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ANALYSIS OF THE OPEN, DUALISTIC ECONOMY: AN APPLICATION TO THE PHILIPPINES

Field Work Report #9

by

John C. H. Fei and Douglas S. Paauw

M-8805
August, 1966

Introduction

The purpose of this paper is to expound a special technique for use in the typology approach to analysis of economic development. The discussion consists of two parts: (1) the evolution of the technique itself as a contribution to development planning methodology and (2) an application of this technique to the Philippine economy to demonstrate the usefulness of the method for analysis of an open, dualistic economy. Development of the technique proceeds first with reference to the open, dualistic economy (here exemplified by the Philippines) in Sections I and II. It is specifically applied to the Philippine economy in Section III and is later generalized by more formal analysis in Section IV so that its applicability to other types of economies becomes apparent. The mathematical proofs of theorems presented in Section IV will be found in the mathematical appendix which is written as a self-contained unit not requiring previous mathematical background.

The analysis is designed to assist in understanding the development dynamics of particular types of less developed economies. Such understanding is required for two related objectives: (1) to provide a sounder basis for formulating development strategies and policies applicable to economies of each type and (2) to serve as a basis for studies of comparative typology. Methods of analysis currently available do not

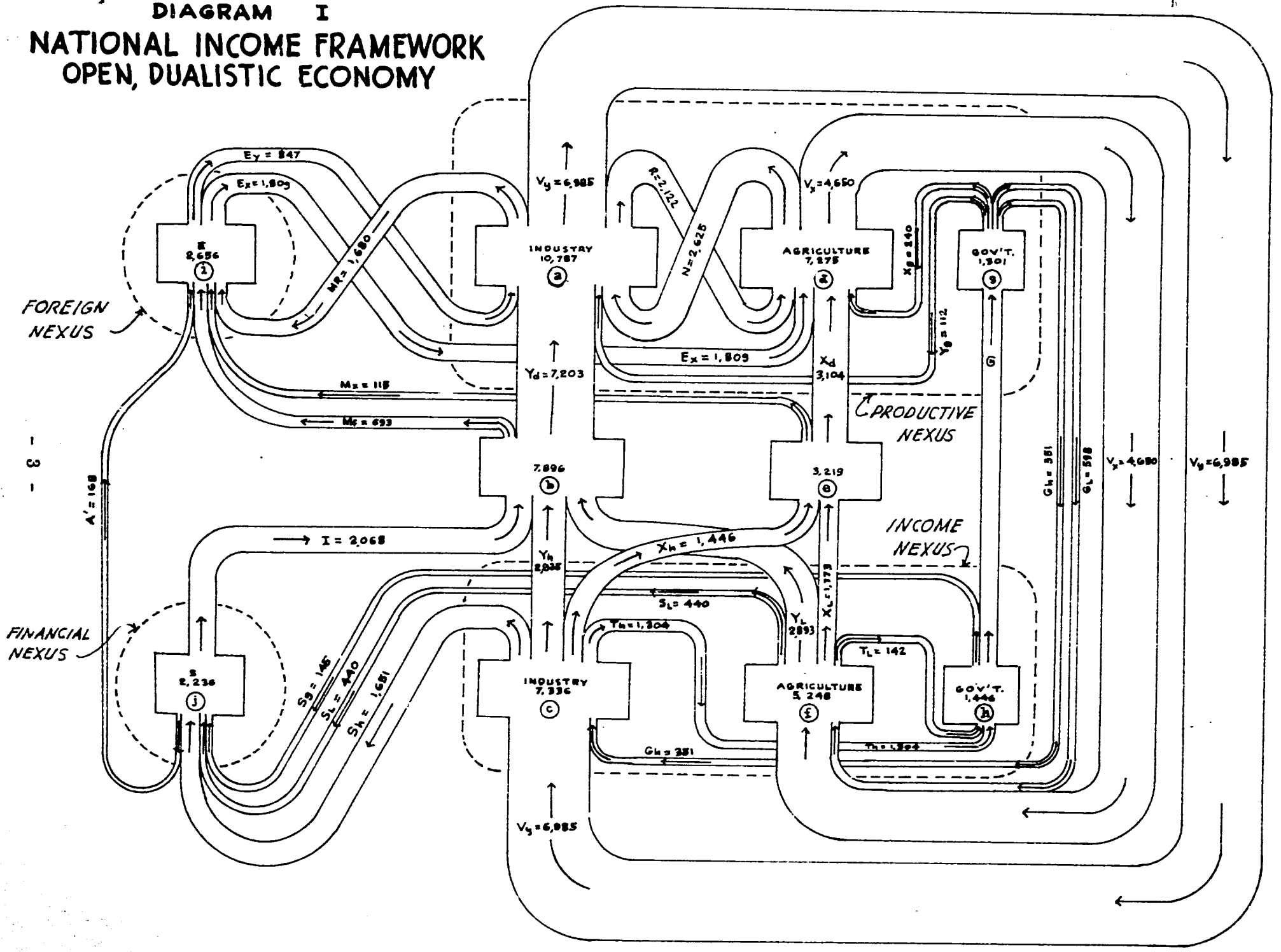
adequately emphasize structural and functional differences among developing economies for either purpose. This paper demonstrates that the techniques presented are capable of differentiating basic structural and functional characteristics, as well as revealing inter-temporal changes in these characteristics.

I. THE NATIONAL INCOME ACCOUNTING FRAMEWORK

The national income accounting system for the open, dualistic economy is presented in Diagram I. This system distinguishes four basic nexuses of economic activity which are shown as dotted circles: production, income disposition, financial, and foreign. Two of the nexuses, production and income disposition, are broken down into three sectors: industry, agriculture, and government. In other words, each of the three domestic sectors appears in both the production and income disposition nexus, facilitating analysis of both production and income for these three sectors.

Government productive activity is defined as the provision of collective consumption services, valued as the total of government expenditures. The private sector is comprised by agriculture and industry. Agricultural production is defined as productive activity which

DIAGRAM I
NATIONAL INCOME FRAMEWORK
OPEN, DUALISTIC ECONOMY



creates organic output by a process in which land is involved in an essential way.[1] This leaves the industrial sector defined to include essentially all non-agricultural private economic activity. The foreign nexus shows the relationship between the three domestic sectors and the rest of the world, while the financial nexus is the balancing sector. Functionally, it provides savings resources for the finance of the economy's investment.

The national income accounting system in Diagram I, therefore, gives a synoptic view of the operation of the open, dualistic economy in terms of relationships among (and in some cases, within) five key centers of economic activity: industry, agriculture, government, foreign, and finance. These relationships are shown as directed flows of monetary payments with arrows in the pipes indicating direction of the flows. All directed flows are national income accounting concepts, referring to magnitudes of monetary payments in a given year. Actual figures shown in the pipes are estimates for the Philippines for the year 1964, in millions of pesos at constant 1955 prices.

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[1] Milled rice and similar processed agricultural products, however, are considered as agricultural output, with services (e.g., milling) rendered by the industrial sector construed as intermediate services purchased from the industrial sector.

Diagram I contains a series of junctures at which several directed flows converge. There are 10 such junctures, represented by the 10 rectangular boxes, indexed (a) through (j). A central property of each juncture is that all directed inflows equal all directed outflows. In fact, as we will elaborate in more technical terms below, this is a necessary feature of any legitimate national income accounting system. Conceptually, we can think of an accounting equation at each juncture stating this equality.

In Table I we present a list of the 27 variables shown as the national income flows in Diagram I. We now introduce each of these flows from the viewpoint of inflows into the junctures, (a) through (j).

Beginning with juncture (a), which shows the industrial production sector, we find four inflows representing the four components of demand for industrial output. Two of these are final demand: Y_d, industrial output for final domestic use, and E_y, industrial exports. The remaining two represent demand for intermediate goods purchased by government, Y_g, and by agriculture, N.

Juncture (b) shows the total supply of industrial goods available for final domestic use. Total spending on these goods includes three components, the three inflows into the juncture: purchases of capital goods for investment, I; industrial goods purchased by industrial

households for consumption, Y_h; and industrial goods purchased by agricultural households for consumption, Y_L.

Juncture (c) represents the income disposition function for industrial households. Virtually all income is from factor income payments from the industrial sector itself, shown as the inflow V_y, value added by industrial production. This reflects the assumption in our framework that virtually all income in the industrial sector derives from employment within the sector itself. This provides a direct link between the production and income functions. (A similar assumption is made for the agricultural sector.) A second, small component of industrial income is shown as the inflow G_h, government purchases of primary factor services from industrial households.

Juncture (d) shows agricultural production. As in the case of industry, there are four inflows comprising the four components of demand for agricultural output. Two are final demand; i.e., X_d, agricultural output for final domestic use, and E_x, agricultural exports. The two other inflows are intermediate good purchases by industry, R, and by government, X_g.

Total supply of agricultural goods available for final domestic use is shown at juncture (e). The two inflows into the juncture are purchases of agricultural consumer goods by industrial households, X_h, and by agricultural households, X_L.

Inflows comprising income of agricultural households are shown at juncture (f). The larger inflow, V_x, value added by agricultural production, represents payments for primary factor services employed in agriculture, while the smaller inflow, G_L, represents government purchases of primary factor services from agriculture.

The government sector's total expenditures on goods and services for collective consumption are shown as the only inflow, G, into juncture (g). Government income is shown at juncture (h) as two inflows: T_h, industrial sector tax payments, and T_L, agricultural sector tax payments.

Juncture (i) represents the foreign nexus. Three of the inflows are payments for imports: M_R, intermediate goods imported by the industrial sector; M_x, imports of agricultural consumer goods by the agricultural sector; and M_f, imported industrial goods for final use. The latter includes both imported capital goods and imported industrial consumer goods. A' is shown as inflow from the financial nexus and represents the rather untypical situation of capital export. This situation has prevailed in the Philippines in the last few years and, hence, is shown in Diagram I. Ordinarily, we would expect to find a reverse flow of foreign savings from the foreign nexus to the developing country.

Finally, juncture (j) shows the financial nexus. In the Philippine case shown, there are three domestic savings inflows: S_g, government savings; S_L, agricultural sector savings; and S_h, industrial sector savings. As noted in the preceding paragraph, we would usually find an inflow of foreign savings into the developing country's financial nexus. For this reason, the outflow of capital, A', is shown at the bottom of the rectangular box rather than at the top.

We have now described each national income flow, shown in Diagram I, in terms of inflows into each of the 10 junctures. Since each inflow is an outflow from another juncture, there are a total of 27 national income flows, related by 10 accounting equations, one for each juncture.[2]

This framework is explicitly designed for the study of the open, dualistic economy. As a national income accounting system it represents a general equilibrium approach to facilitate investigation of the growth dynamics of the economy as a whole. All of the essential economic relationships which relate to the development of this type of economy are included in the system. For this reason, the framework,

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[2]If total inflows were considered as additional variables, we would have 10 additional variables and 10 additional equations.

although working within a national income accounting system, includes relationships not specifically accounted in conventional systems. Examples include the recording of intersectoral intermediate goods flows, the disaggregation of households into three income distribution sectors, and the disaggregation of both exports and imports between industrial and agricultural. Disaggregation is limited to a selected number of intersectoral relationships to keep the number of variables studied within manageable bounds. This choice represents a workable compromise between the relatively high level of aggregation employed in most development models and the excessive amount of disaggregation reflected in input-output analysis. The system is designed to facilitate quantitative analysis of the relationships among a small number of relatively large (aggregated) sectors. The sectors selected for analysis are considered to be strategic for the development of a particular type of economy. The framework used in this paper reflects of this typology approach to the open, dualistic economy.

Table I: Accounting Variables in Open, Dualistic Economy Framework

Juncture	Inflows
a	Industrial Output sold as:
	Y_d : goods for final domestic use
	E_y : exports
	Y_g : intermediate goods purchased by government
	N : intermediate goods purchased by agriculture
b	Total available industrial output for final use purchased as:
	I : investment goods
	Y_h : consumer goods purchased by industrial households
	Y_L : consumer goods purchased by agricultural households
c	Industrial sector income received as:
	V_y : value added by industrial production
	G_h : government primary factor purchases

Table I: Accounting Variables in Open, Dualistic Economy Framework (Continued)

Juncture	Inflows
d	Agricultural output sold as:
	X_d : goods for final domestic use
	E_x : exports
	X_g : intermediate goods purchased by government
	R : intermediate goods purchased by industry
e	Total available agricultural output for final use purchased as:
	X_L : consumer goods purchased by agricultural households
	X_h : consumer goods purchased by industrial households
f	Agricultural sector income received as:
	V_x : value added by agricultural production
	G_L : government primary factor purchases
g	G : total government expenditures
h	Government revenue received as:
	T_h : industrial sector tax payments
	T_L : agricultural sector tax payments

Table I: Accounting Variables in Open, Dualistic Economy Framework (Continued)

Juncture	Inflows
i	Foreign sector receipts from:
	M_R : intermediate goods imported by industrial sector
	M_x : imported agricultural consumer goods
	M_f : imported industrial goods for final use
	A' : capital outflow
j	Total domestic savings as:
	S_h : savings of industrial households
	S_L : savings of agricultural households
	S_g : savings of government

II. LINEAR GRAPH THEORY AND NATIONAL INCOME ACCOUNTING

A. Linear Graph Concepts

To facilitate the later analysis we define two concepts which are basic to the techniques we develop--the concepts of a directed linear graph and a valued linear graph.

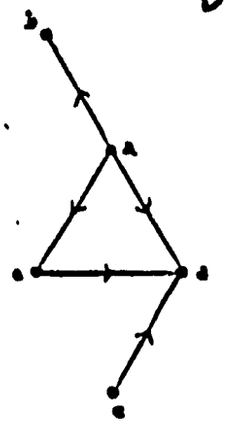
Definition: A directed linear graph, G, is formed of a set of vertices (a, b, c....) and a set of directed edges which are ordered pairs of vertices (x, y). For the directed edge (x, y), x is the initial vertex and y is the terminal vertex.

Diagram IIa is a directed linear graph in which a directed edge is represented by an arrow pointing from the initial to the terminal vertex. The formal mathematical definition of the directed linear graph shown is given just below the diagram. Unless otherwise noted, we will be concerned only with directed edges in this paper. Hence, in several places, we dispense with the adjective "directed."

Definition: A valued linear graph, A, is a directed linear graph, G, with a number written on each edge. If the number x is written on edge (a, b), x is referred to as the value of the edge. The linear graph, G, is called the skeleton of A.

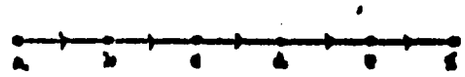
Diagram IIb is a valued linear graph constructed on the skeleton shown in Diagram IIa.

DIAGRAM II: LINEAR GRAPHS

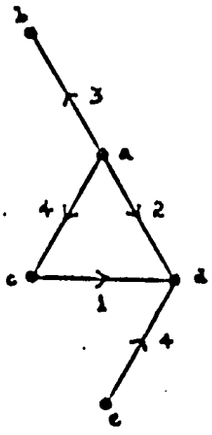


VERTICES: (a, b, c, d, e)
DIRECTED EDGES: (a, b) (a, c)
(a, d) (c, d) (d, e)

a: DIRECTED LINEAR GRAPH

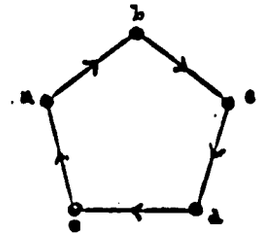


d: PATH

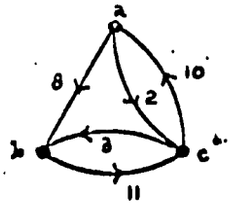


	a	b	c	d	e
a		3	4	2	
b					
c				1	
d					
e				4	

b: VALUED LINEAR GRAPH AND TABLE

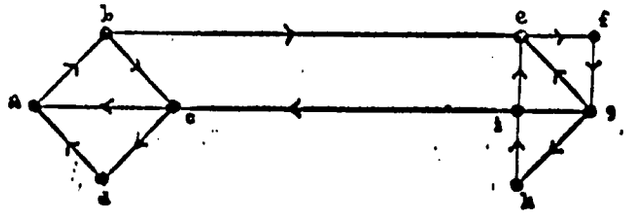


c: CIRCUIT



	a	b	c	
a		8	2	10
b			11	11
c	10	3		13
	10	11	13	

e: EULER GRAPH AND BALANCED TABLE



f: CYCLIC NET

The national income accounting system of Diagram I can be construed as a valued linear graph. The skeleton describes the structure of the small, open economy showing 27 flows (edges) connecting 10 vertices (a, b, c, d, e, f, g, h, i, j).

