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INVESTMENT PROJECT ANALYSIS IN A MODEL OF OPTIMAL GROWTH: THE CASE OF CHILE

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Abstract

This paper describes a set of numerical experiments with a small optimal growth model, aimed at calculating a set of shadow prices for evaluation of investment projects in the Chilean economy. The model is designed to emphasize the dependence of Chile on foreign trade, particularly imports of capital goods. For this reason two state variables are used--one standing for plant-type capital goods which are produced by a non-traded sector and the other representing equipment-type capital goods which must be imported. These imports are paid for by exogenous traditional exports, and exports from a traded goods sector. The model thus determines allocation of factors between the two sectors, and the corresponding pricing policy.

Numerical results show (i) a rate of discount consistent with the results of other rate-of-return studies in Chile; (ii) a very flat transformation surface between the traded and non-traded sectors, with a correspondingly inflexible relative price ratio; (iii) inverse proportionality between tariffs and the exchange rate, which follows from the flat transformation surface and the fact that in the optimal solution the traded goods price must equal the product of one-plus-the-tariff, the marginal revenue from exporting (essentially a constant), and the exchange rate. This last result

is inconsistent with partial equilibrium estimates of the shadow exchange rate, and some doubt is raised as to what is the "true" value of this shadow price.

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Lance Taylor
May 1971

INVESTMENT PROJECT ANALYSIS IN A MODEL OF OPTIMAL GROWTH: THE CASE OF CHILE*

1. Introduction

When the wave of Ramsey analysis was getting underway a few years ago, there was implicit in the literature the idea that these models could be used to calculate interest rates and other shadow prices for use in investment project analysis. More recently, a number of theoretical works discussed this possiblity explicitly.[1/] Nonetheless, empirical tests of the usefulness of this idea have to date been lacking.[2/] The purpose of this paper is to report on one such attempt.

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[1/] The most important of these studies is the book by Arrow and Kurz /1/. See also Little and Mirrlees /13/ and Marglin /14/.

[2/] Newbery /16/ describes some interesting hypothetical calculations of shadow wage rates in a labor surplus model, but stops short of specific empirical applications.

The test case presented here is a two-sector open economy model of Chile, a semi-industrialized country specialized in the export of copper. In sections 3 through 5, (following a general description in section 2 of the economic problems faced by an "externally strangled" economy like Chile's) some of the salient characteristics of the Chilean economy and the model's attempt to capture them are discussed. Subsequent sections summarize the results obtained with numerical solution of the model, and discuss the specific problems of

(i) specification of functional forms and estimation of the relevant parameters for Chile (sections 6 & 7);

(ii) interpretation of the quantity and shadow price solutions of the model, their "reasonableness" in light of other studies of Chile, and their sensitivity to changes in some of the exogenous data and parameters (sections 8, 9 and 10);

(iii) implementation of a computer algorithm based on the differential dynamic programming algorithms of Jacobson and Mayne /10/ to solve the optimal growth model and to check the sensitivity of the shadow prices to second order feedback effects from large investment projects[3/] (appendix).

[3/] A related problem is the election between first order and slower but more accurate second order algorithms for the solution of optimal growth problems. This is also discussed in the appendix.

2. Open Economy Development Constraints

Two coherent theories about the constraints on growth with which developing countries must deal have consolidated themselves in recent years. The first of these (stemming from the seminal work of Lewis, Nurkse, and Rosenstein-Rodan) stresses the institutional barriers to the transformation of "surplus" agricultural labor into an urbanized, industrial work force. The existence of these constraints leads to valuation of new industrial labor at a shadow wage below the institutionally fixed market wage. Using the tools of optimal growth theory, Little and Mirrlees /13/, Marglin /14/, and Newbery /16/ have attacked the problem of calculating the shadow wage rate in a labor surplus economy.

The second theory--with which we will be concerned in this paper--has its intellectual roots in the writings of the Latin American Structuralist economists of the 1950's,[4/]

[4/] Most of these economists were associated with Raul Prebisch at the U.N. Economic Commission for Latin America (ECLA). Their work has been ably summarized by Bianchi /4/.

but received formal treatment only in the 1960's in the two-gap models of Chenery and other economic programmers. The two-gap thesis is that additional foreign exchange has dual effect on a developing economy: it both supplements domestic savings and permits the importation of goods which cannot be produced domestically during the early stages of industrialization for reasons such as economies of scale in production and technological backwardness. Using the mordant terminology often employed around ECLA, one may say that two-gap economies are subject to "external strangulation" resulting from their dependence on a growing foreign exchange supply.

In practice the structuralists' remedy for strangulation was import substitution, although they still wrote about the need to develop exports as a "quasi-capital goods industry". Nonetheless, the policy emphasis on import substitution led to the erection of high protective walls around many Latin and other developing economies, consequent overvaluation of the domestic currency and an associated handicap to exportation of anything but the primary products in which these countries had (and have) overwhelming comparative advantage. The two-gap solution to this problem stressed foreign aid as a short run palliative; a more long-term approach

would be gradual reduction of trade barriers and devaluation to foment exports. In any case, the inability of domestic industry to produce a full range of sophisticated goods will persist in many developing countries (especially the small ones) for decades. The resulting "external dependence" (another ECLA slogan) leads logically to concern about correctly evaluating the foreign exchange gains and losses of investment projects. The shadow exchange rate is the apropriate tool of evalution when it is calculated from an optimizing model taking trade restrictions into account.

Clearly, most developing economies face both surplus labor and external dependence problems to some extent (and a fusion of the two models might therefore be called for). Nonetheless, in the case of Chile, the foreign exchange constraints are more important. The country displays all the classic signs of high tariffs, overvalued currency, extreme export specialization, frequent gluts of excess capacity, etc. Although labor problems exist as well (the unemployment rate is usually around six percent, the agricultural sector has for years been semi-stagnant, and the service sector is unduly large), most economists who have studied Chile agree that the foreign exchange problem is paramount.

For this reason, an open economy model of Chile is developed in this paper--first in general terms in the next three sections, and then in somewhat specialized form for numerical solution in succeeding sections.

3. A Model for Chile

The basic hypothesis (which is most similar to that of McKinnon /15 / among authors published in the English-language literature) is that domestic production depends on two types of capital goods--plant which is produced domestically, and equipment which must largely be imported.[5/]

For purposes of the model, we identify the plant-type capital stock with the output of a non-traded sector of the economy. This sector's product is also used to satisfy final consumption demands (for services, outputs of the utilities, and so on) and exogenous demands such as government consumption and residential housing construction.

The product balance equation for the non-traded sector number one can be written as follows:

$$\dot{x}_1 = f^1(x_{01}, x_{11}, L_1, t) - c_1 - z_1 - \delta_1 x_1 \qquad (3.1a)$$

where $\dot{x}_1$ (= dx_1/dt) is the accumulation of domestic-type capital; $f^1(x_{01}, x_{11}, L_1, t)$ is the production function, dependent

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[5/] In recent years, Chile has imported more than 80% of its equipment.

on the sector's use of foreign-type capital (x_{01}), domestic-type capital (x_{11}), labor (L_1), and technical progress indexed by time (t); c_1 is the consumption of the non-traded product; z_1 is exogenous demand; and δ_1 is the rate of depreciation of domestic-type capital.

In terms of the traditional foreign-trade model, we should now specify two additional sectors, one exporting and one competing with imports. Unfortunately, Chilean data do not permit any real distinction to be drawn among industries which do compete with imports and those (aside from copper and iron mining) which are actual or potential exporters. For this reason, we work with only one traded goods sector, whose output can be consumed, used to satisfy exogenous demands and transformed to foreign exchange through substitution of competitive imports or exportation. In practice, this last "trade-improvement" destination of product will be identified with non-traditional exports, for reasons to be discussed shortly. The balance equation of the traded goods sector is

$$f^2(x_{02}, x_{12}, L_2, t) - c_2 - z_2 - e_2 = 0 \qquad (3.1b)$$

where the notation is as above, with the addition of the

term e_2, the non-traditional export volume.

The foreign exchange constraint faced by the Chilean economy can be written as

$$\dot{k}_0 = (1/\pi_0)[e_{trad} + g(e_2, t) - \pi_{int}(\mu_1 \ell^1 + \mu_2 \ell^2) - \pi_c c_0 - \delta_0 \pi_0 x_0 - r]. \qquad (3.2a)$$

where $\dot{k}_0$ is accumulation of foreign type capital, μ_1 is a coefficient relating non-competitive intermediate imports to production levels in sector i, c_0 is consumption of non-competitive imports, $g(e_2, t)$ is the revenue function giving foreign exchange receipts from exportation of e_2 units of non-traditional exports, e_{trad} is foreign exchange generated by traditional mineral exports (exogenous), δ_0 is the depreciation rate on foreign-type capital, and r is the outflow of capital (calculated basically as repatriation of copper mine profits, less foreign aid).[6/] We assume that world prices of imports of capital goods (π_0), intermediate inputs (π_{int}) and consumption goods (π_c) are fixed, while the export price varies with the volume traded via the function $g(e_2, t)$.

