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Changes in relative factor shares in the United States agricultural and manufacturing sectors may be examined by means of the Hicks coefficient to relate these changes to changes in quantities of different inputs for many input production functions. Except for some recent empirical work, the criterion Hicks coefficient has been, however, mainly theoretical. This work and recent articles by Binswanger and Berndt and Wood are integrated to allow examination of changes in distribution of output among some key participants in United States production, such as laborers and owners of land and capital as the quantities of two key inputs; energy and fertilizer are varied with respect to the quantities of other inputs. Hicks coefficients are derived from the translog cost function used by Binswanger and Berndt and Wood. Results show that the relative factor shares are highly flexible with respect to factor proportions in modern technology such as in the United States and there is substantial scope for altering the distribution of output favorable to laborers by altering factor proportions. However, distribution of output would be skewed against laborers and capital owners if energy output grows slower than growth in employment and capital respectively.

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CHANGES IN RELATIVE FACTOR SHARES IN UNITED STATES AGRICULTURAL AND MANUFACTURING SECTORS

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CHANGES IN RELATIVE FACTOR SHARES IN UNITED STATES AGRICULTURAL AND MANUFACTURING SECTORS

By

C. G. Ranade

In 1932 in The Theory of Wages Sir John Hicks explored a way to relate changes in relative factor shares to input substitution when there are two factors of production by using the elasticity of substitution as a criterion. Since then there have been several attempts to generalize his work for production with many inputs (Meade, Samuelson, and Sato and Koizumi, 1973). 1/ In this respect, Professor Samuelson comments:

> When more than two inputs are involved, the problem of generalizing the Hicks σ coefficient becomes complex. Champernowne, Pigou, and others in pursuit of the question of how the change in a factor price ... affects the amount demanded of a different factor ..., have moved in the direction of defining a square matrix of partial-elasticities of substitution ... Aside from being complex, this approach has led to little in the way of definite results and has moved away from the task of providing a criterion of changes in relative factor shares. (Samuelson, p. 467.)

In a recent paper, Ranade (1977a) has shown that in fact the Hicks coefficient can be used to examine changes in relative factor shares due to changes in quantities of different inputs for many input production functions. For the two input production function Hicks coefficient is identical with partial and direct elasticities of substitution defined by Allen and McFadden, respectively, while for a general production function it is the reciprocal of the partial elasticity of complementarity first defined by Hicks (1970) and later elaborated by Sato and Koizumi (1973a).

Except for some recent empirical work by Ranade (1977b) on rice production in the Philippines, the criterion Hicks coefficient has been, however, mainly theoretical. The purpose of this paper is to examine how relative factor shares would change as different factor proportions are varied for both the United States agricultural and manufacturing sectors by using this criterion.

The paper does not estimate any fresh cost or production functions for this purpose. Instead, it uses the results in two recent articles of Binswanger, and Berndt and Wood which estimate factor demand functions by using the cost function approach. The former article examines derived demand for five inputs, namely, land, labor, fertilizer, machinery and other inputs in the agricultural sector

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1/ See Ranade (1977b) for the survey of literature on this subject.

for the period 1949 and 1964. The latter article examines derived demand for four inputs, namely, capital, labor, energy and intermediate inputs called materials in the manufacturing sector for the period 1947 to 1971. These two papers provide results and data useful for accomplishing the purpose of this paper.

Indeed, the purpose of this paper is not merely to estimate Hicks coefficient for the above two sectors. Instead it is intended that the implications of this paper would add to our existing knowledge on changes in distribution of output among some key participants in U.S. production such as laborers and owners of land and capital as the quantities of two key inputs, energy and fertilizer, are varied with respect to the quantities of other inputs. Very little research is done in this area.

Methodology

First a brief sketch is given of the methodology for using Hicks coefficient to examine changes in relative factor shares. Then Hicks coefficients are derived from the translog cost function used by Binswanger, and Berndt and Wood.

