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MULTI-SECTORAL PROJECT ANALYSIS
USING
MIXED INTEGER PROGRAMMING*

by

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MULTI-SECTORAL PROJECT ANALYSIS USING MIXED INTEGER PROGRAMMING

I. Introduction

The coordination of investment is one of the central tasks of development policy in Korea. At the project level, investment planning in Korea, as in other less-developed economies, is typically carried out through the analysis of individual projects considered more or less in isolation from one another. But the application of benefit-cost investment criteria is unwarranted where industries are linked through the use of intermediate products and when future product prices depend upon the set of projects selected for investment. If there are economies of scale in related sectors, then it is necessary to rely on a planning procedure whereby projects are considered simultaneously. Only in this way is it possible to reflect the dependence of the profitability of one project upon which of the remaining projects are selected to be undertaken.

However, all projects are not equally influential on the course of relative prices in the near future. For example, the decision to build a textiles plant in Korea has a far less significant impact on the future structure of the economy than the decision to build a large steel mill. It is most important to examine a project in a general equilibrium setting when that project, if accepted, would require a large fraction of the investment budget and would result

in a major change in relative prices. Such is often the case where economies of scale are present.

The effects of economies of scale in one sector upon the optimal allocation of investment funds among sectors have been discussed under the category of "external economies."[1] But to date there has been no operational means through which the external economies linked to a particular project can be satisfactorily included in a partial analysis of that project's benefits and costs. To include the external effects of an investment project within the set of factors considered in its evaluation, it is necessary to turn to general equilibrium analysis. In general equilibrium analysis scarce resources are allocated among sectors simultaneously, so there is no need to associate benefits and costs with particular investment projects.

This paper deals with the formulation and use of a dynamic, multi-sectoral optimizing model for investment planning. This model is specifically designed to incorporate both the effects of economies of scale in some sectors and interdependence between sectors within the framework in which the costs and benefits of large investment projects are examined. In this model, external economies are internalized as resources are simultaneously allocated to all of the sectors of the economy.

Dynamic input-output models, such as the one used here, are generally specified at a level of aggregation much above that of

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[1] Footnotes appear at the rear of the paper.

the individual plant or project.[2] The relevance of these models to investment planning is consequently reduced, since it is at the project level that investment decisions are made. To make solution practical and yet retain the operational significance of a dynamic, input-output model, it is necessary to focus upon a small subset of specific investment decisions. The projects most usefully singled out for detailed attention within a multi-sectoral framework are those which, through the competition for scarce investment resources or input-output relationships, are likely to have the greatest impact on the welfare gains from investment in the rest of the economy. Factors impinging upon the investment decisions related to these projects should be specified in considerable detail in the model while the remaining activity in the economy is included with less detail in a manner insuring that competing demands by all sectors for scarce resources are adequately reflected.

This course has been adopted here. The model is applied to planning the timing of investment in a petrochemicals plant and in an integrated iron and steel mill in Korea. Both plants are under serious consideration, both have economies of scale in their construction, and the construction of either would require more than five per cent of the total budget for investment in manufacturing during the Second Five-Year Plan. The existence of economies of scale in the construction of the plants and the expense of constructing either give rise to a strong argument in favor of analyzing the merits of

each within a multi-sectoral framework.

The model is described in Section II, and some solutions to it are analyzed in Section III. Finally, the Conclusion discusses the model's usefulness in determining the extent to which the construction of either plant would contribute to Korea's development goals.

II. The Model

Except for the inclusion of economies of scale and the treatment of import substitution for petrochemicals and various classes of ferrous metals, the model is in the tradition of the models used by Chenery and Kretschmer /2 /, Manne /9 /, Bruno /1 /, Eckaus and Parikh /7 /, and Chenery and MacEwan /3 /. In common with the first four of these, it focuses attention on the choice between domestic production and imports to satisfy a mix of endogenously and exogenously generated demands; like the last two, it also focuses on the optimal savings problem, though over a shorter period of time. In other words, the model is basically formulated as a dynamic input-output model with the inclusion of inequality constraints, import alternatives, and foreign capital inflows. The formulation of the model as a mixed integer programming model draws on the work of Chenery and Westphal /4 /.

Economies of scale in the provision of capacity in the petrochemicals and steel sectors are included through the use of fixed charge cost functions. Engineers have long recognized that there

are economies of scale in capacity construction in the major process industries. Typically, the cost of capacity is related to the scale of the plant constructed through the equation "Total Cost = $a\ y^b$," where "a" is a constant factor, "y" is the scale of plant, and "b" is the elasticity of total cost with respect to scale. In the construction of petrochemicals plants and integrated steel mills, "b" has been found to range between 0.6 and 0.8.[3] To obtain a solvable model, the constant elasticity cost function has been approximated by a fixed charge cost function of the form "Total Cost = $B^a\ \delta + B\ y$," where "B^a" is a fixed charge incurred only if capacity is built and "B" is the variable cost associated with constructing a unit of capacity. "δ" is equal to zero if a plant is not constructed and is equal to one if a plant, regardless of its size, is built. The fixed charge cost function closely approximates the constant elasticity cost function over a wide range of plant sizes.

The model is linear except for the specification of fixed charge capacity cost functions in the two sectors noted above. Consequently, linear programming cannot be used to obtain solutions, and recourse is made to mixed integer programming techniques.[4] A mixed integer programming solution algorithm developed by Davis, Kendrick and Weitzman /6 / has been used to compute solutions.

Treatment of Production and Import Substitution

Central to the classification of sectors in the model is their division into "traditional" sectors and import substituting "complexes."

TABLE I

Sector Classification in the Model

I. Traditional Sectors:

1. Agriculture, Forestry and Fishery (1, 2, 3, 4)
2. Coal and Other Minerals, Coal Products (5, 6, 32210/270, 32900/270)
*3. Processed Foods, Beverages, Tobacco, Forestry Products, Printing and Publishing (7, 8, 12, 13, 14, 15)
4. Fiber Spinning (9)
5. Textile Fabrics and Products (10, 11)
6. Leather and Rubber Products, Miscellaneous Light Manufactured Products (16, 17, 32)
7. Chemicals (18, 19, 20)
8. Fertilizer (21)
9. Petroleum Fuels and Lubricants (32300/270)
*10. Cement and Earthen Products, Construction (23, 24, 33, 34)
11. Iron and Crude Steel (25)
12. Steel Products (26)
13. Finished Metal Products, Non-ferrous Metals (27, 28)
*14. Machinery and Transport Equipment (29, 30, 31)
15. Electricity, Water, Commerce, Transport and Services (35, 36, 37, 38, 39, 40, 41)

II. Important Industrial Complexes:

PC. Petrochemical Complex (includes all stages of petrochemicals processing up to and including the production of plastic resins, synthetic rubber polymers, and synthetic fiber intermediates prior to conversion to filament)

IS. Integrated Iron and Steel Mill (includes both traditional iron and crude steel and steel products and formerly non-competitive types of iron and crude steel and steel products)
IS1: Iron and crude steel produced in the integrated mill
IS2: Steel products produced in the integrated mill

Note: The numbers in parenthesis appearing after each traditional sector are the numbers of the included sectors in the 43 sector aggregation scheme of the Bank of Korea.

A "*" designates a sector which produces capital goods.

(Hereafter the petrochemicals plant and integrated steel mill will often be referred to as "complexes.") To understand this division, it is necessary to briefly examine the structure of the Korean economy. The classification scheme used in the model is presented in Table I.

The fifteen traditional sectors include all production in the economy with which imports were competitive in 1966. Included within these sectors are lines of production which existed in Korea in 1966; the relevant input-output data and investment cost figures are based on historical statistics which have been modified to reflect structural changes thought to have occurred by the end of 1966.[5]

The two projects that have been singled out for special attention represent largely import substituting investments for Korea. The product mix and technology assumed for each plant was predetermined on the basis of engineering studies. The production of petrochemicals--reference is made to plastics, synthetic fibers, and synthetic rubber--from the initial stages on up through the final stage of processing into resin or polymer is not currently a part of the Korean economy. There do exist in Korea, on the other hand, facilities for producing iron and crude steel and for processing crude steel into light steel sheets and bars for construction use. But the Korean steel industry is unbalanced, since the bulk of pig iron production goes into castings so that the tonnage of crude steel production is less than half that of iron, and most of the crude

steel production is based on the use of scrap. The integrated steel mill is proposed as a means of redressing the balance between pig iron and crude steel produced in Korea as well as a means of introducing the production of more specialized steel products and heavier forms of rolled products.

Since the integrated steel mill represents both import substitution and the expansion of an already existing (albeit incompletely rationalized) industry, three classes of unfinished ferrous metal products have been distinguished--traditional iron and crude steel (commodity 11), traditional steel products (12), and formerly non-competitive iron, crude steel, and steel products (IS). Sectors producing traditional iron and crude steel and traditional steel products by the existing techniques are included in the model (sectors 11 and 12). Two production activities are distinguished for the integrated mill: the production of iron and crude steel, and the production of steel products. The integrated mill produces all three forms of unfinished ferrous metal products; the demand for the import substituting output of the mill is met by production in its two components in fixed proportions.