An alternative formulation of the foreign exchange constraint is in terms of domestic prices which take into account

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[6/] Foreign borrowing is not included as an endogenous control variable (or total debt as a state variable) for two reasons--first, to lighten the computational burden, and second, because the supply of funds to Chile (with all its monetary problems) is probably quite inelastic. For an elegant theoretical treatment of a model very similar to the present one where borrowing is endogenous, see Bruno /6/.

the effects of tariffs and subsidies, quotas and dumping on resource allocation. Often planners (particularly in externally strangled countries) inherit all manner of such distortions from the past, and it is impossible to think of moving from a highly protected situation to an optimal commercial policy in the short run. Hence, existing trade restrictions have to be included among the constraints on planners' decisions.

Concentrating on tariffs as proxies for all types of commercial interventions, we can express total tariff collections less subsidies as

$$-t_2\pi_2 e_2 + t_{int}\pi_{int}(\mu_1 f^1 + \mu_2 f^2) + t_c\pi_c c_0 + t_0\pi_0(\dot{x}_0 + \delta_0 x_0)$$

Subtracting this quantity from (3.2a), we express the balance of payments restriction as

$$\dot{x}_0 = (1/\tau_0\pi_0)[e_{trad} + \tau_2 g(e_2, t) - \tau_{int}\pi_{int}(\mu_1 f^1 + \mu_2 f^2) - \tau_c\pi_c c_0 - \tau_0\pi_0\delta_0 x_0 - r] \qquad (3.2b)$$

where $\tau_j = 1 + t_j$ is the "force" of tariff j. This revision amounts to rewriting the balance of payments in domestic prices, and leads to an overall domestic demand-supply balance of the form:

$$P_1(\dot{x}_1 + \delta_1 x_1 + c_1 + z_1) + P_2(c_2 + z_2) + P_0[\tau_0\pi_0(\dot{x}_0 + \delta_0 x_0) + \tau_c\pi_c c_0 + \tau_{int}\pi_{int}(\mu_1 \ell^1 + \mu_2 \ell^2)]$$

$$= P_1 \ell^1 + P_2 \ell^2 + P_0(e_{trad} - r)$$

where P_1 and P_2 are the prices of goods one and two, P_0 is the exchange rate, and it is assumed that $P_2 = P_0\tau_2\pi_2$.

From this equation, we see that using (3.2b) as the balance of payments restriction implies that capital inflows are assumed to be given exogenously in domestic price terms (so that dollar foreign trade balance is implicitly maintained by changes in reserves), and that tariff revenues (along with other non-distortionary taxes) are used to finance the exogenous demands z_1 and z_2. These assumptions could be relaxed with an explicit treatment of the government's intertemporal budget constraint, but both computational and data limitations forbade this extension in this paper. The reader should note, however, that the numerical results are really of a partial equilibrium nature when the "domestic price" balance of apyments equation (3.2b) (with exogenous tariff changes) is imposed.

Adopting the usual optimal growth format, we assume that the planning office is interested in finding the maximum of the following functional

$$\int_{t_0}^{t_f} \exp(-\rho t) L(t) \sum_{i=0}^{2} u_i[c_i(t)/L(t)]dt \tag{3.3}$$

where ρ is the rate of discount of consumer "felicity"; the u_i are components of a separable felicity function, defined per capita, then aggregated again over the whole population (assumed proportional in size to the labor force) at time t; t_0 is the initial time in the planning horizon, and t_f is the terminal time.[7] (In the computations t_0 was set to zero and t_f to 30 years.)

The constraints which the planning office faces are then the supply-demand balances (3.1), the foreign exchange constraint in one of its two forms (3.2a) or (3.2b), three factor use equations,

$$x_1 - \sum_i x_{1i} = 0 \tag{3.4}$$

$$x_0 - \sum_i x_{0i} = 0 \tag{3.5}$$

$$L - \sum_i L_i = 0, \tag{3.6}$$

(where L(t) is the exogenously growing labor force), initial

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[7] The program is phrased in terms of finite time maximization for reasons of computational feasibility, and also because we will not be concerned with the subtle implications of infinite programs in any case.

values of the capital stocks

$$x_0(t_0) \text{ and } x_1(t_0) \text{ given,} \tag{3.7}$$

and the necessity to reach terminal capital stock targets (whose derivation is discussed below):

$$x_0(t_f) \text{ and } x_1(t_f) \text{ specified.} \tag{3.8}$$

In schematic form, these equations complete the statement of the model. Before we enter into discussion of the optimality conditions and explicit specification of the various functions, a few observations are in order:

(i) Treating mineral exports e_{trad} as exogenous appears justified as a first approximation in Chile. The gran Minería, which produces most of the copper, employs less than one percent of the labor force, is geographically isolated from the main population centers, and has few input-output connections with the rest of the economy. Further, copper prices have fluctuated widely in the past, and are largely beyond Chilean control. These reasons, plus the fact that expansion of copper production requires major, non-marginal investments, justifies leaving this sector outside the year-to-year workings of the model.

(ii) Imports, as noted, are all treated as non-competi-

tive. Mainly, this can be blamed on lack of data identifying production characteristics of separate import-competing and export sectors. We might note, however, that import substitution possibilities are often alleged by ECLA and other economists to have "run out" in countries like Chile. If by this they mean that costs of production for new import substitutes are high, they are correct in most cases,[8/] and it is reasonable to assume that imports are not likely to be reduced below their current fifteen percent of GDP in the near future. Conversely, they aren't likely to rise much above fifteen percent, either, since there is too much social consensus against closing or even not expanding already established, though costly, production operations via allowing import competition. For these results, the proportionality assumptions adopted here seem realistic.

(iii) As we will see in the solutions, production from sector 2 for exportation (or trade improvement) grows rapidly. As a general policy prescription, this rapid non-traditional export growth is unassailable. However, the solution does not tell which sectors should have their exports pushed. In part this is a computational problem, but data availability

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[8/] See /20 / for empirical verification.

has also prevented the specification of production and demand functions for a few more disaggregated export sectors.

(iv) The model as stated marks a break with the two-gap tradition from which it is largely derived by not explicitly including a savings constraint. In part, optimization of a non-linear welfare functional replaces this omission, but one would still like some representation of the apparent (from the National Accounts) fact that wage-earners in Chile don't save. This harks back to the problem of duality which we previously decided to ignore, but it still would make a useful extension of this type of model to include behavioral savings restrictions.

(v) The model is set up with sectoral allocations of factors and consumption, investment, and export levels as control variables. In principle, one would want to replace these aggregates with explicit policy variables, controlled by the government. Unfortunately, this is empirically impossible in an aggregate model. However, the government certainly can control investment allocations to sectors (mainly through its control of the foreign exchange market) and has various means of influencing consumption and industry production levels. For this reason, I feel quite certain that

designing policies to implement the model's solutions (assuming that one believes the functional and empirical specification!) would not prove impossible, particularly since the model's "future" is similar to past observed behavior of the Chilean economy.

(vi) Finally, as mentioned previously but worth repeating here, tariffs and other instruments of commercial policy are not treated as control variables. At best, planners can hope to have some control of the changes (hopefully equalizations) of trade restrictions, with more rapid changes presumably leading to higher political costs from plant failures, worker relocations, and so on. There is no conceptual problem in modelling these costs explicitly, but the practical difficulties one foresees point immediately to the alternative of parametric tariff reduction adopted here.

4. Optimality Conditions, and Their Interpretation

Let demand prices for foreign-type capital stock, the domestic good, and the export good be defined respectively as P_0, P_1, and P_2. Also, let the rentals on the two types of capital be r_0 and r_1, let the wage of labor be w, and use the notation $(\tau_i \pi_i)^+$ to mean $(\tau_i \pi_i / \tau_0 \pi_0)$, i.e. the price of any type i of non-capital traded good expressed in terms of

the capital good price.[9] Finally, for the two domestic sectors, let a net price P_i^* be defined as

$$P_i^* = P_i - P_0(\tau_{int}\pi_{int})^+ \mu_i \qquad i=1,2 \tag{4.1}$$

Using this notation, plus dynamic programming, the minimum principle, or simple common sense, we can write out a number of equations describing instantaneous equilibrium in the eonomy. They are as follows:

<u>Consumption equilibrium relationships</u>

$$\begin{aligned} e^{-\rho t}\, Lu_i'(c_i/L) &= P_i \qquad i = 1,2 \\ e^{-\rho t}\, Lu_0'(c_0/L) &= P_0(\tau_c\pi_c)^+ \end{aligned} \tag{4.2}$$

These equations in the first derivatives of the components of the additive felicity function (the u_i' terms) can be viewed as setting a scale for the demand prices in terms of instantaneous felicity.

<u>Production equilibrium relationships</u>

$$P_i^*\, f_0^i = r_0 \qquad i = 1,2 \tag{4.3}$$

$$P_i^*\, f_1^i = r_1 \qquad i = 1,2 \tag{4.4}$$

$$P_i^*\, f_L^i = w \qquad i = 1,2 \tag{4.5}$$

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[9] The definition of $(\tau_i\pi_i)^+$ clearly presupposes that we are considering maximization subject to the balance of payments expressed in terms of domestic prices. Setting τ_i and τ_0 to unity covers the case of maximization subject to world prices.

These relationships involving the marginal products of the three factors would be valid for cost-minimizing firms in a competitive economy, and determine allocation of the three scarce factors in the model.

World price/domestic price relationships

$$P_2 = P_0 \tau_2 g' / (\tau_0 \pi_0) \tag{4.6}$$

where g' is the derivative of the export revenue function.

