ILCA Research Report

PN-AAP-428

Cattle herd dynamics: an integer and stochastic model for evaluating production alternatives

Panos A. Konandreas
and Frank M. Anderson

June 1982

INTERNATIONAL LIVESTOCK CENTRE FOR AFRICA
ADDIS ABABA, ETHIOPIA

ILCA PUBLICATIONS

The International Livestock Centre for Africa (ILCA) is an autonomous, non-profit making research and information centre with a mandate to improve livestock production throughout sub-Saharan Africa. The activities and publications of the centre are financed by the Consultative Group on International Agricultural Research (CGIAR). The CGIAR members which have funded ILCA to date are the International Development Research Centre, the International Fund for Agricultural Development, the United Nations Development Programme, the World Bank, and the governments of Australia, Belgium, Denmark, the Federal Republic of Germany, France, Iran, Ireland, Italy, the Netherlands, Nigeria, Norway, Saudi Arabia, Sweden, Switzerland, the United Kingdom and the United States of America.

Responsibility for ILCA publications rests solely with the centre and with such other parties as may be cited as joint authors. Until 1982, the centre's two main series of scientific publications were *Systems Studies* and *Monographs*. These two series have now been superseded by the present series of *Research Reports*.

PN AAA 428

ERRATUM

Page 16, lines 6-10.

$$E_m = 0.376W^{0.73} + 0.0021W.D.k_m \quad (2.12)$$

Where: E_m = net energy requirements for maintenance (MJ/day)

W = liveweight (kg)
D = distance walked (km/day)
k_m = coefficient of the conversion efficiency of metabolizable energy to net energy for maintenance.

Cattle herd dynamics: an integer and stochastic model for evaluating production alternatives

Panos A. Konandreas
and Frank M. Anderson

ILCA Research Report No. 2
International Livestock Centre for Africa
June 1982

CONTENTS

PREFACE

1. INTRODUCTION 1

2. ANALYTICAL BACKGROUND 7

2.1 Forage intake 7
2.1.1 *Ad libitum* forage intake 7
2.1.2 Forage intake in real systems 10

2.2 Energy utilization 14
2.2.1 Maintenance 15
2.2.2 Lactation 17
2.2.3 Pregnancy 17
2.2.4 Liveweight gain 18
2.2.5 Energy released from mobilization of body reserves 19
2.2.6 Energy available to calves from milk consumed 19

2.3 Animal reproduction 20

3. THE SIMULATION MODEL 25

3.1 Overview of the computer simulation model 25
3.2 Specification of forage on offer and animal activity 28
3.3. Algorithms simulating biological processes 31
3.3.1 Feed intake 31
3.3.2 Growth and milk production 36
3.3.2.1 Liveweight limits and the condition index 36
3.3.2.2 Growth algorithm for adult males 39
3.3.2.3 Milk production algorithm 40
3.3.2.4 Growth algorithm for adult females 46
3.3.2.5 Growth algorithm for calves 49
3.3.3 Reproduction 52
3.3.3.1 Heifer reproductive maturity 52
3.3.3.2 Conceptions 53
3.3.3.3 Births 56
3.3.4 Mortality 57

3.4 Herd management policy options 59
3.4.1 Supplementary feeding 59
3.4.2 Milk offtake 60
3.4.3 Weaning 61
3.4.4 Breeding 62
3.4.5 Purchases 62
3.4.6 Sales 63
3.4.7 Drought policy 64

4. THE SIMULATION PROCESS 65

APPENDIX A: A procedure to estimate liveweight growth curves 69

APPENDIX B: A procedure to estimate the age effect on potential milk yield 73

APPENDIX C: A procedure to estimate the lactation curve 77

APPENDIX D: A procedure to estimate the age effect on fertility 81

APPENDIX E: A procedure to estimate the age effect on mortality 83

APPENDIX F: Example input data set 85

LITERATURE CITED 93

PREFACE

A primary goal of the research programme of the International Livestock Centre for Africa (ILCA) is to identify opportunities for improvements in tropical African livestock systems, and studies on the productivity of cattle herds are an integral part of this research. However, the consequences of changes in cattle productivity at the herd level are manifested over time in a complex way which can only be predicted probabilistically. At the same time, field research on cattle herd productivity requires substantial resources and an extended period before results are available. For this reason, ILCA's field research programme is complemented by mathematical modelling. The objectives of this approach, integrating field research and modelling, are to provide better predictions of the consequences of change than are possible from informal calculations, to maximize the transferability of research results and to engender and support the close integration of research in all the appropriate scientific disciplines.

Due to earlier limitations in manpower and computer facilities, ILCA's initial modelling efforts were based on the use of existing models. The Animal Science Systems Group at Texas A & M University (TAMU) had developed a model of cattle production which represented an appropriate introduction to modelling techniques for livestock researchers in Africa. In 1978, ILCA, TAMU and the Botswana Government's Animal Production Research Unit cooperated in the application of the TAMU model to commercial ranching and traditional cattle systems in Botswana. ILCA Systems Study No. 1 reports on that research.

This exploratory work highlighted the need for a simulation model with stochastic features - particularly for the forage component, as year-to-year variability in the quality and quantity of forage available is a key determinant of herd productivity in nearly all African livestock production systems. It was also considered that an integer-based model, i.e. one which treats animals in the simulated herd as individual entities, was most appropriate for simulating the generally small herd production units in Africa. Additionally, the model had to be structurally flexible, so that it could be readily applied to a range of production situations without major modifications.

This Systems Study presents the rationale and formal specifications of an

operational model with these features. The authors wish to thank J C M Trail for his constructive criticism and substantive comments from the initial conception to the final preparation of this report. They are also grateful to C de Haan, S Westley, C R W Spedding, P Chudleigh, J King, J Durkin, H Kahn, K Milligan and R von Kaufmann for many useful suggestions on an earlier draft, and to G Maloba for typing the final text. However, any errors or deficiencies that may remain are the authors' sole responsibility.

ABSTRACT

A general cattle herd simulation model is presented in which a herd is simultaneously represented as both a biological and an economic unit. The model is time dynamic, stochastic, non-optimizing and integer, that is, it treats animals in the simulated herd as individual entities. The model provides the user with an array of policy options - for weaning, breeding, milking, buying and selling stock, and supplementation to increase production or for strategic reasons during periods of drought - so that herd performance can be evaluated under alternative production regimes.

Five general components in the model account for changes in the biological status of animals during each month of simulation : forage intake, energy requirements, production and growth, mortality and reproduction. The quantity and quality of forage on offer are specified stochastically. The parameters of mathematical relationships of the various biological processes are drawn from the literature and specified for particular systems under study based on observations from these systems. Thus, the model is data based where possible. The FORTRAN computer code is adequately modularized so alterations and refinements can be made easily.

KEY WORDS

Cattle herd, simulation model, stochastic, production alternatives, Africa, feed intake, energy requirements, growth, mortality, reproduction.

RESUME

Le présent document expose les caractéristiques d'un modèle général de simulation dans lequel un troupeau de bovins est considéré à la fois comme unité biologique et unité économique. Le modèle est dynamique dans le temps, stochastique, non-optimisant et numériquement entier dans la mesure où il traite les animaux du troupeau simulé en tant qu'entités individuelles. Il fournit à l'utilisateur toute une gamme de possibilités en ce qui concerne le sevrage, la reproduction, la traite, l'achat et la vente du cheptel, ainsi que la complémentation destinée à accroître la production ou utilisée pour des raisons stratégiques pendant les périodes de sécheresse afin que l'évaluation des performances du troupeau puisse s'effectuer par le système de facteurs alternatifs dans le cadre du système de production

Cinq composantes générales du modèle, à savoir consommation fourragère, besoins énergétiques, production et croissance, mortalité et reproduction, expliquent les changements de l'état biologique des animaux pour chaque mois de simulation. Dans le modèle, la quantité et la qualité du fourrage offert sont spécifiés stochastiquement. Sur la base des données fournies par la documentation disponible, les paramètres des relations mathématiques existant entre les divers processus biologiques sont spécifiés pour un système donné, à la lumière des renseignements tirés de l'observation dudit système. Ce modèle se fonde donc, autant que possible, sur l'utilisation de données. Le code FORTRAN automatisé a été adéquatement modulé pour faciliter les modifications et les améliorations éventuelles.

MOTS-CLES

Troupeau de bovins, modèle de simulation, stochastique, Afrique, alternative de production, besoins énergétiques, croissance, reproduction, mortalité.

1. INTRODUCTION

The long-term survival of traditional livestock production systems within the rapidly evolving national economies of Africa will depend on their capacity to provide products in quantities and at prices which satisfy the subsistence and income needs of the livestock producers. Equally, the survival of these production systems will depend on their impact on land resources, as existing surplus capacity can quickly be eliminated by poor management or over-use. Human populations in the main livestock producing areas of Africa are increasing, and there is a strong positive linkage between the sizes of human and livestock populations due to the need for stock to meet subsistence needs. Thus these systems will be expected to produce even more in future from the same land resource base.

Existing pastoral systems evolved over many years when the pressure on grazing lands was less intense than it is at present. Thus the limitations and fragility of the resource base were not appreciated, either by the pastoralists themselves or others concerned with their welfare. Periodic catastrophes caused by drought or disease were expected, with their attendant impacts on animal and human populations. This situation has changed somewhat over the last few decades, with the introduction of a variety of technical innovations and structural changes. For example, the rinderpest campaign has removed a major source of mortality in domestic cattle across the continent; however, by permitting livestock producers to keep higher average herd sizes over time, the elimination of rinderpest has indirectly contributed towards increasing pressure on grazing resources. The development of water supplies has made grazing areas accessible which were previously little used and changed seasonal grazing patterns. Overall, this development has been favourable, but it has also resulted in overgrazing and non-reversible range degradation in many areas.

Certainly many of the effects of development, both positive and negative, were foreseen, and, on balance, the changes brought about have probably been positive. However there are now few relatively empty areas in the pastoral zones of Africa and pastoral production systems are, according to many observers, already near the limits of their capacity. For this reason, it has become

increasingly important that the consequences of any proposed changes be fully elaborated before they are introduced to the real systems.

Despite important differences among various livestock production systems in Africa, for example in the degree of association with agriculture or the extent and nature of stock movements, all are concerned essentially with the conversion of forage and other feed resources, via the animals and in the context of management regimes, into flows of products of use to man. The management regimes regulate the scheduling of some of the biological processes (such as the breeding season and age of calves at weaning) and the timing and rates of offtake from the herds.

The forage resources of many African pastoral systems are held communally. They are characterized by low levels of productivity per unit area and high variability in yields, both within and across years. Where grazing resources are held in common, the individual herd manager has few choices or opportunities to improve the supply of forage to his herd at any given time, and is limited primarily to moving his animals as the forage resources in one location are depleted. Some systems similar to commercial ranching do exist, with privately owned stock on privately owned and fenced land, as for example in Botswana and Kenya. However, they account for only a small fraction of the total yield of livestock products from the pastoral areas. In both types of production system, there is a substantially high residual variability in the resource base, and this is becoming more severe as livestock and agricultural enterprises expand, restricting the range of herd movements. This variability in the supply and quality of forage on offer compounds the variability inherent in the animal-level biological processes.

However, within the confines of the production environment, managers have important means at their disposal to regulate their herds and the flow of products from them. To a large extent the day-to-day or season-to-season performance of the herd is determined by the management practices followed. The benefits of some of these practices are obvious, but other practices have complex multiplicative and feedback effects which may become apparent only after considerable time. For example, in a particular production system taking a higher proportion of cow's milk for human consumption may be considered desirable. The uninformed observer, judging from a seemingly small short-term effect on calf growth, may be inclined to recommend such a practice on a routine basis. Such a decision, however, would ignore what might be a major effect on the condition of the cows and their subsequent reproductive performance, as well as the long-term effects on the calves of energy deprivation at a crucial stage in their development. These effects are not apparent at the time when the extra milk is taken. Another example is late weaning of calves. If the females of a breed have relatively long and high-yielding lactations, any extension of the calf weaning age will make extra energy demands on the dams. If this extended lactation period coincides with the season when forage is limited and of poor quality, the cows will lose considerable weight and the period to the next conception will be delayed, resulting in lower calving rates. The gains to the calf from the additional milk may or may not compensate for the cost in reproductive performance. Again, the trade-offs are not clear.

These simple examples illustrate the complexity of the interactions between a cattle herd, the forage resource and the management regime, and the importance of having a facility to make forward projections of the consequences of changes in a cattle production system. Such a facility, an analytical model, should be able to project herd performance and productivity, taking into account the essential responses of animals to forage and management practices.

A model of cattle production systems will have several features if it is to be applicable to a range of production situations and of use as a tool to complement more traditional research, particularly by allowing the integration of research results into a more holistic view of the target system. These features are described below:

1. The processes of reproduction, growth and death are fundamental to cattle production systems: together they are the major determinants of overall herd productivity. As such, they should be depicted separately in the model and their effects should, to the extent possible, be determined endogenously as functions of other factors embedded in the model.

2. As there are many components of livestock systems which are inadequately researched and understood, the more complex and comprehensive a model the more appropriate it is to incorporate stochastic features reflecting the limited understanding of the processes involved. This implies that some of the components of a herd model will be stochastic because the determinants of many processes cannot be specified accurately in detail. For example, the complex set of variables which determine forage supplies to a herd are incompletely understood, as well as sufficiently complex to require a major separate modelling effort. Thus forage supplies in a herd-level model are best represented probabilistically (i.e. as stochastic variables), thereby reflecting the observed variability in real production systems.

3. A model must be validated if it is to be useful for studies of a particular production system. Thus, the structures in a model should be based upon data readily observable in the corresponding real systems. Similarly, the output of a model should principally be parameters with real-world counterparts. The use of artificial variables should be minimized because the functioning of any component of the model with artificial variables can only be verified and validated as a logical expectation to which no probability can be attached.

4. The model must be time dynamic in order to schedule correctly events and the responses to events parallelling those in real production systems. The computational cost of specifying time in a model as a continuous variable is massive. Thus a discrete time-step must be specified for calculation, having regard to computational requirements, the availability of data for model specification and validation, and the appropriate interval to account for the important dynamics in the real systems.

5. Models which represent animals as integer entities have a direct correspondence to reality, as in such models animals are born, die, are pregnant or not pregnant, and so on. The mechanisms in the model then manipulate the status of individually simulated animals. This has operational advantages over non-integer formulations as computations are generally more efficient and the output is readily understood. In non-integer formulations, a herd is described by classes of animals which are to a large extent artificial. These classes vary through time, even for the same herd size, as they are fractionated and recombined. The interpretation of output from models with non-integer representations of animals is more difficult, particularly for small herds, which is usually the case in Africa.

6. Where a variety of different management regimes are feasible, and this is the rule rather than the exception in African cattle production systems, the model should permit specification of regimes in sufficient detail to allow simulation of the available options.

7. Optimizing models oblige the analyst to specify an objective function. As herds in Africa are managed to satisfy many objectives, and the ranking of these often varies through time according to the status of the system, it is more appropriate to formulate and use non-optimizing models of livestock production systems. The output from such non-optimizing models can then be evaluated relative to a range of objectives. This approach avoids a common problem of drawing incorrect conclusions because of misspecification in the objective function.

8. To be useful for the analysis of a production situation other than the one for which it was first developed, a model should be designed so that its components can be modified, added to or deleted with a minimum effort. A modular structure is best able to satisfy this requirement. It is also important to document a model fully so that it can be used by persons other than those originally responsible for its design and development. Finally, the computer code should be written in a language routinely available on a wide range of digital computers - for example FORTRAN.

Together, these features imply that the best model of a livestock production system is time dynamic, stochastic, non-optimizing and treats simulated animals as individual entities. Several different models of cattle production systems have been developed for particular purposes over the last decade, but none has included this combination of features. Most have emphasized economic analyses, with insufficient attention to the underlying biological processes. By contrast, most of the changes introduced into livestock production systems in Africa have been technical innovation packages focused primarily on increasing biological performance; only rarely have development efforts focused exclusively on altering the economic context of production.

The World Bank developed a herd projection model (IBRD, 1972) which has been followed by a number of similar models (such as BAE, 1974; IADB, 1975) which simulate future herd numbers in yearly time-steps, with deterministic herd

productivity parameters for each year provided as data. The IADB (1975) model is a derivative of the IBRD one (1972). These models are most often used to calculate the rates of return for various sets of assumptions about the effects of investment. The BAE (1974) model is similar in most respects, but also includes a facility to preset the sequence of year-types occurring throughout the planning period. The different year-types assign different carrying capacities to the range area being considered. In this way, this model represents an important source of variability in the system, albeit simplistically.

The ranch model developed at the Centro International de Agricultura Tropical (CIAT) (Juri et al, 1977) is of more general application. It also uses a yearly time-step and stresses the financial aspects of ranch development, but, in addition, it has several stochastic components which permit evaluation of the risks involved in pasture development. It was designed for application in a ranching area of eastern Colombia where pasture establishment has a significant chance of failure in any year.

Dahl and Hjort (1976, 1979) developed a simple projection model to evaluate long-term herd dynamics on the basis of different assumed calving and mortality rates. This model focuses on aggregates and uses only two parameters to describe herd performance, and therefore can be used only for understanding demographic trends in large populations of stock. The authors have used the model to evaluate the recovery period for cattle, camel, and sheep and goat herds from a hypothetical 2- or 3-year drought.

Researchers in the Animal Science Department of Texas A&M University (Sanders and Cartwright, 1979; Smith, 1979) have developed, tested and applied a cattle production model which focuses on biological responses at the herd level to various sets of production conditions. The specification of a cattle genotype is provided as data to the model. It is a deterministic model and comparisons between runs are made using the results generated when steady-state conditions are reached at a future date in simulated time. This model is an important advance over others which do little to represent the basic biological processes. Furthermore, calculations are made in monthly time-steps. In the original formulation of the model, however, cattle in the simulated herd are not represented as integer entities, which makes it more appropriate to systems where the herding units are relatively large. Development of this model is continuing, including an integer version, and the perceived utility of its application to production problems in Guyana (Davis et al, 1976), Colombia (Cartwright et al, 1977), Venezuela (Ordonez, 1978) and Botswana (ILCA, 1978) has encouraged the group to initiate work on a sheep and goat model with similar features.

The structure of the model described here was influenced by ILCA's experience in the application of the original Texas A&M model in Botswana (ILCA, 1978) and the features considered essential for the evaluation of livestock production alternatives in Africa as discussed above. Chapter 2 provides the analytical background to the biological relationships represented in the model, based on

quantitative evidence reported in the literature. The detailed algorithms used in the computer simulation model are elaborated in Chapter 3. Chapter 4 outlines the necessary steps for the application of the model. The appendices suggest analytical procedures for estimating parameters required as inputs to the model, based on field data.

2. ANALYTICAL BACKGROUND

2.1 FORAGE INTAKE

In general, forage intake by cattle grazing extensively is influenced by the age and physiological status of individual animals, the quality and quantity of the forage on offer, and the environment, including management and climatic factors. Considerable experimental work has been done, particularly in the last two decades, attempting to quantify the separate influences of the individual determinants of intake. This work has suffered from a lack of standardization in the definition of the experimental animals used, the environment of the experiments, the quality and quantity of forage on offer, and above all, a clear definition of the variables being measured. Consequently, it is difficult to synthesize this work into a comprehensible framework; most attempts have only indicated the direction of effects or have achieved quantifications with limited applicability (Balch and Campling, 1962; Baile and Forbes, 1974; Bines, 1976; Cordova et al, 1978).

This section deals first with a synthesis of selected experimental work leading to a specification of *ad libitum* intake by cattle. An approach is then proposed to modify the specification of *ad libitum* intake according to the particular environment of the system under study.

2.1.1 *Ad libitum* forage intake

Voluntary consumption of forage is limited primarily by the rate of passage of food through the digestive tract. This rate is, in turn, a function of the digestibility of the forage consumed. As a consequence, other factors being equal, digestibility should explain variations in voluntary forage consumption.

Several studies have focused on the extent to which digestibility does in fact explain voluntary forage intake. In grazing situations with a highly variable mix of forage on offer and a high portion of roughage in the diet, the digestibility of the forage is recognized as the predominant factor limiting voluntary food intake (Campling et al, 1962; Blaxter and Wilson, 1962; Conrad et al, 1964; Montgomery

and Baumgardt, 1965; Conrad, 1966; Hodgson, 1968; Karue et al, 1973; Baile and Forbes, 1974). As digestibility increases, intake increases substantially, but over a certain level of digestibility (about 65%) voluntary dry matter intake, *ceteris paribus*, becomes negatively related to digestibility (Conrad et al, 1964; Montgomery and Baumgardt, 1965; Conrad, 1966). Thus, on a diet of highly digestible concentrated feeds, voluntary intake is very closely related to the animal's requirements (Coppock et al, 1974).

Although experiments have shown a strong relationship between voluntary feed intake and digestibility, results cannot be generalized beyond the particular contexts of the experiments. Most of the experiments reported related to mature animals at a certain physiological stage, fed on forage whose digestibility was altered, either artificially through mixing with concentrates or naturally as the growing season progressed.

The usual method of estimating the voluntary forage intake of freely grazing animals is by faecal output (for instance, Conrad et al, 1964; Elliot and Fokkema, 1961; Elliot et al, 1961; Corbett et al, 1963; Hodgson, 1968). Daily intake (I) and daily faecal output (F) are related as

$$F = (1 - d)\ I \qquad (2.1)$$

where d is the digestibility fraction of the forage consumed. Faecal output is usually expressed on the basis of metabolic liveweight, i.e.

$$F = a\,W^{0.73} \qquad (2.2)$$

where a is the rate of passage through the digestive tract (kg/kg metabolic weight/day), referred to as the *physical limit*. Other things being equal, this rate is a function of the physiological status of the animal (e.g. pregnancy, lactation, etc). Thus, in general

$$a = f(p) \qquad (2.3)$$

where p is physiological status. Equations (2.1), (2.2) and (2.3) solved for I yield:

$$I = f(p)\,W^{0.73}/(1 - d) \qquad (2.4)$$

where: I = forage intake (kg/day)
W = liveweight (kg)
d = digestibility of forage consumed (fraction)
f(p) = the rate of passage through the digestive tract (kg/kg metabolic weight/day) which is in general a function of the animal's physiological status.

Most reported results fall within this general specification, with differences between the functional forms depending on which variable is held constant. Of studies carried out in tropical Africa, one of the most comprehensive, in terms of number of animal physiological situations analysed, length of observation period, digestibility range covered and completeness of recorded results, is reported in Elliot and Fokkema (1961) and Elliot et al (1961). Observations of intake were made for a continuous period of 2 years under natural pasture grazing without

supplementation. All possible physiological situations were observed, including dry, pregnant and lactating animals. The digestibility of the forage consumed ranged between 42 and 65%, and a highly significant relationship between intake and digestibility was estimated. On the basis of the figures reported by Elliot et al (1961), the derived rate of passage through the digestive tract (coefficient a) for dry cows was 0.042, for cows in the last 3 months of pregnancy it was 0.045, and for lactating cows 0.049. Thus, the intake of pregnant and lactating cows was about 7% and 15% higher than the intake of dry cows. The stage of lactation might also have an effect on intake. Elliot and Fokkema (1961) suggest on the basis of their observations that the lactational stimulus on appetite declines as lactation progresses, so that at weaning the differences between lactating and dry cows are relatively small. However, a comparison by Corbett et al (1963) of intake figures for cows from the 2nd to 4th months of lactation with those for cows from the 5th to 7th months of lactation revealed no apparent relationship between intake and the stage of lactation.

Caution should be used in extrapolating the rates of passage derived from the figures reported by Elliot et al (1961) beyond the digestibility limits of 42 to 65% within which they experimented. Very low quality forage, for instance with a crude protein content below 6% which corresponds to approximately 40% digestibility (Glover et al, 1957), substantially depresses forage intake. To account for this effect, Sanders and Cartwright (1979), drawing from results reported by Campling et al (1962), assumed an additional multiplicative adjustment in voluntary intake for crude protein (CP) less than 0.06 (6%) by the factor $(CP/0.06)^{0.6}$. Assuming a close relationship between crude protein and digestibility, this adjustment can be expressed in terms of digestibility alone, defined as $(d/0.4)^{0.6}$, being in effect when digestibility is below 40%.

Similarly, as digestibility rises above about 65%, chemostatic or thermostatic mechanisms appear to regulate intake (Montgomery and Baumgardt, 1965; Conrad et al, 1964; Conrad, 1966; Baile and Forbes, 1974), implying, *ceteris paribus*, a constant energy intake for these high digestibility levels. Thus, it will be assumed that when the digestibility of forage is greater than 65% the animal's feed intake will be adjusted to maintain metabolizable energy equal to the level at 65% digestibility, as implied by equation (2.4):

$$d.I = 0.65\, f(p)\, W^{0.73}/(1 - 0.65)$$

which yields

$$I = 1.86\, f(p)\, W^{0.73}/d \qquad (2.5)$$

This constraint to voluntary intake at high digestibility levels is referred to as the *physiological limit*.

In addition to the quality of the forage on offer and the size and physiological status of grazing animals, the level of *ad libitum* intake is also a function of the animal's age. Very young calves (1 to 2 months) consume insignificant quantities of forage as they cannot digest a greater intake. The digestive system of ruminants develops gradually, reaching adult capacity at about 18 months.

Relatively few quantitative estimates of voluntary forage intake by young animals are reported in the literature. Hodgson (1968) experimented with calves from 3 to 6 months, grazing on forage of digestibility ranging from about 65 to 80%. Intake was linearly related to digestibility within this range, indicating that the chemostatic mechanisms which limit intake in mature animals are not operating in young, rapidly growing calves. The relationship obtained was:

$$I = (-0.143 + 0.3d)\, W^{0.73}$$

where: I = daily voluntary intake (kg dry matter)
d = digestibility (fraction)
W = liveweight (kg).

Information is apparently lacking on forage intake by young animals at low digestibility levels. An extrapolation of the above function implies that intake would be zero for digestibility below about 48%, which indicates that extrapolation downwards for the full range of digestibilities encountered is not possible. However, Hodgson also reports information on faecal output which provides an indirect measure of intake. He found that daily faecal output had an upper limit of about 22 grams per kg metabolic weight. Thus, voluntary intake by calves can be expressed by the equation (2.4), where coefficient a has the value 0.022 or about 53% of the coefficient obtained for dry cows.

Similarly, although it is recognized that the voluntary intake of old animals is reduced due to loss of teeth and other problems associated with aging, quantified data are lacking. Sanders and Cartwright (1979) assume that voluntary intake is reduced by 3% for each year of age beyond 8. Thus, for animals older than 8 years, voluntary intake as reported earlier is adjusted in a multiplicative fashion by the factor (1 - 0.03 (t - 8)), where t is age in years.

In summary, *ad libitum* intake by freely grazing cattle can be expressed as a function of the quality of the forage on offer and the age, size and physiological status of individual animals. However, it would be incorrect to apply the absolute levels of the intake coefficients as reported above to all production systems. Other factors, such as the palatability of the forage, ambient temperature, regularity and quantity of water on offer and breed adaptability, have not been explicitly included. However, in relative terms, the intake coefficients for various classes of animals should be comparable between different production systems. Thus, as explained later in Section 3.3.1, the intake coefficient for a reference class of animals is estimated, based on observations from the system under study, and from this estimation the intake coefficients for other classes of animals are obtained.

2.1.2 Forage intake in real systems

The estimates of forage intake reported above assume an *ad libitum* feeding regime and also adequate time for grazing. In many systems intake might be reduced because the quantity of forage on offer is limited, or daily grazing time might be restricted due to environmental constraints (such as heat) or considerable time spent walking.

