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9. ABSTRACT
(HOUSING R & D)
In 1962 the University of Michigan received a USAID grant to explore the potential of foam plastics for solving housing problems in developing nations. A feasibility study completed in 1962 (see PN-AAD-174) established the desirability of investigating the use of foam plastics as structural material. The project was conducted in two phases. The first phase involved experimentation to obtain information on a variety of foam plastics. Several structural systems were established as scale models before two structural concepts were selected for full-scale construction and testing. One concept involved application of spray polyurethane foam on a folding wood lattice armature. The other concept involved production of large structural components made of paper-laminated polyurethane foam board. During the second phase of the project, four prototype structures representing different techniques were built. One consisted of seven umbrella-shaped components made of sprayed polyurethane foam. A second was a two-story structure made of triangulated bents of folded paper-laminated polyurethane foam board. A third consisted of four square umbrella-shaped components made of flexible foam coated with reinforced polyester. The fourth was a room-sized unit in which polyurethane foam board served as the mandrel for filament-wound polyester. The research program also investigated requirements for production and marketing of chemicals and foam plastic products for structures and building components in underdeveloped countries. The major recommendation stemming from the project was

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that AID collaborate with the U.S. plastics industry in sponsoring construction and evaluation of suitable prototype dwelling units in developing countries. Interim reports of the project activities during 1964 and 1965 are available as PN-ADD-177 through PN-AAD-182.

SOC H-2
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Reports
PN-AAD-181

ARCHITECTURAL RESEARCH LABORATORY - THE UNIVERSITY OF MICHIGAN

Quarterly report: to the Agency for International Development
on ORA project 05687
for the period January through March 1965

RESEARCH ON STRUCTURAL SYSTEMS UTILIZING THE FOAM PLASTICS AS A STRUCTURAL MATERIAL FOR HOUSING IN UNDERDEVELOPED AREAS

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REVIEW OF WORK PERFORMED

1. Development work on the four prototype structures and their related testing program is progressing according to schedule.

   a. The folded plate system. The upper level of the two-story prototype structure has been enclosed with acrylic panels and zip-on synthetic gaskets (see Appendix A, Plate 1). An expandable folding structure made from the same paper-polyurethane foam board was produced by the Plydom Corporation of Toronto, Canada, and was given to the project (see Appendix A, Plate 1). This structure is scheduled to be erected here in May. The testing program involving the use of paper-polyurethane foam board has been completed and a paper on this subject has been prepared (see Section 3).

   b. The folding armature system. A 16-foot square wooden lattice armature was erected for the production of two test shells (see Appendix A, Plate 2). The two shells were produced by spraying both sides of the armature with a 3.7 pcf polyurethane formulation prepared for this purpose by Wyandotte Chemicals. Equipment and technical assistance for the spraying of these shells was provided by Binks Manufacturing and Wyandotte Chemicals. For the production of the two shells, the top of the armature was first covered with a nylon-reinforced paper skin as in the case of the shell which was erected last year (see Interim Report, July 1963 through June 1964). A sheet of flexible foam was then stretched on top of the paper-covered armature to serve as a mold release and become the inner surface of the top shell. One shell was sprayed from the top and the other from underneath the armature. After being allowed to cure, the two shells were easily separated (see Appendix A, Plate 2). The armature became an integral part of the lower shell, while the upper shell consists entirely of polyurethane foam. The two shells are being set up for comparative testing prior to the erection of a final demonstration shell.

   c. The rigidized flexible system. An investigation was conducted to determine possibilities for obtaining optimal anticlastic shell shapes through the stretching of flexible foam as well as to investigate the relationship between "minimum surface" and "minimum structure" (see Appendix A, Plate 3). Data from this research as well as from similar experiments with synclastic surfaces relating to the folding armature system has been used for the preparation of a paper (see Section 3). At the same time, three 12' x 12' umbrella-shaped components were produced by stretching flexible foam as in

the case of the demonstration made at Rice University. Two components were produced by rigidizing the flexible foam with urethane resin and spraying it on both sides with glass fibers and polyester resin. The third component was also sprayed from both sides but the foam was not rigidized. A fourth component will be produced by spraying on the flexible foam only from one side. A testing program has been set up to compare these three alternatives (see Appendix A, Plate 3). After the completion of testing, the four components will be used for the erection of a 24' x 24' prototype structure.

d. The filament winding system. A model of a mandrel for the production of a room-size component (approximately 12' x 16' x 8') was prepared and sent to the Hercules Powder Company to be filament wound. The system and the process followed are illustrated in a series of photographs (see Appendix A, Plate 4). On the basis of this experiment and further analysis of this system, Hercules has agreed to produce a similar full-scale unit for the project. For economic reasons, in the production of this structure, the plastic ribs will be replaced by 2" square steel tubing. Two inch thick paper-skin polyurethane-foam board, contributed by Armorlite Corporation, is to be placed between the tubular ribs. The inner lining of the structure will be made of pressed hardboard contributed by Masonite Corporation . The structure is scheduled to be wound during the latter part of May.

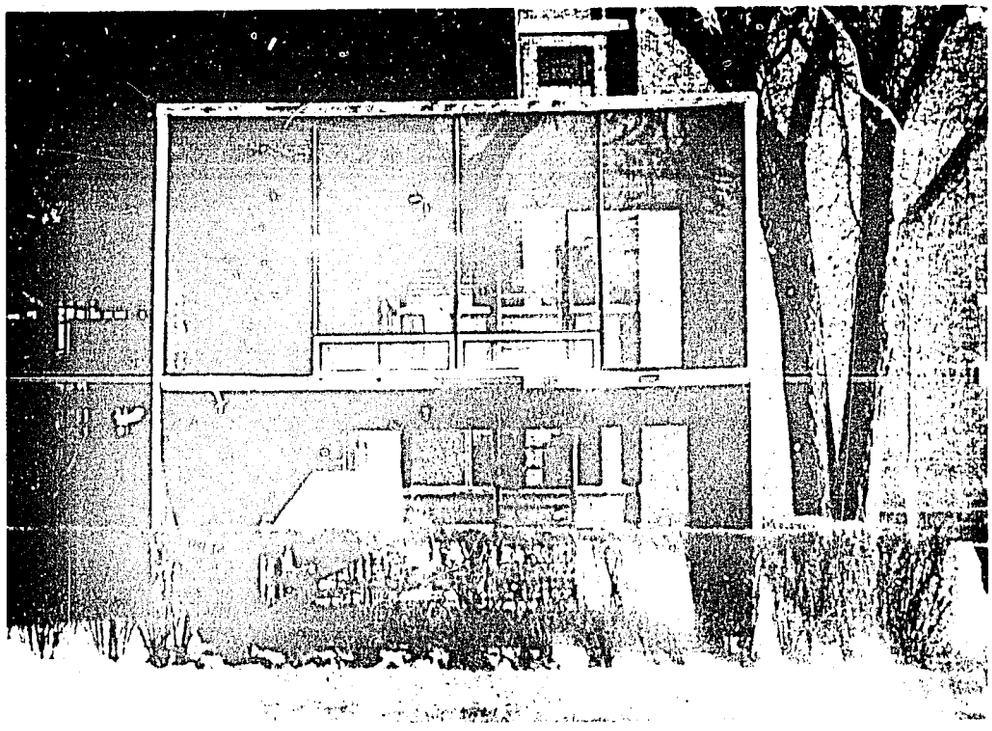
2. Collection of material to determine the market potentials of foam plastics in emerging nations is nearing completion. Information has been obtained from industry and the United Nations. Correlation of this material is expected to begin in June.

