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PRIMARY TWO

VOLUME 1

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TEACHERS' GUIDE

MATHEMATICS

Prepared
at the
Entebbe Mathematics Workshops

TCR/ORA
Official File Copy

PRELIMINARY EDITION

Educational Services Incorporated
Watertown, Massachusetts, U.S.A.

ENTEBBE MATHEMATICS SERIES

Primary I -- Revised Preliminary Edition
Pupil Book: Two volumes
Teachers' Guide: Two volumes

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Secondary II -- Preliminary Edition
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Basic Concepts of Mathematics -- Preliminary Edition
An Introductory Text for Teachers

NOTE

This book was written at a study and writing workshop held during the summer of 1963 at Entebbe, Uganda. More than fifty mathematicians and mathematics teachers from Africa, the United Kingdom, and the United States attended the Workshop.

The book was edited and produced with the utmost speed so that it might be tried out in African primary schools during 1964. Therefore, there may be errors in editing and production. The African Education Program hopes that teachers who use this book will appreciate the circumstances under which it was produced.

TEACHERS' GUIDE VOLUME ONE

MATHEMATICS PRIMARY TWO

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Entebbe Mathematics Workshops

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Educational Services Incorporated
Watertown 72, Mass., U. S. A.

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The Entebbe Mathematics Workshops were directed by Professor Walter Prenowitz of Brooklyn College, Brooklyn, N. Y. and by Professor W. T. Martin of Massachusetts Institute of Technology, Cambridge, Massachusetts.

The books for teachers and pupils of Primary Two Mathematics were written at Entebbe by the following:

Chairman: Professor Clarence Hardgrove (1962, 1963)
Northern Illinois University
De Kalb, Illinois, U. S. A.

Editors: Professor Clarence Hardgrove

Professor Vincent Haag (1963)
Franklin and Marshall College
Lancaster, Pennsylvania, U. S. A.

Professor Shirley Hill (1962, 1963)
University of Missouri
Kansas City, Missouri, U. S. A.

Miss Lucy Addy (1962, 1963)
Achimota School
Achimota, Ghana

Mr. Baranaba Avinyia (1963)
Arua Teacher Training College
Arua, Uganda

Mr. Muhamed Bashraheil (1963)
Bububu School
Zanzibar

Mr. D. A. Fakoya (1963)
Federal Ministry of Education
Lagos, Nigeria

Mrs. Margaret Greene (1963)
Annie Walsh Memorial School
Freetown, Sierra Leone

Miss Helene Junius (1963)
Monrovia Demonstration School
Monrovia, Liberia

Mr. Frank Kazembe (1963)
Ministry of Education
Blantyre, Nyasaland

Mr. Paul Merrick (1963)
Elementary Science Study
Educational Services Incorporated
Watertown, Massachusetts, U. S. A.

Mr. Morton H. Mughogho (1963)
Ministry of Education
Lilongwe, Nyasaland

Mr. Raphael Mwajombe (1963)
Peramiho Teacher Training College
Peramiho, Tanganyika

Mr. George T. Obodai (1963)
King Tackie Memorial Primary School
Accra, Ghana

Mr. Alfred C. Oblitey (1963)
Institute of Education, University of Ghana
Accra, Ghana

Professor B. J. Pettis (1963)
University of North Carolina
Chapel Hill, North Carolina, U. S. A.

Mr. E. L. Thomas (1963)
Charles Lwanga Teacher Training College
Chisekesi, Northern Rhodesia

Dr. Onyerisara Ukeje (1963)
University of Nigeria
Nsukka, Nigeria

Mr. Robert Walsh (1963)
UNESCO, Institute of Education, Makerere College
Kampala, Uganda

The preparation of these books for publication was coordinated at the offices of Educational Services Incorporated by Stanley D. Weinstein. Editorial services were provided by Edward Webster under the direction of Professors Hardgrove, Haag, and Hill. Typing and layout were supervised by Barbara Morse and Sharon E. McGreevey. The cover was designed by Nat Bellantone and Edward Webster. Other art work was prepared at Entebbe by Mrs. Joan Fleet and in the United States by Kay Hildebrand, Margaret Starr O'Leary, and Alice C. Welch. Production was supervised by Edward Webster and Benjamin T. Richards.

PREFACE

The purpose of the Entebbe Mathematics programme for Primary Two is:

To help children learn the best modern mathematics they can learn.

To help them discover the ideas of mathematics for themselves.

We want children to learn good mathematics and to think about mathematics. These two purposes are not attained separately but at the same time.

This book was written to help you help your pupils learn and think about mathematics. The book will help you in these ways:

It tells you about the ideas of mathematics that you are teaching; it refers you to sections in your Mathematics Handbook where you can also read about the ideas.

It suggests things you can use in your classroom to help you teach.

It tells you how to teach each of the units.

It encourages you to let the children think for themselves. (Ask the question, "Why" often and let the children ask questions.)

Testing the Programme

This book and the Pupil Book were written to be tested by you and by your pupils. You and other teachers who use the books will be asked to write your opinions of the units you teach. The books will then be revised with the help of your comments. You are asked to do these things:

Teach the material of a unit as best you can, in the way suggested.

Think about the teaching and learning which you and your pupils have done.

Fill out a Unit Evaluation Form for each unit. (See the form at the end of this section.) Give the form to your supervisor.

How to Use the Book

The first thing you do before teaching from Entebbe Mathematics, Primary Two is to read all the books carefully (this book, the Pupil Book and your Teachers'

Handbook). They will help you

Know the whole programme for the year.

Know what things you need to help you teach.

Know what parts of the programme you wish to discuss with other teachers and supervisors.

Know what parts of the programme you want to read about further in your Teachers' Handbook.

Know how this book and the Pupil Book fit together (Did you find the boxes in this book which tell you the pupil page to use? Did you find the unit number of each pupil page?)

Know how this book, the Pupil Book, and your Teachers' Handbook are planned to help you.

The second thing you do is to study the unit you are to teach and make plans for teaching the unit. This study helps you

Plan in detail your teaching from day to day.

Know what things to collect to help you teach the unit.

Know what further reading and study you and your pupils should do to teach and learn the ideas in the unit.

The third thing you do is teach the unit. Keep reading this book and your Teachers' Handbook. They will help you teach. Remember you teach best when children think things out for themselves. You question and guide. Tell as little as possible.

The fourth thing you do is to think about the unit and its use by you and the children. Think about how the unit could be improved for you and the pupils.

The fifth thing you do is to thoughtfully fill out a Unit Evaluation Form.

The Plan of the Book

This book explains in detail how to teach the thirteen units of the Primary Two programme. A unit is separated into "stages". Each of the stages is organized into several "activities". These activities are the suggestions for teaching.

An activity may be a single lesson; two or three activities may become a single lesson; or, you may continue or revise one activity several days. You must decide about how long your pupils need to spend with an activity. Study your children. Listen to them and watch them and decide when they can move to a new activity or stage.

Each unit uses ideas learned in units which come before it. For that reason, it is important that no mathematics be omitted. Some children may need only some of the activities in the book to help them learn an idea. If so, other activities may be omitted.

Helping Children Learn

Children learn best from things they do and by thinking about what they do. You can help by finding worthwhile things for them to do and to think about and by not telling them what to think.

This method means that you too must be doing and thinking as you help the pupils. These are some of the ways you can help children think about and discover ideas:

Choose good problems and ask pupils to find answers.
Ask questions as pupils work.
Make suggestions as needed.
Suggest the use of things as models for the problem.
Supply a word or words now and then, when a pupil has a good idea he cannot express.
Suggest different methods of work.
Give encouragement at all times.
Help pupils summarize and say what their answers mean.

Outcomes

As you help pupils in mathematics to find their own answers, to think for themselves, you are teaching them mathematics and you are teaching them a way to think about mathematics. You are also helping them to learn a way to solve any problem.

This searching attitude leads to satisfaction and enjoyment for you and your pupils. It is the hope of the writers of the programme that it is not only enjoyed but that much mathematics is learned. Good luck.

PRIMARY I and II UNIT REPORT

1963 - 64

Date: _____

Number of Unit: _____

Name of Unit: _____

Fill out one of these reports for each unit as you complete it. Send it by air mail to Educational Services Incorporated, 108 Water Street, Watertown, Massachusetts, 02172 U.S.A. Your comments are very much needed. They will be used for the revision of the books.

PLEASE ANSWER ALL THE QUESTIONS IN THIS REPORT

Your Name: _____

Name and Address of Your School _____

_____

Date you started teaching the Unit: _____

Date you completed the Unit: _____

Number of periods used in teaching the Unit: _____

Length of daily lesson period: _____minutes.

TEACHERS' GUIDE

Answer each of the following questions and give reasons for your answers.

Questions	Answers and Reasons for Answers
1. Were the teaching suggestions clear?	
2. Were the teaching suggestions sufficient?	
3. Were the instructions in the Guide helpful in developing the ideas?	
4. Were the suggestions for materials to be used in the classroom sufficient?	
5. Was the background information for teachers in this unit clear, helpful and sufficient?	

What suggestions can you give for the improvement of the Teachers' Guide for this Unit?

_____

_____

_____

_____

_____

_____

PUPIL BOOK

Answer each of the following questions and give reasons for your answers.

Question	Answers and Reasons for Answers
1. Did your pupils learn the ideas the unit is meant to develop?	
2. Were the pupil pages helpful in developing the ideas of the unit?	
3. Were the pupil pages for this unit sufficient?	
4. Did you use materials other than those suggested in the book?	
5. Were the illustrations in the pupil book suitable?	
6. Did the pupils enjoy this unit?	
7. Did you enjoy teaching this unit?	

What suggestions can you give for the improvement of this Unit? _____

_____

_____

_____

GENERAL REMARKS and ASSESSMENT of

THE UNIT

To the best of my knowledge, this Unit is _____(excellent, good, fair, poor) because:

(1) _____

_____

_____

(2) _____

_____

_____

(3) _____

_____

_____

(4) _____

_____

_____

(5) _____

_____

_____

(6) _____

_____

_____

_____
Signature

_____
Date

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UNIT 1

NUMBER I

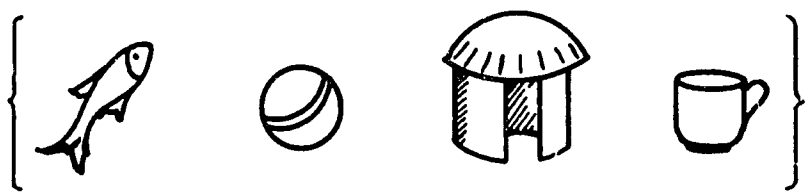
Objectives:

1. To revise the idea of sets and of number as a property of sets.
2. To use matching to compare and order sets.
3. To revise number names and numerals.
4. To revise the idea of place value with numerals to 100.
5. To develop the idea of ordering numbers up to one hundred, using a number line.
6. To revise and expand the idea of ordinal numbers.

Background Information for Teachers:

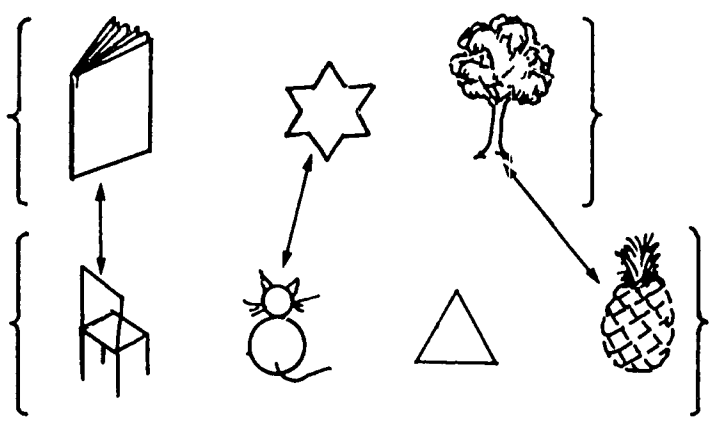
See also Part 2, "Arithmetic", in your Teachers Handbook.

This is a picture of a set:



The word "set" is the special name used for any collection of things. Each thing in a set is called a member of that set. The members of the set pictured above are a fish, a ball, a house, and a cup.

Sets are compared by a process called matching. The members of two sets below, which we will call the second and third sets, can be matched as follows:



The book in the second set is matched with the chair in the third set, the star with the cat, and the tree with the pineapple. There is no member left in the second set to match with the triangle. You can see from this matching that the second set has fewer members than the third set and the third set has more members than the second set. When the third set is compared with the first set (the set of a fish, a ball, a house, and a cup) by matching their members you see that one of these sets has exactly as many members as the other. Sets that have exactly as many members as each other are called equivalent sets.

Equivalent sets have something in common. They have the same number of members. For example, number three is used to describe the set of a book, a star, and a tree, and to describe any set that is equivalent to it. Each of these equivalent sets has three members. The symbol 3 is a numeral for the number three.

A set that has no members is called the empty set. The empty set has zero members. A numeral for the number zero is 0. The set of men with three hands is an example of the empty set. It has zero members.

Set	Number	Numeral
{ }	zero	0
	one	1
	two	2
	three	3
and so on	and so on	and so on

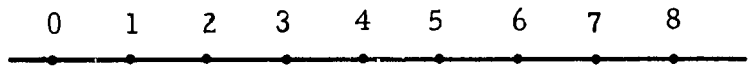
For each set there is only one number, but one number describes many equivalent sets.

When sets are arranged as above, they are said to be ordered. That is, sets are arranged in their natural order when each set has one more member than the set before. When sets are arranged in their natural order, the numbers of the sets are also ordered: 0, 1, 2, 3, and so on.

When a set has more than nine members, the members can be grouped in tens. Then we can write a decimal numeral for the number of the set.

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are called digits. To write the decimal numeral for a number greater than nine, two or more digits must be used. For example, the decimal numeral 25 refers to 2 groups of ten and 5 groups of one. In a decimal numeral for a whole number the digit on the right shows the number of groups of one, the next digit shows the number of groups of ten, the third digit the number of groups of one hundred members in the set, and so on. In the decimal numeral 236 the 2 shows that there are two groups of hundreds. The 3 shows three groups of tens. The 6 shows six ones. This way of writing numerals uses the place values of the digits. The place of the digit in the numeral helps to tell us the meaning of the digit.

When numbers are matched to points on a line segment, the numbers can be compared.



Make a number line by marking any point near the left end of a line segment and label it 0. Mark a point to the right of the 0 point and label it 1. Continue to mark points to the right with the same space between them. Label these points with numerals for whole numbers in their natural order. It is not important what space is chosen between points. It is important that the spaces between points are the same. When any point for a number is chosen on a number line, all the points to the left refer to numbers that are less than that number, while those to the right refer to greater numbers. For example, $2 < 5$ and $7 > 5$. That is, 2 is less than 5, and 7 is greater than 5. On the line, the point for 2 is to the left of the point for 5, and the point for 7 is to the right of the point for 5.

Stage 1: Assigning Numbers to Sets

Vocabulary: set, member, empty, match; zero, one, two, ... ten.

Materials: Stones, seeds, sticks, pencils, bottle tops, or any objects that are easy to get and to use to make sets; 5" x 7" numeral cards with 0, 1, ... 9, written on one side and the corresponding word, zero, one, ... nine, on the back; inch-square numeral (0 - 9) cards, (one set of 10 cards prepared by each child); 10 boxes or tins large enough to hold a set of nine objects; pupil pages 1 - 4.

Teaching Procedure:

Note: The first few activities are for revision. You should use only the activities needed by the children. These activities should move quickly.

Activity 1: Naming the number of a set (0 - 5).

Place six boxes and many objects on the floor so all the children can see them. Use the objects to make sets.

Place each set in a different box. Make sets with one member, two members, three, four, and five members, and a set that has no members. Ask children to come up in turn and point to the set with four members, the set with one member, and so on. Let the children look and say without counting how many objects are in the set. Quickly make still different sets. Let as many children as possible give number names to the sets in the boxes.

Show the children the set of large numeral (0 - 5) cards. Hold up the 4-card. Let the children say, "four". Show them the other side of the card and let them read the word "four". Let them find a set with four members and place the card beside it. Go on in this way with the other numeral cards.

Activity 2: Reproducing sets (0 - 5) of a given number.

Give each child 5 or more small objects. Hold up a numeral card and ask each child to make a set on his desk with the number of members the card says. Go on with the numeral (0 - 5) cards. Vary this activity by showing the numeral on the card in some cases and the word in others.

Let the children work in pairs. Ask each child to make as many different sets as he can on his desk. Let a child in turn ask his partner to point to the set he names. For example, when Kofi calls out "five", Kojo points to his set of five members. Give each child six inch-square pieces of cardboard or paper and let him make a set of numeral (0 - 5) cards. Ask each child to place the correct numeral card on each of the sets that his partner has made.

Activity 3: Naming the number of a set. Pupil Pages 1 to 2

Ask the children to find page 1 in their books. Let them describe the sets. Ask them to point to a set which has the number you call out. For example, when you call out "four" they point to the set of cups. Go on in this way until the children give number names to all the sets. Remember that the idea to be learned is that every set has a number. It will not matter if a child calls the set of umbrellas a set of bushes. Let him call them bushes. Threeness is the idea to be learned.

Get the children to place their inch-square numeral cards on the sets. For example, the 1-card goes on the set with an animal as its member. Go around the room to check that children have matched numeral cards with the right sets.

Ask children to find page 2 in their books. Let them match numeral cards with the sets. Ask the children to place the cards beside the sets.

Activity 4: Making sets of 6 - 9 members and naming the number of the sets.

Put a collection of objects on your table or on the floor. Call the children in turn to make sets of six objects, seven objects, eight, and nine objects. Hold up a 5" x 7" numeral (6 - 9) card and ask a child to come up and point to the set with the number of members shown by the card. Let him read the numeral. Go on until all the sets have been numbered. Let the children in turn choose a numeral card, and place it on the set having the number of members shown by that card. Go on until all the cards have been used. Repeat this activity using the sides of the cards showing number words.

Let the children find other sets of things in the classroom or in pictures on the walls. Let them tell how many members each set has and choose a numeral card which

which shows that number. Use both sides of the cards. For example, let some choose the 7-card and others the seven-card.

Place five boxes or tins on the floor. Put a numeral card (use either side) by each tin or box. Let a child choose a box and put in it the number of objects named by the numeral. Continue until all the boxes or tins have sets in them. Repeat the activity with other children.

Activity 5: Naming the number of a set (0 - 9). Pupil Pages 3 – 4

Give each child five inch-square pieces of cardboard. Let him make numeral (6 - 9) cards to be used with the cards he made in Activity 3. Ask the children to open their books at page 3 and let them talk about the sets they see. For example, "This set of books has eight members." Ask them to point to the set with five members. Check that they are all pointing to the set of boxes. Go on with this activity until all the sets are given numbers.

Ask the children to choose the correct numeral cards and place them on the sets pictured on page 3. Go around the room to check that all children have matched numerals and sets correctly.

Tell the children to turn to page 4. Ask them to match their numeral cards with the sets on this page. Let children work in pairs to check each other's work.

Stage 2: Matching Sets

Vocabulary: Exactly as many as, fewer than, more than, less than, most, fewest, natural order,

Materials: Same as for Stage I; pupil pages 5 - 7

Teaching Procedure:

Activity 1: Comparing sets of objects by matching their members.

Let six children stand in front of the class. Ask, "How many members are there in this set of children?" Give a child a set of six pencils and let him give one to each of the six children. Ask, "Are there more pencils than children? Are there

fewer pencils than children? Are there exactly as many pencils as children?" Collect the pencils. Give a child a set of five pencils and ask him to give one to each member of the set of six children. There will be one child left with no pencil. Ask the three questions asked before. Go on with this activity. Use different sets of children and pencils.

Give a book to each child in the class. Ask questions like those just asked. Get the children to say there are exactly as many members in the set of books as there are in the set of children.

Make two sets of objects on the floor. Ask a child to match the members of one set with the members of the other set. Let him match the members by pointing to a member of one set with his right hand while he points to a member of the other set with his left hand. Ask the same three questions. Make many other pairs of sets, and let the children match them.

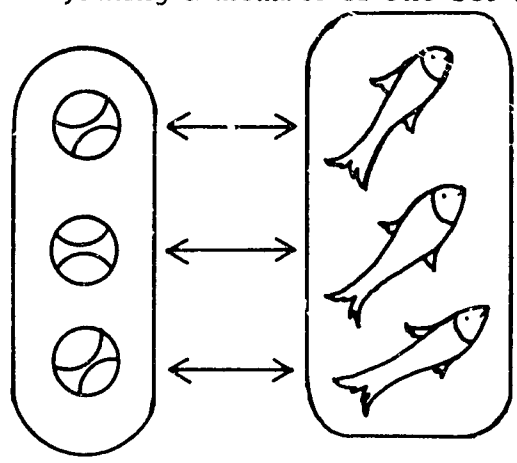
Ask each child to make a set of objects on his desk. Ask the children to use their sets to make two subsets. Let them match the members of one subset with the members of the other subset. Let the children work in pairs. Get them to ask each other the same three questions about the subsets.

Ask each child to form a set and match it with that of his partner. Go on until the children can match sets and say when the members of two sets are matched exactly, "This set has exactly as many members as that set."

Let pairs of children play this game. Give each child a set of objects on his desk. Let one child arrange a set and show it to his partner. The second child looks at the set and arranges a set of his own which he keeps hidden. The first child now guesses whether his partner's set has more members than his own, fewer members than his own, or exactly as many members. Both children can match the members of the sets and say whether the guess is correct. Let them take turns arranging the first set and guessing. The children may keep the score. They get one point for every correct guess.

Activity 2: Comparing sets by matching their members.

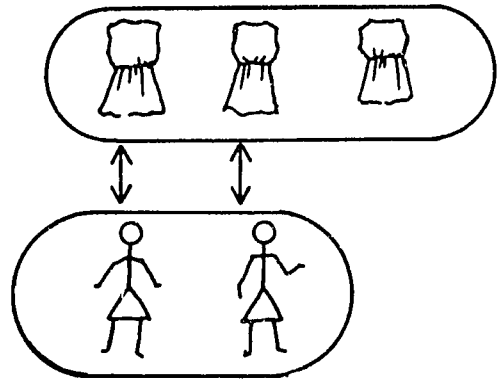
Draw on the blackboard two sets which have the same number of members. Ask a child to come up and match the members of one set to the members of the other set. Show him how to draw a line joining a member of one set with a member of the other set. For example:



Point to one set and ask, "Has this set more members than the other set? Has it fewer members? Has it exactly as many members?"

Draw two other sets with the same number of members and ask another child to match them. Ask the questions again. Make many pairs of sets which children can match exactly. Get the children to say, for example, "The set of balls has exactly as many members as the set of fish."

When children have matched many pairs of sets with exactly the same number of members, draw on the blackboard pairs of sets which do not have the same number of members. For example



Ask the children to match the members of the two sets. They will say that some members are not matched. Ask questions as before, "Does the set of dresses have fewer members than the set of girls?" Let the children draw two sets on their papers and match the members. Go around the class and let each child tell how his sets compare.

Activity 3: Comparing sets by matching their members. Pupil Pages 5 - 6

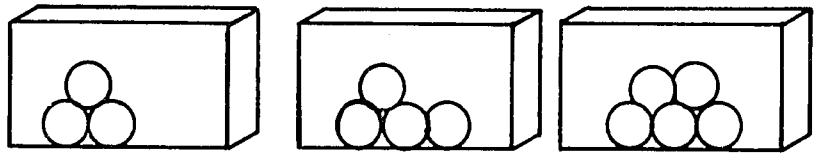
Ask the children to find page 5 of their books. Let them talk about the sets pictured there. Ask a child to point to two sets that have the same number of members, for example, the set of bags and the set of a ball and a boy. Ask the children to match the members in these two sets. Ask them to point to a member of one set with a finger of the right hand and to a member of the other set with a finger of the left hand. Let the children match as many pairs of sets as possible. Let them find the sets which have more members or fewer members than another set. Let them compare the two sets by saying, "The set of birds has more members than the set of baskets." "The set of baskets has fewer members than the set of birds ." "The set of birds has exactly as many members as the set of scissors."

Continue this activity on page 6.

Activity 4: Ordering sets of objects with 0 - 9 members.

Place 10 boxes or tins in front of the class. In the boxes put sets of objects with 0 - 9 members. Put the box with the set of five members on the floor so all can see. Ask a child to find the set with one fewer member. Tell him to place this set beside the set with five members. (Let us suppose he puts it to the left of the first box as the class looks at it).

Ask another child to find a set with one member fewer than the last set and place it to the left of the last set. The boxes will now be placed this way.



Repeat this activity until the empty box has been placed in the row. Ask a child to find a set with one member fewer than the empty set. Let the children talk about this and say there is no set like that.

Ask a child to find a set with one more member than the set on the right. Ask him to place it in the row to the right of the other sets. Go on in this way until all the sets have been arranged in order, or all the sets are ordered. (Note: Sets may also be ordered so that any set has more members than any of the sets to its right. However, the usual way of ordering sets or numbers is as shown above.)

Put your hand on one box, for example, the box containing seven objects. At the same time point with your other hand to another set (for example, of six members). Ask, "Has that set (of six members) more members or fewer members than this set (of seven members)?" (It has fewer members.)

Keep your hand on the set of seven members and point to the set of five members. Ask, "Has that set (of five members) more members or fewer members than this set?"

Keep your hand on the set of seven members and point to other sets in turn. Repeat these questions. Get the children to say, "The sets to the left of the set of seven members have fewer than seven members. The sets on the right of the set of seven members have more than seven members."

Now put your hand on the set with four members. Ask the children to come up in turn and point to sets that have fewer members and sets that have more members. Say, "Point to a set that has more members than this one."

Go on with this kind of questioning until children can say that when sets are ordered like this and you choose two sets. The set on the right has more members and the set on the left has fewer members.

Activity 5: Ordering numbers and comparing numbers.

Use the same sets of objects that were used in Activity 4. Move the boxes so that the sets are not ordered. Give each of the numeral (0 - 9) cards to a child. Ask each child with a card to come in turn and place his numeral card by the box which contains the number of objects shown on this card. Repeat this activity as needed.

Get the children to order the sets from left to right again. Let them match the numeral cards to the boxes.

Let the children read the numerals from the cards by the boxes in order. Get them to begin with zero. Ask whether the numerals are ordered. Point to a box and its numeral card, for example, 9. Ask, "What is the number of this set?" (9).

Say, "What number is one less than this number?"

"What number is one greater than this number?"

"Is there another number which is less than this number?"

"Is there another number which is greater than this number?"

Repeat this activity. Begin with other boxes. Let the children say that the numerals are ordered and that the number of each of the sets to the left of a set is less than the number of that set. Tell the class that when numeral cards are arranged like this, we say that the numbers they show are in natural order.

Activity 6: Ordering sets and their numbers.

Ask each child to draw a set of objects with 0 - 9 members on his paper. Ask each child to write on the other side of the paper the numeral which matches the set.

Choose any child to come up and show his set to the class. Ask, "Who has a set with more members? Who has a set with fewer members? Who has a set with exactly as many members?"

Let another child come up and show his set to the class. Ask, "Has David's set more members than John's? Has David's set fewer members than John's? Has David's set exactly as many members as John's set?"

If David's set has more members than John's, let him stand to the left of John; if fewer, let him stand to the right of John. (This order is a right to left ordering. Children should know the numerals can be ordered many ways.) If David's set has exactly as many members as John's, let David stand behind John.

Ask a third child to show his set to the class. Let the class compare this set with those of John and David.

Ask, "Will Mark stand behind John or David? Will he stand to the right of John? Will he stand to the right of David? Will he stand between John and David?" Always ask the children, "Why?" Help the children to understand that if Mark's set has fewer members than David's but more than John's, Mark must stand between the other boys.

Ask, "Who has the set with the <u>most</u> members? Who has the set with the <u>fewest</u> members?"

Move around the class. Choose children with sets that have different numbers and ask them to show their sets to the class. Let the children compare each new set with those of the children standing in front to the class. Ask where each child should stand in the order. Ask <u>why</u>, for example, "Why must Andrew stand between Mary and John?" (Because Andrew's set has more members than Mary's and fewer members than John's.)

Go on with this activity until at least one set with each of the numbers from 0 - 9 has been used. When no child has a set with one particular number of members, ask the remaining children to change the sets which they have drawn. Choose a child to fill in the line.

Ask all the children standing to show the numerals written on the other side of their papers. Ask, "Are the numerals in their natural order?" Let a child read the numerals. Let him begin with "0" on the right.

Vary this activity by separating the class into groups of at least ten children. Ask each child to draw sets of from 0 to 9 objects on his paper. Ask him to write the numeral which matches the set on the other side of the paper. Let each group in turn stand in a row in any order in front of the class. Ask the group to show their sets. Let the rest of the class say where the children must stand to order the sets.

When the sets have been ordered ask the group to turn their papers over. Let the class tell whether the numerals are in natural order. Go on until each group shows its sets.

Activity 7: Matching numerals to sets with 0 - 9 members. Pupil Page 7

Let each child use his set of ten small numeral (0 - 9) cards. Ask the children to open their books at page 7. Let them tell about the sets they see. Ask them to point to the set with the fewest members. Ask them the number of this set (zero). Ask them to find the numeral card for the set and place it on the set. Ask the children to find the set that has one more member than the empty set, to point to it, and to place on it the numeral card for it. Go on in this way until all the numeral cards have been used. Ask the children to point to the set with the most members.

Stage 3: Numbers and Numerals 1 - 100.

Vocabulary: Number names to one hundred.

Materials: A large supply of counting sticks (about 20 to 30 per child); two sets of 5" x 7" numeral (0 - 9) cards; two number trays with two parts (for TENS and ONES); rubber bands or string for tying sticks into bundles; two charts of one hundred dots each (see Activity 6); pupil pages 8 - 9.

Teaching Procedure:

Activity 1: Separating sets of sticks into subsets of ten members.

Give each child a set of counting sticks and pieces of string or rubber bands to tie the sticks into bundles. Let each child have from twenty to thirty sticks. Do not give each of the children the same number of sticks.

Ask each child to hold up a set of ten sticks. Let them tie these into a bundle. Get the children to tie as many of their sticks as possible into bundles of ten sticks.

Ask one child, for example, John, to come up and hold up his sticks. Let him hold the bundles of ten in his right hand and the separate sticks in his left hand. Get him to tell the class how many sticks he has. (I have two bundles of ten and seven ones.)

Ask another child, for example, Mary, to come up and hold up her sticks in the same way. Let her tell how many sticks she has. (I have three bundles of ten and two ones.) Let the two children stand side by side. Ask, "Has Mary more

sticks than John? Has Mary fewer sticks than John? Who have more sticks than Mary? Who have fewer sticks than John? Who have exactly as many sticks as Mary? Who have exactly as many as John? Who have fewer sticks than Mary, but more sticks than John?"

Ask two other children to hold up their sticks and repeat the activity.

Let the children keep their sticks for other activities.

Activity 2: Ordering sets by the number of their members.

Let the children use the sticks that they had in Activity 1. Ask ten children to come out and show their sticks to the class. Let them hold bundles of tens in their right hands and separate sticks in their left hands. Ask each child to say the number of his set of sticks. Let the rest of the class say how these children must stand so that the numbers of their sets are ordered. Let the child whose set has the fewest members be to the left as the class sees them. When two children have sets with the same number of members, let one child stand behind the other.

Ask each child in the group to say again how many sticks he has. Let the child with the fewest sticks start. For example:

First child: "I have two tens and three ones."

Second child: "I have two tens and five ones." and so on.

Choose ten other children to come up. Let each child show his set of sticks, say how many he has, and stand in the correct order in a row. Repeat the activity as needed. (Let the children keep their sticks.)

Activity 3: Using numerals for tens and ones (1 - 30).

Let the children put out their sticks. Ask one child to come up and show his set of sticks. Ask, "How many bundles of ten has John?" Take the bundles of ten sticks from his right hand and put in their place the numeral card showing that number of bundles. Say, "This card tells how many bundles of sticks John had." Ask, "How many single sticks has John?" Take the single sticks from his left hand and put in their place the numeral card showing that number of sticks. Say, "This card

tells how many single sticks John had." Ask the class to read the number of sticks that John had altogether from the numeral cards in his hands. (For example, "Two tens and three ones.")

Ask another child to come up and show his sticks. Let a second child replace the sticks in his hands by numeral cards showing the correct number of tens and ones. Repeat this with other children.

Vary by letting the children show part of their sticks so they use all of the numbers 1 - 30.

Activity 4: Numerals for numbers 1 - 100.

Lay out two sets of 5" x 7" numeral (0 - 9) cards on your table so that any card may be easily found by a child. Call a boy to stand in front of the class. Say, as you put one bundle of sticks in his right hand and zero sticks in his left hand, "I put some bundles of ten sticks in his right hand and some single sticks in his left hand." Ask, "How many bundles of ten does he have?" (1) "How many single sticks does he have?" (0) Let a girl hold up the numeral cards to show this number of sticks. She held a "1" in her right hand, and a "0" in her left hand. Ask the children to look at the numeral and say the number of sticks. (One ten and zero ones.)

Give the boy another stick to hold in his left hand. Ask other children to change the cards in the girl's hands to show the number of sticks the boy has. She now holds a 1-card in each hand. Let the class read the number of sticks shown by the cards as "one ten and one" and also as "eleven". Continue adding sticks to the set held by the boy. Let the children keep changing the cards of the girl to show the number of members in each new set. Let the class read each new numeral in the two possible ways. After the number 19 has been reached, give the boy another single stick. Let the children try to give him the proper numeral cards. Ask the class what must be done so they can use numeral cards. Let the boy with the sticks tie the ten sticks which he now holds in his left hand into a bundle and put it in his right hand. Ask the girl to show the correct numeral cards. (A 2-card in her right

hand and a 1-card in her left hand.) Let the class say if "20" is right. Let them read "20" as both "two tens and zero ones", and "twenty".

Let another pair of children come up and go on with the activity until 40 is reached.

Revise the number names of the multiples of ten in this way:

Choose a pair of children. Give one child two bundles of ten sticks. Ask, "How many sticks does he have?" Let the other child show this number with numeral cards. Let the class read the numeral as "two tens and zero ones" and "twenty". Go on adding bundles of tens to the first child's set, letting the children change the numeral cards held by the second child, and asking the children to read the numerals in two ways.

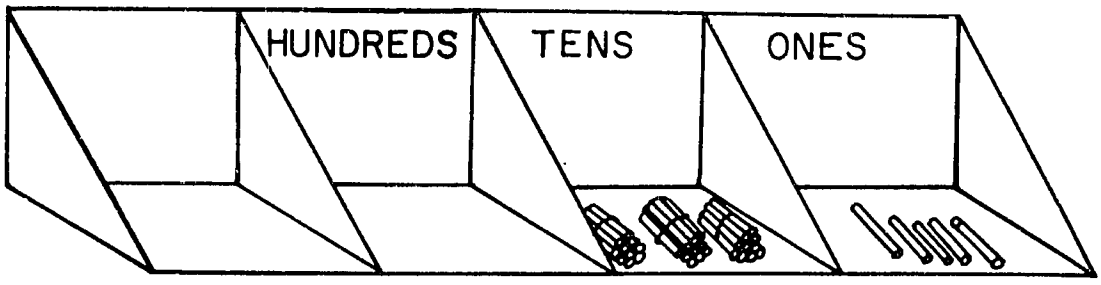
Go on until the first child is holding ten bundles of ten. Let the children try to give the second child the right numeral cards. They cannot. There is no single digit numeral for ten. Let the child with the ten bundles of ten sticks tie them into one large bundle. You take the bundle and say, "We say this is a set of a <u>hundred</u> sticks. Hundred is a name for ten tens." Say, "We used two hands to show tens and over. How many hands do we need to show hundreds, tens and over?" (3). Ask a third child to come up and stand to the left of the boy with the other sticks. Say, "Our new boy will use one hand to hold the bundle of a hundred sticks." Give him the bundle. Ask, "Do we need a third hand to hold the numeral cards?" (Yes). Let a child come up and stand in the proper place.

Let the children look at the sticks in the three hands, left to right, and tell what numeral cards go in the other three hands, left to right. (1, 0, 0). Ask how many sticks the boys have. (One hundred, zero tens and zero ones.) Ask the children to read the numeral made by the cards. (One hundred, zero tens and zero ones or one hundred).

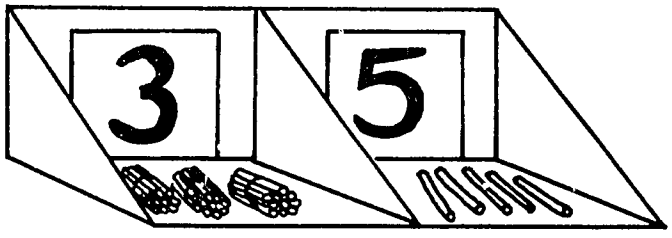
Repeat the activity as needed. Vary the activity by starting with "1 ten and 5 ones" sticks and adding bundles of tens until the numeral 105 is made.

Activity 5: Using a number tray to select numerals for numbers 0 - 100.

Place at least ten bundles of ten and ten separate sticks on your table and the number trays. The number trays are easily made by removing one side from a shoe box so that parts of three sides and the bottom are left. Use some of the cardboard from the side that was removed to make a division that separates the boxes into two parts. Use the boxes alone to show 2-place numerals and together to show 3-place numerals. Write "HUNDREDS", "TENS", and "ONES" on cards to stand in the boxes as shown below.



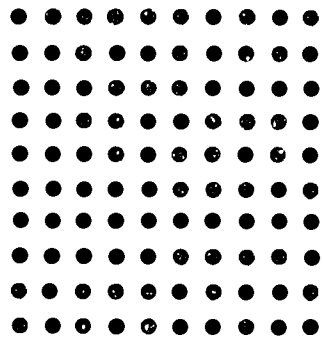
Put out one of the trays with the "TENS" and "ONES" cards in it as shown above. Put some bundles of tens and some single sticks in the tray; tens in the left side as the children see it, ones in the right side. Let the class count the number of bundles of tens and of ones as you place them in the tray, one stick at a time. Ask a child to stand the correct numeral card in each side of the tray to show how many there are in each place. See below. Let the class watch and tell whether these numerals are the right ones.



Let a child change the number of sticks in the tray and tell the class how many tens and ones he has in the tray. Let him ask another child to come up and choose the correct numeral cards to put in the tray. Repeat this activity. Ask the class to read the cards each time. If more than 9 tens and 9 sticks are used, put the second number tray by the first number tray to show a 3-place numeral. Go on as needed.

Activity 6: Describing a set in tens and hundreds.

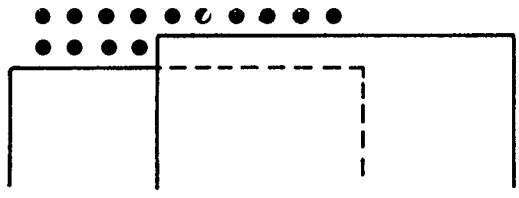
Draw two square patterns of one hundred dots on large sheets of cardboard or brown paper. Have one of these on the wall in front of the class.



Ask, "How many dots are there in one row? How many rows of dots are there? How many dots are there altogether?" (Ten tens , or one hundred).

With a sheet of paper or cardboard hide part of the pattern. (See drawing below.) Leave one row of ten dots uncovered. Ask, "How many dots can you see?" (Ten) Move the paper down so that two rows are uncovered and ask, "How many rows of dots can you see?" (Two) "How many dots can you see?" (Two tens , or twenty.) Keep moving the paper down and asking similar questions until the class has counted to 100 by tens.

Use two sheets of paper to cover part of the pattern, for example, to show 14 dots, as follows:



Ask, "How many dots can you see?" (One ten and four ones, or fourteen). Move the two sheets of paper to show another pattern of dots and repeat the question.

Hang the other pattern of one hundred dots beside the first. Ask two children to come out and have each show a number of dots on these two patterns. Let each child use two sheets of paper as you have done. Ask the other children to compare the two sets of dots. Choose your questions so that the class will tell you, "John is showing more dots than Mary." "Mary is showing fewer dots than John." "John is showing fifty-seven dots." "Mary is showing forty-three dots." "Fifty-seven is greater than forty-three." "Forty-three is less than fifty-seven." Let other children come out to repeat this activity.

(Note: You will think of many ways to use these patterns of one hundred dots with other units during the year.)

Activity 7: Writing numerals from 1 - 100. Pupil Pages 8 - 9

Ask the children to turn to page 8 in their books. Copy the first exercise on the blackboard. Ask a child to write the missing numeral, for example

		TENS	ONES
1 tens and 5 ones	=	1	5

Let the children copy the exercises into their exercise books and write in the missing numerals. Go around the class, giving help as needed. Some children may need to show the numbers with sets of sticks before writing the numerals. When the exercises are completed, let the children read their answers, for example, "Four tens and zero ones equal forty."

Follow the same plan for the use of page 9.

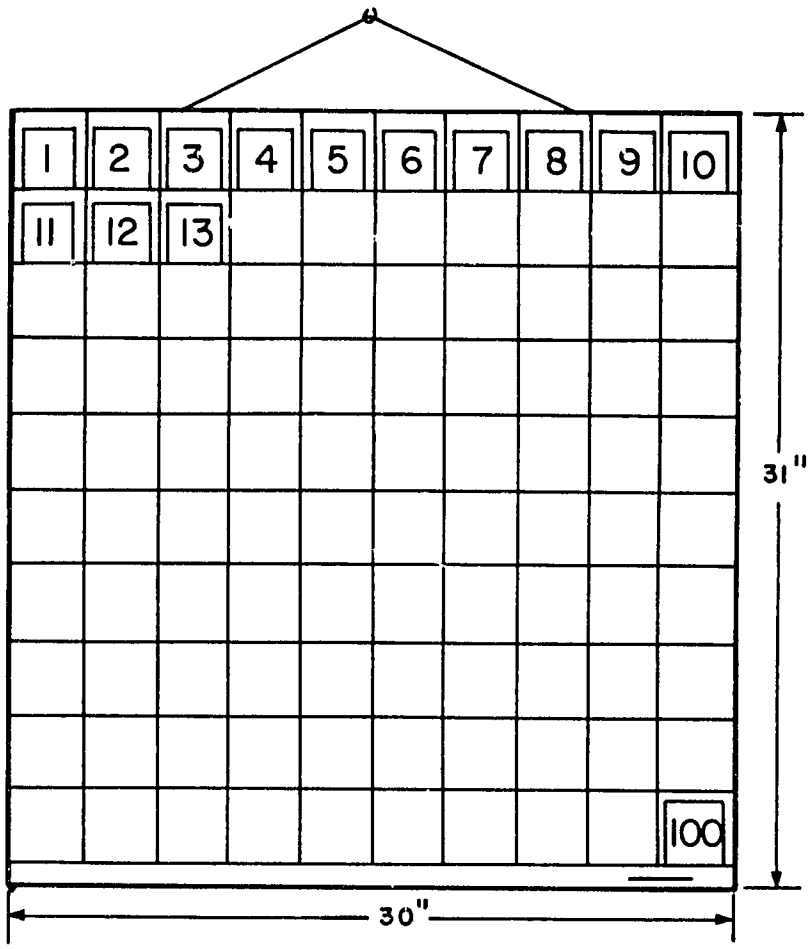
Stage 4: Ordering Numbers

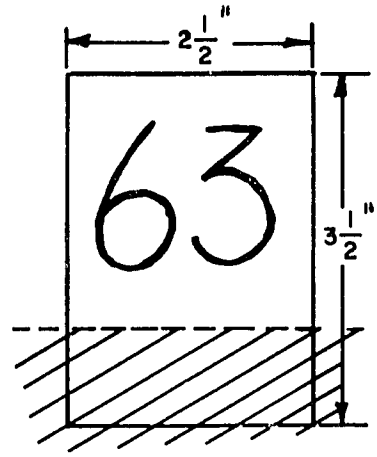
Vocabulary: Number line

Materials: calendar, hundred-pocket chart described below; pupil page 10.

Directions for making a hundred-pocket chart: The chart is made of a large sheet of heavy paper folded to form ten long pockets. Cards bearing the numerals 1 - 100 are placed in these pockets as shown in the diagram below. The numeral

cards measure 2 1/2" x 3 1/2". The numerals are written on the top 2 1/2" x 2 1/2" regions of the cards. The other regions of the cards are hidden when they are placed in the chart.

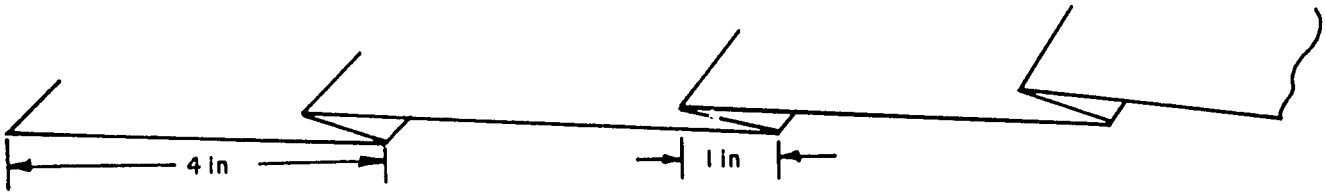




To make the chart, fold a 52" x 32" sheet of heavy paper in the following way:

(1) Draw lines across the sheet of paper to help you fold it in the correct places. (The lines should be drawn 4" from the top, then 5" from this line, and so on.)

(2) Fold on the lines.

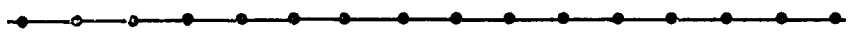


(3) When the paper has been correctly folded, fix the pockets down flatly with staples or pins, or by sewing with heavy thread. Fasten the pockets so that all the numeral cards can be placed in their correct positions. (4) Rule nine heavy vertical lines down the chart to divide it into ten columns as shown.

Teaching Procedure:

Activity 1: A number line with numbers 0 - 10 and beyond.

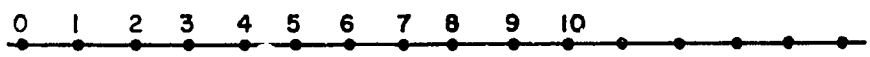
Draw a line segment on the blackboard. Mark points on the line segment at equal intervals like this.



Say, "We are going to give number names to the points marked on the line. We will call this point zero." Write "0" above a point at the left of the line. Ask, "What number comes after zero in the natural order?" (1) Say, "We will give that

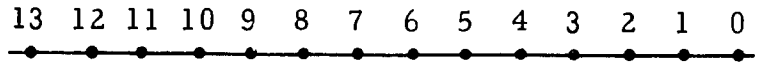
name to the next point marked on the line." Say, "Now we have points named 0 and 1. What number comes next?" (2) "We will give that name to the next point." Let a pupil come up and write "2" above the next point.

Continue to question in this way. Let the children come in turn to write numerals up to 10 above the points marked on the line.



Draw a line segment on the blackboard. Mark it off into equal intervals as before. Mark one of the points on the right "0". Ask a child to come up and point

to the new marked point to the left of "0". Ask him to give that point the name "1". Ask another child to point to the next point to the left. Let him give that point the name "2". Go on until all the marked points have names as shown below.



Ask the children if this number line is like the other one. Let them talk about this. They should say, "On one line the numerals go from right to left in natural order. On the other the numerals go from left to right."

Say, "The numerals may go either way. We will use number line with numerals in natural order from left to right. We will use a number line like the first one."

Let the children draw a line segment on their papers. Ask them to mark points on the line segment. Ask them to name the points with numerals in order. Let them talk about their lines. They should say, "The line segments may have different lengths. Some line segments have more numerals on them than others. We can make longer and longer line segments. Some number lines have points marked far apart. Some number lines have points marked close together."

<u>Activity 2:</u> Number lines using numbers 0 - 100 **Pupil Page 10**

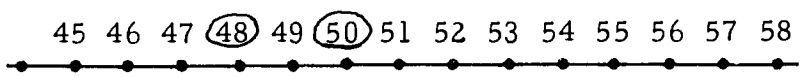
Draw another line segment on the blackboard. Mark points at equal intervals. Ask a child to name a point "0". (It does not matter which point the child chooses to name zero. If he chooses one near the right end draw a larger line segment. Say, "We need a longer line segment.")

Let the children come up in turn to name the other points marked on the number line.

"What number is one greater than zero?" Come and show the point for that number." Then let the child write "1" above that point. "What number is two greater than one? Come and show the point for that number." (3) "Show the point for the number that is two greater than three." (5) "Label the point for the number that is three greater than five." (8) "Show the point for the number that is one less than eight." (7), and so on. Each time let the pupil write the numeral over the point he showed.

Tell the class that when numerals have been written on the number line like this, we say that the numbers are in their natural order. Point to any numeral on the line, for example, 5. Ask the class to tell you the numbers that are greater than five. Ask them which numbers are less than five. Help them to see that the numbers for points to the right of 5 are all greater than 5, and all the numbers for points to the left of 5 are less than 5.

Draw another line segmenton the board. Tell the class that the first point need not always be the point for zero. Write "45" by the point on this line and ask the children to come up and write the other numerals for numbers in natural order.



Draw a ring around two of the numerals on the line, as shown above. Let the children compare the two numbers the numerals show, for example: "48 is less than 50." "50 is greater than 48."

Make a number line on the blackboard. Write a numeral above the last marked point to the right on the line. Let the children in turn put numerals on the other marked points of the line. Make a number line with a numeral at a middle point on the line. Let the children come in turn to put numerals on the line. Draw a long line segment on the blackboard. Let the children build up a number line using all the numerals 0 - 100.

Let the children talk about these things: (1) Any line segment can be used to make a number line. (2) The marked points on the number line are spaced evenly. (3) The points are marked with numerals in natural order. (4) Number lines can have many numerals on them. (5) We write numerals in order from left to right. (6) Sometimes a number line has numerals from 0 to 10, sometimes 45 to 90. We can make many different number lines.

Ask the children to find page 10 in their books. Let them copy the number lines from the page into their exercise books. Ask them to write in the missing numerals. Go around the class giving help as needed. As you talk with the children, point to two numerals on a line. Ask which of the numbers named is greater and which is less.

Note: The other part of page 10 will be used in Activity 5.

Activity 3: Locating numerals on the hundred-pocket chart.

Hang the hundred-pocket chart in front of the class. Let the numeral cards be arranged in the natural order but with the blank sides of the cards showing. Ask, "How many cards in each row?" (Ten). "How many rows are there?" (Ten). "How many cards are there altogether?" (Ten tens or one hundred). Let the children count the cards to be certain.

Use a sheet of paper or cardboard to hide all the cards except the top row. Ask, "How many cards in this one row?" (Ten). Turn the last card in the row over. (10). Move the cardboard down to show two rows. Ask, "How many rows can you see?" (2). "How many cards can you see?" (Two tens or twenty). Turn over the last card in this row. (20). Let the children explain why 2 and 0 are used in the numeral 20. Point to the last card in the third row. Ask, "What numeral do you think is written on this card?" (30). Turn the card over to check the answers. Repeat these questions until the last card in every row has been turned over. Point to each of these cards in turn and let the class count in tens to 100.

With two sheets of cardboard hide all except the first two rows of cards and the first three in the third row (as in Stage 3, Activity 6). Ask, "How many cards

can you see?" (Two tens and three ones, or twenty-three). Turn over the last card to check. (23). Repeat this several times.

Turn all the cards again so that only blank sides show. Point to any card and ask the class to say what is written on it. Let the children write the numeral they guess. Turn the card over to check the answers. Let them tell why they guessed a numeral. Repeat this activity several times. Vary it by placing cards in incorrect places. Let children explain how they know these cards are not in the correct places. Go on as needed.

Activity 4: Deciding which of two numbers is greater.

Ask a child to give any number from 1 - 100 (for example, 35). Let him take the card which he thinks shows the numeral for that number from the hundred-pocket chart. Let him show the numeral to the class and say, "My number is 35." Let the children say whether he has the right card. Let him hold the card at his seat. Continue this activity until each child has a card.

Let pairs of children compare their numbers. Let them decide which number is greater and which is less. For example, let them say, "My number, 57, is greater than 35." and, "Your number, 35, is less than 57."

You hold up a numeral card and say, "Hold up your card if your number is greater than mine." Let the children check to see that all who have a greater number are holding up their cards. "Hold up your card if your number is less than mine." Repeat this activity, holding up a different card each time.

Activity 5: Using the signs for "greater than" and "less than". Pupil Page 10

Let the children use the cards which they chose in the previous activity or others they choose.

Write two numerals on the blackboard; for example, 27 and 35. Point to the numerals in turn and say, "Twenty-seven is ... thirty-five." Let the class repeat this sentence and fill in the missing words, "less than". Tell the class that they can write that sentence in a shorter way. Write the sign, < (less than), between the numerals: (27<35). Let the class read this as, "Twenty-seven is less than thirty-five."

Write the same two numerals in the reverse order. Point to the numerals in turn and let the class say, "Thirty-five is greater than twenty-seven." Write the sign, > (greater than), between the two numerals: (35>27). Let the class read the sentence.

Write several pairs of numerals on the blackboard; for example, 92 and 82. Let the children in turn write the correct sign between the numerals. Let some children write the numerals in the reverse order; for example, 82 and 92, and put in the correct sign. Help the children to see that 92>82 and 82< 92 really mean the same thing.

Draw the following on the blackboard.

Let pairs of children come up. Ask them to hold up their numeral cards and say which is greater and then which is less. Let them arrange their numerals to make true sentences.

For example,

Let the children hold their cards against the blackboard. Let them write the correct sign between the numerals.

Ask the class to read each of the sentences. Allow as many different pairs of children as possible to do this activity.

Let each child put his numeral card back in the correct pocket in the hundred-pocket chart. Ask such questions as: " Does 12 or 27 come first in the chart?"

Ask the children to turn to page 10 of their books. Let them copy the pairs of numerals on the bottom of the page into their exercise books. Ask them to put in the

missing signs, > or < , to make the sentences true. Let them make up sentences to write in their books. Go around the class. Ask each child to read some of the true sentences he has written.

Stage 5: Ordinal Numbers

Vocabulary: First, second, third, ... one hundredth; next; last.

Materials: Hundred-pocket chart

Teaching Procedure:

Activity 1: Revising the ordinals next, last, first - tenth.

Arrange a set of five different objects in a row on your table, for example: pencil, piece of chalk, exercise book, box, stick.

Point to each object in turn and let the class count them. Point to the pencil and say, "This is the first object in the row." Let the children repeat, "This is the first object." Point to the chalk and say, "This is the second object in the row." Point to each object in turn until you have given each object an ordinal number name. Let the class repeat each sentence after you have said it. Point to any object and ask a child to tell you its position in the row. Ask, "Which place has the chalk in the row?" (The chalk is second in the row.) Repeat this type of question several times. Now ask, "Which object is third in the row?" (The book is third in the row.) Vary the activity by asking such questions as, "Which object is next after the second object?" (The third object, which is the book.) "Which object is last in the row?" (stick). Remove the pencil and repeat the activity. Make a row using more objects and repeat the activity.

Arrange a row of children (about 10) from the front of the classroom to the back. Let them face the back. Let the children say their number in the row. The child in the front will say, "One", the next child, "Two", and so on. Ask questions such as, "Who is the first child in the row?" "Who is the tenth child in the row?", and so on. Let the children make up sentences such as, "John is seventh in the row."

Let the first child in the row be seated. Ask, "Who is first now?" "Who is second?" and so on. Let two other children be seated. Ask, "Who is tenth now?", and

so on. Let the children talk about how to decide who is first and that the same person is not always first.

Let the children in the row turn about and face the front. Ask, "Who is first?" (Tom). Ask, "Tom, were you first before you turned?" (No, I was last.) Go on with the other children. Let each say, for example, "Now I am second. Before we turned, I was ninth."

Activity 2: Using the ordinals first - tenth.

Hang the hundred-pocket chart in front of the class, with the blank sides of the cards showing. Point to the rows in turn, starting with the top row. Say, "This is the first row of cards." Let the class say, "That is the second row.", and so on, as you point to each row in turn.

Point to the first card and say, "This is the first card in the first row." Point to another card and say, for example, "This is the third card in the fourth row." Point to other cards and let the children tell you which row it is in and its position in that row.

Vary the activity in these ways:

1. Ask the children to point to cards which you name. Say, "Come and point to the tenth card in the tenth row."
2. Ask the children to read the numeral which is written on any card you name. Ask, "What numeral is written on the fifth card in the first row?" (5). Turn the card over to check the answer. Repeat this type of question.
3. Ask, "Which card has the numeral 75 written on it?" (The fifth card in the eighth row). Ask a child to come out and turn over that particular card to check the answer.
4. Turn the cards so all the numerals show. The children in turn read parts of the chart beginning with the first card in this way: first, second, third,... tenth, eleventh,...hundredth.

Note: Try to use the ordinals throughout the school days in the childrens' activities.

UNIT 2

ADDITION I

Objectives:

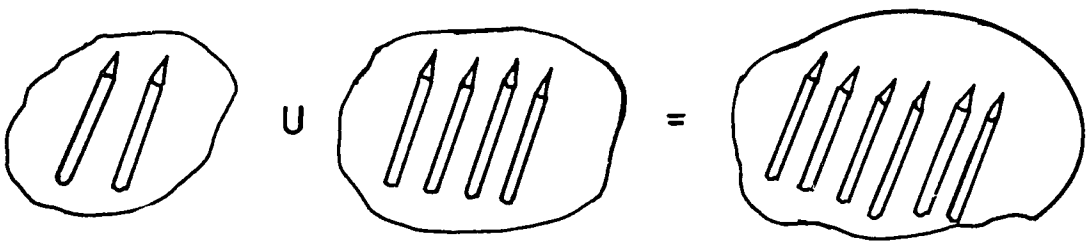
1. To revise the basic addition facts with sums of 18 or less.
2. To understand the meaning of addition as it relates to the union of sets.
3. To extend ideas of addition as an operation on numbers.
4. To learn to read a mathematical sentence involving addition and to complete sentences by finding the missing sum or addend.
5. To learn different names for the same number.
6. To learn that the sum of two numbers is the same regardless of the order of the addends.

Background Information for Teachers:

See also Part 2, "Arithmetic", Sections 3, 5 and 8 in your Teachers' Handbook.

The idea of adding numbers is related to putting together two sets of objects. If we put together the members of the set of boys in a class and the set of girls in the class we form a new set, the set of all children in the class. That is, the set of children is the union of the two sets. We sometimes say that we join the two sets.

The union of a set of two pencils and a set of four other pencils is a new set, a set that has six members. The sentence below states this.



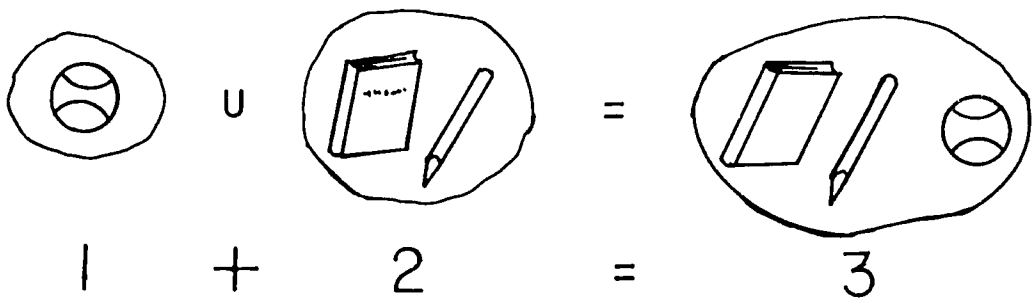
The symbol " $\cup$ " means "joined to". (A set of two pencils joined to a set of four other pencils is a set of six pencils.)

We find that when we join a set of any 2 things with a set of 4 different things the new set has 6 things as members. We express this in the mathematical sentence,

$$2 + 4 = 6.$$

The sentence, $2 + 4 = 6$, can be applied to the union of any set of 2 members and any set of 4 members provided no member is in both sets. In adding the numbers 2 and 4, we do not care what the things in the sets are; we want to know how many things there are.

We <u>join</u> sets; we <u>add</u> numbers. One is an operation on sets; the other is an operation on numbers. We use different signs, "$\cup$" and "+", for these operations. The two sentences below show this.



Addition is an operation on a pair of numbers. It attaches a number to two given numbers. It attaches four to two and two, five to four and one, and so on. In the example given above, $1 + 2 = 3$ tells us that three is the number attached to one and two by addition. We say that the <u>sum</u> of one and two is three. We can also say that "1 + 2" is a numeral for the number three just as "3" is a numeral for the number three.

The children can find the sum of any two whole numbers by joining two different sets that have these numbers of members. The number of members in the union of the sets is the sum. To find the sum of 0 and 3 they can join a set with zero members (the empty set) with a set of three members. They find that the union is a set of three members. They know that 3 is the number attached by addition to 0 and 3. In other words, $0 + 3 = 3$.

A number has many different symbols that name it. For example, we show three with the numerals "0 + 3" and "3". Three may also be shown by "2 + 1", by "1 + 2", by "3 + 0" and by the word "three". We know that the numeral "6" names the number six. But six is also named by "2 + 4", "5 + 1", "3 + 3", and so on. The sentence

$$3 + 3 = 6$$

tells us that "3 + 3" and "6" are different numerals for the same number. The symbol "=" means that the number named by each of the numerals is the same. It is six.

We have been talking about sentences like

$$1 + 2 = 3.$$

This is a <u>number sentence</u>. It is a <u>true</u> mathematical sentence stating that one plus two <u>equals</u> three.

The sentence

$$4 + 2 \neq 8$$

is also a true number sentence. It says that four plus two <u>is not equal to</u> eight. The sentence

$$4 + 2 < 8$$

tells us more about the relation between 4 + 2 and 8. It says that 4 + 2 <u>is less than</u> 8. The children know that this means that 6 is less than 8. This idea may be written "6 < 8". They know that "4 + 2" and "6" are both numerals for the number six. They learned that six is less than eight when they learned about ordering numbers.

Another true number sentence is

$$5 + 1 > 5$$

This says that 5 + 1 <u>is greater than</u> 5. The children know that "5 + 1" is a numeral for six and that six is greater than five. Sentences like

$$1 + 2 = 3$$

are <u>equations</u>. Sentences like

$$4 + 2 < 8 \text{ and } 5 + 1 > 5$$

are called <u>inequalities</u>.

We can present open sentences such as $5 + 3 = \square$ and $\square + 2 = 4$. We call these sentences "open" because they are incomplete. They are neither true nor false. We make them either true or false by writing a numeral in the box. We ask the children to make true sentences by putting a numeral in the box, as in the examples:

$$5 + 3 = \boxed{8}$$

and

$$\boxed{2} + 2 = 4.$$

In these examples there is only one number whose numeral may be written in the box to make a true sentence.

On the other hand, for an open sentence such as

$$4 + 2 > \square$$

we discover that any one of several numbers may be chosen. Different numerals may be written in the box to make the sentence true. We might write "4" or "5" or "0" or "2" and so on.

In this unit the children use sentences like

$$2 + n = 8$$

in which a letter stands for a number. This sentence is an open sentence. The children tell what number n makes the sentence true. In the example

$$2 + n = 8$$

we know that if n is 6, the result is a true sentence,

$$2 + 6 = 8.$$

Addition is an operation on two numbers. These two numbers are called <u>addends</u>. To each pair of numbers addition attaches a number. This third number is the <u>sum</u>. Like other operations of arithmetic, addition has certain properties. This means that there is something about addition that will be true any time we add.

It is always true that no matter what two numbers we add, a change in the order of the two numbers (the <u>addends</u>) does not change the sum. Addition attaches the same number to 2 and 3 as to 3 and 2. That is, the sum of 2 and 3 is the same as the sum of 3 and 2. Also, $5 + 2 = 2 + 5$, $7 + 1 = 1 + 7$, and so on, for any two numbers. We call this property the <u>commutative property of addition</u>. No matter what numbers a and b are, $a + b = b + a$.

You will not use the word "commutative" with the children, but they can understand that the order of the numbers added does not change the sum. Help them discover this for themselves by using many examples. They will say that 7 is the sum both of 3 and 4 and of 4 and 3. In other words, "3 + 4" and "4 + 3" are both numerals for the number seven. They will say that $3 + 4 = 4 + 3$. They will say also that $0 + 6 = 6$ and $6 + 0 = 6$, or that $0 + 6 = 6 + 0$, and so on.

Another property of addition may be discovered by the children. Addition is an operation on two numbers, so if we have a sentence such as

$$3 + 1 + 4 = \square$$

we must decide which two numbers to add first. We could add 3 and 1 and then add 4 to that sum. Or, we could add 1 and 4 and add 3 to that sum. The parentheses in the sentence

$$(3 + 1) + 4 = \square$$

show that 3 and 1 are added first. The parentheses in the sentence

$$3 + (1 + 4) = \square$$

show that 1 and 4 are added first. Compare the two examples.

$(3 + 1) + 4 = \square$ $\qquad$ $3 + (1 + 4) = \square$

$4 + 4 = \square$ $\qquad$ $3 + 5 = \square$

We know that the sum is the same. We know that $(3 + 1) + 4 = 3 + (1 + 4)$. We can try many examples. No matter what three numbers we add, we find that the grouping of the addends does not change the sum. If a, b, and c are any numbers, then $(a + b) + c = a + (b + c)$. We call this property the <u>associative property of addition</u>.

You will not use the word "associative" with the children. They can understand that no matter how they group the addends the sum is the same. You help them discover this by using many examples. They will say that $(1 + 2) + 3 = 1 + (2 + 3)$, that $(2 + 2) + 4 = 2 + (2 + 4)$, and so on.

The children know the basic addition facts. These are all the combinations of addends represented by the numerals 0 - 9. They know also what a 2-digit numeral means. For example, 43 means 4 tens and 3 ones. With this background they can learn to add numbers represented by two-digit numerals. For example, they can use what they know to add

$$\begin{array}{r} 23 \\ +\ \underline{35} \end{array}$$

The children must know that $3 + 5 = 8$ and that $2 + 3 = 5$, and that in one column they add ones and in the other they add tens. They should say that the numeral for the sum, "58", represents 5 tens and 8 ones.

<u>Stage 1: Joining Sets and Adding Numbers</u>.

<u>Vocabulary</u>: putting sets together, joining sets, union, add, plus, equal, not equal, greater than, less than, number names, numeral, number sentence.

<u>Materials</u>: objects of many kinds to form sets, counters, two sets of one-inch numeral (0 - 9) cards for each child, three sets of 5" × 7" numeral (0 - 18) cards; pupils pages 11 - 25.

Teaching Procedure:

Activity 1: Joining sets.

This activity is planned for the revision of the idea of putting together two sets of things. Hold up several objects in one hand, for example, three pencils and a book. Ask the children to describe the set of things in your hand. (Three pencils and a book.) Hold some other objects in your other hand. Ask the children to tell the members of that set.

Put all the objects of the two sets together. Say, "I am putting all the set members together." Show that the result is a new set. Ask, "What are the members of the new set?" Let the children describe this set by telling what its members are. Tell them that you have put together two sets to make a new set.

Put some other sets together. Let one child make a set of sticks and another child can make a set of stones. Ask the two to give all the members to a third child. Say, "You have put your two sets together." Let them describe the new set.

From a collection of many objects on your table, let pairs of children select things to form two sets. Ask them to join the two sets and describe the new set they get. Let them name all its members. Give each child a collection of small objects. Tell him to make two sets. Let the children tell the number of things in each of their sets. Get them to put the sets together and tell the number of things in the new set.

Go on with activities like this until the children know what "putting sets together" and "joining sets" mean. Be careful that in the sets you join there is no object in one set that is also a member of the other set.

Use some examples of joining sets of things that are alike, for example, two books and three other books. Also use examples of joining sets of things that are not alike, for example, the set of a book and the set of a pencil, a piece of chalk, and a tin.

Give some examples in which you or the children do not see the set members joined together. Just talk about and think about putting the sets together. For instance, ask, "The set of all boys in the class joined to the set of all girls in the class is what set?" (The set of all children in the class.) Also ask, "The set of all chairs in the room put together with the set of all tables in the room is what set?"

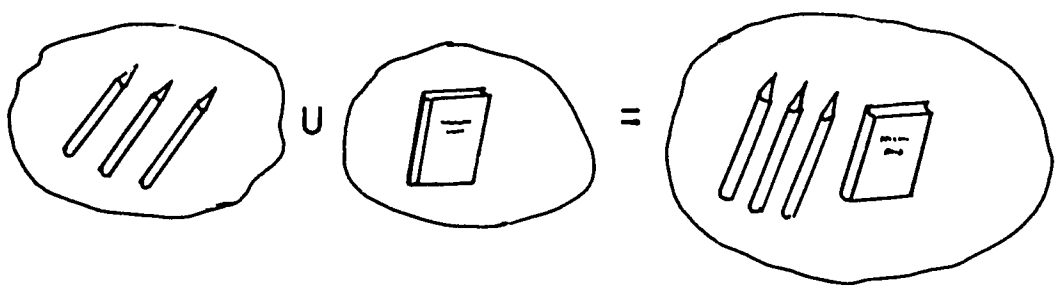
Use some examples in which the empty set is joined with another set. Hold two pieces of chalk in one hand and hold out the other hand with nothing in it. Ask, "Tell the members of the two sets." (One set has 2 pieces of chalk as its members, the other set has no members. It is empty.) Ask the children what the new set will be if the two sets are joined. (A set of 2 pieces of chalk.) They should say after several examples, "The empty set joined with any set is the second set."

Say to the children, "When we join sets we say we are making a <u>union</u> of the sets." Go on as you show with sets, "I am making a union of a set of three pencils and a set of two books. See the new set."

