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AGRICULTURAL PRODUCTION, particularly in developing countries, is generally a risky process, and considerable evidence exists to suggest that farmers behave in risk-averse ways.[1] Yet considerations of risk are rarely incorporated into regional or sectoral planning models; rather, farmers are assumed to behave in a risk-neutral, profit-maximizing way. Explicit representations of uncertain outcomes and farmers' attitudes toward them have appeared mostly in farm-level models. At the aggregate level, more indirect approaches have been used that amount to specifying "conservative" reactions to changes in the light of uncertainty; for example, the flexibility constraints of recursive programming (Day 1963).

Overview

On theoretical grounds, neglect of risk-averse behavior in agricultural planning models can be expected to lead to important overstatements of the output levels of risky enterprises (often reflected in overly specialized cropping patterns), hence also to overestimates of the value of important resources (for example, land and irrigation water). This chapter applies the theoretical framework of chapter 7 and reports results from two case studies in Mexico that were designed to measure the magnitudes of some of these biases; in the process, the chapter attempts to provide a quantification of risk aversion at aggregate farm levels. Other case studies using this methodology are found in chapters 12, 13, and 15 of this volume.

The two models used in this chapter are both regional linear programming models of the production of annual crops grown under irrigated conditions. They are static equilibrium models and, through incorpora-

1. See in particular, the following case studies: Cancian (1973); Dillon and Anderson (1971); Francisco and Anderson (1972); Lin, Dean, and Moore (1974); Officer and Halter (1968); and O'Mara (1971, also chapter 9 of this book).

tion of linear demand functions, simulate competitive market equilibriums in which both prices and quantities are endogenous. The first model, ALPHA, comprises eight of the irrigation submodels of CHAC, and therefore it may be considered reasonably representative of the role of risk in CHAC. The second model, BETA, is a slightly revised and updated version of the model by Pomareda and Simmons (chapter 12).[2]

In the following we briefly review the risk specification of the models and show why ignoring risk-averse behavior in programming models can lead to biases in estimated crop outputs and the values of scarce resources. In the section "Description of the Models," we present the two models in some detail and discuss the methods used to estimate salient risk parameters. The section "Results of the Models" presents findings and demonstrates the numerical importance of incorporating risk-averse behavior into agricultural planning models. The final section contains our brief, concluding remarks.

Method of Incorporating Risk

The underlying behavioral assumption in our models is that farmers maximize expected net income less its standard deviation—an (E,σ) utility function—rather than expected profits. This assumption follows a tradition begun by Markowitz (1959) and Tobin (1958) regarding choice under uncertainty. More precisely, however, our assumption is a variation on the approach of Baumol (1963) who used an $(E,\phi\sigma)$ formulation that gives more reasonable answers than the straight (E,σ) approach in some cases. With Baumol's approach, the decisionmaker is assumed to establish subjectively a confidence limit and a floor on expected returns, to which the limit is applied. In this chapter, parametric programming techniques are used to derive values of the subjective parameter ϕ.

It is well known that a pure (E,σ) function, taken over a significant range of values, has some rather strange behavioral properties, and even the assumption of normally distributed outcomes will not rescue it from that fact. Nevertheless, it has been shown that, over a limited range of values, such a function can be viewed as an approximation (via truncated Taylor's series) to a polynomial function with all the desirable properties. More precisely, the first two terms of a Taylor's series yield an (E,V) utility function, and the (E,σ) efficiency frontier is a tangency to it at the equilibrium point.

2. Several versions of this model have been prepared over time at the request of the vegetable producers' association in northwest Mexico for use in their planting decisions.

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Tsiang (1972, 1974) has provided a careful defense of the (E,σ) approach as a useful approximation in certain classes of choice, and it has proved to be helpful in improving predictions of behavior under risk at the farm level. This chapter builds on Tsiang's defense, with the purpose of showing one way in which it may be used in an aggregate rather than a micro model. One of Tsiang's conditions for (E,σ) analysis to be useful is that the risk be "small" relative to the total wealth of the risk taker. Although this condition may be met for the commercial, irrigated farms covered in our models, it may not be satisfied for the case of rainfed (*temporal*), subsistance farmers confronted with new technologies.

Given the underlying $(E,\phi\sigma)$ behavioral specification, the objective function of an aggregate linear programming model that simulates competitive market equilibria can be written as:

$$(8.1) \qquad \max\ U = X'M\,(A - 0.5BMX) - C'\,X - \Phi(X'\,\Omega X)^{1/2},$$

where

X = A vector of aggregate crop levels

M = A diagonal matrix of average yields

C = A vector of cost coefficients

A,B = The coefficient matrixes of the linear demand structure $EP = A - BMX$, where EP is the expected price and B is assumed to be diagonal[3]

Φ = An appropriate aggregate of individual farm ϕ coefficients

Ω = An appropriate aggregate of individual farm covariance matrixes of activity revenues.

The derivation and justification of this objective function was provided by Hazell and Scandizzo in the preceding chapter. It is equivalent to the sum of expected values of producers' and consumers' surplus over all markets, and it gives the asymptotic values of expected quantities and market-clearing prices in equilibrium.

The usual assumption of profit-maximizing behavior is, of course, equivalent to setting Φ equal to zero in equation (8.1). Thus, the effect of ignoring risk-averse behavior depends on the properties of the term $\Phi(X'\,\Omega X)^{1/2}$, at least within the confines of our behavioral assumptions.

Let the constraint set of the aggregate model be denoted by

$$DX \leq b,$$

where D is a matrix of technical coefficients, and b is a vector of resource supplies.

3. Diagonalization is obtained in the models by grouping commodities into demand independent groups (see chapter 3).

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The Lagrangian of the model is then:

$$L = X'M(A - 0.5BMX) - C'X - \Phi(X'\Omega X)^{1/2} + \nu'(b - DX), \tag{8.2}$$

where ν is a vector of dual values.

Now, from the necessary Kuhn-Tucker conditions, we obtain

$$\frac{\partial L}{\partial x_j} = m_j(a_j - b_j m_j x_j) - c_j - \Phi(X'\Omega X)^{-1/2} \sum_{i=1}^{n} \omega_{ij} x_i - \sum_{k=1}^{s} \nu_k d_{kj} \leq 0, \quad j = 1 \text{ to } n, \tag{8.3}$$

where lower case letters denote elements of the corresponding capital matrixes, n denotes the number of crops, and s denotes the number of constraints.

Complementary slackness conditions further require that, for all nonzero x_j in the solution, equation (8.3) must hold as a strict equality. Thus, we can rewrite equation (8.3) for any nonzero x_j as:

$$EP_j = \frac{1}{m_j}\left[c_j + \sum_{k=1}^{s} \nu_k d_{kj} + \Phi(X'\Omega X)^{1/2} \sum_{i=1}^{n} \omega_{ij} x_i\right], \tag{8.4}$$

where we have used the fact that, since B is diagonal, then

$$EP_j = a_j - b_j m_j x_j.$$

In words, equation (8.4) states that, for each nonzero activity, the expected marginal cost per unit of output must be exactly equal to expected price. The expected marginal cost comprises expected own marginal cost, c_j, plus expected opportunity costs, $\Sigma_k \nu_k d_{kj}$, as reflected in the dual value of the resources used by that activity, plus a marginal risk factor, $\Phi(X'\Omega X)^{-1/2} \Sigma_i \omega_{ij} x_i$.

