BIBLIOGRAPHIC DATA SHEET

1. CONTROL NUMBER: PN-AAH-654

2. SUBJECT CLASSIFICATION (695): JD10-0000-0000

3. TITLE AND SUBTITLE (240)

Policy and program issues raised by the application of compound models

4. PERSONAL AUTHORS (100)

Davis, Russell

5. CORPORATE AUTHORS (101)

Harvard Univ. Ctr. for Studies in Education and Development

6. DOCUMENT DATE (110): 1979

7. NUMBER OF PAGES (120): 34p.

8. ARC NUMBER (170): 379.1.D263b

9. REFERENCE ORGANIZATION (180): Harvard

10. SUPPLEMENTARY NOTES (500)

(In Harvard Institute for International Development, development discussion paper no. 69)

11. ABSTRACT (950)

12. DESCRIPTORS (920)

Models
Planning
Educational planning
Policies
Program planning

13. PROJECT NUMBER (150): 931008900

14. CONTRACT NO.(140): AID/ta-C-1336

15. CONTRACT TYPE (140)

16. TYPE OF DOCUMENT (160)

AID 590-7 (10-79)

379.1
D263b

PN-AAH-654

Development Discussion Papers

Harvard Institute
for International Development

HARVARD UNIVERSITY

POLICY AND PROGRAM ISSUES RAISED BY THE APPLICATION OF COMPOUND MODELS

Russell Davis

DEVELOPMENT DISCUSSION PAPER No. 69

June 1979

HARVARD INSTITUTE FOR INTERNATIONAL DEVELOPMENT
Harvard University,
1737 Cambridge Street,
Cambridge, Massachusetts,
02138, U.S.A.

Development Discussion Papers 59 through 73 were originally prepared for the United States Agency for International Development under a research contract with the Center for Studies in Education and Development of the Harvard Graduate School of Education. The Harvard Institute for International Development collaborated with the Center in this project and the papers included in this series are a sample of the contributions by participants affiliated with HIID.

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Discussion of the policy and program issues that are revealed in the applications of comprehensive educational planning models to planning and policy analysis. Emphasis is on two general, compound models that were actually applied in planning, one in a Latin American country and the other in the Middle East. The model framework and the issues that are raised are not country specific but have relevance to educational planning in any similar country. The discussion in the text is non-technical and the model structures are shown only in an appendix. The focus is on policy and program issues and not model development technicalities.

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1.0 Introductory Note

This paper is designed to complement the previous paper which described and discussed rational approaches to education planning through systems models and analysis procedures. The previous paper described the models and the theory and methods currently used in planning, and went a bit beyond current practice to suggest further extension in model development and systems analysis.

A central point was that systems models tend to take a black box approach to analysis of the central process of teaching-learning that goes on within education systems. Black box analysis is not a fatal flaw in the application of systems analysis to the investigation of the working of physical and social systems, or in the design of physical and social systems based on the investigation. Black box analysis is applied in the study of many social phenomena, in demography, sociology, and economics.

There is always a limit on how deeply analysis can go into the working of a complex social process. The analysis of education by economists or planners may stop at a more molar level than that of the sociologist; and in turn the psychologist may go more deeply into the learning process than the sociologist; but most social analysis does not detail the inner works of a complex social process or human interaction. Compound models, or systems models, are designed to operate at a highly molar level of analysis. The model results can serve to raise policy issues, but they do not always do so at a level of specificity or detail that yields the analyst a single and unambiguous solution to the

problem that the model is designed to analyze.

In this brief paper the discussion will be based on the policy implications of two comprehensive or compound planning models. The first model was developed by Schiefelbein and applied in the planning of the Chilean Educational Reform in the mid-sixties. The design of the model and the analysis of the results of running the model are described in Schiefelbein and Davis (1974). The mathematical structure of the model is given in Appendix I to this paper.

The second model, called the Compound Model, was developed by the World Bank and Saudi Arabian planners and run with Saudi Arabian data in 1977-78. There is nothing new in the model. The structural parts of this model are in a form that is used in many manpower and educational planning models, but the parts are linked together in general or compound form. There will be no discussion of Saudi Arabian data or the particular results of the application. The model is discussed only in terms of its general characteristics and the general planning issues it raises. A general schematic of the model is shown in Appendix II in this paper. The model has been programmed for the computer and there is extensive documentation of the model structure and the computer code. These are the property of the World Bank and the Saudi Arabian government and will not be presented here.

2.1.0 The Chilean Model Used to Trace Policy Issues

This is a brief discussion of several issues taken up during the process of educational reform in Chile. These issues lent themselves to analysis using the Chilean system model. The model or some version of it was used to explore a number of these questions, primarily by simulating changes in the system and studying the future effects of these changes.

2.1.1 Extending Six-Year Primary Schooling to Eight Years Basic Education

One of the earliest and most critical issues discussed by educators was whether to extend primary education from six to eight or nine years. The decision was finally made to create an 8-year primary school. People believed that the social benefits obtained from an additional two years (retention of children in school) would have been even larger with a 9-year school, but felt that the effort to achieve such a reform would have put too much strain on the system. In addition to the question of whether most children would continue on for seven, eight or nine years after six years of primary, the planners had to determine: the capacity of the system to provide facilities, teachers and materials for the increased enrollment; what to do with the secondary school teachers who might be unemployed; the cost of each alternative; and the political resistance that would be provoked. The existing model required a few modifications, because its present structure assumed a six-year primary school, but all except the last question could be studied with the modified model. (Data to answer the last question were available from experimental trials run in Africa and other developing country settings.)

2.1.2 Social Promotion

A second major issue faced by the planners was that of social promotion. Given an overall objective of normalizing the age-grade distribution (which stood as a proxy for educational "quality") it was important to improve the flow of students through the system. Use of the model earlier had showed that official repeater and dropout rates could not be correct. Employing an historical series of enrollment figures, the planners demonstrated that

official rates gave unrealistic projections compared to present real figures. The analysts used a trial and error method to estimate what the real dropout and repeater rates must have been. These rates showed that, contrary to common belief,repetition was a grave problem, while desertion or drop-out was minor. The planners then decided to set aside recommendations to work on the dropout question, and look for policies that could reduce repeater rates.

The simplest solution was that of automatic or social promotion. At least some of the planners believed, on the basis of skimpy evidence, that teachers' grades bore little relationship to either the intellectual ability of children or their academic achievement levels. These planners felt that all children should be promoted automatically at the end of each year, through the first six grades. The model results were used to examine the impact of such a policy: on total enrollments in primary; pressures on secondary; and costs.

Alternatives,such as automatic promotion through the first four, first three, or first two, grades could also be examined. On the assumption that automatic promotion would increase the heterogeneity of abilities in a given classroom, and hence permit fewer students per teacher, analysts studied the pupil/teacher and cost implications for the entire system. Could automatic promotion mean, for example, that fewer children overall could be served, assuming that funds were not unlimited? Or assuming that to maintain the same class size teachers would require some additional training--what would be the implications for teacher training and curriculum reform?

