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TIME PREFERENCE AND DEVELOPMENT PLANNING:

Screening and Selection of Projects

by

BRUCE EDWARDS

M-7883
Dev. Plan. 64-2
May, 1964

PREFACE

It is with some hesitation that the attached manuscript is being circulated in its present form. All research goes through several stages from rough idea, through discarded drafts, to final outline, to exposition. The attached material has been selected from a broader and more comprehensive framework. This part was carried through to this stage because my attention will be diverted to some empirical field work for the next several months. Although I believe the underlying formal argument to be correct and understandable, it is likely that future revisions will be required to produce a well rounded document. Reference is made in this preface to a few problems already apparent.

The paper is addressed to one special problem in the planning field; i.e., the time preference aspect of approving and vetoing projects. For this reason several complicated and important facets have been glossed over. As we mention in the text, for example, problems of projecting costs and revenue under conditions of uncertainty are assumed away <u>only</u> because they are beyond the scope of this paper, not because they are deemed to be solved or unimportant. A similar statement is appropriate with regard to those projects whose true value is not measured in terms of the expected revenue which they will produce.

A caveat also appears to be needed about what one reader referred to as the "picture of the Planning Office." Obviously, the extreme specialization implied in the stages used in this paper should not be taken literally. No bureaucracy is minutely subdivided by specific function, and in practice the time preference dimensions of a project tend to be evaluated simultaneously with estimation of the costs, revenues, and social benefits. The project screening

process is broken up into stages, therefore, only for purposes of exposition. In fact, it is likely that screening functions carried out in the first two of our four stages might well be done by parts of the government bureaucracy which are ordinarily not construed to participate formally in the planning process. Hence, one of the aims of the suggested procedures is to extend the span of the project planning process somewhat beyond the limits implied by conventional methods.

Discussions with colleagues from the Development Planning Project suggest that the comparisons and implications covered in sections III and IV also require brief prefatory comments. The more the treatment of time preference was investigated, the more it became obvious that business procedures, economic theory, mathematical formulations, and ideological issues have tended to become woven together into a familiar set of habitual thought processes and concepts. Each part supports and is supported by the other components, at least in terms of the usual expression. An alternative conceptual framework can be understood only in terms which took on all of these dimensions simultaneously. Sections III and IV are an attempt to cut through this body of circular reasoning to objectively study the alternative procedures exclusively from the viewpoint of their usefulness for making intelligent decisions efficiently and quickly. The procedures being suggested here, like the conventional predecessors, should be equally useful both to a competitive, laissez-faire, market-centered economy guided by market rates of interest and to a highly controlled, centrally administered economy using shadow prices.

BRUCE EDWARDS
April 28, 1964

Explanation of Symbols

A = Gross Annual Return

a = Natural logarithm of (1 + A)

B = Expected annual return on re-invested earnings

R = Normal interest

b = Natural logarithm of (1 + B)

e = The base of natural logarithms

g = R/A = Ratio between normal interest and gross annual return. Used as an intermediate variable.

ln = The natural logarithm of a number.

r = Natural logarithm of (1 + R)

R max. = Maximum expected future value of R

R min. = Minimum expected future value of R

t = Number of years in the future at which a goal will be accomplished.

$x = \frac{B}{B+A-R}$ = An intermediate variable used for simplification.

I. INTRODUCTION

Among economists trained in the West, a surprising amount of energy has been devoted to the computation and comparison of internal rates of return using compound interest tables for many economic applications. When attention is shifted to less developed countries, however, one finds that poor statistics and scarcities of trained personnel limit the applicability of precise planning methods. In such situations the niceties learned in the classroom are likely to be apologetically abandoned. The position adopted in this paper is that the apologies are unnecessary and that most of the energies diverted to training people to use the sophisticated techniques are a misdirection of scarce resources.

Before entering into the main argument of this paper, we begin with a brief introduction to the approach the present writer considers to be relevant for development planning in less developed countries. To some extent the argument relies on the fact that the classroom and the development planner are concerned with two different problems. In a very real sense the classroom is concerned with pure knowledge, and all of the nuances of minor differences under a variety of ideal conditions are appropriate pursuits. The development planner, on the other hand, is essentially in a decision-making environment. When a particular proposal, in the form of a project or a program, comes across his desk, the first decision he is likely to confront is whether to approve it by signing his name or not to approve it. If the second alternative is adopted, a second decision must be made either to reject it completely or to recommend further study on the proposal so that it can be resubmitted later with further modifications or with more adequate supporting evidence. Faced with personal time pressures and scarce professional staff, the planner must

keep the items in the further-study category down to manageable proportions. On the one hand, the decision-maker is hardly interested in the relative importance of projects in the group which are so inferior as to be rejected outright. On the other hand, he will, no doubt, be well aware that among the projects he approves some are more important than others, but he cannot afford the time to dwell on these differences.

Because of this fundamental difference in attention to detail, the academician may at times infer that important considerations are being completely ignored in the body of this article. Many of these more academic issues, however, are discussed in the appendices where they have been placed to prevent their diverting our attention from the major issues confronting the development planner.

The procedure which is developed for project selection can be described as a modified payout-period approach. More specifically, this procedure is concerned with how long it will be before a project has earned back all of the original investment plus a normal return on the investment. If the probable life of the project is less than its payout period in this sense, the project should obviously be rejected. Alternatively, any earnings by the project after original investment and the normal interest have been covered constitutes an excess return, and only this excess return can properly be considered a profit.

It is apparent that such a procedure will make irrelevant computing the internal rate of return and thereby calculating exactly how much better the project is than "normal." In exchange for this, the procedure eliminates the necessity for making a single cardinal estimate of the useful economic life of the project, as is essential if normal depreciation procedures are used. Except for this change, most of the assumptions of conventional procedure are

retained. For example, it is assumed that the time and materials cost of the original investment in real terms can be determined from the engineering data and that the annual physical output can be predicted along with the annual physical input of materials and services required to produce that output. There may be considerable doubt about using physical productivity ratios computed from other societies, but there is no reason to think that errors in this parameter would affect one procedure more than another.

An even more serious objection can be made to the assumption that the relative future values of outputs and inputs are known. Future valuation immediately raises the time preference problem. The Western advisers are likely to have a different preference function than the responsible political leaders in the country, and both of these may differ from the implicit preference function revealed in the marketplace. Such differences are real and should be directly faced and resolved. Although it is possible to camouflage such problems with mathematical hieroglyphics, it is futile to expect numerical hocus-pocus to make a positive substantive contribution in this area.

Finally, we might note the problem of efficient use of planning personnel. One of the major objectives of the procedure presented in this article has been the development of rules of thumb, simple and plausible enough to be understandable to untrained people, for making obvious decisions at lower levels in the bureaucracy. The procedure is described in several levels, the simplest being at level 1. Implicitly it is assumed that as a person shows interest and ability and gains experience at a given level of the planning process, he can be given additional training and responsibilities for promotion to higher levels. In such cases it is desirable that his earlier training and

experience can be simply explained as a special simplified case of the more sophisticated procedures which he is to carry out at a higher level. Only the planning staff need be concerned with the more sophisticated aspects of project selection such as the formal relationship between interest rates and the approved projects. Finally, the senior planning staff alone need be concerned with the type of abstruse relationships discussed in the technical appendices - for example, the setting of guideline interest rates and the general theoretical problems associated with the rationale behind the decisions which have been made.

II. THE SCREENING PROCEDURE

A. Stage I: Pre-Screening: The Payout Period

The first stage in the process of project selection is to be the simplest and most unsophisticated and, accordingly, is termed pre-screening. In this stage, however, preliminary decisions are made on the basis of which all of the rest of the selection process is based. Persons operating at this level will be quite plentiful in comparison with the number of senior planning officials available in the society. The individual concerned with this pre-screening function may be expected to have (1) at least the equivalent of an elementary education in the 3 R's, (2) several years of adult experience in the equivalent of the business world, and (3) be above the national average in native intelligence. We conceive of his primary responsibilities in the day-to-day operation of an economic enterprise. He must have a concept of the cost of inputs of materials and service in producing whatever is produced; he must understand the concept of the value of output; and he must have at least a rough idea of the concept of the original cost of an investment - the cost of materials and other costs involved in getting a new productive process started.

These concepts of cost and value can be either naive or sophisticated, approximate or precise, but they must exist. In most cases they will be derived from the person's experience in the marketplace. Where marketplace values are not appropriate, it is the responsibility of the senior planning official to provide some alternate guidelines. This can be done by instituting a set of subsidies or taxes which "warp" the costs and values perceived by personnel operating at stage one. Or the senior planning officials could establish a set of rough "shadow prices" which are to be used in project proposals submitted to the planning agencies. If the latter course is followed, presumably a set of non-market incentives would also have to be introduced in the form of prizes of one sort or another for proposals which pass the rough criteria.

On the basis of the assigned or market values, the personnel at stage one compute the value A for a proposed project or program according to the following formula:

$$A = \frac{\text{annual value of outputs - annual value of inputs}}{\text{original cost of project}}$$

The symbol A can be called the gross annual rate of return on the original investment and is the key concept used throughout this paper.[1/] It will vary from project to project and from program to program, so in some ways is similar to the internal rate of return. However, it makes no allowance for depreciation, nor for normal rates of interest, nor for any profit either normal or supranormal.