This relationship scales the domestic demand price for sector two in terms of the shadow exchange rate P_0, and the subsidies given. Note that (4.6) is written assuming that planners accept present commercial policy as given i.e. they seek to maximize welfare subject to the foreign exchange constraint in domestic prices, (3.2b). Since the price factor $\tau_2/(\tau_0 \pi_0)$ in Chile exceeds unity, this means that domestic shadow prices of traded goods will exceed the marginal foreign exchange gains from or losses in trade, multiplied by the shadow exchange rate P_0. On the other hand, if planners maximized welfare subject to the world price foreign exchange constraint, (3.2a), the tariff factors would disappear from (2.15), so that the traded good's price P_2 and the exchange rate P_0 would be more nearly equal. Because a neoclassical production technology provides little freedom for relative

price shifts between the traded and non-traded good (section 7), the exchange rate will also rise relative to the non-traded good's price under trade liberation. Thus, the shadow exchange rate from the model solution subject to the domestic price foreign exchange constraint will be undervalued in comparison to its level in the world price constrained solution.

Dynamic price relationships

Finally, the changes in shadow prices of the two capital goods are just equal to their net rentals,

$$-\dot{p}_0 = r_0 - p_0\delta_0 \tag{4.7}$$

$$-\dot{p}_1 = r_1 - p_1\delta_1 \tag{4.8}$$

This is a standard result from optimal growth theory, and can be rewritten in standard fashion in terms of own rates of interest, capital gains, and social rates of discount, by judicious substitutions from (4.2) - (4.5). We leave this to the reader, and will solely concentrate on explicating certain consequances of (4.7) - (4.8) for investment project analysis, in the next section.

5. Project Evaluation in the Optimal Growth Model

An investment project in the context of optimal growth

models is just a perturbation. That is, we can imagine that we have an optimal solution to the model (3.1) - (3.8) and then perturb it in the following way:

(i) Add the amounts $h_i(t)$--which may be of either sign--to the balance equations (3.1) and the foreign exchange constraint (3.2a) or (3.2b). These $h_i(t)$, when positive, are just the quantities of goods and foreign exchange produced by some investment project. When negative, they are the amounts of goods and exchange used up by the project.

(ii) Add the amounts $z_i(t)$ to the three factor balance equations (3.4) - (3.6). These $z_i(t)$, which will normally be negative, are the amounts of capital and labor tied up in the project at any time t.

Now it is reasonable to assume (and is proved in /19 /, among other places), that the project will be economically desirable if its shadow value over its lifetime from its time of initiation t_I to its time of closure t_c is positive, i.e. if

$$0 < \int_{t_I}^{t_c} [\sum_i p_i(t)h_i(t) + \sum_{j=0}^{1} r_j(t)z_j(t) + wz_2(t)]dt \tag{5.1}$$

Economically desirable is here understood in terms of welfare: the value of the welfare functional (3.3) will be increased

if we perturb the without-project model solution by h(t) and z(t), when the criterion (5.1) is satisfied.[10, 11]

Two comments are in order about this rule:

First, the P_i and r_j decline over time, so (5.1) gives more weight to effects of the project/perturbation in the near than in the far future. However, prices and factor rentals in a multi-capital goods model need not (and in general will not) decline in fixed proportion according to simple exponential decay functions. For this reason, "discounting" in the traditional sense is not an appropriate decision rule, although in particular models the decreases in prices and factor rentals over time may be approximated by exponential function. We will see in section 8 to what extent such approximations apply in the present model. For the moment, we stick with writing (5.1) in general form.

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[10] Stictly speaking, this is a first order result since we have not considered the effects of the project itself on the equilibrium prices of the model. However, this effect is likely to be empirically irrelevant, as is shown in the appendix.

[11] The evaluation rule (5.1) is a re-statement of the simplex criterion for a model with differentiable constraints. Hence, (5.1) and simplex-based project evaluation rules such as Domestic Resource Cost (or Bruno-Krueger exchange rates /2/) are equivalent.

Second, (5.1) shows that the project should be evaluated with tariff-ridden shadow prices, when welfare is optimized with respect to the tariff-inclusive foreign exchange constraint, (3.2b). This means--somewhat counter to what is usually recommended--that when project analysts really accept current tariffs as binding, they should value capital inflows or foreign exchange revenues from export projects at a price which is low relative to the prices of currently traded goods.[12/] The low valuation

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[12/] Normally, one expects a relatively high shadow price for foreign exchange when there are numerous trade restrictions. In fact, some calculations of exchange rates for use in investment project are constructed to fit this preconception in the following way: we know from (4.2) that any domestic price index will also be an index of instantaneous felicity gains from additional consumption. But from (4.6) we can express this index as some weighted average of tariffs, multiplied by P_0. If we make a similar index of foreign exchange gains and losses from trade in world prices, and divide it into the domestic price index, then we end up multiplying P_0 by a factor exceeding unity, to "correct" for the tariff-caused undervaluation of the exchange rate. As indicated in the text, this is not an appropriate correction if planners choose to respect tariffs as given. If tariffs aren't to be respected, it is better not to use an ad hoc correction, but to solve the model subject to the world prices of (3.2a), calculate the corresponding P_0, and evaluate traded goods at their marginal foreign exchange cost or revenue times P_0. As mentioned above, P_0 under free trade will be higher relative to traded goods prices, than under restricted trade. As we will see below, the same is true for P_0 relative to the non-traded good's price.

on foreign exchange reflects the fact that consumption patterns and associated marginal welfare valuations adapt themselves to high tariffs and low foreign exchange content. If free-traders in the planning office don't want to accept this tariff-ridden consumption (and marginal utility) pattern, then it is appropriate for them to try to impose the use of world relative prices for traded goods and appropriately scaled domestic prices for non-traded goods, in investment project analyses and elsewhere. One can view a world-price solution of the optimal growth model as an attempt to indicate the appropriate relative prices.

In more general terms, planners can also try to calculate prices corresponding to welfare maximization subject to politically feasible tariff reductions. To my mind, this is more representative of what is possible in planning operations, and some of the solutions presented here are calculated for various plausible patterns of trade liberation in Chile.[13/]

[13/] This can be considered as an attempt to implement a procedure which Bacha and I judged impractical in / 2 /. On concluding this paper, the reader may decide whether the opinion expressed in / 2 / or the attempt here is the more appropriate. One might note in passing that people who write articles proposing formulas for calculating the shadow price of foreign exchange sometimes treat either (3.2a) or (3.2b) as <u>the</u> appropriate restriction, and then steadfastly and unswervingly carry out their analysis and polemics without considering the alternative. Since running the economy according to world prices is not the sort of thing that one usually recommends to the Minister, while blindly accepting tariffs is sub-optimal economically, some compromise--such as shadow-pricing in light of expected future tariff reductions--might seem desirable.

6. Functional Specifications

(i) Most international evidence and some early returns from Chile / 3 / indicate that elasticities of substitution in production between labor and aggregate capital are somewhat less than unity. However, the received wisdom on the substitutability between plant and equipment (or domestic-type and foreign-type capital) is much less developed and persuasive. Two-gap and programming models have assumed that the relevant (direct partial) elasticity of substitution is either zero or infinite. In these circumstances, an agnostic position has been adopted. Production possibilities in each sector are supposed to be described by a two-level C.E.S. production function, the upper level reflecting the trade-offs between aggregated capital stock x_i^* and effective labor L_i

$$f^i = A_i \exp(\xi_i t)[\gamma_{1i}(x_i^*)^{-\beta_{1i}} + (1 - \gamma_{1i})(L_i)^{-\beta_{1i}}]^{(-1/\beta_{1i})}$$

$$i = 1,2 \qquad (6.1)$$

where A_i is the scale parameter in the upper-level C.E.S., ξ_i is the rate of technical progress, γ_{1i} is the distribution parameter and β_{1i}($= (1/\sigma_{1i}) - 1$) represents substitution possibilities between aggregate capital and labor. The capi-

tal aggregate x_i^* is itself based on a C.E.S. index,

$$x_i^* = [\gamma_{2i}(x_{1i})^{-\beta_{2i}} + (1 - \gamma_{2i})(x_{0i})^{-\beta_{2i}}]^{(-1/\beta_{2i})}$$

$$i = 1,2 \qquad (6.2)$$

where the interpretation of the parameters is as in the upper level function.

(ii) Welfare functional: the theoretical literature is none too explicit about the appropriate form for the functions u_i in (4.3). The most natural choice, from the point of view of interpretation at least, is one of the utility functions which have been proposed to justify complete systems of demand functions. Among these, the most readily interpretable is the utility function of the Stone-Geary linear system of demand functions /8 /:

$$\sum_i u_i(c_i/L) = \sum_i a_i \log[(c_i/L) - b_i] \qquad (6.3)$$

$$(c_i/L) > b_i \qquad i = 0,1,2;\ \sum_i a_i = 1;\ a_i > 0, i = 0,1,2$$

If q_0, q_1, and q_2 are the prices faced by the consumer, then his "subsistance income" y^* can be defined as $\Sigma_i b_i q_i$. If his income is just y^* (the utility function is not defined for income levels below y^*), he consumes b_i of each good. Any supernumerary income $y - y^*$ is divided among the goods

in proportions a_i--a_i being larger as the income elasticity of demand for good i is higher.

