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Location Analysis: A New Tool for Health Planners

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TABLE OF CONTENTS

SECTION	TOPIC	PAGE
	Table of Contents	i
	List of Illustrations	ii
	List of Tables	iii
1	Introduction	1
2	An Example of Covering Reasoning - The Set Covering Problem	8
3	Covering and Policy Analysis - Use of the Maximal Location Covering Method	17
4	Notions of Geographical Relations for Use in Health Access Analysis	25
5	Alternate Demand Assumptions	32
6	Alternate Assumptions Concerning Facilities and Health Services	36
7	Alternate Objectives for Health Facility Planning	39
	Acknowledgements	40
	References	41

LIST OF ILLUSTRATIONS

NUMBER	ILLUSTRATION	PAGE
1-1	A Trade-off Curve Between Accessability and Number of Facilities	5
1-2	A Service Map	7
2-1	A Network of Nine Communities	9
2-2	Lines Drawn Around the Dominating Demand Sets	12
2-3	The Trade-off Between Maximum Service Time and the Number of Facilities	14
3-1	Fifty Five Towns	18
3-2	Set Covering Solution for Maximum Time of 15 Minutes	21
3-3	Maximal Covering Solution for S=15, T=10	22
3-4	Maximal Covering Solution with Maximal Service Time of 10 Units (S=10)	24
4-1	A Network of Nine Communities	27

LIST OF TABLES

NUMBER	TABLE	PAGE
1-1	Data Needs for a Sample Problem	2
1-2	Results From a Sample Problem Solution	3
2-1	The Range of Service of Potential Facilities	10
2-2	Coverage of Demand Sets by Potential Facility Sites	13
3-1	Graphic and Population Data for Maximal Covering	19
4-1	Alternate Geographical Assumptions	25
5-1	Geographical Representations of Demand	32
5-2	Alternate Demand Surrogates	33
6-1	Alternate Notions of Services and Facilities	36
7-1	Alternate Public Access Objectives	39

SECTION ONE: INTRODUCTION

Human health is related to many factors, such as the absence of disease vectors, social and economic institutions, environmental stress, nutrition, exercise, employment, genetic influences, preventative and curative health systems, and the access of people to care. While access is but one of many factors, it can be important to health outcomes.

Over the past eight years there have been significant theoretical advances made by quantitative geographers and applied mathematicians on problems relating to the organization of public services [1] (11). There have also been numerous projects to apply this theory to real public sector problems [2]. The nature of these applications involve logistics of providing public services which achieve public objectives of access, utilization, cost, efficiency, and distributional equity.

The logistical issues include:

---the location of facilities in time and space
---the routing of a service through time and space
---the creation of service districts for a public service in time and space

This report describes methods to locate facilities with respect to demand patterns for health services so as to maximize the access of people to health care. This set of methods is called *covering techniques*. They are termed covering techniques because they try to cover people with services within time or distance restrictions. These methods are of interest because they require relatively small quantities of usually-available information, are relatively inexpensive to use, and yield a rich variety of valuable results. Some of these methods are simple enough to be used by a motivated and patient individual, armed only with some basic data on population and distances, a pad of paper, and a pencil. Other methods require a computer to perform the complicated calculations. This report explains what the methods can and cannot do.

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[1] Brackets, [#], denote footnotes which are listed at the bottom of each page. Underscored numbers in parenthesis, (1), refer to references listed at the end of this report.

[2] See references (1) through (24).

Data Needs

The information needed for covering analysis is: (a) the nature of demand for services [3]; (b) the nature of supply of services; (c) a description of the geographical relationships between demands; and (d) a statement of the public objectives to be achieved. Table 1-1 illustrates the data needs for a sample problem.

Table 1-1

DATA NEEDS FOR A SAMPLE PROBLEM

Category of Information	Sample Information
nature of health service	ambulance service for emergencies
nature of demand	a call from a town that there is a need for an ambulance
geographical relationships	nine towns dispersed over a rural area
objectives of system	minimize the number of ambulances [quantity of service] that are required to cover all the towns [access to service] to meet the smallest response time (the time from the call to the arrival of help) [quality of service]

Much of this report describes the variety of problems which involve alternate conceptions of supply, demand, objectives and geographical relationships. The second section describes set covering. Section three discusses maximal covering location (hereinafter known as MCL), the next level of covering complexity. Section four shows many definitions

---

[3] The word *demand* can have several meanings. One meaning is *need*, all those who could benefit from service. Another concept is the number who *would* present themselves for service at a given price. A third notion is the number who *do* seek service in a given place in a specified period of time.

of geographical relationships that can be utilized with MCL analysis. Section five describes alternate concepts of services compatible with the MCL approach. Section six extends MCL applicability to alternate notions of demand for health services. Section seven is concerned with alternate public objectives.

Results of Analyses

This report is designed to show a potential user the wide applicability and utility of covering techique results. The four outputs from covering analysis are numerical statements about (a) the quantity of service; (b) the quality of service [4]; (c) accessibility of service; and (d) maps of geographical region served by each facility. Two forms of the output - tradeoff curves and service maps - are of particular interest, and will be used in later chapters. Table 1-2 illustrates the results from such an analysis.

Table 1-2

RESULTS FROM A SAMPLE PROBLEM SOLUTION

Issues	Sample Results
quantity of service	Two ambulances can serve...
access of service	...an emergency call from any town...
quality of service	...within fifteen minutes maximum.
service districts	the optimal service regions for the two ambulances are: ambulance 1: towns 1,2,3,4,5,6 ambulance 2: towns 4,6,7,8,9
location of service	ambulance 1 located in town 3 ambulance 2 located in town 8

[4] If and only if quality of service can be defined solely in terms of access.

Tradeoff Curves

Between any set of objectives, there can exist a tradeoff surface describing how much of one goal must be sacrificed to achieve given values for other goals [5]. When there are two objectives the tradeoff surface appears as a curve as in figure 1-1 (6) [6]. Figure 1-1 shows the tradeoff between two goals: access and quantity of service. Access is defined as the fraction of the total population covered by the health service within two time units. Quantity of service is defined as the minimum number of facilities that must be built in order that the access will be achieved. A large number of facilities, twelve, can cover 100 percent of the population within the time limit. A small number of facilities, one, covers 30%, a much smaller fraction of the population. There is thus a tradeoff between quantity of service and accessibility.

This tradeoff curve shows the limits of physical feasibility. Any point to the left of the curve, for example point A, is infeasible. Any point to the right and below the curve, for example point B, is inferior to some point on the curve, such as C. Thus the points on the curve and only those points are of interest as policy alternatives.

The choice of one point on the curve implies the imposition of a set of preferences for one objective relative to other goals. The implicit tradeoff can be made explicit by drawing a tangent line to the curve at the point selected as a 'best compromise' solution. For example, assume C is selected as the best compromise solution. The slope of the tangent line to the point is 60°. This implies that the decision maker is indifferent at the margin between the cost of an additional facility and the increased coverage of 15% of the population.

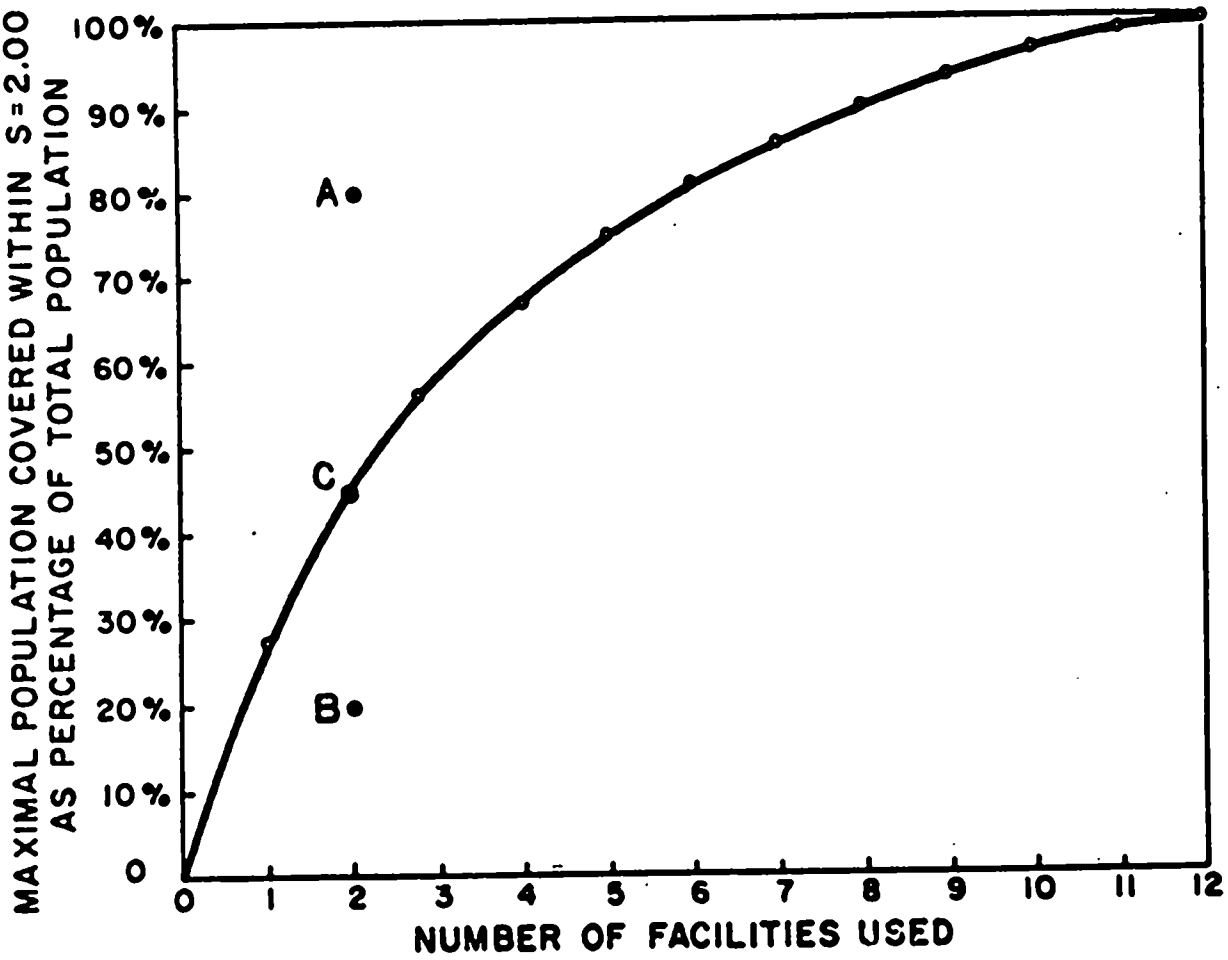
A tradeoff curve not only makes explicit the implications of any decision; it can help structure a decision. It is possible to read off from figure 1-1 the incremental value (in terms of additional population covered) of one extra facility. Moving from two to three facilities

---

[5] The concept of tradeoff curves may or may not be useful to the policy maker. A tradeoff curve is only a statement of the change in the rate at which one objective is improved and another goal suffers. It has meaning to some persons, but may be irrelevant to others.

