MULTIVARIATE SEASONAL TIME SERIES FORECAST

WITH APPLICATION TO ADAPTIVE CONTROL

by

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PREFACE

This report is one of a series of publications which describe various studies undertaken under the sponsorship of the Technology Adaptation Program at the Massachusetts Institute of Technology.

In 1971, the United States Department of State, through the Agency for International Development, awarded the Massachusetts Institute of Technology a grant. The purpose of this grant was to provide support at M.I.T. for the development, in conjunction with institutions in selected developing countries, of capabilities useful in the adaptation of technologies and problem-solving techniques to the needs of these countries. At M.I.T., the Technology Adaptation Program provides the means by which the long-term objective for which the A.I.D. grant was made, can be achieved.

Fred Moavenzadeh

Program Director

ABSTRACT

A general multivariate model for seasonal riverflow is proposed. The formulation relates discharge at a particular station to current discharge at other stations as well as previous discharges at any station. Additionally, the formulation allows for moving average terms and accounts for seasonality in the mean and variance. An identification strategy is suggested and two general parameter estimation algorithms are discussed. A technique to obtain multi-lead forecasts from an identified model and the use of these to obtain approximate conditional Markovian transition matrices is given.

The identification, estimation and validation of univariate and multivariate models is demonstrated using historical monthly discharges of the Nile basin.

A new adaptive reservoir control algorithm which uses the approximate conditional Markovian transition matrices is also derived. It uses a dynamic programming formulation of the value iteration type with previous inflow and present storage as states. The number of stages over which the algorithm must be solved at each decision, and thus the computational burden, is dramatically reduced by using a tabulated boundary value function derived from the stationary control problem.

The control algorithm is not evaluated in this work.

ACKNOWLEDGEMENTS

This study was mostly sponsored by the M.I.T. Technology Adaptation Program, which is funded through a grant from the Agency for International Development, United States Department of State. Sperry Flight Systems of Sperry Corporation provided additional support to Mr. Kevin Curry during the last portion of this work. The views and opinions expressed in this work, however, are those of the authors and do not necessarily reflect those of the sponsors.

Acknowledgement must be given to the complete staff of the Technology Planning Program, which includes personnel at M.I.T. and Cairo University, and the M.I.T. Information Processing Center, where all computations were performed. Their administrative support and cooperation has been very helpful.

The work also benefitted from the many discussions with Messrs. Luis Alarcon and Michael Dettinger.

A grateful word goes to Ms. Anne Clee and Mrs. Liza Drake for patiently typing the drafts of this report. Mr. Ross Buchanan also helped considerably in handling the details of publication.

Work was performed at the Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Department of Civil Engineering, M.I.T.

TABLE OF CONTENTS

LIST OF FIGURES

Figure No.

Figure No.

Figure No.

LIST OF TABLES

Table No.

Chapter 1

INTRODUCTION

1.1 Reservoir Control Using Real Time Information

Since the mid-1960's, a number of papers have been published on the subject of deriving optimal release policies for multiple purpose reservoirs (e.g., Butcher, 1971, Loucks and Falkson, 1970, Su and Deininger, 1972). Stochastic mathematical programming models are necessary due to the complexity of the problem. A release not only affects the present benefits derived from the reservoir but also those from future releases. Furthermore, how the release affects future benefits is dependent on future inflows which are unknown.

Most of these stochastic mathematical programming models determine what may be called a closed-loop reservoir control law. The adjective closed-loop is used since the derived control (i.e., reservoir release) depends only on the present reservoir volume and inflow, the states of the reservoir system. Closed-loop reservoir control is theoretically attractive because it implicitly takes into account the probabilistic distribution of the future state of the reservoir system by maximizing expected utility. It is also attractive because, once derived, the control is extremely simple to implement.

However, there is one serious deficiency to this control. Closed-loop control can only be derived for stationary systems. Thus, the use of real-time information (i.e., knowledge of the future behavior of the system derived from present and near past information of any kind) is precluded.

A new adaptive algorithm, defined in this work, bypasses this drawback in closed-loop control. The algorithm uses a dynamic programming formulation of the value iteration type with states defined as previous inflow and present storage. The inflow is assumed to be characterized by a first order periodic (to account for seasonality) Markov chain. Real time information is introduced via conditional Markovian transition matrices (and conditional benefits function). Each time conditional information is available, the dynamic program is solved. The computational burden is dramatically reduced by using results which are implicitly derived in the computation of the closed-loop control.

The availability of this new adaptive control algorithm provides strong motivation for the development of a technique useful for obtaining conditional Markov transition matrices.

1.2 Scope and Outline of Work

The primary focus of this work is the development of practical techniques useful for the identification, estimation and performance evaluation of seasonal multivariate stochastic streamflow models which may be used to obtain multi-lead forecasts. In addition, a technique for expressing the conditional information inherent in multi-lead forecasts as approximate conditional Markov transition matrices is suggested. An example application of the derived modeling techniques is illustrated using Nile basin gaging stations.

As a secondary effort, the theoretical foundation of the adaptive control algorithm briefly mentioned above is fully developed.

It provides motivation for the investigation of stochastic streamflow models.

Since historical streamflows of the River Nile are used in an applied example, the hydrology of the Nile basin is discussed in Chapter 2. The available historical streamflow data is also described.

Chapter 3 presents the theoretical foundation of the new adaptive control algorithm.

Considerations which affect the decision on forecasting approach are discussed in Chapter 4.

Chapter 5 reviews current literature concerned with univariate and multivariate stochastic streamflow models. The identification, estimation and validation of a proposed general seasonal multivariate stochastic model is also discussed in detail. In addition, the use of a fitted model to obtain multi-lead forecasts is described.

Chapter 6 presents a method by which a model of the type proposed in Chapter 5 may be used to obtain approximate conditional first order Markov transition probabilities required by the adaptive control algorithm of Chapter 3.

The actual identification, estimation and validation of a few univariate and multivariate modesl of the River Nile is presented in Chapter 7. An evaluation of the new adaptive control algorithm is not performed in this work.

Chapter 8 is a summary of this work. Conclusions and suggestions for future studies are given.

Chapter 2

HYDROLOGY OF THE NILE AND DESCRIPTION OF THE HIGH ASWAN DAM

2.1 The Nile River Tributaries and Yield

2.1.1 Introduction

The Nile is the second longest river in the world. It originates in the Equatorial Lakes and Ethiopian Plateau and flows northward to the Mediterranean Sea. The basin area is estimated to be about 2,900,000 km^2. Over this great spatial extent, there are large variations in climate and topography.

The text below gives an account of the main tributaries of the Nile and their respective yields. As an aid to following the discussion, the map and schematic diagram of the River Nile Basin, Figures 2.1 and 2.2, respectively, should be consulted.

The material presented in this chapter was essentially gathered from two sources. They are: Abul-Atta (1978) and Cairo University (1977).

2.1.2 Equatorial Lakes Plateau

The Equatorial Lakes, Victoria, Kioga, George, Edward, and Albert, constitute the beginnings of the Nile River. Lake Victoria contributes flow to Lake Albert via Lake Kioga as does Lake George via Lake Edward. The Nile from Lake Albert up to the city of Nimule, close to the southern border of Sudan, is known as the Albert Nile.

Lake Victoria has an average surface area of 67,000 km^2 and a rainfall catchment basin of 195,000 km^2. Taking into account direct

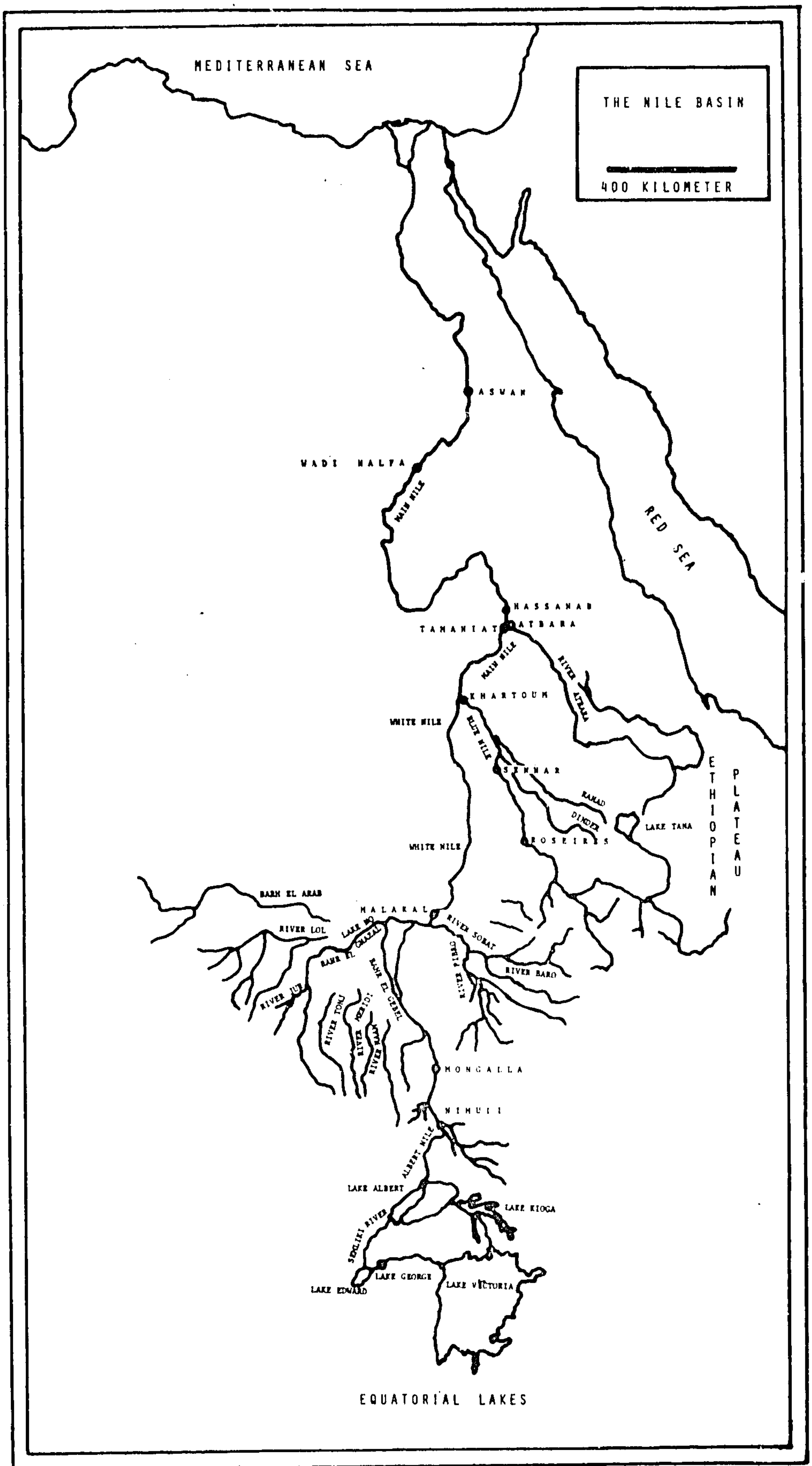


Figure 2.1 Map of Nile Basin

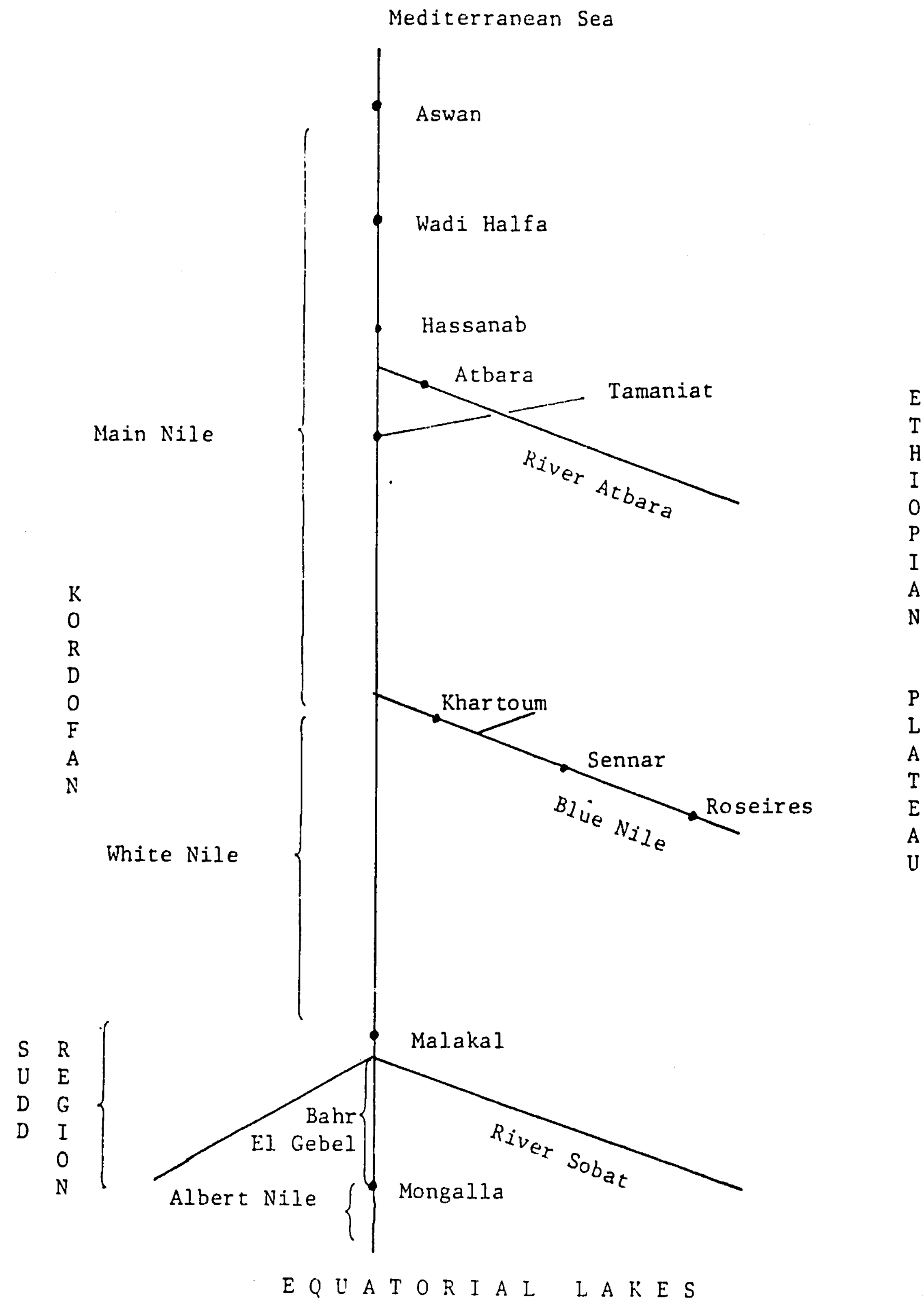


Figure 2.2

SCHEMATIC DIAGRAM OF NILE RIVER BASIN
(not to scale)

rainfall on the Lake, drainage from rainfall on the catchment area, and evaporation, it is estimated that the average annual net water yield to Lake Kioga is between 23.5 and 25.5 milliard m^3 (i.e., 10^9 cubic meter) per annum.

Estimates of the net water yield of Lake Kioga, excluding the contribution of Lake Victoria, range from -1.2 to -1.0 milliard m^3/year, implying water losses. The average outflow from Lake Kioga to Lake Albert is estimated to be between 22.5 and 24.3 milliard m^3/year.

The surface area of Lake George and Lake Edward is 300 and 2200 km^2, respectively. Their combined catchment area is 20,000 km^2. Lake Edward flows into Lake Albert via the Semliki River, which has a catchment area of 8000 km^2. Estimates of annual yield to Lake Albert from these three sources is estimated to be between 4.0 and 4.4 milliard m^3.

Lake Albert has an average surface area of 5,300 km^2 and a basin area of 17,000 km^2. Taking into consideration the average yield from upstream catchment areas, its own catchment area (including surface area), and evaporation, the net average annual yield from Lake Albert to the Albert Nile is estimated to be between 23.7 and 26.5 milliard m^3.

2.1.3 The River from Nimule City to Malakal

Downstream of Nimule City, the river is known as Bahr El Gebel. From Nimule to Mongalla, an additional 4.7 milliard m^3 per year flows into the river from torrential tributaries. Considering losses, the annual discharge at Mongalla is estimated at 27.0 to 30.0 milliard m^3/year.

Beyond Mongalla, the Bahr El Gebel flows into a large swampy area known as the Sudd region. The area is covered by a dense growth of aquatic weeds. Through seepage and evaporation losses, the relative magnitude of which is unknown, the discharge to Malakal is approximately halved. Hence, a discharge of only 14.7 to 15.0 milliard m^3 per year reaches Malakal from the Equatorial Lakes Plateau.

Before Malakal, two sources, the Bahr El Ghazal river basin of Southwest Sudan and a portion of the Ethiopian Plateau, add approximately 0.5 and 13.5 milliard m^3 per year, respectively, to the Bahr-El-Gebel flow. Thus, the estimate of total annual flow to Malakal is 28.0 to 29.0 milliard m^3 per year.

2.1.4 Bahr El Ghazal Basin

The Bahr El Ghazal basin has an area of about 526,000 km^2. It principally lies in the southwestern portion of the Sudan. Its major tributaries are the Bahr El Arab, River Lol, River Jur, River Tonj, River Meridi and River Naam. The Rivers Jur and Lol have the largest estimated average discharge, 5.3 and 4.3 milliard m^3 per year, respectively. The combined discharge of all the tributaries to the Bahr El Ghazal is estimated at 11.8 milliard m^3 per annum. However, almost all the flow is lost in the Sudd swamps before reaching the Bahr El Gebel at Lake No. The net contribution of the Bahr El Ghazal to the White Nile, the name of the Nile downstream of Lake No, is estimated at 0.5 milliard m^3 per year.

2.1.5 Ethiopian Plateau

On the average, the Ethiopian Plateau contributes 85% of the annual flow which eventually reaches Aswan in Egypt. Its three main watersheds are the River Sobat, Blue Nile and River Atbara.

The Sobat is formed by the junction of the Rivers Baro and Pibor. The headwaters of the Baro lie in the southern portion of the Ethiopian Plateau. The origins of the Pibor are divided between the Ethiopian Plateau, the plains of Sudan, east of the Bahr El Gebel, and the northern slope of the Equatorial Lakes Plateau, although the greater part of its discharge comes from the Ethiopian Plateau.

The Sobat loses about a quarter of its flow near Malakal due to seepage and evaporation. Its average annual contribution to the White Nile at Malakal is 13.5 milliard m^3 per year.

As previously given, the average annual discharge of the White Nile at Malakal is estimated to be between 28.0 and 29.0 milliard m^3. The sources of flow are the Bahr El Gebel, Bahr El Ghazal and the River Sobat. Between Malakal and the junction of the Blue Nile, a distance of approximately 960 km, the river is also known as the White Nile. Over this stretch, the average annual discharge is reduced to an estimated 26.0 milliard m^3.

The Blue Nile has its origins at Lake Tana, which yields about 3.8 milliard m^3 per annum. Over the 940 km stretch to Roseires, this flow is supplemented by several tributaries so that the average annual flow there is estimated to be 50.0 milliard m^3. Between Roseires and the intersection of the Blue and White Nile at Khartoum, 660 km, the river is joined by the Dinder and Rahad tributaries. The total discharge of the Blue Nile at Khartoum is estimated to be 52.0 milliard m^3 per annum.

Downstream the junction of the Blue and White Nile at Khartoum, the river is called the Main Nile. Up to the junction with the River Atbara at the City of Atbara (310 km), the flow is reduced by 2.0 milliard m^3, mostly due to evaporation, and thus the average annual flow of the main Nile at this point is estimated to be (26+52-2=) 76.0 milliard m^3.

The source of the River Atbara is the northern portion of the Ethiopian Plateau. The total length of the river is approximately 880 km and has an estimated annual discharge of 12.0 milliard m^3 year. Thus, the total estimated discharge of the River Nile after the junction of the River Atbara is 88.0 milliard m^3 per year.

2.1.6 The Nile from Atbara to Aswan

From Atbara to the Aswan Reservoir, the Nile flows through a region of little rainfall and has no major tributaries. Hence, the average annual flow is reduced from 88.0 to 84.0 milliard m^3 per annum.

2.2 Monthly Variation of Rainfall and Streamflow

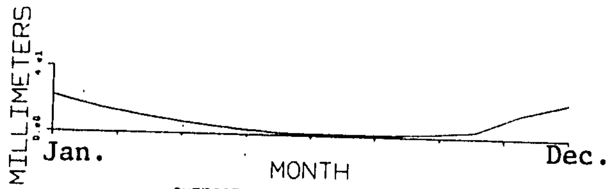
The average monthly variation of rainfall for several sub-areas of the Nile River Basin is illustrated in Figures 2.3 to 2.13. The data was obtained from the Ministry of Public Works, Egypt (1963).

Several observations may be made:

- Practically no rain falls on the main Nile north of Khartoum (Figures 2.3 and 2.4).
- Rainfall increases following the Blue Nile from Sudan to the Ethiopian Plateau (Figures 2.5 and 2.6).
- Rainfall steadily increases from lower Egypt to the Sudd region (Figures 2.3, 2.4, 2.7, 2.8 and 2.11).
- There is a pronounced peak of rainfall over the Ethiopian Plateau and Nile River north of Malakal during August (Figures 2.4 through 2.9).
- Relatively smooth monthly variation of rainfall south of Malakal (Figures 2.8 through 2.13).
- Rainfall reaches its lowest level in December over the entire Nile basin south of Egypt (Figures 2.4 to 2.13).

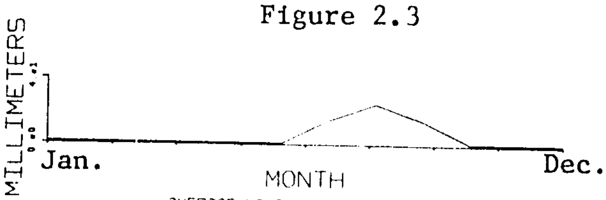
The average monthly variation of streamflow caused by the variation in rainfall may be inspected in Figures 2.14 through 2.23. The streamflow data used for their construction is listed in Appendix 1. The locations of the gaging sites are shown on the Nile Basin Map, Figure 2.1, and the schematic diagram, Figure 2.2.

The average monthly flow at the gaging sites Roseires, Sennar, Khartoum, and Atbara, Figures 2.21, 2.20, 2.19 and 2.17, respectively, mimics the behavior of the monthly variation of rainfall on the Ethiopian Plateau. The flow is very low from December until May. Beginning in June, the river rises rapidly and peaks in August. The recession of the flow is quite rapid until the month of November. From



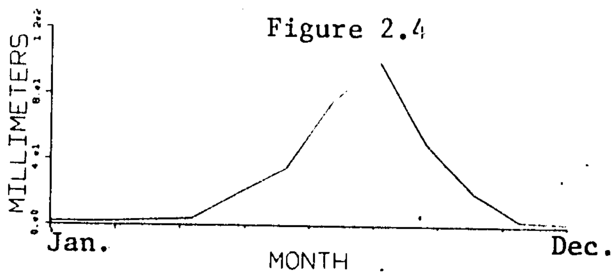
AVERAGE MONTHLY RAINFALL ON
LOWER EGYPT

Figure 2.3



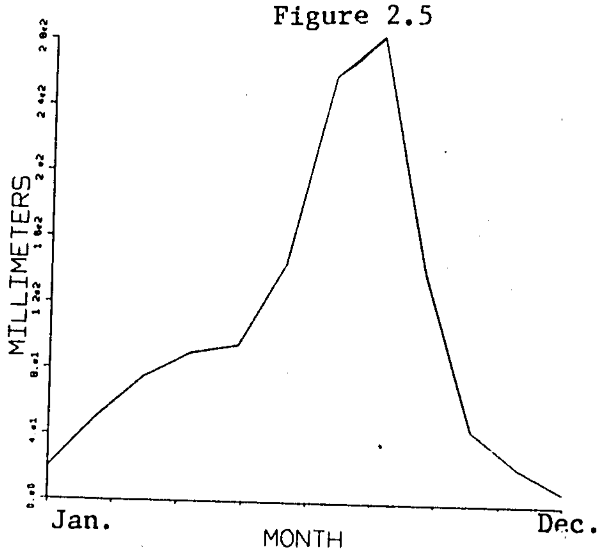
AVERAGE MONTHLY RAINFALL ON
THE MAIN NILE IN SUDAN

Figure 2.4



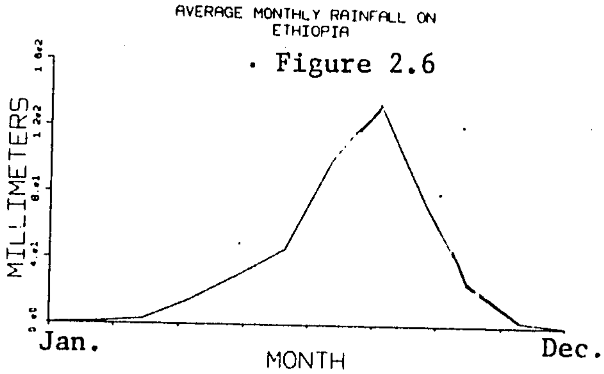
AVERAGE MONTHLY RAINFALL ON
THE BLUE NILE IN SUDAN

Figure 2.5



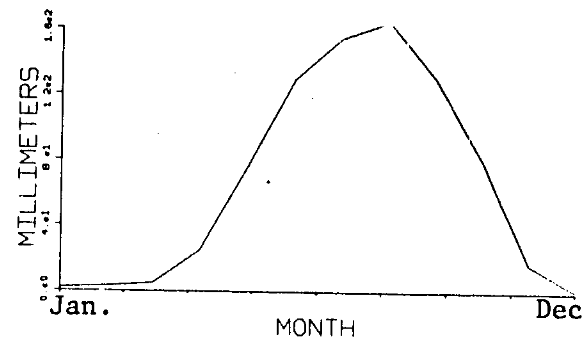
AVERAGE MONTHLY RAINFALL ON
ETHIOPIA

Figure 2.6



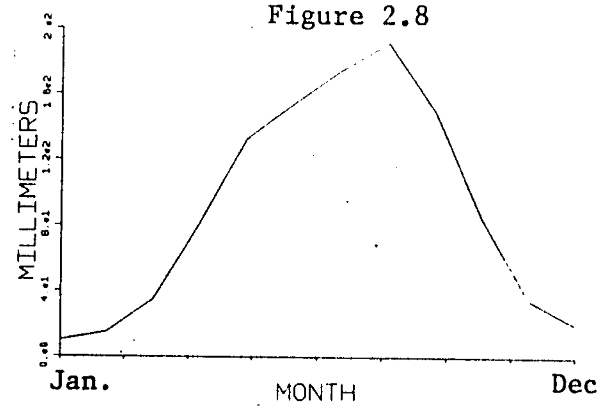
AVERAGE MONTHLY RAINFALL ON
THE NORTHERN WHITE NILE AND KHORDOFAN

Figure 2.7



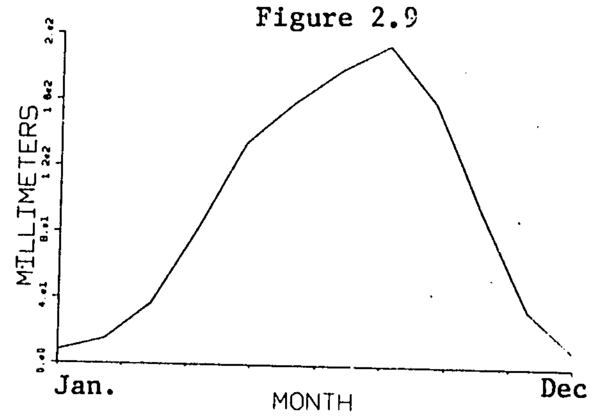
AVERAGE MONTHLY RAINFALL ON
THE SOUTHERN WHITE NILE (RENK TO LAKE NO)

Figure 2.8



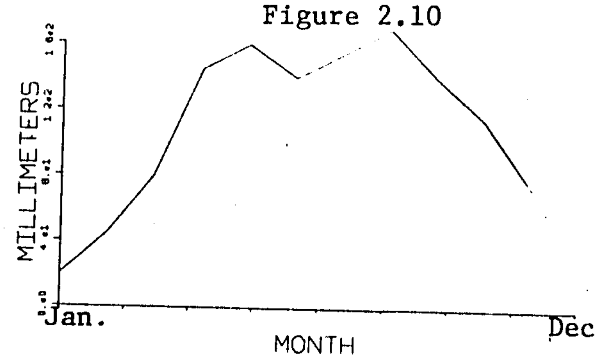
AVERAGE MONTHLY RAINFALL ON
THE RIVER SOBAT BASIN

Figure 2.9



AVERAGE MONTHLY RAINFALL ON
THE BAHR EL GHAZAL BASIN

Figure 2.10



AVERAGE MONTHLY RAINFALL ON
THE BAHR EL GEBEL BASIN

Figure 2.11

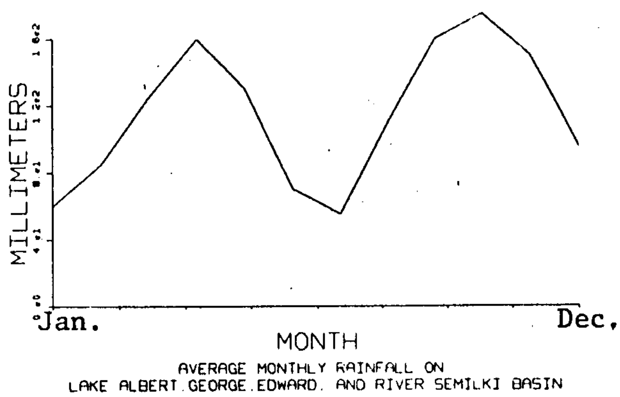


Figure 2.12

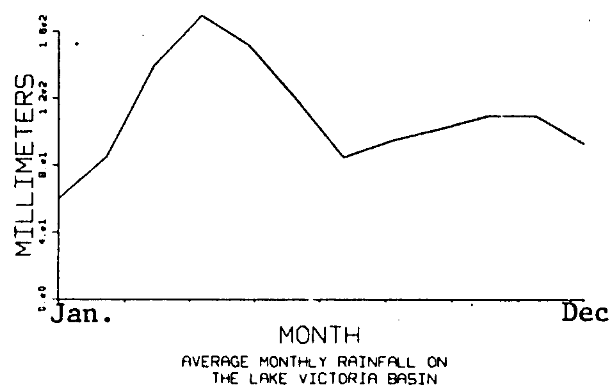
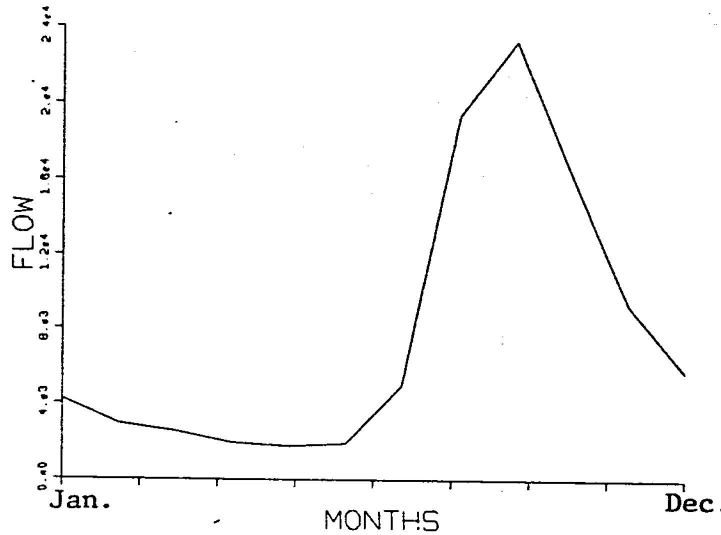


Figure 2.13

November to June, the flow steadily and slowly decreases to a minimum.

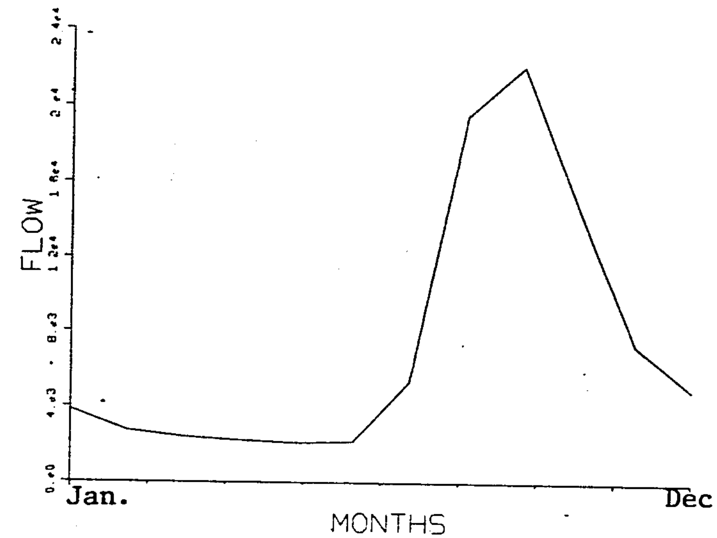
The behavior of the flow at Mongalla doesn't follow the behavior of rainfall over its basin in any obvious manner. The greatest amount of rainfall on the basin, Figures 2.12 and 2.13, occurs in the month of April whereas the greatest discharge occurs in August, Figure 2.23 (the discharge does have a local maximum in May). Also, the lowest amount of rainfall over the basin occurs in July and the lowest discharge in February. A final important observation is the small monthly fluctuation in discharge relative to rainfall. Apparently, the storage ability of the Equatorial Lakes significantly influences the discharge.

The variation of discharge at Malakal does follow the variation of discharge at Mongalla and rainfall over the Sobat basin, Figure 2.9, although it is out of phase with both by about two months. The lowest two month accumulation of rainfall over the Sobat basin occurs in February, as does the lowest flow at Mongalla. The minimum flow at Malakal is two months later in April. The highest flow at Mongalla and



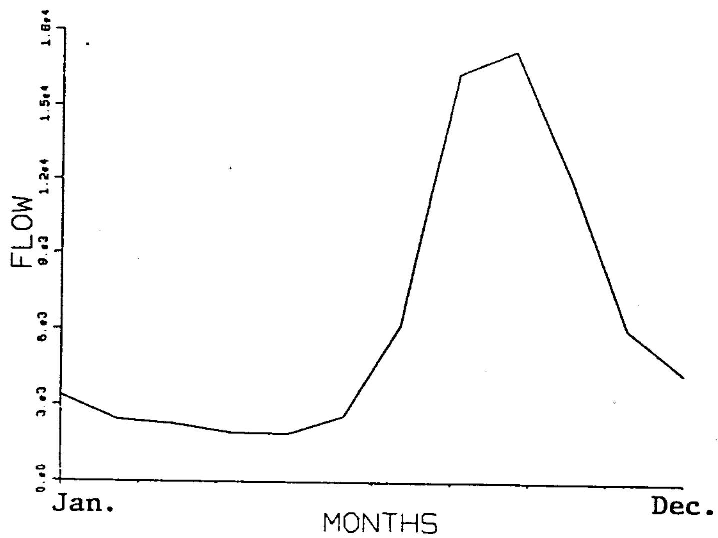
AVERAGE MONTHLY FLOW AT ASWAN (1871-1964. 10**6 M**3)

Figure 2.14



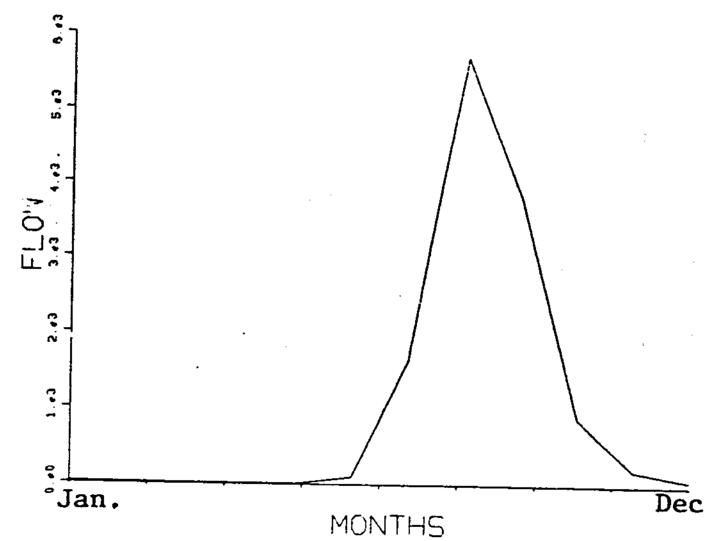
AVERAGE MONTHLY FLOW AT WADI HALFA (1890-1976. 10**6 M**3)

Figure 2.15



AVERAGE MONTHLY FLOW AT HASSANAB (1909-1952. 10**6 M**3)

Figure 2.16



AVERAGE MONTHLY FLOW AT ATBARA (1903-1967. 10**6 M**3)

Figure 2.17

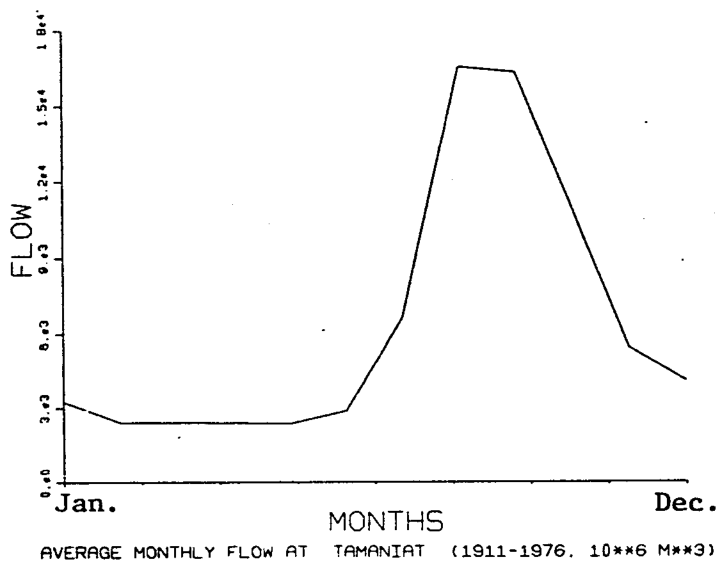


Figure 2.18

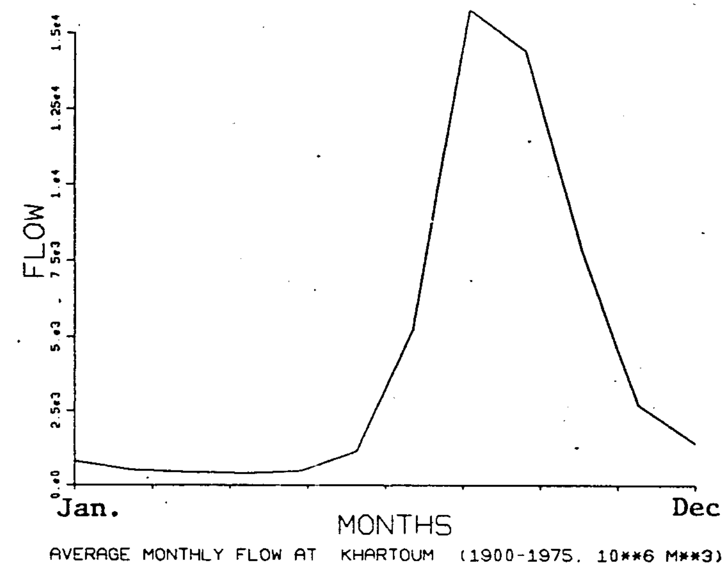


Figure 2.19

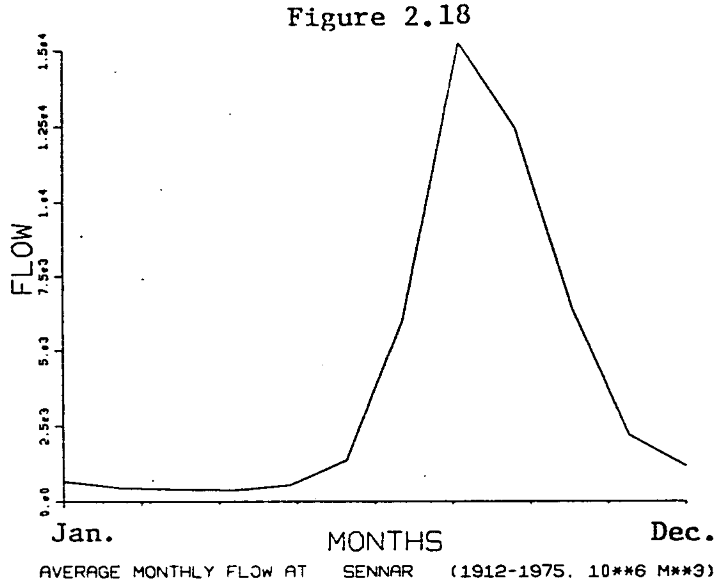


Figure 2.20

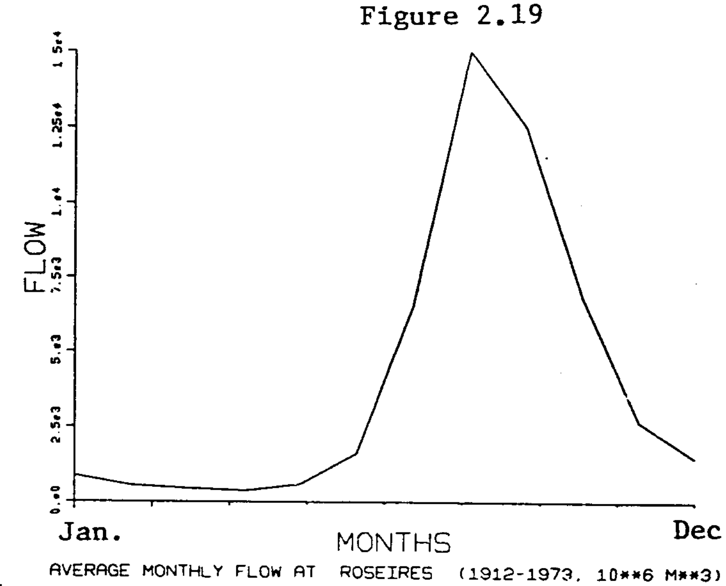
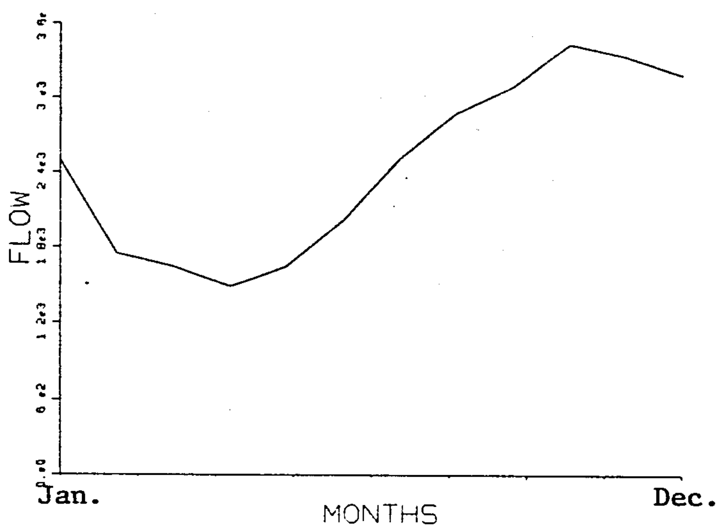
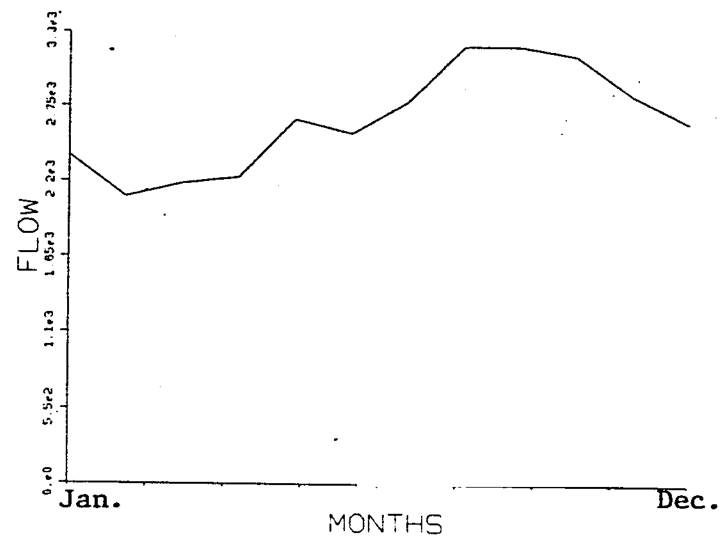


Figure 2.21



AVERAGE MONTHLY FLOW AT MALAKAL (1905-1976, 10**6 M**3)

Figure 2.22



AVERAGE MONTHLY FLOW AT MONGALLA (1905-1975, 10**6 M**3)

Figure 2.23

highest rainfall over the Sobat basin occur two months before the highest flow at Malakal in October. The Sudd swamps probably cause this phase shift by damping flow into Malakal.

Given the average monthly discharge of the White Nile (Malakal, Figure 2.22), Blue Nile (Khartoum, Figure 2.19) and the River Atbara (Atbara, Figure 2.17), the behavior of the discharge of the main Nile at recording stations Tamaniat, Hassanab, Wadi Halfa and Aswan, Figures 2.18, 2.16, 2.15 and 2.14, respectively, is not surprising. From November of one year until May of the next, the flow is slowly and steadily decreasing. During this period, the main source of flow is from Malakal. The flow begins to rise in May and in June rapidly increases until it peaks in September. The flow then rapidly recedes. From July until October, most of the flow in the main Nile has its origins in the Ethiopian Plateau.

Figure 2.24 shows the relative magnitudes (ratios) of the monthly flows at Atbara, Khartoum and Malakal to the monthly flows at Wadi Halfa (the last recording station before Aswan Reservoir). This figure clearly illustrates the dominance of the White Nile from November to June and the Blue Nile from July to October.

Figure 2.25 shows the average amount of time it takes water to travel from Malakal, Sennar and Atbara to Wadi Halfa (Cairo University, 1977). As expected, the time of travel is long during low flow periods and much shorter during high flow periods. The time of travel ranges from 25 to 38 days for Malakal, 9 to 30 days for Sennar,

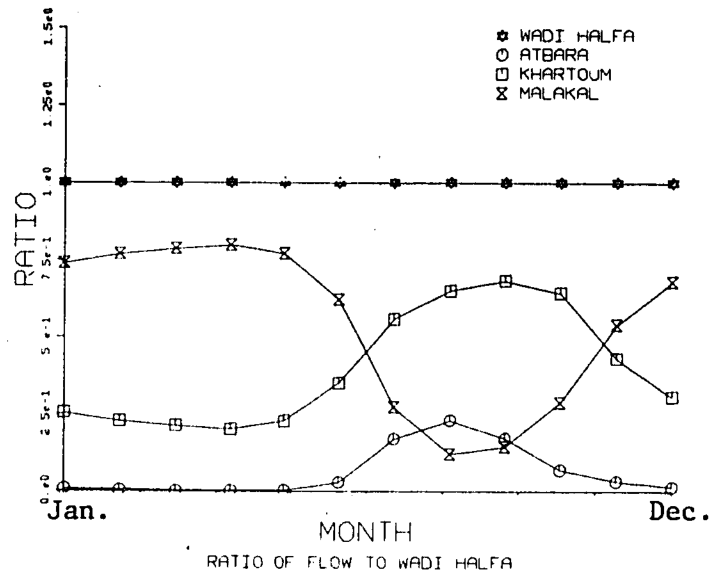


Figure 2.24

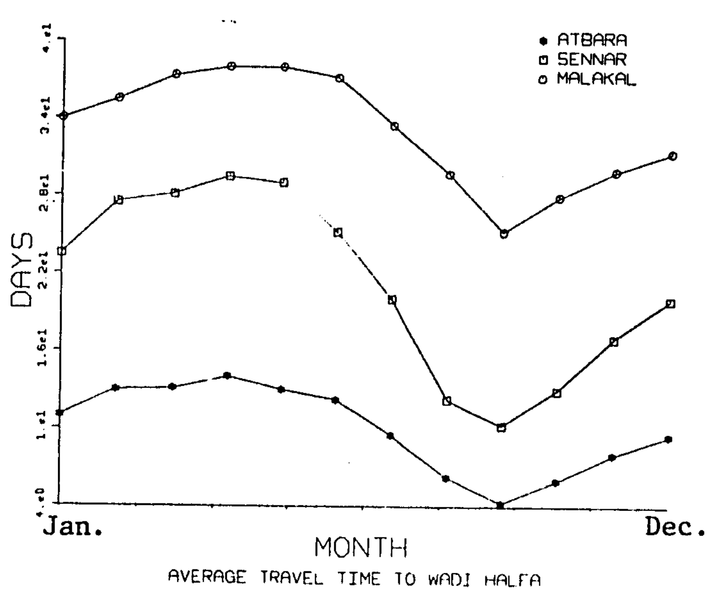


Figure 2.25

and 5 to 15 days for Atbara.

2.3 The High Aswan Dam

The High Aswan Dam is a multipurpose reservoir. It protects Egypt from the violent flood waters of the Ethiopian Plateau by storing the water in the largest man-made reservoir in the world. The water is then released to supply the irrigation and electrical power needs of Egypt.

The dam is located 6 km south of Aswan. It is of the rockfill type. Construction began in 1962 and was completed in 1970.

As noted above, the average annual inflow to the reservoir is estimated at 84 milliard m^3 per annum. At a level of 182 m, the storage capacity of the reservoir is 162 milliard m^3, approximately twice the annual inflow. Dead storage accounts for 30 milliard m^3 of the total capacity and live storage, between levels 147-175 m, accounts for 90 milliard m^3. The remaining 41 milliard m^3 of available storage is used for flood protection.

At present, the operating rules of the High Dam require that the level of the reservoir does not exceed 175 meters on August 1. This is a way to assure that there is sufficient capacity to receive the new flood.

Quoting from Dr. A. Azim Abul-Atta, former Egyptian Minister of Irrigation and Land Reclamation (1978):

"- from August 1st, the water requirements are to be released, the levels to be observed, forecasts about the natural river yield to be carried out - in sequence - to make possible releases from the reservoir which are commensurate with the levels expected upstream of the Dam.

"In carrying out this system, the possibility of having to increase the discharges released from the reservoir so as to make sure that storage does not exceed the maximum determined level [175 m by August 1st] has been taken into account."

It is clear that the Egyptian authorities realize the value of discharge forecasts for the operation of the High Aswan Dam.

Chapter 3

OPTIMAL RESERVOIR CONTROL

3.1 Introduction

The ultimate aim of forecasting the flow of the Nile River is for the optimal operation of the Aswan Reservoir. Thus, even though the major emphasis of this work is forecast, a discussion of some mathematical concepts of stochastic reservoir control and how real time forecast information can be used will be helpful.

This chapter presents two particular reservoir control schemes. Both specify an optimal release relative to an objective function.

The first scheme considered is a closed loop control. A tabulated control law defines the optimal reservoir release solely as a function of the previous inflow and present reservoir volume, the states of the reservoir system since inflow is assumed to be a first order Markov chain. This type of control does not incorporate any real time forecast information. The Markov model of streamflow remains constant throughout all times, independent of observed river flows.

The derivation of a tabulated closed loop control law for the Aswan Reservoir using a dynamic programming model of the value iteration type (Howard, 1960) has been discussed in detail by Alarcon (1979). The approach is briefly discussed in Section 3.2 due to its intimate relation to the second control scheme considered which does use real time forecast information.

The second control scheme also uses a dynamic programming algorithm of the value iteration type to derive the optimal reservoir release. However, since real time forecasts of returns and discharges will be used, a control law that is solely a function of the previous reservoir inflow and current reservoir volume cannot be tabulated and the dynamic programming algorithm must be used each time new forecasts are available. Fortunately, the number of stages over which the dynamic programming algorithm must be solved, and thus the computational burden, may be dramatically reduced by using a boundary value function. The boundary value function is computed from results obtained from the solution of the closed loop control problem as formulated in Section 3.2.

The specifics of the second control scheme and evaluation of the boundary value function will be discussed in Section 3.3.

The presentation of optimal reservoir control using real time forecast information and its relation to optimal closed loop control via the boundary value function is important. The only other similar discussion known is in Verhaegue (1977).

3.2 Closed Loop Reservoir Control in the Absence of Real Time Forecast Information

In the model that follows, it is assumed that the design parameters are fixed, that is, the reservoir capacity, storage and release targets, are predetermined. The problem is to derive an optimal release policy.

The continuous variables of time, inflow, and reservoir storage are discretized. The inflow is assumed to be a first order Markov chain with a periodic transition matrix. The periodicity is due to the annual streamflow cycle. Initial storage at the beginning of the current period and inflow in the previous period specify the state of the reservoir system.

Assume that the periodic Markov chain describing inflow has a known period T. Call each state transition time interval a step and every T steps a cycle. Let t be the number of steps remaining in a cycle, t=0, 1, 2, ..., T-1.

Consider the following notation and definitions.

t and nT are related to physical time as shown in Figure 3.1.

$S_i(t)$ = reservoir storage at the beginning of step t equal to the i^{th} discrete value of the variable

$Q_j(t)$ = inflow during step t equal to the j^{th} discrete value of the variable

$P_{jk}(t)$ = Prob $(Q_k(t-1)|Q_j(t))$ (see Figure 3.1 for relation of t to physical time and Section 6.2 for derivation from historical flow record)

$G_t(i,j,r)$ = $G_t(S_i(t),\ Q_j(t),\ r)$ = return from the reservoir over step t for initial reservoir storage i, inflow j, and release r selected from the predetermined set R(t)

$V_{nT+t}(i,j)$ = $V_{nT+t}(S_i(t),\ Q_j(t+1))$ = maximum total expected value from the next nT+t steps given reservoir volume $S_i(t)$ and previous inflow $Q_j(t+1)$

β = one step discount factor; $0 \leq \beta \leq 1$

$\beta = 1$ corresponds to the undiscounted case

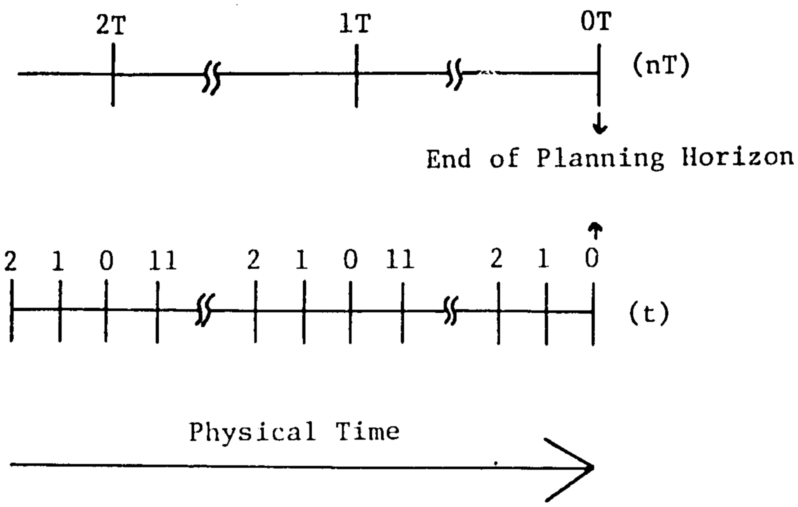


Figure 3.1

RELATION OF t AND T TO PHYSICAL TIME

Because of the periodicity of the Markov chain

$$P_{jk}(t) = P_{jk}(nT + t) \qquad \forall j,k,t,n \tag{3.1}$$

It is assumed that

$$G_t(i,j,r) = G_{nT+t}(i,j,r) \qquad \forall t,i,j,r,n \tag{3.2}$$

Although the functions $P.(\cdot)$ and $G.(\cdot)$ are periodic, they are considered stationary since they depend only on t and not absolute time (nT + t).

The total expected value function, $V.(\cdot)$, may be computed, for $n \geq 0$, from the recursion relation

$$V_{nT+t}(i,j) = V_{nT+t}\,(S_i(t),Q_j(t+1)) = \max_{r \varepsilon R(t)} E\Big[G_t(S_i(t),Q_k(t),r) + \beta V_{nT+t-1}(S_i(t) + Q_k(t) - r,\ Q_k(t))\Big] \tag{3.3}$$

where $E[\cdot]$ denotes the expectation operator. This equation results from the direct application of Bellman's Principle of Optimality (Bellman, 1957).

Notice that the state equation derived from continuity considerations

$$S_\ell(t - 1) = S_i(t) + Q_k(t) - r \tag{3.4}$$

is implicitly enforced in Equation 3.3.

Using the probabilistic characterization of $Q.(\cdot)$, a first order Markov chain, Equation 3.3 may be written as

$$V_{nT+t}(i,j) = \max_{r \in R(t)} \left[\sum_k P_{jk}(t) \left\{ G_t(i,k,r) + \beta\, V_{nT+t-1}(S_i(t)+Q_k(t)-r,k) \right\} \right] \tag{3.5}$$

Let $r^*_{nT+t}(i,j)$ be equal to the optimal release given state i,j with nT+t steps remaining in the operating horizon as determined from Algorithm 3.5.

Su and Deininger (1972) indicate how recursion Algorithm 3.5 may be used to find the optimal closed loop reservoir release policy for the undiscounted, $\beta=1$, and discounted case, $0 \leq \beta < 1$, with $T \leq 1$. Essentially they extended results derived by MacQueen (1966) and Odini (1967) for the discounted and undiscounted case, respectively, with $T = 1$.

Two results proved by Su and Deininger are needed for the present discussion. (Conditions 3.1 and 3.2 are assumed to hold. The transition probability function $P.(\cdot)$ must satisfy certain other conditions. See Su and Deininger for further discussion.)

(a) For arbitrary choice of initial values $V_o(i)$ and $\beta = 1$ (undiscounted case)

$$\lim_{n\to\infty} \left[V_{(n+1)T+t}(i,j) - V_{nT+t}(i,j) \right] = g \qquad \forall\; i,j,t \tag{3.6}$$

where $V.(\cdot)$ is determined from Algorithm 3.5.

(b) For arbitrary choice of initial values $V_o(i)$ and $0 \leq \beta < 1$ (discounted case)

$$\lim_{n\to\infty} V_{nT+t}(i,j) = V_t^*(i,j) \qquad \forall\ i,j,t \tag{3.7}$$

where $V.(\cdot)$ is determined from Algorithm 3.5 and $V_t^*(i,j)$ is interpreted as the unique maximum total expected return over an infinite planning horizon, starting in state (i,j) with t steps remaining in the current cycle.

Thus given the function $V_o(\cdot)$, the sequence $V_1(\cdot)$, $V_2(\cdot)$, ... (and implicitly $r_1^*(\cdot)$, $r_2^*(\cdot)$, ...) is recursively calculated using Algorithm 3.5. For the undiscounted case ($\beta = 1$) monotonically converging upper and lower bounds on g are also recursively calculated (Su and Deininger, 1972). When the bounds on g are sufficiently tight (as defined by the programmer), the optimal closed loop reservoir release policy will be the last cycle of $r.^*(\cdot)$.

For the discounted case ($0 \leq \beta < 1$) monotonically converging upper and lower bounds on $V.^*(\cdot)$ are also calculated (Su and Deininger, 1972) along with $V.(\cdot)$ and $r.^*(\cdot)$. When the bounds on $V.^*(\cdot)$ are adequately tight (as defined by the programmer) the optimal closed loop reservoir release policy will be the last cycle of $r.^*(\cdot)$.

Figure 3.2 is a schematic representation of closed loop control for a reservoir system.

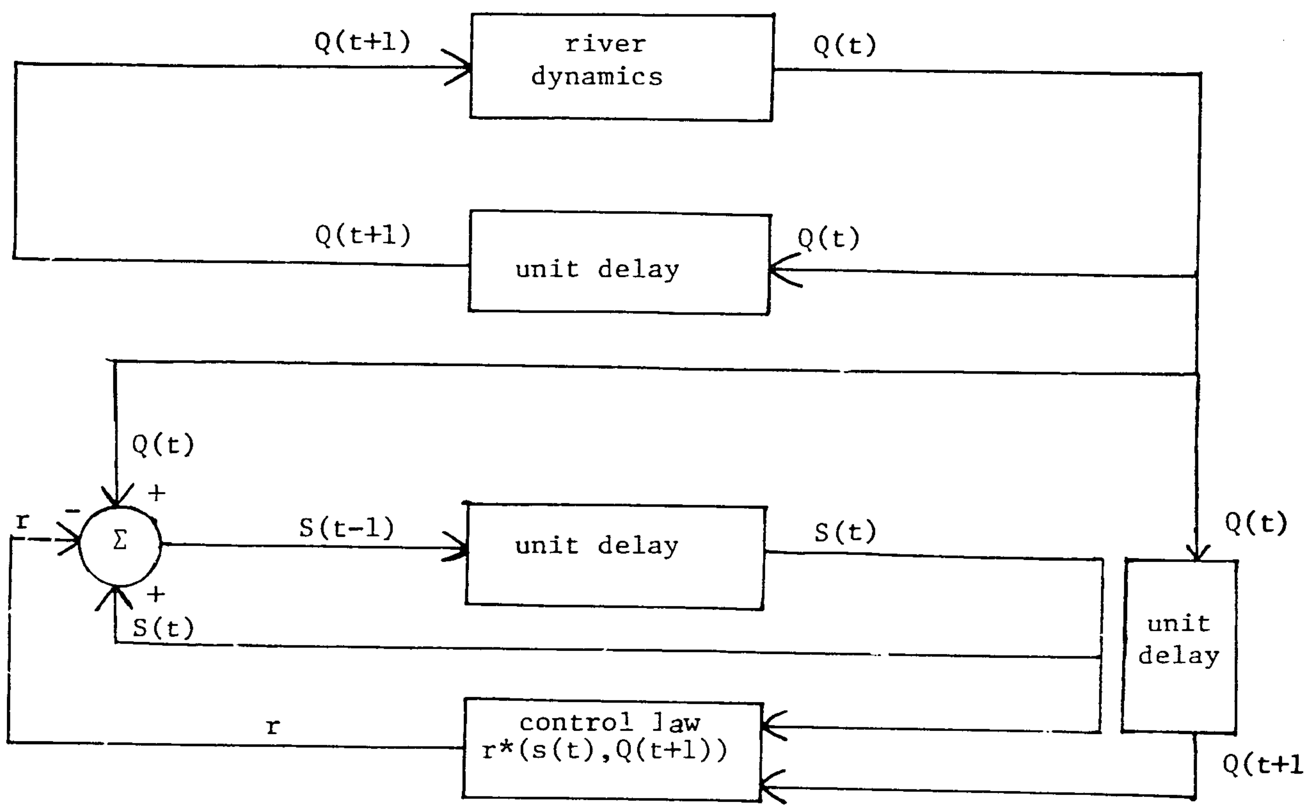


Figure 3.2

SCHEMATIC REPRESENTATION OF CLOSED LOOP CONTROL

3.3 Optimal Reservoir Control with the Use of Real Time Forecast Information

The derivation of the optimal closed loop control policies for the discounted and undiscounted case as described above was done under stationarity Assumptions 3.1 and 3.2.

$$P_{jk}(t) = P_{jk}(nT + t) \qquad \forall\ j,k,t,n \tag{3.1}$$

$$G_t(i,j,r) = G_{nT+t}(i,j,r) \qquad \forall\ i,j,r,n,t \tag{3.2}$$

These conditions were necessary for the convergence of Algorithm 3.5 to an optimal policy.

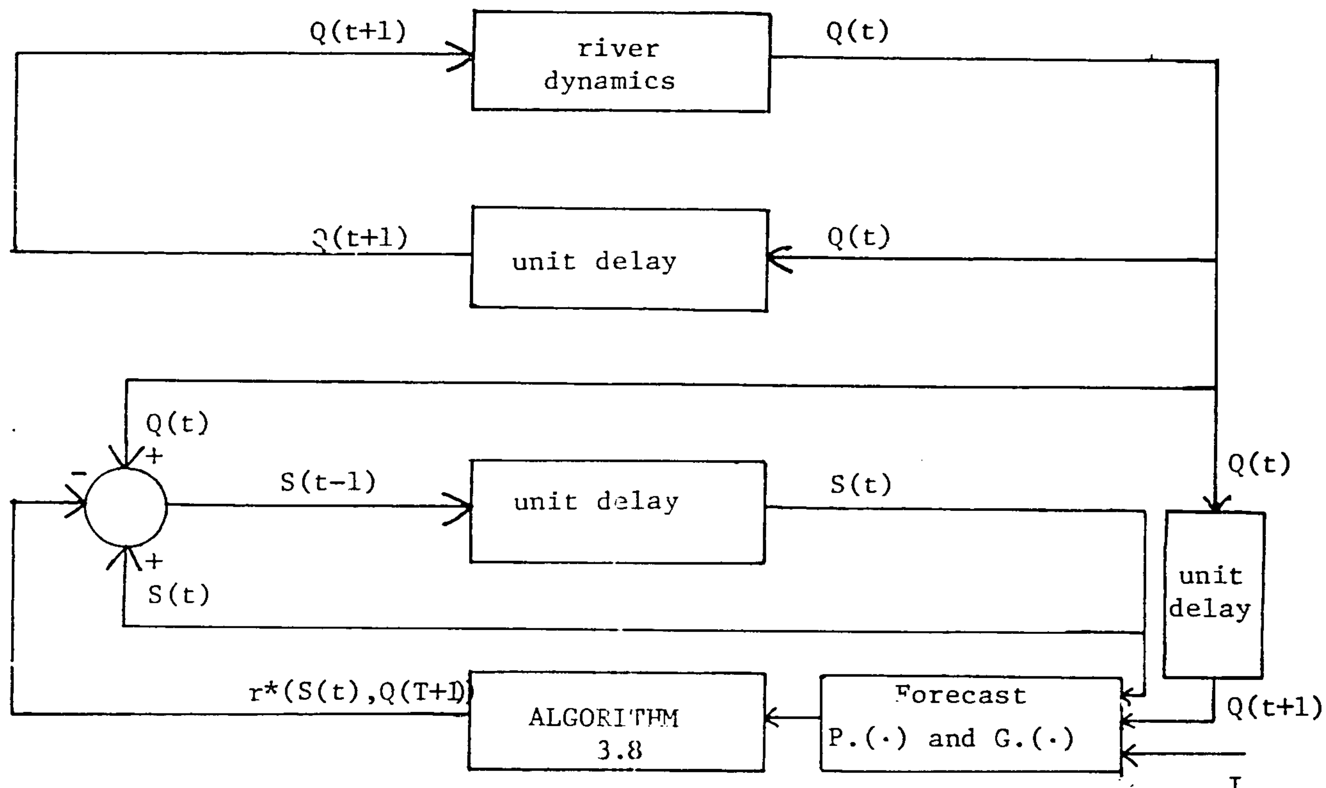
In actual reservoir operation, these stationarity assumptions are not necessarily satisfied. At a given point in time information can be gathered and used to update streamflow transition matrices and return functions such that Conditions 3.1 and 3.2 are no longer valid. For example, at any time, flow into the Aswan reservoir will be affected in a predictable way over a finite horizon by the flows presently observable at upstream stations. Thus, in general, $P.(\cdot)$ will be a function of absolute time. Similarly, the value of hydro-electric generation may be increased for a finite length of time into the future due to a transient period of alternative energy shortages. With the new nonstationary $P.(\cdot)$ and $G.(\cdot)$ functions, the closed loop policy will no longer be optimal.

A natural approach to the determination of the optimal reservoir release when $P.(\cdot)$ and $G.(\cdot)$ are nonstationary and updated at every step would be to apply Algorithm 3.5 repeatedly at every step to determine the optimal release. The logic of this procedure would be as follows:

$$\left.\begin{array}{ll} \text{Step 1:} & \text{use all causal information available to} \\ & \text{determine conditional } P.(\cdot) \text{ and } G.(\cdot) \\ \text{Step 2:} & \text{with M steps remaining in the planning horizon,} \\ & \text{compute the function } \{r_m^*(i,j);\ m=1,2,\ldots M; \\ & \forall\ i,j\} \text{ using Algorithm 3.5 with the conditional} \\ & P.(\cdot) \text{ and } G.(\cdot) \text{ defined in Step 1. Take the imme-} \\ & \text{diate control action } r_M^*(i,j) \text{ where } i,j \text{ is the} \\ & \text{present reservoir volume and previous inflow} \\ \text{Step 3:} & \text{if } M \neq 1, \text{ then } M = M-1 \text{ and go to Step 1} \end{array}\right\} \quad (3.8)$$

Figure 3.3 is a schematic diagram of the suggested adaptive real time control Algorithm 3.8.

A major problem with the above control algorithm is that for the probably large value of M it is computationally burdensome. Fortunately, computation may be dramatically reduced by realizing that at an arbitrary time P.(·) and G.(·) are nonstationary, and hence dependent on absolute time, for relatively few number of steps into the future given the presently available information. For example, the presently observable upstream river flows and probably any other process will affect downstream flows in a predictable way for only a finite number of steps, say M_o, into the future. Therefore, if P.(·) is computed using



I_t = all non state information available at time t

Figure 3.3

SCHEMATIC REPRESENTATION OF RESERVOIR CONTROL USING REAL TIME INFORMATION present information it, in general, will be nonstationary for the next M_o transitions and stationary beyond (see Chapter 6).

Thus, letting M_o denote the number of steps starting from the present (M) during which $P.(\cdot)$ or $G.(\cdot)$ are non-stationary, $r^*_M(i,j)$ in Step 2 of the above adaptive control algorithm can be found by using the recursion Algorithm 3.5 beginning with the boundary function $V'_{M-M_o}(i,j)$ to successively solve for $r^*_{M-M_o}(\cdot),\ldots,r^*_M(\cdot)$. The boundary value function $V'_{M-M_o}(\cdot)$ represents the relative value of being in a particular state over another with $M - M_o$ steps remaining in the planning horizon.

For M sufficiently large the information necessary to readily evaluate $V'_{M-M_o}(\cdot)$ was implicitly obtained when finding the optimal closed loop control for both the discounted and undiscounted case using the technique described in Section 3.2.

For the undiscounted case and stationarity Assumptions 3.1 and 3.2 it was pointed out that

$$\lim_{n\to\infty} (V_{(n+1)T+t}(i,j) - V_{nT+t}(i,j)) = g \qquad \forall\ i,j,t \tag{3.9}$$

where g is the unique maximum gain per cycle, and $V.(\cdot)$ is calculated from Algorithm 3.5. Thus, for large n,

$$V_{nT+t}(i,j) = ng + w_t(i,j) \qquad \forall\ i,j,t \tag{3.10}$$

and

$$V_{nT+t}(i,j) - V_{nT+t}(k,\ell) \doteq w_t(i,j) - w_t(k,\ell) \qquad \forall\ i,j,k,\ell,t \tag{3.11}$$

where $w_t(\cdot)$ is the intercept of the linear asymptote of $V_{nT+t}(\cdot)$. The function

$$B_t^{ud}(i,j) = w_t(i,j) - w_t(k,\ell) \qquad \forall\ i,j,t \tag{3.12}$$

with arbitrary, but fixed, k, ℓ, is the difference in total expected value (over a long future) of presently being in state i, j instead of state k,ℓ with t steps remaining in a cycle. Thus, $B_t^{ud}(i,j)$ is the desired boundary value function $V'_{M-M_o}(\cdot)$ for the undiscounted (ud) case. That the boundary value function is independent of absolute time (nT+t) and dependent on t is expected since the steady state probabilities (i.e., the probability of being in a particular state infinite time in the future) of an ergodic Markov chain are independent of initial state.

The information necessary to evaluate the boundary value Function 3.12 is readily available at the termination of Algorithm 3.5 when it is being used to derive the closed loop control for the undiscounted case. Recall from Section 3.2 that Algorithm 3.5 is terminated if the bounds on the unique maximal gain, g, are sufficiently tight. Hence, if the time of termination is at $n=n_o$, then

$$\hat{B}_t^{ud}(i,j) = V_{n_oT+t}(i,j) - V_{n_oT+t}(k,\ell) \doteq w_t(i,j) - w_t(k,\ell) \qquad \forall\ i,j,t \tag{3.13}$$

where k,ℓ is arbitrary, but fixed. The boundary value function $V'_{M-M_o}(\cdot)$ is evaluated from $B_t^{ud}(\cdot)$ by

$$\hat{V}'_{M-M_o}(i,j) = \hat{B}^{ud}_{MOD_T|M-M_o|}(i,j) \qquad \forall\ i,j,M,M_o \quad (3.14)$$

where $MOD_T|M-M_o|$ is defined as the fractional remainder of $M-M_o/T$ multiplied by T ($MOD_T|M-M_o|\ \varepsilon\{0, 1, 2, \ldots, T-1\}$).

For the discounted case and stationarity Assumptions 3.1 and 3.2, Section 3.2 pointed out that the convergence of $V_{nT+t}(i,j)$ to $V_t^*(i,j)$ for all i,j and t is used as the termination criteria of Algorithm 3.5 when deriving the optimal closed loop control. $V_t^*(i,j)$ is the unique maximum total expected return over an infinite planning horizon given that the present state is i,j and there are t steps remaining in a cycle. Thus, letting n_o denote the time of termination, the function

$$\hat{B}_t^d(i,j) = V_{n_oT+t}(i,j) \qquad \forall\ i,j,t \qquad (3.15)$$

defines the desired boundary value function $V'_{M-M_o}(i,j)$ for the discounted (d) case through the relation

$$\hat{V}'_{M-M_o}(i,j) = \hat{B}^d_{MOD|M-M_o|}(i,j) \qquad \forall\ i,j,M,M_o \qquad (3.16)$$

For both the discounted and undiscounted case, the boundary functions $\hat{B}_t^d(i,j)$ and $B_t^{ud}(i,j)$ may be tabulated over i,j and t using Equations (3.15) and (3.13), respectively. Thus, the calculation of the optimal control, when P.(•) and G.(•) are nonstationary, using the adaptive real time control Algorithm 3.8 described in Step 1 through 3 above, basically only requires the solution of an M_o stage dynamic programming problem each time new forecasts are made available.

Before closing this section, and presenting a summary in the next, the theoretical behavior of $B_t^{\bullet}(\cdot)$ as a function of $V_o(\cdot)$ should be pointed out. The relation is important since $V_o(\cdot)$ is usually unknown and thus must be assumed for reservoir problems.

For the discounted case, the influence of $V_o(\cdot)$ on the boundary value function is geometrically decreasing as the time distance between them increases. This is due to the discount factor. Thus, for large $M-M_o$, the boundary condition

$$V'_{M-M_o}(i,j) = B^d_{MOD|M-M_o|}(i,j) \qquad \forall\ i,\ j \tag{3.17}$$

may be considered independent of $V_o(\cdot)$.

For the undiscounted case, $V_o(\cdot)$ will affect the value of $V_{n_oT+t}(\cdot)$ calculated from Algorithm 3.5 at the time of termination. However, the effect for large n_o is to change the value of being in any particular state for any t by an additive constant.* This relation may be represented by

$$[V^{ud}_{n_oT+t}(i,j)\,|\,V^1_o(\cdot)] \doteq [V^{ud}_{n_oT+t}(i,j)\,|\,V^2_o(\cdot)] + C_t \qquad \forall\ i,j,t \tag{3.18}$$

or

---

*Howard (1960) proves this for an ergodic (aperiodic, single recurrent class) Markov chain and T = 1. Using an approach similar to that of Su and Deininger (1972), the proof for T > 1 is straightforward.

$$\lim_{n_o \to \infty} [V^{ud}_{n_oT+t}(i,j) \mid V^1_o(\cdot)] = \lim_{n_o \to \infty} [V^{ud}_{n_oT+t}(i,j) \mid V^2_o(\cdot)] + C_t \qquad \forall\ i,j,t \tag{3.19}$$

where $V^1_o(\cdot)$ and $V^2_o(\cdot)$ are 2 arbitrary functions and C_t is some constant. This result is intuitively reasonable. Since the steady state probabilities, π_{ijt}, are independent of initial state for ergodic Markov chains, the total expected value at the end of the planning horizon, if far into the future, is independent of present state and is given by

$$\sum_{ij} \pi_{ijt}\, V_t(i,j)\Big|_{t=0} \tag{3.20}$$

Inspecting Equation (3.13)

$$\hat{B}^{ud}_t(i,j) = V_{n_oT+t}(i,j) - V_{n_oT+t}(k,\ell) \tag{3.13}$$

it is clear that the constant of Equation (3.18) is cancelled when deriving the boundary function and hence $B^{ud}_t(i,j)$ and $V'_{M-M_o}(i,j)$ are independent of $V_o(\cdot)$ in the undiscounted case.

3.4 Summary

The previous section discussed a 3-step algorithm (3.8) which may be used for reservoir control. Real time information is incorporated into the control action through the use of the adaptive functions $P.(\cdot)$ and $G.(\cdot)$. Through the use of boundary conditions, $V'_{M-M_o}(\cdot)$, obtained from the calculation of the optimal closed loop reservoir control as outlined in Section 3.2, it was shown that the 3-step real time algorithm does not involve a heavy computational burden.

The remainder of this work is concerned with the real time characterization of riverflow dynamics and how this information may be used to update P.(•) (see Chapter 6). The real time behavior of G.(•) will not be considered.

Chapter 4

PRELIMINARIES OF FORECASTING APPROACH

4.1 Flow Aggregation

The previous chapter briefly outlined a real time reservoir control scheme. In particular, it was shown that for each release decision real time information concerning inflow could be introduced through conditional Markovian transition matrices.

Inherent to this reservoir control approach is the discretization of time, and thus the aggregation of inflow. The degree of aggregation which is desirable is a primary question.

Probably the single most important factor bearing on this determination is the relative magnitudes of inflow rate and reservoir storage capacity. The time scale for which variability of inflows is significant would also be important.

The annual inflow to Aswan Reservoir is approximately one half the reservoir storage volume (see Section 2.3). Also due to the great spatial extent of the Nile River Basin the inflow rate varies smoothly from week to week.

For these reasons it was decided to approach the determination of optimal reservoir release policy on a monthly level of aggregation. (Conveniently, this is the level of aggregation presently used in Egypt, see Alarcon, 1979). Consequently the forecasts of monthly inflow are needed.

4.2 "Black-Box" vs. Conceptual Forecasting Approach

The Nile Basin and its environment may be thought of as a physical system which converts a spatially and temporally varying precipitation process, $p(x,y,t)$, into a temporally varying inflow, $Q(t)$, into Aswan Reservoir (see Figure 4.1).

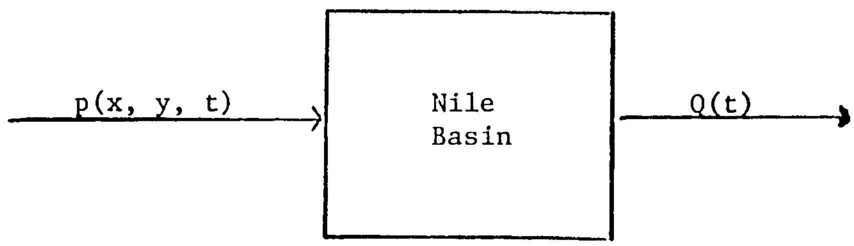


Figure 4.1

NILE BASIN INPUT OUTPUT MODEL

At the extremes there are two approaches which may be used to forecast the inflow.

A detailed moisture accounting model which routes precipitation among the various hydrologic pathways could be constructed. Given the state of the system at t_o, forecasts of future behavior could be made using forecasted precipitation (i.e., $\hat{p}(x,y,t)$, $t > t_o$). This forecasting approach is conceptual since it tries to imitate the response of the various hydrologic units.

To forecast monthly inflow to Aswan Reservoir the conceptual solution would be prohibitively expensive, if even humanly possible. Vast amounts of physical data would be required to build and calibrate a model and even the measurement of $p(x,y,t)$ would be a formidable task.

At the other extreme, statistical techniques with no physical considerations could be used to derive a functional relation between precipitation, or some surrogate measure, and the inflow. Since the physical dynamics relating precipitation to inflow are not explicitly considered, the resulting model is often referred to as a black-box. Forecasts are made by using the forecasted precipitation or surrogate measure.

The black-box modeling approach may actually give better forecasts than a conceptual approach and has the advantage of requiring only historical observations of the precipitation (or surrogate measure) and the inflow for model calibration.

A modeling approach that lies somewhere between the conceptual and the black-box will be taken in this work. Specifically, monthly inflow forecasts into Aswan Reservoir will be made using observations of past monthly inflows and monthly flow at other upstream gaging stations. The structure of the model will be mostly determined by using simple physical principles (this aspect is discussed in detail in Section 5.5) while the coefficients of the model are obtained by statistical techniques. The stochastic nature of the problem will be explicitly accounted for.

In the following section, the decision to use upstream observations of monthly flow and not to use precipitation observations is discussed.

4.3 Exclusion of Precipitation and Use of Upstream Monthly Flow Observations in Forecasting Model

The real time information used for the forecasts of monthly Aswan Reservoir inflows (represented by flow at Wadi Halfa), for the models presented in this work, is observations of past monthly reservoir inflow and past observations of monthly flows at other upstream gaging stations throughout the Basin (i.e., stations listed in Appendix One). The usefulness of the upstream observations of flow for the forecast is obvious considering the travel times between stations, Figure 2.25. Precipitation observations were not used since their usefulness is questionable.

In Section 2.2, it was shown that the monthly discharge at the gaging stations of the Blue Nile and River Atbara closely follow the variation of monthly rainfall over the Ethiopian Plateau. Apparently, the time of delay between rainfall and subsequent realization of river flow is much less than one month and on the order of a few days. Thus, for any given month, the informational content of precipitation and discharge in this area is roughly equivalent and there will be little benefit to using observations of both.

