Technological Change, Distributive Bias and Labor Transfer in a Two Sector Economy

Uma Lele
John W. Mellor

Reprinted from the
Oxford Economic Papers
Vol. 33, No. 3, November 1981

INTERNATIONAL FOOD POLICY RESEARCH INSTITUTE
1776 Massachusetts Avenue, N.W.
Washington, D.C. 20036

An offprint from

OXFORD ECONOMIC PAPERS

Volume 33, Number 3 November 1981

CLARENDON PRESS • OXFORD

TECHNOLOGICAL CHANGE, DISTRIBUTIVE BIAS AND LABOR TRANSFER IN A TWO SECTOR ECONOMY

By UMA LELE *and* JOHN W. MELLOR*

SLOW growth in overall employment and unequal distribution of benefits from the new foodgrain technologies continue to be two of the most pressing current problems of many low income countries. There have been efforts to increase employment rapidly, without substantial increase in the rate of growth of food production, e.g. in India following the 1971 election. However, such attempts have generally been accompanied by high rates of inflation, particularly of food prices. This is because as much as 60 percent of the increase in income of low income wage earners in developing countries is spent on consumption of cereals alone (John W. Mellor and Uma Lele 1973). And yet, the growth in food production in developing countries has barely kept pace with the growth of population. The foodgrain sector has thus not only been a slow generator of additional employment and income; through inadequate supply of wage goods it has also constituted a major constraint to the growth of nonagricultural employment.

The question of labor transfers has, of course, received extensive treatment in development literature and especially in two-sector models[1] (most notably by W. Arthur Lewis, Fei-Ranis, Jorgensen, Todaro and Harris). A few formulations, such as those by Dixit and Hornsby, also deal with increasing production of wage goods, but do not allow for technological change.[2] Various others treat the question of marketed surpluses of food, but do not incorporate it formally in models of growth or relate it to labor supply as a separate but interacting variable.[3] The variations in the distributive bias of the different types of new technologies in foodgrain production have, however, been extensively documented in the empirical literature.[4]

The critical role of the wage goods constraint in creating nonagricultural employment has also been recognized by policymakers, but only implicitly. Consequently, unlike Mainland China, few developing countries have had the political will or the institutional mechanisms to mobilize the limited

*Uma Lele is Senior Economist, the World Bank and John W. Mellor is Director, International Food Policy Research Institute, Washington, D.C., U.S.A. We are grateful to Chandrashekhar Ranade for considerable assistance on the paper particularly in developing the necessary proof. We also acknowledge the contribution of an anonymous reviewer in correcting inaccuracies and improving clarity of presentation.

[1] For a detailed review of two sector models see Mellor (1974).

[2] See Mellor (1974).

[3] See Mellor (1974).

[4] Mellor and Lele (1973). For a detailed analysis of several innovations in two major locations in the Philippines, see Chandrashekhar G. Ranade (1977). See also, for India, C. H. H. Rao (1975).

domestic food surpluses for consumption of wage earners without causing the prices of food to rise in relation to those in the nonagricultural sector. These price increases have discouraged decisionmakers from following a policy of expanding employment.[5] Similarly, few developing countries have relied on rapidly increasing imports of cereals as a way of expanding employment, partly arising out of a perception of inelastic demand for their own exportable surpluses.

In agriculture, as the classic sector of diminishing returns, the production increase necessary to release the wages good constraint is of course achieved largely through technological change. Agricultural technologies, however, vary substantially in their distributive bias. They therefore have important implications for the generation of employment directly in the agricultural sector. In addition, the different demand elasticities among various income classes of food producers also affect the size of the marketable surplus of the wage goods that is generated by the foodgrains sector. The initial employment effect, and the consequent size of the marketed surplus, thus in turn affect the prices of food relative to nonfood output as well as the level of real wages in the nonfoodgrain sector. These factors are thus crucial in determining the rate at which the wages goods constraint is released and off-farm employment is generated.

In this context we analyze the effect of alternative assumptions with respect to distributive bias of technological change in the foodgrain sector on (a) marketable surplus from that sector, (b) the rate of growth of nonfoodgrain sector employment, (c) the price of foodgrain in relation to the nonfoodgrain output and (d) the degree of factor intensity in the nonfoodgrain sector. We examine these relationships with the use of a two-sector model similar to the large family of dualistic models so as to focus on the critical role of food production in influencing labor transfers, and to analyze the complex interactions of the food and the labor markets.

The distinguishing features of the two-sector model developed in this paper are: (1) incorporation of biased technological change in the foodgrain sector and (2) separation of the food and labor markets into two independent but interacting markets. Rather than assuming that food moves commensurately and automatically with labor, we assume the marketable surplus of food to be influenced by the distribution of income and the different price and income elasticities of demand of landowners and laborers in the foodgrain producing sector for domestic consumption of foodgrains. Technologically induced changes in income distribution in the foodgrain sector therefore affect the demand for food in the foodgrain sector, the marketable surplus, the price of foodgrains in terms of nonfoodgrains output and the rate of labor transfers to the nonfoodgrain sector.

[5] For a critical analysis of such policies in India, see Lele (1971).

The model also provides results relating to the factor intensity in the nonfoodgrain sector. It illustrates how the directions of change in these two factors are influenced by the direction of distributive bias and the nature of interaction between the food and the labor markets. These results are substantially different from those in previous models.

