263-0061
PN-AAD-755 62
ISN 46876

CAIRO UNIVERSITY - MASSACHUSETTS INSTITUTE OF TECHNOLOGY

TECHNOLOGY AND DEVELOPMENT PROGRAM

STUDIES OF CONVENTIONAL AND INNOVATIVE SINTERING FOR THE EGYPTIAN IRON AND STEEL COMPANY

by

David Gordon Wilson and Theodosios P. Korakianitis

August, 1985

PREPARED BY THE
TECHNOLOGY AND DEVELOPMENT PROGRAM
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

SUBMITTED TO THE

UNITED STATES AGENCY FOR INTERNATIONAL DEVELOPMENT
CONTRACT NO. 263-0061-C-00-1001-00

TDP
85-5

PREFACE

This report is one of a series of publications which describe various studies undertaken under the sponsorship of the Technology and Development Program at the Massachusetts Institute of Technology.

The United States Department of State, through the Agency for International Development, awarded the Massachusetts Institute of Technology a contract to provide support at M.I.T. for the development, in conjunction with institutions in selected developing countries, of capabilities useful in the adaptation of technologies and problem-solving techniques to the needs of those countries. This particular study describes research conducted in conjunction with Cairo University, Cairo, Egypt. The analysis, conclusions, and recommendations are those of the author and do not necessarily reflect the views of the Agency for International Development.

In the process of making the TDP supported study, some insight has been gained into how appropriate technologies can be identified and adapted to the needs of developing countries per se, and it is expected that the recommendations developed will serve as a guide to other developing countries for the solution of similar problems which may be encountered there.

Fred Moavenzadeh

Program Director

STUDIES OF CONVENTIONAL AND INNOVATIVE SINTERING FOR THE EGYPTIAN IRON AND STEEL COMPANY.

SUMMARY

Two approaches to improved sintering at the Helwan plant are reviewed in the work reported here. The first is the search for improved operation of the present sintering machines, which have been the principal limitation to increased production at present. We have examined the sintering operation through the use of a sophisticated computer model. The results from the model have turned out to be counterintuitive: while practical benefits are likely to result from operation of the present sinter machines with increased bed depth, the optimum from the point of view of minimum coke consumption is apparently given by using an intermediate bed depth, not greatly different from that employed at present.

The second approach is a potential longer-term solution. It is the use of a grate-less vertical-shaft sintering process, with the sinter mixture fed in at the top of the shaft, and air drawn out at the same location, and cool sinter cake taken out at the bottom of the shaft. This process offers the promise of a considerable reduction in coke use and in the proportion of fines that have to be recirculated. It is examined here in the form of a simple analytical model. It is proposed for consideration for further study by more sophisticated analytical modelling and by bench-scale tests in practice at Cairo University or at the Egyptian Iron and Steel Company.

ACKNOWLEDGEMENTS

The overall program, of which the work reported here is a small part, was initiated by Dr. M. G. El-Maghraby of Cairo University. He assembled a team drawn from various disciplines in the university, and from the Egyptian Iron and Steel Company and the Egyptian Aluminium Company, invited the participation of MIT, and made a proposal to the Technology Development Program. This program has been directed at MIT by Professor Fred Moavenzadeh, and supported by funding from the US Agency for International Development, Department of State.

The first part of the work reported here is a continuation of the program started by Mark A. McPeak, and previously given in US report no. 1. The work was based on a dynamic mathematical model of the sintering process developed by Dr. R. M. Young, of British Steel Corporation. He has freely and generously responded with helpful advice whenever we have called upon him. McPeak's work has been taken to a greater degree of refinement by Theodosios P. Korakianitis, with the result that the extrapolated results of the earlier work have been somewhat changed. The study of the vertical sintering process was started by Sean O'Connell, with the major part of the work being carried out by Theo Korakianitis.

Sabina Rataj provided program support, in particular with the production of these reports.

To all of these people I express my grateful thanks.

David Gordon Wilson, co-principal investigator
Massachusetts Institute of Technology,
Cambridge, Massachusetts
August, 1985

CHAPTER 1

THEORETICAL STUDY OF OPTIMUM SINTERING.

It is desirable first to summarize the problems that have been encountered in the operation of the sinter machines at the Egyptian Iron and Steel Company plant at Helwan.

Many of the difficulties with the sintering process, and with others in the plant, stem from the high alkalinity of the ore that has to be used. This condition has led to deposition on and corrosion of the grate bars. When the sinter mixture is spread on the grate, its "angle of repose" allows it to bridge over the normal holes in the grate bars. A burned-out grate bar has, however, a much larger hole, allowing sinter mixture to fall through. Whether a little or a lot falls through, the thickness of sinter mixture over the region of the damaged grate bar will be reduced, and its porosity will very likely be increased.

The sinter mix spread on the grate passes over suction boxes that pull air through from below, and the top surface of the mix is brought up to burning temperature in the ignition hood[1]. The air flow through the bed is not uniform, however. Where there is a clogged grate bar, the air-flow resistance will be very high, and sintering of the mixture cannot take place in its vicinity. Where there is a broken grate bar, there will be a reduced air-flow resistance purely on account of the absence of the normal grate blockage. In addition, there will be a reduced sinter thickness and perhaps an increased porosity. All these aspects combine to produce a much higher air flow and a consequently higher rate of travel of the combustion zone over the defective grate bar than elsewhere. Within a short time after leaving the ignition hood, therefore, bright-red holes appear in the moving sinter bed. The air is pulled through these holes as if in a forge. The great bars are then subjected

1. McGannon, Harold E., editor. THE MAKING, SHAPING AND TREATING OF STEEL. 8th edition, 1964. U.S.Steel corp., Pittsburgh, PA.

to a temperature far higher than elsewhere in the machine, and the combination of high temperature and high alkali content produces an environment that causes further degradation of the grate bars.

The sinter bed is, therefore, in a state of unstable equilibrium. Once a failure in a grate bar has occurred, the failure is likely to spread and become worse.

The rapid burn-through over defective grate bars, and the absence of sintering combustion over blocked bars, produce poor-quality sinter in those areas. Other areas in the bed are also affected because they will have somewhat less air than the bed is designed for, partly because of the diversion of the air to the path of least resistance, and partly because this "leakage" reduces the overall pressure difference driving the flow.