3. Both papers prepared for the London conference on "Plastics in Building Structures" to be held June 14-16 were accepted by the conference board. The first is on "Structural Potentials of Paper Skin Polyurethane Foam Board" and the second on "Application of Minimum Structure to Cellular Plastics." The final drafts of these papers are attached to this report as Appendix B. These papers are also to be presented at the conference on Cellular Plastics to be held in Ann Arbor on September 22-24.

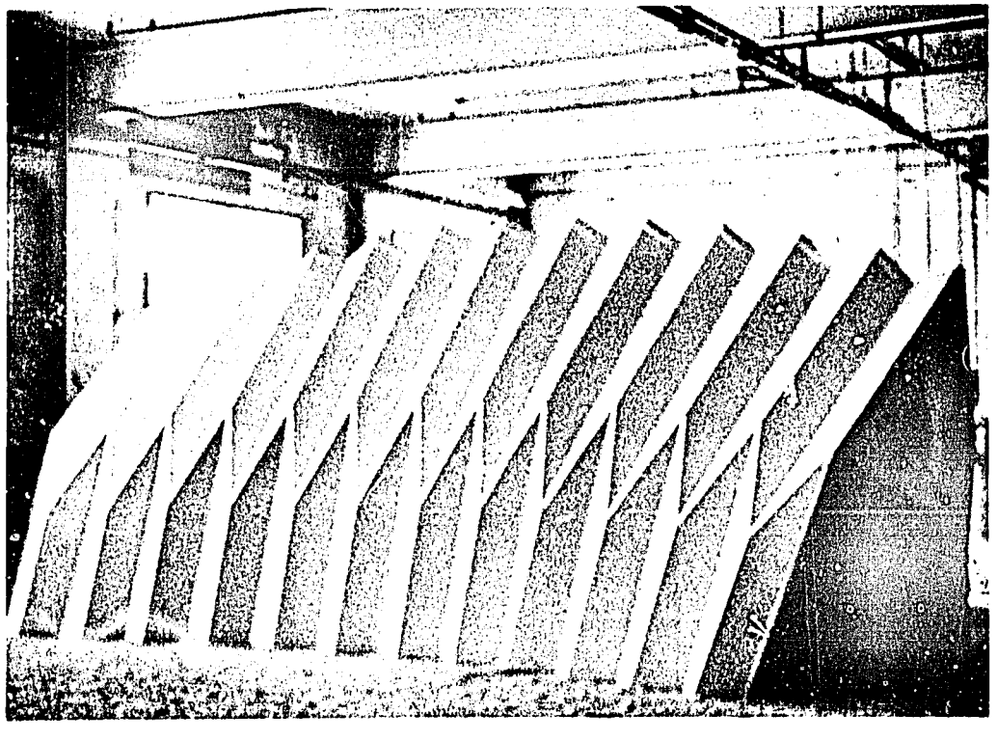
Appendix A

STRUCTURAL RESEARCH AND DEVELOPMENT

1. THE FOLDED PLATE SYSTEM

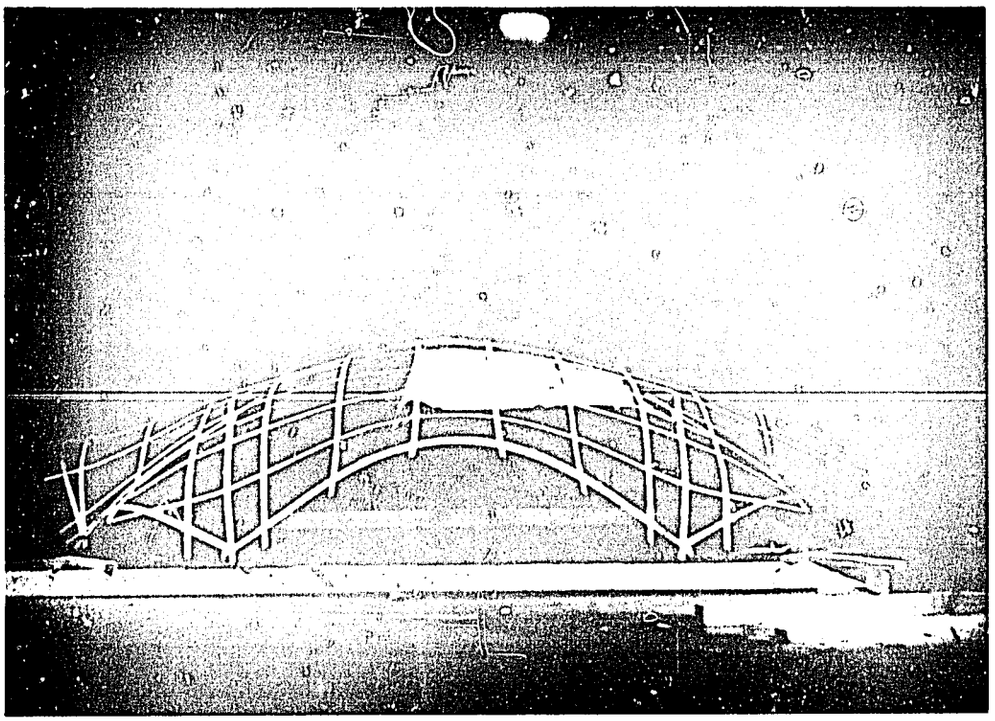


1a Two-story folded plate test structure with enclosure

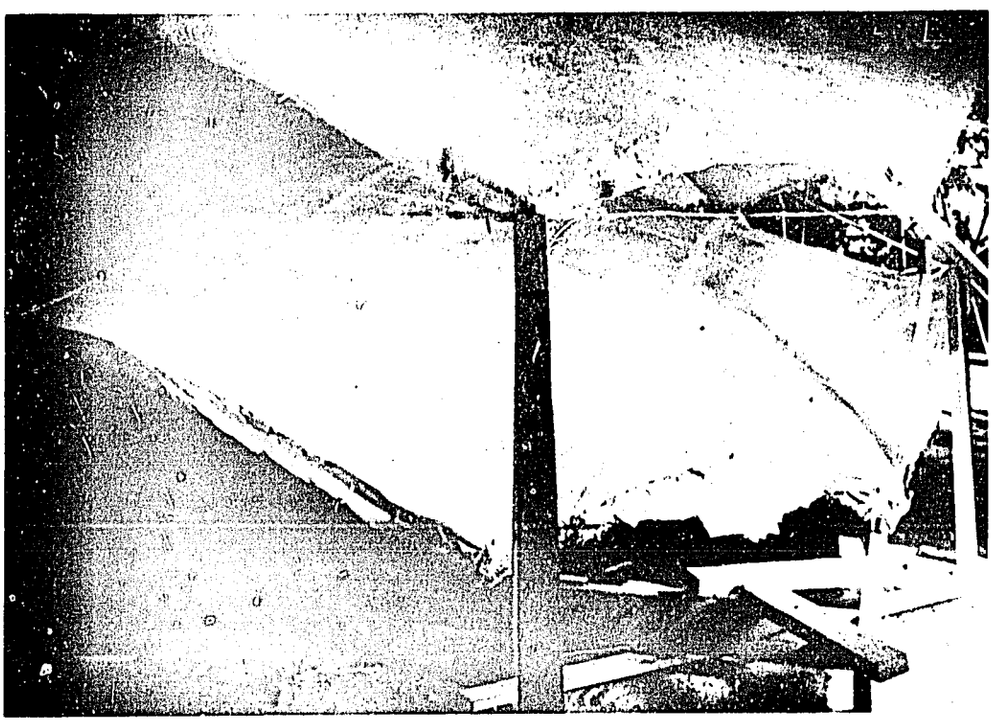


1b Expandable folded plate structure produced by Plydom Corp.

2. THE FOLDING ARMATURE STRUCTURE

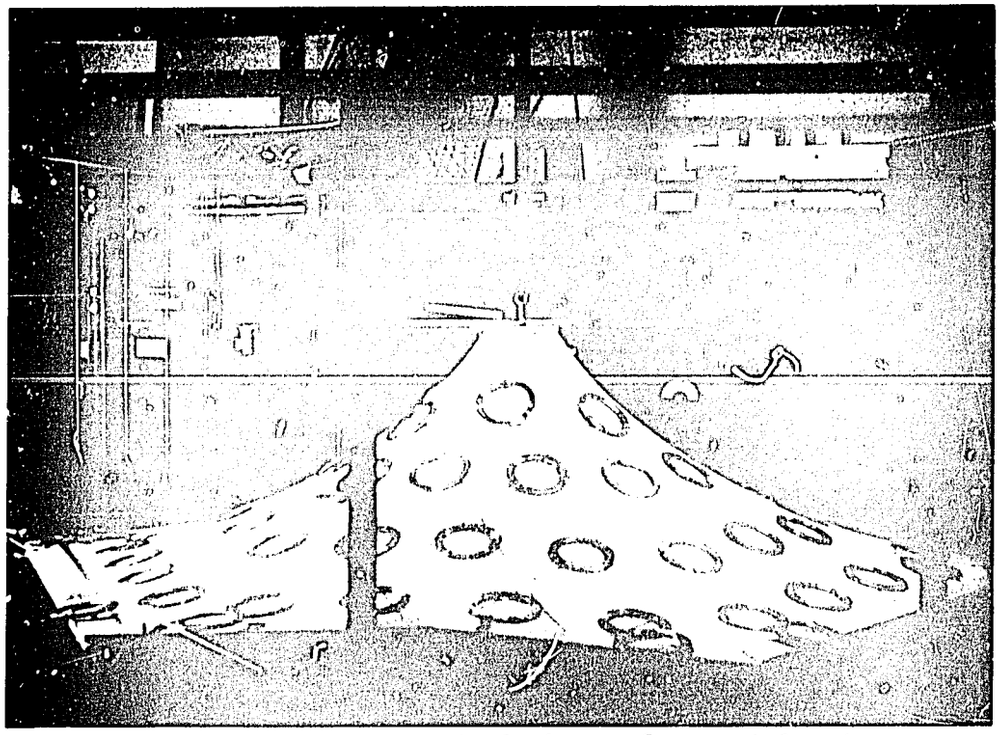


2a Revised shape of folding armature to be sprayed from both sides for the production of two shells