The planners' and their proposals provoked resistance from teachers. The compromise reached was that first and second grade children would be promoted

automatically if they attended at least 75 percent of their classes. With data on average attendance rates, planners could examine whether the promotion policy would have any effect at all. The major counter argument was that standards would be lowered, quality reduced and social chaos result.

2.1.3 Teacher Training Issues and the Model

The planners had several reasons for wanting to upgrade the teaching force, and knew that teachers were strongly in favor of any program that would give them increased status (and income). The model could, therefore, be used to examine the effects of raising primary teacher training requirements from 12 years (6 primary plus 6 normal) to 15 (the most common proposal being a three year program at the university level). Some of the gains from further training would be larger class sizes, and reduced dropout and repetition rates. Some of the costs would be incurred through teacher demands for higher salaries, as well as the costs of providing the training, and the increased time to get a teacher into the system.

Formal teacher training could also be compared with programs to upgrade teachers in-service, or those that proposed taking in para-teachers with lower levels of training, and training them in-service up to the level of current staff. Experience with high school graduates with no formal training had been positive; that is, they did learn how to manage children and they were enthusiastic about teaching.

Some of the variables involved were the pool of available candidates for training, the loss rate to other professions of persons who go through various kinds of training programs, the "ability" levels of persons with different kinds of training, salary levels, time to train, years the teachers will

remain in service, and geographical dispersion. Most of these variables could be handled by the model.

The planners finally came down to deciding between continuing the present normal school system, upgrading teachers with in-service training, or pre-service training of new candidates in the universities. The planners assumed that they could demand more of better trained teachers, and looked to see if improvements in "quality of teaching" (indicated by class size and failure rates) were offset by increased costs of training and salaries.

2.1.4 Other Model Application Possibilities

The model could have been used, although it was never applied to this task, to examine the effect of introducing new teaching methods. Specifically, the Ministry wanted to introduce a reading program that promised much greater achievement for students, but would also require significant inputs into teacher preparation and development of materials. It would have been possible to set some variable values for these factors into the model, using the $\underline{y}$ technological variable, to see if gains offset costs.

The same kind of approach could be used to estimate the cost/benefits of school lunch programs and the provision of self-study materials and books. Later data on the relationship between these inputs and student achievement was made available to the planners, who could have used the additional data to provide estimates that would allow planners to apply the model to study impacts on the system.

3.0.0 Social Promotion as a Policy Option

Social promotion is often viewed as an all or none policy option to be applied over all the grades and without any criteria or supplementary pedagogical

modifications. If carried out in this extreme form it simply means pushing all children through the lower grades toward eventual grief in the upper ones. However the old policy of not-promoting, especially in the first grade, often reflects these discriminatory features:

a. It probably penalizes unduly for the effects of slower maturation and readiness, in that a selection criterion which is largely based on reading skill dominates. When this criterion is applied it selects out those children who have less readiness for reading.

b. The group selected for non-promotion very likely are from lower SES groups-deprived of family stimulus, kindergarten or pre-school experience, and may be from a family cultural background that is totally alien to the school culture. The children also may be affected physiologically by nutrition induced difficulties. They probably are from homes where there is little social organization of the kind that sends children to school regularly, keeps them there, and helps and encourages them while there. If these assumptions about inequality in background and readiness are plausible then a variety of program options might be required.

3.1.1 Readiness Programs

One program might be designed to give disadvantaged children help through readiness programs at kindergarten and pre-school level. This would consist of enrichment through readiness training, possibly supplemented by health and nutrition and socialization measures. If this worked, one option would be to establish social promotion policies at the first grade and to promote all children to higher grades and apply selective promotion later. The evidence from enrichment programs indicates that short-term remediation

is not lasting in its effect and that marked disabilities may persist over several years. Hence later screening might still adversely affect substantial proportions of disadvantaged children. Furthermore, practical experience suggests that there is less enthusiasm for supporting pre-school programs, and that in fact only middle and upper class parents utilize such programs in any substantial numbers. Also many of the pools of disadvantaged children already show substantial pile-ups of over age children who would be out of age-grade phase if kept in pre-school.

3.1.2 Developmental Reading

Another alternative is to continue social promotion (of a modified sort)[1] and remediation through two or three grades, but to emphasize developmental reading only. The IEA studies (1975) indicate that reading is one of the major sources of retardation in the schools of all countries and that much more and longer lasting direct instruction in building reading skills is necessary.[2] Hence the selection might still be useful at the first grade but be misapplied when it simply is used to fail and make the child repeat the first grade. A better policy alternative might be to promote the child to second grade but to screen out those who fail, largely on reading, and to select them into special programs at the second grade.

The requirements for additional teacher numbers would not be vast for this group, since the students would normally have been kept in the school system anyway and served as repeaters in the first grade. Now they would be

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[1] Social promotion need not be mindlessly automatic. In Chile minimal attendance standards (75% of the years classes attended, with exceptions for illness) were maintained.

[2] Purves, Alan and Daniel Levine (eds). Educational Policy and International Assessment: Implications of the IEA Surveys of Achievement. Berkeley: McCutchan, 1975

served by extra teachers assigned to the second grade level for specific work in reading instruction. These teachers would have to be especially trained, perhaps in service, but this could be added to instructional costs as a measure for improving the quality of the system.

3.1.3 Simulating Alternatives with the Model

The model would be used to simulate the difference in costs of the old system, where the enrollment pile up occurs again and again in the first grade, and the new system where students who fail are pushed along to reading remediation in the second. It should also be clear that this would be easier to do and cheaper in large multi-room and multi-class urban schools than in rural primary schools, especially where there is one teacher already covering more than one grade.

3.1.4 Continuing Programs in Higher Grades of Primary School

The same alternative would be continued through second and third grade, although here some criterion for promotion might be introduced, i.e./attendence at a minimum proportion of class sessions. Apart from this students would be pushed along with their regular grade group and given extra reading instruction at third grade.

The program should continue up to the fourth grade, in order to accomodate the fact that the effects of retardation are fairly persistent. At this point regular promotion criteria might be introduced. Those who still fail should receive training for basic literacy, or in the case where there are psycho-motor difficulties or mental retardation special school programs would be developed and made available according to need, and resources

to meet the need.

The model could be used to simulate the affect of such a policy on flow, enrollments by grades, instructional needs by grades, and added costs.

4.1.0 General Comment on Compound Models

The main use of compound models, like the one shown in Appendix II, is not in predicting the future balance of manpower requirements and educational supply. No model, no matter how large and detailed it may be, and no matter how skillfully it is designed and no matter how accurate the information supplied, can predict the future. Even less can running the model make some desired future come to pass. The model once it is understood, can serve policy planners well in these ways:

4.2.1 Future Systems Gaps

Model runs serve to alert the planner to future gaps between manpower requirements and supply. These gaps will appear if the economic and educational changes that are traced from analysis of past and present conditions continue into the future. The Compound Model bases its economic growth forecasts and educational supply projections on past and present economic and educational facts. Model runs do not predict the future, but rather show what can happen if current conditions continue as they are, or change in some pattern that can be deduced from the past and present. The Compound Model will reveal shortages in the supply of educated people for the work force if present entrance rates and promotion rates continue. The model also will reveal the effect of continuation rates from one program to another and the possible effects of work force participation rates, especially of female participation

rates. (We also see that this future is not inevitable in that these numbers and rates could be changed by changing policies and programs).