This equivalence is not observed for the discharge downstream of the Equatorial Lakes at Mongalla, or for the discharge downstream of the Sudd region at Malakal (see Section 2.2). The lakes and swamp most likely act to damp the variation in discharge. Thus, it is possible that observations of recent monthly precipitation will improve the ability to predict the future discharges in this area.

However, due to the tremendous damping ability of the Equatorial Lakes and the Sudd region, discharge in these areas varies smoothly between months and in fact between years (Mongalla [1905-1975] and Malakal [1905-1976] have yearly autocorrelation coefficients of .88 and .78, respectively) and thus the monthly flow is highly predictable without observations of rainfall. Also, taking into consideration that waters flowing through Mongalla and Malakal contribute only 30% of the total annual inflow to the Aswan Reservoir, the possible benefit of a small increase of forecast ability in this region using rainfall observations does not warrant the increased operational complexity of a model that would need real time observations of rainfall.

There is one other reason for not using rainfall in the forecasting model. A model which incorporates precipitation observations would require forecasted monthly precipitation to yield forecasted inflow. Because the monthly autocorrelation between precipitation over the Nile Basin is generally very small, as shown in Table 4.1, the forecasted precipitation would contain little conditional information.

In closing, it is stressed that it is the particular hydrologic behavior of the Nile and the monthly level of aggregation chosen which lead to the decision to exclude observations of precipitation when making inflow forecasts. Good hourly or daily forecasts of reservoir inflow for a small watershed would almost undoubtedly require precipitation observations. Similarly, discharge forecasts for a large watershed in which the source of the spring flow is primarily snowmelt would improve with observations of prior precipitation.

Station	Lat.	Long.	Years	J-F	F-M	M-A	A-M	M-J	J-J	J-A	A-S	S-O	O-N	N-D	D-J
Atbara	17°4 '	33°58'	1908-1957	.00	.00	.00	-.06	.23	-.11	-.02	.44	-.08	-.00	-.00	.00
Khartoum	15°37'	32°33'	1908-1952	.00	.00	-.02	0.23	0.11	-.21	.20	.18	-.19	-.08	.00	.00
Geteina	14°52'	32°22'	1906-1957	.00	.00	.00	.04	.10	-.07	.23	.16	-.08	.22	.00	.00
Sennar	13°33'	34°37'	1915-1957	.00	.00	.03	-.14	-.09	-.20	-.01	-.18	-.28	.06	.00	.00
Nyala	12°04'	24°53'	1920-1957	.00	.00	.49	.23	-.03	-.13	-.02	-.01	.22	-.08	-.03	-.11
Roseires	11°51'	34°23'	1905-1957	-.03	-.07	-.01	-.12	-.03	.04	-.04	.37	-.29	.07	.01	-.02
Renk	11°45'	32°47'	1906-1957	.00	.00	.10	-.00	-.11	.01	-.16	.12	.00	-.09	-.05	.08
Kadugli	11°00'	29°43'	1910-1957	.09	-.05	.53	.07	-.10	-.06	-.19	.12	.04	-.20	-.00	.00
Malakal	9°32'	31°29'	1916-1957	.00	.00	.20	-.12	-.09	-.14	.12	-.24	-.02	.25	-.05	-.33
Raga	8°28'	25°41'	1910-1957	-.03	.03	-.12	-.01	-.18	-.23	-.13	.15	.14	.04	-.06	-.06
Gambeila	8°15'	34°35'	1906-1957	-.07	.02	.09	.00	-.09	.32	-.06	.34	.16	-.16	-.24	.27
Wau	7°42'	28°01'	1904-1957	-.08	.05	-.08	.15	.20	-.08	.01	.04	.16	.09	.14	.14
Bor	6°12'	31°33'	1905-1957	-.07	.11	-.03	-.06	.00	-.03	.09	.25	.28	.07	-.04	.21
Yei	4°05'	30°40'	1916-1957	.21	-.18	-.15	.09	.23	.01	.30	.33	-.18	-.04	-.11	.01
Nimule	3°36'	32°03'	1904-1954	.07	-.08	-.18	-.04	-.22	-.08	.02	.14	-.09	.03	.00	-.20

Table 4.1 Monthly Autocorrelations of Precipitation for Some Nile Basin Measuring Stations (Precipitation data from Nile Control Department, 1950, 1955, 1957 and 1963)

Chapter 5

MONTHLY MULTIVARIATE FORECAST OF RIVERFLOW

5.1 Introduction

This chapter will be concerned with the real time forecast of monthly flow. The primary focus will be the identification, estimation, and validation of stochastic dynamic models. A discussion of the specific technique that is required so that these models may be efficiently used to obtain the conditional Markovian transition matrices needed for the adaptive real time control Algorithm 3.8 will be delayed until Chapter 6.

The previous chapter pointed out that due to the particular nature of the River Nile hydrology and the large storage capacity of the High Aswan Dam, monthly forecasts of inflow are adequate. Also, upstream observations of flow will undoubtably improve forecasts of reservoir inflow and there seems to be little need to explicitly include observations of precipitation. Thus, all models considered will explicitly account for a seasonal variation of flow and multivariate versions will have the capability of using observations of upstream flow.

5.2 Literature Review

5.2.1 Univariate Models

Although there are numerous publications which propose univariate stochastic dynamic models for the simulation of monthly flow, there have been few which address the issue of streamflow forecasting over periods greater than one month. Chapter 3 gave a specific motivation why multi-lead forecasts are desirable.

Rao and Kashyap (1973, 1974) are two of the few researchers who have suggested a method to obtain multi-lead streamflow forecasts. They propose a univariate stochastic dynamic model which has the general form

$$Y(k,i) = \sum_{\ell=1}^{n_1} \alpha_\ell \, Y(k,i-\ell) + U(i) + V(k,i) + \sum_{\ell=1}^{n_2} \alpha_{n_1+\ell} \, V(k,i-\ell)$$

$$i = 0, 1, \ldots, 11 \qquad k = 0, 1, 2, \ldots \tag{5.1a}$$

where

$$Y(k,i) = \text{discharge during the } i^{th} \text{ month of the } k^{th} \text{ year} \tag{5.1b}$$

$$\left.\begin{aligned} Y(k,i-\ell) &= Y(k-1,\ 12+i-\ell) \\ V(k,i-\ell) &= V(k-1,\ 12+i-\ell) \end{aligned}\right\} \text{ if } i-\ell < 0 \tag{5.1c}$$

$$U(i) = \alpha_o + \sum_{j=1}^{n_3} (\alpha_{n_1+n_2+2j-1} \cos\omega_j i + \alpha_{n_1+n_2+2j} \sin\omega_j i) \tag{5.1d}$$

$$\omega_j = 2\pi j/12 \tag{5.1e}$$

$$V(k,i) = \Psi(i)\ W(k,i) \tag{5.1f}$$

$$\Psi(i) = \beta_o + \sum_{j=1}^{n_4} (\beta_{2j-1} \cos\omega_j i + \beta_{2j} \sin\omega_j i) \tag{5.1g}$$

$$\underline{N} = \{n_1, n_2, n_3, n_4\} = \text{structural parameters} \tag{5.1h}$$

$$\underline{\alpha} = \{\alpha_o, \alpha_1, \ldots, \alpha_{n_1+n_2+2n_3}\} = \text{model coefficients} \tag{5.1i}$$

$$\underline{\beta} = \{\beta_o, \beta_1, \ldots, \beta_{2n_4}\} = \text{model coefficients} \tag{5.1j}$$

The random sequence, $W(k,i)$ is assumed to satisfy

$$\left.\begin{array}{l} E[W(k,i)] = 0 \qquad \forall\ k,i \\ E[W(k,i)W(k',i')] = \delta(k-k')\delta(i-i')\sigma_w^{\ 2} \qquad \forall\ k,k',i,i' \\ E[W(k,i)Y(k,i-j)] = 0 \qquad \forall\ k,i\ ;\ j > 0 \\ \delta(\cdot) \text{ is the Kronecker delta function} \end{array}\right\} \tag{5.1k}$$

The deterministic function $U(i)$ has a period of 12. It is introduced to reflect the annual variation of monthly means. The noise term $V(\cdot)$ has been included to account for the part of the flow which is nondeterministic. Its multiplicative factor, $\Psi(i)$ ($V(k,i) = W(k,i) \cdot \Psi(i)$), also has a period of 12 and is present to account for the annual variation of monthly standard deviations.

For simplicity of notation Equation (5.1a) may be written in terms of the one-dimensional sequences $Y(j)$ and $V(j)$.

$$Y(j) = \sum_{\ell=1}^{n_1} \alpha_\ell Y(j-\ell) + U(MOD_{12}(j)) + V(j) + \sum_{\ell=1}^{n_2} \alpha_{n_1+\ell} V(j-\ell) \tag{5.1ℓ}$$

where

$$j = 12k + i \tag{5.1m}$$

$$Y(j) = Y(k,i) \tag{5.1n}$$

$$V(j) = V(k,i) \tag{5.1o}$$

and $MOD_{12}(j)$ is defined as the fractional remainder of j/12 multiplied by $12(MOD_{12}(j) \varepsilon \{0,1,\ldots,11\} \forall j)$.

Given an historical data set of length n+1, $(n+1 > n_1+n_2+2n_3)$ $\{Y(i), i=0,1,2,\ldots,n\}$, the structural parameters, $\underline{N}$, may be identified and the model coefficients, $\underline{\alpha}$, estimated. Given $\hat{\underline{N}}$, and $\hat{\underline{\alpha}}$, Rao and Kashyap indicate that multi-lead forecasts may be made using the following recursion relation.

$$E[Y(n+m)|\ \{Y(i),i=0,1,2,\ldots,n\}] \equiv E[Y(n+m)|n] \equiv \hat{Y}(n+m) =$$

$$\sum_{\ell=1}^{\hat{n}_1} \hat{\alpha}_\ell E[Y(n+m-\ell)|n] + \hat{U}(MOD_{12}(n+m)) +$$

$$\sum_{\ell=1}^{\hat{n}_2} \hat{\alpha}_{n_{1+\ell}} \quad E[V(n+m-\ell)|n] \quad ; \quad m > 0 \tag{5.2a}$$

where

$$E[Y(n+m-\ell)|n] = Y(n+m-\ell) \text{ if } m-\ell \leq 0 \tag{5.2b}$$

$$E[Y(n+m-\ell)|n] = \hat{Y}(n+m-\ell) \text{ if } m-\ell > 0 \tag{5.2c}$$

$$E[V(n+m-\ell)|n] = \hat{V}(n+m-\ell) = Y(n+m-\ell)$$

$$- \left[\sum_{\ell'=1}^{\hat{n}_1} \hat{\alpha}_{\ell'} \ Y(n+m-\ell-\ell') + \hat{U}(MOD_{12}(n+m-\ell)) + \sum_{\ell'=1}^{\hat{n}_2} \hat{\alpha}_{n_1+\ell'} \ \hat{V}(n+m-\ell-\ell') \right] \quad \text{if } m-\ell \leq 0 \tag{5.2d}$$

and

$$E[V(n+m-\ell)|n] = 0 \qquad \text{if } m-\ell > 0 \text{ or } n+m-\ell < \hat{n}_1 \tag{5.2e}$$

Thus, beginning with the calculation of $\hat{Y}(n+1)$, the forecasts $\hat{Y}(n+2)$, $\hat{Y}(n+3)$,... may be recursively calculated.

Rao and Kashyap (1973) fit Model 5.1 to the monthly flows of 3 different rivers. Table 5.1 shows the estimated structural parameters of each model needed for forecast Algorithm 5.2. Note that $\hat{n}_2=0$ for all the models.

River	$\hat{n}_1$	$\hat{n}_2$	$\hat{n}_3$
Godavari	1	0	3
Krishna*	2	0	4
Krishna	4	0	4
Wabash*	1	0	1
Wabash	1	0	2

* Both models pass diagnostic checks (see Rao and Kashyap, 1973).

TABLE 5.1: ESTIMATED STRUCTURAL PARAMETERS OF MODEL 5.1 FOR THREE DIFFERENT RIVERS (from Rao and Kashyap, 1973)

Using Algorithm 5.2 Rao and Kashyap found that monthly forecasts with mean squared error less than historical monthly variance may be obtained for up to three months.

Many other stochastic dynamic models of monthly riverflow presented in the literature are very similar to the general Rao-Kashyap Model 5.1. For example, Rodriguez-Iturbe (1968) proposed the following model for monthly flows of the Orinoco River.

$$Y(k,i) = U(i) + Z(k,i) \tag{5.3a}$$

$$Z(k,i) = \rho Z(k,i-1) + W(k,i) \tag{5.3b}$$

where $Y(\cdot)$, $U(\cdot)$, $W(\cdot)$ are defined in Model 5.1 and $|\rho| < 1$.

Model 5.3 consists of a cyclic deterministic component, $U(\cdot)$, and a stochastic part, $Z(\cdot)$, which is shown to follow a first order autoregressive process with an additive white noise random component. It may be reformulated as

$$Y(k,i) = U(i) + \rho(Y(k,i-1) - U(i-1)) + W(k,i)$$

$$= \rho Y(k,i-1) + U'(i) + W(k,i) \tag{5.4a}$$

where

$$U'(i) = (U(i) - \rho U(i-1)) \tag{5.4b}$$

Model 5.4 is clearly similar to 5.1 with structural parameters $n_1=1$, $n_2=0$, $n_3 \leq 6$ and $n_4=0$.

Clarke (1973) also identified and estimated the parameters of a stochastic model similar to 5.3 for forecasting monthly riverflow. Neither Rodriguez-Iturbe or Clarke evaluate the effectiveness of the model for multi-lead forecasts.

Maissis (1977) demonstrated that a high order (greater than the number of observations per cycle) autoregressive model can be used to forecast monthly riverflow. Specifically, he used a model of the form

$$Y(k,i) = \sum_{\ell=1}^{15} \alpha_\ell Y(k,i-\ell) + W(k,i) \tag{5.5}$$

Obviously Model 5.5 is a particular case of 5.1 with $n_1=15$, $n_2=0$, $n_3=0$ and $n_4=0$. Maissis concludes that

> "the accuracy of short and medium term forecasts computed by this model [5.5] is quite good"

although he does not give any explicit quantitative results in support of the above statement.

Another type of stochastic dynamic equation commonly used for modeling riverflow is the monthly Markov model (Fiering and Jackson, 1971). It has the general form

$$Y(k,i) = \alpha_1(i)\ Y(k,i-1) + U(i) + V(k,i);\ i = 0,\ 1,\ 2,\ \ldots,\ 11 \tag{5.6}$$

This model is similar to 5.1 but is not a particular case since the regression coefficient, $\alpha_1(i)$, explicitly depends on the month.

A generalization of the Markov Model 5.6 is the multi-lag monthly regression model (Fiering and Jackson, 1971)

$$Y(k,i) = \sum_{\ell=1}^{n_1(i)} \alpha_\ell(i)\, Y(k,i-\ell) + U(i) + V(k,i) \quad ; \quad i=0,1,2,\ldots 11 \tag{5.7}$$

This model is always as good or a better characterization of monthly riverflow than the Rao-Kashyap Model 5.1 with the restriction $n_2=0$ imposed. This is easily seen since the dynamics of river flow will probably change with season and in any case Model 5.1 with $n_2=0$ will be a particular case of the more general Model 5.7. However, given a short historical record difficulties may be encountered in estimating the large number of parameters of Model 5.7. In such a case better results may be obtained from the more parsimonious Model 5.1.

A model discussed in detail by Roesner and Yevdyevich (1966) and applied by McKerchar and Delleur (1972) is closely related to Model 5.7. It has the form

$$Z(k,i) = \frac{Y(k,i) - U(i)}{\sigma_Y(i)} \tag{5.8a}$$

$$Z(k,i) = \sum_{\ell=1}^{n_1} \rho_\ell\, Z(k,i-1) + W(k,i) \tag{5.8b}$$

$$\sigma_Y(i) = \gamma_o + \sum_{j=1}^{n_5} (\gamma_{2j-1} \cos\omega_j i + \gamma_{2j} \sin\omega_j i) \tag{5.8c}$$

$$i = 0,1,2.,,,11$$

$$\omega_j = 2\pi j/12 \tag{5.8d}$$

with U(i) as in model 5.1. The seasonal behavior of the monthly means and variances in the above model are preserved by normalizing the flow variable (5.8a). It can be seen that this model is related to Model 5.7 by writing Equation 5.8b as

$$\frac{Y(k,i) - U(i)}{\sigma_Y(i)} = \sum_{j=1}^{n_1} \frac{\rho_j[Y(k,i-j)=U(i-j)]}{\sigma_Y(i-j)} + W(k,i) \tag{5.9}$$

Letting n_1=1 for illustrative purposes, Equation 5.9 can be rearranged to give

$$Y(i) = \frac{\sigma_Y(i)}{\sigma_Y(i-1)} \rho_1 Y(k,i-1) + \left\{U(i) - \frac{\sigma_Y(i)}{\sigma_Y(i+1)} \rho_1 U(i-1)\right\}$$

$$+ \sigma_Y(i)W(k,i)$$

$$= \alpha_1'(i)\, Y(k,i-1) + U'(i) + V'(k,i) \tag{5.10a}$$

where

$$\alpha_1'(i) = \frac{\sigma_Y(i)}{\sigma_Y(i-1)} \rho_1 \tag{5.10b}$$

$$U'(i) = U(i) - \frac{\sigma_Y(i)}{\sigma_Y(i+1)} \rho_1 U(i-1) \tag{5.10c}$$

$$V'(k,i) = \sigma_Y(i)\, W(k,i) \tag{5.10d}$$

McKerchar and Delleur (1972) fit Model 5.8 with n_1=2 to monthly flows of the Ohio River and showed that forecasts with error variance less than historical monthly variances may be obtained for up to 3 months. This result agrees with the previously mentioned results of Rao and Kashyap.

Box and Jenkins (1970) propose that a multiplicative ARIMA (Autoregressive-Integrated-Moving Average) model be used to forecast seasonal time series. They argue that, for example, February of one year is related to February of the previous year so that the following relation should apply.

$$\Phi_P(B^{12})\ \nabla_{12}^{D}\ Y(k,i) = \Theta_Q(B^{12})\ \varepsilon(k,i) \tag{5.11a}$$

$$B^{12} = \text{backward shift operator, } B^{12}Y(k,i)$$
$$= Y(k-1,i) \tag{5.11b}$$

$$\nabla_{12}^{D} = (1-B^{12})^{D} \tag{5.11c}$$

$$\Phi_P(B^{12}) = (1 - \Phi_1 B^{12} - \Phi_2 B^{2\cdot 12} - \ldots - \Phi_P B^{P\cdot 12}) \tag{5.11d}$$

$$\Theta_Q(B^{12}) = (1 - \Theta_1 B^{12} - \Theta_2 B^{2\cdot 12} - \ldots - \Theta_3 B^{Q\cdot 12}) \tag{5.11e}$$

$$\varepsilon(k,i) = \text{disturbance term} \tag{5.11f}$$

Box and Jenkins further argue that a model of the form 5.11 might be used to link the current March with previous March observations, and so on, for each of the twelve months.

The noise term, $\varepsilon(k,i)$, probably would not be independent of $\{\varepsilon(k,j),\ j = i-1,\ i-2...\}$ so that the disturbance terms might be represented by a model of the type

$$\phi_p(B)\nabla^d\varepsilon(k,i) = \theta_q(B)W(k,i) \tag{5.12a}$$

$$\nabla^d = (1-B)^d \tag{5.12b}$$

$$\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - ... -\phi_p B^p) \tag{5.12c}$$

$$\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - ... -\theta_q B^q) \tag{5.12d}$$

$W(k,i)$ as defined in 5.1k

Combining Models 5.11 and 5.12 gives the multiplicative ARIMA Model 5.13

$$\phi_p(B)\Phi_P(B^{12})\nabla^d\nabla_{12}^D Y(k,i) = \theta_q(B)\Theta_Q(B^{12})W(k,i) \tag{5.13}$$

Box and Jenkins denote the structure of the above model by the notation $(p,d,q) \times (P,D,Q)_{12}$.

Little experience has been accumulated in the use of the multiplicative ARIMA for modeling monthly riverflow. Only two applications, McKerchar and Delleur (1972) and Clarke (1973), were located.

McKerchar and Delleur argue that the (2,0,0) X $(0,1,1)_{12}$ ARIMA model which they estimated for monthly Ohio River flows is inadequate for forecasting because it cannot account for the seasonal variability in the historical monthly standard deviations. They demonstrate that a model of the form 5.8 yields superior forecasts.

Clarke fit a (0,1,1) X $(0,1,1)_{12}$ model to a historical record of 14 years. His major emphasis was parameter estimation and he did not evaluate the model adequacy.

One probable reason for the infrequent use of the multiplicative ARIMA is its complexity. Identification of the structure (i.e., specification of p,d,q and P,D,Q) is difficult and the coefficient estimation problem is generally nonlinear. For example, the (0,1,1) X $(0,1,1)_{12}$ model may be written as

$$y(k,i) - y(k,i-1) - y(k-1,i) - y(k-1,i-1) =$$

$$W(k,i) - \theta_1 W(k,i-1) - \Theta_1 W(k-1,i) - \theta_1 \Theta_1 (W(k-1,i-1) \qquad (5.14)$$

which is clearly nonlinear in coefficients due to the term $\theta_1\Theta_1$. Box and Jenkins (1970) do describe an iterative estimation procedure for the general multiplicative ARIMA.

One strong advantage of the multiplicative ARIMA model is its small number of coefficients. For example the (0,1,1) X $(0,1,1)_{12}$ model shown in Equation 5.14 has only 3 unknown parameters, the coefficients θ_1 and Θ_1 and the variance of the noise $W(\cdot)$, σ_W^2.

5.2.2 Multivariate Models

Thus far all the stochastic dynamic models reviewed have been univariate (i.e., the future value of the flow depends only on past values of itself and noise terms). Chapter Four pointed out the availability of upstream observations of flow for the Nile River. Undoubtably, forecasts of reservoir inflow will be considerably improved by using this information and thus a multivariate stochastic dynamic model is needed.

Similar to the univariate case there have been numerous publications which propose stochastic dynamic models for the simulation of multivariate monthly discharges. However, no reference to the use of these models for multi-lead forecasts could be found. Thus, only one multivariate stochastic dynamic model will be discussed. The particular model reviewed was chosen since it is commonly used in practice. Also, it is similar to the model proposed later in this thesis for multi-lead inflow forecasts using upstream observations.

The general form of the model is

$$\underline{Y}(k,i) = \underline{\underline{A}}_i \left[\begin{bmatrix} \underline{Y}(k,i-1) \\ \underline{Y}(k,i-2) \\ \vdots \\ \underline{Y}(k,i-p) \end{bmatrix} - \begin{bmatrix} \overline{\underline{Y}}(i-1) \\ \overline{\underline{Y}}(i-2) \\ \vdots \\ \overline{\underline{Y}}(i-p) \end{bmatrix} \right]$$

$$+ \overline{\underline{Y}}(i) + \underline{\underline{C}}_i \underline{W}(k,i) \qquad i = 0,1,2,\ldots,11 \qquad (5.15a)$$

where

$$\underline{Y}(k,i) = \begin{bmatrix} Y_1(k,i) \\ Y_2(k,i) \\ \vdots \\ Y_{n_o}(k,i) \end{bmatrix} \tag{5.15b}$$

$$Y_j(k,i) = \text{flow at station } j\text{, year } k\text{, month } i \tag{5.15c}$$

$$n_o = \text{number of gaging stations} \tag{5.15d}$$

$$\underline{\overline{Y}}(i) = \begin{bmatrix} \overline{Y}_1(i) \\ \overline{Y}_2(i) \\ \vdots \\ \overline{Y}_{n_o}(i) \end{bmatrix} \tag{5.15e}$$

$$\overline{Y}_j(i) = \text{mean flow of month } i\text{, station } j \tag{5.15f}$$

$$\underline{\underline{A}}_i = (n_o \text{ X } pn_o) \text{ matrix of coefficients, month } i \tag{5.15g}$$

$$\underline{\underline{C}}_i = (n_o \text{ X } n_o) \text{ matrix of coefficients, month } i \tag{5.15h}$$

$$\underline{W}(k,i) = \begin{bmatrix} W_1(k,i) \\ W_2(k,i) \\ \vdots \\ W_{n_o}(k,i) \end{bmatrix} \tag{5.15i}$$

$$\left.\begin{array}{ll} E[W_j(k,i)] = 0 & \forall\ j,k,i \\ E[W_j(k,i)\ W_{j'}(k',i')] = \delta(j-j')\delta(k-k')\delta(i-i')\sigma_W^{\ 2} & \\ \qquad\qquad \forall\ j,j',k,k',i,i' & \\ \text{where } \delta(\cdot) \text{ is the Kronecker delta} & \\ E[W_j(k,i)\ Y_{j'}(k,i-\ell)] = 0 & \forall\ j,k,i,j',\ \ell > 0 \end{array}\right\} \tag{5.15j}$$

The model may be referred to as a p'th order multivariate autoregressive.

The equation for a given element of Model 5.15 is

$$Y_j(k,i) = \sum_{\ell=1}^{p} \sum_{m=1}^{n_o} \left[\underline{\underline{A}}_i\ (j,(m-1)p+\ell)\ \{Y_m(k,i-\ell) - \overline{Y}_m(i)\} \right]$$

$$+ \overline{Y}_j(i) + \sum_{m=1}^{n_o} \underline{\underline{C}}_i\ (j,m)\ W_m(k,i) \tag{5.16}$$

Observing Equation 5.16 it can be seen that the discharge $Y_j(k,i)$ depends upon previous discharges at that station as well as others plus a random component. The random component results from a linear combination of the vector $\underline{W}(k,i)$. Since in general the matrix $\underline{\underline{C}}_i$ is not diagonal the random component of discharge at each station is related to the random component at all other stations.

Fiering (1964), Fiering and Jackson (1971) and Matalas (1967) discuss the estimation and use of the model. Curry and Bras (1978) show the adequacy of the first order multivariate autoregressive model for the simulation of discharge at gaging stations throughout the Nile Basin.

5.2.3 Summary

To summarize the literature review presented, three main points should be made.

(1) Several univariate stochastic dynamic models have been proposed for monthly riverflow. However, little experience has been gained in the use of these models for multi-lead forecasts.

(2) Available data show that univariate models may be used to obtain monthly riverflow forecasts with mean square error less than historical variance for up to three months.

(3) Several multivariate stochastic models have been proposed for monthly riverflows. There is no available data on the usefulness of these models for multi-lead forecasts.

5.3 Iterative Approach to Model Building

There is no unique approach to be followed in model building but there are general guidelines which can aid in the search for an "adequate" model. This section briefly discusses these guidelines and should serve to put the mathematical techniques presented in the rest of this chapter in context.

The first step in model building is the proposal of a general class of models which, in the judgment of the modeler, will contain specific models that are adequate for the purpose at hand. A priori information of the phenomena being modeled and considerations of model coefficient estimation may play important roles in this decision. For example, a priori information indicating the existence of seasonality or other types of deterministic behavior, such as a linear trend, will certainly influence the class of models proposed. Also, if it is felt that a linear (in coefficients) class of models can adequately represent the phenomena a harder to estimate nonlinear class of models may be avoided.

The principle of parsimony (Box-Jenkins, 1970) together with a consideration of the amount of data available to estimate the model may also help to determine the general class of models to be considered. A model built with little data must, out of necessity, contain few model coefficients. However, with more data, a less parsimonious model may be desirable.

Two examples of a general class of models proposed for monthly univariate river discharge were Models 5.1 and 5.13.

The second step to be taken is the identification of a particular model from the general class to be tested for adequacy. This step, as the first, is somewhat subjective.

In general, inspection of the autocorrelation, partial autocorrelation, and spectral density functions should be useful when modeling a univariate time series (Box and Jenkins, 1970; Granger and Newbold, 1977; Kashyap and Rao, 1976). For the multivariate case, cross correlation and cross spectral analysis can be helpful. However, practical consideration of the phenomena being modeled as well as past experience may prove to be the biggest aid.

It should be emphasized that the identification step only implies further analysis of the model which may be discarded at a later time.

The identification of Model 5.1 is equivalent to specifying $\underline{N} = \{n_1, n_2, n_3, n_4\}$ and for Model 5.13 the set $\{p,d,q,P,D,Q\}$.

After a model has been identified its coefficients are estimated. The structure of the model and assumptions of the noise structure will partially determine the estimation scheme employed. The most common practice is estimation using a least squares criteria.

The estimation of Model 5.1 involves finding values of the $\underline{\alpha}$ and $\underline{\beta}$ coefficient vectors. The estimation of Model 5.13 yields values of the coefficients in the polynomial operators $\phi_p(B)$, $\Phi_P(B)$, $\theta_q(B)$ and $\Theta_Q(B)$.

Given parameter estimates the model is checked for adequacy. The first check should be that of parameter significance. Most parameter estimating techniques will give information on the distribution of each parameter so that hypothesis tests may be performed. If all parameters are found to be significant other adequacy tests are warranted.

Some of the most common tests involve residual analysis. Assumptions made on the noise structure, such as whiteness, may be tested and a quantitative measure of the ability of the model to explain the historical data may be calculated using the estimated residuals. If the purpose of the model is for simulation, the statistics of the model output, such as the mean, variance, autocorrelation function, and spectral density, may be compared to the historical statistics by using Monte Carlo techniques. If the model is found to be adequate it may be used for the intended purpose and the model selection process is terminated.

If a model does not pass the adequacy checking state, the cause of the failure should help identify a new model from the general class to be investigated. The same steps explained above are repeated until an adequate model is found.

Figure 5.1 summarizes the iterative approach to model building.

5.4 Proposed General Class of Models

In this section a general class of models will be proposed for the forecast of monthly inflow to Aswan. As the previous section discussed and as illustrated in Figure 5.1 this should be the first step in model building.

Several factors influenced the choice of the proposed class of models. Most important of these are that a specific model from the class

- must be able to account for seasonal variation of mean and variance observed in monthly riverflow
- must be capable of using observations of upstream flow when used for forecasting
- must be capable of yielding multi-lead forecasts

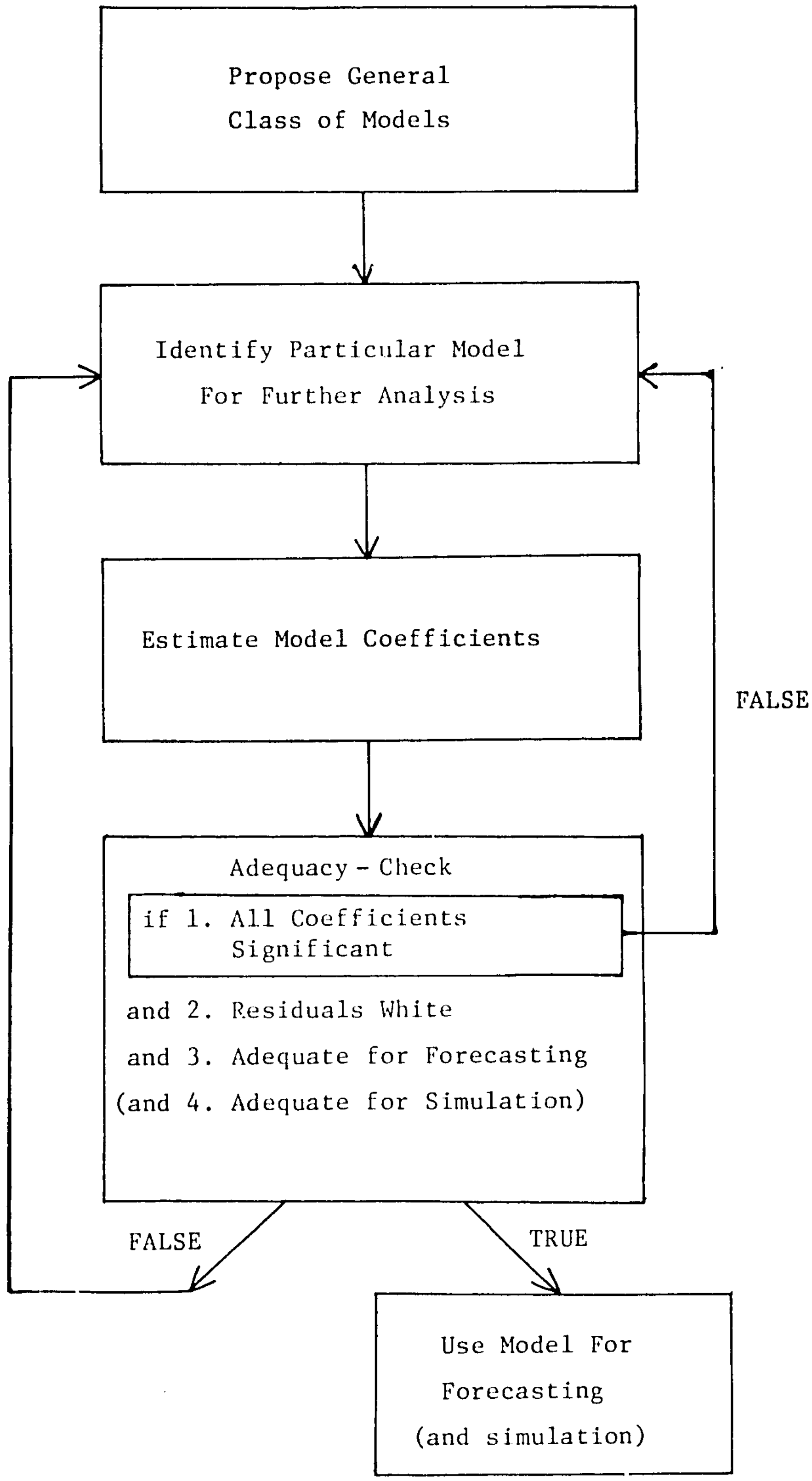


FIGURE 5.1: ITERATIVE APPROACH TO MODEL BUILDING (adapted from Box and Jenkins, 1970)

These factors lead to the proposal of the following class

$$\underline{\underline{B}}_i\ \underline{Y}(k,i) = \underline{\underline{\Gamma}}_i \begin{bmatrix} \underline{Y}(k,i-1) \\ \underline{Y}(k,i-2) \\ \vdots \\ \underline{Y}(k,i-n_1(i)) \end{bmatrix} + \underline{U}(i) + \underline{V}(k,i)$$

$$+ \underline{\underline{G}}_i \begin{bmatrix} \underline{V}(k,i-1) \\ \underline{V}(k,i-2) \\ \vdots \\ \underline{V}(k,i-n_2(i)) \end{bmatrix} \quad i=0,1,2,\ldots 11 \qquad (5.17a)$$

$\underline{\underline{B}}_i$ = $(n_o \times n_o)$ matrix of coefficients for month i (5.17b)

$$\underline{Y}(k,i) = \begin{bmatrix} Y_1(k,i) \\ Y_2(k,i) \\ \vdots \\ Y_{n_o}(k,i) \end{bmatrix} \qquad (5.17c)$$

$Y_j(k,i)$ = discharge station j, year k, month i (5.17d)

n_o = number of stations (5.17e)

$\underline{\underline{\Gamma}}_i$ = $(n_o \times n_o n_1(i))$ matrix of coefficients month i (5.17f)

$$\underline{U}(i) = \begin{bmatrix} U_1(i) \\ U_2(i) \\ \vdots \\ U_{n_o}(i) \end{bmatrix} \tag{5.17g}$$

$U(i)$ = deterministic term month i (periodic with period 12) (5.17h)

$$\underline{V}(k,i) = \begin{bmatrix} V_1(k,i) \\ V_2(k,i) \\ \vdots \\ V_{n_o}(k,i) \end{bmatrix} \tag{5.17i}$$

$$V_j(k,i) = \sum_{m=1}^{n_o} \underline{\underline{C}}_i\,(j,m)W_m(k,i) \tag{5.17j}$$

$\underline{\underline{C}}_i$ = $(n_o \times n_o)$ matrix of coefficients (5.17k)

$\underline{\underline{G}}_i$ = $(n_o \times n_o n_2(i))$ matrix of coefficients month i (5.17ℓ)

$W_j(k,i)$ is a random sequence assumed to satisfy conditions 5.15j.

The equation for flow at a given location is

$$Y_j(k,i) = -\sum_{\substack{m=1 \\ m\neq j}}^{n_o} \underline{\underline{B}}_i\,(j,m)\ Y_m(k,i) + \sum_{\ell=1}^{n_1(i)} \sum_{m=1}^{n_o} \underline{\underline{\Gamma}}_i(j,(m-1)n_1(i)+\ell)Y_m(k,i-\ell)$$

$$+ U_j(i) + V_j(i,k) + \sum_{\ell=1}^{n_2(i)} \sum_{m=1}^{n_o} \underline{\underline{G}}_i\,(j,(m-1)n_2(i)+\ell)\ V_m(k,i-\ell)$$

$$i = 0,1,2,\ldots,11 \tag{5.18}$$

Inspection of Equation 5.18 shows that the general formulation allows the discharge $Y_j(k,i)$ to be dependent on current discharges at other stations as well as previous discharges at any station. In addition the discharge contains a deterministic term, $U_j(i)$, and a random disturbance. Since in general the matrices $\underline{\underline{C}}_i$ and $\underline{\underline{G}}_i$ are not constrained to a particular form, the random disturbance at each station is related to the random disturbance at all other stations. In addition the random component may be serially correlated for finite lag.

The general Model 5.17 may be considered to have the combined features of the n_1'th order multivariate autoregressive Model 5.15 and the natural multivariate extention of the Rao-Kashyap univariate Model 5.1. This can be illustrated by writing the general model in reduced form (Johnston, 1972),

$$\underline{Y}(k,i) = \underline{\underline{\Pi}}_i' \begin{bmatrix} \underline{Y}(k,i-1) \\ \underline{Y}(k,i-2) \\ \vdots \\ \underline{Y}(k,i-n_1(i)) \end{bmatrix} + \underline{U}'(i) + \underline{V}'(k,i) + \underline{\underline{T}}_i' \begin{bmatrix} \underline{V}(k,i-1) \\ \underline{V}(k,i-2) \\ \vdots \\ \underline{V}(k,i-n_2(i)) \end{bmatrix} \quad (5.19a)$$

$$i = 0,1,2,\ldots,11$$

where

$$\underline{\underline{\Pi}}_i' = \underline{\underline{B}}_i^{-1} \underline{\underline{\Gamma}}_i \quad (5.19b)$$

$$\underline{U}'(i) = \underline{\underline{B}}_i^{-1} \underline{U}(i) \quad (5.19c)$$

$$\underline{V}'(k,i) = \underline{\underline{B}}_i^{-1} \underline{V}(k,i) \quad (5.19d)$$

$$\underline{\underline{T}}_i' = \underline{\underline{B}}_i^{-1} G_i \quad (5.19e)$$

By inspection, it is seen that Model 5.19 differs from the multivariate autoregressive model only in that it allows for serially correlated noise. The univariate Rao-Kashyap model is conceptually the same except that it does not allow for periodically varying coefficients (i.e., coefficiencs which depend on month but not year).

Model 5.17 is clearly not parsimonious in its use of parameters. For the case n_o=5, $n_1(i)$=2 and $n_2(i)$=1, that is 5 stations, second order regression and first order moving average, there are 105 parameters in each multivariate monthly equation (this is excluding the $\underline{\underline{C}}_i$ parameters since they are not necessary for forecasting). This amounts to 21 parameters for the model of a single month and station, Equation 5.18. Given the length of most simultaneous multivariate riverflow records and in particular that of the Nile, 56 years (excluding Hassanab, Appendix I), the reliable estimation of parameters seems impossible.

However, the situation is not hopeless. Using physical considerations and certain simplifying assumptions (tested for their validity in so far as possible at the adequacy checking stage) more parsimonious models may be identified. This is the topic of the next section.

5.5 Model Identification and Preliminary Forecasting and Estimation Considerations

5.5.1 Use of Causality: Blue Nile Example

In so far as possible, theoretical considerations of the process being modeled should guide the identification exercise. For the problem at hand, the hierarchical causality structure between the Nile River Basin gaging stations may be exploited for partial identification.

Before discussing how causality is used in identification a definition and a statistical test for causality proposed by Granger and Newbold (1977) will be presented.

The following two rules are assumed true.

(i) The future cannot cause the past. Strict causality can occur only with the past affecting the present or future.

(ii) It is sensible to discuss causality only for a group of stochastic processes. It is not possible to detect causality between two deterministic processes.

Given these rules, Granger and Newbold suggest the following definition of causality. If

$$P(Y_j(k,i) \mid \Omega(k,i)) \neq P(Y_j(k,i) \mid \Omega(k,i) - \psi(k,i)) \tag{5.20a}$$

where

$P(A|B)$ = the conditional distribution function of A given B (5.20b)

$\Omega(k,i)$ = all the information in the universe at time (k,i) apart from $Y_j(k,i)$ (5.20c)

$\psi(k,i)$ = a single element of $\Omega(k,i)$ (5.20d)

holds, then the element $\psi(k,i)$ is causal to $Y_j(k,i)$.

Definition 5.20 is far too general to be of practical use. Rather than dealing with all of the information in the universe, a plausible set of information, I_t, will have to be used. Preferably some underlying theory will guide the selection of I_t. However, I_t may be determined by the available data.

Also, it is impractical to hope to deal with conditional distribution functions when given only a finite set of data. An assumption of a particular multivariate distribution is not generally acceptable.

An alternate route is to use a summary statistic such as the conditional mean. If this route is taken then Definition 5.20 can be approximated in terms of linear predictors.

Let causality be redefined as:

If

$$\mathrm{var}[\varepsilon(I(k,i))] < \mathrm{var}[\varepsilon(I(k,i) - \psi(k,i))] - C_\alpha \qquad (5.21a)$$

where

var[$\cdot$] is the variance operator

$I(k,i)$ = limited information set exclusive of $Y_j(k,i)$ (5.21b)

$\psi(k,i)$ = an element of $I(k,i)$ (5.21c)

$$\varepsilon(I(k,i)) = Y_j(k,i) - L(I(k,i)) \qquad (5.21d)$$

$L(I(k,i))$ = linear predictor of $Y_j(k,i)$ given $I(k,i)$ (5.21e)

$$E[\varepsilon(I(k,i)] = 0 \qquad (5.21f)$$

$$\varepsilon(I(k,i) - \psi(k,i)) = Y_j(k,i) - L(I(k,i) - \psi(k,i)) \qquad (5.21g)$$

$L(I(k,i) - \psi(k,i))$ = linear predictor of $Y_j(k,i)$ given $I(k, i) - \psi(k, i)$ (5.21h)

$$E[\varepsilon(I(k,i) - \psi(k,i))] = 0 \tag{5.21i}$$

α = significance level

C_α = constant which depends on α (as α increases C increases)

then the element $\psi(k,i)$ is causal to $Y_j(k,i)$ or more specifically the element $\psi(k,i)$ is linearly and causally related in the least squares sense to $Y_j(k,i)$, with respect to the information set $I(k,i)$, at significance level α.

Definition 5.21 is a long way from the general Definition 5.20 but it does provide a feasible scenario. However, it must be emphasized that when using Definition 5.21, true causality may be missed or spurious causality observed between two variables, because a third variable, causal to both, has been left out of $I(k,i)$.

The usefulness of causality considerations and Definition 5.21 for the identification of a model from the general class 5.17 will now be illustrated using the Blue Nile as an example. Later in this section the entire Nile Basin will be considered.

Figure 5.2 is a schematic diagram of the Blue Nile gaging stations. Let

$Y_K(k,i)$ = discharge at Khartoum year k, month i

$Y_S(k,i)$ = discharge at Sennar year k, month i

$Y_R(k,i)$ = discharge at Roseires year k, month i

$P_K(k,i)$ = total precipitation on Blue Nile Basin exclusive of $P_S(k,i)$ and $P_R(k,i)$

$P_S(k,i)$ = total precipitation on Blue Nile Basin above (upstream) Sennar exclusive of $P_R(k,i)$

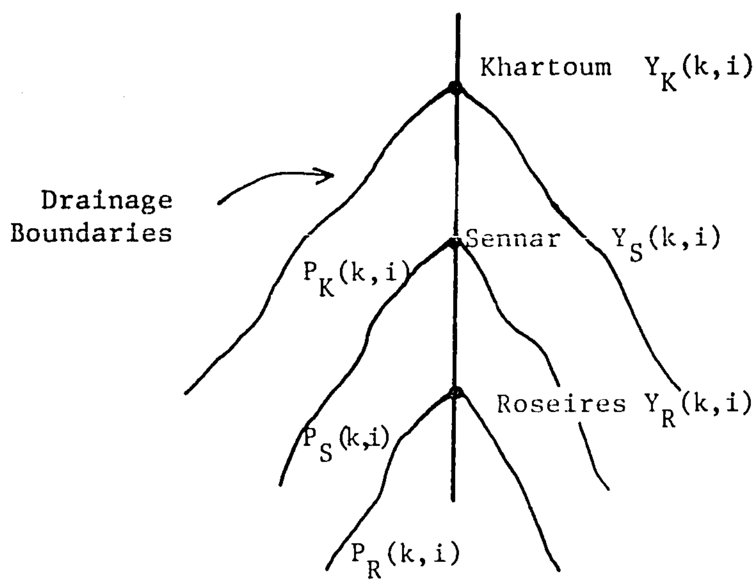


Figure 5.2: SCHEMATIC DIAGRAM OF BLUE NILE BASIN

$P_R(k,i)$ = total precipitation on Blue Nile Basin above (upstream) Roseires

$P_K(k,i) + P_S(k,i) + P_R(k,i)$ = total precipitation on Blue Nile Basin

Consider the discharge at Sennar, $Y_S(k,i)$. Defining the limited information set as:

$$I(k,i) = \{Y_K(k,i-j),\ Y_S(k,i-1-j),\ Y_R(k,i-j)\} \quad j=0,1,2... \tag{5.22}$$

use of Definition 5.21 with the data in Appendix 1 showed that the set

$$\{Y_K(k,i),\ Y_S(k,i-j),\ Y_R(k,i-\ell);\ \ j=1,...n_{SS}(i), \ell=0,1...,n_{SR}(i)\} \tag{5.23}$$

was causal to $Y_S(k,i)$ for all i. The fact that $\{Y_R(k,i-j),\ j=0,1,...,n_{SR}(i)\}$ is causal to $Y_S(k,i)$ is compatible with intuitive reasoning since flow through Roseires becomes flow through Sennar. Also, the causal relation between $Y_S(k,i)$ and $\{Y_S(k,i-j),\ j=1,...,n_{SS}(i)\}$ is reasonable since these values may be surrogate measures of groundwater flow and soil moisture. However, that $Y_K(k,i)$ is causal to $Y_S(k,i)$ defies physical intuition. Luckily a strong physically based argument may be advanced that suggests the existence of a third variable not included in $I(k,i)$ that is correlated to both $Y_S(k,i)$ and $Y_K(k,i)$ and hence is responsible for their apparent two-way causal relationship.

A variable which is undoubtably causal to $Y_S(k,i)$ and not included in $I(k,i)$ is the total precipitation on the Blue Nile Basin above Sennar exclusive of $P_R(k,i)$, denoted by $P_S(k,i)$ (see Figure 5.2). Since $P_S(k,i)$ is causal to $Y_S(k,i)$ and $Y_S(k,i)$ is causal to $Y_K(k,i)$, $P_S(k,i)$ will be correlated with $Y_K(k,i)$ so that $Y_K(k,i)$ is a measure of $P_S(k,i)$. Therefore $Y_K(k,i)$ will be causal to $Y_S(k,i)$ <u>relative to</u> the limited information set $I(k,i)$.

Another observation tends to support the idea that $Y_K(k,i)$ is "causal" to $Y_S(k,i)$ due to the missing observation $P_S(k,i)$. Chapter 4 pointed out that monthly precipitation series have small lag-one autocorrelation coefficients. Thus, although $Y_K(k,i)$ serves as a measure of $P_S(k,i)$, $Y_K(k,i-1)$ will be "noncausal" to $Y_S(k,i)$. This is indeed the observed behavior as shown by Set 5.23.

Based on the previous arguments the set of variables causal to $Y_S(k,i)$ may be reduced to

$$\{Y_S(k,i-j),\ Y_R(k,i-\ell),\ j=1,\ldots,n_{SS}(i),\ \ell = 0,1,\ldots,n_{SR}(i)\} \quad \forall\ i \tag{5.24}$$

Application of the causality Definition 5.21 and the use of the arguments previously presented showed that $Y_K(k,i)$ had the set of causal variables

$$\{Y_K(k,i-j),\ Y_S(k,i-\ell)\ Y_R(k,i-m),\ \ j=1,\ldots,n_{KK}(i),$$
$$\ell = 0,1,\ldots,n_{KS}(i),\ m=0,1,\ldots,n_{KR}(i)\} \quad \forall\ i \tag{5.25}$$

and $Y_R(k,i)$ the set

$$\{Y_R(k,i-j), \quad j=1,\ldots,n_{RR}(i)\} \qquad \forall\ i \tag{5.26}$$

Using these sets (5.24, 5.25, 5.26) of causal variables the general Model 5.17 for the 3 stations of the Blue Nile may be written as (let $n_1(\cdot) = 1$ for simplicity of notation)

$$\underbrace{\begin{bmatrix} 1 & & \underline{\underline{0}} \\ b_{21} & 1 & \\ b_{31} & b_{32} & 1 \end{bmatrix}}_{\underline{\underline{B}}_i} \underbrace{\begin{bmatrix} Y_R(k,i) \\ Y_S(k,i) \\ Y_K(k,i) \end{bmatrix}}_{\underline{Y}(k,i)} = \underbrace{\begin{bmatrix} \gamma_{11} & & \underline{0} \\ \gamma_{21} & \gamma_{22} & \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix}}_{\underline{\underline{\Gamma}}_i} \begin{bmatrix} Y_R(k,i-1) \\ Y_S(k,i-1) \\ Y_K(k,i-1) \end{bmatrix}$$

$$+ \underline{U}(i) + \underline{V}(k,i) + \underline{\underline{G}}_i \underline{V}(k,i-1) \qquad \forall\ i \tag{5.27}$$

As can be seen from Equation 5.27 the consideration of causality has significantly reduced the number of model coefficients to be estimated in the matrices $\underline{\underline{B}}_i$ and $\underline{\underline{\Gamma}}_i$.

5.5.2 Identification of Entire Nile Model

5.5.2.1 Use of Causality

From the information presented in the previous section it may be concluded that in general past and present discharges of upstream stations and past discharges at the given station are "causal" (in the sense of Definition 5.21) to the present discharge at the given station. Using this conclusion the model for the full Nile River Basin will now be considered.

Figure 5.3 is a schematic representation of the Nile Basin and its gaging stations above Aswan reservoir. Gaging station Hassanab has been excluded from the figure and the following analysis due to its short historical record relative to the other gaging stations (see Appendix 1). Using the conclusion stated above, the structure of the $\underline{\underline{B}}_i$ and $\underline{\underline{\Gamma}}_i$ coefficient matrices of the general Model 5.17, shown in Tables 5.2 and 5.3, respectively, may be assumed. To illustrate the structure of $\underline{\underline{\Gamma}}_i$ it was arbitrarily assumed that $n_1(i)$ was equal to 2. The actual value of $n_1(i)$ must be determined by application of causality Definition 5.21.

5.5.2.2 Advantages of Reduced Model Form

The reduced form of the general Model 5.17, Model 5.19, has certain advantages over the full structural form. First of all, forecasting is more convenient. In the full structural form (Model 5.17), any particular discharge may depend on the current discharge at other stations. Thus, in general a one step ahead forecast for a particular station required forecasts of discharge at other stations. In the reduced form (Model 5.19) all discharges depend only upon past values at the various stations and the one step forecast requires only currently known discharges. Similarly, multi-lead forecasts are more easily made from the reduced form model (see Section 5.8).

Another advantage is that coefficient estimation of the reduced form model, as compared to the estimation of the full structural model,

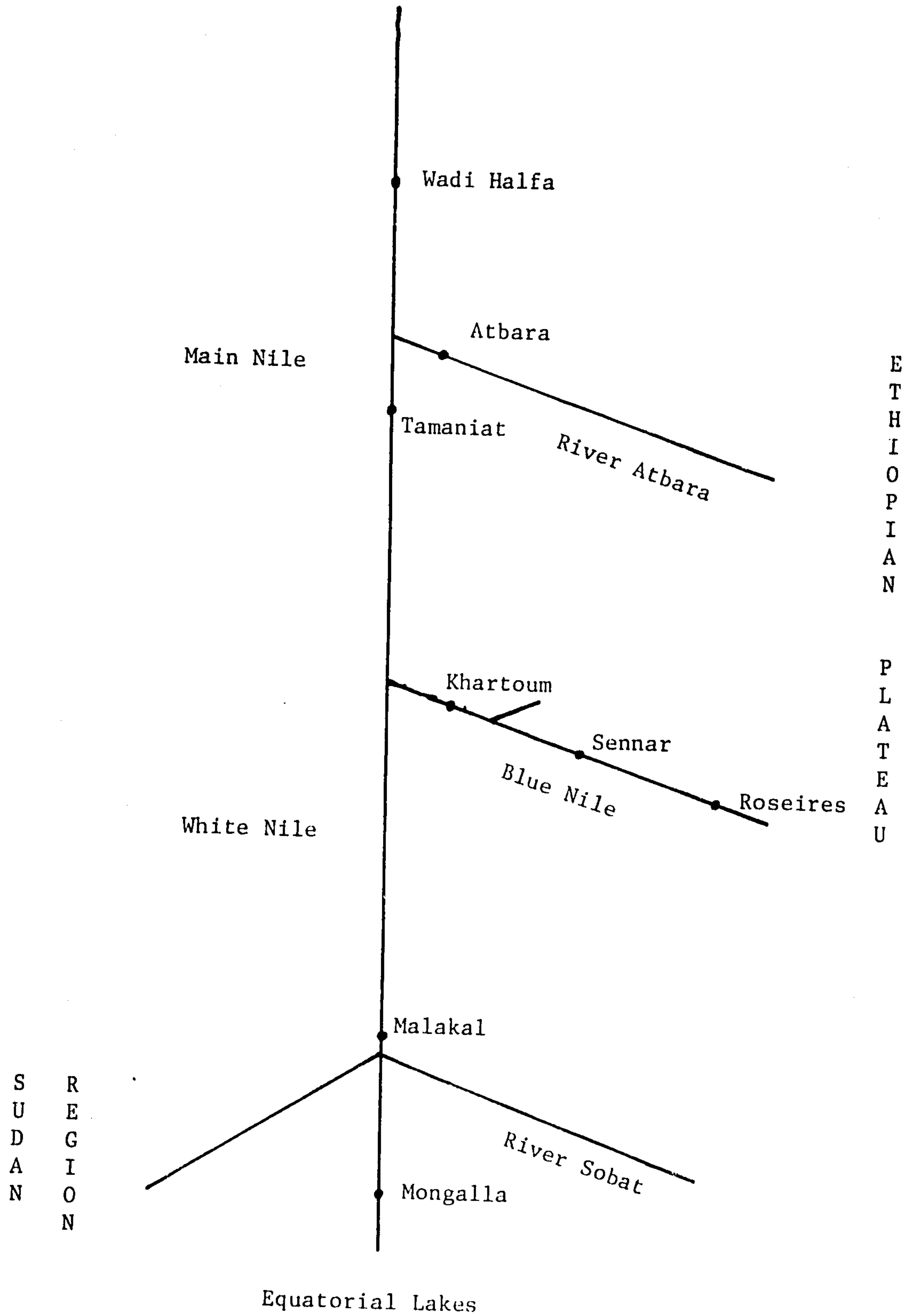


Figure 5.3: SCHEMATIC DIAGRAM OF NILE RIVER BASIN
(not to scale)

$$\underbrace{\begin{matrix} 1 & & & & & & & \\ X & 1 & & & & & & \\ X & X & 1 & \underline{\underline{0}} & & & & \\ 0 & 0 & 0 & 1 & & & & \\ 0 & 0 & 0 & X & 1 & & & \\ X & X & X & X & X & 1 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \\ X & X & X & X & X & X & X & 1 \end{matrix}}_{\underline{\underline{B}}_i} \qquad \underbrace{\begin{matrix} \text{Roseires } (k,i) \\ \text{Sennar } (k,i) \\ \text{Khartoum } (k,i) \\ \text{Mongalla } (k,i) \\ \text{Malakal } (k,i) \\ \text{Tamaniat } (k,i) \\ \text{Atbara } (k,i) \\ \text{Wadihalfa } (k,i) \end{matrix}}_{\underline{Y}(k,i)}$$

X - coefficients which cannot be considered zero a priori

0 - coefficients which can be considered zero zero a priori

Table 5.2: STRUCTURE OF $\underline{\underline{B}}_i$ (GENERAL MODEL 5.17) ASSUMED FOR NILE RIVER BASIN

Roseires (k,i-1)	Roseires (k,i-2)	Sennar (k,i-1)	Sennar (k,i-2)	Khartoum (k,i-1)	Khartoum (k,i-2)	Mongalla (k,i-2)	Mongalla (k,i-2)	Malakal (k,i-1)	Malakal (k,i-2)	Tamaniat (k,i-1)	Tamaniat (k,i-2)	Atbara (k,i-1)	Atbara (k,i-2)	Wadihalfa (k,i-1)	Wadihalfa (k,i-2)
X	X												$\underline{\underline{0}}$		
X	X	X	X												
X	X	X	X	X	X										
0	0	0	0	0	0	X	X								
0	0	0	0	0	0	X	X	X	X						
X	X	X	X	X	X	X	X	X	X	X	X				
0	0	0	0	0	0	0	0	0	0	0	0	X	X		
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

X - coefficients which cannot be considered zero

0 - coefficients which can be considered zero ($n_1(.) = 2$ assumed)

Table 5.3: STRUCTURE OF $\underline{\underline{\Gamma}}_i$ (GENERAL MODEL 5.17) ASSUMED FOR NILE RIVER BASIN

is in general simpler. The required complexity of an algorithm used for coefficient estimation of the full structural model is again the result of a particular discharge depending upon current discharge at other stations. In general, the random disturbance at a given station may be cross-correlated (and is likely to be since precipitation observations are not explicitly in the model) to the random disturbance at all other stations (See Section 5.4). Thus, current discharges which are explanatory variables in a single given equation may be correlated with the disturbance term of the equation. This contemporaneous correlation causes simple least square estimators such as ordinary least squares (OLS) and generalized least squares (GLS), which may be applicable to the estimation of the reduced form of the model with cross-correlated residuals, (see Section 5.6), to be inappropriate. More complicated techniques such as two and three stage least squares or full information maximum likelihood must be used (Johnston, 1972 and Goldberger, 1964).

For the reasons given above, only the reduced form of the Nile Basin model will be considered any further.

The structure of the reduced form may be found by first considering the Nile River Basin Model identified in the previous Section (5.5.2.1).

$$\underline{\underline{B}}_i \underline{Y}_i(k,i) = \underline{\underline{\Gamma}}_i \begin{bmatrix} \underline{Y}(k,i-1) \\ \underline{Y}(k,i-2) \\ \vdots \\ \underline{Y}(k,i-n_1(i)) \end{bmatrix} + \underline{U}(i) + \underline{V}(k,i)$$

$$+ \underline{\underline{G}}_i \begin{bmatrix} \underline{V}(k,i-1) \\ \underline{V}(k,i-2) \\ \vdots \\ \underline{V}(k,i-n_2(i)) \end{bmatrix} \quad i=0,1,\ldots 11 \qquad (5.28)$$

where $\underline{\underline{B}}_i$ and $\underline{\underline{\Gamma}}_i$ have the structural forms as shown in Tables 5.2 and 5.3, respectively, and the structure of $\underline{\underline{G}}_i$ is unspecified.

The reduced form of the Nile Model may be written as

$$\underline{Y}(k,i) = \underline{\underline{B}}_i^{-1} \underline{\underline{\Gamma}}_i \begin{bmatrix} \underline{Y}(k,i-1) \\ \vdots \\ \underline{Y}(k,i-n_1(i)) \end{bmatrix} + \underline{\underline{B}}_i^{-1} \underline{U}(i) +$$

$$\underline{\underline{B}}_i^{-1} \underline{V}(k,i) + \underline{\underline{B}}_i^{-1} \underline{\underline{G}}_i \begin{bmatrix} \underline{V}(k,i-1) \\ \vdots \\ \underline{V}(k,i-n_2(i)) \end{bmatrix}$$

$$i = 0,1,2,\ldots 11 \qquad (5.29)$$

Model 5.29 may be written without explicitly showing $\underline{\underline{B}}_i^{-1}$,

$$\underline{Y}(k,i) = \underline{\underline{\Pi}}_i \begin{bmatrix} \underline{Y}(k,i-1) \\ \vdots \\ \underline{Y}(k,i-n_1(i)) \end{bmatrix} + \underline{U}'(i) + \underline{V}'(k,i) +$$

$$\underline{\underline{T}}_i \begin{bmatrix} V'(k,i-1) \\ \vdots \\ V'(k,i-n_2(i)) \end{bmatrix} \qquad i = 0,1,2,\ldots 11 \qquad (5.30)$$

Table 5.4 shows the structure of $\underline{\underline{\Pi}}_i (= \underline{\underline{B}}_i^{-1} \underline{\underline{\Gamma}}_i)$ which is obtained directly from the forms of the identified $\underline{\underline{B}}_i$ and $\underline{\underline{\Gamma}}_i$.

5.5.2.3 Complete Identification: Noise Structure Assumptions

At this point part of the identification problem has been solved. Specifically several elements of the $\underline{\underline{\Pi}}_i$ matrix of Model 5.30 are required to be zero. However, full identification is far from accomplished.

Consider for example the coefficient matrix $\underline{\underline{\Pi}}_i$. The maximum lag of any flow variable {i.e., $n_1(i)$} up to now has been assumed equal to 2. Also the only restrictions placed on the individual elements, exclusive of those required to be zero, is that they <u>may</u> be non-zero. Full identification of $\underline{\underline{\Pi}}_i$ requires the knowledge of $n_1(i)$ and which elements are non-zero.

In this work the final identification of $\underline{\underline{\Pi}}_i$ will be accomplished by use of information gained during the estimation and adequacy checking steps (see Figure 5.1). As Section 5.3 briefly mentions and as Section 5.7

Roseries (k,i-1)	Roseires (k,i-2)	Sennar (k,i-1)	Sennar (k,i-2)	Khartoum (k,i-1)	Khartoum (k,i-2)	Mongalla (k,i-1)	Mangalla (k,i-2)	Malakal (k,i-1)	Malakal (k,i-2)	Tamaniat (k,i-1)	Tamaniat (k,i-2)	Atbara (k,i-1)	Atbara (k.i-2)	Wadihalfa (k,i-1)	Wadihalfa (k,i-2)
X	X										$\underline{\underline{0}}$				
X	X	X	X												
X	X	X	X	X	X										
0	0	0	0	0	0	X	X								
0	0	0	0	0	0	X	X	X	X						
X	X	X	X	X	X	X	X	X	X	X	X				
0	0	0	0	0	0	0	0	0	0	0	0	X	X		
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

X - coefficients which cannot be considered zero

0 - coefficients which can be considered zero ($n_1(i) = 2$ assumed)

Table 5.4: STRUCTURE OF $\underline{\underline{\Pi}}_i = \underline{\underline{B}}_i^{-1} \underline{\underline{\Gamma}}_i$ FOR NILE RIVER BASIN MODEL: COMPUTED USING $\underline{\underline{B}}_i$ and $\underline{\underline{\Gamma}}_i$ OF TABLES 5.2 AND 5.3, RESPECTIVELY

will show, most coefficient estimation procedures yield information on the distribution of each parameter so that hypothesis tests on significance can be performed. Also, information inferred from the properties of the estimated residuals and tests of performance are useful for identification.

The full identification also requires the specification of the disturbance structure, specifically, the value of $n_2(i)$ and the structural form of $\underline{\underline{T}}_i$ (Equation 5.30). Physical considerations are essentially useless for this task. Thus, the approach taken here will be akin to trial and error. A disturbance structure will be assumed and the model will be estimated. Insofar as possible, the assumed disturbance structure will then be checked to see if it is correct. If no evidence is found to the contrary, the estimated model will be retained as a candidate for the final model.

In the next section, it will be demonstrated that the appropriate estimation procedure is closely related to the assumed error structure. The estimation of a model with lagged variables and autocorrelated noise requires an estimation procedure radically different to that needed for a model with lagged variables and non-autocorrelated noise. Thus, at this time, two different models, one with autocorrelated noise and the other with non-autocorrelated noise, which are particular cases of Model 5.30, will be proposed.

The first model proposed is of the form

$$\underline{Y}(k,i) = \underline{\underline{\Pi}}_i \begin{bmatrix} Y(k,i-1) \\ \vdots \\ Y(k,i-n_1(i)) \end{bmatrix} + \underline{U}'(i) + \underline{V}'(k,i) \qquad (5.31)$$

$$i = 0, 1, 2, \ldots, 11$$

The restriction $n_2(i) = 0$ for all i has been enforced so that the error term is non-autocorrelated (cross correlation may still exist).

The second model proposed is obtained from Model 5.30 by imposing the restriction that $\underline{\underline{\Pi}}_i$, $\underline{\underline{T}}_i$, $n_1(i)$ and $n_2(i)$, and hence $\underline{\underline{B}}_i$, $\underline{\underline{\Gamma}}_i$, and $\underline{\underline{G}}_i$, are time invariate. The model may be written as

$$\underline{Y}(k,i) = \underline{\underline{\Pi}}^* \begin{bmatrix} \underline{Y}(k,i-1) \\ \vdots \\ \underline{Y}(k,i-n_1^*) \end{bmatrix} + \underline{U}^*(i) + \underline{V}^*(k,i) + \underline{\underline{T}}^* \begin{bmatrix} \underline{V}^*(k,i-1) \\ \vdots \\ \underline{V}^*(k,i-n_2^*) \end{bmatrix} \quad (5.32)$$

To stress time independence, the subscripts of $\underline{\underline{\Pi}}_i$ and $\underline{\underline{T}}_i$ and the arguments of $n_1(i)$ and $n_2(i)$ are dropped. Also, to distinguish terms of Model 5.32 from 5.31, a '*' superscript will be used.

Models 5.31 and 5.32 will be repeatedly referred to throughout the rest of this work.

5.6 Model Estimation

5.6.1 Introduction

Thus far, two models, 5.31 and 5.32, have been partially identified for the monthly flow of the Nile River Basin. Discussion will now focus on estimation of model parameters.

A primary difference between Model 5.31 and 5.32 is the noise structure. Model 5.32 allows the disturbance term to be autocorrelated for finite lag whereas Model 5.31 does not. This difference influences the choice of estimation procedure. The specific technique used to estimate Models 5.31 and 5.32 will be discussed in Sections 5.6.3 and 5.6.4, respectively. Both estimation procedures use a least squares criteria and thus a general discussion of least squares estimation will be presented first.

5.6.2 Least Squares Estimation

The following is a general review of least squares parameter estimation of linear models. The material presented here and in Section 5.6.3 has almost entirely come from Goldberger (1964) and Johnston (1972). Both of these books are excellent references on least squares estimation of stochastic models and the reader is referred to these for more detail.

Consider n observations of the variables Z and X_2, X_3, ..., X_K. Assuming that a linear relationship exists between Z and the X's and a disturbance term ε, the observations may be formulated as

$$Z_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \ldots + \beta_K X_{Kt} + \varepsilon_t \qquad t = 1, 2, \ldots, n \tag{5.33a}$$

or using matrix notation

$$\underline{Z} = \underline{\underline{X}}\, \underline{\beta} + \underline{\varepsilon} \tag{5.33b}$$

where

$$\underline{Z} = \begin{bmatrix} Z_1 \\ Z_2 \\ \cdot \\ \cdot \\ \cdot \\ Z_n \end{bmatrix} \tag{5.33c}$$

$$\underline{\underline{X}} = \begin{bmatrix} 1 & X_{21} & \cdots & X_{K1} \\ 1 & X_{22} & \cdots & X_{K2} \\ \vdots & \vdots & & \vdots \\ 1 & X_{2n} & \cdots & X_{Kn} \end{bmatrix} \tag{5.33d}$$

$$\underline{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_K \end{bmatrix} \tag{5.33e}$$

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \tag{5.33f}$$

The vector of coefficients $\underline{\beta}$ and the distribution of $\underline{\varepsilon}$ are unknown. The problem is to estimate them.

To begin, certain assumptions must be made. Later some of these assumptions will be relaxed.

Assumption 1: $\underline{\underline{X}}$ is nonstochastic so that each and every element of $\underline{\underline{X}}$ is independent of each and every element of $\underline{\varepsilon}$

Assumption 2: The expected value of each element of $\underline{\varepsilon}$ is zero (i.e., $E[\underline{\varepsilon}] = \underline{0}$)

Assumption 3: The ε_t have constant variance σ_ε^2 (i.e., homoscedastic) and are pairwise uncorrelated (i.e., $E[\underline{\varepsilon}\,\underline{\varepsilon}^T] = \sigma_\varepsilon^2\,\underline{\underline{I}}$, where $\underline{\underline{I}}$ is the nxn identity matrix)

Assumption 4: $\underline{\underline{X}}$ has rank K (<n) (i.e., the number of observations exceeds the number of parameters being estimated and no exact linear relation exists between the X variables).

Let $\underline{\hat{\beta}}$ denote the vector of estimated coefficients. Using a residual least squares criteria, $\underline{\hat{\beta}}$ is found by minimizing the scalar function

$$\min_{\underline{\hat{\beta}}} J = (\underline{Z} - \underline{\underline{X}}\,\underline{\hat{\beta}})^T (\underline{Z} - \underline{\underline{X}}\,\underline{\hat{\beta}}) \tag{5.34}$$

Minimization of a scalar with respect to a vector is obtained when

$$\frac{\partial J}{\partial \underline{\hat{\beta}}} = \underline{0} \tag{5.35}$$

and

$$\left| \frac{\partial^2 J}{\partial \underline{\hat{\beta}}} \right| \geq 0 \tag{5.36}$$

where $|\cdot|$ represents the determinant of the argument.

Thus, differentiating Equation 5.34 and setting the result to zero gives

$$-2\,\underline{\underline{X}}^T \underline{Z} + 2\,\underline{\underline{X}}^T \underline{\underline{X}}\,\underline{\hat{\beta}} = 0 \tag{5.37}$$

or

$$\underline{\underline{X}}^T \underline{\underline{X}}\,\underline{\hat{\beta}} = \underline{\underline{X}}^T \underline{Z} \tag{5.38}$$

By Assumption 4, $\underline{\underline{X}}^T \underline{\underline{X}}$ is non-singular so that

$$\underline{\hat{\beta}} = (\underline{\underline{X}}^T \underline{\underline{X}})^{-1} \underline{\underline{X}}^T \underline{Z} \tag{5.39}$$

Also by Assumption 4 it is obvious that Equation 5.36 will be satisfied. Note that the least squares estimator $\underline{\hat{\beta}}$ is linear in the observations $\underline{Z}$.

The mean of $\underline{\hat{\beta}}$ is found by first substituting Equation 5.33b into 5.39 which gives

$$\hat{\beta} = (X^T X)^{-1} X^T (X \beta + \varepsilon)$$

$$= (X^T X)^{-1} X^T X \beta + (X^T X)^{-1} X^T \varepsilon$$

$$= \beta + (X^T X)^{-1} X^T \varepsilon \tag{5.40}$$

Taking expected values gives

$$E[\hat{\beta}] = \beta + E[(X^T X)^{-1} X^T \varepsilon] \tag{5.41}$$

By Assumptions 1 and 2

$$E[(X^T X)^{-1} X^T \varepsilon] = 0 \tag{5.42}$$

so that

$$E[\hat{\beta}] = \beta \tag{5.43}$$

Thus, $\hat{\beta}$ is unbiased.

The variance-covariance matrix of $\hat{\beta}$ is given by

$$\Sigma_{\hat{\beta}\hat{\beta}} \equiv \mathrm{cov}(\hat{\beta}) = E[(\hat{\beta} - E[\hat{\beta}])(\hat{\beta} - E[\hat{\beta}])^T] \tag{5.44}$$

$$= E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T]$$

Using Equation 5.40, 5.44 becomes

$$\Sigma_{\hat{\beta}\hat{\beta}} = E[(X^T X)^{-1} X^T \varepsilon \, ((X^T X)^{-1} X^T \varepsilon)^T]$$

$$= E[(X^T X)^{-1} X^T \varepsilon \, \varepsilon^T X (X^T X)^{-1}] \tag{5.45}$$

Using Assumptions 1 and 3, Equation 5.45 may be evaluated as

$$\Sigma_{\hat{\beta}\hat{\beta}} = (X^T X)^{-1} X^T \sigma_\varepsilon^2 I X (X^T X)^{-1}$$

$$= \sigma_\varepsilon^2 (X^T X)^{-1} \tag{5.46}$$

The Gauss-Markov theorem (Goldberger, 1964) asserts that under Assumptions 1 through 4 the variance matrix 5.46 has the minimum variance of any linear estimator of $\underline{\beta}$. Thus, the estimator in Equation 5.39 is the best linear unbiased estimate (BLUE) of $\underline{\beta}$.

An unbiased estimate of the residual variance, σ_{ε}^{2}, can be found from the estimated residual sum of squares. Let

$$\underline{Z} = \underline{\underline{X}}\,\underline{\hat{\beta}} + \underline{\hat{\varepsilon}} \tag{5.47}$$

so that

$$\begin{aligned}
\underline{\hat{\varepsilon}} &= \underline{Z} - \underline{\underline{X}}\,\underline{\hat{\beta}} \\
&= \underline{Z} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}\,\underline{Z} \\
&= (\underline{\underline{X}}\,\underline{\beta} + \underline{\varepsilon}) - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}\,(\underline{\underline{X}}\,\underline{\beta} + \underline{\varepsilon}) \\
&= (\underline{\underline{X}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}\,\underline{\underline{X}})\,\underline{\beta} + \underline{\varepsilon} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}\,\underline{\varepsilon} \\
&= (\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T})\,\underline{\varepsilon}
\end{aligned} \tag{5.48}$$

The estimated residual sum of squares may then be expressed as

$$\underline{\hat{\varepsilon}}^{T}\,\underline{\hat{\varepsilon}} = \underline{\varepsilon}^{T}[\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}]^{T}\,[\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}]\,\underline{\varepsilon}$$

or

$$\underline{\hat{\varepsilon}}^{T}\,\underline{\hat{\varepsilon}} = \underline{\varepsilon}^{T}[\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}]\,\underline{\varepsilon} \tag{5.49}$$

since

$$\begin{aligned}
&(\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T})^{T}\,(\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}) \\
&\qquad = (\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T})\,(\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}) \\
&\qquad = \underline{\underline{I}} - 2\underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T} + \underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}\,\underline{\underline{X}}(\underline{\underline{X}}^{T}\,\underline{\underline{X}})^{-1}\,\underline{\underline{X}}^{T}
\end{aligned}$$

$$= \underline{\underline{I}} - 2\underline{\underline{X}}(\underline{\underline{X}}^T\, \underline{\underline{X}})^{-1}\, \underline{\underline{X}}^T + \underline{\underline{X}}(\underline{\underline{X}}^T\, \underline{\underline{X}})^{-1}\, \underline{\underline{X}}^T$$

$$= \underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^T\, \underline{\underline{X}})^{-1}\, \underline{\underline{X}}^T \tag{5.50}$$

Taking the expected value of Equation 5.49 gives[1,2]

$$\begin{aligned}
E[\hat{\underline{\varepsilon}}^T\, \hat{\underline{\varepsilon}}] &= E[\underline{\varepsilon}^T[\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^T\, \underline{\underline{X}})^{-1}\, \underline{\underline{X}}^T]\, \underline{\varepsilon}] \\
&= \sigma_\varepsilon^2\, \mathrm{Tr}[\underline{\underline{I}} - \underline{\underline{X}}(\underline{\underline{X}}^T\, \underline{\underline{X}}^{-1})\, \underline{\underline{X}}^T] \\
&= \sigma_\varepsilon^2[\mathrm{Tr}(\underline{\underline{I}}) - \mathrm{Tr}(\underline{\underline{X}}(\underline{\underline{X}}^T\, \underline{\underline{X}}^{-1})\, \underline{\underline{X}}^T)] \\
&= \sigma_\varepsilon^2[\mathrm{Tr}(\underline{\underline{I}}) - \mathrm{Tr}((\underline{\underline{X}}^T\, \underline{\underline{X}})^{-1}\, \underline{\underline{X}}^T\, \underline{\underline{X}})] \\
&= \sigma_\varepsilon^2(n - K)
\end{aligned} \tag{5.51}$$

---

[1] If

and

$$\left.\begin{aligned} E[\varepsilon_i^2] &= \sigma_\varepsilon^2 \\ E[\varepsilon_i\, \varepsilon_j] &= \delta(i - j)\, \sigma_\varepsilon^2 \end{aligned}\right\} \quad \forall\ i, j$$

then

$$E[\underline{\varepsilon}^T\, \underline{\underline{A}}\, \underline{\varepsilon}] = \sigma_\varepsilon^2\ \mathrm{Tr}\ (\underline{\underline{A}}) \qquad \forall \quad \underline{\underline{A}}$$

where

$\underline{\varepsilon}$ is (lxn)

$\underline{\underline{A}}$ is (nxn)

Tr (Trace) is the sum of the principle diagonal elements of the argument.

This is easily seen since

$$\begin{aligned}
E[\underline{\varepsilon}^T\, \underline{\underline{A}}\, \underline{\varepsilon}] &= E[\sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_i\, \varepsilon_j\, \underline{\underline{A}}(i, j)] \\
&= \sum_{i=1}^{n} \sigma_\varepsilon^2\, \underline{\underline{A}}(i, i) \\
&= \sigma_\varepsilon^2\ \mathrm{Tr}(A)
\end{aligned}$$

[2] For compatible matrices $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$

$$\mathrm{Tr}(\underline{\underline{A}}\, \underline{\underline{B}}\, \underline{\underline{C}}) = \mathrm{Tr}(\underline{\underline{B}}\, \underline{\underline{C}}\, \underline{\underline{A}}) = \mathrm{Tr}(\underline{\underline{C}}\, \underline{\underline{A}}\, \underline{\underline{B}})$$

Thus,

$$S_{\varepsilon}^{2} = \frac{\hat{\varepsilon}^{T}\ \hat{\varepsilon}}{n-K} \tag{5.52}$$

yields an unbiased estimate of the residual variance σ_{ε}^{2}.

Since $\underline{\underline{X}}$ is nonstochastic, $(\underline{\underline{X}}^{T}\ \underline{\underline{X}})^{-1}$ is nonstochastic and it follows that an unbiased estimate of the covariance matrix of $\hat{\underline{\beta}}$, $\underline{\underline{S}}_{\hat{\beta}\hat{\beta}}$, may be obtained by substituting S_{ε}^{2} for σ_{ε}^{2} in Equation 5.46.

$$\underline{\underline{S}}_{\hat{\beta}\hat{\beta}} = S_{\varepsilon}^{2}\ (\underline{\underline{X}}^{T}\ \underline{\underline{X}})^{-1} \tag{5.53}$$

Due to the fact that some of the necessary Assumptions 1 through 4 may not be fulfilled, the above results serve only as a beginning point in the derivation of procedures required for the estimation of Models 5.31 and 5.32.

5.6.3 Estimation of the Reduced Form of the General Model with Non-Autocorrelated Noise (Model 5.31)

Recall from Section 5.5.2.3, the proposed Model 5.31

$$\underline{Y}(k,\ i) = \underline{\underline{\Pi}}_{i} \begin{bmatrix} \underline{Y}(k,\ i-1) \\ \underline{Y}(k,\ i-2) \\ \cdot \\ \cdot \\ \cdot \\ \underline{Y}(k,\ i-n_{1}(i)) \end{bmatrix} + \underline{U}'(i) + \underline{V}'(k,i) \quad i = 0,\ 1,\ \ldots,\ 11 \tag{5.31}$$

The equation for any particular station j and month i is

$$Y_{j}(k,i) = \sum_{\ell=1}^{n_{1}(i)} \sum_{m=1}^{n_{o}} [\underline{\underline{\Pi}}_{i}(j,\ (m-1)\ n_{1}(i) + \ell) \cdot Y_{m}(k,\ i-\ell)] + U_{j}'(i) + V_{j}'(k,i) \tag{5.54}$$

As discussed in Section 5.5, some elements of $\underline{\underline{\Pi}}_{i}$ may be assumed to be zero a priori. However, to simplify the following discussion, it will be assumed that this information is not available.

Given n/12 observations, Equation 5.54 may be written in the notation of the general linear Model 5.33b (let $n_1(i) = 1$ and $j = 1$ for ease of notation);

$$\underline{Z}_1' = \underline{\underline{X}}_1' \underline{\beta}_1' + \underline{\varepsilon}_1' \tag{5.55a}$$

where

$$\underline{Z}_1' = \begin{bmatrix} Y_1(1, i) \\ Y_1(2, i) \\ \vdots \\ Y_1(n/12, i) \end{bmatrix} \tag{5.55b}$$

$$\underline{\underline{X}}_1' = \begin{bmatrix} 1 & Y_1(1, i-1) & Y_2(1, i-1) & \cdot & \cdot & \cdot & Y_{n_o}(1, i-1) \\ 1 & Y_1(2, i-1) & Y_2(2, i-1) & \cdot & \cdot & \cdot & Y_{n_o}(2, i-1) \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ 1 & Y_1(n/12, i-1) & Y_2(n/12, i-1) & \cdot & & \cdot & Y_{n_o}(n/12, i-1) \end{bmatrix} \tag{5.55c}$$

$$\underline{\beta}_1' = \begin{bmatrix} U_1'(i) \\ \underline{\underline{\Pi}}_i(1, 1) \\ \underline{\underline{\Pi}}_i(1, 2) \\ \vdots \\ \underline{\underline{\Pi}}_i(1, n_o) \end{bmatrix} \tag{5.55d}$$

$$\underline{\varepsilon}_1' = \begin{bmatrix} V_1'(1, i) \\ V_1'(2, i) \\ \vdots \\ V_1'(n/12, i) \end{bmatrix} \tag{5.55e}$$

For the single Equation 5.54 of the proposed multivariate Model 5.31, it can easily be demonstrated that the regressors and the disturbance are contemporaneously uncorrelated.

