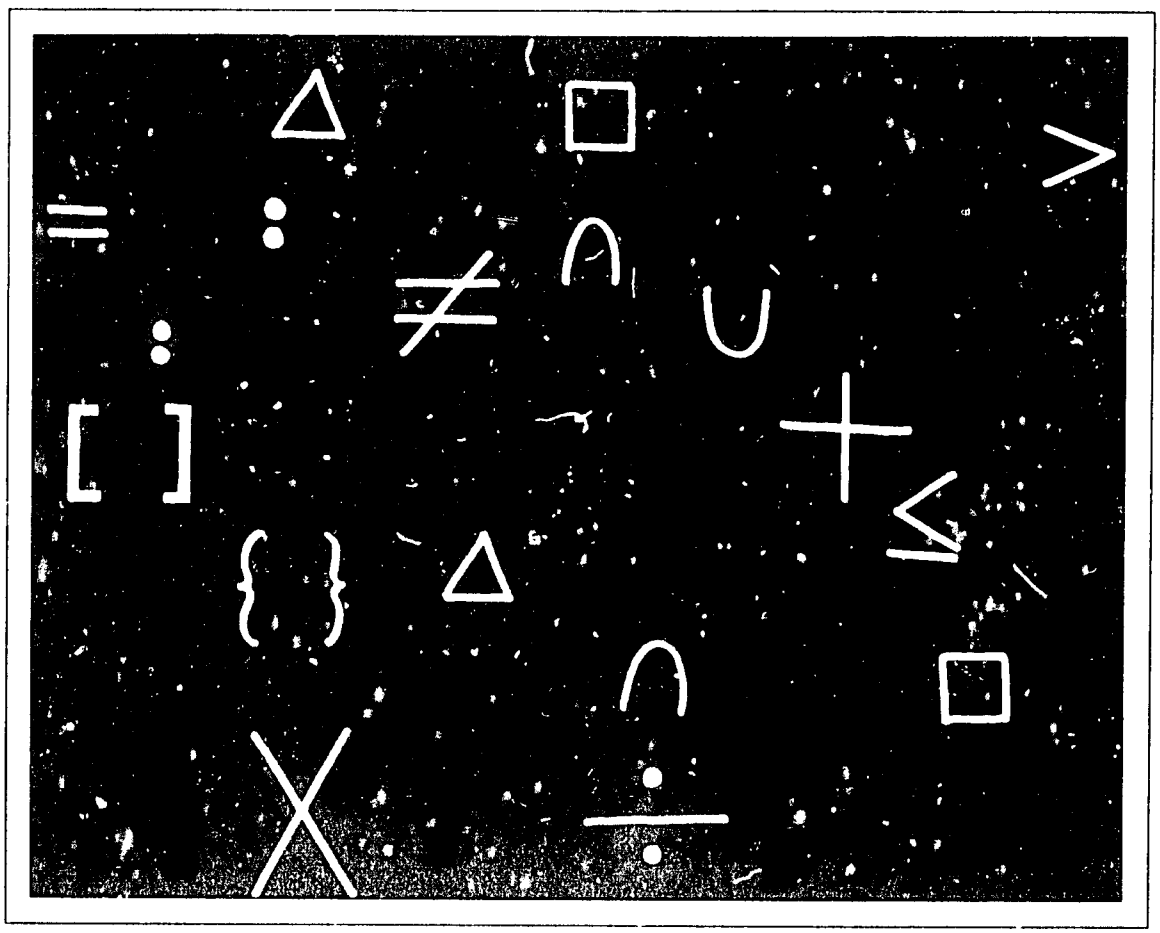
PN-AAT-093 40647

PRIMARY FOUR

teachers'



Entebbe Mathematics Series

preliminary edition

S R A

Educational Services Incorporated
Newton, Massachusetts, U.S.A.

ENTEBBE MATHEMATICS SERIES

Primary One
Revised Preliminary Edition
Pupil Book: One volume
Teachers' Guide: Two volumes

Primary Two
Preliminary Edition
Pupil Book: Two volumes
Teachers' Guide: Two volumes

Primary Three
Preliminary Edition
Pupil Book: Two volumes
Teachers' Guide: Two volumes

Primary Four
Preliminary Edition
Pupil Book: One volume
Teachers' Guide: One volume

Primary Five
Preliminary Edition
To be written at 1966 Workshop

Entebbe Mathematics Teachers' Handbook, Primary I-III, Preliminary Edition

FIVE YEAR COURSE

Secondary One
Preliminary Edition
Student Text: One volume
Teachers' Guide: Three volumes

Secondary Two
Preliminary Edition
Student Text: Three volumes
Teachers' Guide: Three volumes

Secondary Three
Preliminary Edition
Student Text: Algebra—One volume
Geometry—One volume
Teachers' Guide: Algebra—One volume
Geometry—One volume

Secondary Four
Preliminary Edition
Student Text: Algebra—One volume
Geometry—One volume
Teachers' Guide: Algebra—One volume
Geometry—One volume

Secondary Five
Preliminary Edition
To be written at 1966 Workshop

FOUR YEAR COURSE

Secondary C One
Preliminary Edition
Student Text: Algebra—One volume
Geometry—One volume
Teachers' Guide: Algebra—One volume
Geometry—One volume

Secondary C Two
Preliminary Edition
To be written at 1966 Workshop

Basic Concepts of Mathematics, An Introductory Text for Teachers, Preliminary Edition

Volume I —*Structure of Arithmetic*—Chapters 1-21
Volume II —*Structure of Arithmetic*—Chapters 22-46
Volume III—*Foundations of Geometry*—Chapters. 47-57
Volume IV—*Measurement, Functions and Probability*—Chapters 58-68

Teachers'
Guide

ENTEBBE MATHEMATICS SERIES

Primary Four
Preliminary Edition

Prepared at the 1965
Mombasa Mathematics Workshop

Published for the
AFRICAN EDUCATION PROGRAM
of
EDUCATIONAL SERVICES INCORPORATED
Newton, Massachusetts
by
SCIENCE RESEARCH ASSOCIATES, INC.
a subsidiary of International Business Machines Corporation
Chicago, Illinois

Acknowledgements

The Teachers' Guide and the Pupil Book for Primary Four of the Entebbe Mathematics Series were prepared at the 1965 Mombasa Mathematics Workshop. The workshop was a part of a programme of curriculum development of the African Education Program of Educational Services Incorporated.

Funds for the Program have been provided by the United States Government through the International Cooperation Administration, the Agency for International Development, and the Department of State; by the Ford Foundation; and by the Louis and Pauline Cowan Foundation, Inc. The Primary Four books are being published pursuant to A.I.D.-financed contract No. AID/res-21 with Educational Services Incorporated.

The 1965 Mombasa Workshop was directed by Professor W. T. Martin of the Massachusetts Institute of Technology, Cambridge, Massachusetts, and Mr. John Oyelese of the University of Ibadan, Ibadan, Nigeria.

The books for Primary Four of the Entebbe Mathematics Series were written by the following:

Chairman: Professor Clarence Hardgrove
Northern Illinois University
De Kalb, Illinois

Mr. Muhamed Bashraheil
Bububu School
Zanzibar, Tanzania

Professor Edwin F. Beckenbach
University of California
Los Angeles, California

Professor George F. Feeman
Williams College
Williamstown, Massachusetts

Professor Vincent Haag
Franklin and Marshall College
Lancaster, Pennsylvania

Professor Shirley Hill
University of Missouri
Kansas City, Missouri

Mr. Julius E. Jonah
Prince of Wales School
Freetown, Sierra Leone

Mr. Frank Kazembe
Education Offices, Inspectorate
Limbe, Malawi

Mrs. Mary Neville
Ministry of Education (USAID)
Lusaka, Zambia

Mr. Robert L. N. Offurum
King's College
Lagos, Nigeria

Mr. Jameson Shakalima
Chilenje Lower Primary School
Lusaka, Zambia

Dr. Onyerisara Ukeje
University of Nigeria
Nsukka, Nigeria

Mr. Hosea Kasule
Ministry of Education
Kampala, Uganda

PREFACE

The purposes of Primary Four of the Entebbe Mathematics Series are:

1. To help children learn the best mathematics they can learn.
2. To help them discover the ideas of mathematics for themselves.

We want children to learn mathematics and to think about mathematics. These two purposes are attained at the same time, not separately.

This book was written to help you help your pupils learn as they think about mathematics. The book will help you in these ways:

- It tells you about the ideas of mathematics that you teach; it refers you to sections in your Teachers' Handbook where you can read about the ideas.
- It tells you how to teach each of the units and encourages you to think of other methods.
- It suggests things you can use in your classroom to help you teach.
- It encourages you to let the children think for themselves. (Ask the question "Why?" often and let the children ask questions.)

Testing the Programme

This book and the Pupil Book were written to be tested by you and by your pupils. You and other teachers who use the books will be asked to write your opinions of the units you teach. The books will then be revised with the help of your comments. You are asked to do these things:

- Teach the material of a unit as best you can, in the way suggested.
- Think about teaching and learning that you and your pupils have done.
- Fill out a Primary Unit Report for each unit. (See sample Report on pages ix, x, xi, and xii.)

How to Use the Book

The first thing you do before teaching from Primary Four of the Entebbe Mathematics Series is to read all the books carefully (this book, the Pupil Book, and your Teachers' Handbook). Also read Primary Three of the Series. They will help you:

- ▶ Know the whole programme for the year.
- ▶ Know what things you need to help you teach.
- ▶ Know what parts of the programme you wish to discuss with other teachers and supervisors.
- ▶ Know what parts of the programme you want to study about further in your Teachers' Handbook or in other books.
- ▶ Know how this book and the Pupil Book fit together.
- ▶ Know how this book, the Pupil Book, and your Teachers' Handbook are planned to help you.

The second thing you do is to study the unit you are to teach and make plans for teaching the unit. This study helps you.

- Plan in detail your teaching from day to day.
- Know what things to collect to help you teach the unit.
- Know what further reading to do so that you can teach the ideas of the unit.

The third thing you do is teach the unit. Keep reading this book and your Teachers' Handbook. They will help you teach. Remember you teach best when children think things out for themselves. You question and guide. Tell as little as possible.

The fourth thing you do is to think about the unit and its use by you and the children. Think about how the unit could be improved for you and the pupils.

The fifth thing you do is to thoughtfully fill out a Primary Unit Report.

The Plan of the Book

This book explains in detail how to teach the twenty-one units of Primary Four. The teaching plan for a unit is separated into "stages". Each of the stages is organized into several "activities". These activities are suggestions for teaching.

You must decide about how long your pupils need to spend with a stage and an activity. An activity may be a single lesson; two or three activities may become a single lesson; or, you may continue to revise one activity several days. Study your children. Listen to them and watch them and decide when they can move to a new activity or stage.

Each unit uses ideas learned in units that come before it. For that reason, it is important that no mathematics be omitted. Some children may need only some of the activities in the books to help them learn an idea. If so, other activities may be omitted.

Helping Children Learn

Children learn best from things they do and by thinking about what they do. You can help by finding worthwhile things for them to do and to think about and by not telling them what to think.

This method means that you too must be doing and thinking as you help the pupils. These are some of the ways you can help children think about and discover mathematics:

Choose good problems and ask pupils to find answers.
Ask questions as pupils work.
Make suggestions as needed.
Suggest the use of things as models for a problem.
Supply a word or words now and then, when a pupil has a good idea he cannot express.
Suggest new and different methods of work.
Give encouragement at all times.
Help pupils summarize and help them tell what their answers mean.

Outcomes

As you help pupils to find their own answers and to think for themselves about mathematics, you are teaching them mathematics and you are teaching them a way to think about mathematics. You are also helping them to learn a way to solve <u>any</u> problem.

This searching attitude leads to satisfaction and enjoyment for you and your pupils. It is the hope of the writers of the programme that it is not only enjoyed but that much mathematics is learned. Good luck.

ENTEBBE MATHEMATICS SERIES

PRIMARY UNIT REPORT

Date : _____

Number of Unit _____ in Primary Book _____ Volume _____

Name of Unit : _____

Fill out one of these reports for each unit as you complete it. Send it by air mail to Educational Services Incorporated, African Education Program, 55 Chapel Street, Newton, Massachusetts, 02158, U.S.A. Your comments are very much needed. They will be used for the revision of the books.

PLEASE ANSWER ALL THE QUESTIONS IN THIS REPORT

Your Name : _____

Name and Address of Your School : _____

_____

Date you started teaching the Unit : _____

Date you completed the Unit : _____

Number of periods used in teaching the Unit : _____

Length of daily lesson period : _____ minutes.

Answer each of the following questions and *give reasons* for your answers.

Questions	Answers and Reasons for Answers
1. Were the teaching suggestions clear?	
2. Were the teaching suggestions sufficient?	
3. Were the instructions in the Guide helpful in developing the ideas?	
4. Were the suggestions for materials to be used in the classroom sufficient?	
5. Was the background information for teachers in this unit clear, helpful and sufficient?	

What suggestions can you give for the improvement of the *Teachers' Guide* for this Unit?

_____

_____

_____

_____

_____

_____

Answer each of the following questions and *give reasons* for your answers.

Question	Answers and Reasons for Answers
1. Did your pupils learn the ideas the unit is meant to develop?	
2. Were the pupil pages helpful in developing the ideas of the unit?	
3. Were the pupil pages for this unit sufficient?	
4. Did you use materials other than those suggested in the book?	
5. Were the illustrations in the pupil book suitable?	
6. Did the pupils enjoy this unit?	
7. Did you enjoy teaching this unit?	

What suggestions can you give for the improvement of this Unit?

_____

_____

_____

GENERAL REMARKS

and

ASSESSMENT OF THE UNIT

To the best of my knowledge, this Unit is _____ (excellent, good, fair, poor) because:

(1) _____

_____

_____

(2) _____

_____

_____

(3) _____

_____

_____

(4) _____

_____

_____

(5) _____

_____

_____

(6) _____

_____

_____
Signature

_____
Date

CONTENTS

UNIT 1 Numbers and Numerals 1

Revising the Idea of Whole Numbers 5

Place-Value in the Decimal System 9

Base-Five Numerals 13

Introduction to Roman Numerals 17

UNIT 2 Mathematical Sentences 21

Revision of Mathematical Sentences 24

UNIT 3 Describing Geometric Figures 31

Segments, Lines, and Rays 34

Angles 40

Congruent Angles 44

Triangles 46

Rectangles 51

Circles 53

Solid Figures 56

UNIT 4 Operations 60

Sentences with Several Operations 63

Various Operations 67

Properties of Operations 71

Closure of a Set of Numbers Under an Operation 79

UNIT 5 Addition and Subtraction of Whole Numbers I 83

Revision of Techniques of Addition and Subtraction 85

Summary of Properties of Addition 90

Techniques of Addition with Sums Greater than 10,000 92

Techniques of Subtraction with Large Numbers 95

UNIT 6 Measurement of Line Segments and Angles 100

Revising Measurement of Segments 102

Centimetre Unit 106

Angle Measure 110

UNIT 7 Multiplication and Division of Whole Numbers I 133

Revision of Properties of Multiplication 119

Revision of Techniques of Multiplication and Division 122

Estimation of Products and Missing Factors 129

UNIT 8 Fractions I 135

Revision of Fractions 138

The Fractions $\frac{1}{7}, \frac{1}{9}, \frac{1}{20}, \frac{1}{30}, \ldots, \frac{1}{100}$ 144

Decimal Names and Per Cent Names of Fractions 146

UNIT 9 Addition and Subtraction of Whole Numbers II 156

Mathematical Sentences Involving Addition and Subtraction 156

Games and Puzzles Using Addition and Subtraction 160

Word Problems and Applications of Addition and Subtraction 165

UNIT 10 Sets of Numbers 170

Even and Odd Numbers 172

Subsets of Multiples of Five and Ten 174

The Set of Whole Numbers and Operations on Whole Numbers 178

UNIT 11 Relations Among Geometric Figures 187

Intersection of Figures 190

Movements in a Plane 195

UNIT 12 Multiplication and Division of Whole Numbers II 202
Extension of Techniques of Multiplication to Include Factors Greater than 100, with One Factor Less than 1000 202
Estimating Products and Factors 206
Extension of Techniques of Division 210

UNIT 13 Fractions II: Addition and Subtraction. 216
Revision of Addition and Subtraction of Fractions Named with the Same Denominator 220
Addition and Subtraction of Fractions 225
Properties of Addition of Fractions 232

UNIT 14 Integers 237
Extending the Number Line 240
Adding Integers 245

UNIT 15 Measurement of Plane Regions 249
Simple Closed Curves and Plane Regions 253
Revising Tiling of Regions 256
Finding Formula for Area of a Rectangle 261
Area of Triangular Regions 264

UNIT 16 Prime Numbers and Squares of Numbers. 267
Prime Numbers 270
Squares of Whole Numbers 274

UNIT 17 Operations on Whole Numbers. 278
Extension of Division to Include Any Known Factor Less than 100 278
Revision of Operations on Whole Numbers 282

UNIT 18 Fractions III: Multiplication 285
Multiplying a Whole Number and a Fraction 289
Techniques of Multiplication of Fractions 299

UNIT 19 Geometry 307
Using the Straightedge 309
Using a Compass 311
Sketching Solid Figures 314
Movements and Symmetries of Figures 317

UNIT 20 Collecting and Recording Data 324
Simple Experiments in Chance 327
Collecting Data and Reading Graphs 330

UNIT 21 Clock Arithmetic 335
Solving Problems with a Clock 339
Clock Addition 341
Clock Subtraction 346

UNIT 1
Numbers and Numerals

Objectives

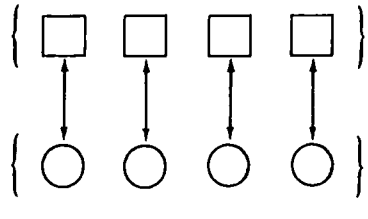
1. To revise the idea of a whole number
2. To revise ideas of place-value in the decimal system
3. To rename numbers and to revise expanded form of decimal numerals
4. To revise counting and to extend counting to include ten thousands, hundred thousands, and millions
5. To prepare for base-five system of numeration by revising counting by fives and renaming numbers with five as a base

Background Information for Teachers

A set is any collection of objects. Each object in a set is called a member of the set. Below is a picture of a set:

The members of the set are the things pictured. We may identify a set of objects by naming its members.

Sets are compared with each other by matching. If the members of two sets can be matched exactly one-to-one, the two sets are called equivalent sets. Equivalent sets have the same number of members. Below is a picture of two equivalent sets showing one of the ways to match the members:



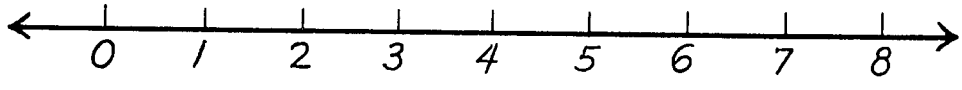
Each of the sets has four members. Each set is described by the number four. There are many other sets equivalent to these sets. Each of them has four members. The numeral "4" names the number four.

Below are some other examples of sets and their numbers:

Sets of Objects	Number	Numeral
{ Δ }	one	1
{ Δ Δ }	two	2
{ Δ Δ Δ }	three	3

The empty set is the set which has no members. For example, the set of green goats is empty. There are no green goats. The number zero describes the empty set. The numeral "0" names the number zero. The numbers zero, one, two, three, and so on, are called whole numbers.

We can use the number line to help talk about numbers. There are many ways in which the number line can be drawn. We will picture it as below and think of it as a horizontal straight line. Each whole number is assigned to a point on the line. Mark any point of the line with a dot and assign "0" to this point. Then mark a second point to the right of "0" and assign "1" to it. Mark a third point to the right of "0" the same distance from "1" as "1" is from "0". Assign "2" to it. The line segment between "0" and "1" is the unit segment. All line segments between consecutive whole number points on the line are the same length as this unit segment. Continue by assigning "3", "4", and so on, to points on the line, moving to the right.



In Primary One, Two, and Three the children learned to write numerals for whole numbers using the decimal system. In this unit, the decimal or base-ten system is revised. Then the base-five system and the Roman system of writing numerals are introduced.

The decimal system has the following properties.

1. For counting objects in a set are grouped by tens, hundreds, thousands, and so on.
2. The digits used to form numerals are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
3. Each numeral is written as a sequence of digits: For example, 123 and 7324.

4. The decimal system is called a <u>place-value</u> system. The value of each digit depends on the digit and on its <u>place</u> in the sequence.

For example, the numeral 7324 names the number seven <u>thousands</u>, three <u>hundreds</u>, two <u>tens</u> and four <u>ones</u>. Another way of writing the numeral is:

$$\begin{aligned} 7324 &= (7 \times 1000) + (3 \times 100) + (2 \times 10) + (4 \times 1) \\ &= 7000 + 300 + 20 + 4 \end{aligned}$$

We call 7000 + 300 + 20 + 4 the <u>expanded form</u> of the numeral 7324. Each numeral can be expanded in this way. Primary Four children are already familiar with this idea. They used it in addition and subtraction.

The base-five system is also a place-value system. It differs from the decimal system in two ways. First, for counting objects are grouped by fives. Second, only the digits 0, 1, 2, 3 and 4 are used to write numerals.

In counting, we often group by fives. For example, suppose there is an election for class monitor. The children vote for one of two children. There are thirty-five children in the class. The vote is recorded by using strokes. Each fifth stroke cuts the first four in each group.

First Child	𝍸 𝍸 𝍸 𝍸 ||	(twenty-two)
Second Child	𝍸 𝍸 |||	(thirteen)

In the decimal system, we record the vote as 22 to 13. Note that twenty-two is four groups of five, plus two. Thirteen is two groups of five, plus three. The base-five numeral for twenty-two is 42_{five}, meaning 4 fives and 2 ones; for thirteen it is 23_{five}, meaning 2 fives and 3 ones. The word "five" is written by the numeral to tell the grouping.

In this Unit, the only base-five numerals used are two-digit numerals. Below are some examples:

Number	Grouping	Base-five numeral
seven	⬭ ••	12_{five}
ten	⬭ ⬭	20_{five}
seventeen	⬭ ⬭ ⬭ ••	32_{five}
twenty-three	⬭ ⬭ ⬭ •••	43_{five}

Many years ago the Romans developed a system of numeration which is different from the decimal and base-five systems. The Roman system uses I for one, V for five, X for ten, L for fifty, and C for one hundred. These numerals are still used on clocks, buildings and other places.

Look at the Roman numeral XXVI. What number does it name? In order to answer this question, we must know some rules for the system.

1. Each numeral I, V, X, L, C always has the <u>same</u> value. The Roman system is not a place-value system.
2. A numeral is read from left to right.
3. Add the numbers the numerals represent if the succeeding numerals name smaller or equal numbers.

$$XXVI = 10 + 10 + 5 + 1 = 26$$

4. If a numeral precedes another numeral which names a greater number, then subtract the number the first numeral represents from the number the second numeral represents.

$$LX\underline{IV} = 50 + 10 + (5 - 1) = 64$$

Below are examples of Roman numerals with the names also given in the decimal system:

$$VII = 5 + 1 + 1 = 7$$
$$IX = 10 - 1 = 9$$
$$XXIV = 10 + 10 + (5 - 1) = 24$$
$$CXLVI = 100 + (50 - 10) + 5 + 1 = 146$$

The names for the first ten counting numbers in each of these three systems are:

Decimal	1	2	3	4	5	6	7	8	9	10
Base-five	1_{five}	2_{five}	3_{five}	4_{five}	10_{five}	11_{five}	12_{five}	13_{five}	14_{five}	20_{five}
Roman	I	II	III	IV	V	VI	VII	VIII	IX	X

STAGE 1: Revising the Idea of Whole Numbers

Vocabulary: Set, whole numbers, counting numbers, number line, numerals

Materials: Objects which can be used to illustrate sets; pupil pages 1-3

Teaching Procedure

ACTIVITY 1: Sets and Whole Numbers | Pupil page 1

The purpose of this activity is to revise these ideas:

1. The whole numbers tell something about sets of things. They tell how many members are in particular sets.
2. All sets which match one another exactly one-to-one have the same number of members.
3. Any set of things has just one whole number which describes it.
4. One whole number can describe many different sets of things.
5. We count the number of members in a set by matching a set of counting numbers with the members of the set.

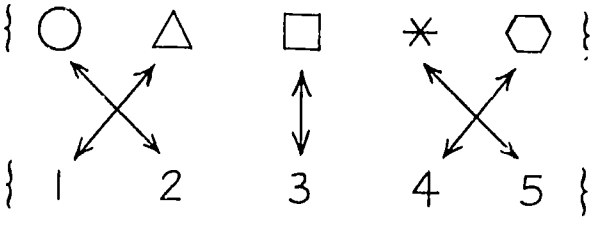
Write the numeral "5" on the blackboard. Ask as you point to the numeral, "What can you tell about this?" The children may answer, "It is five" or "It is the name for five". Ask, "What does it tell us? What do we mean when we say 'five'?" Encourage the children to talk about five as a number which tells how many things are in certain sets. For example, someone might say, "Five tells how many fingers are on my hand". Or, "Five is the number of people in my family", and so on.

Ask the children to think of and describe many different sets which have five members. They may show sets of objects in the classroom, such as five books, five pencils, or five boys.

Draw on the blackboard some pictures of sets that have five members. Ask, "In what way are all these sets alike?" (They all have the same number of members, five.) "How can we show they all have the same number?" A

child may say, "We count all the members". Let him count to show this. Say, "Pretend you cannot count. Is there another way to show that all these sets have the same number of members?" Encourage the children to match the members one-to-one. If two sets match exactly, they have the same number of members.

Remind the children that when we count the members we are also matching sets. We match a set of counting numbers with the set of things. One of the ways of matching the set is:



Write another numeral, such as "3", on the blackboard. Let the children point out many different sets with three members. Emphasize that the number three describes all these sets and that <u>any</u> set which matches each of them exactly one-to-one is described by the number three. Talk of other numbers in the same way.

Finally, hold out an empty hand. Ask, "What number describes the set of books I am holding in my hand?" (Zero) Remind the children that the set of the books you are holding is the empty set. The number of members in the empty set is zero.

Ask the children to turn to pupil page 1. There are many sets of things pictured. First let the children try to find all the sets of things described by the number four. One child can tell the sets he finds. Let others tell of more sets if they find them. Do the same for other numbers such as five and three.

ACTIVITY 2: The Set of Whole Numbers **Pupil page 2**

Draw pictures of several sets of objects on the blackboard. Include one set with three objects, one with six objects, one with seven objects, one

with four objects and one with no objects. The set with no objects is called the empty set. Represent it in this way:

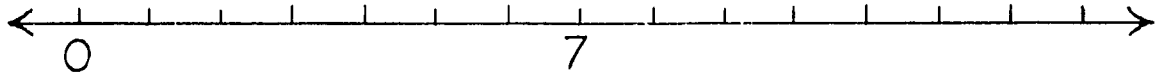
{ }

For each set, ask a child to write the name of the number of its members on the blackboard. The sets need not be in any particular order.

Rub out only the pictures of sets of things and ask, "What do we now have on the blackboard?" Guide them to say that there are names for several different numbers on the blackboard. The names are numerals.

Show on the blackboard the set and the number line below. Mark points on the number line that are equally spaced.

{3, 6, 7, 4, 0 }



First, assign only "0" and "7" to points on the line. Ask a child to choose one number from the set shown above and write its numeral below the proper point. Then let another child choose another number and write its numeral below the proper point. A third child writes the remaining numeral, so that "0", "3", "4", "6" and "7" are assigned to points on the line.

Ask the children to name the other points. They should fill in 1, 2, 5, 8, 9, 10, and so on, as far as the line is marked.

Ask, "What kind of numbers have we assigned to points on the number line?" Emphasize that the numbers are called whole numbers. Ask, "Can you name other whole numbers?" Extend the line to the right and let the children mark other whole number points through "16". Say, "Now we have written the names for a larger set of whole numbers than before". Write the numerals on the blackboard.

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}

Ask, "Are there still other whole numbers in the set of whole numbers?" Let the pupils name a few more and extend and mark the line until it reaches the edge of the blackboard. Ask, "If we had more space, could we go on marking points and assigning names for whole numbers?" Let the children name numbers

until they agree that they could go on and on. Whenever a number is given, ask if a number that is one more can be named.

Ask, "Can you think of one greatest number?" If someone guesses a large number such as one million, say, "I can say one million and one, that is still greater". If they make another guess, show that one can be added to that number to make a greater number. Guide the children to agree finally that whatever number is mentioned, a still larger number can be given, and there is a point on the number line to which it can be assigned.

Revise the idea that the set of whole numbers is the set 0, 1, 2, 3, 4, and so on. Ask, "What numbers are in the set of counting numbers?" Revise the idea that the counting numbers are 1, 2, 3, 4, and so on. Guide the children to see that the set of whole numbers is the set of counting numbers and zero.

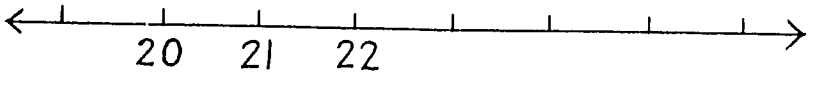
Ask the children to open their books to pupil page 2. Let a child read exercise 1. Say, "In the first exercise there are many names for numbers. Some are whole numbers some are not whole numbers. Write all the names for whole numbers you find". Use the same plan for exercise 2.

After the children have completed exercise 2, help them read exercise 3. Say, "There are three answers given: A, B, and C. Only one is correct. Write in your exercise book the correct answer: A, B, or C". Finally read exercise 4 and ask the children to choose the correct answer: A, B, or C. Help them understand that the set asked for has in it numbers which are greater than 6 but also less than 10. (The answer for exercise 4 is B.)

ACTIVITY 3: Whole Numbers on the Number Line

Pupil page 3

Draw the number line on the blackboard and assign whole numbers to three points like this:



Say, "This is the number line. We have assigned whole numbers to three points. Can you assign whole numbers to the other points shown?" Do a similar example using a different subset of the whole numbers.

Ask the children to open their books to pupil page 3. Help the children read the first sentence. Get them to copy in their exercise books the number line of exercise 1 and name the other points. Let them do the same for exercises 2 and 3.

Call attention to exercise 4. Say, "In exercise 4 you see two pictures of the number line exactly lined up. Notice that points for 0, 1, and 2 are in similar positions for both lines. On the lines there are empty boxes. Names for numbers go in these boxes. Write in your exercise book the number names that go in the boxes. Notice that parts of the line are blocked out, so that you cannot just count to find the answers".

Ask the children to look at exercise 5. Say, "Under the second picture of the number line there are empty boxes. This is almost like the fourth exercise. But notice that number names and marks are not lined up as they were in exercise 4. For example, zero is lined up with one, one is lined up with two, and so on. The first picture of the number line helps you to find the number names missing from the second picture of the number line".

STAGE 2: Place-Value in the Decimal System

Vocabulary: Place, digit, place-value, ones' place, tens' place, hundreds' place, thousand, ten thousand, hundred thousand, million, expanded form

Materials: 3 bundles of small sticks (100 sticks each); 5 bundles of small sticks (10 sticks each); 9 single sticks; pupil pages 4-6

Teaching Procedure

ACTIVITY 1: Place-Value to Thousands

Write the following on the blackboard:

235 304 111

Ask a group of four children to come up and show the meaning of the first

numeral, 235, using sticks and bundles of sticks. For example, 235 is shown by:

2 bundles of 100 sticks,

3 bundles of 10 sticks, and

5 single sticks.

Encourage the children to arrange the sticks in the same order as the digits to recall the idea of place-value.

Let another group of children show 304 with sticks and let a third group show 111.

Write:

235 532

Ask, "What numbers are named on the blackboard?" The children may answer, "Two hundred thirty-five; five hundred thirty-two". Ask, "Do you see something alike in the two numerals?" Let them talk about this until someone says that the numerals have the same digits. Ask, "Why do they not name the same number?" Guide them to say it is because the places of the digits are different. Emphasize that in 235 the "5" means 5 ones because its place-value is one, but in 532 the "5" means 5 hundreds because its place-value is hundred, and so on.

Introduce a few similar examples and discuss the place-value of digits. Then revise the ideas using a place-value table. Write the numerals in the left column and let the children fill in the table as follows:

	Hundreds	Tens	Ones
614	6	1	4
146	1	4	6
90		9	0
744	7	4	4

Stress the idea that the tens' place has a place-value ten times that of the ones' place which is one. The tens' place is one place to the left of the ones' place. The hundreds' place has a place-value ten times that of the tens' place. The hundreds' place is one place to the left of the tens' place, and so on.

Revise also the idea that 0 in any place simply means that there is none of that unit represented. For example, in 90, the "0" tells us that there are no ones. We need to write the digit 0 to know that the "9" refers to nine tens.

ACTIVITY 2: Numbers Greater than One Thousand **Pupil page 4**

This activity extends place-value ideas to numbers beyond one thousand.

Write the following on the blackboard:

10 ones, 10 tens, 10 hundreds

Ask, "What is another name for each of these numbers?" (1 ten, 1 hundred, and 1 thousand.)

Write on the blackboard:

10 thousands

Ask the pupils to give another name for this. Let them make guesses guiding them to see that they may say "1 ten thousand" or "ten thousand". Point out that we do not use a new name for 10 thousands as we did for 10 hundreds, which we call 1 thousand.

Write several 5-digit numerals on the blackboard, for example:

12,304 13,456 34,562

Discuss the numbers shown and ask the children to tell what each digit means. Thus, 34,562 is 3 ten thousands, 4 thousands, 5 hundreds, 6 tens, and 2 ones. Explain that we call this number thirty-four thousand, five hundred sixty-two. Discuss the other examples in the same way.

Before continuing the activity, find the population of a town or district near you which might be in the ten thousands or hundred thousands. Using this example and population of your country, extend the idea of place-value to hundred thousands and then to millions (thousand thousands).

Then let the children turn to pupil page 4 and do the exercises.

ACTIVITY 3: Expanded Notation

Pupil page 5

Write the following on the blackboard:

65 × 5; 314 + 11; 305 + 20;

200 + 100 + 25; 325; 300 + 20 + 5

Let the pupils discuss the numerals and what they name. Ask, "What number does the first example name?" (325) "What number does the second example name?" (325) and so on. Let the children say that each names the same number, 325. Emphasize the idea that any number can be named in many ways. Call out a few numbers and let children think of different ways to name them.

Return to the examples on the blackboard. Say, "These names are all for the number three hundred twenty-five. Which name is the one we usually use?" Get them to decide that "325" is usually the way we show three hundred twenty-five. It is the simplest name for the number.

Ask, "What does each digit in 325 mean? Which of the names on the blackboard shows that 325 means three hundreds, two tens, and five ones?" Guide the children to see that the expanded form

300 + 20 + 5

is used to help make the meaning clear.

Ask the children to open their books to pupil page 5 and complete the exercises. Let the children do one or two exercises of each type together and then complete all the exercises in their exercise books.

ACTIVITY 4: Counting by Thousands

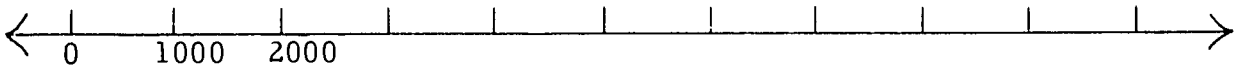
Pupil page 6

Introduce this activity by talking with the class about large groups of people they have seen. For example, you may begin by discussing the school's opening assembly, where all the children in the school are gathered. Other places where children may have been in large groups are sports meetings and large crowds assembled to greet an important visitor. You and the children will think of other examples.

Say, "When there are many thousands of people, someone may try to guess the size of the crowd by groups of hundreds or even thousands. You have learned to count by numbers such as two, five, and ten. You can also

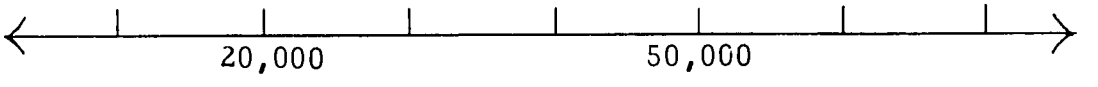
count by thousands. We will show this on the number line".

Draw the number line on the blackboard, marked as shown below:



Ask different children to assign whole numbers to the other points marked, counting from 0 to 10,000. Let the children count aloud by thousands.

Ask, "How can you count by ten thousands?" Let one child count in this way. Say, "Let us try this on the number line to check". Draw the number line with units of ten thousands, with two whole numbers assigned as shown:



Let children fill in the other numerals. Ask one child to count by ten thousands.

Use another picture of the number line to illustrate counting by hundred thousands to one million. Finally, illustrate counting by millions in the same way.

Let the children turn to pupil page 6 and do the exercises. Ask them to copy the pictures shown there in their exercise books and fill in numerals for all the points marked on the line.

STAGE 3: Base-Five Numerals

Vocabulary: Base-five, base-ten, decimal name

Materials: One set of 24 bottle tops for each group of children, pupil pages 7-8

Teaching Procedure

ACTIVITY 1: Grouping and Counting by Fives **Pupil page 7**

Give each group of children 24 bottle tops and get the pupils to arrange the bottle tops in sets of five. Ask, "How many sets of five bottle tops can you make; how many bottle tops are left?" (4 sets of five and 4 single bottle tops left over.) Write on the blackboard and let a child read:

Twenty-four is 4 fives and 4 ones

Say, "Count out just fourteen of your bottle tops and arrange them in sets of five". How many sets of five do you have? (two) How many ones are left over? (four) Write on the blackboard:

Fourteen is 2 fives and 4 ones.

Repeat this procedure for several other examples, grouping fewer than 24 bottle tops into sets of five. Show the results on the blackboard in fives and ones. Remind the children that "2 fives" means 2 sets of five, "3 fives" means 3 sets of five, "4 ones" means 4 single bottle tops, and so on.

Now draw 16 x's on the blackboard:

```
xxx        xx

xx         x

   x  x    xx

  xx  x    x
```

Ask a pupil to separate the x's into sets of five by drawing a ring around a set. Ask, "How many sets of five are there? (three) How many are left over? (one) How many x's in all?" (sixteen)

Write on the blackboard:

Sixteen is 3 fives and 1 one.

Repeat this for several sets of fewer than 25. Let children write the sentence on the blackboard that describes the fives and ones. Let the children revise skip counting by fives to twenty, "five, ten, fifteen, twenty". Point out that this is the same as "1 five, 2 fives, 3 fives, 4 fives".

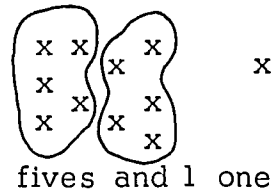
Say, "Sometimes it is easy to count things if we put them in sets of five. We can keep the count in this way". (Make marks as you count.)

Show that every stroke counts one thing and the cross strokes help us to see a group of five strokes quickly. Say, "We need to count only the fives. How many fives are shown? How many ones?" (3 fives and 2 ones) Show that we count quickly. "Five, ten, fifteen, plus two gives us seventeen".

Ask the children to turn to pupil page 7 and do the exercises. If needed, they may use bottle tops grouped in fives to help them with exercises 4 to 11.

ACTIVITY 2: Expressing Numbers in Base-Five Numerals **Pupil page 8**

Ask each child to arrange 11 bottle tops in sets of five. Illustrate this on the blackboard using x's. Ask a child to write the number of fives and the number of ones shown.



2 fives and 1 one

Let them repeat the procedure using 10 bottle tops. Then ask each child to write on a piece of paper the number of fives and the number of ones. Give several sets which have "0" ones.

Say, as you point to the "2 fives and 1 one", "Suppose you leave out the words 'fives' and 'one' and write '21'". Write:

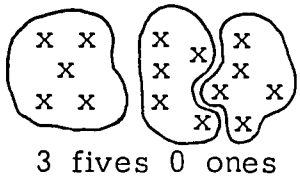
21

Ask what "21" means. If the children say "Twenty-one" remind them that it cannot mean twenty-one. Twenty-one is 2 tens and 1 one. Ask, "How many bottle tops show 2 fives and 1 one?" Guide them to say that this "21" means 2 fives and 1 one. Say, "I will write 2 fives and 1 one like this to show grouping in fives and ones".

21_{five}

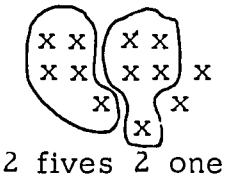
Point to the numeral above and ask, "What does this tell us?" (It tells us there are two fives and 1 one.)

Do other similar examples on the blackboard:



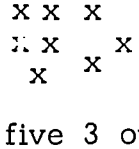
3 fives 0 ones

30_{five}



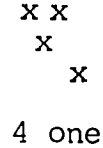
2 fives 2 ones

22_{five}



1 five 3 ones

13_{five}



4 ones

4_{five}

Write on the blackboard the two numerals:

$$13 \qquad\qquad 13_{five}$$

Ask, "Which numeral shows 1 five and 3 ones? (the second) How many bottle tops does it name?" (five plus three or eight bottle tops)

Ask, as you point to "13", "What number is named by this numeral?" (13 is the decimal name of thirteen. It means 1 ten and 3 ones.) Emphasize that the "13" represents thirteen things, but "13_{five}" represents eight things. Remind the children that the numeral "13" shows grouping in tens. Say, "Therefore we call "13" a base-ten or decimal numeral".

Point out that the numeral "13_{five}" shows grouping in fives. Ask, "What will we call this numeral?" (a base-five numeral)

Tell the children that we cannot read "13_{five}" as "thirteen" because it names eight things, not thirteen. Explain that we can read it as "one-three, base five" or as "1 five and 3 ones".

Show some other examples of base-five numerals, using bottle tops to illustrate the number of things named. Include no more than twenty-four things. Write a base-five numeral and let children illustrate with bottle tops. For example, "21_{five}" is illustrated by 2 sets of five bottle tops and 1 single bottle top. Let children tell how the numeral is read and how many bottle tops are named. Write their answers on the blackboard in the form of a table:

Numeral	Read	Number
21_{five}	two-one, base-five	eleven
42_{five}	four-two, base-five	twenty-two
30_{five}	three-zero, base-five	fifteen
3_{five}	three, base-five	three

Discuss these numerals with the children, guiding them to see the following:

1. In a two-digit numeral in base-five, we have a ones' place and a fives' place, just as in a base-ten numeral of two digits we have a ones' place and a tens' place.

   23 means 2 tens and 3 ones, or twenty-three

   23_{five} means 2 fives and 3 ones, or thirteen

2. For base-ten numerals we use ten different digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). For base-five numerals we use five different digits (0, 1, 2, 3, 4).

Ask the children to turn to pupil page 8 and complete the table in their exercise books.

STAGE 4: Introduction to Roman Numerals

Vocabulary: Roman Numerals (I, V, X, L, and C)

Materials: A set of 4" × 3" numeral (I, V, X, L, C) cards; set of 4" × 3" numeral (0-9) cards; pupil pages 9-10

Teaching Procedure

ACTIVITY 1: Roman Numerals I to X

Let the children examine the first pages of certain books that are numbered with Roman numerals. (You may need to get books from the library which have a few pages numbered using Roman numerals.) Also call attention to chapters of some books that are numbered using Roman numerals. Tell the children that these are Roman numerals.

Ask the children if they know other places where Roman numerals are used. One possible answer is a clock face. If one is available, show such a clock or watch face. If not, draw on the blackboard a picture of a clock face showing Roman numerals. Let the children discuss the numerals and the different symbols used. Compare the clock face showing Roman numerals with a clock face showing decimal numerals. Let children compare the two ways of writing numerals.

Show two sets of numeral cards as illustrated below:

1	I
2	II

Ask two children to write the numerals on the blackboard, the first writing 1, the second writing I as shown below:

1 I

Repeat the activity for the symbols 2, II to 10, X. Guide the children to decide

that I, V, and X are the only signs used in Roman numerals to write numerals for one to ten. Discuss the "one more" idea for I, II, and III; and the "one less" idea for IV (IV means 5 - 1); the "one more" idea for VI, VII, and VIII; and the "one less" idea for IX.

Let the children make on the blackboard a chart of decimal and Roman numerals for one to ten. Let them talk about the chart and copy it in their exercise books.

ACTIVITY 2: Roman Numerals I to C

Revise quickly the Roman numerals I to X. Write I on the blackboard and ask a child to write the corresponding Roman numeral for it. Do the same for 2, 3, . . . 10.

Show the Roman numerals for 10 to 15 (X, XI, XII, XIII, XIV, XV). For example, XII shows 10 + 2 and XV shows 10 + 5. Ask the children to guess the Roman numerals for sixteen through twenty. Help them by comparing these numerals to the Roman numerals for six through ten. For example, eighteen (XVIII) uses addition but nineteen (XIX), like nine, shows subtraction. Emphasize that XI to XX are like I to X except that X is put to the left of each symbol I to X.

Put the following chart on the blackboard:

I	II	III	IV	V
VI	VII	VIII	IX	X
XI	XII	XIII	XIV	XV

Let the children talk about the chart and decide how to continue it.

XXI	XXII	XXIII	XXIV	XXV
XXVI	XXVII	XXVIII	XXIX	XXX

Let them write the numerals to XXXIX (39). At this point, show the symbol L for fifty. Say, "If L is fifty what do you think forty is?" Let the children guess. Guide them to see the symbol is XL (50-10), the symbol for forty-one is XLI, and so on.

Say, "You need one other symbol, then you can write all Roman numerals for numbers to one hundred. The symbol for one hundred is C. What is the

Roman numeral for ninety?" (XC for 100 - 10) Give some other examples of decimal numerals and let the children write the corresponding Roman numerals.

Discuss the table shown on the blackboard and help the children to discover that the Romans used grouping by fives as well as grouping by tens. For example, VII means five plus two; XII means ten plus two. They may also discover that the Romans generally did not repeat a symbol more than three times in one numeral. Instead of writing forty as XXXX, for example, they wrote it as XL, (50 - 10).

ACTIVITY 3: Comparing Numeral Systems **Pupil page 9**

Write on the blackboard the Roman numerals I, V, X, L, and C. Ask children to write decimal numerals for the numbers named. (1, 5, 10, 50, and 100) Ask, "How many different symbols are used in writing Roman numerals to one hundred?" The children should decide that there are just five different symbols, those written on the blackboard.

Write the following numerals on the blackboard and let the children write the Roman numerals for the numbers named:

1	(I)	15	(XV)	60	(LX)
4	(IV)	19	(XIX)	70	(LXX)
5	(V)	20	(XX)	80	(LXXX)
9	(IX)	30	(XXX)	90	(XC)
10	(X)	40	(XL)	100	(C)
14	(XIV)	50	(L)		

Let the children talk about the two different numeral systems and compare them. Guide them to bring out the following points.

1. The Roman system uses grouping by both fives and tens. The decimal system uses grouping by tens.
2. The Roman system uses addition as in XI (ten plus one) and subtraction as in IX (ten minus one). The decimal system uses only addition (23 = 20 + 3).
3. The Roman system is <u>not</u> a place-value system. Each numeral always has the same value. The decimal system is a place-value system.

On the third point, get the children to notice the difference by writing pairs of numerals for the same number. Show that in the decimal system the position of the symbol tells its value. In the Roman system this is not true. For example, compare XXX and 30. Guide the children to see that in XXX, 3 tens is shown by repeating the X three times. But in 30, 3 tens is shown by the "3" in the tens' place.

Guide the children to see that there is no symbol for zero in the Roman system and to understand the usefulness of a "0" in a place-value system.

Ask the children to turn to pupil page 9 and write the numeral that belongs in each box. If the numeral is shown in the decimal system, they write the Roman numeral for the number named. If the numeral is shown in the Roman system, they write the decimal numeral for the number named.

ACTIVITY 4: Story Problems

Pupil page 10

The purpose of this activity is to revise the ideas developed in this unit. Let the pupils open their books to pupil page 10. Exercise 1 revises both Roman numerals and the reading of a clock. Recall that in Primary Three the expression "three quarters past" the hour was for the more familiar "one quarter to" the next hour. In exercise 3, "three quarters past twelve" means the same as "one quarter to one".

Exercises 4 and 5 use base-five and base-ten numerals. Exercises 6 to 8 revise the ideas of days, months and years. Exercise 9 is called a many-choice question. Exactly one of the three answers given is correct. The pupils decide which one. (Answer (c) is the correct choice.)

Exercise 10 is a new kind of exercise. Occasionally, exercises on a pupil page will extend ideas discussed in class. Let the pupils try these exercises without class discussion. An exercise of this type will be marked with an asterisk (*). We will call it a "challenge" problem.

In exercise 10, Kofi has 24 sticks. In base-five this is written 44_{five}. His friend gives him another stick, he has 25 sticks. He has 5 bundles of 5 sticks each. Write 100_{five} and say, "One-zero-zero, base-five".

UNIT 2
Mathematical Sentences

Objectives

1. To revise ideas of operations represented in mathematical sentences
2. To revise mathematical sentences showing equalities
3. To revise mathematical sentences showing inequalities

Background Information for Teachers

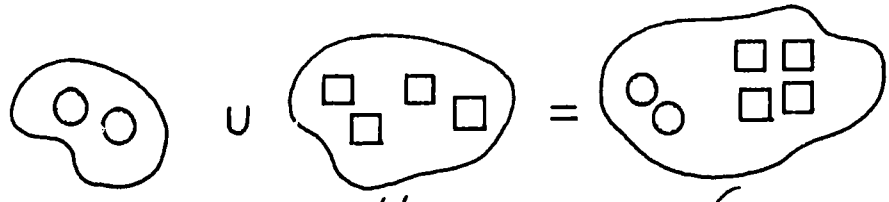
In Unit 1 the idea of a whole number was revised. In this Unit, operations on whole numbers using mathematical sentences are revised. The decimal system will be used throughout the unit.

An operation on pairs of whole numbers assigns to each pair of whole numbers exactly one number. For example, addition is an operation. We know the sum of 3 and 2 is 5. The number assigned to the pair of whole numbers 3 and 2 by the operation of addition is five.

There are many operations on whole numbers. Two operations are very important for primary school children. These are the operations of addition and multiplication. Both of these operations are explained in terms of sets.

For example, the sum of 2 and 4 is shown as 6 by the union of two disjoint sets, one of 4 members, the other of 2 members.

Sets:



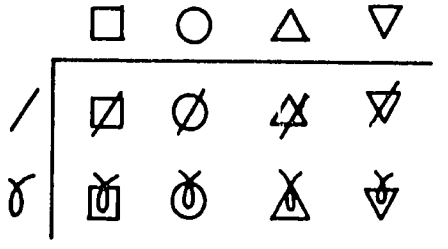
Numbers: 2 + 4 = 6

Recall that "U" is the symbol for the union of two sets. Two sets are disjoint if there is no member that is in both sets.

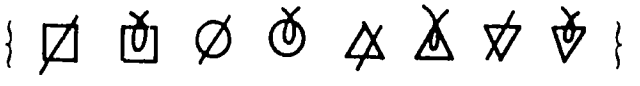
We use "=" to mean that two symbols name the same thing. For example, 2 + 4 and 6 name the same number. Therefore we may write "2 + 4 = 6". The expression "2 + 4 = 6" is an example of a mathematical

sentence. We use a mathematical sentence to express a relation between numbers.

The product of 2 and 4 is shown as 8 by pairing the members of two sets, one of 2 members, the other of 4 members. For example, suppose the sets are { / γ } and { □ ○ △ ▽ }. How many ways can we pair a member of the first set with a member of the second set?



The new set we get is this set of pairs:

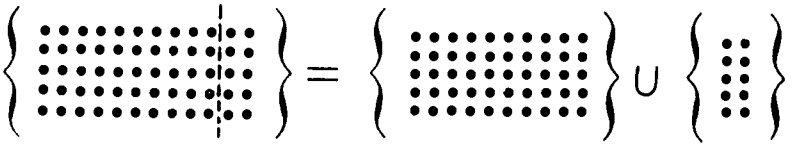


In the picture above are 2 rows of pairs, with 4 pairs in each row. We write $2 \times 4 = 8$. We read, "Two times four equals eight". The expression "$2 \times 4 = 8$" is another example of a mathematical sentence.
The set of pairs has 4 + 4 members, so:

$$2 \times 4 = 4 + 4$$
$$2 \times 4 = 8$$

This example shows that multiplication is different from addition. It also shows that repeated addition can be used to find products.

Multiplication is distributive with respect to addition. The meaning of the property is shown by this example:



$$5 \times (10 + 2) = 5 \times 10 + 5 \times 2$$
$$5 \times 12 = 50 + 10$$
$$5 \times 12 = 60$$

We use the distributive property and expanded notation to find products of whole numbers by using products of smaller numbers.

In this unit, several types of mathematical sentences are used. An example of one type is:

$$3 \times 4 = n$$

This is a sentence with a missing product. To solve the sentence we find the whole number n which makes the sentence true. For this sentence, n is 12. Every other number makes the sentence false.

An example of a second type is:

$$4 + n = 9$$

This is a sentence with a missing addend. What whole number n makes this sentence true? (n is 5) Another way of writing this example is $9 - 4 = n$. This sentence uses the idea of subtraction. Subtraction is the process of finding the missing addend.

Is there a whole number we can assign to $5 - 7$? There is no whole number that can be added to 7 to give the sum 5. Therefore, we cannot subtract a whole number from another and always get an answer which is a whole number.

If n is 2, the mathematical sentence $4 \times n = 8$ is true. Another way of writing this sentence is $8 \div 4 = n$, using the symbol "÷" for division. Division is the process of finding the missing factor. As with subtraction, we cannot divide any two whole numbers and always get an answer which is a whole number. For example, there is no whole number n for which the sentence $5 \div 7 = n$ is true.

Pairs of mathematical sentences make clearer the meanings of subtraction and division:

$$12 \div 3 = 4 \text{ because } 3 \times 4 = 12$$

$$19 - 5 = 14 \text{ because } 14 + 5 = 19$$

"$3 \div 0 = n$" is meaningless. There is no number n that makes the sentence $0 \times n = 3$ true, because zero times any number is zero. The sentence $0 \div 0 = n$ is also meaningless. Every whole number makes this sentence true, since $0 \times n = 0$ for every number. Therefore, we cannot divide by zero.

We also talk about numbers that are not equal. For example, 7 is greater than 5. The mathematical sentence "7 > 5" says the same thing in symbols. This sentence is called an inequality. Some other inequalities are:

$$9 > 4; \qquad 27 > 14; \qquad 4 \times 2 > 3$$

We can show inequalities on the number line. If 7 > 3, the point for 7 is to the right of the point for 3. If a number n is greater than a number m, then the point for n is to the right of the point for m. We write:

$$n > m$$

We may also write sentences such as "5 < 7". This means "5 is less than 7". If a number n is less than a number m, then the point for n is to the left of the point for m. We write:

$$n < m$$

If we think of two whole numbers n and m, then exactly one of these sentences is true:

$$n < m \qquad n = m \qquad n > m$$

Below are more examples of true sentences showing inequalities:

$9 \div 3 \neq 5$ ("$\neq$" means "is not equal to")

$12 \div 4 < 7$

$9 + 5 < 10 \times 5$

$3 \times (4 + 2) > 5 \times (4 \div 2)$

STAGE 1: Revision of Mathematical Sentences

Vocabulary: Mathematical sentence, true sentence, is less than, is greater than

Materials: Pupil pages 11-15

Teaching Procedure

ACTIVITY 1: Mathematical Sentences and Operations **Pupil page 11**

Write the following sentences on the blackboard:

$$6 + 7 = n \qquad 8 + 9 = m \qquad 13 + a = 18$$

Tell the children that you are writing mathematical sentences. Let them talk about the sentences, their meanings and the meanings of the different symbols. For example, some of the things the children may discuss are:

1. In the sentences, the 6, 7, 8, 9, 13, and 18 are names for numbers.
2. The numbers tell how many members are in certain sets.
3. The sign, +, is an addition sign. In the sentence $6 + 7 = n$, + indicates that we add 6 and 7.
4. The sign, =, in the sentence $6 + 7 = n$, shows that $6 + 7$ is equal to n. It shows two names for the same number.
5. We must find what numbers n, m, and a make the sentences true.

Ask, "Can you make the first sentence true?" Revise the idea that n must be 13 to make the sentence $6 + 7 = n$ a true sentence. Help the children remember that $6 + 7$ and 13 are names for the same number.

Continue talking in a similar way about the other sentences. Let the children tell that in the last sentence a is a missing addend.

Write the following sentences on the blackboard:

$15 - 9 = b$	$13 - 0 = c$	$20 - 15 = y$
$9 \times 3 = z$	$6 \times 8 = a$	$10 \times m = 100$
$10 \div 5 = n$	$12 \div 12 = d$	$72 \div 9 = g$

Let the children talk about the signs −, ×, ÷ and their meanings. Revise the meaning of subtraction as related to addition by comparing pairs of sentences such as:

$15 - 9 = b$ means $9 + b = 15$, and $13 - 0 = c$ means $0 + 13 = c$.

Ask children in turn to tell what numbers make the sentence true. For example, since $9 + 6 = 15$ is true, we know that b is 6 and that $15 - 9 = 6$ is true.

Talk about the multiplication sentences and the division sentences in a similar way. For the division sentences, revise the idea that:

$10 \div 5 = n$ means $5 \times n = 10$,

$12 \div 12 = d$ means $12 \times d = 12$,

and $72 \div 9 = g$ means $9 \times g = 72$.

Ask the children to make the sentences on pupil page 11 true sentences. Get them to write the answers in their exercise books.

Write many examples like the following on the blackboard:
(Do not include examples which require renaming.)

$17 + 21 = n$ $\quad$ $422 + 167 = b$ $\quad$ $81 + 18 = a$

Let the children tell what numbers make the sentences true. One child may go to the blackboard and write the addends in vertical form for addition. For example, he may write:

$$\begin{array}{r} 422 \\ \underline{167} \end{array}$$

By using this form, he finds that <u>b</u> is 589. After the children have solved all three examples, write other examples in addition in which renaming and expanded form are revised:

$59 + 33 = m$ $\quad$ $75 + 17 = c$ $\quad$ $59 + 35 = y$

Let the children work out the examples on the blackboard showing all the steps. For example:

$$\begin{array}{rcl} 59 & = & 50 + 9 \\ \underline{33} & = & \underline{30 + 3} \\ & & 80 + 12 = 80 + 10 + 2 = 92 \end{array}$$

<u>m</u> is 92

Then get the children to do a few examples by thinking through the renaming and not writing out the expanded form. The sentence $48 + 37 = n$ may be solved by thinking in this way:

$$\begin{array}{r} 48 \\ \underline{37} \end{array}$$

1. Add the number of ones. (8 + 7) ones = 15 ones. Rename 15 ones as 1 ten and 5 ones. Write 5 in the ones' place and remember the 1 ten.
2. Add the number of tens. (1 + 4 + 3) tens = 8 tens. Write 8 in the tens' place.

Let the children do Exercises 1-18 (Addition) on pupil page 12. In their exercise books they write the number that makes each sentence true.

Write some examples of multiplication sentences on the blackboard. Let the children solve them by renaming and working out the expanded form. After the children have written out the work, go over each step with the class:

$24 \times 6 = n$ $\quad$ $38 \times 3 = n$ $\quad$ $13 \times 7 = n$

The written work for the first example will look like this:

$$\begin{array}{r} 20 + 4 \\ \underline{\qquad 6} \\ 120 + 24 = 144 \end{array}$$

Then show the vertical form for the same example:

$$\begin{array}{r} 24 \\ \underline{6} \\ 24 \\ \underline{120} \\ 144 \end{array}$$

Discuss each step with the children; then show the short form for each sentence. Explain that while it is not necessary to write out all steps, we must think about each one. For example, in multiplying 24 × 6 the thinking goes like this:

$$\begin{array}{r} 24 \\ \underline{6} \\ 144 \end{array}$$

1. Multiply 6 × 4. (6 × 4 = 24) Rename 24 as 2 tens and 4. Write 4 in the ones' place and remember 2 tens.
2. Multiply 6 × 2 tens. (6 × 2) tens = 12 tens. Add the tens. (12 + 2) tens = 14 tens. Write the 14 tens.

Ask the children to open their books to page 12 and do Exercises 19-36 in their exercise books. Let them find the number that makes each sentence true.

ACTIVITY 3: Inequalities in Mathematical Sentences **Pupil page 13**

Write the following sentence on the blackboard:

$$4 > 3$$

Let the pupils talk about it and remember that it is a mathematical sentence. Ask a child to read the sentence. If there is difficulty, do other examples of sentences using the symbol ">" until they say that the sentence 4 > 3 means "4 is greater than 3".

Use the same plan for sentences using the symbol "< " for "is less than". For example, write:

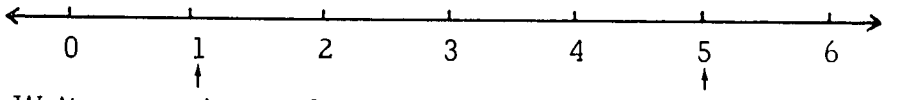
$$5 < 7 \qquad 3 < 4 \qquad 1 < 8$$

Let the children read these sentences as "Five is less than seven", and so on.

Write a few sentences using both < and > symbols and let the children tell whether the sentences are true or false:

$$5 > 1 \qquad 4 < 4 \qquad 2 < 1 \qquad 7 > 6$$

Encourage the pupils to give reasons for their answers. For example, "The first sentence, $5 > 1$, is true because five is greater than one". Let a child show this on the number line, pointing out that 5 is to the right of 1:



Write some incomplete sentences omitting the inequality sign. For example, write:

$$7 _ _ _ 3$$

Ask the children what symbol makes a true sentence.

After a few more examples, ask the children to turn to pupil page 13 and solve the Exercises 1 to 6.

When they have completed these, put the following sentence on the blackboard:

$$(3 \times 2) _ _ _ (4 \times 3)$$

Ask what symbol makes a true sentence. Guide the children to see that they first multiply. Ask, "What number is named by (3×2)? (6) What number is named by (4×3)? (12) What symbol will you write to make the sentence true?" (6 is less than 12; the symbol to use is < .)

$$(3 \times 2) < (4 \times 3)$$

Do several examples of this kind with the class.

Ask the children to look again at page 13 in their books and complete the exercises. Notice that there are a few examples in which the symbol, =, is needed. This reminds the pupils that "=" is used in mathematical sentences to show that the two numerals name the <u>same</u> number.

ACTIVITY 4: Solving Mathematical Sentences — Pupil pages 14-15

Write the following story problem on the blackboard:

> Luka's father owns 234 cattle. He gives 25 cattle to his eldest son. How many cattle does the father now own?

Ask the children to read the problem. Get a child to write a mathematical sentence for the problem. The question to answer is, "How many cattle does Luka's father now own?" Let n represent this number. The sentence to solve is:

$$234 - 25 = n$$

Let the children talk about the problem like this:

> Since Luka's father gave away some of his cattle, we subtract the number he gave away from the number he had. After the cattle are given away, a number of cattle are left.

Do several other story problems with the class. Remind them that in story problems they:

- Know what the problem is about and keep in mind the question asked in the problem.
- Write a mathematical sentence for the problem, with a letter representing the missing number.
- Find a number that makes the mathematical sentence true.
- Use that number to answer the question in the problem.

Illustrate each of these steps in the following example:

> Ayo helps her mother sell oranges. One day she sold oranges to 2 people. Each bought 7 oranges. Then she sold oranges to 4 people. Each bought 6 oranges. She had 6 oranges left. How many oranges did she have at the beginning of the day?

- What is the question asked? How many oranges did Ayo have at the beginning of the day? Let n be that number.
- Write the mathematical sentence for the problem. All the facts needed should be included:

  $$(2 \times 7) + (4 \times 6) + 6 = n$$

  The number of oranges she had at the beginning is the total number sold plus the number remaining. (Some children may write the sentence like this: $(7 \times 2) + (6 \times 4) = n - 6$.)
- To find n, multiply, then add, to find that if n is 44 the sentence is true.
- Answer the question in the problem: Ayo had 44 oranges at the beginning of the day.

When the plan of solving a problem is clear, ask the children to turn to pupil page 14 and work the exercises in their exercise books. Encourage the children to use mathematical sentences to help find the answers. Ask them to write sentences which answer the questions in the problems. Follow the same plan for introducing pupil page 15.

The pupils' written work for Problem 1, page 14, may look like this:

1) $250 + n = 315$ or $315 - 250 = n$

$n = 65$ $\quad$ $65 = n$

$$\begin{array}{r} 315 \\ \underline{250} \\ 65 \end{array}$$

Ali must drive 65 miles more.

UNIT 3
Describing Geometric Figures

Objectives

1. To revise line segment, line, congruence of line segments, and to introduce the idea of a ray
2. To develop the definition of an angle as a pair of rays with a common endpoint, and the idea of congruence of angles
3. To revise and extend the idea of triangles to isosceles, equilateral, and right-angled triangles
4. To revise and extend the idea of rectangles and squares to include ideas of vertices and sides
5. To revise the idea of circles and extend to closed figures that are not circles
6. To revise cubes, spheres, rectangular solids, and extend to cylinders and cones

Background Information for Teachers

In Primary One, Two, and Three the pupils learned to name and describe plane figures such as points, line segments, lines, angles, triangles, and rectangles. They also learned to recognize solid figures such as balls and boxes.

In Primary Four the plan is to:

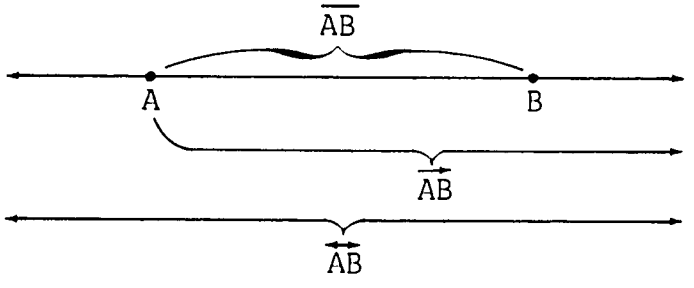
(a) extend the recognition of plane and solid figures to include more special figures such as rays, isosceles and equilateral triangles, cylinders, and cones (with more specific names such as sphere for ball, rectangular solid for box, and so on), and

(b) identify more properties which connect these figures: properties of intersection of figures, properties of symmetries of figures, properties of figures as they are moved about in a plane.

In this Unit, the naming and describing of figures is continued. In Units 11 and 19 the study of properties of these figures is continued. New mathematical ideas included in this unit are the following:

RAYS

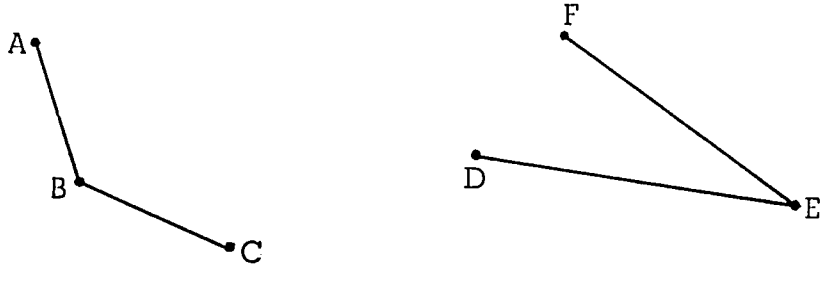
When a line segment is "stretched" without end in both directions the resulting figure is called a line. When a line segment is "stretched" without end in one direction with one endpoint of the segment fixed, the figure formed is called a ray. A familiar example is a ray of light from a torch. In order to keep these three figures - line segment, line, ray – clearly distinct, we use special symbols for them. For example, the line segment with endpoints A and B is denoted by $\overline{AB}$ and read "line segment AB". The line through the points A and B is denoted by $\overleftrightarrow{AB}$ and read "line AB". The ray with endpoint A and passing through B is denoted by $\overrightarrow{AB}$ and read "ray AB".



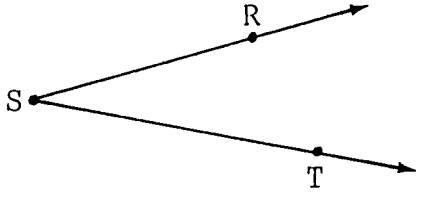
Throughout the study of geometry this distinction between line segments, lines, and rays is important. The symbols used for these figures help the children to remember this distinction.

ANGLES

The idea of an angle was developed earlier by looking at the corner of a table top. Then the children drew an angle by drawing two line segments with a common endpoint. For Primary Three, this idea of an angle was adequate; but when children compare the sizes of two angles this description is not adequate. For example, in the figure below, angle ABC is larger than angle DEF, even though the sides of angle ABC are shorter than the sides of angle DEF.



In this unit, we give a more complete idea of an angle. To do this we now think of an angle as a pair of rays with a common endpoint. For example, the angle RST is the pair of rays SR and ST with the common endpoint S. Another name for this angle is angle TSR.

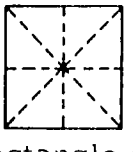


With this idea of an angle, we can compare the sizes of two angles easily by copying one of the angles and laying the copy on the other angle.

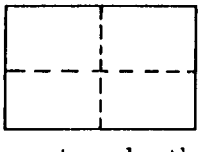
SYMMETRIES

As the pupils learn to identify geometric figures, they notice similarities and differences among the figures. For example, a square has all its sides congruent and a rectangle may have just pairs of opposite sides congruent; an equilateral triangle has all three of its sides congruent and an isosceles triangle may have just two of its sides congruent.

These and other examples of differences among figures can be understood also by a study of symmetries. If we fold a paper cutout of a square, we find that there are several ways to fold the cutout to make the two halves fit exactly. We can fold the square along any of the dotted lines marked on the first figure below and get two parts that fit exactly:



A rectangle that is a square



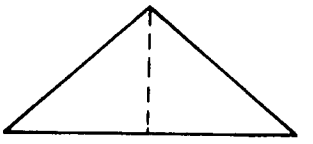
A rectangle that is not a square

We say that the square is symmetric about these dotted lines; that is, the square has four folding symmetries. But when we fold a paper cutout of a rectangle that is not square, we find that this figure has only two folding symmetries. This is shown by the second figure above.

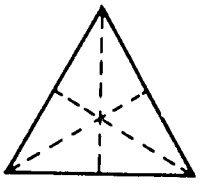
In a similar way we can discover differences among other figures. A

line segment has one folding symmetry; an isosceles triangle has one folding symmetry; an equilateral triangle has three folding symmetries.

Line segment



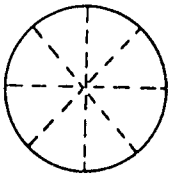
Isosceles triangle



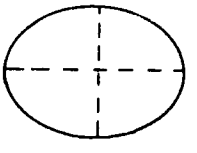
Equilateral triangle

We can use the following symmetries to show differences between a circle and an oval, and between a sphere and an egg-shaped solid.

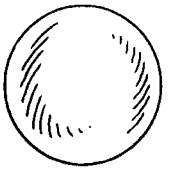
If a cutout of a circle is folded along <u>any</u> diameter, the parts will fit exactly; that is, the circle has as many folding symmetries as it has diameters. But a figure that is not a circle has possibly none, one, two or more folding symmetries, but not an unending number like a circle.



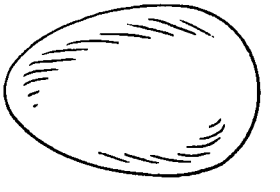
Circle



Oval



Sphere



Egg-shaped solid

To show symmetries of the sphere let a ball roll on a flat smooth surface. No matter in which direction it rolls, it will move along a straight line. This means that if we cut through the centre of the sphere from any direction the two pieces will be the same shape. But if an egg is allowed to roll on a flat surface, it will sometimes move along a crooked path.

The idea of symmetry will be used many times as the study of geometry unfolds.

STAGE 1: Segments, Lines, and Rays

Vocabulary: Curve, line segment, endpoint, line, ray

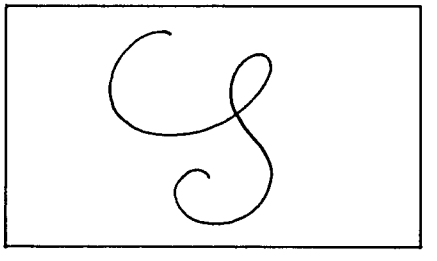
Materials: An 18" piece of string for each child; large rubber bands; drawing pins; pupil pages 16 - 17

Teaching Procedure

Activity 1: Revising Lines

Pupil page 16

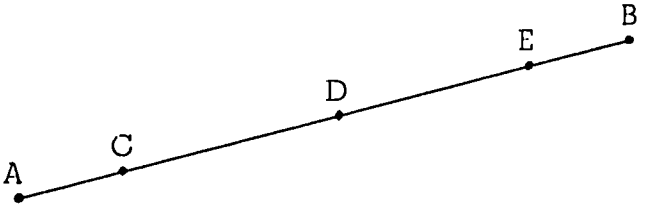
Give each pupil a piece of string. Ask him to place it on a sheet of paper and draw carefully on the paper the curve shown by the string. The curve might look like this:



Ask, "Is this curve a line segment?" (No) Get a pupil to hold the string on the paper to show a line segment. Draw the line segment on the paper. Let the pupil name the endpoints of the segment. (A and B) Say, "He has given the name $\overline{AB}$ to the line segment".

Let the other pupils show line segments with their strings.

Draw a line segment on the blackboard and label the endpoints A and B. Let children come to the blackboard and show and name other points on the line segment, such as C, D, E, and so on. Question the pupils until they agree that they can find and name as many points on a line segment as they please.

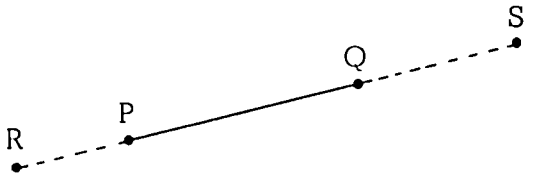


Say, "Line segment $\overline{AB}$ is shown on the blackboard. Are there other line segments shown on the blackboard?" Accept answers such as $\overline{AC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EB}$, $\overline{AD}$, $\overline{AE}$, $\overline{CE}$, $\overline{CB}$, $\overline{DB}$.

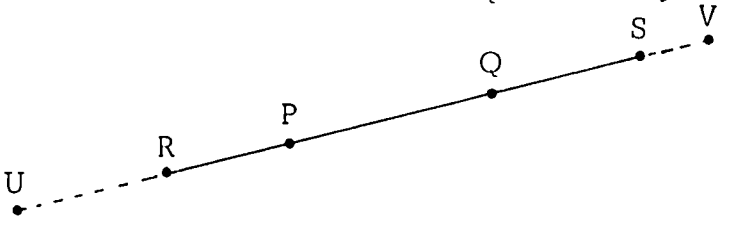
Say, "We name a line segment with the names of its two endpoints. We put a mark over the letters to show that it is a line segment. We name the line segment from A to B like this". Write, $\overline{AB}$.

Mark two points on the blackboard, and name them P and Q. Ask, "How many line segments can we draw with P and Q as endpoints?" Let the children discuss this question. Guide them to stretch a string between points P and Q to revise the fact that only one line segment has the points P and Q as endpoints.

Ask, "How many line segments can you draw so that P and Q are points on the line segment?" The children may not be able to answer this question immediately. Draw the line segment $\overline{PQ}$. Mark two more points R and S in line with P and Q, as in the diagram.

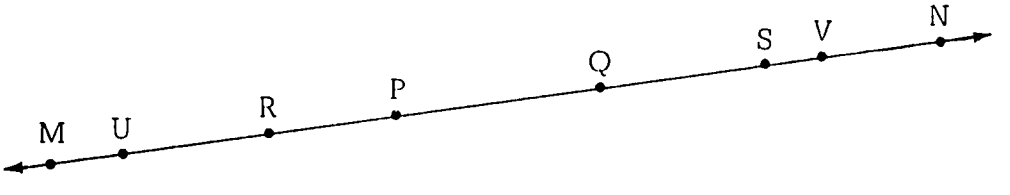


Ask, "Are points P and Q on the line segment $\overline{RS}$?" (Draw the line segment $\overline{RS}$ and let the class see that P and Q are on this segment.) Mark two more points U, V on the same diagram in line with P and Q as shown.



Ask, "Are points P and Q on the line segment $\overline{UV}$?" (Yes) Get pupils to find two more points, say M and N (that are not on $\overline{UV}$), so that P and Q will lie on the line segment $\overline{MN}$. (Let pupils use a stretched string to find M and N.) In this way get the class to draw longer and longer line segments on which points P and Q lie. Finally, ask again, "How many line segments can you draw through points P and Q?" (As many as we please)

Say, "Imagine that you draw all the line segments you can through P and Q, even beyond the blackboard. What is this figure called?" (A <u>line</u>) Ask pupils to mark arrows at the ends to show that a line goes on without end in both directions.



Again let the pupils find line segments on this line. ($\overline{MU}$, $\overline{UR}$, $\overline{RP}$, $\overline{MR}$, $\overline{MP}$, and so on) Help them to understand that there are many segments on the line. Ask "How many <u>lines</u> can be drawn through the two points P and Q?" (Only one)

Continue to question the class until these ideas are revised: one and only one line can be drawn through two points; a line goes on without end in both directions.

Ask questions such as these about the figure on the blackboard:

1. "How many lines can be drawn through P and Q?" (One) Call this line $\overleftrightarrow{PQ}$. Point out that the arrows above the letters show that it goes on without end.
2. "How many lines can be drawn through R and S?" (One) Call this line $\overleftrightarrow{RS}$.
3. "Is line $\overleftrightarrow{PQ}$ the same as line $\overleftrightarrow{RS}$?" (Yes)
4. "What other names can the line have?" ($\overleftrightarrow{MU}$, $\overleftrightarrow{MR}$, $\overleftrightarrow{UR}$, $\overleftrightarrow{UP}$, $\overleftrightarrow{UQ}$, $\overleftrightarrow{MN}$, and so on)

Continue with questions like these until the pupils understand that a line may have many names and that any two points on a line may be used to name the line. (A line segment may have only two names, because the two endpoints name the segment. The names $\overline{MN}$ and $\overline{NM}$ are the only names for one line segment.)