4.3.1 Systems Bottlenecks

Compound Model runs serve the planner in general terms by showing the relationship of different parts of a large, complex system, and the effects of one part of the system on another. Hence the model runs help systems planners identify bottlenecks in the system. An example in the Chilean planning model was the indication of a bottleneck in expansion of systems enrollments because of a limited output of trained teachers. The model suggested the necessity of a fairly massive in-service teacher training effort, run on an accelerated basis in the early years and developed into a slower-paced, higher quality effort in later years. The model runs also indicated a bottleneck in the training of skilled workers, unless shorter in-service training programs were offered. These programs were projected at high unit costs, and the model indicated that in the long run the workers could be trained more inexpensively in existing vocational school programs.

One bottleneck which prevents flow through the system to higher levels is the effect of school examinations and restricted promotions. Another bottleneck in manpower supply in many countries is low female participation rates in the work force. (Yet another may be problems in recruiting or training teachers in science and technical fields to man programs that produce scientists, engineers and technicians). Some bottlenecks, e.g., restricted promotion, can be removed quickly with simple policy and program changes. Some require changes in social patterns, changes in the basic structure of the school system, and greatly increased budgets.

4.3.2 Pre-Views of Systems Futures and Lead Time

Model runs serve by showing possible effects several years before they occur. This pre-view provides planners and policy makers with lead time to develop other options and alternatives to change things for the future.

4.4.1 Arraying Policy Options and Alternatives

The model helps the planner array broad sets of policy and program options in a systematic way. In arraying these options the policy maker or planner may see relationships among them that are not otherwise easy to identify. Examples would be tradeoffs between reducing economic growth targets and manpower allocations policies (recruitment and support of expatriate manpower), or tradeoffs between manpower allocations policies (recruitment and high cost support of expatriate manpower) and investments in education to increase the supply of national manpower. Once these broad options are arrayed and understood it is easier to identify more specific policy and program alternatives within the education and training sector.

4.4.2 Policy Options

Running the model may suggest in broad terms certain of these alternatives. Examples might be social promotion policies, nonformal education and training to increase output in a shorter period; or alternative arrangements to provide more coverage in rural areas at reasonable costs.

4.4.3 Pilot Scale Trial of Options and Alternatives

Once options have been identified they can be tried at pilot scale, and the outcomes evaluated, and the effects of extending programs throughout the system can be simulated. Hence, models can serve for experimentation with both large policy and specific program alternatives. Different outputs and

different resource requirements can be estimated. The effects of these alternatives can be traced throughout the system.

4.5.1 Identifying Information Requirements

Running the model also helps to identify information that is essential for planning and policy development. The model does not produce the information or refine it. The model identifies the information required and specifies the form and currency required of the information. To service the model and to keep it running requires the gathering and up-dating of information.

In working with models, planners will realize that no model is ever perfect in form or complete in information. The model can be constantly disaggregated and refined to help planners understand regional and local realities.

4.6.1 The Model and Social Learning in Organizations

More than any other outcome the model should serve as a learning experience for planners, policy makers and decision makers. In the ideal situation learning should increase for all those groups and for all other individuals and groups who are affected by the economic and social events portrayed in the model. In obtaining more current information the planner increases his knowledge of the education and training system. In turn this knowledge can be diffused more widely to inform others. But the increase in knowledge and learning should go beyond an increase in factual knowledge and cognitive learning.

Discussion of the model should promote dialogue among planners and communication with people outside of planning. In this exchange the most important outcome is social learning. In discussing the design and the results of the model with others the planner should increase his own understanding of the assumptions, values and preferences, even biases, that have shaped the model and the real world it portrays. In the discussion the planner can increase the awareness of others as to their assumptions, values and preferences; and, most importantly, he can increase his own awareness of his own assumptions, values and preferences. In this exchange the true value of a general model is realized through social learning.

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APPENDIX I

Table I *Tabular Summary of the Linear Model*
Equations and Inequations

Page References: Theory	Page References: Example	Brief Description
(1) Minimize: $C = \sum_t c_t$		Objective cost function
(2) $c_t = a_{1t}z_t + a_{2t}y_t + \delta_{1t}v_t + \delta_{2t}u_t + \epsilon_{1t}w_t$		Defining the annual cost of operations
(3) $c_t \leq \hat{c}_t$		Budget boundaries
(4) $X_t\lambda_n \leq \pi_t$		Demographic constraint
(5) $X'_t\lambda_a = z_t$		Defining the total enrollment by level
(6) $\Phi_{1t}z_t - \sum_{s=1}^{t-1} \Omega_{ys}z_s - \sum_{s=1}^{t-1} u_s + \sum_{s=1}^{t-1} \Lambda u_s \leq \hat{\Phi}_1$		Number of teachers required
(7) $\Phi_{2t}z_t - \sum_{s=1}^{t-1} v_s \leq \hat{\Phi}_2$		Capacity requirements
(8) $\Omega_{t-1}z_{t-1} + y_{t-1} = z_t$		Educational Technology relations
(9) $y_t \leq H_t z_{t-1}$		Limit on increments on student promotions
(10) $M_t Z_t z_t + w_t - \Lambda w_t + u_t - \Lambda u_t \geq \psi_{t+1}$		Manpower requirements
(11) $w_t \geq \tilde{w}_t$		Limit on number of workers on-the-job trained
(12) $w_t \leq \hat{w}_t$		Limit on number of workers on-the-job trained
(13) $x_{ijt} \geq x_{ij,t-1}$		Non-decreasing number of students
(14) $x_{ijt} \geq \tilde{x}_{ijt}$		Limit on number of students in each age and level
(15) $X_t\lambda_n \geq P_t\Pi_t$		Limit on number of students in each age.

The Objective Function: Discussion

The objective of this model is to minimize the operating expenses at all levels of the Chilean educational system over a given number of time periods, while meeting the requirements of certain desired levels of economic, social, and educational activities. The model gives one optimal solution each trial but, in reality, Chilean planners were looking for a whole family of optimal results with each such result the outcome of a particular set of assumptions made by planners and educators. This has been discussed in the earlier chapter in the section dealing with the use to which the model results were put in searching for alternative educational policies. *To recapitulate briefly — the model was not so much used to derive a single "optimal" or right answer for future educational development in Chile as it was to show in clear fashion a set of possible developments that could be related to different educational policies and practices.*

For example, operating expenditures depend on the particular set of assumptions being used. In certain cases these expenses may be defined as the sum of the current costs plus depreciation charges plus an interest charge on the capital cost. The current design of the model, however, will provide a framework for studying the allocation over a given period of a fixed amount of resources which are already determined from a national point of view, i.e. the allocation of current and capital expenditures assigned in a national plan and projected budget. The possibility of obtaining national or foreign loans to complement Chilean resources will not be explicitly built into the model. Consequently, the expenses will include both the current costs and the investments required to add new capacity to the system as these must be met out of Chilean resources.