For the regressors and disturbance to be contemporaneously uncorrelated, the condition (see Model 5.55)

$$E[Y_j(k,\ i-\ell)\ V_1'(k,\ i)] = 0 \qquad \begin{array}{l} j = 1,\ 2,\ \ldots,\ n_o \\ k = 1,\ 2,\ \ldots,\ n/12 \\ \ell = 1,\ 2,\ \ldots,\ n_1(i) \end{array} \tag{5.56}$$

must be satisfied. From Section 5.5, it is clear that $V_1'(k,\ i)$ is defined by

$$V_1'(k,\ i) = \sum_{j=1}^{n_o} \left[\underline{\beta}_i^{-1}(1,j)\ \left[\sum_{\ell=1}^{n_o} C_i(j,\ell)\ W_\ell(k,i)\right]\right] \tag{5.57}$$

where $W.(\cdot)$ is a random sequence which satisfies the conditions of Equation 5.15j. Specifically, $W.(\cdot)$ satisfies

$$E[W_j(k,i)\ Y_{j'}(k,\ i-\ell)] = 0 \qquad \forall_j,\ k,\ i,\ j',\ \ell > 0 \tag{5.58}$$

Since $V_1'(k,\ i)$ is a linear combination of $\{W_\ell(k,i),\ \ell = 1,\ \ldots,\ n_o\}$ and since each member of the set is uncorrelated with $\{Y_j(k,\ i-\ell),\ j=1,\ \ldots,\ n_o,\ \ell > 0\}$, Equation 5.56 is satisfied.

Thus, the regressors of Equation 5.55 are contemporaneously uncorrelated with the disturbance term and therefore the application of the OLS estimators to each equation of the general multivariate Model 5.31 will yield consistent estimates of the coefficients.

However, if the disturbance vector of Model 5.31 is cross (inter) correlated (i.e., $E[V'_j(k,i)\ V'_\ell(k,i)] \neq 0,\ \forall\ j,\ell$), as it will almost undoubtedly be (see Section 5.5.2.2), the estimators suggested above, although consistent, are not asymptotically efficient.[1] Given that the error terms between equations are correlated, it is intuitive that the coefficients of a single equation of the multivariate Model 5.31, should be estimated simultaneously with all other coefficients. This may be accomplished as follows.

The n/12 observations of each equation of the multivariate model may be written together in the notation of the general linear Model 5.33b as

$$\begin{bmatrix} \underline{Z}'_1 \\ \underline{Z}'_2 \\ \vdots \\ \underline{Z}'_{n_o} \end{bmatrix} = \begin{bmatrix} \underline{\underline{X}}'_1 & & & \underline{\underline{0}} \\ & \underline{\underline{X}}'_2 & & \\ & & \ddots & \\ \underline{\underline{0}} & & & \underline{\underline{X}}'_{n_o} \end{bmatrix} \begin{bmatrix} \underline{\beta}'_1 \\ \underline{\beta}'_2 \\ \vdots \\ \underline{\beta}'_{n_o} \end{bmatrix} + \begin{bmatrix} \underline{\varepsilon}'_1 \\ \underline{\varepsilon}'_2 \\ \vdots \\ \underline{\varepsilon}'_{n_o} \end{bmatrix} \qquad (5.59a)$$

where $\underline{Z}_i$, $\underline{\underline{X}}_i$ and $\underline{\varepsilon}_i$, $i = 1, 2, \ldots, n_o$, are as defined in Model 5.55. Equation 5.59a may be written as

$$\underline{Z}'' = \underline{\underline{X}}''\ \underline{\beta}'' + \underline{\varepsilon}'' \qquad (5.59b)$$

---

[1] $\hat{\gamma}$ is an asymptotically efficient estimator of γ if $\hat{\gamma}$ is consistent and

$$\lim_{n\to\infty} E[(\hat{\gamma}_n - \gamma)^2] \leq \lim_{n\to\infty} E[(\tilde{\gamma}_n - \gamma)^2]$$

where $\tilde{\gamma}$ is any other consistent estimator and

$\hat{\gamma}_n$ = the estimate of γ computed from $\hat{\gamma}$ with a sample size of n

$\tilde{\gamma}_n$ = the estimate of γ computed from $\tilde{\gamma}$ with a sample size of n

γ = the true value of the coefficient being estimated

Assumptions 1, 2 and 4 of the linear Model 5.33b will be satisfied by 5.59b as just shown. However, Assumption 3, that the elements of $\underline{\varepsilon}$ have constant variance and are pairwise uncorrelated, will now be violated.

The prior assumptions that the disturbance in any single equation is non-autocorrelated and homoscedastic implies

$$E[\underline{\varepsilon}'_i \, \underline{\varepsilon}'^T_i] = \sigma_{\varepsilon'_i \varepsilon'_i} \, \underline{\underline{I}} \tag{5.60}$$

and the assumption of cross correlation implies

$$E[\underline{\varepsilon}'_i \, \underline{\varepsilon}'^T_j] = \sigma_{\varepsilon'_i \varepsilon'_j} \, \underline{\underline{I}} \tag{5.61}$$

where $\underline{\underline{I}}$ is the nxn identity matrix. By definition, the variance-covariance matrix of $\underline{\varepsilon}''$ is

$$E[\underline{\varepsilon}'' \, \underline{\varepsilon}''^T] = \begin{bmatrix} E[\underline{\varepsilon}'_1 \, \underline{\varepsilon}'^T_1] & E[\underline{\varepsilon}'_1 \, \underline{\varepsilon}'^T_2] & \cdots & E[\underline{\varepsilon}'_1 \, \underline{\varepsilon}'^T_{n_o}] \\ E[\underline{\varepsilon}'_2 \, \underline{\varepsilon}'^T_1] & E[\underline{\varepsilon}'_2 \, \underline{\varepsilon}'^T_2] & \cdots & E[\underline{\varepsilon}'_2 \, \underline{\varepsilon}'^T_{n_o}] \\ \vdots & \vdots & & \vdots \\ E[\underline{\varepsilon}'_{n_o} \, \underline{\varepsilon}'^T_1] & E[\underline{\varepsilon}'_{n_o} \, \underline{\varepsilon}'^T_2] & \cdots & E[\underline{\varepsilon}'_{n_o} \, \underline{\varepsilon}'^T_{n_o}] \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{\varepsilon'_1 \varepsilon'_1} & \sigma_{\varepsilon'_1 \varepsilon'_2} & \cdots & \sigma_{\varepsilon'_1 \varepsilon'_{n_o}} \\ \sigma_{\varepsilon'_2 \varepsilon'_1} & \sigma_{\varepsilon'_2 \varepsilon'_2} & \cdots & \sigma_{\varepsilon'_2 \varepsilon'_{n_o}} \\ \vdots & \vdots & & \vdots \\ \sigma_{\varepsilon'_{n_o} \varepsilon'_1} & \sigma_{\varepsilon'_{n_o} \varepsilon'_2} & \cdots & \sigma_{\varepsilon'_{n_o} \varepsilon'_{n_o}} \end{bmatrix} \otimes \underline{\underline{I}} \equiv \underline{\underline{\Sigma}}_{\varepsilon''\varepsilon''} \otimes \underline{\underline{I}} \tag{5.62}$$

where $\otimes$ denotes Kronecker multiplication of matrices.[1]

Thus, clearly Assumption 3 of the linear Model 5.33b is not satisfied and OLS estimation cannot be performed. However, using Generalized Least Squares (GLS) estimators, the coefficients of the more general linear Model 5.59b can be consistently and efficiently estimated with no new assumptions.

Working towards deriving the GLS estimators, let

$$E[\underline{\varepsilon}'' \, \underline{\varepsilon}''^{T}] = \underline{\underline{\Omega}} \qquad (5.63)$$

where $\underline{\underline{\Omega}}$ is the matrix defined in Equation 5.62. Assuming that $\underline{\underline{\Omega}}$ is a positive definite matrix, there exists a nonsingular $\underline{\underline{P}}$ that satisfies

$$\underline{\underline{\Omega}} = \underline{\underline{P}} \, \underline{\underline{P}}^{T} \qquad (5.64)$$

so that[2]

$$\underline{\underline{P}}^{-1} \, \underline{\underline{\Omega}} (\underline{\underline{P}}^{-1})^{T} = \underline{\underline{I}} \qquad (5.65)$$

and

$$(\underline{\underline{P}}^{-1})^{T} \, \underline{\underline{P}}^{-1} = \underline{\underline{\Omega}}^{-1} \qquad (5.66)$$

Pre-multiplying Equation 5.59b by $\underline{\underline{P}}^{-1}$ gives

---

1

$$\underline{\underline{A}} \otimes \underline{\underline{B}} = \begin{bmatrix} a_{11}\underline{\underline{B}} & a_{12}\underline{\underline{B}} & \cdots & a_{1m}\underline{\underline{B}} \\ a_{21}\underline{\underline{B}} & a_{22}\underline{\underline{B}} & \cdots & a_{2m}\underline{\underline{B}} \\ \vdots & \vdots & & \vdots \\ a_{n1}\underline{\underline{B}} & a_{n2}\underline{\underline{B}} & \cdots & a_{nm}\underline{\underline{B}} \end{bmatrix}$$

where $\underline{\underline{A}}$ is an (nxm) matrix and $\underline{\underline{B}}$ is of arbitrary dimension.

2 $(\underline{\underline{P}}^{T})^{-1} = (\underline{\underline{P}}^{-1})^{T} \quad \forall$ nonsingular $\underline{\underline{P}}$

$$\underline{\underline{P}}^{-1}\,\underline{Z}'' = \underline{\underline{P}}^{-1}\,\underline{\underline{X}}''\,\underline{\beta}'' + \underline{\underline{P}}^{-1}\,\underline{\varepsilon}'' \tag{5.67a}$$

or

$$\underline{Z}''' = \underline{\underline{X}}'''\,\underline{\beta}'' + \underline{\varepsilon}''' \tag{5.67b}$$

where

$$\underline{Z}''' = \underline{\underline{P}}^{-1}\,\underline{Z}''$$
$$\underline{\underline{X}}''' = \underline{\underline{P}}^{-1}\,\underline{\underline{X}}''$$
$$\underline{\varepsilon}''' = \underline{\underline{P}}^{-1}\,\underline{\varepsilon}''$$

The covariance of $\underline{\varepsilon}'''$ is given by

$$\begin{aligned}
E[\underline{\varepsilon}'''\,\underline{\varepsilon}'''^{T}] &= \underline{\underline{P}}^{-1}\,E[\underline{\varepsilon}''\,\underline{\varepsilon}''^{T}]\,\underline{\underline{P}}^{-1T} \\
&= \underline{\underline{P}}^{-1}(\underline{\underline{\Omega}})\,\underline{\underline{P}}^{-1T} \\
&= \underline{\underline{P}}^{-1}\,\underline{\underline{\Omega}}\,(\underline{\underline{P}}^{-1})^{T} \\
&= \underline{\underline{I}}
\end{aligned} \tag{5.68}$$

Thus, Model 5.67 satisfies the assumptions required for OLS estimation so that

$$\begin{aligned}
\hat{\underline{\beta}}'' &= (\underline{\underline{X}}'''^{T}\,\underline{\underline{X}}''')^{-1}\,\underline{\underline{X}}'''^{T}\,\underline{Z}''' \\
&= (\underline{\underline{X}}''^{T}\,\underline{\underline{\Omega}}^{-1}\,\underline{\underline{X}}'')^{-1}\,\underline{\underline{X}}''^{T}\,\underline{\underline{\Omega}}^{-1}\,\underline{Z}''
\end{aligned} \tag{5.69}$$

is a consistent estimator of $\underline{\beta}''$ and is efficient. A consistent estimate of its variance-covariance matrix is given by (Goldberger, 1964)

$$\underline{\underline{S}}_{\hat{\underline{\beta}}''\hat{\underline{\beta}}''} = E[(\hat{\underline{\beta}}'' - \underline{\beta}'')(\hat{\underline{\beta}}'' - \underline{\beta}'')] = [\underline{\underline{X}}''^{T}\,\underline{\underline{\Omega}}^{-1}\,\underline{\underline{X}}'']^{-1} \tag{5.70}$$

The difficulty with implementing the Estimators 5.69 and 5.70 is that the matrix $\underline{\underline{\Omega}}^{-1}$ is unknown. However, following the suggestions of Zellner (Johnston, 1972), the OLS estimator may be used on each individual equation (i.e., 5.55a) of the multivariate Model 5.31 to obtain estimates of the coefficients. Using these coefficients, the residuals of each individual equation may be computed and subsequently used to estimate $\underline{\underline{\Omega}}^{-1}$.

Equations 5.62 and 5.63 imply

$$\underline{\underline{\Omega}} = \underline{\underline{\Sigma}}_{\varepsilon''\varepsilon''} \otimes \underline{\underline{I}} \tag{5.71}$$

A given element of $\underline{\underline{\Sigma}}_{\varepsilon''\varepsilon''}$ is estimated from

$$\underline{\underline{S}}_{\varepsilon''\varepsilon''}(i,j) = \frac{(\underline{Z}_i' - \underline{\underline{X}}_i' \hat{\underline{\beta}}_i')^T (\underline{Z}_j' - \underline{\underline{X}}_j' \hat{\underline{\beta}}_j')}{((n/12)-\mathrm{rank}(\underline{\underline{X}}_i'))^{1/2}((1/12)-\mathrm{rank}(\underline{\underline{X}}_j'))^{1/2}} \tag{5.72}$$

where $\hat{\underline{\beta}}_{\cdot}$ is the OLS estimate of $\underline{\beta}'_{\cdot}$ and $\underline{Z}'_{\cdot}$ and $\underline{\underline{X}}'_{\cdot}$ are as defined in Equations 5.55b and 5.55c, respectively. Thus, using Equations 5.71 and 5.72, $\hat{\underline{\underline{\Omega}}}$ may be found. The specific structure of $\hat{\underline{\underline{\Omega}}}$ makes it particularly easy to invert. From Equation 5.71

$$\hat{\underline{\underline{\Omega}}}^{-1} = (\underline{\underline{S}}_{\varepsilon''\varepsilon''} \otimes \underline{\underline{I}})^{-1} = \underline{\underline{S}}_{\varepsilon''\varepsilon''}^{-1} \otimes \underline{\underline{I}} \tag{5.73}$$

As Section 5.8 will show, an estimate of the coefficients of the linear transformation which maps the set $\{W_j(k,i);\ j = 1, 2, \ldots, n_o\}$ to $\{V_j(k,i);\ j = 1, 2, \ldots, n_o\}$ (see Equation 5.57) is not required by the algorithm used to obtain multi-lead forecasts. However, if Model 5.31 were to be used to produce multivariate synthetic series of discharge, the coefficients of the transformation would be required. Since this work is concerned only with the forecast problem, no attempt will be made to derive an appropriate estimation algorithm.

5.6.4 Estimation of the Reduced Form of the General Model with Autocorrelated Noise (Model 5.32)

In Section 5.5.3 Model 5.32 was proposed

$$\underline{Y}(k,i) = \underline{\underline{\Pi}}^{*} \begin{bmatrix} \underline{Y}(k,i-1) \\ \underline{Y}(k,i-2) \\ \vdots \\ \underline{Y}(k,i-n_1^{*}) \end{bmatrix} + \underline{U}^{*}(i) + \underline{V}^{*}(k,i) + \underline{\underline{T}}^{*} \begin{bmatrix} \underline{V}^{*}(k,i-1) \\ \underline{V}^{*}(k,i-2) \\ \vdots \\ \underline{V}^{*}(k,i-n_2^{*}) \end{bmatrix}$$

$$k = 0,1,\ldots$$

$$i = 0,1,2,\ldots,11 \qquad (5.32)$$

The model differs from the reduced general multivariate Model 5.19 in that the coefficient matrices $\underline{\underline{\Pi}}^{*}(=\underline{\underline{B}}_i\underline{\underline{\Gamma}}_i\ ;\ \underline{\underline{B}}_i^{-1} = \underline{\underline{B}}_j^{-1},\ \underline{\underline{\Gamma}}_i = \underline{\underline{\Gamma}}_j,\ \forall i,j)$ and $\underline{\underline{T}}^{*}(=\underline{\underline{B}}_i^{-1}\underline{\underline{G}}_i\ ;\ \underline{\underline{B}}_i^{-1} = \underline{\underline{B}}_j^{-1},\ \underline{\underline{G}}_i = \underline{\underline{G}}_j,\ \forall i,j)$ are time invariant as opposed to being periodic. However, the variance of $\underline{V}^{*}(\cdot)$ is still a function of season i since the matrix $\underline{\underline{C}}_i$ is not assumed to be time invariant,

$$\underline{V}^{*}(k,i) = \underline{\underline{B}}_i^{-1}\,\underline{\underline{C}}_i\,\underline{W}(k,i) \qquad (5.74a)$$

$$= \underline{\underline{C}}_i^{*}\,\underline{W}(k,i)$$

where

$$\underline{\underline{C}}_i^{*} = \underline{\underline{B}}_i^{-1}\underline{\underline{C}}_i \qquad (5.74b)$$

$$\underline{\underline{B}}_i^{-1} = \underline{\underline{B}}_j^{-1} \qquad \forall i,j \tag{5.74c}$$

Assuming that monthly discharges are being modeled the deterministic term $\underline{U}^*(i)$ is of period 12. Thus, a given element of $\underline{U}^*(i)$ may be written as

$$U_j^*(i) = \underline{\underline{\alpha}}(j,1) + \sum_{\ell=1}^{n_3^*} [\underline{\underline{\alpha}}(j, 2\ell) \cos\omega_\ell (i+1) + \underline{\underline{\alpha}}(j,2\ell+1) \sin\omega_\ell i+1] \tag{5.75a}$$

where

$$i = 0, 1, \ldots, 11$$

$$\omega_\ell = \frac{2\pi\ell}{12} \tag{5.75b}$$

and

$$n_3 \leq 6 \tag{5.75c}$$

The periodic nature of $\underline{U}^*(i)$ and $\underline{V}^*(i)$ make possible the preservation of the seasonal behavior of monthly mean and variance observed in historical riverflow records.

The equation of a single element of $\underline{Y}(k,i)$ in Model 5.32 may be written as

$$Y_j(k,i) = \sum_{\ell=1}^{n_1^*} \sum_{m=1}^{n_o} \underline{\underline{\Pi}}^*(j,(m-1)n_o+\ell) Y_m(k,i-\ell)$$

$$+ U_j^*(i) + V_j^*(k,i) + \sum_{\ell=1}^{n_2^*} \sum_{m=1}^{n_o} \underline{\underline{T}}^*(j,(m-1)n_o+\ell) \cdot V_m^*(k,i-1)$$

$$k=0,1,2,\ldots$$

$$i=0,1,2,\ldots,11 \tag{5.76a}$$

or using a one dimensional notation

$$Y_j(t) = \sum_{\ell=1}^{n_1^*} \sum_{m=1}^{n_o} \underline{\underline{\Pi}}^*(j,(m-1)n_o+\ell)Y_m(t-\ell) + U_j^*(MOD_{12}(t))$$

$$+ V_j^*(t) + \sum_{\ell=1}^{n_2^*} \sum_{m=1}^{n_o} \underline{\underline{T}}^*(j,(m-1)n_o+\ell)V_m^*(t-\ell) \qquad (5.76b)$$

where

$t = 12k + i$

Given a set of observations, Equation 5.76b could be written in the notation of the linear Model 5.33b with the disturbance term given by

$$V_j^*(t) + \sum_{\ell=1}^{n_2^*} \sum_{m=1}^{n_o} \underline{\underline{T}}^*(j,(m-1)n_o+\ell)V_m^*(t-\ell) \qquad (5.77)$$

However, in general, the regressors would be contemporaneously correlated with this disturbance term. For example,

$$E[Y_j(t-1)(V_j^*(t)+ \sum_{\ell=1}^{n_2^*} \sum_{m=1}^{n_o} \underline{\underline{T}}^*(j,(m-1)n_o+\ell)V_m^*(t-\ell))] \neq 0 \qquad (5.78)$$

since at least

$$E[Y_j(t-1)V_j^*(t-1)] \qquad (5.79)$$

is nonzero. Thus, the use of OLS estimators is inappropriate (see Section 5.6.3).

Panuska (1969) derived a consistent estimation algorithm for the coeffients of a particular case of the stochastic difference equation given in 5.76b. The algorithm was developed for the univariate case (i.e., only one Y sequence) and for $V^*(\cdot)$ with constant variance. In Kashyap (1971), Kashyap and Rao (1976), and Rao and Kashyap (1973, 1974), theoretical properties of the algorithm are discussed in great detail and its applicability for the estimation of the coefficients of the monthly riverflow Model 5.1 presented in Section 5.2.1 is demonstrated.

An algorithm, essentially like Panuska's, but capable of estimating Model 5.32 is derived in the following paragraphs. The algorithm is closely related to generalized least squared estimation (GLS) discussed in Section 5.6.3.

The single element Equation 5.76b of the multivariate Model 5.32, can be written in the form of the linear Model 5.33b (for convenience, let $n_1^* = 1$ and $n_2^* = 1$, and assume that no elements of $\underline{\underline{\Pi}}^*$ or $\underline{\underline{T}}^*$ are apriori assumed zero)

$$Z_j^*(t) = \underline{X}_j^*(t)\ \underline{\beta}_j^* + \varepsilon_j^*(t) \tag{5.80a}$$

$$Z_j^*(t) = Y_j(t) \tag{5.80b}$$

$$\varepsilon_j^*(t) = V_j^*(t) \tag{5.80c}$$

$$\begin{aligned} \underline{X}_j^*(t) = [&Y_1(t-1)\ Y_2(t-1)\ \ldots\ Y_{n_o}(t-1)\ \ 1 \\ &\cos\omega_1(\mathrm{MOD}_{12}(t))\ \sin\omega_1(\mathrm{MOD}_{12}(t)) \ldots \\ &\cos\omega_{n_3^*}(\mathrm{MOD}_{12}(t))\ \sin\omega_{n_3^*}(\mathrm{MOD}_{12}(t)) \\ &\varepsilon_1^*(t-1)\ \varepsilon_2^*(t-1)\ \ldots\ \varepsilon_{n_o}^*(t-1)] \end{aligned} \tag{5.80d}$$

$$\underline{\beta}_j^{*T} = [\underline{\underline{\Pi}}^*(j,1)\ \underline{\underline{\Pi}}^*(j,2)\ldots\ \underline{\underline{\Pi}}^*(j,n_o)\ \underline{\underline{\alpha}}(j,\ell)\ \underline{\underline{\alpha}}(j,2)\ldots$$

$$\underline{\underline{\alpha}}(j,2n_3+1)\ \underline{\underline{T}}^*(j,1)\ \underline{\underline{T}}^*(j,2)\ \ldots\ \underline{\underline{T}}^*(j,n_o)] \tag{5.80e}$$

Given n+1 observations of $Y_j(\cdot)$ $\{Y_j(i),\ i=0,1,2,\ldots,n\}$, they may be written as

$$\underline{Z}_j^{*(n)} = \underline{\underline{X}}_j^{*(n)}\ \underline{\beta}_j^* + \underline{\varepsilon}_j^{*(n)} \tag{5.81a}$$

where

$$\underline{Z}_j^{*(n)} = \begin{bmatrix} Z_j^*(1) \\ Z_j^*(2) \\ \vdots \\ Z_j^*(n) \end{bmatrix} = \begin{bmatrix} Y_j(1) \\ Y_j(2) \\ \vdots \\ Y_j(n) \end{bmatrix} \tag{5.81b}$$

$$\underline{\underline{X}}_j^{*(n)} = \begin{bmatrix} \underline{X}_j^*\ (1) \\ \underline{X}_j^*\ (2) \\ \vdots \\ \underline{X}_j^*\ (n) \end{bmatrix} \tag{5.81c}$$

$$\underline{\varepsilon}_j^{*(n)} = \begin{bmatrix} \varepsilon_j^*(1) \\ \varepsilon_j^*(2) \\ \vdots \\ \varepsilon_j^*(n) \end{bmatrix} \tag{5.81d}$$

The superscript '(n)' is used to call attention to the number of observations, or row dimension of each vector or matrix. This notation will simplify the subsequent derivation.

From Equation 5.74a it is clear that the variance of $\varepsilon_j^*(t)(=V_j^*(t))$ is a function of season and hence the variance of $\varepsilon_j^*(\cdot)$ has period 12 when modeling monthly discharges. For now, assume that

$$\mathrm{Var}[\varepsilon_j^*(t)] = \Psi_j^*(\mathrm{MOD}_{12}(t))^2 \tag{5.82}$$

Using Equation 5.82 and noting that $\varepsilon_j^*(\cdot)$ is not autocorrelated (i.e., $E[\varepsilon_j^*(i)\ \varepsilon_j^*(\ell)] = 0 \quad \forall i \neq \ell$) the noise covariance matrix is

$$E[\underline{\varepsilon}_j^{*(n)}\ \underline{\varepsilon}_j^{*(n)}] = \begin{bmatrix} \Psi_j^*(1)^2 & & & & & & & & \underline{\underline{0}} \\ & \Psi_j^*(2)^2 & & & & & & & \\ & & \cdot & & & & & & \\ & & & \cdot & & & & & \\ & & & & \cdot & & & & \\ & & & & & \Psi_j^*(11)^2 & & & \\ & & & & & & \Psi_j^*(0)^2 & & \\ & \underline{\underline{0}} & & & & & & \ddots & \\ & & & & & & & & \Psi_j^*(\mathrm{MOD}_{12}(n))^2 \end{bmatrix}$$

$$= \underline{\underline{\Omega}}_j^{*(n)} \tag{5.83}$$

The application of the generalized least squares (GLS, see

Section 5.6.3) estimator of $\underline{\underline{\beta}}_j^*$ to Model 5.81 will yield consistent estimates since all elements of $\underline{\underline{X}}_j^{*(n)}$ are contemporaneously uncorrelated with $\underline{\varepsilon}_j^{*(n)}$ (each sequence $\{V_j^*(i),\ i=0,1,\ldots,n\}\ \forall j$ is a white noise process) and other necessary assumptions are satisfied.[1] Thus, by Equation 5.69

$$\underline{\hat{\beta}}_j^{*(n)} = (\underline{\underline{X}}_j^{*(n)T} (\underline{\underline{\Omega}}_j^{*(n)})^{-1} \underline{\underline{X}}_j^{*(n)})^{-1} \underline{\underline{X}}_j^{*(n)T} (\underline{\underline{\Omega}}_j^{*(n)})^{-1} \underline{\underline{Z}}_j^{*(n)} \tag{5.84}$$

Of course this does not solve the estimation problem since the $\varepsilon^*(\cdot)$'s (and hence $\underline{\underline{X}}_j^{*(n)}$) and $\Psi_j^*(\cdot)$'s (and hence $\underline{\underline{\Omega}}_j^{*(n)}$) are unknown.

Note however that given initial estimates of parameter vectors $\{\underline{\hat{\beta}}_j^{*(1)},\ j=1,2,\ldots,n_o\}$ the residuals $\{\hat{\varepsilon}_j^*(1),\ i=1,2,\ldots,n_o\}$ may be estimated from Equation 5.80a as

$$\hat{\varepsilon}_j^*(1) = Z_j^*(1) - \underline{\hat{X}}_j^*(1)\underline{\hat{\beta}}_j^{*(1)} \qquad j=1,2,\ldots,n_o \tag{5.85}$$

where $Z_j^*(1)$ is as defined in 5.80b and $\underline{\hat{X}}_j^*(1)$ is as defined in 5.80d except that the elements $\{\varepsilon_j^*(0),\ j=1,2,\ldots,n_o\}$ are replaced with their expected value which is zero. The estimated residuals $\{\hat{\varepsilon}_j(1),\ j=1,2,\ldots,n_o\}$

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[1] As noted in Section 5.6.3, although the GLS estimator of $\underline{\beta}_j^*$ yields consistent estimates they are not asymptotically efficient if the error term $\varepsilon_j^*(\cdot)$ is cross correlated between equations (i.e., $E[\varepsilon_j^*(t)\varepsilon_k^*(t)] \neq 0;\ j \neq k$). This subject will be addressed again in the closing of this section.

may be used to form estimated matrices $\{\hat{\underline{\underline{X}}}_j^{*(2)}, j=1,2,\ldots,n_o\}$. Assuming that the estimates exist of the functions $\{\Psi_j^*(\cdot), j=1,2,\ldots,n_o\}$ (this point will be discussed later) the matrices $\{\hat{\underline{\underline{\Omega}}}_j^{*(2)}, j=1,2,\ldots,n_o\}$ may be formed and $\hat{\underline{\beta}}_j^{*(2)}$ for all j may be estimated using Equation 5.84 with $\underline{\underline{X}}_j^{*(n)}$ and $\underline{\underline{\Omega}}_j^{*(n)}$, replaced by $\hat{\underline{\underline{X}}}_j^{*(2)}$ and $\hat{\underline{\underline{\Omega}}}_j^{*(2)}$. This process can be repeated until the parameters $\{\hat{\underline{\beta}}_j^{(n)}, j=1,2,\ldots,n_o\}$ are found.

This is essentially the estimation procedure which will be used to find the coefficients of Model 5.32. However, as formulated above, the algorithm is computationally inefficient. The estimation of $\hat{\underline{\beta}}_j^{*(n)}$ requires the inversion of the matrix

$$\hat{\underline{\underline{X}}}_j^{*(i)T}(\hat{\underline{\underline{\Omega}}}^{*(i)})^{-1}\underline{\underline{X}}_j^{*(i)} \tag{5.86}$$

at each iteration. Essentially, the addition of a new observation requires the problem to be completely reworked and no use is made of the previous estimate of $\underline{\beta}_j^*$. Intuitively this seems to waste effort and in fact it does. A sequential form for the estimate can be determined so that new observations can be incorporated without completely reworking the problem.

To derive the sequential estimator it will be convenient to write the GLS estimator of $\hat{\underline{\beta}}_j^{*(i)}$ as (see Equation 5.84)

$$\hat{\underline{\beta}}_j^{*(i)} = \underline{\underline{S}}_j^{(i)}(\underline{\underline{X}}_j^{*(i)})^T(\underline{\underline{\Omega}}_j^{*(i)})^{-1}\ \underline{Z}_j^{*(i)} \tag{5.87a}$$

where

$$\underline{\underline{S}}_j^{(i)} = [(\underline{\underline{X}}_j^{*(i)})^T\ (\underline{\underline{\Omega}}_j^{*(i)})^{-1}\ \underline{\underline{X}}_j^{*(i)}]^{-1} \tag{5.87b}$$

or

$$(\underline{\underline{S}}_j^{(i)})^{-1} = (\underline{\underline{X}}_j^{*(i)})^T \, (\underline{\underline{\Omega}}_j^{*(i)})^{-1} \, \underline{\underline{X}}_j^{*(i)} \tag{5.87c}$$

Given the new observations $Z_j^*(i+1)$ and $\underline{X}_j^*(i+1)$ (dropping the "^" notation for now) the matrix $(\underline{\underline{S}}_j^{(i+1)})^{-1}$ may be written as

$$(\underline{\underline{S}}_j^{(i+1)})^{-1} = (\underline{\underline{X}}_j^{*(i+1)})^T \, (\underline{\underline{\Omega}}_j^{*(i+1)})^{-1} \, \underline{\underline{X}}_j^{*(i+1)}$$

$$= \left[(\underline{\underline{X}}_j^{*(i)})^T \;\vdots\; \underline{X}_j^*(i+1)^T \right] \left[\begin{array}{c|c} (\underline{\underline{\Omega}}_j^{*(i)})^{-1} & \underline{0} \\ \hline \underline{0} & (\Psi_j^*(MOD_{12}(i+1))^2)^{-1} \end{array} \right] \left[\begin{array}{c} \underline{\underline{X}}_j^{*(i)} \\ \hline \underline{X}_j^*(i+1) \end{array} \right]$$

$$= (\underline{\underline{X}}_j^{*(i)})^T \, (\underline{\underline{\Omega}}_j^{*(i)})^{-1} \, \underline{\underline{X}}_j^{*(i)} + \frac{(\underline{X}_j^*(i+1))^T \underline{X}_j^*(i+1)}{\Psi_j^*(MOD_{12}(i+1))^2}$$

$$= (\underline{\underline{S}}_j^{(i)})^{-1} + \frac{(\underline{X}_j^*(i+1))^T \, \underline{X}_j^*(i+1)}{\Psi_j^*(MOD_{12}(i+1))^2} \tag{5.88}$$

Taking the inverse of both sides of Equation 5.88 yields

$$\underline{\underline{S}}_j^{(i+1)} = \left[(\underline{\underline{S}}_j^{(i)})^{-1} + \frac{(\underline{X}_j^*(i+1))^T \, \underline{X}_j^*(i+1)}{\Psi_j^*(MOD_{12}(i+1))^2} \right]^{-1} \tag{5.89}$$

Using the matrix inversion lemma (Sage and Melsa, 1971) Equation 5.89 may be written as

$$\underline{\underline{S}}_j^{(i+1)} = \underline{\underline{S}}_j^{(i)} - \frac{\underline{\underline{S}}_j^{(i)}(\underline{X}_j^*(i+1))^T\,\underline{X}_j^*(i+1)\,\underline{\underline{S}}_j^{(i)}}{[\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\,\underline{\underline{S}}_j^{(i)}(\underline{X}_j^*(i+1))^T]} \tag{5.90}$$

which gives $\underline{\underline{S}}_j^{(i+1)}$ as a simple expression of $\underline{\underline{S}}_j^{(i)}$, the observation vector $\underline{X}_j^*(i+1)$ and the residual variance $\Psi_j^*(MOD_{12}(i+1))^2$. Most importantly, $\underline{\underline{S}}_j^{(i+1)}$ is found without having to invert a matrix or without the need of $\{\underline{X}_j^*(k),\ k=1,2,3,\ldots,i\}$.

At this point $\hat{\underline{\beta}}_j^{*(i+1)}$ could be calculated by directly using Equation 5.87a. However, the approach would require the availability of $\{\underline{X}_j^*(k),\ Z_j^*(k),\ k=1,2,\ldots,i+1\}$. An alternative which avoids this may be derived.

Substituting Equation 5.90 into 5.87a gives

$$\hat{\underline{\beta}}_j^{*(i+1)} = \left[\underline{\underline{S}}_j^{(i)} - \frac{\underline{\underline{S}}_j^{(i)}\,(\underline{X}_j^*(i+1))^T\,\underline{X}_j^*(i+1)\,\underline{\underline{S}}_j^{(i)}}{[\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\,\underline{\underline{S}}_j^{(i)}(\underline{X}_j^*(i+1))^T]}\right] \cdot$$

$$\left[(\underline{\underline{X}}_j^{*(i+1)})^T\,(\underline{\underline{\Omega}}_j^{*(i+1)})^{-1}\,\underline{Z}_j^{*(i+1)}\right] \tag{5.91}$$

By writing the matrices of the second term of Equation 5.91 in partitioned form (similar to Equation 5.88) the following relation is easily derived

$$(\underline{\underline{X}}_j^{*(i+1)})^T (\underline{\underline{\Omega}}_j^{*(i+1)})^{-1} \underline{Z}_j^{*(i+1)}$$

$$= (\underline{\underline{X}}_j^{*(i)})^T (\underline{\underline{\Omega}}_j^{*(i)})^{-1} \underline{Z}_j^{*(i)} + \frac{\underline{X}_j^{*T}(i+1)\ Z_j^*(i+1)}{\Psi_j^*(MOD_{12}(i+1))^2} \tag{5.92}$$

Substituting Equation 5.92 into 5.91 and performing some algebraic manipulation gives

$$\hat{\underline{\beta}}_j^{*(i+1)} = \left[\underline{\underline{I}} - \frac{\underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1))^T\ \underline{X}_j^*(i+1)}{[\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\ \underline{\underline{S}}_j^{(i)}\ (\underline{X}_j^*(i+1)^T)]} \right] \cdot$$

$$\left[\underline{\underline{S}}_j^{(i)} (\underline{\underline{X}}_j^{*(i)})^T (\underline{\underline{\Omega}}_j^{*(i)})^{-1} \underline{Z}_j^{*(i)} \right] \tag{5.93}$$

$$+ \underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1)) \left[\frac{1}{\Psi_j^*(MOD_{12}(i+1))^2} - \frac{\underline{X}_j^*(i+1)\ \underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1))^T}{[\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\ \underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1))^T] \cdot \Psi_j^*(MOD_{12}(i+1))^2} \right] Z_j^*(i+1)$$

By Equation 5.87a

$$\underline{\underline{S}}_j^{(i)} (\underline{\underline{X}}_j^{*(i)})^T (\underline{\underline{\Omega}}_j^{*(i)})^{-1} \underline{Z}_j^{*(i)} = \hat{\underline{\beta}}_j^{*(i)} \tag{5.94}$$

In Equation 5.93 letting

$$\underline{K}_j^{(i+1)} \equiv \frac{\underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1))^T}{[\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\ \underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1))^T]} \tag{5.95}$$

and substituting in Equation 5.94 yields

$$\hat{\underline{\beta}}_j^{*(i+1)} = (\underline{\underline{I}} - \underline{K}_j^{(i+1)} \underline{X}_j^*(i+1))\ \hat{\underline{\beta}}_j^{*(i)} + \underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1))^T \left[\frac{1}{\Psi_j^*(MOD_{12}(i+1))^2} - \frac{\underline{X}_j^*(i+1)\ \underline{\underline{S}}_j^{(i)}\ (\underline{X}_j^*(i+1))^T}{[\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\ \underline{\underline{S}}_j^{(i)} (\underline{X}_j(i+1))^T] \cdot \Psi_j^*(MOD_{12}(i+1))^2} \right] Z_j^*(i+1)$$

$$= (\underline{\underline{I}} - \underline{K}_j^{(i+1)} \underline{X}_j^*(i+1))\hat{\underline{\beta}}_j^{*(i)} + \left[\frac{\underline{\underline{S}}_j^{(i)}\ (\underline{X}_j^*(i+1))^T}{\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\underline{\underline{S}}_j^{(i)}\ (\underline{X}_j^*(i+1))^T} \right] \cdot$$

$$\left[\frac{\Psi_j^*(MOD_{12}(i+1))^2 + \underline{X}_j^*(i+1)\underline{\underline{S}}_j^{(i)}\ (\underline{X}_j^*(i+1))^T}{\Psi_j^*(MOD_{12}(i+1))^2} - \frac{\underline{X}_j^*(i+1)\underline{\underline{S}}_j^{(i)}\ (\underline{X}_j^*(i+1))^T}{\Psi_j^*(MOD_{12}(i+1))^2} \right] Z_j^*(i+1) \tag{5.96}$$

Substituting Equation 5.95 into 5.96 gives

$$\underline{\hat{\beta}}_j^{*(i+1)} = (\underline{\underline{I}} - \underline{K}_j^{(i+1)}\underline{X}_j^*(i+1)\ \underline{\hat{\beta}}_j^{*(i)} + \underline{K}_j^{(i+1)} \Bigg[\underline{\underline{I}} +$$

$$\frac{\underline{X}_j^*(i+1)\ \underline{\underline{S}}_j^{(i)} (\underline{X}_j^*(i+1))^T}{\Psi_j^*(MOD_{12}(i+1))^2} - \frac{\underline{X}_j^*(i+1)\underline{\underline{S}}_j^{(i)}\ (\underline{X}_j^*(i+1))^T}{\Psi_j^*(MOD_{12}(i+1))^2} \Bigg] Z_j^*(i+1)$$

$$= \underline{\hat{\beta}}_j^{*(i)} + \underline{K}_j^{(i+1)}\ (Z_j^*(i+1) - \underline{X}_j^*(i+1)\ \underline{\hat{\beta}}_j^{*(i)}) \tag{5.97}$$

Equation 5.97 is the desired sequential estimator of $\underline{\beta}_j^*$.

Before summarizing the complete estimation algorithm for Model 5.32, one final simplification is made.

The vector $\underline{K}_j^{(i+1)}$ may also be written as

$$\underline{K}_j^{(i+1)} = \frac{\underline{\underline{S}}_j^{(i+1)} (\underline{X}_j^*(i+1))^T}{\Psi_j^*(MOD_{12}(i+1))^2} \tag{5.98}$$

which is easily verified by a direct substitution of Equation 5.90 for $\underline{\underline{S}}_j^{(i+1)}$. Thus the Estimator 5.97 may be written

$$\underline{\hat{\beta}}_j^{*(i+1)} = \underline{\hat{\beta}}_j^{*(i)} + \frac{\underline{\underline{S}}_j^{(i+1)} (\underline{X}_j^*(i+1))^T}{\Psi_j^*(MOD_{12}(i+1))^2}\ (Z_j^*(i+1) - \underline{X}_j^*(i+1)\underline{\hat{\beta}}^{*(i)}) \tag{5.99}$$

The complete estimation algorithm for Model 5.32 is presented below. For convenience, with no loss of generality, it is assumed that $n_1^*=1$ and $n_2^*=1$.

Two methods may be used to initialize $\underline{\hat{\beta}}_j^*$ and $\underline{\underline{S}}_j$. In the first (Panuska, 1969)

$$\underline{\hat{\beta}}_j^{*(1)} = \text{element of } \Phi_j \quad \forall\ j \tag{5.100a}$$

where Φ_j is a set known to contain $\underline{\beta}_j^*$, and

$$\underline{\underline{S}}_j^{(1)} = \underline{\underline{I}} \qquad \forall\ j \tag{5.100b}$$

The second method is

$$\underline{\underline{S}}_j^{(i)} = [(\underline{\underline{\hat{X}}}_j^{*(i)})^T (\underline{\underline{\hat{\Omega}}}_j^{*(i)})^{-1} \underline{\underline{\hat{X}}}_j^{*(i)}]^{-1} \quad \forall\ j \tag{5.101a}$$

$$\underline{\hat{\beta}}_j^{*(i)} = \underline{\underline{S}}_j^{(i)}(\underline{\underline{\hat{X}}}_j^{*(i)})^T (\underline{\underline{\hat{\Omega}}}^{*(i)})^{-1} \underline{Z}_j^{*(i)} \quad \forall\ j \tag{5.101b}$$

The vectors $\underline{Z}_{\cdot}^{*(i)}$ are constructed as indicated in Equation 5.81b using the available observations $Y_{\cdot}(\cdot)$. The matrices $\underline{\underline{X}}_{\cdot}^{*(i)}$ are constructed as shown in Equations 5.80d and 5.81c using the available observations $Y_{\cdot}(\cdot)$ and the indicated deterministic functions. The residuals $\{\varepsilon_j^*(\cdot),\ j=1,2,\ldots,n_o\}$ are replaced by randomly generated Gaussian deviates with zero mean and variance equal to $\hat{\Psi}_j^*(\ell)^2$ (where $\ell = MOD_{12}(t)$), if available and otherwise the sample variance of

$\{Y_j(k,\ell),\ k=0,1,\ldots\}$ is used. If the functions $\{\hat{\Psi}_j^*(\ell),\ \forall\ j,\ell\}$ are available the $\hat{\underline{\underline{\Omega}}}_j^{*(i)}$ are constructed as indicated in Equation 5.83 and if not $\hat{\underline{\underline{\Omega}}}_j^{*(i)}$ is assumed to be $\underline{\underline{I}}$ for all j.

Given the initial $\hat{\underline{\beta}}_j^{*(i)}$ and $\underline{\underline{S}}_j^{*(i)}$ the following estimation algorithm is used:

STEP 1: For $j = 1,2,\ldots,n_o$ find

$$\hat{\varepsilon}_j^*(i) = Z_j^*(i) - \underline{X}_j^*(i)\hat{\underline{\beta}}_j^{*(i)} \tag{5.102a}$$

STEP 2: For $j = 1,2,\ldots,n_o$ find

2a: Form $\hat{\underline{X}}_j(i+1)$, the row vector of explanatory variables for $Z_j^*(i+1)$ (given in a consistent order). Elements are specified by the structures of $\underline{\underline{\Pi}}^*$ and $\underline{\underline{T}}^*$ and n_3^* assumed a priori and thus are taken from the sets $\{Y_j(i),\ Y_j(i-1),\ \ldots,\ Y_j(i-n_1^*)\}\ \forall_j$, $\{\hat{\varepsilon}_j^*(i),\ \hat{\varepsilon}_j^*(i-1),\ \ldots,\ \hat{\varepsilon}_j^*(i-n_2^*)\}\ \forall_j$, and $\{\cos\omega_1(MOD_{12}(i+1)),\ \sin\omega_1(MOD_{12}(i+1),\ \ldots,\ \cos\omega_{n_3^*}(MOD_{12}(i+1))$ $\sin\omega_{n_3^*}(i+1))\}$ (see Equation 5.80d). (5.102b)

2b:

$$\underline{\underline{S}}_j^{(i+1)} = \underline{\underline{S}}_j^{(i)} - \frac{\underline{\underline{S}}_j^{(i)}(\hat{\underline{X}}_j^*(i+1))^T\hat{\underline{X}}_j^*(i+1)\underline{\underline{S}}_j^{(i)}}{[\hat{\Psi}_j^*(MOD_{12}(i+1))^2 + \hat{\underline{X}}_j^*(i+1)\underline{\underline{S}}_j^{(i)}(\hat{\underline{X}}^*(i+1))^T]} \tag{5.102c}$$

2c: $$Z_j^*(i+1) = Y_j(i+1) \tag{5.102d}$$

2d: $$\hat{\underline{\beta}}_j^{*(i+1)} = \hat{\underline{\beta}}_j^{*(i)} + \frac{\underline{\underline{S}}_j^{(i+1)}(\underline{X}_j^*(i+1))^T}{\hat{\Psi}_j^*(MOD_{12}(i+1))^2}(Z_j^*(i+1)-\underline{X}_j^*(i+1)\hat{\underline{\beta}}_j^{*(i)})$$

(5.102e)

STEP 3: $i = i+1$; if $i > n$ stop and if $i \leq n$ go to step 1.

In general the functions $\{\Psi_j^*(\cdot),\ j=0,1,\ldots,11\}$ will not be available a priori. However, Algorithm 5.102 may be initialized in the manner previously suggested and used with

$$\hat{\Psi}_j^*(i) = 1 \quad ; \quad \forall\ i,j \tag{5.103}$$

to obtain the estimates $\{\hat{\underline{\beta}}_j^{(n)},\ j=1,2,\ldots,n_o\}$. Given these estimates the residuals of all observations may be computed using Equations 5.104a, 5.104b, and 5.109d with $\hat{\underline{\beta}}^{*(i)}$ replaced by $\hat{\underline{\beta}}^{*(n)}$

$$\hat{\varepsilon}_j^{*(n)}(i) = Z_j^*(i) - \underline{X}_j^*(i)\underline{\beta}_j^{*(n)} \quad ;$$

$$j=1,2,\ldots,n_o \tag{5.104}$$

The superscript (n) on $\varepsilon_\cdot^*(\cdot)$ (i.e., $\hat{\varepsilon}_\cdot^{(n)}(\cdot)$) has been added to emphasize that it has been calculated with $\hat{\underline{\beta}}_\cdot^{*(n)}$.

Using $\{\hat{\varepsilon}_j^{*(n)}(i);\ \forall\ j,i\}$ estimates of $\{\Psi_j^*(k)^2\}$ may be obtained from

$$\Psi_j^*(k)^2 = \frac{1}{n_4} \sum_{\ell=1}^{n_4} (\hat{\varepsilon}_j^{*(n)}(k+12\ell))^2 \qquad \forall\ j,k \tag{5.105a}$$

$$n_4 = \frac{n}{12} \tag{5.105b}$$

These estimates may then be used to initialize Algorithm 5.102 according to 5.101a and 5.101b and directly in Algorithm 5.102 to obtain a final estimate of the paramaters $\{\hat{\underline{\beta}}_j^{*(n)},\ j=1,2,\ldots,n_o\}$.

As pointed out earlier in this section, Algorithm 5.102 consistent but not asymptotically efficient estimates of $\underline{\beta}_{\cdot}^{*}$. The inefficienty is a result of not explicitly accounting for the cross correlated error between equations. As in the algorithm defined in Section 5.6.3, asymptotically efficient estimates of $\underline{\beta}_{\cdot}^{*}$ may be obtained from the straightforward extension of Algorithm 5.102 to the simultaneous estimation of $\{\underline{\beta}_j^{*(\cdot)},\ j=1,2,\ldots,n_o\}$ rather than the estimation of each $\underline{\beta}_j^{*(\cdot)}$ separately.[1] However, the computational burden (memory and execution time) would be greatly increased; an especially undesirable side effect since, as formulated, Algorithm 5.102 is suitable

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[1] The essential requirement for the derivation of a recursive least squares estimator is that the matrix $\underline{\underline{\Omega}}^{*(i+1)}$ may be written as (see Equation 5.38)

$$\underline{\underline{\Omega}}^{*(i+1)} = \left[\begin{array}{c|c} \underline{\underline{\Omega}}^{*(i)} & \underline{\underline{0}} \\ \hline \underline{\underline{0}} & \underline{\underline{R}}(i+1) \end{array}\right]$$

where $\underline{R}(i+1)$ is the covariance matrix of the new set of measurements (i.e., $\underline{\overline{Z}}(i+1)$).

for an online procedure. Thus, in this work recursive least squares will be applied separately to each equation of Model 5.32 as defined in Algorithm 5.102.

Since the matrix $\underline{\underline{C}}_i^*$ (see Equation 5.74b) is not needed to obtain multi-lead forecast from Model 5.32 as shown in Section 5.8, no attempt will be made to derive an appropriate estimation algorithm. If Model 5.32 were to be used for multivariate simulation of discharges, estimates of $\underline{\underline{C}}_i^*$ would be required.

5.7 Tests for Model Adequacy

5.7.1 Introduction

Section 5.3 suggested guidelines, broken down to four convenient steps, as shown in Figure 5.1, to be followed in an iterative approach to model building. In accordance with these guidelines, a general model was proposed in Section 5.4. Sections 5.5 and 5.6 discussed model identification and estimation, respectively. The final step suggested for model building, adequacy testing, is the subject of the present section.

Tests for model adequacy may be broken down into three broad areas of concern:

1. significance of coefficients
2. properties of residuals
3. performance

5.7.2 Significance of Coefficients

5.7.2.1 Reduced Form General Model with Non-autocorrelated Noise (Model 5.31)

With little additional effort, the coefficient estimation algorithms suggested in Section 5.6 yield information useful in determining coefficient significance. This information, as discussed in Sections 5.3 and 5.5, is immediately useful in deciding whether a new model need be identified and, if so, what form should be proposed.

Section 5.6.3 defined an appropriate estimation algorithm for the reduced form of the general model with non-autocorrelated noise, Model 5.31. Recall that the first step of the algorithm is the OLS estimation of coefficients in each of the individual equations (i.e., Equation 5.54) of the multivariate Model 5.31. In accordance with Equation 5.53 and using the notation of Equation 5.55, the covariance matrix of the parameter vector $\underline{\beta}'_j$ of an individual equation is given by

$$\underline{\underline{S}}_{\hat{\underline{\beta}}'_j \hat{\underline{\beta}}'_j} = s^2_{\varepsilon'_j} (\underline{\underline{X}}'^T_j \underline{\underline{X}}'_j) \tag{5.106a}$$

where

$$\hat{\underline{\beta}}'_j = (\underline{\underline{X}}'^T_j \underline{\underline{X}}'_j)^{-1} \underline{\underline{X}}'^T_j \underline{Z}'_j \tag{5.106b}$$

$$\hat{\underline{\varepsilon}}'_j = \underline{Z}'_j - \underline{\underline{X}}'_j \hat{\underline{\beta}}'_j \tag{5.106c}$$

$$s^2_{\varepsilon'_j} = \frac{\hat{\underline{\varepsilon}}'^T_j \hat{\underline{\varepsilon}}'_j}{n - \text{rank}(\underline{\underline{X}}'_j)} \tag{5.106d}$$

Since $\hat{\underline{\beta}}'_j$ is an unbiased estimate (see Section 5.6.2), $\hat{\underline{\beta}}'_j(i)$, as a rule of thumb, may be regarded as not being significant if it lies in the range $[-\underline{\underline{S}}^{1/2}_{\hat{\underline{\beta}}'_j \hat{\underline{\beta}}'_j}(i,i), +\underline{\underline{S}}^{1/2}_{\hat{\underline{\beta}}'_j \hat{\underline{\beta}}'_j}(i,i)]$ and thus removed from the model. With additional assumptions about the distribution of the residuals, much more sophisticated significance tests may be constructed. One of the most useful is the partial or sequential F-test (Draper and Smith, 1966).

Recall the general linear Model 5.33b,

$$\underline{Z} = \underline{\underline{X}}\ \underline{\beta} + \underline{\varepsilon} \qquad (5.33b)$$

Let $\hat{\underline{\beta}}$ denote the OLS estimate of $\underline{\beta}$. The residuals are approximately

$$\hat{\underline{\varepsilon}} = \underline{Z} - \underline{\underline{X}}\ \hat{\underline{\beta}} \qquad (5.107)$$

so that the residual sum of squares may be estimated from

$$\hat{\underline{\varepsilon}}^T\hat{\underline{\varepsilon}} = (\underline{Z} - \underline{\underline{X}}\ \hat{\underline{\beta}}^T)\ (\underline{Z} - \underline{\underline{X}}\ \hat{\underline{\beta}})$$

$$= \underline{Z}^T\underline{Z} - \hat{\underline{\beta}}^T\ \underline{\underline{X}}^T\underline{Z} \qquad (5.108)$$

Equation 5.108 may be epxressed in words as

$$\begin{Bmatrix}\text{total sum}\\ \text{of squares}\end{Bmatrix} - \begin{Bmatrix}\text{sum of squares}\\ \text{explained by}\\ \text{regression}\end{Bmatrix} = \begin{Bmatrix}\text{residual or}\\ \text{unexplained}\\ \text{sum of squares}\end{Bmatrix}$$

$$\underline{Z}^T\underline{Z} - \hat{\underline{\beta}}^T\ \underline{\underline{X}}^T\ \underline{Z} = \hat{\underline{\varepsilon}}^T\hat{\underline{\varepsilon}} \qquad (5.109)$$

Now, consider two different models proposed to explain the same $\underline{Z}$,

Model 1: $Z_t = \beta_1 + \beta_2 X_{2t} + \ldots + \beta_p\ X_{pt};\ t = 1, 2, \ldots, n$

Model 2: $Z_t = \beta_1 + \beta_2 X_{2t} + \ldots + \beta_q\ X_{qt};\ t = 1, 2, \ldots, n;\ q > p$

Let $\hat{\underline{\beta}}^1$ denote the OLS estimate of $\underline{\beta}$ (=$\{\beta_1, \beta_2, \ldots, \beta_p\}$ for Model 1 and $\hat{\underline{\beta}}^2$ the OLS estimate for Model 2. The scalar quantity

$$SS(\hat{\beta}_{p+1}, \hat{\beta}_{p+2}, \ldots, \hat{\beta}_q | \hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_p) = \underline{\hat{\beta}}^{T^2} \underline{\underline{X}}^T(2)\ \underline{Z} - \underline{\hat{\beta}}^{T^1} \underline{\underline{X}}^T(1)\ \underline{Z} \tag{5.110a}$$

where

$$\underline{\underline{X}}(1) = \begin{bmatrix} 1 & X_{21} & \cdots & X_{p1} \\ 1 & X_{22} & \cdots & X_{p2} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ 1 & X_{2n} & \cdots & X_{pn} \end{bmatrix} \tag{5.110b}$$

$$\underline{\underline{X}}(2) = \left[\begin{array}{c|ccc} & X_{(p+1)1} & \cdots & X_{q1} \\ & X_{(p+1)2} & \cdots & X_{q2} \\ \underline{\underline{X}}(1) & \cdot & & \\ & \cdot & & \\ & \cdot & & \\ & X_{(p+1)n} & \cdots & X_{qn} \end{array}\right] \tag{5.110c}$$

may be interpreted as the extra sum of squares explained by the regression due to the terms $\beta_{p+1} X_{(p+1)t} + \ldots + \beta_q X_{qt}$. If the errors are independently and identically normally distributed and $\beta_{p+1} = \beta_{p+2} = \ldots = \beta_q = 0$, then $SS(\hat{\beta}_{p+1}, \hat{\beta}_{p+2}, \ldots, \hat{\beta}_q | \hat{b}_1, \hat{\beta}_2, \ldots, \hat{\beta}_p)$ will be distributed as $\sigma_\varepsilon^2 \chi_{q-p}^2$ independently of $(n-q)S_\varepsilon^2$ (= $\underline{\hat{\varepsilon}}^T\underline{\hat{\varepsilon}}$; see Equation 5.52) which is distributed $\sigma_\varepsilon^2 \chi_{n-q}^2$ (Draper and Smith, 1966). (If the errors are not explicitly assumed to be normal, the the Central Limit Theorem may be appealed to justify these distributions as being approximately correct.) Thus, the quantity

$$\frac{[SS(\hat{\beta}_{p+1}, \hat{\beta}_{p+2}, \ldots, \hat{\beta}_q | \hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_p)/q-p]}{S_\varepsilon^2} \tag{5.111a}$$

follows an $F_{q-p,n-q}$ distribution (with q-p and n-q degrees of freedom) and may be used to test the hypothesis $\beta_{p+1} = \beta_{p+2} = \ldots = \beta_q = 0$. For the adjusted extra sum of squares given in Equation 5.111a, solve

$$F_{q-p,n-q}(SS(\cdot)) = 1 - \delta \tag{5.111b}$$

for δ. The hypothesis may be rejected at a $(1 - \delta)$ level of significance. This test is referred to as the F-test.

With this background, the partial F-test and its application to the testing of parameter significance and usefulness in the identification of Model 5.31 may now be explained. To aid the discussion, recall the form of a single equation from the multivariate Model 5.31,

$$Y_j(k,i) = \sum_{\ell=1}^{n_1(i)} \sum_{m=1}^{n_o} [\underline{\underline{\Pi}}_i(j,\ (m-1)n_1(i) + \ell)\ Y_m(k,\ i-\ell)] + U_j'(i) + V_j'(k,i) \tag{5.54}$$

Given n observations, this equation may be written in the notation of the general linear Model 5.33b, as shown by Equation 5.55 and repeated here.

$$\underline{Z}_j' = \underline{\underline{X}}_j'\ \underline{\beta}_j + \underline{\varepsilon}_j' \tag{5.112}$$

From the causality considerations discussed in Section 5.5 and in particular as shown for the Main Nile model (Table 5.4), several of the coefficients in the set $\{\underline{\underline{\Pi}}_i(j,\ (m-1)n_1(i) + \ell);\ \ m = 1,\ \ldots,\ n_o;\ \ell = 1,\ \ldots,\ n_1(i)\}$ (or equivalently $\underline{\beta}_j'$) may, a priori, be required to be zero.

However, as pointed out in Section 5.5.2.3, the maximum lag of any flow variable ($n_1(i)$) has yet to be identified and the only restriction placed on the individual coefficients due to causality considerations, exclusive of those required to be zero, is that they may be non-zero. Using a particular form of the F-test (the partial F-test) in conjunction with a forward selection procedure, $n_1(i)$ and the coefficients which are non-zero in each equation of Model 5.31, may be identified. The procedure, generally referred to as stepwise regression, is explained in the following algorithm. Figure 5.4 illustrates the algorithm flow and should be helpful.

ALGORITHM 5.113

1) Initialization

   Consider the model for the flow at a particular station j during a given month i, $Y_j(k,i)$ (i.e., Equation 5.54). Using $n_1(i) = n_{1,max}$ where $n_{1,max}$ is the maximum lag for which any flow can influence future flows (this value may be checked for appropriateness at a later stage) and causality considerations discussed in Section 5.5, form the set of potential explanatory variables (i.e., variables associated with coefficients which cannot a priori be assumed zero). Find the extra sum of squares explained by the regression due to each of the individual potential explanatory variables (see Equations 5.110a and 5.54),

1) INITIALIZATION

A) CONSIDER MODEL FOR $Y_j(k, i)$
B) FORM SET OF POTENTIAL EXPLANATORY VARIABLES
C) FIND LEVEL OF SIGNIFICANCE FOR EACH EXPLANATORY VARIABLE
D) CHOOSE THE LARGEST LEVEL OF SIGNIFICANCE: IF $> (1 - \alpha)$, ENTER ASSOCIATED VARIABLE AND GO TO 2; IF $< (1 - \alpha)$, GO TO 5

2) DETERMINATION OF SIGNIFICANCE

A) FOR EACH POTENTIAL EXPLANATORY VARIABLE (WHETHER IN MODEL OR NOT), FIND LEVEL OF SIGNIFICANCE, GO TO 3

3) REMOVAL OF VARIABLES

A) CHOOSE THE MINIMUM VALUE FROM THE SET OF SIGNIFICANCES ASSOCIATED WITH THE VARIABLES PRESENTLY IN THE MODEL: IF $< (1 - \gamma)$, DELETE VARIABLE FROM MODEL AND GO TO 2; IF $> (1 - \gamma)$, GO TO 4

4) ENTRY OF VARIABLES

A) CHOOSE THE MAXIMUM VALUE FROM THE SET OF SIGNIFICANCES ASSOCIATED WITH POTENTIAL EXPLANATORY VARIABLES PRESENTLY NOT IN THE MODEL: IF $> (1 - \alpha)$, ENTER VARIABLE AND GO TO 2; IF $< (1 - \alpha)$, GO TO 5

5) TERMINATION

A) NO ENTRY OR DELETION OF VARIABLE INDICATED, TERMINATE PROCESS AND TAKE THE MODEL OF $Y_j(k, i)$ TO BE MADE UP OF THE VARIABLES PRESENTLY IN MODEL

FIGURE 5.4
STEPWISE REGRESSION ALGORITHM FLOW

$$SS(\hat{\beta}_j(\ell) = \hat{\Pi}_j(j, \cdot)|\hat{\beta}_j(1) = \hat{U}_j(i))/S_\varepsilon^2 \tag{5.114}$$

where S_ε^2 is the estimated residual variance of the model containing the explanatory variables associated with $\beta_j(\ell)$ and $\beta_j(1)$. For each F statistic given in Equation 5.116, find the maximum level of significance at which the hypothesis $\beta_j(\ell) \neq 0$ is accepted (see Equation 5.111b). Choose the largest value from these significance levels. If this value is greater than some predetermined (by user) level $(1 - \alpha)$(say .95), enter the variable associated with that significance into the model and go to step 2. If the largest value is less than $(1 - \alpha)$, no variable can enter significantly and thus the procedure is stopped and the model of $Y_j(k,i)$ is taken as

$$Y_j(k,i) = \hat{\beta}_j(1) + \varepsilon(k,i) = \hat{U}_j(i) + \varepsilon(k,i) \tag{5.115}$$

2) Determination of Significance

For each of the potential explanatory variables (whether presently in the model or not) find its associated extra sum of squares

$$SS(\hat{\beta}_j(\ell) = \hat{\Pi}(j,\cdot)|\hat{\beta}_j(1) = \hat{U}_j(i), \phi)/S_\varepsilon^2 \tag{5.116}$$

where ϕ is the set of estimated coefficients associated with explanatory variables presently in the model exclusive of $\hat{\beta}_j(\ell)$ and S_ε^2 is the estimated residual variance of the model containing the explanatory variables associated with the union of $\hat{\beta}_j(\ell)$, $\hat{\beta}_j(1)$ and ϕ. Using these values, find the maximum level of significance at which the hypothesis $\beta_j(\ell) \neq 0$ is accepted. Go to step 3.

3) Removal of Variables

Choose the minimum value from the set of significances associated with the variables presently in the model (i.e., set ϕ). If this value is less than some predetermined (by the user) $(1 - \gamma)(\doteq .95)$, remove the associated variable from the model and return to step 2. If the value is greater than $(1 - \gamma)$, no variable is removed from the model and variables are considered for entry by proceeding to step 4.

4) Entry of Variables

Choose the maximum value from the set of significances associated with the potential explanatory variables presently not in the model. If this value is greater than $(1 - \alpha)$, enter the variable associated with that significance into the model. If the value is less than $(1 - \alpha)$, go to step 5.

5) Termination

No variable can be deleted from the model (i.e., all significances associated with variables presently in model are $> (1-\gamma)$) and no variable can be entered (i.e., all significances associated with variables presently not in model are $<(1-\alpha)$). Thus, take the model of $Y_j(k,i)$ to be made up of the variables associated with $\hat{\beta}_j(1)$ and ϕ.

The complete identification and preliminary OLS estimation of Model 5.31 requires the application of Algorithm 5.113 for each i, $i = 0, 1, \ldots, 11$, and j, $j = 1, 2, \ldots, n_o$ (i.e., for each month and gaging station). The OLS estimates are then used to estimate the residual covariance matrix associated with each of the identified monthly multivariate Models 5.31. These covariance matrices, as discussed in Section 5.6.3, are subsequently used for the GLS estimation of Model 5.31. The GLS estimates could be tested for significance and deleted or added to the model as appropriate, using a stepwise regression procedure simiiar to that defined in Algorithm 5.113. However, the acceptance of the partial F-tests as valid requires the assumption that the estimated residual covariance matrices are accurate up to a scale factor. This causes a dilemma in that an indication from the results of the F-test to add or delete variables from the model is also an indication that the estimated residual covariance matrices are in error. Thus, the form of the model as identified in the OLS stepwise regression procedure, Algorithm 5.113, will be accepted as final.

Johnston (1972) points out that even if the true cross correlations between the single equations of the monthly multivariate Model 5.31 are near zero, the sample residuals computed using the OLS estimates may yield non-negligible covariances. If this is the case, GLS estimates might mistakenly be computed resulting in estimates with somewhat greater variances than those of the OLS estimates. To test for this situation, the covariance matrix of the GLS coefficient estimates for each monthly multivariate model may be estimated from Equation 5.70.

$$\underline{\underline{S}}_{\hat{\underline{\beta}}''\hat{\underline{\beta}}''} = E[(\hat{\underline{\beta}}'' - \underline{\beta}'')(\hat{\underline{\beta}}'' - \underline{\beta}'')] = [\underline{\underline{X}}''^{T} \, \underline{\underline{\Omega}}^{-1} \, \underline{\underline{X}}'']^{-1} \quad (5.117)$$

using $\underline{\underline{X}}''$ and $\hat{\underline{\underline{\Omega}}}$ as defined in Equations 5.59 and 5.73, respectively, and compared to the estimated covariance matrices, Equation 5.106, of the corresponding OLS estimates.

5.7.2.2 Reduced Form General Model with Autocorrelated Noise (Model 5.32)

Unfortunately, tests for parameter significance such as those formulated above cannot, in general, be applied in conjunction with the estimation Algorithm 5.102 for the reduced form of the general model with autocorrelated noise (Model 5.32) presented in Section 5.6.4. For correlated noise (i.e., $n_2 \neq 0$), the covariance matrix of the estimated parameters cannot be calculated since the explanatory variables include the estimated residuals (see Section 5.6.2). Also, the partial F-test is inappropriate since the residuals are heteroscedastic so that the residual sum of squares is not distributed χ^2. In addition, for correlated noise, the extra sum of squares explained by the regression is not independent of the residual sum of squares.

Under the case of non-autocorrelated noise (i.e., $n_2 = 0$) and the assumption that the estimated residual variances (Equation 5.105) are the true variances, the covariance matrix of the estimates is given by

$$\mathrm{Cov}(\hat{\underline{\beta}}_j^{*(n)}) = \underline{\underline{S}}_j^{(n)} \qquad (5.118a)$$

since (see Equation 5.87b)

$$\underline{\underline{S}}_j^{(n)} = [(\underline{\underline{X}}_j^{*(n)})^T \; (\underline{\underline{\Omega}}^{*(n)})^{-1} \; \underline{\underline{X}}_j^{*(n)}]^{-1} \qquad (5.118b)$$

(see Equations 5.80 through 5.87 for definition of notation). Thus, in this special case and as a rule of thumb, a particular estimated coefficient $\beta_j^*(i)$ may be regarded as not being significant if it lies in the range $[-(\underline{\underline{S}}_j^{(n)}(i,i))^{1/2}, + (\underline{\underline{S}}_j^{(n)}(i,i))^{1/2}]$. Rao and Kashyap (1973) suggest that in the more general case of correlated noise, the covariance matrix in Equation 5.118 may be used to obtain some idea of a parameters variance. However, they do caution that this variance may be considerably different from the true variance.

A plot of the evolution of the estimated parameter vector obtained from Algorithm 5.102, $\hat{\underline{\beta}}_j^{*(i)}$, may be useful. A highly unstable parameter suggests that the parameter is insignificant or that the model as a whole is inappropriate.

The highly subjective nature of the two techniques proposed above for deciding the significance of a parameter estimated for Model 5.32 increases the reliance upon other available techniques of deciding model adequacy. As pointed out in the introduction of this section (5.7.1), these include tests of performance and those concerned with the properties of residuals. The next section addresses the latter group.

5.7.3 Residual Analysis

5.7.3.1 Whiteness Tests

A basic assumption of the proposed general Model 5.17 is that each element of the noise vector at time k, $\underline{V}(k,i)$, is a linear combination of the white noise vector $\underline{W}(k,i)$ (see Equation 5.17j). Mathematically,

$$\underline{V}(k,i) = \underline{\underline{C}}_i \, \underline{W}(k,i) \tag{5.119a}$$

where

$$\underline{W}(k,i) = \begin{bmatrix} W_1(k,i) \\ W_2(k,i) \\ \cdot \\ \cdot \\ \cdot \\ W_{n_o}(k,i) \end{bmatrix} \tag{5.119b}$$

and $W_{\cdot}(.)$ has the properties (see Model 5.17)

$$E[W_j(k,i)] = 0 \qquad \forall j,\ k,\ i \tag{5.119c}$$

$$E[W_j(k,i)\, W_{j'}(k',i')] = \delta(j-j')\delta(k-k')\delta(i-i')\ \sigma_w^2 \ \forall j,\ j',\ k,\ k',\ i,\ i' \tag{5.119d}$$

where $\delta(.)$ is the Kronecker delta.

Models 5.31 and 5.32 are obtained from the reduced form of the general model and thus their disturbance vectors are given by

$$\underline{V}'(k,i) = \underline{\underline{B}}_i^{-1} \, \underline{\underline{C}}_i \, \underline{W}(k,i) = \underline{\underline{C}}_i^{*}(k,i)\, W(k,i) \tag{5.120}$$

(recall that for Model 5.32, $\underline{\underline{B}}_i = \underline{\underline{B}}_j$, $\forall i,j$ and $V'(k,i) = V^*(k,i)\ \forall k,i$). Using Equations 5.119c and 5.119d, it can be shown that

$$E[V_j'(k,i)] = 0 \qquad \forall i,\ j,\ k \tag{5.120a}$$

and

$$E[V_j'(k,i)\ V_{j'}'(k',i')] = E\left[\left[\sum_{\ell=1}^{n_o} \underline{\underline{C}}_i^*(j,\ell)\ W_\ell(k,i)\right]\right.$$

$$\left.\left[\sum_{\ell=1}^{n_o} \underline{\underline{C}}_{i'}^*(j',\ell)\ W_\ell(k',i')\right]\right] = \delta(i-i')\ \delta(k-k')$$

$$\left[\sum_{\ell=1}^{n_o} (\underline{\underline{C}}_i^*(j,\ell)\ \underline{\underline{C}}_{i'}^*(j',\ell))^2\right]\ \sigma_w^2 \tag{5.120 b}$$

Thus, due to the assumptions on $W_\cdot(.)$, the disturbance vectors, $\underline{V}'(k,i)$, of Models 5.31 and 5.32 have zero mean and may have non-zero lag-zero cross-correlations but only zero autocorrelations and cross-correlations of lag greater than or equal to one.

The appropriateness of the estimation algorithms proposed in Section 5.6 depend upon the validity of Equations 5.120b. The estimation procedures insure that Equation 5.120a is satisfied.[1] Thus, in this section, methods for checking the whiteness condition 5.120b will be outlined.

In the following explanations, let $\hat{V}_j'(k,i)$ be equal to the estimated residual of $Y_j(k,i)$ (i.e., discharge at station j, year k, month i) obtained from the OLS or GLS estimation of Model 5.31 or the estimation of Model 5.32 by Algorithm 5.102.

[1]However, the estimation procedures suggested for Model 5.32 do not insure $E[V_j'(k,i)|i] = 0$ unless n_3^* (see Equation 5.75a) is equal to 6. Since unbiased monthly forecasts are desired (see Section 5.7.4) n_3^* will be a priori assumed equal to 6.

WHITENESS TEST NO 1: Durbin-Watson

Durbin and Watson (1950, 1951) derived a widely used test for autocorrelation of the residuals. Although the test was derived for non-stochastic regression variables, it is applicable to a model with stochastic regression variables so long as they are serially independent and do not contain lagged values of the regressand or lagged noise terms (i.e., no autoregressive or moving average terms). This being the case, the application of the test is only theoretically appropriate for the residuals of Model 5.31 estimated by OLS methods.[1]

The Durbin-Watson test is applied to the OLS residuals of each equation j of the monthly multivariate Model 5.31 by calculating the "d" statistic as

$$d_{ji} = \sum_{k=1}^{n/12} (\hat{V}_j'(k,i) - \hat{V}_j'(k-1,i))^2 / \sum_{k=0}^{n/12} \hat{V}_j'(k,i)^2 \qquad (5.121)$$

Upper (d_U) and lower (d_L) limits, dependent on the number of explanatory variables, observations (n/12 + 1) and chosen significance level, are used to test the hypothesis of zero autocorrelation against the alternative hypothesis of positive first-order autocorrelation.

If $d_{ji} < d_L$, reject the hypothesis of non-autocorrelated $V_j'(1)$ in

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[1]Assuming $n_1(i)$ is less than 12 for all i, the regression variables of Model 5.31 are not lagged values of the regressand, since they are a different month. If $n_1(i)$ is greater than or equal to 12, the test may not theoretically be applicable. Serial independence of the regression variables is assumed to approximately hold, since they are separated by a lag of 12 months.

favor of the hypothesis of positive autocorrelation.

If $d_{ji} > d_U$, do not reject the null hypothesis

If $d_L < d_{ji} < d_U$, the test is inconclusive.

Values for d_L and d_U, tabulated against their arguments, may be found in Durbin and Watson (1951) or Johnston (1972).

To test the alternative hypothesis of negative first-order autocorrelation, compute $(4 - d_{ji})$ and compare this value to d_L and d_U as if testing for positive autocorrelation.

The above test is not appropriate for the residuals obtained from the GLS estimation of each multivariate monthly Model 5.31. However, if these GLS residuals are grouped according to j and the Durbin-Watson statistic is calculated as indicated in Equation 5.121 so that each monthly multivariate equation yields n_o statistics, the test may be assumed to be approximately correct.

WHITENESS TEST NO. 2:

Durbin (1970) suggested a large sample $(n > 30)$ procedure to test for autocorrelated residuals obtained from a model whose explanatory variables contain some autoregressive terms but no lagged noise (i.e., no moving average terms).

Thus, the application of the test to the residuals of Model 5.32 may be assumed approximately correct when $n_1 \geq 1$ and $n_2 = 0$ and when estimated by either least squares or weighted least squares. (Also, the test may be applicable to some residuals of Model 5.31 if $n_1(i)$ is greater than or equal to 12 for any i.)

The test is applied to each equation j of the multivariate model 5.32 by computing the statistics h_j where

$$h_j = \frac{\sum_{k=0}^{n/12} \sum_{i=0}^{11} \hat{V}'_j(k,\ i+1)\ \hat{V}'_j(k,i)}{\sum_{k=0}^{n/12} \sum_{i=0}^{11} \hat{V}'_j(k,\ i+1)^2} \sqrt{\frac{n}{1 - n\ \mathrm{Var}(\hat{\underline{\beta}}_j(1))}} \qquad (5.122)$$

and

$\mathrm{Var}(\hat{\underline{\beta}}_j(1))$ = the variance for the estimated coefficient of the first autoregressive term $(Y_j(k,\ i-1))$ (see Equation 5.118)

Under the hypothesis of non-autocorrelated noise, the statistic h_j is distributed as a standard normal deviate. Thus, if $|h_j|>1.96$, the hypothesis of zero autocorrelation is rejected at the 5% significance level and if $|h_j|>2.58$, the hypothesis is rejected at the 1% significance level. Notice that the test breaks down if n $\mathrm{Var}(\hat{\beta}_j)$ is greater than one.

WHITENESS TEST NO. 3:

Given n successive samples of residuals from a Gaussian population with constant variance and whiteness properties given in Equation 5.119, results of Anderson (1942) show that the estimated autocorrelations and cross-correlations of lag greater than zero are approximately normally distributed with mean zero and variance 1/n. Furthermore, all correlation coefficients of an auto- or cross-correlation series are independently distributed. Thus, if the true parameter values of a model of type 5.31 or 5.32 are known and the model is of the correct form, the correlation coefficients of the estimated residuals may be expected to be independent and to have theoretical variance of the same magnitude.

In practice, the true parameter values are not known. Box and Pierce (1970) have shown that the estimated coefficients of the autocorrelation function of any univariate ARMA model generally have lower variance than n^{-1} for low lags although for higher lags, the variance does approach n^{-1}.

Thus, although the autocorrelation and cross-correlations of the estimated residuals from Models 5.31 and 5.32 can yield valuable evidence concerning model adequacy, it is dangerous to assess the statistical significance of apparent discrepancies of these correlations from their theoretical zero values on the basis of a standard error $n^{-1/2}$. With this warning in mind, it is suggested that sample correlations be calculated according to 5.123 and subjectively inspected.

$$r_{ij}(\ell) = \frac{\frac{1}{n-\ell}\sum_{i=0}^{11}\sum_{k=0}^{n/12} \hat{V}_i'(k,i+\ell)\,\hat{V}_j'(k,i)}{\frac{1}{n-1}\left[\sum_{i=0}^{11}\sum_{k=0}^{n/12}\hat{V}_i'(k,i)\right]^{1/2}\left[\sum_{i=0}^{11}\sum_{k=0}^{n/12}\hat{V}_j'(k,i)\right]^{1/2}} \qquad (5.123)$$

5.7.3.2 Normal Probability Plot

Section 5.7.1 proposed a significance test (partial F) for OLS estimated parameters of Model 5.31. The validity of the tests rests upon the assumption of normally distributed residuals. Thus, in the section a technique which allows the visual inspection of the normality of a set of residuals is discussed.

A coordinate system may be defined such that the cumulative distribution function (CDF) of any normal density function plots linearly (Cornell, 1970). If the estimated CDF of a set of residuals plots approximately linearly in this coordinate system the residuals may be considered normally distributed.

The estimated CDF of a set of residuals $\{\hat{V}'_j(k,i),\ k=0,\ 1,\ 2,\ \ldots,\ n/12\}$ is easily found. First, the residuals are ordered with respect to magnitude so that $\hat{V}'^{(m)}_j(k,i)$ denotes the m'th smallest value. Values of the CDF are then estimated by

$$\mathrm{CDF}(\hat{V}'^{(m)}_j(k,i)) = \mathrm{Prob}(V'_j(\cdot) \leq \hat{V}'^{(m)}_j(k,i)) \doteq \frac{m}{(n/12)+1} \qquad (5.124)$$

Paper which has this special coordinate system predefined may be obtained and is generally referred to as normal probability paper.

5.7.4 Performance Evaluation

In this work, the ultimate use of the stochastic model is multi-lead forecast of monthly discharge. Thus, it is prudent to test the forecasting ability of a few candidate models before making a selection.

Section 5.8 shows how to obtain multi-lead forecasts from a given validated model. Following the notation presented there, let $\hat{Y}_j(k,\ i|k,i-\ell)$

denote a ℓ month ahead forecast of discharge at station j, year k and month i. The bias (B) and mean square error (MSE) of the forecasts may be tabulated over ℓ, i and j using the following relations

$$B(\ell,i,j) = \frac{1}{(n/12)} \sum_{k=0}^{n/12} [Y_j(k,i) - \hat{Y}_j(k,i|k,\ i-\ell)] \qquad \forall j,\ i,\ \ell \quad (5.125a)$$

$$MSE(\ell,i,j) = \frac{1}{(n/12)} \sum_{k=0}^{n/12} [Y_j(k,i) - \hat{Y}_j(k,i|k,\ i-\ell)]^2 \quad \forall j,\ i,\ \ell \qquad (5.125b)$$

Using the historical standard deviation of the sequence $\{Y_j(k,i),\ k = 0,\ 1,\ \ldots,\ n/12\}$, denoted by $S^2(j,i)$, a scaled statistic $R^2(\ell,i,j)$ may be computed from $MSE(\ell,i,j)$ as

$$R^2(\ell,i,j) = \frac{S^2(j,i) - MSE(\ell,i,j)}{S^2(j,i)} \qquad (5.126a)$$

where

$$S^2(j,i) = \frac{1}{(n/12)} \sum_{k=0}^{n/12} (Y_j(k,i) - \overline{Y}_j(i))^2 \qquad (5.126b)$$

and

$$\overline{Y}_j(i) = \frac{1}{(n/12)+1} \sum_{k=0}^{n/12} Y_j(k.i) \qquad (5.126c)$$

The statistic $SB(\ell,i,j)$ and $R^2(\ell,i,j)$, $\forall i$, j and $\ell = 1, 2, 3, \ldots,$ are useful in evaluating the forecasting performance of a given model. Clearly, an unbiased forecast is desired so that the closer the various values of B are to zero, the better the performance. Similarly, it is desirable to minimize the mean square error of forecasts and thus, values of R^2 close to one reflect good performance. If $R^2(\ell,i,j)$ is equal to zero, the forecast $\hat{Y}_j(k,i|k, i-\ell)$ for all k is essentially useless. For $R^2(\ell,i,j)$ less than zero, it would be better to use $\overline{Y}_j(i)$ as a forecast rather than $\hat{Y}_j(k,i|k, i-\ell)$. Note that $R^2(1,i,j)$ for all i,j is the standard "R^2 statistic" encountered in regression theory (Draper and Smith, 1966; Goldberger, 1964; Johnston, 1972).