The consequence of these and other factors is that a very large proportion of the product leaving the sinter beds at the Helwan plant is not sinter cake but fines. In 1980-81 measurements indicated that associated with every ton of sinter produced were 726 kg of fines[2]. The fines are recirculated, mixed in with new sinter, and the process is repeated. As the sinter machines are the limiters to the capacity of the total Helwan plant, the influence of the poor performance is very great. The same report gave the sinter production rate as 57 percent of the design value.

POSSIBLE IMPROVEMENTS

An obvious palliative to the problem stated above is to fill each hole with sinter mixture as it appears. Although there would be a loss of some of the sinter mix through the damaged grate bar, the grate would be protected from further damage, the air would not be able to short-circuit the remainder of the sinter bed, and the general sinter quality would undoubtedly improve. Filling the holes could be carried out manually. A rather simple device could also be designed to fill holes automatically. It would run on rails two-to-four metres long allowing movement of an overhead track along the direction

2. ANALYSIS OF THE PRESENT STATUS IN THE IRON AND STEEL COMPLEX, HELWAN. Report no. 1 of the TPP project "Energetics in the Egyptian Metal Industries", Cairo University, November 1982.

of the bed movement (figure 1). A truck carrying a hopper with sinter mix could traverse the track. An array of infrared sensors would detect the hot spots that are indicative of holes forming in the bed. The truck would be traversed over each hole, and a quantity of sinter mix added until the normal bed depth had been reached. The truck would then return to a "park" position where its hopper would be periodically refilled.

Another possibility is to bring about temporary repairs to deteriorating grate bars. To replace a grate bar requires that the sinter machine be taken out of commission, shut down, partly dismantled and reassembled. Such an operation is likely to take many hours, and probably days, during which time the blast furnace it is feeding either must also shut down, an extremely expensive proposition, or must operate on alternative feedstock at reduced throughput. It would seem possible to weld pieces over the defective grate bars as the grate passes over the cold-end roller, possibly without affecting the operating speed, possibly with an arrest of movement of a few seconds' duration.

A third possibility that could be implemented more rapidly seemed to be to increase the initial bed depth of the applied sinter mix. This would have at least two effects. One is that the pressure drop through the grate would be a smaller proportion of the total pressure drop. Therefore the influence of a deteriorated grate bar would be smaller. The second effect is that the pressure of the sinter mix on the grate would be larger, the bridge that would form in the sinter mix over holes in the grate would be likely to be smaller, and the reduction in resistance of the sinter mix above a hole in the grate would be less than for a smaller bed depth.

This argument gave the principal stimulus to the analytical modelling of the sinter process. We learned about the work of R. W. Young at British Steel[3], and sought to acquire his computer program. Dr. Young generously provided us with his code, and has also given us advice from time to time.

---

3. Young, R. W. DYNAMIC MATHEMATICAL MODEL OF SINTERING PROCESS. Ironmaking and Steelmaking, 1977, no.6, pp 321-8

SUMMARY OF MCPEAK'S WORK.

In US Report no. 1[4], McPeak gave his preliminary results of the use of the Young code. They followed generally accepted trends. Of particular interest was the effect of increased bed depth. McPeak had time to produce only a few data, generally three points for each variable. His conclusion from the brief study of bed depth was that operation at increased bed depth was possible within limits. Fan pressure would have to be increased, leading to increased fan power; and there would be a reduced maximum bed temperature. This temperature reduction was somewhat unexpected. Sintering occurs as the limestone, coke and iron ore react to produce a temporary flux that solidifies to result in a sinter cake strong enough to withstand high loads at high temperatures in the blast furnace. Too high or too low a temperature would result in sintering not occurring, and in the production of fines. The principal problem in the present Helwan machines is the production of a very high proportion of fines. It was important, therefore, not to make recommendations that would result in a worsening of this severe problem.

RECENT DEVELOPMENTS

The report and the computer code with the sample cases were sent to the Cairo team. At MIT the question of what is the optimum bed depth in a sintering process continually arose. The advantages of greater uniformity of air distribution, coupled with the possibility of a greater production for a given bed area, have been mentioned. It also seemed probable that there would be a greater degree of heat conservation, allowing a smaller proportion of coke to be used. Against these possible advantages is the undoubted disadvantage of the requirement of higher fan power to pull the air through a greater bed thickness. Conceptually the advantages of increasing the bed depth would be greater than the disadvantages until the bed depth had become considerably greater. This tentative conclusion led to a short study, reported later in this document, on the possibility of a sintering process in which the sinter bed would move vertically downward and the combustion zone would stay stationary in space.

---

4. ENERGETICS IN THE EGYPTIAN METAL INDUSTRIES. US report no.1: POTENTIAL IMPROGEMENTS IN the SINTERING, BLAST-FURNACE AND STEEL-FURNACE PROCESSES. MIT, Cambridge, MA, November 1983.

The results of the study of the conventional sintering process follow.

EXTENDED STUDY OF CONVENTIONAL SINTERING.

The work by Young (ref. 3) was based largely on that of Szekely and Carr[5]. Young notes in the introduction to his paper that "Several authors have documented the more complicated equations applied to sinter plants, while others have produced solutions which exhibit the correct trends..... The methods chosen, however, have in some cases relied on analytical techniques and the solutions were thus necessarily limited to simplified cases. In other cases, numerical methods have been used with more success, . . . but the computer models produced were slow in operation, and dynamic simulation of the complete bed was consequently difficult.

"In this paper a dynamic model of the sintering process is described which uses more sophisticated numerical techniques to obtain a faster solution to the equations describing heat and mass changes in sinter beds. The sensitivity of the model to variations in the most important input conditions is investigated. ....."

Young goes on to describe the formulation of the mathematical model. "Based on the heat and mass flows. . . . the mathematical model supplies the solids and gas temperatures and the oxygen, carbon dioxide, water, limestone, and coke levels as functions of time and depth in the bed. Before deriving the equations which constitute the model, certain assumptions are made by which significant gains in speed of computation may be obtained with the generation of acceptably small errrors. These are:

(i) the time-dependent variation of the gas side can be separated from the depth-dependent one, thus changing the gas partial-differential equations to ordinary differential equations;

---

5. Szekely, J. and R. G. Carr. ON NONISOTHERMAL FLOW OF GASES THROUGH PACKED BEDS. Transactions of the A.I.M.E., vol. 242, p. 918-921, 1968.

