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A Population Projection Model: Description of A Computer Program and Illustrations of Its Policy Applications

Kenneth Maurer and T. Paul Schultz

A Report prepared for

AGENCY FOR INTERNATIONAL DEVELOPMENT

Rand
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PREFACE

This report develops and demonstrates some of the applications of a simple model for the projection of populations over time. The computer program and the model it embodies are not an advance in the state of the art, but they do combine several attractive features in a compact, easy to use form. The program is designed to reduce the cost of routine population projections required for long term planning and decisionmaking and may find use in AID and in other agencies concerned with international development, population growth, and family planning. This demographic model was developed by T. Paul Schultz to facilitate research undertaken for the Agency for International Development, Office of Population, under contract csd-2533, and was programmed by Kenneth Maurer. The central objective of this research project is the integration of a theory of fertility and family behavior that should facilitate understanding and resolving of population problems in developing countries.

SUMMARY

The purpose of this report is to make accessible an adaptable program for projecting the age composition and growth of populatio... The user specifies the time path of age-specific birth and death rates and the initial population; the program then computes most generally desired demographic characteristics of the population over time, including the average number of children born and surviving by age to women of various ages. The fertility and mortality trends may be made endogenous to a model of population growth, as is done in Section III, where age-specific birth rates are made a function of the average number of living children the group of women has. The user may flexibly add his own subroutines for linking the projected population structure to labor force participation; migration; household savings; demand for educational, family planning, and health services; and the like, as needed.

A number of examples show the use of the program and the significant differences between "stable" and actual initial population structures for the purpose of generating policy implications about the interactions between the "demographic transition" and the development process.

Starting with a "stable" population, any broad decline in age-specific mortality rates creates a prompt rise in population growth and an extended rise in the dependency ratio. This increased economic burden on the society's adult population reaches a peak after about 20 years if the new lower level of mortality also stabilizes after a few decades. If, on the other hand, any moderately effective measures of birth control are adopted by parents so they will not exceed their traditional surviving family size goals, dependency ratios decline about five percent in two decades.

From the initial "stable" population, a drop in fertility generate similar oscillatory waves in the age structure, but in the reverse direction. A drop in reproductive goals first creates a marked reduction in crude birth rates and population growth. But this brief

respite is followed shortly by a resurgence in crude birth rates as the children born before the change in reproductive goals grow up and enter their childbearing years. This resurgence gradually subsides after 20 years as the smaller, post-transition cohorts begin to enter their childbearing period. The oscillatory behavior created by the slowly changing age composition is more pronounced the greater the speed of changes in birth and death rates and the extent to which they reinforce one another. For example, if a drop in child mortality coincides with a (postwar) baby boom and is then followed by a sharp decline in age-specific fertility, a decade may follow when changes in the age composition will contribute to an increase in crude birth rates and the natural rate of population growth even though age-specific birth rates may continue to fall.

The population of Bangladesh may for many reasons be typical of that of low income countries with very high child mortality and a youthful age structure. In Section III, Bangladesh exemplifies the differences between "stable" and actual population projections. Initially the society bears a very heavy dependency (child to adult ratio) burden that appears likely to grow less burdensome in the future even if only a modest decline in age-specific birth rates occurs. However, this reduction in the dependency ratio is likely to coincide with a substantial increase in the rate of population growth and perhaps even parallel an increase in crude birth rates as the age composition becomes more heavily concentrated in the childbearing years. Although more effective birth control may speed the decline in birth rates and secure a larger ultimate decline in crude birth rates, given the assumptions of the model, a reduction in the surviving family size goal appears to be essential to any substantial slowing of population growth in Bangladesh and countries like it.

ACKNOWLEDGMENTS

We benefited from the comments of A. J. Harman and D. J. O'Hara on an earlier draft of this report. Because of the shortage of time, however, we have not here fully pursued the illustrative and investigative applications of the population projection model that were proposed by our colleagues. This work must await the completion of further research that will permit us to incorporate this demographic model into a more comprehensive characterization of the development process, with specific examination of various policy instruments and their effectiveness.

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I. INTRODUCTION

In this demographic model designed for simulating population change, the user provides the initial female population by age, the time path of age-specific birth and death rates, and, if known, additional optional information. The program then computes the demographic characteristics of the population over the requested planning horizon. A special feature of the model is its estimation of the average number of children borne by women of various age cohorts and the average number and age distribution of those still living. Furthermore, the time path of age-specific fertility (and mortality) can be defined as an endogenous function of such variables as target family size, surviving family size, and the effectiveness of available birth control (that is, contraception combined with abortion).

Section II presents a mathematical description of the demographic model as well as a brief guide on the program's operation. Section III gives a number of examples of how the program can be used to explore the underlying intricacies of hypothetical population change embodied in alternative assumptions. Exercises are also performed using actual data from Bangladesh to illustrate how the program can help isolate deficiencies and inconsistencies in demographic materials. Section IV discusses several policy conclusions that follow from these illustrative exercises. Appendixes report in detail user information and options, a test problem, mathematical approximation procedures, and a full printout of the program.

II. A DEMOGRAPHIC MODEL AND PROJECTION PROGRAM

INTERRELATIONSHIPS OF DEMOGRAPHIC VARIABLES: A MODEL

To describe the growth and change of a population within a mathematical framework, we needed certain restrictive assumptions. The model is limited to a single population, distinguished by age but not by sex. Although the absence of interaction between male and female populations leads to certain inconsistencies, the simplification in terms of mathematical formulation and required data make this assumption desirable.[1] Within this framework a number of conventional demographic terms are defined in Table 1, and their interrelationships are stated here to facilitate a mathematical description of the development of a population over time.[2] First, let $\ell_x(t)$ represent the number of persons aged x at time t, for which it is assumed

$$0 \leq \ell_x(t) < \infty \tag{1}$$

$$\lim_{x \to \omega} \ell_x(t) = 0 \text{ for some fixed } \omega,\ 0 < \omega < \infty \tag{2}$$

$$\ell_{x+u}(t+u) \leq \ell_x(t) \text{ for all } u > 0. \tag{3}$$

Assumption (1) precludes the possibility of "negative" or infinite populations, and assumption (2) assures that no person survives beyond

---

[1] To evaluate the relative composition of a population by male and female components, a separate projection must be performed for each sex. The inconsistencies of such independent projections, the cases in which the independent approximations are accurate, and the necessary data and assumptions for simultaneous projections of both sexes can be found in Keyfitz, 1968, pp. 271-319.

[2] All symbols used are defined in Table 1. The symbols for conventional life table functions, defined by (1) through (9), follow the notation used by Coale and Demeny, 1966, pp. (38)-(39) with two exceptions. First, the argument, t, has been added since vital rates are allowed to vary with time, unlike the case of a stable population. Second, the normalization rule $\ell_x = 100{,}000$ has not been imposed, but rather $\ell_x(t)$ is the actual number of live births at time t.

Table 1

DEFINITION OF SYMBOLS

$\ell_x(t)$	The number of persons of age x living at time t.
${}_nL_x(t)$	The number of person-years lived between ages x and x+n by persons aged x at time t.
${}_nd_x(t)$	The number of persons aged x at time t who die before age x+n
${}_nm_x(t)$	The age-specific mortality rate between ages x and x+n for persons aged x at time t.
${}_nq_x(t)$	The probability that a person aged x at time t will die before age x+n.
$T_x(t)$	The number of person-years remaining to be lived by persons aged x at time t.
$e^o_x(t)$	The average number of years of life remaining for a person aged x at time t.
${}_nA_x(t)$	The number of persons between ages x and x+n at time t.
${}_nB_x(t)$	The number of female children born to women aged x at time t between ages x and x+n.
${}_nf_x(t)$	The age-specific female birth rate for women aged x at time t between ages x and x+n.
${}_n\alpha_x(t)$	An age distribution weight between zero and one, chosen such that

$$ {}_nL_x(t) = n\{\ell_{x+n}(t+n) + {}_n\alpha_x(t)[\ell_x(t)-\ell_{x+n}(t+n)]\} . $$

an arbitrary finite age. Assumption (3) requires that a given cohort will decline in size as time passes and its members age.

From this basic function describing the population we then define:

$$ {}_nL_x(t) = \int_0^n \ell_{x+u}(t+u)\,du \qquad (n > 0) \tag{4} $$

$$ {}_nd_x(t) = \ell_x(t) - \ell_{x+n}(t+n) \qquad (n > 0) \tag{5} $$

$$ {}_nm_x(t) = {}_nd_x(t)/{}_nL_x(t) \qquad (n > 0,\ {}_nL_x(t) > 0) \tag{6} $$

$$ {}_nq_x(t) = {}_nd_x(t)/\ell_x(t) \qquad (n > 0) \tag{7} $$

$$ T_x(t) = \int_x^\omega \ell_u(t+x-u)\,du \tag{8} $$

$$ e_x^o(t) = T_x(t)/\ell_x(t) \tag{9} $$

$$ {}_nA_x(t) = \int_0^n \ell_{x+u}(t)\,du \qquad (n > 0) \tag{10} $$

From (3) and (4) we note that for each set of values of n, x, and t there exists a weight, ${}_n\alpha_x(t)$, between zero and one, such that

$$ {}_nL_x(t) = n\{\ell_{x+n}(t+n) + {}_n\alpha_x(t)[\ell_x(t)-\ell_{x+n}(t+n)]\}\,. \tag{11} $$

If the mortality rate is constant between ages x and x+n, then the value of ${}_n\alpha_x(t)$ will decrease from 0.5 when the mortality rate is zero to approximately 0.475 as the mortality rate approaches 300 per thousand. Values of ${}_n\alpha_x(t)$ outside this range imply that deaths between ages x and x+n are concentrated heavily near age x when ${}_n\alpha_x(t)$ is low or near x+n when ${}_n\alpha_x(t)$ is high. At ages less than five years, when deaths are heavily concentrated in the first days and months of life, ${}_n\alpha_x(t)$ tends to be low.

Using definitions (4) through (6) we then derive the following relationships:

$$ {}_nL_x(t) = \ell_x(t) \frac{n}{1+n[1-{}_n\alpha_x(t)]\,{}_nm_x(t)} \tag{12} $$

$$ \ell_{x+n}(t+n) = \ell_x(t) \frac{1-n\,{}_n\alpha_x(t)\,{}_nm_x(t)}{1+n[1-{}_n\alpha_x(t)]\,{}_nm_x(t)} \tag{13} $$

$$ \ell_{x+n}(t+n) = {}_nL_x(t) \frac{1-n\,{}_n\alpha_x(t)\,{}_nm_x(t)}{n} \tag{14} $$

$$ {}_nL_{x+n}(t+n) = {}_nL_x(t) \frac{1-n\,{}_n\alpha_x(t)\,{}_nm_x(t)}{1+n[1-{}_n\alpha_{x+n}(t+n)]\,{}_nm_{x+n}(t+n)} \tag{15} $$

Having defined the relationship between population and mortality, we now specify the relationship between population and fertility. Let ${}_nB_x(t)$ represent the number of female births to women aged x at time t, between ages x and x+n.[1] The age-specific fertility rate is then defined as

$$ {}_nf_x(t) = {}_nB_x(t)/{}_nL_x(t) \qquad (n > 0,\ {}_nL_x(t) > 0) \tag{16} $$

and the number of persons aged 0 at time t is exactly the number of births at time t

$$ \ell_o(t) = \sum_{x \in s} {}_nB_x(t) = \sum_{x \in s} {}_nf_x(t)\,{}_nL_x(t). \tag{17} $$

The pattern of population growth over time is, therefore, completely determined by the initial population structure, $\{{}_nL_x(0)\}$, and the values of the weights, $\{{}_n\alpha_x(t)\}$, age-specific mortality rates, $\{{}_nm_x(t)\}$, and age-specific fertility rates, $\{{}_nf_x(t)\}$, over time.

---

[1] In order to obtain this value, we have assumed a sex ratio at birth of 106 males per 100 females, or approximately .485 female births per birth of either sex. Where birth rates are available by sex, such an approximation is not necessary.

For purposes of projection, n is assumed to be one year: the cohort interval and the increment for projection. Given an initial population (that is, values of ${}_1L_x(0)$, x=0, 2, ...,), together with complete schedules of age-specific fertility and mortality rates, new values of ${}_1L_x(t)$, t=1, 2, ..., are projected according to equations (12), (15) and (17) as follows:

$$ {}_1L_{x+1}(t+1) = {}_1L_x(t) \frac{1-{}_1\alpha_x(t)\ {}_1m_x(t)_m}{1+[1-{}_1\alpha_x(t)]\ {}_nm_{x+1}(t+1)}, \quad x \geq 0 \qquad (18) $$

$$ \ell_o(t+1) = \sum_{x=o}^{\omega} {}_1f_x(t)\,{}_1L_x(t) \qquad (19) $$

$$ {}_1L_o(t+1) = \ell_o(t+1) \frac{1}{1+[1-{}_1\alpha_o(t+1)]\ {}_1m_o(t+1)} \qquad (20) $$

Although this allows the projection of values of ${}_1L_x(t)$, the actual observed population is more properly represented by the value of ${}_1A_x(t)$. The ratios ${}_1L_x(t)/{}_1L_{x+1}(t+1)$ and ${}_1A_x(t)/{}_1A_{x+1}(t+1)$, however, will differ significantly only if the ratios change substantially in less than one period. In such a case, the second ratio will lag the first in adjusting. To compensate for this lag the program substitutes a combination of the current year and the previous year in place of equation (19):

$$ \ell_o(t+1) = \sum_{x=o}^{\omega} \frac{{}_1f_x(t)\ {}_1L_x(t) + {}_1f_x(t-1)\ {}_1L_x(t-1)}{2} \qquad (21) $$

This has the effect of lagging changes in the values of ${}_1L_x(t)$ to coincide more accurately with changes in the observed population, ${}_1A_x(t)$.

The major assumptions made for the projection of the population by this model are summarized as follows:

A. The growth and development of the entire population can be well approximated by considering the growth and change of a single sex within the population.

B. Except for the earliest ages (less than 2 years), mortality rates remain constant within any single year cohort.[1]

C. The observed population, ${}_1A_x(t)$, is well approximated by the lagged value of ${}_1L_x(t-.5)$, as described above.

Under these assumptions, equations (18), (20), and (21) define the evolution of a population based on a single sex within the population. The data necessary for the projection are the initial age distribution of the population, $\{{}_1A_x(t)\}$, and the age-specific fertility and mortality schedules, $\{{}_1f_x(t)\}$ and $\{{}_1m_x(t)\}$, for the period of the projection.

DESCRIPTION OF THE COMPUTER PROGRAM FOR POPULATION PROJECTION

The program first reads all control cards, which specify the title of the projection; the form of the data on initial population size, fertility, and mortality; the period of the projection; and any optional control parameters. If the user has supplied initial fertility or mortality rates for cohorts broader than a single year, the rates given are assumed to be constant across all ages included in the cohort, with the following exception: if a single mortality rate, ${}_4m_1(0)$, is given for ages one through four, the program assumes that ${}_1m_x(0) = b_x\, {}_4m_1(0)$, $x = 1,2,3,4$ and $b_1 = 1.83$, $b_2 = 1.0$, $b_3 = .65$, $b_4 = .42$.[2] When the

---

[1] It is assumed in the program that ${}_1\alpha_x(t) = 0.5$ for all ages $x > 1$. For ages less than one year and for one year, the program assumes that ${}_1\alpha_0(t) = 0.35$, and ${}_1\alpha_1(t) = 0.42$ unless the user supplies other values for these parameters. Although these default values are arbitrary, they are of plausible magnitude for moderate to high mortality levels and are approximately equal to the values used by Coale and Demeny, 1966, p. (20) in the construction of their life tables.

[2] These values are arbitrarily chosen to approximate the decline in mortality rates between ages one and five. They were chosen such that, when a slowly growing stable population is considered, the aggregate mortality rate ${}_4m_1$ is equal to the sum of the individual year mortality rates ${}_1m_1$, ${}_1m_2$, ${}_1m_3$, ${}_1m_4$, weighted by the corresponding populations.

initial population size is given for cohorts broader than a single year, the population of each cohort is distributed across the individual years of the cohort as if the entire cohort arose from a stable population characterized by the initial mortality rates given and a growth rate R.[1] If the user does not supply a value of R, the program first distributes the population of each cohort as if R = 0, then calculates the growth rate implied by this population distribution and the initial fertility and mortality rates, and finally redistributes the population of each cohort using this derived value of R. For most cases encountered, the estimated value of R changes by less than one percent when additional iterations of this procedure are performed.

The user may enter a description of the "average" family, that is, the average number of children ever born and the average number of surviving children by age per woman of a given age. If these data are not supplied, the program estimates this description on the assumption that initial fertility and mortality rates have prevailed for past generations. After printing the initial data, the program projects the changes in the population by one year intervals until the data have been calculated and printed for the last year of the projection.

The following is a brief description of the actual projection of a population from one year (t) to the next (t+1). The first step in the projection is the evaluation of any user-supplied fertility or mortality functions, fertility functions being evaluated first if both are included.[2] A typical mortality subroutine might redefine mortality

---

[1]A stable population results from constant age-specific mortality rates and a constant growth rate over a long period and can be characterized by

$$ {}_1L_x(t) = {}_1L_{x-1}(t)e^{-R}\,\frac{1-{}_1a_x(t)\,{}_1m_x(t)}{1-[1-{}_1a_{x-1}(t)]\,{}_1m_{x-1}(t)} $$

Lotka referred to such a population as <u>Malthusian</u>, reserving the term <u>stable</u> for the special case in which the constant growth rate results from the prolonged prevalence of constant age-specific fertility rates. See Coale and Demeny, 1966, pp. (9), (10).

[2]For a description of several alternative methods of projecting mortality and fertility schedules, see Shryock <u>et al</u>., 1971, pp. 771 ff.

rates by either an assumed functional relationship of other variables or simply by changing progressively from an initial life table mortality to a new mortality level as shown in Appendix B. Fertility may be directly related to exogenous variables, such as family planning program inputs, or to such endogenous variables as family size, as shown in Appendix B. After new levels of mortality and fertility are determined, the program proceeds to update the description of the "average" family, as described below. The number of women of each age is replaced by the survivors who were one year younger in the previous year, according to formula (18). The total number of births is then calculated by equation (21), and the population aged less than one year is derived by equation (20).

The description of the "average" family includes the total number of live births per woman and the average number of surviving children per woman by the children's ages. The average number of births per woman is calculated by adding the age-specific female birth rate (the number of live female children born per 1,000 women of a given age) to the previous total number of births per 1,000 women of the corresponding age. The average number of children aged less than one year is calculated by reducing the number of new births per woman of a given age, as is done for the entire population in equation (20). For older children, the average number of survivors is calculated as in equation (18). Thus the average family size is estimated, as well as its age distribution and mortality experience. Although the distribution of families about this average can not be inferred directly from the model, the distribution may be approximated by means similar to those described in Appendix C.

The projection for a new year has now been completed, and, if the user has requested output for the year, the program prints data on the projected population. The output includes the title, year of projection, a summary description of the "average" family, and the following data for both the entire population and for each age group: population, percent of the total population, change in population, rate of change, number of births, fertility rate, number of deaths, and the mortality rate. The user may also request that the following summary statistics

be calculated: the percent of the population aged 15 through 44, the average age of the population, the dependency ratio (number of persons aged under 15 or over 60 per person aged 15 through 59), the expectation of life at birth, the birth-woman ratio (the number of births per woman age 15 through 44), and the child-woman ratio (the number of persons less than age 5 per woman aged 15 through 44).[1] The program then continues with the next year's projection until the period of projection is completed, at which point the program either begins to read instructions for a new projection exercise, or terminates.