In this work, the forecasting performance of two or more models will be compared by tabulating $B(\ell,i,j)$ and $R^2(\ell,i,j)$ over all i,j and $\ell = 1, 2, \ldots, 12$. The model which performs "best" will be chosen by subjectively comparing these tabulated statistics. No attempt is made to use an aggregate statistic which reflects the overall forecast performance of a model.

5.8 Multi-Lead Forecasting

Multi-lead forecasts from Model 5.31 or 5.32 are easily obtained. Both models are of the general form (see 5.19 for notation)

$$\underline{Y}(k,i) = \underline{\underline{\Pi}}_i \begin{bmatrix} \underline{Y}(k, i-1) \\ \underline{Y}(k, i-2) \\ \vdots \\ \underline{Y}(k, i-n_1(i)) \end{bmatrix} + \underline{U}'(i) + \underline{V}'(k,i)$$

$$\underline{\underline{T}}_i \begin{bmatrix} \underline{V}'(k, i-1) \\ \underline{V}'(k, i-2) \\ \vdots \\ \underline{V}'(k, i-n_2(i)) \end{bmatrix} \qquad i = 0, 1, 2, \ldots, 11 \quad (5.127)$$

For Model 5.31, the conditions

$$\left.\begin{aligned} \underline{\underline{T}}_i &= 0 \\ n_2(i) &= 0 \end{aligned}\right\} \forall i \qquad (5.128)$$

apply and for Model 5.32

$$\left.\begin{aligned} &\underline{\underline{\Pi}}_i = \underline{\underline{\Pi}}_j = \underline{\underline{\Pi}}^* \\ &n_1(i) = n_1(j) = n_1^* \\ &\underline{U}'(i) = \underline{U}^*(i) = \underline{\underline{\alpha}} \begin{bmatrix} 1 \\ \cos\omega_1 i \\ \sin\omega_1 i \\ \vdots \\ \cos\omega_{n_3} i \\ \sin\omega_{n_3} i \end{bmatrix} \\ &\underline{\underline{T}}_i = \underline{\underline{T}}_j = \underline{\underline{T}}^* \\ &n_2(i) = n_2(j) = n_2^* \end{aligned}\right\} \forall i, j \qquad (5.129)$$

apply. Given estimates of $\underline{\underline{\Pi}}_i$, $\underline{U}'(i)$ $\underline{\underline{T}}_i$, $n_1(i)$, and $n_2(i)$ for all i, all denoted by the " ^ " convention, multi-lead forecasts for either model may be made using the following recursion relation.

$$E[\underline{Y}(k,\ i+\ell)\,|\,\underline{Y}(k,i),\ \underline{Y}(k,\ i-1),\ \ldots,\ \underline{Y}(0,\ 0)] \equiv$$

$$\hat{\underline{Y}}(k,\ i+\ell\,|\,k,i) = \hat{\underline{\underline{\Pi}}}_{MOD_{12}(i+\ell)} \begin{bmatrix} \hat{\underline{Y}}(k,\ i+\ell-1\,|\,k,i) \\ \hat{\underline{Y}}(k,\ i+\ell-2\,|\,k,i) \\ \vdots \\ \hat{\underline{Y}}(k,\ i+\ell-n_1(MOD_{12}(i+\ell))\,|\,k,i) \end{bmatrix}$$

$$+\ \hat{\underline{U}}'(MOD_{12}(i+\ell)) + \hat{\underline{\underline{T}}}_{MOD_{12}(i+\ell)} \begin{bmatrix} \hat{\underline{V}}'(k,\ i+\ell-1\,|\,k,\ i) \\ \hat{\underline{V}}'(k,\ i+\ell-2\,|\,k,i) \\ \vdots \\ \hat{\underline{V}}'(k,\ i+\ell-n_2(MOD_{12}(i+\ell))\,|\,k,i) \end{bmatrix} \qquad (5.130a)$$

where

$$\hat{\underline{Y}}(k,\ i+\ell-j\,|\,k,\ i) = \underline{Y}(k,\ i+\ell-j) \text{ if } (12k + i+\ell-j) \leq (12k + i) \qquad (5.130b)$$

$$\hat{\underline{V}}'(k,\ i+\ell-j\,|\,k,i) = Y(k,\ i+\ell-j)\ -$$

$$\hat{\underline{\underline{\Pi}}}_{MOD_{12}(i+\ell-j)} \begin{bmatrix} \hat{\underline{Y}}(k,\ i+\ell-j-1\,|\,k,i) \\ \hat{\underline{Y}}(k,\ i+\ell-j-2\,|\,k,i) \\ \vdots \\ \hat{\underline{Y}}(k,\ i+\ell-j-n_1(MOD_{12}(i+\ell-j)\,|\,k,i) \end{bmatrix}$$

$$-\hat{\underline{U}}(MOD_{12}(i+\ell-j) - \hat{\underline{\underline{T}}}_{MOD_{12}(i+\ell-j)} \begin{bmatrix} \hat{\underline{V}}'(k,\ i+\ell-j-1\,|\,k,\ i) \\ \hat{\underline{V}}'(k,\ i+\ell-j-2\,|\,k,\ i) \\ \vdots \\ \hat{\underline{V}}'(k,\ i+\ell-j-n_2(MOD_{12}(i+\ell-j))\,|\,k,i) \end{bmatrix}$$

$$\text{if } 0 \le (12k + i + \ell - j) \le (12k + i) \tag{5.130c}$$

$$\underline{\hat{V}}'(k,\ i{+}\ell{-}j \mid k,i) = 0 \quad \text{if} \quad (12k + i + \ell - j) > (12k + 1)$$
$$\text{or} \quad (12k + i + \ell - j) < i_{START} \tag{5.130d}$$

$$i_{START} = \min i \quad \text{s.t.}[n_1(i) - i + 1 \ge 0] \quad \forall i \tag{5.130e}$$

$$(k,\ i + \ell - j) = (k + 1,\ i + \ell - j - 12) \quad \text{if} \quad i + \ell - j > 11 \tag{5.130f}$$

$$(k,\ i + \ell - j) = (k - 1,\ i + \ell - j + 12) \quad \text{if} \quad i + \ell - j < 0 \tag{5.130g}$$

For Model 5.32

$$\underline{\hat{U}}'(i) = \underline{\underline{\hat{\alpha}}} \begin{bmatrix} 1 \\ \cos\omega_1 i \\ \sin\omega_1 i \\ \cdot \\ \cdot \\ \cdot \\ \cos\omega_{n_3} i \\ \sin\omega_{n_3} i \end{bmatrix} \tag{5.130h}$$

Thus, assuming that observations of $\underline{Y}(.)$ are available up to time (k, i), $\underline{\hat{V}}'(.)$ may be calculated recursively beginning with $\underline{\hat{V}}'(0, i_{START})$. Using $\{\underline{\hat{V}}'(k, i), \underline{\hat{V}}'(k, i-1, \ldots\}$, $Y(k, i{+}\ell \mid k, i)$ for $\ell > 0$ may be calculated recursively beginning with $\underline{\hat{Y}}(k, i + 1 \mid k, i)$.

Chapter 6

MULTI-LEAD FORECASTS AND OPTIMAL RESERVOIR CONTROL

6.1 Introduction

Chapter 3 outlined an algorithm (3.8), which may be used for reservoir control. Using this algorithm, each control decision is made using the solution of a dynamic programming model of the value iteration type which incorporates real time information pertaining to inflow (i.e., knowledge of the future behavior of inflow based on present and past information from any source) through the use of conditional first order Markov transition matrices. Because the conditional Markov transition matrices become stationary (in the sense of Equation 3.1) after relatively few number of steps, say M_o, into the future, the dynamic programming model need only be solved over M_o stages. Thus, the algorithm is computationally efficient.

This chapter focuses on the derivation of approximate conditional Markov transition matrices using a validated univariate or multivariate stochastic dynamic model of streamflow, such as those discussed in Chapter 5. Also, the determination of M_o will be discussed. To begin, the derivation of stationary Markov transition matrices, as suggested by Alarcon (1979), will be presented.

6.2 Derivation of Stationary Markov Transition Matrices from Historical Data

Consistent with Chapter 3, the i'th, j'th element of the first order Markovian transition matrix at time nT+t, where T is the period of the periodic Markov chain and t is the number of steps remaining in a cycle (see Figure 3.1) will be denoted by

$$P_{ij}(nT+t) = \text{Prob}(Q_j(nT+t-1) \mid Q_i(nT+t)) \qquad (6.1)$$

where

$Q_i(nT+t)$ = the i'th discrete value of discharge at time nT+t

Assuming stationarity, (i.e., that the Markov chain is the same from one cycle to the next), Equation 6.1 is dependent only on t,

$$P_{ij}(nT+t) = P_{ij}(t) = \text{Prob}(Q_j(t-1) \mid Q_i(t)) \qquad (6.2)$$

Given the continuous joint probability density of Q(t) and Q(t-1) denoted by $f_{Q(t)Q(t-1)}(q(t), q(t-1))$, $P_{ij}(t)$ for each i and j may be calculated as follows.

The marginal probability density (mpd) of each random variable is found by integrating the joint density over the other random variable,

$$f_{Q(t)}(q(t)) = \int_{-\infty}^{\infty} f_{Q(t)Q(t-1)}(q(t), q(t-1))\, dq(t-1) \qquad (6.3)$$

and

$$f_{Q(t-1)}(q(t-1)) = \int_{-\infty}^{\infty} f_{Q(t)Q(t-1)}(q(t), q(t-1))\, dq(t) \qquad (6.4)$$

Using these marginal densities, each random variable is divided into N intervals in such a way that the probability mass over the interval corresponding to the larger values of discharge is β and the masses associated with the remaining intervals are equivalent and given by

$$\frac{1 - \beta}{N - 1} \tag{6.5}$$

The values defining the intervals are denoted in order from the largest to the smalest by $q_j(.)$, $j = 1, 2, \ldots, N-1$, as shown in Figure 6.1. The medians of each interval, denoted by $\{Q_j(.), j = 1, 2, \ldots, N\}$, are chosen as representative values for the discrete model implicit to the dynamic programming formulation.

The transition probabilities, $P_{..}(t)$, are found from the conditional density of $Q(t-1)$ given $Q(t)$, denoted by $f_{Q(t-1)|Q(t)}(q(t-1), q(t))$, using the following relationships

$$P_{ij}(t) = \begin{cases} \displaystyle\int_{q_1(t-1)}^{\infty} f_{Q(t-1)|Q(t)}(q(t-1), Q_i(t))\, dq(t-1) & ; j = 1, \forall i \quad (6.6a) \\ \displaystyle\int_{q_j(t-1)}^{q_{j-1}(t-1)} f_{Q(t-1)|Q(t)}(q(t-1), Q_i(t))dq(t-1) & ; 1 < j < N, \forall i \quad (6.6b) \\ \displaystyle\int_{-\infty}^{q_{N-1}(t-1)} f_{Q(t-1)|Q(t)}(q(t-1), Q_i(t))\, dq(t-1) & ; j = N, \forall i \quad (6.6c) \end{cases}$$

and

$$f_{Q(t-1)|Q(t)}(q(t-1), q(t)) = \frac{f_{Q(t)Q(t-1)}(q(t), q(t-1))}{f_{Q(t)}(q(t))} \tag{6.6d}$$

A graphical illustration of the computation is shown in Figure 6.2.

The only difficulty which arises in the application of the above methodology is finding a joint probability density for the inflows of

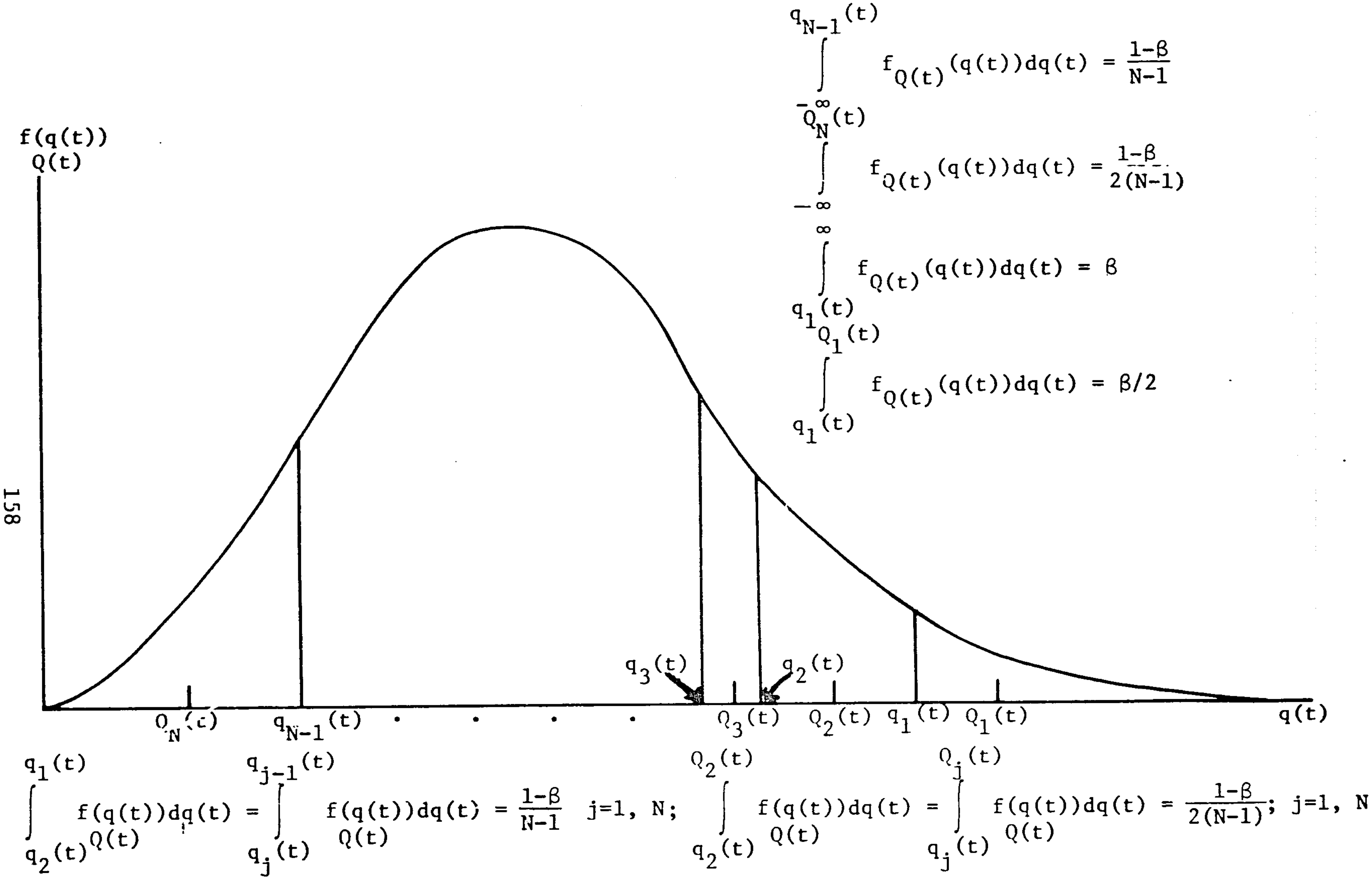
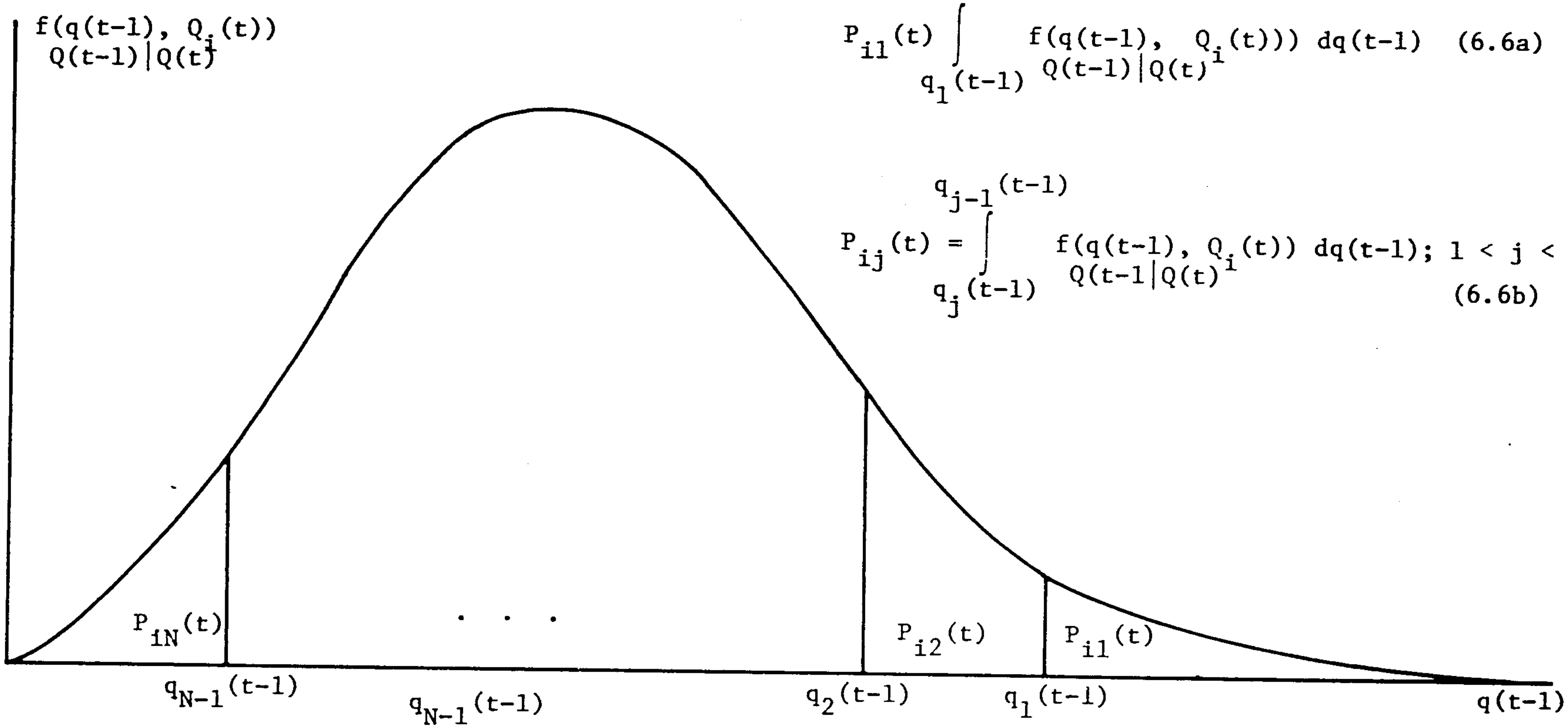


Figure 6.1
Discretization of Inflow
(adapted from Alarcon, 1979)



$$P_{i1}(t) \int_{q_1(t-1)}^{\infty} \underset{Q(t-1)|Q(t)}{f}(q(t-1),\ Q_i(t))\ dq(t-1) \qquad (6.6a)$$

$$P_{ij}(t) = \int_{q_j(t-1)}^{q_{j-1}(t-1)} \underset{Q(t-1|Q(t)}{f}(q(t-1),\ Q_i(t))\ dq(t-1);\ 1 < j < N \qquad (6.6b)$$

$$P_{iN}(t) = \int_{-\infty}^{q_{N-1}(t-1)} \underset{Q(t-1)|Q(t)}{f}(q(t-1),\ Q_i(t))\ dq(t-1) \qquad (6.6c)$$

Figure 6.2
Derivation of First Order Markov Transition Matrices

consecutive months which is rigorously justifiable. Alarcon (1979), for the sake of tractable mathematics, assumes that the marginal probability density for the inflow of any given month is either normal or lognormal and that the joint density of inflow (or transformed inflow for lognormal marginal density) between months is multivariate normal. Under the assumptions of large β (say > .05) and relatively coarse discretization ($N \doteq 10$), the sensitivity of the transition probabilities to different joint probability functions is weakened. Thus, under these conditions, the a priori assumptions of normal densities is appropriate.

Given historical data, the decision of which marginal density function to use for inflow during any given month is easily made using the χ^2 goodness of fit test (Bendat and Piersal (1971) or Benjamin and Cornell (1970)). Burges, et al. (1975) and Stedinger (1979) discuss parameter estimation for the normal and lognormal density functions.

6.3 Derivation of Approximate Conditional Markov Transition Matrices from Multi-lead Forecasts

Section 5.8 showed how multi-lead forecasts may be obtained from a validated univariate or multivariate stochastic dynamic model of streamflow. First order Markov transition probabilities must be derived from these point forecasts over the period of nonstationarity (in the sense of Equation 3.1) if they are to be of use in the adaptive control algorithm (3.8) presented in Chapter 3.

In this section, a technique to identify the period of nonstationarity and derive approximate transition matrices from point forecasts is offered for consideration. A rigorous theoretical justification does not exist, although arguments can be given definding its "reasonableness". In any case, the ultimate justification lies in the usefulness of the methodology in conjunction with control Algorithm 3.8. Unfortunately, the scope of this work does not involve this evaluation. (The evaluation may be done by using the forecasting model developed for the Nile in this study and the dynamic programming model of the Aswan Reservoir developed by Alarcon (1979)).

Towards defining the proposed technique, let j denote the station which measures the inflow of interest. Given observations of inflow at station j and discharge of other upstream stations inclusive of and prior to year k (=n/12) and month i (e.g., $\{\underline{Y}(k,i), \underline{Y}(k, i-1), \ldots\}$), the following assumptions about the conditional joint probability densities of future inflows are made.

> ASSUMPTION 1: The conditional joint probability density of inflow (station j) for year k and months $i+\ell$ and $i+\ell+1$, $0 < \ell < M_o$, is multivariate normal with means $\hat{Y}_j(k, i+\ell|k, i)$ and $\hat{Y}_j(k, i+\ell+1|k, i)$, computed from 5.133, variances $MSE(\ell, i, j)$ and $MSE(\ell+1, i, j)$, computed from 5.125b, and correlation coefficient given by ($Y_j(.)$ is historical data for station j)

$$\frac{\sum_{k'=0}^{n/12}(Y_j(k', i+\ell) - \overline{Y}_j(i+\ell))(Y_j(k', i+\ell+1) - \overline{Y}_j(i+\ell+1))}{\left[\sum_{k'=0}^{n/12}(Y_j(k', i+\ell) - \overline{Y}_j(i+\ell))^2\right]^{1/2}\left[\sum_{k'=0}^{n/12}(Y_j(k', i+\ell+1) - \overline{Y}_j(i+\ell+1))^2\right]} \tag{6.7a}$$

where

$$\overline{Y}_j(i') = \frac{1}{n/12} \sum_{k'=0}^{n/12} Y_j(k', i') \tag{6.7b}$$

ASSUMPTION 2: The conditional joint probability of inflow between contiguous months beyond year k and month $i+M_o$ is only a function of month.

Assumptions 1 and 2 imply that M_o represents the number of months, starting from k, i during which the observed discharges affect inflow at station j. It may be taken as

$$M_o = \max \ell \quad \text{s.t. } R^2(\ell, i, j) > \varepsilon \tag{6.8}$$

where $R^2(.)$ is computed from 5.129a and ε is some small positive number (say .05). Approximate Markov transition matrices over the interval (k, i) to (k, $i+M_o$) are calculated as indicated (for both inflows distributed normal) in the previous section. A different discretization scheme may be defined at the user's discretion.

The Markov transition matrices beyond k, $i+M_o$ are stationary since the underlying joint density is assumed stationary. Recall from Chapter 3 that this stationarity allows the boundary condition to be invoked at k, $i+M_o$ in control Algorithm 3.8.

Chapter 7

APPLICATION

7.1 Introduction

This chapter will discuss an application of the iterative approach to model building outlined in Chapter 5. The two proposed general models, Models 5.31 and 5.32, will be fitted to the historical Nile basin flows given in Appendix 1. As Figure 7.1 shows, results from the univariate and multivariate models estimated using the various schemes proposed in Chapter 5 will be presented. The univariate models considered will be for Wadi Halfa, the last gaging station before the High Aswan Dam. It will be estimated and evaluated using the 1890-1976 historical record. The multivariate models will be for the eight gaging stations discussed in Section 5.5.2. They will be estimated and evaluated using the largest simultaneous historical record, 1912 through 1967.

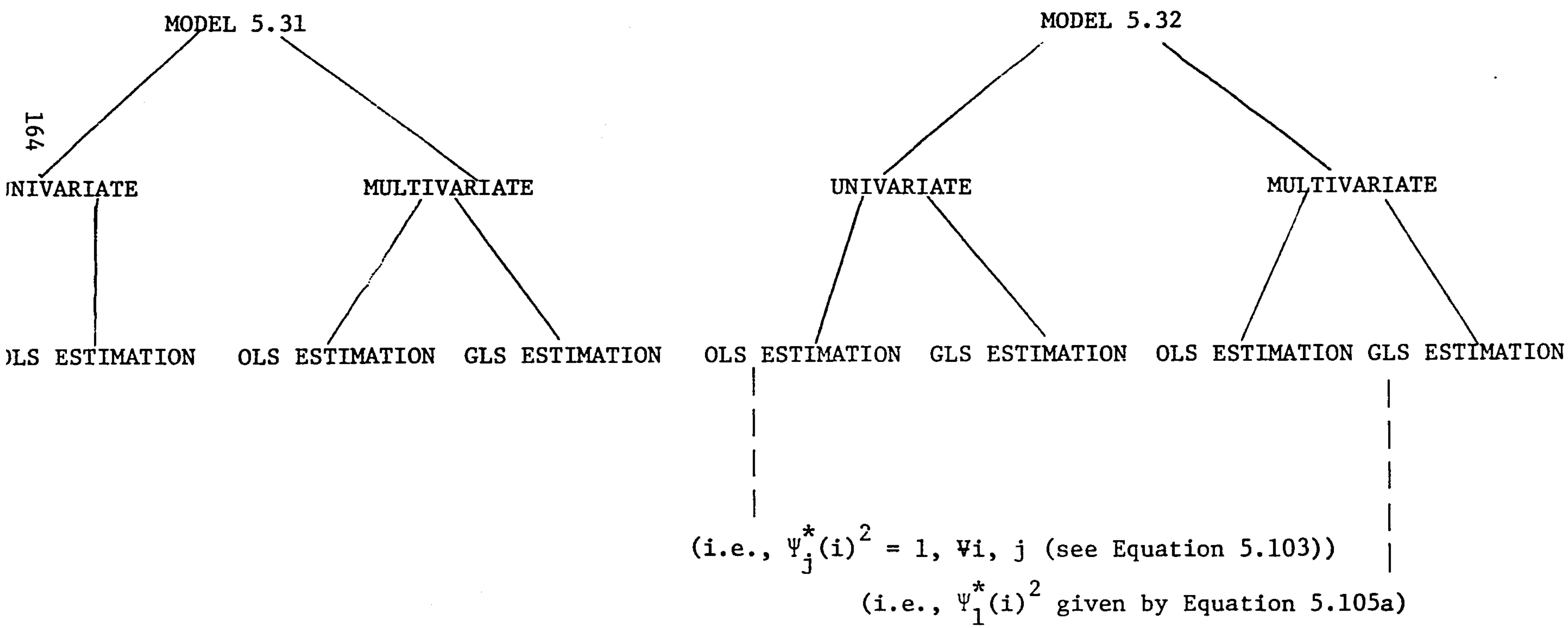
7.2 Identification

Section 5.5 discussed how the hierarchical causality structure between the Nile River basin gaging stations was exploited for partial identification. Several elements of the $\underline{\underline{\Pi}}_i$ and $\underline{\underline{\Pi}}^*$ coefficient matrices of Models 5.31 and 5.32, respectively, were a priori required to be zero.

For Model 5.31, complete identification required the knowledge of the maximum lag ($n_1(i)$) for which past flows of the station of concern and downstream stations give information about the present flow. Also, the elements of the regression matrices ($\underline{\underline{\Pi}}_i$) which are not a priori assumed

FIGURE 7.1

CASES TO WHICH THE ITERATIVE APPROACH TO MODEL BUILDING IS APPLIED



zero but should, in fact, be zero because the associated explanatory variable does not give additional information (relative to the "best" alternative model) about the present flow had to be identified.

These identification tasks were accomplished using the stepwise regression Algorithm 5.113, which was implemented using the Honeywell Multics Consistent System developed by Klensin (1979). First, the algorithm was applied to determine a reasonable $n_1(i)$. Letting $n_1(i)$ be equal to 24, a number which intuitively seems too large, several models for the flow of a given month at a single station were identified.

A 0.95 significance level was used as the cutoff for both entry and removal of variables from an equation. The greatest lag of any explanatory variable which did enter into the final model (i.e., which did give "significant" additional information about the present flow relative to the best alternative model) was 12. Thus, $n_1(i)$ was assumed to be 12 in all subsequent identification. The results, that is the final identified univariate and multivariate models for each month, will be given in the next section, 7.3.

The complete identification of Model 5.32 also required finding n_1^* and the elements of $\underline{\underline{\Pi}}^*$ which are not a priori assumed zero but should be zero. Additionally, n_2^* and the structure of $\underline{\underline{T}}^*$ had to be found.[1]

As Chapter 5 pointed out, there was no objective algorithm like 5.113 which could be used on Model 5.32 that is theoretically appropriate for identification. Thus, trial and error had to be relied on to a large degree. Possible insignificant elements of $\underline{\underline{\Pi}}^*$ and a likely value of n_1^*

[1] The parameter n_3^* was a priori assumed equal to 6 so that the condition $E[V_j'(k,i)|i]=0$ is enforced (see Section 5.7.3.1).

were preliminarily identified by applying Algorithm 5.113 to the individual equations of the model. It is stressed that this application is not theoretically justifiable but was only used to provide a good starting point for further identification. Using this initial structure of $\underline{\underline{\Pi}}^*$ and letting n_2^* be equal to zero, the model was checked for adequacy. Following the iterative approach to model building, the results from this stage were used to identify a new model which was subsequently tested for adequacy. After several iterations, n_1^* was found to be 12 and n_2^* zero. No moving average terms were found that improved the performance of the model. The final model (univariate and multivariate) which was identified will be given in the next section. Section 7.4 will briefly indicate some of the alternative models which were evaluated.

7.3 Estimation and Significance of Coefficients

7.3.1 Model 5.31

The final structure identified for univariate Model 5.31 is indicated in Table 7.1. For each monthly equation, the variables which enter the model at a significance level greater than 0.95 are listed. The OLS estimated coefficient, standard error, partial F statistic, degrees of freedom and significance level are also given.

Table 7.2 gives the same information for each equation of the multivariate Model 5.31.[1] Following the recommendations of Section 5.6.3, the OLS estimated coefficients were used to estimate the variance-covariance matrix of the disturbance terms for each multivariate monthly model via Equation 5.72. The normalized matrices (i.e., correlation matrices) corresponding to these estimated matrices are given in Table 7.3. The generally high correlations, especially for August through November, implied the appropriateness of GLS estimation. The GLS estimated coefficients and standard errors are given in Table 7.2. Notice that in almost all cases the standard error of the GLS estimated coefficient is significantly less than that of the OLS estimated coefficient.

---

[1]Notice that some equations for Malakal flows include observations of Roseires flows as explanatory variables, contrary to the identified structure given in Section 5.5 (see Table 5.4). An explanation is given in Section 7.4.1.

Model of	Variable	Lag	Estimated Coefficient	Standard Error	Partial F Statistic	Degree of Freedom	Significance Level
January	Dec	1	0.822	0.051	257.520	80	100.0
	Nov	2	-0.127	0.041	9.461	80	99.7
	Oct	3	0.059	0.016	13.064	80	99.9
	May	8	0.293	0.050	34.525	80	100.0
	Mar	10	-0.137	0.047	8.431	80	99.5
	Constant		-428.144	176.998	5.851	80	98.2
February	Jan	1	0.969	0.095	103.939	81	100.0
	Dec	2	-0.191	0.071	7.170	81	99.1
	Sep	5	0.020	0.009	4.939	81	97.1
	May	9	0.177	0.041	18.781	81	100.0
	Constant		-855.997	184.710	21.477	81	100.0
March	Feb	1	1.193	0.139	73.963	81	100.0
	Jan	2	-0.536	0.130	16.970	81	100.0
	Oct	5	0.047	0.017	8.018	81	99.4
	Mar	12	0.120	0.056	4.652	81	96.6
	Constant		216.911	212.728	1.040	81	68.9
April	Mar	1	0.893	0.068	172.128	81	100.0
	Dec	4	-0.200	0.040	24.673	81	100.0
	Jun	10	0.225	0.084	7.242	81	99.1
	May	11	0.504	0.077	42.591	81	100.0
	Constant		486.385	186.457	6.805	81	98.9
May	Apr	1	0.757	0.057	174.411	83	100.0
	May	12	0.261	0.057	21.318	83	100.0

Table 7.1
Results of OLS Estimation of Univariate Model 5.31; Monthly Equations for Wadi Halfa

Model of	Variable	Lag	Estimated Coefficient	Standard Error	Partial F Statistic	Degree of Freedom	Significance Level
	Constant		-112.789	119.528	0.890	83	65.2
June	May	1	1.297	0.117	122.133	82	100.0
	Apr	2	-0.973	0.148	42.985	82	100.0
	Mar	3	0.389	0.081	22.855	82	100.0
	Constant		668.879	142.258	22.107	82	100.0
July	June	1	1.126	0.196	32.911	84	100.0
	Constant		2879.584	448.557	41.212	84	100.0
August	Jul	1	2.146	0.240	79.961	83	100.0
	Jun	2	-2.173	0.509	18.224	83	100.0
	Constant		12736.649	1204.349	111.842	83	100.0
September	Aug	1	0.911	0.104	77.172	81	100.0
	Jul	2	-0.596	0.298	4.00	81	95.1
	May	4	-1.344	0.422	10.126	81	99.8
	Jan	8	1.304	0.313	17.401	81	100.0
	Constant		5364.884	1954.193	7.537	81	99.3
October	Sep	1	0.627	0.048	169.522	81	100.0
	Jun	4	0.947	0.439	4.665	81	96.6
	May	5	-1.994	0.402	24.571	81	100.0
	Feb	8	0.702	0.272	6.645	81	98.8
	Constant		832.030	1342.282	0.395	81	46.8
November	Oct	1	0.461	0.028	265.170	83	100.0
	Feb	9	0.420	0.120	12.162	83	99.9
	Constant		-569.883	508.771	1.255	83	73.4

Table 7.1 (cont'd)
Results of OLS Estimation of Univariate Model 5.31;
Monthly Equations for Wadi Halfa

Model of	Variable	Lag	Estimated Coefficient	Standard Error	Partial F Statistic	Degree of Freedom	Significance Level
December	Nov	1	0.461	0.031	215.461	82	100.0
	Aug	4	0.049	0.015	10.133	82	99.8
	Jan	11	0.383	0.055	48.972	82	100.0
	Constant		-889.457	316.410	7.902	82	99.4

Table 7.1 (cont'd)
Results of OLS Estimation of Univariate Model 5.31;
Monthly Equations for Wadi Halfa

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Wadi Halfa	Dec	1	0.457	0.064	51.549	47	100.0	0.468	0.055
	Tamaniat	Dec	1	0.273	0.071	14.949	47	100.0	0.0268	0.062
	Malakal	Dec	1	0.393	0.039	103.792	47	100.0	0.390	0.034
	Sennar	Nov	2	-0.180	0.025	49.853	47	100.0	-0.189	0.023
	Roseires	Jun	7	0.083	0.024	11.894	47	99.9	0.078	0.021
	Tamaniat	Mar	10	-0.141	0.025	31.896	47	100.0	-0.145	0.022
	Khartoum	Mar	10	0.257	0.107	5.753	47	98.0	0.269	0.095
	Constant			-372.251	108.342	11.805	47	99.9	-357.115	96.522
February	Wadi Halfa	Jan	1	0.419	0.043	92.821	52	100.0	0.405	0.036
	Malakal	Jan	1	0.420	0.034	152.367	52	100.0	0.435	0.030
	Constant			-9.647	103.870	0.009	52	7.4	4.554	89.308
March	Atbara	Feb	1	-3.611	1.548	5.441	46	97.6	-3.562	1.185
	Tamaniat	Feb	1	0.922	0.092	101.589	46	100.0	0.960	0.071
	Malakal	Feb	1	0.507	0.113	20.045	46	100.0	0.456	0.092
	Tamaniat	Jan	2	-0.437	0.058	57.020	46	100.0	-0.417	0.046
	Mongalla	Nov	4	-0.503	0.077	43.232	46	100.0	-0.389	0.058
	Roseires	Nov	4	0.225	0.057	15.583	46	100.0	0.164	0.044
	Atbara	Oct	5	-0.218	0.097	5.010	46	97.0	-0.132	0.074
	Mongalla	Oct	5	0.316	0.077	16.910	46	100.0	0.237	0.061
	Constant			625.061	133.407	21.953	46	100.0	588.348	111.500
April	Tamaniat	Mar	1	0.860	0.047	334.866	46	100.0	0.868	0.041

Table 7.2a

RESULTS FOR OLS AND GLS ESTIMATION OF MULTIVARIATE MODEL 5.31; MONTHLY EQUATIONS FOR WADI HALFA

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
April (contd)	Malakal	Mar	1	0.160	0.046	12.289	46	99.9	0.147	0.040
	Khartoum	Mar	1	-1.714	0.202	72.104	46	100.0	-1.518	0.176
	Roseires	Mar	1	2.113	0.198	113.347	46	100.0	1.896	0.174
	Atbara	Feb	2	-7.616	1.323	33.149	46	100.00	-6.370	1.121
	Roseires	May	10	-0.422	0.124	11.628	46	99.9	0.389	0.171
	Khartoum	May	10	0.441	0.203	4.714	46	96.5	-0.349	0.104
	Khartoum	Apr	12	0.480	0.154	9.772	46	99.7	0.389	0.130
	Constant			-400.029	111.060	12.974	46	99.9	-390.171	96.824
May	Tamaniat	Apr	1	0.593	0.035	287.089	45	100.0	0.618	0.029
	Malakal	Apr	1	0.764	0.118	41.885	45	100.0	0.806	0.094
	Khartoum	Apr	1	0.385	0.172	5.014	45	97.0	0.447	0.138
	Khartoum	Mar	2	-1.729	0.347	24.813	45	100.0	-1.280	0.272
	Sennar	Mar	2	0.919	0.272	11.399	45	99.8	0.613	0.213
	Tamaniat	Feb	3	-0.308	0.063	24.092	45	100.0	-0.272	0.049
	Roseires	Feb	3	0.685	0.208	10.881	45	99.8	0.453	0.162
	Malakal	Jan	4	0.347	0.052	44.175	45	100.0	0.279	0.041
	Mongalla	Oct	7	-0.123	0.037	11.274	45	99.8	-0.107	0.029
	Constant			-302.098	97.250	9.650	45	99.7	-354.327	80.777
June	Tamaniat	May	1	0.703	0.046	236.361	50	100.0	0.768	0.038
	Roseires	May	1	0.655	0.093	49.095	50	100.0	0.547	0.080
	Atbara	Feb	4	11.982	2.963	16.352	50	100.0	9.218	2.201

Table 7.2a (cont'd)
RESULTS FOR OLS AND GLS ESTIMATION OF MULTIVARIATE MODEL 5.31; MONTHLY EQUATIONS FOR WADI HALFA

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
June (Contd)	Atbara	Jan	5	-4.743	1.847	6.596	50	98.7	-4.583	1.386
	Constant			164.115	96.321	2.903	50	90.5	104.608	84.533
July	Atbara	Jun	1	7.344	1.137	41.715	44	100.0	5.537	0.841
	Sennar	Jun	1	-1.230	0.378	10.571	44	99.8	-0.909	0.280
	Roseires	Jun	1	2.103	0.412	26.009	44	100.0	1.854	0.309
	Atbara	May	2	-9.597	4.618	4.319	44	95.6	-6.662	3.415
	Mongalla	May	2	0.374	0.103	13.046	44	99.9	0.370	0.077
	Khartoum	May	2	1.561	0.469	11.097	44	99.8	1.243	0.347
	Atbara	Apr	3	-23.426	7.361	10.127	44	99.7	-16.482	5.444
	Roseires	Dec	7	1.952	0.355	30.303	44	100.0	1.542	0.262
	Khartoum	Nov	8	-0.501	0.126	15.953	44	100.0	-0.364	0.093
	Atbara	Jul	12	-0.497	0.124	16.005	44	100.0	-0.401	0.092
	Constant			715.067	446.619	2.563	44	88.3	995.165	341.805
August	Atbara	Jul	1	1.410	0.383	13.534	50	99.9	1.282	0.249
	Sennar	Jul	1	1.762	0.211	69.519	50	100.0	1.411	0.151
	Sennar	Jun	2	-1.305	0.545	5.739	50	98.0	-0.302	0.346
	Sennar	Feb	6	-4.799	1.829	6.888	50	98.9	-2.738	1.213
	Constant			10102.008	1175.671	73.832	50	100.0	10150.064	902.921
September	Atbara	Aug	1	0.949	0.211	20.286	50	100.0	0.771	0.111
	Sennar	Aug	1	0.591	0.145	16.538	50	100.0	0.571	0.085
	Khartoum	Feb	7	12.715	3.380	14.152	50	100.0	4.596	1.834

Table 7.2a (cont'd)

RESULTS OF OLS AND GLS ESTIMATION OF MULTIVARIATE MODEL 5.31; MONTHLY EQUATIONS FOR WADI HALFA

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
September (contd)	Khartoum	Nov	10	-1.004	0.448	5.019	50	97.0	-0.310	0.247
	Constant			4584.462	1737.609	6.961	50	98.9	7657.633	1172.785
October	Sennar	Sep	1	1.073	0.086	156.919	51	100.0	1.040	0.071
	Atbara	Apr	6	-34.304	10.616	10.442	51	99.8	-18.518	5.760
	Sennar	Apr	6	-2.864	1.193	5.758	51	98.0	-1.536	0.730
	Constant			1891.081	1210.145	2.442	51	87.6	1760.970	968.057
November	Sennar	Oct	1	0.677	0.033	422.734	50	100.0	0.622	0.025
	Wadi Halfa	Dec	11	0.612	0.142	18.451	50	100.0	0.609	0.083
	Atbara	Dec	11	-5.709	2.186	6.820	50	98.8	-4.255	1.277
	Roseires	Nov	12	-0.460	0.183	6.291	50	98.5	-0.485	0.106
	Constant			1326.222	506.258	6.863	50	98.8	1687.365	310.970
December	Tamaniat	Nov	1	0.480	0.032	229.332	48	100.0	0.502	0.028
	Malakal	Oct	2	0.203	0.067	9.223	48	99.6	0.160	0.059
	Tamaniat	Sep	3	0.121	0.035	12.099	48	99.9	0.134	0.029
	Khartoum	Sep	3	-0.086	0.034	6.362	48	98.5	-0.100	0.029
	Atbara	May	7	2.526	1.042	5.878	48	98.1	2.240	0.877
	Atbara	Mar	9	-18.341	7.871	5.429	48	97.6	-17.660	6.623
	Constant			426.739	234.214	3.320	48	92.5	427.699	208.979

Table 7.2a (cont'd)
Results for OLS and GLS Estimation of Multivariate Model 5.31;
Monthly Equations for Wadi Halfa

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Atbara	Dec	1	0.283	0.073	14.935	62	100.0	0.259	0.081
	Atbara	Jan	12	0.265	0.109	5.964	62	98.3	0.248	0.115
	Constant			-2.546	6.006	0.180	62	32.7	0.163	6.587
February	Atbara	Jan	1	0.588	0.040	214.754	60	100.0	0.583	0.036
	Atbara	Dec	2	-0.177	0.036	23.713	60	100.0	-0.199	0.034
	Atbara	Oct	4	0.010	0.004	7.313	60	99.1	0.016	0.004
	Atbara	Mar	11	0.725	0.280	6.683	60	93.8	0.738	0.204
	Constant			-3.283	2.470	1.767	60	81.1	-6.859	2.532
March	Atbara	Jan	2	0.040	0.015	7.344	61	99.1	0.042	0.014
	Atbara	Nov	4	0.021	0.005	16.239	61	100.0	0.016	0.005
	Atbara	Sep	6	-0.001	0.000	12.677	61	99.9	-0.001	0.000
	Constant			0.999	1.150	0.754	61	61.1	0.220	1.304
April	Atbara	Apr	12	0.581	0.127	21.070	63	100.0	0.505	0.125
	Constant			2.148	2.025	1.126	63	70.7	2.715	2.372
May	Atbara	Apr	1	1.119	0.140	64.093	62	100.0	1.203	0.136
	Atbara	May	12	0.372	0.123	9.097	62	99.6	0.287	0.119
	Constant			1.989	2.424	0.673	62	58.5	2.332	2.803
June	Atbara	Jun	12	0.299	0.199	6.300	63	98.5	0.158	0.121
	Constant			68.843	21.698	10.066	63	99.8	67.490	16.992
July	Constant			1627.323	*	278.235	64	100.0	1656.237	109.672
August	Atbara	Jul	1	1.701	0.234	52.663	63	100.0	1.388	0.187
	Constant			2842.716	422.919	45.181	63	100.0	3345.475	359.204

Table 7.2b
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Atbara

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
September	Atbara	Aug	1	0.466	0.062	56.604	62	100.0	0.373	0.047
	Atbara	Dec	9	7.421	2.679	7.671	62	99.3	4.826	1.716
	Constant			619.304	395.224	2.455	62	87.8	1293.848	321.783
October	Atbara	Sep	1	0.221	0.025	78.717	63	100.0	0.182	0.022
	Constant			41.038	99.004	0.172	63	32.0	204.453	90.107
November	Atbara	Oct	1	0.215	0.018	140.788	63	100.0	0.221	0.017
	Constant			10.144	17.380	0.341	63	43.8	-1.077	16.996
December	Atbara	Nov	1	0.312	0.030	109.200	60	100.0	0.329	0.030
	Atbara	Jul	5	0.009	0.004	5.375	60	97.6	0.011	0.003
	Atbara	Jan	11	0.214	0.106	4.072	60	95.2	0.221	0.092
	Atbara	Dec	12	0.187	0.074	6.435	60	98.6	0.227	0.068
	Constant			-28.504	8.310	11.765	60	99.9	-37.203	7.971

Table 7.2b (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31;
Monthly Equations for Atbara

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Malakal	Dec	1	1.372	0.118	134.026	49	100.0	1.234	0.115
	Sennar	Dec	1	0.657	0.089	55.045	49	100.0	0.663	0.090
	Malakal	Nov	2	-0.760	0.209	13.280	49	99.9	-0.573	0.189
	Roseires	Nov	2	-0.171	0.059	8.515	49	99.5	-0.132	0.054
	Malakal	Sep	4	0.463	0.156	8.810	49	99.5	0.447	0.134
	Tamaniat	Aug	5	0.080	0.013	37.250	49	100.0	0.054	0.019
	Roseires	Aug	5	-0.075	0.011	50.025	49	100.0	-0.060	0.018
	Mongalla	Jul	6	-0.310	0.056	30.543	49	100.0	-0.260	0.052
	Tamaniat	Mar	10	-0.207	0.045	21.302	49	100.0	-0.200	0.046
	Khartoum	Mar	10	0.490	0.179	7.479	49	99.1	0.300	0.175
	Mongalla	Feb	11	0.290	0.060	23.377	49	100.0	0.210	0.066
	Constant			27.844	254.029	0.012	49	8.7	67.381	220.849
February	Malakal	Jan	1	0.614	0.038	262.424	59	100.0	0.671	0.038
	Constant			878.665	99.824	77.477	59	100.0	877.850	97.892
March	Tamaniat	Feb	1	0.673	0.058	134.899	56	100.0	0.645	0.065
	Roseires	Feb	1	0.330	0.155	4.566	56	96.3	0.397	0.331
	Tamaniat	Mar	12	0.670	0.067	99.728	56	100.0	0.639	0.062
	Malakal	Mar	12	-0.470	0.070	44.716	56	100.0	-0.434	0.064
	Constant			-221.884	150.965	2.160	56	85.3	-168.365	157.237
April	Tamaniat	Mar	1	0.814	0.079	105.464	52	100.0	0.842	0.062
	Roseires	Mar	1	0.588	0.197	8.852	52	996	0.519	0.235
	Tamaniat	Feb	2	-0.622	0.130	22.930	52	100.0	-0.535	0.103

Table 7.2c
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Tamaniat

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
August (contd)	Sennar	Feb	6	-5.014	1.818	7.605	58	99.2	-1.841	0.999
	Constant			11147.903	1238.987	80.957	58	100.0	10898.557	796.936
September	Tamaniat	Aug	1	0.784	0.076	105.041	57	100.0	0.664	0.064
	Sennar	Dec	9	4.205	0.861	23.825	57	100.0	2.193	0.515
	Roseires	Nov	10	-1.477	0.530	7.763	57	99.3	-0.732	0.269
	Constant			2376.590	1619.305	2.154	57	85.2	4916.657	1282.797
October	Sennar	Sep	1	0.800	0.088	82.542	58	100.0	0.893	0.079
	Khartoum	Apr	6	-2.983	1.442	4.276	58	95.7	-1.213	0.592
	Constant			2184.817	1325.638	2.716	58	89.5	274.927	1053.779
November	Sennar	Oct	1	0.734	0.075	96.749	55	100.0	0.625	0.054
	Roseires	Oct	1	-0.329	0.078	17.700	55	100.0	-0.258	0.058
	Malakal	Sep	2	1.060	0.157	45.679	55	100.0	1.001	0.077
	Tamaniat	Mar	8	-0.430	0.123	12.190	55	99.9	-0.368	0.062
	Roseires	Feb	9	0.805	0.355	5.142	55	97.3	0.412	0.321
	Constant			248.905	468.889	0.282	55	40.2	707.979	291.801
December	Tamaniat	Nov	1	0.290	0.087	11.069	50	99.8	0.405	0.066
	Malakal	Nov	1	0.637	0.082	60.441	50	100.0	0.501	0.066
	Khartoum	Nov	1	-0.765	0.160	22.869	50	100.0	-0.875	0.120
	Sennar	Nov	1	0.435	0.135	10.399	50	99.8	0.512	0.100
	Roseires	Nov	1	0.634	0.118	29.037	50	100.0	0.555	0.087
	Malakal	May	7	-0.311	0.132	5.577	50	97.8	-0.140	0.107

Table 7.2c (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Tamaniat

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
April (Contd)	Malakal	Jan	3	0.438	0.090	23.743	52	100.0	0.356	0.069
	Roseires	Jan	3	0.868	0.199	18.967	52	100.0	0.836	0.183
	Mongalla	Dec	4	0.125	0.050	6.128	52	98.3	0.127	0.040
	Tamaniat	Nov	5	-0.118	0.033	12.545	52	99.9	-0.128	0.026
	Tamaniat	Sep	6	-0.031	0.014	5.000	52	97.0	-0.016	0.011
	Constant			623.700	230.965	7.292	52	99.1	402.007	186.272
May	Tamaniat	Apr	1	0.584	0.102	32.974	55	100.0	0.603	0.079
	Malakal	Apr	1	1.235	0.144	73.511	55	100.0	1.164	0.105
	Roseires	Apr	1	1.446	0.313	21.413	55	100.0	0.946	0.255
	Tamaniat	Mar	2	-0.271	0.104	6.834	55	98.8	-0.208	0.079
	Roseires	Mar	2	-1.051	0.294	12.807	55	99.9	-0.649	0.312
	Constant			-293.890	176.268	2.780	55	89.9	-366.595	140.418
June	Tamaniat	May	1	-0.514	0.111	21.281	56	100.0	0.894	0.072
	Khartoum	May	1	-1.424	0.475	8.965	56	99.6	-1.283	0.309
	Sennar	May	1	1.446	0.306	22.384	56	100.0	1.231	0.230
	Tamaniat	Apr	2	-0.514	0.111	21.281	56	100.0	-0.316	0.073
	Constant			1699.484	178.462	90.687	56	100.0	1511.771	133.805
July	Roseires	Jun	1	1.883	0.290	42.144	58	100.0	1.263	0.197
	Sennar	Sep	10	0.114	0.054	4.505	58	96.2	0.028	0.029
	Constant			2156.098	818.511	6.939	58	98.9	4335.810	510.691
August	Sennar	Jul	1	1.274	0.168	57.735	58	100.0	1.121	0.113

Table 7.2c (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Tamaniat

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
December (contd)	Tamaniat	Feb	10	-0.281	0.067	17.658	50	100.0	-0.303	0.052
	Roseires	Feb	10	-0.318	0.132	5.764	50	98.0	-0.361	0.316
	Tamaniat	Dec	12	0.581	0.072	64.827	50	100.0	0.474	0.056
	Roseires	Dec	12	-0.463	0.129	12.842	50	99.9	-0.271	0.115
	Constant			-721.990	201.806	12.800	50	99.9	-633.540	153.777

Table 7.2c (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31;
Monthly Equations for Tamaniat

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Malakal	Dec	1	0.815	0.085	91.170	57	100.0	0.810	0.089
	Mongalla	Oct	3	0.059	0.017	11.897	57	99.9	0.180	0.054
	Roseires	Oct	3	0.168	0.048	12.482	57	99.9	0.049	0.018
	Constant			-1031.643	167.999	37.709	57	100.0	-977.268	173.341
February	Malakal	Jan	1	0.465	0.053	77.440	58	100.0	0.439	0.052
	Mongalla	Oct	4	0.219	0.037	34.839	58	100.0	0.255	0.040
	Constant			-77.772	77.257	1.013	58	68.2	-103.792	74.549
March	Malakal	Feb	1	0.979	0.097	102.142	57	100.0	0.984	0.099
	Malakal	Jan	2	-0.258	0.060	18.796	57	100.0	-0.261	0.057
	Mongalla	Oct	5	0.094	0.035	7.460	57	99.2	0.104	0.042
	Constant			284.682	57.476	24.533	57	100.0	259.626	58.720
April	Malakal	Mar	1	0.396	0.037	114.672	57	100.0	0.372	0.031
	Mongalla	Feb	2	0.069	0.025	7.915	57	99.3	0.104	0.026
	Malakal	Oct	6	0.168	0.037	21.007	57	100.0	0.127	0.029
	Constant			122.791	88.591	1.921	57	82.9	235.215	70.571
May	Malakal	Apr	1	0.575	0.074	59.927	58	100.0	0.629	0.049
	Mongalla	May	12	0.093	0.028	11.135	58	99.9	0.064	0.019
	Constant			558.534	69.215	65.118	58	100.0	550.129	48.598
June	Malakal	May	1	0.481	0.077	38.607	53	100.0	0.402	0.067
	Roseires	May	1	0.183	0.040	21.038	53	100.0	0.185	0.034
	Malakal	Feb	2	0.093	0.034	7.456	53	99.1	0.107	0.026

Table 7.2d
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations of Malakal

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
June (contd)	Roseires	Oct	8	0.018	0.006	8.899	53	99.6	0.022	0.005
	Malakal	Sep	9	-0.128	0.051	6.407	53	98.6	-0.097	0.040
	Malakal	Jul	11	0.409	0.071	33.595	53	100.0	0.389	0.056
	Roseires	Jun	12	-0.087	0.024	12.786	53	99.9	-0.101	0.018
	Constant			344.228	122.932	7.841	53	99.3	395.520	93.498
July	Malakal	Jun	1	0.854	0.042	422.147	56	100.0	0.832	0.037
	Roseires	May	2	-0.059	0.025	5.477	56	97.7	-0.063	0.022
	Malakal	Oct	9	0.113	0.021	29.892	56	100.0	0.113	0.018
	Roseires	Jul	12	-0.012	0.004	8.199	56	99.4	-0.011	0.004
	Constant			510.338	51.984	96.379	56	100.0	546.762	45.428
August	Malakal	Jul	1	1.150	0.213	29.244	54	100.0	0.989	0.219
	Mongalla	Jul	1	0.112	0.031	12.721	54	99.9	0.100	0.031
	Malakal	Jun	2	-0.464	0.172	7.280	54	99.1	-0.279	0.173
	Malakal	Apr	4	0.360	0.089	16.358	54	100.0	0.373	0.093
	Roseires	Mar	5	-0.174	0.065	7.100	54	99.0	-0.087	0.046
	Mongalla	Mar	5	-0.096	0.047	4.182	54	95.4	-0.288	0.102
	Constant			362.770	205.575	3.114	54	91.7	427.931	210.971
September	Malakal	Aug	1	1.761	0.089	392.427	56	100.0	1.677	0.075
	Mongalla	Aug	1	0.095	0.019	26.168	56	100.0	0.101	0.017
	Malakal	Jul	2	-0.708	0.114	38.302	56	100.0	-0.692	0.095
	Mongalla	Mar	6	-0.095	0.029	10.938	56	99.8	-0.074	0.024

Table 7.2d (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations of Malakal

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
September	Constant			-296.655	142.939	4.307	56	95.7	-154.923	117.537
October	Malakal	Sep	1	1.640	0.155	112.454	56	100.0	1.551	0.111
	Malakal	Aug	2	-0.807	0.185	19.044	56	100.0	-0.739	0.134
	Mongalla	Aug	2	0.089	0.027	10.734	56	99.8	0.085	0.022
	Roseires	Aug	2	-0.016	0.006	8.870	56	99.6	-0.003	0.005
	Constant			650.723	206.444	9.935	56	99.7	539.796	156.355
November	Malakal	Oct	1	0.499	0.091	30.083	55	100.0	0.824	0.143
	Malakal	Sep	2	0.578	0.126	21.049	55	100.0	0.168	0.180
	Roseires	Sep	2	0.039	0.006	44.365	55	100.0	0.026	0.006
	Mongalla	Apr	7	0.169	0.032	27.846	55	100.0	0.162	0.033
	Malakal	Mar	8	-0.213	0.047	20.624	55	100.0	-0.187	0.047
	Constant			-681.872	126.657	28.983	55	100.0	-380.022	127.697
December	Malakal	Nov	1	0.957	0.073	171.580	56	100.0	0.984	0.071
	Roseires	Nov	1	0.165	0.040	17.133	56	100.0	0.168	0.039
	Roseires	Sep	3	0.037	0.010	14.229	56	100.0	0.041	0.011
	Mongalla	Aug	4	0.088	0.038	5.419	56	97.6	0.047	0.041
	Constant			-1196.090	152.166	61.786	56	100.0	-1227.089	159.324

Table 7.2d (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31;
Monthly Equations of Malakal

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Mongalla	Dec	1	0.589	0.045	170.320	66	100.0	0.685	0.049
	Mongalla	Oct	3	0.158	0.035	20.664	66	100.0	0.125	0.033
	Mongalla	Jul	6	0.185	0.045	16.741	66	100.0	0.130	0.040
	Constant			-165.771	49.181	11.361	66	99.9	-147.707	43.865
February	Mongalla	Jan	1	0.917	0.013	4753.672	68	100.0	0.881	0.010
	Constant			-101.910	34.421	8.766	68	99.6	-45.533	23.635
March	Mongalla	Feb	1	0.302	0.132	5.219	66	97.4	1.476	0.247
	Mongalla	Jan	2	0.784	0.137	32.979	66	100.0	-0.255	0.230
	Mongalla	Jul	8	-0.136	0.045	9.291	66	99.7	-0.097	0.040
	Constant			65.955	45.078	2.141	66	85.2	45.164	38.441
April	Mongalla	Mar	1	1.082	0.054	409.038	67	100.0	1.192	0.059
	Mongalla	Oct	6	-0.102	0.041	6.257	67	98.5	-0.159	0.042
	Constant			182.384	50.227	13.186	67	99.9	143.203	50.503
May	Mongalla	Apr	1	1.515	0.155	95.760	67	100.0	1.505	0.149
	Mongalla	Jan	4	-0.436	0.142	9.419	67	99.7	-0.422	0.140
	Constant			311.125	84.318	13.615	67	100.0	320.282	83.589
June	Mongalla	May	1	0.786	0.068	135.686	67	100.0	0.793	0.079
	Mongalla	Dec	6	0.157	0.063	6.275	67	98.5	0.148	0.080
	Constant			55.237	83.011	0.443	67	49.2	58.562	94.063
July	Mongalla	Jun	1	0.831	0.075	124.100	66	100.0	0.771	0.080
	Mongalla	Feb	5	-0.409	0.176	5.435	66	97.7	-0.386	0.287

Table 7.2e
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Mongalla

Model of	Variable	Lag		OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
July (contd)	Mongalla	Dec	7	0.500	0.148	11.445	66	99.9	0.540	0.230
	Constant			217.124	89.410	5.897	66	98.2	225.946	96.962
August	Mongalla	Jul	1	1.327	0.123	115.790	67	100.0	1.313	0.109
	Mongalla	May	3	-0.317	0.122	6.767	67	98.9	-0.341	0.106
	Constant			342.159	122.010	7.864	67	99.3	422.340	109.841
September	Mongalla	Aug	1	1.059	0.046	541.912	68	100.0	1.027	0.043
	Constant			-201.713	154.357	1.708	68	80.4	-130.044	139.379
October	Mongalla	Sep	1	0.929	0.045	435.776	67	100.0	0.977	0.045
	Mongalla	Oct	12	0.105	0.043	5.907	67	98.2	0.061	0.042
	Constant			-168.208	117.639	2.045	67	84.3	-122.296	116.513
November	Mongalla	Oct	1	0.649	0.053	148.115	67	100.0	0.667	0.044
	Mongalla	Mar	8	0.312	0.071	19.247	67	100.0	0.278	0.060
	Constant			119.834	113.764	1.110	67	70.4	93.860	99.404
December	Mongalla	Nov	1	0.758	0.044	295.871	67	100.0	0.773	0.030
	Mongalla	Jun	6	0.254	0.050	25.301	67	100.0	0.205	0.034
	Constant			-170.645	70.027	5.938	67	98.3	-109.021	49.561

Table 7.2e (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31;
Monthly Equations for Mongalia

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Khartoum	Dec	1	0.518	0.040	165.737	55	100.0	0.484	0.029
	Roseires	Oct	3	-0.017	0.008	5.199	55	97.4	-0.004	0.005
	Sennar	Aug	5	0.018	0.006	9.909	55	99.7	0.010	0.004
	Sennar	Apr	9	-0.246	0.082	8.877	55	99.6	-0.103	0.055
	Roseires	Feb	11	0.258	0.064	16.405	55	100.0	0.034	0.103
	Constant			-132.643	87.061	2.321	55	86.7	1.494	73.606
February	Sennar	Jan	1	0.441	0.076	33.362	54	100.0	0.300	0.067
	Khartoum	Dec	2	0.279	0.080	12.097	54	99.9	0.214	0.056
	Sennar	Dec	2	-0.323	0.086	14.049	54	100.0	-0.157	0.066
	Roseires	Oct	4	0.011	0.005	4.827	54	96.8	0.004	0.003
	Sennar	Aug	6	0.025	0.005	27.456	54	100.0	0.021	0.007
	Roseires	Aug	6	-0.014	0.003	17.721	54	100.0	-0.011	0.006
	Constant			-57.130	52.175	1.199	54	72.2	-26.478	39.016
March	Khartoum	Feb	1	0.959	0.127	57.475	55	100.0	0.856	0.164
	Khartoum	Jan	2	-0.238	0.082	8.314	55	99.4	-0.159	0.077
	Khartoum	Oct	5	0.021	0.006	10.857	55	99.8	0.011	0.006
	Sennar	Sept	6	-0.020	0.007	8.574	55	99.5	-0.003	0.006
	Sennar	Jul	8	0.018	0.008	4.814	55	96.8	0.004	0.006
	Constant			120.506	56.067	4.620	55	96.4	67.116	55.059
April	Khartoum	Mar	1	0.753	0.144	27.207	56	100.0	0.895	0.105
	Roseires	Mar	1	0.234	0.094	6.257	56	98.5	0.164	0.094

Table 7.2f

Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Khartoum

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
April (contd)	Sennar	Feb	2	-0.461	0.139	10.977	56	99.8	-0.442	0.095
	Roseires	Jan	3	0.225	0.084	7.154	56	99.0	0.119	0.058
	Constant			-23.559	53.296	0.195	56	34.0	25.682	38.140
May	Sennar	Apr	1	0.855	0.157	29.777	58	100.0	0.597	0.089
	Roseires	Mar	2	-0.272	0.112	5.907	58	98.2	-0.171	0.096
	Constant			288.311	56.640	25.911	58	100.0	340.202	40.852
June	Sennar	May	1	1.055	0.153	47.284	59	100.0	0.783	0.134
	Constant			553.047	98.782	31.345	59	100.0	721.025	95.158
July	Khartoum	Jun	1	2.122	0.363	34.191	59	100.0	1.269	0.227
	Constant			2923.787	453.201	41.621	59	100.0	3998.384	328.107
August	Sennar	Jul	1	1.533	0.154	99.150	57	100.0	1.278	0.122
	Sennar	Jun	1	-1.421	0.451	9.940	57	99.7	-0.832	0.252
	Roseires	Aug	12	-0.183	0.060	9.329	57	99.7	-0.085	0.047
	Constant			11292.589	1107.451	103.977	57	100.0	10622.106	1013.346
September	Khartoum	Aug	1	0.571	0.121	22.168	56	100.0	0.419	0.101
	Roseires	Aug	1	0.253	0.094	7.186	56	99.0	0.189	0.104
	Roseires	Apr	5	-4.496	1.327	11.490	56	99.9	-1.405	0.674
	Khartoum	Dec	9	2.237	0.519	18.563	56	100.0	0.779	0.327
	Constant			87.506	1582.551	0.003	56	4.4	4442.117	1117.669
October	Sennar	Sep	1	0.753	0.086	76.189	59	100.0	0.786	0.070
	Constant			-1623.928	1098.143	2.187	59	85.5	-2042.153	935.131

Table 7.2f (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Khartoum

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
November	Sennar	Oct	1	0.516	0.060	73.576	58	100.0	0.449	0.039
	Roseires	Oct	1	-0.186	0.066	7.858	58	99.3	-0.137	0.044
	Constant			426.780	192.618	4.909	58	96.9	513.356	172.258
December	Sennar	Nov	1	0.364	0.030	146.432	57	100.0	0.423	0.024
	Sennar	Sep	2	0.027	0.011	6.180	57	98.4	0.011	0.008
	Khartoum	Dec	12	0.121	0.053	5.153	57	97.3	0.082	0.034
	Constant			-24.759	112.731	0.048	57	17.3	92.656	102.461

Table 7.2f (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31;
Monthly Equations for Khartoum

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Sennar	Dec	1	0.551	0.030	328.874	58	100.0	0.529	0.024
	Roseires	Feb	11	0.350	0.059	34.973	58	100.0	0.098	0.110
	Constant			-175.645	58.576	8.992	58	99.6	-29.981	63.854
February	Sennar	Jan	1	0.411	0.048	72.928	56	100.0	0.383	0.032
	Sennar	Aug	6	0.028	0.006	21.705	56	100.0	0.020	0.008
	Roseires	Aug	6	-0.013	0.004	8.419	56	99.5	-0.005	0.007
	Roseires	Feb	12	0.116	0.053	4.872	56	96.9	0.007	0.062
	Constant			-129.969	69.278	3.520	56	93.4	-66.228	54.444
March	Sennar	Feb	1	1.268	0.178	50.645	55	100.0	0.903	0.139
	Roseires	Feb	1	-0.437	0.119	13.563	55	99.9	-0.417	0.148
	Sennar	Oct	5	-0.073	0.022	11.262	55	99.9	-0.031	0.016
	Roseires	Oct	5	0.096	0.020	23.039	55	100.0	0.050	0.016
	Sennar	Sep	6	-0.022	0.007	9.968	55	99.7	-0.002	0.007
	Constant			166.240	70.846	5.506	55	97.7	91.951	63.111
April	Sennar	Mar	1	0.637	0.095	45.000	58	100.0	0.630	0.073
	Roseires	Jul	9	0.025	0.010	6.510	58	98.7	0.012	0.006
	Constant			-43.467	63.456	0.469	58	50.4	47.988	42.426
May	Sennar	Apr	1	1.665	0.343	23.517	58	100.0	0.823	0.177
	Roseires	Apr	1	-0.979	0.306	10.255	58	99.8	-0.425	0.167
	Constant			306.574	80.066	14.661	58	100.0	437.833	53.364
June	Sennar	May	1	1.262	0.178	50.014	59	100.0	0.877	0.160

Table 7.2g
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Sennar

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
June	Constant			683.296	114.882	35.376	59	100.0	917.629	113.564
July	Roseires	Jun	1	1.896	0.336	31.794	59	100.0	1.137	0.221
	Constant			2895.110	578.267	25.065	59	100.0	4247.254	407.480
August	Sennar	Jul	1	1.461	0.163	80.402	57	100.0	1.152	0.118
	Roseires	Jun	2	-1.461	0.515	8.041	57	99.4	-0.600	0.285
	Roseires	Aug	12	-0.215	0.060	13.023	57	99.9	-0.040	0.046
	Constant			12096.317	1140.771	112.437	57	100.0	9867.280	959.867
September	Roseires	Aug	1	0.351	0.075	22.021	56	100.0	0.322	0.064
	Sennar	Jul	2	0.585	0.169	11.955	56	99.9	0.301	0.081
	Sennar	Apr	5	-3.519	1.457	5.830	56	98.1	-1.308	0.613
	Sennar	Dec	9	2.023	0.547	13.707	56	100.0	0.840	0.273
	Constant			2508.847	1243.511	4.071	56	95.2	5455.730	947.347
October	Sennar	Sep	1	0.651	0.080	65.858	59	100.0	0.648	0.066
	Constant			-1586.124	1020.887	2.414	59	97.4	-1505.889	888.085
November	Sennar	Oct	1	0.592	0.056	104.486	58	100.0	0.526	0.034
	Roseires	Oct	1	-0.277	0.062	20.119	58	100.0	-0.245	0.039
	Constant			402.900	179.209	5.054	58	97.2	464.171	158.574
December	Sennar	Nov	1	0.370	0.025	217.420	57	100.0	0.405	0.019
	Roseires	Sep	3	0.034	0.009	13.916	57	100.0	0.021	0.006
	Roseires	May	7	0.131	0.065	4.020	57	95.0	0.055	0.037
	Constant			-145.668	101.698	2.052	57	84.2	3.229	83.675

Table 7.2g (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Sennar

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
January	Roseires	Dec	1	0.503	0.031	255.203	58	100.0	0.477	0.026
	Roseires	Feb	11	0.620	0.530	136.319	58	100.0	0.202	0.107
	Constant			-188.823	54.858	11.848	58	99.9	38.232	62.173
February	Roseires	Jan	1	0.453	0.066	46.752	55	100.0	0.369	0.054
	Roseires	Oct	4	0.019	0.006	8.402	55	99.5	0.009	0.005
	Roseires	Aug	6	-0.008	0.004	4.294	55	95.7	0.006	0.004
	Roseires	Mar	11	0.249	0.087	8.238	55	99.4	-0.039	0.079
	Roseires	Feb	12	0.243	0.093	6.830	55	98.8	0.177	0.104
	Constant			-109.608	74.897	2.142	55	85.1	-52.558	68.753
March	Roseires	Feb	1	0.836	0.081	105.469	59	100.0	0.771	0.123
	Constant			-6.467	48.069	0.031	59	13.9	19.045	60.873
April	Roseires	Mar	1	0.571	0.065	76.694	58	100.0	0.550	0.074
	Roseires	Oct	6	0.019	0.007	7.881	58	99.5	0.009	0.004
	Constant			-11.563	49.710	0.054	58	18.3	67.056	35.176
May	Constant			606.938	*	193.114	60	100.0	620.800	47.570
June	Roseires	May	1	0.897	0.168	28.375	59	100.0	0.529	0.141
	Constant			1090.548	116.973	86.919	59	100.0	1315.851	107.096
July	Roseires	Jun	1	1.908	0.329	33.543	59	100.0	1.180	0.215
	Constant			3438.942	566.341	36.872	59	100.0	4682.497	399.717
August	Roseires	Jul	1	1.053	0.219	23.133	58	100.0	0.971	0.142
	Roseires	Mar	5	-6.998	1.499	21.788	58	100.0	-0.248	1.192

Table 7.2h
Results of OLS and GLS Estimation of Multivariate Model 5.31; Monthly Equations for Roseires

Model of	Variable		Lag	OLS Estimated Coefficient	OLS Standard Error	Partial F Statistic	Degree of Freedom	Significance Level	GLS Estimated Coefficient	GLS Standard Error
August	Constant			11265.273	1644.560	46.923	58	100.0	9277.227	1084.062
September	Roseires	Aug	1	0.404	0.078	26.802	58	100.0	0.419	0.076
	Roseires	Jul	2	0.489	0.175	7.852	58	99.3	0.200	0.109
	Constant			3278.444	1204.629	7.407	58	99.1	5105.776	979.033
October	Roseires	Sep	1	0.492	0.086	32.467	59	100.0	0.535	0.064
	Constant			682.742	1112.915	0.376	59	45.8	107.520	870.687
November	Roseires	Oct	1	0.265	0.022	148.238	59	100.0	0.252	0.019
	Constant			868.305	158.045	30.185	59	100.0	959.844	144.540
December	Roseires	Nov	1	0.405	0.042	91.300	57	100.0	0.462	0.035
	Roseires	May	7	0.204	0.092	4.850	57	96.8	0.127	0.060
	Roseires	Dec	12	0.244	0.079	9.665	57	99.7	0.181	0.051
	Constant			-102.854	181.919	0.320	57	42.6	-133.121	126.329

Table 7.2h (cont'd)
Results of OLS and GLS Estimation of Multivariate Model 5.31;
Monthly Equations for Roseires

WADI HALFA	1.000	0.047	-0.155	0.127	-0.181	0.205	0.124	0.142
ATBARA	0.047	1.000	-0.138	0.133	-0.048	0.060	-0.033	-0.020
TAMANIAT	-0.155	-0.138	1.000	0.167	0.053	0.211	0.196	0.119
MALAKAL	0.127	0.133	0.167	1.000	0.178	0.004	0.001	-0.048
MONGALLA	-0.181	-0.048	0.053	0.178	1.000	-0.164	-0.090	0.008
KHARTOUM	0.205	0.060	0.211	0.004	-0.164	1.000	0.670	0.433
SENNAR	0.124	-0.033	0.196	0.001	-0.090	0.670	1.000	0.710
ROSEIRES	0.142	-0.020	0.119	-0.048	0.008	0.433	0.710	1.000

TABLE 7.3a: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED JANUARY MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.074	0.477	0.030	0.199	0.222	0.255	0.139
ATBARA	0.074	1.000	0.007	0.150	-0.228	-0.293	-0.378	-0.067
TAMANIAT	0.477	0.007	1.000	0.188	0.015	0.283	0.266	0.214
MALAKAL	0.030	0.150	0.188	1.000	-0.217	0.153	0.092	0.167
MONGALLA	0.199	-0.228	0.015	-0.217	1.000	0.451	0.219	0.157
KHARTOUM	0.222	-0.293	0.283	0.153	0.451	1.000	0.522	0.322
SENNAR	0.255	-0.378	0.266	0.092	0.219	0.522	1.000	0.528
ROSEIRES	0.139	-0.067	0.214	0.167	0.157	0.322	0.528	1.000

TABLE 7.3b: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED FEBRUARY MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	-0.118	0.409	0.415	0.014	0.115	0.191	0.080
ATBARA	-0.118	1.000	-0.170	0.191	0.055	0.144	0.195	0.438
TAMANIAT	0.409	-0.170	1.000	0.159	0.024	-0.006	0.064	-0.047
MALAKAL	0.415	0.191	0.159	1.000	-0.051	0.176	0.214	0.169
MONGALLA	0.014	0.055	0.024	-0.051	1.000	0.077	0.073	0.138
KHARTOUM	0.115	0.144	-0.006	0.176	0.077	1.000	0.729	0.494
SENNAR	0.191	0.195	0.064	0.214	0.073	0.729	1.000	0.521
ROSEIRES	0.080	0.438	-0.047	0.169	0.138	0.494	0.521	1.000

TABLE 7.3c: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED MARCH MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.141	0.336	-0.118	-0.190	0.095	0.066	0.060
ATBARA	0.141	1.000	0.526	0.026	-0.004	-0.034	0.054	0.029
TAMANIAT	0.336	0.526	1.000	0.015	0.004	0.262	0.206	0.257
MALAKAL	-0.118	0.026	0.015	1.000	0.293	0.432	0.619	0.624
MONGALLA	-0.190	-0.004	0.004	0.293	1.000	0.253	0.269	0.372
KHARTOUM	0.095	-0.034	0.262	0.432	0.253	1.000	0.746	0.766
SENNAR	0.066	0.054	0.206	0.619	0.269	0.746	1.000	0.805
ROSEIRES	0.060	0.029	0.257	0.624	0.372	0.766	0.805	1.000

TABLE 7.3d: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED APRIL MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.005	0.471	0.354	0.061	0.255	0.256	0.209
ATBARA	0.005	1.000	0.027	-0.162	0.169	0.093	0.214	0.083
TAMANIAT	0.471	0.027	1.000	0.583	0.245	0.647	0.514	0.502
MALAKAL	0.354	-0.162	0.583	1.000	0.346	0.702	0.571	0.564
MONGALLA	0.061	0.169	0.245	0.346	1.000	0.526	0.510	0.485
KHARTOUM	0.255	0.093	0.647	0.702	0.526	1.000	0.874	0.784
SENNAR	0.256	0.214	0.514	0.571	0.510	0.874	1.000	0.833
ROSEIRES	0.209	0.083	0.502	0.564	0.485	0.784	0.833	1.000

TABLE 7.3e: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED MAY MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.228	0.579	0.402	0.266	0.325	0.244	0.224
ATBARA	0.228	1.000	0.436	0.169	0.098	0.402	0.393	0.372
TAMANIAT	0.579	0.436	1.000	0.426	0.282	0.761	0.665	0.673
MALAKAL	0.402	0.169	0.426	1.000	0.164	0.416	0.484	0.426
MONGALLA	0.266	0.098	0.282	0.164	1.000	0.214	0.160	0.104
KHARTOUM	0.325	0.402	0.761	0.416	0.214	1.000	0.840	0.810
SENNAR	0.244	0.393	0.665	0.484	0.160	0.840	1.000	0.847
ROSEIRES	0.224	0.372	0.673	0.426	0.104	0.810	0.847	1.000

TABLE 7.3f: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED JUNE MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.423	0.515	0.240	0.190	0.486	0.508	0.533
ATBARA	0.423	1.000	0.541	0.396	0.189	0.601	0.478	0.516
TAMANIAT	0.515	0.541	1.000	0.422	0.277	0.845	0.865	0.861
MALAKAL	0.240	0.396	0.422	1.000	0.188	0.308	0.235	0.242
MONGALLA	0.190	0.189	0.277	0.188	1.000	0.220	0.260	0.310
KHARTOUM	0.486	0.601	0.845	0.308	0.220	1.000	0.908	0.915
SENNAR	0.508	0.478	0.865	0.235	0.260	0.908	1.000	0.956
ROSEIRES	0.533	0.516	0.861	0.242	0.310	0.915	0.956	1.000

TABLE 7.3g: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED JULY MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.548	0.670	0.217	0.266	0.636	0.616	0.643
ATBARA	0.548	1.000	0.429	0.211	0.320	0.404	0.533	0.526
TAMANIAT	0.670	0.429	1.000	0.189	0.385	0.725	0.726	0.716
MALAKAL	0.217	0.211	0.189	1.000	0.309	0.096	0.069	0.123
MONGALLA	0.266	0.320	0.385	0.309	1.000	0.254	0.354	0.426
KHARTOUM	0.636	0.404	0.725	0.096	0.254	1.000	0.807	0.740
SENNAR	0.616	0.533	0.726	0.069	0.354	0.807	1.000	0.744
ROSEIRES	0.643	0.526	0.716	0.123	0.426	0.740	0.744	1.000

TABLE 7.3h: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED AUGUST MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.672	0.792	-0.065	0.216	0.705	0.711	0.720
ATBARA	0.672	1.000	0.604	-0.095	0.490	0.671	0.725	0.723
TAMANIAT	0.792	0.604	1.000	0.152	0.419	0.847	0.800	0.738
MALAKAL	-0.065	-0.095	0.152	1.000	0.432	0.147	0.159	0.052
MONGALLA	0.316	0.490	0.419	0.432	1.000	0.494	0.492	0.516
KHARTOUM	0.705	0.671	0.847	0.147	0.494	1.000	0.883	0.791
SENNAR	0.711	0.725	0.800	0.159	0.492	0.883	1.000	0.841
ROSEIRES	0.720	0.723	0.738	0.052	0.516	0.791	0.841	1.000

TABLE 7.3i: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED SEPTEMBERMULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.475	0.801	0.015	0.163	0.720	0.696	0.613
ATBARA	0.475	1.000	0.454	-0.094	-0.176	0.579	0.581	0.641
TAMANIAT	0.801	0.454	1.000	0.052	0.235	0.913	0.881	0.819
MALAKAL	0.015	-0.094	0.052	1.000	0.238	-0.094	-0.098	-0.069
MONGALLA	0.163	-0.176	0.235	0.238	1.000	0.175	0.177	0.148
KHARTOUM	0.720	0.579	0.913	-0.094	0.175	1.000	0.963	0.932
SENNAR	0.696	0.581	0.881	-0.098	0.177	0.963	1.000	0.935
RUSEIRES	0.613	0.641	0.819	-0.069	0.148	0.932	0.935	1.000

TABLE 7.3J: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED OCTOBER MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.679	0.593	0.327	-0.104	0.581	0.460	0.564
ATBARA	0.679	1.000	0.583	0.224	0.089	0.607	0.521	0.474
TAMANIAT	0.593	0.583	1.000	0.335	0.070	0.822	0.775	0.598
MALAKAL	0.327	0.224	0.335	1.000	-0.218	0.155	0.119	0.285
MONGALLA	-0.104	0.089	0.070	-0.218	1.000	0.317	0.486	0.359
KHARTOUM	0.581	0.607	0.822	0.155	0.317	1.000	0.922	0.784
SENNAR	0.460	0.521	0.775	0.119	0.486	0.922	1.000	0.814
ROSEIRES	0.564	0.474	0.598	0.285	0.359	0.784	0.814	1.000

TABLE 7.3K: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED NOVEMBER MULTIVARIATE MODEL 5.31

WADI HALFA	1.000	0.181	0.035	0.183	0.214	0.301	0.302	0.221
ATBARA	0.181	1.000	0.341	-0.005	0.201	0.254	0.215	0.489
TAMANIAT	0.035	0.341	1.000	-0.004	0.072	0.430	0.364	0.362
MALAKAL	0.183	-0.005	-0.004	1.000	0.078	0.050	-0.094	0.102
MONGALLA	0.214	0.201	0.072	0.078	1.000	0.098	0.056	0.143
KHARTOUM	0.301	0.254	0.430	0.050	0.098	1.000	0.763	0.675
SENNAR	0.302	0.215	0.364	-0.094	0.056	0.763	1.000	0.696
ROSEIRES	0.221	0.489	0.362	0.102	0.143	0.675	0.696	1.000

TABLE 7.3L: RESIDUAL COVARINACE MATRIX OF OLS ESTIMATED DECEMBER MULTIVARIATE MODEL 5.31

7.3.2 Model 5.32

Table 7.4 indicates the final structure identified for univariate Model 5.32. The estimated parameter values and standard errors obtained from Algorithm 5.102 and Equation 5.118a for $\Psi_1^*(i)^2 = 1 \;\; \forall i$ and $\Psi_1^*(i)^2$ from Table 7.5 are given. Notice that $n_2^* = 0$ so that the standard error estimates are theoretically appropriate (see Section 5.7.2.2).