The petrochemicals complex is easily specified as a new input-output sector (sector PC). The picture of production activity which emerges from the approach taken here is given in Figure I which shows, in tableau format, the input-output functions for the nineteen production activities. From left to right, there are fifteen for the

FIGURE I

Production Activity Tableau

From \ To	Traditional Sectors	Petro Chemicals	Iron & Crude Steel	Steel Products	Non-traditional Ferrous Produc
			Iron and Steel Mill		
Traditional Commodities	$1-A_{1,1}$ — — — — — $-A_{1,15}$	$-A_{1,PC}$	$-A_{1,IS1}$	$-A_{1,IS2}$	0
	$-A_{i,j}$				0
			$1-A^*_{11,IS1}$	$1-A^*_{12,IS2}$	$-Q_{IS1}$ $-Q_{IS2}$
	$-A_{i,j}$				0
	$-A_{15,1}$ — — — — — $1-A_{15,15}$	$-A_{15,PC}$	$-A_{15,IS1}$	$-A_{15,IS2}$	0
Petro-Chemicals	$-X_{PC,1}$ — — — — — $-X_{PC,15}$	$1-A_{PC,PC}$	0	0	0
Non-traditional Iron & Steel	$-X_{IS,1}$ — — — — — $-X_{IS,15}$	0	1	0	1

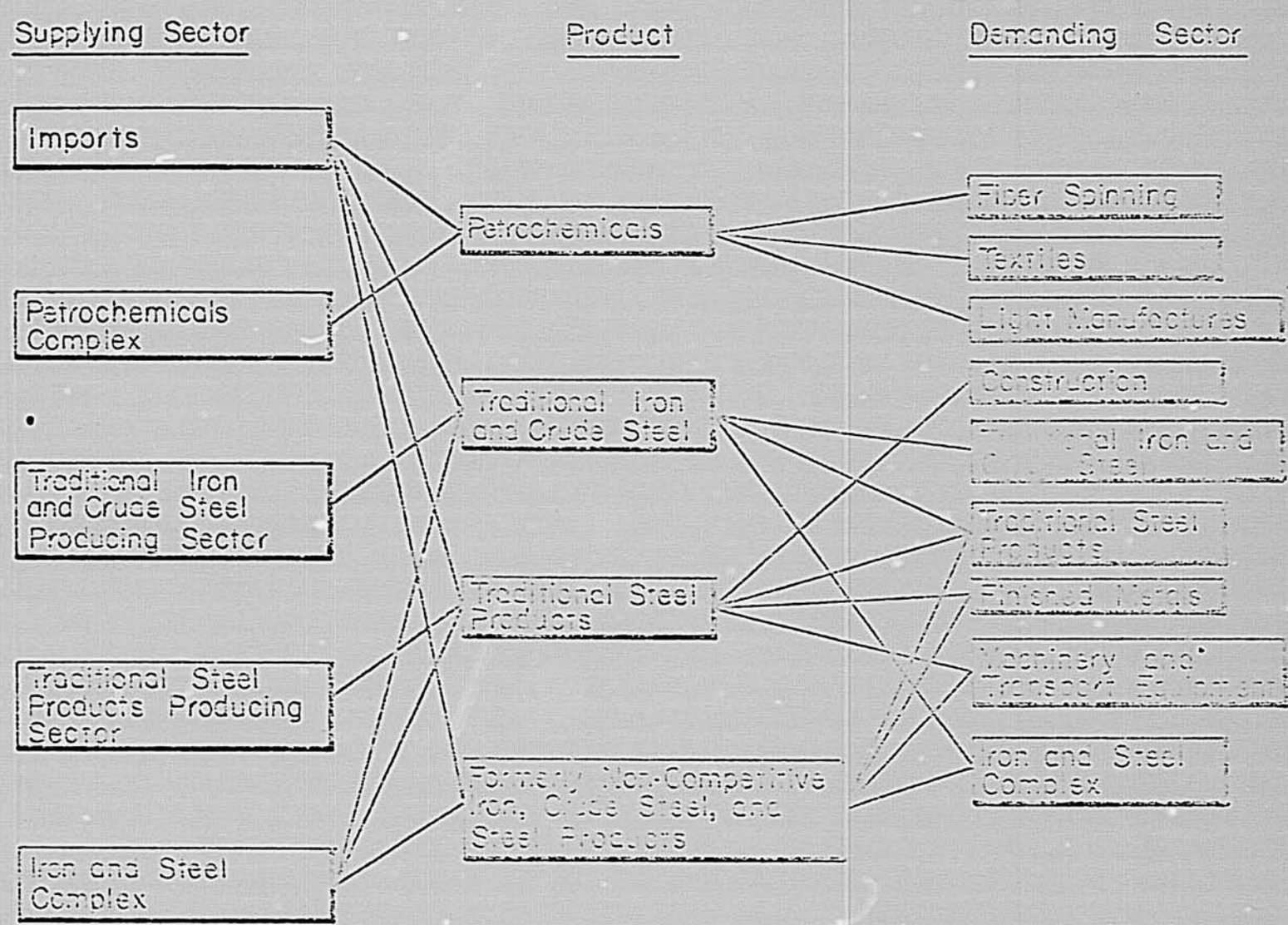

FIGURE II

Flow Diagram for Petrochemicals and Ferrous Metals

traditional sectors, one for petrochemicals, two for production in the integrated mill, and the last activity is the provision of non-traditional ferrous metals by the integrated mill. The "A_{ij}'s" and "N_{ij}'s" are input-output coefficients; "Q_{ISi}" determines the proportion of iron and crude steel (i=1) and steel products (i=2) present in import substituting production in the new mill. The input-output matrix has nineteen columns (production activities) but only seventeen rows (commodity classes).[6] Figure II indicates the major supply and demand relationships between production in the complexes and production in the traditional sectors.

Activities in the Model

The model is stated algebraically in Table II, and Figure III presents a tableau of the activities present in each period of the model. The usage of the symbols is described in the Glossary at the end of this paper.

The model is run over a three-period span; each period includes two years. The periods are 1967-68, 1969-70, and 1971-72. Each period or cycle of activity is composed of six sets of activities:

1) production, designated by "x_n;"
2) increments to capacity, denoted by "y_n;"
3) imports, symbolized by "m_n;"
4) exports, given by "e_n;"
5) accumulation of foreign exchange reserves, "u_t;" and,
6) consumption, given by "c_t^T."

All production and capacity creation in the sectors of the model are endogenous. Investment in each of the complexes requires two variables, "δ_n" as the zero-one integer variable determining the fixed charge cost of capacity creation, and "y_n" as the amount of capacity created and the determinant of the variable charge investment cost. A common capacity constraint is shared by the activities representing production in the integrated steel mill.

Imports of all products (with the exception of Overhead and Services which are not traded) are endogenous, as are exports of petrochemicals; all other exports are exogenously determined by projections of export demand and commodity availability. This course was followed because of insufficient computational capacity to handle the bounds which would have to be placed on endogenous export activities. Foreign exchange accumulation is specified relative to the initial level of foreign exchange reserves in the economy. There is no activity representing the borrowing of foreign exchange in the basic specification for reasons that will become clear. Thus foreign capital inflows are specified exogenously.

Like exports, government demands for goods and services (g_n) are exogenous. The only major components of production activity not reflected in the model are the direct employment of persons by the government and household sectors. With these two exceptions, the sum total of production activity in the model yields gross domestic product. Income earned by Koreans working abroad is exogenously

FIGURE [illegible]

One Period Tableau of Model I: The Capital Accumulation Model

	Production						Investment					Imports		Exports/h		C	RHS	No. of Rows
No. of Activities	15	1	1	1	1	15	1	1	1	1	14	1	1	1	1	1		
Activity Vector	x_t	$x_{PC,t}$	$x_{IS1,t}$	$x_{IS2,t}$	$F^s_{IS,t}$	V_t	$Y_{PC,t}$	$Y_{IS,t}$	$I_{PC,t}$	$I_{IS,t}$	F_t	$M_{PC,t}$	$M_{IS,t}$	$E_{PC,t}$	E_t	c^T_t		
Commodity Balances	I-A	$-A_{PC}$	$-A_{IS1}$	$-A_{IS2}$	$-C_{IS}$	-J-E	$-B_{PC}$	$-B_{IS}$	$-B^s_{PC}$	$-B^s_{IS}$	-I					-S	$\geq \frac{\lambda}{P_c} S^a + e_t + S_t$	15
PC Balance	$-N_{PC}$	1-*										1		-1			$\geq -E_{PC} x_{PC,t-1}$	1
IS Balance	$-N_{IS}$		$-E_{IS1}$	$-E_{IS2}$	1								1				$\geq$ (1)	1
Foreign Exchange	N	N^s_{PC}	N^s_{IS1}	N^s_{IS2}		M	M_{PC}	M_{IS}	M^s_{PC}	M^s_{IS}	I	1	1	-F	1		$\geq f_t + F^T_t x_{1,t} - u_{t-1}$	1
Capacity Constraints	I																$\geq K_t$	15
PC Capacity		1															$\geq K_{PC,t}$	1
IS Capacity			1	1													$\geq K_{IS,t}$	1
Import Bounds											I						$\geq -(I_t - I^T_{1,t} F)$	14
Consumption Bound																1	$\geq (P_t/P_c) c^T_t (1-c)^{-1} t^T$	1
Savings Bound	-rV	$-rV^s_{PC}$	$-rV^s_{IS1}$	$-rV^s_{IS2}$		B^T	B^T_{PC}	B^T_{IS}	B^{sT}_{PC}	B^{sT}_{IS}					1	E_{t-1}	$\geq$ (2)	1
IS Complex Constraints			1	1	-1												≥ 1	1
			1														$\geq x_{IS1,t-1}$	1
				1													$\geq x_{IS2,t-1}$	1
Integer Constraints PC Complex																		
		1					-1		$+V_{PC}$								≥ 0	1
									1								$= 1$ or 0	1
IS Complex								-1		V_{IS}							≥ 0	1
										1							$= 1$ or 0	1

Notes:

*: $-A_{PC,PC} - E_{PC}$

=: Includes $-E_{PC}$, E_{IS1}, E_{IS2} respectively

(1) $-E_{IS1} x_{IS1,t-1} - E_{IS2} x_{IS2,t-1}$

C is a 15 x 14 matrix having $C_{ii}=1$ for $i=1,14$, and zeroes elsewhere.

h: Foreign exchange reserve accumulation

c: Consumption

(2) $f_t - f_{t-1} + I_{t-1} - u_{t-1} - I^T V_{t-1} - B^{sT}_{PC} Y_{PC,t-1}$

$- B^{sT}_{PC} V_{PC,t-1} - B^{sT}_{IS} Y_{IS,t-1} - B^{sT}_{IS} V_{IS,t-1} + B_{IS,hous.} c^T_{t-1}$

$-$ slack for $ex._t$ $-$ slack for $ex._{t-1}$ $- rVx_{t-1}$

$- rV^s_{PC} x_{PC,t-1} - rV^s_{IS1} x_{IS1,t-1} - rV^s_{IS2} x_{IS2,t-1}$

TABLE II

Algebraic Statement of the Model

(a) Output determination; $t = 1, \ldots, \tau$

(a.1) $$x_{i,t} + m_{i,t} - \sum_{j=1}^{15} A_{ij}\, x_{j,t} - A_{i,PC}\, x_{PC,t} - A^*_{i,IS1}\, x_{IS1,t} - A^*_{i,IS2}\, x_{IS2,t} - in_{i,t} - c_{i,t} = g_{i,t} + e_{i,t}$$

$$i = 1, \ldots, 10, 13, 14, 15$$

(a.2) $$x_{i+10,t} + x_{ISi,t} + m_{i+10,t} - \sum_{j=1}^{15} A_{i+10,j}\, x_{j,t} - A_{i+10,PC}\, x_{PC,t} - A^*_{i+10,IS1}\, x_{IS1,t} - A^*_{i+10,IS2}\, x_{IS2,t} - Q_{ISi}\, ms_{IS,t} - in_{i+10,t} - c_{i+10,t} = g_{i+10,t} + e_{i+10,t}$$

$$i = 1, 2$$

(a.3) $$ms_{PC,t} + m_{PC,t} - \sum_{j=1}^{15} N_{PC,j}\, x_{j,t} - A_{PC,PC}\, x_{PC,t} - e_{PC,t} - in_{PC,t} \geq 0$$