---

[1]Some sources define the dependency ratio, birth-woman ratio, and child-woman ratio in terms of different denominator age groups. The convention adopted here is that used by Coale and Demeny, 1966, pp. (38)-(39).

III. USE OF THE POPULATION PROJECTION MODEL

To illustrate several of the possible uses of the projection model described in the previous section, we report two sets of exercises here First, an initially "stable" population structure is projected for 100 years under alternative assumptions analogous to those associated with the "demographic transition" apparently occurring today in most low income countries. Second, contemporary data for Bangladesh are used to generate an initial population structure and vital rates from which 50-year projections are traced out. Recall that the model retains only the female population; hence, death rates are for women, and birth rates are for female births per thousand woman-years lived in the specific age group.

STABLE POPULATION PROJECTIONS

The method of achieving the initial stable population structure is described in Appendix B. Briefly, the regime of mortality is from Coale and Demeny "East" life table level 11, and fertility is estimated for East Pakistan circa 1960.[1] These values were projected for 200 years to obtain an approximately "stable" population age structure.[2] The initial population is summarized in Table 2. The salient features are a 3.17 percent annual rate of population growth, a high crude birth rate of about 52 per thousand, and a high crude death rate of 20 per

---

[1]These life table mortality figures are used to obtain a single consistent schedule of mortality approximating the mortality experience of East Pakistan. Most life tables, however, are drawn largely from American and European experiences, because of the availability of more reliable records for these regions. Such mortality schedules, even when matched for overall levels of mortality, may seriously underestimate the child and infant mortality rates actually found in less developed countries. For a comparison of both UN and Coale and Demeny life tables with such high mortality rates, see Adlakha, 1971.

[2]The percentage distribution of the female population by age changed over time only in the third significant digit after a century. Changes in the projected age distributions may therefore be attributed only to assumed changes in vital rates during the projection period.

Table 2

PROJECTION OF INITIALLY STABLE POPULATION[a]

Age Group	Total Population (percent)	Mortality Rate (per thousand females)	Fertility Rate[b]
<1	4.472	205.00	0
1-4	14.488	23.53	0
5-9	15.033	5.45	0
10-14	12.581	3.20	0
15-19	10.558	4.45	120
20-24	8.802	5.87	135
25-29	7.293	6.88	140
30-34	6.016	7.66	135
35-39	4.942	8.53	104
40-44	4.043	9.30	65
45-49	3.289	10.81	25
50-54	2.641	14.57	0
55-59	2.066	21.40	0
60-64	1.544	33.20	0
65-69	1.070	52.42	0
70-74	.657	82.36	0
75-79	.337	127.93	0
80+	.169	229.54	0

[a]Population Growth Rate: 3.174
Crude Birth Rate: 51.47
Crude Death Rate: 20.18
Dependency Ratio: 1.014

[b]Female births per thousand woman-years lived in specific age group.

thousand. The ratio of dependents to adults of economically active ages is about one, there being some nine children 14 years of age or younger for every ten adults between the ages of 15 and 59.

The first set of projections of this initially stable population, reported in Table 3, presume no change in the severe initial regime of mortality summarized in Table 2. The initial level of fertility is consistent with average completed fertility of about seven live births, with only five children, on average, surviving to the mother's 50th birthday (see Table 7, line 3-A). It is arbitrarily assumed that parents shift from their past accomplished family size goal of five living children to a new goal of four living children. Alternative assumptions are explored concerning the availability (and use) of birth control methods to reduce the initial high fertility levels. The no birth control case (Table 3, case A) reflects essentially no change in initial fertility behavior (or family size goal). The 50 percent effective birth control case (case B) represents the probable contribution of traditional folk methods of contraception and abortion. The 99 percent effectiveness level (case D) is intended to represent the opposite extreme of the highest probable level of modern birth control effectiveness. Most populations would fall somewhere between these extremes of 50 and 99 percent birth control effectiveness, depending on their education, private sector response, public policies to subsidize the spread of birth control information, services and supplies, and, of course, the legal and cultural attitudes toward fertility regulation.[1]

The population projection model integrates the cohort estimates of the number of children living, surviving family size goal, and birth control effectiveness to compute for each year a birth rate for each age cohort of women. Specifically, the average number of living children for each cohort is assumed to be the mean of a size distribution

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[1]The value of birth control effectiveness is actually the product of two distinct values: the technical effectiveness of the particular contraceptive method and the proportion of women who use the particular method. Thus a particular level of birth control effectiveness could be obtained either through the introduction of more effective contraceptive techniques or through the use of existing techniques by more women.

Table 3

PROJECTION OF INITIALLY STABLE POPULATION: ASSUMING CONSTANT MORTALITY AND SURVIVING FAMILY SIZE GOAL OF FOUR CHILDREN

	Elapsed Time (years)									
	0	5	10	15	20	30	40	50	75	100
Population Growth Rate (percent per year)										
A. No Birth Control	3.174	3.174	3.174	3.174	3.174	3.174	3.174	3.174	3.174	3.174
B. 50% Effective Birth Control	3.174	2.686	2.781	2.847	2.779	2.697	2.670	2.647	2.635	2.636
C. 90% Effective Birth Control	3.174	2.320	2.502	2.620	2.485	2.341	2.297	2.248	2.225	2.226
D. 99% Effective Birth Control	3.174	2.240	2.444	2.572	2.421	2.264	2.217	2.160	2.135	2.136
Crude Birth Rate (per thousand population per year)										
A. No Birth Control	51.47	51.47	51.47	51.47	51.47	51.47	51.47	51.47	51.47	51.47
B. 50% Effective Birth Control	51.47	45.60	46.82	47.75	46.93	46.11	45.97	45.89	45.89	45.89
C. 90% Effective Birth Control	51.47	41.19	43.53	45.18	43.52	42.14	41.96	41.75	41.78	41.77
D. 99% Effective Birth Control	51.47	40.24	42.84	44.65	42.79	41.29	41.10	40.86	40.90	40.89
Dependency Ratio (Persons less than 15 and over 59 to persons 15 to 59)										
A. No Birth Control	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014	1.014
B. 50% Effective Birth Control	1.014	.966	.925	.893	.911	.913	.906	.913	.915	.916
C. 90% Effective Birth Control	1.014	.929	.861	.808	.841	.841	.832	.845	.849	.851
D. 99% Effective Birth Control	1.014	.921	.847	.790	.826	.826	.817	.831	.835	.838

of numbers of living children to differing frequencies of women in that cohort. This size distribution is approximated according to formulas described in Appendix C. Only the fraction of women with more living children than the specified family size goal are assumed to adopt the available means of birth control. Hence, the model assumes no birth spacing or delay of marriage, but only the application of birth control to prevent all further births to a growing share of each cohort as it ages.[1] Birth control effectiveness is defined to include any combination of institutional (for example, marital separation), biological (for example, prolonged lactation), contraceptive, and abortifacient measures that reduce the initial age-specific birth rates by the particular percentage. For example, if half of the women aged 35 to 39 had more than the target four living children and adopted a 50 percent effective birth control method, the entire cohort would report a birth rate 25 percent below the initial level, that is, 78 female births per thousand woman-years (see Table 2).

With no birth control (case A) the initial stable population is preserved by definition, but as more effective birth control is adopted, the birth rate and population growth rate diminish. Extremely effective birth control (99 percent) leads to a reduction in the birth rate of about a fifth, similar to the shift in family size goal. Moreover, population growth slows by about one-third. A traditional birth control method (50 percent effective) achieves about half of these declines in vital rates.

The time paths of slowing population growth and reducing the birth rate are, however, predictably irregular. A sharp decline occurs in the first five years as fertility is curbed in response to the modified family size goal. After five years, birth rates rebound for another decade and only thereafter slowly subside to the trough

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[1] It is possible, with very effective birth control and severe child mortality, that a large fraction of a cohort of women would adopt birth control as their family size goal changed, and then as they experience losses among their children they would return to childbearing to replace these losses from, say, a cholera epidemic.

achieved in the earlier period. The logical consequence of the age composition changes after a sharp drop in fertility can be attributed to the relatively rapidly increasing numbers of young women reaching childbearing ages from the cohorts born before the decline in fertility.[1] Fifteen years after the change in fertility occurred, the relative size of the cohort of women entering into the childbearing years begins to decrease, sending the birth rate down again toward its new stable level. The dependency ratio lags behind the birth rate, reach- a low 15 years after the change in fertility goals, and thereafter oscillates toward its new equilibrium value

The second and third sets of projections trace through the implications of a reduction in the regime of mortality. In Table 4 the decrease in death rates is limited to the child population less than age 5,[2] whereas Table 5 reports the perhaps more typical broad reduction in mortality regime observed in Europe, which benefits persons of all ages. For an approximate replication of the substantial changes in death rates that have occurred in low income countries from 1930 to 1960, the initial regime (Coale and Demeny, 1966, "East" life tables, level 11) is assumed to improve in equal proportionate steps in the first decade to "East" life tables, level 16 and in the second decade stabilizes at level 18. This implies, for example, that infant mortality drops from 205 per thousand in the first year of the projection to 111 in the tenth year and to 81 after the 19th year. Initially, three out of ten children die before reaching their fifth birthday, whereas nine out of ten reach this age based on the regime of mortality obtained after the 20th year of the projection.

With no change in the initial age-specific fertility rates the crude death rate drops from 20 per thousand to less than 15 in ten

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[1]This delayed consequence of the drop in fertility can be seen today in Taiwan where changes in only the age composition would account for a 23 percent increase in crude birth rates from 1970 to 1980 if age-specific birth rates would not change from 1970 levels. See Freedman and Sun, 1971.

[2]This pattern of improvement in only child mortality might occur if preschool and infant nutrition and health care were given priority in low income countries.

Table 4

PROJECTION OF INITIALLY STABLE POPULATION: ASSUMING MORTALITY DECLINES ONLY FOR CHILDREN, SURVIVING FAMILY SIZE GOAL OF FOUR CHILDREN

		Elapsed Time (years)									
		0	5	10	15	20	30	40	50	75	100
		Population Growth Rate (percent per year)									
A.	No Birth Control	3.174	3.359	3.460	3.465	3.494	3.531	3.571	3.588	3.602	3.601
B.	50% Effective Birth Control	3.174	2.846	3.028	3.095	3.046	2.994	3.002	2.997	3.000	2.999
C.	90% Effective Birth Control	3.174	2.462	2.723	2.838	2.712	2.596	2.583	2.550	2.543	2.542
D.	99% Effective Birth Control	3.174	2.378	2.659	2.785	2.64	2.511	2.493	2.453	2.444	2.443
		Crude Birth Rate (per thousand population per year)									
A	No Birth Control	51.47	51.17	50.56	49.89	49.61	49.93	50.25	50.33	50.36	50.35
B	50% Effective Birth Control	51.47	45.30	45.86	46.04	44.93	44.47	44.59	44.59	44.61	44.61
C	90% Effective Birth Control	51.47	40.90	42.53	43.40	41.43	40.45	40.48	40.35	40.38	40.37
D	99% Effective Birth Control	51.47	39.95	41.84	42.85	40.67	39.59	39.61	39.44	39.48	39.47
		Dependency Ratio (Persons less than 15 and over 59 to persons 15 to 59)									
A	No Birth Control	1.014	1.026	1.051	1.078	1.093	1.095	1.099	1.100	1.096	1.095
B	50% Effective Birth Control	1.014	.976	.956	.945	.975	.976	.971	.976	.975	.975
C	90% Effective Birth Control	1.014	.938	.887	.852	.895	.892	.884	.894	.894	.896
D.	99% Effective Birth Control	1.014	.930	.873	.832	.878	.874	.866	.878	.878	.880

Table 5

PROJECTION OF INITIALLY STABLE POPULATION: ASSUMING DECLINE IN MORTALITY AT ALL AGES, SURVIVING FAMILY SIZE GOAL OF FOUR CHILDREN

		Elapsed Time (years)									
		0	5	10	15	20	30	40	50	75	100
		Population Growth Rate (percent per year)									
A.	No Birth Control	3.174	3.441	3.600	3.623	3.666	3.685	3.717	3.733	3.746	3.745
B.	50% Effective Birth Control	3.174	2.929	3.168	3.252	3.217	3.147	3.146	3.137	3.136	3.135
C.	90% Effective Birth Control	3.174	2.545	2.862	2.995	2.883	2.748	2.725	2.686	2.671	2.671
D.	99% Effective Birth Control	3.174	2.461	2.797	2.942	2.811	2.663	2.634	2.588	2.570	2.569
		Crude Birth Rate (per thousand population per year)									
A.	No Birth Control	51.47	51.18	50.56	49.88	49.55	49.78	50.07	50.15	50.18	50.18
B.	50% Effective Birth Control	51.47	45.30	45.82	45.97	44.81	44.24	44.31	44.29	44.31	44.30
C.	90% Effective Birth Control	51.47	40.89	42.47	43.28	41.26	40.16	40.12	39.95	39.97	39.96
D.	99% Effective Birth Control	51.47	39.93	41.77	42.73	40.49	39.29	39.23	39.02	39.04	39.03
		Dependency Ratio (Persons less than 15 and over 59 to persons 15 to 59)									
A.	No Birth Control	1.014	1.025	1.049	1.076	1.090	1.093	1.097	1.098	1.094	1.094
B.	50% Effective Birth Control	1.014	.975	.954	.943	.972	.973	.969	.975	.974	.975
C.	90% Effective Birth Control	1.014	.937	.886	.850	.892	.889	.882	.894	.894	.896
D.	99% Effective Birth Control	1.014	.929	.871	.830	.875	.871	.864	.877	.878	.881

years and approximately stabilizes at 13 per thousand after 20 years. The rate of population growth, therefore, increases gradually from 3.2 to 3.7 percent per year, an increase of almost 16 percent, and the dependency ratio increases sharply by about eight percent in 20 years. At the family level this increase in dependency burden is evident (see Table 7) for women between the ages of 30 and 39, who experience a 12 percent increase in the number of living children they must support solely because of the decline in child death rates.[1]

If the improvement in mortality is limited to children under age five (Table 4) the increase in population growth is three-fourths the magnitude of that for a reduction of all death rates. The child-specific decline in mortality leads to a slightly more adverse rise in the dependency ratio, but the differences are not substantial. These exercises indicate that the significant differences in mortality that exist between high and low income countries and occur during the process of development mainly benefit children and substantially increase the survival chances for the preschool population.[2]

Where a 20 percent reduction in fertility goal occurs simultaneously with the reduction in death rates, the 50 percent effective birth control is sufficient to offset the mortality decline, ultimately slow population growth, and clearly reduce the dependency ratio. Highly effective birth control (99 percent), of course, permits birth rates to fall further, by 20 percent, and reduces the rate of population growth by about one-fifth in five years. An intermediate-term resurgence of crude birth and population growth rates is followed again by a fall to about this trough by the end of a century.

A more extreme example is illustrated in case D of Table 6. In this instance the same decline in mortality is assumed to occur for all persons as in Table 5, but the family size goal of parents shifts now

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[1] For example, women age 30 to 34 had borne 4.739 children of which only 3.532 survived under the initial regime of death rates, whereas 3.942 were living after 30 years of the projected mortality decline.

[2] See evidence of this point for Punjab communities of India in Wyon and Gordon, 1971, Chapter 7.

Table 6

PROJECTION OF INITIALLY STABLE POPULATION: ASSUMING MORTALITY DECLINES AT ALL AGES AND SURVIVING FAMILY SIZE GOAL OF THREE CHILDREN

		Elapsed Time (years)									
		0	5	10	15	20	30	40	50	75	100
		Population Growth Rate (percent per year)									
A.	No Birth Control	3.174	3.441	3.600	3.623	3.666	3.685	3.717	3.733	3.746	3.745
B.	50% Effective Birth Control	3.174	2.693	2.969	3.087	3.018	2.913	2.893	2.873	2.865	2.865
C.	90% Effective Birth Control	3.174	2.125	2.513	2.703	2.522	2.330	2.267	2.200	2.162	2.164
D.	99% Effective Birth Control	3.174	2.002	2.417	2.623	2.414	2.205	2.131	2.052	2.005	2.008
		Crude Birth Rate (per thousand population per year)									
A.	No Birth Control	51.47	51.18	50.56	49.88	49.55	49.78	50.07	50.15	50.18	50.18
B.	50% Effective Birth Control	51.47	42.57	43.64	44.24	42.71	41.84	41.80	41.76	41.77	41.76
C.	90% Effective Birth Control	51.47	36.02	38.64	40.22	37.44	35.92	35.63	35.39	35.41	35.39
D.	99% Effective Birth Control	51.47	34.59	37.58	39.39	36.30	34.65	34.31	34.02	34.05	34.03
		Dependency Ratio (Persons less than 15 and over 59 to persons 15 to 59)									
A.	No Birth Control	1.014	1.025	1.049	1.076	1.090	1.093	1.097	1.098	1.094	1.094
B.	50% Effective Birth Control	1.014	.953	.912	.884	.921	.923	.917	.926	.926	.928
C.	90% Effective Birth Control	1.014	.898	.814	.752	.805	.805	.797	.815	.819	.823
D.	99% Effective Birth Control	1.014	.886	.793	.725	.782	.780	.772	.793	.799	.803

to three living children.[1] With highly effective birth control (99 percent) the crude birth rate drops sharply from 51.5 to 34.6 in five years and then rises to 39.3 in the subsequent ten years, only to fall again to 34.4 after another 25 years. The dependency ratio also drops by almost 30 percent in 15 years and thereafter increases about ten percent to a new stable level after the passage of another 35 years.

Table 7 summarizes the program estimates of the outcome in terms of surviving family size and completed fertility for different aged cohorts during several of the projections. For example, with the broad drop in death rates and the moderate reduction in family size goal, a 90 percent effective regime of birth control reduces completed fertility for women between the ages of 35 and 39 from six to five births in a decade (cases A through C of Table 3); the number of living children these women must provide for on the average decreases from 4.33 to 3.63. Several decades must elapse before older cohorts are able to scale down their completed reproduction toward the new goal.

The implied age-specific birth rates that underlie some of the stable population projections are summarized in Table 8. As assumed in the model, fertility reduction occurs by means of adopting birth control to terminate further childbearing, rather than by delaying the initiation of childbearing and spacing of births. The resultant reductions in age-specific birth rates are concentrated at older ages and parallel declines in Taiwan and the United States rather than those in countries where marriage has been delayed (for example, Ireland or Korea) or where married women have played only a limited role in the nonfamily labor market (for example, The Netherlands).

PROJECTIONS OF THE BANGLADESH POPULATION

The second set of population projections is based on estimates of the population structure and vital rates for East Pakistan in the years 1962-1963. Table 9 summarizes the initial population statistics.

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[1] The no birth control or no change in fertility case (case A) is identical to that shown in case A of Table 5.