The sharp differences between low and high income consumers in their elasticities of demand for food are well documented. In India, for example, cross-sectional estimates of income elasticities of demand indicate levels of about 0.8 and 0.2 for bottom two and top two deciles respectively.[6] On the whole, income elasticities of demand for foodgrains are, however, observed to be less than one and are assumed to be so in this model.[7]

In order to focus on the most important relationships from the point of view of development policy, some additional assumptions have been made. For instance, the sum of the absolute magnitudes of income elasticity of demand (n) and the elasticity of budget share with respect to the change in relative price of foodgrains (ε) is assumed to be less than 1, as empirically the absolute magnitude of ε is usually expected to be small, i.e. closer to zero than to 1.

In the labor market, the formulation assumes perfect mobility between sectors so that, at equilibrium, the ratio between the wage rate in the nonfoodgrain sector and the average labor income in the foodgrain sector is constant. The average labor income in the foodgrain sector is determined by the total labor income generated by the *flow* of labor in the foodgrain sector divided equally among the total *stock* of labor. Per capita income of the workers in the foodgrain sector then maintains a constant relationship to the level of real wages in the nonfoodgrain sector. We assume an underemployment equilibrium in the foodgrain sector at a given wage $\bar{W}$ as depicted in Fig. 1. The conditions of low productivity and the labor–leisure choices in traditional agriculture which lead to such an underemployment equilibrium have been well analyzed in the literature (Nakajima, 1961, Mellor, 1963 and Sen, 1966). The assumption of underemployment equilibrium should not be confused with an assumption of zero marginal productivity of labor.[8] Rather our assumption reflects the widely noted reality of highly elastic supply of labor from agriculture, if the wage goods constraint is relaxed.

[6] Mellor and Lele (1973). For the Philippines, Goldman and Ranade (1976) find that income elasticity of demand for cereals, mainly rice, in the lowest income decile is 1.05 while it is 0.41 for the top decile.

[7] The results of the model remain unchanged irrespective of whether wage rate in the nonfoodgrain sector is a multiple of or equal to the average labor income in the foodgrain sector. It should be noted, much conventional wisdom to the contrary, that when the physical environment dictates a short, peak work period, the wage rate in agriculture at that season may be higher than that in nonagriculture at that or any other season, while concurrently the average product or total yearly income is lower in agriculture than nonagriculture. For empirical evidence, see Ranade (1977), p. 108.

[8] For a full analysis of this important distinction, see Mellor (1963) and Sen (1966).

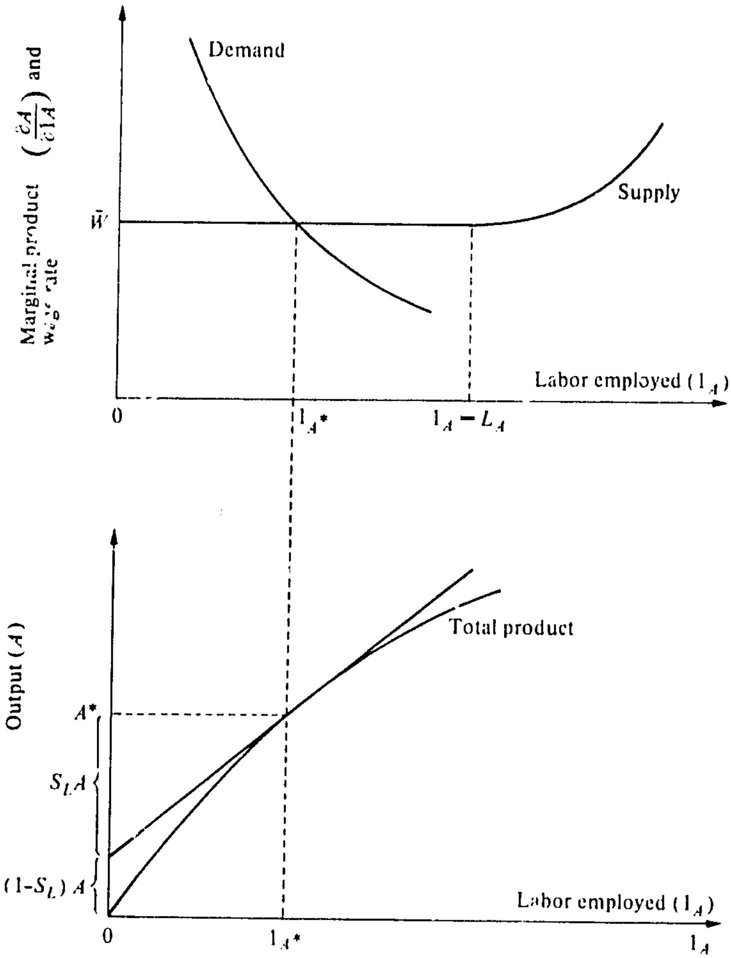

FIG. 1. Equilibrium in foodgrain sector labor market.

1. Analytical framework

The production function for food grains, assumed to have constant returns to scale and diminishing marginal rates of substitution, is as follows:

$$A = F(N, E) \tag{1}$$

such that

$$F_N = \frac{\partial F}{\partial N} > 0, \qquad F_E = \frac{\partial F}{\partial E} > 0 \quad \text{and} \quad \frac{\partial^2 A}{\partial N^2}, \frac{\partial^2 A}{\partial E^2} < 0$$

where A is the foodgrains output, and N and E are the levels of land and labor inputs, respectively. Both land and labor are measured in efficiency units such that $N = xZ$ and $E = yl_A$, where x and Z are respectively the efficiency and the fixed amount of land, and y and l_A are respectively the efficiency and the amount of labor employed. Both x and y are exogenously given and depend upon technology t.[9]

It is assumed that technological change increases the efficiency of land faster than that of labor, that is,

$$\frac{dx}{dt}\frac{1}{x} = \lambda_Z > \lambda_L = \frac{dy}{dt}\frac{1}{y}, \tag{2}$$

where λ_Z and λ_L are rates of growth of the efficiency of land and labor respectively.