(a.4) $$x_{PC,t} = ms_{PC,t}$$

(a.5) $$ms_{IS,t} + m_{IS,t} - \sum_{j=1}^{15} N_{IS,j}\, x_{j,t} - in_{IS,t} \geq 0$$

(b) Private consumption demand; $t = 1, \ldots, \tau$

(b.1) $$c_{i,t} = (P_t/P_0)\, S^a_i + S_i\, c^T_t$$

$$i = 1, \ldots, 15$$

(b.2) $$c^T_t = \sum_{i=1}^{15} c_{i,t}$$

(c) Demand for investment goods; $t = 1, \ldots, \tau$

(c.1) $$in_{i,t} = \sum_{j=1}^{15} B_{i,j}\, y_{j,t} + B^a_{i,PC}\, \delta_{PC,t} + B_{i,PC}\, y_{PC,t} + B^a_{i,IS}\, \delta_{IS,t} + B_{i,IS}\, y_{IS,t} + E_i\, y_{i,t}$$

$$i = 1, \ldots, 15$$

(c.2) $in_{PC,t} = E_{PC}\ (x_{PC,t} - x_{PC,t-1})$

(c.3) $in_{IS,t} = E_{IS1}\ (x_{IS1,t} - x_{IS2,t-1}) + E_{IS2}\ (x_{IS2,t} - x_{IS2,t-1})$

(d) Foreign exchange use; $t = 1,\ldots,\tau$

(d.1) $$\sum_{i=1}^{15} N_i\ x_{i,t} + N^*_{PC}\ x_{PC,t} + N^*_{IS1}\ x_{IS1,t} + N^*_{IS2}\ x_{IS2,t} + \sum_{i=1}^{15} M_i\ y_{i,t} + M^a_{PC}\ \delta_{PC,t} + M_{PC}\ y_{PC,t} + M^a_{IS}\ \delta_{IS,t} + M_{IS}\ y_{IS,t} + \sum_{i=1}^{14} m_{i,t} + m_{PC,t} + m_{IS,t} + u_t - F\ e_{PC,t} - u_{t-1} \leq f_t + F \sum_{i=1}^{14} e_{i,t}$$

(e) Capacity constraints; $t = 1,\ldots,\tau$

(e.1) $x_{i,t} - k_{i,t} \leq 0$ $\qquad i = 1,\ldots,15$

(e.2) $x_{PC,t} - k_{PC,t} \leq 0$

(e.3) $x_{IS1,t} + x_{IS2,t} - k_{IS,t} \leq 0$

(f) Capacity determination; $t = 1,\ldots,\tau$

(f.1) $k_{i,t} = \nu_t k_{i,0} + \sum_{s=1}^{t-1} U_{i,s,t}\ y_{i,s} - d_{i,t-1}$ $\qquad i = 1,\ldots,15$

(f.2) $d_{i,t} = \nu_t(1 - D_i)^{\nu_t t}\ k_{i,0}$ $\qquad i = 1,\ldots,15$

(f.3) $k_{PC,t} = \sum_{s=1}^{t-1} U_{PC,s,t}\ y_{PC,s}$

(f.4) $k_{IS,t} = \sum_{s=1}^{t-1} U_{IS,s,t}\ y_{IS,s}$

(g) Competitive import bounds; $t = 1,\ldots,\tau$

(g.1) $m_{i,t} \leq \mu_i\ (f_t + F \sum_{i=1}^{14} e_{i,t})$ $\qquad i = 1,\ldots,14$

(h) Private consumption and savings bounds; $t = 1,\ldots,\tau$

(h.1) $c^T_t \geq (P_t/P_0)\ c^T_0\ (1+\mathcal{C})^{\sum_{s=1}^{t-1} \nu_s}$

(h.2) $-\sigma \sum_{j=1}^{15} V_j (x_{j,t} - x_{j,t-1}) - \sigma V_{PC} (x_{PC,t} - x_{PC,t-1}) - \sigma V_{IS1} (x_{IS1,t} - x_{IS1,t-1}) - \sigma V_{IS2} (x_{IS2,t} - x_{IS2,t-1}) +$

$\sum_{j=1}^{15} B_j^T (y_{j,t} - y_{j,t-1}) + B_{PC}^{aT} (\delta_{PC,t} - \delta_{PC,t-1}) +$

$B_{PC}^T (y_{PC,t} - y_{PC,t-1}) + B_{IS}^{aT} (\delta_{IS,t} - \delta_{IS,t-1}) + B_{IS}^T$

$(y_{IS,t} - y_{IS,t-1}) + E_{PC} (x_{PC,t} - 2 x_{PC,t-1} + x_{PC,t-2})$

$+ E_{IS1} (x_{IS1,t} - 2 x_{IS1,t-1} + x_{IS1,t-2}) + E_{IS2} (x_{IS2,t} -$

$2 x_{IS2,t-1} + x_{IS2,t-2}) + u_t - 2 u_{t-1} + u_{t-2} + s1_{d.1,t}$

$- s1_{d.1,t-1} + B_{10,hous.} (c_t^T - c_{t-1}^T) \leq f_t - f_{t-1}$

(i) Iron and steel complex bounds; $t = 1, \ldots, \tau$

(i.1) $ms_{IS,t} - x_{IS1,t} - x_{IS2,t} \leq 0$

(i.2) $x_{ISi,t} - x_{ISi,t-1} \geq 0 \qquad i = 1,2$

(j) Terminal growth constraint; $t = \tau$

(j.1) $\sum_{i=1}^{15} V_i k_{i,\tau+1} + V_{PC} k_{PC,\tau+1} + V_{IS}^* k_{IS,\tau+1} - (1+\gamma)^{\nu_\tau} (\sum_{i=1}^{15} V_i x_{i,\tau}$

$+ V_{PC} x_{PC,\tau} + V_{IS1} x_{IS1,\tau} + V_{IS2} x_{IS2,\tau}) \geq 0$

(j.1a) $\sum_{i=1}^{15} V_i k_{i,\tau+1} + V_{PC} k_{PC,\tau+1} + V_{IS}^* k_{IS,\tau+1} + u_\tau - (1+\gamma)^{\nu_\tau}$

$(\sum_{i=1}^{15} V_i x_{i,\tau} + V_{PC} x_{PC,\tau} + V_{IS1} x_{IS1,\tau} + V_{IS2} x_{IS2,\tau}$

$+ u_{\tau-1} - u_\tau) \geq f_{\tau+1} - (1+\gamma)^{\nu_\tau} f_\tau$

(k) Terminal capital stock bounds; $t = \tau$

(k.1) $(1+\gamma)^{\nu_\tau} \phi_j x_{i,\tau} - k_{i,\tau+1} \geq 0 \qquad j = 1, \ldots, 15$

(k.1a) $-(1+\gamma)^{\nu_\tau} x_{15,\tau} + k_{15,\tau+1} \geq 0$

(k.2) $(1+\gamma)^{\nu_\tau} \phi_{PC} \, x_{PC,\tau} - k_{PC,\tau+1} \geq 0$

(k.3) $(1+\gamma)^{\nu_\tau} \phi_{IS} \, (x_{IS1,\tau} + x_{IS2,\tau}) - k_{IS,\tau+1} \geq 0$

(l) Integer constraints; $t = 1, \ldots, \tau$

(l.1) $y_{PC,t} - Y_{PC} \, \delta_{PC,t} \leq 0$

(l.2) $\delta_{PC,t} = 1$ if $y_{PC,t} > 0$; $\delta_{PC,t} = 0$ if $y_{PC,t} = 0$

(l.3) $y_{IS,t} - Y_{IS} \, \delta_{IS,t} \leq 0$

(l.4) $\delta_{IS,t} = 1$ if $y_{IS,t} > 0$; $\delta_{IS,t} = 0$ if $y_{IS,t} = 0$

(m) Non negativity constraints; $t = 1, \ldots, \tau$

(m.1) $x_{i,t}$, $m_{i,t}$, $x_{PC,t}$, $x_{IS1,t}$, $x_{IS2,t}$, $in_{i,t}$, e^T_t, $ms_{IS,t}$, $ms_{PC,t}$, $e_{PC,t}$, $in_{PC,t}$, $m_{PC,t}$, $m_{IS,t}$, $in_{IS,t}$, $y_{i,t}$, $\delta_{PC,t}$, $y_{PC,t}$, $\delta_{IS,t}$, $y_{IS,t}$, $u_t \geq 0$, all relevant "i"

Objective Function

$$\max W_I = \sum_{t=1}^{\tau} \frac{1}{(1+\rho)^{\sum_{s=1}^{t-1} \nu_s}} \sum_{s=2}^{\nu_t} \frac{1}{(1+\rho)^{s-1}} \frac{e^T_t}{\nu_t} + \sum_{s=1}^{\infty} \frac{(1+\gamma)^s}{(1+\rho)^{s-1}} \frac{e^T_\tau}{\nu_\tau}.$$

determined and important only in the foreign exchange constraint. Both exogenous foreign capital inflow and net factor income from abroad are included in "f_t."

All magnitudes in the model are measured in billions of Korean won (270 ₩ = $1). Capacity is stated in terms of the yearly output available from the normal operation of plant and equipment. Each period in the model covers two years.

Constraints in the Model

The constraints will be discussed in the order in which they appear in Table II. The major constraints are on the use of capacity in each sector, the use of foreign exchange, and the availability of domestic savings. There are no labor constraints in the model. Output determination (a) gives the commodity balances for the seventeen commodities appearing in the model. The first fifteen equations, (a.1) and (a.2), pertain to the supply and demand for the traditional sector outputs; equations (a.3) and (a.5), to the demand and supply for formerly non-competitive goods produced in the two complexes, and equation (a.4) stipulates that all production in the petrochemicals complex is used for import substitution.

Supply consists of two components, domestic production and imports. Demand is divided into five parts:

1) intermediate demand for use in production, reflected through the input-output coefficients, the "A_{nm}'s;"

2) investment demand for capacity creation, "in_n;"

3) private consumption demands, "c_i;"

4) exogenous demand for exports, "e_n;" and,

5) exogenous demand on government account, "g_i."

In equations (a.2), "ms_{IS}" is the use of the iron and steel complex's output for import substitution.