Greenhalgh et al (1966) studied the effect of grazing intensity on herbage consumption and estimated that about 60% excess feed should be available for maximum consumption to be achieved. Similarly, Zemmelink (1980) studied the effect of selectivity on voluntary intake and estimated that 15 to over 40% excess feed was required to achieve maximum intake, depending on the quality of the forage. The general model suggested is:

$$y = m\left[1 - e^{-(px/m)^h}\right]^{1/h}$$

where: y = level of voluntary intake
x = quantity of forage on offer
m = upper limit for y
p = fraction of the forage which may be considered edible or acceptable
h = a parameter indicating the shape of the relationship between x and y.

This specification assumes no limitation on the time available for feeding and thus might not apply in many situations. Wilson and Flynn (1974), for example, report that beef cattle offered grass silage *ad libitum* required at least 6 hours access for maximum intake. Access times for extensively grazing cattle would be expected to be much higher for maximum intake, depending on the spatial concentration of the forage on offer. Thus, a more complete specification determining actual intake should take into consideration both quality and quantity of forage on offer and the time during which animals have access to this forage. The following is an attempt to provide such a general specification, starting with the assumption that the rate of forage consumption per unit of time is a function of the quantity of the forage on offer, provided the forage quality is adequate and the forage can be consumed by cattle:

$$r = f(Q_1) \qquad (2.6)$$

where: r = rate of forage consumption (kg dry matter/hour)
Q_1 = quantity of consumable forage on offer acceptable to the grazing animals (kg dry matter/ha).

It should be expected that

$$\partial r/\partial Q_1 \geq 0$$

That is, the rate of consumption is higher, the higher the quantity of acceptable forage on offer. The general shape of the relationship between r and Q_1 is depicted in Figure 2.1.

Whether the level of voluntary intake is achieved as estimated from the digestibility fraction depends on the quantity of acceptable forage on offer and the length of grazing time allowed in the system under consideration. The forage on offer will be characterized by a quality distribution function as shown in Figure 2.2 which provides the relative quantities of forage of different digestibilities. From this distribution function, it is possible to obtain a forage supply curve as shown in Figure 2.3 relating quantities of forage on offer above a given digestibility level. Thus, if $\bar{d}$ is the average digestibility of the total forage on offer (Q_T) and if the total forage is consumed, then its digestibility will be equal to $\bar{d}$.

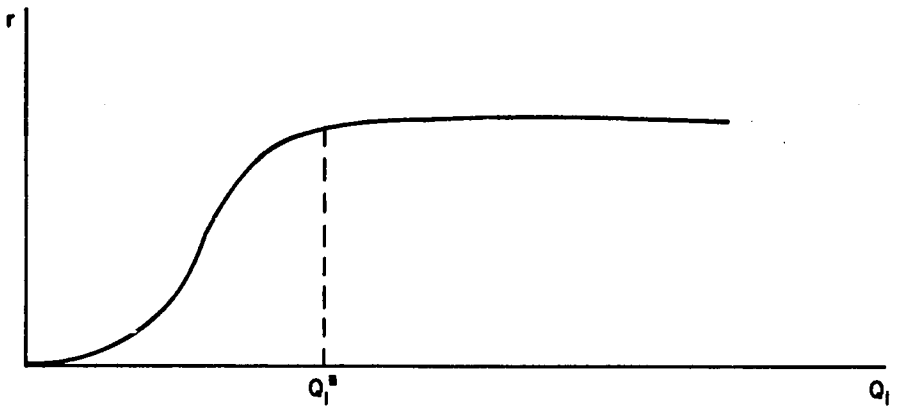


Figure 2.1 Rate of consumption (r) in kg/hour as a function of the density of acceptable forage on offer (Q_1) in kg/ha

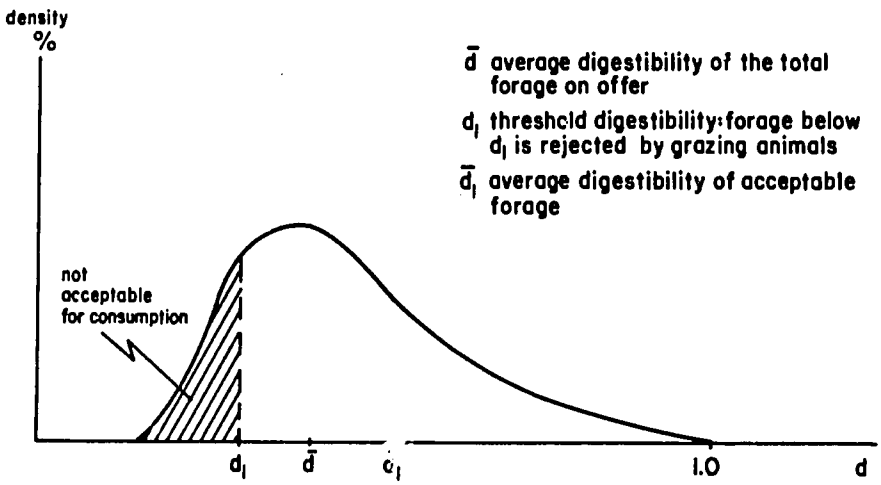


Figure 2.2 Density distribution of the quantity of forage on offer according to its digestibility (d)

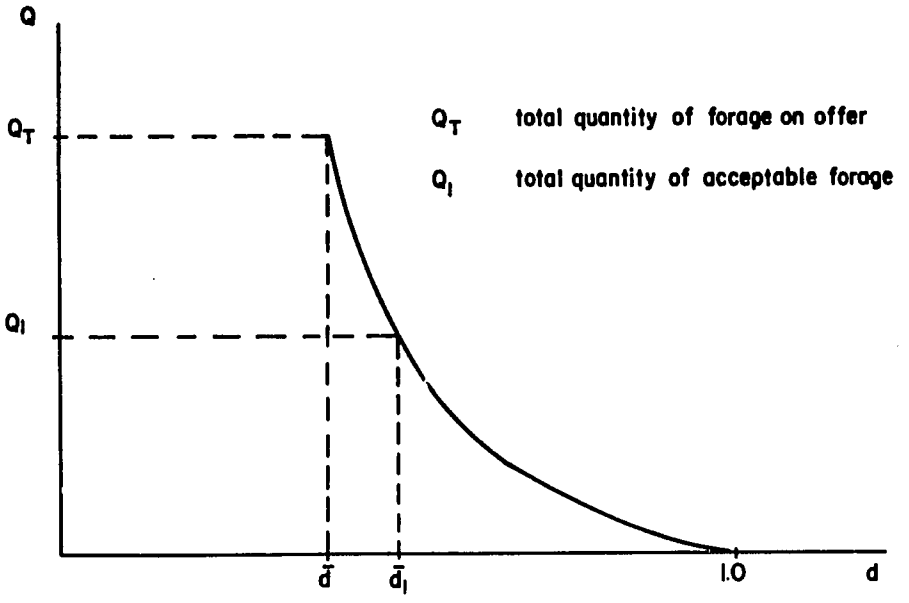


Figure 2.3 Forage supply curve: relationship between quantity of forage grazed (Q) and its average digestibility (d)

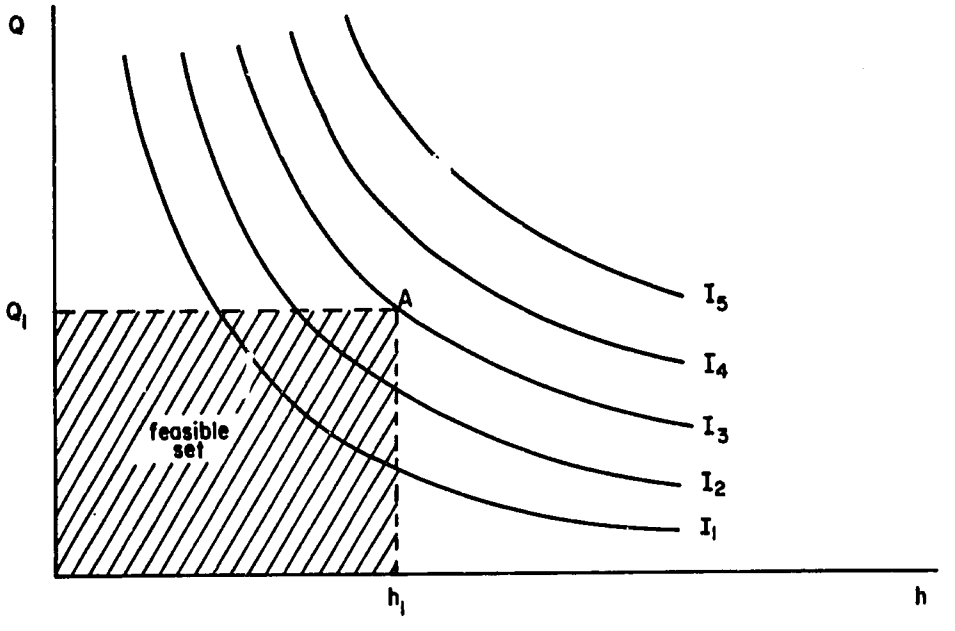


Figure 2.4 Isointake curves: relationship between quantity of acceptable forage on offer (Q) and length of grazing time (h) for different levels of intake (I_1, I_2, ...)

In general, animals will reject forage below a certain minimum digestibility (d_1) which implies an average digestibility of acceptable forage $\bar{d}_1$, and a corresponding quantity Q_1, such as $\bar{d}_1 > \bar{d}$ and $Q < Q_T$.

Given relationship (2.6) between the rate of consumption and the density of the forage on offer, the total intake during h hours of grazing time will be

$$I = r.h = f(Q_1).h \qquad (2.7)$$

where: I = total intake (kg)
h = grazing time (hours).

This relationship suggests a trade-off curve between the quantity of forage on offer in a given area and the length of grazing time. A given intake level can be obtained from dense forage in a short time or from less dense forage over a longer period. These trade-off curves will be called isointake curves, representing the different combinations of forage on offer and length of grazing time that will yield the same intake. Figure 2.4 shows a family of isointake curves for different intake levels. In any real situation, the time actually available for grazing (h_1) is limited by management and environmental factors, while forage on offer is limited to the quantity of a minimum acceptable digestibility level (Q_1). Thus the feasible set of forage quantity and grazing time is shown in the shaded area of the figure.

Thus, the level of voluntary intake as estimated from the digestibility fraction is achieved if its corresponding *ad libitum* isointake curve crosses the feasible set of Figure 2.4. The maximum actual intake corresponds to the level of the isointake passing through point A. Conversely, if the *ad libitum* isointake does not cross the shaded area, then actual intake is less than the estimated voluntary intake.

2.2 ENERGY UTILIZATION

Organic nutrients obtained from the different sources of feed available to an animal are used for a variety of purposes, including the maintenance of body functions, the construction of body tissues, the synthesis of milk, and the conversion to mechanical energy used for walking and other work. These diverse functions all require the transfer of considerable quantities of energy, so that in most situations when the energy requirements of the animals' different functions are met it may be assumed that the animal's non-energy requirements (protein, minerals and vitamins) are also met. Hence, the nutritive value of different feeds can be expressed by their energy content, or more precisely by their ability to supply energy with a high coefficient of conversion into usable energy for the different body functions.

The gross energy contents of different forages are very similar, at about 18 MJ/kg (Hunt, 1966). As shown in Figure 2.5, a portion of this energy is lost

as faeces, while the remaining digestible energy (DE), proportional to the digestibility (d) of the consumed feed, is converted into metabolizable energy (ME) after additional losses of about 19% of DE as urine and methane (Armstrong, 1964; MAFF, 1975). Metabolizable energy expressed as a fraction of gross energy is referred to as metabolizability (q), and relates to the digestibility of the consumed forage approximately as:

$$q = 0.81\ d \qquad (2.8)$$

Thus, metabolizable energy obtained from a given intake can be expressed as:

$$ME = 18I.q \qquad (2.9)$$

or equivalently

$$ME = 14.6\ I.d$$

where: ME = metabolizable energy (MJ)
I = feed intake (kg)
d = digestibility of feed intake (fraction).

The conversion of metabolizable energy into net energy is also associated with some losses as heat, which depend on the quality of the consumed feed and the body function for which the energy is utilized. In the most general case, energy use in an animal can be accounted for by its requirements for maintenance, lactation, pregnancy and growth. Thus, conversion of metabolizable energy to net energy is described by the relationship:

$$E_i = k_i.ME = 14.6\ k_i.I_i.d \qquad (2.10)$$

where: E_i = net energy (MJ) required for the i'th body function
k_i = coefficient of the efficiency of conversion of metabolizable into net energy for the i'th body function
I_i = quantity of feed (kg) required to meet the energy requirements of the i'th body function
d = digestibility of consumed feed.

Solving (2.10) for I_i provides the quantity of forage needed to meet the energy needs of the i'th body function:

$$I_i = E_i/(14.6\ k_i.d) \qquad (2.11)$$

Thus, for the calculation of intake required for the i'th body function, net energy requirements (E_i) and the efficiency coefficients (k_i) need to be specified. This is done in the next sections, followed then by formulae specifying the net energy released from the mobilization of body tissue in the case of inadequate feed intake and the energy available to calves from milk consumption.

2.2.1 Maintenance

Maintenance can be defined as that state of the animal in which there is neither a net gain nor loss of nutrients (Kay, 1976). Maintenance requirements are estimates of the amount of nutrients required to achieve such an equilibrium. One component of the energy requirements for maintenance is referred to as basal

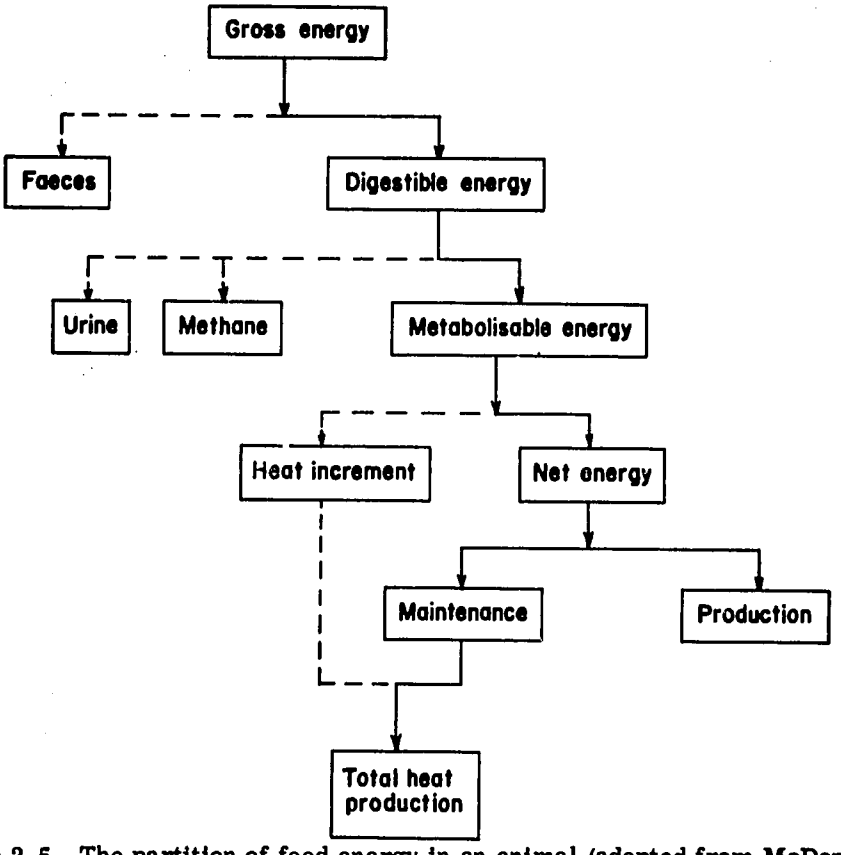


Figure 2.5 The partition of feed energy in an animal (adapted from McDonald et al, 1978)

metabolism, and is proportional to the metabolic body size of the animal. The second component of the energy requirements for maintenance relate to the level of the animal's activity and can be expressed approximately by liveweight and the daily distance walked. Thus, following Blaxter (1969) and Webster (1978), total net energy requirements for maintenance can be obtained from the relationship:

$$E_m = 0.376\ W^{0.73} + 0.0021\ W.D \qquad (2.12)$$

where: E_m = net energy requirements for maintenance (MJ/day)
W = liveweight (kg)
D = distance walked (km/day).

The efficiency with which metabolizable energy is used for maintenance (k_m) can be expressed as a function of the metabolizability of the consumed forage (see, for example, Blaxter, 1974; van Es, 1976; MAFF, 1975; Pigden et al, 1979):

$$k_m = 0.55 + 0.3\ q \qquad (2.13)$$

Substituting the values of (2.12) and (2.13) into equation (2.11) yields the feed requirements needed to meet energy for maintenance. In general, the quantity of feed required increases for heavier animals and greater activity and decreases for feed of higher quality.

2.2.2 Lactation

Net energy requirements for lactation are approximately proportional to the quantity of milk produced:

$$E_1 = e_1 \cdot M \tag{2.14}$$

where: E_1 = net energy requirements for lactation (MJ/day)
e_1 = energy content of milk (MJ/kg)
M = milk yield (kg/day).

The energy content of milk is approximately given by the relationship (MAFF, 1975):

$$e_1 = 0.0386\ BF + 0.0205\ SNF - 0.236 \tag{2.15}$$

where: BF = butter fat content (g/kg)
SNF = solids-not-fat content (g/kg).

For example, for a BF of 54 g/kg and an SNF of 85 g/kg, which approximately corresponds to milk from Zebu cattle (Williamson and Payne, 1978), the calculated energy content is 3.6 MJ/kg.

The efficiency with which metabolizable energy is converted into net energy for lactation (k_1) is less than that for maintenance. The suggested relationship (Pigden et al, 1979) is:

$$k_1 = 0.463 + 0.24\ q \tag{2.16}$$

The significance of the metabolizability coefficient is not very high (van Es, 1976), and a value for k_1 of about 0.60 is usually suggested (e.g. MAFF, 1975).

Substituting the values of (2.14), (2.15) and (2.16) into equation (2.11) yields the feed requirements to produce a given quantity of milk of a certain energy content. In general, the quantity of feed required increases for higher milk yields and higher energy contents and decreases for feed of higher quality.

2.2.3 Pregnancy

Net energy requirements for pregnancy involve the energy deposited in the uterus and associated tissues (E_{p1}), the energy associated with synthetic processes involved in foetal growth (E_{p2}), the energy required for foetal maintenance and the increased maternal fasting metabolism occurring during pregnancy (E_{p3}). Thus total net energy for pregnancy as given by MAFF (1975) is defined as:

$$E_p = E_{p1} + E_{p2} + E_{p3} \tag{2.17}$$

where: $E_{p1} = 0.03\ e^{0.0174t}$

$E_{p2} = E_{p3} = 0.452\ e^{0.01t}$

t = number of days after conception
e = base of the natural logarithms (approximately 2.718).

The efficiency with which metabolizable energy is converted into net energy for pregnancy depends on the different end uses of this energy. Thus, associated with the three net energy expenditures as described above there are corresponding efficiency coefficients as follows (MAFF, 1975):

$$k_{p1} = k_{p2} = 1.0 \qquad (2.18)$$
$$k_{p3} = k_m$$

where: k_m = the coefficient of the efficiency of converting metabolizable energy into net energy for maintenance given by equation (2.13).

2.2.4 Liveweight gain

Net energy requirements for liveweight gain depend in general on both the liveweight gain achieved and the animal's present liveweight. A relationship suggested by MAFF (1975) is:

$$Eg = DW.\ \frac{(6.28 + 0.0188\ W)}{(1 - 0.3\ DW)}$$

where: Eg = net energy requirements for liveweight gain (MJ/day)
DW = liveweight gain (kg/day)
W = present animal liveweight (kg).

Conversely, the liveweight gain obtained from the availability of net energy Eg is given by the inverse of the above relationship, i.e.

$$DW = Eg/(6.28 + 0.0188W + 0.3\ Eg) \qquad (2.19)$$

The efficiency with which metabolizable energy is converted into energy for weight gain (k_g) is lower than that for lactation and is given approximately by the relationship (Blaxter, 1973):

$$k_g = 0.03 + 0.81\ q \qquad (2.20)$$

However, for liveweight gain during lactation, k_g is much higher and approximately equals the efficiency coefficient for lactation:

$$k_g = k_l \qquad (2.21)$$

or is even higher, according to experiments by Moe et al (1971).

Substitution of relationships (2.19) and either (2.20) or (2.21) according to pregnancy status into relationship (2.11) yields the feed requirements for live-

weight gain. In general, the quantity of feed required increases for higher live-weight gains and heavier animals and decreases for feed of higher quality.

2.2.5 Energy released from mobilization of body reserves

When energy derived from feed intake is insufficient to meet the animal's total energy requirements, then the energy balance is achieved by the mobilization of body reserves. This is common soon after parturition, but in situations of low quality and quantity forage on offer and without adequate supplementation an animal may need to mobilize body reserves to meet pregnancy and maintenance requirements.

Body tissue has an energy value of about 20 MJ/kg. Energy released by tissue mobilization is lower than the energy that originally went into its synthesis. Moe et al (1971) obtained the following relationship for energy utilized for milk production and originating in body tissue loss:

$$E_1 = 0.632\ ME - 0.84\ TL - 79.7$$

where: E_1 = net energy deposited as milk (Kcal/kg metabolic weight)
ME = metabolizable energy (Kcal/kg metabolic weight)
TL = tissue loss energy (Kcal/kg metabolic weight).

This expression indicates that the coefficient of conversion of energy from body tissue mobilization to net energy for lactation is 0.84. MAFF (1975) suggests a value of 0.82. Thus, for example, if 10 kg of milk are produced, with an energy content of 3.6 MJ/kg, drawing exclusively on body tissue mobilization, this would imply a weight loss of 2.19 kg computed as (10 kg x 3.6 MJ/kg)/(0.82 x 20 MJ/kg). The same efficiency coefficient is assumed when energy released from body tissue mobilization is used to meet pregnancy or maintenance requirements.

In general, assuming an energy value of 20 MJ/kg for body tissue and a coefficient for its utilization of 0.82, the net energy available from the mobilization of body tissue is:

$$E = 16.4\ DW \qquad (2.22)$$

where: E = net energy available for maintenance, pregnancy or lactation (MJ)
DW = mobilized body tissue (kg).

The amount of body tissue needed to meet a net energy deficit E is given by the inverse:

$$DW = E/16.4 \qquad (2.23)$$

2.2.6 Energy available to calves from milk consumed

Milk consumption is the main source of energy for young calves. The gross energy content of milk is given by equation (2.15) as a function of butter fat and

solids-not-fat content. The fat fraction of milk accounts for about 60% of its energy content and is highly digestible (almost 98% for young lambs) (Owen, 1976). The overall digestibility of cow milk is nearly 93% (MAFF, 1975), which implies a digestibility level of around 86% for its solids-not-fat content. Metabolizable energy from milk is also used with great efficiency, at about 75% for lambs (Owen, 1976).

Based on these conversion factors, and assuming a close correspondence between lambs and calves, the net energy derived by calves from milk consumption is about 70% of the gross energy contained in the consumed milk (0.93 x 0.75 = 0.70). Thus the net energy available from a given quantity of consumed milk is given by:

$$E = 0.70 \; e_1 M \qquad (2.24)$$

where: e_1 = gross energy of milk (MJ/kg) as given by equation (2.15)
M = milk consumption (kg).

The quantity of milk needed to meet a net energy requirement is given by the inverse:

$$M = 1.43 \; E/e_1 \qquad (2.25)$$

2.3 ANIMAL REPRODUCTION

Reproduction in a cattle herd is a complex phenomenon. A number of factors have been identified in the literature as determining the rate of reproduction of individual females in a herd, including age, liveweight level and change, and post-partum period (Lamond, 1970; Buck et al, 1976; Newton et al, 1980; Pleasants and Ginindza, 1980).

In estimating reproduction rates, the females in a herd are classified in two categories, heifers and mature cows. The timing of a heifer's first conception is a major factor influencing its lifetime productivity, and is usually subject to some control by management to ensure that conception does not occur when an animal is in poor condition and that subsequent calving will occur at an appropriate time. Primarily, however, the timing of the first conception is determined by age and liveweight (Lamond, 1970; Pinney et al, 1972; Siebert and Field, 1975; Drennan, 1977; Anderson, 1977). According to these and other studies, puberty is reached at an average liveweight which is largely genetically determined, though the standard deviations for age and liveweight at puberty are large. If animals are inadequately nourished, their growth rate is reduced and the onset of puberty is delayed (Joubert, 1963). This is particularly evident in harsh environments where nutrition levels for young stock may be seriously inadequate after weaning. In such cases, heifers may need to reach higher weights than normal for their breed at the time of first conception (Sparke and Lamond, 1968). The variability in age or liveweight at puberty is probably greater than normal in undernourished animals (Lamond, 1970). At the other extreme, heifers which have grown very rapidly tend to conceive less readily than heifers which have grown at a moderate rate (Reid et al, 1963).

Assuming that age and liveweight account for most of the variability in heifer conception rates, then the above observations lead to the construction of a probability surface representing heifer conception, as presented in Figure 2.6. The relationship represented in this figure has the following features:

1. There is a minimum age/liveweight combination at which conception can occur (point A).

2. There is an 'ideal' liveweight at any age with a maximum probability of conception (segment (AD), the projection of the segment (AE) onto the liveweight-age plane).

3. The non-zero probabilities of conception correspond to weights above a minimum liveweight boundary ($W_{min, t}$) and below a maximum liveweight boundary ($W_{max, t}$) characteristic of the breed and the production system.

4. The surface is regular and smooth. Thus, it permits the derivation of isoprobability of conception curves, defining liveweight/age combinations with equal probabilities of conception.

Such a formulation of the determinants of conception in heifers is well suited to use in a computer simulation model. The use of this formulation in regard to the other components of the model is discussed in detail in Section 3.3.3.1.

For mature cows after first calving, conception rates are basically a function of age, with maximum calving rates achieved in the middle age range and lower observed conception rates for both younger and older animals. The general age effect on average conception rates is shown in Figure 2.7. Such relationships are usually derived from an aggregation of data from several years (e.g. Buck et al, 1976). While they usually explain a high portion of observed fertility for different age groups, some unexplained variance arises principally from the particular management system used and genetic differences between animals. Shifts upwards or downwards in fertility levels would be expected for herds with higher or lower average planes of nutrition. Buck et al (1976) show such a curve for the total animal population observed, but not for each age class. Their results show substantially higher calving rates for cows in good condition relative to those in poorer condition. At the extreme, however, the reproductive performance of very heavy cows is marginally lower than that of cows in average to good condition.

When sufficient data on conception rates exist and are disaggregated by age and liveweight, a family of curves can be obtained, with each curve corresponding to a given age class. An exemplary curve, for a given age class, is shown in Figure 2.8. The relationship represented in this figure has the following features which are common to all age classes:

1. The curve is truncated on the left at liveweight W_{min} and on the right at liveweight W_{max} representing the lowest and highest liveweights observable for this age class.