(ii) the thermal conductivity within individual particles in the bed is infinitely high and the particles thus have a uniform temperature;

(iii) radiation from the top and bottom layers is negigible, and in other layers convective heat transfer alone is significant;

(iv) moisture does not condense at a lower point in the bed;

(v) the temperature of the solids cannot rise until a known critical moisture content is reached;

(vi) particle sizes of coke, limestone, and ore have each only one representative initial value, which is taken to be that of the specific size of the micropelleted feed;

(vii) the reactions of coke and limestone particles start at the surface of the particles and take place at a definite moving boundary, which leaves a core of unreacted material gradually shrinking with time.

(viii) in the combustion of coke, only the C, O_2, CO_2 reaction has been considered, and this has been taken to be irreversible; and

(ix) each constituent of the mixture exists in discrete particles and is not agglomerated with other materials."

We ran cases for various bed depths, from approximately the conventional depth, 295 mm, to approximately double this depth, 595 mm, and for each we used a range of coke content and air mass flow. To eliminate other effects in comparing the results, we kept all other input variables constant through these calculations. Some of these were the reaction constants, densities, particle radii, ignition temperatures and times, ore-melting temperatures, limestone content, moisture content, shape factor, void ratio, and depth and time increments. For each bed depth and coke content the gas flow was varied until the program output, of solid and gas temperatures and particle radii after combustion, was "acceptable". For the particle radii to be acceptable the coke had to be completely consumed, and the coke particle radii were, therefore, zero. The final radii of the limestone particles had to be either zero or small: it was not necessary to aim for exact complete limestone combination. The temperatures had to reach a maximum of at

least 1500K and no more than 1800K, this being the temperature range at which sintering occurs.

The results are shown in figures 2-4. They show some interesting and highly nonlinear effects that appear to represent real phenomena.

Our calculations indicate that at each bed depth locally high sintering temperatures could be produced with low proportions of coke - for instance, at a bed depth of 295 mm, a temperature of over 2000K could be obtained with two-percent (0.02) coke. However, an extremely precise range of airflow would be required, too precise to be obtainable in practice. This seems to be associated with locally stoichiometric conditions. Therefore these high-temperature regions are theoretically and practically unstable. As the air flow is increased, a wide region of stable sintering temperature is reached, this stable temperature (ie one that is tolerant to a wide variation in air flow) being dependent principally on coke proportion. Whereas the bed temperatures in the unstable regions of air flow differ for different bed depths, the bed temperature in the stable region is a function only of coke proportion and is unaffected by bed depth. Thus to reach a temperature of 1550K, four-percent coke (0.04) is required in all cases.

Increasing bed depth, therefore, will not reduce the consumption of coke from any fundamental consideration. There will, in fact, be an increased power consumption in the air suction fans. To increase the bed depth might therefore seem unattractive. However, if increasing bed depth also increases the uniformity of the air flow, decreasing the effect of blocked or burnt-out grate bars, and if the proportion of fines is thereby reduced, a considerable saving in the use of coke and electric power per unit of acceptable sinter produced should be achieved.

CHAPTER 2

PRELIMINARY STUDY OF VERTICAL COUNTERFLOW SINTERING.

SUMMARY

A form of iron-ore sintering in which the sinter solids are fed vertically downwards in a circular tower and the combustion air is drawn vertically upwards was subjected to a simple thermodynamic analysis.

The results indicate that a considerable saving in coke, perhaps up to 50 percent over the typical consumption in conventional horizontal-bed sintering, could be realized. Both the coke and the combustion gases would be discharged at a relatively cool temperature. A further expected advantage is a more unifrom degree of sintering and a lower proportion of fines and recirculation than in conventional processes. Once set into operation, no ignition fuel is required for such a system. There is no grate, and capital and operating costs would be expected to be considerably lower than for present plants.

THE COUNTERFLOW SINTERING CONCEPT

As the bed thickness is increased in a horizontal sintering machine, the bed speed must be reduced for the same air flow. The limiting case of this trend is one in which the bed speed is zero and the thickness infinite. A more useful concept is one where the bed thickness is finite and is moving counterflow to the air, rather than crossflow (figure 5). We do not know if this concept is novel. It seems obvious, but we have not been able to locate references to such arrangements being tried or analyzed. Accordingly, we have carried out a preliminary study of the thermodynamic relationships to be expected in such a device.

The model we investigated was as follows. Sinter mixture is fed to and spread on the top of a vertical duct of constant diameter. Two forks that can be inserted, lowered, removed, and recycled back to the first position sequentially maintain a full condition in the duct. Air is pulled upwards through the duct at a controlled rate. When the bed has reached a predetermined depth it is ignited by the temporary application of, for instance, natural-gas burners. These are kept burning for a

period as the sinter is allowed to fall downwards, and a height of fused sinter is built up below the combustion zone. This zone is kept at a constant height by adjustment to the sinter feed rate, or the air flow, or both. At some point, to be found empirically, the burners are shut off and the combustion becomes self-sustaining. The reason the ignition burners need to be kept in operation for longer than required merely to ignite the coke is that a considerable reduction in coke content is possible because of the intrinsic conservation in energy in this process. Therefore, if the steady-state proportion of coke were used to start the process, the temperature reached would be too low to produce fusing temperatures, at least until an operating time approximately equal to that necessary for a layer of solids to traverse the whole vertical extent of the bed has passed. By that time the counterflowing air and solids will both be fully preheated, and sintering temperatures will be reached.

The air is preheated automatically by heat exchange with the cooling sinter emerging from the combustion zone. Likewise the sinter is preheated by heat exchange with the hot combustion gases drawn upwards from the combustion zone. The heat exchange can be very effective: the only penalties for using a longer length of counterflow heat exchange above and below the combustion zone are increaed fan power from the increased flow resistance, and increased capital costs from the greater height of the apparatus. Eventually the sinter will reach a point below the combustion zone at which its temperature will fall to, say, 450K (assuming that an appropriate air and coke rate are used - see below). At this point, the fork blades could be programmed to cut off a layer of sinter and to discharge it to a blast-furnace-feed conveyor.

RESULTS OF PRELIMINARY ANALYSIS

We chose some specifications for the counterflow-sintering model and investigated the relationship among coke content, excess air, and maximum solids temperature reached. Among the specifications were that the sinter mixture and the air would enter the sinter duct at ambient temperature, about 300K. The sinter mixture would contain 13 percent lime by mass. The gas-solid heat-exchanging regions above and below the combustion zone were specified as being long enough to produce a thermal effectiveness of 90 percent each. (Thermal effectiveness of a heat exchanger is the ratio of the quantity of heat transferred to the maximum theoretically possible.) Our calculations show that this level of effectiveness would be very easy to reach in practice.