In the model solutions described below, parameters in the base year were estimated to correspond to observed Chilean income and price elasticities of demand. However, the parameters b_i were generally increased as a function of time. This increase in (psychological) subsistence income corresponds to rising expectations on the part of consumers, and is appropriate for a growing economy.

(iii) Export revenue is assumed to be determined by a demand function with constant elasticity η:

$$g(e_2, t) = C \exp(r_{exp} t) e_2^{(1/\eta) + 1} \tag{6.4}$$

where e_2 is the volume of non-traditional exports, and the time trend allows for growth of world trade.

(iv) World prices are inferred using base year tariff factors τ, on the assumption that base year domestic prices are all equal to unity:

$$\pi_j = 1/\tau_j \tag{6.5}$$

where j indexes types of traded goods. The τ_j's (including τ_e, which represents the cost differential the domestic producers face in placing their goods in the world market) are

assumed to decline over time in radioactive form:

$$\tau_j(t) = 1 + [\tau_j(t_0) - 1] \cdot \exp[-r_j(t - t_0)]. \qquad (6.6)$$

In some of the numerical exercises described below, it is assumed that tariffs are equal to zero throughout the planning period (the world price foreign exchange constraint binds). When the domestic-price foreign exchange constraint is used, the τ_j's are assumed to decline from their initial levels with halving times of five or fifteen years.

7. Base Year Specification and Parameter Values

Data for the model were to a large extent drawn from projections made by ODEPLAN, the Chilean National Planning Office, for the economy in a "full employment" 1970 /17 /. In reality, 1970 was far from being a full employment year because of pre- and post-election economic uncertainty. However, the ODEPLAN projections are still a fairly realistic indicator of what a full employment base year would look like, and were used (stylistically) for this reason.

The first non-traded sector of the model was identified with services, commerce, transportation and communication, and the utility industries. Of course, some trade is carried out in these sectors (notably in transportation) but to a first approximation, they do not play a major role in

the international commerce of most countries. Remaining sectors were treated as traded, and to a large extent this is realistic. (A number of manufacturing industries in Chile are not in fact traded, because of prohibition of imports of their products. However, they are tradeable and were aggregated into sector two for this reason.)

Table 1 shows production conditions corresponding roughly to the ODEPLAN base year. About two-thirds of gross output is assume traded (or tradeable), the rest non-traded. Capital-labor ratios in the traded sector are clearly higher than in the non-traded sector, and average labor productivity is much higher as well. These relationships correspond to what one expects in a developing country.

Table 2 shows the parameter values assumed for the model. These again are stylized, but based as far as possible on empirical studies, as follows:

(i) Jere Behrman's production function estimates /3 / show that elasticities of substitution in Chile are rather low, as is the rate of technical progress. Hence rates of technical progress of one percent and elasticities of substitution of two-thirds (traded sector) and one-third (non-traded sector) are probably not far from the mark. The value 0.5 for

TABLE 1
BASE YEAR PRODUCTION CONDITIONS[1/]

	non-traded goods (sector 1)	traded goods (sector 2)
output	6.15	11.70
employment	1.618	1.324
domestic capital		
amount	14.19	11.61
relative to output	2.31	0.99
relative to labor	8.77	8.77
foreign capital		
amount	8.74	9.46
relative to output	1.42	0.81
relative to labor	5.40	7.15
capital aggregate		
amount	11.96	10.61
relative to output	1.94	0.91
relative to labor	7.39	8.01
average labor productivity	3.80	8.84

[1/] Monetary magnitudes in billions of 1965 escudos. Employment in millions of workers.

the elasticity of substitution between the two types of capital was simply assumed. Sato's estimates for the U.S.A. /18 / are higher, but the lower value seemed appropriate in view of the lower flexibility demonstrated by the Chilean economy in most respects. The distribution parameters for the C.E.S. functions were calculated using observed labor shares and estimates of the gross rates of return to domestic and foreign capital of none and eleven percent (non-traded sector) and twenty-four and twenty-six percent (traded sector). The latter rates in turn follow from a study of Chilean manufacturing returns /5 /; the former were based on rough calculations using ODEPLAN value-added and production data. Depreciation rates were based on ODEPLAN estimates, as were the non-competitive intermediate import coefficients.

(ii) The a_i parameters for the Stone-Geary function were chosen on the basis of shares of the three consumption goods (from sectors one and two, and non-competitive imports) in total consumption, and estimated expenditure elasticities of 0.625, 1.05, and 1.45 respectively. The b_i parameters and their growth rates were chosen initially to correspond to ODEPLAN's crudely estimated own-price elasticities, and

TABLE 2
VALUES OF THE PARAMETERS FOR THE BASIC SOLUTION

Definition	Parameter Symbol	Value
rate of technical progress in non-traded sector	ξ_1	0.01
rate of technical progress in traded sector	ξ_2	0.01
upper level elasticity substitution: non-traded sector	σ_{11}	0.3333
upper level elasticity of substitution: traded sector	σ_{12}	0.6667
elasticity of substitution between foreign and domestic capital	$\sigma_{21} = \sigma_{22}$	0.50
non-competitive intermediate import coefficients	μ_1 μ_2	0.05 0.20
capital depreciation rates domestic type foreign type	 δ_1 δ_0	 0.02 0.04
supernumerary income allocation parameters in Stone-Geary expenditure function (1: non-traded good; 2: traded good; 0: non-competitive imports)	a_1 a_2 a_0	0.13 0.75 0.12
"subsistence" consumption levels in Stone-Geary function (base year, multiplied by base year labor force of 2.942 million)[1]	$b_1 \cdot L$ $b_2 \cdot L$ $b_0 \cdot L$	2.971 10.885 0.530

TABLE 2 (cont.)

rates of growth of all "subsistence" levels b_i		0.011
social rate of discount	ρ	0.06
demand elasticity of sector 2 exports	η	-10.0
time trend in export demand function	r_{exp}	0.01
world prices:[2]		
foreign-type capital goods	π_0	0.75
non-competitive consumption goods	π_c	0.45
non-competitive intermediate inputs	π_{int}	0.62
non-traditional exports (base year)	π_2	0.6667
initial levels of exogenous demands:[1]		
sector 1		1.75
sector 2		0.20
growth rate of exogenous demands		0.05
rate of decline of all tariffs per year		0.13863(five-year halving time)
capital inflows from abroad ("foreign aid")		U.S. $50 million per year
labor force growth rate		0.02

[1] billions of 1965 escudos

[2] domestic prices in base year are all assumed to be one

then adjusted along with their growth rates to give a relatively credible evolution of aggregate savings in the model solutions. The value of 0.06 for the discount rate was chosen in line with values used in a number of economic programming models, but variations in its value had little effect on the solution.

(iii) The export demand elasticity of -10 is arbitrary (although similar values have been used in other studies). In any case, it is the subject of sensitivity analysis below. World prices were inferred by dividing domestic unit-values by estimated aggregate price differentials. The actual differentials are arbitrary, although based on ODEPLAN estimates and a variety of other estimates described in /20/.

(iv) The exogenous demands correspond mainly to residential construction (assumed exogenous because of the difficulties involved in incorporating it into static utility functions) and government consumption in the non-traded sector, and to government consumption in the traded sector. The assumed growth rate of five percent in these demands is consistent with Chilean experience in the 1960's. The other exogenous entries in the balance of payments, mineral exports and capital outflows, were set equal to ODEPLAN estimates for 1970-80, and extrapolated past 1980 according to

their growth rates in the latter part of the decade.

(v) Finally, terminal capital stock levels were chosen by an informal iteration, with different values being tried until significant decumulations or rapid accumulations of capital near the terminal time were avoided. Although this model does not strictly satisfy the conditions under which consumption turnpike theorems are proved (technical progress is not Harrod neutral, and the exogenous variables grow at non-exponential or differing exponential growth rates), steady exponential growth in the capital stocks was usually observed after a three- to five-year initial adjustment period.[14/] The steady growth path was used as a reference point, and terminal capital stocks (and their shadow prices) manipulated until growth in the last five years of the 30-year horizon was "quite" steady, on a graph. The initial ten to twenty years of the optimal price and quantity programs were essentially unaffected by these terminal adjustments.

[14/] Similar results have been observed for activity analysis models of the Japanese economy, solved by methods essentially the same as those used here /21 /. It is extremely difficult to prove that a turnpike exists in the Japanese models with trending input-output and productivity coefficients, but steady-state disaggregated growth is usually observed in the numerical solutions.

There are two additional features of these data that merit explicit comment.

First and not surprising, the base year production data do not conform to the principles of static neoclassical resource allocation embodied in equations (4.1) - (4.6). The average productivity of labor and rates of return to capital in the non-traded sector are clearly far too low relative to the same magnitudes in the traded sector for there to be any equalization of marginal products of factors. In the real world, this is an expected occurrence for these particular sectors. In an optimizing model without a built-in distortion prohibiting equalization of marginal products in the two sectors, the discrepancy must vanish. This can happen via a resource shift from the non-traded to traded sector, or an increase in the relative price of non-traded goods from our assumed value of unity. We can see immediately which of these two possible shifts away from the base year situation provides more flexibility from the second salient feature of the technological specification, the relative flatness of the model's production possibility curve.