The arguments for a discounted cost criterion must also be discussed in terms of the specific policies to be pursued. The present value criterion provides a measure of the total resource used in terms of initial year resources. Implicit in this criterion is a time preference.[3] The use of cost parameters expressed in monetary terms of each year (usually, increasing costs coefficients) forces the fullest utilization possible of initial resources before use of second period resources, exhaustion of second period resources before third period, and so on. In any event, it is necessary to express all variables homogeneously, i.e. if we use the present value criterion, the budget constraint must also be discounted.

The Objective Function: Statement

The present example will focus on how the educational system can use a given flow of annual resources. Exploring the effects of discounting, therefore, seems advisable especially given the large capital investments involved. A consequence of these statements is that the minimization of the total costs for the educational system can be written:

Minimize

(1) $$C = \sum_t c_t$$

where C = the present value (discounted to year one) of the operational expenses of all levels in the educational system during t periods under the below-stated set of constraints. The expenses include

[3] In certain cases it might be necessary to discount the building capacity and the stock of teachers beyond the model time horizon. For example, assuming constant nominal costs and a positive discount rate, the rationale of the model would be to attain the goals using minimal resources from the initial year. In an extreme situation, the solution might show that many students must not attend classes even if teachers, room, and budget are available. The introduction of a minimum budget constraint may, of course, solve the problem, as would adequate social constraints.

current costs and the investment required to add new capacity to the system.

c_t = the present value cost of operating the educational system in year t under the set of constraints for all the periods.

The c_t's represent the aggregate results of choice variables.[4]

The type of cost function used depends upon the specific aims of a given design. The applicability of a linear cost function in this situation has already been discussed briefly. The market prices may change from period to period, but they are not influenced by the educational expenditures in each period. If the market prices are being estimated exogenously, we can define the operational expenses of the system in each period as the sum of the different types of expenses. Therefore, to each period corresponds a linear step of the "piecewise" total cost function.

The total cost of operating the educational system in period "t" is now defined as the sum of the inner products of the following pairs of vectors:

$$c_t = a_{1t} \cdot z_t + a_{2t} \cdot y_t + \delta_{1t} \cdot v_t + \delta_{2t} \cdot u_t + \epsilon_{1t} \cdot w_t \tag{2}$$

where: t = time index of the period. The unit is one year.

c_t = cost of operating the educational system in year t under the set of constraints for all the periods.

z = column vector, $(n \times 1)$, of activity levels z_j. Each activity level represents the number of students in educational level j.

j = educational level index. There are "n" educational levels.

a_1 = row vector, $(1 \times n)$, of coefficients a_{1j}. Each coefficient represents[5] the annual current costs per student in level j.

y = column vector, $(n \times 1)$, of activity levels y_j. Each activity level represents the number of students promoted to educational level j in excess of the historical trend.

a_2 = row vector, $(1 \times n)$, of coefficients a_{2j}. Each coefficient represents the monetary annual costs per student (promoted in excess of the historical trend) in level j. This vector denotes the costs of introducing improvements in the system in order to get a higher rate of promotion in the corresponding level.

v = column vector, $(n \times 1)$, of activity levels v_j. Each activity level represents the number of square meters built in the period for the educational level j.

δ_1 = row vector, $(1 \times n)$, of coefficients δ_{1j}. Each coefficient represents the costs per square meter built in the period in level j.

u = column vector, $(n \times 1)$, of activity levels u_j. Each activity level represents the number of new teachers trained on the job that are required for educational level j in addition to those teachers trained in the formal educational system.

δ_2 = row vector, $(1 \times n)$, of coefficients δ_{2j}. Each coefficient represents the annual costs per teacher trained on the job for level j.

w = column vector, $(n \times 1)$, of activity levels w_j. Each activity level represents the number of workers trained on the job (except teachers trained on the job) at level j in the period "t".

ϵ_1 = row vector, $(1 \times n)$, of coefficients ϵ_{1j}. Each coefficient represents the annual costs per worker trained on the job for level j.

[4] The choice variables include enrollments, increments in enrollments, teachers, buildings, other resources, and on-the-job training activities. They may be rewritten in terms of the choice of variables if the computer storage capacity is exhausted.

[5] A summary explanation of the terms of the cost equation may be helpful. The first term (a_1z) represents the per-student cost multiplied by the number enrolled in that educational level; the second term (a_2y) represents the cost per student of improving the system multiplied by the number of additional students promoted because of the improvement; the third term (δ_1v), represents the building costs; the next term (δ_2v) the costs of training new teachers on the job; and the last term (ϵw) cost for training workers on the job.

The total cost of each period is, therefore, defined as the sum of the costs of enrollments in different levels plus the costs of the increments in enrollments over the historical rates, the costs of adding space capacity, and the costs of training workers and teachers on the job.

The use of the a_{1t} vector evades a detailed treatment of the whole problem of technology of education. Each component of the cost vector includes teacher's time, salaries, costs of teaching aids, overhead, and the rest of the input, costs.[6] There would be no computational problem to disaggregate the a_{1t} vector.[7] Average historical costs are available for estimating the a_{1t} vectors in the Chilean situation. As most countries also have cost figures, the estimation of these parameters is not a major probelm in applying the model to other countries.

The vector y_t incorporates the estimated effects of improvement through more intensive education which aims at increasing student flows through the system. Examples in Chile are providing specialized teachers, guidance services to assist in the solution of personal problems, use of better teaching aids, and specially designed curricula. All of these steps have increased the promotion rates in Chilean schools during the period of the educational reform. *The exact form of this improvement has yet to be assessed, and the model will not serve to make this assessment. It merely takes into account in general terms what can be observed. Hence the results can not be precise.*

The vector y_t can also be expressed as a partitioned vector, each y_{it} element representing a vector of increments in the promotion of students of age level i from educational level $j-1$ to level j. The costs of the devices for improving the promotion, e.g. remedial reading, teaching aids, or supervision can be different in each age group. If this is the case, the a_{2t} vector must be partitioned in the same way. The assumption that the level of a "y" activity can be varied is an assumption of fact which is generally true though not precisely determined. Direct knowledge obtained through an evaluation of the results of the Chilean educational reform will be useful to test the reality of these assumptions and to improve future use of the model. Improvement in educational technology might have a high price.[8]

Data for the other coefficient vectors (δ_1, δ_2, ϵ_1) are available for Chile. Average cost of general construction (δ_1) is available in most countries. Variability increases when there are many regional differences and tends to decrease when standard pre-assembly procedures are used. In most countries there are on-the-job training programs for up-grading workers, so their costs (ϵ_1) can also be computed. Application of the model to other countries would present problems in the determination of the costs of training teachers on the job (δ_2). Other countries have accelerated teacher formation, but few have gathered data in a form in which costs can be computed.[9]

[6] Davis (1966) presents a good number of operational approaches to derive coefficients of costs to be used in more detailed models.

[7] The lack of knowledge of the effects of inputs on the educational outcomes, however, proves to be a serious hurdle.

[8] Data for estimating the a_{2t} vectors is usually poor. Information is lacking, even in the developed countries (Abt, 1967). In the meantime, educated guesses have to be used.