Table 7

AVERAGE NUMBER OF CHILDREN EVER BORN AND NOW LIVING ASSOCIATED WITH POPULATION PROJECTIONS[a]

Table, Projection, and Time		Age of Women						
		15-19	20-24	25-29	30-34	35-39	40-44	45-49
3-A	any	.703	1.990	3.369	4.739	5.908	6.720	7.136
		(.573)	(1.544)	(2.553)	(3.532)	(4.327)	(4.826)	(5.003)
3-C	10	.694	1.893	3.072	4.238	5.349	6.278	6.898
		(.565)	(1.465)	(2.259)	(2.955)	(3.562)	(4.095)	(4.501)
3-C	20	.705	1.886	2.990	3.927	4.639	5.148	5.567
		(.564)	(1.460)	(2.255)	(2.908)	(3.373)	(3.664)	(3.861)
3-C	30	.700	1.892	2.989	3.925	4.642	5.096	5.326
		(.570)	(1.465)	(2.255)	(2.907)	(3.375)	(3.603)	(3.701)
5-A	10	.703	1.990	3.369	4.739	5.908	6.720	7.136
		(.609)	(1.634)	(2.668)	(3.655)	(4.446)	(4.926)	(5.080)
5-A	20	.699	1.990	3.369	4.740	5.908	6.720	7.136
		(.621)	(1.696)	(2.805)	(3.856)	(4.671)	(5.149)	(5.277)
5-A	30	.701	1.987	3.364	4.739	5.908	6.720	7.136
		(.625)	(1.706)	(2.839)	(3.942)	(4.831)	(5.368)	(5.516)
5-C	10	.693	1.879	2.983	3.981	4.899	5.752	6.468
		(.600)	(1.538)	(2.346)	(3.035)	(3.633)	(4.155)	(4.554)
5-C	20	.700	1.869	2.944	3.859	4.577	5.105	5.545
		(.622)	(1.590)	(2.439)	(3.113)	(3.574)	(3.848)	(4.022)
5-C	30	.691	1.864	2.937	3.822	4.503	4.965	5.233
		(.616)	(1.597)	(2.469)	(3.162)	(3.654)	(3.925)	(3.986)
6-C	10	.660	1.722	2.687	3.628	4.582	5.526	6.340
		(.572)	(1.407)	(2.106)	(2.752)	(3.381)	(3.976)	(4.453)
6-C	20	.672	1.709	2.627	3.408	4.040	4.573	5.114
		(.596)	(1.452)	(2.172)	(2.745)	(3.146)	(3.430)	(3.687)
6-C	30	.657	1.702	2.623	3.371	3.956	4.359	4.606
		(.585)	(1.457)	(2.201)	(2.785)	(3.207)	(3.440)	(3.498)
6-D	10	.656	1.699	2.630	3.534	4.465	5.416	6.264
		(.568)	(1.388)	(2.059)	(2.677)	(3.287)	(3.888)	(4.393)
6-D	20	.670	1.685	2.567	3.305	3.898	4.404	4.944
		(.594)	(1.431)	(2.121)	(2.659)	(3.030)	(3.295)	(3.553)
6-D	30	.652	1.677	2.563	3.266	3.810	4.184	4.417
		(.581)	(1.435)	(2.150)	(2.697)	(3.086)	(3.297)	(3.348)
6-D	100	.660	1.687	2.555	3.262	3.788	4.128	4.296
		(.587)	(1.443)	(2.145)	(2.704)	(3.100)	(3.329)	(3.406)

[a]Average number of children now living given in parentheses.

Table 8

AGE-SPECIFIC BIRTH RATES ASSOCIATED WITH POPULATION PROJECTIONS[a]

Table, Projection, and Time		Age of Women						
		15-19	20-24	25-29	30-34	35-39	40-44	45-49
3-A	any	247	278	288	278	214	134	52
5-B	10	244	258	243	214	156	94	35
5-D	10	242	240	206	166	109	60	22
6-D	10	226	206	171	134	85	45	15

[a]Birth rates are expressed in conventional terms of births of both sexes per thousand women of the specified ages. The birth rates used in the program are female births per thousand women of the specified ages. It is assumed that the sex ratio of live births is 106 males per hundred females.

Table 9

INITIAL BASIS FOR BANGLADESH POPULATION PROJECTION

Ages	Population Structure[a] (thousands)	(percent of total)	Death Rate[b] (per thousand)	Birth Rate[b] (per thousand)
Less than 1	1,437.7	5.06	194.40	
1-4	4,187.3	14.73	24.40	
5-9	4,381.5	15.41	5.86	
10-14	3,549.5	12.48	2.35	
15-19	2,319.5	8.16	4.69	135.7
20-24	1,943.5	6.83	6.76	169.6
25-29	2,018.5	7.10	8.89	163.7
30-34	1,656.5	5.83	8.25	145.7
35-39	1,437.5	5.06	8.20	86.7
40-44	1,206.0	4.24	12.32	28.2
45-49	989.5	3.48	10.97	13.0
50-54	822.5	2.89	13.10	
55-59	679.0	2.39	22.49	
60-64	557.0	1.96	47.75	
65-69	447.0	1.57	47.61	
70-74	341.0	1.20	82.28	
75-79	187.8	.66	121.38	
80 and over	268.7	.95	273.88	

[a]Population structure is 1962-1963 average for East Pakistan female population from Brackett and Akers, 1965, Table E3, p. 32 (Assuming Constant Fertility and Mortality-Series A).

[b]Vital rates are 1962-1963 averages for East Pakistan female population from Population Growth Estimation Experiment (Report of The): Description and Some Results for 1962 and 1963, 1968. Birth rates are estimated as the number of female children born to women of each age (Table A-18) divided by the number of women of that age (Table PC-3). Death rates are estimated as the number of female deaths at a given age (Table B-1) divided by the number of women of that age. Assuming that the sample population given in Table PC-3 is a good estimate of ${}_nL_x$, these ratios are estimates of the mortality rates and fertility rates as defined in Section II of this report.

The notes to Table 10 describe assumptions underlying these initial data and derivative projections shown in Table 10. The crude birth rate is initially 49 per thousand persons, the crude death rate is 25, and the rate of population increase is 2.4 percent per year. The dependency ratio in this population is substantially higher, 1.18, than in the preceding "stable" population projection.

In case A (Table 10), age-specific birth and death rates estimated for 1962-1963 from the Population Growth Experiment are assumed to remain unchanged. The initial age composition, which reflects very high fertility of the 1950s, contributes to a sharp increase in the projected crude birth rate for the first 20 years, stabilizing thereafter at about 55 births per thousand population; the crude death rate falls to about 21 per thousand. With constant age-specific birth and death rates, Bangladesh experiences a 40 percent acceleration in its rate of population growth from 2.4 to 3.4 percent per year within 20 years. Nonetheless, the extraordinarily high initial dependency ratio declines by one-tenth in the first decade of the projection as the population structure moves toward a more "stable" configuration.

All subsequent projections of the Bangladesh population assume that the regime of mortality estimated for 1962-1963 systematically improves until 1982-1983 and remains unchanged thereafter (see notes to Table 10). Case B of Table 10 is based on the assumption that this change in age-specific mortality rates does not elicit any change in age-specific birth rates. In this case the rate of population growth increases more than 60 percent in 20 years, crude death rates drop to about 14 per thousand, and the dependency ratio, after first falling, returns in 40 years to the very high initial levels observed in 1962-1963.

We now consider alternative assumptions regarding how reproduction goals change and the effectiveness of birth control adopted to accomplish these goals. Implicit in the initial population statistics summarized in Table 9 is the "stable" surviving family size of about five children when the mother is 50 years of age. In the case C projections, it is assumed that this "traditional" surviving family size goal remains in effect, but in case C-1 only 50 percent effective

Table 10

PROJECTION OF CONTEMPORARY POPULATION OF BANGLADESH ASSUMING DIFFERENT TRENDS IN MORTALITY AND FERTILITY BEHAVIOR[a]

Assumptions for Models	Elapsed Time (years) 0	5	10	20	30	40	50
Population Growth Rate (percent per year)							
A	2.393	2.868	3.132	3.432	3.434	3.493	3.515
B	2.393	3.138	3.566	3.942	3.968	4.057	4.089
C-1	2.393	2.754	3.263	3.577	3.529	3.613	3.617
C-2	2.393	2.470	3.053	3.310	3.213	3.294	3.271
C-3	2.393	2.409	3.009	3.252	3.146	3.226	3.197
D-1	2.393	2.715	3.184	3.404	3.376	3.424	3.422
D-2	2.393	2.401	2.914	3.004	2.949	2.958	2.927
D-3	2.393	2.333	2.857	2.918	2.859	2.859	2.821
Crude Birth Rate (per thousand per year)							
A	48.81	50.23	52.49	55.22	54.69	55.00	55.13
B	48.81	49.95	51.55	53.10	52.93	53.54	53.68
C-1	48.81	45.62	48.30	49.26	48.45	49.08	48.99
C-2	48.81	42.43	46.07	46.45	45.25	45.90	45.59
C-3	48.81	41.75	45.60	45.85	44.57	45.23	44.87
D-1	48.81	45.14	47.39	47.37	46.87	47.14	47.04
D-2	48.81	41.58	44.45	43.09	42.53	42.48	42.20
D-3	48.81	40.81	43.84	42.17	41.62	41.49	41.18
Dependency Ratio (persons less than 15 and over 59 to persons 15-59)							
A	1.175	1.124	1.083	1.094	1.115	1.085	1.083
B	1.175	1.135	1.119	1.173	1.200	1.175	1.174
C-1	1.175	1.095	1.045	1.084	1.095	1.065	1.068
C-2	1.175	1.064	.992	1.024	1.023	.991	.996
C-3	1.175	1.057	.981	1.011	1.008	.976	.982
D-1	1.175	1.092	1.037	1.052	1.051	1.023	1.022
D-2	1.175	1.060	.977	.970	.949	.922	.921
D-3	1.175	1.053	.965	.953	.928	.901	.901

[a]Initial population age structure, fertility rates, and mortality rates are those of Table 9.

Case A: Initial mortality and fertility rates continue unchanged.

Case B: Initial fertility rates continue unchanged. Mortality declines over the first ten years by a constant schedule of rates. The schedule of rates is defined such that, if the initial mortality rates were those of "East" life tables, level 11 females of Coale and Demeny (1966), the mortality rates would be reduced to those of "East" life tables, level 16 females. Over the

Table 10, continued

second ten years, mortality rates are similarly reduced by rates that would take "East" life tables, level 16 female mortality rates to level 18 female mortality rates. After 20 years, mortality rates remain constant at their new levels.

Case C: Mortality rates decline as for case B. Fertility rates for all women with more than a desired family size of five surviving children are reduced as described in Appendix C. The value of ε is 50 percent for case C-1, 90 percent for case C-2, and 99 percent for case C-3.

Case D: Identical to case C, except the desired family size is assumed to decline linearly from five to four children over the first 20 years and remain constant at four children thereafter. The three variants assume 50, 90, and 99 percent effective birth control as in case C.

birth control is adopted to accomplish this goal. In case C-2 birth control is assumed to be 90 percent effective and in case C-3, 99 percent effective. With extremely effective birth control (C-3), crude birth rates decrease about 16 percent in five years compared with the control case (projection B), whereas the rate of population growth is about ten percent less; and the dependency ratio falls first ten, and eventually 20, percent below the control projection (B). Reproductive behavior in the control case is assumed not to respond to the mounting pressures of the dependency ratio or to the increasing number of living children older parents must support.

The final set of projections (D) postulate the same mortality changes that occur in B and C, but in this case the surviving family size goal is assumed to decline linearly from five to four over the first 20 years of the projection. Three levels of birth control effectiveness are considered as in the earlier examples. With 99 percent effective birth control (D-3), earlier noted patterns are exaggerated. However, relative to the case where fertility goals did not change (C-3), the 20 percent decline in reproductive goals causes only a seven percent drop in crude birth rates, and the rate of population growth and the dependency ratio fall about ten percent below these values associated with the larger family size goal but similarly effective birth control (C-3).

From this specific examination of the case of Bangladesh several policy implications emerge. The population in 1962-1963 is much younger than the "stable" population associated with current estimated birth and death rates. This would appear to follow from a sharp drop in death rates in the decades preceding 1960 and perhaps an unusual rise in fertility during the decade following the partition of India and Pakistan with its unsettling aftermath. From the high dependency ratio for 1962-1963, the population composition of Bangladesh must become more favorable from an economic point of view (the dependency ratio must fall) regardless of subsequent trends in fertility or mortality.

However, the rate of population growth is also likely to accelerate after 1962-1963 both because of evolving age composition effects and because of the effect of probable continuing declines in age-specific

death rates. This latter source of further acceleration in the rate of population growth would be approximately offset by a compensating decrease in fertility that was motivated by parents trying to hold their completed surviving family size constant (projection C-1) in the face of the declining child death rates if their birth control were somewhat more than 50 percent effective. Unless there is a further reduction in the number of living children parents want or a great increase in the effectiveness of birth control that they adopt, the rate of population growth is likely to increase for Bangladesh to about three percent per year. Although it does not seem realistic to project a decline in the rate of population growth, it seems highly reasonable to assume that some combination of birth control effectiveness and change in reproductive goals will induce a substantial reduction in the dependency ratio with its improved implications for the economic productive capabilities of the population relative to its consumption requirements.

IV. CONCLUSIONS

This report makes accessible an adaptable program for projecting the age composition and growth of populations. The user specifies the time path of age-specific birth and death rates and the initial population; the program then computes most generally desired demographic characteristics of the population over time, including the average number of children born and surviving by age to women of various ages. The fertility and mortality trends may be made endogenous to a model of population growth, as was done in the preceding section, where age-specific birth rates were made a function of the average number of living children the group of women had. The user may flexibly add his own subroutines for linking the projected population structure to labor force participation; migration; household savings; demand for educational, family planning, and health services; and so on, as needed.[1]

In the preceding section a number of illustrations showed the use of the program and the significant differences between "stable" and actual initial population structures for the purpose of generating policy implications about the interactions between the "demographic transition" and the development process.

Starting with a "stable" population, any broad decline in age-specific mortality rates creates a prompt rise in population growth and an extended rise in the dependency ratio. This increased economic burden on the society's adult population reaches a peak after about 20 years if the new lower level of mortality also stabilizes after a few decades. If, on the other hand, any moderately effective measures of birth control are adopted by parents not to exceed their traditional surviving family size goals, dependency ratios decline about five percent (see Table 5, case B) in two decades.

---

[1]Obviously, if the target for a family planning program is the population of women of reproductive age and particularly those aged 30 to 44, these target groups could be projected with ease under alternative assumptions about future birth rates for any period beyond a couple of decades into the future.

From the initial "stable" population a drop in fertility generates similar oscillatory waves in the age structure, but in the reverse direction. A drop in reproductive goals first creates a marked reduction in crude birth rates and population growth. But this brief respite is followed shortly by a resurgence in crude birth rates as the large number of children born before the change in reproductive goals grow up and enter upon their childbearing years. This resurgence of crude birth rates and population growth rates gradually subsides after 20 years as the smaller, post-transition cohorts begin to enter their childbearing period. The oscillatory behavior created by the slowly changing age composition is more pronounced the greater the speed of changes in birth and death rates and the extent to which they reinforce one another. For example, if a drop in child mortality coincides with a (postwar) baby boom, and is then followed by a sharp decline in age-specific fertility, a decade will follow (for example, the 1970s in Taiwan) when changes in the age composition will contribute to an increase in crude birth rates and the natural rate of population growth, even though age-specific birth rates may continue to fall (Freedman and Sun, 1971).

The Bangladesh population may for many reasons be typical of that in low income countries with very high child mortality and a youthful age structure. The choice of Bangladesh exemplifies the differences between "stable" and actual population projections. Initially the society bears a very heavy dependency (child to adult ratio) burden that appears likely to grow less burdensome in the future even if only a modest decline in age-specific birth rates occurs. However, this reduction in the dependency ratio is likely to coincide with a substantial increase in the rate of population growth and perhaps even parallel an increase in crude birth rates as the age composition becom more heavily concentrated in the childbearing years. Although more effective birth control may speed the decline in birth rates and secure a larger ultimate decline in crude birth rates, given the assumptions of the model, a reduction in the surviving family size goal appears to be essential to any substantial slowing of population growth in countries like Bangladesh.

Appendix A

INSTRUCTIONS FOR USING THE PROGRAM

The projection program requires two sets of information: a Control Section and a Data Section. The Control Section supplies the program with all necessary information for performing the projection, such as the title, location, and format of the data to be read; what output to print; and what options to use. The Data Section supplies the program with the initial population distribution, fertility and mortality rates, and optionally a description of the "average" family. These two sections may be repeated as often as necessary. The variable names used by the program are defined in Table A-1. The program will check for possible errors and, if one is encountered, return a descriptive error message. The error messages used are listed in the foldout table at the end of this report.

CONTROL SECTION

Card 1 -- Title Card

This card contains a title of up to 80 characters identifying the projection. It will be printed at the top of each page of output.

Card 2 -- Projection Card (I4,6X,4I2,2X,20I1,F10.0,2(5X,F5.0)).

col. 1-4: Base year of the projection.

col. 11-12: Data set reference number for the data set containing the initial population distribution. Any legitimate data set reference number (1 through 99, except for 6, the printer) may be used. If none is supplied, the program assumes that the data set is on cards, unit (5).

col. 13-14: Data set reference number for the data set containing the initial fertility rates. If no value is provided, the program assumes that these data are in the same data set as the population distribution.

Table A-1

DEFINITION OF VARIABLES IN PROGRAM

Variable	Definition
POP(I,J)	The population aged J - 1 years for J = 1,...,81; I = 1,2 for the current and previous year respectively.
BIRTHS(I,J)	The number of female births to women aged J - 1 for J = 1,...,81; I = 1,2 for the current and previous years respectively.
DEATHS(I,J)	The number of deaths of persons aged J - 1 for J = 1,...,81; I = 1,2 for the current and previous years respectively.
ASFR(I,J)	The age-specific fertility rate for women aged J - 1 for J = 1,...,81 in births per woman per year; I = 1,2 for the current and previous year respectively.
ASMR(I,J)	The age-specific mortality rate for persons aged J - 1 for J = 1,...,81 in deaths per person per year; I = 1,2 for the current and previous year respectively.
FAMILY(I,J)	Description of "average" family: Mother's age = I + 14 for I = 1,...,35; Female child's age = J - 1 for J = 1,...,35; Total number of live female births to mother for J = 36.
IYEAR	Year of projection.
AZERO	${}_1\alpha_0$(IYEAR)
AONE	${}_1\alpha_1$(IYEAR)
R	The value initially calculated (or read) for the growth rate used to disaggregate the population of multiple year cohorts (if Option 1 is less than 3).
IOPT(I)	The value of Option number I for I = 1,...,20.

col. 15-16: Data set reference number of the data set containing the initial mortality rates. Same default condition applies as for the initial fertility rates.

col. 17-18: Data set reference number for the data set containing the description of the "average" family, if such a description is read. Same default condition applies as for the initial fertility and mortality rates.

col. 21-40: Values assigned to the array IOPT(I), I = 1,...,20. These values determine which options will be used for the projection. The available options and the corresponding values of IOPT are listed in Table A-2.

col. 51-60: Value of R required for disaggregating multiple year cohorts in the population distribution. This value is used only if IOPT(9) = 1.

col. 66-70: Initial value of AZERO. This value is used only if IOPT(7) = 1.

col. 76-80: Initial value of AONE. This value is used only if IOPT(7) = 1.

Card 3 -- Output Card (20I4)

This card contains the list of years for which output is to be printed. The years need not be in order, but the list is assumed to end once a year less than or equal to the base year is encountered. As many as 100 years of printout are allowed, and additional cards may be used as necessary. Note that if less than 100 years of output is desired the list must end with a year less than or equal to the base year, even if a new card is required for only this year.

DATA SECTION

POPULATION (8F10.0)

The initial population distribution is the first data read by the program. The population is read from ages 0 through 80+. The data may be aggregated in any one of four forms, as determined by IOPT(1). These four forms are listed in Table A-2.

