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ESTIMATES OF LABOUR AGGREGATION FUNCTIONS

by

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ABSTRACT

In the present paper, both cross-section and time-series data are used to estimate aggregation functions for labour. Two separate classifications of labour are used, one defining categories by occupation, the other defining them by educational level. The elasticities of substitution are estimated for both occupational classifications from cross-section data for the states of the U.S.

Results of the tests run indicate that, for the purpose of estimating total growth in output due to labour, a time-dependent CES production with a fine classification of labour should yield the most accurate predictions, providing that the shift parameters can be estimated.

For educational planners, the most obvious implication of these results is that the manpower requirements approach should be abandoned. In fact, rate of return analysis with constant relative wage rates, implying an infinite elasticity of substitution, would be a better tool for planning, at least in the short run.

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C R S Dougherty
February 1971

ESTIMATES OF LABOUR AGGREGATION FUNCTIONS *

The progressive sophistication of the theory of production has inevitably generated dissatisfaction with the treatment of labour as a homogeneous input and has stimulated attempts to disaggregate it. At the same time the surge of interest in educational planning has contributed an additional powerful impetus to research in this field through the need to improve understanding of the inter-relationships between education, productivity, and the structure of the labour force.

All studies of labour disaggregation automatically belong to one or the other of two general approaches according to their use of the criteria chosen for identifying different types of labour. In one approach the criteria are used to classify the labour force into a number of mutually exclusive categories which are then treated as separate and independent factors of production. In the second, the criteria themselves are quantified and treated as factors of production.

The first approach, which has almost invariably employed either occupation or, in an ordinal sense, years of schooling, as a basis for classification, has been developed with most enthusiasm by educational planners in an effort to relate the growth of an economy in a tangible and uncomplicated way to the derived demands imposed on the educational system. The second approach, which has usually employed wage-weighted measures of skill or education as criteria, embraces the construction of indexes of labour efficiency and the theory of human capital, and has been used primarily for the purpose of accounting for growth.

So little empirical work on aggregate production functions has been done with either approach that it would be premature to speculate which is more satisfactory or promising. The only research to date using the second approach

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* I am indebted to Yoram Ben-Porath, Zvi Griliches, Richard Layard, George Psacharopoulos and Adrian Wood for helpful criticism and suggestions.

has been a series of studies by Griliches,[1] who has estimated Cobb-Douglas functions for the agricultural and manufacturing sectors of the United States with a labour quality index, usually based on education, as an argument of the function in addition to labour and capital.

Within the first approach, Leontief input-output analysis gained widespread and, on the whole, uncritical acceptance when Parnes (1962) used it as the basis for the manpower requirements approach to the planning of education, and until recently it has not suffered serious challenge in spite of the fact that no evidence has ever been produced to support the assumptions implicit in this specification of the production function. The most important assumption, of course, is that the elasticity of substitution between different types of labour is equal to zero. It is natural to question whether this is in fact the case and to hypothesize that the more general constant elasticity of substitution (CES) function might be more realistic.

A pioneering step towards testing this hypothesis has been taken by Bowles (1969). Classifying labour into three categories (persons with less than eight, eight to eleven, and more than eleven, years of schooling), he used employment and wage data from twelve countries to derive estimates of the elasticity of substitution between each pair of categories. The elasticity between the higher two categories was not significantly different from infinity, and hence they could be aggregated linearly into a single category. Bowles estimated the elasticity between this and the remaining category to be eight. Hence he constructed an aggregation function for the original three categories in the form of a two-level CES function.

In the experiments described in this paper, which belongs to the first approach, Bowles' techniques were developed for the purpose of constructing more detailed aggregation functions. Two separate classifications of labour were used, one defining categories by occupation, the other defining them by educational level. For both classifications employment and wage data for American states were used to estimate pairwise elasticities of substitution between the categories, and hence the parameters of multilevel CES functions

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1 Notably Griliches (1963) and Griliches (1967). For a summary of the whole series see Griliches (1970).

were estimated. Needless to say, there arise a number of theoretical issues in the estimation of such aggregation functions, particularly in the choice of assumptions and in the interpretation of the results. Since many of these issues are carefully discussed by Bowles, there would be little point in covering them exhaustively here. Instead the more important problems will be discussed when considering the results of the experiments.

Cases can be argued for defining labour categories either by occupation or by educational level. These are examined in Section I. Sections II and III describe the estimation of aggregation functions using the occupational and the educational definitions respectively. Section IV gives an account of sensitivity analysis designed to assess the consequences of taking one specification of the aggregation function instead of another. Section V discusses the behaviour of the wage structure over time and the problem of relative complementarity between skilled labour and capital. The concluding section discusses the implications of the results.

I The classification of labour inputs

Ideally one would like to identify all the characteristics responsible for the differences in productivity between individuals and to devise a classification system based upon them. At present this objective remains far out of reach. In the first place, in the absence of a direct measure of the productivity of an individual, his wage has to serve as a surrogate, entailing well-known but often exaggerated problems. Secondly, none of the growing number of studies on income generating functions has accounted for more than a modest proportion of the variance in the wage. Perhaps the most comprehensive study to date is that of Hanoch (1965), who, using data from the 1 in 1,000 sample of the 1960 U.S. Census of Population, investigated the effects of such variables as education, occupation, age, race, geographical location, type of residence, origin of birth, mobility, marital status, number of children, as many as 49 variables (mostly dummies) in all, and could account for at most 35% of the variance in the earnings of males aged 25-64.

Thirdly, as Wood (1969) has emphasized, there is little understanding of

the roles of the variables which make a statistically significant contribution to the income generating functions. These variables are usually restricted to the easily measurable characteristics of the individual, and each may be merely an instrument for one or several true underlying characteristics directly related to productivity. This problem of course arises in any form of statistical analysis. In general the significance of the coefficient of the instrumental variable should rule out the possibility that the association between it and the true characteristics is fortuitous. So long as the true characteristics and the instrument maintain a stable relationship with one another, such substitution matters only to the extent that it impairs perception of the nature of the economic process under investigation. However if the relationship is not necessarily fixed then the coefficient of the instrument means little. In this case the consequence of being satisfied with the instrument, if one or more of the true characteristics can be controlled, must be inefficient policy.

In view of the low proportion of the variance of the wage accounted for by income generating functions, it is clear that any aggregation function for labour based on conventional characteristics will be a poor predictor of the wage at the level of the individual. Nevertheless, through the simple operation of the law of Large Numbers, the prediction of the average wage of a category as a whole may be relatively accurate. Therefore in the empirical investigation below, the aggregation functions and the properties derived from them are intended to apply only to categories and not to individuals.

Of all the measured characteristics of labour, the only ones which regularly appear cross-classified by wages, outside of such specialized surveys as the 1 in 1,000 sample, are occupation, education, and industry of employment. For lack of data other characteristics will not be taken into account when choosing criteria for a labour classification. Furthermore industry, being tenuously connected with the personal attributes of an individual, will also be excluded. There remain therefore the possibilities of classifications based on education, occupation, or a combination of the two.

Both education and occupation appear prominently in income generating functions. In order to obtain some prior indication of the relative promise for the estimation of aggregation functions of the two classifications, the variance in

the wage accounted for by each separately and for them both together was estimated, using the same data and holding constant as many other variables as possible. Suitable data for this purpose were found in Tables 2 and 3 of Subject Report PC (2) 7B, "Occupation by Earnings and Education", of the 1960 U.S. Census of Population, where earnings of male workers are cross-classified by age, race, and geographical location (North and West / South) in addition to occupation and education. Since the proportion of the variance accounted for increases with the fineness of the classification, the same number of categories, six, were used for both criteria in order to improve the comparability of the results. In the case of occupation the categories were (1) professionals and technicians; (2) managers and proprietors; (3) clerical workers; (4) sales workers; (5) craftsmen; (6) operatives, service workers and labourers. For education they were (1) 4 or more years of college; (2) 1 to 3 years of college; (3) 4 years of high school; (4) 1 to 3 years of high school; (5) 8 years of elementary school; (6) 0 to 7 years of elementary school. In both cases farm managers and workers were excluded. Table 1 summarizes the results. Further details of the treatment of the data may be found in Appendix 3.

It can be seen from the one-way analysis that occupation and education account for roughly the same proportion of the variance in the wage. In both cases the proportion is seldom much greater than 15%. As has been argued above, this low figure implies not that the criteria are unsuitable as classifications for aggregation functions, but that the latter are likely to be less sharply defined than might be desirable.

The proportion of the variance explained by the two-way analysis was on the average about a third higher than that explained by either of the one-way analyses.[2] Since the improvement was thus modest in comparison with the increase in the complexity in the formulation of an aggregation function, the possibility of using both classifications together was discarded.

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2 It was not possible to allocate the variance explained to the two classifications separately and to their interaction, as may sometimes be done in two-way analysis, because there were empty cells in the cross-classification. See, for example, Scheffé (1959), pp. 112 - 119.

TABLE 1

Proportion of the variance in the earnings of males in the labour force of the United States in 1960 accounted for by occupation and education

	age	one-way analysis: occupation	one-way analysis: education	two-way analysis: occupation & education
North				
Whites				
	25-34	.077	.064	.104
	35-44	.141	.140	.193
	45-54	.165	.161	.231
	55-64	.166	.142	.223
Non-Whites				
	25-34	.063	.079	.102
	35-44	.169	.173	.207
	45-54	.257	.250	.281
	55-64	.110	.071	.117
South				
Whites				
	25-34	.101	.120	.155
	35-44	.149	.184	.225
	45-54	.161	.195	.247
	55-64	.158	.169	.235
Non-Whites				
	25-34	.124	.117	.154
	35-44	.109	.125	.154
	45-54	.123	.118	.154
	55-64	.025	.008	.029

In view of the fact that, according to the one-way analysis, there is little to choose between the criteria, estimates of aggregation functions were derived for both separately. A classification based on occupation is likely to be more attractive to the econometrician interested in explaining productivity and growth, since there exist in most countries far more plentiful data on the occupational structure of the labour force than on its educational structure. On the other hand educational planners, given a choice of a direct link between education and productivity, and an indirect link via occupation, should find the former preferable if education is not inferior to occupation as a determinant of productivity, especially since the connection between education and occupation is not well understood.