[6] Data from 30 node problem in (20).

FIGURE 1-1: A TRADE-OFF CURVE BETWEEN ACCESSIBILITY AND NUMBER OF FACILITIES



increases coverage from 46 to 57 percent. One additional facility is worth eleven percent incremental coverage.

As might be expected, the first increments of population coverage are less costly than the last. Moving from one to two facilities, the incremental population covered is 18% (28% to 46%). But moving from eleven to twelve facilities, the coverage gained is only 2% (98% to 100%). Thus it is possible to gain insight into the cost-effectiveness (in terms of population coverage) of a new facility.

Service Maps

Access analysis does more than calculate coverage; these techniques can also site a facility and determine its service district. Associated with any point in figure 1-1 is a service map, such as figure 1-2, which shows how people and services are related in geographical space [7].

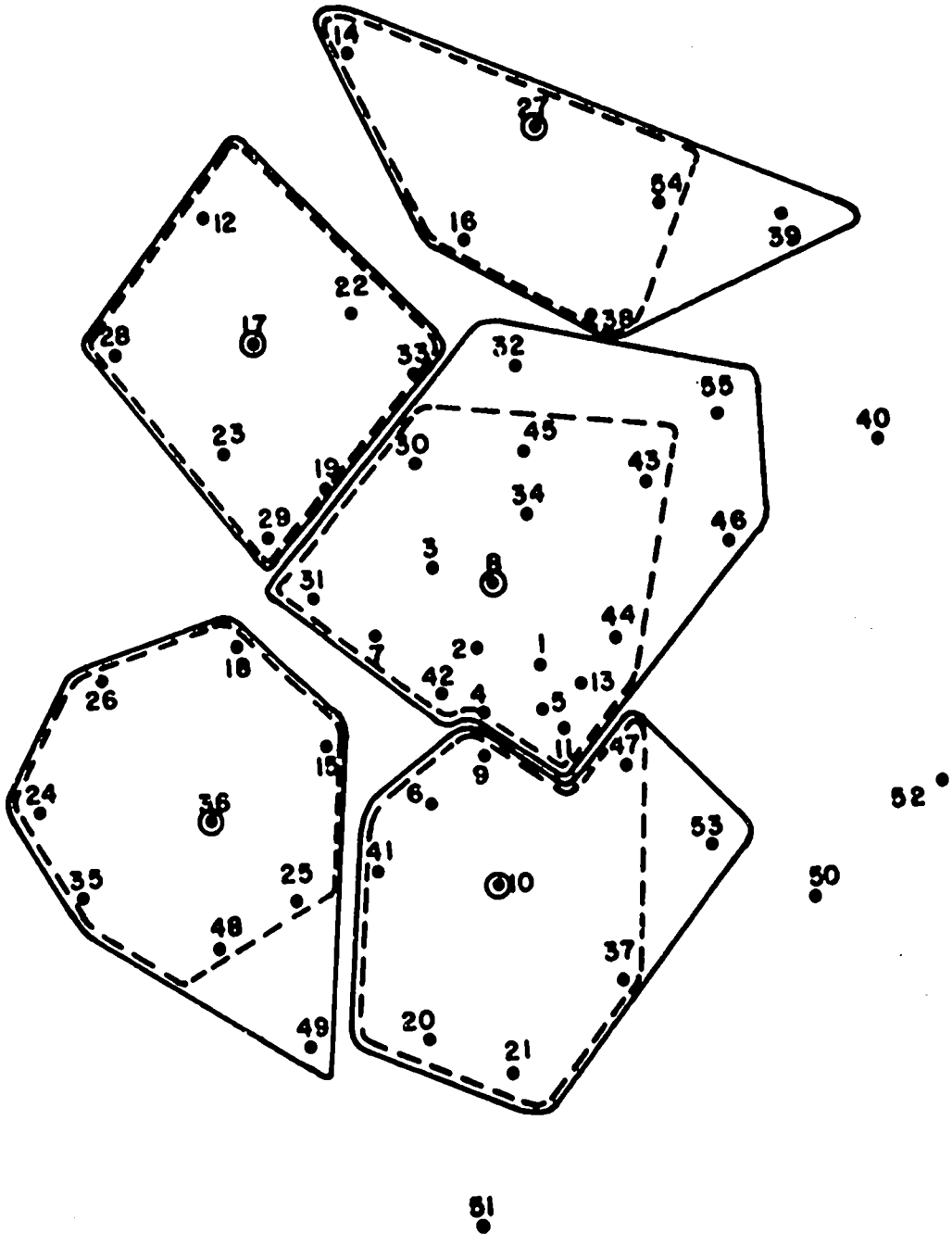
A service map provides information concerning the location of facilities and the service districts of those facilities. There are five facilities in figure 1-2. One is located at each of the points surrounded by a circle - nodes numbered 8, 10, 17, 27, and 36. There are two different districts which are shown around each facility. The districts denoted by the dashed lines include communities served within ten time units from the facility. For example, the facility at point 27 serves five communities within 10 time units - numbers 27, 14, 16, 38, and 54. A second type of district, denoted by the continous line, shows the communities served within 15 time units. The facility at node 27 can serve points 27, 14, 16, 38, 54, and 39 within 15 time units. The point 39 is outside the district for ten units, but inside the district for 15 units.

Some of the points are not within the service districts. For example, point 50 cannot be served (is not enclosed in a district) within either 10 or 15 time units. This point is said to be *uncovered* within 15 time units, even though it can be shown that it is covered in 20 time units. The meaning of the term 'covering techniques' is thus related to the ability of designated facilities to cover demand within a specified maximum time limit.

---

[7] Results from (6). Figures 1-1, 1-2, and tables 1-1 and 1-2 relate to several problems; no relation between illustrations is intended.

FIGURE 1-2: A SERVICE MAP



SECTION TWO: AN EXAMPLE OF COVERING REASONING - THE SET COVERING PROBLEM

The purpose of this section is to show how to formulate and solve a simple location problem - *set covering*. The problem is to find the minimum number, the location, and the service districts of facilities, such that all demand points find service within some specified period of time.

This problem is solved here by drawing circles and using common sense. The analytical procedures are physical and intelligible. More complicated location problems are solved by computer. When the computer solves these problems, it is only following a comparable line of reasoning, termed an *algorithm*.

Consider a region of nine separated communities, as shown in figure 2-1. The numbered points in the diagram represent the communities. The times of travel between adjacent communities are attached to each link of the network (enclosed by semi-circles). Facilities may be located in any community. All communities must be served within a maximum time.

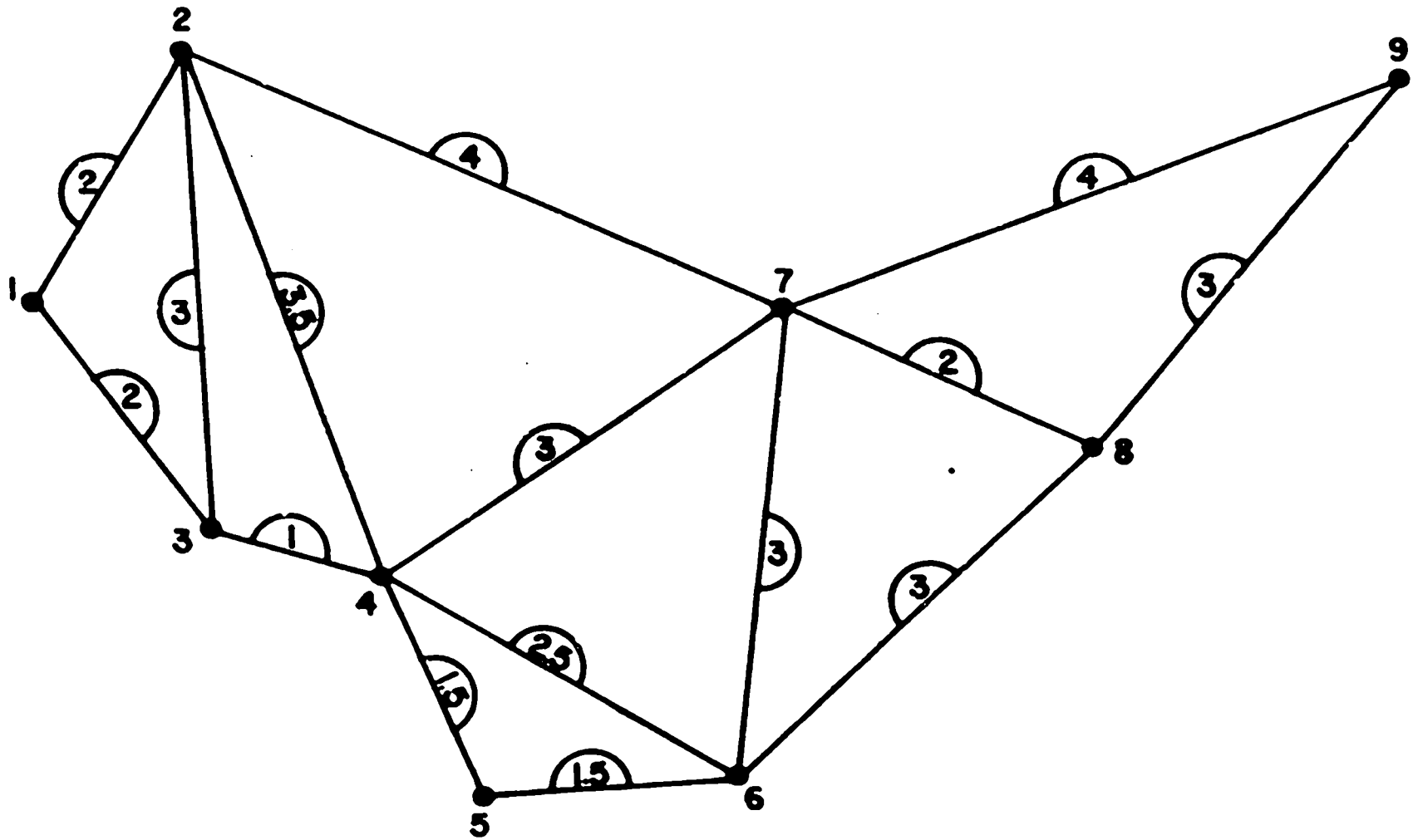
Suppose the maximum time of service is three time units. It is possible to begin at some point, select an arc leading from it, and see which points can be reached within three time units. For example, begin at community 1 and travel in the direction of points 3 and 4. It is possible to serve communities 1, 3, and 4 with a facility at point 1, since each can be reached within three time units. There exist only a finite number of arcs from any community. Search along each arc leaving any point. Such searches identify the complete set of all communities which can be served by a facility if it were to be located at that given point. For example, a facility located in community 9 could only serve points 9 and 8. Only two arcs leave from 9, and 8 is within three time units traveled along one of those arcs.

