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DETERMINATION OF THE OPTIMUM LOCATION AND CAPACITY OF A STORAGE TANK ON A TYPICAL WATERSHED

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OCCASIONAL PAPER 3

ECONOMICS & STATISTICS UNIT

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DETERMINATION OF THE OPTIMUM LOCATION AND CAPACITY OF A STORAGE TANK ON A TYPICAL WATERSHED

by

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Occasional Paper No. 3

Research on water harvesting for gravity irrigation[1/] involves the question of where to locate the storage tank on the watershed. The following tries to answer this question by means of formulating the problem in mathematical terms and solving it as well as testing the solution on the assumed set of parameters.

1. Mathematical Derivation of Optimal Solution

We take a watershed area, TA, of typical shape with no sub-watersheds and of given unit size. The position of the tank for harvesting the water determines the catchment area, CA, and the remaining area, RA, which is assumed to be irrigated from the tank (see Figure 1). Thus

(1) $TA = CA + RA$, where $TA = 1$, and
$0 < CA < 1$, $0 \leq RA < 1$.

The amount of irrigation water, IW, to be collected from the catchment area depends upon the runoff, RO, and a parameter, c:

(2) $IW = cRO \cdot CA$,

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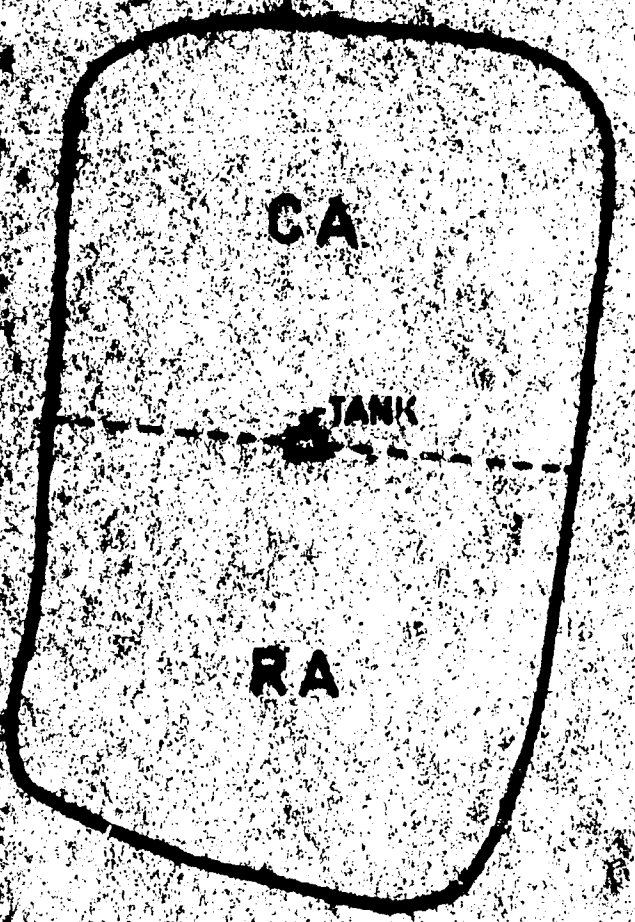
* The author thanks J.G.Ryan for helpful comments, suggestions and moral support.

1/ For reasons to be concluded in the last section of this paper, we include pumping out of the tank, but exclude piping uphill as it appears not feasible.

[illegible] area (RA) by the [illegible]



where the runoff is known to be a function of absolute rainfall, RF, and other factors, OF, such as intensity and seasonal distribution of rainfall, and soil type and condition, slope and cover of the land:

$$(3)\quad RO = f(RF, OF).$$

The parameter c represents the net amount of catchment water available for irrigation after allowing for evaporation and seepage losses.

Distribution of the irrigation water over the remaining area determines the irrigation per hectare, ID:

$$(4)\quad ID = \frac{IW}{RA}$$

By inserting equation (2) into equation (4) it follows:

$$(5)\quad ID = \frac{CA}{RA}\, cRO.$$

Irrigation increases yields, IY, such that:

$$(6)\quad IY = f(ID),\ \text{ceteris paribus other inputs.}$$

We assume (6) to be a quadratic function of the following type:

$$(7)\quad IY = NY + g(ID) - h(ID)^2,$$

where NY represents the yield without irrigation.