The success of the program depends on motivating the junior level

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1/ See Appendix I, p.A3 for elaboration of the concept of gross annual rate of return.

operating level personnel to consider large numbers of potential projects or programs and to compute an "A value" for each of them. The next step is to compute a payout period $t_o = 1/A$ and to determine whether or not the probable life of the project or program exceeds t_o. If the result of this test is positive, the project should be brought to the attention of supervisory personnel since the project will obviously cover depreciation while interest and profits can be ignored at this stage. If the probable life of the project is less than t_o, i.e., less than 1/A, a second question can be raised; i.e., is the probable life of the project greater or less than .70 t_o, i.e., .70/A.[1/] If the probable life is under $.70t_o$, the project should be summarily dropped. In the other case, where the probable life is between $.70t_o$ and t_o, the project might be brought to the attention of supervisory personnel. Although this would constitute an unlikely project for approval, there is a small possibility that reinvesting the earnings would make the project worthwhile.

B. Stage II: Interest Rate Screening

The personnel operating at stage two will still be primarily concerned with day-to-day operating decisions but at a higher level. Although it is impossible to draw precise analogies, we may think of them as representing the equivalent of plant managers, owners, and operators of small- or medium-sized firms, county agents, officials of a chamber of commerce, and similar positions. Presumably they would have some form of business education, either in a commercial high school, junior college, or through some adult education

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1/ For justification of the number .70, see Appendix II, p. A6

program. The important qualification is that personnel operating at this stage need a concept of an interest rate. It need not be a point estimate; in fact, concepts of a minimum rate of return and a maximum rate of return are appropriate to this stage of the selection process. These concepts will be employed for analysis of the screening function at this stage. The minimum rate of return will be denoted R_{min} and the maximum rate of return R_{max}. These concepts apply not to internal rates of return on individual projects but rather to the problem of forecasting and measuring the relevant rate which is applicable in the society - the range which is almost sure to include the normal rate of return even though the latter is not precisely known. Where heavy reliance in the society is being placed on market mechanisms, it is probably appropriate to develop either some official forecasts made by the planning organization or a set of forecasts which become widely accepted and are made by various non-official, financial and economic organizations. If reliance is not placed on market mechanisms, the planning organization should specify the minimum and maximum interest rates which are to be used in evaluating projects and programs for approval. The setting of these guidelines rates will be discussed further below, but we note here that their use will make necessary the development of non-market incentives to encourage the submission of projects and programs as previously indicated.

The first step in the screening process at stage 2 is to identify all projects which are associated with a value of A which is less than the minimum interest rate discussed immediately above. These projects can be placed on a "temporarily dead" shelf. They might be worth consideration in a society with a zero effective rate of interest, but in the existing situation the gross annual return would be unable to meet the most favorable interest requirements

even if the project had perpetual life and no depreciation were necessary. Although circumstances eventually may change, these projects can be ignored for the foreseeable future.

Screening against the minimum interest rate is likely to eliminate some of those projects which had been labelled as possibilities for reasons of their internal reinvestment of earnings. The remaining projects qualifying on this basis should be reviewed to assure that such reinvestment possibilities were conceivable and then passed on to the junior planning staff at the next screening stage (Stage III).

The remaining projects which have come to stage II personnel from the stage I screening should be tested by computing the value for $t_i = 1/A-R_{max}$. We recall that the value of R_{max} is given to the stage II supervisory personnel either explicitly by the planning authorities or implicitly through the widespread acceptance of an independent non-official estimate of these values.[1/] If the probable useful life of the project is longer than t_i, the stage II personnel should send the project to the junior planning staff at stage III with a recommendation that it be approved and preliminary processing be initiated.

After the three-step procedure at stage II has been completed, a small class of borderline cases will remain. These are the items which will have an expected useful life greater than 1/A but less than $1/A-R_{max}$. These "borderline cases" are also submitted to stage III personnel as such for the

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1/ Throughout this paper the symbol R is used for the interest rate. For a refined discussion of minor points in this connection, see [redacted] Appendix III.

recommendation that they be studied using more sophisticated techniques.

C. Stage III: Analysis of "Marginal" Projects and Length of Life Screening

The personnel at stage III might be referred to as junior professional planning staff. To some extent their qualifications would consist of more advanced formal education, specifically for the field of economic planning. Some, however, perhaps the majority, would be former operating personnel who had functioned earlier at stages I and II. They would have shown some interest and ability in the planning process as opposed to the making of daily operating decisions, at least in the sense of having a comparative advantage in planning. At this level we envisage the beginning of professional specialization. Those who have come up through operating positions from stages I and II are likely to be most interested in the projects which have been sent up to stage III for further study when stage I and stage II procedures were not strong enough to generate absolute decisions.

The first step at this stage is to sort out those projects associated with an A value which falls in between the minimum and maximum figures given for R. Projects of this type can be collected together into a single group and labelled marginal. Inability to reach an earlier decision in these cases arose from uncertainty as to what the relevant normal rate of return within the society is going to be, and no amount of mathematical precision can remove this intrinsic uncertainty. When a more precise notion of the range of the normal rate of return becomes possible, the earlier procedures can be used again with the modified values of R_{min} and R_{max}. This is likely to produce a decision without the use of unnecessarily complex mathematics.

The other group of projects sent up for further study will be those which have useful lives between t_o and t_i, where t_i is used as a first

approximation of t_u, an upper bound for the minimum probable economic life for a project to be accepted. As more sophisticated approaches are introduced, the value of t_u can be decreased in a variety of ways in accordance with specific directions given by the senior planning officials to the stage III personnel. This is discussed in greater detail in Appendix III, but a particular case might be considered here which is almost universally applicable. The first step in the variation is to define $g = R_{max}/A$ where R_{max} is the maximum value of the normal rate of return. Then a value of t_u is obtained by the following equation:

$$t_u = t_i \left(1 - \frac{g}{2} - \frac{g^2}{6} - \frac{g^3}{12} - \frac{g^4}{20} \ldots\right). \qquad (1)$$

The dotted lines inside of the parenthesis indicate that the entire term in the parenthesis is an infinite decreasing series. The stage III personnel need only carry the series far enough to reduce the value of t_u below the expected life of the project. When this has been accomplished, the project is approved and referred to stage IV for processing. The stage IV personnel would presumably instruct those working at stage III that if a project cannot be passed after computing a specified number of terms (e.g., four), then the project should be tested against a value of t_ℓ as follows:

$$t_\ell = t_o \left(1 + \frac{g}{2} + \frac{g^2}{3} + \frac{g^3}{4} \ldots\right) \qquad (2)$$

which is complementary to t_u in that if the expected useful life of the project is less than t_ℓ, the project is rejected as not covering the normal interest. In this case the lowest probable value of the normal interest should be used so that $g = R_{min}/A$. If a project cannot be passed after computing a specified number of terms (e.g., four) in equation (2), the project should then be referred to stage IV for final consideration.

The simplest explanation to offer the stage III personnel is to introduce them to the concept of reinvesting the funds which have been generated on the basis that the depreciation reserves will not be needed until the equipment and other assets need replacement. It is not recommended that conventional compound interest procedures be introduced at this stage.

Another section of the stage III personnel will be concerned with checking projects which have been referred from stage II with a definite recommendation for approval. Junior planners working in this section will have several tasks. One of these will be straightforward review of the reasonableness of the assumptions and the accuracy of mathematical computations involved in earlier processing. This would be a useful introduction to the procedures for those who came into the system with conventional Western training and orientation but who had had no previous experience at the operating levels. It might also be useful to assign former operating personnel temporarily to these tasks with which they would already be familiar while they were becoming oriented to other aspects of working in the planning bureaucracy. This section could also consolidate and formalize the various projects into programs so that they could be included in the plan in more manageable groupings.

A third section of the junior planning staff would, at least in some cases, calculate the internal rates of return and/or net profits over and above normal rates of return using procedures with which Western economists and businessmen are more familiar. This process would be both educational and substantively useful. It would be educational in the sense that stage III personnel, regardless of their previous backgrounds, would become convinced by experience that the two procedures were in fact completely equivalent as they had been told by their planning superiors. The procedure would be substantively

useful in those cases where the project or program involved significant elements of foreign aid or participation by private Western capital, since those providing foreign capital would probably insist on having the programs translated into terminology and concept with which they were familiar.

D. Stage IV: The Role of the Professional Planner

Stage IV personnel might aptly be defined as the senior planning officials. In a small country there might be only a single individual in this role, while in most cases there would likely be a handful of relatively well trained people. In the larger countries they might be augmented by a group of assistants but, even so, they would operate as a "face-to-face group" except when they were geographically separated by large distances. As the top technicians in the planning process, they would be held responsible by the political authorities. Some or all of them would likely have a political role to play within the country, but we might consider that as part of their function as intelligent, well informed citizens rather than as an *ex officio* part of the planning office job.

Before considering the professional aspects of the senior planners' role, a caveat is in order. There is much more to the planning process than the time preference dimension, and the planning bureaucracy would be likely to carry out evaluations of the projects and programs in accordance with other criteria in addition to those imposed by time preference. Many, if not all, of these other criteria would also lend themselves to the type of organization we envisage. Since this article is concerned only with the time preference dimension, we will consider only that aspect of the senior planners' function.

Looking at the process we describe, the senior planner would have much to be thankful for. A large proportion of the unsatisfactory proposals will be screened out at stages I and II. Thus, planners at stage IV need not waste their scarce time and energies on these. In addition, at stages II and III a substantial number of projects will be identified which, in fact, are worth undertaking. These can be channeled into the plan with a minimum of difficulty and consideration at the top. It is likely that these projects will be successful, and the top planners can claim credit for them.

There will remain the group of residual projects which are borderline or marginal in a variety of senses of those words. If he is so inclined, the senior planning official can bring to bear his most sophisticated techniques and procedures in order to decide which of these should be included in the plan and which should be excluded. Perhaps of even more importance, he can rest assured that the inclusion or exclusion of a limited number of these marginal projects is not likely to make the plan significantly better or worse. Assuming that the plan has some influence on the eventual performance of the society, the inclusion or exclusion of these projects should have a minimal difference on welfare of the society as a whole. This is especially valuable information because when the planner feels the need to play politics, as most planners do, he can effectively use these projects as rewards and punishments without suffering severe pangs of conscience.