There are two aspects of the valued linear graph which have significance for our typology approach to development planning and strategy. First, there is the feature of a specific structural image of the economy (the skeleton) which reveals a particular pattern of economic relationships emphasized for a given type of economy. Second, there is the quantitative aspect, an attribute associated with the values assigned to each edge of the skeleton.

The former aspect, the structural skeleton of a system, is of primary significance for the typology approach. It is the skeleton which describes the totality of economic relationships which exist among the economy's key nexuses and sectors. With reference to the small, open economy, for example, the central features of dualism and openness can be defined mathematically from the characteristics of the skeleton. Basic features of other types of economies should be similarly defined from the skeleton.

The second aspect, values of the edges in the skeleton, allows us to apply the skeleton to a specific economy by assigning concrete numbers to each relationship. It is this aspect which brings to the

skeleton the unique numerical substance for individual cases. In combination, therefore, the skeleton and values provide a basis for comparative typology. Different economies can be compared by analysis of different skeletons as well as by analysis of numerical values on the same basic skeleton. In the discussion that follows, we develop techniques for both the analysis of the skeleton and the analysis of values.

B. The Linear Graph and the Square Table

A valued linear graph can always be cast into the form of a square table under the following rules:

(1) The number of sectors in the square table (i.e., the number of similarly indexed columns and rows) equals the number of vertices in the linear graph. Each column and row must be indexed by a vertex notation.

(2) If the value of the edge (a, b) is X, the value of the cell in the a-th row (i.e., the initiating vertex) and the b-th column (i.e., the terminating vertex) is x.

As an illustration, the square table corresponding to a valued linear graph is shown beside the graph in Diagram II. It is apparent that a square table can always be interpreted as a valued linear graph under the above rules. We apply this principle to the valued linear graph in Diagram I by putting it in the form of a square table, Table III.

Definition: A square table is a <u>balanced</u> table if the sum of all entries in any row is the same as the sum of all entries in the like-indexed column.

A valued linear graph corresponding to a balanced table is called an <u>Euler graph</u>. In an Euler graph, the value of total inflows into each vertex equals the value of total outflows. Diagram IIc shows an Euler graph and its corresponding balanced table. It can be seen that the table is in balance from the equality between row sums (written at the right-hand margin of the table) and the column sums (written at the bottom of the table). We shall say that a sector of a square table is <u>in balance</u> if the sum of all the entries in a row equals the sum of all entries in the like-numbered column.

We now state an elementary theorem (which can be easily proved) to be referred to as the "Balanced Table Theorem":

Theorem: <u>A square table with n sectors is a balanced table when n-1 sectors are in balance.</u>

Graphically, the interpretation of this theorem is that, if a valued linear graph has n vertices and if n-1 vertices are in balance, it is an Euler graph. The reasonableness of the theorem is apparent from this graphic interpretation.

An economic application of the Balanced Table Theorem can now be made. Referring to Table II, which has 10 sectors, the first 9 sectors are in balance by definition.[3] The theorem then implies that the last sector is in balance, showing the familiar equality between savings and investment. Thus, the national income accounting system of the open, dualistic economy is represented by a balanced table (i.e., an Euler graph).

C. The Linear Graph as a National Income Accounting Skeleton

We have seen that an Euler graph may be interpreted as the skeleton of a national income accounting system. This is based on the fact that any meaningful national income accounting system can be put into the form of a balanced table with positive entries. We now investigate what properties a linear graph must have to serve as the skeleton of such a meaningful national income accounting system. For this purpose we introduce the following definitions:

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[3]Each of these 9 sectors is in balance because for each sector a residual item is <u>defined</u> to balance the sector. This national income accounting principle will be more systematically explored in Section IV.

TABLE II: BALANCED TABLE DERIVED FROM DIAGRAM I

	TERMINAL VERTEX / INITIAL VERTEX	INDUSTRY			AGRICULTURE			GOVERNMENT		FOREIGN	FINANCE	TOTAL
		Production a	Total Supply b	Income Disposition c	Production d	Total Supply e	Income Disposition f	Production g	Income Disposition h	i	j	
INDUSTRY	Production a			V_y 6,985	R 2,122					M_R 1,680		10,787
	Total Supply b	Y_d 7,203								M_f 693		7,896
	Income Disposition c		Y_h 2,935			X_h 1,446			T_h 1,304		S_h 1,651	7,336
AGRICULTURE	Production d	N 2,625					V_x 4,650					7,275
	Total Supply e				X_d 3,104					M_x 115		3,219
	Income Disposition f		Y_L 2,893			X_L 1,773			T_L 142		S_L 440	5,248
GOV'T	Production g	Y_g 112		G_h 351	X_g 240		G_L 598					1,301
	Income Disposition h							G 1,301			S_g 145	1,446
FGN	i	E_y 847			E_x 1,809							2,656
FIN	j		I 2,068							A' 168		2,236
TOTAL		Y 10,787	Y_a 7,896	P_h 7,336	X 7,275	X_a 3,219	P_L 5,248	G 1,301	T 1,446	M 2,656	S' 2,236	

Definition: A set of edges form a path connecting "a" and "z" if they can be written in the form (a, b) (b, c) (c, d). . . . (x, y) (y, z) where a, b, c. . . . z are distinct. A path is a circuit if "a" is "z."

Diagram IId is a path with a length of five edges, and Diagram IIe is a circuit of five edges. This circuit can be written in the form (a, b, c, d, e), in which the vertices appear in the specified sequence.

Definition: A linear graph is a cyclic net if for every pair of vertices x, y there is a path from x to y and a path from y to x.

Diagram IIf is a cyclic net.

Visual inspection of Diagram I, the skeleton of the accounting system for the open dualistic economy, reveals that this skeleton is a cyclic net. We shall prove in Section IV that the skeleton of a "legitimate" national income accounting system is always a cyclic net.[4] For the analysis in this section, we assume that this is true.

D. Aggregation

Let the linear graph, G, in Diagram IIIa be given and let the vertices be classified in the following sub-sets:

---

[4] A linear graph, G, is defined to be the skeleton of a legitimate national income accounting system if a strictly positive Euler graph can be written on G (i.e., strictly positive values must be attached to all edges of G to form an Euler graph).

$A = (a) \quad B = (b, c, d) \quad C = (e, f) \quad D = (g, h, i)$

such that each vertex belongs to one and only one of these sub-sets. An <u>aggregated linear graph,</u> A(G), based on this classification is a linear graph with vertices:

A, B, C, D

and contains an edge (X, Y) if, and only if, there is an edge (u, v) in G such that us is in X and v is in Y.

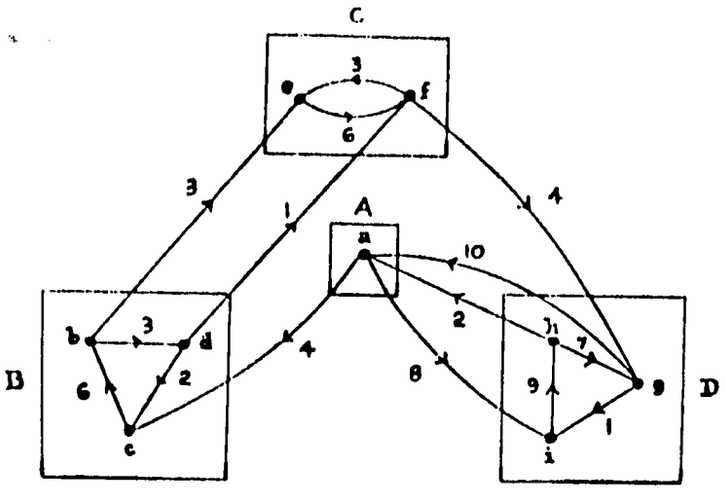
An example is given in Diagram IIIa and IIIb. The vertex classification of Diagram IIIa is indicated by the four squares, and the aggregated linear graph derived from IIIa is shown in IIIb.

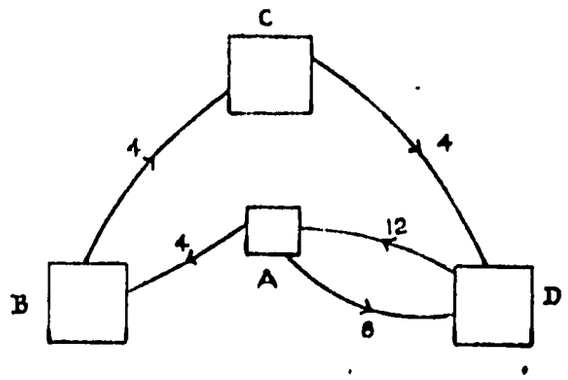
When a <u>valued</u> linear graph is defined on G, the valued linear graph can be aggregated by valuing the edge (X, Y) as the <u>sum</u> of the values of all edges (u, v) where us is in X and v is in Y. The aggregation of the valued linear graph shown in Diagram IIIa can be seen from the valued edges in Diagram IIIb.

These rules of aggregation can be generalized to apply to any linear graph and any valued linear graph.

The valued linear graph in Diagram IIIa is an Euler graph. One can verify that the aggregated linear graph shown in IIIb is also an Euler graph. This is an example of the following theorem which can be easily proved:

DIAGRAM III: AGGREGATES





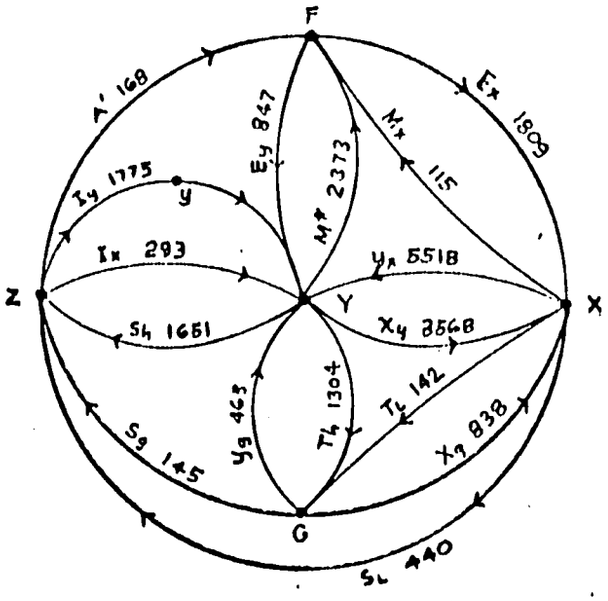
		A	B			C		D		
		a	b	c	d	e	f	g	h	i
A	a			4						
B	b				3	3				
	c		6							
	d			2						
C	e						6			
	f					3		4		
D	g	10								
	h	2						7		
	i								9	

(a)

	A	B	C	D
A	X	4		8
B		X	4	
C			X	4
D	12			X

(b)

DIAGRAM IV: AGGREGATED LINEAR GRAPH OPEN DUALISTIC ECONOMY



	Y	X	G	F	Z
Y		$X_y = R + X_h$ 3568	T_h 1304	$M^a = M_R + M_f$	S_h 1651
X	$y_x = N + Y_L$ 5518		T_L 142	M_x 115	S_L 440
G	$y_g = Y_g + G_h$ 463	$x_g = X_g + G_L$ 838			
F	E_y 847	E_x 1809			
Z	$I = I_y + I_x$ 2068			A' 168	

(a) (b)

Theorem: <u>The aggregation of an Euler graph leads to an Euler graph.</u>

The operation of aggregation can be represented in table form, as indicated in Diagram IIIa and IIIb. To perform this operation, let the vertices belonging to the same sub-set be listed adjacently, as in the table in Diagram IIIa. In this diagram the heavier lines mark off the cells in the aggregated table in IIIb. The sum of all entries in each cell in the disaggregated table (Diagram IIIa) is computed and recorded in the aggregated table, as shown in Diagram IIIb.

When this aggregation technique is applied to the skeleton of the open, dualistic economy shown in Diagram I (and the Balanced Table shown in Table II),[4a] we obtain the aggregated linear graph and aggregated table shown in Diagram IVa and IVb.

The economic significance of such aggregation is the simplification of economic relationships into a limited number of aggregates which can be subjected to intensive study. This is essential for understanding the operation of the economy as a whole and the relationship of the parts to the dynamics of the whole system. It is apparent that there are many ways to aggregate a detailed table similar to the one presented in Table II. The choice of a particular way will depend on the parts of the whole to be emphasized in the analysis. In the case of the aggregation results shown in Diagram IV we have selected an approach which stresses <u>intersectoral</u> relationships among industry (Y),

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[4a] For this aggregation, the vertex classifications (a, b, c), (d, e, f), (g, h), (i), (j) so that the aggregated table is a 5 x 5 table.

agriculture (X), government (G), foreign trade (F), and finance (Z), while neglecting internal (intrasectoral) relationships within each sector. This emphasis is based on our premise that key development phenomena are likely to be found in changing flows among the strategic sectors listed.

Given the importance of the intersectoral relationships in the open, dualistic system, the flows (edges) shown in Diagram IVa require elaboration. The previous system (Diagram I) has been reduced to 5 sectors (vertices) with 16 directed edges (flows). This has been done partly by combining the production and income distribution components of each domestic sector (agriculture, industry, government) into one sector, thus eliminating flows within these sectors and partly by combining flows among the five sectors into aggregates. Each of the 16 resulting flows (with numbers representing estimates for the Philippines, 1964) can be seen in the aggregated linear graph presented in Diagram IVa and in the aggregated table given in Diagram IVb. Flows in the table should, of course, be read from columns to rows; i.e., the first entry from column Y to X, (xy), represents payments by industry (Y) to agriculture (X). At each vertex we can identify inflows of money payments to the sector represented by the vertex from other sectors and

outflows to other sectors.[5] The same flows can also be read from the balance table, reading vertically for each sector to obtain inflows and horizontally to obtain outflows. Beginning with vertex Y, the industrial sector, the following inflows are shown: (1) payments from agriculture, y_x, composed of payments for both intermediate goods (N in Diagram I) and for consumers goods (Y_L in Diagram I); (2) payments from government, y_g, including both payments for intermediate goods (Y_g in Diagram I) and for primary factors (G_h in Diagram I); (3) payments for exports, E_y; and (4) payments from the finance sector for investment goods, I, broken down into two flows to facilitate later analysis; i.e., I_y, industrial investment, and I_x, agricultural investment.

At vertex X, the agricultural sector, three inflows are shown: (1) payments from industry, x_y, including both industrial purchases of intermediate goods (R in Diagram I) and consumers goods (X_y in Diagram I); (2) payments from government, x_g, composed of purchases of intermediate goods (X_g in Diagram I) and primary factors (G_L in Diagram I); and (3) payments for exports, E_x.

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[5] Since every outflow from one sector is an inflow to another, we will discuss only inflows.

At vertex G, the government sector, inflows are T_h, tax payments from the industrial sector and T_L, tax payments from the agricultural sector. At vertex F, the foreign sector, inflows comprise payments for imported agricultural consumer goods, I_x, imported industrial goods, M* (composed of M_R, industrial intermediate goods, and M_f, finished industrial goods, in Diagram I), and A', capital export. At vertex Z, the finance sector, the three inflows represent three domestic savings flows: S_h, industrial savings; S_L, agricultural savings, and S_g, government savings.

E. Economic Circulations

Applied to national income accounting, a circuit (defined above) depicts a particular type of <u>economic circulation</u> revealing a complete and consistent set of events which assumes an independent accounting existence. When a skeleton, such as the one in Diagram IVa is given, we can systematically identify all circuits implied in the structure. The resulting set of circuits represents all possible individual circulations found in the economic structure described by the skeleton.

If the skeleton is a cyclic net, it can be easily shown that:

Theorem: <u>In a cyclic net every edge is included in a circuit.</u>

For this reason a large number of circuits can generally be found in a cyclic net. For the cyclic net shown in Diagram Va, for example, we can identify five circuits designated in Diagram Vb as C_1, C_2, C_3, C_4, and C_5.