Now, had we made the usual assumption of risk neutrality, the marginal risk term in equation (8.4) would disappear (because $\Phi = 0$). Consequently, incorporating risk-averse behavior leads to different output levels in the model solution, and the direction of change from risk neutrality depends critically upon the sign of $\Sigma_{i=1}^{n} \omega_{ij} x_i$. Crops which have large variances in revenues or positively correlated revenues with most other crops, or both, will tend to have a positive marginal risk term, and this will lead to a lower output under risk-averse behavior. In contrast, crops that have negatively correlated revenues with most other crops will tend to have a negative marginal risk term, and hence their output will be increased under risk-averse behavior.

To show the effect of risk-averse behavior on the valuation of scarce resources, we can rearrange equation (8.4) as:

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$$(8.5) \qquad \nu_s = \left[m_j\, E(P_j) - c_j - \Phi(X'\Omega X)^{-1/2} \sum_{i=1}^{n} \omega_{ij}\, x_i - \sum_{k=1}^{s-1} \nu_k\, d_{kj}\right] / d_{sj},$$

where, by suitable rearrangement over k, ν_s can be the shadow price of any selected resource.

Clearly, the imputed value of the sth resource for the jth nonzero activity will be greater or smaller than its value under risk neutrality depending upon the sign of $\Sigma_i\, \omega_{ij}\, x_i$. But the outcome also depends on the valuations ν_k, $k \neq s$. Equation (8.5) must, of course, hold for all nonzero activities, and so we have a simultaneous model in the ν_k. The overall effect of risk-averse behavior on the value of ν_s is not therefore apparent from equation (8.5). It depends on the total risk effect of the complete portfolio of farm crops.

We do know, however, that since $\Phi(X'\Omega X)^{1/2} \geq 0$ in the model objective function (8.1), then the value of the objective is smaller under risk-averse behavior than under risk neutrality. Euler's theorem then implies that the total valuation of the scarce resources must be smaller. This, however, still permits the possibility of some resources' increasing in value, providing that others are reduced by sufficiently large amounts. For those resources whose imputed value is reduced by the inclusion of risk (and very likely that means most, if not all, resources), how do we interpret the new valuation? To the extent that the model's structure faithfully reflects individual and market decisions, it can be said that an adequate treatment of risk normally reduces the price that farmers would be willing to pay for their production inputs.

Description of the Models

The two models to be described both deal with annual crop production in selected irrigation districts in Mexico.

The ALPHA model

The first model (hereafter called the ALPHA model) is in actuality part of CHAC and encompasses eight of the more than one hundred administrative districts of the Mexican Ministry of Water Resources (Secretaría de Recursos Hidráulicos, SRH). These selected districts are not contiguous but are scattered throughout the arid agricultural areas of Mexico; they are among the largest districts in their respective regions:

	District
Pacific Northwest	Culiacán, Comisión del Fuerte, Guasave, Río Mayo, Santo Domingo
North Central	Ciudad Delicias, La Laguna
Northeast	Bajo Río San Juan

Table 8-1. *Average District Cropping Patterns, 1967–68 to 1969–70, ALPHA Model*
(harvested hectares)

Crop	District								Aggregate	Percentage of national production
	El Fuerte	*Culiacán*	*Río Mayo*	*Guasave*	*Cuidad Delicias*	*Bajo Río San Juan*	*Santo Domingo*	*La Laguna*		
Dry alfalfa	1,988	—	2,144	—	6,510	—	285	5,498	16,425	34
Cotton	46,364	—	15,535	—	7,903	1,190	17,585	67,964	156,541	25
Green alfalfa	—	543	—	—	—	—	—	5,224	5,767	2
Rice	11,335	23,568	—	3,480	—	—	—	—	38,383	25
Sugarcane	12,706	24,172	—	—	—	—	—	—	36,878	12
Safflower	4,790	13,374	10,435	3,737	—	—	1,098	—	33,434	29
Barley	—	—	112	—	—	—	—	—	112	1
Chile	386	1,570	—	48	—	—	—	—	2,004	9
Beans	16,224	11,024	—	202	—	—	—	—	27,450	3
Chickpeas	561	938	—	271	—	—	—	—	1,770	1
Tomatoes	3,049	9,563	—	581	—	—	—	—	13,193	37
Sesame	3,010	2,815	8,390	144	—	—	—	—	14,359	4
Maize	10,792	4,302	4,071	2,420	10,053	54,269	1,038	6,213	93,158	2
Cantaloupe	231	387	—	722	—	—	—	—	1,340	4
Potatoes	1,320	—	—	—	—	—	—	—	1,320	5
Cucumbers	—	—	—	8	—	—	—	—	8	0
Watermelons	775	325	—	41	—	74	—	—	1,197	5
Sorghum	24,238	22,795	10,616	1,238	7,719	19,876	—	5,592	92,074	11
Soybeans	16,264	4,392	11,886	—	—	—	—	—	32,542	20
Wheat	23,561	3,057	29,969	5,742	29,668	1,048	11,738	16,159	120,933	16
Total	177,576	122,825	93,158	18,634	61,853	76,457	31,744	106,641	688,888	
Number of farms	16,484	6,224	9,185	2,984	10,710	4,480	647	48,341	99,055	
Available hectares per farm	10	12	8	6	4	16	47	2	5.8	

— Zero.

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Taken together, the eight districts account for significant shares of the national production of cotton, tomatoes, dry alfalfa, rice, soybeans, and safflower (see table 8-1). They also produce a wide range of cereal crops and vegetables, and some sugarcane. Some double cropping is practiced in all the districts, particularly in the vegetable-growing areas. The average district cropping patterns for the crop years 1967–68 and 1969–70 are given in table 8-1; not covered is a small percentage of land devoted to crops that are not included in the models. Crop production is almost entirely dependent on irrigation in all eight districts, and any small areas of rainfed land have been excluded.

In total, the eight district models cover 99,000 farms, 5.8 hectares in average size, and a district breakdown is included in table 8-1. For modeling purposes, each district is treated as a single large farm. The farms are presumed to be sufficiently homogeneous so that this procedure is unlikely to lead to any serious problems of aggregation bias. The model activities provide for the production, in each district, of crops in table 8-1 grown by that district, each with a choice of three mechanization levels and two planting dates. A set of labor activities provides flexiblity in selecting seasonal combinations of family and hired day labor. Family labor is charged a reservation wage of half the hired day-labor rate, a value derived from CHAC (see chapter 2). Purchasing activities provide for the supplies of mules, machinery, and irrigation water. Seasonal constraints are imposed on land and labor, and an annual constraint is imposed on water supplies.[4] Technical coefficients and costs are taken at average levels for 1967–68 to 1969–70. The model constraints are also based on this period. Average yields are based on the six-year period from 1966–67 to 1971–72, and risk parameters were estimated from time-series data spanning the period from 1961–62 to 1970–71.