Ask the children to make the union of two sets and tell about it. Let them use the objects on their desks. Continue as needed.

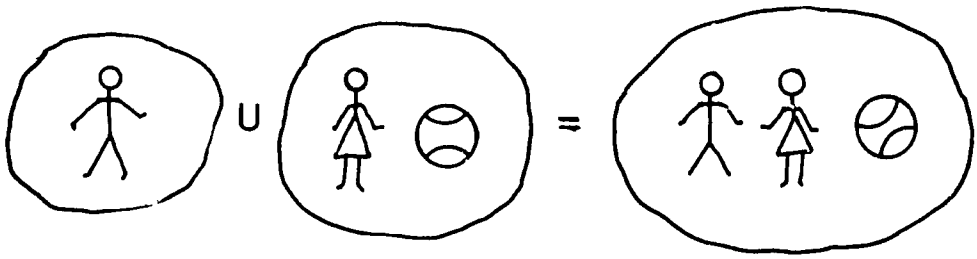
<u>Activity 2</u>: Writing sentences about unions of sets.

Show some examples of joining two sets of objects in the room. Show on the blackboard the sentence describing the union of sets. For example, after putting together a set of 3 pencils and a set of 1 book, show this on the blackboard as:

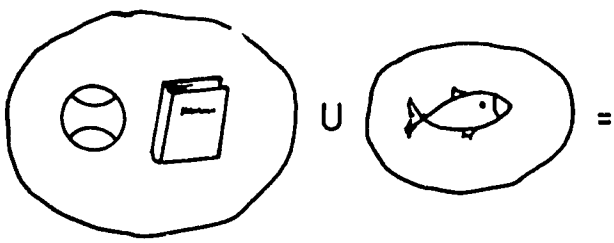


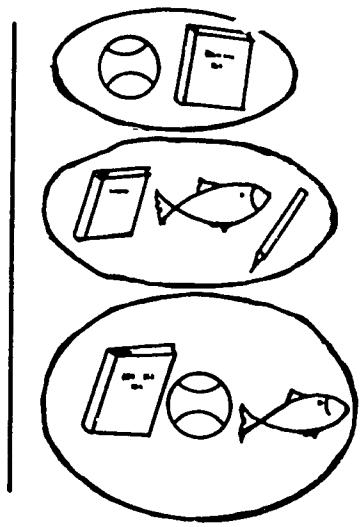
Say that the sign "$\cup$" means that the two sets are joined or put together; a union of two sets has been made. Say that the "=" sign means that the two sets joined are equal to or the same as the set of three pencils and a book. The children should know that "equals" in the sentence means that the things pictured on one side of the sign "=" are exactly the things pictured on the other side.

Show the joining of two other sets of things. Picture the sets on the blackboard in a sentence such as



Let the children read the sentence. (A set of a boy joined with a set of a girl and a ball is a set of a boy, a girl, and a ball.) Put examples on the blackboard of incomplete sentences showing the joining of two sets. Show several possible pictures that might complete the sentences. For example:





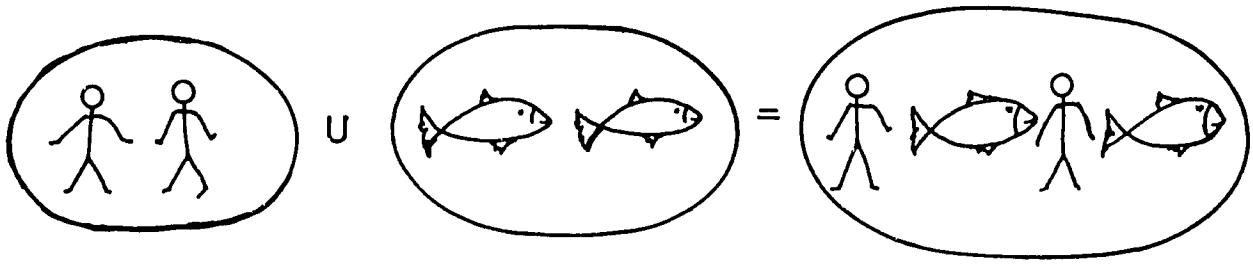
Ask a child to come to the blackboard and point to the set picture that makes the sentence true. He says that the set picture must have as members everything that is a member of either one or the other of the two sets that are joined.

Draw many sentences like this on the blackboard. Let the children read the sentences. Use the empty set in some of them. Continue as needed.

Activity 3: Union of sets and addition.

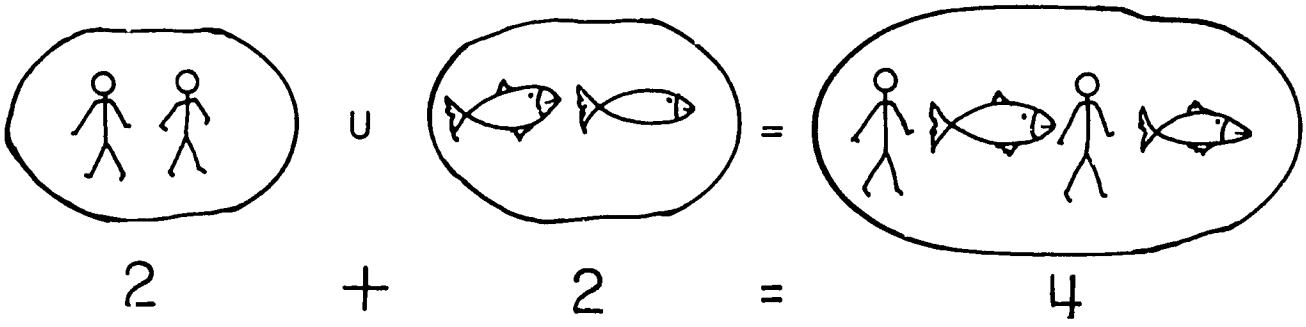
Ask the children to give examples of joining sets. Get the children to do these things: (1) tell how many members are in the first set; (2) tell how many members are in the second set; (3) join the two sets; and (4) tell how many members there are in the new set.

After a few examples of this kind write on the blackboard a sentence expressing the union of two sets.



Point to the first set and ask, "How many members are in this set?" Write the numeral "2" under the set. Point to the second set and ask the number of its members. (2.) Write "2" under the picture of that set. Point to the third picture and say, "When we join the two sets we have this set. How many members are in it?" (4.) Write "4" under the picture of the set.

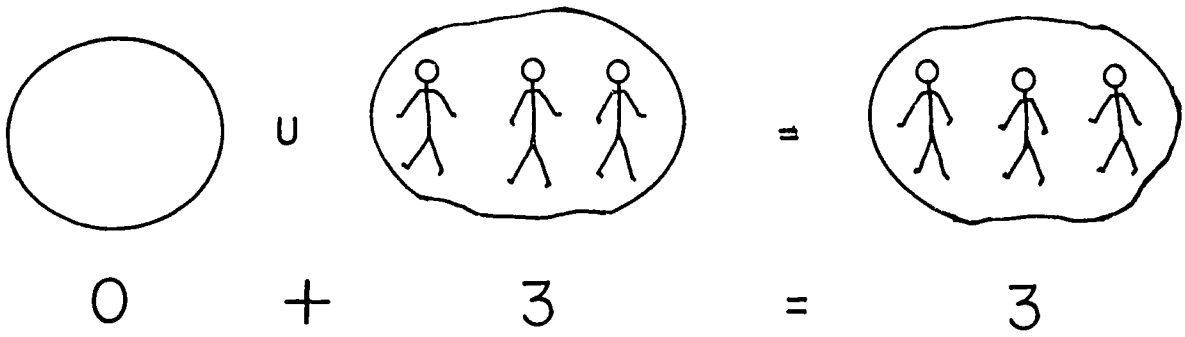
Put the "+" and "=" in the sentence. Say, "When we add numbers we use the sign '+'." The blackboard should now look like this:



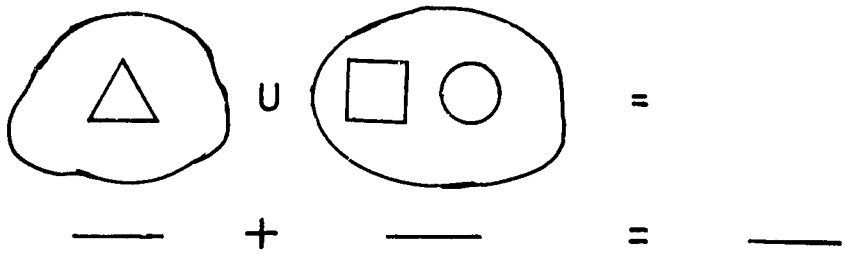
Let the children read the sentences. (A set of two things joined with a set of two things equals a set of four things. Two plus two equals four.) Point to the

sentence 2 + 2 = 4 and say, "The number of members in the union of the sets is 4. We can write this as 2 + 2 = 4."

Use many other examples. Include some with the empty set as shown below. For each sentence let the children tell how many in each set. Write the addition sentence under the union sentence. Let the children read each sentence.



Draw many pictures of the union of sets on the blackboard with the third set missing. See the sentence below.



Ask one child to draw a picture of the set we get when the sets are joined. (You may wish to draw the set as the child tells what the members of the set are.) Let him call another child to write the numerals in the spaces under the pictures. Let all the children help as this child writes 1 + 2 = 3. Get him to call a third child to read his addition sentence. (One plus two equals three.)

Tell the children, "In the first sentence we show that a set with one member is joined with a set with two members. One sentence tells us that the new set we made has three members. In the second sentence we show that one number is added to another number. One sentence tells us that the sum of one and two is three." Point out the sign for addition and the equal sign. Let the children talk about what they mean in sentences.

Go on in this way with other sets.

Activity 4: Making addition sentences. Pupil Pages 11-12

Ask each child to put out two sets of one-inch numeral (0 - 9) cards. If the children do not have these from Unit 1 let them make them.

Let the children open their books to page 11. Ask them to read the first sentence to themselves. Tell them to think what the members of the union of the sets are. Ask a child to tell what the new set will have as members. Let the other children tell if he is right. (It is not necessary that the children name the members in the order pictured. It is necessary that they name each member.) Ask the children to tell the number of members in the sets and use their numeral cards to fill the boxes. Say, "You have made an addition sentence."

Let the children talk about the sentence they made. They will say: The numeral card under the picture of each set tells the number of members in the set. The third numeral card tells the number of members in the union of the two sets; it also tells the sum. The sentence about joining sets helped us write the addition sentence. The sum of the two numbers is the number of members in the two sets put together.

Let the children make sentences from the other sets pictured on pages 11-12. Go around the class. Ask questions like these: What do the sentences say? What do the two symbols $\cup$ and + mean? What number is the sum? How did you find the sum?

Activity 5: Revising basic addition facts. Pupil Pages 13-15

Write many sentences on the blackboard like those on pages 13-15. You and the children make the sentences true by writing numerals in the boxes. Let the children show how they can find the sum of two numbers with sets they make. Say, "If we do not know a sum we can make two sets and join them."

Use sentences like this: $2 + 7 = \square$; and like this: $\square = 4 + 3$. For example, show these two sentences:

$$7 + 2 = \square$$
$$\square = 7 + 2$$

Ask the children to tell the sum of 7 and 2. Tell them that the "=" symbol means that the same number is named by the numerals on both sides. So if 7 + 2 = 9, then 9 = 7 + 2. The numeral for the sum can be to the left of the symbol "=" or it can be to the right.

Ask the children to copy each of the sentences from pages 13-15 in their exercise books. Tell them to make the sentences true by writing the numerals for the sums. Let them put out counters to use if they do not know the sum.

Some children may need more exercises. Some children may not need all the exercises.

Activity 6: Revising basic addition facts. Pupil Pages 16-17

Ask the children to put out their one-inch numeral (0-12) cards. If they do not have them let them make a set. Let the children find pages 16-17 in their books. These pages are to be used together.

Ask the children to find the sets that have a certain number of members. Say, "Put your finger on the picture of a set of 9 things. Put your 9-card on that set." Go on in this way until the number of each set has been told and a numeral card is on each set picture.

Write on the blackboard numerals like these that name the numbers 0 - 12.

4 + 3	0 + 8	10 + 2	2 + 4	6 + 3
2 + 2	8 + 0	0 + 0	2 + 8	5 + 6
0 + 1	5 + 4	1 + 1	5 + 5	3 + 2
10 + 1	6 + 6	3 + 3	3 + 1	5 + 2
6 + 1	4 + 1	0 + 3	2 + 0	2 + 1

Point to one of the numerals, for example, 4 + 3. Let a child read it. Ask all the children to point to the picture in their books showing that number of things. They will point to the set of seven keys. Their 7-card will help them find the right set picture.

Go on pointing to the numerals on the blackboard and asking the children to find a set with that number of members.

Change the activity. Say, "Look at the set of keys. How many members does it have? (7.) Come to the blackboard and find a number name for the set of keys." The children may point to "4 + 3" or "6 + 1" or "5 + 2" or other numerals you may have written.

Go on naming the sets on pages 16-17 and asking the children to find a numeral on the blackboard that tells the number of the set.

Activity 7: Addition - sums of 10-18. Pupil Pages 18 - 19

Let the children put 18 counters on their desks. Ask them to make a set of 10 counters and a set of 4 counters. Write the numerals 10 and 4 on the blackboard like this:

10 4

Let them put the sets together. You put "+" between the numerals on the blackboard. Say, "This numeral is ten plus 4." Ask the children to tell how many members the new set has. When the answer "fourteen" has been given, complete the sentence on the blackboard.

$$10 + 4 = 14$$

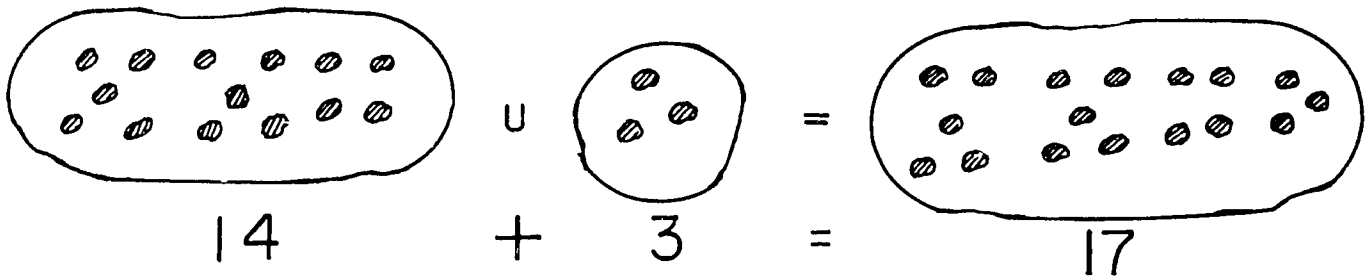
Ask someone to read the sentence. You say, "The numerals 10 + 4 and 14 both tell us how many are in your set."

Repeat the activity with sets so you can write: 10 + 6 = 16, 10 + 8 = 18, 10 + 3 = 13, and so on.

After many of these examples ask the children to make a set of 14 counters and another set of 3 counters. Put these numerals on the blackboard. Ask the children to join their two sets and tell how many members. Write on the blackboard the sentence that described the operation on numbers.

$$14 + 3 = 17$$

To be sure that the children know the relationship between the union of two sets and addition of numbers, show this on the blackboard. Let the children help you by telling what members are in the union of the two sets, telling the number of each set and by telling how 14 and 3 relate to 17.



Use a few more examples of this kind. Make the sum a number from 10 to 18.

Let the children find pages 18-19 in their books. Ask them to copy the sentences into their exercise books and complete each sentence by writing a numeral in the box. Some children may need to use counters to find sums. Do not let them use counters if they know the sums.

It is not necessary for any or all the children to do all the exercises in one lesson. Let them do only as many as you think should be done at one time.

<u>Activity 8</u>: Finding numbers whose sum is a given number.

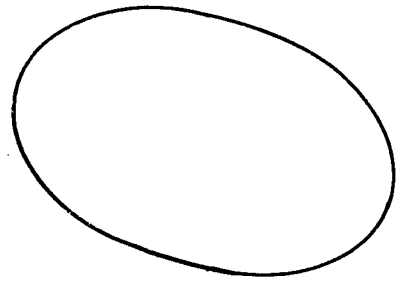
Put out two sets of 5" × 7" numeral (0-18) cards. Give a card to each child. Tell the children they each are the number their card shows. Call one child to show his card to the class. If he has the 7-card let him say, "I am number seven." Ask the child to choose two addends of 7. Say, "Addends are two numbers to add." Let him choose two children from the class holding cards for numbers whose sum is seven. For example, if Tunde has "5" and Ngozi has "2", he calls out "Tunde, Ngozi." Ask these two children to come to the front and show their cards. Ask the other children if he has chosen correctly. Let him tell you the addition sentence which uses the numbers he has chosen. Let the children say whether he is right. Let them read the sentence.

$$7 = 5 + 2$$

Go on with the activity with other children.

Activity 9: The empty set and zero. Pupil Page 20

Draw a picture of the empty set on the blackboard like this:



Point to the set and ask, "What are the members of this set?" (It has no members.) "What is the set called?" (The empty set.) "How many members has the empty set?" (0.) Tell a child to write this numeral. Let the children say, "The number of members in the empty set is zero."

Put some open sentences like these on the blackboard.

$\square = 1 + 0$	$6 + 0 = \square$	$0 + 3 = \square$	$0 + 0 = \square$
$10 + 0 = \square$	$4 + 0 = \square$	$\square = 2 + 0$	$\square = 5 + 0$

Let the children in turn write the numerals for the sums to make the sentences true. Ask the other children to say whether each sentence is a true sentence.

Use these examples to help the children discover that zero added to any number is equal to that number. Get them to notice this pattern in the examples given. Ask questions like these: "Six added to zero is what number? Four added to zero is what number?"

Put this sentence on the blackboard:

$$\square + 0 = \square$$

Let a child put a numeral in one of the boxes. Then ask another child to tell what numeral goes in the other box to make the sentence true. Do this several times. The children should say that the same numeral must be written in each box.

Let the children open their books to page 20. Ask them to cover the boxes with their numeral cards to make the sentences true.

Activity 10: Revision of addition in vertical form. Pupil Pages 21 - 22

The purpose of this lesson is to revise the vertical form of addition and sums of 10-18.

Put the sentence $10 + 6 = \square$ on the blackboard. Let the children talk about the sentence: "It is an addition sentence. The addends are 10 and 6. The sum is missing." Let a child write the numeral for the sum in the box. Ask the class to tell whether he is right.

Say, as you write, "I am writing this addition sentence in another way." The blackboard will look like this:

$$10 + 6 = \boxed{16} \qquad \begin{array}{r} 10 \\ +\ \underline{6} \\ \square \end{array}$$

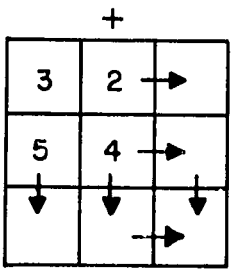
Ask another child to write a numeral in the second box. Tell the class again, "The two exercises say the same thing." Let the children read the sentence. Say, "We can show addition in either form."

Let some children write an addition sentence in one form on the board. Let other children write the same sentence in the other form. Ask the children to turn to page 21 in their books. Let them show the sums with their numeral cards.

Let the children copy the exercise from page 22 into their exercise books and write the sums.

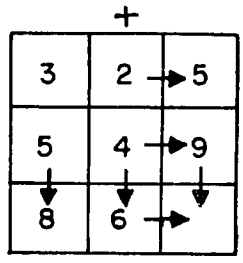
Activity 11: Addition. Pupil Page 23

Put a chart like this on the blackboard:



Say, "The signs tell us to add. Some of the numbers we add are in a row. Some of the numbers we add are in a column. What numerals go in the boxes?" Let the children talk about this. You say, "3 + 2 = 5. I will put 5 in this box." Write "5" in the top box. Ask a child to write a numeral that he thinks should go in another box.

He can add 3 and 5, 2 and 4, or 5 and 4. If he put a "9" in the chart, get him to say, "5 + 4 = 9. A 9 goes in the chart here." Go on until the chart looks like this:



Finally, the last box can be filled by adding either 5 and 9 or 8 and 6. The children find that whichever pair they choose to add, the sum is 14.

Put several other charts like this on the blackboard for the children to fill in. The children find that to put a number in the last box they can either add "across" or "down". Let them talk about why this is true.

Let the children find page 23 in their books. Ask them to fill in the boxes. Each empty box may be filled by placing a numeral card on the box. (Since the same numeral must be used in several places, give each child some blank, one-inch square cards. As he needs a numeral, he can take a card, write the numeral on it, and place it on the proper box.)

<u>Activity 12</u>: Making an addition table. | Pupil Pages 24 - 25 |

The purpose of this activity is to revise addition combinations of addends (0-9). To do this the children make an addition table. Let them put out several sets of one-inch numeral (0-18) cards, with some blank ones to make cards as needed.

Put an addition table on the blackboard like the one below. The numerals which are dotted are to be written in by the children.

+	0	1	2	3	
0	0	1	2	3	
1	1	2	3	4	
2	2	3	4	5	
3	3	4	5	6	

Go over each of the combinations with the class. Let the children tell the sums. They add each of the numbers shown down the left side to each of the numbers shown across the top. Begin with 0 and 0. Let a child point with one hand to an "0" and with his other hand to an "0". Let him write the sum, 0, in a box in the same row and column as the addends. Go on with other addends. Let a child point to each of the two addends, tell the sum and show where to write the numeral. Get another child to write the numeral. For example, when the children add 2 and 3 they point to the "2" on the left side, and to the "3" on the top. Then move across the row and down the column. Their fingers meet. Here they write the numeral "5" for the sum.

Let the children find pages 24 and 25 in their books. Ask them to complete the tables by placing their numeral cards in the empty boxes.

Stage 2: Finding Missing Addends

Vocabulary: Same as Stage 1.

Materials: Same as Stage 1; pupil pages 26-32.

Teaching Procedure:

Activity 1: Missing addends. Pupil Pages 26 — 27

Let the children open their books to page 26. Ask them to look at the sentence about sets at the top of the page. The sentence shows that if the children join some set with the first they get a set of five balls. Ask, "How many balls are in the missing set?"

Let the children use counters to show the sentence. If they make a set of 2 counters they must join a set of 3 other counters to it to make a set of 5 counters. They find also that they add 3 to 2 to get 5.

Therefore, to make the sentence $2 + \square = 5$ a <u>true</u> sentence, they put "3" in the box.

Say, "We made a true sentence about sets. We made a true sentence about addition. Look at our addition sentence. I will write it on the blackboard." Do so.

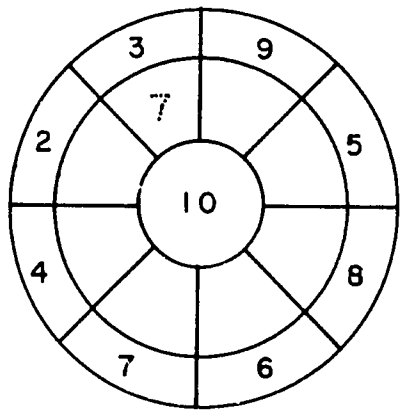
$$2 + \boxed{3} = 5$$

Ask, "What number is the sum?" (5.) "What are the addends?" (2 and 3.) "What number did we find to make the sentence true?" (3.) Say, "This is an exercise in finding a missing addend."

Ask the children to make the other sentences on pages 26-27 true. Let them use counters to make the set sentences true and numeral cards for the addition sentences. Go around the class and give help as needed. Ask often, "What is the missing set? What is the missing addend?"

<u>Activity 2</u>: Finding missing addends. Pupil Page 28

Put one or two number wheels like the one below on the blackboard and let the children talk about them. You can call them "missing addend wheels".



Ten is the sum on this wheel. Ask, as you point, "Three is one addend of 10. What is the other addend?" or "Three plus what number is equal to ten?"

Let a child answer by writing a numeral in the space (shown by the dotted line "7"). Write this as a sentence, $3 + \square = 10$. Let another child show this with counters. A child who makes an error can see that his answer does not make a true sentence. He can write another numeral on the wheel. Go on until the wheel is filled in.

After the wheel is completed, revise each of the sentences. The children know there are many different numeral names for the number ten. Guide them to say that "3 + 7", "9 + 1", "5 + 5", "8 + 2", "6 + 4", "7 + 3", "4 + 6", and "2 + 8" are all numerals for ten.

Let the children complete the wheels on page 28. Ask them to show the missing addends by placing numeral cards in the blank spaces. When the wheels are completed ask the children to write in their exercise books the addition sentences the wheels show. For example, the sentences that the top wheel shows are:

$6 + \boxed{5} = 11$ $\quad 9 + \boxed{2} = 11$ $\quad 4 + \boxed{7} = 11$ $\quad 8 + \boxed{3} = 11$

Activity 3: Finding missing addends to make sentences true. Pupils Pages 29-30

Put this sentence on the blackboard:

$$4 + \square = 9.$$

Point to the "9" and ask what it says. (It names the number nine.) Say, "The = sign tells us that both parts of the sentence should name the number nine." Use your hands to circle the "$4 + \square$" and ask, "What can we put in the box so that this says nine?" (5.) Let a child write "5" in the box.

Ask another child to read the sentence. Ask, "Is the sentence true?" (It is true. Both 4 + 5 and 9 name the same number.)

Ask the children to copy the sentences from pages 29-30 into their exercise books. Let them put a numeral in the box to make the sentence true. Go around the class. Ask the children to read their sentences and tell if they are true sentences. Also get them to point to the sum in a sentence and to the addends. Ask, "What are you finding in this sentence, a sum or an addend?"

Activity 4: Different names for a number. Pupil Page 31

Ask a child to write a numeral for fourteen on the blackboard. He will write "14". Ask another child to write another numeral for fourteen. He may write "7 + 7". Then ask other children to write still different numerals for fourteen. Go on until many numerals have been written.

Make a chart on the blackboard for fourteen, like the ones for 15-18 on page 31. Say, "We can put some of our numerals for fourteen in this chart. We can put any pair of numbers in the chart if they have 14 as a sum." Let the children in turn put their numerals in the chart. Make the chart longer if the class names many pairs of numbers.

Ask the children to complete the charts on page 31. Let them put numeral cards on the blank boxes. Let the children make charts like these on their papers and write different names for a number. Let a child read the names he wrote for 18. Let other children tell other names for 18.

Activity 5: Addition sentences. Pupil Page 32

Put this sentence on the blackboard.

$$4 + 1 = \boxed{} + 3$$

Ask a child to point to the equal sign. Ask the children what it means. Let them talk about this question. Help them say that it means: the number named on one side of the sign is the same as the number named on the other side.

Ask, "Do we know what the number is?" (Yes. It is five.) Ask, "Why is it 5?" (4 + 1 = 5.)

Ask, "How can we make the sentence true?" (Write a numeral in the box.) Let a child write a numeral in the box. (He may write "2" or some other numeral.) Put your hand around "$\boxed{2}$ + 3" and ask, "Does this show five?" If "2" is written in the box the children will say, "Yes." They will also say the sentence is true. If some other numeral is written, for example 3, they will say, "No. $\boxed{3}$ + 3 does not show five. The sentence is not true." Then rub off the numeral and try again to make the sentence true.

Ask the children to open their books to page 32. Let them copy the sentences into their exercise books. Ask them to find the numbers which make the sentences true. Tell the children they make the addition sentences true by finding an addend.

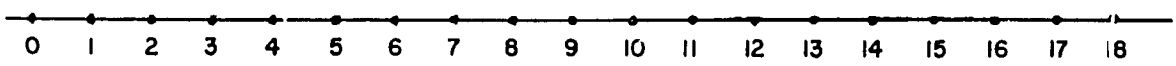
Stage 3: Using the Number Line for Addition.

Vocabulary: number line, forward, space.

Materials: Numeral cards, counters, stick-figure pictures; pupil pages 33-34.

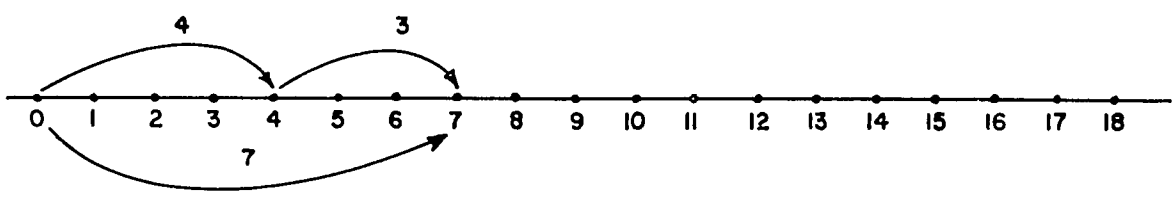
Activity 1: Using a number line to complete a sentence.

Draw a line segment on the blackboard. Mark points at equal intervals. Name some of the points with numerals. For example, write in just the 1, the 4, the 9, and the 16. Let the children write in the other numerals. The number line should look like this when the children write in the numerals:



Tell the children, "This is not the only number line we could make. We could use a longer line segment. We could put the marked points closer together or farther apart. Today you will use this one. Later you will use other number lines."

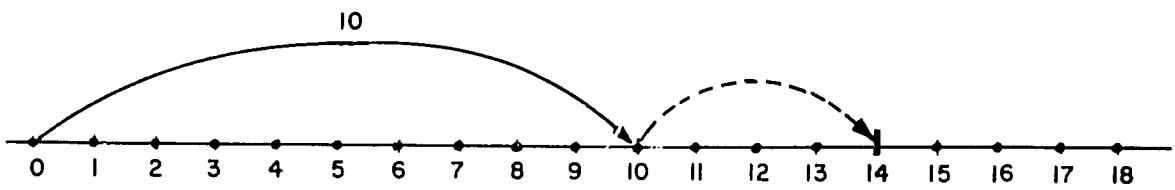
Write the sentence $4 + 3 = \boxed{}$ on the blackboard. Say, "I will use the number line to find the numeral to write in the box. I start at '0'. This piece of chalk will show my trip. I move 4 spaces on the line. I am at '4'. I move 3 more spaces on the line. I am at '7'. I moved 4 spaces and then 3. That is the same as moving 7 spaces." Make marks like the ones below on the number line as you talk. Let the children talk about how this shows that $4 + 3 = \boxed{7}$.



Write these open sentences on the blackboard. Ask children to show them on the blackboard number line. Let them tell how they use the line to find the numeral to write in the box. Continue as needed.

$3 + 5 = \square$ $\quad 1 + 6 = \square$ $\quad 5 + 6 = \square$ $\quad \square = 8 + 6$

Write the sentence $10 + \square = 14$ on the blackboard. Say, "I will use the number line to find the numeral to write in the box. This piece of chalk will again show how I move. I start at '0'. I move 10 spaces. I am at '10'. I must now move some more spaces to arrive at '14'. I move to '14'. I count the spaces I move to get there. I count 4 spaces. I will write '4' in the box. $10 + \boxed{4} = 14$." Make marks like the ones below on the number line as you talk. Let the children talk about how this shows that $10 + \boxed{4} = 14$.



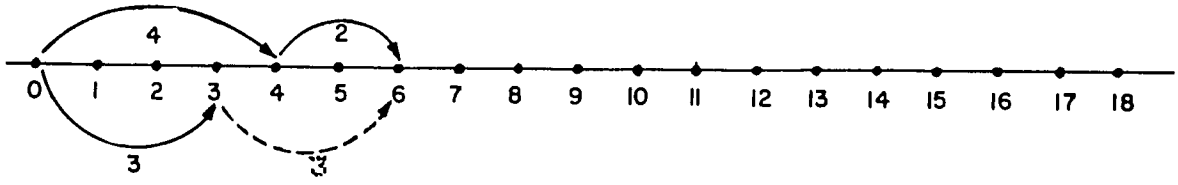
Write these open sentences on the blackboard. Ask children to show them on the blackboard number line. Let them tell how they use the line to find the numeral to write in the box. Continue as needed.

$6 + \square = 10$ $\quad 8 = 4 + \square$ $\quad 9 + \square = 14$ $\quad 7 + \square = 15$

Let the children try to show this sentence on the blackboard number line:

$$4 + 2 = 3 + \square$$

Suggest, if necessary, that the children show two equal trips. One trip shows $4 + 2$. Another trip shows $3 + \square$. Let them make marks like the ones below on the blackboard number line. Ask, "What numeral do we write in the box?" (We can write "3". We moved three spaces to make the trips equal. The numeral "3" makes the sentence true.)



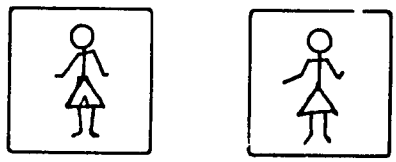
Get the children to show other sentences like this. Let them show both the open sentence and the number-line diagram. Each time the children should make the sentence true with what they learn from the number-line diagram.

Activity 2: Using a number line. Pupil Page 33

To introduce this activity, use the blackboard number line marked with the numerals 0 - 18. Put these open sentences on the blackboard.

$$11 + 3 = \square \qquad 9 + 6 = \square$$

Put out two 3" × 5" cards with stick figures of a boy and girl sketched on them as shown below. Say, "This is a picture of Joseph. This is a picture of Margaret. They are going to take trips on our blackboard number line today." Let several children show trips which Joseph and Margaret could take.



Call on a boy to come up. Say, "Here is the picture of Joseph. He is going to take a trip. (Point to the first sentence, $11 + 3 = \square$.) This tells us about his trip. You show us how Joseph moves along the number line."

Let the child move the stick-figure picture 11 spaces forward, starting from 0. Let him move the figure 3 more spaces. Ask, "On what number is Joseph now?" (He moved 11 + 3 spaces. He is now at 14.) Say, "Our sentence $11 + 3 = \square$ was not finished. Can you now write a numeral in the box?" After a child does this say, "Now the sentence about the trip is a true sentence."

Let a girl show the next sentence. Get her to show the second sentence as a trip for Margaret along the number line. Let another child tell about the trip and the sentence as was done for Joseph.

Write other sentences on the blackboard. Let the children work them out as a game on the picture shown on page 33. Ask the children to use small seeds, beans, or other counters to move along the road. Say, "The road is a number line. The seed is a rabbit hopping along the road." Let the children begin at 0 and move

the rabbit along the road. The number sentence tells about the trip. When the moves are made, ask, "What numeral do we write in the box?"

Include sentences with missing sums and missing addends, like these:

$5 + 6 = \square$ $\square = 9 + 9$ $10 + \square = 16$

$12 = 7 + \square$ $8 + 5 = \square$

Let the children also find the number which makes a sentence like this true:

$$3 + 9 = 6 + \square$$

Let each child move 2 different rabbits along the road like this. Say, "There were two rabbits, a black one and a white one. The rabbits want to take a trip. They start at zero. The white rabbit goes 3 spaces, then 9 more. He is on 12. The black rabbit moves 6 spaces along the road. He wants to go as far as the white rabbit. How many more spaces must he move?" (6.) Say, "Look at our sentence, we will write '6' in the box so the sentence is the true story of the trip of the rabbits."

Make up stories about trips on the number-line road pictures. Let the children also tell stories about this picture. For example:

"Mr. Rabbit hops very fast. He laughs at Mr. Tortoise because he goes so slowly. Mr. Rabbit says, 'See how much faster I can go!' And he hops 5 spaces, then makes a big hop of 7 spaces. But Mr. Tortoise keeps moving slowly along the road, one space at a time. He finally catches up with Mr. Rabbit. How many spaces does he move?"

Let the children move counters along the road to act out the stories. Get them to write a mathematical sentence to tell about each story.

<u>Activity 3</u>: Making mathematical sentences. Pupil Page 34

Under each of the number line examples on page 34, a sentence is shown. Let the children look at the diagrams. Ask them to write under each number line a true sentence which tells about the picture on the line. Let them make these sentences by putting numeral cards on the boxes.

Stage 4: Discovering the Commutative Property of Addition

Vocabulary: Order.

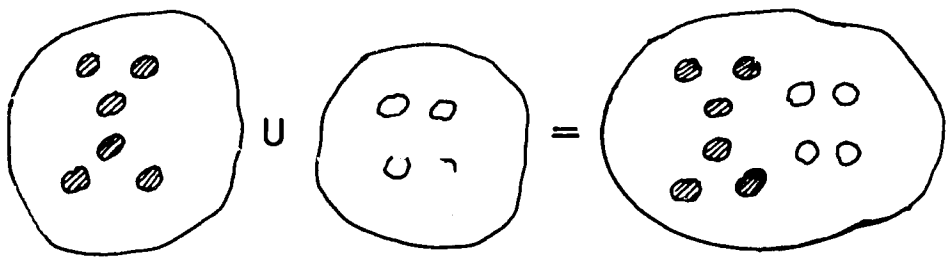
Materials: Counters, numeral cards, cardboard strips with numerals, addition combination cards, pupil pages 35-37.

Teaching Procedure:

Activity 1: Addition has the order property.

The aim of this lesson is to help the children discover that a change in order of two addends does not change the sum. Let them add many pairs of numbers. Let them add the same numbers with the order changed. For every pair of numbers they try, they find that the sum is the same. Finally they may say that this seems to be true for all pairs of numbers.

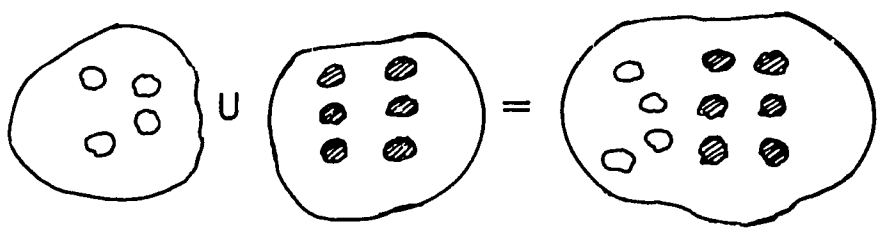
Ask the children to make a set of six counters on their desks and then a set of four other counters. Let them join the sets. Write this sentence on the blackboard as you say, "I will draw a picture of your sets on the blackboard."



Ask a child to write a sentence about adding the numbers of members in the two sets. He will write

$$6 + 4 = 10.$$

Then let the children use the same set of ten counters to form a set of four and then a set of six. Ask them to join the sets. Say, "I will draw a picture of your sets." Let a child put the addition sentence below the picture.



Go on with other examples of this kind. Let the children talk about this question, "Does it matter in which order we put the two sets together?" The children say that the union of two sets is the same regardless of the order of the sets joined.

Activity 2: Changing the order of addends. Pupil Page 35

Put some pairs of exercises on the blackboard. For example, write

$$\begin{array}{r} 3 \\ +\ \underline{14} \end{array} \qquad \begin{array}{r} 14 \\ +\ \underline{3} \end{array} \qquad \begin{array}{l} 11 + 5 = \square \\ 5 + 11 = \square \end{array} \qquad \begin{array}{r} 12 \\ +\ \underline{5} \end{array} \qquad \begin{array}{r} 5 \\ +\ \underline{12} \end{array} \qquad \begin{array}{l} \square = 4 + 15 \\ \square = 15 + 4 \end{array}$$

Let the children first complete these exercises in a way they know. Leave the exercises on the blackboard. Ask the children to complete the exercises on page 35 by covering the boxes with their numeral cards. When the children are finished, ask them to point to a pair of exercises that are similar. Talk about why the pairs are similar. The children will say, "Each pair has the same addends. The addends are in different order." Ask, "Is this true for all the sentences of the blackboard?" Try to get the children to discover that if the order of two numbers added is changed there is no change in the sum.

Activity 3: Changing the order of the addends. Pupil Page 36

The exercises on this page use the fact that the sum of two numbers is not changed by a change in the order of the addends.

Ask the children to make the sentences on page 36 true. Let them show the sum for the pairs of sentences by placing a single numeral card in the box.

Ask the children to read the first pair of sentences. Let them talk about this question: "What do you notice about the two sentences?" They may say, "The same numbers are added. The numbers are added in different order. The sum is the same both times." Ask, "Is this true of the sentences in the other spaces?" They should say, "The same numbers are added. They are added in a different order. The sum is the same." Ask, "Do you think this is true for all the numbers you know?"

Activity 4: Using the order property to make true sentences. Pupil Page 37

Ask the children to make the sentences on page 37 true sentences by placing numeral cards in the boxes. Talk with the children about the order property. Ask them if they can complete the first sentence without adding. Let the children talk about this. Let them say what the missing addend is.

Help the children to use what they discovered about the order of the addends. They may say, "We know that both parts of the sentence name the same number. If one addend in each part is the same, then we know the other addend is the same."

Let the children make the other sentences true. Discourage the use of addition.

Activity 5 (Optional Activity): Matching numeral game.

Put out thirty-four 6" × 3" numeral (0 + 1 - 8 + 9) cards. Put the following in one pile:

0 + 1	1 + 3	3 + 4	4 + 6	6 + 7	7 + 9
0 + 2	2 + 3	3 + 5	5 + 6	6 + 8	8 + 9
1 + 2	2 + 4	4 + 5	5 + 7	7 + 8	

In another pile, put cards showing the same numerals but with the order of the addends changed.

1 + 0	3 + 1	4 + 3	6 + 4	7 + 6	9 + 7
2 + 0	3 + 2	5 + 3	6 + 5	8 + 6	9 + 8
2 + 1	4 + 2	5 + 4	7 + 5	8 + 7	

The cards in the piles should not be in the order given above.

Give out the cards in one pile to half of the children. Give the cards in the other pile to the other half. If there are fewer than 34 pupils then give cards from one pile to half the children. Put the other cards away. Also put away the matching cards from the second pile. For example, if you take out $\boxed{9 + 7}$ and $\boxed{9 + 8}$

then you must also take out [7 + 9] and [8 + 9] from the other pile. If you have more than 34 pupils, you can play the game twice.

Ask the two groups of children to line up. Let each child stand opposite the child of the other group holding the card matching his card, (5 + 4 opposite 4 + 5, 1 + 2 opposite 2 + 1, and so on). The two lines should be some distance apart. Halfway between them place an object, for example, a duster. At the start of the game the teams will be lined up in this way:

[4 + 5] [5 + 7] [1 + 2] [0 + 2] [2 + 4] [7 + 9] and so on,

duster

[5 + 4] [7 + 5] [2 + 1] [2 + 0] [4 + 2] [9 + 7] and so on.

Call out a number. If you call "9" then both "4 + 5" and "5 + 4" can run to pick up the duster and back to their places. If "5 + 4" gets the duster then his team scores a point.

If you call out "6" then "4 + 2" and "2 + 4" should run for the duster. If "2 + 4" takes the duster back to his place then his team scores a point. Continue until all sums shown on the cards have been called. You may call a number more than once.

Stage 5: Using Addition in Solving Word Problems.

Vocabulary: Add, more, total, sum, all together, in all, how many, plus.

Materials: Pupil page 38.

Teaching Procedure:

Activity 1: Using addition. Pupil Page 38

Tell the children to open their books to page 38. Ask them to look at the pictures. Say, "I will tell a story about the pictures."

Read the following word problems. Let the children answer by telling the sum. Let the children talk about each problem. Ask them how they know the answer.

1. (First picture) In a corner of the forest there lived a family of four snakes. One day a cousin from another part of the forest came to visit them. "I would like to stay with you if you have room in your house," he said. The father snake answered, "We have room for five snakes all together." Was there room enough for the cousin to stay? Why?

2. (Second picture) One shelf in the classroom has books about science. William took three of the books to look at. This left five books still on the shelf. How many science books are there all together?

3. (Third picture) Asha's mother has six bananas in her basket. Asha put two more bananas into the basket. How many bananas are now in the basket?

4. (Fourth picture) Two children brought flowers today to make the room look pretty. Amu brought three flowers and Efua put four flowers with them. How many flowers are there?

5. (Fifth picture) Grace has a pencil box with five pencils. Her brother gives her two pencils. How many pencils does she have now?

6. (Sixth picture) Helena was helping her mother wash the cups. She washed five cups and then she said to her sister, Maria, "Will you finish washing the cups?" Maria washed the other three cups. Can you tell how many cups there were to be washed?

7. (Seventh picture) Elimo and Wanjau want to play a game, but they must have three children to play. How many more children must they get to join the game?

8. (Eighth picture) James helps sell bottles of cooking oil in the market. He first puts three bottles on the table. Then he adds one more bottle. How many bottles are now on the table?

Let the children make up other stories about the pictures.

Activity 2: Using addition to solve problems.

Read the word problems below as in the last activities. The children do not now have pictures about the stories. They must do more thinking.

As the stories are read, let the children have their exercise books open. Let them write the sentence for each story or write the addends in vertical form to be added. Some children, however, will be able to write the numeral for the answer as the story is told. Do not ask the children to write more than they need to write.

1. A monkey tossed down five coconuts from a tree. Then he climbed another tree and tossed down four coconuts. How many coconuts did he toss down all together?

Let the children talk about this and each of the problems. Some of the talk may be like this: "We know the number of coconuts the monkey tossed down from the first tree. How many were there? (There were five.) So we write '5' on the board. What other number are we given? (We know how many coconuts he tossed from the second tree.) What is that number? (4) We write '4' on the board. Now, what are we asked to find? (The total number of coconuts he tossed down.) How can we find this number? (By adding the two numbers.) So we write, $5 + 4 = \square$. Now, can you tell how many coconuts the monkey tossed down all together?" (It is five plus four.)

2. Mother gave Kimba four oranges this morning. This afternoon she gave him four more oranges. How many oranges does he have in all?

3. Today we read six pages in our reading book. We also read three pages in our mathematics book. How many pages did we read today?

4. If we draw ten pictures this morning and then draw eight pictures this afternoon, how many pictures will we finish today?

5. Yusufu and his father went fishing. His father caught eleven big fish and Yusufu caught five big fish. They took them all to market and sold every one. How many fish did they sell at the market?

6. Alice has nine bananas in her basket. She puts them all in Mother's empty basket. Robert has eight bananas and he puts all of them in Mother's basket too. How many bananas are in Mother's basket now?

7. Majuma's father works in an office. He works five hours in the morning and two hours in the afternoon. How many hours does he work in a day?

8. John has three older brothers. But he also has three brothers who are younger than he is. How many brothers does John have?

You and the children tell other stories. For each story, talk about the number sentence used to find the answer.

UNIT 3

GEOMETRY

Objectives:

1. To revise and extend the ideas of line segment, four-sided figure, rectangle, and circle.
2. To introduce the idea of point, straight line, angle, right angle, triangle, right-angled triangle, square, sphere, cylinder, and prism.

Background Information for Teachers.

Also see Part 2, "Geometry", in your Teachers' Handbook and Primary One Unit 13.

The Primary I program has an introduction to the ideas of line segment, four-sided figure, rectangle, and circle. Each of these is a geometric figure, and each was developed by asking the children to do these things:

1. Find physical objects which show the geometric figure. For example, a straight edge of a table shows a line segment; the edges of a book show a rectangle; and the edge of a round coin shows a circle.

2. Make new objects which show the geometric figure. For example, a stretched string shows a line segment; paper cutouts show line segments, triangles, four-sided figures, or circles.

3. Trace these objects on the blackboard and on paper; the tracings are geometric figures.

4. Name these figures "line segments", "circles", and so on.

In this unit follow the same steps, as the children revise the figures of Primary One program (Objective 1) and as the children learn about new figures (Objective 2).

In Primary One the children made line segments by tracing straight edges, made circles by tracing round objects such as coins and tins, and made rectangles by tracing books, boxes, and so on. In this unit they learn to show a point by tracing

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the tip of a corner where two straight edges meet, and an angle by tracing two straight edges that make a corner. If the corner is square the angle is a right angle. They learn to show triangles by tracing three-sided objects made of sticks, paper, or cardboard. They learn to call a triangle right-angled if it has a square corner. At the end of the unit they meet for the first time three simple geometric figures in space. These are the surfaces of balls, boxes, and tins, and are named spheres, prisms, and cylinders.

The children also learn and use some new ideas about the figures they met in Primary I. They learn of line segments that fit each other exactly and about a special kind of rectangle called a square. By folding paper figures they discover some properties of rectangles and squares. They begin to use letters to name points, line segments, and four-sided figures. They also show a straight line by learning to extend a line segment.

Stage 1: Points, Line Segments and Straight Lines.

Vocabulary: Tip of a corner, point, endpoints of a line segment, congruent line segments, extending a line segment, straight line.

Materials: Books, rectangular pieces of cardboard, paper, string; pupil page 39.

Teaching Procedure:

Activity 1: Finding the tip of a corner; naming the tip, point.

Move your hand over the top of your table and say, "This is the top of my table." Ask, "Where are the corners of my table top?" Get some pupils to point out the corners. Run your fingers along the two edges to the corner. Say, "This is a corner of my table." Put a pencil tip at the place where the edges meet and say, "This is the tip of the corner." Repeat this for another corner of the top.

Point to a third corner and run your finger along its edges. Say, "This is another corner of the table top." Get a child to put his finger on the tip of the corner. Ask, "Is there another corner on the table top?" (Yes.) Let another pupil point it out. Get a third pupil to put his finger on the tip of the corner. Each child should say, "This is the tip of the corner."

Point out corners of other objects in the room, for example, of books, blackboards, and pieces of paper. Ask children to find still other corners. Ask each child to put a finger on the tip of any corner he has chosen.

Ask the pupils to look at objects on their desks. Say, "Point to a corner." Walk around the room and help where needed. Say, "Put your finger on the tip of the corner." The pupils may choose corners of their desks, of books, of papers, and of slates. They should not choose corners that are rounded.

Pick up a book or rectangular piece of cardboard. Point out one of its corners and the tip of the corner. Say, "I am going to draw on the blackboard a picture of just the tip of this corner." Emphasize "tip". Place the book or cardboard corner against the blackboard. Where the tip of the corner is on the blackboard put a chalk dot. Make the dot large enough to be seen by all the class but not larger than necessary. Say, "We will call this dot on the blackboard a point. Whenever we make a dot on our blackboard or paper or slates it is a picture of a point.

Get each child to choose an object with corners. Let him trace the tips on his paper. Ask some of the children to hold up their papers, point to the dots they have made, and say, for each, "This is a point."

Activity 2: Drawing the point made by the tip of a paper corner.

Fold a piece of paper to make a straight edge. Fold it again to make a square corner. (See Primary I, Unit 13, Stage 3, Activity 4.) Ask, "What does this show?" (A square corner.) "Does it have a tip?" (Yes.) Ask a child to point to the tip. Say, "This tip shows a point." Ask, "How can I draw that point?" Put the square corner flat on the blackboard and make a chalk dot at the tip. Say, "I made a square corner. The corner has a tip. I made a picture of the tip of the corner. The picture is a point."

Let each child make a square corner of paper and draw on paper a picture of the tip. Ask the children to say, "This is a point."

(Let them keep their square corners of folded paper for Activity 4.)

Activity 3: Revising line segments.

Point to objects that have edges that are straight; such as tables, books, paper and walls. Run your finger along the edges that are straight and say, "This is a straight edge." Hold a string stretched tightly between your hands and say, "This is straight. We use a stretched string to see if an edge is straight." Hold the stretched string along some edges to test their straightness.

Trace some of the straight edges on the blackboard. Tell the pupils that when a straight edge is traced on the blackboard, or on paper, "We made a line segment." Let a few of the children trace straight edges on the blackboard and ask them what figure they have drawn. (Line segment.) Test these for straightness with a stretched string.

Ask the children to find some objects on their desks that have straight edges. Let them trace these on their papers. Ask them to say, "This is a line segment."

Activity 4: Finding the endpoints of a line segment.

Take the square corner you used in Activity 2 and unfold it once. Say, "I do not have a square corner. I have one straight edge." Run your finger along the straight edge and ask, "What is this?" (A straight edge.) Trace the straight edge on the blackboard and ask, "What have I made?" (A line segment.)

Ask, "Does the straight edge have corners?" Let the children talk about this. Try to get them to say that at each end of the paper straight edge there is a corner and the corner has a tip. Hold the paper straight edge in position against the line segment, put your finger at one end of the straight edge, and ask "Can I draw a picture of the tip of this corner?" Draw this dot.

Remove the paper and repeat for the other corner. Say, "I have drawn a picture of the straight edge and a picture of each corner tip." Run your finger along the line segment and ask, "What is this?" (A line segment.) Point to each of the dots in turn and ask, "What is this?" (A point.) As you point again at one dot, say, "At this end of our line segment is a _____." Get the children to say the word "point". Repeat for the other end. Say, "At each end of our line segment there is a point."

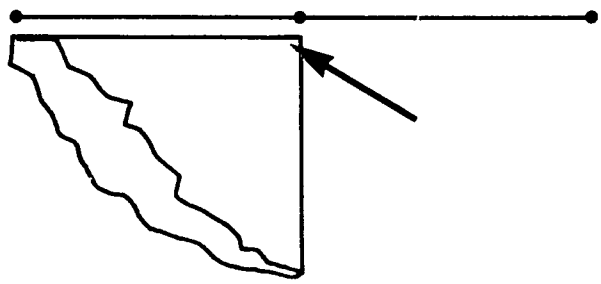
Repeat the above, tracing the edge of a book. Say, "When we draw a line segment it has a point at each end. These points have names. They are called the endpoints of the line segments."

Let the children in turn, choose objects with straight edges and trace the edges on the blackboard. Ask them what they have drawn. (Line segments.) Ask them what are at the ends of their line segments. (Endpoints.) Have them draw these points.

Get each child to draw several line segments and their endpoints on his paper.

Activity 5 (Optional Activity): Other points on a line segment.

Using the paper straight edge you used in Activity 4, draw a line segment on the blackboard. Draw its endpoints. Say, as you point, "This line segment has two endpoints, one here and one here." Holding the paper in place on the blackboard, fold it over to make the square corner you made in Activity 2. The children should see this:



Point to the square corner and ask if it has a tip. Say, "I am drawing a picture of the tip. Draw the picture, making a dot that is on the line segment. Ask, "What is the dot?" (It is a picture of the tip of the square corner. It is a point.) "Is it on the line segment or off the line." (On.) "Is it one of the endpoints of the line segment?" (No.)

Ask, "How many points can we see on the line segment?" (Three.) Unfold the square corner and ask, "Can I make any more points on the line segment?" (Yes. Make a different square corner with the paper.) Fold the paper over to make a square corner at a different place on the paper straight edge.

Draw the picture of the tip of this corner. Say, "This dot gives another point on the line segment. How many points can you see on the line segment now?" (Four.) "Can you find any more points on the line segment?" (Yes.) Let the children draw several more points. Let them talk about how many points they could make on the line. (More than we could count.)

Activity 6: "Someone took away my line segment."

Hold a book in your hand. Use it to draw a line segment on the blackboard. Mark the endpoints with dots. Say, "These are the endpoints of the line segment."

Say, "One day I drew a line segment just like this. I left it and went home to sleep. When I cam back the next morning, Mr. Spider had been here. He took away my line segment, like this. (Rub off all of the line segment except the two dots at the ends.) All that he left me was the two endpoints. How can I make my line segment come back?" The children may see the book still in your hand, and tell you to use the book to trace the line segment again. If not, ask questions and get them to tell you to fit an edge of the book on the two endpoints on the blackboard and trace the line segment. Ask if they think this will bring it back. Trace the line segment again , and ask if they think it has come back. Remove the book to show it.

Ask a child to draw a line segment on the blackboard and to mark the endpoints. Get a second child to play Mr. Spider and to rub off all but the two endpoints. Ask the first to make the line segment come back. Help both children if needed. Try to make a game of it. Repeat with several pairs of children. With some, get the first child to turn his back before Mr. Spider does the erasing.

Rub the figures off the blackboard. Say, "Now I will not show you the line segment. I will show you only what Mr. Spider left." Make dots for the endpoints of the line segment. Ask, "Can you make the line segment come back?" Let the children tell you to fit a book to the two dots and trace along it from one point to the other. Get a child to do this. Help him to do the fitting and drawing. Ask, "Do you think this is just like the line segment Mr. Spider took away?"

Rub the figures off the blackboard again and say, "Another day I drew a line segment and marked the endpoints. Mr. Spider took away that line segment and left only these." Draw two different dots on the board. Ask, "How can we make the line segment come back?" Again they should say that you fit a straight edge to the two points and trace along the straight edge between the points.

Let the children work in pairs. Tell one child in each pair to draw two dots on the paper of the other child. Let the second child "put back the line segment" between the two points. Ask the second child to draw two dots, and the first child to put back the line segment" between the two points. Repeat this a few times more.

Draw two points on the blackboard. Say, "If I draw <u>any</u> two points like these on the blackboard we can play a game. I can pretend a line segment was there and someone took it away. Can you make it come back?" Let the children talk about this. Ask a child to draw the line segment. Say, "If anyone draws two points for you, you can think, "These are the endpoints of a line segment. I know how to find the line segment. I put a straight edge between the two points and I draw from one point to the other along the edge. This makes the line segment between the two points."

Repeat with other pairs of points on the blackboard. Go around the room drawing pairs of points on pupils' papers. Let them find the line segments between the points. Ask the pupils to start at one point and end at the other point.

<u>Activity 7</u>: Naming endpoints and line segments. Pupil Page 39

Let the children talk about these things: (1) A line segment has two endpoints; (2) If two points are drawn we can draw the line segment between the two points.

Draw several line segments on the blackboard. Make them of different lengths and going in different directions. Point to one of the line segments. Say, "I want to talk about this line segment. Point to another line segment and say the same thing. Point to a child and say, "If you are not looking, how do I talk about

this boy so you know which boy I mean?" Go on with this type of question. Get the children to say that line segments need names just as people need names.

Say, "We do not name points with the same names as we name people. We could call this point Bill and this one Ade, but we do not. We use letters to name the endpoints of the line segment, one letter for each endpoint."

Draw a line segment on the blackboard and mark its endpoints with dots. Say, "I will name this endpoint A." Write A by the endpoint. Ask someone to suggest a letter name for the other endpoint. Write the letter over the point. Be sure the two endpoints get different names. Suppose they are A and D. Say, "Now we have names for the two endpoints. One is named A and the other is named D. Could we have used other names for these two points?" Let the children give other letter-names, say that these names could have been used, but we will use A and D.

Say, "Now the endpoints of this line segment have names. What name can we give the line segment?" Let the pupils talk about this. Say, "This line segment has two endpoints, named A and D. It is the line segment between the points A and D. We call it 'AD'." Write "AD" over the line segment. Ask several children to repeat this name. Tell them that the name "DA" is also a name for the line segment.

Draw another line segment on the blackboard. Get the children to name its endpoints and to name the line segment. Get several children to repeat the name of this line segment.

Draw two points on the blackboard. Get the children to name them, for example, G and S. Say, "We have two points. Can we use them to find a line segment?" (Yes. Use a straight edge to draw a line segment from G to S.) Ask, "Can we name the line segment?" (GS)

Repeat this with other pairs of points.

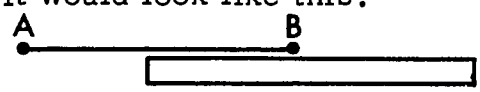
Now ask the pupils to open their books to page 39. Ask what is pictured on the page. (Lines, line segments and their endpoints.) You name line segments

and let the children find them. Let the children point to each line segment and name it. (Notice that one figure is not a line segment.)

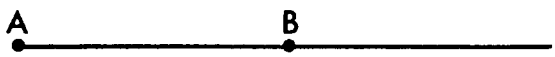
Activity 8: Extending a line segment to make a straight line.

Draw a line segment on the blackboard. Name the endpoints A and B. Say, "I have drawn a line segment, AB." Pick up an object with a straight edge longer than AB. Run your finger along the straight edge and say, "I can use this to draw another line segment."

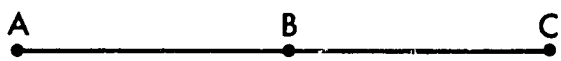
Fit the straight edge along line segment AB but let the right half extend to the right of B. It would look like this.



Trace along the straight edge. Say as you trace, "I am drawing a line segment with this straight edge." Remove the straight edge. On the blackboard the pupils can see this picture.



Say as you trace along the line segment with your finger, "This is a new line segment." Ask a child to name its right hand endpoint. (Suppose he chooses C.) The blackboard will look like this.



Trace again from A to C. Ask a child to name the line segment. (AC) Say, "We have extended line segment AB to make line segment AC." Use the same plan and extend line segment AC to the right again. Repeat again and again until the line segment goes near the edge of the blackboard. When this is done you have another line segment, say, AE.

Use the same activity to extend the line segment to the left. Fit the straight edge under AB with part of the edge extending to the left of A. Trace from A to

the left end of the straight edge. If you name the new left hand endpoint M the picture should look like this:

Let the children extend the line segment again and again to the right and left until there is no more room on the blackboard. Ask, "Can you extend the line segment now?" (There is no blackboard.) Ask, "Could you extend it by drawing on the wall? (Yes.) Could the line segment go through the wall?"

By questions like these help the children to imagine that the line segment could be extended many many times to make new line segments.

Say, "Think of a line segment which is extended on and on many times. We begin with line segment AB. We extend it many times today. We go home and sleep. We begin again tomorrow. If we went on and on we could extend it forever. A line segment extended like this becomes a straight line.

Stage 2. Congruent line segments.

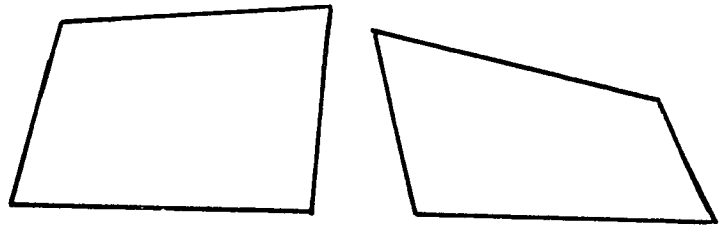
Vocabulary: Congruent, not congruent.

Materials: Two specially chosen cardboard cutouts ; two specially chosen straight edges about 2 feet long, one about 2 inches longer than the other; paper strips, string, pupil page 40.

Teaching Procedure:

Activity 1: Straight edges that fit each other exactly.

Make two cardboard cutouts that do not fit exactly, but which have some edges that fit each other. Here are two, for example:

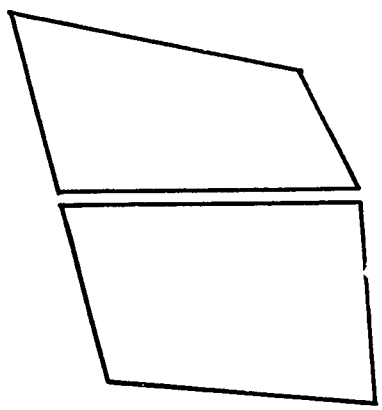


(The two lefthand edges fit each other, and so do the two bottom edges.)

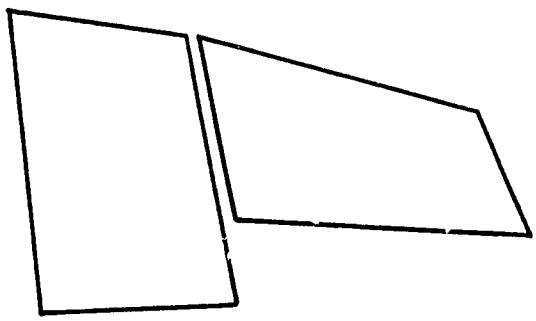
Talk with the children about one thing "fitting" another. For example, a shoe fitting a foot, a button fitting a button-hole, a banana skin fitting a banana, and a door fitting a doorway.

Hold up the two cardboards in your hands and ask, "Does one of these cardboards fit the other?" Let the children talk about this. They could talk of fitting one cardboard on top of another to see if they fit exactly or not. Try fitting the two cardboards this way.

Since the two do not fit, the children may say, "Try to fit one edge of one cardboard to an edge of the other." Ask them to show this. They may fit them as shown below.



Then say, "These two edges fit each other." Two edges that do not fit are these:



Let the children try to fit different edges of one cardboard to the edges of the other. They find some pairs that fit each other and some that do not. Choose one pair that fit. Say, "Here are two straight edges that exactly fit each other, from one end to the other." Show another pair that do not fit each other exactly and say, as you point to the edges and the ends, "These two edges do <u>not</u> fit

exactly. When we put them together and fit this end of this edge with this end of the other edge, the other two ends here do not fit. There is a left-over part of one edge."

Pick out two children seated on opposite sides of the classroom. Hand one cutout to one child, and the other cutout to the other child. Say, "Suppose the first cardboard belongs to Ben and the other to Helen. Suppose we wanted to see if this edge of Ben's cardboard fitted this edge of Helen's." Mark with crayon two edges that you know will fit together. "But suppose Ben and Helen are angry with each other and will not let us fit their cutouts together. Can you think of a way we could find out if those two edges really fit each other?" Remind the children that Ben and Helen will not let their cutouts be fitted together, but they will let us look at and handle their cutouts separately.

Let the children talk about this and give their ideas. They may say, "Let someone go look at the edge of Ben's cardboard and then go look at the edge of Helen's. Let them then say whether they will fit or not." If they say this, point out that this would be guessing. If the children do not suggest a "go-between", you suggest it. A "go-between" is something that can go from the edge of one cut-out to the other. A person going back and forth is not enough; but perhaps a person with "something else" is.

Ask what "something else" could be used. (Paper or string.) The go-between person could take a piece of paper whose edge fits exactly along one edge on Ben's cutouts. The "go-between" person could now fit the folded strip of paper to the edge of Helen's cardboard. If they fit exactly, just as the paper strip did with the edge of Ben's cutout then the two cutout edges fit exactly.

Choose one child to act as the "go-between" person. Help another child choose a strip of paper for the "go-between" edge (paper strip). Let the "go-between" child fit the "go-between" edge first to the edge of Ben's cutout and then to the edge of Helen's. Let the class decide whether the two edges fit each other exactly.

Repeat this, using string instead of paper. (String is better than paper. It doesn't need to be folded.)

Say, "Suppose Ben and Helen won't let us fit paper or string to the edges of their cardboards, but they will let us trace the edges." Take Ben's cardboard to the blackboard, draw a line segment along the marked edge, and return the cardboard to Ben. Repeat with Helen's cardboard. Say, "That's all that Ben and Helen will let us do with their cardboards. We have only pictures of the two edges. How can we use these pictures to see if the edges fit exactly?" This time some child should suggest using paper or string on the pictures. If they do not, you suggest this. Use paper and string to see if the pictures fit exactly. Say, "If the paper or string fits the two pictures exactly then we know that the two edges fit each other exactly. We know this in spite of Ben and Helen not letting us fit the edges together."

Activity 2: Congruent line segments Pupil Page 40

Take two straight edges, both about 2 feet long but one being 2 inches longer than the other. Trace the longer one vertically on the left of the blackboard and the shorter one horizontally on the right of the blackboard. Put the two straight edges at opposite ends of your desk. Say, "We're going to play a game about Mr. Spider. We are going to suppose Mr. Spider comes in and takes away these two straight edges." Have a child play Mr. Spider. Ask him to hide the straight edges in the room. Say, "Mr. Spider is laughing at us. He said we do not know whether the two straight edges fit exactly. But I played a trick on Mr. Spider. I made these pictures (point to the line segments on the blackboard) of the straight edges. Can we show Mr. Spider that we know whether the edges fit exactly?" The children may suggest matching a string which fits one line segment exactly to the other line segment. Let another child do this. They will find the string does not exactly fit the second line segment. Say, "Mr. Spider was fooled. What do we know?" (The two straight edges do not fit exactly.)

Say, "If we have two line segments on the blackboard we can not move one next to the other to see if they fit exactly. We can not move chalk marks across

the blackboard. And we can not bend the blackboard over to try to fit them together. But we can use paper or string, just as we did to fool Mr. Spider."

Repeat the above with two straight edges which fit exactly. Tell the children that the two line segments which fit exactly are congruent line segments. Let them repeat the word. Ask them to say, "The two line segments will fit exactly. They are congruent line segments."

Get the pupils to find page 40 in their books. Give each child a piece of string about 7 inches long. Tell them to use their string in each exercise to see whether the two line segments will fit each other exactly.

Tell them to put a bean on each pair that is congruent. Walk around helping, checking and correcting. Get the children to point to congruent pairs and say, "This pair of line segments is congruent."

Stage 3: Introduction to Angles

Vocabulary: Two line segments end at the same point, angle, right angle, comparing corners.

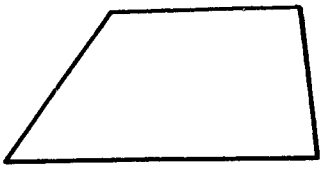
Materials: Paper straight edges, paper square corners, sticks, a specially cut piece of cardboard, classroom objects, a rectangular piece of paper for each child, scissors, four-sided cardboard cutouts , pupil pages 41 and 42.

Teaching Procedure:

Note: In the activities which follow have the children make angles by tracing the corners of cardboard cutouts , by tracing a pair of sticks laid end to end, and by tracing a straight edge in two different positions that are end to end; make angles by tracing square corners, and name this kind of angle "right angle"; decide which figures are angles and which ones are not, and which angles are right angles and which are not.

Activity 1: Introducing angles. Pupil Page 41

Hold up a large piece of cardboard shaped like this:

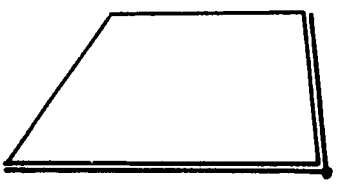


Run your finger along the bottom edge and say, "This is an edge of this piece of cardboard. Is it straight? What do we call it?" (A straight edge.) Trace the straight edge on the blackboard and say, "When I trace the straight edge on the blackboard, what figure have I made?" (A line segment.)

Run your finger along the right-hand edge of the cardboard. Say, "Here is another straight edge." Ask what figure you would get if you traced it. Trace it on the blackboard but some distance away from the first line segment.