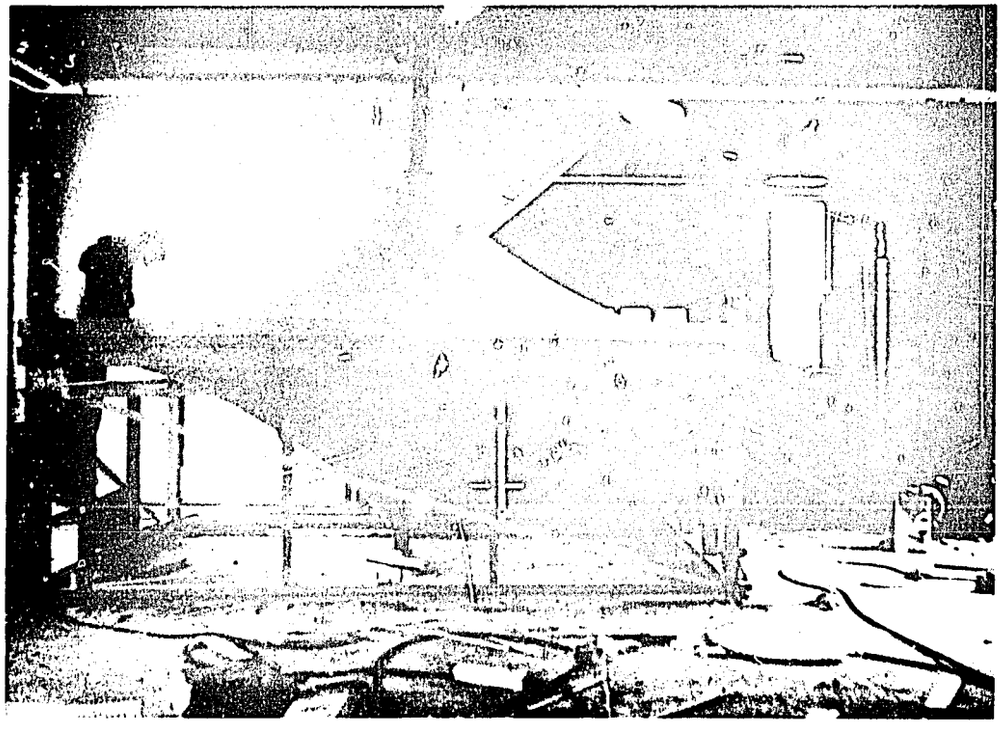


2b After spraying with polyurethane foam the two shells are separated in preparation for testing

3. THE RIGIDIZED FLEXIBLE SYSTEM

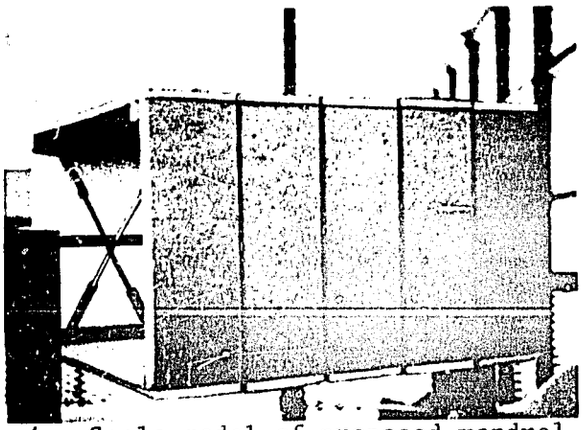


3a Research to determine optimal shapes for anticlastic surfaces

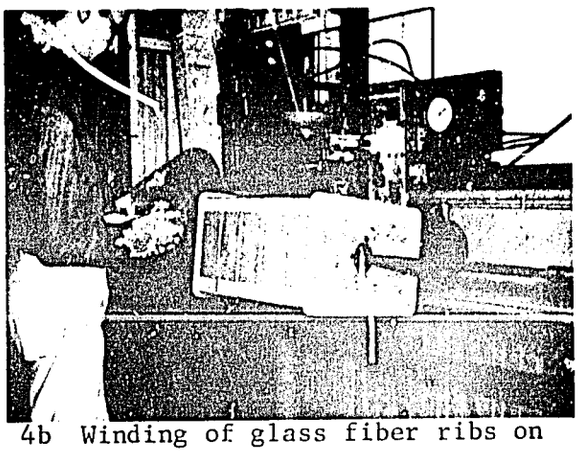


3b Load testing of two 12' x 12' umbrella shaped components with air pressure

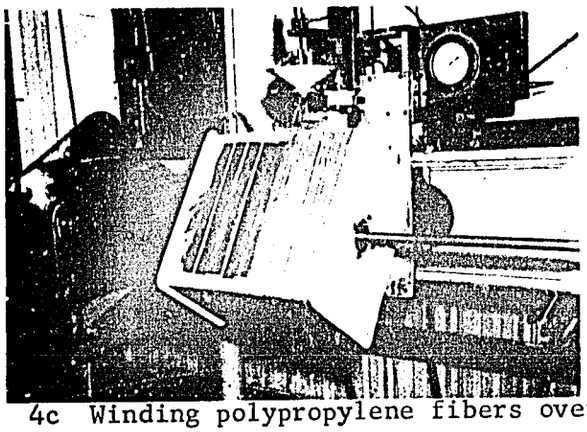
4. THE FILAMENT WINDING SYSTEM



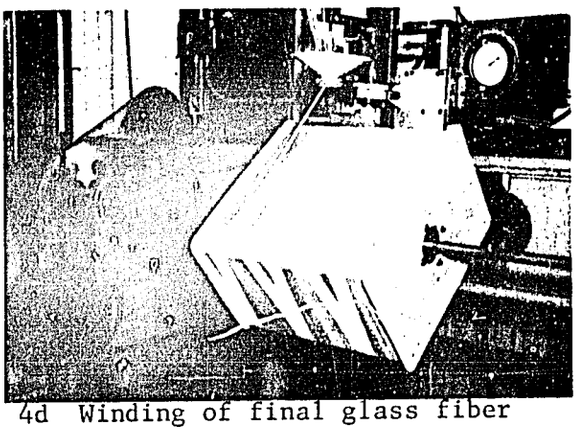
4a Scale model of proposed mandrel showing rib spacing between foam-board liner



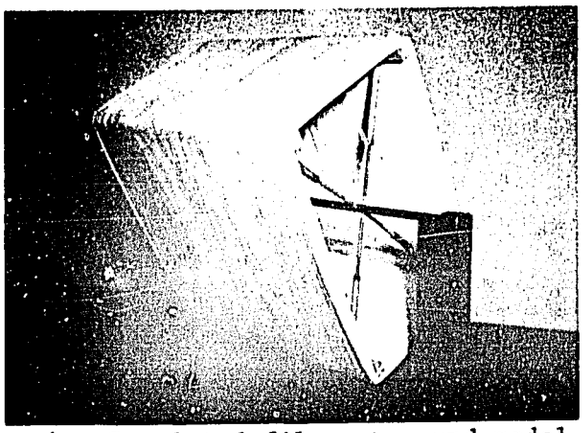
4b Winding of glass fiber ribs on inner liner