In the foodgrain labor market an equilibrium is reached at a constant real wage ($\bar{W}$) equalizing the marginal physical productivity of labor and hence,

$$\bar{W} = \frac{\partial A}{\partial l_A} = yF_E \tag{3}$$

such that $l_A < L_A$ where L_A is the total foodgrain labor force. Equilibrium in the foodgrain sector labor market is shown in Fig. 1.

Then the relative share of foodgrain labor is

$$S_L = \frac{l_A \bar{W}}{A} = \frac{EF_E}{A} \tag{4}$$

Further, the average income of laborers in the foodgrain sector is,

$$y = \frac{l_A \bar{W}}{L_A} = \frac{S_L A}{rL} \tag{5}$$

where r = proportion of foodgrain labor force in total labor force L, that is $r = L_A/L$.

Marketed supply of foodgrains, M_s, to the nonfoodgrain sector is the difference between output and consumption in the foodgrain sector so that

$$M_s = A - \bar{C} - bS_L A \tag{6}$$

where, $\bar{C}$ = constant consumption of foodgrains by landlords, and b = budget share of foodgrains for laborers such that,

$$b = b(P, y) \tag{7}$$

where P is the relative price of foodgrain output with the price of nonfoodgrain output as the "numeraire". Further,

$$\frac{\partial b}{\partial p}\cdot\frac{p}{b} = \varepsilon < 0 \quad \text{and} \quad \frac{\partial b}{\partial y}\cdot\frac{y}{b} = \eta - 1 < 0$$

[9] For convenience, time and technology are denoted by the same variable t.

where ε is the elasticity of budget share with respect to change in price and η is income elasticity of demand for foodgrains. Note that the model thus allows for different income elasticities of demand for landlords (assumed to be equal to zero) and laborers (assumed to be less than one).

The production function for the nonfoodgrain sector is a Cobb–Douglas linear homogeneous of the first degree as follows:

$$Q = K^{\alpha} L_I^{1-\alpha} \tag{8}$$

where, Q = nonfoodgrain output, K = exogenously given capital stock, L_I = labor input in the nonfoodgrain sector, and α = relative share of capital (constant).

In the nonfoodgrain sector laborers are employed at a wage rate W equalling marginal productivity of labor, i.e.,

$$W = (1-\alpha)\frac{Q}{L_I} = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}\frac{1}{(1-r)^{\alpha}} \tag{9}$$

Labor migrates from the foodgrain sector to the nonfoodgrain sector until the wage rate in the nonfoodgrain sector is equal to a constant proportion β of per capita income of foodgrain laborers.

$$W_I = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}\frac{1}{(1-r)^{\alpha}} = \beta P\frac{l_A \bar{W}_A}{L_A} \quad \text{where} \quad \beta \lesseqgtr 1 \tag{10}$$

depending upon marginal productivities of labor in the two sectors.

Market demand for food in the nonfoodgrain sector, M_D, is equal to the budget share allocated to food consumption out of wage income by the nonfoodgrain laborers, i.e. $b(W/p)L_I$. Thus in the foodgrain market, equilibrium is attained when

$$M_s = A - \bar{C} - bS_L A = b\frac{W}{P}L_I = M_D, \tag{11}$$

That is,

$$A - \bar{C} - \frac{l_A \bar{W}}{r} b = 0 \tag{12}$$

This describes the general equilibrium system. The formulation consists of six predetermined variables, namely, capital (K), total labor (L), quantity of land (Z), foodgrain wage ($\bar{W}$), and efficiencies of land (x) and labor (y). It can be shown that given these variables all the endogenous variables (l_A, A, S_L, r, P, M_s and W/P) can be uniquely determined. Note, given $\bar{W}$, Z, x and y, one can uniquely determine the labor input (l_A), output (A) and the share of labor (S_L) from equation (3), (1) and (4) respectively (Fig. 1).

Further, differentiating (10) and (12) partially with respect to r, we get, respectively, the following

$$\frac{\partial P}{\partial r}=\frac{P}{r}\left(1+\frac{\alpha r}{1-r}\right)\text{... for labor market, and} \qquad (13)$$

$$\frac{\partial P}{\partial r}=\frac{P}{r}\frac{\eta}{\varepsilon}\text{... for foodgrain market.} \qquad (14)$$

All the terms on the right hand side of (13) are positive i.e. $\partial P/\partial r>0$, and hence the price of foodgrain relative to nonfoodgrain output declines when the proportion of population in the foodgrain sector declines; both with respect to the labor market. This is explained by the fact that, *ceteris paribus*, as the proportion of population in the foodgrain sector declines, per capita income in that sector increases, and for the equilibrium in the labor market to be maintained the adjustment has to come from a decline in the price of foodgrain relative to nonfoodgrain output. Additionally, since $\eta>0>\varepsilon$ the right hand side of equation (14) is negative. Therefore the price of foodgrain relative to nonfoodgrain output increases as r declines with respect to the foodgrain market. Again, this is explained by the fact that *ceteris paribus*, as