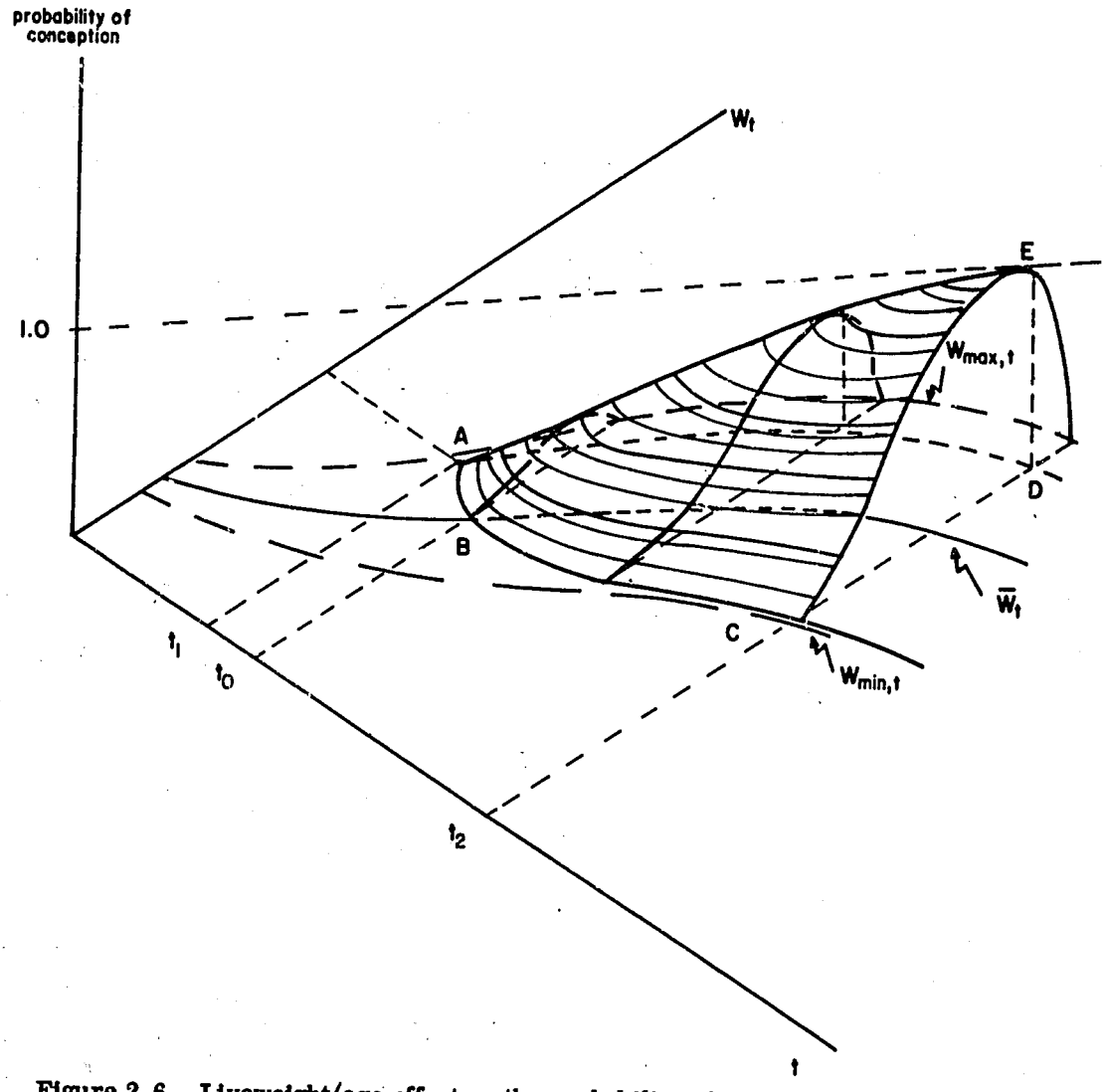


Figure 2.6 Liveweight/age effect on the probability of conception of heifers

2. Between these limits, the curve is smooth and continuous.

3. The maximum conception rate corresponds to an 'ideal' liveweight for conception, intermediate between W_{min} and W_{max} and, in general, closer to the upper limit of liveweight.

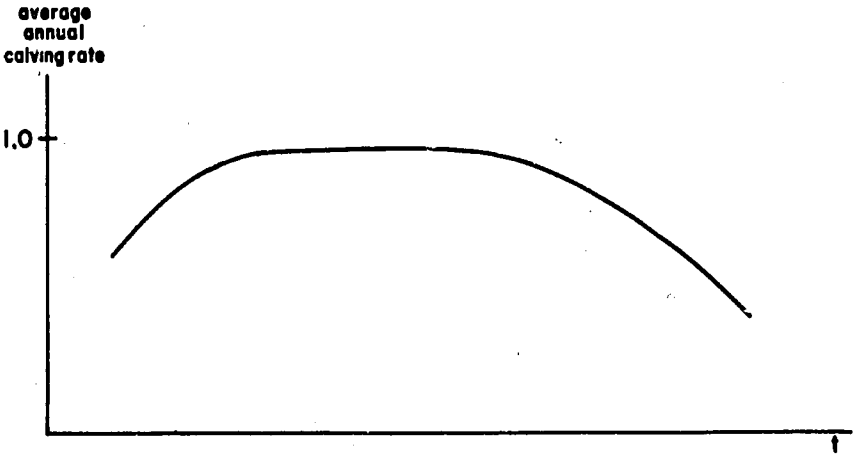


Figure 2.7 Effect of age on cow fertility

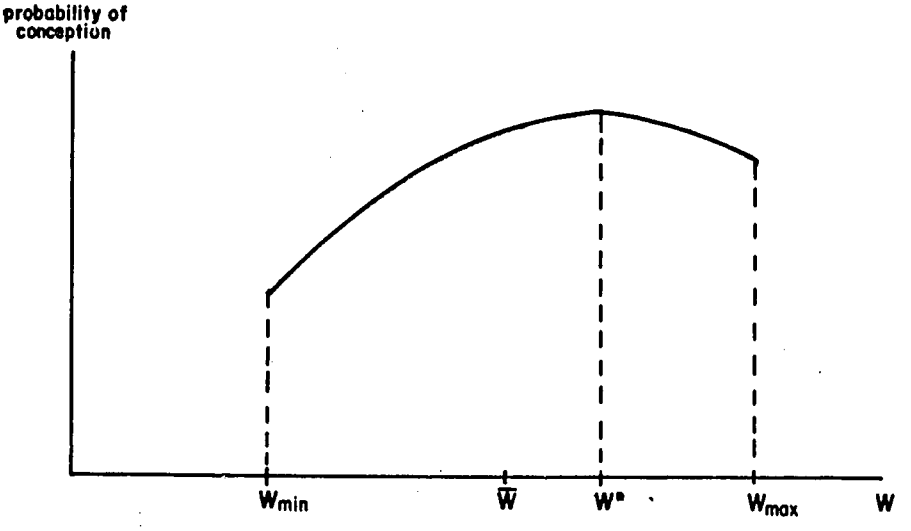


Figure 2.8 Effect of liveweight on conception

Conception in cows is also influenced by the length of the post-partum period. Conception virtually never occurs in the first month post-partum, but subsequently this effect diminishes rapidly. It is not usually considered significant by the 2nd or 3rd month post-partum.

3. THE SIMULATION MODEL

3.1 OVERVIEW OF THE COMPUTER SIMULATION MODEL

Computer simulation of biological systems requires two basic simplifications. First, a system's processes have to be approximated with mathematical expressions involving only the subset of all possible variables that can be identified as determinants of each process. Second, although the links between the different interrelated processes of the system are time-continuous phenomena, they must be specified in discrete time-steps.

A plethora of explanatory variables might be suggested on theoretical grounds as determinants of the outcomes of a complex system. However, if they are to be incorporated in a mathematical model they must be readily measurable, with some known degree of accuracy, and their individual impact on the system must be well understood. For example, the quality and quantity of forage on offer, animal activity and the physiological status of animals can be used as the major determinants of voluntary feed intake. Other factors, such as environmental temperature, humidity, palatability and toxicity, are known to play some role in influencing intake, but their quantitative impacts on intake are not adequately known. Thus, they are excluded from the simulation of forage intake.

The selection of an appropriate time-step is also determined by practical considerations. A short time-step, say 1 day, would require the detailed quantitative description of numerous micro-processes which might not be important in explaining the system's evolution over longer periods, which are more relevant in practical terms. A short time-step also requires a correspondingly frequent collection of a large volume of field data. However, such frequent data collection is generally not possible, or even justifiable. Finally, a short time-step substantially increases computational requirements. These increased requirements will pose some limitation on the use of a stochastic model as large and complex as the one presented here, even given the high speed of modern computers.

Based on these considerations a time-step of one month was chosen for the

model. Herd-level field data are often collected at monthly intervals, so the use of this time-step facilitates model validation. Furthermore, a monthly time-step is generally adequate for the specification of management practices, such as breeding season, weaning age and supplementation.

The transition in the status of each animal from one calendar month to the next during the simulated time period is determined by the set of biological processes and decision rules embedded in the model. These are summarized in Figure 3.1. At the beginning of each month of simulation, the model determines the forage quality and quantity on offer and the activity level of animals in the herd. Forage on offer is simulated independently, based on historical time-series data. The details of the forage component of the model are elaborated in Section 3.2. After determining the forage on offer, the model determines the changes taking place in each animal's status during the month of simulation, using endogenous biological processes regulated by exogenous management policies. Each animal in the herd is processed independently, except for cows with suckling calves. Suckling calves secure at least a fraction of their energy intake in the form of milk from their dams, and the balance between energy demand and energy requirements is determined simultaneously for the cow and her calf to avoid over- or under-charging the energy costs to either of them.

Five general components in the model account for changes in the biological status of animals in a month of simulation: these are the forage intake, energy requirements, production and growth, reproduction, and mortality. The algorithms determining forage intake are based on the analytical relationships presented in Section 2.1, adjusted to specific systems by the procedure elaborated in Section 3.3.1. Similarly, energy requirements for the different body functions are obtained from the relationships given of Section 2.2, depending on the level of each activity.

Intake of feed energy is exactly balanced by its utilization for maintenance (plus pregnancy and lactation as required) and weight gain or loss. Separate production and growth algorithms are used to determine liveweight changes for adult males, calves and non-lactating cows, as well as milk production and liveweight change for lactating cows. As shown in Figure 3.1, the level of production and growth for individual animals may be influenced by exogenous management practices such as supplementary feeding, calf weaning, milk offtake and responses to drought. The details of the production and growth algorithms are elaborated in Section 3.3.2, and those of supplementary feeding, milking, weaning and drought options are described in Sections 3.4.1, 3.4.2, 3.4.3 and 3.4.7.

The reproduction component of the model determines whether a non-pregnant cow conceives or a pregnant cow calves during a given month. Conception is modelled as a probabilistic process, qualified by the distinction between heifers and mature cows. Conception is influenced by nutritional status and management practices, described as breeding policy in Figure 3.1, specifying the breeding season and a minimum weight and/or age to be attained before animals are bred. Details of the reproduction algorithms are presented in Section 3.3.3, and those of breeding policy in Section 3.4.4.

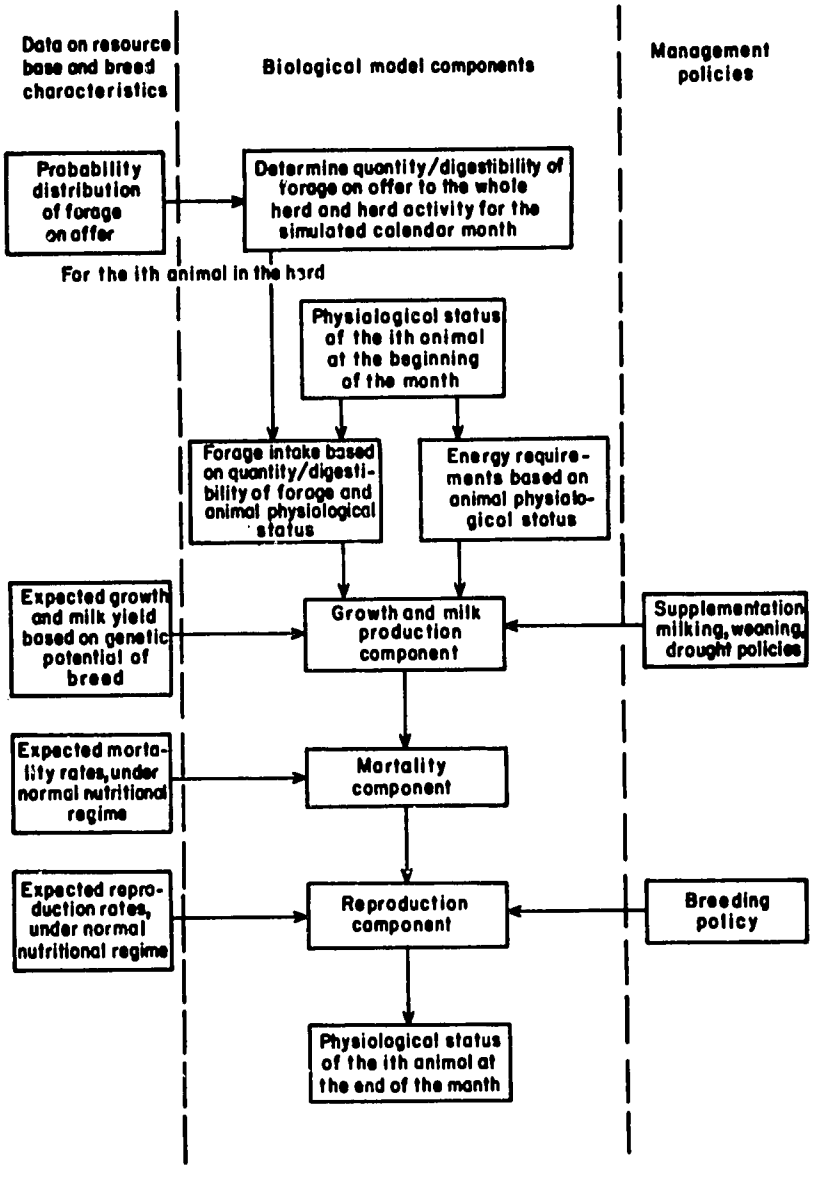


Figure 3.1 Basic components of the model

Finally, the mortality component of the model determines whether an animal dies during a month of simulation. Mortality is modelled as a probabilistic process qualified by the age of an animal and its nutritional status. Details of these algorithms are given in Section 3.3.4.

At the end of each simulated month, the model determines sales and purchases of animals, depending on individual animal attributes and on aggregate herd size targets, as specified by management. These components are discussed in detail in Sections 3.4.5 and 3.4.6. As extended periods of feed shortage are characteristic of semi-arid environments, the model includes a component to specify various selling or supplementation options to mitigate against the effects of drought. These are described in detail in Section 3.4.7.

3.2 SPECIFICATION OF FORAGE ON OFFER AND ANIMAL ACTIVITY

The quantity and quality of forage on offer to a herd under extensive grazing conditions vary substantially from season to season and from one year to the next. There are basically two approaches to incorporating this variability into a herd-level simulation model. First, the biological processes determining primary production can be described in quantitative terms as a subsystem, with forage output (quantity and quality composition) predicted from the driving variables of the primary production subsystem (see for example, van Keulen, 1975). Such an approach requires a statistical description of the variables of the primary production system, including rainfall intensity and seasonal distribution, soil fertility, solar radiation and grazing pressure. In practice, the amount of information required to drive the primary production model is overwhelming, even in cases where the biological processes are well understood. Thus, this approach is impractical for a general herd simulation model with emphasis on secondary production.

The second approach, which is employed here, bypasses the underlying processes of the primary production subsystem. Instead, the model uses a statistical description of the quantity and digestibility of forage on offer, based on field observations from the production system under study. Of course, this specification of primary production as an exogenous factor means that there is no feedback to the primary production system of the effects of its exploitation,for instance of overgrazing.

Like forage production, the seasonal level of activity of cattle herds is associated with environmental variability. Herd managers respond to the variability of forage on offer by seasonal migrations. Even in systems where forage is adequate throughout the year, the usual water sources might be exhausted for some periods so that animals must be moved to distant watering points. Excessive animal activity implies a significant additional energy cost and might also reduce forage intake due to the reduced time available for grazing. Both of these impacts are incorporated in the appropriate components of the model.

The specification of the level of animal activity is directly related to the specification of forage on offer. A year of good rainfall implies adequate forage and water resources, so that relatively low animal activity levels are expected. Conversely, a year of poor rainfall implies limited forage and water resources, requiring higher levels of animal activity.

Statistical information on primary production and animal activity must be available on a monthly basis. However, monthly observations of forage quantity and digestibility are not independent of each other within a given year, as the environmental conditions prevailing during the growing season mainly determine primary production for the whole year. Thus, field observations of primary production have to encompass the full annual cycle. Analytically, a particular year outcome can be denoted by three 12-element vectors, Q, d and D, where Q describes the quantity of forage on offer (t/ha) for the 12 months of the annual cycle, d describes the corresponding average digestibility of the forage, and D gives the corresponding average daily distances walked (km). Several years' observations of these variables provide a basis for their statistical description as a multivariate probability distribution function:

$$\begin{bmatrix} Q \\ d \\ D \end{bmatrix} \sim \Phi \left[\begin{bmatrix} \bar{Q} \\ \bar{d} \\ \bar{D} \end{bmatrix}, \Sigma \right] \qquad (3.1)$$

where:

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{12} \end{bmatrix} \quad \text{and } \bar{Q} = \begin{bmatrix} \bar{Q}_1 \\ \bar{Q}_2 \\ \vdots \\ \bar{Q}_{12} \end{bmatrix}$$

the vectors of monthly (1, 2, ... , 12) quantities and average quantities for a given year (t/ha).

$$d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{12} \end{bmatrix} \quad \text{and } \bar{d} = \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \vdots \\ \bar{d}_{12} \end{bmatrix}$$

the vectors of monthly (1, 2, ... , 12) digestibilities and average digestibilities for a given year (fraction)

$$D = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_{12} \end{bmatrix} \quad \text{and } \bar{D} = \begin{bmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \vdots \\ \bar{D}_{12} \end{bmatrix}$$

the vectors of monthly (1, 2, ... , 12) distances and average distances walked for a given year (km/day)

$$\Sigma = \left[\begin{array}{c|c|c} \Sigma^{QQ} & \Sigma^{Qd} & \Sigma^{QD} \\ \hline \Sigma^{Qd} & \Sigma^{dd} & \Sigma^{dD} \\ \hline \Sigma^{QD} & \Sigma^{dD} & \Sigma^{DD} \end{array}\right]$$

the estimated symmetric variance-covariance matrix, derived from observed quantities and qualities of forage on offer and associated distances walked.

In general it is possible to obtain estimates for $\bar{Q}$, $\bar{d}$, $\bar{D}$ and Σ from a limited number of yearly observations. However, these estimates will not be statistically significant unless information exists for a longer time series, of say 20 to 30 years. Thus, to make the model applicable to systems where time series records have not been kept consistently for a long period, an additional simplification is introduced. It is assumed that it is possible to construct K representative year types which characterize the range of the possible outcomes of primary production in the system under study. Additionally, it is assumed that each outcome has a known relative probability of occurrence thereby allowing for the specification of a probability distribution where the sum of K probabilities equals unity. Formally:

$$\begin{bmatrix} Q \\ d \\ D \end{bmatrix}_i$$ = the quantity, quality and activity vector for year type i (i = 1, 2, ..., K), and

P_i = the probability of occurrence of year type i (i = 1, 2, ..., K)

such that

$$\sum_{i=1}^{K} P_i = 1.$$

At the minimum, K equals one, in which case this specification implies that primary production and animal activity are known with certainty and do not exhibit any year-to-year variability. A value of three for K implies that the primary production and associated animal activity of the system under study can be characterized in terms of 3 discrete year-types corresponding, say, to poor, average and good years. A maximum of 5 year-types is likely to be permitted by the data base in any one production context. To construct a more refined year-type classification, adequate information must be available to estimate primary

production variability in the more accurate form as specified by equation (3.1).

An example of a data set required for the specification of forage on offer, covering 3 possible year-types, is presented in Appendix Table F.1.

3.3 ALGORITHMS SIMULATING BIOLOGICAL PROCESSES

3.3.1 Feed intake

The general relationship determining forage intake by extensively grazing cattle, which emerges from the discussion in Section 2.1, is restated here as follows:

$$I = m(d,t).m(Q).m(D).m(t).m(x,t).m(p,t).a.\bar{W}_t^{0.73} \qquad (3.2)$$

where I = dry matter forage intake (kg/day)

- $m(d,t)$ = digestibility-of-forage-correction multiplier, a function of the digestibility of forage on offer (d) and for high digestibility levels also of age (t)
- $m(Q)$ = quantity-of-forage-correction multiplier, a function of the quantity of forage on offer (Q)
- $m(D)$ = grazing-time-correction multiplier, a function of the distance walked per day (D), taken as a proxy for grazing time limitations
- $m(t)$ = age-correction multiplier, a function of the age (t) of individual animals
- $m(x,t)$ = sex-correction multiplier, a function of the sex (x) and age (t) of individual animals
- $m(p,t)$ = physiological-status-correction multiplier, a function of the physiological status (p) and age (t) of individual animals
- a = intake coefficient (kg dry matter/kg metabolic weight/day) of the reference class of animals
- $\bar{W}_t$ = expected liveweight (kg) for animal's age and sex (see Section 3.3.2.1).

The *digestibility-of-forage-correction multiplier* accounts for the effect of digestibility on the voluntary intake of animals (physical limit) and the adjustments to intake at very low and very high digestibility levels. Its specification is as follows:

(a) for $d \leq 0.40$

$$m(d,t) = (d/0.40)^{0.6}/(1-d) \qquad \text{for all } t$$

(b) for $0.40 < d \leq 0.65$

$$m(d,t) = 1/(1-d) \qquad \text{for all } t$$

(c) for $d > 0.65$

$$m(d,t) = 1/(1-d) \qquad \text{for } t \leq 1.5 \text{ years}$$

$$m(d,t) = 1.86/d \qquad \text{for } t > 1.5 \text{ years.}$$

The *quantity-of-forage-correction multiplier* and the *grazing-time-correction multiplier* account for adjustments in voluntary intake due to limitations in the quantity of acceptable forage on offer and the time available for grazing. During certain seasons, estimated *ad libitum* intakes might not be realized because of insufficient forage on offer and/or excessive walking requirements, such as to distant watering points or for migrations. Section 2.1.2 explained in principle how these constraints could reduce voluntary intake below the *ad libitum* level. As the data required to estimate the relationships proposed in Section 2.1.2 are normally not available, a simplified formulation is employed here, based on a minimum of data. Assume:

Q^* = the minimum quantity (t/ha) of acceptable forage on offer, below which a reduction in *ad libitum* forage intake is observed

D^* = the maximum distance (km) walked per day, above which a reduction in *ad libitum* forage intake is observed.

These two threshold levels imply that forage intake will equal the *ad libitum* level when both the quantity of forage on offer (Q) is greater than Q^* and the distance walked per day (D) is less than D^*. If either of these conditions is not met, then a reduction in the voluntary intake is assumed, though the extent of this reduction is an empirical question. The assumed effects of these two factors are shown in Figures 3.2 and 3.3, subject to validation for each particular system under study. Figure 3.2 implies a reduction to 50% of the *ad libitum* intake level when the quantity of acceptable forage on offer is 50% of the threshold level. Similarly, Figure 3.3 implies a 5% reduction intake below the *ad libitum* level for each km walked over the threshold level D^*. Formally, the assumed multiplicative effect of the forage on offer (Q) on *ad libitum* intake is:

(a) for $Q \leq Q^*$

$$m(Q) = Q/Q_*$$

(b) for $Q > Q^*$

$$m(Q) = 1.0$$

and the assumed multiplier effect of distance walked daily (D) is:

(a) for $D \leq D^*$

$$m(D) = 1.0$$

(b) for $D > D^*$

$$m(D) = 1.0 - 0.05\ (D - D^*).$$

The *age-correction multiplier* accounts for the reduction in voluntary intake of animals older than 8 years and the zero forage intake of animals younger than 2 months. Its specification is:

(a) for $t \leq 0.17$ years (+ 2 months)

$$m(t) = 0$$

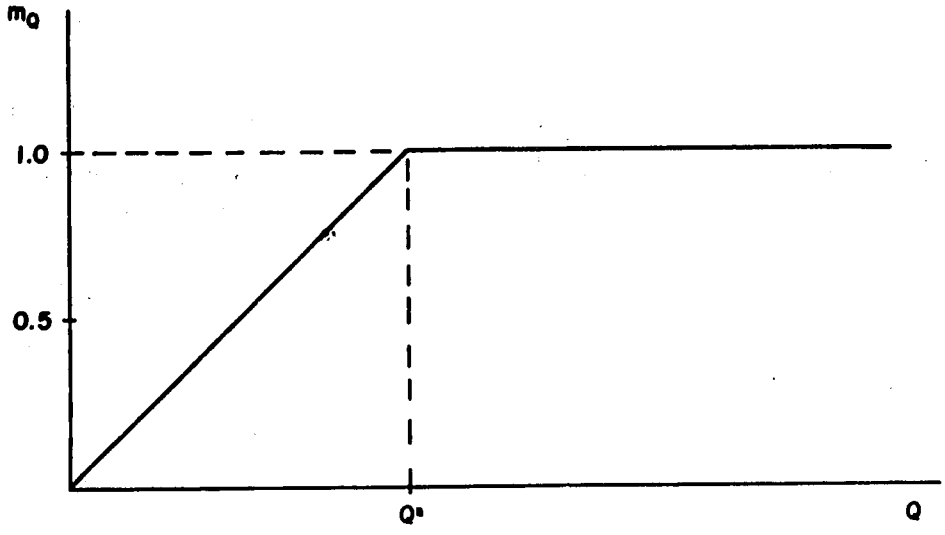


Figure 3.2 Assumed multiplicative effect (m_Q) of quantity of acceptable forage on offer (Q) on voluntary intake

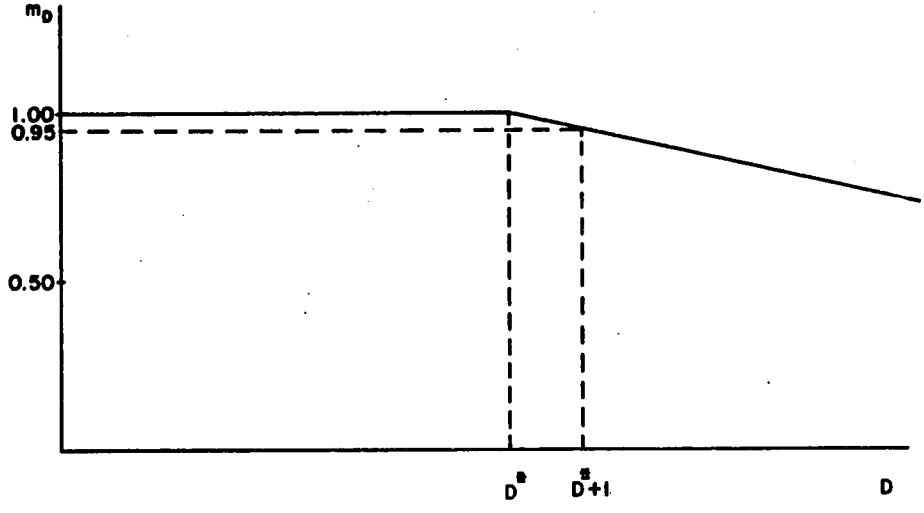


Figure 3.3 Assumed multiplicative effect (m_D) of daily distance walked (D) on voluntary intake

(b) for 0.17 years < t ≤ 8 years

$$m(t) = 1.0$$

(c) for t > 8 years

$$m(t) = 1.0 - 0.03(t - 8).$$

The *sex-correction multiplier* accounts for the apparent higher appetite of young males (x = 1) relative to young females (x = 2). It is assumed that males up to 18 months have a 10% higher appetite than females of a corresponding age. Formally, this multiplier is specified as:

(a) for x = 1

$$m(x, t) = 1.1 \quad \text{for } t \leq 1.5 \text{ years}$$
$$m(x, t) = 1.0 \quad \text{for } t > 1.5 \text{ years}$$

(b) for x = 2

$$m(x, t) = 1.0 \quad \text{for all } t.$$

For the specification of the *physiological-status-correction multiplier*, the following animal classes (p) are considered:

1. very young calves (3 to 6 months)
2. young and rapidly growing animals (7 to 18 months)
3. males and dry females (older than 18 months)
4. pregnant cows (last 3 months of pregnancy)
5. lactating cows, including cows which are both lactating and pregnant.