11 Aggregation functions based on occupation

It was assumed that the aggregate production function for each state could be written in the form

$$(1) \qquad Y = f_s(L, Z)$$

where Y is output, L is a function of the individual categories of labour, Z is the vector of the other factors of production, and f_s is the form of the function in state s.

The aggregation function for labour was assumed to be a CES function whose arguments could either be the individual categories or be themselves CES functions of the categories. In general the aggregation functions estimated had two levels of aggregation along the lines suggested by Sato (1967), but in principle a hierarchy of aggregations was conceivable, limited only by the number of basic categories and the quality of the data. The advantage of using a tiered instead of a single level CES function is that it relaxes the assumption of the latter that the partial elasticities of substitution between the basic categories are constant and equal. However even a tiered function implies an exaggeratedly simple view of the structure of the labour force. Clearly more sophisticated functions

could have been considered; but, as will be demonstrated, even at this crude level there are diminishing returns to functional complexity.

The basic problem therefore was that of estimating the parameters of functions of the type

$$(2) \qquad N = \left(\sum_i a_i N_i^{\phi} \right)^{\frac{1}{\phi}}$$

where N is an aggregation of labour, which, if not the overall labour aggregation, would serve as an argument of a higher aggregation; the N_i are component subcategories which may or may not be basic occupational categories; and ϕ is equal to $(\sigma - 1)/\sigma$ where σ is the Allen partial elasticity of substitution between pairs of categories.[3]

The data used for estimating such functions were taken from the 1950 U.S. Census of Population. It was difficult to specify in advance the appropriate fineness of the occupational classification. If the classification were too coarse, there would be the risk that a single category might encompass several heterogeneous occupations, with the result that it would be unlikely to possess strongly identifiable properties of its own. On the other hand the finer the classification, the more difficult it would be to determine plausible groupings of categories within the tiers of the aggregation function, and the greater would be the problems imposed by deficiencies in the quality of the data. Accordingly three occupational classifications were defined. The first utilized the eight basic non-farm occupations: (1) professionals and technicians; (2) administrators and proprietors; (3) clerical workers; (4) sales workers; (5) craftsmen; (6) operatives; (7) service workers; (8) labourers. The second classification had four categories obtained by combining the categories of the first two at a time: category (9) combined (1) and (2); (10) combined (3) and (4); (11) combined (5) and (6); and (12) combined (7) and (8). In the same way the third classification was obtained from the second: category (13) combined (9) and (10), and (14) combined (11) and (12).

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3 Allen (1938) pp. 503 - 505. For a demonstration that equation (2) corresponds to his definition, see Uzawa (1962).

Other relatively coarse classifications were considered, notably one with three categories grouping (1) and (2); (3), (4) and (5); and (6), (7) and (8); but these did not yield an improvement in the results.

For each of the 28 states with more than 500,000 males employed, earnings and "effective employment", the latter being defined to be male employment plus female employment weighted by the ratio of female earnings to male earnings, were calculated for each basic occupation. Effective employment for categories (9) - (12) was measured by summing the effective employment in the two component categories, and the wage was the average of their wages weighted by effective employment. In the same manner effective employment and wages were calculated for categories (13) and (14) from the data for their component categories. Details of the sources and treatment of the data may be found in Appendix 1.

The parameters of equation (2) were estimated via the marginal productivity relationship derived from it:

$$(3) \qquad w_{is} = a_i N_{is}^{\phi-1} N_s^{1-\phi} \frac{\partial f_s}{\partial N_s}$$

where

w_{is} = marginal productivity of category i, assumed equal to earnings

$\frac{\partial f_s}{\partial N_s}$ = marginal productivity of category N as a whole

and the subscript s refers to the state furnishing the observation.

Hence

$$(4) \qquad y_{is} = c_i + (\phi - 1) M_{is} + b_s$$

where

$$y_{is} = \log w_{is}$$

$$c_i = \log a_i$$

$$M_{is} = \log N_{is}$$

$$b_s = \log \left(N_s^{1-\phi} \frac{\partial f_s}{\partial N_s} \right)$$

and $-1/(\phi - 1)$ is a maximum likelihood estimate of σ.

If there were k categories among the N_i, equation (4) would have as independent variables the variable M and k+27 dummy variables, and 28k observations would be available for estimating the parameters. The dummy variables comprise the c_i for the k categories of labour and the b_s for the 28 states, with one of the variables, arbitrarily chosen, dropped in order to eliminate linear dependence among them.

In the special case when k is equal to two, the dummy variables for the states can be eliminated by dividing the expression for w_1 given by (3) by that for w_2 :

$$\frac{w_{1s}}{w_{2s}} = \frac{a_1}{a_2} \left(\frac{N_{1s}}{N_{2s}} \right)^{\phi - 1} \tag{5}$$

The parameters are estimated by taking logarithms as before.

In these formulations of the regression equation it has been assumed that in each state the structure of wages is dependent on the structure of the labour force. Of course, to the extent that interstate wage differentials induce migration, the reverse is also true, and the resulting identification problem implies that ordinary least squares regression analysis yields biased estimates of the coefficients of equations (4) and (5). For two reasons this possibility has not been taken seriously in the following analysis. In the first place labour migration is likely

to be a very slow process in comparison with wage adjustment,[4] and hence it is not likely that any such bias is severe. Indeed if wage adjustments are not fairly rapid one is not justified in writing equation (3) at all. In the second place, it is by no means agreed in the literature that wage differentials exert a significant influence on migration,[5] and even in the study which is most favourable to the hypothesis, that of Gallaway, Gilbert and Smith (1967), the income variable explains at most 25%, and usually much less, of the variance in interstate migration in the U.S. between 1955 and 1966.

Occupational mobility gives rise to a similar identification problem to the extent that it is encouraged by occupational wage differentials. It is certainly biased in the direction of increasing income,[6] but this may simply reflect the fact that work experience enables an individual to upgrade his socioeconomic status, and such movements are not necessarily responsive to the actual sizes of wage differentials. Even if they are responsive, they are probably slow compared with wage adjustment, and hence the biases caused by neglecting the simultaneity of the system are unlikely to be large.

It should be noted that, since the wages of categories (9) - (14), and of categories aggregated from the basic categories, are averages of the wages of

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4 On average about three to four percent of the population ages more than one year moved from one state to another in the U.S. in each year during the 1950's. (Raimon (1963), p. 429)

5 Raimon (1963) found highly significant Spearman rank correlation coefficients between net migration to U.S. states and their average per capita income during the period 1950-1958, and Gallaway, Gilbert and Smith (1967), using a more detailed interstate migration matrix, found relative income to be a highly significant variable in addition to relative unemployment and distance. On the other hand Nelson (1959) states (p.58) that he found no significant relationship between interstate migration and relative income in the U.S. during the years 1935-1940 and 1949-1950, and argues that information, transmitted by relatives and friends, is a more important determinant. Greenwood (1969), in a similar but more formal study, found that the migrant stock, defined to be the number of persons born in one state and living in another, was a key determinant of the flow of migration between them in the period 1955-1960, in addition to other variables such as unemployment, education and distance.

6 In Table 3 below the mobility ratio for a movement from a lower paid occupation to a higher paid occupation is almost invariably higher than the ratio for the reverse movement between the same occupations.

their subcategories weighted by effective employment in them, the variables w_{is} and N_{is} in equation (3) are not estimated independently of each other. The estimate of the parameter ϕ is therefore likely to be upwardly biased, and this will increase the estimate of the elasticity of substitution. No attempt was made to evaluate the bias quantitatively. It is improbable that the bias could have been serious, since weighted averages are not usually particularly sensitive to their weights. In any case the bias would be conservative in the sense that it would reduce the apparent significance of the results.

In order to obtain a feel for the possible structure of the aggregation function, the pairwise elasticities of substitution were computed between all the eight basic occupational categories, using equation (5).[7] Since ϕ is equal to $(\sigma - 1)/\sigma$, the estimates of the coefficient $(\phi - 1)$ are equivalent to estimates of $-1/\sigma$. Table 2 gives the derived maximum likelihood estimates of σ.[8]

A major problem with such estimates arises from the fact that an estimate of $(\phi - 1)$ not significantly different from zero, which implies an estimate of σ not significantly different from infinity, may be due to either of two contrary causes: such an estimate is likely to be obtained both when the two categories of labour are excellent substitutes, and when they play such dissimilar roles in the productive process that there exists no relationship between them at all. It is impossible to distinguish between these cases on statistical grounds, and it is thus necessary to make use of such extraneous evidence as may be available.

On the hypothesis that those occupations which are good substitutes in production possess common characteristics which are likely to encourage substitution

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7 The pairwise elasticities were computed using the model

$$Y = f_s(L, Z)$$

$$L = (a_i N_i + a_j N_j)$$

where the subscripts i and j are those of the two categories in the pairwise regression. All the other labour categories, together with the non-labour factors of production, have now been bundled into Z.

8 Table 20 in Appendix 4 gives for each of the elasticities the t-ratio against the hypothesis that it is equal to infinity, this ratio being just the t-ratio against the hypothesis that the coefficient $(\phi - 1)$ is equal to zero.

TABLE 2

Pairwise elasticities of substitution for the eight basic occupational categories

	2	3	4	5	6	7	8
1	9.8*	5.3*	51.9	9.6	8.2	3.0**	3.0**
2		3.4**	5.3*	-15.2	10.8	2.8**	3.1**
3			3.0**	3.3**	5.5**	2.2**	2.4**
4				-2.5**	42.4	2.7**	3.4**
5					11.8	12.3	3.5**
6						95.1	9.0*
7							7.1

* significantly different from infinity at 5% level ** at 1% level

TABLE 3

Mobility of the non-agricultural American labour force from first employment to employment in 1962: actual flow as a ratio of the flow expected with perfect mobility

		1	2	3	4	5	6	7	8
Prof.	1	4.35	0.96	0.63	0.60	0.19	0.13	0.17	0.09
Admin.	2	1.19	2.77	0.73	1.33	0.37	0.29	0.34	0.36
Cler.	3	1.04	1.34	2.48	1.68	0.57	0.56	0.85	0.47
Sales	4	0.79	1.69	1.39	2.81	0.51	0.69	0.50	0.37
Craft.	5	0.62	1.04	0.56	0.71	1.93	0.68	0.65	0.66
Oper.	6	0.44	0.80	0.77	0.73	1.26	1.49	1.03	1.06
Serv.	7	0.54	0.66	0.71	0.49	0.80	1.21	3.86	1.40
Lab.	8	0.42	0.61	0.72	0.64	1.09	1.37	1.32	2.43

Source: Table J2.3 of Blau and Duncan (1964). In their table some of the occupations were subdivided. These have been recombined in order to improve comparability with Table 2. Table 3 is thus an aggregated version of Table 2.7 of Blau and Duncan.

in supply, information on intragenerational mobility would be relevant. In their study of the American labour force, Blau and Duncan (1964) obtained data on its occupational distribution in 1962 cross-classified by the occupational distribution of the first employment of each worker. Table 3, prepared from these data, shows for each pair of occupations the number of workers who had moved from one occupation to the other as a ratio of the number who would have moved had there been perfect mobility, that is, no association between first employment and employment in 1962.