In order to emphasize the economic significance of circuits as economic circulations, we study the skeleton shown in Diagram IVa, which is also a cyclic net. We recall that this diagram shows economic flows among five sectors: foreign (F), agriculture (X), industry (Y), government (G), and finance (Z). By systematic listing of all possible circuits we can identify twenty-nine, including seven of two edge lengths, ten of three edge lengths, seven of four edge lengths, and five of five edge lengths. To assist in understanding the economic meaning of these circuits, we have listed all but the complex five length circuits in Table III.[6]

Table III is designed to classify the circuits in terms useful for economic analysis. The first broad distinction is between <u>current</u> and <u>financial</u> (capital) transactions, depending on whether circuits touch the financial sector, Z. A further sub-classification distinguishes between

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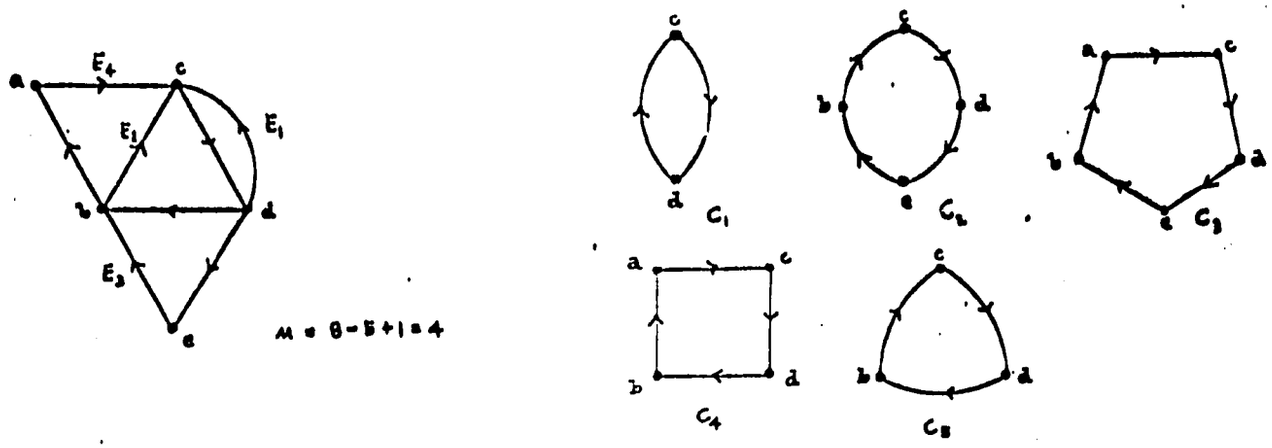
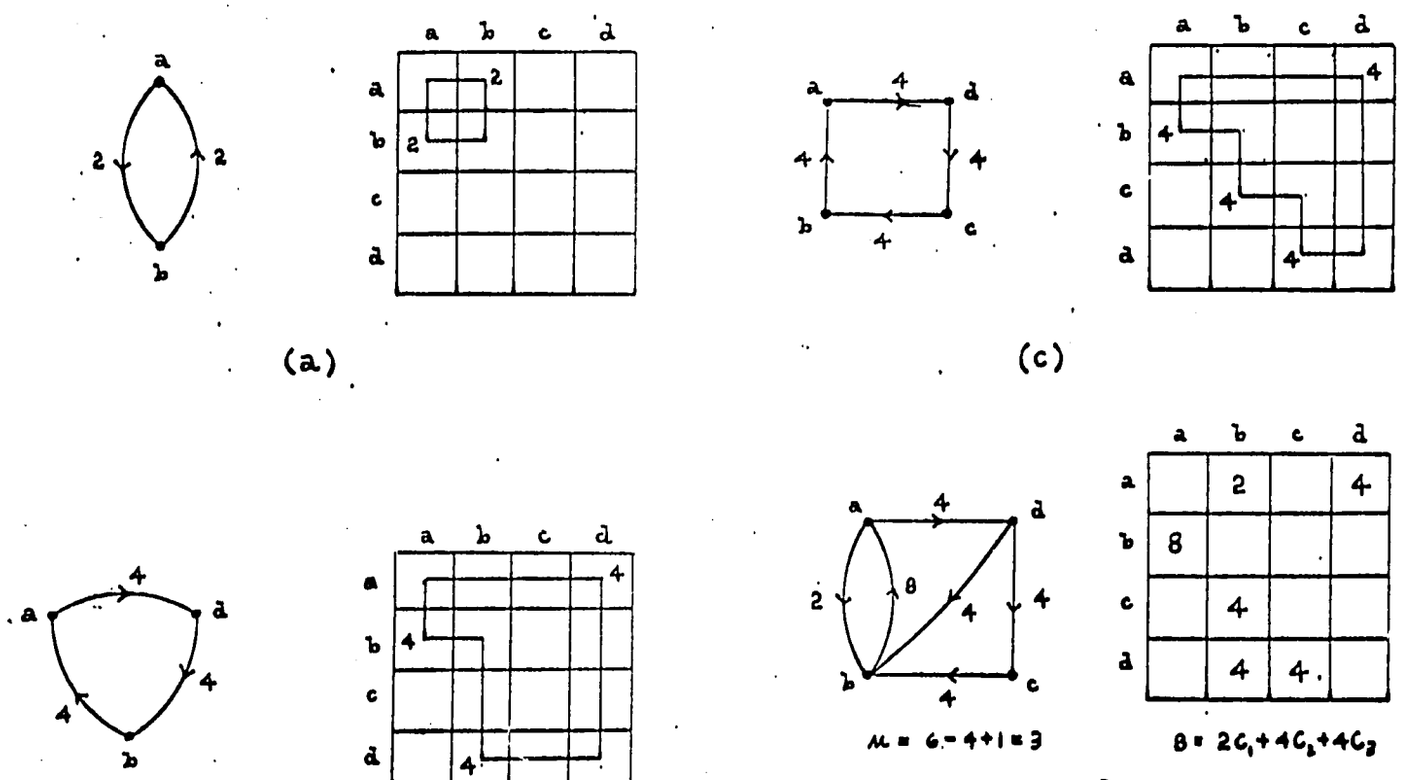
[6] Five length circuits are likely to be too cumbersome to be convenient for economic analysis.



(a) (b)

DIAGRAM V: CIRCUITS OF LINEAR GRAPH



(a) (c) (b) (d) (e)

DIAGRAM VI: CIRCULATION

TABLE III: ECONOMIC CIRCULATIONS IN OPEN, DUALISTIC ECONOMY (DIAGRAM IVa)

	CURRENT (Z excluded)				FINANCIAL (Z included)			
	Private (G excluded)		Public (G included)		Private		Public	
	Bilateral Exchange	Triangular Exchange	Bilateral Transfer	Triangular Transfer	Intra-sectoral	Inter-sectoral	Intra-sectoral	Inter-sectoral
DOMESTIC	$C_2 = (XY)$		$C_3 = (XG)$ $C_5 = (YG)$	$C_9 = (XYG)$ (XGY)	$C_3 = (Z_yY)$ $C_{13} = (XZY)$	$C_{14} = (ZY)$ (XZ_yY)	$C_{15} = (GZ_yY)$	(GZY)
FOREIGN	$C_{11} = (FX)$ $C_6 = (FY)$	$C_{12} = (FXY)$ (FYX)			$C_{16} = (FXZ)$ (ZFY)			

Quadrangular Transfers (4 edges): Current: (FXGY) (FYGX); and Financial: (ZFXY) (XGZF) (XGZY) (YGZF) (XGZ_yY)

private and public circulations, depending on whether circuits involve the government sector, G. Both private and public current circulations are further broken down between bilateral exchange and triangular exchange for private and bilateral transfers and triangular transfers for public. Bilateral exchange involves only two sectors while triangular exchange involves accommodation through a third sector. A similar interpretation holds for the distinction between bilateral and triangular transfers. For financial circulations, both public and private, we distinguish between intrasectoral and intersectoral finance. Intrasectoral finance represents circulations in which a sector's own real savings finance its own investment, while intersectoral finance involves transferring one sector's real savings to finance investment in another sector. Finally, all circulations are classified into foreign and domestic, depending on whether the foreign sector, F, is involved.

Only the relatively simple two and three length circulations are classified according to these principles in Table III. Four length circuits, referred to as quadrangular transfers, are listed at the bottom of the table and separated into current and financial.

In the first cell in Table III we see a current circulation $C_2 = (XY)$ which is also domestic, private, and bilateral. This circulation represents the important phenomenon of domestic commodity

trade between the two dualistic sectors of the economy--agriculture, X, and industry, Y. Referring to Diagram IV, we observe the flow y_x, representing agricultural purchases of commodities from industry, and including both intermediate goods (shown as N in Diagram I) and final consumer goods (shown as Y_L in Diagram I). These may conveniently be thought of as "fertilizer and cloth" purchases from industry by agriculture. We also observe a reverse flow of payments from industry to agriculture, x_y, representing industrial purchases of agricultural commodities, including both intermediate goods (R in Diagram I) and final consumer goods (X_h in Diagram I). These may be thought of as payments for "raw materials and food."

Other current domestic circulations shown are C_8 = (XG), C_5 = (YG), C_4 = (XYG) and (XGY). Both C_8 = (XG) and C_5 = (YG) are bilateral circulations, the former representing current flows between agriculture and government and the latter between industry and government. Since these are public circulations, not involving the principle of exchange, the bilateral aspect is referred to as bilateral transfers. In the case of C_8 (=XG), for example, there is a bilateral transfer of taxes from agriculture to government with the reverse flow representing government acquisition of real agricultural goods and services (X_g and G_L, respectively, in Diagram I) with these resources. A similar interpretation holds for C_5 (=YG), the industry-government circulation.

C_9 = (XYG) is an example of a triangular transfer involving agriculture, industry, and government. This circulation involves tax transfers from industry to government, government purchase of agricultural goods and services from these tax resources, and agricultural purchase of industrial goods to complete the circulation. This may be thought of as industry providing fertilizer and cloth to agriculture to compensate agriculture for its providing raw materials, food, and services to the government. The adjustment between industry and government is the industrial sector's transfer of money taxes enabling the government to purchase from agriculture. Essentially the same mechanism operates, mutatis mutandis, for the circulation shown as (XGY), with agriculture paying taxes to accommodate government purchase of industrial goods and industry receiving compensation in the form of agricultural goods.

Current circulations are also shown involving the foreign sector. The simplest, bilateral cases are C_{11}(FX) and C_6(FY) representing, respectively, trade between agriculture and the rest of the world and between industry and the rest of the world. In C_{11}, for example, in real terms agriculture exports in exchange for a reverse flow of imports to agriculture. The triangular exchange cases (C_{12} = FXY and FYX) are a bit more complicated. In C_{12} = (FXY), we find agriculture exporting

goods, a compensatory flow of goods from industry to agriculture and, finally, imports into the industrial sector to complete the circulation. It is this circulation, incidentally, which makes both domestic and foreign terms of trade relevant to the study of the distribution of gains from trade between agriculture and industry.

We turn now to the financial circulations shown in Table III. Two domestic, private, intrasectoral circulations are shown, $C_3 = (Z_y Y)$ and $C_{13} = (XZY)$. The former is interpreted as the industrial sector's own financing of industrial investment, representing a flow of industrial savings, S_h, to the finance sector and back to industry to purchase capital goods for industrial investment. The second, $C_{13} = (XZY)$ is interpreted as agricultural finance of investment in agriculture, representing a flow of agricultural savings, S_L, to the finance sector, and thence to industry to purchase capital goods for agricultural investment. This is construed as an intrasectoral circulation since agricultural savings are employed to finance agricultural investment.

Two domestic, private intersectoral financial circulations, $C_{14} = (ZY)$ and (X, Z_y, Y), are also shown. The former represents industrial finance of agricultural investment, I_x, and the latter, agricultural finance of industrial investment, I_y. These involve an original flow of savings from industry, S_h, and from agriculture, S_L, respectively.

Two domestic, public, financial circulations are shown, one intrasectoral and one intersectoral. $C_{15} = (GZ_yY)$ represents industrial savings forced by government taxation flowing to the government as tax revenues, thence from government to the finance sector as government savings, S_g, and finally to industry to finance industrial investment, I_y. This circulation is construed as <u>intrasectoral</u> finance since the industrial sector itself is forced to save to provide the real capital goods for investment. The other domestic, public, financial circulation, GZY, is construed to be <u>intersectoral</u> finance since government taxation forces savings upon the industrial sector; these savings are transferred through government to finance investment in agriculture, I_x, through a flow from finance to industry for purchase of capital goods to be added to capital stock in agriculture.

Finally, we observe two financial circulations, both private, which involve the foreign sector, $C_{16} = (FXZ)$ and (ZFX). Both are intrasectoral. The former represents agricultural exporting to finance capital export, the circulation flowing through agriculture in the form of agricultural savings, S_L, and finally through finance to the foreign sector in the form of capital export, A. The second foreign financial circulation involves the same process beginning from industrial exports.

The circulations of four lengths, representing quadrangular transfers, are shown at the bottom of Table III. They will not be discussed in detail, but it should be noted that each can be given economic interpretation as a complete and independent set of events--which may assume importance for analysis. For example, the financial circulation, ZFXY, represents a quadrangular pattern of capital export. In this circulation, the industrial sector saves (S_h), these savings flow to finance to accommodate capital export, the agricultural sector exports to provide foreign exchange, industry transfers real goods to agriculture in return, and industrial units acquire foreign assets. Another quadrangular example is the financial circulation, XGZF, which represents a quadrangular pattern among agriculture, government, finance, and the foreign sector. In this case, tax transfers from agriculture to government allow government to obtain foreign exchange from agricultural exports. These resources flow through finance as capital exports to allow government acquisition of foreign assets.

It should be emphasized that these circulations have been derived from the structure of the skeleton shown in Diagram IVa. Of all possible economic circulations, only a given number (29 in this case) can be identified from a particular skeleton. If, for example, the skeleton were changed to show an inflow of foreign savings rather than

capital export (as in Diagram IVa), it would be possible to trace circulations showing flows generated by foreign savings not included in the circulations discussed here. Such circulations are, in fact, used in Section III for analysis of the Philippine economy during certain years.

F. Decomposition Technique

In the previous section we have given the economic interpretation of circulations to circuits. An economic circulation, in addition to a qualitative aspect (represented by the circuit), has a quantitative aspect, i.e., the monetary value of the particular circulation. In Diagram VIa, b, c, for example, we show monetary values for three circulations: 2 dollars for C_1, 4 dollars for C_2, and 4 dollars for C_3. This quantitative aspect of a circulation is the coefficient of the circuit. More precisely:

Definition: A circulation is a valued linear graph obtained by assigning a fixed number "a" to every edge of a circuit. The number "a" is called the coefficient of the circulation.

A circulation is obviously an Euler graph and can be represented by a balanced table, each cell of which contains zero or "a" numbers. The balanced tables corresponding to the three circulations in Diagram VIa, b, c are shown on the same diagram. When these tables

are added (in the matrix sense), we obtain a balanced table B which is shown with the Euler graph interpretation in Diagram VId.

Since B is the sum of the circulations, it can be denoted by:

$$B = 2\,C_1 + 4\,C_2 + 4\,C_3$$

This can be generalized as follows:

Definition: A valued linear graph, B, is a <u>sum of circulations</u> if there exists circuit c_i and coefficient a_i ($i = 1, 2, \ldots . r$) so that:[7]

$$B = a_1c_1 + a_2c_2 + \ldots . + a_rc_r \text{ and}$$

B is <u>decomposable</u> into circulations with coefficients a_i ($i = 1, 2, \ldots . r$).

We observe that B is obviously an Euler graph when it is a sum of circulations. However, an Euler graph is not necessarily decomposable. For example, the Euler graph in Diagram VIe cannot be decomposed. In fact, the skeleton does not even contain a single circuit.

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[7] In matrix notation c_i is a square matrix containing "1" for a cell corresponding to an edge of a circuit and "0" elsewhere.

To investigate the phenomenon of decomposition we defined the cyclomatic number[8] for any directed linear graph:

Definition: Let G be any directed linear graph. The <u>cyclomatic number</u> of G is $u = E - V + 1$ where E is the number of edges and V is the number of vertices.

An example is given in Diagram VId where the number of edges is 6, the number of vertices 4, and the cyclomatic number is 3.