Assume that any point of demand can serve as a potential facility site. To find by construction the set of communities which can be served by a facility at some site, identify the facilities which serve that community [1].

---

[1] A caution is needed here. These sets are identical by the assumption that a facility could only be located in one of the communities. See chapter 4 for cases where facilities can be sited on arcs or at points on the plane but off the network.

FIGURE 2-1: A NETWORK OF NINE COMMUNITIES



numbers within semi-circles are the times of travel between adjacent communities

For example, a facility at 1 can serve communities 1, 2, 3, and 4 since all these points are within three time units of 1. An equivalent statement is that a facility at sites 1, 2, 3, or 4 could serve community 1. This set of points {1,2,3,4} constitutes the potential sites that could cover community 1 with service within three time units. For community 8, the set of potential covering sites is {6,7,8,9}.

A search emanating from each network point (node) yields table 2-1. The *service set* is the collection of communities which can be served by a facility site. The facility sites which cover each community within three time units are also listed [2].

TABLE 2-1: THE RANGE OF SERVICE OF POTENTIAL FACILITIES

POTENTIAL FACILITY SITE	SERVICE SET OF THAT FACILITY [maximum time of service=3]	FACILITIES AT WHICH LOCATIONS COULD SERVE THIS SITE [maximum time of service=3]
Community 1	1,2,3,4	1,2,3,4
Community 2	1,2,3	1,2,3
Community 3	1,2,3,4,5	1,2,3,4,5
Community 4	1,3,4,5,6,7	1,3,4,5,6,7
Community 5	3,4,5,6	3,4,5,6
Community 6	4,5,6,7,8	4,5,6,7,8
Community 7	4,6,7,8	4,6,7,8
Community 8	6,7,8,9	6,7,8,9
Community 9	8,9	8,9

Concentrate upon the third column, the set of potential sites that could cover a given community. Certain patterns

---

[2] The service set and the set of potential facility sites are identical for each community in this example through the assumption that a facility can be placed at any demand point. Under alternate assumptions, these sets can be different.

occur in the data. For example, the potential covering sites for communities 1, 2, and 3 are

community	sites which could serve that community within three time units
1	1,2,3,4
2	1,2,3
3	1,2,3,4,5

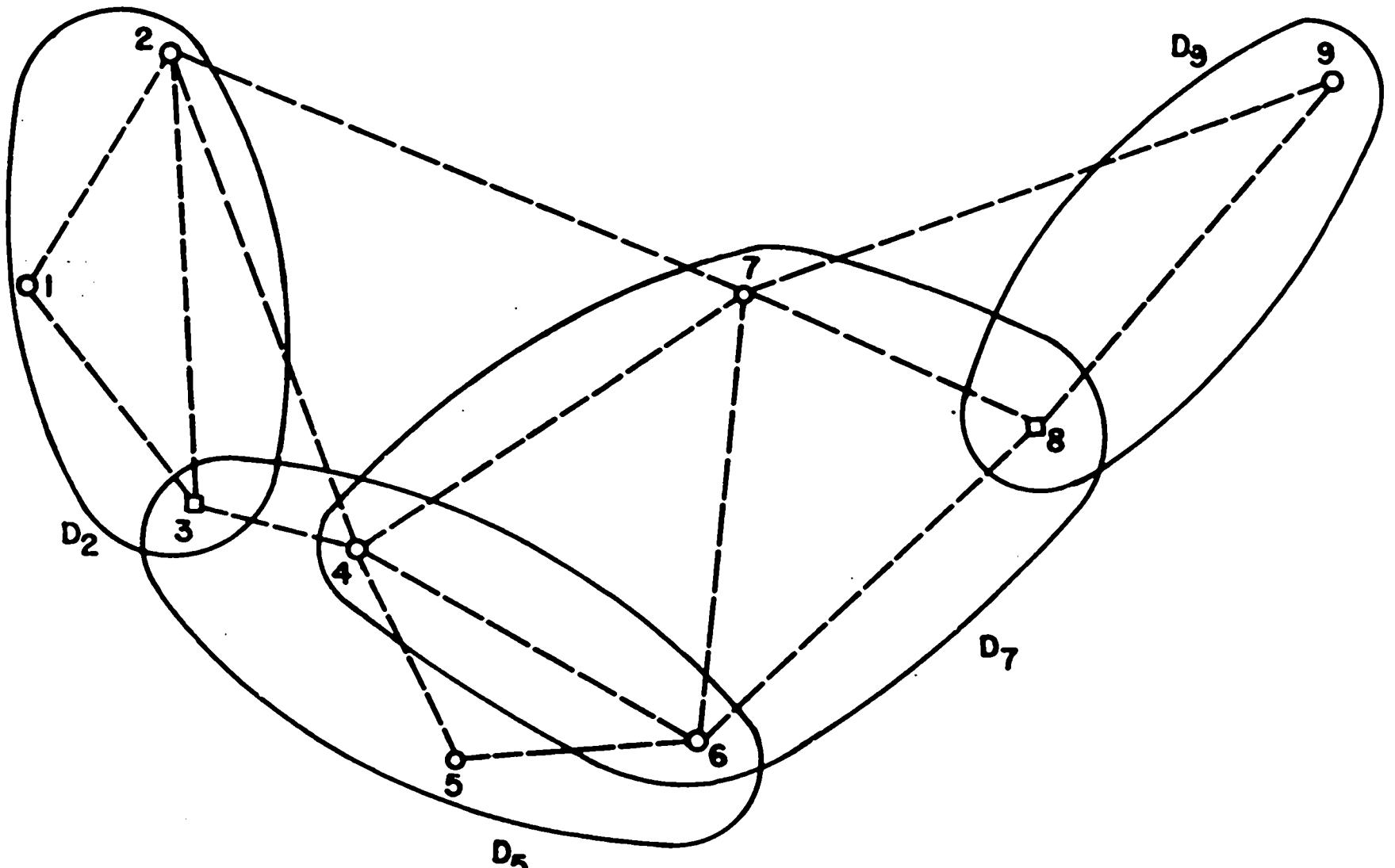
If a facility is not located at points 1, 2, 3, or 4, then it will not be possible to serve community 1 within three time units. If there is no facility in 1, 2, or 3, community 2 will not be covered. Likewise, unless a facility is located at either 1, 2, 3, 4, or 5, it will not be possible to cover community 3 in three time units. Such sets of potential covering sites restrict the set of points among which at least one facility must be located. Notice that if the restriction is satisfied for community 2 [one facility at either 1, 2, or 3], then the restrictions for communities 1 and 3 are automatically satisfied, since a facility will be located at either 1, 2, or 3. Restriction 2 is said to *dominate* restrictions 1 and 3, and is enclosed by a rectangle in table 2-1.

A parallel analysis can be made for communities 8 and 9. The set {8,9} dominates the set {6,7,8,9}. If a facility must be located at either point 8 or 9, then by definition one shall be located at one of the nodes 6, 7, 8, or 9. Each of the sets which *cannot* be dominated by any other are enclosed in rectangles in table 2-1.

The set covering problem is reduced in size by eliminating from further analysis all those sets that are dominated. If only the undominated restrictions are satisfied, then by definition the dominated sets will be satisfied. When all restrictions are satisfied, each community will be served by a facility within three time units.

The undominated sets shall be labelled with the number of their corresponding community: D2 = {1,2,3}; D5 = {3,4,5,6}; D7 = {4,6,7,8}; and D9 = {8,9}. Lines can be drawn around each of these sets and labelled, as in figure 2-2.