If there had been perfect mobility, all the numbers in Table 3 would be equal to unity. In fact it may be seen that the table is dominated by large numbers on the diagonal, which indicate that workers tend to remain in the occupation of first employment. Most of the other cells contain numbers lower than unity. If these are taken as indicators of poor scope for substitution, then all but three of the estimates of σ in Table 2 not significantly different from infinity may be considered to imply no relationship. The three remaining estimates are those between categories (6) and (7) (mobility ratios 1.03 and 1.21 in the two directions), categories (7) and (8) (mobility ratios 1.40 and 1.32), and categories (5) and (6) (mobility ratios 0.65 and 1.26). The last is equivocal, since the mobility ratio from (5) to (6) is lower than unity.

A revised version of Table 2 with the insignificant results eliminated would thus just contain the diagonal elements and those off-diagonal elements with asterisks. Looking at the pattern of the results, it does appear, as might be expected, that categories which are close to one another on socioeconomic orderings are good substitutes.[9] A notable exception is the elasticity of substitution between sales workers and craftsmen, which is negative, implying complementarity. This result

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9 Two of the best known socioeconomic orderings are those of North and Hatt, and of the U.S. Bureau of the Census. The former, described in Reiss (1961), is based on the results of a sample survey designed to evaluate the prestige attached to the occupations. The latter was calculated using the average levels of education and income of the occupation (see Subject Report PC (2) 5B of the 1960 U.S. Census of Population for the methodology employed). Both orderings put professionals and managers first, and operatives, service workers and labourers last, in that order, and thus agree substantially with the conventional census ordering. The North-Hatt scale places craftsmen just above clerical and sales workers, which are close together. The Bureau of the Census puts sales workers above clerical workers and both above craftsmen.

may just reflect the fact that the two categories straddle the boundary between the nonmanual and manual occupations. The importance of the nonmanual/manual distinction is underlined by the tendency of the manual occupations to have high elasticities of substitution with respect to one another and low elasticities of substitution with respect to the nonmanual occupations. These observations suggest that when the categories are grouped for aggregation to a higher level, there should be a clear division in the structure between the nonmanual and the manual occupations.

TABLE 4

Pairwise elasticities of substitution and t-tests of the difference of the estimates from infinity for occupational categories (9) - (12)

	elasticity			t-ratio		
	10	11	12	10	11	12
9	4.8	6.6	2.4	2.6	1.9	5.2
10		4.9	2.0		2.3	7.3
11			6.8			2.8

Proceeding to the coarser, four-category classification, one observes much the same pattern, but more clear cut. In Table 4 adjacent categories have high pairwise elasticities. Category (12) has low elasticities with respect to the categories (9) and (10). Category (9) appears to have a higher elasticity with respect to category (11) than with respect to category (12), when the reverse might have been anticipated, but this result may simply be due to the greater sampling error indicated by a lower value for the t-ratio.

Given the abundance of significant results using the finer classifications, the two-category classification (nonmanual and manual workers) became redundant. Nevertheless, in order to check some of the aggregation functions constructed using the other two classifications, the pairwise elasticity of substitution

between categories (13) and (14) was estimated, being 4.1 with a t-ratio 2.5.

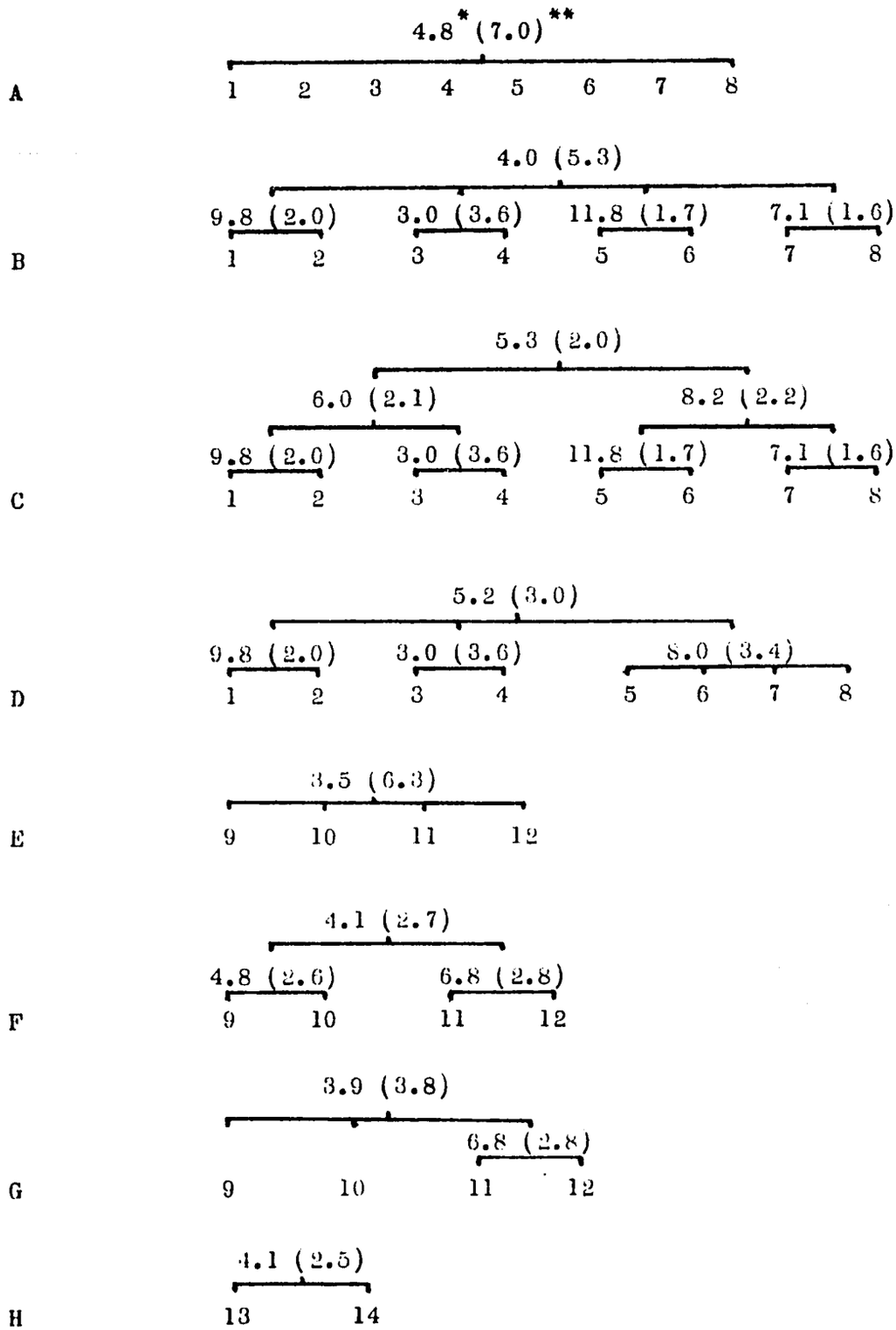
In view of the results gathered in Tables 2 and 4, there appear to be four possible candidates for aggregation structures using the eight basic categories: (A) using a single level CES function to aggregate all eight categories simultaneously; (B) aggregating the categories pairwise using the elasticities estimated in Table 2, and then using another CES function to aggregate the four supercategories; (C) aggregating the categories pairwise, estimating the pairwise elasticities between the four supercategories, using these estimates to aggregate the supercategories two at a time, and then finally aggregating the two derived categories; and (D) aggregating categories (1) and (2), and (3) and (4), pairwise, and aggregating the four manual categories together, the three supercategories thus derived being aggregated together at the second level.

In the case of the coarser classification three structures were considered: (E) aggregating categories (9), (10), (11) and (12) in a single level CES function; (F) aggregating (9) and (10), and (11) and (12), pairwise using the elasticities estimated in Table 4, and aggregating the two derived categories in another CES function; and (G) aggregating (11) and (12) pairwise, and then aggregating the derived category with (9) and (10). Structures (E) - (G) thus correspond directly to structures (B) - (D). Indeed the only difference is that in structures (B) - (D) the original eight categories were aggregated pairwise using the estimated pairwise elasticities of substitution, whereas in (E) - (G) they were directly combined in a process implicitly using in each case an infinite elasticity of substitution.

In Figure 1 the various structures are assembled, together with the aggregation of categories (13) and (14), labelled (H). In order to demonstrate how the diagram should be interpreted, the results for structure (D) will be explained. Using the logarithmic form of equation (5), the pairwise elasticities of substitution between categories (1) and (2), and (3) and (4), were estimated to be 9.2 and 3.0 respectively, with t-ratios 2.0 and 3.6. These estimates appear in Table 2. Using equation (4), the elasticity of substitution between categories (5) - (8) was estimated to be 8.0. with t-ration 3.4. The three groups were then aggregated into three supercategories using the estimates of the elasticities and the values of the parameters a_i also estimated by equations (4) and (5). Using equation (4) again, the elasticity of substitution between the three supercategories was

FIGURE 1

Aggregation structures based on occupational classifications



* elasticity of substitution ** t-ratio

estimated to be 5.2 with t-ratio 3.0.

It can be seen that almost all the estimates of the elasticities of substitution are significant at the 5% level, and a number of them are significant at the 1% level. The results for the structures as wholes are not directly comparable, and there would appear to be little justification for preferring one structure to another. This conclusion was perhaps predictable. The true relationship between the variables is obviously involved and complicated by other factors of production. Thus any CES structure is bound at best to be a crude approximation, and if one such structure exists then a number of equally plausible variations are also likely to exist, especially since the high elasticities of substitution indicate relatively flat aggregation isoquants.