Private consumption demand (b) for traditional commodities is determined by a set of marginal propensities to consume out of total consumption expenditure, the "S_i's." The intercepts in these relations can be varied along with the slopes to approximate any desired elasticity of demand with respect to consumption expenditure. Since the elasticities from which these demand functions are derived pertain to per capita consumption, a correction for anticipated population changes is applied to the intercept term, "S_i^a." "P_t" is the sum over the period of the population in each year of that period.

Demand for investment goods (c) is related by means of input-capacity coefficients, "B_{in}'s," to the gross addition to capacity in a sector, "y_n." Stock accumulation is endogenous and tied to the producing, rather than the using, sector.

Equation (c.1) defines the demand for the traditional sector outputs for use on investment account. Equations (c.2) and (c.3) define stock accumulation in the two complexes. Stock accumulation in the traditional sectors takes place when capacity is created, while stock accumulation in the complexes occurs at the point when output increases. The latter would seem to be the better specification,

but the fact that the model is solved in a reduced form makes the former a more convenient formulation for the traditional sectors.

Foreign exchange use (d) restricts the use of foreign exchange to the supply provided by exports plus the sum of the net foreign capital inflow and the net factor income from abroad. The foreign exchange derived from exports is the sum of exports measured in domestic prices multiplied by a conversion factor, "F," which translates domestic prices into f.o.b. export prices. A different conversion factor should be used for each sector's commodity, since freight, insurance, and tariff rates depend upon the sector as well as the destination of the exports. Unfortunately, the data to distinguish between these prices at the sectoral level do not exist.

Non-competitive imports are required to produce all of the commodities. Investment also requires non-competitive imports--machinery and skilled technicians for installation and labor training. Added to these demand components are competitive imports and the net foreign exchange accumulation of the period.

Capacity constraints (e) require that output in each sector not exceed the available capacity to produce, "k_n."

Capacity determination (f) updates initial capacity by the investment occurring within the plan period. Capacity in a sector in period "t" equal to last period's capacity less any depreciation occurring at period plus the capacity expansion due to last period's investment.

"$U_{i,s,t}$" is a factor translating normal yearly output, the terms in which capacity is measured, into the increase in output actually possible in a sector "i" in period "t" as a result of investment in period "s." The gestation period between initial investment and usable capacity is the length of the period in which the investment takes place. This treatment gives an adequate reflection of gestation lags without the necessity to distinguish different gestation lags for different capital goods inputs.

The inclusion of several years within a single period gives "U" an important time dimension. "$U_{i,s,t}$" has been taken as a constant equal to the number of years in period "t" as a matter of convenience. Thus it is assumed that plants come on line and are immediately capable of producing at full "normal" capacity.

Depreciation, determined in equation (e.2), occurs only in the fifteen traditional sectors and is related to the capacity existing at the beginning of the planning period. "D_i" is the yearly depreciation rate of capacity, and "ν_t" is the number of years in a period. This formulation of depreciation is used, since the planning horizon is too short for plant installed within it to fall victim to sufficient deterioration or obsolescence to cause normal output to decline. <u>Competitive import bounds (g)</u> restrict competitive imports of the traditional sectors' commodities to a fraction of total imports. In reality these bounds are introduced in place of non-linear or step functions for production in the traditional sectors. Without the

bounds and with constant costs, it could occur that all of a sector's supply is provided through imports.[7] The possibility that a broadly-defined commodity aggregate be wholly imported denies the existence of individual products within the aggregate which are at varying distances from the margin of importation. Rather than using step functions to represent the production function within a sector or further disaggregating the sectors, bounds on competitive imports are used because of the lack of empirical knowledge necessary to specify step production functions and because of the ever-present problem of computational capacity.[8]

Private consumption and savings bounds (h) respectively place a floor under consumption in any period and a ceiling on the marginal propensity to save out of gross domestic product. Both of these are substitutes for a non-linear objective function based on consumption. The first constraint (h.1) stipulates that per capita consumption must grow at a rate at least equal to "ε." The second holds the marginal propensity to save to less than "σ." In (h.2), the first four terms determine gross domestic product by means of the value-added coefficients "V_n." The remaining terms define total investment and introduce a correction due to foreign capital inflows and foreign exchange accumulation necessary to determine domestic savings.

Iron and steel complex bounds (i) are required because of the treatment of the steel complex. Equation (i.1) is sufficient to insure that only the output of the integrated mill is used to meet the

demand for formerly non-competitive imports of iron and steel. The second equation requires that the composition of the output of the integrated mill not change over time in a perverse fashion.

Integer constraints (l) are used to obtain the desired fixed charge cost functions for investment in the two complexes.

Non-negativity constraints (m) are a self-explanatory feature of the programming model.

Terminal Conditions and the Objective Function

The longer one works with dynamic planning models, the clearer becomes the realization that terminal conditions cannot be chosen independently of the objective function and vice versa. It is a trite but true observation that the world, or in our case, Korea, does not die with the final period of the model. Thus the impact on the future of activity within the finite horizon of the planning model must in some fashion be included in the welfare function. And, to be consistent, the influence of future production upon plan period activity, especially investment, must also be reflected. This is generally accomplished through conditions on the amount and composition of investment in the terminal period of the model (or, if it is a single period model, on investment in that period) without a correction in the objective function. An exception is the Chenery-MacEwan model /4/. Its approach is followed here.

Terminal growth constraint (j) is required, as in all models of this

gender, to assure that investment takes place in the terminal period of the planning horizon. Equation (j.1) provides for sufficient investment to assure a post-terminal rate of growth of gross domestic product of not less than "γ" per cent per annum. Assuming full utilization of capacity in the post-terminal period, gross domestic product in that period will equal the sum of the value-added coefficients times the sectoral capacities available in the period (the first terms in the equation). The term in parentheses to the left of the inequality sign is simply gross domestic product in the terminal period. A feature of this specification which is notable, particularly in a model with economies of scale, is provision for the growth in output either from investment or from excess capacity in the terminal period.

Equation (j.1a) which replaces (j.1) in some runs of the model, states that it is total sources (equal to gross domestic product plus foreign capital inflow and net factor income from abroad less net foreign exchange accumulation) which must grow by at least "γ" per cent. This slight change has the effect of keeping the model from using foreign exchange accumulations for consumption in the terminal period without having to compensate for the increased consumption by additional investment.

<u>Terminal capital stock bounds (k)</u> are introduced to avoid edge effects which would lead to terminal investment in one or, at most, a few sectors. By placing upper bounds on the terminal capital stocks

and relating them to the capital stock that would exist if all capital stocks were to grow at "γ" per cent, this potential problem is avoided. The multiplicative factor in this relation is "φ_n" which is always greater than one.

In the course of solving the model, it became necessary to use a lower bound on terminal investment in services and overhead to insure an a priori appropriate amount of investment in that sector. In this case, constraint (k.1a) replaces the fifteenth constraint in equation (k.1), and production is required to grow at the post-terminal growth rate of gross domestic product.

The Objective Function values a plan of activity in terms of its consumption over an infinite horizon. The objective function is presented as the sum of two terms; the first includes consumption during the plan period which enters the objective function discounted at rate "ρ." All discounting is to the first year of the plan period. The awkward appearance of the term is due to the use of a one-year discount factor to discount consumption occurring in periods of greater than one year's length.

The second expression in the objective function is the value of post-terminal consumption that can be obtained by virtue of the terminal capital stocks. The terminal growth constraint can be interpreted as an injunction that the economy embark on a growth path along which consumption grows at rate "γ," starting from the level of consumption achieved in the terminal period. Post-terminal consumption

TABLE III

Summary Information -- Korean Planning Models

Data	SECTORS 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	PC	IS1	IS2
Input-Output Coefficients (A_{in})																		
1. Agriculture	.09	.03	.24	.36	.01	.02	.03			.02								
2. Mining		.17	.01	.01		.01	.01	.03		.04	.06	.01	.07	.01	.02		.09	.01
3. Food	.03	.01	.13	.01	.01	.04	.05	.06		.10	.01		.01	.03	.04	.01		
4. Fiber Spinning				.05	.37	.03												
5. Textiles	.01	.01	.01		.10	.05							.01		.01			
6. Light Manufacturing		.01	.01	.01	.01	.09	.01	.01						.02	.02			
7. Chemicals	.01	.03	.01		.01	.06	.06	.01	.01	.01		.01	.01	.02	.02	.05		.01
8. Fertilizer	.06							.01										
9. Petroleum		.01				.01	.01	.08	.03	.02	.05	.02	.01	.01	.02	.06	.04	.01
10. Construction and Cement		.01	.01			.01	.02			.13	.03		.01	.02	.02	.01	.03	
11. Iron and Steel											.14	.36	.02	.01			.21	.40
12. Steel Products		.01								.05	.02	.07	.03	.08			.01	.11
13. Finished Metal Products		.01	.01			.03	.02			.04	.01	.03	.13	.09				.02
14. Machinery		.03	.01	.01	.01	.01	.01	.02		.03	.01	.01	.01	.12	.02		.01	.01
15. Overhead and Service	.03	.18	.12	.08	.09	.14	.17	.16	.04	.12	.06	.09	.12	.12	.12	.15	.11	.12
Petrochemicals				.16	.04	.07												
Iron-Steel Complex										.02	.01	.10	.08	.06				
Non-competitive Imports (N_n)		.01	.09	.01	.01	.11	.17	.01	.43	.02	.34	.09	.15	.08	.01	.12	.23	.09
Value Added Coefficients (V_n)	.77	.48	.34	.29	.35	.30	.43	.59	.47	.37	.26	.21	.34	.30	.70	.59	.26	.21
Consumption Demand (S_i)	.26	.01	.18		.10	.04	.02		.02	.03			.01	.01	.33			
Capital/Output (B_i^T, B_n^{T*})	1.17	.97	.40	.73	.39	.29	.47	1.89	.93	.80	.95	.52	.78	.50	1.54	1.84+	.95=	.95=
Imported Mach./Output (M_i)	.20	.19	.10	.44	.15	.09	.21	1.46	.67	.40	.19	.27	.21	.20	.39	1.53	.72	.72
Capital/Value Added (B_i^T/V_i, B_n^{T*}/V_n)	1.54	2.04	1.16	2.35	1.11	.96	1.11	3.17	1.95	2.15	3.69	2.48	2.26	1.68	2.21	3.12	3.65	4.52

NOTES: All coefficients have been rounded to the nearest 100th. Totals may not reconcile due to rounding.

* Based on a petro-chemicals complex of size 25 million won, double the planned size

= Based on an iron and steel complex of size 44 billion won, the planned size

\+ Includes induced investment in housing.

$C_{IS,11} = .18$; $C_{IS,12} = .82$

is assumed to grow at this rate and is discounted relative to the immediately post-terminal year at a rate "ω," which may differ from the plan-period rate of discount due to a willingness of the nation to endure sacrifice for a short period to achieve a high plateau from which to initiate a path of constant growth.