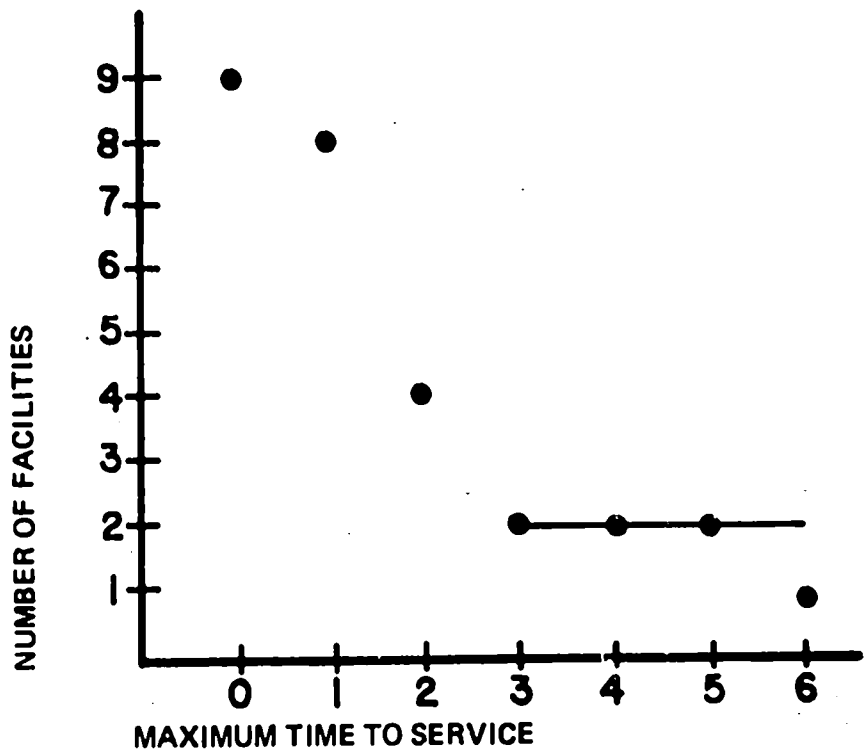
FIGURE 2-2: LINES DRAWN AROUND THE DOMINATING DEMAND SETS



There may be several sets of optimal facility sites. Although there are no alternate solutions for the above problem with only two facilities, such solutions are not only possible but frequent. For example, set the maximum time equal to four units. There is not an unique set of sites, but rather six pairs of facility locations that can serve all communities within four time units: {3,6}, {3,8}, {3,9}, {4,7}, {4,8}, and {4,9}. There are six possible solutions. Notice that even though the maximum time limit was relaxed from three to four units, no reduction in the number of facilities required is possible. Sites {3,8} are one of the solutions for the [maximum time equal to four] problem - the identical solution for a [maximum time equal to three] problem.

What would happen if the maximum time was set equal to two units? Four facilities are now required to serve all communities within two time units. Alternate site configurations exist, each of which solve this problem. As the maximum time for access is reduced, more facilities are needed to cover all the communities. The relationship between the *maximum time restriction* and the *minimum number of facilities needed to cover* is the tradeoff curve of figure 2-3. Increased performance requires a larger number of facilities.

FIGURE 2-3: THE TRADE-OFF BETWEEN MAXIMUM SERVICE TIME AND THE NUMBER OF FACILITIES



By visual inspection, we can determine which potential facility site covers (is an element of) which D set. Site 1 covers set D2. Site 3 covers demand set D2 and D5. Table 2-2 is a list of the sets covered by each potential facility site.

TABLE 2–2:
COVERAGE OF DEMAND SETS
BY POTENTIAL FACILITY SITES

Facility Site	Demand Sets Covered
1	D2
2	D2
3	D2 and D5
4	D5 and D7
5	D5
6	D5 and D7
7	D7
8	D7 and D9
9	D9

Facility site 3 dominates facility sites 1 and 2 in that it covers what they cover and more. Likewise facility sites 4 and 6 dominate site 5. Site 8 dominates sites 7 and 9, since it covers as much as they do, and more. We can eliminate all the dominated sites from consideration, leaving only sites 3, 4, 6, or 8 as candidate locations for a facility.

Consider set D2. There is currently only one site under consideration to cover that demand, namely point 3. This point must be included; it is essential to covering D2. Note, however, it also covers demand set D5.

Consider set D9. Point 8 is the only remaining community that can serve D9. Point 8 must become a facility site since it is essential to cover D9. Note, however, that 8 also covers demand surface D7.

All dominant demand sets are now covered, and hence all communities are now covered. The selection of site 3 and 8 as the locations of the facilities will insure that all communities can be served within 3 time units. The facility located at community 3 will serve a region composed of communities 1, 2, 3, 4, 5, and 6. The service region of the facility at community 8 will be 4, 6, 7, 8, and 9. The minimum number, the location, and the service districts of facilities have been found. All communities have access to service within three time units.

If this problem were to be solved for all values of the maximum time to service, the trade-off curve would be continuous. When solved for zero through six time units by integer, a finite number of points can be plotted as in figure 2-3. To move from a [maximum of two time units] to a [maximum of one time unit] increases the number of required facilities from four to eight. A comparable improvement in the time standard, from four units to three units, requires no increase in the number of facilities, since in both cases two facilities suffice. This tradeoff curve graphs the number of incremental facilities needed to improve service through reduction in the maximum time of service.

A flat portion exists between three and nearly six time units in figure 2-3. The covering pattern of facility locations need not change over the interval. In other words, facility sites {3,8} are a solution over the range of three to nearly six time units. Such sites might be natural ones to select for a facility.

This section has shown how a graphical technique solves the problem of finding the minimum number, location, and service districts for facilities. A motivated and patient individual can use this method given only the network of communities and the times of travel between them [3].

The problem chosen for solution was quite simple. Only a small number of communities were involved. The objective was simply to cover all the communities within the maximum time standard. Only a single service is involved. Knowledge of travel times is assumed. More complex problems involving larger groups of communities may be tedious to solve by hand. This method can easily be computerized. Different methods to solve larger, more complex problems are almost always solved by computer.

One caution is that the graphical method may not always solve. It is possible to arrive at forms which cannot be further reduced, where inherent dominance relations do not exist. Ninety percent of the problems have terminated with an optimal solution; the remaining ten percent of the cases are termed cyclic. Procedures for resolving cyclic cases exist and are described elsewhere (22).

---

[3] Maximal distance restrictions can be substituted for maximum time units. The technique only requires numbers on the network. Such numbers may be interpreted as either time or distance units.

This section portrays the kind of reasoning that lies behind computer algorithms for covering analysis. In the following sections, we shall discuss more realistic access problems.

SECTION THREE: COVERING AND POLICY ANALYSIS - USE OF THE MAXIMAL COVERING LOCATION METHOD

This section introduces maximal covering location analysis (6). Maximal covering is similar to set covering in terms of three decisions - location of facilities, quality of service (maximal time to service), and number of facilities. Both set and maximal covering solutions produce a map of the sites and service districts of facilities. Tradeoff curves can be constructed from the sequence of solutions for different maximal times.

The change from set covering to maximal covering permits a modification in the definition of the nature of demand. In set covering, demand was represented by points of human settlement. In maximal covering the surrogate for demand becomes population (or people requesting service) at a point in space, rather than the point as such. Figure 3-1 can be viewed as a representation of 55 towns on a map [1]. Set covering would treat demand as being the towns themselves. Maximal covering treats the population of the towns, or more generally the demand for services at those towns.

Associated with each town is a location and a population. The location can be described as a point on a graph, with x-axis and y-axis coordinates. Table 3-1 represents the population and location data for figure 3-1, where

a_i = the population of the town

x_i = the x-coordinate of the town on a map

y_i = the y-coordinate of the town on a map

In maximal covering, there are four decision variables: location, quality of service (maximal time), number of facilities, and the percent of the total population covered.

Suppose that a decision maker is interested in covering all demand within a particular time limit of 15 time units with the fewest possible facilities (5). Assume that the 55-node network of table 3-1 represents his particular problem. The decision maker is implicitly structuring the location set covering problem for S=15, where S stands for the maximum service time. Solution will yield a minimum number of facilities that cover all demand sites within the specified maximum time. Figure 3-2 gives a location set

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[1] Data from (20).