III Aggregation functions based on education

The same methodology was used for constructing aggregation functions based on educational classifications. Again the data for estimating the functions were taken from the 1960 U.S. Census of Population, and the same 28 states were used. Details on the sources and treatment of the data can be found in Appendix 1. It was again convenient to define eight basic categories: (1) 4 years or more of college; (2) 1 - 3 years of college; (3) 4 years of high school; (4) 1 - 3 years of high school; (5) 8 years of elementary school; (6) 5 - 7 years of elementary school; (7) 1 - 4 years of elementary school; (8) no school years completed. Once more it seemed worthwhile to define a coarser classification by combining the categories pairwise: category (9) covered the whole of college; (10) covered the whole of high school; (11) covered the last four years of elementary school; and (12) covered the first four years of elementary school and no education. Finally, as before, two more categories, (13) and (14), were constructed by combining categories (9) and (10), and (11) and (12), respectively.

The pairwise elasticities of substitution for the eight basic categories and for the four broader categories appear in Tables 5 and 6 respectively. The elasticity of substitution between categories (13) and (14) was 3.3 with a t-ratio 5.0.

TABLE 5

Pairwise elasticities of substitution for the eight basic educational categories

	2	3	4	5	6	7	8
1	-17.7*	-8.2*	10.1	-25.5	5.4**	6.0**	5.1**
2		46.3	6.9*	-26.9	6.9**	6.5**	5.9**
3			7.7**	215.1	9.9**	7.9**	7.0**
4				18.1**	13.9*	9.0**	7.4**
5					52.5*	17.0**	10.4**
6						9.0**	10.0*
7							49.5

* significantly different from infinity at 5% level ** at 1% level

TABLE 6

Pairwise elasticities of substitution and t-tests of the difference of the estimates from infinity for educational categories (9) - (12)

	elasticity			t-ratio		
	10	11	12	10	11	12
9	34.0	5.9	5.8	0.3	2.6	7.3
10		5.9	8.5		3.6	5.0
11			21.6			2.6

Looking at Table 6, one may observe that categories (9) and (10) are good substitutes for one another, as are categories (11) and (12), but that the remaining pairwise elasticities are relatively low. These results, and the low elasticity of substitution between categories (13) and (14), suggest that there should be a sharp break in the aggregation structure between the upper four basic categories and the lower four. Looking at Table 5, the pairwise elasticities of substitution between the upper four categories are all high, two of them with estimates which are greater than infinity, i.e. negative, caused by the estimate of the coefficient ($\phi - 1$) in equation (5) falling below zero. There would seem to be little justification for hypothesizing any particular substructure for these categories other than straightforward aggregation by means of a single level CES function. Likewise the bottom four categories all have high pairwise elasticities and a single level CES function would seem to be adequate for aggregating them.

With these considerations in mind, the following structures were estimated: (A) aggregating the eight basic categories in a single CES function; (B) aggregating the upper four categories with one CES function and the lower four with another, and then aggregating the two derived categories at a second level; (C) aggregating categories (9) - (12) in a single CES function; and (D) aggregating categories (9) and (10) pairwise, and (11) and (12) pairwise, and the two derived categories pairwise. Finally the structure consisting of a simple CES function aggregating the two categories (13) and (14) was labelled (E).

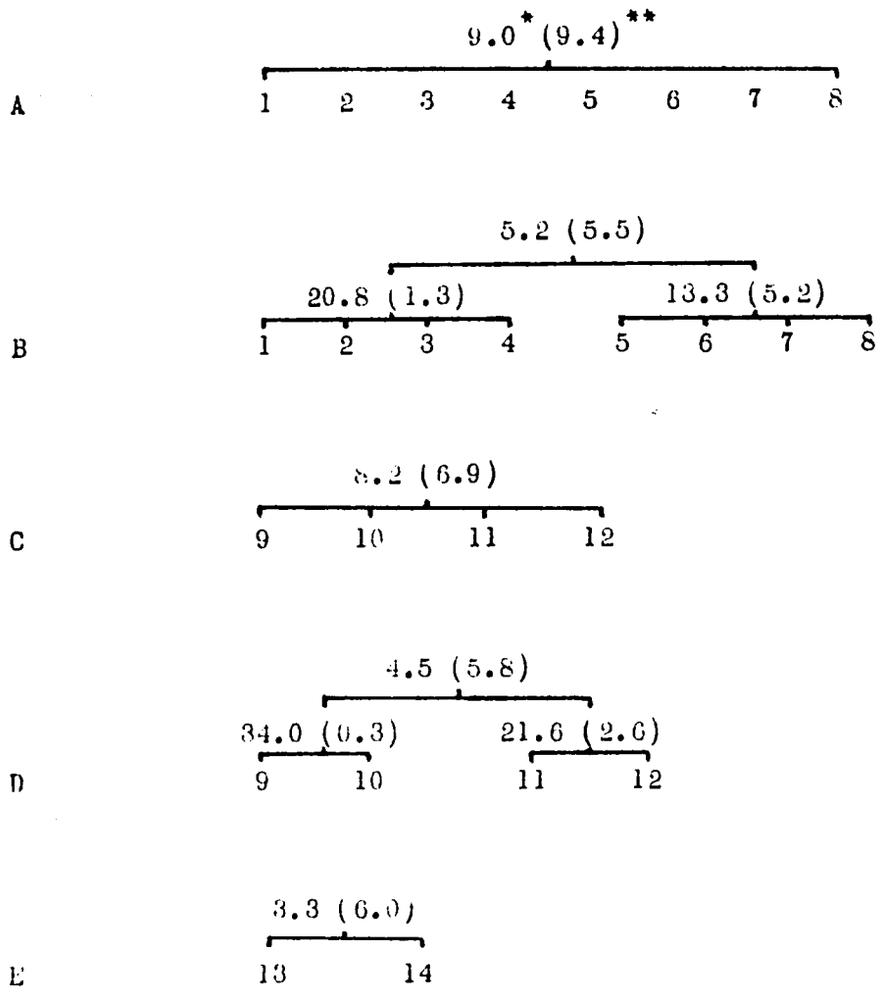
The results are shown in Figure 2, which should be interpreted in the same way as Figure 1. As can be seen by comparing the two figures, the structures for education are similar to those previously estimated for the occupational classification. Most of the elasticities are high and significant, but some are very high indeed and not significantly different from infinity.

IV Sensitivity analysis

In the previous two sections a variety of aggregation structures were estimated for both the occupational and the educational classifications, and it was not possible to discriminate among them on statistical grounds. Accordingly it is of

FIGURE 2

Aggregation structures based on educational classifications



* elasticity of substitution ** t-ratio

interest to discover whether the specification of the function makes any difference in practice.

For this purpose data for the United States on employment by occupation in 1951 and 1967, and on wages by occupation in 1951, were used to estimate relative wages by occupation in 1967, and the aggregate labour index for that date, for each specification of the aggregation structure in Figure 1.[10] In order to derive the absolute level of wages in 1967, it was necessary to introduce an estimate of the marginal product of the aggregate labour index at that date, and this was calculated on the assumption that the elasticity of total wages with respect to the labour index was equal to zero.

Before discussing the results it should be emphasized that the estimates for 1967 cannot be expected to be realistic. The aggregation structures were estimated from cross-section data, and thus several important factors which are likely to influence the structure of wages over time were held constant. The estimates for 1967 should therefore be interpreted as representing the structure of wages and the labour index that would have been in existence in 1951 if the 1951 labour endowments had been replaced at that date by the 1967 endowments. The actual figures for 1967, which are irrelevant to the present analysis, are given in the last line of each table, and they are in fact quite different from any of the "projections". The reasons for this divergence are discussed in Section V.

Tables 7, 8 and 9 give the wages of each category and the aggregate labour index in 1967 as proportions of their levels in 1951, for each structure in each fineness of occupational classification. In addition the results of using linear aggregation are shown for each fineness (labelled (A1), (E1) and (H1) for the eight, four and two category classifications respectively).

Looking at the tables, it is immediately clear that the estimate of the increase in the labour index is hardly affected by the specification of the aggregation structure, and it is little different from that obtained using linear aggregation.

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10 The data were taken from the Current Population Reports, Series P-60, of the U.S. Bureau of the Census. See Appendix 2 for a description of their treatment. Wages are expressed in current dollars throughout this section.

TABLE 7

Estimates of wages and the labour index in the United States for 1967 employment data, as ratios of their levels in 1951, for structures using the eight category labour classification

category / structure	1	2	3	4	5	6	7	8	aggregate labour index
A	1.90	2.08	2.03	2.09	2.16	2.15	2.11	2.24	1.324
B	1.94	2.02	2.01	2.10	2.17	2.17	2.13	2.22	1.325
C	1.96	2.05	2.02	2.11	2.15	2.14	2.11	2.20	1.326
D	1.94	2.02	2.01	2.10	2.17	2.16	2.14	2.22	1.325
A1	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	1.330
actual	2.32	2.30	1.89	1.98	1.99	1.93	1.95	1.84	-

TABLE 8

Estimates of wages and the labour index in the United States for 1967 employment data, as ratios of their levels in 1951, for structures using the four category labour classification

category / structure	9	10	11	12	aggregate labour index
E	1.96	2.03	2.17	2.17	1.329
F	1.98	2.03	2.16	2.16	1.330
G	1.97	2.03	2.16	2.15	1.330
E1	2.07	2.07	2.07	2.07	1.335
actual	2.29	1.91	1.96	1.93	-

TABLE 9

Estimates of wages and the labour index in the United States for 1967 employment data, using the two category labour classification, for selected values of the elasticity of substitution

structure	σ	wages of (13)[a]	wages of (14)[a]	ratio of wages	wages of (13) as ratio of wages in 1951	wages of (14) as ratio of wages in 1951	labour index as ratio of index in 1951
1951	-	4449	3100	1.43	1.00	1.00	1.000
H	4.1	8939	6721	1.33	2.01	2.17	1.322
H1	∞	9248	6445	1.43	2.08	2.08	1.326
H2	1.0	7969	7584	1.05	1.79	2.45	1.310
H3	0.5	6703	8712	0.77	1.51	2.81	1.295
actual	-	9702	6041	1.61	2.18	1.95	-

[a] current dollars per year

Similarly the estimates of the wage structures are not particularly sensitive to the specification of the aggregation structure. However the estimates obtained using linear aggregation, which implies constant relative wages, frequently differ by more than 5%.