Say, as you point to the proper places on the blackboard and cardboard, "This is a line segment. It is a picture of this straight edge. This is another line segment. It is a picture of my other straight edge. Did I trace the line segments as I should have? They may reply that the two line segments should come together. If they do not say this, go back to the cardboard edges and say, "The two edges meet at the tip of a corner." Get the pupils to tell you to trace the two edges again so that the ends of the line segments touch.

Hold up the cardboard, point to the bottom edge and the right-hand edge, and say, "There are two straight edges that meet at this tip. I am going to trace the edges and the tip, all together, on the board." Holding the cardboard in one position, trace the two edges and mark a dot at the tip.

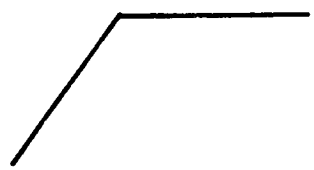


Remove the cardboard. Ask the children whether they think this picture is better than the first picture. They will agree that it is.

Put the cardboard against the blackboard again. Run your finger along the two edges you traced and say, "Here are two straight edges. I traced them and made these line segments." Indicate the segments. "The two edges meet at the tip of this corner. Do the two line segments meet?" (Yes.) "Where?" (At the dot.) "When we make a dot on the blackboard or paper, what do we call it?" (A point.)

Remove the cardboard and say, "I have made a figure. Tell me about it." They will say they see two line segments and a point. Run your finger along one line segment towards the point and say, "This line segment ends at this point." Run your finger along the other and say, "And this other line segment also ends at that point. So our figure is made of <u>two line segments</u> that end at the same point. There is a name for this figure. It is called an angle." Get the children to repeat, "Angle." Leave the figure on the blackboard.

Pick up the cardboard cutout. Point to the left hand edge and the top edge. Trace along the edges with your finger. Ask if the straight edges meet. (Yes.) Ask where they meet. (At the corner tip.) If I put this cardboard on the blackboard and trace these two edges, I will make a figure. What do we call the figure?" (An angle.) Trace the two edges and make this figure.

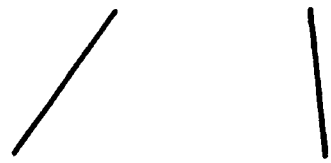


Ask, "What is the name of this figure?" (Angle.) What makes an angle?" (Two line segments that end at the same point.) Leave this figure on the blackboard.

Repeat this plan using the bottom edge and the left edge of the cardboard.

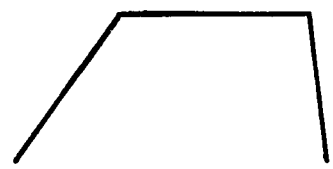
Point to the last figure you have drawn and ask, "How many line segments are in this figure?" (Two.) "Do they end at the same point?" (Yes.) "What kind of a figure is this?" (Angle.) Repeat with the other two figures. Let the children say again that an angle is made of two line segments that end at the same point.

Trace the left side and the right side of the cardboard cutout getting this figure:



Ask, "How many line segments are here?" (Two.) "Do they end at the same point?" (No.) "Is it an angle?" (No. The two line segments do not end at the same point.)

Draw the left edge, top edge, and right edge together. See below.



Ask, "Is this figure an angle?" (No. It is made of three line segments. It is not an angle.) Some child may say, "But I see an angle in the picture." or "There are two angles in the picture." Say, "You are right. Each of these two angles is a part of the figure. The whole picture is not an angle."

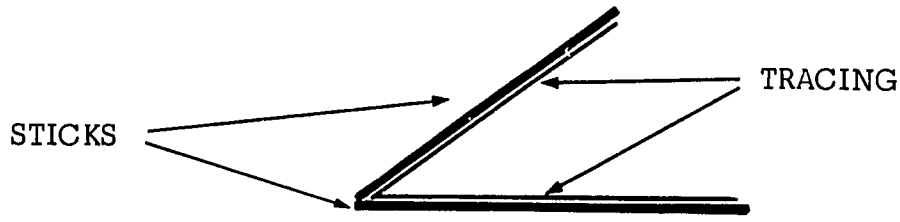
Tell the children to turn to page 41 in their books. Let them decide which figures on the page are angles. Ask them to put markers on the figures that are angles.

<u>Activity 2</u>: Making angles using sticks and cardboard cutouts.

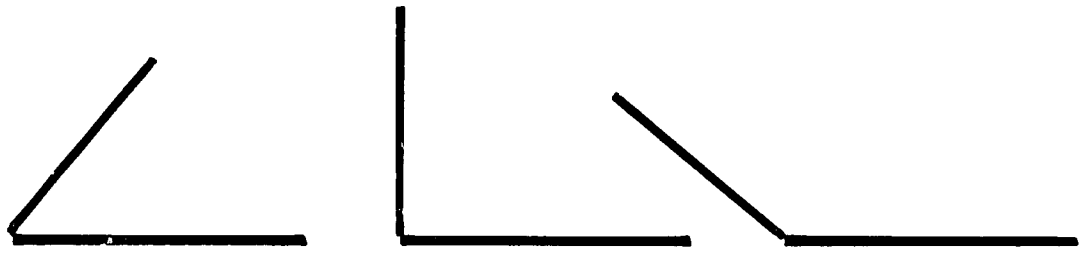
Ask the children what classroom objects can be used to trace angles. They may say books, desk tops, table tops and pieces of paper show pairs of straight edges (perhaps several pairs) that end at the same point. Pick up an object mentioned. Trace along two edges which meet at their end points and say, "These two edges show an angle."

Ask if a book could be used to trace an angle. Ask if a table top could be used to trace an angle. Put some sticks on the desk and ask if these could be used to make an angle. Ask how many sticks you would need. When there is an

answer "Two," hold two sticks against the blackboard not end to end and ask if you could make an angle from them if you traced the sticks as they are. (No. They do not end at the same point.) Put an end of one stick against an end of the other stick and ask if you can now make an angle by tracing the sticks. Get a child to help you trace the angle made by the two sticks on the blackboard like this:



Hold one stick in place against the blackboard. Put the other stick as it ends at the same point as the first. Keep the two ends together; move the second stick into different positions. For example, like this:

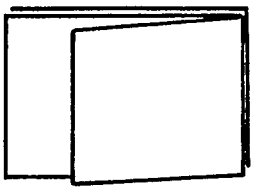


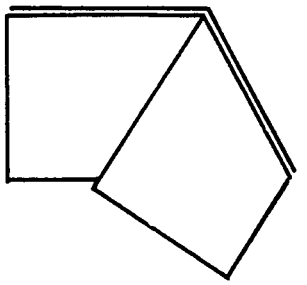
Let the children talk about the many angles that could be made by putting the second stick in different positions so that the two sticks end at the same point. Trace some of these angles on the blackboard.

Separate the children into two groups. In one group give each child some sticks and ask him to put the sticks on his desk to show angles. Walk around the room making sure the sticks are laid down in pairs so they end at the same point. Get each child to trace on paper one of the pairs of sticks. Give each child in the other group a cardboard cutout and ask him to make on his paper as many angles as he can.

Activity 3: Drawing angles, using folded paper or a straight edge.

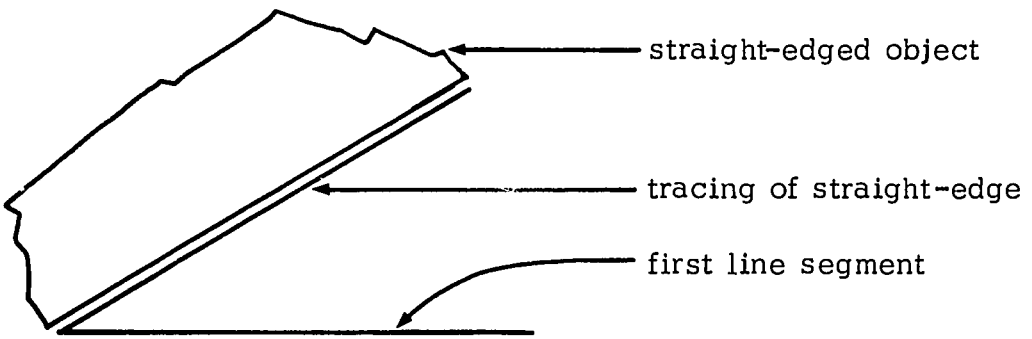
Put a square corner made from folded paper against the blackboard. Trace the corner. (See the left picture below.) Ask, "What figure is this?" (Angle.) "What goes together to make an angle?" (Two line segments which end at the same point.) Unfold the paper and fold again as in the right picture below.





Trace the top edge and the right edge on the blackboard to show another example of an angle. Let the children talk about different ways paper can be folded and traced to make angles. Let them draw angles on their papers by folding paper and tracing the angles shown.

Hold up a paper which shows a straight edge and ask, "Can I use just one straight edge to make an angle?" The children may answer, "No." Trace the straight edge on the blackboard and say, "There is one line segment. Does it make an angle?" (No.) What else is needed?" (Another line segment which ends at the same point as the first line segment.) Fit one end of the straight edge to one end of the line segment and trace the straight edge. Ask as you point, "Is this an angle?"



Show the children that whenever the straight edge has one end fitted to the line segment, tracing the straight edge will make an angle.

Get each child to use a straight edge to draw angles on his paper. Go around the classroom giving help as needed.

Activity 4: Introducing right angles. Pupil Page 42

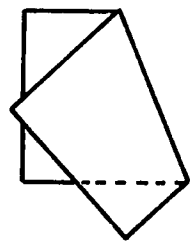
Hold up a square corner made from folded paper. Ask, "What kind of corner is this?" (Square.) "Can you use it to make an angle?" (Yes.) Trace the angle on the blackboard and say, "This is a special kind of angle. It is called a right angle." Get the children to point out square corners in the classroom. Ask them to run their fingers along the edges which form the corners and say, "This is a right angle." Let each child fold a piece of paper to show a square corner and a right angle.

Tell some children to choose objects that show square corners. Get them to trace the corners on the blackboard. Let others show the angles are right angles by fitting their square corners made of paper to them.

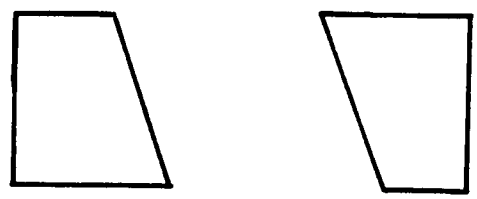
Ask the children to find page 42 in their books. Tell them to use their square corners to find the right angles on the page. Ask them to put a bean on each right angle. Go around the class to help. Ask some children to move their fingers along the line segments and say, "These line segments make a right angle," or "This angle is a right angle," or "This angle is not a right angle."

Activity 5: Finding right angles on paper cutouts.

Give each child a rectangular piece of paper and ask him to fold down one corner like this:

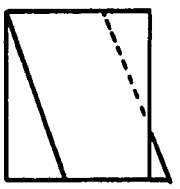


Get him to cut the paper along the edge of the fold. They each now have two pieces of paper like these:



Ask several children to hold up their two pieces of paper and show the corners that are square and the corners that are not square. Point to corners of some papers and let the children run their fingers along the two edges of the corners. Ask for each, "Can you use this to trace a right angle?" The children should use their paper square corners to help them decide. Let the children trace the corners after they say whether the corners can be used to trace right angles.

Show the children how to compare a corner of one piece with a corner of the other. For example, suppose the two pieces are like the ones in the figure above. Fit the top left corner of the second piece to the top left corner of the first, slide the second piece straight across onto the first, giving this picture:



This shows that the corners do not fit exactly. The top left corner of the second piece is inside the top left corner of the first. Get the children to say the corners do not fit. Let the children compare the angles on their two cutouts. Let them try to fit one corner to another. First put one corner on top of the other so that the tip of the second corner fits on the tip of the first corner, and one edge of the second corner fits on an edge of the first. Get the children to say, "The two corners fit exactly," or "They do not fit exactly."

Let the children compare the other angles on their cutouts. Each time they say, "The two corners fit exactly," or "The two corners do not fit exactly." Also get them to use their square corners to tell which corners can be used to trace right angles.

Stage 4: Introducing Triangles.

Vocabulary: Three-sided figure, four-sided figure, five-sided figure; triangle, right-angled triangle.

Materials: Three-edged, four-edged, and five-edged cardboard cutouts, both large for your use and small for the children; paper, scissors; three-edged

cardboard cutouts, some having a square corner, others having two or three edges of equal length; pupil pages 43-45.

Teaching Procedure:

Activity 1: Tracing three-sided objects.

Put out a pile of cardboard cutouts with edges that are straight. (Make four or five cutouts with three edges, four or five with four edges, and two with five edges.) Hold up a cutout with three straight edges, run your finger along one of the edges. Say, "This is a straight edge. If I trace it on the blackboard, what will I make?" (Line segment.) Repeat with each of the other two edges. Say, "How many edges does this cutout have? (Three.) Each edge makes a line segment when I trace it. How many line segments can I make from this cutout?" (Three.) Trace the cutout on the blackboard. As you trace each edge, say, "This makes a line segment." Remove the cutout. Say, "I have made a figure. How many line segments are in the figure?" (Three.) Run your finger along the line segments and say, "There are three line segments in this figure. Each pair ends at the same point. The figure has three sides. It is a three-sided figure."

Repeat with another cutout that has three edges. There are now two three-sided figures on the blackboard. Point to them and say, "Here are two figures. Each has three sides."

Hold up a four-sided cutout. Run your finger along its edges and ask, "How many edges are on this cutout?" (Four.) "I will trace this cutout on the blackboard. This will make a figure. How many sides will the figure have?" (Four.) Trace the cutout and say, "This is a four-sided figure."

Ask a child to point out a three-sided figure on the blackboard. Ask another to point out a four-sided figure. Ask a third to point out another three-sided figure, and so on. Get each of these three children to tell the number of sides of the figure he chooses.

Hold up a cutout with five edges. Get the children to tell the number of edges. Ask, "If I traced a figure from this cutout, how many sides would it have?"

Trace the cutout and tell the number of line segments you draw. Remove the cutout and ask the pupils to tell the number of sides the figure has.

Say, as you point to the different figures, "Some figures have three sides, some have four sides, and some have five sides. A three-sided figure has a special name, triangle." Let the children repeat, "Triangle." Point to one of the three-sided figures on the blackboard and ask, "How many sides in this figure?" (Three.) Is it a triangle? (Yes.) Why?" (It has three sides.)

Point to the four-sided figure and ask a child how many sides it has. Ask another child, "Is this a triangle?" (No.) Ask him why it is not a triangle. Point to the five-sided figure and ask if it is a triangle. (No.) Now point to the second three-sided figure and ask, "Is this a triangle? (Yes.) Why?" Point in turn to each figure on the blackboard. Let the children say, "This is a triangle," or "This is not a triangle." Each time let some child tell why it is or is not a triangle.

Pick up a three-sided cutout and ask, "If I trace this cutout I will make a figure. How many sides will it have? (Three.) What do we call a figure with three sides?" (Triangle.) Run your finger along the three edges of this cutout and say, "I can trace a triangle from this. It has three edges. I am going to put it on this end of my table."

Pick up a four-sided cutout and ask, "Can I use this cutout to make a triangle?" (No.) "Why?" (It has four edges, not three.) Say, "Cutouts that cannot be used to make triangles I will put on the other end of the table." Hold up another cutout and ask, "Can we use this to make a triangle?" Ask where it goes on the table. Repeat this with other cutouts.

Now ask a child to come up and choose a cutout. Ask him in which pile it should be placed and tell him to put it there. Repeat this until there are no more cutouts in the pile. Let the children help any child who chooses the wrong pile.

Trace on the blackboard some cutouts from each of the two piles. Trace the triangles on one end of the blackboard; trace the four-sided and five-sided figures on the other end. Let the children talk about the two sets of figures. Let the children point to the figures and say if each is a triangle.

Give some small cardboard cutouts to each group of four children. Ask them to trace each cutout on their papers. Ask the children how many sides their figures have and if any are triangles.

Activity 2: Identifying triangles. Pupil Page 43

Tell the children to find page 43 in their book. Let them find the triangles on the page and place a marker on them. Go around the class helping as needed.

Activity 3: Introducing right-angled triangles. Pupil Page 44

Put out some cardboard cutouts with three edges each. Include two or three cutouts with square corners. Hold up one of the cutouts and ask, "How many edges? (Three.) How many corners?" (Three.) Ask, "Are any of the corners square corners?" Let the children talk about this. Get them to test to find out whether any of the corners are square corners. Let them fit a square corner to each of the three corners to test them for squareness.

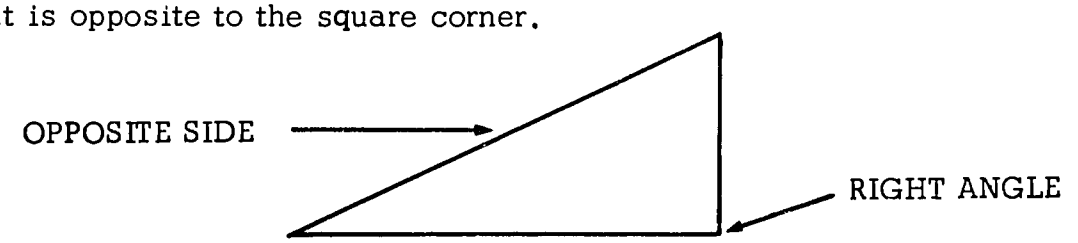
Repeat the plan above with two other cutouts. Be sure one of the three cutouts has a square corner. When the children talk about the cutout with a square corner be sure they say that it has one square corner.

Say, "I am going to trace these three cutouts on the blackboard. I will make three triangles on the blackboard. Will one of the triangles have a square corner?" Let the children talk about this and say, "One of the cutouts has a square corner. The triangle traced from it will have a square corner." Trace the three cutouts. Ask, "Do you see a square corner in the figures? (Yes.) Which figure has a square corner? Which corner is square? How can we make sure the corner is square?" (It was traced from a square corner of the cutout. Or, we can fit a square corner to the corner.)

Pick up two different cutouts, one with a square corner and one without a square corner. Trace these on the blackboard. Ask, "Are there square corners in these figures? Where?" Let the children point to the square corner. "What do we call these angles?" (Right angles.)

There are now five triangles on the blackboard. Point to the two that have square corners and say, "This triangle has a right angle here. This other triangle has a right angle here. We have a special name for a triangle with a right angle. It is called a right-angled triangle." Point to one of the other three triangles and say, "This triangle does not have a square corner. It is not a right-angled triangle." Point to a second one that is not right-angled and ask a child, "Is this triangle a right-angled triangle?" Point to other triangles and get the children in turn to say, "That is a right-angled triangle," or "That is not a right-angled triangle." Let the class say whether each answer is right and why.

Ask the class if they know why a triangle with a square corner is called "right-angled". Let the children talk about this. Help by pointing to a right-angled triangle and ask, "Which corner is a square corner?" Cover with your arm the side that is opposite to the square corner.



Say, "I have covered one side of the triangle. How many sides are left?" (Two.) Run your finger along these two sides and say, "Here is a line segment, and here is another line segment. They end at the same point. When two line segments end at the same point what is the name of the figure they make?" (Angle.) "Is this angle a special angle?" (It is a right angle, because it makes a square corner.)

Rub the figure off the blackboard. Choose and trace around a cutout that shows a right-angled triangle. Put the cutout with the others and hide them. Say as you do this, "I am going to play a game. I am going to play that I made this triangle and then Mr. Spider hid my cutouts. How can I find out if this is a right-angled triangle?" Lead them to say, "Use a paper square corner to test each angle. If one angle is a right angle, the triangle is right-angled. If no angle is a right angle, the triangle is not a right-angled triangle." Let the children do this and say, "The square corner fits one angle. The triangle is right-angled." Bring

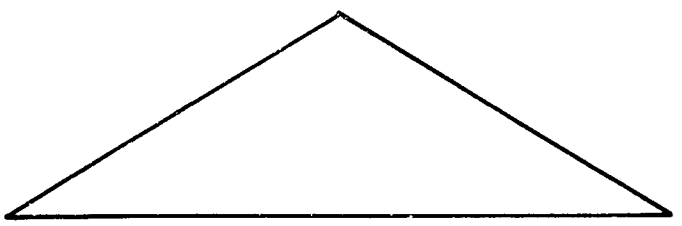
out the cutouts and say, "We know this triangle is right-angled. Mr. Spider did not keep us from finding out."

Repeat the above with a cutout that shows a triangle that is not right-angled.

Tell the pupils to open their books to page 44. Ask them to tell which of the triangles on the page are right-angled triangles. Let them use their paper square corners to test the angles of the triangles. Get them to put a marker on each triangle that is right-angled.

Activity 4: Some triangles have congruent sides. Pupil Page 45

Choose a cutout with two congruent edges. Hold it up. It may be shaped like this:



Say, "Here is a cutout. Tell about its edges." The children may say some or all of these things: There are three edges; the edges meet at three corners; if we trace the cutout we will make a triangle; two edges look alike. Get them to talk about the edges. They may say: The two edges meet at the top corner; the edges might fit each other. Ask questions until the last idea is said.

Ask, "How could we be sure the two edges fit exactly?" (Fold one edge over on the other.) "Is there any other way?" Let the class talk about this. A child may say that we can use paper or string. If not, follow the plan of Stage 2 and suggest a "go between". Let the children help you fit a string to the edges.

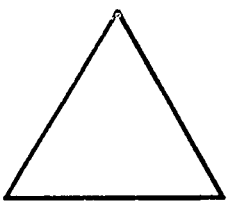
The children will say, "This figure has two edges that fit each other exactly. We fitted a string exactly to this edge and then, when we moved the string to the other edge, it fitted there exactly."

Trace the cutout on the blackboard. Ask, "What is the name of this figure? (Triangle.) How many sides has the triangle? (Three.) Tell me about the sides."

(The left side and the right side fit each other exactly.) Suggest that they use a string to test the sides. Let them do this and say, "This string fits each side exactly. That means that the two sides fit each other exactly. When two line segments fit each other exactly, what do we call them?" (Congruent.)

Say, "This triangle has two sides that fit exactly. It has two sides that are congruent." (The name for such a triangle is isosceles, but do not use it with the children.)

Now hold up a cutout like this with three congruent sides.



Repeat the plan above but fit the string to all three sides. Be brief when you can.

At the end of the activity tell the class, "This triangle has all three sides alike. Each side exactly fits each of the other sides exactly. The sides are all congruent. There are many triangles like this. (Such triangles are called equilateral, but do not use the term with the children of Primary II.)

Tell the pupils to find page 45 in their books. Ask them to tell which of the triangles on the page have two congruent sides, which have three congruent sides and which have none. Let them use string to see which sides are congruent. Tell them to put a bean on each of the congruent sides of a triangle. Some triangles will have beans on no sides, some on two sides and some on three sides. Let the children say which triangles have no congruent sides, two congruent sides or three congruent sides.

Stage 5: Revising Rectangles and Introducing Squares.

Vocabulary: Square.

Materials: String; a few cardboard cutouts with three edges, a few with five edges, and twelve or more with four edges. (The four-edged cutouts should be of three

kinds: those that do not show rectangles, those that show rectangles that are not squares, and those that show squares); pupil pages 46-49.

Teaching Procedure:

Activity 1: Introducing rectangles.

Put cardboard cutouts on your table. Make some of the cutouts with three edges, some with four edges, and some with five edges.

Hold up a cutout with three edges. Ask, "How many edges? (Three.) If I traced this on the blackboard, what kind of figure would I get?" (A three-sided figure or a triangle.) Repeat this with a four-edged cutout, and with a cutout having five edges.

Hold up a four-edged cutout. Run your fingers along its edges. Ask, "Does this cutout show a four-sided figure? (Yes.) Would I make a four-sided figure if I traced this cutout? (Yes.) I will put the cutouts that show four-sided figures at this end of the table." Hold up a three-edged, cutout and ask a child, "How many edges does this have? (Three.) What kind of figure does it show? (A triangle.) Should I put it on the end of the table with the four-edged cutout? (No.) I will put all the cutouts that do not show four-sided figures at the end of the table."

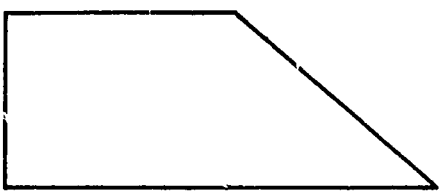
Go through the pile of cutouts this way. For each cutout ask a child, "How many edges does it have? Does it show a four-sided figure?" Put each cutout in the proper pile.

Keep the pile of four-edged figures on your table. Put away the other pile. Tell the class, "We are going to talk about four-sided figures. I put the other cutouts away."

Hold up a four-edged cutout that has all four corners square. Ask, "How many edges? (Four.) How many corners? (Four.) Can we say anything else about the corners?" (They look alike; or, they are all square.) Pick up a paper square corner and say, "Here is a square corner. We can test the corners of this cutout to see which ones are square." Let the children test each corner with the paper square corner. Get them to say that all four corners are square.

Now trace the cutout on the blackboard. Ask, "What kind of figure do we have? (Four-sided.) How many angles does it have?" (Four.) Ask, "Do you see anything special about the angles?" Since the class knows that the four corners of the cutout were all square, someone will say that each angle of the blackboard figure is a right angle. Let the children check each angle with a paper square corner. The children will say, "The four angles are right angles." Tell the children, "There is a special name for a four-sided figure that has four right angles. The name is '<u>rectangle</u>'." Let the children repeat the name "rectangle" several times.

Repeat the plan. This time use a cutout shaped like this:



Note: This cutout does <u>not</u> show a rectangle. Two of its corners are not square corners. The tracing will have two angles that are not right angles.

Separate the class into small groups. Give each group some four-edged cutouts. Be sure the children have paper square corners. Ask each group to put their cutouts into two piles: those that show rectangles and those that do not. Ask the children of each group why certain cutouts show rectangles. Get them to use the word "rectangle" in their answers.

<u>Activity 2</u>: Finding rectangles. Pupil Pages 46 to 47

Tell the children to turn to pages 46 and 47 in their books. Ask them to find all the figures that are rectangles and to put a marker on each rectangle. Go around the class giving help as needed. Keep asking, "Why is this a rectangle?" and "Why is this not a rectangle?"

<u>Activity 3</u>: Introducing squares.

Hold up a 2 feet by 1 foot cardboard cutout that shows a rectangle. Ask, "What figure does this show? (A rectangle.) Why?" (The cutout has four edges,

and the four corners are all square corners.) Get some children to test the corners for squareness. Get the children to say, "This cutout shows a rectangle." Let two children trace the cutout on the blackboard. Say, "Here is a rectangle. It has four angles. Are the angles all alike? (Yes.) Why?" (They are all right angles.)

Say, "The rectangle has four sides. The sides are line segments. Do the sides all look alike?" Let the children talk about the sides. Guide them to talk about the pairs of opposite sides. Ask, "Do the left and right sides look alike? Do the top and bottom sides look alike? (Yes.) How do they look alike?" Get the children to say the sides are congruent. Ask, "When are two line segments congruent?" Let the children say, "Two line segments are congruent if they fit each other exactly. Ask, "How can we find out if the top and bottom sides are congruent?" (Fit a stretched string to one side and then to the other side to see if the same string exactly fits both sides.) Let the children do this and say, "The top and bottom sides of this rectangle are congruent."

Repeat the above activity to answer the question, "Do the right and left sides look alike?"

Let the class tell what they know about the sides of the rectangle: (1) the top and bottom sides are congruent; (2) the left and right sides are congruent; (3) the top side and the right side are not congruent. Ask, "Are all four sides of this rectangle congruent?" (No. Some are congruent, but not all four.)

Hold up a large square cutout, and trace it on the blackboard. Get the children to tell about the rectangle. Guide them to talk about these things: the number of angles, whether the angles are all alike, whether it is a rectangle and what sides are congruent. If congruent sides are not mentioned ask, "Do the sides all look alike? (Yes.) Are all four sides congruent?" Get the children to use a stretched string to decide. Let them say, "This string fits all four sides exactly. This means all four sides are congruent."

Say, "This rectangle has four sides that are alike. The four sides are congruent. There is a special name for this kind of rectangle. It is called a square." Let the class repeat the word "square" several times.

Hold up, in turn, five or six cutouts, some showing rectangles that are not squares, the rest showing squares. Ask, as you hold up each, "Does this show a square." As the children answer, put the cutouts into two piles, one which shows squares, one which does not. Trace on the blackboard one cutout from each pile. Get the children to test the sides of each of these figures with stretched string to tell whether the figure is or is not a square.

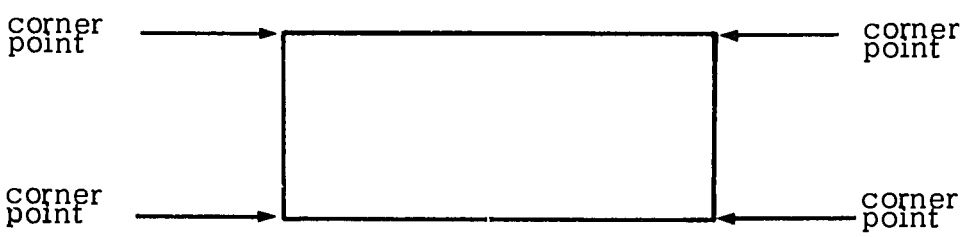
Separate the class into groups. Give each group some cutouts that show rectangles. Use some that show squares, and some that do not show squares. Ask each group to test the edges of each cutout to see whether it shows a square. Tell the children to put those cutouts that show squares into one pile, those that do not into another. Be sure each group has some string to use to test the edges.

Activity 4: Identifying squares. Pupil Pages 48-49

Tell the children to turn to pages 48 and 49. Ask them to find the squares and put a marker on each square. Go around the class giving help as needed. Ask often, "Why is this figure a square?" or "Why is this figure not a square?"

Activity 5: Naming a rectangle.

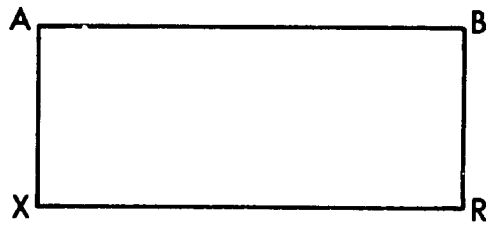
Draw a rectangle on the blackboard. Say, "We have talked about the four angles in a rectangle, and the four sides in a rectangle. Do you see four of anything else in this rectangle?" Let the class talk about this. Some one may say, "There are four corner points." They mean that at each of the four corners is a point. Point to one of the four points in the rectangle.



Say, "Here is a corner point." Ask children to point to other corner points. Ask, "How many corner points in this rectangle?" (Four.)

Ask, "What names do we use for these points?" (Letters. We use a letter for each point.) Let the children who pointed out the corner points give names to

their points. Put the letters on the blackboard next to the points they name. Suppose your figure now looks like this:



Say, "We have names for the four corner points of our rectangle. We put all these names together to make a name for our rectangle. We name it rectangle ABRX." Get the children to repeat this name several times.

Repeat with another rectangle and then with a square.

Stage 6: The Circle.

Vocabulary: round, circle.

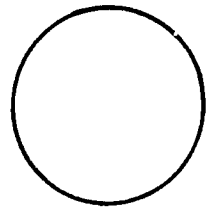
Materials: Ten or more cardboard cutouts, some showing rectangles, some showing three- and four-sided figures and about five showing circles; a book; three or four round objects such as plates and round tins; a piece of string about four feet long; many pieces of string about seven inches long; beans or other markers; pupil page 50.

Teaching Procedure:

Activity 1: Introducing round objects.

Hold up a large cardboard cutout that shows a rectangle. Run your finger around the edges. As you trace each side and each corner, get the children in turn to tell you what you are tracing. (A side or straight edge; a corner.) Get the children to say that the cutout has four straight edges and it has four corners.

Hold up a circular cardboard cutout like this:



(This can be made by placing a round plate, or the bottom of a large round tin, on a piece of cardboard, tracing around the plate or tin, and cutting along the mark.) Let the children talk about its shape. To get the talk started you may ask: "Is the cutout four-sided? (No.) Is it three-sided? (No.) Does it have any corners? (No.) Does it have any straight edges? (No.) Will a piece of stretched string fit anywhere along the edge? (No.)" Be sure that the children know that this cutout has no straight edges and no corners. Trace around the cutout with your finger. Show the children that your finger never turns a corner and never goes straight; it just goes around. Let some of the children run their fingers around the cutout and say, "This cutout has no corners and no straight edges."

Get them to say that some cutouts were called "four-sided" and some "three-sided". Ask, as you hold up the round cutout, "How do you describe a cutout like this?" Some child may know the word "round" and suggest it. If not, you say, "This is a round cutout."

Now hold up several cutouts, one after the other. Each time let the children, in turn, tell whether the cutout is three-sided, four-sided or round. Ask other children to come to your table and choose round cutouts from the pile of cutouts.

Choose two objects, one a book, the other a round object such as a plate or the base of a round tin. Hold these up. Ask questions like these about each object: (1) How many sides? (2) How many corners? (3) What figure does the book show? (4) Is this object round? Talk with the class about how the shape of the two objects differ. Get the children to use the words "four-sided" and "round".

Let the children name some objects that are round. They may name such objects as wheels, buttons, coins, and rings. Ask them to bring one round object each to school the next school day.

Activity 2: Recognizing round objects.

When the children bring to school objects they believe are round, go around the room holding up each object. Let the children say whether or not the object is round. Let them tell why it is round.

Let the children feel the edges of coins, buttons, and round tins. Get them to trace around the edges with their fingers. Ask them to roll some of the objects. If an object rolls smoothly on its edge it is round.

Let each child who brought a round object trace this object on his paper. Let others share the round objects and trace them on their papers. Ask the children to look at the figures they have made. Say, "This is a special figure." Let the children put their finger on the mark and trace the figure with their finger around and around. Let them say, "This figure has no straight sides. It has no angles."

Activity 3: Introducing circles by tracing.

Use the same cardboard cutouts that you used in Activity 1. Hold up a cutout that shows a rectangle. Ask what figure it shows. Trace the cutout on the blackboard and let the children tell you the kind of figure you have drawn.

Hold up a round cutout. Ask, "Does this show a figure?" Let the children talk about this. Some may say it does; others may say it does not. Ask, "Can I trace this on the blackboard?" Let the children help you trace the cutout. Say, "The cutout does show a figure. It is the figure we traced. Do you know a name for this figure?" If they do not, say, "This is a special figure. It is a circle."

Repeat this activity with several other cutouts. Use some cutouts that show circles and others that do not. Let some of the children help you choose cutouts to be traced. Let them look at the cutouts, feel them, and tell what figures the cutouts show.

Also trace books, plates, tins and other round objects on the blackboard. Ask the children to name the figures that are traced.

Activity 4: Making circles without tracing round objects.

Say, "Can you fold paper to show a circle." Give some of the children sheets of exercise paper and let them try. You take a sheet of exercise paper and try to do it yourself. The children will say that a circle can not be shown by folding a piece of paper.

Show the children a piece of string about four feet long. Ask if they can show a circle with the string. Guide them to say that you lay the string on the floor, move it so it goes around and one end of the string reaches the other end. Let some of the children do this. Ask if they think a circle is shown. (The string figure may be almost round. It will not be perfectly round.) Let the children talk about this. They may move the string to try to make the figure as round as possible. Say, "I will tie the ends together." There is now a string loop lying on the floor. Move the loop to make it as round as you can. Trace the string with chalk. Remove the string. Ask, "Have we made a circle?" Let the children say the circle is not as good as the circles they made by tracing plates and tins.

Say, "Let us make a better circle. Paper folding did not give us a good circle. String did not give us a good circle. I know how we can use paper <u>and</u> string together to make a good circle. (A child may suggest a string compass before you suggest it.) I want you to help me. I'll use the blackboard instead of paper so that you can see."

Follow this plan and draw a circle. Put the loop of string over one forefinger (of left hand) and press that fingertip against the blackboard. Let the loop hang against the blackboard.

Ask, "What do I do now?" Try any suggestion that is made. If none are made or if none work, put the tip of your other forefinger through the other end of the string loop. Stretch the loop as far as you can. Do not move the forefinger of the left hand. Ask the children again for suggestions. If none are given, move your right forefinger a little way along the blackboard. <u>Keep the string stretched and your left forefinger in place</u>. Again ask for suggestions. If none are made, move your right forefinger in longer and longer swings until someone says, "Swing it <u>all</u> the way around." Do this. Ask if you have made a circle. (You have. It cannot be seen on the blackboard.) Hold a piece of chalk in the end of the loop where your right forefinger was. Swing the chalk all around the blackboard until the chalk comes back to its starting point. Remove your hands and the string and ask, "Did I make a circle?" Say, "You can draw circles like this too." Put a

small loop of string, a piece of paper and a pencil on your table. Ask a child to come up and draw a circle. Put the loop over one of your fingers and place the tip of this finger firmly in the middle of the paper. Put the pencil tip inside the other end of the loop on the paper, and stretch the loop. Ask the child to hold the pencil and move it on the paper around your finger, just as you did on the blackboard. Help him to keep the string stretched and to move the pencil all the way around your finger. Tell the class about each thing you do. Show them the circle.

Separate the class into pairs. Give each pair a piece of string about 6 or 7 inches long. Get them to make a loop. Ask one child in each pair to hold one fingertip firmly in the center of a piece of paper with the loop around his finger. Let the other child trace a circle. He does these things: (1) put a pencil tip inside the loop; (2) pull the loop tight with the pencil; and (3) move the pencil on the paper all the way around the first child's finger tip. Tell the children to keep the loop stretched.

Go around the class and give help where needed. Let each pair repeat this several times. Be sure each of the children do some tracing.

Activity 5: Revision of figures. Pupil Page 50

Tell the pupils to turn to page 50 in their books. Ask them to put a marker on each figure that is a circle.

Stage 7: Objects Showing Roundness.

Vocabulary: Solid sphere, ball.

Materials: Ball, orange, egg, round tin, coin, button, balloon, box, round stick; objects showing roundness that are brought by the children; clay.

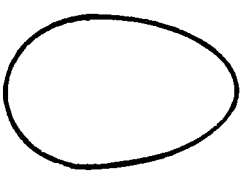
Teaching Procedure:

Note: In Primary I and in Stage 6 of this unit the idea of "roundness" was introduced by talking about objects such as coins, buttons, and tins, that have round edges. When these objects are traced we draw the geometric figure called a circle. A circle can be drawn on flat surfaces (blackboard and paper). In this

stage the children talk about other physical objects that show roundness. They will not trace these objects. The children learn only the shape of the objects.

First they talk of objects that can be called "perfectly round" (balls and marbles). Second, they talk of objects that show no straightness and no flatness but are not perfectly round (eggs, oranges, pineapples, seeds, ground nuts, and pebbles). Third, they talk of objects that are partly round and partly straight or flat (coins, buttons, round tins, bottles, round pans, pieces of bamboo and round sticks).

Before going to school, children learn about the roundness or straightness or flatness shown in objects around them. They learn about this in three ways: they <u>see</u> how the object looks; they <u>feel</u> it; and they watch how it <u>moves</u> (how it rolls or bounces or tumbles or floats). Any perfectly round object, such as a ball, always looks round from every viewpoint, it feels round every way it is held, and no matter how it is moved on a level surface, it rolls easily, smoothly, and straightly. An object, such as an egg, that has no straightness and no flatness but is not perfectly round, always looks "rounded" every way you look at it. But the kind of roundness you see changes. From end on, an egg looks round; from the side, an egg looks "rounded" but not round, as shown below.



If an egg is rolled on its side it will roll easily; but if it is rolled end over end it wobbles or stops rolling. A face of a coin shows roundness at its edge; but a coin looked at edge on shows straightness. The edge of a coin feels round, but its two faces feel flat. A coin rolled on its edge rolls easily and straightly; placed flat, a coin does not roll. The statements about a coin are true of a round tin.

In this stage, the children study the shape of <u>whole</u> physical objects. They think about objects that show roundness. They think about many objects that show perfect roundness, such as balls and marbles. They <u>see</u>, <u>feel</u>, and <u>roll</u> objects to learn that objects have different kinds of roundness. They learn that a ball always

looks round, always feels round, and always rolls easily, smoothly, and straightly. They learn that any other kind of object does not do at least one of these.

An object that is perfectly round is called a solid sphere. It is also called a ball.

Activity 1: Finding roundness by looking, feeling, and rolling.

Ask each child to bring to school something that shows some roundness. Some examples are balls, ground nuts, oranges, seeds, small wheels, bamboo, pebbles, round tins and round sticks. Put out on your table the first group of objects mentioned above in Materials.

Ask children in turn to show the objects they brought. Let the class talk about each of these objects in these ways: what parts of the object are round; what parts are straight or flat; whether one object has more roundness than another.

If a ball has been brought by some child, hold it up and ask the child if it looks round. (If no ball has been brought, use the one from your table.) Hold up the ball where you find it. Ask children in a different part of the room if it looks round from where they sit. Turn the ball around. Ask children if it looks round. Put the ball in a hand of a child and ask if it feels round. Turn the ball in his hand and ask the same question. Let others feel the ball and tell you whether it feels round every way they hold it. Let the class talk about how a ball looks and feels. Help them to say that every way you see a ball it looks round, and every way you hold a ball it feels round. Move the ball several times on the table and floor. Get the children to tell you it rolls easily, straightly, and smoothly.

Hold up the box from your collection and ask if anyone can see any roundness in it. Let some children feel it. Ask if they can feel any roundness. Show the box will not roll. Let some of the children try to roll it.

Repeat this activity with a round tin brought by some child. (If no child brought a tin use the one from your table.) Show the tin to different children in many ways. Each time ask, "Does it look round?" Help them to see that from the end it looks round, and from the side it shows straightness. Let the children feel

the tin. Let them feel the rounded side, the ends, the edges. Get some children to roll the tin on its sides and to try to roll it end over end. Get them to tell how the tin looks, feels and moves.

Get children in turn to tell how the rounded objects they brought show roundness and straightness and flatness as they look at their objects, as they feel them, and as they try to roll them.

Tell the children that they have seen objects that show roundness in different ways. Some are perfectly round, others are partly round, and some, like boxes, have no roundness. Tell them that perfectly round objects are called balls or solid spheres. Let them place their objects into two piles, one of objects that are perfectly round, and the other of objects that are not round.

Activity 2: Making balls from clay or mud.

Get each child to bring to school some clay or mud shaped into a ball or to be shaped into a ball. Let the children shape these as roundly as they can. Show them how to use seeing, feeling, and rolling to help them shape the balls. Let the children tell which balls are the roundest. Get the children to test the balls by seeing, feeling, and rolling them.

UNIT 4
ADDITION II

Objectives:

1. To learn that the sum of three numbers is the same any way the addends are grouped for addition.
2. To learn to find the sum of three or more numbers.
3. To learn to find the sum of numbers with 2-digit numerals (no renaming of the sum).
4. To learn the meaning of equality and inequality in addition sentences.
5. To learn to solve word problems using addition.
6. To understand the relationship of word sentences and number sentences.

Background Information for Teachers: See Unit 2, Addition I.

Stage 1: Addition of Numbers with Sums Greater than Eighteen

Vocabulary: Tens, ones, place, numeral, addend, sum

Materials: Counters, small objects of many kinds, bundles of sticks; number tray; pupils pages 51-56.

Activity 1: Tens and ones

Briefly revise the idea of place value. Use bundles of ten sticks to represent tens and single sticks to represent ones.

Show the children three bundles of sticks. Ask how many bundles there are. (3) Write "3" on the board. Ask, "Does the numeral mean three sticks?" (It means three tens, not three ones.) "How can we change this to read 3 tens?" Let a child show the numeral for thirty. (30)

Previous Page Blank

Say, "We know that the '3' means 3 tens. It is in the tens place. And what does the zero mean?" The children should recall that it means no single units or ones. Put the set of 3 bundles on your table.

Show the children two more bundles of sticks. Ask, "How many bundles of ten are here?" Ask a child to write a numeral on the board to represent the 2 tens (20). If a child writes "2", ask him to show what it means with sticks. Help him to see that "2" means two single sticks or 2 ones. He should understand that a "20" means 2 tens.

Pick up the 3 bundles of tens and the 2 bundles of tens. Join the two sets. Say, "We can put the set of three bundles together with the set of two bundles. When we do, we know that we add the number of members. How can we show this in a sentence?" Let a child write the sentence on the blackboard.

$$30 + 20 = \boxed{}$$

Say, "When we joined the two sets, how many bundles did we find in the new set? (5) Each bundle shows ten. We have 5 tens. Can you write the numeral for 5 tens in the box?"

The sentence should be completed as

$$30 + 20 = \boxed{50}$$

Let the children read the sentence. They may say, "Three tens plus two tens equal five tens". This helps them see the relation of the sentence 30 + 20 = 50 to the sentence 3 + 2 = 5. They may also read the sentence, "Thirty plus twenty equals fifty."

Give other examples of this kind. Join the sets of bundles of sticks. Show the mathematical sentences on the blackboard. Include some exercises in vertical form. For example, join a set of 4 bundles with a set of 4 other bundles and write this on the blackboard:

$$\begin{array}{r} 40 \\ +\ \underline{40} \end{array}$$

Continue as needed.

Activity 2: Pictures and number sentences | Pupil Page 51 |

Let the children study the pictures of the unions of sets on page 51. Ask them to think of the sentences the pictures show. Let them write the sentences in their exercise books.

Activity 3: Adding tens to make sentences true | Pupil Pages 52 to 53 |

The exercises on pages 52 and 53 let the children use what they learned in the last activity.

Let the children copy the sentences in exercise books. Ask them to make the sentences true. Some children may need their counting sticks to show the sentence. Put the number tray on the table too. Some children may need it.

Activity 4: Joining sets of sticks to show tens and ones

Put out some bundles of ten sticks and some single sticks on your table and the number tray.

Hold up three bundles of ten sticks and four single sticks. Ask, "How many bundles of ten sticks in this set?" (3) "How many single sticks?" (4) "How many sticks in the set?" (3 tens and 4) Ask the children to write a numeral for this on their papers. Then you write "34" on the blackboard.

Separate the set of sticks into bundles and single sticks. Hold the 3 bundles in one hand. Say, "Write the numerals for this many sticks on your papers." Hold up the 3 single sticks. Ask the children to write the numeral for this set of sticks. You write the numerals on the blackboard. Say, "We will put an addition sign between the numerals. Now we have another numeral for 3 tens and 4 ones." The blackboard looks like this. (Put the set of sticks in the number tray for use later.

34 30 + 4

Hold up a set of 1 bundle of sticks and 2 single sticks. Follow the plan just used to get the children to write the numeral "12" to show the number of sticks in

the set. Then follow the plan just used to get the children to write the numeral "10 + 2" to show the number of sticks in the set. You write these numerals on the blackboard under the "34" and "30 + 4".

Point to the number tray. Say, "In the number tray is our set of 3 tens and 4 sticks. In my hand is a set of 1 ten and 2 sticks. Will someone make a union of these two sets? Put them together in the number tray."

After a child does this, let another child tell about the action. (He put the 1 ten with the 3 tens in the TENS part of the tray. He put the 2 sticks with the 4 sticks in the ONES part of the tray.) Ask the children to write the mathematical sentence on their papers that they showed with the sticks. You write it on the blackboard. The blackboard now looks like this.

$$\begin{array}{r} 34 \\ \underline{+12} \end{array} \qquad \begin{array}{r} 30 + 4 \\ \underline{10 + 2} \end{array} \qquad 34 + 12 = \boxed{}$$

Ask, "How many single sticks in the new set?" (6) "How many tens?" (4) "How many sticks in the new set?" (4 tens and 6 ones) "How can we write that as a numeral?" (46 and 40 + 6) Ask the children to make their sentences true. You write "46" in the box on the blackboard and "40 + 6" under the "10 + 2".

Let the children show the addition of other numbers in the same way. (Do not have the sum of the ones or tens greater than 9.)

Activity 5: Addition

Pupil Pages 54 - 55

Revise Activity 4. Work out one or two examples on the blackboard with the pupils. Show one example in both vertical and horizontal form. First you could write

$$\begin{array}{r} 21 \\ \underline{+18} \end{array} \qquad \begin{array}{r} 20 + 1 \\ \underline{10 + 8} \end{array}$$

Ask the children to add the ones and to add the tens. Then show the same exercise as

$$21 + 18 = \square$$

Say, "How many ones are there? (1 + 8) How many tens are there? (2 + 1) Then we have 3 tens and 9 ones. Can you write a numeral for 3 tens and 9 ones?" The children should be able to write "39" and to read this as "thirty-nine."

Let the children copy the exercises from pages 54 and 55 in their exercise books. Ask them to copy the sentences as true sentences. (It is not necessary that all the sentence be completed as one lesson. Ask the children to do as many as you think they can do in one lesson.)

Activity 6: Finding missing addends **Pupil Page 56**

Put out the counting sticks and the number tray.

Write the sentence $10 + \square = 40$ on the blackboard. Ask, "How can I show this sentence with our sticks?" By questions get the children to say, "We put 4 bundles of sticks in the number tray. We separate this set into two sets. One of the sets is 1 bundle and the other set is what is left. It is 3 bundles. It is 3 tens. We write '30' in the box to make the sentence true." Continue with similar sentences as needed.

Let the children copy the sentences from page 56 into their exercise books. Ask them to copy the sentences as true sentences.

Stage 2: Addition of More Than Two Addends and the Associative Property

Vocabulary: grouping

Materials: Objects for sets; 5" × 7" numeral (0-9) cards, cards with + sign on them, 1-inch numeral (0-9) cards for the children; pupil pages 57-64.

Teaching Procedure:

Activity 1: Three addends

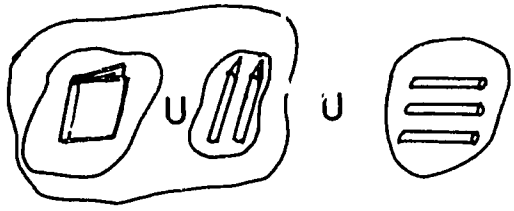
Put out a book, 2 pencils, 3 pieces of chalk, other objects to make sets and the numeral (0-9) cards and several cards with the plus sign on them.

Show a set of one book, a set of two pencils, and a set of three pieces of chalk. Draw a picture of the three sets on the blackboard. See the drawing below.

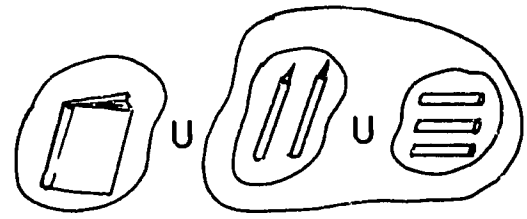
Say, "We have three sets. Which two do we put together first?" A child may say, "Join the set of one book with the set of two pencils." Do this, and then join this new set with the set of the three pieces of chalk. Ask the children to describe the new set. (Its members are a book, two pencils, and three pieces of chalk.) Show the joining as in drawing below on the left.

Make the same three sets again. Say, "We will join the sets in another way. We join the set of pencils and the set of chalk first." Do this. Say, "Now we put the set of one book with the other set." Do this. Let the children describe the set. Show the joining as in the drawing below on the right.

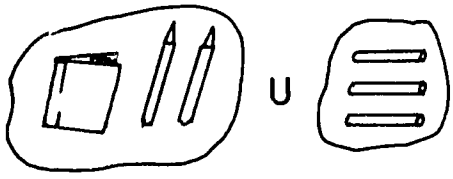
The blackboard should now look like the drawing below without the numerals. Say, as you point to the blackboard drawings, "We had three sets. We joined them in two different ways. Did we get the same set each time or did we get a different set each time?" (The same set)



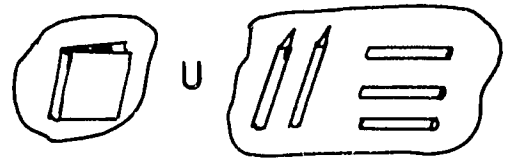
1 + 2 + 3



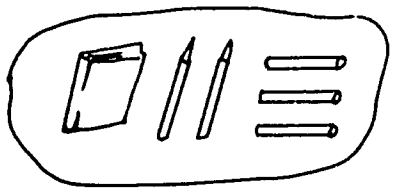
1 + 2 + 3



3 + 3



1 + 5



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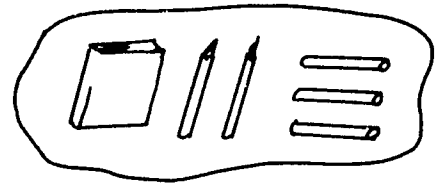


6

Go over the blackboard drawings and talk with the children about them. As you go from drawing to drawing write the number of each set under it. Say, for example, "We joined a set of 1 book with a set of 2 pencils. We can write 1 + 2." Later you or the children say, "We now have a set of 1 book and 2 pencils. That is a set of 3. 1 + 2 = 3."

Your aim is to get the children to notice two things: (1) when three sets are joined they may be grouped for joining in any way; (2) when three numbers are added they may be grouped for adding in any way. Do not ask the children to say these words. They may say, "Both ways we joined the sets we got the same set", and "Both ways we added the numbers we got the same number."

Let three children choose a single card each. Ask them to stand in a row along the front of the room. Say, "We wish to add three numbers. We can add any two of the cards which are side by side. Which two shall we add first?"

Suppose the children with the numeral cards are lined up, with children holding cards showing the + between them, like this:

$$\boxed{5}\ \boxed{+}\ \boxed{2}\ \boxed{+}\ \boxed{4}$$

If a child says to add 5 + 2 first, let the children with cards showing this move together. Let another child find the 7-card and take the place of the children showing 5 + 2. Let the children go on telling that 7 and 4 are added. Ask a child to find the 11-card and take the place of the children showing 7 + 4.

Show this in sentences on the blackboard as shown below. Talk as you write. As you put parentheses around 5 + 2 say, "You added these together. The sum is 7. I write 7 for 5 + 2. . . ." End by saying, "The sum of 5 + 2 + 4 is 11. We found 11 by adding two numbers at a time in groups." The blackboard looks like this:

$$5 + 2 + 4 = \boxed{}$$
$$(5 + 2) + 4 = \boxed{}$$
$$7 + 4 = \boxed{11}$$

Go over the same plan but begin by grouping 2 and 4 together. The blackboard looks like this when you are finished.

$5 + 2 + 4 = \square$	$5 + 2 + 4 = \square$
$(5 + 2) + 4 = \square$	$5 + (2 + 4) = \square$
$7 + 4 = \boxed{11}$	$5 + 6 = \boxed{11}$

Let the children talk about what they have done. You and the children point to the work on the blackboard as you talk. They will say among other things, "We added three numbers. The sum is 11. We added the numbers two different ways. The sum is 11 each time. One way we added 5 and 2 first. The other way we added 2 and 4 first. We can add either way."

Talk with the children again about the sets and what they showed. Write the sentences for $1 + 2 + 3$ on the blackboard in the same two ways you did for $5 + 2 + 4$. Let the children notice that the way numbers are grouped for adding does not change their sum.

Use other examples as needed.

Activity 2: Adding three numbers **Pupil Page 57**

Write two sentences such as these on the blackboard.

$(2 + 5) + 3 = \square$ $2 + (5 + 3) = \square$

Point to the first sentence and ask which of the addends are added first. They should say 2 and 5. Ask, "What number is $2 + 5$?" Write "7" under the "$2 + 5$". Ask, "Now, what two numbers do we add?" They say that $7 + 3 = 10$, so "10" is the numeral in the box.

Go through a similar plan for the second sentence. The children should say that $5 + 3$ is 8 and that the sentence may be rewritten as

$2 + 8 = \square$

Write another pair of sentences like these

$(1 + 3) + 4 = \square$ $1 + (3 + 4) = \square$

Ask a child to tell which two addends are added first in the first sentence. "What do we write in the box?" (4 + 4 = 8. Write "8" in the box.)

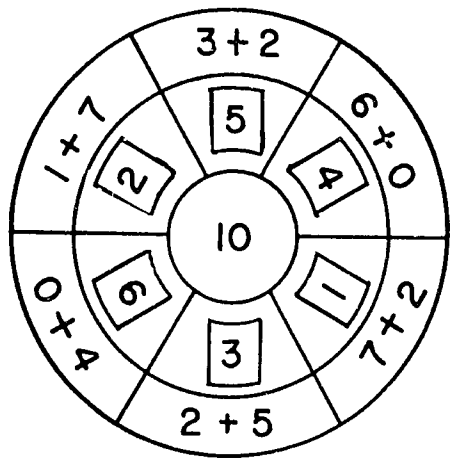
Ask, "How are the two sentences alike?" (They have the same addends.) Say, "We have found the sum in the first sentence. Do you know the sum in the second sentence without adding?" Some of the children may have discovered that for the different grouping of the addends the sum is the same. So, without adding, they know that the sum is eight.

Let the children make the sentences true on page 57 by putting numeral cards on the boxes. Let them talk about why they can show one sum for two sentences.

Activity 3: Addition wheel. Pupil Page 58

Each of the sections of the wheel on page 58 show an addition sentence. The "10" in the middle is the sum of the numbers in each section.

Ask the children to put a numeral card in the empty space to show the missing addend. When the sentences are all completed the circle will look like the one below:



Let the children write on their papers the true sentences which they show on the wheel.

$(3 + 2) + 5 = 10$	$3 + (2 + 5) = 10$
$(1 + 7) + 2 = 10$	$1 + (7 + 2) = 10$
$(6 + 0) + 4 = 10$	$6 + (0 + 4) = 10$

Help the children to see that the sentences opposite each other on the wheel have the same addends. Ask them to look at the sentences they wrote on their papers. Ask if any have the same addends. You write the sentences in pairs on the blackboard. Again help the children to notice that numbers may be grouped in any way for adding.

Activity 4: (Optional Activity): Addition sentences. Pupil Page 59

Let the children put their numeral cards on the boxes in the second sentence in each section of page 59. Ask them to make the sentences true.

Again help the children to notice that three numbers may be grouped in any way for adding.

Activity 5: Addition game. Pupil Page 60

Let the children play the game by putting numeral cards on all the spaces. They pick cards so that each column and each row has the sum shown by the numeral next to the game box. Each column and each row contains three spaces. There are three addends in each. One addend is in the middle row and the middle column. All others may be found by the children.

Ask the children to look at the first game on page 60. Say, "You are to try to put numeral cards on all the spaces so that if you add down or across you get 20 as a sum. Do not put a card in the middle space. You know that number. It is ten.

Go on with the second game in the same way.

Activity 6: Addition of three numbers each greater than 10.

Pupil Pages 61 to 62

Put out bundles of sticks and some single sticks and the number tray. Make these three different sets of sticks:

1 bundle of ten and 2 single sticks,
1 bundle of ten and 1 single stick,
1 bundle of ten and 5 single sticks.

Show the sets one at a time. Ask children in turn to tell how many tens and how many ones there are in each set. Put the sets in a number tray. Ask children in turn to write on the blackboard a numeral for the number. You write the exercise on the blackboard as shown below. At this time do not write the sum.

Say, "We put all the sticks together in the number tray. How many tens are in the tray? How many ones? (3 tens and 8 ones)

"Can we add the numbers to find the sum?" Ask a child to come to the blackboard. Let him add the number of ones. Ask another child to add the number of tens shown. Write in the sums. The blackboard should look like this:

12 = 1 ten and 2 ones
11 = 1 ten and 1 one
15 = 1 ten and 5 ones
38 = 3 tens and 8 ones

Tell the children that we used sets to show 12 + 11 + 15. Ask, "Do you need to use sets?" Let the children talk about this. They may say, "We can add ones; then add tens. We can add. We do not need sets of sticks."

Say, "Let us try to find another sum without the sticks. You add the numbers to find the sum." Write the sentence 42 + 10 + 23 = $\square$ on the blackboard as shown below on the left and write the numerals in vertical form.

Say, "We can write each addend as tens and ones." You write the addends as shown below to the right as the children tell how many tens and ones. The blackboard should look like this:

$42 + 10 + 23 = \square$

42 = 4 tens and 2 ones
10 = 1 ten and 0 ones
23 = 2 tens and 3 ones

Let the children add the ones and add the tens. Ask, "What is the sum?" (7 tens and 5 ones or seventy-five).

Write the sentence below on the blackboard . Ask the children to read the addends and tell how many ones there are altogether. (None) Ask, "How many tens?" (7) Write "70" in the box. Say, "We can write this exercise in another way. Write it as it is shown to the right below. Write the sum under the addends. The blackboard looks like this.

$20 + 40 + 10 = \boxed{70}$

20
40
10
70

Let the children copy some or all of the exercises from pages 61 and 62 into their exercise books. Ask them to find the sums.

Note: The children may put pieces of paper under the row of exercises and write the sums on them. This saves the time of copying.

Activity 7: Addition of four addends. Pupil Pages 63 to 64

Write this sentence on the blackboard:

$4 + 6 + 3 = \square$

Ask the children to tell how they would group the addends. A child may say, " 4 + 6" and another, "6 + 3". Ask one of the children to tell why he grouped the addends as he did. Help the children to say that they could use either

(4 + 6) + 3 or 4 + (6 + 3). Help them also to say that the first way makes addition easier. It is easier because they made a ten.

Write this exercise on the blackboard in two forms:

$$4 + 6 + 3 + 5 = \square \qquad \begin{array}{r} 4 \\ 6 \\ 3 \\ \underline{5} \end{array}$$

Say, "You have four numbers to add. How will you group them?" Let the children talk of all the ways they can group the addends. They can group 4 and 6 and then 3 and 5. They can group 6 and 3 and then 4 and 5. The aim is to help children look at an exercise and tell the best way to group addends. Some ways of grouping the addends make addition easier.

Note: The exercises on page 63 have addends which are easier to add if the addends are grouped into multiples of ten. For example, $7 + 3 + 6 = \square$ is easier to add if we group it as (7 + 3) + 6 or 10 + 6. And, 5 + 9 + 1 is easier to add if we group it as 5 + (9 + 1) or 5 + 10.

Let the children copy the sentences from page 63 into their exercise books. Ask them to find the sums. Get them also to show with parentheses the way they grouped the addends. For example, they write the first example like this:

$$(7 + 3) + 6 = \boxed{16}$$

Let the children find the sums for the exercises on page 64.

Stage 3: Using the Number Line with Addition; Equalities and Inequalities

Vocabulary: greater than, less than

Materials: Small cardboard figure of a frog, a 3" x 5" card with a stick figure drawn on the card; pupil pages 65 - 67.

Teaching Procedure:

<u>Activity 1</u>: Using the number line to show addition.

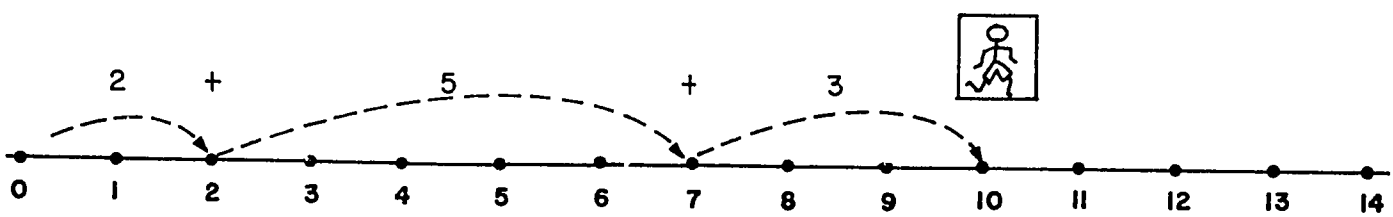
Put out the stick-figure picture and draw a number line on the blackboard. Say, as you write the sentence below, "We will use the number line to help us make this sentence true. We will find a numeral to write in the box."

$$2 + 5 + 3 = \square$$

Say, as you hold up the stick-figure, "You can use this picture of (Use a name of one of the children) to show the sentence. We will pretend Kofi is walking along the number line. He will start at the 0-point. Let the numbers in the sentence tell how many steps he takes. Read the sentence and tell me about Kofi's trip." Let the children say, "Kofi will take 2 steps, then 5 steps and then 3 steps."

Let a child pretend to be Kofi and show how he takes steps along the line. Let another child take the trip described by the sentence. Let the class tell him how many steps to take. Draw a dotted line to show the 2 steps, the 5 steps, and the 3 steps. See the number line below.

Let other children show the same sentence on the line as they pretend to be Kofi. Get them to tell about the trip like this. "Kofi starts at 0. He takes 2 steps to the 2 point. He takes 5 steps to the 7 point. He takes 3 steps. He ends at the 10 point. $2 + 5 + 3 = \square$

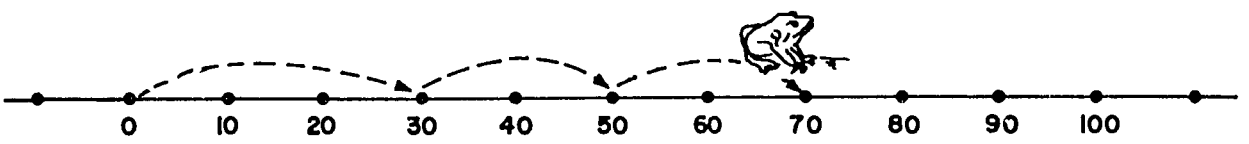


Write other sentences for the children to make true sentence. Let them all have a part in showing the sentence on the number line and in saying what number

goes in the box. Let some children move the stick figure. Let some children draw the lines to show the trip. Let some children say how many steps to take. Let some children tell about the whole trip. Let some children tell the numeral to write in the box.

Activity 2: Using a number line with multiples of ten.

Put out a cardboard cutout of a frog. Write "10" on his back. (Or, use an object which the children pretend is a frog.) Make a number line on the blackboard marked in tens as shown below.



Say as you hold up the frog or an object, "I want you to know Mr. Ten Frog. (Point to the number line.) This is Mr. Ten Frog's number line. It is marked in tens. Mr. Ten Frog always jumps along the line by tens."

Ask a child to show how Mr. Ten Frog jumps along the line. Let him take the frog and show the frog hopping along.

Let children show number sentences with Mr. Ten Frog the way they did with the stick figure in Activity 1. The sentence 30 + 20 + 20 = $\square$ is shown on the number line above. Sentences which the pupils may show are these:

$\square$ = 10 + 30 + 50	30 + 30 + 30 = $\square$	50 + $\square$ = 90
$\square$ = 20 + 0 + 70	30 + 40 + 20 = $\square$	$\square$ + 20 = 50

Let the children make moves along the line and answer these questions (You can think of others):

1. Suppose Mr. Ten Frog can only hop one space of ten at a time. How many hops must he take to reach 70?

2. Mr. Ten Frog first hops 5 spaces, then 3 spaces. He wants to get to number 90. How many more spaces must he hop?

3. Mr. Ten Frog hops 4 spaces in one hop. If he hops again and goes the same distance, on what number does he stop?

4. Can you write a number sentence that shows that Mr. Ten Frog made three hops and ended on 90? The children may write sentences, such as 30 + 30 + 30 = 90 and 20 + 40 + 30 = 90.

<u>Activity 3:</u> Making sentences true with "=" or "≠" Pupil Page 65

Put the open sentence below on the blackboard. Ask the children what numeral goes in the box to make the sentence true. (16) Let a child write 16 in the box.

$$6 + 10 = \square$$

Say, as you point to the "16", "The number 16 makes the sentence true. Can you think of any other number that makes the sentence true?" A child may suggest a number, say 15. If no one says another number, you say a number. "Let us see if 15 makes the sentence true."

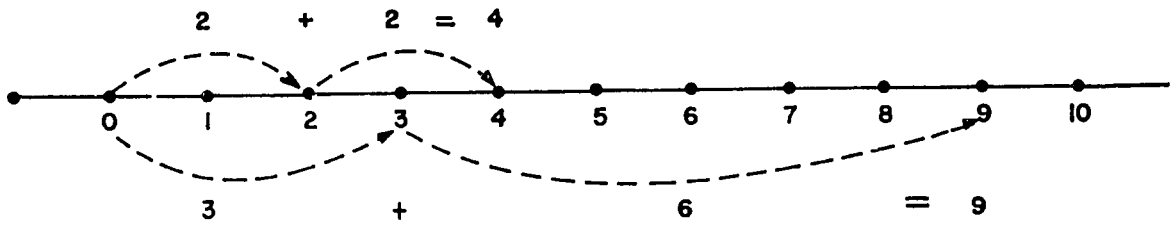
Rub out the "16" in the box and put "15" in its place. Ask, "Is the sentence true now?" (No) Make a line through the "=" to make it "≠". Say, "Now the sentence says that 6 + 10 <u>is not equal to</u> 15. I have made the sentence true. It is true that 6 + 10 does not equal 15."

Put several sentences on the blackboard in which the sign (either = or ≠) is needed to make the sentences true. For example,

4 + 2 --- 5	6 + 3 --- 9	2 + 2 --- 3 + 6
5 + 6 ---12	3 + 1 --- 8	2 + 2 --- 3 + 1

Ask the children each to read a sentence. Ask them to tell whether "=" or "≠" makes the sentence true.

Let them use the number line to show the sentence is true. For example, they may show the sentence 2 + 2 ≠ 3 + 6 like this:



Let the children say, "The line shows 2 + 2 = 4 and 3 + 6 = 9. The numbers 4 and 9 are not equal (≠). We know that the sentence <u>is</u> true." Let the children go on with the other sentences.