A major task of the senior planning officials is the setting of the values of R_{min} and R_{max}, either by objective forecasting techniques or by fiat. The wider apart these two figures are, the more projects - and even whole programs - become identified as marginal. This can be either an asset or a liability, depending on whether the marginal projects are viewed as desirable

instruments of political power or alternatively as a large group of problems which have to be solved objectively. In any case, the maintenance of a gap between the two values of R provides the needed slack for making adjustments which cannot be accurately predicted. If the two values are too close together, the de facto normal rate of return might fall outside of the expected band leading either to critical bottlenecks or underutilization of resources. If such a breakdown occurs, ad hoc solutions will have to be adopted on a semi-rational basis because too much information has been eliminated by the lower level screening processes. Except for this danger the amount of work passed up to the professional planning staff at stages III and IV will be controlled by the accepted values of R_{max} and R_{min}.

In addition to these specific tasks, the senior planning staff at stage IV will have responsibility for monitoring and reviewing the entire project-selection process proceeding at the lower three stages. For example, it was mentioned that at stage III a variety of formulae and procedures are available for estimating alternative values of t_u. Each of these will catch many but not all of the projects which should be summarily passed and approved at stage III and will leave a residual to be processed at stage IV. Only practical day-to-day operating experience will indicate which screening procedures are effective enough to warrant their institutionalization at stage III. In other cases, such procedures are better left for the stage IV technicians to use in making individual decisions or for examples to be used in educating and broadening the junior planning staff who will eventually become senior planning officials.

E. Application of the Screening Procedure: A Hypothetical Example

Before turning to an evaluation of the suggested procedure and investigation of its strong and weak points in comparison with more conventional techniques, it might be useful to see what happens as we follow an example through the steps in the screening process. There is no loss of generality in assuming that all values are measured in dollars and that these values are determined by a forecast of the probable market values which will prevail in the future.

The process is begun, for example, by a junior official in one of the local manufacturing plants who conceives of an expansion of facilities which would have a first cost of $20 million. The new addition to the plant would have an expected output which could be sold at the rate of $35 million per year and the costs of labor and materials required to operate the plant at "normal full capacity" would be $32 million. Putting this into the stage I formula produces a value of A = .15:

$$A = \frac{\$35 \text{ Million} - \$32 \text{ Million}}{\$20 \text{ Million}} = .15$$

Alternatively, an intelligent but uneducated farmer discussing matters with the other farmers in the neighborhood refers to an idea dreamed up by his brightest son who has just graduated from the sixth grade. The idea is for a small dam and minor irrigation canals to improve the water supply on their plots. The resulting increase in crops is estimated to have a market value of about $2,000 a year; in fact, most of the crops would probably be directly consumed within households rather than sold. The maintenance work on the project would cost about $500 a year if it were done by hired labor; in fact, it will probably be done by the farmers themselves, half willingly and half as

a form of a payment of taxes in kind. In like fashion, the original cost of the work is estimated at approximately $10,000, part of which must come from governmental financial sources to pay for needed materials while most of the labor will be supplied locally on a semi-voluntary basis. Again using the standard stage I procedures, the value of A is computed as:

$$A = \frac{\$2,000 - \$500}{\$10,000} = .15$$

These two extreme cases have been chosen to show how much is really assumed in all procedures which try to make comparisons by reducing complex social proposals and phenomena to a few dimensions. As these examples come out of stage I, they are judged to be completely equivalent in terms of the gross annual rate of return, namely, A = .15, and the only remaining question is whether or not the productive life of the project is long enough to warrant its being included in the development plan. (It may be entered either as a separate chapter or as one of many statistics behind a one-line item.) We note that focusing on one aspect, length of productive life of the project, is neither more nor less overwhelming than the statement that the relevant variable is the rate of profit or the rate of interest on a particular investment.

To return to the two examples which will now be treated as one, the next step is to compute the value of $t_o = 1/A$ which is 6 2/3 years. A priori we would suspect both projects to have a productive life exceeding 6 2/3 years, so we would expect the stage I personnel to pass the suggestions on to their superiors. If, however, one or the other of these projects or another equivalent project with a value of A = .15 were to have a life of less than 6 2/3 years, the stage I personnel could compute .70 times t_o = 4 2/3 years. If the anticipated life of the project were less than that (less than 56 months),

the project could be summarily rejected regardless of any arguments about consideration of reinvestment of earnings or taking account of compounding procedures in other ways.

As the project is passed up to the stage II personnel, the concept of an interest rate is introduced. Again making an arbitrary assumption, the planners have informed us that the interest rate (the normal rate of return on the marginal projects) will lie between 8 and 12 per cent. A is obviously greater than R_{min} (.15 is greater than .08), and we saw previously that the expected life of the project was over 6 2/3 years. The next step is to check the value of

$$t_i = 1/A\text{-}R_{max} = 1/.15 - .12 = 1/.03 = 33\ 1/3 \text{ years}$$

and find that t_i = 33 1/3 years. A priori, we might expect the irrigation project to have a useful life at least that long or longer although it would be more difficult to make a judgment in the case of the manufacturing plant. In any case, the decision cannot be made here on an a priori basis but must, in fact, be made as circumstances dictate in the particular society. If, following accepted rules of thumb, the expected useful life of either or both projects is over 33 1/3 years, the project is summarily recommended for approval at stage II and passed on up to stage III. If the expected life is under 33 1/3 years, it must be passed on up on an undecided basis.

Assuming the project has reached stage III on an undecided basis, the personnel at this stage can rather rapidly estimate a value for t_u as follows:

$$g = \frac{R_{max}}{A} = \frac{.12}{.15} = .8$$

$$t_u = (t_i)(1 - \frac{g}{2} - \frac{g^2}{6} - \frac{g^3}{12} - \frac{g^4}{20})$$

$$= 33.33(1 - \frac{.8}{2}...) = 20 \text{ years}$$

$$= 33.33(1 - \frac{.8}{2} - \frac{.64}{6}) = 16 \text{ years}$$

$$= 33.33(1 - \frac{.8}{2} - \frac{.64}{6} - \frac{.512}{12} - \frac{.4096}{20}) = 15 \text{ years}$$

If it is deemed likely that either or both of the projects still are not summarily approved at 15 or 16 years, it is possible to compute a value of t_ℓ in the following fashion:1/

$$g = \frac{R_{min}}{A} = \frac{.08}{.15} = .5333 \approx .5$$

$$t_\ell = (t_o)(1 + \frac{g}{2} + \frac{g^2}{3} + \frac{g^3}{4}....)$$

$$= (6.67)(1 + \frac{.5}{2}....) = 8\ 1/3 \text{ years}$$

$$= (6.67)(1 + \frac{.5}{2} + \frac{.25}{3} + \frac{.125}{4} = 9+ \text{ years}$$

In computing the value of t_ℓ, the value of g has been rounded down to .5 for convenience in computation. In this fashion, the range of indecision has been substantially reduced. If the projects are judged to have an expected economic lifetime of under 9 years, they will be summarily rejected; if the expected economic lifetime is over 15 years, they will be summarily approved; and if the expected lifetime is in the range 9 - 15 years, they become marginal projects which are referred to the senior personnel at stage IV for decisions.

The extent to which the suggested procedure has been "useful" is a concept which itself defies universally accepted "objective" measurement. It

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1/ Note that the value of g is different in this case because R_{min} is used instead of R_{max}.

is most probable that some projects or programs in this situation would have A values of .15 and expected useful lifetimes of 11 or 12 or 13 years. These would have to be brought to the attention of the senior personnel. It is also likely that some projects will be so short-lived as to be rejected at early stages and others will be so fundamentally sound that they are quickly given a green light and (hopefully) speeded on their path through the bureaucratic red tape. Beyond this, little can be said on a general basis.

III. A COMPARATIVE EVALUATION OF THE METHOD

A. Some Preliminary Problems

Most of the purely technical matters including the formal mathematical proofs and illustrations have been placed in the technical appendices at the end of the article. The appendices also compare technical aspects of the project selection method proposed here with other methods in general use. Although the topics discussed in this section can be regarded as "theoretical," they are theoretical in the sense used by generalists trying to fit a particular item into an over-all context.

We digress briefly to consider the concepts of time preference and interest as they have emerged in Western economic thought. A few essential points will be raised without specific references to the vast literature that has been written on the subject. Historically, the concept of interest arose from observing that in the market place those who had extra money could lend it out at a positive rate of return. This simple transaction has two quite separate and distinct elements in it. One element which occurs again and again in discussions of interest is the fact that capital, in every sense of the word, is scarce and must be rationed. The existence of a positive rate of interest

means that "less important" uses for capital are rationed out of the market because they find that they cannot pay the going rate of return. This leaves the available financial resources to be used by projects which will pay a rate of return equal to or greater than the normal interest rate. This we might think of as the rationing function of the interest rate.

A quite separate, distinct, and essentially different thread also runs through the historical discussions of interest. This is the idea relating interest to income distribution and ownership of wealth. Even if it is (or were) necessary for the efficient allocation of resources to require that the users of capital pay a positive interest rate, there has long been a feeling that the recipient of interest income has not earned it. This expressed itself in the medieval usury law and in more modern times in treatment of interest payments and interest earnings for income taxation purposes. The socialists, and particularly the Communists who came into power in Russia, were so set on wiping out the unearned elements of income that they "abolished" interest by fiat. In failing to distinguish between the two quite distinct functions of the interest rate, however, they found it necessary to devise other ad hoc techniques for allocating capital. Presently the Soviet planners have realized that the interest rate is an amoral tool which they can completely control for political goals but at the same time use effectively as a technical device to increase the efficient operation of their economy.