The cyclomatic number is an important concept in linear graph theory, with useful applications to our analysis of the open, dualistic economy. In a national income accounting skeleton, for example, since E is the number of variables and V-1 the number of independent accounting equations, u is the number of exogenous variables whose value can be arbitrarily assigned. The planning implications of this important deduction are discussed in a later section.

In Section IIC above, we pointed out that the skeleton of every legitimate national income accounting system is a cyclic net. For cyclic nets, we have the following "decomposition theorem":

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[8]Ordinarily this definition is applied to a linear graph which is <u>connected</u>, a concept discussed later.

Theorem: <u>If N is a cyclic net with a cyclomatic number, u, there exist u circuits C_i(i=1, 2, . . . u) in N such that any Euler graph, E, defined on N is uniquely decomposable into C_i, i.e., there exists a unique set of coefficients a_i (i=1, 2, . . . u) so that:</u>

$$E = a_1C_1 + a_2C_2 + \ldots\ldots a_uC_u$$

The decomposition theorem contains three parts. The first part asserts that there exists a set of u circuits, C_i, which may be identified from the skeleton. For later reference, these comprise a <u>set of basic circuits</u>. The second part asserts that any Euler graph defined on this cyclic net can be decomposed into these circuits. The third part asserts that the decomposition is unique; i e., there are no alternative sets of coefficients to the one specified. The theorem will be re-examined in Section IV and proved in the mathematical appendix. We shall also develop the actual procedure for decomposition in Section IV, where the more general applicability of our approach will be discussed.

III. ANALYSIS OF THE PHILIPPINE ECONOMY

The technique of decomposition can be applied to any type of economy at any level of aggregation. The application here is to the Philippine economy as an example of the open, dualistic type. The level of aggregation for our analysis in this section (shown in Diagram IV) emphasizes <u>intersectoral</u> relationships among agriculture, industry, government, finance, and the foreign sector.

We have shown above that there are 29 possible circulations which can be identified from the national income accounting skeleton of Diagram IV. This number is too large to provide a grasp of the dynamics of an economy. The decomposition theorem of the previous section offers a convenient device to reduce the number of circulations for analytical purposes. Since the cyclomatic number for the skeleton in Diagram IV is $u=12$, we know that there exists among the 29 circuits a set of 12 circuits (economic circulations) into which any national income accounting system defined on the skeleton can be uniquely decomposed.

To analyze the Euler graph showing the accounting flows for 1964 (presented in Table IVB), we proceed by identifying a set of 12 basic circuits,[9] and these are shown in Diagram VII, labelled C_2, C_3, C_5, C_6, C_8, C_9, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, and C_{16}. It will be noted that this is the same set of basic circuits classified in Table III, and the economic interpretation of each of these circulations has been given in Section IIE. We now proceed to derive the value (coefficient) of each of these 12 circulations by decomposing the Euler graph.[10] In Diagram VII we present the results of the decomposition procedure. To show the nature

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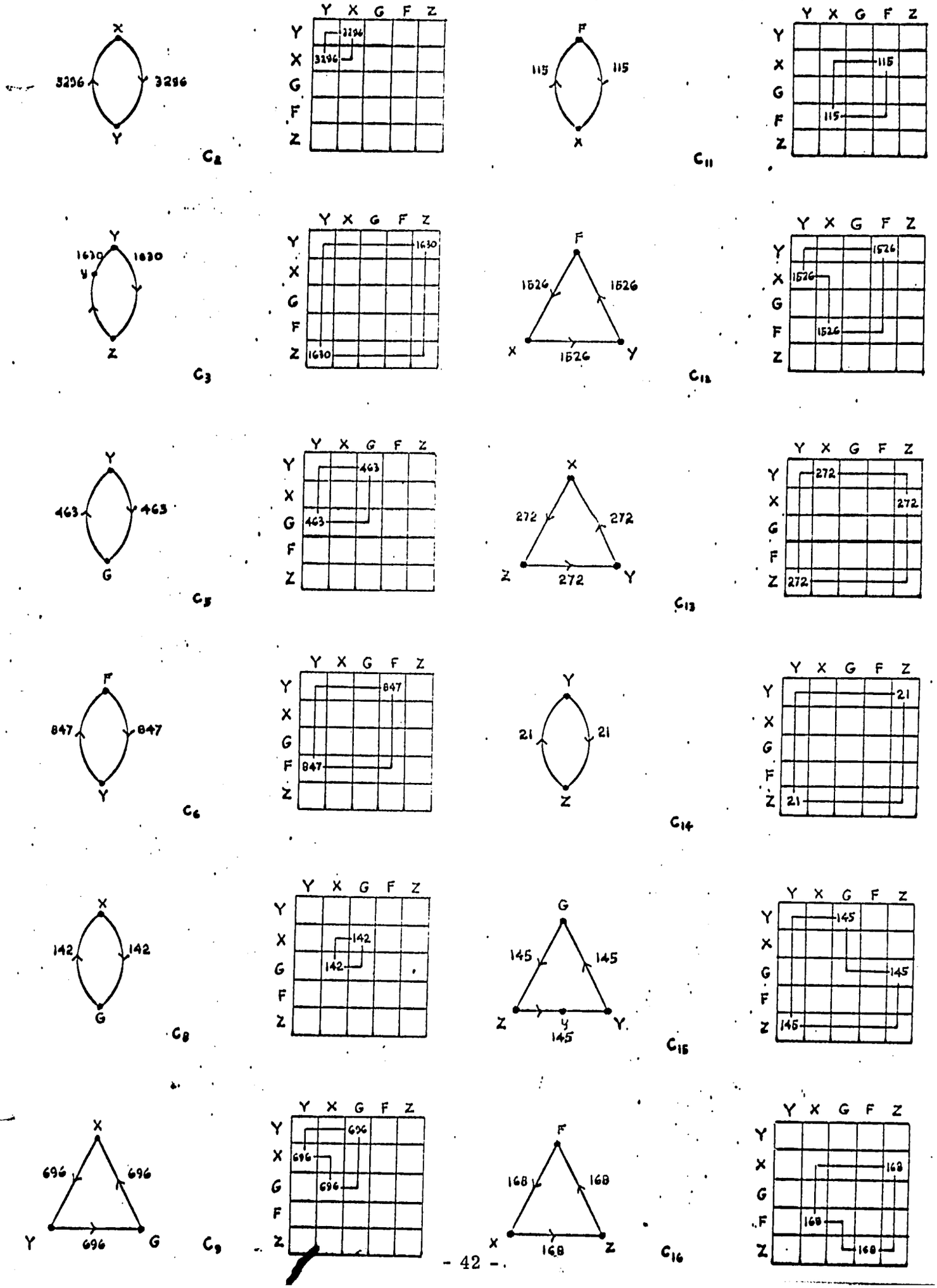
[9] Techniques for identifying a set of basic circuits are described in Section IV.

[10] The procedure for decomposing an Euler graph is presented in Section IV.

TABLE IV: CLASSIFICATION OF VALUED ECONOMIC CIRCULATIONS FOR THE PHILIPPINES: 1964
(in millions of pesos, 1955 prices)

	CURRENT				FINANCIAL				TOTAL
	Private		Public		Private		Public		
	Bilateral	Triangular	Bilateral	Triangular	Intra-sectoral	Inter-sectoral	Intra-sectoral	Inter-sectoral	
DOMESTIC	3,296 C_2		142 C_8 463 C_5	696 C_9	1,630 C_3 272 C_{13}	21 C_{14}	145 C_{15}		6,665
FOREIGN	115 C_{11} 847 C_6	1,526 C_{12}			168 C_{16}				2,656
Sub-total	5,784		1,301		2,091		145		9,321
Total: Current & Financial	7,085				2,236				

DIAGRAM VII: ECONOMIC CIRCULATIONS, PHILIPPINES, 1964



and value of each economic circulation, a valued circuit and balanced table are given. Each circulation now has a specific coefficient, uniquely determined according to the decomposition theorem. All values shown are in million pesos at 1955 prices.

Decomposition of the Euler graph of Diagram IVb has yielded results, therefore, which can be used to describe the economy by writing an equation of linear form:

$$E = 3296\ C_2 + 1630\ C_3 + 463\ C_5 + 847\ C_6 + 142\ C_8 + 696\ C_9 + 115\ C_{11} + 1526\ C_{12} + 272\ C_{13} + 21\ C_{14} + 145\ C_{15} + 168\ C_{16}$$

This allows us to visualize the economy's operation in the year analyzed (1964) in terms of these 12 circulations, each with a specific value. The dualistic and open features of the economy, the central characteristics of this type, can now be analyzed in quantitative and qualitative terms. To assist in interpreting the meaning of these results, we present in Table IV the set of basic circulations with their coefficients classified according to the system introduced in Table III.

We recall that the 12 circulations just classified represent <u>intersectoral</u> relationships. The totals shown in Table IV, therefore, represent the sum of intersectoral relationships of particular types as well as a grand total, T, for all intersectoral relationships described by

the 12 intersectoral circulations. Using these totals to describe the Philippine economy in 1964, we find, for example, that 24 per cent of all intersectoral circulations were financial in nature, showing considerable financial integration among sectors.[11] The other major breakdown in Table IV, domestic-foreign, is also of interest: approximately 29 per cent of all intersectoral flows involved the foreign sector, a relationship which may be used to measure the degree of openness of the economy.[12]

Another distinction, public versus private intersectoral relationships, is shown; public flows (those touching the government sector) represent about 16 per cent of the total. The government's share in financial relationships is particularly small, only about 6 per cent of their total. This represents the volume of government saving which, under our definition of the government sector, must be transferred to finance investment in either agriculture or industry (or, as a third possibility, used to accommodate capital export).

A unique aspect of the 1964 circulations, which will emerge more clearly from the inter-temporal comparisons below, is the accommodation of capital export by the Philippine economy. Circulation C_{16} in

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[11] The significance of these examples will be more apparent from the intertemporal comparisons presented below.

[12] Here, too, intertemporal and international comparisons would yield interesting conclusions.

Diagram VII and Table IV shows this phenomenon in the amount of P168 million. The export of capital is shown as a flow from the finance sector, Z, to the foreign sector, F. This capital transfer required both domestic saving and available foreign exchange. We see that both were accommodated through the agricultural sector. Agricultural exports provided the foreign exchange resources (a part of the large surplus of agricultural exports of 1,809 million over agricultural imports, 115 million). These resources were transferred to the finance sector in the form of agricultural savings. The necessity for agriculture to accommodate this outflow of capital, however, caused a shortfall in agricultural savings financing agricultural investment. Of total agricultural savings of P440 million, P168 million were diverted to accommodate the economy's capital export leaving P272 million for financing agricultural investment which totalled P293 million. This shortfall of P21 million led to another circulation, C_{14}, showing industrial savings accommodating agricultural investment. This circulation represents the only example of intersectoral finance in 1964, reflecting a transfer of real savings accomplished by the industrial sector to accommodate real capital accumulation in agriculture. This occurred despite the industrial sector's relatively large deficit (P124 million) in financing industrial investment (P1,775 million) from its own voluntary savings (P1,651 million). The adjustment is shown in

circulation C_{15} in which the entire amount of government savings (P145 million) was employed to cover the industrial sector's deficit in financing from voluntary savings its own investment and capital transfer to agriculture (a shortfall of P124 million for investment and P21 million for capital transfer). It will be noted in Table IV that this circulation is classified as intrasectoral finance. This is done since the real savings to accommodate this circulation were forced upon the industrial sector itself by government taxation. In the final analysis, therefore, the industrial sector covered slightly more (by P21 million) than its own investment (P1,775 million) through a combination of voluntary (P1,651 million) and forced saving (P145 million), while the agricultural sector provided a net outflow of savings of P147 million, the difference between the sector's saving and its investment. The adjustments shown by these circulations reveal, therefore, that the agricultural sector bore the brunt of the economy's capital export (P168 million) in 1964.

Inter-temporal Comparisons: The Philippine Economy

We now proceed to apply our techniques to analyze the changing structure of an open, dualistic economy over time. In addition to the results presented for the Philippine economy in 1964, three additional years (1949, 1953, and 1960) have been studied to give us a time span of 15 years.

We find, however, that inter-temporal comparisons present additional difficulties. In the Philippine case, several of the flows shown in Diagram IV reversed direction between 1949 and 1964. Examples include capital export, A', which did not appear until the 1960's. Prior to 1963 a reverse flow, A, capital import (foreign savings) existed. Similarly, sectoral savings for two of the three domestic sectors, agriculture and government, were negative in 1949 while our estimates for 1964 show positive savings for both sectors. Such reversing directions in the flows analyzed imply that economic circulations existing at one point in time do not exist at another.[13] Partly for this reason, we have found it necessary in applying our techniques to the four years to select different sets of basic circuits for different years. We know, however, that every set of basic circuits will comprise 12 circulations. This is true because the same skeleton (Diagram IVa) has been consistently employed for analysis of the four years. Hence, the number of edges and vertices remain the same, and the cyclomatic number is 12 in all cases.

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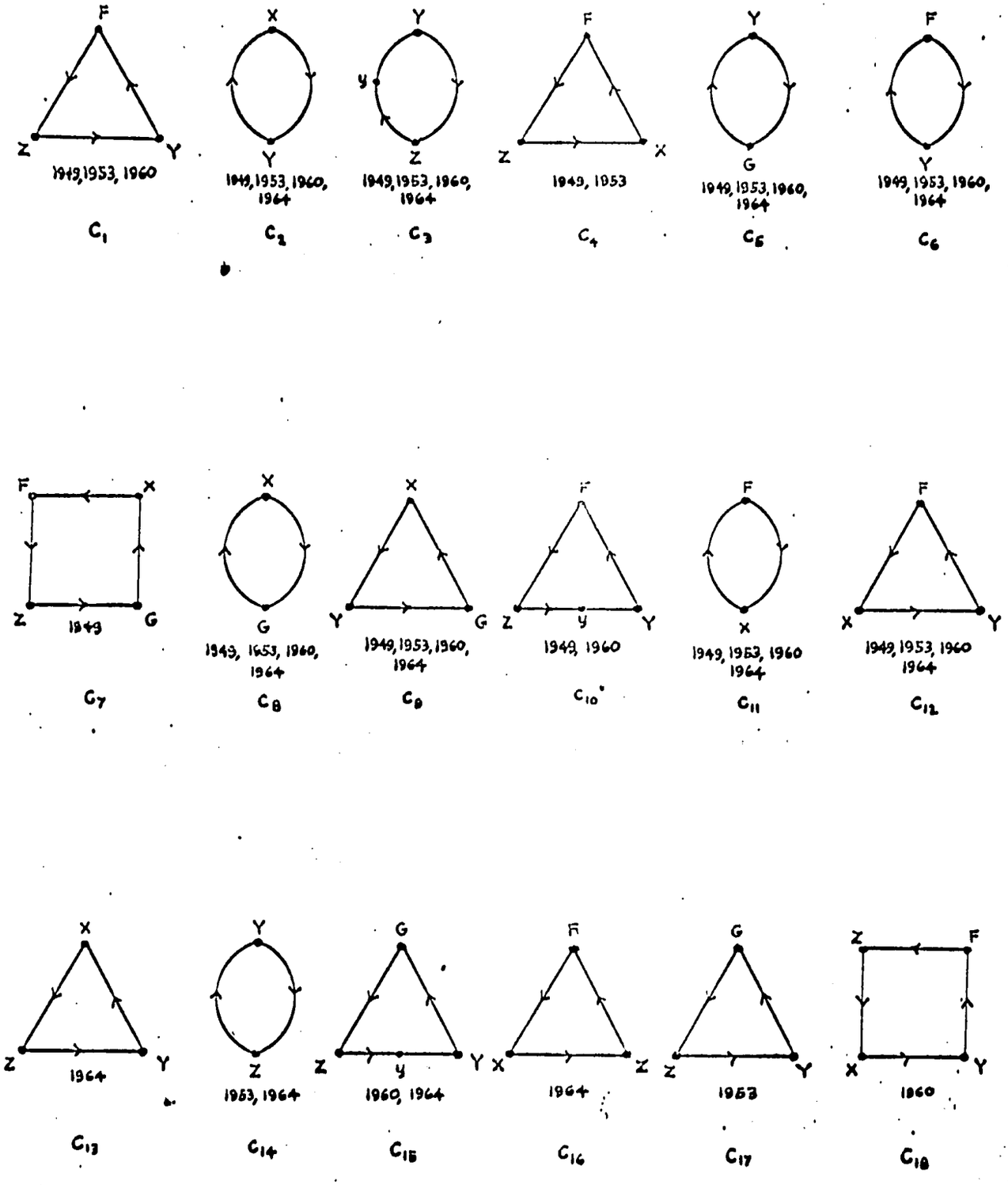
[13] This problem, leading to the necessity to shift bases, is more fully investigated in Section IV.