Assume that both agricultural and manufacturing sectors operate under conditions of perfect competition in both product and factor markets. Let the production function for a sector be

$$Q = F(X_1, \ldots, X_i, \ldots, X_n) \tag{1}$$

$$\text{with } F_i = \frac{\partial Q}{\partial X_i} > 0 > \frac{\partial^2 Q}{\partial X_i^2} = F_{ii}, \quad i=1,\ldots,n,$$

Where Q is output and X_i is the ith input. The production function is assumed to be quasi-concave, that is, the contour set $C(Q) = \{X_i, \ldots, X_n | F \geq Q\}$ is convex for every Q.

Hicks coefficient, σ_{ij}, between i^{th} and j^{th} input is defined as follows

$$\sigma_{ij} = \frac{F_i F_j}{F F_{ij}}, \tag{2}$$

and due to perfect competition the relative share of ith input, S_i, is given as,

$$S_i = \frac{X_i F_i}{Q} \tag{3}$$

Then the following is the relationship between the rate of change in the relative share of ith factor and changes in the quantities of all inputs 2/ (Ranade, 1977a).

$$\frac{\dot{S}_i}{S_i} = \Sigma \; S_j \left(\frac{1}{\sigma_{ij}} - 1\right) \left(\frac{\dot{X}_j}{X_j} - \frac{\dot{X}_i}{X_i}\right), \; i, j=1,\ldots,n. \tag{4}$$

Thus, if the pairwise Hicks coefficients between i^{th} and other inputs are all less than unity and if the quantity of ith input grows slower than the quantity of other inputs, its relative share in output would increase over time. Note that Hicks coefficient is a sufficient criterion of changes in relative shares and not a necessary criterion. That is, given the magnitudes of Hicks coefficients and changes in input quantities, the changes in the relative factor shares can be inferred. However, changes in relative factor shares do not necessarily help in knowing the magnitude of Hicks coefficients.

Now let us see how to derive Hicks coefficient from a cost function. The cost function is derived from the first order conditions of the minimization of the cost of production, $C = \Sigma P_i X_i$, subject to a given output given by (1); where P_i = price of the ith input. Let such cost function be written as follows:

$$C = g\,(P_i, \ldots, P_n, Q). \tag{5}$$

Using the Shephard lemma it can be shown that

$$S_i = \frac{g_i P_i}{C}, \tag{6}$$

where $g_i = \frac{\partial C}{\partial P_i} = X_i$. Furthermore, Sato and Koizumi (1973b) have shown that the partial elasticity of complementarity between i^{th} and j^{th} inputs, c_{ij}, is

$$c_{ij} = \frac{FF_{ij}}{F_i F_j} = \frac{C}{P_i P_j} \frac{G_{ij}}{G} = \frac{1}{\sigma_{ij}} \tag{7}$$

where $g_{ij} = \frac{\partial^2 C}{\partial P_i \partial P_j}$, and G and G_{ij} are the determinant and $(i,j)^{th}$ cofactor of the following bordered Hessian matrix

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2/ The dot over a variable refers to change in that variable over time.

$$\begin{bmatrix} 0 & g_1 & \cdots & g_n \\ g_1 & g_{11} & \cdots & g_{1n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ g_n & g_{n1} & \cdots & g_{nn} \end{bmatrix}$$

Note that Hicks coefficient σ_{ij} is simply the reciprocal of c_{ij}.

In particular, Binswanger, and Berndt and Wood considered the following linear homogeneous translog cost function 3/

$$\ln C = \alpha_o + \Sigma \alpha_j \ln P_j + 1/2\ \Sigma\Sigma\gamma_{ij}\ \ln P_i \ln P_j (i,j=1,\ldots,n), \tag{8}$$

subject to

$$\sum_j \alpha_j = 1,\ \sum_j \gamma_{ij} = 0 \text{ for all } i, \text{ and } \gamma_{ij} = \gamma_{ji} \text{ for } i \neq j. \tag{9}$$

where α's and γ's are the parameters to be estimated. It can be shown that for the translog cost function (8) subject to conditions (9) the elements of the bordered Hessian are the following: 4/

$$g_i = \frac{C}{P_i} S_i \tag{10}$$

$$g_{ii} = \frac{C}{P_i^2} (S_i^2 - S_i + \gamma_{ii}) \tag{11}$$

$$g_{ij} = \frac{C}{P_i P_j} (S_i S_j + \gamma_{ij}) \text{ for } i \neq j, \tag{12}$$