Income distribution aspects are explicitly raised here to point to the possibility of conflict with the rationing function of the interest rate. In this paper, however, it is assumed that the redistribution of income and wealth or, alternatively, the defense of the status quo, should be accomplished by means other than the manipulation of the interest rate being used in the

planning process. In this way the interest rate becomes simply defined as a procedure for rationing scarce resources available for investment for a time span exceeding one time period.[1/]

Capital-using projects submitted to the decision-making process typically have many dimensions. By a variety of techniques the number of dimensions are reduced; in this paper each project has only two: (1) the value of A, the gross annual rate of return, and (2) the expected productive life of the project. It is obvious, perhaps, that between two projects which have equal expected lifetimes, the one with the higher gross annual rate of return would be considered superior; between two projects which have the same gross annual rate of return, the one with the longer lifetime would be considered superior. The "tough" decision arises in comparing two projects, one having a higher gross annual return and the other having a longer expected lifetime. The choice in this case inevitably involves implicitly pure value judgments, issues concerning who really speaks for the society, questions of defining the social preference function, and similar intangibles. Given these difficulties, it is important that the set of such projects be defined as accurately as possible. Pairs of projects in which one is demonstrably inferior to the other should not be included unnecessarily in the set of all projects dependent on arbitrary value judgments. In a similar fashion, when a choice between projects has been made on subjective normative grounds, it is desirable to avoid giving the impression that the decision was reached using only objective criteria. With this as background, we can turn now to comparisons between the suggested

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1/ Presumably one time period in this context is equivalent to one year.

procedures and those more normally used in the Western business and academic circles in Europe and the United States.

B. Theoretical Justification

The problem of theoretical justification behind specific project selection methods deserves attention to clarify issues. The first issue concerns whether or not complexity necessarily makes for better theory. The present writer has encountered a fairly widespread reaction to his suggested procedures which implies that operationally simple methods are inevitably inferior in terms of theoretical considerations to the more complex standard formulae for project selection.

It is obvious that one must define what is meant by theory to explore this issue. The author prefers to think of "theory" as a group of interrelated generalizations which are believed to be useful as first approximations for describing the relationships which exist within a broad class of real phenomena. Under this or a similar definition, the question of superiority might depend on the usefulness of the idea, on the closeness of the approximation when the general rules are applied to specific instances, and/or the ease with which the idea can be fitted into the general body of theory of which it will be a part. Given this interpretation of theory, a surprisingly plausible case can be made for the proposition that accepted mathematical formulae for proper selection are in fact theoretically inferior to the procedures suggested here.

For purposes of simplification, economic theorists frequently make assumptions about perfect knowledge, mobility of resources, and in the field of time preference, perfect predictability of future events. No one argues that such assumptions are realistic. They are adopted, rather, as necessary

evils to keep from cluttering up the main analysis. Any realistic approach would prefer to avoid these assumptions if it were possible at a zero cost. The major perceived weakness in the conventional procedures for project selection is precisely at this point. The first assumption ordinarily made is that it is possible to find a single cardinal number which is a useful estimate of the economic life of the asset under consideration. This number is then used in making allowances for depreciation and in other ways becomes a foundation stone for the whole subsequent analysis.

It is not difficult to show in examples that a relatively small and quite plausible change in the expected life of the asset may have a substantial effect on the estimated rates of return or estimated profits over and above the normal interest charges. Thus the project selection criterion may in some cases be peculiarly susceptible to the judgments which have been made here. More seriously, the fact that these are pure judgments tends to be buried rather than brought into the fore. In many cases, a consistent over-estimate or under-estimate of lifetimes for all projects within a given society will create excessive optimism or pessimism but will not affect ordinal rankings. This will be particularly true if the society has been stabilized for a considerable period of time so that a sizeable body of experience has been accumulated and codified in manuals of normal operating procedure. It is this fact which probably explains why normal procedures work as well as they have in the developed countries. It is an entirely different matter, however, when these traditional Western methods are applied to the very different conditions found in less developed countries. Not only are physical and social conditions unlike those in countries with modern, advanced economies but also political and social stability are less likely to produce standard values for the

critical parameter employed, i.e., normal lifetime. Finally, the backgrounds and training given to the decision-making personnel are likely to vary considerably. This would be complicated even more seriously if a variety of manuals of normal lifetimes were used, particularly if some of the manuals were inconsistent with each other.

The final weakness in assuming a precise estimate of the life of an asset is that there is no substitute for experience. This means, in effect, that whether the figures used are right or wrong, they will have to be accepted or rejected largely on faith, without empirical verification for a considerable number of years. It seems much more appropriate to build such an unstable and unreliable figure into the process of decision making as late as possible rather than as one of the first assumptions. This will increase the opportunity for flexibility in consciously changing the "assumed" values of the critical parameters.

C. Mathematical Complexity

A second basis for comparing the procedures is the matter of mathematical complexity. The standard formulae have been developed over a considerable span of time. A person's first introduction to time preference and interest rates usually takes place in terms of the concept of simple interest. The total expected future gain is divided by the original investment, and this in turn is divided by the number of years involved to give an annual rate of return. As operating techniques become more sophisticated, it is realized that the initial returns may be reinvested so the idea of compound interest is introduced. The evolvement of bond markets with regular periodic payments has led to the construction of annuity formulae and tables, and the

habits of keeping books on an annual basis has led to their application to problems of expected profits and other gains. The net result has been that the vast majority of people who use these formulae and tables have learned to develop confidence in the answers they yield because they work rather than from an understanding of the mathematics and the assumptions underlying their computations. Much of the material in the first technical appendix showing the equivalence between the two procedures seems mathematically intricate or complex. This complexity arises not because of any inherent difficulty in the suggested procedures but rather from the inherent complexity of the accepted formulae. The mathematical complexity of the procedures we suggest is evolved in stages in the selection process, enabling the operating personnel at each level to understand the methods they employ and the assumptions necessary for their decisions.

D. Definition of Profits

A third basis of comparison might be summed up in the question, "When are profits really earned?" It is quite apparent that if all expectations are realized a variety of ex post rationalizations will all be found to have been fulfilled. A more interesting question is what happens when one or more of the expectations "goes sour." Suppose that the expected life of an asset is 25 years and that using straight line methods four per cent per year is set aside for depreciation. To adopt the other figures from the example presented in Appendix IV on accounting procedures, we further assume that the market interest rate is 10 per cent and the expected gross annual return will be 15 per cent. In this case costs will be defined as 4 per cent depreciation plus 10 per cent interest or 14 per cent. If an annual return is realized at 15 per cent, the profit each year will be recorded in a dollar amount which is

equivalent to a 1 per cent additional return on capital. Thus, in a real sense, depreciation, interest, and profits are all averaged over the expected life of the asset. If, in fact, the asset were to last thirty years, profits would suddenly jump to 5 per cent of the original capital during the last five years when the asset had been fully depreciated. Looking back, the accountant would have regrets for not having set the appropriate depreciation allowance at only 3 1/3 per cent per year. Had he done so, realized profits in the earlier 25 years would have been 1 2/3 per cent of investment capital instead of only 1 per cent (all of this over and above the normal rate of return, or interest on capital of 10 per cent).

A more interesting question is what would happen if the asset were suddenly and unexpectedly to become useless from wear or obsolescence after 17 years. At this point the accumulated depreciation reserves would be 68 per cent of the original investment. No one would seriously expect the investor to describe this project as a desirable one. Although an 11 per cent return (1 per cent above normal) had been earned for all of these years, the investor would hardly regard the loss of 32 per cent of his capital as a minor inconvenience. It is much more likely that a reassessment would be made, showing the appropriate depreciation to have been about 6 per cent per year (rather than 4 per cent). Now the return received would turn out to be slightly over 9 per cent a year involving a loss of about 1 per cent per year instead of a profit when compared with the normal interest return of 10 per cent.

It would appear to be clearly preferable, therefore, to assume that the first requirement of any project or program is the recouping of the original investment. If that is the case, all of the return should be allocated to

this purpose until it has been fulfilled. Only then, to the extent that additional returns are earned, is it possible for the project to begin to earn interest. Presumably this interest is expected to reach or surpass a normal rate of return or the project would not be undertaken in the first place. If and when these expectations are realized, the computed internal rate of return can be permitted to continue to rise or the excess over and above normal can be denoted to be a profit. In one sense, it is quite amazing that the economists who have been so diligent in insisting on using marginal rather than average concepts in both micro- and macro-economics have so uncritically accepted the implicit averaging over time involved in almost all time preference models used for project selection.

In summary, a comparison of the suggested procedures for handling time preference in project selection with the more widely employed methods suggests a case for theoretical and operational superiority of the former. The procedures presented in this paper are based on assumptions which are at least as realistic as those underlying the conventional methods; they involve simpler mathematics, at least from the viewpoint of the non-college graduate; and they are superior in recognizing the marginal nature of profits and their extreme uncertainty.