The district models are linked in block diagonal form and are integrated into an aggregate market structure, similar to that in CHAC. That is, the market comprises linear domestic demand functions of the form $EP = A - BMX$, and has import and export possibilities at fixed prices. For simplicity, and to approximate cross-elasticity relations in demand, the crops are classified into demand independent groups, and linear substitution is allowed between products within each group at rates fixed by base-year relative prices.[5] The definition and characteristics of these demand groups are summarized in table 8-2. The demand curves for each

4. We assume the input costs and the resource constraint values to be nonstochastic; this assumption can be relaxed, using one of several available techniques to handle stochastic constraints. See, for example, Charnes and Cooper (1959), Madansky (1962), and Maruyama (1972).

5. For a more detailed description, see chapter 3. Income effects are ignored in this procedure because this is a partial equilibrium model.

Table 8-2. *Characteristics of Demand Groups, ALPHA Model*

Demand group	Commodity	Base-period price (pesos per metric ton): Commodity	Group-price index[a]	Own-price elasticity
1	Sugarcane	70	70	−0.25
2	Tomatoes	1,150	1,150	−0.4
3	Chile	1,500	1,500	−0.2
4	Cotton fiber	5,770	5,770	−0.5
5	Dry alfalfa	400	446	−0.3
	Green alfalfa	100		
	Barley	930		
	Chickpeas	990		
	Maize	860		
	Sorghum	630		
6	Rice	1,220	1,285	−0.3
	Beans	1,830		
	Chickpeas	990		
	Potatoes	930		
7	Maize	860	817	−0.1
	Wheat	800		
8	Cantaloupe	680	741	−2.0
	Watermelons	780		
9	Safflower	1,550	1,164	−1.2
	Sesame	2,410		
	Cottonseed oil	830		
	Soybeans	1,600		
10	Cucumbers	590	590	−0.6

a. Group-price indexes are computed using base-year quantity weights.

group have the same price elasticities as in CHAC but are located at mean output levels appropriate for the eight district aggregates according to the procedure in chapter 11. Export and import constraints are also taken from CHAC and are prorated according to the ratio of output from the eight districts to national output for each product.

The BETA model

The second model (hereafter called the BETA model) was developed by Pomareda and Simmons (see chapter 12) and, for the purposes of this study, can be viewed as a model of the irrigation districts of Culiacán, Humaya, El Fuerte, and Guasave in the state of Sinaloa, with a simplified representation of competitive supplies from Guatemala. Taken together, these four districts accounted, in 1973–74, for about 90 percent of the national exports of tomatoes, green peppers, and cucumbers, and about 40 percent of the national exports of melons.

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For modeling purposes, Culiacán and Humaya are grouped as a single region [elsewhere in this volume, they are grouped with San Lorenzo under the name "Culmaya"], as are El Fuerte and Guasave. This grouping permits satisfactory consideration of water transfers between district irrigation authorities. The two regions are then treated as single decision-making units.

Although all short-cycle crops in each region are included in the model, special emphasis was given to the modeling of activities representing vegetable production. Each vegetable is allowed several planting dates; and the yields for each planting date are disaggregated by months (the harvest period of a hectare of tomatoes planted in September, for example, may last as long as three and a half months). Yields are also disaggregated into exportable and nonexportable quality according to U.S. Department of Agriculture regulations on the quality of imports. Nonexportable qualities are channeled into domestic Mexican markets.

The principal resource constraints for each region are monthly supplies of land and labor and the annual supply of water. Labor requirements are specified by three categories: labor for cultivation, harvesting, and packing vegetables. Hiring activities provide for unlimited supplies of labor for each category of work, but at different wage rates. All input costs are at 1973–74 prices.

The market structure in the BETA model is rather more complicated than that in the ALPHA model, although prices again are made endogenous through the incorporation of linear demand schedules. The complexity arises because Mexican vegetable exports (produced mainly in Sinaloa) have significant price effects in the U.S. market and because they compete directly with Guatemalan exports. Consequently, to model export demands adequately, the U.S. and Guatemalan vegetable markets are incorporated directly into the model, along with the supply of melons produced competitively in the Apatzingan district of Mexico. A further source of complexity arises because U.S. prices are treated endogenously on a monthly basis.

Mexican domestic demands are treated according to CHAC, with the demand schedules and price elasticities given in table 8-3. Prices of vegetables other than tomatoes (peppers, cucumbers, and cantaloupes) are fixed with perfectly elastic demands. The supply of melons from Apatzingan is simply incorporated through four seasonal linear programming activities.

Vegetable supplies from Guatemala (peppers, cucumbers, cantaloupes, and honeydews) are incorporated through another linear programming submatrix, which simply contains a single production vector per crop. Domestic demands in Guatemala are assumed to be perfectly elastic at fixed prices.

The U.S. market is incorporated by means of monthly linear supply

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Table 8-3. *Demand Functions for Tomatoes and Traditional Crops in Mexico, BETA Model*

Crop	Demand equation[a]	Direct-price elasticity
Tomatoes	$P = 2.993 - .00008372Q$	−0.5
Sesame	$P = 3.068 - .00011210Q$	−1.2
Cotton	$P = 5.276 - .00000537Q$	−0.5
Rice	$P = 2.960 - .00001536Q$	−0.3
Safflower	$P = 2.069 - .00000531Q$	−1.2
Beans	$P = 5.573 - .00008800Q$	−0.3
Chickpeas	$P = 3.448 - .00007409Q$	−0.3
Maize	$P = 1.126 - .00001938Q$	−0.2
Sorghum	$P = 1.185 - .00000208Q$	−0.3
Soybeans	$P = 2.334 - .00000423Q$	−1.2
Wheat	$P = 0.936 - .00000107Q$	−0.5

Note: Price elasticities were taken from CIAC. Mean prices and quantities were taken from SRH (1972).

a. Quantity (Q) in thousands of kilos; price (P) in pesos per kilo.

and demand functions (see table 8-4). It is assumed that supplies and demands are both independent through time, and the U.S. prices in other periods are fixed (Mexican exports do not compete in these months).

The model maximand is a simple generalization of the objective function of the CIAC kind, in that the producers' and consumers' surplus are summed by product, by season, and by country.[6] Risk is introduced only in the two regional models for Sinaloa, again using an $(E, \phi\sigma)$ utility formulation.

Estimation of risk parameters

To incorporate risk-averse behavior into the models in accordance with equation (8.1), it is necessary to have estimates of Φ and Ω for each irrigation district. In principle, Φ and Ω should be suitable weighted averages of the parameters of risk aversion and the covariance matrixes of revenue of individual farms. Such information is not, of course, available—even if it were, suitable aggregation procedures have not yet been developed. Consequently, in formulating the ALPHA and BETA models, more approximate and aggregate procedures were used.