Thus once the values of C, P_i, S_i, α's and γ's are known, by substituting g_i and g_{ij} in the bordered Hessian, Hicks coefficient σ_{ij} can be estimated from equation (7). However Binswanger's paper does not provide the estimates of P_i's and

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3/ In their paper the logarithm of output also appeared on the right hand side of (8). However in their actual estimation output was held constant and therefore in deriving Hicks coefficient for the translog cost function output is not explicitly mentioned.

4/ See the appendix for these derivations.

therefore the formula (7) is not directly applicable. This problem can be overcome as follows.

Let $y_i = \frac{P_i}{C} g_i$ and $y_{ij} = \frac{P_i P_j}{C} g_{ij}$. Substituting these values in the bordered Hessian it can be shown that

$$\sigma_{ij} = \frac{Y}{Y_{ij}} \tag{13}$$

where Y and Y_{ij} are the determinant and cofactor of the following matrix

$$\begin{bmatrix} 0 & y_1 & \cdots & y_n \\ y_1 & y_{11} & \cdots & y_{1n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ y_n & y_{n1} & \cdots & y_{nn} \end{bmatrix}$$

Thus by substituting the estimates of S_i, α_i and γ_{ij}, given in the two papers, into equation (13) changes in relative factor shares are examined by making some alternative assumptions about changes in factor-proportions.

Note that for the translog cost function (8) the relative share of ith input can be computed from equation (6) as follows:

$$S_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j \qquad (i,j=1,\ldots,n) \tag{14}$$

From the above equation changes in relative factor shares due to changes in factor prices can be deduced from the estimates of parameters α_i and γ_{ij} presented in the papers of Binswanger, and Berndt and Wood. However, the same is not true about the changes in relative factor shares according to changes in factor proportions. Hicks coefficient is useful for this purpose.

Results and Implications

For the agricultural production there are five factor inputs, namely, land, machinery, fertilizer, labor, and other inputs such as livestock, poultry, feeds, seeds and plants (Binswanger, p. 385). For the manufacturing sector there are four factor inputs, namely, capital, labor, energy, and materials such as agricultural goods, nonfuel mining, and construction; manufacturing excluding petroleum products; and transportation (Berndt and Wood, p. 262).

Table 1. Hicks Coefficients and Direction of Changes in Relative Factor Shares in the United States Agricultural Sector 1/

Pair of inputs		Hicks coefficient	Change in relative share of				
i^{th}	j^{th}	σ_{ij}	Land	Fertil-izer	Mach-inery	Labor	Other
Land	Fertilizer	-0.1879	+	-	0	0	0
Land	Machinery	0.7403	-	0	+	0	0
Land	Labor	0.2373	-	0	0	+	0
Land	Other	0.2812	-	0	0	0	+
Fertilizer	Machinery	0.4301	0	-	+	0	0
Fertilizer	Labor	0.2396	0	-	0	+	0
Fertilizer	Other	1.0131	0	+	0	0	-
Machinery	Labor	1.3009	0	0	+	-	0
Machinery	Other	2.1482	0	0	+	0	-
Labor	Other	-5.0454	0	0	0	+	-

1/ Hicks coefficient is computed from the average relative factor shares for the period 1947-64 (Binswanger, p. 383).