By inserting equation (5) into equation (7) form:

$$(8)\quad IY = NY + gcRO\frac{CA}{RA} - h\left(cRO\frac{CA}{RA}\right)^2.$$

Total returns, TR, from the watershed are the sum of the yield without irrigation, NY, on the catchment

area, CA, plus the yield due to irrigation, IY, on the irrigated area, RA:

(9) $TR = NY\ CA + IY\ RA.$

By inserting equation (8) into equation (9) and rearranging, the following ensues:

(10) $TR = NY\ CA + NY\ RA + gcRO\ CA - hc^2(RO)^2\frac{(CA)^2}{RA}.$

From equation (1) it follows that $NY\ CA + NY\ RA = NY\ TA$, and we note that TA of a given watershed is a constant, which to simplify is set equal to one.

The total costs, TC, of irrigation are determined by the cost of digging and maintaining the tank plus the cost of lifting the water[2/]. Both costs are a function of the amount of water stored and used, and this function is assumed to have the form:

(11) $TC = dIW$, which in terms of equation (2) we can write:

(12) $TC = d\ cRO\ CA.$

Net returns, NR, are the difference between total returns and total costs, or

(13) $NR = TR - TC$

(14) $NR = NY\ TA + gcRO\ CA - hc^2(RO)^2\frac{(CA)^2}{1-CA} - dcRO\ CA$

If total costs and total returns are correctly specified then the location of the tank is optimal when net returns are maximized.

2/ These costs are further explained in the next section where an application of the result is demonstrated.

If we set the first derivative of NR with respect to CA equal to zero we find the maximum of this function, providing the second derivative is negative:

$$(15) \quad \frac{\partial NR}{\partial CA} = gcRO - hc^2(RO)^2 \frac{2CA-(CA)^2}{(1-CA)^2} - dcRO = 0$$

$$(16) \quad \frac{\partial^2 NR}{\partial CA^2} = -\frac{2hc^2(RO)^2}{(1-CA)^3}$$

Since by definition of equation (1) $CA < 1$ it follows that (16) is negative and (15) represents a maximum.

From (15) we derive:

$$(17) \quad 1 - CA = \frac{hcRO}{g-d+hcRO}$$

Thus the optimum proportion of RA over TA which determines the optimum position of the storage tank can be expressed as:

$$(18) \quad RA = \frac{hcRO}{g-d+hcRO}$$

2. Application of Optimal Solution

In an attempt to apply equation (18) we have tried to approximately determine the values of the parameters involved:

c - this parameter was postulated in equation (2). It expresses the percentage of the water originally harvested, that is not lost during storage and irrigation. Values between .6 and .8 appear to be reasonable assumptions.

g and h - these parameters were introduced with the water production function (7). They are estimated by assuming an average function for all crops. The values[3/] for four of such functions (see Figure 2) may vary as follows:

Parameters	Function 1	Function 2	Function 3	Function 4
NY	1000	700	500	500
g	290	270	240	230
h	3.6	4.0	4.0	2.8