Let the children turn to pupil page 16. After they read the questions on the page and answer them, let them discuss the answers. In the first figure there are three lines; other names for $\overleftrightarrow{AC}$ are $\overleftrightarrow{AE}$, $\overleftrightarrow{EC}$, $\overleftrightarrow{EA}$, $\overleftrightarrow{CE}$, $\overleftrightarrow{CA}$; and other names for $\overleftrightarrow{BD}$ are $\overleftrightarrow{AD}$, $\overleftrightarrow{AB}$, $\overleftrightarrow{DA}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{DB}$. In the second figure, the line segments shown are $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{BC}$, $\overline{BD}$, $\overline{CD}$; there are six different line segments shown. In the third figure, there are 4 lines ($\overleftrightarrow{PW}$, $\overleftrightarrow{TW}$, $\overleftrightarrow{PR}$, $\overleftrightarrow{QR}$, and other names the lines may have); and there are 12 line segments ($\overline{PQ}$, $\overline{PW}$, $\overline{QW}$, $\overline{PT}$, $\overline{PR}$, $\overline{RT}$, $\overline{RV}$, $\overline{RQ}$, $\overline{VQ}$, $\overline{TV}$, $\overline{TW}$, $\overline{VW}$).

If pupils have difficulty in making the distinction between a line segment and a line, hold a piece of rubber band along a drawing of a line segment on the blackboard. Stretch the rubber band several times. Help the children imagine that the rubber band can be stretched on forever. As you do this, question

the pupils in a way that suggests to them that a line is formed by extending a line segment without end in both directions.

ACTIVITY 2: Rays **Pupil page 17**

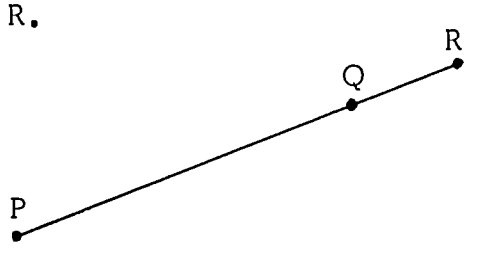
Briefly revise the ideas of line segments and lines in a way similar to this:

Draw a line segment. How many endpoints does it have? (Two)

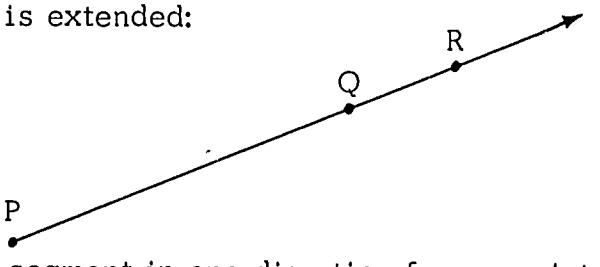
Call the endpoints A and B. What are names for the line segment? ($\overline{AB}$ or $\overline{BA}$)

Think about the line $\overleftrightarrow{AB}$. Does it have any endpoints? (No, a line goes on without end in both directions.)

Mark a point, P, on the blackboard. Invite a pupil to draw a line segment with one endpoint at P. (Let him use a straightedge.) Call the other endpoint of the segment Q. Get a second pupil to draw another line segment with one endpoint at P so that Q is a point on that line segment. Call the other endpoint of his segment R.



Ask, "Can the line segment be extended to longer and longer segments, always with one endpoint at P?" Let the class discuss this until they decide that the segments can be extended without end. Show this fact by drawing an arrow at the end that is extended:

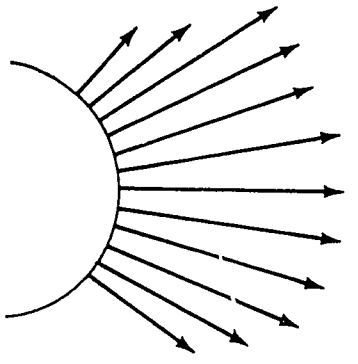


Say, "When we extend a segment in one direction from a point we get a <u>ray</u>. A ray has one endpoint".

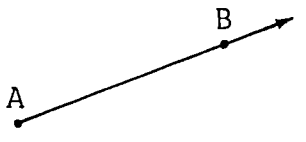
Let the pupils show rays by holding a piece of rubber band with one end held against a point on the blackboard and the rubber band stretched further and further. Help the children imagine that the band can be stretched on

forever. As you do this, guide the pupils to think that a ray is formed by extending a line segment without end in one direction.

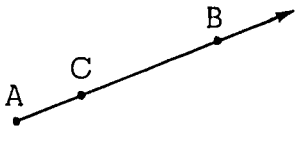
Ask the class to give examples of rays. Replies may include, among others: The light from an electric torch (the light bulb is the endpoint and the light moves along a ray) and a ray from the sun. (The sun is the endpoint.)



Draw a ray on the blackboard from point A through point B and say, "There is a special symbol for a ray. The ray shown on the blackboard is named $\overrightarrow{AB}$. (Write "$\overrightarrow{AB}$") We name a ray by naming its endpoint first then any other point on it. The name has an arrow above it". ($\overrightarrow{AB}$)



Go on, "The arrow shows that the ray starts at A and goes on through B without end". Name another point C on the ray and ask, "Is ray $\overrightarrow{AB}$ the same ray as $\overrightarrow{AC}$? (Yes) Is $\overrightarrow{AB}$ the same ray as $\overrightarrow{BA}$?" (No, $\overrightarrow{BA}$ has its endpoint at B and $\overrightarrow{AB}$ has its endpoint at A.)

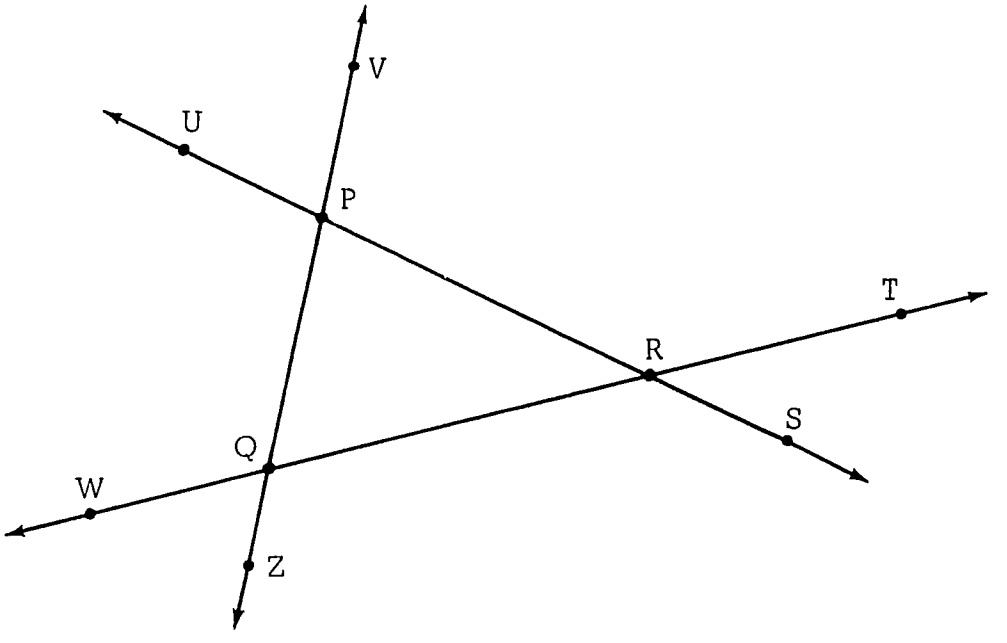


Continue by letting the class discuss the difference between line segments, lines and rays. Let a pupil draw a line segment on the blackboard; let another pupil extend this line segment and draw an arrow at one end to show a ray; and let still another extend this ray in the other direction and draw an arrow on that end to show a line. Let each pupil name the figure he draws.

For example, if the first pupil names his line segment $\overline{DE}$, then the second pupil may name his ray $\overrightarrow{DE}$ and the third name his line $\overleftrightarrow{DE}$.

Question the pupils about names, "Do $\overline{DE}$ and $\overline{ED}$ name the same line segment? (Yes) Do $\overrightarrow{DE}$ and $\overrightarrow{ED}$ name the same ray? (No) What is the difference between $\overrightarrow{DE}$ and $\overrightarrow{ED}$? Do $\overleftrightarrow{DE}$ and $\overleftrightarrow{ED}$ name the same line?" (Yes)

On a figure such as the one below, let pupils name all the rays, line segments, and lines they see:



(In the figure there are eighteen segments, each with two names; eighteen rays, some with several names; and three lines, each with many names.)

Get the pupils to answer the questions on pupil page 17.

STAGE 2: Angles

Vocabulary: Angle, vertex

Materials: Objects such as rulers, set squares, compasses, T-squares, cubes, balls, bottles; pieces of wire; a few sticks; large rubber bands; pupil pages 18-21

Teaching Procedure

ACTIVITY 1: Revision of Angles **Pupil pages 18-19**

Ask the children to use objects in the classroom to show angles. Get them to show an angle by placing a finger of each hand at the corner of an object (vertex of an angle) and then moving the two fingers along the edges of the object (sides of the angle). They can use objects such as blackboards, tables, chairs, rulers, set squares, compasses, T-squares, boxes, classroom door, windows, and so on. Let them point out objects which do not show angles, for example, balls and eggs.

Ask some of the children to bend pieces of wire to show angles. Let other children show angles with two sticks. Get them to trace out each of the angles using a finger of each hand, starting always at the vertex and moving along the sides of the angle.

Mark and name three points A, B, and C on the blackboard. Ask the children to use these points to draw two line segments and tell what figure is shown. (Angle) Get them to name the angle. (They may have drawn one of three angles: angle ABC, angle BAC, or angle ACB.)

B.

A. C.

Suppose the children draw and name angle ABC. Tell them that point B is the vertex of the angle. Let them tell what line segments form the angle. ($\overline{AB}$ and $\overline{BC}$)

Ask them to draw two other angles on the same figure, using the points A, B, and C. Get them to name the vertex of each and the line segments which form the angles.

Get the children to cross two sticks to show angles. Let them trace out the four angles formed and point to the vertex of each.

Let the children draw other angles on the blackboard, name them, trace out each angle, point to and name each vertex.

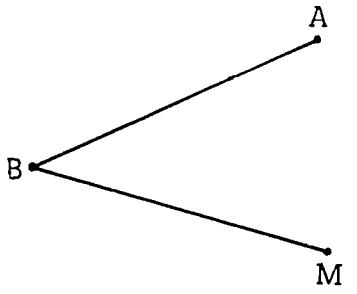
Let the children follow the same plan with the angles on pupil pages 18-19.

ACTIVITY 2: Angles

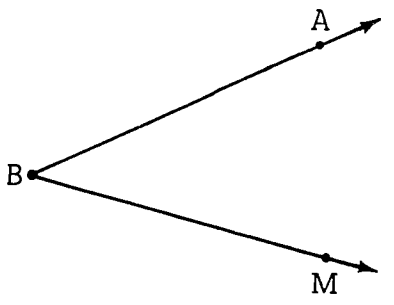
Pupil pages 20-21

The purpose of this activity is to introduce the idea of an angle as a pair of rays with the same endpoint.

Mark a point, B, on tne blackboard. Get a child to draw a line segment with B as an endpoint. Ask, "Does this show an angle?" (No) Let another child draw a second line segment with B as an endpoint. Ask, "Does this show an angle?" (Yes) Let the children name the two segments, the angle and the vertex of the angle.

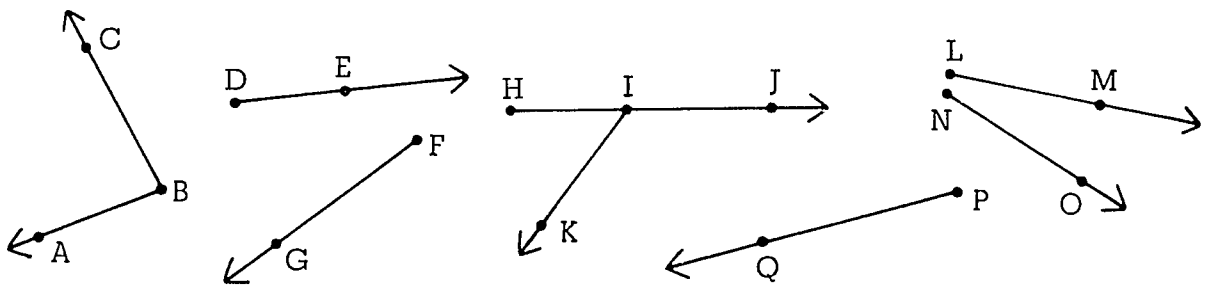


Hold two pieces of rubber bands each with an end at B, and let two children stretch the rubber bands along $\overline{BA}$ and $\overline{BM}$ to show two rays $\overrightarrow{BA}$ and $\overrightarrow{BM}$. Get the pupils to imagine that the bands can stretch forever. Let them tell what the pieces of rubber bands show. (Two rays with endpoints at B) Get other children to trace along the pieces of rubber bands to draw pictures of the rays.



Say, as you point to the last picture, "This is a picture of an angle. <u>An angle is formed by two rays that have the same endpoint".</u> Ask, "What is the name of the angle? (Angle ABM or angle MBA) What is its vertex?" (B)

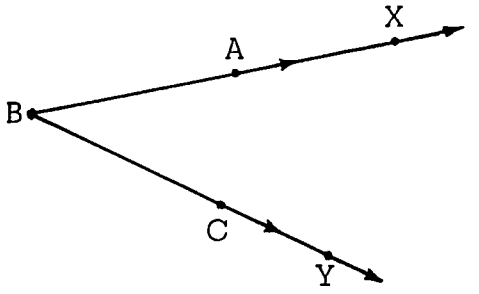
Draw several rays on the blackboard. Make some with the same endpoints:



Get the children to trace out the angles, to name the rays that form them, and to name the endpoints of the rays.

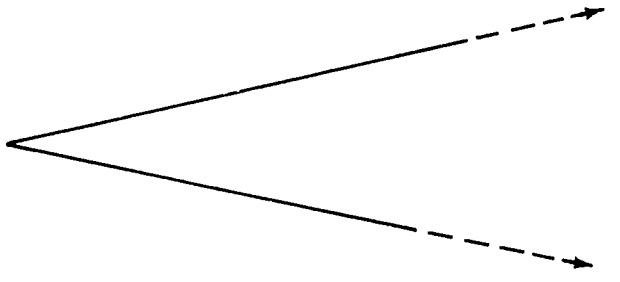
Let the children show other angles by drawing pairs of rays in different directions and with different names.

Draw the figure below on the blackboard:



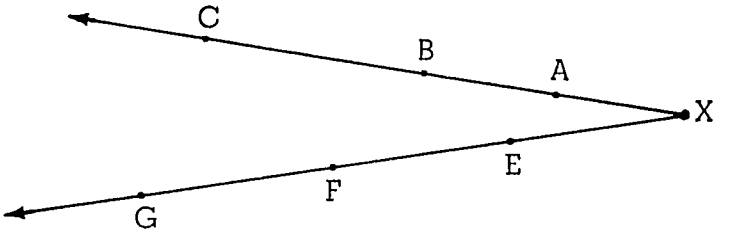
Ask, "Are angle ABC and angle XBY the same angle?" (Yes) Help the children decide that $\overrightarrow{BA}$ and $\overrightarrow{BX}$ are the same ray, that $\overrightarrow{BC}$ and $\overrightarrow{BY}$ are the same ray, and that angle ABC and angle XBY are the same angle.

Get two pupils to hold against the blackboard the ends of a piece of rubber band and a third child to hold the middle of the band against the blackboard, so that the band forms an angle. Ask the children to look at the angle formed. Let a child trace it on the blackboard.



Let the two pupils holding the ends stretch the rubber. Say, "The rubber band now shows an angle. Is this angle the same angle you showed before?" (Yes) Let a child trace the rays on the blackboard as they are now shown. Continue to stretch the rubber band and let the children decide each time that they show the same angle. Help them decide that the rays go on and on but the angle formed is the same angle.

Ask a child to draw two rays with a common endpoint. Ask another child to name the endpoint and three other points on each ray.



Ask, "How many angles are formed by these two rays? (One) What are some names for the angle? (AXE, AXF, CXG, CXE, FXC, and so on.) Which letter occurs in every name? (X) Can you name the angle without this letter? (No) What do you call the point named by that letter?" (Vertex)

Let the class turn to pupil pages 20–21 and answer the questions about angles.

STAGE 3: Congruent Angles

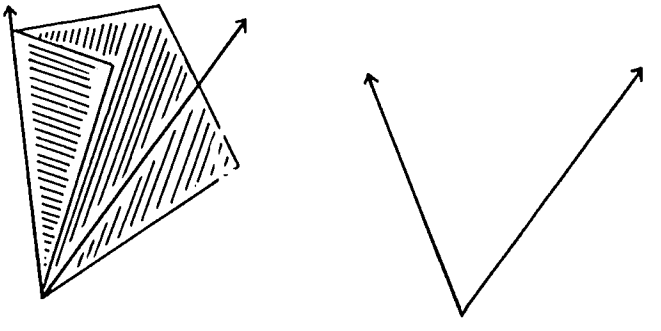
Vocabulary: Greatest angle, smaller angle, congruent angles

Materials: A small sheet of paper (or a piece of stiff wire) for each pupil or group of pupils, pupil pages 22-23

Teaching Procedure

ACTIVITY 1: Comparing Sizes of Angles **Pupil page 22**

Draw two angles of different sizes on the blackboard. Choose the one which appears smaller and fold a sheet of paper to fit this angle.

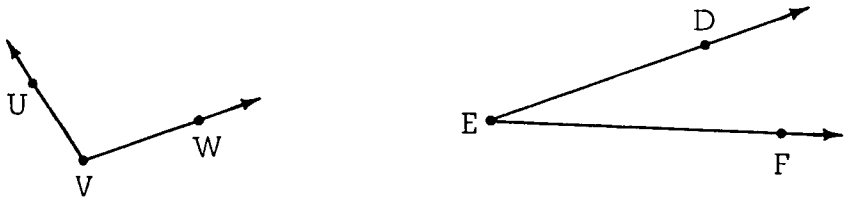


Let the children talk about the fact that the edges of the folded paper show an angle of the same size as the first angle drawn on the blackboard. Move the folded paper to the other angle and let the class decide which of the two angles is greater, which is smaller. (Let the vertex of one fall on the

vertex of the other.)

Note: Bent wire, if available, is a better illustration of the angle than folded paper. The wire shows more clearly the two rays that form the angle.

Let the pupils compare the angles shown on pupil page 22. Get them to make copies of the angles with either folded paper or bent wire. Pairs of children may work together. Help them to understand that changes in the lengths of the rays drawn do not change the size of the angle. For example, in the picture below, angle UVW is greater than angle DEF, even though its rays as drawn appear to be shorter.



After the pupils have written answers to the four questions on pupil page 22 let them talk about their answers.(Angle UVW is greater than each other angle; Angles RPQ, DEF, and XYZ are the same size; Angle CAB is smaller than each other angle.)

ACTIVITY 2: Congruence of Angles **Pupil page 23**

In previous activities the pupils described two line segments of equal length as congruent segments. To help them decide whether two segments were congruent they measured the segments with a piece of string or with a ruler.

In this activity, guide the pupils to describe two angles of the same size as congruent angles. On pupil page 23 there are several figures shown. Get the pupils to name angles that are congruent. Let them fold papers or bend wires to help them decide.

After the pupils have answered the questions in their books, ask them to name the greatest angle and the smallest angle in the figures of each exercise.

STAGE 4: Triangles

Vocabulary: Triangle, side of triangle, right angle, right-angled triangle, isosceles triangle, equilateral triangle

Materials: A sheet of paper for each child; paper cutouts of triangles, some equilateral, some isosceles, some neither; pupil pages 24-26

Teaching Procedure

ACTIVITY 1: Revision of Triangles and Right Angles **Pupil page 24**

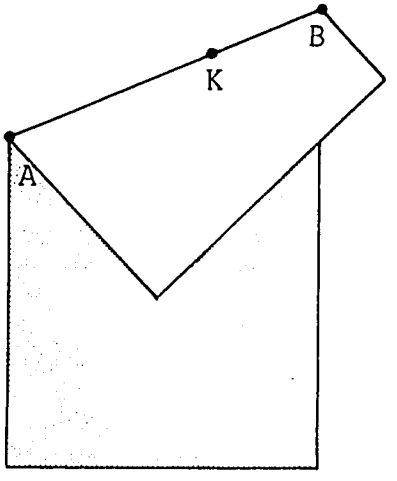
Mark three points on the blackboard (not all on a line) and get the pupils to draw line segments joining the three points. Ask, "What is the figure formed?" (Triangle) Let the children point out the vertices, letter the vertices (A, B, C) and name the triangle (Triangle ABC). Ask, "How many angles has the triangle?" (Three) Get pupils to use a finger of each hand to trace each angle. (Guide them to begin at the vertex.) Let them name the angles. Ask, "How many line segments form the triangle?" (Three) "Name them". ($\overline{AB}$, $\overline{BC}$, $\overline{AC}$). Explain that these are called the sides of the triangle. Guide the children to say, "A triangle has three sides".

Ask the children to draw several triangles in their exercise books, name the vertices and name the triangles.

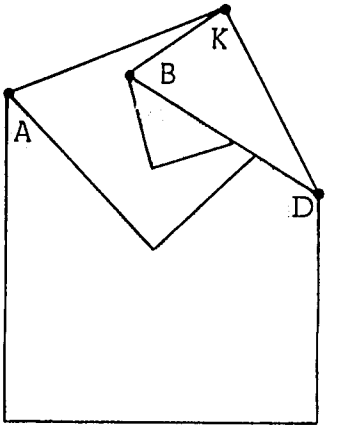
Fold a large piece of paper twice so the two creases show right angles. Then let each child fold a piece of paper to show creases at right angles. See the figure on page 47.

Fold the paper like this: first, fold the paper to form any crease AB; second, choose any point K on AB, and fold again so that KB falls along KA. Unfold the paper.

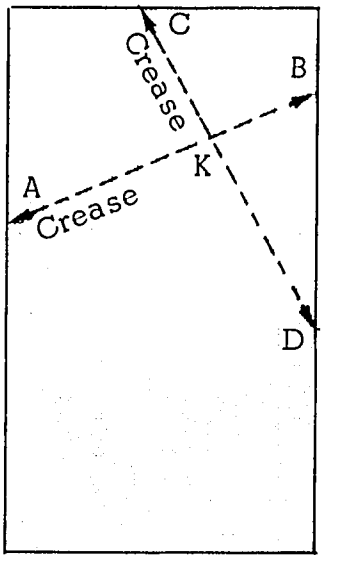
Get the pupils to trace with two fingers the angles formed by the creases. Ask, "How many angles are formed?" (Four) Let the pupils draw the rays which form the angles.



First Fold



Second Fold



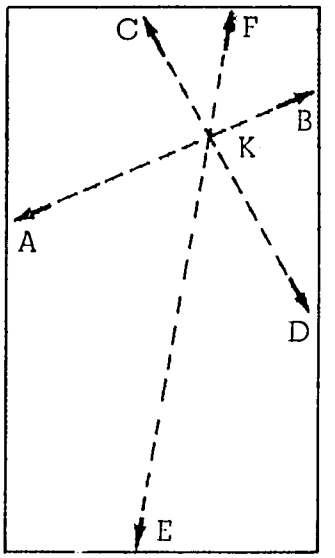
Unfolded

Let the children compare the four angles formed by the creases. Ask, "Which angle is largest? (They are the same size.) What do we call angles that are the same size?" (Congruent angles) Guide the children to decide that there are four congruent angles with vertex K. Say, "Angles like each of these have a special name. Do you know the name?" (They may recall from Primary Two and Three that the angle shown by a square corner is called a right angle.) If not, tell them that each angle like these is called a right angle.

Let a child fold his paper to show a right angle. Get him to fit this right angle formed from folded paper to each of the angles formed by the creases of another pupil's unfolded paper. Get other pairs of children to do the same activity.

Name the creases $\overline{AB}$ and $\overline{CD}$ on your large paper as shown in the picture above. Show by arrows and get the pupils to tell that the angles are formed by pairs of rays.

Make other creases in your paper through point K. (For example, FE) Get the children to do the same with their papers. Unfolded, the papers may look like this:



Ask the children to name some angles shown and compare them to right angles. (Let them use a folded-paper right angle.)

Question the children until they say that some angles are smaller than a right angle and some angles are greater than a right angle. Get the children to trace out angles shown on objects in the classroom and to tell whether the angles are right angles, greater than a right angle, or smaller than a right angle.

Ask the pupils to do the exercises on pupil page 24.

ACTIVITY 2: Some Special Triangles **Pupil pages 25-26**

Ask each child to fold a paper to show a right angle. On the blackboard draw a triangle, ABC, that has one right angle, and a triangle, MNO, that does not have a right angle.

Get the children to trace out the line segments that form triangle ABC. Let them trace out the three angles formed. Ask, "Are any of the three angles right angles?" If they guess, let them show with their folded papers that the angle is or is not a right angle. If they do not know, suggest that they use their folded papers to decide.

Let them go through the same activity using triangle MNO.

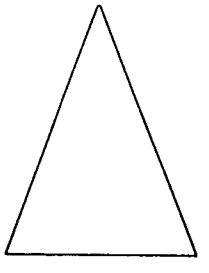
Tell the children that one of the angles of triangle ABC is a right angle. The triangle is called a right-angled triangle. Ask, "Is triangle MNO a right-angled triangle?" (No. It has no right angle.)

Mark several sets of three points on the blackboard. Let the children draw line segments to make triangles. (Some of the points should be chosen so that right-angled triangles are drawn.) Ask whether each triangle is a right-angled triangle. Suggest that the pupils place a folded-paper right angle over the angles of each triangle to decide whether the triangle has a right angle.

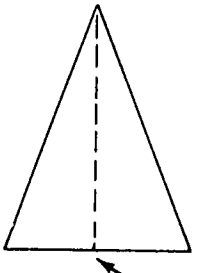
On pupil page 25 there are many figures containing right-angled triangles. Let the pupils find and name each right-angled triangle.

Show the class several paper cutouts of triangles. Be sure that the cutouts have been made so that some of the triangles have no sides congruent, some have two sides congruent, and some have all three sides congruent.

Say, "Some triangles are different from others. You can find the differences by folding cutouts of the triangles". Hold up a cutout that has two sides congruent and ask a child to help you fold it. Say, "Fold the cutout so that you show two parts that fit exactly". Let the pupil try several folds until he finds a fold that gives two parts exactly the same size. Unfold the cutout and draw a line segment along the crease.



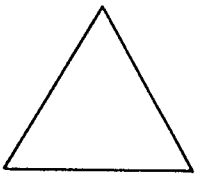
Two sides congruent



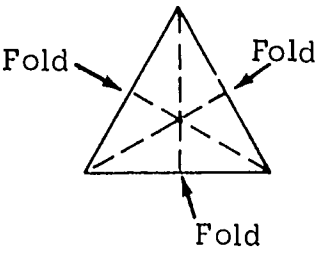
Fold

Ask, "Can you make a different fold that makes two other parts that fit exactly?" (No) Say, "This triangle has only one fold that makes two parts that fit exactly". (Later the children will use the words "folding symmetry" to mean a folding that gives two parts that fit exactly.)

Hold up a triangular cutout with three sides congruent. Let pupils in turn try to find as many folds as they can which give two parts that fit exactly. They will find three such folds. Draw a line segment along each of these three creases. (As shown on page 50.)



Three sides congruent



Hold up a triangular cutout with none of its sides congruent to any other. Let the pupils try to fold this cutout to show two parts that fit exactly. They will discover that there is no way to fold the triangle to make two parts that fit exactly.

Let the children discuss the differences in the three triangles; repeat the folding on other triangular cutouts. Ask questions about the sides of the triangles, such as, "If a triangle has one fold how many sides of the triangle are congruent?" (Let the pupils look at the folded cutout and see that two sides fit each other exactly. They will say, "This triangle has two congruent sides".)

Say, "Any triangle with two congruent sides is an isosceles triangle". Let the class decide which of the cutouts show isosceles triangles and put these cutouts together in a pile. Ask, "If a triangle has three folds (three folding symmetries) how many sides of the triangle are congruent?" Let the pupils look at the cutout with three folds and decide that the triangle has all three sides congruent. Say, "A triangle with all three sides congruent is called an equilateral triangle". Ask the class to decide which of the cutouts show equilateral triangles and put these cutouts together in a pile.

The cutouts are now in three piles: isosceles triangles, equilateral triangles, and all the other triangles. Let the children talk about the triangles in the last pile and decide, "No triangle in this pile has any pair of its sides congruent".

On pupil page 26 there are many triangles shown. Let the pupils use pieces of string to decide which triangles have two sides congruent (isosceles triangles), which have all three congruent (equilateral triangles), and which have no pairs of sides congruent. Notice that all the triangles that are equilateral are also isosceles.

STAGE 5: Rectangles

Vocabulary: Rectangle, square

Materials: Cutouts of four-sided figures, including many rectangles and squares; papers folded to form right angles; pupil page 27

Teaching Procedure

ACTIVITY 1: Revising Rectangles and Squares

Pupil page 27

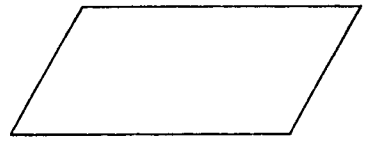
Draw several four-sided figures on the blackboard. Make at least one rectangle, one square, and include the figure shown in the drawing below. Ask, "How many line segments are in each of these figures?" (Four. These are four-sided figures.) "Are any of the figures rectangles?"

Let a pupil point out a rectangle and tell how he knows it is a rectangle. Let other pupils decide whether the answer is complete. Guide them to agree that a four-sided figure is a rectangle if:

- each side is congruent to the opposite side, and
- all four angles are right angles.

Let a pupil test the figure to show that it is a rectangle. Let him compare the lengths of its sides with a piece of string. Let him test its angles with a folded-paper right angle to see that all its angles are right angles.

Point to the figure like this one on the blackboard:



Ask, "Is this a rectangle?" Let a pupil test with a string and find that the opposite sides are congruent. Ask, "Is it a rectangle?" If anyone says "Yes", let him test the angles with a folded-paper right angle. (He finds that its angles are not right angles and it is not a rectangle.)

Point to a square on the blackboard and ask whether it is a rectangle. If someone says "No", let him compare opposite sides and check the angles. Keep questioning until the class agrees that: the figure is a rectangle; the

figure is a square; and every square is a rectangle. Let the pupils point out other figures that are not rectangles and tell why they are not rectangles.

Continue by asking:

How many sides does a rectangle have? (Four)

How many vertices does a rectangle have? (Four)

How many angles does a rectangle have? (Four)

What kind of angles does a rectangle have? (All the angles are right angles)

Is every square a rectangle? (Yes)

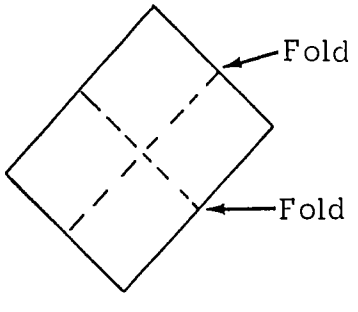
What figures in the classroom show rectangles?

Continue the revision by letting the class turn to pupil page 27 and complete the sentences about four-sided figures. The children may not need to measure the figures for congruent parts. Let them decide by inspection and by checking with folded-paper right angles.

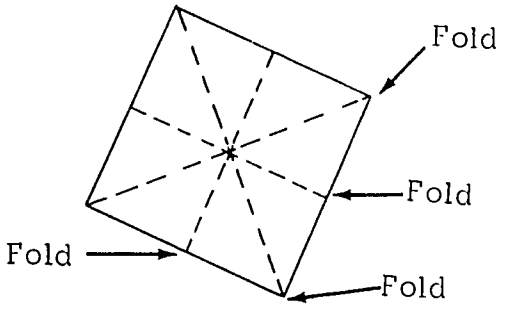
ACTIVITY 2: Symmetries of Rectangles

Show the class some paper cutouts of four-sided figures. Include cutouts of squares, of rectangles that are not squares, and of other four-sided figures.

Ask the children to tell which cutouts show rectangles. Say, "We can fold the cutouts and find some difference between the rectangles and the other four-sided figures". Let a pupil try to fold a rectangular cutout that is not a square so that the two parts formed fit exactly. When he makes the fold, draw a line segment along the crease. Ask a child to make a different fold. When this has been done, let other children try to make still different folds, until the class decides that only two folds can be made.



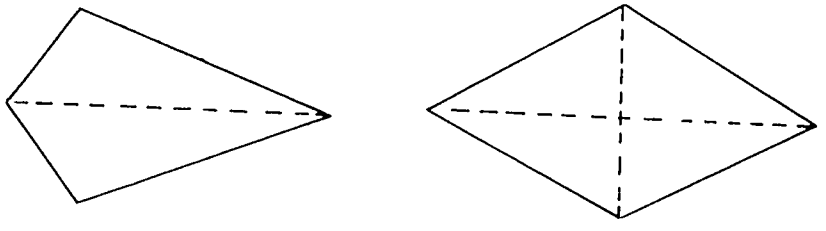
Rectangle



Square

Let the pupils find all the folds (symmetries) they can make in a square. Help them until they decide that a square has four folds (as shown in the previous figure). Encourage the class to talk about the different number of folds in the rectangle (2) and the square (4).

Let pupils try to fold other four-sided figures that are not rectangles. Usually there are no folds (no symmetries). It is possible, as shown below, to have one or two folds even though the figure is not a rectangle.



When the figure has two folds (two symmetries) as in the rectangle (or in a figure like the one shown above), guide the children to notice that the two creases form right angles. Let the children continue to discover such facts as:

- When the rectangle is folded each time, it shows the opposite sides are congruent and pairs of angles are congruent.
- When the rectangle is folded twice and unfolded, the two creases separate the figure into four congruent rectangles.
- When the square is folded four times and unfolded, the four creases separate the figure into eight congruent right-angled triangles.

STAGE 6: Circles

Vocabulary: Circle, centre, radius, diameter

Materials: A piece of string for each pupil (about 7" to 10" long); a short stick (about 3" long) for each pupil; pupil pages 28-29

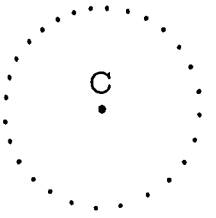
Teaching Procedure

ACTIVITY 1: Revising Circles

Pupil page 28

Give each pupil a short straight stick. Ask the pupils to mark a point in their exercise books and name it C. With one end of the stick at C let

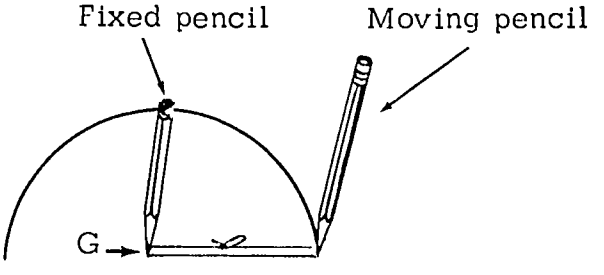
them mark a point at the other end of the stick. Ask them to do this for many positions of the stick with one end at C, getting many points on the paper like the picture below.



Ask the pupils to sketch a curve through these points and to tell what the figure is. (Circle) Explain that point C is called the centre of the circle. Guide the children to say that each point they marked was a stick length from the centre.

Let the class tell what objects in the classroom and in their homes show circles.

Say, "You can make a better drawing of a circle". Give each pupil two pencils and a piece of string about 7" to 10" inches long. Get them to tie it into a loop. Guide them to draw a circle in this way: mark a point G in their exercise books; place one pencil at G and hold it fixed; put the loop of string around the pencil; stretch the string with a second pencil; and move the second pencil around with the string tight. A circle is drawn with point G as the centre.



Let the children practice drawing several circles. They may exchange loops of string so they draw circles with different radii.

Draw a circle on the blackboard. Draw a line segment from the centre to any point on the circle. Say, "This line segment is called a radius of the circle". Let the pupils draw several radii on the circles they drew in their

exercise books. Get them to tell how many radii they can draw in a circle. (As many as we please) Ask, "Which radius in a circle is the longest?" Let them talk about this until it is decided that every radius in a circle is just as long as another. That is, all radii of a circle are congruent.

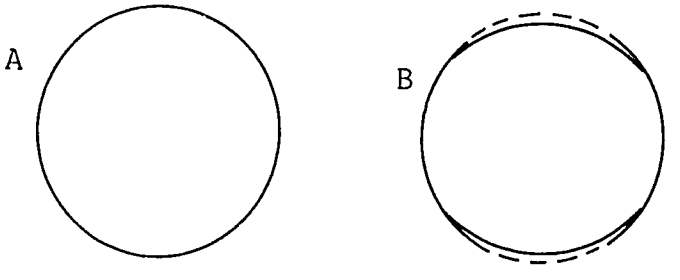
Let a child draw on the blackboard a line through the centre of a circle. Mark the points where this line crosses the circle. Explain that any line segment through the centre with its endpoints on the circle is called a diameter of the circle. Let the children draw several other diameters. Ask, "How many diameters can you draw in a circle? (As many as we please) Which diameter of a circle is longest?" (They are all congruent.)

Let the pupils practice drawing radii and diameters in the circles they drew in their exercise books. A pupil may point out that a diameter is twice as long as a radius.

Ask the class to complete the sentences on pupil page 28.

ACTIVITY 2: Symmetries of Circles

Make large cutouts of the two figures shown below. Figure A is a circle and figure B is a slightly flattened oval figure. (This can be drawn by trimming off the part of the circle shown by the dotted lines.)



Ask, "Are these figures circles?" Let the children think about this question. Encourage them to tell ways of deciding whether either figure is a circle.

Guide the questioning until a pupil (or you) suggests trying to fold the figures to show two parts that fit exactly. They will find that figure A can be folded many times. In fact, it can be folded in any direction so that two parts fit exactly. Draw line segments along several creases and let the class notice that all the line segments pass through a single point.

Ask, "Is figure A a circle? (Yes) What are the lines drawn along the creases? (Diameters) Where is the centre of the circle?" (At the point where the diameters cross.)

Let the pupils try to fold figure B to show two parts that fit exactly. They will find that figure B has only two folds. There are some directions in which the figure cannot be folded to form two parts that fit exactly. Ask, "Is figure B a circle?" (No)

Let the pupils talk about these two figures and say that a circle can be folded in any direction and every crease shows a diameter. You may wish to let the children fold a sheet of paper several times and cut designs from the folded paper. When the paper is unfolded, the cutout will have as least as many symmetries as the number of folds.

ACTIVITY 3: Revising Plane Figures **Pupil page 29**

Let the class revise the geometric figures they studied in this unit. Let them do this by turning to pupil page 29 and pointing out and describing all the figures they recognize:

Line segments, lines, angles, curves, triangles, right-angled triangles, isosceles triangles, rectangles, squares, circles.

If the pupils notice some solid figures in the picture let them describe the figures in preparation for the next stage.

STAGE 7: Solid Figures

Vocabulary: Cube, sphere, rectangular solid, cylinder, cone, edge, face, vertex

Materials: Match boxes, other boxes, balls, round tins, sugar cubes, and other objects showing the shapes of the solid figures mentioned in the vocabulary; pupil pages 30-31

Teaching Procedure

ACTIVITY 1: Names of Solid Figures **Pupil page 30**

The pupils have learned the names of some solid figures such as cube, ball, and box in Primary One, Two, and Three. Revise these figures as you give pupils objects to hold and feel. Let them choose all the objects with the shape of a box and place them on one section of a table, and on another section place all the objects with the shape of a ball. Among the box-shaped objects let them pick those that show cubes and place these to one side. Ask them to take from the table other objects that do not show boxes or balls.

Explain that the objects shaped like balls show a figure called a sphere. Place an egg on the table and ask whether this also shows a sphere. (No) Let the pupils watch a ball roll down a smooth desk top and notice that it rolls in a straight line. Then let the egg roll down the desk top and notice that it rolls in a crooked path, showing that the egg is not a sphere.

Guide the children to tell about the difference between boxes that show cubes and boxes that do not show cubes. (All the faces of a box show rectangles. All the faces of a cube show rectangles that are squares.) Explain that objects shaped like boxes are called rectangular solids because their faces all show rectangles.

Let the pupils place all the round tins and objects of the same shape at one spot on the table. Explain that all objects shaped like these show a figure that is called a cylinder.

Finally, show the class objects shaped like a cone and revise the name cone.

As each of these solid figures is named, let pupils tell about objects around them or in their homes that show these figures.

Many examples are possible, depending on the region of Africa in which you live. Some examples are:

Sphere: balls, map globe, marbles, oranges, some seeds

Rectangular solid: some books, match boxes, chalk boxes, bricks, some bars of soap, some mud bricks

Cube: some ice cubes, sugar cubes, dice, some mud bricks

Cylinder: oil cans, logs, pieces of sugar cane, water pipes, round pillars, some chimneys

Cone: paper cones for nuts, caps of a coffee kettle, nose cone of a rocket, roofs of round huts

On pupil page 30 let the pupils name each of the solid figures shown. (Figures A and H are spheres; E, G and J are cylinders; C and I are cones; D and F are cubes; B, D, F and K are rectangular solids.)

(Note: In geometry we agree that a square is a special kind of a rectangle with all sides congruent, and that a cube is a special kind of a rectangular solid with all square faces.)

ACTIVITY 2: Describing Solid Figures **Pupil page 31**

In Activity 1, the pupils revised the names of some solid figures and learned other names. In this activity, let the pupils describe the solid figures in terms of faces, vertices, and edges.

Hold up a rectangular solid, say a chalk box, and ask the class to describe the figure it shows. A pupil may say that it has corners; let him then point to the corners and tell what the corners show. (Right angles) A child may say it has edges; let him run a finger along the edges and tell what the edges show. (Line segments) A child may say that the sides of the box show rectangles; let him show a side of the box and say, "The sides are called faces". Let the children tell how many faces there are, and that the faces have edges. A child may say the box shows points at the corners; Let him show a point at a corner and say, "The points at the corners are called vertices". Each tip is called a vertex. Let them continue to talk about the faces, the edges along the faces, and the vertices at the corners of the faces.

Hold up a round tin and ask the class to describe this solid figure. Accept any reasonable comments and use them in the discussion. For example, a child may say that it has two flat ends shaped like circles. Another may say that it has no corners and does not show any angles or vertices.

Ask, "How are the cylinder and the sphere different from each other?" (The cylinder has two flat ends, but the sphere has no flat parts.)

Ask, "How are the cylinder and the cone different from each other?" (The cylinder has two flat ends, but the cone has only one. Both show circles. The cylinder will roll in a straight line, but a cone will roll in a circle.)

Let the class then turn to pupil page 31 and answer the questions about solid figures. They may use solid figures like those shown on the page to help them answer the questions.

UNIT 4
Operations

Objectives

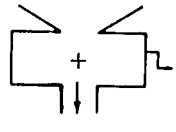
1. To solve mathematical sentences which use several operations
2. To revise the meaning of an operation
3. To revise the meaning of a property of an operation
4. To revise the properties of addition and multiplication: associative property; commutative property; distributive property; properties of zero and one
5. To learn the idea of closure of a set of numbers under an operation

Background Information for Teachers

In Unit 1 we discussed the idea of a whole number and different ways of writing numerals for whole numbers. In Unit 2 we discussed operations on whole numbers. The operations used were addition, subtraction, multiplication, and division. These are sometimes called binary operations because they assign to each pair of whole numbers exactly one number.

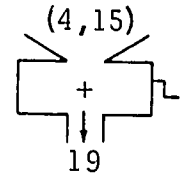
The purpose of this Unit is to help the pupils understand better the meaning of an operation. They are familiar with some properties of operations. For example, they can use the distributive property of multiplication to find products of two numbers. In this unit other properties of operations are discussed. We will emphasize that not all operations have the same properties.

There is a simple way to show that an operation assigns one number to a pair of numbers. Pretend to have a machine which performs the operation. An addition machine may look like this:



We put first one number and then another into the machine, turn the handle,

and a number pops out at the bottom. Here is an example shown for the sentence: $4 + 15 = 19$.



First the number 4 and then the number 15 are put into the machine. We turn the handle. The number 19 pops out. We will use operation machines in several of the activities of this unit.

<u>Commutative Property</u> An operation on whole numbers is commutative if it assigns the same number to the pair (a, b) as it assigns to the pair (b, a) where <u>a</u> and <u>b</u> are any whole numbers. For example:

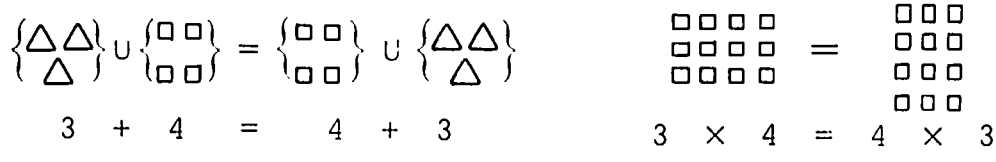
Addition assigns 7 to (3, 4) and also to (4, 3);

Multiplication assigns 12 to (3, 4) and also to (4, 3).

We say,

$$3 + 4 = 4 + 3 \quad \text{and} \quad 3 \times 4 = 4 \times 3$$

We can show these sentences with sets of objects:



Addition and multiplication are commutative operations.

Subtraction and division do <u>not</u> have the commutative property. For example:

$$9 \div 3 \neq 3 \div 9$$

and

$$7 - 3 \neq 3 - 7$$

<u>Associative Property</u> Suppose we wish to add three numbers. Since addition is a binary operation, we first add a pair of numbers. Then we add this sum to the third number. For example, $5 + 4 + 7$ can have these two meanings:

1. $(5 + 4) + 7 = 9 + 7$ and $9 + 7 = 16$
2. $5 + (4 + 7) = 5 + 11$ and $5 + 11 = 16$

Notice that the result is the same in both 1 and 2. This is an example of the associative property. The result is the same with either grouping so:

$$(5 + 4) + 7 = 5 + (4 + 7)$$

Both addition and multiplication have the associative property. Let a, b, and c be any whole numbers. Then these sentences are always true:

$$a + (b + c) = (a + b) + c$$

and

$$a \times (b \times c) = (a \times b) \times c$$

Subtraction and division do not have the associative property. For example:

$$(9 - 5) - 3 \neq 9 - (5 - 3)$$

$$(12 \div 6) \div 3 \neq 12 \div (6 \div 3)$$

Distributive Property This property was discussed in the background for Unit 2. It is mentioned again here because of its importance.

Multiplication is distributive over addition.

If a, b, and c are whole numbers, then

$a \times (b + c) = (a \times b) + (a \times c)$ is always true.

For example:

$$7 \times (10 + 7) = 7 \times (17) \quad \text{and} \quad (7 \times 10) + (7 \times 7) = 70 + 49$$
$$= 119 \qquad\qquad\qquad\qquad = 119$$

Hence,

$$7 \times (10 + 7) = (7 \times 10) + (7 \times 7)$$

Addition Property of Zero For any whole number a, the sentence $a + 0 = a$ is true. For example:

$$5 + 0 = 5 \quad \text{and} \quad 0 + 5 = 5$$

Multiplication Property of Zero For any whole number a, the sentence $0 \times a = 0$ is true. For example:

$$5 \times 0 = 0 \quad \text{and} \quad 0 \times 5 = 0$$

Multiplication Property of One For any whole number a, the sentence $a \times 1 = a$ is true. For example:

$$5 \times 1 = 5 \quad \text{and} \quad 1 \times 5 = 5$$

Closure Property We say that the set of whole numbers is closed under

a certain operation if the operation assigns to every pair of whole numbers one whole number.

The set of whole numbers is closed under addition. If we add two whole numbers, the sum is always a whole number.

The set of whole numbers is closed under multiplication. If we multiply two whole numbers, the product is always a whole number.

The set of whole numbers is not closed under subtraction. For example, there is no whole number n that makes the sentence $5 - 7 = n$ true. Later the idea of the opposite of a whole number is introduced to get a set of numbers that is closed under subtraction.

The set of whole numbers is not closed under division. For example, there is no whole number n that makes the sentence $5 \div 7 = n$ true. The number that makes the sentence true is $\frac{5}{7}$, since $5 \div 7$ is another name for the fraction $\frac{5}{7}$. But $\frac{5}{7}$ is not a whole number.

The emphasis in this Unit is on understanding the meaning of these properties and not on the names of the properties. In the activities, strange operations will be used to extend the pupils' understanding of the properties discussed.

STAGE 1: Sentences with Several Operations

Vocabulary: Array

Materials: Counters such as bottle tops; pupil pages 32-35

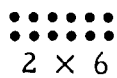
Teaching Procedure

ACTIVITY 1: Regrouping Sets and Renaming Numbers

Give each of the pupils 12 counters. Ask them to arrange the counters in a rectangular array of 3 rows of 4 counters each. Help the children write the mathematical sentence the array shows:

$$12 = 3 \times 4$$

Get the children to show other arrays with their 12 counters and for each help them to write a mathematical sentence. Some of the arrays and sentences may be these:



$12 = 2 \times 6$

$12 = (3 \times 2) + (3 \times 2)$

$12 = (3 \times 3) + (1 \times 3)$

$12 = (2 \times 4) + (4 \times 1)$

Get the children to agree that each of the sentences gives a different name for 12. Ask, "What are some other names for 12?" Get the pupils to write their answers in the form of sentences like these:

$12 = 10 + 2$; $12 = 24 \div 2$; $12 = 3 + 4 + 5$;

$12 = (4 \times 6) - 12$; $12 = 14 - 2$; $12 = (5 \times 4) - (2 \times 4)$

Emphasize that there are many ways to name 12, and that there are many ways to name other members. Get the children to write in their exercise books mathematical sentences that show other names for numbers such as 20, 36, and 24.

ACTIVITY 2: Mathematical Sentences **Pupil page 32**

Write on the blackboard:

$$4 \times (5 + 2) \quad _ _ _ \quad (4 \times 5) + 2$$

Point out that the dotted line shows that a symbol is missing in the sentence. Ask, "What symbol can you write to make the sentence true?" Guide the children to decide that the missing symbol is > because

$$4 \times (5 + 2) = 4 \times 7 \quad \text{and} \quad (4 \times 5) + 2 = 20 + 2$$
$$= 28 \qquad\qquad\qquad = 22$$
$$\text{and} \qquad 28 > 22$$

Emphasize the importance of first finding the numbers shown in the brackets. Get a child to complete the sentence.

$$4 \times (5 + 2) > (4 \times 5) + 2$$

Write another sentence such as:

$$8 + (14 \div 2) _ _ _ (14 \div 2) \times 8$$

Guide the children to see that the missing symbol is < because:

$$8 + (14 \div 2) = 8 + 7 \quad \text{and} \quad (14 \div 2) \times 8 = 7 \times 8$$
$$= 15 \qquad\qquad\qquad = 56$$
$$\text{and} \qquad 15 < 56$$

Get a child to complete the sentence:

$$8 + (14 \div 2) < (14 \div 2) \times 8$$

Use other sentences to help the children understand that the numeral inside the bracket is a name for one number. Emphasize again the importance of first finding the numbers shown inside the brackets and then doing the other operations.

Ask the children to copy the sentences on pupil page 32 in their exercise books and make the sentences true by writing the missing symbols.

ACTIVITY 3: Familiar Operations **Pupil page 33**

Write several sentences like these on the blackboard:

(a) $5 + (e \times 5) = 10$

(b) $12 = 2 \times (12 \div n)$

(c) $(16 - a) + 6 = 13$

Get the children to solve the sentences and explain their work. They may explain their work like this:

(a) $(e \times 5)$ is 5 because $5 + 5 = 10$. If $(e \times 5) = 5$, then e is 1.

(b) $(12 \div n)$ is 6 because $12 = 2 \times 6$. If $(12 \div n) = 6$, then n is 2.

(c) $(16 - a)$ is 7 because $7 + 6 = 13$. If $(16 - a) = 7$, then a is 9.

Get the children to open their books to pupil page 33 and find the numbers that make the sentences true.

ACTIVITY 4: Story Problems **Pupil pages 34-35**

Before doing the exercises on pupil pages 34 and 35 give a few problems as examples and discuss ways to solve them.

Example 1: Kwami bought 2 packets of oranges. Each packet contained 6 oranges. He shared these oranges equally among 4 boys. How many oranges did each boy get?

Ask, "What question does the problem ask?" (How many oranges did each boy get?)

Guide the children to write a mathematical sentence for the problem

by helping them: decide how many oranges Kwami bought (Two packets of 6 oranges are shown as (2×6)); think about what Kwami did with the oranges (He shared them equally among 4 boys); and discuss the problem and its mathematical sentence until a sentence is written.

$$(2 \times 6) \div 4 = b$$

Let the pupils find what number b makes the sentence true. (Since $(2 \times 6) \div 4 = 3$, b is 3.) Ask, "What is the answer to the question in the problem?" (Each boy gets 3 oranges.)

> Example 2: Nina's family invited Ame's family and Haji's family for a birthday party. There were 22 people in all. If 10 people were members of Nina's family and 8 were members of Ame's family, how many were members of Haji's family?

Ask the children what question the problem asks. They can let s be the number of people in Haji's family. Guide the pupils to write the mathematical sentence by helping them decide that at the party there were $(10 + 8 + s)$ people in all. Let someone write the sentence:

$$(10 + 8) + s = 22$$

Ask, "What is $(10 + 8)$? (18) What number s makes the sentence true? (4) What is the answer to the question in the problem?" (There were 4 people from Haji's family at the party.)

Let the pupils open their books to pupil pages 34 and 35 and answer the questions about the story problems. Get the pupils to write in their exercise books the following for each exercise: the mathematical sentence, the missing number, and the answer to the question asked in the problem. This is the way their exercise books may look for pupil pages 34 and 35:

1. $(7 \times 4) - 6 = n$
   n is 22
   22 animals were put back on the mud pile.

2. $(n + 7) \div 13 = 2$
   n is 19
   There were 19 coconuts on the ground at first.

3. $(32 \div 4) + k = 15$
   k is 7
   Each friend had 7 marbles at first.

4. $(m \times 8) - 14 = 42$
   m is 7
   There were 7 boys.

5. $(n \div 2) - 2 = 8$

   n is 20

   The boys have 20 beans together.

6. $(4 \times p) + (3 \times p) = 56$

   p is 8

   The teacher drew 8 squares and 8 triangles.

7. $(5 \times s) = 30$

   s is 6

   Each boy polished 6 pairs of shoes. Each boy earned 12 shillings.

8. $(7 \times t) - (7 \times 3) = 42$

   t is 9

   Each boy gathered 9 bananas.

STAGE 2: Various Operations

Vocabulary: No new vocabulary

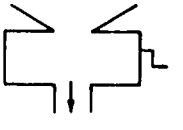
Materials: Pupil pages 36 - 38

Teaching Procedure

ACTIVITY 1: Recognising Operations

Pupil page 36

Draw an operation machine on the blackboard:



Ask a pupil to name a pair of whole numbers. He may name (12, 4). Say, "I put this pair of numbers one at a time into the machine, first the 12, then the 4". Pretend to turn the handle and then write 16 at the bottom. Ask, "What operation has the machine done?" (Addition) Ask a pupil to write the symbol for addition on the picture of the machine. Help the class to make this sentence about this operation:

$$12 + 4 = 16$$

Rub out the "16" and the "+" on the picture of the machine. Pretend to turn the handle and then write "8" at the bottom. Ask, "What operation has the machine done?" (Subtraction) Let a child write the "-" sign on the

machine and ask him to make a sentence about putting the ordered pair (12, 4) in the subtraction machine.

$$12 - 4 = 8$$

Repeat the activity for multiplication (×) and division (÷) with the same pair of numbers. Do a few more examples with other pairs of numbers using +, −, ×, ÷. In each case, let a child make a sentence about the pair of numbers, the operation and the number it assigns. Help the children understand that when operating on two whole numbers, the operations assign exactly one number to the ordered pair of whole numbers. (The number assigned may or may not be a whole number.)

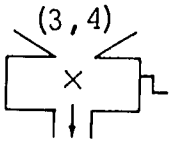
Ask the pupils to look at the operation machines on pupil page 36, and find the missing numerals or symbols. Point out that in some exercises, the pair of numbers are missing; in others the number given by the operation is missing; in still others the operation is missing. Get the pupils to notice also that when they find the pair of numbers to put into the machine there may be more than one possible pair. Encourage them to find several pairs of whole numbers in these exercises.

ACTIVITY 2: Operation Machines

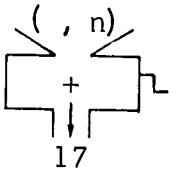
Pupil page 37

Draw these two diagrams on the blackboard:

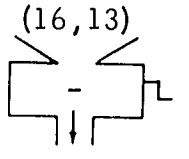
(1) (a)

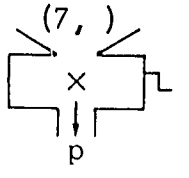


(b)



(2)





Let the pupils study the diagrams one at a time and talk about them. Pretend to turn the handle of machine a and ask, "What number pops out of the machine? (12) Why? (3 × 4 = 12) What type of machine is b? (Addition machine) What numbers are put into the machine? (First we put in 12; we do not know the other number, n.) What else do you know? (17 pops out of the machine.) What mathematical sentence helps you find the missing number?"

Guide the children to make the sentence:

$$(3 \times 4) + n = 17$$

Ask what number n makes the sentence true. (5) Say, "Machine a works before machine b works". Discuss in a similar way the second example.

Talk with the pupils about a short way of showing the operations of the two machines. Guide them to see and agree that the operations may be shown like this:

$$(1)\quad (3,\ 4) \xrightarrow{\times} (\ ,\ n) \xrightarrow{+} 17 \qquad\qquad (2)\quad (16,\ 13) \xrightarrow{-} (7,\ ) \xrightarrow{\times} p$$

Write these two examples on the blackboard:

$$(3)\quad (3,\ 4) \xrightarrow{\times} (\ ,\ 5) \xrightarrow{\times} q \qquad\qquad (4)\quad (4,\ 5) \xrightarrow{\times} (3,\ ) \xrightarrow{\times} t$$

Let the children talk about the examples. Guide them to write sentences about the operations. In (3) the sentence is $(3 \times 4) \times 5 = q$. In (4) the sentence is $3 \times (4 \times 5) = t$. Let them find the number which makes each sentence true.

Let the pupils turn to pupil page 37 of their books. Ask them to write a mathematical sentence for each exercise, and then find the number that makes the sentence true.

ACTIVITY 3: Other Operations

Pupil page 38

Remind the pupils that there are operations other than addition, multiplication, subtraction, and division. Ask the children to think of an operation machine. Say "When we put this pair of numbers into the machine this number pops out". Say this each time you write these pairs and the numbers assigned to the pairs:

(a) $(3,\ 6) \longrightarrow 6$

(b) $(13,\ 3) \longrightarrow 3$

(c) $(15,\ 7) \longrightarrow 7$

(d) $(20,\ 11) \longrightarrow 11$

(e) $(4,\ 8) \longrightarrow 8$

Ask, "What operation is this?" If the pupils seem puzzled, remind them that for every ordered pair of whole numbers that is put into the machine a single whole number pops out. Ask, "Can you tell what the machine is doing?" Get them to think of the results again and to talk about the results. Guide them to say that in each case the machine chooses the second number of every pair. Place an s above each arrow as shown below: (Call this operation "seconding".)

(a) $(3, 6) \xrightarrow{s} 6$

(b) $(13, 3) \xrightarrow{s} 3$

(c) $(15, 7) \xrightarrow{s} 7$

(d) $(20, 11) \xrightarrow{s} 11$

(e) $(4, 8) \xrightarrow{s} 8$

Change the order of the numbers in each pair. Ask the pupils for the result in each case when "seconding" is the operation:

(f) $(6, 3) \xrightarrow{s} 3$

(g) $(3, 13) \xrightarrow{s} 13$

(h) $(7, 15) \xrightarrow{s} 15$

(i) $(11, 20) \xrightarrow{s} 20$

(j) $(8, 4) \xrightarrow{s} 4$

Guide the pupils to make mathematical sentences for each case. The mathematical sentences (f), (g), (h), (i), and (j) are:

(f) $6 \text{ s } 3 = 3$

(g) $3 \text{ s } 13 = 13$

(h) $7 \text{ s } 15 = 15$

(i) $11 \text{ s } 20 = 20$

(j) $8 \text{ s } 4 = 4$

Ask four pupils in turn to give you pairs of numbers. Suppose they give (3, 5), (6, 1), (7, 0), and (2, 11). Tell them that when each pair is put into a machine in the order given, you get the following results:

$(3, 5) \longrightarrow 3$

$(6, 1) \longrightarrow 1$

$(7, 0) \longrightarrow 0$

$(2, 11) \longrightarrow 2$

Ask, "What is this operation? What operation does this machine do?" Let the pupils think about and talk about the results. Guide them to see that the machine chooses the smaller of the two numbers. As before, let a child write a mathematical sentence for each example. (If they call this operation k then 6 k 1 = 1, 2 k 11 = 2, and so on.) Let the children make up other operations until the idea of an operation is clear.

Ask the children to turn to pupil page 38. Talk with them about the exercises and work some of them.

Answers for pupil page 38:

1. Operation ▫ is "the greater number minus the smaller number", (greater - smaller);
2. Operation · is "one more than the sum", (sum + 1) ;
3. Operation ₂ is "twice the sum", (2 × sum);
4. Operation ▽ is "one more than the greater number"; (greater number + 1);
5. Operation △ is "twice the smaller number", (2 × smaller number);
6. Operation ₖ is "twice the first number plus the second", ((2 × first number) + second).

STAGE 3: Properties of Operations

Vocabulary: Commutative property, associative property, distributive property, property of one, property of zero

Materials: Counters, pupil pages 39-43

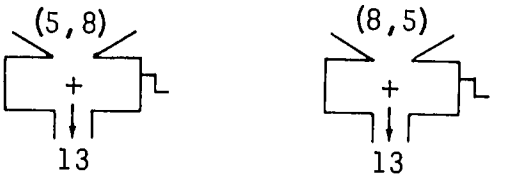
Teaching Procedure

ACTIVITY 1: Commutative Properties of Addition and Multiplication

Pupil page 39

Draw an addition machine on the blackboard. Ask a pupil to name a pair of whole numbers, say (5, 8). Write the numerals for the numbers on the picture. Say, "I put in the number 5, then the number 8". Pretend to turn the handle. Ask, "What number pops out of the machine?" (13) Draw another

addition machine. Get a child to put the same numbers in this machine but change the order. (The 8 goes in first, then the 5.) Let another child pretend to turn the handle and show the number that pops out. (13) The blackboard will look like this:



Say, "You changed the order of the addends, but the same number popped out. (13) Is this true for every other pair of numbers you add?" (Yes) Let the children try other pairs of numbers. Guide the children to show this idea in mathematical sentences. For example:

$$8 + 5 = 5 + 8$$

Repeat the activity with other pairs of whole numbers, if necessary, and finally lead them to see that for any two whole numbers a and b :

$$a + b = b + a$$

Tell them that this is a property of addition. It is called the <u>commutative property of addition</u> . That is, the sum of two whole numbers is the same no matter in which order you add them.

Repeat the above activity but use a multiplication machine. In a similar way lead the children to see that for any two whole numbers a and b ,

$$a \times b = b \times a$$

Remind them that this is a property of multiplication and is called the <u>commutative property of multiplication</u>.

Let the pupils turn to pupil page 39. Let them talk about a few of the exercises. Guide the pupils to see that you do not have to do the operations each time to find the missing number. Let them use the commutative property to find the missing number by inspection. For example, in $9 + 7 = 7 + b$, if b is 9, the sentence is true. (Commutative property of addition)

After the children have finished the exercises let them talk about their answers. Get them to make true sentences for those sentences that are false.

ACTIVITY 2: Associative Properties of Addition and Multiplication

Pupil page 40

Write on the blackboard:

$$(1) \quad (4 + 3) + 5 = n$$

Ask, "What number n makes the sentence true?" (12)

Then say, "Suppose we group the numbers another way". Write:

$$(2) \quad 4 + (3 + 5) = n$$

Say, "The addends in the two sentences are the same numbers but the brackets show a different grouping. What number n makes the second sentence true?" (12) Guide the children to see that they begin by finding the number named in the brackets.

Ask, "Was the sum the same as in the first sentence?" (Yes. In both cases it was 12.) Use other examples to decide that no matter which of the two ways we group three whole numbers the sum is the same. Say, "If we have any three whole numbers a, b, and c, then:

$$(a + b) + c = a + (b + c)".$$

Remind the children that this is another property of addition. It is called the associative property.

Ask, "Do you think multiplication has the associative property? Is this sentence always true for any three whole numbers?"

$$(2 \times 5) \times 6 = 2 \times (5 \times 6)$$

Use the same procedure with several other examples until the children decide that multiplication is associative. Ask the pupils to do the exercises on pupil page 40. Guide them to use the associative property to help them answer the questions.

ACTIVITY 3: The Distributive Property

Pupil page 41

Write the following story problem on the blackboard:

> Musa bought 3 packets of biscuits on Monday. He bought 4 packets on Tuesday. Each packet contained 5 biscuits. How many biscuits did Musa buy on Monday and Tuesday together?

Let the pupils talk about the story problem. Ask, "How many biscuits did

Musa buy on Monday? (15) How do you know it is 15?" The children may say something like, "There are 5 biscuits in each packet. Musa bought 3 packets. He bought 5 × 3 or 15 biscuits". Guide them to find in a similar way the number of biscuits Musa bought on Tuesday. (5 × 4) Ask, "How many biscuits did Musa buy altogether?" Help them to make the sentence:

$$n = (5 \times 3) + (5 \times 4)$$

Let the children say what number n is. (35)

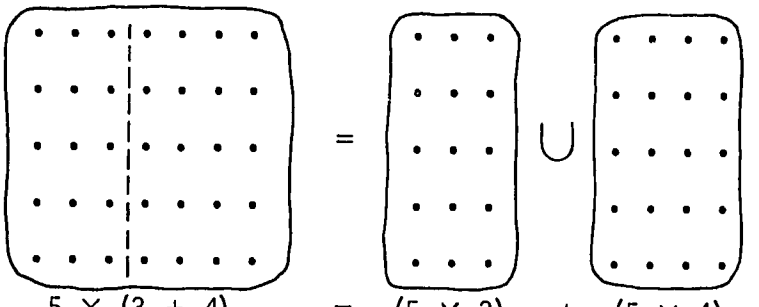
Ask, "Can you think of another way of finding how many biscuits Musa bought?" Help the pupils by asking questions such as, "How many packets did Musa buy altogether? (3 + 4 or 7) How many biscuits were in each of these packets?" (5) Guide them to write:

$$n = 5 \times (3 + 4)$$

Help the children discover that in both sentences n is the same number. Guide them to write:

$$5 \times (3 + 4) = (5 \times 3) + (5 \times 4)$$

Draw a rectangular array to show the sentence:



$5 \times (3 + 4) \quad = \quad (5 \times 3) \quad + \quad (5 \times 4)$

Write on the blackboard:

$$3 \times (5 + 2) = (3 \times 5) + (3 \times 2)$$

Ask, "Is this sentence true?" (Yes) Let the children tell why. Do other similar examples until the children are convinced that for any three whole numbers a, b, and c,

$$a \times (b + c) = (a \times b) + (a \times c)$$

Remind the pupils that this is called the distributive property of multiplication over addition. We also say that multiplication is distributive over addition.

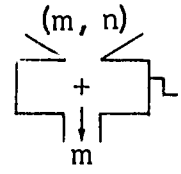
Let the children do the exercises on pupil page 41. Guide them to use

the distributive property in working out the exercises. For example, in Exercise 1 they will see by the distributive property that a is 8 without doing the operations.

ACTIVITY 4: Properties of Zero and One

Pupil page 42

Draw an operation machine for addition on the blackboard with m and n as the ordered pair of numbers put into the machine.



Say, "Let us choose any number m. Imagine that the same number m is assigned by addition to the ordered pair of numbers (m, n). Do we then know what number n is?" If no one can tell, suggest that they try a number for m and see what n must be. If a child says, "Let m be 5", then 5 is assigned to the pair (5, n). Ask someone to show a sentence for this:

$$5 + n = 5$$

Let children suggest other numbers and write the resulting sentences. Continue until there are four or five examples, such as:

$$5 + n = 5; \quad 8 + n = 8; \quad 3 + n = 3; \quad 19 + n = 19$$

Ask, "What number n makes these sentences true?" They should see that n is 0 in each sentence. Guide the children to say, "When 0 is added to a number, the sum is that number." Tell them this is called the property of zero. Ask, "If 0 is an addend what can you say about the sum?" (It is the other addend.)

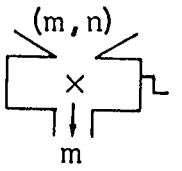
Put several sentences on the blackboard to illustrate the property of zero:

$$32 + 0 = a \qquad n = 195 + 0 \qquad 513 + 0 = t$$

$$17 + 0 = x \qquad 0 + c = 62 \qquad m = 0 + 12$$

Let the children tell what numbers make the sentences true. Draw a multiplication machine on the blackboard and say, "Here is another operation machine. It is a multiplication machine. When you put in the pair of numbers (m, n), m comes

out. What number do you think n is? Try a few examples and see".



Again, let children suggest different numbers for m. If a child says "eleven", then pretend to turn the handle and write "11" as the number assigned to (11, n). Let the child write a sentence to show this:

$$11 \times n = 11$$

After a few more examples ask, "What number is n? (One) This is called the property of one. What is the property of one? (When any number and 1 are multiplied the product is the same as the number.) You can say this another way. If 1 is a factor, then the product is the same as the other factor".

Write several sentences to illustrate the property of one. Let the children tell what numbers make the sentences true:

$19 \times 1 = n$	$43 \times 1 = m$	$r = 1 \times 613$
$m = 1 \times 367$	$930 \times 1 = t$	$s = 20 \times 1$

Ask the children to turn to pupil page 42 and answer the questions in their exercise books. Read over the questions with the class. Be certain the directions for Exercises 7 to 20 are clear. The children are to solve the sentence, write what number makes the sentence true, and then tell what property the sentence shows. The properties shown in the exercises are these:

7. distributive property
8. property of one for multiplication
9. property of one for multiplication
10. commutative property of addition
11. associative property of multiplication
12. property of zero for addition
13. associative property of addition
14. commutative property of addition
15. property of zero for addition
16. commutative property of addition

17. commutative property of multiplication
18. associative property of addition
19. commutative property of addition
20. distributive property

ACTIVITY 5: Discovering the Properties of Operations **Pupil page 43**

This is a class activity with discussion of each step.

In earlier activities in this Unit the children discussed the properties of the operations of addition and multiplication. They also worked with operations that are not used often. The purpose of this activity is to get the children to look at a few of these less used operations and decide whether the operations also have certain properties.

Ask the children to open their books to pupil page 43. Tell them to look at the exercises in Part I and read the sentence at the top. Ask, "Can you tell what number the F operation assigns to a pair of numbers? (The first number of the pair) You are given answers in two exercises. In sentence 1, the pair of numbers is (7, 3). If the F operation assigns the first number, what number will it assign to this pair? (7) For the pair (4, 9) the F operation assigns what number?" (4) Go over Exercises 3 and 4 in the same way. Let the children complete Exercises 1 to 12. Go over the answers with the class to see that everyone has the correct results.

Then say, "We know that addition and multiplication are operations which have the commutative property. What does this mean?" Guide the children to say that if we change the order of the two numbers the result of the operation is still the same.

Say, "Do you think the F operation has the commutative property? Look carefully at Exercises 1-12. Is the result of the operation the same when you change the order of the numbers?"

Give the children a chance to think about the exercises and then tell whether they think the operation is commutative. After several have expressed opinions, compare some of the pairs in which the numbers are the same but in different order. For example, write these pairs on the blackboard:

(2, 10) ; (10, 2) and (8, 7) ; (7, 8).

Let the children tell the result of the F operation for each pair. Say, "The pairs (2,10) and (10,2) have the same two numbers but in different order. Is the same number assigned to each?" (No. For the first pair it is 2; for the second pair it is 10.)

Compare the other two pairs in the same way, Ask, "Is the F operation commutative?" (No) Give the children a chance to agree it is not and to tell why.

Ask the children to study the sentence in Exercise 13 and decide whether the sentence is true or false. (It is false because $4 \neq 6$.)

Revise briefly the associative property. Let the children say that if an operation is associative, three numbers can be grouped in either of two ways and the result is the same. For example:

$$(2 + 3) + 4 = 2 + (3 + 4) \quad \text{and} \quad (2 \times 3) \times 4 = 2 \times (3 \times 4)$$

Get the children to study the sentence in Exercise 14 and decide whether the sentence is true or false. (It is true because the first number is always 2.)

This may be shown in a diagram on the blackboard. Point to the numerals as the children talk about each step:

$$\begin{array}{ccccc} (2 & F & 3) & F & 4 \\ & \downarrow & & & \\ & 2 & F & & 4 \\ & & \downarrow & & \\ & & 2 & & \end{array} \qquad \begin{array}{ccccc} 2 & F & (3 & F & 4) \\ & & & \downarrow & \\ 2 & & F & & 3 \\ & & \downarrow & & \\ & & 2 & & \end{array}$$

Say, "If the F operation is associative we could put any three numbers in place of 2, 3, and 4 in the sentence and it would always be true. Do you think it is associative?" (Yes) Let the children think about this. Let them try a few examples. They will see that the first number is the same no matter how the numbers are grouped.

Say, "We know that addition and multiplication are both commutative and associative. Now we have discovered an operation that is associative but <u>not</u> commutative". The F operation is <u>not commutative</u> but it is <u>associative</u>.

Follow a similar procedure for the two other operations on page 43. This is planned as a class activity with discussion of each step. Some suggestions for leading the discussion for Parts II and III follow.

II. The G Operation (The greater number)

First let the children do the exercises to see which number is assigned each pair. Then compare some pairs where the same numbers are in different order, for example, (1, 6) and (6, 1), (9, 3) and (3, 9), and so on. Ask, "Are the results of the G operation the same in each case? (Yes) Do you think this is always true? Is the G operation commutative?" The children may decide it is commutative because no matter what the order of the numbers, the greatest number of the pair is assigned. From this conclusion, they see that the sentence 7 G 5 = 5 G 7 is true.

Ask if the G operation is associative. Let them try the example (0 G 2) G 1 = 0 G (2 G 1). Show a diagram as they discuss the sentence:

$$\begin{array}{ccccc} (0 & G & 2) & G & 1 \\ & \downarrow & & & \\ & 2 & G & & 1 \\ & & \downarrow & & \\ & & 2 & & \end{array} \qquad \begin{array}{ccccc} 0 & G & (2 & G & 1) \\ & & & \downarrow & \\ 0 & & G & & 2 \\ & & \downarrow & & \\ & & 2 & & \end{array}$$

The pupils find that the sentence is true.

Let them use other numbers. They will decide that the G operation is associative because in whatever way the numbers are grouped, the greatest number is assigned.

Guide them to say, "We have discovered that the G operation is both commutative and associative".

III. The $\triangle$ operation (Half the sum)

Use the same type of questions to help the children decide that the $\triangle$ operation is commutative but is not associative.

Remind the children that they have found that not all operations have the same properties. Addition and multiplication have both the commutative and associative properties but other operations were found that have just one of these properties.

STAGE 4: Closure of a Set of Numbers Under an Operation

Vocabulary: Closed, not closed

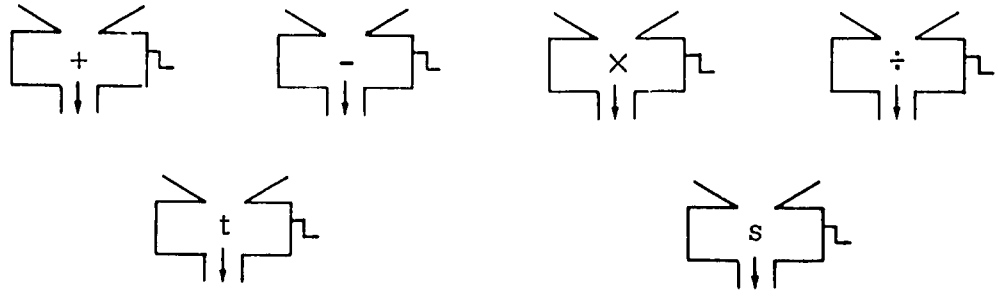
Materials: Pupil page 44-45

Teaching Procedure

ACTIVITY 1: Operations Under Which the Set of Whole Numbers is Closed

Pupil pages 44-45

Draw several operation machines on the blackboard as shown:



Say as you draw, "These machines use only whole numbers. Only whole numbers are put in; only whole numbers pop out". Let operation t be "add one to the greater of the two numbers. (If the pair are equal, use the number given. For example, $(3, 3) \xrightarrow{t} 4$). Let operation s be "$\frac{1}{2} \times$ (greater number minus smaller number)" (If the numbers in the pair are equal, then the greater minus the smaller is zero. For example, $(3, 3) \xrightarrow{s} 0$).

Ask a child to name a pair of whole numbers, for example, (10, 7). Say, "Put this pair of numbers, first 10 and then 7, into each of the six machines. Write the results like this:

$(10, 7) \xrightarrow{+} 17$ $\qquad$ $(10, 7) \xrightarrow{-} 3$

$(10, 7) \xrightarrow{\times} 70$ $\qquad$ $(10, 7) \xrightarrow{\div}$

$(10, 7) \xrightarrow{t} 11$ $\qquad$ $(10, 7) \xrightarrow{s}$

The children find that two of the machines did not work. There is no answer in two examples. Guide the children to say that $10 \div 7$ is not a whole number and that $\frac{1}{2} \times (10 - 7)$ is not a whole number.