The values $\Psi_1^*(i)^2$ given in Table 7.5 are related by a scale factor to the $\Psi_1^*(i)^2$ calculated as suggested in Section 5.6.4, Equation 5.105. The scale factor was taken as the average $\Psi_1^*(i)^2$ obtained from Equation 5.105,

$$\frac{1}{12} \sum_{i=0}^{11} \Psi_1^*(i)^2 \tag{7.1}$$

Theoretically, the multiplication of the $\Psi_1^*(i)$ function by any scale factor will not affect the final results of the estimation Algorithm 5.102. However, in this particular case, the numerical stability of Algorithm 5.102 required that the $\Psi_1^*(i)^2$ obtained from Equation 5.105 be scaled. No explanation could be found for this observation.

Notice from Table 7.4 that the standard errors of the coefficients increase when using the $\Psi_1^*(i)^2 = 1 \;\; \forall i$. This behavior is the opposite of the expected. Possibly it is due to the large range in the calculated $\Psi^*(.)^2$, 2 orders of magnitude.

Figure 7.2 shows the evolution plots of the coefficients in the final univariate Model 5.32. The smoothness of the plots indicates

Variable	$\Psi^*_1(i)^2=1\ \Psi_i$ Value	Standard Error	$\Psi^*_1(i)^2$ as given in Table 7.5 Value	Standard Error
Wadi Halfa LAG1	0.822	0.032	.749	0.069
Wadi Halfa LAG2	-0.180	0.032	-.226	0.049
Wadi Halfa LAG11	0.077	0.023	.194	0.047
1	2071.894	232.586	2489.285	376.610
$\cos\omega_1 i$	-1869.744	181.206	-1522.555	234.148
$\sin\omega_1 i$	-2561.332	233.910	-3235.809	461.604
$\cos\omega_2 i$	-1516.685	184.039	-1876.501	356.592
$\sin\omega_2 i$	2880.544	128.939	2609.674	241.476
$\cos\omega_3 i$	2663.769	96.622	2666.437	237.997
$\sin\omega_3 i$	631.463	133.336	796.130	256.615
$\cos\omega_4 i$	-123.148	86.059	-32.431	199.682
$\sin\omega_4 i$	-1433.934	81.581	-1599.854	199.035
$\cos\omega_5 i$	-571.341	75.082	-680.462	132.543
$\sin\omega_5 i$	855.911	81.336	852.442	235.542
$\cos\omega_6 i$	472.756	54.731	499.558	134.233

Table 7.4

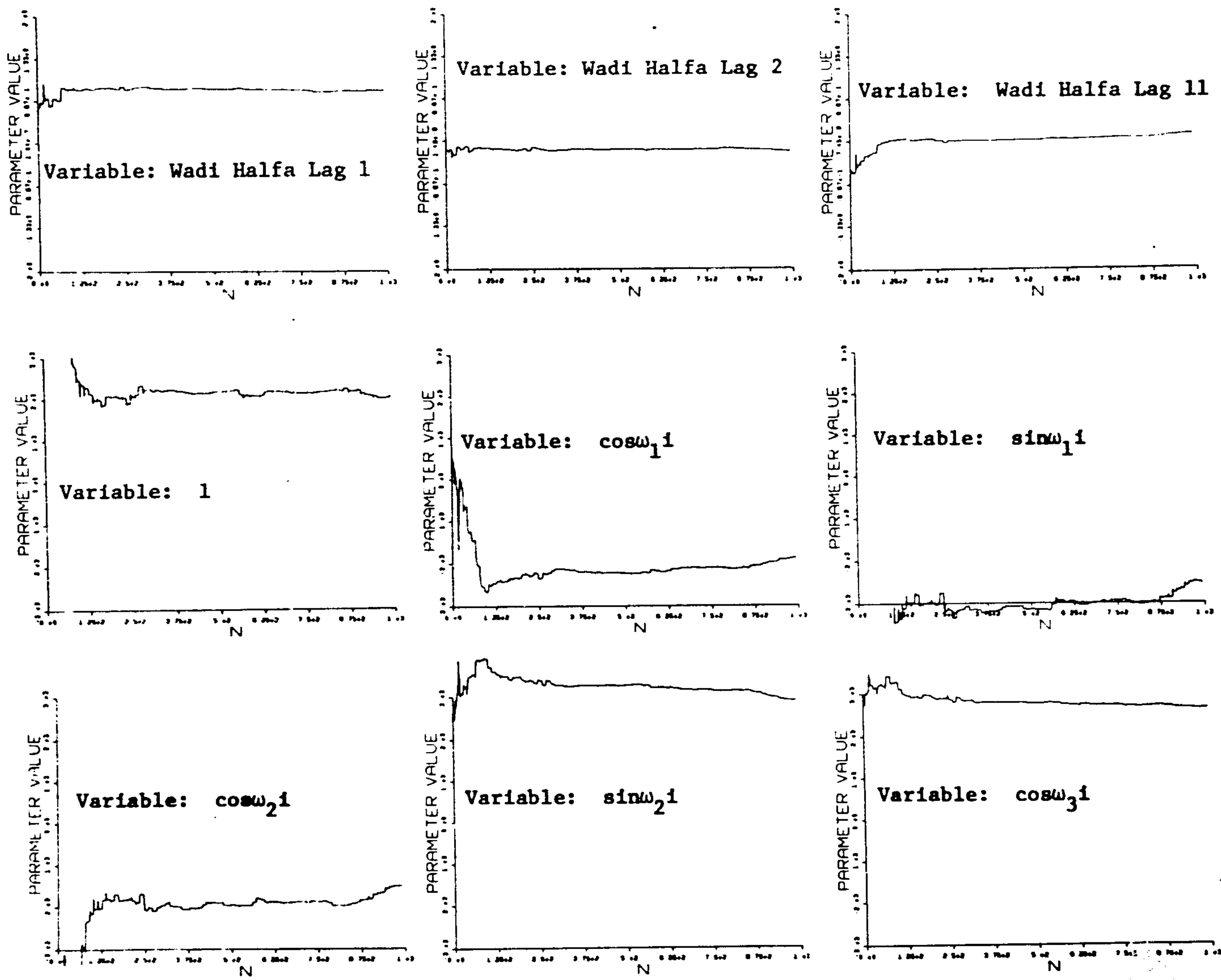
Standard Errors of the Coefficients in the Final Univariate Model 5.32 for Wadi Halfa; Estimated with $\Psi^*_1(i)^2=1\ \Psi_i$ and $\Psi^*_1(i)^2$ as given in Table 7.5

	$\Psi_1^*(i)^2$
i = 0	0.061
= 1	0.044
= 2	0.068
= 3	0.101
= 4	0.055
= 5	0.082
= 6	0.677
= 7	4.512
= 8	3.755
= 9	1.781
=10	0.695
=11	0.169

Table 7.5

$\Psi_1^*(i)^2$ calculated via Equations 5.105a and 7.1 for Univariate Model 5.32

Figure 7.2
Evolution Plots of the Coefficients in the Final Univariate Model 5.32 for Wadi Halfa



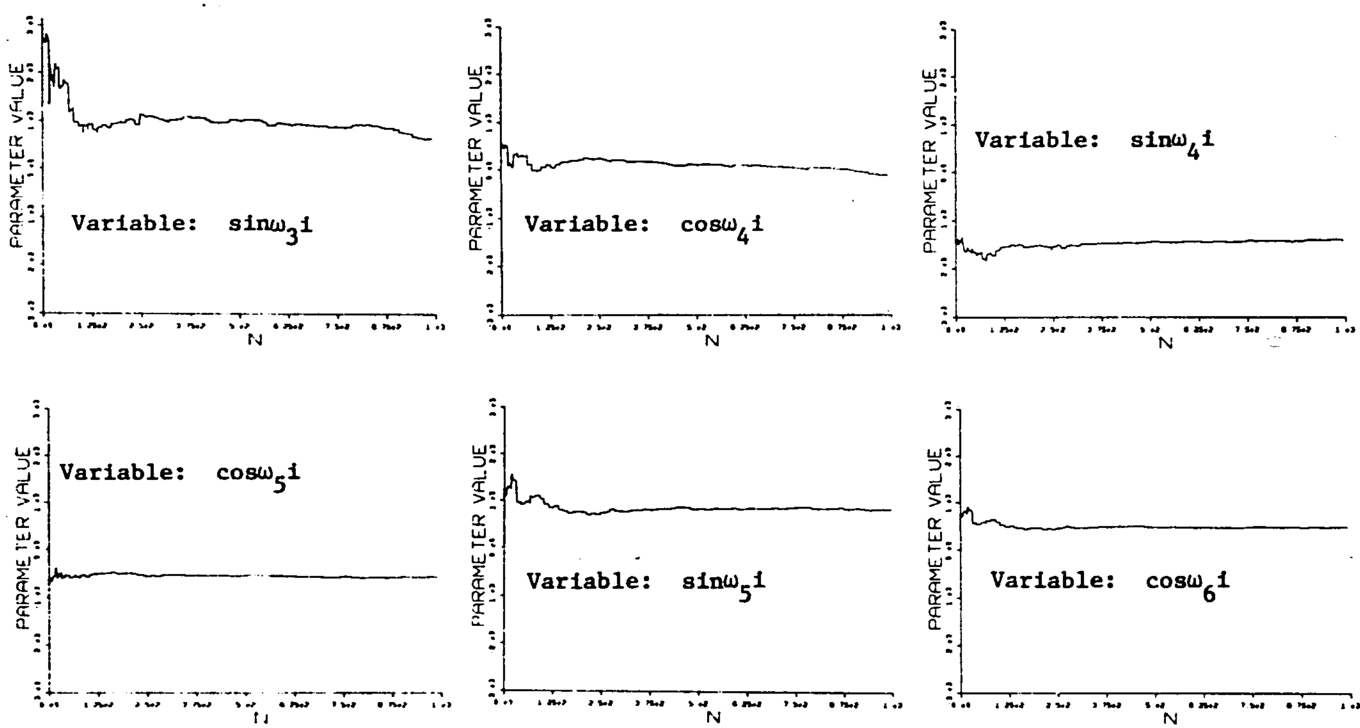


Figure 7.2 (cont'd)
Evolution Plots of the Coefficients in the Final Univariate Model 5.32 for Wadi Halfa

adequacy of the model.

The values of the coefficients and their standard errors in the final multivariate Model 5.32 are given in Table 7.6. The coefficients were estimated using Algorithm 5.102 with $\Psi_j^*(i)^2 = 1 \ \forall i, j$. The coefficients estimated with $\Psi_j^*(i)^2$ obtained via Equations 5.105 and 7.1 had larger standard errors, similar to the univariate case, and thus are not given.

Notice that the equation for Malakal flows include observations of Roseires as an explanatory variable, contrary to the identified structure given in Section 5.5. An explanation is given in Section 7.4.1.

Figure 7.3 shows the evolution plots of the coefficients in the final multivariate equation for Wadi Halfa. Plots for the other stations are not given due to space considerations. The smoothness of the plots does not indicate structural problems.

7.4 Model Adequacy

7.4.1 Residual Analysis

7.4.1.1 Model 5.31

Tables 7.7 through 7.9 report the results of the application of whiteness tests 1 through 3 to the residuals of the final univariate Model 5.31 given in the previous section. The upper (d) and lower (d_L) limits of the Durbin Watson test, test number 1, Table 7.7, are given at the 5% level and were obtained from Johnston (1972). The hypothesis that the residuals are uncorrelated was rejected only for the month of

Model of	Variable		Value	Standard Error
WADI HALFA	WADI HALFA	LAG1	-0.102	0.047
	ATBARA	LAG1	0.946	0.012
	SENNAR	LAG1	0.851	0.062
	MALAKAL	LAG2	0.779	0.140
	MONGALLA	LAG5	-0.111	.078
	ATBARA	LAG10	0.129	.073
	WADI HALFA	LAG11	0.190	0.043
	ATBARA	LAG11	-0.421	.100
	ROSEIRES	LAG12	-0.183	0.046
	1		1685.048	281.768
	$\cos\omega_1 i$		-1450.651	212.453
	$\sin\omega_1 i$		-2389.630	271.395
	$\cos\omega_2 i$		-1407.699	176.971
	$\sin\omega_2 i$		1961.505	163.464
	$\cos\omega_3 i$		2090.654	119.444
	$\sin\omega_3 i$		381.164	114.134
	$\cos\omega_4 i$		-503.493	89.958
	$\sin\omega_4 i$		-1117.911	76.925
	$\cos\omega_5 i$		-550.870	74.754
	$\sin\omega_5 i$		1010.540	69.132
	$\cos\omega_6 i$		530.128	49.220

Table 7.6a

Values and Standard Errors of the Coefficients in the Final Multivariate Model 5.32; Model for Wadihalfa

Model of	Variable		Value	Standard Error
ATBARA	ATBARA	LAG1	.549	0.032
	ATBARA	LAG2	-.115	0.032
	ATBARA	LAG11	-.093	0.028
	1		674.309	48.194
	$\cos\omega_1 i$		-782.651	58.254
	$\sin\omega_1 i$		-998.321	68.204
	$\cos\omega_2 i$		-289.750	47.141
	$\sin\omega_2 i$		1092.863	58.812
	$\cos\omega_3 i$		901.232	36.769
	$\sin\omega_3 i$		-230.119	47.553
	$\cos\omega_4 i$		-381.498	37.682
	$\sin\omega_4 i$		-516.723	30.364
	$\cos\omega_5 i$		-204.300	29.014
	$\sin\omega_5 i$		327.957	21.247
	$\cos\omega_6 i$		142.654	20.625

Table 7.6b

Values and Standard Errors of the Coefficients in the Final Multivariate Model 5.32; Model for Atbara

Model of	Variable		Value	Standard Error
TAMANIAT	MALAKAL	LAG1	0.353	0.145
	MONGALLA	LAG1	0.305	0.093
	KHARTOUM	LAG1	-0.241	0.099
	SENNAR	LAG1	0.994	0.106
	MALAKAL	LAG3	0.475	0.149
	MONGALLA	LAG5	-0.364	0.115
	MALAKAL	LAG6	-0.206	0.117
	TAMANIAT	LAG12	0.385	0.065
	KHARTOUM	LAG12	-0.410	0.069
	1		1374.75	282.493
	$\cos\omega_1 i$		-2539.573	160.527
	$\sin\omega_1 i$		-1982.217	275.011
	$\cos\omega_2 i$		-1178.173	151.751
	$\sin\omega_2 i$		2388.420	107.760
	$\cos\omega_3 i$		1802.707	83.636
	$\sin\omega_3 i$		264.822	80.979
	$\cos\omega_4 1$		-344.473	63.475
	$\sin\omega_4 i$		-708.427	64.859
	$\cos\omega_5 i$		-147.788	59.487
	$\sin\omega_5 i$		798.930	63.401
	$\cos\omega_6 i$		413.210	42.668

Table 7.6c

Values and Standard Errors of the Coefficients in the Final Multivariate Model 5.32; Model for Tamaniat

Model of	Variable	Value	Standard Error
MALAKAL	MALAKAL LAG1	0.967	0.036
	MALAKAL LAG2	-0.338	0.051
	MONGALLA LAG2	0.119	0.019
	MALAKAL LAG3	0.169	0.050
	ROSEIRES LAG3	0.029	0.005
	MALAKAL LAG4	-0.106	0.035
	MONGALLA LAG4	0.053	0.024
	ROSEIRES LAG5	0.009	.006
	MONGALLA LAG6	-0.113	0.029
	MONGALLA LAG7	0.081	0.027
	ROSEIRES LAG7	-0.015	0.006
	1	299.063	39.716
	$\cos\omega_1 i$	-371.006	39.377
	$\sin\omega_1 i$	-230.562	43.136
	$\cos\omega_2 i$	-114.560	24.791
	$\sin\omega_2 i$	-93.892	27.931
	$\cos\omega_3 i$	-2.300	13.954
	$\sin\omega_3 i$	-73.131	18.247
	$\cos\omega_4 i$	89.043	11.774
	$\sin\omega_4 i$	24.912	11.979
	$\cos\omega_5 i$	99.146	12.606
	$\sin\omega_5 i$	86.575	11.237
	$\cos\omega_6 i$	12.880	7.145

Table 7.6d

Values and Standard Errors of the Coefficients in the Final Multivariate Model 5.32; Model for Malakal

Model of	Variable	Value	Standard Error
MONGALLA	MONGALLA LAG1	0.900	0.020
	MONGALLA LAG5	0.073	0.021
	1	71.262	27.093
	$\cos\omega_1 i$	-256.602	14.749
	$\sin\omega_1 i$	-103.739	22.101
	$\cos\omega_2 i$	-54.508	14.410
	$\sin\omega_2 i$	-10.419	14.318
	$\cos\omega_3 i$	112.300	14.293
	$\sin\omega_3 i$	30.405	14.280
	$\cos\omega_4 i$	-58.080	14.303
	$\sin\omega_4 i$	-19.323	14.304
	$\cos\omega_5 i$	88.404	14.311
	$\sin\omega_5 i$	135.191	14.578
	$\cos\omega_6 i$	-32.932	10.095

Table 7.6e

Values and Standard Errors of the Coefficient in the Final Multivariate Model 5.32; Model for Mongalla

Model of	Variable	Value	Standard Error
KHARTOUM	SENNAR LAG1	0.801	0.037
	SENNAR LAG2	-0.121	0.037
	KHARTOUM LAG12	-0.060	0.026
	1	1810.420	171.815
	$\cos\omega_1 i$	-1964.029	146.049
	$\sin\omega_1 i$	-2645.063	233.997
	$\cos\omega_2 i$	-1193.938	150.478
	$\sin\omega_2 i$	2555.630	124.350
	$\cos\omega_3 i$	1941.343	79.364
	$\sin\omega_3 i$	90.380	100.999
	$\cos\omega_4 i$	-476.273	69.824
	$\sin\omega_4 i$	-910.139	61.724
	$\cos\omega_5 i$	-199.412	58.753
	$\sin\omega_5 i$	832.477	64.640
	$\cos\omega_6 i$	464.113	43.696

Table 7.6f

Values and Standard Errors of the Coefficients in the Final Multivariate Model 5.32; Model for Khartoum

Model of	Variable	Value	Standard Error
SENNAR	SENNAR LAG1	0.691	0.036
	SENNAR LAG2	-0.094	0.036
	1	1632.380	129.154
	$\cos\omega_1 i$	-2087.572	135.931
	$\sin\omega_1 i$	-2249.465	165.082
	$\cos\omega_2 i$	-812.628	117.209
	$\sin\omega_2 i$	2566.610	109.635
	$\cos\omega_3 i$	1855.761	66.931
	$\sin\omega_3 i$	-142.580	91.433
	$\cos\omega_4 i$	-534.548	66.765
	$\sin\omega_4 i$	-829.568	56.142
	$\cos\omega_5 i$	-153.240	55.839
	$\sin\omega_5 i$	741.351	61.037
	$\cos\omega_6 i$	391.896	41.376

Table 7.6g

Values and Standard Errors of the Coefficients in the Final Multivariate Model 5.32; Model for Sennar

Model of	Variable	Value	Standard Error
ROSEIRES	ROSEIRES LAG1	.592	0.029
	1	1728.487	129.135
	$\cos\omega_1 i$	-2359.880	79.341
	$\sin\omega_1 i$	-2506.043	176.370
	$\cos\omega_2 i$	-997.019	104.248
	$\sin\omega_2 i$	2717.365	72.308
	$\cos\omega_3 i$	1839.352	62.615
	$\sin\omega_3 i$	-32.518	70.584
	$\cos\omega_4 i$	-482.809	61.127
	$\sin\omega_4 i$	-801.581	58.750
	$\cos\omega_5 i$	-184.500	58.368
	$\sin\omega_5 i$	658.114	59.760
	$\cos\omega_6 i$	322.677	41.684

Table 7.6h

Values and Standard Errors of the Coefficients in the Final Multivariate Model 5.32; Model for Roseires

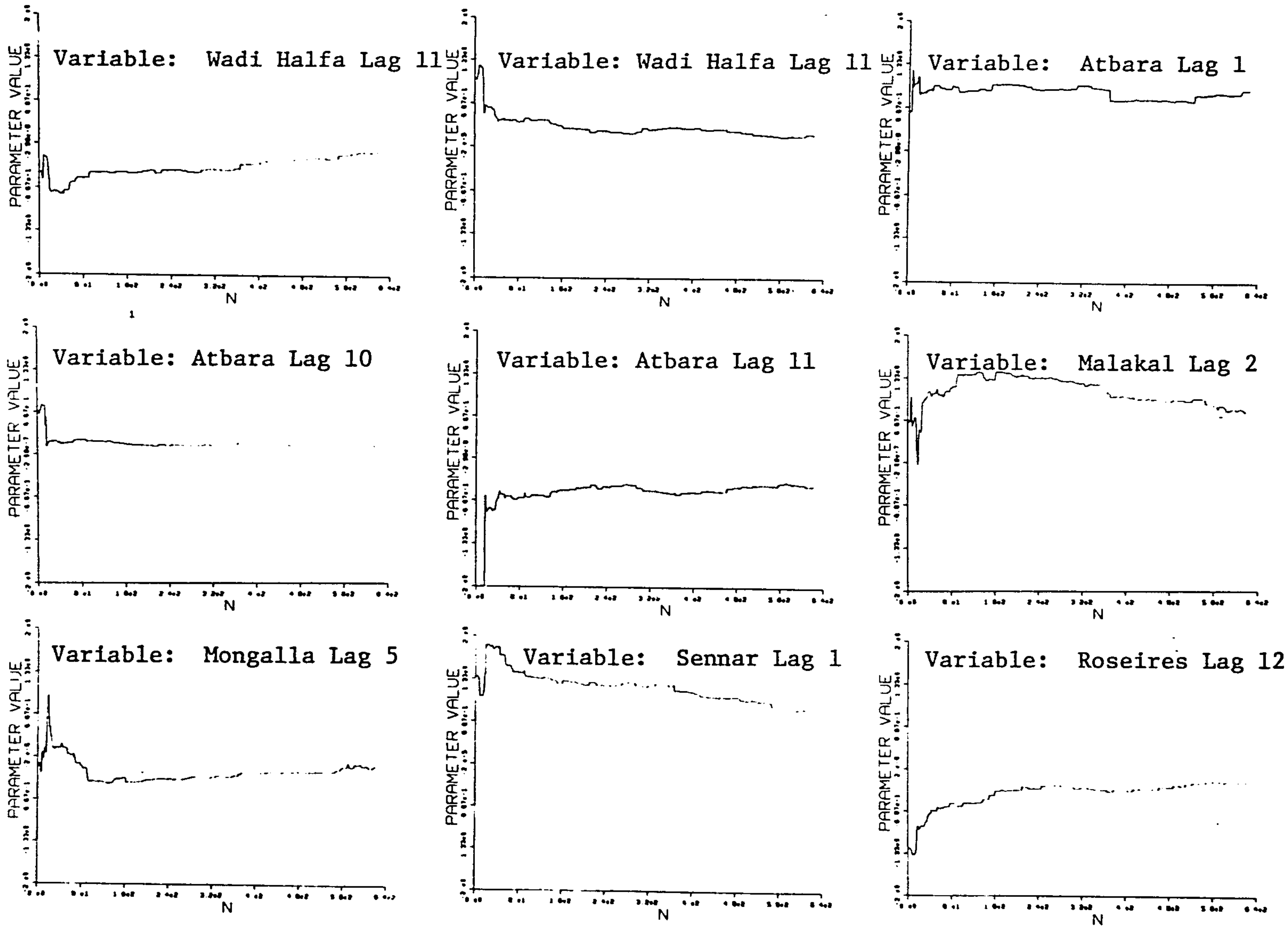


Figure 7.3
Evolution Plots of the Coefficients in the Final Multivariate Model 5.32 (Wadi Halfa only)

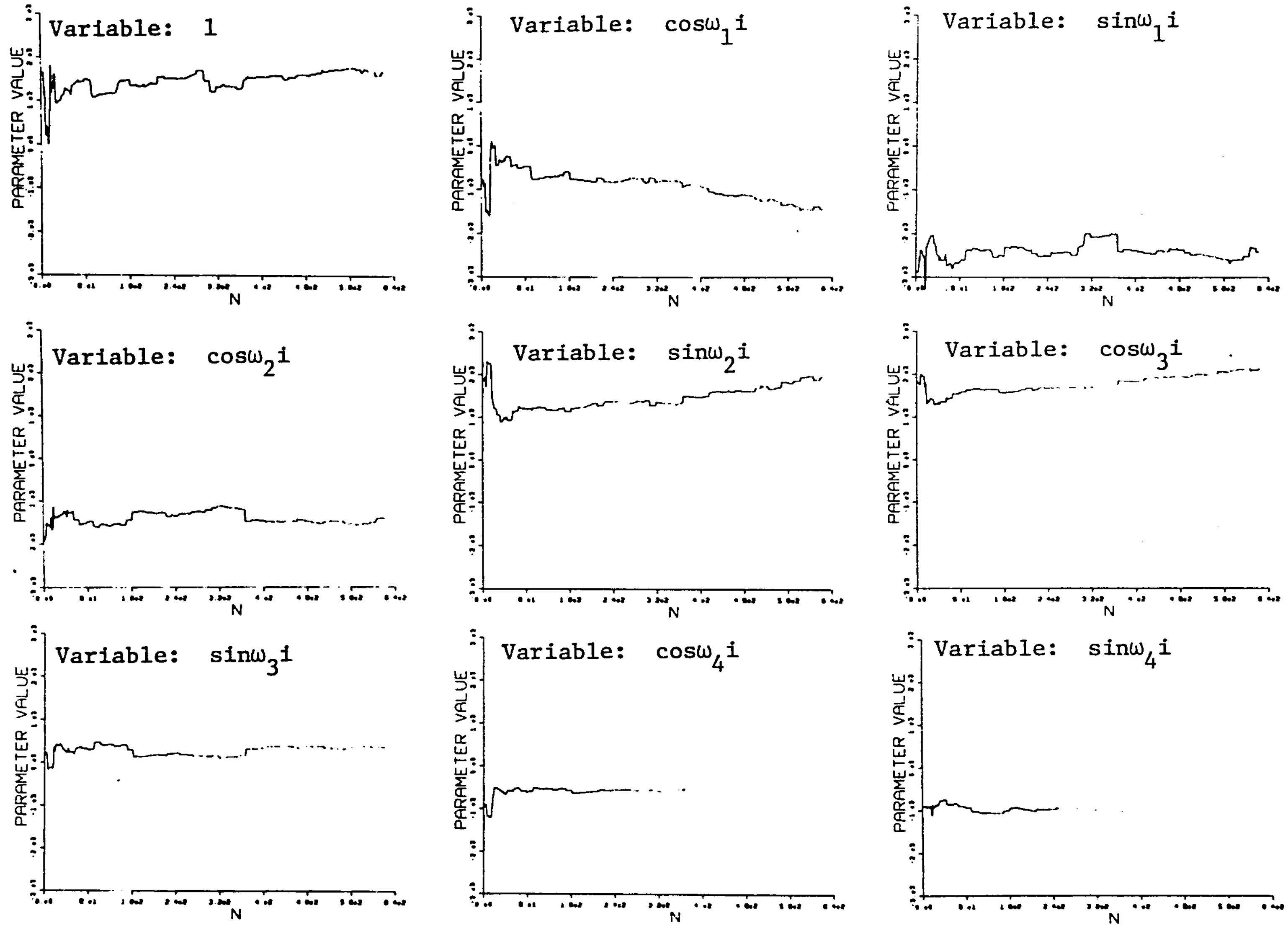


Figure 7.3 (cont'd)
Evolution Plots of the Coefficients in the Final Multivariate Model 5.32 (Wadi Halfa only)

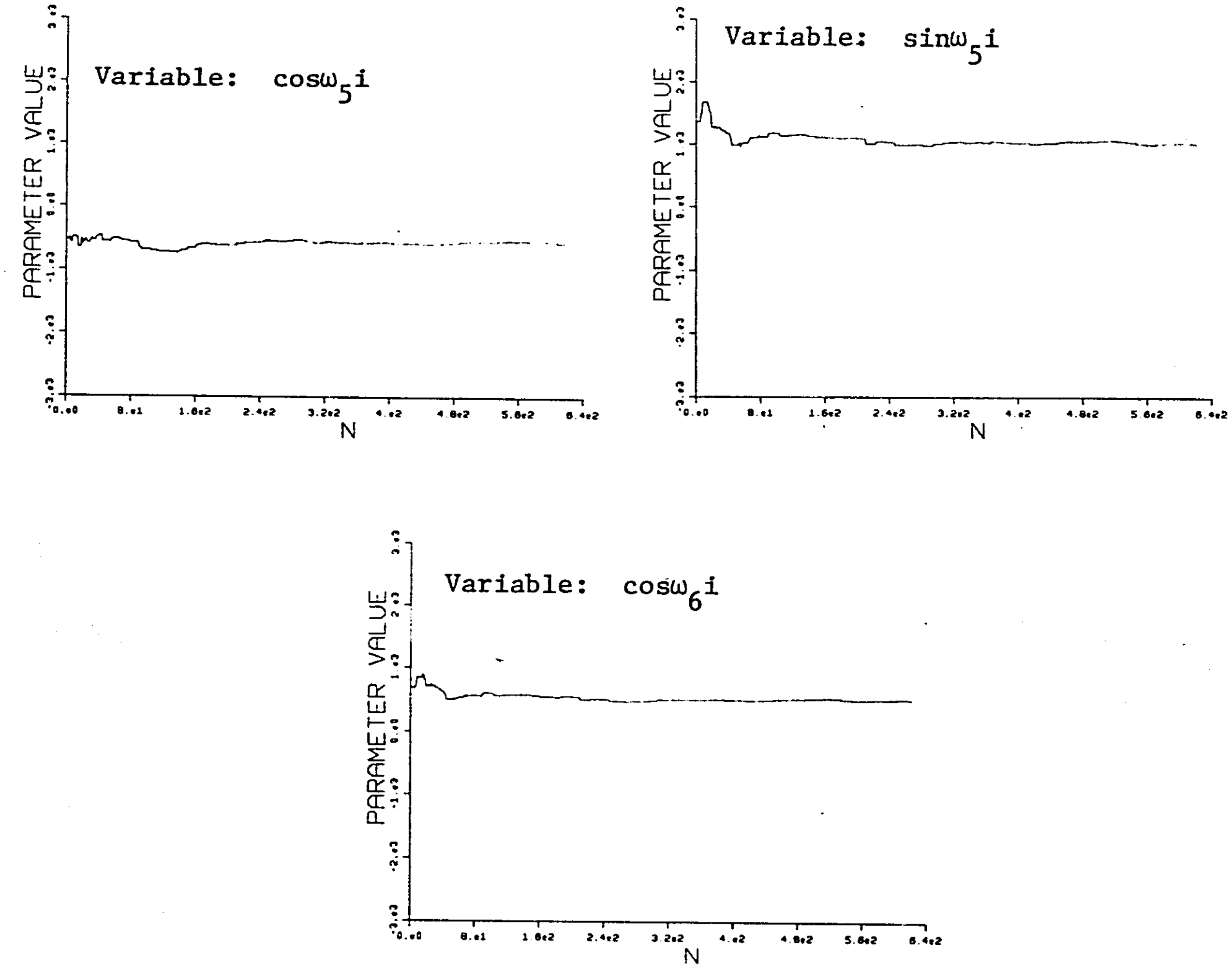


Figure 7.3 (cont'd)
Evolution Plots of the Coefficients in the Final Multivariate Model 5.32 (Wadi Halfa only)

Station	Month	N	K	d	4-d	d_L	d_U	H: ρ=0
Wadi Halfa	Jan.	86	6	2.037	1.963	<1.52-1.54	>1.77-1.78	NR
	Feb.	86	5	1.734	2.255	1.52-1.54	1.77-1.78	I
	Mar.	86	5	NA	–	–	–	–
	Apr.	86	5	1.831	2.169	1.52-1.54	1.77-1.78	NR
	May	86	3	NA	––	–	–	–
	Jun.	86	4	1.715	2.285	1.55-1.57	1.75	I
	Jul.	86	2	1.858	2.142	1.60-1.61	1.70	NR
	Aug.	86	3	2.486	1.541	1.57-1.59	1.72-1.73	R
	Sep.	86	5	2.183	1.817	1.52-1.54	1.77-1.78	NR
	Oct.	86	5	1.972	2.028	1.52-1.54	1.77-1.78	NR
	Nov.	86	3	2.299	1.701	1.57-1.59	1.72-1.73	I
	Dec.	86	4	1.972	2.028	1.55-1.57	1.75	NR

Table 7.7

Results of Durbin Watson Test on Residuals of Univariate Model 5.31

N = Number of observations; K = number of explanatory variables.

NR → Hypothesis ρ=0 not rejected; I → results of Durbin Watson test inconclusive. R → Hypothesis ρ=0 rejected.

Model for	Test Statistics	H: $\rho=0$
March	.03247	NR
May	.20379	NR

Table 7.8

Results of Whitness Test No. 2 Applied

to Univariate Model 5.31;

NR → hypothesis $\rho=0$ not rejected

LAG WADI HALFA VS WADI HALFA

LAG																
1-16	-0.008	-0.008	0.006	0.016	-0.004	0.003	0.010	-0.005	0.002	0.001	0.014	-0.121	0.077	-0.007	-0.017	-0.021
17-32	0.003	0.007	-0.027	-0.002	0.012	0.046	-0.027	0.029	0.069	0.019	0.012	0.016	-0.006	0.004	-0.003	-0.009
33-48	-0.022	0.035	0.090	-0.090	-0.020	-0.024	-0.025	0.012	0.002	0.004	0.000	0.012	-0.008	-0.074	0.084	-0.027

Table 7.9
Autocorrelation Function of Residuals Obtained from Univariate Model 5.31

of August. The Durbin Watson test was not applicable to the residuals obtained from the March and May univariate equations due to the presence of a first order autoregressive term (see Section 5.7.3.1).

Whiteness test number 2 is applicable to the March and May residuals and the results given in Table 7.8 show that the hypothesis of uncorrelated residuals is not rejected at a 5% level.

The autocorrelation function of the residuals, calculated as indicated by Equation 5.123, is given in Table 7.9. Following from the discussion of whiteness test number 3, the individual coefficients may be expected to have a standard error of approximately .03 $(=(1/(86 \times 12)^{1/2}))$. Using this criteria, the coefficient at lag 12, - 0.121, is unusually large. However, no alternative model could be found that clearly had a more desirable autocorrelation function.

Figure 7.4 shows the normal plots of the residuals obtained from each univariate monthly model. There does not appear to be any pronounced violation of the normality assumption implicit in the partial F test used to test coefficient significance (see Section 5.7.2.1).

Tables 7.10 through 7.13 and Figure 7.5 give results of analysis done on the residuals of OLS estimated multivariate Model 5.31. The results of the Durbin Watson test, whiteness test number 1, on the residuals from the final OLS multivariate model given in the previous section are presented in Table 7.10. The limits used in the testing are the 5% level and were obtained from Johnston (1972). The hypothesis of uncorrelated residuals was rejected infrequently.

Station	Month	N	K	d	4-d	d_L	d_U	H: $\rho=0$
WADI HALFA	Jan.	55	8	2.258	1.742	< 1.38	> 1.77	I
	Feb.	55	3	1.761	2.239	1.45	1.68	NR
	Mar.	55	9	2.117	1.883	< 1.38	> 1.77	NR*
	Apr.	55	9	1.923	2.077	< 1.38	> 1.77	NR*
	May	55	10	2.172	1.826	< 1.38	> 1.77	NR*
	Jun.	55	5	2.180	1.820	1.38	1.77	NR
	Jul.	55	11	1.875	2.125	< 1.38	> 1.77	NR*
	Aug.	55	5	1.901	2.099	1.38	1.77	NR
	Sep.	55	5	2.189	1.811	1.38	1.77	NR
	Oct.	55	4	2.154	1.846	1.41	1.72	NR
	Nov.	55	5	2.618	1.382	1.38	1.77	I
	Dec.	55	7	2.241	1.759	< 1.38	> 1.77	I

Table 7.10a

Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31. Monthly Equations for Wadi Halfa

Station	Month	N	K	d	4-d	d_L	d_U	H: ρ=0
ATBARA	Jan.	64	3	NA	-	-	-	NR
	Feb.	64	5	1.121	2.879	1.41-1.44	1.77	R
	Mar.	64	4	1.888	2.112	1.44-1.47	1.73	NR
	Apr.	64	2	NA	-	-	-	-
	May	64	3	NA	-	-	-	-
	Jun.	64	2	NA	-	-	-	-
	Jul.	64	1	-	-	-	-	-
	Aug.	64	2	1.632	2.368	1.51-1.54	1.65-1.66	I
	Sep.	64	3	2.516	1.484	1.48-1.50	1.69-1.70	I
	Oct.	64	2	1.743	2.257	1.51-1.54	1.65-1.66	NR
	Nov.	64	2	2.813	1.187	1.51-1.54	1.65-1.66	R
	Dec.	64	5	NA	-	-	-	-

Table 7.10b

Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31. Monthly Equations for Atbara

Station	Month	N	K	d	4-d	d_L	d_U	H: ρ=0
Tamaniat	Jan.	61	11	2.161	1.839	< 1.41	> 1.77	NR*
	Feb.	61	2	1.735	2.265	1.51-1.54	1.65-1.66	NR
	Mar.	61	4	NA	-	-	-	-
	Apr.	61	9	2.428	1.572	< 1.41	> 1.77	I
	May	61	6	1.901	2.099	< 1.41	> 1.77	NR
	Jun.	61	5	2.471	1.529	< 1.41	> 1.77	I
	Jul.	61	3	1.795	2.205	1.48-1.50	1.69-1.70	NR
	Aug.	61	3	2.207	1.793	1.48-1.50	1.69-1.70	NR
	Sep.	61	4	2.345	1.646	1.44-1.47	1.73	I
	Oct.	61	3	2.219	1.781	1.48-1.50	1.69-1.70	NR
	Nov.	61	6	2.246	1.754	< 1.41	> 1.77	I
	Dec.	61	11	NA	-	-	-	-

Table 7.10c

Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31. Monthly Equations for Tamaniat

		N	K	d	4-d	d_L	d_U	H: ρ=0
Malakal	Jan	61	4	2.101	1.899	1.44-1.47	1.73	NR
	Feb	61	3	1.538	2.462	1.48-1.50	1.69-1.70	I
	Mar	61	4	2.562	1.438	1.44-1.47	1.73	R
	Apr	61	4	1.701	2.299	1.44-1.47	1.73	I
	May	61	3	2.228	1.772	1.48-1.50	1.69-1.70	NR
	Jun	61	8	2.143	1.857	<1.41	>1.77	NR
	Jul	61	5	1.875	2.125	1.41-1.44	1.77	NR
	Aug	61	7	2.234	1.766	<1.41	>1.77	NR
	Sep	61	5	2.413	1.587	1.41-1.44	1.77	I
	Oct	61	5	2.048	1.952	1.41-1.44	1.77	NR
	Nov	61	6	1.566	2.434	<1.41	>1.77	I
	Dec	61	5	1.655	2.345	1.41-1.44	1.77	I

Table 7.10d Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31; Monthly Equations for Malakal

Station	Month	N	K	d	4-d	d_L	d_U	H: ρ=0
Mongalla	Jan.	70	4	2.310	1.69	1.49	1.74	I
	Feb.	70	2	.620	3.380	1.55	1.67	R
	Mar.	70	4	1.537	2.463	1.49	1.74	I
	Apr.	70	3	1.739	2.261	1.52	1.70	NR
	May	70	3	1.962	2.038	1.52	1.70	NR
	Jun.	70	3	1.800	2.200	1.52	1.70	NR
	Jul.	70	4	2.534	1.466	1.49	1.74	R
	Aug.	70	3	2.121	1.879	1.52	1.70	NR
	Sep.	70	2	1.860	2.14	1.55	1.67	NR
	Oct.	70	3	NA	-	-	-	-
	Nov.	70	3	1.664	2.336	1.52	1.70	I
	Dec.	70	3	1.832	2.168	1.52	1.70	NR

Table 7.10e

Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31. Monthly Equations for Mongalla.

Station	Month	N	K	d	4-d	d_L	d_U	H: ρ=0
Khartoum	Jan.	61	6	1.717	2.283	<1.41	>1.77	I
	Feb.	61	7	2.190	1.810	<1.41	>1.77	NR*
	Mar.	61	6	1.750	2.250	<1.44	>1.77	I
	Apr.	61	5	1.778	2.222	1.41-1.44	1.77	NR
	May	61	3	1.911	2.089	1.48-1.50	1.69-1.70	NR
	Jun.	61	2	2.070	1.93	1.51-1.54	1.65-1.66	NR
	Jul.	61	2	1.936	2.064	1.51-1.54	1.65-1.66	NR
	Aug.	61	4	1.917	2.083	1.44-1.47	1.73	NR
	Sep.	61	5	2.816	1.184	1.41-1.44	1.77	R
	Oct.	61	2	2.380	1.620	1.51-1.54	1.65-1.66	I
	Nov.	61	3	2.086	1.914	1.48-1.50	1.69-1.70	NR
	Dec.	61	4	NA	-	-	-	-

Table 7.10f

Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31. Monthly Equations for Khartoum.

Station	Month	N	K	d	4-d	d_L	d_U	H: ρ=0
Sennar	Jan.	61	3	1.613	2.387	1.48-1.50	1.69-1.70	I
	Feb.	61	5	2.175	1.825	1.41-1.44	1.77	NR
	Mar.	61	6	1.740	2.260	<1.41	>1.77	I
	Apr.	61	3	1.743	2.257	1.48-1.50	1.69-1.70	NR
	May	61	3	2.064	1.936	1.48-1.50	1.69-1.70	NR
	Jun.	61	2	2.161	1.839	1.51-1.54	1.65-1.66	NR
	Jul.	61	2	1.635	2.365	1.51-1.54	1.65-1.66	I
	Aug.	61	4	1.868	2.132	1.44-1.47	1.73	NR
	Sep.	61	5	2.325	1.675	1.41-1.44	1.77	I
	Oct.	61	2	2.366	1.634	1.51-1.54	1.65-1.66	I
	Nov.	61	3	2.040	1.960	1.48-1.50	1.69-1.70	NR
	Dec.	61	4	1.856	2.144	1.44-1.47	1.73	NR

Table 7.10g

Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31. Monthly Equations for Sennar

Station	Month	N	K	d	4-d	d_L	d_U	H: ρ=0
Roseires	Jan.	61	3	1.919	2.081	1.48-1.50	1.69-1.70	NR
	Feb.	61	6	NA	-	-	-	-
	Mar.	61	2	2.342	1.658	1.51-1.54	1.65-1.66	NR*
	Apr.	61	3	1.755	2.245	1.48-1.50	1.69-1.70	NR
	May	61	1	-	-	-	-	-
	Jun.	61	2	2.081	1.919	1.51-1.54	1.65-1.66	NR
	Jul.	61	2	1.794	2.206	1.51-1.54	1.65-1.66	NR
	Aug.	61	3	1.951	2.049	1.48-1.50	1.69-1.70	NR
	Sep.	61	3	2.016	1.984	1.48-1.50	1.69-1.70	NR
	Oct.	61	2	2.424	1.576	1.51-1.54	1.65-1.66	I
	Nov.	61	2	2.100	1.900	1.51-1.54	1.65-1.66	NR
	Dec.	61	4	NA	-	-	-	-

Table 7.10h

Results of Durbin Watson Test on Residuals Obtained from OLS Estimated Multivariate Model 5.31. Monthly Equations for Roseires

K = number of explanatory variables

N = number of observations

NR = hypothesis not rejected ($H:\rho=0$)

I = inconclusive results

R = hypothesis rejected ($H:\rho=0$)

NA = test not applicable (due to lagged autoregressive term)

superscript * = conclusion is tentative since upper and lower limits are not precisely known

Table 7.10i

Results of the Durbin Watson Test on Residuals obtained from OLS Estimated Multivariate Model 5.31; Key to abbreviations used in Tables 7.10a through 7.10h

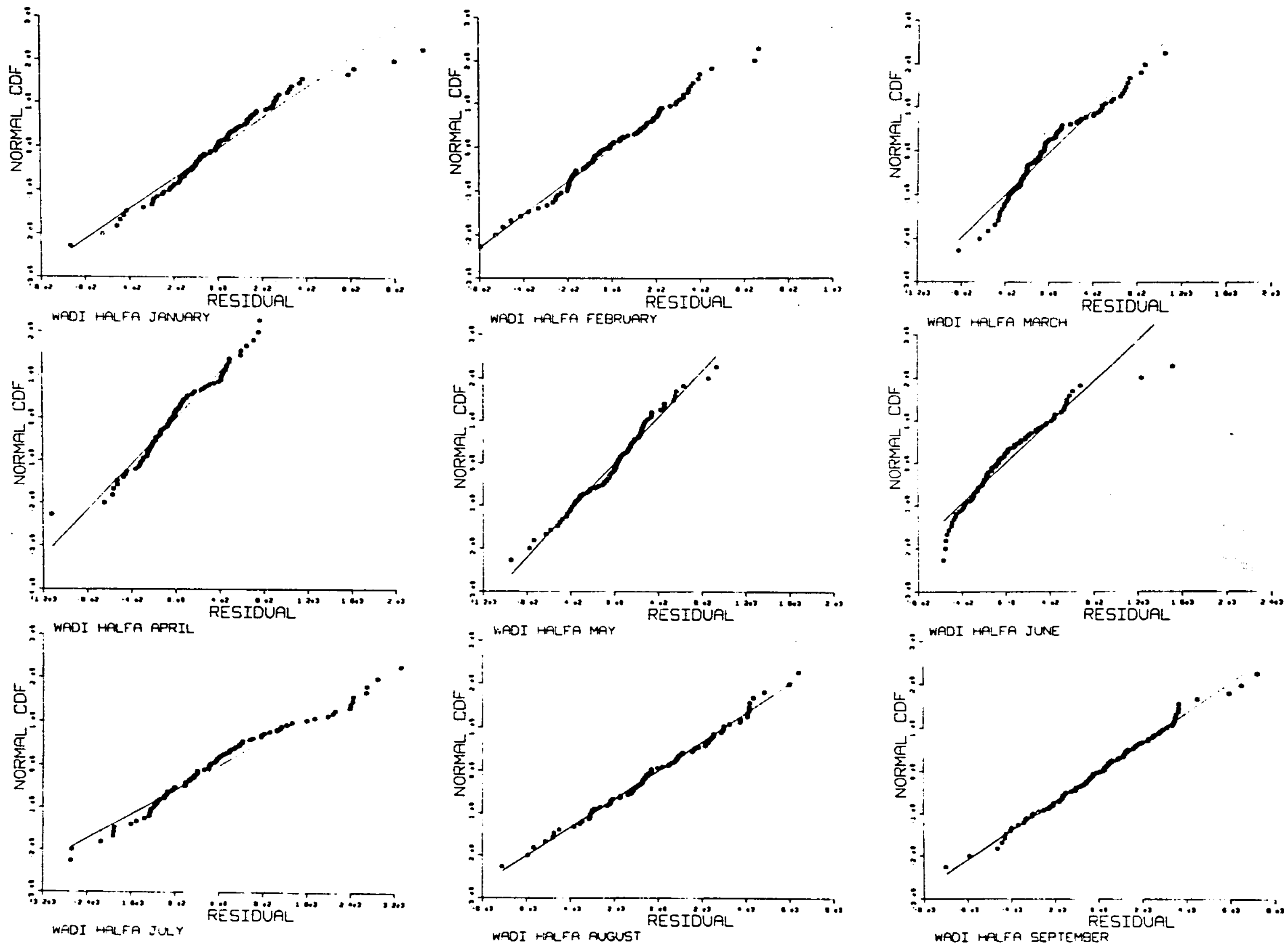


Figure 7.4
Normal Plots of Residuals Obtained from each Monthly Equation of Wadi Halfa Univariate Model 5.31

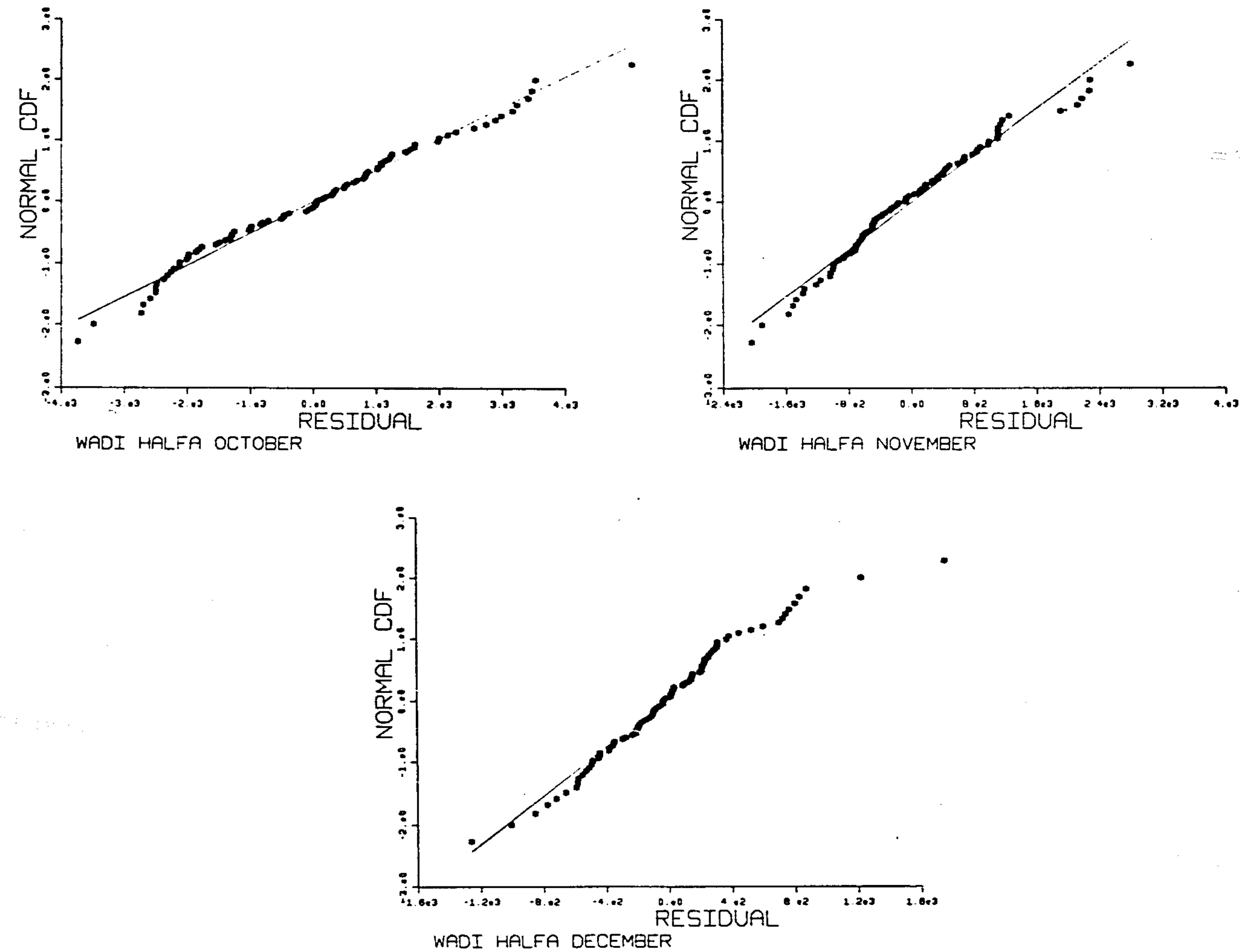


Figure 7.4 (cont'd)
Normal Plots of Residuals Obtained from each Monthly Equation of Wadi Halfa Univariate Model 5.31

Model for	Test Statistics	H: $\rho=0$
Atbara January	-0.040	NR
Atbara April	-0.121	NR
Atbara May	-2.038	R(5%); NR(1%)
Atbara June	NA	NR
Atbara December	-0.474	NR
Tamaniat March	0.331	NR
Tamaniat December	-0.755	NR
Mongalla October	0.015	NR
Khartoum December	1.073	NR
Roseires February	0.315	NR
Roseires December	0.839	NR

NR = Hypothesis $\rho=0$ not rejected

R = Hypothesis $\rho=0$ rejected

NA = Test not applicable since $n\mathrm{Var}(\beta_j(1))$ is greater than one (see Section 5.7.3.1)

Table 7.11

Results of Whitness Test No. 2 on Residuals Obtained from OLS Estimated Multivariate Model 5.31

LAG MALAKAL VS KHARTOUM

1-16	0.052	0.080	0.073	-0.017	0.030	0.077	-0.001	-0.024	0.015	-0.018	0.015	-0.036	-0.064	-0.029	0.081	0.152
17-32	0.003	-0.021	0.041	0.026	0.019	0.014	0.035	-0.026	0.017	-0.024	0.007	-0.020	0.014	0.063	-0.122	-0.015
33-48	0.041	-0.055	-0.008	-0.004	-0.028	0.093	0.020	0.077	0.042	-0.027	0.052	0.055	-0.038	-0.011	-0.014	-0.035

LAG MALAKAL VS SENNAR

1-16	0.077	0.077	0.084	0.013	0.037	0.080	0.034	-0.018	-0.003	0.010	0.021	-0.033	-0.053	-0.024	0.066	0.168
17-32	-0.002	-0.043	0.043	0.023	0.010	0.004	0.025	-0.023	0.007	-0.017	-0.006	-0.004	0.072	0.041	-0.087	-0.005
33-48	0.034	-0.038	-0.001	-0.017	-0.041	0.081	0.027	0.080	0.082	0.009	0.029	0.019	-0.014	-0.018	-0.027	-0.025

LAG MALAKAL VS ROSEIRES

1-16	0.052	0.043	0.125	0.016	0.054	0.107	-0.002	-0.041	0.017	-0.009	0.015	-0.030	-0.057	-0.011	0.054	0.158
17-32	0.009	-0.005	0.069	0.005	-0.001	0.013	0.025	-0.021	0.013	-0.016	-0.036	-0.008	0.116	0.063	-0.108	-0.028
33-48	0.059	-0.027	-0.009	-0.010	-0.026	0.091	0.050	0.092	0.038	-0.022	0.030	0.041	-0.013	-0.037	-0.014	-0.010

Table 7.12
Results of Whiteness Test No. 3 Applied to Residuals of OLS Estimated Multivariate Model 5.31; Malakal Assumed Independent of Roseires

LAG WADI HALFA VS WADI HALFA

1-16	-0.118	0.025	0.011	0.011	-0.016	-0.005	-0.017	-0.001	0.014	0.008	-0.028	-0.081	0.078	-0.009	0.005	-0.002
17-32	-0.011	-0.002	-0.014	-0.009	-0.002	0.112	-0.112	0.100	-0.037	0.002	0.018	-0.016	0.004	0.006	0.012	-0.014
33-48	0.006	0.038	0.020	-0.062	0.089	0.017	-0.034	0.003	-0.007	-0.010	-0.001	-0.006	-0.010	-0.049	0.016	0.024

LAG WADI HALFA VS ATBARA

1-16	-0.051	0.013	0.059	0.006	-0.002	0.000	-0.004	0.003	-0.010	0.021	-0.005	-0.046	0.037	0.065	0.017	-0.008
17-32	0.001	0.008	0.005	-0.003	0.009	0.029	-0.072	-0.033	-0.010	-0.005	0.007	-0.005	0.013	0.006	0.004	0.004
33-48	-0.002	0.015	0.028	-0.114	0.047	0.040	-0.062	0.014	0.002	-0.003	0.007	-0.005	-0.009	-0.008	0.047	0.085

LAG WADI HALFA VS TAMANIAT

1-16	-0.055	0.030	-0.013	0.016	-0.009	-0.014	-0.008	-0.006	-0.005	0.023	-0.028	-0.092	0.109	0.052	-0.003	-0.019
17-32	-0.017	0.009	-0.026	-0.013	0.029	0.094	-0.107	0.083	-0.078	-0.006	0.037	0.003	-0.001	-0.003	0.003	-0.004
33-48	0.012	0.031	0.075	-0.016	0.113	-0.031	0.021	0.006	-0.005	0.000	-0.022	-0.013	-0.016	-0.037	0.029	0.105

LAG WADI HALFA VS MALAKAL

1-16	0.030	-0.008	-0.000	0.034	0.000	-0.051	-0.025	0.016	0.043	0.020	0.004	0.023	-0.004	0.031	-0.008	-0.003
17-32	-0.047	-0.005	-0.031	-0.120	-0.032	0.001	-0.019	-0.083	-0.010	0.022	-0.012	0.009	-0.020	-0.035	-0.020	0.009
33-48	-0.085	-0.026	0.022	0.017	-0.019	-0.007	-0.000	0.028	0.011	0.035	0.086	0.010	0.043	-0.020	0.010	0.010

LAG WADI HALFA VS MONGALLA

1-16	0.073	0.083	-0.031	0.029	-0.012	-0.000	-0.002	-0.049	0.031	0.024	-0.009	-0.022	0.013	0.094	-0.028	-0.007
17-32	-0.034	0.038	-0.020	0.019	-0.001	0.029	0.065	-0.083	-0.110	-0.004	0.007	-0.058	-0.040	-0.006	-0.019	-0.036
33-48	-0.014	0.107	-0.026	0.062	-0.010	-0.055	-0.055	0.023	0.018	-0.002	-0.010	-0.009	-0.090	-0.110	0.094	0.026

LAG WADI HALFA VS KHARTOUM

1-16	-0.074	-0.007	0.000	0.028	-0.017	0.008	-0.000	-0.004	-0.012	0.027	0.010	-0.178	0.070	0.028	-0.017	-0.008
17-32	-0.011	0.007	0.000	-0.008	0.013	0.076	-0.086	0.084	-0.116	-0.089	0.012	-0.006	0.003	0.000	-0.009	0.007
33-48	0.021	0.034	0.046	-0.057	0.093	-0.051	-0.019	0.008	0.004	-0.004	-0.002	-0.005	-0.018	-0.023	-0.041	0.056

LAG WADI HALFA VS SENNAR

1-16	-0.020	-0.005	-0.002	0.029	-0.018	0.010	0.005	-0.003	-0.004	0.033	-0.005	-0.125	0.037	0.021	0.003	-0.019
17-32	-0.008	0.005	-0.002	-0.009	0.017	0.070	-0.086	0.039	-0.089	-0.044	0.016	-0.021	0.004	-0.005	-0.009	0.001
33-48	0.024	0.038	0.078	-0.049	0.044	-0.066	-0.004	0.006	0.005	0.000	0.004	-0.014	-0.014	-0.036	0.002	0.040

Table 7.13
Results of Whiteness Test No. 3 on Residuals Obtained from OLS Estimated Multivariate Model 5.31

LAG WADI HALFA VS ROSEIRES

1-16	-0.064	-0.010	-0.004	0.029	-0.010	0.003	0.008	0.006	-0.007	0.039	-0.042	-0.106	0.037	0.016	-0.013	-0.019
17-32	-0.011	0.002	-0.004	-0.003	0.003	0.060	-0.063	0.082	-0.094	-0.064	-0.014	-0.010	0.008	0.000	-0.004	0.003
33-48	0.015	0.040	0.027	-0.111	0.079	-0.079	0.002	-0.004	0.005	-0.006	-0.002	-0.002	-0.011	-0.029	0.024	0.038

LAG ATBARA VS ATBARA

1-16	-0.052	0.002	0.002	0.004	0.001	0.001	-0.001	-0.002	-0.006	0.027	-0.062	-0.104	0.057	0.111	-0.008	-0.006
17-32	0.000	-0.002	0.000	0.002	-0.004	0.017	-0.013	-0.054	-0.028	-0.047	0.018	-0.002	0.002	0.001	0.000	-0.001
33-48	0.002	0.036	0.016	-0.166	-0.004	0.016	-0.009	0.002	-0.003	0.002	-0.001	-0.001	-0.021	-0.041	0.009	-0.057

LAG ATBARA VS TAMANIAT

1-16	0.054	0.001	-0.001	-0.007	-0.007	-0.010	0.019	-0.014	-0.051	-0.023	-0.113	-0.113	0.083	0.077	-0.009	-0.011
17-32	0.003	0.008	-0.008	0.014	0.012	0.053	-0.090	-0.043	-0.108	-0.017	0.015	0.013	-0.017	0.012	0.007	0.014
33-48	0.007	0.005	0.087	-0.129	0.076	-0.023	0.016	-0.009	0.008	0.007	-0.012	0.001	-0.043	-0.078	0.024	-0.032

LAG ATBARA VS MALAKAL

1-16	-0.016	0.001	-0.033	-0.014	-0.045	-0.136	0.008	-0.018	0.044	-0.022	0.005	0.021	-0.009	0.041	-0.025	-0.008
17-32	-0.061	0.005	-0.035	-0.130	0.029	0.013	-0.019	-0.053	-0.024	-0.002	-0.054	0.027	-0.020	0.041	-0.027	-0.025
33-48	-0.041	-0.011	0.001	-0.001	-0.021	-0.037	0.017	-0.007	0.012	0.041	0.009	-0.003	0.008	-0.042	0.044	-0.010

LAG ATBARA VS MONGALLA

1-16	0.013	0.077	0.054	-0.003	-0.022	0.001	-0.011	-0.018	0.050	-0.010	-0.111	-0.038	-0.021	0.106	-0.014	-0.010
17-32	-0.013	-0.001	-0.019	0.039	-0.026	0.020	0.033	-0.062	-0.091	-0.030	-0.047	-0.022	-0.018	-0.017	0.007	-0.005
33-48	0.016	0.019	-0.026	0.028	0.021	-0.039	-0.028	0.021	0.002	-0.024	-0.021	-0.038	-0.091	-0.120	0.102	-0.094

LAG ATBARA VS KHARTOUM

1-16	0.036	0.028	0.004	0.007	-0.002	-0.001	0.005	0.002	-0.042	-0.004	-0.047	-0.211	0.039	0.046	0.002	-0.008
17-32	-0.002	0.001	-0.003	0.006	0.009	0.038	-0.072	0.018	-0.089	-0.046	0.019	-0.002	-0.000	0.001	0.001	0.007
33-48	0.021	0.053	0.067	-0.122	0.066	-0.025	0.021	-0.003	0.001	-0.001	0.001	-0.007	-0.055	-0.036	-0.045	-0.018

LAG ATBARA VS SENNAR

1-16	0.084	0.028	-0.016	-0.002	-0.003	-0.003	0.003	0.003	-0.026	0.037	-0.087	-0.172	0.023	0.044	-0.000	-0.013
17-32	-0.001	0.000	-0.005	0.019	0.027	0.048	-0.029	-0.044	-0.062	-0.041	0.023	-0.010	-0.003	-0.001	-0.003	-0.003
33-48	0.013	0.043	0.023	-0.125	-0.006	-0.031	0.027	-0.001	-0.002	0.000	0.004	-0.010	-0.062	-0.079	-0.026	-0.052

Table 7.13 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from OLS Estimated Multivariate Model 5.31

LAG ATBARA VS ROSEIRES

1-16	0.002	0.020	-0.007	0.000	-0.000	-0.005	0.003	0.007	-0.037	0.044	-0.125	-0.139	0.061	0.041	-0.008	-0.006
17-32	0.001	0.001	-0.006	0.008	0.010	0.023	-0.046	-0.018	-0.106	-0.041	0.012	-0.001	0.002	-0.000	0.002	-0.010
33-48	0.007	0.023	-0.013	-0.148	0.056	-0.007	0.019	-0.006	-0.001	-0.001	0.001	-0.002	-0.053	-0.068	0.066	-0.095

LAG TAMANIAT VS TAMANIAT

1-16	0.024	0.022	-0.006	0.001	0.007	-0.022	0.006	0.007	-0.034	-0.029	-0.034	-0.087	0.075	0.102	0.006	-0.034
17-32	-0.001	0.001	-0.017	-0.016	0.028	0.062	-0.013	0.014	-0.004	0.053	0.017	0.009	0.005	-0.009	-0.014	-0.013
33-48	0.037	-0.040	0.051	-0.060	0.064	-0.022	0.013	0.014	-0.014	0.011	-0.020	0.005	-0.036	-0.006	-0.037	0.008

LAG TAMANIAT VS MALAKAL

1-16	0.011	-0.048	-0.002	0.004	-0.015	-0.060	-0.048	0.015	0.036	0.028	0.022	0.018	0.009	0.055	0.019	-0.021
17-32	0.016	-0.004	-0.046	-0.064	0.006	-0.005	-0.033	-0.055	0.010	0.032	-0.016	-0.014	-0.020	-0.023	0.011	-0.030
33-48	-0.005	-0.054	-0.015	0.005	-0.029	-0.017	-0.023	0.049	0.005	-0.013	0.059	0.019	0.034	-0.015	0.000	0.013

LAG TAMANIAT VS MONGALLA

1-16	0.151	0.104	-0.018	0.060	0.017	0.013	-0.028	-0.032	-0.021	-0.021	-0.037	-0.013	0.044	0.108	-0.026	0.020
17-32	-0.016	0.049	-0.014	0.006	0.030	0.042	0.090	-0.090	-0.096	-0.001	0.060	-0.031	-0.021	-0.021	0.011	-0.013
33-48	0.060	0.054	0.002	0.009	0.005	-0.050	-0.021	0.060	0.023	-0.005	0.001	0.001	-0.108	-0.012	0.104	0.040

LAG TAMANIAT VS KHARTOUM

1-16	0.018	-0.012	0.037	0.011	0.001	0.004	0.003	0.004	-0.015	-0.005	0.011	-0.189	0.018	0.044	-0.017	-0.017
17-32	-0.010	-0.005	-0.008	-0.014	0.026	0.066	0.013	0.022	-0.041	-0.028	-0.005	0.012	0.003	-0.003	-0.013	0.003
33-48	0.058	-0.018	0.062	-0.088	0.064	-0.034	-0.016	0.006	-0.002	0.005	-0.010	0.013	-0.022	0.005	-0.048	0.002

LAG TAMANIAT VS SENNAR

1-16	0.068	0.009	0.007	0.013	-0.002	0.005	0.008	0.009	-0.011	0.001	-0.001	-0.124	0.037	0.044	0.001	-0.027
17-32	-0.011	-0.007	-0.004	-0.015	0.036	0.089	0.025	0.028	0.015	0.026	0.006	-0.005	0.005	-0.008	-0.013	0.005
33-48	0.052	0.022	0.061	-0.098	0.003	-0.075	-0.000	0.001	-0.001	0.009	-0.008	-0.002	-0.022	-0.007	-0.014	-0.020

LAG TAMANIAT VS ROSEIRES

1-16	0.023	-0.005	-0.000	0.009	-0.003	0.001	0.007	0.011	-0.028	0.018	0.004	-0.116	0.033	0.018	-0.011	-0.034
17-32	-0.010	0.003	-0.005	-0.016	0.032	0.084	0.021	0.031	-0.023	-0.008	-0.018	-0.001	-0.001	-0.007	-0.003	-0.004
33-48	0.046	-0.001	-0.017	-0.129	0.047	-0.044	-0.005	-0.007	-0.000	-0.003	-0.001	0.013	-0.023	-0.011	0.005	0.004

Table 7.13 (cont'd)
Results of Whiteness Test No. 3 on Residuals Obtained from OLS Estimated Multivariate Model 5.31

LAG MALAKAL VS MALAKAL

LAG																
1-16	0.076	-0.104	0.021	0.044	0.061	-0.045	-0.024	-0.040	-0.039	-0.013	0.010	-0.022	-0.065	-0.001	-0.002	0.027
17-32	-0.025	-0.029	-0.029	-0.000	-0.030	-0.042	-0.010	-0.019	-0.044	0.027	-0.000	0.000	-0.017	0.012	-0.015	-0.009
33-48	0.048	-0.018	-0.013	-0.018	-0.036	-0.092	-0.033	0.003	0.055	0.020	-0.061	0.002	-0.026	0.022	0.007	-0.011

LAG MALAKAL VS MONGALLA

LAG																
1-16	0.019	0.012	0.015	-0.009	0.122	-0.054	0.020	0.039	0.037	0.035	-0.035	-0.037	-0.020	-0.030	0.032	-0.002
17-32	0.061	0.057	-0.015	0.029	0.004	0.034	0.003	0.014	0.017	0.015	0.057	0.035	0.083	-0.018	-0.068	-0.040
33-48	0.047	-0.019	-0.032	-0.006	-0.029	0.014	-0.011	0.027	0.025	0.015	0.063	0.014	-0.054	-0.104	0.014	-0.031

LAG MALAKAL VS KHARTOUM

LAG																
1-16	0.073	0.018	-0.035	-0.026	0.007	0.067	0.016	-0.023	0.032	0.004	0.001	-0.030	-0.047	-0.011	0.065	0.152
17-32	-0.030	-0.049	0.053	0.025	-0.000	0.022	0.005	-0.017	0.013	-0.027	-0.006	-0.011	-0.011	0.062	-0.119	-0.019
33-48	0.050	-0.072	0.004	0.002	-0.030	0.053	-0.005	0.077	0.066	-0.035	0.056	0.026	-0.014	-0.020	-0.004	-0.043

LAG MALAKAL VS SENNAR

LAG																
1-16	0.087	0.021	-0.011	-0.011	0.009	0.080	0.049	-0.012	0.019	0.022	0.002	-0.034	-0.045	-0.007	0.059	0.153
17-32	-0.036	-0.076	0.055	0.026	-0.009	0.011	-0.004	-0.015	0.004	-0.029	-0.021	-0.015	0.028	0.048	-0.077	-0.006
33-48	0.026	-0.054	-0.005	-0.002	-0.037	0.014	-0.017	0.080	0.099	0.003	0.031	-0.008	-0.002	-0.041	-0.011	-0.025

LAG MALAKAL VS ROSEIRES

LAG																
1-16	0.052	0.001	0.003	-0.034	0.051	0.106	0.012	-0.029	0.017	0.009	0.005	-0.031	-0.053	0.008	0.035	0.135
17-32	-0.039	-0.034	0.086	0.000	-0.005	0.016	0.009	-0.018	-0.002	-0.028	-0.031	-0.023	0.085	0.074	-0.105	-0.023
33-48	0.048	-0.059	-0.008	0.002	-0.030	0.048	0.007	0.071	0.050	-0.033	0.035	0.021	-0.017	-0.042	0.003	-0.005

LAG MONGALLA VS MONGALLA

LAG																
1-16	0.018	0.026	0.069	0.014	0.047	0.028	-0.017	0.019	-0.008	0.065	0.017	0.005	0.082	-0.015	-0.003	0.026
17-32	-0.034	0.021	0.020	0.008	0.012	-0.015	-0.065	-0.004	0.015	-0.126	-0.038	-0.012	-0.039	0.029	-0.004	-0.012
33-48	0.027	0.041	-0.014	-0.032	-0.004	0.003	-0.026	-0.008	-0.011	-0.023	0.011	-0.073	-0.095	-0.033	0.021	-0.041

LAG MONGALLA VS KHARTOUM

LAG																
1-16	0.038	0.023	-0.014	0.042	0.005	0.007	0.020	0.030	-0.012	0.018	0.012	-0.054	-0.063	0.021	-0.011	-0.033
17-32	-0.012	0.011	0.010	0.029	0.024	-0.090	0.052	-0.022	-0.059	-0.055	0.002	-0.017	-0.025	-0.003	0.065	-0.005
33-48	0.025	-0.016	0.010	0.027	0.023	0.000	-0.013	0.026	-0.003	0.021	-0.019	0.016	-0.080	-0.006	-0.035	-0.063

Table 7.13 (cont'd)
Results of Whiteness Test No. 3 on Residuals Obtained from OLS Estimated Multivariate Model 5.31

LAG MONGALLA VS SENNAR

1-16	0.079	-0.021	-0.045	0.018	-0.001	0.005	0.023	0.025	-0.027	-0.002	0.047	-0.035	-0.032	0.018	-0.065	-0.050
17-32	-0.011	0.020	0.028	0.063	0.008	-0.076	0.049	0.010	-0.037	-0.059	-0.026	-0.007	-0.021	-0.005	0.071	0.007
33-48	0.033	-0.021	0.013	-0.008	-0.004	-0.007	-0.033	0.010	-0.007	0.016	-0.030	-0.027	-0.057	-0.052	-0.031	-0.082

LAG MONGALLA VS ROSEIRES

1-16	0.024	-0.030	0.008	0.027	0.011	0.009	0.019	0.027	-0.029	0.002	0.036	-0.067	-0.040	0.021	-0.069	-0.037
17-32	-0.013	0.016	0.024	0.041	0.018	-0.055	0.028	0.001	-0.016	-0.068	-0.003	0.006	-0.028	-0.001	0.056	0.020
33-48	0.004	-0.041	0.006	-0.030	0.027	-0.002	-0.034	0.002	0.005	0.011	-0.017	0.002	-0.082	-0.031	-0.055	-0.031

LAG KHARTOUM VS KHARTOUM

1-16	-0.039	-0.023	0.024	0.015	-0.002	-0.000	0.001	-0.009	-0.039	-0.010	-0.042	-0.180	0.060	0.039	-0.022	-0.026
17-32	-0.002	-0.001	-0.004	-0.008	0.011	0.035	0.066	0.078	-0.100	-0.013	-0.004	0.002	-0.007	0.003	-0.004	0.004
33-48	0.046	-0.054	0.096	-0.013	0.079	0.007	-0.009	0.015	-0.002	-0.011	-0.009	-0.006	-0.026	-0.046	-0.089	-0.011

LAG KHARTOUM VS SENNAR

1-16	0.021	-0.012	-0.018	0.017	-0.009	-0.004	-0.002	-0.009	-0.043	-0.001	-0.053	-0.107	0.073	0.022	-0.004	-0.031
17-32	0.000	0.004	-0.001	-0.005	0.017	0.050	0.086	0.054	-0.044	0.029	-0.000	-0.017	-0.009	-0.003	-0.000	0.007
33-48	0.045	-0.023	0.072	-0.031	0.008	-0.044	0.003	0.012	-0.006	-0.010	-0.009	-0.019	-0.028	-0.043	-0.053	-0.030

LAG KHARTOUM VS ROSEIRES

1-16	-0.008	-0.049	-0.002	0.009	-0.013	-0.007	-0.001	-0.003	-0.051	0.029	-0.031	-0.080	0.080	0.022	-0.006	-0.036
17-32	-0.007	0.003	-0.002	-0.006	0.011	0.070	0.084	0.037	-0.071	0.009	-0.015	-0.012	-0.009	-0.002	0.001	0.002
33-48	0.035	-0.031	0.014	-0.021	0.048	-0.004	0.013	-0.002	-0.009	-0.013	-0.010	-0.009	-0.031	-0.040	-0.014	-0.032

LAG SENNAR VS SENNAR

1-16	0.045	-0.015	-0.037	0.018	-0.010	-0.000	-0.001	-0.016	0.002	0.012	-0.023	-0.069	0.089	0.012	-0.019	-0.046
17-32	-0.006	0.004	0.006	0.005	0.008	0.062	0.077	-0.044	-0.062	0.002	0.004	-0.023	-0.008	-0.008	0.001	-0.004
33-48	0.053	0.016	0.022	-0.054	-0.005	-0.066	0.017	0.014	-0.001	-0.003	-0.007	-0.003	-0.020	-0.038	-0.072	-0.041

LAG SENNAR VS ROSEIRES

1-16	-0.009	-0.051	-0.017	0.011	-0.012	-0.002	-0.004	-0.009	-0.012	0.023	-0.025	-0.045	0.073	0.024	-0.024	-0.047
17-32	-0.011	0.007	0.001	0.001	0.000	0.069	0.039	-0.063	-0.105	-0.006	-0.009	-0.016	-0.007	-0.004	-0.000	-0.002
33-48	0.040	-0.013	-0.022	-0.079	0.070	-0.024	0.016	-0.000	-0.004	-0.006	-0.009	0.003	-0.020	-0.047	-0.048	-0.060

LAG ROSEIRES VS ROSEIRES

1-16	0.056	-0.056	0.008	0.021	-0.004	-0.007	0.001	-0.013	-0.015	0.119	0.030	-0.092	0.057	0.076	-0.016	-0.042
17-32	-0.012	0.007	-0.006	0.003	-0.005	0.056	0.062	0.037	-0.071	0.021	-0.024	-0.009	-0.002	-0.004	-0.003	0.000
33-48	0.034	0.020	0.010	-0.041	0.044	-0.009	0.030	0.005	-0.003	-0.006	-0.002	0.002	-0.030	-0.019	0.005	-0.015

Table 7.13 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from OLS Estimated Multivariate Model 5.31