[9] It will nevertheless be shown later that even if precise data are unknown the model may be a useful framework for the study of the effects of possible new educational policies.

Financial Identities and Constraints

The operational expenses in any one year cannot exceed the total amount of resources available. This condition may be expressed:

(3) $c_t \leq \hat{c}_t$

where $\hat{c}_t$ = maximum amount of resources the educational system can expend in year t.

This boundary condition limits the choice variables to values which are judged to be politically and administratively feasible. These constraints, therefore, can also be considered as social constraints. One way of estimating the $\hat{c}_t$ would be to relate them to the amount of GNP in each year.[10]

$$\hat{c}_t = \beta_t \, \mathrm{GNP}_t$$

where β_t = percentage of GNP devoted to education.

Constraint 3 can be disaggregated into the different types of costs that make up the total cost of operation described in constraint 2. This disaggregation might show the costs of the various conventional inputs, teachers salaries, instructional materials, and the like that go into education. Operational expenses can also be disaggregated by educational levels. In this case, it might be stated that the total cost devoted to primary education cannot exceed a given percentage of total costs.

Demographic Relations

The total number of students of a given age must be less than or equal to the population of the same age. The constraint can be expressed:

(4) $X_t \lambda_n \leq \Pi_t$

where t = time index of the period. The unit used is one year.

X = matrix, $(a \times n)$, of activity levels x_{ij}. Each activity level represents the number of students of age level i in educational level j in the period "t".

i = age level index. There are "a" age levels.

j = educational level index. There are "n" educational levels.

λ_n = column vector, $(n \times 1)$, of unit elements = $[1, 1, \ldots, 1, 1]'$.[11]

Π = column vector, $(a \times 1)$, of π_i values. Each value indicates the number of children in age group i in the period.

This boundary constraint reflects the population balance in each year. The column vector, however, could be premultiplied by a diagonal matrix in order to reflect geographical peculiarities of some sectors of the population or to reflect social goals, e.g., to educate a given percentage of children in the primary education entrance age group in period t.

The disaggregation of enrollments by age is given as an example of working in more than one dimension. The age dimension was chosen because it will be used in the numerical implementation. In other situations it might make sense to disaggregate by sex, regions, source of financing, specialization, religion, main language, and urban-rural difference.

[10] Other ratios can be computed, e.g. setting c_t as a percent of the total budget. Rates of growth can also be incorporated to reflect changes in social decisions.

[11] Multiplication by λ_n simply serves to collect the grade enrollments across one age group i which must be less than or equal to the number in the group i as (4) states.

Identities for Enrollments by Levels

The activity level of each one of the x_{ijt} variables is chosen in order to minimize the objective function. In each solution of the model many of the x_{ijt} variables will be included. For the sake of simplicity a "z" variable was used previously. The z variable represents the number enrolled in each grade. The students of different age levels in educational level "j" add up to the z_j value. Thus the X matrix is related by definition with the vector z:

$$X'_t \lambda_a = z_t \tag{5}$$

where λ_a = column vector, $(a \times 1)$, of (identical) unit elements.[12]

Constraint 5 is an identity. The equation states that the z vector can be substituted for the left hand side of equation 5.[13]

Stock of Teacher's Relations

The relationship between annual flows and stocks must be carefully examined between time periods. In each period there must be available a sufficient number of teachers to handle the enrollments which means a number that equals the product of the enrollments in the different levels and the corresponding teacher-student ratio which appears as a coefficient in the model. The constraint that the total number of teachers required must be less than or equal to the total number available can be written as:

$$\Phi_{1t} z_t \leq \hat{\Phi}_1 \qquad \text{for } t = 1$$

(6)

$$\Phi_{1t} z_t - \sum_{s=1}^{t-1} \Omega_{ps} z_s - \sum_{s=1}^{t-1} u_s + \sum_{s=1}^{t-1} \Lambda u_s \leq \hat{\Phi}_1 \qquad \text{for } t \geq 2$$

where t = time index of the period. The unit used is one year.

Φ_1 = diagonal matrix, $(n \times n)$, of coefficients ϕ_{1j}. Each coefficient represents the number of teachers per student in educational level j. Each coefficient, therefore, corresponds to the reciprocal of the student-teacher ratio.

z = column vector, $(n \times 1)$, of activity levels z_j. Each activity level represents the number of students in educational level j. There are "n" educational levels.

Ω_p = matrix, $(n \times n)$, of coefficients ω_{pj}. Each coefficient represents the graduation rate for teachers going to teach in educational level j. Most of these coefficients are usually going to be zero.

u = column vector, $(n \times 1)$, of activity levels u_j. Each activity level represents the number of additional teachers trained on the job required for educational level j in addition to those teachers trained in the formal educational system.

Λ = matrix, $(n \times n)$, with 1's in the diagonal above the main one and zeros elsewhere. The product of this matrix and a column vector is a new vector with the j element being the $j+1$ element of the previous vector.[14] The last element of the new vector is zero:

[12] The λ_a collects the number in various ages across one grade group just as λ_n collected the number in various grades across one age group.

[13] If the computer capacity is exhausted, constraint 5 can be eliminated by rewriting the constraints where the "z" variable was used.

[14] Multiplication by the matrix Λ simply shifts teachers up to the next higher level meaning they are on-job trained in one level and pass on to the next highest.

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & 0 & & 1 & 0 \\ 0 & 0 & 0 & & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$\hat{\Phi}_1$ = column vector, $(n \times 1)$, of coefficients $\hat{\phi}_{ij}$. Each coefficient represents the number of teachers teaching in educational level "j" in the beginning of the initial period.

The first term of the left hand side of constraint 6 expresses the total number of teachers required in the period at level j. Usually, the number of teachers' categories will be less than "n". In this case, the first term of the left hand side must be adjusted in order to group all the teachers required in each category.

The second term of the left hand side expresses the total number of teachers graduated since the beginning of period $(t=0)$. The summation starts from year 1, that is, the graduates of previous years have been considered in the initial stock $\hat{\Phi}_1$. The final limit, $t-1$, implies a one year lag. Different lags, however, can be assumed for the time required to upgrade each level. In certain cases the lags may be differentiated by the training level of the active teachers in a given educational level (Beeby, 1966).

The third term of constraint 6 shows the number of teachers that reach a given category through on-the-job training. The fourth term shows the number of teachers that left the j category for the $j+1$ category through a similar upgrading process. This process presumes that teachers' advancement is a step-by-step event through an ordered sequence of levels.

The whole expression can be adjusted to represent different situations. For example, in order to take into account the fraction of the graduates that do not join the teaching staff, the second term of the expression would become:

$$\sum_{s=1}^{t-1} M_{ys}\Omega_{ys} z_s \qquad \text{for } t \geq 2$$

where M_y = matrix, $(n \times n)$, of coefficients μ_{yj}. Each coefficient represents the proportion of the total number of graduates going to teach in educational level j.

Those who fail to become graduate teachers in a given year remain in the student flow. They will be taken into account through inequality 10.