Ask for other pairs of whole numbers and repeat the procedure used with the number pair (10, 7). Each time list the results. Do this for about six or seven pairs of numbers.

Ask, "With which of these operations do we <u>always</u> get a whole number for an answer?" After some discussion, the children will say that the operations +, ×, and t always assign a whole number to every pair of numbers. The number

pair (10, 7) showed that ÷ and s do not always assign a whole number to every pair of numbers. For example, $(7,10) \xrightarrow{-} n$ shows that subtraction does not assign a whole number to (7, 10).

Explain to the pupils that they have discovered a property of an operation. Say, "We say the set of whole numbers is closed under an operation if the operation on any pair of whole numbers always gives a whole number."

Guide the children to decide that:

- the set of whole numbers is closed under the operation +, ×, and t;
- the set of whole numbers is not closed under the operations −, ÷, and s.

Ask, "How do we show that the set of whole numbers is not closed under a certain operation?" We find a pair of whole numbers for which the operation does not give a whole number.

Let the pupils turn to pages 44 and 45 in their books and talk about the exercises. In each case, they find the operation from the results given; then use the operation to find the missing numbers. Only whole number results are used. In Exercise 9, let the pupils choose those operations under which the set of whole numbers is closed; in Exercise 10, get them to choose those operations under which examples are given to show the set of whole numbers is not closed.

Note that there are many ways to describe the operations. Let the pupils work with their own descriptions. One possible set of answers for each exercise is given below.

Answers for pupil page 44:

1. Operation A is "addition".
2. The missing numbers are 21, 17, and 26.
3. Operation B is "choose first number".
4. The missing numbers are 9, 10, 13.
5. Operation C is "choose smaller number if they are different, and the number if they are the same".
6. The missing numbers are 9, 7, and 13.
7. Operation D is "$\frac{1}{2}$ the sum".

8. The missing numbers are 11, no answer, and 13.

9. A, B, C.

10. D (There is no answer for the pair (10,7)).

Answers for pupil page 45:

11. Operation E is "multiplication".

12. The missing numbers are 5, 24, and 49.

13. Operation F is "$\frac{1}{2}$ of the larger if the numbers are different and of the number if they are the same".

14. The missing numbers are no answer, 4, and no answer.

15. Operation G is "one less than the smaller if the numbers are different and of the number if they are the same".

16. The missing numbers are 2, no answer, and 6.

17. Operation H is "2 × (larger minus smaller)" (If the numbers are the same subtract them).

18. The missing numbers are 10, 8, and 0.

19. E, H.

20. F, G.

UNIT 5

Addition and Subtraction of Whole Numbers I

Objectives

1. To achieve mastery of the techniques of addition and subtraction of whole numbers
2. To extend techniques of addition and subtraction of whole numbers to any sum
3. To continue the development of the ability to express mathematical relationships in story problems as mathematical sentences
4. To use addition and subtraction in solving problems about money, weights, and measures

Background Information for Teachers

See "Background Information for Teachers" for Unit 1, Unit 2, and Unit 4 in this book. In particular, read again the suggestions for solving word problems on pages 28, 29, and 30 of Unit 2.

We will discuss briefly the ideas on which the techniques of addition and subtraction are based.

<u>Addition</u> The expanded form of numerals together with the properties of addition are used in a short form for addition. For example:

475 = 400 + 70 + 5		(Expanded form)
367 = 300 + 60 + 7		(Expanded form)
700 + 130 + 12	(add ones; add tens; add hundreds)	
= 700 + (100 + 30) + (10 + 2)		(Rename 130 and 12)
= (700 + 100) + (30 + 10) + 2		(Associative property)
= 800 + 40 + 2		(add hundreds, add tens)
= 842		(Rename 800 + 40 + 2)

The process used in the short form may be described as follows:

$$\begin{array}{r} 475 \\ \underline{367} \\ 842 \end{array}$$

1. Think of 475 as 4 hundreds, 7 tens and 5. Think of 367 as 3 hundreds, 6 tens and 7.
2. Add the ones. 5 + 7 = 12. Rename 12 as 1 ten and 2 ones. Write 2 in the ones' place of the sum. Remember 1 ten.

3. Add the tens. (1 + 7 + 6) tens = 14 tens. Rename 14 tens as 1 hundred and 4 tens. Write 4 in the tens' place of the sum. Remember 1 hundred.

4. Add the hundreds. (1 + 4 + 3) hundreds = 8 hundreds. Write 8 in the hundreds' place of the sum.

5. The sum is 842.

<u>Subtraction</u> As an example, find the number that makes this sentence true:

$$475 - 367 = n$$

Using the expanded form we have:

$$\begin{array}{rcl} 475 & = & 400 + 70 + 5 \\ \underline{367} & = & \underline{300 + 60 + 7} \end{array}$$

We first find the ones' digit in the numeral for <u>n</u> . There are not enough ones in the ones' place of 475 to subract 7. So we rename 475 to give:

$$\begin{array}{rcl} 475 & = & 400 + 60 + 15 \\ \underline{367} & = & \underline{300 + 60 + \ \ 7} \\ & & 100 + \ \ 0 + \ \ 8 \\ & = & 108 \end{array}$$

The process used in the short form may be described as follows:

$$\begin{array}{r} 475 \\ \underline{367} \\ 108 \end{array}$$

1. Think of 475 as 4 hundreds, 7 tens and 5. Think of 367 as 3 hundreds, 6 tens and 7.

2. There are fewer ones in the ones' place of 475 than in the ones' place of 367. Rename 475 as 4 hundreds, 6 tens and 15 ones. Subtract the ones. 15 - 7 = 8. Write 8 in the ones' place of the missing addend.

3. Subtract the tens. (6 - 6) tens = 0 tens, Write 0 in the tens' place of the missing addend.

4. Subtract the hundreds. (4 - 3) hundreds = 1 hundred. Write 1 in the hundreds' place of the missing addend.

5. The missing addend is 108.

STAGE 1: Revision of Techniques of Addition and Subtraction

Vocabulary: No new vocabulary

Materials: Pupil pages 46-55

Teaching Procedure

ACTIVITY 1: Revision of the Techniques of Addition Using Expanded Form **Pupil pages 46-47**

Write this problem on the blackboard:

Tamu's father made bricks for a new house. He made 316 bricks in the first week, 248 bricks in the second week, and 122 bricks in the third week. How many bricks did he make in the three weeks?

Ask, "What question is asked in the problem? (How many bricks did Tamu's father make in the three weeks?) What do you know about the problem?" (We know the number of bricks Tamu's father made in each of the three weeks.) Help the children to make a mathematical sentence for the problem. Let a child write this sentence on the blackboard:

$$b = 316 + 248 + 122$$

Guide the children to write the numerals in vertical form to find the sum of the numbers. Get them to write the expanded form of the numerals and then to add the ones, the tens, and then the hundreds. The work on the blackboard will look like this:

$$\begin{array}{rcl} 316 & = & 300 + 10 + 6 \\ 248 & = & 200 + 40 + 8 \\ \underline{122} & = & \underline{100 + 20 + 2} \\ & & 600 + 70 + 16 \\ & = & 600 + 70 + (10 + 6) \\ & = & 600 + 80 + 6 \\ & = & 686 \end{array}$$

Point to the mathematical sentence and ask, "What is $\underline{b}$? ($\underline{b}$ is 686) Can you answer the question in the problem?" (Tamu's father made 686 bricks in the three weeks.)

Ask the pupils to open their books to pupil pages 46 and 47 and do the exercises. If needed, help the children work one or two exercises on the blackboard. Let the pupils, who can, add the numbers without writing the numerals in expanded form.

ACTIVITY 2: Revision of the Techniques of Addition Using Short Form

Pupil pages 48-50

Write the following on the blackboard:

There are four schools in a town. In the first school the total number of pupils is 684, the second has 360 pupils, the third has 844 pupils, and the fourth has 285 pupils. How many pupils are there altogether in the four schools?

Ask, "What question does the problem ask?" (How many pupils are in all four schools?) Guide the children to see that they can find the answer by adding the numbers of pupils enrolled in the schools. Let a child write this mathematical sentence on the blackboard:

$$684 + 360 + 844 + 285 = m$$

Say, "It is easier to add these numbers if you write the numerals in vertical form". Write the numerals in the form to the left below and guide the work as shown to the right:

$$\begin{array}{r} 684 \\ 360 \\ 844 \\ \underline{285} \\ 2173 \end{array}$$

1. Think of 684 as 6 hundreds, 8 tens and 4 ; think of 360 as 3 hundreds and 6 tens; and so on.
2. Add the ones. 4 + 0 + 4 + 5 = 13. Rename 13 as 1 ten and 3. Write 3 in the ones' place of the sum. Remember 1 ten.
3. Add the tens. (1 + 8 + 6 + 4 + 8) tens = 27 tens. Rename 27 tens as 2 hundred and 7 tens. Write 7 in the tens' place of the sum. Remember 2 hundreds.
4. Add the hundreds. (2 + 6 + 3 + 8 + 2) hundreds = 21 hundreds. Rename 21 hundreds as 2 thousands and 1 hundred. Write 2 in the thousands' place and 1 in the hundreds' place of the sum.
5. The sum is 2173 .

Point to the mathematical sentence and ask, "What is m? (m is 2173) Can you answer the question in the problem?" (There are 2173 pupils in the four schools.)

Let the pupils do the exercises on pupil pages 48, 49, and 50.

ACTIVITY 3: Revision of the Techniques of Subtraction **Pupil pages 51-53**

Write this on the blackboard:

$$\begin{array}{r} 325 \\ \underline{148} \end{array}$$

Ask the children to work together to subtract the numbers. Say, "Think of the numbers in expanded form and then subtract the number of ones and then tens and then hundreds". Get the children to write the expanded form of the two numerals:

$$\begin{array}{rcl} 325 & = & 300 + 20 + 5 \\ \underline{148} & = & \underline{100 + 40 + 8} \end{array}$$

Ask, "Are there enough ones in the ones' place of 325 to subtract 8? (No) What can you do?" (Rename the sum)

$$\begin{array}{rcl} 325 & = & 300 + 20 + 5 = 300 + 10 + 15 \\ \underline{148} & = & \underline{100 + 40 + 8} = \underline{100 + 40 + \ \ 8} \end{array}$$

Let a child subtract the ones (15 - 8 = 7) and write 7 in the ones' place. Ask, "Are there enough tens in the tens' place of the sum to subtract 40? (No) What can you do?" (Rename the sum)

$$\begin{array}{rcl} 325 & = & 300 + 20 + 5 = 300 + 10 + 15 = 200 + 110 + 15 \\ \underline{148} & = & \underline{100 + 40 + 8} = \underline{100 + 40 + \ \ 8} = \underline{100 + \ \ 40 + \ \ 8} \end{array}$$

"Can you subtract the tens?" (Yes. (11 - 4) tens = 7 tens.) Let the child who answers write 70 in the missing addend. Ask, "Can you subtract the hundreds?" (Yes. (2-1) hundreds = 1 hundred.) Write 100 in the missing addend. Say, "The missing addend is 100 + 70 + 7. What is its simplest name?" (177)

Tell the children that there is a shorter form for writing their work. Let the pupils go over the example again. As they do so, guide them to write the form to the left and to explain the work to the right as shown in the

following example. Point to the proper numerals as the children talk.

$$\begin{array}{r} 325 \\ \underline{148} \\ 177 \end{array}$$

1. Think of 325 as 3 hundreds, 2 tens and 5. Think of 148 as 1 hundred, 4 tens and 8.
2. There are not enough ones in the ones' place of the sum to subtract 8. Rename 325 as 3 hundreds, 1 ten and 15. Subtract the ones. 15 - 8 = 7. Write 7 in the ones' place.
3. There are not enough tens in the tens' place of the sum to subtract 4 tens. Rename 3 hundreds and 1 ten as 2 hundreds and 11 tens. Subtract the tens. (11-4) tens = 7 tens. Write 7 in the tens' place.
4. Subtract the hundreds. (2-1) hundred = 1 hundred. Write 1 in the hundreds' place.
5. The missing addend is 177.

For the next example, write the following story problem on the blackboard:

In two days, trains carried 9724 tons of copper to the coast. If 2428 tons were carried on the second day, how many tons were carried on the first day?

Ask, "What question does the problem ask? (How many tons were carried on the first day?) What do you know? (9724 tons were carried altogether and 2428 of these tons were carried the second day.) How do you find how many tons were carried the first day?" Help the children to see that they subtract to find the answer. Guide the children to write a mathematical sentence for the problem. They may write either of the following sentences:

$$9724 - 2428 = d$$

$$d + 2428 = 9724$$

Before talking about the method of subtraction, let the children make guesses as to <u>about</u> what number <u>d</u> makes the sentence true. They may think: the sum is about 9000: one addend is about 2000; the missing addend is about 7000.

Say, "You guessed that <u>d</u> is about 7000. Now subtract the numbers and find what <u>d</u> is". Write the numerals for the numbers on the blackboard.

Let the children go through the steps of renaming and then subtracting. You point to the numerals and write the digits in the answer as they talk.

$$\begin{array}{r} 9724 \\ \underline{2428} \\ 7296 \end{array}$$

Let the children say that d is 7296, and that 7296 tons of copper were carried the first day. Ask, "Did you make a good guess? (Yes. We guessed 7000.) Can you check the answer?" (Yes. We can add 7296 and 2428. The sum should be 9724. There were 9724 tons of copper carried in two days.) Let them check their answers.

Let the children do the exercises on pupil pages 51, 52, and 53. It is not necessary to do all three pages during one lesson. Spend only as much time on these pages as the children need.

ACTIVITY 4: Inequalities Using Addition and Subtraction Pupil pages 54-55

Write the following problem on the blackboard:

> There were 213 people waiting for buses. Five buses arrived. Each bus seats 43 people. No one is allowed to stand. Can everyone be seated? If so, are there any seats not used?

Let the pupils talk about the problem. Guide the children to say, "We find which of two numbers is larger. One number 213 tells how many people there are. The other number, 43 + 43 + 43 + 43 + 43, tells how many seats there are". In sentence form, this will be:

$$213 ___ 43 + 43 + 43 + 43 + 43$$

Get the pupils to decide which symbol, $<$, $>$, or $=$, makes the sentence true. Ask one of the pupils to add 43 five times. The sum is 215. (If a pupil notices that $43 + 43 + 43 + 43 + 43 = 5 \times 43$, let him do the multiplication.) Guide the children to decide: the correct symbol is $<$; there are enough seats for everyone; two seats are not used.

Below are other examples of this kind of exercise. You or the pupils may make up story problems to go with the sentences. In each exercise get the pupils to decide which symbol, $<$, $>$, or $=$, makes the sentence true:

342 + 576 _ _ _ 1000 - 81

876 + 754 _ _ _ 726 + 871

948 - 385 _ _ _ (474 + 117) - 28

8 + (728 - 432) _ _ _ 654 - 349

Let the pupils open their books to pupil pages 54 and 55 and do the exercises.

STAGE 2: Summary of Properties of Addition

Vocabulary: No new vocabulary

Materials: Pupil pages 56-57

Teaching Procedure

ACTIVITY 1: Commutative Property of Addition and the Property of Zero

Pupil page 56

The purpose of this activity is to revise the commutative property of addition and the property of zero. In Unit 4, operation machines were used to get the pupils to discover the properties of addition. This method may be used again to start the discussion.

Get the pupils to recall the meaning of the commutative property of addition: We may add two numbers in either order and we always get the same sum. (In symbols, a + b = b + a, where a and b are any whole numbers.) Get the pupils to recall also that the sum of any number and zero is that number. (In symbols, a + 0 = a, for any whole number a .)

Put an addition chart like this one the blackboard. (Use any five numbers.)

+	5	19	32	6	11
5					
19					
32					
6					
11					

First ask the pupils for the sums that go in the boxes that are filled in the following example. Remind them that the sums are obtained by adding the number

to the left in a row to the top number in a column. For example, 5 + 32 = 37.

+	5	19	32	6	11
5	10	24	37	11	16
19		38	51	25	30
32			64	38	43
6				12	17
11					22

Ask, "Can you fill in the other boxes without doing any other addition?" Guide the children to say that because of the commutative property, they know all the sums. Let the children fill the other boxes.

Let the pupils open their books to pupil page 56. Chart 1 can be completed without any additions. Also use this exercise to revise the property of zero. Chart 2 requires two additions. Chart 3 requires three additions.

In Exercise 4, Fatu's way uses both commutative property and the associative property. The commutative property is used several times. This exercise may be too difficult for some pupils. It is a challenge problem and is marked with an *.

Exercise 5 is solved in this way:

$$(2 + 23) + (5 + 20) + (8 + 17) + (11 + 14) = m$$
$$4 \times 25 = m$$
$$m \text{ is } 100$$

You may give this sentence to some of the children to solve if they find Exercises 4 and 5 a challenge.

$$n = 5 + 10 + 15 + 20 + 25 + 30$$

ACTIVITY 2: Associative Property of Addition **Pupil page 57**

The purpose of this activity is to revise the associative property of addition. Again, operation machines may be used to start the discussion. Get the pupils to recall the meaning of the associative property: If three numbers are added, the sum is the same no matter which pair is added first. (In symbols, $a + (b + c) = (a + b) + c$, where $\underline{a}$, $\underline{b}$, and $\underline{c}$ are any whole numbers.)

You may want the children to revise one of the operations introduced in Unit 4 for which this property did not hold. One operation used was: "Take half the sum". It was called the "Δ" operation. For example,

$$(2 \Delta 6) \Delta 10 = 4 \Delta 10 = 7$$

$$2 \Delta (6 \Delta 10) = 2 \Delta 8 = 5$$

The operation Δ is not associative. You may want to use it for contrast with addition. (Choose carefully the numbers you use with operation Δ so the children can find $\frac{1}{2}$ of all the sums.)

Let the pupils open their books to pupil page 57. Let them read and think about Wakabi's way of adding. Help them to decide why it works. They should tell where the associative property is used. Let the children use Wakabi's method to work these easier examples:

a. $19 + 12 = 19 + (1 + 11)$
$= (19 + 1) + 11$
$= 20 + 11$
$= 31$

b. $57 + 17 = 57 + (3 + 14)$
$= (57 + 3) + 14$
$= 60 + 14$
$= 74$

Get the children to solve Exercises 1 to 4 on pupil page 57. Then discuss Wakabi's way of subtracting. Below are more examples you and the children may work together. The work may look like this:

c.
$49 + n = 145$
$(45 + 4) + n = 145$
$45 + (4 + n) = 145$
$4 + n$ is 100
n is 96

d.
$19 + n = 64$
$(14 + 5) + n = 64$
$14 + (5 + n) = 64$
$5 + n$ is 50
n is 45

Then get the pupils to solve Exercises 5 and 6 in their exercise books.

STAGE 3: Techniques of Addition with Sums Greater than 10,000

Vocabulary: No new vocabulary

Materials: Pupil pages 58-63

Teaching Procedure

ACTIVITY 1: Adding Three or Four Numbers **Pupil pages 58-59**

In this activity, the children add three or four numbers, some with sums that exceed 10,000. Encourage the use of the vertical form in writing the exercises. Revise often the ideas of place-value in decimal names and renaming. Addition in this activity includes renaming in two places. (Note that because of the use of renaming the word "carrying" is not necessary. The words used in this program help to develop meaning; "carrying" does not.)

Write this story problem on the blackboard:

> Songo town is divided into three sections. In the first section, there are 1134 people; in the second, there are 1376 people; and in the third, there are 2244 people. How many people are there in Songo town?

Let the pupils talk about the problem and tell what it asks. Say, "How can you find the number of people in Songo town?" (Add the number of people in the three sections.) Let the children make a sentence about the number of people in the town and let a child write:

$$1134 + 1376 + 2244 = n$$

Guide the children to find n by writing the exercise in the form to the left below and thinking as to the right:

$$\begin{array}{r} 1134 \\ 1376 \\ \underline{2244} \\ 4754 \end{array}$$

1. Think of 1134 as 1 thousand, 1 hundred, 3 tens and 4; of 1376 as 1 thousand 3 hundreds, 7 tens and 6; and so on.
2. Add the ones. 4 + 6 + 4 = 14. Rename 14 as 1 ten and 4. Write 4 in the ones' place of the sum. Remember 1 ten.
3. Add the tens. (1 + 3 + 7 + 4) tens = 15 tens. Rename 15 tens as 1 hundred and 5 tens. Write 5 in the tens' place of the sum. Remember 1 hundred.
4. Add the hundreds. (1 + 1 + 3 + 2) hundreds = 7 hundreds. Write 7 in the hundreds' place of the sum.

5. Add the thousands. (1 + 1 + 2) thousands = 4 thousands. Write 4 in the thousands' place of the sum.

6. The sum is 4754.

Guide the class to see that to add they always start by adding the number of ones, then the tens, then the hundreds, and so on. Renaming is done whenever necessary. Go over the process again, adding up if the children added down. Ask them to do the renaming and to give the sums. Guide them to see that this change in the order of adding is a check. When the addition has been checked, ask, "What is n ? (n is 4754.) Can you answer the question in the problem?" (Yes. There are 4754 people in Songo town.) Do another example with the children in a similar way. You or the children may make up a problem which gives a sentence like this:

$$4823 + 3406 + 1143 + 2952 = n$$

Let the pupils do the exercises on pupil pages 58 and 59. Encourage them to write the numerals in each exercise in vertical form before adding. If they have difficulty, help them do a few more examples.

ACTIVITY 2: Adding More than Four Numbers **Pupil pages 60-63**

In this activity only the vertical form is used in addition. Renaming may be needed in any or all places. If more help is needed use examples with three addends in which renaming is done in only one place, then two places, and so on. Some examples you may use for revision are:

1. 14763	2. 7466	3. 21376
11004	6120	4174
4114	3349	7212

Write the following story problem on the blackboard:

A farmer has five farms. At harvest time his children picked the fruits. In the first farm they picked 5423 fruits; in the second farm they picked 4916 fruits; in the third farm they picked 7013 fruits; in the fourth they picked 11246; and in the fifth they picked 8129 fruits. How many fruits did they pick altogether?

Ask, "What does the problem ask? (How many fruits did the children pick altogether?) Let n be that number. How do you find n?" (Add the number picked in each farm.) Let a child write a sentence to show this:

$$5423 + 4916 + 7013 + 11246 + 8129 = n$$

Guide the children to see that to add so many numbers it helps to arrange the numerals in vertical form as shown below:

$$\begin{array}{r} 5423 \\ 4916 \\ 7013 \\ 11246 \\ \underline{8129} \end{array}$$

Let a pupil show and explain the work at the blackboard. Ask, "What is the sum?" (36,727) Encourage the children to check the sum by adding in the other direction also.

Let the children answer the question in the word problem. (n is 36,727. Since n is the total number of fruits picked by the children, the answer to the question is this: 36,727 fruits were picked by the children.)

Ask the children to do the exercises on pages 60 to 63. These exercises may be done in several different lessons.

In story problems, encourage the children to begin by writing a mathematical sentence about the story. Then they add the number with the numerals arranged in vertical form. The final step is to answer the question asked in the problem.

STAGE 4: Techniques of Subtraction with Large Numbers

Vocabulary: Estimate

Materials: Pupil pages 64-70

Teaching Procedure

ACTIVITY 1: Revising Renaming in Subtraction Pupil pages 64-65

In this activity revise quickly the idea of renaming in subtraction and

give practice in subtracting large numbers.

Before asking the pupils to work the exercises in their books, go over a few examples to illustrate the use of expanded form and renaming.

Write the following example on the blackboard. Do it only in short form but discuss each step with the children. Let them tell what the steps in renaming are but write the numerals yourself to keep the activity moving as rapidly as possible. The thinking of the children will be similar to that described to the right below:

$$\begin{array}{r} 5211 \\ \underline{2489} \\ 2722 \end{array}$$

1. Think of 5211 as 5 thousands, 2 hundreds, 1 ten and 1. Think of 2489 as 2 thousands, 4 hundreds, 8 tens and 9.
2. There are not enough ones in the ones' place of the sum to subtract 9. Rename 5211 as 5 thousands, 2 hundreds, 0 tens and 11. Subtract the ones. 11-9 = 2. Write 2 in the ones' place.
3. There are not enough tens in the tens' place of the sum to subtract 8 tens. Rename 5 thousands, 2 hundreds and 0 tens as 5 thousands, 1 hundred and 10 tens. Subtract the tens. (10-8) tens = 2 tens. Write 2 in the tens' place.
4. There are not enough hundreds in the hundreds' place of the sum to subtract 4 hundreds. Rename 5 thousands and 1 hundred as 4 thousands and 11 hundreds. Subtract the hundreds. (11-4) hundreds = 7 hundreds. Write 7 in the hundreds' place.
5. Subtract the thousands. (4-2) thousands = 2 thousands. Write 2 in the thousands' place.
6. The missing addend is 2722.

Do several other subtraction examples and go over the steps as before. Some you may use are these:

$$\begin{array}{r} 9514 \\ \underline{1855} \end{array} \qquad \begin{array}{r} 6013 \\ \underline{3489} \end{array}$$

After the subtraction in the second example, say, "Remember that

subtraction is finding the missing addend. You can check your answer by thinking of the addition sentence for the example". Get the children to write the sentence 3489 + n = 6013 for the second example. Go on, "By subtracting you found that the missing addend is 2524. If n is 2524, is the addition sentence true?" Let the children add (3489 + 2524) to show that the sum is 6013. Say, "We call this 'checking our answers' ".

Ask the children to work in their exercise books the exercises on pupil pages 64 and 65.

ACTIVITY 2: Extending Subtraction to Numbers Greater than 10,000

Pupil page 66

The ideas and methods used in this activity are the same as in the last activity. Here the ideas are extended to numbers greater than 10,000 so that renaming and subtracting in the ten thousands' place are included.

Begin with an example like this:

$$\begin{array}{r} 32,921 \\ \underline{18,365} \end{array}$$

Ask the children to copy it in their exercise books and subtract. When they have completed the exercise, work it on the blackboard and discuss each step.

After the example is completed, guide the children to see that they can check the subtraction by adding:

$$\begin{array}{r} 18,365 \\ \underline{14,556} \\ 32,921 \end{array}$$

Do a few other examples as needed and then let the children do the exercises on pupil page 66. Encourage them to use addition to check their answers. If some have difficulty, go over several exercises step-by-step with them.

ACTIVITY 3: Story Problems with Large Numbers

Pupil pages 67-68

Ask the children to turn to pupil page 67. Let a child read the first problem aloud. Go over the procedure for solving the problem. The procedure

for problem 1 is:

a. Compare the three numbers telling the height of the three mountains. Which is the largest number? (19,340) Therefore, Kilimanjaro is the highest.

b. Compare the other two numbers. Which is larger, 17,058 or 14,178? (17,058) Therefore, Mount Kenya is higher than Mount Elgon.

c. Answer the questions: How can we find how much higher Kilimanjaro is than Mount Elgon? (We subtract the height of Mount Elgon from the height of Kilimanjaro.) How can we find how much higher Kilimanjaro is than Mount Kenya?

d. Show the mathematical sentences for these.

e. Answer the questions after doing the subtractions.

Read through the other problems with the class and let the children do the work in their exercise books. Tell them to write the mathematical sentences for the questions asked, solve the sentences and write the answers to the questions.

Notice that in the second problem they are asked to show a mathematical sentence comparing a greater number with a lesser number, using population figures. They may show this as:

$$315{,}000 > 179{,}000$$

If there is difficulty with reading the problems let the pupils help each other read the problems one at a time, giving the children sufficient time to do each before reading the next. Follow the same procedure for the problems on pupil page 68.

ACTIVITY 4: Addition and Subtraction of Large Numbers Pupil page 69

Ask the pupils to turn to pupil page 69 and do the exercises in their exercise books. Remind the children to read the exercises carefully because there are both addition and subtraction exercises.

Say, "Remember that it usually helps in solving word problems to write

a mathematical sentence. Then you can solve the sentence to find the answer to the question asked".

ACTIVITY 5: Estimating Sums **Pupil page 70**

Write on the blackboard:

329 + 512

Ask, "How many hundreds are shown in 329? (3) How many hundreds in 512? (5) How many hundreds in the sum?" (8)

Ask, "If I say that 329 + 512 is 800, is this correct? (No) Is 800 close to the sum?" Let the children discuss this and tell you that the sum is 841, and that 800 is close to the sum but not correct.

Explain that we estimate a number when we give another number close to the first number. Let the children talk about examples of situations in which they estimate numbers. For example:

- We say the population of Nigeria is 55 million. This is an estimate of the population. It is close to the correct number. It is not exact.
- We say there are 8000 miles of roads in a country. This is an estimate of the correct number of miles. It is not exact.

Let the pupils turn to page 70 and estimate the sums orally. Encourage them to think of an estimate of each number and then give an estimate of the missing number by adding or subtracting the estimates. Accept estimates that are reasonably close. For example, one pupil may say that in Exercise 1, the sum is about 700, because 420 is about 400 and 373 is about 300. Another may say that the sum is about 800 because 373 is about 400. Accept either of these.

In Exercise 7, for example, one pupil might say a is about 15,000. Each time a pupil gives an estimate, get him to tell how he got it.

UNIT 6

Measurement of Line Segments and Angles

Objectives

1. To revise the meaning of a measure as a number of units and the use of standard units of inch, foot, yard in measuring segments; to extend standard units to mile and to $\frac{1}{2}$ inch and $\frac{1}{4}$ inch
2. To learn to convert between inches, feet, and yards
3. To introduce centimetre unit and notice the difference in measure when using different units (without conversions)
4. To find perimeters of triangles and other polygons with standard units of measure
5. To approximate perimeters of circles and other closed figures by measuring with a string
6. To introduce the idea of angle measure by using a small unit angle

Background Information for Teachers

The children have learned to measure a line segment in standard units of inches, feet, and yards. They have chosen the unit of measure according to the length of the object to be measured: inches for small segments and yards for long segments. In this Unit the standard unit of mile for great distances is introduced as well as some smaller units, such as half-inch and quarter-inch, to give more precise measurements than before.

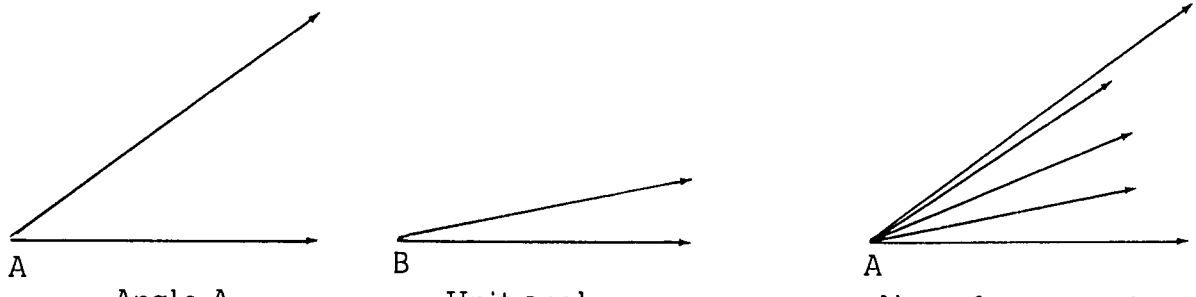
When the idea of measurement of length is revised it is important to think of a measurement as having two parts: a number (sometimes called the <u>measure</u>), and a <u>unit</u> of measure. For example, when we say the pencil is 8 inches long, this measurement has a number, 8, and a unit, inch. The number 8 tells us how many of the units will cover the pencil, and the word "inch" tells us what unit is used. The number alone or the unit alone will not give a measurement. Both must be given before we can tell how long the pencil is.

When one segment is measured twice with different units, the measures we get will also be different. For example, a certain segment that has length 2 feet also has length 24 inches. We notice that the two measurements are the same, although the two numbers are different because the units are different. We can write:

2 feet = 24 inches

This tells us that the two measurements are the same. (That is, they refer to the same length.) We know, for example, that 1 foot is the same measurement as 12 inches, and 1 yard is the same measurement as 3 feet. With this information we convert between these units.

When the centimetre unit is introduced in this Unit the conversion between centimetres and inches is not included. It is enough in Primary Four for the pupils to recognize that a centimetre is also a standard unit and to get practice in using it. A quantity, such as length, is measured by choosing a unit of this quantity (length) and deciding how many units "make up" the quantity. For example, an amount of liquid is measured by choosing a unit of the liquid such as a pint and deciding how many pints "make up" the total amount. Thus, when we attempt to measure an angle, we choose a small angle as a unit and decide how many unit angles, side by side, make up the angle.



Angle A Unit angle About 3 unit angles

We then say that the size of angle A in the figure is about 3 units (or we can say the size of angle A is between 3 units and 4 units).

In Primary Four we are not yet ready to introduce a standard unit of angle, such as a degree. This will be done in Primary Five. In the meanwhile, the idea of angle measure can be made clear by using any small angle as a unit.

STAGE 1: Revising Measurements of Segments

Vocabulary: Measure, unit of measure, mile

Materials: Several pieces of string or cord more than 30 feet long; a ruler for each child marked in inches; pupil pages 71-74

Teaching Procedure

ACTIVITY 1: Measurement of Segments **Pupil pages 71-72**

Let the class talk about finding lengths of objects and distances between points. Ask, "How can a farmer measure the side of his field without using a tape measure?" Let children discuss this until someone says that the farmer can measure by counting paces across the field. Let a pupil (Tamu) show how to find the width of the classroom by counting paces. Ask, "What unit of measure is Tamu using to measure across the room?" If no one answers, explain that he is using a pace as a unit. Say, "Tamu found that the room is 12 paces wide. How many units?" (12) Explain that the measure of the room is 12 and the unit of measure is Tamu's pace.

Let the children tell about other ways of measuring objects without using a tape measure. For example:

1. Measuring the length of a book by counting spans of Fatu's hand.
2. Measuring the width of a doorway by counting John's foot steps.
3. Measuring the length of the table by counting the lengths of Foley's pencil.

Each time a suggestion is given, ask what the unit of measure is. Let the child show how to measure the object using this unit of measure. In the above cases, the measurements might be:

1. The book is 2 units long, if the unit is Fatu's hand span.
2. The doorway is 5 units wide, if the unit is John's foot.
3. The table is 14 units long, if the unit is Foley's pencil.

(In each case the measurement will probably not be exactly a whole number of units. Help the children to explain, for example, that the length of the book is greater than 2 units long and less than 3 units long.)

Let three different children measure the width of the doorway, each with a different unit, say Mary's foot, Ike's hand span, and Foley's pencil.

When each reports his measurement of the doorway, write the results on the blackboard. They might be:

Measure	Unit of Measure
5	Mary's feet
7	Ike's hands
8	Foley's pencil

Ask, "Are these measurements all the same?" (Yes) Why? Let the children explain that all three measurements were made of the same doorway, but the numbers (the measures) are different because the units of measure were different. Again compare the three units of measure to see that they are different lengths. Also compare lengths of children's feet to see that even different feet are different units.

Ask, "What do people use as a unit so that the unit will always be the same length for each person?" If no one answers "foot" or "inch", hold up a foot ruler and ask, "What is this? (A foot ruler) How long is it? (A foot) Are all foot rulers the same length?" (Yes) Children may say that an inch is used as a unit (let a child show an inch on your ruler), or that a yard is used as a unit (let a child show how long a yard is on the blackboard). In this way the children revise the fact that 1 foot is 12 inches long, 1 yard is 3 feet long, and 1 yard is 36 inches long.

Give each child a ruler. Let the children practice using the inch marks on their rulers by turning to pupil page 71 and measuring the line segments in Exercise 1. Go around the class and help the pupils to place the left end of the ruler at the left endpoint A of the line segment $\overline{AB}$, and find the inch mark on the ruler near the right endpoint B. Let a child report the measurement:

The length of $\overline{AB}$ is 3 inches.

Let the pupils measure each segment in Exercise 1 and write a statement about the measurement in their exercise books.

In Exercise 2 the segments are not a whole number of inches long. Let the pupils follow the example and report their measurement between two whole numbers of inches.

On pupil page 72 there is more practice if needed.

At the end of the activity ask, "If you want to measure the length of the classroom, would you use an inch unit? (No. It is too small.) What unit would you use? (A foot or a yard.) If you want to measure the distance from (name your town) to (name another town several miles away) would you use a foot unit?" Let the children discuss this. Ask whether they know any longer units than a foot to use in measuring the distance between towns. If a child suggests miles, explain that we measure very long distances using a mile unit. A mile is 1760 yards long.

ACTIVITY 2: Practicing Measuring

Revise the idea of unit and measure. Let several pupils find the lengths of large objects in the room (such as the blackboard), using a foot as a unit. Ask them to report the measurements between two whole numbers of feet. For example, the blackboard is more than 9 feet long and less than 10 feet long. Let some pupils find the lengths of small objects in inches.

Say that large objects are measured more closely if we use both feet and inches. Again let a child find the length of some object, for example, the blackboard. When he measures the last whole foot, say the ninth foot, let him measure the remaining part in inches. He can then report the length as "9 feet and 5 inches" or "More than 9 feet and 5 inches and less than 9 feet and 6 inches". Let the children continue measuring several large objects in feet and inches.

Lay out a piece of the long cord. Mark or knot the cord at each yard mark. Let the children agree after measuring again, that the marks are three feet apart and that they are a yard apart.

Take the class outdoors and let them use the marked cord to measure distances between objects, for example, two trees. First, let them measure in yards; then in yards and in feet.

Ask two children to stretch the marked cord to show ten yards. Let the class look at this distance. Say, "We can use this length of cord as a unit. How many of these units do you think make a mile?" Let them guess. Then tell them that a mile is 176 of these lengths. Ask, "How many yards in our unit cord? (10) How many yards in a mile?" (176 × 10 = 1760)

ACTIVITY 3: Changing from One Unit to Another **Pupil page 73**

Draw a line segment two feet long on the blackboard. Let a child measure the line segment in feet and another child measure it in inches. Let the class decide that these two measurements are the same since they give the length of the same line segment. Write:

24 inches = 2 feet

Ask, "How many inches long is 1 foot?" (12) Write:

12 inches = 1 foot

Then write:

36 inches = _____ feet

Let a child write in 3.

Give several other examples with inches, feet, and yards, with at least one example in yards and miles. For example:

If,

1 mile = 1760 yards

then:

2 miles = _____ yards

The exercises on pupil page 73 can be used to give the pupils practice in changing from one unit to another.

ACTIVITY 4: Measuring with $\frac{1}{2}$ and $\frac{1}{4}$ Inch Units **Pupil page 74**

Let the class talk about the sizes of the units they use for measuring lengths. Guide them to revise that they use large units to measure long distances and small units to measure short distances. Let them also tell that to get better measurements they use smaller and smaller units to measure the remaining parts.

Ask, "What is the smallest unit of length we have used? (Inch) Look at your rulers and see whether any smaller unit than an inch is marked on it". Let the children point to marks on the ruler between the inch marks and tell what they represent. ($\frac{1}{2}$ inch and $\frac{1}{4}$ inch) Suggest that their rulers look like the number line with all the $\frac{1}{2}$'s and $\frac{1}{4}$'s marked.

On pupil page 74 there are some line segments to be measured. Help the children to measure with inches and $\frac{1}{2}$ inches or with inches and $\frac{1}{4}$ inches. Let them write their answers in sentences. For example, the length of $\overline{AB}$ is $2\frac{1}{2}$ inches.

STAGE 2: Centimetre Unit

Vocabulary: Centimetre, perimeter, closed figure

Materials: Rulers with inch and centimetre scales (one for each child); piece of cardboard about 1" x 12"; round tins, round buckets, pots and other objects showing circles; pins; piece of string about 12" long for each pupil; pupil pages 75-76

Teaching Procedure

ACTIVITY 1: Measuring in Centimetres with a Ruler **Pupil pages 75-76**

Ask the pupils to tell the units of measure they already know. (Inch, foot, yard, mile) Let them explain that they use a yard or a foot to measure long segments and they use an inch to measure short segments.

Tell the pupils that many people use a smaller unit than an inch to find the length of small things. This unit is called a centimetre. Show them the centimetres marked on a foot ruler. Let the pupils cut a piece of paper or stick one centimetre long. Let them tell whether the unit centimetre is longer or shorter than the inch unit, the foot unit, the yard unit, the mile unit.

Take a long piece of cardboard and mark points along its edge so that the points are one centimetre apart. Call this a centimetre ruler. Ask a pupil to use this ruler to measure his pencil and tell how many centimetres it measures. (For example, "The pencil is longer than 8 centimetres and shorter than 9 centimetres".) Let other pupils measure objects such as their exercise books, text books, and table tops.

Choose a pupil who has a foot ruler which is also marked off in centimetres. Let him match his foot ruler with the cardboard centimetre ruler. Ask whether any marks on the ruler show centimetres. Do this with several pupils in the class.

Go around the class and let each pupil show you centimetres on his ruler if it is marked in centimetres. Let them use their rulers to find the length in centimetres of different things in the room.

Put a pencil or other object on the table and call out two pupils. Let each hold a ruler. Let the first pupil measure the object and tell its length in inches. Then tell the second pupil to measure the same object and tell its length in centimetres. Do the same with other pairs of pupils and other objects. Let them tell the lengths; for example, one child says the Pupil Book is more than 21 centimetres wide and less than 22 centimetres wide; the other child says the Pupil Book is more than 8 inches wide and less than 9 inches wide.

Draw line segments of different lengths on the blackboard. Ask some pupils to measure the line segments in centimetres. Let other pupils measure them in inches. Let the children discuss the reason why the measure in centimetres is always different from the measure in inches, even though the lengths are the same. (The centimetre unit is shorter than the inch unit, so there are more centimetres in the length than there are inches.)

Ask, "If you measure the wall of the classroom do you use centimetres? (No. A centimetre unit is too small.) What unit of measure that you know is better?" (Foot or yard)

Say, "You may measure line segments in inches or in centimetres".

Draw a triangle and a rectangle on the blackboard. Ask, "How do you find the 'perimeter' of this triangle?" If the pupils do not remember the meaning of perimeter, say, "Perimeter is the total length of all sides of a closed figure". Get the pupils to tell how they find the perimeter of the triangle and the perimeter of the rectangle. Their answers might include any of these:

1. Find the measure of each side of the figure and add the measures.
2. Lay a string around the figure so that it matches the figure. Then measure the length of the string.
3. Roll the figure carefully along a line until it is completely turned around. Then measure the distance on the line from the beginning to the end of the rolling.

Ask the pupils to do the exercises on pupil pages 75 and 76. (Explain that a short way to write centimetre is cm., and a short way to write inch is in.)

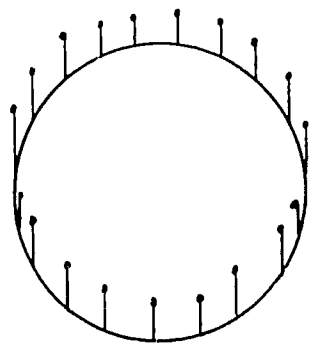
ACTIVITY 2: Finding Perimeters **Pupil page 77**

Show the pupils a round tin. Use your finger to trace the circle it shows. Say, "We want to find the perimeter of this circle". Ask the children to guess the perimeter of the circle in inches. Give several children each a string. Tell them, "You have a string. How can you use it to find the perimeter of the circle?" Let them in turn fit the string around the tin and cut it so it fits exactly. Say, "Now the length of the string is the perimeter of the circle. How many inches is that?" Let them use their rulers to measure the string and tell the perimeter in inches. Let them compare the length with their guesses.

Get the pupils to measure the same string in centimetres and tell the perimeter of the circle in centimetres.

Let other pupils find the perimeter of other round objects in inches and centimetres in the same way. They can measure other round tins, round buckets, cooking pots, round table tops, and so on.

Let some pupils use string to find the perimeter of a large circle drawn on paper. If they find it difficult to keep the string around the circle, give them pins and let them fix the pins around the circle. Put the pins near to each other so that the string almost fits the circle. Let the pupils cut the string so it fits along the circle, and then measure the string in both inches and centimetres. Let them say, "The measurement of the length of this string gives the perimeter of the circle".



Ask the pupils to measure the diameter of the same circle in inches and centimetres. Let them use the string or draw a straight line from one edge of the circle to another through the centre point. Ask, "Is the diameter shorter than the perimeter?" (Yes) Note: Do not expect them at this time to make any other comparison between diameter and circumference.

Separate the pupils into groups, go outside and draw circles on the ground. Get them to use long cords to find the perimeter of the circle first in inches and then in centimetres. Help them to decide that the foot unit is a better unit than the centimetre unit for measuring large circles.

Pupil page 77 can be used for more practice in measuring perimeters of figures. Give each pupil a piece of thin string about 12 inches long. Help the pupils to lay the string on a figure and mark the point on the string where it meets the other end. Let them write the measurements in centimetres and compare their results with each other.

STAGE 3: Angle Measure

Vocabulary: Unit angle

Materials: Sheets of paper; cutter (razor blade); pieces of wire; semi-transparent papers; pupil page 78

Teaching Procedure

ACTIVITY 1: Angle Measure

Draw a line segment on the blackboard and ask a pupil to measure it. Ask, "What units do we use to measure a line segment?" (Centimetre, inch, foot, yard, mile) Let the children talk about when each is used.

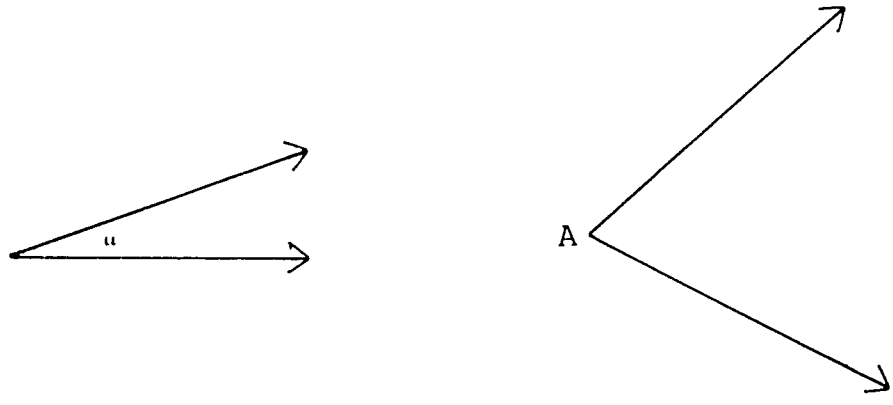
Draw several angles each on a sheet of semi-transparent paper. (Be sure that two of the angles are congruent.) Compare the congruent angles by placing one sheet of paper on the other so the children can see that the angles fit exactly. Ask, "Are these angles congruent?" (Yes) Compare two angles that are not congruent by placing one sheet of paper on the other so that the vertex and one ray fit. Ask, "Are these angles congruent? (No) Which is greater than the other? Do they have the same measure? (The angles are not congruent; their measures are different.) Which has the greater measure?" (The bigger one has the greater measure; the smaller one has the smaller measure.)

Ask, "Do you think we can use a line segment as a unit to measure an angle?" Let the pupils discuss this until it is decided that they cannot. Say, "We must find a unit we can use. Remember that to measure a line segment we use a unit which is a line segment. To measure a region what type of unit is used?" (We used a tile. A tile is a unit which is a region.)

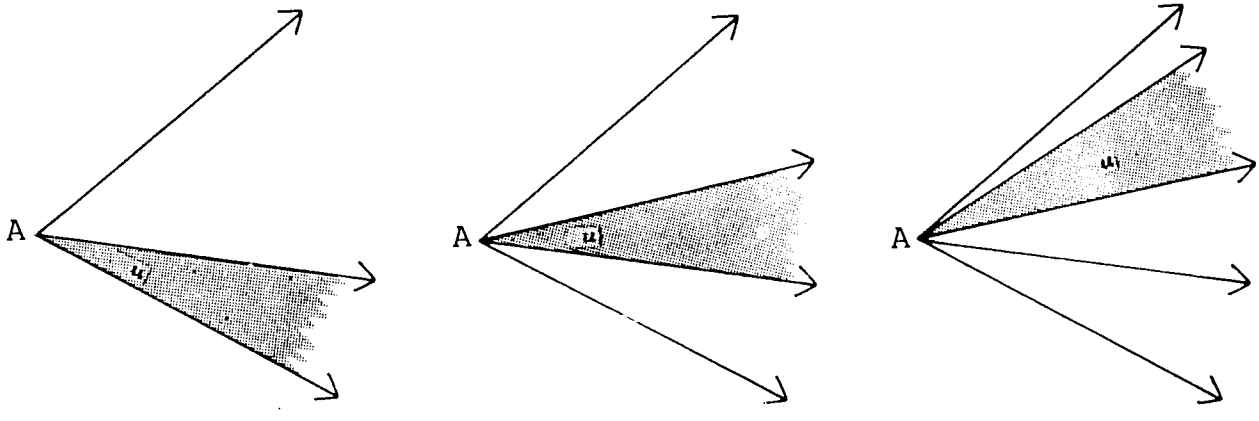
Ask, "To measure an angle, what type of unit must we use? (An angle) Why?" (Because to measure anything we must use a unit which is of the same kind as the thing to be measured.)

Draw a large angle on the blackboard. Ask each child to draw on a sheet of paper a small angle to use as his unit angle. Let him cut it out. (Or fold the paper to show the angle.) Get him to think of how to use his unit angle to measure the large angle on the blackboard. (If wire is available let one child use a short piece of wire bent in the shape of a small angle to show a small angle. This shows an angle better than the paper cutout.) Invite a pupil to show the class how the measurement of the angle on the blackboard is made with his unit angle. Continue with other pupils. Each time a pupil shows his method ask, "Did he measure the angle properly?"

Get the children to see that if the angle below on the left is the unit angle chosen by the pupil and he wants to measure the angle on the right we must find how many times this unit angle must be used to "make up" the angle A.



First lay one side of the cutout unit angle (or bent wire unit angle) on one side of angle A with the vertex of the unit angle on the vertex of angle A.



Now draw a ray along the other side of the unit angle. Then place the cutout unit angle with its first side on the ray just drawn and draw a ray along the other side. Continue until the second side of the unit angle comes close to the second side of angle A. Then count the number of times the unit was used to make up angle A. In the figure above, the size of angle A is greater than 3 units and less than 4 units.

Since the units used by the children will be of different sizes, they will get different measures of angle A. Let them discuss the reason for this.

Let the children repeat this exercise by drawing a large angle on a paper and measuring it with any smaller angle they choose as their unit angle. Ask each child to tell the measure he obtained, for example, "My angle is larger than 5 units and smaller than 6 units".

ACTIVITY 2: Measuring Angles **Pupil page 78**

Ask each child to draw on a paper an angle to use as a unit angle. Let him cut it out. (If you can supply pieces of stiff wire to make the angle, do so.)

Ask the pupils to use their unit angle to measure angles shown by objects in the room such as the tops of tables and the corners of walls of the room. Get them to do the exercises on pupil page 78. Copy the unit angle by tracing it and make a cutout (or by bending a stiff wire to the shape of the unit angle). Let them tell the measures of angles A, B, C, and D.

UNIT 7

Multiplication and Division of Whole Numbers I

Objectives

1. To revise techniques of multiplication and division
2. To develop the ability to estimate products and missing factors
3. To revise the commutative and associative properties of multiplication and the distributive property
4. To continue the development of the ability to solve word problems by using mathematical sentences
5. To show the application of multiplication and division to money, weights, and measures

Background Information for Teachers

See "Background Information for Teachers" for Unit 1, Unit 2, and Unit 4 of this book.

MULTIPLICATION TECHNIQUES

In the sentence below 20 is called the product and 4 and 5 are called factors:

$$4 \times 5 = 20$$

Since 20 is obtained by multiplying 4 by a whole number, 20 is called a multiple of 4. A multiple of a whole number n is any number obtained by multiplying n by a whole number: $0 \times n$, $1 \times n$, $2 \times n$, $3 \times n$, $4 \times n$, and so on. Twenty is also a multiple of 5. Multiples of 5 are 0, 5, 10, 15, 20, 25, 30, and so on.

The associative and commutative properties of multiplication are useful in finding products where one factor is a multiple of 10. Following are some examples:

(1) $6 \times 30 = 6 \times (3 \times 10)$
$= (6 \times 3) \times 10$
$= 18 \times 10$
$= 180$

(2) $40 \times 30 = (4 \times 10) \times (3 \times 10)$
$= (4 \times 3) \times (10 \times 10)$
$= 12 \times 100$
$= 1200$

(3) $5 \times 700 = 5 \times (7 \times 100)$
$= (5 \times 7) \times 100$
$= 35 \times 100$
$= 3500$

The techniques of finding products also use the idea of place-value and the distributive property of multiplication over addition. Two examples follow:

Example 1. Find the product, 7×132.

(a) $7 \times 132 = 7 \times (100 + 30 + 2)$ (Rename 132)
$= (7 \times 100) + (7 \times 30) + (7 \times 2)$ (Distributive property)
$= 700 + 210 + 14$
$= 924$

(b) In the vertical form, the process is shown like this:

$$\begin{array}{r} 132 \\ \underline{7} \\ 14 \\ 210 \\ \underline{700} \\ 924 \end{array} \qquad \left.\begin{array}{l} (7 \times 2 = 14) \\ (7 \times 30 = 210) \\ (7 \times 100 = 700) \end{array}\right\} \text{(Distributive property)}$$

(Think of 132 as 100 + 30 + 2)

(c) The short form of the process is shown below to the left. The process is described to the right.

$$\begin{array}{r} 132 \\ \underline{7} \\ 924 \end{array}$$

1. Think of 132 as 100 + 30 + 2.
2. $7 \times 2 = 14$. Rename 14 as 1 ten and 4 ones. Write 4 in the ones' place of the product. Remember 1 ten.
3. 7×3 tens $= 21$ tens. $(21 + 1)$ tens $= 22$ tens. Rename 22 tens as 2 hundreds and 2 tens. Write 2 in the tens' place. Remember 2 hundreds.
4. 7×1 hundred $= 7$ hundreds. $(7 + 2)$ hundreds $= 9$ hundreds. Write 9 in the hundreds' place.
5. The product is 924.

Example 2. Find the product, 36×172.

(a) $36 \times 172 = (30 + 6) \times 172$ (Rename 36)

$= (30 \times 172) + (6 \times 172)$ (Distributive property)

These last two products, written in vertical form, are:

$$\begin{array}{rl} 172 & \\ \underline{30} & \\ 60 & (30 \times 2 + 60) \\ 2100 & (30 \times 70 = 2100) \\ \underline{3000} & (30 \times 100 = 3000) \\ 5160 & \end{array} \quad \text{and} \quad \begin{array}{rl} 172 & \\ \underline{6} & \\ 12 & (6 \times 2 = 12) \\ 420 & (6 \times 70 = 420) \\ \underline{600} & (6 \times 100 = 600) \\ 1032 & \end{array}$$

Therefore, $36 \times 172 = 5160 + 1032$

$= 6192$

(b) A short form that includes all the above is shown below.

A description of the process used is similar to that given for Example 1.

$$\begin{array}{rl} 172 & \\ \underline{36} & \\ 1032 & (6 \times 172 = 1032) \\ \underline{5160} & (30 \times 172 = 5160) \\ 6192 & \end{array} \quad \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{(Distributive property)}$$

The process of finding other products is similar. With understanding and practice, children are able to use the short form at all times. They will always be thinking of the product as the sum of simpler products.

DIVISION TECHNIQUES

Division is the process of finding the missing factor when the product and one factor are known. For example, $6 \div 3 = n$ means the same as $3 \times n = 6$. The missing factor is $\underline{n}$. If $\underline{n}$ is 2, both sentences are true.

The techniques of division use the idea of place-value and the distributive property of division over addition. This form of the distributive property is shown with an easy example.

Suppose we wish to solve the sentence $n = 42 \div 3$. We can rewrite the sentence and use the distributive property.

$n = 42 \div 3$

$3 \times n = 42$

$3 \times n = 30 + 12$ (Rename 42 as 30 + 12)

$3 \times n = (3 \times 10) + (3 \times 4)$ (Rename 30 and 12 as multiples of 3)

$3 \times n = 3 \times (10 + 4)$ (Distributive property)

n is 10 + 4 (Multiplication assigns only one number to a pair of whole numbers.)

This shows that if $n = 42 \div 3$, then the following is true:

$n = (30 + 12) \div 3$

and $n = (30 \div 3) + (12 \div 3)$

and $n = 10 + 4$

n is 14

That is, division is distributive over addition.

The ideas used in the techniques of division are shown with the following examples.

Example 1. Find n that makes $272 \div 8 = n$ a true sentence.

```
   20 + 10 + 4 = 34
8) 272
   160  (160 ÷ 8 = 20)
   112
    80  (80  ÷ 8 = 10)
    32
    32  (32  ÷ 8 = 4)
```

- The product 272 is renamed as 160 + 80 + 32. These addends are chosen because they are each a multiple of 8.
- The distributive property is used. Each of the addends chosen is divided by 8.
- The missing factor is 20 + 10 + 4 = 34

In the division process addends are chosen one at a time. In this example, 160 was chosen first because it is a multiple of 8 and also of 10. Then 160 was subtracted from 272 to get 112. Addends of 112 that are multiples of 8 were then chosen and the process continued.

The choice of the addends 160, 80, 32 is not the best one. Children will discover this as they become skillful at choosing multiples.

Since 160 is not the <u>largest</u> multiple of 8 and of 10 that is also less than 272, a better choice of addends would be 240 and 32. Then the missing factor, 30 + 4 = 34, is quickly obtained. A short form showing this is:

```
     30 + 4 = 34
8) 272
   240      (240 ÷ 8 = 30) }
    32                     } (Distributive property)
    32      (32  ÷ 8 = 4) }
```

Example 2. Find <u>n</u> for which n = 928 ÷ 8 is true. The short form for dividing 928 by 8 is shown on the left below. The process is described on the right. Notice that 928 is renamed with addends that are multiples of 8 and the distributive property is used.

```
     100 + 10 + 6 = 116
3)  928
    800  (800 ÷ 8 = 100)
    128
     80   (80 ÷ 8 =  10)
     48
     48   (48 ÷ 8 =   6)
```

1. Find an addend of 928 that is a multiple of 8, a multiple of 100 and as large as possible but less than 928. This addend is 800.
2. 800 ÷ 8 = 100. Write 100 as part of missing factor.
3. Subtract. 928 - 800 = 128.
4. Find an addend of 128 that is a multiple of 8, a multiple of 10, and as large as possible but less than 128. This addend is 80.
5. 80 ÷ 8 = 10. Write 10 as part of missing factor.
6. Subtract. 128 - 80 = 48
7. 48 is a multiple of 8. 48 ÷ 8 = 6. Write 6 as part of missing factor.
8. The missing factor is 100 + 10 + 6 or 116.

We can check our work. If 116 is the missing factor then 116 × 8 = 928 is a true sentence. We can check this by multiplying 116 × 8.

In Example 2, if the product is less than 800, step 1 is meaningless.

We then move directly to step 3. Example 1 shows this. There we first chose an addend which is a multiple of 8 and a multiple of 10 (not a multiple of 100). In the same way, if the product is less than 80, we look immediately for multiples of 8, only.

Estimating answers is helpful to the division process. For example, in $272 \div 8 = n$, the number n is between 30 and 40, because $30 \times 8 = 240$ and $40 \times 8 = 320$, and the product 272 is between 240 and 320.

It is not always true that the product is a multiple of the known factor. For example, there is no whole number n that makes $14 \div 3 = n$ a true sentence. 14 is not a multiple of 3. But,

$$14 = (4 \times 3) + 2$$

Since 2 is less than 3, 2 has no addend which is a multiple of 3. This leads us to a way of doing problems in which the product is not a multiple of the known factor. As an example, find $728 \div 3$.

```
   200 + 40 + 2 = 242
3) 728
   600     (600 ÷ 3 = 200)
   128
   120     (120 ÷ 3 = 40)
     8
     6     (6 ÷ 3 = 2)
     2          ?
```

- Carry on the division process as before.
- Notice that there is no way to rename 2 so that the process can continue.

We write the results in this way:

$$728 = (242 \times 3) + 2$$

Notice that (242×3) is the largest multiple of 3 less than 728 and that 2 is less than 3. (The 2 is sometimes called a "division remainder" and 3 a "divisor". We will not use these terms.) Below are simple examples showing the form in which the results of similar exercises are written.

Exercise	Result
$17 \div 5$	$17 = (3 \times 5) + 2$
$25 \div 7$	$25 = (3 \times 7) + 4$
$37 \div 8$	$37 = (4 \times 8) + 5$

Example 3. $5963 \div 34 = n$. The number $\underline{n}$ is between 100 and 200, since $100 \times 34 < 5963$ and $5963 < 200 \times 34$. The short form for the division process is:

```
     100 + 70 + 5 = 175
34) 5963
    3400          (3400 ÷ 34 = 100)
    2563
    2380          (2380 ÷ 34 = 70)
     183
     170          (170  ÷ 34 = 5)
      13
```

$\underline{n}$ is not a whole number. The result is written: $5963 = (175 \times 34) + 13$. We can check the result by multiplication and addition.

STAGE 1: Revision of Properties of Multiplication

Vocabulary: No new vocabulary

Materials: Pupil pages 79-80

Teaching Procedure

ACTIVITY 1: Revision of the Commutative Property of Multiplication and the Property of One

Pupil page 79

Operation machines were discussed in Unit 4 when properties of multiplication were revised. If you believe they are needed, use the machines again to show examples of the fact that $a \times b = b \times a$ and $a \times 1 = a$, where $\underline{a}$ and $\underline{b}$ are any whole numbers.

Another way to revise the commutative property of multiplication is to let the pupils make on the blackboard a chart like the one on the top of the next page. Get them to fill in the products as shown.

×	1	5	6	8	9
1	1	5	6	8	9
5		25	30	40	45
6			36	48	54
8				64	72
9					81

← Diagonal

Ask, "How can the rest of the boxes be filled in without multiplying?" Guide the pupils to notice that the boxes below the diagonal will show the same numbers that are shown in the boxes above the diagonal. The numbers are the same because of the commutative property of multiplication. Get the pupils to show the correct box for each number and complete the chart. Each time get them to tell why they chose the number. For example, "40 goes in this box because $8 \times 5 = 5 \times 8$".

Ask pupils to turn to pupil page 79 and work the exercises. Go around the class and let pupils tell why they write a numeral in a box without multiplying.

ACTIVITY 2: Revision of the Associative and Distributive Properties of Multiplication

Pupil page 80

Write a sentence like the following on the blackboard:

$$2 \times (4 \times 6) = (2 \times 4) \times 6$$

Ask whether this is a true sentence. Let the children decide that the sentence is true because:

$$2 \times (4 \times 6) = 2 \times 24 = 48 \quad \text{and} \quad (2 \times 4) \times 6 = 8 \times 6 = 48$$

Guide the children to find a product such as 7×40, by use of the associative property in this way:

$$\begin{aligned} 7 \times 40 &= 7 \times (4 \times 10) \\ &= (7 \times 4) \times 10 \\ &= 28 \times 10 \\ &= 280 \end{aligned}$$

Revise the distributive property by helping the children complete a chart on the blackboard like the one shown below to the left:

×	5	7	
6			
9			

+ ↓

+ →

×	5	7	12
6	30	42	72
9	45	63	108
15	75	105	180

+ ↓

+ →

Get the pupils to:

1. Multiply 6 × 5 and write 30 on the chart as shown above to the right.
2. Multiply 6 × 7, 9 × 5, and 9 × 7 and write the products on the chart as shown.
3. Add 5 + 7 and 6 + 9 and write the sums on the chart as shown.
4. Multiply 6 × 12 and write the product as shown.
5. Add 30 + 42 and write the sum as shown.

Point to the 72 on the chart and ask what it means in the chart. (It is 6 × 12 and also 30 + 42.) Write as you point to the chart:

$$6 \times 12 = 6 \times (5 + 7) = 30 + 42$$

or

$$6 \times 12 = 6 \times (5 + 7) = (6 \times 5) + (6 \times 7)$$

Ask, "What property has been used on the table to find 6 × (5 + 7) ?" (Distributive property)

Discuss in a similar way 9 × (5 + 7), 15 × (5 + 7), 5 × (6 + 9), 7 × (6 + 9), and 12 × (6 + 9) both on the chart and in sentence form. Finally, let the pupils show that:

$$\begin{aligned}(6 + 9) \times (5 + 7) &= 6 \times (5 + 7) + 9 \times (5 + 7)\\ &= 72 + 108\\ &= 180\end{aligned}$$

Let the children do the exercises on pupil page 80. Get the children to tell how the charts show the use of the distributive property.

STAGE 2: Revision of Techniques of Multiplication and Division

Vocabulary: No new vocabulary

Materials: Pupil pages 81-86

Teaching Procedure

ACTIVITY 1: Multiplication — One Factor a Multiple of 10; Other Factor Less than 100

Pupil page 81

Write sentences like these on the blackboard:

1. $a = 6 \times 10$
2. $6 \times 40 = c$
3. $30 \times 40 = d$
4. $e = 60 \times 25$

Ask what process is needed to solve the sentences. As the children talk about each sentence get them to estimate the product, do the multiplication and tell the product. (Let the children use arrays if needed, but encourage them to learn a process of multiplication.)

Guide the children to use the properties of multiplication in the examples in this way:

1. $c = 6 \times 40$
   $= 6 \times (4 \times 10)$ (Rename 40)
   $= (6 \times 4) \times 10$ (Associative property)
   $= 24 \times 10$
   c is 240

2. $d = 30 \times 40$
   $= (3 \times 10) \times (4 \times 10)$ (Rename 30 and 40)
   $= (3 \times 4) \times (10 \times 10)$ (Commutative and associative property)
   $= 12 \times 100$
   d is 1200

3. $e = 60 \times 25$
   $= 60 \times (20 + 5)$ (Rename 25 as 20 + 5)
   $= (60 \times 20) + (60 \times 5)$ (Distributive property)
   $= 1200 + 300$
   e is 1500

Guide the children to study these examples and to devise a way of

multiplying by a multiple of 10. Encourage them to give the properties used at each stage.

Let the pupils open their books to pupil page 81 and work the exercises in their exercise books.

ACTIVITY 2: Revising Techniques of Multiplication with One or Both Factors Less than 100

Pupil pages 82-83

Write on the blackboard sentences like these:

1. $p = 6 \times 58$ 2. $q = 45 \times 73$

Let the pupils talk about methods they can use to find the products. Guide them to show the multiplication in sentence form and then in the short form.

1. $p = 6 \times 58$

$= 6 \times (50 + 8)$ (Rename 58)

$= (6 \times 50) + (6 \times 8)$ (Distributive property)

$= 300 + 48$

$= 348$ p is 348

The short form of this example is:

$$\begin{array}{rl} 58 & \\ \underline{6} & \\ 48 & (6 \times 8) \\ \underline{300} & (6 \times 50) \\ 348 & \end{array}$$

(Rename 58 as 50 + 8)

(Distributive property)

p is 348

2. $q = 45 \times 73$

$= (40 + 5) \times (70 + 3)$ (Rename 45 and 73)

$= (40 + 5) \times 70 + (40 + 5) \times 3$ (Distributive property)

$= (40 \times 70) + (5 \times 70) + (40 \times 3) + (5 \times 3)$ (Distributive property)

$= 2800 + 350 + 120 + 15$

$= 3285$ q is 3285

Guide the pupils to see that in this example the product has been expressed as the sum of four simple products and that the above procedure is shown by

the short form below:

$$\begin{array}{rl} 45 & \\ \underline{73} & \\ 15 & (3 \times 5) \\ 120 & (3 \times 40) \\ 350 & (70 \times 5) \\ \underline{2800} & (70 \times 40) \\ 3285 & \end{array}$$

q is 3285

Help the children to understand and use a still shorter form. It is shown below to the left and the process is described to the right.

$$\begin{array}{r} 45 \\ \underline{73} \\ 135 \\ \underline{3150} \\ 3285 \end{array}$$

1. Think of 45 as 40 + 5, and of 73 as 70 + 3.
2. Multiply by 3.
   (a) $3 \times 5 = 15$. Rename 15 as 1 ten and 5. Write 5 in the ones' place. Remember 1 ten.
   (b) 3×4 tens = 12 tens. (12 + 1) tens = 13 tens. Rename 13 tens as 1 hundred and 3 tens. Write 1 in hundreds' place and 3 in tens' place.
3. Multiply by 7 tens.
   (a) 7 tens $\times$ 5 = 35 tens. Rename 35 tens as 3 hundreds and 5 tens. Write 5 in the tens' place and 0 in the ones' place. Remember the 3 hundreds.
   (b) 7 tens $\times$ 4 tens = 28 hundreds. (28 + 3) hundreds = 31 hundreds. Rename 31 hundreds as 3 thousands and 1 hundred. Write 3 in the thousands' place and 1 in the hundreds' place.
4. Add. 135 + 3150 = 3285. The product is 3285.

Guide the children to do other examples as needed on the blackboard.

Let the pupils open their books to pupil pages 82 and 83 and do the exercises there. Let them use the form of multiplication they understand. Encourage them finally to use the shorter form. In the story problems, get the children to write mathematical sentences before finding the missing number.

ACTIVITY 3: Revising Division **Pupil page 84**

Help the pupils recall the relationship between multiplication and division. Write on the blackboard either a multiplication sentence or a division sentence and let the children supply the other. For example, let them discuss the fact that these sentences tell the same thing about factors and products:

$7 \times 6 = 42$ and $6 = 42 \div 7$

Below are other examples of this type:

$6 \times 8 = 48$	$10 = 80 \div 8$
$6 = 48 \div 8$	$8 = 80 \div 10$
$8 = 48 \div 6$	$80 = 10 \times 8$
$8 \times 40 = 320$	$60 = 1200 \div 20$
$40 = 320 \div 8$	$20 = 1200 \div 60$
$8 = 320 \div 4$	$1200 = 60 \times 20$

Write a number of sentences with missing factors like those below and get the children to solve them:

$7 \times n = 84$	$n \times 30 = 330$	$20 \times n = 800$
n is 12	n is 11	n is 40

Now write a sentence such as this on the blackboard:

$$n = 216 \div 9$$

Ask the pupils what the sentence means. They may think of <u>n</u> as a missing factor in the multiplication sentence, $n \times 9 = 216$. Let them talk about how to find <u>n</u> and guide them to revise the following method.

$n = 216 \div 9$

$= (180 + 36) \div 9$ (216 renamed as 180 + 36)

$= (180 \div 9) + (36 \div 9)$ (Distributive property)

$= 20 + 4$

n is 24

As they do the work above on the blackboard, question them about each

step in a way similar to this:

- Why did you rename 216 as 180 + 36? (Because 180 is a multiple of 10 and a multiple of the known factor 9.)
- Can you rename 216 other ways? (Yes, other ways to rename 216 are 90 + 72 + 54 and 90 + 90 + 36.)
- Why is it better to rename 216 in the example as 180 + 36 than as 90 + 72 + 54? (Because 180 is the largest addend of 216 that is a multiple of 10 and a multiple of 9.)

Guide the class to use a short form to show the work.

```
     20 + 4 = 24
9) 216
   180          (180 ÷ 9 = 20)
    36
    36          (36 ÷ 9 = 4)
```

Let the children try another example, such as:

$$n = 385 \div 11$$

Ask the pupils for two addends of 385 each of which is a multiple of 11. Get them to name several pairs of addends but talk in particular about 330 and 55. At this stage, guide them to estimate n. Keep in mind that n is such that $n \times 11 = 385$. Decide that, since:

$$30 \times 11 = 330,$$
$$n \times 11 = 385,$$
$$\text{and } 40 \times 11 = 440,$$

then n is between 30 and 40. Estimating n helps the children to choose 330 as one addend of 385.

Let the pupils help you to work the exercise on the blackboard using a short form.

```
     30 + 5 = 35
11) 385
    330         (330 ÷ 11 = 30)
     55
     55         (55 ÷ 11 = 5)
```

Thus n is 35. Let them check by multiplication. $35 \times 11 = 385$.

Do a few more examples with the pupils working on the blackboard. Ask them to open their books to pupil page 84 and do the exercises. Let them check each by multiplication.

ACTIVITY 4: Revising Division When the Product Is Not a Multiple of the Known Factor

Pupil pages 85-86

Write the following story problem on the blackboard:

Modu brought home 76 oranges from his farm.

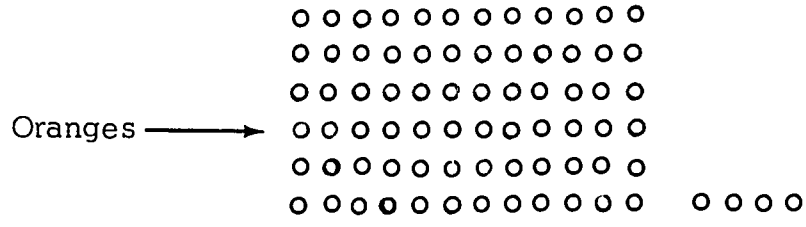
He shared them equally among his 6 friends.

How many did each receive?

Let the pupils discuss the problem and tell what question is asked. Get them to write a mathematical sentence for the problem. They might give any of these:

$$n = 76 \div 6, \qquad n \times 6 = 76, \qquad \text{or} \quad 6 \times n = 76$$

Ask, "How can we find n so that the sentence is true?" One pupil might suggest that the oranges can be placed in 6 rows, one for each friend. Ask that pupil with the help of other pupils to draw an array of 6 rows on the blackboard:



He will find that the oranges fill six rows with 12 in each row, but that there are 4 oranges left over. Ask, "Can Modu share the oranges equally among his friends? (No) Is 6 a factor of 76? (No , because there is no whole number that makes the sentence $6 \times n = 76$ true.) Is 76 a multiple of 6? (No) How can we write a sentence for the array and the oranges that are left?" Guide the pupils to write:

$$76 = (6 \times 12) + 4$$

If no pupil suggests placing the oranges in an array, some pupil may say he can solve the sentence $6 \times n = 76$ by dividing 76 by 6. Invite this pupil to show the class how to do this. He will rename 76 so that one of the addends is a multiple of 6. He might choose:

$$76 = 60 + 16$$

Ask him to write addends of 16 that are multiples of 6. He might write:

$$76 = 60 + 12 + 4$$

Tell him to go on with division:

```
     10 + 2
   ---------
6) 76
   60          (60 ÷ 6 = 10)
   --
   16
   12          (12 ÷ 6 = 2)
   --
    4
```

Let the class discuss what was done and decide that they cannot divide 76 by 6 and get a whole number. There is 4 left and 4 is not a multiple of 6. The pupils will say things like this:

- There is no whole number n that makes $76 \div 6 = n$ true;
- 6 is not a factor of 76;
- 76 is not a multiple of 6;
- The oranges cannot be shared equally among the 6 boys;
- We can write:

$$76 = (6 \times 12) + 4$$

Each friend will receive 12 oranges. There are 4 oranges left over. Continue by asking the class to solve this sentence:

$$318 \div 9 = n$$

Guide the pupils to use a short form as follows:

```
     30 + 5
   ---------
9) 318
   270         (270 ÷ 9 = 30)
   ---
    48
    45         (45 ÷ 9 = 5)
    --
     3
```

From the work the pupils will see that in the example 318 was renamed in this way:

$$318 = 270 + 45 + 3$$

Ask, "Is 9 a factor of 318? (No, because 3 is left over and 9 is not a factor of 3.) Is 318 a multiple of 9?" (No) Guide the pupils to say that n is not a whole number. Help them to see that although 318 is not a multiple of 9 it can be renamed as follows:

$$318 = (35 \times 9) + 3$$

Let the pupils find that (35×9) is the greatest addend of 318 which is a multiple of 9.

Let the pupils open their books to pupil page 85. Help them do a few of the exercises on the blackboard using a short form. They should think of the product as the sum of two addends. One addend is the greatest multiple of the known factor which is less than the product, the other addend is less than the known factor. Guide them to fill in the blank spaces in Exercises 1 to 6. In Exercise 1:

$$482 = (43 \times 11) + 9$$

43×11 is the greatest multiple of 11 that is less than 482; 9 is the other addend; it is less than 11.

Let the children also do the exercises on pupil page 86.

STAGE 3: Estimation of Products and Missing Factors

Vocabulary: Exact, estimate

Materials: Pupil pages 87-91

Teaching Procedure

The purpose of this stage is to encourage the children to make an estimate of products by thinking of the two factors. Estimating differs from wild guessing. It uses known facts to decide that the result of a given operation comes close to or is about a certain number.

It is important for children to understand that there are times when it is enough to know an estimate of a number. For example, population is usually reported as estimates. It is not necessary, nor even possible, to know exactly how many people there are in a city but only to know about how many. For other purposes, it is important to know an exact number. For example, it is important for airlines to know the exact number of plane seats available on every flight so too many tickets are not sold. Children need to learn when to be exact and when estimation is enough.

ACTIVITY 1: Estimating Products **Pupil page 87**

Write the following on the blackboard:

Malawi	4,000,000
Sierra Leone	2,150,000
Zambia	4,000,000

Include also the population of your own country if it is not given above.

Say, "These numbers are the reported population figures of several African countries". Let the children read the numbers aloud. Ask, "Do you think these numbers tell the exact number of people in these countries?"

Let the pupils talk about this. Bring out the ideas that:

- the numbers do not represent exactly the number of people;
- population is always changing;
- at any one time we do not know just how many people are in a country;
- the figures are taken to the nearest million or ten-thousand; and so on.

Say, "Many times we do not need to know an exact number. We need only know about how many. We can use what we know to make guesses that are close to the number. When we do this we estimate the number". Write the following story problem on the blackboard:

Two villages, A and B, grew very fast and became busy towns. Ten years ago village A had 515 people and now has 8 times that number. Ten years ago village B had 420 people and now has 9 times that number.

Ask, "Can you quickly tell which town has more people now?" Encourage the children to answer without doing any multiplication. Ask them to tell the reasons for their answers. One good answer is: Town A has more people because it has about 4000 people (8 × 500) and Town B has about 3600 people (9 × 400). (Accept as good estimates numbers which are close to the exact answer, if the children have good reasons for making these estimates.)

