT AGE

ESTIMATION FOR DATA SET # 1
HALIBUT, 1 STRATUM, LENGTH INTERVAL = 1 CM

OUTPUT CATCH AT AGE IN NUMBER

AGE	CATCH AT AGE	VARIANCE	STDEV
1	.1186285E+03	.1405781E+05	.1185656E+03
2	.8897136E+03	.1488923E+06	.3858657E+03
3	.1745816E+05	.1552390E+08	.3940038E+04
4	.5093117E+05	.3837092E+08	.6194427E+04
5	.2708684E+05	.2757564E+08	.5251251E+04
6	.1162559E+05	.7383872E+07	.2717328E+04
7	.2135313E+04	.7855074E+06	.8862885E+03
8	.1621256E+04	.5373474E+06	.7330399E+03

MEAN WEIGHT PER FISH = 0.9703
EMPIRICAL VARIANCE OF MEAN WEIGHT PER FISH = .3049852E-03
TOTAL CATCH IN WEIGHT = .1086637E+06
ESTIMATED TOTAL CATCH IN NUMBER = .1119853E+06
VARIANCE OF TOTAL CATCH IN NUMBER = .4062129E+07

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
6

OPTIMUM ALLOCATION OF SAMPLING EFFORT

Select one of the following options:
-1 : RETURN TO MAIN PROGRAM
1 : MINIMIZE VARTOT SUBJECT TO GIVEN TOTAL COST
2 : MINIMIZE TOTAL COST SUBJECT TO GIVEN VARTOT

1

MINIMIZING VARTOT SUBJECT TO A GIVEN TOTAL COST

Give the UNIT COST for OBSERVING LENGTH OF A FISH
C1 --->
1.0

INPUT UNIT COST FOR MEASURING LENGTH LENGTH OF A FISH

Give the UNIT COST for AGEING A FISH
C2 --->
10

INPUT UNIT COST FOR AGEING A FISH

Give the TOTAL COST
TC --->
2000

INPUT TOTAL COST

```
IF YOU USE A FIXED AGE SUBSAMPLING SCHEME
      OPTIMUM SAMPLE SIZE FOR LENGTH =      419
      OPTIMUM SAMPLE SIZE FOR AGE    =      158
      MINIMUM VARTOT OBTAINED        =  0.79519E-02
SUBJECT TO A GIVEN TOTAL COST OF    2000.00
   WITH  UNIT COST FOR OBSERVING LENGTH OF A FISH =  1.00
         UNIT COST FOR AGEING A FISH =   10.00
```

OUTPUT RESULTS OF OPTIMUM ALLOCATION FOR SAMPLING EFFORT

```
IF YOU USE A RANDOM AGE SUBSAMPLING SCHEME
    OPTIMUM SAMPLE SIZE FOR LENGTH =      596
    OPTIMUM SAMPLE SIZE FOR AGE    =      140
    MINIMUM VARTOT OBTAINED        =  0.39340E-02
SUBJECT TO A GIVEN TOTAL COST OF    2000.00
   WITH  UNIT COST FOR OBSERVING LENGTH OF A FISH =  1.00
         UNIT COST FOR AGEING A FISH =   10.00
```

Select one of the following options:
-1 : RETURN TO MAIN PROGRAM
1 : MINIMIZE VARTOT SUBJECT TO GIVEN TOTAL COST
2 : MINIMIZE TOTAL COST SUBJECT TO GIVEN VARTOT

2

MINIMIZING TOTAL COST SUBJECT TO A DISIRED VARTOT

Give the UNIT COST for OBSERVING LENGTH OF A FISH
C1 --->
1.0

Give the UNIT COST for AGEING A FISH
C2 --->
10

Give the DESIRE PRECISION for estimated age compostion in fraction.
Enter 0.05 for 5%, i.e., VARTOT = 0.0025 or 0.25%
D = SQRT (VARTOT) --->
0.05

INPUT DESIRED PRECISION

```
IF YOU USE A FIXED AGE SUBSAMPLING SCHEME
    OPTIMUM SAMPLE SIZE FOR LENGTH =     1332
    OPTIMUM SAMPLE SIZE FOR AGE    =      503
    MINIMUM TOTAL COST REQUIRED    =  0.63620E+04
SUBJECT TO DESIRED PRECISION OF .50000E-01
   WITH  UNIT COST FOR OBSERVING LENGTH OF A FISH =  1.00
         UNIT COST FOR AGEING A FISH =   10.00
```

OUTPUT RESULTS OF OPTIMUM ALLOCATION FOR SAMPLING EFFORT

```
IF YOU USE A RANDOM AGE SUBSAMPLING SCHEME
    OPTIMUM SAMPLE SIZE FOR LENGTH =      936
    OPTIMUM SAMPLE SIZE FOR AGE    =      220
    OPTIMUM TOTAL COST REQUIRED    =  0.31360E+04
SUBJECT TO DESIRED PRECISION OF .50000E-01
   WITH  UNIT COST FOR OBSERVING LENGTH OF A FISH =  1.00
         UNIT COST FOR AGEING A FISH =   10.00
```