Table 2. Hicks Coefficient and Direction of Changes in Relative Factor Shares in the United States Manufacturing Sector 1/

Pair of inputs		Hicks coefficient	Change in relative share of			
i^{th}	j^{th}	σ_{ij}	Capital	Labor	Energy	Intermediate
Capital	Labor	3.8052	+	-	0	0
Capital	Energy	0.0544	-	0	+	0
Capital	Materials	0.5018	-	0	0	+
Labor	Energy	0.4718	0	-	+	0
Labor	Materials	0.5909	0	-	0	+
Energy	Materials	0.9808	0	0	-	+

1/ In computing σ_{ij} the estimates of the relative share for 1959 are used. The directions of changes in relative factor shares, however, do not change if the estimates of relative factor shares for other years during 1949 to 1971 are used.

Hicks coefficients between different pairs of the above inputs in each sector are presented in Tables 1 and 2. Those tables also present the direction of changes in relative factor shares as a particular ratio of input quantities increases, holding all other ratios of quantities of inputs constant. Such directions are derived by using the estimated Hicks coefficient in equation (4). Those tables can be read as follows. Consider the first pair of inputs land (i^{th}) and fertilizer (j^{th}) in Table 1. Then the signs in the first row of that table mean that as the quantity of land per unit of the quantity of fertilizer (land-fertilizer ratio) is increased and if all other factor proportions are held constant, the relative share of land would decrease, that of fertilizer would increase while the relative shares of labor, machinery and other inputs would remain unchanged. In the following discussion, unless specified, a particular ratio of the input quantities is varied holding all other ratios of input quantities constant.

Consider first the relative share of labor. In both the sectors this share decreases as capital (machinery) per unit of labor increases. In contrast, this share increases when fertilizer and energy grow faster than labor. Thus increase in capital intensity, on the one hand, distributes less output to labor while increase in fertilizer and energy intensity is beneficial to laborers in terms of their relative share in output in each sector.

Increase in land per unit of machinery, labor or other inputs results in a decline in the relative share of land in agricultural output. Increase in fertilizer per unit of land also results in decline in the relative share of land. Increase in materials in terms of all other inputs in the manufacturing sector results in their decline in share in output. Note that the bulk of these materials is comprised of agricultural goods. Thus it appears that a relative increase in use of agricultural goods in manufacturing production results in decline in their relative benefits from manufacturing production.

Increase in energy per unit of capital increases the relative share of capital in manufacturing while increase in fertilizer per unit of machinery increases the relative share of machinery in agriculture.

Thus in the agricultural sector, if fertilizer and land grow faster than labor the relative share of labor increases, while in the manufacturing sector, if energy and materials grow faster than labor the relative share of labor increases.

From equation (4) it can be verified that if the magnitudes of Hicks coefficients are closer to unity, relative factor shares are less sensitive to changes in factor proportions. Since the estimates of the majority of pairwise Hicks coefficients are much different from unity (Tables 1 and 2), technology in both the sectors seems to have highly flexible relative factor shares with respect to changes in factor proportions.

In their well-known paper on intercountry CES production functions, Arrow, Chenery, Minhas and Solow concluded that the relative share of labor would <u>increase</u> over time if capital-labor ratio increases over time. Their conclusion was based upon their estimate of elasticity of substitution between capital and

labor which was less than unity. In contrast, this paper finds that as capital-labor ratio increases, holding other factor proportions constant, the relative share of labor would decrease over time.

Conclusions

This paper examines changes in relative factor shares by using Hicks coefficient for the United States agricultural and manufacturing sectors with many input production. It shows that the distribution of output would be skewed against laborers and capital owners if energy input grows slower than growth in employment and capital, respectively. Similar is the effect of fertilizer on the relative share of labor and machinery in agriculture. In view of the recent energy crisis around the world this finding is important in setting policies for controlling energy use in production and for increasing employment. The finding would be operationally meaningful especially when relationship between changes in relative factor shares and changes in personal income distribution is known.

The findings show that the relative factor shares are highly flexible with respect to factor proportions in modern technology such as in the United States and there is substantial scope for altering the distribution of output favorable to laborers by altering factor proportions.

In contrast to this in traditional agricultures of LDC's where there are usually two relatively less substitutable inputs such as land and labor and where labor input grows faster than land, the relative share of labor invariably declines over time. However in view of the current technological change in LDC's which has brought fertilizer and new cultural practices in agriculture, it would be useful to study how the relative share of labor varies as the amount of fertilizer per unit of land is changed.