IV. IMPLICATIONS FOR DEVELOPMENT STRATEGY

In addition to the advantages discussed above, our suggested procedures might well have significant implications for a country's over-all strategy of development. Presumably the funds and other resources devoted to a project selected according to our criteria could have been devoted to other worthwhile uses. These alternative uses can be thought of as being postponed until they

can be financed from the accumulating depreciation reserves of the projects which have been adopted. Thus, in a dynamic sense, the crucial decision is not whether a particular project is adopted but rather whether it is adopted now. In this context, the price of gaining immediate command of investible resources is a commitment to repayment of the original capital in full as rapidly as is feasible. Over and above the repayment by means of the depreciation reserves, the society may be thought of as having an additional claim subsequent to complete accumulation of the depreciation reserve. This is the normal interest. It also is part of the price which the project must pay to secure resources now instead of in the future. The society could permit this interest to be used in a number of ways. The government might wish to allow such resources to be expatriated in fulfillment of earlier promises used to lure foreign capital into the country; it might consider such interest to be peculiarly subject to governmental taxation so that the resources it represents can be administratively directed by government; it might permit these funds and the resources represented thereby to be, at least partially, diverted to increasing consumption levels so as to encourage domestic savings; or it might insist that the funds and resources involved be used to create new capital expansion to further the general growth of the society under a wide variety of alternative conditions. In any case, a meeting of these prescribed conditions can be properly regarded as part of the price paid for a high priority consideration. If this project had not been undertaken, presumably another one would have been which would have been willing to pay the prescribed price.

Once the depreciation and normal return have been recovered, further returns until the end of the useful life of the project can be properly regarded as profits. Although, in this instance also, the society is perfectly free to

set the rules governing the use of claims to resources, these profits are more directly associated with the project since they would apparently not exist if one of the marginal projects had been substituted in its stead. For this reason, perhaps, perfect consistency in the use of profits generated by different projects is not necessary, and under certain conditions a case could be made for allowing the resources they command to be diverted to frivolous or luxurious consumption expenditures or any alternative that might be conceived. Here the choice depends on how the political winds are blowing and perhaps on the expected incentive effects.

A final word is appropriate to avoid a misconception of the general approach represented in this paper. It might be thought that the suggested procedures are temporary rules of thumb which give approximate decisions but which eventually must be superseded by the "normal procedures" which have been widely adopted in the developed world. In our view, however, the suggested procedures are merely the beginnings of what could be developed into a fully self-sufficient system for evaluating the time preference dimensions of all decisions which would be made at all levels within any particular society or group of societies.

Admittedly, this point of view is likely to create initial difficulties for economists and businessmen trained in the West. Such difficulties are inevitable; all of us tend to regard the familiar as normal. If it is true, however, that our framework is more appropriate to less developed countries than the conventional one, the basis for effective operational decisions in project selection would become sounder as the methods were more generally applied and improved. Moreover, the necessity of learning to translate accepted business and economic principles into a slightly new conceptual framework would itself be an educational process for foreigners participating

in the development of less developed countries. There is much room for critical re-evaluation of the assumptions on which our thinking about less developed countries is based. Re-evaluation might help to create a willingness to reject assumptions which work well at home but are not applicable in the quite different circumstances of less developed countries. We note that a comparison of the suggested new procedure with the normal more conventional ones indicates both procedures will approve of a good idea and will reject a bad one under conditions of full knowledge.

The major circumstances in which it will be necessary to translate a project or program into more conventional terms arise when substantial contributions of foreign capital are needed. Whether these are supplied by private foreign investment or a request is being made to include them in some foreign aid program, the rationale must be explained in "developed country" terminology. In this case the junior staff personnel in the planning bureau should be encouraged to treat the conventional procedures as a sort of folk custom, widely accepted, by Western economists and businessmen. Exposure of this kind would acquaint these officials-in-training with a sample of the problems faced by all people as they move from one culture into another. As a side effect, it might contribute to a worldwide redistribution of humility and self-confidence to the benefit of all concerned.

APPENDIX I: GENERAL THEORETICAL CONSIDERATIONS

The main part of this work is concerned with the decision-making process in choosing between alternative projects or programs involving expenditures in the present in anticipation of a stream of benefits which will be received over a period of time stretching into the future.

In discussing this material the model used will strike some as being rather unnatural in comparison with the standard formulae used for computing interest, annual returns, and annuities, etc. The problems which are raised in this manner are likely to seem most serious to those readers who are most familiar with the technical aspects of these questions and who are, therefore, best equipped to follow a discussion carried out in more technical terminology. Since one of the major arguments for the procedures discussed is that they greatly simplify the subject matter for the non-specialist, it seemed preferable to relegate the pedantic minitiae to a technical appendix. This also provides a place to discuss best certain other criticisms which may seem to be potentially serious at first glance but which on further consideration do not turn out to be substantively important.

The first appropriate step is to review briefly the assumptions on which the argument is based. The major set of assumptions relates to the ability to measure the capital costs involved in creating the project, the value or cost of the current inputs of materials or services during each time period in the future, and the value of the resulting output of the project during each time period in the future. All of these are to be valued without any consideration given to time preference or interest payments. The analysis then proceeds to introduce the appropriate modifications to take care of time preference and/or interest. To many people these assumptions may seem unrealistic - a point of

view shared by this author. The point being argued is that if these costs and values are known, most of the differences between simple and complicated techniques for handling the time preference problems are insignificant; and if these costs and values are not known, most of the expenditure of time and effort on refining time preference computations involves a misdirection of scarce technical manpower and ability.

In addition to the foregoing, conventional procedures for treating time problems make two other assumptions. These are unnecessary and/or not valid and hence are not used in this analysis. The first is the assumption that at the time a decision is made to accept or veto a particular project it is possible to select a single cardinal number which is a useful and meaningful estimate of the economic life of the project. Once this number has been selected, it is possible to amortize the depreciation over the "useful life of the project," to calculate an internal rate of return on the investment which the project will earn, and to compute an anticipated rate of "profit" which the project will earn over and above the "normal rate of return" or the "internal rate of return on the marginal project." In practice this approach usually assumes that the depreciation, the interest, and the profit are all equally spread over the life of the project.

The analysis used here assumes that the economic life of the project is not uniquely determined in advance although something is known about the range and probability distribution of the expected economic lives. A major purpose of this appendix is to show that in the special case where it can be assumed that this expected lifetime can be precisely forecast, the decisions resulting from this analysis will agree in all important respects with those reached through more conventional procedures. It is also true that when ex post

analysis shows that the original forecasts of useful economic life are wrong and the conventional calculations have been revised to include this new, more accurate, information, again the two analyses will be in full agreement in all important respects.

As mentioned above, the third set of assumptions used in conventional analysis is that depreciation, interest, and profit are all earned proportionately over the life of the project. An alternative assumption which is used here is that the funds which ordinarily go to these three accounts are pooled in a concept called the gross annual return. The gross annual return is credited to the depreciation reserve until that reserve is equal to the original investment. This occurs at a time designated t_o. After t_o all of the gross annual return plus any interest earned by the reinvested depreciation reserve is allocated to payment of the back interest accrued while the depreciation reserve was being accumulated. The back interest will have been fully paid at time t_i. After time t_i the entire gross annual return is available for profits because presumably the reinvested depreciation reserve adequately covers the current interest liability. Without repeating all of the arguments justifying this assumption, perhaps two points should be made. The first point is that there is no "correct" method of identifying what funds are attributable to depreciation, interest, and profits; the conventional rule of thumb is one of a vast number of possibilities which can be theoretically justified. It also has a long history of acceptance in the business mores of Western society. The second point is that in almost all situations it is foolish to talk about a positive rate of return when the capital itself is lost or to speak of a "profitable" investment which did not earn at least normal interest on the capital invested in addition to building up an adequate depreciation reserve.

Turning to the formal aspects of the theory, conventional procedures attempt to define a precise point or boundary line (on a graph) separating the projects which are approved from those which are not approved. Only those projects which are exactly on that line are defined as marginal. Here, however, we have chosen to define marginal projects as a set which would show up on a graph as a band having a "width" which is small but non-zero. Thus the limit or line which separates the accepted projects from the marginal projects is not identical with the limit or line which separates the marginal projects from the rejected projects. The analysis is carried out in terms of separate estimates of these two limits. It is shown that the "conventional" answers always lie between these two limits and that the distance between the limits is strictly controllable. If desired, by gradually making the analysis more detailed, these limits can be made essentially identical to each other and, therefore, to the conventional answers. Since knowledge and trained manpower are not free goods, presumably the analysis would never be carried to this extreme.

APPENDIX II: DEPRECIATION

The key variable in this analysis is called the gross annual return per unit of investment. Gross, because no account has been taken of time preference or interest; annual return signifying the value of the output during one year less the value of the physical inputs (including labor) received and used up during one year all divided by the initial investment value. A unit of investment thus becomes the numeraire or standard of value in which output, inputs, and returns are measured. As indicated in the definitions, the gross annual return per unit of investment is designated by the symbol A. It will presumably always be less than 1, since any item that can pay for itself in less than a year will be expensed rather than capitalized. It would be possible to put a time subscript on the symbol A so that it would have a distinct value for each year in the future - positive until the original investment was scrapped and zero after that. We will assume A is constant over time in accordance with usual practice. A complementary variable, a, is defined as the natural logarithm of (1 + A) so that $e^a = (1 + A)$. "a" thus becomes the internal rate of return (gross) under assumption of instantaneous compounding computed on the basis of a single year of operation.

As a first step, we might assume that the market rate of return is zero so that the entire gross annual return for the life of the project is divided between depreciation reserve and profits. As indicated earlier, it is assumed here that all of A is allocated to depreciation reserves until a time, designated t_o, when the accumulated reserves are equal to 1. After t_o the entire gross annual return is available as profits.

Our initial estimate of t_o is made by

$$t_o = 1/A \qquad (1.1)$$

which is essentially the old "pay out period" approach. This formula is consistent with the assumption that as the money is earned, it is not reinvested but is held idle. Under these extreme assumptions A need only be an arithmetic average of the annual returns during the period in question as there is no penalty or premium attached to promptness or lateness in realizing gains.