So far the large weight in the objective function on terminal consumption has been rationalized in terms of giving post-terminal consumption value in the welfare function. By virtue of the terminal growth constraint it can also be approached as a means of giving value to the terminal capital stocks. It is a substitute procedure for putting terminal capital stocks directly into the welfare function.

III. Use of the Model in Project Analysis

The parameter values actually used in the model are based on much the same data as is used in the Korean Sectoral Planning Model /12/. Table III presents the values of some of the major parameters. The model has been solved under a variety of assumptions concerning the availability of foreign capital, the required post-terminal growth rate, and the parameter values in the objective function. The optimality of building either complex has been found to be most sensitive to the assumed amount and timing of foreign capital inflows over the period.

The solutions to the model exhibit very high growth rates of gross domestic product--rates as high as 13 per cent per annum are

not unusual. Two factors are largely responsible: 1) the low unit cost of capacity in many of the sectors coupled the absence of capacity absorption limits or decreasing returns to investment activity as a whole (the incremental capital output ratio runs around 2.1); and 2) the high value of consumption in the terminal period makes investment in the first two periods so valuable that it is usually at its upper bound in these periods. In other words, consumption growth is at its lower bound in the first two periods.

The use of the model has demonstrated that if sectors are ranked by comparative costs, then fiber spinning, fertilizer, petroleum, the traditional iron and crude steel and steel products sectors, finished metals, and machinery and transport equipment head the list having the highest costs. Production in the complexes is also near the top of the list. Consumer goods and intermediates to consumer goods production are at the bottom of the list. Which products are actually imported depends upon the availability of foreign exchange.

In this paper we shall examine alternative solutions to two specifications of the model. In both specifications, the in-plan discount rate, "ρ," has been set to zero (reflecting the desire of Korean government officials for investment now, consumption later), the post-plan discount rate, "μ," is twenty-five per cent, and the minimum growth rate of per capita consumption is two per cent.[9] In one specification foreign capital inflows are exogenous (as in the

statement of the model above), but in the other borrowing is included in the model so that foreign capital inflows are endogenously determined on the basis of the interest rate on borrowing and the productivity of foreign exchange.

The treatment of foreign capital inflows as exogenous is admittedly unrealistic, though in the early experiments with the model it had the virtue of making interpretation easier. Korea does have control over the amount and timing of foreign capital inflows: one important means whereby that control is exercised is through the licensing of projects, since the foreign exchange costs of large projects are generally financed abroad. Roughly seventy per cent of the total cost of both the integrated steel mill and the petrochemicals plant will be financed by foreign capital.

This aspect of the country's control over foreign capital inflows is best reflected in the borrowing activity incorporated in the second specification (though the magnitude of foreign capital inflows is not directly related to the construction of the plants). Since no less-developed country can obtain all of the foreign capital it might profitably use, it is necessary to impose a limit on the amount of terminal debt when borrowing is included in the model.[10]

Even though the borrowing specification is the more realistic, it is still worth while to look at solutions in which borrowing is not permitted and foreign capital inflows are exogenous. For it is in the contrast between the optimal solutions when borrowing is

permitted and those when foreign capital inflows are exogenous that the major determinant of the conditions under which it is profitable to build one or both of the complexes is found.

It is worth while to temporarily set aside the specific case of Korea along with the petrochemicals plant and integrated steel mill to consider, in more general terms, some of the issues connected with lumpy investment in decreasing cost industries. We use the word "lumpy" to characterize those cases in which the construction of a single plant requires a high proportion of the total, economy-wide investment during the period of its construction.

In the first place, it is well to distinguish between economies of scale in the provision of capacity on the one hand, and the lumpiness of investment on the other. So far as the lumpiness of investment in a given sector is concerned, the questions to be asked in determining the optimality of that investment revolve around the accommodation that must be made in the rest of the economy to undertake the construction of a plant. Can the plant be financed without requiring a major reduction of investment elsewhere in the economy (e.g., by foreign financing); and if so, is its return high enough to repay the cost expended in its construction? If not, are the returns from the investment sufficient to compensate for the large sacrifice of investment elsewhere in the economy?

The issues related to economies of scale are somewhat different and revolve around the question of how much excess capacity should

be permitted when the plant comes on line. By over-building capacity when plant construction exhibits economies of scale, it is possible to decrease average unit capacity costs. But over-capacity generally requires the sacrifice of capacity and hence production somewhere else in the economy.

The interesting feature of the petrochemicals plant and the integrated iron and steel mill in the case of Korea is that they incorporate both lumpiness and economies of scale in their capacity costs. But we are not considering a unique case here. Other "middle-sized" economies like Korea face the same kind of investment problem.

Where lumpiness and economies of scale are combined in the investment cost for a single plant, the return to the concentration of investment in that plant and in its related industries (related through supply and demand links) over a short period of time is obvious. If enough investment resources can be mustered, a large plant with consequently low unit capacity costs can be built and immediately utilized due to the simultaneous investment in the suppliers of its intermediate inputs and the users of its output. But the cost of such a pattern of investment is the necessity to obtain large amounts of investment resources, either through increased domestic savings or foreign capital inflows, or at the sacrifice of capacity expansion elsewhere.

If the plant is financed through the reduction of investment in other sectors of the economy, then the potential domestic demand for

the output of the plant must fall. In this case either a smaller plant must be built with less reduction of investment in the rest of the economy, or greater excess capacity must be experienced when the plant comes on line--both of these mean greater costs of production in the plant and lower profit (at least in the first periods of its operation).

There is an alternative pattern which is sometimes optimal. Prior to its construction, some of the industries for which the plant is a supplier can be built up through imports of the plant's output. Then, when the plant comes on line, the demand for its output exists without the necessity for as much concentration of investment in the plant and its related industries. But if the cost of importing the plant's output greatly exceeds domestic production costs, this pattern may be less attractive than the concentration of investment in all the plant's related sectors at the point at which it is built or than simply building up excess capacity in the plant's related sectors prior to the plant's coming on line. Such excess capacity could be built up in both the supplying and demanding sectors related to it.

Finally, though the rationale for having capacity in the plant's using sectors available when the plant comes on line is clear, it is not so obvious why it is necessary to simultaneously or previously increase capacity in it supplying sectors. It would be possible to simply import the required intermediate goods. But this, too, is not

without cost: either imports of other commodities must be sacrificed, or exports must be expanded at the expense of providing capacity in the export industries (assuming that foreign exchange reserves are not used).

In the discussion so far, we have assumed tacitly that production in all sectors of the economy is increasing (or at least not falling) during the period in which the plant comes on line. This need not be true: the intermediate products required for production in the plant can be obtained through cutting back production in the sectors requiring the same intermediate goods. Indeed, this sometimes seems to occur either through the death of old and inefficient industries or through a lack of sufficient planning.

The above discussion was meant to be suggestive of the things to look for which determine the optimality of investment in projects which require large amounts of investment resources. We have assumed that the project is basically sound, in the sense that the opportunity cost of domestic production in the plant is less than that of imports, and have focused on questions of timing and plant size. Returning to Korea: it is necessary to determine whether investment in petrochemicals or non-traditional iron and steel will be at the expense of investment elsewhere, and if so, if the expense is justified. It is justified to the extent that the foreign exchange no longer needed for imports of petrochemicals or non-traditional iron and steel can be used to permit a more rapid growth of supply in

other sectors than would have been the case had either or both of the complexes not been built.

In summary, the things to watch for are:

1) what happens to the level of total investment should a complex be built;

2) what happens to investment in the using and supplying industries related to the complex;

3) what is the effect of the above on the size of the complex that is built and its utilization; and

4) how is the growth of supply in the rest of the economy affected?

In the last point we emphasize supply rather than production, since in a world with trade, the supply of goods and services is more relevant to welfare determination than the production of these.

The answers to these questions can be provided only by a multi-sectoral, inter-temporal model of the kind formulated above. In fact, the issues themselves are suggested by the dynamic general equilibrium system from which such a model is derived. Keeping the alternatives outlined above in mind, we now turn to the solutions obtained for the two specifications. The economy's accommodation to investment in the two complexes is best explored by examining alternative solutions to a given specification in which the pattern of investment in the complexes is varied. A pattern of investment in the complexes is distinguished by its timing. We may omit the

TABLE IV

Alternative Solutions: Non-Borrowing Specification

	Solution			
	A[1]	B	C	D
Period of investment in:				
PC	np	1	1	2
IS	np	np	1	1
Period	**Average Yearly Gross Domestic Product**			
1967/68	982	979	979	n.c.
1969/70	1280	1252	1268	n.c.
1971/72	1807 (13.8)	1712	1653	n.c.
	Average Yearly Consumption			
1967/68	765	765	765	765
1969/70	833	833	833	833
1971/72	1445 (12.4)	1410	1389	1417
	Average Yearly Investment			
1967/68	326	323	323	n.c.
1969/70	510	464	409	n.c.
1971/72	472 (13.8)	433	416	n.c.
	Total Imports Minus Exports-Yearly Average			
1967/68	180	180	180	180
1969/70	149	135	64	175
1971/72	211	245	256	182
Sector	**Capital Stocks in 1971-1972**			
1	823 (12.7)	755	719	775
2	60 (13.5)	40	40	59
3	457 (16.3)	439	378	438
4	87 (14.1)	85	63	85
5	267 (18.4)	262	205	262
6	134 (25.0)	131	127	131
7	77 (22.0)	76	73	78
8	19 (31.0)	19	19	19
9	39 (3.8)	42	43	40
10	352 (19.8)	274	265	285
11	6 (1.5)	6	6	6
12	23 (5.8)	21	21	21
13	21 (6.6)	22	22	22
14	143 (21.0)	132	123	139
15	831 (15.9)	808	781	819
PC	0	75	75	75
IS	0	0	55	66

Notes: All magnitudes in billions of won

np: Plant not constructed during plan

1: The numbers in parantheses in the third column are compound growth rates for 1966 to 1972 for solution A.

n.c.: Not computed

third period from the description of the pattern, since investment in that period is not optimal in any of the solutions obtained to the model. In addition, the construction of more than one plant is never optimal. Thus there are nine possible patterns of investment that are relevant. Of these nine, we shall consider only the ones generated by the solution algorithm. The solution for each pattern is the optimal solution given the pattern of investment in the complexes.

Capacity Expansion Paths With Exogenous Foreign Capital Inflows

The foreign capital inflow plus net factor income from abroad in each period for the first specification is 180 billion won per year.[11] The terminal growth constraint related to gross domestic product is used, and the required post-terminal growth rate is 13 per cent. Imports of any single commodity cannot exceed in value ten per cent of total imports. The marginal savings propensity is unbounded, i.e., constraint (h.2) is omitted.