For each year analyzed, a set of 12 basic circuits could be found in accordance with the principles described in Section IV. In all, 18 circuits, C_1, C_2,.....C_{18}, were used for analysis of the four years. These circuits are presented in Diagram VIII, which shows the year(s) for which each circuit was employed in the decomposition analysis. The circuits not previously described are: C_1 = (FZY), C_4 = (FZX), C_7 = (FZGX), C_{10} = (FZyY), C_{17} = (GZY), and C_{18} = (ZXYF). It will be noted that all but one of these, C_{17} = (GZY), touch the foreign sector, F, and, hence, represent foreign circulations. In addition, they <u>all</u> touch the finance sector, Z. This gives some insight into the reason for their relevance to the years prior to 1964; foreign savings played a role in financing the economy's investment (as well as dissaving by some sectors) in the three years, 1949, 1953, and 1960. This condition had been reversed by 1964, when capital was exported and all three domestic sectors showed positive savings.

Circulation C_1 = (FZY) which was used for 1949, 1953, and 1960 represents foreign savings, A, financing agricultural investment, I_x, with an accommodating flow of industrial payments for imports. Circulation C_4 = (FZX), used for 1949 and 1953, shows foreign savings, A, financing dissaving by the agricultural sector, S_L', allowing that sector to purchase foreign goods. Circulation C_7 = (FGZX), which is

DIAGRAM VIII: CIRCUITS USED FOR ANALYSIS OF PHILIPPINE ECONOMY, 1949, 1953, 1960, 1964



quadrangular, represents a relatively complex phenomenon existing in only 1949, i.e., foreign savings, A, financed government sector dissaving, S_g', allowing the government sector to purchase domestic agricultural output, with an accommodating flow compensating the agricultural sector through import of goods. Circulation C_{10} = (FZY), used for 1949 and 1960, represents foreign savings, A, financing industrial investment, I_y, with accommodation through imports into the industrial sector. Circulation C_{17}, used only for 1953, represents government sector savings, S_g, from industrial sector tax payments, T_h, financing agricultural investment, I_x. This implies forced saving on the industrial sector for an intersectoral transfer of finance. Finally, Circulation C_{18} = (ZXYF), used for 1960, shows a quadrangular relationship with foreign savings, A, financing agricultural dissaving, S_L', allowing the agricultural sector to purchase domestic industrial output and providing compensation to the industrial sector through an accommodating flow of industrial sector payments for imports.

The results of the decomposition analysis using the basic circuits described are presented in the classification tables V (1949), VI (1953), VII (1960), and VIII (1964). The table for each year shows the values of the 12 basic economic circulations which were employed to decompose uniquely all intersectoral flows for that particular year. We now have a basis to analyze inter-temporal structural change in the Philippine economy, 1949-1964, by using the four classification tables.

TABLE V: CLASSIFICATION OF VALUED ECONOMIC CIRCULATIONS FOR THE PHILIPPINES: 1949
(in millions of pesos, 1955 prices)

	CURRENT				FINANCIAL				Total 3,729	Ratio to GNP .6673
	Private		Public		Private		Public			
	Bilateral	Triangular	Bilateral	Triangular	Intra-sectoral	Inter-sectoral	Intra-sectoral	Inter-sectoral		
DOMESTIC	1,876 C_2		43 C_8 179 C_5	153 C_9	153 C_3				2,404	.4302
FOREIGN	320 C_{11} 246 C_6	236 C_{12}				34 C_4 292 C_{10} 162 C_1 35 C_7			1,325	.2371
Total	2,678		375		676		0		3,729	
Ratio to GNP	.4792		.0671		.1210				.6673	

$GNP_{49} = 5,588$

TABLE VI: CLASSIFICATION OF VALUED ECONOMIC CIRCULATIONS FOR THE PHILIPPINES: 1953
(in millions of pesos, 1955 prices)

	CURRENT				FINANCIAL				Total 4,580	Ratio to GNP .6198
	Private		Public		Private		Public			
	Bilateral	Triangular	Bilateral	Triangular	Intra-sectoral	Inter-sectoral	Intra-sectoral	Inter-sectoral		
DOMESTIC	2,291 C_2		241 C_5 73 C_8	269 C_9	431 C_3	53 C_{14}		72 C_{17}	3,430	.4642
FOREIGN	287 C_6 108 C_{11}	659 C_{12}				52 C_1 44 C_4			1,150	.1556
Total	3,345		583		580		72		4,580	
Ratio to GNP	.4527		.0789		.0785		.0097		.6198	

$GNP_{53} = 7,389$

TABLE VII: CLASSIFICATION OF VALUED ECONOMIC CIRCULATIONS FOR THE PHILIPPINES: 1960
(in millions of pesos, 1955 prices)

	CURRENT				FINANCIAL				Total 7,103	Ratio to GNP .6877
	Private		Public		Private		Public			
	Bilateral	Triangular	Bilateral	Triangular	Intra-sectoral	Inter-sectoral	Intra-sectoral	Inter-sectoral		
DOMESTIC	3,108 C_2		350 C_5 108 C_8	471 C_9	659 C_3		150 C_{15}		4,846	.4692
FOREIGN	514 C_6 94 C_{11}	1,406 C_{12}				199 C_1 21 C_{10} 23 C_{18}			2,257	.2185
Total	5,122		929		902		150		7,103	
Ratio to GNP	.4959		.0899		.0874		.0145		.6877	

GNP_{60} = 10,329

TABLE VIII: CLASSIFICATION OF VALUED ECONOMIC CIRCULATIONS FOR THE PHILIPPINES: 1964
(in millions of pesos, 1955 prices)

	CURRENT				FINANCIAL				Total 9,321	Ratio to GNP .7407
	Private		Public		Private		Public			
	Bilateral	Triangular	Bilateral	Triangular	Intra-sectoral	Inter-sectoral	Intra-sectoral	Inter-sectoral		
DOMESTIC	3,296 C_2		142 C_8 463 C_5	696 C_9	1,630 C_3 272 C_{13}	21 C_{14}	145 C_{15}		6,665	.5296
FOREIGN	115 C_{11} 847 C_6	1,526 C_{12}			168 C_{16}				2,656	.2111
Total	5,784		1,301		2,091		145		9,321	
Ratio to GNP	.4596		.1034		.1662		.0115		.7407	

GNP_{64} = 12,584

Extensive analysis of inter-temporal changes will be undertaken in later papers when the entire time series for the period 1949-1964 has been decomposed. In this paper, discussion is limited to a few conclusions suggested by the results for the four benchmark years studied here. To assist in the analysis, several types of circulation (foreign-domestic, current-financial, and private-public) have been totalled for each of the years. We have then computed the ratio of each type of circulation to GNP. The results of these operations for each of the years studied are shown in Tables V through VIII.

Turning first to the financial circulations, we see a general trend toward greater financial integration in the economy reflected in a rising ratio of financial circulations to GNP. We note first, however, that the year 1949 was untypical; virtually all financial circulations involved inflows of foreign savings. Foreign savings, in fact, represented about 9 per cent of GNP and P523 million of total financial circulations of P676 million. This reflected the fact that in 1949 the Philippine economy was still in the throes of rehabilitation and was heavily dependent on foreign savings for financing of both investment and consumption. Domestic financial circulations, therefore, represented only 2.7 per cent of GNP. By 1953 this situation had changed dramatically, with domestic financial circulations rising to P556 million of a total of

P652 million and representing 7.5 per cent of GNP, compared to foreign financial circulations equivalent to 1.3 per cent of GNP. The ratio of domestic financial circulations to GNP rose slightly above this figure (to 7.8 per cent) in 1960 and showed considerable improvement by 1964 when it rose to 16 per cent. Moreover, by 1964, the one remaining foreign financial circulation, represented capital export rather than foreign savings.

Public financial circulations, however, have been consistently small. They were non-existent in 1949 and in the three other years hovered near 1 per cent of GNP. Hence, progress toward financial integration of the economy has been primarily a matter of private sector activity.

The current circulations reflect non-financial intersectoral relationships, i.e., exchange of current goods and services. The startling conclusion emerging from the results in this case is that, unlike financial circulations, current circulations have remained relatively unchanged as a fraction of GNP. Private current circulations showed a ratio of 48 per cent of GNP in 1949, 45 per cent in 1953, 50 per cent in 1960, and 46 per cent in 1964. At first glance, one hypothesizes, therefore, that private intersectoral linkages to overcome the basic problem of domestic dualism have not been strengthened over the

15-year period, 1949-1964. This conclusion becomes even more apparent when we study the domestic-foreign breakdown in current circulations. Private domestic current circulations represented 34 per cent of GNP in 1949, 31 per cent in 1953, 30 per cent in 1960, and 26 per cent in 1964. For the predominantly private sector economy of the Philippines, these results show a surprising lack of progress in expanding private current intersectoral relationships in the domestic economy.

While private domestic current circulations have decreased, however, private foreign current circulations have shown a rapid rise relative to GNP. Equivalent to about 14 per cent of GNP in both 1949 and 1953, these circulations rose to approximately 20 per cent of GNP in the two later years, 1960 and 1964. This suggests that current integration with the foreign sector is rapidly outpacing current integration within the domestic economy. This is indeed a central characteristic of the development of the Philippine open, dualistic type economy during the fifteen-year period.

Some slight progress has been achieved in raising the level of public current circulations. At the beginning of the period studied, in 1949, the ratio of public current circulations to GNP was about 7 per cent, rising to 8 per cent in 1953, 9 per cent in 1960, and 10 per cent in

1964. It should be pointed out, however, that the growth of this type of current circulation is unlikely to have any direct effect on alleviating the basic problem of domestic dualism between agriculture and industry. In a basically private enterprise economy, this problem must be solved primarily by strengthening the direct linkages between industry and agriculture.

To provide a general index of progress toward intersectoral integration, we present in Diagram IX a graphic representation of the ratios of various types of intersectoral circulations to GNP for the four years studied. Total circulations may be construed as a measure of the integration ratio for the economy as a whole. This measure is shown as comprising four broad types of economic circulations: private current, private financial, public current, and public financial. The first two, private current and private financial, represent a measure of private sector integration while the latter two, public current and public financial, represent a measure of integration between the government and the private sector. Private current circulations are separated into their domestic and foreign components.

The major conclusions stated may be seen by visual inspection of Diagram IX. The total integration ratio of the economy has risen gently from below .70 in 1949 to slightly above .70 in 1964. The break shown for

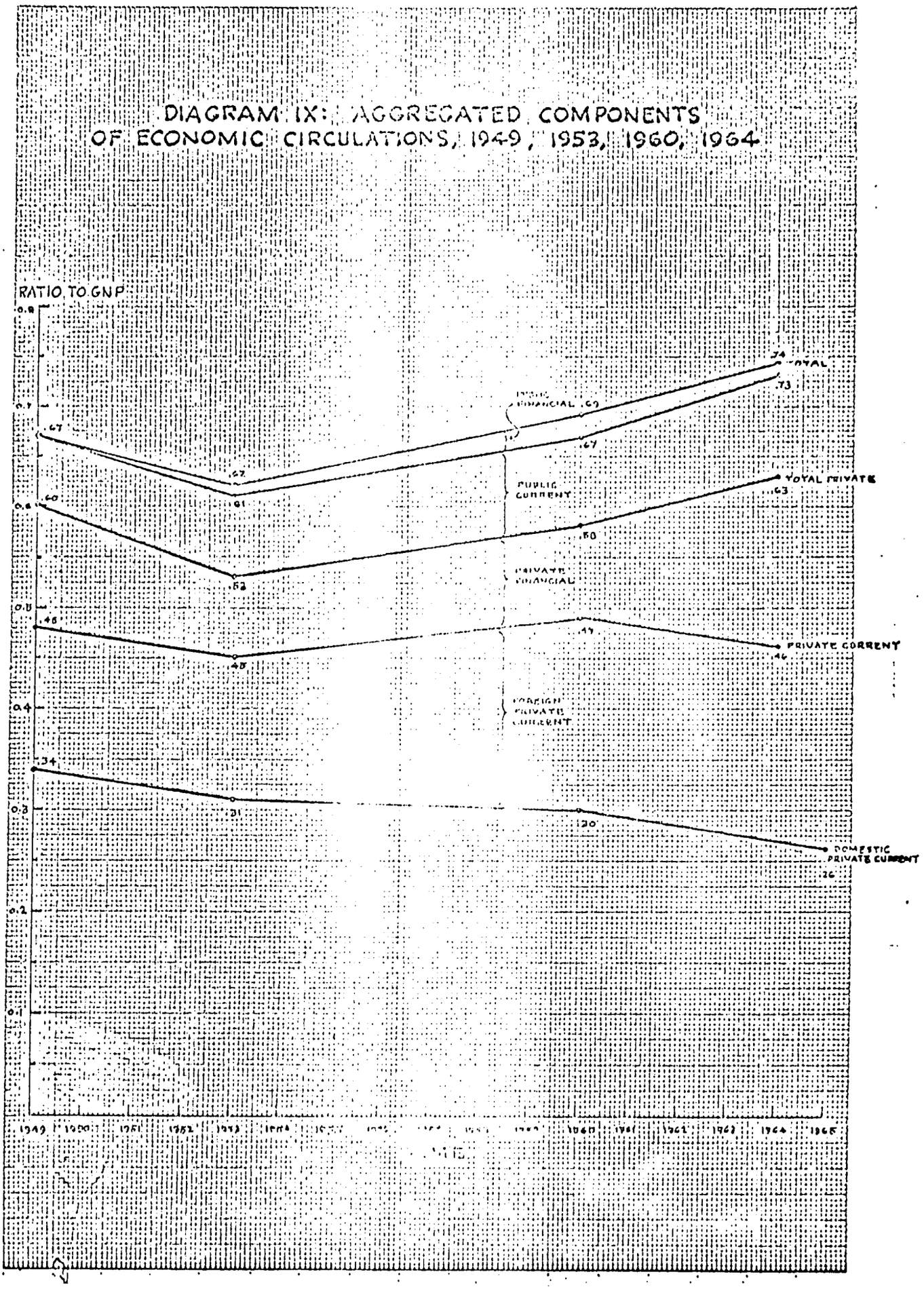


DIAGRAM IX: AGGREGATED COMPONENTS OF ECONOMIC CIRCULATIONS, 1949, 1953, 1960, 1964
RATIO TO GNP
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
.67
.67
.67
.67
.60
.52
.50
.63
.48
.45
.49
.46
.34
.31
.30
.26
.74
.73
TOTAL
PUBLIC FINANCIAL
PUBLIC CURRENT
TOTAL PRIVATE
PRIVATE FINANCIAL
PRIVATE CURRENT
FOREIGN PRIVATE CURRENT
DOMESTIC PRIVATE CURRENT
1949
1950
1951
1952
1953
1960
1961
1962
1964
1965

1953 is explained by a sharp reduction in private financial circulations between 1949 and 1953, caused by a rapidly declining rate of foreign savings inflows. The gradual rise in the aggregate ratio between 1953 and 1964 was caused primarily by a combination of growing private financial circulations (particularly after 1960) and public current circulations. Between 1960 and 1964, these circulations offset a decline in private current circulations.

It is also apparent that the decline in private current circulations is explained by the consistently falling domestic component of these circulations. Domestic private circulations fell by 8 percentage points (relative to GNP), offsetting the significant rise in private financial circulations.

IV. LINEAR GRAPH THEORY AND COMPARATIVE TYPOLOGY

In the preceding discussion we have employed linear graph techniques to analyze intersectoral relationships in a particular open, dualistic economy--the Philippines. In this section, we investigate the principles under which these techniques can be applied to other types of economies at various levels of aggregation. Mathematical proofs of the basic propositions in the argument are presented in the mathematical appendix.