ESTIMATION OF Ω MATRIXES. The only available data on revenue variations were time-series data on prices and yields at the level of irrigation districts. Using these data (thirteen years in the case of the ALPHA model,

6. For a mathematical description, see chapter 12 and Pomareda and Simmons (1977).

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Table 8.4. *Supply and Demand Functions for Vegetables, BETA Model*

Product	Month	Equation[a] Supply	Demand
Tomatoes	December	$P = .0000770Q$	$P = 7.241 - .000043755Q$
	January	$P = .0001052Q$	$P = 6.116 - .000036722Q$
	February	$P = .0001864Q$	$P = 4.719 - .000020254Q$
	March	$P = .0001705Q$	$P = 7.098 - .000044305Q$
	April	$P = .0001278Q$	$P = 5.398 - .000019006Q$
	May	$P = .0000541Q$	$P = 8.248 - .000039815Q$
Peppers	December	$P = .0002715Q$	$P = 10.935 - .00046591Q$
	January	$P = .0003822Q$	$P = 11.041 - .00040690Q$
	February	$P = .0007818Q$	$P = 9.075 - .00026592Q$
	March	$P = .0006631Q$	$P = 10.835 - .00035203Q$
	April	$P = .0004780Q$	$P = 12.203 - .00042629Q$
Cucumbers	December	$P = .0003792Q$	$P = 4.046 - .00016589Q$
	January	$P = .0007304Q$	$P = 3.905 - .00011539Q$
	February	$P = .0010917Q$	$P = 3.968 - .00009907Q$
	March	$P = .0009442Q$	$P = 4.852 - .00015937Q$
	April	$P = .0002798Q$	$P = 3.562 - .00008637Q$
Cantaloupes	January	$P = .240380Q$	$P = 5.126 - .0004930Q$
	February	$P = .048850Q$	$P = 5.126 - .0004930Q$
	March	$P = .024020Q$	$P = 7.389 - .0001910Q$
	April	$P = .016450Q$	$P = 5.699 - .0000411Q$
	May	$P = .000110Q$	$P = 6.125 - .0000205Q$
Honeydews	January	n.a.	$P = 5.110 - .0013200Q$
	February	n.a.	$P = 5.110 - .0013200Q$
	March	n.a.	$P = 2.843 - .0002630Q$
	April	n.a.	$P = 3.170 - .0002240Q$

n.a. Not available.

Note: For supply, an elasticity of 1.00 is assumed.

a. Quantity (Q) in thousands of kilos; price (P) in pesos per kilo.

but only six years for the BETA model), covariance matrixes of crop revenues were calculated for each district modeled after detrending the original crop revenues by linear regressions.

There are two potential problems with these estimated Ω matrixes. First, the appropriate Ω matrixes should be aggregates of the covariance matrixes subjectively perceived by farmers at market equilibrium, and these may well differ from observed statistical relations. The estimated matrixes are therefore good to the extent that farmers' perceived values have converged to the observed statistical relations and that the time-series data covered an equilibrium period. The latter requirement may not be too unrealistic, in that few crops showed statistically significant or numer-

ically important trends in their cropping areas during the time periods under consideration.

Second, the Ω matrixes should be direct aggregates of covariance matrixes of individual farms and should not incorporate any covariance relations that may exist between farms within a district. This is because it is required that:[7]

$$\Phi_h \, (X_h' \, \Omega_h \, X_h)^{1/2} = \sum_{k \in h} \Phi_k \, (x_k' \, \Omega_k \, x_k)^{1/2},$$

Where h denotes the hth irrigation district and k denotes the kth farm. The estimated Ω_h matrixes are therefore good to the extent that the Ω_k matrixes are the same for all farms in the hth district, and that the covariance between the revenues of any two crops i and j are the same between farms as within farms. That is,

$$\text{cov}\,(r_{ki}, r_{kj}) = \text{cov}\,(r_{\ell i}, r_{\ell j}) = \text{cov}\,(r_{\ell i}, r_{kj}),$$

where r denotes activity revenue and k, ℓ are farms.[8] Unfortunately, suitable cross-sectional samples of farm data could not be found to test these requirements.

ESTIMATION OF ϕ. In the absence of any empirical data from which to estimate the ϕ coefficients, the basic procedure followed was to search, through postoptimality techniques, values of ϕ that enabled the models to best describe a set of base-year prices. In both models we make the simplifying assumption that ϕ is the same for all irrigation districts. Various measures of "goodness of fit" were tried, but a simple average of the absolute values of the deviations in prices served as well as any (hereafter called the MAD, mean absolute deviation).

This method of estimating ϕ poses the obvious difficulty that it will pick up errors in model misspecification and data, and that there is no way

7. See chapter 7.

8. Under these assumptions, and letting R_j denote the average revenue of the jth crop observed at the district over all farms, and K the number of farms in the district, we obtain:

$$\begin{aligned}
\text{cov}\,(R_i \, R_j) &= \frac{1}{K^2} \sum_k \sum_\ell E(r_{ki} \, r_{\ell j}) - \frac{1}{K^2} \sum_k \sum_\ell E(r_{ki}) \, E(r_{\ell j}) \\
&= \frac{1}{K^2} \sum_k \sum_\ell \text{cov}\,(r_{ki}, r_{\ell j}) \\
&= \frac{1}{K^2} \sum_k \sum_\ell \text{cov}\,(r_{ki}, r_{kj}) \\
&= \frac{1}{K} \sum_k \text{cov}\,(r_{ki}, r \quad) \\
&= \text{cov}\,(r_{ki}, r_{kj}), \text{ for all } k.
\end{aligned}$$

Table 8-5. *Price Solutions by Commodity Group for Different Values of Φ, ALPHA Model*
(pesos per metric ton)

Demand group	Values of Φ						Base-period index
	0	0.5	1.0	1.5	2.0	2.5	
1. Sugarcane	68	70	72	68	68	68	70
2. Tomatoes	321	703	988	1,200	1,494	1,772	1,150
3. Chile	1,031	1,125	1,124	1,352	1,330	1,547	1,500
4. Cotton fiber	5,770	5,770	5,770	5,770	5,770	5,770	5,770
5. Forage crops	499	475	445	449	444	443	446
6. Food crops	1,373	1,316	1,241	1,233	1,227	1,217	1,285
7. Cereals	1,044	1,043	992	970	958	989	817
8. Melons	446	416	420	468	476	416	741
9. Vegetable oils	1,014	1,004	1,022	1,058	1,133	1,200	1,164
10. Cucumbers	516	436	284	148	148	148	590
MAD[a]	219	175	153	123	146	172	

Note: Prices are reported as group indexes, using base-year quantity weights (these weights are discussed in chapter 3).

a. The MAD is the mean absolute deviation of the solution value from the base-period values.

of knowing how serious these errors might be other than to judge the "reasonableness" of the estimated parameter.

Results of the Models

The model Φ coefficients are a direct representation of risk-averse behavior at the aggregate farm level. Thus, solving the models[9] for different values of Φ provides direct information about the effects of different degrees of risk aversion on equilibrium prices and quantities and gives a basis for quantifying the actual value of Φ.