Also write sentences like these on the blackboard.

$$10 + 3 = \square \qquad 20 + 5 = \square$$
$$10 + 3 \neq \square \qquad 20 + 5 \neq \square$$

Ask the children what numerals can be written in the boxes to make the sentences true. Let a child write a numeral in each box. He might write:

$$10 + 3 = \boxed{13}$$
$$10 + 3 \neq \boxed{16}$$

Let the other children say whether the sentences are true. Ask, "Do you know any other numbers that make the first sentence true?" (No) Ask, "Do you know any other numbers that make the second sentence true?" If a child suggests a number, rub out the numeral in the box and write the numeral for the number named. "Is the sentence true now?" Ask for other numbers and continue to ask if the numbers make the sentence true. The children will say, "There are many numerals that make the second sentence true.

We can write any numeral in the box except "13" to make the second sentence true. The numeral "13" is the only numeral that makes the first sentence $10 + 3 = \square$ true.

Ask the children to copy the exercises on page 65 in their exercise books. Let them make each sentence true. For those on the left of the page they write either "=" or "≠" in the space. For those on the right of the page they write a numeral in the box.

In some of the sentences on page 65 the children will use what they know about the commutative and associative properties of addition. For example, the children will know without adding that "=" goes in the space in this sentence:

5 + 2 --- 2 + 5

They know that 5 + 2 and 2 + 5 are both numerals which name 7. They know that 5 and 2 are addends in each numeral. The order of the addends is changed. So "=" is the symbol which makes the sentence true. Another example from page 65 is this sentence:

(2 + 3) + 1 --- 2 + (3 + 1)

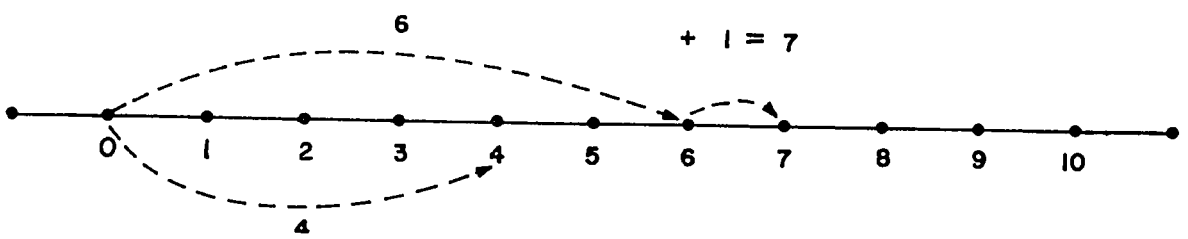
In this sentence the children will see two numerals with the same three addends. The grouping of the three addends is changed. They know that does not change the sum. They know at once that the symbol needed is "=".

Activity 4: Using the number line to show inequalities. Pupil Pages 66 to 67

Put this sentence on the blackboard:

6 + 1 --- 4

Ask, "Will the sign "=" make this a true sentence?" (No; 6 + 1 is not equal to 4.) Say, "Show the two numbers 6 + 1 and 4 on the number line." Let the children show the numbers in this way:



Ask, "What does the number line show about the numeral (6 + 1)? (It is a name for 7.) Which is greater, 7 or 4?" (7) What sign should be used so we have a true sentence? (Since 7 is greater than 4, we write 6 + 1 is greater than 4). Write in the space:

6 + 1 _>_ _ 4

Point to the sentence below and say, "Suppose the sentence is turned this way. Do you use the ">" sign ?"

4 --- 6 + 1

Let the children look at the number line. It shows 4. It shows 6 + 1. Ask, "What can you say about 4 and 6 + 1 to make the sentence true?" (4 is less than 7. We know that 4 is less than 6 + 1. We use a "<" sign.) Let a child put "<" in the statement. Ask the children to read the statement and say whether it is a true sentence.

Put the following sentences on the blackboard. Show some of them on the blackboard number line. Make them true by writing in the sign "=" the sign " < " or the sign " > ". Let the children copy the other sentences on their papers. Let them use the number line to help them make the sentences true.

9 --- 6 + 2	18 --- 18 + 1	7 + 7 --- 7 + 0
8 --- 6 + 2	30 --- 20 + 5	5 + 2 --- 5 + 5

Let the children go on with the sentences on page 66 in their books. Let the children write true sentences in their exercise books. Let the children show the sentences by moving two different markers along the number line on page 67 of their books.

Stage 4: Using Letters to Name Numbers in Addition Sentences.

Vocabulary: a (or other letter) as the name of a number.

Materials: 5" by 7" numeral (0-9) cards; one-inch square cards with letters a, b, c, m, and n written on them; pupil pages 68 - 70.

Teaching Procedures:

Activity 1: Using letters to name numbers

Revise the use of open sentences. Write these on the blackboard:

$$4 + \square = 8 \qquad 6 + 3 = \square$$

Ask the children to make the sentences true.

Say, "We are going to play a game. I will think of a number. I will not tell its name. You will call it m. You are to find the number I am thinking. I will help you by writing an open sentence on the blackboard. The sentence will help you tell what the number m is."

Ask the children to close their eyes. You write this sentence on the blackboard.

$$7 + 8 = 15$$

Cover the "8" with a card on which is written the letter m. Let the children open their eyes to see this:

$$7 + \boxed{m} = 15$$

Say, "This sentence is true. Read the sentence. What number is m ?" When the children give an answer, remove the card so that the children see the "8" that was covered. Ask, "What number is m in this sentence?" (8) Write the sentence

$$m = 8$$

Go on, "I am thinking of another number, and I will call it a. Go through the same plan as with the other sentence. Write a true sentence. Cover a number in the sentence with an a-card. Ask the children to look at it.

$$\boxed{a} + 10 = 10$$

Ask the children to read the sentence. Let them say what number a is. Remove the card and show "0". Say, "a is zero." Write this under the sentence.

$$a = 0$$

Go on with the game. Use other letters of the alphabet for numbers in the sentences. Do not always use the same letter to name the same number.

Write these sentences on the blackboard:

$7 + y = 14$	$3 + 2 = n + 0$	$a + 7 = 15$
$y =$ ___	$n =$ ___	$a =$ ___
$6 + 4 = n$	$9 + 1 = 5 + a$	$b + 0 = 18$
$n =$ ___	$a =$ ___	$b =$ ___

Let the children tell what number the letter in each sentence names. Let them check their answers. Ask them to rub out the letter and write a numeral. Ask them to read the sentence. Let the other pupils say whether it is true.

For example, in the first sentence a child says that y is 7. Say, "Let us check to see if y is 7." Let another child rub out the letter "y" and write "7" in its place. Let a child read the sentence. Ask, "Is the sentence $7 + 7 = 14$ true? (Yes) Is it true that $y = 7$? (Yes)" Go on with the other sentences.

<u>Activity 2</u>: Puzzle sentences. Pupil Page 68

Tell the following story to help the children use page 68. Let the children have their closed mathematics books on their desks. Tell the story this way:

"Once there was a little spider who lived in (say the name of your country or town). He was really not a bad little spider but he loved to play tricks on people and especially on children.

"One day he was passing a school. It was the (Use the name of your school) school. He looked in a window and he saw the children working with number sentences.

"' Well,' he said to himself, 'those children seem to know a great deal about numbers. See how easily they read the sentences. We'll see about that. I shall have to play a little trick on them. I shall fix those sentences so the children cannot read the numbers.'

"So he got some paint and some paint brushes, and after the children had gone he sneaked into the classroom, opened all the children's books, and went to work with paint and brushes. He painted very, very fast because, of course, like all spiders he had eight legs and he could hold a brush in each one.

"Open your books at page 68. You see what Mr. Spider's trick was. He has painted a letter over one of the numerals in each of the true sentences.

"But this is not the end of the story. You can play a trick on Mr. Spider. He does not know it but you can tell what the numbers are. You play a trick on Mr. Spider. Read the sentences and write in your exercise books the numeral Mr. Spider painted over.

"We will do the first sentence together. Copy the sentence $12 + m = 18$ in your exercise books. We know that m is a number. What is it? (6). Write '$m = 6$' under the sentence in your exercise books. Go on with the other sentences on the page. You will fool Mr. Spider. Remember to make all the sentences true."

Activity 3: Number guessing game.

Separate the children into two teams.

Put a stack of numeral cards on a table face down. One child from the first team goes to the table and picks a card. He looks at the card. It tells him his number. He shows the card to the other team. He does not show it to his own team. His team must find his number. Let him say to his team, "My number is m. I will use my number m in a sentence. You tell me my name." Ask him to write a true sentence on the blackboard using the letter "m". For example, if his number is 6 he may write

$$m + 1 = 7$$

His team tries to say what his number is. If they do not say the number after the first clue, he can write a second sentence. He again asks, "What is my number?" He may write this sentence.

$$m + 0 = 0 + 6$$

When they guess his number, a child from the other team has a turn. Go on until each child has had a turn to choose a numeral card and write a sentence using his number.

Activity 4: Solving number sentences. Pupil Pages 69 to 70

Let the children tell what number n is in each sentence on page 69 and 70. Ask them to place a numeral card on the box in each exercise.

Stage 5: Solving Word Problems Using Mathematical Sentences

Vocabulary: No new vocabulary

Materials: None

Teaching Procedure:

Activity 1: Word sentences and their number sentences.

The aim of this activity is to let children relate a number sentence to a word sentence. They first read a problem. They answer the question asked in the problem. They write a number sentence for the problem.

Read the sentences below to the children. Let them answer the question asked in each problem after it is read.

1. In the classroom there are twelve boys and six girls. There is a teacher in the classroom too. How many people are there altogether?

2. In a football match, Peter's team scored 9 goals. The other team scored 7 goals. How many goals were scored in the match altogether? Which team won the match? How many more goals did the winning team score than the losing team?

Go back to each problem and help the children write a number sentence which tells the story of the problem.

Problem 1: Read the problem again. Say, "I want to write this story with numbers. Will you help me? The story says there were 12 boys, 6 girls, and 1

teacher. The story asks how many people there were altogether. We will call that number c. There are three sets. Put together they make another set." As they say the number in each set write the numbers on the blackboard like this:

12 6 1 c

Say, "We joined three of the sets to make another set. What do we do with the numbers 12, 6, and 1? (Add them) Write "+" between the numerals on the blackboard. Ask, as you point to the writing on the blackboard, "How can I make a sentence from this? What do you know about 12 + 6 + 1 and c?" (They are equal.) Write "=" to make a sentence. Say, "We have written a number sentence which tells the same story as the problem.

Let one child tell the problem. Let another child read the number sentence. Let the children talk about the problem and the number sentence and say, "They tell the same story."

Ask, "What number is c? (19) Write "19 = c" on the blackboard. The blackboard looks like this:

$$12 + 6 + 1 = c$$
$$19 = c$$

The other problem can be talked about in the same way. Let the children write on the blackboard the numbers in the sentence.

Ask the children to try to make a number sentence. Let them talk about the relation of the story in the problem and the story in the number sentence.

Use the problems below in the same way. Be sure the children know the story of the problem. Let them tell what numbers are in the story. Let them tell how these numbers are thought about in the story. From this, let them write a number sentence.

1. Robert and Obi went into the bush to pick mangoes. Robert picked ten mangoes. Obi picked two more mangoes than Robert picked. How many mangoes did Obi pick?

2. John's team won three races in the morning and three races in the afternoon. Paul's team won four races in the morning. At the end of the day Paul's team had won the same number of races as John's team. How many races did Paul's team win in the afternoon?

Activity 2: Writing a story for a number sentence.

Write the number sentence below on the blackboard as you say, "This number sentence tells some stories. I know one it tells."

$$5 + 2 = n$$

Say, "One story it tells is this: There were 5 children playing a game. Two children joined them. How many children are playing?" Let the children talk about the two stories, the one you told and the one told by the number sentence.

Say, "This sentence also tells this story: Five books were on a table. Tom put 2 more books on the table. How many books are on the table? Let the children talk about this story and that told by $5 + 2 = n$.

Let the children tell other stories for the sentence $5 + 2 = n$. Get them to say that a number sentence may tell many stories.

Write the sentence below on the blackboard. Ask, "Do you know a story this sentence tells?"

$$5 - 2 = b$$

Give each child time to think of a story. Then let the pupils tell their stories. Let them talk about each story. Let them say whether the sentence tells or does not tell the same story as that told by the child. Let him change his story if they do not tell the same story. Stories they may tell are these: "There were five birds in the tree. Two birds went away. How many birds are in the tree?" "Mary had 5 bananas. She ate 2 bananas. She gave Okello the others. How many did he get?"

Go on in the same way with these number sentences:

$6 + n = 11$ $\quad$ $5 - b = 2$ $\quad$ $4 + 5 + m = 10$

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Obodai, G. T., Accra City Council Educational Unit, Accra, Ghana

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Polome, E. G. C., University of Texas, Austin, Texas

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STAFF

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Prepared
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Entebbe Mathematics Workshops

PRELIMINARY EDITION

Educational Services Incorporated
Watertown, Massachusetts, U.S.A.

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UNIT 5

SUBTRACTION

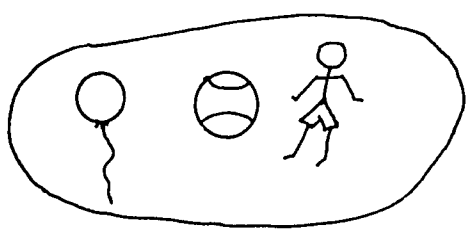
Objectives:

1. To learn to separate sets into subsets.
2. To revise the basic subtraction facts.
3. To understand subtraction as the inverse of addition.
4. To learn to read a number sentence involving subtraction and to complete a number sentence by finding the number that makes it true.
5. To learn the meaning of equality and inequality in a sentence involving subtraction.
6. To learn to solve word problems involving subtraction and to understand the relationship of word sentences and mathematical sentences.

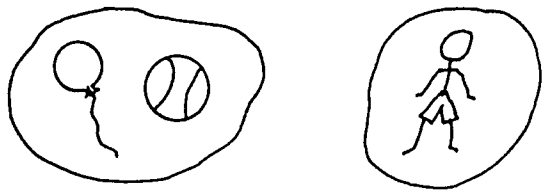
Background Information for Teachers:

Also see Part 2, "Arithmetic", Sections 4, 5, and 7 of your Teachers' Handbook.

The separation of sets of objects into subsets illustrates subtraction. Think of a set of three members:



The set can be separated into subsets in several ways. One way is to take the balloon and the ball aside and leave the boy. Each of the sets below is a subset of the original set.



We can also form the following pairs of subsets: boy and ball, the balloon alone; boy and balloon, the ball alone.

If we think of just the number of members in the set, we find that when we pick out any two of the members, the remaining subset has one member. Or if we pick out one member, the remaining subset has two members. We can think of this in terms of two number sentences.

$$2 + 1 = 3$$
$$1 + 2 = 3$$

We could also separate the set of three members into three subsets each of which has one member. We then think of this number sentence:

$$1 + 1 + 1 = 3$$

But we still have not considered all subsets of the set of a balloon, a ball, and a boy. Suppose we pick out all the members of the set and call this a subset. What members are in the remaining subset? There are none, of course. In other words, the subsets here are:

1. the set of a balloon, a ball and a boy and
2. the empty set.

We see that every set has itself as one of its subsets and every set has the empty set as one of its subsets. If we think of just the number of members in each of the subsets we have the sentences

$$3 + 0 = 3 \qquad \text{and} \qquad 0 + 3 = 3.$$

If we know the members of the set and also the members of one subset, then we can easily tell what the members of the other subset are. For example, if we know that the set we are considering has as its members a ball, a boy, and a balloon, and one subset nas the boy and balloon as its members, then we know that the remaining set has as its members the ball. We could join these two subsets to form the set we started with.

Also, if we know the sum and one addend we can find the other addend. The following sentence is related to the example about subsets just given:

$$2 + \square = 3$$

We see that joining two sets is related to separating a set into two subsets. In the same way, addition is related to subtraction. When we know the sum and one addend, the problem of finding the missing addend is called a <u>subtraction</u> problem. If the sum is 3 and one addend is 2, we can write "3 - 2" for the missing addend.

We say that subtraction is the inverse of addition. To subtract 2 from 3 we find the number to be added to 2 to get 3. When we write sentences, the number n that makes "2 + n = 3" true is the same number n that makes "3 - 2 = n" true.

The separation of a set of four things into two subsets leads to the true sentences:

$$4 - 2 = 2,$$

$$4 - 1 = 3 \quad \text{and} \quad 4 - 3 = 1,$$

$$4 - 4 = 0 \quad \text{and} \quad 4 - 0 = 4.$$

Each of these sentences is related to a sentence about addition. For example, the sentence

$4 - 2 = 2$ is true because $2 + 2 = 4$ is true.

You write the sentences about addition that are related to the other sentences above.

Stage 1: Separating Sets and Subtracting

Vocabulary: Subset, minus, subtract

Materials: Objects to form sets; 1-inch square numeral (0-18) cards; strips of cardboard of 1-9 unit lengths (see Activity 5); pupil pages 71-73

Activity 1: Separating sets of objects into subsets.

Put several sets of objects on a table and two pieces of paper or cardboard. Ask the children to describe the sets. Let a child separate one set into two subsets. Ask him to put each subset on a piece of paper. Ask him to describe each subset. For example, a child may separate a set consisting of a book, a pencil, and a piece of chalk into two subsets. On one piece of the paper he may put the book and the pencil and on the other paper the piece of chalk.

Say, "He has separated the set into two subsets. The book and pencil are members of one subset. The piece of chalk is a member of the other subset. We join the two subsets and get the set with which we started." Ask someone else to separate the same set into two different subsets.

Go on until several different ways of forming pairs of subsets have been shown. If no one suggests it, pick up all the members of the set and place them on one paper. Say, "This is a subset too. This subset has the same members as the set. What things are left in the other subset on this paper?" (None) "What do we call that set?" (The empty set) "The empty set is a subset. Whenever we talk about one subset having as members all the members that are in the set then the other subset is the empty set."

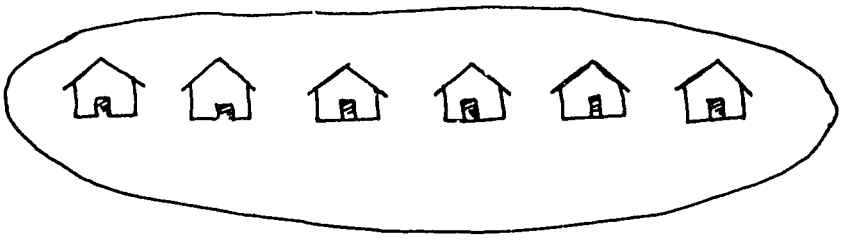
Each time a set is separated into two subsets, let the children show that

if they join the pair of subsets they have the set with which they started. They do this by moving the objects back together in the middle of the table.

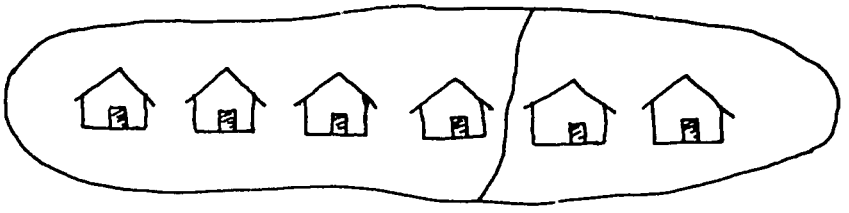
Use enough examples so that many sets with different numbers of members are included.

Activity 2: Separating sets. Pupil Page 71

Draw a set of houses on the blackboard.



Ask, "What are the members of the set?" Separate the set into two subsets by drawing a line in this way:



Ask, "What have I done to the set?" (Separated it into two subsets) Move a finger around the subset with the most members and say, "This is one of the subsets. How many members has this subset?" (4) Move a finger around the other subset and ask, "How many members are in this subset?" (2)

Rub off the line separating the houses and let a child draw another line separating the set into two different subsets. Ask the children to tell the number of members in each subset. For example, there may be three members in each subset. Tell the class, "We began with a set of 6 houses. It was separated into two subsets, three houses in one and three houses in the other. If we join the set of three houses with the set of three houses what set do we have?" (A set of 6 houses) Show this by rubbing off the line separating the pictures.

Draw other pictures of sets and continue in the same way until the idea is understood.

Ask the children to open their books at page 71. Let them describe the sets they see. Then give each child some small sticks or a piece of string to put on the page to separate the sets pictured into subsets.

Ask the pupils to look at the first set. Say, "There are light-coloured balls and dark-coloured balls. Can you separate the set so that the light-coloured balls are in one subset and the dark-coloured balls are in another? Put a stick (or string) between the subsets. How many members are there in the subset of light-coloured balls? How many are there in the subset of dark-coloured balls?"

Write the numerals "2" and "3" on the blackboard as the children answer. Say, "Can you join the subsets again?" (The children remove the stick or string) "How many balls are in the set?" (5) Complete this sentence on the blackboard ,

$$5 = 2 + 3$$

Ask, "How many bananas are in the set of bananas?" (3) Ask a child to write the numeral for three on the blackboard. Then let the children separate the set into two subsets with sticks. Ask, "How many members are in one of your subsets?" If a child says "Two," write "2" on the blackboard.

Say, "We had a set of three things and separated it into two subsets. One subset has two things in it." Write on the blackboard the open sentence,

$$3 = 2 + \square \quad \text{or} \quad 3 = \square + 2$$

Let a child write on the blackboard the number which makes the sentence true. Say, "The other subset should have one member. Look at your books and see if this is true."

Say, "Suppose we begin with the subset of one banana. Can someone show us a sentence that tells that we have a set of 3 members and one of its two subsets has one member?" Help the children to write

$$3 = 1 + \square \quad \text{or} \quad 3 = \square + 1$$

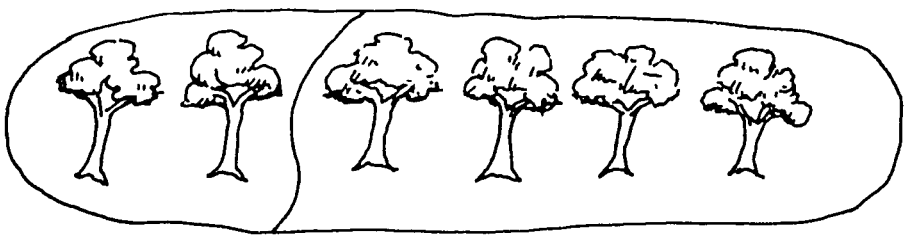
Go on with the other exercises on the page. Write open sentences on the blackboard for each exercise after the sets are separated. Ask a child to tell the missing addend to make the sentence true. Let the children check this by looking at the pictures of the subsets. For example, if a pupil puts "1" in the box,

$$3 = 1 + \boxed{1} \quad \text{or} \quad 3 = \boxed{1} + 1$$

let the class decide whether the sentence is true. (It is false) Emphasize that we are trying to find a number that makes the sentence true.

Activity 3: Writing sentences about sets and subsets. Pupil Page 72

Draw a set of trees on the blackboard. Ask a child to separate the set into two subsets.



Let a child tell about what was done to the set of trees. For example, he may say, "There is a set of six trees. It is separated into a subset of two trees and a subset of four trees."

Rub off the line and say, "If we join the set of two trees with the set of four trees what set do we have?" Put the sentence for this on the blackboard below the picture.

$$6 = 2 + 4$$

Say that we can write a sentence showing the separation into subsets. We have six members and one of the subsets has four members. This sentence shows this:

$$6 - 4 = \boxed{}$$

We complete the sentence to make it true like this:

$$6 - 4 = \boxed{2}$$

The sentence tells us that the other subset has 2 members. We read the sentence, "Six minus four equals two." We say that we subtract the numbers 6 and 4.

Give another example orally. Say, "I have a set of eight books and I separate it into two subsets. One of the subsets has three members. We know that when the two subsets are joined we get a set of eight members. We write this as

$$8 = 3 + \square$$

Or, we can subtract to answer the question. Who can write a sentence showing that we subtract?" Help one of the children to say and write,

$$8 - 3 = \square$$

Ask, "How many members do we begin with?" (8) "What numbers do we subtract?" (8 and 3)

Go on in the same way with a few more examples which help the children to write these sentences:

$9 = 1 + \square$	$17 = 9 + \square$
$6 = 3 + \square$	$14 = \square + 6$
$8 = \square + 7$	$10 = 2 + \square$

Let the children write the subtraction sentences beside them. For example, help them write

$$9 - 1 = \square \quad \text{by the sentence} \quad 9 = 1 + \square$$

Ask the children to open to page 72. Ask them to write two sentences for each example, one about addition and one about subtraction. Go over the first example with the class in this way. "How many members are in the set of cups?" (5) "We write 5. When the set is separated, how many members are in one of the subsets?" (4 or 1) "How many members are in the other subset? Can you write a sentence showing that if a set of 5 things is separated into two subsets and one subset has 4 members, then the other must have 1 member?" ($5 - 4 = 1$) "Can you write a sentence showing that if you join the set of 4 things with the set of 1 thing you have a set of five things?" ($5 = 4 + 1$)

Let the children work alone writing two sentences for each picture. You help as needed.

Activity 4: Related addition and subtraction sentences. Pupil Page 73

Revise addition and its inverse, subtraction, by giving one or two examples at the blackboard. For example, write

$$18 = 9 + \square$$

Ask a child to complete the sentence to make it true. Then write

$$18 - 9 = \square$$

Say, "We know the same things in each of these sentences. We know the sum, 18, and one addend, 9. We are to find the missing addend." Let another child complete the second sentence to make it true. Ask the children to read each sentence. (Eighteen equals nine plus nine. Eighteen minus nine equals nine.) Let the children talk about the sentences.

Let the children complete all the sentences on page 73 by putting numeral cards in the boxes to make the sentences true. Point out that the pairs of sentences are related. One is an addition sentence and the other is a subtraction sentence. Counters may be used to form and separate sets if necessary.

Activity 5: Subtracting by finding the missing addends.

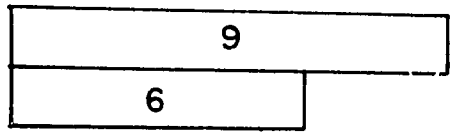
For this activity you will need strips of cardboard of different lengths that are marked with numerals 1 - 9. The 1-strip may be of any length but the 2-strip must be twice that length, the 3-strip three times that length, and so on.

Let children show some subtraction exercises with the strips. For example, put the following sentence on the blackboard:

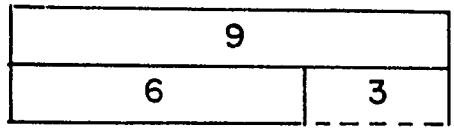
$$9 - 6 = \square$$

Put the 9-strip on the table. Then ask a child what number the sentence told

us to subtract. (6) Let another child select the 6-strip and place it along the 9-strip in this way.



Ask another child to find the strip needed to add to the 6-strip so that the whole bottom strip is as long as the 9-strip. To complete the exercise he should choose the 3-strip. By placing it properly the child sees whether or not his answer is correct. The strip will fit only if he chooses the correct numbered strip as shown below.



When the child is satisfied that the 3-strip is the correct choice, let him go to the blackboard and write the numeral in the box. Continue with other examples of the same type.

Stage 2: Subtraction, Equalities and Inequalities

Vocabulary: no new words

Materials: objects to form sets; 1-inch numeral (0-18) cards; eleven 3-inch square cards with patterns of dots; eleven $2\frac{1}{2}$" × 6" pieces of cardboard (see Activity 5); pupil pages 74-81.

Activity 1: Related addition and subtraction sentences.

Give each child 20 counters. Ask the children to form a set of 18 counters. Then ask them to take 7 of the counters and place them to one side on their desks. Say, "We have separated the set of 18 into two subsets. We have used 7 of the counters to form a subset. Without counting, can you tell me the number of counters in the other subset?"

After an answer has been told, say, "We can check the answer by writing a number sentence. If we put all the counters together again, how many will we have? (18) We put a subset of 7 counters with the other subset and we have 18 counters."

Get some pupils to look at the set and to write the sentence about them. They will write,

$$7 + \square = 18 \qquad \text{or} \qquad \square + 7 = 18$$

Ask a child to make the sentences true. (Point out that the order of adding numbers does not change the sum. This means that these two sentences are true for the same number.) When the child has written a numeral in the boxes ask the children to count the members of the other subset they have to see if the sentences are true.

Ask, "Can someone write a subtraction sentence about your sets?" A child should write

$$18 - 7 = 11$$

Ask the child to read the sentence to the class. (Eighteen minus seven equals eleven)

Let the children talk about what the sentence means. (If we have a set of 18 things and separate a subset of 7 things from the set there is also a subset of 11 things left.) Ask, "If we have seven things, how many things must we put with them to have eighteen?" Point to the addition sentence, $7 + \square = 18$, and ask, "What subtraction sentence asks the same question?" When a child answers, write the sentence $18 - 7 = \square$ under the sentence $7 + \square = 18$. Ask the children to look at the other addition sentence, $\square + 7 = 18$. Ask for a subtraction sentence to match this. Try to get the reply, $18 - \square = 7$, and write it under $\square + 7 = 18$. Get someone to tell what this sentence asks. (It asks to what number must 7 be added to get 18?)

Write several subtraction sentences on the blackboard, such as

$14 - 10 = \square$	$16 - 3 = \square$	$14 - \square = 9$
$17 - 14 = \square$	$15 - 13 = \square$	$\square - 7 = 3$

Ask the children to write under each the addition sentence that says the same thing. For example,

$14 - 10 = \square$	$16 - 3 = \square$	$14 - \square = 9$
$10 + \square = 14$	$16 = 3 + \square$	$\square + 9 = 14$
$17 - 14 = \square$	$15 - 13 = \square$	$\square - 7 = 3$
$14 + \square = 17$	$15 = 13 + \square$	$7 + 3 = \square$

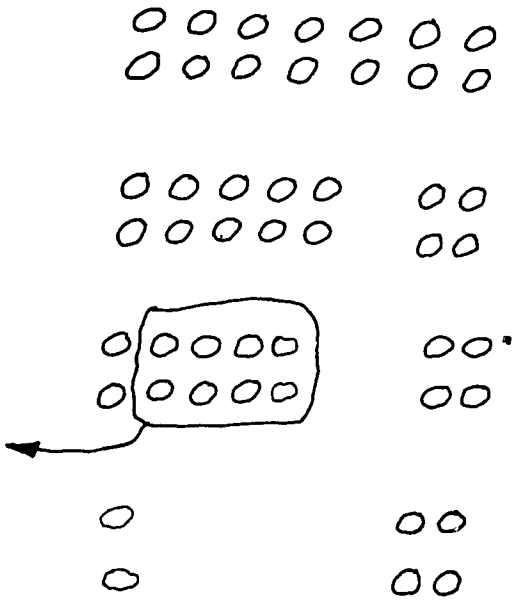
Let the children discuss each pair of related sentences. Keep talking about the meaning of "-". For example, 14 - 10 means that one addend of 14 is 10 and we are to find the missing addend. Then, for example, the sentence $14 - \square = 9$ says that one addend of 14 is unknown and the other addend is 9. The children should then see that the sentence may also be written as $\square + 9 = 14$. Let the pupils make all the sentences true. They can use counters if necessary to help them in completing a sentence or to check an answer.

Activity 2: Related subtraction and addition sentences. Pupil Page 74

Let the children copy the sentences from page 74 in their exercise books. Ask them to write the numerals in the boxes that make the sentences true.

Activity 3: Showing subtraction. Pupil Page 75

Let the children count out 14 objects. Ask, "How can we rename 14 as the sum of ten and another number?" (14 = 10 + 4) Get them to arrange their 14 objects in two subsets of 10 and 4. Say, "Take away 8 objects from your set of ten." (10 - 8 = 2). Ask the pupils to count the objects in the subset which is left after the set of 8 was removed. (2 + 4 = 6) The steps of the action may be shown on the blackboard like this:



Set of 14 objects

Think of 14 as 10 + 4

Take 8 away; subtract 8 from 10

Objects remaining; 2 + 4 = 6

Show this example in the same way: 16 - 9 = $\square$. Let the children open their books to page 75. Get them to use counters to help in completing sentences to make them true. They do this by renaming a number greater than ten as ten plus another number. They should regroup their counters into a set of ten and a set of the remaining number of objects.

Let the children copy the sentences in their exercise books.

<u>Activity 4:</u> Subtraction sentences. Pupil Page 76

Let the children find page 76. Let them talk about the wheel they find on the page. There are eight subtraction sentences represented on the wheel. For each the number ten is needed to make the sentence true.

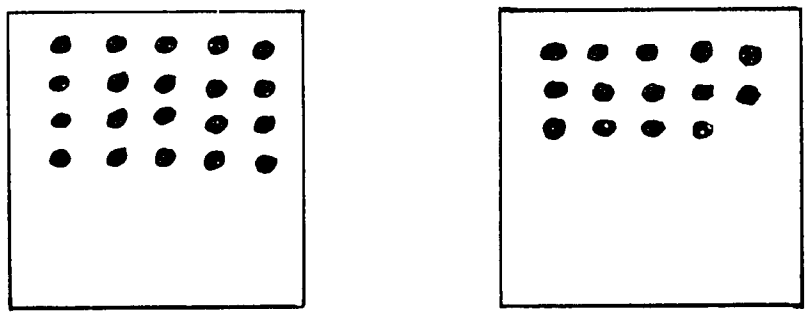
Ask the children to look at each of the sections of the wheel in turn. Read one of the sentences with the children, for example, 11 - 1 = 10. Say that we begin with the number named on the outer ring and subtract from it the number named on the inner ring. For example, subtract one from eleven, two from twelve, and so on.

As the children think about each sentence, they find that the number needed to make a true sentence is ten. When they have agreed that ten is the missing number in <u>every</u> case, they place a 10-card on the center of the wheel.

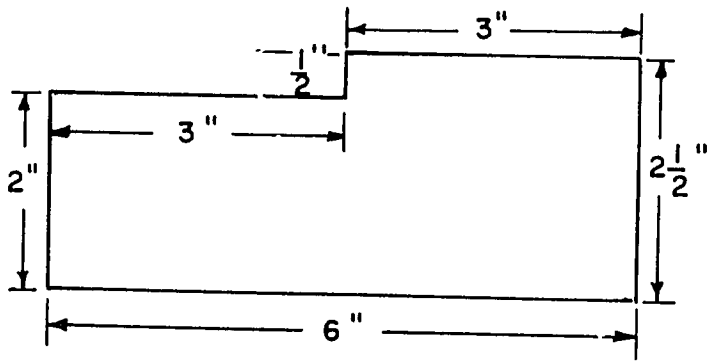
Then let the children complete all the sentences at the bottom of the page to make them true. They place a numeral card in each box.

<u>Activity 5:</u> Subtraction game.

For this activity you will need eleven 3-inch square cards with patterns of dots as shown below. Make the dots in rows of five or fewer. The number of dots should be different on each card, with totals from ten to twenty. For example, the pattern of 20 and the pattern of 14 will look like this:



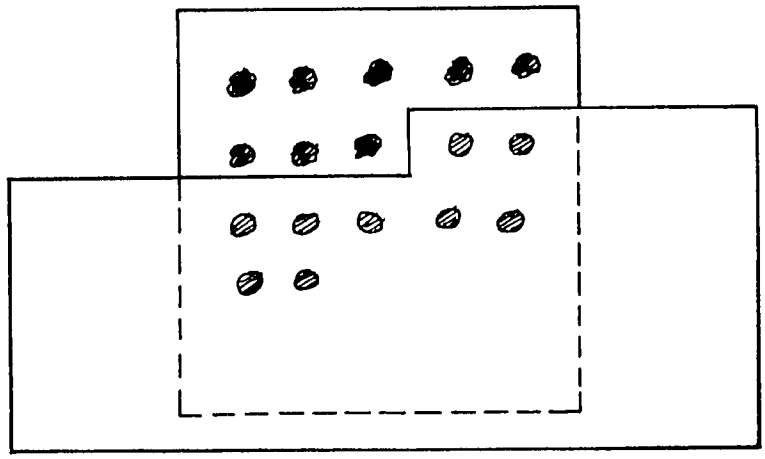
Then cut eleven pieces of cardboard in the following dimensions.



Two children at a time can use a pattern card and cardboard. Four to six children may work in one group with two children playing the game in turns.

First, the two children playing decide upon the total number of dots on the card. Let us suppose that they agree that there are 17 dots. They both write "17" in their exercise books. One child covers a number of the dots with the cardboard while the other closes his eyes. The second child then opens his eyes and decides

upon the number of dots he thinks are covered. He does so by finding the number left uncovered and subtracting this number from the total number. (17 in this example)



In the sketch above, the child finds eight dots uncovered. He may subtract 8 from 17 and answer "9" immediately. If he finds it helpful he may write in his exercise book either the sentence

$$17 - 8 = \square \qquad \text{or} \qquad 17 = 8 + \square$$

After the answer is given, the first child removes the cardboard and the answer is checked. If the child has answered correctly, then it is his turn to cover the card. Another child can enter the game and try to tell the number of dots covered on another card.

Any number of dots may be covered. It may be all of the dots, it may be none, it may be any part.

Continue the activity as needed.

<u>Activity 6:</u> Subtraction in vertical form.

Write a subtraction sentence on the blackboard and then show the same subtraction in vertical form.

$$17 - 8 = \square \qquad \begin{array}{r} 17 \\ -\ 8 \\ \hline \end{array}$$

Say, as you point, "Both of these say the same thing." Ask a child to complete both exercises so that the sentence is true. Let him read the sentence. Both examples may be read "Seventeen minus eight equals nine."

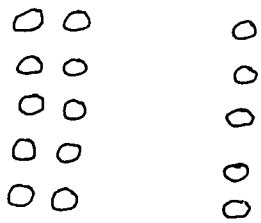
Point out that we may also think of the examples as saying that the number added to eight to get seventeen is the number nine. Let the children give several other examples showing subtraction in both forms.

Then put a number of exercises such as the following on the blackboard.

15	18	14	11	13
- 7	- 9	- 8	- 6	- 8

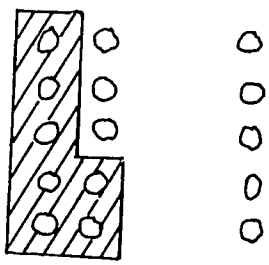
Say, as you point to the first exercise, "We wish to subtract seven from fifteen. Let us make a picture of fifteen things on the blackboard." Point to the numeral "15" and say, "How many tens are there? (1) We will draw one set of ten things on the board. How many ones? (5) We draw five more things."

Draw fifteen discs in this pattern, with ten discs separated from the five discs.



15 = 10 + 5

"What number is subtracted?" (7) Cover seven of the group of ten discs with a piece of cardboard.



Say, "Now let us find the number of discs left uncovered. We subtracted 7 from the ten. How many are left on this side?" (3) "How many more are left on this side?" (5) The number left is 3 + 5. How many all together?" (8)

Let a child make the first sentence true by writing "8" on the blackboard. Use the same plan for the other exercises.

The purpose of this method is to help the children see that 15 - 7 = $\square$ may

be expressed as $10 - 7 + 5 = \square$ or as $3 + 5 = \square$. Do not write these sentences but help the children to see that we subtract seven from ten to get three and then add the remaining five.

Activity 7: Subtraction in vertical form. Pupil Page 77

Get the children to copy the exercises from page 77 into their exercise books and make the sentences true. First introduce one or two examples in the way you used in Activity 6.

Activity 8: Different names for numbers. Pupil Page 78

Put the example $9 - 5$ on the blackboard. Ask the children to tell what number $9 - 5$ names. (4) Ask if anyone can write another name for four using the minus symbol. Many answers could be given such as $10 - 6$; $5 - 1$; and $19 - 15$. Let many children come to the blackboard and write other names for the number four. Do the same for other numbers, for example, $16 - 11$.

Ask the children to find page 78. Get them to show three different names for each number using numeral cards.

Activity 9: Subtraction sentences. Pupil Page 79

Write a sentence of the type shown on page 79 on the blackboard. For example, write

$$6 - 1 = 8 - \square$$

Tell the children that they are to make a true sentence by writing a numeral in the box. Ask a child to read the sentence and then complete a true sentence. After he has written a numeral, ask him to tell the number named by "6 - 1". (5) Then point to "8 - $\square$" and ask, "Does this part of the sentence name five also?" He should understand that if so the sentence is true. If he has written a numeral other than "3", ask him to tell what number is named. Then he should change his answer.

Let the pupils work a number of similar examples. Then ask them to copy and complete the exercises on page 79.

Activity 10: Equality and nonequality. Pupil Pages 80-81

Revise the meaning of the symbol for "is equal to" (=) and the symbol for "is not equal to" (≠). Put some examples on the blackboard, such as

18 - 8 ___ 12	16 - 7 ___ 10
15 - 10 ___ 5	10 - 10 ___ 0
19 - 2 ___ 17	14 - 7 ___ 5

Let the children in turn write either "=" or "≠" in the spaces so that the sentences are true. After a child writes a symbol let him read the sentence and say whether it is true.

Write some pairs of sentences on the blackboard like the following

14 - 6 = □ 14 - 6 ≠ □

Ask a child to make each sentence true by writing a numeral in each box. After completing the sentences, let the children read both sentences and decide whether they are true.

Ask if anyone can put different numerals in the boxes so that the sentences are still true. Let several children try different numerals and read the new sentences.

The pupils will say after trying different numerals that only one number will make the first sentence true, but that many numerals can be written in the box in the second sentence to make it true.

Ask whether anyone can think of a numeral that can be written in both boxes so that both sentences will be true. If suggestions are made let the children try them out. They should see that there is no number which makes both sentences true.

Assign as many of the exercises on pages 80-81 as can be done by the children in a single lesson. Continue with the others in the next lesson if the exercises are needed.

Stage 3: Using the Number Line to Subtract

Vocabulary: no new vocabulary

Material: Cardboard with stick-figure pictures, Pupil Pages 82-85

Teaching Procedure:

Activity 1: Addition and subtraction with the number line. Pupil Page 82

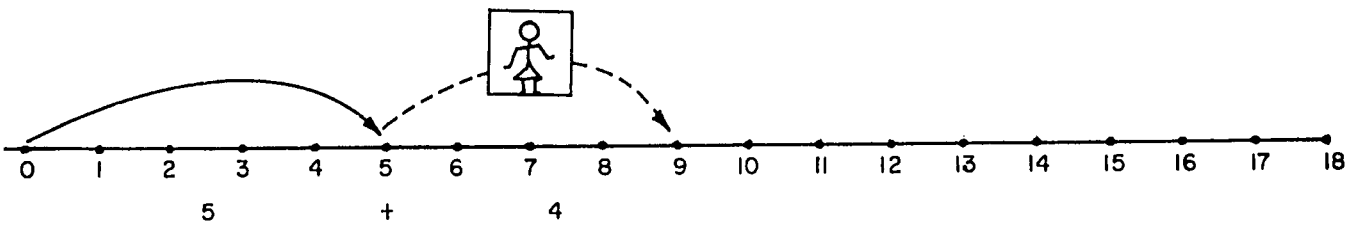
Draw a number line on the blackboard and put in points numbered from 0 - 18. Put in a few of the numerals, say 5, 12, and 15, then ask different children to fill in the others.

To show moves on the line, use two cards. On one draw a stick figure of a girl and on the other a boy.

Write a sentence on the board:

$$9 - 5 = \square$$

Say, "We may think of this sentence as asking us, 'What number added to five gives nine?' We must find the missing addend. To find the missing addend we will use the number line in this way." Get one of the children to show 9 - 5 by moving one of the stick figures along the line. Give the figure the name of the child showing the movement. For example, if you call on Akua you can say, "Akua is going to jump along the line. What mark must she end on? (9) How many spaces does she move in her first jump?" (5) "She is at 5; how many more spaces must she jump to get to 9?" As you and the pupils talk Akua should make jumps on the line like this.

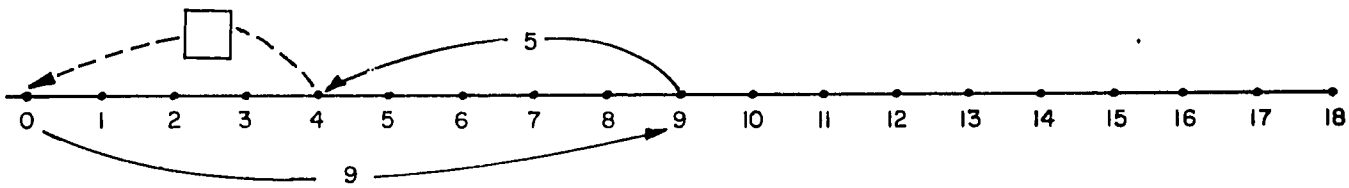


Ask Akua to make the sentence true by writing "4" in the box.

Say, "We can show subtraction another way. When we separated sets of counters and talked about subtraction, we took away part of the counters. We can

find the number which makes a subtraction sentence true by moving along the line, then moving back the number of spaces to be subtracted."

Put the same sentence, "9 - 5 = □", on the blackboard. Move a stick figure card to the right from 0 to 9, and move back 5 spaces. Say, "We first move 9 spaces. Then we subtract what number? (5) So how many spaces do we move back? (5) What mark are we on? (4) What number is 9 - 5?" (4)



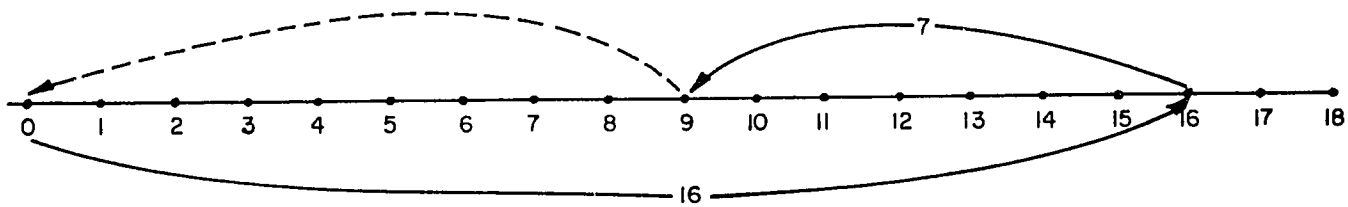
The dotted line in the figure shows the number added to five to get nine.

Let the children talk about the two ways of showing this sentence on the line. They may say, "If we move 9 spaces, then move five in the other direction, we end on the 4 mark." (9 - 5 = 4) "We also see that if we first move 4 spaces, then 5 more, we end on the 9 mark." (4 + 5 = 9)

Let the children show subtraction sentences on the number line on page 82. Let them show how Mr. Frog hops on the number line to show a number sentence. Ask the children to use a small object (bean) and pretend it is Mr. Frog. Let them write the sentence in their exercise books.

Begin with this example on the blackboard,

16 - 7 = □



Ask, "At what point does Mr. Frog start to make the jumps?" (0) "How many jumps will he make in order to get to 16?" (16) "In what direction shall he jump?" (To the right.) "We want to subtract 7 from 16. How many jumps will Mr. Frog make

from 16 and in what direction?" (7 jumps to the left) "Where does he land?" (9) Show these jumps on the number line as the children talk. Ask, "What number goes in the box to make the sentence true?" (9)

Show another way to find the number on the number line. Say, "Mr. Frog wants to get from 0 to 16. First he hops from 0 to 7. How far must he hop next time?" (9)

You may also draw the number line on the floor and let the pupils take turns solving problems by making jumps on the floor.

Write several sentences on the blackboard. Ask the children to find page 82 in their books. Let them move a small object along the number line as they talk about each of the sentences, for example, 14 - 8 = $\square$. Say, "How many spaces does Mr. Frog hop first?" (14) "Let him hop from 0 to 14. To subtract 8 what hops must he make?" (He hops 8 spaces back) "Where does he land?" (6) "What number makes the sentence true?" (6) Or, "Mr. Frog wants to get from 0 to 14. First he hops from 0 to 8. How far must he hop next time?" (6) Continue as needed.

Activity 2: Showing subtraction using the number line **Pupil Page 83**

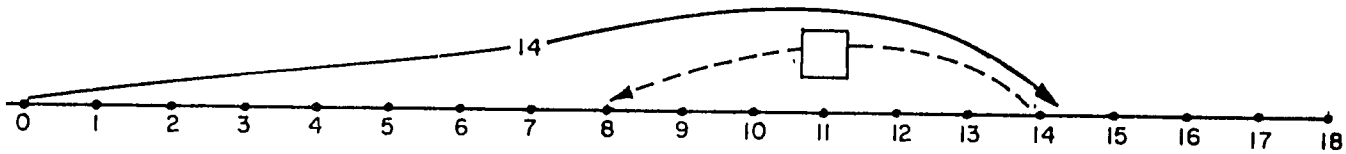
Ask the children to copy the subtraction sentences on page 83 in their exercise books. Let them complete the sentences to make them true. The number line pictures may be used as an aid in finding the missing number. Go over the first sentence with the class.

Ask a child to read the sentence. (17 minus 8 is equal to some number) Ask, "Where do we begin on the number line?" (0) "How many spaces do we move first?" (17) "How many spaces do we move next and in which direction?" (8 spaces back) "Where do we land?" (9) "What number is 17 - 8 equal to?" (9)

Let the children work out the other sentences for themselves. Give help as needed.

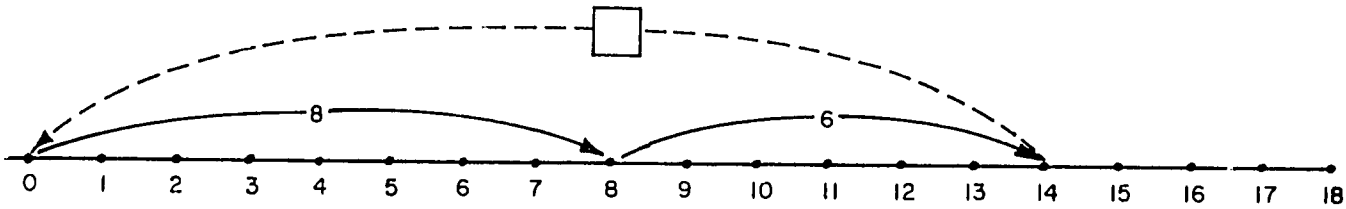
Activity 3: Sentences with missing numbers. Pupil Page 84

Write on the blackboard this sentence: 14 - $\square$ = 8. Show the sentence on a blackboard number line like this as you talk with the children.



Ask these questions: "At what point do we begin?" (0) "How many jumps do we make first?" (14 forward) "The sentence says that when a certain number is subtracted from 14 we get 8. If we move backward that number of spaces from 14, where will we land?" (At 8) "In order to move back to the 8 from mark 14, how many spaces must we jump?" (6) "What then is the missing number in the sentence?" (6)

Write an example of a different type on the blackboard: $\square$ - 6 = 8. If the children think of this in terms of addition, the solution is a simple one. Let them say, "The sentence says that the number added to 6 to get a number is 8. We will show that 6 and 8 are added. The number in the box is 14." This may be shown on the number line more easily than it can be talked about.



Do not tell the children that to find the number that makes the sentence true we add 8 to 6. Solving the problem should not become just a trick. If children know the relation of subtraction and addition sentences they will discover this method for themselves. Ask the children to complete the sentence so it is true. Let them use the number line to work it out.

Give other similar examples. Let the children in turn complete the sentences to make them true. They may use the number line to illustrate their sentences.

After several examples, tell them to open their books to page 84 and complete

the exercises. Get them to copy the sentences in their exercise books. If it is helpful they can use the number line shown on page 82 to work out their answers. Let them move small objects along the line or trace along the line with their fingers.

Note: Some children may still need to use counters and combine and separate sets to solve these exercises. Some may need to think about addition as they work. For example, $\square - 8 = 7$ means that one addend of a sum is 8 and the other is 7; $15 - \square = 8$ means that one addend of 15 is unknown and the other is 8.

Activity 4: Subtraction and inequalities. Pupil Page 85

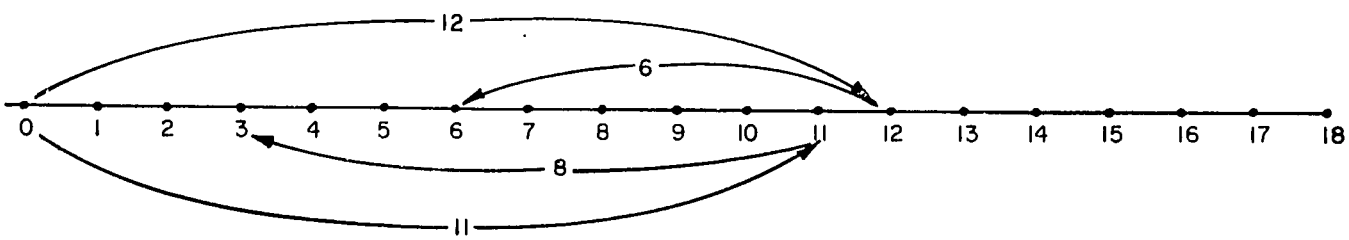
Put an incomplete sentence such as the following on the blackboard

12 - 6 ___ 11 - 8

Explain that the blank space in the sentence means that a part of the sentence is missing. Say, "We need to write something in the space to make the sentence true."

First, let the children tell the numbers named by "12 - 6" and "11 - 8". Ask, "Are the numbers 12 - 6 and 11 - 8 the same?" (No) "What can you tell me about these two numbers? Are they equal?" Go on asking questions until the children say that 12 - 6 is greater than 11 - 8.

Ask a child to show the two numbers on the number line and to show that one is greater.



Ask, "Which of the numbers is greater?" (12 - 6) Let someone write the sign that means "is greater than " (>) in the blank space in the sentence. Follow this same plan with this sentence,

10 - 4 ___ 15 - 6

In this sentence, the missing symbol is < , because 10 - 4 is less than 15 - 6. Show this sentence on the number line.

Let the children open their books to page 85. For each exercise let them decide what symbol goes between the numerals to make a true sentence. Encourage the children to locate points on the number line to help them. Let the children copy the sentences in their exercise books and use either "<" or ">" to make true sentences.

Stage 4: Using Letters to Name Numbers in Subtraction Sentences

Vocabulary: no new vocabulary

Materials: one-inch numeral (0-18) cards; a few 2" x 3" cards with letters such as a, b, c, m, n, y on them; Pupil Page 86.

Teaching Procedure:

Activity 1: Letters used to name numbers in subtraction sentences.

Say, "Do you remember the story in which Mr. Spider came and played a trick on us? He painted letters over the numerals in our addition sentences. You know too that we found that we could tell what the missing numerals were. If he played the same trick on us with numerals in our subtraction sentences, do you think we could still find the missing numeral? Let us try a subtraction sentence."

Write this sentence on the blackboard

$$15 - 10 = n$$

"What number is n in this sentence?" When someone suggests a number, rub off the n and replace it with the numeral suggested. Ask, "Is the sentence true?" If the answer is 5, say, "We know that if n is 5 the sentence is true." Write this sentence on the blackboard

n is 5

The following plan may be used for other examples. Tell the children to close their eyes while you write on the blackboard the true sentence,

$$6 = 12 - 6 \quad .$$

Cover the last "6" with a card with a letter on it. For example

$$6 = 12 - m$$

Then the children open their eyes and try to guess what m is. Say, "This sentence is true. What is m?" When an answer is given, remove the card so that the pupils can see immediately whether their answer makes the sentence true. If correct, write "m is 6".

Give many similar examples and let the children take turns at writing true sentences covering a numeral with a letter-card and calling on other children to tell what number the letter stands for to make the sentence true.

Use different letters so that the children understand that we do not always use the same letter. They should also understand that one letter may stand for different numbers in different sentences.

Activity 2: Letter names for numbers. Pupil Page 86

Put some examples like those on page 86 on the blackboard. For example, write

$11 - 9 = m$	$14 - a = 6$
m is ___	a is ___

Ask a child to write the numeral in the space naming the number which makes the sentence true. In the first example, he should know that if m is replaced by the numeral 2 the sentence is true. Therefore, he writes "m is 2". Call on different children to do the examples.

Ask all the children to open their books at page 86. Let them tell what numbers make the sentences true by placing numeral cards in the boxes.

Stage 5: Subtraction from Numbers Twenty and Beyond

Vocabulary: no new vocabulary

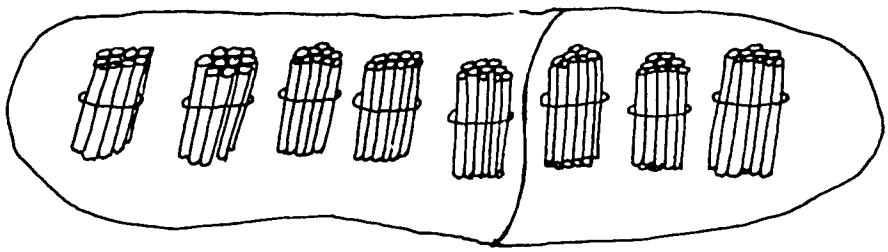
Materials: single sticks and bundles of tens; Pupil Pages 87-92.

Activity 1: Subtracting multiples of ten. Pupil Pages 87-88

Give several examples of separating sets of bundles of ten sticks. This will help show subtraction of multiples of ten. For example, begin with 8 bundles of sticks, ten in each bundle. Ask, "How many tens are there?" (8 tens) "How do we write the numeral for 8 tens?" (80) Let a child write the numeral on the blackboard.

Take away 3 of the bundles as you say, "I am separating 3 tens sticks and putting them here. How can we show in a sentence that we separated 3 tens from a set of 8 tens?" Let someone write on the blackboard the sentence, 80 - 30 = $\square$

It may be helpful to draw a picture of the set of bundles on the blackboard.



Say, "If we put all the bundles back together, we have 8 tens or 80 sticks. How many do we add to 30 to get 80? How many tens do we add to three tens to get eight tens?"

Show the same exercise in vertical form.

$$\begin{array}{r} 80 \\ -30 \\ \hline 50 \end{array}$$

Ask the children to read this. (Eighty minus thirty equals fifty; or 8 tens minus 3 tens equals 5 tens.)

After several examples, let the children copy the exercises from pages 87-88 in their exercise books and do the subtraction. It may be wise to use the pages on two different days.

Activity 2: Subtraction of numbers with 2-digit numerals. Pupil Page 89

Write this sentence on the blackboard, 38 - 16 = $\square$.

Show the sentence with some bundles of ten sticks and some single sticks. Hold up 3 bundles of sticks and 8 single sticks. Ask how many sticks there are. (3 tens and 8 ones) Place these on the table. Tell a child to move 16 of the sticks (1 bundle and 6 loose sticks) away from the set. Ask the class how many sticks are left. (2 bundles and 2 loose sticks, or 2 tens and 2 ones, or 22)

Write the following on the blackboard to show these steps:

38	=	3 tens and 8 ones		= 30 + 8
- 16	=	1 ten and 6 ones	or	= 10 + 6
22		2 tens and 2 ones		= 20 + 2

Work another example on the blackboard in the same way.

Let the children open their books to page 89. Ask each child to copy the first exercise and find the number that makes the sentence true. Let them use the bundles of sticks, if necessary. Go around the class and give help to the children who need it.

Ask a child with the correct answer to tell the number. Let him tell how he got the number. Give further help to those who made mistakes.

Let the children work the other exercises on page 89 in their exercise books.

Activity 3: Subtraction of numbers with 2-digit numerals. Pupil Page 90

Put this example on the blackboard

$$\begin{array}{r} 25 \\ -13 \\ \hline \end{array}$$

Ask, "How many tens and how many ones does '25' show?" Write "2 tens and 5 ones". Then ask, "How many tens and how many ones does '13' show?" Complete the example on the board as

25	=	2 tens and 5 ones
- 13	=	- 1 ten and 3 ones

Ask a child to subtract the ones and then the tens. Say, "We have 1 ten and 2 ones. How can we write this?" (12) "Twelve is the number added to 13 to get 25."

Ask if someone can write the same example as a mathematical sentence. (25 - 13 = 12).

Give several similar examples in both horizontal and vertical form. Then let the children open their books to page 90 and complete the exercises by placing numeral cards in the boxes.

Activity 4: Subtraction sentences. Pupil Page 91

Talk with the children about the example 28 - 14 = $\square$ to introduce the exercises on page 91. Say, "We are to write a numeral to make a true sentence. We use the same plan we used before and write it like this."

$$\begin{array}{r} 28 \\ -14 \\ \hline \end{array}$$

"We think of 28 as 2 tens and 8 ones and of 14 as 1 ten and 4 ones." Let the pupils subtract and say, "We get 1 ten and 4 ones or 14."

Activity 5: Subtraction. Pupil Page 92

This is a revision page. Go over one or two examples at the blackboard and then let the children work alone. They can copy the exercises in their exercise books and subtract the numbers.

Stage 6: Using Subtraction in Word Problems

Vocabulary: difference

Materials: Pupil Pages 93-94

Activity 1: Using subtraction to answer questions. Pupil Page 93

Ask the children to look at the pictures on page 93. Let them tell about the

different things in the pictures. Then ask some questions such as the following:

How many more dogs are there than cows? How could we write this as a subtraction sentence?

What is the difference between the number of ducks and the number of cows? Write the subtraction sentence.

How many fish do you see in the stream? If the boy catches two, how many will be left? Can you write a sentence about this?

What is the difference between the number of flowers and the number of rabbits? Write the sentence that tells us.

Ask many more questions and let the children make up questions and stories.

Activity 2: Using subtraction in word problems. Pupil Page 94

Pupil page 94 is a page of pictures. Many questions could be asked about them. Some are suggested below for your to use. You will think of others. The pupils can think of still others.

1. Mother bought some milk from the market. When she came home she found some of the bottles were broken in the basket.
   How many bottles of milk did mother buy?
   How many bottles were broken?
   How many bottles were left unbroken?
2. I saw some rabbits in the compound. Another rabbit came along.
   How many rabbits did I see at first?
   How many more rabbits came?
   How many rabbits did I see altogether?
3. My friend has some dogs. Some of them ran away.
   How many dogs did my friend have?
   How many ran away?
   How many dogs were left?
4. There were some balls on a table. The children took some for their game.
   How many balls were on the table?

How many were taken for the game?

How many balls are left on the table?

5. Okosi's mother gave him some sweets. He ate some of them.

How many sweets did Okosi have?

How many did he eat?

How many were left?

Activity 3: Putting word problems into subtraction sentences.

The following problems are to be read to the children. Talk about each problem and let the children try to write a number sentence about the problem. Then use the sentence to solve the problem. Get them to notice that putting the words into mathematical sentences helps them to get an answer.

1. Football is played by two teams. Each team has the same number of players. One team has eleven players. How many players are in the other team. (11) How many players are in both teams together? (11 + 11 = b)

2. In the Headmaster's office there is a set of balls. The set of balls has 28 members. Fourteen of the balls are red. The other balls are yellow. How many balls are yellow? (28 - 14 = y)

3. A farmer planted 40 coffee trees. Thirty of the trees grew up well, and the other trees died in the hot sun.
How many coffee trees died? (40 - 30 = n)

4. A boy's exercise book had 60 pages. He used 40 pages writing stories. How many pages were left unused? (60 - 40 = n)

5. A girl had 70 ground nuts. She gave her brother 30 of her nuts. How many nuts did she have for herself? (70 - 30 = n)

6. In a classroom cupboard there are 90 exercise books. Forty of them have blue covers, and the rest have red covers.
How many books have red covers? (90 - 40 = n)

7. Two girls each had a set of beads. The first girl had yellow beads and

the second girl had black beads. They put their beads together and made a set of 70 beads. There were 30 yellow beads.
How many beads were black? (70 - 30 = n)

8. Two boys brought bundles of sticks. The first boy brought a bundle of 44 sticks. The second boy brought a bundle of 33 sticks.
How many sticks did the two boys bring together? (44 + 33 = n)

9. A fisherman had 50 small fish in his basket. He sold 30 fish in the market. He took the rest home.
How many fish did he take home? (50 - 30 = n)

10. A man took 80 eggs to the market. He broke 40 of them on the way.
How many eggs were not broken? (80 - 40 = n)

Stage 7: Addition and Subtraction Sentences

Vocabulary: no new vocabulary

Materials: one-inch numeral (0-18) cards, sticks in bundles of tens and some single sticks; pupil pages 95-100

Activity 1: Making sentences true. Pupil Page 95

The sentences on page 95 are open sentences. They use letters to name numbers. Some are subtraction sentences and some are addition sentences. Go over a few examples with the children before they open their books.

Put this sentence on the blackboard

$$3 + 5 = 10 - n$$

Say, "We are to tell what number n is to make the sentence true." Point to "=" and say, "Remember that this symbol means that '3 + 5' is the same number named by '10 - n'. What is 3 + 5?" (8) "What number is n, so that 10 minus n equals 8?" (2)

Get the children to read each symbol carefully. Tell them that in some sentences there is a "+" and in others there is a "-".

Let the children find page 95. Ask them to place numeral cards in the boxes to make the sentences true.

Activity 2: Making sentences true. Pupil Page 96

In the sentences on page 96 there are two numbers named by letters. The pupils are told one of the numbers. They are to find the other.

Work several examples on the blackboard before the children open their books. Put these sentences on the board.

$$m + n = 8$$
$$m \text{ is } 5$$
$$n \text{ is } \square$$

Ask a child to read the first sentence. Say, "We know that m and n are both numbers. What number is their sum?" (8) "We are told what m is. What number is m?" (5) "If we rub off the m in the sentence we could put in the numeral 5." Write the numeral 5 in place of m. The sentence now is $5 + n = 8$. Ask, "What number is n?" (3) Let the child who answers write the numeral in the box.

Do some other addition sentences and some subtraction sentences. Then let the children use numeral cards to make the sentences true on page 96.

Activity 3: Addition and subtraction revised. Pupil Pages 97 - 99

The exercises on pages 97-99 are like ones the children have done before. Put this exercise on the blackboard.

$$\begin{array}{r} 36 \\ \underline{+42} \end{array}$$

Point to the addition symbol and ask, "What does this say?" Say, "Look carefully at the symbol in any exercise. The symbol tells you whether you add or

subtract." Let a child work out the exercise with bundles of ten sticks and single sticks. He first puts down three bundles and six sticks. Then he puts down four bundles and two sticks. Let him tell how many tens (7) and how many ones (8) there are together. Ask, "How do we write this?" (78) Let him write 78 on the blackboard. Let the children talk about this exercise and how 78 was found.

Ask the children to open their books to page 97. Get a child to read the first exercise. Let all the children write the sentence in their exercise books and complete it to make true sentences. Let a pupil tell his answer and then show the sentence with sticks. Let the children copy and complete the other exercises to make true sentences.

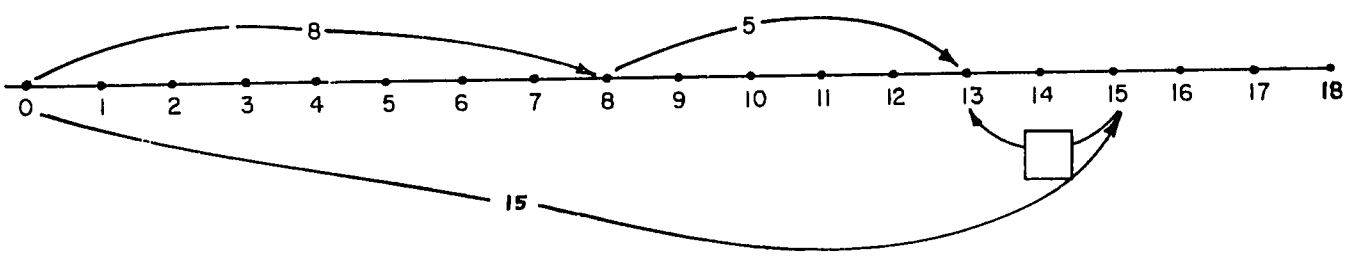
Follow the same procedure for page 98 and 99 as needed. Use sticks often to work out examples. Let the children work alone as soon as they can.

Keep asking the children if there is a "+" or "-" symbol in a sentence. Then ask what the symbol tells them to do to numbers.

Activity 4: Addition and subtraction on the number line. Pupil Page 100

The number line may be used to solve problems involving addition and subtraction. Draw a number line on the blackboard and mark off points from 0-18. Write this sentence on the blackboard

$$8 + 5 = 15 - \square$$

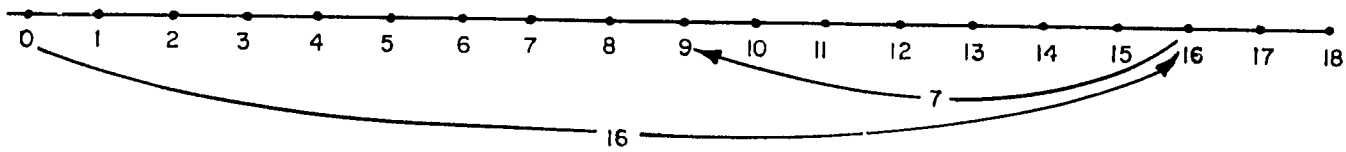


Let a child show 8 + 5 on the number line. He will show two jumps, one of 8 followed by one of 5. The second jump ends at 13. The sentence says that this sum (13) is the same number as 15 minus a certain number. To find that number, first mark point 15. Ask a child how many jumps it takes to go from 15 to 13 (2).

Let a child write the numeral 2 in the box and decide whether the sentence is true. Let the class talk about the exercise and how it was solved.

Follow the same plan with the following sentence

$$5 + \boxed{} = 16 - 7$$



Let a child show (16 - 7) on the number line. This uses two jumps. The second jump ends at point 9. Let the children say, "Another name for 16 - 7 is 9. Nine is equal to 5 plus a certain number." To find the number, first let a child mark the point 5. Ask the children how many jumps from 5 to 9. (4) Say, "Four is the number that should be in the box, $5 + \boxed{4} = 16 - 7$. Work a few more examples.

Let the pupils open their books to page 100 and work the problems there in their exercise books in the same way.