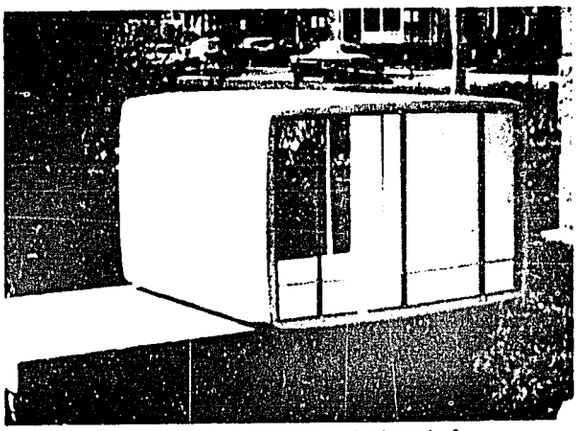
4c Winding polypropylene fibers over inner liner and ribs



4d Winding of final glass fiber skin



4e Completed filament wound model of structure



4f Filament wound model with proposed enclosure

Appendix B

FINAL DRAFTS OF PAPERS FOR THE LONDON CONFERENCE ON "PLASTICS IN BUILDING STRUCTURES"

ARCHITECTURAL RESEARCH LABORATORY - THE UNIVERSITY OF MICHIGAN

Final draft: for presentation at the Conference on 'Plastics in Building Structures,' London, 14 to 16 June 1965.

STRUCTURAL POTENTIALS OF PAPER SKIN POLYURETHANE FOAM BOARD

J Sterling Crandall, Research Associate

Summary

Experimental structures have been built and tested which utilize polyester impregnated, paper-laminated urethane foam board as the principle stress-resisting material. These structures have been found adequately strong for the construction of light industrial and residential buildings. Material properties are in some instances an advantage; in others a restriction. However, problems met thus far have been amenable to conventional engineering solutions. A two-story prototype building has been constructed which shows performance characteristics similar to conventional wood-frame structures.

STRUCTURAL POTENTIALS OF PAPER-SKIN POLYURETHANE FOAM BOARD

J. Sterling Crandall, B. Arch., AIA, Research Associate
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The development of efficient equipment for the production of paper-skin polyurethane foam board on a continuous basis has brought this material to the foreground from the standpoint of its possible uses in building. In addition to being well suited for standard wall board and decking applications, this material, because of its unique properties, appears to warrant an investigation for the establishment of further structural potentials.

The ease with which this board can be creased and folded suggests the possibility of using it in a great variety of folded plate structures. Because of its mechanical properties, it appears to permit the design of more highly stressed structural components than is usually the case with foam plastic materials, Finally, the increasing interest of manufacturers in this system of producing slab stock, stemming from its high production efficiency and low capitalization cost, is reflected in the availability of higher quality material at decreasing costs.

These factors have stimulated the architectural research staff at The University of Michigan to explore in some depth the structural potentials of paper-laminated polyurethane foam board as part of its research program on the "Structural Uses of Foam Plastics for Housing in Underdeveloped Areas"[1].

A variety of materials obtainable in rolls may be used as skins for this foam board. However, paper seems to be one of the best suited for this purpose, because structural properties can easily be added through secondary applications depending on performance requirements within the structure[2].

PRODUCTION AND PRESENT-DAY APPLICATIONS

Paper-laminated polyurethane foam board is being produced today by a number of companies throughout the world. Although there are some variations in the system of production, it basically involves a continuous process in which the skins, introduced in rolls, go through the machine at the same rate as the foam that is being produced. The liquid foam components are uniformly distributed between the two skins. The foam expands and adheres to the skins as the material moves through a form or a nip roller. The board after passing through a heat-curing operation, is then cut to the desired size[3].

Since the status of paper-laminated foam as a construction industry resource will depend on its present and anticipated production, one phase of investigation has been aimed at identifying present and potential producers and to establish the nature of their products. This survey is being carried on by questionnaire, and is expected to provide a fairly accurate picture of the state-of-the-art, when completed. At this time, it has been possible to identify eight companies in the United States which are actively engaged in the commercial production of some type of paper-laminated foam board. Two other companies are pursuing an active program of research and development in this area, while two additional concerns suggest that their entry into the field is only contingent on developing markets. At present, five United States firms are prepared to supply production machinery for this material, either as a standard item or as custom-built equipment. The full-time production capacity of United States firms is estimated to be well in excess of 10,000,000 board feet per month. There are reasons to believe that European, Asian and Australian activity in this area exceeds that encountered in the United States.

Many of the problems which have plagued early developers have been largely overcome. Panel flatness and thickness can be controlled within acceptable limits and it is only a matter of time and demand before inconsistencies in surface appearance will be eliminated also. In most commercially available board, skin adhesion has been found to exceed the cell strength of the foam. Thus, foam formulation, foam density, and the type of paper become the parameters for materials to satisfy differing structural criteria.

Paper-laminated urethane foam board is available in thicknesses from 0.125 to 2.0 inches, and in densities from 1.8 to 2.5 lb per cubic foot. It has been used in building as roof insulation, plaster base, and ceiling material. It has also found structural applications in vacation shelters, emergency housing, and the like (Figure 1).

EVOLUTION OF STRUCTURAL SYSTEMS

The structural systems considered by the project staff have been based on the unique properties of the material and its method of production. The ease of scoring and bending the material, and its continuous, linear production system, became important determinants of the resulting structural forms. Ideally, the material can move in a linear progression from the foam-dispensing equipment and the paper rolls to the finished structural component by the addition of the