Help the class to recognize this quick way to make an estimate. Guide them to say that in this problem it is easy to multiply quickly if they take the nearest multiple of 100.

Write on the blackboard examples like the following and let the children estimate the products. The column at the right gives some possible ways to estimate. These are not the only ways. Accept other estimates if the children have good reasons for making them.

Examples	Some Estimates for n
1. 49 × 11 = n	About 550 because 50 × 11 = 550; or about 500 because 50 × 10 = 500; or about 490 because 49 × 10 = 490.
2. 28 × 4 = n	About 100 because 25 × 4 = 100; or about 120 because 30 × 4 = 120.
3. 79 × 8 = n	About 640 because 80 × 8 = 640.

Give a few other examples, guiding the children to see that it is often easy to use the nearest multiple of 10 or 100 in estimating and to see that there are many possible ways to make a good estimate.

Ask the children to open their books to pupil page 87. Ask them to read sentence 1 and the answer, "n is about 260". Let a child tell how he thinks this estimate was made. (26 × 10) Let the pupils talk about why this is a good estimate. (9 is close to 10. We can multiply 26 × 10 quickly.) Let them do the other exercises in their exercise books.

Go over the story problems with the class. In discussion, guide the children to see that there are several ways to estimate. For example, in Exercise 12, they can think of any one of these products and get a good estimate:

$$42 \times 20,\ 40 \times 20,\ 40 \times 24,\ \text{or } 40 \times 25.$$

ACTIVITY 2: Estimating Missing Factors **Pupil page 88**

Ask the children to open their books to pupil page 88 and to read the title of the page. Say, "On page 87 of your book you estimated products. You know the product and one of the factors. You are to estimate the missing factor or tell about what it is".

Go over Exercise 1 with the class. Let a child read the exercise aloud. Say, "The sentence says that 5 multiplied by some number equals 52. Five multiplied by what whole number comes close to 52? The answer given is 10. We know that $5 \times 10 = 50$ and 50 is a little less than 52. Suppose we try 11?" Let the children decide that 11 is also a good estimate because $5 \times 11 = 55$ and 55 is a little more than 52.

Go over Exercise 2 with the class. Let the pupils try several factors and decide which comes closest to the missing factor. For example, $6 \times 8 = 48$ and $6 \times 9 = 54$. Eight is the better estimate of the missing factor because 48 is closer to 50 than 54. Ask the pupils to complete the exercises in their exercise books, estimating the whole number closest to the missing factor. Go over the story problems with the class after they have completed the other exercises. More than one answer is acceptable for a problem. For example, in Exercise 12, the answer may be 10 or 11. In Exercise 13 the answer may be any number between 20 and 30.

ACTIVITY 3: Low and High Estimates **Pupil page 89**

Note: This activity extends the ideas of estimating products and missing factors by letting the children learn to find a range of estimate. For example,

they decide on a least number they think a product is and a greatest number they think it is. Then they know the actual product is between these numbers. Gradually they learn to narrow the range by deciding whether a product is closer to the low or the high estimate.

Discuss a problem like the following. (You may not need to write part of the problem on the blackboard.)

> In a town there are 2230 children in primary school. The government officials want to plan some new school buildings. They need to estimate the number of children who will be in primary school ten years from now. The officials decide that the best estimate is almost 4 times the present number. For about how many children do they plan?

In the discussion let the children talk about what is known (there are 2230 pupils now in school) and what the estimate is. (In 10 years there will be almost 4 times as many pupils.) Stress the idea that the officials cannot know now the exact number of pupils in school ten years from now. The officials can only get a number close to the actual number of pupils.

Ask, "Can you quickly give an estimate of how many pupils there will be? You need not give an exact product". Guide the children to see that the number is a little greater than 2000×4. Therefore a good first estimate is about 8000. Children may give other first estimates such as $3000 \times 4 = 12{,}000$ or $2500 \times 4 = 10{,}000$. Accept all such answers for which children have good reasons. Any estimate between 8000 and 12,000 is a good first estimate.

Suppose 8000 is given as an estimate. Say, "Yes, we know the number of pupils will be close to 8000. Will it be greater or less than 8000? (Greater) Why?" Guide the children to suggest that since 2230 is greater than 2000 the product of 2230 and 4 is greater than 8000.

Let $\underline{n}$ be the product and write:

$$n > 8000$$

Let a child read the sentence and then say, "We can also say that 8000 is less than the product. Can someone write the sentence that expresses this?"

$$8000 < n$$

Ask, "What is a number that we know must be greater than the product?" Help the children to see that since 2230 is between 2000 and 3000, a high estimate is 3000 × 4 or 12,000. (If someone says, "10,000, because 2500 × 4 is 10,000 and 2500 is greater than 2230", accept this as a high estimate.)

Show a sentence to express the high estimate, for example:

$$n < 12{,}000$$

Say, "We know that 8000 is less than n and that n is less than 12,000".

Show this on the number line (with marks spaced 2000 apart).

Get a pupil to write:

$$8000 < n \qquad \text{and} \qquad n < 12{,}000$$

Give a few more examples and discuss them with the class.

Examples	Some Estimates for n
$142 \times 6 = n$	n is between 100 × 6 and 200 × 6; $600 < n$ and $n < 1200$
$515 \times 3 = n$	n is between 500 × 3 and 600 × 3; $1500 < n$ and $n < 1800$
$9 \times n = 79$	79 is between 9 × 8 and 9 × 9, $8 < n$ and $n < 9$

Let the children open their books to pupil page 89. Go over Exercises 1 and 8. Ask the children to complete the other exercises by finding a low and a high estimate.

For some of the children who grasp the idea of estimation easily you may wish to encourage closer estimates. For example, one estimate in Exercise 11 is: $10 < n$ and $n < 20$. A closer estimate is: $12 < n$ and $n < 13$.

ACTIVITY 4: Inequalities and Story Problems

Pupil pages 90-91

As a revision of the work in this Unit, ask pupils to open their books to pupil pages 90 and 91 and do the exercises. If help is needed, do one or two examples on the blackboard. If needed, revise yards, feet, and inches.

UNIT 8

Fractions I

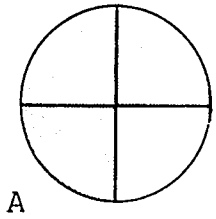
Objectives

1. To revise and extend the concept of fractions to $\frac{1}{7}$ and $\frac{1}{9}$ and to include $\frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \cdots \frac{1}{100}$
2. To revise the ordering of fractions and comparing of pairs of fractions
3. To introduce decimal names for selected fractions and to use per cent names for these fractions
4. To revise the renaming of fractions and extend to decimal names and per cent names
5. To increase, through counting by fractional units, facility in the use of numbers

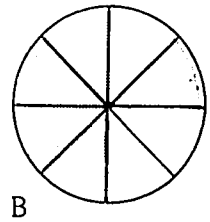
Background Information for Teachers

See Teachers' Handbook, pages 52-59; Primary Three, Teachers' Guide, Unit 5, pages 108-125.

In Primary Three, the pupils learned that fractions are numbers that describe parts of wholes: parts of whole regions, parts of whole objects, or parts of sets. For example, we assign the number three-fourths to each of the shaded regions shown below.



A

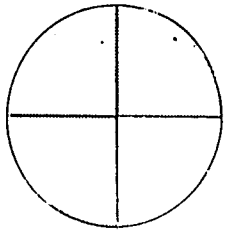


B

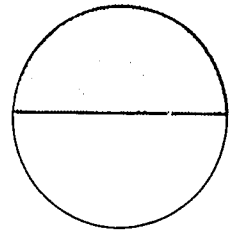
Figure A suggests that the number three-fourths be named by "$\frac{3}{4}$" since the shaded region is 3 of 4 congruent parts. Figure B suggests that the number three-fourths be named by "$\frac{6}{8}$" since the shaded region is 6 of 8 congruent parts. Thus, $\frac{3}{4}$ and $\frac{6}{8}$ are names for the same number since the shaded parts of A and B are congruent regions. In this way, many names are given to a number that is a fraction.

Figures like these also give us a way to compare fractions. The figures below help us decide whether one fraction is greater than another. For example, we can show with shaded regions that $\frac{3}{4} > \frac{1}{2}$.

$\frac{3}{4}$



$\frac{1}{2}$



The whole number twenty-eight has the name "28", which means 2 tens and 8 ones, or:

$$(2 \times 10) + (8 \times 1)$$

Twenty-eight has many other kinds of names, for example, XXVIII or 4×7. We can also use several kinds of names for a fraction. For example, one kind of name for the number three-fourths is the common name $\frac{3}{4}$. Another common name for three-fourths is $\frac{75}{100}$. Since $\frac{75}{100}$ is $(7 \times \frac{1}{10}) + (5 \times \frac{1}{100})$, $\frac{75}{100}$ can be written as ·75, meaning 7-tenths and 5-hundredths. Thus, the <u>decimal name</u> ·75 is another kind of name for the number three-fourths. Still a third kind of name is used for the number three-fourths. It is a <u>per cent name</u>. We can write three-fourths as 75%, which means $75 \times \frac{1}{100}$. The symbol "75%" is read "75 per cent" and is called the per cent name for the number three-fourths.

For some numbers we use a mixture of a whole number name and a common name. For example, the number three-halves has these names:

common name	$\frac{3}{2}$
mixed name	$1 + \frac{1}{2}$, usually written $1\frac{1}{2}$
	(Some fractions do not have mixed names.)
decimal name	1·5, meaning $1 + (5 \times \frac{1}{10})$
per cent name	150%, meaning $150 \times \frac{1}{100}$

It is important to remember that some numbers that are fractions are also whole numbers. For example, the number four-halves is the whole number two.

If a number is a fraction, then it has a common name. The common name consists of two parts: one part names the whole number numerator; the other part names the counting number denominator. Thus, zero is a fraction and six-fifths is a fraction. (Zero has the common name $\frac{0}{1}$ and six-fifths has the common name $\frac{6}{5}$.) Later in the primary programme the children will learn about some numbers that are not fractions because they have no common name with whole number numerator and counting number denominator. In the meanwhile all the numbers we deal with in <u>Primary Four</u> are fractions.

The decimal name of a fraction is an extension of the name used for a whole number. Each digit in the decimal name has a value depending on its position in relation to the ones' place. A point, called a decimal point, helps us find the ones' place. The ones' place is just to the left of the decimal point. The places to the right of the ones' place represent tenths, hundredths, and so on. Notice the names for the number 38·75.

mixed name	$38 + \frac{75}{100}$
expanded form	$(3 \times 10) + (8 \times 1) + (7 \times \frac{1}{10}) + (5 \times \frac{1}{100})$
decimal name	38·75

Tens' place
Ones' place
Tenths' place
Hundredths' place

We use a decimal name when we write 3·40 shillings to mean 3 shillings and 40 cents or when we write 5·20 dollars to mean 5 dollars and 20 cents. Decimal names have been used in France for many years for the measurement of length, weight, and so on. In other countries the decimal names for numbers are being introduced for these same measurements.

In reading 14·5, two forms are used. Some people read 14·5 as "fourteen and five-tenths"; other people read it as "fourteen point five".

"Fourteen and five-tenths" helps pupils understand the meaning of the number; "fourteen point five" does not. Meaning is important to the introduction of decimal names. We will always read numbers like 3·42 as "three and forty-two hundredths".

When working with the per cent name for a fraction, we understand that "per cent" means "hundredths". Thus 1% means $\frac{1}{100}$; 14% means $14 \times \frac{1}{100}$ or $\frac{14}{100}$; 100% means $\frac{100}{100}$ or 1; 150% means $\frac{150}{100}$, or $\frac{3}{2}$; and so on.
Since 100% = 1, we can name any fractions with a per cent name. For example:

$$\begin{aligned} \frac{3}{4} &= \frac{3}{4} \times 1 \\ &= \frac{3}{4} \times (100\%) \\ &= (\frac{3}{4} \times 100)\% \\ &= \frac{300}{4}\% \\ &= 75\% \end{aligned}$$

Per cent names for fractions are used in many ways; for example, in stating interest rates and in presenting statistical information.

STAGE 1: Revision of Fractions

Vocabulary: Fraction, numerator, denominator, common name for a fraction, mixed name for a fraction

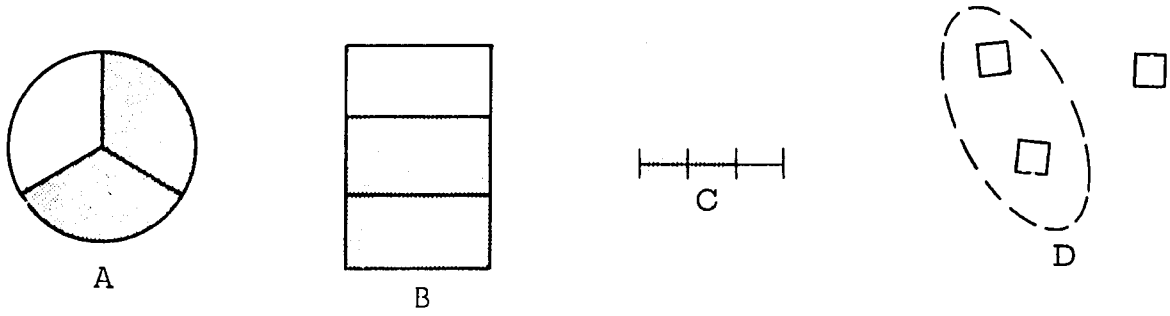
Materials: Pupil pages 92-96

Teaching Procedure

ACTIVITY 1: Revising Fractions as Numbers **Pupil page 92**

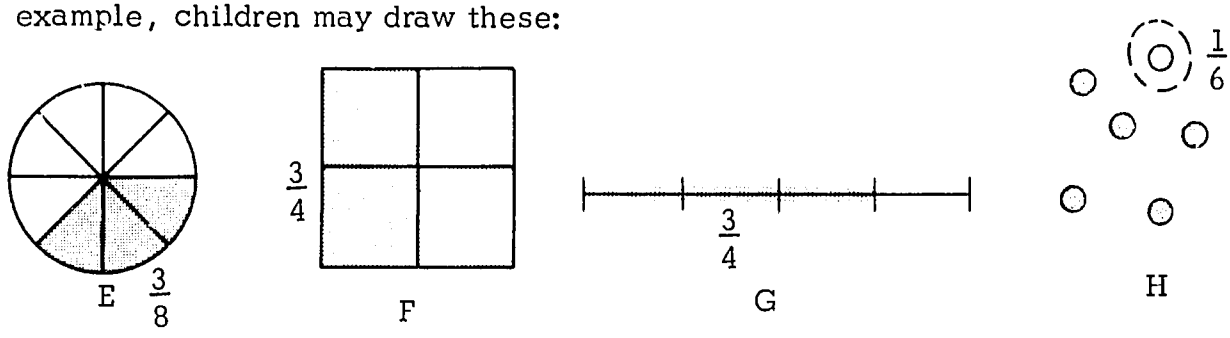
Remind the children that we use numbers called fractions when we talk about parts of whole objects or parts of sets.

Draw these figures on the blackboard:



Say, "Use a fraction to describe the part of A, of B, and of C that is shaded. ($\frac{2}{3}$ of the region A is shaded. $\frac{2}{3}$ of region B is shaded. $\frac{2}{3}$ of line segment C is shaded.) What fraction describes the part of set D that has a ring around it?" ($\frac{2}{3}$)

Let the children draw regions,,line segments and sets on the blackboard, show parts of each thing drawn and assign a fraction to each part. For example, children may draw these:



Revise some or all of the numbers that the children have used to describe parts of things. Ask, "What number did you use to describe the shaded part of circle A? Part of region B?" and so on.

$\frac{2}{3}$ of region A was shaded;

$\frac{2}{3}$ of line segment C was shaded;

We shaded $\frac{3}{4}$ of region G ;

We drew a ring around $\frac{1}{6}$ of set H; and so on.

Ask, "What numbers did you use? ($\frac{2}{3}$, $\frac{3}{4}$, . . . , $\frac{3}{4}$, $\frac{1}{6}$) What do we call numbers like these? (Fractions) Fractions are numbers that we can use to describe a part of a whole thing or part of a set. What does the 3 mean in

the statement, '$\frac{2}{3}$ of region A is shaded'? (It is the number of congruent parts into which we separated region A.) What does the 2 mean in the statement '$\frac{2}{3}$ of region A is shaded'?" (It is the number of congruent parts of region A that are shaded.)

Continue to let the children draw regions, line segments, and sets to show fractions such as $\frac{1}{5}$, $\frac{5}{8}$, and $\frac{2}{6}$.

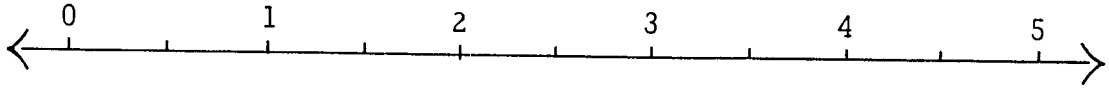
If more practice is needed, let the children tell what parts of their figures are unshaded. Emphasize that they are using numbers called fractions.

Ask the pupils to do the exercises on pupil page 92. While the pupils work or after they finish, talk with them about the exercises. Exercises 10 and 11 help the children summarize the lesson.

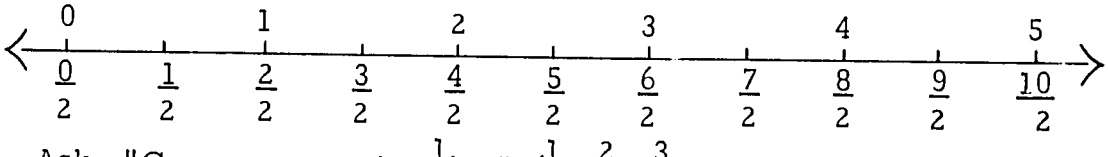
ACTIVITY 2: Fractions and the Number Line

Pupil pages 93-94

Draw on the blackboard the number line as shown below. As you draw the line, get the children to say that each unit segment is separated into halves:



Put a finger on the first point to the right of the 0-point that separates a unit segment. Ask, "What number should we assign to this point?" ($\frac{1}{2}$) Let a child write $\frac{1}{2}$ under this point. Say as the child writes $\frac{1}{2}$, "He is assigning a number to the point. Its name is $\frac{1}{2}$". Put a finger on the next point, the 1-point. Say, "This is the 1-point. What fraction can you assign to the point?" ($\frac{2}{2}$) Continue in this way until all the marked points to the right of the 0-point have fractions assigned to them. Ask, "How many halves is 0?" (Zero halves) Write $\frac{0}{2}$ under the 0-point. The number line now looks like this:



Ask, "Can you count by $\frac{1}{2}$'s?" ($\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, ...) Let the children imagine the number line continuing to the right as they count. Vary the activity by

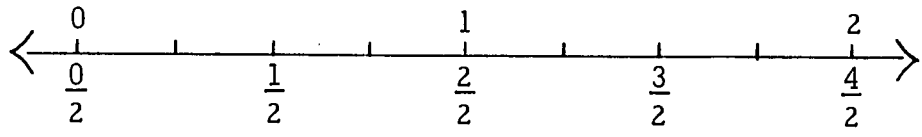
letting the pupils skip count backwards from say $\frac{8}{2}$ to $\frac{1}{2}$, by $\frac{1}{2}$'s and then by $\frac{2}{2}$'s. Sometimes let the pupils put their fingers on points on the number line, sometimes let them use their eyes, and other times suggest that the children not look at the number line as they count.

Revise the meaning of the words denominator and numerator. Ask, "What are the denominators of $\frac{0}{2}$, $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, ...? (2) What does the 2 mean in $\frac{3}{2}$ and $\frac{9}{2}$? (It tells the number of congruent parts of the unit segment.) What are the numerators? (0, 1, 2, ...) What does the numerator 3 mean in $\frac{3}{2}$?" (It tells the number of congruent parts we counted.)

Get the children to open their books to pupil page 93 and work the exercises there. Let them continue with pupil page 94. (Help the children understand that there is only one number line. The five pictures of the number line on page 94 just show different points on the number line.)

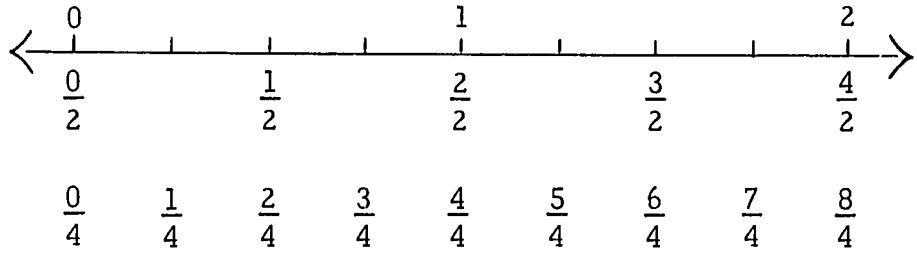
ACTIVITY 3: Names for Fractions; Ordering Fractions **Pupil pages 95-96**

Draw on the blackboard the number line as shown below:



Ask, as you put your finger on the 1-point, "What number is assigned to this point? (One) What are some names for one?" (1 and $\frac{2}{2}$) Ask similar questions about numbers for the 2-point and 0-point.

Let the pupils make marks separating the unit segments into 4 congruent parts. Let the children in turn tell the numbers for all these points and write the number names of the points on the lines. The blackboard may now look like this:



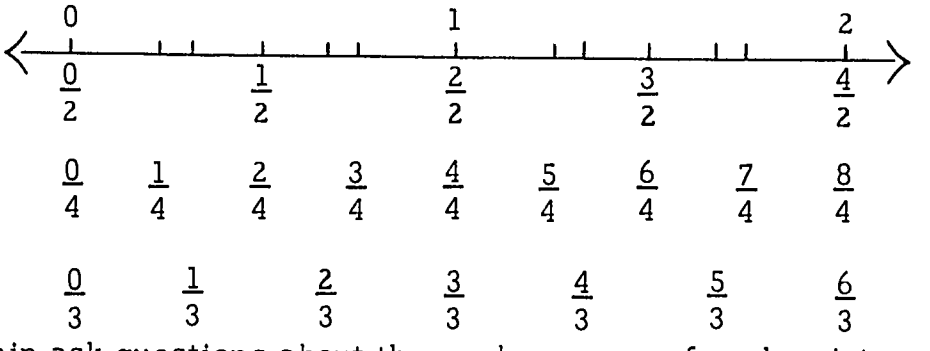
Again ask as you point to the 1-point, "What are some names of the number for this point?" ($1, \frac{2}{2}, \frac{4}{4}, \ldots$) Ask similar questions about the 2-point, 0-point, $\frac{1}{2}$-point, and $\frac{3}{2}$-point.

Say, "The $\frac{2}{2}$-point has a whole number name. What is it? (1) The 2-point has other names. What are some? ($\frac{4}{2}, \frac{8}{4}$) We call names like $\frac{4}{2}$ and $\frac{8}{4}$ the common names for fractions. What are some common names for the fraction one half?" ($\frac{1}{2}, \frac{2}{4}$) Let the children in turn write on the blackboard some common names for the fractions one-half and three-halves.

Say, "I know another name for three-halves." Write $1 + \frac{1}{2}$. Go on, "This is called a mixed name for the fraction. (It is called mixed because part of it is a whole number name and part a common name for a fraction.) We write $1 + \frac{1}{2}$ in a shorter way. We write $1\frac{1}{2}$. This means $1 + \frac{1}{2}$". Part of the blackboard will look like this:

Fraction:	three-halves
Common names:	$\frac{3}{2}, \frac{6}{4}$
Mixed names:	$1\frac{1}{2}, 1\frac{2}{4}$

Let the pupils make marks separating the unit segments of the number line into three congruent parts. Let them in turn tell the names for all these points and write the number names of the points on the line. The number line now looks like this:



Again ask questions about the number names of each point marked on the line. For example, the 2-point is named $2, \frac{4}{2}, \frac{8}{4}$, and $\frac{6}{3}$. Go on with other points.

Write this sentence on the blackboard and ask if it is a true sentence.

$$\frac{3}{2} = \frac{7}{4}$$

Whatever answer is given to the question, let a child find the $\frac{3}{2}$-point and the $\frac{7}{4}$-point on the number line. Again ask, "Is the sentence true?" (No) Why? (Another name for $\frac{3}{2}$ is $\frac{6}{4}$. But $\frac{6}{4}$ is not a name for $\frac{7}{4}$, because $\frac{7}{4}$ is $\frac{1}{4}$ more than $\frac{6}{4}$.) Make the sentence true".

$$\frac{3}{2} \neq \frac{7}{4}$$

Write several other sentences on the blackboard like these:

$$\frac{6}{4} = \frac{3}{2} \qquad \frac{4}{3} = \frac{4}{2} \qquad \frac{3}{3} = \frac{2}{2} \qquad \frac{2}{3} = \frac{3}{4}$$

Ask whether the sentences are true. Guide the children to study the number line, to tell whether the numbers are assigned to the same point, and to tell whether the numbers are equal.

Ask, "Which is greater, $\frac{1}{2}$ or $\frac{1}{3}$?" Let the children discuss this question, and guide them to rename the fractions until they decide that:

$$\frac{1}{2} = \frac{3}{6} \text{ and } \frac{1}{3} = \frac{2}{6}$$

Then ask, "Which is greater, $\frac{1}{2}$ or $\frac{1}{3}$?" ($\frac{1}{2}$, because $\frac{3}{6}$ is greater than $\frac{2}{6}$) Let a child write this on the blackboard as a sentence:

$$\frac{1}{2} > \frac{1}{3} \text{ or } \frac{1}{3} < \frac{1}{2}$$

Let another child show the points for $\frac{1}{2}$ and $\frac{1}{3}$ on the number line and decide that the $\frac{1}{2}$-point is to the right of the $\frac{1}{3}$-point.

Continue with sentences such as:

$$\frac{2}{2} > \frac{4}{4} \qquad \frac{3}{4} < \frac{2}{3} \qquad \frac{1}{2} < \frac{2}{4} \qquad 2 > \frac{2}{3}$$

Guide the children to think about each sentence by asking questions such as these: Is the sentence true? Why? If it is not true, make it true.

Write sentences such as the following on the blackboard:

$$\frac{1}{2} = \frac{n}{4} \qquad \frac{t}{2} = \frac{3}{3} \qquad \frac{4}{2} = \frac{6}{g}$$

Help the children to tell what numbers make these sentences true. (For example, $\underline{n}$ is 2 because $\frac{1}{2}$ and $\frac{2}{4}$ are names for the same fraction.)

Ask the pupils to work the exercises on pupil pages 95 and 96. Let them use the number line on pupil page 94 if they need it.

STAGE 2: The Fractions $\frac{1}{7}, \frac{1}{9}, \frac{1}{20}, \frac{1}{30}, \ldots, \frac{1}{100}$

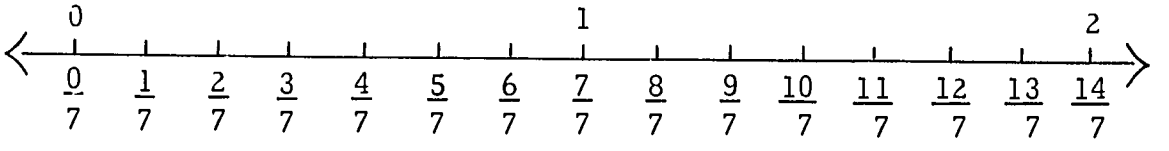
Vocabulary: One-seventh, one-ninth, one-twentieth, . . . , one-hundredth

Materials: Three 2" × 20" strips of paper, one folded into 7 congruent parts, one into 9 congruent parts, and one into 10 congruent parts; squared paper for each child: pupil pages 97-100

Teaching Procedure

ACTIVITY 1: The Fractions $\frac{1}{7}, \frac{1}{9}$, and $\frac{1}{10}$ **Pupil pages 97-99**

Draw a line segment a little more than 40 inches long. Mark a 0-point near the left end of the line. Show the children the 2" × 20" strip of paper that is folded into seven congruent parts. Let them use the strip first to mark off unit segments. Let the class examine the strip and say it shows seven congruent parts and that each part is one-seventh of the strip. Let other children use the strip to mark the seventh-points along the number line from 0 to 2. Help them to mark and name the points as shown below:

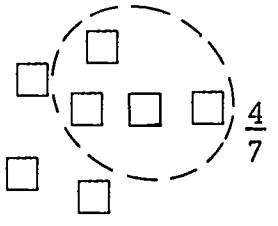


Guide the children to study the line and decide things like:

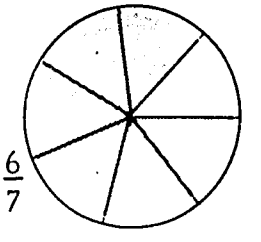
- $\frac{8}{7} > \frac{5}{7}$ because the $\frac{8}{7}$-point is to the right of the $\frac{5}{7}$-point.
- $\frac{8}{7} > \frac{5}{7}$ because 8 congruent parts of a line segment are more than 5 of the same congruent parts.
- $\frac{7}{7}$ is another name for 1; $\frac{14}{7}$ is another name for 2; $1\frac{1}{7}$ is another name for $\frac{8}{7}$.

- The denominators used are all 7. There are 7 congruent parts of the line segment from 0 to 1.
- The numerators used are 0, 1, 2, ... 14. The numerator 5 in $\frac{5}{7}$ means that we are thinking of 5 of the congruent parts of the line segment from 0 to 1.

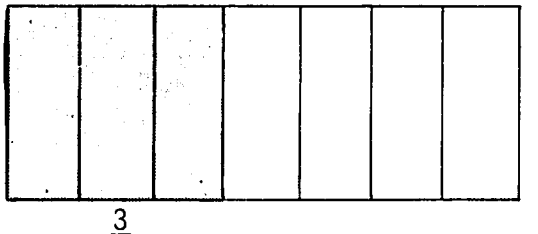
Ask the children to draw regions and to draw sets on the blackboard to show $\frac{1}{7}$, $\frac{5}{7}$, and so on. Some pictures they may draw are these. (Let them estimate $\frac{1}{7}$ of a region.)



$\frac{4}{7}$



$\frac{6}{7}$



$\frac{3}{7}$

Continue in the same way as above for the development of the fractions one-ninth and one-tenth. Get the children to prepare a number line for each; study it and find out things for the fractions; draw pictures of regions and sets to show the fractions.

Ask, "Can you use the number line to show fraction names with denominator 20? With denominator 30?" Let the pupils talk about this. Guide them to say that they can draw the number line (and pictures of regions and sets) to show fraction names with any denominator.

Get the children to answer the question about regions, sets, and the number line on pages 97 to 99.

ACTIVITY 2: The Fraction $\frac{1}{100}$

Pupil page 100

Let the children work together to make 100 dots on the blackboard. Ask, "How many dots are shown on the blackboard?" (100) Point to one dot and ask, "What fraction tells what part this 1 dot is of all the 100 dots?" (One-hundredth) Get a child to show 7 dots. Ask, "What fraction tells what part 7 dots is of all the 100 dots?" ($\frac{7}{100}$) Go on with other fraction names with denominator 100. Each time, get a child to show, for example, 25 dots; get another child to tell what part this set of dots is of the 100 dots; get still another child to write the common name for the fraction. ($\frac{25}{100}$)

Let the pupils make marks around a 10 × 10 array of squares on a sheet of squared paper. Let them shade $\frac{5}{100}$, of it, then $\frac{10}{100}$, and so on.

Be sure the pupils understand ideas like these about fractions named with denominator 100:

- The meaning of 100 in $\frac{17}{100}$.
- The meaning of 17 in $\frac{17}{100}$.
- $\frac{17}{100} < \frac{50}{100}$; $\frac{50}{100} > \frac{17}{100}$.
- $\frac{50}{100}$ is greater than $\frac{17}{100}$ because 50 of the congruent parts are more than 17 of the parts.
- $\frac{100}{100}$ is a name for 1; $\frac{50}{100}$ is a name for $\frac{1}{2}$; numbers have many names.
- $\frac{51}{100}$ is a common name for the fractions fifty-one hundredths.

Let the children answer the questions on pupil page 100.

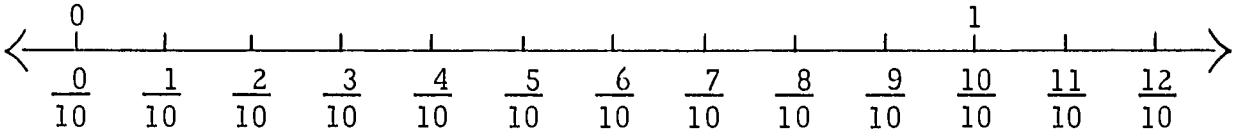
STAGE 3: Decimal Names and Per Cent Names of Fractions

Vocabulary: Decimal name of a fraction, decimal point, per cent name of a fraction

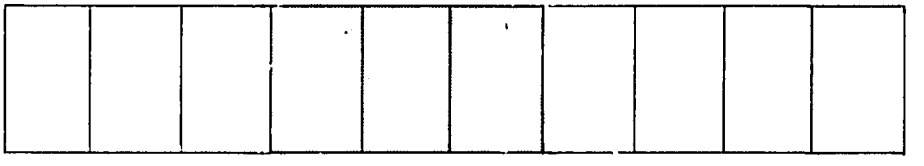
Materials: Squared paper for each pupil, pupil pages 101-106

ACTIVITY 1: Decimal Names for Tenths | Pupil page 101

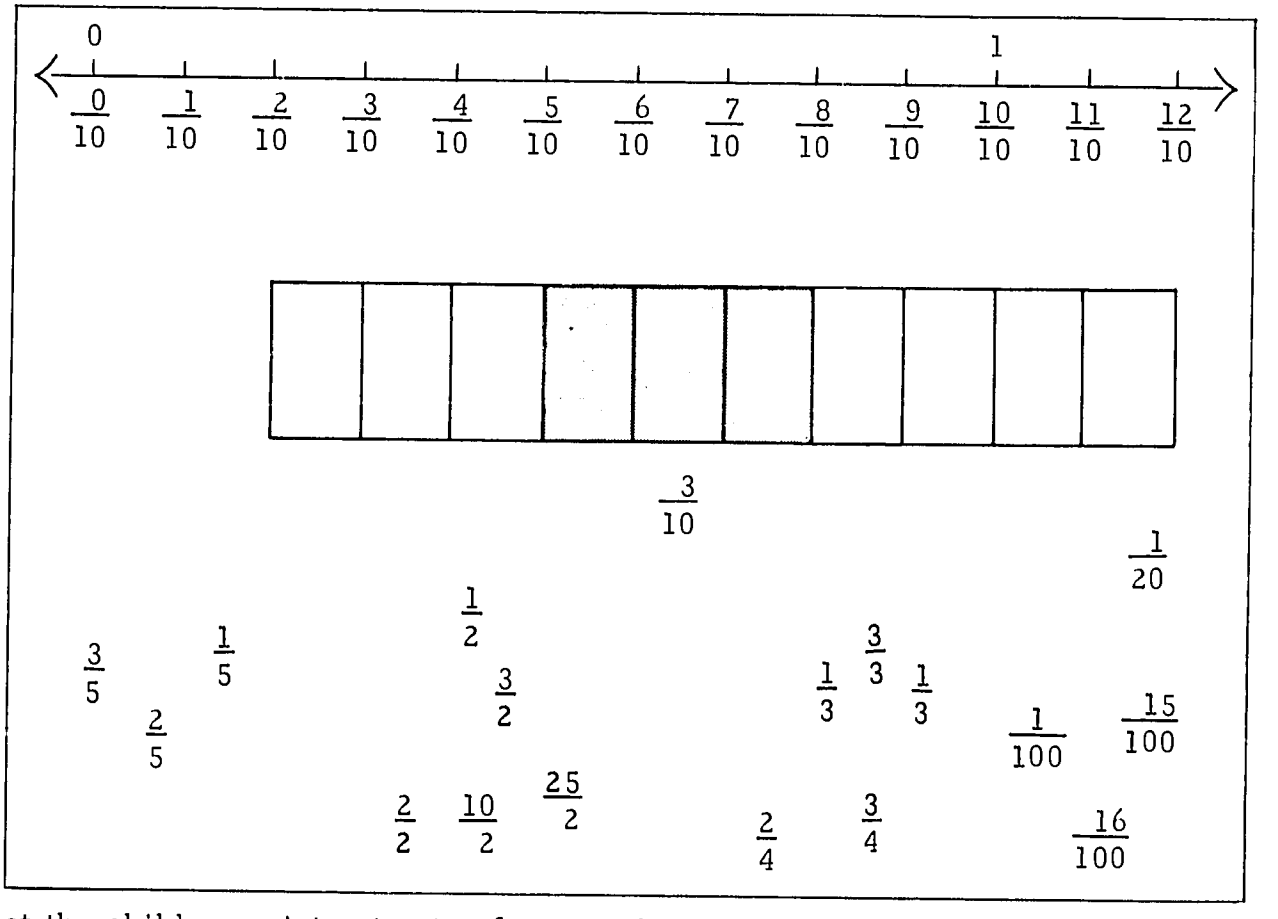
Draw on the blackboard a rectangular region separated into ten congruent regions (see figure below) and a number line showing a line segment from 0 to 1 separated into ten congruent segments. Guide the children to say that: each of these ten congruent segments on the number line shows one-tenth of the line segment from 0 to 1; and each of the small regions in the rectangular region shows one-tenth of the large region. Let them write the common name $\frac{1}{10}$ for one-tenth, and then name all the tenth-points on the number line. The number line will look like this:



Say, "You have written common names for fractions". Point to the shaded part of the following figure. Ask, "What fraction describes the shaded region? (Three-tenths) Write a common name for the fraction. ($\frac{3}{10}$) What is its denominator? (10. The region is separated into 10 congruent parts.) What is its numerator?" (3. Three of the congruent parts are shaded.)



Get the children to write on the blackboard some common names with other denominators. Put the names with the same denominators all in the same part of the blackboard. The blackboard might look like the drawing on the top of page 148.

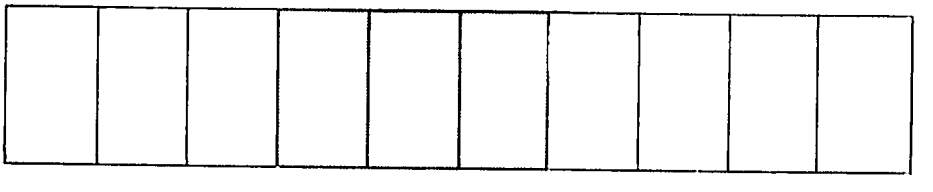


Let the children point out sets of names that have the same denominator. Say, as you point out the set of names with denominator 10, "Fractions named with a denominator 10 also have another kind of name". Point to $\frac{3}{10}$ on the blackboard and ask, "What fraction does this name? (Three-tenths) Another name for this fraction is this". Write:

·3

Go on, "We read this 'three-tenths'. This name is the decimal name for three-tenths".

Go over this again. As you do, write the different names for three-tenths. Part of the blackboard will look like this:

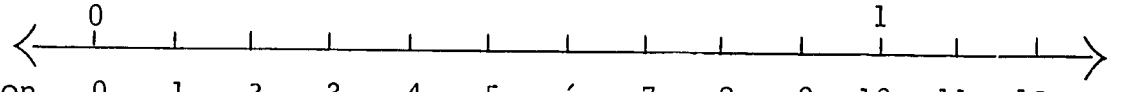


Common name: $\frac{3}{10}$

Decimal name: ·3

Say, "This dot to the left of '3' in the decimal name is called the decimal point. It is part of the decimal name for the fraction".

Get the children to look again at the number line on the blackboard. Ask, "How are the points named? (They are named with tenths.) What are the denominators?" (They are all 10) Guide the children in turn to read each name, beginning with $\frac{0}{10}$, and to write the decimal name for the fraction. The number line will look like this:



Common name	$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$	$\frac{11}{10}$	$\frac{12}{10}$
Decimal name	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	1·1	1·2

Help the children to:

write $\frac{10}{10}$ as 1·0; and read 1·0 as "ten-tenths"

write $\frac{11}{10}$ as 1·1; and read 1·1 as "eleven-tenths"

write $\frac{12}{10}$ as 1·2; and read 1·2 as "twelve-tenths"

Ask, "What is a mixed name for $\frac{12}{10}$? ($1\frac{2}{10}$) Write:

$$\frac{12}{10} = 1\frac{2}{10} = 1\cdot 2$$

Say, "All these name the same point on the number line. Do they name the same number? (Yes) We read '1·2' as 'one and two-tenths'". Get the children to read the following names for fractions: (Extend the number line if you wish.)

1·1 (one and one-tenth, or eleven-tenths)

1·3 (one and three-tenths, or thirteen-tenths)

.
.
.

2·0 (two and zero-tenths, or twenty-tenths)

2·1 (two and one-tenth or, twenty-one tenths)

.
.
.

4·7 (four and seven-tenths, or forty-seven tenths)

Ask, "Which is greater, $\frac{5}{10}$ or $\frac{7}{10}$? Which is greater, ·5 or ·7?" Guide the children to write the following sentences and tell why the sentences are true:

$$\frac{5}{10} < \frac{7}{10} \text{ and } \cdot 5 < \cdot 7$$

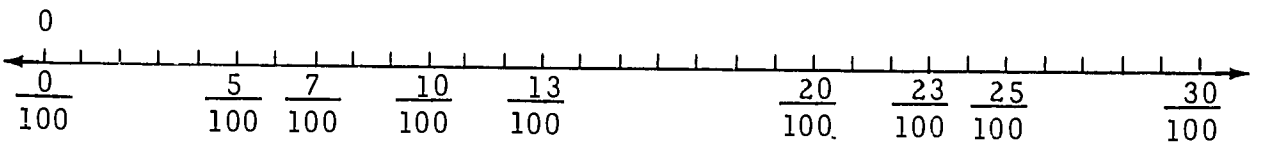
Continue by getting the pupils to order pairs of fractions like these:

1·5 and 1·8; ·7 and 1·4; 4·1 and 1·4; ·3 and ·2

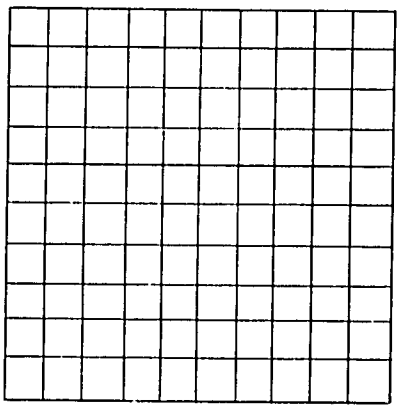
Ask the children to complete the chart and answer the questions on pupil page 101. They may need to use the number line they drew on the blackboard.

ACTIVITY 2: Decimal Names for Hundredths **Pupil pages 102-103**

Put out for use in this activity sheets of squared paper for each pupil. Draw on the blackboard a 10 × 10 array of squares and the picture of the number line below. (If you have a large blackboard make the line longer.)



Let each child draw line segments to enclose a 10 × 10 array of squares on a sheet of squared paper. Get the children either to cut out their arrays or to fold the papers to show the array. It will look like this:



Ask, "How many small regions are there in the large region? (10 × 10 = 100) What fraction is one small region of the whole square region?" ($\frac{1}{100}$)

Get the pupils to count some of the small regions in their array by $\frac{1}{100}$'s. As they point to their small regions, you point to the small regions in the 10×10 array on the blackboard. Let them count:

$$\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \frac{4}{100}, \frac{5}{100}, \frac{6}{100}, \text{ and so on.}$$

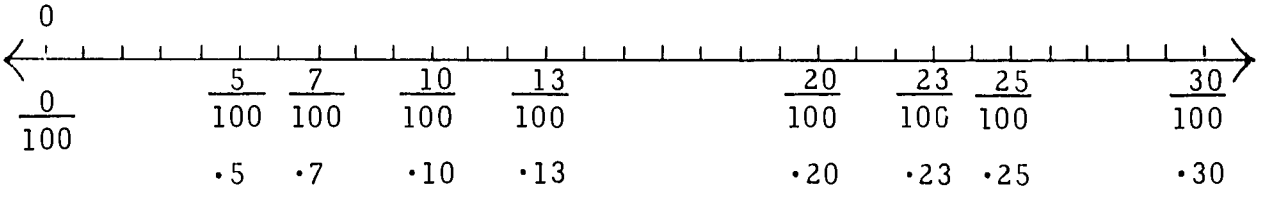
Get them in turn to write common names for the fractions from $\frac{1}{100}$ to at least $\frac{25}{100}$.

You name a fraction (say $\frac{30}{100}$) and ask a child to show a part of his 10×10 array that shows the fraction. Go on with other fractions in this same way.

Say, as you write $\frac{25}{100}$, "This is a common name for twenty-five hundredths. It also has a decimal name. It is ·25. The two names are read the same". As you talk, write:

$$\text{twenty-five hundredths: } \frac{25}{100} = \cdot 25$$

Point to the number line. Get the children in turn to read names of points and to write the decimal names under the common names. You may extend the line and mark more points as needed. The line will look like this:



Point to the 10×10 array on the blackboard and ask, "How many $\frac{1}{100}$'s does the whole array show?" (100) Get a child to write $\frac{100}{100}$.

Get about half the pupils to shade all, or $\frac{100}{100}$, of their arrays. Get the other pupils to shade some of the small squares of their arrays, say $\frac{25}{100}$, $\frac{17}{100}$, $\frac{58}{100}$, and so on. Let a child with all of his array shaded and a child with part of his array shaded come out and hold the arrays side by side. Guide the children to say:

"I shaded $\frac{100}{100}$ of my array;

I shaded $\frac{45}{100}$ of my array; and

The two arrays together have $\frac{145}{100}$ of an array shaded".

Get them to write a common name, a mixed name, and a decimal name for each fraction they used.

Call out several other pairs of children, one with his whole array shaded and the other with part of his array shaded. Get them to follow the same plan as the first pair of children. The blackboard may look like this:

Common name:	$\frac{145}{100}$	$\frac{190}{100}$	$\frac{107}{100}$
Mixed name:	$1\frac{45}{100}$	$1\frac{90}{100}$	$1\frac{7}{100}$
Decimal name:	1·45	1·90	1·07

You may also call out three children or more with all their arrays shaded and a child with part of his array shaded. Get them to tell the fraction shown by the four arrays. Ask them to write names for the fraction. For example, they may write:

$$\frac{365}{100} \qquad 3\frac{65}{100} \qquad 3{\cdot}65$$

Write pairs of names for fractions on the blackboard and guide the children to show the fractions with their arrays and tell which fraction is larger. You may use numbers like these:

·25 and ·37; 1·25 and 1·11; ·07 and ·70; ·8 and ·3

Get the children to write sentences like these:

·25 < ·37; 1·25 > 1·11; ·07 < ·70; ·8 > ·3

Say, "Read the number names ·35 and ·27. Do not use your arrays. Tell which number is greater. (·35) How do you know?" (When I count hundredths, 35-hundredths is more than 27-hundredths.) Go on with this type of activity with other children as needed. Emphasize that if the two names show hundredths, the numerator helps them decide which number is larger.

Write 3·6 and 728 on the blackboard as shown on page 153. Point in turn to the digits of "728" and ask, "What is the place value of this digit?" As the children answer, write in "one", "ten", and "hundred" as shown.

Tell the children as you point to the 6 of "3·6", "The place-value of the 6 is a tenth". Write tenth as shown. Guide the children to say the place-value of the 3 is one.

			3	·6
7	2	8	·	
hundreds	ten	one		tenth

Write on the blackboard other number names such as 35, ·9, 3·4, 75·2. Let the children read the number name and tell the place value of each digit.

On the chart that is on the blackboard, write ·75 above 3·6 as shown below. Get the children to read ·75 (75-hundredths) and you write "hundredth" as shown in the chart. Point to the 5 in ·75, follow down to the word "hundredth", and say, "The place value of 5 is a hundredth". In a similar way, get the children to say, "The place value of 7 (in ·75) is a tenth".

				·7	5
			3	·6	
7	2	8	·		
hundreds	ten	one		tenth	hundredth

Write on the blackboard other number names such as ·25; 8; 45; 3·25. Let the children read the number name and tell the place value of each digit.

Ask the pupils to do the exercises on pupil pages 102 and 103. If needed let them use the number line, 10 × 10 array, and chart which are on the blackboard, and their own 10 × 10 arrays.

ACTIVITY 3: Per Cent Names for Fractions **Pupil pages 104-106**

Write on the blackboard "thirty-five hundredths", "ten-hundredths", and so on as shown on page 154. Ask the children to write a common name, a mixed name if there is one, and a decimal name for each. (Use other fractions if the children need the practice.) The blank column will be used later in the activity.

	Common Name	Mixed Name	Decimal Name	
thirty-five hundredths	$\frac{35}{100}$	none	·35	
ten-hundredths	$\frac{10}{100}$	none	·10	
one hundred-hundredths	$\frac{100}{100}$	1	1·00	
seven-hundredths	$\frac{7}{100}$	none	·07	
two hundred thirty-five hundredths	$\frac{235}{100}$	$2\frac{35}{100}$	2·35	

After that part of the chart is completed, say, "There is another name we give fractions. There is a space in the chart for the name. The fraction $\frac{25}{100}$ has the name 25 per cent". (Write "25 per cent" on the blackboard and then "25%".)

$\frac{25}{100}$ 25 per cent 25%

Point to the three names. Say, "These are all names for fractions. The first is the common name, and the second is the per cent name. Per cent means hundredths. The symbol for per cent is '%'".

Write these names for fractions on the blackboard.

$\frac{7}{100}$, $\frac{14}{100}$, $\frac{37}{100}$, $\frac{65}{100}$, $\frac{100}{100}$, $\frac{125}{100}$

Ask the children in turn to read the common name, write the per cent name under it, and read the per cent name. Guide them to say that

- per cent means hundredths;
- a common name with denominator 100 can be changed to a per cent name;
- a per cent name of a fraction can be changed to a common name with denominator 100.

Write "Per Cent Name" in the fourth column of the chart that is on the blackboard and let the children in turn write per cent names for the fractions.

Let the children do as many of the exercises on page 104 and 106 as they need. It may not be necessary to ask all the children to do all of the exercises. For some of the questions, the children may need their 10 × 10 arrays and the number line on pupil page 94.

UNIT 9

Addition and Subtraction of Whole Numbers II

Objectives

1. To achieve mastery of the techniques of addition and subtraction
2. To use addition and subtraction in solving story problems, in puzzles, and in application to money, weights and measures

Background Information for Teachers

See "Background Information for Teachers" for Units 1, 2, 4, and 5 in this book.

STAGE 1: Mathematical Sentences Involving Addition and Subtraction

Vocabulary: No new vocabulary

Materials: Pupil pages 107-110

Teaching Procedure

ACTIVITY 1: Addition Sentences | Pupil page 107

The purpose of this activity is to help the pupils to achieve mastery of the addition process for any sum with any number of addends.

Let the class work out an exercise in which they revise place-value up to millions. Write a five-digit, a six-digit, and a seven-digit numeral on the blackboard. Let the pupils tell the numbers named, and the meaning of each digit in the numeral. Revise also the expanded form of decimal numerals. Make sure that the pupils are familiar with all of these ideas.

Let the pupils solve Sentences 1 to 6 on pupil page 107. Help them to understand that when they write addends for addition in a vertical form they use place-value.

Discuss the following examples with the pupils in the way suggested.

Example 1: n = 246,018 + 94,324 + 515,651

Ask the pupils to give the expanded form for each numeral, and then add the numbers in this form. The work may be as follows:

$$
\begin{array}{rcl}
246{,}018 & = & 200{,}000 + 40{,}000 + 6{,}000 + 10 + 8 \\
94{,}324 & = & 90{,}000 + 4{,}000 + 300 + 20 + 4 \\
515{,}561 & = & 500{,}000 + 10{,}000 + 5{,}000 + 500 + 60 + 1 \\
\hline
n & & 700{,}000 + 100{,}000 + 15{,}000 + 800 + 90 + 13
\end{array}
$$

$$
\begin{array}{rl}
 & 700{,}000 \\
 & 100{,}000 + 40{,}000 \\
 & \quad + 10{,}000 + 5{,}000 + 800 + 90 \\
 & \qquad\qquad\qquad\qquad\qquad + 10 + 3 \\
\hline
n = & 800{,}000 + 50{,}000 + 5{,}000 + 800 + 100 + 3
\end{array}
$$

$$n = 800{,}000 + 50{,}000 + 5{,}000 + 900 + 3$$

n is 855,903

Example 2: n = 8,069 + 345,678 + 93,762 + 876 + 134,704 + 7,769

To find <u>n</u>, let the pupils arrange the numerals as shown below to the left and tell about the process in a way similar to that described to the right.

$$
\begin{array}{r}
8{,}069 \\
345{,}678 \\
93{,}762 \\
876 \\
134{,}704 \\
7{,}769 \\
\hline
590{,}858
\end{array}
$$

1. Think of each addend in its expanded form.
2. Add the ones. (9 + 8 + 2 + 6 + 4 + 9) ones = 38 ones. Rename 38 ones as 3 tens and 8 ones. Write 8 in the ones' place in the sum. Remember 3 tens.
3. Add the tens. (3 + 6 + 7 + 6 + 7 + 0 + 6) tens = 35 tens. Rename 35 tens as 3 hundreds and 5 tens. Write the 5 in the tens' place in the sum. Remember the 3 hundreds.
4. Add the hundreds. (3 + 0 + 6 + 7 + 8 + 7 + 7) hundreds = 38 hundreds. Rename 38 hundreds as 3 thousands and 8 hundreds. Write the 8 in the hundreds' place and remember the 3 thousands.
5. Add the thousands. Add the ten thousands and then the hundred thousands. In each continue as before.
6. The sum is 590,858.

Let the pupils check the addition by adding upward in the columns. Encourage them to decide that it is important to check their work. Get them to do other examples on the blackboard. Some pupils may need to add using the expanded form. If so, give them examples with three or four addends but encourage them to use the short vertical form as soon as they can.

Let the pupils then do Exercises 7 to 9 on page 107. Encourage them to write a mathematical sentence for a story problem before carrying out the addition. Check the pupils' work with them.

ACTIVITY 2: Subtraction Sentences **Pupil pages 108-109**

The purpose of this activity is to help the pupils achieve mastery of the subtraction process for sums up to millions with renaming of the sum in any place.

Write on the blackboard an example like the one following. Guide the children to revise the method of subtraction by using the expanded form as shown. Let them tell, as they subtract, about the renaming needed at each step.

n = 854,805 - 136,256

$$\begin{array}{rcl} 854{,}805 & = & 800{,}000 + 50{,}000 + 4{,}000 + 800 + 5 \\ \underline{136{,}256} & = & \underline{100{,}000 + 30{,}000 + 6{,}000 + 200 + 50 + 6} \end{array}$$

$$\begin{array}{rcl} 854{,}805 & = & 800{,}000 + 40{,}000 + 14{,}000 + 700 + 90 + 15 \\ \underline{136{,}256} & = & \underline{100{,}000 + 30{,}000 + 6{,}000 + 200 + 50 + 6} \\ & = & 700{,}000 + 10{,}000 + 8{,}000 + 500 + 40 + 9 \end{array}$$

n is 718,549

Guide the children to do the same exercise using the short form. This form is shown below to the left. Guide them to tell about the process in a way similar to that described to the right.

$$\begin{array}{r} 854{,}805 \\ \underline{136{,}256} \\ 718{,}549 \end{array}$$

1. Think of the sum and known addends in expanded form.
2. There are fewer ones in the ones' place of 854,805 than in the ones' place of 136,256. Rename 854,805 as

854 thousands, 7 hundreds, 9 tens and 15 ones. Subtract the ones. (15 - 6) ones = 9 ones. Write 9 in the ones' place.

3. Subtract the tens. (9 - 5) tens = 4 tens. Write 4 in the tens' place.
4. Subtract the hundreds. (7 - 2) hundreds = 5 hundreds. Write 5 in the hundreds' place.
5. There are fewer thousands in the thousands' place of 854,805 than in the thousands' place of 136,256. Rename 854 thousands as 8 hundred thousands, 4 ten thousands and 14 thousands. (14 - 6) thousands = 8 thousands. Write 8 in the thousands' place.
6. Subtract the ten thousands. (4 - 3) ten thousands = 1 ten thousand. Write 1 in the ten-thousands' place.
7. Subtract the hundred thousands. (8 - 1) hundred thousands = 7 hundred thousands. Write 7 in the hundred-thousands' place.
8. The missing addend is 718,549.

Ask, "What is n ? (n is 718,549) Is the sentence 718,549 + 136,256 = 854,805 true?" Help the pupils to understand that addition is a check for subtraction. Let them add to show the sentence is true.

Let the pupils do other examples on the blackboard. Then ask them to turn to pupil page 108 and do the exercises. Encourage the pupils to use a short form for addition. However, if some pupils need to use the expanded form, let them do so. Encourage the pupils to check their results using additions. Note that in some of the exercises they find the missing addend in an addition sentence. Encourage the pupils to write these addition sentences as subtraction sentences. For example:

n + 14,362 = 25,181 means n = 25,181 - 14,362

When the exercises on pupil page 108 are completed let the pupils do the exercises on page 109. In these exercises the pupils use both addition and subtraction.

ACTIVITY 3: Inequalities **Pupil page 110**

Ask the pupils to open their books to pupil page 110 and do the exercises. Guide them to decide by inspection or by estimation which symbol to use. In the story problems get the pupils to write mathematical sentences to help them answer the questions in the problems. Talk with the children about Exercise 6. Note that Exercise 11 asks for an estimate.

STAGE 2: Games and Puzzles Using Addition and Subtraction

Vocabulary: No new vocabulary

Materials: 30 small stones, pupil pages 111-115

Teaching Procedure

ACTIVITY 1: Cryptograms **Pupil page 111**

Write this example (cryptogram) on the blackboard:

$$\begin{array}{r} E\ 9\ K\ 6 \\ 2\ 8\ L\ 2 \\ 6\ 4\ 8\ K \\ 8\ k\ 2\ 4 \\ \hline 2\ 3\ 9\ 8\ 9 \end{array}$$

Let the pupils study it. Ask them to tell what the letters stand for. Each letter is a code letter for some numeral. Remind them that any letter stands for just one digit. For example, wherever we find K in the exercise, K is always the same digit. (In this example K is 7. If K is 7 in one place, it is 7 in every other place.)

Write the answers as the children discover them.

K is 7 L is 0 E is 5

Give them an example of a cryptogram for subtraction. Then let them do the exercises on pupil page 111. They write the exercise, replacing the code letters with the numerals they stand for. They begin work in the ones' place just as in other examples of addition and subtraction.

Answers for pupil page 111:

1. M is 2 D is 0 J is 4	2. S is 5 T is 8	3. A is 3 K is 1
4. A is 9 B is 8	5. R is 7 G is 9 Z is 6	6. F is 0 P is 7 E is 5
7. R is 9 N is 8 U is 3	8. C is 4 A is 9 B is 3	9. X is 8 Z is 0 Y is 5
10. N is 7 P is 8 M is 9	11. G is 3 F is 4 E is 6	12. D is 1 E is 9 G is 2

ACTIVITY 2: Riddles **Pupil page 112**

Ask the children to open their books to pupil page 112 and read the riddles. The exercises may be done individually in exercise books or orally as a class activity. Even though this Unit deals primarily with addition and subtraction, some multiplication is used. For some exercises it is helpful if the children first write mathematical sentences. To clarify Exercises 4 and 6, ask one of the pupils to write any 3-digit numeral, for example, 467. Ask someone else to write the numeral obtained from this by reversing its digits. (764)

Exercise 8 is a challenge problem because the pupils are asked to find two numbers. It will take two sentences to describe the exercise. Let m and n be the 2 numbers. The sentences are:

$$m + n = 50$$

$$m - n = 24$$

The second sentence may also be written $m = 24 + n$. Then $(24 + n) + n = 50$. So n is 13 and m is 37.

Answers for pupil page 112:

(1) 15; (2) 11; (3) 13; (4) 324; (5) 22; (6) 75; (7 14; (8) 37, 13

Hint for Exercises 4 and 6. Suggest that the children solve them with these cryptograms:

(4)
```
 3 2 A
   9 9
 -------
 A 2 3
```
A is 4

(6)
```
 7 B
 1 8
 -----
 B 7
```
B is 5

ACTIVITY 3: Choosing the Subset **Pupil page 113**

On pupil page 113 let pupils choose a subset from the set {0, 6, 7, 8, 9} so that all the members of the subset make each of the sentences 1 to 6 true. For example, for 99 + □ = 99, the subset is {0}, because 0 is the only number in the set that makes the sentence true. (Notice that four numbers in the set make the sentence in Exercise 6 true.)

Let the pupils continue with the other exercises in a similar way. In Exercises 11 to 16, remind the pupils that the sign ≠ means "is not equal to".

Answers for pupil page 113:

1. {0}	2. {9}	3. {7}	4. {8}
5. {8}	6. {6, 7, 8, 9}	7. {75}	8. {500}
9. {75}	10. {25, 75}	11. {84}	12. {48, 84}
13. {84}	14. {48}	15. {84}	16. {84}

ACTIVITY 4: Cross-Numeral Puzzle **Pupil page 114**

Let the children open their books to pupil page 114. Ask them to copy the puzzle in their exercise books and use the clues beneath the puzzle to help them complete it. You may want to go over one or two of the clues. For example clue 9-across gives the numeral 78. Write "78" in the two squares beginning at 9. Clue 3-down is the beginning of a set of whole numbers with the second member missing. Ask the pupils if

they can recognize the set. Someone may recognize it as a set of even whole numbers. The missing one is 2. Write 2 in the 3-square.

Let the pupils discuss clue 12-across and clue 14-down. Let them guess a rule to find the members. The rule in 12-across is "add 3". The rule in 14-down is "add the 2 previous members". The missing members are 5 in 12-across and 8 in 14-down.

You may want to make a game of doing the puzzle. There are 13 clues in each column. Let half the class complete the puzzle by going down, the other half by going across. After both groups have finished, compare results.

Answers for pupil page 114:

3	7	2		7	4
2	4		9	6	3
9		7	8		7
	5	3		5	
8	1		8	4	4
3	6	2		9	1

ACTIVITY 5: A Game with Stones

Let the pupils collect 30 stones. Arrange them in ten piles of 3 each to play a game. The rules of the game are:

1. Two pupils play. Decide who plays first.
2. In turn, each pupil picks up either 1, 2, or 3 piles of stones, but not more than 3.
3. The player who picks up the last pile wins the game.

You may make a game that uses addition out of this game by having each pupil announce the total number of stones picked up after his turn. The winner is the player who announces "30".

You may make a game that uses subtraction by having each pupil announce the number of stones remaining to be picked up after his turn. The winner is the player who announces "0".

You may repeat the game, but change the total number of stones, or the number of piles, or the number of stones in each pile. The rules remain the same.

ACTIVITY 6: Message and Square Pupil page 115

Let the pupils open their books to pupil page 115. At the top of the page is a puzzle in which the pupils are to find the hidden message using the code given below the message. You may want to help with one or two of the parts to make the exercise clear. (For example in the first region 16 - 9 + 0 = 7. 7 is the code for T. T goes in the region.) The message is:

T	H	E		Q	U	I	C	K
B	R	O	W	N		F	O	X
		J	U	M	P	S		
O	V	E	R			T	H	E
	L	A	Z	Y		D	O	G

Help the pupils to notice that the decoded message makes use of every one of the 26 letters of the alphabet.

Exercise 2 is a 4 by 4 magic square. The sixteen regions are to be filled with the numerals from 1 to 16, each used once. The sum of the numbers named along each row, each column and each diagonal is 34. Before asking the pupils to complete this 4 x 4 square, use the two examples of 3 by 3 magic squares shown below as a class activity. In each case the nine regions are to be filled with the numerals from 1 to 9, each used once. The sum along each row, each column and each diagonal is 15. Start by filling only two or three regions and let the children decide what numerals go in the other regions. Notice that 5 is in the centre region in each magic square.

8	1	6
3	5	7
4	9	2

Sum: 15

8	3	4
1	5	9
6	7	2

Sum: 15

Let the pupils complete the magic square on the pupil page. Then ask them to make another 4 by 4 magic square similar to the one on the pupil page. Help them begin by filling several regions. One possibilitity is:

15	10	3	6
4	5	16	9
14	11	2	7
1	8	13	12

Sum: 34

STAGE 3: Word Problems and Applications of Addition and Subtraction

Vocabulary: No new vocabulary

Materials: Pupil pages 116-119

Teaching Procedure

The activities in this stage give practice in solving story problems and applying mathematics to problems of money, weight and measure. You may introduce these activities in the way that is best for your class. For example, if the children have difficulty in reading, read a problem with the entire class, let the children solve it independently, then read the next problem, and so on. If some children have no reading difficulties those children may work alone while you help others.

You make the solution of some problems an activity the class does together with your guidance. The solution of other problems the children may try without so much help.

Notice that in may of the problems several questions are asked. Some require addition or subtraction or both and the answer is a sum or difference. Others ask which of two numbers is greater. Some ask for an estimate and not an exact number. Point these things out to the children before asking them to do the exercises.

Encourage children to make mathematical sentences for the story problems. This is important information for children who have difficulty relating the numbers of the problem in the way the story suggests. The machematical sentence is a way of stating the essential facts.

ACTIVITY 1: Story Problems in Geography **Pupil page 116**

Go over and revise the steps in solving a story problem as you discuss the first problem on pupil page 116 with the children. The steps are:

1. Decide what the problem is about and keep in mind the question asked in the problem.

The problem is about the number of people in Zambia, how many of them live in one of the four largest towns and how many do not. The question asked is, "How many people do <u>not</u> live in one of the four towns?"

2. Write a mathematical sentence for the problem. There are two ways of writing the sentence. Either may be used:

$$3{,}600{,}000 - (118{,}000 + 114{,}500 + 88{,}600 + 33{,}900) = n$$

or

$$(118{,}000 + 114{,}500 + 88{,}600 + 33{,}900) + n = 3{,}600{,}000$$

3. Find a number that makes the mathematical sentence true. The number shown in brackets in the sentence (the total population of the four towns) is found first. The children can write the numerals in vertical form to find the sum, 355,000. Then subtract this number from 3,600,000 to find <u>n</u>. <u>n</u> is 3,245,000.
4. Use the number to answer the question in the problem.

   The number of people who do not live in one of the four largest towns is 3,245,000.

Let a child read the incomplete statement in Exercise 2, about estimation. Revise briefly the idea of estimation. (An exact number is not required. Facts are used to make a good guess.)

Put these facts on the blackboard:

People in Zambia	3,600,000
People in four largest towns	355,000
People not in four largest towns	3,245,000

Guide the children to say that to answer the question they need only the first two facts. Let a child write a mathematical sentence for Problem 2.

$$3{,}600{,}000 = n \times 355{,}000$$

Let the children make an estimate of <u>n</u>. By inspection they may say that the best estimate is 10. The product is about 10 times the known factor.

Use a similar procedure with the other problems or let the children work independently and then discuss their answers.

Notice that the method of estimation in Exercise 7 is very much like that of Exercise 2. It also asks for an estimate of the missing factor. (The best estimate is about 10.)

Exercises 3 and 5 use ideas of altitude. You may wish to precede the exercises with a brief discussion of altitude, the meaning of "sea level" and "feet above sea level", and so on. In Exercise 4 a distance expressed in feet is compared to 1 mile.

ACTIVITY 2: Using Addition and Subtraction

Pupil page 117

The exercises on page 117 may require more than one operation. You may wish to do some or perhaps all of the exercises with the whole class. Or, you may let the children work in small groups, with perhaps some working independently. Go over at least one problem with the whole class, and talk about all of the problems when they are completed. Point out that several problems ask more than one question.

Show the children that in some problems it may be best to get all measurements expressed in one unit before writing a mathematical sentence. For example, in Exercise 1 if 50 pounds is changed to 1000 shillings (50 × 20); the sentence is:

$$1000 - (220 + 220 + 200 + 200) = n$$

Also, emphasize that in other problems it is better to relate some of the measurements before changing to the same unit. For example, in Exercise 5 it is better not to change 10 tons and 12 tons to pounds. The problem asks that the measurements be compared. 12 tons is 2 tons more than 10 tons. Then 2 tons is changed to 4480 pounds (2240 x 2) and the sentence is:

$$2600 ___ 4480$$

To make it true write:

$$2600 < 4480$$

Therefore, 10 tons and 2600 pounds is less weight than 12 tons.

Guide the children to see that they should think about the problem carefully in order to decide how to plan their work. The best place to start will not be the same for all problems or for all children.

ACTIVITY 3: Story Problems in Science **Pupil page 118**

Let the children solve the problems on pupil page 118 by just thinking about the relationships described. It is not necessary to write mathematical sentences.

Go over the first and second exercises with the pupils. They may use the clock face to help answer the questions. For example, to find the time 6 hours after 10 o'clock they think of the hour hand beginning at 10 and moving 6 spaces along. It stops at 4. So 6 hours after 10 o'clock in the morning it is 4 o'clock in the afternoon. Since Haji spends $\frac{1}{2}$ hour walking to the beach and stops $\frac{1}{2}$ hour on the way he must start 1 hour before 4 o'clock. ($\frac{1}{2} + \frac{1}{2} = 1$) Again, let the children find this on the clock face. One space before 4 (moving backwards) is 3. He must start at 3 o'clock to get to the beach at 4 o'clock. If he spends 1 hour on the beach and $\frac{1}{2}$ hour walking home, the hour hand moves from 4 on the clock face, $1\frac{1}{2}$ spaces to $5\frac{1}{2}$ ($4 + 1\frac{1}{2} = 5\frac{1}{2}$). Haji arrives home at half past five.

Give help as needed as the children do the other problems on page 118.

ACTIVITY 4: Using Properties of Addition **Pupil page 119**

The children can solve some of the exercises on page 119 more easily if they use the properties of addition. Do not tell them how to apply the properties but guide the discussion so that the pupils discover the use themselves. You may say that some problems are easier if they use what they know about addition.

Introduce this idea by discussing Exercise 1. Let the suggestions for each step come from the children but help them in discovering a simple solution such as the following. As the steps are suggested write them on the blackboard:

$$(400 + 96) + 300 = 700 + n$$

$$(96 + 400) + 300 = 700 + n \qquad \text{(Commutative property)}$$

$$96 + (400 + 300) = 700 + n \qquad \text{(Associative property)}$$

$$96 + 700 = 700 + n \qquad \text{(Rename 400 + 300 as 700)}$$

It is now easy for the pupils to know, without adding, that n is 96 because addition is commutative.

Go over Exercise 2 in a similar way. The work on the blackboard may look like this:

$$2251 + 49 = 100 + n$$

$$(2200 + 51) + 49 = (49 + 51) + n \quad \text{(Rename 2251 and 100)}$$

$$220 + (51 + 49) = (49 + 51) + n \quad \text{(Associative property)}$$

n is 2200 because addition is commutative.

Help the children solve the sentence in Exercise 3 without adding.

Ask the pupils to go on with the story problems. You may wish to discuss each problem with the class or let the children work them alone. Notice that Exercise 4 may be solved very much like Exercises 1 to 3. The mathematical sentence is:

$$1200 + 75 = 275 + n$$

Renaming and the associative and commutative properties are used in the solution.

In Exercise 5, the children find the perimeter first in yards.

$$60 + 60 + 20 + 20 = n \qquad n \text{ is } 160$$

They change 160 yards to 480 feet (160 x 3) and then compare 480 feet with 500 feet. Since 500 - 480 = 20, they find that Daniel has 20 feet of fence left.

UNIT 10
Sets of Numbers

Objectives

1. To revise even and odd numbers and define even numbers as multiples of two
2. To classify whole numbers into subsets of multiples and into other subsets
3. To summarize the properties of addition and multiplication of whole numbers

Background Information for Teachers

See Teachers' Handbook, pages 25-29 and pages 34-36; Teachers' Guide, Primary Three, "Background Information for Teachers", pages 38-40, 222-226; and "Background Information for Teachers", Unit 7 of this book.

In multiplication and division, we find products of pairs of numbers and factors of numbers. For example, the product of 3 and 4 is 12, since $3 \times 4 = 12$; 3 and 4 are factors of 12, since $3 \times 4 = 12$.

The emphasis in this Unit is on sets of multiples of whole numbers. For example, the set of multiples of three that are less than 20 is:

$$\{0, 3, 6, 9, 12, 15, 18\}$$

The children will discover properties of multiples of whole numbers and of other subsets of whole numbers. For example, the decimal numerals for the multiples of five all end in 0 or 5, and if a decimal numeral ends with 0 or 5 then the number it represents is a multiple of five.

The multiples of two are called even numbers. The set of even whole numbers to 20 is:

$$\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

Every even number can be expressed in the form:

$2 \times n$, in which n is some whole number.

The sum of any two even numbers is an even number. To show this, think of two even numbers, $2 \times m$ and $2 \times n$. Then:

$$(2 \times m) + (2 \times n) = 2 \times (m + n).$$

Since $m + n$ is a whole number whenever <u>m</u> and <u>n</u> are whole numbers, then $2 \times (m + n)$ is some even number.

The whole numbers that are not even are called <u>odd</u> numbers. An odd number is one more than an even number. For example, 7 is an odd number because it is one more than an even number, 6. Every odd number can be expressed in the form:

$(2 \times n) + 1$, in which <u>n</u> is some whole number.

The sum of any even number and any odd number is an odd number. $(6 + 5 = 11)$ We can show this by thinking of an even number, $2 \times m$, and an odd number, $2 \times n + 1$. Then:

$$\begin{aligned}(2 \times m) + (2 \times n + 1) &= (2 \times m) + (2 \times n) + 1 \\ &= 2 \times (m + n) + 1\end{aligned}$$

We know that $2 \times (m + n)$ is an even number; so $2 \times (m + n) + 1$ is an odd number.

In the same way we can show that the sum of any two odd numbers is an even number. The set of whole number multiples of three, to 21, is:

$$\{0, 3, 6, 9, 12, 15, 18, 21\}$$

Every multiple of three can be expressed in the form:

$3 \times n$, in which <u>n</u> is some whole number.

Notice that each whole number is either a multiple of three, or one more than a multiple of three, or two more than a multiple of three. The sum of any two multiples of three is a multiple of three. To show this think of the multiples of three as $3 \times m$ and $3 \times n$. Then

$$(3 \times m) + (3 \times n) = 3 \times (m + n).$$

Since $(m + n)$ is a whole number, $3 \times (m + n)$ is a multiple of three.

STAGE 1: Even and Odd Numbers

Vocabulary: Even, odd

Materials: Pupil page 120

Teaching Procedure

ACTIVITY 1: Multiples of Two

Quickly revise even and odd numbers. For example, you may ask a pupil to name a whole number less than 50. Then ask another pupil to tell whether the number is <u>even</u> or whether the number is <u>odd</u>.

Unless the pupils have difficulty in recognizing even and odd numbers, make this part of the activity short. If there is some difficulty, draw a number line on the blackboard, marked from 0 to 50, and let children skip count by twos. (Beginning at zero and skip counting by twos, they will name the even numbers. Beginning at one and skip counting by twos, they will name the odd numbers.)

The next part of the activity is designed to teach the children that the even whole numbers are multiples of two. Write on the blackboard:

$2 \times n$ (<u>n</u> is a whole number.)

Ask questions like the following: (To help the children answer suggest that they choose numbers for <u>n</u> and find $2 \times n$.)

1. We know that <u>n</u> is a whole number. Is $2 \times n$ a whole number? (Yes)
2. If <u>n</u> is 7, what is $2 \times n$? (14) Is 14 a multiple of two? (Yes) Is 14 an even number? (Yes)
3. If <u>n</u> is 13, what is $2 \times n$? (26) Is 26 a multiple of two? (Yes) Is 26 an even number? (Yes)
4. If <u>n</u> is any whole number, is $2 \times n$ a multiple of two? (Yes) If <u>n</u> is any whole number, is $2 \times n$ an even number? (Yes)
5. Is 18 a multiple of two? (Yes, because $18 = 2 \times 9$) Is 18 an even number? (Yes)
6. Is every even number a multiple of two? (Yes)

Help the children decide from questions like those above that:

- $2 \times n$ is an even number when $\underline{n}$ is a whole number, and
- every even number is a multiple of two.

Then write on the blackboard:

$(2 \times m) + 1$ ($\underline{m}$ is a whole number.)

Ask questions like the following: (To help the children answer suggest that they choose numbers for $\underline{m}$ and find $(2 \times m) + 1$.)

1. If $\underline{m}$ is 5, what is $(2 \times m) + 1$? (11) Is 11 an even number? (No, it is odd.) Is 11 a multiple of two? (No) Is 11 one more than some even number? (Yes, 11 is one more than 10.)
2. If $\underline{m}$ is 8, what is $(2 \times m) + 1$? (17) Is 17 an odd number? (Yes) Is 17 a multiple of two? (No) Is 17 one more than an even number? (Yes, 17 is one more than 16.)
3. If $\underline{m}$ is any whole number, is $(2 \times m) + 1$ an odd number? (Yes)
4. Is $(2 \times m) + 1$ one more than an even number? (Yes, it is one more than $2 \times m$.)

From these kinds of questions, help the pupils decide that:

- $(2 \times m) + 1$ is an odd number when $\underline{m}$ is a whole number, and
- every odd number is one more than some even number.

A pupil may ask whether 0 is an even number. Ask him whether 0 is a multiple of two. (He will agree that 0 is a multiple of two because $0 = 2 \times 0$.) Then ask whether every multiple of two is an even number. (Yes) Let him then decide that 0 is an even number. He can see this also by skip counting by twos from an even number such as 10 to the left on the number line.

ACTIVITY 2: Addition of Odd and Even Numbers Pupil page 120

Get the pupils to copy the three charts from pupil page 120 into their exercise books and to write in the sums. Get them to choose other pairs of numbers to add and show them in the charts.