The results are shown in a graph in figure 6. The maximum temperature reached by the solids is plotted against the excess air used, and the proportion of coke in the solids is shown as a parameter. Perhaps counterintuitively, the proportion of coke needed for any given temperature to be reached goes down as the excess-air rate is increased. The reason is that the air acts as a heat-exchange medium as well as the bearer of combustion oxygen. If the thermal-capacity rate of the air is below that of the sinter mixture, the sintered solids cannot be cooled down to near ambient temperature by the air, and they will be discharged at a high temperature. On the other hand, if the air-flow rate is high enough to give the gas flow a higher thermal-capacity rate than the solids, the sinter will be discharged cold and the gases will come through the top of the bed at a high temperature. Minimum energy loss, and therefore minimum coke usage, is given by setting the excess-air rate to give approximately equal thermal-capacity rates for the gases and solids.

If a minimum sintering temperature is set, for instance 1600K, the middle curve of figure 6 would be used. The minimum viable proportion of coke is shown to be given at an excess-air rate of about 120 percent. In this condition the longitudinal temperature profile through the bed would be as shown in figure 7. Further reduction in coke rate seems to be limited by intrinsic variations in specific heats of the air and gases, and the solids before and after sintering, together with the change in mass-flow rate of the solids as the carbon is burned to CO_2.

FUTURE PROGRAM

We hope that a more detailed analysis will be carried out in Cairo, Helwan or Cambridge, together with an experiment aimed at modelling the full three-dimensional conditions. A laboratory-scale model of a gas-solid counterflow process using surrogate low-temperature materials that may be viewed through glass tubes could be run to verify the theory.