The procedure begins with acceptance of a national income accounting flow diagram (e.g., Diagram I) as the quantitative framework for the analysis. Such a specific framework is viewed as a special case of a more general and systematically defined framework involving a large number of basic sectors. Table IX presents this general accounting framework in a square table of the input-output variety. To illustrate the general principles involved, the table includes three domestic sectors (1, 2, and 3), a foreign sector, F, and a finance sector, Z. Each domestic sector (1, 2, and 3) has three components, each representing a particular aspect of the sector's functioning: (1) a production aspect (a, b, c); (2) an income disposition aspect (d, e, f); and (3) an investment aspect (g, h, i). This breakdown yields 11 columns and rows in the square table.

The first three rows (a, b, c) contain the familiar accounting variables found in a dynamic input-output system: x_{ij} is the inter-industry payment (from the j-th to the i-th sector) on current account while C_{ij}, I_{ij}, and E_i are the three components of final demand for i-th sector output representing, respectively, consumption, investment (from the j-th sector), and export. The row sums X_i are the total outputs of the i-th sector.

TABLE IX: NATIONAL INCOME ACCOUNTING TABLE

		DOMESTIC									Foreign	Finance	
		Production			Income Disposition			Investment					
		1	2	3	1	2	3	1	2	3	F	Z	Total
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	
1	(a)	x_{11}	x_{12}	x_{13}	c_{11}	c_{12}	c_{13}	I_{11}	I_{12}	I_{13}	E_1		X_1
2	(b)	x_{21}	x_{22}	x_{23}	c_{21}	c_{22}	c_{23}	I_{21}	I_{22}	I_{23}	E_2		X_2
3	(c)	x_{31}	x_{32}	x_{33}	c_{31}	c_{32}	c_{33}	I_{31}	I_{32}	I_{33}	E_3		X_3
1	(d)	v_{11}	v_{12}	v_{13}	t_{11}	t_{12}	t_{13}						Y_1
2	(e)	v_{21}	v_{22}	v_{23}	t_{21}	t_{22}	t_{23}						Y_2
3	(f)	v_{31}	v_{32}	v_{33}	t_{31}	t_{32}	t_{33}						Y_3
1	(g)	d_1										I_1	
2	(h)		d_2									I_2	
3	(i)			d_3								I_3	
F	(j)	M_1	M_2	M_3									M
Z	(k)				S_1	S_2	S_3				A		
Total								I'_1	I'_2	I'_3			

The first three columns (a, b, c) represent costs of production. They include current intermediary goods costs (x_{ij}), costs of imported goods (M_i), capital consumption allowances (d_i), and value added (v_{ij}), constituting primary factor payments (wages, rent, interest, profit) from the i-th production sector to the j-th households. The first three sectors are in balance since v_{ij} includes profits which are defined to balance "cost and revenues."

In the next three sectors, (d, e, f), t_{ij} represents unilateral transfer payments from the j-th households to the i-th household. These payments assume particular importance when one of the domestic sectors is government, in which case t_{ij} includes such items as taxes, social security contributions and payments, interest on public debt, and subsidies. The sum of a row, Y_i, is the personal income of the i-th sector (i.e., the sum of value added income and transfer income). Savings (S_i) is defined as the balancing item for these sectors and is interpreted as the difference between personal income and expenditures of both the consumption and transfer payment types.

For the next three sectors (g, h, i) the column I_i' is gross investment in the i-th sector. Net investment (I_i) is defined to balance these sectors; i.e., $I_i = I_i' - d_i$ where d_i represents capital consumption allowances in the i-th sector.

In the foreign sector (F), capital inflow (A) is defined to balance total imports (M) and total exports. Thus, the first ten sectors (a to j) are in balance by definition. The last sector is also in balance (total investment (I) equals total savings plus capital import) by the balance theorem.

This national income accounting system is a general framework, including not only the variables from the dynamic input-output system but also those emphasized in the typical Keynesian system. Variables of the dynamic input-output type are X_{ij}, C_{ij}, I_{ij}, E_i, and M_i, while Keynesian variables include income payments (v_{ij}), transfer payments (t_{ij}), savings (S_i), and gross, as well as net, investment (I_i' and I_i).

The national income accounting framework for the open, dualistic economy is only a special case of this general system. It is derived by making the following specific assumptions:

(1) Industry, agriculture, and government are identified as sectors 1, 2, and 3, respectively.

(2) All encircled entries in row c are deleted, and C_{33} represents total government expenditure ($G = X_3$), construed as a measure of total government output in accord with current accounting convention.

(3) All encircled entries in row b are deleted, showing that we assume that agriculture does not produce investment goods. Deletions in (2) above also exclude government from production of investment goods, leaving us with the special assumption that only the industrial sector produces investment goods.

(4) The encircled v_{ij} items are deleted by our special assumptions that there are no intersectoral income payments between the two private sectors and that the government sector does not supply primary factors of production.

(5) The encircled t_{ij} entries are deleted on the basis of our special assumption that private transfer payments are negligible.

(6) All d_i items are neglected by our assumption ignoring capital consumption allowances.

Thus, the general accounting system presented in this section allows us to clearly identify all of the special assumptions we have made to study the structure of the open, dualistic economy. The general system obviously provides for construction of more complicated frameworks at all possible levels of aggregation and is adequate for applying the typology approach to all possible economies.[14]

[14]As pointed out in the text, such systems as input-output and Keynesian are special cases of our system. In particular, when (3) is interpreted as the government sector, we obtain an aggregated table representing the most familiar national income accounting system, i.e.: (ab)(de)(c)(f)(ghi)(j)(k). In our analysis of the open, dualistic economy, the aggregation used is (ad) (be) (cf) (ghi) (j) (k).

Positive Balanced Table

In an earlier section, it was shown that a national income accounting system can be put into the form of a balanced table. In order to apply such a system to the study of long-run economic growth, we make the further assumption that a national income accounting system is a positive balanced table; i.e., that all entries are positive. This assumption is justified by three considerations:

(1) Very few entries can take on negative values intrinsically. (In Table IV, for example, the only items which can be negative are the balancing items.)

(2) When an economy is growing over the long-run, a negative value is not likely to occur indefinitely.

(3) A negative value (-x) in a cell, or on an edge (x, y), can be replaced by a positive value (x) to form a positive balanced table.

For these reasons, we define a national income accounting structure abstractly, as follows:

Definition: A national income accounting structure is a linear graph, G, on which a strictly positive Euler graph can be defined (i.e., strictly positive numbers can be assigned to every edge of G to form an Euler graph).

We now state a theorem to be proved in the appendix:[15]

---

[15] The definition of "connected" is given below.

Theorem 1: <u>A connected linear graph is a national income accounting structure if and only if it is a cyclic net.</u>

It is precisely this reason that leads us to investigate the special technique for decomposition of an Euler graph defined on a cyclic net.

Exogenous Versus Endogenous Variables

When a skeleton of an accounting structure is represented by a linear graph with a cyclomatic number, u, the total number of variables (edges) exceed the total number of independent accounting equations (i.e., V-1) by u. This suggests that we can choose u variables as exogenous, leaving the remaining variables ($n-u=V-1$) as endogenous variables. When the values of the exogenous variables are given arbitrarily, therefore, the endogenous variables can be determined residually by the accounting equations. This knowledge is of special relevance for <u>consistent planning</u> for economic development--in essence requiring acceptance of a specific accounting framework (skeleton) and planning all variables to form a balanced table.

It is not true, given an accounting skeleton, that any set of u edges can be arbitrarily chosen to form a set of exogenous variables. For example, in Diagram Xa where $u=2$, the set of edges (E_1, E_2) cannot

be a set of exogenous variables because the value of E_1 is always equal to the value of E_2. Hence, two numbers cannot be arbitrarily assigned to them. Similarly, in Diagram Xb, E_1 and E_2 cannot both be exogenous variables at the same time (see below). Thus, only certain sets of u edges, to be called a basic edge set, can qualify as a set of exogenous variables.

Understanding the properties of basic edge sets is crucial both for our decomposition technique and the planning applications referred to above. The following linear graph definitions are presented to analyze the basic edge sets.

Definition: A set of directed edges on a linear graph form a link between x and y if they form a path from x to y after the direction of some of these edges is reversed.

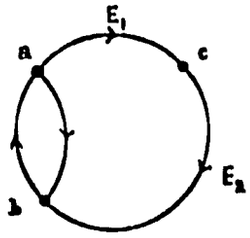
An example of a link is given in Diagram Xc where a path can be formed by reversing the direction of the edge (dc) to (cd).

Definition: A set of directed edges on a linear graph form a circle if they form a circuit after the direction of some of the edges of the circle is reversed.

An example of a circle is given in Diagram Xd where a circuit can be formed by reversing the direction of the edge (dc) to (cd).

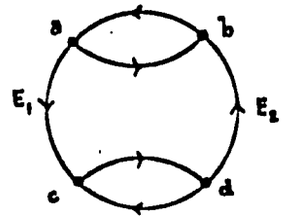
We note that a path is a special case of a link and a circuit is a special case of a circle.

DIAGRAM X: LINK, CIRCLE & CONNECTED LINEAR GRAPH



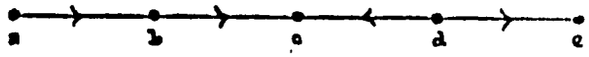
$\mu = 2$

(a)



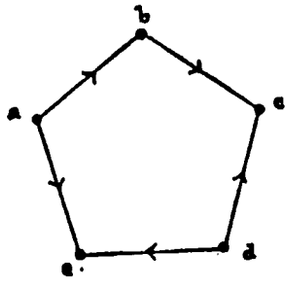
$\mu = 3$

(b)



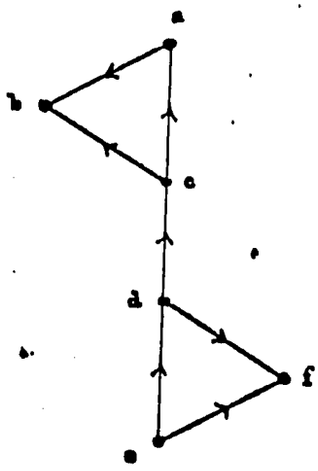
(c)

LINK a TO e



(d)

CIRCLE



(e)

CONNECTED

(f)

CIRCLE FREE

Definition: A linear graph is <u>connected</u> if for every pair of vertices (x, y) there is a link between x and y. (See Diagram Xe)
A linear graph is <u>circle-free</u> if it has no circle. (See Diagram Xf.)
A linear graph is a <u>tree</u> if it is circle-free and connected.

Understanding the concept of a tree is essential to our analysis. If a tree has any edge at all, it has at least one <u>end vertex</u>, i.e., a vertex touched by only one edge, known as an <u>end edge</u>. When the end vertices and end edges are removed from a tree, what remains is still a tree. In Diagram XIa, the end vertices and end edges are, respectively, a, b, c, d, and T^1_a, T^1_b, T^1_c, T^1_d. When these are removed, we have the tree shown in Diagram XIb. The end vertices and end edges of this tree are, respectively, e, f, g, and T^2_e, T^2_f, T^2_g, which, when removed, leave the tree shown in Diagram XIc. This process can be repeated until finally only one edge (or one vertex) is left.

Symbolically, the end edges removed successively may be assigned a <u>causal order</u>, as follows:

Causal order 1: T^1_a, T^1_b, T^1_c, T^1_d (end edges removed in first step)

Causal order 2: T^2_e, T^2_f, T^2_g (end edges removed in second step)

Causal order 3: T^3_h, T^3_i (end edges removed in third step)

Causal order 4: T^4_j (end edge removed in fourth step)

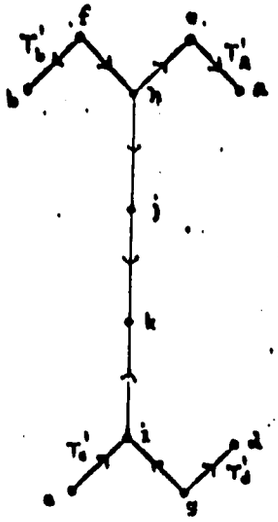
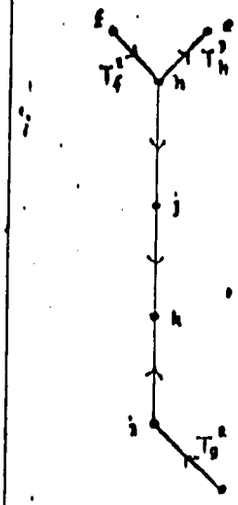
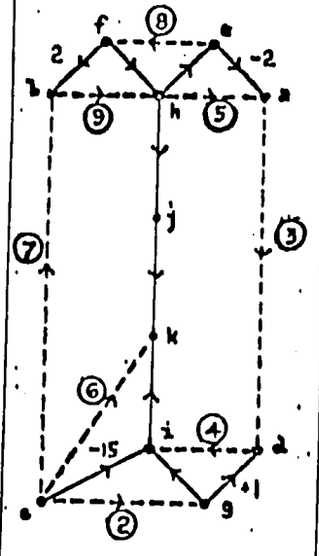
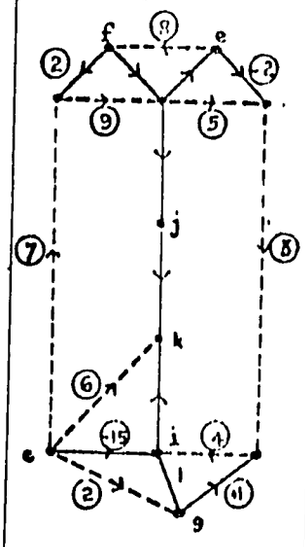
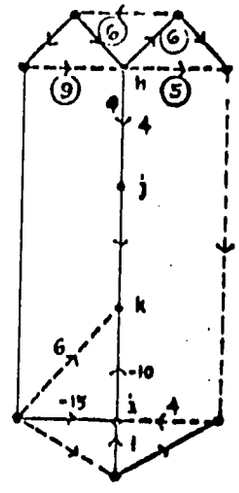
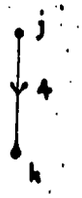
	(a)	(b)	(c)	(d)
TREE				
END VERTICES	a, b, c, d	e, f, g	h, i	j, k
END EDGES	T^1_a, T^1_b, T^1_c, T^1_d	T^2_e, T^2_f, T^2_g	T^3_h, T^3_i	T^4_j
MAXIMUM TREE (solid) AND BASIC EDGE SET (dotted)	(a')  $\mu = 10 - 11 + 1 = 8$	(b') 	(c') 	(d') 
EDGE VALUE DETERMINED	T^1_a (-2) T^1_b (2) T^1_c (-15) T^1_d (1)	T^2_e (6) T^2_f (6) T^2 (1)	T^3_h (3) T^3_i (-8)	T^4_j (4)

DIAGRAM XII DETERMINATION OF ENDOGENOUS VARIABLES BY CAUSAL ORDER

When any tree is given, a causal order can be assigned to every edge in this way.

A connected linear graph which is not a tree will contain some sub-graphs which are trees. The following definition emphasizes this phenomenon.

Definition: A sub-set of edges, T, of a linear graph, G, is a <u>maximum tree</u> if T is a tree which touches every vertex of G. The edges of G not in T form a <u>basic edge set</u>.

In Diagram XIa', the solid edges form a maximum tree (in fact, the same tree shown in Diagram XIa for which a causal order analysis has been made above). All vertices, a to k, are touched by edges in this tree. We observe that the cyclomatic number of this linear graph, $u=8$, is equal to the number of basic (dotted) edges. This is an example of the following theorem:

Theorem 2: If a connected linear graph, G, has V vertices and a cyclomatic number u, every maximum tree of G has V-1 edges and, hence, every basic edge set has u edges.

Returning to the planning application, let G be a linear graph with E edges corresponding to E planning variables. We present the following definition:

Definition: A set of variables (edges) of a linear graph, G, form a set of <u>exogenous variables</u> and the other edges form a set of <u>endogenous variables</u> if, after arbitrary values are assigned to all exogenous variables, unique values can be assigned to all endogenous variables to form an Euler graph.

With this background, we can state the following theorem:

Theorem 3: <u>Let a connected linear graph G be given. A set of edges of G form a set of exogenous variables if, and only if, they form a basic edge set (i.e., if and only if the endogenous variables form a maximum tree of G).</u>

We discuss the sufficient condition of this theorem by using an example while the necessary condition is given in the mathematical appendix.