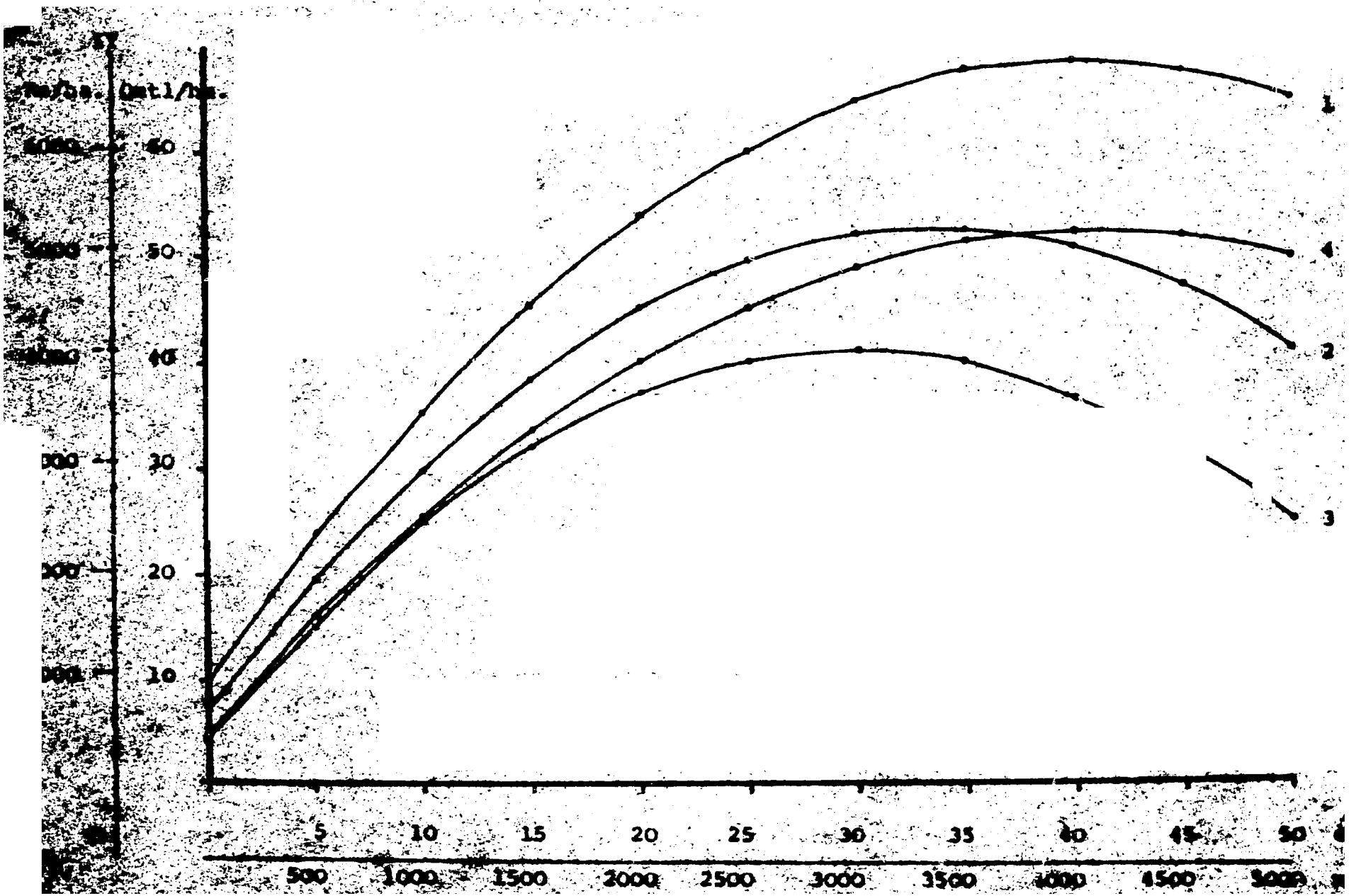
d - is the parameter that contains information about the marginal costs of irrigation (equation 11). These costs are mainly a direct function of the quantities of water stored and pumped respectively. It appears reasonable as a first approximation to assume a linear relationship between cost and capacity for all tank sizes.[4/]. In detail the costs (as derived from BWSB tank costs) consist of:

	Annual costs
1. Tank costs Rs.160 to Rs.200/cm.ha. of effective storage with a tank life of 10 years and an annual cost of 10% for interest and maintenance of tank and watershed; this implies:	Rs.32 to Rs.40/cm.ha.

3/ The function is expressed in terms of RS./ha. and cm/ha. (= 100 m^3/ha.)

4/ Assuming optimum shapes for different tank capacities.

Figure 2 - Average Production Function of Water



2. Pump costs:

   Rs.3000 per 5 H.P.engine and pump over 10 years plus 10% interest implies Rs. 600/year. Assuming conservatively 5 to 10 cm. ha.of water pumped annually the costs are: Rs.60 to 120/cm.ha.