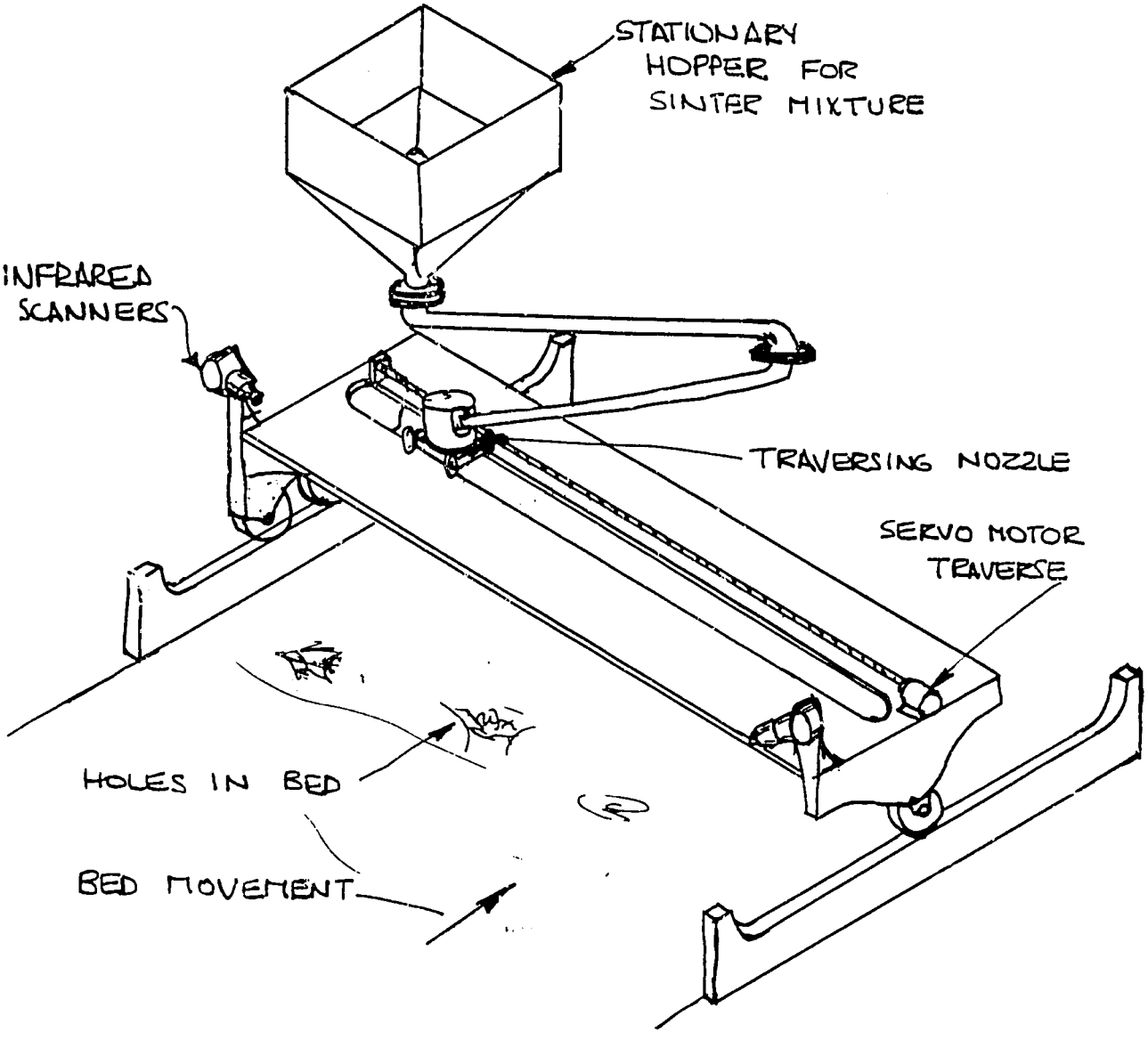


FIGURE 1 CONCEPT OF AUTOMATIC SINTER-BED HOLE FILLER

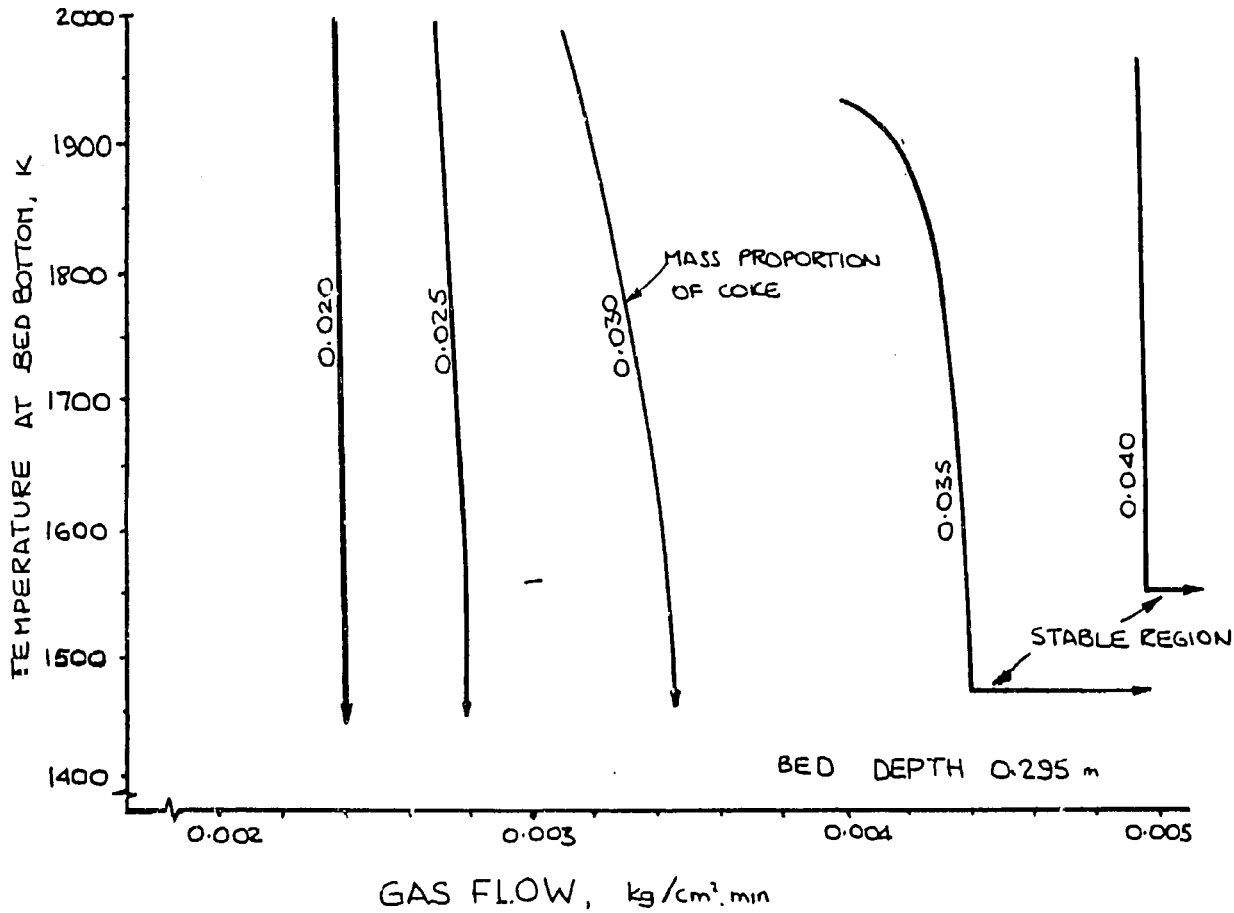