In this respect, contrary to the changes in the relative share of labor in United States agriculture, Ranade (1977b) has found that in Philippine rice production the relative share of labor declines as the quantity of fertilizer per unit of land is increased using modern technology. Is this because farmers apply less than optimum amounts of fertilizer in Philippine rice production? How do the relative factor shares vary as the amount of factor proportions are varied in production in other LDC's?

One could criticize the whole approach of this paper by asserting that the effect of factor prices on factor shares is more relevant than the effect of factor proportions. Such criticism appears to be more valid in sectoral analysis where prices act as exogenous variables while factor proportions become endogenous variables, than in a macro analysis at national level where prices could be treated endogenously. However, considering the substantial concern of policymakers about depletion of resources around the world, excessive preoccupation of economists with the price rather than the quantity of resources might result in policies irrelevant to current issues.

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Appendix

(A) To Derive the Elements of Bordered Hessian:

Differentiating the translog cost function partially with respect to P_i we get

$$\frac{\partial C}{\partial P_i} = \frac{C}{P_i}(\alpha_i + \Sigma\gamma_{ij} \ln P_j), \quad i,j=1,\ldots,n \tag{1}$$

Thus

$$S_i = \frac{P_i}{C} g_i = (\alpha_i + \Sigma\gamma_{ij} \ln P_j),$$

and therefore

$$g_i = \frac{C}{P_i} S_i \tag{2}$$

Differentiating (1) partial with respect to P_i,

$$\frac{\partial^2 C}{\partial P_i^2} = \left(\frac{\partial C}{\partial P_i}\frac{1}{P_i} - \frac{C}{P_i^2}\right)(\alpha_i + \sum_j \gamma_{ij} \ln P_j) + \frac{C}{P_i^2}\gamma_{ii}$$

Substituting (2) and S_i in the above equation and rearranging the terms on the right hand side,

$$\frac{\partial^2 C}{\partial P_i^2} = \frac{C}{P_i^2}(S_i - S_i^2 + \gamma_{ii}) = g_{ii} \tag{3}$$

Differentiating (1) partially with respect to P_j,

$$\frac{\partial^2 C}{\partial P_i \partial P_j} = \frac{\partial C}{\partial P_j}\frac{1}{P_i}(\alpha_i + \Sigma\gamma_{ij} \ln P_j) + \frac{C}{P_i P_j}\gamma_{ij}$$

$$\frac{\partial^2 C}{\partial P_i \partial P_j} = \frac{C}{P_i P_j}(S_i S_j + \gamma_{ij}) = g_{ij} \tag{4}$$

(B) To Show: $\sigma_{ij} = \frac{Y}{Y_{ij}}$

Substitute $g_i = \frac{C}{P_i} y_i$ and

$g_{ij} = \frac{C}{P_i P_j} y_{ij}$ in the bordered Hessian matrix. Then the

determinant of that matrix (G) and $(i,j)^{th}$ cofactor (G_{ij}) can be written as follows:

$$G = Y\, C^{n+1} / \prod_{i=1}^{n} P_i^2 \tag{5}$$

$$G_{ij} = Y_{ij}\, C^n / \left(P_i P_j \prod_{k=1}^{n} P_k^2\right) \text{ where } k \neq i \neq j \tag{6}$$

It was shown that the $(i,j)^{th}$ Hicks coefficient is

$$\sigma_{ij} = \frac{G}{G_{ij}} \frac{P_i P_j}{C} \tag{7}$$

Substituting (5) and (6) in (7),

$$\sigma_{ij} = \frac{Y}{Y_{ij}} \frac{C^{n+1}}{\prod_{i=1}^{n} P_i^2} \frac{P_i P_j \prod_{k=1}^{n} P_k^2}{C^n} \frac{P_i P_j}{C} = \frac{Y}{Y_{ij}}$$