FIGURE 3-1: 55 TOWNS

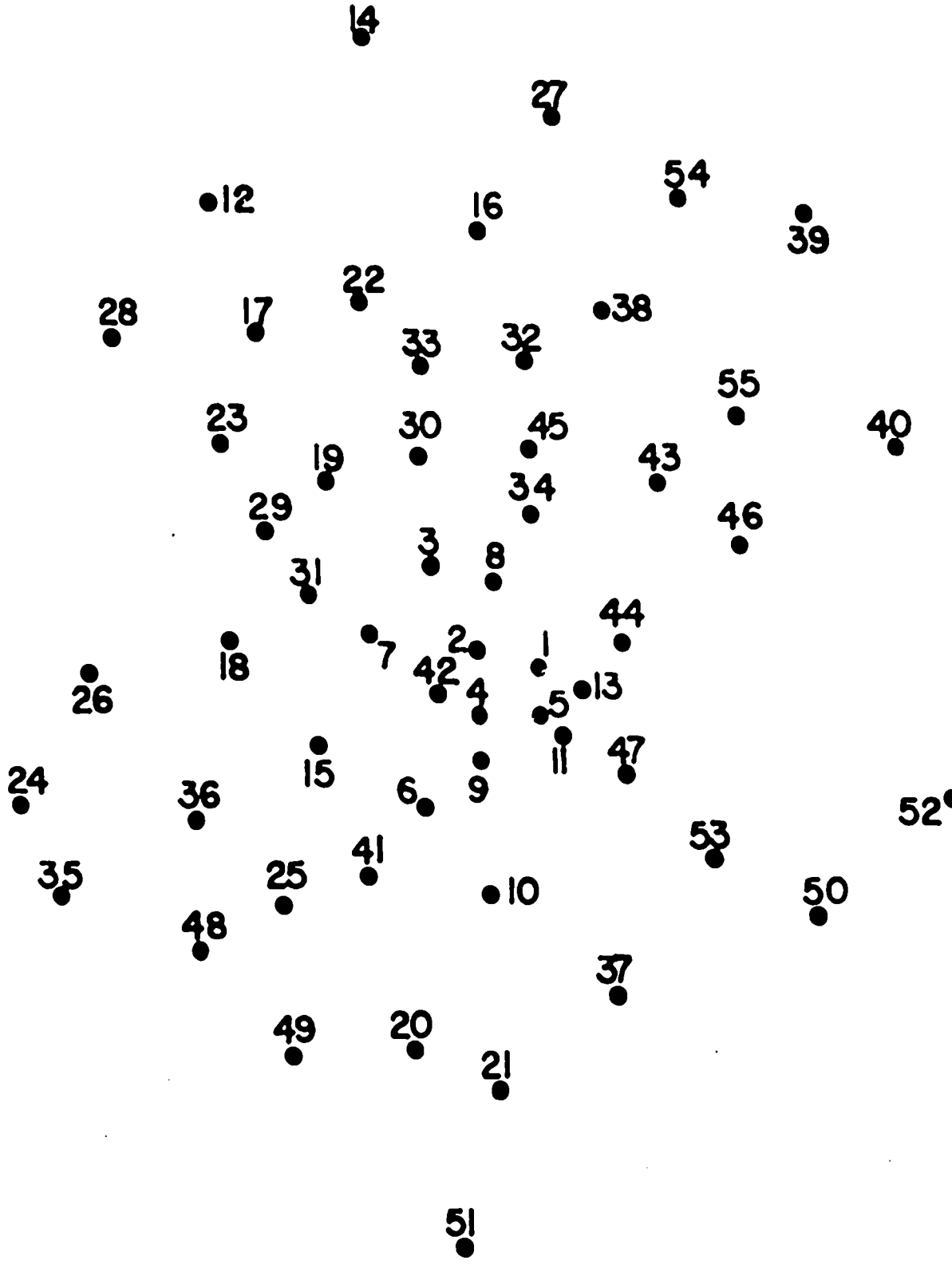


TABLE 3-1: GEOGRAPHIC AND POPULATION DATA FOR MAXIMAL COVERING

i	a_i	x_i	y_i	i	a_i	x_i	y_i
1	710	32	31	28	60	12	47
2	620	29	32	29	60	19	38
3	560	27	36	30	60	27	41
4	390	29	29	31	60	21	35
5	350	32	29	32	50	32	45
6	210	26	25	33	50	27	45
7	200	24	33	34	50	32	38
8	190	30	35	35	50	8	22
9	170	29	27	36	50	15	25
10	170	29	21	37	50	35	16
11	160	33	28	38	40	36	47
12	150	17	53	39	40	46	51
13	140	34	40	40	40	50	40
14	120	25	60	41	40	23	22
15	120	21	28	42	40	27	30
16	110	30	51	43	40	38	39
17	100	19	47	44	40	36	32
18	100	17	33	45	30	32	41
19	90	22	40	46	30	42	36
20	90	25	14	47	30	36	26
21	90	29	12	48	30	15	19
22	80	24	48	49	30	19	14
23	80	17	42	50	30	45	19
24	80	6	26	51	30	27	5
25	80	19	21	52	20	52	24
26	70	10	32	53	20	40	22
27	60	34	56	54	20	40	52
				55	20	42	42

covering solution for a service limit of S=15 on the 55-node network. In this case five facilities are required to cover all nodes within a limit of 15 minutes.

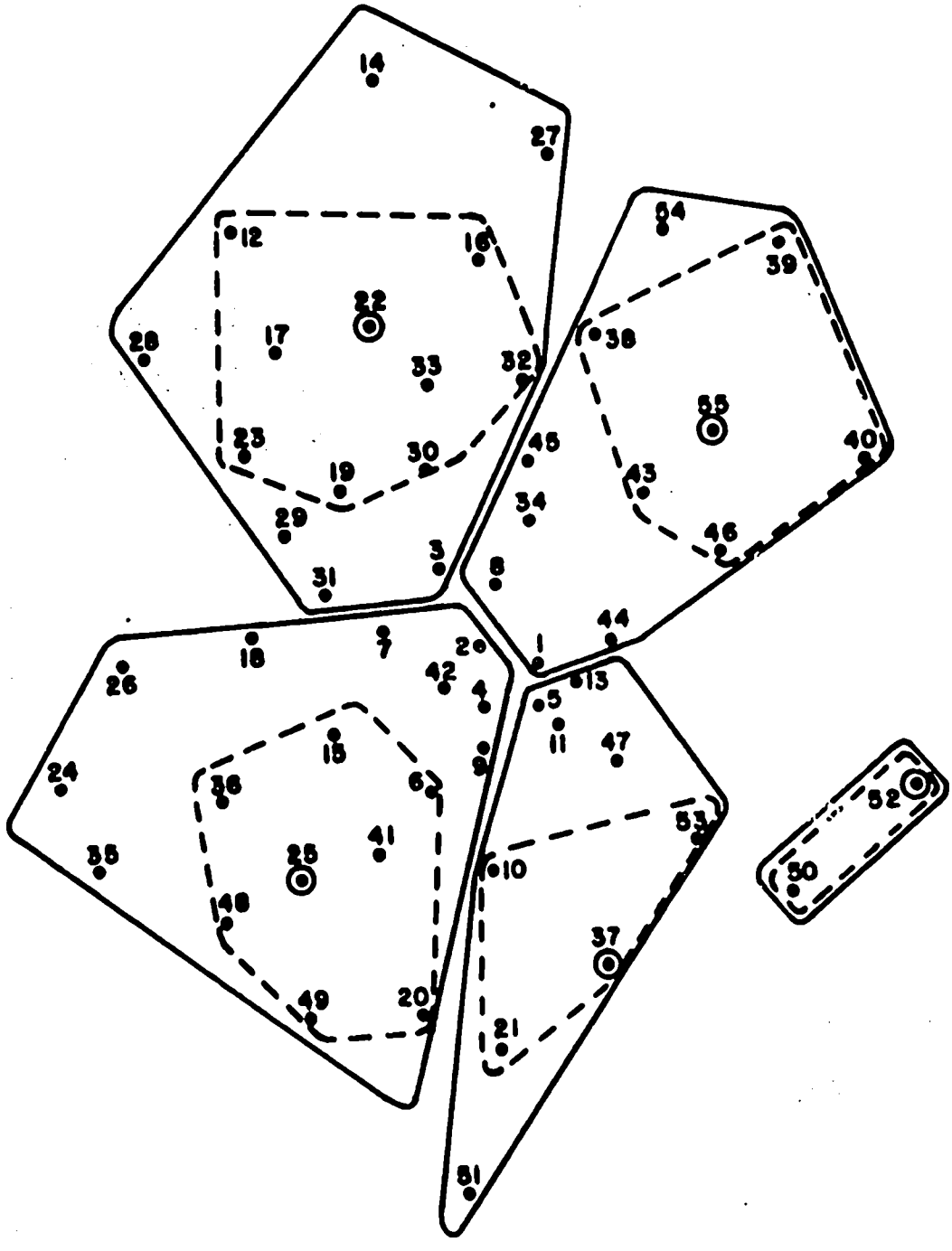
The circled nodes on the graph given in figure 3-2 represent the five sites chosen to house facilities. The nodes on the graph are partitioned into sets, each node being assigned to its closest facility. In figure 3-2 there are two kinds of partitions: those enclosed by solid lines and those indicated by dashed lines. The solid line partitions indicate which nodes are covered within S=15. The dashed line partitions denote which nodes are covered within S=10.

For example, node 42 is in the solid line partition surrounding a facility located at node 25. This means that the closest facility to node 42 is located at node 25 and that the time of travel from node 42 to that facility is less than 15. Furthermore, node 42 is not in the dashed line partition for the facility located at node 25, indicating that node 42 lies at location greater than 10 time units from its closest facility, node 25. Figure 3-2 also shows, for this particular location set covering solution, that 201 of the total population are covered within 10 time units; the rest of the population (439) is located between 10 and the maximal service time of 15.

This particular optimal solution is but one of many alternate optima, each characterized by a minimum number of facilities (five) which cover all communities within fifteen time units maximum. The maximal covering location method can be used to choose among the alternative optima of the location set covering problem to achieve the best coverage for some more "desirable" inner distance. The outer maximum time limit is S=15. Let there also be an inner maximum time limit of T=10. By solving the maximal covering location problem with S=15, T=10, and still using five facilities, it is possible to determine the solution which provides the best coverage within T=10 while maintaining complete coverage within S=15.

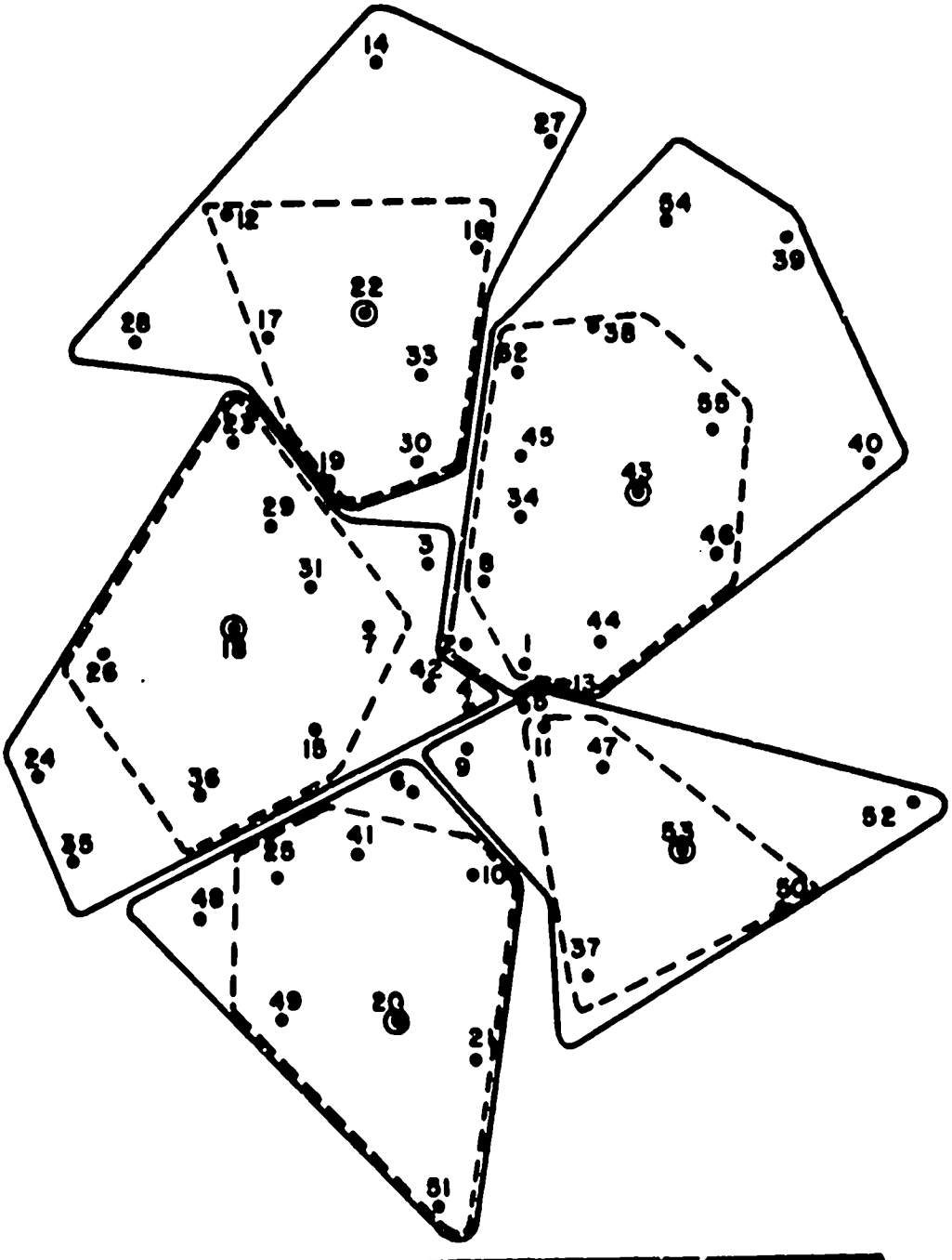
It is entirely possible that the decision maker may feel it desirable to cover as much population as possible within 10 if he can still provide complete coverage within 15. After all, he is giving up nothing. The maximal covering location problem was solved for this network, with S=15 and T=10 and the solution is presented in figure 3-3. Essentially, figure 3-3 depicts the alternative optimum of the location set covering problem for S=15 which covers the maximum population within 10. Notice that the coverage within 10 for this solution amounts to 354 out of 640. By the nature