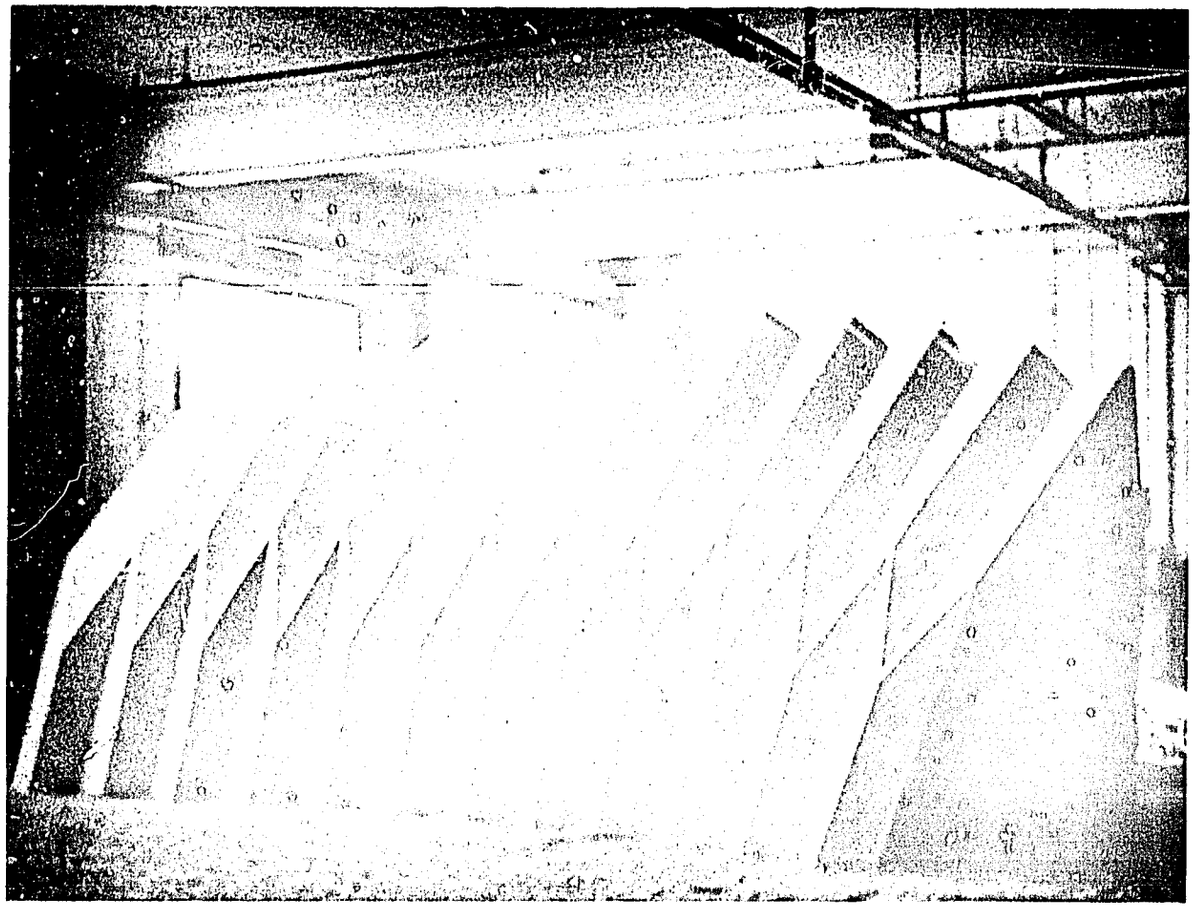


Figure 1: A recent commercial application of paper-laminated polyurethane foam board as a structural material. (Courtesy of Plydom Corporation, Ltd., Woodbridge, Ontario).

necessary scoring and forming equipment to the end of the board-producing machine.

Consideration in the design of structural systems has also been given to component versatility. In the interest of reducing fabrication and erection cost, it was felt desirable that each system should utilize as much as possible the same basic structural component for wall, floor, and roof sections. With these ends in mind, three structural systems were developed. Their sections are illustrated in Figure 2. Each system utilizes a single structural component which can be used vertically as well as horizontally to form a bent requiring the joining between bays to complete the structural enclosure. Initial studies conducted by the project staff indicated that the paper fibers of the skins were likely to buckle and fail at relatively low stress levels. Impregnation of the paper skins with polyester resin was found to resolve this problem and to enhance considerably the impact resistance of the surface as well. The surfaces of the three structural components were impregnated to varying degrees.

The results of preliminary load testing of these three components indicated that the triangular section had the greatest structural capacity and thus might also be considered as a floor section (Figure 3). Since the system could thus be used for two-story as well as one-story shelters, the triangular section was chosen for further investigation. In the interest of exploring the potential of this system, a testing program was set up to parallel the development of a two-story prototype structure. The board material used for the test sections and for the prototype structure had a thickness of 3/8 of an inch, a foam density of 2.5 lb per cubic foot, and skins consisting of 69 lb kraft liner board.

DEVELOPMENT TESTING

In connection with the testing program which has been conducted by the project staff, Professor Paraskevopoulos reported the following in a paper presented at the International Conference on Cellular Plastics, which was held in New York in September 1964:

> "Through the erection of test structures and other experimentation with materials we have been trying to determine the influence of certain unknown factors in foam plastics on a structural form. This has been done in order to identify areas where additional research and development work is needed to improve performance standards. At the same time we are trying to establish the extent to which the present weaknesses and inconsistencies of the materials can be compensated through corrective measures within a structure."