In Table 9 it may be observed that the estimates of the labour index obtained using elasticities of substitution equal to 1.0 and 0.5 are not much lower than the linear aggregation estimate, but the estimates of wages are very dissimilar.

The implications of these results depend upon the intended use of the aggregation function. They indicate that for the purpose of measuring the contribution of labour to economic growth, linear aggregation should provide estimates virtually

equivalent to those obtained using more complicated aggregation structures. On the other hand the results suggest that for the other obvious application, analysis of the rate of return to education, the specification of the aggregation structure is of appreciable importance, since the estimates of wage differentials vary considerably among the different structures, even though, in percentage terms, the estimates of the wages themselves do not. The sensitivity of the estimates of wage differentials is demonstrated in Table 9. The differential between the wages of nonmanual and manual workers would be 38% in 1967 using the elasticity of substitution calculated in Section III. If σ were assumed to be equal to infinity, the differential would remain the same as it was in 1951, 43%. If a Cobb-Douglas function were used, it would be 5%. If σ were equal to 0.5, it would be negative.

The educational structures were not subjected to sensitivity analysis of this kind because they resemble the occupational structures closely and it was presumed that such analysis would yield similar conclusions.

V Time series estimates of aggregation functions

In all countries in advanced stages of industrialization, the supply of the more skilled and educated types of labour has been growing much more rapidly than the labour force as a whole. In view of the experiments described in Sections II and III, one would expect their wages to have fallen relatively to those of the rest as a result. Over long periods of time this has indeed been the case, as a large number of studies testify. Ober (1948) divided the U.S. labour force into five classes by skill and found that, in the period 1907-1947, the ratio of the median earnings of the most skilled class to that of the least skilled class fell from 2.05 to 1.55. Knowles and Robertson (1951) examined the time rates for skilled and unskilled occupations in four industries in the U.K. and found that, in the period 1880-1950, the premium for skill fell by about two thirds. Dunlop and Rothbaum (1955) reported that in the interval between 1933-1940 and 1952-1953 the skill differential fell by a half in Italy and by a quarter in France. Keat (1960), using data on 141 manual occupations in 17 industries in the U.S., found

that the dispersion of hourly wages, measured by the coefficient of variation (the standard deviation divided by the mean), fell by about a third between 1903 and 1956. Bry (1960) showed that skill differentials rose marginally in Germany from 1871 until the late 1880's, remained constant until the end of the century, and (ignoring a period of sharp contraction and dispersion during the decade 1914-1924) declined gradually until 1928, with little change between that date and 1943. Woods and Ostry (1962) constructed time series on 15 skill differentials in Canada, which, on the average, show a marked decline between the period 1923-1933 and the late 1940's, and a further decline by the late 1950's.

Two studies which stand out against this tide should be noted. Long (1960) quotes three government surveys which showed that the ratio of skilled to unskilled wage rates remained roughly constant in the U.S. from 1860 to 1890, and Ozanne (1962), examining the records of a large manufacturing plant in Chicago, found large cycles but no pronounced trend in the ratio between 1858 and 1959. One should however be most cautious in interpreting such statistics for the U.S. before 1914 because one has no means of estimating the extent to which the improvement in the quality of the labour force due to the growth of the educational system was offset by the relative reduction in the flow of skilled immigrants.

On balance, therefore, the findings of these long-term studies appear at least superficially consistent with the aggregation functions of Figure 1. However the available evidence on the recent behaviour of wage rates is much less favourable, for the relative decline of skill differentials seems to have ceased in the more industrialized countries since 1945. According to Gunter (1964), there was little contraction of relative wages between 1950 and 1960 in the U.S., the U.K., France, Italy and Germany, although it appears to be continuing in a sample of developing countries. Lydall (1968), using estimates of mean wages and salaries for 116 occupations in the U.S. calculated by Miller (1966), found that the coefficient of variation fell by just over a third from 1939 to 1949 but changed little between 1949 and 1959. Papola and Bharadwaj (1970) estimated that in the period 1949-1965 the coefficient of variation of the wage structure has been approximately constant in the U.S., Germany and Sweden, and that it appears

to have risen in the U.K. and France. Griliches (1970) found that in the U.S. the ratios of the mean income of high school graduates to those of college graduates and elementary school graduates both remained fairly stable between 1939 and 1966. Finally, Tables 7 - 9 in Section IV demonstrate that the wages of the more skilled occupations in the U.S. grew faster than those of the less skilled over the period 1951-1967. Table 10 presents some more detailed statistics derived from the same source (Current Population Reports, Series P-60, of the U.S. Bureau of the Census: see Appendix 2).

TABLE 10

Mean wages of the major occupations, as a ratio of the mean wages of operatives, for the U.S. 1951 - 1967

	1951	1953	1955	1957	1959	1961	1963	1965	1967
Prof. + tech.	1.62	1.71	1.74	1.81	1.95	1.89	1.87	1.85	1.94
Administrators	1.73	1.78	1.89	1.84	1.95	1.97	1.88	1.91	2.06
Clerical wkrs	1.10	1.13	1.11	1.10	1.15	1.11	1.10	1.08	1.08
Sales workers	1.30	1.17	1.41	1.23	1.37	1.24	1.32	1.40	1.33
Craftsmen	1.21	1.23	1.25	1.22	1.28	1.24	1.24	1.25	1.25
Service wkrs	0.82	0.83	0.85	0.78	0.85	0.81	0.81	0.80	0.83
Labourers	0.74	0.73	0.71	0.74	0.75	0.69	0.69	0.70	0.71
Unweighted coef. of variation	0.28	0.26	0.30	0.28	0.31	0.30	0.32	0.31	0.33
Weighted coef. of variation	0.32	0.35	0.33	0.34	0.32	0.34	0.32	0.36	0.34

The first seven rows give the mean income of males in the major occupational groups of the U.S. labour force as a ratio of the mean income of operatives, the largest group numerically, for the period 1951-1967. The next two rows give the coefficient of variation of the wages of the eight major occupations, unweighted, and weighted by employment, respectively. It may be seen that the mean incomes of

professionals and technicians, and administrators, rose relatively to those of operatives; those of clerical workers, sales workers, craftsmen and service workers maintained stable ratios; and those of labourers declined relatively. The unweighted coefficient of variation rose slightly, but the fluctuations in the weighted coefficient make it difficult to detect a trend. On the whole it thus appears that the dispersion of incomes increased a little during the period.

A number of rival explanations have been offered for the observed stability of the wage structure, and no attempt will be made to evaluate them here. Instead attention will be confined to two of the more popular and plausible. According to one hypothesis, technical progress is biased in such a way as to increase the demand for skilled labour relatively to that of unskilled labour, and has sustained its price in the face of the relative increase in its supply. According to the other, there exists relative complementarity between capital and skill, and this, together with the fact that the stock of capital has been growing much faster than the labour force, has had the same effect.[11]

The analysis will be restricted to the estimation of a single level CES aggregation function for the eight major occupations, and hence the marginal productivity relationship represented by equation (3) may be written

$$(6) \qquad w_{it} = a_i L_{it}^{\phi-1} L_t^{1-\phi} \frac{\partial Y_t}{\partial L_t}$$

where

Y_t = GDP for the U.S. in year t

L_t = aggregate labour index in year t

L_{it} = employment in occupation i in year t

w_{it} = wage of occupation i in year t

and $\phi = (\sigma - 1)/\sigma$ as before.

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11 For example, see Griliches (1969).

According to the first hypothesis this relationship shifts as time passes, and according to the second it shifts as the capital-labour ratio increases. A simple way of modelling these effects is to make the parameter a_i a function of time and a function of the capital-labour ratio, k , respectively:

$$(7) \qquad w_{it} = a_i(t)\, L_{it}^{\phi-1}\, L_t^{1-\phi}\, \frac{\partial Y_t}{\partial L_t}$$

$$(8) \qquad w_{it} = a_i(k_t)\, L_{it}^{\phi-1}\, L_t^{1-\phi}\, \frac{\partial Y_t}{\partial L_t}$$

Even if a satisfactory series for the capital stock had been available, the inevitable multicollinearity would almost certainly have made it impossible to distinguish between the effects of the accumulation of capital and of time, acting as a proxy for technical progress. Hence no attempt was made to estimate equation (8).

As a first approximation it was assumed that the proportional rate of growth of the shift factor $a_i(t)$ was constant, so

$$(9) \qquad a_i(t) = \alpha_i\, e^{\lambda_i t}$$

where α_i and λ_i are constants.

The labour aggregation function may now be written

$$(10) \qquad L = \left(\sum_i \alpha_i\, e^{\lambda_i t}\, L_i^{\phi} \right)^{\frac{1}{\phi}}$$

TABLE 11

Estimates of the parameters of equation (10)

	$\lambda_i - \lambda_6$ (% per year)	t-ratio	α_i	a_i
Prof. and tech.	1.31	3.21	0.171	0.189
Administrators	0.93	3.35	0.185	0.201
Clerical workers	0.01	0.02	0.118	0.112
Sales workers	0.49	1.87	0.124	0.126
Craftsmen	0.14	0.54	0.133	0.132
Operatives	-	-	0.111	0.109
Service workers	0.03	0.12	0.086	0.070
Labourers	-0.48	1.74	0.074	0.060

The parameters of the function were estimated using time series on employment and wages by occupation derived from the Current Population Reports, Series P-60 (see Appendix 2).[12] The estimate of the value of $(\phi-1)$ was -0.092, with t-ratio 1.07, implying an elasticity of substitution equal to 10.9. The estimates of the remaining parameters for the different occupations are given in Table 11. In the absence of a time series for $\partial Y/\partial L$ it was not possible to estimate the absolute values of the λ_i, but it was possible to estimate their differences. Table 11 gives the difference between the value of λ_i for each occupation and that for operatives, λ_6, and the t-ratio for this difference. The fourth column gives the estimates of the parameters a_i in aggregation function (A) of Figure 1, for comparison with the estimates of the α_i.[13] It may be observed that those time series weights which in 1951 were lower than their cross-section counterparts had above average rates of growth during the period, and those above their cross-section counterparts had lower than average rates of growth. The two sets of

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12 t was put equal to the calendar year minus 1951.