Select one of the following options:
-1 : RETURN TO MAIN PROGRAM
1 : MINIMIZE VARTOT SUBJECT TO GIVEN TOTAL COST
2 : MINIMIZE TOTAL COST SUBJECT TO GIVEN VARTOT

-1

FINISHING OPTIMUM ALLOCATION

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
2

REGROUP LENGTH INTERVAL

LENGTH INTERVAL TO REGROUP AGE-LENGTH KEY
3

BY 3cm

DO YOU WANT TO SEE THE REGROUPED AGE-LENGTH KEY?
YES(Y)/NO(N).
N

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
3

CALCULATE AGE PROPORTION & STANDARD DEVIATION

GIVE THE TITLE OF THIS ESTIMATION, UP TO 60 CHARACTERS
HALIBUT, 1STRATUM, LENGTH INTERVAL=3cm

OUTPUT

ESTIMATION FOR DATA SET # 1
HALIBUT, 1 STRATUM, LENGTH INTERVAL = 3 CM

AGE	PROPORTION	VARIANCE	STDEV
1	.1059322E-02	.2241949E-05	.1497314E-02
2	.8625909E-02	.2533159E-04	.5033050E-02
3	.1641277E+00	.2885899E-02	.5372057E-01
4	.4557775E+00	.6597268E-02	.8122357E-01
5	.2365819E+00	.4561612E-02	.6753971E-01
6	.9969398E-01	.1186280E-02	.3444241E-01
7	.1989172E-01	.1374773E-03	.1172507E-01
8	.1424200E-01	.9092782E-04	.9535608E-02

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
4

CALCULATE CATCH AT AGE.

OUTPUT

ESTIMATION FOR DATA SET # 1
HALIBUT, 1 STRATUM, LENGTH INTERVAL = 3 CM

AGE	CATCH AT AGE	VARIANCE	STDEV
1	.1167904E+03	.2724668E+05	.1650657E+03
2	.9510076E+03	.3081011E+06	.5550685E+03
3	.1809510E+05	.3517310E+08	.5930691E+04
4	.5024953E+05	.8098258E+08	.8999032E+04
5	.2608319E+05	.5564930E+08	.7459846E+04
6	.1099127E+05	.1445381E+08	.3801817E+04
7	.2193064E+04	.1672066E+07	.1293084E+04
8	.1570182E+04	.1105677E+07	.1051512E+04

MEAN WEIGHT PER FISH = 0.9856
EMPIRICAL VARIANCE OF MEAN WEIGHT PER FISH = .3147653E-03
TOTAL CATCH IN WEIGHT = .1086637E+06
ESTIMATED TOTAL CATCH IN NUMBER = .1102501E+06
VARIANCE OF TOTAL CATCH IN NUMBER = .3938531E+07

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
2

REGROUP LENGTH INTERVAL

LENGTH INTERVAL TO REGROUP AGE-LENGTH KEY
5

BY 5cm

DO YOU WANT TO SEE THE REGROUPED AGE-LENGTH KEY?
YES(Y)/NO(N).
N

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
3

ALCULATE AGE PROPORTION

GIVE THE TITLE OF THIS ESTIMATION, (UP TO 60 CHARACTERS)
HALIBUT, 1 STRATUM, LENGTH INTERVAL=5cm

OUTPUT

ESTIMATION FOR DATA SET # 1
HALIBUT, 1 STRATUM. LENGTH INTERVAL = 5 CM

AGE	PROPORTION	VARIANCE	STDEV
1	.1059322E-02	.3362923E-05	.1833827E-02
2	.9859844E-02	.4700634E-04	.6856117E-02
3	.1767330E+00	.4919321E-02	.7013787E-01
4	.4390428E+00	.1048925E-01	.1024170E+00
5	.2331215E+00	.7051416E-02	.8397271E-01
6	.1057910E+00	.1890977E-02	.4348537E-01
7	.2104520E-01	.2287645E-03	.1512496E-01
8	.1334746E-01	.1323469E-03	.1150421E-01

WHAT IS YOUR CHOICE OF AN OPTION NUMBER?
4

OUTPUT

ESTIMATION FOR DATA SET # 1
HALIBUT, 1 STRATUM. LENGTH INTERVAL = 5 CM

AGE	CATCH AT AGE	VARIANCE	STDEV
1	.1167904E+03	.4086782E+05	.2021579E+03
2	.1087049E+04	.5715641E+06	.7560186E+03
3	.1948483E+05	.5989844E+08	.7739409E+04
4	.4840453E+05	.1282156E+09	.1132323E+05
5	.2570167E+05	.8589690E+08	.9268058E+04
6	.1166347E+05	.2302163E+08	.4798086E+04
7	.2320236E+04	.2781497E+07	.1667782E+04
8	.1471559E+04	.1608870E+07	.1268412E+04

```
MEAN WEIGHT PER FISH:                                  0.9856
EMPIRICAL VARIANCE OF MEAN WEIGHT PER FISH:  .3147653E-03
TOTAL CATCH IN WEIGHT:                        .1086637E+06
ESTIMATED TOTAL CATCH IN NUMBER:              .1102501E+06
VARIANCE OF TOTAL CATCH IN NUMBER:            .3938531E+07


WHAT IS YOUR CHOICE OF AN OPTION NUMBER??
-1
FORTRAN STOP
```