Terms can also be adjusted for retirements and deaths. If necessary, inequality (6) can be rewritten to account for them. The stock in period t is presented on the right hand side. For a given level j the constraint for $t \geq 2$ would be:

$$\phi_{ijt} z_{jt} \leq \hat{\phi}_{1j}(1-\nu_0)^t + \sum_{s=1}^{t-1} \omega_{ijs} z_{js}(1-\nu_1)^{t-s-1} + \sum_{s=1}^{t-1} u_{js}(1-\nu_2)^{t-s-1} - \sum_{s=1}^{t-1} u_{(j+1)s}(1-\nu_3)^{t-s-1}$$

where ν = annual rate of retirement of the teaching force over the planning period for each one of the different sources of the stock of teachers. The rates of retirement are functions of the age composition of each group and can change over time.

Building Capacity Relations

The total spatial need in each year is defined as the number of students in educational level j multiplied by a specified area per student. The result must be less than or equal to the available space. The constraint may be expressed as:

$$\Phi_{2t} z_t \leq \hat{\phi}_2 \qquad \text{for } t = 1$$

(7)

$$\Phi_{2t} z_t - \sum_{s=1}^{t-1} v_s \leq \hat{\phi}_2 \qquad \text{for } t \geq 2$$

where t = time index of the period. The unit used is one year.

Φ_2 = diagonal matrix, $(n \times n)$, of coefficients ϕ_{2j}. Each coefficient represents the number of square meters of space per student in educational level j. Expressed in terms of the space of the standard classroom, the coefficient, therefore, corresponds to the reciprocal of the student-classroom ratio.

v = column vector, $(n \times 1)$, of activity levels v_j. Each activity level represents the number of square meters of classroom space built in the period for the educational level j.

$\hat{\phi}_2$ = column vector, $(n \times 1)$, of coefficients $\hat{\phi}_{2j}$. Each coefficient represents the number of square meters available in the beginning of the initial period for the educational level j.

As in the previous constraint, if the number of different building types is less than "n", the Φ_2 matrix must be adjusted. A one year lag has also been considered. The "v" terms can be adjusted for depreciation. The next expression shows the adjusted constraint for a given level j and for $t \geq 2$:

$$\phi_{2jt} z_{jt} \leq \hat{\phi}_{2j}(1-i_0)^t + \sum_{s=1}^{t-1} v_{js}(1-i_1)^{t-s-1}$$

where i = annual rate of depreciation of the buildings stock for each one of the different sources of the stock. If the rates of depreciation are functions of the age distribution of each building type source, they can also change over time.

Educational Technology: Relations and Constraints

Educational technology and other school conditions are reflected by repetition, promotion, and drop-out rates which include the effects of mortality. These rates completely determine the pattern of movement of enrollments through the system. The historically derived relationships between the ratios and enrollment, however, have been modified to allow for economically efficient improvements, as follows:

(8) $$\Omega_{t-1} z_{t-1} + y_{t-1} = z_t$$

where t = time index of the period. The unit used is one year.

Ω = matrix $(n \times n)$, of transition ratios (probabilities).

The ω_{jj} elements of the matrix are the repetition ratios of the educational level j, while the $\omega_{j+1,j}$ elements correspond to the promotion ratios from educational level j to the level $j+1$. (The drop-out and graduation ratios from each level are included in equation 10. Part of them are also included in equation 6.): [15]

[15] If p equals promotion rate and r equals repetition rate, the fraction of original entrants that will eventually graduate from a system of j levels is given by the continuous product:

$$\frac{p_1}{1-r_1} \cdot \frac{p_2}{1-r_2} \cdots \frac{p_j}{1-r_j}$$

$$\Omega = \begin{bmatrix} \omega_{11} & 0 & \dots & 0 & 0 \\ \omega_{21} & \omega_{22} & \dots & 0 & 0 \\ 0 & \omega_{32} & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & \omega_{n-1,n-1} & 0 \\ 0 & 0 & \dots & \omega_{n,\ n-1} & \omega_{nn} \end{bmatrix}$$

y = column vector, $(n \times 1)$, of y_j activity levels. Each activity level represents the number of students promoted to educational level j in addition to the historical trend (overflow).

This equation provides one of the dynamic links between consecutive periods. The expression, being a recurrent formula, transforms short-term student achievements into the long-run distribution of student educational attainment. The transition matrix propagates present improvements in the promotion rate to the student distributions in future years. These links can be modified according to the situation that is being assessed.

The transition ratios depend on age and to reflect this equation 8 can be expressed as:

$$\Omega_{1t}\, x'_{1t} + \Omega_{2t}\, x'_{2t} + \dots + \Omega_{at}\, x'_{at} + y_t = z_{t+1}$$

where Ω_i = matrix, $(n \times n)$, of transition ratios of the age level i student population. The definition of each of these matrices is similar to that of the Ω matrix described previously.

x'_{it} = $(n \times 1)$ column vectors corresponding to the rows in matrix X.

The y-vector which reflects improved promotion rates permits one to introduce some flexibility into the solutions of the model and to achieve useful solutions to linear programming problems which had been previously intractable. This vector can, for instance, be disaggregated into age categories. The level of activity shown by these variables would then give the increment in the proportion of students historically promoted at each age. The associated improvement in educational technology would be described by the cost weights given to these variables. Given a wide spread of ages within grades, it is helpful to disaggregate because students at the extreme points reflect retardation, deprivation, and other special problems. Hence they may need educational treatment, which is different from that provided to the cohorts making average progress. For most cases this special treatment or enrichment is more costly.

If the "y" variables are considered as policy instruments, the improvements implicit in their values are valid only in a given range of these values. Remedial courses, for example, may solve the problems of a fraction of the participating students in the courses. The same is true for other improvements in educational technologies. In practice in Chile as elsewhere, the relationship between application of intensive educational technologies and their quantitative effects has not been adequately rsearched and measured. There are two sources for establishing the grounds that there are such measurable effects although there are no servicible results to guide the planner in the general case. The voluminous literature of educational research, in the United States, Europe and elsewhere, provides partial answer for specific circumstances and modalities. Lately, following the Coleman study (1966), economists have been studying the relationships between what goes in and what comes out of an educational process through analysis of educational production functions (Levin, 1968).

A subjective statement of the upper limits on these effects can be expressed in either absolute terms or as a percentage of the enroll-

ments of the previous period. The next constraint corresponds to the latter case:

(9) $y_t \leq H_t z_{t-1}$

where H = diagonal matrix, $(n \times n)$, of coefficients η_j. Each coefficient represents the maximum improvement on promotions from educational level j.

The coefficients η_j are related to the cost coefficients a_{2j} in the objective function; however, the annual cost of student promotions above the number predicted by the historical trend, equals the sum of a_{1j} and a_{2j}. It may be hypothesized that the a_{2j} parameters tend to vary as the y-variables increase their values which implies that the larger the improvement in technology, the larger the cost coefficient. This is similar to most input-output type relationships in that the inputs eventually grow at an increasing rate.[16] The practical consequences of this are known to educators although the fact that beyond a certain point you get a less clear-cut return for your money is not a popular subject for discussion.