When they have finished the charts, ask questions like these:

1. In which chart did you add pairs of odd numbers? (Chart A)
2. Look at the numerals under "Sum" in Chart A. Are these names for odd numbers or for even numbers? (Even numbers)
3. Think of two other odd numbers. What is their sum? Is the sum an odd number? (No) Go on with other pairs of odd numbers.
4. Do you think the sum of any two odd numbers is always an even number? (Yes)

Ask questions of a similar type about Charts B and C. Get the children then to complete Exercises 1 to 12 on pupil page 120.

STAGE 2: Subsets of Multiples of Five and Ten

Vocabulary: Divisible

Materials: Pupil pages 121-123

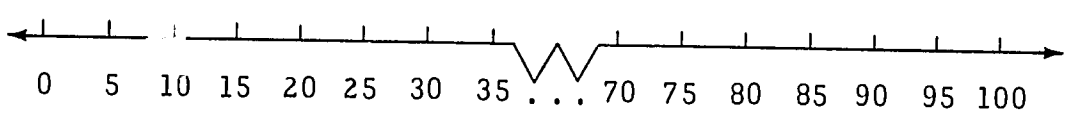
Teaching Procedure

ACTIVITY 1: Sets of Multiples of Five and Ten **Pupil page 121**

Get the children to begin at zero and skip count by fives to 100. Say, "Each of these numbers is a multiple of five." Get the children to show why these numbers are multiples of five, using sentences like these: $0 = 5 \times 0$, $5 = 5 \times 1$, $10 = 5 \times 2$, $15 = 5 \times 3$, and so on.

The children will say, for example, "30 is a multiple of five because $30 = 5 \times 6$".

Draw a number line on the blackboard and let children mark and name points for the multiples of five from 0 to 100.



Follow the same plan for multiples of ten. Let the children skip count, write mathematical sentences showing the multiples of ten and prepare the number line with the numerals for the multiples of ten from 0 to 200.

Get the children to think about the set of <u>all</u> the multiples of five:

$$\{0, 5, 10, 15, 20, 25, \ldots\}$$

Question them and guide them to decide:

- Each number in the set is 5 less than the number that follows it.
- Each number in the set can be expressed as $5 \times n$, where $\underline{n}$ is some whole number.
- Each numeral for a multiple of 5 has a "0" or a "5" in the ones' place.

Say, "If a number is a multiple of five we say the number is divisible by five". Ask, "Is 17 divisible by 5? (No. 17 is not a multiple of 5, or "17" does not have "0" or "5" in the ones' place.) Is 55 divisible by 5? (Yes) Why?" ($55 = 5 \times 11$, or "55" has "5" in its ones' place.)

Get the children to think in a similar way about the set of all multiples of ten.

$$\{0, 10, 20, 30, \ldots\}$$

Help them to decide:

- Each number in the set is ten less than the number that follows it.
- Each numeral for a multiple of ten has "0" in the ones' place.
- Each multiple of ten can be named $10 \times n$, where $\underline{n}$ is some whole number.
- Each number in the set is divisible by ten.

Get the children to complete the statements about multiples of two, of five, and of ten on pupil page 121.

ACTIVITY 2: Subsets of Multiples of Other Numbers **Pupil page 122**

Get the pupils to skip count by threes from zero and write on the blackboard the numerals for the set of numbers they name:

$$\{0, 3, 6, 9, 12, 15, \ldots\}$$

Ask the children to write mathematical sentences for some of the numbers, showing that the numbers are multiples of three.

$0 = 3 \times 0$, $3 = 3 \times 1$, $6 = 3 \times 2$, $9 = 3 \times 3$, and so on.

Guide the children to decide:

- Each number in the set is 3 less than the number that follows it.

- Each multiple of three can be expressed as $3 \times n$, where $\underline{n}$ is some whole number.
- Each multiple of three is divisible by 3.

Let the pupils make a multiplication chart on the blackboard. Let them first write in multiples of three as shown on the chart below and then write in multiples of other numbers.

×	0	1	2	3	4	5	6	7	8	9
0				0						
1				3						
2				6						
3	0	3	6	9	12	15	18	21	24	27
4				12						
5				15						
6				18						
7				21						
8				24						
9				27						

Ask the pupils to name the set of multiples of each of the numbers zero to nine and write the numerals in the chart. Then let them choose a number, say 12, shown on the chart and tell what numbers it is a multiple of. (12 is in the chart in four places. The chart shows that 12 is a multiple of two, of three, of four, and of six. 12 is also a multiple of one and of twelve, since $12 = 12 \times 1$.)

Let the children work the exercises about multiples on pupil page 122.

ACTIVITY 3: A Way to Classify Multiples **Pupil page 123**

Ask a pupil to write the set of counting numbers that are multiples of three and are less than 40.

{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39}

Let the children revise some of the things they noticed about multiples of three from the last activity.

Revise quickly the set of even numbers, odd numbers, multiples of 4, of 5, of 6, . . . of 10.

Use the following examples to help the pupils find sets of numbers that have several properties. In each case, let the children list the sets and then find numbers that are in all the sets.

Example 1 What is the set of whole numbers between 0 and 30 that are even and are multiples of three?

Even numbers: {2, 4, 6, 8, 10, 12, 14, 16, 18, . . . 30}

Multiples of three: {3, 6, 9, 12, 15, 18, 21, 24, 27}

Even numbers that are multiples of three: {6, 12, 18, 24}

(Notice that the answer to the question is the set of numbers common to the first two sets. The set of numbers common to the first two sets is called the intersection of the two sets.)

Example 2 What is the set of numbers between 0 and 30 that are odd and are multiples of three?

Odd numbers: {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, . . . 29}

Multiples of three: {3, 6, 9, 12, 15, 18, 21, 24, 27}

Odd and multiples of three: {3, 9, 15, 21, 27}

Example 3 What is the set of numbers between 0 and 30 that are multiples of three and are multiples of six?

Multiples of three: {3, 6, 9, 12, 15, 18, 21, 24, 27}

Multiples of six: {6, 12, 18, 24}

Multiples of three and multiples of six : {6, 12, 18, 24}

Example 4 What is the set of numbers between 0 and 25 that are multiples of four and are multiples of five?

Multiples of four: {4, 8, 12, 16, 20, 24}

Multiples of five: {5, 10, 15, 20}

Multiples of four and of five: {20}

Go on with examples of this type. When you think the class or some of the pupils can work by themselves, ask them to find the sets in the exercises on pupil page 123.

STAGE 3: The Set of Whole Numbers and Operations on Whole Numbers

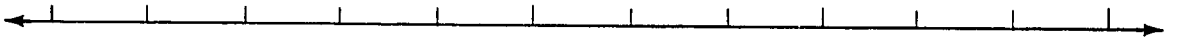
Vocabulary: Truth set

Materials: Pupil pages 124-128

Teaching Procedure

ACTIVITY 1: The Set of Whole Numbers **Pupil page 124**

Construct this number line on the blackboard, marking points, but do not assign numbers. Mark at least twelve points.



Say, "What is the smallest whole number in the set of whole numbers?" (Zero) Write 0 below the mark furthest to the left. Say, "What whole number is one more than zero?" (One) Let a pupil write 1 below the next mark. Ask, "What number is one more than one?" (Two) Let a child write 2. Continue until all the marked points have numbers assigned to them.

Suppose the numbers 0 to 11 have been assigned. Write on the blackboard:

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

Ask, "Is this the set of whole numbers?" Guide the children to say that the set is not complete, that the line segment could be extended and they could go on and on assigning whole numbers to points. Stress the idea that each number in the set is one more than the number before it and that we can always name a number which is one more than any number mentioned.

Put a few more numbers in the set shown on the blackboard and say, "You could go on and on writing the names for whole numbers. Would you ever finish? (No, we could always name another number.) There is a way to show the set of whole numbers".

Put three dots after the last numeral in the series:

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots\}$$

Explain that the dots show that the numbers go on and on. Write:

$$\{0, 1, 2, 3, 4, \ldots\}$$

Say, "This also shows the set of whole numbers. The dots mean the set goes on to 5, 6, 7, 8, and so on, for all whole numbers".

Write:

$$\{8, 9, 10, 11, 12, \ldots\}$$

Ask the pupils to tell what this set is. Let them talk about the set until they decide: that the set shown is the set of whole numbers that are greater than 7; and that the set is a subset of the set of whole numbers. Give a few similar examples, such as, the set of whole numbers greater than 4:

$$\{5, 6, 7, 8, 9, 10, \ldots\}$$

Write:

$$\{0, 1, 2, 3, 4\}$$

Say, "Here is another subset of the whole numbers. What can you tell about this set?" (It is the set of whole numbers less than 5.) Write on the blackboard:

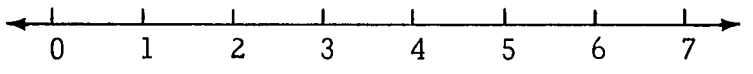
The whole numbers less than five

Say, "Name the members of the set of whole numbers less than 8". Then write:

The whole number less than eight

Let a pupil show on the blackboard the set of whole numbers less than 8 and the number line with only points marked with numbers of the set:

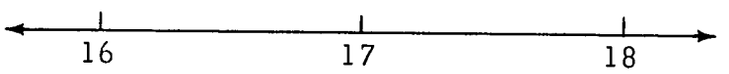
$$\{0, 1, 2, 3, 4, 5, 6, 7\}$$



Write:

$$15 < n \text{ and } n < 19.$$

Let a child read the sentence. (Fifteen is less than n and n is less than 19.) Say, "n is a whole number greater than 15 and less than 19". Let a child draw the number line and assign those whole numbers that make the sentence true and another child show the set:



$\{16, 17, 18\}$

Give other similar examples such as: $151 < n$ and $n < 159$, $2 < n$ and $n < 14$, and so on. Let children draw the number line with points marked for the whole numbers which make the sentences true.

Write:

The whole numbers less than 2×3.

Let the children decide that this set of numbers is the set of whole numbers less than 6. Let them show the set and draw the number line assigning 0, 1, 2, 3, 4, and 5 to points. Give other similar examples.

Ask the children to do the exercises on pupil page 124. They are to draw the number line for each subset of whole numbers described, and assign numbers to the points marked.

ACTIVITY 2: Truth Sets Pupil page 125

Write on the blackboard:

$$25 + 32 = n$$

Ask, "What numbers make the sentence true? (n is 57) Are there other numbers that make the sentence true? (No) There is just one number n that makes the sentence true".

Write:

$$\{57\}$$

Write:

$n < 3 + 2$ and n is a whole number.

Ask, "What numbers make the sentence true?" Let the children suggest numbers. ($\underline{n}$ is 0, or $\underline{n}$ is 1, or $\underline{n}$ is 2, or $\underline{n}$ is 3, or $\underline{n}$ is 4.) Try the numbers as they are named to check the truth of the sentence. Say as you write, "The members of the set of numbers that make the sentence true is this set":

$$\{0, 1, 2, 3, 4\}$$

Go on, "For each sentence there is a subset of whole numbers whose members make the sentence true. We call this set the truth set for the sentence What is the truth set for this sentence?"

$$n < 3 \times 3$$

Let the children name members of the truth set. When all the members are suggested show the set.

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

Ask as you write the sentence below, "What is the truth set for this sentence?"

$$n = 15 \div 3$$

(The truth set is $\{5\}$.) Ask, "Are there other members of this truth set? Does any number other than 5 make the sentence true?" (No) Remind the children that some truth sets have one member, some have many members. Use other examples. Include one example such as:

$$10 < n$$

For this sentence, the truth set is the entire set of whole numbers greater than 10. Let the children name members of the truth set until they see that it goes on and on without end. Guide them to show the truth set as:

$$\{11, 12, 13, 14, 15, \ldots\}$$

Ask the children to open their books to pupil page 125 and tell what the page is about. Discuss how to complete the chart. Go over the example. Remind the pupils that they find the truth set for each sentence and that the members of the truth set are all whole numbers. The children may need help with exercises like Exercise 4. ($\{6, 7, 8, 9\}$)

ACTIVITY 3: Operations on Whole Numbers

Pupil page 126

Write the following on the blackboard:

A) (2, 4) → 8	B) (6, 4) → 2	C) (2, 9) → 11	D) (6, 3) → 2
(3, 3) → 9	(15, 9) → 6	(4, 3) → 7	(19, 19) → 1
(5, 3) → 15	(20, 11) → 9	(12, 19) → 31	(10, 2) → 5
(1, 6) → 6	(19, 8) → 11	(7, 0) → 7	(56, 7) → 8

Say, "The arrows show that an operation assigns one whole number to each pair of whole numbers. For example, in A the operation assigns 8 to the pair (2, 4), it assigns 9 to the pair (3, 3), and so on. What is the operation?" (Multiplication) Let a child put the symbol for multiplication (×) above the arrows.

$$(2, 4) \xrightarrow{\times} 8$$

Let the children discover what the other operations are. (Remind them that the numbers are operated on in the order given.)

Emphasize the following in discussing the operations of addition, subtraction, multiplication and division:

- The operations are on two numbers;
- An operation assigns exactly one number to each pair of numbers;
- The pair of numbers is in a particular order.

Illustrate the importance of the third point by an example of division and subtraction:

$4 \div 2$ is not the same as $2 \div 4$; and

$5 - 1$ is not the same as $1 - 5$.

Let the children complete the exercises on pupil page 126 by telling what operation is shown for each sentence. When they have finished the exercises, go over their answers. Someone may notice that there are two possible answers

Write on the blackboard:

For all whole numbers a and b, $a + b = b + a$.

Ask the children what the statement means. Guide the discussion to bring out these ideas:

- We can substitute any whole numbers for a and b in the sentence above and the sentence will be true.
- We can change the order of the pair of numbers, add, and the sum remains the same.
- We say that addition has the commutative property.

Let the children suggest some numbers for a and b and write the sentences on the blackboard. For example:

$3 + 4 = 4 + 3$;

$15 + 9 = 9 + 15$, and so on.

Ask, "Are all these sentences true?" (Yes, because $a + b = b + a$ for all whole numbers.)

Lead a similar discussion for the sentences below expressing properties of multiplication and addition. After each discussion let the children suggest some numbers for a, b, and c and write sentences.

For all whole numbers a and b, $a \times b = b \times a$	Multiplication is commutative.
For all whole numbers a, b, and c, $(a \times b) \times c = a \times (b \times c)$	Multiplication is associative.
For all whole numbers a, b, and c, $(a + b) + c = a + (b + c)$	Addition is associative.
For all whole numbers a, b, and c, $a \times (b + c) = (a \times b) + (a \times c)$	Multiplication is distributive over addition.

Write:

$a + 0 = a$

$a \times 1 = a$

Ask, "What properties do these show?" (They are the property of zero for addition and the property of one for multiplication.) Emphasize these ideas: when zero is added to a number the sum is the number, and, when a number is multiplied by one the product is the number.

Let the children do the exercises on pupil page 127 by writing the name of the property suggested by each sentence in Exercises 1 to 12. List the properties on the blackboard to help the children as they work.

- Commutative property for addition
- Commutative property for multiplication
- Associative property for addition
- Distributive property
- Property of zero for addition
- Associative property for multiplication
- Property of one for multiplication

Notice that Exercises 13 and 14 are marked challenge problems. Exercise 13 illustrates the property of zero for addition. The first operation is subtraction, so n is 8 - 8 or 0. Since zero plus any number gives that number, the result of the addition operation is whatever one chooses m to be. Exercise 14 illustrates the property of one in the same way. In Exercises 15 to 20, the sentences may be solved easily by inspection and by applying certain properties. For example, since addition is commutative, n in Exercise 15 is 10,943. Applying the distributive property in exercise 20, n is $86 \times (6 + 4)$ or 86×10, so n is 860.

Give a few examples of similar sentences on the blackboard before asking the children to complete Exercises 15 to 20.

ACTIVITY 5: Closure of the Set of Whole Numbers

Pupil page 128

Draw two operation machines on the blackboard, one for addition and one for subtraction. Say, "These machines will only pop out whole numbers. Try some pairs of numbers in each of the machines".

Let the children suggest pairs of numbers, pretend to put the first of the numbers into the machine and then the second into the machine. If the result is a whole number let the child write the name for that number. For some pairs the machine does not work, so no number name is written.

Try every pair the pupils suggest in both machines. When a whole number results, let a child show a mathematical sentence. In some pairs given, the first number is greater than the second. Such a pair works in both machines. But be sure some examples have pairs for which the first number is smaller than the second, such as (2, 3). Pairs like this give a whole number for addition but not for subtraction.

Draw two other operation machines, one for multiplication and one for division. Explain that these machines also pop out only whole numbers. Try many pairs of numbers in these machines.

The children will discover that for any pair of whole numbers the addition and multiplication machines will pop out a whole number. For some pairs, the subtraction machine will not pop out a whole number. For some pairs, the division machine will not pop out a whole number.

Ask the children what this tells us about the set of whole numbers and the operations. Guide them to say that when we add two whole numbers we always get a whole number and when we multiply two whole numbers we always get a whole number.

Say, "We say the set of whole numbers is closed under addition and under multiplication. Is it closed under subtraction? If we have any pair of whole numbers, can we always subtract the second from the first and get a whole number?" (No) Let the child who answers give an example of a pair to which subtraction does not assign a whole number.

Ask, "Is the set of whole numbers closed under division? If we have any pair of whole numbers, can we always divide the first by the second and get a whole number?" (No) Again, let a child give an example.

Let the children open their books to pupil page 128. For Exercises 1 to 10 they choose the names of whole numbers. (Exercises 1, 2, 3, 5, 8, and 10 name whole numbers. The others do not.) When they have completed Exercises 1 to 10, let a child read the sign above the operation machines. Say, "Exercises 11 to 17 give pairs of whole numbers to be put into the four machines. Try the pair in each machine. If you get a whole number from the machine write its name in the proper box. If the machine does not give a whole number, leave that box empty".

When the table is completed ask, "For which operations do you have empty boxes?" (Subtraction and division) Let someone read the question at the bottom of the page. Discuss the answers to the question. (18, Yes; 19, No; 20, Yes; 21, No.)

UNIT 11

Relations Among Geometric Figures

Objectives

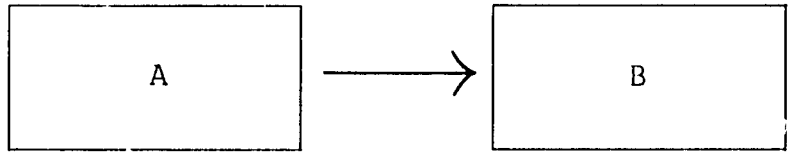
1. To revise intersection properties of points and lines and extend to intersection of lines with plane figures and with solid figures
2. To introduce ideas of parallel lines and movements of figures — parallel movement and turning

Background Information for Teachers

In Unit 3 certain geometric figures were named and described. They were line segments, rays, lines, triangles, rectangles, circles, rectangular solids, spheres, cones, and cylinders. In this Unit some relations among the figures, such as intersections of lines with points, of lines with lines, and of lines with other figures are revised and the idea of moving figures on a plane by parallel and turning movements is introduced.

MOVEMENT OF FIGURES

Since we can move objects from one place to another, it is easy to imagine moving geometric figures around on a flat surface. A geometric figure is an idea that is suggested by the shape of an object. For example, a line segment is our idea of the edge of a ruler or the part of a string stretched between two points. We do not really move geometric figures. All we do is move a copy of a geometric figure from one position on a plane to another.



For example, when we say that rectangle A is moved to rectangle B, in the figure above, we think of tracing rectangle A onto a sheet of paper and moving the paper on the plane so that the tracing will lie on B. This means that rectangles A and B are congruent. There are two kinds of movement that can

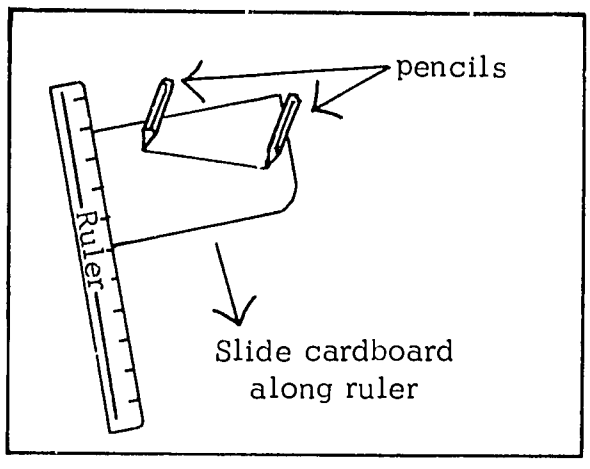
be made on a plane — parallel movement and turning movement.

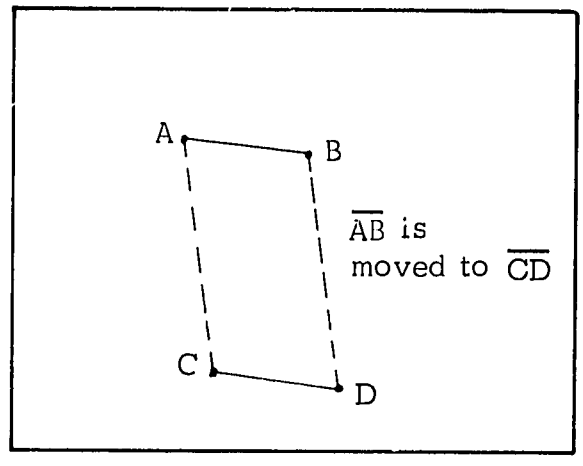
Parallel movement We say that a figure is moved by a parallel movement if its points are moved along parallel lines. The meaning of parallel lines develops at the same time as the idea of parallel movement.

In order to make a parallel movement, we think of moving a figure in one direction without any turning. We do this as follows:

Given the line segment $\overline{AB}$ drawn on a piece of paper, trace the line segment and then copy it from the tracing onto a thick piece of cardboard. (Be sure that the piece of cardboard has at least one edge that is straight.) Punch a narrow hole in the cardboard at each endpoint of the line segment and insert a pencil point in each hole. Place the carboard on the paper so that the two pencil points are on the endpoints of $\overline{AB}$.

Place a ruler against the straight edge of the piece of cardboard. Let someone hold the ruler firmly so that it does not move. We are now ready to make a parallel movement of $\overline{AB}$. (See to the left below.)

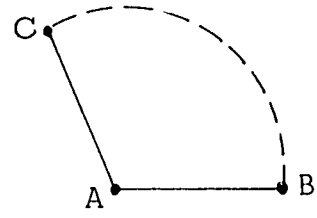




Slide the cardboard along the ruler. Keep the pencils in place so they mark paths of motion. When the motion is stopped, remove the cardboard and the new position of the line segment $\overline{AB}$ is $\overline{CD}$, as shown in the picture to the right. A has been moved to C along $\overline{AC}$ and B has been moved to D along $\overline{BD}$. We say that the dotted lines of motion are parallel lines and that this kind of motion is parallel motion. It is also true that the first line segment $\overline{AB}$ is parallel to the line segment $\overline{CD}$ into which it is moved and that the line segments $\overline{AB}$ and $\overline{CD}$ are congruent.

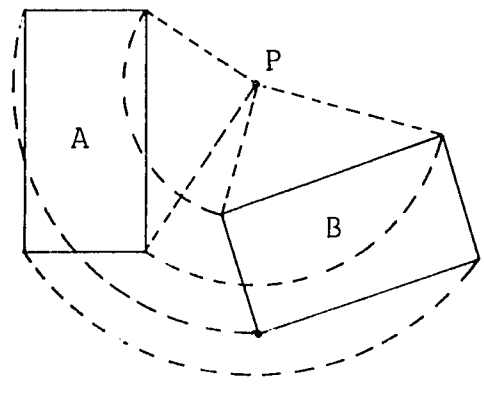
Whenever a figure is moved by a parallel movement, each point of the figure is moved along a line and all the lines of motion are parallel.

Turning movement A parallel movement of a figure is along parallel lines. Turning movement is about a point. For example, if we imagine one endpoint of a line segment to be fixed and if we turn the line segment about this point (like a door turns on its hinges), a new position of the line segment is obtained.



In the figure above, endpoint A is fixed and we think of line segment $\overline{AB}$ turning about point A. Line segment $\overline{AB}$ is moved to line segment $\overline{AC}$. We say that the line segment is moved to the new position by a turning movement about one endpoint.

We can also think of moving a rectangle by a turning movement about a point outside the rectangle. Imagine that each vertex of the rectangle A in the figure below is connected by a string to a turning point P. Then imagine that the figure is turned with all the strings held tight. The rectangle B in the final position is congruent to the rectangle A in the former position. Each vertex of the rectangle is moved along a circle with centre at P.



Later in Entebbe Mathematics, we will use the word translation to describe a parallel movement and the word rotation to describe a turning movement. At this time the words are not important. However, it is important

to understand that (1) these movements are along parallel lines or along circles with the same centre and (2) either kind of movement of a figure takes us to another figure that is congruent to the first.

In this unit the following ideas are also developed: (1) any two opposite sides of a rectangle are parallel, since one side can be moved to an opposite side by a parallel movement; (2) two parallel lines cannot intersect because the lines can be formed by moving two points by a parallel movement and the points are always the same distance apart.

STAGE 1: Intersection of Figures

Vocabulary: Intersect, intersection

Materials: Straight piece of wire; rectangular cardboard boxes, balls, cardboard, cones and cylinders, pupil pages 129-136

Teaching Procedure

ACTIVITY 1: Revision of Points, Line Segments, and Lines Pupil pages 129-130

Get a pupil to mark two dots on the blackboard. Ask, "What do the two dots show?" (Some pupils may say positions, others points.) Whichever word is used, guide the children to agree that a point is a position. Get another pupil to name the points, say A and B.

Ask the children to show other points in the classroom. They may show tips of corners, pencil tips, a dot at any position on any surface, and so on.

Ask the pupils to turn to pupil pages 129 and 130. Let the pupils read the questions in class and follow the instructions as a class activity.

ACTIVITY 2: Intersection of Lines Pupil pages 131-132

Use pupil pages 131 and 132 for revision. Help the pupils to read Exercises 1 to 4 on pupil page 131. They may need to read the exercises several

times in order to learn any new words. Encourage the class to discuss their answers as they follow the instructions given.

Let the pupils read the other exercises on pupil pages 131 and 132 silently and work the exercises in their exercise books.

When the exercises are completed ask:

(a) In how many points can two lines intersect?
(Two lines can intersect in only one point.)

(b) How many lines can be drawn through two points?
(Only one line can be drawn through two points.)

(c) How many lines can be drawn through one point?
(Very many lines can be drawn through one point.)

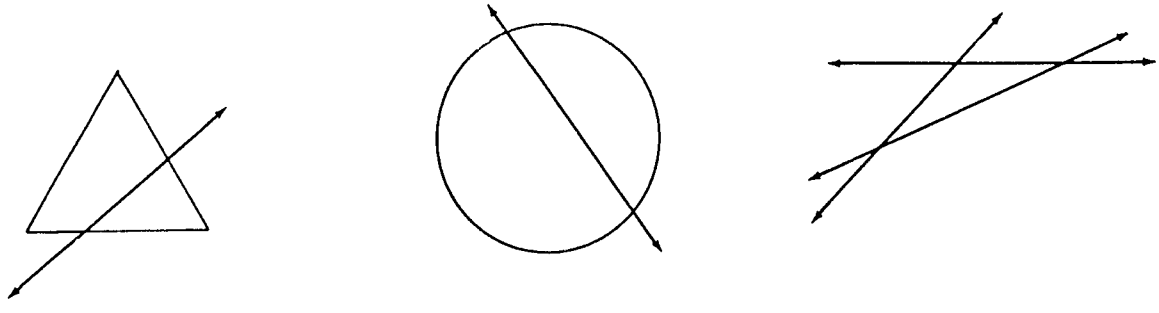
(d) How many points are there on a line?
(Very many points are on a line.)

Get the children to answer these questions in their own language. Help them to understand that the statements are true for all lines and points and that these are properties of lines and points.

ACTIVITY 3: Intersection of Lines with Other Figures Pupil pages 133-134

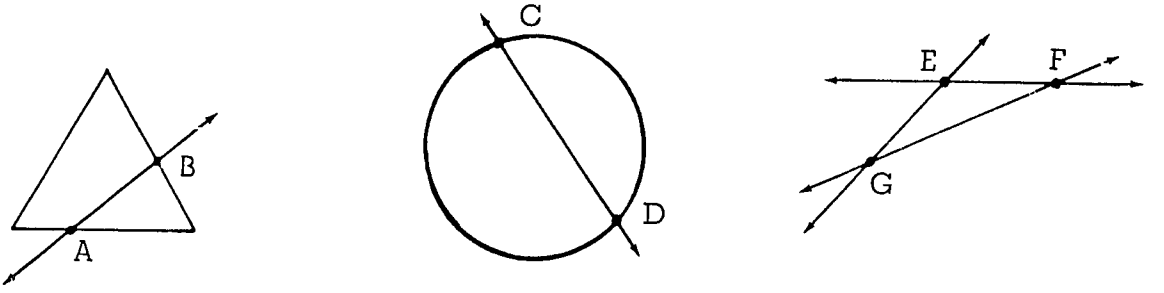
Ask a pupil to draw on the blackboard two lines that intersect. Get the children to mark and name the intersection, say P. Guide them to say, "The two lines intersect. The intersection is point P".

Draw figures similar to these on the blackboard:



Get the children in turn to mark the intersections and to name the intersections.

(The points below which are named A through G are the intersections.)



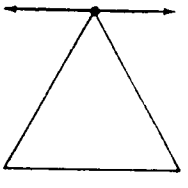
Ask, "What is the intersection of two figures?" (The part of one that is also a part of the other.)

Let the pupils look at pupil page 133 and tell what the intersections of the figures are. Ask them to complete the statements to make true sentences. Exercise 4 may need explanation. Line $\overleftrightarrow{HM}$ and triangle HMG have line segment $\overline{HM}$ in common. Get them to say the figures intersect in $\overline{HM}$ or the intersection is $\overline{HM}$.

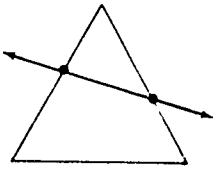
After the work on pupil page 133 is finished, use pupil page 134 as a class activity. Let pupils read the questions in class and follow the instructions. The object of the activity is to revise the idea of the intersection of two figures as the part the figures have in common.

Answers for pupil page 134:

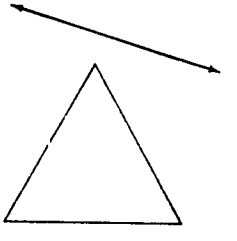
1.



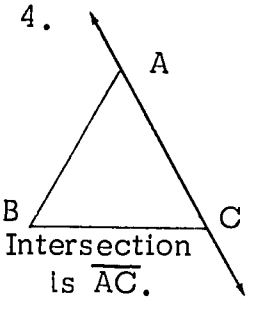
2.



3.



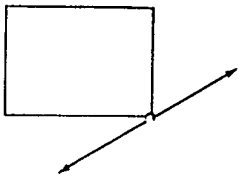
4.



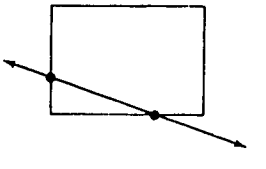
Intersection is $\overline{AC}$.

5. No

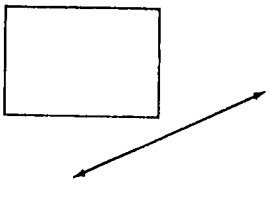
6.



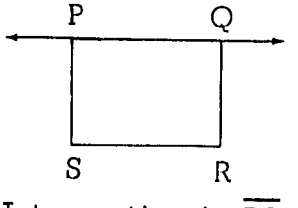
7.



8.



9.



Intersection is $\overline{PQ}$.

10. No

ACTIVITY 4: Intersection of Solid Figures

Pupil pages 135-136

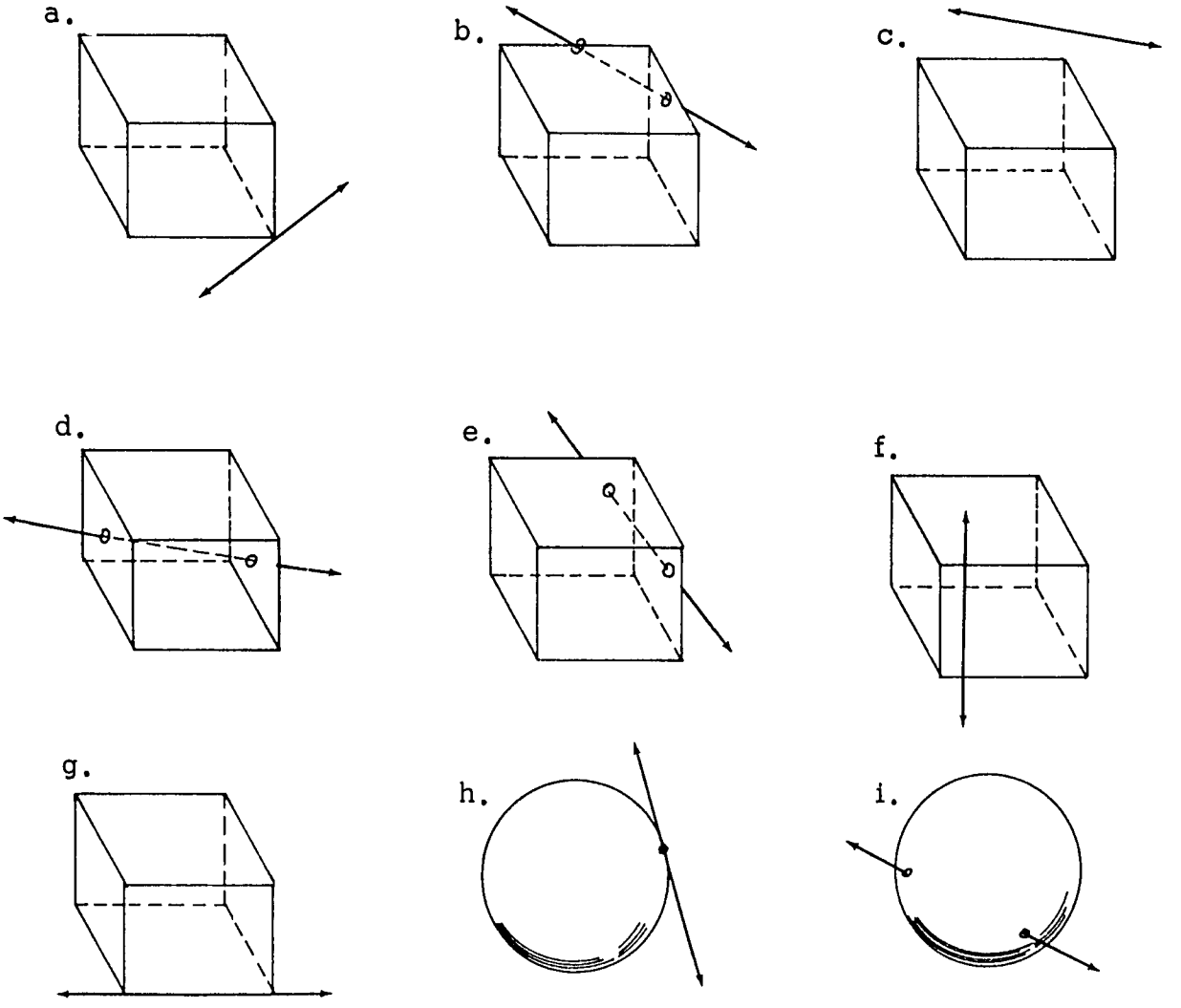
The pupils have used only figures whose pictures are easy to draw. Solid figures can be handled but are not easy to draw on the plane of a sheet of paper. In this activity the pupils use objects which show the solid figures.

Put out straight pieces of wire, cardboard boxes, and balls. Ask the children to show with a wire and a rectangular cardboard box the figures described below. To show figures b, d, and e the children can pierce boxes with wires. (For drawings of how these figures look see the top of page 194.)

1. A straight wire and a box can intersect in one point.
   (See Figure a)
2. A straight wire and a box can intersect at no point.
   (See Figure c)
3. A straight wire and a box can intersect in two points.
   (See Figures b, d, and e)
4. A straight wire and a box can intersect in a line segment.
   (See Figures f and g)
5. Can they intersect in exactly three points? (No)

Each time a child shows one of the intersections with the box and wire, get other children to point out the place of intersection.

Get a child to use the straight wire and a sphere to show how the wire and the sphere can intersect at one point. (See Figure h) Ask, "If the wire pierces the sphere, in how many points do the wire and sphere intersect?" (Two points: See Figure i)



Let the class answer the questions on pupil pages 135 and 136.

Answers for pupil page 135:

1. One point: A
2. Two points: D and E
3. No intersections
4. Two points: P and Q
5. Two points: T and W
6. Line segment: $\overline{XY}$

For pupil page 136 put out straight wires and cardboard models of cones and cylinders for the children to use if they need them. If you have no cardboard boxes shaped like cones and cylinders make models from paper.

Answers for pupil page 136:

1. One point: A
2. Two points: C and D
3. Two points: E and F
4. Two points: X and Y
5. One point: W

STAGE 2: Movements in a Plane

Vocabulary: Movement, parallel, turning

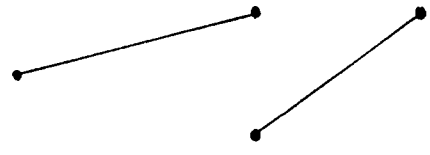
Materials: Rulers, stick about 3 ft. long, a piece of stiff cardboard for each group of pupils, drawing pins, pupil pages 137-139

Teaching Procedure

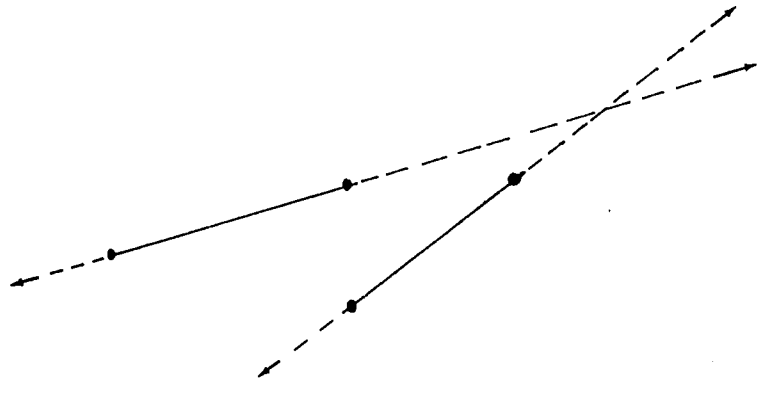
ACTIVITY 1: Parallel Lines **Pupil page 137**

Ask a pupil to show an example of two lines that intersect. He can draw two intersecting lines on the blackboard, or point to two edges. For example, he can point to the side edge and the bottom edge of a book and say that these edges show two lines that intersect at the corner of the book.

After several pupils show examples of lines that intersect, ask a child to show an example on the blackboard of two lines that do not intersect. He can draw two line segments, such as the ones below, and say the line segments do not intersect.



Say, "We have drawn line segments that do not intersect". Ask another child to extend the line segments to lines and to decide whether the lines he draws intersect.



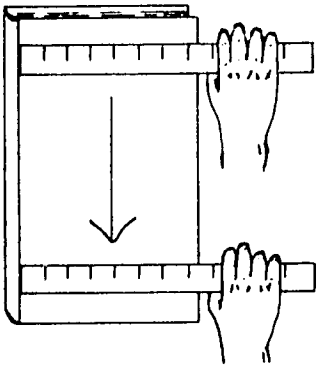
If the lines intersect, ask another child to try to draw two lines that do not intersect. Let the class discuss each drawing and try to decide whether the lines intersect or not, even if the lines are drawn on and on without end.

Let pupils mention objects that show two lines that do not intersect. They may mention these among others:

1. The lines along opposite edges of their Pupil Book.
2. The lines along the bottom and the top of a door.
3. The lines along the rails of a railroad track.
4. The lines along two telephone poles.
5. The line along the bottom of the right wall of the room and the line along the top of the front wall.

(Suggestions such as (1) to (4) show two lines on the same plane. If a child gives an example like (5), accept it, but do not suggest it yourself. Lines such as in (5) do not intersect because they are not on the same flat surface. Try to think only of lines that lie on the same flat surface and do not intersect.)

Each time a pupil suggests two lines that do not intersect, ask, "Can you show that they do not intersect?" Let the children try to answer this question until someone shows with a stick or a ruler that the lines are the same distance apart everywhere. Help them to show, for example, that the sides of a book are the same distance apart all along the book by holding a ruler as shown in this figure and letting the ruler slide along the book.



Do the same with a stick to show that the sides of a doorway are the same distance apart from top to bottom. This tells us that the two lines along the sides of the doorway do not meet.

Explain to the class that two lines that do not intersect because they are the same distance apart everywhere are called parallel lines. Write the word "parallel" on the blackboard.

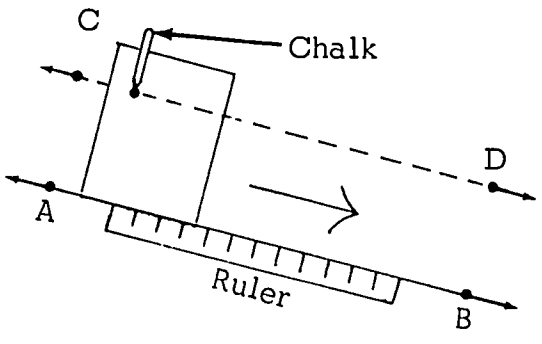
Ask questions such as, "Why are the rails of a railraod tract parallel? (Because the distance between the rails is everywhere the same.) Look at the lines along the bottom of the front wall and the side of the same wall. Are these lines parallel?" (No, because they are not everywhere the same distance apart.)

Let the pupils then turn to pupil page 137 and decide which pairs of lines are parallel. They can use their rulers to help them decide.

ACTIVITY 2: Parallel Movements | Pupil page 138

Revise the idea of parallel lines as lines that do not meet and are the same distance apart everywhere.

Draw a line $\overleftrightarrow{AB}$ on the blackboard and say, "We will draw a line parallel to $\overleftrightarrow{AB}$". Take a stiff piece of cardboard with straight edges. Make a small hole (large enough for a piece of chalk) near one corner of the cardboard. Ask a pupil to hold a ruler or long straightedge tightly along $\overleftrightarrow{AB}$. Then place the chalk in the hole, place the edge of the cardboard against the ruler, and slowly move the cardboard along the ruler.



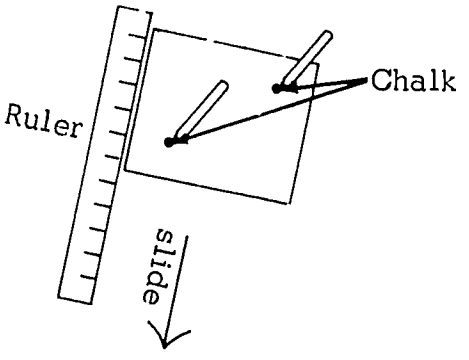
The chalk traces out a line. Call it $\overleftrightarrow{CD}$.

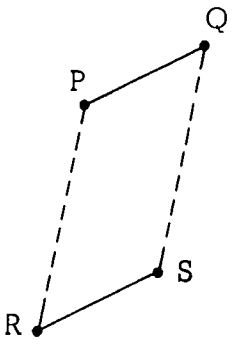
Let the children decide that the new line $\overleftrightarrow{CD}$ is parallel $\overleftrightarrow{AB}$ and discuss why the lines are parallel. They will decide that at every point along the new line the distance between $\overleftrightarrow{CD}$ and $\overleftrightarrow{AB}$ is the same. This means that $\overleftrightarrow{CD}$ will never intersect $\overleftrightarrow{AB}$ no matter how far it is drawn.

Give each group of children a piece of cardboard and a ruler. Let each group draw a line on a sheet of paper and draw another line parallel to it by the same construction you did at the blackboard. (Help the children to make holes to fit a pencil point. Go around the classroom to see that the pupils are holding the ruler steady as they move the cardboard.)

Explain that as the cardboard moves along the ruler, point C is moving to point D. Say, "This is called a parallel movement because C is moved to D along a line parallel to $\overleftrightarrow{AB}$".

Make another hole in your demonstration cardboard to fit another piece of chalk. Hold the cardboard against the blackboard and make dots through the two holes. Mark the dots P and Q. Say, "We can move the line segment $\overline{PQ}$ by a parallel movement". Place the cardboard again on the blackboard with the holes at P and Q as shown below. Ask a pupil to hold a ruler tightly against an edge of the cardboard. Then with the pieces of chalk in the holes, move the cardboard slowly along the ruler. When you stop the motion, name the two stopping points R and S.



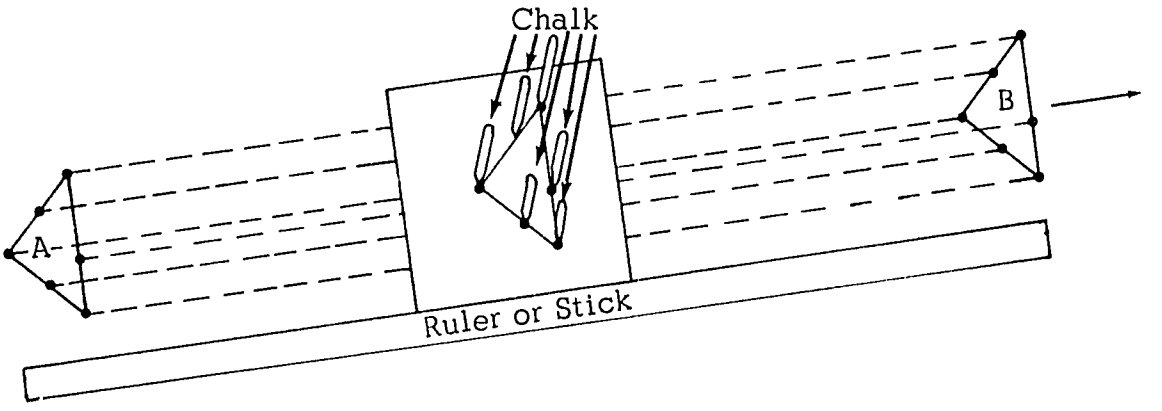


Say, "We have moved the line segment $\overline{PQ}$ to the line segment $\overline{RS}$. Along what line did we move P to R?" (Along $\overleftrightarrow{PR}$) Ask, "Along what line did we move Q to S? (Along $\overleftrightarrow{QS}$) Are these lines of motion parallel? (Yes) Is $\overline{RS}$ congruent to $\overline{PQ}$?" (Yes, because the distance between the two holes in the cardboard is always the same.)

Draw a closed figure on the blackboard (a square, a rectangle, a triangle, or a circle). Call it figure A. Ask the class to imagine that you have copied this figure on the cardboard and that you have punched holes at many points of the

tracing. Then let them imagine that you place the cardboard over figure A on the blackboard and put pieces of chalk through the holes. Ask them to imagine that you move the cardboard along a ruler so that the pieces of chalk trace lines from figure A and stop after moving a short distance. See picture below.

Ask, "After the movement is stopped, the pieces of chalk will form the outline of another figure. Call it figure B. What is the shape of figure B?" Let the children decide that figure B will be congruent to figure A. Also let them tell about the lines of motion made by the chalks. (Parallel lines) The picture below shows the parallel movement of a triangle.



On pupil page 138 there are some figures. Let the pupils decide which segments of the figure can be moved to other segments of the figure by parallel movements. For example, in the rectangle in Exercise 1, $\overline{XY}$ can be moved to $\overline{UV}$ and $\overline{XU}$ can be moved to $\overline{YV}$ by parallel movements.

Ask the class to tell what figure XYVU is. (A rectangle) "Are the sides $\overline{XU}$ and $\overline{YV}$ on parallel lines? (Yes) Are the sides $\overline{XY}$ and $\overline{UV}$ on parallel lines?" (Yes) Let the class say that opposite sides of a rectangle are on parallel lines.

In figure 2, no segment can be moved to any other.

In figure 3, $\overline{AB}$ can be moved to $\overline{ED}$.

In figure 4, each side can be moved to the opposite side by a parallel movement. Ask, "Is the figure in exercise 4 a rectangle, since its opposite sides are parallel?" (No, because the angles are not right angles.)

In exercise 5, no side can be moved to another.

In exercise 6, $\overline{KT}$ can be moved to $\overline{LZ}$ by a parallel movement.

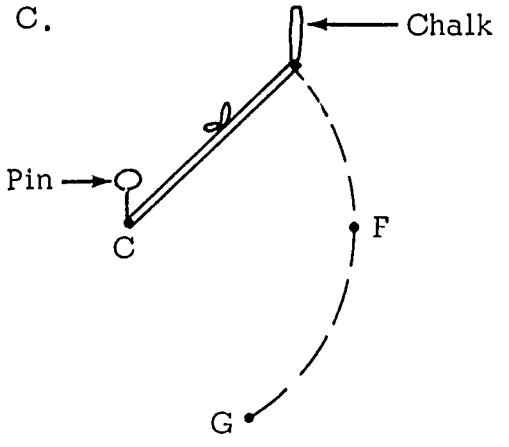
Let the children tell which sides of the figures are congruent. They should understand that if one segment can be moved to another then the two segments are on parallel lines and also are congruent segments.

ACTIVITY 3: Turning Movements **Pupil page 139**

Mark a point A on the blackboard and hold a ruler against the point. As you draw a chalk line from A along the ruler, say, "I am moving point A". Stop the chalk line and mark point B at the end. Say, "I have moved point A to point B. What kind of movement was this?" (A parallel movement)

Let the children discuss this and decide that a parallel movement from one point to another is always along a line.

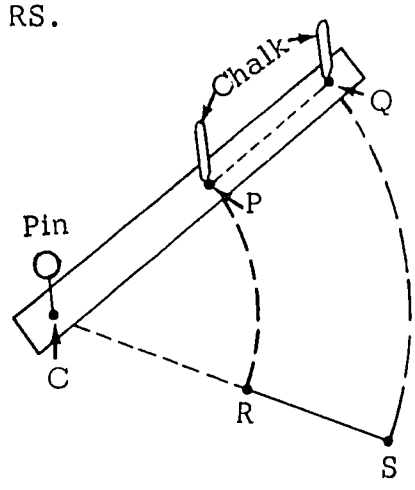
Mark points C and D on the blackboard and place a pin at C. With a looped string stretched around the pin and around a piece of chalk at D, start to draw a circle about C.



As you move the chalk from D, say, "I am moving point D". After drawing part of the circle, stop and mark the stopping point F. Say,"Point D moved to point F. Was this a parallel movement?" (No, it was not along a line.)

Explain that this movement is a turning movement about a point. Ask, "What is the path from D to F?" (A part of a circle) Continue drawing the circle from point F to another point, say G. Let the class tell about the movement from F to G. (Along a circle with centre at C) Ask, "Can we move point D until it comes back to its first position?" (Yes, by making a complete turn.) Let the children discuss the fact that with a turning movement a point can be moved along a circle until it comes back to its first position.

Make three holes along a line on a piece of cardboard as shown below. Put a pin in one hole C and pieces of chalk in the other two, P and Q. Say, "Let us move segment $\overline{PQ}$ with a turning movement about C". The class will see that $\overline{PQ}$ is moved to $\overline{RS}$.



Ask, "Along what curve is P moved to R? (Along part of a circle with centre at C) What is the radius of this circle? ($\overline{CP}$) Along what curve is Q moved to S? (Along part of a circle with centre at C and radius $\overline{CQ}$.) Is $\overline{PQ}$ congruent to $\overline{RS}$?" (Yes)

Ask the class for examples of things that move by turning movements. Some they may give are these:

1. The bottom of a door turns about a hinge. (The corner of the door is an endpoint.)
2. The hand of a clock turns about the centre. (This is another example of turning movement of a line segment about its endpoint.)
3. The spokes of a bicycle wheel turn about the centre of the wheel.
4. The blades of a fan turn about the centre of the fan.
5. A motor car tire at the end of a rope moves with a turning motion about the other end of a rope to form a swing.
6. The circular pendulum of a clock moves with a turning motion.

On pupil page 139 there are points and line segments with their paths of motion. Let the class decide whether the motion is a parallel movement, a turning movement, or neither.

UNIT 12

Multiplication and Division of Whole Numbers II

Objectives

1. To extend the techniques of multiplication to include factors greater than 100
2. To extend the techniques of division to include known factors greater than 12
3. To use estimation in finding products and missing factors
4. To use multiplication and division in applications to money, weights and measure

Background Information for Teachers

See "Background Information for Teachers" for Units 1, 2, 4, and 7 in this book.

STAGE 1: Extension of Techniques of Multiplication to Include Factors Greater than 100, with One Factor Less than 1000

Vocabulary: No new vocabulary

Materials: Pupil pages 140-145

Teaching Procedure

ACTIVITY 1: Techniques of Multiplication with One Factor a Multiple of 10 or 100

Pupil page 140

First revise multiplication by 10 and by 100 with work on the blackboard. Here are some examples you may use.

$6 \times 10 = 60$	$125 \times 10 = 1250$	$20 \times 10 = 200$
$28 \times 100 = 2800$	$127 \times 100 = 12{,}700$	$200 \times 100 = 20{,}000$

Make sure the pupils understand the techniques of multiplication by 10 or by 100 and can multiply rapidly.

Revise the technique of multiplication where one factor is a multiple of 10 or 100 using the associative property. Encourage the pupils to explain each step. Use examples like these:

1. $n = 35 \times 60$
   $n = 35 \times (6 \times 10)$
   $= (35 \times 6) \times 10$ (Associative property)
   $= 210 \times 10$
   n is 2100

2. $q = 85 \times 700$
   $= 85 \times (7 \times 100)$
   $= (85 \times 7) \times 100$ (Associative property)
   $= 595 \times 100$
   q is 59,500

Let the pupils do other examples in a similar way. Guide them to write out the products in only one step.

3. $p = 45 \times 80$
   $p = 3600$

4. $m = 68 \times 400$
   $m = 27{,}200$

Let the pupils open their books to pupil page 140 and do the exercises there.

ACTIVITY 2: Techniques of Multiplication with One Factor Greater Than 100 But Less Than 1000

Pupil pages 141-142

Go over with the class multiplication examples like the ones below to help the children use the technique of multiplication. Encourage the pupils to explain each step of the operation.

(1a) $t = 7 \times 268$
$= 7 \times (200 + 60 + 8)$
$= (7 \times 200) + (7 \times 60) + (7 \times 8)$ (Distributive property)
$= 1400 + 420 + 56$
$= 1876$
t is 1876

Guide the children to use a short form of multiplication. Again get them to explain each step. When they have finished the example, ask them to tell how their work in (1a) is like that in (1b) and how their work in (1a) is like that in (1c).

(1b)

$$\begin{array}{rl} 268 & \\ \underline{7} & \\ 56 & (7 \times 8) \\ 420 & (7 \times 60) \\ \underline{1400} & (7 \times 200) \\ 1876 & \end{array}$$

or

(1c)

$$\begin{array}{r} 268 \\ \underline{7} \\ 1876 \end{array}$$

Go on with another example in a similar way.

(2a)

$$\begin{aligned} q &= 38 \times 376 \\ &= (30 + 8) \times 376 \\ &= (30 \times 376) + (8 \times 376) \quad \text{(Distributive property)} \\ &= 11{,}280 + 3{,}008 \\ &= 14{,}288 \end{aligned}$$

q is 14,288

(2b)

$$\begin{array}{rl} 376 & \\ \underline{38} & \\ 3008 & (8 \times 376) \\ \underline{11280} & (30 \times 376) \\ 14288 & \end{array}$$

Guide the pupils to see that in (2a) and (2b) only one factor is renamed in expanded form. They can do this because they know how to multiply 8×376 and 30×376.

Let the pupils open their books to pupil pages 141 and 142 and work the exercises. In each case encourage the children to use a short form when they understand the process.

ACTIVITY 3: Extension of Techniques of Multiplication Pupil pages 143-145

In this activity help the children to extend the techniques of multiplication to the multiplication of two numbers, each greater than 100. Help the children to see how they use the distributive property.

Write the following mathematical sentence on the blackboard:

$$n = 348 \times 652$$

Discuss the meaning of the sentence with the pupils and ask them for a way of finding n. Guide them to express 348 in expanded form and then to use the distributive property as follows:

$$\begin{aligned} n &= 348 \times 652 \\ &= (300 + 40 + 8) \times 652 \\ &= (300 \times 652) + (40 \times 652) + (8 \times 652) \text{ (Distributive property)} \\ &= 195600 + 26080 + 5216 \\ &= 226{,}896 \\ &\text{n is } 226{,}896 \end{aligned}$$

Guide the children to do this exercise in a short form. Let them tell how the steps of one form relate to the steps of the other form.

$$\begin{array}{r} 652 \\ \underline{348} \\ 5216 \\ 26080 \\ \underline{195600} \\ 226896 \end{array} \qquad \left.\begin{array}{l} (8 \times 652) \\ (40 \times 652) \\ (300 \times 652) \end{array}\right\} \text{(Distributive property)}$$

Guide them to see that the short form above uses the same process as the sentence form but is easier to use.

You and the pupils work a few more examples on the blackboard. Try to help each pupil understand the techniques and to work independently.

Let the pupils do the exercises on pupil page 143. (Some of the exercises

have a factor greater than 1000.) Help the pupils to use the written form that they understand. Also let them solve the exercises on pupil pages 144 and 145. Go around the class and give help as needed. Use only those exercises needed by the children.

STAGE 2: Estimating Products and Factors

Vocabulary: No new vocabulary

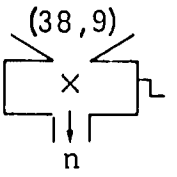
Materials: Pupil pages 146-150

Teaching Procedure

ACTIVITY 1: Estimating Products **Pupil page 146**

In this activity, the idea of estimating products is revised. (The idea was introduced in Unit 7.) Now the children estimate products when the factors are shown in sentences, in diagrams, and as the pair of numbers to be put into multiplication machines.

Illustrate each type of exercise shown on pupil page 146 with an example before the children open their books. Draw this operation machine on the blackboard:



Say, "Remember. You can estimate products. You look at the factors and guess the product. You can estimate the numbers that pop out of multiplication machines. If you put first 38 and then 9 into the machine, a number n pops out. Can you estimate the number n?"

Let the children give answers but discourage wild guessing. There are several ways to estimate this product. Accept answers for which children can

give good reasons. For example, a good estimate of n is 380, because $38 \times 10 = 380$. Another good estimate is 360 because $40 \times 9 = 360$. Good estimates are also 35×10, 36×10 and 37×10.

Write a multiplication sentence expressing the idea the machine shows:

$$38 \times 9 = n$$

If, for example, 360 is given as an estimate, write:

n is about 360

Continue writing good estimates in this form. Emphasize that we cannot say that n is 360 or 380, and so on. These are not the exact products. We can only say that they come close to the product.

Write this example on the blackboard:

$(8, 19) \xrightarrow{\times} n$, n is about_____

Point out that this is just another way of showing what happens to the numbers in the operation machine. Let the children give estimates for n.

Ask the pupils to open their books to pupil page 146 and write estimates for the products in their exercise books. They are not to use the multiplication process to find exact products; they are to give a number they can quickly see is close to the product.

ACTIVITY 2: Estimating Missing Factors **Pupil page 147**

Let the children open their books to page 147, and ask a child to read the direction. Emphasize that for the exercises on this page the children are to estimate one factor. Again, they tell by inspection a number that comes close to the missing factor.

Do several examples on the blackboard before the children begin the exercises.

For example, in the sentence:

$$n \times 7 = 50,$$

the children will see at once that since $7 \times 7 = 49$, a good estimate of n is 7. The missing factor is about 7.

Let the children write in their exercise books estimates for each missing factor on pupil page 147.

ACTIVITY 3: Estimating the Answers in Story Problems **Pupil page 148**

Let the children open their books to pupil page 148. Go over the first problem with the class. Ask, "In this problem, are you to find a factor or a product?" (Factor) Let the pupils write a mathematical sentence about the problem:

$$450 \times n = 4400$$

Let the children estimate the missing factor. (A good estimate is 10, since $450 \times 10 = 4500$.) Say, "Yes, n is about 10. We can say that the River Nile is about 10 times as long as the Rufiji River".

You may wish to read through all the problems with the class or let the children go on with no further help. You will know whether they need help in reading the stories.

Encourage the children first to write a mathematical sentence for the problem and then to estimate n. Finally, get them to use their estimate of n to answer the question asked in the problem. Point out that in some problems the missing factor is to be estimated; in others, the missing product is to be estimated.

ACTIVITY 4: Finding High and Low Estimates **Pupil pages 149-150**

Write the following sentence on the blackboard:

$$28 \times 4 = n$$

Let the children give some estimates for n. Ask, "What is an estimate for n that you know is a little less than n?" Ask the children to give reasons for their answers. For example, 80 is a low estimate because $20 \times 4 = 80$ and 20 is less than 28. Say, "We know that 80 is less than n. We can say 80

is a low estimate for n". Write on the blackboard:

$$80 < n$$

Go on, "What is an estimate that you know is greater than n?" A pupil may answer, "120, because $30 \times 4 = 120$ and 30 is greater than 28". Say, "Yes, we know that 120 is greater than n. We can say 120 is a high estimate for n". Write on the blackboard:

$$120 > n \text{ or } n < 120.$$

Let the children read both sentences. Say, "We know two things about n. We know $80 < n$ and $n < 120$. We know n is between 80 and 120".

$$80 < n \text{ and } n < 120$$

If the pupils need to think about the idea further, let them use the number line. Help them to assign n to a point somewhere between the points for 80 and 120.

Go over a few other examples of this type, letting children express low and high estimates for n in sentences of the form:

$$80 < n \text{ and } n < 120.$$

Write a mathematical sentence with a missing factor, such as:

$$7 \times n = 143.$$

Let the pupils give some estimates for n. Ask, "Can you give a low estimate for n?" One possible answer is 20 because $7 \times 20 = 140$ and 140 is less than 143. Let a pupil write this on the blackboard:

$$20 < n$$

Guide the pupils to see that since 7×20 is less than 143, they may try 21 as an estimate for n. Ask, "Is 21 a low or a high estimate for n? (High) Why? (Because $7 \times 21 = 147$ and 147 is greater than 143.) Write a sentence that shows us 21 is a high estimate".

$$21 > n, \text{ or } n < 21$$

Say, "You have found that n is between 20 and 21". Let a pupil write the mathematical sentences showing low and high estimates:

$$20 < n, \text{ and } n < 21$$

Let the children read the sentence. (20 is less than n and n is less than 21.)

Give other similar examples of finding low and high estimates for missing factors.

Ask the children to open their books to pupil page 149 and complete the chart. Read over the story problems and let the class solve them together. In Exercise 13 they find that 5 buses are needed. Four (the low estimate) is not enough. Some seats in the fifth bus will be empty.

In Exercise 14 they will find 4 boxes is a low estimate and 5 is a high estimate. Four boxes will not provide enough pencils for all the children; the teacher must open five boxes.

On pupil page 150 the story problems involve giving low and high estimates. Notice that in Problem 2, however, the pupils are asked to choose one of two answers. The number to be estimated is either more than or less than 2. The children will choose the answer and write it in their exercise books. In Exercises 1, 3, and 4 they find both a low estimate and a high estimate.

You may talk about each of the exercises with all of the children together or with some children while others work alone. If the children work independently, go over the answers with the entire class when the problems are finished. Let children give reasons for their answers and tell how they solved the problems.

STAGE 3: Extension of the Techniques of Division

Vocabulary: No new vocabulary

Materials: Pupil pages 151-158

Teaching Procedure

ACTIVITY 1: Revision of the Techniques of Division

Pupil pages 151-152

The emphasis in this activity is on the use of the distributive property. Get the children to solve the following problem:

> Twelve countries sent 576 people to a meeting. Each country sent the same number of people. How many people were sent from each country?

Let the pupils discuss what the problem is about, what question is asked, and how to go about answering it. Help them to write a mathematical sentence for the problem and find n by division. Guide them to rename 576 with two addends that are both multiples of 12 and one of the addends is a multiple of 10. (480 + 96) The mathematical sentence and two forms of the work are shown below. Encourage the use of the short form.

$$n = 576 \div 12$$

$n = (480 + 96) \div 12$

$n = (480 \div 12) + (96 \div 12)$

$n = 40 + 8$

$n = 48$

or

```
      40 + 8 = 48
12) 576
    480        (480 ÷ 12 = 40)
     96
     96        (96 ÷ 12 = 8)
```

Get the children to answer the question in the problem. (48 people were sent from each country.)

Ask, "How can you check the division?" Help the pupils to remember that $n = 576 \div 12$ means $12 \times n = 576$. Let them multiply 12×48 and check that the product is 576.

Ask the children to open their books to pupil pages 151 and 152 and do the exercises.

ACTIVITY 2: Revision of Techniques of the Division Process Pupil pages 153-154

Begin the activity with an example like this:

$$27 \div 4 = n$$

Ask, "Is n a whole number? (No) What is the largest multiple of 4 that is less than 27? (24 because $4 \times 6 = 24$.) How can you finish this mathematical sentence, $27 = (4 \times 6) + ___$?" $(27 = (6 \times 4) + 3)$

Do a few more examples of this type until the pupils recall the ideas which were developed in Unit 7. Some other examples are:

1. $33 \div 7$,	$(4 \times 7) + 5 = 33$
2. $47 \div 8$,	$(5 \times 8) + 7 = 47$
3. $75 \div 9$,	$(8 \times 9) + 3 = 75$

Write the following story problem on the blackboard:

A group of 249 people were going on a visit to a park. Buses holding 9 passengers each were to be used. How many 9-passenger buses will the group need?

Get the pupils to discuss the problem, what question is asked and how to go about answering it. They will see that the sentence to be solved is:

$$249 \div 9 = n$$

Ask a pupil to write this as a multiplication sentence. $(9 \times n = 249.)$ Ask, "About how large is n?" Guide the pupils to see that n is between 20 and 30. $(20 < n$ and $n < 30)$ Then guide them to use a short form of the division process. The discussion will follow the steps shown at the top of page 213.

```
    20 + 7 = 27
9) 249
   180     (180 ÷ 9 = 20)
    69
    63     (63 ÷ 9 = 7)
     6
```

1. Estimate n. $20 < n$ and $n < 30$.
2. The largest addend of 249 that is both a multiple of 10 and a multiple of 9 but less than 249 is 180.
3. $180 \div 9 = 20$. Write 20.
4. Subtract. $249 - 180 = 69$.
5. The largest addend of 69 that is a multiple of 9 but less than 69 is 63.
6. $63 \div 9 = 7$. Write 7.
7. Subtract. $69 - 63 = 6$.
8. $6 < 9$. The result of the division process is: $249 = (27 \times 9) + 6$.

Ask, "What was the question asked in the problem? (How many buses are needed?) Can you answer the question?" Let the pupils discuss the answer until they decide that 28 buses are needed. 27 of these will be filled. Ask, "How many vacant seats are there in the last bus to leave the park?" (6 are filled; 3 are vacant.)

Now let the pupils open their books to pupil pages 153 and 154 and do the exercises.

ACTIVITY 3: Extension of Techniques of Division Pupil pages 155-158

The purpose of this activity is to extend the techniques of division to include exercises where the known factor is 13, 15, 25, or other selected numbers less than 100. The ideas are the same as those revised in Activities 1 and 2 of this stage.

Write the following story problem on the blackboard:

A group of thirteen fishermen put some nets out in the ocean. When they pulled in the nets and counted the fish, they found 702 fish in the nets. The fish were divided equally among the 13 men. How many fish did each man get?

Ask one of the pupils to write on the blackboard a mathematical sentence for the problem.

$$n = 702 \div 13$$

Get the children to write the sentence another way. ($13 \times n = 702$) Ask, "About how large is n?" Let the pupils discuss this until they see that n is between 50 and 60. ($50 < n$, and $n < 60$) Guide the pupils in the division process. The written work and thinking will be similar to this:

```
     50 + 4 = 54
13) 702
    650      (650 ÷ 13 = 50)
     52
     52      (52 ÷ 13 = 4)
```

1. Estimate n. $50 < n$ and $n < 60$.
2. The largest addend of 702 that is both a multiple of 10 and a multiple of 13 but less than 702 is 650.
3. $650 \div 13 = 50$. Write 50.
4. Subtract. $702 - 650 = 52$.
5. 52 is a multiple of 13.
   $52 \div 13 = 4$. Write 4.
6. The missing factor is 54.

Ask, "What is the missing factor? (54) Answer the question asked in the problem". (Each man got 54 fish.)

Continue with this story problem:

The next week two other men joined the group of thirteen fishermen. This time when they pulled in the nets, they had 1297 fish. Again they decided to divide the fish equally. How many fish did each man get?

Ask one of the pupils to write on the blackboard a mathematical sentence for the problem. ($n = 1297 \div 15$) Ask, "Why do you divide by 15? (There are now 15 fishermen.) Write the sentence another way. ($15 \times n = 1207$) About how large is n?" Let the children discuss this. (n is between 80 and 90.) As before, guide the pupils in the division process. The written work and thinking will be similar to this:

```
      80 + 6 = 36
15) 1297
    1200      (1200 ÷ 15 = 80)
      97
      90      (90 ÷ 15 = 6)
       7
```

1. Estimate n.
   $80 < n$ and $n < 90$.
2. The largest addend of 1297 that is both a multiple of 10 and a multiple of 15 but less than 1297 is 80×15 or 1200.
3. $1200 \div 15 = 80$. Write 80.
4. Subtract. $1297 - 1200 = 97$.
5. The largest addend of 97 that is a multiple of 15 is 90.
6. $90 \div 15 = 6$. Write 6.
7. Subtract. $97 - 90 = 7$.
8. $7 < 13$. The result of the division process is:
   $1297 = (15 \times 86) + 7$.

Get the children to answer the question in the problem. (Each man gets 86 fish.) "Are there fish left over? (Yes) How many?" (7)

Let the pupils open their books to pupil pages 155 to 158 and do the exercises as needed.

UNIT 13

Fractions II: Addition and Subtraction

Objectives

1. To revise the ideas and techniques of addition and subtraction of fractions named with the same denominator
2. To develop the techniques of addition and subtraction of selected fractions named with different denominators
3. To illustrate the properties of addition of fractions

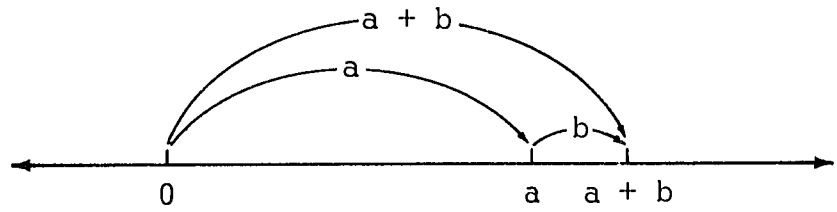
Background Information for Teachers

See "Background Information for Teachers" for Unit 8 in this book.

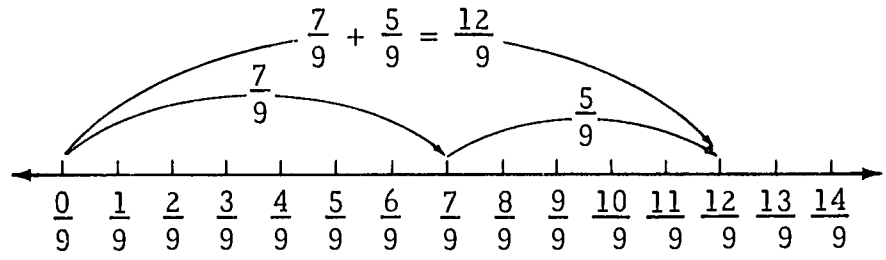
The number line can be used to show which of two numbers, a and b, is the greater. If a is greater than b, then the point for a is to the right of the point for b, and we write $a > b$.

If a and b are whole numbers we can tell directly which number is greater. For example, $7 > 5$ because 7 is 2 more than 5, $7 = 5 + 2$. For fractions that are given by common names, the situation is similar if the denominators are the same. For example, although it is not immediately clear which one of $\frac{3}{10}$ and $\frac{2}{7}$ is greater, we know at once that $\frac{7}{9} > \frac{5}{9}$ because $7 > 5$.

If a and b are whole numbers, we use addition facts and place-value to find their sum, for example, $7 + 5 = 12$ and $72 + 63 = 135$. We can also use the number line to show the sum of a and b. First we go from 0 to a. Then we go to the right from a a distance b, ending at $a + b$.



For fractions the situation is similar <u>if the denominators are the same</u>. For example, although the sum $\frac{3}{10} + \frac{2}{7}$ is not immediately clear, we know at once that $\frac{7}{9} + \frac{5}{9} = \frac{12}{9}$ because $7 + 5 = 12$. On the number line, this is shown like this:



Subtraction is much the same. If two numbers are whole numbers, then we use subtraction facts and place-value to find the missing addend, for example, $72 - 63 = 9$. For fractions, the situation is similar, <u>if the denominators of the fractions</u> are the same. Thus we know that $\frac{7}{9} - \frac{5}{9} = \frac{2}{9}$, because we know that $7 - 5 = 2$.

In the first stage of this unit ordering, and adding and subtracting fractions named with the <u>same</u> denominator, are revised.

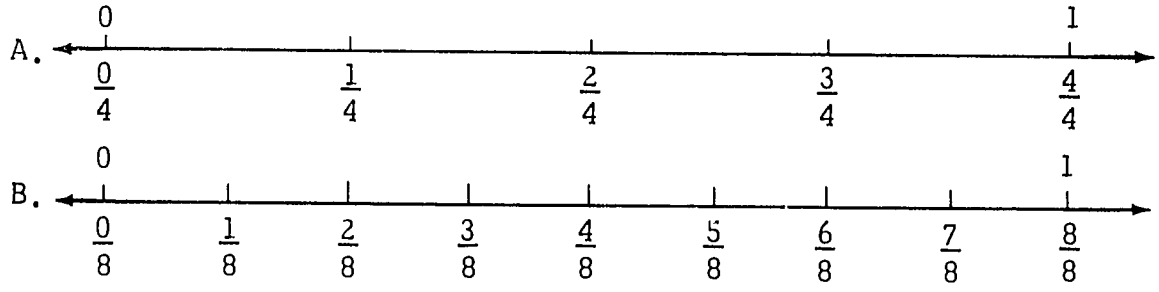
Let $\frac{a}{b}$ and $\frac{c}{b}$ be common names for fractions. Here the numerators <u>a</u> and <u>c</u> are whole numbers. The denominator <u>b</u> of each is a counting number.

From the above discussion, we can discover the following facts about fractions named with the same denominator:

1. $\frac{a}{b} > \frac{c}{b}$, if $a > c$;	or	$\frac{5}{3} > \frac{2}{3}$, if $5 > 2$.
2. $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$;	or	$\frac{5}{3} + \frac{2}{3} = \frac{5 + 2}{3}$.
3. If $\frac{a}{b} > \frac{c}{b}$, then $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$;	or	If $\frac{5}{3} > \frac{2}{3}$, then $\frac{5}{3} - \frac{2}{3} = \frac{5 - 2}{3}$

Rules could be developed for ordering, adding and subtracting fractions that are named with <u>different denominators</u>. Fortunately, such rules are made unnecessary by <u>renaming</u> the fractions that are to be ordered, added, or subtracted. This is done in such a way that the fractions have common names with the <u>same denominator</u>. Then the methods already discovered are sufficient. In the second stage of this unit, pupils learn how to rename fractions in this way.

The number line in picture A below has the unit segment from 0 to 1 separated into four congruent parts. In picture B, the unit segment from 0 to 1 is separated into eight congruent segments.



If we begin at the 0-point of line A and count off three of the congruent parts the name for the point is $\frac{3}{4}$. However, if we begin at the 0-point of line B and count off six of the congruent parts we are at the same point on the number line as when we counted before. Now the name for the point is $\frac{6}{8}$; $\frac{3}{4}$ and $\frac{6}{8}$ are both names for the same point. On line B we separated the unit segment into twice as many parts $(8 = 4 \times 2)$, but there are twice as many $(6 = 3 \times 2)$ of the parts.

We say that $\frac{3}{4}$ and $\frac{6}{8}$ are common names for the <u>same fraction</u>*, and we write:

$$\frac{3}{4} = \frac{6}{8} \text{ (because they name the same numbers);}$$

$$\frac{6}{8} = \frac{3 \times 2}{4 \times 2} \text{ (because } 6 = 3 \times 2 \text{ and } 8 = 4 \times 2\text{).}$$

---

*The numbers which we call fractions here are sometimes called non-negative rational numbers. In this language, the common names of these numbers are called fractions, and the numeral $\frac{6}{8}$ is a fraction that names the rational number three-fourths.

Therefore,

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2}$$

This example illustrates the following general principle:

If $\frac{a}{b}$ is a common name of a given fraction, and $\underline{c}$ is a counting number, then $\frac{a \times c}{b \times c}$ is also a common name of this same fraction. That is,

$$\frac{a}{b} = \frac{a \times c}{b \times c}$$

For example: $\frac{3}{12} = \frac{3 \times 5}{12 \times 5}$; $\frac{3}{12} = \frac{15}{60}$

The principle given above can be used in the opposite direction. For example:

$$\frac{8}{20} = \frac{2 \times 4}{5 \times 4}$$

$$\frac{8}{20} = \frac{2}{5}$$

Since the 2 and 5 of the fraction $\frac{2}{5}$ have no common factor other than 1, we say that $\frac{2}{5}$ is the simplest common name of $\frac{8}{20}$.