FIGURE 3-2: SET COVERING SOLUTION FOR MAXIMUM TIME OF 15 MINUTES



POPULATION INSIDE 10----------	201
POPULATION OUTSIDE 10 BUT INSIDE 15 ---	439
POPULATION OUTSIDE 15 ----------	0

FIGURE 3-3: MAXIMAL COVERING SOLUTION–S = 15*, T = 10**



POPULATION INSIDE 10 ----------	354
POPULATION OUTSIDE 10 BUT INSIDE 15 ---	286
POPULATION OUTSIDE 15 ----------	0

*Outer maximal service time of 15

**Inner maximum service time of 10

of the maximal covering problem, it is possible to state that no more than 354 can be covered within 10 time units while maintaining complete coverage within 15, given no more than five facilities. The solution in figure 3-3 is clearly a better solution than the one given in figure 3-2, for the coverage within 10 time units is 75 percent better than the solution given in figure 3-2. Therefore, by using maximal covering, it is possible to determine the most desirable location set covering solution.

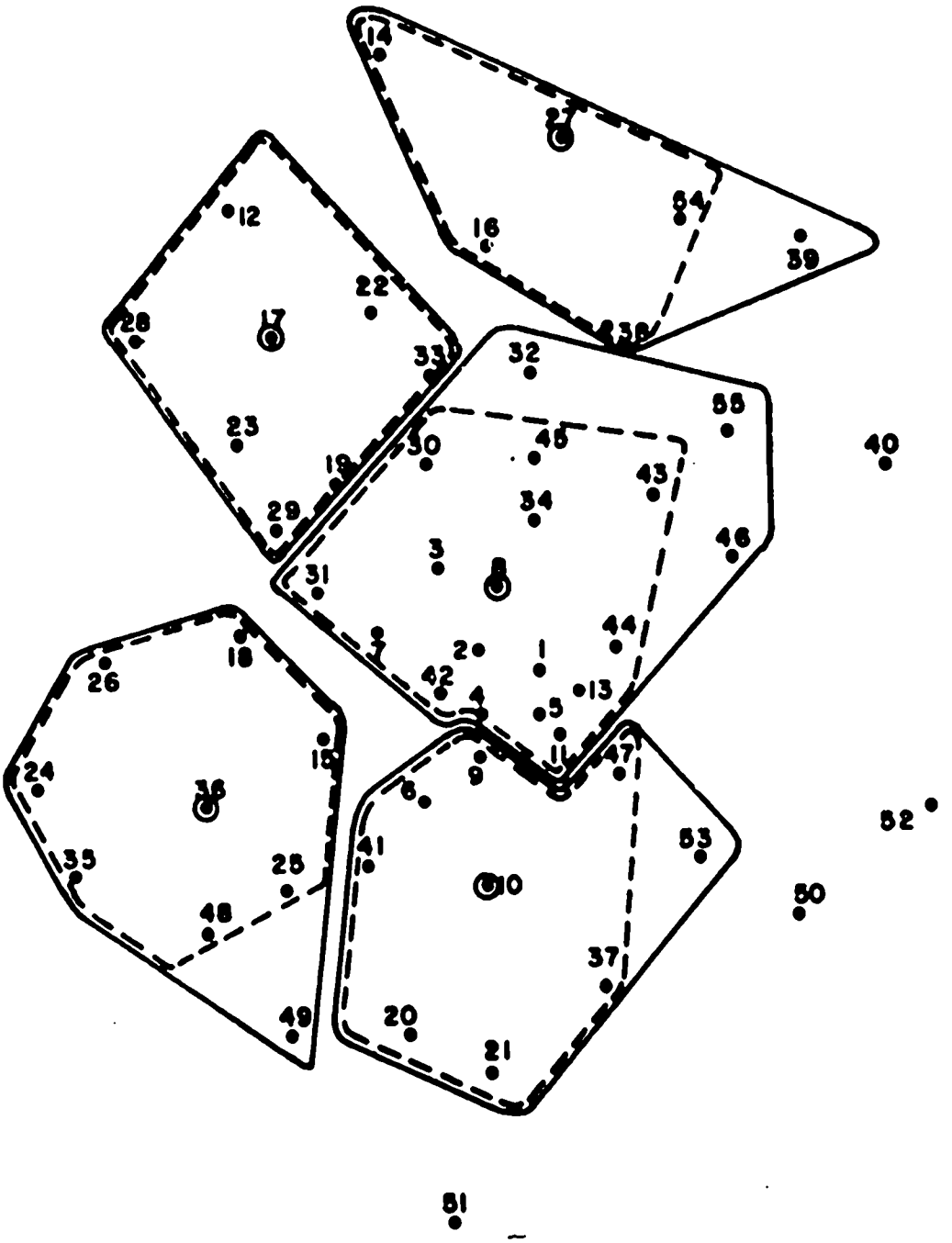
We know that it is impossible to cover more than 354 within 10 while maintaining complete coverage within 15 time units using five facilities. Thus, it would be futile to try to cover more than 354 within 10 unless either more facilities were allowed or the outer maximum travel time limit (of 15) were relinquished.

In the example problem it is in fact possible to achieve a very large maximal cover within S=10 using five facilities if one relaxes the mandatory constraint that all persons be served within 15 time units. From the viewpoint of the decision maker, achieving maximal coverage by five facilities within a time limit of 10 could be desirable. The tradeoff is whether the improvement of coverage within 10 can justify some number of people who will now be more than 15 time units from a facility.

Figure 3-4 gives a maximal covering solution for S=10. A node in the graph which is not covered within S=15 is not included in any partition. Notice that the population covered within S=10 has reached 609 out of a total population of 640. In this case, only four nodes with a combined population of 12 cannot be reached within 15 time units from their closest facilities.

In order to cover all demand within 15 time units, the amount covered within 10 must decrease from 609 (given by maximal covering with S=10) to 354 (with T=10, S=15). The reduction of 255 in the population covered within 10 can be viewed as one of the costs of maintaining that all demand must be serviced within 15. That is, the requirement S=10 forces 12 persons of 640 (at four out of 55 communities) to be more than 15 time units distant from service. At the same time, 255 persons experience reduced access time, from 15 to 10 time units to service.

FIGURE 3-4: MAXIMAL COVERING SOLUTION WITH MAXIMUM SERVICE TIME OF 10 UNITS (S = 10)



POPULATION INSIDE 10 ----------	609
POPULATION OUTSIDE 10 BUT INSIDE 15 ---	19
POPULATION OUTISDE 15 ----------	12

SECTION FOUR: NOTIONS OF GEOGRAPHICAL RELATIONS FOR USE IN HEALTH ACCESS ANALYSIS

The sole notion of geographical relations used in previous sections is a set of points on a network. These networks - points connected by line segments - implicitly assumed that demand is expressed only at nodes. Each point is located in relation to others by its coordinates on a Cartesian plane. Travel occurs along the line segments connecting adjacent points. Associated with each line segment is a known time of travel between a point and its adjacent neighbor. These assumptions are listed in table 4-1.

TABLE 4–1: ALTERNATE GEOGRAPHICAL ASSUMPTIONS

Subject	Strict Assumption	Relaxed Alternative Assumptions
possible demand locations	location only at nodes on a network	location on line segments
		location at any point in the plane
possible health service locations	location only at nodes	location on line segments
		location at any point in the plane
geographical relation between service and demands	known travel time between points	estimated travel time
		known travel distance
		estimated travel distance
	single set of time/ distance relations	travel time or distance depends upon mode

This conceptualization of geographical relations is not the only approach that is consistent with location access analysis. The purpose of this section is to describe the alternate notions of geographical relations which can be used with covering methods. This discussion is not intended to be all inclusive. This section will use the words 'health service' and 'demand for the health service' without defining them; different concepts of service and demand are discussed in sections five and six.

Three strict assumptions have been implicit in previous sections - demand expressed at nodes, services located at network points, and one set of nodes for services and demands. In a real problem any or all three of these assumptions might be relaxed. Reference to figure 4-1 will often be made during this section to illustrate the modifications of assumptions.