UNIT 6

FRACTIONS I

Objectives:

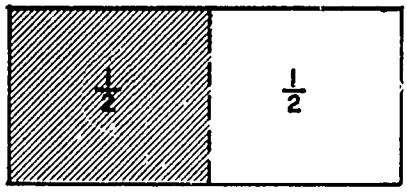
1. To revise the meaning of the fractions one-half and one-fourth.
2. To extend the concept of fractions to include one-third, one-sixth, one-eighth, and to learn their names and numerals.
3. To locate $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{6}$ on the number line.
4. To separate sets into subsets with the same number of objects and to separate regions and line segments into congruent parts.
5. To compare pairs of fractions.

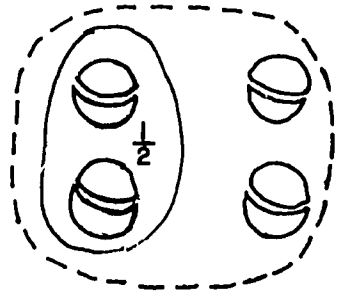
Background Information for Teachers:

Also see Part 2, "Arithmetic," in your Teachers' Handbook.

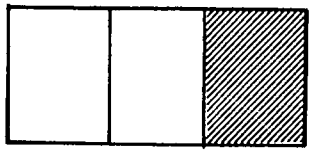
Children have many experiences with parts of objects. They need numbers to think about these parts. The numbers 0, 1, 2, 3, ... are used for sets of whole objects. That is why we call these numbers whole numbers. We use other numbers to describe parts (or fractions) of an object, parts of sets of objects, parts of regions. We call these numbers fractions.

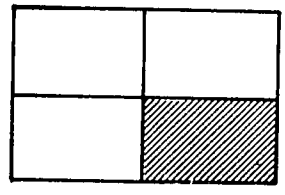
The idea of a fraction is approached in Units 6 and 11 through the use of physical objects. An object or set of objects is called a whole. If the whole is separated into two equal parts, then each part is called one-half of the whole. The symbol "$\frac{1}{2}$" is used to name the number for one of these two equal parts. For example:

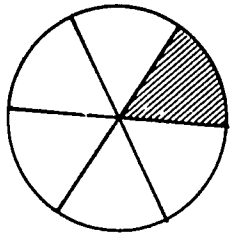


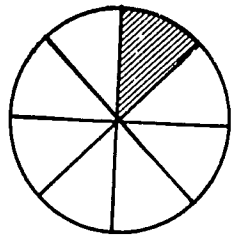


Some sets of objects can be separated into three subsets with the same number of objects. Also, regions of various kinds can be separated into three parts of the same size. Each part or each subset is called one-third of the whole. The symbol for the number one-third is "$\frac{1}{3}$". When a region is separated into four parts of the same size or a set of objects is separated into four subsets with the same number of objects, we call each part one-fourth of the whole. The symbol for the number one-fourth is "$\frac{1}{4}$". Similarly, one-sixth, $\frac{1}{6}$, is one of six equal parts and one-eighth, $\frac{1}{8}$, is one of eight equal parts.









There is an order among fractions, just as there is among whole numbers. This order is best shown on the number line. For example, we know that one-half of an object is greater than one-fourth of the object. That is, " $\frac{1}{2} > \frac{1}{4}$ " is a true sentence. We can show on a number line which of any two fractions is greater. The one whose point on the number line is to the right of the other is the greater fraction.

$\frac{1}{2}$ is to the right of $\frac{1}{4}$; $\frac{1}{2} > \frac{1}{4}$

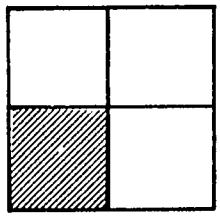
When we talk about the order of numbers, we use the words "greater than" or "less than". The symbols for these words are ">" for "is greater than" and "<" for "is less than". For example:

" $\frac{1}{2} > \frac{1}{3}$ " is a true sentence

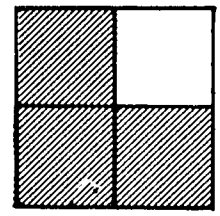
" $\frac{1}{4} < \frac{1}{2}$ " is a true sentence

" $\frac{1}{8} > \frac{1}{6}$ " is a false sentence

In this unit we also introduce fractions such as $\frac{2}{3}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{2}{6}$, and so on. Since $\frac{1}{3}$ refers to one of three equal parts of a whole, the fraction $\frac{2}{3}$ refers to two of three equal parts of a whole. In general, if a is a counting number and b is a counting number, then the fraction $\frac{a}{b}$ refers to a parts of the whole, and each part is obtained by separating the whole into b equal parts. For example,



$\frac{1}{4}$, or 1 of 4 equal parts



$\frac{3}{4}$, or 3 of 4 equal parts

A number can be named by many fractions. Two fractions that are names for the same number are called <u>equal fractions.</u> For example, in this unit we see that:

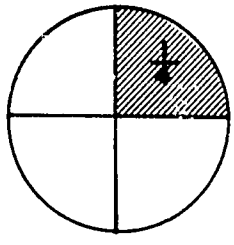
one-half is named by $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$;

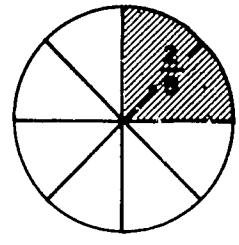
one-third is named by $\frac{1}{3}$, $\frac{2}{6}$;

one-fourth is named by $\frac{1}{4}$, $\frac{2}{8}$.

The children will learn many more names for these fractions as they continue their study of mathematics.

The true sentence " $\frac{1}{4} = \frac{2}{8}$ " tells us that the fractions $\frac{1}{4}$ and $\frac{2}{8}$ are the same number, one-fourth. The two fractions $\frac{1}{4}$ and $\frac{2}{8}$ are the same part of a whole. This may be shown by separating the same object in different ways. For example:





In the first picture, the shaded region is one of four equal parts, and in the second picture the shaded region of the same size is two of eight equal parts.

The counting number one can be named by fractions. One is two-halves, $\frac{2}{2}$; one is three-thirds, $\frac{3}{3}$; one is four-fourths, $\frac{4}{4}$; and so on. Thus, one is represented by all these equal fractions $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$.

Stage 1: Revision of One-Half and One-Fourth; Introduction of One-Eighth

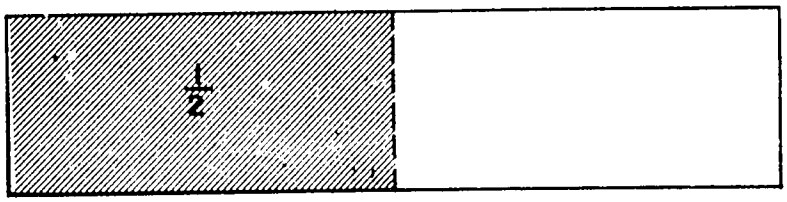
Vocabulary: one-half, halves, one-fourth, fourths, fraction, whole, part, one-eighth, unequal.

Materials: Oranges, knife; small objects to form sets, rectangular pieces of paper for you and pupils to fold; paper cutouts in shape of discs, rectangles, triangles and squares; three 5" × 7" cards with " $\frac{1}{2}$ ", " $\frac{1}{4}$ ", and " $\frac{1}{8}$ " written on them; 1-inch square cards on which pupils can write " $\frac{1}{2}$ ", " $\frac{1}{4}$ ", and " $\frac{1}{8}$ "; pupil pages 101-104

Teaching Procedure

Activity 1: Folding paper to show halves and fourths of the paper.

Give each child a rectangular piece of paper. Use one big rectangular paper for demonstration. Ask the children to watch while you fold the paper into two equal parts. Unfold it and ask the children, "Into how many parts have I folded the paper?" (2)



Ask, "Are the parts equal?" What do you call one part of it?" (One-half)

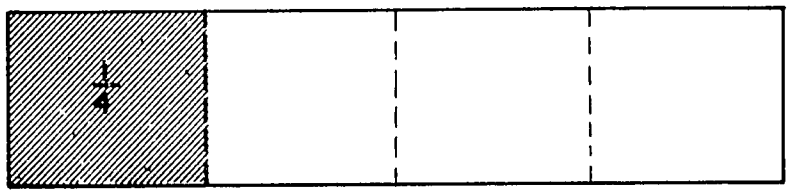
Let the children fold their papers into halves. Go around the class and make sure that they fold the paper into two equal parts. Ask them what they call each of the two equal parts.

Note: Let the children show their answers with their papers. If some children do not show halves let them check the parts and talk of why the parts are not halves.

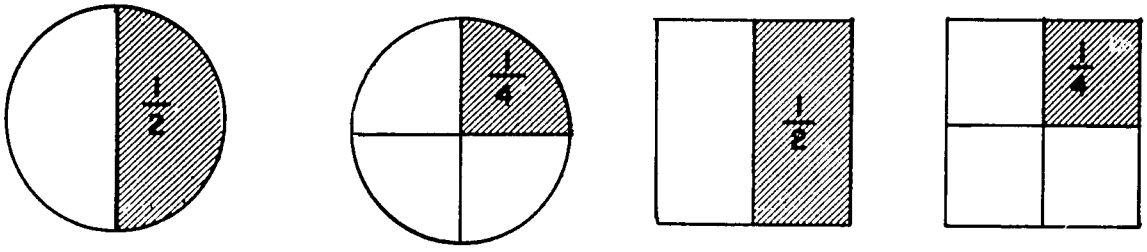
Write the numeral " $\frac{1}{2}$ " on the blackboard. Say as you write, "This is the way we write one-half." Hold up the numeral card on which " $\frac{1}{2}$ " is written and ask "What does this say?" (one-half) Let the children repeat the word several times. Ask the children to show one-half of another whole object. For example, half a page of an exercise book, half a piece of chalk, half of any folded paper, half of a window, half of a door, and so on.

Go around the class and see whether the pupils are holding or pointing to halves of objects or regions. Let the children write the numeral " $\frac{1}{2}$ " several times in their exercise books.

Fold your paper into <u>fourths</u>. Unfold it and ask, "Into how many parts did I fold the paper? (4) Are the parts equal?" Let the children watch as you match the parts.



Ask the children to fold their papers into fourths. Go around the class and see that the children fold the paper into four equal parts. Ask the children to unfold the paper. Ask, "How many equal parts can you see? (four) What do we call one part of the folded paper?" <u>(One-fourth)</u> "How many one-fourths can you see?" (4) Help the children to see that when <u>a</u> <u>whole</u> object is separated into two equal parts, one part is called one-half. When a whole object is separated into four equal parts, each part is called one-fourth. Repeat this activity using cutouts of other regions such as:

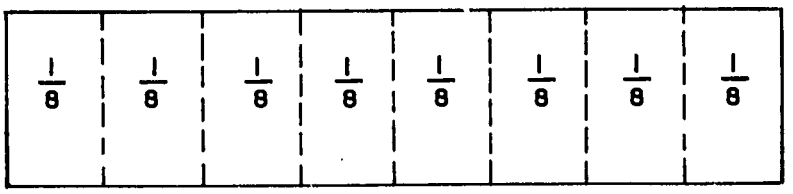


Introduce the numeral " $\frac{1}{4}$ " in the same way " $\frac{1}{2}$ " was revised earlier in the activity.

Activity 2: Introduction of the idea of eighths.

Let the children take out the paper cutouts which they folded into four equal parts in Activity 1. Revise the idea that one-half of an object is one of two equal parts and one-fourth of an object is one of four equal parts.

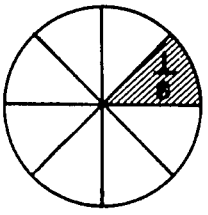
Hold up the big rectangular paper which you folded into four equal parts. Refold it into eight equal parts. Crease firmly. Unfold and show it to the children.

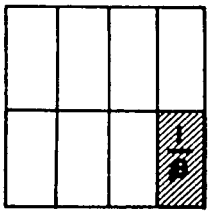


Ask, "How many parts do you see? Are these eight parts equal?" Let the children decide that the whole piece of paper has been folded into eight equal parts. Tell them that each of these parts is called one-eighth. Let them repeat the words "one-eighth" several times.

Ask the children to fold their papers to show eight equal parts. Go around the class and see that they fold their papers into equal parts. Let them say that one of the eight equal parts of the whole paper is one-eighth.

Repeat the above activity, using other paper cutouts of discs and squares as shown below.





Introduce the numeral " $\frac{1}{8}$ " in the same way " $\frac{1}{2}$ " and " $\frac{1}{4}$ " were introduced in Activity 1.

Activity 3: Matching numerals to parts of things. Pupil Pages 101-102

Give each child some blank one-inch square cards. Ask him to write $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ on the cards as needed.

Get the children to find page 101 in their books and look at the pictures. Ask, "What part of each of these figures is shaded?" Let the children place a numeral card that tells what part of a figure is shaded under each figure.

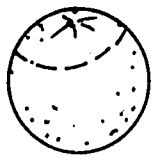
Go around and check answers. Help those who need help. If a child does not choose the correct numeral card to match the part shaded, ask him the number of parts into which each figure has been separated. Ask if the parts are equal.

Let the children turn to page 102 and look at the shaded parts of the figures. Ask them to place the correct numeral card under the shaded part of the figure the numeral describes.

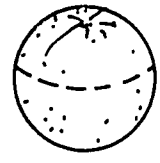
Activity 4: Using one-half, one-fourth and one-eighth.

Show the children a whole orange or some other object. Cut it into two equal parts. Ask, "How many pieces of orange do we have? Are the pieces the same size?" Say, "This orange is cut into two equal parts." Hold up one piece. Ask, "Is this part all of the orange? (No) What part of the orange is it?" (One-half)

Cut another orange into two unequal parts. Show the two parts to the children and and ask, "Are these two parts the same size?" They should be able to say that the two parts are not equal. Hold up one piece. Ask, "Is this one-half of the orange?"

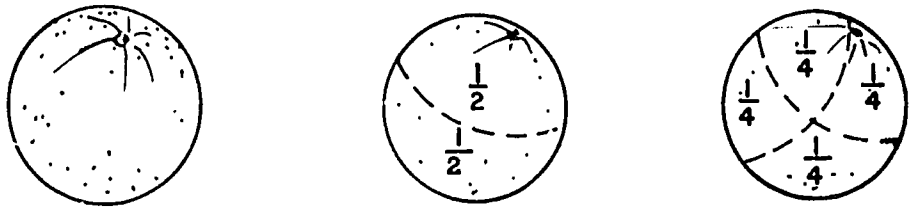


unequal parts

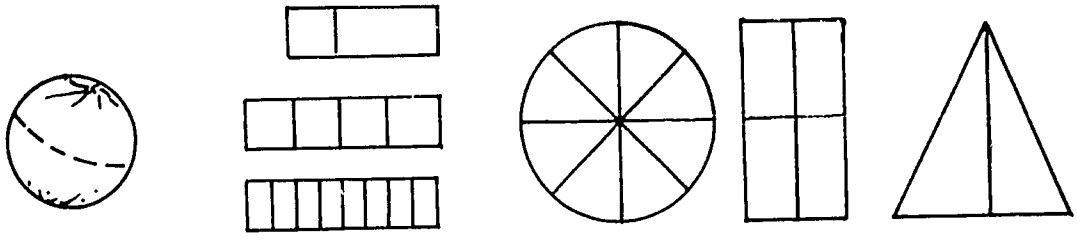


two halves

Pick up the two equal parts of the orange. Ask, "What can I do to get four equal parts of this orange?" (Cut each half into two equal parts; there will then be four equal parts of the orange.) Cut the orange and show the four equal parts to the class. Ask, "What do we call each part?" (One-fourth) Ask a child to show the other pupils the numeral card that says one-fourth.

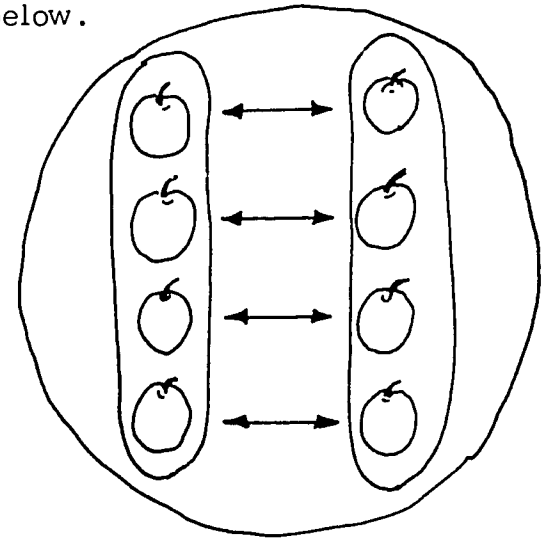


Show the four equal parts of the orange to the class. Ask, "How can we get eight equal pieces of this orange?" (By cutting each fourth into two equal parts) Cut the orange. Hold up one part and ask, "What part of the whole orange is this?" (One-eighth) Ask a child to show the other pupils the numeral card that says one-eighth. Draw on the blackboard the figures below. Ask the children to point to figures separated into halves, into fourths, and into eighths.

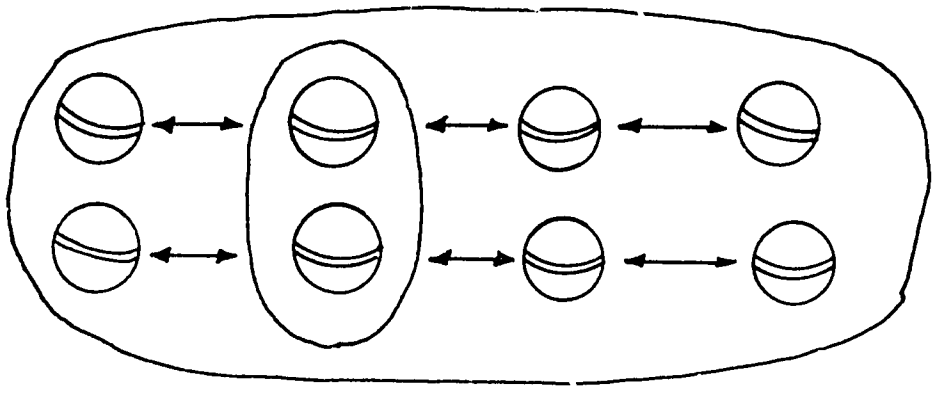


<u>Activity 5:</u> Separating sets into halves, fourths and eighths.

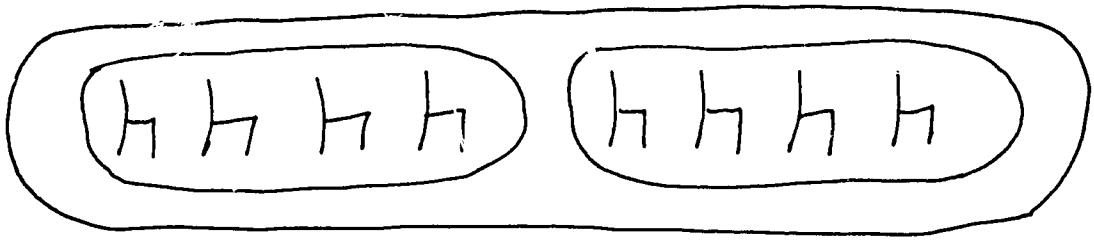
Put a set of eight objects on your table. Separate the set into two subsets so that each has the same number of members. Check this by matching the members of the subsets as shown below.



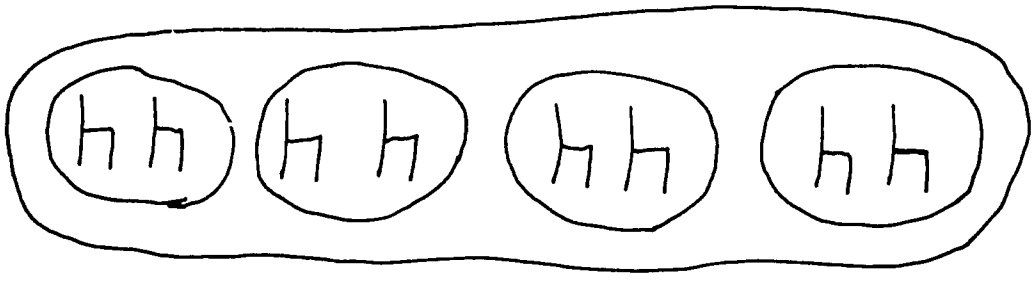
Ask a child to point to one subset and tell what part it is of the whole. (One-half of the set of objects) Ask, "How many objects are in each subset?" (4) Ask a child to separate the set of eight objects into four subsets so that each subset has the same number of objects.



Ask the children, "What part is each subset of the whole set?" (One-fourth of the set of balls) Let a child separate the set of objects into eight subsets with the same number of objects in each subset. Ask, "What part is one subset of the whole set?" (One-eighth) Repeat this activity, using different objects such as chairs, desks in a row, balls in a basket, books, bottle tops, bundles of sticks, and so on.



2 subsets; each one-half of set



4 subsets; each one-fourth of set

8 subsets of set; each one-eighth of set

Activity 6: Matching numeral cards to subsets of a set. Pupil Pages 103-104

Let the children turn to page 103 of their books. Ask the children to look at the pictures of sets and their subsets. Ask them to choose a numeral card to place on the picture of each set to show the part the subset is of the whole set. Go around the class and check answers.

Let the children turn to page 104 and continue in the same way

Stage 2: Locating the Position of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ on the Number Line

Vocabulary: No new vocabulary

Materials: 2" × 18" strips of paper; blank 1-inch square cards for the children to write the symbols $<$, $>$, $=$ on them; 1-inch square numeral cards with the numerals $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and 0 and 1 on them; pupil pages 105-106

Teaching Procedure:

Activity 1: Locating " $\frac{1}{2}$ ", " $\frac{1}{4}$ ", and " $\frac{1}{8}$ " on the number line.

Draw a number line on the blackboard. Say, "This is a number line. There are some points marked with numerals. Help me mark the other points on the line."

Take a strip of paper the same length as the line segment between 0 and 1. Hold up the strip of paper and ask, "Is this the same length as the line segment between 0 and 1?" Give a child the strip of paper to place on the line segment between 0 and 1 and 1 and 2 and so on. Get the child to say whether the strip of paper is the same length as the line segment between the points on the number line.

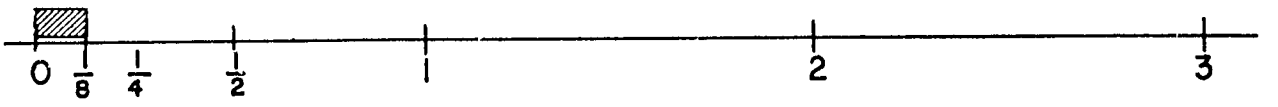
Ask another child to fold the strip of paper into halves. Give the folded strip of paper to still another child to check whether it is separated into two equal parts. Place the paper on the number line with the left end at "0". Mark the point at the fold. See the drawing below. Ask the children to name the part of the paper used. (One-half) Then say, "We call this point $\frac{1}{2}$ because it marks the end of the $\frac{1}{2}$ part." As you say this, write " $\frac{1}{2}$ " below the point.

Ask a child to fold each half of the strip of paper again into two equal parts. Ask, "Into how many equal parts is the strip of paper folded?" (Four) Place the folded paper on the number line with the left end at "0". Mark the point at the new fold. See the drawing below. Ask a child to name the part of the paper used. (One-fourth) Write $\frac{1}{4}$ at its proper place and say, "We call this point $\frac{1}{4}$ because it marks the end of the $\frac{1}{4}$ part."

Get several children to place the one-fourth strip of paper on the number line to check the position of one-fourth.

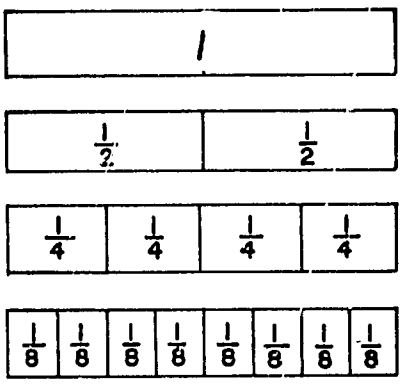
Ask, "How can we get one-eighth of the strip of paper?" (Fold the strip again) Give the one-fourth strip of paper to a child to refold to show eighths. Take the one-eighth strip of paper. Place it on the number line and mark the end point. Ask a

child to name the point. (One-eighth) Call a child to write the numeral $\frac{1}{8}$ at its proper place on the number line. Say, "We call this point $\frac{1}{8}$ because it marks the end of the $\frac{1}{8}$ part."



Activity 2: Preparing strips to show different kinds of parts.

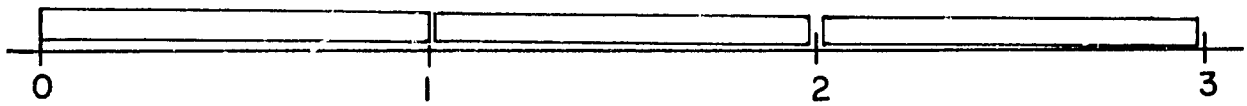
Give each child four 2" × 18" strips of paper. (These strips may be cut from used paper or newsprint) Write "1" on the blackboard. Ask the children to hold up one whole strip of paper. Let them write "1" on that strip of paper. Ask them to take another strip of paper and fold it into halves, another into fourths, and the last into eighths. Let them colour one-half of the second strip, one-fourth of the third, and one-eighth of the last strip.



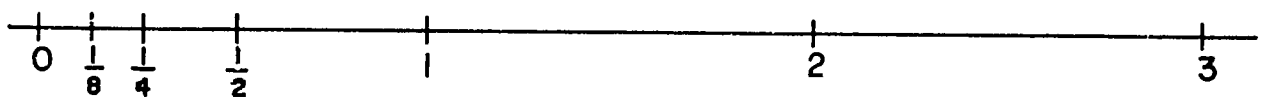
Ask the children to write the correct numeral in each of the parts as shown above.

<u>Activity 3</u>: Ordering the numbers 0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$ and 1 on the number line.

Draw a number line on the blackboard. Ask a child to take his one whole strip of paper, place it along the number line and mark as many points as possible. Help him to mark points "0" and "1" at the ends of his strip and to continue marking "2", "3", and "4", placing his strip as shown below.



Ask a child to use the strip of paper on which he coloured one-half to mark the position of " $\frac{1}{2}$ ". Guide him to put the left end of his strip at "0" and mark the other end. Then let other children check this point with their $\frac{1}{2}$ strips. Ask another child to use the strip of paper on which he coloured one-fourth to mark the position of " $\frac{1}{4}$ ". Let other children check this. Call another child to use his strip on which one-eighth was coloured to mark the position of " $\frac{1}{8}$ ". Let other children check this with their $\frac{1}{8}$ strips. The number line will look like this.



Ask the children to look at the positions of 0 and 1 on the number line. (0 is to the left of 1 and 1 is to the right of 0) Say, "One whole strip of paper is longer than no strip of paper." Help the children to see that 1 is greater than 0. One is to the right of 0 on the number line. Ask the children to look at a strip with one-half coloured and another with one-fourth coloured. Let them find out whether $\frac{1}{2}$ of a strip is greater than $\frac{1}{4}$ of a strip by comparing the lengths of the strips. Ask the children to look at the position of the $\frac{1}{2}$ and $\frac{1}{4}$ points on the number line. Ask which number is greater? Let a child place a finger of his left hand on the $\frac{1}{4}$ point and a finger of his right hand on the $\frac{1}{2}$ point. Let the children look at the number line. Ask such questions as: Which is greater, $\frac{1}{2}$ or $\frac{1}{4}$? Is $\frac{1}{2}$ to the right of $\frac{1}{4}$?

Which is greater, $\frac{1}{8}$ or $\frac{1}{4}$? Is $\frac{1}{4}$ to the right of $\frac{1}{8}$?

Let the children use their coloured strips if they find it difficult to answer the questions.

As a child answers each question, let him point to the numeral on the number line to show that the smaller of two numbers is always to the left of the other and the greater of two numbers is always to the right of the other.

Write the following sentences on the blackboard. Get the children to read them. As each is read get the children to decide whether it is a true or false sentence. For example, "$\frac{1}{4} > 0$" is read "$\frac{1}{4}$ is greater than 0." The sentence is true. Some children may need to use parts of objects as they decide which fraction is greater.

$\frac{1}{4} > 0$	$\frac{1}{2} < 1$	$\frac{1}{8} < \frac{1}{4}$
$\frac{1}{2} > \frac{1}{4}$	$\frac{1}{4} > \frac{1}{2}$	$\frac{1}{4} > \frac{1}{8}$
$1 < \frac{1}{2}$	$0 < \frac{1}{4}$	$\frac{1}{8} < 0$
$\frac{1}{2} > 0$	$\frac{1}{4} > 1$	$\frac{1}{2} > 0$

<u>Activity 4:</u> Making sentences true. Pupil Page 105

Ask the children to turn to page 105 of their books. Let them look at the incomplete sentences. Ask, "Which symbols (=, >, or <) must we put between the pairs of numerals to make true sentences?" For example:

$$\frac{1}{4} \text{ --- } \frac{1}{2}$$

If a child puts "<" between the $\frac{1}{4}$ and $\frac{1}{2}$ ask, "Is the sentence true or false?" Give each of the children three blank 1-inch cards. Get them to write <, >, and = on the cards.

Ask them to place a symbol between each pair of numerals to make the sentence true.

Activity 5 (Optional): Naming the shaded part of a whole. Pupil Page 106

Ask the children to turn to page 106 of their books. Ask them to look carefully at the figures. Let the children place the correct numeral card on each figure to tell what part of it is shaded.

Stage 3: Introduction of One-Third and One-Sixth

Materials: 5" × 7" numeral cards (0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, $\frac{1}{3}$, and $\frac{1}{6}$); sets of one-inch square numeral and symbol cards for each child (1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, 0, $\frac{1}{3}$, $\frac{1}{6}$, <, >, =); 2" × 18" strips of paper (strips used in previous stages may be used); set of large objects (boxes) to form set; counters; cardboard or paper discs and parts of discs; string and coloured beads; pupil pages 107-110

Note: You may use all or part of the following activities as needed.

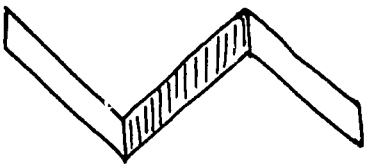
Teaching Procedure:

Activity 1: Introduction of one-third and one-sixth.

Give each child three strips of paper, each about 2" × 18". Ask the children to hold up one of the strips of paper. Ask, "What part of the strip do we have?" (One-whole) Ask the children to write "1" on that strip.

Let the children take another of the strips. Ask them to fold it into three equal

parts. Let each child work out his own way of doing this. Remind children to check that the three parts are equal.



Note: There is no simple way of folding the paper, as there was when the strips were folded into halves, fourths, and eighths. Children will have to use a trial-and-error method of folding, of testing and then refolding.

Ask a child who has been able to fold his strip into three equal parts to demonstrate to the rest of the class. Let the children compare the lengths of the three parts of his strip. When all the strips have been correctly folded, let the children cut or tear them along the folds.

Ask each child to hold up one of his pieces. Ask, "What do we call one part of four equal parts?" (One-fourth) "What do we call one part of eight equal parts?" (One-eighth) "What can we call one part of three equal parts?" (One-third) Let the children repeat "one-third" several times and match the words with the one-third part of the strip of paper.

Write the numeral " $\frac{1}{3}$ " on the blackboard. Let each child write the numeral on each of the three equal parts of his strip. Let the children place the three parts of the strip on top of the strip marked "1". Ask, "How many thirds make one whole strip?"

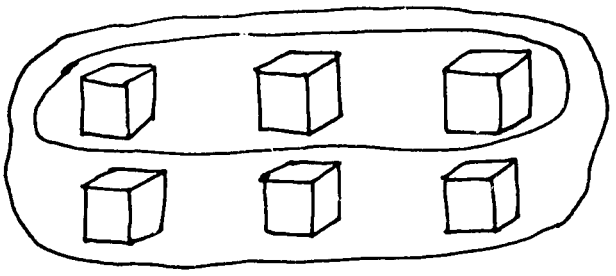
Hold up a strip of paper. Say, "This is one whole strip of paper. I am folding it into three equal parts." Fold into thirds. Unfold and ask, "Into how many parts have I folded the paper?" (three) "Are the parts equal?" (Yes) Ask a child to check by matching the parts. Ask, "How can I make six equal parts?" (Fold each one-third piece into two equal parts) Fold the strip into six equal parts. Unfold and show the strip to the class. Ask, "Into how many equal parts have I folded the paper?" (Six) "What can we call one part of it?" (One-sixth) Write the numeral

" $\frac{1}{6}$ " on the blackboard and let the children repeat it several times.

Ask the children to take their last strip of paper and fold into sixths. Go around the class and see that each strip is folded into six equal parts. Let the children write " $\frac{1}{6}$ " on each one-sixth part of the paper. (Save the strips for use later)

Activity 2: Separating sets of objects into thirds and sixths.

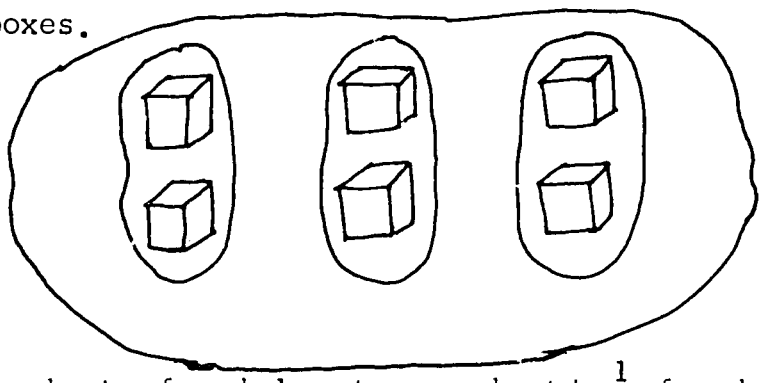
Place a set of six boxes or other easily seen objects on your table. Get a child to count and tell the class the number of boxes. (6) Ask a child to separate the set into two subsets, each with the same number of boxes. Say, "We had a set of 6 boxes. The set is now separated into 2 subsets. Do the subsets have the same number of boxes? Is each subset $\frac{1}{2}$ of the whole set?" (Yes)



Two subsets of a whole set; one subset is $\frac{1}{2}$ of the whole set

Call a child to match the subsets exactly and say whether the subsets have the same number of members.

Ask a child to separate the set of six boxes into three subsets, each with the same number of boxes.

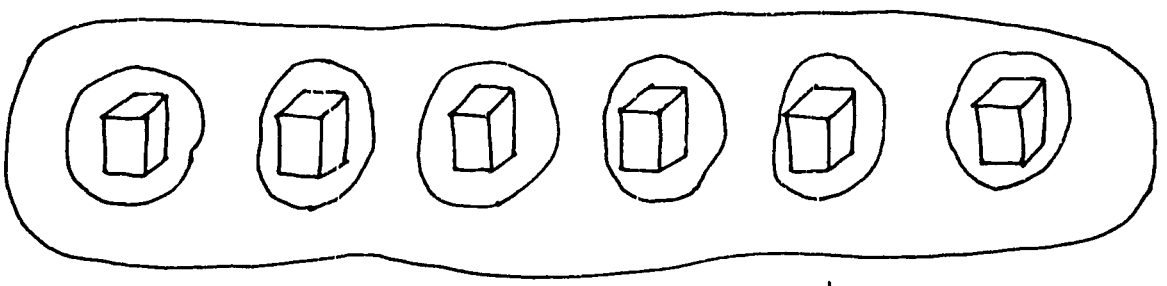


Three subsets of a whole set; one subset is $\frac{1}{3}$ of a whole set.

Say, "We had a set of 6 boxes. The set is now separated into 3 subsets. Do the subsets have the same number of objects? Is each subset $\frac{1}{2}$ of the whole set? (No) Is each subset $\frac{1}{3}$ of the whole set?" (Yes) "How many objects are in each subset?" (2) "Do the subsets have the same number of members?" (Yes) Call a child to match the subsets exactly and check whether they have the same number of members. Ask, "What part of the whole set is one of the three subsets?" (One-third) Let a child write the numeral for one-third on the blackboard.

Let each child separate a set of six bottle tops into thirds and write $\frac{1}{3}$ on his paper several times.

Call a child to separate the set of six boxes into six subsets with the same number of members.



Six subsets of a whole set; one subset is $\frac{1}{6}$ of the whole set

Ask, "How many boxes have you in each subset?" (1) "What part is one of the subsets of the whole set of six boxes?" (One-sixth) Get a child to write the numeral for one-sixth on the blackboard.

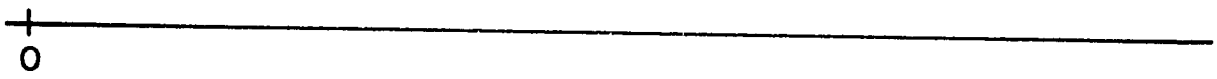
Let each child separate a set of six bottle tops into sixths and write the numeral for one-sixth on his paper several times.

Give children practice as needed in separating sets of objects into thirds and sixths.

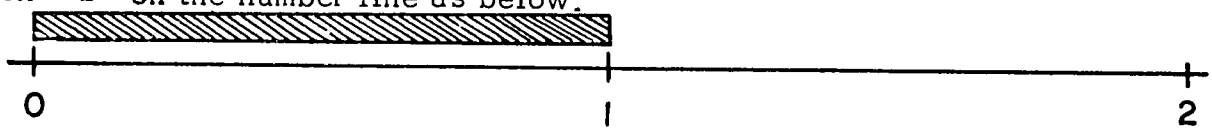
Show the 5" × 7" numeral cards ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$) to children and let them match the cards to parts of objects in the classroom. Objects may be such things as strips of paper, pieces of chalk, fruits and paper cutouts.

Activity 3: Locating the position of $\frac{1}{3}$ and $\frac{1}{6}$ on the number line.

Draw a line on the blackboard and mark the zero point on the left.



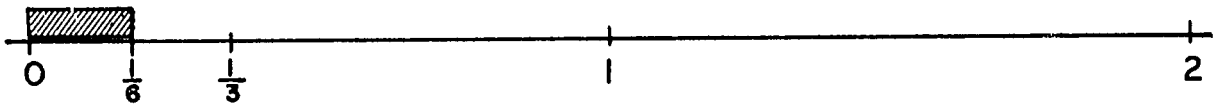
Take a 2" × 18" strip of paper, show it to the class and ask, "What part of this strip of paper do you see?" (One whole). Say, "I am going to place the strip along the number line so that one end falls on the point marked zero, and I will make a point at its other end. I will mark it '1'." Follow the same plan and locate and mark "2" on the number line as below.



Give a whole strip of paper to a child. Let him fold it into three equal parts. Let the other children watch as he folds. Give it to another child to check whether the paper has been folded into three equal parts. Place the one-third strip along the line from "0" and mark the point for one-third as shown below. Ask a child to write "$\frac{1}{3}$" at the point.

Ask the children, "How can we fold the strip into six equal parts?" (Fold each one-third piece into two equal parts.) Give the strip of paper to a child to fold. Unfold it and ask, "Into how many equal parts is the paper folded? (Six) What do we call one part of it?" (One-sixth) Give the paper to a child and let him place it along the number line and mark the point for one-sixth. Get another child to write the numeral

$\frac{1}{6}$ at the point.



Hold up a one-third strip and a one-sixth strip. Ask children, "Which strip is longer? (One-third) Is $\frac{1}{3}$ greater than $\frac{1}{6}$?" (Yes) Say, "Then we can write '$\frac{1}{3} > \frac{1}{6}$'. Is the point for $\frac{1}{3}$ to the right of the point for $\frac{1}{6}$?" (Yes)

Write other sentences on the blackboard and let the children read them and decide if each sentence is true by looking at the number line. Continue to question them about the connection between "to the right of" and "is greater than".

$\frac{1}{6} < \frac{1}{3}$; $\frac{1}{6} > 0$; $0 < \frac{1}{6}$ $1 < \frac{1}{3}$

$1 > \frac{1}{6}$; $1 < 0$: $0 < \frac{1}{3}$.

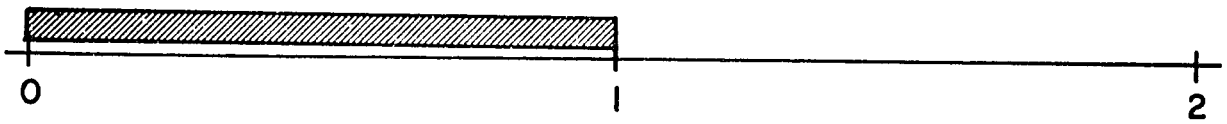
Help the children to use parts of objects in the classroom to illustrate each of the sentences. Objects that can be used are strips of paper, fruits, pieces of chalk and paper cutouts.

<u>Activity 4:</u> Locating $0, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ on the number line.

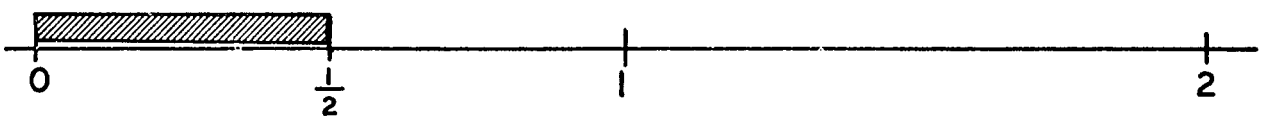
Give each child six 2" × 18" strips of paper. The children may use the strips they used in previous activities. Let them fold one into halves, one into fourths, one into eighths, one into thirds, one into sixths, and let one remain whole. Ask the children to mark each whole and each part with a numeral.

Draw a line on the blackboard and mark the point "0" near the left end of the line. Place the whole strip of paper on the line with one end on zero. Mark the point on which the other end of the strip falls with "1". Continue marking 2, 3, and

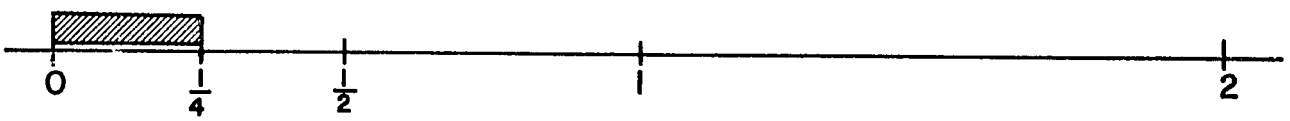
so on, along the line as convenient. (See the drawing below.)



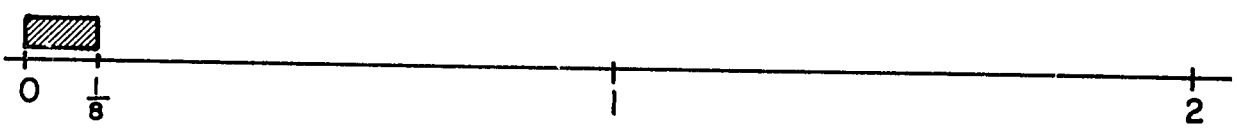
Ask the children to hold up their strips folded into halves. Call one child to use his strip to mark the one-half point on the number line. Let another child write the numeral $\frac{1}{2}$ at the point.



Call a child to mark the one-fourth point on the number line. Ask another child to write $\frac{1}{4}$ at the point.



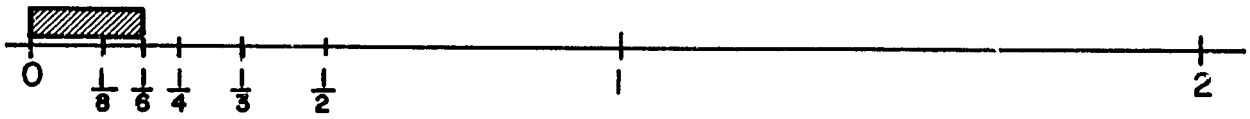
Call a child to place his one-eighth strip on the number line and mark the point. Ask another child to write $\frac{1}{8}$ at the point.



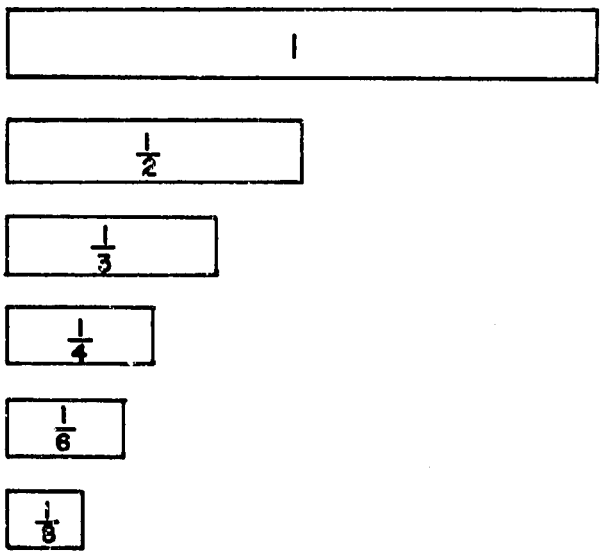
Get a child to mark the one-third point on the number line. Ask another child to write $\frac{1}{3}$ at the point.

Ask a child to mark the one-sixth point on the number line. Ask another child

to write $\frac{1}{6}$ at the point.



Ask the children to look at their strips and compare the pieces formed by the folds. Let them arrange on their desks the strips of paper from largest to smallest. Ask the children questions such as "Which is greater, $\frac{1}{2}$ or $\frac{1}{4}$? Which is less, $\frac{1}{8}$ or $\frac{1}{3}$?" The strips should be arranged like this:



<u>Activity 5:</u> Completing sentences to make them true. Pupil Page 107

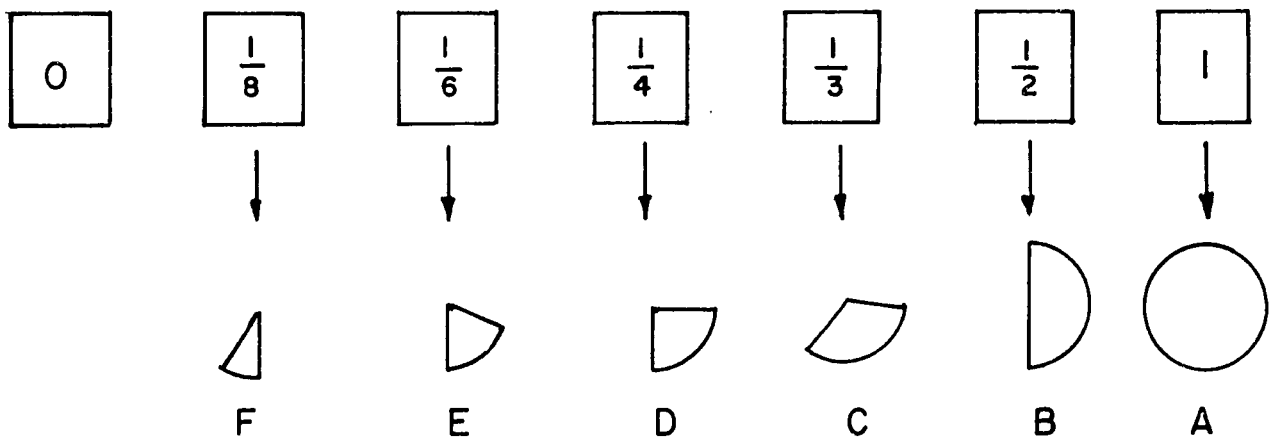
Ask the children to turn to page 107 of their books. Give them their cards with 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$, and the symbols $<$, $>$, and $=$ on them. Let the children look at page 107 and talk about the incomplete sentences they find, for example, $\frac{1}{2} > \square$. Read the sentence and ask a child to complete it. Ask the pupils to select a card to complete the sentence to make it true. Many cards may be used to make this a true sentence . One child may choose the $\frac{1}{3}$ card, another the $\frac{1}{4}$ card. Discuss why several different numbers may be used. Go on with a few other sentences in the same way. Then let the children complete the other sentences to make them true. Go

around the class, helping the children and checking work. If other sentences are needed write them on the blackboard or on exercise cards for the children to copy.

Let the children use their folded strips or the number line to help them choose a card if help is needed.

Activity 6: Ordering the numbers 0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$ and 1.

Show the children the 5" × 7" numeral cards, the disc and parts of discs as shown below.



Stand the parts of discs upright on a table. Give the numeral cards to seven children. Let them look at the six parts of discs (A - F) and choose the appropriate part of a disc to match the numeral card they hold. The child with numeral card "1" may possibly choose disc 'A'. The child with numeral card "$\frac{1}{2}$" may choose region 'B', and so on.

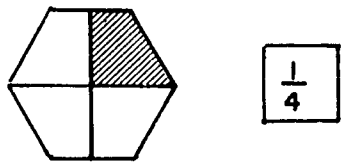
Let a pair of children compare their parts of discs by placing one on the other. Ask the child with the largest part to stand at the right front of the classroom holding his numeral card and part of disc. Ask the other child to stand to the left front of the classroom. Get another child to compare his part of a disc with that of each of the children at the front. Let him find the place where his numeral card goes in the line. Go on until all have found their places. The child with the "0" card and no disc will be on the far left of the front of the classroom. The cards will be held in

the order shown above.

Repeat the activity as needed. Vary the activity by giving the children the parts of discs and letting them find the numeral card that matches their cutout. Then get them to order the numerals on the cards.

Activity 7: Matching parts of regions and numerals. Pupil Page 108

Give the children their numeral cards with 0, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ on them. Let the children turn to page 108 of their books and look carefully at the regions pictured on the page. Ask them to choose the numeral card that tells what part of each region is shaded. Draw the figure below on the blackboard as an example.

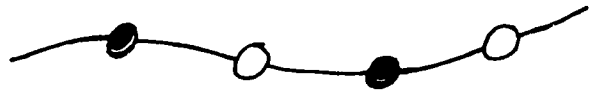


Use page 108 to test each child's understanding by moving around the class asking each child questions. For example: "Show me a region that has one-fourth part shaded," and so on.

Activity 8: Parts of sets. Pupil Page 109

Ask children to look at page 109 which shows sets of beads strung together. Let children point to the set of beads named "A". Ask, "How many beads are in set A? How many black beads are in set A? What part of set A is white? What part of set A is black?" Go on in the same way with the other sets. Name the sets B, C, D, and so on. Ask the children to point to each set and name the part of the set which is black or white. For example:

One-half is black;
One-half is white.



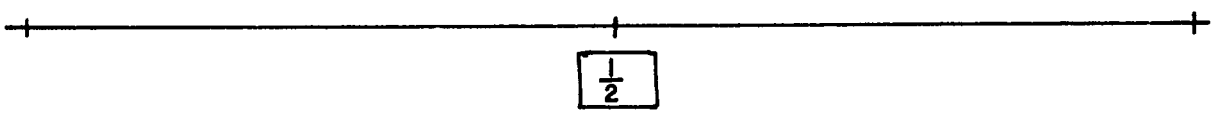
One-half is black;
One-half is white.

Help children understand that when one-half of a set is white, it does not matter how the members are arranged.

If children have difficulty in matching a number to the parts of the sets, let them work with real beads, or with two shades of counters. Get them to rearrange the beads so that all the beads of one kind are together.

Activity 9: Revising locating of fractions on the number line. Pupil Page 110

Give the children their numeral cards. Ask them to turn to page 110 of their books. Get them to look at the first number line. Ask children to show the points marked 0 and 1. Ask "Can you find another point on the line? Put your finger on it." Ask, "Is the distance from 0 to this point the same as the distance from the point to 1?" (Yes) Let them check by using a strip of paper that they fold into halves. Ask, "What can we call this point?" (One-half) Ask the children to place their $\frac{1}{2}$-card in the box by the point.



Follow the same plan and let the children place numeral cards on the other lines. When needed let them check their work by using strips of paper to find what part of the distance between "0" and "1" is marked.

UNIT 7

WEIGHT, CLOCK AND MONEY

Objectives:

1. To revise the comparison of the weight of two objects. (Heavier than and lighter than.)
2. To introduce the pound as a unit of weight. (This object weighs less than a pound, about a pound, or more than a pound.)
3. To revise reading the clock by the hour and extend to reading the clock to the half and quarter hour.
4. To know what happens at particular times during the day.
5. To compare two periods of time. (This is a longer time or a shorter time than that.)
6. To revise the recognition of coins and to know something each coin will buy.
7. To know what sets of coins will buy the same article.

Background Information for Teachers:

Children should be able to lift two objects, one in one hand and one in the other and tell whether one object is "heavier than" another, is "lighter than" another, or weighs about the same as another. Comparison of the objects in this way is best done at first when the objects cannot be seen. If children cannot see the objects they will not make judgments about weight because of size.

A standard unit of weight, for example, pound, is a weight to which the weights of other objects are compared. You can choose an object that the children know weighs a pound as an object to which other things are compared. The object you choose may be a pound bag of sugar, a box of sugar cubes, or anything the children know is bought in a one pound package. Then they compare other objects to the one pound object and say, "This weighs more than a pound," "This weighs less than a pound," or "This weighs about a pound."

The activities suggested for reading the clock are ones that help children relate things they do to a specific time of day. This gives them a reason to read the clock to the hour, half hour, or quarter hour.

Since there is no common set of coins used in the countries of Africa, the teaching activities concerned with money in this book are not specific. You may follow the suggestions made and plan others which are appropriate to your country. It is important in Primary II that children know the different sets of coins they can use to buy the same article.

You should centre all activities for the teaching of weight, the clock, and money around things the pupils do from day to day.

Stage 1: Weight

Vocabulary: weighs, pound

Materials: stones, tins of sand, small packets of sand; objects which weigh a pound (bag of sugar or flour, block of wood); string; a large cardboard box.

Teaching Procedure:

Note: The general plan of this stage is (1) to revise "heavier than" and "lighter than" by lifting pairs of objects; (2) to introduce the pound as a standard unit of weight; (3) by lifting to compare the weight of objects and the weight of an object that weighs a pound.

Activity 1: Revising "heavier than" and "lighter than".

Arrange various tins, marked A, B, C, and so on, that are filled with stone or sand and crumpled waste paper on your table. Fill the tins so that when they are lifted by the children there is a great difference in the weights of the tins. For example, fill tin A almost full of sand and cover with crumpled paper; fill tin B half full of sand and cover with crumpled paper; and fill tin C with crumpled paper. Cover the tins if the crumpled paper does not hide the contents.

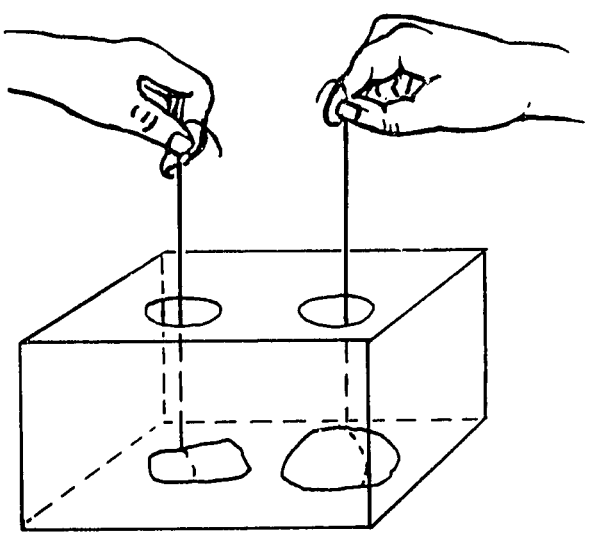
Let several of the pupils in turn lift a pair of the tins, one in each hand. They should move their hands up and down to feel the weight. Let them talk about which is heavier and which is lighter. Let several other children lift another pair of the tins and then talk about their weight and decide which is heavier. Ask other children to check whether they are right. Give each child a chance to lift a pair of tins and decide which is heavier.

After the children have lifted the tins remove the cover so they may see what the tins contain. Let the children talk about why you covered the tins. (We might have known which was heavier by looking.)

Activity 2: Comparing the weight of two hidden objects.

Prepare a box like the one pictured below to hide the objects. The box should measure at least 1 foot by 1 foot by 1 foot and have no bottom. Cut two 4-inch holes in the top and put a string through each. Tip the box forward and tie objects of different weights to each string. Do not let the children see what you have tied to the strings. This way they must rely only on lifting to decide which object is heavier.

Let several children lift the objects by the strings. Then let them together decide which object is heavier, and which is lighter. After they decide show them the two objects and let them lift them and again compare the weight of the objects.



Continue as needed with other pairs of objects. Use at least one pair of objects that weigh the same. Try to let all children have a chance to compare the weight of two objects in this way. More children can take part if you let several children lift the same objects before there is a decision about which object is heavier.

Activity 3: Lifting objects to decide which is heavier.

Let the children compare the weights of pairs of objects in the classroom. This time they can see what objects are being compared. The pupils should lift objects and then say, "This object is heavier than that one," or "This object is lighter than that one," or "These objects weigh about the same."

Activity 4: Introducing the pound as a unit of weight.

Arrange on your table several articles such as a pound of sugar, a wooden block weighing a pound, other blocks of wood, stones of different weight, pencil, sheet of paper, and a large book. Among these articles should be some which weigh much more than a pound and some which weigh much less than a pound.

Hold up a pound bag of sugar (or other object which the pupils know weighs a pound). Say, "This is a bag of sugar. It weighs a pound." Ask the class to repeat the word "pound" and to say, as they lift the bag, "The bag of sugar weighs a pound."

Let the children talk of articles in the shops which are sold by the pound. (Sugar, flour, butter, salt, rice) Let them also talk of their own weight.

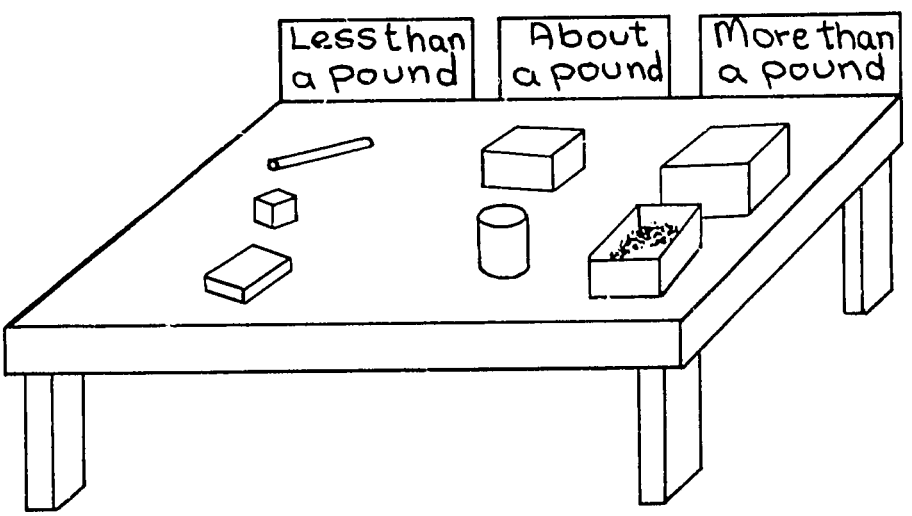
Ask several pupils to lift the bag of sugar in one hand and another object, say a stone, in another. Let them decide together whether the stone weighs more than a pound or less than a pound or about a pound. Continue in a similar way to let the children compare the weights of the objects on your table with the pound bag of sugar. Each time they should say, "This object weighs more (or less) than a pound," or

"This object weighs about a pound."

Activity 5: Sorting objects by comparing to an object that weighs a pound.

Again arrange many objects on your table, some which weigh more than a pound, some which weigh less and some which weigh about a pound.

Ask the children to compare the weight of each object with the weight of the pound bag of sugar by lifting. Let them separate the objects into three subsets. Let them put the objects that weigh about a pound in the center of the table, the objects that weigh more than a pound on one end, and the objects that weigh less than a pound on the other end of the table. See the figure below.



Stage 2: The Clock

Vocabulary: half past, quarter past, longer time, shorter time, minute hand, hour hand.

Materials: large cardboard clock, a real clock, cards; pupil pages 111-115.

Teaching Procedure:

Activity 1: When we do things.

Show the class a large demonstration clock or a real clock. Move the hands around the clock. You and the children describe the clock.

(1) It has a long hand which shows the passing of minutes. It is called the minute hand.

(2) It has a short hand which shows the passing of the hours. It is called the hour hand.

(3) The clock face has numerals from 1 to 12. The hands move around the clock from 12 to 1, to 2, to 3, . . . and to 12.

(4) The minute hand moves faster than the hour hand.

Move the hour hand around the clock beginning at 7 o'clock in the morning. Let the children tell what they usually do at each hour of the day. For example, "At 7 o'clock I get up and eat. At 8 o'clock I come to school," and so on.

Activity 2: What can I do in an hour; what can I do in a minute.

Let the children watch the hands of a real clock as they move. Talk of which hand moves faster, and that one hand tells us about hours and one hand tells us about minutes.

Ask the pupils to look at the hour hand when it is at, say, 9 o'clock and to remember everything they do in the hour from 9 o'clock to 10 o'clock. Remind them about every ten minutes that the hour is passing and let them say it is not yet 10 o'clock. When the clock reads 10 o'clock let the children tell all they have done in the hour, for example, "We read from a book, we drew some pictures, we got a drink, and so on."

Ask them to watch the minute hand of the clock and to remember what they do in the next minute. When a minute has passed let them tell what they have done.

Let them talk about these questions: "Can you do more things in an hour or in a minute?" "Which is a longer time, a minute or an hour?" (They should decide an hour is a longer time than a minute) We can do many things in an hour and few things in a minute."

Activity 3: Longer time and shorter time.

Ask pairs of children to do two different things. Let them begin at the same time and then discuss with the class which child took the longer time and which child took the shorter time. (Do not have a race.) Some of the things two children might do are these:

(1) One child walk around the school building, another child walk around a chair.

(2) One child go outside to get a stick, and another child get a pencil from your table.

(3) One child takes 20 steps, another child takes one step.

After each pair of children has done as told let the pupils decide, "He took a longer time than she took," "She took a shorter time than he took," or "They took the same time." Try to get all the children to take part in the activity.

Activity 4: Reading the clock to the half hour.

Show the class a real clock. (A demonstration clock will do, but it is not as good as a real clock because the hands do not move together.) Move the hands from one hour to another as the class watches.

Say, "Pretend one hour passes as the hands move like this." Move the hands from one hour to the next. Discuss these questions: How far did the hour hand move? How far did the minute hand move? (The hour hand moved from 9 to 10. The minute hand moved all the way around the clock.) Continue in the same manner to move the

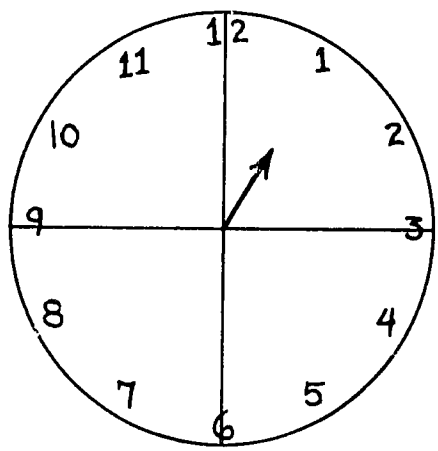
hands from 10 o'clock to 11 o'clock and on as needed.

Then move the hands from 9 o'clock to half past nine. Ask, "Did this show that an hour passed?" (No) "How can you tell?" (The hour hand is half way between 9 and 10; the minute hand is half way around the clock.) Say, "When the hands are here we say it is half an hour past 9 o'clock, or half past 9 o'clock." Continue moving the hands around the clock. Let the children tell when it is 10 o'clock, half past 10 o'clock, 11 o'clock, half past 11 o'clock, and so on.

Ask the children, in turn, to fix the hands of the demonstration clock at 10 o'clock and then half past 1 o'clock, and so on. Each time the hands are fixed call another child to tell what they do at that time of day.

Activity 5: Reading the clock to the quarter hour.

Show the class a real clock. Move the hands from, say, 9 o'clock to half past 9 o'clock and then to 10 o'clock, and so on. Each time let the children tell the time the clock shows. Begin again and this time stop the hands at quarter past 9 o'clock. Ask, "Is it 10 o'clock?" (No) "Is it half past 10 o'clock?" (No) "What part of the hour has passed?" (One fourth) "We say it is a quarter past 9 o'clock." Draw a clock face on the blackboard like this:



Let the children talk about the clock face being separated into four parts or into quarters. Draw in the hour hand as shown. Use a stick for a minute hand. Place the stick to point to 12 and say, "This is the minute hand. What time does the clock read?"

(1 o'clock) Move the hand to show half past the hour and ask, "What time does the clock read?" (Half past 1 o'clock.) Move the hands to show three quarters past the hour and ask, "What time does the clock show?" Let the children talk about the time the clock shows. Move the hands back to 1 o'clock and then around the clock again. With the children say, "1 o'clock; quarter past 1 o'clock; half past 1 o'clock; three quarters past 1 o'clock." Change the hour hand to 2 o'clock and go through the plan again. Change the hour hand and continue around the clock reading each quarter hour as needed. Change to the use of a real clock so the pupils can see the hour hand and minute hand move together.

Activity 6: Using the clock. Pupil Pages 111-113

Ask the children to turn to page 111 in their books. Give each of them two sticks of different lengths to use as hands for the clock. You name times of day and let the children show these times by placing the hands on the clock face. Move around the room giving help as needed. Name times of day like these: 3 o'clock; quarter past 3 o'clock; half past 3 o'clock; three quarters past 3 o'clock; half past 7 o'clock.

Let children name times of day each quarter hour beginning at seven o'clock to noon, place their sticks so that the clocks show that time, and tell what they do at that time of day. Ask children to turn to page 112-113 in their books. Name a clock face, for example, clock A. Let several children in turn read the time of day the clock shows. Let the pupils decide the correct time. Go on with the other clocks pictured.

Activity 7: A shopping trip. Pupil Pages 114-115

Tell the children that they are to pretend that today is Saturday and Okella and her mother are going on a shopping trip. Let the children find pictures about the trip on pages 114 and 115. Let them read the story the pictures tell and read the clocks which show the time.

The pictures were planned to tell the story below. However, allow the children to see other stories in the pictures and tell them.

(1) At 7 o'clock Okella and her mother got ready to go to market.

(2) At half past 8 o'clock they leave the house.

(3) At quarter past 9 o'clock they are at the market.

(4) At 1 o'clock they wait for the bus to go home.

(5) At three quarters past 1 o'clock they arrive home.

(6) At half past 4 o'clock Okella plays.

Let the pupils tell what Okella did before leaving home for the market, what was bought at the market, what games they played, what they do after play, and so on. Let the pupils tell as many stories as possible from the pictures. They can also tell of their own shopping trips.

Stage 3: Money

Vocabulary: the names of the coins in your country.

Materials: real coins; paper coins; inch-square cards; picture cutouts of toys, sweets, and so on; any small inexpensive articles which could be bought in a shop; pupil page 116.

Teaching Procedure:

Note: The plan for this stage is (1) to revise the name of each coin and to know some things which each coin will buy, (2) to know what sets of coins will buy the same thing.

The teaching activities suggested for this stage cannot be specific because there is no one set of coins used in the countries using this book.

Recognizing and using coins: Distribute real coins or paper coins to children. Let them show each coin and tell some things it will buy. Continue as needed.

Arrange a set of coins on your table. Ask children to choose a coin which will buy articles you name, for example, a mango, a pencil, or an exercise book.

Shopping games. Set up a classroom shop. Collect real objects or pictures of objects to sell in the shop. Choose children to be shopkeepers and buyers. Let a customer come and select two articles from the shop and pay for each, one at a time. Let the shopkeeper find the total cost of the two articles and hold up a card showing the cost.

Let a customer select an article that can be bought with one coin and pay for it with that coin. Let another customer choose the same article and pay for it with two or more coins. Let the shopkeeper hold up the two sets of coins and get the children to see that each set will buy the same article. Repeat this activity as needed.

Shopping: Make up simple stories of shopping which children can think about. Some examples are:

(1) Joe wants to buy a fish and an egg. What coin does he need to buy a fish? What coin does he need to buy an egg? Which costs more, a fish or an egg?

(2) Lucy has a coin to buy a pencil and a coin to buy an eraser. What two coins does she have?

(3) What coin will you need to buy a balloon? Kofi wants to buy a piece of sugarcane. What coin does he need to buy it? Does Kofi need more money or less money than you?

(4) Fatu bought a boat with a six-pence coin. Sorie bought a boat like Fatu's with two three-pence coins. Did one boat cost more than the other? (The coin names and the object names may be changed for use in your country.)

Shopping Activity

Pupil Page 116

Ask the children to find page 116 in their books. Let them tell what coin or set of coins will buy each article on the page.

Name a coin and let the children tell what things on the page can be bought with that coin. For example, "I can use a ____ coin to buy a banana or a mango."

Ask questions about buying two of the articles. For example, "What coins do you need to buy an orange and a fish?" "How much money will I need to buy both balloons?"

Let the children make up their own story problems for other children to think about and answer.

UNIT 8

NUMBER II

Objectives:

1. To expand the idea of place value to numerals up to 1,000.
2. To expand the idea of ordering numbers (1-1,000) using a number line.
3. To develop counting by twos, fives, tens, twenties, fifties, and hundreds up to 1,000.
4. To introduce the idea of odd and even numbers from counting by twos.

Background Information for Teachers:

See "Background Information" for Unit 1 and Part 2, "Arithmetic", of your Teachers' Handbook.

Stage 1: Group Counting of from 1-100 Objects

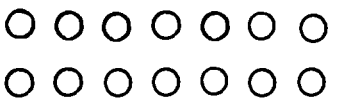
Vocabulary: odd, even

Materials: small objects for counting (bottle tops); hundred pocket chart; pupil pages 117-118.

Teaching Procedure:

Activity 1: Separating sets into subsets.