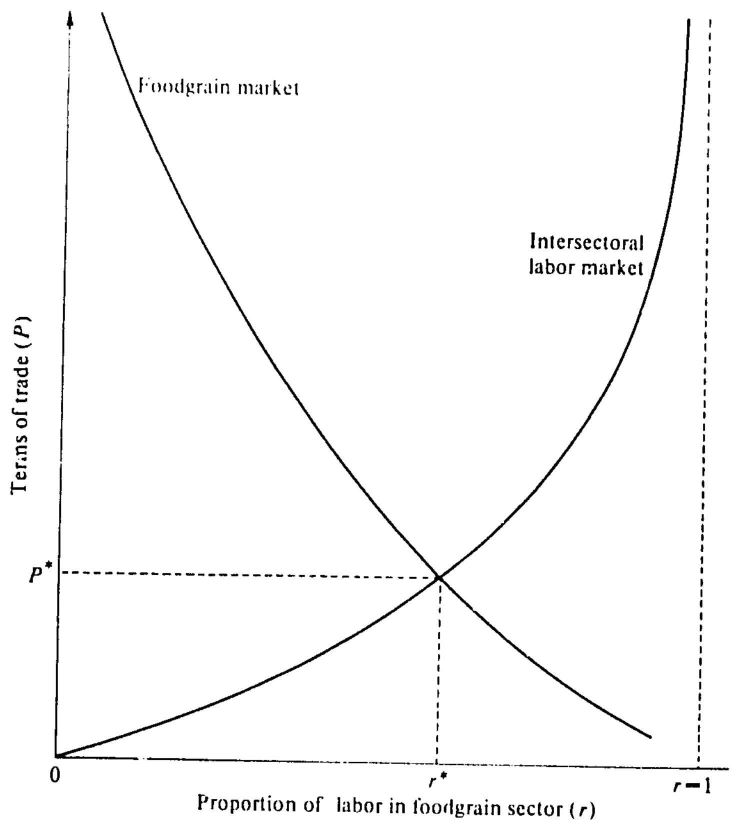

FIG. 2. General equilibrium.

the proportion of population in the foodgrain sector declines and per capita income in that sector increases, the wage rate in the industrial sector also increases, raising effective demand for foodgrain and their price relative to nonfoodgrain output. These opposite phenomena lead to the unique values of P and r given the predetermined variables and the values of l_A, A and S_L as shown in Fig. 2. Then, finally, W/P and M_s can be determined from equations (10) and (6). Stability of this equilibrium is shown in Appendix A.

II. Sensitivity analysis

The technological change affects first the efficiencies of land and labor. Change in the efficiencies would in turn affect the relative share of labor depending upon the nature of substitution between land and labor as follows:

$$\frac{dS_l}{dt}\frac{1}{S_l} = (\sigma - 1)\frac{dy}{dt}\frac{1}{y}, \tag{15}$$

where σ is the elasticity of substitution between land and labor. This equation implies that the relative share of labor would decrease, remain constant, or increase depending whether σ is less than, equal to, or greater than one.[10]

The sensitivity of each of the endogenous variables such as foodgrain labor, price of foodgrains in relation to nonfoodgrain output, marketed surplus and real wages with respect to effect of technological change on labor's share is shown in the following sensitivity matrix. It also shows the sensitivity of these variables to population growth and growth of nonfoodgrain capital separately.

The most interesting results are obtained in the case of an increase in foodgrain output that is accompanied by a change in relative factor shares. The results obtained for a constant labor share are reinforced when labor's share declines as a result of an increase in foodgrain output. In the case of W/P, the real wage rate in the nonfoodgrain sector, the effect of increased foodgrain output accompanied by decline in labor's share directly depresses per capita income of the labor force in the foodgrain sector while decline in labor's share causes a decrease in the proportion of population in the foodgrain sector. This latter phenomenon acts to increase per capita income of the existing population in the foodgrain sector. Thus, the direction of change of the equilibrium level depends upon the relative magnitudes of these opposite influences.

When an increase in foodgrain output is accompanied by an increase in labor's relative share, the effect on the proportion of the labor force in

[10] This relation can be derived by using equations (3) and (4).

TABLE 1
Sensitivity matrix[a]

Endogenous variable	Increase in foodgrain output (A) when relative share of labor (S_l): Increases	Constant	Decreases	Growth of: Capital stock (K)	Population (L)
Proportion of foodgrain labor in total labor (r)	±	−	−	−	+
Price of foodgrains relative to nonfoodgrain output (P)	±	−	−	+	+
Real wage in nonfoodgrain sector (W/P)	±	+	+	+	−
Marketable surplus (M_s)	±	+	+	+	−

[a] See Appendix B for the mathematical steps in deriving the sensitivity matrix on the basis that $0 < \eta < 1$, $\epsilon < 0$ and $0 < \eta - \epsilon < 1$. Negative (positive) sign means decline (increase) in that variable. "±" means the direction of change in that endogenous variable is indeterminate.

agriculture (r), on the price of foodgrains relative to nonfoodgrains (P) and on marketable surplus (M_s) may take either sign. If labor's relative share increases only slightly, relative to the increase in foodgrain output, the effect of increased foodgrain output on r, P and M_s will be greater relative to that of increased labor's share. However, if the labor's share increases substantially as a result of the increase in foodgrain output, the effect on r, P and M_s may be opposite to that when increased foodgrain output is not accompanied by changing labor share.

These interactions are discussed in the dynamic analysis in the next section. The preceding discussion does suggest that in the context of growth the most interesting results in the sensitivity matrix are those relating to labor's share in foodgrain output. They show that with an increased labor share, as exemplified by production increases in a traditional foodgrain sector, the marketed surplus of foodgrain may decline and the real wage in the nonfoodgrain sector may increase. Converse changes may be expected when technological change decreases labor's share in foodgrain output. The factor shares in the foodgrain sector are thus of crucial importance in the growth of the nonfoodgrain sector in a dualistic economy.