The amount of investment in this specification is bounded from above in the first two periods by the consumption requirements constraints and by the fixed amount of foreign capital inflow, and bounded from below in the terminal period by the terminal growth constraint. Due to the high welfare value of terminal consumption, investment in each period in all solutions to this specification is at its bound regardless of its composition. Consequently, capacity expansion in the complexes takes place at the cost of growth in

traditional sectors' capacity. This is strikingly illustrated in Table IV which presents selected details for four alternative solutions (corresponding to four patterns of investment in the complexes) to this specification.

The welfare values of the four patterns differ by virtue of differences in consumption in the terminal period; solution A is the optimal solution having the greatest terminal consumption. The level of investment is nearly the same for all patterns in the first period but varies in the second and third periods due to two factors: variations in foreign exchange reserves accumulation in the second and in the composition of investment in the first two periods.

With respect to the latter, the composition of investment within a period determines the incremental capital output ratio. It, in turn, along with the level of investment, establishes the upper bound to the growth of gross domestic product between the present period and the next. And since the growth of domestic savings is limited by the growth of gross domestic product, the growth of investment depends upon the composition of investment in prior periods. The effects of a suboptimal composition of investment in one period are thus compounded through time by a lower growth rate of investment. The pattern of investment in the complexes in a given period not only influences investment in the traditional sectors in that period but also has a profound impact on the future course of total investment.

A comparison of solutions A, B, and C illustrates the determinants

of the accumulation of foreign exchange reserves in period two. As more import substituting investment takes place in the first period through the construction of the complexes, less foreign exchange is needed to support production in the second period. This permits the release of foreign exchange for reserves accumulation. The increase in foreign exchange reserves accumulation in period two moving from solution A to B and then from B to C is equal to the foreign exchange savings permitted by the construction of first the petrochemicals plant and then, in addition, the integrated steel mill. The accumulated reserves are used to increase consumption in the terminal period through competitive imports. More foreign exchange is required in the second period in solution D due to less import substitution in the first period and the foreign exchange cost of the petrochemicals plant constructed in that period.

If we now turn our focus to the composition of the capital stocks in period three, the effects on capacity expansion by sector of a rigid upper bound on investment in the first two periods are clearly seen. Reference to Figure II is helpful in comparing these solutions.

In solution B a petrochemicals plant is constructed in the first period. Comparing the capacity levels of the third period for solution B with those for solution A, in which neither complex is built during the plan period, the cost of building the petrochemicals plant is quite clearly a decline in the capacity of its using sectors. To build the plant, capacity in the sectors using its output had

to be cut back. Consequently, the profit from building the plant is restricted, since it cannot be utilized as completely as it would have been had its construction not been responsible for a decline in capacity expansion in the rest of the economy. To accommodate construction of the plant, capacity expansion in the consumption goods industries also had to be restricted.

In solution C both complexes are constructed in the first period with a further cutback in the capacity of the using sectors of both plants and in the consumption goods industries. In solution D, the construction of the petrochemicals plant is delayed to the second period while the integrated mill continues to be built in the first. Of the four solutions this is the second best--by delaying construction of the petrochemicals facility, greater capacity in the using sectors of both complexes can be constructed, and capacity in the major supplier of the integrated mill can be expanded. Consequently a larger mill is constructed; the largest permissible petrochemicals plant is always built, since there is no bound on its exports (in this specification) and since excess terminal capacity in petrochemicals contributes to the terminal growth requirement. Terminal consumption in solution D falls short of that in solution A, since less foreign exchange can be passed into the terminal period when the petrochemicals complex is built in the second.

In conclusion, there appear to be three causes for the absence of investment in the complex sectors in the optimal solution for

TABLE V

Alternative Solutions: Borrowing Specification National Income Accounts*

Solution		Period of Investment PC	IS	W_I	GDP	C^T	I	E	M	M^1	M^2	M^3	M-E
A	1967/68	np	np	42048.1	982	765	311	102	268	65	92	112	166
	1969/70				1258	944	541	179	492	91	150	251	313
	1971/72				1757	1279	373	280	275	136	73	66	-5
B	1967/68	1	np	42818.6	981	765	341	102	299	64	124	110	197
	1969/70				1263	948	535	179	485	92	151	242	306
	1971/72				1788	1306	347	280	248	137	71	40	-34
C	1967/68	2	np	42867.9	981	765	306	102	264	65	89	111	162
	1969/70				1265	948	573	179	521	93	188	240	344
	1971/72				1790	1306	347	280	249	137	71	40	-34
D	1967/68	2	1	42884.0	980	765	324	102	283	64	109	109	181
	1969/70				1255	942	584	179	537	95	191	250	358
	1971/72				1787	1309	305	280	210	146	64	0	-72
E	1967/68	2	2	42936.0	980	765	295	102	263	65	88	110	161
	1969/70				1264	949	607	179	558	91	214	253	379
	1971/72				1789	1310	306	280	211	147	147	0	-71

Notes: * All national income accounts are yearly averages for the period in billions of won.

Column headings are:

W_I objective values;

GDP gross domestic product;

C^T consumption;

I investment;

E exports;

M total imports;

M^1 non-competitive imports for production;

M^2 non-competitive investment goods imports;

M^3 competitive imports;

M-E total imports less exports;

TABLE VI.A

Production and Supply: Borrowing Specification

Sector	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	PC	IS
							Average Annual Total Supply (Production Plus Imports)										
1967-1968																	
Solution A	481	51	228	52	104	62	38	30	32	186	12	23	29	97	441	17	13
B	481	51	228	52	104	61	37	29	32	186	11	22	29	96	441	17	13
C	481	51	227	52	104	61	37	29	32	186	12	23	29	96	440	17	13
D	481	51	228	52	104	60	37	29	32	186	12	22	29	95	440	17	13
E	481	51	227	52	104	61	37	29	32	186	11	22	29	96	440	17	13
1969-1970																	
Solution A	582	63	304	86	164	86	49	37	42	338	17	34	43	143	573	23	19
B	583	63	303	85	163	86	50	37	43	330	13	33	42	141	574	23	17
C	585	64	305	87	165	87	51	37	43	332	16	34	44	142	577	23	19
D	580	66	302	84	162	85	49	36	43	336	0	33	43	143	574	23	54
E	585	64	306	87	165	87	51	37	42	337	12	33	43	143	577	23	18
1971-1972																	
Solution A	773	81	411	127	246	125	72	50	61	310	23	41	59	156	770	40	26
B	779	79	416	128	250	126	75	50	63	282	20	39	58	159	785	43	26
C	780	80	417	128	250	127	75	50	63	283	20	40	58	159	786	45	26
D	776	82	413	129	250	126	75	50	64	250	6	40	57	155	788	45	54
E	777	82	414	129	250	127	75	50	65	251	6	40	57	155	789	45	58
							Average Annual Production										
1967-1968																	
Solution A	472	51	228	52	104	62	38	25	22	158	7	23	27	73	441	0	0
B	473	51	228	52	104	61	37	25	22	158	7	22	27	73	441	0	0
C	473	51	227	52	104	61	37	25	22	158	7	23	27	73	440	0	0
D	473	51	228	52	104	60	37	25	22	158	7	22	27	73	440	0	0
E	473	51	227	52	104	61	37	25	22	158	7	22	27	73	440	0	0
1969-1970																	
Solution A	546	47	304	50	164	82	49	22	42	302	6	34	24	107	573	0	0
B	547	47	303	49	163	69	51	22	43	294	6	23	24	105	574	23	0
C	549	47	305	51	165	87	51	22	43	296	6	34	24	110	577	0	0
D	544	47	302	48	162	51	49	22	43	300	0	33	24	107	574	0	55
E	549	47	306	51	165	87	51	22	42	302	6	23	24	107	577	0	0
1971-1972																	
Solution A	773	81	411	127	246	125	72	50	61	310	23	41	59	156	770	0	0
B	779	79	416	128	250	126	75	50	63	282	5	39	58	159	785	43	0
C	780	80	417	128	250	127	75	50	63	283	6	40	58	159	786	45	0
D	776	82	413	129	250	126	75	50	64	250	6	40	57	155	788	45	55
E	777	82	414	129	250	127	75	50	65	251	6	40	57	155	789	45	58

TABLE VI.B

Capacity and Imports: *Borrowing Specification*

Sector		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	PC	IS
capacity:	1967/68	473	54	246	55	137	62	42	25	22	158	7	26	27	73	444	0	0
Solution A																		
capacity:	1969/70	546	47	304	50	164	106	76	22	42	302	6	34	24	107	573	0	0
	1971/72	773	81	411	127	246	125	72	50	61	310	23	41	59	156	804	0	0
excess capacity:[1]	1967/68	0	5.3	7.5	4.9	23.9	0	8.6	0	0	0	0	13.4	0	0	.7	0	0
	1969/70	0	0	0	0	0	23.1	35.3	0	0	0	0	0	0	0	0	0	0
	1971/72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.1	0	0
imports:[2]	1967/68	1.6	0	0	0	0	0	0	15.4	31.2	15.2	40.1	0	8.3	24.5	0	100.0	100.0
	1969/70	6.1	26.3	0	41.7	0	4.7	0	40.1	0	10.6	62.6	0	44.4	25.0	0	100.0	100.0
Solution B																		
capacity:	1969/70	547	47	303	49	163	69	51	22	43	294	6	23	24	105	547	58	0
	1971/72	779	79	416	128	250	126	75	50	63	291	6	39	58	159	785	56	0
excess capacity:[1]	1967/68	0	5.4	7.3	5.1	23.9	2.4	11.3	0	0	0	0	14.9	0	0	.7	0	0
	1969/70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60.0	0
	1971/72	0	0	0	0	0	0	0	0	0	3.0	0	0	0	0	0	24.9	0
imports:[2]	1967/68	1.6	0	0	0	0	0	0	15.4	31.1	15.2	39.4	0	8.0	23.8	0	100.0	100.0
	1969/70	6.1	25.8	0	41.9	0	19.9	0	40.1	0	10.8	49.5	28.4	42.8	25.4	0	0	100.0
Solution C																		
capacity:	1969/70	549	47	305	51	165	87	51	22	43	296	6	34	24	110	577	0	0
	1971/72	780	80	417	126	250	127	75	50	63	292	6	40	58	159	785	59	0
excess capacity:[1]	1967/68	0	5.4	7.5	4.9	23.9	1.2	11.2	0	0	0	0	0	13.4	0	.9	0	0
	1969/70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1971/72	0	0	0	0	0	0	0	0	0	3.3	0	0	0	0	0	24.9	0
imports:[2]	1967/68	1.6	0	0	0	0	0	0	15.4	31.1	15.2	40.1	0	8.1	24.0	0	100.0	100.0
	1969/70	6.1	26.2	0	41.3	0	0	0	40.3	0	10.8	61.2	0	44.8	22.4	0	100.0	100.0
Solution D																		
capacity:	1969/70	544	47	302	49	162	55	49	22	43	300	6	33	24	107	574	0	73
	1971/72	776	82	413	129	250	126	75	50	64	297	6	40	57	155	788	59	73
excess capacity:[1]	1967/68	0	5.4	7.5	5.2	24.0	3.3	11.6	0	0	0	0	13.6	0	0	.8	0	0
	1969/70	0	0	0	1.2	0	8.3	0	0	0	0	100.0	0	0	0	0	0	25.0
	1971/72	0	0	0	0	0	0	0	0	15.7	0	0	0	0	0	0	24.9	24.9
imports:[2]	1967/68	1.6	0	0	0	0	0	0	15.4	31.1	15.2	40.1	0	7.9	23.3	0	100.0	100.0
	1969/70	6.2	29.1	0	42.5	0	40.3	0	39.9	0	10.6	100.0	0	43.5	25.1	0	100.0	0
Solution E																		
capacity:	1969/70	549	47	306	51	165	87	51	22	42	302	6	23	24	107	577	0	0
	1971/72	777	82	414	129	250	127	75	50	65	298	6	40	57	155	789	59	78
excess capacity:[1]	1967/68	0	5.4	7.5	4.9	23.9	1.3	11.2	0	0	0	0	14.9	0	0	.9	0	0
	1969/70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1971/72	0	0	0	0	0	0	0	0	0	15.3	0	0	0	0	0	24.9	24.9
imports:[2]	1967/68	1.6	0	0	0	0	0	0	15.4	31.2	15.3	39.4	0	6.1	23.8	0	100.0	100.0
	1969/70	6.1	26.4	0	41.4	0	.7	0	40.3	0	10.6	49.9	30.0	44.8	25.1	0	100.0	100.0