Quantification of risk aversion at the aggregate level

Pertinent results from the ALPHA model are presented in tables 8-5 and 8-6. Table 8-5 shows the effect of different Φ values on the domestic equilibrium prices for the commodity groups delineated in table 8-2. On

9. The models are solved with a linear programming algorithm using the linearization techniques of Duloy and Norton (chapter 3 of this volume) and Hazell (1971).

Table 8-6. *Quantities Produced for Domestic Market for Different Φ Values, ALPHA Model*
(metric tons)

Crop	Values of Φ						Base-year quantities
	0	0.5	1.0	1.5	2.0	2.5	
Dry alfalfa	137,323	139,120	140,916	140,916	140,916	140,916	179,019
Cotton	484,488	476,111	445,869	358,485	285,965	268,476	243,454
Green alfalfa	173,445	175,714	177,983	177,983	177,983	177,983	226,109
Rice	115,369	115,784	118,774	118,774	141,885	143,695	126,197
Sugarcane	2,659,859	2,659,859	2,659,859	2,659,859	2,659,859	2,659,859	2,627,020
Safflower	75,777	74,467	72,752	67,437	86,449	70,914	72,490
Barley	510	517	523	523	523	523	665
Chile	15,409	15,161	15,161	14,789	14,789	14,294	14,459
Beans	30,278	30,278	31,060	31,060	26,428	26,765	33,001
Chickpeas	1,239	1,264	1,272	1,272	1,250	1,251	1,585
Tomatoes	223,682	202,712	185,237	171,257	153,781	136,241	174,752
Sesame	9,643	9,475	9,258	8,581	8,161	7,662	9,224
Maize	194,989	200,993	198,093	163,852	163,851	163,406	210,801
Cantaloupe	9,966	10,589	10,589	9,967	9,966	10,589	6,935
Potatoes	45,065	45,065	46,228	46,228	21,734	22,010	27,139
Cucumbers	20	33	35	359	2,195	2,301	19
Watermelons	22,213	23,601	23,601	22,213	22,213	23,601	10,850
Sorghum	336,856	341,263	345,670	345,669	345,670	345,669	285,818
Soybeans	59,815	58,781	63,359	75,595	63,982	68,706	57,220
Wheat	326,064	320,453	328,585	369,565	369,566	367,959	343,979

the one hand, the prices of groups 2 (tomatoes), 3 (chile), and 9 (vegetable oils) all increase with Φ, indicating corresponding reductions in the quantities produced for the domestic market. On the other hand, the prices for groups 5 (forage crops), 6 (food crops), 7 (cereals), 8 (melons), and 10 (cucumbers) decrease as Φ increases, indicating that production of these crops for a domestic market increases as producers become more risk averse. The quantity effects are shown in detail in table 8-6 at the individual commodity level. The prices of groups 1 (sugarcane) and 4 (cotton fiber) show no response to the risk-aversion parameter.

These results confirm the ambiguities involved in predicting the effect of risk-averse behavior on the supplies of individual crops as discussed in the section "Method of Incorporating Risk," above. They also suggest a useful definition of riskiness in crop production that takes intercrop relations into account. High- (low-) risk crops can be defined as those in which production decreases (increases) as producers become more risk averse, whereas risk-neutral crops are those whose production is unaffected by Φ.

The last columns in tables 8-5 and 8-6 contains the base-year values (1967–68 to 1969–70 averages) of prices and quantities. By comparing the

model solutions for different Φ values with these base-year values, we have a basis for selecting the "best-fitting" value of Φ.

Clearly, the solution corresponding to risk neutrality ($\Phi = 0$) is quite unsatisfactory. It predicts unrealistically high levels of production of cotton, tomatoes, cantaloupes, potatoes, watermelons, and sorghum, and particularly low prices for groups 2, 3, and 8. There is a definite improvement in both the price and quantity fits as Φ increases, but this deteriorates again as Φ approaches 2.5. In the latter solution, the quantities produced of tomatoes, maize, and potatoes become unrealistically low, whereas those for rice, cucumbers, sorghum, and wheat become too large.

In selecting a value of Φ, we have chosen to concentrate on the commodity group prices because the market structure of the model can only be expected to work best at the level of the demand group. The last row of table 8-5 reports the simple MAD's (mean absolute deviations) of the price fits and clearly demonstrates the superiority of the solution for $\Phi = 1.5$.

We conclude that introducing risk-averse behavior into the ALPHA model significantly improves its predictive power compared with the more usual assumption of risk neutrality (the price MAD is reduced by 44 percent when Φ is increased from 0 to 1.5), and that a reasonable measure of the risk parameter at the aggregate farm level may be about 1.5.[10]

Table 8-7 reports the comparable price solutions from the BETA model. The commodities were not grouped in this model (demand independence is assumed), and, because some prices are fixed exogenously to the model, these are not reported in table 8-7. Table 8-8 shows the effect of different Φ values on the cropping patterns in the state of Sinaloa. We reported production in metric tons for the ALPHA model because there are important yield differences between districts in that model, but this is not a problem with the BETA model. The figures in hectares incorporate production for both the export and domestic markets, which is also an essential feature given the importance of vegetable exports in the BETA model.

The model shows considerable flexibility in its response to different Φ values, a feature reflected in the wide range of hectarage values for each crop. Tomatoes, peppers, cotton, safflower, and soybeans exhibit high-risk behavior in that their production falls rapidly as risk aversion increases, whereas wheat, sesame, and maize are clearly low-risk crops.

Comparison of the model solutions with base-period values (1973–74 actual figures in this case) again shows that introducing risk-averse behavior leads to significant improvements in the model's predictive powers as compared with risk neutrality. In this case, the best price fit occurs when $\Phi = 0.5$ (the price MAD is reduced by 52 percent as Φ is increased from 0 to 0.5).

10. Assuming normal distributions, this Φ value corresponds to a 6.7 percent confidence band, which is very close to the sort of risk levels accepted by statisticians!

Table 8-7. *Price Solutions for Different Values of Φ, BETA Model*
(pesos per kilo)

Crop	Values of Φ: 0	0.5	1.0	1.5	Base-year values
Vegetables[a]					
Tomatoes	3.493	3.879	4.034	4.226	3.850
Peppers	4.002	4.747	4.759	4.978	4.852
Cucumbers	2.824	2.888	2.900	2.676	2.764
Other[b]					
Cotton	2.414	2.524	2.560	2.619	2.470
Safflower	1.371	1.526	1.658	1.839	1.600
Soybeans	1.485	1.490	1.534	1.622	1.450
Chickpeas	1.427	1.447	1.449	1.451	2.100
Rice	1.082	0.987	0.871	0.884	1.075
Beans	2.001	1.993	1.979	2.026	1.980
Wheat	0.825	0.775	0.743	0.725	0.800
Sorghum	0.655	0.637	0.589	0.616	0.625
Sesame	3.055	2.842	2.340	2.286	2.778
Maize	0.995	0.873	0.745	0.654	0.844
MAD	0.210	0.101	0.164	0.215	

a. Export prices; that is, average U.S. wholesale prices for winter season.
b. Domestic prices.