Arrange the class into small groups (about 5 - 6 per group). Give each group about 100 small objects to count. Ask the children to arrange their sets of objects in subsets, each of two members.



and so on

Let each group count their objects in twos as one child in the group moves the counters together two at a time.

Move around the room as the pupils work. Give help as needed.

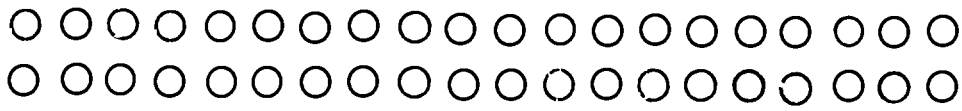
Let the class count aloud by twos without using the counters.

Ask the groups to rearrange their counters together in subsets of five members. Let one child move the groups of five counters together as the rest of the group counts by fives, "Five, ten, fifteen, twenty, . . .". Let them count by fives without using the counters.

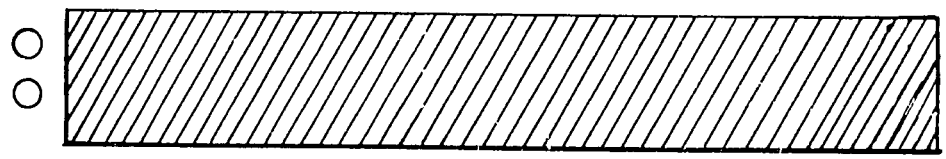
Repeat this activity using subsets of tens and twenties.

<u>Activity 2:</u> Odd and even numbers.

Draw a pattern of counters on the blackboard as follows or have it prepared on a strip of cardboard:



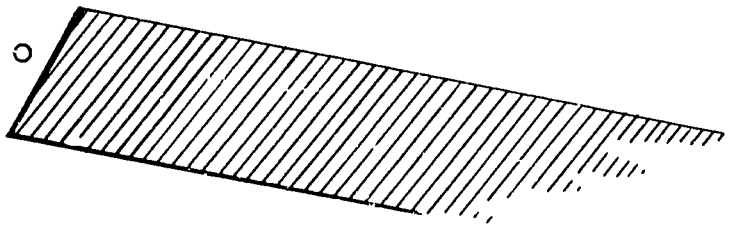
Use a strip of paper to cover all the counters except the first two.



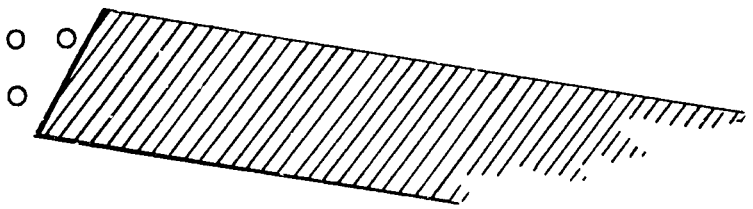
Ask the children to tell you how many counters they can see. (2) Move the paper to show two more counters and ask the children to tell you how many they now see. Keep showing two more counters each time as the class counts by twos.

Tell the class that the numbers that they use when they count by twos are called <u>even</u> numbers. Ask children to tell you any even number. Reverse the activity by starting with the whole pattern uncovered. Keep on hiding two counters at a time while the children count back by twos. (They will be counting backward by twos.)

Use the same pattern of counters drawn on the blackboard. Cover all except one counter with the strip of paper.



Ask the class to say how many counters they can see. Move the paper to uncover two more counters.



Ask, "How many counters can you see now?" (3) Keep moving the paper to show two more counters each time while the children count: "One, three, five, seven," and so on. Tell the class that the numbers that they use when they count in this way are called <u>odd</u> numbers.

Ask children to tell you any odd numbers they can think of. Reverse the activity as with the activity for even numbers. Let the children count back by twos beginning with an odd number.

<u>Activity 3</u>: Identifying sets which have an even or odd number of members.

Let children name sets in their classroom and count the members of the sets by twos. Get them to tell which sets have an even number of members and which have an odd number of members.

<u>Activity 4:</u> Counting by twos and fives on the Hundred Pocket Chart.

Hang the hundred pocket chart in front of the class. Let the blank sides of the cards show. Get the children to say what numeral is written on each second card as you point to it. Point to the second card; turn it over. In a similar way turn over

the fourth, sixth, and so on. Continue until you reach the 20-card. The chart will look like this:

	2		4		6		8		10
	12		14		16		18		20

Let the children read what they see on the cards. Ask, "What do we call these numbers?" (Even) Ask, "What is the next even number?" (22) Let a child turn that card over. Ask the children, in turn, to turn over any card that has a numeral for an even number. They will see that the numerals for even numbers form a pattern. They may say that all the numerals for even numbers end with 0, 2, 4, 6, or 8.

Let the class tell the numerals on the cards that are hidden. Point to the cards in turn as the class counts: "1, 3, 5," and so on. Ask, "What are these numbers called?" (Odd)

Again turn all the cards in the chart so that the blank sides show. Begin with the 1-card and point to each card, in turn, as the class counts, "1, 2, 3, 4, 5". Stop and turn the 5-card over. Point to the next card and say, "One". Let the class count "1, 2, 3, 4, 5" again as you point to the cards in turn. Stop when they say, "five," and turn the 10-card over. Continue in this way, turning over every fifth card with the children counting "1, 2, 3, 4, 5" each time. When you have reached "40", allow children to come out and turn over cards in the same way.

				5					10
				15					20
				25					30

Let the class read aloud what they see on the cards. Say, "We use these numbers to count by fives." With their eyes closed get the children to repeat the numbers used to count by fives, both forward and backwards.

Let some of the children who can, repeat this activity counting by other groups, for example, by threes or fours.

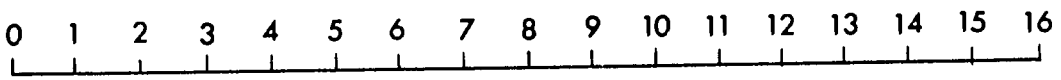
Let them find patterns formed by the numerals on the chart. For example, when they count in threes they will make a chart like this.

		3			6			9	
	12			15			18		
21			24			27			30
		33			36			39	

Here the numerals form sloping lines. They will remember that the numerals for even numbers formed vertical lines.

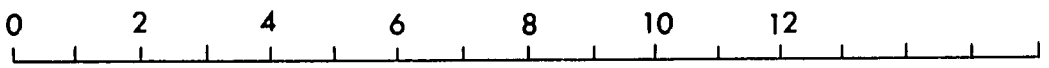
<u>Activity 5:</u> Counting by twos on a number line.

Draw a number line on the blackboard. Let the children write the numerals on it.



Tell the class that Mr. Two Frog is jumping along this line. He jumps two spaces every time. He starts at point "0". Ask a child to show the class where Mr. Two Frog lands with each jump. (2, 4, 6, 8, and so on)

Draw a second number line below the first. Let the children write in the numerals on which the frog lands as he jumps.



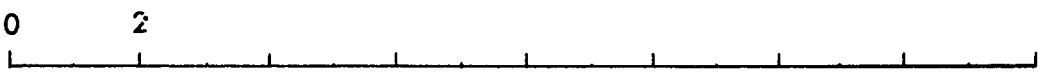
Let the pupils together say the numbers used with the line. Ask them what these numbers are called, odd or even.

Draw another number line. Write "1" above the second point. Tell the children, "This time Mr. Two Frog is going to start at one." Let the children write in the numerals where Mr. Two Frog lands. Ask what the numbers these numerals represent are called, odd or even.

Draw another number line. Write "25" above the first point. Tell the class that Mr. Two Frog is still jumping two spaces each time but he is starting from the 25-point. Let children fill in the numerals as before. Help them to understand that the part of the number line they use can be started at 2, at 25, or at any number.

Activity 6: Counting by twos and fives on a number line.

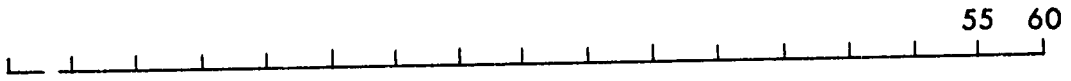
Draw a number line on the blackboard. Tell the children that this time they will mark only those points where the frog lands each time.



Mark the first point "0" and the next point "2". Tell the class that the frog started from "0" and jumped to "2". Point to the next mark, "2", and say that is where he landed. Ask the class what numeral should be written above the next point. (4) Let the children fill in the rest of the numerals for even numbers.

Draw another number line. Mark the first point "0" and the next point "5". Say, "Mr. Five Frog uses this number line." Ask, "What numeral must we write above the next point?" (10) Let the children, in turn, write other numerals on the line.

Draw other number lines. Sometimes start with numbers other than zero. Sometimes start from the right end, as below.



Let the children fill in numerals on the line as they count back by fives.

Activity 7: Filling in missing numbers on number lines. Pupil Page 117

Ask the children to turn to page 117 in their books. Let them copy the number lines into their exercise books and fill in the missing numerals. Move around the class and help as needed. Ask the children as you talk with them to tell you what number they are counting by as they work on each number line (that is, how far the frog jumps each time). Ask others to point to the numerals for odd and even numbers.

Activity 8: Filling in missing number in number sequences. Pupil Page 118

Ask the children to turn to page 118 in their books. Let the children talk about the sequences of numbers on the page. Let them copy the sequences in their exercise books and fill in the missing numbers. Move around the class and give help as needed. Suggest the children use the number chart on page 118. Ask each child questions about his work. For example, "Are you counting forward or back in this row of numerals?" "By what number are you counting each time?" Help children having difficulty by showing them how to use the number chart to fill in a sequence.

Stage 2: Counting by Tens and Hundreds

Vocabulary: thousand

Materials: a large supply of sticks used in previous activities; string or rubber bands to tie the sticks in bundles; cardboard cutouts (one per child, see Activity 2); pupil page 119.

Teaching Procedure:

Activity 1: Separating sets into subsets of ten members and one hundred members.

Arrange the class in groups of 4 or 5 children. Place a large heap of sticks (about 20 per child) on a table in front of the class. Ask, "Can you guess how many sticks are in this set?" Let the children write their guesses on the blackboard. Ask, "How can you find out how many sticks there are?" Tell the class they all can help find the number of sticks.

Ask one child to separate the set of sticks into subsets, one for each group. Tell him to put about the same number of sticks in each set but not to count. Let each group separate their set of sticks into smaller subsets and give one subset to each child. It is better if the subsets do not have the same number of members. Let the children separate the sets without counting.

Let each child count his sticks and tie them into bundles of ten. Ask how many each child has. Tell the children to write their answers in their exercise books. Let children check each others' work. Ask questions such as: "Which person in this group has the most sticks?" "Who has the least number of sticks in this group?" "Has Joseph more sticks than Mary?" Let one child in each group count his sticks while the others in the group watch, listen and check. Ask children to use two possible ways of counting:

"One ten, two tens, three tens, three tens and one, three tens and two, . . ." And then,

"Ten, twenty, thirty, thirty-one, thirty-two, . . ."

Let the children in each group place all their sticks in one large heap. Let them discuss how they can find the number of sticks in this set, count the sticks, and write the number of sticks on a paper. Choose one child from each group. Let him explain to the class how his group has found the number of sticks. Some of the children will be able to say the number of sticks is, for example, "16 tens and 8 ones", and be able to write correctly "168".

Ask questions to help the children know that ten bundles of ten sticks each can be bundled together to form a larger bundle of one hundred sticks. Ask one group how many bundles of ten they had when they put all the sticks together?" Ask, "How many single sticks did you have when you put all your sticks together?" If the group says that there were, for example, 15 sticks, ask, "What must you do if there are more than ten sticks when all the sets are joined together?" (Tie them into bundles of ten)

"What is another name for ten ones when they are bundled together?" (One ten)

"What is another name for two tens?" (Twenty)

"What is another name for three tens?", and so on.

"What is another name for ten tens?" (One hundred)

"What can you do when you have ten single sticks?" (Make a bundle of ten)

"What can you do when you have ten bundles of sticks?" (Make a big bundle of ten bundles)

"How many sticks in this big bundle?" (One hundred)

Let each group tie their bundles of ten into hundreds and count their sticks in both these possible ways:

"One hundred, one hundred and ten, . . ., one hundred, two tens and four ones, . . .,"

"One hundred, one hundred and ten, . . ., one hundred and twenty-four, . . .,"

Let all of the groups join their sets together on the table in front of the class. Let them place the set of hundreds, the set of tens and the set of ones in separate heaps. There will probably be more than ten single sticks and more than ten bundles of ten. Ask a child to tie these into bundles of ten and bundles of one hundred.

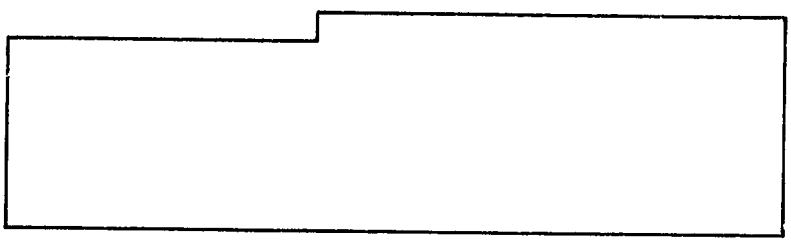
While one child points to the bundles in each heap, let the class count together,

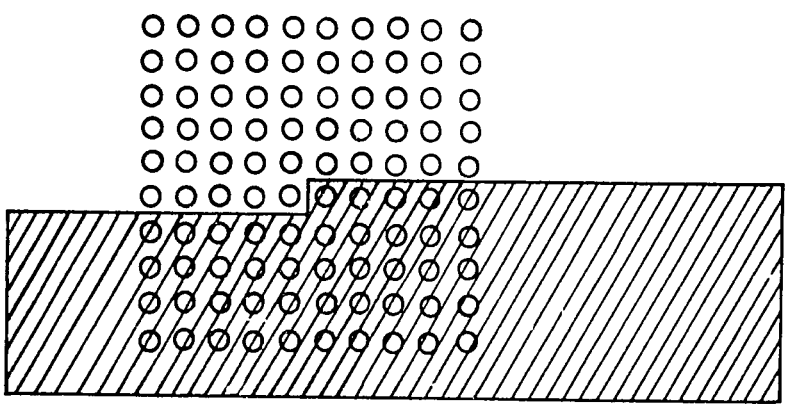
for example, "One hundred; two hundreds; . . .; eight hundreds, one ten and two ones, . . ." Let them also count the sticks as: "One hundred; two hundred, . . ., eight hundred, eight hundred and ten; eight hundred and eleven; eight hundred and twelve, . . ., and so on."

Check now on the guesses of the number of sticks that were made at the beginning of the activity. Ask the class to tell how they found the number of sticks.

Activity 2: Recognizing patterns of counters, 1 to 1,000. Pupil Page 119

Each child requires a small piece of cardboard or stiff paper cut in this shape:





Ask the class to find page 119 in their books. Let the pupils talk about what they find.

Below are some of the many simple activities which you can help the pupils do with this page as they cover and uncover part of the dots.

a. Ask the children how many dots there are on the whole page. Help them to see that there are ten dots in each row, a hundred dots in each square, and a thousand dots on the page.

b. Let the children count by hundreds as they uncover the appropriate number of squares of a hundred dots with their counters.

c. Ask such questions as, "How many tens in one hundred?" "How many tens in two hundreds?" . . . "How many tens in one thousand?"

d. Let the pupils count by fives, tens, twenties, fifties, as they move their cardboard cutouts to cover parts of the page.

e. Say a number and ask the children to show that number of dots. Let children work in pairs, one child checking the answer of the other.

f. Let several children stand together in front of the class showing their pages of one thousand dots. Let the children count by hundreds.

Note: A large sheet of paper with a pattern of one thousand counters can be pinned on the wall for class use. Make a large cutout to use with it. For example, show a certain pattern of counters and ask the class to tell you the number of counters.

Stage 3: Writing Numerals for Sets Greater Than One Hundred

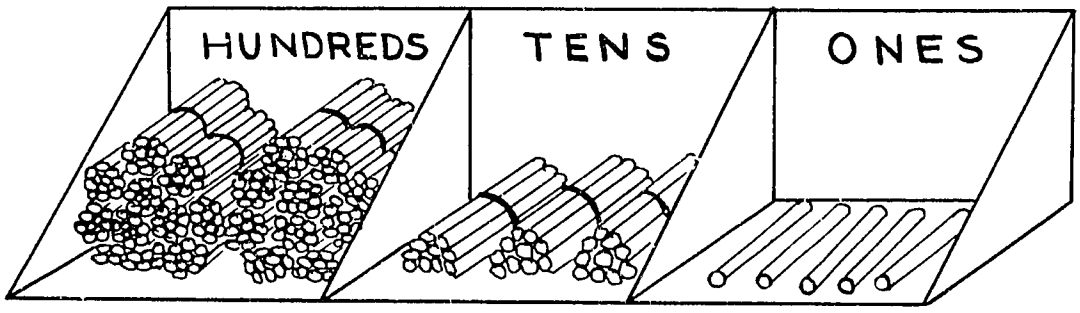
Vocabulary: no new vocabulary.

Materials: counting sticks; a number tray (see Unit 1 page 17); three sets of 5" × 7" numeral cards; pupil pages 120-121.

Teaching Procedure:

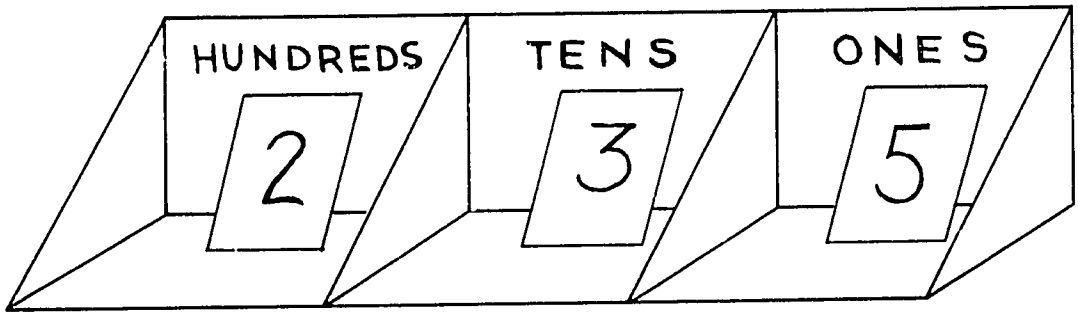
Activity 1: Writing numerals 100-1,000.

Repeat the experiences of Stage 2 by letting children organize a large set of sticks into bundles of tens and bundles of hundreds. Place the number tray in front of the class. Ask in which places the single sticks go, and where the bundles of ten sticks are placed. Review previous work with tens and ones if necessary. Now ask where the hundreds bundles must be placed.



Ask a child how many ones sticks there are in the tray. Tell him to choose the numeral card which shows that number and put it in the tray. Let other children place numeral cards in the other two places to show the number of tens and hundreds.

The number tray should now look like this:



Change the number of sticks in the tray many times and repeat the activity. Be sure to arrange the sticks so that the numeral "0" must be used, for example, when there are no tens or ones in the tray. After each new set of numeral cards has been placed

in the tray, let the class read numerals on the cards aloud. Ask them how many sticks are in the tray. Get someone to write this on the blackboard.

Reverse the activity.

Write a three-digit numeral, for example, 370, on the blackboard. Ask a child to set up the number tray as in the second diagram above, using the numeral cards "3", "7", and "0". Let another child replace the numeral cards by the correct number of bundles and single sticks.

Activity 2: Writing the numerals, 1-1,000. Pupil Page 120

Ask the children to turn to page 120 in their books. Let them set up the number tray with numeral cards to show the first exercise. Ask a child to replace the cards with the correct number of sticks. Let the pupils copy the exercises into their exercise books and write the correct numerals under the headings HUNDREDS, TENS, AND ONES. Talk about the exercises where no number of tens and ones are given. When necessary, use the number tray to show that the numeral "0" must be used when these numbers are named.

Let the children copy the other exercises in their exercise books. Let them rename each number as they make the sentences true.

Decide whether children really understand this activity by asking them to show their answers on the number tray.

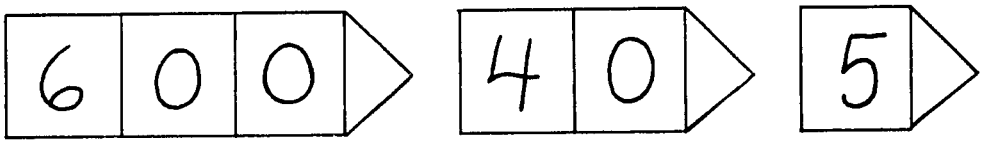
Activity 3: Writing three-digit numerals. Pupil Page 121

Ask the children to turn to page 121 in their books. (This activity uses only the first part of the page.)

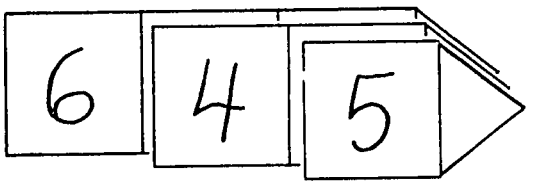
Ask them to read the first exercise as "six hundred and forty and five", and then as "six hundred and forty-five", and then as "six hundreds and four tens and 5 ones" Let them tell what goes under "hundreds", "tens" and "ones" in the exercise. Let the children write the numerals for the numbers in their exercise books, and let them set up the number tray as in Activity 1.

Activity 4 (Optional activity): Showing numerals with cards.

Let the children make sets of cards for 100-900, 10-90, and 1-9 like the ones shown below. The squares should be about 1 inch by 1 inch.



Let the children show numerals by combining the cards. For example, by placing the "5" card on top of these as shown below "645" can be formed.



When the cards are combined in other ways, the numerals 45 and 605 also can be formed. With the set of twenty-seven cards, 1-9, 10-90, and 100-900, the children can build any numeral from 1 to 999. Let them work in pairs with one child naming a number and the other showing the numeral for the number with his cards.

Activity 4: Changing tens to hundreds to tens. Pupil Page 121

Ask children to do the rest of the examples on page 121 of their books. These examples require children to understand how to regroup tens into hundreds and hundreds into tens.

Place 17 bundles of ten sticks in the number tray in front of the class. Put all these bundles in the tens' part of the tray. Ask, "How can we regroup these bundles?" (Tie ten bundles of tens into one bundle of one hundred) Ask, "Where does the bundle of one hundred go?" (In the hundreds' part) This will help the children to see that "17 tens" names the same number as "1 hundred and 7 tens".

The exercise, "1 hundred and 3 tens = _____ tens," can be shown as follows:

Place a one hundred bundle and three tens bundles of sticks in the number tray in the correct parts. Ask the children how many tens there are altogether. Remove the string from the hundred bundle and place the resulting ten bundles of tens in the tens' part of the tray. Ask the children how many tens there are now.

Emphasize that: (1) The number of sticks has not changed but the way in which the sticks are grouped has been changed; (2) Each time the set of sticks is regrouped, the number of sticks in the set is renamed.

Show with a number tray that the number of sticks in a set can be given several different number names. Use the example below.

316 =	3 hundreds and	1 ten	and	6 ones
316 =		31 tens and		6 ones
316 =	3 hundreds and			16 ones
316 =				316 ones

Let the class say, "These are all names for the same number." Go on with the renaming of sets as needed.

Stage 4: Building up Sequences of Numbers 1-1,000

Vocabulary: no new vocabulary.

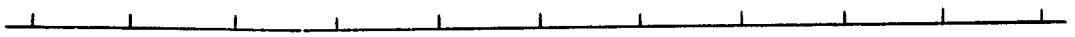
Materials: same as for Stage 3; pupil pages 119, 122-123.

Teaching Procedure:

Activity 1: Building up number lines by twos, fives, tens, hundreds, and so on. Pupil Page 122

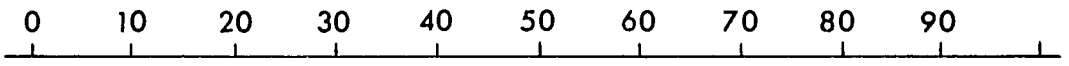
Place the number tray on your table. Put out the collection of sticks including at least 9 bundles of a hundred, 10 bundles of ten and 20 separate sticks.

Draw a number line on the blackboard. Put one stick in the ones' place of the number tray. Ask, "How many members has this set of sticks?" (1) Let the child write "1" on the line.



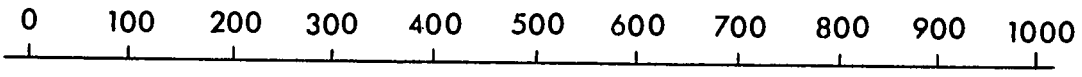
Put another stick in the number tray. Ask, "How many members in the set now?" Let another child write "2" on the line. Continue in this way until "9" is reached. Remove all the sticks from the tray. Ask, "How many members has this set of sticks now?" (None) "What do you call this set?" (Empty set) "What numeral do we write to show how many members this set has?" (0) Let a child write "0" at the point to the left of "1"?

Draw another line. Place one bundle of ten sticks in the tens' part of the number tray. Ask, "How many members has this set of sticks?" (10) Write "10" above the second point on the line. Continue the activity by adding more bundles of ten to the tray and writing numerals on the line until "90" is reached. As before, remove all the bundles and ask the same question about the empty set. Let a child write "0" above the first point to the left of "10" so that the line will look like this:



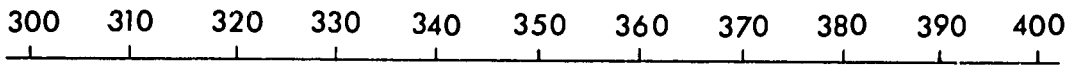
Note: Vary the way in which you draw a number line on the blackboard. Write the numerals below the line as often as you do above. Sometimes it is more convenient for them to be written below the line, sometimes above.

Build up another number line in the same way, putting bundles of hundreds in the number tray.

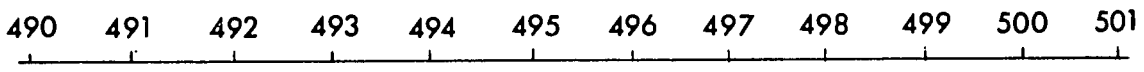


Let the class read the numerals 0, 100, 200, . . ., that is, let them say the numbers they use to count by hundreds.

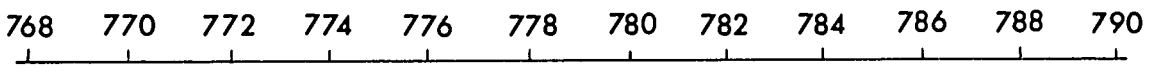
Draw another number line. Put 3 bundles of hundreds in the tray. Write "300" above the first point. Put a bundle of ten sticks in the tray and ask, "How many sticks are there now?" (310) Let a child write "310" above the next point. Continue putting bundles of ten in the tray until 390 is reached. When another ten is put in, there will be ten bundles of ten in the tens' part of the tray. Ask the class what regrouping must be done. (Tie them into a bundle of a hundred sticks) Repeat the question, "How many sticks are in the set now?" (400) Write "400" in the line.



Repeat this activity, using the number tray to help build up number lines like the ones below.



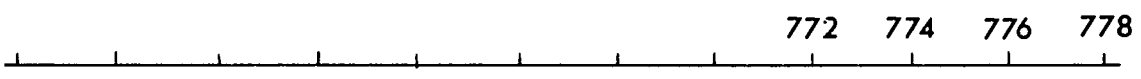
(Begin with 490 sticks and put one stick in the tray at a time. When 500 is reached, ten sticks must be tied into a bundle and ten tens must be tied into a bundle of a hundred.)



(Begin with 768 sticks and put two sticks in the tray at a time.)

After each number line is made, let the children read the numbers from the line.

Vary the activity by starting at the last point on the line and removing sticks from the tray. For example,



Vary the activity by starting in the middle of a number line and build the line in both directions.

Note: It is not necessary to use the number tray to help build the number lines if the children understand the number line without the use of the tray.

Ask the children to turn to page 122 in their books. Let them copy the number lines into their exercise books and fill in the missing numerals.

Activity 2: Recognizing patterns of counters. Pupil Page 119

Ask the children to turn to page 119 in their books. Let them use the cutouts as they did in Stage 2, Activity 2.

Ask the pupils to use their cutouts to cover part of the dots to show a pattern of 300 dots. Ask them to move their cutouts to show two more dots. Ask, "How many dots can you see now?" Let them keep moving their cutouts to show two more dots each time. Let them say the number of dots which are uncovered after each move. They will now be counting by twos, 300, 302, 304, and so on.

Vary this activity by starting with another number of dots and by changing the number of dots to be uncovered each time. Each time let the children count aloud. Let them also count backwards by covering up a certain number of dots each time.

Activity 3: Filling in missing numerals. Pupil Page 123

Ask children to turn to page 123 in their books. Let them copy out the sequences of numerals and fill in the numerals that are missing. Allow them to use the number tray, the 100 chart on page 118, the 1,000 dots on page 119, or any other method they need.

UNIT 9

ADDITION AND SUBTRACTION

Objectives:

1. To extend ideas of place value and decimal numerals to renaming (carrying) in addition.
2. To extend ideas of place value and decimal numerals to renaming (borrowing) in subtraction.
3. To extend addition and subtraction to sums of 999.

Background Information for the Teacher:

Also see Part 2, Section 7, "Arithmetic", of your Teachers' Handbook.

We have used the ideas of place value and the base ten (decimal) system in adding and subtracting tens and ones. Now that we have extended the system to include hundreds, we can add and subtract hundreds, tens, and ones.

Until now we have not asked the children to add tens or ones if the sum exceeded nine. We have not included subtraction sentences in which the numeral for the number to be subtracted has more units in the ones place than does the numeral for the number from which we are subtracting. We will now use the children's understanding of the decimal system to develop techniques of renaming numbers which will help the children add and subtract any number.

Let us look at the technique for finding the sum in a sentence such as

$$38 + 28 = \square$$

It is a long process to join a set of 38 members with a set of 28 members and then count the members of the union.

We make the process of adding easier by grouping by tens. We think of 38 as 3 tens and 8 ones. We think of 28 as 2 tens and 8 ones. We can write this as

$$38 = 30 + 8$$

$$28 = 20 + 8$$

These numerals are now written in expanded form. They are renamed.

We can now proceed to add the tens and to add the ones. The properties of addition allow us to do this. For example, the numerals in expanded form may be written like this:

$$38 + 28 = (30 + 8) + (20 + 8)$$

Since we know that neither the order nor the grouping of the addends affects the sum (addition is commutative and associative), we can order, group and add tens and ones like this:

$$(30 + 8) + (20 + 8) = (3 \text{ tens} + 2 \text{ tens}) \text{ and } (8 \text{ ones} + 8 \text{ ones})$$

If we add the tens we get 5 tens and if we add the ones we get 16. Sixteen may be <u>renamed</u> as 1 ten and 6 ones. The sum then becomes

$$50 + 16 = 50 + (10 + 6) = (50 + 10) + 6 = 66.$$

The sum, therefore, is 66.

We use these same ideas when we work with the shortened form.

$$\begin{array}{r} 38 \\ +\ \underline{28} \\ 66 \end{array}$$

Let us summarize the steps:

(1) Rename 38 as 3 tens and 8 ones.

(2) Rename 28 as 2 tens and 8 ones.

(3) Add the ones: $(8 + 8)$.

(4) Rename 16 as 1 ten and 6 ones.

(5) Add the tens: $(30 + 20) + 10 = 60$.

(6) Write the sum, 6 tens and 6 ones, as 66.

Let us now look at the technique for renaming (borrowing) in subtraction. Again, renaming numbers makes the process easier. Instead of forming a set of many members, then separating a subset and counting the members of the other subset, we can use a shortened form. For example,

$$45 - 27 = (\qquad)$$

Since subtraction is the inverse of addition, we know that the sentence can be rewritten in this way.

$$27 + (\quad) = 45$$

This sentence may be read "27 added to what number equals 45?" The number which we need to find in the sentence is an addend. To find the missing addend we first rename the sum and known addend. We think of 27 as 20 + 7; we think of 45 as 40 + 5. The sentence with the numerals in expanded form is

$$20 + 7 + (\quad) = 40 + 5.$$

To find the missing addend, let us think about the ones. We cannot add a number to 7 to get 5 so we rename the sum 40 + 5 as 30 + 15. The steps in the renaming are these:

$$40 + 5 = (30 + 10) + 5 = 30 + (10 + 5) = 30 + 15$$

The sentence now becomes

$$20 + 7 + (\quad) = 30 + 15$$

We must add 8 to 7 to get 15; we must add 10 to 20 to get 30. The missing addend is (10 + 8) or 18.

$$20 + 7 + (10 + 8) = 30 + 15$$

Because 27 + (18) = 45 we know that 45 - 27 = 18. Another way to write this is:

$$\begin{array}{rcl} 45 & = & 30 + 15 \\ -\underline{27} & = & \underline{20 + \ \ 7} \\ & & 10 + \ \ 8 = 18 \end{array}$$

In shortened form, we write just

$$\begin{array}{r} 45 \\ -\underline{27} \\ 18 \end{array}$$

In summary, the steps are:

(1) Rename 45 as 4 tens and 5 ones.

(2) Rename 27 as 2 tens and 7 ones.

(3) Rename 45 as 3 tens + 15.

(4) Find the number which added to 7 equals 15.

(5) Find the number of tens which added to 2 tens equals 3 tens (20 + 10 = 30).

(6) Write the missing addend (1 ten and 8 ones) as 18.

(7) The missing addend equals 45 - 27.

Stage 1: Addition With Sums to 999 Without Carrying

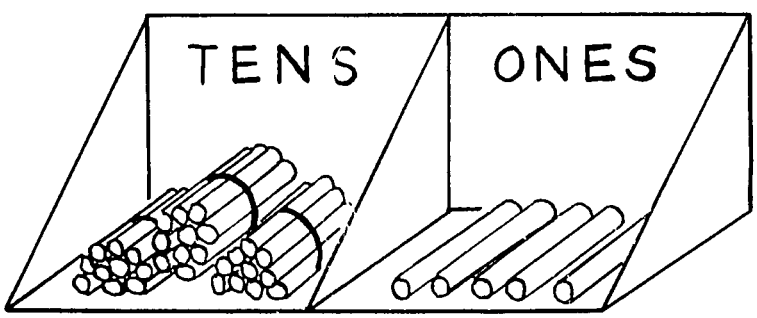
Vocabulary: no new vocabulary.

Materials: bundles of ten sticks, single sticks; blank 1-inch square cards for making numeral cards; number tray; pupil pages 124-130.

Teaching Procedure:

Activity 1: Place value. Pupil Page 124

Use the number tray with two sections as shown below.



Take some single sticks and ask a child to place a stick in the ones space. Then ask the child to continue to put single sticks in the ones space. As he places a stick he should tell how many there are. Continue until there are ten sticks in the ones space. Add one more stick and ask one of the pupils to count the number of sticks in this space. (11) Ask him whether we should keep all the sticks in there. (No) Let another child tie ten of the sticks together and put the bundle in the tens space. If the children do not understand this, ask questions such as: How many tens are there in 11? Help the children to see that 11 could be written as 10 + 1.

Write this sentence on the blackboard:

$$11 = 10 + 1.$$

Ask a child to count out ten sticks. Ask another child to tie the ten sticks with a string. Ask, "How many sticks do we have?" (Ten sticks; one bundle of ten sticks; or 1 ten sticks) Tell the child to put the bundle in the proper space in the number tray. Call out another child to say how many sticks there are in the tens space of the tray. (2 tens) Ask, "How many sticks are there in the ones space?" (1) "How many sticks there are altogether?" (2 tens and 1 one) Ask, "Can you tell another way to name the number?" (21 or 20 + 1) Write this on the blackboard:

$$21 = 20 + 1.$$

Put other sets of sticks in the tray. Ask the children to write numerals for the number of the sets. If you put 4 bundles in the tens space write "40" on the blackboard. If you put 6 sticks in the ones space, complete the numeral as "40 + 6". Help the children to say that this shows 4 tens and 6 ones. They should then be able to tell that we also write this number as "46".

Ask the children to open their books to page 124. Go over the first example with the class. Ask, "How many bundles of ten sticks are there?" (1 bundle of ten sticks) "How many single sticks are there?" (3) "How can we write this number?" (10 + 3) Say, "This shows that we have 1 ten and 3 ones. How can we write the numeral in a simpler way?" (13)

Go over the second example in the same way. Let the children place in the space to the right a 34-numeral card which they make from a blank card. Let them go on with the other exercises making numeral cards to show the number of sticks. When they have finished, go over each exercise. Let the children tell the number of sticks shown. Let them say a number of tens and ones and the simpler number name.

Activity 2: Place Value. Pupil Page 125

Put several numerals in a column on the blackboard, such as

86
74
39

Ask the children to tell the number of tens and the number of ones represented by each numeral. Mark another column "Tens" and a third column "Ones". See pupil page 125 for a model. Ask, "How many tens and ones are shown by 86?" (8 tens and 6 ones) Write the "8" in the tens column and the "6" in the ones column. Ask, "How do we write 8 tens? (80) If we write 8 tens as '80', how do we write the number name to show that we also have 6 ones?" (80 + 6) Let the children talk about the way to write the simplest names for a number.

Get the children to copy the numerals from page 125 in their exercise books. Ask them to fill in the columns of numerals shown.

Activity 3: Hundreds, tens and ones. Pupil Page 126

Help the children through questions and the use of the number tray and sticks to say:

(1) "We put single sticks in the ones space of the number tray until we have ten sticks. We tie the ten sticks into a bundle of ten and put it in the tens space. We write the numeral '10'. The '1' in the numeral is in the tens place. It means 1 ten."

(2) "If we put bundles of ten sticks in the tray until there are ten bundles, we have 10 tens. We tie the ten bundles into a bundle of a hundred and put it in the hundred space. We have 1 hundred sticks. We write the numeral 100. The '1' in the numeral is in the hundreds place. It means 1 hundred."

Write "615" on the board. Ask, "How many ones are shown in the ones place? (5) How many tens are there in the tens place? (1) How many hundreds are there? (6)" Let the children show this number with sticks in the number tray.

Write a column of numerals on the blackboard. Let the children tell the number of hundreds, tens, and ones. Write the expanded form for the numerals like this.

$$116 = 100 + 10 + 6$$
$$204 = 200 + 0 + 4$$
$$443 = 400 + 40 + 3$$
$$721 = 700 + 20 + 1$$

Write some other numerals on the blackboard. Let the children, in turn, write the numerals in expanded form.

Get the children to copy the numerals on page 126 into their exercise books and write them in expanded form as hundreds, tens, and ones.

Activity 4: Addition of hundreds, tens, and ones. Pupil Pages 127-128

Write this addition exercise on the blackboard.

$$\begin{array}{r} 241 \\ +\ \underline{125} \end{array}$$

Ask the children to tell the number of hundreds, tens and ones for each addend. As they give the number, you write the expanded form of the numerals on the blackboard as shown below. (This is the form used on pupil page 127 and pages following. Try to use the form.)

$$\begin{array}{r} 200 + 40 + 1 \\ +\ \underline{100 + 20 + 5} \end{array} \qquad \begin{array}{r} 241 \\ +\ \underline{125} \end{array}$$

Let the children add the hundreds, and then the tens and then the ones to get the sum, 300 + 60 + 6. Write this under the expanded form. Ask, "Can you write 300 + 60 + 6 in a simpler form ?" (366) As the work is done write the numerals on the blackboard as shown below. Let a child complete the exercise by writing the numeral "366" as the sum.

$$\begin{array}{r} 200 + 40 + 1 \\ \underline{100 + 20 + 5} \\ 300 + 60 + 6 = \end{array} \qquad \begin{array}{r} 241 \\ +\ \underline{125} \end{array}$$

Get the children to say that both forms are ways of adding. Give some other examples. Include some with both 3-digit and 2-digit numerals for addends. For example, you may use these:

$$\begin{array}{r} 162 \\ +\ \underline{27} \end{array} \qquad \begin{array}{r} 31 \\ +\ \underline{218} \end{array} \qquad \begin{array}{r} 103 \\ +\ \underline{50} \end{array}$$

Let the children copy the exercises on pages 127 and 128. Get them to find the sum for the expanded form of the exercises. They can then write these sums in simpler form.

Activity 5: Addition. **Pupil Page 129**

Write some exercises on the blackboard like those on pupil page 129. Let the children find the sums. Help them to say, "We think of the addends as hundreds, tens and ones; we add the ones, and then the tens and then the hundreds."

Ask the children to copy the exercises from page 129 on their papers and write the answers. They should be able to use the short form.

Stage 2: Addition up to 99 with Carrying

Vocabulary: renaming, carrying.

Materials: number tray with three spaces (hundreds, tens, ones); pupil pages 130-132.

Teaching Procedure:

Activity 1: The grouping of sticks into tens and hundreds.

Put out some sticks. You and the children make many bundles of ten sticks and leave some single sticks. Bundle some of the bundles of ten sticks into bundles of hundred sticks.

Call out three children Give one child two bundles of tens and five single sticks. Tell him to put them in the spaces of the number tray. Give the second

pupil one bundle of ten and seven single sticks. Tell him to put them in the number tray. The bundles of ten should be in the tens space and the single sticks in the ones space.

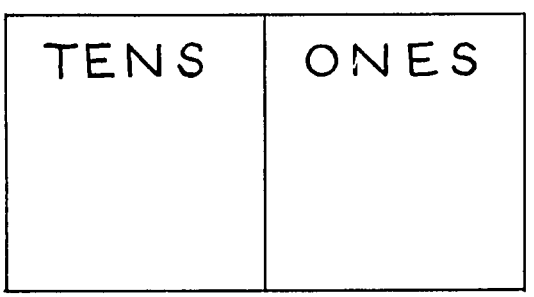
Ask the third child to count the loose sticks. (5 + 7 or 12) Ask, "How many tens and ones do you have?" (1 ten and 2 ones) Tell him to tie the 10 sticks together to make one bundle of ten and leave the 2 single sticks in the ones space. Tell him to put the bundle where it belongs. He will put it in the tens space. In the tens space there is now:

2 bundles from the first pupil
1 bundle from the second pupil
1 bundle which the third pupil made from the sticks in the ones space.

Ask, "How many bundles of ten are in the tray?" (4 bundles of ten) Ask another child to tell how many sticks in all. (4 bundles of ten and 2 loose sticks, or 42)

Let the pupils show the addition 25 + 17 using the same plan. Use other examples, as needed. Try to get many students to take part.

Give groups of children exercises similar to the above and sticks to show them. If you do not have enough number trays for each group to have one, mark regions on papers like the one pictured below. Let the groups use these in the same way as they used the number trays.



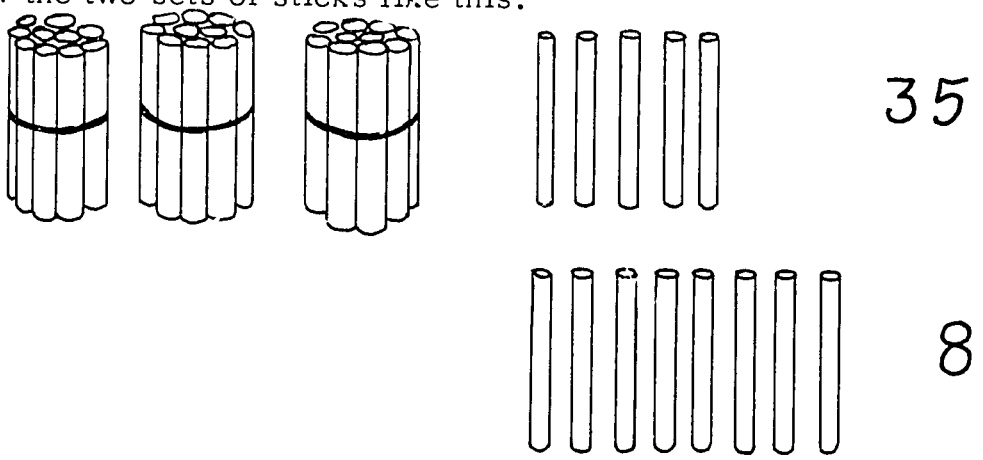
Activity 2: Addition using expanded notation. Pupil Page 130

Put out the counting sticks. Write this exercise on the blackboard.

$$35 + 8 = \square$$

Give a child some bundles of sticks and some single sticks. Ask him to read the numbers to be added and rename each as tens and ones. (3 tens and 5 ones plus 8 ones) Let another child put out sticks to show the numbers. (3 bundles of sticks and 5 single sticks, and 8 single sticks)

Let him show the two sets of sticks like this.



Get someone to put the sets together and tell how many single sticks there are. (13) Ask a child to form bundles of ten sticks with the single sticks. Ask him how many bundles and how many single sticks there are. (1 ten and 3 ones) Ask, "How many bundles of tens do we have?" (30 + 10 or 40) "How many sticks are there altogether?" (40 + 3 or 43)

Write the same exercise in vertical form. Let the children rename each of the addends. You write the addends in expanded form as shown below. (This form is the same as that used on pupil page 130.) Get the children to add the ones and then the tens. (30 + 13) Ask them to rename the 13. You write 30 + 13 as 30 + 10 + 3 and then as 43. Relate each step to the childrens' work with the sticks. The blackboard will look like this.

$$35 + 8 = \square \qquad \begin{array}{r} 30 + 5 \\ 8 \\ \hline 30 + 13 \end{array} = 30 + 10 + 3 = \begin{array}{r} 35 \\ + \ 8 \\ \hline 43 \end{array}$$

Go over the exercise again. Be sure each of the children understands it.

Put this exercise on the blackboard.

$$\begin{array}{r} 28 \\ +\ \underline{19} \end{array}$$

Let the children say that the addends must be renamed, and that the ones are added and the tens are added. As they tell you what to do record the work in this form.

$$28 + 19 = \square \qquad \begin{array}{r} 20 + 8 \\ \underline{10 + 9} \\ 30 + 17 \end{array} = 30 + 10 + 7 = \begin{array}{r} 28 \\ +\ \underline{19} \\ 47 \end{array}$$

Let the children open their books to page 130. The first problem on this page has been worked. Let the children study it. Ask them to copy and add the numbers of each of the problems.

<u>Activity 3</u>: Addition using expanded form. Pupil Pages 131-132

Put this exercise on the blackboard.

$$35 + 49 = \square$$

Ask a child to rename the numbers 35 and 49 in tens and ones. Write the expanded form of the numbers and the addition like this.

$$35 + 49 = \square \qquad \begin{array}{r} 30 + 5 \\ \underline{40 + 9} \\ 70 + 14 \end{array} = 70 + 10 + 4 = \begin{array}{r} 35 \\ +\ \underline{49} \\ 84 \end{array}$$

Get the children to understand that addition has the following basic steps: (1) renaming the addends 35 and 49; (2) adding the numbers of ones (5 + 9 = 14); (3) renaming 14 in tens and ones (10 + 4); (4) adding the number of tens (30 + 40 + 10 = 80); (5) writing the sum 84.

Get the children to think of the work written in this form.

$$\begin{array}{r} 35 \\ +\ \underline{49} \\ 84 \end{array}$$

<u>Note</u>: The addition of 1 ten from the units column to the number of tens in the tens column is often called "carrying". We do not use the word "carrying" in this book. "Rename" has more meaning for the children.

Write this example on the blackboard.

$$49 + 26 = \square$$

Ask a child to rename the addends 49 and 26 in tens and ones. Write the work on the blackboard in this way.

$$49 + 26 = \square \qquad \begin{array}{r} 40 + 9 \\ \underline{20 + 6} \\ 60 + 15 \end{array} = 60 + 10 + 5 = \begin{array}{r} 49 \\ + \underline{26} \\ 75 \end{array}$$

Again let the children tell the basic steps: (1) renaming 49 and 26; (2) adding 9 and 6 of the one column; (3) renaming 15 as 10 + 5; (4) adding all the tens (40 + 20 + 10 = 70); (5) writing the sum 75.

Tell the children that if the process is understood they need not write out all the steps. The work can be shown like this.

$$\begin{array}{r} 49 \\ + \underline{26} \\ 75 \end{array}$$

Write the exercise below on the blackboard. Let the children try to add the numbers without writing all of the steps.

$$\begin{array}{r} 26 \\ + \underline{19} \end{array}$$

You decide when the children need to stop writing all the steps and to use the short form.

When they are ready for the short form, ask them to copy the exercises on pages 131 and 132 on their papers and add the numbers. The exercises should be used as needed. They may be used on several days.

Stage 3: Addition with Sums to 999

Vocabulary: no new vocabulary.

Materials: pupil pages 133-137.

Teaching Procedure:

Activity 1: Renaming units as tens in addition. Pupil Pages 133

Write the exercise 273 + 108 = ☐ on the blackboard as a sentence and in vertical form. Ask a child to rename both addends in hundreds, tens, and ones. (200 + 70 + 3 and 100 + 0 + 8)

With the help of the pupils work out the example in this way:

$$273 + 108 = \begin{array}{r} 200 + 70 + 3 \\ \underline{100 + 0 + 8} \\ 300 + 70 + 11 \end{array} = 300 + 80 + 1 = \begin{array}{r} 273 \\ + \underline{108} \\ 381 \end{array}$$

Go over each step. Let the children say, "We add the hundreds, tens and ones. We have a sum of 300 + 70 + 11. Then we rename 11 as 1 ten and 1. We add the 1 ten of 11 to 7 tens. We now have 300 + 80 + 1. We write this as 381." Complete the exercise as

$$\begin{array}{r} 273 \\ + \underline{108} \\ 381 \end{array}$$

Let the children work other problems of this same type together. For example, 326 + 48 = ☐. The children will work it out like this.

$$326 + 48 = \square \qquad \begin{array}{r} 300 + 20 + 6 \\ \underline{40 + 8} \\ 300 + 60 + 14 \end{array} = 300 + 70 + 4 = \begin{array}{r} 326 \\ + \underline{48} \\ 374 \end{array}$$

Ask the pupils to open their books to page 133. The first exercise on this page is worked. Let the children study it. Ask them to complete the other exercises on the page.

Activity 2: Renaming tens as hundreds in addition. Pupil Page 134

Write this addition exercise, 164 + 273 = ☐ on the blackboard in vertical form. Ask a child to express the numbers 164 and 273 in hundreds, tens and ones (100 + 60 + 4 and 200 + 70 + 3). With the help of the pupils put work on the blackboard like this.

$$164 + 273 = \square \qquad \begin{array}{r} 100 + 60 + 4 \\ \underline{200 + 70 + 3} \\ 300 + 130 + 7 \end{array} = 300 + 100 + 30 + 7 = 400 + 30 + 7 = \begin{array}{r} 164 \\ + \underline{273} \\ 437 \end{array}$$

From this expansion the children should see why the answer is 437. Work another example such as 476 + 282 = ☐ on the blackboard. Then let the children open books to page 134. Let them study the first example, which has been worked. Then let them complete the rest of the examples on this page.

Activity 3: Renaming in addition. Pupil Pages 135-137

Write this exercise, 247 + 138 = ☐, on the blackboard in vertical form and let the children tell how to find the sum. Put the work on the blackboard like this.

$$247 + 138 = \square \qquad \begin{array}{r} 200 + 40 + 7 \\ \underline{100 + 30 + 8} \\ 300 + 70 + 15 \end{array} = 300 + 70 + 10 + 5 = 300 + 80 + 5 = \begin{array}{r} 247 \\ + \underline{138} \\ 385 \end{array}$$

Help the children to see that the process of addition has these basic steps: (1) renaming the addends 247 and 138; (2) adding the numbers in the hundreds, tens and ones columns (200 + 100 = 300; 40 + 30 = 70; 7 + 8 = 15); (3) renaming the 15 of the ones column in tens and ones (10 + 5); (4) adding the 10 of the 15 to the sum of the number of tens in the ten column (10 + 70 = 80); writing the sum 385. The work may be shortened to this form:

$$\begin{array}{r} 247 \\ + \underline{138} \\ 385 \end{array}$$

Work the example shown below to show renaming in the tens place. Put the work on the blackboard like this.

$$174 + 462 = \square \qquad \begin{array}{r} 100 + 70 + 4 \\ \underline{400 + 60 + 2} \\ 500 + 130 + 6 \end{array} = 500 + 100 + 30 + 6 = 600 + 30 + 6 = \begin{array}{r} 174 \\ \underline{+\ 462} \\ 636 \end{array}$$

Help the children to say that the process of addition has these basic steps: (1) renaming the addends 174 and 462; (2) adding the numbers in the hundreds, tens and ones columns (100 + 400 = 500; 70 + 60 = 130; 4 + 2 = 6); (3) renaming the 130 of the tens column in hundreds and tens (100 + 30); (4) adding the 100 of the 130 to the sum of the hundreds column (100 + 500 = 600); (5) writing the sum 636.

Ask a child to show how the form may be shortened:

$$\begin{array}{r} 174 \\ \underline{+\ 462} \\ 636 \end{array}$$

Write this example of the same type on the blackboard.

$$173 + 293 = \square$$

Ask the children to work the exercise on their papers. Go around the room and help those who need help.

Call a child who added the numbers correctly to tell his answer. Ask him to tell how he got the sum. Show his work on the blackboard. Ask another child to show how the work could be written in a shorter form.

$$\begin{array}{r} 173 \\ \underline{293} \\ 468 \end{array}$$

Let the children open their books to page 135. Ask them to work the problems by the short method. Let them continue with pages 136 and 137 on other days or as needed.

Stage 4: Addition with Sums to 999; Renaming in Two Columns

Vocabulary: no new vocabulary.

Materials: pupil pages 138-139.

Teaching Procedure:

Activity 1: Addition using renaming. Pupil Page 138

Write this exercise on the blackboard as a sentence and in vertical form.

$$375 + 167 = \square$$

Ask a child to rename the numbers 375 and 167 as hundreds, tens and ones. (300 + 70 + 5; 100 + 60 + 7) Write these names on the blackboard. Let them add each of the columns in the expanded form. Ask, "Which of the sums 400, 130, and 12 will you rename?" Let the children talk about this question. They will say, "We will rename 130 and 12. The new names will help us write the sum in a simpler way." As they find the simpler name for the sum, you put their work on the blackboard in this form.

$$\begin{array}{l} 300 + 70 + 5 \\ \underline{100 + 60 + 7} \\ 400 + 130 + 12 = 400 + 130 + 10 + 2 = 400 + 140 + 2 = 400 + 100 + 40 + 2 \end{array}$$

$$\begin{array}{r} 375 \\ +\ \underline{167} \\ = 500 + 40 + 2 = 542 \end{array}$$

Help the children see that the basic steps of addition are: (1) renaming 375 and 167; (2) adding the numbers in the ones column (5 + 7 = 12); (3) renaming the 12 (10 + 2); (4) adding the 10 to the numbers of the tens column (10 + 70 + 60 = 140); (5) renaming 140 (100 + 40); (6) adding the 100 to the number of hundreds (100 + 300 + 100 = 500); (7) writing the sum 542.

Tell the children, "To get the 1 ten of the ones column to add to the tens column and to get the 1 hundred from the tens column to add to the hundreds column we rename the addends and the sums. The steps may be written in a shortened form like this:"

$$\begin{array}{r} 375 \\ \underline{167} \\ 542 \end{array}$$

When the basic steps are understood by the children, it will not be necessary for them to use the expanded form.

Work this example with the pupils.

$$265 + 437 = \square$$

Ask a child to rename the numbers 265 and 437 in expanded form. Let the children go on with the addition using the basic steps in addition. You write on the blackboard as they work. The blackboard will look like this.

$$\begin{array}{l} 200 + 60 + 5 \\ \underline{400 + 30 + 7} \\ 600 + 90 + 12 = 600 + 90 + 10 + 2 = 600 + 100 + 2 = 700 + 0 + 2 = \end{array} \quad \begin{array}{r} 265 \\ + \underline{437} \\ 702 \end{array}$$

Note: You may wish to let all or some of the children show this exercise with sticks in the number tray.

Let the children name the basic steps used in the exercise: (1) renaming the addends 265 and 437; (2) adding the numbers in the ones column (5 + 7 = 12); (3) renaming the 12 (10 + 2); (4) adding the 1 ten to the numbers of the tens column (10 + 60 + 30 = 100); (5) renaming the 100 or 10 tens as 1 hundred and 0 tens; (6) adding the 1 hundred to the numbers of the hundreds column (100 + 200 + 400 = 700); (7) writing the sum of 700 + 2 as 702. The form above may be shortened to this:

$$\begin{array}{r} 265 \\ \underline{437} \\ 702 \end{array}$$

Put another example on the blackboard. Let the children work it on their papers using the short method. Go around the class and help those who need help.

Let the children open their books to page 138 and copy and work the examples from their book in their exercise books.

<u>Activity 2</u>: Addition using renaming. Pupil Page 139

Work this example.

$$26 + 74 = \square$$

Ask a child to rename the numbers 26 and 74 as tens and ones. Let the children go on with the addition using the basic steps of addition. Write the work on the blackboard in this form:

$$26 + 74 = \square \qquad \begin{array}{r} 20 + 6 \\ \underline{70 + 4} \\ 90 + 10 \end{array} = \begin{array}{r} 26 \\ + \underline{74} \\ 100 \end{array}$$

Let the children name the basic steps in the addition above; (1) renaming 26 and 74; (2) adding the numbers in the ones column (6 + 4 = 10); (3) renaming 10 as 1 ten; (4) adding 1 ten to the numbers in the tens column (10 + 20 + 70 = 100); (5) writing the sum 100. Ask a child to write a shorter form on the blackboard:

$$\begin{array}{r} 26 \\ \underline{74} \\ 100 \end{array}$$

Note: You may wish to have this exercise shown with sticks in the number tray. Help the children work this example:

$$146 + 78 = \square$$

Ask one of the children to rename 146 and 78 in hundreds, tens and ones. Write the work with the help of the children on the blackboard in this way.

$$\begin{array}{r} 100 + 40 + 6 \\ \underline{70 + 8} \\ 100 + 110 + 14 \end{array} = 100 + 110 + 10 + 4 = 100 + 120 + 4 = 200 + 20 + 4 = \begin{array}{r} 146 \\ + \underline{78} \\ 224 \end{array}$$

Let the children name the basic steps in the addition above: (1) renaming the addends 146 and 78; (2) adding the numbers in the ones column (6 + 8 = 14); (3) renaming the 14 (10 + 4); (4) adding the 1 ten to the numbers of the tens column (10 + 40 + 70 = 120); (5) renaming 120 (100 + 20); (6) adding the 1 hundred to the numbers of the hundreds column (100 + 100 = 200); (7) writing the sum of 200 + 20 + 4 as 224.

Ask one of the children to show the exercise in a shorter form:

$$\begin{array}{r} 146 \\ \underline{78} \\ 224 \end{array}$$

Ask the pupils to open their books to page 139 and copy the examples in their exercise books. Let them work the examples using the short form. At this point, you should use the expanded form only to review the basic steps.

Stage 5: Addition of Three Addends Using Renaming

Vocabulary: no new vocabulary.

Materials: pupil page 140.

Teaching Procedure:

Activity 1: Addition of three addends.

Tell the children that we also can find the sum of three numbers. Write this exercise on the blackboard:

$$24 + 30 + 13 = \square$$

Ask one of the children to rename the numbers in expanded form. Tell another child to add the ones and the tens. (4 + 0 + 3 = 7; 20 + 30 + 10 = 60) Let another child tell the sum, (6 tens and 7 ones or 67). As they work you show the work on the blackboard like this.

$$24 + 30 + 13 = \square \qquad \begin{array}{rcr} 20 + 4 & & 24 \\ 30 + 0 & & 30 \\ \underline{10 + 3} & & \underline{+\ 13} \\ 60 + 7 & = & 67 \end{array}$$

Continue with other examples which do not require renaming of the sum. Let the children tell the basic steps in this addition example.

Activity 2: The addition of three addends. Pupil Page 140

Write this addition sentence on the blackboard.

$$36 + 25 + 18 = \square$$

Ask, "How do we think of 36, 25, and 18 to add them?" Let the children talk about this question. Help them say, "We rename them as tens and ones."

Say, "You rename them and I will write what you say." The form should be like this.

$$36 + 25 + 18 = \square \qquad \begin{array}{r} 30 + 6 \\ 20 + 5 \\ \underline{10 + 8} \end{array} \qquad\qquad \begin{array}{r} 36 \\ 25 \\ \underline{18} \end{array}$$

Say, "We will first add the number of ones." (6 + 5 + 8 = 19) Show this sum on the blackboard. Ask, "What is the sum of the tens?" (20 + 30 + 10 = 60) Write this under the tens. Ask a pupil to rename the 19 in tens and ones. (10 + 9) Ask, "What shall we do with the 1 ten?" (We will add it to the other tens. 60 + 10 = 70) Ask, "What is the simplest name for the sum, 70 + 9? (79) The work you write on the blackboard should look like this.

$$36 + 25 + 18 = \square \qquad \begin{array}{r} 30 + 6 \\ 20 + 5 \\ \underline{10 + 8} \\ 60 + 19 \end{array} = 60 + 10 + 9 = 70 + 9 = \begin{array}{r} 36 \\ 25 \\ \underline{+\ 18} \\ 79 \end{array}$$

Let the children notice that when they add three numbers they add them two at a time. For example, in the addition of the ones in the last example (6 + 5 + 8), the 6 and 5 may be added first as in: (6 + 5) + 8 = 11 + 8 = 19; or the numbers may be added like this: 6 + (5 + 8) = 6 + 13 = 19. The sum is the same for both. The same is true of the addition of the number of tens. The children can add like this (30 + 20) + 10 = 50 + 10 = 60 or like this 30 + (20 + 10) = 30 + 30 = 60.

Help the children to shorten the form of the written work to this.

$$\begin{array}{r} 36 \\ 25 \\ \underline{18} \\ 79 \end{array}$$

Ask the children to find the exercises on page 140. Work one of them with the children. Let them copy the other exercises in their exercise books and solve them.

Stage 6: Subtraction

Vocabulary: no new vocabulary.

Materials: number tray; counting sticks; nine 6" × 2" cards, nine 4" × 2" cards, ten 2" × 2" cards; pupil pages 141-145.

Teaching Procedure:

Activity 1: Writing 3-digit numerals in expanded form.

Write the numeral "423" on the blackboard. Ask one of the children to write this in expanded form. (400 + 20 + 3) Say, "We can read this as 4 hundreds, 2 tens and 3 ones. Give another child 4 bundles of a hundred sticks, 2 bundles of ten sticks and 3 single sticks. Ask, "How many sticks does he have?" (4 hundreds, 2 tens, and 3 ones, or 423) Tell him to put the sticks in the spaces in the number tray. Ask another child to write the number of sticks in the tray in expanded form.

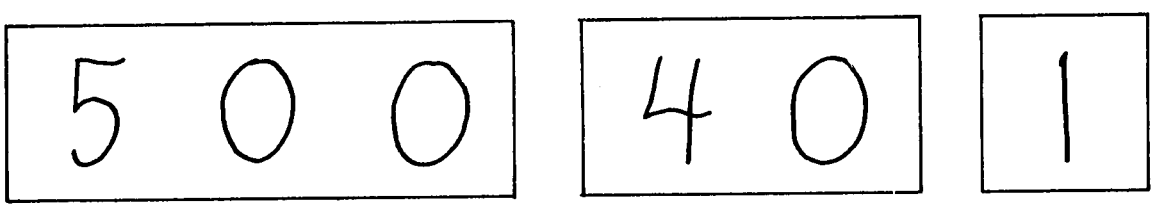
$$423 = 400 + 20 + 3$$

Be sure that all the children understand this. Repeat this same plan with other numbers, say, 256, 541, 303, and so on.

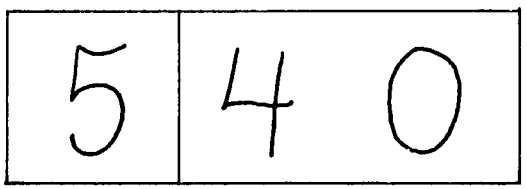
Activity 2: Expanded form of 3-digit numeral. Pupil Page 141

Make a set of nine 2" × 6" cards. Write each of the following numerals which show multiples of 100 on one of the cards: 100, 200, ...900. Make a set of nine 2" × 4" cards. Write each of the following numerals which show multiples of ten on one of the cards: 10, 20, ...90. Make a set of ten 2" × 2" cards. Write each of the following numerals on one of the cards: 0, 1, 2, ...9.

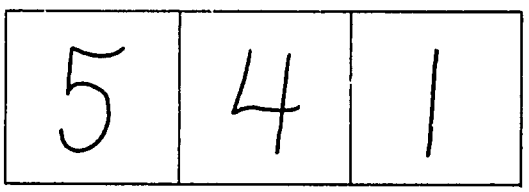
Pass out the cards to 28 children. Ask the children with cards showing 500, 40, and 1, to come out. Below is a picture of the three cards.



Ask, "How can we show the sum of the numbers named by these numerals with your cards?" Let the children talk about this. Ask the child with the 500-card to lay his card on your table, with the numeral up. Ask the child with the 40-card to put his card on top of the 500 card so that the numeral "4" falls on the "0" in the tens place like this:



Then ask the child with the 1-card to place his card so the numeral "1" falls on the "0" as shown below.



Ask, "What number do you show?" (541) Say, "This shows that 500 + 40 + 1 = 541 and that 541 = 500 + 40 + 1." Repeat this activity with three other numeral cards as many times as needed to make the expanded notation of a number clear.

Let the children open their books to page 141 and work the examples from the page in their exercise books.

Activity 3: Using place value to order numerals. Pupil Page 142

Note: We wish children to get practice in comparing numbers to help them use the idea of place value. In this activity we will let them look at the hundreds place of two three-digit numerals and say which of the two numbers is greater.

Write the numerals 842 and 482 on the blackboard. Let the children read the numbers and talk about sets of 842 and 482 the children may have seen.

Ask, "How many hundreds do these show?" (800 and 400) Which of these two hundreds is greater?" (800) "Which of these two numbers 842 and 482 is greater? (842) Write on the blackboard 842 > 482.

Let a child use the number tray to show that 842 > 482.

Write 842 and 852 on the blackboard. Ask, "If we look at the hundreds places of these two numerals, can we tell which is greater?" (No) "What is the number of hundreds in the first number?" (800) "What is the number of hundreds in the second number?" (800) "What does this mean?" (The two numbers have the same number of hundreds.) "How can we tell which of the two numbers is greater?" (Look at the number of tens) "What are the number of tens?" (4 tens and 5 tens) "Which number of tens is greater?" (5 tens) "What does this mean?" (This means that the second number with 5 tens is greater than the first number with 4 tens.) "Which number is greater?" (852 > 842) "Can the number of ones in the numbers make any difference?" (No)

Write these numerals on the blackboard: 842 and 843. Ask, "Which is greater?" At this stage the children should be able to say 843 and tell why. If not follow the plan above. Repeat with some other numbers. Then let the children open their books to page 142. Ask them one by one to do the examples orally. The first one has been worked as a sample. The numbers should be listed least to greatest, left to right.

Activity 4: Subtracting without renaming. Pupil Page 143-144

Give a child 1 bundle of hundred sticks and 4 bundles of ten sticks. Ask, "How many sticks does he have?" (1 hundred and 4 tens or 140) Let him put the sticks in the correct spaces in the number tray. Tell another child to take 20 sticks from the tray. Ask, "How many bundles did he remove?" (2 tens) Ask another child to count the sticks still in the tray. (120) Show this operation on the blackboard:

$$140 - 20 = \square \qquad \begin{array}{r} 100 + 40 \\ 20 \\ \hline 100 + 20 \end{array} = \begin{array}{r} 140 \\ -\ 20 \\ \hline 120 \end{array}$$

Ask, "What operation have we shown?" (Subtraction) Say, "In this subtraction we are given a sum, 140, and an addend, 20. What do we call the 120?" (Missing addend)

Remove all the sticks from the tray. Give a child 3 bundles of a hundred sticks and 7 bundles of tens. Ask, "How many sticks does he have?" (370) Ask him to put the sticks in the correct spaces in the number tray. Tell another child to remove 230 sticks from the tray. Ask, "How many bundles did he take from the hundreds space?" (2 hundreds) "How many bundles did he take from the tens space?" (3 tens) Ask another child to count the sticks still in the tray. (1 bundle of hundred and 4 bundles of tens, or 140) Say, "In this subtraction, what is the missing addend?" (140) Represent the work on the blackboard in this way.

$$370 - 230 = \square \qquad \begin{array}{r} 300 + 70 \\ \underline{200 + 30} \\ 100 + 40 \end{array} \quad = \quad \begin{array}{r} 370 \\ \underline{-\ 230} \\ 140 \end{array}$$

Help the children to see that the work may be shortened to this form.

$$\begin{array}{r} 370 \\ \underline{-\ 230} \\ 140 \end{array}$$

Help the children work another example in the same way.

Ask the children to open their books to page 143 and copy the exercises and work them in their exercise books. Let the pupils continue with problems on page 144 in another lesson or as needed.

Activity 5: Subtracting without renaming. Pupil Pages 145-146

Give one of the children 2 bundles of hundreds, 9 bundles of tens, and 7 single sticks. Ask, "How many sticks does he have?" (2 hundreds, 9 tens and 7 ones, or 297) Ask a child to write this number in expanded form. (200 + 90 + 7) Tell the first child to put the sticks in the number tray. Tell another child to take 25 sticks from the tray (2 tens and 5 ones). Ask another child to count the number of sticks still in the tray (2 hundreds, 7 tens, and 2 ones, or 272). Say, "We can show this as subtraction." Write the subtraction sentence and then the exercise in vertical form. Then in steps, as you and the children talk of the action with the sticks, show the subtraction like this.