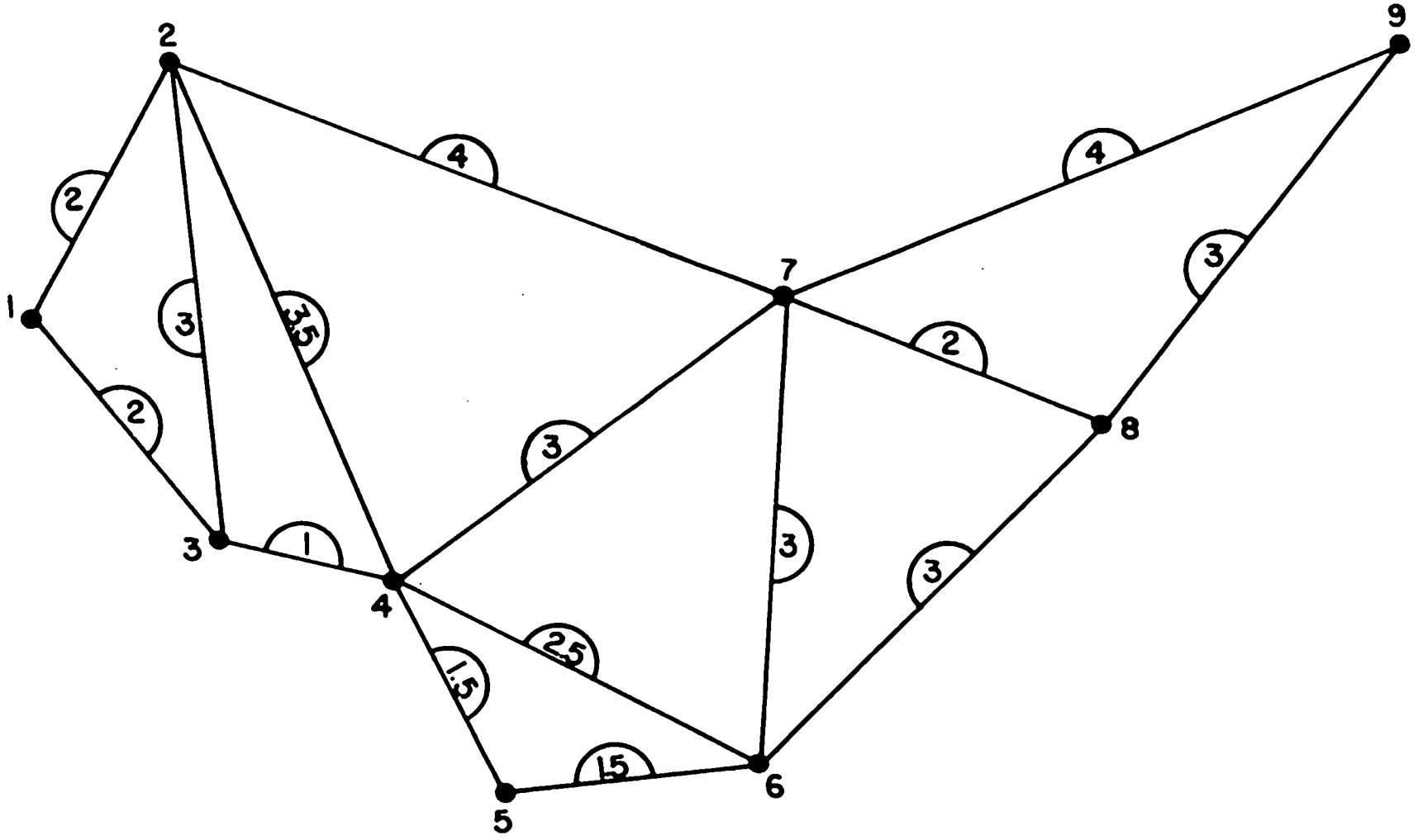
Possible Demand Locations

In previous sections demands for health services had been assumed to be located strictly at the points (nodes) of a defined network. Points 1 through 9 in figure 4-1 are an example. Demand exists at those points, and at those points alone. No demand was permitted at points on the line segments (the arcs) or at other unidentified points on the general plane. This network conception of geographical space is realistic for many problem settings. Many rural areas are well represented by a set of discrete towns (points), inter-connected by roads (line segments), along which travel between adjacent towns can occur.

However, for many other problems this geographical structure of demand - as existing at points in space - is unrealistic. For example, a nearly continuous string of settlements along a highway may be represented by a line segment. A dense city or isolated rural region may be thought of as population spread over a plane. One planar example would be evenly distributed population over the area in figure 4-1 bounded by the line segments 1-3, 3-4, 4-5, 5-6, 6-8, 8-9, 9-7, 7-2, and 2-1.

It is possible for covering methods to utilize either of these alternate views of demand. If demand were viewed as spread upon the line segments, the problem is known as *the arc-covering problem.* If demand is viewed as spread uniformly over a plane, the problem becomes *infinite solution space covering.* There is a rich literature in both the

FIGURE 4-1: A NETWORK OF NINE COMMUNITIES



arc-covering and infinite solution space areas. The best summary of arc-covering techniques is a book by Berlin, Liebman, and Male that is in preparation (2). There are many sources of information for infinite solution space methods (2, 11, 14).

While methods exist to solve non-network problems, it does not follow that a user should automatically opt for their use. These methods have limitations too. For example, infinite solution space methods require explicit solution using calculus - something that requires sophisticated analysts. Many solutions are complicated enough to be problems at the boundary of applied mathematics research. As tools for operational use in day-to-day planning, they are not as practical as network methods. Network formulation also accomodates a wider variety of objective functions (11, 14).

Possible Health Facility Locations

In previous sections only points on a network could be potential sites for health facilities. In section 2, the same set of points (nos. 1 through 9 in figure 4-1) are the potential sites and the sources of demand. The question naturally arises whether analytical techniques exist to solve problems where this restriction that facilities be built at network nodes is relaxed.

Two relaxations of this 'facility on nodes' constraint are that a facility be permitted at (a) any point on any of the line segments connecting the points of the network or (b) at any point at all on the plane. In a recent paper Church and Meadows demonstrated that maximal location covering could solve these relaxed problems (4). Thus, it is possible to allow the set of potential sites for health facilities to include any point on the plane, whether at a node or on an arc of a network or not.

Geographical Relation Between Service and Demands

Known travel times between the points of a network have, up to this section, been assumed. A single set of known travel time relations may not always exist or be appropriate. An example is travel between points a and b, where car takes 10 minutes, burro requires 40 minutes and by foot, 60 minutes. The alternate notions of geographical relations include *known distances* and *estimates* of time or distance.

time vs. distance

Time and distance are treated in an identical fashion in analysis. In figure 4-1, numbers are listed in semicircles on the line segments between adjacent points. In section 2 these were interpreted as 'the times of travel between adjacent communities'. Analytically, it would be perfectly acceptable to view these numbers instead as the distance of travel between communities. The covering methods require numbers as data; it is up to human interpretation as to whether the numbers stand for time or distance relationships.

estimation

Times of travel from one point to another have been assumed to be known in advance - perhaps through measurement. Approximation is an alternative to measurement.

One way to generate information concerning the times or distances of travel between points is by measurement. Distance between points can be read from a map. Times of travel can be measured by stop watch. In a recent access study of the Baltimore Fire Department, actual times of travel between facilities (the fire stations) and demand points (fires) were measured with stop watches (17). Measurement yields travel time or distance information along only the arcs measured. Analytical methods called shortest path techniques can calculate the shortest arc between any two points given coordinates, as in table 3-2.

Measurement of travel times or distances may be difficult. In such cases, estimation methods, such as the x-y method, the x-y-z method; and Delphi estimation may be helpful. X-y estimation requires the placing of points on a map. The map is placed on graph paper, and the x and y coordinates of each point are read from it; table 3-2 could be an example of such data. Using a shortest path technique, the complete set of time/distance relations can be generated in terms of the x-y coordinate system. These values can then be transformed into realistic data by multiplication by the scale factor. This x-y method implicitly assumes that the real geographic relationship between points is not distorted by reduction to two dimensions and that travel is by the most direct route. The x-y-z method refers to any of several techniques for estimating distances from information concerning longitude, latitude, and altitude. These are analogous to the x-y method, but using three dimensions. Delphi estimation is accomplished by using perceived distances by asking people how long it takes them to travel (or how far is the distance) between points.

The use of any measurement or estimation method involves assumptions. Measurement assumes that a single sample of 'the time that one vehicle requires from one point to another' can be generalized to all vehicles at all times. Different travel times for rush hours and for off-hours introduce further complexity. Use of x-y or x-y-z graph estimation approaches must assume that shortest path calculations do not distort geographical relationships. Delphi estimation presumes some correspondence between *perceived* time/distance and the *measurable* time/distance of travel. The user should be careful in selecting the method that is most effective within the given problem context.

multi-modal travel

Another assumption is that only one mode of travel exists, implying one time or distance measurement between any two points. What if this assumption is relaxed, and multiple means of travel between health facilities and demand points can exist?

For example, when an emergency arises in Maryland, either of two parellel emergency health systems can come to the rescue: a helicopter system and an ambulance system. The helicopter travels by air. Time and distance between any two points in the state is defined in terms of its ground-air-ground travel. The ambulances travel by land; time and distance are defined by the road network in the state. Between any two points there exist two different shortest times of travel.

The addition of an extra set of time/distance relationships between all points on a network is only a minor complication to the solution techniques. Each point will be represented by two points, creating an air network and a land network. A developing nation example might be travel by foot, pack animal, or automobile.

One complication of multi-mode travel is that the costs of solving a problem will rise more than proportionately. Any time the size of a problem doubles, the time and effort needed for solution more than doubles (5).

the fringe problem

The fringe problem arises where a complete set of geographical relations exist but are not necessary. For example, a matrix of shortest distance of travel may exist between the nine points of figure 4-1. However if concern over access is relevant only when travel time exceeds some minimum, some of the data may be unnecessary. Procedures have been developed to have the computer ignore certain time or distance data if it is irrelevant (5). Such data reduction can be quite cost-effective, since a smaller problem can be solved for a greater-than-proportional savings versus a larger problem.