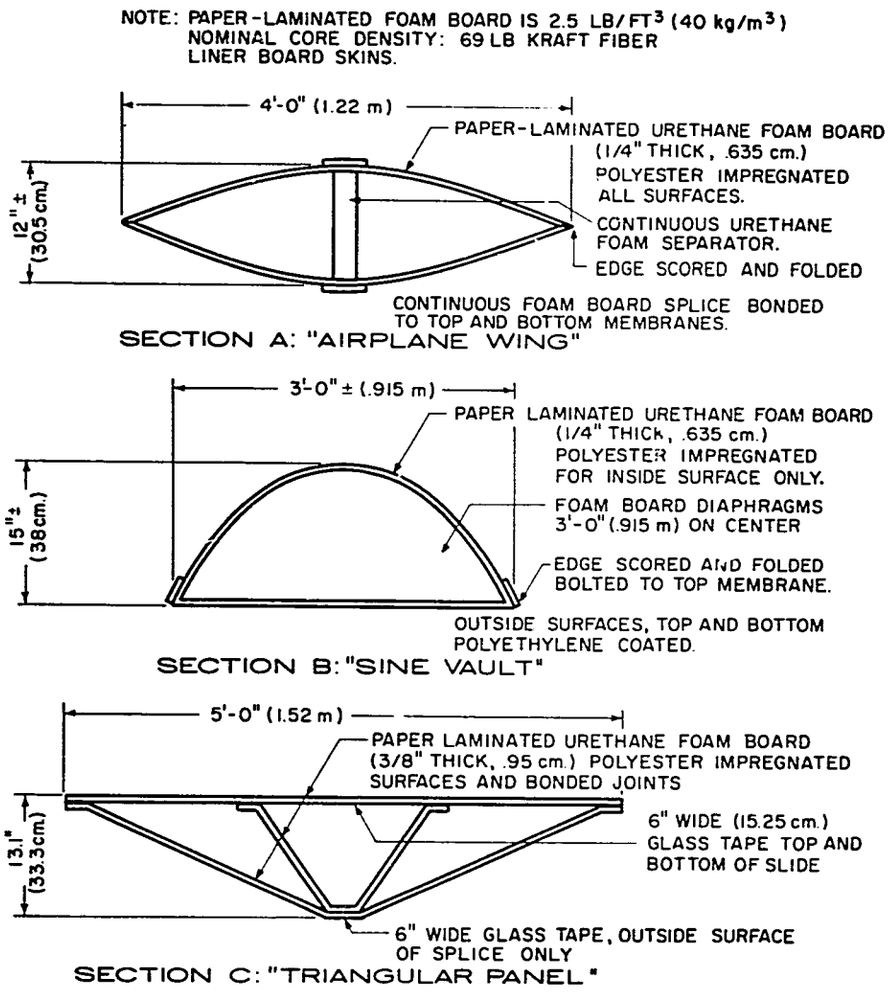


Figure 2: Cross sections of three test structures built to investigate the utilization of paper-laminated polyurethane foam board as a structural material.

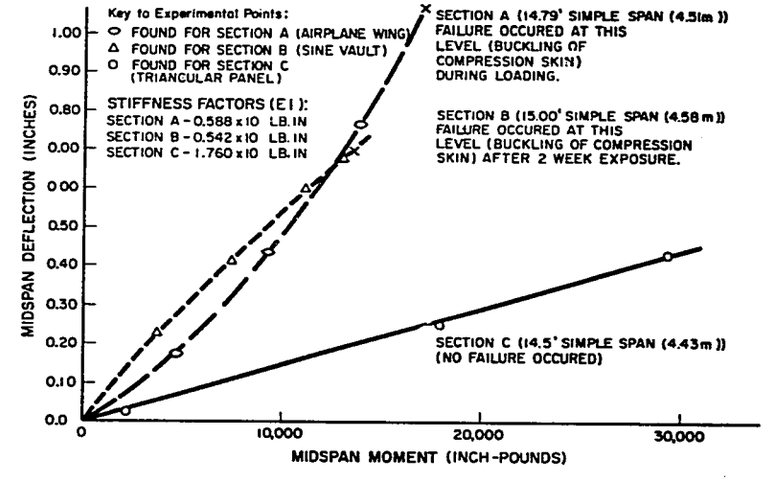


Figure 3: Relative performance of test structures shown in figure 2.

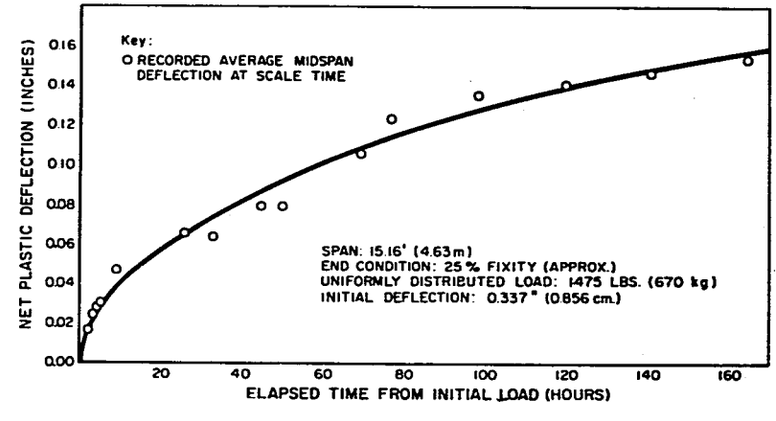


Figure 4: Long-term loading behavior of triangulare test structure shown in figure 2.

Thus, the testing of the paper-laminated foam board has been designed to clarify four general areas related to the structural performance of the prototype structure.

a. The Effect of Creep. In foam plastics the effect of creep is often so pronounced that it imposes a major restriction on design of structures based on the use of these cellular materials. With the paper-laminated foam composite there was concern that shear creep in the foam might drastically reduce the available moment capacity of the section for long-term loading imposed by furniture, fixtures and the like. Before proceeding with the erection of a prototype structure, the original test section was reloaded to about 25% of its estimated capacity. The section remained loaded for a period of time during which deflections were recorded.

The results of this experiment are summarized in Figure 4. Conclusions drawn from this experience were as follows: (1) the rate of creep was decreasing rapidly and could be expected to approach negligible amounts; (2) the effect of shear creep in the foam, while not measurable by this technique, imposed no serious restrictions on the capacity of this section; and (3) through appropriate design, deflections of the prototype structure could be held to an acceptable level for any given period of time. Consequently, work was begun on the prototype structure (Figure 5), while in the meantime other experiments were conducted to explore the subject of creep in greater depth.

A box beam section was constructed and supported as shown in Figure 6. Aside from the relative simplicity of analysis provided by this section, an important factor in its construction was the transfer of stress to the top and bottom skins through a layer of urethane foam. Any differential creep of the foam with respect to the polyester impregnated paper elements of the section would be reflected in the relative behavior of these skins. Loads of 60 lb (27.25 kg) have been imposed at the third points of the beam for a period of 6 months. During this period deflections have been recorded, and strain measurements have been taken at various points of the beam. The results are shown in Figure 7.

Plastic deformation has been substantial; more than 100% of the elastic deformation experienced. However, there has been no evidence indicating a difference in behavior between the cellular material and the impregnated paper. Also, the deformations plotted against time show the typical logarithmic pattern which enables prediction of the apparent modulus of elasticity at the end of any period of time, and permits the design of structures which will not exceed allowable deflections over this period. Indications are that a flexural modulus of

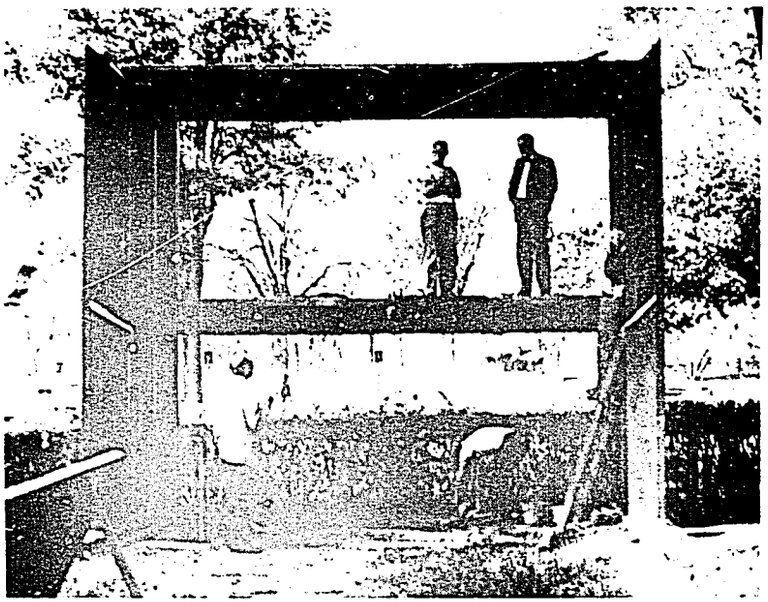


Figure 5: Prototype structure principally of paper-laminated polyurethane foam board.