Effects on supply response behavior

Our results have already shown that the introduction of risk-averse behavior into the models leads to different levels of production for most crops compared with the assumption of risk neutrality. These results are, of course, estimates of single points on the product supply functions, but they amply demonstrate the biases inherent in ignoring risk-averse behavior in aggregate linear programming models. We turn now to an exploration of the broader effects of risk-averse behavior on both the slope and location of the domestic supply response functions for selected crops.

Given that most domestic prices are endogenous to our two models, it is not possible to derive directly the effects of price changes on domestic supplies. Rather, the location of the domestic demand curves must be shifted, and the model allowed to determine the new equilibrium values of both prices and the quantities supplied. Supply response functions derived in this way also allow for price and quantity adjustments in all other markets; they are not, therefore, the partial supply functions described in economic texbooks but must be considered as total supply response relations.

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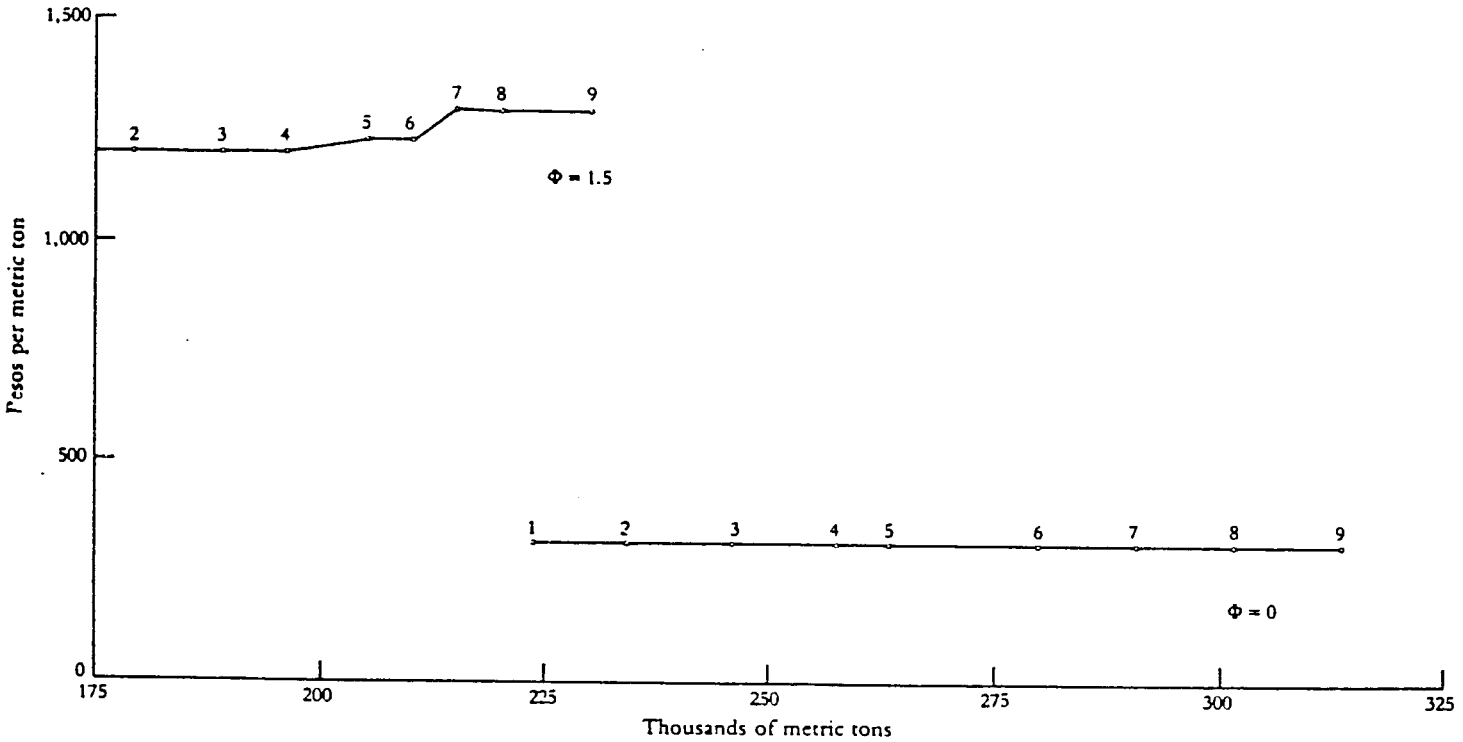
Table 8-8. *Cropping Area in Sinaloa for Different Values of Φ, BETA Model*
(hectares)

Crop	Values of Φ				Base-year quantities
	0	0.5	1.0	1.5	
Vegetables					
Staked tomatoes	21,713	15,117	11,996	8,595	14,200
Ground tomatoes	4,459	0	0	0	1,400
Green peppers	11,984	3,379	3,379	2,409	2,909
Cucumbers	2,107	1,869	1,697	2,531	3,643
Other					
Cotton	64,232	53,409	51,904	46,210	48,075
Safflower	98,976	72,983	52,751	27,187	81,471
Soybeans	104,685	103,267	102,005	89,111	117,827
Chickpeas	17,115	16,646	15,396	15,396	25,844
Rice	31,308	33,047	35,576	34,733	51,505
Beans	32,165	32,165	32,187	32,180	35,524
Wheat	28,685	48,776	54,142	68,697	49,989
Sorghum	51,376	55,266	57,005	55,717	45,798
Sesame	218	2,597	8,558	9,155	2,584
Maize	7,087	14,875	18,769	18,889	17,740

To simplify the presentation, we have selected two commodities from each model: commodity groups 2 (tomatoes) and 7 (cereals) from the ALPHA model, and sorghum and safflower from the BETA model. In each experiment, the demand curve for the relevant group or commodity was rotated to the right at discrete intervals of 5 percent each on the quantity axis while holding all other demand curves at their initial positions. The experiments were repeated for both risk neutrality and the best-fitting values of Φ. The supply response functions so derived are reported in figures 8-1 through 8-4. The points on these functions depict model solutions, and these are numbered so that 1 represents the base-period solution, 2 represents the solution with a 5 percent quantity shift in the relevant demand, 3 represents a 10 percent shift, and so on. Thus, points labeled with the same numbers in each figure correspond to solutions with different Φ values but with identical demand structures.

The two models show markedly different supply response behavior. The response relations obtained from the ALPHA model (figures 8-1 and 8-2) are all highly elastic, with supplies to the domestic market increasing by about the same percentage as the corresponding shifts in demand. Apparently, relative equilibrium prices in the ALPHA model are such that production of tomatoes and cereals is constrained by the volume of domestic demand rather than by the marginal profitability of these crops.

Figure 8-1. *Domestic Supply Response for Tomatoes, ALPHA Model*



Note: Solutions are numbered as follows: 1, base period; 2, with 5 percent quantity shift in relevant demand; 3, with 10 percent quantity shift; 4, with 15 percent quantity shift; and so on.