The educational technology constraints are especially important in countries that are semi-developed such as Chile. In these countries, most of the children in the primary age population attend school, so the magnitude of student inputs to the educational system cannot increase substantially. This necessitates the use of more refined methods to meet the growing requirements of the country, e.g. improvement of promotion and reduction of drop-outs. Because of the limitations on quantitative solutions to this problem one is dealing, for the first time, with "quality" problems, but in their simplest aspects. The qualitative solutions will be expanded on and refined in the quadratic model to be developed in the second half of this book.

Manpower Requirements Relations

The labor force requirement estimates can be made using different methods. The manpower approach and the rate of return correspond to the primal and dual expressions of the same problem (Blaug, 1966 and Daniere, 1965). In both cases unemployment problems and historical increments in productivity or in learning-by-doing effects on productivity must be considered.

The demand for labor, being exogenous, does not allow for substitutions between number and level-of-skills of workers in each run of the model. The inflexibility of the manpower approach itself, where lack of substitutibility among classes of education and training is assumed, has been generally criticized in Chile as elsewhere However, in Chile as elsewhere, planners are aware of this and treat their results accordingly. Running for different sets of assumptions may be a partial answer to this problem, while reassigning surpluses generated in higher levels to lower labor force levels will be suggested as a second possibility within the present structure of the model.

Though analysis of labor demands is beyond the mandate of most educational planners, some attention must be paid to the interaction effects between decisions made within the structure of the model and the demand for various classes of labor. For example, manpower forecasts do not take into account the relatively fixed relationships between enrollments in two adjoining educational levels in consecu-

[16] As was mentioned before the choice of the type of relationships depends on more adequate data than that actually available. Through piecewise functions, however, the model provides a good substitute for non-linear expressions (Marowitz and Manne, 1957).

tive years nor the almost constant drop-out rates. The enrollment in any level j is caught between a manpower demand signal of some future year and the inflexibility introduced by the limitations on enrollment in level j minus one. These rigidities send out waves that may disturb the whole system. Because of the fixed and cumulative nature of education it is not easy to shift transition coefficients rapidly. Most of the flexibility comes at the first year of entry point of any level when there is a pool or a backlog of potential entrants and the number admitted is under policy control. A quick policy response at the upper levels may not be possible, however, and in such situations there are what some planners call bottle-necks — either not enough potential entrants or not enough potential places open for entrants. Drop-out rates also have an effect when they interact with future manpower demands. One must, therefore, be wary of accepting the forecasts without checking the gaps observed in each one of the levels caused by the excluded relationships.[17] As this model provides a framework for disciplined judgment, the educational planner can utilize it to make the forecasting of manpower requirements more meaningful.

The labor supply is generated by two sources. The first is the proportion of the leavers from the educational system that want to join the labor force. As some jobs require higher skills and more experience than the educational leavers possess, the second portion of supply comes from on-the-job training of labor force members. The constraint can be written:

(10) $$M_t Z_t z_t + w_t - \Lambda w_t + u_t - \Lambda u_t \geq \psi_{t+1}$$

where M = diagonal matrix, $(n \times n)$, of the proportions u_i of the total leavers of the j educational level actually joining the labor force.

Z = diagonal matrix, $(n \times n)$, of the drop-out rates ζ_j from the j level of the educational system.

z = column vector, $(n \times 1)$, of activity levels z_j. Each activity level represents the number of students in educational level j. There are "n" educational levels.

w_t = column vector, $(n \times 1)$, of activity levels w_{jt}. Each activity level represents the number of workers (not including teachers) trained on-the-job for level j in period t. For $t=0$, it represents the initial stock of workers trained for level j.

Λ = matrix, $(n \times n)$, with 1's in the diagonal-above the main one and zeros elsewhere. It is the same shift mechanism described on page 72.

u = column vector, $(n \times 1)$, of activity levels u_j. Each activity level represents the number of additional teachers trained on-the-job that are required for educational level j in addition to those teachers trained in the formal educational system.

ψ = column vector, $(n \times 1)$, of coefficients ψ_j. Each coefficient represents the additional number of workers (including teachers) with educational level j required in the period.

The left hand side of equation 10 describes the labor supply. The first term of the left hand side shows the number of leavers from educational level j that join the labor force. The second term adds to level j the number of workers up-graded from level j–1. The third term subtracts the number of workers from level j on-the-job trained for level j+1. Similarly, the fourth and fifth terms express the effect of teachers' on-the-job training. The right hand side corresponds to the manpower requirement vector. A one year lag is assumed.

[17] In order to match the higher manpower level requirements surpluses may be generated in the lower levels. Once the educational demand has been created it is almost impossible to reduce it.

If supply matched demand in each level and in each period, no adjustment would be necessary; however, many times there appears a surplus of manpower in a given level. In most cases the manpower planner should check the forecasts, but in certain cases it is interesting to observe the effects of a reallocation of the surplus to jobs requiring lower qualifications. The next equation shows how to do this:

$$m_t Z_t z_t + w_t - \Lambda w_t + u_t - \Lambda u_t + \Lambda s_t - s_t = \psi_{t+1}$$

where s = column vector, $(n \times 1)$, of activity levels s_j. Each activity level represents the surplus in educational level j devoted to level $j-1$ except s_1 which corresponds to a slack activity. The "s" variable then functions to adjust and balance the equation between supply and demand.

In this scheme, the use of an equation instead of inequality 10 may not introduce a stronger constraint. In fact, this special use of the slack variables tends to reduce the requirements. All the qualified labor force will be employed before the unskilled workers. The model cannot, however, take into account the fact that the surpluses in each level will move salaries downward and, consequently, more people will be in demand.[18]

If the economic requirements are smaller than the social requirements, there would be overqualification at certain levels of the labor force or more skilled people unemployed. There would also be some incentive to emigrate.

Institutional Constraints on Manpower Training

The start-up problem of an on-the-job training system is one source of institutional constraints. The supply of trained workers is limited by the need for a large initial investment in specialized instructors and equipment and, in underdeveloped countries, an initial lag of three to four years for training the instructors is also common. Technological change, on the other hand, requires a continuous retraining of some sectors of the labor force. Given these facts it seems prudent to set a minimum activity level in the training establishment that ensures the maintenance of a core staff with the capability of training the workers, since operations cannot be built up rapidly in a short time. The feasible range for the on-the-job training capacity will, therefore, be:

(11) $w_t \geq \bar{w}_t$

(12) $w_t \leq \hat{w}_t$

where $\bar{w}$ = column vector, $(n \times 1)$, of coefficients $\bar{w}_j$. Each coefficient represents the minimum number of workers it is necessary to train in any period to maintain a core staff of trainers and supervisors and maintain equipment to permit a rapid expansion of operations up to full capacity.

$\hat{w}$ = column vector, $(n \times 1)$, of coefficients $\hat{w}_j$. Each coefficient represents the maximum number of workers it is possible to train in any period given the assumed average available capacity.