We can find common names with the same denominator for $\frac{2}{3}$ and $\frac{3}{4}$ as follows:

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} \qquad \frac{3}{4} = \frac{3 \times 3}{4 \times 3}$$

and

$$\frac{2}{3} = \frac{8}{12} \qquad \frac{3}{4} = \frac{9}{12}$$

When $\frac{2}{3}$ and $\frac{3}{4}$ are written as $\frac{8}{12}$ and $\frac{9}{12}$, these names of the two fractions have the <u>same denominator</u>. Thus, we have the following principle:

Any two fractions $\frac{a}{b}$ and $\frac{c}{d}$ have common names with the same denominator. That is,

$$\frac{a}{b} = \frac{a \times d}{b \times d} \quad \text{and} \quad \frac{c}{d} = \frac{c \times b}{d \times b}$$

For example, we find common names with the same denominator for $\frac{4}{5}$ and $\frac{3}{4}$ as follows:

$$\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20} \quad \text{and} \quad \frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

We can now:

(a) choose the greater of the two fractions ($\frac{4}{5} > \frac{3}{4}$ because $\frac{16}{20} > \frac{15}{20}$);

(b) add the fractions ($\frac{4}{5} + \frac{3}{4} = \frac{31}{20}$ because $\frac{16}{20} + \frac{15}{20} = \frac{31}{20}$);

(c) subtract the fractions ($\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$ because $\frac{16}{20} - \frac{15}{20} = \frac{1}{20}$).

As a final example, we find the sum: $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{5}$

$$(\frac{1}{2} + \frac{1}{3}) + \frac{1}{5} = (\frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2}) + \frac{1}{5}$$ (first add the fractions in the brackets)

$$= (\frac{3}{6} + \frac{2}{6}) + \frac{1}{5}$$

$$= \frac{5}{6} + \frac{1}{5}$$

$$= \frac{5 \times 5}{6 \times 5} + \frac{1 \times 6}{5 \times 6}$$

$$= \frac{25}{30} + \frac{6}{30}$$

$$= \frac{31}{30}$$

To show that the addition of fractions is commutative we name two fractions with the same denominator. Suppose the fractions have names of the form $\frac{a}{b}$ and $\frac{c}{b}$. We add the fractions:

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \quad \text{and} \quad \frac{c}{b} + \frac{a}{b} = \frac{c + a}{b}$$

Then $\frac{a + c}{b} = \frac{c + a}{b}$ ($a + c = c + a$ for whole numbers)

and $\frac{a}{b} + \frac{c}{b} = \frac{c}{b} + \frac{a}{b}$

This shows that a change in the order of adding two fractions does not change the sum. The addition of fractions is commutative.

STAGE 1: Revision of Addition and Subtraction of Fractions Named with the Same Denominator

Vocabulary: No new vocabulary

Materials: Pupil pages 159-161

Teaching Procedure

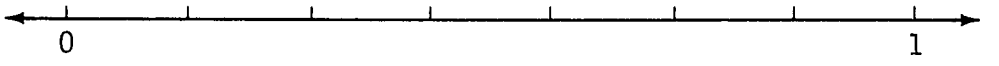
ACTIVITY 1: Revision of Addition of Fractions Named with the Same Denominator

Pupil page 159

Write the following sentence on the blackboard and discuss it with the children:

$$\frac{1}{7} + \frac{5}{7} = n$$

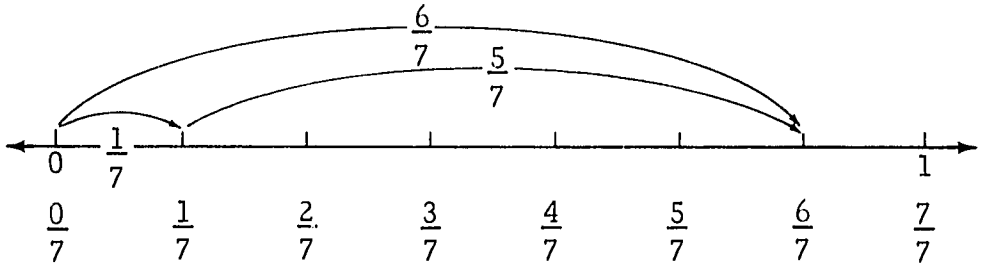
Ask the children how to find the number n that makes the sentence true. Help them to draw a number line and mark it in sevenths from 0 to 1, as shown below:



Get the children to state the number of congruent parts into which the unit segment has been separated. (7) Ask one child to write another name for the 0-point. ($\frac{0}{7}$) Ask the children to write the names for the points after $\frac{0}{7}$.

Get a child to show $\frac{1}{7} + \frac{5}{7}$ on the number line and to write on the blackboard the true sentence.

$$\frac{1}{7} + \frac{5}{7} = \frac{6}{7}$$



Call attention to the denominators of $\frac{1}{7}$ and $\frac{5}{7}$ and that they are the same denominator. Ask, "How do you add fractions named with the same denominator?" Let the pupils discuss this question and decide that they add the numerators to find the numerator for the sum. Let the children talk again about the addition of the fractions $\frac{1}{7}$ and $\frac{5}{7}$. As they talk you record the work.

$$\frac{1}{7} + \frac{5}{7} = \frac{1 + 5}{7}$$
$$= \frac{6}{7}$$

Write this sentence on the blackboard: $\frac{2}{9} + \frac{4}{9} = y$. Let the children find $\underline{y}$. If necessary, again draw the number line and mark the unit segment with ninths. Proceed as for sevenths. Help the children to see that:

$$\frac{2}{9} + \frac{4}{9} = \frac{2 + 4}{9}$$
$$= \frac{6}{9}$$

Write the following sentence on the blackboard and ask a child to find $\underline{m}$ without using a number line:

$$\frac{2}{10} + \frac{3}{10} = m$$

Guide him to write this on the blackboard:

$$\frac{2}{10} + \frac{3}{10} = \frac{2 + 3}{10}$$
$$= \frac{5}{10} \qquad \underline{m} \text{ is } \frac{5}{10}$$

Ask the children to write decimal names for $\frac{2}{10}$ and $\frac{3}{10}$. (·2 and ·3) Then let a pupil write the sentence above with decimal names.

$$\cdot 2 + \cdot 3 = \cdot 5$$

(Revision of decimal names for fractions may be needed at this point.) As another example write this on the blackboard:

$$\frac{8}{100} + \frac{5}{100} = n$$

Let a child find the number $\underline{n}$ that makes the sentence true. ($\frac{13}{100}$) Ask the children to write decimal names for $\frac{8}{100}$ and $\frac{5}{100}$. (·08 and ·05) Let a child write the sentence:

$$\cdot 08 + \cdot 05 = \cdot 13$$

Also ask for per cent names. ($\frac{8}{100} = 8\%$, $\frac{5}{100} = 5\%$) Let a child write the same sentence using per cent names.

$$8\% + 5\% = 13\%$$

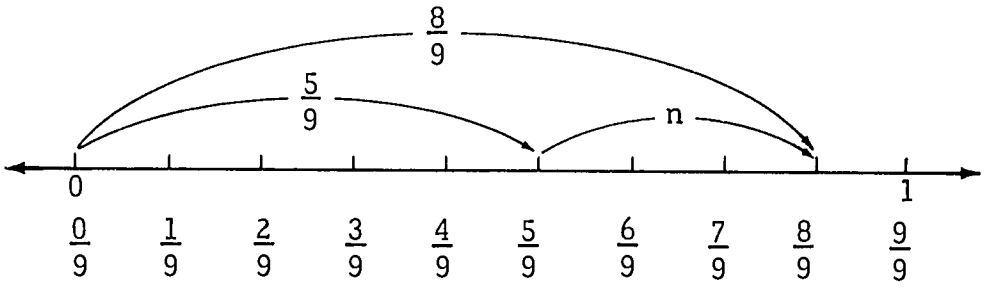
For more examples of this kind let the children turn to pupil page 159 and work the exercises. Do not require that the children rewrite the answers in simple form. For example, the sum in Exercise 15 is $\frac{40}{60}$. Do not expect the answers $\frac{4}{6}$ or $\frac{2}{3}$. If they are given accept them.

ACTIVITY 2: Finding the Missing Addends, Using Number Line

Write this sentence on the blackboard and ask the children to discuss its meaning. (It asks what number added to $\frac{5}{9}$ equals the sum $\frac{8}{9}$.)

$$\frac{5}{9} + n = \frac{8}{9}$$

If needed, draw the number line on the blackboard to show the sentence. Ask a child to mark the number line to show the sum $(\frac{8}{9})$ and the known addend. $(\frac{5}{9})$ Ask another child to draw arrows from 0 to $\frac{8}{9}$, and from 0 to $\frac{5}{9}$, and to show the missing addend with an arrow. The number line should then look like this:



Ask a child to find the number <u>n</u> by skip counting by ninths from $\frac{5}{9}$ to $\frac{8}{9}$. (<u>n</u> is $\frac{3}{9}$.) Let another child check this result by adding $\frac{5}{9} + \frac{3}{9}$ to show that the sum is $\frac{8}{9}$.

Write this sentence on the blackboard:

$$\frac{4}{100} + n = \frac{8}{100}$$

Ask a child to find the number that makes the sentence true. (<u>n</u> is $\frac{4}{100}$.) Let him explain how he found <u>n</u>. Ask other children to write this same sentence using other names for the fractions:

$\cdot 04 + n = \cdot 08$; n is $\cdot 04$.

$4\% + n = 8\%$; n is 4%.

Let the children solve the following sentences in their exercise books. (Ask them to write several names for each number that makes the sentence true.)

$n + \frac{3}{9} = \frac{7}{9}$	$\frac{4}{7} + m = \frac{6}{7}$
$\cdot 5 + a = \cdot 8$	$\cdot 03 + b = \cdot 09$
$25\% + y = 75\%$	$45\% + t = 80\%$

Allow the children who still have difficulties to draw pictures of the number line to show the sentences.

ACTIVITY 3: Subtraction of Fractions Named with the Same Denominator

Pupil pages 160-161

Write this sentence on the blackboard:

$$25 - 10 = n$$

Ask a child to read the sentence and another child to rewrite the sentence as an addition sentence. Let him read it. ($25 = 10 + n$) Ask the children to tell how the two sentences are related. (The same number n makes both sentences true; or the two sentences tell us the same thing about n; or n is 15 in both sentences.)

Write this sentence on the blackboard and discuss it with the children:

$$\frac{7}{9} - \frac{3}{9} = n$$

Let one child read the sentence. Ask another child to rewrite the sentence on the blackboard as an addition sentence and then read it. ($\frac{7}{9} = \frac{3}{9} + n$) Let the children solve the two sentences. Discuss in both sentences what the sum is ($\frac{7}{9}$) and what the known addend is. ($\frac{3}{9}$) Guide the children to find the missing addend. (n is $\frac{4}{9}$ in each sentence.)

Help the children discover that to subtract fractions named with the same denominator they may subtract the numerators. For example:

$$\frac{7}{9} - \frac{3}{9} = \frac{7 - 3}{9}$$
$$= \frac{4}{9}$$

Work more examples on the blackboard with the children, such as:

$\frac{5}{10} - \frac{3}{10} = n$ $\qquad$ $\cdot 6 - \cdot 4 = n$

$75\% - 40\% = n$ $\qquad$ $\cdot 09 - \cdot 02 = n$

Let the children work the exercises on pages 160 and 161 in their exercise books. Do not ask the children to rename their answers in simpler forms. For example, in Exercise 1 on page 160, n is $\frac{3}{9}$. Do not ask that $\frac{3}{9}$ be renamed as $\frac{1}{3}$. Accept $\frac{1}{3}$ if it is given.

STAGE 2: Addition and Subtraction of Fractions

Vocabulary: No new vocabulary

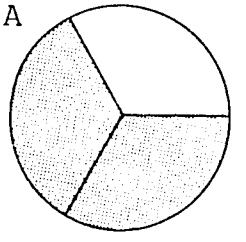
Material: Pupil pages 162-166

Teaching Procedure

ACTIVITY 1: Different Names for the Same Fractions **Pupil page 162**

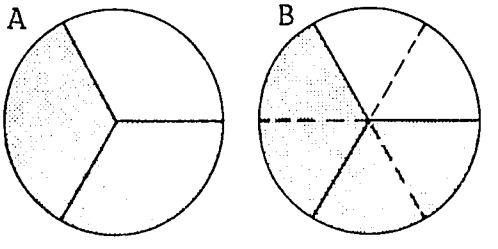
Note: As preparation for the technique of adding and subtracting any fractions, first revise the writing of different names for the same fraction.

Draw on the blackboard a circular region. Separate it into three congruent regions. Shade two of the three regions:



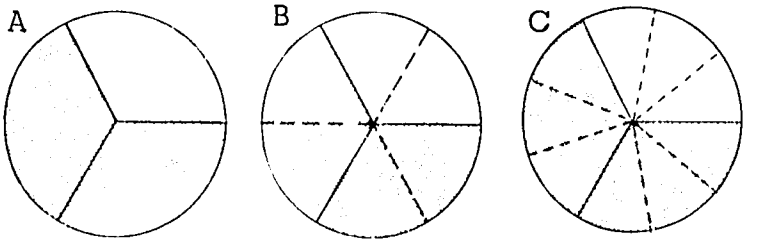
Ask, "What part of the circular region is shaded? (Two-thirds) What common name do you write for this?" ($\frac{2}{3}$)

Draw another circular region B congruent to A. Separate it into six congruent regions. Shade B as shown:



Guide the pupils to say that the shaded region in Figure A is congruent to the shaded region in figure B. Ask, "What common name do you write for the shaded part of B? ($\frac{4}{6}$) Do $\frac{4}{6}$ and $\frac{2}{3}$ name the same number?" (They name the same number because the shaded region of A and B are the same part of the circular regions.)

Draw another circular region C congruent to B. Separate it into nine congruent regions. Shade C as shown.



Ask, "What common name do you write for the shaded part of C? ($\frac{6}{9}$) What can you say about the shaded regions of A, B and C? (They are congruent.) What can you say about the common name $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{6}{9}$?" (They name the same number.) Write on the blackboard:

$$\frac{2}{3} = \frac{4}{6}$$
$$\frac{2}{3} = \frac{6}{9}$$

Ask, "Can you give other names for two-thirds?" (Yes, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{12}{18}$, and so on.) Continue writing these facts on the blackboard:

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{2}{3} = \frac{6}{9}$$

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{2}{3} = \frac{10}{15}, \text{ and so on.}$$

Ask the children to tell how to find these names for two-thirds. Help them to see this pattern and write the factors of the numerators and denominators like this:

$$\frac{2}{3} = \frac{4}{6} = \frac{2 \times 2}{3 \times 2}$$

$$\frac{2}{3} = \frac{6}{9} = \frac{2 \times 3}{3 \times 3}$$

$$\frac{2}{3} = \frac{8}{12} = \frac{2 \times 4}{3 \times 4}$$

$$\frac{2}{3} = \frac{10}{15} = \frac{2 \times 5}{3 \times 5}, \text{ and so on.}$$

Let the children tell in their own words what they notice. (Another common name of a fraction can be found by multiplying the numerator and the denominator by the same counting number.) Let pupils point to the work on the blackboard to show this. For example, another name for $\frac{2}{3}$ can be found by multiplying both the 2 and 3 by 5; the new name is $\frac{10}{15}$.

As an example of the above principle, ask the pupils to rename $\frac{2}{3}$ and $\frac{1}{5}$ so that they have the same denominators. Let the class think about this. Ask questions like these to help them, "What are some names for $\frac{2}{3}$? What are some names for $\frac{1}{5}$?" As the pupils give these names, write them on the blackboard:

Names for $\frac{2}{3}$: $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \ldots$

Names for $\frac{1}{5}$: $\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \ldots$

Ask, "Can you find names for $\frac{2}{3}$ and $\frac{1}{5}$ with the same denominator?" If someone says $\frac{10}{15}$ and $\frac{3}{15}$ draw a ring around these names in the lists, and write:

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}$$

If someone says $\frac{20}{30}$ and $\frac{6}{30}$, write:

$$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$$

$$\frac{1}{5} = \frac{1 \times 6}{5 \times 6} = \frac{6}{30}$$

Let the pupils practice renaming several other pairs of fractions. Get them to do the exercises on pupil page 162.

ACTIVITY 2: Ordering of Fractions **Pupil page 163**

Write $\frac{2}{5}$, $\frac{3}{5}$ on the blackboard.

Ask, "Which is the greater number? ($\frac{3}{5}$) Why?" (Because 3 parts of 5 is more than 2 parts of 5; or because $\frac{3}{5}$ is $\frac{1}{5}$ more than $\frac{2}{5}$.) Accept either of these

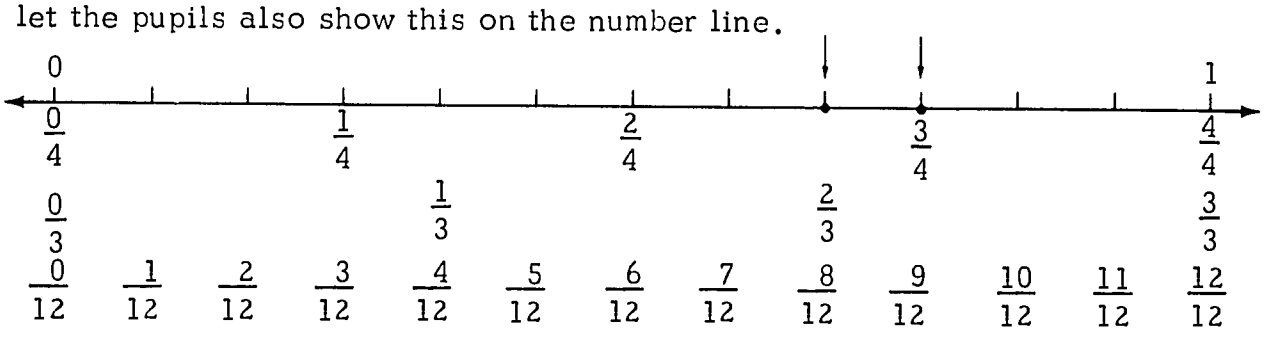
answers. Draw a number line to show that $\frac{3}{5}$ is to the right of $\frac{2}{5}$ on the line.

Write $\frac{2}{3}$, $\frac{3}{4}$ on the blackboard. Again ask, "Which number is greater?" Some of the pupils may know the answer. If so, ask them to tell why. Then ask the pupils to find names for $\frac{2}{3}$ and $\frac{3}{4}$ with the same denominator. Let a child write:

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{88}{12}$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Ask, "Which is greater, $\frac{9}{12}$ or $\frac{8}{12}$? ($\frac{9}{12}$) Which is greater, $\frac{2}{3}$ or $\frac{3}{4}$?" ($\frac{3}{4}$) Help the pupils decide that it is easy to tell which of a pair of fractions is greater when the fractions are named with the same denominator. If needed, let the pupils also show this on the number line.



Guide the pupils to say, "$\frac{3}{4}$ is to the right of $\frac{2}{3}$ on the number lines; $\frac{3}{4}$ is greater than $\frac{2}{3}$; $\frac{2}{3}$ is less than $\frac{3}{4}$". Let a child write:

$$\frac{3}{4} > \frac{2}{3} \quad \text{and} \quad \frac{2}{3} < \frac{3}{4}$$

Ask the pupils to do the exercises on pupil page 163. For example, in exercise 1 they write:

$$\frac{3}{4} = \frac{15}{20} \quad \text{and} \quad \frac{2}{5} = \frac{12}{20}$$

$$\frac{3}{4} > \frac{2}{5} \quad \text{because} \quad \frac{15}{20} > \frac{12}{20}$$

ACTIVITY 3: Techniques of Adding Fractions Named with Different Denominators

Pupil page 164

Write on the blackboard the sentence:

$$\frac{2}{5} + \frac{1}{5} = m$$

Ask, "What number m makes the sentence true?" ($m = \frac{3}{5}$) Point out to the pupils that they can add these numbers easily because the denominators are the same.

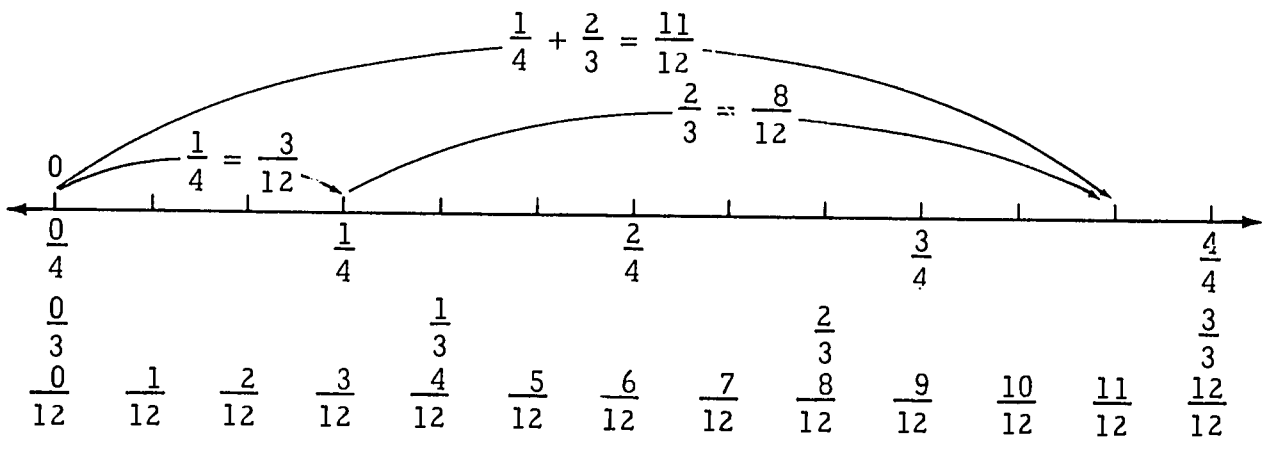
Write another sentence on the blackboard:

$$\frac{1}{4} + \frac{2}{3} = n$$

Say, "Here is another addition sentence. Are the denominators the same? (No) Can we find the sum by adding the numerators? (No) Why? (The parts we are adding are not the same.) Can we make the parts or the denominators the same? (Yes) How? (Rename the fractions.) What are some names for the number $\frac{1}{4}$? ($\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$ . . .) What are some names for the number $\frac{2}{3}$? ($\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$ . . .) Do any of these names for the fractions have the same denominator?" (Yes, $\frac{3}{12}$ and $\frac{8}{12}$.) Lead the pupils to see that:

$$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12}$$
$$= \frac{3 + 8}{12}$$
$$= \frac{11}{12} \qquad n \text{ is } \frac{11}{12}$$

If needed, let a pupil illustrate this on the number line as follows:



Use the same procedure as above and let the class work other examples such as: $\frac{2}{3} + \frac{3}{4} = m$. In an example such as this let the pupils decide that the sum $\frac{17}{12}$ may be written with a mixed name, $1\frac{5}{12}$.

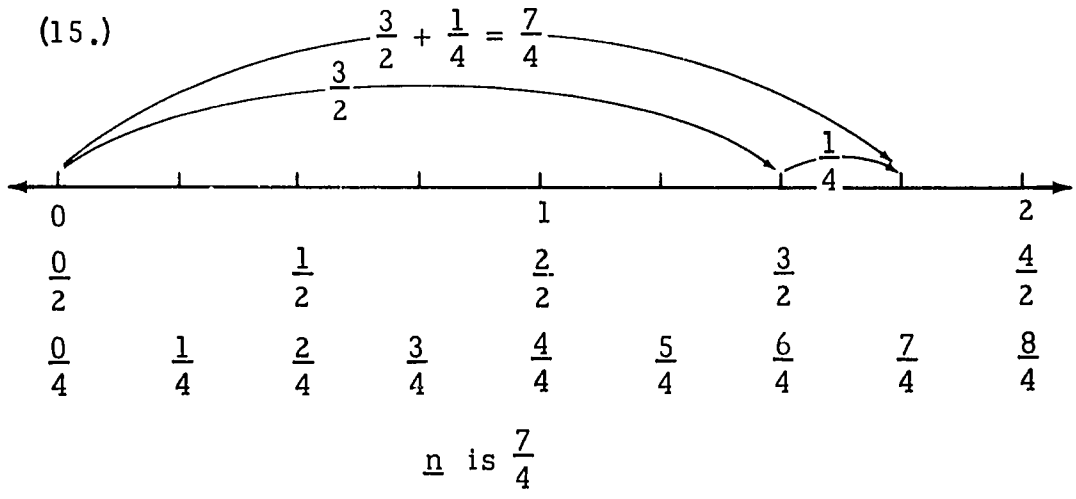
Ask the pupils to do the exercises on pupil page 164.

<u>Answers to Exercises 7 and 15, pupil page 164:</u>

(7.) $\frac{1}{2} + \frac{5}{6} = \frac{3}{6} + \frac{5}{6}$

$= \frac{8}{6}$, <u>m</u> is $\frac{8}{6}$

(15.)



<u>n</u> is $\frac{7}{4}$

ACTIVITY 4: Subtraction of Fractions **Pupil page 165**

Write on the blackboard the sentence, $\frac{2}{5} + \frac{1}{3} = n$. Let one of the pupils rewrite the sentence so the fractions are named with the same denominator. Let the children work together to find the sum:

$$\begin{aligned} n &= \frac{2}{5} + \frac{1}{3} \\ &= \frac{6}{15} + \frac{5}{15} \\ &= \frac{6 + 5}{15} \\ &= \frac{11}{15} \end{aligned}$$

Guide them to say, "The sum of $\frac{2}{5}$ and $\frac{1}{3}$ is $\frac{11}{15}$. $\frac{2}{5}$ and $\frac{1}{3}$ are addends; $\frac{11}{15}$ is the sum". Say, "Suppose you know the sum, $\frac{11}{15}$, and one addend, $\frac{2}{5}$, can

you find the other addend? (Yes) What do you call the process of finding a missing addend when the sum and one addend are given?" (Subtraction)

Write a subtraction sentence on the blackboard, $m = \frac{4}{5} - \frac{2}{3}$. Let the pupils rename the fractions with the same denomination and subtract:

$$\begin{aligned} m &= \frac{4}{5} - \frac{2}{3} \\ &= \frac{12}{15} - \frac{10}{15} \\ &= \frac{12 - 10}{15} \\ &= \frac{2}{15} \end{aligned}$$

m is $\frac{2}{15}$

Ask, "What is the sum? ($\frac{4}{5}$) What is the known addend? ($\frac{2}{3}$) What is <u>m</u>?" (The missing addend, the number that we add to $\frac{2}{3}$ to get the sum $\frac{4}{5}$. <u>m</u> is $\frac{2}{15}$.) Let the children check the subtraction by adding $\frac{2}{3} + \frac{2}{15}$ to get $\frac{4}{5}$.

$$\begin{aligned} \frac{2}{3} + \frac{2}{15} &= \frac{10}{15} + \frac{2}{15} \\ &= \frac{10 + 2}{15} \\ &= \frac{12}{15} \text{ (or } \frac{4}{5}) \end{aligned}$$

Let the children try other examples. Then ask them to work the exercises on pupil page 165. In Exercise 1 their work will look like this:

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$$

<u>m</u> is $\frac{3}{6}$ (or <u>m</u> is $\frac{1}{2}$)

ACTIVITY 5: Story Problems

Pupil page 166

Write a story problem like the following on the blackboard:

> A father has a boy and a girl in boarding schools. He spends $\frac{1}{6}$ of his salary for the boy's school fee and $\frac{1}{12}$ of his salary for the girl's school fee. What fraction of his salary does he spend on both school fees?

Help the pupils to read the problem. Ask, "What question is asked in the problem?" (What fraction of the father's salary does he spend on the school fees?) Let the pupils choose a letter, say n, to represent the fraction or number required. Guide the pupils to make this mathematical sentence:

$$n = \frac{1}{6} + \frac{1}{12}$$

Let the children find the number that makes the sentence true. Get one of them to show the work on the blackboard like this:

$$\begin{aligned} n &= \frac{1}{6} + \frac{1}{12} \\ &= \frac{2}{12} + \frac{1}{12} \\ &= \frac{2 + 1}{12} \\ &= \frac{3}{12} \text{ or } \frac{1}{4} \\ n \text{ is } & \frac{1}{4} \end{aligned}$$

Let the children answer the question in the problem. (The father spent $\frac{3}{12}$ (or $\frac{1}{4}$) of his salary on the school fees.)

Do other examples with the children. Then ask them to solve the story problems on pupil page 166. Remind the pupils to follow these steps in solving story problems:

- Know what the problem is about and what question the problem asks.
- Write a mathematical sentence for the problem, using a letter to represent the missing number.
- Find the number that makes the sentence true.
- Use the number to answer the question in the problem.

STAGE 3: Properties of Addition of Fractions

Vocabulary: No new vocabulary

Materials: Pupil pages 167-168

Teaching Procedure

ACTIVITY 1: Commutative Property of Addition

The pupils know that the questions of addition and multiplication are commutative in the set of whole numbers. For example, they know that these are true sentences:

$$4 + 9 = 9 + 4$$

and

$$8 \times 7 = 7 \times 8$$

They also know that the operations of subtraction and division are <u>not</u> commutative in the set of whole numbers. For example:

$$3 - 2 \neq 2 - 3,$$

$$3 \div 2 \neq 2 \div 3.$$

In this activity, help the children answer the question, "Is addition commutative in the set of fractions?"

Draw on the blackboard an addition machine. Ask the pupil to name a pair of fractions, say, $(\frac{1}{2}, \frac{1}{3})$. (Note that $\frac{1}{2}$ is the <u>first</u> number of the pair and $\frac{1}{3}$ is the <u>second</u> number of the pair.) Say, "Let us put this pair of fractions into the addition machine. We first put in $\frac{1}{2}$ and then $\frac{1}{3}$. (Let a child pretend to turn the handle.) What number pops out?" $(\frac{5}{6})$

Now get the pupils to add the <u>same pair</u> of fractions but to <u>change the order</u>. Say, "Let us put the pair of fractions $(\frac{1}{3}, \frac{1}{2})$ into the addition machine, this time put in $\frac{1}{3}$ first. (Let a child pretend to turn the handle.) What number pops out?" $(\frac{5}{6})$ Ask, "Is the sum the same in both examples?" (Yes, the sum is $\frac{5}{6}$ in each.)

Write on the blackboard:

$$\frac{1}{2} + \frac{1}{3} = \frac{1}{3} + \frac{1}{2}$$

Use the same procedure with the pairs of fractions $(\frac{1}{4}, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{4})$. The blackboard work should appear as follows:

$$\frac{1}{4} + \frac{1}{2} = \frac{3}{4} \quad \text{and} \quad \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\frac{1}{4} + \frac{1}{2} = \frac{1}{2} + \frac{1}{4}$$

Ask the pupils to think of the results they got when they added $\frac{2}{5} + \frac{1}{5}$ and then added $\frac{1}{5} + \frac{2}{5}$. (The result is the same. The sum in each is $\frac{3}{5}$.)

Guide the children to think about the above examples and say that the examples show:

- The sum of two fractions is the same in either order they are added.
- Addition of fractions is commutative.

Pupils may bring up the question, "What about subtraction? Does it matter if we change the order of the numbers?" Guide the children to think about this question. Ask them to think of the results they obtain if they subtract $\frac{5}{7} - \frac{3}{7}$ and $\frac{3}{7} - \frac{5}{7}$. Help them decide that:

$$\frac{5}{7} - \frac{3}{7} \neq \frac{3}{7} - \frac{5}{7}, \text{ and}$$

subtraction of fractions is not commutative.

ACTIVITY 2: Addition Property of Zero **Pupil page 167**

Ask the class to solve sentences such as the following:

$$\frac{5}{8} + 0 = n; \qquad \frac{3}{4} + t = \frac{3}{4};$$

$$0 + \frac{5}{8} = m; \qquad t + \frac{3}{4} = \frac{3}{4}.$$

Let the pupils suggest four or five more examples in which one of the addends is zero. Guide the class to decide that if one of the addends is zero, then the sum is equal to the other addend. This fact is called the <u>addition property of zero</u>.

Tell the pupils to turn to pupil page 167 and work the exercises in their exercise books. This page includes exercises on the commutative property of addition and the addition property of zero.

ACTIVITY 3: Associative Property of Addition **Pupil page 168**

Write this sentence on the blackboard:

$$\left(\frac{3}{11} + \frac{1}{11}\right) + \frac{5}{11} = y$$

Ask, "What number $\underline{y}$ makes the sentence true?" Get the pupil who replies $\frac{9}{11}$ to tell how he got $\frac{9}{11}$. If he answers, "I first added $\frac{3}{11} + \frac{1}{11}$", say, "What number is named in the brackets?" ($\frac{4}{11}$ because $\frac{3}{11} + \frac{1}{11} = \frac{4}{11}$.) Write on the blackboard another sentence showing the next step:

$$\frac{4}{11} + \frac{5}{11} = y$$

Ask, "What number $\underline{y}$ makes the sentence true?" ($\frac{9}{11}$ because $\frac{4}{11} + \frac{5}{11} = \frac{9}{11}$.) Say, "Your work shows that if $\underline{y}$ is $\frac{9}{11}$ the sentence $(\frac{3}{11} + \frac{1}{11}) + \frac{5}{11} = y$ is true". Emphasize that in sentences like these we first find the number named in the brackets.

Say, "Suppose the numbers are grouped another way". Write on the blackboard:

$$\frac{3}{11} + (\frac{1}{11} + \frac{5}{11}) = t$$

Go on, "This shows the same numbers are added, but the brackets show a different grouping. What number $\underline{t}$ makes the sentence true?" Get the pupil who replies $\frac{9}{11}$ to tell how he got $\frac{9}{11}$. If he answers, "I first added $\frac{1}{11} + \frac{5}{11} = \frac{6}{11}$", then write a sentence showing:

$$\frac{3}{11} + \frac{6}{11} = t$$

Ask, "What number $\underline{t}$ makes the sentence true?" ($\frac{9}{11}$ because $\frac{3}{11} + \frac{6}{11} = \frac{9}{11}$.) Say, "Your work shows that if $\underline{t}$ is $\frac{9}{11}$ the sentence $\frac{3}{11} + (\frac{1}{11} + \frac{5}{11}) = t$ is true". Emphasize again that in sentences like these we begin work by finding the number named in the brackets.

Call attention to the fact that in the two examples, the results show:

$$(\frac{3}{11} + \frac{1}{11}) + \frac{5}{11} = \frac{9}{11} \text{ and}$$
$$\frac{3}{11} + (\frac{1}{11} + \frac{5}{11}) = \frac{9}{11}.$$

Ask, "Was the sum the same in both cases?" (Yes, $\frac{9}{11}$.) Write:

$$(\frac{3}{11} + \frac{1}{11}) + \frac{5}{11} = \frac{3}{11} + (\frac{1}{11} + \frac{5}{11}).$$

Use other examples to help pupils decide that no matter which of the two ways three fractions are grouped, the sum is the same. Tell the pupils that this is a property of the addition of fractions. It is called the Associative Property.

Ask, "What about subtraction? Is subtraction of fractions associative?"

Write on the blackboard the following sentence:

$$(\frac{5}{11} - \frac{2}{11}) - \frac{1}{11} = y$$

Ask, "What number y makes this sentence true? What number is named in the brackets?" ($\frac{3}{11}$ because $\frac{5}{11} - \frac{2}{11} = \frac{3}{11}$.) Write on the blackboard a sentence showing the next step:

$$\frac{3}{11} - \frac{1}{11} = y$$

Ask, "What number y makes this sentence true?" ($\frac{2}{11}$ because $\frac{3}{11} - \frac{1}{11} = \frac{2}{11}$.) The work above shows that if y is $\frac{2}{11}$ the sentence $(\frac{5}{11} - \frac{2}{11}) - \frac{1}{11} = y$ is true.

Write on the blackboard the sentence $\frac{5}{11} - (\frac{2}{11} - \frac{1}{11}) = t$. Ask, "What number t makes this sentence true?" ($\frac{4}{11}$ because $\frac{2}{11} - \frac{1}{11} = \frac{1}{11}$ and $\frac{5}{11} - \frac{1}{11} = \frac{4}{11}$.) The work above shows that if t is $\frac{4}{11}$ the sentence $\frac{5}{11} - (\frac{2}{11} - \frac{1}{11}) = t$ is true.

Ask, "Are these two results the same?" (No, one result is $\frac{2}{11}$, the other is $\frac{4}{11}$.) Write on the blackboard:

$$(\frac{5}{11} - \frac{2}{11}) - \frac{1}{11} \neq \frac{5}{11} - (\frac{2}{11} - \frac{1}{11})$$

Emphasize that subtraction of fractions is not associative.

Get the pupils to do the exercises on pupil page 168. These exercises help the children to use these ideas: in the set of fractions addition is associative; and subtraction is not associative. Urge the children to find answers without adding or subtracting.

UNIT 14
Integers

Objectives

1. To introduce negative numbers through skip counting on the number line
2. To name positive and negative integers as opposites of each other
3. To identify even and odd integers
4. To introduce addition of integers

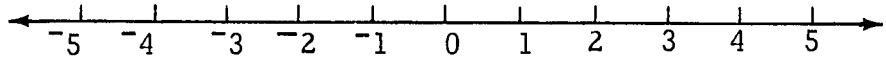
Background Information for Teachers

We use the number line to help us think about whole numbers and fractions.

We will use it now to help us think about other numbers. It is natural to think of possible numbers for points on the left side of the 0-point and also to use the numbers for the points on the number line to represent losses as well as gains. We can, for example, show a loss of two by a number for a point two units to the left of the 0-point and a gain of three by a number for a point three units to the right of the 0-point.

We used counting numbers for the points marked on the right of the 0-point. The numbers for points on the left of the 0-point also have names. We cannot use the counting number names again for they are already assigned to points. We need new names. Names that tell <u>how far</u> the points are from the 0-point and also tell that the points are <u>on the left</u> of the 0-point. The name of the number for the point two spaces to the left of the 0-point is "$^{-}2$". The raised dash tells that the point is on the left side of the 0-point and the "2" tells how many spaces it is from the 0-point. "$^{-}2$" is read "negative two".

The number line then looks like this:



The $^{-}5$ point, for example, is the same distance from 0 as the 0 is from the 5-point, but on the opposite side of 0. We say that $^{-}5$ is the <u>opposite</u> of 5. Of course, this means also that 5 is the <u>opposite</u> of $^{-}5$. Each of the numbers has an opposite number. (If we agree that 0 is its own opposite.) We give the name <u>integers</u> to the numbers for all the points we can mark in this way on the number line.

$$\{\underbrace{\underbrace{\ldots\ {}^{-}4,\ {}^{-}3,\ {}^{-}2,\ {}^{-}1}_{\text{Negative integers}},\ \underbrace{0,\ 1,\ 2,\ 3,\ 4,\ \ldots}_{\text{Positive integers}}}_{\text{Integers}}\}$$

The counting numbers are called the <u>positive integers</u>. The opposites of the counting numbers are called the <u>negative integers</u>. Thus, the set of integers consists of:

- the positive integers (counting numbers),
- zero, and
- the negative integers.

The positive integers are classified as even or odd. If we start at 0 on the number line and skip count by twos to the right, the numbers we count are called the <u>even</u> positive integers. If we begin at 1 and skip count by twos to the right, the numbers obtained are the <u>odd</u> positive integers.

In the same way we classify all the integers as even or odd. If we start at 0 on the number line and skip count by twos either to the right or to the left, the set of numbers we get is the set of <u>even integers</u>:

$$\{\underbrace{\ldots,\ {}^{-}8,\ {}^{-}6,\ {}^{-}4,\ {}^{-}2}_{\text{Even negative integers}},\ \underbrace{0,\ 2,\ 4,\ 6,\ 8,\ \ldots}_{\text{Even positive integers}}\}$$

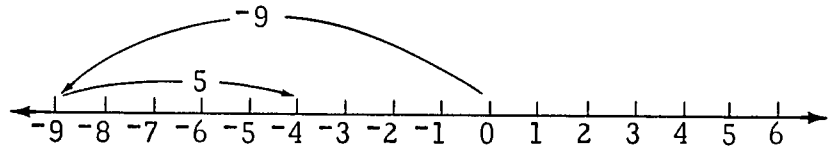
Starting at 1 and skip counting by twos either to the right or to the left gives the set of <u>odd integers</u>.

$$\{\underbrace{\ldots,{}^{-}7,\ {}^{-}5,\ {}^{-}3,\ {}^{-}1}_{\text{Odd negative integers}},\ \underbrace{1,\ 3,\ 5,\ 7,\ \ldots}_{\text{Odd positive integers}}\}$$

There are many uses for the integers. Losses of any kind are described by negative numbers. For example, a loss of 9 shillings is described by the integer $^{-}9$. Gains of any kind are described by positive numbers. For example, a gain of 2 grams in weight is described by the integer 2. Putting gains and losses together suggests the addition of integers. For example, if a boy has a debt of 9 shillings ($^{-}9$) and pays 5 shillings (5), then he still has a debt of 4 shillings ($^{-}4$). This is shown by the sentence:

$$^{-}9 + 5 = {}^{-}4$$

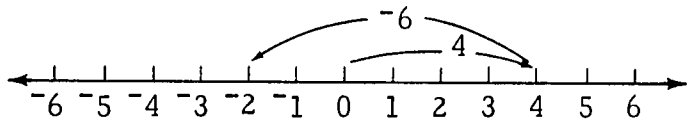
On the number line this sentence is shown by moving from 0 to $^{-}9$ and then moving to the right 5 units. (Remember that we always add a counting number on the number line by moving to the right.) The final position is at $^{-}4$.



As another example, think of winning 4 points in a game (4) and then losing 6 points ($^{-}6$). The result is that we are in debt 2 points ($^{-}2$). The sentence for this is:

$$4 + {}^{-}6 = {}^{-}2$$

This is shown on the number line by starting at 0, moving 4 units to the right, and then 6 units to the left. The final position is at $^{-}2$.



We show the addition of integers on the number line by starting at 0 and moving to the right for positive integers and moving to the left for negative integers. The integer for the final position is the sum.

In this unit only an introduction to integers is made. Later in Entebbe Mathematics these ideas are included: addition of integers; the properties of the addition of integers; and the multiplication of integers.

STAGE 1: Extending the Number Line

Vocabulary: Negative, positive, opposite integer, even integer, odd integer

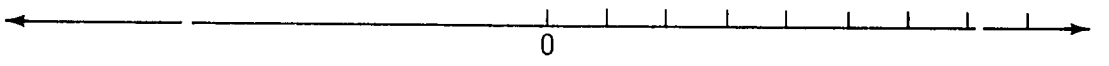
Materials: Pupil pages 169-173

Teaching Procedure

ACTIVITY 1: Naming Negative Numbers **Pupil page 169**

In this activity the children learn to assign numbers to the points on the number line to the left of the 0-point. Make it a discovery activity. Do not tell how the points are named. The children decide for themselves during the activity.

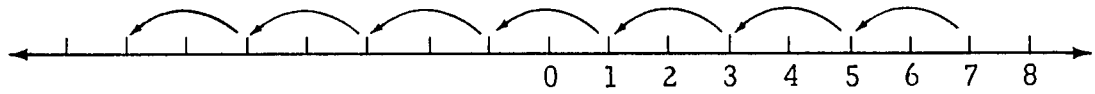
Draw a long line segment on the blackboard and mark the 0-point about the <u>middle</u> of the line. Mark equally spaced points to the right of the 0-point as shown:



Get the pupils to label the points. 1, 2, 3, 4, ... and so on.

Quickly revise such things as skip counting forwards and backwards from a point. For example, ask a pupil to begin at 0 and skip count to the right by twos. Then ask another pupil to begin at 7 and skip count by twos to the left. As he does this get him to touch each point and say, "Seven, five, three, one".

Ask, "Can you keep going to the left?" Let the class discuss this. Ask if there are points on the left of the 0-point. (Yes, there are many points.) Let them mark some points on the left end of the number line, using the same spacing as between the points to the right of 0. Let the pupil at the blackboard show the points he comes to as he continues to jump 2 spaces to the left.



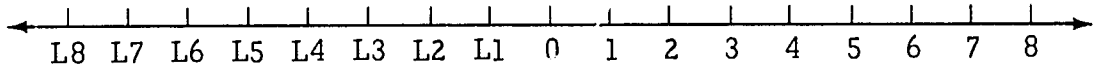
Say, "We assigned numbers to the points on the right side of the 0-point. What do we call these numbers?" (The counting numbers) "Do you think we can assign numbers to the points on the left side?"

Get many suggestions from the pupils. If a child suggests that we also assign the counting numbers to the points on the left, let the class talk about this. They will decide that there must be only one point for each number and that they cannot use the same counting number for a point on the right and another point on the left. As they discuss the problem, they will agree that they need new numbers for the points on the left of the 0-point.

Place your finger on the point one unit to the left of the 0-point and ask, "What number shall we assign to this point?" Help by asking questions like these:

- How far is this point from the 0-point? (1 unit)
- Must the name of the number tell us how far the point is from the 0-point? (Yes)
- Is the point on the left of the 0-point? (Yes)
- Must the name of the number tell us that the point is on the left? (Yes)
- What name can we give this point so the name tells us these two things?

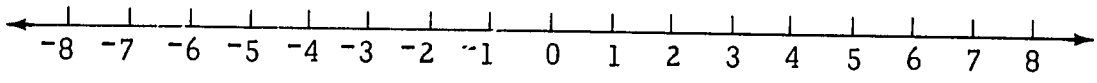
Let the children make suggestions. One pupil may say, "Call it 'left one'". Say, "Very good. How shall we write that on the number line?" He might say, "Write L1 to mean left 1", or "Write $\bar{1}$". Let the pupil that suggests a way, use his new kind of name to label the points on the left on the number line. For example:



Whatever kind of names the pupils give these numbers make sure the names show:

1. how far the points are from the 0-point, and
2. that the points are on the left of the 0-point.

When this is done, explain that in mathematics we use a dash before the numeral to show that it stands for one of our new numbers. Write numerals like these on the number line. Explain that $^{-}1$ is read "negative one", $^{-}2$ is read "negative two", and so on. Say, "These new numbers are called **negative numbers**". Now the number line should look like this:



Again question the children to be sure they understand that:

- our new numbers are for points on the left of the 0-point;
- the new numbers are called negative numbers;
- the names we give negative numbers have dashes to show that the points are to the left of the 0-point;
- the names for negative numbers also show how far the points are to the left of the 0-point.

Ask the children to open their books to pupil page 169 and look at the first picture of the number line. Say, "Tell where to find the point for $^{-}7$. (7 units to the left of 0.) Where is the point for 4?" (4 units to the right of 0.) Let the pupils copy the number line and write names for the other points marked on it.

The other exercises on page 169 can be done by the pupils in their exercise books or at the blackboard. Some exercises require skip counting.

ACTIVITY 2: Naming Integers **Pupil page 170**

Draw the number line on the blackboard. Mark the 0-point. Ask the pupils to name points on the right and points on the left of the 0-point. Revise terms such as "negative six", "negative four", and numerals such as $^{-}6$, $^{-}4$, and so on. Revise skip counting to the left and to the right from a given point.

Say, "All these numbers for the points marked on the number line are called integers". Get the children to use the word "integer" by saying such things as, "$^{-}7$ is an integer; 9 is an integer; or $^{-}2$ is an integer; 0 is an integer; 5 is an integer; and $^{-}83$ is an integer".

Let the number line remain on the blackboard. Ask the pupils to turn to page 170 in their books and look at the first ten exercises. Ask them in turn to read the questions aloud and answer them. At first they will need to look at the number line on the blackboard to help them.

When these exercises are completed, ask the class to tell about problems in which integers can be used. Guide them to think of gains and losses, feet above sea level and feet below sea level, floors above ground level and floors below ground level, having money and owing money, miles north of the equator and miles south of the equator, points won and points lost in a game, and so on. In each example, let the pupils tell which condition is shown by a negative number. (Losses are shown by negative numbers and gains by counting numbers; feet below sea level are shown by negative numbers; and so on.)

Let the class look at Exercises 11 to 20 on pupil page 170 and decide what integers are used.

Answers for pupil page 170:

11. $^{-}7$;	12. $^{-}30$;	13. 5;	14. $^{-}3$;	15. 12;
16. $^{-}15$;	17. 17;	18. 7;	19. $^{-}16$;	20. 105.

ACTIVITY 3: Opposites of Integers **Pupil pages 171-172**

Help the children draw the number line on the blackboard with points named from $^-8$ to 8. Place your finger on a point, say the 6-point. Ask, "How far is this point from the 0-point? (6 units) Is there another point on the line the same distance from the 0-point? (The $^-6$-point). Is the $^-6$-point on the same side of 0 as the 6^-point?" (No, it is on the opposite side of the 0-point.)

Repeat this series of questions for other points, such as the $^-3$-point, the 8-point, the $^-4$-point. Say, "We call $^-6$ the opposite of 6 because the $^-6$-point is on the opposite side of 0 from the 6-point and the same distance from 0". Ask, "What is the opposite of 4? ($^-4$) What is the opposite of $^-5$?" (5)

Let the children give the opposites of many integers. Then ask, "Does each integer have an opposite?" Children will probably say, "Yes". When they do, ask, "Does 0 have an opposite?" Let the class discuss this question then you explain that we call 0 the opposite of 0 and then every integer has an opposite.

For more experience in finding opposites, let the class turn to page 171 in their books and work the exercises.

When the exercises are completed, revise the idea of integers. Let the pupils talk about the kinds of integers: the negative integers are numbers for the points to the left of the 0-point; the counting numbers are numbers for the points to the right of the 0-point. Explain that we can call the counting numbers the positive integers. This word tells us that the counting numbers are integers for the points on the right of 0.

Pupil page 172 helps the children revise all the ideas learned in the first three activities.

ACTIVITY 4: Even and Odd Integers **Pupil page 173**

Draw on the blackboard the number line marked from $^-12$ to 12. Ask a pupil to point to and name the even counting numbers. (He skip counts and names 2, 4, 6, 8, 10, and 12.)

Say, "He has named the even counting numbers. Has he named the even positive integers?" (Yes. The positive integers are the counting numbers.) Write on the blackboard:

$\{2, 4, 6, 8, 10, 12, \ldots\}$: Even positive integers

Ask another pupil to name the opposite of each number in this set. He will say, "$^{-}2, {}^{-}4, {}^{-}6, {}^{-}8$, and so on". Write this set on the blackboard and ask, "What should we call this set?" Guide the children to call this the set of even negative integers. Write:

$\{{}^{-}2, {}^{-}4, {}^{-}6, {}^{-}8, {}^{-}10, {}^{-}12, \ldots\}$: Even negative integers

Ask, "Is $^{-}6$ an even integer?" (Yes) Is 8 an even integer? (Yes) Is 0 an even integer?" Let the class discuss this and decide that 0 is an even whole number and thus is an even integer. Say as you write, "This is the set of all even integers".

$\{\ldots {}^{-}8, {}^{-}6, {}^{-}4, {}^{-}2, 0, 2, 4, 6, 8, \ldots\}$: Even integers

Repeat this activity for odd integers. First let a pupil list all odd counting numbers and identify them as the set of odd positive integers. Then let another pupil list the opposites of the odd positive integers and identify them as the set of odd negative integers. Finally, let a pupil list the whole set of odd integers.

Finish the activity by asking the pupils to work the exercises on pupil page 173.

STAGE 2: Adding Integers

Vocabulary: No new vocabulary

Materials: Pupil pages 174-179

Teaching Procedure

ACTIVITY 1: Adding Gains and Losses Pupil pages 174-175

Ask the pupils to turn to pupil pages 174 and 175 and in turn read a few of the story problems and talk about them. Help the children to understand what questions are asked. Each time a pupil gives an answer, let him explain how he got it. Then ask him to give numbers for parts of the problem. Record the answers on the blackboard. For example, in Exercise 1 on page 174 the answers are:

Ike <u>owes</u> 9 shillings.	$^{-}9$
Ike <u>paid</u> 6 shillings.	6
Then Ike still <u>owes</u> 3 shillings.	$^{-}3$

After Exercises 1 to 5 are completed, say, "In each of these problems some numbers are added". Go back to the work shown on the blackboard and write in the addition sentence as shown below for Exercise 1:

Ike owes 9 shillings.	-9	
Ike paid 6 shillings.	6	$^{-}9 + 6 = {}^{-}3$
Then Ike still owes 3 shillings.	$^{-}3$	

For each of the exercises on page 175 let the pupils write in their exercise books a sentence that tells the answer to the problem.

<u>Answers for pupil page 174:</u>

1. $^{-}9 + 6 = {}^{-}3$
   Ike owes 3 shillings.
2. $^{-}5 + 8 = 3$
   Nume is 3 miles north.
3. $18 + {}^{-}22 = {}^{-}4$
   Fatu's father had a loss of 4 leones.
4. $^{-}6 + {}^{-}8 = {}^{-}14$
   Makulu owes 14 shillings.
5. $4 + {}^{-}6 = {}^{-}2$
   Ada was 2 floors below ground level.

<u>Answers for pupil page 175:</u>

1. $7 + {}^{-}4 = 3$	2. $^{-}5 + {}^{-}3 = {}^{-}8$	3. $^{-}4 + 10 = 6$
4. $7 + 5 = 12$	5. $^{-}5 + 0 = {}^{-}5$	6. $8 + {}^{-}8 = 0$

7. Apia had the most points; Odum had the lowest score.

Question the pupils about the sentences. Make sure that they understand how to use an integer for each part of a problem.

ACTIVITY 2: Adding Integers on the Number Line — Pupil pages 174-177

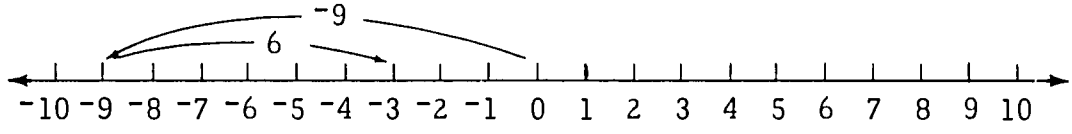
Revise the exercises on pupil page 174 as follows. Help the pupils draw

the number line on the blackboard marked from $^-10$ to 10. Then ask a pupil to read Exercise 1 again about Ike's money. Let a pupil give the sentence he wrote for the problem. (Write "$^-9 + 6 = {}^-3$" below the number line.)

Ask another pupil to show on the number line what each number in the sentence means. Help by asking questions like these:

- How do we get from 0 to $^-9$? (Go left 9 spaces from 0.)
- How many spaces does the next number, 6, tell us to go? (6 spaces) To the right or to the left? (To the right)
- If we go from $^-9$ a distance of 6 spaces to the right, at what point are we? ($^-3$)

As the pupils answer these questions, draw arrows on the number line like this:



Then let a pupil tell the answer to the story problem. (Ike still owes 3 shillings.)

Repeat this activity for some of Exercises 2, 4, and 5 on page 174 and 1 to 6 on page 175. Each time let a pupil start from the 0-point and draw arrows to show the sentence for the problem. Keep questioning the pupils until they see that we move to the left for a negative integer and the right for a positive integer. The sum is given by the numeral at the end of the last arrow.

Ask the pupils to turn to page 176 in their books and read about the gecko climbing the pole. Each time a pupil answers a question, ask him to explain how the gecko moved on the pole and ask him to write a sentence which tells the answer.

<u>Answers for pupil page 176:</u>

1. $3 + {}^-5 = {}^-2$
2. $^-5 + {}^-8 = {}^-13$
3. $4 + 3 = 7$
4. $10 + {}^-8 = 2$
5. $^-11 + 6 = {}^-5$
6. $^-10 + 10 = 0$
7. $^-12 + 20 = 8$
8. $11 + -11 = 0$
9. $(10 + {}^-18) + 9 = 1$
10. $(^-7 + 4) + {}^-3 = {}^-6$
11. $(2 + 0) + {}^-9 = {}^-7$
12. $(^-1 + 11) + {}^-10 = 0$

On pupil page 177 the children get more experience in writing addition sentences by following the arrows shown on the number line (Exercises 1 to 5) and from drawing arrows on number lines to show addition sentences (Exercises 6 to 15).

Answers for pupil page 177, Exercises 1-5:

1. $^{-}3 + 5 = 2$
2. $4 + {}^{-}8 = {}^{-}4$
3. $-6 + 4 = {}^{-}2$
4. $^{-}10 + 16 = 6$
5. $9 + {}^{-}21 = {}^{-}12$.

ACTIVITY 3: Adding Integers | Pupil pages 178-179

Revise the naming of integers and adding of integers on the number line. Then write these exercises on the blackboard:

$^{-}8 + 5$ $\qquad$ $^{-}7 + {}^{-}4$ $\qquad$ $6 + {}^{-}3$

Let the pupils find the sums in any way they can. They may find $^{-}8 + 5$, for example, by thinking of a debt of 8 shillings and a payment of 5 shillings, which gives a debt of 3 shillings remaining, that is:

$$^{-}8 + 5 = {}^{-}3$$

Or they may use the number line and draw arrows from 0 to $^{-}8$ and then from $^{-}8$ five units to the right, ending at $^{-}3$.

Allow the pupils to find the sum of two integers in any of these ways but encourage them to give the sum without using the number line or gains and losses, if they can.

Use pupil pages 178 and 179 for more experience in adding integers if the pupils need them. Notice that on pupil page 179 Exercises 1 to 10 prepare the way for later work in finding missing addends and in recognizing the associative and commutative properties of addition of integers.

UNIT 15

Measurement of Plane Regions

Objectives

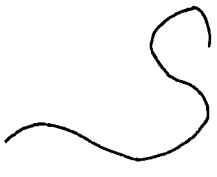
1. To learn the idea of a plane and simple closed curves on a plane
2. To describe plane regions
3. To revise tiling of rectangular regions and measuring of regions with square-unit regions, and extend to use of square-inch unit regions
4. To develop a formula for finding area of a rectangular region
5. To approximate the measure of a right-angled triangular region by tiling with square-inch tiles and with square-centimetre tiles, and to estimate areas of other regions

Background Information for Teachers

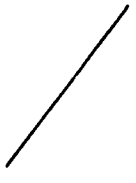
See Entebbe Mathematics Primary Three, Unit 8, pages 190-196.

Plane Regions A plane curve is any figure on a plane or flat surface that can be shown by drawing a path with a pencil without lifting the pencil. It is the kind of figure shown by a piece of string lying in any position on a sheet of paper. Some curves are line segments, some curves are angles, some are triangles, and so on.

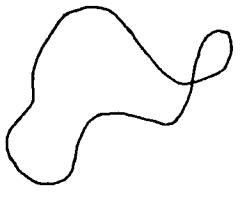
Some plane curves are closed curves; that is, they can be shown by a string that has its two ends tied together.



Curve



Curve that is a line segment

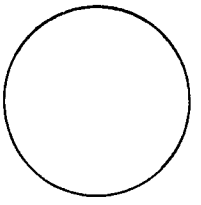


Closed curve

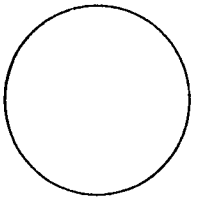
Simple closed curve

Some closed curves are called simple closed curves because they are closed and do not cross themselves. A figure eight ∞ is a closed curve that is not simple. A circle, a rectangle, and a triangle are examples of simple closed curves.

Simple closed curves form plane regions. A simple closed curve together with the part of the plane inside the curve is a <u>plane region</u>. For example, a circular region is the part of the plane on the circle together with the part inside the circle:

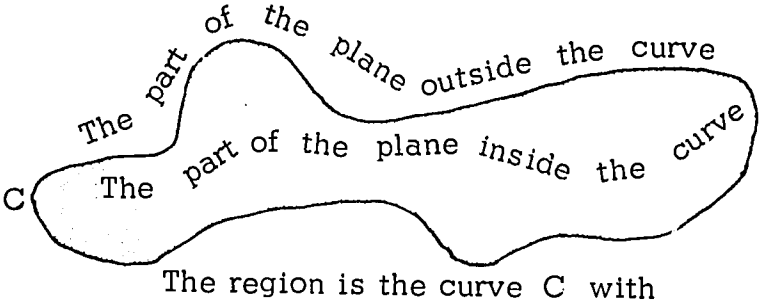


Circle



Circular region

Every simple closed curve cuts off the portion of the plane inside the curve. The rest of the plane is outside the curve.



The region is the curve C with the part of the plane inside C

Any two points inside the simple closed curve can be connected with a curve that does not cross C. But to connect an inside point to an outside point we must cross C.

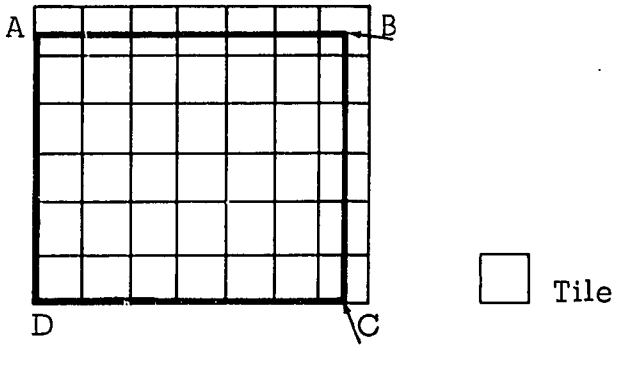
<u>Measurement</u> When we make a measurement of something we make a comparison of it with a small unit of the same kind. Measurement is a process of assigning a number to a quantity which tells how many fixed units make up the quantity. A measurement consists of two parts: a number, called the <u>measure</u>, and a <u>unit</u> of measure.

We measure a segment by comparing it with a unit segment. We measure an angle by comparing it with a unit angle. We measure a plane region by comparing it with a unit plane region.

The <u>area</u> of a closed figure is the measurement of its plane region. For example, the area of a rectangle is the measurement of the region enclosed by the rectangle. The formula for the area of a rectangle is developed in this Unit.

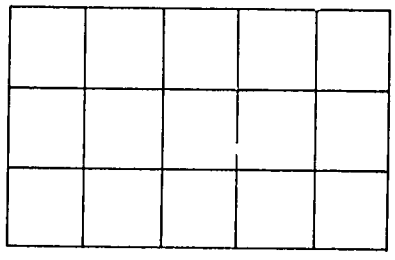
<u>Areas</u> If two regions are congruent they are the same size. When we look at two regions we sometimes can tell whether one region is larger than the other.

For example, rectangular region A is larger than rectangular region B, because we can fit a copy of B entirely inside A. But our eyes alone cannot always tell which of two regions is larger and we need to find the size of the regions. To find the size of a region, we compare it with a unit region. A square is a convenient region to use as the unit of measure. A square unit of measure is sometimes called a <u>tile</u>. We arrange unit tiles (all congruent to each other) in <u>arrays</u> to cover the region and then count the number of tiles used. In this way we tell approximately the measure of the region in tiles. For example, in the figure below the size of region ABCD is greater than 30 tiles and is less than 42 tiles.



The larger array has 6 rows of 7 tiles each. ($6 \times 7 = 42$) Forty-two tiles are needed to cover ABCD. The smaller array has 5 rows of 6 tiles each. ($5 \times 6 = 30$) Thirty tiles can fit inside ABCD.

Some <u>standard units</u> for measuring regions are <u>square inch</u> (a square region with sides 1 inch long), <u>square centimetre</u> (a square region with sides 1 centimetre long), <u>square foot</u>, <u>square yard</u>, and <u>square mile</u>. Small regions are measured best with small units, and large regions with large units. For example, we would probably use square centimetres to measure the region shown by a rectangular postage stamp. But we would probably use square yards to measure the region shown by a football field, and square miles to measure the region occupied by a country. When we measure a line segment we find the length of the segment; when we measure a plane region we find the area of the region. Thus, the word <u>area</u> means the measurement of a plane region. For example, the area of the rectangle shown below is 15 square centimetres. (15 sq. cm.)



This means that the unit is a square centimetre and that 15 units cover the region of the rectangle.

There is no need actually to place tiles on a rectangular region to find its area. We can visualize the array of tiles needed. We measure the lengths of the sides of the rectangle to tell how many rows of tiles are needed and how many tiles are in each row. Then, without making the array, we multiply to find the number of tiles needed.

We use a sentence called a <u>formula</u> to find areas of all rectangles. If <u>l</u> and <u>w</u> are names for the measures of the sides of the rectangle (in a unit of length) and <u>A</u> is the name for the measure of the area (in square units), then:

$$A = l \times w.$$

Other regions are measured approximately by fitting tiles over the region. In this way we decide that, for example, a certain triangular region is larger than 10 sq. in. and smaller than 15 sq. in.

STAGE 1: Simple Closed Curves and Plane Regions

Vocabulary: Flat surface, plane, closed curve, simple closed curve, region

Materials: Piece of string about 3" long for each child; pupil pages 180-181

Teaching Procedure

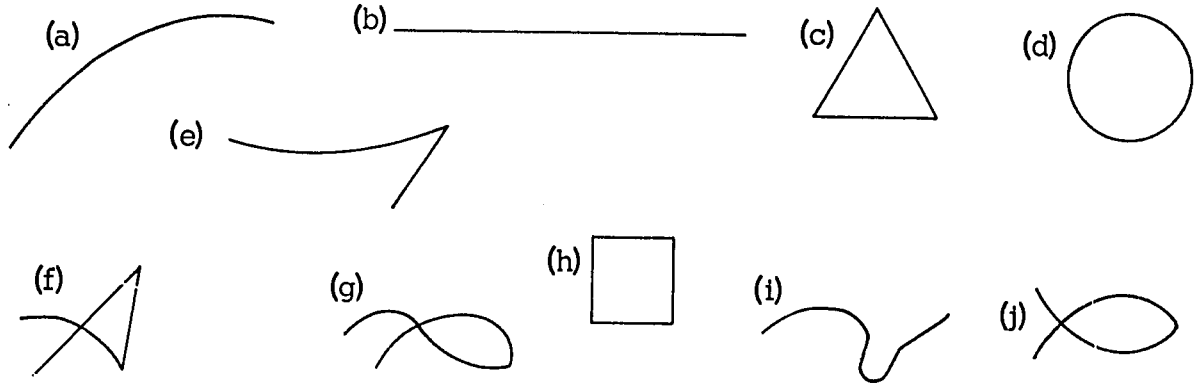
ACTIVITY 1: Curves on Planes | Pupil page 180

Ask pupils to point out flat surfaces in the classroom. (Table top, sheet of paper, surface of the blackboard, floor) Let the children move their hands over each surface as they say, "This shows a flat surface".

Move your hand over one of the flat surfaces (for example, the side of a box) and say, "This flat surface is part of a plane. The plane goes on and on without end". Show with your hands that the plane extends beyond the side of the box in all directions. Let children give other examples and say, "This flat surface is part of a plane. The plane goes on and on without end".

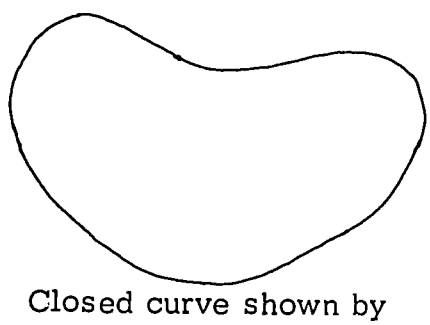
Say, "Show some surfaces that are not flat". (Surface of a ball, side of a round tin, side of a tea cup) Let the children move their hands over each surface and say, "This is not flat. It is not part of a plane!"

Give each pupil a sheet of paper and a string about 4 inches long. Get them to say, "This sheet of paper is a flat surface. It is part of a plane". Ask them to arrange the string on the sheet of paper and trace along the string. Get them to arrange the string many other ways and trace along the string. Some tracings they may make are shown at the top of page 254.

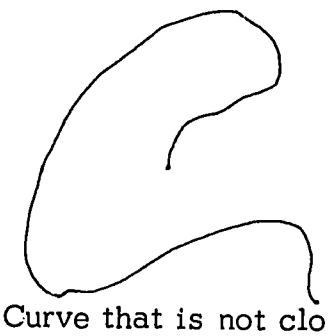


Say, "You have made pictures of curves. We can draw pictures of curves on planes". Let the children point to a curve and say, "This is a curve". For some pictures they can say, "This curve is a circle", or "This curve is a line segment", or "This curve is a triangle". You may wish to draw some curves like some of those shown. Guide the children to say, "These are curves. These curves are on a plane".

Move around the room to observe the figures drawn by the children. As you move, point to figures like (c), (d), and (h) and say for each, "This is a closed curve". Point to figures like (a), (b), (e), (f), (g), (i), and (j) and say for each, "This is not a closed curve". Let the children, in turn, run their finger along the curves they draw and say, "This is a closed curve", or "This is not a closed curve". Let them show with their string other figures which are closed and still others which are not closed.

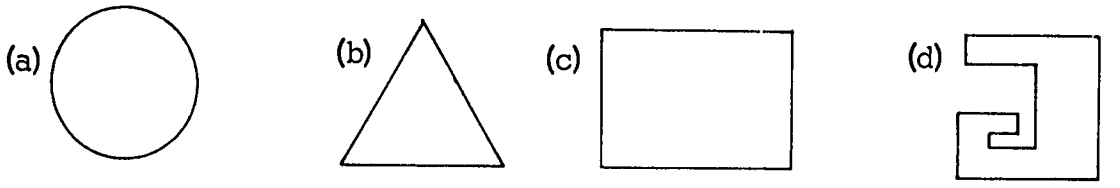


Closed curve shown by string

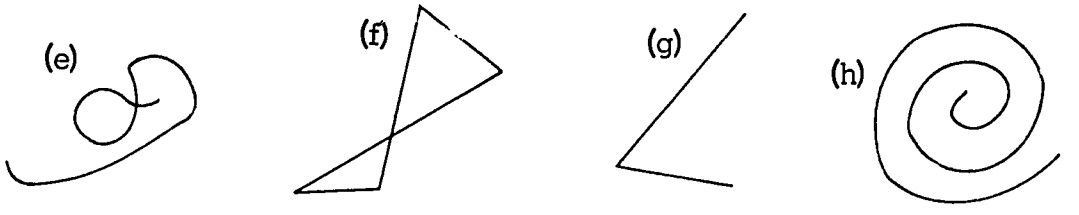


Curve that is not closed shown by string

Tell pupils that a closed curve which does not cross itself is called a simple closed curve. Ask pupils to show some examples of simple curves and some examples of curves that are not simple closed curves.



Examples of simple closed curves



Examples of curves that are not simple closed curves

Ask whether the picture of a figure (f) shows a simple closed curve. (No. It intersects itself.) Get the children to do the exercises about curves on pupil page 180.

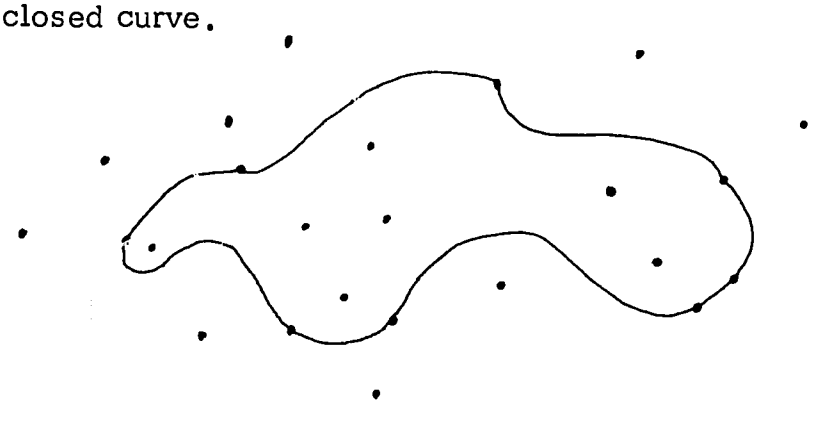
<u>Answers for pupil page 180</u>:

2), 4), 8), 9) show simple closed curves.

12), 13), 15), 17), 18) do not show simple closed curves.

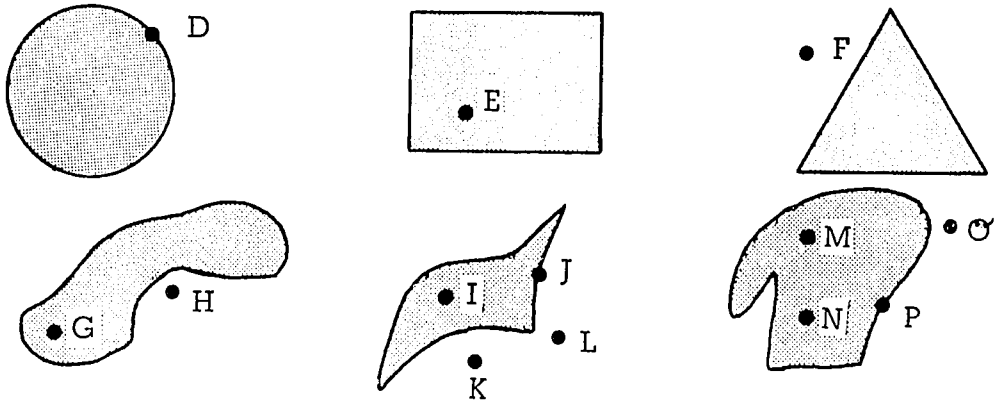
ACTIVITY 2: Plane Regions **Pupil page 181**

Run your hand over the surface of the blackboard. Guide the children to say that the flat surface of the blackboard is part of a plane. Get a child to draw a closed curve on this plane similar to the one below. Ask children in turn to mark points outside the closed curve, points inside the closed curve and points on the closed curve.



Guide the children to say as they locate points, "There are many points outside the closed curve. There are many points on the curve. There are many points inside the curve. We could go on and on marking points".

Explain that a simple closed curve together with the part of a plane inside the curve is called a region. Let the pupils in turn draw simple closed curves on the blackboard. Let them shade the regions of the curves.



Mark points similar to those shown. Let the children in turn tell whether the points are inside, on, or outside the curve. Get them to identify a circular region, a triangular region, and a rectangular region.

Ask whether each of the points D to P is a part of the region shown. For example, D is on the circle and is a point of the circular region, F is outside the triangle and is not a point of the triangular region. Let the pupils turn to pupil page 181 and answer the questions about simple closed curves.

STAGE 2: Revising Tiling of Regions

Vocabulary: Tile, measure, unit, area, square inch, square centimetre

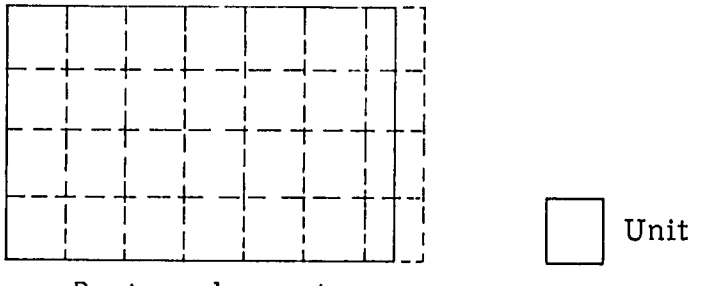
Materials: Ten inch-square cutouts for each group of pupils; about 25 square-centimetre cutouts; rectangular (4 in. by 6 in.) cutouts; rectangular (3 cm. by 7 cm.) cutouts; square (25 cm. by 25 cm.) cutouts; pupil pages 182-185

Teaching Procedure

ACTIVITY 1: Measuring a Region by Tiling **Pupil pages 182-183**

Revise measurement of line segments briefly. Let a child measure a line segment drawn on the blackboard, using inch units. Ask him to explain what unit of measure he uses and what the measure is. For example, if the length is 11 inches, the unit is an inch and the measure is 11.

Draw and shade a rectangular region on the blackboard. Ask, "If you measure this plane region what unit do you use?" If the class does not remember this from Primary Three, let them suggest units. They should remember that we use units of the same kind as the thing we are measuring. Continue with questions until they decide that a small square region is the unit. Carefully draw a small square region beside the rectangular region on the blackboard. As the children explain how to measure the rectangular region with the unit square, draw unit squares on the region until there are as many units inside the figure as possible. Let the class count the units and say, for example, "This rectangular region is larger than 24 units". Then draw more unit squares until the region is completely covered. (See below.) Let the class say, "The rectangular region is smaller than 28 units".



Rectangular region

Ask the class to open their books to pupil page 182. Give each group of pupils 10 inch-square cutouts or get the children to make them. Ask, "What do the cutouts show? (Square regions) Are all your cutouts congruent?" (Yes, because one fits exactly on each of the others.) Ask the pupils to measure the

sides of the cutouts. They will find that the sides are one inch long. Say, "The cutouts show square inches. We will use a square inch as a unit to measure regions. Which region shown on page 182 is the largest?" Some pupils will guess, some will suggest that they can tell by measuring the regions. Let each group lay the square-inch units or inch tiles on the regions and tell the measures of the regions. In their exercise books they will write, for example:

The size of rectangular region A is 10 square inches;

The size of rectangular region B is 6 square inches.

Let someone tell which region is largest. (Region A, because its measure is greater than the measures of the other regions.)

On pupil page 183, there are regions shown which are not rectangular. Ask the children to use their square-inch tiles to decide how many tiles can be fitted entirely inside the figures.

(6 in G; 3 in H; 1 in J; 4 in K; 5 in L; 2 in M)

Ask how many tiles are needed to cover each region completely?

(8 cover G; 6 cover H; 4 cover J; 6 cover K; 11 cover L; 6 cover M)

Let the pupils write in their exercise books the results, for example:

Region G is larger than 6 square inches and smaller than 8 square inches;

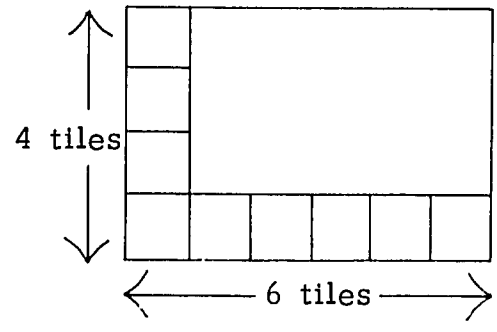
Region H is larger than 3 square inches and smaller than 6 square inches.