Animal class 3 is taken as a reference class (i.e. m(3, t) = 1.0). On the basis of this class, together with the literature reviewed in Section 2.1.1, the correction multipliers for the other animal classes are specified as:

(a) for p = 1 (0.25 ≤ t ≤ 0.5 years)

$$m(p, t) = 0.53$$

(b) for p = 2 (0.5 < t ≤ 1.5 years)

$$m(p, t) = 0.53 + 0.47\ (t - 0.5)$$

(i.e. an interpolation between classes 1 and 3)

(c) for p = 3 (t > 1.5 years)

$$m(p, t) = 1.0$$

(d) for p = 4 (last 3 months of pregnancy)

$$m(p, t) = 1.07 \quad \text{for all } t$$

(e) for p = 5 (lactating cows)

$$m(p, t) = 1.15 \quad \text{for all } t.$$

Finally, the *intake coefficient of the reference class of animals* (a) is calculated on the basis of observations from the particular system under con-

sideration. In any production system, there exists a calendar month of the most probable year type during which the reference class of animals is in equilibrium as regards liveweight, that is, neither gaining nor losing weight. This implies that the daily dry matter intake during this equilibrium month is just sufficient to maintain body weight for the reference animal at the level of activity for the month.

The liveweight of the reference animal (for example, a 7-year-old dry female) is known, as are the quantity and digestibility of the forage on offer and the level of animal activity for the equilibrium month. The computation of the intake coefficient (a) is achieved by equating intake requirements, as specified by functional forms (2.11) to (2.13), sufficient to allow exactly for maintenance and the animal's level of activity, with the functional form (3.2).

For example, consider the forage on offer as specified in Appendix Table F.1 and the breed as specified by the parameters given in Appendix Table F.2. Also assume that during an average year-type the reference class (7-year-old dry females) maintains liveweight during the month of April, when the digestibility of forage on offer is 0.50 and the animals walk 12 km/day. The intake requirements to maintain a 350 kg animal at this activity level, as obtained from equations (2.11) to (2.13), are 7.32 kg dry matter/day. The coefficient of intake for the reference class of animals is then computed by equating this equilibrium intake requirement with functional form (3.2):

$$1/(1-0.50) \,.\, 1.1.1.1.1.a.350^{0.73} = 7.32$$

which yields an intake coefficient a = 0.0509.

In addition to forage, the feed available to an animal could include supplements. Provisions are made in the model for two types of supplementation: concentrates and high quality forage. In both cases, the quality and the quantity of the supplements are specified by management (Section 3.4.1). Aside from their different nutritive values, the two types of supplements are distinguished in the model in terms of their impact on voluntary forage intake through grazing. Because concentrates are generally supplied in small quantities and are highly digestible, it is assumed that they do not have any positive or negative effect on the level of voluntary intake through grazing, although it has been reported in some studies, such as Campling et al (1972), that the provision of urea results in increased voluntary intake of very low quality forage. When supplementation consists of high quality forage, it is assumed that the quantity consumed reduces the amount of standard forage which would have otherwise been consumed through grazing. For each kg of high quality forage consumed, voluntary intake of the standard forage is assumed to be reduced by 0.5 kg. In summary, concentrates are treated as net additions to voluntary intake through grazing, whereas high-quality forage is assumed to substitute for lower-quality forage which would have otherwise been consumed by a ratio of 2 to 1.

3.3.2 Growth and milk production

3.3.2.1 Liveweight limits and the condition index. A simulation model of herd productivity must include an adequate representation of the liveweight potential of the breed under study. Observed liveweights of individual animals of a particular breed are a function of the genetic characteristics of the breed and the production system in which they are maintained. When sufficient liveweight data are available for the system under study, the expected values of liveweights can be estimated, as well as the distribution of liveweights according to age and sex categories. These statistics can then be used to estimate extreme upper and lower limits of fluctuations in liveweights such that observed liveweights will lie within these limits with a specified degree of confidence.

Assume that the observed average liveweight evolution curve of animals of a given sex in the system under study is described by the following function:

$$\bar{W}_t = f(t) \qquad (3.3)$$

where: t = animal's age (months)
$\bar{W}_t$ = the average observed liveweight (kg) at age t

and that the distribution of liveweights around $\bar{W}_t$ is given by the general function Φ:

$$W_t \sim \Phi\ (\bar{W}_t,\ \sigma_t^2) \qquad (3.4)$$

where σ_t = the estimated standard deviation of liveweights at age t.

This is, in general, a function of t:

$$\sigma_t = g(t) \qquad (3.5)$$

Estimated parameters of functional forms (3.3) through (3.5) will depend on the production system in which the observations were made and the number of years over which data were recorded. If the observations were from a few years with below average performance, then the mean liveweights calculated will be below those which would be estimated if the data were recorded from another sequence of years more representative of the system under study. Similarly, the range of liveweights at any age estimated from a short time series will tend to be smaller than that estimated from a longer time series. Thus, ideally, these relationships should be estimated from data collected over an adequately long and representative time period, to express the full range of liveweight possibilities for the breed and system under study. The actual period required depends on the variability of environmental conditions in the system; where forage availability varies substantially from year to year, a longer period is necessary.

Although statistical estimates of functions (3.3) through (3.5) could theoretically be possible, in practice they would require considerable cross-sectional and time-series data to guarantee high levels of significance. Because

such data are often not available, less stringent assumptions than those in (3.3) to (3.5) have to be made for the specification of liveweight evolution and associated distributions. Thus, it is assumed that liveweights are distributed normally around their mean for a sex and age category, with a coefficient of variation k_t which implies a standard deviation of liveweight variability:

$$\sigma_t = k_t . \bar{W}_t = k_t . f(t) \qquad (3.6)$$

A procedure is presented in Appendix A to permit an approximate estimation of function (3.3) from a minimum set of data, when time-series and cross-sectional data are not sufficient for direct estimation. The coefficient of liveweight variability is in general a function of age, and, for a given breed, of environment, reflecting adaptations to relatively favourable or harsh conditions. For modelling purposes, three categories of animals are considered: calves up to 6 months for which the major source of energy intake is milk from their dams, relatively mature animals (over 18 months), and animals in a transition period from 7 to 18 months. For the first category, the value of k is taken as equal to 0.30, and it is assumed to be independent of breed and environmental variability. This value was found to reflect calf liveweight variability for a number of breeds and production systems, but it can be changed readily if it does not depict the situation for a particular application of the model. For mature animals, the coefficient of variation is provided as data, derived from observed liveweight variability for the system and breed under study. Finally, for animals in the transitional category, the coefficient of variation is computed by interpolation between 0.30 and the coefficient for mature animals.

Under these assumptions concerning average liveweight evolution and associated variability around the expected values for each age group, it is possible to construct an upper and lower boundary of liveweight variability. Thus, using a normal distribution, the 95% confidence boundaries for liveweight are defined as follows for each sex:

$$W_{max,t} = \bar{W}_t (1 + 1.96\, k_t) \qquad (3.7)$$

$$W_{min,t} = \bar{W}_t (1 - 1.96\, k_t) \qquad (3.8)$$

where: $\bar{W}_t$ = average estimated liveweight for age t

$W_{max,t}$ and $W_{min,t}$ = the upper and lower liveweight limits respectively, at the 5% probability level.

The estimated range for each sex defines a feasible set of simulated liveweights. Figure 3.4 depicts the general shape of the average liveweight evolution curve and associated liveweight boundaries and presents two exemplary cases of liveweight paths. Case 1 shows an animal with a below average nutritional status until it dies due to starvation at the time its liveweight drops below the lower boundary, $W_{min,t}$ at age t_1. The liveweight evolution of the second animal (case 2)

depicts a more typical situation, where liveweight changes follow seasonal forage fluctuations. In this case, the liveweight of the animal oscillates above and below the mean for its sex and age, and it survives at least until age t_2.

The upper boundary for each sex can be regarded in practical terms as the maximum liveweight of animals of the breed under study. Similarly, the lower boundary can be regarded as the minimum liveweight, below which death occurs due to starvation. Thus, no animals should be observed below that weight. It is recognized that this is a simplification of the complex set of factors which regulate liveweight and limit liveweight fluctuations, as substantial differences are observed among individuals of a given breed. However, since data are often inadequate to ascribe particular genetic potentials to each animal in the simulated herd, it is assumed that all animals of the breed under study are genetically equivalent as regards their potential for liveweight gain and tolerance to fluctuations in liveweight over time. Similarly, it is assumed that animals of the same class and physiological status have identical constraints on intake and identical feed conversion efficiencies.

For an animal of a certain age and weight, a liveweight condition index can be defined depending on the relative position of its current liveweight vis-a-vis the corresponding upper and lower liveweight limits:

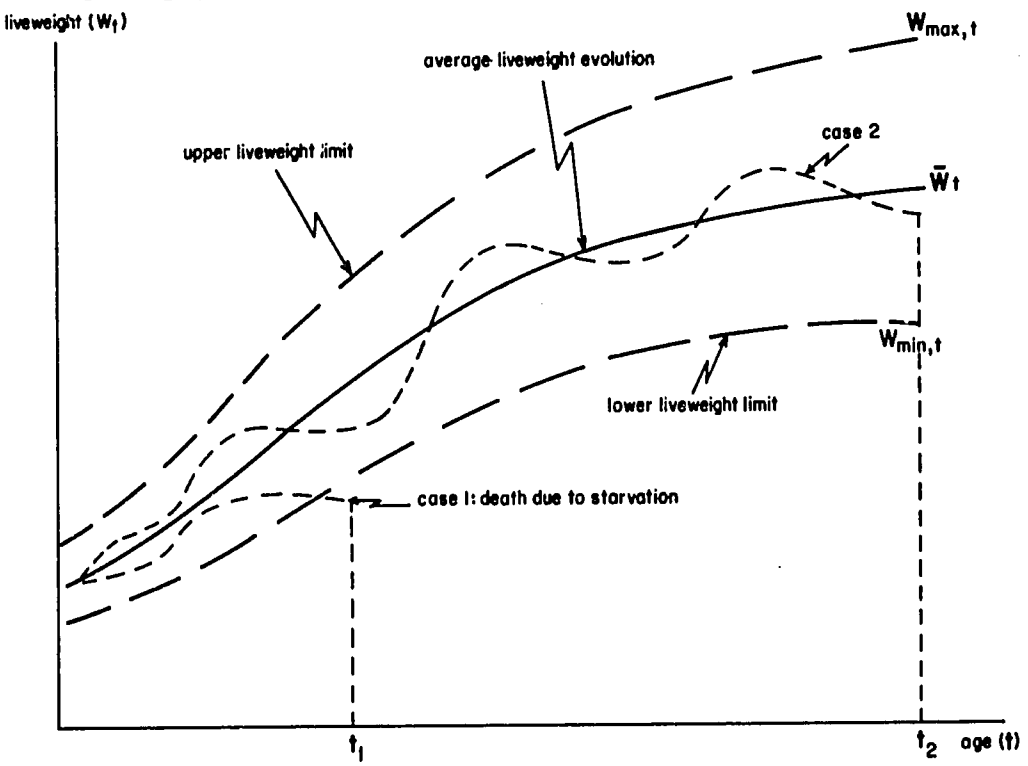


Figure 3.4 General shape of the average liveweight evolution curve and associated boundary curves, and exemplary illustrations of two simulated liveweight paths

$$c_t = (W_t - W_{min,t})/W_{max,t} - W_{min,t}) \quad (3.9)$$

where $W_{max,t}$ and $W_{min,t}$ are as in (3.7) and (3.8).

The condition index ranges from 1.0 (when W_t equals $W_{max,t}$) to 0.0 (when W_t equals $W_{min,t}$). This index is used in the model to modify lactation yields and reproductive performance.

The liveweight limits are also used to determine the maximum permissible liveweight gain or loss during a simulated month. At the end of the simulated month an animal of age t in months can at most increase its liveweight to $W_{max,t+1}$ or at most decrease it to $W_{min,t+1}$. Thus, the maximum daily liveweight increase (DW^i_{max}) and decrease (DM^d_{max}) are:

$$DW^i_{max} = (W_{max,t+1} - W_t)/30 \quad (3.10)$$

$$DW^d_{max} = (W_t - W_{min,t+1})/30 \quad (3.11)$$

where again $W_{max,t+1}$ and $W_{min,t+1}$ are obtained from equations (3.7) and (3.8) and where a month has 30 days. These two permissible liveweight changes play an important role in the specification of the growth algorithms as described in subsequent sections.

The growth algorithms that follow determine average daily change in liveweight over a month of simulation based upon the energy content of the total feed available to each animal, and its requirements as specified in Section 2.2. Intake of feed energy is used for maintenance (plus pregnancy and lactation when appropriate), with any surplus or deficit balanced by liveweight gain or loss. In a situation of energy surplus, if the resulting daily liveweight gain is in excess of DW^i_{max} then the feed intake is reduced to the level which gives a gain exactly equal to DW^i_{max}. In an energy-deficit situation, if the resulting daily liveweight loss is greater than DW^d_{max}, then for non-lactating animals the weight loss will exceed DW^d_{max}, and the animal dies due to starvation in the subsequent month if it fails to increase its liveweight above the lower liveweight boundary. For lactating animals, the energy expended on lactation is reduced (implying a reduction in milk yield) to the level where weight loss does not exceed DW^d_{max}. If the weight loss at zero milk yield still exceeds DW^d_{max} then, as for non-lactating animals, the cow dies due to starvation in the subsequent month of simulation. The calculations to determine the balance between available energy and energy requirements are made on a daily basis, with productivity over a month calculated as 30 times the daily changes.

3.3.2.2 Growth algorithm for adult males. Energy intake by adult males in the simulated herd is used first to satisfy maintenance and second for weight gain. If the energy intake is not sufficient to meet maintenance requirements then body resources are mobilized as required and the animal loses weight. Energy is available from the basic forage on offer and from food supplements if they are provided.

A descriptive flowchart of the growth algorithm for males is presented in Figure 3.5, indicating the computational procedures followed. Forage intake (I_f^a) and the forage requirements for maintenance (I_{fm}^r) are determined first. If forage intake is more than adequate for maintenance the animal will gain weight equivalent to the excess energy in the forage intake over the energy required for maintenance, plus the energy from any food supplements provided. If the energy from forage is not sufficient for maintenance but a food supplement is provided, then the total intake is compared with maintenance requirements: if intake is greater than requirements, then weight gain occurs, if less, then body reserves are mobilized to meet the energy deficit and the animal loses weight.

Table 3.1 illustrates the computational procedures involved in the growth algorithm for males. The exact relationships used in the calculations are shown in the footnotes to the table. Out of a total forage intake of 7.58 kg/day, 3.55 kg/day are available for weight gain above maintenance requirements. This results in a potential daily gain of 0.89 kg or 26.7 kg for the month of simulation. Since this potential monthly gain is well below the maximum permissible, actual daily gain equals potential daily gain, i.e. DW = 0.89 kg/day.

3.3.2.3 Milk production algorithm. The milk production algorithm used in this model is shown in Figure 3.6. Three factors determine the daily milk yield: the age of the cow, the number of months post-partum and the liveweight condition index. The maximum potential yield (Y_{max}^p) is usually realized by middle aged cows. The extent to which the milk yields of younger and older cows are reduced below this maximum depends on the breed and the particular production system. In general, the potential milk yield of a cow at age t can be expressed as:

$$Y_t^p = Y_{max}^p \cdot y_t \qquad (3.12)$$

where: Y_t^p = potential milk yield per lactation (kg) of a cow at age t

Y_{max}^p = maximum potential milk yield per lactation (kg) of middled aged cows

y_t = potential milk yield realized at age t relative to the maximum (fraction).

If sufficient data are available, it is possible to specify relative potential milk yields at all ages. Appendix B presents an analytical procedure for estimating the age effect from a minimum of data.

The distribution of milk yield over a given lactation is primarily a function of breed and management environment. In general, the daily milk yield increases to a maximum by the 4th to 6th week post-partum and declines steadily thereafter. Exponential formulations have often been used to estimate lactation curves from weekly or daily data (Cobby and le Du, 1978; Wood, 1969). The time-step in this model is one month, so there is only need to approximate the lactation curve in monthly intervals. Thus the maximum potential daily milk yield during the n'th month post-partum can be expressed as:

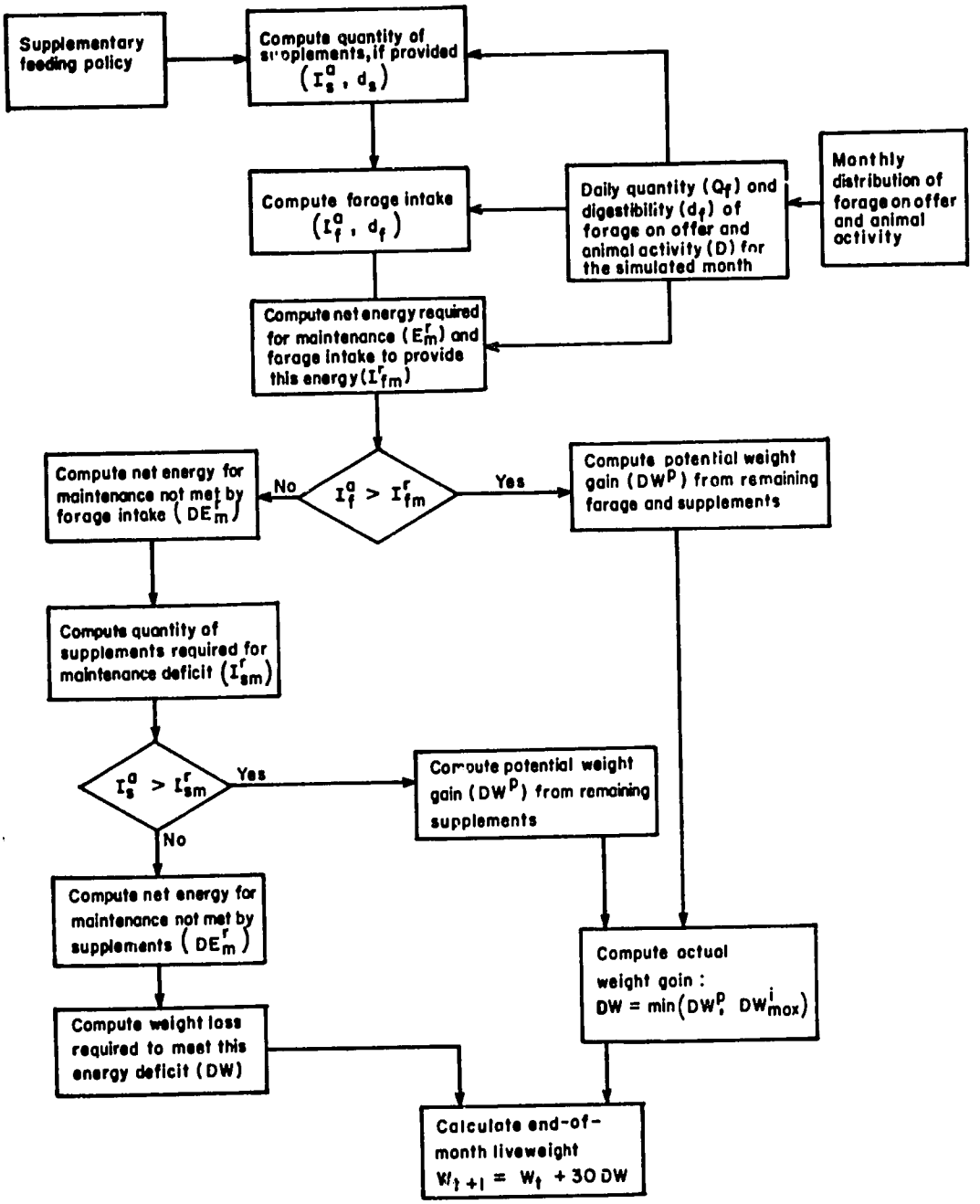


Figure 3.5 Flowchart of the growth algorithm for adult males

Table 3.1 Illustration of calculations involved in the growth algorithm for adult males[a]

Variable[b]		Value
System-related variables		
1.	Quantity of forage on offer (Q_f)	2.7 t dry matter/ha
2.	Digestibility of forage on offer (d_f)	0.60
3.	Threshold quantity of forage below which voluntary intake is reduced (Q^*_f)	0.8 t dry matter/ha
4.	Distance walked per day (D)	8 km/day
5.	Threshold distance above which voluntary intake is reduced (D^*)	14 km/day
6.	Intake coefficient for reference class (a)	0.0509
Animal-related variables		
7.	Age (t)	2 years
8.	Mean liveweight for age ($\bar{W}_t$)	270 kg
9.	Coefficient of variation for liveweight (k_t)	0.25
10.	Current liveweight (W_t)	240 kg
11.	Upper liveweight limit ($W_{max,t}$)	402 kg
12.	Lower liveweight limit ($W_{min,t}$)	138 kg
13.	Voluntary intake of forage (I^a_f)	7.58 kg dry matter/day
14.	Net energy required for maintenance (E^r_m)	24.58 MJ/day
15.	Forage efficiency coefficient for maintenance (k_m)	0.696
16.	Forage intake required for maintenance (I^r_{fm})	4.03 kg dry matter/day
17.	Forage available for weight gain ($I^a_f - I^r_{fm}$)	3.55 kg dry matter/day
18.	Forage efficiency coefficient for weight gain (k_g)	0.424
19.	Net energy available for weight gain (E^a_g)	13.18 MJ/day
20.	Potential weight gain (DW^p)	0.89 kg/day
21.	Actual weight gain (DW)	0.89 kg/day

[a] The production system in this example is as described in Appendix Tables F.1 and F.2. The calculations reported here correspond to the values given for November (month 11) during a poor year.

[b] The sources for the values of different variables are:

1 to 5 from Appendix Table F.1
6 as computed in Section 3.3.1
7, 9 and 10 data for illustrative purposes
8 obtained as in Appendix A using values from Appendix Table F.2
11 and 12 from relationships (3.7) and (3.8) respectively
13 from relationship (3.2)
14, 15 and 16 from relationships (2.12), (2.13) and (2.11) respectively
18, 19 and 20 from relationships (2.20), (2.10) and (2.19) respectively.

$$M^p_{t,n\,max} = (Y^p_t/30).h_n \qquad (3.13)$$

where:

$M^p_{t,n\,max}$ = maximum potential daily milk yield of a cow at age t in the n'th month post-partum (kg/day)

Y^p_t = as obtained from equation (3.12)

n = period post-partum (months)

h_n = maximum potential milk yield at the n'th month of lactation, as a fraction of total yield.

In practice, the average monthly yields over the 1st and 2nd months post-partum are approximately equal. For modelling purposes, it is assumed here that milk yields are equal for the first 2 months post-partum and decline linearly thereafter. This simplification of the lactation curve considerably reduces the field data required for its estimation, while retaining its essential attributes. Appendix C presents an analytical procedure and the data required for estimation based on these assumptions.

The cow's energy balance during the current month and its nutritional status during previous months are principal determinants of the actual current milk yield (Broster, 1972; Lowman et al, 1979). The cow's energy balance during the current month is based on total available energy minus energy requirements; the algorithm dealing with this aspect is discussed in detail in the following section. The nutritional status of previous months can be summarized by the current liveweight condition index as given by (3.9). When this condition index is above 0.3, it is assumed that liveweight condition does not have any effect on potential milk yield. For a condition index below 0.3, a linear effect is assumed, as depicted in Figure 3.7. Formally, the assumed multiplicative effect of liveweight condition on potential milk yield is defined as $m_c = 1$ for $c \geq 0.3$, and $m_c = c/0.3$ for $c < 0.3$. Thus, potential daily milk yield is:

$$M^p_{t,n} = M^p_{t,n\,max}.m_c \qquad (3.14)$$

where $M^p_{t,n}$ = potential daily milk yield of a cow at age t in the n'th month post-partum (kg/day)

$M^p_{t,n\,max}$ = as obtained from equation (3.13)

c = current liveweight condition index

m_c = assumed multiplicative effect of liveweight condition on maximum potential daily yield.

Table 3.2 illustrates the sequence of calculations to determine daily potential milk yields. Based on the parameters of the system under consideration and the specific parameters of the cow considered in this example, the potential milk yield is calculated as 2.44 kg/day. This potential daily milk yield is used as

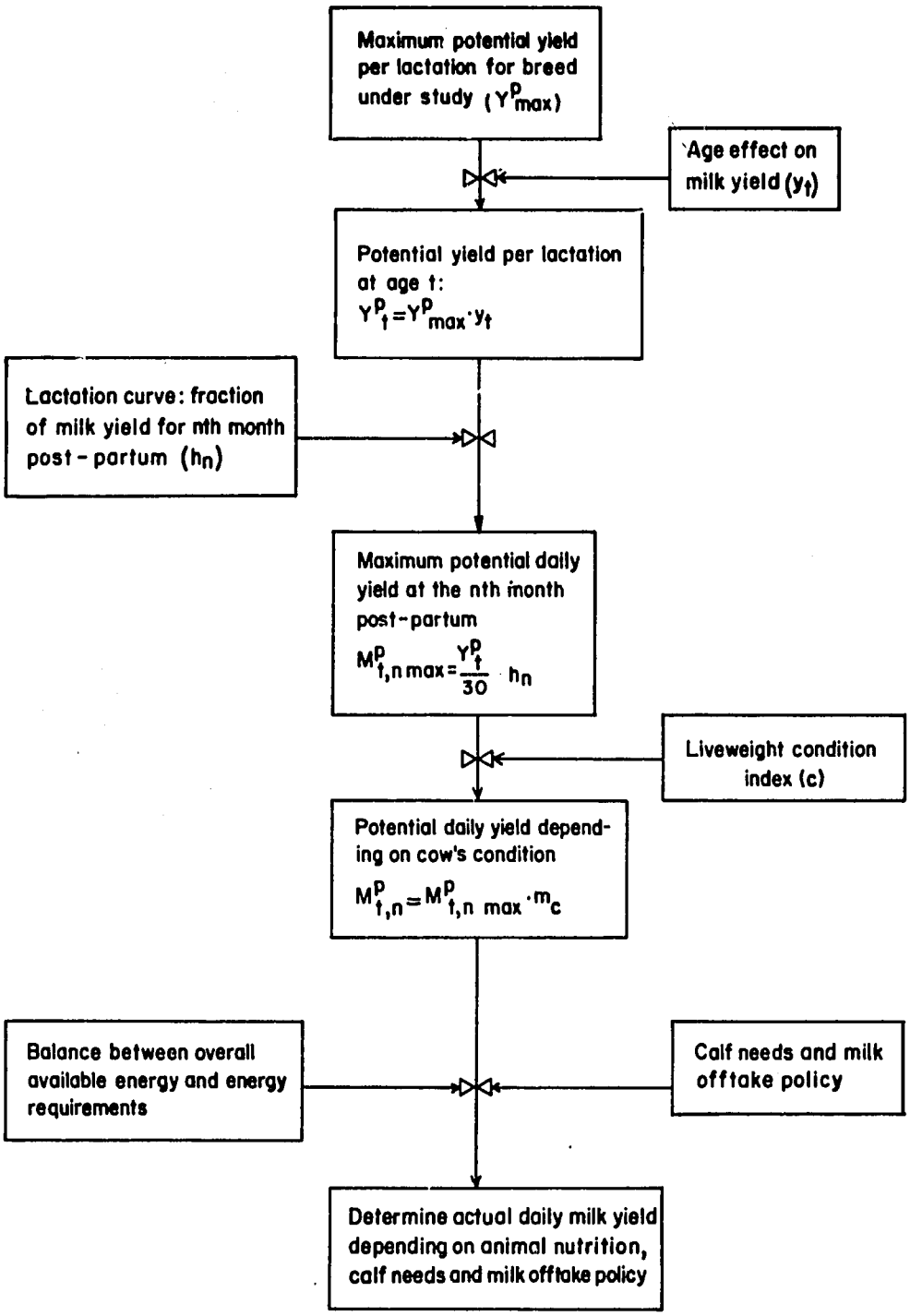


Figure 3.6. Flowchart of the algorithm for milk production

Table 3.2 Illustration of calculations involved in the milk production algorithm[a]

Variable[b]	Value
System-related variables	
1. Maximum milk yield per lactation (Y^p max)	800 kg
2. Length of lactation (N)	9 months
3. Fraction of milk produced in the first 2 months of lactation ($h_{1,2}$)	0.35
Animal-related variables	
4. Age (t)	5 years
5. Expected liveweight for age ($\bar{W}_t$)	350 kg
6. Coefficient of liveweight variation (k_t)	0.25
7. Current liveweight (W_t)	340 kg
8. Upper liveweight limit (W max, t)	521 kg
9. Lower liveweight limit (W min, t)	178 kg
10. Liveweight condition index (c_t)	0.47
11. Fraction of maximum yield for a 5-year-old cow (y_5)	0.985
12. Potential milk yield per lactation for a 5-year-old cow ($Y^p{}_5$)	788 kg
13. Period post-partum (n)	6 months
14. Maximum daily milk yield during the 6th month post-partum as a fraction of total (h_6)	0.093
15. Maximum daily milk yield during the 6th month post-partum ($M^p{}_{5,6\ max}$)	2.44 kg/day
16. Multiplier effect of liveweight condition on milk yield (m_c)	1.0
17. Potential daily milk yield during the 6th month post-partum ($M^p{}_{5,3}$)	2.44 kg/day

[a] The production system considered in this example is as described in Appendix Tables F.1 and F.2.