$297 - 25 = \square$

$$\begin{array}{r} 200 + 90 + 7 \\ - \quad 20 + 5 \\ \hline 200 + 70 + 2 \end{array} \quad = \quad \begin{array}{r} 297 \\ - \ 25 \\ \hline 272 \end{array}$$

Help the children see that the work may be shortened to this form.

$$\begin{array}{r} 297 \\ - \ 25 \\ \hline 272 \end{array}$$

Work the exercise below by the same plan used above, first with sticks and then on the blackboard. The work on the blackboard will look like this.

$$\begin{array}{r} 597 \\ - \ 235 \\ \hline 362 \end{array} \qquad \begin{array}{r} 500 + 90 + \\ - \ 200 + 30 + 5 \\ \hline 300 + 60 + 2 \end{array} \quad = \quad \begin{array}{r} 597 \\ - \ 235 \\ \hline 362 \end{array}$$

Help the children to see that the written form of the subtraction may be shortened.

$$\begin{array}{r} 597 \\ - \ 235 \\ \hline 362 \end{array}$$

Let the children tell the steps they followed in the exercise: (1) renaming each number in hundreds, tens and ones; (2) subtracting the ones, tens and hundreds ($7 - 5 = 2$, $90 - 30 = 60$, $500 - 200 = 300$); (3) writing the missing addend 362.

Ask the children to open their books to page 145. Let them copy and work the exercises in their exercise books. Let them go on with the exercises on page 146 as needed.

Stage 7: Subtraction Using Renaming

Vocabulary: renaming.

Materials: same as for Stage 6; pupil pages 147-149.

Teaching Procedure:

Activity 1: Renaming numbers. Pupil Page 147

Give one of the children 7 bundles of ten sticks and 5 single sticks. Ask, "How many sticks does he have?" (7 tens and 5 ones, or 75) Tell him to put the sticks in the correct spaces in the number tray. Ask another child to write "75" on the blackboard and then write it in expanded form. (70 + 5) Tell still another child to take 1 bundle of ten sticks from the tens space, untie it and place the single sticks (10) in the ones space. Ask the pupils to tell how many sticks are in the ones section. (15) Let a child count the sticks. Ask, "How many sticks are in the number tray?" (6 tens and 15 ones) Ask, "Is this number the same number or a different number than 75?" Let the children talk about this and decide it is the same number. Let a child write the numeral for the number of sticks now in the tray in the expanded form. (60 + 15) Ask, "Are 70 + 5 and 60 + 15 names for the same number?" (Yes) Write on the blackboard:

$$70 + 5 = 60 + 15.$$

Let the children talk about why these are equal. (They are both names for 75) Get the children to rename several numbers in these two ways, for example, 36 = 30 + 6 = 20 + 16.

Let the children open their books to page 147. Ask them to rename in two ways the numbers on the page. Let them write the numerals in their exercise books.

Activity 2: Subtraction using renaming. Pupil Pages 148-149

Give 5 bundles of ten sticks and 4 single sticks to a child. Ask, "How many sticks does he have?" (54) Get him to put the sticks in the number tray. Ask another child to write the numerals for the number in expanded form on the blackboard. (54 = 50 + 4) Ask still another child to take 1 bundle from the tens section, untie it, and put the single sticks (10) in the ones space. Ask, "How can we tell the number of sticks in the tray as we see them now?" (40 + 14) Ask, "Is it easier to take 8 sticks from the tray, when there are 50 + 4 sticks in the tray or when there are 40 + 14 sticks in the tray?" Let the children talk about

this. They may need to try it. They will decide that it is easier to take 8 from the tray when they see 40 + 14 sticks in the tray. Ask a child to take 8 sticks from the tray. Ask, "How many are left?" (46)

Revise the exercise with the sticks. As you and the children talk about the exercises you write this subtraction on the blackboard in this form.

$$54 - 8 = \square \qquad \begin{array}{r} 40 + 14 \\ 8 \\ \hline 40 + 6 \end{array} = \begin{array}{r} 54 \\ -\ 8 \\ \hline 46 \end{array}$$

Get the children to see that the work may be shortened to:

$$\begin{array}{r} 54 \\ -\ 8 \\ \hline 46 \end{array}$$

Let the children work another example of the same type in the same way first with sticks and then with numerals on the blackboard. You may use this example.

$$72 - 56 = \square$$

Give one of the children 7 bundles of ten sticks and 2 single sticks and ask, "How many sticks are there?" (72) Get a child to put the sticks in the number tray. Ask one of the children to rename 56 in tens and ones. (50 + 6) Say, "I wish to take 56 sticks from the tray." How many single sticks would you have to take from the ones space to take 56 sticks from the tray. (6 single sticks) How many single sticks are in the tray?" (2) Ask, "How can we take 6 single sticks from the tray?" Let the children talk about this and decide this: untie one bundle of ten in the tens space and put the single sticks in the ones space. Ask, "If the sticks from 1 bundle of ten are untied and put in the ones space, how many single sticks will there be in the number tray?" (12) Let one of the children show this with the sticks and tray. Let him count the number of single sticks in the ones space. Ask another child to name the number of sticks in the tray in the way they now see them. (60 + 12) Tell still another child to take 56 sticks from the tray. Ask, "How many sticks are now in the tray?" (16)

Revise the exercise with the sticks. As you do so, show the exercise with numerals on the blackboard as shown below.

$72 - 56 = \square$

$$\begin{array}{r} 60 + 12 \\ \underline{50 + \ \ 6} \\ 10 + \ \ 6 \end{array} \quad = \quad \begin{array}{r} 72 \\ -\ \underline{56} \\ 16 \end{array}$$

Again help the children to see that this could be shortened to:

$$\begin{array}{r} 72 \\ -\ \underline{56} \\ 16 \end{array}$$

Help the children to work this exercise:

$83 - 48 = \square$

Ask one of the children to rename 83 so they may subtract 48. (70 + 13) Ask the children to work this problem on their paper. Let them use sticks if they need sticks. Go around the class and help those that need help. Give them enough time to think out the exercises. When they have finished, work out the example with the children. Show this work on the blackboard.

$83 - 48 = \square$

$$\begin{array}{r} 70 + 13 \\ \underline{40 + \ \ 8} \\ 30 + \ \ 5 \end{array} \quad = \quad \begin{array}{r} 83 \\ -\ \underline{48} \\ 35 \end{array}$$

Let the children show the shortened form for the exercise.

$$\begin{array}{r} 83 \\ -\ \underline{48} \\ 35 \end{array}$$

Ask the children to open their books to page 148 and work the exercise in their exercise books. Let them use both the expanded form and the shortened form. Go on with the exercises on page 149 on another day or as needed.

UNIT 10

MULTIPLICATION

Objectives:

1. To learn the meaning of multiplication.
2. To find products of two whole numbers (limit to products of 18).

Background Information for the Teacher:

Also see Part 2, "Arithmetic," of your Teachers' Handbook.

There are two basic operations on whole numbers--addition and multiplication. When we form the union of two sets of objects which have no members in common and we count the members of the union, we are finding a sum. This operation on the numbers of the sets is called addition. For example:

Union of sets: $\{\square, O, \Delta\} \cup \{V, \ast, \varphi, \ominus\} = \{\square, O, \Delta, V, \ast, \varphi, \ominus\}$

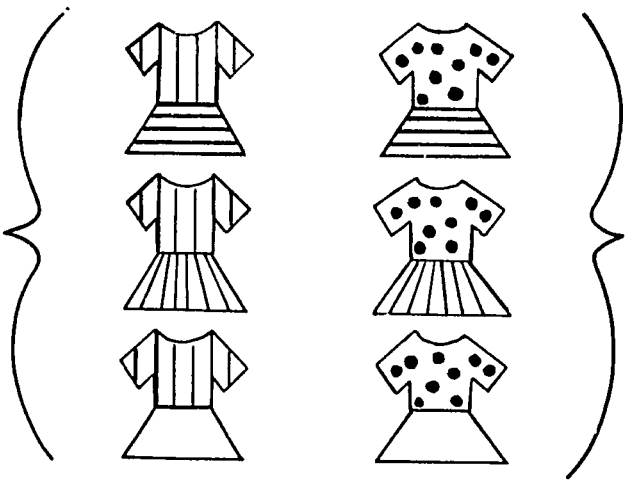
Adding of numbers: $3 + 4 = 7$

In this unit we introduce the operation of multiplication. The plan is to show multiplication first with real objects that are familiar to the children. We let them match the objects of two sets in every possible way. They pair every member of the first set with every member of the second set and form a new set of all possible pairs. The pupils keep a record of the new set formed. We call this "mixing" the two sets to form the new set of all possible pairs.

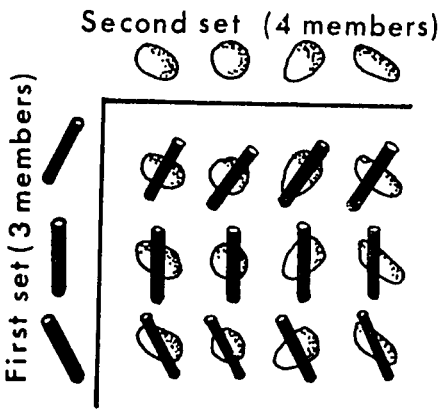
The goal of this activity is to get, in a natural way, a set of objects that are arranged in rows. The children can then count the number of pairs in several ways. They can count by rows. And they can count the members in each row and add this number as many times as there are rows.

For example, suppose the pupils are asked to mix a set of three skirts and a set of two blouses. They are to form every possible costume. Each costume is made

by matching a skirt to a blouse. The mixed set is the set of all possible costumes or pairs of skirts and blouses. A picture of the set of custumes looks like this.



The picture of the mixed set is arranged in three rows. Each row has two members. In this form, the set is easy to count by twos. It can also be counted by adding 2, three times: 2 + 2 + 2. We say that a set of three members mixed with a set of two members gives a new set of 2 + 2 + 2 members. As another example, let us mix a set of three sticks with a set of four stones. This is a picture of the set formed.



The Mixed Set: 3 rows of 4 pairs each

We wish to add the total number of pairs formed when we mix the sets shown above. Since there are 3 rows of 4 pairs each in the new set we add 3 fours,

$$4 + 4 + 4.$$

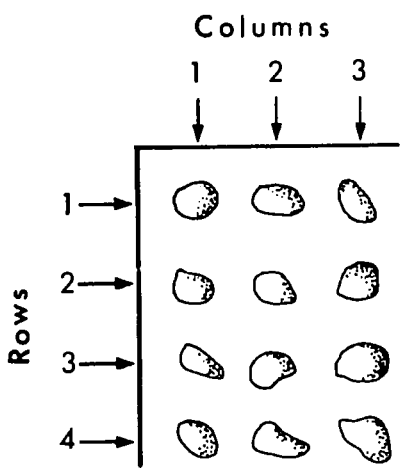
In this way the children are introduced to sets of objects arranged in rows. The process of finding the total number of members in all the rows is called multiplication. If there are three rows of members with four in each row, that is, three fours, we use the symbols 3×4 to write the total number. "3×4" is read "3 fours", or "3 <u>times</u> 4", or "3 multiplied by 4", or "the <u>product</u> of 3 and 4".

The children discover, for example, that 3×4 is $4 + 4 + 4$, or 12, from many sets of objects arranged in rows. They must learn first some of the multiplication facts; then they find that the total number of objects can be found by multiplication. In this unit, the class learns these things:

1. To use the multiplication facts up to products of 12 and possibly beyond.
2. To use multiplication language such as "3 and 4 are <u>factors</u> of 12, because 12 is the <u>product</u> of 3 and 4."
3. To write multiplication facts in true sentences, such as "$3 \times 4 = 12$".
4. To find numbers which make multiplication sentences true, such as $3 \times \square = 12$.

In a later unit, more multiplication facts will be learned through activities and problems. In this unit, our object is to help children develop meaning for multiplication.

Note: In this book, we use the words "row" and "column" in the following way:



This is a picture of 12 stones arranged in 4 rows and 3 columns.

Stage 1: Revision of Counting

Vocabulary: no new words

Materials: objects such as seeds of sticks to form sets; pupil page 150.

Teaching Procedure:

Activity 1: Counting by twos, threes, fours and fives.

Play a game with the pupils. Let them form a line. Assign to each pupil a number, in natural order, starting from 1. To make sure that each child knows his name, call out a number and let the pupil with that name say, "Here". Go on in this way as needed.

Ask the class to count by twos using their own number names. Pupil-2 calls out, "Two"; Pupil-4 calls out "Four"; and so on, to the end of the line. Let the pupils count in the same way by fives (5, 10, 15,...), by threes (3, 6, 9,...), and by fours (4, 8, 12,..).

Ask the last pupil in line to step out and find Pupil-12. Get him to do this by counting by twos along the line of pupils. As he touches the pupils he should say their number names, "Two, four,...", stopping at Pupil -12. Let the class correct any wrong names. Let the second from last pupil come forward and find Pupil-15. To do this let him decide to count by threes. Go on with this activity. Ask a child to find another child with a given number name by counting by twos, threes, fours, or fives.

Activity 2: Counting on a number line. Pupil Page 150

Separate the class into groups and give each group 30 or more small objects, for example, seeds. Ask them to count their seeds by twos, then count by fives, and then count by tens. Let them do this by arranging the seeds into subsets as they count.

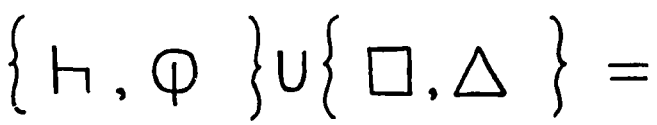
Draw several lines on the blackboard like the number lines shown on Pupil Page 150. Ask the class to open their books to this page. Let a pupil come to the blackboard and count to 24 by twos on the straight number line, using a stick to touch each point as he says the number. Ask another pupil to count by twos on the winding number line, also touching each point as he says the number. Get the pupils to find page 150 in their books. Ask them to follow in their books by touching the points with their fingers.

Get the children to do the same activity, this time counting by fives, and then counting by tens.

Rub off the number lines and let the class close their books. Let a pupil count by twos as far as he can, without any help from objects or a number line. Let others count by fives; others by tens. If any of the pupils have difficulty with the counting, let them again count objects or count spaces on the number lines on pupil page 150.

<u>Activity 3</u>: Relating counting by groups to repeated addition.

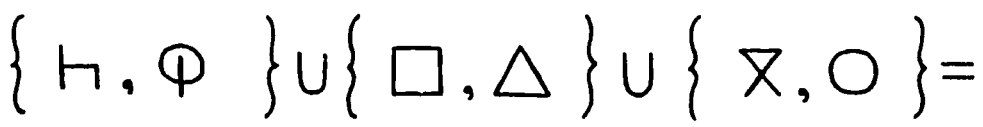
Draw on the blackboard pictures of two sets of objects such as these:



Ask a pupil to draw the picture of the union of the sets on the right to make the picture sentence complete. Then ask for a number sentence telling about these sets.

$$2 + 2 = 4$$

Draw another picture of three sets, each with two members, such as:



Again ask a pupil to complete the picture sentence. Let another child write the number sentence shown by the sets.

$$2 + 2 + 2 = 6$$

Draw four sets, each with two objects, and again ask for the picture to be completed and the number sentence which the picture shows to be written.

$$2 + 2 + 2 + 2 = 8$$

Ask pupils to form their own sets of two objects each at their tables. Let them write number sentences for their joined sets. In this way, get them to see that counting by twos is the same as adding twos.

Ask the class: "If you start at zero and count by twos 3 times, at what number will you stop?" (6) "Is this the same number as 2 + 2 + 2? (Yes) "At what number will you stop if you count by twos 4 times?" (8) "Is this the same number as 2 + 2 + 2 + 2?" (Yes)

Ask these kinds of questions about counting by fives 2 times, 3 times, 4 times, and so on. Also ask these questions about counting by tens. Let the pupils decide that counting by fives 3 times gives the same number as 5 + 5 + 5, and so on. Also ask these questions about counting by threes.

Stage 2: Mixing Sets

Vocabulary: mixing sets, pairs, rows, columns.

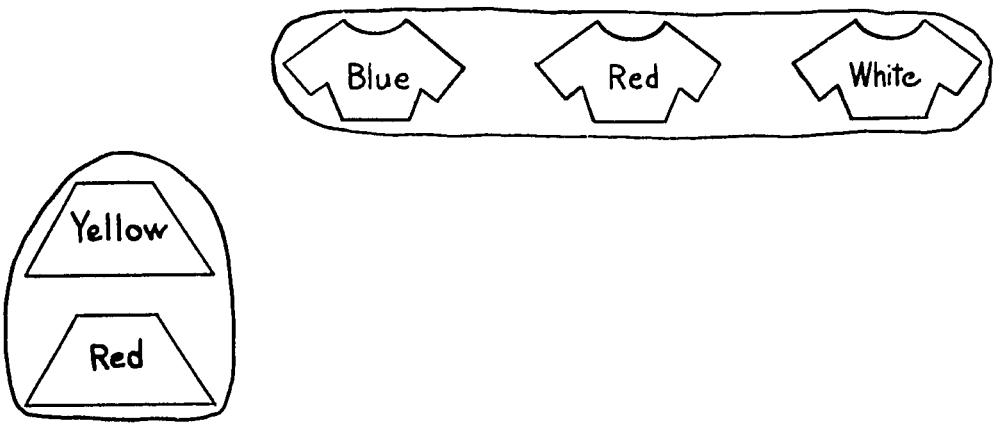
Materials: flat stones of various sizes; sticks of various lengths; seeds or other objects for counting; three differently coloured blouses or shirts; two differently coloured skirts or knickers. (These garments should be large enough to fit over the children's clothes); books of different sizes and colours; pencils of different lengths; tins of different colours; no pupil pages.

Teaching Procedure:

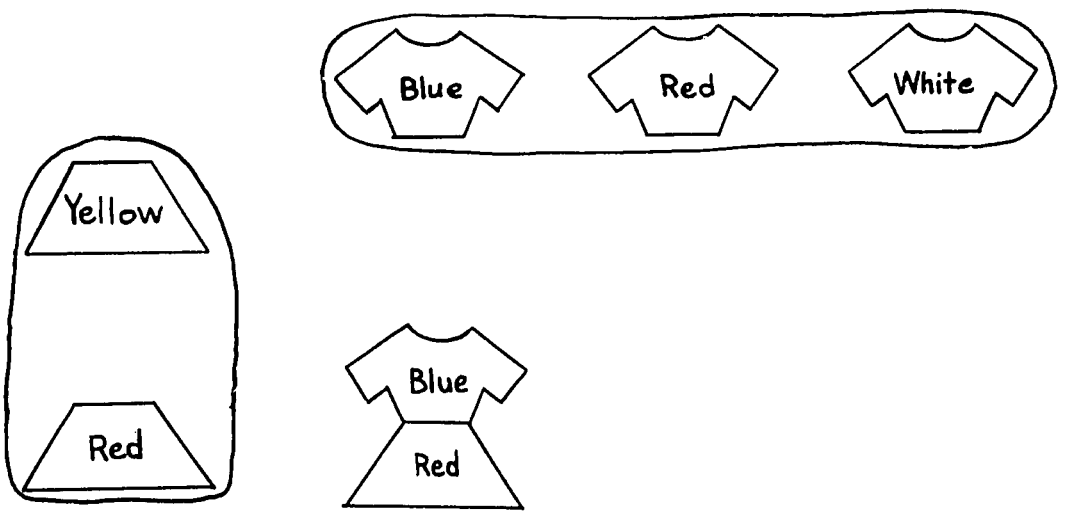
Activity 1: Pairing members of one set with all members of another set.

Say to the class, "You are going to learn a new way of putting sets together." Show them a set of two skirts (or knickers). Use two skirts of different colours; for example, a yellow skirt and a red skirt. Place the skirts in a column on a table for the class to see. Ask them to describe the set and tell its number. (Two)

Show the class a set of blouses (or shirts). Use, for example, a blue blouse, a red blouse, and a white blouse. Place the blouses in a row on your table. Ask the pupils to describe this set and tell its number. (Three) The sets on your table look like this:



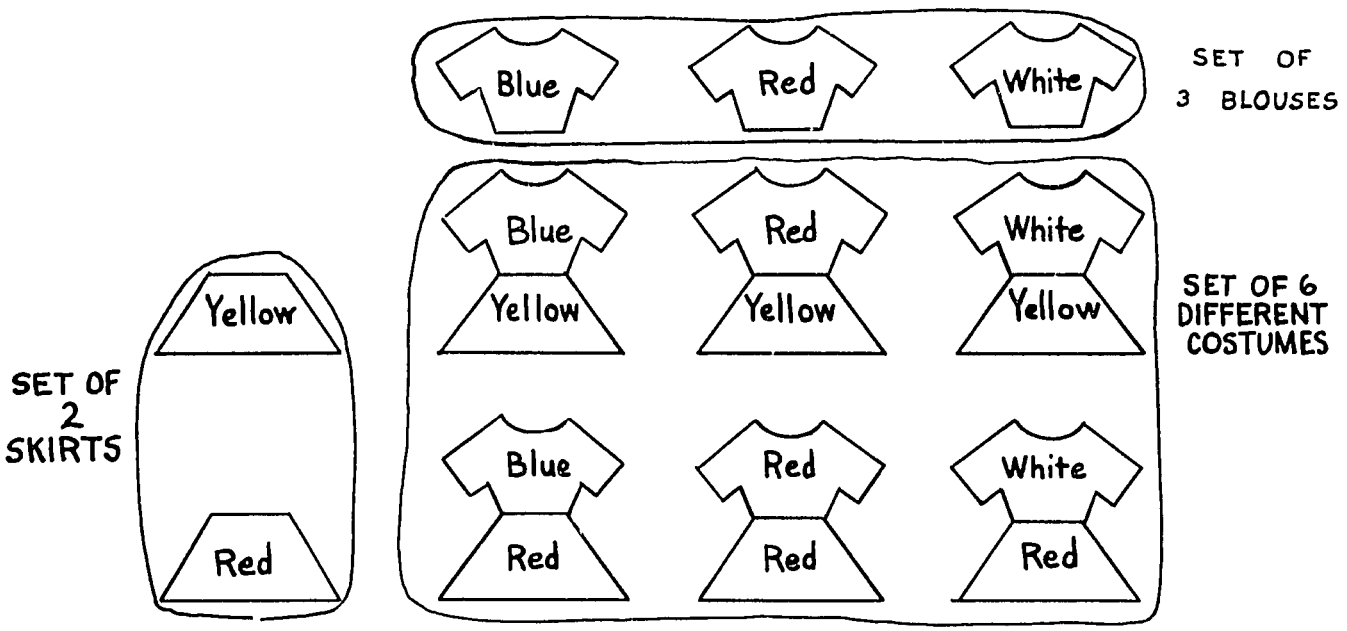
On the blackboard draw a picture of the sets as they appear on your table, using coloured chalk or writing the names of the colours. Say to the class, "Think of all the ways a girl (boy, if you use knickers and shirts) can dress in a skirt and blouse (knickers and shirt)." Invite a pupil to come up and put on a costume. She may choose any skirt and any blouse. Draw on the blackboard the costume she chooses. Use coloured chalk or write the names of the colours, and draw the costume in its position on the chart. For example, if the pupil chooses a red skirt and a blue blouse, you will draw a picture of the costume on the blackboard in this position:



Then tell the pupil to put the skirt and blouse back in the sets on the table.

Invite another pupil to come forward and put on a different costume. Let the class decide whether it is a different costume from the one shown on the blackboard. For example, this child might choose a yellow skirt and a blue blouse. Let the children think about the questions, "Are these costumes different?" Let them talk until they say that the two costumes are different if either the skirts or the blouses are different or if both are different. Then draw a picture of this costume on the blackboard, in its proper place. Again put the clothes back in their sets.

Go on asking pupils to come up and dress in a costume different from the other costumes. Each time, draw a picture of the costume in its proper place on the chart. Stop when a pupil tells you that <u>every</u> possible costume has been made and that all are shown on the blackboard. The picture on the blackboard may then look like this:

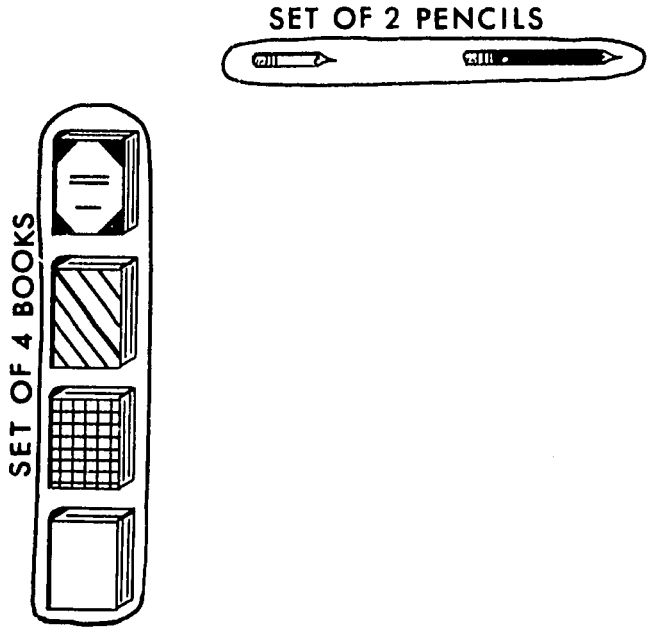


(Notice that skirts of the same colour are in the same row, and blouses of the same colour are in the same column.)

Ask the class to tell how many costumes can be made when two skirts and three blouses are mixed. They will count and say, "Six." Say, "When we mix a set of 2 members with a set of 3 members, we get a new set. The new set is a set of pairs." Explain that in this example, the pairs are costumes. Each costume is a pair made up of a skirt and a blouse. There are 6 pairs (costumes). Ask the pupils to point to and describe each pair (costume) in the drawing on the blackboard. Let them tell why each costume is a pair.

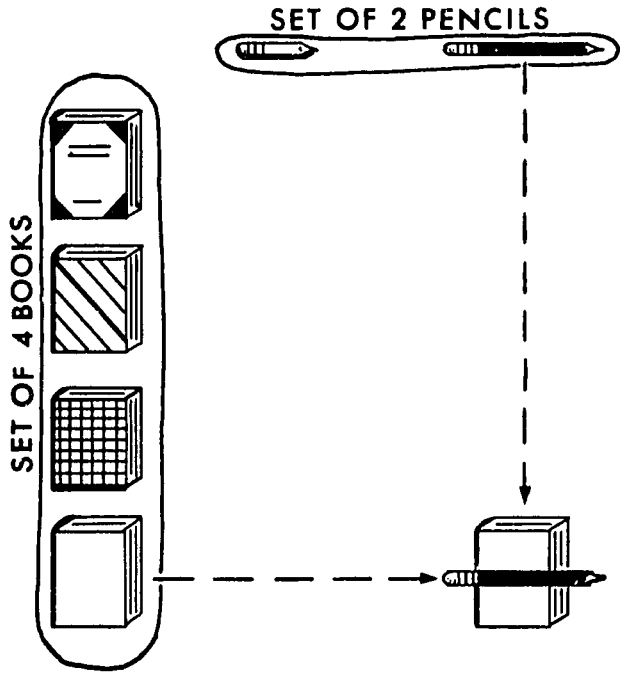
Activity 2: Mixing sets of books and pencils.

Tell the class that they are going to see another example of mixing sets. Show them a set of four books of different colours or sizes and a set of two pencils of different colours and lengths. Place them on a table like this:



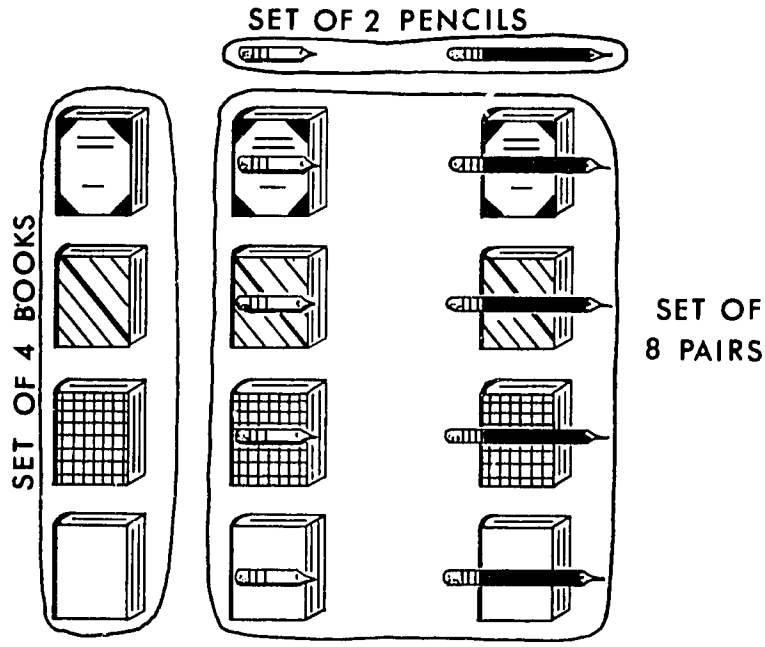
Also make a drawing on the blackboard showing the books and pencils in these positions.

Ask the pupils to put a pencil on a book in every possible combination. One pupil, for example, could put the second pencil on the fourth book like this.



When he does, draw this on the blackboard, showing the book in its proper row and the pencil in its proper column.

Put the book and pencil back in their sets. Ask another pupil to put a pencil on a book to form a different pair. Go on in this way letting the pupils make and draw pairs on the blackboard, until all pairs are made. The blackboard looks like this:



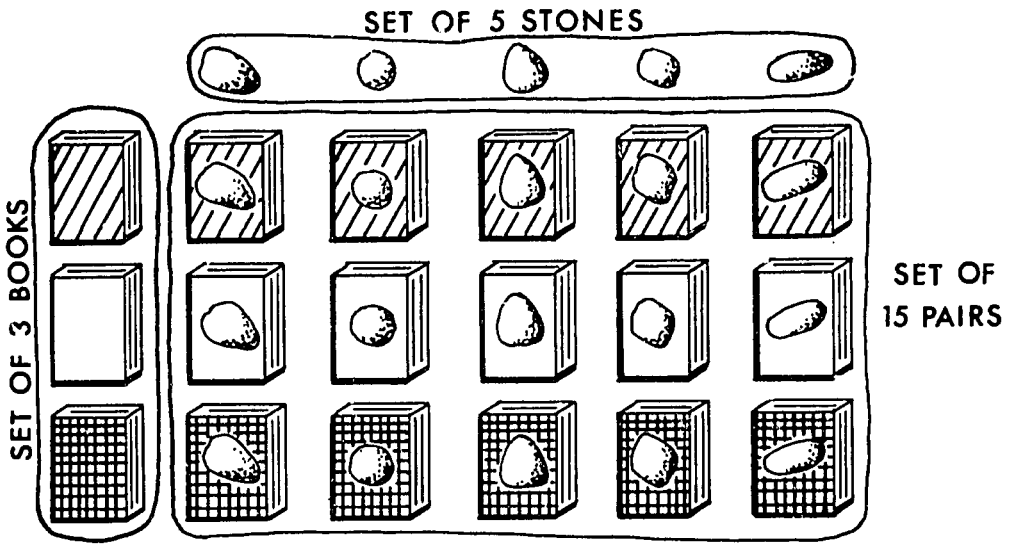
Talk about whether <u>every</u> possible pair of a book and a pencil have been made. Count the number of pairs of a book and a pencil. Get the pupils to say, "A set of four books mixed with a set of two pencils gives a set of eight pairs of a book and a pencil."

Ask, "How many rows are there in this new set of pairs?" (4) "How many pairs are in each row?" (2)

Ask, "Is there an easy way to count the members of this set?" Try to get the answer, "We can count the pairs by twos, because there are two in each row", or, "We can add 2 and 2 and 2 and 2."

Revise the results of mixing a set of two skirts with a set of three blouses. Let the class remember from the picture on the blackboard that there were two rows of costumes, with three in each row. Ask how they can count the costumes. Let the pupils tell how they count the costumes by threes, or how they add 3 + 3.

Arrange 3 books and 5 stones on the table in the same way as you did the books and pencils. Again ask pupils to come forward and mix stones with books to form all possible pairs. Draw pictures of these pairs. The picture on the blackboard will look like this:



Ask, "How many rows of pairs are there? (3) "How many pairs are in each row?" (5) "How many pairs of books and pencils do we have?" (15) "How did you get this number?" ("Counted by fives: 5, 10, 15," or, "Added 5 and 5 and 5.") Let the children talk about what they get when they mix sets. They will say, for example, "A set of 3 books mixed with a set of 5 pencils gives a set of 15 pairs of a book and a pencil." Go on talking of other examples of mixed sets.

Activity 3: Mixing sets.

Separate the class into groups, each with a leader who will draw the pictures of mixed sets on a paper for his group. Give each group some books of different colours and pencils of different lengths. Tell each group to form a set of 2 books and a set of 3 pencils. Ask them to arrange the books in a column and the pencils in a row, as they did in Activity 2. Let each group mix its sets, with the leader drawing each pair on a paper. The leader will report to you when his group has found the number of all the pairs. Let each group answer these questions: How many rows are

there in the mixed set? How many are in each row? Ask them to count by groups in rows to get the number and to add to get the number.

Tell each group to mix a set of 3 books and 3 pencils and find the number of pairs in the new set.

Each time two sets are mixed ask someone to explain what they found out. They may say, for example, "A set of three stones mixed with a set of three sticks gives a set of nine pairs of a stone and a stick." Ask for the number of rows of pairs and the number of pairs in each row and for the total number of pairs in the mixed set. Go on with other sets as needed.

Note to Teacher: By this time the students should know these things:

1. Two sets can be mixed to get a new set.
2. The new set is a set of all pairs of members.
3. A way to show all the pairs in the mixed set is to arrange the pairs in rows and columns.

If some pupils learn faster than others, let the pupils who learn faster mix other pairs of sets. For example, they may mix these sets:

1. Two tins of different colour mixed with four sticks of different lengths.
2. Five stones of different colours and size mixed with two sticks of different lengths.

While these children are working you help the children who learn slowly.

Stage 3: Meaning of Multiplication

Vocabulary: multiply, multiplication, times, multiplication sign

Materials: four bottles and 3 small boxes; sticks and flat stones of various sizes; pupil page 151.

Teaching Procedure:

Activity 1: Using counting to multiply; using the multiplication sign.

Revise the mixing of sets by getting the children to mix a set of 4 with a set of 3 to get a set of 12 pairs. For example, let them mix a set of 4 bottles and a set of 3 small boxes. Choose the bottles and the boxes so the bottles fit into the boxes. Revise ways of telling how many pairs are in the new set.

On the blackboard draw a picture which shows four rows of objects (pairs) with three objects (pairs) in each row. Ask, as you point to the blackboard, "How many rows are there?" (4) "How many objects are in each row?" (3) Get a pupil to count all the objects in all the rows. When he is finished, ask another pupil to count the objects by threes.

Ask, "Can we count the objects in the rows another way?" Try to get the pupils to say, "We can get 12 by adding 3 + 3 + 3 + 3," or, "We can get to 12 by counting by threes 4 times," or, "There are 4 threes."

Revise the mixing of a set of 3 stones and a set of 5 sticks. After a picture is drawn of all the pairs, ask, "How many rows?" (3) "How many in each row?" (5) Again ask the pupils to count all the objects by fives, 5, 10, 15. Ask another child to add to find the number of objects, 5 + 5 + 5. (15) Say., "Three fives equals fifteen. We have another way of saying this. We say, three times five equals fifteen."

Let the class repeat this several times. Ask what number is named. (Three fives are counted: 5, 10, 15. We know 3 times 5 equals 15.)

Repeat this activity for 4 times 2. Let the class answer, "4 twos is 2, 4, 6, 8; so 4 times 2 equals 8. Repeat this activity for 2 times 3, 3 times 2, and 3 times 4. Be sure to draw a picture of the mixed sets each time.

Tell the pupils that we write a sign for "times" like this, "×". Write

"3 × 4"

on the blackboard and say, "3 times 4". Let the class repeat, "3 times 4 " several

times as they look at "3 × 4". Ask the class to write "3 times 2" on their papers, using the sign for times. They write "3 × 2". Go on with other examples as needed.

When you think the children understand the use of the "×" sign, ask them to think of some number sentences which use "×" and you write them on the blackboard. For example, they may say and you write,

$$3 \times 5 = 15$$
$$4 \times 3 = 12$$
$$2 \times 4 = 10$$

Let pupils read each of these. For example, "Three times five equals fifteen." After a pupil reads the sentence, ask him if the sentence is true. If he says "Yes", ask, "Why?" (3 times 5 is 3 fives. 5 and 5 and 5 is 15. So 3 times 5 equals 15.) If this reply is not given, show an example of mixing sets of 3 objects and 5 objects.

When someone reads the sentence, "2 times 4 equals 10," ask him, "Is this sentence true?" If he says, "No," ask, "Why?" (2 fours is 4 and 4. 4 and 4 is 8, not 10.) Let them read other number sentences of this kind and tell whether they are true. Do not use sentences in which the result is greater than 15. Some children may still need pictures of mixed sets. Use a few sentences which are not true.

Say, "When we find 3 times 5 we <u>multiply</u> 3 by 5. Sometimes we read "3 × 5" as "3 <u>multiplied</u> by 5". Let the children say these new words a few times.

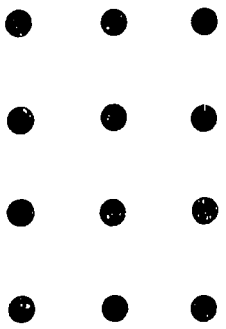
<u>Activity 2:</u> Multiplying three and four.

Say, "We are going to <u>multiply</u> 4 by 3. What is a way to write this?"

$$4 \times 3$$

Explain that no matter what sets of 4 members and 3 members we use, there will be

4 rows in the mixed set, with 3 in each row. Make marks on the blackboard to show this:



four rows, 3 in each row:
4 x 3 in all.

Ask the class to count the marks and complete this number sentence to make it true:

$$4 \times 3 = \square$$

Again, ask for several ways to answer this question: counting by rows of threes, 3, 6, 9, 12 or by adding threes, 3 + 3 + 3 + 3. Let the children say that "4 times 3 is 4 threes."

Call a pupil to the blackboard and ask him to draw some rows of marks to show this.

$$3 \times 4 = \square$$

Help him to draw 3 rows of marks with 4 marks in each row. Let another pupil count the marks by counting in groups or by adding. Ask still another pupil to put a numeral in the box to make the sentence true.

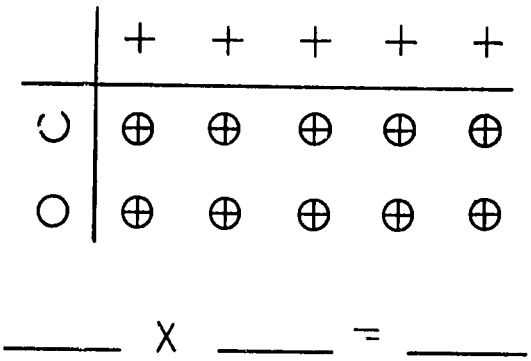
$$3 \times 4 = \boxed{12}$$

Go on with this activity to develop these sentences: $2 \times 5 = \square$, $6 \times 2 = \square$ $3 \times 3 = \square$, $2 \times 7 = \square$.

Activity 3: Mixing sets

Pupil Page 151

Tell pupils to turn to page 151 in their books. Ask them to copy the first diagram. Then tell them to draw all the circles and crosses. Their work will look like this:



Ask them to count the crossed circles by counting in groups or by adding, and to write a multiplication sentence. The sentence should be 2 × 5 = 10.

Let them do the same for each diagram on the page.

Stage 4: Learning Products and Factors

Vocabulary: product, factor

Materials: small objects for counting; about thirty 3" × 5" multiplication sentence cards (See Activity 1); one 12" × 12" cardboard with 25 marks arranged in a 5 × 5 pattern (See Activity 1); inch square numeral cards or blank cards; pieces of paper and string to make number tags (Activity 5); pupil pages 152-154.

Teaching Procedure:

Activity 1: Multiplication sentences with products less than 18

Revise the multiplication in this way. Arrange a set of four sticks and a set of two stones on a table. Let the class give names to the sticks and stones. For example, call the sticks from shortest to longest Mr. A, Mr. B, Mr. C, and Mr. D.

Call the smaller stone Miss X and the larger stone Miss Y. On the blackboard show this diagram:

SET OF 4 STICKS	SET OF 2 STONES X	Y
A		
B		
C		
D		

Ask the pupils to tell you all the names of pairs they get when they mix the two sets. If a pupil says: "Mr. B and Miss X," write "BX" in the proper place in the diagram. If a pupil says, "Mr. D and Miss Y", write "DY" in the proper place. Go on in this way until the diagram is filled with all pairs.

SET OF 4 STICKS	SET OF 2 STONES X	Y
A	AX	AY
B	BX	BY
C	CX	CY
D	DX	DY

SET OF 8 PAIRS

Let the pupils count the pairs by rows. Ask them to tell the number of rows and the number of pairs in each row. Then ask for a way to write this in symbols or in a number sentence.

$$4 \times 2 = 8$$

Repeat this same activity with other sets as needed. Give names to the objects in the sets and use these names in a diagram, showing the name of each possible pair.

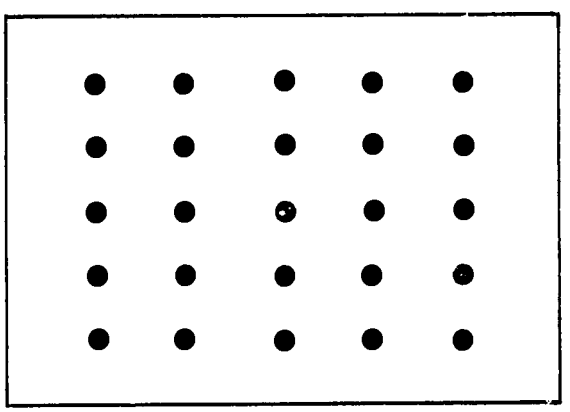
Ask the class to remember as many multiplication sentences which are true sentences as they can. Each time a pupil gives a true sentence, such as "$2 \times 3 = 6$"

write "2 × 3" on one side of a slate, and write "6" on the other side. Then hold up the side showing "2 × 3" and ask what number we get when we multiply these two numbers. When they say "6", turn the slate around to check their answer. For each sentence, let the class suggest sets to be mixed to show the product.

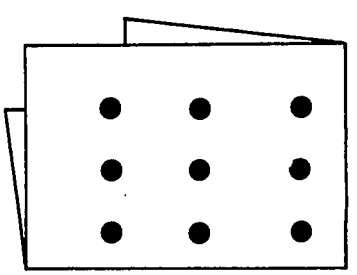
In this way, try to get pupils to discover each of the following.

2 × 3 = 6	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10	6 × 2 = 12
2 × 4 = 8	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15	7 × 2 = 14
2 × 5 = 10	3 × 4 = 12	4 × 4 = 16		
2 × 6 = 12	3 × 5 = 15			
2 × 7 = 14				

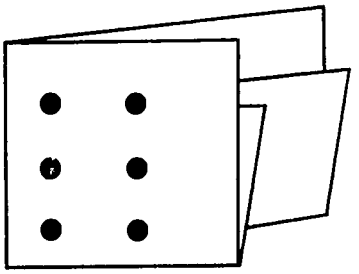
Help the pupils to discover these products by showing them this 12" × 18" diagram.



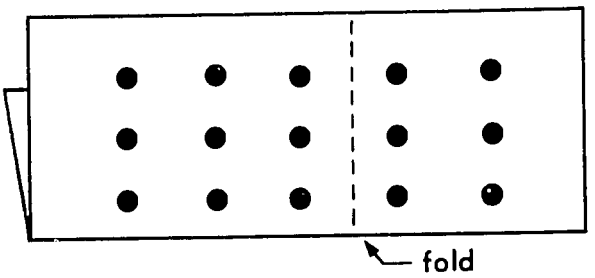
For example, to help the children find 3 × 3, fold the paper twice to show 3 rows of 3 dots.



Let the pupils count by threes to get 9. Turn the paper around to show 3×2. Let them count to get 6.



To help the pupils discover $3 \times 5 = 15$ open the folded paper to show that 3×5 in two parts. One part is 3×3, the other is 3×2.



Let them say that 3×3 and 3×2 are $9 + 6$, which is 15. Let the children show other multiplication sentences by folding this diagram many ways.

Make 3" × 5" sentence cards for each of the results. For example, write $\boxed{3 \times 4}$ on one side of the card; write $\boxed{12}$ on the other side of the card.

If some of these multiplication facts are not suggested by the class, for example, 3×5, form 3 rows of 5 objects each and let the class count them by rows of fives or let them add 5 three times. Go on with this activity as needed.

<u>Activity 2:</u> Using 1 as a factor in multiplication.

Let the pupils think about the meaning of "1×3". Question them until someone says it means "1 row of 3 things," or "1 three." Get them to show this with objects and with a diagram on the blackboard. Help a child write this sentence.

$$1 \times 3 = 3$$

Let them also think about "3 × 1." Try to get the answer: "This means 3 rows with one in each row," or "3 ones," or "1 added three times." Get them to show this with objects and with a diagram. Help a child write this sentence for this multiplication.

$$3 \times 1 = 3$$

Go on in the same way to get the children to write these sentences.

1 × 2	=	2	2 × 1	=	2
1 × 3	=	3	3 × 1	=	3
1 × 5	=	5	5 × 1	=	5
1 × 6	=	6			

Each time get pupils to say, for example, "1 × 4", means "1 row of 4", or "5 × 1" means "5 rows of ones", and then ask, "What does '1 × 1' mean?" Let the class decide that

$$1 \times 1 = 1$$

Put, for example, "5 × 1" on one side of a slate and "5" on the other side. Hold up one side and ask what is on the other. Let the children make sentence cards which show 1 × 1, 1 × 2, ..., 1 × 10, 2 × 1, 3 × 1, ... 10 × 1 on one side and the products of these numbers on the other.

Ask the class what they think 35 × 1 should be. Why? What about 1 × 62? Why?

<u>Activity 3:</u> Multiplication sentences. Pupil Page 152

Ask the class to turn to page 152 in their books. Call on one pupil to tell what number makes the first sentence true. If he cannot answer, let him show 3 rows of 5 seeds on your table and count the seeds by fives.

$$3 \times 5 = \boxed{15}$$

If he answers immediately, let him show his answer by forming 3 rows of five seeds and counting by fives. Let the other children talk about his answer and agree that "15" makes the sentences true.

Ask another pupil to tell what number makes the second sentence true. Follow the plan used above.

Write "12" on the blackboard and ask what number this numeral names. Write "4 × 3" on the blackboard and ask what number this numeral names. Tell the children that when we write "4 × 3 = 12" we mean that "4 × 3" and "12" each name the same number, twelve.

Ask the pupils to look at the other exercises. For each exercise, tell them to make a numeral card or use a card they have and place it on the box to make the sentence true. Go around the class and check the answers. Let the pupils show their answers by drawing rows and columns of marks on their papers.

Let the children talk about the exercise at the top of the right column ($4 \times \square = 8$) Do not discourage any kind of thinking about this problem. Some pupils may experiment with numbers in the box, starting with 1, until a number makes the sentence true. Others may remember that "4 × 2" is another name for "8" and in this way find the answer. Others may use 8 seeds and form 4 equal rows and see there are 2 seeds in each row. Others may think of counting by some groups four times to end at 8 and find they must count by twos. <u>Do not tell</u> anyone the answer. Let each child find the number for himself in any way he can. You ask helpful questions and give encouragement.

Ask each pupil to look at the other exercises and try to find the numbers that make the sentences true. Allow any kind of thinking that leads to the answers. Let them put numeral cards on the boxes to make the sentences true.

Talk to the class about the last sentence. Ask if anyone knows a whole number that can be multiplied by 2 to get 7. Or, ask if anyone can count by twos and end at 7? Or, take seeds and try to arrange them in rows with two in each row. Can you do this? Get the class to agree that there is no whole number we can put in the box

to make the sentence true.

Note: There are addition and subtraction sentences on this page. They are put there to make sure the children are alert and for revision of the sentences.

Activity 4: Using the words "factor" and "product".. Pupil Page 152

Ask the children to find page 152 in their books. Let them again put numeral cards on the boxes to make the sentences true. Ask them to look at the first sentence. Say, "If I multiply 3 by 5, what number do I get?" (15) "What is another name for 3 × 5?" (15) Tell the children we may say this another way. We may say,

"The product of 3 and 5 is 15."

or

"15 is the product of 3 and 5."

Let the class repeat these sentences with you. Get them to call the answer to a multiplication exercise the product of the numbers being multiplied. Let the class and individuals read the results of the other exercises using the word "product". (Note: there are three sentences which have no products. They are not multiplication sentences. Let the children tell this. Do not tell them.)

Ask the class to look again at the first sentence. Tell the children that another way to say this is:

"3 and 5 are factors of 15."

Explain that the numbers multiplied are called factors of the product.

$$\underbrace{3 \times 5}_{\text{factors}} = \underbrace{15}_{\text{product}}$$

Let pupils, in turn, tell for the other exercises which numbers are factors and which are products. Be sure that they always say "factors of ____" and "product of

_____and_____". For example, 5 and 2 are factors of 10. Ten is the product of 5 and 2. (Note: there are three sentences which have no factors. Let the children use the words "addends" and "sum".)

Ask the class these questions:

What is the product of 2 and 2 ?

What is the product of 1 and 5 ?

What is another name for the product of 4 and 2?

What are factors of 12?

Encourage several answers to the last question. The children may answer 3 and 4, 4 and 3, 2 and 6, 6 and 2, 1 and 12, and 12 and 1. Keep asking for other factors until the children name them all.

Activity 5: Factor game.

Play a game in which 12 children are given number names from "one" to "twelve," in any order. Hang number tags around their necks. Let the pupils make the tags from paper and string. Let pupils whose names are pairs of factors of 8 find each other and take hands. Pupil-1 will take the hand of Pupil-8. Pupil-4 will take the hand of Pupil-2.

Call out 12 and let the pairs of factors of 12 find each other. Call out 7. Call out 10. Give the number tags to other children and continue the activity as needed.

Activity 6: Describing sets with multiplication sentences. Pupil Page 153

Give each of five children 3 objects of any kind, for example, pencils, books or seeds. Let them stand in front of the class. Ask another pupil to find the total number of objects held by all of the five children. When he answers, ask him how he found the number. Let the other children decide whether he is right. Say, "You should find the number as quickly as you can." If he counted the objects one at a time, ask whether anyone knows a faster or an easier way to count the objects.

Someone may answer that he can count them by threes, 3, 6, 9, 12, 15. Ask if anyone knows still a faster way to find five threes. A pupil may say, "Since there are 5 threes, this means 5 times 3." Ask if anyone remembers a simpler name for 5×3. If no one suggests the importance of remembering the name, you suggest it.

Continue this activity. Let the class work in groups. Give each of 3 children in a group 4 objects to hold. Let the rest of the group talk about ways of counting the number of objects. (Counting by fours, 4, 8, 12; recognizing that there are 3 fours; or remembering that $3 \times 4 = 12$.)

Do this with 6 children each holding 2 objects; with 4 children each holding 4 objects; and so on. Do not use more than 16 objects in this activity.

Ask the pupils to turn to page 153 in their books. Get them to look at the first picture. Ask "What do you see in this picture?" (A window) How many panes are in the window?" (12) Say, "You find 12 by multiplying numbers." Ask a child to write a number sentence for this problem. He may write either $3 + 3 + 3 + 3 = 12$ or $3 \times 4 = 12$ or $4 \times 3 = 12$. Let the pupils read the sentence and say whether the child is right. Get children to write the other sentences. They should understand that all these sentences tell the same thing about the number of panes in the window.

Let the class talk about each picture on the page in this same way. They should for each write a multiplication sentence telling the total number of objects in the picture.

After they have talked about the sentences, let each pupil write on paper the multiplication sentences which the pictures show.

Activity 7: Remembering the multiplication facts. Pupil Pages 154-155

Hold up the sentence card showing 2×4. Ask, "What is the product of 2 and 4?" After an answer is given, turn the card over to show the product on the back. Talk about why their answer is the same as the number on the back of the card or why it is different. Go on asking questions in this way, using all the sentence cards several times. Change the language to "What is 3 multiplied by 5?" "What is 7 times 2?" "What is the product of 2 and 4?"

Let a child find all the cards which show "6" as a product. Stand them against a wall so that all may see them. Ask a child to tell the factors of 6. If he gives a correct answer, for example, 2×3, turn that card over. Keep asking for other factors of 6 until all the cards are turned over.

Let the children talk about how many multiplication names 6 has. They may give other names for six, such as $5 + 1$, and $8 - 2$.

Follow the same plan for factors of eight, ten, twelve, fifteen, and sixteen.

Let children work in groups with leaders. Give a group points as each pupil of the group gives a correct answer.

Ask the class to turn to page 154 in their books. Let them put numeral cards on the boxes to make the sentences true. If any of the pupils have trouble let them arrange seeds in rows and columns and count the total. (Notice that one of the exercises involves addition.) Tell the children they should remember factors and products.

Let the class find page 155 in their books. Ask them to look at the pictures. Get a pupil to tell a story problem about the first picture. For example, he may say, "A boy gathered some fruit. He had three baskets. He put five fruits in each basket. How many fruits did he gather?" Ask for a multiplication sentence for the problem.

$$3 \times 5 = \square$$

Get a child to complete the sentence to make it true. (15) "How many fruits did the boy collect?" (15) Let the pupils decide whether "15" makes the sentence true.

Repeat this activity for the other pictures on the page. Make up other story problems if the class needs and enjoys this activity.

UNIT 11

FRACTIONS II

Objectives:

1. To revise the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$.
2. To understand that the same fraction has many different names.
3. To introduce the multiples of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$.
4. To compare fractions and decide which are equal.

Background Information for Teachers:

See "Background Information" for Unit 6 and Part 2, "Arithmetic," of your Teachers' Handbook.

Stage 1: Revision of the Fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{6}$

Vocabulary: no new vocabulary.

Materials: five 2" x 18" strips of paper for each child (the strips may be those used in Unit 6); no pupil pages.

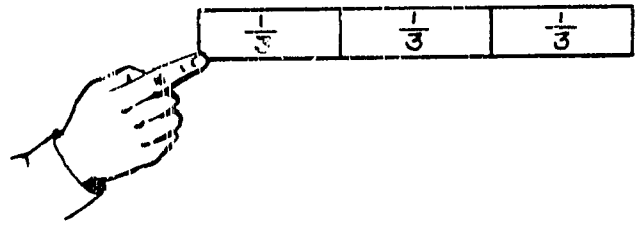
Teaching Procedure:

Activity 1: Revising the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{6}$.

Give each child five 2" x 18" strips of paper. Ask the children to fold one of the strips into two equal parts. Let them write the numeral " $\frac{1}{2}$ " on each part of the strip. Ask them to fold another strip into four equal parts and write numeral " $\frac{1}{4}$ " on each equal part. Let the children follow the same plan and fold the other strips of paper into eighths, thirds, and sixths. Go around and check the children's work. Give help where necessary.

Ask individuals to hold up their strips and show the class a part you name, for

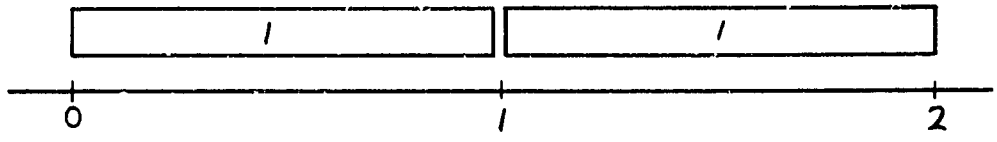
example, one third. Let them use their fingers to trace around the parts as they show it. See below.



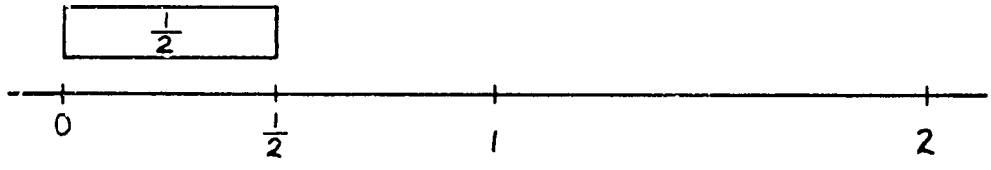
Activity 2: Locating the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{6}$ on the number line.

Put out the 2" x 18" strips of paper the children folded in Activity 1.

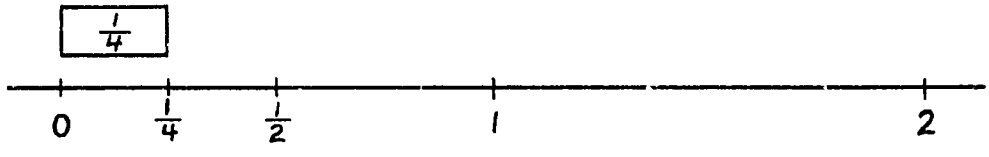
Draw a number line on the blackboard. Place the strip of paper marked "1" along the left part of the number line. Mark the point at the left end of the strip "0" and the point at the right end of the strip "1". Place the same strip along the line again with the left end at "1". Mark the point at the other end "2". See below. Continue marking the points on the line as convenient. Let children use their strips to mark some points.



Ask the children to hold up their strips that are folded and marked in halves. Call a child to place his folded strip along the line with the left end at "0". Let him mark the point at the fold "$\frac{1}{2}$". Let another child place his strip on the line to check the placing of the mark.



Ask the children to hold up their strips that have been folded into fourths. Call one child to place his folded strip along the number line with the left end at "0" and mark the point at the fold "$\frac{1}{4}$". Let other children check the point with their strips.



Get the children to locate the numerals $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{6}$ on the number line in the same way. When the numerals have been located, the number line will look like the one below.

Let children look at the number line and answer such questions as the following:

Which fraction is greater: $\frac{1}{3}$ or $\frac{1}{4}$? ($\frac{1}{3}$)

Which fraction is less: $\frac{1}{6}$ or $\frac{1}{8}$? ($\frac{1}{8}$)

Let children give reasons for their answers. For example, they may say, " $\frac{1}{8}$ is to the left of $\frac{1}{6}$; $\frac{1}{8}$ of a strip is shorter than $\frac{1}{6}$ of a strip; $\frac{1}{6}$ of a strip is longer than $\frac{1}{8}$ of a strip." Let the two children compare the parts of strips marked $\frac{1}{6}$ and $\frac{1}{8}$.

Write incomplete sentences on the blackboard, for example,

$$\frac{1}{6} - - - \frac{1}{4}$$

Ask the children to complete the sentence to make it true using the symbol > or < .

$$\frac{1}{6} < \frac{1}{4}$$

Let the children read each sentence aloud and talk about why it is true.

Let children complete sentences like the following to make them true. Always ask for reasons why a certain symbol is chosen.

$\frac{1}{2} - - - \frac{1}{8}$ $\qquad$ $\frac{1}{3} - - - \frac{1}{2}$ $\qquad$ $\frac{1}{8} - - - \frac{1}{6}$

$\frac{1}{4} - - - \frac{1}{2}$ $\qquad$ $\frac{1}{3} - - - \frac{1}{4}$ $\qquad$ $\frac{1}{2} - - - \frac{1}{4}$

$\frac{1}{6}$ - - - $\frac{1}{2}$	$\frac{1}{3}$ - - - $\frac{1}{6}$	$\frac{1}{4}$ - - - $\frac{1}{6}$
$\frac{1}{3}$ - - - $\frac{1}{8}$	$\frac{1}{6}$ - - - $\frac{1}{6}$	$\frac{1}{3}$ - - - 1.

Stage 2: Introducing the Multiples of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{6}$

Vocabulary: no new vocabulary

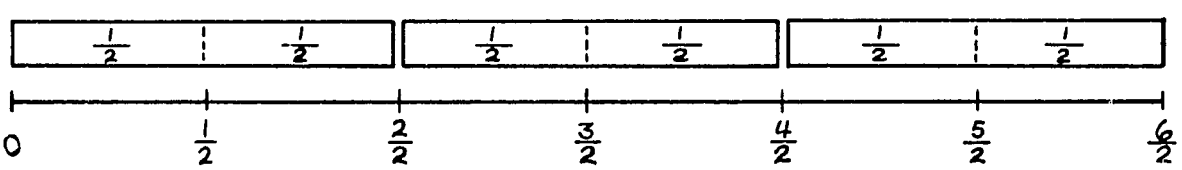
Materials: 2" x 18" strips of paper used in Stage 1; 4-inch square of paper for each child; inch-square symbol (< , >, =) cards; pupil pages 156-157.

Teaching Procedure:

Activity 1: Introducing the idea of $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, and so on.

Briefly revise the meaning of each of the following numbers: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$. Let children find parts of objects in the classroom that might help to illustrate the numbers; for example, one-half of a stick. Ask children to compare pairs of fractions to tell which is greater. For example, ask "Which is greater: $\frac{1}{2}$ or $\frac{1}{3}$? Which is less: $\frac{1}{6}$ or $\frac{1}{8}$?"

Ask three children to come up and hold their strips of paper folded into halves along a number line as shown below.



Let the children count the halves. They will say, "one half, two halves, three halves, ... six halves." Mark the left end of the strip on the left "0"; mark the point at the fold of this strip " $\frac{1}{2}$ ". As the children again count the half strips locate the numerals " $\frac{2}{2}$ ", " $\frac{3}{2}$ " ... " $\frac{6}{2}$ " as shown above.

Note: If the children find it difficult to say two-halves and write "$\frac{2}{2}$", ask them such questions as these: "Into how many equal parts did we fold the papers?" (2) "What do we call one of these two equal parts?" (One-half) "How do we write one-half?" ($\frac{1}{2}$) "What do we call two of the two equal parts?" (Two halves) "How do we write two-halves?" ($\frac{2}{2}$)

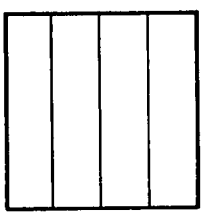
Write the following sentences on the blackboard and get the children to read them and ask whether the sentences are true and why.

$\frac{3}{2} > \frac{1}{2}$ $\frac{3}{2} < \frac{2}{2}$; $\frac{5}{2} < \frac{3}{2}$; $\frac{5}{2} > \frac{2}{2}$.

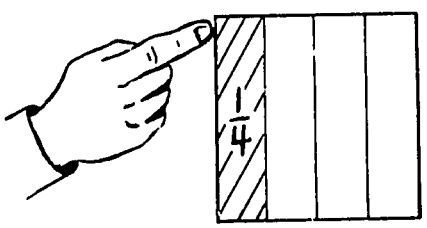
Activity 2: Introducing multiples of $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, and $\frac{1}{6}$.

Give each child a 4-inch square of paper and let him fold it into fourths as shown below. Ask children to unfold and count the number of fourths. (One fourth two fourths, three fourths, four fourths).

Draw a picture of one of the squares on the blackboard and mark the folds with lines.

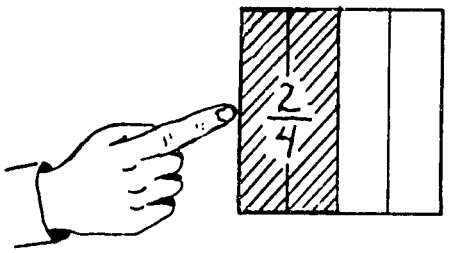


Trace around one-fourth of the square with your finger. Ask a child to write the numeral for that part in the diagram.



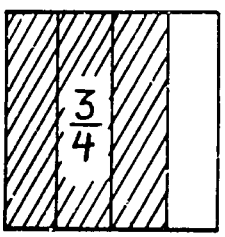
Let the children trace around $\frac{1}{4}$ of their square.

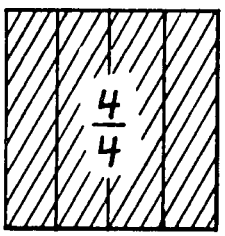
Draw another square, the same size as before, and mark it off in fourths. With your finger trace around two-fourths of the square. See the figure below.



Ask, "How many fourths of the square is this?" (Two fourths) Let a child write the numeral on the part. See the figure above. The child may first write the numeral " $\frac{1}{4}$ ". Since he now needs to write the numeral for "two fourths" help him to erase the "1" of the " $\frac{1}{4}$ " and write a "2." Let the children talk about what $\frac{2}{4}$ means. Ask the children to trace around $\frac{2}{4}$ of their square.

Repeat the same plan with folded squares to help the children understand $\frac{3}{4}$ and $\frac{4}{4}$ and write the numerals; see the figures below of the blackboard drawings.





Use the same plan to help children to understand the meaning of $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{8}{8}$; $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$; $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$.

Let children talk about what $\frac{4}{4}$, $\frac{3}{3}$, $\frac{8}{8}$ and $\frac{6}{6}$ mean. ($\frac{4}{4}$ of a square paper is all of the paper; $\frac{3}{3}$ of a strip of paper is all of the strip.) The children may need objects to show their answers.

Let the children talk about the relationship between fractions such as $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$. Let them count the parts of the paper strips or squares which they folded into fourths as, "One fourth, two fourths and three fourths." They will say that $\frac{3}{4}$ is greater than $\frac{1}{4}$ because one fourth is one of four equal parts and three fourths represents three of four equal parts.

Use this method to compare $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{3}{3}$; to compare $\frac{1}{6}$, $\frac{2}{6}$, ... $\frac{6}{6}$; to compare $\frac{1}{8}$, ... $\frac{8}{8}$.

Write the following sentences on the blackboard and let the children read them:

$$\frac{1}{3} < \frac{2}{3}; \quad \frac{2}{3} < \frac{1}{3}; \quad \frac{3}{3} > \frac{2}{3}; \quad \frac{2}{3} < \frac{3}{3}.$$

Ask whether the sentences are true and why. Let the children point to the parts of folded strips or squares to decide which sentences are true.

Activity 3: Making sentences true by using the symbols =, >, and <. **Pupil Page 156**

Give each child some 1-inch square symbol (>, <, =) cards. Ask the children to turn to page 156 of their book. Read the first incomplete sentence and ask a child to complete it. For example, $\frac{1}{2}$ - - - $\frac{2}{2}$, "One half is less than two halves." Let the class decide whether the sentence is true.

Ask the children to place a symbol card between each pair of numerals on the page to make the sentences true. Some of the children may need to use objects to decide which number is larger or which is smaller.

Activity 4: Deciding whether a sentence is true or false. **Pupil Page 157**

Ask the children to turn to page 157 of their books. Get them to talk about the number line on the page. Let a child read the first sentence aloud and point on the number line to the two numerals in the sentence to tell whether the sentence is true. Ask another child why the sentence is true (or false). Let other children in turn read the sentences, look at the number line and decide which fraction is greater or smaller.

Ask the children to write a few similar sentences in their exercise books.

Stage 3: To Locate the Fractions $\frac{1}{2}, \frac{2}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{1}{4}, \ldots, \frac{1}{6}, \ldots \frac{1}{8}, \ldots$ on the number line

Vocabulary: no new vocabulary

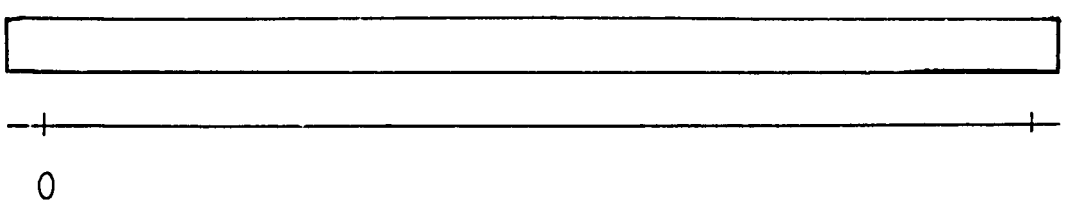
Materials: 2" x 18" strips of paper used in Stage 1; inch-square numeral cards and symbol (>, < , =) cards, inch-square blank cards or paper; six 6" x 24" pieces of brown paper on which number lines may be drawn; pupil pages 158-161.

Teaching Procedure:

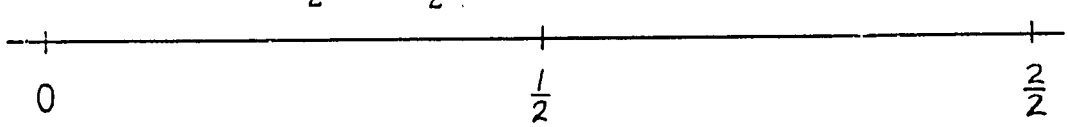
Activity 1: Locating fractions on number lines.

Give each child his 2" x 18" strips of paper which he has folded into halves, thirds, fourths, sixths and eighths.

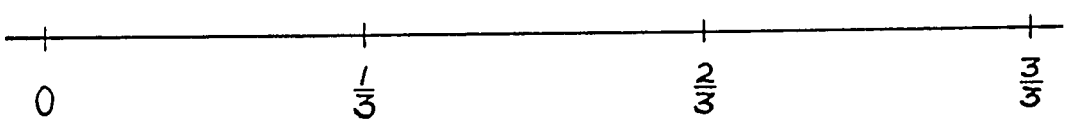
Draw a line on each of the six 6" x 24" pieces of brown paper. Fasten each of the pieces to the wall or the blackboard, one below the other. Hold up a 2" x 18" strip of paper and ask, "Is this the same length as one of the strips you folded?" (Yes) Match your strip with one of the children's strips. Place the strip along the line you drew on the top piece of brown paper and mark the points at the ends of the strip "0" and "1". Do this on every one of the six lines on the wall.