This analysis suggests not only that change in factor shares may be a particularly important feature of current "green revolution" agricultural technology, but also helps remove a growing anomaly in the perception of

Japanese economic history. Recent downward revision of estimates of the growth rate for agricultural output in the early Meiji period are consistent with retention of the earlier estimates of growth in nonagricultural employment if one takes into account the acceleration in agricultural marketings associated with change in agricultural technology (See Thomas Smith 1959 and James Nakamura 1966). The yield increasing agricultural technology associated with the Meiji period shifted factor shares away from labor as compared to the highly labor-intensive methods of production increase in the preceding Tokugawa period (Sen 1966). Thus we see agriculture's contribution to overall Japanese growth as arising from the effect of technological change on both the level of output and the change in factor shares arising from that increased output.

III. Dynamic analysis

The dynamic analysis involves the simultaneous effect of change in factor shares through change in factor efficiencies, population and capital stock on nonfoodgrain employment, real wages, terms of trade and marketable surplus. These results are presented in the following equations.

$$\frac{\mathrm{d}r}{\mathrm{d}t}\frac{1}{r}=c_1\frac{\mathrm{d}S_L}{\mathrm{d}t}\frac{1}{S_L}-c_2\left(\frac{\mathrm{d}A}{\mathrm{d}t}\frac{1}{A}-\frac{\mathrm{d}L}{\mathrm{d}t}\frac{1}{L}\right)-c_3\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\frac{1}{Q}-\frac{\mathrm{d}L}{\mathrm{d}t}\frac{1}{L}\right)-c_4 \tag{16}$$

$$\frac{\mathrm{d}W_l}{\mathrm{d}t}\frac{1}{W_l}=\alpha\left(\frac{\mathrm{d}K}{\mathrm{d}t}\frac{1}{K}-\frac{\mathrm{d}L_l}{\mathrm{d}t}\frac{1}{L_l}\right) \tag{17}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t}\frac{1}{P}=\mathrm{d}_1\frac{\mathrm{d}S_L}{\mathrm{d}t}\frac{1}{S_L}-\mathrm{d}_2\left(\frac{\mathrm{d}A}{\mathrm{d}t}\frac{1}{A}-\frac{\mathrm{d}L}{\mathrm{d}t}\frac{1}{L}\right)-\mathrm{d}_3\left(\frac{\mathrm{d}A}{\mathrm{d}t}\frac{1}{A}-\frac{\mathrm{d}Q}{\mathrm{d}t}\frac{1}{Q}\right)-\mathrm{d}_4 \tag{18}$$

$$\frac{\mathrm{d}M_s}{\mathrm{d}t}=e_1-e_2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{1}{r} \tag{19}$$

where c's, d's and e's are all positive given that $0<\eta<1$ and $0<\eta-\varepsilon<1$.[11]

From equation (16), the influence of various factors on the rate of growth of nonfoodgrain employment can be derived. For example, the greater the rate of growth of foodgrain output, the faster the rate of growth of nonfoodgrain employment. The rate of growth of employment in the nonfoodgrain sector is inversely related to the rate of change of labor's share in foodgrain output.

Technological change in the foodgrain sector which increases labor's share in output dampens the rate of growth of nonfoodgrain employment. This occurs through: (1) decreasing the marketed supply of foodgrain, and (2) increasing the level of wages in the nonfoodgrain sector required to withdraw labor from foodgrain production. Technological change that reduces

[11] See Appendix C for derivation of the table.

labor's share of foodgrain output may increase the growth of nonfoodgrain employment. Equation (19) shows the identity between the rate of growth of nonfoodgrain employment and marketable surplus. Thus it can be seen that the same factors shown on the right hand side of equation (16) determine in the same manner the rate of growth of marketable surplus.

Equation (17) shows that there is a monotonically increasing relation between the capital–labor ratio in the nonfoodgrain sector and per capita income in the foodgrain sector. Also, since $\alpha < 1$ the capital–labor ratio increases more rapidly than the rate of growth of per capita income. It is interesting to note here that since $Y = \frac{S_L \cdot A}{rL} = \frac{W}{P}$ per capita income in the foodgrain sector may increase, not only because of an increase in foodgrain output, but also because of an increase in labor's share or a decline in the labor force in the foodgrain sector. It, therefore, seems highly probable that the capital–labor ratio in the nonfoodgrain sector would rise over time, for even if foodgrain output increases only as rapidly as the population growth, and even if labor's share does not increase, just the withdrawal of population from the foodgrain sector would cause an increase in per capita income of foodgrain sector laborers. However, the faster foodgrain production grows and the more labor augmenting technological change in the foodgrain sector, by keeping the capital–labor ratio in the nonfoodgrain sector from rising as rapidly as it would otherwise, the more likely is the comparative advantage to continue in the production and export of labor-intensive commodities in a dualistic economy such as that depicted here.

Equation (18) shows that the movement of relative prices of food and nonfoodgrain output is dependent upon the relative share of labor and growth of foodgrain production relative to that of population and nonfoodgrain output, and may move in either direction depending upon the magnitudes of these several parameters and variables. It should be noted that the relative prices between sectors are determined by the price and income elasticities on the one hand and by the factor shares in the foodgrain sector and average propensities to consume of the two income classes on the other hand. However, it can be seen that a foodgrain output increase accompanied by a reduced factor share to labor will certainly turn the relative price against the foodgrain sector.