THE END

REMARKS:

Some general results after regrouping the length interval:

1. The smaller the length interval (increased number of length categories), the smaller the standard deviations of the age proportions and the catch at age. This is true as long as there are adequate numbers of fish in each of the length intervals. (Also, reported by Tanaka, 1953)

2. The smaller the length interval the smaller the estimated variances of the mean weight per fish, and the smaller the variance of the total catch in numbers. Remember that only total catch in weight was entered, not in numbers.

3. The bias of the estimates of catch at age increases when larger length intervals are used.

References

Kimura, D.K. 1977. Statistical assessment of age-length key. J. Fish. Board Can. 31:317-324.

Lai, H.L. 1987. Optimum allocation for estimating age composition using age-length key. Fish. Bull. U.S. 85:179-185.

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Appendix I. Overview of Age-Length Key Statistics

A. Basic equations for age-length key

Age-length keys (ALK) are widely used in fisheries to estimate the age compositions of catches. The theory of ALK is based on a double sampling technique with stratification (Tanaka 1953). The first stage involves a simple random sampling from a relatively large, less costly, length sample. The second stage involves a stratified random sampling for a smaller, more costly, age subsample from each length stratum. The proportion of fish in the ith age class (p_i) and the variance of p_i are estimated (Lai 1987) as

$$(1) \qquad p_i = \sum_{j=1}^{L} \ell_j q_{ij}$$

$$(2) \qquad Var(p_i) = \sum_{j=1}^{L} \left[\frac{\ell_j^2 q_{ij} (1-q_{ij})}{n_j} + \frac{\ell_j (q_{ij} - p_i)^2}{N} \right]$$

where, ℓ_j is the proportion of fish that fall into the jth length stratum,

N is total length sample size,

n_j is the size of the age subsample in the jth length stratum,

q_{ij} is the proportion of n_j fish classified into the ith age class,

A is the number of age classes, and

L is the number of length strata.

The total catch at age in numbers is estimated by

$$(3) \qquad C_i = C\, p_i$$

where, C is the total catch in numbers ($= \Sigma C_i$). Its estimated variance (Quinn et al. 1983b) is

(4) $$Var(C_i) = C_i^2 \{C.V.^2(C_i) + C.V.^2(p_i) - C.V.^2(C_i)C.V.^2(p_i)\}$$

where $C.V.^2$ is the squared coefficient of variation.

To estimate $C.V.^2(C_i)$ the following the procedures are used. The average weight of individual fish in the sample is calculated (the Equations (5) and (6) are different from Quinn's) by

(5) $$W = \frac{\Sigma W_j \cdot N_j}{\Sigma N_j}$$

where W_j is the mean weight of individual fish in length group j and N_j is the number of length sample in length group j.

The variance of W is

(6) $$Var(W) = (\Sigma W_j^2 - (\Sigma W_j)^2 / \Sigma N_j)) / (\Sigma N_j)(\Sigma N_j - 1)$$

For a total catch in weight, T, the total catch in numbers is calculated as

(7) $$C = T/W$$

Using the delta method, the variance and C.V. of C are

(8) $$Var(C) = Var(W)\, C^2/W^2$$

(9) $$C.V.(C) = \sqrt{Var(C)/C^2}$$

Similarly, the C.V. of p_i is calculated by

$$(10) \quad C.V.(p_i) = \sqrt{Var(p_i)/p_i^2}$$

B. Optimum sampling for an age-length key

Two subsampling schemes related to ALK are frequently used by fisheries biologists. These are: (i) Fixed age subsampling, in which the size of the age subsample in all length strata is constant (i.e., $n_j = n/L$, where $n = \sum n_j$ is the total age subsample size), and (ii) Random age subsampling, in which the size of the age subsample in each length stratum is proportional to the length sample size for all length strata (i.e., $n_j = n\,\ell_j$).

For fixed age subsampling, substituting $n_j = n/L$ into (1), the $Var(p_i)$ is

$$(11) \quad Var(p_i) = \sum_{j=1}^{L} \left[\frac{L\,\ell_j^2 q_{ij}(1-q_{ij})}{n} + \frac{\ell_j (q_{ij}-p_i)^2}{N} \right]$$