3. Operating the pump costs about Rs.80 to 120/ha. 5 cm., thus Rs.16 to 24/cm.ha.

Total estimated costs: Rs.108 to 184/cm.ha.

RO - the runoff, varies with absolute rainfall, and with intensity and seasonal distribution of rainfall as well as with soil type, slope, cover and condition of the land. As a rough estimate we assume a runoff range of between 10% to 20% of annual rainfall of say 700 mm; i.e. we propose RO to vary between 7 and 14 cm.ha. per annum.

Inserting arbitrarily all lower (a), upper (b), and medium (c) estimates of variables and parameters into equation (18) we find:

$$\text{(a)}\quad RA_a = \sqrt{\frac{2.8 \times .6 \times 7}{230-108+2.8 \times .6 \times 7}} = \sqrt{\frac{11.76}{133.76}} = .30$$

$$\text{(b)}\quad RA_b = \sqrt{\frac{3.6 \times .8 \times 14}{290-184+3.6 \times .8 \times 14}} = \sqrt{\frac{40.32}{146.32}} = .52$$

$$\text{(c)}\quad RA_c = \sqrt{\frac{4.0 \times .7 \times 10.5}{270-146+4.0 \times .7 \times 10.5}} = \sqrt{\frac{29.4}{153.4}} = .44$$

These results show that for data set (a) the watershed would best be split into a 70% catchment area with only 30% of irrigated area. This proportion is due to relatively low water costs and a "flat" production function (Function 4), which for optimum output requires a relatively large amount of water, while because of low runoff rates and high losses, water is scarce.

Data set (b), on the other hand, implies a share of 48% catchment area and 52% irrigated area. In this case fairly high water costs coincide with large quantities of water available, and a "steep" production function (Function 1) which shows a strong response of output to water.

Data set (c) suggests that about 56% of the area be catchment area, while 44% be irrigated area. Higher water costs would reduce the catchment area, especially if also larger quantities of water were available and vice versa.

3. CONCLUSION

These results reveal the sensitivity of the system to its various parameters, especially to the costs of water. As soon as these costs in Rs. per cm./hectare exceed the g-value of the production function of say 290 plus the h-value times cRO, water harvesting and irrigation is no longer justified (the expression under the square root becomes negative and there is no solution to RA.) Any expensive pumping device can easily drive the solution to this point, piping of water being one.

On the other hand, as long as there is a reasonab'y cheap way to harvest and redistribute the water, the optimum solutions for our data tend to allocate about two thirds to one half of the area for catchment in order to allow for a fairly intensive irrigation of the remaining area.

It is likely that the production function for water becomes steeper the better the timing of irrigation. In such a case the catchment area would tend to be decreased and the irrigated area expanded. This, as such, is a very desirable strategy, but it should be kept in mind that probably this requires relatively more sophisticated and consequently more expensive irrigation equipment (stronger pump, better channels). The relationships among the factors involved are such that increases in water costs d are quite likely to eventually outrun the possible increase in the value of g+h.cRO, whatever techniques are applied, and it is at this point, that water harvesting would disappear as an optimum solution altogether.

It should be emphasized that this paper does not intend to prejudice current experiments on the ICRISAT watersheds. The data used in this analysis are by no means qualified to do this. However, the above result indicate the degrees to which the various parameters included are contributing to the optimal solutions, and consequently where further additional research is required. It is strongly felt that a highly critical value like that of water costs, especially the costs

for lifting water out of the tank, must be given more attention, than presently is done at ICRISAT. Also more experiments specifically designed to determine crop-water-production functions should be carried out.

The analysis does not take into consideration the fact that most of the variables and parameters included are stochastic in nature, e.g. rainfall. When attaching stochastic properties to these variables, the optimum solution may turn out to be different. This will have to be investigated further and field experiments should also embrace consideration of crop response to both the timing and quantity of irrigation as related to varying periods of moisture stress and their probability of occurrence. Ryan is at present doing some work in this area.

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