Table A-2

CONTROL OPTIONS

Option	Results
1	Controls level of aggregation for initial population distribution to be read as follows: Value — Cohorts 0 — 0,1-4,5-9,10-14,...,75-79,80+ 1 — 0,1,2,3,4,5-9,10-14,...,75-79,80+ 2 — 0,1,2,3,4,5,6,7,8,9,10-14,...,75-79,80+ 3 — 0,1,2,3,4,5,6,7,8,9,10,11,...,78,79,80+
2	Controls level of aggregation and limits of initial fertility schedule to be read as follows: Value — Cohorts 0 — 15-19,20-24,25-29, 30-34,35-39,40-44,45-49 1 — Cohort size and limits read as data. See Data Section: Fertility.
3	Controls level of aggregation for initial mortality schedule to be read. Values and resulting cohorts are the same as for Option 1.
4	Program generates initial description of "average" family if value is zero (0); description read as data if value is one (1).
5	Fertility rates constant if value is zero (0); fertility rates supplied by ENFERT if value is one (1).
6	Mortality rates constant if value is zero (0); mortality rates supplied by ENMORT if value is one (1).
7	AZERO = 0.35, AONE = 0.42 if value is zero (0); values of AZERO and AONE read if value is one (1).
8	Projection output printed for single years of age through age nine (9) if value is one (1); output printed for ages 0,1-4, 5-9 if value is zero (0).
9	Initial value of R calculated by program if value is zero (0); value of R read if value is one (1).
10	Average age, percent aged 15-44, dependency ratio, expected lifespan at birth, birth-woman ratio, and child-woman ratio printed if value is one (1).
11-20	Available for user options in INFERT and ENMORT.

FERTILITY (8F10.0)

The initial fertility rates are read second by the program. If the value of IOPT(2) = 0, fertility rates are read for ages 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, and 45-49. If the value of IOPT(2) = 1, the program reads values N1, N2, and N3 (3I2) from the fertility data set before reading any fertility rates. These values represent the lower limit of the first cohort, the width of each cohort, and the lower limit of the last cohort for which fertility rates are to be read. N3 must be greater than or equal to N1, and the difference N3-N1 must be evenly divisible by N2. If, for example, IOPT(2) = 1 and values of N1 = 10, N2 = 10, and N3 = 50 were supplied, fertility rates would be read for ages 10-19, 20-29, 30-39, 40-49, and 50-59.

MORTALITY (8F10.0)

The program next reads the initial mortality rates for ages 0 through 80+. The levels of aggregation allowed are the same as those for the initial population distribution shown in Table A-2. The level of aggregation is determined by IOPT(3).

FAMILY (8F10.0)

If the value of IOPT(4) = 1, the program next reads a description of the "average" family. This description includes the number of children currently living, by age, by age of mother, and the total number of children ever born by age of mother. These data are stored in the array FAMILY as shown in Table A-1. The data will be read in the following order:

((FAMILY(I,J), J = 1, 36), I = 1, 35),

that is, the number of children alive for ages 0 through 35 and the total number of children ever born according to the age of the mother, from 15 through 49. Eight values are read per record until all entrie have been read.

Although a fixed format is specified for population, fertility, mortality, and family description, any decimal points coded in the

data will override the specifications of this format. The following assumptions are made for the data read by the program:

1. If an initial growth rate, R, is read, the value is per 1,000 per year.
2. Fertility and mortality rates are per 1,000 per year.
3. Fertility rates and the description of the "average" family are in terms of female children per woman, not total children per woman.

USER SUPPLIED FERTILITY AND MORTALITY FUNCTIONS

If levels of fertility and mortality remain constant, the projected population will assume a stable configuration characteristic of the particular mortality and fertility schedules supplied as input. The program, however, allows the user to vary mortality and fertility rates over time. The variation may be either a simple function of time or include the current family size and age distribution and various measures of birth control and family planning and policy options, such as levels of expenditure on health or family planning programs. Any of these possibilities is permitted by the use of two subroutines, ENFERT and ENMORT.

ENFERT is called if the value of IOPT(5) = 1, and ENMORT is called if the value of IOPT(6) = 1. These subroutines may use any of the values of IOPT(I), I = 11,...,20 to supply additional parameters for the functions. All data are supplied to and returned by the subroutines through COMMON statements as shown in Appendix D, p. 76. If both ENFERT and ENMORT are used, ENFERT is called first. ENFERT should return the new values of the age-specific fertility rate as ASFR(1,I), I = 1,...,81; ENMORT should return the values of the age-specific mortality rate as ASMR(1,I), I = 1,...,81. All values returned should be in terms of per person per year, not per 1,000 per year as the input values are. If IOPT(3) = 0 (if a single mortality rate is read initially for the combined ages one through four), the program will assume that the values returned for ages one through four by ENMORT represent such a combined average. The program will then set ASMR(1,2) = ASMR(1,3) * 1.83, ASMR(1,3) = ASMR(1,3), ASMR(1,4) = 0.65 * ASMR(1,3), and ASMR(1,5) = 0.42 * ASMR(1,3) as is done with the initial values.

Appendix B

A SAMPLE PROBLEM

The following pages contain a complete sample problem, including the user supplied fertility and mortality functions, control section input, initial population, fertility rates, and mortality rates, together with all output generated by the program. The initial mortality rates are taken from Coale and Demeny, 1966, p. 448, Female Population "East" life tables, level 11. The initial fertility rates are estimated from Schultz and DaVanzo, 1970 and the U.S. Bureau of the Census.[1] The initial population is the stable population distribution implied by these fertility and mortality rates.[2] By starting with this stable population, any changes in population or vital rates over the projection period can be attributed directly to changes in age-specific fertility and mortality rates determined by subroutines ENFERT and ENMORT.

In our example, ENMORT reduces the initial age-specific mortality from "East" life tables, level 11 to level 16 over the first ten years of the projection, and from level 16 to level 18 over the second ten years, after which time they are assumed to remain constant. If IOPT(11) = 1, mortality rates are reduced for all ages less than five years; if IOPT(11) = 2, mortality rates are reduced for all ages. The subroutine ENFERT reduces fertility rates by ε for all women with IOPT(13) or more living children (IOPT(13)/2 or more living female children), where ε is determined by the value assigned to IOPT(12) as follows:

IOPT(12) = 1, ε = 50%, IOPT(12) = 2, ε = 90%; IOPT(12) = 3, ε = 99%

---

[1] Brackett and Akers, 1965.

[2] This stable population was generated by allowing the program to run for a period of 200 years with constant fertility and mortality rates, at which point the population distribution showed no measurable changes over time.

Surviving family sizes are assumed to follow a truncated normal distribution as described in Appendix C. The function QTRUNC calculates the normal distribution function according to approximations 5704 and 5722 (Hart et al., 1968).

The following option values were specified for the sample problem that follows:

IOPT(5) = 1	User supplied fertility function,
IOPT(6) = 1	User supplied mortality function,
IOPT(8) = 1	Extended output for ages 0 through 9,
IOPT(10) = 1	Population summary statistics printed,
IOPT(11) = 2	Mortality reduced for all ages,
IOPT(12) = 2	90 percent effective response of fertility to changes in surviving family size,
IOPT(13) = 4	Desired family size of 4 children.

All data were read from cards (Data Set number 5). Output was requested for years 5, 10, 15, 20, 30, 40, 50, 75, and 100. The problem was run on Rand's IBM 360/65 (OS Release 19, MVT), and required 32.3 seconds of Central Processing Unit (CPU) time and 56K-bytes of core.

FORM 20
8 FEB 66

```
MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION
  0     5   5   5     00001101012240000000
  5   10  15   20   30   40   50   75  100
  44720 0.   144880 0.   150330 0.   125810 0.   105580 0.   88020 0.   72930 0.   60160 0.
  49420 0.   40430 0.   32890 0.   26410 0.   20660 0.   15440 0.   10700 0.   6570 0.
  3370 0.   1690 0.
 120.   135.   140.   135.   104.   65.   25.
 205.00   23.53   5.45   3.20   4.45   5.87   6.88   7.66
  8.53   9.30   10.81   14.57   21.40   33.20   52.42   82.36
 127.93   229.54
```

CONTROL SECTION INPUT FOR SAMPLE PROBLEM

```
      SUBROUTINE ENMORT
      DIMENSION POP(2,81),BIRTH(2,81),DEATH(2,81),ASFR(2,81),ASMR(2,81)
     $ ,FAMILY(35,36),IOPT(20)
      COMMON POP,BIRTH,DEATH,ASFR,ASMR,FAMILY,IYEAR,AZERO,AONE,R,IOPT
      DIMENSION RATE(18,2)
      DATA RATE /.940292363,.918173130,.925396221,.932448199,.936131335,
     & .937543142,.937803854,.940323212,.945103280,.952056959,.960175632
     &,.964115711,.965282695,.967383322,.970910113,.976004045,.980637421
     &,.987285705,.969551226,.952976722,.957159093,.962948226,.964961095
     &,.966472141,.965946647,.967657333,.970635287,.974838409,.979784490
     &,.981991304,.982753132,.984046906,.986090689,.988853482,.991210617
     &,.993642006/
      IF(IOPT(11).EQ.0.OR.IYEAR.GT.20) RETURN
      MAX = 5
      IF(IOPT(11).EQ.2) MAX = 81
      K = 1
      IF(IYEAR.GT.10) K = 2
      L = 1
      DO 1 I = 1,MAX
      IF(I.EQ.2) L = 2
      IF(I.GT.5.AND.I-5*(I/5).EQ.1) L = L + 1
    1 ASMR(1,I) = ASMR(1,I) * RATE(L,K)
      RETURN
      END
```

```
      SUBROUTINE ENFERT
      IMPLICIT REAL*8 ($)
      REAL*8 QTRUNC
      DIMENSION BR(35)
      COMMON POP(2,81),BIRTH(2,81),DEATH(2,81),ASFR(2,81),ASMR(2,81),
     & FAMILY(35,36),IYEAR,AZERO,ACNE,R,IOPT(20)
      IF(IYEAR.GT.1) GO TO 3
      $DFS = IOPT(13)
      $DFS = $DFS * 0.5D0
      DO 4 I = 1,35
    4 BR(I) = ASFR(1,I+15)
    3 EPS = 0.0
      IF(IOPT(12).EQ.1) EPS = 0.50
      IF(IOPT(12).EQ.2) EPS = 0.90
      IF(IOPT(12).EQ.3) EPS = 0.99
      IF(EPS.EQ.0.0) RETURN
      DO 1 I = 16,50
      I1 = I - 15
      FS = 0.0
      DO 2 J = 1,35
    2 FS = FS + FAMILY(I1,J)
      $A = I - 13
      $A = 0.2D0 * $A
      $F = FS
      $M = .7317D-1 - .1433D0 * $A + 1.276D0 * $F
      $S = .5490D0 + .1282D0 * $A + .2589D0 * $F
      $Q = QTRUNC($DFS,$M,$S)
      Q = $Q
    1 ASFR(1,I) = BR(I1) * ( 1.0 - EPS * Q )
      RETURN
      END
```

```
      FUNCTION QTRUNC(X,M,S)
      IMPLICIT REAL*8 (A-H,M-Z)
      Q = 1.0D0
      IF(X.LE.0.0D0) GO TO 1
      Q = 0.0D0
      IF(S.LE.0.0D0) GO TO 1
      Q = QNORM( - M / S )
      IF(Q.LE.0.0D0) GO TO 2
      Q = QNORM( ( X - M ) / S )  /  Q
    1 QTRUNC = Q
      RETURN
    2 Q = 0.0D0
      GO TO 1
      END
```

```
      FUNCTION QNORM(X)
      IMPLICIT REAL*8 (A-H,O-Z)
      IS = 1
      F = 0.5D0
      IF(X.EQ.0.0D0) GO TO 4
      F = 0.0D0
      IF(X.GT.100.0D0) GO TO 4
      F = 1.0D0
      IF(X.LT.-100.0D0) GO TO 4
      Z = X * 0.707106781186547
      IF(X.GE.0.0D0) GO TO 1
      Z = - Z
      IS = -1
    1 IF(Z.LT.8.0D0) GO TO 2
      P = 1.63271618512628 + Z * ( 2.35360143283568 + Z * (
     $    3.03185804944393 + Z * ( .895157182255506 + Z *
     $    .564189583547937 ) ) )
      Q = 1.29314873038423 + Z * ( 5.08080210486990 + Z * (
     $    4.96496300826808 + Z * ( 5.87382846427043 + Z * (
     $    1.58662479494698 + Z ) ) ) )
      GO TO 3
    2 P = 883.478942608496 + Z * ( 1549.67931240372 + Z * (
     $    1347.19413409759 + Z * ( 723.040002777530 + Z * (
     $    255.500494694958 + Z * ( 59.2400101129141 + Z * (
     $    8.37653108141970 + Z * .564189559442610 ) ) ) ) ) )
      Q = 883.478942608500 + Z * ( 2546.57854580975 + Z * (
     $    3337.22136998927 + Z * ( 2606.71201526511 + Z * (
     $    1333.56997567996 + Z * ( 460.285123691601 + Z * (
     $    105.500254397689 + Z * ( 14.8470122375235 + Z ) ) ) ) ) ) )
    3 F = 0.5D0 * DEXP( - Z * Z )  * P / Q
      IF(IS.LT.0) F = 1.0D0 - F
    4 QNORM = F
      RETURN
      END
```

****

POPULATICN PROJECTION UTILITY MODEL, VERSION OF 1 OCTOBER 1971. PROGRAMMING BY K. M. MAURER PAGE 1

MORTALITY CECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

PROJECTICN FROM YEAR C TO YEAR 100

WITH OUTPUT FOR THE FOLLOWING YEARS 5 10 15 20 30 40 50 75 100

CCNTROL OPTION VALUES USED -

1 : 0	6 : 1	11 : 2	16 : 0
2 : 0	7 : 0	12 : 2	17 : 0
3 : 0	8 : 1	13 : 4	18 : 0
4 : 0	9 : 0	14 : 0	19 : 0
5 : 1	10 : 1	15 : 0	20 : 0

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

SUMMARY OF INPUT DATA FOR POPULATION, FERTILITY, AND MORTALITY YEAR 0

COHORT	POPULATION (1)	% OF TOTAL (2)	BIRTH RATE* (3)	DEATH RATE* (4)
TOTAL	10000047.	100.000	51.474	20.179
LESS THAN 1	447200.	4.472	0.0	205.000
1 - 4	1448796.	14.488	0.0	23.656
5 - 9	1503296.	15.033	0.0	5.450
10 - 14	1258098.	12.581	0.0	3.200
15 - 19	1055794.	10.558	120.000	4.450
20 - 24	880201.	8.802	135.000	5.870
25 - 29	729301.	7.293	140.000	6.880
30 - 34	601600.	6.016	135.000	7.660
35 - 39	494200.	4.942	104.000	8.530
40 - 44	404299.	4.043	65.000	9.300
45 - 49	328899.	3.289	25.000	10.810
50 - 54	264100.	2.641	0.0	14.570
55 - 59	206600.	2.066	0.0	21.400
60 - 64	154400.	1.544	0.0	33.200
65 - 69	107000.	1.070	0.0	52.420
70 - 74	65700.	0.657	0.0	82.360
75 - 79	33700.	0.337	0.0	127.930
80 AND OVER	16900.	0.169	0.0	229.540

* PER THOUSAND PER YEAR

AVERAGE AGE	21.132	PERCENT AGED 15-44	41.6536	DEPENDENCY RATIO	1.0141
EXP (BIRTH)	44.922	BIRTH - WOMAN RATIO	0.1236	CHILD - WOMAN RATIO	0.4552

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.10589	0.18057	0.0	0.0	0.0	0.0	0.0	0.0	0.35154	0.28646	81.48770
20 - 24	0.11913	0.39291	0.25992	0.0	0.0	0.0	0.0	0.0	0.99510	0.77196	77.57610
25 - 29	0.12354	0.42418	0.47485	0.25378	0.0	0.0	0.0	0.0	1.68445	1.27634	75.77232
30 - 34	0.11913	0.42460	0.50849	0.46461	0.24906	0.0	0.0	0.0	2.36961	1.76590	74.52280
35 - 39	0.09177	0.37022	0.50530	0.49758	0.45583	0.24282	0.0	0.0	2.95381	2.16351	73.24493
40 - 44	0.05736	0.26259	0.43069	0.49450	0.48817	0.44424	0.23525	0.0	3.35977	2.41279	71.81407
45 - 49	0.02206	0.14082	0.29920	0.42163	0.48514	0.47574	0.43030	0.22674	3.56790	2.50164	70.11504

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 5

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	11276716.	100.000	279833.	2.5447	461060.	40.886	153406.	13.604
0	412339.	3.657	17347.	4.3918	0.	0.0	62133.	150.684
1	360771.	3.199	16177.	4.6944	0.	0.0	10137.	28.099
2	335785.	2.978	15965.	4.9918	0.	0.0	5156.	15.355
3	314488.	2.789	-36911.	-10.5039	0.	0.0	3139.	9.981
4	347591.	3.082	-35718.	-9.3184	0.	0.0	2242.	6.449
5	380711.	3.376	13333.	3.6293	0.	0.0	1408.	3.699
6	365700.	3.243	11866.	3.3536	0.	0.0	1353.	3.699
7	352219.	3.123	11190.	3.2814	0.	0.0	1303.	3.699
8	339471.	3.010	10654.	3.2402	0.	0.0	1256.	3.699
9	327316.	2.903	11831.	3.4222	0.	0.0	1211.	3.699
1 - 4	1358635.	12.048	-40487.	-2.8937	0.	0.0	20674.	15.217
5 - 9	1765416.	15.655	57875.	3.3894	0.	0.0	6530.	3.699
10 - 14	1473630.	13.068	46236.	3.2392	0.	0.0	3324.	2.256
15 - 19	1236622.	10.966	38682.	3.2290	145367.	117.552	3956.	3.199
20 - 24	1031444.	9.147	32421.	3.2452	122249.	118.522	4386.	4.252
25 - 29	855064.	7.583	27014.	3.2624	87558.	102.399	4267.	4.991
30 - 34	705491.	6.256	22348.	3.2714	57161.	81.023	3973.	5.631
35 - 39	579644.	5.140	18394.	3.2773	30536.	52.680	3728.	6.432
40 - 44	474183.	4.205	15047.	3.2773	13990.	29.503	3449.	7.274
45 - 49	385707.	3.420	12228.	3.2741	4200.	10.890	3403.	8.822
50 - 54	309873.	2.748	9860.	3.2865	0.	0.0	3761.	12.137
55 - 59	242722.	2.152	7812.	3.3254	0.	0.0	4353.	17.934
60 - 64	181760.	1.612	5960.	3.3902	0.	0.0	5112.	28.127
65 - 69	126256.	1.120	4238.	3.4736	0.	0.0	5710.	45.226
70 - 74	77865.	0.690	2705.	3.5984	0.	0.0	5680.	72.941
75 - 79	39980.	0.355	1415.	3.6699	0.	0.0	4638.	116.015
80 +	20116.	0.178	732.	3.7786	0.	0.0	4331.	215.314

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 21.907 PERCENT AGED 15-44 43.2966 DEPENDENCY RATIO 0.9373
EXP (BIRTH) 48.506 BIRTH - WOMAN RATIO 0.0944 CHILD - WOMAN RATIO 0.3627

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.10707	0.18424	0.0	0.0	0.0	0.0	0.0	0.0	0.34680	0.29131	83.99738
20 - 24	0.10795	0.37757	0.26131	0.0	0.0	0.0	0.0	0.0	0.94591	0.74683	78.95366
25 - 29	0.09326	0.34681	0.47697	0.25430	0.0	0.0	0.0	0.0	1.53501	1.17135	76.30919
30 - 34	0.07380	0.27976	0.51075	0.46547	0.24951	0.0	0.0	0.0	2.11848	1.57929	74.54829
35 - 39	0.04798	0.19932	0.50752	0.49850	0.45666	0.24338	0.0	0.0	2.67396	1.95336	73.05110
40 - 44	0.02687	0.12308	0.43253	0.49541	0.48906	0.44529	0.23590	0.0	3.13884	2.24813	71.62306
45 - 49	0.00992	0.06202	0.30045	0.42240	0.48603	0.47687	0.43151	0.22744	3.44911	2.41664	70.06543