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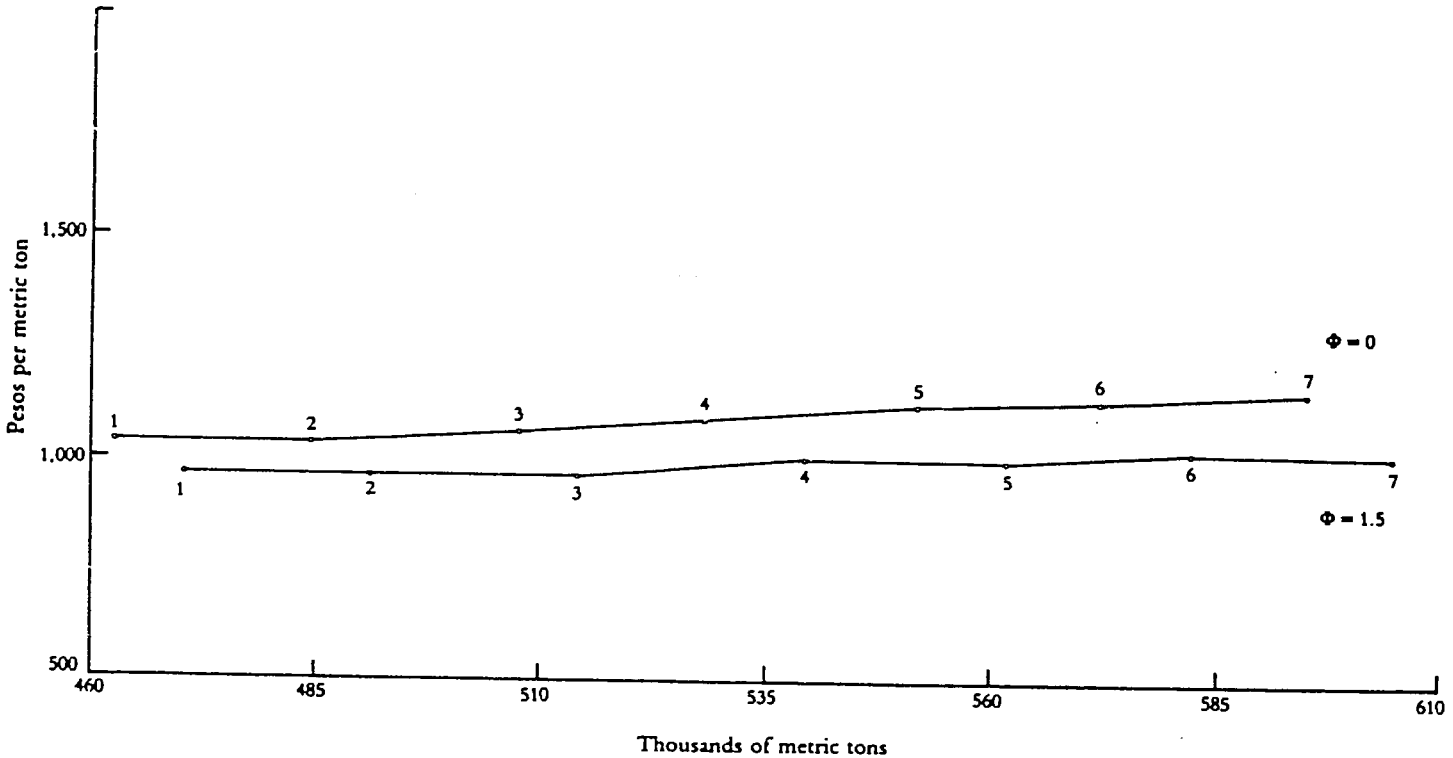
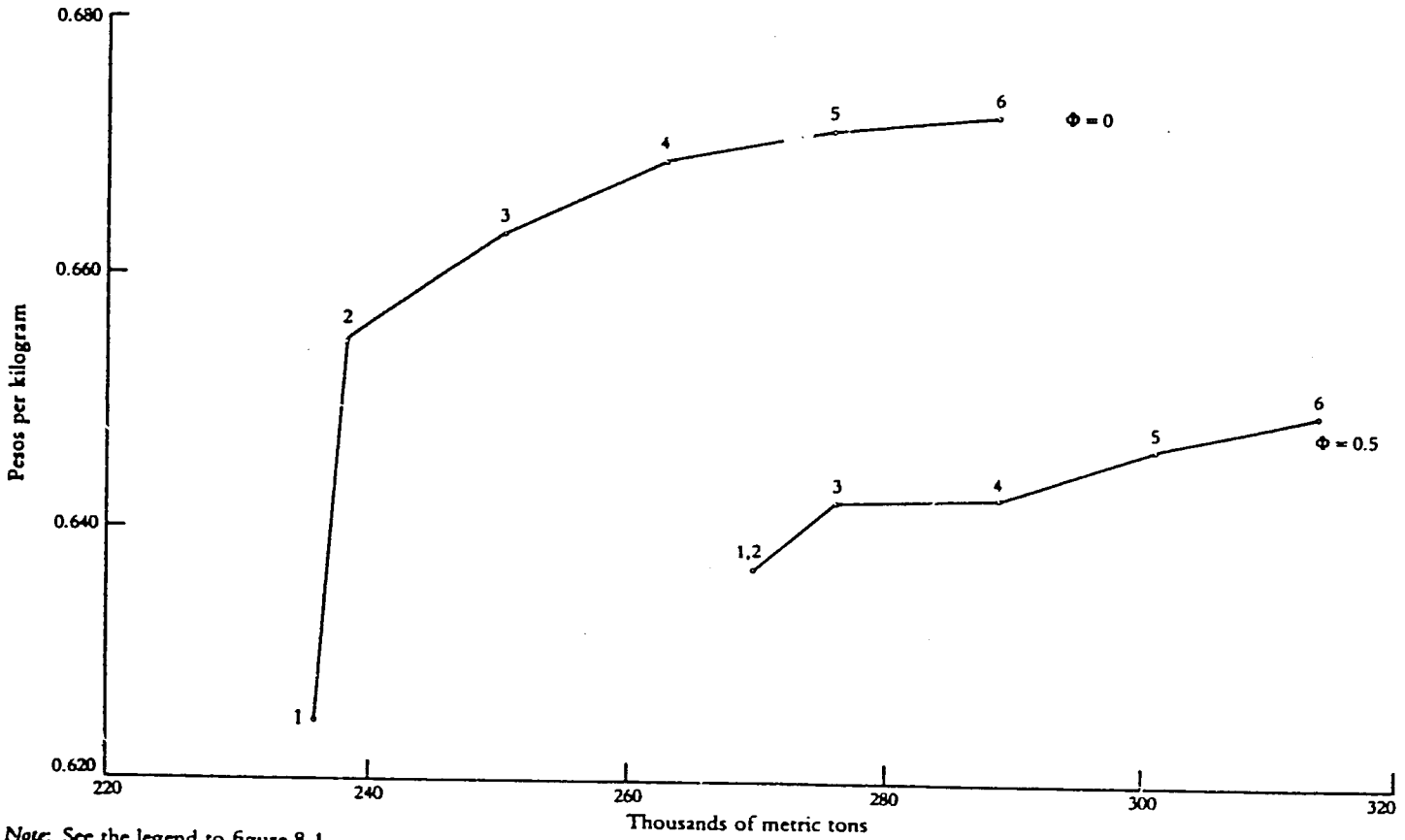
Figure 8-2. *Domestic Supply Response for Cereals, ALPHA Model*