IV. Conclusions

By assuming the existence of labor and food markets as two separate but interacting markets in a dualistic economy, the model highlights the adverse effect of the wages good constraint on growth of employment in the non-agricultural sector in a situation of traditional low productivity agriculture faced in many developing countries. Further, it demonstrates the relationship of increased agricultural production and especially of factor shares with growth of employment in the nonagricultural sector. This it does by showing that technological change which increases labor's share in agriculture may well lead to a decline in the marketed surplus of foodgrains and an increase in the real wages in the nonfood sector. On the other hand, in situations of biased technological change even if the direct employment effect of new technology in agriculture is limited, by generating a marketed surplus of foodgrains, such technological change may relax the wages goods constraint, thus facilitating an increase in employment in the nonagricultural sector.

World Bank, Washington, D.C.
International Food Policy Research Institute, Washington D.C.

APPENDIX A: STABILITY CONDITIONS

Let us hypothesize that the terms of trade increase over time if demand for the marketable surplus exceeds its supply,

$$\dot{P} = H[M_D - M_S] \tag{A.1}$$

such that $H' > 0$ and that labor migrates to the nonfoodgrain sector when the demand price for nonfoodgrain sectbr labor exceeds its supply price.

$$\dot{r} = G\left[\frac{W}{P} - \frac{l_A \bar{W}}{rL}\right] \tag{A.2}$$

such that $G' < 0$.

A necessary and sufficient condition for local stability of the system (A.1) and (A.2) are that[1]

$$\frac{\partial \dot{P}}{\partial P} + \frac{\partial \dot{r}}{\partial r} < 0 \quad \text{and} \quad \frac{\partial \dot{P}}{\partial P}\frac{\partial \dot{r}}{\partial r} - \frac{\partial \dot{P}}{\partial r}\frac{\partial \dot{r}}{\partial P} > 0 \tag{A.3}$$

Differentiating equations (12) and (10) with respect to t nd r we get

$$\frac{\partial \dot{P}}{\partial P} + \frac{\partial \dot{r}}{\partial r} = H'\frac{S_l bA}{r}\frac{\varepsilon}{P} + G'P\frac{S_L A}{rL}\left[1 + \frac{r\alpha}{1-r}\right]\frac{1}{r} < 0 \tag{A.4}$$

$$\frac{\partial \dot{P}}{\partial P}\frac{\partial \dot{r}}{\partial r} - \frac{\partial \dot{P}}{\partial r}\frac{\partial \dot{r}}{\partial P} = -H'G'\frac{WbA}{r^2}\left[S_L b\eta - S_l b\varepsilon\left(1 + \frac{r\alpha}{1-r}\right)\right] > 0 \tag{A.5}$$

When $\eta > 0$, $\varepsilon < 0$, $H' > 0$ and $G' < 0$. Note that these are sufficient conditions for the system o satisfy (A.3) and hence they are the sufficient conditions for local stability of the system.

APPENDIX B: TO DERIVE SENSITIVITY MATRIX

The effect of changes in exogenous variables x, v, K or L on endogenous variables l_A, r, P, M_S and (W/P) can be determined as follows: let $\theta = t$, K or L. Note that change in t, technological change, implies change in x and y.

[1] These conditions are derived by using the theoretical discussion in P. A. Samuelson, *Foundations of Economic Analysis*, New York 1947 pp. 266–67.

Differentiate (3) logarithmically with respect to θ and note $\lambda_Z > \lambda_L$. Then

$$\frac{\partial l_A}{\partial \theta}\frac{1}{l_A} = \begin{cases} \lambda_Z - \lambda_L + \dfrac{\sigma}{S_Z}\lambda_L > 0 & \text{when } \theta = t. \\ 0 & \text{when } \theta = K \text{ or } L. \end{cases}$$

Further, substitute the value of P from equation (10) in (12) and then differentiate (12) partially with respect to θ. After rearranging terms,

$$\frac{\partial r}{\partial \theta}\frac{\theta}{r} = \frac{1}{|\Delta|}\left\{S_L b\varepsilon\psi_1(\theta) + \psi_2(\theta) - S_L[r - b(\eta - \varepsilon)]\frac{\partial l_A}{\partial \theta}\frac{\theta}{l_A}\right\} \tag{B.2}$$

where ε and η are, respectively, the elasticity of budget share with respect to price and income elasticities of demand for foodgrains by laborers; $\psi_1(\theta)$ and $\psi_2(\theta)$ are functions of θ; and

$$|\Delta| = S_L b\left[\eta - \varepsilon\left(1 + \frac{r\alpha}{1-r}\right)\right] > 0, \tag{B.3}$$

since $0 < \eta < 1$ and $\varepsilon < 0$.

Differentiating (10) logarithmically with respect to θ and then rearranging terms gives the following two equations:

$$\frac{\partial P}{\partial \theta}\frac{\theta}{P} = \psi_1(\theta) - \frac{\partial l_A}{\partial \theta}\frac{\theta}{l_A} + \left(1 + \frac{r\alpha}{1-r}\right)\frac{\partial r}{\partial \theta}\frac{\theta}{r}, \tag{B.4}$$

and

$$\frac{\partial (W/P)}{\partial \theta}\frac{\theta}{(W/P)} = \psi_3(\theta) + \frac{\partial l_A}{\partial \theta}\frac{\theta}{\lambda_A} - \frac{\partial r}{\partial \theta}\frac{\theta}{r}, \tag{B.5}$$

where $\psi_3(\theta)$ is the function of L.