This is illustrated in Figure 1, which shows transformation surfaces corresponding to initial and final factor

endowments in the basic solution to the model. Clearly, in any given year, only a small range of relative prices is permitted if we insist, as we must, on a solution with both goods produced. Hence, factor shifts between the two sectors must bear the main burden of static adjustment in this model.[15, 16] Moreover, we can also see that supply of the traded good is highly price elastic. This has implications for the numerical solution of the model (see appendix) and also for the magnitude of the adjustments the exchange rate must make from its tariff-ridden value when the model is solved subject to a world price foreign exchange constraint.

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[15] Harry Johnson pointed out long ago that the transformation surface generated by linearly homogeneous neoclassical production functions with at least as many factors as goods is likely to be quite flat /11/. (Of course, the surfact is literally flat, i.e. contains hyperplanes with more goods than factors.) No one who works with planning models has, as far as I know, pointed out explicitly that restricted relative price variability and tendencies toward specialization are inherent in constant returns specifications (despite the ad hoc constraints usually added to avoid them), although Chenery and Raduchel /7/ do note that large factor shifts were made in their models in response to modest parametric changes.

[16] Emmanuel Drandakis pointed out to me that the transformation surface of Figure 1 is only one of a family which go together to define an envelope surface satisfying the dynamic efficiency conditions (4.7) and (4.8). Hence, in the efficient long run, relative prices lying along the efficient frontier are even more restricted in range than the Figure indicates. Over time, of course, prices can shift: the final year transformation curve in Figure 1 is noticeably steeper than the initial year curve.

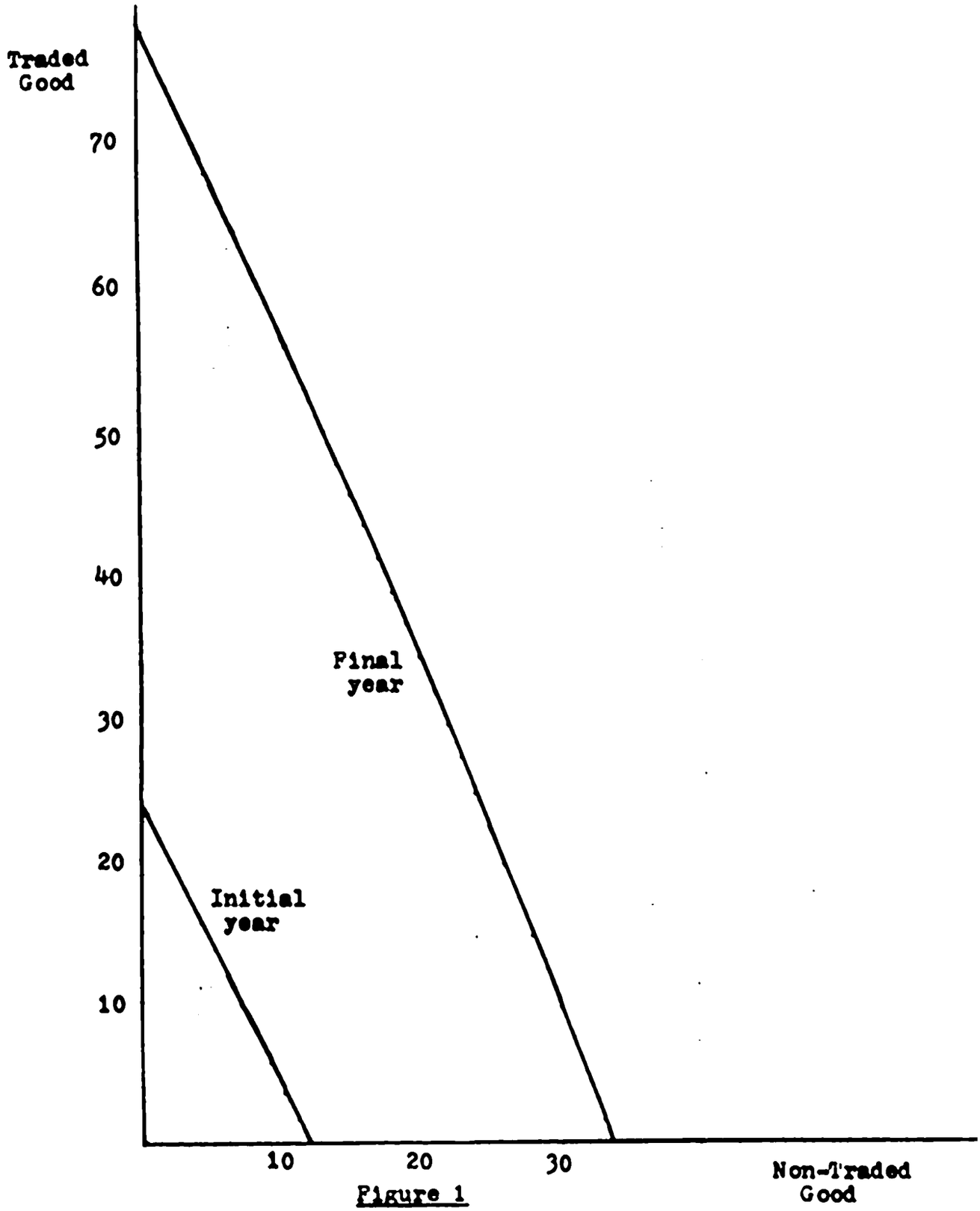

Figure 1

Product transformation curves for basic solution
(Billions of 1965 Escudos)

8. A Basic Solution

In a low order system of the type considered here, probably the best way to get a feel for the characteristics of the solution is via graphical presentation. In our particular case, Figures 2 through 4 illustrate a number of interesting characteristics of the model's basic 30-year solution (subject to the specifications of Table 2).

It is apparent from Figures 2 and 3 that growth of the major aggregate variables is remarkably steady in the basic solution. After a slight initial spurt, Gross Domestic Product in 1965 prices grows steadily at 3.4 percent per year from year 6 until year 30. Over the same period, both types of capital stock (Figure 3) grow at 3.7 percent. From Figures 2 and 3, it can be seen that the arrival on stream of new copper production capacity in 1971 (which causes the big jump in Mineral Exports) is used to finance a substantial increase in the amount of foreign-type capital, relative to the domestic type. After this readjustment of the capital stocks to "turnpike proportions", their aggregate amounts grow proportionately.[17]

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[17] Naturally, savings also grow steadily, after the initial spurt in investment in foreign-type capital goods. The steady state savings share of GNP is between 19 and 20 percent, high by historical standards in Chile, but by no means inconceivable under a growth-oriented socialist government. One might add the GDP growth rate also lies within the range which history would indicate as "reasonable" for Chile.

Many of the flow variables, however, do not show steady growth. In particular, the volume of non-traditional exports first drops off in response to the spurt in mineral exports, and then grows rapidly until it amounts to nine percent of GDP (8.6 percent in value terms) by the final year. Although the total export share in GDP is approximately the same in the initial and terminal years, the large proportionate rise of non-traditional exports marks a substantial structural change in the economy. If realistic (and it clearly depends on the accuracy of the copper export forecasts), this projection of export growth leads naturally to questions about <u>which</u> specific industries should grow to make up the aggregate total. We can not pursue such issues here, but they seem well worth investigating.

Figure 4 indicates that growth of non-traditional exports and other products from the traded goods sector requires fairly substantial resource shifts. Capital-labor ratios in sector two more than double over the 30-year planning period. In addition, there is an initial shift of resources from the ODEPLAN projections (which as noted above do not embody a fully efficient neoclassical allocation); both capital-labor ratios in the traded goods sector increase relatively to the