[b] The sources for the values of different variables are:

1 to 4 data for illustrative purposes
5 obtained from Appendix A using values from Appendix Table F.2
6 and 7 data for illustrative purposes
8, 9 and 10 from relationships (3.7), (3.8) and (3.9) respectively
11 obtained as in Appendix B using values from Appendix Table F.3
12 from relationship (3.12)
13 data for illustrative purposes
14 obtained as in Appendix C using values from Appendix Table F.3
15, 16 and 17 from relationship (3.13), Figure 3.7 and relationship (3.14) respectively.

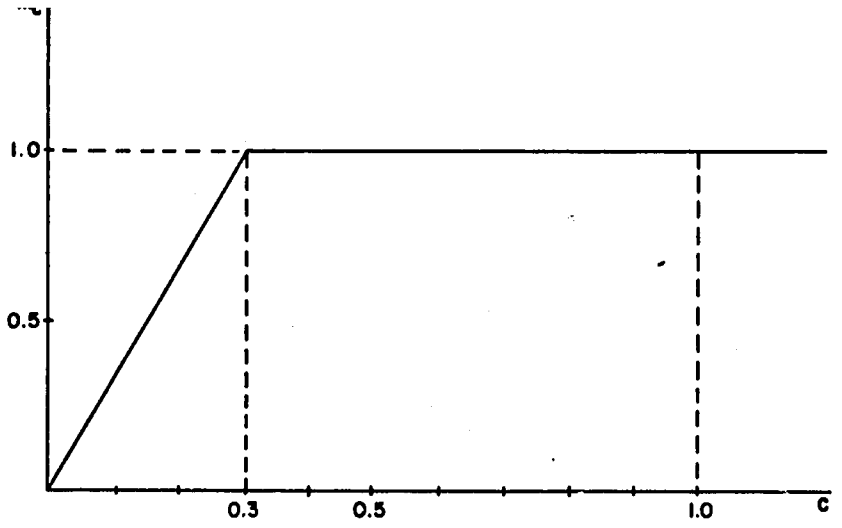


Figure 3.7 Assumed multiplicative effect (m_c) of liveweight condition index (c) on potential milk yield

an input to the calculations of the cow growth algorithm described in the following section. Depending on the total energy available to the animal, this potential daily milk yield may or may not be achieved.

3.3.2.4 Growth algorithm for adult females. In general, cows use energy for maintenance, pregnancy, lactation and weight gain. In this model it is assumed that energy needs for maintenance and pregnancy are satisfied first. Maintenance and pregnancy are treated as joint and inseparable energy demands. If additional energy is available it is used first for lactation and then, only when full milk production potential is realized, for weight gain. Energy needs for maintenance and pregnancy have to be met at all times, either from the food on offer or by mobilizing body reserves. Body reserves are also mobilized to meet lactation requirements.

Figure 3.8 is a flowchart of the growth algorithm for cows. The potential milk production of a lactating cow is provided to the algorithm, based on the cow's age, stage of lactation and condition (Section 3.3.2.3). Forage intake (I_f^a) is determined, and compared with forage requirements for maintenance (I_{fm}^r) and pregnancy (I_{fp}^r). The flowchart details the subsequent operation of the algorithm based on this comparison, with the calculations leading to predicted daily weight gain or loss. In summary, if overall energy intake is adequate, lactation requirements are met and any residual energy intake is deposited as weight gain. Body reserves are mobilized as required if the energy intake is inadequate for maintenance and pregnancy needs. If intake is sufficient for these functions but not

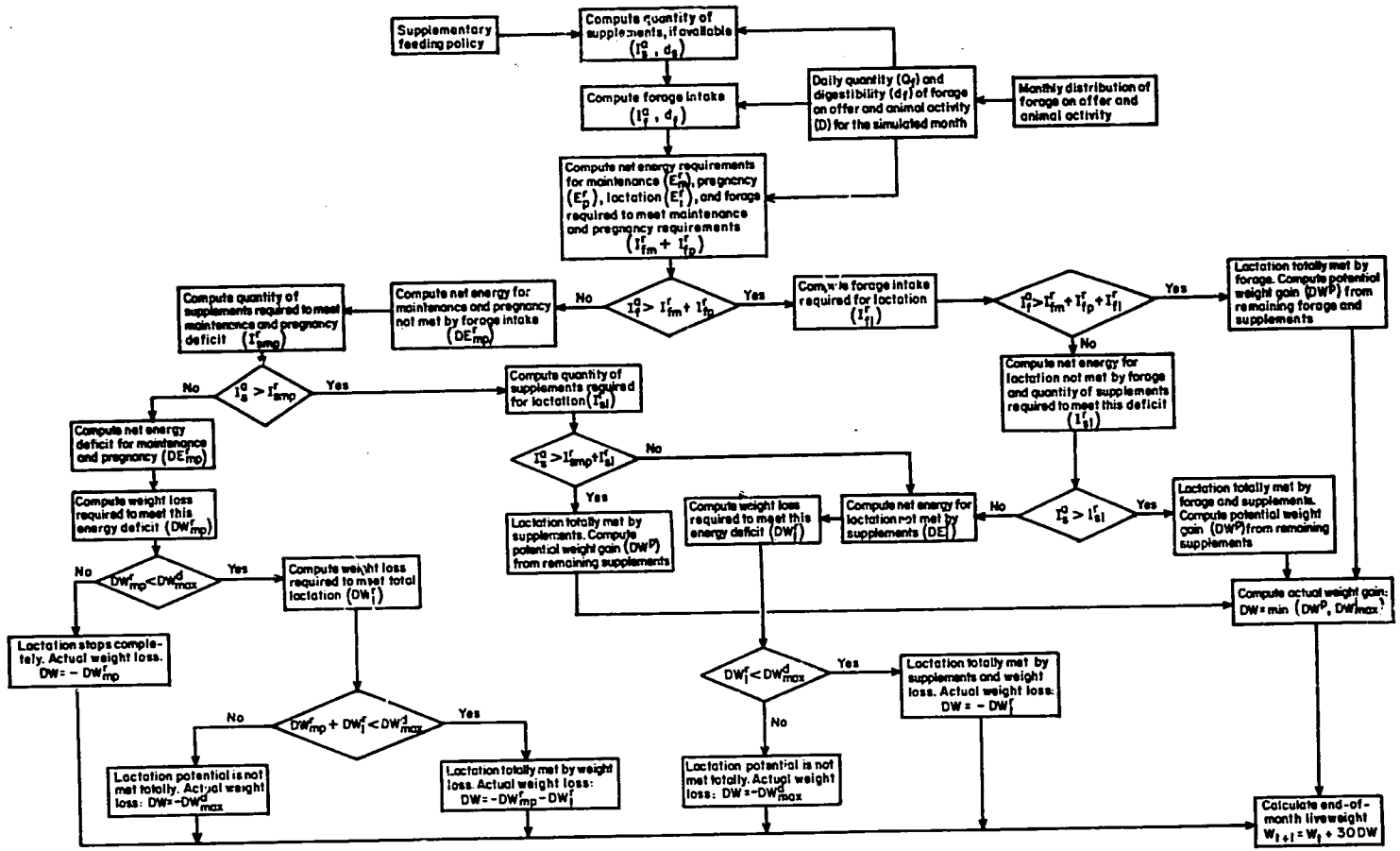


Figure 3.8 Flowchart of the growth algorithm for adult females

Table 3.3 Illustration of calculations involved in the growth algorithm for cows[a].

Variable[b]	Value
System-related variables (additional to Table 3.2)	
1. Quantity of forage on offer (Q_f)	1.3 t dry matter/ha
2. Digestibility of forage on offer (d_f)	0.42
3. Threshold forage quantity below which voluntary intake is reduced (Q_{f*})	0.8 t dry matter/ha
4. Distance walked per day (D)	15 km/day
5. Threshold distance above which voluntary intake is reduced (D^*)	14 km/day
6. Intake coefficient for reference class (a)	0.0509
7. Quantity of supplements provided (I^a_s)	1.29 kg dry matter/day
8. Digestibility of supplements (d_s)	0.86
Animal-related variables (additional to Table 3.2)	
9. Voluntary intake of forage (I^a_f)	6.90 kg dry matter/day
10. Net energy required for maintenance (E^r_m)	37.20 MJ/day
11. Forage efficiency coefficient for maintenance (k_m)	0.652
12. Forage intake required for maintenance (I^r_{fm})	9.30 kg dry matter/day
13. Forage deficit for maintenance ($I^r_{fm}-I^a_f$)	2.40 kg dry matter/day
14. Net energy deficit for maintenance (DE^r_m)	9.60 MJ/day
15. Supplements efficiency coefficient for maintenance (k_m)	0.76
16. Supplements required to meet maintenance deficit (I^r_{sm})	1.01 kg dry matter/day
17. Supplements available for lactation ($I^a_s-I^r_{sm}$)	0.28 kg dry matter/day
18. Net energy required per unit of milk produced (e_1)	3.6 MJ/day
19. Potential daily milk yield ($M^p_{5,6}$)	2.44 kg
20. Net energy required for potential lactation (E^r_1)	8.87 MJ/day
21. Supplements efficiency coefficient for lactation (k_1)	0.63
22. Supplements required for potential lactation (I^r_{s1})	1.11 kg dry matter/day
23. Supplements deficit for lactation ($I^r_{sm}+I^r_{s1}-I^a_s$)	0.83 kg dry matter/day
24. Net energy deficit for lactation (E^r_1)	6.57 MJ/day
25. Weight loss required to meet lactation (DW^r_1)	0.40 kg/day
26. Actual weight loss (DW)	0.40 kg/day

[a]The production system considered in this example is as described in Tables F.1, F.2 and Table 3.2. Calculations correspond to June (month 6) of an average year.

[b]The sources for the values of different variables are:
1 to 5 from Appendix Table F.1
6 as computed in Section 3.3.1
7 and 8 data for illustrative purposes
9 from relationship (3.2)
10, 11 and 12 from relationships (2.12), (2.13) and (2.11) respectively
14 from relationship (2.10)
15 and 16 from relationships (2.13) and (2.11) respectively
18 data for illustrative purposes, see relationship (2.15)
19 from Table 3.2
20, 21 and 22 from relationships (2.14), (2.16) and (2.11) respectively
24 and 25 from relationships (2.10) and (2.23) respectively.

sufficient to meet calculated needs for lactation, then body reserves can be mobilized up to a maximum level corresponding to a weight loss DW^d_{max} (Section 3.3.2.1). A downwards adjustment in milk yield is in effect when required body reserves for lactation are in excess of DW^d_{max}.

The computation procedures of this algorithm are illustrated in Table 3.3. The production system and the status of the cow used in this example are the same as in Table 3.2. In addition to forage intake, the cow is supplemented by 1.5 kg of barley daily, which corresponds to 1.29 kg dry matter as the dry matter content of barley is 860 g/kg.

Forage intake by the cow is calculated as 6.90 kg/day which is insufficient to meet the cow's maintenance and pregnancy requirements, due to its low digestibility and the cow's excessive activity. The energy deficit for maintenance and pregnancy is fully met by supplements, with an excess 0.28 kg/day of supplements available for lactation. As this is not sufficient to meet total lactation requirements, the animal will lose a calculated 0.40 kg/day liveweight. Total potential weight loss for the current month of simulation equals 12.0 kg (calculated as 0.40 x 30) and is well within permissible weight loss limits. Thus the lactation requirements of the cow are fully met, the animal loses 12.0 kg over the month, and at the end of the month has a simulated liveweight of 328 kg (calculated as 340 - 12).

3.3.2.5 Growth algorithm for calves. In the context of the model, a calf is defined as an unweaned animal whose energy requirements are met totally or partially by milk from its dam. In general, both forage and supplements may also be available to a calf. The algorithm in the model specifies that milk consumption has first priority for calves, forage is consumed next, and supplements are then consumed if they are available and can be utilized. In the model, the voluntary forage intake by calves up to 2 months is zero.

The growth algorithm for calves is depicted in Figure 3.9. Milk available to a calf (M^a) is taken as equal to the residual of the milk yield of its dam minus any offtake for human consumption. The quantity of milk required to meet the calf's maintenance (M^r_m) is calculated and compared with the quantity of milk available. If sufficient milk is available for maintenance, then any excess milk plus any forage or supplements consumed results in weight gain. If insufficient milk is available for maintenance, then calculations are made to determine if the forage available can meet the maintenance deficit. If the available forage is also inadequate, then supplements (if available) are used to meet the remainder of the maintenance deficit. If available forage and/or supplements are inadequate, body reserves are mobilized to meet the residual energy deficit for maintenance and the calf loses weight. The calf gains weight when the total energy intake is greater than maintenance requirements, subject to the maximum permissible level of weight gain.

The operational procedures of the algorithm are illustrated in Table 3.4. A 6-month-old male calf is taken as an example. In addition to 3.5 kg of milk available from its dam, it also consumes 2.03 kg dry matter of forage daily. Milk alone is not adequate to meet its maintenance requirements, leaving

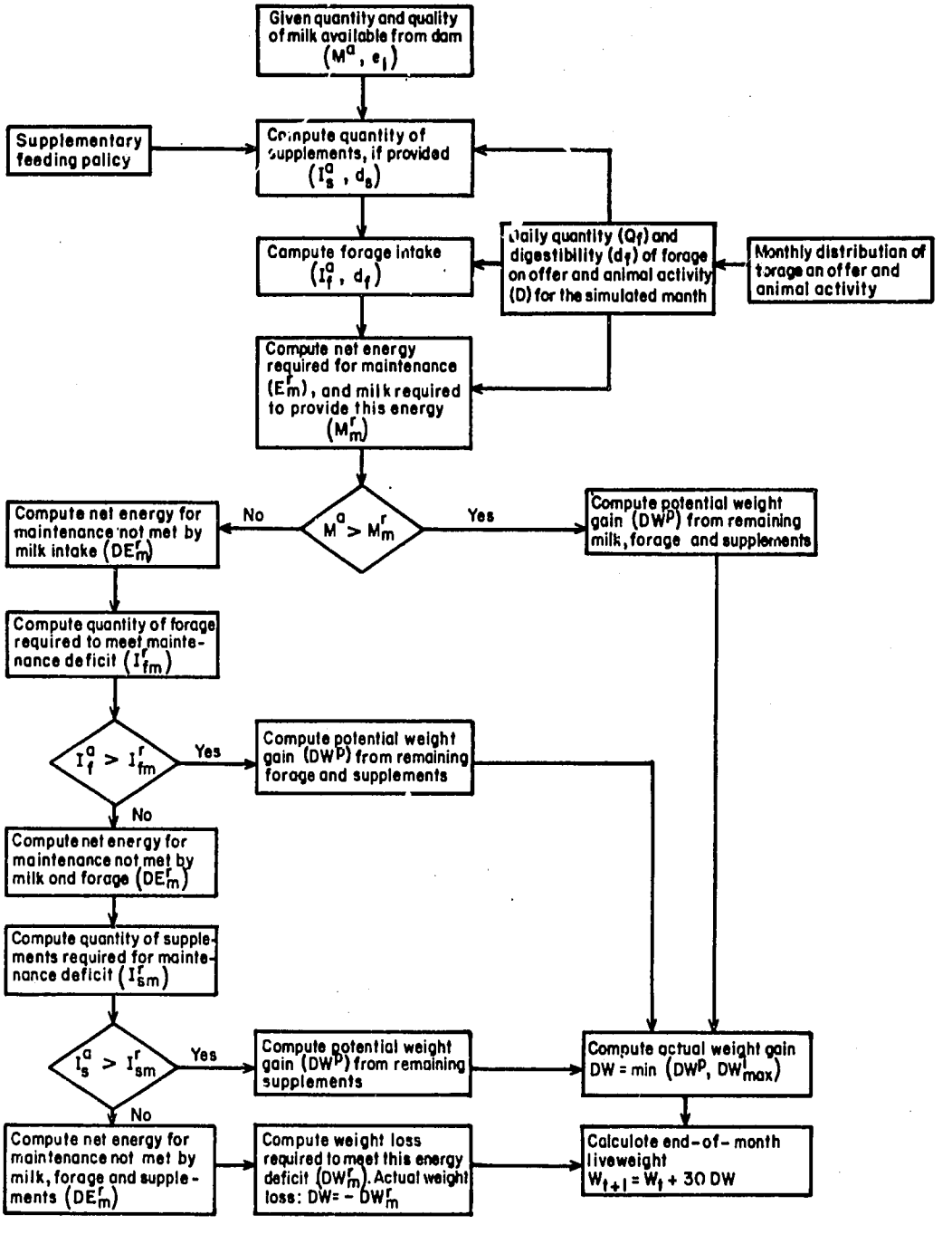


Figure 3.9 Flowchart of the growth algorithm for calves

Table 3.4 Illustration of calculations involved in the growth algorithm for calves[a]

Variable[b]	Value
System-related variables	
1. Quantity of forage on offer (Q_f)	3.4 t dry matter/ha
2. Digestibility of forage on offer (d_f)	0.62
3. Threshold forage quantity below which voluntary intake reduced (Q^*_f)	0.8 t dry matter/ha
4. Distance walked per day (D)	8 km/day
5. Threshold distance above which voluntary intake reduced (D*)	14 km/day
6. Intake coefficient for reference class	0.0509
Animal-related variables	
7. Age (t)	0.5 years
8. Sex (x)	1 (male)
9. Expected liveweight for age and sex ($\bar{W}_t$)	87 kg
10. Coefficient of liveweight variation (k_t)	0.30
11. Current liveweight (W_t)	100 kg
12. Upper liveweight limit (W max, t)	138 kg
13. Lower liveweight limit (W min, t)	36 kg
14. Milk intake available from dam (M^a)	3.50 kg/day
15. Net energy per unit of milk available (e_1)	3.60 MJ/kg
16. Voluntary intake of forage (I^a_f)	2.03 kg dry matter/day
17. Net energy required for maintenance (E^r_m)	12.52 MJ/day
18. Milk intake required for maintenance (M^r_m)	4.97 kg/day
19. Milk deficit for maintenance ($M^r_m - M^a_r$)	1.47 kg/day
20. Net energy deficit for maintenance (DE^r_m)	3.70 MJ/day
21. Forage efficiency coefficient for maintenance (k_m)	0.70
22. Forage intake required for maintenance deficit (I^r_{fm})	0.58 kg dry matter/day
23. Forage available for weight gain ($I^a_f - I^r_{fm}$)	1.45 kg dry matter/day
24. Forage efficiency coefficient for weight gain (k_g)	0.44
25. Net energy available for weight gain (E^a_g)	5.78 MJ/day
26. Potential weight gain (DW^p)	0.58 kg/day
27. Actual weight gain (DW)	0.58 kg/day

[a]The system considered in this example is as described in Tables F.1 and F.2. Calculations correspond to November (month 11) of an average year.

[b]The sources for the values of different variables are:
1 to 5 from Appendix Table F.1
6 as computed in Section 3.3.1
7, 8, 10, 11, 14 and 15 data for illustrative purposes
9 obtained as in Appendix A using values from Appendix Table F.2
12 and 13 from relationships (3.7) and (3.8) respectively
16 from relationship (3.2)
17 and 18 from relationships (2.12) and (2.25) respectively
20, 21 and 22 from relationships (2.24), (2.13) and (2.11) respectively
24, 25 and 26 from relationships (2.20), (2.10) and (2.19) respectively.

an energy deficit of 3.70 MJ/day to be met by forage. The available forage covers this deficit fully, with an excess of 1.45 kg dry matter/day which results in a potential weight gain of 0.58 kg/day or 17.4 kg for the month. As this potential monthly gain is below the maximum permissible gain, the actual gain will also equal 0.58 kg/day, and at the end of the month the simulated liveweight will be 117.4 kg (computed as 100 + 17.4).

3.3.3 Reproduction

3.3.3.1 Heifer reproductive maturity. As indicated in Section 2.3, aside from management interventions, a heifer's first successful conception is determined principally by age and liveweight. This process is stochastic, as for any given age there exist different transitional probabilities associated with different liveweights, with higher probabilities applying for heifers in relatively better condition.

This formulation requires considerable data from the system under study for estimating the probability surface, as postulated in Figure 2.6. Such data are seldom available, however, even for production systems which have been relatively thoroughly studied. Usually, an age/liveweight relationship must be inferred from data on the age distribution of heifers at first calving, which can be translated into an age distribution at first successful conception. This distribution is usually skewed, as shown in Figure 3.10 where t_o corresponds to the average age at first successful conception of heifers of average liveweight. Similarly t_1 and t_2 are the minimum and maximum observed ages at first conception, corresponding to heifers of very good and very poor liveweight condition, respectively.

Operationally, the conditions for heifer reproductive maturity can be approximately specified from ages t_1, t_o and t_2 in conjunction with the breed's average liveweight evolution curve and the associated minimum and maximum boundaries, as specified in Section 3.3.2.1. Liveweight $W_{min,\ t_2}$ in Figure 3.10 corresponds to the minimum liveweight of heifers at t_2 required for reproductive maturity. Similarly, liveweights $\overline{W}_{t_o}$ and $W_{max,\ t_1}$ correspond to ages t_o and t_1 respectively. The linear segments (AB) and (BC) define approximately the age/liveweight combinations required for heifer reproductive maturity. The shaded area then defines the feasible age/liveweight set for the breeding herd. Whether a heifer of age t and liveweight W_t has reached reproductive maturity can be ascertained by checking on whether point (W_t, t) is in the feasible set. The relevant checks are:

$$t_1 (\overline{W}_{t_o} - W_t) + t(W_{max,\ t_1} - \overline{W}_{t_o}) + t_o(W_t - W_{max,\ t_1}) \geq 0$$

$$\text{for } t_1 \leq t \leq t_o \qquad (3.15)$$

$$t_o (W_{min,\ t_2} - W_t) + t(\overline{W}_{t_o} - W_{min,\ t_2}) + t_2 (W_t - \overline{W}_{t_o}) \geq 0$$

$$\text{for } t_o < t \leq t_2 \qquad (3.16)$$

where: $W_{max,\ t_1}$, $\bar{W}_{t_0}$ and $W_{min,\ t_2}$ are obtained from equations (3.7), (3.3) and and (3.8) after substituting t_1, t_0 and t_2 respectively.

In addition to these physiological conditions for reproductive maturity, conceptions are only possible within the conditions of the breeding regime in effect, as specified in Section 3.4.4.

3.3.3.2 Conceptions. In any month of the breeding season, a fraction of cows in the breeding herd will conceive. Since the mechanisms underlying the loss of foeti before full term are difficult to model, it is assumed that conception results in a birth 9 months later, as the gestation period of cows is 284 days. Thus, since birth rates are perfectly correlated with conception rates in the model, annual calving rates and conception rates are synonymous. The model calculates the energy cost for a 284 day gestation, although the simulated time is 9 months or 270 days.

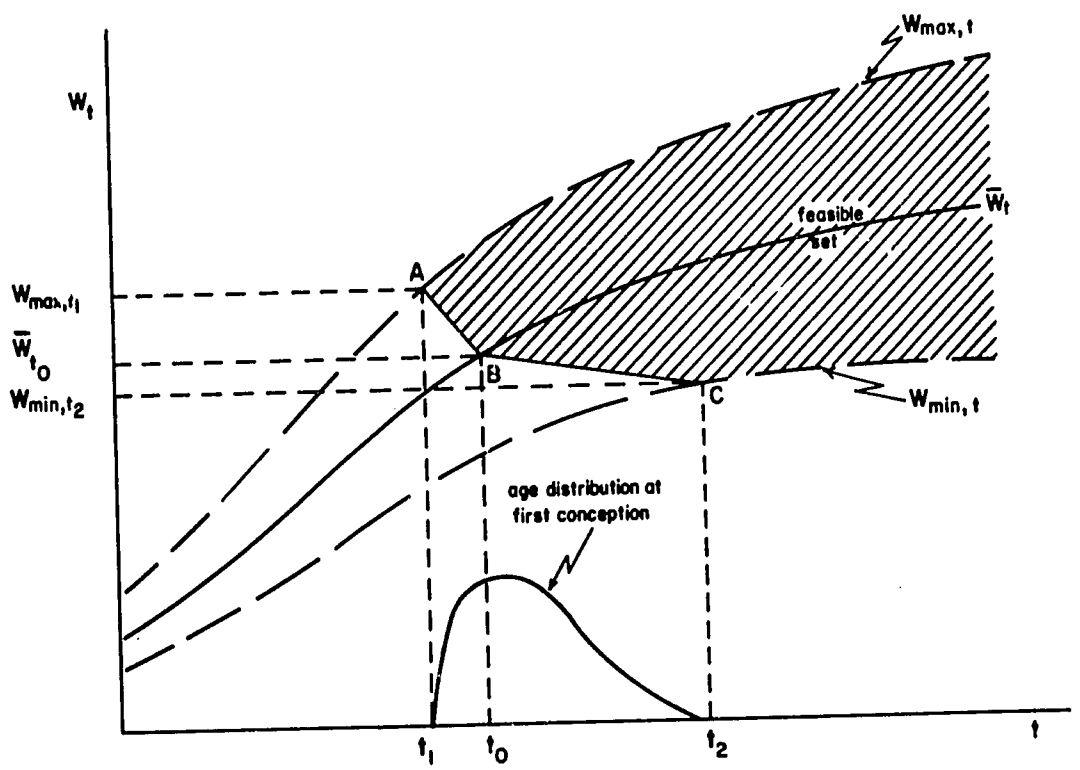


Figure 3.10 Age and liveweight combinations for heifer reproductive maturity

Three variables were identified in Section 2.3 as the main determinants of conception for mature cows: age, liveweight and the interval since previous calving. The individual effects on conception of the first two of these factors were shown in Figures 2.7 and 2.8 respectively. Though these factors affect conception simultaneously, data do not usually exist to allow a multivariate estimation. At best, data exist to estimate the age effect on fertility, which notationally can be represented by:

$$\bar{r}_t = f(t)$$

where: t = age of cow (years)

$\bar{r}$ = average annual conception rate corresponding to age t (fraction).

For situations where adequate data are not available, Appendix D describes a procedure for approximate estimation using a minimum data set.

Observed herd fertility corresponds to a particular breeding regime, ranging from controlled seasonal breeding to breeding year-round. The specification of the breeding regime in the model is described in Section 3.4.4. It is assumed here that, other things being equal, a cow is equally likely to conceive in any month of the breeding season. This assumption implies the following relationship between the expected probability of conception in a single month of the breeding season and the annual conception rate:

$$\bar{P}_t + (1 - \bar{P}_t)\,\bar{P}_t + \ldots (1 - \bar{P}_t)^{L-1}\,\bar{P}_t = \bar{r}_t \qquad (3.17)$$

where: $\bar{P}_t$ = the expected probability of conception during a given month of the breeding season of a cow of age t

L = the length of the breeding season in months.