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 10

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	12913780.	100.000	359252.	2.8615	548483.	42.473	138213.	10.703
0	503206.	3.897	18779.	3.8764	0.	0.0	55735.	110.760
1	444938.	3.445	17350.	4.0576	0.	0.0	8159.	18.337
2	417763.	3.235	16867.	4.2073	0.	0.0	4186.	10.020
3	394893.	3.058	16481.	4.3554	0.	0.0	2572.	6.513
4	374730.	2.902	16139.	4.5007	0.	0.0	1577.	4.208
5	356372.	2.760	15857.	4.6567	0.	0.0	894.	2.510
6	339162.	2.626	15670.	4.8442	0.	0.0	851.	2.510
7	322206.	2.495	15619.	5.0943	0.	0.0	809.	2.510
8	305369.	2.365	-35595.	-10.4395	0.	0.0	766.	2.510
9	339608.	2.630	-34676.	-9.2646	0.	0.0	852.	2.510
1 - 4	1632323.	12.640	66837.	4.2694	0.	0.0	16494.	10.104
5 - 9	1662715.	12.876	-23125.	-1.3717	0.	0.0	4173.	2.510
10 - 14	1735113.	13.436	57633.	3.4357	0.	0.0	2759.	1.590
15 - 19	1451794.	11.242	46109.	3.2802	170453.	117.408	3339.	2.300
20 - 24	1211734.	9.383	38520.	3.2833	142738.	117.796	3732.	3.080
25 - 29	1005657.	7.787	32233.	3.3113	103560.	102.977	3640.	3.620
30 - 34	830535.	6.431	26816.	3.3365	70023.	84.311	3438.	4.140
35 - 39	682705.	5.287	22136.	3.3510	38688.	56.668	3311.	4.850
40 - 44	558622.	4.326	18155.	3.3590	17799.	31.863	3179.	5.690
45 - 49	454370.	3.518	14771.	3.3602	5222.	11.494	3271.	7.200
50 - 54	365211.	2.828	11903.	3.3691	0.	0.0	3692.	10.110
55 - 59	286713.	2.220	9469.	3.4154	0.	0.0	4309.	15.030
60 - 64	215633.	1.670	7318.	3.5128	0.	0.0	5139.	23.830
65 - 69	150723.	1.167	5323.	3.6609	0.	0.0	5881.	39.020
70 - 74	93627.	0.725	3465.	3.8427	0.	0.0	6048.	64.600
75 - 79	48534.	0.376	1899.	4.0714	0.	0.0	5106.	105.209
80 +	24596.	0.190	1011.	4.2885	0.	0.0	4968.	201.970

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 22.329 PERCENT AGED 15-44 44.4566 DEPENDENCY RATIO 0.8860
EXP (BIRTH) 51.368 BIRTH - WOMAN RATIO 0.0955 CHILD - WOMAN RATIO 0.3720

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.10952	0.19070	0.0	0.0	0.0	0.0	0.0	0.0	0.34652	0.30022	96.63995
20 - 24	0.10989	0.39099	0.26832	0.0	0.0	0.0	0.0	0.0	0.93943	0.76919	81.87860
25 - 29	0.09606	0.36399	0.45633	0.25640	0.0	0.0	0.0	0.0	1.49133	1.17278	78.64017
30 - 34	0.07865	0.30534	0.41423	0.46878	0.25058	0.0	0.0	0.0	1.99058	1.51758	76.23785
35 - 39	0.05286	0.22567	0.33295	0.50202	0.45855	0.24453	0.0	0.0	2.44971	1.81658	74.15480
40 - 44	0.02972	0.13980	0.23335	0.49888	0.49108	0.44744	0.23729	0.0	2.87598	2.07756	72.23831
45 - 49	0.01072	0.06871	0.14179	0.42530	0.48804	0.47918	0.43412	0.22899	3.23397	2.27685	70.40416

POPULATION PROJECTION UTILITY MODEL, VERSION OF 1 OCTOBER 1971. PROGRAMMING BY K. M. MAURER PAGE

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 15

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	14930843.	100.000	434186.	2.9951	646228.	43.281	142520.	9.545
0	598895.	4.011	20271.	3.5032	0.	0.0	56831.	94.893
1	532656.	3.567	18270.	3.5517	0.	0.0	7677.	14.412
2	503102.	3.370	17471.	3.5976	0.	0.0	3962.	7.875
3	478693.	3.206	16839.	3.6460	0.	0.0	2450.	5.119
4	457565.	3.065	16314.	3.6972	0.	0.0	1513.	3.308
5	438628.	2.938	16786.	3.9791	0.	0.0	884.	2.016
6	420270.	2.815	16615.	4.1160	0.	0.0	847.	2.016
7	402150.	2.693	16386.	4.2477	0.	0.0	811.	2.016
8	384325.	2.574	16148.	4.3858	0.	0.0	775.	2.016
9	366805.	2.457	15884.	4.5264	0.	0.0	740.	2.016
1 - 4	1972014.	13.208	68893.	3.6200	0.	0.0	15603.	7.912
5 - 9	2012177.	13.477	81818.	4.2385	0.	0.0	4058.	2.016
10 - 14	1636611.	10.961	-22694.	-1.3677	0.	0.0	2155.	1.316
15 - 19	1711523.	11.463	57127.	3.4530	200805.	117.325	3294.	1.924
20 - 24	1424897.	9.543	45555.	3.3027	166765.	117.037	3701.	2.597
25 - 29	1183812.	7.929	37937.	3.3107	120406.	101.710	3604.	3.044
30 - 34	979021.	6.557	31665.	3.3425	82457.	84.224	3439.	3.512
35 - 39	805672.	5.396	26267.	3.3701	46949.	58.272	3366.	4.178
40 - 44	659611.	4.418	21602.	3.3858	22269.	33.760	3304.	5.009
45 - 49	536661.	3.594	17619.	3.3946	6579.	12.259	3489.	6.501
50 - 54	431495.	2.890	14193.	3.4012	0.	0.0	3983.	9.232
55 - 59	339261.	2.272	11234.	3.4247	0.	0.0	4674.	13.778
60 - 64	256185.	1.716	8653.	3.4956	0.	0.0	5633.	21.989
65 - 69	180300.	1.208	6311.	3.6273	0.	0.0	6559.	36.381
70 - 74	113040.	0.757	4154.	3.8149	0.	0.0	6904.	61.079
75 - 79	59213.	0.397	2299.	4.0395	0.	0.0	5961.	100.666
80 +	30487.	0.204	1282.	4.3911	0.	0.0	5964.	195.630

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 22.520 PERCENT AGED 15-44 45.3057 DEPENDENCY RATIO 0.8497
EXP (BIRTH) 52.604 BIRTH - WOMAN RATIO 0.0955 CHILD - WOMAN RATIO 0.3801

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.11051	0.19336	0.0	0.0	0.0	0.0	0.0	0.0	0.34596	0.30437	87.97950
20 - 24	0.11024	0.39766	0.27809	0.0	0.0	0.0	0.0	0.0	0.93660	0.78598	83.91867
25 - 29	0.09580	0.36802	0.47289	0.26371	0.0	0.0	0.0	0.0	1.47926	1.20042	81.14975
30 - 34	0.07933	0.31231	0.43496	0.44918	0.25296	0.0	0.0	0.0	1.94745	1.52874	78.49933
35 - 39	0.05489	0.23804	0.36324	0.40778	0.46239	0.24598	0.0	0.0	2.33247	1.77232	75.98456
40 - 44	0.03180	0.15218	0.26400	0.32779	0.49516	0.45003	0.23888	0.0	2.66486	1.95984	73.54399
45 - 49	0.01155	0.07540	0.16100	0.22980	0.49207	0.48195	0.43709	0.23086	2.97807	2.11972	71.17746

POPULATION PROJECTION UTILITY MODEL, VERSION OF 1 OCTOBER 1971. PROGRAMMING BY K. M. MAURER PAGE

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 20

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	17258272.	100.000	483591.	2.8829	712071.	41.260	143361.	8.307
0	669576.	3.880	13971.	2.1309	0.	0.0	54436.	81.300
1	604593.	3.503	12066.	2.0363	0.	0.0	6849.	11.328
2	580017.	3.361	10959.	1.9258	0.	0.0	3590.	6.190
3	561234.	3.252	12498.	2.2777	0.	0.0	2258.	4.023
4	543832.	3.151	16520.	3.1329	0.	0.0	1414.	2.600
5	524282.	3.038	18140.	3.5839	0.	0.0	849.	1.620
6	504355.	2.922	17515.	3.5977	0.	0.0	817.	1.620
7	485120.	2.811	16990.	3.6294	0.	0.0	786.	1.620
8	466476.	2.703	16514.	3.6702	0.	0.0	756.	1.620
9	448373.	2.598	16072.	3.7177	0.	0.0	726.	1.620
1 - 4	2289674.	13.267	52044.	2.3259	0.	0.0	14111.	6.163
5 - 9	2428604.	14.072	85230.	3.6371	0.	0.0	3934.	1.620
10 - 14	1982057.	11.485	80887.	4.2546	0.	0.0	2160.	1.090
15 - 19	1615508.	9.361	-22037.	-1.3457	189350.	117.207	2601.	1.610
20 - 24	1681471.	9.743	56423.	3.4721	196111.	116.631	3682.	2.190
25 - 29	1393746.	8.076	44862.	3.3259	140405.	100.739	3568.	2.560
30 - 34	1154035.	6.687	37268.	3.3371	95895.	83.095	3439.	2.980
35 - 39	951126.	5.511	31019.	3.3713	55236.	58.074	3424.	3.600
40 - 44	779633.	4.517	25640.	3.4005	26900.	34.504	3438.	4.410
45 - 49	634703.	3.678	20975.	3.4176	8175.	12.880	3726.	5.870
50 - 54	510607.	2.959	16942.	3.4319	0.	0.0	4304.	8.430
55 - 59	401870.	2.329	13410.	3.4521	0.	0.0	5076.	12.630
60 - 64	304272.	1.763	10286.	3.4986	0.	0.0	6174.	20.290
65 - 69	215360.	1.248	7488.	3.6021	0.	0.0	7305.	33.920
70 - 74	136208.	0.789	4951.	3.7720	0.	0.0	7866.	57.750
75 - 79	72160.	0.418	2776.	4.0014	0.	0.0	6950.	96.319
80 +	37829.	0.219	1591.	4.3906	0.	0.0	7168.	189.489

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 22.695 PERCENT AGED 15-44 43.8949 DEPENDENCY RATIO 0.8918
EXP (BIRTH) 53.688 BIRTH - WOMAN RATIO 0.0940 CHILD - WOMAN RATIO 0.3906

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.11132	0.19945	0.0	0.0	0.0	0.0	0.0	0.0	0.34985	0.31077	88.82947
20 - 24	0.11078	0.40158	0.28275	0.0	0.0	0.0	0.0	0.0	0.93465	0.79510	85.06932
25 - 29	0.09568	0.36935	0.48070	0.27356	0.0	0.0	0.0	0.0	1.47183	1.21929	82.84134
30 - 34	0.07892	0.31193	0.43958	0.46586	0.26036	0.0	0.0	0.0	1.92940	1.55666	80.69102
35 - 39	0.05516	0.24022	0.37137	0.42854	0.44338	0.24855	0.0	0.0	2.28835	1.78722	78.10101
40 - 44	0.03277	0.15764	0.27830	0.35790	0.40252	0.45424	0.24058	0.0	2.55266	1.92396	75.37082
45 - 49	0.01223	0.08035	0.17513	0.26019	0.32358	0.48643	0.44016	0.23273	2.77239	2.01082	72.53003

POPULATION PROJECTION UTILITY MODEL, VERSION OF 1 OCTOBER 1971. PROGRAMMING BY K. M. MAURER PAGE

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 30

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	22698672.	100.000	607152.	2.7483	911507.	40.157	190842.	8.408
0	853686.	3.761	23538.	2.8354	0.	0.0	69404.	81.300
1	765556.	3.373	20624.	2.7685	0.	0.0	8672.	11.328
2	729203.	3.213	19118.	2.6924	0.	0.0	4514.	6.190
3	700322.	3.085	17869.	2.6184	0.	0.0	2818.	4.023
4	676355.	2.980	16820.	2.5503	0.	0.0	1758.	2.600
5	655745.	2.889	15720.	2.4562	0.	0.0	1062.	1.620
6	637764.	2.810	14527.	2.3308	0.	0.0	1033.	1.620
7	621036.	2.736	13327.	2.1929	0.	0.0	1006.	1.620
8	605563.	2.668	12426.	2.0950	0.	0.0	981.	1.620
9	591042.	2.604	11889.	2.0529	0.	0.0	957.	1.620
1 - 4	2871435.	12.650	74431.	2.6611	0.	0.0	17762.	6.196
5 - 9	3111147.	13.706	67889.	2.2308	0.	0.0	5040.	1.620
10 - 14	2759634.	12.158	61753.	2.2889	0.	0.0	3008.	1.090
15 - 19	2362864.	10.410	82924.	3.6371	277065.	117.258	3804.	1.610
20 - 24	1923385.	8.474	78484.	4.2541	223926.	116.423	4212.	2.190
25 - 29	1553530.	6.844	-20950.	-1.3306	155225.	99.917	3977.	2.560
30 - 34	1604796.	7.070	53852.	3.4722	131549.	81.972	4782.	2.980
35 - 39	1321316.	5.821	42529.	3.3257	75212.	56.922	4757.	3.600
40 - 44	1086110.	4.785	35073.	3.3370	36940.	34.011	4790.	4.410
45 - 49	886661.	3.906	28915.	3.3711	11590.	13.071	5205.	5.870
50 - 54	714682.	3.149	23501.	3.4001	0.	0.0	6025.	8.430
55 - 59	563321.	2.482	18612.	3.4168	0.	0.0	7115.	12.630
60 - 64	427450.	1.883	14178.	3.4305	0.	0.0	8673.	20.290
65 - 69	303955.	1.339	10136.	3.4496	0.	0.0	10310.	33.920
70 - 74	194091.	0.855	6552.	3.4939	0.	0.0	11209.	57.750
75 - 79	104459.	0.460	3623.	3.5927	0.	0.0	10061.	96.319
80 +	56527.	0.249	2095.	3.8491	0.	0.0	10711.	189.489

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 23.188 PERCENT AGED 15-44 43.4034 DEPENDENCY RATIO 0.8889
EXP (BIRTH) 53.688 BIRTH - WOMAN RATIO 0.0925 CHILD - WOMAN RATIO 0.3781

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.11137	0.19640	0.0	0.0	0.0	0.0	0.0	0.0	0.34544	0.30777	89.09607
20 - 24	0.11058	0.40280	0.28525	0.0	0.0	0.0	0.0	0.0	0.93188	0.79863	85.70064
25 - 29	0.09490	0.36826	0.48645	0.28464	0.0	0.0	0.0	0.0	1.46864	1.23425	84.03976
30 - 34	0.07786	0.30893	0.44081	0.47839	0.27477	0.0	0.0	0.0	1.91112	1.58076	82.71355
35 - 39	0.05407	0.23621	0.36883	0.43483	0.46771	0.26553	0.0	0.0	2.25126	1.82716	81.16168
40 - 44	0.03230	0.15588	0.27837	0.36567	0.42774	0.45201	0.25040	0.0	2.48258	1.96237	79.04561
45 - 49	0.01242	0.08175	0.18135	0.27693	0.36141	0.41581	0.42636	0.23704	2.61650	1.99308	76.17342

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 40

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	29755056.	100.000	789168.	2.7245	1193792.	40.121	256181.	8.610
0	1119648.	3.763	28280.	2.5912	0.	0.0	91027.	81.300
1	1006450.	3.382	25836.	2.6347	0.	0.0	11401.	11.328
2	959909.	3.226	25159.	2.6915	0.	0.0	5942.	6.190
3	921898.	3.098	24421.	2.7210	0.	0.0	3709.	4.023
4	889458.	2.989	23491.	2.7127	0.	0.0	2312.	2.600
5	860991.	2.894	22900.	2.7323	0.	0.0	1395.	1.620
6	835131.	2.807	22637.	2.7861	0.	0.0	1353.	1.620
7	809625.	2.721	22411.	2.8468	0.	0.0	1312.	1.620
8	784434.	2.636	22041.	2.8910	0.	0.0	1271.	1.620
9	759701.	2.553	21301.	2.8847	0.	0.0	1231.	1.620
1 - 4	3777714.	12.696	98908.	2.6886	0.	0.0	23364.	6.185
5 - 9	4049878.	13.611	111287.	2.8256	0.	0.0	6561.	1.620
10 - 14	3472586.	11.671	91200.	2.6971	0.	0.0	3785.	1.090
15 - 19	3026971.	10.173	66047.	2.2306	354818.	117.219	4873.	1.610
20 - 24	2677736.	8.999	59897.	2.2880	311294.	116.252	5864.	2.190
25 - 29	2272360.	7.637	79747.	3.6371	227278.	100.018	5817.	2.560
30 - 34	1835712.	6.169	74901.	4.2538	150255.	81.851	5470.	2.980
35 - 39	1472734.	4.950	-19678.	-1.3185	83392.	56.624	5302.	3.600
40 - 44	1510348.	5.076	50686.	3.4724	50895.	33.698	6661.	4.410
45 - 49	1231758.	4.140	39644.	3.3255	15868.	12.882	7230.	5.870
50 - 54	995622.	3.346	32146.	3.3365	0.	0.0	8393.	8.430
55 - 59	786936.	2.645	25657.	3.3702	0.	0.0	9939.	12.630
60 - 64	598286.	2.011	19665.	3.3987	0.	0.0	12139.	20.290
65 - 69	426062.	1.432	14068.	3.4145	0.	0.0	14452.	33.920
70 - 74	272650.	0.916	9033.	3.4267	0.	0.0	15745.	57.750
75 - 79	147403.	0.495	4907.	3.4435	0.	0.0	14198.	96.319
80 +	81072.	0.272	2765.	3.5314	0.	0.0	15362.	189.489

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 23.462 PERCENT AGED 15-44 43.0040 DEPENDENCY RATIO 0.8820
EXP (BIRTH) 53.688 BIRTH - WOMAN RATIO 0.0933 CHILD - WOMAN RATIO 0.3827

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.11134	0.19904	0.0	0.0	0.0	0.0	0.0	0.0	0.34849	0.31037	89.06201
20 - 24	0.11042	0.40285	0.28921	0.0	0.0	0.0	0.0	0.0	0.93657	0.80248	85.69271
25 - 29	0.09500	0.36853	0.48644	0.28175	0.0	0.0	0.0	0.0	1.46464	1.23171	84.09630
30 - 34	0.07774	0.30846	0.43996	0.47939	0.27720	0.0	0.0	0.0	1.90665	1.58276	83.01247
35 - 39	0.05378	0.23439	0.36605	0.43313	0.47331	0.27626	0.0	0.0	2.24227	1.83692	81.92227
40 - 44	0.03201	0.15440	0.27510	0.36190	0.42895	0.46416	0.26425	0.0	2.45832	1.98076	80.57365
45 - 49	0.01224	0.08049	0.17810	0.27217	0.35893	0.42190	0.44974	0.25324	2.57382	2.02681	78.74702