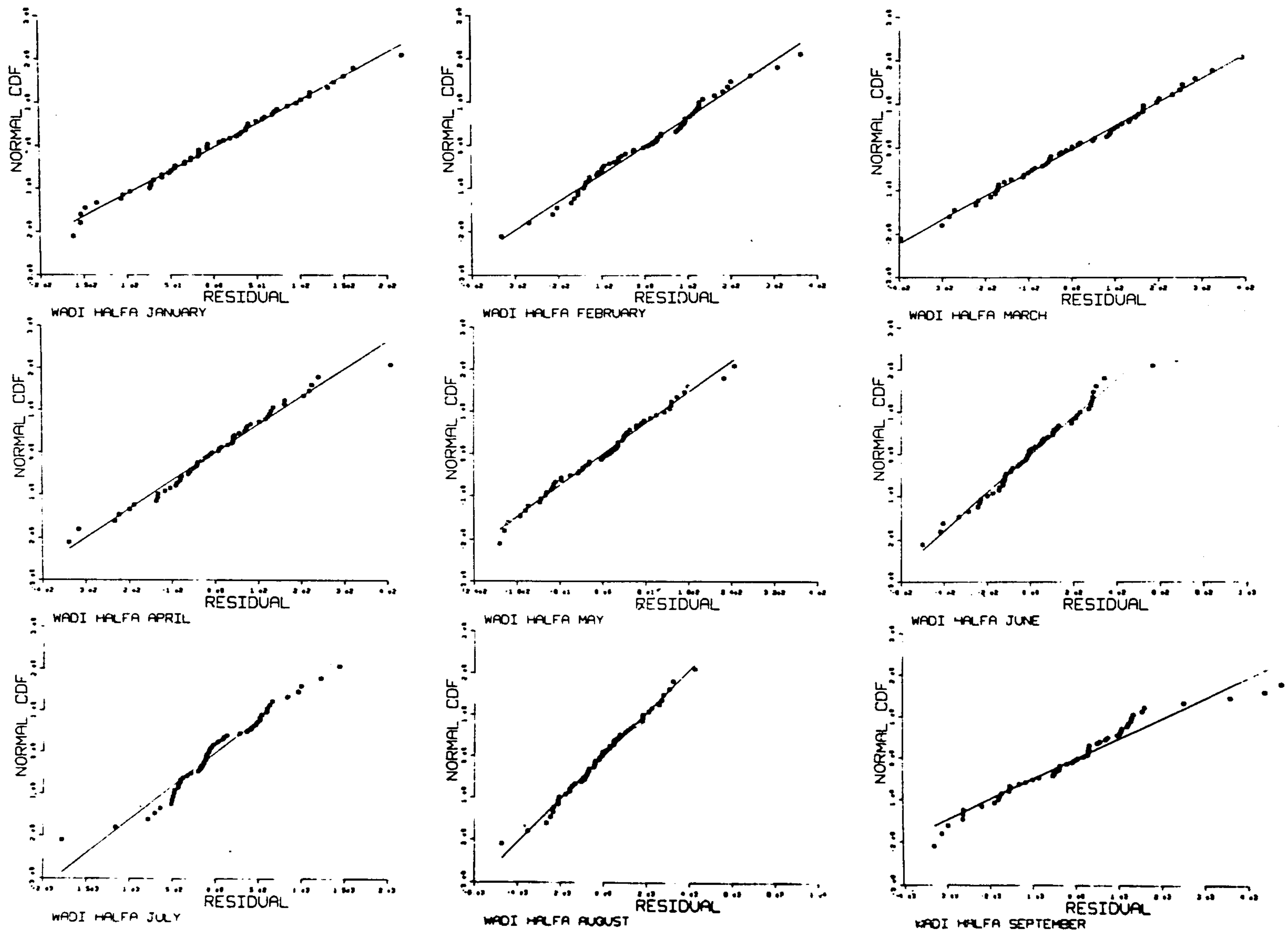


Figure 7.5
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

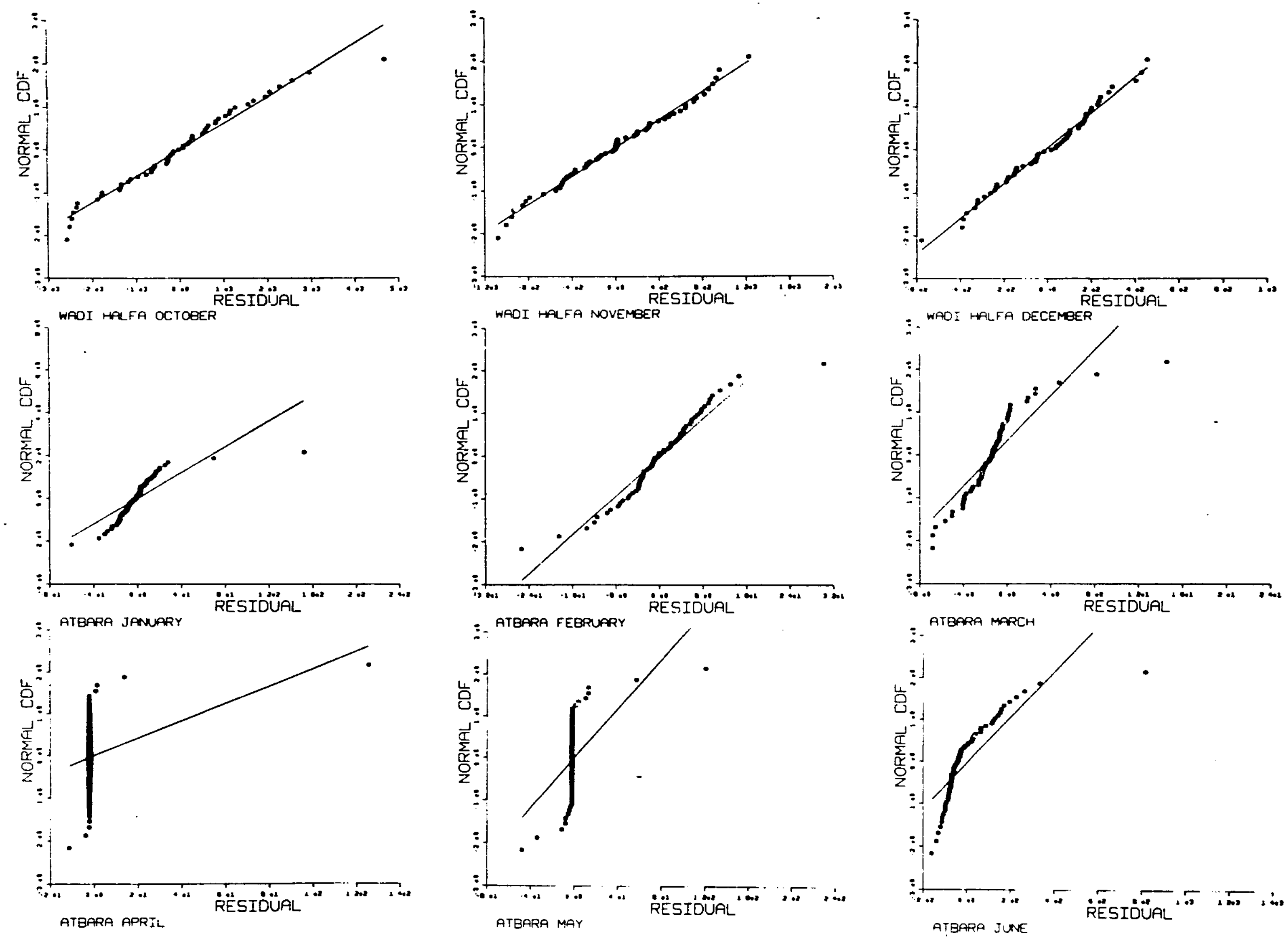


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

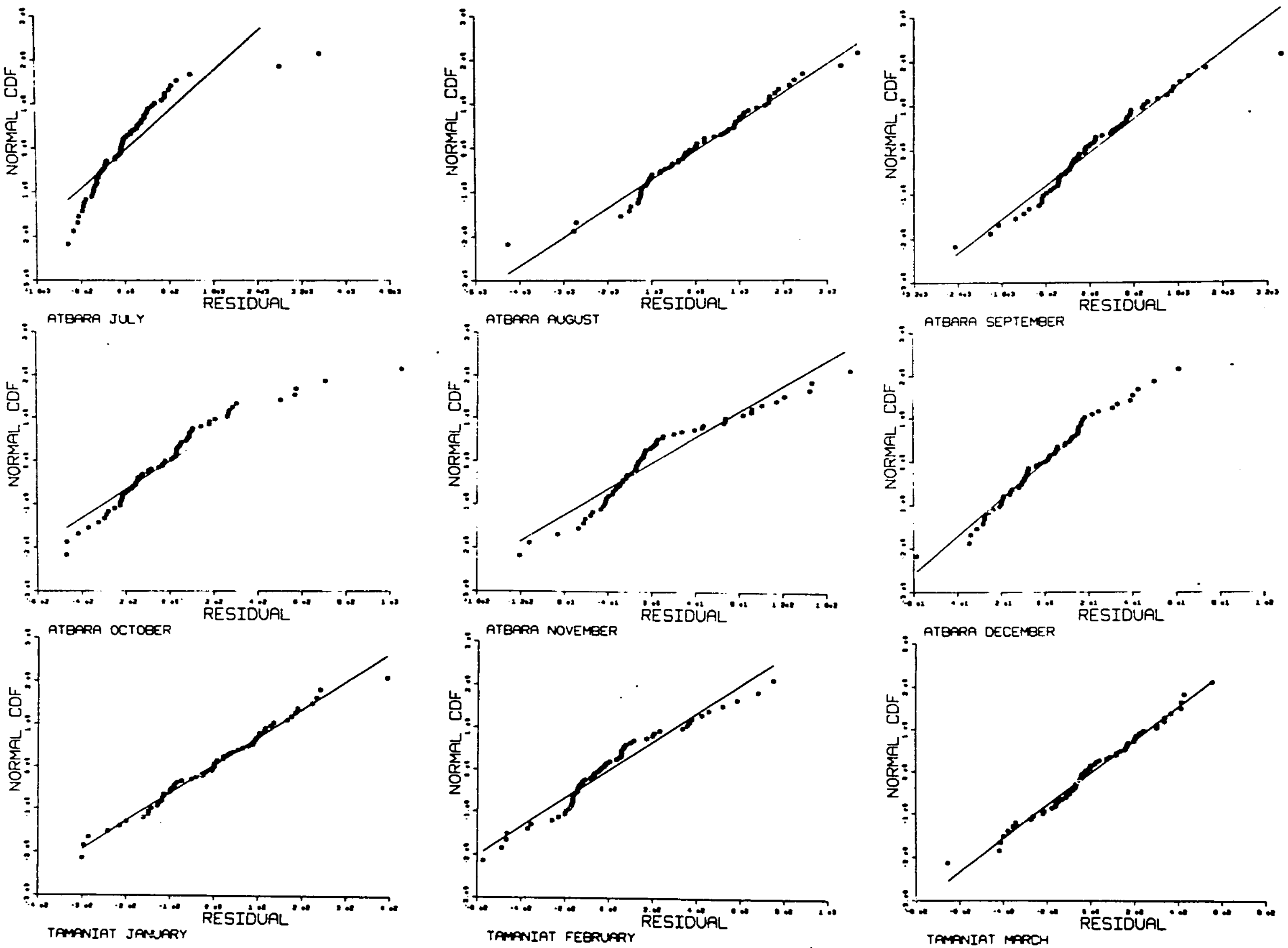


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

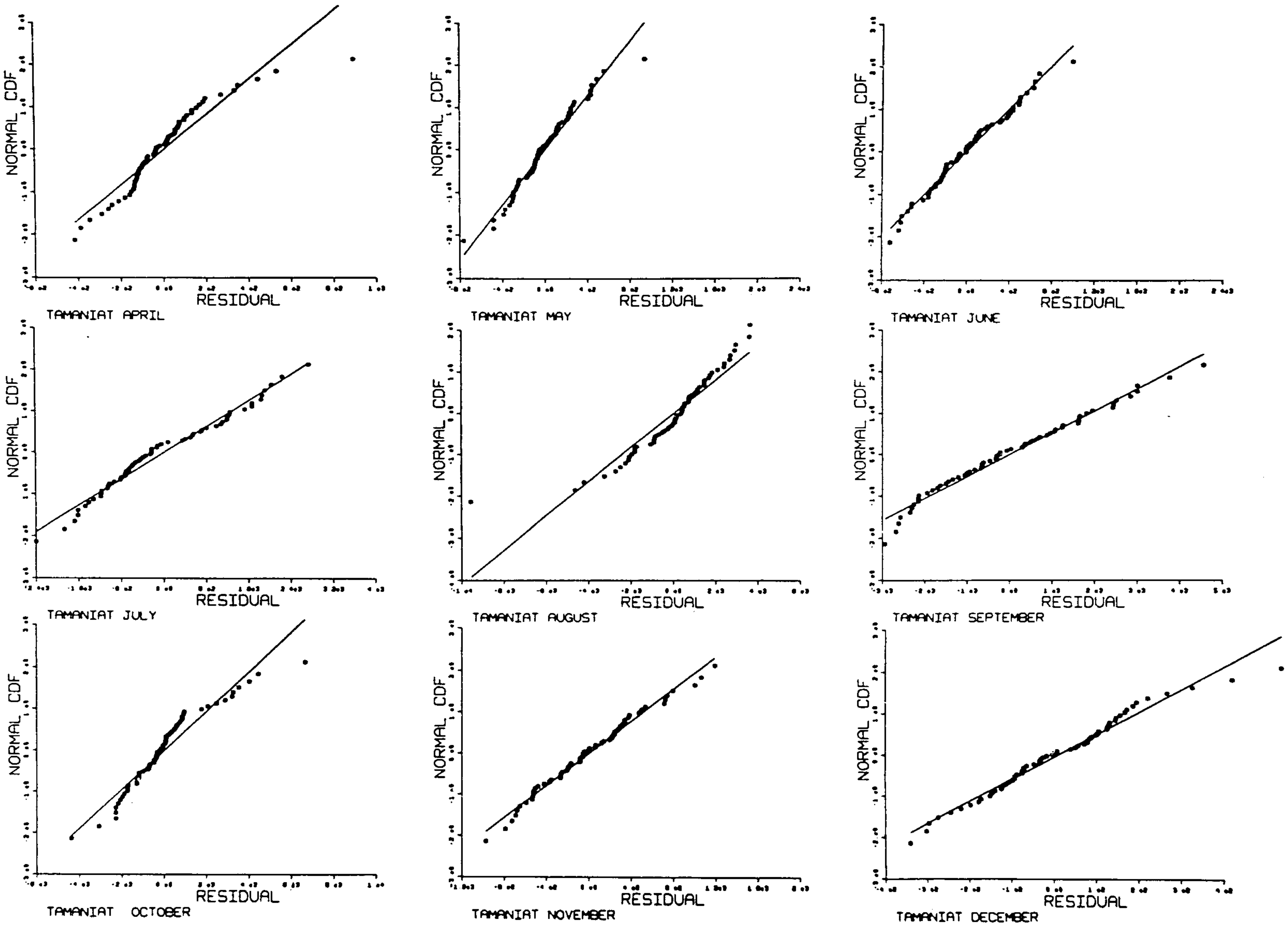


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

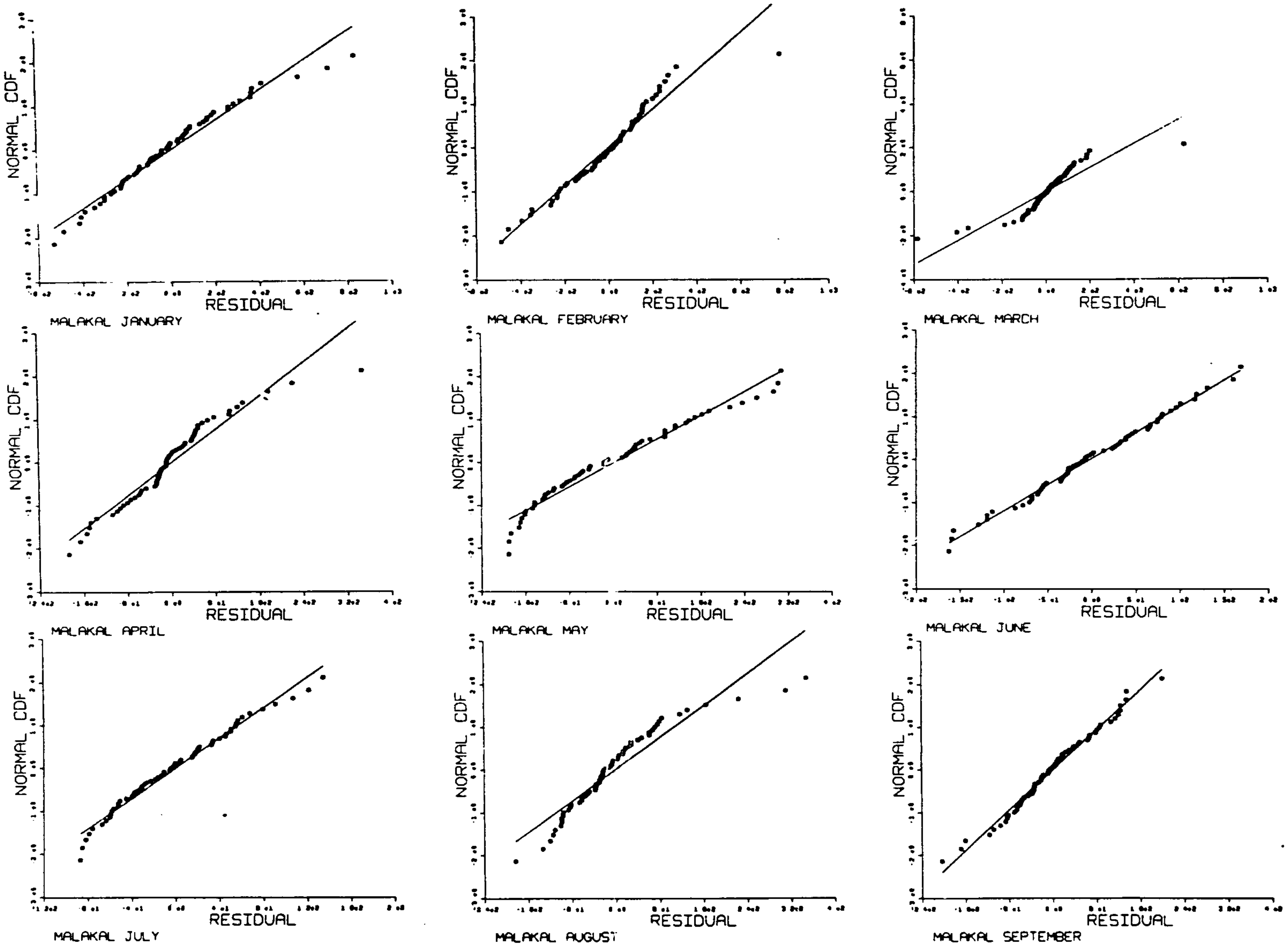


Figure 7.5 (cont'd)

Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

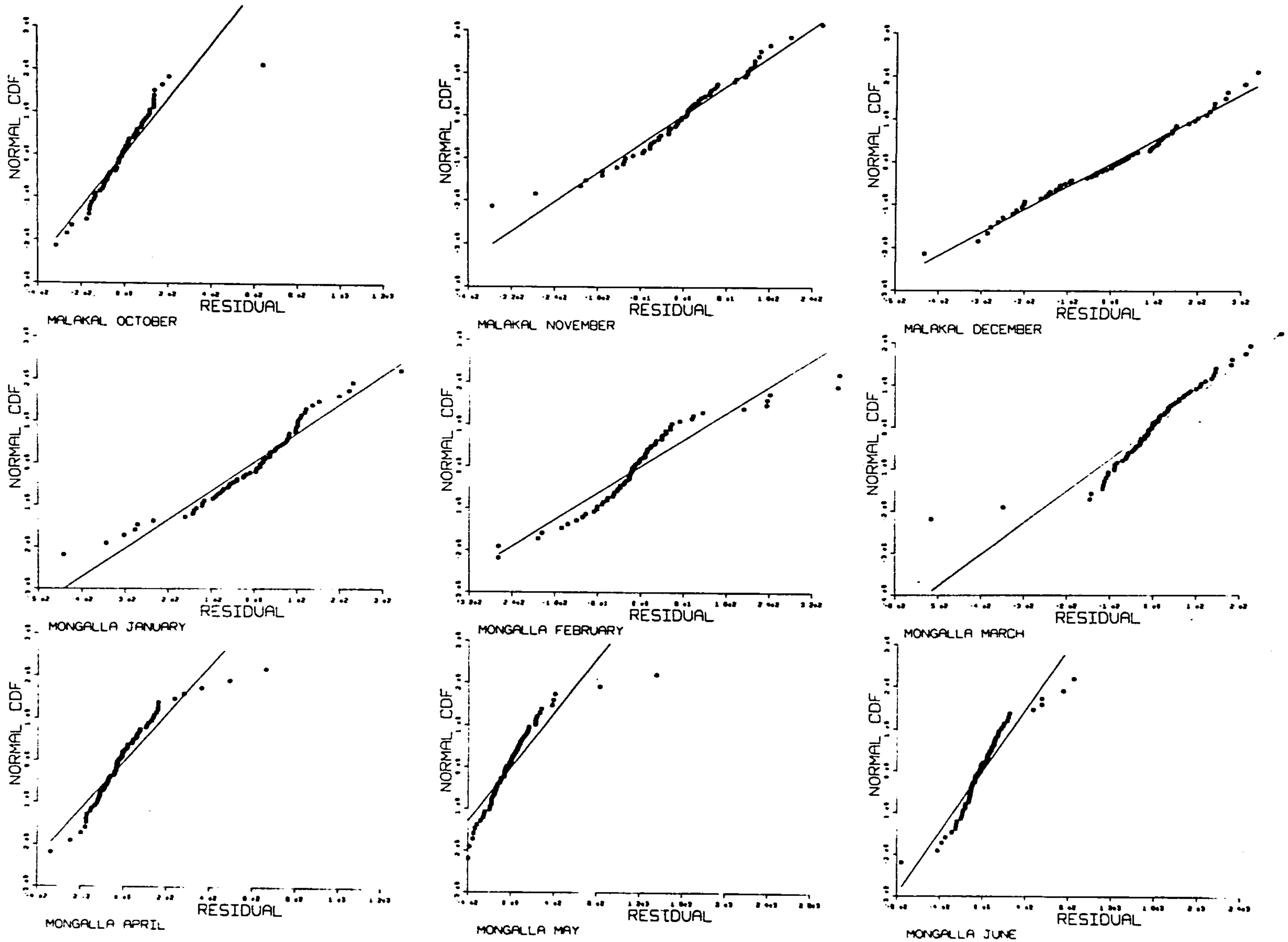


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

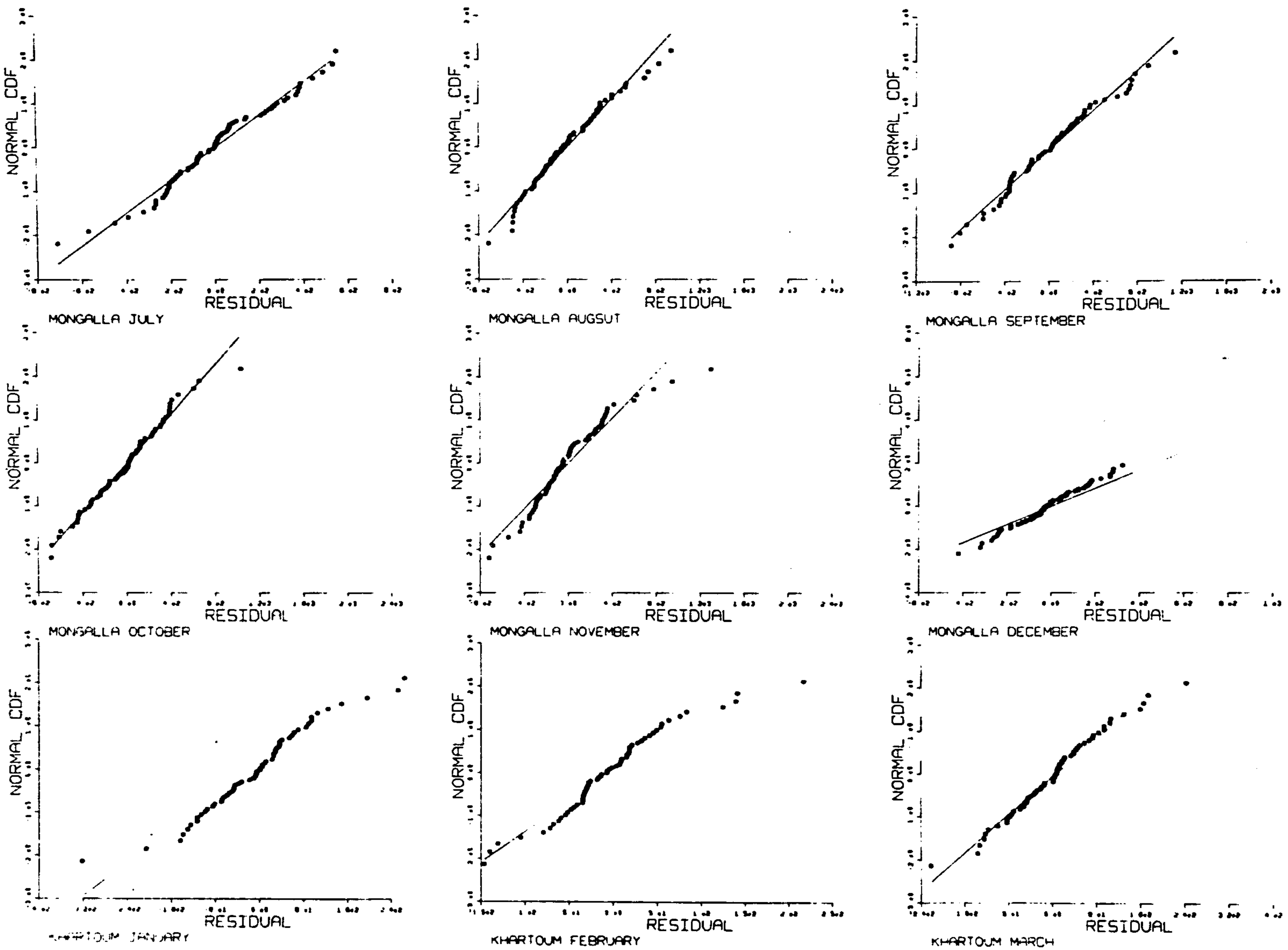


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

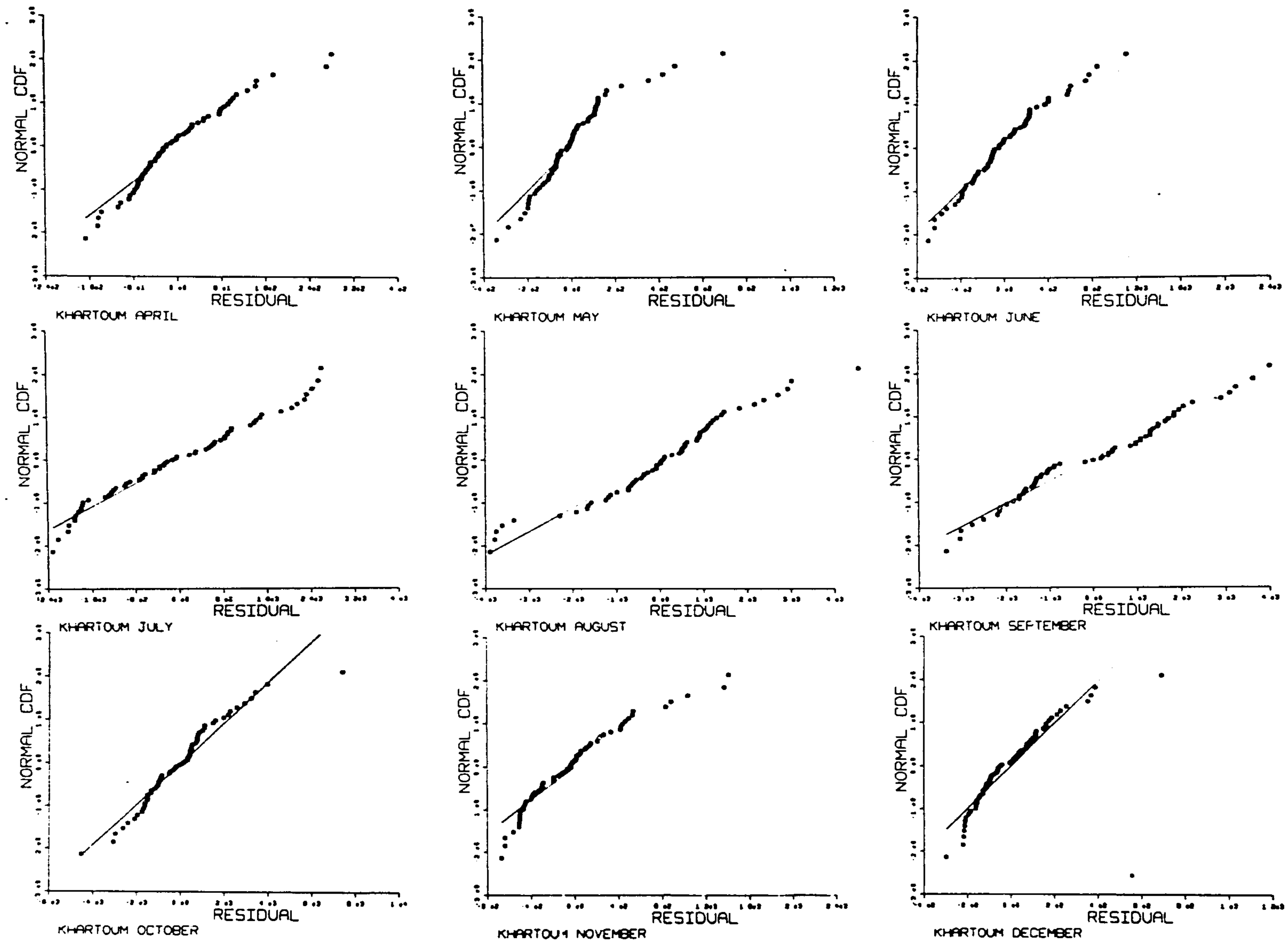


Figure 7.5
Normal Plots of Residuals Obtained from Each Equation of OLS Estimated Multivariate Model 5.31

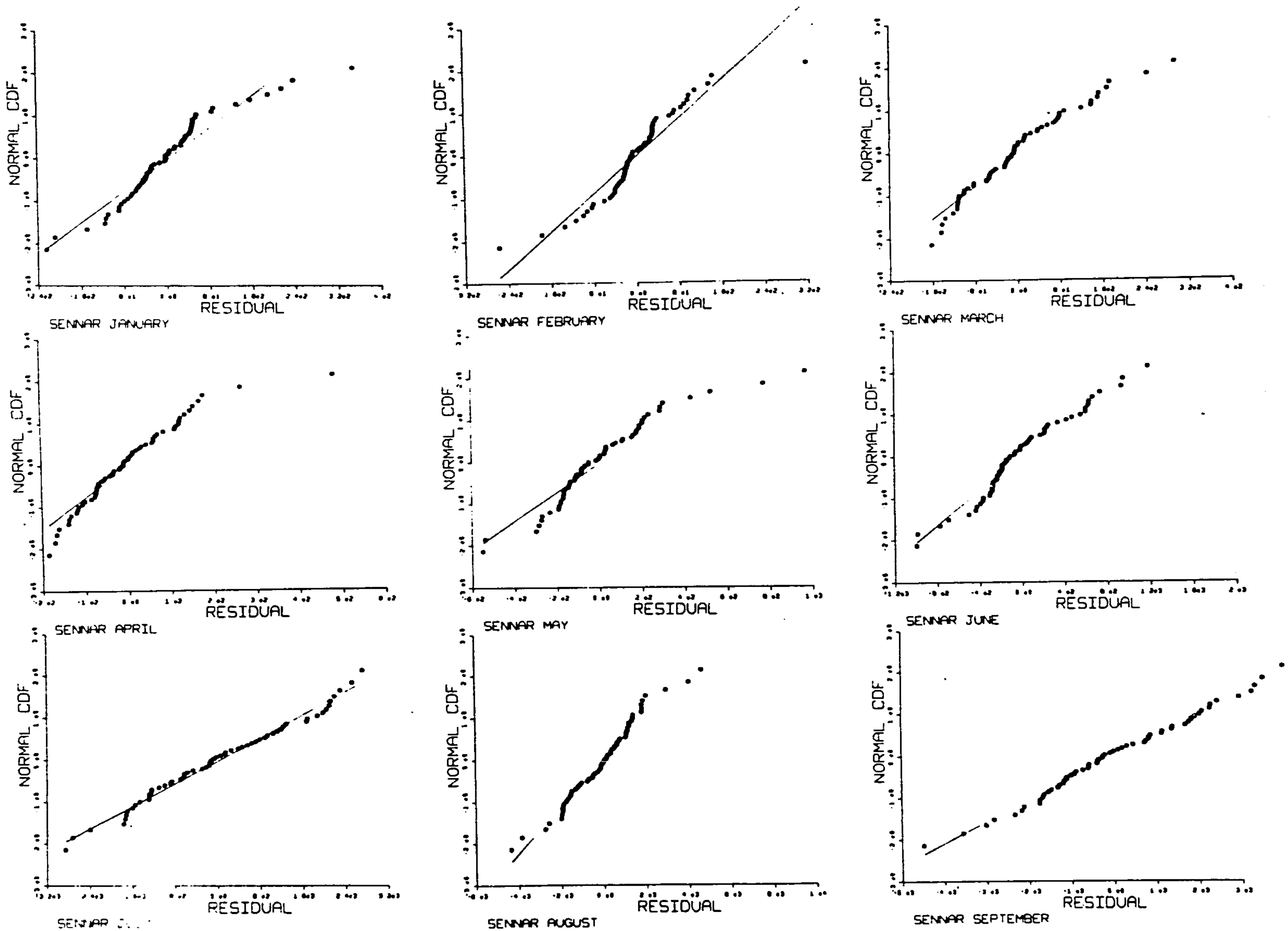


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

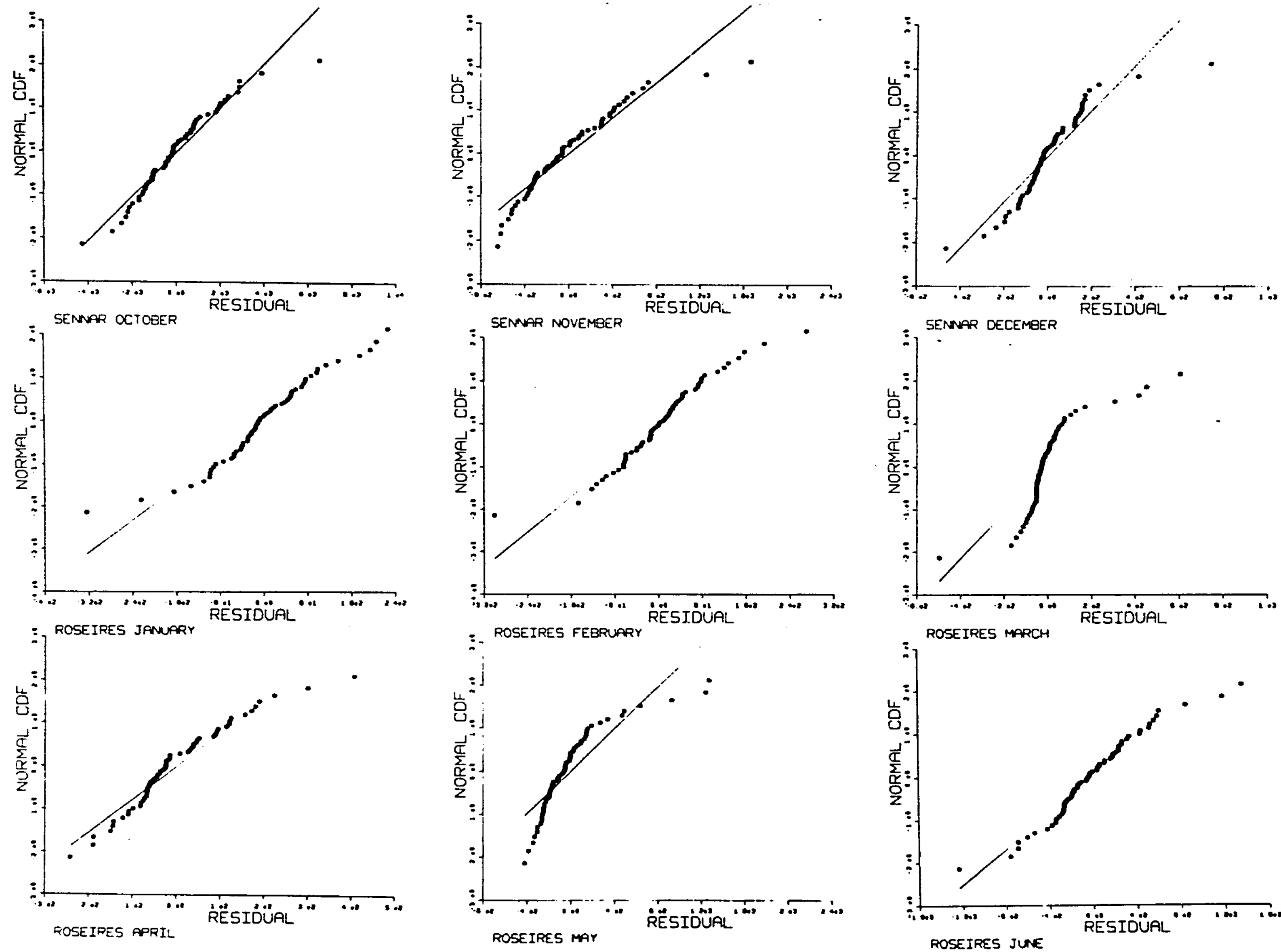


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

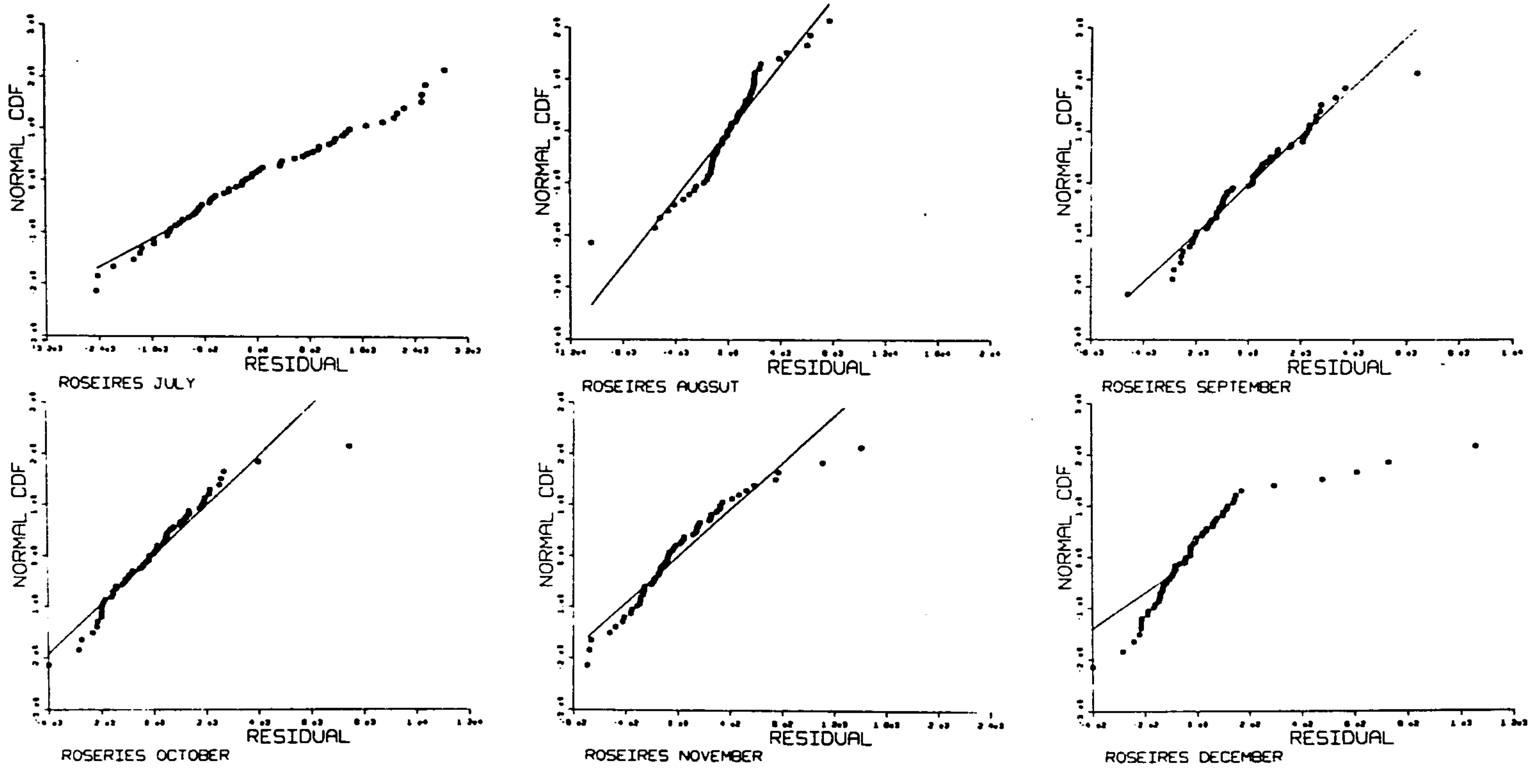


Figure 7.5 (cont'd)
Normal Plots of Residuals Obtained from each Equation of OLS Estimated Multivariate Model 5.31

For those residuals from equations for which the Durbin Watson test was not applicable, due to the presence of a first order autoregressive term, whiteness test number 2 was applied. The results are given in Table 7.11. The hypothesis of zero first order correlation in the residuals was not rejected at the 5% significance level in all cases except Atbara-May. However, for the Atbara-May case, the hypothesis was not rejected at the 1% level.

Tables 7.12 and 7.13 present correlation coefficients obtained from the application of whiteness test number 3 to two different OLS multivariate Models 5.31. Table 7.12 gives the cross-correlation function between Malakal and the Blue Nile gaging stations (i.e., Khartoum, Sennar, Roseires) residuals obtained from a multivariate model that assumed Malakal flows to be independent of Blue Nile flows. The assumption is consistent with the identified structure obtained in Section 5.5, which was based on causality arguments. However, the unusually large cross-correlation coefficients (the approximate standard error is .04) suggest that Blue Nile flows should be included as explanatory variables in Malakal equations. A good physical explanation which supports this action was found. Using the Nile Basin map of Figure 2.1, notice that the River Sobat contributes ungaged flow to Malakal. Chapter 2 estimated the Sobat yield to the 13.5 milliard m^3 per year as compared to a total yield at Malakal of 28.0 to 29.0 milliard m^3. The River Sobat originates in the Ehtiopian Highplains as does the Blue Nile. This, even if the Blue Nile flows are not physically causal to the River Sobat flows, and hence Malakal flows, they will be causal relative to the limited information set of observed flows (see Section 5.5) since they are a surrogate measure of the precipitation which influences Malakal flow via the Sobat flows at Roseires. Thus, the Blue Nile stations were incorporated as explanatory variables in the Malakal equations. The resulting final structure and OLS estimated coefficien's were previously given in Table 7.2 Table 7.13 presents the full set of correlation coefficients obtained for the application of whiteness test number 3 to the residuals from this multivariate model. Notice that in comparison to Table 7.12, the cross-correlation coefficients between Malakal and the Blue Nile gaging stations residuals have been significantly reduced.

Comparing the various correlation coefficients of Table 7.13 to their approximate standard error (.04), it can be seen that some values are somewhat greater than would be expected for white noise.

Figure 7.5 shows the normal plots of the residuals obtained from each monthly model of each gaging station of the OLS estimated multivariate Model 5.31 given in Table 7.2. The majority of the plots do not indicate the assumption of normality is violated. However, some plots for the residuals obtained from Atbara, Malakal, Mongalla and Roseires flows, especially Atbara, are clearly non-Gaussian. The implication is that the validity of the partial F test of Algorithm 5.113 used for identification is questionable.

Results of residual analysis done on the GLS estimated multivariate Model 5.31 (given in Table 7.2) are presented in Tables 7.14 through 7.16. The Durbin Watson test for correlated residuals is given

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Wadihalfa	2.293	1.738	2.006	1.876	2.065	2.188	1.872	2.016	2.108	2.272	2.615	2.310
H: ρ=0	I	NR	NR*	NR*	NR*	NR	NR*	NR	NR	NR	I	I
Atbara		1.125	1.928					1.962	2.360	1.570	2.230	
H: ρ=0	NA	R	NR	NA	NA	NA	-	NR	I	I	NR	NA
Tamaniat	2.029	1.732		2.297	1.742	2.307	1.568	2.299	1.989	2.375	2.222	
H: ρ=0	NR*	NR	NA	I	I	I	I	NR	NR	I	NR*	NA
Malakal	2.106	1.670	2.535	1.719	2.277	2.036	1.786	2.197	2.386	1.956	1.421	1.757
H: ρ=0	NR	I	I	I	NR	NR*	NR	NR*	I	NR	I	I
Mongalla	2.479	1.577	1.736	1.583	1.965	1.745	2.543	2.145	1.878		1.631	1.931
H: ρ=0	I	I	NR	I	NR	NR	I	NR	NR	NA	I	NR
Khartaum	1.366	1.791	1.551	1.783	1.877	1.916	1.818	2.264	2.314	2.405	2.078	
H: ρ=0	R*	NR*	I	NR	NR	NR	NR	NR	I	I	NR	NA
Sennar	1.282	1.296	1.665	1.673	1.998	1.864	1.688	2.180	2.026	2.362	2.080	1.923
H: ρ=0	R	R	I	I	NR	NR	NR	NR	NR	I	NR	NR
Roseires	2.160		2.033	1.757	1.957	1.824	1.767	2.231	2.015	2.412	2.100	
H: ρ=0	NR	NA	NR	NR	-	NR	NR	NR	NR	I	NR	NA

Table 7.14a
Results of Durbin Watson Test on Residuals Obtained from GLS Estimated Multivariate Model 5.31

K	5% Level d_L	d_U
2	1.49	1.64
3	1.45	1.68
4	1.41	1.72
5	1.38	1.77
>5	<1.38	>1.77

N = 55

K = number of explanatory variables

N = number of observations

NR = hypothesis not rejected (H: ρ=0)

I = inconclusive results

R = hypothesis rejected (H: ρ=0)

NA = test not applicable (due to lagged autoregressive term)

Superscript * = conclusion is tentative since upper and lower limits are not precisely known

Table 7.14b

Results of Durbin Watson Test on Residuals Obtained from GLS Estimated Multivariate Model 5.31; Upper (d_U) and Lower (d_L) Limits

Model for	Test Statistics	H: ρ=0
Atbara January	-0.030	NR
Atbara April	1.273	NR
Atbara May	-2.324	R(5%); NR(1%)
Atbara June	-0.526	NR
Atbara December	-0.430	NR
Tamaniat March	0.300	NR
Tamaniat December	-0.363	NR
Mongalla October	-0.020	NR
Khartaum December	-0.914	NR
Roseires February	-0.813	NR
Roseires December	0.871	NR

NR = hypothesis of $\rho=0$ not rejected

R = hypothesis of $\rho=0$ rejected

Table 7.15

Results of Whitness Test No. 2 on Residuals Obtained from GLS Estimated Multivariate Model 5.32

LAG WADI HALFA VS WADI HALFA

1-16	-0.000	0.049	0.008	0.012	-0.013	-0.011	-0.015	-0.003	-0.004	-0.019	-0.040	-0.093	0.130	0.022	-0.002	-0.004
17-32	-0.009	0.000	-0.015	-0.008	0.002	0.086	-0.046	0.065	-0.086	-0.005	0.021	-0.014	0.006	0.001	0.014	-0.011
33-48	0.002	0.037	0.059	-0.063	0.111	-0.027	-0.021	0.002	-0.004	-0.011	0.002	0.003	-0.010	-0.075	0.028	0.011

LAG WADI HALFA VS ATBARA

1-16	0.079	0.081	0.075	0.011	-0.003	0.001	-0.003	0.000	-0.011	0.020	-0.018	-0.038	0.078	0.085	0.016	-0.009
17-32	0.003	0.007	0.006	0.001	0.009	0.019	-0.058	-0.045	-0.023	-0.010	0.013	-0.009	0.012	0.004	0.004	0.003
33-48	-0.002	0.027	0.014	-0.129	0.044	0.002	-0.057	0.012	0.003	-0.002	0.005	-0.003	-0.010	-0.011	0.049	0.044

LAG WADI HALFA VS TAMANIAT

1-16	0.071	0.067	0.010	0.019	-0.004	-0.022	-0.005	-0.001	-0.026	-0.014	-0.060	-0.096	0.113	0.076	-0.007	-0.016
17-32	-0.013	0.008	-0.024	-0.011	0.024	0.089	-0.075	0.058	-0.115	-0.025	0.022	-0.002	-0.001	-0.004	0.005	-0.001
33-48	0.013	0.042	0.109	-0.024	0.136	-0.065	0.005	0.008	0.001	-0.004	-0.015	-0.002	-0.024	-0.064	0.042	0.051

LAG WADI HALFA VS MALAKAL

1-16	0.029	-0.004	-0.010	0.035	-0.022	-0.066	-0.020	0.032	0.021	0.008	-0.004	0.027	0.006	0.043	-0.001	-0.016
17-32	-0.044	-0.019	-0.043	-0.128	-0.030	0.008	-0.028	-0.089	-0.020	0.010	-0.028	0.000	-0.031	-0.029	-0.032	0.004
33-48	-0.078	-0.031	0.002	0.013	-0.020	-0.030	-0.012	0.026	0.009	0.020	0.077	0.011	0.058	-0.024	0.016	0.019

LAG WADI HALFA VS MONGALLA

1-16	0.113	0.103	0.010	0.039	0.005	0.002	0.012	-0.057	0.017	0.002	-0.044	-0.025	0.005	0.091	-0.017	0.009
17-32	-0.024	0.042	-0.020	0.012	-0.009	0.039	0.061	-0.058	-0.131	-0.045	-0.002	-0.050	-0.045	-0.012	0.001	-0.031
33-48	-0.005	0.116	-0.015	0.067	0.010	-0.066	-0.075	0.024	0.023	-0.012	-0.008	-0.013	-0.106	-0.118	0.089	0.028

LAG WADI HALFA VS KHARTOUM

1-16	0.051	0.040	0.021	0.033	-0.012	0.004	-0.002	0.003	-0.024	0.002	-0.051	-0.141	0.070	0.049	-0.023	-0.013
17-32	-0.011	0.006	-0.004	-0.012	0.012	0.081	-0.054	0.044	-0.140	-0.073	-0.004	-0.009	0.000	-0.000	-0.010	0.008
33-48	0.020	0.055	0.073	-0.048	0.087	-0.076	-0.016	0.008	0.003	-0.006	0.001	0.001	-0.015	-0.046	0.020	0.003

LAG WADI HALFA VS SENNAR

1-16	0.100	0.040	0.021	0.030	-0.015	0.002	0.002	-0.000	-0.019	0.012	-0.043	-0.116	0.081	0.056	-0.013	-0.021
17-32	-0.012	0.002	-0.003	-0.009	0.017	0.073	-0.042	0.007	-0.112	-0.038	-0.001	-0.026	-0.001	-0.004	-0.011	0.002
33-48	0.021	0.051	0.099	-0.065	0.051	-0.090	-0.012	0.005	0.002	-0.003	0.004	-0.008	-0.022	-0.066	0.038	0.031

Table 7.16

Results of Whiteness Test No. 3 on Residuals Obtained from GLS Estimated Model 5.31

LAG WADI HALFA VS ROSEIRES

1-16	0.064	0.025	0.019	0.031	-0.006	0.002	0.004	0.009	-0.017	0.023	-0.075	-0.130	0.066	0.047	-0.016	-0.022
17-32	-0.012	0.006	-0.001	-0.006	0.000	0.067	-0.025	0.019	-0.136	-0.061	-0.035	-0.018	0.006	-0.001	-0.005	0.001
33-48	0.012	0.050	0.054	-0.101	0.075	-0.089	-0.012	-0.002	0.006	-0.004	0.002	0.000	-0.015	-0.052	0.018	-0.004

LAG ATBARA VS ATBARA

1-16	0.079	0.040	0.007	0.003	0.000	0.000	-0.002	-0.002	-0.009	0.016	-0.068	-0.099	0.079	0.112	-0.004	-0.005
17-32	-0.000	-0.002	-0.000	0.002	-0.003	0.013	-0.016	-0.060	-0.043	-0.043	0.014	-0.001	0.002	0.000	0.001	-0.000
33-48	0.001	0.023	0.009	-0.179	-0.012	0.005	-0.010	0.001	-0.001	0.002	-0.001	-0.002	-0.023	-0.039	0.001	-0.069

LAG ATBARA VS TAMANIAT

1-16	0.097	0.028	0.005	-0.008	-0.008	-0.011	0.015	-0.009	-0.061	-0.036	-0.151	-0.118	0.059	0.065	-0.009	-0.011
17-32	0.003	0.008	-0.008	0.013	0.019	0.063	-0.096	-0.060	-0.119	-0.032	0.008	0.007	-0.015	0.010	0.008	0.013
33-48	0.009	0.005	0.065	-0.142	0.060	-0.039	0.008	-0.011	0.008	0.008	-0.010	0.002	-0.055	-0.080	0.005	-0.041

LAG ATBARA VS MALAKAL

1-16	-0.007	-0.007	-0.039	-0.020	-0.045	-0.141	-0.022	-0.012	0.031	-0.016	-0.013	0.026	-0.003	0.038	-0.024	-0.012
17-32	-0.060	-0.006	-0.048	-0.130	0.018	0.003	-0.012	-0.061	-0.030	-0.013	-0.065	0.018	-0.018	0.044	-0.022	-0.030
33-48	-0.045	-0.014	-0.012	-0.014	-0.023	-0.042	0.017	-0.004	0.013	0.033	0.010	0.004	0.030	-0.045	0.048	-0.005

LAG ATBARA VS MONGALLA

1-16	0.047	0.089	0.071	0.012	-0.017	-0.006	-0.012	-0.021	0.041	-0.009	-0.125	-0.049	-0.024	0.098	0.000	-0.004
17-32	-0.008	-0.001	-0.027	0.032	-0.019	0.030	0.032	-0.060	-0.097	-0.040	-0.048	-0.027	-0.015	-0.007	0.009	-0.005
33-48	-0.012	0.031	-0.036	0.009	0.022	-0.036	-0.032	0.015	0.004	-0.018	-0.021	-0.036	-0.094	-0.135	0.100	-0.079

LAG ATBARA VS KHARTOUM

1-16	0.101	0.048	0.009	0.005	-0.003	-0.001	0.003	0.002	-0.046	-0.017	-0.102	-0.160	0.029	0.043	-0.003	-0.010
17-32	-0.001	0.001	-0.006	0.005	0.011	0.053	-0.081	-0.014	-0.101	-0.043	0.009	-0.004	-0.001	0.000	0.000	0.008
33-48	0.018	0.034	0.050	-0.114	0.050	-0.024	0.014	-0.004	0.001	0.001	0.004	-0.006	-0.056	-0.067	-0.011	-0.046

LAG ATBARA VS SENNAR

1-16	0.136	0.044	-0.012	-0.006	-0.003	-0.004	0.000	0.000	-0.031	0.023	-0.108	-0.135	0.030	0.039	-0.010	-0.015
17-32	-0.003	-0.000	-0.007	0.017	0.029	0.054	-0.025	-0.059	-0.088	-0.043	0.008	-0.011	-0.003	-0.000	-0.003	-0.002
33-48	0.006	0.019	0.008	-0.141	-0.007	-0.038	0.013	-0.006	-0.002	0.000	0.006	-0.010	-0.062	-0.102	-0.000	-0.054

Table 7.16 (cont'd)
Results of Whiteness Test No. 3 on Residuals Obtained from GLS Estimated Model 5.31

LAG ATBARA VS ROSEIRES

1-16	0.090	0.047	0.001	-0.003	-0.002	-0.005	-0.001	0.004	-0.039	0.032	-0.126	-0.132	0.050	0.040	-0.010	-0.010
17-32	-0.000	0.001	-0.007	0.007	0.011	0.031	-0.040	-0.051	-0.106	-0.042	0.000	-0.005	0.000	-0.001	-0.000	-0.011
33-48	0.003	0.011	0.009	-0.151	0.043	-0.017	0.010	-0.008	-0.001	-0.000	0.004	-0.005	-0.050	-0.090	0.017	-0.089

LAG TAMANIAT VS TAMANIAT

1-16	0.081	0.051	0.007	0.004	0.003	-0.024	0.003	0.002	-0.024	-0.023	-0.036	-0.083	0.076	0.099	-0.014	-0.025
17-32	-0.002	-0.000	-0.019	-0.010	0.031	0.054	0.009	0.057	-0.021	0.040	0.025	0.007	-0.001	-0.008	-0.016	-0.011
33-48	0.028	-0.029	0.071	-0.055	0.097	-0.039	-0.004	0.009	-0.010	0.004	-0.015	0.009	-0.036	-0.059	-0.016	-0.036

LAG TAMANIAT VS MALAKAL

1-16	0.033	-0.026	-0.001	0.014	-0.031	-0.074	-0.019	0.019	0.032	0.021	0.013	0.031	0.018	0.060	0.022	-0.030
17-32	0.008	-0.012	-0.044	-0.075	0.006	-0.007	-0.022	-0.049	0.001	0.029	-0.019	-0.009	-0.039	-0.022	0.004	-0.028
33-48	-0.010	-0.043	-0.012	0.006	-0.027	-0.019	-0.019	0.039	-0.009	-0.023	0.032	0.000	0.039	-0.023	0.001	0.018

LAG TAMANIAT VS MONGALLA

1-16	0.164	0.108	-0.003	0.058	0.015	0.014	-0.013	-0.032	-0.016	-0.005	-0.063	0.006	0.028	0.107	-0.019	0.029
17-32	-0.011	0.050	-0.014	0.005	0.014	0.052	0.092	-0.054	-0.106	-0.023	0.046	-0.017	-0.018	-0.021	0.020	-0.011
33-48	0.064	0.068	-0.003	0.021	0.024	-0.056	-.032	0.069	0.019	-0.011	0.004	0.010	-0.113	-0.039	0.107	0.041

LAG TAMANIAT VS KHARTOUM

1-16	0.063	0.020	0.028	0.013	0.001	0.007	-0.002	-0.000	-0.028	0.001	-0.025	-0.138	0.016	0.049	-0.039	-0.014
17-32	-0.007	-0.003	-0.008	-0.015	0.027	0.064	0.023	0.045	-0.055	-0.016	-0.006	0.005	-0.004	-0.006	-0.021	0.000
33-48	0.049	-0.013	0.051	-0.076	0.064	-0.042	-0.024	0.000	-0.004	0.001	-0.005	0.015	-0.015	-0.049	-0.032	-0.052

LAG TAMANIAT VS SENNAR

1-16	0.115	0.036	0.009	0.014	-0.003	0.008	0.002	0.003	-0.027	0.007	-0.013	-0.105	0.053	0.060	-0.024	-0.020
17-32	-0.011	-0.006	-0.004	-0.012	0.032	0.075	0.049	0.054	-0.005	0.027	0.005	-0.010	-0.002	-0.008	-0.023	0.000
33-48	0.049	0.013	0.061	-0.092	0.033	-0.074	-0.018	-0.003	-0.006	0.003	-0.005	0.000	-0.019	-0.052	-0.006	-0.040

LAG TAMANIAT VS ROSEIRES

1-16	0.088	0.014	0.005	0.010	-0.002	0.004	0.005	0.004	-0.034	0.020	-0.034	-0.122	0.039	0.030	-0.031	-0.027
17-32	-0.010	0.003	-0.007	-0.015	0.017	0.070	0.045	0.048	-0.063	-0.004	-0.025	-0.009	-0.004	-0.009	-0.013	-0.006
33-48	0.035	0.008	0.002	-0.100	0.062	-0.062	-0.021	-0.012	-0.001	-0.003	-0.002	0.011	-0.019	-0.050	-0.021	-0.065

Table 7.16 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from GLS Estimated Model 5.31

LAG	MALAKAL VS MALAKAL															
1-16	0.086	-0.102	0.026	0.043	0.069	-0.044	-0.029	-0.037	-0.036	-0.003	0.019	-0.021	-0.063	-0.003	0.007	0.030
17-32	-0.038	-0.045	-0.039	0.001	-0.026	-0.038	-0.004	-0.021	-0.057	0.035	0.002	0.002	-0.014	-0.003	-0.018	-0.014
33-48	0.036	-0.010	-0.004	-0.017	-0.040	-0.084	-0.039	0.016	0.041	0.016	-0.062	0.002	-0.034	0.017	0.009	-0.020

LAG	MALAKAL VS MONGALLA															
1-16	0.019	0.018	0.001	-0.011	0.126	-0.049	0.025	0.044	0.038	0.013	-0.037	-0.028	-0.024	-0.008	0.028	-0.018
17-32	0.050	0.061	-0.016	0.034	0.011	0.029	0.015	0.004	0.004	0.025	0.053	0.023	0.078	-0.017	-0.064	-0.033
33-48	0.060	-0.016	-0.014	-0.002	-0.016	0.019	-0.011	0.010	0.027	0.013	0.057	0.022	-0.051	-0.107	0.013	-0.008

LAG	MALAKAL VS KHARTOUM															
1-16	0.075	0.022	-0.029	0.005	0.045	0.060	0.002	-0.020	0.038	0.005	-0.014	-0.040	-0.043	0.012	0.050	0.138
17-32	-0.021	-0.032	0.055	0.018	0.013	0.031	0.005	-0.016	0.007	-0.037	-0.018	-0.028	0.001	0.022	-0.134	-0.019
33-48	0.044	-0.067	0.006	-0.001	-0.008	0.051	0.017	0.056	0.076	-0.009	0.039	0.046	-0.016	-0.026	-0.008	-0.037

LAG	MALAKAL VS SENNAR															
1-16	0.090	0.018	-0.009	0.019	0.050	0.081	0.023	-0.019	0.025	0.022	-0.009	-0.040	-0.034	0.009	0.045	0.147
17-32	-0.041	-0.065	0.059	0.021	-0.001	0.017	-0.007	-0.015	0.007	-0.035	-0.021	-0.039	0.031	0.028	-0.115	-0.017
33-48	0.024	-0.052	0.003	-0.007	-0.018	0.024	0.011	0.051	0.095	0.031	0.022	0.012	-0.013	-0.038	-0.013	-0.026

LAG	MALAKAL VS ROSEIRES															
1-16	0.074	-0.003	0.021	-0.000	0.055	0.103	0.006	-0.028	0.031	0.018	-0.009	-0.029	-0.045	0.012	0.039	0.144
17-32	-0.036	-0.048	0.088	0.011	-0.009	0.020	0.015	-0.008	-0.002	-0.036	-0.023	-0.038	0.062	0.052	-0.123	-0.023
33-48	0.045	-0.048	0.001	0.001	-0.013	0.047	0.024	0.049	0.075	-0.008	0.026	0.034	-0.008	-0.033	-0.012	-0.022

LAG	MONGALLA VS MONGALLA															
1-16	0.008	-0.037	0.057	0.007	0.036	0.015	-0.023	0.025	-0.013	0.071	0.013	-0.004	0.099	-0.008	0.004	0.020
17-32	-0.040	0.015	0.011	0.009	0.012	-0.002	-0.060	-0.011	0.032	-0.122	-0.035	-0.007	-0.034	0.040	-0.067	-0.014
33-48	0.034	0.063	-0.010	-0.051	0.007	0.019	-0.023	0.013	-0.007	-0.024	0.018	-0.073	-0.086	-0.028	0.044	-0.043

LAG	MONGALLA VS KHARTOUM															
1-16	0.020	-0.023	-0.015	0.032	-0.008	0.010	0.027	0.027	0.005	0.014	-0.014	-0.039	-0.033	0.015	-0.018	-0.034
17-32	-0.008	0.009	0.010	0.015	-0.001	-0.071	0.028	-0.023	-0.043	-0.084	0.013	-0.014	-0.023	0.007	0.052	-0.000
33-48	0.021	-0.017	-0.013	-0.004	0.038	-0.005	-0.002	0.029	0.002	0.016	-0.012	-0.002	-0.101	-0.022	-0.041	-0.071

Table 7.16 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from GLS Estimated Model 5.31

LAG MONGALLA VS SENNAR

1-16	0.052	-0.056	-0.049	0.003	-0.015	0.002	0.024	0.015	-0.007	0.005	0.019	-0.030	-0.012	0.014	-0.067	-0.049
17-32	-0.006	0.014	0.027	0.043	-0.003	-0.067	0.028	0.001	-0.012	-0.089	-0.013	-0.010	-0.016	0.001	0.056	0.011
33-48	0.035	-0.008	-0.001	-0.047	0.024	-0.008	-0.022	0.015	-0.001	0.003	-0.027	-0.035	-0.074	-0.055	-0.044	-0.078

LAG MONGALLA VS ROSEIRES

1-16	0.025	-0.039	-0.004	0.021	-0.002	0.007	0.018	0.012	-0.013	0.008	0.006	-0.033	-0.010	0.013	-0.058	-0.032
17-32	-0.001	0.019	0.022	0.029	-0.012	-0.035	0.037	-0.008	-0.008	-0.087	0.005	0.005	-0.022	0.008	0.046	0.013
33-48	0.016	-0.022	0.018	-0.035	0.058	-0.003	-0.026	0.013	0.012	0.006	-0.014	-0.013	-0.083	-0.041	-0.048	-0.063

LAG KHARTOUM VS KHARTOUM

1-16	0.051	0.029	0.020	0.014	0.001	-0.001	0.003	-0.003	-0.037	0.008	-0.050	-0.140	0.010	0.041	-0.038	-0.026
17-32	-0.002	0.002	-0.005	-0.011	0.014	0.055	0.057	0.048	-0.121	-0.020	-0.008	-0.005	-0.009	0.001	-0.002	0.005
33-48	0.041	-0.033	0.060	-0.020	0.059	-0.023	-0.012	0.010	-0.005	-0.010	-0.008	-0.001	-0.027	-0.073	-0.056	-0.081

LAG KHARTOUM VS SENNAR

1-16	0.110	0.041	-0.007	0.011	-0.008	-0.007	0.000	-0.007	-0.042	0.009	-0.041	-0.097	0.051	0.042	-0.027	-0.029
17-32	-0.005	0.003	-0.001	-0.007	0.017	0.057	0.090	0.031	-0.071	0.009	-0.005	-0.024	-0.011	-0.002	-0.001	0.006
33-48	0.043	-0.012	0.059	-0.043	0.020	-0.062	-0.010	0.008	-0.010	-0.010	-0.008	-0.016	-0.035	-0.074	-0.030	-0.075

LAG KHARTOUM VS ROSEIRES

1-16	0.088	0.021	0.005	0.005	-0.010	-0.008	0.003	-0.003	-0.048	0.028	-0.046	-0.106	0.054	0.032	-0.029	-0.035
17-32	-0.009	0.006	-0.002	-0.008	0.003	0.062	0.091	0.034	-0.108	-0.016	-0.027	-0.022	-0.012	-0.003	-0.000	0.003
33-48	0.028	-0.018	0.028	-0.025	0.063	-0.035	-0.006	-0.002	-0.009	-0.010	-0.008	-0.008	-0.038	-0.061	-0.033	-0.088

LAG SENNAR VS SENNAR

1-16	[illegible]	0.035	-0.009	0.016	-0.007	-0.005	0.002	-0.014	-0.005	0.029	-0.004	-0.073	0.052	0.031	-0.045	-0.044
17-32	-0.008	0.005	0.004	-0.000	0.019	0.059	0.074	-0.056	-0.084	-0.017	-0.006	-0.028	-0.010	-0.007	-0.002	-0.003
33-48	0.052	0.019	0.029	-0.052	0.012	-0.063	0.006	0.010	-0.003	-0.005	-0.005	-0.002	-0.021	-0.066	-0.044	-0.094

LAG SENNAR VS ROSEIRES

1-16	0.075	0.017	0.003	0.011	-0.007	-0.005	0.002	-0.011	-0.012	0.025	-0.028	-0.087	0.033	0.025	-0.048	-0.048
17-32	-0.011	0.008	0.002	-0.002	-0.002	0.053	0.057	-0.055	-0.122	-0.037	-0.025	-0.024	-0.010	-0.007	-0.001	-0.004
33-48	0.034	0.001	-0.001	-0.050	0.067	-0.036	0.000	-0.002	-0.005	-0.005	-0.007	0.001	-0.023	-0.063	-0.048	-0.123

LAG ROSEIRES VS ROSEIRES

1-16	0.093	-0.000	0.022	0.025	-0.006	-0.002	0.003	-0.017	-0.022	0.062	-0.016	-0.102	0.060	0.061	-0.031	-0.042
17-32	-0.007	0.011	-0.002	0.001	-0.007	0.044	0.073	-0.001	-0.095	-0.006	-0.028	-0.017	-0.001	-0.002	-0.005	0.001
33-48	0.025	0.018	0.019	-0.025	0.068	-0.012	0.032	0.005	-0.001	-0.005	0.001	-0.004	-0.030	-0.059	-0.010	-0.101

Table 7.16 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from GLS Estimated Model 5.31

in Table 7.14. The test was performed at the 5% significance level and in only four cases was the hypothesis of zero correlation rejected. Table 7.15 gives the results of whiteness test number 2 which was applied to the residuals from monthly equations which have an autoregressive term. The hypothesis of uncorrelated residuals was tested at the 5% significance level and was not rejected in all cases except for Atbara-May. However, the hypothesis of zero correlation in the residuals from the Atbara-May equation was not rejected at the 1% significance level.

The full set of correlation coefficients obtained from the application of whiteness test number 3 to the residuals of the GLS estimated multivariate Model 5.31 is given Table 7.16. Assuming whiteness, the approximate standard error should be about .04. It can be seen that a few more values exceed the approximated 2 sigma level (.08) than expected. However, considering the limit is only an approximation, the comparison does not clearly indicate correlated residuals.

7.4.1.2 Model 5.32

Whiteness Test No. 2 could not be applied to the residuals of the OLS estimated (i.e., $\Psi_1^*(i)^2 = 1 \ \forall i$) univariate Mode. 5.32 (specified in Table 7.4) since the term $nVar(\hat{\beta}_j(1))$ (see Equation 5.122) exceeded one. Table 7.17 reports the autocorrelation coefficients of the residuals calculated via Equation 5.123. Although more values exceed the approximate 2σ level (.06) than expected, the hypothesis of white noise is not clearly violated.

LAG	WADI HALFA VS WADI HALFA															
1-16	-0.002	-0.014	0.001	0.035	0.013	0.007	0.009	-0.007	0.013	0.031	0.054	-0.088	0.101	-0.037	0.000	-0.018
17-32	-0.006	0.005	-0.024	-0.014	0.010	0.034	0.044	0.077	0.071	-0.021	0.015	0.002	-0.006	-0.000	-0.021	-0.015
33-48	-0.008	0.029	0.110	-0.001	0.033	-0.005	-0.000	0.002	-0.013	-0.013	-0.013	-0.014	0.018	-0.067	0.051	0.051

Table 7.17
Autocorrelation Function of Residuals Obtained from Univariate Model 5.32 Estimated with

$$\Psi_1^*(i)^2 = 1 \quad \forall i$$

Following the iterative approach to model building, the autocorrelation function was used to identify other models to be tested. Several models with an autoregressive or moving average term with lag corresponding to a lag for which the autocorrelation function has a high value (e.g., lag 12 and 13) were evaluated. No model was found with better properties.

Table 7.18 presents the results from the application of Whiteness Test No. 2 to the residuals of multivariate Model 5.32 as given in Table 7.6. Notice that the test was not performed on the residuals of the Wadi Halfa, Tamaniat or Khartoum models. Tamaniat and Khartoum residuals could not be tested since the model does not contain a first order lagged autoregressive term. The residuals from the Wadi Halfa model could not be tested due to the term $nVar(\underline{\beta}_j(i))$ (see Equation 5.122) exceeding one. For the models tested, the hypothesis of zero correlation in the residuals was rejected at the 5% level only for Malakal. The hypothesis was not rejected at the 1% level for any model.