13 There was no straightforward way of estimating confidence intervals for the α_i and the a_i.

of estimates thus appear to be consistent with each other.

It is worth mentioning that an attempt to estimate the parameters of equation (10) omitting the shift factors yielded a nonsense estimate for the elasticity of substitution, as might be expected. The value of $(\phi - 1)$ was 0.146, implying a value of -6.9 for σ.

As in the cross-section case, an estimate of the CES function was made with the occupations aggregated linearly pairwise into four broader categories, and a further estimate was made with these new categories in turn aggregated pairwise to form the groups of nonmanual and manual occupations. The functions thus corresponded to structures (E) and (H) of Figure 1. The value of $(\phi - 1)$ for the four category function was close to zero and insignificant, and implied a value of 17.8 for σ. In the case of the two category function, $(\phi - 1)$ was positive, implying a negative value for σ. The relative rates of growth of the shift factors were lower for the four category function than one would have expected given their values for the eight category function. For the two category function the relative shift of the nonmanual occupations was actually negative. The results were thus poor for the four category function and totally counterintuitive for the two category function. It therefore appears that one must disaggregate at least as far as the major occupations of the census classification in order to obtain plausible results with time-series data.[12]

Summary and conclusions

In the experiments described in this paper both cross-section and time-series data were used to estimate aggregation functions for labour. The estimates derived from the time-series data may be expected to be of greater pertinence for the more obvious applications of such functions, but they can at best be described as tentative, given the problems of estimation and the small number of observations. Most of the statistical analysis was therefore concerned with the relatively straightforward estimation of aggregation functions from the cross-section data.

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12 It should be noted that there were only 20 and 6 degrees of freedom in the estimation of the four and two category functions, respectively, compared with 48 for the eight category function.

The first issue to be investigated was that of choosing an appropriate criterion for disaggregating the labour force. The results of Section I indicated that both occupation and education provide economically relevant classifications, since in both cases the proportion of the variance of wages accounted for by the classification was statistically significant, and that therefore either could serve as a basis for an aggregation function. However in both cases the proportion was small, as has been found in previous studies, implying that neither classification could be expected to be particularly efficient in the sense that it would yield a sharply defined aggregation structure.

In Sections II and III the elasticities of substitution between the basic categories, and between categories aggregated from the basic categories, were estimated for both the occupational and the educational classifications from cross-section data for the states within the U.S., and it was found that they were always significantly higher than unity at the 1% level but generally significantly lower than infinity at the 5% level. These findings cast serious doubts over the validity of the extension of input-output analysis known as the manpower requirements approach for the projection of the structure of the labour force, since it depends on the assumption of zero elasticities of substitution between different kinds of labour. Even the Cobb-Douglas function would imply unrealistically little scope for substitution and hence would also be inappropriate for aggregating labour. On the other hand linear aggregation would imply excessive scope for substitution. Thus it appeared that the general CES function would be an improvement on simpler forms of aggregation, at least from a theoretical point of view.

However it was possible to estimate, for both the occupational and the educational classifications, several equally plausible CES structures and none could be regarded as being more satisfactory than the rest. Figures 1 and 2 show them schematically. In general the elasticities at the lower levels of the structures are higher than those at the upper levels, as one might expect: adjacent categories in the lower levels are likely to be better substitutes than adjacent categories in the higher levels, since the latter represent greater proportions of the labour force and are more heterogeneous. If anything, the numerical results understate the relative reduction of the elasticities in the higher levels, since, for reasons discussed in Section II, they may be upwardly biased.

Because the relationship between education and occupation is complex, it is difficult to test whether the structures based on the two classifications are consistent with each other. However, since there exists some correlation between them, persons belonging to the upper half of the educational classification tend to belong to the upper half of the occupational classification, and vice versa. One might therefore expect that the elasticity of substitution between their upper and lower halves would be similar for both classifications. Among the occupational structures, (C), (F) and (H) contain splits between the upper and lower four basic categories, and the elasticities of substitution between the halves are 5.3, 4.1, and 4.1 respectively. Among the educational structures, (B), (D), and (E) contain such splits and the elasticities are 5.2, 4.5, and 3.3 respectively. The similarity of the ranges for the elasticity suggests that the structures are at least in this respect mutually consistent.

All these estimates were obtained from the cross-section data. In Section V it was observed that in recent years skill differentials have not been falling in the U.S. and the more industrialized European countries in spite of the continued relative increase in the supply of skilled labour. It was hypothesized that this might be due either to relative complementarity between capital and skilled labour or to technical progress biased in the direction of increasing the demand for skilled labour. The CES function approach was extended accordingly and tested statistically using time-series data for the U.S. as a whole. The quality of the results varied with the fineness of the occupational classification. Those for the eight category classification appeared plausible and were consistent with their cross-section counterparts. Those for the four category classification were less reasonable, and those for the two category classification were unrealistic.

The results assembled in this paper indicate that, for the purpose of estimating total growth in output due to labour, a time-dependent CES function with a fine classification of labour should yield the most accurate predictions, providing that the shift parameters can be estimated. Otherwise there is little to be gained by going beyond wage-weighted linear aggregation, as the sensitivity analysis of Section IV shows.

For educational planners the most obvious implication of the results is that the manpower requirements approach should be abandoned. Indeed rate of return

analysis with constant relative wage rates, implying an infinite elasticity of substitution, should be a much better tool for planning, at least in the short run. However, since such analysis is sensitive to changes in wage differentials, and since these in turn are sensitive to the specification of the aggregation structure,[13] it would be much better still to use the CES function approach to obtain a feedback relationship between the growth of the different levels of the educational system and the rates of return to them, via the growth of the labour force and the structure of wages.

Throughout this paper the word "experiment" has been employed when referring to the various forms of statistical analysis undertaken, in order to emphasize the spirit of the investigation. The CES function approach has been chosen because it is an obvious and convenient starting point, and because it appears to survive the appropriate statistical tests. However the underlying relationships between the different types of labour are undoubtedly much less simple, even without taking into account the effects of capital intensity and technical progress, and of course both of these bring additional dimensions to the problem. Thus the most that can be hoped is that the results are suggestive and help to develop a feel for the problem.

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13 The sensitivity analysis of Section IV was of course confined to structures estimated from the cross-section data. It seems reasonable to suppose that the estimates of wage differentials would also be sensitive to the specification of the structure in the case of functions estimated from the time-series data, which would be more relevant to the planning of education. In view of the wide dispersion in the shift parameters in Table 11, it is certainly true that the single level time-series function yields estimates of wage differentials very different from those obtained by linear aggregation, except in the short run.

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Appendix 1

Sources and treatment of the cross-section data used for estimating aggregation structures

The data for the occupational and educational classifications in Sections II and III were taken from Tables 124 and 138 respectively of Chapter D of the state volumes, PC (1), of the 1960 U.S. Census of Population.

Table 124 gives, for each state, the earnings in 1959 of males and females, separately, in the experienced civilian labour force for eleven major occupational headings, most of them broken down into a number of subcategories. It was decided to exclude two of the headings, "farmers and farm managers", and "farm laborers and foremen", on the ground that farm workers are relatively immobile and tend to form a separate labour market. The headings "private household workers" and "service workers, excluding private household", were amalgamated into a single category labelled "service workers". The remaining seven categories, and the category "service workers", were used as the basic occupational categories and are listed in Section II.

Table 138 gives the income in 1959 of males and females 25 years old and over, by years of school completed. The data for education are thus less satisfactory than those for occupation in two respects: income includes, in addition to earnings, such items as rents, royalties, interest, dividends, social security benefits and other government transfers, and contributions for support from persons outside the household, which are not related to the productivity of the individual; furthermore the data only cover persons aged 25 or more, and therefore exclude a significant proportion of the labour force. The eight educational categories given in the table were used as the basic categories and are listed in Section III.

1. wages

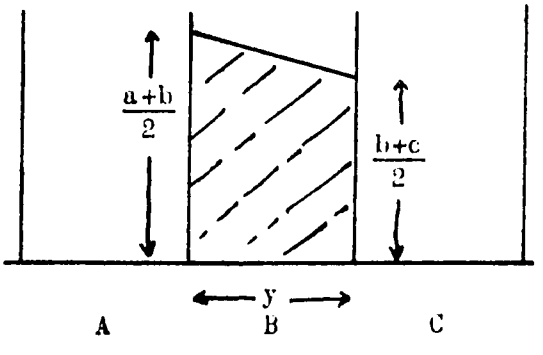
In neither table was the mean wage for each labour category given directly. Instead, nine wage intervals were defined and the number of persons belonging to each given. The mean wage was calculated, for both sexes separately, by taking a

representative wage for each of the intervals and then computing the average of these weighted by employment within the intervals. Choosing a representative wage for each interval was complicated by the shape of the income distribution, which made taking the midpoint of the interval inappropriate. Several methods of allowing for the non-uniformity were considered, and the following, which appeared to give plausible results, was adopted.

Given three equal adjacent intervals, A, B and C, with employment a , b and c respectively, the mean wage of interval B was taken to lie

$$(a + 3b + 2c)/(3a + 6b + 3c)$$

of the interval above its lower bound. This expression is the result of supposing that the employment frequency distribution within interval B is linear, that the value of this distribution at the lower limit of the interval is proportional to the average employment in intervals A and B, and that the value of the distribution at the upper limit of interval B is proportional to the average employment in intervals B and C.



assumed frequency distribution for interval B

If interval B has size y , and the frequency distribution has values $\frac{1}{2}(a+b)z$, $\frac{1}{2}(b+c)z$ at its lower and upper limits (z being an unknown constant), then

$$\text{total employment} = b = \frac{1}{4}(a + 2b + c)\,y\,z$$

and

$$\text{mean wage} = \text{lower limit} + \frac{1}{12}(a + 3b + 2c)\,y\,z\,/\,(\text{total employment})$$

$$= \text{lower limit} + (a + 3b + 2c)\,y\,/\,(3a + 6b + 3c)$$

When the wage intervals were unequal, say when A, B and C had lengths v , y and w respectively, the mean wage of B was calculated to be

$$\text{lower limit} + \frac{\frac{y}{v}a + 3b + 2\frac{y}{w}c}{3\frac{y}{v}a + 6b + 3\frac{v}{w}c}\;y$$

The first wage interval ($0 to $999 or loss) was always given a mean of $700. The last ($10,000 and over) was always given a mean of $21,100, using the value of 1.9 for the Pareto constant estimated by Houthakker (1959). The mean of the second highest wage interval, $7,000 - $9,999 , was calculated on the assumption that it had the same skewness as the interval just below it. The means of the remaining intervals were calculated by the method outlined above.