FIGURE 2 HORIZONTAL-BED SINTERING: COMPUTED RESULTS FOR 295mm DEPTH

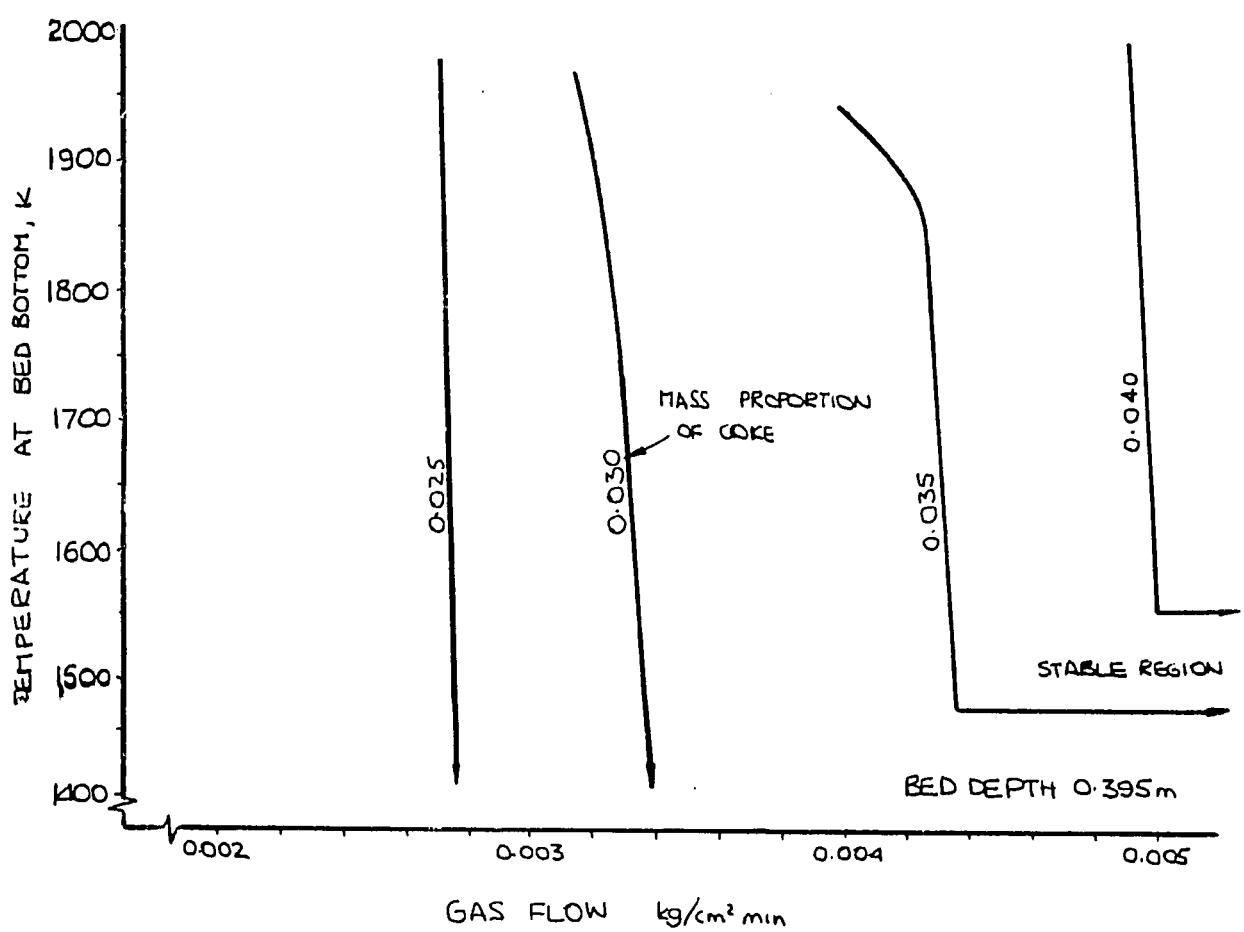


FIGURE 3 HOZONTAL-BED SINTERING: COMPUTED