SECTION FIVE: ALTERNATE DEMAND ASSUMPTIONS

Demand for health services is a reflection of the interaction between health needs, incomes, and health delivery institutions. Access methods usually treat demand as a known or estimated parameter. This section explores the geography, timing, and assumption structure of demand information.

Geography

Four alternate geographical representations may be used to describe demand - weighted or unweighted points on a graph, and points along an arc or in continuous space (see table 5-1). While much of the section is written with weighted network points in mind, the reader may extend the discussion in a parallel fashion to continuous space.

TABLE 5-1: GEOGRAPHICAL REPRESENTATIONS OF DEMAND

set covering	discrete network	uniform weights at points on a graph
maximal covering	discrete network	weighted points on a graph on a graph
arc covering	discrete network	uniform weights over arcs
infinite solution space	continuous two dimensional space	uniform distribution over a plane

Static Demand Surrogates

Real *demand* for services is unknown. Policy makers often use some surrogate for demand. Table 5-2 lists alternate concepts of demand for the siting of a defined set of facilities on a defined network of points with respect to demand at one moment in time.

TABLE 5–2: ALTERNATE DEMAND SURROGATES

Surrogate	Type of Estimate	Methodology	Degree of Effort
estimate	sample statistic	population sample	great effort
expressed demand	observed statistic	count bodies	intermediate effort
a community	assumption	identify a community	very easy
population	assumption	use census data	easy
population cross-section	assumption	use census data	minor effort

What is demand? A clinician's notion of demand may be *need* or some measure of the people who actually require treatment. To identify the needy, survey a small sample of a population; sample information can be used to make inferences concerning the population. In some cases, information will already exist in health records concerning the frequency of a health condition. Sampling provides excellent information, but can be expensive and time consuming. Information based upon small, biased, or irrelevant samples can be misleading. For example, demand data based upon a population sample in one local area may have no relationship to demand in a community somewhere else.

An economist's notion of demand might be *all those who present themselves for treatment*. Such expressed demand is countable at existing facilities. If such information is not already available, it may be costly to collect. If available, such data may be biased because those who do not have access will not present themselves for treatment.

Two surrogates noted previously in section two are a community as a point of demand or some measure of population. It is possible to use census data to weight the points representing human habitation, and locate facilities to cover these weighted population nodes. It is also possible to take various cross sections of the population, and use these numbers to weight the nodes. Cross sections can be made by age, sex, income class, employer, race, or other factors.

Many health planning problems involve certain sectors of a population. Maternal and child care involves women within a certain age group. Heart attacks occur primarily

among people of particular ages. Persons without access to health care can often be classified by level of income, place of residence, race, occupation or other factors included in census data.

Location analysis can site facilities to cover such racial, income, occupation, sex, or age groups by using such data to weight the nodes. If a hierarchy of needs exists (e.g., treat the indigent first, next the poor, etc.), then it is possible to weight various cross-sections of populations and thus arrive at a composite weight for a point on a network.

Dynamic Demands

The location of facilities to cover demands that change over time is amenable to access analysis. Age stratification, social group populations, and income distribution will vary with time. However a facility site, once determined, may be difficult to change. Dynamic location of facilities is possible, but rests upon some definition of what will be the service demand in the future.

Several strategies of dynamic location can be employed to weight the future versus the present. One procedure is to find the best locations with regard to present demands and then determine the implications for the future. Alternately, each time period, such as today, 5 years from now, 10 years from now, etc., can be designated with a weight/significance. The siting problem becomes determination of locations with respect to the distribution of demand over time. For example, future demand may be discounted in a manner similar to discounting future benefits or costs in a traditional cost-benefit analysis. Such discounting could reflect a social time preference, in that service to a person today is worth more than serving someone in ten years. Future demands could also be discounted for uncertainty; presumed demand of ten years hence may or may not materialize. A final strategy to demand over time is to determine tradeoffs between locations that optimize coverage today and those that optimize coverage at some point in the future. Tradeoff curves similar in spirit to those previously displayed can show what present degree of coverage must be sacrificed to achieve greater coverage in the future.

All these methods of dynamic location require some information concerning future demand. Typical sources of such information are population projections, government planning documents, or estimates of experts.

Summary

This section has discussed the variety of definitions of demand that can be used with access/covering methods to locate facilities. The solution techniques only require some weighting factor. Whether this factor is survey data, expressed demand statistics, uniform weights (communities), population, cross-sections of population, or expert guess-timates is of no difference to the algorithms. It is up to the policy maker in any particular application to define what is the nature of the demand which he hopes to cover through facility location. This discussion has used networks and points as the representation of geographical relationships. The reader may extend the results of this chapter to other spatial relationships, including arcs and the continuous plane.

SECTION SIX: ALTERNATE ASSUMPTIONS CONCERNING FACILITIES AND HEALTH SERVICES

This section discusses different notions of health services and health facilities, given prior assumptions of spatial relationships and demand [1]. Three issues relating to the definition of health service will be explored: location vs. relocation; single vs. multiple services; and one-level covering vs. heirarchical covering. The implicit assumption of previous sections and their relaxed alternatives are listed in table 6-1.

TABLE 6-1: ALTERNATE NOTIONS OF SERVICE AND FACILITIES

Issue	Previous Assumptions	Related Alternate Assumptions
location vs. relocation	location of all new facilities	location of some new facilities; others previously located cannot be relocated
		location of some new facilities; relocation of old facilities possible
		a relocation of facilities problem
service/facility equality vs. divergence	one set of facilities to provide one health service	one set of facilities to provide many health services
		several sets of facilities to provide many health services
		several parallel sets of facilities to provide a single health service
hierarchy of covering	one level of service and facilities	a hierarchy of levels of service and facilities. For example:
		(1) the lowest level of facilities should cover demand (population)
		(2) secondary facilities should cover referrals from the primary ones
		(3) a pattern of higher levels covering the next lower level referrals.

[1] An implicit assumption in this section is that demand is represented by populations at points in a network. However, analysis can also proceed with alternate geography and demand assumptions.

Location vs. Relocation

In sections two and three, the location technique implicitly found a set of new facility sites where none before existed. In practice this situation of all new facilities rarely arises. Usually some facilities already exist which are providing health services. The planning problem may be where to locate an incremental facility or two, with the sites of the previous facilities as fixed. Or, perhaps one or more old facilities may be relocated over time.

The procedure for treating an existing facility which cannot move is to assure its continued existence through a constraint. If there exists an old facility which can be relocated, it is possible to solve once with a constraint that the facility remain at its site and once without such a requirement. The comparison of solutions shows the implications of relocating the facility. Location/relocation implies slight data modifications which are not burdensome.

Single Vs. Multiple Services

Another implicit assumption in previous sections has been that each facility shall provide one and only one health service. This may be an accurate description of the policy problem; for example, the siting of centers to disperse family planning information and perform no other service. For other problems, a 'one service' assumption is too restrictive. One facility may house several services. Several parallel facilities could provide one service. The following paragraphs will discuss these and other notions.

One facility may be called upon to provide several health services. For example, a rural health post in an isolated area might be called upon to provide information about health, first aid services for emergencies, primary care for endemic diseases, etc. A facility which is *sited* to provide access to one set of users may or may not provide optimal access for a different clientele. For example, imagine that a rural health post is called upon to provide first aid for a large region and pre-natal care for a smaller, more compact region. The optimal set of locations for access to pre-natal services may diverge from the best sites to cover first aid. One strategy is to show the degree of coverage of pre-natal needs which must be sacrificed to provide a designated level of first aid services, or vice-versa. Results are not expressed as a single set of optimal sites, but rather as a tradeoff curve. The selection of the best compromise solution will be up to the decision makers.

Another case is when many facilities provide a single service in parallel. An example is the emergency medical system of Maryland. Two parallel systems, ambulances and helicopters, can bring victims from wherever the emergency occurs to a hospital. Both helicopter pads and ambulance stations need to be sited with respect to a single form of demand (emergency calls) and to each other. This category of problem was treated in section four. The analytical approach is to construct augmented networks including both sources of service.

A final type of problem is the case where services are offered in sequence. Some facilities cover demand; others provide access only to referrals from another level. The problem is how to cover a demand with a first rung of facilities, how to cover this first rung of facilities with referral centers, and so on, up a ladder of increasing complexity. An example of such a problem is the health regionalization plans in Colombia. There the intention is to site a lowest level of facility, the rural health posts, so that a large fraction of the population can be covered. Each health post in turn is to be covered by a first level of local clinics. Each local clinic is covered by a regional clinic, and so on, up to university hospital centers in urban areas. This area of *hierarchical covering* is currently a very fertile area of research.

SECTION SEVEN: ALTERNATIVE OBJECTIVES OF HEALTH FACILITY PLANNING

The definition of objectives is central to the task of site selection. Some typical goals which are used in location analysis are listed in table 7-1. Multiple objectives imply solutions that show tradeoffs between goals. An improvement in one objective (e.g., population covered) can only be obtained by a decrease in another. More coverage can only be purchased by a larger number of facilities.

TABLE 7–1: ALTERNATE PUBLIC ACCESS OBJECTIVES

Minimization of maximum time or distance

Minimization of total (or, equivalently, average) time/distance

Minimization of number of facilities

Minimization of cost

Maximization of population/demand covered

Maximization of utilization of service

Analysis in section three implicitly considered three objectives, population coverage, number of facilities, and maximum time/distance. Varying the value of S (the maximum time/distance permitted) from small to large generates a tradeoff curve between a time dimension (maximal service time or distance), the number of required facilities, and population coverage.

Given a cost function, (a statement of how costs increase with the number of facilities) costs can be traded off. P-median techniques locate p (a number) of central facilities so as to minimize average or total time/distance traveled in a health delivery system (6, 14). Use of p-median methods allow tradeoffs of average distance objectives versus others. One issue is determining whether average time/distance better represents the public purpose of health facility than maximal time to service. Other techniques attempt to maximize the utilization of services through the location of facilities (11, 14).

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