Given $\bar{r}_t$ (annual conception rate) and L, equation (3.17) can be solved for $\bar{P}_t$ using, for example, Newton's approximation method.

The actual probability of conception during a given month of the breeding season is also a function of the liveweight condition of the animal (c) and the number of months post-partum (n). As data on the effects of these two factors are usually limited, the simplest formulation of their effects is considered here, subject to validation for particular systems. Their respective effects are incorporated into the model in a multiplicative fashion as shown in Figures 3.11 and 3.12. The condition multiplier (m_c) implies a reduction in the probability of conception to 60% of its expected value when the animal has lost considerable weight ($c = 0$). Conception is more favourable than normal for animals at higher than average weights, with a maximum probability of conception at $c = 0.90$. For over-

fat cows, with a condition index greater than 0.90, the probability of conception is assumed to be slightly depressed. Analytically, the assumed impact of liveweight on the probability of conception is:

$$m_c = 0.60 + 1.5\,c \qquad \text{for } c \leq 0.20$$

$$m_c = 0.833 + 0.33\,c \qquad \text{for } 0.20 < c \leq 0.90$$

$$m_c = 3.233 - 2.333\,c \qquad \text{for } c > 0.90.$$

The post-partum period multiplier (m_n) implies a complete prohibition of conception during the 1st month post-partum and no effect thereafter. Analytically, this multiplier is defined as:

$$m_n = 0.0, \qquad \text{for } n < 2$$

$$m_n = 1.0 \qquad \text{for } n \geq 2.$$

In summary, the probability of conception for an animal of age t, of condition c, at the n'th month post-partum is given by the relationship:

$$P_t = \bar{P}_t . m_c . m_n \tag{3.18}$$

where: $\bar{P}_t$ is derived from the solution of equation (3.17), given $\bar{r}$ and L.

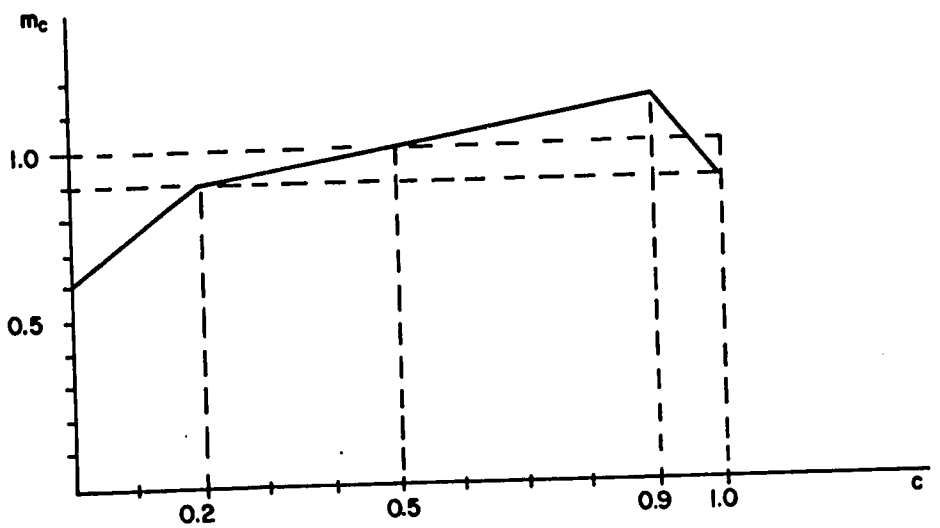


Figure 3.11 Assumed multiplicative effect (m_c) of liveweight condition index (c) on the probability of conception

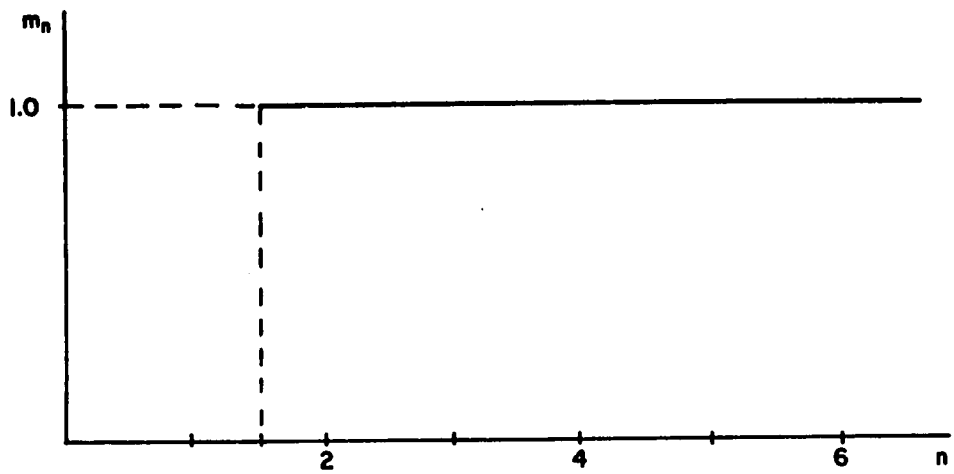


Figure 3.12 Assumed multiplicative effect (m_n) of the period in months post-partum (n) on the probability of conception

3.3.3.3 Births. Conception in the model leads to a birth after a 9-month gestation unless the cow dies during this period. The sex of the newborn calf is determined probabilistically, with equal probability for either sex. If the calf is a male, no other attribute is determined at the time of its birth. However, for female calves three attributes are determined probabilistically at birth, based on the *a priori* probabilities of their occurrence in the simulated herd due to chance or genetic inheritance. These are conformation, fertility and milk let-down capacity.

From experience, and according to his preference and standards, a herd manager will judge a female calf as having an acceptable or unacceptable conformation for retention as a breeding cow. The probability of a female calf actually being born with bad conformation is provided as data. A random number between 0.0 and 1.0 is drawn for each female calf born; if the number drawn is less than the bad conformation probability, the calf is identified as having bad conformation and is designated to be culled according to the particular policy followed for the herding system under study.

Some fraction, usually small, of a population of females of any particular breed will never conceive because of a variety of disorders. This intrinsic infertility rate is one of the determinants of the overall fertility of a herd; its importance relates directly to the cost of maintaining an unproductive female until the herd manager can detect her infertility. While there are diseases which might result in infertility of previously fertile cows, it is assumed that such occurrences are of minor importance and accounted for in the specification of overall fertility rates. Hence the concern here is only infertility relating to probabilistic intrinsic factors.

In the simulated herd this intrinsic infertility is determined at the time of a female's birth. The fraction of barren heifers out of the total number of heifers that reach reproductive age can usually be established from records made over a period of years. When a female calf is born, a random number between 0.0 and 1.0 is drawn from a uniform distribution. If this number is less than the given fraction of barren females, then that female is identified as barren. Until the point when they would be identified by management, all barren females are treated as any other females, subject to the same management policies. Barren females are disposed from the herd only when a culling policy is applied in regard to reproductive performance; if such a policy is absent then a barren cow will stay in the simulated herd for an extended period until it dies or is sold for some other reason.

The capacity of a lactating cow to let down milk for human offtake relates to its breed type, and within a breed to the particular psychosynthesis of individual dams. In a female calf this particular attribute can be considered inherited from its dam or attributed totally to chance. In either case, it is assumed that the fraction of cows that let down milk is known for the herd under study, as well as the portion of the milk yield that they let down. Whether a newborn female will be able to let down milk during her reproductive life is again determined probabilistically by drawing from a uniform probability distribution between 0.0 and 1.0 and comparing the number drawn with the specified probability that a female in the herd will be able to let down milk. If the number drawn is greater than this value then milk can be taken in the absence of a suckling calf. The maximum fraction of milk which can be taken in the absence of a calf is provided as data to the model.

3.3.4 Mortality

This cattle model focuses on the dynamics of herd-level productivity in response to different nutritional regimes and management practices. Because the principal source of variability in the model is in the quantity and quality of forage on offer, in any run of the model the simulated nutritional status of animals could vary considerably, and some losses could occur in the simulated herd due to nutritional stress. Thus, the model is structured so that deaths can occur due directly to nutritional stress if harsh production environments are being simulated. Additionally, the model allows for 'normal' losses caused by a complex set of factors not directly related to nutritional status.

As detailed in Section 3.3.2.1, the liveweight of any animal in the simulated herd is bounded from below according to its age and sex. The lower boundary corresponds to the lowest observed liveweights of animal alive in the herd. If an animal loses sufficient liveweight or does not gain liveweight as it should in accordance with its increasing age, its age/liveweight combination may fall below this lower boundary, in which case death occurs with certainty due to starvation.

In addition, in each month of the simulation each animal has a chance of death independent of its nutritional status. In this way the model accounts for the wide range of causes of death in real production systems. The specification of this

source of mortality is based on observed annual 'normal' mortality rates for each age class in the absence of nutritional stress. Annual mortality rates are different for different animal classes. In some systems, a high mortality rate is observed for calves to 1 year of age. After the animals pass their early growth stage, their chances of survival increase rapidly. The lowest death rates are typically observed for animals from about 4 to 9 years. After that, problems associated with old age appear which are manifested in progressively increasing death rates.

These general observations suggest that to simulate mortality with a reasonable degree of accuracy, age-related differences in mortality rates must be taken into account. This is done in the model by specifying two age groups, animals younger and older than 1 year. Within the first group calves up to 3 months are treated separately. Thus, if S_1 is the survival rate to 3 months and S_2 the survival rate to 1 year, then the monthly probability of death to 3 months (P_1) and the monthly probability of death from 4 months to 1 year can be calculated as:

$$P_1 = 1 - \sqrt[3]{S_1} \tag{3.19}$$

$$P_2 = 1 - \sqrt[9]{S_2/S_1} \tag{3.20}$$

The monthly mortality rates obtained from (3.19) and (3.20) are the test values in binomial trials where a random number between 0.0 and 1.0 is drawn for each animal from a uniform distribution. If the number drawn is less than the test value corresponding to the animal's age, then death occurs and the animal is removed from the herd.

Annual mortality rates for animals over 1 year old are disaggregated by age. In cases where age-specific information is not available, Appendix E provides a procedure for estimating age-related mortality rates from a minimum of data. Annual mortality rates are used to calculate monthly rates, such that when compounded over 12 months they give the annual rates. As in the case of calves, these monthly mortality rates are the test values in binomial trials. For example, if the annual mortality rate for a given age is 5% (i.e. survival probability of 95% over a year), then the appropriate test value in the binomial trial is .0043 (where $(1 - .0043)^{12} = 0.95$). A random number between 0.0 and 1.0 is drawn and if it is less than the test value death occurs.

3.4 HERD MANAGEMENT POLICY OPTIONS

3.4.1 Supplementary feeding

Costs are incurred directly or indirectly when livestock are fed with supplements. Purchased feeds incur direct costs. Farm-produced supplements may be available, but in the context of the productivity of the whole farm it might be more advantageous to use them in another way, rather than feeding them to the cattle herd. In both cases, the type, quality and quantity of feed supplements available for livestock are limited. Thus, the level of supplementation, its allocation between different classes of stock and its seasonal distribution are important economic questions.

As indicated in Section 3.3.1, two distinct kinds of supplementary feeds are considered: concentrates and improved quality forages. Additionally, it is assumed that supplements are only given when the digestibility of forage on offer is below a certain specified threshold level. The maximum quantities of supplements available per head are also specified by management.

Supplements are usually provided only to particular classes of animals in the herd. Animals with higher energy needs, higher production potential or those considered essential to the long-term viability of the herd are commonly given preference. In the model a limited number of different animal classes can be supplemented. These are:

1. female calves prior to weaning when the quantity of milk available from their dams is below a certain level (X_1)
2. male calves prior to weaning when the quantity of milk available from their dams is below a certain level (X_2)
3. female calves for a specified period in months (X_3) after weaning
4. male calve for a specified period in months (X_4) after weaning
5. lactating cows for a specified period in months (X_5) after calving
6. pregnant cows for a specified period in months (X_6) before calving
7. non-pregnant cows for a specified period in months (X_7) before the beginning of the breeding season. This class only applies in a controlled breeding situation (see Section 3.4.4).
8. steers for a specified period in months (X_8) before the planned sale date
9. all animals whose weight drops a certain percentage (X_9) below their expected weight.

In addition to these specifications for particular animal classes, threshold levels of forage digestibility (d^*_{fi}) must be specified for each animal class below which supplementation takes place, as well as the quantities of concentrates (I^a_{ci}) and/or improved forage (I^a_{si}) available to each class and their respective digestibilities (d_{ci}) and (d_{si}).

An example of a supplementation policy is given in Table 3.5, with three classes of animals specified for supplementation. Female calves (class 1) are supplemented prior to weaning with a maximum of 0.75 kg dry matter/day of concentrates of 83% digestibility if less than 2 kg/day of milk are available to them and if the digestibility of the forage on offer is below 60%. Lactating cows (class 5) are supplomented for the first 4 months after calving with a maximum of 2.5 kg dry matter/day of improved forage of 65% digestibility if the digestibility of the forage on offer is below 50%. Steers (class 8) are supplemented for 3 months before sale with a maximum of 2.0 kg dry matter/day of improved forage of 65% digestibility, if the digestibility of the forage on offer drops below 45%. An appropriately high threshold level for any class will cause the animals in that class to be supplemented independently of the digestibility of the available forage, if so desired.

Table 3.5 Example specification of supplementation options[a]

Class to be Supplemented (i)	X_i	d^*_{fi}	I^a_{ci}	d_{ci}	I^a_{si}	d_{si}
1	2	0.60	0.75	0.83	-	-
5	4	0.50	-	-	2.5	0.65
8	3	0.45	-	-	2.0	0.65

[a] See text for variable definitions.

3.4.2 Milk offtake

Some cattle breeds require stimulation by their calves before they will let down milk for human offtake. In these cases, only cows with a calf at foot can be milked, and lactation will cease when the calf is weaned. Other breeds can be milked with or without a calf, with no difference in total daily yield. The capacity of a breed to let down milk without a calf is incorporated in the model with the specification of the milk let down fraction, as described in Section 3.3.3.3. After calves are weaned, only this fraction of the total potential milk yield can be removed for human consumption.

Milk offtake for human consumption reduces the energy available to the suckling calf. It can also make additional energy demands on the lactating cow if milking together with suckling results in more milk taken than by the calf alone. A rational manager milks cows selectively, taking into account the condition of both cows and calves. Thus, lactating cows are grouped into a limited number of classes for modelling the milking process. A milking policy is based on a set of rules provided as data to a simulation run, comprising any combination of the following options:

1. A maximum percentage (X_1) of a cow's daily milk yield can be taken from cows with a female calf at foot.

2. A maximum percentage (X_2) of a cow's daily milk yield can be taken from cows with a male calf at foot.

3. For female calves, no milk offtake is allowed if the weight of the calf drops below a certain percentage (X_3) of its expected weight ($\overline{W}_t$ in Section 3.3.2.1).

4. For male calves, no milk offtake is allowed if the weight of the calf drops below a certain percentage (X_4) of its expected weight ($\overline{W}_t$ in Section 3.3.2.1).

5. A maximum percentage (X_5) of the cow's daily milk yield can be taken from cows in their first lactation.

6. No milk offtake is allowed if the total daily yield is below a certain level (X_6).

7. No milk offtake is allowed if the weight of the lactating cow drops below a certain percentage (X_7) of its expected weight ($\overline{W}_t$ in Section 3.3.2.1).

8. No milk offtake is allowed from a cow more than a specified number of months (X_8) pregnant.

A milking policy specified, for example, with X_1 = 40%, X_2 = 60% and X_3 = 80% implies a maximum milk offtake of 40% for a cow with a female calf and 60% for a cow with a male calf. Additionally, milk offtake from cows with female calves will cease whenever calf weights drop below 80% of expected levels.

3.4.3 Weaning

The weaning component of the model takes into consideration explicit policies specified by management (controlled weaning) and particular circumstances when the dams cannot provide sufficient milk for their calves (enforced weaning). Controlled weaning can be specified in two ways. All suckling calves may be weaned in a particular calendar month regardless of their age or liveweight. Such an option is only relevant in a controlled breeding situation when the calves are all approximately the same age. Otherwise, calves may be weaned individually when they reach prespecified ages and liveweights.

Enforced weaning occurs before the appropriate calf age and liveweight levels have been reached if the dam dies, if her milk production ceases, or if she reaches the 8th month of her next pregnancy. These checks are made before the controlled weaning algorithm is executed.

For both controlled and enforced weaning the linkages between the dam and her calf are broken. Thereafter in the simulation, the calf is treated as a separate animal.

3.4.4 Breeding

The breeding policy used in the model includes control over the breeding season and the condition and age of the animals to be bred. The breeding season is specified in terms of the calendar months each year when conception can take place. In this way, two or more separate breeding seasons could be specified. If no particular breeding season is specified, the model assumes year-round breeding. Within any month of the breeding season, the liveweight and age of each individual female must exceed prespecified limits. If such limits are not specified, then every mature female has access to breeding regardless of liveweight and age.

3.4.5 Purchases

Herds are often built up in ranch development schemes by purchasing heifers as breeding stock. Because these purchases are usually financed by loans, they are scheduled in advance over a period of years, with the type of animal purchased and the time of purchase each year detailed in the ranch development programme. Because the elapsed time between the acquisition of breeding stock and the sale of their progeny is typically at least 3 or 4 years, the ranches will, overall, have negative cash flows during this period. For this reason, short-term steer-raising enterprises are often included in the first years of a ranch development programme to provide a high rate of capital turnover and a cash surplus needed to finance the negative cash flow of the breeding enterprise. So that the model can be used in ranch development schemes, it permits detailed specification of both heifer and steer purchases.

It is recognized that opportunistic purchases of stock can and do take place. However, such purchases depend upon many factors, including the manager's perception of risk, the availability of cash and the relative and absolute prices of stock and forage, so they are considered beyond the scope of the current model.

In this model, stock purchases can be made only during the first 5 years of a run. Heifers and steers purchases are specified as follows:

1. Heifers can be bought during a given calendar month (d_1) at a certain age (t_1) and weight (W_1), and in numbers N_{11}, $N_{12}, \ldots,$ N_{15} for the first 5 years of operation respectively.

2. Steers can be bought during a given calendar month (d_2) of a certain age (t_2) and weight (W_2), and in numbers N_{21}, $N_{22}, \ldots,$ N_{25} for the first 5 years of operation respectively.

An example of a buying policy is given in Table 3.6. In this example, 46 heifers are bought in the first 5 years, all during the 5th calendar month (May), at an age of 36 months and weighing 240 kg: 15 are bought in year 1, 12 in year 2, 8 in year 3, 6 in year 4, and 5 in year 5. Thirty-five steers are bought in July (month 7), at an age of 24 months and weighing 200 kg: 20 are bought in year 1, 10 in year 2, 5 in year 3 and none in years 4 or 5.

Purchased animals are treated exactly the same way as other animals in the simulated herd; they are subject to the same breeding, supplementary feeding and selling policies.

Table 3.6 Example specification of a buying policy

Class of Animals Acquired	Calendar Month of Acquisition	Age of Acquired Animals (months)	Liveweight of Acquired Animals (kg)	Number Acquired During First 5 Years of Run				
				1	2	3	4	5
Heifers	5	36	240	15	12	8	6	5
Steers	7	24	220	20	10	5	-	-

3.4.6 Sales

As with buying policy, only prespecified selling rules are included in the model; opportunistic sales are not modelled. Animals are sold when they are no longer considered productive, when they reach planned market weights or ages or if the total female herd exceeds the target size, giving a marketable surplus of heifers.

Sales of individual animals depend on liveweight and age conditions which have been specified for different classes of stock. These conditions are entered as data and apply throughout a run:

1. Males after they reach a certain age (t_1) and weight (W_1) are sold in a specified calendar month (d_1).
2. Females above a certain weight (W_2) which do not calve by a certain age (t_2) are sold in a specified calendar month (d_2).
3. Females with poor conformation are sold in a specified calendar month (d_3) after reaching a certain age (t_3) and weight (W_3).
4. Cows which have not conceived for an extended period (t_4) since their last calving are sold in a specified calendar month (d_4).
5. Cows which are neither lactating nor pregnant and have reached old age (t_5) are sold in a specified calendar month (d_5).

An example of a sales policy is given in Table 3.7. Males, barren females and females with poor conformation are sold during the 5th calendar month (May) when they reach 42, 60 and 42 months of age and weigh at least 350, 270 and 250 kg respectively. Cows are sold during the 9th calendar month (September) in either of two cases: if they have not conceived for at least 30 months after their last calving, or when they reach old age, specified in this example as 120 months.

Table 3.7 Example specification of a selling policy for individual animals in different classes

Animal Class (i)[a]	Calendar Month of Sale	Critical Age in Months (t_i)	Critical Weight in Kg (W_i)
1	5	42	350
2	5	60	270
3	5	42	250
4	9	30	-
5	9	120	-

[a]As specified in text.

A second set of sale policy options available in the model specifies the conditions under which excess heifers are sold. Based on resource carrying capacity or other considerations, a target size of the breeding herd may be specified. The breeding herd includes all females which have reached reproductive age and have had at least one exposure to breeding. If this target size is exceeded, then the excess heifers are sold.

3.4.7 Drought policy

In systems where the quantity and quality of forage on offer vary widely from year to year, a sequence of years of poor forage may occur, leading to a drought situation. The effect on the herd of such a situation will depend on many factors, including the prior condition of the individual animals, the age and sex composition of the herd and the management response, which could include strategic emergency sales of animals and/or supplementary feeding.

In the context of the model, a situation is defined as a drought when the average liveweight condition index of the herd at the beginning of a simulated month is below a certain threshold value and the digestibility of the forage on offer during that month is not sufficient for maintenance of the reference animal. The threshold condition index for the herd used in the model is 0.10.

In such a drought situation, the model assumes that all animals other than those in the breeding herd are sold. Calves at foot are weaned and sold and all lactating cows are dried off. All remaining animals in the breeding herd receive sufficient supplementation to maintain liveweight, with allowance for pregnancy requirements where appropriate. The quality of the drought supplement is provided as data to the model. The occurrence and duration of drought situations and the total quantity of strategic supplements provided are reported at the end of each run.

4. THE SIMULATION PROCESS

Any computer simulation model, however general, is restricted by its design to specific classes of problems. To use a model to help solve real-life problems, they must be formulated to fit into the set of problems accounted for by the existing model. If this is not possible, the model will have to be altered, the problem at hand restated, or perhaps even a new model constructed.

The first and most difficult stage in the simulation process is to construct a clear definition of a problem and to formulate explicit objectives of the simulation experiments to give answers to the problem. These objectives usually take the form of hypotheses to be tested or effects to be estimated. The second stage is to determine whether the specific problem to be solved falls within the set of problems accounted for by an existing model. Application of an existing model may be straightforward, but, more usually, it would require some revision of the problem definition or the model or both.

If an existing model is judged appropriate for the type of problem at hand, the third stage in the simulation process is the derivation of data on the particular production system under study to validate and run the model. Every simulation model requires a certain amount of data of adequate quality to operate. If the data requirements of a particular model cannot be met, then a less demanding model may be identified or a different approach to the research problem selected. Validation of the model for the particular system under study is the fourth-stage in the simulation process. At this point specific parameters in the computer model may need to be altered to replicate the processes of the real system. The fifth and last stage in the simulation process is experimentation with the model, leading to the testing of stated hypotheses and the derivation of measures of the effects to be estimated.

Experimentation with a herd simulation model consists of a quantitative description of the productivity of the livestock system under study, evolving over a finite time horizon. Figure 4.1 is a schematic representation of the livestock production process in an input/output framework. The production process starts at the beginning of the simulated period with an initial herd (specified as given in

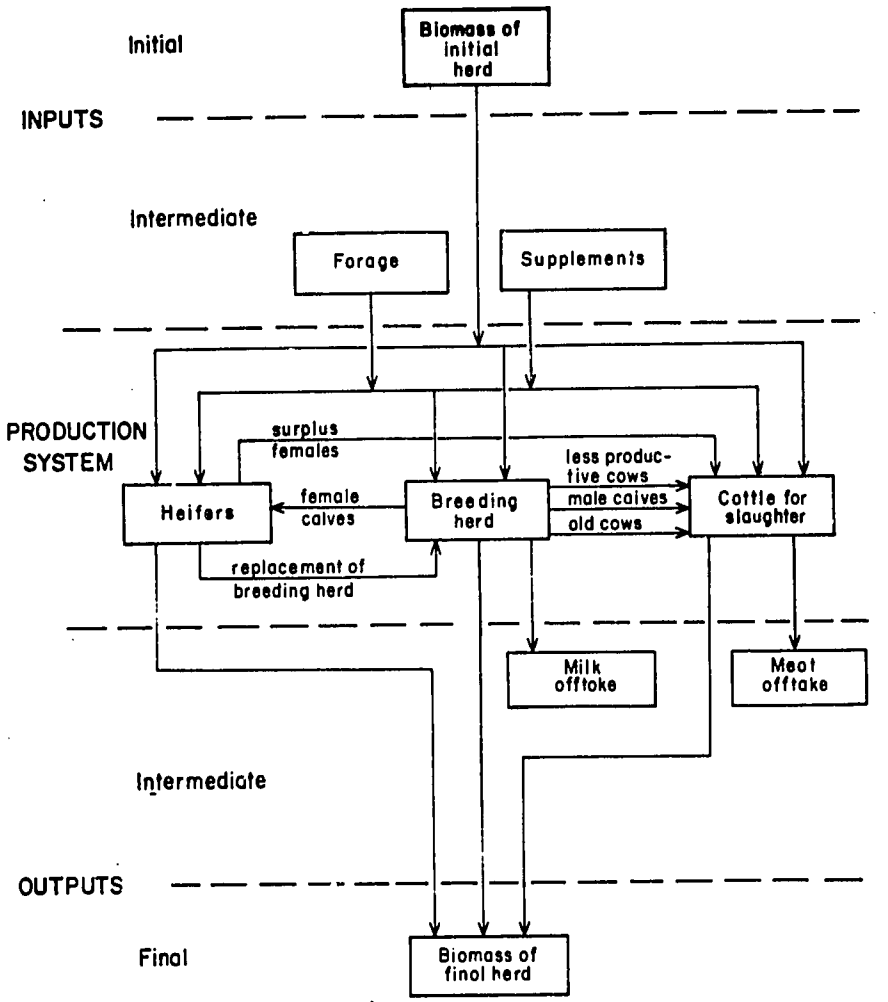


Figure 4.1 An input/output configuration of a livestock production system.

the example of Appendix Table F6) which evolves over simulated time into the final herd at the end of the run. The intermediate inputs and outputs during this period are the feeds of various types consumed and the milk and meat produced.

The production process specified in this model is stochastic. Starting with the same initial conditions and management regime, an infinite number of different outcomes may result over a finite simulated time period. The variety of possible outcomes occurs because of the variability in the forage regime from year to year and in the animal-level processes of conception, sex of calves and mortality. Alternative production strategies can be compared and evaluated by generating a sufficient number of realizations from each strategy, or replications, to permit the calculation of statistically significant probability distributions for each of the critical variables. Less than 20 replications are usually adequate to identify significant differences between alternative production strategies. A larger number of replications results in progressively higher computational requirements.

This model has been applied successfully in the comparison and evaluation of Tswana and Simmental x Tswana cows in Botswana as milk producers under alternative milking and supplementation regimes (Konandreas et al, 1981). Other applications to traditional herding situations in different environments of Nigeria and Mali have also been initiated.

The computer model is written in standard FORTRAN language and is operational on a Hewlett-Packard 3000 series III mini-computer. The model is highly modularized so that it is readily transferable to other systems. Changes in the FORTRAN code to account for peculiarities in particular production systems can be done without major changes in the programme's overall structure. The data required to drive the model are entered interactively. Consistency checks are made on much of the data as they are entered. A complete description of the structure of the computer model, the input data interactive dialogue, and the output of the model as currently formulated is available on request.

APPENDIX A: A PROCEDURE TO ESTIMATE LIVEWEIGHT GROWTH CURVES

Sufficient data are not always available to estimate statistically the average liveweight evolution from birth to maturity of animals in a herd. A procedure is described here to allow estimation of liveweight evolution from a minimum of data.