Then, the Vartot (Kimura 1977; Lai 1987), an error index for assessing the precision of the age-length key, is calculated as

$$(12) \quad Vartot = \sum_{i=1}^{A} Var(p_i)$$

$$= \sum_{i=1}^{A} \sum_{j=1}^{L} \left[\frac{L\,\ell_j^2 q_{ij}(1-q_{ij})}{n} + \frac{\ell_j (q_{ij}-p_i)^2}{N} \right]$$

$$= \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [L\,\ell_j^2 q_{ij}(1-q_{ij})]}{n} + \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [\ell_j (q_{ij}-p_i)^2]}{N}$$

$$= \frac{a_1}{n} + \frac{a_2}{N}$$

For Random age subsampling, substituting $n_j = n\,\ell_j$ into (1), the Vartot is

$$(13)\text{Vartot} = \sum_{i=1}^{A} \sum_{j=1}^{L} \left[\frac{\ell_j q_{ij}(1-q_{ij})}{n} + \frac{\ell_j (q_{ij}-p_i)^2}{N} \right]$$

$$= \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [\ell_j q_{ij}(1-q_{ij})]}{n} + \frac{\sum_{i=1}^{A} \sum_{j=1}^{L} [\ell_j (q_{ij}-p_i)^2]}{N}$$

$$= \frac{b_1}{n} + \frac{b_2}{N}$$

The program calculates the cost function as a simple linear equation relating the unit cost of observing the length and age of a fish to N and n

$$(14) \quad C = c_1 N + c_2 n$$

where C is the total cost, c_1 is the unit cost of observing the length of a fish, and c_2 is the unit cost of determining the age of a fish.

Generally, survey designs are based on two constraints (Lai 1987). The first, as derived previously, relates to obtaining the precision of p_i, viz., to minimize Vartot at a fixed total cost. The second determines the total cost required to achieve a given precision, that is, to minimize total cost at a desired level of Vartot.

For a fixed age subsample, using equations (12) and (14) and the Cauchy-Schwarz inequality (see Lai 1987 for derivation), we obtain

$$(15) \quad r^* = (n/N)^* = \sqrt{a_1 c_1 / a_2 c_2}$$

This quantity is the optimum subsampling ratio (r^*) required to reach the minimum (min.) Vartot subject to the cost function given in (14). Therefore, N^* and n^* are dependent on (14):

(16) $N^* = C/(c_1 + c_2 r^*)$

(18) $n^* = r^* N^*$

(19) $$\text{min. Vartot} = \frac{a_1}{n^*} + \frac{a_2}{N^*}$$

To minimize the total cost subject to a given $D=\sqrt{\text{Vartot}}$, substitute r^* into (12) and obtain:

(20) $N^* = (a_1/r^* + a_2) / D^2$

(21) $n^* = r^* N^*$

(22) $\text{min. } C = c_1 N^* + c_2 n^*$

Similarly, the optimum allocation of a random age subsample can be obtained using (13) and (14) and the Cauchy-Schwarz inequality. The solutions of r^*, N^*, n^* and minimum Vartot subject to a given total cost are:

(23) $r^* = (n/N)^* = \sqrt{b_1 c_1 / b_2 c_2}$

(24) $N^* = C/(c_1 + c_2 r^*)$

(25) $n^* = r^* N^*$

(26) $$\text{min. Vartot} = \frac{b_1}{n^*} + \frac{b_2}{N^*}$$

Optimum allocation of a random subsampling effort subject to a desired level of $D=\sqrt{Vartot}$ is obtained as:

(27) $N^* = (b_1/r^* + b_2) / D^2$

(28) $n^* = r^*N^*$

(29) $\min. C = c_1N^* + c_2n^*$

Because $0 < r^* \leq 1$, equations (15) and (23) indicate that

(30) $c_1/c_2 \leq a_2/a_1$ for Fixed age sumsampling and

$c_1/c_2 \leq b_2/b_1$ for Random age subsampling

must be met for the optimum allocation.