ACTIVITY 2: Square-Inch Units and Square-Centimetre Units

Pupil pages 184-185

Revise the idea of measuring a rectangular region with square-inch units. Show a rectangular cutout 4 inches by 6 inches and let a pupil measure this region with square-inch tiles. As he lays the tiles on the rectangular cutout, ask the class questions such as:

- How many rows of square-inch tiles does he need? (4)
- How many square-inch tiles in each row? (6)
- How can you find the number of square-inch tiles he used? (Accept any answer such as: Count by sixes, 6, 12, 18, 24; Add, 6 + 6 + 6 + 6; Multiply, 4 × 6)
- Does he need to put all the tiles on the region to find how many tiles he needs? (Guide the children to decide that he needs only the tiles along the edges to show how many rows and how many in each row.)



- Does he need to lay tiles on the region to find how many tiles he needs? Can he find the number another way? (Guide the children to decide that he could measure the two sides of the rectangle in inches in order to find how many rows of square inches and how many square inches in each row.)

Explain that we use the word area for the measurement of a region. Say, "The area of this rectangle (hold up the cutout that was measured) is 24 square inches".

Note: The word area is used for the measurement of a plane region. It is correct to say, for example, that the area of the rectangular region is 24 square inches. It is not quite correct to say "the area of the rectangle", since area is a measurement of a region. But this is such a common expression that we will use it without misunderstanding. Thus, "area of the rectangle" will mean "area of the rectangular region".

Ask, "What small units did we use to measure line segments?" (Inches and centimetres) Explain that people often use a smaller square unit than a

square inch. They sometimes use a square centimetre. Show the class a square-centimetre tile and let a pupil measure its sides with a centimetre ruler. He will say, "Its sides are each 1 centimetre long". Guide the class to say, "It is a square centimetre".

Hold up a rectangular cutout (3 cm. by 7 cm.) and ask the class how to find the area of this region in square centimetres. Let the class watch as you give a pupil some square-centimetre tiles and he lays these tiles on the region. When he does this, again ask questions similar to those asked about the 4 in. by 6 in. cutout.

Guide the class to decide, "We can find the area by finding the number of rows of tiles needed (3) and the number of tiles needed in each row (7). The area of this rectangle is 21 square centimetres".

On pupil page 184 let the pupils use their rulers to make sure what size units are shown in the figures (square centimetre). Then ask the children to answer the questions. Explain that they should imagine that square-centimetre tiles are laid on the regions (as shown by the crossed lines). In Exercises 4 to 6 the children imagine extra tiles that are needed to cover the regions. (Region C has an area greater than 24 sq. cm. and less than 30 sq. cm. Region D has an area greater than 25 sq. cm. and less than 36 sq. cm.) The number of square centimetres is found either by counting tiles or by multiplying the number of rows by the number of tiles in each row.

On pupil page 185 are more exercises in finding areas of rectangular regions. In Exercises 4, 5, and 6 the children will find that multiplication is the easiest way to find the area, because parts of the regions are covered. Enough of each region is shown to give the number of rows and the number of units in each row.

Answers for pupil page 185:

1. 20 square units	2. 18 square units	3. 35 square units
4. 70 square units	5. 48 square units	6. 39 square units

STAGE 3: Finding Formula for Area of a Rectangle

Vocabulary: Length of rectangle, width of rectangle

Materials: 20 square-inch tiles, yard stick, pupil pages 186-187

Teaching Procedure

ACTIVITY 1: Finding Area of a Rectangle by Measuring its Sides

Pupil page 186

Revise the method of finding the area of a rectangle used in the previous activity. Let a pupil lay square-inch tiles on a surface of his Pupil Book to find its area. Ask him, "Do you need to cover the whole book with inch squares to find its area?" (No, I need to find only the number of rows of squares and the number of square inches in each row.) Let him tell what the array of tiles is like that would cover the surface of the book:

9 rows of square inches, with

11 square inches in each row.

Say, "How can you find the area of the surface of the book? ($9 \times 11 = 99$) "The area of the book is less than 99 square inches". Let the class discuss how the area was found. Let them decide that to find the area of a rectangle they multiply two numbers. The first number is the number of rows of squares placed on the region. The second number is the number of squares in each row.

Show the children another rectangular surface, such as the top of a crayon box. Ask how they can find the area of this rectangle in square centimetres without laying square cutouts on the region. Question the class until someone suggests that he can measure the sides of the rectangle with centimetre units and tell how many square centimetres are needed. Let him do this, and get him to tell:

the number of rows of square centimetres, and

the number of square centimetres in each row.

Let others tell how to find the area in square centimetres by multiplying these two numbers. Finally, guide them to say, "The area of the rectangle is ____ square centimetres".

On pupil page 186 are several rectangles. Get the children to find the areas. For each rectangle let the pupils use their rulers to measure two sides of the rectangles in centimetres and then multiply the two measures to find the area in square centimetres.

As further activity, let pupils find the area of such things as a table top in square feet, the area of the classroom in square yards, the area of the top of a match box in square centimetres, the area of the blackboard in square feet, and so on. In each case, expect the children to find the measures of two sides, multiply the measures, and report the area in the form:

The area of the ____ is ____ square ____.

ACTIVITY 2: Formula for Area of a Rectangle

Pupil pages 187-188

Revise the method of finding the area of a rectangle by multiplication. Draw a rectangle on the blackboard with its sides 7 inches and 9 inches long. Ask a pupil to find the area of this rectangle and to explain to the class how he found the area. Guide him to explain that:

- he measures one side in inches to find how many rows of square inches are needed,
- he measures another side to find how many square inches are in each row,
- he multiplies these two numbers to find the number of square inches in the area.

Let him report, "The area of the rectangle is 7 × 9 or 63 square inches".

If needed, let pupils repeat this activity with other rectangles that you draw on the blackboard. Say, "We call the measurement of the longer side the length of the rectangle and we call the measurement of the shorter side the width of the rectangle".

Point to the rectangle drawn on the blackboard and ask, "What is the length of this rectangle? (9 inches) What is the width of this rectangle? (7 inches) How do we find the area of the rectangle? " (We multiply 7 and 9 to find the number of square inches in the area.)

Ask the class how we write a mathematical sentence for finding the area. "What letter should we use for the number of units in the length? (Suggest the letter l because it is the first letter of length.) What letter should we use for the number of units in the width? (Suggest the letter w.) What letter should we use for the number of square units in the area? (Suggest the letter A.) What is the mathematical sentence, using these letters?" Let the class discuss this and help them decide that the sentence is:

$$l \times w = A$$

Ask, "What does this sentence tell us?" (The number A of square units in the area of a rectangle is found by multiplying the number l of units in its length by the number w of units in its width.)

Repeat some measurements of rectangular objects from the previous activity, such as the floor of the classroom, the table top, and so on. Make a chart like this on the blackboard:

Rectangle	Unit	l	w	A	Area
Floor	foot	30	25	750	750 sq. ft.
Table top	foot	4	3	12	12 sq. ft.
Match-box top	centimetre	6	4	24	24 sq. cm.,

and so on.

Let the pupils use this method with the rectangles on pupil page 187. (Measure the sides of the rectangles and use the mathematical sentence, $l \times w = A$, to find the areas.)

Get the pupils to continue with pupil page 188 as an extra activity in finding areas. Each region can be separated into rectangular regions and the total area found by adding the separate areas of the rectangle.

STAGE 4: Area of Triangular Regions

Vocabulary: No new vocabulary

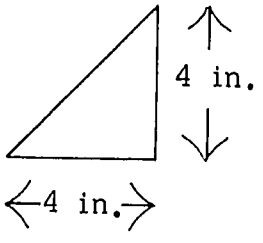
Materials: 12 square-inch tiles for each group of three or four pupils; 60 square-centimetre tiles; sheet of paper for each child to fold to show a right angle; circular cutout; pupil pages 189-192

Teaching Procedure

ACTIVITY 1: Finding the Area of a Right-Angled Triangle

Pupil pages 189-190

Help the children fold sheets of paper to show right angles. (See pages 47-48) Let the children use these to help them draw right-angled triangles in their exercise books. Ask the children to make the two sides forming the right angle of length 4 inches.



Say, "Show the triangle formed by tracing along the triangle with your fingers. Show the triangular region by moving your hands over the surface. We want to find the area of this region". Give each group of three or four pupils about 12 square-inch tiles. Say, "Find how many square-inch tiles you can put on the triangular region, begin from the right angle. (6) Is 6 square inches the area of the whole region? (No, the 6 tiles do not cover the whole region.) Is the area greater than or less than 6 square inches?" (Greater)

Get the pupils to cover all the region with as few tiles as possible. Ask, "How many tiles cover the region? (10) Is 10 square inches the area of the region?" (No. Some of the tiles extend outside the region.) Guide the children

to decide that the area is greater than 6 square inches and less than 10 square inches.

Repeat the activity using square-centimetre tiles on the same right-angled triangle. Give one group of children about 60 square-centimetre tiles. Guide the children to decide after placing the tiles on the region that the area is greater than 45 square centimetres and less than 55 square centimetres. Let another group check the work and agree on the area.

Ask, "Why are the two numbers for the area of this triangle different?" (We used different units to measure the region.)

Note: If any pupils say that they can find the area exactly by cutting some tiles in half, let them explain how to do this. Ask the pupils to explain how they know they are using half tiles. (By folding a cutout, they can see that the parts are congruent, so that each triangular part is half the area of a tile.) In this way it is found that the area of the region is 8 square inches.

Pupil pages 189 and 190 show two triangular regions. Let the pupils decide by counting squares what the areas of those regions are.

Answers for pupil page 189:

The area of the right-angled triangle CDE is between 10 sq. in. and 20 sq. in.

Answer for pupil page 190:

The area of the right-angled triangle KRS is between 78 sq. cm. and 102 sq. cm.

ACTIVITY 2: Areas of Other Regions **Pupil pages 191-192**

Revise the method of finding the area of a right-angled triangle. Then use the same method of finding the area of a circular region. Let a pupil place square-inch tiles on a circular cutout and report that the area of the region is between _____ sq. in. and _____ sq. in.

On pupil pages 191 and 192 there are two regions with dotted lines showing positions of tiles on the regions. Let the pupils count tiles to find the areas. They may find a shorter method by noticing a rectangle inside the

region. They can get the area of this rectangle by multiplication and then count the remaining tiles.

Answers for pupil page 191:

The area of the circular region is between 49 sq. cm. and 77 sq. cm.

Answers for pupil page 192:

The area of the region is between 14 sq. in. and 34 sq. in.

UNIT 16

Prime Numbers and Squares of Numbers

Objectives

1. To revise factors and to extend to prime factors
2. To classify counting numbers into prime numbers and numbers that are not prime
3. To factorise counting numbers into products of prime numbers
4. To introduce squares of numbers

Background Information for Teachers

In a sentence such as $12 = 3 \times 4$, 12 is called the product, and 4 and 3 are called factors of 12. We say that 12 is <u>factorised</u> when 12 is expressed as a product of factors. Other ways to factorise 12 are:

$$12 = 1 \times 12 \text{ and } 12 = 2 \times 6$$

In this unit we factorise counting numbers so we can identify <u>prime</u> numbers and <u>squares</u> of numbers.

<u>PRIME NUMBERS</u>

Every counting number can be factorised. Some can be factorised in several ways. The set of all counting numbers appearing in the factorisations of a given number is called the <u>set of factors</u> of the number. For example: since $12 = 1 \times 12$, $12 = 2 \times 6$, $12 = 3 \times 4$, the set of factors of 12 is:

$$\{1, 2, 3, 4, 6, 12\}$$

Another way to say this is: 12 is a multiple of each member of the set $\{1, 2, 3, 4, 6, 12\}$.

Let us look at the sets of factors of the first few counting numbers and try to find a pattern.

Counting Number		Set of Factors
1		1
2	P	1, 2
3	P	1, 3
4		1, 2, 4
5	P	1, 5
6		1, 2, 3, 6
7	P	1, 7
8		1, 2, 4, 8
9		1, 3, 9

Notice that the counting number one has only <u>one</u> factor. Some of the counting numbers, such as 2, 3, 5, 7, and so on, have <u>two</u> factors. The factors are 1 and the counting number itself. All the other counting numbers have <u>more than two</u> factors. These are 4, 6, 8, 9, and so on.

The counting numbers with two factors are of unusual interest in mathematics. We call them the <u>prime numbers</u>. (In the table above, the prime numbers are marked with P.) The numbers with more than two factors are called composite numbers, but in <u>Primary Four</u> we will simply say they are <u>not prime</u>.

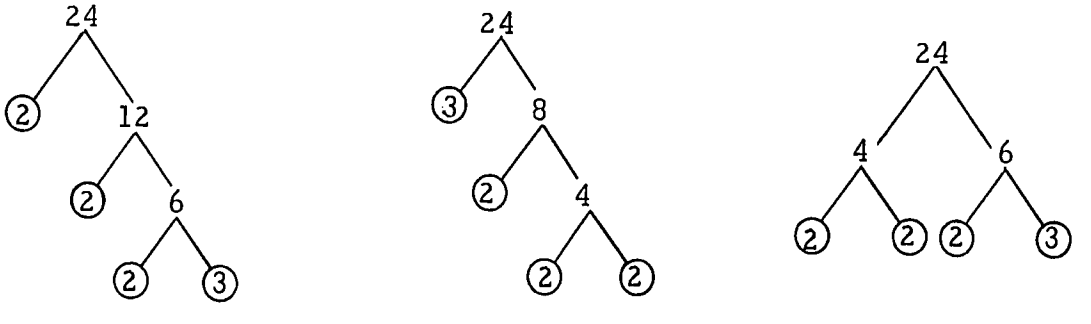
Why do we call certain numbers prime? We can find the answer by looking at some examples. Suppose we factorise 24 in every possible way and then continue to factorise each factor as far as possible.

$$24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3$$

$$24 = 3 \times 8 = 3 \times 2 \times 4 = 3 \times 2 \times 2 \times 2$$

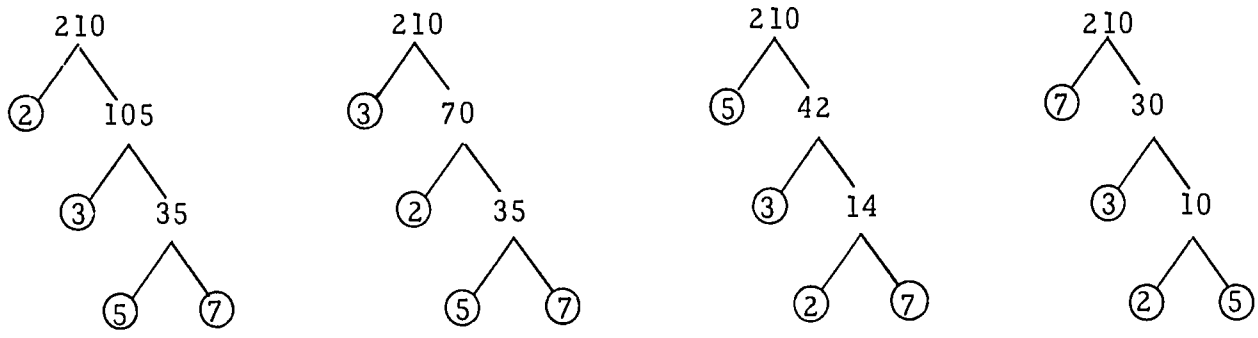
$$24 = 4 \times 6 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3$$

We show how each factor is further factorised by drawing arrows. Simpler diagrams, called "factor trees", can also be drawn to show the factorisation.



In every diagram we draw for the factorisation of 24, the final set of factors is the same set. They are the factors at the ends of the branches with rings around them. In the factorisation of 24 there are three 2's and one 3. Furthermore, every factor in this set is a <u>prime</u>.

Let us try another example:



$$210 = 2 \times 3 \times 5 \times 7$$

Again we find that every set of factors is the same, and the set again has all <u>primes</u> in it.

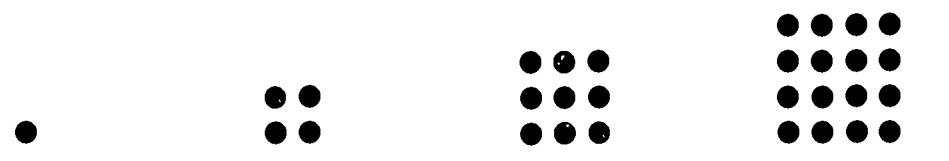
It is true that each counting number greater than 1 is either:

- a prime number, or
- it can be expressed in only one way as a product of prime numbers.

These examples show that the prime numbers are "prime" in their use in arithmetic. Prime factorisation of numbers is the key to many mathematical processes.

<u>SQUARES OF NUMBERS</u>

Some counting numbers can be expressed as the <u>square</u> of a number. For example: $1 = 1 \times 1$, $4 = 2 \times 2$, $9 = 3 \times 3$, $16 = 4 \times 4$, and so on. We say that 1 is the square of 1, 4 is the square of 2, 9 is the square of 3, and so on. Such numbers are called squares of numbers because they can be represented by a square array.



$1 \times 1 = 1$, $2 \times 2 = 4$, $3 \times 3 = 9$, $4 \times 4 = 16$, and so on.

Squares of numbers are shown by special symbols. We write:

2^2 for 2×2; 3^2 for 3×3; 4^2 for 4×4, and so on.

We read "2^2" as "two squared", and "3^2" as "three squared". Later these symbols are useful when we write:

$$281 = (2 \times 10^2) + (8 \times 10) + 1 \text{ and}$$

$$2{\cdot}81 \times 10^2 = 281, \text{ and so on.}$$

STAGE 1: Prime Numbers

Vocabulary: Prime number, numbers that are not prime, prime factor

Materials: Pupil pages 193-195

Teaching Procedure

ACTIVITY 1: Revision of Factors **Pupil page 193**

Write some multiplication sentences on the blackboard such as these:

$2 \times 3 = 6$ $\quad$ $3 \times 4 = 12$

$2 \times 4 = 8$ $\quad$ $3 \times 5 = 15$

Guide the children to tell that:

The 6, 8, 12, and 15 are products;

Since $2 \times 3 = 6$, 2 and 3 are factors of 6;

Since $3 \times 4 = 12$, 12 is a multiple of 3 and a multiple of 4.

Get the children to write on the blackboard all the multiplication sentences that show a product of 12. They will write these sentences:

$3 \times 4 = 12$ $\quad$ $2 \times 6 = 12$ $\quad$ $1 \times 12 = 12$

Ask, "What are the factors of 12?" Write on the blackboard:

The factors of 12 are 1, 2, 3, 4, 6, and 12.

Ask the pupils to write all the multiplication sentences with 6 as a product:

$$2 \times 3 = 6 \qquad 1 \times 6 = 6$$

Guide the children to name the factors of 6. Write on the blackboard:

The factors of 6 are 1, 2, 3, and 6.

Follow the same plan in naming the factors of 5. The children will find they can write only one sentence, $1 \times 5 = 5$; and that 5 has only two factors, 1 and 5. Write on the blackboard:

The factors of 5 are 1 and 5.

Let the children find the factors of 2, of 8, of 7, of 11, and so on. Draw a chart on the blackboard showing the results to at least 15.

Counting Number	Set of Factors
1	{ 1 }
2	{ 1, 2 }
3	{ 1, 3 }
4	{ 1, 2, 4 }
5	{ 1, 5 }
6	{ 1, 2, 3, 6 }
7	{ 1, 7 }
8	{ 1, 2, 4, 8 }

Let the children tell which number has only one factor (1), which numbers have exactly two factors (2, 3, 5, 7, 11, and so on), which numbers have more than two factors (4, 6, 8, 9, 10, 12, 14, 15, and so on). (If possible save the chart for use in Activity 2.)

For more revision of factors and multiples of numbers let the children turn to the pupil page 193 and work the exercises.

ACTIVITY 2: Prime Numbers **Pupil page 194**

Revise the work of Activity 1 by again guiding the pupils to think about the chart showing counting numbers and their sets of factors. If the chart from

Activity 1 was not saved get the children to prepare it again on the blackboard. Let them study it. Ask, "What do you notice about the sets of factors?" Guide them to say:

- Every counting number has 1 in its set of factors;
- Every counting number has itself as one of its factors;
- One counting number has one factor;
- Some counting numbers have only two factors;
- Some counting numbers have more than two factors.

Let a child tick each number that has only two factors. Say, "The counting numbers with only two factors are called prime numbers". (The numbers 2, 3, 5, 7, 11, 13, 17, and so on, are ticked.)

Let pupils write "prime" beside each numeral ticked on the chart. Ask for the next prime number after the numbers listed on the chart. Let pupils list more counting numbers until they come to the next prime. Get them to say that these are not prime numbers. Ask for the next prime number after that, and let the children tell what it is without listing more numbers that are not primes.

Let the children do the exercises on pupil page 194.

Answers for pupil page 194:

1) two 2) one 3) 1, 17 4) one 5) 23, 29, 31, 37 6) two
7) 2, 3 8) 42, 44, 45, 46, 48, 49 9) 1, 9, 15 10) 12, 14, 16, ..., 24

ACTIVITY 3: Finding Prime Factors Pupil page 196

Write this multiplication sentence on the blackboard:

$$12 = 3 \times 4$$

Ask, "What factors of 12 does this sentence show? (3, 4) Are the factors both prime numbers?" (No, 4 is not a prime.) Get the children to write a multiplication sentence which shows 12 as a product of prime numbers. Help them to name 4 as 2×2, so that the work looks like this:

$$12 = 3 \times 2 \times 2$$

Say, "This sentence names 12 as the product of its prime factors". Ask, "Can you think of 24 as the product of its prime factors?" Help the pupils write either:

$24 = 3 \times 8$
$24 = 3 \times 2 \times 4$
$24 = 3 \times 2 \times 2 \times 2$

or

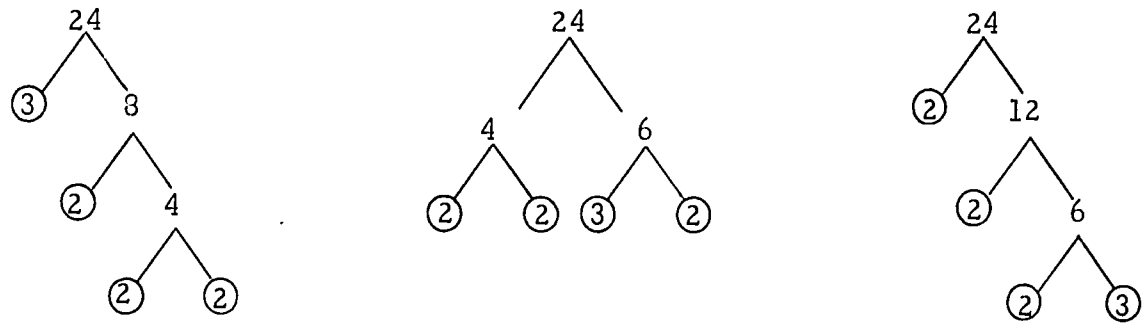
$24 = 4 \times 6$
$24 = 2 \times 2 \times 6$
$24 = 2 \times 2 \times 2 \times 3$

or

$24 = 2 \times 12$
$24 = 2 \times 2 \times 6$
$24 = 2 \times 2 \times 2 \times 3$

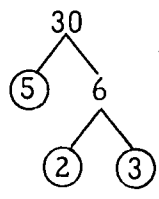
They will decide that any of these sentences shows that 24 can be expressed as a product of prime factors.

Show the pupils a "factor tree" diagram for finding the prime factors of 24:



Say, "We find two factors of each factor until we get prime factors. The prime factors are the numbers at the ends of the branches. $24 = 2 \times 2 \times 2 \times 3$".

Let the pupils try another example, such as finding prime factors of 30.



$30 = 5 \times 2 \times 3$

Ask, "Are all the factors at the ends of the branches prime factors?" (Yes, 5, 2, and 3 are prime numbers.)

After several more examples, get the pupils to work the exercises on pupil page 195.

Answers for pupil page 195:

1) $26 = 2 \times 13$ 2) $18 = 2 \times 3 \times 3$ 3) $36 = 2 \times 2 \times 3 \times 3$

4) 2, 3 5) 2, 5 6) $48 = 2 \times 2 \times 2 \times 2 \times 3$

7) $2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 = 77$ 8) No. 9 is not prime.

9) Yes. Both 3 and 17 are prime. 10) $3 + 7 = 10$, $3 \times 7 = 21$

11) $7 - 5 = 2$, $7 + 5 = 12$ 12) $13 + 7 = 20$, $13 - 7 = 6$

STAGE 2: Squares of Whole Numbers

Vocabulary: Square of a number

Materials: 25 counters for each group of pupils; numeral (1-50) cards; pupil pages 196-198

Teaching Procedure

ACTIVITY 1: Introduction to Squares of Numbers **Pupil page 196**

Help the children make on the blackboard a multiplication chart, like the one on page 275, with products of numbers from 1 to 6. Get them to count up four and across two. Ask, "What number do you find?" (8) Draw a ring around the numeral 8. Get the pupils to count up two and across four. Ask, "What number do you find? (8) Why did you get 8 both times?" (Because $4 \times 2 = 2 \times 4$.) Give several similar examples, such as five up and three across; and then three up and five across. Each time the children get the same product. Ask, "What property of multiplication does this show?" (The commutative property of multiplication.)

Finally, get the pupils to count the same number up as across, for example, five up and five across to 25 or two up and two across to 4. Get the children to draw boxes around the numerals for these numbers. Continue with one up and one across, three up and three across, and so on. The multiplication chart will look like this:

6	6	12	18	24	30	$\boxed{36}$
5	5	10	15	20	$\boxed{25}$	30
4	4	8	12	$\boxed{16}$	20	24
3	3	6	$\boxed{9}$	12	15	18
2	2	$\boxed{4}$	6	8	10	12
1	$\boxed{1}$	2	3	4	5	6
×	1	2	3	4	5	6

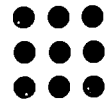
Write on the blackboard:

$1 \times 1 = 1$ $\quad$ $2 \times 2 = 4$ $\quad$ $3 \times 3 = 9$

$4 \times 4 = 16$ $\quad$ $5 \times 5 = 25$ $\quad$ $6 \times 6 = 36$

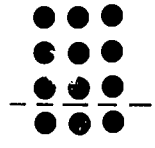
Ask the pupils to look at these products and to tell if they see a pattern. (Each of the numbers has two equal factors.)

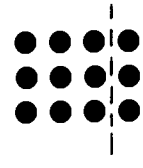
Give groups of children some counters. Ask each group to count out nine counters and to try to arrange them in arrays with the same number of rows as columns. They will form the array:



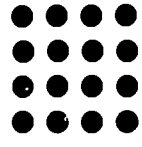
Say, "This is a square array. There are as many rows as columns".

Ask each group of pupils to count out 12 counters and try to arrange them in a square array. (They cannot.) Their work may look like this:





Ask the pupils whether they can arrange 16 counters in a square array. They will make this array:



Say, "We can make a square array with 1 counter, with 4 counters, with 9 counters, with 16 counters, and so on.

These numbers 1, 4, 9, 16, 25, 36, and so on are called squares of counting numbers. What is the next square of a number after those that are written on the blackboard?" (49, $7 \times 7 = 49$.)

Get the pupils to turn to pupil page 196 and do the exercises there. Exercise 17, for example, is a true sentence if a is 8. If the pupils do not immediately think of $64 = 8 \times 8$, they find this number by trying several numbers for a.

ACTIVITY 2: Symbols for Squares of Numbers Pupil pages 197-198

Revise the squares of the counting numbers from 1 to 10. Let pupils write on the blackboard:

$1 = 1 \times 1$ $4 = 2 \times 2$ $9 = 3 \times 3$ $16 = 4 \times 4$ $25 = 5 \times 5$

$36 = 6 \times 6$ $49 = 7 \times 7$ $64 = 8 \times 8$ $81 = 9 \times 9$ $100 = 10 \times 10$

Allow pupils to extend this list farther if they wish.

Explain that there is another way to show the square of a number. Write on the blackboard:

$$3 \times 3 = 3^2$$

Say as you point to the symbols, "Three times three equals three squared". Let a child write "3^2" beside "3×3" on the list of sentences on the blackboard. In the same way let pupils decide that:

$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, and so on.

Ask, "What is 0^2?" ($0^2 = 0 \times 0$, so $0^2 = 0$.)

Get the pupils to find the numbers that make these sentences true:

$n = 5^2$, $n = 3^2 + 4^2$, $t \times t = 36$ $p = 6^2 - 5^2$

Now get the pupils to turn to pupil page 197 and do the exercises there. Let them continue with pupil page 198 as needed.

ACTIVITY 3: (Optional) Games with Subsets of Counting Numbers

Give each child one of the set of 2" × 3" numeral cards with numerals for the counting numbers from 1 to the number of children in the class. Tell the pupils that the cards they have tell their names. Say, "I will think of a subset of the counting numbers. I will tell you about the set. If your name is in the set of numbers I am thinking of, stand up". Describe sets like these:

1. Twenty is a multiple of each of the numbers in the set I am thinking of. (Children with names of 1, 2, 4, 5, 10, and 20 stand.)
2. Each of the numbers in the set I am thinking of is a factor of 27. (Children with names of 1, 3, 9, and 27 stand.)
3. Twenty-five is a multiple of each of the numbers in the set I am thinking of. (1, 5, 25)
4. I am thinking of a set of numbers that have 4 as a factor. (4, 8, 12, 16, 20, ...)
5. I am thinking of a set of numbers that are prime numbers. (2, 3, 5, 7, ...)
6. I am thinking of a set of numbers that are multiples of 3. (3, 6, 9, ...)
7. I am thinking of a set of numbers that are prime numbers and factors of 28. (2, 7)
8. Each of the numbers I am thinking of is the square of a number. (1, 4, 9, 16, 25, ...)
9. I am thinking of the set of odd numbers that are multiples of four. (No children stand. The set is the empty set.)
10. I am thinking of a set of prime numbers that are also factors of 15. (3, 5)
11. Each of the numbers in the set I am thinking of is a factor of 30. (1, 2, 3, 5, 6, 10, 15, 30)
12. Each of the numbers in the set I am thinking of is a multiple of 4. (4, 8, 12, 16, ...)

UNIT 17
Operations on Whole Numbers

Objectives

1. To extend the techniques of division to include any known factor less than 100
2. To revise the techniques of addition, subtraction, multiplication, and division

Background Information for Teachers

See "Background Information for Teachers" for Units 1, 2, 4, 5, and 7 of this book.

STAGE 1: Extension of Division to Include Any Known Factor Less Than 100

Vocabulary: No new vocabulary

Materials: Pupil pages 199-204

Teaching Procedure

ACTIVITY 1: Revision of Techniques of Division Pupil pages 199-200

Write the following problem on the blackboard:

> A farmer collected 756 pineapples from his garden and put them in boxes. He put 42 pineapples in each box. How many boxes did he use?

Guide the pupils to make a mathematical sentence for the problem. If they make a division sentence ask them to write it as a multiplication sentence. ($n = 756 \div 42$ and $n \times 42 = 756$)

Using the multiplication sentence help the pupils estimate the number n that makes the sentence true. (Since $10 \times 42 = 420$ and $20 \times 42 = 840$ then n is between 10 and 20, because 420 is less than 756 and 840 is greater than 756.) Write on the blackboard:

$$10 < n \text{ and } n < 20$$

Ask pupils to work the division on the blackboard. The written work and the thinking will be similar to this:

```
      10 + 8 = 18
42)  756
     420
     336
     336
```

1. Estimate n. $10 < n$ and $n < 20$. n is between 10 and 20.
2. The largest addend of 702 that is both a multiple of 42 and a multiple of 10 is 42×10 or 420.
3. $420 \div 42 = 10$. Write 10.
4. Subtract. $756 - 420 = 336$.
5. 336 is a multiple of 42. $336 \div 42 = 8$. Write 8.
6. The missing factor is $10 + 8$ or 18.

Ask, "How many boxes did the farmer use?" (He used 18 boxes.) Let the children check the result by multiplication. ($18 \times 42 = 756$)

Help pupils to work this problem on the blackboard:

At a sugar factory 637 pounds of sugar is to be put into 25-pound bags. How many bags can be filled?

Ask one of the pupils to write a mathematical sentence for the problem:

$$n = 637 \div 25$$

Let another pupil write this sentence showing a missing factor. Guide him to write:

$$637 = n \times 25$$

Let the children estimate n. They may see that if n is 20, $20 \times 25 = 500$ and that if n is 30, $30 \times 25 = 750$. Therefore n is between 20 and 30 because 500 is less than 637 and 750 is greater than 637. Ask one of the pupils to write this in a sentence: $20 < n$ and $n < 30$. Help the pupils with the division process. The written work and the thinking will be similar to this:

```
      20 + 5 = 25
25)  637
     500
     137
     125
      12
```

1. Estimate n. 20 < n and n < 30. n is between 20 and 30.
2. The largest addend of 637 that is both a multiple of 25 and a multiple of 10 is 20 × 25 or 500.
3. 500 ÷ 25 = 20. Write 20.
4. Subtract. 637 - 500 = 137.
5. The largest addend of 137 that is a multiple of 25 is 125.
6. 125 ÷ 25 = 5. Write 5.
7. Subtract. 137 - 125 = 12.
8. 12 is less than 25. The result of the division process is 637 = (25 × 25) + 12.

Ask the pupils, "How many bags can be filled? (25) Is any sugar left? (Yes) How many pounds of sugar?" (12)

Ask pupils to open their books to pages 199 to 200 and do the exercises as needed.

ACTIVITY 2: Extension of Techniques of Division **Pupil pages 201-204**

The purpose of this activity is to extend the techniques of division to include exercises with known factors of any number less than 100. Begin the activity with the estimation of missing factors.

Ask one of the pupils to give an example of a mathematical sentence with a missing factor and with the known factor any number less than 100. For example: 23 × n = 541. Ask, "What is a low estimate for n?" Someone may estimate that n is about 20. Ask, "Why is this a low estimate?" (23 × 20 = 460 and 460 is less than 541) Ask, "What is a high estimate for n? (n is about 30.) Why is this a high estimate? (23 × 30 = 690 and 690 is greater than 541.) How can we write these estimates using the symbol for less than?" Guide the children to write the sentence below and say that n is between 20 and 30.

$$20 < n \text{ and } n < 30$$

Let the pupils suggest other sentences of this kind that give them practice in estimation. Write this mathematical sentence on the blackboard:

$$37 \times n = 2072$$

Let the pupils make up a story problem that this sentence fits. Ask, "How can we find n? (We use the division process.) What is the first step in the process? (We estimate n.) About how large is n?" Guide the discussion until the pupils decide that n is between 50 and 60. ($50 < n$ and $n < 60$.) Guide the pupils with the division process. The written work and the thinking will be similar to this:

```
      50 + 6 = 56
37)  2072
     1850
      222
      222
```

1. Estimate n. $50 < n$ and $n < 50$. n is between 50 and 60.
2. The largest addend of 2072 that is both a multiple of 10 and a multiple of 37 is 50×37 or 1850.
3. $1850 \div 37 = 50$. Write 50.
4. Subtract. $2072 - 1850 = 222$.
5. 222 is a multiple of 37. $222 \div 37 = 6$. Write 6.
6. The missing factor is 56. (Check by multiplication.)

Let the pupils answer the question asked in their story problem. Ask, "What property of division did you use in the division process? (The distributive property) How do you use it?" (We thought of 2072 as $1850 + 222$. We divided each addend by 37. $1850 \div 37 = 50$. $222 \div 37 = 6$. The missing factor is $50 + 6$ or 56.)

Write this sentence on the blackboard:

$$n \times 17 = 1875$$

Again let the pupils make up a story problem that this sentence fits. Guide the pupils in estimating n until they decide that n is between 100 and 200. ($100 < n$ and $n < 200$.) Let the pupils do the division process. The written work and the thinking will be similar to this:

```
      100 + 10 = 110
17)  1875
     1700
      175
      170
        5
```

1. Estimate n. 100 < n and n < 200. n is between 100 and 200.
2. The largest addend of 1875 that is both a multiple of 17 and a multiple of 100 is 100 × 17 or 1700.
3. 1700 ÷ 17 = 100. Write 100.
4. Subtract. 1875 - 1700 = 175.
5. The largest addend of 175 that is both a multiple of 10 and a multiple of 17 is 170.
6. 170 ÷ 17 = 10. Write 10.
7. Subtract. 175 - 170 = 5.
8. 5 is less than 17. The result of the division process is 635 = (17 × 110) + 5.

Let the pupils answer the question asked in their story problem.

Let the pupils open their books to pupil pages 201 to 204 and do the exercises as needed. If there are no story problems on a particular pupil page, choose one or two of the exercises and let the pupils make up their own story problems for these exercises. Encourage estimation and checking by multiplication in division exercises.

STAGE 2: Revision of Operations on Whole Numbers

Vocabulary: No new vocabulary

Materials: Pupil pages 205-217

Teaching Procedure

ACTIVITY 1: The Four Basic Operations | Pupil page 205

This stage emphasizes basic skills in operations. It also stresses writing mathematical sentences that illustrate the basic operations.

Let the pupils open their books to page 205 and talk about the exercises. Let them tell the mathematical sentence if there is only one operation involved. When more than one operation is involved, let them write the required mathematical sentence on the blackboard.

Help the children to work inside the brackets first as shown below for Exercises 6 and 14.

(6) $(143, 11) \xrightarrow{\div} \underline{13}$

$143 \div 11 = 13$

(14) $(7, 6) \xrightarrow{\times} (80, \underline{42}) \xrightarrow{-} \underline{38}$

$80 - (7 \times 6) = 38$

ACTIVITY 2: Revising Techniques of Addition and Subtraction | Pupil pages 206-209

To help the children do the exercises on pupil pages 206 to 209, revise the following:

- place value in decimal numerals, and
- renaming numbers.

Let the children open their books to pupil pages 206 to 209 and do the exercises and story problems as needed. Talk with the children about a few of the exercises and then let them work on their own. Let them go on to the next page when they complete the work on a page. Get them to write out mathematical sentences for each story problem to guide them in their work. Encourage the pupils to check their addition by adding first in one direction and then in the other and to check subtraction by addition.

ACTIVITY 3: Revising Techniques of Multiplication and Division

Pupil pages 210-219

Revise the following:

- finding the product of any two numbers when one is less than 10;
- finding the product of any two numbers when one is a multiple of 10 or 100; and
- estimating products and missing factors.

Let the pupil do the exercises and story problems on pupil pages 210 to 215 as suggested in the last activity. In the exercises with products that are not multiples of the known factors let the pupils write the results in the form for the exercise below.

```
      30 + 3 = 33
13) 437
    390
     47
     39
      8
```

$437 = (33 \times 13) + 8$

Let the pupils do the exercises on pupil pages 216 to 217 as supplementary exercises <u>as needed</u>.

UNIT 18

Fractions III: Multiplication

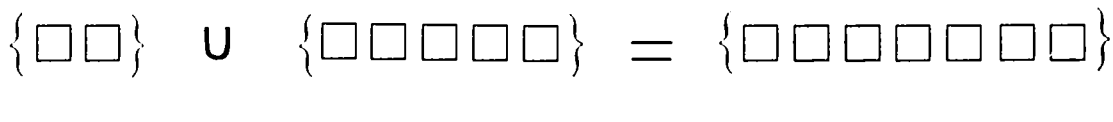
Objectives

1. To revise the technique of multiplying a whole number and a fraction
2. To develop techniques of multiplying two fractions
3. To illustrate the properties of multiplication of fractions

Background Information for Teachers

See "Background Information for Teachers" of Units 10 and 13, pages 257-259 and 324-325 in Entebbe Mathematics, Primary Three, and of Units 9 and 16 in this book.

When we add fractions, it helps to think of the way we add whole numbers. For example, we can use unit squares to show the sum of 2 + 5, like this:



$$2 + 5 = 7$$

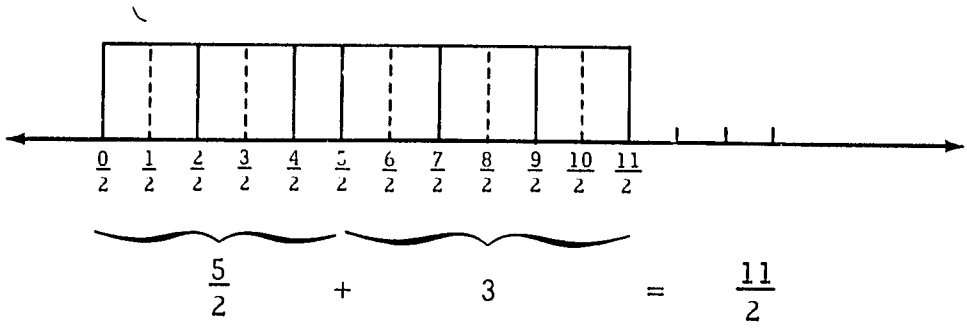
We also can show the sum with unit squares on the number line:

In a similar way we can use unit squares to show the sum of $\frac{5}{2}$ + 3 like this:

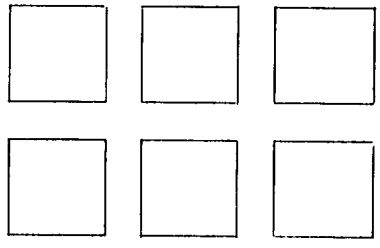
$$\frac{5}{2} + 3 = n$$

$$\frac{5}{2} + \frac{6}{2} = \frac{11}{2}$$

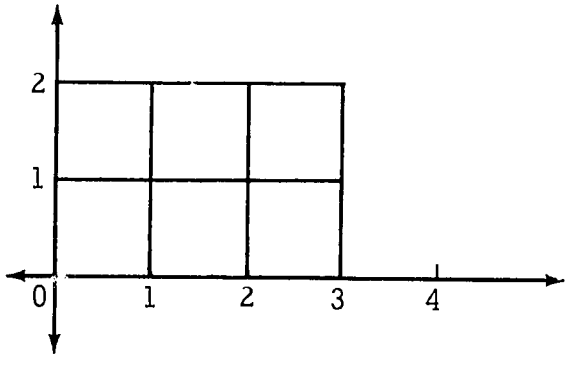
We can also show the sum with unit squares on the number line in this way:



When we find products of fractions, we again think of the way we operate on whole numbers. For example, the product of 2×3 is shown with two rows of three unit squares each:

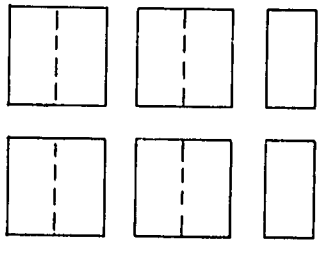


This product is also shown by <u>two</u> pictures of the number line like this:

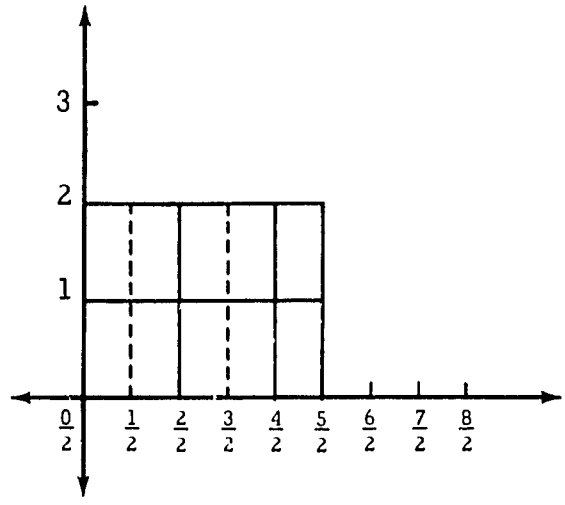


There are six squares in all; so we write $2 \times 3 = 6$.

In a similar way, we can show the product of $2 \times \frac{5}{2}$ as two rows of squares, with $\frac{5}{2}$ squares in each row.



This product is also shown by two pictures of the number line like this:



Counting halves of squares in either drawing above there are 2×5 halves of squares. We can write:

$$2 \times \frac{5}{2} = \frac{2 \times 5}{2}$$

$$= \frac{10}{2}$$

After seeing several such examples, we are able to find numbers that make such sentences as these true:

$2 \times \frac{1}{3} = n$ ($\underline{n}$ is $\frac{2}{3}$, because $2 \times \frac{1}{3} = \frac{2 \times 1}{3}$; $2 \times \frac{1}{3} = \frac{2}{3}$)

$3 \times \frac{5}{4} = p$ ($\underline{p}$ is $\frac{15}{4}$, because $3 \times \frac{5}{4} = \frac{3 \times 5}{4}$; $3 \times \frac{5}{4} = \frac{15}{4}$)

$2 \times \frac{1}{2} = t$ ($\underline{t}$ is 1, because $2 \times \frac{1}{2} = \frac{2}{2}$; $2 \times \frac{1}{2} = 1$)

Thus, we see that for any whole number $\underline{a}$ and any fraction $\frac{b}{c}$:

$$a \times \frac{b}{c} = \frac{a \times b}{c}$$

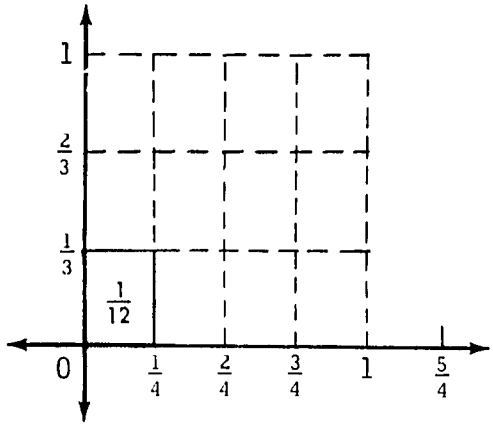
Also, for any counting number $\underline{a}$:

$$a \times \frac{1}{a} = 1$$

The same type of drawing, shown on page 288, helps us to find a product such as:

$$\frac{1}{3} \times \frac{1}{4}$$

We show $\frac{1}{4}$ of a unit square and then $\frac{1}{3}$ of $\frac{1}{4}$ of the unit square. $\frac{1}{3}$ of $\frac{1}{4}$ of a unit square is 1 of 12 congruent parts of the unit square, or $\frac{1}{12}$ of the square. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.



$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

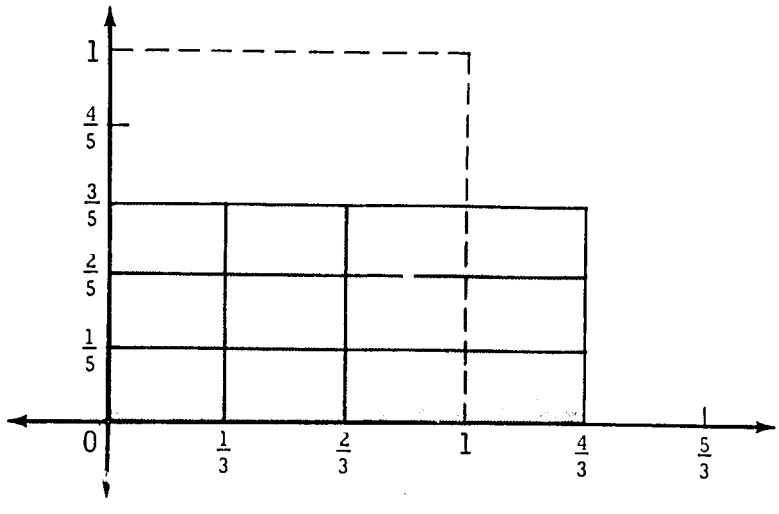
After many examples of this kind we find that, for example:

$$\frac{1}{2} \times \frac{1}{5} = \frac{1}{2 \times 5} = \frac{1}{10},$$

$$\frac{1}{4} \times \frac{1}{5} = \frac{1}{4 \times 5} = \frac{1}{20}, \text{ and that}$$

$$\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}, \quad \text{for any fractions } \frac{1}{a} \text{ and } \frac{1}{b}$$

We can also use two pictures of the number line to show the product of any two fractions. For example, the product of $\frac{3}{5} \times \frac{4}{3}$ is shown as $\frac{3}{5}$ of $\frac{4}{3}$ of the unit square. The array has 3 rows with four $\frac{1}{15}$'s in each row. There are 3×4 or twelve $\frac{1}{15}$'s in the array.



$$\frac{3}{5} \times \frac{4}{3} = \frac{3 \times 4}{5 \times 3} = \frac{12}{15}$$

We know then that for any fractions $\frac{a}{b}$ and $\frac{c}{d}$:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

A fraction such as $\frac{5}{4}$, for example, can be expressed in several ways:

$\frac{5}{4} = 5 \times \frac{1}{4}$,

$\frac{5}{4} = 5 \div 4$, and

$\frac{5}{4}$ is the number that makes the sentence $4 \times n = 5$ true.

We can also write many names for a fraction. For example:

$$\frac{5}{4} = \frac{10}{8} = \frac{15}{12} = \ldots$$

$$\frac{5}{4} = 1\frac{1}{4}$$

$$\frac{5}{4} = 1 \cdot 25$$

$$\frac{5}{4} = 125\%$$

Finally, we know that for any fraction $\frac{a}{b}$ (not zero) there is a fraction $\frac{b}{a}$ such that:

$$\frac{a}{b} \times \frac{b}{a} = 1.$$

STAGE 1: Multiplying a Whole Number and a Fraction

Vocabulary: No new vocabulary

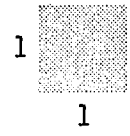
Materials: Pupil pages 218-221

Teaching Procedure

ACTIVITY 1: Product of a Counting Number and a Unit Fraction

Pupil page 218

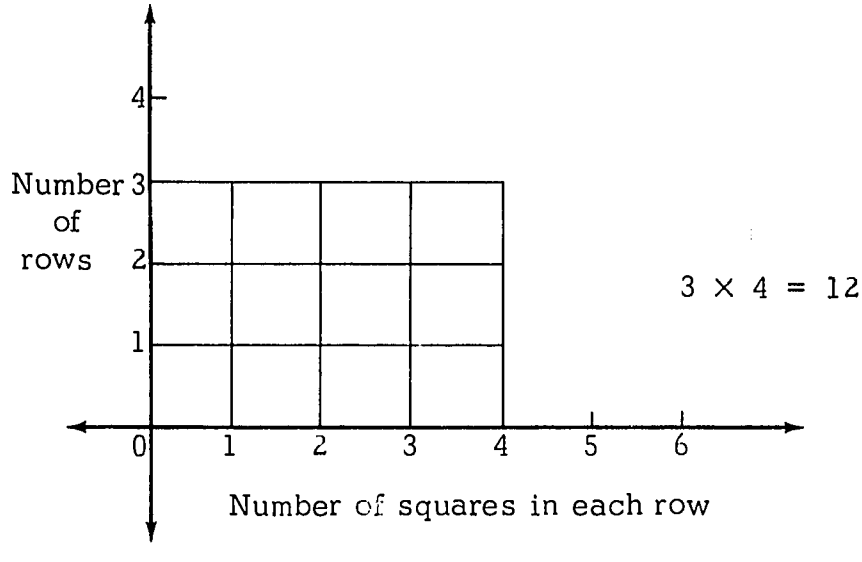
Ask, "What is a square unit?" (It is a square region with sides of length 1 unit.) Draw a unit square on the blackboard.



Draw an array of three rows of unit squares with four in each row. (See the following figure.) Ask, "How many unit squares are there in the array of squares? (12) How did you find this?" Write:

$$3 \times 4 = 12$$

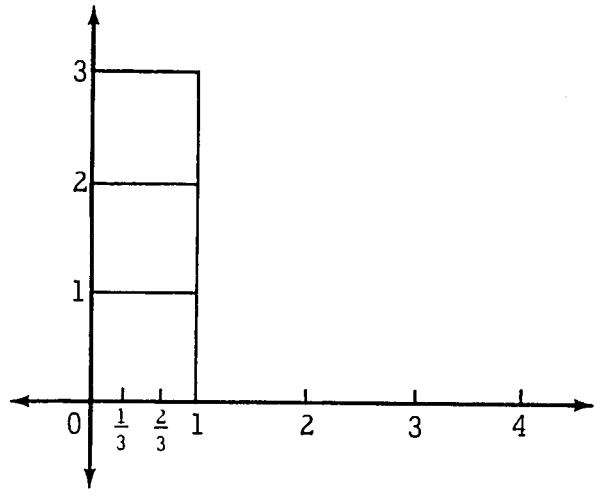
Note: When you draw the array, as in the following figure, extend the edges to show two pictures of the number line. Mark the horizontal line to show the number of squares in each row. Mark the vertical line to show the number of rows. Let the pupils look at the figure and count along one line to find how many squares are in each row of the array and along the other line to find how many rows of squares.



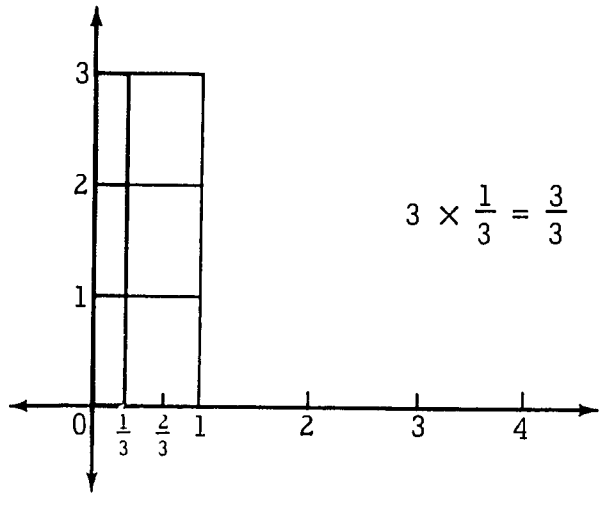
On the same figure, let a pupil draw more squares to make a new array that shows the product 4 × 5. (He will draw a horizontal line through the

4 point and a vertical line through the 5 point. The array will have 4 rows, with 5 squares in each row.) Continue with other arrays, until the pupils understand that the two pictures of the number line help them count the number of rows and the number of squares in each row.

Draw three unit squares on the blackboard with pictures of the number line along the edges as shown below:



Mark points to separate the horizontal line segment from 0 to 1 in thirds. Ask a pupil to name the points. ($\frac{0}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$) Ask another child to draw a line to show that the squares are separated into thirds of squares. (Help him to draw a vertical line through the $\frac{1}{3}$ point.) Shade the diagram so that it looks like this.



Ask, "How many thirds of squares are shaded? (3 thirds) What fraction of

a unit square is shaded? ($\frac{3}{3}$ or 1) What product is shown by the shaded array?" (3 rows with $\frac{1}{3}$ in each row):

$$3 \times \frac{1}{3} = \frac{3}{3} \text{ or } 1$$

Guide the pupils to understand that they count on one number line to 3 to find the number of rows and count on the other number line to $\frac{1}{3}$ to find the number in each row. The product of 3 and $\frac{1}{3}$ is 3 thirds or $\frac{3}{3}$ or 1.

Repeat this activity with several other products of a whole number and a unit fraction. For example:

$$4 \times \frac{1}{5} = \frac{4}{5} \qquad 2 \times \frac{1}{4} = \frac{2}{4}$$

$$4 \times \frac{1}{4} = \frac{4}{4} \qquad 5 \times \frac{1}{3} = \frac{5}{3}$$

Note: This is a revision for most pupils. It is also a new way to think about products by the use of the number line. We will use this method throughout this Unit.

Ask the pupils to work the exercises on pupil page 218. In some of the exercises they will find that for any counting number, say 5, there is a fraction, in this case $\frac{1}{5}$, such that their product is 1. In symbols:

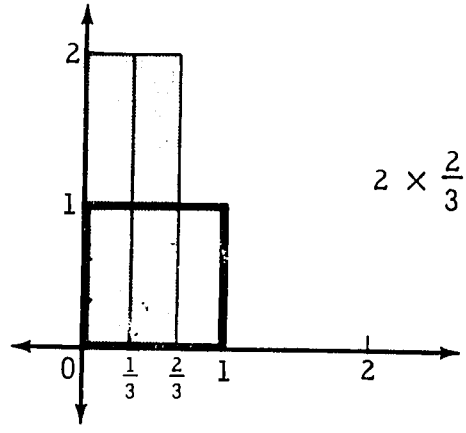
$$a \times \frac{1}{a} = 1$$

ACTIVITY 2: Product of a Whole Number and a Fraction **Pupil page 219**

Write this sentence on the blackboard:

$$2 \times \frac{2}{3} = n$$

Draw two pictures of the number line on the blackboard as in Activity 1. Let a pupil tell how to show an array for the product of $2 \times \frac{2}{3}$. As he tells you, draw horizontal lines through 1 and 2 to show two rows. Then mark the horizontal number line in $\frac{1}{3}$'s and count to $\frac{2}{3}$. Draw vertical lines through $\frac{1}{3}$ and $\frac{2}{3}$ to show 2 thirds in each row. Shade the array, as in the figure at the top of page 293. Let the pupils again tell how many rows, (2) and how many $\frac{1}{3}$'s in each row. (Two $\frac{1}{3}$'s)



$$2 \times \frac{2}{3} = \frac{4}{3}$$

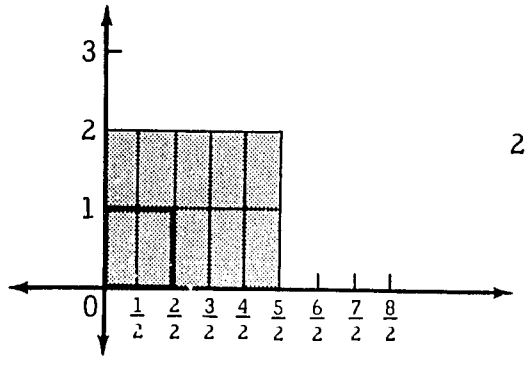
Ask, "How many thirds of a square are shaded? (4 thirds) What multiplication is shown by the shaded region? ($2 \times \frac{2}{3}$) How many unit squares are shaded?" ($\frac{4}{3}$) Help the children to decide:

$$2 \times \frac{2}{3} = \frac{2 \times 2}{3} \qquad 2 \times \frac{2}{3} = \frac{4}{3} \qquad \underline{n} \text{ is } \frac{4}{3}$$

Write this sentence on the blackboard:

$$2 \times \frac{5}{2} = n$$

Draw two pictures of the number line, as before. Let the pupils tell how many rows of squares to draw (2); how many $\frac{1}{2}$'s in each row. (Five $\frac{1}{2}$'s)



$$2 \times \frac{5}{2} = \frac{10}{2}$$

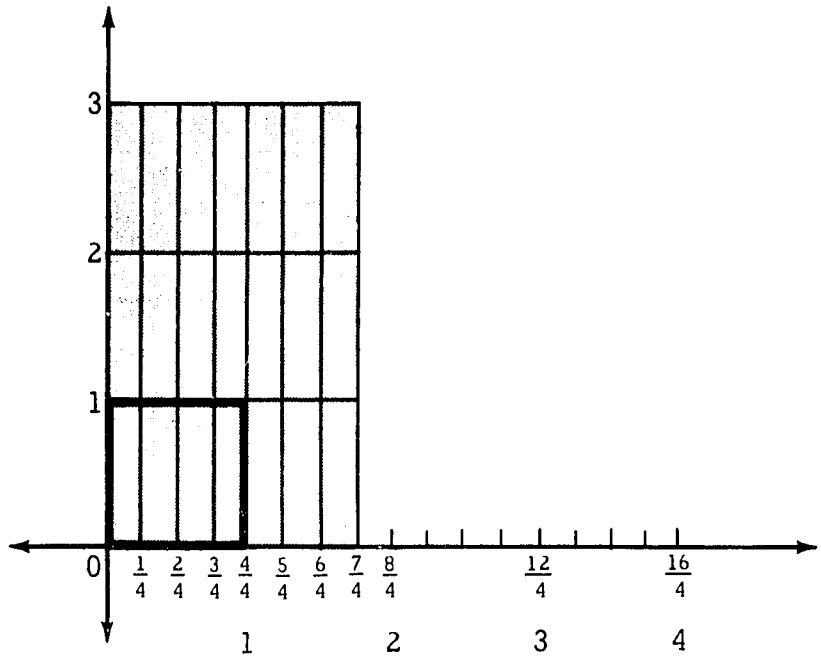
Ask a child to count half squares in the figure. (10 halves) Let another child tell what multiplication is shown by the figure. ($2 \times \frac{5}{2}$) Write the sentence on the blackboard and get the children to read it:

$$(2 \times \frac{5}{2} = \frac{10}{2})$$

Ask, "How many unit squares are shaded?" ($\frac{10}{2}$ or 5 unit squares) Help them to see that:

$$2 \times \frac{5}{2} = \frac{2 \times 5}{2} \qquad 2 \times \frac{5}{2} = \frac{10}{2} \qquad \underline{n} \text{ is } \frac{10}{2} \text{ or } 5$$

If the children need more practice, draw a diagram as shown below:



Ask, "What product does the array show? How many rows in the array? How many $\frac{1}{4}$'s in each row? How many fourths of squares are in the array?" (3 × 7 fourths) Help the children to see that:

$$3 \times \frac{7}{4} = \frac{3 \times 7}{4}, \text{ and } \quad 3 \times \frac{7}{4} = \frac{21}{4}.$$

Give other examples such as $2 \times \frac{3}{5}$, and ask the children to find the products directly. If they do not understand that $2 \times \frac{3}{5} = \frac{2 \times 3}{5}$; or $2 \times \frac{3}{5} = \frac{6}{5}$, draw an array to show the products.

Help the children solve this story problem:

> Lima made five shirts. She used $\frac{2}{3}$ of a yard of cloth for each shirt. How many yards of cloth did she use in all?

Ask the children to write a sentence for the problem. Guide them to write:

$$5 \times \frac{2}{3} = n$$

Ask the children to find the number n that will make the sentence true:

$$5 \times \frac{2}{3} = \frac{5 \times 2}{3}, \quad \text{and} \quad 5 \times \frac{2}{3} = \frac{10}{3}. \qquad \underline{n} \text{ is } \frac{10}{3}$$

Let the children use $\frac{10}{3}$ to answer the question asked in the problem. (Lima used $\frac{10}{3}$ yards of cloth; or Lima used $3\frac{1}{3}$ yards of cloth.)

Work other story problems on the blackboard with the children. For example:

> Loti can read $\frac{3}{5}$ of a page every minute. He read for 20 minutes. How many pages did he read?

Follow the method above. The sentence is $20 \times \frac{3}{5} = n$.

$$20 \times \frac{3}{5} = \frac{20 \times 3}{5}, \text{ and } 20 \times \frac{3}{5} = \frac{60}{5}, \text{ and } \frac{60}{5} = 12. \qquad \underline{n} \text{ is } 12$$

Guide the children to say, "Loti read 12 pages". Let the children open their books to pupil page 219 and work the exercises in their exercise books. (In Exercises 3 and 6 change the decimal names to common names. In Exercise 12, change 20% to $\frac{20}{100}$ or $\frac{1}{5}$.)

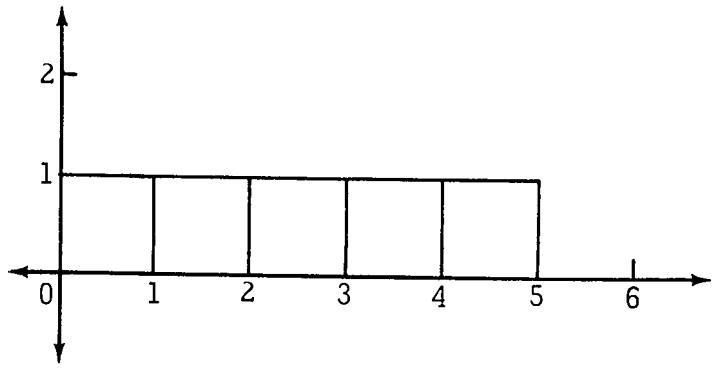
ACTIVITY 3: Product of a Fraction and a Whole Number

Pupil page 220

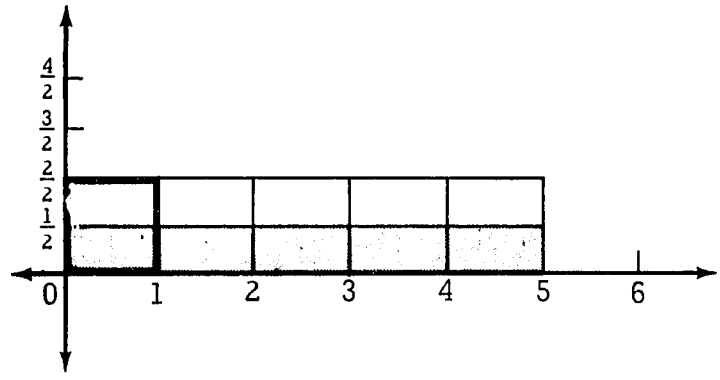
Write this sentence on the blackboard:

$$\frac{1}{2} \times 5 = n$$

Draw two pictures of the number line and five unit squares as shown below:



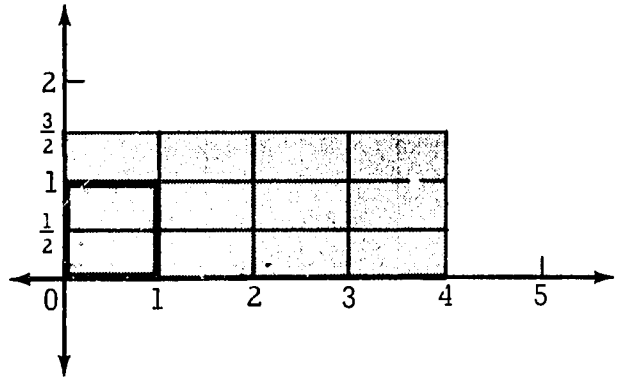
Say, "We will show one half of each square". Draw a horizontal line through the $\frac{1}{2}$ point and shade the region as shown at the top of page 296.



Ask, "How many rows of half squares are there? (1) How many in each row? (5) How many halves have been shaded?" (5 halves) Guide the children to write:

$$\frac{1}{2} \times 5 = \frac{5}{2}$$

Draw another figure on the blackboard as shown below:



Ask the children if they can write the product shown by the shaded region. Guide them to decide that it shows $\frac{3}{2} \times 4$. Get a child to write this on the blackboard:

$$\frac{3}{2} \times 4 = \frac{3 \times 4}{2}$$

$$= \frac{12}{2} = 6$$

Solve similar sentences on the blackboard. For example:

$$\frac{5}{4} \times 3 = n.$$

If some children are not able to find n, draw arrays as before.

Help the children work on the blackboard some story problems such as:

> Numi had a rope 16 feet long. He used $\frac{3}{4}$ of it to tie a bundle of sticks. How many yards of rope did he use?

Guide the children to write a mathematical sentence, find n, and answer the question asked in the problem:

$$\frac{3}{4} \times 16 = n$$

$$\frac{3}{4} \times 16 = \frac{3 \times 16}{4}$$

$$= \frac{48}{4} \text{ or } 12$$

n is 12. Numi used 12 yards of rope.

Let the children open their books to pupil page 220 and do the exercises in their exercise books.

ACTIVITY 4: Dividing Counting Numbers Pupil page 221

Revise these ideas:

- When a whole number is divided by a counting number, the result is a fraction;
- Every fraction can be found by dividing a whole number by a counting number.

Write on the blackboard a sentence such as:

$$5 \div 3 = n$$

Ask a pupil to write the sentence as a multiplication sentence. He will write:

$$3 \times n = 5, \quad \text{or} \quad n \times 3 = 5$$

Let the class discuss these sentences, saying such things as: the product is 5, and one factor is 3, the missing factor is n. Ask, "Is the number that

makes these two sentences true a whole number? (No, there is no whole number that can be multiplied by 3 to give 5.) Is $\underline{n}$ a fraction? (Yes) What fraction?" Guide the pupils to decide that $\underline{n}$ is $\frac{5}{3}$, because:

$$3 \times \frac{5}{3} = \frac{3 \times 5}{3}$$

$$= \frac{15}{3} \text{ or } 5$$

Ask, "What is $5 \div 3$?" ($5 \div 3 = \frac{5}{3}$) Talk about other examples of this kind until the pupils have revised the ideas that:

$$3 \div 2 = \frac{3}{2},\ 4 \div 5 = \frac{4}{5}, \text{ and so on.}$$

Let pupils explain the ideas in their own words, for example:

$\frac{3}{2}$ is the number we get when we divide 3 by 2,

$\frac{4}{5}$ is the number we get when we divide 4 by 5, and so on.

If a child suggests that:

$\frac{a}{b}$ is the number we get when we divide $\underline{a}$ by $\underline{b}$,

praise him, but do not require this statement of any child.

Give another example with larger numbers, for example:

$$99 \div 7 = n$$

Help the pupils to decide:

$$99 \div 7 = \frac{99}{7} \text{ and say, "}\underline{n}\text{ is } \frac{99}{7}\text{".}$$

Ask, "Can $\frac{99}{7}$ be written with a mixed name?" Guide the class to think of doing the division problem $99 \div 7$ by techniques they learned in Unit 17.

$$(99 \div 7) = (70 + 28 + 1) \div 7 \qquad \text{(Renaming 99)}$$

$$= (70 \div 7) + (28 \div 7) + (1 \div 7) \qquad \text{(Distributive property)}$$

$$= 10 + 4 + \frac{1}{7}$$

$$= 14\frac{1}{7} \qquad \underline{n} \text{ is } 14\frac{1}{7}$$

Guide the children to say that $\frac{99}{7}$ has the mixed name $14\frac{1}{7}$:

$$99 \div 7 = \frac{99}{7}$$

$$= 14\frac{1}{7}$$

Let other children do the division process by the short form:

$$\begin{array}{r|l} & \underline{10} + 4 + \frac{1}{7} = 14\frac{1}{7} \\ 7 & 99 \\ & \underline{70} \quad (70 \div 7 = 10) \\ & 29 \\ & \underline{28} \quad (28 \div 7 = 4) \\ & 1 \quad (1 \div 7 = \frac{1}{7}) \quad \underline{n} \text{ is } 14\frac{1}{7} \end{array}$$

If more practice is needed give other exercises in changing common names to mixed names by the division process. Let the pupils use the exercises on pupil page 221 to help revise the idea.

STAGE 2: Techniques of Multiplication of Fractions

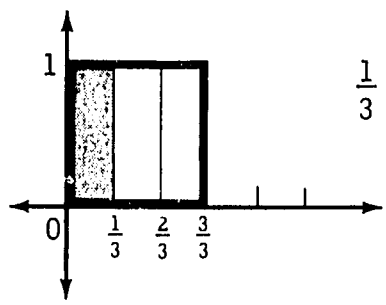
Vocabulary: No new vocabulary

Materials: Pupil pages 222-227

Teaching Procedure

ACTIVITY 1: Products of Unit Fractions **Pupil pages 222-223**

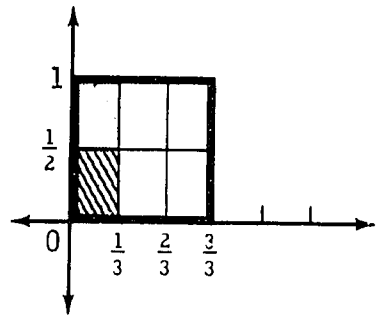
Draw on the blackboard a unit square region. Separate it into three congruent regions as shown below. Shade one of the parts.



$\frac{1}{3}$ of unit square is shaded

Ask, "What part of the square region has been shaded?" ($\frac{1}{3}$) Draw a horizontal line through the $\frac{1}{2}$ point, separating the shaded region into two congruent parts and shaded as shown below.

Ask, "What part of the shaded region is shaded again?" ($\frac{1}{2}$ of the shaded region. $\frac{1}{2}$ of $\frac{1}{3}$ of the unit square is shaded.)



Ask, "What part of the unit square has been shaded twice?" ($\frac{1}{6}$) Guide the pupils to understand that $\frac{1}{6}$ of the unit square has been shaded twice because there are six of these small regions in the unit square. Write:

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{2 \times 3}; \quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Let the pupils do other examples like this if needed. From the examples the pupils should discover that to find the product of two unit fractions (fractions named with numerator 1) it is only necessary to find the product of the denominators:

$$\frac{1}{3} \times \frac{1}{5} = \frac{1}{3 \times 5} \quad \text{or} \quad \frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$$

$$= \frac{1}{15}$$

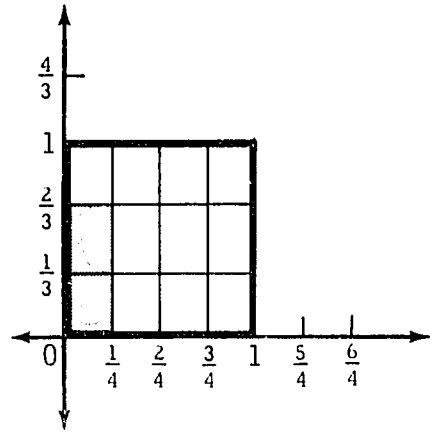
Let the pupils work the exercises on pages 222 and 223. For the story problems, get the pupils to write a mathematical sentence, solve the sentence, and answer the question in the problem. Notice that Exercises 10 to 15 on page 223 also revise addition and subtraction of fractions.

ACTIVITY 2: Products of Fractions Pupil pages 224-225

Draw a unit square on the blackboard with pictures of the number line along its sides. Ask the class to tell how to draw an array for the sentence:

$$\frac{2}{3} \times \frac{1}{4} = n$$

Guide the children to say: the vertical number line has its unit segment marked in $\frac{1}{3}$'s and the horizontal number line in $\frac{1}{4}$'s; count off $\frac{2}{3}$'s and $\frac{1}{4}$'s and draw lines as shown below, to form a $\frac{2}{3}$ by $\frac{1}{4}$ array:

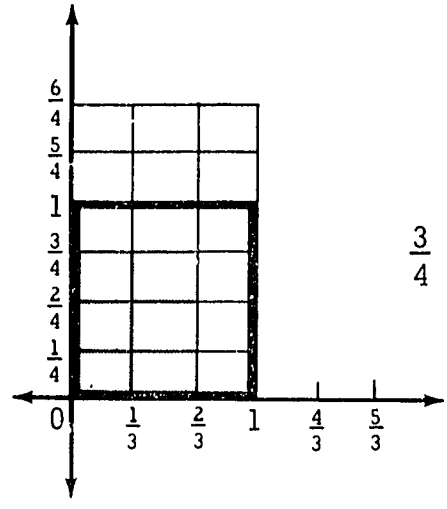


Ask, "What part of the unit square is shaded? ($\frac{2}{3}$ of $\frac{1}{4}$ of the unit square.) What part of the unit square is each small region? ($\frac{1}{12}$) How many $\frac{1}{12}$'s are shaded? (Two $\frac{1}{12}$'s) What is $\frac{2}{3} \times \frac{1}{4}$?" ($\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$)

Let pupils count again, if necessary, to see that 2 out of 12 regions, or $\frac{2}{12}$ of the unit square is shaded.

If needed, use the same plan with other examples, for example:

$$\frac{3}{4} \times \frac{2}{3}$$



$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Write on the blackboard this mathematical sentence:

$$\frac{2}{3} \times \frac{3}{5} = n$$

Ask, "What number n makes the sentence true?" ($\frac{6}{15}$) Let the pupils try other examples like this. Guide them to decide how to find the product of two fractions. (Find the product of the numerators and the product of the denominators to form the common name of the product.) Get a pupil to show the work:

$$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5}$$

$$= \frac{6}{15}$$

Give an example written with decimal names. For example:

$$\cdot 3 \times \cdot 5 = n$$

The pupils will write these numerals with common names:

$$\frac{3}{10} \times \frac{5}{10} = n$$

$$\frac{3}{10} \times \frac{5}{10} = \frac{3 \times 5}{10 \times 10}$$

$$= \frac{15}{100}$$

$$= \cdot 15$$

Thus, $\cdot 3 \times \cdot 5 = \cdot 15$; n is $\cdot 15$.

Let the pupils try other examples including some story problems. Then let them work the exercises on pupil pages 224 and 225. For the story problems, they write the mathematical sentence, solve the sentence, and answer the questions in the problem.

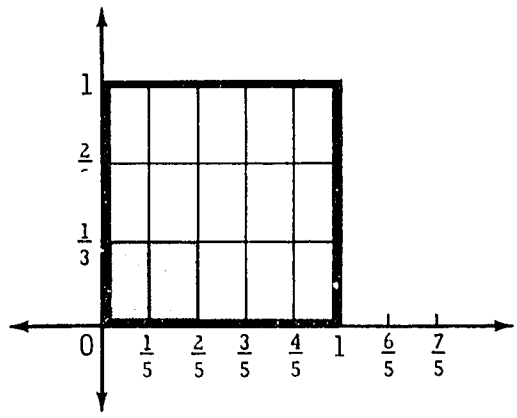
ACTIVITY 3: Multiplying Fractions **Pupil page 226**

Revise products of fractions by letting the class tell how to solve these sentences:

(a) $4 \times \frac{4}{5} = a$ (b) $\frac{1}{4} \times \frac{1}{5} = n$

(c) $\frac{1}{3} \times \frac{2}{5} = b$ (d) $\frac{3}{4} \times \frac{5}{2} = m$

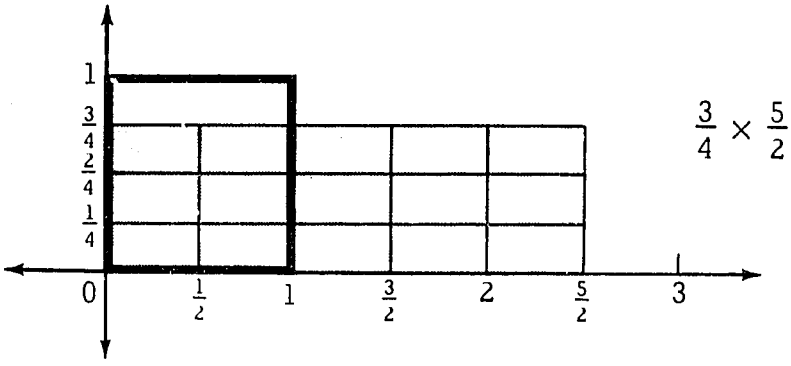
Question the class about several ways of thinking about these sentences. For example, in Sentence (c), $\frac{1}{3} \times \frac{2}{5}$ represents $\frac{1}{3}$ of $\frac{2}{5}$ of a whole. Picture this with an array:



The shaded region is $\frac{1}{3}$ of $\frac{2}{5}$ of the whole unit square. The shaded region is 2 parts of the unit square. It is $\frac{2}{15}$ of the unit square. Help the children remember that:

$$\frac{1}{3} \times \frac{2}{5} = \frac{1 \times 2}{3 \times 5}$$

In Sentence (d), one of the factors $(\frac{5}{2})$ is greater than 1. Let a pupil explain how to show this product with an array. Draw the diagram, shown at the top of page 304, as he talks.



$$\frac{3}{4} \times \frac{5}{2} = \frac{3 \times 5}{4 \times 2}$$

Count to $\frac{3}{4}$ by fourths on the vertical line and count to $\frac{5}{2}$ by halves on the horizontal line. The shaded region shows $\frac{3}{4}$ of $\frac{5}{2}$ of the unit square. Let the pupils find the part each small region is of the unit square ($\frac{1}{8}$ of the unit square) and how many small regions are shaded ($3 \times 5 = 15$). Guide the pupils to decide that $\frac{15}{8}$ of the unit square is shaded:

$$\frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

Ask, "Does the method used before for multiplying fractions work here?" (Yes, we find <u>n</u> in the sentence $\frac{3}{4} \times \frac{5}{2} = n$.)

$$\frac{3}{4} \times \frac{5}{2} = \frac{3 \times 5}{4 \times 2}$$

$$= \frac{15}{8}$$

$$\underline{n} \text{ is } \frac{15}{8}$$

Let the children find other products, such as:

$$\frac{4}{3} \times \frac{4}{5}; \quad \frac{5}{3} \times \frac{1}{4}; \quad \frac{6}{5} \times \frac{4}{3}; \quad \text{and so on.}$$

Each time a pupil gives a product, let him explain how he found it. If a pupil suggests a general plan for multiplying, let him try to write it on the blackboard:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Otherwise, accept his explanation about multiplying the numerators and multiplying the denominators. If needed, show arrays on the blackboard for some examples.