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 50

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	38834224.	100.000	1015792.	2.6860	1551440.	39.950	340443.	8.767
0	1454363.	3.745	38009.	2.6836	0.	0.0	118239.	81.300
1	1306149.	3.363	34301.	2.6969	0.	0.0	14796.	11.328
2	1244993.	3.206	32829.	2.7083	0.	0.0	7706.	6.190
3	1195498.	3.078	31273.	2.6862	0.	0.0	4810.	4.023
4	1153822.	2.971	29561.	2.6294	0.	0.0	3000.	2.600
5	1117800.	2.878	28389.	2.6059	0.	0.0	1811.	1.620
6	1085563.	2.795	27715.	2.6199	0.	0.0	1759.	1.620
7	1054111.	2.714	27134.	2.6421	0.	0.0	1708.	1.620
8	1023350.	2.635	26272.	2.6349	0.	0.0	1658.	1.620
9	993557.	2.558	25113.	2.5931	0.	0.0	1610.	1.620
1 - 4	4900462.	12.619	127964.	2.6813	0.	0.0	30312.	6.185
5 - 9	5274381.	13.582	134623.	2.6192	0.	0.0	8544.	1.620
10 - 14	4565613.	11.757	118660.	2.6683	0.	0.0	4977.	1.090
15 - 19	3940286.	10.146	108280.	2.8257	461940.	117.235	6344.	1.610
20 - 24	3369612.	8.677	88503.	2.6973	391906.	116.306	7379.	2.190
25 - 29	2910902.	7.496	63526.	2.2310	290862.	99.922	7452.	2.560
30 - 34	2555542.	6.581	57150.	2.2875	208930.	81.756	7615.	2.980
35 - 39	2154290.	5.547	75601.	3.6370	122033.	56.646	7755.	3.600
40 - 44	1727712.	4.449	70467.	4.2533	58153.	33.659	7619.	4.410
45 - 49	1372816.	3.535	-18054.	-1.2980	17625.	12.839	8058.	5.870
50 - 54	1384534.	3.565	46471.	3.4730	0.	0.0	11672.	8.430
55 - 59	1093212.	2.815	35178.	3.3248	0.	0.0	13807.	12.630
60 - 64	833454.	2.146	26898.	3.3350	0.	0.0	16911.	20.290
65 - 69	595177.	1.533	19391.	3.3677	0.	0.0	20188.	33.920
70 - 74	381607.	0.983	12529.	3.3946	0.	0.0	22038.	57.750
75 - 79	206609.	0.532	6811.	3.4087	0.	0.0	19900.	96.319
80 +	114172.	0.294	3786.	3.4297	0.	0.0	21634.	189.489

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 23.618 PERCENT AGED 15-44 42.8960 DEPENDENCY RATIO 0.8935
EXP (BIRTH) 53.688 BIRTH - WOMAN RATIO 0.0931 CHILD - WOMAN RATIO 0.3815

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.11135	0.19797	0.0	0.0	0.0	0.0	0.0	0.0	0.34726	0.30932	89.07607
20 - 24	0.11047	0.40283	0.28797	0.0	0.0	0.0	0.0	0.0	0.93510	0.80127	85.68920
25 - 29	0.09491	0.36825	0.48638	0.28423	0.0	0.0	0.0	0.0	1.46720	1.23376	84.08969
30 - 34	0.07765	0.30794	0.43941	0.47931	0.28105	0.0	0.0	0.0	1.90996	1.58535	83.00456
35 - 39	0.05380	0.23506	0.36646	0.43347	0.47329	0.27347	0.0	0.0	2.23959	1.83556	81.95959
40 - 44	0.03197	0.15447	0.27494	0.36135	0.42812	0.46513	0.26659	0.0	2.45370	1.98257	80.79901
45 - 49	0.01219	0.07968	0.17680	0.27018	0.35623	0.42026	0.45512	0.26346	2.56225	2.03392	79.38020

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 75

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	75082432.	100.000	1953232.	2.6709	3001026.	39.970	668705.	8.906
0	2813221.	3.747	73401.	2.6790	0.	0.0	228714.	81.300
1	2526638.	3.365	65952.	2.6802	0.	0.0	28621.	11.328
2	2408729.	3.208	62889.	2.6809	0.	0.0	14910.	6.190
3	2313587.	3.081	60409.	2.6811	0.	0.0	9309.	4.023
4	2233044.	2.974	58298.	2.6807	0.	0.0	5805.	2.600
5	2162249.	2.880	56422.	2.6793	0.	0.0	3503.	1.620
6	2098389.	2.795	54692.	2.6761	0.	0.0	3399.	1.620
7	2036479.	2.712	52977.	2.6709	0.	0.0	3299.	1.620
8	1976496.	2.632	51303.	2.6648	0.	0.0	3202.	1.620
9	1918393.	2.555	49727.	2.6611	0.	0.0	3108.	1.620
1 - 4	9481998.	12.629	247548.	2.6807	0.	0.0	58645.	6.185
5 - 9	10192006.	13.574	265121.	2.6707	0.	0.0	16511.	1.620
10 - 14	8806293.	11.729	227614.	2.6533	0.	0.0	9599.	1.090
15 - 19	7627684.	10.159	197948.	2.6643	894198.	117.231	12281.	1.610
20 - 24	6567145.	8.747	172808.	2.7025	763786.	116.304	14382.	2.190
25 - 29	5625103.	7.492	146904.	2.6816	562241.	99.952	14400.	2.560
30 - 34	4813091.	6.410	122840.	2.6190	393620.	81.781	14343.	2.980
35 - 39	4109794.	5.474	106802.	2.6681	232736.	56.630	14795.	3.600
40 - 44	3478147.	4.632	95570.	2.8254	117052.	33.653	15339.	4.410
45 - 49	2913966.	3.881	76554.	2.6980	37407.	12.837	17105.	5.870
50 - 54	2449015.	3.262	53480.	2.2325	0.	0.0	20645.	8.430
55 - 59	2054650.	2.737	45848.	2.2824	0.	0.0	25950.	12.630
60 - 64	1600838.	2.132	56158.	3.6356	0.	0.0	32481.	20.290
65 - 69	1117288.	1.488	45483.	4.2436	0.	0.0	37898.	33.920
70 - 74	696587.	0.928	-5235.	-0.7459	0.	0.0	40228.	57.750
75 - 79	473448.	0.631	15949.	3.4862	0.	0.0	45602.	96.319
80 +	262762.	0.350	8437.	3.3175	0.	0.0	49790.	189.489

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 23.630 PERCENT AGED 15-44 42.9139 DEPENDENCY RATIO 0.8942
EXP (BIRTH) 53.688 BIRTH - WOMAN RATIO 0.0931 CHILD - WOMAN RATIO 0.3816

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.11135	0.19827	0.0	0.0	0.0	0.0	0.0	0.0	0.34760	0.30961	89.07219
20 - 24	0.11047	0.40283	0.28801	0.0	0.0	0.0	0.0	0.0	0.93515	0.80131	85.68803
25 - 29	0.09494	0.36833	0.48640	0.28345	0.0	0.0	0.0	0.0	1.46639	1.23311	84.09175
30 - 34	0.07768	0.30808	0.43955	0.47933	0.28004	0.0	0.0	0.0	1.90909	1.58467	83.00662
35 - 39	0.05379	0.23469	0.36620	0.43322	0.47325	0.27518	0.0	0.0	2.24064	1.83633	81.95576
40 - 44	0.03196	0.15396	0.27441	0.36098	0.42777	0.46507	0.26898	0.0	2.45460	1.98314	80.79271
45 - 49	0.01219	0.08002	0.17714	0.27050	0.35639	0.42036	0.45508	0.26232	2.56137	2.03400	79.41080

MORTALITY DECLINES FOR ALL AGES ; D.F.S. = 4.0 ; 90% EFFECTIVE CONTRACEPTION.

POPULATION, BIRTHS, AND DEATHS PROJECTED DATA YEAR 100

COHORT	POPULATION (1)	% OF TOTAL (2)	CHANGE (3)	RATE * (4)	BIRTHS (5)	RATE ** (6)	DEATHS (7)	RATE ** (8)
TOTAL	145159584.	100.000	3775808.	2.6706	5800125.	39.957	1291294.	8.896
0	5437349.	3.746	141454.	2.6710	0.	0.0	442054.	81.300
1	4883828.	3.364	127067.	2.6713	0.	0.0	55322.	11.328
2	4656324.	3.208	121169.	2.6718	0.	0.0	28822.	6.190
3	4472802.	3.081	116411.	2.6722	0.	0.0	17996.	4.023
4	4317464.	2.974	112377.	2.6724	0.	0.0	11224.	2.600
5	4180923.	2.880	108825.	2.6725	0.	0.0	6773.	1.620
6	4057716.	2.795	105621.	2.6725	0.	0.0	6573.	1.620
7	3938136.	2.713	102510.	2.6726	0.	0.0	6380.	1.620
8	3822079.	2.633	99491.	2.6726	0.	0.0	6192.	1.620
9	3709440.	2.555	96544.	2.6722	0.	0.0	6009.	1.620
1 - 4	18330416.	12.628	477024.	2.6719	0.	0.0	113365.	6.185
5 - 9	19708288.	13.577	512992.	2.6725	0.	0.0	31927.	1.620
10 - 14	17021600.	11.726	442649.	2.6699	0.	0.0	18553.	1.090
15 - 19	14733016.	10.150	382789.	2.6675	1727161.	117.231	23720.	1.610
20 - 24	12692363.	8.744	330400.	2.6727	1476137.	116.301	27796.	2.190
25 - 29	10883493.	7.498	284100.	2.6803	1087816.	99.951	27862.	2.560
30 - 34	9300616.	6.407	241939.	2.6708	760634.	81.783	27716.	2.980
35 - 39	7927040.	5.461	204888.	2.6532	448905.	56.630	28537.	3.600
40 - 44	6732938.	4.638	174725.	2.6642	226583.	33.653	29692.	4.410
45 - 49	5679060.	3.912	149441.	2.7026	72903.	12.837	33336.	5.870
50 - 54	4732786.	3.260	123599.	2.6816	0.	0.0	39897.	8.430
55 - 59	3870211.	2.666	98776.	2.6191	0.	0.0	48881.	12.630
60 - 64	3052788.	2.103	79295.	2.6667	0.	0.0	61941.	20.290
65 - 69	2247094.	1.548	61679.	2.8223	0.	0.0	76221.	33.920
70 - 74	1480291.	1.020	39014.	2.7069	0.	0.0	85486.	57.750
75 - 79	835521.	0.576	18416.	2.2538	0.	0.0	80477.	96.319
80 +	495217.	0.341	12572.	2.6049	0.	0.0	93838.	189.489

* PERCENT PER YEAR
** PER THOUSAND PER YEAR

AVERAGE AGE 23.639 PERCENT AGED 15-44 42.8971 DEPENDENCY RATIO 0.8962
EXP (BIRTH) 53.688 BIRTH - WOMAN RATIO 0.0931 CHILD - WOMAN RATIO 0.3817

STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES

PARENT'S COHORT	CHILDREN SURVIVING AT AGES : 0	1 - 4	5 - 9	10 -14	15 - 19	20 - 24	25 - 29	30 +	TOTAL LIVE BIRTHS	TOTAL SURVIVORS	SURVIVAL RATE (%)
15 - 19	0.11135	0.19826	0.0	0.0	0.0	0.0	0.0	0.0	0.34759	0.30960	89.07225
20 - 24	0.11046	0.40283	0.28808	0.0	0.0	0.0	0.0	0.0	0.93525	0.80138	85.68771
25 - 29	0.09493	0.36833	0.48640	0.28347	0.0	0.0	0.0	0.0	1.46642	1.23314	84.09169
30 - 34	0.07768	0.30809	0.43956	0.47933	0.27995	0.0	0.0	0.0	1.90901	1.58461	83.00681
35 - 39	0.05379	0.23468	0.36620	0.43322	0.47325	0.27519	0.0	0.0	2.24064	1.83633	81.95576
40 - 44	0.03196	0.15391	0.27435	0.36093	0.42773	0.46507	0.26925	0.0	2.45470	1.98320	80.79204
45 - 49	0.01219	0.08001	0.17713	0.27049	0.35639	0.42036	0.45508	0.26236	2.56137	2.03400	79.41064

THIS PROGRAM WAS DEVELOPED AND PROGRAMMED BY KENNETH M. MAURER AND T. PAUL SCHULTZ OF THE

POPULATION RESEARCH PROGRAM

OF THE

```
RRRRRRRR      AAAAAA    NN      NN  DDDDDDDD          CCCCCC      OOOOOO    RRRRRRRR    PPPPPPPP
RRRRRRRRR    AAAAAAAA   NNN     NN  DDDDDDDDD        CCCCCCCC    OOOOOOOO   RRRRRRRRR   PPPPPPPPP
RR      RR  AA      AA  NNNN    NN  DD      DD       CC      CC  OO      OO  RR      RR  PP      PP
RR      RR  AA      AA  NNNNN   NN  DD      DD       CC          OO      OO  RR      RR  PP      PP
RRRRRRRRR   AA      AA  NN NNN  NN  DD      DD       CC          OO      OO  RRRRRRRRR   PPPPPPPPP
RRRRRRRR    AAAAAAAAAA  NN  NNN NN  DD      DD       CC          OO      OO  RRRRRRRR    PPPPPPPP
RR   RR     AAAAAAAAAA  NN   NNNNN  DD      DD       CC          OO      OO  RR   RR     PP
RR    RR    AA      AA  NN    NNNN  DD      DD       CC      CC  OO      OO  RR    RR    PP
RR     RR   AA      AA  NN     NNN  DDDDDDDDD         CC    CC    OOOOOOOO   RR     RR   PP          ..
RR      RR  AA      AA  NN      NN  DDDDDDDD          CCCCCC      OOOOOO    RR      RR  PP          ..
```

TO FACILITATE RESEARCH UNDERTAKEN FOR

```
AAAAAA        IIII      DDDDDDDD
AAAAAAAA      IIII      DDDDDDDDD
AA      AA     II       DD      DD
AA      AA     II       DD      DD
AA      AA     II       DD      DD
AAAAAAAAAA     II       DD      DD
AAAAAAAAAA     II       DD      DD
AA      AA     II       DD      DD
AA      AA ..  IIII  ..  DDDDDDDDD  ..
AA      AA ..  IIII  ..  DDDDDDDD   ..
```

UNDER AGENCY FOR INTERNATIONAL DEVELOPMENT CONTRACT CSD-2533.

A DESCRIPTION OF THE PROGRAM IS AVAILABLE IN :

KENNETH M. MAURER AND T. PAUL SCHULTZ
A POPULATION PROJECTION MODEL
R-953-AID (JANUARY 1972)

THE RAND CORPORATION
1700 MAIN ST.
SANTA MONICA,
CALIF. 90406

Appendix C

ESTIMATION OF FAMILY SIZE DISTRIBUTIONS

The sample problem included in Appendix B assumes that fertility is related to surviving family size. The actual projection program, however, supplies only the mean family size and not the distribution of family sizes. In order to use such a fertility function it is necessary to estimate the distribution of family sizes, given the age of the mother and the mean number of children. Such a distribution was estimated using two sets of age-specific data for East Pakistan[1] and the United States.[2] A truncated normal distribution, defined by

$$\text{Prob}(x < 0 \mid \mu, \sigma^2) = 0$$

$$\text{Prob}(x < X \mid \mu, \sigma^2) = \int_0^X e^{(t-\mu)^2/2\sigma^2} dt \Big/ \int_0^\infty e^{-(t-\mu)^2} dt, \quad X > 0$$

approximates the distributions of family sizes. This distribution allows the complete distribution of family sizes to be summarized by two parameters, μ and σ^2. These parameters were estimated by maximum likelihood using a modification of the Davidon algorithm[3] to maximize the likelihood function with respect to μ and σ^2. Once these parameters were obtained for each of the 14 populations (seven age groups for each set of data), a simple linear model was used to relate μ and σ^2 to the mothers' age and the average family size. The ordinary least squares estimates of this relationship were as follows:

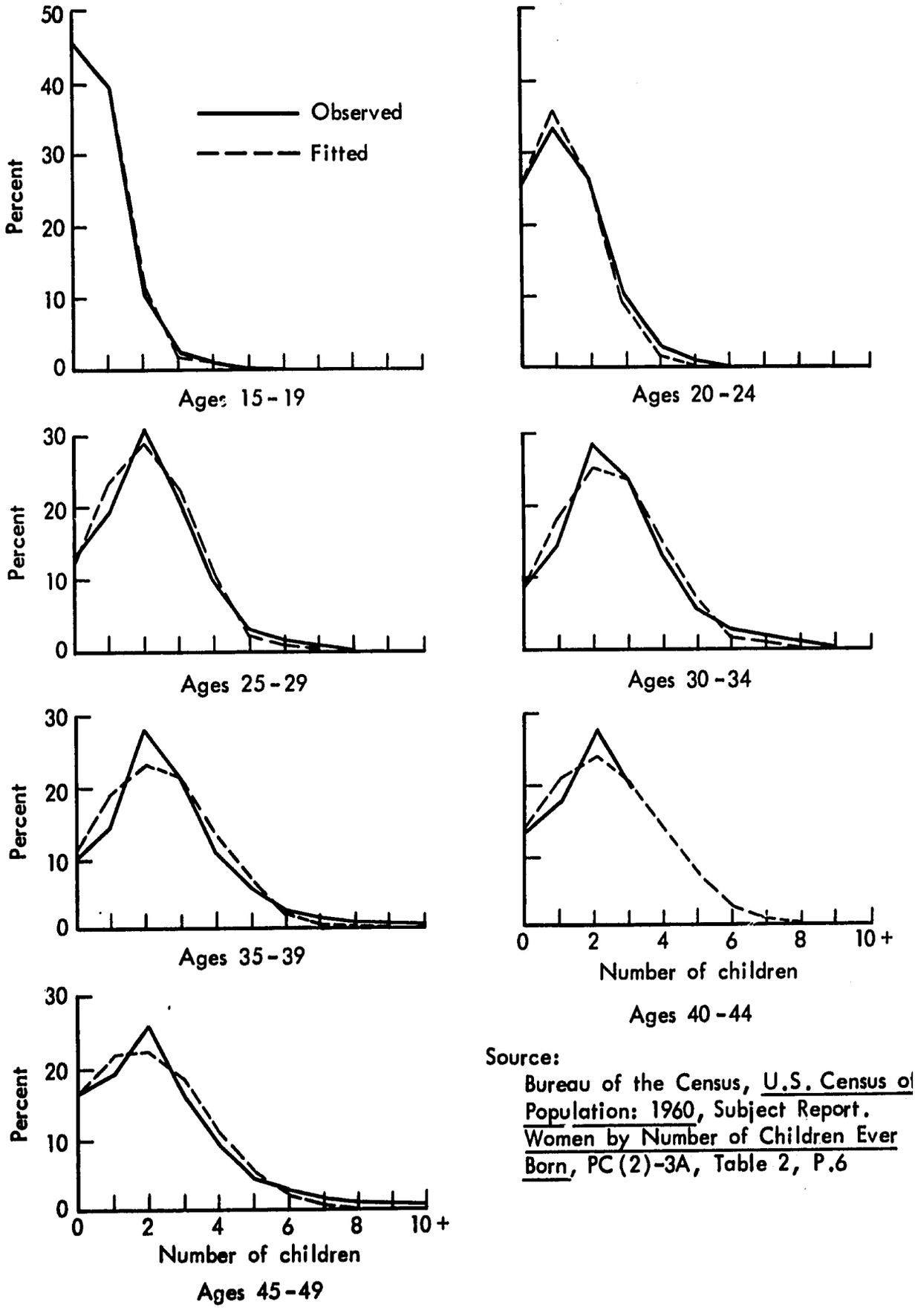
$$\mu = \underset{(1.05)}{.07317} - \underset{(7.03)}{.01433}\ \text{AGE} + \underset{(34.5)}{1.276}\ \text{MEAN} \qquad R^2 = .994,$$

---

[1] Schultz and DaVanzo, 1970.