Ask a child to place his strip, that he folded into halves, on the second number line and mark the points $\frac{1}{2}$ and $\frac{2}{2}$.

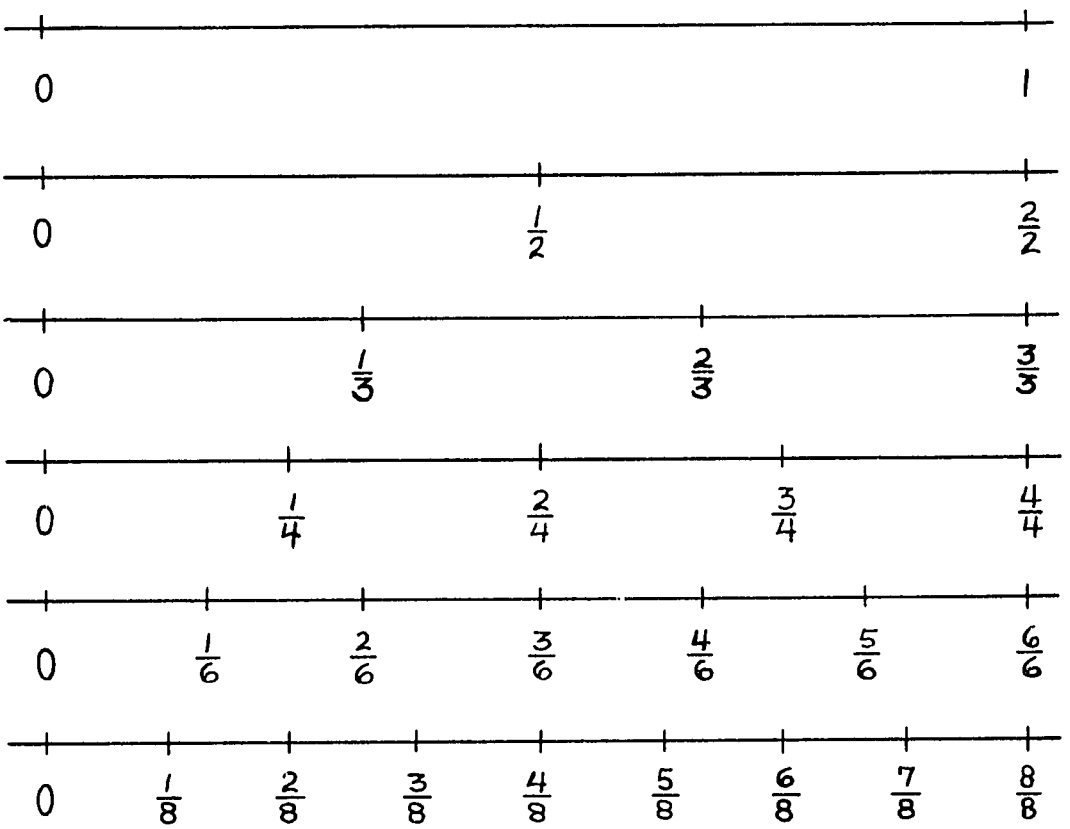


Ask another child to place the strip, that he folded into thirds, along the number line and mark the points $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{3}{3}$.



Let the children mark the fourth line to locate $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$; and the fifth line to locate $\frac{1}{6}$, $\frac{2}{6}$, ... $\frac{6}{6}$ and the sixth line to locate $\frac{1}{8}$, $\frac{2}{8}$, ... $\frac{8}{8}$.

At the end of the activity the six lines on the wall should be marked like this.



Ask the children to compare pairs of fractions. For example, "Which is greater, $\frac{2}{8}$ or $\frac{1}{6}$?" Why? Let the children look at the places where the fractions are on the number lines to help them find which is greater. Ask similar questions about other fractions. Continue as needed.

Let the children talk about how one number line could be made from their six number lines.

Activity 2: Naming the shaded and unshaded parts of wholes. Pupil Pages 158-159

Ask the children to turn to pages 158-159 in their books. Ask them to point to the first picture. Ask, "Into how many equal parts has this square been cut?" (4) "How many of these equal parts are shaded?" (3) "What part of the square is shaded?" ($\frac{3}{4}$) "What part of the square is unshaded?" ($\frac{1}{4}$) Let the children do this for all the shaded parts of the pictures. Give each child several 1-inch squares of paper. Ask them to write the correct fraction on one of the pieces and and place it beside the figure.

Activity 3: Locating fractions on the number line. Pupil Page 160

Let the children turn to page 160 of their books. Give each child a set of 1" x 8" strips of paper that he can fold into halves, thirds, fourths, sixths and eighths. Also give each child his set of numeral cards.

Ask the children to place numeral cards in the boxes on the number lines. Tell them to fold strips to help them choose the correct numeral card.

Activity 4: Making sentences true by using the symbols. Pupil Page 161

Give each child a set of symbol (<, >, =) cards. Ask them to turn to page 161 of their books. Read the first sentence: "One half is greater than three eighths." Ask the children to show > symbol for greater than. Ask whether you have made a true sentence. Why? Let the children complete the sentences by placing a symbol card between each pair of numerals to make true sentences. Go around and check answers. Get the children to read their sentences and tell why the sentences are true.

UNIT 12

MULTIPLICATION II

Objectives:

1. To revise the meaning of multiplication and find products to 18 and beyond.
2. To find missing factors of products to 18 and beyond.
3. To solve story problems by finding products and by finding missing factors.

Background Information for Teachers:

Also see Part 2, "Arithmetic", in your Teachers' Handbook.

Addition and multiplication are the two basic operations of arithmetic. After the pupils learn the meaning of addition, they begin to find sums of numbers. They also find missing addends of sums. For example, if one addend of 15 is 8, the other addend is 7. They know this because $8 + 7$ is 15. In this way, they solve problems in which sets of objects are separated into subsets and they need to find the number of members in one of the subsets.

When a child finds a missing addend of a sum, he is really subtracting numbers. After he finds many missing addends, he learns a symbol and a name for subtraction. For example, if one addend of 15 is 8, the process of finding the missing addend, 7, is called subtraction. It is written: $15 - 8 = 7$.

The children of your class have begun to understand multiplication. They can find the product of two factors. They began this in Unit 10. They also have found some missing factors of products. For example, if one factor of 12 is 3, they know that the missing factor is 4. They know this because they know that 3×4 is 12.

In this unit, the pupils will get a better understanding of multiplication. They will be better able to find products of factors and factors of products. They will solve word problems by finding products and by finding missing factors.

For example, a problem in products is: "There are two children in a family and each child is given six oranges. How many oranges do the children get together?" The answer to this problem is the product of 6 and 2. A problem in missing factors is: "There are 12 oranges to share. Two children share them equally. How many oranges does each child get?" The answer to this problem is the missing factor of 12 since one of the factors is 2.

When a child finds a missing factor of a product, he is doing a division problem, although he does not call the process division. Later (Primary III) when he learns more about products and factors, he learns a symbol and a name for the process. He calls the process of finding the missing factor division. He writes division sentences, for example, $12 \div 2 = 6$.

In this unit, the children do many division problems. They do not use the division symbol or the word "division". When they do use the division symbol later, they will know that division is the process of finding a missing factor.

Stage 1: Revision of the Meaning of Multiplication

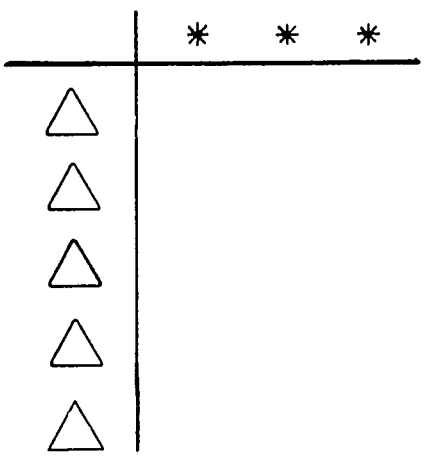
Vocabulary: mixing sets, pairs, product, factor, multiply.

Materials: small objects for counting; no pupil pages.

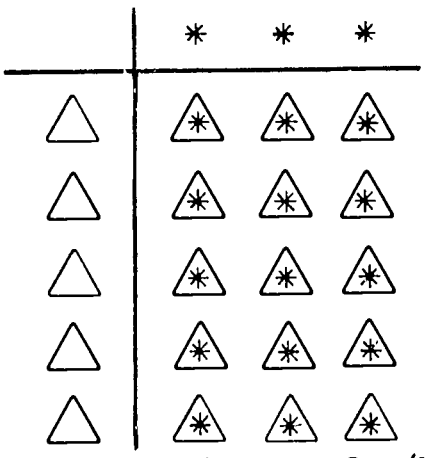
Teaching Procedure:

Activity 1: Revision of the mixing of sets.

Draw this diagram on the blackboard.



Say, "Let us mix two sets." Ask, "How many members are in the set of triangles?" (5) "How many members in the set of stars?" (3) Get the class to tell how to mix the two sets. Ask, "How can we show pictures of all the pairs of triangles and stars we get when we mix these two sets?" Let the pupils tell of the mixing of sets. Draw the pairs on the blackboard as they talk.



Ask, "How many pairs of triangles and stars do we get?" (15) "How did you find the number of pairs?" (Counted by threes: 3, 6, 9, 12, 15; added, 3 + 3 + 3 + 3 + 3) "What does the picture on the blackboard tell about mixing the two sets?" (A set of 5 triangles mixed with a set of 3 stars is a set of 15 pairs of a triangle and a star.) Ask the pupils to tell a number sentence the picture shows. Let them write the number sentence on their papers. You write the sentence on the blackboard:

$$5 \times 3 = 15.$$

Ask the pupils to tell all the ways they know to read the sentence. The answers may be:

1. 5 times 3 equals 15.
2. 5 multiplied by 3 equals 15.
3. The product of 5 and 3 is 15.
4. 5 threes is 15.
5. 5 rows of 3 pairs is 15 pairs.
6. Two factors of 15 are 5 and 3.
7. 5 and 3 are factors of 15.
8. 3 added 5 times is 15.
9. "5 times 3" and "15" are names for the same number.

Finally, ask the pupils to write on their papers a sentence for the mixing of a set of 3 and a set of 5. Let them use a box for the product. Look at the papers and write on the blackboard:

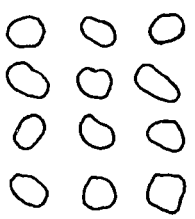
$$3 \times 5 = \square$$

Get a child to tell what number makes the sentence true. (15)

Do this same activity with 3×4, 2×6, 4×4, and so on. Each time draw a diagram to show the mixing of sets. Ask for a sentence, and for ways to read the sentence. Finally, ask for a number sentence to be written. Let the children use a box for the product.

Activity 2: Finding products.

Draw this picture of four rows of stones on the blackboard:



Ask the class how many stones there are. Get them to tell about this in many ways. Some of the answers they may give are:

1. There are four rows of stones. Each row has three stones.
2. There are 4 threes.
3. There are 3 plus 3 plus 3 plus 3 stones.
4. There are 4 times 3 stones.

Ask the class to write on their papers a number sentence the picture shows. Let them use a box for the number of stones. They will write this sentence.

$$4 \times 3 = \square$$

Ask, "What number makes the sentence true?" (12)

Give each group of pupils at least 18 seeds or beans. Write this sentence on the blackboard.

$$3 \times 6 = \square$$

Ask each group to find the number that makes the sentence true. Tell them to find the number by laying seeds in rows on their desks. Go among the groups and help them arrange the seeds like this:

Let one child explain why his group laid the seeds this way. He should be helped to say, " 3×6 means 3 rows of 6 seeds." Ask, "What is the product of 3 and 6?" (18) "Why?" (Because 3 rows of 6 seeds is 6, 12, 18 seeds; or, because $6 + 6 + 6$ is 18.)

In this same way let the groups complete these sentences to make them true:

$2 \times 9 = \square$	$8 \times 2 = \square$	$3 \times 5 = \square$	$4 \times 5 = \square$
$4 \times 4 = \square$	$5 \times 2 = \square$	$4 \times 3 = \square$	$3 \times 5 = \square$

Each time let the class tell why they arranged the seeds as they did and how they counted the seeds to find the product. Write the completed sentences on the blackboard:

$2 \times 9 = \boxed{18}$, $8 \times 2 = \boxed{16}$, and so on.

<u>Activity 3</u>: Solving problems by finding products.

Make up a story problem and tell it to the class. For example, "A boy went to look for eggs. He found seven nests. There were two eggs in each nest. How many eggs did he find?" Ask each pupil to write on his paper a sentence for this problem. Tell him to use a box for the product. Look at the papers and write on the blackboard the sentence that you see.

$7 \times 2 = \square$ or $\square = 7 \times 2$.

Ask, "What number makes the sentence true?" Let the group of pupils use seeds to help them answer the problem. They can imagine the seeds are eggs and arrange them in 7 nests with two in each nest. Help them say, "$7 \times 2 = 14$. The boy found 14 eggs."

Make up and tell another story problem for the children. Go through the plan of letting the children (1) write a sentence for the problem, (2) find the number that makes the sentence true and (3) answer the question in the problem.

The problem may be about 3 baskets with 6 bananas in each basket, or about 4 rooms with 4 people in each room, or about 2 fishermen who each caught 8 fish, and so on.

Repeat this activity several times with different story problems. Write this sentence on the blackboard.

$$6 \times 2 = \square$$

Ask a child to tell a story problem about the sentence. (The answer to the problem should be the number that makes the sentence true.) If a pupil can tell a problem, let the children solve it as they did the other problems. Ask for story problems for other sentences, such as these.

$$3 \times 4 = \square, \quad 5 \times 3 = \square, \quad 2 \times 5 = \square, \quad 9 \times 2 = \square$$

If no one gives a story problem, help the pupils to tell one you think of. Then write a new sentence and again ask for a story.

All through this activity, stress that the problems ask the children to find the <u>product</u> of two numbers. Say, "We call these product problems."

<u>Stage 2: Finding Missing Factors</u>

<u>Vocabulary</u>: missing factors.

<u>Materials</u>: small objects for counting; no pupil pages.

<u>Teaching Procedure</u>:

<u>Activity 1</u>: Sentences with missing factors.

Say, "I am going to give this boy some beans." Count out 12 beans into a pupil's hand, and ask the class this question, "How many rows of beans can he lay on the table with 3 beans in each row?" Let the class think about this question,

and let them talk about the answer given. If they cannot agree on an answer, let the pupil who has the beans lay them out on a table in rows, with 3 beans in each row. Let the class count the number of rows to answer the question. (4) Let a pupil prove this answer by finding the product of 4 and 3.

Separate the class into groups with leaders. Give 12 beans to each group. Ask them if they can tell how many rows of beans they can form with 6 beans in each row. Let them think about this question. Get them to answer without arranging the beans. Ask, "How can you think about multiplication to answer this question?" Help them to say, "There will be two rows, because 2 rows of 6 is 12." Do not tell them this. Let them find it out for themselves.

If the class cannot answer this question, let the groups arrange their 12 beans into rows of 6 beans and count the rows. Ask someone to prove that 2 is the correct number. He may say, "There are two rows because 2 times 6 equals 12," or "2 sixes is 12," or "The product of 2 and 6 is 12."

Ask the class to write a number sentence on their paper for the problem they just solved. You write it on the blackboard

$$2 \times 6 = 12.$$

Let a child read the sentence. Say, "Since 2 times 6 is 12, we know that there are two rows of beans." Rub off the factor 2 and replace it with a box like this.

$$\square \times 6 = 12$$

Ask, "What number makes this sentence true?" (2) Say, "You see that the number 2 makes the sentence true, and 2 is the answer to the question in our problem."

Repeat this entire activity with a similar problem. Give 15 beans to each group. Ask, "How many rows of beans can be formed with 5 beans in each row?" After there is an answer (either without actually arranging the beans, or with the help of the beans), ask the class to write a sentence for the problem. (Ask them to use a box for the number of rows.) Let the children write the number sentence on their paper; you write it on the blackboard:

$$\square \times 5 = 15$$

Ask for the number that makes the sentence true. Say, "We found a missing factor. It answers our question."

Continue this kind of activity. Give each group 16 beans and ask how many rows of 8 beans can be formed. Get them to think of the missing factor of 16 if one factor is 8. Let the class write a number sentence for the problem with a box for the missing factor. Let them find the number that makes the sentence true.

Do this with 16 beans, asking for the number of rows with 4 in each row; and for 18 beans, with 6 in each row; and for 18 beans, with 9 in each row.

The object of this activity is to help children:

1. Recognize problems in which a missing factor is to be found.
2. Write a multiplication sentence for the problem with a box for the missing factor.

Activity 2: Missing factor problems.

Count out 16 beans in your hand. Ask, "How many beans will there be in each row if I put the beans in 4 equal rows?" Let them think about the question. Arrange the beans on the table. Help them to say, "There will be 4 beans in each row, because 4 rows of 4 beans is 16 beans, or because 4 fours is 16." Let the class write a number sentence for this problem on their papers. Let them use a box for the missing factor.

$$4 \times \square = 16.$$

Ask a child to tell what he wrote. Write it on the blackboard. Let the class talk about the sentence you wrote. If it is the correct sentence, they may say, "It says the product of two factors is 16. One of the factors is 4. What is the missing factor?" Let the children decide what number makes the sentence true.

Say, "If I lay these 16 beans in two equal rows, how many beans will be in each row?" Again let each child write on his paper a number sentence for this problem, with a box for the missing factor.

$$2 \times \square = 16$$

Ask, "What does this sentence ask?" They may say all or some of these.

1. 2 rows with how many in each row is 16?
2. 16 beans are shared equally by two rows. How many are in each row?
3. The product of two factors is 16. One of the factors is 2. What is the missing factor?
4. What number makes the sentence $2 \times \square = 16$ true?

Ask, "Are these all ways of stating the same problem?" Let the pupils say they are. Help them also to say that this kind of problem is a <u>missing factor problem</u>.

Do the same activity, this time arranging the 16 beans in 8 rows. Ask the same questions, and again let the class write a number sentence for the problem:

$$8 \times \square = 16$$

Again get the children to notice that this is a problem in which we find a missing factor.

Follow the same plan with 10 beans in 5 equal rows; with 12 beans in rows with 4 in each row; and so on until the class is able to write a sentence for such a problem. Always let the class decide that the problem is a missing factor problem.

<u>Activity 3</u>: Finding problems for sentences.

Write a sentence on the blackboard with a missing factor:

$$5 \times \square = 15$$

Point to the sentence and to a pile of beans on your desk. Say, "Tell me a story problem about rows of beans that the sentence shows." Help the class to say, "The sentence tells of 15 beans put in 5 equal rows." Ask, "What else do we need to know?" (The number of beans in each row) "What is the answer?" (3) "Why is it 3?" (Because 5 threes is 15, or, because 5 times 3 equals 15)

If the children have any trouble with this question, let a pupil arrange the 15 beans as he follows the sentence. Ask questions like these:

1. Does the sentence tell us we must use <u>beans</u>?
2. Could we use 15 bananas instead? (Yes)
3. Could we put the beans in equal piles instead of rows? (Yes)
4. Would the same number make the sentence true? (Yes)
5. What is that number? (3)
6. What do we call this kind of problem? (A missing factor problem)
7. Why is it called a missing factor problem? (Because one factor of 15 is 5 and we want to find the missing factor)

These questions about the sentence should be asked in the order given. Try to get the suggested answers by giving hints or by asking other questions. Do not give the answers yourself.

Repeat this entire activity for the sentences $9 \times \square = 18$, $\square \times 3 = 18$, $7 \times \square = 14$, and so on, as needed.

For contrast, write this sentence on the blackboard.

$$6 \times 3 = \square$$

Ask, "What kind of a sentence is this?" Point to a pile of beans on your desk. Ask a pupil to make up a problem about rows of beans which this sentence shows. Help the pupil say, "We make 6 rows of beans with 3 beans in each row. We find how many beans there are." Ask, "What does 6×3 mean?" (6 threes, or 6 rows of 3) Let a pupil lay out 6 rows of 3 beans. Help the class to understand that the sentence asks for the total number of beans in 6 rows if there are 3 in each row.

Ask as you point to the sentence on the blackboard, "Could we make this problem about bananas or sticks or bottle caps instead of beans?" (Yes) "Would the answer be different?" (No) "Is the answer different if we put the beans in piles or in baskets instead of in rows?" (No) "What do we call this kind of problem?" (A product problem) "Why?" (Because we must find the <u>product</u> of 6 and 3)

Do the same activity with the sentences $4 \times 3 = \square$, $5 \times 2 = \square$ $2 \times 7 = \square$, and so on. Ask the same questions, and get the children to say the sentences are about <u>products</u>.

Stage 3: Product Problems and Missing Factor Problems

Vocabulary: no new words.

Materials: multiplication sentence cards with products to 18; small objects for counting; pupil pages 162-164.

Teaching Procedure:

Activity 1: Revising factors and products. Pupil Page 162

Hold up a sentence card showing "6 × 2". Say, "Two factors are 6 and 2. What is their product?" If there is a correct reply, turn the card around to verify the reply by showing "12". Then ask the pupil who answered how he found the product. (6 × 2 is 6 twos, or 2 added 6 times, and 6 twos is 12) If no one gives the product, ask how "6 × 2" can be shown with objects in rows. Let the class arrange counters on their desks with 6 rows of 2 counters.

Repeat this activity, with variations. Show sentence cards and ask for the product. Ask questions like the following. For example, when you show "3 × 6" you can ask any of these questions:

1. What is the product of 3 and 6?
2. What is 3 times 6?
3. What is 3 multiplied by 6?
4. What is the number that makes this sentence true?
   (Then write on the blackboard: 3 × 6 = $\square$)
5. What is the answer to this product problem?

Show the sentence cards with products and ask for factors. Ask questions like the following, for example, when you hold up a card that shows 18:

1. The product of two factors is 18. What are the factors? (Keep asking for factors until every pair is given and checked.)
2. Here is a product. If one of the factors is 3, what is the missing factor? (Check by turning around the card with "6 × 3" or "3 × 6" on it.)
3. Here is a product. One of its factors is 9. What is the answer to this missing factor problem? (Check by turning around the card with "2 × 9" or "9 × 2" on it.)

Go on showing cards and asking for products or for missing factors. Let the children use counters and form rows whenever necessary to help them. Include all of these:

2×1,	2×2,	2×3,	2×4,	2×5,	2×6,	2×7,	2×8,	2×9,
3×1,	3×2,	3×3,	3×4,	3×5,	3×6,			
4×1,	4×2,	4×3,	4×4,					
5×1,	5×2,	5×3,						
6×1,	6×2,	6×3,						
7×1,	7×2,							
8×1,	8×2,							
9×1,	9×2.							

Also ask for factors of 2, 3, 4, ..., 18. Do not factor the numbers in this order. Mix the product and factor problems, and mix the easy and the difficult.

Ask the class to turn to page 162 in their books. For each exercise ask whether it is a product problem or a missing factor problem. Let the pupils copy the exercises in their exercise books, and find the numbers that make the sentences true. If there are any doubts about the answers, let the children arrange counters in rows and columns.

Activity 2: Sentences for story problems.

Say, "I am going to tell you a story problem. A boy went looking for nuts. He found 3 trees. He found 6 nuts lying under each tree. How many nuts did he find?" Let the class think about this for awhile. Say, "What is a sentence for this problem?" Someone may reply: "$3 \times 6 = \square$". Ask, "Can we use a letter in the sentence instead of a box?" Suggest a letter, for example, n. Then write on the blackboard:

$$3 \times 6 = \square \qquad 3 \times 6 = n.$$

Ask, "What number goes in the box to make the sentence true?" "What number n makes the sentence true?" (18) "Why?" (3 sixes is 18) "What do we call this

kind of problem?" (A product problem) "Why?" ($\underline{n}$ is the product of 3 and 6) "What does the 18 mean in the problem?" (The boy found 18 nuts.)

Say, "I am going to tell you another story problem. Three boys found a tree with 18 nuts under it. They shared the nuts equally. How many nuts did each boy get?" After the pupils think about this for awhile, ask them to give a number sentence for this problem. Use a letter for the missing factor in the sentence.

$$3 \times n = 18$$

Ask, "What number makes the sentence true?" "What kind of problem do we call this?" (A missing factor problem) "Why do we call this a missing factor problem?" (We know one factor of 18 and want to find the missing factor.)

Repeat these activities with some other story problems. Use some product problems and some missing factor problems. Keep the story problems simple and about things that children would ordinarily do or see. The missing factor problems can be about many kinds of equal sharing.

Activity 3: Story problems for sentences.

The object of this activity is to help children relate a problem to its number sentence, and to show how the product problems and missing factor problems differ.

Write this sentence on the blackboard:

$$3 \times 4 = a$$

Ask the pupils to think of a story problem for this sentence. Let them try to tell one, with your help. For example: "A girl packed eggs in a box. There was enough room in the box for 3 rows of eggs with 4 eggs in each row. How many eggs could she put in the box?" Or, "A boy sorted some fruit. He put the fruit in 3 piles, with 4 in each pile. How many fruit did he have?" Let several children make problems.

Get a pupil to give the answer to one of the problems. Let the children agree that this number answers all the problems.

Ask, "What kind of a problem are these?" (Product problems) "Why are they called product problems?" (The product of 3 and 4 is missing.)

Write this sentence on the blackboard:

$$3 \times n = 15$$

Ask the children to think of a story problem for this sentence. Help them with hints, if necessary, until several problems are told. For example, "Three boys shared 15 nuts that they found. How many nuts did each boy get?" "A house has three rooms, and there are 15 people who want to sleep in the house. How many should sleep in each room if there should be the same number in each room?"

Let the children find the answer to the problems. Ask, "What kind of story problem is this?" (A missing factor problem) "Why are they called missing factor problems?" (We know one of the factors of 15 is 3 and we want to find the missing factor.)

Repeat this entire activity for the sentences $3 \times n = 9$, $4 \times 4 = a$, $3 \times 6 = b$, and $4 \times c = 8$. Each time ask for a story problem and emphasize the kind of problem involved. Let the children write the sentence and find the number that makes it true. Let them answer the question asked in each problem.

Activity 4: Story problems for sentences. Pupil Page 163

Let the class turn to page 163 in their books. There are three pictures shown on the page. For each picture let the class tell a story problem.

For the first picture there are many possible stories. For example: Fifteen fish can be shared equally by 3 people, or by 5 people, or put in 3 equal piles, or in 5 equal piles. For each story, let the class write a sentence. Use a letter for the missing number. Let them find the number that makes the sentence true, and tell what kind of problem it is. Get them to answer the question asked in the problem.

The second picture may suggest: How many pencils can be put on each book if an equal number is put on each book? Five children share the pencils and books; how many does each child get?

The third picture may suggest: How many rows of flags can you put on a field with two in each row?, and so on.

Try to get many different stories for each picture. You may suggest addition or subtraction problems.

Activity 5: Story problems for sentences. Pupil Page 164

Let the class turn to page 164 in their books. Ask the children to look at the three pictures. Let them make up story problems for the pictures and also fill in the two factors in the blanks below the pictures. For example, the first picture suggests that 4 boys share 8 mangos. The sentence below it should be completed like this.

$$\underline{4} \times \underline{2} = 8$$

The second picture suggests putting 16 bananas into 4 baskets equally. Then the sentence should be completed like this.

$$\underline{4} \times \underline{4} = 16$$

The third picture suggests three girls each getting an equal number of the 15 dresses shown. The sentence is:

$$\underline{3} \times \underline{5} = 15$$

UNIT 13
METRIC GEOMETRY

Objectives:

1. To revise the idea of a line segment and its endpoints, an angle, a right angle, a triangle, a four-sided figure, a square, and a rectangle.
2. To revise the use of an inch as a unit of length measure and to introduce units of foot and yard.
3. To introduce the idea of the inside of a geometric figure.
4. To compare the insides of pairs of geometric figures (larger than, smaller than or the same size).
5. To compare the inside of a geometric figure to a unit square.

Background Information For Teachers: See "Background Information for Teachers," Primary I, Units 16 and 17, and Part 2, "Geometry and Measurement" of your Teachers' Handbook.

Stage 1: Revision of Geometric Figures

Vocabulary: no new vocabulary

Materials: straight edge; paper for folding; string; nail board; rubber bands; pupil page 165

Teaching Procedure:

Note: In this stage as before, let the children do as much of the work as

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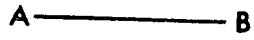
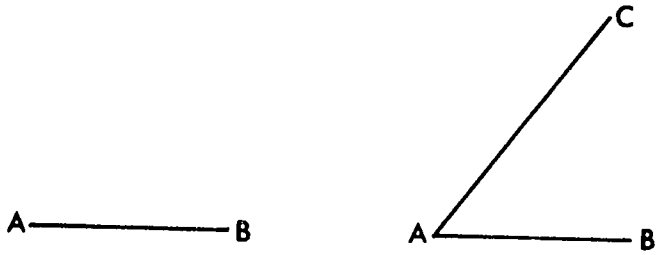
possible. Let them build different geometric figures from line segments. The five drawings below show how they start with a line segment and make five different figures. Notice that each figure is an extension of previous figures.

(1)



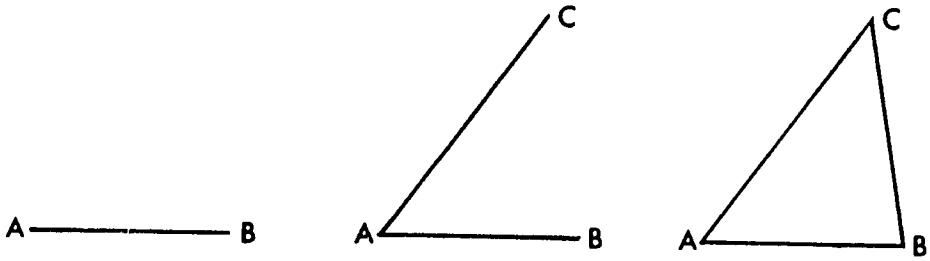
Begin with a line segment. Label its endpoints A and B.

(2)



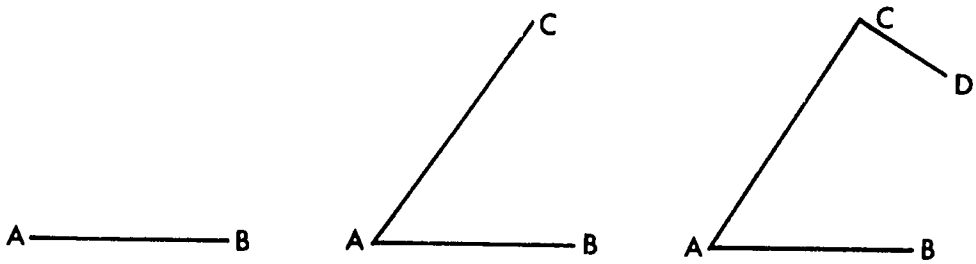
Line segment AC is drawn making an unclosed figure, angle BAC.

(3)



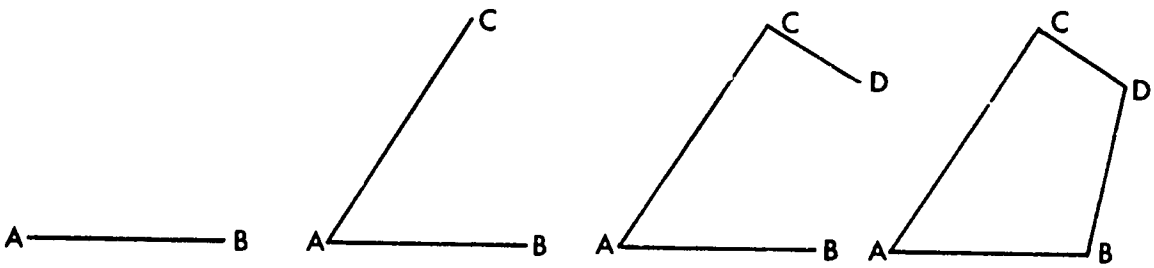
Line segment CB is drawn making a closed figure, triangle ABC.

(4)



Line segment CD is drawn, making an unclosed figure, BACD.

(5)



Line segment BD is drawn making a closed figure, quadrilateral BACD.

Other geometric figures may be drawn using a similar plan.

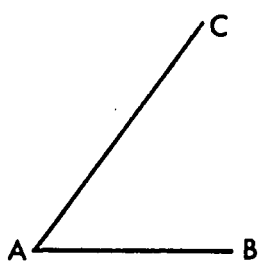
Activity 1: Building up geometric figures from line segments.

Draw an 18-inch line segment on the blackboard. Ask, "What geometric figure is this? (Line segment) Let one of the children point to the endpoints of the line segment. Get another to label the endpoints (A and B). Ask, "What can we name this line segment?" (Line segment AB)

Cut a piece of string 24 inches long. Coil it on the floor so all of the children can see it. Ask, "Does this show a line segment?" (No. It is not straight.) Pick up the piece of string by one end. Lift it up so that the other end is about 1 foot off of the floor. Ask, "Does this show a line segment?" The children will probably say, "No." The weight of the string is not enough to stretch it tightly. If the children say it is a line segment, hold a straight edge next to the string. Ask, "Is the edge of this straight?" Then say, "Is the string straight?" Get them to see that the straightness of the straight edge and the wavy piece of string is not the same.

Hold the piece of string against the blackboard. Slowly stretch the string until it is taut. Move your hands until the taut string fits perfectly along line segment AB. Say, "Now both are straight, both show line segments."

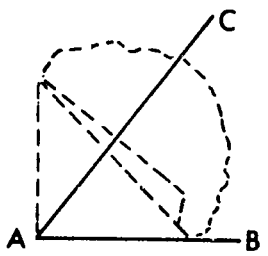
Draw another line segment about 20 inches long. Make one of its endpoints an endpoint of line segment AB forming an angle.



Ask, "How many line segments are drawn on the blackboard?" "Can you name them?" Say, as you run your finger along the second line segment, "Name this line segment." Get the children to see that one endpoint is not labelled. If a child does not suggest a letter name for the endpoint, say, "We will label this endpoint C."

Be sure that the children know that there are two line segments drawn on the blackboard. Let them name the line segments several times.

Ask, "What figure is drawn on the blackboard?" If the children do not say angle, you say, "Is this an angle or is it a four-sided figure?" "Does it have a corner?" "Is its corner square?" "How can we tell whether the corner is square?" If the children do not suggest folding a paper to make a square corner and fit the paper to the corner, you show this method. After you fold a square corner, fit it over the angle on the blackboard, say, "Is this corner square?" (No) "It is not a right angle."



Ask, "What name can we give this angle?" As you ask this question, trace out the angle by running your finger along line segment BA and AC. Get the children to name the angle BAC or CAB.

Ask, "Can you draw line segment CB?" Let the children talk about this. Get one of the children to draw the line segment. Help him hold the straight edge as he draws the line segment. Ask, "Does this new line segment have endpoints?" Get one of the children to point to the endpoints C and B. Run your finger along line segment CB and say, "This is line segment CB." "How many line segments are drawn on the blackboard?" Point to each line segment and get the children to count with you, "One, two, three."

Ask, "What figure is drawn on the blackboard?" Get them to say that the figure is a triangle. Say, as you point, "We name this line segment, AB, this line segment, BC, this line segment, CA; what name can we give the triangle?"

If the children have difficulty doing this, run your finger over the line segments that form the triangle and say, "This is a triangle." "This is triangle BAC. Ask, "Does this triangle have any corners?" Count the number of corners with the children, pointing to each as they count. Ask, "Are any of the corners square corners?" Let the children test each corner with a piece of paper folded into a right angle or square corner.

Repeat this activity, making a four-sided figure instead of a triangle. Make other three and four-sided figures until the children can talk about and name line segments, triangles and four-sided figures. Tell the children to make some triangles and four-sided figures on their papers. Get them to label the endpoints of the line segments and name the figures they draw.

If you have nail boards available (See Unit 9, Primary I), let the children make triangles and four-sided figures by stretching rubber bands around the nails. They can show more than twenty triangles and four-sided figures on a nail board. Make drawings of a nine-point nail board on the blackboard and let the children draw the figures they have made.

Activity 2: Recognizing geometric figures. Pupil Page 165

Note: Pupil Page 165 is a page of eleven geometric figures. This activity helps the children through questions to revise some facts about geometric figures. The list of questions which follow is not a complete one. You think of other questions that help the children recall what they learnt in Primary I, Unit 9, and Primary II, Unit 3.

Tell the children to open their books to page 165. Let them talk of what they see. You ask questions like the following. Also let the children ask each other questions about the figures.

1. How many figures do you see on the page?

2. Do you see any four-sided figures? Point to them.

3. How many four-sided figures are there?

4. Are there any circles? Point to them.

5. How many circles are there?

6. How many triangles do you see? Point to them.

7. How many line segments do you see on the page? (25)

8. Can you give a name for Figure 11?

9. Is Figure 2 a line segment? Why?

10. Do you see any figures on the page that have square corners? (Let them fold paper and fit to the corners of each figure.)

11. What kind of a triangle is Figure 9? (Right-angled triangle)

12. Is Figure 4 a right-angled triangle?

13. Figure 1 and Figure 8 each have four corners and four sides; can you tell how they differ?

14. Figure 1 and Figure 3 each have four corners and four sides; can you tell how they differ?

15. What is a special name for Figure 3?

16. What is a special name for Figure 1?

Stage 2: Measurement of Length, Line Segments, Triangles and Four-sided Figures

Vocabulary: yard, distance around

Materials: one-inch square papers, or cardboards; strips of paper a little longer than 12 inches; strips of paper a little longer than 1 yard; pupil pages 166-167.

Teaching Procedure:

Note: The children learnt in Unit 17 of Primary I to make comparisons of weight and comparisons of length. The idea of a unit of length was developed, and

the children started to use number in the measurement of length. The need for standard units of length was brought out, and the common units of length, inch and foot, were used. The children learnt to use the inch to measure short distances and the foot to measure longer distances. They learnt that 12 inches and 1 foot measure the same distance. The objective of this stage is to revise and extend these ideas and to include the unit, yard.

Activity 1: Revision of unit of length, 1 inch.

Put out two sticks and some inch-square cardboards. Make many inch-squares of paper for the children to use.

Hold up two sticks, one about 14 inches long and the other about 10 inches long. Ask, "Which stick is longer?" "How much longer is this stick than the other one?" "How can you tell how much longer this stick is than the other one?" Let the children talk about this question. Pick up one of the inch square cardboards and say, "I am going to count the number of these that are as long as this stick." Hold the long stick in one hand and let the children help you place inch-square cardboards along the stick. Get the children to count as you do this. Ask, "How many small cardboards are as long as the long stick?"

Hold up the short stick. Repeat the measuring process as before using the inch-square cardboards. Get the children to count the small cardboards that are the same length as the short stick.

Say, "The short stick is 10 of these." "The long stick is 14 of these." Hold up the sticks together so that a pair of ends are even. Run your finger along the part of the long stick that extends beyond the short stick. As you do this, ask, "Do you know how much longer the long stick is than the short one?" Let the children talk about this and the suggestions. Pick up one of the inch squares and fit it at the end of the short stick and along the leftover piece of the long stick.

Get the children to see that the long stick is 4 inch squares longer than the short stick.

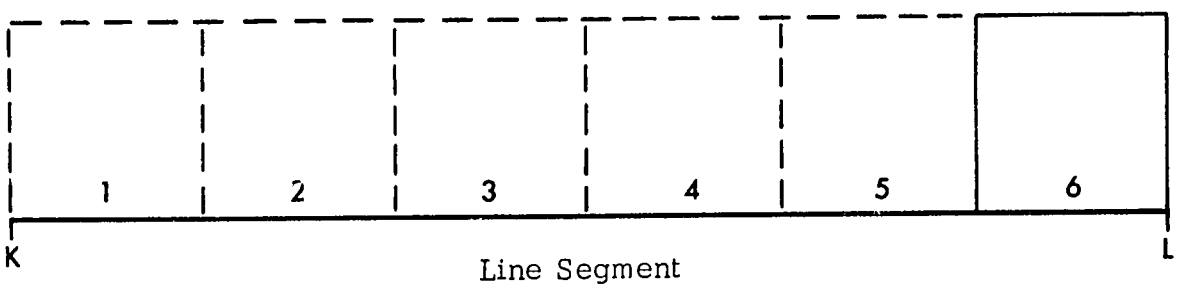
Hold up one of the one-inch cardboards, run your finger along an edge. Ask, "Do you remember the name we gave to this length?" Say, "This is our special small measuring cardboard. We use it to measure small objects. It measures 1 inch. It is 1 inch long." Say that this kind of measure is used over a large part of the world to measure the length of small objects.

Give each group of children some inch-square cardboards. Tell them to measure the edges of their mathematics books. Get them to measure in inches the edges of other objects such as their desks or long sticks.

Activity 2: Measuring the length of line segments in inches.

Pupil Page 166

Tell the children to turn to page 166 in their books. Let them measure the length of the line segments drawn on this page, in inches. This can be done by laying the one inch cardboards along the line segment and counting.



Ask questions like those below as the children measure the length of the line segments in the book. The list which follows is not complete. You think of others. Also let the children ask questions of each other about the line segment.

1. How long is line segment KL?

2. How long is line segment GH?

3. Can you find a line segment that is 3 inches long? (MN)

4. Can you find a line segment that is shorter than CD, but is longer than GH? (KL)

5. Can you find a line segment that is twice as long as AB? (EF)

6. Which is longer, line segment IJ or line segment QR? (IJ)

7. Can you find a line segment that is three times as long as line segment EF? (LK)

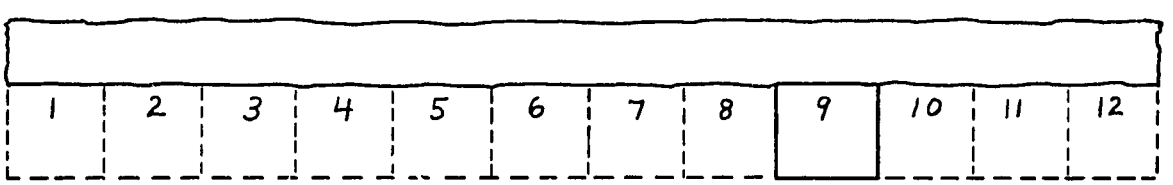
8. Which line segment on page 166 is longer than any of the other line segments? (IJ)

9. Which line segment on page 166 is shorter than any other line segment on the page? (AB)

Activity 3: Making a 1 foot ruler.

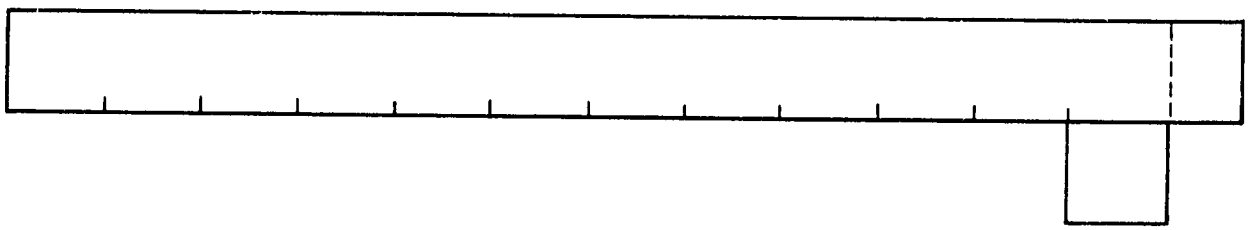
Hold up the special 1-foot stick. Ask, "Do you know the name for this length?" As you say this, run your finger along the edge of the stick. Say, "This is our special stick. It is a 1 foot stick. It is 1-foot long." Tell the children that this kind of measuring stick is used over a large part of the world.

Hold up one of the inch-square cardboards. Ask, "How many inches are as long as 1 foot?" Hold up the special 1 foot stick and say, "We are going to count the number of inches we can put along this 1-foot stick." Let the children place the one-inch cardboards along the edge of the stick. Get the children to count as they do this.



Say, "This stick is one foot long. It is also 12 inches long. There are 12 inches in one foot."

Say, "Each of you can make your own 1-foot measuring stick from paper." Give each of the children a 1-inch by 12 or more inch strip of paper. Let them use a 1-inch cardboard to mark off inches on the paper strip. Each time they move the cardboard 1 inch along the paper strip, tell them to make a mark on the paper strip. When they have marked 12 inches tell them to cut off the extra paper.



Each child now has a 1-foot measuring paper. Let them use the papers to measure distances of more than 5 feet. Get them to put their measuring papers along the edge at your table top. They will say, for example, "The table is more than 5 feet and less than 6 feet long." None of the distances will be exactly a whole number of feet long, say, 6 feet. Get the children to measure many distances in whole feet. Let them work together using as many of their measuring papers as they need.

Let them measure the edge of your table again. Get them to tell the number of whole feet in the distance. Then, ask, "How much is left over?" Let them talk about this question and then ask, "Is it one inch..., 2 inches...?" "How many inches are left over?" Get them to measure the left over part and to say how many inches in the distance.

Activity 4: A new length, the yard.

Let some of the children begin to measure the distance across the room with 1-inch square cardboards. Tell them to put inch squares down on the floor and to count the number of inches as they measure. Let others use their 1-foot papers to measure the distance across the room. When the second group has finished, ask, "Is it better to measure the distance across the room in feet or inches?" (Feet are better; it takes too long when we use inches.) "Is it better to measure your book in

feet or inches?" Get them to see that it is easier to measure some distances in inches and other distances in feet.

Take the class outside. Find two trees that are about 50 feet apart. Ask, "How far is it from this tree to the other tree?" Ask, "Should we measure this distance in inches or feet?" Measure the distance between the trees in feet. Get the children to count the number of feet as they lay their 1-foot strips of paper down.

Go back into the classroom with the pupils. Pick up a strip of paper a little longer than 3 feet. With a 1-foot stick measure off three feet on the long paper strip. Make a mark for each foot. Cut off the extra paper. Say, "How many feet long is this paper?" (3 feet) "This is 3-foot measuring paper. It is 1 yard long. The yard is used to measure some long distances." Let the children make some more yard strips of paper.

Take the children outside again. Get some of the children to measure off the distance between the two trees with the yard strips. Tell the others to count as each yard strip of paper is put down.

Ask, "How many feet is the same as 1 yard?" "How many inches is the same as 1 foot?" "How many inches is the same as 1 yard?"

Activity 5: Measuring the lengths of sides of triangles and rectangles.

Pupil Page 167

Tell the children to open their books to page 167. Get the children to put out their inch-square papers and their 12-inch strip. Let them talk about the seven drawings of geometric figures on the page. They will do the things listed below. If not, question them to complete the activity.

1. Name the kind of geometric figures. (Figure 2 is a triangle.)
2. Name the line segments in each figure. (Figure 5 is made of line segments ST, TU, and US.)
3. Name the geometric figures from the line segments that form the figures. (Figure 5 is a triangle, STU.)

4. Measure the length of the line segments with their inch-squares or their 12-inch strips.

To help the children think about the distances around geometric figures, tell a story and ask questions about a "walking ant." The following list of questions and statements are suggestions. You think of others. (Get the children to find page 167 in their books and find Figure 1.)

1. An ant went walking around triangle ABC. How many line segments did he walk along? How long is line segment AB? (If they do not remember let them measure it again with their 12-inch strips.) How long is line segment BC? How long is line segment CA?

2. The ant starts at point A and walks along line segment AB until he reaches point B. How many inches does he walk as he goes from A to B?

3. When the ant reaches B, he turns the corner of the triangle and walks along line segment BC until he reaches C. How many inches does he walk as he goes from B to C?

4. How many inches has he walked since he started at A?

5. When the ant reaches point C, he turns the corner of the triangle and walks along line segment CA until he reaches A again. How many inches does he walk as he goes from C to A?

6. How many inches did he travel as he walked all the way around the triangle?

Repeat the "walking ant" story for all or some of the other geometric figures on page 167. Each time, let the children tell what the figure is. Let them find the total distance the ant walks around each figure.

Let the children ask questions of each other about the ant and the figures. When the ant story for each figure is finished, talk about these questions:

1. The ant walked around Triangle 1. His friend walked around Triangle 6. Did they walk the same distance?

2. The ant walked around Triangle 1 and then around Triangle 5. Did he walk a longer distance around Triangle 5? (No. Is it 9 inches around?)

3. The ant walked around each of the three triangles. Which walk was longest? Which walk was shortest?

4. The ant walked around each of the seven figures. Which walk was longest? Which walk was shortest?

5. The ant walked around Rectangle 2 and Triangle 5. How many inches did he walk altogether?

6. The ant walks around Triangle 5 and along Line Segment KN. How many inches does he walk altogether?

You can think of other questions. If the children like to talk about the "walking ant", let them make up other questions about the figures.

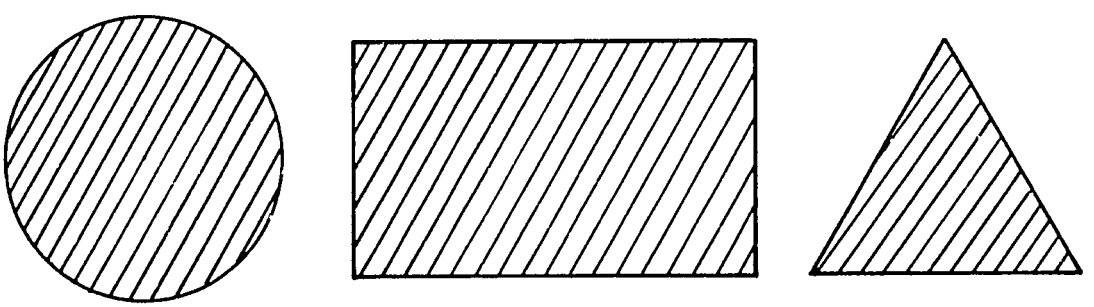
Stage 3: The Inside of a Geometric Figure

Vocabulary: inside of a figure, inside of a circle, a triangle, a rectangle.

Materials: five 8-inch square cards marked as described in Activity 1; circles, triangles and rectangular objects; pupil page 168

Teaching Procedure:

Note: When we draw pictures of geometric figures such as circles, quadrilaterals, and rectangles, a region is formed inside the figure. The drawings below show a circle, a rectangle, and a triangle; the shading shows the inside of the figure.



In this stage, we want the children to become familiar with the inside of geometrical figures. In the activities which follow, let the children use the phrase inside of the circle, or rectangle, or triangle.

Activity 1: The inside of a circle.

Put out some circular objects and coloured chalk.

Trace the edge of a circular object on the blackboard. Run your finger around the chalk tracing. Ask: "What is the name of this figure?" Let the children talk about circles.

Move your hand around the inside of the circle. As you do this, say, "This is the inside of the circle." Say as you act, "I am colouring the inside of the circle with chalk." Again, move your hand over the inside of the circle and repeat, "This is the inside of the circle." Let several of the children move their hands over the inside of the circle and say, "This is the inside of the circle." Let others trace along the circle with their fingers and say, "This is a circle."

Trace the outline of a different circular object on the blackboard. Let the children name it. Again, show them the inside of the circle, and then colour it with chalk. Let some of the children move their hands over the inside of the circle and say, "This is the inside of the circle." Also let them trace along the circle.

Trace the outline of still another circular object on the blackboard. Say, "Show the circle," and "Show the inside of this circle." Let a child colour the inside of the circle. Make several more tracings of circular objects on the blackboard. Let different children show the circles and the insides of the circles and colour the insides of the circles with chalk.

Get the children to make tracings of small circular objects at their desks. Tell them to color the inside of their circles with crayons. Let them hold up their papers to show others how they have made pictures of circles and colored the insides of these circles with crayons. Let them show the inside of their circles with their hands and trace along their circles with their figures.

Activity 2: The inside of a rectangle.

Trace the outline of a rectangular object on the blackboard. With your finger trace along the chalk mark and ask, "What is the name of this figure?" Let the children talk about rectangles and other four-sided figures.

Move your hand around the inside of the rectangle and ask, "What do we call this?" (The inside of the rectangle) Let some of the children show the inside of the rectangle. Let a child colour the inside of the rectangle. Trace other rectangles on the blackboard. Let children come to the blackboard and trace out the rectangle, move their hands over the insides of the rectangles, and colour the insides of the rectangles.

Repeat the same plan with triangles.

Let the children trace small rectangular and triangular objects at their desks. Tell them to colour the inside of their figures with crayons. Let them show the insides of their rectangles and triangles with their hands and trace along the rectangles and triangles with their fingers.

Activity 3: Identifying the inside of geometric figures. Pupil Page 168

Note: There are eleven drawings of geometric figures on page 168. The inside of each figure is described by five different kinds of markings. The inside of each figure is marked either spotted, white, cross-lined, wavy-lined, or black. See below. In this activity, get the children to think about the inside of the figures.

Get the children to describe the inside of each of the figures by asking questions like the following:

1. Can you find a triangle with its inside marked with spots? (Yes)

2. Are there other figures with their insides marked with spots? (Yes) What kind of figure is it? (Rectangle)

3. Is there a circle with its inside marked with spots? (No)

4. How is the inside of circle A marked? (Cross-lined)

5. Is there another circle with its inside marked like circle A? (No)

6. Is there another figure with its inside marked like circle A? (Yes) What kind of figure is this? (Four-sided figure)

7. Are there any other four-sided figures? (Yes, C, F, and J)

8. How is the inside of F marked? (Wavy-lined)

9. Do you see a five-sided figure? (Yes. K)

10. Are there other figures whose insides are marked like K? (Yes, I or E)

11. Are there other figures whose insides are marked like F? (Yes, H)

Ask other questions about the figures and the insides of the figures. If the children ask about Figure E, let them talk about it like this: It is a geometric figure. It is not a circle. It is not a triangle. It is not a four-sided figure...We can show it with a string. It has an inside..."

Stage 4: Comparing the Areas of Geometric Figures

Vocabulary: size

Materials: circular cardboard cutouts; rectangular cardboard cutouts; pupil page 169

Note: In this stage, the children are to compare the areas of geometric figures. The term "area" describes the size of the region of a figure.

The children will use the phrase "inside of the circle", or other figures.

The children used the phrases "longer than" and "shorter than" when they compared lengths.

They used the phrases "heavier than" and "lighter than" when they compared weights.

Now when they compare the insides of geometric figures they will say the inside of one figure is "larger than" or "the same size as" that of another figure.

Activity 1: Comparing the areas of circles.

Make some cardboard circular cutouts of different sizes. Trace the outline of two of the cutouts of different sizes on the blackboard. Colour the inside of the larger circle with coloured chalk. Say, "The inside of this circle is larger than the inside of the other circle." Colour the inside of the smaller circle with chalk. Say as you point to the smaller circle, "The inside of this circle is smaller than the inside of the other circle."

Pick up the smaller cutout and fit it inside the smaller circle on the blackboard. Say, "This cutout fits exactly inside the circle." Take the same cutout and fit it inside the larger circle. Say, "The inside of the cutout is smaller than the inside of this circle."

Put the smaller cutout over the larger one and hold them up, the smaller one in front. Say, pointing to the smaller one, "The inside of the circle this cutout shows is smaller than the inside of the circle shown by the other cutout."

Repeat this activity with another pair of circular cutouts.

Trace the outline of another circular cutout on the blackboard. Right next to it, make a second tracing of the same cutout. Colour the inside of each circle with chalk. Ask, "Which circle has the larger inside?" (The insides of these circles are the same size.) Let the children tell why. Get them to fit the circular cutout over each of the circles to show that the insides are the same size.

Repeat this activity with other pairs of cutouts that are not the same size and some that are the same size. Continue until the children can say that the inside of one circle is larger than, smaller than, or is the same size as the inside of another circle.

Activity 2: Comparing the areas of circles (children's activity). Pupil Page 169

Give each child or each pair of children a two-inch circular cutout. Ask the children to find Page 169 in their books. Let them talk about the figures they see.

Let each child use his cutout to compare the inside of circle A with the inside of circle B. Ask, "Is the inside of circle A larger than the inside of circle B?" Repeat the work with other pairs of circles.

Ask, "Are there any circles on the page that have insides that are the same size?" (A and F) Ask, "Is the inside of circle D larger than the inside of circle B?" Ask, "Is the inside of circle E smaller than the inside of circle B?" Continue as needed to compare the insides of each pair of circles.

Ask these final questions:

1. Which circle on the page has an inside larger than the inside of any of the other circles?
2. Which circle has an inside smaller than the inside of any of the other circles?

Activity 3: Comparing the areas of rectangles.

Prepare a series of rectangular cutouts that have the following dimensions:

5" × 6"	6" × 8"	8" × 10"	9" × 13"
5" × 9"	6" × 10"	8" × 12"	10" × 14"
7" × 11"	8" × 9"	8" × 15"	12" × 16"

Trace the outlines of a large rectangular cutout and a small rectangular cutout on the blackboard. Colour the inside of each figure with chalk. Ask, "Which rectangle has the larger inside?" Let one of the children come to the blackboard and point to the picture of the rectangle that has the larger inside. Ask him to show why with the cutouts.

Rub out one of the rectangles and trace the outline of a different rectangular cutout on the blackboard. Colour the inside of this figure with chalk. Again, let the children decide which rectangle has the larger inside. Let them say, "The inside of this rectangle is larger than the inside of that rectangle."

Repeat this activity until the children can decide which of two rectangles has the larger inside.

Trace the outline of a 12" × 8" rectangle on the blackboard. Say, "Find a rectangular cutout on my table that has a smaller inside than the inside of the rectangle on the blackboard?" When one of the children is successful, start a pile of cutouts that are smaller than the one on the blackboard. Ask, "Can someone find a cutout that shows a rectangle whose inside is larger than the inside of the rectangle on the blackboard?" When one of the children is successful, start a pile of cutouts that show rectangles whose insides are larger than the inside of the rectangle on the blackboard. Hold up another cutout. Ask, "Does this cutout show a rectangle that has a larger or smaller inside than the inside of the rectangle on the blackboard?" Let a child place it in the proper pile. Continue this procedure until all of the cutouts have been compared. At some time you will pick up the cutout that shows a rectangle that has an inside that is the same size as the one shown by the figure on the blackboard. When this happens, let the child who discovers this start a third pile of cutouts.

Let the children, in turn, pick up each cutout and show that it is in the proper pile. A child should say for each, for example, "This cutout shows a rectangle. It has a larger inside than the figure on the blackboard."

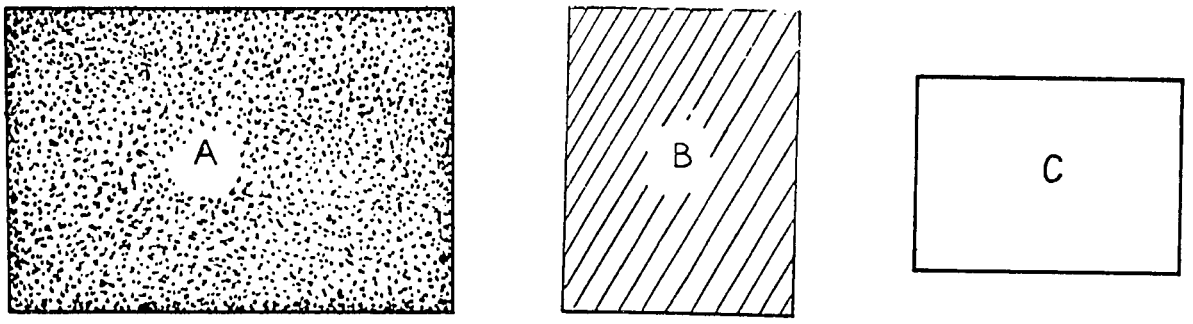
Stage 5: Measuring the Areas of Rectangles

Vocabulary: largest, smallest

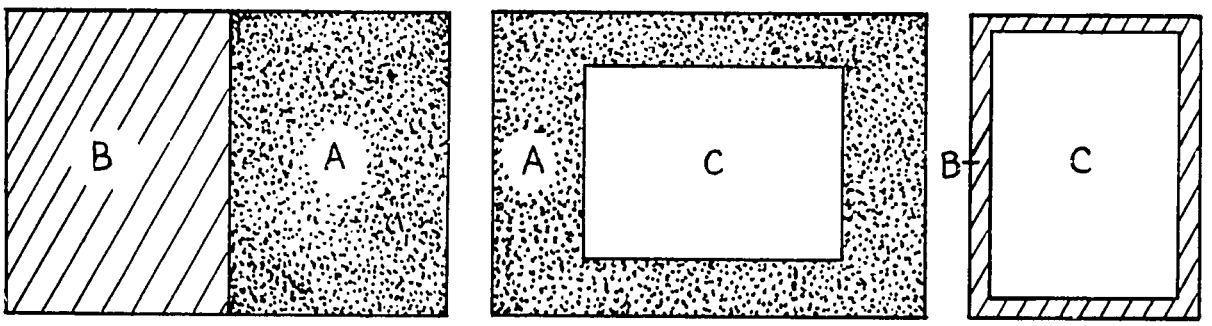
Materials: eight rectangular cutouts (See Activity 1); many rectangular cutouts (See Activity 2); eight 3" × 5" cards (See Activity 3); one-inch square cardboards or papers; pupil page 170.

Teaching Procedure:

Note: The purpose of this activity is to help the children compare the insides of two rectangles. The rectangles we compare are either the same in all dimensions; the same in one dimension; or, one rectangle is larger in both dimensions than the rectangle to which it is compared. The three rectangles in the drawing below illustrate this point:



When any pair of models of these rectangles are fitted one over the other, it is clear which one has the larger area. The drawings below illustrate this point.



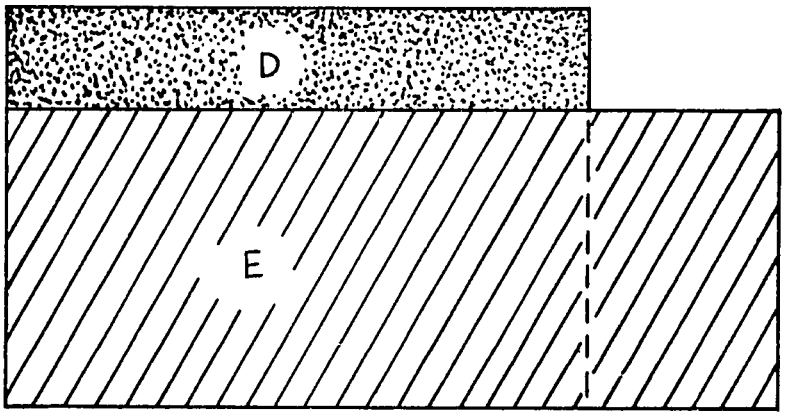
Activity 1: Comparing the areas of rectangles.

Prepare two rectangular cutouts of each of the following dimensions:

Cutout D:	8" × 12"	Cutout F:	4" × 24"
Cutout E:	6" × 16"	Cutout G:	9" × 11"

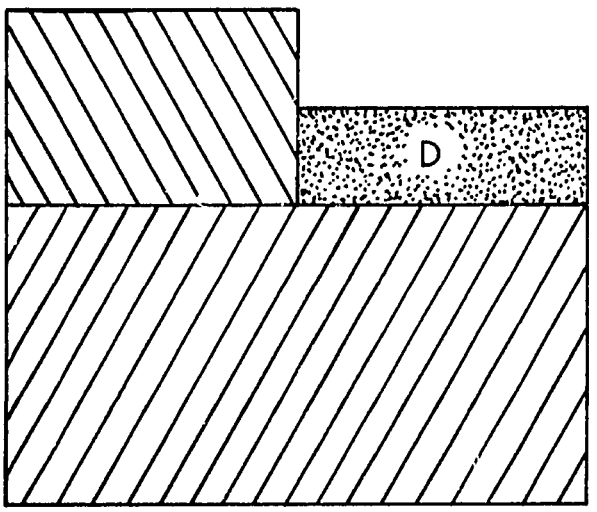
Trace around cutout D on the blackboard. Right next to it, trace around cutout E. Colour the inside of each rectangle with chalk. Ask, "Which of these rectangles has the larger inside?" One of the children may say, "We can tell if we fit cutout D over rectangle E", or "We can tell if we fit cutout E over rectangle D." If they do not suggest this, you fit cutout E over rectangle D on

the blackboard. The drawing below shows the result.

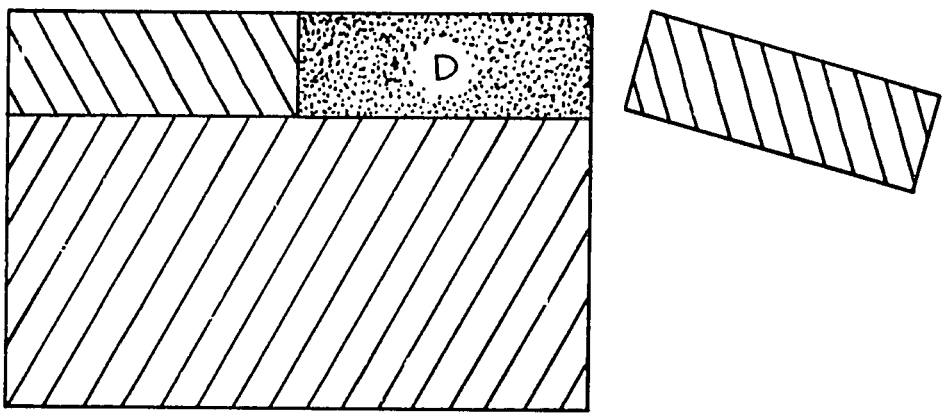


Ask, "Which do you think has the larger inside?" Let them talk about this and guess which has the larger inside. Let them point out that part of D sticks out above E and that part of E sticks out to the right of D. Let a child run his finger over the extra part of E. Let another child mark the extra part of E with a pencil, along the edge of D. Note the dotted line above. Cut off the extra part of E with scissors.

Fit the extra part of cutout E over the extra part of D on the blackboard as shown below.



Ask, "Can you guess which one has the larger inside, D or E?" Let the children talk about this. If they guess, ask how they can tell. Let the children decide what to do next. Point to the extra part of D that is not yet covered by part of E. Run your finger along the extra part of E. Mark off the extra part of the small piece of E with a pencil, along the edge of D. Cut this piece off. Let a child fit the last piece of E over the part of D that is not covered. Ask, "Which rectangular cutout has the larger inside, D or E?"



If the children have any difficulty "visualizing" what was done, put cutout E together and go through the procedure again.

Repeat the procedure to compare the areas of other pairs of rectangular cutouts. The inside of rectangles D, E, and F are the same size. The inside of rectangle G is a larger area than the others.

<u>Activity 2:</u> Comparing the areas of rectangles by fitting. Pupil Page 170

Give each child or group of children five rectangular cutouts, of these dimensions:

2" × 6"	3" × 3"	3" × 4"
2" × 4"	$2\frac{1}{2}$" × 3"	

Let the children look at the rectangles on page 170 of their books. Ask, "Can you fit your paper cutouts over the rectangles?" Let them experiment to find out that each paper cutout fits on exactly one of the figures. Give them help when they need it. Say, "You have a paper cutout which fits each of the figures." "Mark your paper cutouts A, B, C, D, or E so that you know each figure matches in book."

Ask, "Which figure do you think is larger, A or B?" (Hold up your book and point.) Let the children pick up cutouts A and B and experiment with fitting. Let them cut the cutouts to help them fit the cutouts on the rectangles in the book. Help the children who need help. You may need to show the whole class how to fit a cutout on a figure. Say, "The insides of rectangle A and rectangle B are the same size." "This paper cutout B fits exactly inside rectangle A."

Get the children to find whether the inside of rectangle C is larger than the inside of rectangle D? Let the children cut and fit the rectangular cutout on the other and then decide.

Continue as needed with other pairs of rectangles. End with these problems:

(1) Is the inside of any rectangle on page 170 larger than the inside of rectangle B?

(2) Is the inside of any rectangle on page 170 smaller than the inside of rectangle E?"

<u>Activity 3:</u> Measuring areas of rectangles with small squares. Pupil Page 171

Give each child 20 one-inch paper squares. Get the class to find pupil page 171 in their books. Let the pupils talk about the figures on the page. Say, "Find a figure whose inside can be covered exactly with one of your small squares?" "How many small squares does it take to cover the inside of rectangle F?" Let the children cover the inside of each rectangle with squares. Then ask, "Is the inside of any rectangle larger than the inside of rectangle H?"

After the children have covered all of the rectangles and counted the number of small squares used for each ask, "The inside of what rectangle is larger than any other rectangle on the page?" (G) "How many small squares does it take to cover the inside of this large rectangle?" "Which rectangle has a smaller inside than any of the others?" (D) "How many small squares does it take to cover the inside of this rectangle?" (1)

Make eight cards, about 3 inches by 5 inches. Write the letters A, B, C, D, E, F, G, and H on the cards. These letters correspond to the letters used for the rectangles in the book.

A	B	C	D	E	F	G	H
3	6	5	1	4	2	15	9

Ask the children to tell you the number of squares used to cover the inside of rectangle A. Write the number down on the A-card, and stand it up where the children can see it. Do this for eachcard. Then ask, "Which figure has a larger inside than any of the others?" When they say, "G", move this card to the far left. Then ask, "Which figure has a smaller inside than any of the others?" When they say, "D", mover the D-card to the far right. Ask, "Which figure has an inside that is larger than all of the figures other than G?" When they say, "H", move the H-card to the far left, and to the right of the G-card. Continue asking questions of this kind until all of the cards have been placed in order acording to the size of their insides.

After all the cards have been arranged in proper order (G, H, B, C, E, A, F, D) let the children say as you question, "The inside of rectangle G is larger than the inside of any of the others; it is the largest." Say, "The inside of rectangle D is smaller than the inside of any of the others; it is the smallest."