RESULTS FOR 395mm DEPTH

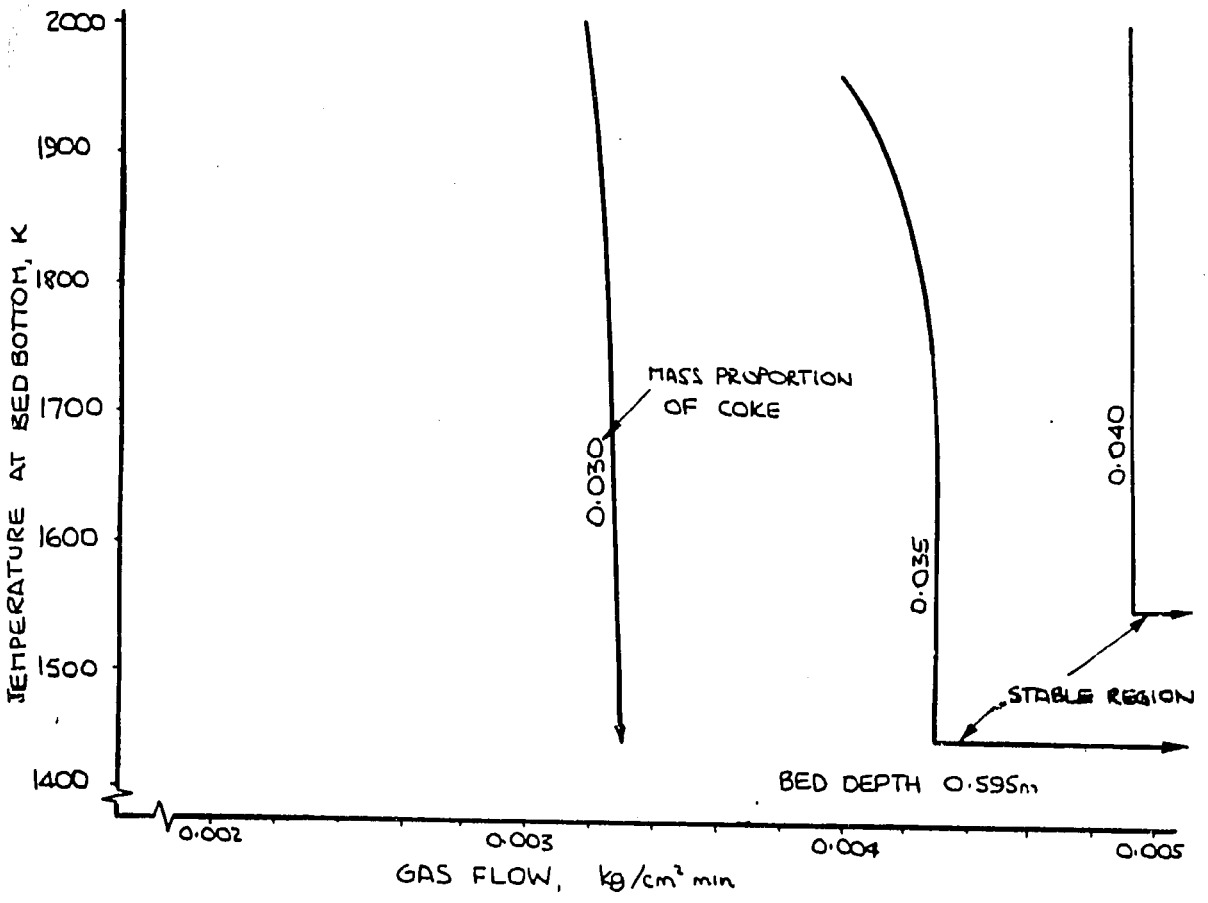
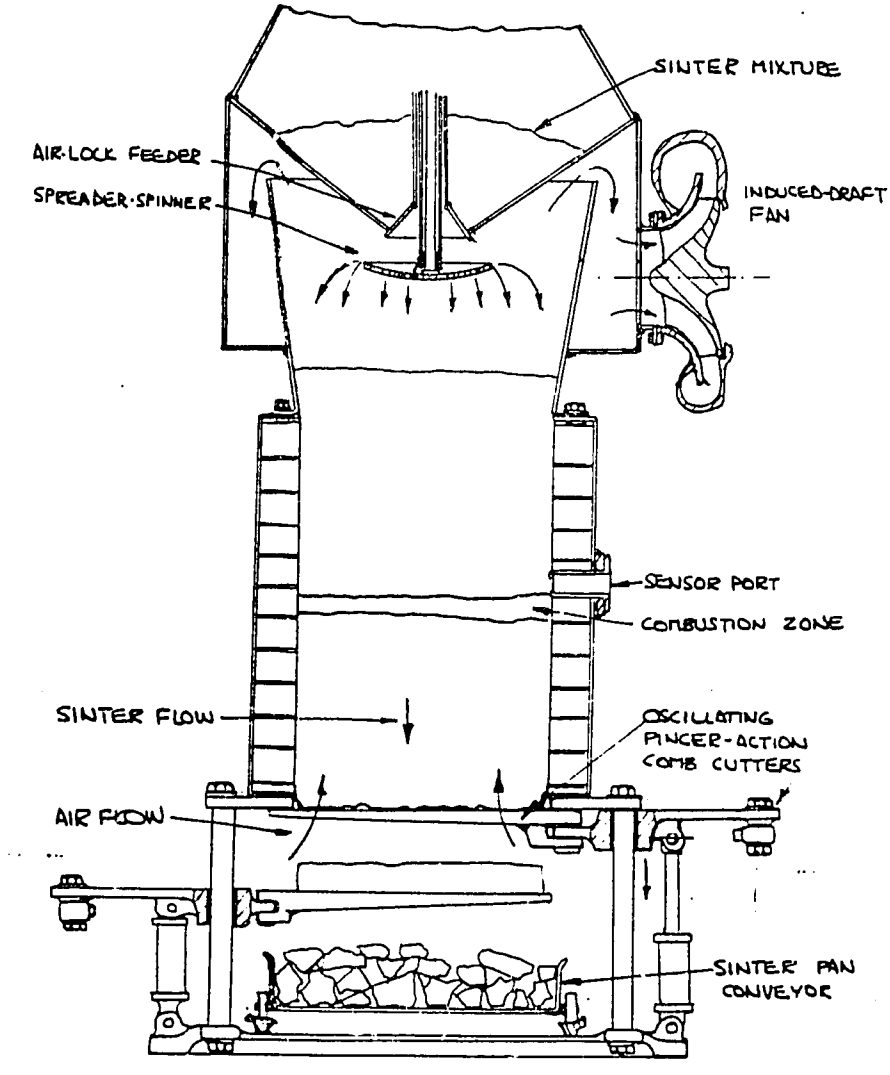
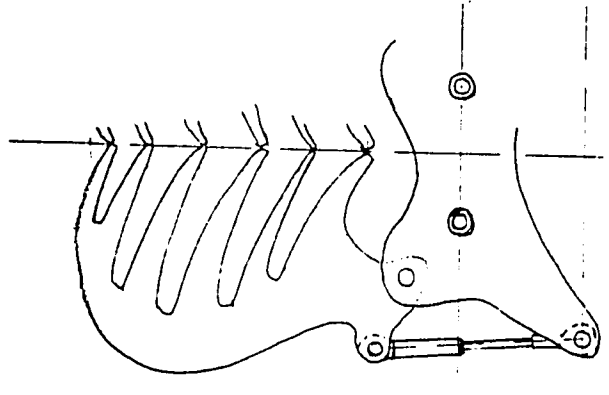