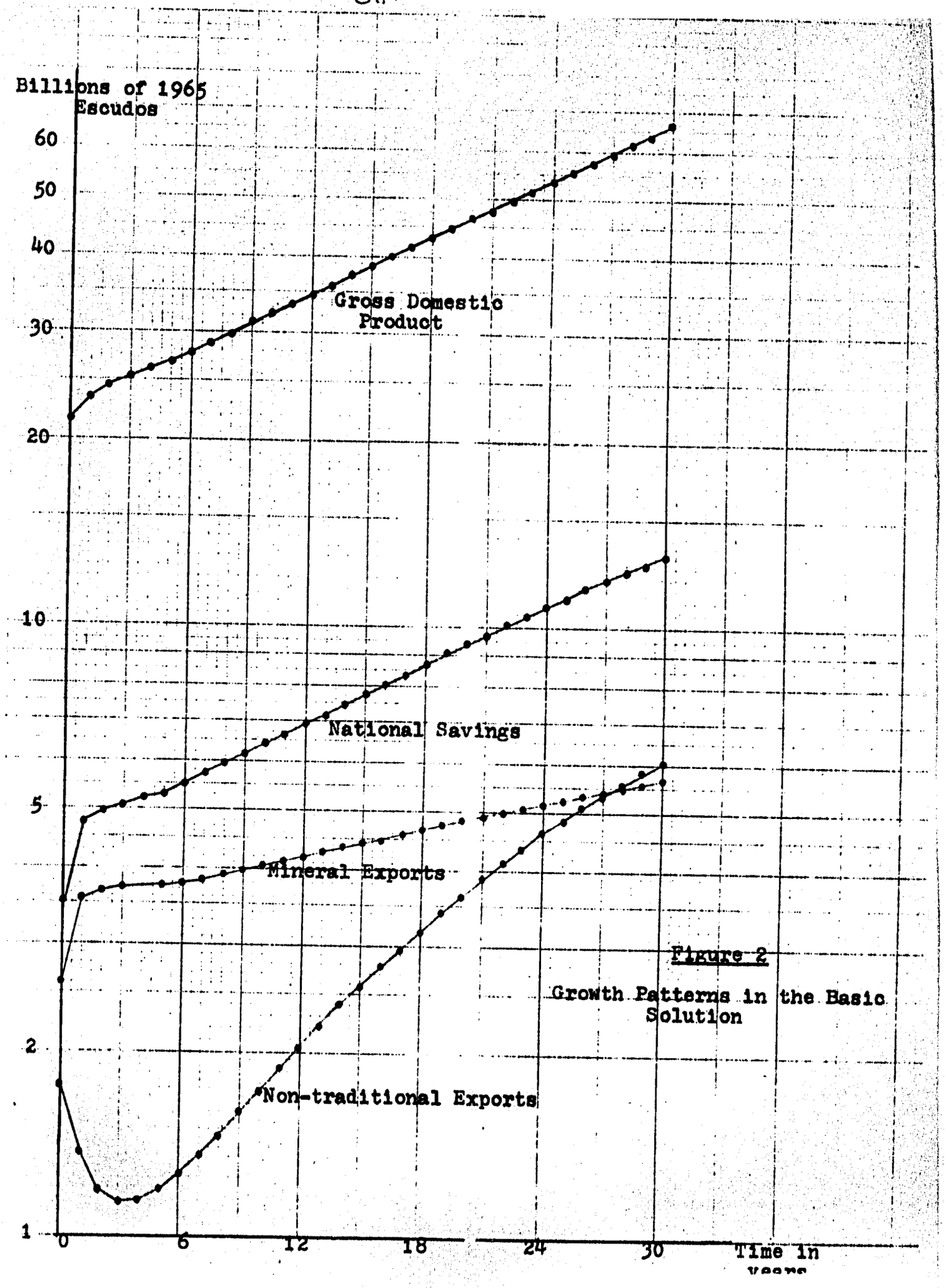

Figure 2

Growth Patterns in the Basic Solution

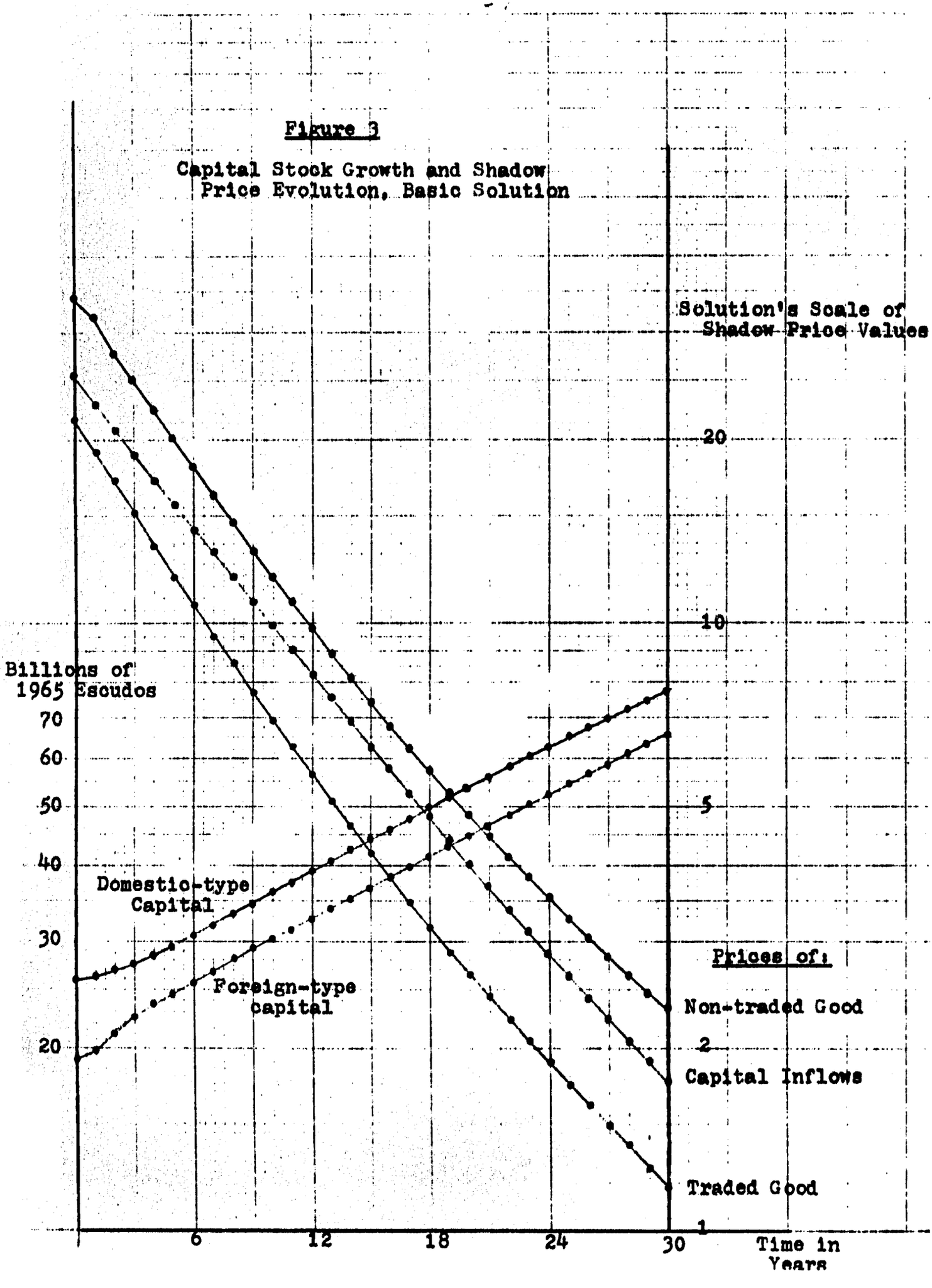
Figure 3
Capital Stock Growth and Shadow Price Evolution, Basic Solution
Solution's Scale of Shadow Price Values
20
10
5
2
1
Billions of 1965 Escudos
70
60
50
40
30
20
Domestic-type Capital
Foreign-type capital
Prices of:
Non-traded Good
Capital Inflows
Traded Good
6
12
18
24
30
Time in Years

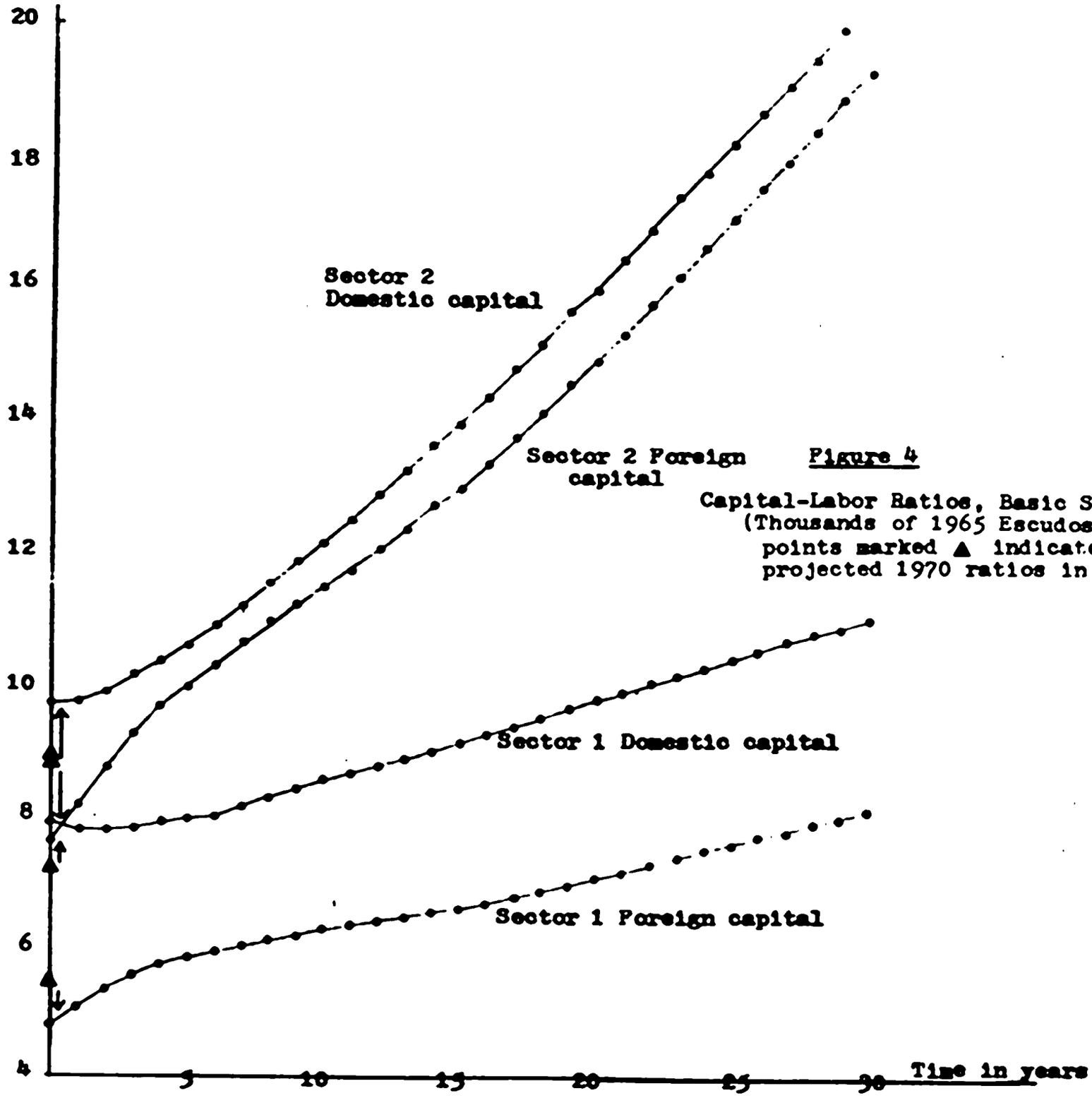

Figure 4

Capital-Labor Ratios, Basic Solution
(Thousands of 1965 Escudos per man; points marked ▲ indicate ODEPLAN's projected 1970 ratios in Table 1.)