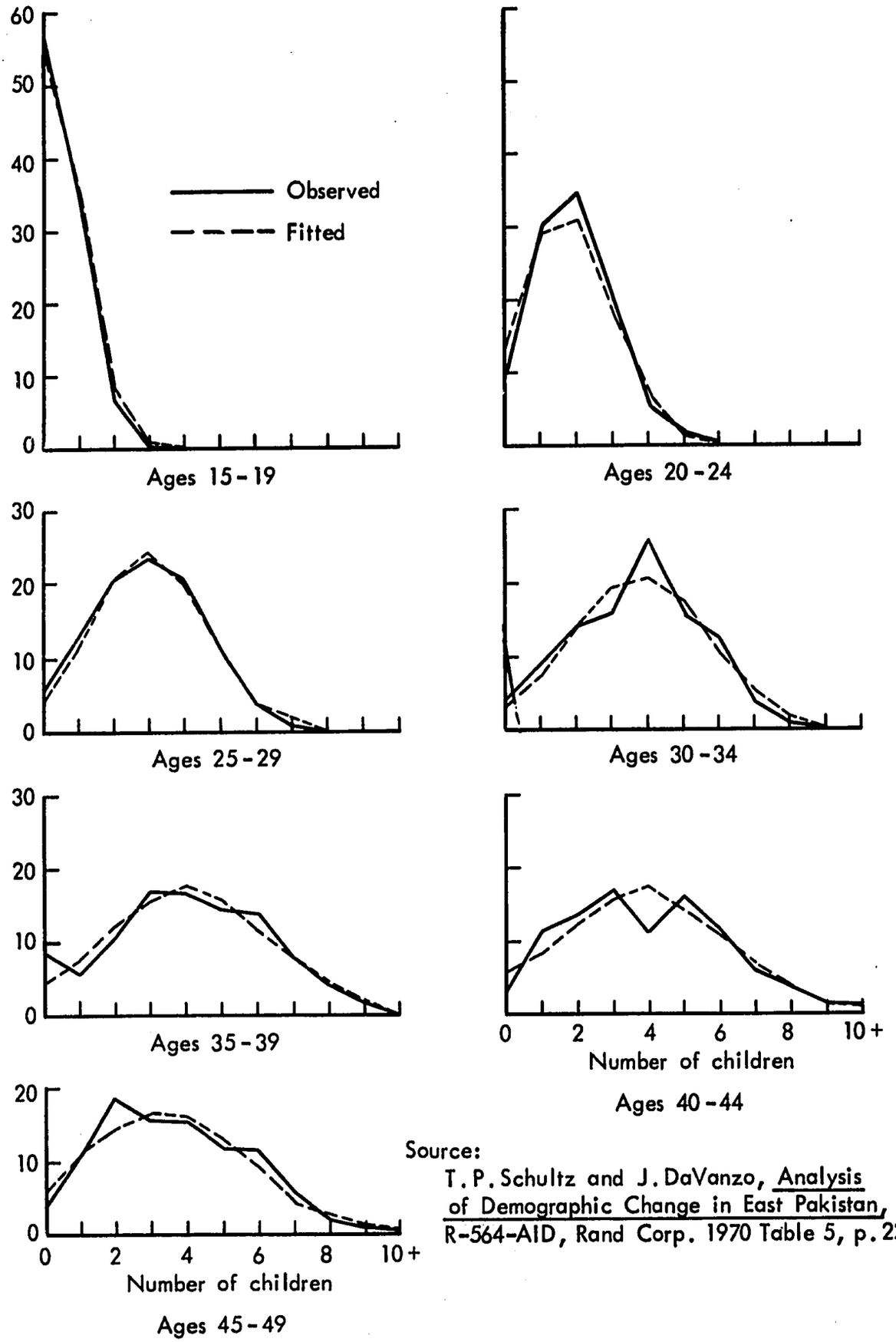
[2] U.S. Bureau of the Census, 1964, Table 2, p. 6.

[3] See Davidon, 1959; Fletcher and Powell, 1963; and Stewart, 1967 for the actual technique used. The results of fitting this function to the known family size distribution are shown in Fig. C-1 and Fig. C-2.



Source:
Bureau of the Census, U.S. Census of Population: 1960, Subject Report. Women by Number of Children Ever Born, PC(2)-3A, Table 2, P.6

Fig. C-1 — Frequency distribution of number of children ever born — observed and fitted: U.S. white 1960



Source:
T. P. Schultz and J. DaVanzo, Analysis of Demographic Change in East Pakistan, R-564-AID, Rand Corp. 1970 Table 5, p. 22

Fig. C-2 — Frequency distribution of number of children now alive — observed and fitted: Central East Pakistan, 1957-1962

$$\sigma^2 = \underset{(4.53)}{.5490} + \underset{(3.62)}{.1282}\ \text{AGE} + \underset{(4.03)}{.2589}\ \text{MEAN} \qquad R^2 = .910,$$

where AGE is the mothers' age in years, MEAN is the mean number of children, and the numbers in parentheses beneath the coefficients are t-ratios (d.f. = 11). Using this relationship, the values of μ and σ^2 can then be estimated from the mothers' age and the mean number of children, and with μ and σ^2 any portion of the family size distribution can be estimated.