It is also possible to rewrite the upper limit in terms of the number of workers trained in the earlier period. If a 20 percent increase is considered the limit, constraint 12 becomes:

(12a) $w_t \leq 1.2\, w_{t-1}$

Similar constraints can be stated in order to take into account institutional problems found in other aspects of the educational system such as unemployed teachers, unused plant capacity, level of operations of the building industry or the teaching aids industry, or teachers unavailable for on-the-job training. Even the constraints on the "y"

[18] This movement along the labor demand curve could be examined through an iterative use of the model if the price elasticity of labor were known or if it could be subjectively estimated.

variables are, in part, due to institutional rigidities. If one of these institutional constraints operates to reduce the space of feasible solutions, the set of assumptions must be reconsidered.

Social Goals Constraints

The social demand is one of the two sources of educational demand included in the model. These social demands engender a set of constraints that can reflect a social point of view. Social goals can be represented in the model. For example, if a number of students of "normal" age i in grade j must continually increase over time or at least remain non-decreasing, the constraint would be:

(13) $$x_{ijt} \geqq x_{ij(t-1)}$$

The social goal can be more precise and reflect system improvement, e.g. the number of students of normal age i in grade j must be greater than the estimations for year t based on the historical trends:

(14) $$x_{ijt} > \bar{x}_{ijt}$$

where $\bar{x}_{ijt}$ = estimated number of students of age level i (normal age for grade) in educational level j in period t if historical trend continues.

For some ages it would be reversed, again to reflect system performance improvement over trend, i.e. fewer over-age children in the grades:

$$x'_{ijt} < x'_{ij(t-1)}$$

This inequality states that for those groups above the normal age (x') the number of students has to be smaller than the estimations based on historical trends, e.g. the students that are three years retarded over the normal age i in grade j must be less than the historical trend in year t.

Another social goal could be to use all the available teacher capacity. This goal can be stated by expressing constraint 6 as an equation instead of an inequality. Social goals related with better student-teacher ratios may also be represented through the ϕ parameters of constraint 6.

Some social goals are expressed as at least a given percentage of children in age group i must attend school. The constraint may be written as:

(15) $$X_t\lambda_n \geqq P_t\Pi_t$$

where P = diagonal matrix, $(a \times a)$, of p_i coefficients. Each coefficient represents the minimum percentage of the i age group in the population that must attend school in the period.

This constraint corresponds to the lower bound of constraint 4. If the boundaries tend to approach each other, the social goal may be expressed at the limits as to educate a given percentage of children in each age group, in a given range of ages. In this case the constraint becomes:

$$X_t\lambda_n = \hat{P}_t\Pi_t$$

where $\hat{P}$ = diagonal matrix, $(a \times a)$, of the coefficients $\hat{p}_i$. Each coefficient represents the percentage of the ith age group of the population that must attend school in the period.

As a final example of a social goal it would be possible to represent the decision to devote to a given type of education, e.g. university, not less than 15 percent of total costs. The inequality would be:

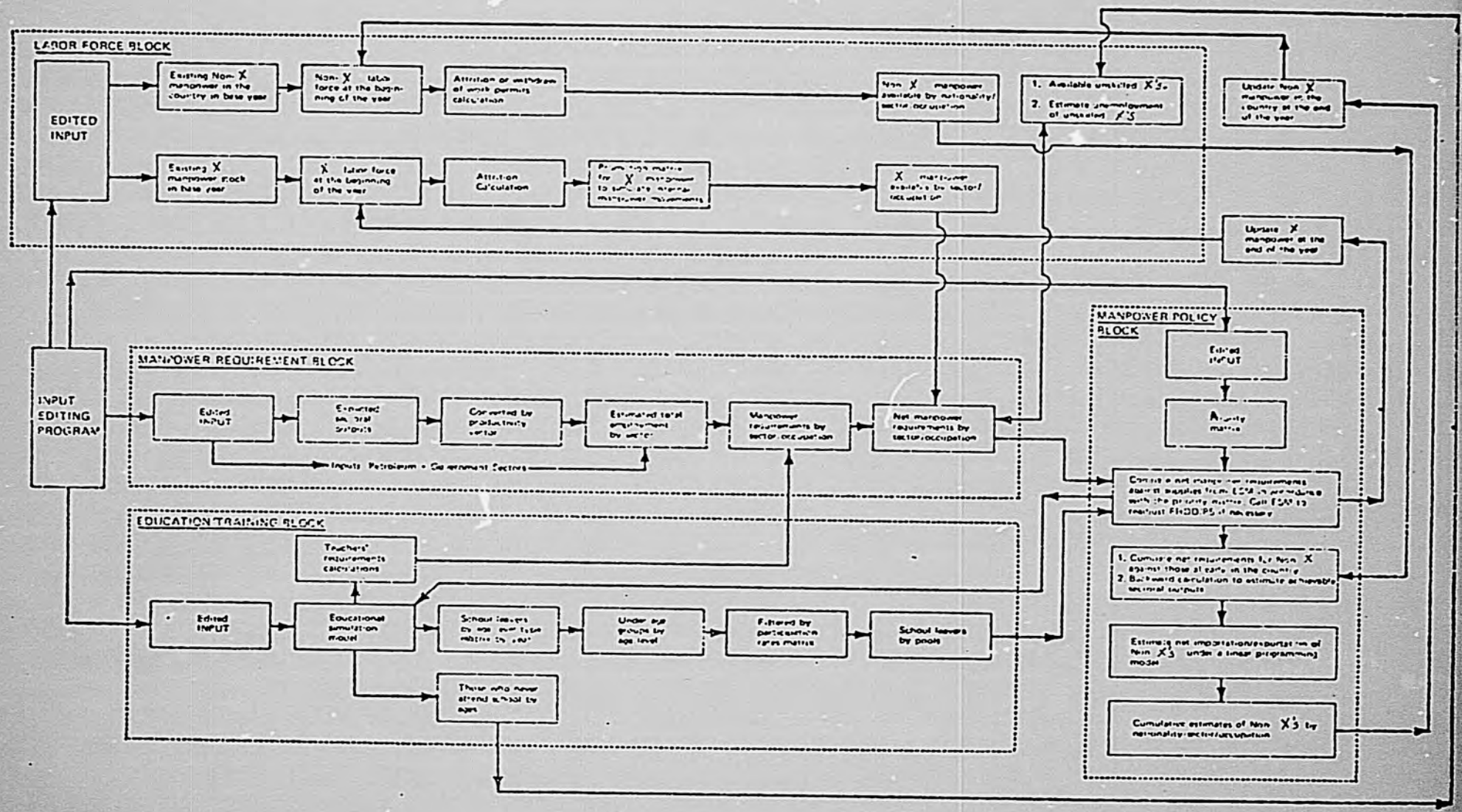
$$c_{nt} \geqq .15c_t$$

where c_t = cost of operating the educational system in year t under the set of constraints for all the periods.

c_{nt} = cost of operating the nth level of the educational system in year t.

In the actual Chilean planning exercise some of these possibilities of using the model were explored peripherally to the main running of the model; others were not. Whether they were actually incorporated into the analysis or not, they are sketched out here for their general interest in case administrators or planners later had some specific interest in exploring them.

APPENDIX II

MANPOWER PLANNINNG STUDY (II)

SKELETON OF THE COMPOUND MODEL