In general, assume that the average liveweight at age t for each sex is expressed by the functional form:

$$\overline{W}_t = f(t) \qquad \text{(A.1)}$$

where: t = age (months).

This relationship is presented graphically in Figure A.1. Point B on the curve is the inflection point, corresponding to that point in the animal's development when maximum liveweight gains occur. Before the liveweight curve can be estimated, however, its mathematical properties must be formally established.

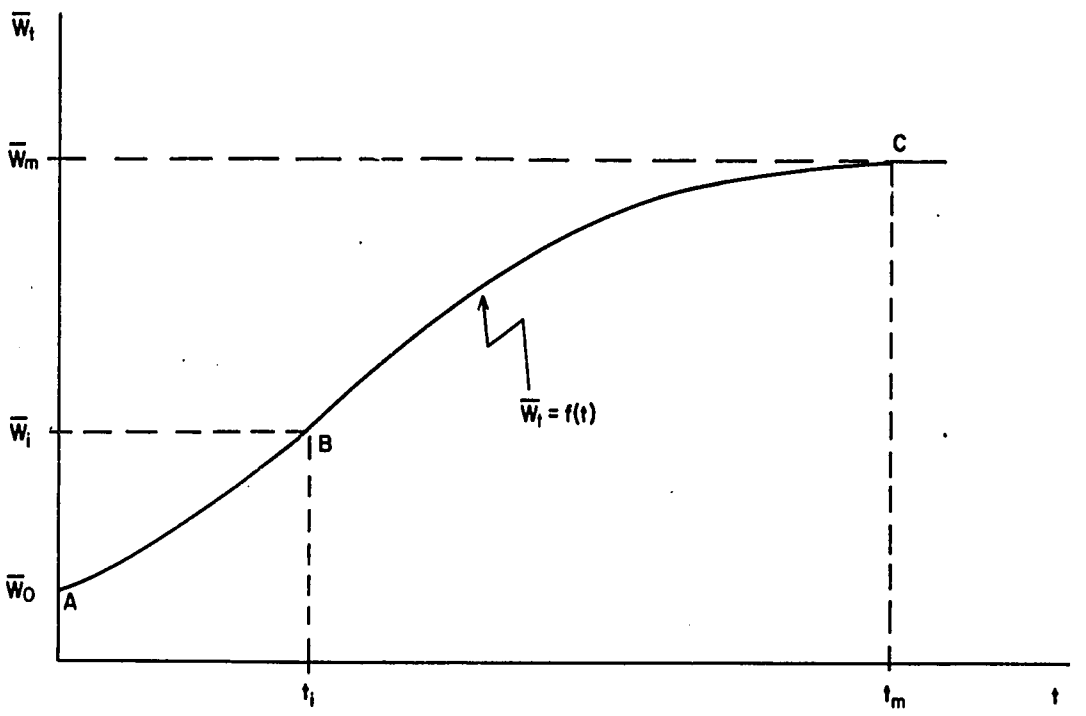


Figure A.1 Average liveweight ($\overline{W}_t$) growth curve

Previous Page Blank

Define:

$\bar{W}_o$ = average liveweight at birth (kg)

t_i = age (months) of maximum liveweight gains, i.e. age at the point of inflection

$\bar{W}_i$ = average liveweight (kg) at the time of maximum weight gain, i.e. at age t_i

t_m = age at maturity (months)

$\bar{W}_m$ = average liveweight at maturity (kg).

A liveweight evolution curve must have the following properties:

1. The curve must pass through point $(\bar{W}_o, 0)$, corresponding to average liveweight at birth.

2. The curve must increase monotonically:

$$d\bar{W}_t/dt \geq 0 \qquad \text{for all } t$$

3. The curve must be concave from below up to the point of inflection:

$$d^2\bar{W}_t/dt^2 \geq 0 \qquad \text{for } t \leq t_i.$$

4. The curve must be convex from below after the point of inflection:

$$d^2\bar{W}_t/dt^2 \leq 0 \qquad \text{for } t \geq t_i.$$

5. The slope of the curve at the age of maturity must equal zero:

$$\bar{W}_t = \bar{W}_m \text{ and } d\bar{W}_t/dt = 0 \qquad \text{for } t = t_m.$$

6. A curve satisfying conditions 1 to 5 can be approximated by two curvilinear segments (AB) and (BC) with the same slope at point B:

$$\bar{W}_t^{(AB)} = \bar{W}_t^{(BC)} = \bar{W}_i \text{ and } d\bar{W}_t^{(AB)}/dt = d\bar{W}_t^{(BC)}/dt \text{ for } t = t_i.$$

Assume that quadratic functions give adequate representations of these two segments. Then the functional forms of segments (AB) and (BC) are:

$$\bar{W}_t = a_1 + b_1 t + c_1 t^2 \quad \text{for segment (AB), where } t \leq t_i$$

$$\bar{W}_t = a_2 + b_2 t + c_2 t^2 \quad \text{for segment (BC), where } t \geq t_i.$$

Application of properties 1, 5 and 6 yields the following equations:

$$a_1 = \bar{W}_o$$

$$a_2 + t_m \; b_2 + t_m^2 \; c_2 = \bar{W}_m$$

$$b_2 + 2t_m \; c_2 = 0$$

$$a_1 + t_i \; b_1 + t_i^2 \; c_1 = \bar{W}_i$$

$$a_2 + t_i \; b_2 + t_i^2 \; c_2 = \bar{W}_i$$

$$b_1 + 2t_i \; c_1 - b_2 - 2t_i \; c_2 = 0$$

These can be solved for a_1, b_1, c_1 and a_2, b_2, c_2, yielding equations (A.2), (A.3) and (A.4) as follows:

$$\bar{W}_t = \bar{W}_o + 2 \frac{t_m (\bar{W}_i - \bar{W}_o) - t_i (\bar{W}_m - \bar{W}_o)}{t_i (t_m - t_i)} t$$

$$+ \frac{t_i (\bar{W}_m - \bar{W}_i) + t_i (\bar{W}_m - \bar{W}_o) - t_m (\bar{W}_t - \bar{W}_o)}{t_i^2 (t_m - t_i)} t^2$$

$$\text{for } t \leq t_i \qquad \text{(A.2)}$$

$$\bar{W}_t = \frac{t_i^2 \bar{W}_m + t_m^2 \bar{W}_i - 2 t_m t_i \bar{W}_m}{(t_m - t_i)^2}$$

$$+ \frac{2t_m (\bar{W}_m - \bar{W}_i)}{(t_m - t_i)^2} t - \frac{(\bar{W}_m - \bar{W}_i)}{(t_m - t_i)^2} t^2$$

$$\text{for } t_i < t \leq t_m \qquad \text{(A.3)}$$

$$\bar{W}_t = \bar{W}_m \qquad \text{for } t > t_m. \qquad \text{(A.4)}$$

The requirement of a convex curve after the point of inflection (property 4) is always met, as shown by taking the second derivative of equation (A.3). Additionally, for the curve to be concave from below up to the point of inflection and monotonically increasing (properties 3 and 2), the following relationship must

also hold true:

$$\frac{\bar{W}_m - \bar{W}_o}{\bar{W}_i - \bar{W}_o} \leq \frac{t_m}{t_i} \leq \frac{\bar{W}_m - \bar{W}_o}{\bar{W}_i - \bar{W}_o} + \frac{\bar{W}_m - \bar{W}_i}{\bar{W}_i - \bar{W}_o} \qquad \text{(A.5)}$$

In equation A.5 the left side of the inequality guarantees a monotonically increasing function, while the right side guarantees concavity up to the point of inflection. Equality on the right side implies a curve degenerated to a straight line up to the point of inflection. If the inequality on the right side is reversed, then the liveweight growth curve is convex from below throughout.

Preliminary applications of known situations failed to pass the test for concavity. In most cases, growth up to the point of inflection can be approximated by a straight line and in some cases a convex growth curve results. Thus, compelled by evidence from field situations, the usually accepted concave property of the liveweight growth curve up to the point of inflection is not enforced in the model.

As an example, consider females in a particular system which have an average weight at birth of 25 kg ($\bar{W}_o$ = 25) and reach their maximum rate of weight gain at 18 months (t_i = 18), weighing at this age on average 200 kg ($\bar{W}_i$ = 200). They mature at 4.5 years (t_m = 54) with an average mature weight of 350 kg ($\bar{W}_m$ = 350). Substitution of these values into the inequalities (A.5) satisfies the left side, implying a monotonically increasing function, but not the right side, implying a convex curve throughout. The estimated equations (A.2), (A.3) and (A.4) are:

$$\bar{W}_t = 25 + 11.1\ t - 0.0772\ t^2 \qquad \text{for } t \leq 18$$

$$\bar{W}_t = 12.50 + 12.50\ t - 0.1157\ t^2 \qquad \text{for } 18 < t \leq 54$$

$$\bar{W}_t = 350 \qquad \text{for } t > 54.$$

APPENDIX B: A PROCEDURE TO ESTIMATE THE AGE EFFECT ON POTENTIAL MILK YIELD

In general, a breed's milk yield potential is highest for cows of middle age, and is lower for younger and older cows. Milk yield potential at age t expressed as a fraction of the maximum is denoted by:

$$y_t = y(t) \qquad \text{(B.1)}$$

where: t = age (years).

This function can be approximated by two curvilinear segments (AB) and (CD) tangent to the horizontal line at points B and C, as shown in Figure B.1 where points B and C correspond to the age range of maximum milk yield potential.

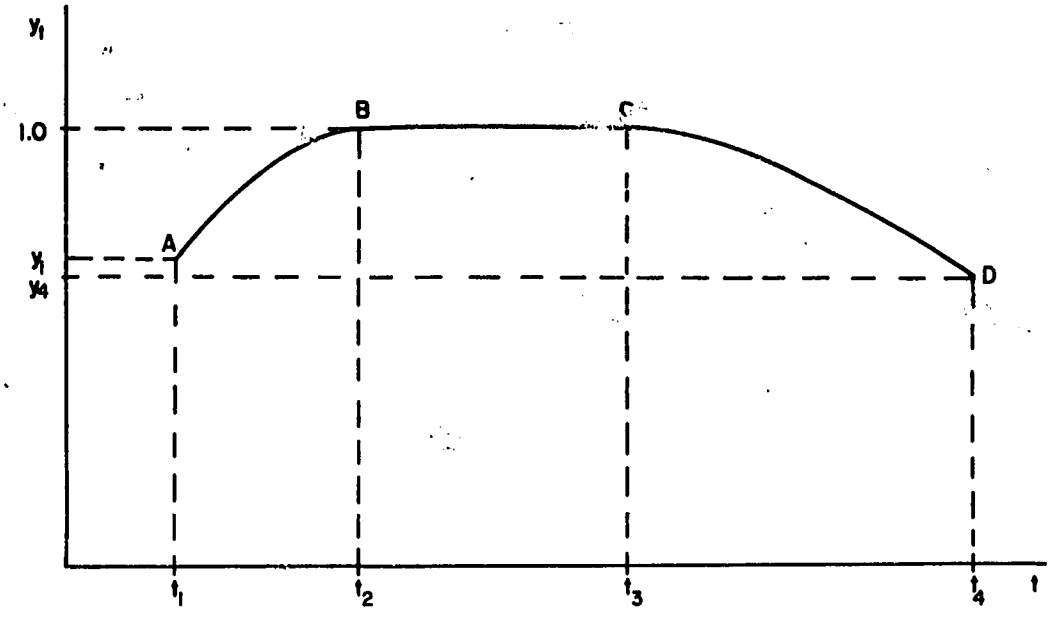


Figure B.1 Relative potential milk yields (Y_t) of cows of different ages (t)

The minimum data necessary to estimate segments (AB) and (CD) of the curve as postulated here are: the fraction of potential milk yield (y_1) realized at a young age (t_1), the fraction (y_4) realized at an old age (t_4) and the intermediate ages (t_2) and (t_3) when milk yield is at its maximum potential. Assume further that quadratic functions can be used to approximate these two segments. The equations for the two quadratic functions are:

$$y_t = a_1 + b_1 t + c_1 t^2 \qquad \text{for segment (AB) where } t_1 \leq t \leq t_2$$

$$y_t = a_2 + b_2 t + c_2 t^2 \qquad \text{for segment (CD) where } t_3 \leq t \leq t_4 .$$

Based on these assumptions concerning the shape of the milk yield potential curve, coefficients a_1, b_1, c_1, a_2, b_2 and c_2 can be derived analytically, yielding the following functional forms:

$$y_t = \frac{t_1 (t_1 - 2t_2) + t_2^2 \; y_1}{(t_2 - t_1)^2} + \frac{2t_2 \; (1-y_1)}{(t_2 - t_1)^2} \; t$$

$$+ \frac{(y_1 - 1)}{(t_2 - t_1)^2} \; t^2 \qquad \text{for } t_1 \leq t \leq t_2 \qquad \text{(B.2)}$$

$$y_t = 1 \qquad \text{for } t_2 < t \leq t_3 \qquad \text{(B.3)}$$

$$y_t = \frac{t_4 \; (t_4 - 2t_3) + t_3^2 \; y_4}{(t_4 - t_3)^2} + \frac{2t_3 \; (1-y_4)}{(t_4 - t_3)^2} \; t$$

$$+ \frac{(y_4 - 1)}{(t_4 - t_3)^2} \; t^2 \qquad \text{for } t_3 < t \leq t_4 . \qquad \text{(B.4)}$$

For lactating cows younger than t_1 and older than t_4, values y_1 and y_4 are assumed to apply respectively.

As an example, consider the case when potential milk yield is highest for 6- to 8-year-old cows ($t_2 = 6$ and $t_3 = 8$). At 3 years, potential milk yield is 80% of the highest potential yield ($t_1 = 3$ and $y_1 = 0.80$), and at 13 years it is 60% of the highest potential yield ($t_4 = 13$ and $y_4 = 0.60$). Substituting these values into formulae (B.2) and (B.4) yields:

$$y_t = 0.2 + 0.267\, t - 0.022\, t^2 \qquad \text{for } 3 \leq t \leq 6$$

$$y_t = 1.0 \qquad \text{for } 6 < t \leq 8$$

$$y_t = -0.024 + 0.256\,t - 0.016\,t^2 \qquad \text{for } 8 < t \leq 13.$$

APPENDIX C: A PROCEDURE TO ESTIMATE THE LACTATION CURVE

It is assumed in the model that the average monthly milk yield is constant over the first 2 months of lactation and declines linearly thereafter. This pattern is depicted in Figure C.1.

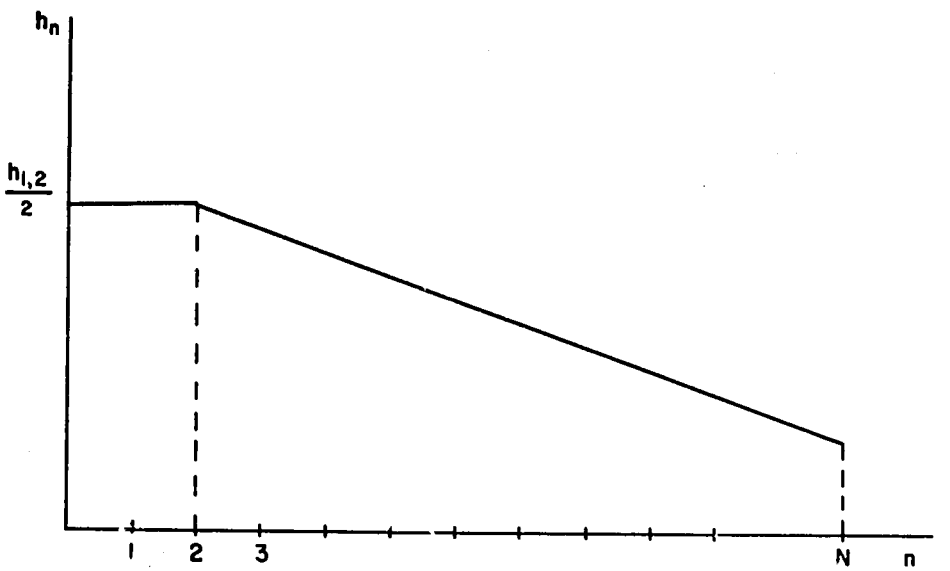


Figure C.1 Distribution of milk yield (h_n) over the lactation period (n)

Lactation yield for the n'th month post-partum can be expressed as a fraction of the potential yield for the whole lactation:

$$h_n = h(n) \qquad \text{(C.1)}$$

Previous Page Blank

such that:

$$h_n = h_{1,2}/2 \qquad \text{for } n = 1,2 \qquad (C.2)$$

$$h_n = a + bn \qquad \text{for } n \geq 2 \qquad (C.3)$$

where: h_n = fraction of milk produced during the n'th month post-partum

$h_{1,2}$ = fraction of milk produced during the first 2 months post-partum.

For a given breed and production system under a normal feeding regime, the fraction of milk yield produced during the first 2 months is approximately constant (a typical value of this fraction is about 0.35). Assuming that this fraction and the length of lactation (N) are known, then it is possible to estimate a simplified specification of a lactation curve comprised of two linear segments as in Figure C.1.

Equation (C.3) applied for n = 2 gives the known fraction of milk yield during the 2nd month of lactation:

$$a+2b = h_2 = h_{1,2}/2 \qquad (C.4)$$

In addition, the sum of h_n's for all months of the lactation period equals unity:

$$\sum_{n=1}^{N} h_n = h_{1,2} + \sum_{n=3}^{N} (a+bn) = 1$$

which yields: $a(N-2) + b\,\dfrac{N(N+1) - 6}{2} = 1 - h_{1,2}$. (C.5)

Equations (C.4) and (C.5) solved for the unknowns a and b yield the lactation curve for $n \geq 2$:

$$h_n = \frac{(N^2 + N + 2)\,h_{1,2} - 8}{2(N^2 - 3N + 2)} - \frac{Nh_{1,2} - 2}{N^2 - 3N + 2}\,n. \qquad (C.6)$$

For h_n to be positive throughout and monotonically decreasing, the following relationship between $h_{1,2}$ and N must also hold:

$$2/N \leq h_{1,2} \leq (4N - 8)/(N^2 - N - 2). \qquad (C.7)$$

For example, consider a lactation lasting 9 months (N = 9) and a yield for the first 2 months post-partum equal to 35% of the total lactation ($h_{1,2} = 0.35$). Relationship (C.7) holds true and the lactation curve in this case is:

$$h_n = \frac{h_{1,2}}{2} = 0.175 \qquad \text{for } n = 1,2 \qquad (C.8)$$

$$h_n = 0.216 - 0.0205\,n \quad \text{for } n = 3, 4, \ldots, 9 \qquad \text{(C.9)}$$

Equations (C.8) and (C.9) imply that of the total milk produced per lactation, 17.5% is produced during each of the 1st and 2nd months post-partum, and this fraction drops to 15.5% in the 3rd month, 11.4% in the 5th, 7.3% in the 7th, and 3.2% in the 9th month.

APPENDIX D: A PROCEDURE TO ESTIMATE THE AGE EFFECT ON FERTILITY

The average annual calving rate ($\bar{r}_t$) of cows of age t can be expressed by a general function:

$$\bar{r}_t = f(t) \qquad (D.1)$$

where: t = age (years).

Cows reach maximum fertility around the middle of their reproductive lives. Function (D.1) can be approximated, in the absence of complete data, as in Figure D.1 with two curvilinear segments (AB) and (CD) tangent to a horizontal line (BC), which corresponds to the range of ages of highest fertility.

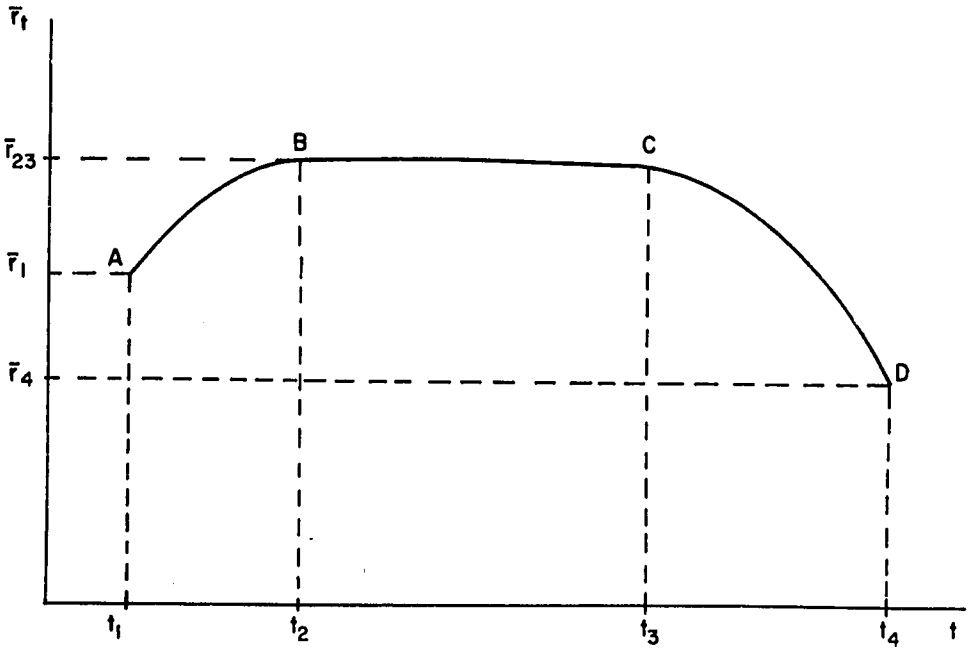


Figure D.1 Average annual calving rate ($\bar{r}_t$) as a function of cow's age in years (t)

Previous Page Blank

The minimum data necessary to estimate segments (AB) and (CD) of the curve are the average calving rate ($\bar{r}_1$) realized by animals at the youngest reproductive age (t_1), the maximum calving rate ($\bar{r}_{23}$) realized between ages t_2 and t_3, and the calving rate ($\bar{r}_4$) realized at the oldest reproductive age (t_4). Assuming that quadratic functions are acceptable representations of the segments (AB) and (CD), then the following equations describe analytically the effect of age on average annual calving rates:

$$\bar{r}_t = \frac{t_1 (t_1 - 2t_2) \bar{r}_{23} + t_2^2 \bar{r}_1}{(t_2 - t_1)^2} + \frac{2t_2(\bar{r}_{23} - \bar{r}_1)}{(t_2 - t_1)^2} t + \frac{(\bar{r}_1 - \bar{r}_{23})}{(t_2 - t_1)^2} t^2 \qquad \text{for } t_1 \leq t \leq t_2 \qquad \text{(D.2)}$$

$$\bar{r}_t = \bar{r}_{23} \qquad \text{for } t_2 < t \leq t_3 \qquad \text{(D.3)}$$

$$\bar{r}_t = \frac{t_4 (t_4 - 2t_3) \bar{r}_{23} + t_3^2 \bar{r}_4}{(t_4 - t_3)^2} + \frac{2t_3(\bar{r}_{23} - \bar{r}_4)}{(t_4 - t_3)^2} t + \frac{(\bar{r}_4 - \bar{r}_{23})}{(t_4 - t_3)^2} t^2 \qquad \text{for } t_3 < t \leq t_4 \qquad \text{(D.4)}$$

For cows conceiving at ages younger than t_1 and older than t_4, values $\bar{r}_1$ and $\bar{r}_4$ are assumed respectively.

As an example, consider a production system where 3-year-old cows have a 50% average annual calving rate ($t_1 = 3$ and $\bar{r}_1 = 0.50$), 5- to 9-year-old cows have a 75% average annual calving rate ($t_2 = 5$, $t_3 = 9$ and $\bar{r}_{23} = 0.75$), and cows 12 years old and older have a 40% average annual calving rate ($t_4 = 12$ and $\bar{r}_4 = 0.40$). Substituting these values into formulae (D.2), (D.3) and (D.4) yields:

$$\bar{r}_t = -0.8125 + 0.625\,t - 0.0625\,t^2 \qquad \text{for } 3 \leq t \leq 5$$

$$\bar{r}_t = 0.75 \qquad \text{for } 5 < t \leq 9$$

$$\bar{r}_t = -2.4 + 0.7\,t - 0.03889\,t^2 \qquad \text{for } 9 < t \leq 12.$$

APPENDIX E: A PROCEDURE TO ESTIMATE THE AGE EFFECT ON MORTALITY

To estimate the age effect on mortality rates from a minimum of field data, a procedure is used similar to that used to estimate age effect on fertility. Average natural mortality rate $\bar{r}_t$ at age t can be represented by a functional form:

$$\bar{r}_t = f(t) \qquad \text{(E.1)}$$

where: t = age (years).

Mortality tends to be lowest for animals in the middle age group, as shown in Figure E.1. This function can be approximated by two curvilinear segments (AB) and (CD) tangent to a horizontal line (BC).

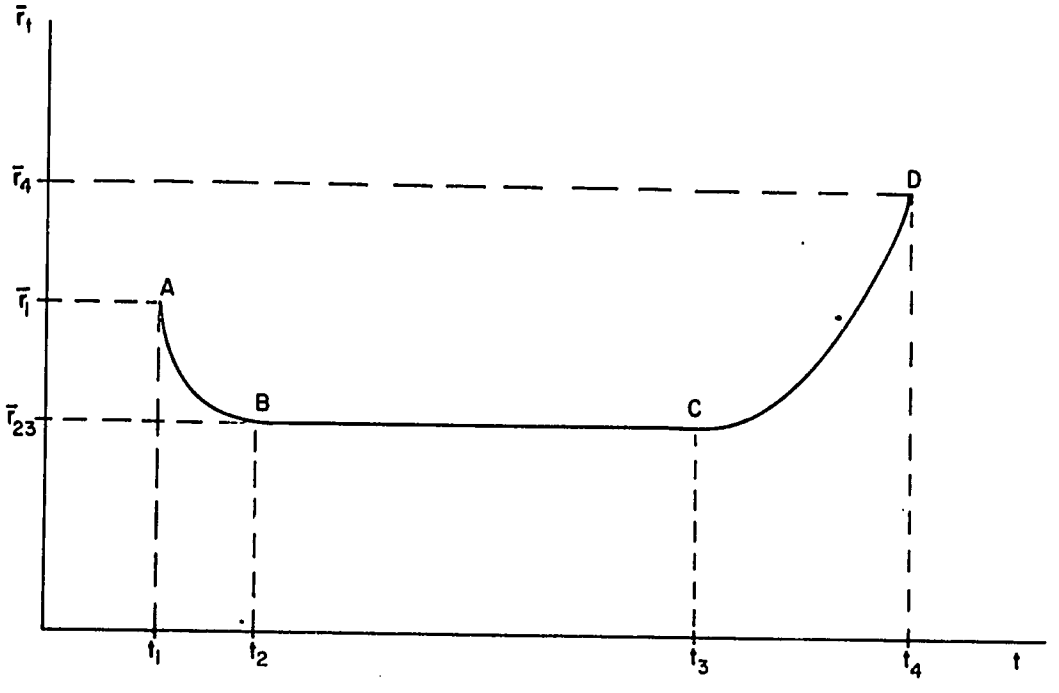


Figure E.1 Average annual mortality rate ($\bar{r}_t$) as a function of age in years (t)

The minimum data necessary to estimate segments (AB) and (CD) of the mortality curve are the average annual mortality rate ($\bar{r}_1$) of animals at a young age (t_1), the minimum annual mortality rate ($\bar{r}_{23}$) of animals between ages t_2 and t_3, and the average annual mortality rate ($\bar{r}_4$) of animals at an old age (t_4). Assuming that quadratic functions can be used to represent segments (AB) and (CD), the estimated functional forms are given by relationships (D.2), (D.3) and (D.4) of Appendix D.