APPENDIX – II

PROGRAM LIST

```
C***********************************************************************
C
C                              AGECOMP
C                     Estimation of Age Composition
C                                and
C     Optimum Allocation of Sampling Effort Using an Age-Length Key
C
C-----------------------------------------------------------------------
C
C                          VAX-II Version 2.0
C
C-----------------------------------------------------------------------
C
C      NOTATION.
C       A      : MATRIX OF AGE-LENGTH KEY (IN NUMBER)
C       XA     : MATRIX OF REGROUPED AGE-LENGTH KEY (IN PROPORTION)
C       L      : LENGTH FREQUENCY (IN NUMBER)
C       XL     : REGROUPED LENGTH FREQUENCY (IN PROPORTION)
C       LC     : VALUE OF LENGTH INTERRVALS
C       NL     : NUMBER OF LENGTH CATEGORIES, DEFAULT=200
C       NA     : NUMBER OF AGES, DEFAULT=50
C       LL1    : LOWER LENGTH CATEGORY OF INTEREST
C       LL2    : UPPER LENGTH CATEGORY OF INTEREST
C       IA1    : LOWEST AGE OF AGE-LENGTH KEY
C       IA2    : HIGHEST AGE OF AGE-LENGTH KEY
C       LINT   : LENGTH INTERVAL TO GROUP THE AGE-LENGTH KEY
C       PA     : ESTIMATED PROPORTION OF AGE (AGE COMPOSITION)
C       C      : ESTIMATED CATCH AT AGE
C       WL     : TOTAL WEIGHT OF SAMPLES AT LENGTH
C       XWL    : MEAN WEIGHT AT LENGTH AFTER REGROUPING
C       WBAR   : MEAN WEIGHT PER FISH IN CATCH
C       VWBAR  : VARIANCE OF MEAN WEIGHT PER FISH
C       TC     : TOTAL CATCH IN NUMBER
C                IN SUBROUTINE OPTA, TC IS TOTAL COST
C       VTC    : VARIANCE OF TOTAL CATCH IN NUMBER
C       CVTC   : C.V. (=VTC/TC**2)
C       WTC    : TOTAL CATCH IN WEIGHT
C       VPA    : VARIANCE OF AGE COMPOSITION (PA)
C       CVPA   : C.V. (=VPA/PA**2)
C       VC     : VARIANCE OF CATCH AT AGE (C)
C       SUML   : TOTAL NUMBER OF LENGTH SAMPLE
C       NSET   : NUMBER OF DATA SETS (IE, STRATA)
C
C-----------------------------------------------------------------------
C
C      NOTE:
C          To run this program, you need the file called
C      "COMMON.FOR".   Check your directory to make sure
C      that the file exists.
C
C-----------------------------------------------------------------------
C
C       HAN-LIN LAI
C
C       Version 1.0       05/28/1985       VAX-II Interactive
C
C       Version 2.0       08/06/1987       VAX-II, MS-FORTRAN Interactive
C
C***********************************************************************

      INCLUDE 'COMMON.FOR'
      CHARACTER*1 OPTION
      NSET=0
      SUML=0.
1000  TYPE *, ' Do you want to see the OPTIONS?  YES(Y)/NO(N)'
```

```
      READ 2, OPTION
2     FORMAT(A1)
      IF(OPTION .EQ. 'Y') TYPE 3
3     FORMAT(//2X,' LIST OF OPTIONS : '//
     >       1x,'0  : LIST THE OPTIONS'/
     >       1X,'1  : INPUT AGE-LENGTH KEY AND LENGTH FREQUENCY'/
     >       1X,'2  : REGROUP LENGTH INTERVAL'/
     >       1X,'3  : ESTIMATING PROPORTION OF AGE'/
     >       1X,'4  : ESTIMATING CATCH AT AGE'/
     >       1X,'5  : SAVE CATCH AT AGE AND AGE PROPORTION'/
     >       1X,'6  : OPTIMUM ALLOCATION FOR AGE-LENGTH KEY'/
     >       1X,'-1 : STOP THE JOB')
999   TYPE 4
4     FORMAT(///' WHAT IS YOUR CHOICE OF AN OPTION NUMBER?')
      READ *,IND
      IF(IND .LT. 0) STOP
      IF(IND .GT. 6) GO TO 1001
      GO TO (10,100,200,300,400,500,600),IND+1
10    TYPE 3
      GO TO 999
100   NSET=NSET+1
      CALL INPUT
      GO TO 999
200   CALL REGROUP
      GO TO 999
300   CALL AGECOMP
      GO TO 999
400   CALL CAGE
      GO TO 999
500   CALL SAVE
      GO TO 999
600   CALL OPTA
      GO TO 999