Notes:

1. Excess capacity as per cent of total capacity in period.

2. Imports as per cent of total supply in period. Not given for 1971-1972 since competitive imports in that period seldom appear.

this specification. First, investment in either complex leads to a higher incremental capital output ratio which reduces the growth of domestic savings and hence of gross domestic product and consumption. Second, the demand for the output of the complexes is diminished by their construction, since it is necessary to reduce investment in their using sectors. As a result the plants cannot be utilized sufficiently to justify their construction. Lastly, and perhaps most importantly, construction of the plants is possible only by lowering capacity expansion in the consumption goods industries. This fact alone is rather insignificant except when coupled with the additional result that the saving in foreign exchange from building the plants does not permit sufficiently high imports of consumption goods to offset the decline in domestic production.

Alternative Capacity Expansion Paths in the Case of More Flexible Foreign Exchange Inflows

The alternative solutions to the first specification nicely demonstrated one set of adjustments required to permit investment in the increasing returns sectors when the total resources devoted to investment could not be increased as a means of accommodation. The various solutions to the specifications in which borrowing is permitted demonstrate an alternative adjustment--increasing the level of total investment.

Since investment is equal to domestic savings plus the gap between imports and exports, permitting flexible foreign exchange

inflows through borrowing gives the model more choice over the amount of investment in any period. This choice is present in only a single direction in the basic specification; foreign exchange reserves can be accumulated for future use.

Tables V and VI present selected information from the solutions for alternative patterns of investment in the complexes for the borrowing specification. In this specification the total sources terminal growth constraint (equation j1.a) is used, and the required post-terminal growth rate is ten per cent.[12] The limit on discounted total borrowing less repayment is equal to the six-year foreign capital inflow in the first specification, 1080 billion won.[13] The rate of interest on foreign capital is five per cent per annum. The upper limit to the marginal propensity to save, "σ," is .30. As in the previous specification, imports of a single commodity cannot exceed ten per cent of total imports.

In solution A for this specification there is no investment in either complex. Relative to the optimal solution in which both complexes are constructed in the second period, this investment pattern results in a loss of 88,716 units of welfare, or slightly over two per cent of the optimum value. This pattern is also less attractive than the others enumerated in the table, all of which involve the construction of a plant in at least one of the complex sectors.

Examination of the five solutions to this specification indicates

that the paths of gross domestic product and of consumption are far less sensitive to the pattern of investment in the complexes when borrowing is permitted than was the case in the absence of borrowing in the prior specification. The path of the gap between imports and exports displays the most sensitivity to changes in the pattern of investment. And of the determinants of this gap, competitive imports and non-competitive imports of capital goods display the greatest variability. The reason is straightforward--borrowing is timed to accommodate the construction of the complexes.

The petrochemicals complex costs about 47.2 billion won on an annual basis for the size of plant constructed in these solutions while the integrated steel mill costs approximately 30.7 billion won (again on an annual basis). Moving from solution A to solution B, where a petrochemicals complex is constructed in the initial period, borrowing increases in the first period by 31 billion won (annual basis). Thus two thirds of the cost of the plant is financed by an increase in the foreign capital inflow. The higher inflow in period one is offset by a reduction in borrowing in the second period and by increased debt repayment in the third. That part of the plant's financing not provided for by increased borrowing is obtained by a reduction in investment in the traditional sectors. Non-competitive imports of capital goods in the first period increase between the two solutions by the full amount of the increase in borrowing which is used to finance the foreign exchange component of the petrochemicals

plant; competitive imports bear the brunt of the reduction of net foreign capital inflows in the latter two periods. The reduction of competitive imports is possible due to import substitution in petrochemicals.

This same pattern of accommodation to investment in the complexes is repeated in solutions C through E. The level of borrowing rises in the period of investment in a complex relative to the situation in which the complex is not built in that period and subsequently falls as competitive imports are cut back along with borrowing in the following periods. In short, the amount of total investment over the plan period rises by nearly the full amount of the cost of the complex (or complexes) constructed. The increased borrowing necessary to finance the rise in investment is repaid, using the foreign exchange saved by import substitution in petrochemicals and/or non-traditional iron and steel. Because the timing of investment is no longer as rigidly set as it was in the basic specification, construction of both complexes in the second period is the optimal pattern. Of the total investment in the second period in solution E (the optimal solution), nearly thirteen per cent goes into the two complexes. They account for six and one-half per cent of total investment over the plan period even though they together account for only 2.25 per cent of gross domestic product in the terminal period.

Foreign capital worth 379 billion won on an annual basis, or 1.4 billion dollars, is needed to finance investment in the second

period. Though it is highly doubtful that Korea will receive such a large inflow of foreign capital (plus net factor income from abroad) in 1969 or 1970, she does have sufficient foreign exchange reserves to permit a wedge this size to be drive between imports and exports. Investment in the second period in the solutions rises as high as 607 billion won (annual basis) even though the marginal propensity to save is limited to .30. The high level of investment is possible due to large foreign capital inflows. It is profitable because of the high value given terminal consumption.

Table VI presents details of the capacity expansion paths for the solutions to the borrowing specification. Except for sectors six, ten, eleven, and twelve, there are only marginal differences between the solutions with regard to the paths of commodity supply and production in each sector. Certainly there are not the major changes which we saw reflected in the terminal capacity levels for alternative solutions to the basic specification. Production and total supply in the major consumption goods sectors is nearly the same for each solution in each period. Since total investment can be increased to accommodate investment in the complex sectors, there is no need to sacrifice consumption to finance either a petrochemicals plant or an integrated steel mill.

The differences between the solutions in the production and total supply of sector ten's output are related to differences in the amount

of terminal investment. Terminal investment is not the same in all solutions because repayment of debt is a substitute for investment in the terminal growth constraint, and, as we have already observed, the level of repayment in the terminal period of the solutions depends upon whether and when the complexes are built. The differences in sectors eleven and twelve are related to the timing of investment in the integrated steel mill which produces a substitute for the output of that sector.

Investment in sector six (light manufactures) is sensitive to the pattern of investment in the complex sectors. When there is investment in either complex in the first period, capacity expansion in sector six is cut back in that period to provide part of the financing. In the case of solution B this reaction in sector six helps explain why investment in petrochemicals is best delayed to the second period while in the case of solution D, it aids in explaining the optimality of the same timing for the integrated iron and steel mill. In both cases it is better to build up the using facilities of the petrochemicals plant gradually prior to its construction (recall that light manufactures are a major user of petrochemicals). In solution B the failure to do so results in idle capacity equal to sixty per cent of total capacity in the petrochemicals plant in period two. Here we have a clear case of the use of imports to delay the construction of a plant whose costs decrease with size so that a larger plant can be built and so that it can be more completely utilized in the

first period of its operation.

IV. Conclusion

In order to plan lumpy investments in decreasing cost industries, it is necessary to simultaneously plan investment in the rest of the economy. One-sector models of capacity expansion in industries with scale economies concentrate only on the role of decreasing unit costs and conclude that over-investment is optimal as a means of lowering unit capacity costs.[14] But multi-sectoral models are able to capture as well the fact that capacity expansion in some decreasing cost industries requires large chunks of investment resources. Investment elsewhere in the economy must adjust to the lumpiness of investment in the decreasing cost industries, and it is largely on the basis of these adjustments that the optimal timing of investment in the complex sectors is to be determined.

Whether the economy can successfully accommodate the lumpy investment in decreasing cost industries depends on a host of factors. Key among these seems to be the amount of foreign capital inflow and its timing. Foreign capital inflows must be timed to accommodate concentrated investment in the decreasing cost industries as well as in the industries related to them. But even here the issue is not simply whether a large part of either complex will be financed by foreign capital inflows contingent upon its construction. In the absence of direct foreign financing, the lumpy investment required

to build a plant in a decreasing cost industry may yet be profitable because, without this import substitution, foreign exchange supplies in later periods may be insufficient to permit as great an increase in the supplies of other sectoral commodities (either through increased production requiring non-competitive imports or through imports).

A clear example of this is seen by comparing the optimal solution to the first specification with the solution to a specification that is comparable but for the fact that the growth of agriculture is limited to six per cent annually. In the first specification, the concentration of investment needed to build and use either of the complexes requires too great a sacrifice of consumption goods. But, when the growth potential of agriculture is limited, concentration of investment in the petrochemicals plant and its related industries _is_ profitable because, as import substituting investment, it releases foreign exchange without which neither equally large imports of agricultural goods nor equally rapid growth in other sectors is possible.