The full set of correlation coefficients obtained from the application of Whiteness Test No. 3 to the residuals of the estimated Multivariate Model 5.32 (specified in Table 7.6) is given in Table 7.19. Under the hypothesis of white noise, the coefficients compare favorably the approximate 2 sigma level (.08). Motivated by some of the coefficients with unusually large values (e.g., Wadi Halfa vs. Wadi Halfa lag 13, Atbara vs. Atbara lag 12, Atbara vs. Roseires lag 12) models were evaluated with a regression or moving term of the indicated station and

	Test Statistics	H: $\rho=0$
Model for Atbara	-0.125	NR
Malakal	2.202	R(5%),NR(1%)
Mongalla	1.077	NR
Sennar	-0.079	NR
Roseires	1.599	NR

Table 7.18 Results of Whitness Test No. 2 on Residuals Obtained from Multivariate Model 5.32 Estimated with $\Psi_j^*(i)2 = 1 \; \forall_{i,j}$

NR = Not Rejected at the 5% Level

R = Rejected at the 5% Level

LAG	WADI HALFA VS WADI HALFA															
1-16	0.034	0.002	-0.052	-0.025	-0.036	-0.064	-0.048	-0.007	0.006	-0.051	0.001	-0.038	0.206	0.030	-0.058	-0.024
17-32	-0.031	-0.061	-0.078	-0.041	0.019	0.119	0.039	0.063	-0.075	0.003	0.001	-0.010	-0.015	-0.051	-0.061	-0.049
33-48	-0.009	0.044	0.153	-0.021	0.156	-0.039	-0.025	0.009	-0.043	-0.046	-0.049	0.003	-0.013	-0.036	0.001	0.062

LAG	WADI HALFA VS ATBARA															
1-16	-0.005	0.023	-0.009	-0.027	0.006	0.010	0.005	0.002	0.003	-0.007	0.030	-0.102	0.123	0.041	-0.024	-0.019
17-32	-0.000	0.001	0.008	0.020	0.018	0.040	-0.013	-0.013	-0.055	-0.002	-0.003	-0.008	0.005	0.009	-0.009	-0.014
33-48	-0.011	0.038	0.063	-0.126	0.079	-0.027	-0.030	0.010	-0.002	0.006	-0.000	0.014	0.002	-0.010	0.032	0.060

LAG	WADI HALFA VS TAMANIAT															
1-16	0.062	0.010	-0.020	0.007	-0.010	-0.045	-0.024	0.013	0.025	-0.081	-0.044	-0.063	0.167	0.080	-0.036	-0.017
17-32	0.008	-0.026	-0.075	-0.051	0.004	0.077	0.013	0.036	-0.088	0.029	0.026	0.011	0.012	-0.023	-0.066	-0.067
33-48	-0.032	-0.015	0.097	-0.010	0.124	-0.033	0.018	0.026	0.005	-0.016	-0.026	0.012	-0.013	-0.052	-0.041	0.011

LAG	WADI HALFA VS MALAKAL															
1-16	0.044	-0.036	-0.035	0.047	0.022	0.033	-0.043	0.056	0.050	-0.068	-0.065	-0.031	-0.049	0.002	0.005	0.020
17-32	-0.009	0.002	-0.016	-0.065	-0.007	-0.034	-0.053	-0.058	-0.016	0.001	-0.028	0.036	0.020	0.014	-0.026	0.025
33-48	-0.005	-0.017	0.031	0.044	0.016	-0.023	0.005	0.037	0.057	-0.026	0.028	0.018	0.009	0.007	0.011	0.009

LAG	WADI HALFA VS MONGALLA															
1-16	0.068	0.065	-0.003	-0.000	-0.035	-0.002	0.005	0.012	0.053	-0.004	-0.054	-0.029	0.019	0.085	0.021	0.001
17-32	-0.034	0.013	-0.014	0.046	0.048	0.045	0.087	-0.090	-0.099	-0.030	0.005	-0.032	-0.056	-0.017	0.006	-0.011
33-48	0.011	0.082	-0.002	0.078	0.040	-0.030	-0.066	0.016	-0.029	-0.061	0.013	0.030	-0.069	-0.085	0.039	0.024

LAG	WADI HALFA VS KHARTOUM															
1-16	0.032	-0.023	-0.017	-0.004	0.006	0.006	-0.002	0.018	0.026	-0.061	-0.032	-0.104	0.123	0.056	-0.035	-0.010
17-32	0.012	0.005	-0.009	-0.012	0.017	0.070	-0.004	-0.001	-0.133	-0.016	-0.001	-0.000	0.008	0.001	-0.025	-0.016
33-48	0.009	0.004	0.093	-0.041	0.063	-0.055	-0.016	0.009	0.002	-0.003	-0.003	0.039	0.023	-0.030	-0.039	0.009

LAG	WADI HALFA VS SENNAR															
1-16	0.042	-0.041	-0.015	0.001	0.011	0.004	0.001	0.012	0.017	-0.050	-0.019	-0.108	0.127	0.059	-0.027	-0.010
17-32	0.016	-0.002	-0.015	-0.012	0.017	0.058	-0.012	-0.032	-0.117	-0.005	0.009	-0.006	0.013	0.003	-0.020	-0.015
33-48	0.013	0.010	0.095	-0.074	0.057	-0.051	0.003	0.019	0.007	-0.003	-0.004	0.024	-0.001	-0.065	-0.032	0.052

Table 7.19

Results of Whiteness Test No. 3 on Residuals Obtained from Multivariate Model 5.32

Estimated with $\Psi_1^*(i)^2 = 1 \quad \forall i, j$

LAG WADI HALFA VS ROSEIRES

1-16	0.007	-0.034	-0.019	-0.004	0.003	0.010	0.016	0.040	0.034	-0.028	-0.056	-0.090	0.127	0.031	-0.035	-0.015
17-32	0.012	0.006	-0.002	0.003	0.029	0.083	-0.002	-0.023	-0.134	-0.017	-0.028	-0.009	0.007	0.011	-0.004	-0.007
33-48	0.021	0.037	0.072	-0.065	0.052	-0.061	-0.019	-0.003	0.012	0.005	0.003	0.041	0.024	-0.035	-0.017	-0.001

LAG ATBARA VS ATBARA

1-16	-0.003	0.012	0.011	0.009	0.004	0.001	-0.002	-0.000	-0.015	0.037	0.040	-0.142	0.180	0.039	-0.025	0.007
17-32	0.001	-0.001	-0.001	0.001	-0.003	0.009	-0.031	0.004	-0.085	-0.000	0.017	-0.006	0.002	0.000	0.001	-0.002
33-48	0.002	-0.009	0.037	-0.230	0.035	-0.012	-0.012	0.004	0.001	0.001	-0.000	0.002	-0.008	-0.024	-0.015	-0.047

LAG ATBARA VS TAMANIAT

1-16	0.034	0.018	0.004	0.021	0.002	-0.003	0.017	0.018	-0.027	-0.070	-0.028	-0.115	0.094	0.021	-0.028	0.002
17-32	0.024	0.007	-0.017	-0.013	0.019	0.054	-0.096	0.019	-0.105	0.003	0.004	-0.006	0.010	0.012	-0.002	0.014
33-48	-0.002	0.012	0.017	-0.176	0.034	-0.057	0.010	-0.006	0.020	0.021	0.013	0.026	-0.030	-0.066	-0.065	-0.012

LAG ATBARA VS MALAKAL

1-16	0.005	0.003	0.022	0.020	0.015	-0.043	-0.101	0.013	0.058	-0.057	-0.021	0.015	-0.020	0.042	0.013	0.011
17-32	-0.019	-0.053	-0.012	-0.122	0.013	-0.031	-0.054	-0.034	-0.007	0.030	-0.051	0.008	0.017	0.034	-0.049	-0.047
33-48	-0.042	-0.028	-0.010	0.020	0.016	-0.054	0.030	-0.000	0.083	0.032	-0.003	-0.025	-0.009	-0.016	0.035	-0.010

LAG ATBARA VS MONGALLA

1-16	0.011	0.061	0.033	0.005	-0.025	0.020	-0.010	-0.012	0.023	-0.027	-0.090	0.013	0.016	0.060	0.002	-0.021
17-32	0.002	0.009	-0.025	0.034	0.036	0.010	-0.023	-0.055	-0.069	-0.017	-0.021	-0.054	0.000	0.009	0.024	0.000
33-48	0.003	0.058	-0.077	-0.030	0.065	-0.032	-0.062	-0.010	-0.010	-0.029	-0.000	-0.009	-0.059	-0.084	0.080	-0.066

LAG ATBARA VS KHARTOUM

1-16	0.066	0.025	-0.002	0.008	0.002	-0.000	-0.003	0.014	-0.016	-0.050	0.019	-0.130	0.078	0.031	-0.014	-0.006
17-32	0.000	0.003	-0.009	-0.018	-0.000	0.039	-0.111	0.015	-0.106	0.004	0.010	-0.010	0.002	-0.001	-0.003	0.013
33-48	-0.004	0.019	0.035	-0.143	0.038	-0.034	0.016	-0.005	0.001	0.003	0.015	0.025	-0.015	-0.060	-0.059	-0.004

LAG ATBARA VS SENNAR

1-16	0.075	0.004	-0.017	0.006	0.002	-0.003	-0.001	0.010	-0.008	0.005	0.043	-0.132	0.071	0.026	-0.019	-0.007
17-32	-0.001	0.004	-0.009	-0.004	0.025	0.055	-0.073	-0.002	-0.097	-0.011	0.007	-0.014	0.002	-0.001	-0.005	0.007
33-48	-0.004	0.020	-0.001	-0.203	0.001	-0.041	0.006	-0.010	-0.002	-0.002	0.008	0.013	-0.028	-0.096	-0.046	0.006

Table 7.19 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from Multivariate Model 5.32 Estimated on $\Psi_1^*(i)^2 = 1 \quad \forall i, j$

LAG ATBARA VS ROSEIRES

1-16	0.029	0.024	-0.006	0.005	0.001	-0.003	-0.002	0.011	-0.010	0.005	0.004	-0.120	0.088	0.013	-0.013	-0.005
17-32	0.000	0.002	-0.006	-0.007	0.018	0.039	-0.071	-0.011	-0.105	0.000	-0.002	-0.009	0.004	-0.002	-0.002	-0.005
33-48	0.002	0.024	-0.011	-0.204	0.032	-0.039	0.009	-0.012	0.001	0.000	0.011	0.016	-0.021	-0.087	-0.020	-0.047

LAG TAMANIAT VS TAMANIAT

1-16	0.073	0.012	-0.034	-0.034	-0.014	-0.041	-0.001	0.023	0.024	-0.049	-0.003	-0.070	0.064	0.070	-0.062	-0.034
17-32	0.003	-0.025	-0.051	-0.036	-0.010	0.049	0.046	0.066	-0.017	0.028	0.025	0.016	-0.007	-0.027	-0.056	-0.041
33-48	0.010	-0.057	0.066	-0.016	0.036	-0.065	-0.033	-0.013	-0.012	-0.013	-0.001	0.029	-0.005	-0.063	-0.071	-0.057

LAG TAMANIAT VS MALAKAL

1-16	0.042	0.007	-0.023	0.011	-0.002	0.012	-0.016	0.012	0.055	-0.053	-0.046	-0.013	-0.034	-0.008	0.020	-0.013
17-32	-0.006	-0.026	-0.044	-0.055	0.002	-0.005	-0.009	-0.034	0.000	0.021	-0.017	0.020	-0.027	-0.026	-0.010	-0.044
33-48	0.037	-0.018	0.010	0.055	0.022	-0.013	-0.019	0.030	0.010	-0.020	0.020	0.007	0.004	0.011	0.019	0.024

LAG TAMANIAT VS MONGALLA

1-16	0.070	0.024	-0.069	-0.024	0.003	0.011	0.008	0.015	0.020	-0.005	-0.059	0.008	0.033	0.096	-0.010	-0.018
17-32	-0.037	0.031	-0.040	-0.004	-0.033	0.002	0.115	-0.055	-0.051	0.001	0.030	0.023	-0.044	-0.010	0.006	-0.036
33-48	0.052	0.022	-0.003	0.036	0.012	-0.031	-0.036	-0.003	-0.042	-0.045	0.028	0.031	-0.081	-0.055	0.047	0.020

LAG TAMANIAT VS KHARTOUM

1-16	0.041	-0.010	-0.005	-0.033	0.011	0.012	0.007	0.020	0.015	-0.021	0.020	-0.056	0.051	0.054	-0.065	-0.027
17-32	0.003	-0.014	-0.018	-0.020	-0.012	0.030	0.021	0.029	-0.047	-0.002	0.023	0.019	0.001	-0.004	-0.028	-0.011
33-48	0.031	-0.047	0.053	-0.036	-0.000	-0.055	-0.033	-0.012	-0.004	-0.007	0.011	0.042	0.028	-0.028	-0.069	-0.050

LAG TAMANIAT VS SENNAR

1-16	0.037	-0.018	-0.017	-0.035	0.007	0.013	0.008	0.015	0.003	-0.013	0.034	-0.049	0.074	0.051	-0.068	-0.032
17-32	0.002	-0.021	-0.023	-0.026	-0.019	0.026	0.046	0.043	0.004	0.014	0.014	0.002	-0.000	-0.004	-0.023	-0.009
33-48	0.029	-0.031	0.060	-0.072	-0.019	-0.086	-0.026	-0.010	-0.006	-0.008	0.006	0.027	0.008	-0.038	-0.038	-0.006

LAG TAMANIAT VS ROSEIRES

1-16	0.029	-0.028	-0.022	-0.032	0.007	0.018	0.017	0.028	0.002	0.009	-0.008	-0.052	0.071	0.030	-0.069	-0.031
17-32	0.006	-0.004	-0.012	-0.025	-0.023	0.031	0.037	0.042	-0.039	-0.007	-0.006	0.012	0.001	0.005	-0.004	-0.002
33-48	0.025	-0.014	0.021	-0.064	-0.029	-0.084	-0.047	-0.022	0.007	0.000	0.017	0.040	0.022	-0.027	-0.037	-0.046

Table 7.19 (cont'd)
Results of Whiteness Test No. 3 on Residuals Obtained from Multivariate Model 5.32
Estimated on $\Psi_1^*(i)^2 = 1 \ \forall i, j$

LAG MALAKAL VS MALAKAL

1-16	0.030	-0.003	0.018	0.005	0.005	-0.021	-0.018	0.014	0.028	0.041	0.096	0.113	0.023	0.048	-0.009	-0.018
17-32	-0.091	-0.113	-0.077	0.029	0.010	-0.007	-0.011	-0.010	-0.031	0.002	0.003	-0.045	-0.109	-0.037	-0.056	-0.007
33-48	0.015	-0.056	0.008	0.006	-0.021	-0.038	-0.029	-0.010	0.041	0.024	-0.052	-0.041	-0.011	0.021	0.056	-0.050

LAG MALAKAL VS MONGALLA

1-16	0.037	-0.033	0.048	0.002	0.058	-0.014	-0.027	-0.047	-0.014	-0.038	-0.077	-0.021	-0.045	-0.029	0.034	0.057
17-32	0.047	0.011	-0.060	-0.008	-0.044	0.001	-0.012	0.007	0.033	0.050	0.045	0.081	0.068	0.010	-0.050	-0.066
33-48	0.045	0.004	-0.001	0.012	0.006	0.108	-0.009	0.062	0.041	-0.014	0.031	-0.009	0.004	-0.044	0.040	0.011

LAG MALAKAL VS KHARTOUM

1-16	0.028	0.050	0.012	-0.019	0.057	0.017	-0.024	0.051	0.015	0.009	-0.011	-0.045	-0.033	-0.106	0.001	0.085
17-32	0.003	-0.031	0.005	-0.008	0.003	0.004	0.009	-0.004	0.011	-0.039	-0.005	-0.019	0.041	-0.025	-0.135	0.033
33-48	0.012	-0.039	-0.007	-0.027	-0.032	0.050	0.035	0.044	0.084	-0.109	0.051	-0.005	-0.018	-0.018	-0.006	-0.012

LAG MALAKAL VS SENNAR

1-16	0.058	0.062	0.020	0.009	0.059	0.028	-0.009	0.037	0.015	0.036	0.020	-0.008	-0.007	-0.097	0.001	0.091
17-32	-0.020	-0.039	0.029	-0.014	-0.013	0.010	0.007	-0.002	0.008	-0.042	-0.020	-0.034	0.044	-0.017	-0.147	0.019
33-48	0.017	-0.019	0.007	-0.030	-0.059	0.020	0.041	0.025	0.077	-0.094	0.025	-0.013	-0.019	-0.011	-0.010	-0.020

LAG MALAKAL VS ROSEIRES

1-16	0.052	0.039	0.028	0.000	0.028	0.052	-0.032	0.029	0.029	0.030	0.014	-0.011	-0.030	-0.082	-0.007	0.080
17-32	-0.037	-0.034	0.037	-0.032	-0.019	-0.004	0.013	0.016	0.010	-0.028	-0.024	-0.027	0.042	-0.014	-0.147	0.000
33-48	0.030	-0.022	-0.002	-0.025	-0.054	0.051	0.057	0.041	0.081	-0.125	0.012	-0.002	-0.018	-0.023	-0.012	-0.014

LAG MONGALLA VS MONGALLA

1-16	0.036	-0.071	0.026	0.004	-0.015	-0.020	-0.051	0.011	0.004	0.033	-0.005	0.040	0.100	0.009	-0.019	0.028
17-32	-0.042	0.042	0.044	-0.022	0.015	-0.007	-0.038	0.014	0.043	-0.095	-0.030	0.000	-0.027	0.046	-0.017	-0.057
33-48	-0.003	0.082	-0.006	-0.071	0.025	0.062	0.003	0.005	-0.004	-0.014	0.005	-0.053	-0.093	-0.013	0.005	-0.045

LAG MONGALLA VS KHARTOUM

1-16	0.049	-0.070	-0.049	-0.023	-0.025	-0.002	0.013	0.042	0.003	-0.035	-0.033	-0.034	-0.047	0.051	-0.031	-0.036
17-32	0.001	0.003	0.027	0.016	0.019	-0.066	0.016	-0.030	-0.037	-0.082	0.042	-0.005	-0.031	-0.028	0.032	-0.004
33-48	0.001	0.017	-0.032	0.004	0.024	0.011	0.009	0.007	0.005	0.022	-0.004	0.007	-0.063	-0.025	-0.033	-0.043

Table 7.19 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from Multivariate Model 5.32 Estimated on $\Psi_1^*(i)^2 = 1 \quad \forall i, j$

LAG	MONGALLA VS SENNAR															
1-16	0.092	-0.088	-0.102	-0.045	-0.032	-0.007	-0.001	0.019	-0.007	-0.016	0.003	-0.023	-0.033	0.043	-0.073	-0.043
17-32	0.007	0.015	0.032	0.029	0.003	-0.062	0.030	-0.08	-0.021	-0.099	0.018	-0.001	-0.026	-0.022	0.038	0.003
33-48	0.005	0.028	-0.002	-0.039	0.003	-0.017	0.005	0.005	0.002	0.012	-0.020	-0.018	-0.056	-0.041	-0.041	-0.027

LAG	MONGALLA VS ROSEIRES															
1-16	0.068	-0.090	-0.061	-0.030	-0.029	0.004	0.011	0.029	-0.004	-0.011	-0.009	-0.033	-0.009	0.023	-0.070	-0.024
17-32	0.015	0.022	0.044	0.032	-0.012	-0.025	0.046	0.003	-0.047	-0.102	0.036	0.006	-0.030	-0.012	0.039	0.011
33-48	0.012	0.011	0.010	-0.015	0.023	-0.001	-0.009	-0.003	0.021	0.013	0.003	0.005	-0.058	-0.024	-0.032	-0.033

LAG	KHARTOUM VS KHARTOUM															
1-16	-0.018	0.026	0.023	-0.025	0.004	0.005	0.004	0.014	-0.009	-0.019	0.000	-0.106	0.075	0.052	-0.036	-0.003
17-32	0.011	0.007	-0.002	-0.011	-0.020	0.037	0.036	0.027	-0.118	-0.007	0.006	0.000	-0.007	-0.000	-0.008	-0.003
33-48	0.046	-0.044	0.049	-0.028	0.029	-0.035	-0.020	0.006	-0.002	-0.007	0.002	0.027	-0.016	-0.046	-0.057	-0.083

LAG	KHARTOUM VS SENNAR															
1-16	-0.003	0.017	0.006	-0.032	-0.005	-0.001	0.003	0.006	-0.024	-0.016	0.021	-0.084	0.111	0.050	-0.030	-0.004
17-32	0.016	0.007	0.002	-0.008	-0.021	0.037	0.065	0.027	-0.074	0.001	-0.001	-0.014	-0.002	0.001	-0.005	0.000
33-48	0.048	-0.025	0.050	-0.056	0.012	-0.069	-0.018	0.001	-0.006	-0.010	-0.005	0.008	-0.031	-0.055	-0.030	-0.049

LAG	KHARTOUM VS ROSEIRES															
1-16	-0.001	0.014	0.009	-0.026	-0.012	-0.001	0.003	0.010	-0.028	0.006	-0.018	-0.083	0.112	0.037	-0.031	-0.005
17-32	0.014	0.010	-0.000	-0.014	-0.028	0.043	0.066	0.037	-0.102	-0.010	-0.020	-0.005	-0.008	0.000	-0.005	-0.004
33-48	0.036	-0.023	0.014	-0.018	0.015	-0.051	-0.028	-0.009	0.001	-0.007	-0.001	0.016	-0.029	-0.037	-0.032	-0.080

LAG	SENNAR VS SENNAR															
1-16	-0.001	0.011	0.007	-0.015	-0.001	0.004	0.008	0.003	-0.010	0.005	0.058	-0.092	0.086	0.034	-0.062	-0.024
17-32	0.004	0.004	-0.001	-0.006	-0.007	0.051	0.050	-0.032	-0.088	-0.027	-0.008	-0.015	-0.001	-0.001	-0.006	-0.007
33-48	0.051	-0.001	0.045	-0.052	0.021	-0.051	0.000	0.001	-0.002	-0.006	-0.003	0.012	-0.012	-0.053	-0.041	-0.077

LAG	SENNAR VS ROSEIRES															
1-16	-0.028	0.001	0.010	-0.012	-0.004	0.005	0.005	0.007	-0.011	0.014	0.007	-0.096	0.070	0.014	-0.065	-0.027
17-32	0.003	0.008	-0.002	-0.009	-0.014	0.050	0.043	-0.037	-0.124	-0.040	-0.026	-0.006	-0.006	-0.003	-0.007	-0.011
33-48	0.039	-0.005	0.011	-0.035	0.028	-0.034	-0.014	-0.010	0.003	-0.004	-0.000	0.019	-0.009	-0.040	-0.040	-0.122

LAG	ROSEIRES VS ROSEIRES															
1-16	0.042	-0.046	-0.017	-0.016	-0.018	0.005	0.001	-0.003	-0.029	0.045	-0.002	-0.108	0.090	0.052	-0.049	-0.031
17-32	0.004	0.016	-0.004	-0.006	-0.026	0.040	0.064	0.010	-0.124	-0.012	-0.023	-0.000	0.003	0.003	-0.010	-0.009
33-48	0.023	0.014	0.008	-0.009	0.049	-0.002	0.009	-0.009	0.000	-0.005	0.007	0.012	-0.021	-0.046	-0.020	-0.103

Table 7.19 (cont'd)

Results of Whiteness Test No. 3 on Residuals Obtained from Multivariate Model 5.32 Estimated on $\Psi_1^*(i)^2 = 1 \quad \forall i, j$

lag. No alternative model was found with significantly better performance or better residual properties.

7.4.2 Performance Evaluation

7.4.2.1 Model 5.31

The bias and "R^2" performance statistics, as defined by Equations 5.125 and 5.126, respectively, for the univariate Model 5.31, (presented in Table 7.1) are given in Table 7.20. The statistics were evaluated using the entire historical record of Wadi Halfa (1890-1976). The "lead" row and the "month" column numbers correspond to the indices ℓ and i, respectively. The station under consideration, in this case Wadi Halfa corresponds to the index j. Notice that the bias is generally less than one percent. The trends of the R^2 table are as expected. The statistic in general decreases descending any column which reflects better forecasting performance (mean square error criteria) as lead time decreases. Notice also that periods of high flow which result from near recent precipitation (e.g. month 7) are harder to predict than periods of low flow (e.g. months 1 or 2) which result from the inertia of the system.

Table 7.21 presents the bias and "R^2" performance statistics for the GLS estimated Multivariate Model 5.31 as given in Table 7.2. The statistics were evaluated using the historical record for the period 1912-1967. Similar to the univariate case, the bias is generally less than one percent. The performance of the multivariate model, as reflected in the R^2 statistics, shows significant improvement relative to the univariate model.

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.572E+01	0.800E+01	-.191E+01	0.130E+01	0.667E+00	0.155E+00	-.105E+01	-.190E+01	0.949E+00	0.598E+01	-.523E+01	-.740E+01
2	0.159E+00	0.135E+02	0.763E+01	-.412E+00	0.165E+01	0.102E+01	-.881E+00	-.416E+01	-.778E+00	0.657E+01	-.248E+01	-.982E+01
3	-.242E+01	0.167E+02	0.112E+02	0.811E+01	0.355E+00	0.103E+01	0.932E-01	-.412E+01	-.221E+01	0.549E+01	-.220E+01	-.854E+01
4	-.430E+00	0.152E+02	0.203E+02	0.113E+02	0.680E+01	0.272E+00	0.106E+00	-.391E+01	-.228E+01	0.459E+01	-.270E+01	-.842E+01
5	0.356E+01	0.166E+02	0.198E+02	0.161E+02	0.920E+01	0.406E+01	-.749E+00	-.391E+01	-.357E+01	0.470E+01	-.312E+01	-.874E+01
6	0.378E+01	0.203E+02	0.210E+02	0.165E+02	0.997E+01	0.546E+01	0.351E+01	-.409E+01	-.489E+01	0.338E+01	-.307E+01	-.904E+01
7	0.629E+01	0.205E+02	0.243E+02	0.172E+02	0.103E+02	-.904E+01	0.510E+01	-.317E+01	-.281E+01	0.604E+00	-.368E+01	-.902E+01
8	0.715E+01	0.226E+02	0.245E+02	0.191E+02	0.108E+02	-.924E+01	-.586E+01	-.283E+01	-.132E+02	0.377E+01	-.496E+01	-.929E+01
9	0.755E+01	0.232E+02	0.261E+02	0.192E+02	0.123E+02	-.877E+01	-.608E+01	-.215E+02	-.958E+01	-.639E+01	-.350E+01	-.988E+01
10	0.694E+01	0.214E+02	0.265E+02	0.201E+02	0.124E+02	-.746E+01	-.554E+01	-.216E+02	-.219E+02	-.369E+01	-.482E+01	-.922E+01
11	0.715E+01	0.216E+02	0.251E+02	0.187E+02	0.130E+02	-.740E+01	-.408E+01	-.215E+02	-.257E+02	-.190E+02	-.125E+01	-.978E+01
12	0.674E+01	0.217E+02	0.253E+02	0.145E+02	0.120E+02	-.676E+01	-.401E+01	-.211E+02	-.239E+02	-.234E+02	-.334E+02	-.593E+01

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.9364	0.9231	0.7828	0.8127	0.8270	0.7200	0.2863	0.5108	0.6458	0.7770	0.7840	0.8784
2	0.7957	0.8348	0.6282	0.6788	0.6556	0.3057	0.1643	0.0394	0.3320	0.6005	0.6330	0.7496
3	0.7237	0.7392	0.5651	0.5283	0.6820	0.3040	0.1084	0.0343	0.1806	0.4278	0.5109	0.7115
4	0.6850	0.6891	0.5313	0.4692	0.6086	0.3422	0.1076	0.0357	0.1804	0.3233	0.4136	0.6318
5	0.5866	0.6481	0.5176	0.4658	0.5629	0.3229	0.1113	0.0357	0.1716	0.3141	0.3296	0.5231
6	0.4554	0.5347	0.4713	0.4711	0.5593	0.2835	0.1268	0.0349	0.1633	0.2930	0.3244	0.4348
7	0.3509	0.3607	0.3495	0.4360	0.5585	0.2989	0.1100	0.0382	0.1648	0.2522	0.3117	0.4332
8	0.3467	0.2710	0.2029	0.3703	0.5283	0.2991	0.1005	0.0264	0.1276	0.2563	0.2596	0.4253
9	0.3476	0.2673	0.1212	0.3250	0.4755	0.2725	0.1032	0.0280	0.1113	0.2267	0.2499	0.3968
10	0.3398	0.2629	0.1189	0.2939	0.4432	0.2126	0.0882	0.0283	0.1202	0.2132	0.2449	0.3789
11	0.3225	0.2650	0.1281	0.3061	0.4189	0.2041	0.0651	0.0276	0.1259	0.2201	0.2259	0.3751
12	0.3194	0.2607	0.1319	0.3390	0.4251	0.1665	0.0594	0.0273	0.1135	0.2348	0.2292	0.3443

Table 7.20
Forecasting Performance of OLS Estimated Univariate Model 5.31, B and R^2 Statistics

BIAS

*****MONTH*****

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.668E-02	-.710E-04	0.117E-02	-.336E-03	-.457E-03	0.508E-04	0.128E-02	0.206E-02	0.761E-02	-.302E-02	0.873E-03	0.727E-03
2	-.397E+01	0.277E-02	0.136E-02	0.159E-03	0.819E-03	0.325E-04	0.121E-02	0.301E-02	0.619E-02	-.343E-02	0.246E-02	0.157E-02
3	-.245E+01	-.204E+01	-.261E-01	-.738E-03	0.820E-03	0.952E-03	0.266E-02	0.294E-02	0.648E-02	-.351E-02	0.230E-02	0.370E-02
4	-.395E+01	-.609E+00	-.190E+01	-.389E-03	0.303E-03	0.715E-03	0.278E-02	0.302E-02	0.645E-02	-.399E-02	0.225E-02	0.482E-02
5	-.264E+01	-.319E+01	0.526E+01	0.592E+00	0.519E-03	0.849E-03	-.220E-01	0.310E-02	0.653E-02	-.404E-02	0.208E-02	-.455E-04
6	-.861E+01	-.299E+01	0.418E+00	0.242E+01	-.634E+00	0.842E-03	-.221E-01	0.314E-02	0.653E-02	-.403E-02	0.206E-02	-.227E-02
7	-.703E+01	-.849E+01	-.985E-01	-.948E+00	-.442E+00	-.281E+01	-.223E-01	0.305E-02	0.653E-02	-.336E-02	0.207E-02	-.174E-02
8	-.645E+01	-.739E+01	-.328E+01	-.187E+01	-.537E+01	-.435E+01	0.314E+02	0.290E-02	0.113E-01	-.280E-02	0.218E-02	-.127E-02
9	-.756E+01	-.710E+01	-.332E+01	-.457E+01	-.686E+01	-.727E+01	0.302E+02	0.215E+03	0.114E-01	-.273E-02	0.229E-02	-.142E-02
10	-.491E+01	-.762E+01	-.336E+01	-.481E+01	-.104E+02	-.709E+01	0.236E+02	0.217E+03	0.153E+03	-.281E-02	0.229E-02	-.514E-02
11	-.470E+01	-.492E+01	-.368E+01	-.476E+01	-.105E+02	-.107E+02	0.207E+02	0.215E+03	0.150E+03	0.118E+03	0.228E-02	-.509E-02
12	-.586E+01	-.369E+01	-.125E+01	-.592E+01	-.105E+02	-.106E+02	0.190E+02	0.215E+03	0.151E+03	0.116E+03	0.612E+02	-.509E-02

R SQUARED STATISTIC

*****MONTH*****

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.9864	0.9569	0.9281	0.9589	0.9822	0.9255	0.8358	0.7502	0.7180	0.7724	0.8918	0.9282
2	0.9507	0.9089	0.7298	0.8299	0.9238	0.6928	0.4002	0.1037	0.4571	0.4262	0.4285	0.8707
3	0.8992	0.8891	0.6603	0.8166	0.9013	0.5959	0.2291	0.0279	0.0533	0.3944	0.2111	0.6297
4	0.8174	0.8637	0.6679	0.8289	0.8649	0.6018	0.2045	0.0132	0.0451	0.1818	0.1881	0.4598
5	0.6503	0.7822	0.6191	0.8162	0.8284	0.5423	0.2132	0.0140	0.0466	0.1345	0.0826	0.3796
6	0.5565	0.6277	0.6113	0.7985	0.7989	0.5154	0.2202	0.0146	0.0466	0.1343	0.0426	0.2725
7	0.4489	0.5273	0.4940	0.7609	0.7962	0.4942	0.1986	0.0214	0.0466	0.0541	0.0465	0.2191
8	0.3617	0.4194	0.4218	0.6992	0.7261	0.4791	0.1872	0.0107	0.0326	0.0475	0.0353	0.2049
9	0.3355	0.3379	0.3595	0.7034	0.6512	0.4440	0.1678	0.1827	0.0080	0.0471	0.0298	0.2139
10	0.3118	0.3137	0.2344	0.6788	0.6207	0.4066	0.1822	0.1759	0.0916	0.0349	0.0301	0.1997
11	0.3035	0.2628	0.1861	0.6281	0.5835	0.3774	0.1734	0.1677	0.0897	0.1027	0.0130	0.1913
12	0.2779	0.2684	0.1481	0.5849	0.5394	0.3631	0.1792	0.1651	0.0875	0.0989	0.0915	0.1700

Table 7.21a

Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics; Wadi Halfa

BIAS

LEAD	*****MONTH*****											
	1	2	3	4	5	6	7	8	9	10	11	12
1	-.465E-05	-.293E-05	0.120E-03	-.132E-05	-.208E-06	-.347E-07	-.192E-03	0.550E-03	0.267E-02	0.160E-03	-.620E-04	0.191E-04
2	-.245E+00	-.556E-05	0.120E-03	-.132E-05	-.203E-05	-.347E-07	-.192E-03	0.300E-03	0.287E-02	0.646E-03	-.268E-04	-.130E-05
3	-.663E-01	-.257E-01	0.120E-03	-.132E-05	-.203E-05	-.347E-07	-.192E-03	0.300E-03	0.277E-02	0.682E-03	0.809E-04	0.105E-04
4	-.667E-01	-.590E-01	0.964E-02	-.132E-05	-.203E-05	-.347E-07	-.192E-03	0.300E-03	0.277E-02	0.663E-03	0.889E-04	0.457E-04
5	-.377E-01	-.593E-01	0.517E-01	0.494E-01	-.203E-05	-.347E-07	-.192E-03	0.300E-03	0.277E-02	0.663E-03	0.850E-04	0.485E-04
6	-.975E-01	-.729E-01	0.516E-01	0.494E-01	0.102E+00	-.347E-07	-.192E-03	0.300E-03	0.277E-02	0.663E-03	0.850E-04	0.452E-04
7	-.912E-01	-.793E-01	0.518E-01	0.494E-01	0.102E+00	0.159E+01	-.192E-03	0.300E-03	0.277E-02	0.663E-03	0.850E-04	0.452E-04
8	-.912E-01	-.784E-01	0.519E-01	0.494E-01	0.102E+00	0.159E+01	0.193E+02	0.300E-03	0.277E-02	0.663E-03	0.850E-04	0.452E-04
9	-.912E-01	-.784E-01	0.520E-01	0.494E-01	0.102E+00	0.159E+01	0.193E+02	0.599E+02	0.277E-02	0.663E-03	0.850E-04	0.452E-04
10	-.912E-01	-.784E-01	0.520E-01	0.494E-01	0.102E+00	0.159E+01	0.193E+02	0.599E+02	0.354E+02	0.663E-03	0.850E-04	0.452E-04
11	-.912E-01	-.784E-01	0.520E-01	0.494E-01	0.102E+00	0.159E+01	0.193E+02	0.599E+02	0.388E+02	0.119E+02	0.850E-04	0.452E-04
12	-.912E-01	-.122E+00	0.520E-01	0.494E-01	0.102E+00	0.159E+01	0.193E+02	0.599E+02	0.387E+02	0.125E+02	0.289E+01	0.442E-04

R SQUARED STATISTIC

LEAD	*****MONTH*****											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.2577	0.8442	0.3694	0.2456	0.6364	0.0357	0.0031	0.4862	0.4417	0.5115	0.6896	0.7600
2	0.3103	0.1205	0.3694	0.2456	0.4416	0.0357	0.0031	0.0091	0.2449	0.3190	0.3027	0.6186
3	0.2187	0.0963	0.2701	0.2456	0.4416	0.0357	0.0031	0.0091	0.0661	0.1855	0.2391	0.4342
4	0.1843	0.1163	0.2813	0.2316	0.4416	0.0357	0.0031	0.0091	0.0661	0.0745	0.1351	0.4009
5	0.1904	0.0940	0.1405	0.2319	0.4312	0.0357	0.0031	0.0091	0.0661	0.0745	0.0319	0.3388
6	0.1729	0.1000	-.0207	0.2319	0.4317	0.0178	0.0031	0.0091	0.0661	0.0745	0.0319	0.1445
7	0.1070	0.0853	-.0216	0.2319	0.4317	0.0312	-.0154	0.0091	0.0661	0.0745	0.0319	0.1445
8	0.1070	0.0757	-.0221	0.2319	0.4317	0.0312	0.0168	-.0092	0.0661	0.0745	0.0319	0.1445
9	0.1070	0.0757	-.0274	0.2319	0.4317	0.0312	0.0168	0.0475	0.0488	0.0745	0.0319	0.1445
10	0.1070	0.0757	-.0274	0.2319	0.4317	0.0312	0.0168	0.0475	0.0472	0.0573	0.0319	0.1445
11	0.1070	0.0757	-.0274	0.2319	0.4317	0.0312	0.0168	0.0475	0.0645	0.0791	0.0140	0.1445
12	0.1070	0.0141	-.0274	0.2319	0.4317	0.0312	0.0168	0.0475	0.0424	0.1018	0.0392	0.1627

Table 7.21b
Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics; Atbara

BIAS

	*****MONTH*****											
LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.664E-01	0.860E-05	-.510E-03	0.259E-02	-.583E-05	-.749E-04	-.280E-02	0.451E-03	-.594E-02	0.153E-03	0.167E-02	-.226E-01
2	0.161E+01	0.196E-03	-.419E-03	0.218E-02	0.119E-02	-.172E-04	-.288E-02	0.138E-02	-.565E-02	-.200E-03	0.728E-02	0.196E-02
3	0.712E+00	0.376E+01	-.306E-03	0.233E-02	0.882E-03	0.405E-04	-.287E-02	0.128E-02	-.501E-02	-.282E-03	0.105E-01	0.377E-04
4	0.692E+00	0.491E+01	0.334E+01	0.240E-02	0.109E-02	-.144E-03	-.287E-02	0.126E-02	-.509E-02	-.688E-03	0.332E-02	0.254E-02
5	0.222E+00	0.212E+01	0.415E+01	0.423E+00	0.109E-02	-.244E-04	-.287E-02	0.126E-02	-.511E-02	-.721E-03	-.325E-03	-.429E-02
6	-.760E+01	0.165E+01	0.207E+01	0.758E+00	-.369E+01	-.383E-04	-.287E-02	0.126E-02	-.511E-02	-.724E-03	0.604E-03	-.736E-02
7	-.517E+01	-.272E+01	0.154E+01	-.376E+01	-.423E+01	0.111E+02	-.287E-02	0.116E-02	-.511E-02	-.681E-03	0.125E-02	-.675E-02
8	-.478E+01	-.207E+01	-.122E+01	-.559E+01	-.803E+01	0.103E+02	0.623E+02	0.113E-02	-.516E-02	0.226E-03	0.102E-02	-.607E-02
9	-.440E+01	-.199E+01	-.751E+00	-.797E+01	-.776E+01	0.847E+01	0.623E+02	0.152E+03	-.518E-02	-.728E-03	-.588E-03	-.629E-02
10	-.218E+01	-.209E+01	-.679E+00	-.804E+01	-.124E+02	0.932E+01	0.623E+02	0.153E+03	0.111E+03	-.557E-03	-.504E-03	-.127E-01
11	-.114E+01	0.206E+00	-.712E+00	-.805E+01	-.122E+02	0.589E+01	0.622E+02	0.152E+03	0.103E+03	0.100E+03	-.513E-03	-.128E-01
12	-.226E+01	0.184E+01	0.768E+00	-.805E+01	-.123E+02	0.613E+01	0.623E+02	0.152E+03	0.113E+03	0.982E+02	0.359E+02	-.129E-01

R SQUARED STATISTIC

	*****MONTH*****											
LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.9703	0.8111	0.8671	0.9117	0.8798	0.7376	0.3965	0.6228	0.6004	0.6186	0.8582	0.9647
2	0.8789	0.7790	0.7917	0.8917	0.7740	0.4991	0.1030	0.0942	0.4446	0.3164	0.4822	0.8527
3	0.8518	0.7832	0.7917	0.8861	0.7735	0.4147	0.0076	0.0131	0.1211	0.3052	0.2648	0.7160
4	0.7728	0.7528	0.7993	0.8667	0.7441	0.3976	0.0076	0.0055	0.0818	0.1255	0.2521	0.5440
5	0.5761	0.7185	0.7881	0.8398	0.7120	0.3625	0.0076	0.0055	0.0783	0.0738	0.1672	0.4510
6	0.4469	0.5993	0.7507	0.8264	0.6975	0.3508	0.0076	0.0055	0.0783	0.0763	0.1143	0.3674
7	0.3437	0.5012	0.6766	0.7743	0.6845	0.3628	-.0108	0.0139	0.0783	0.0606	0.1124	0.3163
8	0.2941	0.4118	0.6532	0.7245	0.6353	0.3601	0.0756	-.0013	0.0836	0.0484	0.0983	0.2991
9	0.2847	0.3439	0.5840	0.6993	0.5721	0.3243	0.0756	0.1757	0.0696	0.0508	0.0837	0.2927
10	0.2666	0.3216	0.5423	0.6753	0.5243	0.2938	0.0756	0.1680	0.1144	0.0334	0.0771	0.2822
11	0.2520	0.2801	0.5249	0.6341	0.4908	0.2780	0.0750	0.1619	0.1334	0.0903	0.0626	0.2509
12	0.2365	0.2834	0.5020	0.6279	0.4706	0.2820	0.0761	0.1607	0.1015	0.0899	0.1123	0.2434

Table 7.21c
Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics; Tamaniat

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.307E-03	0.384E-03	-.356E-03	-.513E-03	0.170E-03	0.267E-02	-.369E-02	-.416E-02	0.241E-02	-.429E-02	0.454E-01	-.958E-03
2	0.670E+00	0.517E-03	0.230E-04	-.645E-03	-.154E-03	0.273E-02	-.147E-02	-.784E-02	-.471E-02	-.553E-03	0.418E-01	0.436E-01
3	0.253E+01	-.222E+01	0.741E-04	-.512E-03	-.234E-03	0.260E-02	-.140E-02	-.637E-02	-.835E-02	-.862E-02	0.451E-01	0.395E-01
4	-.199E+01	-.141E+01	-.375E+01	-.530E-03	-.151E-03	0.256E-02	-.151E-02	-.607E-02	-.741E-02	-.116E-01	0.373E-01	0.423E-01
5	-.276E+01	-.411E+01	-.343E+01	-.894E+00	-.165E-03	0.264E-02	-.154E-02	-.633E-02	-.676E-02	-.112E-01	0.342E-01	0.345E-01
6	-.984E+01	-.312E+01	-.521E+01	-.138E+01	-.456E+01	0.265E-02	-.148E-02	-.727E-02	-.710E-02	-.102E-01	0.347E-01	0.314E-01
7	-.879E+01	-.903E+01	-.348E+01	-.254E+01	-.487E+01	0.604E+00	-.147E-02	-.722E-02	-.901E-02	-.106E-01	0.356E-01	0.319E-01
8	-.866E+01	-.841E+01	-.860E+01	-.137E+01	-.560E+01	0.569E+00	0.494E+00	-.721E-02	-.898E-02	-.155E-01	0.353E-01	0.328E-01
9	-.882E+01	-.842E+01	-.820E+01	-.462E+01	-.486E+01	-.542E+00	0.465E+00	0.114E+01	-.894E-02	-.155E-01	0.238E-01	0.325E-01
10	-.510E+01	-.865E+01	-.827E+01	-.437E+01	-.691E+01	-.699E+00	-.801E+00	0.505E+00	0.350E+01	-.154E-01	0.237E-01	0.198E-01
11	-.244E+01	-.593E+01	-.851E+01	-.442E+01	-.675E+01	-.163E+01	-.661E+00	-.719E+00	0.198E+01	0.579E+01	0.237E-01	0.197E-01
12	-.280E+01	-.466E+01	-.637E+01	-.459E+01	-.678E+01	-.132E+01	-.221E+01	0.358E-01	0.787E+00	0.308E+01	0.176E+02	0.196E-01

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.9084	0.9253	0.9379	0.9374	0.8510	0.9415	0.9686	0.9253	0.9710	0.9694	0.9736	0.9534
2	0.8776	0.8756	0.8455	0.9125	0.8038	0.8096	0.9238	0.8946	0.8160	0.9175	0.9527	0.9203
3	0.8767	0.8791	0.8323	0.8858	0.8153	0.7942	0.8650	0.8770	0.7680	0.7718	0.8798	0.8827
4	0.8310	0.8799	0.8372	0.8741	0.7856	0.7935	0.8535	0.8669	0.7484	0.7059	0.7641	0.7565
5	0.6819	0.8450	0.8368	0.8769	0.7763	0.7538	0.8530	0.8303	0.7423	0.6913	0.7085	0.6451
6	0.5759	0.7151	0.7657	0.8766	0.7872	0.7379	0.8226	0.8213	0.6901	0.6717	0.6777	0.5539
7	0.4746	0.6687	0.6616	0.8387	0.7834	0.7405	0.8134	0.7999	0.6821	0.6118	0.6607	0.4975
8	0.3898	0.5865	0.6580	0.7563	0.7670	0.7394	0.8161	0.7862	0.6688	0.6191	0.6027	0.4706
9	0.3689	0.4802	0.5987	0.7188	0.7296	0.7006	0.8136	0.7885	0.6459	0.6118	0.5955	0.4288
10	0.3305	0.4441	0.4968	0.6573	0.7140	0.6459	0.7739	0.7813	0.6514	0.5907	0.5958	0.4210
11	0.3258	0.3997	0.4488	0.5908	0.6779	0.6341	0.7223	0.7656	0.6351	0.6018	0.5847	0.4178
12	0.3153	0.4030	0.4105	0.5551	0.6534	0.5750	0.7093	0.7192	0.6317	0.5696	0.6252	0.4054

Table 7.21d
Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics;
Malakal

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	-.402E-03	-.868E-04	-.373E-01	0.635E-04	0.415E-02	0.141E-03	-.215E-03	-.135E-02	0.438E-04	0.119E-03	-.116E-03	0.161E-03
2	0.578E+01	-.441E-03	-.375E-01	-.444E-01	0.424E-02	0.343E-02	-.107E-03	-.164E-02	-.135E-02	0.163E-03	-.350E-04	0.694E-04
3	-.790E+00	0.306E+01	-.379E-01	-.446E-01	-.627E-01	0.350E-02	0.243E-02	-.150E-02	-.164E-02	-.120E-02	-.832E-05	0.134E-03
4	-.214E+01	-.273E+01	0.294E+01	-.451E-01	-.629E-01	-.496E-01	0.249E-02	0.414E-03	-.149E-02	-.148E-02	-.913E-03	0.155E-03
5	0.366E+00	-.392E+01	-.392E+01	0.143E+01	-.635E-01	-.498E-01	-.384E-01	0.459E-03	0.469E-03	-.133E-02	-.110E-02	-.547E-03
6	-.490E+01	-.171E+01	-.534E+01	-.675E+01	-.293E+01	-.502E-01	-.385E-01	-.304E-01	0.515E-03	0.577E-03	-.100E-02	-.695E-03
7	-.455E+01	-.635E+01	-.271E+01	-.798E+01	-.125E+02	-.431E+01	-.388E-01	-.305E-01	-.312E-01	0.623E-03	0.270E-03	-.587E-03
8	-.474E+01	-.604E+01	-.822E+01	-.569E+01	-.138E+02	-.133E+02	-.294E+01	-.306E-01	-.313E-01	-.303E-01	0.303E-03	0.107E-02
9	-.528E+01	-.621E+01	-.790E+01	-.105E+02	-.114E+02	-.145E+02	-.128E+02	0.513E+01	-.314E-01	-.304E-01	-.308E-01	0.111E-02
10	-.148E+01	-.668E+01	-.808E+01	-.102E+02	-.164E+02	-.122E+02	-.141E+02	-.457E+01	0.143E+02	-.305E-01	-.308E-01	-.338E-01
11	-.105E+01	-.334E+01	-.856E+01	-.104E+02	-.161E+02	-.171E+02	-.117E+02	-.581E+01	0.430E+01	0.146E+02	-.310E-01	-.339E-01
12	0.366E-01	-.296E+01	-.514E+01	-.109E+02	-.163E+02	-.168E+02	-.167E+02	-.351E+01	0.303E+01	0.490E+01	0.659E+01	-.341E-01

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.9863	0.9950	0.9941	0.9713	0.9225	0.9247	0.9243	0.9067	0.8841	0.9384	0.9019	0.9836
2	0.9742	0.9808	0.9842	0.9706	0.8621	0.8728	0.8940	0.7747	0.8219	0.8469	0.8473	0.9147
3	0.9386	0.9651	0.9648	0.9535	0.8699	0.8373	0.8599	0.7144	0.7119	0.8003	0.8234	0.8818
4	0.9067	0.9236	0.9445	0.9331	0.8345	0.8364	0.8323	0.6963	0.6280	0.7009	0.7759	0.8769
5	0.8882	0.8914	0.8921	0.9091	0.8114	0.8043	0.8301	0.6783	0.6128	0.6340	0.7146	0.8456
6	0.8681	0.8743	0.8530	0.8344	0.7845	0.7907	0.8093	0.6785	0.5929	0.6156	0.6721	0.7876
7	0.7911	0.8470	0.8339	0.8087	0.6909	0.7590	0.8035	0.6664	0.5945	0.5902	0.6658	0.7394
8	0.7354	0.7698	0.8015	0.8005	0.6845	0.6612	0.7738	0.6590	0.5791	0.5961	0.6548	0.7313
9	0.7251	0.7103	0.7314	0.7666	0.6667	0.6617	0.6503	0.6373	0.5737	0.5837	0.6530	0.7139
10	0.6980	0.7031	0.6674	0.7112	0.6544	0.6230	0.6612	0.4870	0.5714	0.5794	0.6374	0.7081
11	0.6895	0.6756	0.6615	0.6495	0.6113	0.5980	0.6273	0.4965	0.4250	0.5803	0.6316	0.6864
12	0.6616	0.6657	0.6388	0.6461	0.5502	0.5483	0.6061	0.4571	0.4420	0.4504	0.6284	0.6770

Table 7.21e
Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics; Mongalla

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.120E-02	0.104E-02	-.621E-03	0.182E-03	-.834E-04	0.656E-04	-.866E-04	0.637E-03	0.278E-02	0.754E-02	-.958E-03	-.341E-03
2	-.492E+01	0.107E-02	0.269E-03	-.370E-03	-.223E-03	0.411E-04	-.222E-05	0.170E-02	0.300E-02	0.723E-02	0.233E-02	0.148E-04
3	-.670E+01	-.185E+01	0.104E-03	0.431E-03	-.340E-03	-.190E-03	-.422E-04	0.163E-02	0.303E-02	0.716E-02	0.270E-02	0.221E-02
4	-.503E+01	-.264E+01	-.500E+00	0.259E-03	-.383E-03	-.317E-03	-.334E-03	0.168E-02	0.299E-02	0.681E-02	0.269E-02	0.252E-02
5	-.450E+01	-.204E+01	-.894E+00	0.750E+00	-.365E-03	-.364E-03	-.493E-03	0.189E-02	0.301E-02	0.678E-02	0.265E-02	0.251E-02
6	-.440E+01	-.191E+01	-.725E+00	0.794E+00	-.159E+01	-.345E-03	-.552E-03	0.200E-02	0.276E-02	0.678E-02	0.265E-02	0.252E-02
7	-.364E+01	-.206E+01	-.734E+00	0.418E+00	-.188E+01	0.130E+02	-.525E-03	0.204E-02	0.279E-02	0.702E-02	0.265E-02	0.251E-02
8	-.353E+01	-.139E+01	-.862E+00	0.194E+00	-.196E+01	0.127E+02	0.658E+02	0.204E-02	0.267E-02	0.722E-02	0.272E-02	0.251E-02
9	-.348E+01	-.133E+01	-.281E+00	0.155E+00	-.209E+01	0.127E+02	0.654E+02	0.155E+03	0.269E-02	0.723E-02	0.280E-02	0.256E-02
10	-.294E+01	-.131E+01	-.232E+00	0.411E+00	-.214E+01	0.126E+02	0.653E+02	0.155E+03	0.107E+03	0.719E-02	0.281E-02	0.260E-02
11	-.303E+01	-.119E+01	-.209E+00	0.439E+00	-.192E+01	0.125E+02	0.652E+02	0.155E+03	0.104E+03	0.888E+02	0.280E-02	0.260E-02
12	-.321E+01	-.122E+01	-.175E+00	0.465E+00	-.189E+01	0.128E+02	0.652E+02	0.155E+03	0.108E+03	0.869E+02	0.254E+02	0.260E-02

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.8885	0.8770	0.6509	0.6435	0.3243	0.3867	0.2985	0.6509	0.5425	0.5546	0.7808	0.8261
2	0.7183	0.8505	0.5099	0.3131	0.1185	0.1042	0.0980	0.0770	0.4087	0.2655	0.3333	0.6635
3	0.6300	0.7953	0.5300	0.2465	0.0814	0.0675	0.0271	0.0134	0.0854	0.2490	0.1382	0.4202
4	0.4803	0.7173	0.5236	0.2343	0.0764	0.0384	0.0226	0.0053	0.0463	0.0960	0.1273	0.1962
5	0.3136	0.5454	0.4868	0.2223	0.0540	0.0280	0.0089	0.0119	0.0473	0.0543	0.0497	0.1755
6	0.2824	0.4106	0.2777	0.2199	0.0530	0.0092	0.0053	0.0175	0.0501	0.0626	0.0283	0.0856
7	0.1821	0.3522	0.2383	0.0622	0.0646	0.0406	-.0155	0.0183	0.0373	0.0514	0.0342	0.0608
8	0.1488	0.1781	0.1999	0.0790	0.0527	0.0386	0.0668	0.0007	0.0394	0.0456	0.0179	0.0592
9	0.1446	0.1515	0.0578	0.0691	0.0597	0.0381	0.0676	0.1689	0.0219	0.0439	0.0105	0.0530
10	0.1320	0.1495	0.0236	0.0260	0.0525	0.0407	0.0691	0.1684	0.0942	0.0279	0.0102	0.0435
11	0.1156	0.1457	0.0229	0.0006	0.0344	0.0382	0.0694	0.1681	0.0955	0.0795	-.0064	0.0419
12	0.1136	0.1436	0.0214	0.0037	0.0193	0.0375	0.0698	0.1678	0.0870	0.0791	0.0321	0.0261

Table 7.21f
Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics; Khartoum

BIAS

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
						*****MONTH*****						
1	0.104E-03	0.487E-04	-.299E-03	-.234E-03	-.363E-04	-.483E-04	0.826E-03	-.320E-02	-.402E-03	0.253E-02	0.842E-03	0.126E-03
2	-.285E+01	0.883E-04	-.342E-03	-.423E-03	-.331E-03	-.780E-04	0.751E-03	-.226E-02	-.484E-03	0.228E-02	0.602E-02	0.466E-03
3	-.471E+01	0.461E+00	-.299E-03	-.449E-03	-.492E-03	-.337E-03	0.751E-03	-.231E-02	-.937E-03	0.223E-02	0.676E-02	0.256E-02
4	-.308E+01	-.249E+00	0.145E+01	-.422E-03	-.551E-03	-.480E-03	0.751E-03	-.218E-02	-.977E-03	0.193E-02	0.675E-02	0.272E-02
5	-.296E+01	0.373E+00	0.732E+00	0.306E+01	-.526E-03	-.530E-03	0.751E-03	-.218E-02	-.974E-03	0.191E-02	0.678E-02	0.271E-02
6	-.277E+01	0.419E+00	0.272E+00	0.261E+01	0.225E+01	-.508E-03	0.751E-03	-.218E-02	-.677E-03	0.190E-02	0.677E-02	0.270E-02
7	-.221E+01	0.330E+00	-.259E+00	0.232E+01	0.185E+01	0.209E+02	0.751E-03	-.218E-02	-.422E-03	0.209E-02	0.676E-02	0.269E-02
8	-.210E+01	0.100E+01	-.353E+00	0.198E+01	0.183E+01	0.206E+02	0.759E+02	-.218E-02	-.406E-03	0.227E-02	0.685E-02	0.268E-02
9	-.197E+01	0.107E+01	0.130E+00	0.192E+01	0.173E+01	0.206E+02	0.759E+02	0.153E+03	-.444E-03	0.228E-02	0.695E-02	0.272E-02
10	-.172E+01	0.112E+01	0.177E+00	0.232E+01	0.165E+01	0.205E+02	0.759E+02	0.153E+03	0.917E+02	0.225E-02	0.696E-02	0.276E-02
11	-.175E+01	0.122E+01	0.200E+00	0.239E+01	0.194E+01	0.204E+02	0.759E+02	0.153E+03	0.893E+02	0.807E+02	0.695E-02	0.276E-02
12	-.154E+01	0.120E+01	0.210E+00	0.242E+01	0.199E+01	0.207E+02	0.759E+02	0.153E+03	0.923E+02	0.792E+02	0.279E+02	0.275E-02

R SQUARED STATISTIC

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
						*****MONTH*****						
1	0.8729	0.8106	0.6167	0.5062	0.2336	0.3685	0.3176	0.6097	0.4848	0.4815	0.7901	0.8641
2	0.7022	0.7675	0.4404	0.3141	0.1293	0.0919	0.0693	0.0534	0.4060	0.2335	0.3489	0.6954
3	0.5704	0.7578	0.4474	0.2907	0.1012	0.0541	0.0026	0.0102	0.1031	0.2227	0.1541	0.4492
4	0.4188	0.6871	0.4152	0.2733	0.0715	0.0268	0.0026	0.0069	0.0651	0.0888	0.1356	0.2069
5	0.2094	0.5282	0.3869	0.2806	0.0544	0.0163	0.0026	0.0069	0.0584	0.0463	0.0577	0.1738
6	0.1852	0.4022	0.2422	0.2963	0.0495	0.0010	0.0026	0.0069	0.0557	0.0520	0.0401	0.0727
7	0.0913	0.3177	0.2005	0.2028	0.0608	0.0636	-.0159	0.0069	0.0534	0.0404	0.0440	0.0450
8	0.0574	0.0886	0.1711	0.1970	0.0583	0.0634	0.0940	-.0115	0.0557	0.0341	0.0270	0.0418
9	0.0506	0.0497	0.0588	0.1814	0.0596	0.0647	0.0940	0.1706	0.0400	0.0333	0.0175	0.0384
10	0.0442	0.0335	0.0164	0.0742	0.0515	0.0642	0.0940	0.1706	0.1073	0.0170	0.0179	0.0296
11	0.0359	0.0337	0.0128	0.0128	0.0407	0.0605	0.0940	0.1706	0.1042	0.0758	0.0017	0.0297
12	0.0307	0.0338	0.0127	0.0084	0.0268	0.0590	0.0940	0.1706	0.0914	0.0739	0.0536	0.0129

Table 7.21g
Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics; Sennar

BIAS

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
	*****MONTH*****											
1	-.375E-04	0.205E-03	0.234E-04	0.240E-03	-.219E-04	-.660E-04	-.225E-02	-.231E-03	-.666E-02	-.157E-01	-.294E-03	-.147E-03
2	-.198E+01	0.192E-03	0.182E-03	0.253E-03	-.219E-04	-.716E-04	-.234E-02	-.244E-02	-.677E-02	-.193E-01	-.425E-02	-.283E-03
3	-.148E+01	-.543E+00	0.171E-03	0.340E-03	-.219E-04	-.716E-04	-.231E-02	-.252E-02	-.814E-02	-.193E-01	-.515E-02	-.211E-02
4	-.284E+01	-.360E+00	-.813E-01	0.334E-03	-.219E-04	-.716E-04	-.231E-02	-.250E-02	-.818E-02	-.201E-01	-.516E-02	-.253E-02
5	-.390E+01	-.108E+01	0.599E-01	-.255E+00	-.219E-04	-.716E-04	-.231E-02	-.250E-02	-.814E-02	-.201E-01	-.535E-02	-.253E-02
6	-.375E+01	-.164E+01	-.492E+00	-.178E+00	0.305E+00	-.716E-04	-.231E-02	-.250E-02	-.814E-02	-.200E-01	-.535E-02	-.262E-02
7	-.360E+01	-.149E+01	-.923E+00	-.705E+00	0.305E+00	0.227E+02	-.231E-02	-.253E-02	-.814E-02	-.200E-01	-.535E-02	-.262E-02
8	-.355E+01	-.137E+01	-.810E+00	-.112E+01	0.305E+00	0.227E+02	0.782E+02	-.255E-02	-.816E-02	-.200E-01	-.535E-02	-.262E-02
9	-.337E+01	-.132E+01	-.716E+00	-.103E+01	0.305E+00	0.227E+02	0.782E+02	0.164E+03	-.816E-02	-.201E-01	-.535E-02	-.262E-02
10	-.337E+01	-.124E+01	-.677E+00	-.956E+00	0.305E+00	0.227E+02	0.782E+02	0.164E+03	0.125E+03	-.201E-01	-.535E-02	-.262E-02
11	-.335E+01	-.124E+01	-.617E+00	-.924E+00	0.305E+00	0.227E+02	0.782E+02	0.164E+03	0.125E+03	0.895E+02	-.535E-02	-.263E-02
12	-.263E+01	-.909E+00	-.617E+00	-.887E+00	0.305E+00	0.227E+02	0.782E+02	0.164E+03	0.126E+03	0.895E+02	0.363E+02	-.262E-02

R SQUARED STATISTIC

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
	*****MONTH*****											
1	0.8674	0.8279	0.3418	0.5311	0.0199	0.2653	0.3195	0.4944	0.4791	0.3973	0.7309	0.6717
2	0.5786	0.8037	0.4361	0.2676	0.0199	0.0022	0.0652	0.0613	0.3702	0.1990	0.2921	0.4464
3	0.3940	0.7635	0.3902	0.2928	0.0199	0.0022	0.0028	0.0140	0.0906	0.1929	0.1563	0.3458
4	0.3255	0.6699	0.3507	0.2692	0.0199	0.0022	0.0028	0.0040	0.0543	0.0750	0.1335	0.2150
5	0.1987	0.5293	0.3078	0.2498	0.0018	0.0022	0.0028	0.0040	0.0481	0.0337	0.0606	0.1783
6	0.1753	0.3983	0.1161	0.2567	0.0018	-.0163	0.0028	0.0041	0.0481	0.0420	0.0342	0.1140
7	0.1032	0.3356	0.0831	0.0961	0.0018	0.0789	-.0156	0.0043	0.0466	0.0420	0.0397	0.0936
8	0.0743	0.1854	0.0942	0.0949	0.0018	0.0789	0.0981	-.0142	0.0467	0.0411	0.0397	0.0897
9	0.0768	0.1458	0.0731	0.0834	0.0018	0.0789	0.0981	0.1704	0.0292	0.0409	0.0388	0.0897
10	0.0768	0.1286	0.0368	0.0530	0.0018	0.0789	0.0981	0.1700	0.1580	0.0231	0.0388	0.0891
11	0.0762	0.1286	0.0384	0.0279	0.0018	0.0789	0.0981	0.1703	0.1582	0.1020	0.0209	0.0891
12	0.0642	0.1156	0.0384	0.0277	0.0018	0.0789	0.0981	0.1701	0.1581	0.1021	0.1481	0.0722

Table 7.21h

Forecasting Performance of GLS Estimated Multivariate Model 5.31, B and R^2 Statistics; Roseires

7.4.2.2 Model 5.32

The forecasting performance of the univariate Model 5.32 (estimated with $\Psi_1^*(i)^2 = 1\ \forall i$ and given in Table 7.4), as indicated by the bias and "R^2" statistics, is given in Table 7.22. The statistics were evaluated using the 1890 to 1976 historical record. Notice that the bias is generally less than one percent and the trend behavior of the "R^2" statistics are as expected.

The bias and "R^2" statistics for the multivariate Model 5.32, estimated with $\Psi_j^*(i)^2 = 1\ \forall i, j$ and specified in Table 7.6, are given in Table 7.23. The 1912 to 1967 historical record was used to evaluate the statistics. The bias is generally insignificant. The "R^2" statistics in general behave as expected. However, some negative values are present. Also, notice that for Wadi Halfa the lead one "R^2"'s for months 1 through 6 are smaller than the univariate Model 5.31 indicating an unexpected degradation in performance (the companion is not strictly valid since the period of evaluation is different).

The next section (7.5) will present some observations and conclusions.

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.245E-03	0.220E-03	0.287E-03	0.459E-03	-.402E-03	-.944E-03	-.537E-03	-.784E-03	-.232E-02	-.230E-02	-.796E-03	-.388E+00
2	0.250E-01	0.421E-03	0.470E-03	0.698E-03	-.692E-05	-.127E-02	-.132E-02	-.124E-02	-.294E-02	-.417E-02	-.269E-02	-.389E+00
3	-.104E+02	0.441E+01	0.592E-03	0.810E-03	0.148E-03	-.103E-02	-.151E-02	-.171E-02	-.322E-02	-.456E-02	-.387E-02	-.390E+00
4	-.572E+01	-.103E+01	0.757E+01	0.873E-03	0.209E-03	-.949E-03	-.138E-02	-.180E-02	-.349E-02	-.465E-02	-.408E-02	-.391E+00
5	0.303E+01	0.119E+01	0.496E+01	0.797E+01	0.242E-03	-.913E-03	-.135E-02	-.174E-02	-.353E-02	-.481E-02	-.411E-02	-.391E+00
6	0.457E+01	0.510E+01	0.595E+01	0.681E+01	0.491E+01	-.909E-03	-.133E-02	-.172E-02	-.351E-02	-.485E-02	-.420E-02	-.391E+00
7	0.468E+01	0.574E+01	0.759E+01	0.722E+01	0.443E+01	-.139E+02	-.132E-02	-.171E-02	-.347E-02	-.478E-02	-.419E-02	-.391E+00
8	0.469E+01	0.578E+01	0.784E+01	0.787E+01	0.459E+01	-.141E+02	-.841E+01	-.172E-02	-.348E-02	-.476E-02	-.417E-02	-.391E+00
9	0.465E+01	0.579E+01	0.786E+01	0.796E+01	0.482E+01	-.141E+02	-.848E+01	-.115E+02	-.349E-02	-.479E-02	-.416E-02	-.391E+00
10	0.465E+01	0.577E+01	0.786E+01	0.796E+01	0.485E+01	-.140E+02	-.846E+01	-.115E+02	-.222E+02	-.482E-02	-.417E-02	-.391E+00
11	0.465E+01	0.577E+01	0.785E+01	0.796E+01	0.486E+01	-.140E+02	-.844E+01	-.115E+02	-.222E+02	-.283E+02	-.421E-02	-.391E+00
12	0.465E+01	0.577E+01	0.785E+01	0.796E+01	0.486E+01	-.140E+02	-.844E+01	-.115E+02	-.222E+02	-.283E+02	-.353E+02	-.391E+00

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.8633	0.8642	0.7384	0.6041	0.7952	0.6059	0.2707	0.3035	0.5459	0.7015	0.6067	0.7753
2	0.5537	0.5976	0.5033	0.4319	0.3791	0.2902	0.1482	0.0302	0.1287	0.4090	0.4778	0.4273
3	0.4794	0.3674	0.3009	0.2518	0.3463	0.2104	0.0978	0.0246	0.0457	0.1370	0.3509	0.5375
4	0.4901	0.3542	0.1892	0.1437	0.2498	0.2022	0.0878	0.0281	0.0439	0.0910	0.1838	0.4282
5	0.3628	0.3425	0.2261	0.1216	0.1883	0.1604	0.0826	0.0296	0.0474	0.0858	0.1449	0.2653
6	0.2339	0.2535	0.2071	0.1207	0.1769	0.1236	0.0786	0.0297	0.0510	0.0886	0.1412	0.2330
7	0.2133	0.1773	0.1593	0.1259	0.1768	0.1479	0.0648	0.0305	0.0513	0.0912	0.1425	0.2311
8	0.2121	0.1679	0.1260	0.1226	0.1782	0.1493	0.0651	0.0191	0.0517	0.0914	0.1441	0.2312
9	0.2118	0.1675	0.1224	0.1166	0.1776	0.1486	0.0662	0.0197	0.0405	0.0915	0.1442	0.2318
10	0.2120	0.1673	0.1223	0.1157	0.1760	0.1473	0.0658	0.0199	0.0425	0.0808	0.1443	0.2319
11	0.2120	0.1673	0.1223	0.1158	0.1758	0.1469	0.0656	0.0198	0.0425	0.0854	0.1342	0.2320
12	0.2120	0.1673	0.1223	0.1158	0.1758	0.1468	0.0656	0.0198	0.0425	0.0854	0.1582	0.2229

Table 7.22
Forecasting Performance of Univariate Model 5.32 Estimated with $\Psi_1^*(i)^2 = 1 \;\; \forall i$,
B and R^2 Statistics

BIAS

*****MONTH*****

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.108E-02	0.675E-03	0.808E-03	-.860E-04	0.168E-02	0.200E-02	0.193E-02	0.312E-02	0.424E-02	0.477E-02	0.325E-02	0.144E-02
2	-.167E+02	0.528E-03	0.483E-03	0.205E-04	0.254E-02	0.266E-02	0.195E-02	0.352E-02	0.428E-02	0.409E-02	0.226E-02	0.362E-03
3	-.148E+02	-.166E+02	0.831E-03	0.902E-04	0.251E-02	0.304E-02	0.223E-02	0.347E-02	0.456E-02	0.447E-02	0.244E-02	0.323E-03
4	-.155E+02	-.155E+02	-.821E+01	0.387E-03	0.263E-02	0.298E-02	0.241E-02	0.350E-02	0.443E-02	0.459E-02	0.271E-02	0.333E-03
5	-.161E+02	-.167E+02	-.862E+01	-.256E+01	0.280E-02	0.306E-02	0.237E-02	0.360E-02	0.444E-02	0.450E-02	0.277E-02	0.329E-03
6	-.163E+02	-.178E+02	-.101E+02	-.240E+01	0.612E-01	0.321E-02	0.244E-02	0.356E-02	0.446E-02	0.449E-02	0.271E-02	0.324E-03
7	-.155E+02	-.189E+02	-.112E+02	-.443E+01	-.733E+00	-.418E+01	0.254E-02	0.361E-02	0.447E-02	0.451E-02	0.269E-02	0.339E-03
8	-.152E+02	-.187E+02	-.128E+02	-.551E+01	-.291E+01	-.559E+01	0.459E+02	0.365E-02	0.447E-02	0.450E-02	0.269E-02	0.313E-03
9	-.149E+02	-.135E+02	-.126E+02	-.639E+01	-.405E+01	-.722E+01	0.451E+02	0.205E+03	0.451E-02	0.451E-02	0.270E-02	0.202E-02
10	-.142E+02	-.184E+02	-.126E+02	-.647E+01	-.530E+01	-.763E+01	0.440E+02	0.203E+03	0.159E+03	0.453E-02	0.270E-02	0.208E-02
11	-.146E+02	-.178E+02	-.130E+02	-.623E+01	-.495E+01	-.970E+01	0.440E+02	0.203E+03	0.157E+03	0.128E+03	0.265E-02	0.202E-02
12	-.131E+02	-.151E+02	-.133E+02	-.931E+01	-.822E+01	-.165E+02	0.382E+02	0.198E+03	0.160E+03	0.129E+03	0.510E+02	0.209E-02

R SQUARED STATISTIC

*****MONTH*****

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.8245	0.8448	0.6248	0.5749	0.6067	0.5622	0.3685	0.5663	0.6788	0.7329	0.7425	0.8420
2	0.7421	0.8540	0.5520	0.5699	0.5668	0.4730	0.1853	0.0332	0.2953	0.3770	0.2879	0.1716
3	0.5261	0.7241	0.5573	0.5663	0.5223	0.4759	0.0798	0.0179	-0.0157	0.1898	0.1979	0.4104
4	0.6473	0.7388	0.5745	0.5389	0.6349	0.4723	0.0380	0.0156	-0.0166	0.0150	0.1113	0.3579
5	0.4418	0.6288	0.6197	0.5071	0.5466	0.5102	0.0493	0.0027	-0.0167	0.0009	0.0073	0.2583
6	0.3448	0.4683	0.5797	0.5559	0.4839	0.4516	0.0474	0.0080	-0.0237	0.0001	-0.0150	0.1453
7	0.2212	0.3362	0.4753	0.5166	0.5349	0.4285	0.0306	0.0026	-0.0180	-0.0026	-0.0157	0.1058
8	0.1860	0.2198	0.3423	0.4139	0.4924	0.4412	0.0813	-0.0151	-0.0206	0.0028	-0.0127	0.0962
9	0.1589	0.1894	0.2652	0.3381	0.4148	0.3944	0.0855	0.1409	-0.0384	0.0013	-0.0044	0.1018
10	0.1551	0.1581	0.2221	0.3143	0.3736	0.3524	0.0965	0.1376	0.0627	-0.0166	-0.0054	0.1109
11	0.1565	0.1453	0.1808	0.2614	0.3385	0.2890	0.0673	0.1435	0.0602	0.0751	-0.0217	0.1149
12	0.1645	0.1889	0.1249	0.1489	0.2683	0.2413	0.1192	0.1629	0.0672	0.0707	0.0485	0.0820

Table 7.23a

Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = 1 \quad \forall i, j$,
B and R^2 Statistics; Wadi Halfa

BIAS

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
	*****MONTH*****											
1	-.154E-03	-.154E-03	-.148E-04	-.323E-04	0.104E-03	-.187E-03	0.000E+00	-.321E-03	-.439E-03	-.237E-03	0.144E-03	0.236E-04
2	-.133E+00	-.238E-03	-.992E-04	-.368E-04	0.862E-04	-.129E-03	-.102E-03	-.330E-03	-.619E-03	-.472E-03	0.129E-04	0.229E-04
3	0.215E+00	0.147E+00	-.128E-03	-.667E-04	0.843E-04	-.135E-03	-.818E-04	-.366E-03	-.625E-03	-.530E-03	-.632E-04	0.233E-04
4	0.275E+00	0.220E+00	0.484E-01	-.737E-04	0.785E-04	-.135E-03	-.843E-04	-.358E-03	-.633E-03	-.533E-03	-.726E-04	0.289E-04
5	0.275E+00	0.220E+00	0.485E-01	0.900E-01	0.774E-04	-.135E-03	-.838E-04	-.358E-03	-.629E-03	-.534E-03	-.735E-04	0.289E-04
6	0.290E+00	0.224E+00	0.417E-01	0.817E-01	0.869E-01	-.134E-03	-.821E-04	-.360E-03	-.628E-03	-.534E-03	-.736E-04	0.289E-04
7	0.278E+00	0.232E+00	0.439E-01	0.779E-01	0.823E-01	0.862E+00	-.832E-04	-.361E-03	-.629E-03	-.533E-03	-.736E-04	0.289E-04
8	0.276E+00	0.228E+00	0.468E-01	0.787E-01	0.810E-01	0.861E+00	0.168E+02	-.360E-03	-.633E-03	-.532E-03	-.729E-04	0.289E-04
9	0.276E+00	0.228E+00	0.459E-01	0.792E-01	0.812E-01	0.861E+00	0.168E+02	0.592E+02	-.635E-03	-.533E-03	-.728E-04	0.240E-04
10	0.276E+00	0.228E+00	0.459E-01	0.792E-01	0.812E-01	0.861E+00	0.168E+02	0.592E+02	0.373E+02	-.536E-03	-.729E-04	0.292E-04
11	0.276E+00	0.227E+00	0.458E-01	0.793E-01	0.813E-01	0.861E00	0.168E+02	0.592E+02	0.373E+02	0.127E+02	-.740E-04	0.292E-04
12	0.274E+00	0.252E+00	0.187E+00	0.372E+00	-.175E+00	0.392E+00	0.171E+02	0.593E+02	0.372E+02	0.125E+02	0.317E+01	0.292E-04

R SQUARED STATISTIC

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
	*****MONTH*****											
1	0.0884	0.7809	-2.6583	-0.0826	0.2001	-0.5009	0.0987	0.2877	0.3570	-0.4025	-0.2319	0.1169
2	-0.1663	-0.1233	-0.9562	-0.1611	-0.2717	-0.5767	0.0989	0.0401	0.0539	0.2005	-1.9906	-1.5257
3	-0.7201	-0.1788	-0.4763	-0.1389	-0.2895	-0.5978	0.1004	0.0403	-0.0440	0.0079	0.1709	-0.4796
4	-0.3393	-0.2604	-0.5562	-0.1626	-0.2878	-0.5981	0.1008	0.0404	-0.0452	-0.0507	-0.0967	-0.1827
5	-0.3394	-0.2599	-0.5557	-0.1614	-0.3117	-0.5981	0.1008	0.0404	-0.0452	-0.0507	-0.0970	-0.1846
6	-0.1373	-0.2075	-0.5423	-0.1657	-0.3121	-0.6274	0.1007	0.0404	-0.0452	-0.0507	-0.0966	-0.1131
7	-0.1000	-0.0773	-0.3087	-0.1582	-0.3137	-0.6233	0.0841	0.0404	-0.0452	-0.0507	-0.0968	-0.1113
8	-0.1006	-0.0675	-0.2288	-0.1682	-0.3111	-0.6232	0.1083	0.0226	-0.0452	-0.0507	-0.0969	-0.1115
9	-0.1006	-0.0678	-0.2261	-0.1736	-0.3134	-0.6229	0.1083	0.0780	-0.0646	-0.0507	-0.0969	-0.1115
10	-0.1006	-0.0678	-0.2261	-0.1736	-0.3134	-0.6229	0.1083	0.0780	-0.0193	-0.0702	-0.0969	-0.1115
11	-0.1006	-0.0678	-0.2261	-0.1736	-0.3133	-0.6232	0.1083	0.0780	-0.0193	-0.0112	-0.1172	-0.1115
12	-0.0986	-0.0711	-0.0289	-0.1344	-0.1655	-0.1128	0.0645	0.0835	-0.0120	-0.0233	-0.0275	-0.1409

Table 7.23b

Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = 1 \ \forall i, j,$

B and R^2 Statistics; Atbara

BIAS

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.201E-02	0.199E-02	0.205E-02	0.158E-02	0.605E-03	0.380E-03	0.745E-03	0.242E-02	0.778E-02	0.719E-02	0.374E-02	0.399E-02
2	-.209E+02	0.224E-02	0.185E-02	0.146E-02	0.129E-02	0.102E-02	0.114E-02	0.304E-02	0.840E-02	0.748E-02	0.375E-02	0.399E-02
3	-.234E+02	-.133E+02	0.204E-02	0.145E-02	0.136E-02	0.145E-02	0.145E-02	0.314E-02	0.869E-02	0.791E-02	0.386E-02	0.399E-02
4	-.260E+02	-.154E+02	-.746E+01	0.176E-02	0.146E-02	0.140E-02	0.161E-02	0.318E-02	0.856E-02	0.800E-02	0.417E-02	0.424E-02
5	-.266E+02	-.186E+02	-.959E+01	-.525E+01	0.172E-02	0.152E-02	0.155E-02	0.323E-02	0.850E-02	0.783E-02	0.418E-02	0.424E-02
6	-.264E+02	-.197E+02	-.109E+02	-.460E+01	-.629E+01	0.170E-02	0.164E-02	0.316E-02	0.853E-02	0.781E-02	0.409E-02	0.424E-02
7	-.255E+02	-.204E+02	-.120E+02	-.597E+01	-.699E+01	0.578E+01	0.169E-02	0.320E-02	0.854E-02	0.783E-02	0.411E-02	0.424E-02
8	-.256E+02	-.201E+02	-.132E+02	-.715E+01	-.858E+01	0.456E+01	0.554E+02	0.322E-02	0.854E-02	0.785E-02	0.413E-02	0.424E-02
9	-.256E+02	-.202E+02	-.132E+02	-.876E+01	-.948E+01	0.288E+01	0.538E+02	0.147E+03	0.853E-02	0.783E-02	0.415E-02	0.439E-02
10	-.250E+02	-.204E+02	-.134E+02	-.889E+01	-.109E+02	0.243E+01	0.527E+02	0.146E+03	0.120E+03	0.781E-02	0.415E-02	0.440E-02
11	-.254E+02	-.199E+02	-.136E+02	-.913E+01	-.112E+02	0.107E+01	0.528E+02	0.145E+03	0.118E+03	0.104E+03	0.410E-02	0.424E-02
12	-.253E+02	-.202E+02	-.131E+02	-.930E+01	-.114E+02	0.677E+00	0.513E+02	0.146E+03	0.118E+03	0.102E+03	0.351E+02	0.424E-02

R SQUARED STATISTIC

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
1	0.8177	0.7127	0.6331	0.7469	0.7032	0.5940	0.3194	0.5361	0.5666	0.6212	0.4396	0.7433
2	0.7219	0.6640	0.5471	0.7270	0.6486	0.4116	0.1692	0.0328	0.3398	0.3140	0.2589	-0.1134
3	0.5595	0.6226	0.5148	0.7147	0.6188	0.3465	0.0780	0.0043	0.0442	0.1987	0.1366	0.5107
4	0.6627	0.6254	0.5383	0.7123	0.6809	0.2823	0.0359	-0.0009	0.0227	0.0538	0.1238	0.4138
5	0.4479	0.6404	0.6945	0.6640	0.5964	0.3163	0.0434	-0.0311	0.0111	0.0254	-0.0009	0.3244
6	0.3385	0.5749	0.6610	0.6875	0.5415	0.2839	0.0248	-0.0221	0.0038	0.0271	-0.0164	0.2223
7	0.2393	0.4149	0.5923	0.6574	0.5820	0.2851	0.0051	-0.0253	0.0064	0.0284	-0.0180	0.1956
8	0.2159	0.3002	0.4846	0.5853	0.5459	0.3114	0.0733	-0.0494	0.0064	0.0306	-0.0112	0.1783
9	0.1796	0.2663	0.4081	0.5150	0.4620	0.2889	0.0797	0.1129	-0.0129	0.0301	-0.0010	0.1839
10	0.1758	0.2272	0.3703	0.4897	0.4093	0.2602	0.0906	0.1099	0.0809	0.0134	0.0011	0.1821
11	0.1676	0.2208	0.3479	0.4622	0.3901	0.2505	0.0856	0.1185	0.0781	0.0837	-0.0140	0.1849
12	0.1699	0.2133	0.3424	0.4445	0.3812	0.2623	0.0984	0.1209	0.0816	0.0823	0.0298	0.1745

Table 7.23c
Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = 1 \quad \forall i, j$,
B and R^2 Statistics; Tamaniat

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.423E-03	0.261E-03	-.174E-03	-.160E-03	-.276E-03	-.328E-03	-.125E-03	0.144E-03	0.295E-03	0.291E-03	0.341E-03	0.385E-03
2	-.472E+01	0.664E-03	0.746E-04	-.328E-03	-.428E-03	-.597E-03	-.442E-03	0.205E-04	0.435E-03	0.575E-03	0.621E-03	0.474E-03
3	-.511E+01	-.959E+01	0.315E-03	-.186E-03	-.533E-03	-.696E-03	-.612E-03	-.185E-03	0.348E-03	0.650E-03	0.777E-03	-.505E+01
4	-.844E+01	-.108E+02	-.122E+02	-.277E-05	-.427E-03	-.758E-03	-.682E-03	-.314E-03	0.184E-03	0.577E-03	0.831E-03	-.505E+01
5	-.950E+01	-.146E+02	-.141E+02	-.852E+01	-.305E-03	-.696E-03	-.709E-03	-.359E-03	0.104E-03	0.451E-03	0.770E-03	-.505E+01
6	-.132E+02	-.158E+02	-.183E+02	-.116E+02	-.974E+01	-.634E-03	-.692E-03	-.351E-03	0.782E-04	0.414E-03	0.680E-03	-.505E+01
7	-.135E+02	-.184E+02	-.196E+02	-.146E+02	-.120E+02	-.129E+01	-.664E-03	-.352E-03	0.882E-04	0.402E-03	0.681E-03	-.505E+01
8	-.136E+02	-.188E+02	-.224E+02	-.151E+02	-.143E+02	-.382E+01	-.948E+00	-.356E-03	0.738E-04	0.409E-03	0.670E-03	-.505E+01
9	-.134E+02	-.191E+02	-.232E+02	-.184E+02	-.140E+02	-.570E+01	-.395E+01	-.602E+00	0.366E-04	0.377E-03	0.678E-03	-.505E+01
10	-.118E+02	-.190E+02	-.234E+02	-.193E+02	-.175E+02	-.497E+01	-.558E+01	-.374E+01	0.136E+01	0.336E-03	0.631E-03	-.505E+01
11	-.124E+02	-.174E+02	-.233E+02	-.195E+02	-.184E+02	-.863E+01	-.464E+01	-.526E+01	-.185E+01	0.323E+01	0.582E-03	-.505E+01
12	-.114E+02	-.179E+02	-.217E+02	-.194E+02	-.186E+02	-.953E+01	-.830E+01	-.425E+01	-.333E+01	0.239E-01	0.147E+02	-.505E+01

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.8933	0.8776	0.9281	0.7760	0.8264	0.8085	0.9008	0.8652	0.9036	0.9398	0.9536	0.9089
2	0.8070	0.8462	0.7791	0.7590	0.4755	0.6410	0.7407	0.7332	0.7027	0.8096	0.8614	0.8519
3	0.8110	0.8652	0.7631	0.6263	0.5549	0.3650	0.7199	0.6682	0.6107	0.6813	0.7702	0.7860
4	0.7443	0.8783	0.7944	0.6608	0.4086	0.4056	0.5685	0.7404	0.5833	0.5874	0.6820	0.6674
5	0.5898	0.8234	0.8285	0.6940	0.4675	0.2377	0.6100	0.6915	0.6395	0.5830	0.5878	0.5497
6	0.4462	0.6574	0.7758	0.6542	0.4844	0.2055	0.4850	0.7212	0.6027	0.5784	0.5712	0.4480
7	0.3598	0.5719	0.6386	0.7326	0.3144	0.2123	0.4446	0.6960	0.6137	0.5439	0.5443	0.4130
8	0.3148	0.5031	0.6024	0.7262	0.4639	0.0509	0.4525	0.6880	0.6177	0.5445	0.5136	0.3698
9	0.2749	0.4376	0.5594	0.6796	0.5822	0.2564	0.2850	0.6790	0.6294	0.5581	0.5070	0.3463
10	0.2560	0.3723	0.4916	0.6501	0.6085	0.3679	0.4498	0.5839	0.6194	0.5736	0.5204	0.3374
11	0.2471	0.3442	0.4079	0.5830	0.6264	0.4514	0.5385	0.6849	0.5518	0.5701	0.5345	0.3455
12	0.2526	0.3331	0.3785	0.5273	0.6039	0.5186	0.6036	0.7058	0.6164	0.5045	0.5623	0.3557

Table 7.23d

Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = 1 \;\; \forall i, j$, B and R^2 Statistics; Malakal

BIAS

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
	*****MONTH*****											
1	-.777E-04	-.108E-03	0.943E-05	-.379E-04	-.400E-04	-.108E-03	-.824E-04	-.882E-04	-.200E-03	-.171E-03	-.185E-03	0.399E-03
2	-.170E+01	-.179E-03	-.845E-04	-.291E-04	-.760E-04	-.145E-03	-.178E-03	-.156E-03	-.282E-03	-.350E-03	-.340E-03	-.122E+02
3	-.996E+01	-.370E+01	-.148E-03	-.113E-03	-.682E-04	-.178E-03	-.210E-03	-.244E-03	-.343E-03	-.421E-03	-.506E-03	-.122E+02
4	-.141E+02	-.111E+02	-.287E+01	-.170E-03	-.145E-03	-.172E-03	-.244E-03	-.270E-03	-.421E-03	-.477E-03	-.574E-03	-.122E+02
5	-.126E+02	-.149E+02	-.956E+01	-.460E+01	-.196E-03	-.241E-03	-.234E-03	-.299E-03	-.442E-03	-.550E-03	-.623E-03	-.122E+02
6	-.204E+02	-.133E+02	-.134E+02	-.114E+02	-.886E+01	-.291E-03	-.307E-03	-.290E-03	-.473E-03	-.577E-03	-.696E-03	-.122E+02
7	-.222E+02	-.211E+02	-.118E+02	-.152E+02	-.156E+02	-.103E+02	-.359E-03	-.366E-03	-.461E-03	-.604E-03	-.723E-03	-.122E+02
8	-.226E+02	-.229E+02	-.195E+02	-.136E+02	-.194E+02	-.170E+02	-.883E+01	-.414E-03	-.536E-03	-.600E-03	-.754E-03	-.122E+02
9	-.220E+02	-.233E+02	-.212E+02	-.212E+02	-.179E+02	-.207E+02	-.154E+02	-.779E+00	-.589E-03	-.675E-03	-.753E-03	.122E+02
10	-.185E+02	-.228E+02	-.216E+02	-.229E+02	-.252E+02	-.192E+02	-.190E+02	-.719E+01	0.874E+01	-.721E-03	-.825E-03	-.122E+02
11	-.196E+02	-.193E+02	-.211E+02	-.233E+02	-.269E+02	-.265E+02	-.175E+02	-.107E+02	0.248E+01	0.997E+01	-.867E-03	-.122E+02
12	-.172E+02	-.204E+02	-.177E+02	-.228E+02	-.273E+02	-.281E+02	-.246E+02	-.928E+01	-.959E+00	0.383E+01	0.196E+01	-.122E+02

R SQUARED STATISTIC

LEAD	1	2	3	4	5	6	7	8	9	10	11	12
	*****MONTH*****											
1	0.9803	0.9760	0.9838	0.9664	0.8998	0.9212	0.9159	0.8804	0.8738	0.9299	0.8639	0.9773
2	0.9595	0.9567	0.9724	0.9473	0.8379	0.8593	0.8848	0.7661	0.7730	0.8193	0.8278	0.8589
3	0.8867	0.9283	0.9594	0.9302	0.8116	0.8175	0.8364	0.7010	0.6842	0.7395	0.8214	0.8519
4	0.8813	0.8025	0.9363	0.9224	0.8002	0.7936	0.7971	0.6750	0.6116	0.6627	0.7628	0.8741
5	0.8852	0.8227	0.8320	0.8937	0.7938	0.7810	0.7725	0.6435	0.5797	0.6093	0.7057	0.8373
6	0.8610	0.8669	0.8239	0.7570	0.7655	0.7807	0.7637	0.6227	0.5513	0.5710	0.6591	0.7814
7	0.7911	0.8381	0.8358	0.7687	0.6344	0.7532	0.7771	0.6084	0.5308	0.5439	0.6481	0.7329
8	0.7344	0.7709	0.7968	0.8014	0.6583	0.6256	0.7506	0.6219	0.5209	0.5274	0.6265	0.7116
9	0.7046	0.7012	0.7360	0.7670	0.6596	0.6639	0.5974	0.6022	0.5413	0.5295	0.6022	0.6826
10	0.6636	0.6907	0.6687	0.7214	0.6541	0.6359	0.6424	0.4348	0.5417	0.5495	0.5982	0.6525
11	0.6310	0.6471	0.6498	0.6583	0.6227	0.6050	0.6193	0.4796	0.4028	0.5577	0.6113	0.6478
12	0.6166	0.6179	0.6123	0.6374	0.5669	0.5674	0.5995	0.4521	0.4435	0.4546	0.6153	0.6560