non-traded sector. After this initial jump, the rapid build-up of foreign-type capital in the first years of the solution is also diverted to sector two. Once the build-up is complete, capital-labor ratios remain in approximately fixed ratio in both sectors, with foreign capital being more important for the traded good. However, by far the largest proportions of incremental labor and capital stocks are assigned to sector two at any time. Rapid export growth has its costs in terms of domestic resources.

Figure 3 also shows the behavior of the shadow prices over time. Their dimensions are in terms of the total welfare integral, and are difficult to interpret. However, only relative prices are of interest for investment project analysis, since equation (5.1) shows that a shadow price evaluation of a project would be unaffected by an arbitrary scaling constant.

The three prices shown in Figure 3 are those of the non-traded good, of capital inflows (the price P_0 divided by $\tau_0\pi_0$) and of the traded good. All decline in fairly smooth fashion over time, although the rate of decline is <u>not</u> constant; strictly speaking, discounting with a constant interest rate is not admissible according to this solution. Depending on your point of view, however, it might be a pretty

good approximation. The rentals on the two types of capital for selected years (relative to the price of the non-traded good) are as follows:

Year	Domestic Type Capital	Foreign Type Capital
0	0.121	0.141
5	0.129	0.104
10	0.120	0.096
15	0.110	0.090
20	0.101	0.083
25	0.093	0.075
30	0.087	0.069

Given that the worthwhileness of an investment project is often insensitive to a narrow range of interest rates, using a capital rental of, say, ten percent might not be completely inappropriate.

We have already seen that given the lack of curvature of the transformation curve, relative net prices cannot vary much in this model (see equations (4.1) - (4.6) and section 7). This is confirmed by Figure 3, which indicates a slight decline in the price of the non-traded good relative to that of the traded good at the beginning of the solution (corresponding to the rapid build-up of the traded capital stock), and then a very slight relative decline of the traded good prices thereafter (resulting from falling marginal revenues as exports expand). However, the overall vari-

ation is small.

More interesting is the fact that the two foreign prices (the traded good and capital inflows) are less than the non-traded good's price, contrary to the base year specification and the normal supposition that the foreign exchange shadow price should be "high". We have already pointed out two reasons for this--the undervaluation of the exchange rate resulting from protection and the fact that both resource reallocations and price shifts from the ODEPLAN base year specification (which had both prices equal to one) were made necessary by the latter's inefficient resource allocation. As Figure 1 indicates, the goods prices must stick fairly closely to their ratios in the basic solution. However, there is room for exchange rate shifts relative to the home good's price as levels of protection change. We explore the importance of this and other responses to alternative specifications in the following section.

9. Variant Solutions

Subject to the search procedure described in section 7 for terminal capital stock levels, a number of variant solutions were run. Their specific parameter changes are as follows:

(i) setting all τ_j to unity throughout the solution period, i.e. generating a "free trade" solution;

(ii) making the halving time of the τ_j 15 years instead of the five years of the basic solution;

(iii) setting exogenous capital inflows to $200 million per year instead of $50 million;

(iv) changing the export demand elasticity from -10 to -5;

(v) setting the social rate of discount to 0.03, instead of 0.06.

In general the effects of the variant specifications on the real variables are not important. This is demonstrated in Table 3, which shows the values of a number of macro-quantities in years 10 and 20 of the various solutions. The changes between the solutions are all quite small, and usually comprehensible in terms of the different specifications involved.[18] By all counts, the biggest changes take place in the solution permitting $200 million of capital inflow, but even here the differences relative to the basic solution

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[18] For example, Table 3 indicates that the effect of free trade is to permit more rapid capital accumulation without great consumption sacrifices relative to the restricted trade solutions. Along the same lines, less accumulation takes place in the solution with five-year halving times. High capital inflow permits both higher consumption and more rapid accumulation, while the lower export demand elasticity leads to welfare loss and a shift in accumulation toward the domestic-type capital. The lower discount rate raises consumption and lowers accumulation, but not much.

TABLE 3

AGGREGATE MAGNITUDES FOR VARIANT SOLUTIONS

		basic solution	"free trade" solution	15-year tariff halving time	$200 million capital inflow	export elasticity of -5.0	discount rate of 0.03
gross domestic product	year 10	32.18	32.31	32.19	32.70	32.06	32.21
	year 20	46.30	46.73	46.10	46.99	45.63	46.25
total consumption	year 10	19.91	19.85	19.89	20.67	19.81	19.97
	year 20	27.31	27.09	27.24	28.52	27.03	27.63
welfare integral	year 10	-85.55	-92.21	-91.17	-56.83	-96.38	-99.8
	year 20	-136.62	-147.35	-143.83	-87.37	-155.29	-169.72
domestic capital	year 10	36.02	37.25	35.91	36.72	37.25	36.09
	year 20	53.80	57.05	52.39	54.80	56.23	52.88
foreign capital	year 10	30.28	30.15	30.51	31.65	29.15	30.32
	year 20	44.71	46.25	44.63	46.24	41.03	44.18

are not large, except that the welfare integral is much less negative with the extra dollars. In general, we conclude along with Harberger /9/ and others that changes in the amount of distortion in a system with smoothly varying demand and production functions don't have a large percentage impact on macro-economic welfare measures. This is not to deny, of course, that the absolute gains from abolishing tariffs or getting more foreign aid may be large.

Table 4 illustrates the effects on the shadow prices of the different specifications. In all cases, the relative shifts of the two goods prices are extremely small. However, the value of capital inflows (the shadow exchange rate P_0 divided by the foreign capital good's post-tariff price) does move relatively to the home good's price P_1. The movements subject to the different specifications can be understood easily in light of the non-shifting net prices P_i^*. At any time point during the solutions, the flat production possibility surface means that the two net prices will have to be in some almost constant ratio K:

$$\frac{P_1^*}{P_2^*} = \frac{P_1 - P_0(\tau_{int}\pi_{int}/\tau_0\pi_0)\mu_1}{P_2 - P_0(\tau_{int}\pi_{int}/\tau_0\pi_0)\mu_2} = K. \qquad (9.1)$$

Substituting equation (4.6), which expresses P_2 in terms of P_0 and the export marginal revenue g', into (9.1) and re-

arranging gives

$$P_1 = \frac{K[\tau_2 g' - \tau_{int}\pi_{int}\mu_2] + \tau_{int}\pi_{int}\mu_1}{\tau_0\pi_0} P_0. \qquad (9.2)$$

Since μ_1 is quite small (non-traded goods require few imported intermediate inputs), (9.2) shows that the home goods price is essentially a constant times the value added of the traded goods sector (taking account of whatever tariffs exist, and expressed relatively to the capital goods price) times the exchange rate. As marginal effective protection of the traded good falls, the exchange rate must rise.

The values of P_0 relative to P_1 in the "free trade" and 15-year tariff halving time solutions illustrate this point, being respectively higher and lower than the values in the basic solution (with a five-year tariff halving time). The free trade and basic solutions converge over time, as the tariffs in the latter approach zero at the end of the 30-year planning period.

A similar increase in the value of the exchange rate shows up in the solution where the elasticity of export demand is reduced to -5.0. Here the exchange rate is higher than in the basic solution, and markedly appreciates as marginal revenue falls with export expansion.