1001  TYPE 5
5     FORMAT(//' If you want to quit, enter -1')
      GO TO 1000
      END


      SUBROUTINE INPUT
      INCLUDE 'COMMON.FOR'
      CHARACTER*80  DOC1,DOC2
      CHARACTER*1 ICHK
      TYPE *,'  New files for age-length key and length frequency?'
      TYPE *,'  YES(Y)/NO(N).'
      READ 8, ICHK
8     FORMAT(A1)
      IF(ICHK .NE. 'Y')GO TO 900
      TYPE *,'  Enter file name for LENGTH FREQUENCY'
      ACCEPT  7, DOC1
      OPEN(UNIT=7,FILE=DOC1,STATUS='OLD')
      TYPE *,'  Enter file name for AGE-LENGTH KEY'
      ACCEPT 7, DOC2
7     FORMAT(A80)
      OPEN(UNIT=8,FILE=DOC2,STATUS='OLD')
      REWIND 7
      REWIND 8
900   WRITE(6,2)NSET
2     FORMAT(1X,'  DATA SET # ',I2)
      NL1=0
C......INPUT LENGTH FREQUENCY DATA
      READ (7,*)WTC
999   NL1=NL1+1
      READ(7,*,END=998)LC(NL1),L(NL1),WL(NL1)
      SUML=SUML+L(NL1)
      GO TO 999
```

```
998     NL=NL1-1
        TYPE *,'Enter LOWEST and HIGEST AGES IN AGE-LENGTH KEY'
        TYPE *,' LOWEST AGE ---> '
        READ *,IA1
        TYPE *,' HIGHEST AGE ---> '
        READ *,IA2
        NA=IA2-IA1+1
C......CHECK THAT THE RANGE OF LENGTHS ARE THE SAME IN BOTH AGE-LENGTH
C......KEY AND LENGTH FREQUENCY DATA. IF NOT,  ADJUST THEM
C......BEFORE READING THE AGE-LENGTH KEY DATA.
        READ(8,*)LL
        REWIND 8
        LD=LL-LC(1)
        IF(LD)100,200,300
100     LD=ABS(LD)
        DO 20 I=1,NL
        J=NL-I+1+LD
        K=NL-I+1
        LC(J)=LC(K)
        L(J)=L(K)
        WL(J)=WL(K)
20      CONTINUE
        DO 21 I=1,LD
        LC(I)=LL+I-1
        WL(I)=0.
21      L(I)=0.
        I=0
        GO TO 200
300     I=LD
        GO TO 200
200     I=I+1
        READ(8,*,END=1000)LX,(A(I,J),J=1,NA)
        GO TO 200
1000    KL=LC(1)
        LL1=MINO(LL,KL)
        LL2=MAXO(I-1,J)
        LL2=LL2+LL1-1
        RETURN
        END


        SUBROUTINE REGROUP
        INCLUDE 'COMMON.FOR'
        CHARACTER*1 ICH
        DIMENSION LG(100)
C......COMBINE LENGTH INTERVAL
        N=1
        DO 15 I=1,NL
        XL(I)=0.
        XWL(I)=0.
        TOT(I)=0.
        DO 15 J=1,NA
        XA(I,J)=0.
15      CONTINUE
        TYPE *,'LENGTH INTERVAL to regroup age-length key'
        READ*,LINT
        DO 10 I=1,NL
        LG(N)=LC(I)
        XL(N)=XL(N)+L(I)
        XWL(N)=XWL(N)+WL(I)
        DO 11 J=1,NA
        XA(N,J)=XA(N,J)+A(I,J)
        TOT(N)=TOT(N)+A(I,J)
11      CONTINUE
C......USE REMAINDER TO ASSIGN COMBINE LENGTH GROUP
        ICK=LC(I)/LINT
```

```
      CHK=FLOAT(LC(I))/FLOAT(LINT)
      RES=CHK-FLOAT(ICK)
      IF(RES.NE.0.) GO TO 10
      IF(I.EQ.NL)GO TO 10
      N=N+1
10    CONTINUE
      TYPE *,'Do you want to see age-length key and length frequency?'
      TYPE *,' YES(Y)/NO(N)'
      READ 2,ICH
2     FORMAT(A1)
      IF(ICH .NE. 'Y') GO TO 120
      DO 20 I=1,N
      LG(I)=LG(I)-LINT+1
      WRITE(6,1)LG(I),(XA(I,J),J=1,NA),TOT(I),XL(I)
1     FORMAT(I3,12F5.0)
20    CONTINUE

C......COMPUTE PROPORTION OF LENGTH FREQUENCY AND AGE-LENGTH KEY
120   DO 30 I=1,N
      IF(XL(I).EQ.0)GO TO 301
      XWL(I)=XWL(I)/XL(I)
301   XL(I)=XL(I)/SUML
      DO 30 J=1,NA
      IF(TOT(I).EQ.0.)GO TO 30
      XA(I,J)=XA(I,J)/TOT(I)
30    CONTINUE
      RETURN
      END


      SUBROUTINE AGECOMP
      INCLUDE 'COMMON.FOR'
C......ESTIMATING AGE PROPORTION AND VARIANCE
      TYPE *,'Give the TITLE of this estimation (UP TO 60 CHARACTERS)'
      READ(5,3)(TITLE(K),K=1,60)
3     FORMAT(60A1)
      WRITE(6,4)NSET,(TITLE(K),K=1,60)
4     FORMAT(////'  ESTIMATION FOR DATA SET # ',I3/1X,60A1)
      WRITE(6,2)
2     FORMAT('  AGE',3X,'  PROPORTION',3X,'    VARIANCE',10X,'STDEV'/