This set of computations was carried out for every basic occupation for both sexes in every state. Hence, for example, the mean wage of the $4,000 - $4,999 interval for males in occupation 1 was $4,534 in California, $4,497 in Mississippi.

	$3,000 - $3,999	$4,000 - $4,999	$5,000 - $5,999
California	25,365	55,698	56,707
Mississippi	4,022	4,121	3,762

employment of male professionals

The mean wage of occupation 7 ("Private household workers" and "Service workers, excluding private household" combined) for each wage group, and thus for the wage distribution as a whole, was calculated using the wage/employment distribution for "Service workers, excluding private household" only. It was assumed that domestic workers would receive part of their income in kind, and therefore that their

earnings data would be misleading.

The wages for males were used as the dependent variable in equations (4) and (5), and are given for occupation and education in Tables 12 and 13 respectively. Similar tables were calculated for females.

2. effective employment

Tables 124 and 138 gave both male and female employment for each category for occupation and education. It was necessary to aggregate together the sexes, and this was accomplished by assuming that the ratios of female to male wages could be used as efficiency factors for converting female employment into equivalent male employment. The efficiency factor takes into account both the fact that fewer women than men work full time and the fact that the productivity of a woman may differ from that of a man in the same occupational or educational category. The effective employment thus calculated served as the independent variable in equations(4) and (5), and are given for occupation and education in Tables 14 and 15 respectively.

TABLE 12

Mean earnings in 1959 of males in the experienced civilian labour force, by occupational category and state (current dollars)

	occupational category							
	1	2	3	4	5	6	7	8
Alabama	7648	8017	4686	5148	4532	3351	2609	2164
California	9771	10485	5390	7270	6523	5138	4221	3829
Connecticut	9525	11160	4987	7193	6044	4817	3958	3609
Florida	7932	8127	4483	6069	4724	3463	3063	2420
Georgia	7820	8132	4431	5762	4253	3080	2579	1964
Illinois	9274	10775	5191	7784	6561	5070	3965	3868
Indiana	8356	9305	5045	6308	5889	4800	3480	3334
Iowa	7847	8626	4782	5996	5075	4642	3101	3298
Kansas	8013	8747	4849	6258	5149	4475	3073	3056
Kentucky	7416	8178	4556	5390	4707	3757	2866	2581
Louisiana	7895	8494	4654	5585	4835	3788	2787	2364
Maryland	9889	10371	5384	6605	5622	4282	3707	3072
Massachusetts	8649	9841	4611	6517	5535	4355	3732	3372
Michigan	9445	10100	5415	6398	6521	4925	3989	3357
Minnesota	8218	9199	4932	6435	5432	4519	3518	3210
Missouri	8126	9128	4763	6510	5389	4278	3209	3185
New Jersey	9972	10778	5272	7579	6299	4958	4006	3816
New York	9545	10196	4837	7331	6001	4646	3863	3851
North Carolina	7246	7736	4228	5123	3873	3071	2493	1392
Ohio	8910	10024	5166	6707	6119	5007	3747	3544
Oklahoma	7870	8075	4800	5670	4691	3967	2694	2526
Pennsylvania	8672	9492	4853	6199	5561	4428	3379	3474
South Carolina	7169	7431	4142	4754	4031	2952	2308	1829
Tennessee	7690	8144	4309	5390	4350	3326	2536	2207
Texas	8236	8686	4757	6060	4933	3971	2795	2491
Virginia	9215	8972	5215	5433	4868	3459	3539	2340
Washington	8811	9511	5234	6519	6006	4987	3820	3797
Wisconsin	8248	9433	5054	6352	5746	4850	3765	3596

TABLE 13

Mean income in 1959 of males aged 25-64 in the experienced civilian labour force, by educational category and state (current dollars)

	educational category							
	1	2	3	4	5	6	7	8
Alabama	9745	7186	5687	4363	3458	2718	1832	1360
California	11087	8201	7206	6184	5052	4315	3329	2677
Connecticut	11918	8775	6988	5944	5054	4485	3457	2738
Florida	9269	7076	5783	4765	3771	3169	2378	1919
Georgia	10033	7424	5698	4402	3537	2850	1882	1383
Illinois	11263	8635	7076	6223	4974	4190	3292	2837
Indiana	10227	7639	6508	5541	4337	3746	2959	2532
Iowa	9382	6812	5741	4958	3860	3226	2381	2202
Kansas	9611	7349	5883	4973	3944	3274	2433	2210
Kentucky	9315	6865	5751	4881	3583	2880	2004	1485
Louisiana	9939	7589	6161	4975	4115	3347	2269	1615
Maryland	12049	8655	6784	5744	4917	4091	2934	2307
Massachusetts	10627	7682	6245	5213	4303	3830	3050	246[illegible]
Michigan	11008	8469	7036	5978	4907	4120	3225	2728
Minnesota	9923	7506	6214	5355	3967	3174	2232	2050
Missouri	9847	7425	6028	5241	4021	3329	2269	1627
New Jersey	11974	8968	7101	6191	5175	4516	3[illegible]	2682
New York	11247	8360	6701	5868	4793	4047	3248	2863
North Carolina	9028	6700	5091	4037	3273	2757	1934	1450
Ohio	10930	8186	6721	5808	4584	3999	3126	2502
Oklahoma	9546	6857	5799	4723	3674	2957	2054	1449
Pennsylvania	10699	7713	6098	5258	4364	3803	2865	2346
South Carolina	9014	6455	5107	4099	3254	2744	1885	1382
Tennessee	9534	6994	5698	4422	3308	2703	1921	1472
Texas	9994	7513	6251	5088	4062	3398	2549	1790
Virginia	11302	8200	6002	4931	4166	3301	2335	1682
Washington	10118	7559	6656	5616	4480	3704	2612	2211
Wisconsin	10063	7478	6322	5607	4537	3765	2731	2475

TABLE 14

Effective employment in the experienced civilian labour force in 1959, by occupational category and state (equivalent males)

	occupational category							
	1	2	3	4	5	6	7	8
Alabama	70176	72215	80177	48590	145296	199781	76770	80101
California	645653	492496	670850	343796	833805	793109	403963	266288
Connecticut	101370	76180	115020	51700	164340	191108	61608	34151
Florida	136201	167403	153041	102214	239097	191328	147901	101279
Georgia	84628	102887	118574	67260	163236	263442	105506	87865
Illinois	324886	277079	473667	202592	554124	665409	259883	181916
Indiana	131082	113018	157394	82671	258901	347287	105573	90076
Iowa	71258	72085	77775	52870	116539	136106	61563	43682
Kansas	67322	63634	70643	41829	106811	97254	49714	35381
Kentucky	60352	62019	73786	46536	122455	177369	57421	50672
Louisiana	77971	82445	85139	46563	130738	159382	84965	52833
Maryland	121948	85916	136623	58781	166860	161277	74107	59910
Massachusetts	197526	142265	238370	108372	285270	373948	141051	74658
Michigan	247251	183468	267169	146274	437224	600496	183591	122[illegible]6
Minnesota	105882	94679	114779	64934	157205	166629	80239	55001
Missouri	115247	111385	166281	86324	198336	250838	102705	74731
New Jersey	234078	189908	280969	127704	352322	428664	147646	97603
New York	658579	550848	940118	374969	838016	1065255	558656	253311
North Carolina	91780	97847	108764	76692	189341	331595	103467	84097
Ohio	295448	240080	354978	182112	555777	696863	215439	193974
Oklahoma	70664	67124	73094	42392	110585	104460	54508	37389
Pennsylvania	338216	267541	440363	222938	640066	840534	275371	266285
South Carolina	44002	46058	50843	37424	94334	181677	54722	51790
Tennessee	82112	79977	101975	62349	155399	229188	86390	70111
Texas	273659	284201	308351	175070	440885	464147	248192	188501
Virginia	113572	96250	132041	68127	181099	221074	67795	81170
Washington	103157	87675	99675	56270	155042	136378	70604	55222
Wisconsin	110550	96578	124982	71681	204883	278848	86303	65336

TABLE 15

Effective employment in the experienced civilian labour force in 1959, by educational category and state (equivalent males)

	educational category							
	1	2	3	4	5	6	7	8
Alabama	63577	59191	168432	179147	98177	204463	137614	40363
California	626265	721323	1371236	1038172	717999	488785	221084	100894
Connecticut	100353	75783	199960	170646	168425	113017	33187	22105
Florida	155902	163221	381774	315653	241253	233480	134068	38688
Georgia	84600	85023	205526	204037	107759	261721	191681	46290
Illinois	303698	301142	760297	652936	728061	413518	166944	53836
Indiana	113713	110738	386082	287754	305140	184719	63124	12622
Iowa	68269	73342	236064	135275	220056	105033	25722	3915
Kansas	67795	70650	187928	113457	156112	72173	24420	4426
Kentucky	54197	56586	139646	126133	213412	178042	118202	24114
Louisiana	75790	63524	160571	134299	87873	184799	152042	69622
Maryland	113940	80039	201453	184477	133462	194371	66731	13642
Massachusetts	189240	160976	457231	362026	278445	212167	58897	46964
Michigan	203894	196559	563245	526157	475480	298511	114299	33660
Minnesota	97025	97821	245640	159165	295636	126592	39595	8130
Missouri	108761	108903	299846	233436	355261	215485	95013	20138
New Jersey	221937	168441	453300	424860	371055	290434	96610	46600
New York	650449	503842	1287507	1232936	1085252	707056	258689	136580
North Carolina	95675	90607	236653	226218	126420	346268	207393	46638
Ohio	268370	242225	759904	647195	586048	394756	137153	39140
Oklahoma	70309	73464	155305	125573	132273	112997	62676	13325
Pennsylvania	305448	240763	880759	742871	765160	567135	184726	79773
South Carolina	49010	40438	106338	110724	59056	155315	115967	32560
Tennessee	72152	71753	185339	168136	195473	217820	149542	31481
Texas	280704	288674	572852	534813	306793	484640	289355	113463
Virginia	120592	97918	227971	203044	101169	279679	139542	31829
Washington	103399	104841	259886	178207	169080	86702	27843	7266
Wisconsin	100697	101996	299609	198328	319364	170063	59492	11970

Appendix 2

Sources and treatment of the time-series data on wages and employment by occupation in the United States

The data used for estimating the time-series aggregation function in Section V were taken from the Current Population Reports, Series P-60, "Incomes of Families and Persons in the United States", published by the U.S. Bureau of the Census. These give the income received by persons aged 14 or more in eleven major occupational categories. For the same reasons as in the treatment of the cross-section data, the two farm occupations were excluded, and the categories "private household workers" and "service workers, except private household" were combined into a single category, "service workers".