In the two other variant solutions, effects are much

TABLE 4

PRICES RELATIVE TO THE NON-TRADED GOOD'S PRICE, VARIOUS SOLUTIONS

solution and year	capital inflow price	export good's price	rent on domestic capital	rent on foreign capital
basic solution				
0	0.747	0.629	0.121	0.141
5	0.777	0.596	0.129	0.104
10	0.831	0.581	0.120	0.096
15	0.848	0.565	0.110	0.090
20	0.835	0.549	0.101	0.083
25	0.803	0.531	0.093	0.075
"free trade" solution				
0	1.055	0.618	0.121	0.139
5	0.960	0.593	0.124	0.108
10	0.933	0.579	0.114	0.098
15	0.901	0.562	0.103	0.089
20	0.859	0.543	0.092	0.079
25	0.810	0.522	0.084	0.070
15-year tariff halving time				
0	0.743	0.628	0.121	0.140
5	0.703	0.598	0.129	0.103
10	0.722	0.583	0.121	0.095
15	0.731	0.569	0.113	0.089
20	0.728	0.553	0.106	0.083
25	0.715	0.538	0.100	0.076

TABLE 4 (cont.)

solution and year	capital inflow price	export good's price	rent on domestic capital	rent on foreign capital
$200 million capital inflow				
0	0.723	0.625	0.121	0.140
5	0.742	0.588	0.127	0.098
10	0.798	0.573	0.118	0.090
15	0.823	0.558	0.107	0.084
20	0.815	0.544	0.099	0.079
25	0.795	0.532	0.094	0.075
discount rate of 0.03				
0	0.748	0.630	0.121	0.141
5	0.778	0.596	0.129	0.103
10	0.831	0.580	0.120	0.096
15	0.848	0.566	0.109	0.090
20	0.834	0.550	0.104	0.085
25	0.805	0.535	0.099	0.079
export elasticity of -5.0				
0	0.829	0.641	0.120	0.137
5	0.912	0.615	0.122	0.109
10	1.022	0.605	0.112	0.103
15	1.099	0.596	0.100	0.099
20	1.131	0.585	0.091	0.096
25	1.126	0.572	0.085	0.093

as might be expected. Quadrupling capital inflows reduces the exchange rate by about 2.5%, another example of the great stability of neoclassical models. And reducing the social rate of discount from six to three percent has virtually no effect on either the price or quantity solution--a result which has been observed in other models.[19/]

10. Comparisons to Other Approaches and Conclusions

There are two other studies of shadow prices in Chile to which the results of this exercise may be directly compared--the rate of return estimates of Bitar and Trivelli /5 / and the foreign exchange shadow price estimates made by Bacha and me /20 /. The former study gives a number of different estimates of the rate of return to capital in Chile, mostly lying in the 10 to 20 percent range. This can be compared to the rental rates (relative to the non-traded good's price) shown in Table 4, and also to the rate at which the shadow prices decline in Figure 3. These in general are around 10 percent, two points higher in the ini-

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[19/] The slight differences between the low discount rate and basic solutions toward the end of the period are due to the fact that I stopped running the former model before generating a smoothly growing solution at the end of the planning horizon.

tial years of the solution and two points lower at the end. Given the rough nature of all calculations involved, I would say that Bitar-Trivelli and the current study tend to confirm each other's results. In both cases, rental rates are determined from production conditions and appear to be far from the guesses as to the utility (and steady-state "social") rate of discount made herein. For a developing country, this is a reassuring result.

Bacha and I estimated from a partial equilibrium model (developed in /2/) a value for the exchange rate which would rule under free trade conditions in Chile. Competitive imports played the role of a protected traded good that exports play in this model, and our estimating formula was

$$\bar{P}_0 = P_0 \tau^{\eta_m/(a+\eta_m)} \tag{10.1}$$

where $\bar{P}_0$ is the free trade exchange rate, P_0 is the current exchange rate, τ is the force of protection given imports, a is a constant and η_m is the demand elasticity of imports. We used a value of -2.0 for η_m, and concluded that the exponent in (10.1) had a value of something like one-half. Thus, the exchange rate rose less (in percentage terms) than τ fell in its approach to its equilibrium value of unity. With a higher value of η_m (resulting, for example, from highly elastic domestic supply of the import good and "normally" elas-

course would not be true, for (10.1) would in the limit become

$$\bar{P}_0 = {}^{T}P_0. \tag{10.2}$$

According to this limiting formula, the product of exchange rate and tariff is constant. But this is just what (9.2) says about the product of value added in the traded good sector and the exchange rate. Thus, in both approaches, a highly elastic ~~domestic~~ supply of the traded good (implicit in the present model in the almost constant ratio K of net prices in (9.2)) leads to an inverse proportionality relationship like (10.2) between exchange rate and tariff (or effective tariff). In econometric terms, the lower trade elasticities undoubtedly have more support. But the present formulation surely is neater theoretically, and may be more valid in the long run. It does carry with it a flat transformation surface, however, and the non-responsiveness of the present model to parameter variations and its strong implications for relative price relationships are part of this nexus.

I suppose I should say that only further research will demonstrate which specification better approximates the "true" state of the world, and in the long run I believe it. For the

moment, however, I would stick with a discount rate of 10% and an exchange rate more-or-less inversely proportional to tariffs, for Chile.

APPENDIX

Computation and Second Order Effects

The model stated here can be solved quite easily with existing control theory (or dynamic decomposition) algorithms. The algorithm actually used follows Jacobson and Mayne /10 /, and can be summarized as follows:

(i) At the beginning of the solution procedure, an initial nominal control history $\bar{u}_i(t)$ (where i indexes control variables) is used to integrate equations (3.1a), (3.2a) or (3.2b), and a differential equation version of (3.3) forward in time, using a fourth-order Runge-Kutta procedure with half-year time steps.

(ii) Values of the final costate variables, corresponding to P_1 and P_0 in previous notation, and also to V_{x_1} ($\partial V/\partial x_1$ where V is the value of the optimized welfare functional (3.3)) and V_{x_0} are chosen, and used as initial conditions to integrate equations (4.7) and (4.8) backward (for P_0 and P_1, or the vector V_x) in time. Equations for V_{xx}, the matrix of second derivatives of V with respect to the state variables, are also integrated backwards. These integrations are made with state variables set to their nominal values from the forward integration, but with new control variables $u_i^*(t)$ calculated at

each time point by maximizing the Hamiltonian evaluated using the V_x.[20] Finally, the difference between the Hamiltonian functions evaluated at the maximizing $u_i^*(t)$ and the nominal $\bar{u}_i(t)$ is also integrated backwards. The result at time t_0 of this quadrature can be shown to be a prediction of the gain in welfare resulting from using controls u_i^* in place of $\bar{u}_i$.

(iii) A new forward integration is made, using new nominal control variables calculated by expanding about u_i^* to second order in the changes of state variables from the previous nominal solution, using the vector V_x and matrix V_{xx} as the basic ingredients for deriving the weights in the expansion.

(iv) This forward/backward integration sequence is continued until the predicted welfare gain mentioned in point (ii) is small. Another second order procedure is then used

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[20] The Hamiltonian is maximized by (i) evaluating output levels along the transformation curve at a number of points close to the previous time point's output vector by varying factor allocations subject to the equations (4.3) - (4.5) expressed in ratio form; (ii) numerically differentiating the transformation curve to find the ratio between the net prices P_1^* and P_2^* of equations (4.1); (iii) solving the ratio explicitly for P_2, using the current values from the integration of P_0 and P_1; (iv) using P_2 to calculate the consumption demand for the traded good from (4.2) and export demand from (4.6); (v) comparing total demand for the traded good with total supply from step (i), and iterating further by expanding the transformation curve about the demand point if the values are not close. The stability of this cobweb-like iteration in the export good market is guranteed by the very high supply elasticity of the export good noted in section 7, and convergence is fast.

to modify the final values of the V_x vector to try to bring the terminal capital stock levels closer to their specified levels, and the process continues until this latter convergence is achieved.

The first part of this algorithm usually converged within two or three backward integrations, after a successful choice of the terminal values of P_0 and P_1. This choice was initially not simple, given the very small range of relative net prices permitted by the flat transformation surfaces exemplified in Figure 1. Moreover, the algorithm's correction of the final prices often led to corner solutions, and it was often necessary to guess the final price ratio several times to get solutions without rapid accumulations or decumulations of capital as the terminal time was approached. (This problem was no doubt exacerbated by the flatness of the transformation curve, but it often occurs in any case, according to the oral tradition of control theorists.)

I think that the good overall performance of the algorithm was due to minimizing the Hamiltonian at all times. This was one of two major differences between the present algorithm and a more slowly converging one used in some previous numerical analysis of similar problems with David Kendrick /12 /.

The other difference was incorporation of second order information via the matrix V_{xx}. The numerical values of this matrix were relatively small (as is shown in Table 5). It can be shown[21] that the product of the V_{xx} terms and the vector of capital rentals measures the impact which one unit of additional capital stock would have on the vector of capital rentals--in effect the interest on the interest generated by a new investment project. Given this economic interpretation, it is not surprising that the second order terms in the expansion for a new $\bar{u}_i(t)$ have little effect. The same is of course true regarding the importance of (second order) effects of investments on interest rates in project evaluations. In a neoclassical world, such niceties need not be taken into account.

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[21] See /19/, equation (27) and surrounding text.

TABLE 5

VALUES OF THE FIRST AND SECOND DERIVATIVES OF THE OPTIMIZED WELFARE FUNCTIONAL WITH RESPECT TO STATE VARIABLES (BASIC SOLUTION)

Time	$\frac{\partial V}{\partial x_1}(=P_1)$	$\frac{\partial V}{\partial x_0}(=P_0)$	$\frac{\partial^2 V}{\partial x_1 \partial x_1}$	$\frac{\partial^2 V}{\partial x_1 \partial x_0}$	$\frac{\partial^2 V}{\partial x_0 \partial x_0}$
0	34.120	25.512	-8.263	-5.727	-5.009
5	20.115	13.675	-2.906	-1.721	-1.485
10	11.924	8.050	-0.946	-0.538	-0.512
15	7.413	4.913	-0.312	-0.166	-0.184
20	4.829	3.086	-0.100	-0.046	-0.067
25	3.277	1.995	-0.026	-0.008	-0.023
30	2.305	1.325	-0.000	-0.000	-0.000

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