     >       ' ====',3(3X,12H============))
      DO 9 J=1,NA
      PA(J)=0.
      VPA(J)=0.
9     CONTINUE
      DO 10 J=1,NA
      DO 11 I=1,N
      PA(J)=PA(J)+XA(I,J)*XL(I)
      IF(TOT(I).LE.0.)GO TO 11
      W(J)=W(J)+(XL(I)**2*XA(I,J)*(1.-XA(I,J)))/TOT(I)
11    CONTINUE
      DO 12 I=1,N
      B(J)=B(J)+(XL(I)*(XA(I,J)-PA(J)**2))/SUML
12    CONTINUE
      VPA(J)=W(J)+B(J)
      SPA(J)=SQRT(VPA(J))
      WRITE(6,1)J+IA1-1,PA(J),VPA(J),SPA(J)
1     FORMAT(I5,3(3X,E12.7))
10    CONTINUE
      RETURN
      END


      SUBROUTINE CAGE
      INCLUDE 'COMMON.FOR'
      DATA SUMW1/0./,SUMW2/0./
```

```
      IF (WTC .GT. 0.) GO TO 100
      TYPE  *,' TOTAL CATCH IN WEIGHT IS NOT AVAILABLE'
      TYPE  *,' COMPUTATION SUSPENDED'
      TYPE  *,' CHECK THE 1ST RECORD OF YOU LENGTH FREQUENCY DATA FILE'
      RETURN
C......ESTIMATING NUMBER OF CATCH AT AGE AND ITS VARIANCE
100   DO 30 I=1,N
      SUMW1=SUMW1+XWL(I)*L(I)
      SUMW2=SUMW2+XWL(I)*XWL(I)*L(I)
30    CONTINUE
      WBAR=SUMW1/SUML
C......THIS IS EMPIRICAL VARIANCE OF WBAR
      VWBAR=(SUMW2-SUMW1*SUMW1/SUML)/(SUML*(SUML-1))
      TC=WTC/WBAR
      VTC=VWBAR*TC*TC/(WBAR*WBAR)
      CVTC=VTC/(TC*TC)
      WRITE(6,4)NSET,(TITLE(K),K=1,60)
4     FORMAT(////'  ESTIMATION FOR DATA SET # ',I3/1X,60A1)
      WRITE(6,1)
1     FORMAT('  AGE',3X,'CATCH AT AGE',3X,'    VARIANCE',10X,'STDEV'/
     >        ' ====',3(3X,12H============))
      DO 31 J=1,NA
      IF(PA(J).EQ.0.)GO TO 311
      C(J)=TC*PA(J)
      CVPA=VPA(J)/(PA(J)*PA(J))
      VC(J)=C(J)**2*(CVTC+CVPA-CVTC*CVPA)
      SC(J)=SQRT(VC(J))
311   WRITE(6,2)J,C(J),VC(J),SC(J)
2     FORMAT(I5,3(3X,E12.7))
31    CONTINUE
      WRITE(6,3)WBAR,VWBAR,WTC,TC,VTC
3     FORMAT(/' MEAN WEIGHT PER FISH                          = ',F12.4/
     >        ' EMPIRICAL VARIANCE OF MEAN WEIGHT PER FISH = ',E12.7/
     >        ' TOTAL CATCH IN WEIGHT                         = ',E12.7/
     >        ' ESTIMATED TOTAL CATCH IN NUMBER               = ',E12.7/
     >        ' VARIANCE OF TOTAL CATCH IN NUMBER             = ',E12.7)
      RETURN
      END


      SUBROUTINE SAVE
      INCLUDE 'COMMON.FOR'
C......SAVE THE NECESSARY DATA FOR FUTURE USES
      CHARACTER*1 ICHK
      CHARACTER*5 STAT
      CHARACTER*80 FNAME
      TYPE *,' Save in a new outputfile? YES(Y)/NO(N)'
      READ 1,ICHK
1     FORMAT(A1)
      IF(ICHK .NE. 'Y') GO TO 100
      TYPE *,' Enter the FILENAME to save results'
      TYPE *,' FILE NAME ---> '
      READ 2,FNAME
2     FORMAT(A80)
      TYPE *,' Status of this file? NEW/OLD'
      READ 3,STAT
3     FORMAT(A5)
      OPEN(UNIT=10,FILE=FNAME,STATUS=STAT)
100   WRITE (10,5)WTC
5     FORMAT(F10.2)
      DO 50 J=1,NA
      IAGE=J+IA1-1
      WRITE(10,4)NSET,PA(J),VPA(J),W(J),B(J),C(J),VC(J)
4     FORMAT(I5,6E12.7)
50    CONTINUE
      RETURN
```

```
      END

      SUBROUTINE OPTA
      INCLUDE 'COMMON.FOR'
      DATA A1,A2,B1,B2/0.,0.,0.,0./

      DO 10 J=1,NA
      DO 11 I=1,N
      A1=A1+(FLOAT(N)*XL(I)**2*XA(I,J)*(1.-XA(I,J)))
      A2=A2+(XL(I)*(XA(I,J)-PA(J)**2))
      B1=B1+(XL(I)*XA(I,J)*(1.-XA(I,J)))
 11   CONTINUE
 10   CONTINUE
      B2=A2
 1000 TYPE *, ' Select one of the following options:'
      TYPE *, '  -1 : RETURN TO MAIN PROGRAM'
      TYPE *, '   1 : MINIMIZE VARTOT SUBJECT TO GIVEN TOTAL COST'
      TYPE *, '   2 : MINIMIZE TOTAL COST SUBJECT TO GIVEN VARTOT'
      READ *, IOPT
      IF(IOPT .LT. 0)RETURN
      IF(IOPT .LT. 1 .OR. IOPT .GT. 2)GO TO 1001
      GO TO (100,200),IOPT
 100  CALL OPT1(A1,A2,B1,B2)
      GO TO 1000
 200  CALL OPT2(A1,A2,B1,B2)
      GO TO 1000