The dotted edges in Diagram XIa' are a set of basic edges, i.e., the exogenous variables. Let values be assigned to these variables as indicated by the encircled numbers. We now seek to determine the values of the endogenous variables (the solid edges which form a maximum tree), according to the causal order shown in Diagram XIa, b, c, d.

We begin with edge T_a^1 in causal order 1. T_a^1 is the <u>only</u> endogenous variable touching the end vertex "a." By requiring that vertex "a" be in balance, the value of T_a^1 is uniquely determined as -2(=3-5). In this way, the values of all edges in causal order 1 (T_a^1, T_b^1, T_c^1, T_d^1) can be determined, as indicated at the bottom of Diagram XIa'.

We then proceed to causal order 2 (shown in Diagram XIb). All endogenous variables in this order (T_e^2, T_f^2, T_g^2) are determined by the same principle; i.e., of all the endogenous variables of causal order 2 or higher, each T_i^2 is the only endogenous variable touching the i-th end vertex. Proceeding through each causal order in this way, we can determine uniquely the values of all endogenous variables in r steps, where r is the maximum causal order. We can be certain that an Euler graph is formed since the endogenous variables form a maximum tree and, hence, <u>every</u> vertex is in balance.

The theorem stated above implies that every set of exogenous variables contains u variables, the cyclomatic number. It also explains why not all edge sets qualify as exogenous variables. Referring back to Diagram Xa, we now see that the set of (E_1, E_2) edges (though equal to the cyclomatic number) is not a set of exogenous variables because the other edges do not form a maximum tree. Similarly, in Diagram Xb, the two edges (E_1, E_2) cannot belong to any set of exogenous variables since the remaining variables would be disconnected.

Decomposition Procedure

We have referred earlier to the concept of a <u>set of basic circuits</u> which we now define precisely:

Definition: Let G be a directed linear graph with cyclomatic number u. A set of circuits $C_i (i=1, 2, 3 \ldots u)$ is a <u>set of basic circuits</u> if there exists a basic edge set $E_i (i=1, 2 \ldots u)$ such that E_i is included in C_i and is not included in C_j if $j > i$.

Thus, a set of basic circuits is defined as having a particular relationship to a given basic edge set. We use Diagram V to exemplify this relationship. Diagram Va is a linear graph with a cyclomatic number of 4. In Vb, a number of circuits of the linear are shown, of which the set C_1, C_2, C_3, C_4 (in this particular order) form a set of basic circuits. This is true because the set of edges E_1, E_2, E_3, E_4, form a basic edge set (i.e., the remaining edges form a maximum tree). One can verify that the basic edge E_i is included only in the basic circuit C_1; E_2 is included in C_2 but not in C_3 or C_4; E_3 is included in C_3 but not in C_4.

This example shows the phenomenon of edge-monopolization. E_1 <u>is</u> included in C_1 and is not included in any other basic circuit, emphasizing that edge E_1 is monopolized by circuit C_1. When E_1 is removed from the linear graph, all other basic circuits (C_2, C_3, C_4) are

still intact. Among the remaining circuits, E_2 is monopolized by C_2. Similarly, when the first r basic edges are removed, only the first r basic circuits are destroyed. Of the circuits which remain, the basic edge E_{r+1} is monopolized by C_{r+1}.

We can now restate the decomposition theorem as follows:

Theorem 4: If N is a cyclic net with a cyclomatic number, u, there exist a set of basic circuits, C_i, $(i=1, 2, \ldots u)$ in N such that any Euler graph, E, defined on N is uniquely decomposable into C_i; i.e., there exist a unique set of coefficients a_i $(i=1, 2, \ldots u)$ so that:

$$E = a_1C_1 + a_2C_2 + \ldots + a_uC_u$$

Based on this edge-monopolizing property, we are able to design a practical procedure for the decomposition of an Euler graph, once a set of basic circuits is given. This procedure is illustrated in Table X, the decomposition table which we have used to obtain the decomposition results for the Philippines in section III above. Columns and rows are numbered to facilitate reference. The top margin shows all the edges of Diagram IVa. In column 1 the twelve circuits from C_1 to C_{12} are indicated.[16] A number "1" is written in a cell if and only if the

[16] These 12 circuits are those shown as numbers C_1 through C_{12} in Diagram VII.

TABLE X: DECOMPOSITION PROCEDURE

(1) CIRCUIT	(2) EDGE VALUE / COEFFICIENT	(3) ZY	(4) YX	(5) YZ	(6) ZX	(7) GY	(8) FY	(9) ZG	(10) XG	(11) GX	(12) $Z_y Y$	(13) XF	(14) YF	(15) XY	(16) YG	(17) FX	(18) FZ
(1) C_1	I_x	1											1				1
(2) C_2	x_y		1											1			
(3) C_3	S_h			1							1						
(4) C_4	S_L'				1							1					1
(5) C_5	y_g					1									1		
(6) C_6	E_y						1						1				
(7) C_7	S_g'							1		1		1					1
(8) C_8	T_L								1	1							
(9) C_9	$x_g - S_g' - T_L$									1				1	1		
(10) C_{10}	$I_y - S_h$										1		1				1
(11) C_{11}	$M_x - S_L' - S_g'$											1				1	
(12) C_{12}	$M^* - I_x - E_y - I_y + S_h$												1	1		1	
EULER GRAPH EDGE VALUE	(13) ORIGINAL	I_x	x_y	S_h	S_L'	y_g	E_y	S_g'	T_L	x_g	I_y	M_x	M^*	y_x	T_h	E_x	A
	(14) 1ST ROUND	$-I_x$	$-x_y$	$-S_h$	$-S_L'$	$-y_g$	$-E_y$	$-S_g'$	$-T_L$	$-S_g'$ $-T_L$	$-S_h'$	$-S_L'$ $-S_g'$	$-I_x$ $-E_y$	$-x_y$	$-y_g$		$-I_x$ $-S_L'$ $-S_g'$
	(15) 2ND ROUND												$-I_y$ $+S_h$	$-x_g$ $+S_g'$ $+T_L$	$-x_g$ $+S_g'$ $+T_L$	$-M_x$ $+S_L'$ $+S_g'$	$-I_y$ $+S_h$

circuit specified in the row contains the edge listed in the column. When this step is completed, the table is referred to as a circuit-edge table.

Once an Euler graph is defined on the skeleton, we can write the values associated with each edge in row 13. We are now ready to begin the decomposition procedure. The entries with a single circle represent monopolized edges (ZY), (YX), (YZ), (ZX), (GY), (FY), (ZG), (XG) and the circuits which monopolize these edges (C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8). The very definition of basic circuits implies that at least one edge is monopolized; i.e., E_1 is in C_1 but not in any other circuit. The coefficient of every circuit monopolizing an edge is uniquely determined and is equal to the value on the monopolized edge. Hence, we can now write the coefficient of $C_1 \ldots\ldots C_8$ in column 2.

Next, for every $C_i (i=1, 2, \ldots 8)$, the values of the coefficient C_i are subtracted from the values on all edges of the Euler graph contained in C_i. This operation is indicated in row 14. For example, the coefficient of C_7 is S_g and, hence, $-S_g$ appears in row 14 under columns 9, 11, and 13, corresponding to the three edges of C_7. Thus, the sum of rows 13 and 14 constitutes a new Euler graph with fewer edges (as the value on every monopolized edge is zero) and containing fewer circuits since C_1, $C_2 \ldots C_8$ are no longer relevant and can be disregarded in later rounds.

We now begin the second round of decomposition, again proceeding by identifying monopolized edges and the circuits which monopolize them. These are indicated by the double-circled entries. The coefficients for these circuits, C_9, C_{10}, C_{11}, are now determined by reference to the new Euler graph (sum of rows 13 and 14) and recorded in column 2.

We now proceed to the third round, the final round in our example, and perform the same operation. In this final round, we determine the coefficient of C_{12}, indicated by the triple-circled entries. This circuit monopolizes the edge (YF). Any Euler graph can be decomposed by this method in a maximum of u (the cyclomatic number) steps.

To re-emphasize the concept of basic circuits, we note that for the 12 circuits ($C_1 \ldots C_{12}$) in the decomposition table (Table X), we can produce a basic edge set which are the 12 monopolized edges, (ZY), (YX) ....(YF), indicated in the given order as the first 12 edges in Table X. One can confirm the fact that C_{r+1} monopolizes E_{r+1} when the first r edges are removed. Finally, we can verify that when all the basic edges are removed, the remaining edges form a maximum tree of the original skeleton. Hence, C_1, $C_2 \ldots C_{12}$ form a set of basic circuits.

The Economics of Decomposition

Once a set of basic circuits has been identified, the decomposition of a particular national income account (Euler graph) to determine the coefficients of each economic circulation is merely a matter of routine calculations. We have presented a procedure for such routine calculation in the previous section. It is apparent that this procedure does not involve economic knowledge or judgment. However, there is a critical input of economic knowledge at an earlier stage when the set of basic circuits is selected for decomposition. This is especially pertinent when, as we usually find for a large national accounting skeleton, there are alternative sets of basic circuits. It is then necessary for decomposition to select one of several sets of basic circuits, B_1, $B_2 \ldots B_m$, on the basis of assessing their economic significance. Such choice requires an intimate knowledge of the economy for each year for which national income flows are to be decomposed. Knowledge of national income accounting data alone is inadequate to assure that the proper set of basic circuits will be selected.

If the national income accounting system is to be analyzed over a number of years, detailed knowledge of the economy and its performance over the entire period is required. There is a problem in changes in sets

of basic circuits, B_i, through time. One B_i appropriate to illuminate details of intersectoral relationships at one point in time may not be germane for another year. While only quantitative changes are shown when only one given base is employed for a period of years, we have confronted this problem in the previous section in connection with the necessity of shifting bases for analyzing the Philippine economy.

Shifting bases is an important part of decomposition analysis which can highlight sources of change in the growth dynamics of an economy. When bases are shifted, we see that the composition of the economic circulations are altered, revealing changes in the intersectoral patterns of resource utilization. Qualitative changes in the structure of the economy can be reflected by this device.

There are no hard and fast rules to assist in making the difficult choice among alternative sets of basic circuits, B_i, as the basis for decomposition analysis for a particular year. Economic understanding supported by experimentation, must be relied upon since the mathematics of linear graph analysis (although helpful as a tool) cannot replace economic knowledge and judgment. In a fundamental sense, the decomposition techniques we have presented in this paper are descriptive in nature. The results are suggestive of new problems whose analysis, however, will require the application of more formal methods, e.g., the postulation of behavioristic relationships.

Some suggestions that may prove helpful for choosing a set of basic circuits for a specific year may be offered. Given a national income accounting system (i.e., an Euler graph) for a particular year, a set of basic circuits, B_i, can be provisionally selected on a trial basis. Decomposition of this B_i yields a set of coefficients, C_i, which represent the values of the economic circulations. However, the decomposition theorem presented in the previous section gives no assurance that all C_i will be non-negative. If some of the values for the C_i turn out to be negative, the result of the decomposition will be unsatisfactory for economic analysis. Negative circulations of real resources lack economic meaning. Thus, when any negative coefficient appears, it is a signal that the set of basic circuits provisionally chosen is inappropriate and an alternative set, B_j, must be sought. This trial-and-error method was successfully employed in the base shifting described in the previous section. The basic set of circuits used for decomposition of the 1964 Philippine data would have produced negative coefficients for the 1949 data, so a new and more meaningful set of basic circuits yielding no negative coefficients was adopted for 1949. We find, therefore, that a mathematical fact (the sign of the coefficient) provides some guidance in choosing an appropriate set of basic circuits.

The preceding discussion raises another problem. Given a <u>positive</u> Euler graph, E, representing an actual national income accounting system, can we always find a set of basic circuits, B_i, that will yield non-negative coefficients when E is decomposed into B_i? An affirmative answer to this question is essential if the trial-and-error method suggested above is to be successful. The following theorem (proved in the mathematical appendix) asserts that the answer is affirmative:

Theorem 5: If U is any strictly positive Euler graph (i.e., a balanced table with all strictly positive entries defined on a directed linear graph, G), there exists a set of basic circuits, B, of G, which will yield non-negative coefficients when E is decomposed according to B.

It should be noted that this theorem differs from the decomposition theorem of the previous section, which merely guaranteed the existence of a set of basic circuits. The decomposition theorem did not, however, assure that decomposition would yield non-negative coefficients. The theorem now stated begins with a <u>non-negative</u> Euler graph, E, and asserts the existence of a base, B, which, when employed for decomposition of E, will yield non-negative coefficients. However, if another Euler graph (even with strictly positive coefficients) were to be decomposed according to B, some negative coefficients may occur. When this phenomenon actually occurs, base shifting becomes unavoidable.

Mathematical Appendix

Basic Edge Set and Basic Circle

We accept the definitions of link, circle, connected, end vertex end edge, cyclomatic number, circle free, tree, maximum tree, basic edge set and causal order of the edges of a tree as given in the text. (For those who are familiar with elementary linear graph theory, it may be added that these are non-directed linear graph concepts which will be used in the appendix first.)

Making use of the facts that a tree (with more than one edge) has at least one end edge and that when an end edge is deleted from a tree what remains again forms a tree, we can easily prove:

Lemma 1 The Cyclomatic Number of a Tree is Zero (i.e., $E=V-1$ for a tree)

Let G be a connected linear graph with cyclomatic number u. Let T be a maximum tree of G and let B be the set of edges of G which are not in T. We now want to prove theorem 2 in the text (i.e., we want to prove B has u-edges). Let the edges of B be written as E_i ($i=1, 2, \ldots r$). If B is empty (i.e., $r=0$), then G (which is the same as T) is a tree and Lemma 1 implies the proof. If B is not empty (i.e., $r > 0$), the edges

E_i may be added, one by one, to T until T finally becomes G. Since T is maximum, the addition of an E_i at each step will increase the cyclomatic number by one (as the number of vertices will not be increased). Since T is a tree, its cyclomatic number is zero (Lemma 1); and, hence, the cyclomatic number of G is r (i.e., u=r). This proves theorem 2. QED.

We shall next prove the following:

<u>Lemma 2</u> <u>A connected linear graph, G^1, with a cyclomatic number u has at least one set of u-edges which form a basic edge set (i.e., when these edges are deleted what remains is a maximum tree).</u>

Proof: If G^1 is a tree, lemma 1 implies that the theorem is true. If G^1 is not a tree, let E_1 be an edge contained in a circle C_1 in G^1. Delete the edge E_1 from G^1, the remaining edge is denoted by G^2. It is obvious that G^2, with a cyclomatic number, u-1, is connected and maximum in G^1. If G^2 is not a tree, the same process can be repeated; i.e., an edge E_2 can be identified which is contained in a circle C_2 or G^2. When E_2 is deleted from G^2, the linear graph formed by the remaining edges is denoted by G^3. In this way we have:

G^1 , G^2 , $G^3 \ldots\ldots G^r$, G^{r+1} (a set of subgraphs)

C_1 , C_2 , $C_3 \ldots\ldots C_r$ (a set of circles)

E_1 , E_2 , $E_3 \ldots\ldots E_r$ (a set of edges)

such that E_i is in C_i (which is in G^i) and such that G^{i+1} is obtained from G^i by deleting E_i. Furthermore, G^i, with cyclomatic number $u-i+1$, is connected and maximum in G^1. Since G^1 has only a finite number of edges, the process must stop, in a finite number of steps, at G^{r+1} which is circle free and is thus a maximum tree of G^1. Since the cyclomatic number of G^{r+1} is zero, we have $u-(r+1)+1=0$ or $u=r$. Thus E_i $(i=1, 2, \ldots u)$ form a basic edge set. QED.

The above proof immediately shows:

Corollary 1: <u>The cyclomatic number u of a connected linear graph G is non-negative; u is zero if and only if G is a tree.</u>

The above proof suggests the definition of a <u>set of basic circles</u> by merely changing the word "circuit" into "circle" in the definition of <u>a set of basic circuits</u> in the text. In other words:

Definition: Let G be a linear graph with a cyclomatic number u. A set of circles C_i $(i=1, 2, \ldots u)$ form <u>a set of basic circles</u> if there exists a set of basic edges E_i $(i=1, 2, \ldots u)$ such that E_i is included in C_i and is not included in any C_j if $j > i$.