FIGURE 4 HORIZONTAL-BED SINTERING: COMPUTED RESULTS FOR 595mm DEPTH





PARTIAL VIEW OF COMB CUTTERS

FIGURE 5 CONCEPT OF VERTICAL GRATE-LESS COUNTERFLOW SINTERING

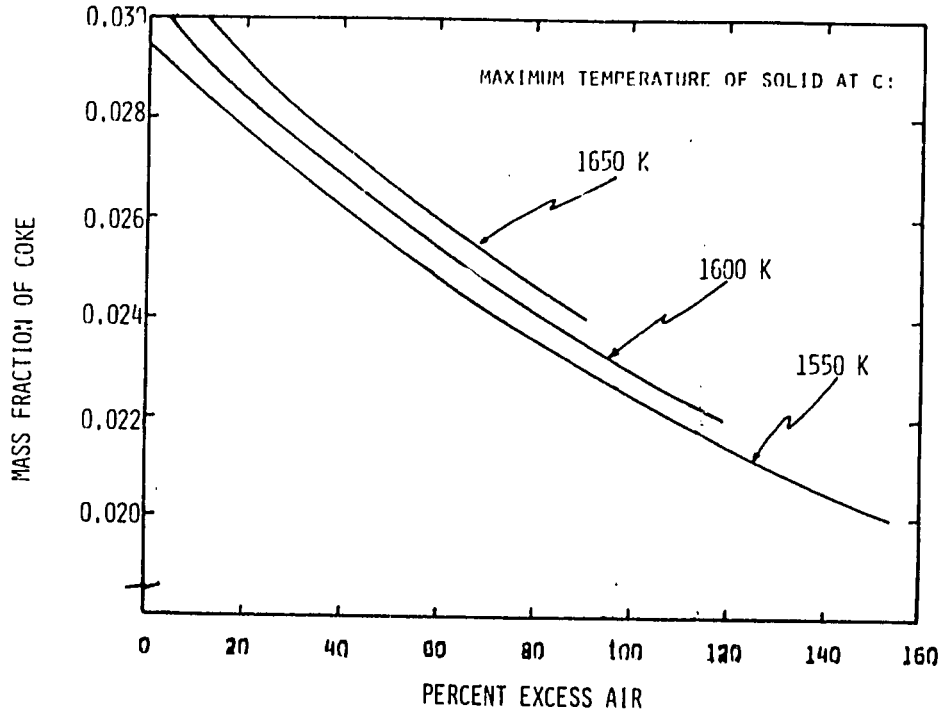


FIGURE 6 VERTICAL COUNTERFLOW SINTERING: EFFECT OF EXCESS AIR

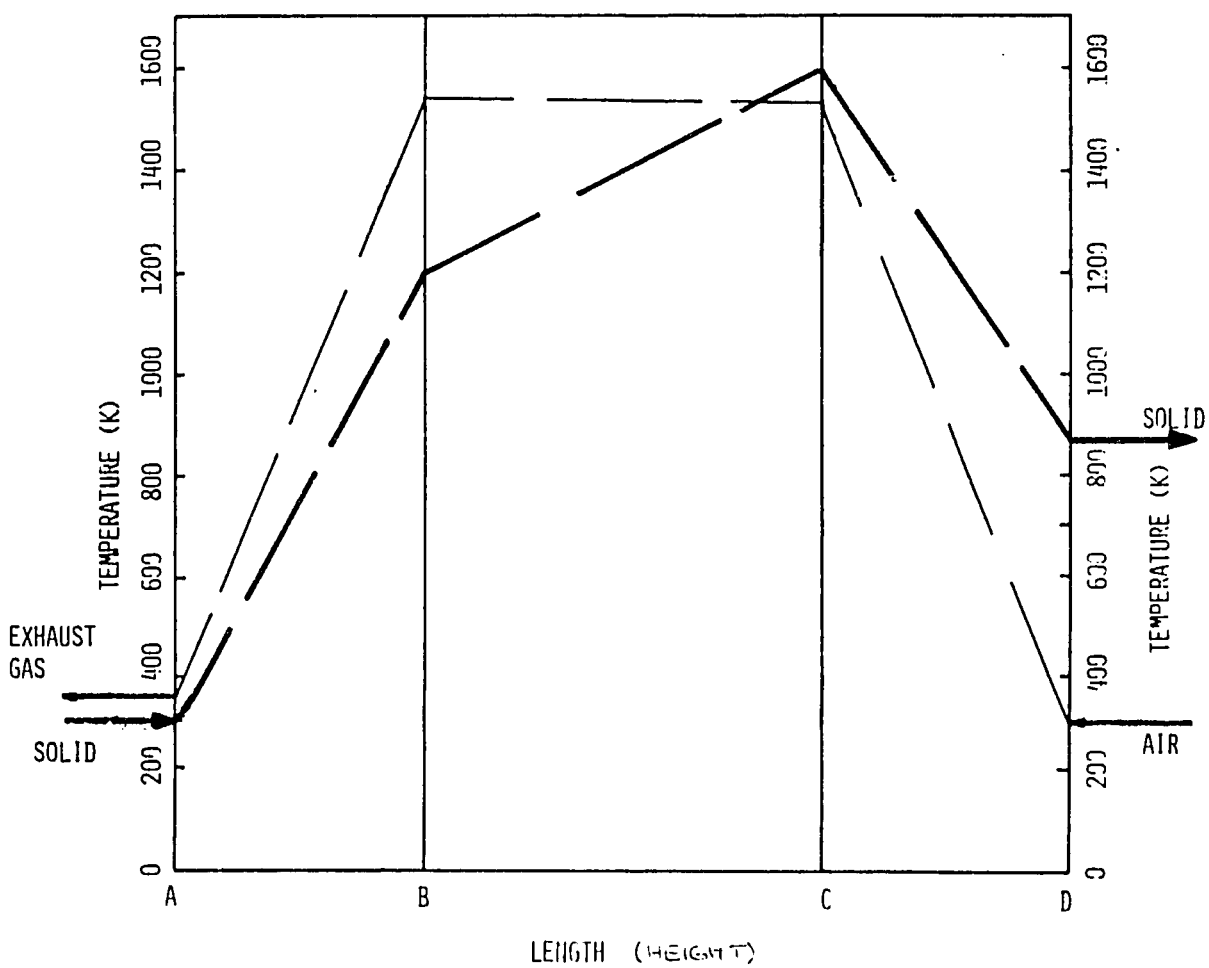


FIGURE 7 VERTICAL COUNTERFLOW SINTERING: TEMPERATURE DISTRIBUTIONS

APPENDIX

The thermodynamic model of the counterflow sintering machine is illustrated in figure 5. It consists of two counterflow heat exchangers separated by a combustion zone. Solid ore enters from the bottom and is warmed up in the first heat exchanger (point A to point B) by the hot exhaust gas that results from combustion. In the combustion zone (point B to point C) the carbon contained in the ore fuels the combustion reaction that warms up both the ore and the gas. In this zone changes in composition occur in both the solid and the gas. In the second heat exchanger (point C to point D) the hot solid warms up the air which enters from the top.

The specific-heat capacities of the solid and the gas, Cps and Cpg respectively, are modelled by the same functions of temperature that Young (ref. 3) used in his sintering program for the horizontal machine:

solid Cps = (0.016+0.922e-4*T-2570./T**2)*4183. [J/(kg.K)]; and

gas Cpg = (0.2174+0.7247e-4*T-0.16e-7*T**2)*4183. [J/(kg.K)],

where T is absolute temperature, K.

Additionally we specified the following.

Temperature of solid at point A	Tsa = 300 K
Temperature of gas at point D	Tgd = 300 K
Temperature of solid at point B	Tsb = 1200 K
Mass fraction of lime at point A	XMlime = 0.13
Effectiveness of first heat exchanger	E1 = 0.95
Effectiveness of second heat exchanger	E2 = 0.95

Variables are: XMcoke, the mass fraction of coke entering at point A; XSair, the amount of excess air from that required for the complete combustion of coke entering at point D; and Msint, the mass flow of sinter entering at point A in kg.

The mass flow of coke entering at point A is Mcoke = Msint*XMcoke.
The mass flow of lime entering at point A is Mlime = Msint*XMlime.
The energy released in the reaction Qin is equal to the energy released by the coke minus the energy absorbed by the lime. The numbers are taken from Young (ref. 3)

Qin = Mcoke*32.84e6 - Mlime*1.78e6 [J]

The dominant reactions during combustion are:

$$C + O_2 = CO_2 \text{ ; and}$$

$$CaCO_3 = CaO + CO_2$$

During the first reaction the amount of solid is reduced by Mcoke and during the second reaction the amount of solid id reduced by 0.44*Mlime. During combustion the amount of solid is reduced by Mdiff and the amount of gas is increased by Mdiff, where

$$Mdiff = Mcoke+0.44Mlime$$

The mass flow of solid through the first heat exchanger is

$$MS1 = Msint.$$

The mass flow of air through the second heat exchanger is

MG2 = Mcoke*11.49*(1.+XSair) (from combustion equation of carbon).

The mass flow of solid through the second heat exchanger is

$$MS2 = MS1-Mdiff.$$

The mass flow of gas through the first heat exchanger is

$$MG1 = MG2+Mdiff.$$

The minimum heat capacity in the first and second heat exchanger, HCMIN1 and HCMIN2, depend on Xmcoke, Xmlime, XSair, and the temperature of the gas and the solid in each heat exchanger.

Five unknowns remain:

Temperature of solid at point C	Tsc
Temperature of solid at point D	Tsd
Temperature of gas at point C	Tgc
Temperature of gas at point B	Tgb
Temperature of gas at point A	Tga

Three equations are obtained from the energy balances of the three zones.

In the first heat exchanger : HCS1*(Tsb-Tsa) = HCG1*(Tgb-Tga) (1)

In the combustion zone :

$$MS2*Cpsa*(Tsd-273) + MG1*Cpga*(Tga-273) = Qin + MS1*Cpsa*(Tsa-273) + MG2*Cpga*(Tgd-273) \quad (2)$$

In the second heat exchanger : HCS2*(Tsc-Tsd) = HCG2*(Tgc-Tgd) (3),

where HCS1, HCS2, HCG1, HCG2 are the heat capacities (mass-flow rate times mean specific heat) of the solid and the gas in the first and second heat exchanger respectively, and Cpga, Cpsa are the average specific-heat capacities of the gas and solid over the two temperatures in the parentheses that follow it.

Two more equations are obtained from the expressions of the heat-exchanger effectivenesses.

For the first heat exchanger:

$$E1 = \frac{HCS1*(Tsb-Tsa)}{HCMIN1*(Tgb-Tsa)} \qquad (4).$$

For the second heat exchanger:

$$E2 = \frac{HCS2*(Tsc-Tsd)}{HCMIN2*(Tsc-Tgd)} \qquad (5).$$

The above equations provide the solution for the five unknown temperatures.