 1001 TYPE *, ' If you want to quit, enter -1'
      GO TO  1000
      END

      SUBROUTINE OPT1(A1,A2,B1,B2)
      INCLUDE 'COMMON.FOR'
      TYPE *,  ' Give the UNIT COST for observing LENGTH of a fish'
      TYPE *,' C1 ---> '
      READ *, C1
      TYPE *,  ' Give the UNIT COST for AGEING a fish'
      TYPE *,' C2 ---> '
      READ *,C2
      TYPE *, ' Give the TOTAL COST'
      TYPE *,' TC ---> '
      READ *,TC

C..................... CHECK FOR CONDITION OF OPTIMUM ALLOCATION

      RC=C1/C2
      RA=A2/A1
      IF(RC .LE. RA) GO TO 990
      TYPE  6, RA
6     FORMAT(//1X,'CONDITION OF OPTIMUM ALLOCATION FOR FIXED AGE',
     >        ' SUBSAMPLING SCHEME IS NOT VALID, SEE TECHNICAL REPORT'
     >        ' FOR DETAILS'/
     >        ' YOUR COST RATIO BETWEEN LENGTH AND AGE MUST BE LESS'
     >        ' THAN ',F7.5)
990   RB=B2/B1
      IF(RC .LE. RB) GO TO 999
      TYPE 7, RB
7     FORMAT(//1X,'CONDITION OF OPTIMUM ALLOCATION FOR RANDOM AGE',
     >        ' SUBSAMPLING SCHEME IS NOT VALID, SEE TECHNICAL REPORT'
     >        ' FOR DETAILS'/
     >        ' YOUR COST RATIO BETWEEN LENGTH AND AGE MUST BE LESS'
     >        ' THAN ',F7.5)
      RETURN

C....................FIXED AGE SUBSAMPLING
```

```
999   R2=(A1*C1)/(A2*C2)
      R=SQRT(R2)
      XNO=C1+C2*R
      NLF=TC/XNO + 0.5
      NAF=R*FLOAT(NLF) + 0.5
      OPTVF=A1/FLOAT(NAF) + A2/FLOAT(NLF)

      WRITE(6,4)NLF,NAF,OPTVF,TC,C1,C2
4     FORMAT(///1X,'IF YOU USE A FIXED AGE SUBSAMPLING SCHEME'/
     >          7X,'OPTIMUM SAMPLE SIZE FOR LENGTH = ', I7/
     >          7X,'OPTIMUM SAMPLE SIZE FOR AGE    = ', I7/
     >          7X,'MINIMUM VARTOT OBTAINED        = ', E12.5/
     >          1X,'SUBJECT TO A GIVEN TOTAL COST OF ',F10.2/
     >          4X,'WITH  UNIT COST FOR OBSERVING LENGTH OF A FISH = '
     >          ,F5.2/
     >          10X,'UNIT COST FOR AGEING A FISH = ',F7.2)

C...................RANDOM AGE SUBSAMPING

      R=SQRT((B1*C1)/(B2*C2))
      NLR = 0.5 +TC/(C1+C2*R)
      NAR=R*FLOAT(NLR)+0.5
      OPTVR=B1/FLOAT(NAR) + B2/FLOAT(NLR)

      WRITE(6,5)NLR,NAR,OPTVR,TC,C1,C2
5     FORMAT(///1X,'IF YOU USE A RANDOM AGE SUBSAMPLING SCHEME'/
     >          5X,'OPTIMUM SAMPLE SIZE FOR LENGTH = ', I7/
     >          5X,'OPTIMUM SAMPLE SIZE FOR AGE    = ', I7/
     >          5X,'MINIMUM VARTOT OBTAINED        = ', E12.5/
     >          1X,'SUBJECT TO A GIVEN TOTAL COST OF ',F10.2/
     >          4X,'WITH  UNIT COST FOR OBSERVING LENGTH OF A FISH = '
     >          ,F5.2/
     >          10X,'UNIT COST FOR AGEING A FISH = ',F7.2)

      RETURN
      END


      SUBROUTINE OPT2(A1,A2,B1,B2)
      INCLUDE 'COMMON.FOR'
      TYPE *,  ' Give the UNIT COST for observing LENGTH of a fish'
      TYPE *,' C1 ---> '
      READ *, C1
      TYPE *,  ' Give the UNIT COST for AGEING a fish'
      TYPE *,' C2 ---> '
      READ *,C2
      TYPE *, ' Give the DESIRED PRECISON for estimated age composition
     >in fraction.'
      TYPE *,' Enter 0.05 for 5%, which means VARTOT = 0.0025 or 0.25%'
      TYPE *,' D=SQRT(VARTOT) ---> '
      READ *,D

C...................FIXED AGE SUBSAMPLING

      R2=(A1*C1)/(A2*C2)
      R=SQRT(R2)
      NLOPTF=(A1/R+A2)/(D*D)+0.5
      NAOPTF=R*FLOAT(NLOPTF)+0.5
      OPTCF=C1*FLOAT(NLOPTF) + C2*FLOAT(NAOPTF)

      WRITE(6,4)NLOPTF,NAOPTF,OPTCF,D,C1,C2
4     FORMAT(///1X,'IF YOU USE A FIXED AGE SUBSAMPLING SCHEME'/
     >          5X,'OPTIMUM SAMPLE SIZE FOR LENGTH = ', I7/
     >          5X,'OPTIMUM SAMPLE SIZE FOR AGE    = ', I7/
     >          5X,'MINIMUM TOTAL COST REQUIRED    = ', E12.5/
```