Table 7.23e

Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = 1 \quad \forall i, j$,

B and R^2 Statistics; Mongalla

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.307E-04	0.504E-03	0.123E-02	0.110E-02	0.461E-03	-.281E-03	0.163E-03	0.692E-03	-.244E-04	-.109E-02	-.100E-02	-.388E-02
2	-.157E+02	0.612E-03	0.113E-02	0.129E-02	0.129E-02	0.412E-03	0.546E-03	0.121E-02	0.535E-03	-.901E-03	-.124E-02	-.983E-02
3	-.159E+02	-.781E+01	0.118E-02	0.123E-02	0.139E-02	0.858E-03	0.918E-03	0.142E-02	0.821E-03	-.542E-03	-.118E-02	-.939E-02
4	-.148E+02	-.789E+01	-.292E+01	0.127E-02	0.136E-02	0.912E-03	0.115E-02	0.161E-02	0.926E-03	-.382E-03	-.995E-03	-.939E-02
5	-.153E+02	-.736E+01	-.296E+01	0.724E+00	0.138E-02	0.896E-03	0.117E-02	0.173E-02	0.103E-02	-.340E-03	-.908E-03	-.910E+01
6	-.152E+02	-.761E+01	-.269E+01	0.702E+00	-.192E+01	0.902E-03	0.117E-02	0.175E-02	0.107E-02	-.277E-03	-.871E-03	-.910E+01
7	-.146E+02	-.753E+01	-.282E+01	0.838E+00	-.193E+01	0.130E+02	0.117E-02	0.174E-02	0.109E-02	-.251E-03	-.851E-03	-.910E+01
8	-.145E+02	-.722E+01	-.278E+01	0.773E+00	-.186E+01	0.130E+02	0.652E+02	0.176E-02	0.110E-02	-.242E-03	-.839E-03	-.910E+01
9	-.145E+02	-.717E+01	-.262E+01	0.793E+00	-.189E+01	0.130E+02	0.652E+02	0.152E+03	0.109E-02	-.251E-03	-.842E-03	-.910E+01
10	-.145E+02	-.717E+01	-.260E+01	0.873E+00	-.188E+01	0.130E+02	0.652E+02	0.152E+03	0.112E+03	-.229E-03	-.831E-03	-.910E+01
11	-.145E+02	-.717E+01	-.259E+01	0.886E+00	-.184E+01	0.130E+02	0.652E+02	0.152E+03	0.112E+03	0.965E+02	-.842E-03	-.910E+01
12	-.145E+02	-.717E+01	-.260E+01	0.886E+00	-.184E+01	0.130E+02	0.652E+02	0.152E+03	0.112E+03	0.965E+02	0.272E+02	-.910E+01

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.8077	0.6995	0.5274	0.5000	0.3122	0.3643	0.1831	0.5197	0.4991	0.5453	0.0247	0.5640
2	0.4368	0.6973	0.2815	0.0821	0.1103	0.0550	0.0317	0.0326	0.2670	0.2276	0.1845	-1.4020
3	-1.0687	0.4769	0.2934	0.0930	0.0334	0.0198	-0.0094	0.0218	0.0207	0.1334	0.0573	0.1803
4	0.2760	-0.4786	0.2532	0.0610	0.0207	-0.0091	-0.0135	0.0218	0.0191	0.0277	0.0847	0.0053
5	0.1726	0.4149	0.3315	0.0084	-0.0019	-0.0162	-0.0160	0.0200	0.0189	0.0258	-0.0051	0.0814
6	0.1655	0.3161	0.2009	0.1487	-0.0272	-0.0331	-0.0172	0.0193	0.0190	0.0261	-0.0077	-0.0231
7	0.0615	0.2023	0.1486	0.0256	0.0124	0.0042	-0.0359	0.0192	0.0191	0.0264	-0.0061	-0.0287
8	0.0557	0.0739	0.0584	0.0406	-0.0066	0.0054	0.0455	0.0011	0.0191	0.0266	-0.0053	-0.0279
9	0.0555	0.0703	-0.0105	0.0106	0.0074	0.0036	0.0451	0.1642	0.0009	0.0266	-0.0050	-0.0275
10	0.0561	0.0705	-0.0158	-0.0134	-0.0013	0.0072	0.0461	0.1641	0.0886	0.0086	-0.0050	-0.0271
11	0.0564	0.0708	-0.0158	-0.0166	-0.0067	0.0061	0.0462	0.1643	0.0885	0.0747	-0.0236	-0.0270
12	0.0565	0.0708	-0.0157	-0.0163	-0.0078	0.0055	0.0461	0.1644	0.0885	0.0747	0.0155	-0.0461

Table 7.23f

Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = 1 \;\; \forall i, j$, B and R^2 Statistics; Khartoum

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.136E-03	-.129E-03	0.235E-03	0.103E-02	0.860E-03	0.486E-03	0.686E-03	0.763E-03	0.215E-03	-.310E-03	-.971E-04	-.665E-02
2	-.101E+02	-.359E-04	0.144E-03	0.119E-02	0.157E-02	0.108E-02	0.103E-02	0.124E-02	0.757E-03	-.199E-03	-.323E-03	-.867E+01
3	-.102E+02	-.432E+01	0.197E-03	0.114E-02	0.166E-02	0.148E-02	0.135E-02	0.142E-02	0.102E-02	0.108E-03	-.258E-03	-.867E+01
4	-.927E+01	-.439E+01	-.233E+00	0.117E-02	0.164E-02	0.152E-02	0.156E-02	0.159E-02	0.111E-02	0.243E-03	-.960E-04	-.867E+01
5	-.971E+01	-.392E+01	-.271E+00	0.253E+01	0.165E-02	0.151E-02	0.158E-02	0.169E-02	0.122E-02	0.301E-03	-.133E-04	-.867E+01
6	-.957E+01	-.414E+01	-.299E-01	0.251E+01	0.156E+01	0.152E-02	0.158E-02	0.171E-02	0.126E-02	0.343E-03	0.144E-04	-.867E+01
7	-.902E+01	-.407E+01	-.144E+00	0.263E+01	0.155E+01	0.206E+02	0.159E-02	0.171E-02	0.127E-02	0.363E-03	0.400E-04	-.867E+01
8	-.893E+01	-.379E+01	-.108E+00	0.257E+01	0.161E+01	0.206E+02	0.757E+02	0.172E-02	0.127E-02	0.367E-03	0.488E-04	-.867E+01
9	-.893E+01	-.374E+01	0.331E-01	0.259E+01	0.158E+01	0.207E+02	0.757E+02	0.152E+03	0.128E-02	0.378E-03	0.433E-04	-.867E+01
10	-.893E+01	-.374E+01	0.562E-01	0.266E+01	0.159E+01	0.207E+02	0.757E+02	0.152E+03	0.964E+02	0.370E-03	0.499E-04	-.867E+01
11	-.893E+01	-.375E+01	0.567E-01	0.267E+01	0.163E+01	0.207E+02	0.757E+02	0.152E+03	0.964E+02	0.838E+02	0.477E-04	-.867E+01
12	-.892E+01	-.375E+01	0.546E-01	0.267E+01	0.163E+01	0.207E+02	0.757E+02	0.152E+03	0.964E+02	0.838E+02	0.296E+02	-.867E+01

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.8710	0.6972	0.4478	0.4679	0.2167	0.3172	0.1863	0.4694	0.4296	0.4741	0.1994	0.6893
2	0.5402	0.6370	0.2083	0.1439	0.1109	0.0511	0.0458	0.0155	0.2686	0.2067	0.2071	-0.9150
3	-0.7238	0.4425	0.2107	0.1026	0.0631	0.0274	0.0095	0.0029	0.0230	0.1273	0.0910	0.3039
4	0.3514	-0.3558	0.1751	0.0698	0.0445	0.0096	0.0064	0.0022	0.0197	0.0265	0.0847	0.1316
5	0.1633	0.4266	0.3253	0.0273	0.0240	0.0028	0.0038	0.0009	0.0177	0.0228	0.0022	0.1240
6	0.1338	0.3398	0.1403	0.2048	0.0116	-0.0122	0.0027	0.0002	0.0180	0.0233	0.0005	0.0111
7	0.0272	0.1704	0.1280	0.0945	0.0336	0.0551	-0.0156	0.0001	0.0180	0.0237	0.0016	0.0054
8	0.0204	0.0275	0.0630	0.0834	0.0259	0.0550	0.0937	-0.0184	0.0180	0.0239	0.0024	0.0053
9	0.0201	0.0200	0.0094	0.0520	0.0310	0.0564	0.0933	0.1612	-0.0002	0.0239	0.0027	0.0057
10	0.0205	0.0186	0.0026	0.0265	0.0254	0.0568	0.0940	0.1611	0.0839	0.0058	0.0027	0.0061
11	0.0208	0.0186	0.0021	0.0232	0.0228	0.0557	0.0941	0.1612	0.0838	0.0692	-0.0158	0.0061
12	0.0208	0.0186	0.0021	0.0230	0.0221	0.0553	0.0941	0.1611	0.0838	0.0691	0.0344	-0.0123

Table 7.23g
Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = 1 \;\; \forall i, j$,
B and R^2 Statistics; Sennar

BIAS

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.805E-03	0.608E-03	0.310E-03	0.159E-03	0.183E-04	-.871E-04	-.102E-03	0.422E-03	0.104E-02	0.144E-02	0.124E-02	0.485E-04
2	-.135E+02	0.109E-02	0.672E-03	0.340E-03	0.111E-03	-.816E-04	-.158E-03	0.331E-03	0.130E-02	0.206E-02	0.207E-02	-.235E-03
3	-.842E+01	-.845E+01	0.955E-03	0.555E-03	0.221E-03	-.316E-04	-.155E-03	0.300E-03	0.125E-02	0.221E-02	0.243E-02	-.240E+00
4	-.132E+02	-.543E+01	-.426E+01	0.722E-03	0.347E-03	0.350E-04	-.134E-03	0.297E-03	0.124E-02	0.218E-02	0.252E-02	-.935E+01
5	-.178E+02	-.826E+01	-.247E+01	-.223E+01	0.446E-03	0.113E-03	-.888E-04	0.326E-03	0.123E-02	0.218E-02	0.253E-02	-.144E+02
6	-.165E+02	-.110E+02	-.415E+01	-.117E+01	-.192E+01	0.165E-03	-.466E-04	0.337E-03	0.125E-02	0.217E-02	0.253E-02	-.144E+02
7	-.160E+02	-.102E+02	-.576E+01	-.217E+01	-.129E+01	0.214E+02	-.122E-04	0.366E-03	0.126E-02	0.217E-02	0.253E-02	-.144E+02
8	-.160E+02	-.994E+01	-.530E+01	-.312E+01	-.188E+01	0.217E+02	0.774E+02	0.382E-03	0.128E-02	0.218E-02	0.253E-02	-.144E+02
9	-.159E+02	-.990E+01	-.514E+01	-.285E+01	-.245E+01	0.214E+02	0.777E+02	0.163E+03	0.129E-02	0.221E-02	0.253E-02	-.144E+02
10	-.159E+02	-.986E+01	-.512E+01	-.275E+01	-.229E+01	0.211E+02	0.774E+02	0.163E+03	0.125E+03	0.221E-02	0.255E-02	-.144E+02
11	-.159E+02	-.986E+01	-.509E+01	-.274E+01	-.223E+01	0.212E+02	0.772E+02	0.163E+03	0.125E+03	0.892E-02	0.255E-02	-.144E+02
12	-.159E+02	-.988E+01	-.509E+01	-.273E+01	-.222E+01	0.212E+02	0.773E+02	0.163E+03	0.125E+03	0.893E+02	0.362E+02	-.144E+02

R SQUARED STATISTIC

LEAD	*****MONTH***** 1	2	3	4	5	6	7	8	9	10	11	12
1	0.8341	0.6420	0.3277	0.5009	0.0329	0.2818	0.1967	0.3861	0.4571	0.3966	-0.2263	0.4917
2	0.2112	0.5106	0.4133	0.1802	0.0143	0.0158	0.0418	0.0321	0.2731	0.1743	-0.1509	-1.5861
3	-2.4075	0.1584	0.3753	0.1946	0.0145	0.0083	0.0052	0.0090	0.0648	0.1501	-0.0395	-0.1848
4	-0.3606	-2.7310	0.3032	0.1816	0.0099	0.0064	0.0027	0.0065	0.0504	0.0532	0.1530	-0.0369
5	-0.1418	-0.2885	-0.0688	0.1565	-0.0166	0.0028	0.0036	0.0044	0.0479	0.0382	0.0551	0.1796
6	0.2038	0.0262	-0.0727	0.2580	-0.0132	-0.0120	0.0030	0.0044	0.0477	0.0401	0.0339	0.0650
7	0.0427	0.3084	-0.0495	0.0537	-0.0248	0.0765	-0.0149	0.0044	0.0482	0.0405	0.0370	0.0404
8	0.0023	0.0548	0.0680	0.0772	-0.0179	0.0703	0.0967	-0.0139	0.0484	0.0409	0.0372	0.0386
9	0.0016	0.0143	0.0184	0.0472	-0.0023	0.0807	0.0956	0.1694	0.0308	0.0410	0.0383	0.0377
10	0.0010	0.0104	-0.0075	0.0049	0.0043	0.0837	0.0982	0.1696	0.1584	0.0234	0.0383	0.0383
11	0.0018	0.0086	-0.0071	-0.0078	0.0039	0.0809	0.0989	0.1692	0.1583	0.1015	0.0207	0.0385
12	0.0017	0.0093	-0.0076	-0.0075	0.0019	0.0797	0.0986	0.1693	0.1584	0.1012	0.1469	0.0209

Table 7.23h

Forecasting Performance of Multivariate Model 5.32 Estimated with $\Psi_j^*(i)^2 = .1 \quad \forall i, j$, B and R^2 Statistics; Roseires

7.5 Observations and Conclusions

The overall forecasting performance of the multivariate version of Models 5.31 or 5.32 is better than their univariate versions. In addition, the univariate and multivariate Model 5.31 forecasting performance is superior to the corresponding univariate or multivariate Model 5.32. Considering forecasting performance only, the multivariate Model 5.31 of the Nile River basin would be the preferred choice.

The identification of univariate or multivariate Model 5.31 is much more straightforward and objective as compared to Model 5.32. The principal disadvantage of Model 5.31, relative to 5.32, is its unparsimonious use of parameters. Some parameters of multivariate Model 5.31 are estimated in a model that has a ration of 5 observations per parameter.

As reported in the literature review, Section 5.2, Rao and Kashyap and McKerchar and Delleur found that forecasts with error variance less than the historical variance may be obtained for up to 3 months. Because these authors do not break up the error variance according to month, a comparison to this work is not clear cut. However, inspecting the various "R^2" statistic tables obtained from models of Wadi Halfa (i.e., Table 7.20, 7.21a, 7.22 and 7.22a), it appears that for the Nile the valuable forecast lead times are somewhat greater than 3 months.

The finding that $n_2^* = 0$ in the univariate Model 5.32 agrees with results reported by Rao and Kashyap (1973). In this work, it was also found that $n_2^* = 0$ in the multivariate Model 5.32.

The residual analysis of univariate and multivariate Models 5.31 and 5.32 did not clearly indicate a violation of the assumption of white residuals. However, whiteness of residuals was also not conclusively shown. This irresolute position indicates a weakness in the whiteness tests.

Another problem with the whiteness tests is that even if some correlation in the residuals is implied, no information is given as to what new model should be formulated to alleviate the condition. This situation was particularly troublesome when Model 5.32 was being identified.

Chapter 8 presents overall conclusions.

Chapter 8

REVIEW, CONCLUSIONS, AND SUGGESTIONS FOR CONTINUED RESEARCH

8.1 Review and Conclusions

The major emphasis of this work has been the identification, estimation and validation of seasonal univariate and multivariate stochastic streamflow models appropriate for obtaining multi-lead forecasts. A theoretical development and solution of these issues has been presented as well as a practical application motivated by the Nile River basin. The forecast performance results of the application indicate the usefulness of the suggested approach.

An original contribution of this work is the theoretical treatment and application of the multivariate case.

As a secondary effort, undertaken to provide a motivation for forecasting, a new adaptive reservoir control algorithm was derived. Information in streamflow forecasts is incorporated into the control decision via approximate conditional first order Markov transition matrices. This control algorithm may be the most significant contribution of this work.

8.2 Suggestions for Continued Research

Section 7.5 pointed out that, as formulated, the residual analysis portion of the adequacy checking stage of the iterative approach to model building provides little information to the identification stage. Possible a better feedback loop can be found.

Data transformations, such as the general Box-Cox (Box and Jenkins, 1970), have the potential to improve forecast performance. Using the performance of models for the Nile basin documented in this work as a baseline, this possibility might be investigated.

Naturally, the identification, estimation and evaluation of models for an entirely new river basin will provide useful and possibly suggestive data.

However, probably the most fruitful area of continued research would be the implementation of the suggested adaptive control algorithm. This may be accomplished by using one of the final stochastic models of the Nile basin given in Chapter 7 in conjunction with the stochastic dynamic programming model for the operation of the High Aswan Dam prepared by Alarcon (1979). Alternatively, hypothetical models may be used.

The fundamental question of the value of forecasts and the sensitivity of the value to various hydrologic and structural parameters can be investigated. The results of the research may indicate when it is beneficial to undertake a forecasting program.

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Appendix 1

MONTHLY FLOW DATA FOR THE NILE RIVER

A1.1 Contents

This appendix contains a listing of historical monthly flows at various stations throughout the Nile River basin. The data listed is that which was actually observed. No attempt was made to deregulate the data to obtain the naturalized flow. A description which gives the time of construction, capacity, and approximate operating policies of various storage works along the Nile River, as well as a listing of the flow data, may be found in Cairo University - M.I.T. Technological Planning Program (1977, 1978).

Data sets contained in this appendix, listed in order, are:

HISTORICAL MONTHLY STREAMFLOWS AT ASWAN ; 1871-1904; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1871	6970	4870	4060	2450	1900	1930	6650	24800	27200	17700	9100	6500
1872	4540	2640	1900	1470	1370	2260	8340	23700	28600	21800	12800	8300
1873	5900	4540	3390	1920	1380	3590	6150	19100	22600	15100	7950	5540
1874	3670	2050	1700	1230	1100	2540	8180	30200	33600	22200	11100	7760
1875	5940	4140	2910	1960	1600	1760	6430	25200	28700	20900	11400	7580
1876	5700	4560	3940	2430	1880	2290	8290	23800	30800	18200	9770	6760
1877	4970	3150	7560	1940	1860	2570	7050	15600	16500	12500	73904	9308
1878	3760	2320	1750	1300	1110	1170	5750	22300	34200	29500	14300	10100
1879	7690	5830	5740	4850	4540	5210	10500	26000	29500	19700	11200	8580
1880	7040	5710	4930	3530	2940	3000	9440	22600	24200	16900	8420	6380
1881	4840	3270	2710	1920	1800	1960	4990	18000	27700	17700	9630	6560
1882	4950	3330	2380	1620	1360	1270	4120	18200	23300	15400	11000	6930
1883	5070	3760	3450	2250	1710	1790	7270	24600	27700	17400	9940	7310
1884	5610	4450	3950	2730	2260	2320	4800	17600	21700	16900	10700	6940
1885	5230	3620	2580	1750	1390	1380	8190	25300	23900	15700	8180	5740
1886	4100	2530	2150	1730	1710	1980	5100	20300	26000	15700	8470	6910
1887	5040	3300	2470	1080	1800	2550	8190	29900	31800	18300	10000	7220
1888	5570	3460	2580	1910	1690	1770	4270	17400	19000	11700	5910	4420
1889	3110	1850	1570	1240	1140	1190	5030	22400	27400	17700	8600	5770
1890	4450	2620	1900	1330	1130	1500	5880	26400	29200	21800	12200	7850
1891	5700	3470	2330	1630	1520	3160	6730	21900	27000	19900	12200	7550
1892	5260	3450	2310	1500	1210	1230	5650	24200	34100	24600	12300	8340
1893	6670	5220	5100	4430	2720	2090	4960	22500	24400	20200	10300	7050
1894	5440	3580	2480	1780	1790	2150	7640	27200	31100	25300	12300	8700
1895	6790	5080	4780	3490	2890	2660	7740	30500	29000	17500	10500	8440
1896	6440	4870	3850	2730	2400	2360	7110	22200	29800	18900	13000	11000
1897	7000	4930	4110	2830	2430	2760	5790	19000	24600	16400	8230	5860
1898	4720	3370	2450	1740	1490	1420	4060	25200	28900	20400	11300	8130
1899	6070	4690	4380	3070	2200	2100	4830	14600	16700	10200	5020	3440
1900	2110	1340	1110	857	885	1260	3880	22900	22100	15800	7210	5090
1901	3840	2390	1870	1340	1380	1530	5330	20700	24300	13200	7230	4990
1902	3380	1960	1580	1280	1200	1330	3480	11000	18300	14100	6820	4470
1903	3620	2240	1660	1140	989	1740	4980	17700	24800	18600	10200	5020
1904	4500	3430	2530	1700	1620	2080	5120	18900	19000	13000	6550	4920

HISTORICAL MONTHLY STREAMFLOWS AT ASWAN ; 1905-1938; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1905	3850	2500	1960	1400	1170	1110	2640	12400	19600	13200	5660	4640
1906	3690	2680	2110	1880	1590	1510	3460	17800	24800	17500	8760	5750
1907	4290	2890	2170	1690	1660	1540	3160	11700	17600	11200	6660	4280
1908	3600	2250	1790	1320	1190	1740	3240	21900	29300	20500	9380	5990
1909	4530	3430	2580	1700	1990	2700	5730	20700	25800	19400	9560	6400
1910	5000	3890	3220	1910	1450	1700	3290	16400	24500	18900	10700	5980
1911	4340	3200	2350	1470	1280	1660	3160	15000	23300	14500	7400	5360
1912	3380	2470	1890	1410	1180	984	3170	18500	18200	11100	5690	4170
1913	3260	2040	1480	1280	1170	1470	1740	6500	12200	7540	4120	2830
1914	1720	1150	1070	947	998	975	2010	19400	20000	16500	10700	6930
1915	4510	3090	2130	1360	1170	1460	2850	10700	14900	14700	8090	5270
1916	3850	2400	1610	1180	1090	1340	5010	25000	26600	22400	13300	7880
1917	5370	3910	3580	2320	1680	1860	4830	17500	27800	22800	11600	7460
1918	5270	3990	4330	4090	3960	3230	5190	13600	17700	11100	6210	4580
1919	3440	2180	1910	1520	1350	1520	3880	16600	20800	12600	6140	4410
1920	3320	2090	1600	1370	1220	1710	5370	18200	17800	15000	8960	5370
1921	3820	2370	1760	1330	1180	1300	2980	15400	19600	14800	7590	4700
1922	3530	2090	1460	996	801	900	3280	18800	23600	15900	8240	4880
1923	3030	2350	1540	1200	1380	2050	4440	20100	21400	16200	7230	5170
1924	4000	2650	1850	1320	1470	1630	5050	18000	22300	14300	7990	5590
1925	3840	2540	1830	1370	1260	1580	3910	13400	16600	12500	6450	4480
1926	3440	2150	1640	1340	1330	2390	4360	19200	20600	14500	8330	5230
1927	3940	2910	2030	1440	1300	1290	4180	15600	18300	13300	6090	4010
1928	2600	1630	1310	1160	1320	2560	6290	18200	20000	12400	7300	4820
1929	3670	2400	1770	1340	1390	3250	8770	23200	24500	18000	9910	5580
1930	4250	2950	2120	1540	1530	1660	4400	18300	17600	11800	5640	4020
1931	2350	1790	1500	1220	1090	1140	2850	15900	21300	15000	8620	4840
1932	3580	2260	1680	1320	1270	1700	4280	18900	22900	15800	7720	5050
1933	4170	3470	2850	1680	1600	1800	3280	13100	22100	15300	9150	5940
1934	4340	2960	2110	1550	1440	1700	6050	20500	23700	16400	8370	5250
1935	4170	2990	2170	1530	1520	2330	7100	21300	24100	17700	8220	5200
1936	4000	2800	2180	1560	1450	1530	5490	19000	23900	15800	7420	4550
1937	3340	2100	1740	1360	1130	1500	3950	19800	22300	14300	6050	4590
1938	3540	2160	2160	1560	1280	1450	3710	23200	27800	19500	9520	5520

HISTORICAL MONTHLY STREAMFLOWS AT ASWAN ; 1939-1972; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1939	4120	3240	2430	1960	2220	1960	3920	13100	18400	13000	7980	4770
1940	3420	2130	2430	1810	1310	1330	3050	15300	18000	10900	4630	3370
1941	2260	1830	2070	1630	1100	1550	4320	12000	14400	11100	8040	4540
1942	3200	2330	2460	2310	1930	1710	4530	20300	20300	14900	6360	4240
1943	3230	1910	2570	2080	1670	1480	2670	15600	24700	14200	6840	4270
1944	3070	2000	2660	2270	1890	1810	4180	16700	18100	12600	5240	3850
1945	2590	1340	2550	2150	1580	1610	3580	14900	18900	16400	8410	5360
1946	3840	2550	2360	2310	2120	1340	6130	25100	27800	15800	9100	5980
1947	4420	3480	3180	2520	2770	2850	3170	14900	22500	15000	6550	4640
1948	3710	2950	2510	2370	2330	1530	5430	16600	19300	15700	9790	5190
1949	3610	2830	2440	2400	2510	1900	4080	18100	20400	14300	7490	4840
1950	3860	2900	2500	2390	2530	2280	4610	20800	22500	14800	6610	4460
1951	3390	2240	2270	2340	2150	1180	2850	15400	17800	12000	7790	4940
1952	3280	2380	2680	2100	1610	1690	3210	14900	21100	12500	6240	3710
1953	2790	1890	2180	2410	1600	1460	3860	22000	20800	13900	6330	4120
1954	2870	1830	2550	2270	1820	1180	5630	24800	28300	20500	8550	5050
1955	3820	2710	2080	2280	2490	2180	4320	19200	22700	18000	7410	4580
1956	3580	2800	2350	2310	2550	2370	5780	19700	20700	18400	12400	5760
1957	3990	3110	2380	2630	3010	2960	4450	17600	21400	9920	4890	3380
1958	2560	1850	2080	2150	1700	1300	5000	24600	24500	15700	8410	4870
1959	3430	2420	2250	2420	2340	1620	3370	18700	29600	16600	9670	5090
1960	3650	2420	2220	2460	2370	1730	3790	17100	20900	14500	6610	3740
1961	2940	1980	2210	2540	2220	1310	5400	22600	27000	18600	9410	5680
1962	4140	3080	2780	2900	2840	3140	4690	15800	22500	16700	7260	4730
1963	3710	2960	2700	2550	2880	3310	5310	20400	21800	13200	6150	5110
1964	4770	3570	3040	2550	3480	3840	5840	24900	27000	18300	12300	6980
1965	5880	5250	4940	3660	3660	4540	5600	13400	17300	12400	9770	5750
1966	4380	3140	2510	2580	3570	2330	5480	12300	16900	11400	5020	4870
1967	2745	3390	3888	3557	4480	5869	6618	8921	7096	12710	5598	4670
1968	3281	4228	4063	3700	4785	6350	6665	6105	4180	3795	3390	3240
1969	2900	3700	3970	3660	4870	6570	6710	5890	4200	3810	3640	3390
1970	3060	3980	4090	3920	5420	6520	6720	6110	4285	3750	3595	3245
1971	3554	3785	4275	3940	5475	6490	6965	6225	4445	3775	3670	3295
1972	3430	4020	4240	4040	5290	6535	6990	6290	4245	3745	3590	3025

HISTORICAL MONTHLY STREAMFLOWS AT WADI HALFA; 1890-1923; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1890	3680	2030	1510	1130	1030	1360	6500	26800	29300	20500	10300	6960
1891	5140	2970	1950	1490	1530	3200	6020	21000	25000	18100	11300	7170
1892	4960	3120	2030	1370	1210	1400	6520	24500	33100	23700	11200	8100
1893	6900	5310	5240	4310	2520	1970	5860	21500	23100	19400	10100	6910
1894	5400	3590	2650	2000	2010	2670	8310	27300	33000	25300	12100	8840
1895	7170	5540	5140	4000	3510	3240	9830	29800	27000	15700	10100	8420
1896	6510	5080	3970	2860	2580	2500	8130	22100	29100	18100	12700	10300
1897	7040	5030	4170	2820	2520	3020	6310	19400	23000	15300	7590	5600
1898	4500	3180	2260	1660	1470	1610	4510	24900	27500	19400	10300	7680
1899	5850	4610	4260	2880	2130	2110	5470	15100	16200	9390	4760	3280
1900	2120	1490	1360	1200	1280	1630	4550	23300	21500	14700	6450	4740
1901	3630	2330	1850	1400	1560	1740	5880	20900	23100	12000	6610	4660
1902	3200	2010	1730	1500	1460	1730	4000	11900	18800	14000	7060	4940
1903	3690	2220	1700	1310	1270	2120	5450	18100	24800	17900	9450	5800
1904	4440	3460	2600	1860	1770	2190	5620	18300	18800	12600	6510	4850
1905	3730	2330	1850	1410	1270	1300	3070	13600	20400	12700	5790	4680
1906	3520	2410	1920	1720	1510	1530	4200	19400	25600	16200	6400	4960
1907	3970	2710	2050	1770	1840	1750	3900	12800	17800	10900	6250	4410
1908	3370	2170	1810	1420	1350	1490	3940	21700	27800	19700	8710	5790
1909	4620	3500	2850	2040	2320	3240	6900	22100	27000	19300	9540	6730
1910	5570	4410	3530	2280	1820	2210	4200	17700	25500	19600	10500	6420
1911	5000	3630	2780	2000	1820	2250	4280	16500	23900	15000	8050	5760
1912	3880	2540	1920	1450	1230	1080	3700	20800	19600	11100	6100	4580
1913	3650	2310	1870	1480	1360	1800	2230	7680	13400	7860	4140	3020
1914	2100	1420	1290	1100	1180	1140	2900	22900	22300	18000	10500	6260
1915	4190	2930	2050	1380	1290	1600	3080	12300	16400	15000	7840	4910
1916	3510	2170	1500	1180	1130	1320	6100	27100	31700	23200	12200	7060
1917	5340	4130	3790	2410	1760	2080	5830	19600	31500	24200	11600	6880
1918	5350	4350	4810	4540	4340	3460	6300	15600	18800	11900	6670	5010
1919	3700	2390	2070	1620	1470	1680	5070	19200	22400	13000	6260	4500
1920	3430	2050	1590	1310	1220	1970	6230	20700	19100	15800	8710	5180
1921	3800	2180	1680	1300	1120	1380	3300	17900	21500	15200	7140	4580
1922	3420	1960	1380	1050	880	1000	3820	19300	25500	17000	8130	5060
1923	3650	1970	1260	1130	1520	2370	5000	22500	23100	16700	6970	4790

HISTORICAL MONTHLY STREAMFLOWS AT WADI HALFA; 1924-1957; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1924	3730	2400	1650	1240	1400	1550	5730	19600	23300	14500	7900	5290
1925	3760	2470	1820	1370	1290	1630	4440	15700	17900	12800	5940	4090
1926	3390	2230	1820	1690	1760	2700	4960	21600	23100	14900	7500	4810
1927	3850	2740	1960	1590	1570	1500	4750	18300	20300	13700	5510	3670
1928	2590	1690	1460	1330	1720	2710	7520	19900	21000	12200	6610	4350
1929	3480	2230	1810	1480	1790	3770	10000	25400	25800	18700	9320	5210
1930	4210	2870	2210	1730	1740	1910	5590	19700	18500	11500	5230	3660
1931	2720	1640	1470	1260	1340	1200	3370	16900	21600	14800	7920	4420
1932	3440	2010	1670	1360	1390	1980	5070	20200	23400	15900	7300	4680
1933	4200	3380	2730	1670	1760	1930	3810	14400	22800	15100	8690	5660
1934	4190	2710	2010	1670	1660	1820	7600	22100	24400	16500	7400	5020
1935	4200	2850	2140	1700	1840	2680	8000	22900	24200	17200	7520	4950
1936	3950	2640	2230	1770	1730	1800	6230	20000	25600	15600	6890	4200
1937	3170	1900	1650	1450	1400	1800	4840	21600	23800	13500	5790	4310
1938	3350	2090	2210	1670	1470	1600	4520	23800	27000	19800	9850	5260
1939	4120	3160	2340	2140	2220	2170	4230	14000	18400	12400	7580	4360
1940	3160	2080	2500	1830	1350	1550	3430	16200	18200	9780	4280	2990
1941	2040	1820	2150	1670	1150	1930	4430	13300	15100	11100	7430	3990
1942	2990	2080	2540	2520	1810	1920	5450	22400	21000	14500	5520	3810
1943	3030	1850	2500	2220	1610	1560	3280	17000	24300	12900	6090	3530
1944	2670	1700	2570	2450	1800	2130	5190	18300	18500	12100	5040	3640
1945	2610	1890	2750	2280	1500	2010	3890	16000	19600	15900	8000	4890
1946	3700	2290	2510	2470	2130	1610	7370	26900	27700	14600	8230	5200
1947	3990	3080	3060	2600	2860	2920	3480	16400	22900	13900	5760	4060
1948	3410	2680	2500	2560	2200	1830	5860	17500	19200	15500	8690	4560
1949	3490	2590	2410	2520	2550	2010	4630	18500	20200	13200	6960	4380
1950	3750	2720	2540	2510	2720	2270	5430	22400	23300	14100	5910	3780
1951	3110	1890	2320	2420	1990	1300	3400	16500	18200	11900	7610	4420
1952	3040	2320	2760	2250	1590	1490	3930	15790	20450	11400	5300	3280
1953	2490	1830	2320	2420	1510	1740	5200	23000	20400	12700	5430	3620
1954	2560	1790	2550	2350	1650	1440	6870	25200	27600	19000	7370	4490
1955	3520	2580	2150	2480	2530	2390	5030	19900	22900	17200	6400	4090
1956	3410	2730	2470	2590	2720	2630	6660	20100	20800	17900	11100	5260
1957	3750	3020	2480	2920	3070	3390	4770	18600	19500	8530	4360	3020

HISTORICAL MONTHLY STREAMFLOWS AT WADI HALFA; 1958-1976; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1958	2320	1880	2170	2400	1770	1610	5920	25200	22700	15200	7150	4540
1959	3220	2310	2280	2540	2270	1860	3970	19500	27800	14800	8470	4350
1960	3120	2220	2230	2500	2500	1800	4480	17400	20400	13500	5370	3140
1961	2550	1780	2310	2600	2030	1510	6550	23200	26800	17400	7560	4920
1962	3670	2760	2590	2990	2980	3220	5000	17400	22600	15500	5830	4020
1963	3350	2820	2560	2820	3250	3590	6990	22100	19700	10600	4880	4560
1964	4300	3240	2860	2700	3630	4100	7920	24900	25700	16400	9450	5850
1965	5750	5080	4390	3580	4170	4520	5820	14800	15600	11000	6600	4910
1966	3930	3020	2540	3210	3550	3110	5570	12800	17500	7290	4470	4530
1967	4650	3306	2108	2985	3317	3030	5890	19902	22050	13919	6240	5270
1968	4092	3422	2883	3150	3255	2988	7006	17825	11400	9951	4590	3689
1969	2954	2294	2635	3840	3782	2571	5890	19778	17400	6262	3960	3348
1970	3255	2575	2145	3480	3193	2199	3441	19499	20760	9114	4860	3751
1971	3472	3422	2297	3660	3502	2352	5487	19096	20160	9300	5550	4185
1972	3472	3103	2378	3660	3348	2490	5084	11129	8820	7378	3630	2558
1973	2192	1494	1559	2568	2858	2769	4092	17174	16560	10199	4860	3131
1974	2992	2271	1969	3030	3162	2868	8490	22568	19470	11346	5130	3534
1975	3286	2567	1938	3060	3193	2049	4991	22754	29550	15190	6480	4681
1976	3751	2842	2458	2661	3658	3210	5487	17236	14730	7347	4440	3565

HISTORICAL MONTHLY STREAMFLOWS AT HASSANAB ; 1909-1942; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1909	5090	3820	2880	2620	3270	5200	12300	22000	23800	18500	9440	6970
1910	5570	4360	3320	2430	2320	3130	6210	17600	21700	18400	9620	6490
1911	5270	3670	2990	2400	2710	3220	6770	16800	21400	13600	8220	6570
1912	4350	2970	2440	1980	1820	2250	6780	18500	16700	9890	6260	4810
1913	3710	2370	2090	1830	1830	1860	2670	8030	10500	5650	3360	2370
1914	1710	1070	1050	949	1110	1460	4610	17100	16300	14700	7290	5110
1915	4100	2810	2010	1520	1690	2250	4470	11200	14100	12200	6670	4470
1916	3300	2000	1520	1130	1240	2020	7150	19000	21500	18800	10100	6190
1917	4710	3620	3150	1890	1680	2560	7670	18200	23100	18900	8990	6000
1918	4790	3790	4070	4000	3560	3450	7230	13800	14600	8900	5210	3970
1919	2800	1930	1740	1420	1270	2090	6520	16100	18800	9250	5020	3860
1920	2580	1630	1460	1170	1130	2720	7320	16300	14300	12500	6700	4330
1921	3260	2120	1530	1030	1070	1890	4570	15200	18100	12000	5630	4030
1922	2970	1800	1270	857	862	1700	5220	17400	18400	13200	6130	4160
1923	3200	1840	1520	1470	1620	3230	6780	19000	18400	12600	5940	4520
1924	3610	2290	1750	1560	1780	2260	6660	16400	18300	11900	7010	4660
1925	3420	2030	1690	1380	1640	2440	5390	14400	12900	9260	5120	4060
1926	2950	1810	1680	1560	2040	3390	5970	18900	18400	11900	6010	4350
1927	3520	2390	1860	1620	1620	2160	5300	15300	14100	9770	4390	3260
1928	2060	1520	1460	1600	2560	3120	8550	16600	16200	9820	5540	4250
1929	3240	2050	1850	1640	2440	4800	10900	20100	21400	16200	7240	4720
1930	3800	2490	2000	1650	2010	2520	6410	16800	15200	8930	4450	3340
1931	2260	1490	1420	1340	1330	1880	4530	15200	17200	12700	5950	3920
1932	2940	1770	1580	1400	1630	2460	6040	17800	18600	12800	5860	4320
1933	3940	3050	2240	1690	1800	2350	4340	14200	17100	12700	7120	4860
1934	3840	2350	2000	1600	1830	2560	8300	18400	19100	13200	6310	4550
1935	3860	2410	1960	1860	2040	3480	9610	20000	20600	14700	6310	4450
1936	3610	2350	2160	1810	1820	2320	7850	17600	19700	11800	5490	3780
1937	2730	1770	1750	1430	1580	2370	6620	18200	18100	9740	4860	3920
1938	2920	2080	2080	1600	1580	2060	6640	19800	20600	16100	6560	4590
1939	3690	2880	2210	2370	2300	2690	5030	13100	15100	10300	5890	3890
1940	2620	2300	2500	1620	1510	1940	3780	15400	1[illegible]400	7390	3610	2700
1941	1850	2090	2180	1460	1300	2950	5050	12000	12200	10200	6010	3480
1942	2880	2200	2690	2580	1700	2550	6570	18300	15800	11500	4560	3560

HISTORICAL MONTHLY STREAMFLOWS AT HASSANAB ; 1943-1951; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1943	2580	2340	2560	2180	1650	1710	4560	13600	16400	10000	4920	3210
1944	2130	2380	2650	2410	1880	2560	5480	14500	13400	8620	3960	2810
1945	2090	2150	2590	2030	1300	2430	4393	13800	15900	12900	6050	4340
1946	3210	2020	2840	2390	1810	2310	7340	20800	21300	12100	6920	4840
1947	3930	3070	3060	2900	2970	2960	4120	15900	18200	11100	4750	3970
1948	3300	2550	2920	2690	2200	2840	7130	16000	16100	13600	6760	4200
1949	3390	2490	2720	2750	2550	2660	6630	17000	17300	11300	5600	4160
1950	3560	2590	2750	2770	2960	2770	5430	16300	16800	10100	4890	3670
1951	3010	1860	2810	2650	1740	1950	4440	14100	12600	10500	5750	3750

HISTORICAL MONTHLY STREAMFLOWS AT ATBARA ; 1903-1936; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1903	0	0	0	0	0	265	1290	6430	5240	1250	324	115
1904	0	0	0	0	0	35	1890	3300	1660	251	50	0
1905	0	0	0	0	0	143	1610	4280	4090	724	212	66
1906	0	0	0	0	0	20	1560	5700	4320	897	268	106
1907	0	0	0	0	0	106	1850	4710	3530	607	207	85
1908	38	9	0	0	0	916	2130	8160	5790	1350	327	121
1909	48	17	0	3	16	513	1570	7650	4990	997	244	91
1910	36	5	0	0	0	62	1160	8130	5730	1880	371	115
1911	48	10	0	0	0	115	923	5300	4920	711	308	88
1912	30	5	0	0	0	126	1990	6990	3330	407	136	46
1913	34	3	0	0	0	0	593	2350	1690	210	32	5
1914	0	0	0	0	0	10	1570	7180	3280	1030	331	82
1915	25	3	0	0	0	136	858	3060	2750	684	146	35
1916	5	0	0	0	0	79	5160	13200	6440	1540	454	173
1917	68	10	0	0	0	221	1240	4[illegible]30	7410	1210	270	95
1918	35	9	0	0	3	22	1050	3620	1200	390	67	8
1919	0	0	0	0	0	100	1790	4650	2910	457	79	2
1920	0	0	0	0	0	219	1860	6040	2600	922	200	43
1921	0	0	0	0	0	50	1150	5520	3430	776	147	12
1922	0	0	0	0	0	43	1990	7900	6470	1000	169	17
1923	0	0	0	0	0	72	1280	6100	3680	725	139	45
1924	12	0	0	0	0	64	2480	6080	4460	774	259	58
1925	0	0	0	0	0	48	1120	3950	2020	466	74	0
1926	0	0	0	0	0	0	2020	4580	3340	624	142	0
1927	0	0	0	0	0	55	1610	4350	2750	615	75	0
1928	0	0	0	0	0	173	1300	4670	2350	476	105	0
1929	0	0	0	0	0	236	2580	7240	3760	974	165	36
1930	0	0	0	0	0	41	2370	4170	2510	449	110	19
1931	0	0	9	0	0	22	1060	5520	3550	931	186	54
1932	0	0	0	0	0	7	1930	6330	4360	813	145	49
1933	0	0	0	0	0	0	874	3640	4190	994	290	119
1934	8	0	0	0	0	333	2830	7420	3300	1040	171	66
1935	0	0	0	0	6	232	1460	5040	4850	1130	192	66
1936	0	0	0	0	0	0	1740	5580	4750	810	176	64

HISTORICAL MONTHLY STREAMFLOWS AT ATBARA ; 1937-1967; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1937	0	0	0	0	0	9	1560	6400	3780	780	154	47
1938	0	0	0	0	0	0	1590	7360	4820	1380	260	93
1939	0	0	0	0	0	13	1270	3970	2580	820	190	39
1940	0	0	0	0	0	0	1070	4700	2230	390	78	28
1941	0	0	0	0	0	37	789	3090	1710	562	274	60
1942	0	0	0	0	0	0	1600	5420	3220	807	150	59
1943	0	0	0	0	0	0	1270	6010	5640	998	234	69
1944	25	8	0	0	0	0	2360	5390	2720	593	125	50
1945	17	0	0	0	123	295	1210	4510	3810	1390	290	82
1946	16	0	0	0	0	316	2440	10680	3750	901	294	102
1947	37	17	1	0	0	34	893	4940	3350	826	145	70
1948	39	13	0	0	0	231	1650	4580	2990	990	191	89
1949	50	13	0	0	0	92	1090	4110	3180	834	183	81
1950	59	0	0	0	0	0	2180	6550	4930	832	182	92
1951	54	2	0	0	0	31	1680	4570	2860	856	314	115
1952	46	0	0	0	0	0	1120	4950	3260	637	137	44
1953	18	1	0	0	0	212	2060	7350	3710	950	216	112
1954	62	19	2	0	0	171	2310	9020	6880	2140	384	163
1955	0	0	0	0	0	0	1030	5250	5010	1860	298	76
1956	0	0	0	0	0	0	1940	6020	3790	1940	607	158
1957	112	61	29	16	9	72	774	5270	3140	410	108	71
1958	57	37	5	0	0	97	1540	7360	4020	1110	207	81
1959	44	14	2	0	0	0	1740	8240	5550	1100	337	129
1960	59	24	5	0	0	8	1510	4200	3330	805	170	55
1961	22	4	0	0	0	47	4440	6130	4830	1200	253	155
1962	67	25	6	0	0	0	1180	6050	4490	1000	230	158
1963	44	18	0	0	15	17	1450	5050	3350	812	149	103
1964	37	23	7	0	0	0	2040	7230	4760	917	191	18
1965	165	135	0	0	15	40	1120	3960	2260	324	35	0
1966	6	0	0	128	117	415	692	3490	1820	217	42	31
1967	8	3	3	78	190	111	1580	5960	4000	934	90	14

HISTORICAL MONTHLY STREAMFLOWS AT TAMANIAT ; 1911-1944; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1911	4550	3030	2200	1750	2090	2680	6450	16400	20200	11800	6790	4940
1912	3320	2280	1830	1450	1380	2010	6500	16800	14800	7820	5030	3900
1913	2960	1810	1650	1410	1870	2050	3270	8740	10800	5710	3470	2370
1914	1710	1280	1310	1230	1420	1820	5970	20100	17500	14900	9170	5520
1915	4240	2830	2100	1470	1820	2650	4960	10000	12800	11000	6270	4600
1916	3360	2140	1690	1510	1630	2460	8430	21800	23800	20500	10300	6440
1917	4940	4090	3380	2140	2100	2910	8280	18100	25400	21000	8980	6200
1918	5050	3920	4110	4390	4480	4490	7780	14800	16500	9690	5820	4390
1919	3220	2270	2100	1700	1710	3020	7620	17900	19700	9780	5110	3750
1920	2690	1890	1720	1410	1440	3150	7890	15900	15200	13100	6490	4580
1921	3410	2010	1860	1540	1500	2440	5100	15800	16400	11500	5500	4140
1922	2930	1750	1340	950	1050	2060	5920	18100	18800	13400	5980	4330
1923	3360	1670	1420	1330	1700	3440	7320	18300	18100	10900	5410	4410
1924	3320	1980	1530	1560	1810	2360	6990	16600	18900	10300	6450	4410
1925	3370	1980	1600	1280	1680	2840	5820	14500	13300	9310	4860	3790
1926	2800	1720	1700	1630	2230	3320	6410	18700	17500	11300	5810	4510
1927	3600	2340	1900	1680	1660	2490	6440	16300	14400	9700	4350	3300
1928	2070	1510	1520	1670	2660	3380	9460	18200	17000	9690	5110	4120
1929	3050	1890	1700	1590	2980	5510	11800	20200	21200	15300	6620	4630
1930	3670	2330	2020	1750	2090	2830	7630	18900	17100	8210	4110	3190
1931	2130	1460	1410	1350	1300	2150	5580	16500	18300	11600	5500	3890
1932	2780	1710	1610	1390	1740	2880	6260	17300	18000	11700	5530	4400
1933	4070	3150	2300	1860	2060	2660	4960	14600	17800	12400	6600	4940
1934	3730	2180	1890	1720	1980	2830	8180	18800	18500	12100	5840	4620
1935	3790	2420	1980	1840	2170	3760	10200	20000	20000	13800	5940	4350
1936	3350	2300	2160	1740	1840	2440	7960	18000	20100	11400	5200	3770
1937	2640	1720	1770	1520	1710	2580	7460	18300	18000	9050	4650	3810
1938	2780	2080	2070	1600	1610	2200	7740	19600	21300	15200	5990	4320
1939	3460	2630	2170	2380	2300	2840	5230	13100	14600	9630	5450	3900
1940	2580	2410	2500	1620	1620	2150	4110	15900	14300	7030	3380	2580
1941	1960	2110	2180	1370	1420	3160	5310	12900	12100	10000	5880	3370
1942	2730	2120	2620	2430	1710	2580	7120	18200	14800	11200	4360	3510
1943	2310	2340	2590	2120	1720	1860	4950	15400	16500	8970	4450	3140
1944	2010	2610	2800	2380	1930	2710	6190	14800	12800	7590	3860	2810

306

HISTORICAL MONTHLY STREAMFLOWS AT TAMANIAT ; 1945-1976; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1945	2000	2270	2580	2140	1550	2670	5250	13800	15700	12300	6370	4220
1946	3180	2120	2930	2520	1860	2570	8840	24100	20400	11100	6130	4700
1947	3860	3040	3030	2950	3090	3030	4660	16900	17600	10100	4580	3900
1948	3310	2570	2950	2660	2170	3170	7890	16600	15900	13700	6460	4020
1949	3200	2110	2830	2840	2450	3030	7120	16600	16300	10800	5290	4070
1950	3460	2620	2840	2850	2980	2750	6160	17100	17000	10700	4660	3600
1951	3060	1880	2940	2790	1660	2120	4460	14600	12200	10200	5240	3680
1952	2530	2650	2640	2080	1590	2160	5780	15400	14100	8910	3940	3070
1953	2270	1980	2830	2160	1740	2400	7040	17800	13900	8810	4280	3320
1954	2150	2350	2760	2420	1510	2460	8450	20700	20600	13500	5430	4160
1955	3400	2360	2650	2920	3120	2740	7190	18100	18000	12900	5070	3920
1956	3310	2480	2810	2710	2950	3290	6900	18000	15900	17000	7510	4370
1957	3620	2660	2850	3090	3350	3970	5790	17100	13700	5800	3510	2720
1958	2140	2060	2380	2380	1620	2700	7560	20200	16100	12000	5470	3720
1959	3160	2100	2730	2690	2230	2410	5510	15800	19900	11400	5990	3910
1960	2960	2240	2820	2820	2520	2370	6240	16300	15600	10600	4090	3050
1961	2400	2000	2890	2800	1840	2110	8160	18900	20200	14100	5770	4590
1962	3700	2770	3060	3310	3440	4080	5560	16000	16500	11200	4700	3920
1963	3220	2720	2640	3050	3830	3660	8010	19000	15300	8590	4140	4460
1964	4150	3360	3100	3370	4240	4490	9330	21200	20800	15000	8050	5910
1965	5800	4830	4270	3660	4910	4570	5860	13600	12300	10900	5640	4680
1966	3710	2900	2650	3870	3510	3820	6200	13300	15400	5730	4610	5200
1967	4670	2900	1990	3490	3450	3270	5850	16400	16800	11800	5580	5090
1968	3844	3538	3193	3630	3689	3360	7192	6262	9270	9145	4230	4061
1969	2635	2436	3224	4740	3968	2934	5797	17949	13020	5394	3720	3503
1970	5022	2477	2691	3870	3072	2451	4061	15531	14070	8122	5220	3255
1971	3565	3190	2883	4080	3379	2544	6572	15097	13410	7223	4770	3751
1972	3844	2807	2920	3840	3056	2566	5177	9455	6900	6200	3120	2508
1973	2136	1668	2027	3060	3224	3360	4557	13702	12840	8122	4110	3091
1974	3044	2016	2313	3360	3286	3510	8308	15717	3770	8277	4740	3565
1975	3441	2158	2471	3510	3023	2100	6324	15345	20490	10447	5940	4681
1976	3751	3074	2511	3690	4588	2420	5890	15376	11610	6262	4410	3379

HISTORICAL MONTHLY STREAMFLOWS AT KHARTOUM ; 1900-1933; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1900	680	327	252	245	412	914	4510	19200	14500	7710	2980	1920
1901	1110	665	481	430	584	1630	6140	17600	15700	6050	2760	1560
1902	790	478	424	359	424	1060	3190	10800	13300	7900	3150	1760
1903	1040	537	405	306	410	1640	4410	16400	18900	10900	4530	2280
1904	1570	1100	633	461	525	1130	5200	14600	13100	6450	3130	1840
1905	1210	644	483	351	334	802	2850	11000	13600	6180	2980	2010
1906	1220	644	638	540	400	852	4740	16300	20300	9780	4030	2400
1907	1420	668	556	477	501	795	3380	9490	11300	5540	2700	1620
1908	730	381	285	233	292	575	3460	17400	19500	12900	5150	3160
1909	1700	904	417	527	1050	3430	8400	25600	17900	11500	4850	2850
1910	1950	1350	915	441	459	1100	4420	15300	18700	13500	4540	2190
1911	1320	766	543	419	556	873	5080	17500	18600	8320	4170	2370
1912	1340	824	576	324	207	848	5910	15900	11400	4390	2210	1230
1913	870	473	351	217	566	466	1880	7520	8620	3000	1140	590
1914	340	203	166	158	246	734	5250	19000	14300	11300	5720	2260
1915	1250	680	478	318	461	971	3320	8060	11300	8430	3510	1840
1916	942	537	358	250	368	828	6760	20200	21500	16100	6050	2960
1917	1730	1020	835	551	645	1410	7300	17700	22900	15800	4820	2500
1918	1290	780	625	337	400	1170	5370	12900	11300	5000	2000	1150
1919	726	438	321	175	241	1120	6200	16800	17300	5180	2220	1230
1920	646	368	358	308	313	1650	6170	13200	11400	8750	3170	1870
1921	888	433	347	202	259	865	3410	14600	14500	7350	2310	1180
1922	727	388	297	171	175	801	4570	16000	16100	8990	2560	1350
1923	782	507	375	454	569	2000	5890	16500	13900	7290	2850	1700
1924	848	572	420	414	497	1000	5390	15000	15600	7200	3800	1830
1925	860	493	436	304	417	1260	3640	13300	10700	6010	2070	1110
1926	644	454	507	535	1090	1710	5590	18900	15100	7950	2440	1480
1927	813	456	464	416	391	1290	5450	16200	12900	6900	1660	860
1928	500	299	301	420	1110	1720	8100	17100	12800	6090	2170	1180
1929	632	386	348	355	1380	3610	10300	18600	18400	11300	3240	1700
1930	1020	622	535	374	578	1370	6330	14700	12200	4750	1680	859
1931	460	273	282	350	215	868	3570	14800	15100	8630	2440	1100
1932	577	345	316	235	516	1240	4890	15600	15900	8070	2090	1040
1933	649	349	384	251	495	1030	3220	13200	15100	8400	2940	1470

HISTORICAL MONTHLY STREAMFLOWS AT KHARTOUM ; 1934-1967; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1934	721	361	395	351	442	1260	7020	18200	14900	8530	2620	1470
1935	891	483	440	464	756	2190	9450	20300	18400	9140	2760	1540
1936	914	621	583	513	547	913	7170	17400	16700	7580	2200	1050
1937	611	387	464	365	433	1050	6470	17500	14800	5600	1950	1100
1938	588	397	426	316	373	995	7380	18800	18200	11700	3290	1460
1939	771	496	489	397	550	1170	4680	12100	11800	7160	2920	1280
1940	655	393	401	302	353	851	2940	16000	12700	4070	1120	697
1941	383	261	302	163	359	1940	4830	11400	10900	7110	2900	1100
1942	557	330	537	409	511	1010	6560	18000	14200	8210	1860	912
1943	477	352	327	257	422	439	4050	15800	15200	6750	2290	974
1944	509	377	280	239	542	778	4880	13800	11400	4910	1310	634
1945	398	266	276	210	392	841	3810	13400	14800	9600	3490	1380
1946	656	387	329	338	380	1250	7750	23000	18000	7970	2950	1500
1947	788	507	533	747	570	799	3630	16400	16300	7280	1990	1270
1948	597	424	591	343	332	1760	7320	14600	14400	10800	3670	1440
1949	777	483	453	438	268	1750	6190	15900	15200	7910	2310	1430
1950	824	446	368	436	781	1330	5390	16500	16300	8270	1880	916
1951	518	348	394	356	289	889	3570	14400	11000	8180	3170	1450
1952	738	461	428	293	410	770	4860	15400	13500	7050	1810	860
1953	533	321	338	233	496	1100	6230	18000	12600	6260	1930	1070
1954	710	440	389	310	278	1090	7800	20000	18200	10500	2600	1300
1955	925	635	425	457	613	1160	5980	17600	16200	10200	2570	1450
1956	896	467	401	365	541	1230	6500	15700	13100	14500	4490	1690
1957	997	587	797	1140	759	1500	4270	16400	11800	3570	1270	758
1958	450	389	215	275	344	1330	6600	19000	14400	9370	2960	1410
1959	887	482	475	297	540	810	3970	14200	17800	9210	3890	1740
1960	950	563	462	438	498	807	5190	17000	14300	7930	1940	966
1961	450	324	354	402	460	850	7340	17400	18300	11000	2920	1990
1962	883	480	392	383	427	1280	3870	15300	14500	8830	1830	978
1963	450	280	168	300	870	1120	5180	16700	12900	4780	1320	1040
1964	658	284	136	168	453	984	7460	19300	17300	11200	3380	1340
1965	748	433	205	204	466	637	3120	12400	9690	6980	2020	1100
1966	486	285	207	272	265	1400	3580	12300	12200	2050	1070	1410
1967	1110	384	170	443	472	825	4320	15000	14500	8210	2270	1500

HISTORICAL MONTHLY STREAMFLOWS AT KHARTOUM ; 1968-1975; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1968	850	740	536	495	450	1038	5766	14167	6420	5952	1386	1063
1969	388	722	1020	921	651	1071	4743	17887	10230	2037	906	626
1970	704	580	487	711	419	582	2142	15407	12360	4619	1773	527
1971	691	693	446	606	391	591	5084	14260	11310	3968	2106	725
1972	896	447	347	641	239	462	3286	9641	3780	2858	549	242
1973	198	168	229	444	384	1041	3224	14539	10710	6014	1425	564
1974	493	255	261	459	481	1401	6200	15438	10710	4619	1998	465
1975	289	307	386	456	341	408	5115	15934	18420	7161	2271	1231

HISTORICAL MONTHLY STREAMFLOWS AT SENNAR ; 1912-1945; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1912	1060	608	401	231	143	1210	5460	15500	10400	3960	1950	994
1913	622	328	222	140	463	280	1940	6940	7580	2230	705	305
1914	164	70	63	113	84	638	5860	17700	12900	10700	5730	2250
1915	1160	634	437	276	570	1170	3540	9880	12400	8440	3250	1490
1916	779	394	211	140	312	861	7640	20000	18300	13200	5610	2790
1917	1540	874	657	444	600	1500	8320	19400	21200	13300	4690	2530
1918	1450	947	719	584	982	2010	6180	14100	10600	4720	2150	1130
1919	654	429	276	146	343	1450	6850	14500	13000	3970	1790	966
1920	606	416	370	233	497	1860	7130	13300	10800	8330	3030	1450
1921	788	439	295	164	307	1050	3730	14100	12300	6250	2360	1230
1922	702	389	265	173	247	1020	5090	15600	13700	7840	2680	1300
1923	731	445	403	463	811	1920	6800	14500	13200	6030	2800	1540
1924	783	511	369	489	518	1280	6490	15100	13300	6010	3450	1650
1925	953	527	393	261	460	1730	4350	12900	9670	5120	1810	1100
1926	619	405	580	510	1240	1810	6630	17200	14000	7030	2250	1370
1927	706	455	465	480	331	1700	6240	14300	10700	5240	1310	804
1928	426	318	393	627	1470	2240	8440	16700	11400	5440	2040	1180
1929	669	505	483	572	1860	3960	10500	18700	16600	10000	2920	1660
1930	952	658	594	539	722	1520	6580	14000	11100	3940	1480	844
1931	440	289	307	395	226	1140	4360	15100	13000	7520	2010	1010
1932	473	361	331	222	779	1390	5920	15800	14900	6770	1810	1020
1933	537	387	374	335	573	1170	3890	13000	13800	7320	2540	1340
1934	630	362	424	360	496	1540	7150	18000	13100	7050	2200	1340
1935	719	440	420	529	1020	2600	10100	17900	15500	8010	2340	1460
1936	843	639	540	542	603	1330	7940	16300	14400	5640	1820	1040
1937	573	400	426	353	560	1320	7170	17200	12800	4460	1710	1050
1938	506	354	421	294	421	1460	8560	18700	16400	9910	2750	1430
1939	749	486	489	424	687	1530	4980	11400	10700	6140	2490	1180
1940	621	417	371	300	388	1050	3490	14900	9900	3440	934	572
1941	267	235	233	152	626	2420	5610	11700	9520	6420	2540	1100
1942	501	310	687	363	660	1330	7450	16400	12100	6340	1610	875
1943	434	360	348	242	498	614	4910	15100	13300	5560	1880	998
1944	471	415	289	280	820	1410	5180	13700	10500	4210	1280	801
1945	369	304	244	216	776	1380	5398	12900	13900	8140	3210	1570

HISTORICAL MONTHLY STREAMFLOWS AT SENNAR ; 1946-1975; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1946	782	439	430	362	391	1800	9060	24000	15000	7060	2730	1380
1947	671	465	516	701	502	922	4230	15990	13200	5320	1670	1150
1948	490	388	544	260	357	2310	7150	13600	13100	9310	3140	1340
1949	648	423	413	446	417	1870	6650	15600	13900	6400	1910	1240
1950	712	411	333	498	852	1570	5970	15000	13800	5190	1490	859
1951	410	347	368	342	368	1000	4000	14400	9150	7180	2760	1370
1952	648	476	429	324	438	959	5530	14500	11000	5800	1560	805
1953	471	302	309	230	554	1010	6290	16400	10400	5070	1700	998
1954	649	408	363	284	343	120	7950	18900	16200	8600	2330	1200
1955	852	554	404	460	688	1310	6930	16500	15500	8280	2190	1250
1956	720	434	380	398	510	1540	6660	14700	11600	12500	3510	1530
1957	862	479	866	1170	719	1580	5400	16200	10400	3200	1090	740
1958	486	400	252	330	398	1780	7320	18200	13200	9020	2680	1340
1959	939	590	492	294	673	1030	4830	14700	15100	8040	3150	1530
1960	939	563	427	486	607	1180	6160	15300	13000	6350	1760	862
1961	434	331	266	457	437	951	6800	14900	15900	9710	2980	1980
1962	869	344	360	338	477	1700	4350	13400	12800	7180	1740	818
1963	376	213	166	318	1200	1420	5690	16000	11300	4220	1320	1050
1964	533	281	84	155	365	1350	8090	18200	15600	9430	2580	1290
1965	687	330	117	401	360	973	3880	12400	8570	5870	1780	1020
1966	411	257	210	200	433	1960	4320	11300	10900	1230	887	1380
1967	1130	299	154	353	590	1060	4690	14200	12500	6420	2100	1340
1968	769	777	675	537	254	1050	6727	14477	5550	4247	1074	644
1969	458	713	1023	726	635	1296	5332	17515	8940	1754	723	709
1970	657	658	322	246	280	822	3720	16771	10320	3875	1362	313
1971	709	727	520	399	381	735	5890	14415	9450	3162	1539	647
1972	725	429	362	420	282	843	3999	8742	3360	2046	510	272
1973	254	220	232	240	381	1239	3689	15717	9480	4216	1056	403
1974	483	290	356	360	424	1482	7657	16213	10800	3720	1032	347
1975	263	382	660	393	313	576	5797	16151	17940	4712	1914	713

HISTORICAL MONTHLY STREAMFLOWS AT ROSEIRES ; 1912-1945; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1912	856	573	358	238	248	1450	5930	14200	8690	3440	1620	871
1913	576	352	272	260	604	396	2320	6620	6090	2140	743	319
1914	190	143	146	232	187	975	5660	16900	11400	8850	4610	1740
1915	873	468	337	227	560	1250	3640	8190	11100	6940	2960	1350
1916	726	410	240	219	490	1390	7450	19400	16800	10900	4500	2210
1917	1190	670	500	372	632	1760	8210	18400	21100	10900	3910	2030
1918	1160	753	649	540	949	1980	5850	12600	8670	3860	1720	883
1919	700	464	318	162	500	1600	7200	15100	13100	4280	1800	942
1920	625	393	396	227	882	2210	7230	12700	9910	7350	2860	1320
1921	778	468	306	204	428	1290	4330	15000	12300	5690	2190	1100
1922	640	384	268	184	371	1460	5510	14200	12200	6850	2330	1220
1923	707	428	412	478	1080	1930	6890	18000	13100	5160	2380	1530
1924	804	535	385	530	564	1830	6960	15100	13900	5930	3540	1630
1925	922	524	399	291	701	2030	4890	12900	9900	5280	2380	1160
1926	722	427	398	354	1850	1990	7320	17300	14000	7060	2670	1550
1927	797	436	400	242	266	1850	6130	12500	9080	5220	1840	990
1928	573	315	254	413	1540	2250	8900	17600	11900	5760	2560	1330
1929	752	467	334	373	1880	4130	11000	19300	16600	9890	3230	1890
1930	1070	622	459	526	608	1610	7090	14300	11300	4280	2020	1080
1931	630	344	257	192	221	1450	4890	15300	12700	7520	2570	1200
1932	675	383	259	203	747	1520	6260	16000	15500	7140	2240	1210
1933	707	419	327	248	512	1210	4510	13300	13400	7130	2900	1510
1934	805	452	317	280	452	1700	7620	17900	12700	7390	2750	1590
1935	878	485	360	380	1100	2590	10900	18900	16700	8100	2830	1590
1936	999	746	471	445	535	1510	8410	16400	14700	5760	2330	1360
1937	824	478	371	236	598	1370	7360	17000	13100	4810	2280	1270
1938	696	383	384	217	370	1660	9120	19000	16500	10000	3230	1610
1939	932	541	405	364	650	1630	5620	11800	10800	6500	2910	1390
1940	792	470	334	243	345	1090	3950	15100	10200	3870	1580	816
1941	475	291	208	126	733	2350	6170	12200	10100	7040	3070	1350
1942	661	370	715	288	556	1420	8070	16600	12900	6870	2210	1210
1943	764	419	289	222	419	864	5070	14800	13800	5940	2410	1230
1944	684	400	281	246	765	1580	6370	14400	11000	4190	1900	1040
1945	616	344	225	180	756	1390	5660	12700	13700	8490	3530	1690

HISTORICAL MONTHLY STREAMFLOWS AT ROSEIRES ; 1946-1973; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1946	945	502	326	292	335	1820	9480	25200	15200	7227	3030	1580
1947	906	517	461	688	415	1040	4920	17100	14900	6580	2250	1380
1948	734	500	438	219	348	2580	7730	14500	14000	10200	3530	1590
1949	893	500	419	354	439	2070	7740	16000	13900	6600	2490	1680
1950	937	474	359	508	802	1780	5940	15100	13900	5570	2090	1190
1951	718	405	350	243	382	1170	4540	15500	9670	7440	3160	1540
1952	763	425	321	245	361	1150	6020	15300	11500	6510	2170	1110
1953	616	336	280	248	569	912	6850	17700	11200	5840	2330	1290
1954	768	431	320	252	304	1630	8690	18100	15000	8570	3040	1620
1955	1100	599	379	459	669	1500	7500	16800	15300	8750	3050	1640
1956	911	514	382	467	440	2330	7160	15000	12000	14100	4320	1950
1957	1090	605	1100	1290	706	1780	5660	17200	10500	3720	1720	1000
1958	590	398	256	319	382	1870	7780	19000	13800	9670	3410	1700
1959	1050	642	456	283	610	1100	5380	16200	16100	9290	3850	1980
1960	1190	714	536	405	547	1310	7040	16300	13600	6980	2450	1400
1961	794	509	366	502	361	1410	8990	17300	16800	11100	3550	2230
1962	1150	602	489	289	558	1700	5350	15100	14200	8940	2560	1460
1963	881	483	400	416	1250	1520	6140	16900	12500	4860	2620	2300
1964	944	565	340	396	443	1670	8870	16900	14600	10600	3720	1950
1965	1150	666	448	483	277	1200	4760	13400	9420	7200	3070	1750
1966	880	563	467	376	608	2190	5170	12300	11300	2260	2230	2410
1967	1600	512	725	489	487	1420	5920	14200	14200	9050	3120	2020
1968	1321	1122	1057	771	353	1647	7440	1299	5460	7409	2166	1485
1969	1293	1299	1528	978	753	1944	6479	1820	10470	3100	1929	1804
1970	1575	1491	741	327	302	1095	4650	16957	11190	6200	2541	1262
1971	1448	1183	1085	537	412	1290	6851	15314	10680	4526	2646	1500
1972	1494	966	970	654	409	1404	5084	9021	4800	3441	1674	927
1973	899	707	415	447	638	1926	5332	15655	10800	6138	2346	1274

HISTORICAL MONTHLY STREAMFLOWS AT MALAKAL ; 1905-1938; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1905	2380	1670	1520	1240	1370	1660	2200	2530	2760	3040	2960	2830
1906	2140	1440	1580	1400	1550	2050	2530	2860	3130	3470	3430	3250
1907	2310	1590	1660	1580	1550	1920	2380	2810	3030	3280	3150	2690
1908	2050	1570	1440	1300	1490	1720	2380	2850	3070	3440	3470	3580
1909	3110	1820	1570	1710	1990	2400	2950	3500	4060	4610	4170	4190
1910	3960	2230	1800	1450	1690	2130	2580	2910	3120	3440	3420	3540
1911	2830	1660	1570	1370	1570	2020	2540	2790	2900	3090	3000	2660
1912	1880	1460	1370	1220	1190	1580	2310	2920	3160	3310	3070	2710
1913	1810	1390	1460	1330	1710	1720	2250	2620	2730	2950	2220	1630
1914	1450	1230	1310	1260	1310	1630	2170	2730	3230	3590	3380	3340
1915	2600	1460	1390	1300	1510	1850	2440	2820	2980	3270	3270	3050
1916	1950	1410	1360	1300	1460	1840	2500	2910	3400	3990	4080	4280
1917	4280	3240	2220	1860	2010	2390	2880	3230	3520	4140	4400	4810
1918	4970	4620	4840	2880	2450	2910	3330	3800	4010	4000	3610	2930
1919	2190	1810	1800	1580	1760	2260	2810	3160	3220	3500	3460	2880
1920	1870	1[illegible]90	1420	1240	1450	1990	2380	2630	2740	3040	2960	2760
1921	1750	1240	1230	1130	1430	1720	2210	2500	2580	2870	2710	2600
1922	1500	1040	963	860	1040	1560	2110	2440	2540	2860	2860	2820
1923	1710	1040	1060	1300	1410	2010	2460	2940	3160	3360	3070	2960
1924	1830	1320	1270	1310	1540	1680	2180	2490	2620	2940	2840	2810
1925	1860	1270	1290	1210	1470	1830	2360	2660	2750	2990	2880	2590
1926	1650	1180	1340	1240	1590	1970	2310	2680	2880	3330	3150	3090
1927	2500	1400	1390	1310	1340	1700	2250	2560	2650	2850	2710	1810
1928	1400	1170	1200	1320	1820	2070	2490	2850	2980	3300	3240	3080
1929	1920	1410	1410	1360	1960	2300	2680	2870	2990	3310	3150	3080
1930	2180	1430	1400	1340	1530	1830	2340	2630	2670	2830	2670	2040
1931	1550	1190	1250	1200	1270	1690	2310	2640	2870	3180	3160	2810
1932	1680	1360	1360	1310	1510	2000	2470	2840	3120	3690	3720	3870
1933	3420	1820	1670	1600	1670	1960	2420	3130	3450	3730	3660	3420
1934	2300	1510	1540	1460	1680	1970	2590	2940	3150	3420	3380	3410
1935	2420	1520	1520	1510	1720	2230	2700	2930	2970	3180	3120	3070
1936	1980	1570	1470	1310	1570	2030	2390	2620	2750	2980	2880	2400
1937	1640	1320	1340	1240	1540	1980	2320	2820	2960	3320	3080	2750
1938	1800	1350	1430	1380	1530	1900	2430	2710	2860	3300	3290	3400

HISTORICAL MONTHLY STREAMFLOWS AT MALAKAL ; 1939-1972; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1939	3170	1780	1600	1480	1750	2160	2540	2710	2790	3030	3000	2530
1940	1650	1350	1370	1280	1370	1730	2190	2490	2590	2790	2630	1900
1941	1460	1190	1240	1160	1340	1940	2360	2620	2710	2880	2840	2910
1942	2050	1330	1440	1210	1480	1940	2380	2690	2960	3140	3000	2730
1943	1640	1259	1340	1310	1510	1730	2210	2590	2740	2950	2900	2510
1944	1600	1320	1320	1320	1600	2040	2420	2670	2810	3080	2860	2400
1945	1670	1270	1270	1070	1230	1810	2260	2570	2810	3240	3150	3110
1946	2290	1310	1220	1060	1220	1700	2260	2950	3510	3780	3740	3880
1947	3780	2340	1460	1420	1560	1930	2410	2720	2920	3270	3210	3350
1948	2700	1510	1440	1320	1480	2000	2450	2720	2890	3140	3150	3340
1949	2940	1730	1520	1450	1450	1920	2370	2720	3010	3440	3280	3340
1950	3080	1710	1480	1420	1710	1960	2430	2820	3090	3420	3300	3180
1951	2210	1350	1350	1160	1190	1630	2110	2330	2460	2690	2650	2620
1952	1780	1270	1220	1180	1420	1740	2190	2470	2570	2780	2750	2560
1953	1670	1260	1280	1250	1430	1750	2200	2600	2770	3040	2970	2450
1954	1710	1250	1290	1290	1360	1780	2320	2770	3130	3490	3360	3220
1955	2610	1590	1400	1400	1500	1930	2350	2590	2790	3080	3040	3090
1956	2840	1970	1580	1540	1820	2080	2480	2750	3000	3320	3180	3240
1957	3040	1780	1650	1790	1600	2060	2450	2740	2790	2960	2800	2340
1958	1600	1280	1290	1180	1380	1750	2340	2670	2840	3110	2960	3010
1959	2220	1360	1380	1240	1560	1910	2300	2530	2680	2920	2860	2940
1960	2320	1430	1400	1320	1610	1950	2390	2640	2680	2840	2780	2730
1961	1820	1320	1380	1380	1380	1730	2310	2850	3220	3730	3430	3400
1962	3300	2700	2420	1810	1900	2260	2720	3060	3240	3530	3500	3730
1963	3760	2950	2270	1880	2350	2610	3020	3450	3880	4630	4680	4760
1964	3930	3100	2890	2480	2440	2560	3170	4150	5200	6090	6210	6420
1965	6060	4460	3800	3070	2800	2800	3500	4050	4200	4560	4400	4130
1966	3200	2320	2190	1990	2320	2780	3250	3610	3930	4410	4520	4400
1967	3560	2390	2140	1900	1870	2170	2740	3250	3560	4210	4090	4060
1968	3720	2674	2266	1848	1888	2214	2706	3097	3330	4689	3720	3255
1969	2592	2158	2189	2040	2130	2508	3063	3286	3420	3813	3870	3813
1970	2806	2216	2167	1929	1975	2373	2861	3255	3570	3968	3990	4154
1971	3672	2500	2372	2067	1993	2181	2775	3255	3570	3968	3960	4092
1972	3255	2248	2068	1836	2306	2451	3013	3255	3210	3317	3030	2492

HISTORICAL MONTHLY STREAMFLOWS AT MALAKAL ; 1973-1976; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1973	2043	1682	1693	1581	1916	2283	2691	2979	3150	3379	3360	3286
1974	2437	1972	1739	1605	1773	2247	2750	3131	3540	4061	3870	3844
1975	2644	1885	1854	1719	1817	2145	2654	2995	3360	3906	3990	3875
1976	3658	2410	2058	1848	1956	2358	2765	3029	3150	3441	3450	3317

HISTORICAL MONTHLY STREAMFLOWS AT MONGALLA ; 1905-1938; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1905	3060	2570	2640	2590	3110	2670	2840	3040	3500	3200	3450	3500
1906	2880	2520	2780	2800	2910	3040	3550	3640	4070	3710	3670	3360
1907	3160	2660	2750	2710	3000	2990	3020	3350	3390	2930	3180	2770
1908	2480	2100	2130	1990	2190	2250	2630	3380	2630	2430	3140	2470
1909	2270	1910	2020	2480	2760	2680	3000	3060	3780	2920	2460	2560
1910	2270	1930	1990	2020	2470	2150	2340	2970	3660	3160	3130	2330
1911	2000	1630	1720	1720	1980	1910	2210	2270	2520	2540	2490	2200
1912	1670	1380	1380	1430	1610	1550	2300	2920	3000	2200	2040	1950
1913	1480	1370	1470	1660	2340	2320	2480	2440	1880	1810	1930	1820
1914	1680	1400	1530	1480	1860	1700	2040	2810	2590	2660	3270	2500
1915	2010	1710	1870	1900	2330	2340	2260	2620	2820	2970	2790	2270
1916	1990	1730	1780	1910	2490	2920	3300	4080	5250	4810	4040	3590
1917	3200	2850	3060	3060	4380	4910	4990	5420	6430	7350	5300	4850
1918	4850	4080	4410	4110	4320	3970	3950	4010	3610	3650	3180	2990
1919	2770	2330	2390	2360	2660	2380	3010	2780	2860	2700	2570	2380
1920	2320	1760	1660	1860	2200	2310	2340	2560	2190	2480	2140	1990
1921	1530	1220	1200	1120	1190	1200	1630	1830	1580	1650	1290	1180
1922	1070	891	999	1050	1250	1160	1240	1560	1950	1550	1460	1080
1923	983	801	853	913	1430	1330	2090	2850	1930	2200	2170	1780
1924	1640	1440	1420	1650	1910	1550	1600	1720	1930	2020	1920	1660
1925	1540	1300	1430	1450	1670	1520	1560	1860	1610	1520	1750	1650
1926	1390	1180	1300	1470	1880	1660	2320	3130	2750	3020	2400	2350
1927	2230	1940	2090	2150	2240	2230	2260	2340	2220	2260	2090	1990
1928	1850	1620	1640	1960	3830	2620	2520	2330	2060	2380	2000	1830
1929	1700	1440	1480	1510	2240	1760	1780	1950	1930	2000	1880	1660
1930	1540	1320	1500	1750	1940	1800	1820	2040	2120	2370	2440	2060
1931	1960	1680	1880	1940	2300	2180	2760	3300	3130	3020	2470	2410
1932	2250	1940	2180	2060	2680	2380	2980	3720	3570	3480	2760	2650
1933	2480	2180	2350	2260	2500	2280	2570	2680	3410	3040	2470	2350
1934	2180	1810	1910	1980	2420	2130	2480	3080	2460	1990	1960	1890
1935	1760	1500	1590	1670	2340	2170	2280	2130	2260	2240	1820	1720
1936	1600	1420	1560	1600	1860	2040	2210	2490	2450	2310	1940	1900
1937	1810	1630	1730	1890	2500	2250	2910	3170	2350	2650	2730	2510
1938	2310	2000	2100	2060	2430	2550	2570	3470	3110	2810	2470	2270

HISTORICAL MONTHLY STREAMFLOWS AT MONGALLA ; 1939-1972; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1939	2140	1830	1940	2120	2190	2020	2220	2300	2140	2010	2120	1900
1940	1720	1530	1610	1710	2230	1730	2050	2500	2100	1760	1630	1680
1941	1580	1320	1560	1520	2150	2760	2100	1980	1970	1970	1920	2040
1942	1890	1650	1940	1960	2600	2650	3100	3780	3990	3130	2720	2730
1943	2610	2210	2270	2170	2460	2530	2760	2750	2640	2270	1950	1840
1944	1710	1450	1510	1540	2140	1610	1950	1830	1970	1950	1600	1450
1945	1320	1070	1070	986	1510	1550	1790	2300	2200	1920	1520	1500
1946	1280	1100	1120	1190	1600	2100	1870	3260	2890	2280	1900	1620
1947	1510	1340	1500	1850	2210	2000	2590	3300	3350	3310	2440	2510
1948	2340	2090	2150	2060	2350	2540	2770	3130	3430	3620	2940	2480
1949	2220	1890	1960	1860	2160	2050	2600	2890	2890	2470	1910	1770
1950	1630	1350	1410	1530	1660	1560	1880	2720	2580	2880	1710	1450
1951	1350	1130	1190	1310	1510	1560	1510	1890	1450	1870	2120	2100
1952	1870	1640	1680	1910	2400	2180	2420	3530	3180	3150	2400	2250
1953	1990	1660	1690	1610	1910	1950	2130	2340	1880	1920	1900	1650
1954	1510	1280	1380	1520	1910	1800	2020	2690	3010	2230	1870	1700
1955	1640	1440	1510	1500	1740	1530	1690	2200	2860	3060	2370	1780
1956	1620	1430	1490	1640	1950	1850	1940	2500	3060	3010	2100	1860
1957	1780	1570	1780	1940	2380	2690	2170	2480	2090	2080	1960	1910
1958	1810	1570	1700	1730	2080	2100	2770	2870	2520	2470	1940	1880
1959	1760	1490	1580	1550	2120	1910	1900	2440	2390	2220	2030	1820
1960	1630	1490	1680	1830	2090	1860	2210	2500	2640	2780	2350	2040
1961	1900	1620	1770	1790	2010	2040	2590	3550	3730	4250	4730	4080
1962	3450	3040	3560	3660	4330	4090	4700	4890	5000	5040	4520	4370
1963	4420	3890	4280	4790	6050	5640	5500	5540	5310	4970	5010	5070
1964	4390	3740	3860	4510	5290	4940	5720	6320	6860	7340	5760	5270
1965	5260	4530	4780	4570	4740	4430	4580	4810	4440	5040	4900	4840
1966	4290	3780	4130	4130	4360	4060	4250	4510	4680	4730	4730	4310
1967	3990	3410	3620	3370	3780	3710	4070	5080	4790	5000	5120	4560
1968	3720	3190	3503	3480	4030	4140	4557	5115	4620	4774	4620	4650
1969	4309	3944	4309	4020	4650	4320	4495	5363	5040	4526	4410	4340
1970	4061	3813	3720	3810	4185	4200	4557	5766	6600	6200	5250	4557
1971	4420	4185	4030	3900	4185	3960	4216	4557	4920	4898	4140	3844
1972	3596	3565	3286	3060	3317	3390	3503	3534	3570	3968	4320	4247

HISTORICAL MONTHLY STREAMFLOWS AT MONGALLA ; 1973-1975; 10**6 CUBIC METERS

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
1973	3844	3658	3193	3240	3937	3630	3689	4278	4140	4030	3960	4844
1974	3565	3534	3441	3360	3596	3690	4185	4495	4380	4340	3900	3715
1975	3534	3379	3286	3180	3441	3450	3441	4650	4980	4836	4170	4092