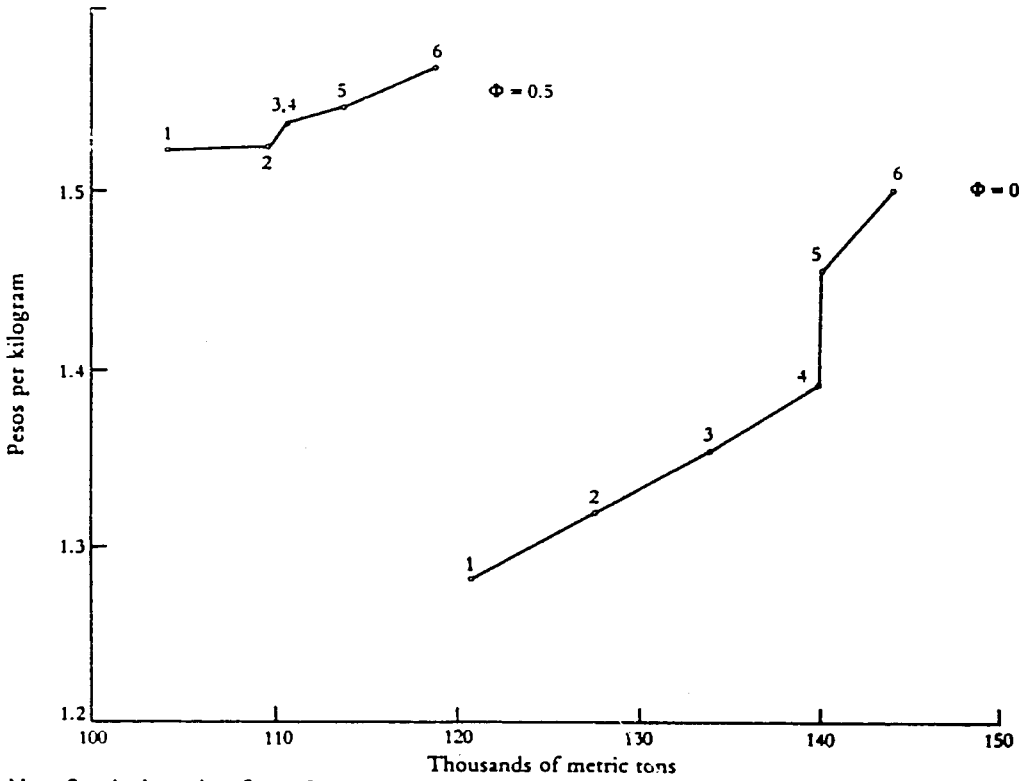
Note: See the legend to figure 8-1.

Figure 8-3. *Domestic Supply Response for Sorghum, BETA Model*



Note: See the legend to figure 8-1.

Figure 8-4. *Domestic Supply Response for Safflower, BETA Model*



Note: See the legend to figure 8-1.

The BETA model, in contrast, consistently yields more inelastic supply response functions. In this case, production increases less than proportionally with demand, causing prices to increase above their initial equilibrium values. Clearly, there is keener competition for resources from other crops in this model.

Generally, the risk parameter Φ does not have a pronounced or consistent effect on the slope of the supply response functions, but it does shift their location quite dramatically. By our earlier definition of high-risk crops, tomatoes in the ALPHA model and safflower in the BETA model fall in that category because their production in the base solutions (points labeled 1) is smaller with the selected Φ values than when $\Phi = 0$. In these cases, Φ shifts the whole supply response function up and to the left (figures 8-1 and 8-4). In contrast, the supply response function for a low-risk crop (for example, sorghum in the BETA model) is shifted down and to the right when risk-averse behavior is introduced (figure 8-3). Cereals (maize and wheat) in the ALPHA model are, for all practical purposes, risk neutral. In this case, the location of the entire supply response function is barely affected by introducing risk-averse behavior (figure 8-2), although the introduction of risk does give some slope to an otherwise perfectly elastic supply function.

Effects on resource valuation

One of the claimed benefits of constructing aggregate linear programming models is that they may provide shadow prices for scarce resources that can be useful in guiding investment decisions. This feature is particularly attractive in the Mexican context for valuing irrigation water.[11] Few models used for the purpose, however have been specified with risk-averse behavior, so that it is pertinent to explore the biases inherent in the approach when assuming risk-neutral behavior.

Tables 8-9 and 8-10 contain the shadow prices of irrigation water at district levels obtained from our models for different values of Φ. These values are taken from annual water constraints and measure the marginal annual return from an additional unit of water when it is used in an optimal seasonal pattern determined by the model.

Except for the El Fuerte-Guasave region in the BETA model, the shadow prices of water consistently decline as Φ increases. Thus, assuming risk neutrality in model specification will most likely lead to an upward bias in the marginal valuation of irrigation water. As we argued theoretically in the section "Method of Incorporating Risk," however, and as the El Fuerte-Guasave region demonstrates, such consistent results need not

11. See, for example, chapter 15.

Table 8-9. *Shadow Prices for Water with Different Values of Φ, ALPHA Model*
(pesos per thousand cubic meters)

	Values of Φ					
District	*0*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
El Fuerte	313	0	0	0	0	0
Culiacán	0	0	0	0	0	0
Río Mayo	1,516	1,248	1,123	1,033	851	845
Guasave	782	355	0	0	0	0
Ciudad Delicias	719	714	523	346	182	115
Bajo Río San Juan	842	598	172	0	0	0
Santo Domingo	2,380	1,934	1,672	1,285	943	656
La Laguna	816	620	487	478	418	352

hold. Assuming risk neutrality in the BETA model would actually lead to an underestimation of the value of water in the El Fuerte-Guasave region when compared with assuming the "best-fitting" value of Φ. This result occurs because the incorporation of risk happens to shift the optimal cropping pattern toward the more water-intensive crops.

Conclusions

This chapter has demonstrated that: (1) reasonable estimates of risk-aversion coefficients can be obtained at aggregate levels through programming techniques; (2) biases in estimates of supply response and resource valuation may be quite significant in planning models that ignore risk-averse behavior; (3) the descriptive preformance of agricultural planning models can be considerably improved by introducing risk-averse behavioral assumptions, even when such assumptions are based on the theoretically problematic (E,σ) utility function. The additional data re-

Table 8-10. *Shadow Prices for Water with Different Values of Φ, BETA Model*
(pesos per thousand cubic meters)

	Values of Φ			
Region	*0*	*0.5*	*1.0*	*1.5*
Culiacán-Humaya	732	404	0	0
El Fuerte-Guasave	523	558	196	42

quirements for the incorporation of risk are time-series observations on prices and yields, by crop, for the relevant producing regions. Unfortunately, this requirement is least likely to be met for the more backward regions. The next two chapters explore techniques that may be more applicable for these areas, although more research is required to incorporate the approaches into mathematical programming models.

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