Differentiating both the sides of the marketable surplus equation (6) with respect to θ and then rearranging the terms gives:

$$\begin{aligned}\frac{\partial M_s}{\partial \theta} = {} & \psi_4(\theta) + S_L A(1 - b\eta)\left(\frac{\partial l_A}{\partial \theta}\frac{\theta}{l_A}\right) - S_L Ab\varepsilon\left(\frac{\partial P}{\partial \theta}\frac{\theta}{P}\right) \\ & + S_L Ab(\eta - 1)\left(\frac{\partial r}{\partial \theta}\frac{\theta}{r}\right)\end{aligned}$$

where $\psi_4(\theta)$ is a function of θ. Substituting (B.2) and B.4) in the above equation and then rearranging terms gives:

$$\frac{\partial M_s}{\partial \theta} = \psi_4(\theta) + A\psi_2(\theta) + S_L A(1 - r)\left(\frac{\partial l_A}{\partial \theta}\frac{\theta}{l_A}\right) - S_L Ab\left(\frac{\partial r}{\partial \theta}\frac{\theta}{r}\right) \tag{B.6}$$

TABLE B.1
Different Values of ψ_i's

Value of θ	$\psi_1(\theta)$	$\psi_2(\theta)$	$\psi_3(\theta)$	$\psi_4(\theta)$
$\theta = t$	0	$-rS_Z\lambda_Z - rS_L\lambda_L$	0	$-S_Z\lambda_Z - S_L\lambda_L$
$\theta = K$	α	0	0	0
$\theta = L$	$1 - \alpha$	$-S_L b(\eta - 1)$	-1	$AS_L b(\eta - 1)$

Substitute ψ_t's for different θ and (B.1) in (B.2), (B.4), (B.5) and (B.6). Then,
When $\theta = t$:

$$\frac{\partial r}{\partial t}\frac{1}{r}|\Delta| = -rS_Z\lambda_Z - rS_L\lambda_L - S_L[r - b(\eta - \varepsilon)]\left[\lambda_Z - \lambda_L + \frac{\sigma}{S_Z}\lambda_L\right]$$

$$= -rS_Z\lambda_Z - rS_L\lambda_L - [r - rS_Z - S_L b(\eta - \varepsilon)]\left[\lambda_Z - \lambda_L + \frac{\sigma}{S_Z}\lambda_L\right]$$

$$= \frac{r}{S_Z}\lambda_L(\sigma - 1) - [r - S_L b(\eta - \varepsilon)]\left[\lambda_Z - \lambda_L + \frac{\sigma}{S_Z}\lambda_L\right]. \tag{B.7}$$

Since $\bar{C} > 0$, from (12) we get, $r - S_L b > 0$. Further, since $0 < \eta < 1$ and $0 < \eta - \varepsilon < 1$ we get $r - \eta S_t b > 0$ and $r - (\eta - \varepsilon)S_L b > 0$. Using these inequalities and (B.1) in (B.7) we get

$$\frac{\partial r}{\partial t}\frac{1}{r} \lesseqgtr 0 \quad \text{when} \quad \sigma \lesseqgtr 1, \text{ that is, when } \frac{dS_L}{dt}\frac{1}{S_L} \lesseqgtr 0 \tag{B.8}$$

This gives the first three elements in the first row of the sensitivity matrix. Using (B.8) the first three elements of the remaining row of the sensitivity matrix can be derived from (B.4), (B.5) and (B.6).
When $\theta = K$:

$$\frac{\partial r}{\partial K}\frac{K}{r} = \frac{\alpha S_L b\varepsilon}{|\Delta|} < 0, \qquad \frac{\partial P}{\partial K}\frac{K}{P} = \frac{\alpha S_L b\eta}{|\Delta|} > 0, \qquad \frac{\partial(W/P)}{\partial K}$$

$$\frac{K}{(W/P)} = -\frac{\partial r}{\partial K}\frac{K}{r} > 0 \quad \text{and} \quad \frac{\partial M_s}{\partial K} = -S_L Ab\left(\frac{\partial r}{\partial K}\frac{1}{r}\right) > 0.$$

These inequalities give the fourth column of the sensitivity matrix.
When $\theta = L$:

$$\frac{\partial r}{\partial L}\frac{L}{r}|\Delta| = S_L b\varepsilon(1 - \alpha) - S_L b(\eta - 1) = -\alpha S_L b\varepsilon + S_L b[1 - (\eta - \varepsilon)] > 0,$$

$$\frac{\partial P}{\partial L}\frac{L}{P} = (1 - \alpha) + \left(1 + \frac{r\alpha}{1 - r}\right)\left(\frac{\partial r}{\partial L}\frac{L}{r}\right) > 0,$$

$$\frac{\partial(W/P)}{\partial L}\frac{L}{(W/P)} = -1 - \left(\frac{\partial r}{\partial L}\frac{L}{r}\right) < 0 \quad \text{and} \quad \frac{\partial M_s}{\partial L} = -S_L Ab\frac{\partial r}{\partial L}\frac{1}{r} < 0.$$

From these inequalities the last column of the sensitivity matrix is derived.