Perhaps at the opposite extreme is the assumption that all reserves can be instantaneously reinvested at the internal rate of return which is presumably at or above the rate available on marginal investments. Under these assumptions the initial investment of 1 is allowed to grow internally until it reaches the level of 2, as shown below.

$$(1 + A)^t = e^{at} = 2 = e^{.69315} \tag{1.2}$$

$$at = .69315 \simeq .70 \tag{1.21}$$

$$t_o = \frac{.70}{a} \tag{1.22}$$

$$t_o = \frac{.70}{A} \cdot \left(\frac{A}{a}\right) = \frac{.70}{A} \cdot C_a \tag{1.23}$$

where $C_a = \frac{A}{a}$

The above formulae are largely self-explanatory. The natural logarithm of 2 is .69315. Here and elsewhere in this paper this is rounded out to .70 resulting in a slight overestimation of the value of t_o.

We will return to the point shortly, but let us initially assume that C_a is equal to 1 and can, therefore, be ignored. Then the estimate of t_o obtained from formula (1.23) is .70 as large as the estimate obtained from (1.1). These extremes determine the probable maximum and minimum values for t_o under zero interest rate conditions. For example, if A is equal to .10, using formula (1.1) we find that it would pay for itself in approximately ten years.

This assumes that there are no gains at all from reinvestment of earnings or other types of compounding. From formula (1.23) we know that this estimate could potentially be as low as seven years. This would be on the assumption of instantaneous reinvestment and compounding at the very favorable internal rate of return.

In this example the actual value of a is .09531 which is the natural logarithm of .10. Therefore, the appropriate value of C_a is approximately 1.05 giving a formula (1.23) estimate of t_o at 7.35 years. If this computation were carried out more precisely with less round off, the value would be 7.27.

We might turn now and consider the factor C_a. The easiest way to do this is to begin with its inverse $\frac{1}{C_a}$ which is equal to the natural logarithm of (1 + A) divided by A as shown below:*

$$\frac{1}{C_a} = \frac{a}{A} = \frac{\ell n\ (1 + A)}{A} \tag{1.3}$$

$$\frac{1}{C_a} = \frac{1}{A}\left[A - \frac{A^2}{2} + \frac{A^3}{3} - \frac{A^4}{4} + \frac{A^5}{5} \ldots\right] \tag{1.31}$$

$$\frac{1}{C_a} = 1 - \frac{A}{2} + \frac{A^2}{3} - \frac{A^3}{4} + \frac{A^4}{5} \tag{1.32}$$

These formulae are valid as long as the value of A is between zero and 1. This covers all relevant cases because a value of A greater than 1 implies that the investment pays for itself in less than one year. In fact, the maximum relevant value of A might be set at .50 or at .3333. The logic of this is that such investments would pay for themselves in two or three years. If that type of project is truly marginal from the time preference or interest cost

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* The symbol ℓn denotes the natural logarithm of a number; i.e., ℓn A means the natural logarithm of A.

standpoint, it is likely that the relevant analysis should be carried on on a monthly rather than an annual basis. Formula (1.32) is an absolutely converging series. For values of A close to zero, $\frac{1}{C_a}$ will be very close to 1 because all terms after the first will be close to zero. If A is set at .33, the actual value of $\frac{1}{C_a}$ is .86304. The rapidity with which formula (1.32) converges is shown by the fact that the first two terms give a value of .835; the first three terms give a value of .871; and the first four terms a value of .862. In each case, of course, the remaining error is less than the value of the last term used. For all values of A between zero and .333 the value of $\frac{1}{C_a}$ in formula (1.32) will lie in the range of .8634 and 1.0. Consequently, the values of C_a will lie between the inverses of these two limits, that is, between 1.0 and 1.16. Placing these values into formula (1.23) and simplifying we get the two estimates in formula (1.41) and (1.42) below.

$$t_o = \frac{1}{A}(.70)(1.0) = \frac{1}{A}(.700) \qquad (1.41)$$

$$t_i = \frac{1}{A}(.70)(1.16) = \frac{1}{A}(.812) \qquad (1.42)$$

Compare these formulae with the original one, namely (1.1). Taking account of instantaneous compounding reduces the effective pay-off period by twenty to thirty per cent, the percentage within this range depending upon the particular internal rate of return used. Restricting the analysis still further to values of A of .10 and below (accrued pay out periods of over ten years) permits formula (1.43) below to be substituted for (1.42).

$$t_o = \frac{1}{A}(.70)(1.05) = \frac{1}{A}(.735) \text{ for } A = .10 \qquad (1.43)$$

If this is done, the "bias" from neglecting compounding becomes quite predictable at 26 to 30 percentage points. Compared to the accrued pay out period approach used in formula (1.1), the appropriate adjustment is almost

entirely a matter of whether or not gross annual returns are or can be reinvested so that earnings are compounded. Once that decision is made, the internal rate of return is practically negligible in terms of its effect on the adjustment factor, although, of course, it entered into the original estimation of t_o in (1.1). Where the nature and degree of reinvestment is highly uncertain, it is appropriate to retreat to the earlier broad limits in equations (1.1) and (1.22). Although this gives a less precise answer, it is an answer in which we can place confidence. This contrasts with more conventional procedures which imply that the deviations from the smooth flow assumptions are small enough to be safely ignored.

It is perhaps interesting to note that the items with the high internal rate of return and, hence, the rapid pay offs are the ones which create the most trouble in this type of analysis. They are also the ones which are most likely to be summarily included by any criteria because the interest element is so relatively unimportant. On the other hand, we might initially suspect that the long-term investments involving relatively low gross annual returns would produce the most difficulty when compared with conventional "precise" procedures. It is, however, in this very area that the two procedures are likely to lead to almost identical decisions. This is on the assumption a single procedure is consistently used for all decisions throughout the particular society. It is uncritical switching between the pay-off period approach and the conventional procedures which are likely to lead to inconsistent and wrong decisions. Even this type of switching is possible, however, if appropriate correction factors are used. In this section we have developed one and found that it varies within surprisingly narrow limits. In later sections it will be shown that other corrections are appropriate as the

analysis becomes more sophisticated and more complex. Working with the idea of bounded limits rather than a single precise figure greatly simplifies these corrections. It also makes the degree of complexity and precision itself a variable subject to precise control by the senior operating economists.

At this point, it is perhaps appropriate to introduce the assumption of a non-zero rate of return on the marginal investment. We turn to that in the following section.

APPENDIX III: INTEREST

At this point we can drop the assumption of a zero interest rate. This does not necessarily mean that the model assumes a Western style financial market. Bearing in mind that the emphasis here is on decision making, the major concern is with the interest rate as a rationing price. In the real world capital is not a free good, and it is not possible to undertake every investment which will eventually cover its depreciation. In conventional terms we might think of computing the internal rate of return of all feasible or possible investments. If these were written down in a single long list, we would place the investment with the highest internal rate of return at the top of the list and the one with the lowest rate of return on the bottom of the list so that all of the investments were rank ordered. Then we would commit funds to each of the investments in order until the capital resources were exhausted. There would exist two items on the list next to each other, the last one which was included and the one immediately below it which was excluded. Presumably, the internal rate of return on these two items would be almost identical and this is the internal rate of return on the marginal investment or the marginal rate of return. In a perfectly competitive laissez-faire type capital market, presumably the market rate of interest should also approach this figure.

The rationing function of the interest rate is to assure that none of the scarce capital is employed on submarginal investment projects. Since all capital is assumed to be employed, such expenditure would mean that a shortage or bottleneck would be created somewhere and that some more desirable project would be eliminated or unnecessarily delayed. By introducing the interest rate as a cost - even a fictitious or shadow cost - it appears that the submarginal

projects are operating at a "loss," and therefore they will not be undertaken. As a result, sufficient resources would be available so that all of the more worthwhile projects will be completed. Thus, the rationing function of the interest rate is completely negative, a disincentive, and applicable to analysis of all types of economy. The important thing is that the resources represented by the interest be taken away from each project. At this point, what is done with these resources is completely irrelevant. Passing these resources on to selected individuals or institutions within the society is likely to have important effects on the matrices of income distributions and incentive effects. It is in these areas, which have nothing to do with capital rationing, that the most important differences among various economic systems occur.

There are a variety of definitions of the interest rate. The definition used here will be the gross annual return on a perpetuity which has a present value of 1. Being a perpetuity, no allowance will have to be made for depreciation. To maintain comparability with other symbols being used the interest rate on an annual basis will be given the symbol R, and r will be the natural logarithm of the term (1 + R). In this section both the depreciation and the interest are treated as costs and, therefore, the entire gross annual return is allocated to these two items until they are fully covered. Only then is any of the return available for profits. The task here is to find the value for t_i which is the point of time in the future at which profits begin to become available.

Throughout this section the term (A - R) will frequently appear. The easiest verbal interpretation of this is that in each year the interest charge is deducted first and the remainder is available for depreciation. This might seem to reverse the earlier assumption that depreciation has the first priority.

This is, however, only a mathematical quirk. At t_i both the depreciation and the interest have been fully covered and, as is shown in the appendix on accounting, taking out the depreciation before anything is taken out for interest has no effect on the value of t_i.

As the first approximation to t_i we again assume that depreciation reserves cannot be reinvested and, therefore, earn no return. If R of the gross annual return A must be set aside each year for interest payments, then the sum A - R is available for depreciation. The estimate of t_i is then given by $t_i = \frac{1}{A-R}$ (3.1)

For example, if A is .27 and R is .22, the estimate for t_i will be 20 years. It is now apparent why the term (A - R) is so crucial. When A and R are close together, very little will be left to cover depreciation, and the project will be worthwhile only if it has a long lifetime. At this level of analysis only the difference matters and the result would be the same if A were equal to .12 and R equal to .07. It is also obvious that A must be larger than R, or equal to it in the case of a non-depreciating investment. The relationship between these two variables is so critical that not only the difference but also the ratio between the two frequently enters into the formulae. By definition, R/A is equal to g.