Consider for example a production system in which 2-year-old animals have an 8% annual mortality rate ($t_1 = 2$ and $\bar{r}_1 = 0.08$), 4- to 9-year-old animals have a 5% annual mortality rate ($t_2 = 4$, $t_3 = 9$ and $\bar{r}_{23} = 0.05$), and 13-year-old animals have a 10% annual mortality rate ($t_4 = 13$ and $\bar{r}_4 = 0.10$). Substituting these values into formulae (D.2), (D.3) and (D.4) yields:

$$\bar{r}_t = 0.17 - 0.06 + 0.0075\ t^2 \qquad \text{for } 2 \leq t \leq 4$$

$$\bar{r}_t = 0.05 \qquad \text{for } 4 < t \leq 9$$

$$\bar{r}_t = 0.303125 - 0.05625\ t + 0.003125\ t^2 \qquad \text{for } 9 < t \leq 13.$$

APPENDIX F: EXAMPLE INPUT DATA SET

The description of the simulation model in Chapter 3 identified the data requirements of each individual component.

This appendix presents a complete example data set for the model. Much of the data are entered into the model in an interactive mode, and the computer code prompts the user to enter a certain data item or items. If a data item is inconsistent with previously entered data the user is prompted accordingly. Table F.1 presents an example of forage data and the associated distances walked. Three year-types are used to represent the range of forage conditions occurring for this illustrative system. Although they have been labelled 'poor', 'average' and 'good' according to dry matter yield and timing of plant growth, such labels have no operational significance in the model. The sum of the probabilities of the three year-types must equal unity.

Table F.2 illustrates the minimum data needed to estimate average liveweight growth curves and associated liveweight boundaries using the procedure presented in Appendix A. This procedure need not be used if sufficient data are available to specify these curves more precisely. The data presented in Table F.2 are consistent and satisfy the restrictions on the functions given in Appendix A. If data are entered which are not consistent, the user is prompted that the data must be revised.

Table F.3 presents example data for an approximate specification of the relationships associated with milk production, using the procedures in Appendices B and C which are appropriate when sufficient data are not available for exact specification of these functions. The data are entered interactively and checked for consistency.

Table F.4 presents data on reproduction, assuming that relevant field data are inadequate and the procedures given in Appendix D are applied to estimate the necessary relationships. Table F.5 presents the data required to estimate the mortality functions used in the model for deaths due to natural causes, such as stillbirths, predators, etc. Deaths due directly to starvation are accounted for separately in the model. The age effect on mortality for the whole range of ages is estimated using this minimum data set according to the procedure given in Appendix E.

F.1 Sample data for the specification of forage on offer and distances walked

a. Monthly details for each year-type

Calendar Month	Year-Types								
	'poor'			'average'			'good'		
	Q[a]	d[b]	D[c]	Q	d	D	Q	d	D
1 = Jan.	2.5	0.56	11	3.0	0.62	10	3.5	0.62	9
2	2.3	0.54	11	2.8	0.56	11	3.3	0.59	10
3	2.0	0.46	12	2.5	0.53	12	2.9	0.54	11
4	1.7	0.44	15	2.1	0.50	12	2.6	0.50	12
5	1.3	0.42	15	1.7	0.48	12	2.2	0.48	12
6	1.0	0.40	17	1.3	0.42	15	1.8	0.47	12
7	0.7	0.40	17	0.9	0.40	16	1.5	0.40	12
8	0.6	0.38	17	1.4	0.40	16	1.7	0.55	11
9	1.2	0.40	17	1.9	0.55	11	2.5	0.60	10
10	2.1	0.60	10	2.7	0.60	10	3.0	0.62	10
11	2.7	0.60	8	3.4	0.62	8	3.9	0.65	8
12 = Dec.	2.8	0.60	8	3.3	0.62	8	3.7	0.65	8

b. Other details for each year-type

Parameter	Year-types		
	'poor'	'average'	'good'
Probability of occurrence	0.25	0.50	0.25
Month of start of growing season	9	9	8
Threshold forage quantity on offer (t/ha) below which intake is reduced (Q*)	0.8	0.8	0.8
Threshold distance walked (km) above which intake is reduced (D*)	14	14	14

[a]Q = quantity of dry matter on offer (t/ha).

[b]d = digestibility (fraction)

[c]D = distance walked (km/day).

Table F.2 Sample data for the estimation of average liveweight growth curves and associated liveweight boundaries

Parameter	Males	Females
Average liveweight at birth (kg)	25	22
Age at the point of inflection (months)	20	18
Average liveweight at the point of inflection (kg)	240	200
Age at maturity (months)	52	48
Average liveweight at maturity (kg)	400	350
Coefficient of liveweight variation for mature animals[a]	0.25	0.25

[a]The coefficient of liveweight variation is defined as the ratio between the standard deviation of liveweight and average liveweight at a given age. The value given here is applicable for animals older than 18 months. For younger animals, see Section 3.3.2.1 for the values of this coefficient assumed in the model.

Table F.3 Sample data for the estimation of relationships associated with milk production

Parameter	Value
Age interval (t_2-t_3)[a] during which cows achieve their maximum milk yield (years)	6-8
Maximum milk yield per lactation for cows within this age interval (kg)	1 000
Age less than t_2 for which milk yield is known (t_1)[b] (years)	3
Fraction of maximum milk yield achieved at age t_1	0.8
Age greater than t_3 for which milk yield is known (t_4)[c] (years)	13
Fraction of maximum milk yield achieved at age t_4	0.6
Maximum length of lactation (months)	9
Fraction of total lactation milk yield produced during first 2 months post-partum	0.35
Fraction of cows in the population that let down milk for human consumption in the absence of a suckling calf	0.7
Fraction of potential milk yield let down by these cows	0.8
Average energy content of milk (MJ/kg)[d]	3.6

[a]Age t_2 must be less than t_3.

[b]For best estimation of the age effect on milk yield, this should be the youngest age (i.e. age at first calving) for which milk yield is known.

[c]For the best estimation of the age effect on milk yield, this should be the oldest age for which milk yield is known.

[d]Computed according to relationship (2.15), given the butter fat and solids-not-fat contents of milk.

Table F.4 Sample data for the estimation of relationships associated with reproduction

Parameter	Value
Minimum age at first calving of heifers in best liveweight condition (months)	36
Average age at first calving of heifers in average liveweight condition (months)	42
Maximum age at first calving of heifers in poorest liveweight condition (months)	52
Age interval $(t_2-t_3)^a$ during which cows achieve maximum fertility (years)	5-8
Average annual calving rate for cows within this age interval (fraction)	0.80
Age less than t_2 for which the calving rate is known $(t_1)^b$ (years)	4
Average calving rate at age t_1 (fraction)	0.60
Age greater than t_3 for which the calving rate is known $(t_4)^c$ (years)	12
Average calving rate at age t_4 (fraction)	0.50
Fraction of barren females in the population	0.05
Fraction of females in the population with bad conformation	0.03

[a] Age t_2 must be less than t_3.

[b] For best estimation of the age effect on reproduction, this age should be the youngest age (i.e. age at first calving) for which calving rates are known.

[c] For best estimation of age effect on reproduction, this should be the oldest age for which calving rates are known.

Table F.6 illustrates the data needed to specify an initial herd. In this example, the initial herd consists of six animals, including two (animals 2 and 6) calves at foot. This data set must be consistent with the calendar date at the start of the simulation, in this example September 1981.

The data set for each animal in Table F.6 can best be illustrated by elaboration of the details for animal 5. This animal is in class 15, so it is a cow with a suckling calf at foot to which it is linked. The calf is number 6 as indicated in the 2nd column of row 5. Dams and suckling calves are linked forwards and backwards because they are treated together in terms of their food energy demands. Animal 5 weighs 305 kg at the beginning of September 1981 and has had one calf, the calf currently at foot which was born in May 1981 (date 8105). This cow conceived again in August 1981 (date 8108) and therefore is expected to calve again in May 1982. Finally, the last column of row 5 specifies that the cow would be expected to let down 75% of her milk yield if she were milked after her calf is weaned.

Table F.5 Sample data for the estimation of relationships associated with mortality due to natural causes other than starvation

Parameter	Value
Average calf survival rate up to 3 months (fraction)	0.85
Average calf survival rate up to 1 year (fraction)	0.80
Age interval (t_2-t_3)[a] of lowest average mortality rate (years)	3-9
Average annual mortality rate within this interval (fraction)	0.03
Age less than t_2 for which the mortality rate is known (t_1)[b] (years)	2
Average annual mortality rate at age t_1 (fraction)	0.04
Age greater than t_3 for which the mortality rate is known (t_4)[c] (years)	13
Average annual mortality rate at age t_4 (fraction)	0.10

[a]Age t_2 must be less than t_3.

[b]For best estimation of the age effect on mortality, this should be 2 years.

[c]For best estimation of the age effect on mortality, this should be the oldest age for which mortality rates are known.

Table F.6 Attributes of animals in herd at the start of simulation, as of September 1981 (8109)

Animal No.	No. of Linked Dam or Calf[a]	Birth Date	Animal Class[b]	Current Live-weight (kg)	No. of Known Partu-ritions	Date of Last Calving	Date of Concep-tion	Let-Down Fraction[c]
1	2	7303	15	340	4	8107	-	0.0
2	1	8107	1	50	-	-	-	-
3	-	7501	14	358	2	7908	8107	0.0
4	-	7912	11	180	0	-	-	0.75
5	6	7610	15	305	1	8105	8108	0.75
6	5	8105	8	65	0	-	-	0.75

[a] In the case of a cow with a calf at foot, provides the corresponding serial number of the calf and vice-versa.

[b] The codes used in the model for different classes of cattle are:

1 = male calf at foot
4 = weaned male (after class 1)
6 = barren female calf at foot
7 = female calf at foot with bad conformation
8 = fertile female calf at foot with good conformation
9 = barren heifer (after class 6)
10 = heifer with bad conformation (after class 7)
11 = fertile heifer with good conformation (after class 8)
12 = barren female in the breeding herd (after class 9)
13 = bad conformation female in the breeding herd (after class 10)
14 = non-lactating female in the breeding herd (after class 11)
15 = lactating female with calf at foot
16 = lactating female without calf at foot.

[c] In this example, some cows let down milk when a calf is not suckling; others, such as no. 1, do not. The let-down fraction of 0.75 for cow no. 5 implies that 75% of her potential milk production can be extracted for human consumption after the weaning of her calf.

LITERATURE CITED

Agricultural Research Council (1965). *The nutrient requirements of farm livestock No. 2: Ruminants.* London, HMSO.

Anderson, W J (1977). Influence of plane of nutrition on the productivity of Angus heifers. M.Ag.Sci. thesis. Palmerston, New Zealand, Massey University.

Armstrong, D G (1964). Evaluation of artificially dried grass as a source of energy for sheep: 2. *J. Agric. Sci. Camb.* 62:399-417.

Arnold, G W and Dudzinski, M L (1967). Studies on the diet of the grazing animal. 2: The effect of physiological status in ewes and pasture availability on herbage intake. *Aust. J. Agric. Res.* 18: 349-359.

Baile, C A and Forbes, J M (1974). Control of feed intake and regulation of energy balance in ruminants. *Physiol. Rev.* 54 (1): 160-214.

Balch, C C and Campling, R C (1962). Regulation of voluntary food intake in ruminants. *Nutr. Abst. Rev.* 32(3): 669-686.

Bines, J A (1976). Factors influencing voluntary food intake in cattle. In H Swan and W H Broster, eds. *Principles of cattle production.* London, Butterworths: 287-305.

Blaxter, K L (1969). The efficiency of energy transformations in ruminants. In K L Blaxter, J Kielanowski and G Thorbek eds. *Energy metabolism of farm animals.* London, Routledge and Kegan Paul: 21-28.

Blaxter, K L (1973). Metabolizable energy and feeding systems for ruminants. In H Swan and D Lewis, eds. Proceedings of Nutrition Conference for Feed Manufacturers. No. 7. Held at University of Nottingham. London, Butterworths.

Blaxter, K L and Wilson, R S (1962). The voluntary intake of roughages by steers. *Anim. Prod.* 4: 351-358.

Broster, W H (1972). Effect on milk yield of the cow of the level of feeding during lactation. *Dairy Sci. Abstr.* 34: 265-288.

Buck, N G, Light, D, Rutherford, A, Miller, M, Rennie, T W, Pratchett, D, Capper, B S and Trail, J C M (1976). Environmental factors affecting beef cow reproductive performance in Botswana. *Anim. Prod* 23: 357-363.

Bureau of Agricultural Economics (BAE) (1974). A computer model simulating extensive beef cattle production systems. Occasional Paper No. 21. Canberra.

Campling, R C, Freer, M and Balch, C C (1962). Factors affecting the voluntary intake of food by cows. 3: The effect of urea on the voluntary intake of oat straw. *Brit. J. Nutr.* 16: 115-124.

Cartwright, T C, Gomez, F G, Sanders, J O and Nelsen, T C (1977). Simulated milk-beef production systems in Colombia. *J. Anim. Sci.* 45 (Supplement 1).

Cobby, J M and le Du, Y L P (1978). On fitting curves to lactation data. *Anim. Prod.* 26 (2): 127-133.

Conrad, H R (1966). Symposium on factors influencing the voluntary intake of herbage by ruminants: Physiological and physical factors limiting feed intake. *J. Anim. Sci.* 25: 227-235.

Conrad, H R, Pratt, A D and Hibbs, J W (1964). Regulation of feed intake in dairy cows. 1: Change in importance of physical and physiological factors with increasing digestibility. *J. Dairy Sci.* 47: 54-62.

Coppock, C E, Noller, C H and Wolfe, S A (1974). Effect of forage-concentrate ratio in complete feeds fed *ad libitum* on energy intake in relation to requirements by dairy cows. *J. Dairy Sci.* 57: 1371-1380.

Corbett, J L, Langlands, J P and Reid, G W (1963). Effects of season of growth and digestibility of herbage on intake by grazing dairy cows. *Anim. Prod.* 5 (2): 119-129.

Cordova, F J, Wallace, J D and Pieper, R D (1978). Forage intake by grazing livestock: A review. *J. Range Manage.* 31 (6): 430-438.

Crampton, E W (1957). Interrelations between digestible nutrient and energy content, voluntary dry matter intake, and the overall feeding value of forages. *J. Animal Sci.* 16: 546-552.

Crampton, B W and Harris, L E (1969). *Applied animal nutrition.* San Francisco, W H Freeman.

Dahl, G and Hjort, A (1976). *Having herds: Pastoral herd growth and household economy.* Stockholm Studies in Social Anthropology No. 2. University of Stockholm.

Dahl, G and Hjort, A (1979). Pastoral change and the role of drought. Report No. R2. Stockholm, Swedish Agency for Research Cooperation with Developing Countries.

Davis, J M, Cartwright, T C and Sanders, J O (1976). Alternative beef production systems for Guyana. *J. Animal Sci.* 43: 235 (abstract).

Drennan, M J (1977). Effect of plane of nutrition during late pregnancy on the incidence of calving problems in beef cows and heifers. *Anim. Breed. Abstr.* 48.

Elliott, R C (1967). Voluntary intake of low-protein diets by ruminants. 1: Intake of food by cattle. *J. Agric. Sci. Camb.* 69: 375-382.

Elliot, R C and Fokkema, K (1961). Herbage consumption studies on beef cattle. *Rhod. Agr. J.* 58: 49-57.

Elliot, R C, Fokkema, K and French, C H (1961). Herbage consumption studies on beef cattle. *Rhod. Agr. J.* 58: 124-130.

Glover, J, Duthie, D W and French, M H (1957). The apparent digestibility of crude protein by the ruminant. 1: A synthesis of the results of digestibility trials with herbage and mixed feeds. *J. Agric. Sci. Camb.* 48: 373-378.

Greenhalgh, J F D, Reid, J W, Aitken, J N and Florence, E (1966). The effects of grazing intensity on herbage consumption and animal production. 1: Short-term effects in strip-grazed dairy cows. *J. Agric. Sci. Camb.* 67: 13-23.

Hodgson, J (1968). The relationship between the digestibility of a sward and the herbage consumption of grazing calves. *J. Agric. Sci. Camb.* 70: 47-51.

Holmes, W, Jones, J G W and Drake-Brockman, R M (1961). The feed intake of grazing cattle. 2: The influence of size of animal on feed intake. *Anim. Prod.* 3: 251-260.

Hunt, L A (1966). Ash and energy content of material from seven forage grasses. *Crop Science.* 6: 507-509.

Inter-American Development Bank (IADB) (1975). HERDSIM simulation model: User manual. Project Analysis Paper No. 2. Economic and Social Development Department. Washington, D C.

International Bank for Reconstruction and Development (IBRD) (1972). A user's guide to BLIVOO herd development simulation with cash flow and rate of return. Computing Activities Department. Washington D C.

International Livestock Centre for Africa (ILCA) (1978). *Mathematical modelling of livestock production systems: Application of the Texas A & M University beef cattle production model to Botswana.* Systems Study No. 1. Addis Ababa.

Joubert, D M (1963). Puberty in female farm animals. *Anim. Breed. Abstr.* 31: 295-306.

Juri, P, Gutierrez, N F and Valdes, A (1977). Modelo de simulation por computador para fincas ganaderos. Cali (Colombia), CIAT.

Karue, C N, Evans, J L and Tillman, A D (1973). Voluntary intake of dry matter by African Zebu cattle: Factors influencing dry matter intake. *E. Afr. Agr. For. J.* 38: 352-360.

Kay, M (1976). Meeting the energy and protein requirements of the growing animal. In H Swan and W H Broster, eds. *Principles of cattle production.* London, Butterworths.

Konandreas, P A, Anderson, F M and Trail, J C M (1981). Economic trade-offs between milk and meat production under various supplementation levels in Botswana. Paper contributed to the IASTED Conference on Simulation Modelling and Development, held in Cairo, 1-3 September.

Lamond, D R (1970). The influence of undernutrition on reproduction in the cow. *Anim. Breed. Abstr.* 38 (3): 359-372.

Lofgreen, G P, Meyer, J H and Hull, J L (1957). Behaviour patterns of sheep and cattle being fed pasture or silage. *J. Animal Sci.* 16: 773-780.

Lowman, B G, Edwards, R A and Somerville, S H (1979). The effect of plane of nutrition in early lactation on the performance of beef cows. *Anim. Prod.* 29: 293-303.

McDonald, P, Edwards, R A and Greenhalgh, J F O (1978). *Animal nutrition.* London, Longman.

Meyer, J H, Lofgreen, G P and Hull, J L (1957). Selective grazing by sheep and cattle. *J. Animal Sci.* 16: 766-772.

Ministry of Agriculture, Fisheries and Food, United Kingdom (MAFF) (1975). *Energy allowances and feeding systems for ruminants.* Technical Bulletin No. 33. London.

Moe, P W, Tyrrell, H F and Flatt, W P (1971). Energetics of body tissue mobilization. *J. Dairy Sci.* 54 (4): 548-553.

Moe, P W and Tyrrell, H F (1972). Metabolizable energy requirements of pregnant dairy cows. *J. Dairy Sci.* 55 (4): 480-483.

Montgomery, M J and Baumgardt, B R (1965). Regulation of food intake in ruminants. 1: Pelleted rations varying in energy concentration. *J. Dairy Sci.* 48: 569-574.

National Research Council (1978). *Nutrient requirements of dairy cattle.* Fifth edition, revised. Washington, D C, National Academy of Sciences.

Newton, J E, Betts, J E and Wilde, R (1980). The effect of body condition and time of mating on the reproductive performance of Masham ewes. *Anim. Prod.* 30: 253-260.

Ordonez, J (1978). Systems analysis of beef production in the western high plains of Venezuela. Ph. D. thesis, College Station, Texas A & M University.

Owen, J B (1976). *Sheep production.* London, Bailliere Tindall.

Oyenuga, V A and Olubajo, F A (1975). Pasture productivity in Nigeria. 2: Voluntary intake and herbage digestibility. *J. Agric. Sci. Camb.* 85: 337-343.

Pigden, W J, Balch, C C and Graham, M, eds. (1979). *Standardization of analytical methodology for feeds: Proceedings of a workshop.* Ottawa, International Development Research Centre.

Pinney, D O, Stephens, D F and Pope, L S (1972). Lifetime effects of winter supplemental feed level and age at first parturition on range beef cows. *J. Anim. Sci.* 34: 1067-1074.

Pleasants, A B and Ginindza, M M (1980). The effect of pre-calving nutrition on the productivity of beef breeding cows in southern Africa. Palmerston, New Zealand, Sheep Husbandry Department, Massey University.

Reid, J T, Loosli, J K, Trimberger, G W, Turk, K L, Asdell, S A and Smith, S E (1963). Causes and prevention of reproductive failures in dairy cattle. 4: Effect of plane of nutrition during early life on growth, reproduction, production, health and longevity of Holstein cows. 1: Birth to calving. Bulletin No. 987. Ithaca, Cornell University Agricultural Experimental Station.

Sanders, J O and Cartwright, T C (1979). A general cattle production systems model. 2: Procedures used for simulating animal performance. *Agricultural Systems*. 4 (4): 289-309.

Siebert, B D and Field, J B F (1975). Reproductive activity in beef heifers following post-weaning feeding on spear grass hay alone or with supplements. *Aust. J. Exp. Agric. Anim. Husb.* 15: 12-16.

Smith, G M (1979). The TAMU beef cattle production model. College Station, Animal Science Department, Texas A & M University.

Sparke, E J and Lamond, D R (1968). The influence of supplementary feeding on growth and fertility of beef heifers grazing natural pastures. *Aust. J. Exp. Agric. Anim. Husb.* 8: 425-433.

Vadiveloo, J and Holmes, W (1979). The prediction of the voluntary feed intake of dairy cows. *J. Agric. Sci. Camb.* 93: 553-562.

van Es, A J H (1976). Factors influencing the efficiency of energy utilization by beef and dairy cattle. In H Swan and W H Broster, eds. *Principles of cattle production*. London, Butterworths: 237-253.

van Keulen, H (1975). *Simulation of water and herbage growth in arid regions*. Wageningen, Centre for Agricultural Publishing and Documentation.

Webster, A J F (1978). Prediction of the energy requirements for growth in beef cattle. *Wld. Rev. Nutr. Diet.* 30: 189-226.

Williamson, G and Payne, W J A (1978). *An introduction to animal husbandry in the tropics*. London, Longman.

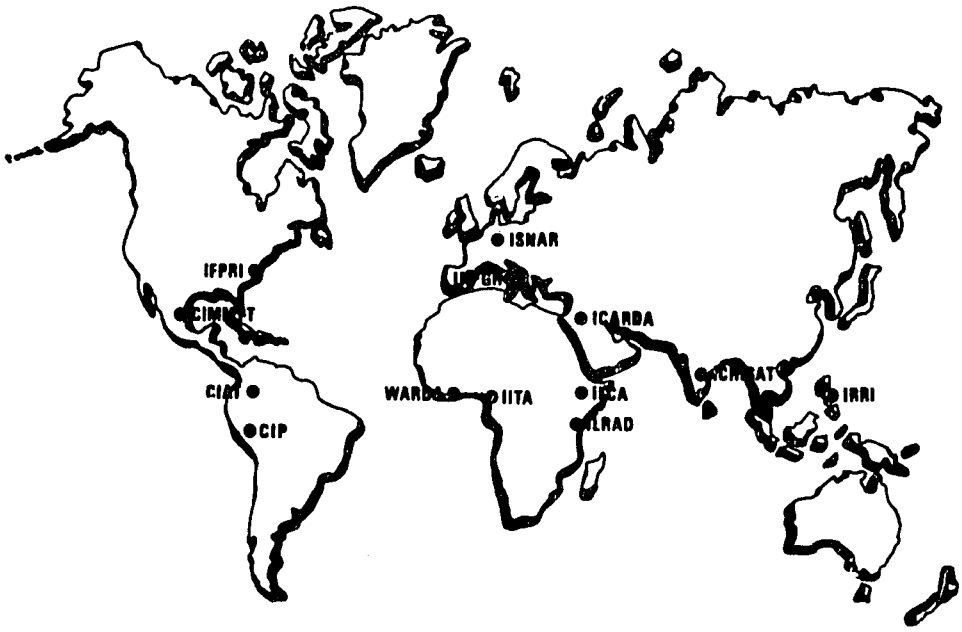
Wilson, R K and Flynn, A V (1974). Observations on the eating behaviour of individually fed beef cattle offered grass silage *ad libitum*. *Irish J. Agric. Res.* 13: 347-349.

Wood, P D P (1969). Factors affecting the shape of the lactation curve in cattle. *Anim. Prod.* 11: 307-316.

Zemmelink, G (1980). *Effect of selective consumption on voluntary intake and digestibility of tropical forages*. Agricultural Research Report No. 896. Wageningen, Centre for Agricultural Publishing and Documentation.

THE CONSULTATIVE GROUP ON INTERNATIONAL AGRICULTURAL RESEARCH

The International Livestock Centre for Africa (ILCA) is one of the 13 international agricultural research centres funded by the Consultative Group on International Agricultural Research (CGIAR). The 13 centres, located mostly within the tropics, have been set up by the CGIAR over the last decade to provide long-term support for agricultural development in the Third World. Their names, locations and research responsibilities are as follows :



Centro Internacional de Agricultura Tropical (CIAT), Colombia: cassava, field beans, rice and tropical pastures.

Centro Internacional de Mejoramiento de Maíz y Trigo (CIMMYT), Mexico: maize and wheat.

Centro Internacional de la Papa (CIP), Peru: potato.

International Centre for Agricultural Research in the Dry Areas (ICARDA), Lebanon: farming systems, cereals, food legumes (broad bean, lentil, chickpea), and forage crops.

International Board for Plant Genetic Resources (IBPGR), Italy.

International Crops Research Institute for the Semi-Arid Tropics (ICRISAT), India: chickpea, pigeon pea, pearl millet, sorghum, groundnut, and farming systems.

International Livestock Centre for Africa (ILCA), Ethiopia: African livestock production.

International Rice Research Institute (IRRI), the Philippines: rice.

International Institute of Tropical Agriculture (IITA), Nigeria: farming systems, maize, rice, roots and tubers (sweet potatoes, cassava, yams), and food legumes (cowpea, lima bean, soybean).

International Laboratory for Research on Animal Disease (ILRAD), Kenya: trypanosomiasis and theileriosis of cattle.

West Africa Rice Development Association (WARDA), Liberia: rice.

International Service for National Agricultural Research (ISNAR), the Netherlands.

International Food Policy Research Institute (IFPRI), USA: analysis of world food problems.

Systems Studies

1. *Mathematical modelling of livestock production systems: Application of the Texas A&M University beef cattle production model to Botswana*, by F.M. Anderson and J.C.M. Trail. 1978.
2. *Livestock production in the subhumid zone of West Africa: A regional review*, ed. S.B. Westley. 1979.
3. *Small ruminant production in the humid tropics*, eds C. de Haan, S.B. Westley and S. Chater. 1979.

Monographs

1. *Evaluation of productivity of Maure and Peul cattle breeds at the Sahelian station, Niono, Mali*, undertaken by the International Livestock Centre for Africa and the Institut d'Economie Rurale du Mali. 1978.
2. *Trypanotolerant livestock in West and central Africa: Vol. 1, General study; Vol. 2, Country studies*, by J.C.M. Trail, C.H. Hoste and Y.J. Wissocq, and Ph. Lhoste in collaboration with I.L. Mason. 1979.
3. *Sahiwal cattle: An evaluation of their potential contribution to milk and beef production in Africa*, by J.C.M. Trail and K.E. Gregory. 1981.
4. *Low-level aerial survey techniques: Report of an international workshop held 6-11 November 1979, Nairobi, Kenya*, eds J.J.R. Grimsdell and S.B. Westley. 1981.
5. *The camel (C. Dromedarius): A bibliographical review*, by E. Mukasa-Mugerwa. 1981.

In 1982 the above two series were discontinued and replaced by the present series of Research Reports.

Research Reports

1. *Trends and prospects for livestock and crop production in tropical Africa*, by C. de Montgolfier-Kouévi and A. Vlavonou. 1982.