The definition of income covers all money income and includes rents, royalties, interest, dividends, social security benefits, etc., in addition to earnings, and is thus much broader than would be desirable. As in the case of the cross-section data, the mean income of each occupation is not given directly. Instead, for each occupation, the percentage distribution over a series of income intervals is tabulated. Hence it was again necessary to estimate a representative income for each interval, taking into account the shape of the income distribution for the occupation, and to calculate the weighted average over all the intervals. For this purpose the procedure described in Appendix 1 was followed without modification. The estimates of mean income by occupation for males for the years 1951-1967 are shown in Table 16. A similar table was calculated for females.

Effective employment for each occupation for each year was calculated in the same way as for the cross-section data, that is, by combining male employment and female employment weighted by the ratio of mean female income to mean male income. The estimates are shown in Table 17.

TABLE 16

Mean income of males in the United States, 1951-1967, by occupational category (current dollars)

	1951	1953	1955	1957	1959	1961	1963	1965	1967
Prof. & Tech.	5003	5986	6283	7340	8222	8762	9248	9838	11594
Administrators	5346	6248	6837	7482	8242	9134	9337	10152	12291
Clerical wkrs	3418	3944	4002	4454	4860	5132	5454	5729	6448
Sales workers	4028	4108	5104	4988	5773	5770	6525	7409	7966
Craftsmen	3760	4316	4518	4951	5405	5746	6133	6608	7481
Operatives	3098	3507	3612	4060	4220	4639	4958	5307	5971
Service wkrs	2548	2902	3051	3165	3593	3751	4004	4265	4955
Labourers	2283	2556	2559	2984	3162	3193	3401	3699	4210

TABLE 17

Effective employment in the United States, 1951-1967, by occupational category (thousand equivalent males)

	1951	1953	1955	1957	1959	1961	1963	1965	1967
Prof. & Tech.	4048	4430	4865	5378	5845	6490	6745	7301	8244
Administrators	5327	5386	5670	5978	6286	6803	6819	6637	7045
Clerical wkrs	5759	5675	6341	6459	6765	7652	7561	8250	8601
Sales workers	2581	2831	2805	2937	2992	3024	3612	3383	3375
Craftsmen	8419	7924	7957	8068	8136	8083	8350	8909	9353
Operatives	10329	10674	11231	9792	10397	9988	10889	11691	11775
Service wkrs	5047	5622	5678	6256	6282	6842	6898	7757	6327
Labourers	3363	3250	3545	3063	3032	2934	3044	3125	3154

Appendix 3

The contributions of occupation and education to the variance in wages : source and treatment of data

Tables 2 and 3 of Subject Report PC (2) 7B, "Occupation by Earnings and Education", of the 1960 U.S. Census of Population give the earnings of male workers cross-classified by occupation, education, age and race, for the North and West, and the South, of the U.S. respectively. In order to improve the comparability of the results of the analysis of variance, the nine occupational categories were reduced to the same number as the educational categories, six, by eliminating the data for "farmers and farm managers" and "farm laborers and foremen", and by combining "operatives","service workers" and "laborers, except farm and mine" into a single category. The six categories are listed for both occupation and education in Section I.

Earnings for each category were given in the form of the number of persons belonging to each of eleven earnings intervals. Because the highest two cells were in general small they were merged. The resulting ten intervals were: (1) $1 - $999 or less; (2) $1,000 - $1,999; thousand dollar intervals until (8) $7,000 - $9,999; (9) $10,000 - $14,999; and (10) $15,000 and over. Representative earnings were taken for each of the intervals: (1) $500; rising by $1,000 for each category until (8) $8,200; (9) $12,000; and (10) $33,000. The last figure was calculated using a value of 1.9 for the Pareto constant.

In order to abstract as far as possible from the effects of age, race and region, the analysis of variance was calculated for each of sixteen groups of observations formed by the cross-classification of these variables. The estimates of mean incomes and their standard deviations obtained from the one-way analysis of variance are given for occupation and education in Tables 18 and 19 respectively. Similar estimates were obtained for the 36 categories in two-way analysis of variance based jointly on occupation and education.

TABLE 18

Means and standard deviations of the earnings of the occupational categories

	Age	1	2	3	4	5	6	R^2
Whites								
North	25-34	6900	7861	5044	6680	5771	4723	.077
		(113)	(155)	(164)	(173)	(99)	(81)	
	35-44	10456	10671	5871	8733	6362	5167	.141
		(183)	(182)	(250)	(248)	(134)	(119)	
	45-54	11449	11204	5802	8507	6075	4856	.165
		(244)	(199)	(293)	(296)	(155)	(136)	
	55-64	10893	10750	5457	7556	5615	4314	.166
		(310)	(238)	(339)	(344)	(184)	(159)	
South	25-34	6487	6949	4697	5829	4735	3790	.161
		(191)	(223)	(262)	(257)	(151)	(126)	
	35-44	9698	9067	5427	6871	5278	4058	.149
		(303)	(265)	(415)	(376)	(206)	(197)	
	45-54	10159	9413	5230	6965	4793	3738	.161
		(422)	(299)	(526)	(468)	(248)	(236)	
	55-64	9345	9065	5021	5885	4400	3200	.158
		(573)	(382)	(641)	(589)	(337)	(313)	
Non-Whites								
North	25-34	5076	4750	3978	4000	4259	3595	.063
		(299)	(858)	(253)	(858)	(224)	(105)	
	35-44	7909	6446	4889	4500	4631	3951	.169
		(454)	(604)	(358)	(1258)	(260)	(130)	
	45-54	13040	3450	4735	-	4623	3741	.257
		(958)	(874)	(519)		(312)	(146)	
	55-64	7350	2000	4625	-	4123	3525	.110
		(1139)	(805)	(569)		(343)	(139)	
South	25-34	4169	-	3786	-	2670	2290	.124
		(311)		(332)		(181)	(71)	
	35-44	4870	2000	4071	-	2740	2517	.109
		(434)	(970)	(367)		(194)	(80)	
	45-54	5450	2000	4125	-	2691	2388	.123
		(549)	(672)	(475)		(207)	(84)	
	55-64	3500	1500	3000	500	2262	2176	.025
		(1308)	(755)	(925)	(1308)	(285)	(108)	

- no estimate for lack of observations

TABLE 19

Means and standard deviations of the earnings of the educational categories

	Age	1	2	3	4	5	6	R^2
Whites								
North	25-34	7556	6222	5747	5278	4622	4049	.064
		(115)	(133)	(83)	(102)	(157)	(195)	
	35-44	12142	8785	6956	6014	5303	4633	.140
		(183)	(205)	(118)	(142)	(190)	(247)	
	45-54	13836	9343	7308	6305	5399	4776	.161
		(246)	(260)	(166)	(166)	(170)	(211)	
	55-64	13386	9226	7377	6327	5360	4676	.142
		(336)	(346)	(257)	(225)	(177)	(193)	
South	25-34	7294	5761	5171	4526	3782	3221	.120
		(185)	(213)	(139)	(160)	(242)	(185)	
	35-44	11445	7925	6410	5288	4504	3721	.184
		(292)	(332)	(206)	(234)	(320)	(230)	
	45-54	12931	9086	6735	5363	4566	3711	.195
		(422)	(432)	(293)	(283)	(329)	(240)	
	55-64	12228	8466	6943	5489	4475	3571	.169
		(595)	(606)	(479)	(406)	(394)	(286)	
Non-Whites								
North	25-34	5215	4356	4067	3719	3478	3208	.079
		(328)	(273)	(160)	(152)	(254)	(211)	
	35-44	8578	5389	4752	4243	3944	3720	.173
		(512)	(426)	(221)	(202)	(272)	(212)	
	45-54	13040	4500	4617	4055	3972	3662	.250
		(963)	(761)	(364)	(296)	(283)	(189)	
	55-64	7350	4500	4000	3950	3625	3501	.071
		(1163)	(1163)	(520)	(368)	(291)	(161)	
South	25-34	4213	3167	2750	2489	2411	2123	.117
		(322)	(294)	(167)	(129)	(186)	(101)	
	35-44	4927	3900	3000	3000	2682	2347	.125
		(410)	(608)	(220)	(167)	(205)	(94)	
	45-54	5450	3500	3222	2786	2706	2312	.118
		(550)	(953)	(318)	(228)	(231)	(90)	
	55-64	3500	-	2500	2046	2333	2162	.008
		(1319)		(1319)	(398)	(311)	(110)	

- no estimate for lack of observations

Appendix 4

TABLE 20

t-ratios for the estimates of the pairwise elasticities of substitution between the eight basic occupational categories

	2	3	4	5	6	7	8
1	2.0	2.1	0.2	0.9	1.7	4.6	4.3
2		5.8	2.0	-0.8	1.5	3.0	3.4
3			3.6	3.0	2.6	6.2	0.2
4				-3.8	0.3	2.9	3.2
5					1.7	1.3	4.7
6						0.2	2.2
7							1.6

TABLE 21

t-ratios for the estimates of the pairwise elasticities of substitution between the eight basic educational categories

	2	3	4	5	6	7	8
1	1.9	2.3	1.1	-1.1	6.8	7.3	4.9
2		0.4	2.4	-1.0	4.9	6.5	4.4
3			3.0	0.1	3.4	5.3	4.2
4				3.4	2.4	4.9	3.9
5					1.9	4.0	4.5
6						4.5	2.4
7							0.6