The close resemblance of (3.1) to (1.1) suggests the interesting question, what is the equivalent to (1.23)? If we assume that to some extent, at least, the accumulating depreciation reserves can be reinvested, the question is at what rate of interest will this reinvestment take place? In order to avoid the argument for the time being we will specify a third variable B, the annual rate of return on reinvested earnings, and its complement b, the natural logarithm of (1 + B). On this basis the general formulae can be developed.

Later B can be set equal to A, or to R, or to any other value or bounded set of values, to create a variety of special cases.

If the project is obligated to make an interest payment of R each year, then the remainder of the gross annual return, namely the amount A - R, is available for investment at the rate B.

$$\text{Amount of Annuity} = \frac{(1 + B)^t - 1}{B} = \frac{e^{bt} - 1}{B} \tag{3.21}$$

Formula (3.21) gives the amount of an annuity of unit value per period after a term of t periods at a rate of interest of B per period. Since in fact the amount available is (A - R), formula (3.21) is multiplied by this term to give the left side of formula (3.22) below. When this term has grown to equal one, the depreciation reserve will be equal to the original investment and the gross annual return will be available for profits. Therefore, the right side of (3.22) has been set equal to one and in the following equations equation (3.22) has been solved for the value of t. t is an alternative figure for t_i based on the above assumptions.

$$\left(\frac{e^{bt}-1}{B}\right)(A-R) = 1 \tag{3.22}$$

$$e^{bt} = \frac{B}{A-R} + 1 = \frac{A-R+B}{A-R} \tag{3.23}$$

$$bt = \ln\left(\frac{A-R+B}{A-R}\right) \tag{3.24}$$

$$-bt = \ln\left(\frac{A-R}{A-R+B}\right) \tag{3.25}$$

$$t = \frac{1}{b}\left[\ln\left(\frac{A-R}{A-R+B}\right)\right] = -\frac{1}{b}\left[\ln\left(1 - \frac{B}{A-R+B}\right)\right] = -\frac{1}{b}\left[\ln\left(1 + \frac{-B}{A-R+B}\right)\right] \tag{3.26}$$

If a table of natural logarithms is available and the values of A, R, and B are all known, it is not a difficult task to solve equation (3.24) or

(3.25) to obtain a precise value for t. For reasons which will become apparent as we progress, this is not our major objective here. In (3.25) use has been made of the fact that the logarithm of an inverse is the negative of the logarithm of the original term. When this is done, the term $(\frac{A-R}{A-R+B})$ must lie in the range zero to one for all admissible values for the variables. Use is made of this fact in (3.26) and (3.27) in which the logarithms are evaluated by means of a convergent infinite series. The final estimate is given in (3.28).

$$-bt = (\frac{-B}{B+A-R}) - \frac{1}{2}(\frac{-B}{B+A-R})^2 + \frac{1}{3}(\frac{-B}{B+A-R})^3 \ldots\ldots \tag{3.27}$$

$$bt = [(\frac{B}{B+A-R}) + \frac{1}{2}(\frac{B}{B+A-R})^2 + \frac{1}{3}(\frac{+B}{B+A-R})^3 \ldots \frac{1}{n}(\frac{-B}{B+A-R})^n] \tag{3.28}$$

$$t_\ell = \frac{1}{b}(\frac{B}{B+A-R})[1 + \frac{1}{2}(\frac{B}{B+A-R}) + \frac{1}{3}(\frac{B}{B+A-R})^2 \ldots \frac{1}{n}(\frac{B}{B+A-R})^{n-1}]$$

$$= \frac{B}{b}(\frac{1}{B+A-R})[1 + \frac{1}{2}(\frac{B}{B+A-R}) + \frac{1}{3}(\frac{B}{B+A-R})^2 + \ldots \frac{1}{n}(\frac{B}{B+A-R})^{n-1}] \tag{3.29}$$

The left-hand term has been given the notation t_ℓ to indicate that it is a lower limit. As additional terms in the series within the brackets are included, the value of t_ℓ gradually increases approaching the value in equation (3.22) "from below." Thus equation (3.29) is a lower limit on the minimum life the project must have to "pay for itself."

(3.28) and (3.29) are at the same time very useful and potentially dangerous. As is illustrated in many subsequent examples, they frequently converge quite rapidly. In these cases only a few terms in the series in the brackets need to be computed. However, the series is not absolutely convergent and we are not sure at any point how large the remaining approximation error is. Where B is large relative to the expression (A-R), the value of the

expression $(\frac{B}{B+A-R})$ will be very close to one. If this expression were one, the series within the brackets would be harmonic series which do <u>not</u> converge but expand without limit. For this reason it is necessary to derive an equation for t_u which is an upper limit for the value of t in (3.22). In this way the residual approximation errors will be brought completely under control.

The first step is to introduce an intermediate variable

$$x = (\frac{B}{B+A-R}) \tag{3.30}$$

With this simplified notation (3.28) becomes

$$\underset{n \to \infty}{bt} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} \ldots\ldots \frac{x^n}{n} \tag{3.31}$$

It is now possible to define a set of U variables, each of which will be larger than bt providing only that x is less than one. The first of these is

$$\underset{n \to \infty}{U_1} = x + x^2 + x^3 + x^4 \ldots x^n \approx \frac{x}{1-x} \tag{3.32}$$

The value of U_1 is determined by the well known formula for the sum of a geometric progression, as shown. Every term in (3.32) is greater than or equal to the corresponding term in (3.31) and therefore U_1 will always be greater than bt in (3.31). The difference between the expressions in (3.32) and (3.31) is quite large except for the first term. Therefore, a new expression U_2 is defined as

$$\underset{n \to \infty}{U_2} = x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{2} \ldots\ldots \frac{x^n}{2}$$

$$U_2 = U_1 - \frac{x}{2}(U_1) = (1 - \frac{x}{2})\, U_1 \tag{3.33}$$

U_2 will also always be larger than bt in (3.31) but will be smaller than U_1 in (3.32) as is evident from a term-by-term comparison. As shown, it can be

expressed as a function of U_1 and x.

In similar fashion it is possible to define U_3 and U_n as follows:

$$U_3 = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{3} + \frac{x^5}{3} \ldots \frac{x^n}{3} = U_2 - (\frac{1}{2} - \frac{1}{3})x^2(U_1) = (1 - \frac{x}{2} - \frac{x^2}{6})U_1 \qquad (3.34)$$

$n \to \infty$

$$U_n = U_1 \; [1 - (1 - \frac{1}{2})x - (\frac{1}{2} - \frac{1}{3})x^2 - (\frac{1}{3} - \frac{1}{4})x^3 \ldots - (\frac{1}{n-1} - \frac{1}{n})x^{n-1}] \quad \text{i.e.,}$$

$$U_n = U_1 \; [1 - \frac{x}{2} - \frac{x^2}{6} - \frac{x^3}{12} \ldots - \frac{x^{(n-1)}}{n(n-1)}] \qquad (3.35)$$

As n gets large in (3.35), U_n will approach bt in (3.31) as a limit. Since U_n is always greater than bt, it functions as a decreasing upperbound. t_u is defined as U_n divided by b producing the desired upperbound for the value of t in (3.22).

$$t_u = \frac{U_n}{b} = \frac{1}{b} \cdot U_1 \; [1 - \frac{x}{2} - \frac{x^2}{6} - \frac{x^3}{12} - \frac{x^4}{20} - \frac{x^5}{30} - \frac{x^6}{42} - \frac{x^7}{56} - \frac{x^8}{72} - \frac{x^9}{90} - \frac{x^{10}}{110} \ldots]$$

$$t_u = \frac{1}{b} \frac{(\frac{B}{B+A-R})}{1 - \frac{B}{B+A-R}} \; [1 - \frac{1}{2}(\frac{B}{B+A-R}) - \frac{1}{6}(\frac{B}{B+A-R})^2 - \frac{1}{12}(\frac{B}{B+A-R})^3 \ldots] \qquad (3.36)$$

$$t_u = \frac{1}{b}(\frac{B}{A-R})[1 - \frac{1}{2}(\frac{B}{B+A-R}) - \frac{1}{6}(\frac{B}{B+A-R})^2 \ldots]$$

$$t_u = (\frac{B}{b})(\frac{1}{A-R}) \; [1 - \frac{1}{2}(\frac{B}{B+A-R}) - \frac{1}{6}(\frac{B}{B+A-R})^2 \ldots] \qquad (3.37)$$

$$t_u = (\frac{B}{b}) \; [1 - \frac{1}{2}x - \frac{1}{6}x^2 - \frac{1}{12}x^3 \ldots]t_i \qquad (3.38)$$

In (3.38) x is again introduced to simplify notation and t_i is taken from (3.1). As already shown expressions of the type B/b tend to be predictably close to one, the difference being related to the difference between annual and instantaneous compounding. Interpreting t_i as a "pay out period" estimate, the expression in the brackets in (3.38) describes the reduction in the pay out period which is made possible by reinvesting .

accumulated depreciation reserves at a compounded rate of interest.