Appendix D

COMPLETE PRINTOUT OF SOURCE DECK OF POPULATION PROJECTION PROGRAM

```
      DIMENSION POP(2,81),BIRTHS(2,81),DEATHS(2,81),ASFR(2,81),
     1 ASMR(2,81),FAMILY(35,36),IOPT(20),TITLE(20),IYEAR(100)
      COMMON POP,BIRTHS,DEATHS,ASFR,ASMR,FAMILY,ITIME,AZERO,AONE,R,
     1 IOPT
      IPAGE = 0
      GO TO 902
  900 WRITE(6,901) IERR
  901 FORMAT(13H0ERROR NUMBER,I3,27H CAUSED TERMINATION OF RUN.)
      CALL EXIT
  903 WRITE(6,905)
  905 FORMAT('1')
      DO 907 I = 1,60
      READ(4,904) TITLE,IOPT
  904 FORMAT(40A3)
  907 WRITE(6,906) TITLE,IOPT
  906 FORMAT(1X,40A3)
      CALL EXIT
  902 CONTINUE
      DO 32 I1 = 1,35
      DO 32 I2 = 1,36
   32 FAMILY(I1,I2) = 0.0
      DO 641 I1 = 1,81
      DO 641 I2 = 1,2
      ASMR(I2,I1) = 0.0
      ASFR(I2,I1) = 0.0
      BIRTHS(I2,I1) = 0.0
      DEATHS(I2,I1) = 0.0
  641 POP(I2,I1) = 0.0
C
C     THE FOLLOWING SECTION READS THE CONTROL DATA FOR THE PROGRAM
C
      IERR = 1
      READ(5,100,END=903,ERR=900) TITLE,IBASE,ND1,ND2,ND3,ND4,IOPT,R,
     $ AZERO,AONE
  100 FORMAT(20A4/I4,6X,4I2,2X,20I1,10X,F10.0,2(5X,F5.0))
      R = 1.0 + 0.001 * R
      IF(ND1.LE.0.OR.ND1.EQ.4.OR.ND1.EQ.6.OR.ND1.GT.99) ND1 = 5
      IF(ND2.LE.0.OR.ND2.EQ.4.OR.ND2.EQ.6.OR.ND2.GT.99) ND2 = ND1
      IF(ND3.LE.0.OR.ND3.EQ.4.OR.ND3.EQ.6.OR.ND3.GT.99) ND3 = ND1
      IF(ND4.LE.0.OR.ND4.EQ.4.OR.ND4.EQ.6.OR.ND4.GT.99) ND4 = ND1
      IYMAX = -10000
      IERR = 3
      I1 = -19
    1 I1 = I1 + 20
      I2 = I1 + 19
      READ(5,101,ERR=900) (IYEAR(I),I=I1,I2)
  101 FORMAT(20I4)
      DO 2 I3 = I1,I2
      IMAX = I3
      IF(IYEAR(I3).GT.IYMAX) IYMAX = IYEAR(I3)
```

```
      IF(IYEAR(I3).LE.IBASE) GO TO 3
    2 CONTINUE
      IF(I2.LT.100) GO TO 1
    3 CONTINUE
      IERR = 2
      IF(IYMAX.LE.IBASE) GO TO 900
      IMAX = IMAX - 1
      IB = IBASE + 1
      IERR = 4
      IF(IOPT(7).EQ.0) GO TO 601
      IF(AZERO.GT.1.OR.AZERO.LT.0.OR.AONE.GT.1.OR.AONE.LT.0) GO TO 900
C
C     THE FOLLOWING SECTION READS THE INITIAL DATA FOR POPULATION,
C     FERTILITY RATES, AND MORTALITY RATES
C
  601 IERR = 6
      IF(IOPT(1).EQ.0) READ(ND1,300,ERR=900) POP(1,1),POP(1,2),(POP(1,I)
     1,I=6,81,5)
      IF(IOPT(1).EQ.1) READ(ND1,300,ERR=900) (POP(1,I),I=1,5),(POP(1,I),
     1 I=6,81,5)
      IF(IOPT(1).EQ.2) READ(ND1,300,ERR=900) (POP(1,I),I=1,10),(POP(1,I)
     1,I=11,81,5)
      IF(IOPT(1).GE.3) READ(ND1,300,ERR=900) (POP(1,I),I=1,81)
  300 FORMAT(8F10.0)
      NF1 = 16
      NF2 =  5
      NF3 = 46
      IF(IOPT(2).EQ.0) GO TO 602
      IERR = 7
      READ(ND2,301,ERR=900) NF1,NF2,NF3
  301 FORMAT(3I2)
      NF1 = NF1 + 1
      NF3 = NF3 + 1
      IERR = 5
      IF(NF1.LE.0.OR.NF2.LE.0.OR.NF3.LE.NF1) GO TO 900
      NNN = NF3 - NF1
      IF(NNN.NE.(NNN/NF2)*NF2) GO TO 900
  602 IERR = 7
      READ(ND2,300,ERR=900) (ASFR(1,I),I=NF1,NF3,NF2)
      IERR = 8
      IF(IOPT(3).EQ.0) READ(ND3,300,ERR=900) ASMR(1,1),ASMR(1,2),
     1 (ASMR(1,I),I=6,81,5)
      IF(IOPT(3).EQ.1) READ(ND3,300,ERR=900) (ASMR(1,I),I=1,5),
     1 (ASMR(1,I),I=6,81,5)
      IF(IOPT(3).EQ.2) READ(ND3,300,ERR=900) (ASMR(1,I),I=1,10),
     1 (ASMR(1,I),I=11,81,5)
      IF(IOPT(3).GE.3) READ(ND3,300,ERR=900) (ASMR(1,I),I=1,81)
      IERR = 9
      IF(IOPT(4).GE.1) READ(ND4,300,ERR=900) ((FAMILY(I,J),I=1,35),J=1,
     1 36)
```

```
C
C     THE FOLLOWING SECTION TRANSFORMS THE INITIAL DATA INTO THE FORM
C     USED INTERNALLY BY THE PROGRAM
C
      IF(NF2.EQ.1) GO TO 603
      NNN = NF2 - 1
      DO 604 I1 = NF1,NF3,NF2
      I2 = I1 + 1
      I3 = I1 + NNN
      DO 604 I4 = I2,I3
  604 ASFR(1,I4) = ASFR(1,I1)
  603 IF(IOPT(3).GE.1) GO TO 605
      ASMR(1,3) = ASMR(1,2)
      ASMR(1,4) = ASMR(1,3) * 0.65
      ASMR(1,5) = ASMR(1,3) * 0.42
      ASMR(1,2) = ASMR(1,2) * 1.83
  605 IF(IOPT(3).GE.2) GO TO 606
      ASMR(1, 7) = ASMR(1,6)
      ASMR(1, 8) = ASMR(1,6)
      ASMR(1, 9) = ASMR(1,6)
      ASMR(1,10) = ASMR(1,6)
  606 IF(IOPT(3).GE.3) GO TO 607
      DC 608 I1 = 11,76,5
      I2 = I1 + 1
      I3 = I1 + 4
      DO 608 I4 = I2,I3
  608 ASMR(1,I4) = ASMR(1,I1)
  607 IF(IOPT(7).GE.1) GO TO 609
      AZERO = 0.35
      AONE = 0.42
  609 DO 46 I1 = 1,81
      ASFR(1,I1) = 0.001 * ASFR(1,I1)
      ASMR(1,I1) = 0.001 * ASMR(1,I)'
      ASFR(2,I1) = ASFR(1,I1)
      ASMR(2,I1) = ASMR(1,I1)
   46 CONTINUE
C
C     THE FOLLOWING SECTION DISTRIBUTES THE POPULATION WITHIN COHORTS
C     IF THE INITIAL POPULATION IS GROUPED INTO MULTIPLE - AGE COHORTS
C
      IF(IOPT(1).GE.3) GO TO 610
      IR = 0
      IF(IOPT(9).GE.1) GO TO 611
      IR = 2
      R = 1.0
  611 IF(IOPT(1).GE.1) GO TO 701
      XO = R*R*R + R*R*TRANS(2) + R*TRANS(2)*TRANS(3) +
     1 TRANS(2)*TRANS(3)*TRANS(4)
      TPOP = POP(1,2)+POP(1,3)+POP(1,4)+POP(1,5)
      POP(1,2)=TPOP*R*R*R/XO
```

```
      POP(1,3) = POP(1,2) * TRANS(2) / R
      POP(1,4) = POP(1,3) * TRANS(3) / R
      POP(1,5) = POP(1,4) * TRANS(4) / R
  701 I1 = 6
      IF(IOPT(1).GE.2) I1 = 11
      DO 702 I2 = I1,76,5
      XO = R*R*R*R + R*R*R*TRANS(I2) + R*R*TRANS(I2)*TRANS(I2+1) +
     1 R*TRANS(I2)*TRANS(I2+1)*TRANS(I2+2) +
     2 TRANS(I2)*TRANS(I2+1)*TRANS(I2+2)*TRANS(I2+3)
      TPOP=POP(1,I2)+POP(1,I2+1)+POP(1,I2+2)+POP(1,I2+3)+POP(1,I2+4)
      POP(1,I2)=TPOP*R*R*R*R/XO
      POP(1,I2+1) = POP(1,I2)   *TRANS(I2)   / R
      POP(1,I2+2) = POP(1,I2+1) *TRANS(I2+1) / R
      POP(1,I2+3) = POP(1,I2+2) *TRANS(I2+2) / R
  702 POP(1,I2+4) = POP(1,I2+3) *TRANS(I2+3) / R
      IR = IR - 1
      IF(IR.LT.0) GO TO 610
      TBR = 0.0
      TDR = 0.0
      TPUL = 0.0
      DO 720 I1 = 1,81
      TPUL = TPUL + POP(1,I1)
      TBR = TBR + POP(1,I1) * ASFR(1,I1)
  720 TDR = TDR + POP(1,I1) * ASMR(1,I1)
      R = 1.0 + ( TBR - TDR ) / TPUL
      GO TO 611
  610 CONTINUE
C
C     THE FOLLOWING SECTION PRINTS THE INITIAL DATA SUMMARY
C
      IPAGE = IPAGE + 1
      WRITE(6,201) IPAGE,(TITLE(I),I=1,20)
  201 FORMAT(96H1POPULATION PROJECTION UTILITY MODEL, VERSION OF 1 OCTOB
     $ER 1971.      PROGRAMMING BY K. M. MAURER,10X,4HPAGE,I5//1H ,20A4/)
      WRITE(6,202) IBASE,IYMAX,(IYEAR(I),I=1,IMAX)
  202 FORMAT(21H0PROJECTION FROM YEAR,I5,8H TO YEAR,I5//36HOWITH OUTPUT
     1FOR THE FOLLOWING YEARS,10I7/6(1X,15I7/))
      WRITE(6,7003)
 7003 FORMAT(29H0CONTROL OPTION VALUES USED -)
      DO 7001 I1 = 1,5
      I2 = I1 + 15
 7001 WRITE(6,7002) ((J,IOPT(J)),J=I1,I2,5)
 7002 FORMAT(30X,4(I3,2H :,I2,5X))
      IF(IOPT(9).LT.1.OR.IOPT(1).GE.3) GO TO 7006
      TPOP = 1000.0 * ( R - 1 )
      WRITE(6,7007) TPOP
 7006 IF(IOPT(7).GE.1) WRITE(6,7008) AZERO,AONE
 7007 FORMAT(21H0INITIAL VALUE OF R =,F8.4)
 7008 FORMAT(9H0AZERO = ,F6.3,8H, AONE =,F6.3)
      IF(IOPT(4).GE.1) GO TO 612
```

```
 24 DO 26 I1 = 1,35
    FAMILY(I1,1) = ASFR(1,I1+15) * TRANS(0)
    FAMILY(I1,36) = ASFR(1,I1+15)
    IF(I1.EQ.1) GO TO 26
    FAMILY(I1,36) = FAMILY(I1,36) + FAMILY(I1-1,36)
    DO 27 I2 = 2,I1
 27 FAMILY(I1,I2) = FAMILY(I1-1,I2-1) * TRANS(I2-1)
 26 CONTINUE
612 CONTINUE
    IPAGE = IPAGE + 1
    WRITE(6,201) IPAGE,(TITLE(I),I=1,20)
    WRITE(6,207) IBASE
207 FORMAT(63H SUMMARY OF INPUT DATA FOR POPULATION, FERTILITY, AND MO
   1RTALITY,20X,5H YEAR,I5,      //3X,6HCOHORT,12X,10HPOPULATION,4X,
   210H% OF TOTAL,3X,11HBIRTH RATE*,3X,11HDEATH RATE*/ 24X,5H( 1 ),8X,
   35H( 2 ),10X,5H( 3 ),9X,5H( 4 )//)
    TPOP = 0.0
    TBR = 0.0
    TDR = 0.0
    DO 28 I1 = 1,81
    TPOP = TPOP + POP(1,I1)
    TBR = TBR + ASFR(1,I1) * POP(1,I1)
 28 TDR = TDR + ASMR(1,I1) * POP(1,I1)
    TBR = 1000.0 * TBR / TPOP
    TDR = 1000.0 * TDR / TPOP
    WRITE(6,208) TPOP,TBR,TDR
208 FORMAT(15H      TOTAL      ,F15.0,5X,8H 100.000,5X,F9.3,5X,F9.3/)
209 FORMAT(15H  LESS THAN 1  ,F15.0,5X,F8.3,5X,F9.3,5X,F9.3)
210 FORMAT(1H ,I5,2H -,I5,2X,F15.0,5X,F8.3,5X,F9.3,5X,F9.3)
211 FORMAT(15H  80 AND OVER  ,F15.0,5X,F8.3,5X,F9.3,5X,F9.3)
    X1 = 100.0 * POP(1,1) / TPOP
    X2 = 1000.0 * ASFR(1,1)
    X3 = 1000.0 * ASMR(1,1)
    WRITE(6,209) POP(1,1),X1,X2,X3
    I2 = 1
    I3 = 4
    X0 = POP(1,2) + POP(1,3) + POP(1,4) + POP(1,5)
    X1 = 100.0 * X0 / TPOP
    X2 = 1000.0 * ( ASFR(1,2) * POP(1,2) + ASFR(1,3) * POP(1,3) +
   1 ASFR(1,4) * POP(1,4) + ASFR(1,5) * POP(1,5) ) / X0
    X3 = 1000.0 * ( ASMR(1,2) * POP(1,2) + ASMR(1,3) * POP(1,3) +
   1 ASMR(1,4) * POP(1,4) + ASMR(1,5) * POP(1,5) ) / X0
    WRITE(6,210) I2,I3,X0,X1,X2,X3
    DO 29 I1 = 6,76,5
    I2 = I1 - 1
    I3 = I1 + 3
    X0 = POP(1,I1) + POP(1,I1+1) + POP(1,I1+2) + POP(1,I1+3) +
   1 POP(1,I1+4)
    X1 = 100.0 * X0 / TPOP
    X2 = 1000.0 * ( ASFR(1,I1) * POP(1,I1) + ASFR(1,I1+1) *
```

```
     1 POP(1,I1+1) + ASFR(1,I1+2) * POP(1,I1+2) + ASFR(1,I1+3) *
     2 POP(1,I1+3) + ASFR(1,I1+4) * POP(1,I1+4) ) / X0
      X3 = 1000.0 * ( ASMR(1,I1) * POP(1,I1) + ASMR(1,I1+1) *
     1 POP(1,I1+1) + ASMR(1,I1+2) * POP(1,I1+2) + ASMR(1,I1+3) *
     2 POP(1,I1+3) + ASMR(1,I1+4) * POP(1,I1+4) ) / X0
      WRITE(6,210) I2,I3,X0,X1,X2,X3
   29 CONTINUE
      X1 = 100.0 * POP(1,81) / TPOP
      X2 = 1000.0 * ASFR(1,81)
      X3 = 1000.0 * ASMR(1,81)
      WRITE(6,211) POP(1,81),X1,X2,X3
      WRITE(6,212)
  212 FORMAT(24H0* PER THOUSAND PER YEAR///)
      DO 613 I1 = 1,81
      BIRTHS(1,I1) = ASFR(1,I1) * POP(1,I1)
  613 DEATHS(1,I1) = ASMR(1,I1) * POP(1,I1)
      IF(IOPT(10).GT.0) CALL STATS
      CALL FAMOUT
C
C     THE FOLLOWING SECTION PERFORMS THE ANNUAL PROJECTION OF THE
C     POPULATION , CALCULATING PROJECTED BIRTHS, DEATHS, AND SURVIVORS
C
      DO 33 ITIME = IB,IYMAX
      IF(IOPT(5).GE.1) CALL ENFERT
      IF(IOPT(6).EQ.0) GO TO 88
      CALL ENMORT
      IF(IOPT(3).GE.1) GO TO 88
      ASMR(1,2) = 1.83 * ASMR(1,3)
      ASMR(1,4) = 0.65 * ASMR(1,3)
      ASMR(1,5) = ASMR(1,3) * 0.42
   88 CONTINUE
      DO 38 I2 = 1,34
      I3 = 36 - I2
      FAMILY(I3,36) = FAMILY(I3-1,36) + ASFR(1,I3+15)
      DO 38 I4 = 1,34
      I5 = 36 - I4
   38 FAMILY(I3,I5) = FAMILY(I3-1,I5-1) * TRANS(I5-1)
      FAMILY(1,36) = ASFR(1,16)
      DO 39 I2 = 1,35
   39 FAMILY(I2,1) = ASFR(1,I2+15) * TRANS(0)
      DO 40 I2 = 1,81
      POP(2,I2) = POP(1,I2)
      BIRTHS(2,I2) = BIRTHS(1,I2)
   40 DEATHS(2,I2) = DEATHS(1,I2)
      POP(1,1) = 0.
      DO 41 I2 = 2,80
      POP(1,I2) = POP(2,I2-1) * TRANS(I2-1)
      BIRTHS(1,I2) = POP(1,I2) * ASFR(1,I2)
      DEATHS(1,I2) = POP(1,I2) * ASMR(1,I2)
   41 POP(1,1) = POP(1,1) + BIRTHS(1,I2) + BIRTHS(2,I2)
```

```
      POP(1,81) = POP(2,81) * TRANS(81) + POP(2,80) * TRANS(80)
      BIRTHS(1,81) = POP(1,81) * ASFR(1,81)
      DEATHS(1,81) = POP(1,81) * ASMR(1,81)
      POP(1,1) = 0.5 * ( POP(1,1) + BIRTHS(1,81) + BIRTHS(2,81) )
      POP(1,1) = POP(1,1) * TRANS(0)
      BIRTHS(1,1) = POP(1,1) * ASFR(1,1)
      DEATHS(1,1) = POP(1,1) * ASMR(1,1)
      IF(ITIME.EQ.IBASE) GO TO 33
      DO 34 I1 = 1,IMAX
      IF(ITIME.EQ.IYEAR(I1)) GO TO 35
   34 CONTINUE
      GO TO 33
C
C     THE FOLLOWING SECTION PRINTS THE PROJECTED DATA FOR A GIVEN YEAR
C     IF SPECIFIED BY THE USER
C
   35 CONTINUE
      IPAGE = IPAGE + 1
      WRITE(6,201) IPAGE,(TITLE(I),I=1,20)
      WRITE(6,205) ITIME
  205 FORMAT(46H POPULATION, BIRTHS, AND DEATHS PROJECTED DATA,27X,5H YE
     1AR,I5  //120H   COHORT          POPULATION  % OF TOTAL    CHANGE
     2  RATE *          BIRTHS    RATE **          DEATHS     RATE **
     3 /20X,5H( 1 ),7X,5H( 2 ),7X,5H( 3 ),6X,5H( 4 ),11X,5H( 5 ),4X,5H(
     46 ),12X,5H( 7 ),5X,5H( 8 )/)
  206 FORMAT(11H   TOTAL    ,F15.0,F11.3,F12.0,F11.4,2(F16.0,F10.3)/)
  213 FORMAT(1H ,3X,I3,4X,F15.0,F11.3,F12.0,F11.4,2(F16.0,F10.3))
  214 FORMAT(1H ,I3,2H -,I3,2X,F15.0,F11.3,F12.0,F11.4,2(F16.0,F10.3))
  215 FORMAT(11H    80 +    ,F15.0,F11.3,F12.0,F11.4,2(F16.0,F10.3))
      TPOP = 0.0
      TPOPL = 0.0
      TB = 0.0
      TBL = 0.0
      TD = 0.0
      TDL = 0.0
      DO 47 I2 = 1,81
      TPOP = TPOP + POP(1,I2)
      TPOPL = TPOPL + POP(2,I2)
      TB = TB + BIRTHS(1,I2)
      TBL = TBL + BIRTHS(2,I2)
      TD = TD + DEATHS(1,I2)
   47 TDL = TDL + DEATHS(2,I2)
      X1 = 100.0
      X2 = TPOP - TPOPL
      X3 = 100.0 * X2 / TPOPL
      X5 = 1000.0 * TB / TPOP
      X8 = 1000.0 * TD / TPOP
      WRITE(6,206) TPOP,X1,X2,X3,TB,X5,TD,X8
      IL = 1
      IF(IOPT(8).GE.1) IL = 10
```

```
      DO 99 IJ = 1,IL
      JI = IJ - 1
      X0 = POP(1,IJ)
      X1 = 100.0 * X0 / TPOP
      X2 = X0 - POP(2,IJ)
      X3 = 100.0 * X2 / POP(2,IJ)
      X5 = 1000.0 * ASFR(1,IJ)
      X8 = 1000.0 * ASMR(1,IJ)
      WRITE(6,213) JI,X0,X1,X2,X3,BIRTHS(1,IJ),X5,DEATHS(1,IJ),X8
   99 CONTINUE
      IF(IOPT(8).GE.1) WRITE(6,227)
  227 FORMAT(1H )
      X0 = POP(1,2) + POP(1,3) + POP(1,4) + POP(1,5)
      X10 = POP(2,2) + POP(2,3) + POP(2,4) + POP(2,5)
      X1 = 100.0 * X0 / TPOP
      X2 = X0 - X10
      X3 = 100.0 * X2 / X10
      X4 = BIRTHS(1,2) + BIRTHS(1,3) + BIRTHS(1,4) + BIRTHS(1,5)
      X5 = 1000.0 * X4 / X0
      X7 = DEATHS(1,2) + DEATHS(1,3) + DEATHS(1,4) + DEATHS(1,5)
      X8 = 1000.0 * X7 / X0
      I3 = 1
      I4 = 4
      WRITE(6,214)I3,I4,X0,X1,X2,X3,X4,X5,X7,X8
      DO 48 I2 = 6,76,5
      I3 = I2 - 1
      I4 = I2 + 3
      X0 = POP(1,I2) + POP(1,I2+1) + POP(1,I2+2) + POP(1,I2+3) +
     1 POP(1,I2+4)
      X10 = POP(2,I2) + POP(2,I2+1) + POP(2,I2+2) + POP(2,I2+3) +
     1 POP(2,I2+4)
      X1 = 100.0 * X0 / TPOP
      X2 = X0 - X10
      X3 = 100.0 * X2 / X10
      X4 = BIRTHS(1,I2) + BIRTHS(1,I2+1) + BIRTHS(1,I2+2) +
     1 BIRTHS(1,I2+3) + BIRTHS(1,I2+4)
      X5 = 1000.0 * X4 / X0
      X7 = DEATHS(1,I2) + DEATHS(1,I2+1) + DEATHS(1,I2+2) +
     1 DEATHS(1,I2+3) + DEATHS(1,I2+4)
      X8 = 1000.0 * X7 / X0
   48 WRITE(6,214)I3,I4,X0,X1,X2,X3,X4,X5,X7,X8
      X0 = POP(1,81)
      X10 = POP(2,81)
      X1 = 100.0 * X0 / TPOP
      X2 = X0 - X10
      X3 = 100.0 * X2 / X10
      X5 = 1000.0 * ASFR(1,81)
      X8 = 1000.0 * ASMR(1,81)
      WRITE(6,215) X0,X1,X2,X3,BIRTHS(1,81),X5,DEATHS(1,81),X8
      WRITE(6,216)
```

```
216 FORMAT(20H0*  PERCENT PER YEAR/ 25H ** PER THOUSAND PER YEAR/)
    IF(IOPT(10).GT.0) CALL STATS
    CALL FAMOUT
 33 CONTINUE
    GO TO 902
    END
```

```
      SUBROUTINE FAMOUT
      DIMENSION POP(2,81),BIRTHS(2,81),DEATHS(2,81),ASFR(2,81),
     1 ASMR(2,81),FAMILY(35,36),IOPT(20)
      COMMON POP,BIRTHS,DEATHS,ASFR,ASMR,FAMILY,ITIME,AZERO,AONE,R,
     1 IOPT
C
C     THE FOLLOWING PROGRAM PRINTS THE PROJECTED FAMILY DESCRIPTION
C
      WRITE(6,203)
  203 FORMAT(1H0,25X,46H STATISTICAL DESCRIPTION OF "AVERAGE" FAMILIES/
     1  /9H PARENT'S,26X,29H CHILDREN SURVIVING AT AGES :,27X,29HTOTAL L
     2IVE   TOTAL   SURVIVAL,/120H  COHORT        0        1 - 4      5 - 9
     3    10 -14    15 - 19    20 - 24    25 - 29      30 +       BIRTHS   SU
     4RVIVORS RATE (%)/)
      DO 30 I1 = 16,46,5
      X0 = 0.0
      X1 = 0.0
      X2 = 0.0
      X3 = 0.0
      X4 = 0.0
      X5 = 0.0
      X6 = 0.0
      X7 = 0.0
      X8 = 0.0
      I2 = I1 + 4
      TPOP = POP(1,I1)+POP(1,I1+1)+POP(1,I1+2)+POP(1,I1+3)+POP(1,I1+4)
      DO 31 I3 = I1,I2
      I4 = I3 - 15
      X0 = X0 + FAMILY(I4,1) * POP(1,I3)
      X1 = X1 + POP(1,I3) * ( FAMILY(I4,2) + FAMILY(I4,3) + FAMILY(I4,4)
     1 + FAMILY(I4,5) )
      X2 = X2 + POP(1,I3) * ( FAMILY(I4,6) + FAMILY(I4,7) + FAMILY(I4,8)
     1 + FAMILY(I4,9) + FAMILY(I4,10) )
      X3 = X3 + POP(1,I3) * ( FAMILY(I4,11) + FAMILY(I4,12) + FAMILY(I4,
     113) + FAMILY(I4,14) + FAMILY(I4,15) )
      X4 = X4 + POP(1,I3) * ( FAMILY(I4,16) + FAMILY(I4,17) + FAMILY(I4,
     118) + FAMILY(I4,19) + FAMILY(I4,20) )
      X5 = X5 + POP(1,I3) * ( FAMILY(I4,21) + FAMILY(I4,22) + FAMILY(I4,
     123) + FAMILY(I4,24) + FAMILY(I4,25) )
      X6 = X6 + POP(1,I3) * ( FAMILY(I4,26) + FAMILY(I4,27) + FAMILY(I4,
     128) + FAMILY(I4,29) + FAMILY(I4,30) )
      X7 = X7 + POP(1,I3) * ( FAMILY(I4,31) + FAMILY(I4,32) + FAMILY(I4,
     133) + FAMILY(I4,34) + FAMILY(I4,35) )
      X8 = X8 + FAMILY(I4,36) * POP(1,I3)
   31 CONTINUE
      X0 = X0 / TPOP
      X1 = X1 / TPOP
      X2 = X2 / TPOP
      X3 = X3 / TPOP
      X4 = X4 / TPOP
```

```
      X5 = X5 / TPOP
      X6 = X6 / TPOP
      X7 = X7 / TPOP
      X8 = X8 / TPOP
      X9 = X0 + X1 + X2 + X3 + X4 + X5 + X6 + X7
      X10 = 100.0 * X9 / X8
      I2 = I1 - 1
      I3 = I1 + 3
      WRITE(6,204) I2,I3,X0,X1,X2,X3,X4,X5,X6,X7,X8,X9,X10
  204 FORMAT(I3,2H -,I3,2X,11F10.5)
   30 CONTINUE
      RETURN
      END
```

```
      FUNCTION TRANS(I)
      DIMENSION POP(2,81),BIRTHS(2,81),DEATHS(2,81),ASFR(2,81),
     1 ASMR(2,81),FAMILY(35,36),IOPT(20)
      COMMON POP,BIRTHS,DEATHS,ASFR,ASMR,FAMILY,ITIME,AZERO,AONE,R,
     1 IOPT
C
C     THE FOLLOWING PROGRAM CALCULATES SURVIVAL RATES FROM ANNUAL
C     MORTALITY RATES FOR EACH AGE COHORT
C
      IF(I.GT.0) GO TO 401
      TRANS = 1.0 / ( 1.0 + ASMR(1,1) * ( 1.0 - AZERO ) )
      RETURN
  401 Z0 = ASMR(2,81)
      IF(I.LT.81) Z0 = ASMR(2,I)
      B1 = 0.5
      B0 = 0.5
      IF(I.EQ.2) B0 = AONE
      IF(I.EQ.1) B1 = AONE
      IF(I.EQ.1) B0 = AZERO
      IF(I.LE.80) Z1 = ASMR(1,I+1)
      IF(I.GT.80) Z1 = ASMR(1,81)
      TRANS = (1.0-B0*Z0)/(1.0+Z1*(1.0-B1))
      RETURN
      END
```

```
      SUBROUTINE STATS
      DIMENSION POP(2,81),BIRTHS(2,81),DEATHS(2,81),ASFR(2,81),
     1 ASMR(2,81),FAMILY(35,36),IOPT(20),TITLE(20),IYEAR(100)
      COMMON POP,BIRTHS,DEATHS,ASFR,ASMR,FAMILY,ITIME,AZERO,AONE,R,
     1 IOPT
C
C     THE FOLLOWING PROGRAM CALCULATES AND PRINTS SUMMARY POPULATION
C     STATISTICS. IT IS USED WHENEVER IOPT(10) > 0
C
      BIR = 0.0
      TPOP = 0.0
      DO 1 I = 1,81
      BIR = BIR + BIRTHS(1,I)
    1 TPOP = TPOP + POP(1,I)
      AGE = AZERO * POP(1,1) + ( 1.0 + AONE ) * POP(1,2)
      XI = 1.5
      DO 2 I = 3,81
      XI = XI + 1.0
    2 AGE = AGE + XI * POP(1,I)
      AGE = AGE / TPOP
      POPMID = 0.0
      DO 3 I = 16,45
    3 POPMID = POPMID + POP(1,I)
      BWR = BIR / POPMID
      PCTMID = 100.0 * POPMID / TPOP
      CWR = ( POP(1,1)+POP(1,2)+POP(1,3)+POP(1,4)+POP(1,5) ) / POPMID
      DO 6 I = 46,60
    6 POPMID = POPMID + POP(1,I)
      DR = ( TPOP - POPMID ) / POPMID
      X = 1.0
      EXPECT = 0.0
      DO 4 I = 1,91
      X = X * TRANS(I-1)
    4 EXPECT = EXPECT + X
      WRITE(6,5) AGE,PCTMID,DR,EXPECT,BWR,CWR
    5 FORMAT(12H AVERAGE AGE,F8.3,10X,20HPERCENT AGED  15-44 ,F8.4,10X,
     $19HDEPENDENCY RATIO    ,F8.4/12H EXP (BIRTH),F8.3,10X,20HBIRTH - WO
     $MAN RATIO ,F8.4,10X,19HCHILD - WOMAN RATIO,F8.4)
      RETURN
      END
```

```
      SUBROUTINE ENMORT
      DIMENSION POP(2,81),BIRTHS(2,81),DEATHS(2,81),ASFR(2,81),
     1 ASMR(2,81),FAMILY(35,36),IOPT(20),TITLE(20),IYEAR(100)
      COMMON POP,BIRTHS,DEATHS,ASFR,ASMR,FAMILY,ITIME,AZERO,AONE,R,
     1 IOPT
C
C     DUMMY PROGRAM : ENMORT
C
      RETURN
      END
```

```
      SUBROUTINE ENFERT
      DIMENSION POP(2,81),BIRTHS(2,81),DEATHS(2,81),ASFR(2,81),
     1 ASMR(2,81),FAMILY(35,36),IOPT(20),TITLE(20),IYEAR(100)
      COMMON POP,BIRTHS,DEATHS,ASFR,ASMR,FAMILY,ITIME,AZERO,AONE,R,
     1 IOPT
C
C     DUMMY PROGRAM : ENFERT
C
      RETURN
      END
```

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*****           THIS PROGRAM WAS DEVELOPED AND PROGRAMMED BY KENNETH M. MAURER
AND T. PAUL SCHULTZ OF THE                **********
                                                                           *****
                                                    POPULATION RESEARCH PROGRAM
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*****                                   UNDER AGENCY FOR INTERNATIONAL DEVELOPMENT CONTRACT
 CSD-2533.                              **********                              *****

*****                                   ********************************************
*****************************************************************************
*****                                   **********                                  A
DESCRIPTION OF THE PROGRAM IS AVAILABLE IN :                                    *****
*****                                   **********                  KENNETH M. MAURER
AND T. PAUL SCHULTZ                           THE RAND CORPORATION                  *****
*****                   A POPULATION PROJECTION MODEL
 1700 MAIN ST.                          **********                  R-953-AID ( JANUAR
Y 1972 )                                      SANTA MONICA,                         *****
*****
 CALIF.   90406                         **********                              *****

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ERROR MESSAGES

Error	Cause
1	Input error attempting to read Projection Card in Control Section.
2	No years specified for output before end of list.
3	Input error attempting to read Output Card(s) in Control Section.
4	Illegal value supplied for AZERO or AONE.
5	Illegal value supplied for N1, N2, or N3 in Data Section: Fertility.
6	Input error attempting to read initial population distribution Data Section: Population.
7	Input error attempting to read N1, N2, N3, or initial fertility rates in Data Section: Fertility.
8	Input error attempting to read initial mortality rates in Data Section: Mortality.
9	Input error attempting to read initial description of "average" family in Data Section: Family