So, the profitability of investment in the complexes is also a function of the resource endowment of the country. Since the lumpy investment in the decreasing cost industries is import substituting investment, it is profitable only if there is insufficient foreign exchange in its absence to sustain as rapid a growth of the supplies of other goods and services. But, if investment resources are at a premium due to the economy's limited ability to save, then the high investment cost of these decreasing cost industries may argue against

the expansion of their capacity.

The advisability of investing large amounts in the decreasing cost industries is consequently a function of the relative prices of the resources they require and the resources they save, as well as of the manner in which investment elsewhere in the economy is timed to accommodate the concentrated use of investment resources required to provide capacity in these industries.

Implications for Korean Development

The borrowing specification best reflects the conditions likely to prevail over the next six years in Korea. The optimal solution to that specification indicates that both projects are profitable--but not spectacularly so. It has been demonstrated that it is profitable to build the plants only if there is, at the same time, a large amount of investment in their using and supplying industries along with continued investment in the consumption goods industries.[15] This can be accomplished only with careful coordination of investment plans for the various sectors of the economy.

Planning is presently proceeding on the basis of a petrochemicals plant with an annual capacity of between twelve and twenty-four billion won output and a one million-ton capacity integrated steel mill. But the solutions to the model (including many not reported here) indicate that, if constructed, the capacity of the petrochemicals facility should be upwards of 50 billion won of annual output and that of the integrated iron and steel mill should be around a million and one-half

tons annually. It appears also that both plants should be built with considerable excess capacity--at the least, about twenty per cent of the capacity of each should be idle in 1972 due to the construction of over-capacity.

The growth rates obtained in the solutions are unrealistically high--for this reason the sizes of the plants constructed in the solutions may well be in excess of the sizes that are optimal when the factors which would reduce the growth rates to more reasonable levels are included in the analysis. But the large discrepancy between the presently planned and "optimal" (as here determined) capacity of the petrochemicals plant can be partly explained by the fact that heretofore there has been no reliable inter-sectoral estimate of the demands for petrochemicals--most estimates to date have been based on international comparisons.

With regard to the timing of construction, the optimal solution to the borrowing specification suggests that investment in both facilities simultaneously is the best course. But in that solution, investment is nearly fifty per cent of gross domestic product during the period of construction while the gap between imports and exports is over thirty per cent of gross domestic product. Whether or not Korea could efficiently absorb so much foreign capital during such a short period deserves serious attention.[16] If the plants are not built simultaneously, then the petrochemicals plant should be built first, since it results in a higher saving of imports per won of investment

than does the integrated steel mill.[17]

The clearest conclusion to emerge from the model's use is that the optimal timing of investment in the complexes depends ultimately upon the availability and cost of foreign capital. Not all of the elements of the cost of foreign capital have been included in the model: for example, the effect of a large deficit on the future availability of foreign capital inflows, or the influence of a large deficit on the attitudes of international lending agencies. The final decision should not be made until there is a more careful examination of all of the elements of the cost of foreign capital.

Finally, it is well to realize that no single planning technique can be used in isolation to formulate rational decisions; each technique fails to take some elements of consequence into account. Among the elements omitted from this model, the following deserve mention.

1) There is no allowance for the fact that a plant's capacity is never fully utilized when it first comes on line. There is always a shakedown period during which the plant's staff learns to operate the equipment and the bugs are worked out of the plant.

2) The fact that a plant's construction can be phased over time has been neglected. For example, the integrated steel mill could be built in three stages of 500,000 tons each with production and capacity advancing in step. The mill's cost would then be only slightly more than if it were built all at once.

3) Capacity absorption limits or decreasing returns to investment

as a whole have not been specified. In large part this is responsible for the high rates of growth and high investment rates.

4) Exports have been determined exogenously. Rather than build the complexes to save foreign exchange, it may be better to expand exports to obtain it.

These elements could be included in the model with only minor modifications. They were omitted only because of limited computational capacity and the time constraints under which this study was made. Not easily incorporated in this type of model, however, are the details of resource base, technology. and so forth, which go into the engineering calculations from which come the precise specifications for a plant's product mix, technological characteristics, and location.

FOOTNOTES

1. See Scitovsky /13/.

2. For example, consider the Korean Sectoral Planning Model /12/ used in formulating the Second Five Year Plan.

3. See Haldi and Whitcomb /8/.

4. See Dantzig /5/, p. 545, for the formulation of a fixed charge cost function in mixed integer-continuous variable form.

5. The derivation of the structural parameters used in the model is discussed in Westphal /14/, Appendix C.

6. The model includes constraints which insure that only the integrated mill, and not the traditional ferrous metals producers, meets the demand for formerly non-competitive iron and steel.

7. This possibility is to a large extent mitigated by existence of costless initial capacity.

8. For an interesting example of a model using step production functions, see Nugent /11/.

9. $\gamma_i = 1.10$ for all sectors.

10. Borrowing is incorporated in the model by including two additional activities and one additional constraint: Borrowing and repayment of foreign funds, "b_t" and "rp_t," respectively, are added to the the model in each period. The necessary additions are made to equations (d), (h.2), and (j.1a). The following constraint is added:

$$\sum_{t=1}^{T} (1+r)^{\nu_t(T-t)} b_t - \sum_{t=1}^{T} (1+r)^{\nu_t(T-t)} rp_t \leq tdb.$$

In this equation "tdb" is the maximum discounted value of debt that can remain in the terminal period and "r" is the rate of interest on foreign funds. For simplicity it is assumed that all foreign exchange inflows are financed either by exports or through borrowing. The slacks in equations (a.3) and (a.5) are also suppressed. Repayment of debt in the terminal period is included in the terminal growth constaint as an alternative to investment, since repayment during the plan period releases resources in post-terminal periods which would otherwise be used for debt repayment.

11. On the basis of discussions with David Cole, it appears this is unrealistically high.

12. The coefficient of "u_T" is set to $(1+\gamma)^{\nu_T}$ in equation (j1.a), which is equivalent to saying that foreign exchange accumulation in the terminal period cannot be used to increase total sources in the post-terminal period.

13. Note that this means that, after repayment, less foreign exchang is available from abroad for imports, since interest charges are accumulated in the debt constraint.

14. See Manne /10/ for the application to investment planning of single sector models with scale economies.

15. In another specification with borrowing included, the debt limit was 540 billion won--half that used here. Both complexes are built in the optimal solution though the petrochemicals plant's construction is shifted to the first period. Even with the lower debt limit, each complex is built simultaneously with its using and supplying facilities.

16. If the construction of the plants is spaced over four rather than two years and the net foreign capital inflow over the perio is held to 540 billion won, then during those four years investment and the foreign capital inflow would be thirty and twenty per cent of gross domestic product, respectively.

17. Based on a number of solutions not examined here.

GLOSSARY OF SYMBOLS

Symbol	Usage
Activities	
x_i	production in sector "i"
m_i	imports of the "i"th sectoral commodity
in_i	investment demand for the "i"th sectoral commodity
c_i	private consumption demand for the "i"th sectoral commodity
g_i	government demand for the "i"th sectoral commodity
e_i	exports of the "i"th sector's output
ms_i	use of production in complex "i" for import substitution
y_i	increment to capacity in the "i"th sector, measured in terms of normal yearly full capacity output
δ_i	integer variable for investment in complex "i"
u	accumulation of foreign exchange reserves relative to initial year of model
f	net foreign capital inflow plus net factor income from abro
k_i	capacity in sector "i" measured in terms of capacity actually available in period
d_i	depreciation measured in terms of normal full capacity output
$sl_{eq.}$	slack variable in equation noted in subscript
Structural Parameters	
A_{ij}, A^*_{ij}	input-output coefficient, use of traditional sector output ($A_{11,IS1}$ is minus the net output of iron and crude steel, and $A_{12,IS2}$ is minus the net output of steel products obtained by operating the respective activities of the mill; $A^*_{11,IS1} = 1 - A_{11,IS1}$, $A^*_{12,IS2} = 1 - A_{12,IS2}$, $A^*_{i,ISj} = A_{i,ISj}$ for all other i, j)
$Q_{i,IS}$	proportion of formerly non-competitive imports of iron, crude steel, and steel products that are iron and crude steel (i=11) and steel products (i=12); $Q_{i,IS}$ equals zero for all other "i"

Structural Parameters (Continued)

Q_{ISi}	$Q_{ISi} = Q_{IS,i+10}$; $i = 1,2$
N_{ij}	input-output coefficient, use of complex sector output
P	sum of population estimates in years spanned by the period
S_i^a	intercept coefficient in the consumption demand function for the "i"th sectoral output
S_i	marginal propensity to consume the "i"th sectoral commodity
B_{ij}	variable charge input-capacity coefficient
E_{ii}, E_i	stock accumulation coefficient
M_i	variable charge non-competitive imports/capacity coefficier
B_j^T	$= \sum_{i=1}^{15} B_{ij} + E_{jj} + M_j$, for traditional sectors
	$= \sum_{i=1}^{15} B_{ij} + M_j$, for complex sectors
B_{ij}^a	fixed charge input-capacity coefficient for complex sectors
M_i^a	fixed charge non-competitive imports/capacity coefficient for complex sectors
B_j^{aT}	$= \sum_{i=1}^{15} B_{ij}^a + M_j^a$, for complex sectors
N_i	non-competitive imports/output coefficient for traditional sector "i"
N_i^*	non-competitive imports/output coefficient for complex sectors
F	conversion factor for exports from won value in domestic prices to won value of foreign exchange
D_i	depreciation coefficient for the "i"th traditional sector
$U_{i,s,t}$	capacity utilization factor, $U_{i,s,t}$ is the capacity utilization factor for the use of capacity in sector "i" in period "t" depending upon the period of investment "s"
V_i, V_{IS}^*	value-added per unit of output in sector "i" (V_{IS}^* is a weighted average for the steel complex)

Structural Parameters (Continued)

Y_i	capacity of largest allowable plant that can be constructe in a single period in complex sector "i"
$B_{10,hous.}$	investment demand for construction for housing per unit of consumption
L	conformably dimensioned vector of ones
I	conformably dimensioned identity matrix
E^*_{IS}	weighted average of E_{IS1} and E_{IS2}

Plan Parameters

ν	number of years spanned by "t"
τ	number of planning periods
μ_i	proportional coefficient used in import bound
ϵ	minimum allowable growth rate of per capita consumption
σ	maximum marginal propensity to save
γ	target post-terminal growth rate of gross domestic product or total sources (depending upon specification)
ρ	discount rate on within plan consumption in welfare function
ω	discount rate on post-plan consumption in welfare function
φ_i	factor of proportionality in post-terminal capital stock bound

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