We now have developed two alternative estimates for a sophisticated pay out period - a lower bound in (3.29) and an upperbound in (3.37). In many, perhaps most, situations these estimates will rapidly converge, and it would make relatively little difference whether the lower bound or upperbound estimate were used. However, in certain cases, namely those for which x is close to one, the difference between the upper and lower bound can be quite great. To take an extreme case, which will be investigated further, assuming B = .50 and (A-R) = .01, the "one term" estimate of t_ℓ is 2.5 years and the "one term" estimate of t_u is 123 years. These two estimates rapidly approach each other as more terms are considered. The example is introduced at this point only to indicate that the difference between the two estimates is, at least potentially, serious. Rather than pursue the example further, we can attack the problem in a more general fashion. One way to do this is compute the ratio $\frac{t_\ell}{t_u}$ which, after simplification, becomes (3.39).

$$\frac{t_\ell}{t_u} = \left(\frac{A-R}{B+A-R}\right) \left(\frac{1 + \frac{1}{2}x + \frac{1}{3}x^2 \ldots\ldots}{1 - \frac{1}{2}x - \frac{1}{6}x^2 \ldots\ldots}\right) = (1 - x) \left(\frac{1+}{1-}\right) \qquad (3.39)$$

In the right side of (3.39) the first factor (within parenthesis) is the ratio of the "one term" estimates of the two bounds respectively. The second factor (within parenthesis) is the ratio of the two infinite series in the two estimates respectively. As these series are expanded, the entire product in (3.39) becomes equal to one since the upper and lower bounds approach each other. In the case of a one term estimate, however, the expression within the second set of parentheses is equal to 1 and the value of (3.39) is at its minimum, indicating the maximum discrepancy between the upper and lower bounds.

Thus, if the value of x in (3.39) were equal to .01, we could be confident that the value of the lower bound would begin at 99% of the value of the upper bound and the discrepancy between the two would diminish. In the extreme example just used, however, the value of x becomes .50/.51 which is equal to .98+. In this case, the first lower bound estimate is only 2% of the upper bound estimate, and this information is so imprecise as to be of little value at all.

Before proceeding with the numerical analysis of this extreme case, a brief look at its economic characteristics may be useful. In the first place, the assumed value of B is very high, .50 indicating a pay out period of only two years. In most realistic cases, it will not be possible to reinvest the depreciation reserves at a rate of return higher than the internal rate of return on the original investment. Therefore, the value of A is also likely to be .50 or greater. For simplicity we can set A = to .50. We have assumed (A-R) equal to .01 which then means that R = .49. This implies that the typical investment earned a rate of return of 49% per year over and above its necessary depreciation. This means that the typical investment is likely to have a gross annual return, a value of A, which is .60, .65, or greater. This would seem to imply a society in which capital was very scarce, and the current project under consideration is apparently truly a marginal project which will not create an available profit over and above depreciation and interest for some years. The question is whether it will be 2 1/2 or 123 1/2 years before such profits become available. Since this is only an example, by making use of the natural logarithm tables and (3.24) we can determine the value of t as follows:

$$t = \frac{1}{b}\left[\ln\left(1 + \frac{B}{A-R}\right)\right] = \frac{\ln 51}{\ln 1.50} = 9.65 \text{ years} \qquad (3.40)$$

It might seem at first glance surprising that the value of t is as low as 9.65 years. However, this is not a short period of time when set in a society where the normal rate of interest is 50%, implying accrued pay-out periods of 2 years or less.

It might be of some interest to see the pattern which emerges as the estimated values of t_u and t_ℓ approach each other in this case. Eliminating the intermediate calculations, Table 1 summarizes the results which would be obtained.

TABLE 1

Estimates of Pay-Out Periods as Number of Terms in Estimate is Increased

No. of Terms	t_u	t_ℓ	$\frac{t_u + t_\ell}{2}$
1	123.3	2.4	62.8
2	62.9	3.6	33.2
3	43.2	4.4	23.8
4	33.5	5.9	19.2
5	27.8	5.4	16.6
6	14.2	5.8	10.0
7	11.7	6.1	8.9

B = .50 A = .50 R = .49

The first column specifies the number of terms used in making the estimates; the second column is the estimate of t_u based on the number of terms in the first column; the third column is the estimate of t_ℓ based on

that number of terms; and the fourth column is the average of the upper and lower limits based on the number of terms in the first column. For example, if five terms are used, the upper limit estimate is 27.8 years; the lower limit estimate is 5.4 years; and the average of the two is 16.6 years. In the early stages, the average is above the actual value of 9.65 years which we obtain in (3.40), but in the seventh and later stages it does not hold true. As pointed out in the main part of the article, for the project which is truly marginal, the final decision will probably have to be made by highly trained personnel.

By way of contrast with the previous example, we might consider a case which is somewhat more typical. Table 2 is similar to Table 1 but applies to a case with an internal rate of return (gross) of 25% and a normal rate of 15%. Depreciation is assumed to be reinvested at the normal rate of return. As shown, a "one term" estimate of the pay-out period is from 4 to 10 years. Using only three terms, this is reduced to the range 5.6 to 6.4 years; and using six terms, the range becomes 6.05 to 6.15 years. Strictly speaking, each of these must also be multiplied by 1.073, the value of $(\frac{.15}{\ln 1.15})$.

TABLE 2

No. of Terms	t_u	t_ℓ	$\frac{t_u + t_\ell}{2}$
1	10	4	7
2	7	5.2	6.1
3	6.4	5.6	6.0
6	6.15	6.05	6.1

B = .15 R = .15 A = .25

In practice, it is quite likely that a specific "rule of thumb" using a small number of terms would take care of most cases. The exact specification of what the rule of thumb should be must, however, depend on experience.

The development of such "rules of thumb" would simplify the necessary mathematics well below the university level. This would make it possible to utilize the grass roots know-how of the intelligent worker with practical knowledge of the job but with more limited formal education.

APPENDIX IV: ACCOUNTING AND BOOKKEEPING

Although the precise details would have to be worked out by persons qualified in the field, some idea can be given of the effect the suggested procedures would have on the bookkeeping done by the individual firm or decision-making unit. To this end, Tables 3 and 4 have been constructed. They are largely self-explanatory. In order to maintain comparability, a gross annual return of 15 per cent, a normal or market interest rate of 10 per cent, and an expected life of 25 years has been maintained throughout both tables. In Table 3, it is assumed that the accumulated depreciation reserves do not earn a positive rate of return. Interest liability attaches not only to the original but also to any accumulated back interest. In Table 4, it is assumed that the depreciation reserves are reinvested at the market rate of interest, namely, 10 per cent. In the "normal accounting procedures" these additional earnings are immediately added to profits; in the proposed procedures they are used to hurry up the repayment of capital. The concept of repayment of capital has been introduced in Table 4 as being more general than a depreciation reserve. There is no reason why this "repayment" cannot be made into a special reserve fund. This fund could then be reinvested at the market rate of 10 per cent which would mean that the accumulating back interest in column 3 and the interest payments in column 5 would be appropriately larger.

As indicated elsewhere, if all flows are smooth and all expectations are realized, a choice can be made at random among many formerly equivalent procedures. The real test comes in terms of how much readjustment must be made when the unexpected happens. On this test the suggested procedures will perform quite well.

TABLE 3

Comparison of Alternative Implicit Accounting Procedures

	Suggested Accounting Procedures					Normal Accounting Procedures		
Time	Depreciation Reserve	Accumulated Back Interest	Depreciation Payments	Interest Payments	Profits	Depreciation Payments	Interest Payments	Profits
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	.00							
1	.15	.10	.15	.00	.00	.04	.10	.01
2	.30	.21	.15					
3	.45	.33	.15					
4	.60	.46	.15					
5	.75	.61	.15					
6	.90	.77	.15	.00				
7	1.00	.80	.10	.05				
8	.00	.73	.00	.15				
9		.65		.15				
10		.57		.15				
11		.47		.15				
12		.37		.15				
13		.26		.15				
14		.13		.15				
15		.00		.15	.00			
16		.00		.10	.05			
17				.10	.05			
18				.10	.05			
19				.10	.05			
20				.10	.05			
21				.10	.05			
22				.10	.05			
23				.10	.05			
24				.10	.05			
25				.10	.05	.04	.10	.01
26				.10	.05	.00	.10	.05
27				.10	.05		.10	.05
....								

A = .15 = Gross Annual Return

R = .10 = Normal or Market Interest Rate

25 years = Expected Economic Life of the Project

Table 4

Comparison of Alternative Implicit Accounting Procedures with Re-Investment of Accumulated Reserves

	Suggested Accounting Procedures							Normal Acctg. Procedures		
Time	Inv.Capital	CumInterest Due (compounded)	Acc.Back Interest	Repayment of Capital	Int.Payments	Profits	Dep.Reserves	Deprec.	Interest	Profits
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	1.00									
1	1.00	.10	.10	.15			.00			.010 + .000 = .010
2	.85	.095	.195	.15			.04	.04	.10	.01 + .004 = .014
3	.70	.090	.285	.15			.08	.04	.10	.01 .018
4	.55	.084	.369	.15			.12	.04	.10	.022
5	.40	.077	.446	.15			.16			.026
6	.25	.070	.516	.15			.20			.30
7	.10	.062	.578	.10	.05		.24			
			.528							
8	.000	.053	.581	.00	.15		.28			
9	.000	.043	.431	.000	.15		.32			
10		.033	.474		.15		.36			
11		.021	.324		.15		.40			
12		.008	.357		.086	.064	.44			
13		.000	.207		.000	.15	.48			
14		.000	.228				.52			
15			.078				.56			
16			.086				.60			
17			.000				.64			
18							.68			
19							.72			
20							.76			
21							.80	.04	.10	
22							.84	.04	.10	
23							.88	.04	.10	.098
24							.92	.04	.10	.102
25							.96	.04	.10	.106
26							1.00	.00	.10	.15
27							1.00	.00	.10	.15
28							1.00	.00	.10	.15

A = .15 Gross Annual Return R = .10 Normal or Market Interest Rate 25 years = Expected Economic Life of Project