APPENDIX C: TO DERIVE GROWTH RATES OF r, P, W AND M_s

Equation (10) and (12) can be written respectively as follows:

$$P\frac{S_L A}{rL} = (1 - \alpha)\frac{Q}{L}\frac{1}{(1 - r)} = (1 - \alpha)\left(\frac{K}{L_t}\right)^{\alpha} = W_t$$

and

$$A - \bar{C} - \frac{S_L A}{r}b = 0. \tag{C.2}$$

Substitute the value of P from (C.1) in (C.2) and then differentiate (C.2) totally with respect to t. After rearranging the terms,

$$\frac{dr}{dt}\frac{1}{r} = -\frac{1}{|D|}S_L b\varepsilon[\alpha_1 + \alpha_2 - \alpha_3] + \frac{1}{|D|}\left[S_L b\eta\alpha_1 + S_L b(\eta - 1)\alpha_2 - (r - S_L b)\frac{dA}{dt}\frac{1}{A}\right]$$

where

$$\alpha_1 = \frac{dS_L}{dt}\frac{1}{S_L}, \qquad \alpha_2 = \frac{dA}{dt}\frac{1}{t} - \frac{dL}{dt}\frac{1}{L}, \qquad \alpha_3 = \frac{dQ}{dt}\frac{1}{Q} - \frac{dL}{dt}\frac{1}{L},$$

and

$$|D| = S_L b\left(\eta - \varepsilon\frac{1}{1-r}\right) > 0.$$

$$\frac{dr}{dt}\frac{1}{r} = \frac{1}{|D|}\left[S_l b(\eta-\varepsilon)\alpha_1 - S_l b(1-\eta+\varepsilon)\alpha_2 + S_L b\varepsilon\alpha_3 - (r - S_L b)\frac{dA}{dt}\frac{1}{A}\right]$$
$$= C_1\alpha_1 - C_2\alpha_2 - C_3\alpha_3 - C_4, \tag{C.3}$$

where $C_i > 0 (i = 1, ..., 4)$ because $0 < \eta < 1$, $0 < \varepsilon$, $0 < \eta - \varepsilon < 1$, $r - S_L b > 0$.

Differentiating (C.1) logarithmically with respect to t and then substituting (C.3),

$$\frac{dP}{dt}\frac{1}{P} = -\alpha_1 - \alpha_2 + \alpha_3 + \frac{1}{|D|(1-r)}[S_L b(\eta-\varepsilon)\alpha_1 - S_L b(1-\eta+\varepsilon)\alpha_2$$
$$+ S_L b\varepsilon\alpha_3] - \frac{1}{1-r}C_4 = \frac{S_l b\eta}{(1-r)|D|}\alpha_1 = \frac{S_L b(1-\eta)}{(1-r)|D|}\alpha_2$$
$$- \frac{S_l b\eta}{|D|}(\alpha_2 - \alpha_3) - \frac{1}{1-r}C_4$$
$$= d_1\alpha_1 - d_2\alpha_2 - d_3(\alpha_2 - \alpha_3) - d_4, \tag{C.4}$$

where all d_i's > 0.

Differentiating the marketable surplus equation (6) with respect to t,

$$\frac{dM_s}{dt} = A(1 - S_L b)\frac{dA}{dt}\frac{1}{A} - AS_l b\eta\alpha_1 - AS_l b(\eta - 1)\alpha_2 - AS_L b\varepsilon\left(\frac{dP}{dt}\frac{1}{P}\right)$$
$$+ AS_L b(\eta - 1)\left(\frac{dr}{dt}\frac{1}{r}\right) \tag{C.5}$$

Substituting (C.4) in (C.5) and rearranging the terms it can be shown that

$$\frac{dM_s}{dt} = A(1-r)\frac{\dot{A}}{A} - AS_L b\left(\frac{dr}{dt}\frac{1}{r}\right) = e_1 - e_2\frac{dr}{dt}\frac{1}{r} \tag{C.6}$$

where e_1 and $e_2 > 0$.

Finally, differentiating (C.1) logarithmically with respect to t,

$$\frac{dW}{dt}\frac{1}{W} = \alpha\left(\frac{dK}{dt}\frac{1}{K} - \frac{dL_l}{dt}\frac{1}{L_l}\right) \tag{C.7}$$

REFERENCES

GOLDMAN H. W. and RANADE, C. G. "Analysis of Income Effect on Food Consumption in Rural and Urban Philippines". *Journal of Agricultural Economics and Development*, Vol. II, No. 2, July 1977, pp. 150–165.

LELE, U. J. *Food Grain Marketing in India*, Ithaca 1971.

MELLOR, J. W. "Models of Economic Growth and Land-Augmenting Technological Change in Foodgrain Production," in N. Islam, ed., *Agricultural Policy in Developing Countries*, London 1974.

——, "The Use and Productivity of Farm Family Labor in Early Stages of Agricultural Development," *Journal of Farm Economics*, Vol. XLV, No. 3, Aug. 1963, pp. 517–534.

MELLOR J. W. and LELE, U. J. "Growth Linkages of the New Foodgrain Technologies," *Indian Jour. Agr. Econ.*, Jan–Mar. 1973, *28*, 35–55.

NAKAJIMA, CHIHIRO "Technological Innovation and Subjective Equilibrium of Family Farm", *Osaka Daigata*—Keizoigaku, II, Nos. 1 and 2, October 1961.

NAKAMURA, J. I. *Agricultural Production and the Economic Development of Japan*, 1873–1922, Princeton 1966.

RANADE, C. G. "Distribution of Benefits from New Agricultural Technologies—A Study at Farm Level", unpublished doctoral dissertation, Cornell Univ. 1977.

RAO, C. H. H. *Technological Change and Distribution of Gains in Indian Agriculture*, Delhi, Macmillan Co. of India, 1975.

SEN, A. K. "Peasants and Dualism with or without Surplus Labor", *Jour. Pol. Econ.*, Oct. 1966, *74*, 425–450.

SMITH, T. C. *The Agrarian Origins of Modern Japan*, Stanford 1959.