The above proof then implies:

Corollary 2: <u>A connected linear graph with a cyclomatic number u has at least one set of u-circles which form a set of basic circles.</u>

Exogenous and Endogenous Variables

Let G be a directed linear graph which is connected. A sub-set of edges, X, of G will be referred to as a set of exogenous variables (the set of edges not in X, denoted by the set notation G-X, referred to as a set of endogenous variables) according to the definition given in the text. We want to prove the necessary condition of theorem 3 in the text. Suppose G-X is not a maximum tree of G, then there are two cases:

Case One: G-X is not circle free. In this case, the endogenous variables contain a circle C. According to the definition of a "circle," the edges of C can be classified into two classes C^+ and C^- (one may be empty) such that a circuit can be formed when the direction of all edges in C^- is reversed. Letting k be any number, we can construct a square table B by:

i) assigning the value "k" to every cell of B corresponding to an edge of C.

ii) assigning the value "-k" to every cell of B corresponding to an edge of C^-.

Then B is a balanced table. Let us assign a particular set of arbitrary values to X (the set of exogenous variables) and determine a set of values for the endogenous variables in G-X.

These values then form a balanced table (i.e., Euler Graph), J, by definition. Notice that B+J is also a balanced table in which the values of the exogenous variables (X) are the same as those in J. This proves that the values of the endogenous variables cannot be determined uniquely by the accounting equation (i.e., by the requirement that an Euler graph be formed).

Case Two: G-X is circle free. This implies that G-X is either not connected or not maximum in G. In this case, it is easy to see that we can take some (at least one) edges from X which, when added to G-X, would have converted the latter into a maximum tree of G. Thus, according to the sufficient condition of theorem 3 (proved in the text), a proper sub-set of X constitutes a set of exogenous variables. This implies that arbitrary values cannot be assigned to all values of X to form an Euler graph. QED.

National Income Accounting Structure

We accept the definitions of a path, circuit, cyclic net, aggregation operation (on a directed linear graph) as given in the text which are proper "directed" linear graph concepts. In addition, let us introduce:

Definition: A sub-graph N of a directed linear graph G is a maximum cyclic net (MCN) of G if N is a cyclic net and is not a proper sub-graph of a larger cyclic net of G.

Definition: A vertex, V, of a directed linear graph is a sink (source) if there is no edge initiating from (terminating at) V.

Definition: A directed linear graph is circuit-free when it contains no circuit.

The validity of the following lemmas can be easily verified:

Lemma 3: A circuit free directed linear graph has at least one source and one sink.

Lemma 4: Every edge of a cyclic net is included in a circuit.

Lemma 5: A circuit of a directed linear graph is always included in an MCN; two MCN cannot have a common vertex.

Let us construct an aggregated linear graph, A(G), by classifying the vertices of a directed linear graph, G, in the following way: a vertex of A(G) either corresponds to a single vertex, x, if x is not in an MCN of G or corresponds to a sub-set of vertices belonging to an MCN of G. (This aggregation operation is indicated in Diagram III.)

Lemma 5 implies that A(G) is circuit-free. If G is connected, then A(G) is also connected. If a connected G is a cyclic net, then A(G) is merely one vertex; otherwise G has at least one edge. This may be summarized as:

Lemma 6: a) A(G) is circuit free.
b) If G is connected, then A(G) is connected.
c) If G is connected and is not a cyclic net, then A(G) has at least one edge.

We shall now accept the idea of a "skeleton" discussed in the text (i.e., a linear graph on which an Euler graph can be constructed). However, in a rigorous treatment, the concept of "skeleton" can be dispensed with as is illustrated in the following lemma:

Lemma 7: If G is a circuit-free, directed linear graph, the only non-negative Euler Graph which can be constructed on G is the trivial one, i.e., the zero-Euler graph in which the value zero is assigned to every edge of G.

Proof: We shall prove the lemma by induction on the number of vertices. Let G be circuit-free with n-vertices. If $n=1$, the lemma is trivial. Let us assume that the lemma is proved for all Euler graphs with n-1 or fewer vertices. If G has any "isolation vertex" (i.e., a vertex touched by no edge at all), the vertex can be deleted and the inductive hypothesis completes the proof. Thus, we may assume G has no isolation vertex. By lemma 3, we can find a vertex "s" which is a source of G. There is at least one edge (s, x) that initiates from "s." For any non-negative Euler graph defined on G, the value on (s, x)--as well as on every edge initiating from "s"--is zero. Let a new linear graph, M, be formed by deleting "s," as well as all edges initiating from "s," from G. Any

non-negative linear graph defined on G can now be defined on M. Since M is circuit-free and has one less vertex than G, the inductive hypothesis completes the proof. QED.

Let us now accept the definition of the <u>national income accounting structure</u> and now prove theorem 1 in the text. Let a directed linear graph G be given. To prove the sufficient condition, suppose G is a cyclic net with edges E_i ($i=1, 2, \ldots n$). By lemma 4, we can find a set of circuits C_i ($i=1, 2, \ldots n$) such that E_i is in C_i. Let k_i ($i=1, 2, \ldots n$) be any set of strictly positive numbers; the valued linear graph $k_1C_1+k_2C_2 \ldots + k_nC_n$ is then obviously an Euler graph with strictly positive values defined on every edge of G. Thus G is a national income accounting system, and the sufficient condition of theorem 1 is proved.

To prove the necessary condition, let U be a strictly positive Euler graph defined on G which is assumed to be connected. Let us construct A(G) (according to lemma 6) and aggregate U accordingly. In this way, we obtain an Euler Graph A(U) (as the aggregation of an Euler graph is an Euler graph) which is strictly positive (as only "additions" are involved). By lemma 6, A(G) is connected and circuit free. Furthermore, if G is not a cyclic net, A(G) has at least one edge on which a strictly positive number is defined by A(U). This contradicts lemma 7, and hence G must be a cyclic net. This proves the necessary condition of theorem 1. QED. This proof implies the following corollary:

Corollary 3: For any non-negative Euler graph U defined on a linear graph G:

a) all values on edges not included in an MCN of G are zero.
b) the edges of G corresponding to strictly positive values of U form a union of disjoint cyclic nets.

Decomposition:

Let G be a linear graph with cyclomatic number u and let C_i ($i=1, 2, \ldots u$) be a set of basic circuits--as defined in the text. The validity of the following lemma can be easily verified:

Lemma 8: The only Euler graph which can be defined on a tree is the trivial one (i.e., the value "0" is assigned to every edge of the tree).

The following lemma constitutes a part of theorem 4 in the text:

Lemma 9: If G^0 is a linear graph (with cyclomatic number u) and if C_i ($i=1, 2, \ldots u$) is a set of basic circuits, then any Euler graph, A, defined on G^0, can be decomposed uniquely in C_i; i.e., there exists a set of unique coefficients a_i ($i=1, 2, \ldots u$) such that

$$A = a_1C_1 + a_2C_2 + \ldots\ldots + a_uC_u$$

Proof: In the definition of C_i, let the set of basic edges be E_i and let G^r denote the linear graph when the first "r" E_i are deleted from G^0. Thus G^u is a maximum tree of G^0. To prove uniqueness, suppose b_i ($i=1, 2, \ldots n$) is another set of coefficients, then denoting $c_i = a_i - b_i$, we have:

$$O = c_1C_1 + c_2C_2 + \ldots\ldots + c_uC_u$$

where "O" on the left-hand side is a square table of zeros. We will prove $c_i = 0$ (i.e., $a_i = b_i$) inductively on $i = 1, 2, \ldots u$. Since C_1 is the only circuit containing E_1, thus $c_1 = 0$. Now we assume $c_1 = c_2, \ldots = c_r = 0$ and, hence, $0 = c_{r+1}C_{r+1} + c_{r+2}C_{r+2} \ldots\ldots + c_uC_u$. Since of all the circuits which appear on the right-hand side C_{r+1} is the only circuit which contains E_{r+1}, thus $C_{r+1}=0$ and the proof is complete.

To prove the possibility of decomposition, let x_1 be the value of A, now denoted by A^1, on the edge E_1. The valued linear graph:

$$A^2 = A^1 - x_1c_1 \text{ (or } A^1 = x_1c_1 + A^2)$$

is an Euler graph now defined on G^1 as C_1 is the only circuit containing E_1 (i.e., the value of A^2 on E_1 is zero). Letting x_2 be the value of A^2 on E_2, the valued linear graph:

$$A^3 = A^2 - x_2C_2 \text{ (or } A^2 = x_2C_2 + A^3)$$

is an Euler graph defined on G^2 as C_2 is the only circuit (among C_i $i \geq 2$) containing E_2. Proceeding in this way, after u-steps, we have:

$$A^{u+1} = A^u - x_uC_u \text{ (or } A^u = A^{u+1} + x_uC_u = X_uC_u)$$

where A^{u+1} is an Euler graph defined on G^u. Since G^u is a tree, $A^{u+1} = 0$ is thus a 0-table by Lemma 8. Then we have:

$$A = A^1 = x_1C_1 + x_2C_2 + \ldots + x_uC_u$$

QED.

We shall now prove the following lemma:

Lemma 10: <u>Every cyclic net, N, has at least one set of basic circuits.</u>

Proof: Let us assume that N has n-edges and a cyclomatic number u. We shall prove by induction on n. Let (x, y) be an edge of N which, by lemma 4, is included in a circuit C_u in N. Let (x, y) be deleted from G and denote the remaining edges by G'. Obviously, G' has n-1 edges and a cyclomatic number u-1. There are two cases:

<u>Case One</u>: <u>G' is a cyclic net</u>. The inductive hypothesis is applied to locate a set of basic circuits C_i ($i=1, 2, \ldots u-1$) in G'. When C_u is added to this set, a set of basic circuits for G is found.

<u>Case Two</u>: <u>G' is not a cyclic net</u>. In this case, there is no path in G' from x to y. There are two subcases:

<u>Sub-case One</u>: <u>(y, x) is not in G (and G')</u>. Form a new directed linear graph, G'', by changing the notations for the vertices "y" and "x" into "z"--a new vertex notation not previously used in G. G'' is a cyclic net with cyclomatic number u and n-1 edges.

The inductive hypothesis leads to a set of basic circuits C_i ($i=1, 2, \ldots u$) in G''. If all the edges in a particular C_i are not in G', then C_i is a path from y to x. Thus, when the edge (x, y) is added to C_i, a circuit C_i^* in G can be formed. (If C_j is in G', take $C_j^* = C_j$.) In this way, a set of basic circuits C_i^* ($i=1, 2, \ldots u$) of G can be found.

<u>Sub-case Two</u>: <u>(y, x) is in G and G'</u>. Delete (y, x) from G' and then construct G'' as in the last case. G'' is now a cyclic net, with a cyclomatic number u-1, on which the inductive hypothesis can be applied to produce a set of basic circuits C_i($i=1, 2, \ldots$ u-1). As in the last sub-case, a set of u-1 circuits C_i^* ($i=1, 2, \ldots u-1$) can now be constructed in G. When the two-edge circuit (y, x) (x, y) is added, a set of basic circuits is found for G. QED.

Lemma 9 and 10 imply theorem 4 in the text. To prove theorem 5, we need the following lemma:

Lemma 11: <u>If U is a non-negative Euler Graph defined on a cyclic net, N, with a cyclomatic number u, there exists a set of basic circuits C_i ($i=1, 2, \ldots u$) in N such that U can be decomposed in C_i with non-negative coefficients.</u>

We will prove this lemma by induction on the number of edges "n," in the cyclic net N. If n=1, N contains a single edge forming a loop and, hence, u=1. The theorem is seen to be true. We assume that the

theorem is proved for all cyclic nets with n-1 or fewer edges. Let N be a cyclic net with n edges and with a cyclomatic number u. Let (x, y) be an edge in N where U takes on a minimum value $m \geq 0$ (i.e., m is not greater than the value of U at any other edge in N). Let G be the linear graph constructed by deleting (x, y) from N. G is connected and contains all vertices of N. Hence, G has n-1 edges and has a cyclomatic number u-1. There are two cases:

Case One: <u>G is a cyclic net</u>. Let C be a circuit of N which contains the deleted edge (x, y). Define a new valued linear graph, U', by subtracting "m" from U at every edge in C. Thus U' is a non-negative Euler graph defined on G. Applying the inductive hypothesis, we can decompose U' into a set of basic circuits of G, i.e.:

$$U' = c_1C_1 + c_2C_2 \ldots\ldots + c_{u-1}C_{u-1}$$

where all the coefficients are non-negative. Then we have

$$U = U' + mC = c_1C_1 + c_2C_2 \ldots\ldots + c_{u-1}C_{u-1} + mC$$

It is easy to see that the circuits on the right-hand side form a set of basic circuits of N and, hence, the theorem is proved.

Case Two: <u>G is not a cyclic net</u>. In this case there is no path from x to y in G. There are two sub-cases:

Sub-case One: <u>(y, x) is not in N (and not in G)</u>. Aggregate N by grouping the two vertices "x" and "y" while keeping all other vertices of N distinct. Denote the aggregated linear graph by A(N) which is a cyclic net with n-1 edges and with a cyclomatic number u. Aggregate U accordingly and obtain a non-negative Euler graph A(U) defined on A(N). Apply the inductive hypothesis, we can decompose A(U):

$$A(U) = c_1C_1 + c_2C_2 + \ldots\ldots + c_uC_u \quad (c_i \geqq 0)$$

where C_i are basic circuits of A(N). If a particular C_i is not in N, it is a path from y to x; the addition of the edge (x, y) to C_i will form a circuit C_i^* in N. (If C_i is in N, take C_i^* to be the same as C_i.) In this way, it is easy to see that C_i^* (i=1, 2, ... u) form a set of basic circuits of N.

The expression:

$$B = c_1C_1^* + c_2C_2^* + \ldots\ldots + c_uC_u^*$$

defines an Euler graph which is decomposed into a set of basic circuits of N with non-negative coefficients. Consider the difference B-U which is an Euler graph and which, by

construction, takes on the value zero on all edges of N other than the edge (x, y). (This is due to the fact that A(U), U, and B have the same value at every one of these edges.) Lemma 8 then implies that B-U also takes on a zero value at (x, y) and, hence, B = U.

Sub-case Two: <u>(y, x) is in N (and G)</u>. Perform the aggregation as before and obtain A(U) defined on A(N) where A(N) now has n-2 edges and has a cyclomatic number u-1. Applying the inductive hypothesis, we have:

$$A(U) = c_1C_1 + c_2C_2 \ldots\ldots + c_{u-1}C_{u-1} \quad (c_i \geq 0)$$

where C_i is a set of basic circuits of A(N). Construct C_i^* as before and let C_u^* be the two-edge circuit [(x, y), (y, x)]. We construct

$$B = c_1C_1^* + c_2C_2^* + \ldots\ldots + c_{u-1}C_{u-1}^* + q \quad C_u^*$$

where $q \geq 0$ is the value of U at the edge (y, x). B is an Euler graph decomposed into a set of basic circuits, C_i^*, of N with non-negative coefficients. Consider the Euler graph B-U where, by construction, the value defined on every edge [except (x, y) and (y, x)] is zero. Since none of the

C_i^* $(i=1, 2, \ldots u-1)$ contains the edge (y, x), the value of B-U at this edge is also zero. Lemma 8 then implies B=U as in the last case. QED.

Notice the proof for theorem 5 is similar to that of Lemma 10. Indeed, we can prove Lemma 10 as a corollary of theorem 5 as follows:

When a cyclic net N is given, we can first apply the sufficient condition of theorem 1 in the text by constructing a non-negative linear graph U on N. We can then apply theorem 5 on U and N and prove Lemma 10 directly. It is interesting to note that while theorem 1 is a directed linear graph theorem (i.e., it is <u>not</u> a valued linear graph theorem), it can, nevertheless, be proved as a <u>valued</u> linear graph theorem. It is for this reason that an independent proof of Lemma 10, based only on (none-valued) directed linear graph concepts, was given above.

We are now ready to prove theorem 5, in the text, by induction on the number of edges of G. By corollary 3b, G is the union of disjoint cyclic nets. If there are more than one cyclic nets in G, the inductive hypothesis, applied to the individual cyclic nets, immediately completes the proof. Thus, we may assume G is a cyclic net. Lemma 11 then immediately implies theorem 5. QED.