Note: Do not insist on simplified names for the products. For example, a child may write:

$$\frac{2}{3} \times \frac{6}{5} = \frac{12}{15}$$

Do not expect him to name $\frac{12}{15}$ as $\frac{4}{5}$. If he does, accept the name. Skill in renaming will develop later.

Give further practice, in which various kinds of names are used for the numbers. For example:

$\cdot 3 \times \frac{1}{4}$; $\quad 2\cdot 5 \times \frac{3}{5}$; $\quad 4 \times 20\%$;

$\frac{7}{4} \times \cdot 6$; $\quad \cdot 3 \times \cdot 2$; $\quad \frac{2}{3} \times 18\%$; and so on.

Guide the pupils in each example to change the names of the numbers to common names before multiplying:

$$\cdot 3 \times \frac{1}{4} = \frac{3}{10} \times \frac{1}{4}; \qquad 2\cdot 5 \times \frac{3}{5} = \frac{25}{10} \times \frac{3}{5}; \qquad 4 \times 20\% = 4 \times \frac{20}{100}$$

$$= \frac{3 \times 1}{10 \times 4} \qquad = \frac{75}{50} \qquad = \frac{80}{100}$$

$$= \frac{3}{40} \qquad = 80\%$$

$$\frac{7}{4} \times \cdot 6 = \frac{7}{4} \times \frac{6}{10}; \qquad \cdot 3 \times \cdot 2 = \frac{3}{10} \times \frac{2}{10}; \qquad \frac{2}{3} \times 18\% = \frac{2}{3} \times \frac{18}{100}$$

$$= \frac{7 \times 6}{4 \times 10} \qquad = \frac{6}{100} \qquad = \frac{36}{300}$$

$$= \frac{42}{40} \qquad = \cdot 06 \qquad = \frac{12}{100} \text{ or } 12\%$$

Let pupils get more experience in multiplying fractions named in many ways by working the exercises on pupil page 226.

ACTIVITY 4: (Optional) Missing Factors

Pupil page 227

If the class enjoys work with fractions, use this activity to prepare for the work of Entebbe Mathematics, Primary Five.

Let the pupils solve some of the sentences on page 227 as a class activity. Write the first exercise on the blackboard and ask, "What number must be multiplied by 5 to give the product $\frac{5}{3}$?" ($\frac{1}{3}$) Let a pupil check by multiplying $5 \times \frac{1}{3}$. Continue in this way. For Exercise 3 guide the children to reason as follows:

What must I multiply 5 by to get 1?	($\frac{1}{5}$)
What must I multiply 1 by to get 3?	(3)
If I multiply 5 by ($\frac{1}{5} \times 3$), I get 3.	($5 \times \frac{1}{5} \times 3$)
Then a is $\frac{3}{5}$.	

Let the pupils test this missing factor by multiplying $5 \times \frac{3}{5}$. After the pupils work together on several of the exercises let them work alone to finish the exercises. In Exercises 13 to 18 they learn that for any fraction $\frac{a}{b}$ (other than 0) there is another fraction $\frac{b}{a}$ so that:

$$\frac{a}{b} \times \frac{b}{a} = 1$$

This idea will be used many times in the work with fractions.

ACTIVITY 5: Revision

Pupil page 228

Let the pupils use pupil page 228 to revise renaming fractions in many ways and writing fractions for arrays shown on the number line.

Answers for pupil page 228, Exercises 9-12:

9. $\frac{5}{6} \times \frac{7}{5} = \frac{35}{30}$

10. $\frac{5}{3} \times \frac{7}{4} = \frac{35}{12}$

11. $\frac{3}{5} \times \frac{4}{3} = \frac{12}{15}$

12. $\frac{6}{7} \times \frac{9}{4} = \frac{63}{28}$

UNIT 19
Geometry

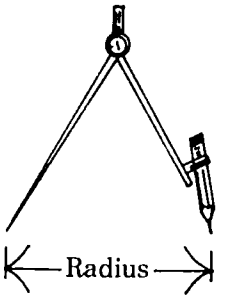
Objectives

1. To learn the use of a straightedge in drawing line segments, angles, and other figures
2. To learn the use of a compass in drawing circles and in laying off distances
3. To learn to construct models of solid figures — pyramids, cubes, and rectangular solids; to describe their faces, edges, and vertices; and to make sketches of these figures
4. To extend ideas of intersections of lines with solid figures
5. To revise ideas of movements in a plane and to discover that figures are moved into congruent figures by these movements
6. To discover some turning symmetries and revise folding symmetries of certain plane figures — square, rectangle, circle, and equilateral triangle (through paper rotating and folding)

Background Information for Teachers

Some of the study of geometry is devoted to drawing figures. For example, lines are drawn parallel to given lines, lines are drawn perpendicular to given lines, and angles are drawn congruent to given angles. These constructions are done using only a straightedge and a compass.

The fact that only one line can be drawn through two points is the basic principle of using the straightedge. The fact that every radius of a circle is congruent to every other radius is the basic principle of using the compass. The distance between the point tip and the pencil tip of the compass can be held fixed.



This distance is called the radius of the compass because it is the length of the radius of the circle drawn by the compass.

The compass can be fixed to have any radius (from zero radius to the largest spread of the compass). Once the radius is fixed, this distance can be laid off on any lines to form congruent line segments with this length. Thus, we can construct a line segment congruent to a given line segment: the compass is used to lay off the distance (that is, to fix the two endpoints), and the straightedge is used to draw the line segment between the endpoints.

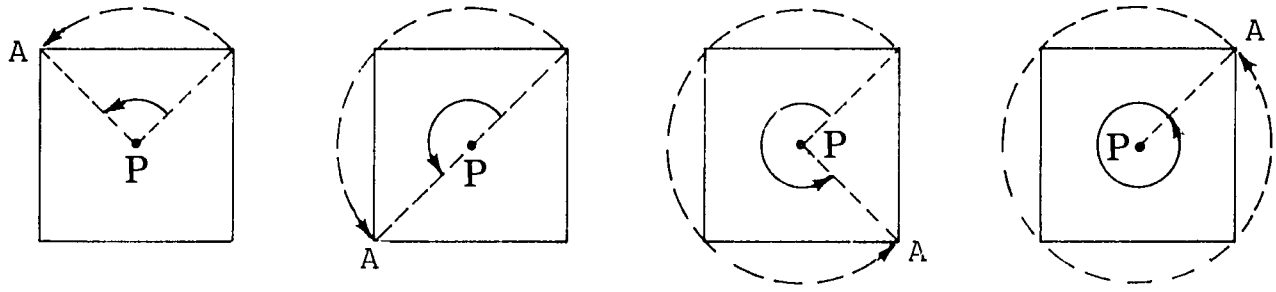
We recognize certain solid figures, such as boxes, cubes, spheres, cylinders, and cones. A more difficult skill is to draw sketches of these solid figures on the flat surface of a sheet of paper. We can help children develop this skill. They can cut out patterns and fold them to build paper models of some of the solid figures and with these models in front of them, begin to make rough sketches and begin to "see" the solid in a plane drawing.

In this unit, the following ideas are revised: the ideas of parallel movements and turning movements; and the idea of folding the cutout of a figure to see how the parts can be made to fit. (Such a folding is called a folding symmetry.)

Figures can be classified by the number of their folding symmetries:

Figure	Folding Symmetries
Rectangle (not square)	2
Square	4
Isosceles triangle (not equilateral)	1
Equilateral triangle	3
Circle	Any number (about any diameter)
Oval	2 or more

Another kind of symmetry is a turning symmetry. For example, a cutout of a square can be turned about a point P until it is moved to the same position as before. This will happen if it is turned $\frac{1}{4}$ of the way around, $\frac{1}{2}$ of the way around, $\frac{3}{4}$ of the way around, or all of the way around (follow point A):



Each of these turns moves the square to the same position as before.

We can classify figures by the number of their turning symmetries:

Figure	Turning symmetries
Rectangle (not square)	2
Square	4
Isosceles triangle (not equilateral)	1
Equilateral triangle	3
Circle	Any number

STAGE 1: Using the Straightedge

Vocabulary: No new vocabulary

Materials: Rulers; pupil pages 229-231

Teaching Procedure

ACTIVITY 1: Drawing Line Segments, Angles, and Polygons Pupil pages 229-231

The purpose of this activity is to help children use a straightedge to draw figures made up of lines, rays, and line segments. (Help the children learn to hold straightedges firmly in place as they draw line segments.)

Let the pupils follow the directions on pupil pages 229, 230, and 231. Each pupil will need a straightedge. You help with the reading of the exercises and with the drawing of the figures. Let the pupils use thin paper to trace the points on page 231 and draw line segments connecting point 1 to point 2, point 2 to point 3, and so on. They will see a picture when they finish.

Answers for pupil pages 229 and 230:

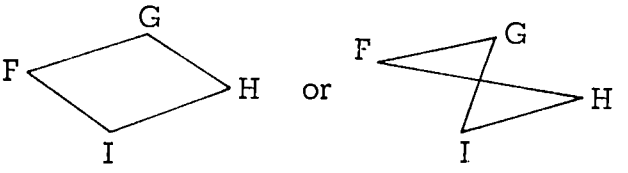
3. No

6. Three

7. Angle ECD; angle CDE; angle CED

8. Triangle

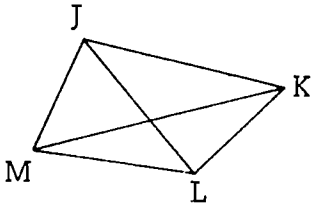
10. Two types of figures may be drawn:



11. Answer will depend on the figure drawn in (10). Those drawn above may be named FGHIF and FGIHF.

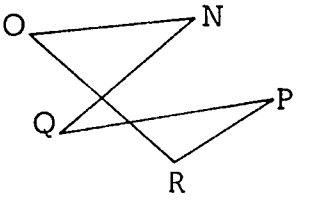
12. Answer will depend on figure drawn in (10). Yes for FGHIF. No for FGIHF.

14.



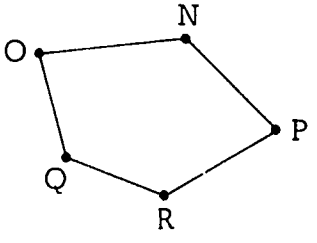
15. Six

17.

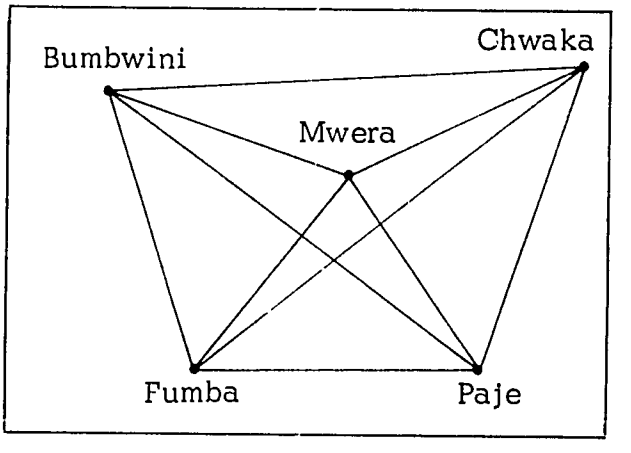


18. No. It crosses itself.

19.



20.



21. There are 10 roads.

22. B to F to M to C to P; B to F to C to M to P;
B to C to M to F to P; B to C to F to M to P;
B to M to C to F to P; B to M to F to C to P;
B to M to C to P; B to M to F to P;
B to F to M to P; B to F to C to P;
B to C to M to P; B to C to F to P;
B to F to P; B to M to P;
B to C to P; B to P.

23. Sixteen different paths.

STAGE 2: Using a Compass

Vocabulary: Compass, radius of compass

Materials: Ruler and compass for each pupil (or each group of pupils); looped string (about 6" long); and three drawing pins for each group of pupils; pupil pages 232-234

Note: If your school does not have compasses, make cardboard compasses like this: Cut a narrow strip of stiff cardboard, 1 inch by 7 inches; make a small hole about $\frac{1}{2}$ inch from one end; put a drawing pin through this hole (the hole is the pin point of this cardboard compass); make other holes in the strip at various distances from the pin point (1 inch, 2 inches, 3 inches, ... 6 inches); make each hole large enough for a pencil point.

Teaching Procedure

ACTIVITY 1: Drawing Figures with Looped Strings Pupil page 232

Give each group of pupils a piece of looped string and three drawing pins. Ask them to open their books to pupil page 232. Get them to press a drawing pin in the middle of their exercise books and place the looped string around the pin. Let them read the first exercise and follow the directions. Go around the class and help pupils to keep the strings stretched as they draw circles with their pencils.

Note: If your school has no drawing pins, get the children to use two pencils. Help them to hold one pencil point firmly on the exercise books and use the other pencil to draw a circle.

Let the pupils continue reading the exercises and following the directions. They will need rulers for Exercises 3 to 6. Talk about Exercise 6 and guide the children to say that $\overline{CA}$, $\overline{CB}$, and $\overline{CD}$ all have the same length (all are congruent) because each is a radius of the circle.

For Exercise 7, help the pupils to place two drawing pins (less than 3 inches apart, so that the looped string will fit loosely around them) on their exercise books. Show them how to hold a pencil against the string to draw a closed curve. Let them discuss Exercise 8, but do not tell the answer. They will learn the answer for themselves in Exercises 12 and 16. (A circle can be folded along any diameter to make the parts fit. But the figure drawn in Exercise 7 can be folded along only two lines to make the parts fit. It is not a circle.)

Note: The figure drawn in Exercise 8 is called an oval. Do not introduce this word to the pupils.

If the children enjoy this activity, let them place three pins on their exercise books so that the looped string will fit loosely around them. Then a pencil held tightly against the string will draw a closed curve that is not a circle and not an oval. (The children will find that the cutout of this figure can be folded once if the pins are vertices of an isosceles triangle that is not equilateral; three times if the pins are vertices of an equilateral triangle; and no fold if the pins are not vertices of an isosceles triangle.)

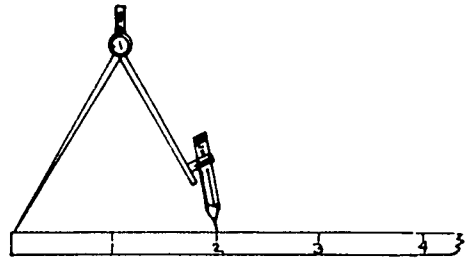
ACTIVITY 2: Constructing Congruent Line Segments

Pupil page 233

Give each pupil a compass. (If compasses are not available, make cardboard compasses as described in "Materials".) Show the pupils how to hold the compass: hold the top of the compass between the thumb and forefinger; spread the compass to form a gap between the pin point and pencil point; press down lightly to set the pin point in the paper; and twist your fingers to turn the pencil point on the paper.

Let the children practice drawing figures on their exercise books. Let them draw short arcs and long arcs with the same distance between the pin point and pencil point. Let them discuss the figures they draw. They will decide that (1) if the pencil is turned about the pin point to form a closed curve, the figure is a circle; (2) if the pencil turns part way around, the figure is part of a circle; (3) the distance between the points of the compass is the same as the length of the radius of the circle drawn. Say, "The distance between the points of the compass is called the radius of the compass".

Show the class how to set the radius of their compasses: hold the pin point against the end of a ruler and then set the pencil point against any desired mark on the ruler:



Radius set at 2 inches

Let the class turn to pupil page 233 and follow the instructions in the exercises.

Exercises 3 and 4 will help the pupils learn that with a fixed radius of the compass they can lay off congruent line segments, each as long as the radius. In Exercises 5 to 9 they learn how to mark off congruent line segments along a given line segment. They separate the 16 centimetre line segment $\overline{CD}$ into four 4 centimetre line segments. In Exercises 10 to 12, they measure $\overline{PQ}$ using $\overline{RS}$ as a unit and find that the length of $\overline{PQ}$ is 5 units.

ACTIVITY 3: Drawing a Circle with a Compass | Pupil page 234

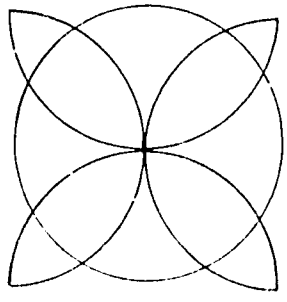
Ask the children to set the radius of their compasses at 3 inches and in their exercise books draw part of a circle and then all of a circle. Continue with radii of 3 inches, 4 inches, 5 centimetres, and so on as needed.

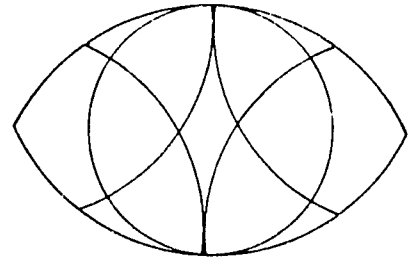
Get the children to do the exercises on pupil page 234.

In Exercises 1 to 6 the pupils revise the fact that the radii of a circle are all congruent and that a diameter of a circle is twice as long as a radius.

In Exercises 10 to 18, the pupils learn that two circles can intersect in one point, as in (12), in two points, as in (17), or not intersect, as in (15).

For more practice in the use of a compass, let the class draw designs of their own. There is no end to the number and the variety of figures they can draw, for example:





STAGE 3: Sketching Solid Figures

Vocabulary: Face, edge, vertex, rectangular solid, cube, cone, pyramid

Materials: Five sheets of tracing paper and five sheets of stiff paper for each group of pupils; scissors; objects showing solid figures (rectangular solid, cube, cone, sphere, cylinder, pyramid); pupil pages 235-239

Teaching Procedure

ACTIVITY 1: Making Models of Solid Figures Pupil pages 235-239

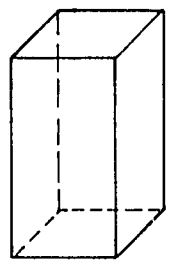
Get out your collection of objects showing solid figures and put them on the table. Let the class talk about the figures, their names, how they differ in the number of faces, vertices, and edges or in their roundness or flatness. Do not spend much time on this part of the activity. Its purpose is to get the pupils thinking about shapes of solid figures.

Say, "We are going to make some solid figures. Then you can decide what figures they are".

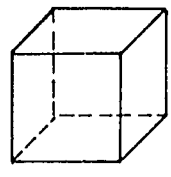
Tell the class to turn to pupil pages 235 to 239. Let groups of pupils copy the five figures on sheets of stiff paper as follows:

1. Trace the figures (solid lines and dotted lines) on pieces of tracing paper.
2. If there is carbon paper at hand, use it to copy the figures on pieces of stiff paper. (If not, copy the figures on tracing paper and punch through the tracing paper at corners of the figure on to the stiff paper.)
3. Cut the stiff paper along the outlines of the figures.
4. Fold along all the dotted lines so that the edges come together to form a solid.
5. Use gum on the shaded tabs to hold the figures together.

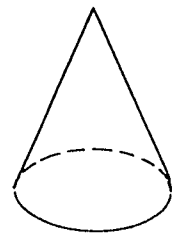
Let each pupil have a part in making these figures. When each group has made five figures, ask them to line the figures up on their table tops. The figures they make are these:



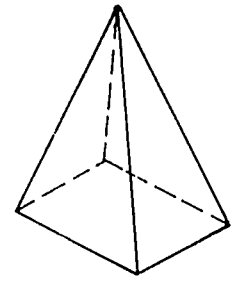
Page 235
Rectangular Solid



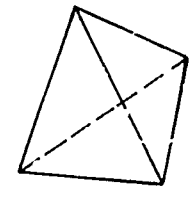
Page 236
Cube



Page 237
Part of Cone



Page 238
Pyramid
(five faces)



Page 239
Pyramid
(four faces)

Ask, as you point to the figure from pupil page 235, "What is this figure? (A rectangular solid) Why do we call it a rectangular solid? (Because all its faces show rectangles.) How many faces? (6) How many edges? (12) How many vertices?" (8)

As you ask these questions let the pupils use their own rectangular solids to help them with the answers.

Ask the same questions about the other figures.

Pupil page 236: Figure is a cube (all faces show squares) with 6 faces, 12 edges, and 8 vertices.

Pupil page 237: Figure is part of a cone. It has one vertex. (We cannot say it has any faces or edges. There is no bottom on the figure the children made. The bottom would show a circular region.)

Pupil page 238: The class will probably not know a name for this figure. Tell them it is called a pyramid. Let them tell about its faces. (Four faces show triangles and one face shows a square.) It has 8 edges and 5 vertices.

Pupil page 239: Tell the class that this is another pyramid. Ask how it is different from the other pyramid? (All its faces show triangles. It has 4 faces, 6 edges, and 4 vertices.)

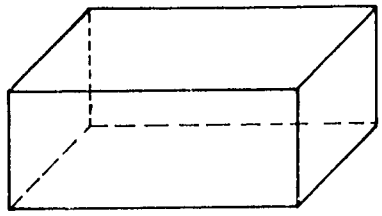
ACTIVITY 2: Sketching Solids

Ask each child to look at the rectangular solid he helped make. Ask, "How many faces has the rectangular solid? (6) How many vertices? (8) How many edges?" (12)

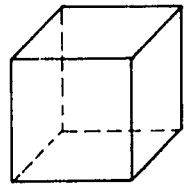
Place one of the rectangular solids on the table and invite all the children to look at it. Say, "Watch as I make a picture of this on the blackboard".

Imagine that you are looking at the model from the same position as the pupils. Begin to sketch on the blackboard the faces they can see most clearly. Each time you draw a line segment show them the edge of the solid it represents. When a rectangle is drawn, let them show the face of the solid it represents. Use dotted lines to show the faces, edges, and vertices which you cannot see. Let pupils sketch a picture of their rectangular solid as they see it.

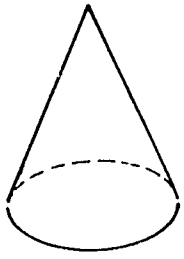
Follow the same procedure for sketching the solids shown by the other models. The finished sketches will appear as shown at the top of page 317. (Do not expect their sketches to be perfect.) Ask them to criticise each other's pictures and to make suggestions for improving them.



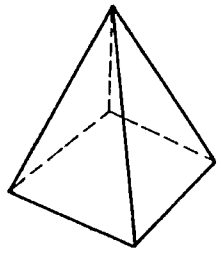
Rectangular solid



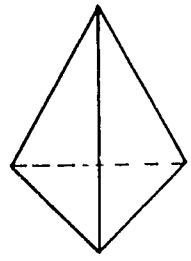
Cube



Cone



Pyramid



Pyramid

STAGE 4: Movements and Symmetries of Figures

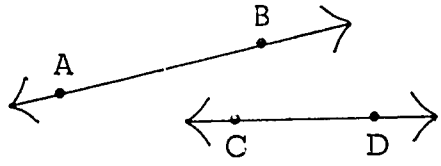
Vocabulary: Parallel lines, parallel movement, turning movement, turning symmetry, folding symmetry

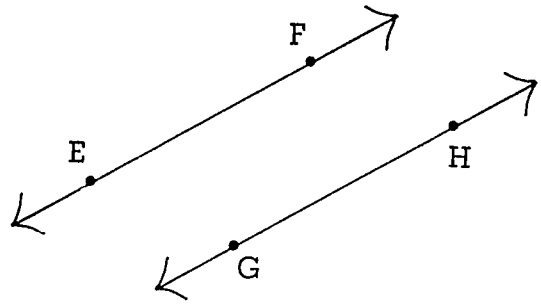
Materials: Large newsprint cutouts of four-sided figure (not a rectangle), rectangle (not a square), square, triangle (not isosceles), isosceles triangle, equilateral triangle, circle, and oval; large sheet of paper; pupil pages 240-243

Teaching Procedure

ACTIVITY 1: Revising Parallel Lines | Pupil page 240

Draw on the blackboard two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ that will intersect if they are extended, and two lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ that are parallel.





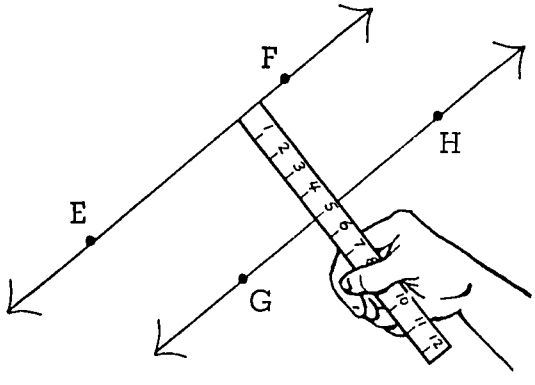
Ask, "Are $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel lines?" Let the children decide that they are not, because the lines intersect. (This gives the class an opportunity to discuss the fact that lines go on in both directions without end.)

Ask, "Are $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ parallel lines?" As children try to decide, lead them to suggest a way to test whether or not the lines are parallel. The discussion should revise these ideas:

(a) Two lines in a plane are parallel if they do not intersect;

(b) Parallel lines are the same distance apart all along the lines; and

(c) To test that the lines are parallel slide a ruler between the lines.



Let a pupil test lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ by sliding a ruler between them, as in the above drawing. Ask other pupils to do the same thing until everyone agrees that the lines are parallel. The lines are the same distance apart all along.

Repeat this activity by letting pupils test opposite edges of a book to see if they are on parallel lines. Draw more lines on the blackboard, some pairs parallel and other pairs not parallel, and let pupils test them.

Continue this activity by letting pupils turn to pupil page 240 and decide which pairs of lines are parallel. They will write in their exercise books:

$\overleftrightarrow{CD}$ and $\overleftrightarrow{KL}$ are parallel;

$\overleftrightarrow{AB}$ and $\overleftrightarrow{PR}$ are parallel;

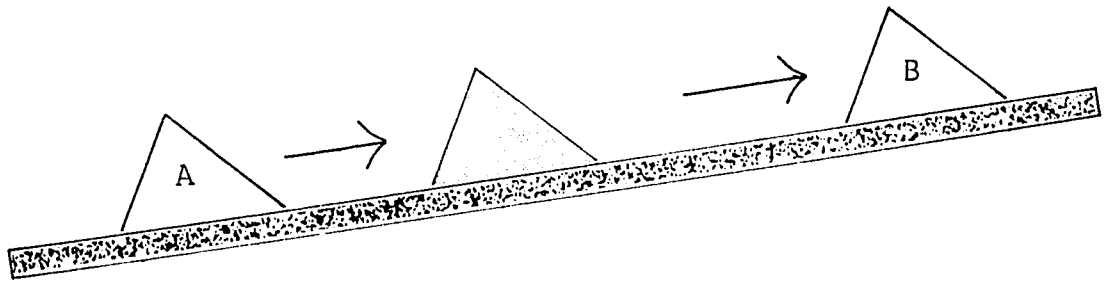
$\overleftrightarrow{ST}$ and $\overleftrightarrow{MN}$ are parallel.

ACTIVITY 2: Revising Parallel Movements and Turning Movements

Pupil page 241

Revise the idea of a parallel movement as a movement along parallel lines. Hold a cutout of a triangle against the blackboard and trace it. Mark A on the picture of the triangle as shown below. Ask, "How can we move this triangle by a parallel movement?" Let the children decide that you can hold a straightedge along one side of the triangle and then slide the cutout of the triangle along the straightedge to a new position.

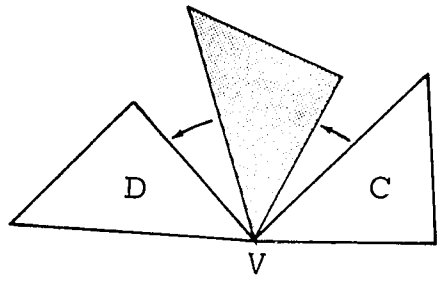
Do this and then trace the cutout in the new position. Call the new picture of the triangle B.



Say, "Triangle A was moved to triangle B by a parallel movement. Will one fit exactly on the other? (Yes, because they both fit the same cutout.) Are triangles A and B congruent?"

If needed, move other figures, say rectangles or other four-sided figures, by parallel movements and ask the same question about congruence.

Revise the idea of turning movement as a movement about a point or along circles. Trace a cutout of a triangle on the blackboard and call the picture triangle C. Say, "We will move triangle C by a turning movement about a point". Mark a vertex of the triangle with a V and hold the cutout over the triangle. Put a pin at V and slowly turn the cutout about the point V. After turning about a half revolution, stop and again trace around the cutout. Call the new triangle D.



Say, "We moved triangle C to triangle D by a turning movement about point V. Is triangle C congruent to triangle D?" (Yes, because they both fit the same cutout.)

If needed, move other figures with turning movements.

Trace a cutout on the blackboard (any figure from your collection of cutouts). Label it X. Put the cutout over the drawing. Then slowly move the cutout parallel to its first position for some distance, stop, and turn the cutout about a vertex of the cutout or some other point of the cutout. Stop the motion and again trace the cutout. Label the new figure Y.

Ask, "How did I move figure X to figure Y? (By a parallel movement and then a turning movement.) Is figure X congruent to figure Y?" (Yes, because the figures we get by these movements fit the same figure.)

Let pupils practice this activity by use of pupil page 241. Ask the children to trace a figure, say N, and make a cutout. Then place the cutout on N. Ask the children to slide the cutout on the page of the book with parallel and turning movements only and try to fit the cutout on another figure.

N will fit on F (but not on P).

Let them do a similar activity with B and then with K.

B will fit on R (but not on E).

K will fit on D (but not on C).

<u>Answers for pupil page 241</u>:

N is congruent to F

B is congruent to R

K is congruent to D

ACTIVITY 3: Folding Symmetries and Turning Symmetries

Pupil pages 242-243

Put out your large newsprint cutouts of:

Triangle (not isosceles),
Isosceles triangle (not equilateral),
Equilateral triangle,
Four-sided figure (not a rectangle),
Rectangle (not a square),
Square,
Circle,
Oval (rounded figure, not a circle).

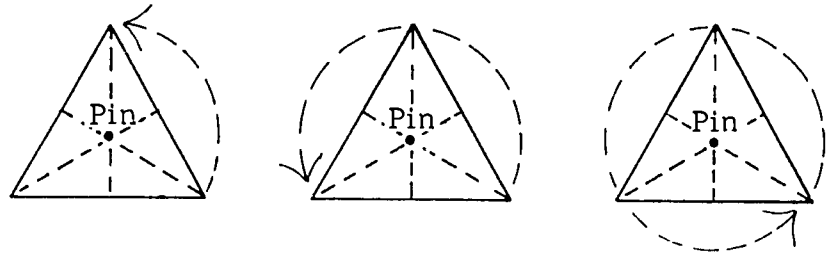
Choose one of the cutouts, say the isosceles triangle, and trace it on the blackboard. Then hold up the cutout and ask, "How many ways can we fold the cutout so the two parts formed will fit each other?"

Let the child try to fold the cutout until he succeeds in making the two parts fit. Then ask another child to try to find a different way of folding. Let the class decide that there is only one way to fold this figure so the two parts will fit. Say, "When we can fold the cutout so the parts fit, we say the figure has a folding symmetry". (The word "symmetry" will need to be used several times until the pupils learn it.)

Put the folded cutout against the figure drawn on the blackboard and trace a dotted line along the fold, showing that the figure has one folding symmetry. Ask, "What kind of triangle has one folding symmetry?" (An isosceles triangle, because the fold shows it has two congruent sides.)

Unfold the cutout and again hold it against the triangle drawn on the blackboard. Ask, "Can we turn the cutout so that it fits the figure again?" As you say this, place a pin through the centre of the cutout and turn the cutout slowly about the pin. Let the pupils watch the cutout and the drawing under it to see whether they fit. They will find that it does not fit until the cutout has been turned around completely to the original position.

Repeat these same activities with the cutout of the equilateral triangle. The class will decide that the equilateral triangle has three folding symmetries. The pupils will also see that if a pin is placed in the cutout at the intersection of the folds, the cutout can be turned about the pin to three positions where the figures fit.



Say, "Each time the cutout fits the figure as we turn it, we have found a turning symmetry. How many turning symmetries does the equilateral triangle have?" (Three)

Continue this activity with each of your cutouts. Make a chart on the blackboard like the one below. Make a record of the results. (Let the children copy the chart in their exercise books.)

Figure	Folding Symmetries	Turning Symmetries
Isosceles triangle	1	1
Equilateral triangle	3	3
Rectangle	2	2
Square	4	4
Circle	Any number	Any number

The children will find that the cutout of a circle can be folded along any diameter so that the parts will fit and it can be turned any amount around its centre and it will always fit. If the children fold and turn cutouts of a triangle that is not isosceles or a four-sided figure that has no sides congruent, they will find no folding symmetries. All figures have at least one turning symmetry (by turning through one complete revolution).

On pupil pages 242 and 243 there are six figures shown. Let the children trace these figures and make cutouts. Ask the children to fold the cutouts and decide how many folding symmetries each has. Then ask the children to turn the figures to decide how many turning symmetries each has.

Answers for pupil pages 242 and 243:

		Figure	Folding Symmetries	Turning Symmetries
Page 242	(1)	F	4	4
	(2)	N	0	1
	(3)	D	2	2
Page 243	(4)	V	5	5
	(5)	T	3	3
	(6)	S	1	1

Note: Guide the pupils to place pins at the intersection of the folds of the cutouts to find turning symmetries. If there is only one fold, the pin can be put anywhere along the fold. If no fold, put the pin anywhere on the figure.

UNIT 20
Collecting and Recording Data

Objectives

1. To prepare for the idea of probability by collecting data in simple games of chance
2. To show how information can be set out in tables and in graphs
3. To introduce the reading of graphs of data

Background Information for Teachers

We do many things each day without being sure what the outcome will be. When we leave home for school, we cannot be sure we will arrive on time. When children play a game of snakes and ladders or ludo, we cannot be sure of the number that will show when they roll the die. When we select a goal at the start of a football game by tossing a coin, we do not know whether the coin will show heads or tails. When a farmer plants his seed, he cannot be sure of the size of the crop he will harvest.

Often the things we do have uncertain outcomes. Whether we arrive at school on time depends on the weather, on our state of health, perhaps on the mechanical condition of the bus or bicycle that transports us, and on many other things. When we roll a die, we know it will fall with its face showing one of the numerals 1, 2, 3, 4, 5, or 6, but we do not know in advance which particular numeral will show. Also, when we toss a coin, we cannot know before we toss the coin whether it will fall heads or tails. The farmer's harvest depends on the quality of his seed, the amount of sunshine and rainfall, the fertility of his soil, and his care in warding off weeds and insect pests. Our world is full of uncertainty and risks.

Probability is a branch of mathematics that helps us to measure uncertainty and risks. In Entebbe Mathematics, Primary Five and Six, the pupils will learn to find the probabilities of some events. In this Unit, the

pupils will prepare for their study of this important branch of mathematics. They will begin by studying sets of a new kind. These are sets of information, or data.

The children will learn to:

(a) collect data (or sets of information),

(b) organize data,

(c) picture data in tables and graphs,

(d) interpret data.

There are many sources of data in which the children are interested and which they can study. For example, your school might keep a record of daily rainfall. If not, you may get the pupils to record the number of inches of rain that fall each day during a month.

Other sources of interesting data are your school's records of temperature, attendance, and the number of pupils in each class, the daily growth of a plant, the length of the shadow of a stick at different hours of a day, the number of goals scored in a series of football games, the number of brothers and sisters in each of the pupils' families, or their own heights and weights. From newspapers and books, there is sporting data, the population of different villages in their country or of different countries of Africa, tax data, and the values of imports and exports.

We can also <u>collect</u> data by playing simple games of chance, such as tossing coins or throwing dice. For example, suppose we throw a die twenty times and record the results. The data may look something like this:

5, 3, 2, 2, 1, 6, 4, 3, 5, 6.

2, 1, 4, 2, 3, 5, 1, 6, 6, 2.

There is no set pattern in this data as it is recorded. The data can be organized as shown below. Write the six numerals and make a stroke under a numeral for each piece of the data.

1	2	3	4	5	6
///	~~////~~	///	//	///	////

The data is now organized so that we can easily tell which number occurred the greatest number of times, and which occurred the least number of times.

To picture the data, we may also record the results on squared paper, like this:

	×				
	×				×
×	×	×		×	×
×	×	×	×	×	×
×	×	×	×	×	×
1	2	3	4	5	6

To interpret the data we ask ourselves questions like this:

Why does 2 occur more times than other numerals?

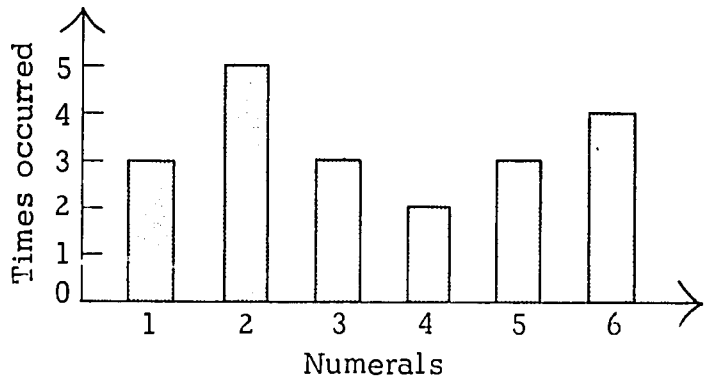
Why does 4 occur fewer times than other numerals?

If the experiment is repeated will 2 occur the most times?

If the experiment is repeated will 4 occur the fewest times?

To test these questions, we can repeat the experiment several times. Finally, we conclude that any of the numerals is about as likely to appear as any other.

The picture made on squared paper is just one way of picturing data. It can be changed into a bar graph. For the data pictured above, the bar graph looks like this with each bar the same width, and the length of a bar showing the number of times a numeral occurred.



In this Unit and in other units of Entebbe Mathematics, Primary Five and Six, the pupils will use these and other ways of picturing data.

STAGE 1: Simple Experiments in Chance

Vocabulary: Tossing coin, head, tail, runs, experiment, die

Materials: Coins, cardboard model of cube from Unit 19 (or a die)

Teaching Procedure

ACTIVITY 1: Tossing a Coin

Say, "You do many things without knowing what the results will be". Give an example and let the pupils give others. Examples are:

- We do not know who will win the next football match.
- We do not know when we kick a ball whether it will go through the goal posts.
- We do not know whether a seed will grow when we plant it.

Say, "We are going to do some experiments. We do not know what the results will be".

Show the class a large coin. Ask what is on the two sides of the coin. Tell them we call one side the head and the other side the tail. Say, "I will throw the coin. Can you tell which side the coin will show when it lands?" (No) Toss the coin and get the children to tell the side it shows when it lands.

Get two children, say Juma and Lima, to come out. Give the coin to Juma and a piece of chalk to Lima. Let Juma toss the coin and let Lima write the results on the blackboard. If the coin shows the head, Lima writes H for head; if it shows the tail, he writes T for tail. Let Juma toss the coin ten times. The results Lima writes on the blackboard may look like this:

H, T, T, H, T, H, H, H, T, H

Ask, "Did the results of the toss go like this, H, T, H, T, H, T, ...? (No) How many heads were there? (6) How many tails? (4) What is the

largest run of heads? (3) (Explain that a run is the number of consecutive times the head or tail falls.) What is the largest run of tails?" (2) Underline the runs like this:

H, T, T, H, T, H, H, H, T, H

run of 2 T's run of 3 H's

Get another pair of children to repeat the experiment. The result will be different. It may be this:

T, T, H, H, T, H, H, H, H, T

Ask the children questions like those above. Let other pairs of children do the experiment with the coin until there are five lists of ten marks each on the blackboard. Ask questions like these about the experiments:

- Which experiment has the most heads? The most tails?
- What is the largest run of heads? Of tails?
- What is the total number of heads in the 50 tosses?
- How many tails are there?
- If we repeat the experiment once more, about how many heads would we expect to show?

Discuss any answer given to the last question. Encourage children to give reasons for their answers. Talk about the reasons given. Let the class decide that there is no reason to believe that heads will occur more often than tails, or that tails will occur more often than heads.

ACTIVITY 2: Rolling a Die

Show the children a cardboard cube (die) with numerals 1 to 6 marked on the faces. You can mark numerals on a cardboard model of a cube made in Unit 19 or get a regular die as used in games like ludo. Say, "You threw a coin and recorded the results. Now you will roll the die and record the results".

Get two children, say Ben and Frank, to come out. Give the die to Ben and a piece of chalk to Frank to record the results on the blackboard. Tell the class that when Ben rolls the die it will show one of its numerals, 1, 2, 3, 4, 5, or 6, on top. Ask, "Can you tell what numeral will show each time he

rolls the die?" (No) Let Ben and other children roll the die 60 times. The results Frank writes may be these:

5, 3, 2, 2, 1, 6, 3, 5, 6, 1,
2, 1, 4, 2, 3, 1, 6, 6, 2, 3,
4, 3, 4, 5, 2, 3, 5, 2, 5, 1,
6, 2, 1, 3, 2, 5, 6, 1, 3, 6,
3, 2, 5, 4, 4, 4, 6, 3, 4, 5,
3, 4, 2, 4, 1, 2, 6, 1, 4, 6.

Ask, "Do you notice any pattern?" The pupils will agree there does not appear to be any. Tell two other children, say Goma and Ruth, to come out. Ask Ruth to write the numerals 1, 2, 3, 4, 5, 6 on the blackboard and to make strokes under a numeral each time Goma calls out a number from the list. When they are finished, the result may look like this:

1	2	3	4	5	6
卌	卌	卌	卌	卌	卌
𝍸	卌	卌	卌	𝍸	卌
	𝍸	𝍸			

Say, "Which numeral showed the greatest number of times? (2, it showed 12 times.) Which numeral showed the least number of times?" (5, it showed only 8 times.)

Say, "You can show this same set of information by means of a picture". Write the numerals 1, 2, 3, 4, 5, 6 in order on the blackboard. Above each numeral, write an × for each time the numeral showed. For example, above the 1 make 9 ×'s. The final result looks like this:

	×				
	×	×			
	×	×	×		×
×	×	×	×		×
×	×	×	×	×	×
×	×	×	×	×	×
×	×	×	×	×	×
×	×	×	×	×	×
×	×	×	×	×	×
×	×	×	×	×	×
×	×	×	×	×	×
×	×	×	×	×	×
1	2	3	4	5	6

Let the children look at the figure and tell which column of X's is highest, (2) and which is shortest. (5) Ask, "If we rolled the die another 60 times, do you think 2 would show most often again? Can you tell beforehand which will show most often?" Let them discuss this question and decide that no one can tell what will happen. There is no reason to believe that any one of the numerals will show more often than any other. It seems likely that one numeral will show as often as any other.

Note: Allow any kind of discussion about this experiment. In Entebbe Mathematics, Primary Five, the children will learn that in an experiment like this each outcome is just as likely to happen as any other. But in other kinds of experiments some outcomes are more likely than others. The pupils will then learn to measure this likelihood. In this unit pupils are not ready to do more than list the outcomes and put them in table form.

STAGE 2: Collecting Data and Reading Graphs

Vocabulary: Data, graph, bar graph, circle graph

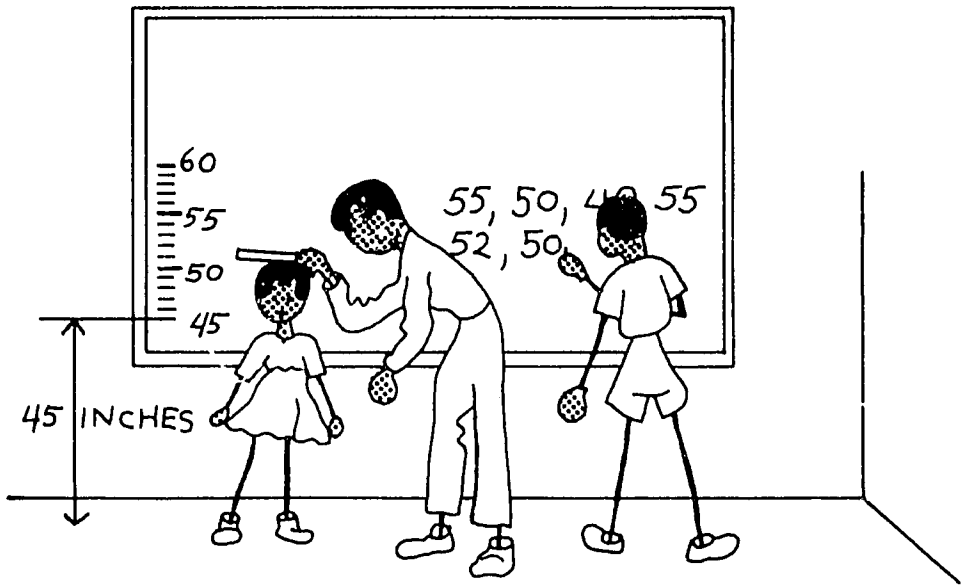
Materials: Pupil pages 244-245

Teaching Procedure

ACTIVITY 1: Collecting Data

Say, "There are many kinds of experiments we can do to learn about the world around us. You did experiments to learn about the chances of a coin falling in certain ways or a die rolling in certain ways. Now you will do another experiment to learn about the heights of the pupils in the class".

With a measuring stick mark points on the blackboard or the wall at these distances from the floor: 45 inches, 46 inches, ... 60 inches. (See the picture on page 331.) Then ask each pupil, in turn, to come to the blackboard to have his height measured.



Get a pupil to write the heights in inches on the blackboard as you measure heights of the pupils. (Let the class watch as you do the measuring. You can hold a ruler flat on the child's head and against the measuring scale. Use the lower inch measure. For example, if the height is between 50 and 51 inches, write it as 50 inches.)

Tell each child to remember his height in inches. A pupil will write each height, and the record on the blackboard may look like this:

53, 50, 49, 53, 51, 50, 49, 51, 55, 53,
55, 52, 53, 59, 52, 54, 52, 52, 53, 51,
51, 50, 50, 51, 47, 50, 58, 53, 55, 52, 56

Let the class look at the numerals on the blackboard. Discuss with them the fact that this is a set of <u>data</u> (a set of information) about their heights. The set tells many things about their heights. Let them discuss what the data tells them. They may say that it shows:

- Who is tallest;
- Who is shortest;
- How many pupils are 51 inches tall;
- Which height occurs most often;
- Which height is the middle height, from the shortest to tallest;
- How tall we expect a Primary Four pupil to be in any school.

Help by asking questions such as:

1. What is the greatest height listed on the blackboard? (59)
   Who is the tallest pupil? (Obi. He is 59 inches tall.)
2. What is the least height listed on the blackboard? (47)
   Who is the shortest pupil? (Ada. She is 47 inches tall.)
3. How many pupils are 51 inches tall? (At this point let the class count the number of times the 51 appears in the list. There are 5 pupils who are 51 inches tall.)

Suggest to the class that there are better ways to show the data listed on the blackboard. The list is mixed up. Guide them to put it in order. Write on the blackboard, below the record the pupil made, each height in order from the smallest to largest. (See below.) Let a pupil stand at the blackboard and make a stroke under a numeral each time you call out a number from the list. The final result may look like this:

47	48	49	50	51	52	53	54	55	56	57	58	59
/		//	~~////~~	~~////~~	~~////~~	~~////~~/	/	///	/		/	/

The children will see that this list of heights is easier to think about than the mixed list. Ask questions like these:

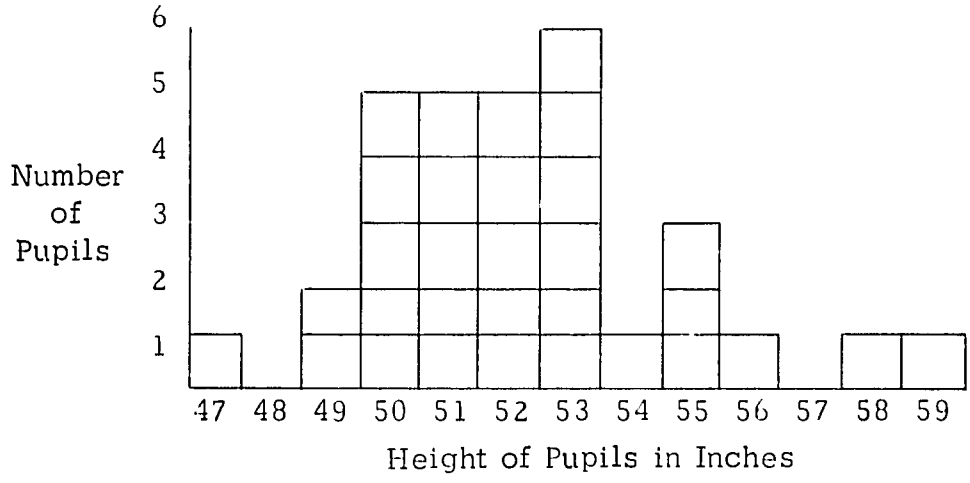
1. "Which height appears the most times?" (53 inches. More pupils are 53 inches tall than any other height. There are 6 pupils who are 53 inches tall.)

2. "If you stand in line from shortest to tallest, how tall is the middle pupil?" Let the class think about this. Help them decide that in the class of 31 pupils the 16th pupil in line is the middle one. (This is only an example, and your class will probably have a different number of pupils. If there are an even number of pupils, there will be two middle heights.) Let them count the marks to find the 16th mark. It is in the 52-inch group. The middle height is 52 inches.

3. "If a new pupil comes into our class, how tall do you think he will

be? Would you be surprised if he were much less than 47 inches tall? Would you be surprised if he were much over 59 inches tall?" Let the class decide that they would expect a new pupil to be about as tall as someone in the class.

Say, "There is another way to show the data about heights. We can show the heights with a graph". On a large piece of squared paper, make a graph like this (or draw it on the blackboard):



Let the children check the graph to see that it shows the same information as the table. (There are 31 squares, one for each pupil. Above each height there is a certain number of squares to show how many pupils are that height.) Encourage the children to read information from the graph. For example, the graph shows that 3 pupils are 55 inches tall, no children are 57 inches tall, and so on.

As an additional activity do the following, if you have the necessary information. Get a record of the heights of pupils in a Primary Two or a Primary Three class. Prepare a graph of these heights like the graph above, and show it to the class. (Do not tell the children from what class the information was taken.) Ask whether they think the new graph shows heights of pupils in a Primary Four class? A Primary Three class? Let them give reasons for their guesses. They should see that the heights shown by the new graph are from, say 43 inches to 55 inches. The children are younger than they. Let the pupils read information from the new graph and discuss what it tells them about heights of pupils in the other classes.

ACTIVITY 2: Reading Graphs Pupil pages 244-245

In Activity 1, the children made a graph and read the information given in the graph. In this activity they will have more practice in reading graphs of different kinds.

Ask the pupils to open their books to pupil page 244. Tell the pupils that this picture is a graph, called a bar graph. The graph shows Secondary Grammar School enrollment in Nigeria for the years 1956 to 1961. Let the pupils study the graph for some minutes. Point out that the enrollments are shown on the vertical line, and the years are shown on the horizontal line. Ask, "What was the enrollment in the year 1958?" Reading from the top of the 1958 bar across to the vertical number line the pupils read that the enrollment was approximately 90,000. The pupils may use a ruler to help them. Let them answer the other questions on page 244.

Pupil page 245 shows a circle graph of the time Onyerisara spent doing certain things during a day. Allow the pupils several minutes to study the graph. Ask, "What can you read from the graph?" Let the pupils talk about the graph and answer the questions on page 245.

Answers for pupil page 245:

1. 25% 2. $\frac{1}{4}$ 3. 6 hours 4. $\frac{3}{8}$ 5. 9 hours 6. 9 - 3 = 6

7. 6 - 6 = 0 8. $\frac{1}{8}$

UNIT 21
Clock Arithmetic

Objectives

1. To use clock arithmetic to introduce sets of numbers with only 4 or 5 members
2. To introduce a new operation and to find some of its properties

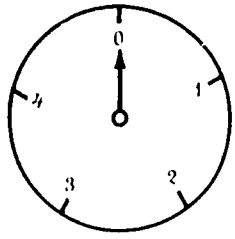
Background Information for Teachers

The purpose of this Unit is to introduce sets of numbers which have only a few members. A new kind of addition will be defined on these sets of numbers. We will determine properties of the operation and find relationships between numbers in each set.

There are two reasons for introducing this topic. First, we are familiar with several important sets of numbers: the whole numbers, the fractions, and the integers. We can operate with these numbers and use the properties of the operations. The set of whole numbers has many members. There are only a small number of members in the sets we consider in this Unit. Because of this, it will be easy to study the operations on members of these sets. The properties of these operations are the same as those we know. Since this is the last unit in the Primary Four programme, it provides a summary of some important ideas developed earlier. The setting, however, is quite different.

Second, there is a close relationship between the ideas developed here and the ideas developed in the geometry units. These relationships will be discussed in Entebbe Mathematics, Primary Five and Primary Six. In this background section, the mathematical ideas which form the basis for the topic are discussed.

Suppose we have a circle with 5 marks on it, as on the picture:



Assign the first five whole numbers as shown. To mark positions, we put in a single hand which turns about the centre of the circle like a clock hand. To the children this will look like a dial or a clock face with one hand. We will call it a "5-clock" because it has 5 marks labeled.

The hand of the clock can move either clockwise ( ) or anti-clockwise ( ). The usual movement is clockwise. We write 4 to mean that the hand starts at 0 and moves 4 spaces clockwise to the position 4. The other numbers, 0, 1, 2, and 3, are shown in a similar way.

<u>Clock Addition</u> is defined as two consecutive movements of the hand. The first movement begins at 0, the second begins at the mark where the first movement ends. For example, "4 + 3" means "the hand starts at 0, moves 4 spaces to the 4-position, from there, 3 spaces to the 2-position". The "sum" is shown by the final position of the hand. Therefore:

$$4 + 3 = 2$$

Similarly, $3 + 1 = 4$, $3 + 3 = 1$, $2 + 4 = 1$, $3 + 0 = 3$, and so on. All possible sums of these numbers are shown in the addition chart below:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

The 5-clock is used to define the operation and to make the addition chart. After we make the addition chart, and if we wish, there is no need to refer to the 5-clock. (For purposes of games and for introduction of sets with other than 5 members, the clock is still useful.) Since this addition

chart contains all possible sums, we can use it to study the properties of the operation of clock addition. The properties of clock addition are the following.

Closure The set of numbers $\{0, 1, 2, 3, 4\}$ used with the 5-clock is closed under the operation of clock addition. If we choose any two numbers from the set and add them, their sum is a member of the set. Looking at the addition chart, we see all sums are members of the set.

Commutative property If a and b are any two numbers in the set, then $a + b = b + a$. We can use the addition chart to show the commutative property. The numbers in the boxes below the diagonal are the same as those above the diagonal and are similarly placed. Another way to check this property is to try all possible cases. For example, since $2 + 4 = 1$ and $4 + 2 = 1$, then $2 + 4 = 4 + 2$.

Associative property If a, b, and c are any three numbers in the set, then $a + (b + c) = (a + b) + c$. For example:

$$2 + (3 + 4) = 2 + 2 \text{ or } 4$$

$$(2 + 3) + 4 = 0 + 4 \text{ or } 4$$

The other examples can be checked in the same way. (This property is not included in the activities of this Unit for the pupils.)

Property of zero If a is any number in the set, then $a + 0 = a$. Adding zero on the 5-clock is the same as not moving the hand at all.

Opposites For each member of the set $\{0, 1, 2, 3, 4\}$ there is another member such that their sum is 0. We say that b is the opposite of a, if $a + b = 0$.

0 is its own opposite, since $0 + 0 = 0$.

1 is the opposite of 4; and

4 is the opposite of 1, since $4 + 1 = 0$.

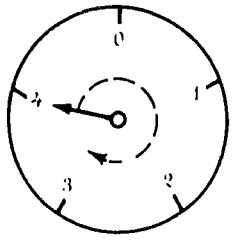
2 is the opposite of 3, and

3 is the opposite of 2, since $3 + 2 = 0$.

So far we have talked only about clock addition. Other "clock operations" can be defined. Subtraction is discussed below. Extensions to clock multiplication and clock division will be discussed in Entebbe Mathematics, Primary Five and Primary Six.

Clock subtraction is the process of finding the missing addend. For example, for what number $\underline{n}$ in the set $\{0, 1, 2, 3, 4\}$ is the sentence $4 + n = 3$ true? There are two ways we can think about this.

First, we can use a 5-clock:



We place the hand at the 4 position and find through how many units in a clockwise direction it is moved to get to 3. The missing addend is 4 because $4 + 4 = 3$. Then, $\underline{n}$ is 4.

Second, we can look at the addition chart. Find the sum 3 in the "4" row of the chart. Notice what column it is in. Since it is in the 4 column, this means that $4 + 4 = 3$, and $\underline{n}$ is 4.

In working with whole numbers we gave meaning to the symbol "-" for subtraction. For example, $9 - 4 = 5$ because $5 + 4 = 9$. This same symbol can be used to show clock subtraction. In clock arithmetic, "3 - 4" means "the number that must be added to 4 to give the sum 3". The result is shown on a clock by moving the hand to 4 and then finding how much farther it must be moved to get to the final position "3". The result on a 5-clock is:

$$3 - 4 = 4$$

The relationship between "+" and "-" is shown by these sentences:

$$3 - 4 = 4 \text{ because } 4 + 4 = 3.$$

$$4 - 3 = 1 \text{ because } 1 + 3 = 4.$$

Some of the ideas discussed here for 5-clocks are also true for clocks with other than five units. The number of members in the set is changed; all the properties are the same, except in some cases the property of opposites is no longer true.

STAGE 1: Solving Problems with a Clock

Vocabulary: Clockwise, clock addition

Materials: A small stick for each child, pupil page 246

Teaching Procedure

ACTIVITY 1: Once Upon a Time

Pupil page 246

Construct on the blackboard a clock face, with the circle having a radius of about 8 inches. Mark the points on the circle for 1 to 12 and write all the numerals. Mark the centre point clearly. (Use the picture on pupil page 246 as a model.) Tell the following story:

> Once there was a boy named Ali who lived in Zanzibar. This is the way he set his watch. When the sun came up in the morning he set his watch at 12 o'clock. (Illustrate by placing a pencil with one end at the centre of the clock face and letting the other end point to 12.)

Say, "Pretend this stick (or pencil) is the hour hand (the shorter hand) on Ali's watch. When the hand points to 12 the day begins. The sun is just rising". Write:

1 hour later - Ali is eating breakfast

Ask, "Where will the clock hand point then?" (To the 1) Let a child help you move the stick so that the end points to the 1. Emphasize the direction of the movement. Say, "We say that we are moving clockwise, because this is the way the hands on the clock move". Write:

3 hours after his breakfast - Ali is doing mathematics in school.

Ask, "Where will his clock hand point?" (To the 4) Let a pupil move the stick clockwise 3 spaces to the 4.

Say, "Think of what we have done as addition. You moved the clock hand twice. How many spaces did you move first? (1) How many spaces did you move next?" (3, because 3 hours after breakfast Ali is doing mathematics.) Show this in the sentence:

$$1 + 3 = 4$$

Say, "This is called clock addition. Each move you make gives an addend. The number of spaces moved is the addend".

Write the following on the blackboard. Get children to move the stick around the clock face, tell where the clock hand is, and write a mathematical sentence for each:

a. 4 hours after sunrise, Ali is in mathematics class; 3 hours after mathematics class, Ali is eating lunch. Where is the clock hand? $4 + 3 = n$. $(4 + 3 = 7)$

b. Ali leaves school when his clock says 9 o'clock; 4 hours later he is eating dinner. Where is the clock hand? $9 + 4 = n$. $(9 + 4 = 1)$

c. When the sun came up the clock hand was at 12; when the sun went down the clock hand was again at 12. How far had the hand moved? Where is the clock hand? $12 + 12 = n$. $(12 + 12 = 12)$

Get the children to notice by moving the stick around the clock face that if we start at, say 9 and move 4 spaces we arrive at 1. In clock addition, using Ali's clock:

$$9 + 4 = 1, \text{ and } 11 + 3 = 2.$$

Write the following sentences on the blackboard:

d. $6 + 4 = n$ e. $8 + 7 = n$ f. $8 + 10 = n$

Let the children open their books to page 246. Give each child a small stick to use as a clock hour hand. He can use the clock face pictured. Put one end of his stick on the centre point and move the other end around the clock face to help in finding what number n makes the sentence true. (The answers to the sentences above are: (d) 10, (e) 3, and (f) 6.)

Let the children continue to use the sticks and the picture of the clock face in solving the problems on page 246. Remind the children that when the story begins at, say 8 o'clock, they place the stick pointing to 8, then move it clockwise as the story suggests. You may go over each problem with the class solving it together to avoid difficulties in reading.

STAGE 2: Clock Addition

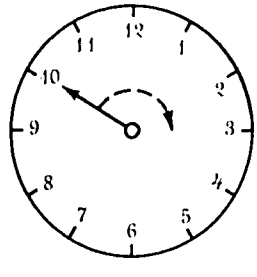
Vocabulary: No new vocabulary

Materials: 6" × 6" squared papers for pairs of children to construct 5-clock addition charts; pupil pages 247-249

Teaching Procedure

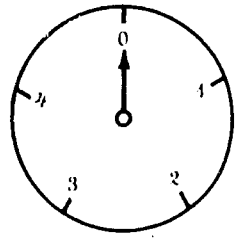
ACTIVITY 1: Addition with a 5-Clock Pupil page 247

Show or draw a clock face. Say, "I came to school at 10 o'clock. I spent 5 hours here. What time did I leave school?" Guide the pupils to find the answer by counting 5 spaces from the number 10 on the clock face, moving clockwise as shown below. (They stop at 3.)



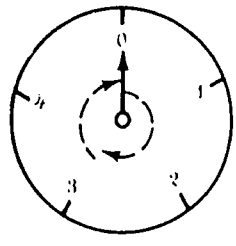
Say, "When you add whole numbers, 10 + 5 = 15; but when you add numbers shown by this clock face, 10 + 5 = 3. This is called clock addition". Emphasize that whatever number we add on this clock we do not get a sum greater than 12. The largest number shown on the clock is 12.

Say, "Our time clock has 12 numerals because we tell time with 12 hours. We can make clock faces for any set of numbers. Then we can use clock addition by moving the hand in a clockwise direction". Draw a clock face on the blackboard with only five marks and number them from 0 to 4 as shown below. Say, "This is called a 5-clock".



Point out that on this clock we always start from the 0-point and move around and on the number line we start from the 0-point and move to the right or left.

Say, "Add on this 5-clock by counting the spaces from the 0-point moving clockwise. Find the sum of 3 and 2". Write, 3 + 2. Let one pupil put a stick as a clock hand on the 0-point and move the stick 3 spaces clockwise. He will stop at the 3-point. Then let him move on clockwise 2 spaces from the 3-point and tell where the hand stops.



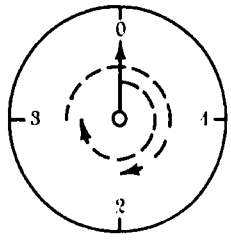
The pupils will see that beginning from the 0-point and counting three spaces, then counting again clockwise two spaces from the 3-point, takes them to the 0-point. So for the 5-clock, 3 + 2 = 0.

Let pupils do other examples such as:

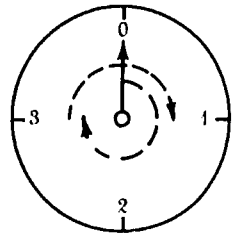
3 + 2 = 0	2 + 2 = 4	0 + 2 = 2	3 + 2 = 0
4 + 2 = 1	1 + 2 = 3	4 + 4 = 3	3 + 3 = 1

Let the children open their books to pupil page 247 and do the exercises. Explain that there are both 5-clocks and 4-clocks on the page. The 4-clock is used in the same way as the 5-clock but it has only 4 numerals. Guide the

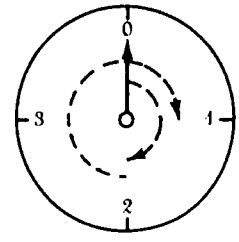
pupils to work out the examples by counting on the 4-clock as on the 5-clock. For example:



3 + 3 = 2



3 + 2 = 1

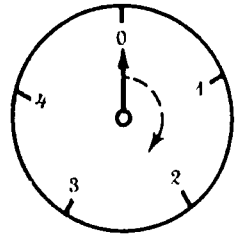


2 + 3 = 1

ACTIVITY 2: Making Addition Tables

Pupil page 248

Begin this activity by drawing a 5-clock on the blackboard:



Ask, "What movement shows 2? (The hand starts at 0, moves clockwise to the 2-point.) What movement shows 4?" (Start at 0, move around to position 4.)

Ask, "How do we show clock addition of two numbers?" Let the pupils discuss this and say that clock addition is shown by making two movements of the hand, starting at 0. Ask the children to use the 5-clock to find the sum of these numbers:

3 + 3; 3 + 4; 3 + 2; 3 + 1; 3 + 0.

Ask, "Is it correct for us to write 3 + 2 = 4 + 1?" (Yes, 3 + 2 and 4 + 1 name the same number. They both name 0.)

Say, "You can show all possible sums in a chart. We call it an addition chart for the 5-clock". Draw the chart on the blackboard and let the children fill in the boxes with the correct numerals.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Say "Think about the set {0, 1, 2, 3, 4}. If you add two numbers in the set using clock addition do you get a sum which is a number in the set? (Yes) How can you tell this by looking at the addition chart?" Guide the pupils to see that all possible sums are given in the chart and that the sums are all numbers in their set. Ask, "Have you used this property before? Did the addition of whole numbers have this property? (Yes) How did you describe the property?" Help the pupils remember the property and say, "The set of whole numbers is closed under addition"; and to say, also, "The set {0, 1, 2, 3, 4} is closed under clock addition".

Let the pupils open their books to pupil page 248. A 4-clock, a 3-clock, and a 7-clock are drawn there. Get the children to complete the addition charts. Sums on these clocks are found in the same way as for the 5-clock.

Answers for pupil page 248:

1.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

2.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

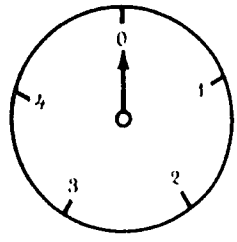
3.

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

ACTIVITY 3: Commutative Property

Pupil page 249

Draw on the blackboard a picture of a 5-clock. Ask the children what set of numbers is used on this clock. ({0, 1, 2, 3, 4})



Revise with the pupils the addition chart for the 5-clock. Guide them to make the chart. Let the pupils, in turn, come up to the blackboard, do an operation using the clock, and then fill in a space of the chart. The chart when completed looks like this:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Say, "We will use this addition chart to solve mathematical sentences". Write these sentences on the blackboard:

(a) $3 + 0 = n$ (b) $4 + 0 = n$ (c) $1 + 0 = n$

Ask the pupils to think of addition on the 5-clock and use the chart to solve the sentences. Help them to see that they do not need to use the chart. They know the sum of any number and zero is the number itself. Therefore in (a) n is 3, in (b) n is 4, and in (c) n is 1. Ask, "What property is this? (It is the property of zero for addition.) Is it a property for clock addition? (Yes) Is the property of zero also true for addition of the set of whole numbers?" (Yes)

Write the following pairs of sentences on the blackboard:

(d) $4 + 1 = a$ (e) $c = 3 + 2$ (f) $2 + 4 = n$

$1 + 4 = a$ $c = 2 + 3$ $4 + 2 = n$

Ask, "How many sums do you find for (d)? for (e)? for (f)? (Only one for each pair.) Why? (When we add the same two numbers the sum is always the same.) Write a mathematical sentence about each pair of numbers in each pair of sentences". Guide them to write:

$4 + 1 = 1 + 4$, $3 + 2 = 2 + 3$, $2 + 4 = 4 + 2$

Through questions help the pupils to see that in each sentence the

order in which two numbers are added is changed but the sum is the same and that this is a property of clock addition called the commutative property.

Get each pair of children to prepare an addition chart for the 5-clock on squared paper. (It will be like the one on the blackboard. Draw a diagonal in the chart on the blackboard as shown below.) Let the children fold their charts along the diagonal. Ask, "What do you notice about the numerals on either side of the diagonal?" Lead them to see that the 2 falls on 2, 3 on 3, 1 on 1, and so on.

+	0	1	2	3	4
0	1	2	3	4	0
1	2	3	4	0	1
2	3	4	0	1	2
3	4	0	1	2	3
4	0	1	2	3	4

Help the children to see that this shows that clock addition is commutative.

Get the children to make an addition chart for the 4-clock on the blackboard. Use this chart to discuss the property of zero for clock addition and the commutative property of clock addition. Let the pupils do the exercises on pupil page 249. The answer to exercise 10 is: No. There is no "4" in the 4-clock chart.

STAGE 3: Clock Subtraction

Vocabulary: Clock subtraction

Materials: A small stick for each child; one $1\frac{1}{4}" \times 1\frac{1}{4}"$ cardboard and a pin for each child; pupil pages 250-251

Teaching Procedure

ACTIVITY 1: Clock Subtraction **Pupil page 250-251**

Draw a diagram of a 5-clock on the blackboard. Use it to help the children

make an addition chart. Write the following sentences on the blackboard:

(a) $2 + n = 4$; (b) $3 + n = 0$

Remind the children that the addition in these sentences is clock addition on a 5-clock. Let the pupils talk about the mathematical sentences and what they mean; let them recall that in each sentence n is a missing addend; and get them to find the missing addend using the 5-clock. Encourage the pupils to show how the missing addends are found. For example, a child may say:

I start with the hand of the clock at 0. To show

$$2 + n = 4$$

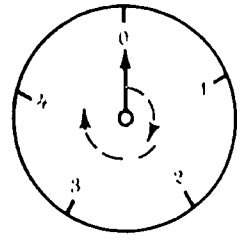
I move the hand clockwise two units. It is now at 2.

I keep moving the hand clockwise until it is at 4.

I count the number of spaces the hand moved from 2 to 4.

I count 2.

The missing addend is 2. n is 2. $(2 + 2 = 4)$



Go on in a similar way with the other examples. Say, "We are doing clock subtraction".

Write this sentence on the blackboard: $2 - 3 = n$. Get a child to write the sentence as an addition sentence. The blackboard looks like this:

$$2 - 3 = n \qquad 3 + n = 2$$

Let the pupils find n in $3 + n = 2$ to make the sentence true for the 5-clock. (n is 4) Let them say what n makes $2 - 3 = n$ a true sentence. Ask the pupils questions to help them relate subtraction sentences to addition sentences. Emphasize that subtraction is the process of finding a missing addend. Continue with other sentences such as:

$$3 - 3 = n, \quad 4 - 2 = n, \quad 1 - 4 = n \quad 2 - 3 = n.$$

Let children discuss the movement of the clock hand which illustrates these sentences.

Continue to relate missing-addend sentences to subtraction sentences. For example, $3 - 3 = n$ may be written as $3 + n = 3$; $4 - 2 = n$ may be written as $2 + n = 4$, and so on.

Ask the children to do the exercises on pupil pages 250 and 251. Let each child use a small stick with the pictures of a 5-clock and a 4-clock. Point out that there are both addition and subtraction sentences in 5-clock arithmetic on pupil pages 250 and 251.

Discuss Exercises 13 to 17 on pupil page 251 with the class. Say, "This is a picture of a roundabout where four roads come together. Cars enter from north, south, east, or west and leave at north, south, east, or west. They move clockwise around the roundabout. Pretend you are driving a car and enter from N (north) and leave at S (south). After entering at N, when you come to E you are $\frac{1}{4}$ of the way around. (Let the children move their fingers around the picture.) We call this $\frac{1}{4}$ turn. From E to S is another $\frac{1}{4}$ turn. How many $\frac{1}{4}$ turns of the roundabout have you made? (2) We say you have moved 2 spaces on the roundabout. Exercises 13 to 17 tell you where a car enters and where it leaves. For each exercise tell how many spaces the car travels, that is, how many $\frac{1}{4}$ turns it made". (The answers are: (13) 2, (14) 4, (15) 3, (16) 4, (17) 1.)

For Exercises 18 to 25 on pupil page 251 the children each need a $1\frac{1}{4}'' \times 1\frac{1}{4}''$ piece of cardboard and a pin. Let the pupils make a paper cutout the same size as the dotted line square ABCD on page 251. Ask them to mark the vertices of their square with A, B, C, and D and to draw in the arrow at A as is shown for square ABCD. Let the pupils fit their square on square ABCD of their book and fix it to the book by putting a pin in the centre dot. Let A fall on A, B on B, and so on. Ask the children to turn their cardboard squares so that the arrows at A move clockwise from 0 to 1, to 2, to 3, and on to 0. Let the children experiment with turning their squares. Then give these directions:

- Place the cardboard so that the A-arrow points to 0. Move the square clockwise until the arrow points at 1. This is called a <u>Turn 1</u>.

- Move the arrow back to 0. Move the square clockwise until the arrow points at 2. This is called a <u>Turn 2</u>.
- Make 1 more turn from 2. This is called a <u>Turn 1</u>. Where is the arrow? (At 3)

Say, "You have just made a Turn 2 and then a Turn 1. Where is the arrow?" (The arrow is at 3.) Write on the blackboard this sentence and let the children tell about it:

Turn 2 + Turn 1 = Turn 3.

Exercises 18 to 23 show Turns the way we performed clock addition. Let the pupils tell where the arrow ends for each exercise and complete the addition chart. (Each number tells how many $\frac{1}{4}$ turns are made. For example, 2 + 2 means a Turn 2, then a Turn 2 again.)

When the chart is completed, let the children compare this chart with the addition chart they made for a 4-clock. (Pupil page 248) Point out that turning the square clockwise is like adding on the 4-clock.

AFRICAN EDUCATION PROGRAM
MATHEMATICS STEERING COMMITTEE

Professor W. T. Martin, Chairman
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dr. Iya Abubakar
Ahmadu Bello University
Zaria, Nigeria

Mr. James L. Aldrich
Educational Services Incorporated
Newton, Massachusetts

Professor Edward G. Begle
Stanford University
Stanford, California

Professor David H. Blackwell
University of California
Berkeley, California

Mr. Hugh P. Bradley
Educational Services Incorporated
Newton, Massachusetts

Professor Jerome S. Bruner
Harvard University
Cambridge, Massachusetts

Professor Robert B. Davis
Webster College
Webster Groves, Missouri

Professor Robert P. Dilworth
California Institute of Technology
Pasadena, California

Professor Roy Dubisch
University of Washington
Seattle, Washington

Professor Vincent Haag
Franklin and Marshall College
Lancaster, Pennsylvania

Professor Clarence Hardgrove
Northern Illinois University
DeKalb, Illinois

Mr. D. K. Abbiw-Jackson
Kwame Nkrumah University of Science and Technology
Kumasi, Ghana

Mr. Christopher Modu
West African Examinations Council
Yaba, Lagos, Nigeria

Mr. Paul E. Mugambi
Makerere University College
Kampala, Uganda

Mr John O. Oyelese
University of Ibadan
Ibadan, Nigeria

Professor David A. Page
Educational Services Incorporated
Newton, Massachusetts

Professor B. J. Pettis
University of North Carolina
Chapel Hill, North Carolina

Mr. J. E. Phythian
University College
Dar es Salaam, Tanzania

Professor Walter Prenowitz
Brooklyn College
Brooklyn, New York

Professor Alfred L. Putnam
University of Chicago
Chicago, Illinois

Professor Donald E. Richmond
Williams College
Williamstown, Massachusetts

Professor Patrick Suppes
Stanford University
Stanford, California

Dr. Onyerisara Ukeje
University of Nigeria
Nsukka, Nigeria

Mr. Stephen White
Columbia Broadcasting System, Inc.
New York, New York

Dr. Awadagin Williams
University of Sierra Leone
Freetown, Sierra Leone

Dr. Grace Alele Williams
University of Lagos
Lagos, Nigeria

Professor Jerrold R. Zacharias
Massachusetts Institute of Technology
Cambridge, Massachusetts

Ex Officio:
Dr. James R. Killian, Jr.
Chairman of the Board
Educational Services Incorporated
Newton, Massachusetts

Printed in the United States of America

35D