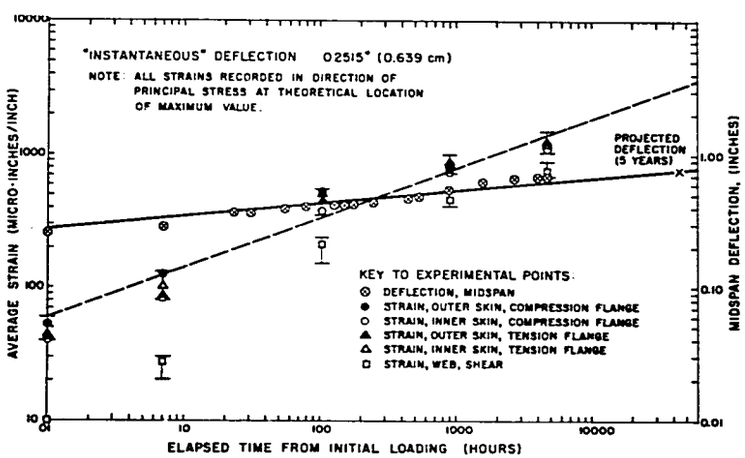
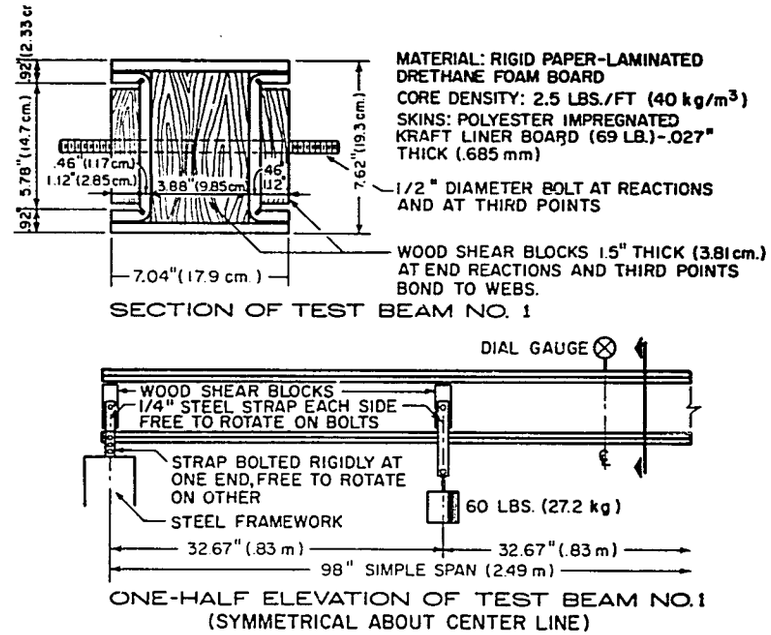


Figure 7: Summary of results of long-term load testing of beam shown in figure 6.



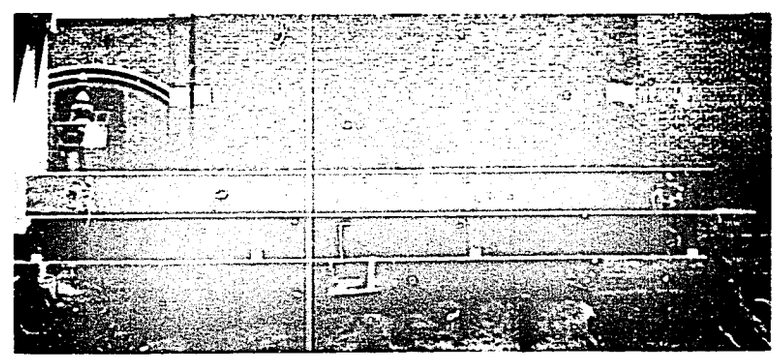


Figure 6: Test beam constructed to study creep behavior in paper-laminated foam structures. Detail of beam (top); elevation of beam and testing apparatus (middle); and photograph of strain gage instrumentation (bottom)

one-quarter the value found instantaneously will yield satisfactory results over an extended period of time, perhaps as much as fifty years. Higher values can, of course, be used for relatively short loading durations, since much of the time related deformation experienced during loading has been found to be reversible. The recovery of the triangular test bent following removal of the loads is shown in Figure 8. Deflections arising from sustained loads, such as furniture and structural dead load, should be computed using an E value reduced in accordance with the expected life span of the structure.

b. Glass Fiber Tape Reinforcement. Because of limitations in the width of board material at the time when the triangular test section was constructed, narrower sections had to be joined to construct the top and bottom membranes. To do this, glass fiber tape was used, impregnated and bonded to the paper skins with polyester resin (see Figure 2). As the testing of the section proceeded, a suspicion grew that this expedient measure might be contributing significantly to the performance of the section. Since the cost of this tape was nominal, and to prevent possible failure of the prototype structure, it was decided to proceed with its construction by using glass fiber tape in all of the joints as a precautionary measure.

At the same time a second box beam was set up for testing paralleling the first and identical in detail, excepting that one-inch-wide glass fiber tapes were bonded into the longitudinal joints between the web and the flange membranes. This beam was loaded, as in the first case, with 60 lb (27.25 kg) at the third points. However, in this case only the center span deflection has been recorded. Figure 9 indicates the performance of this reinforced beam as compared to the early performance of the unreinforced one. A minor refinement has been introduced into this experiment in that the unreinforced beam has remained loaded during the entire period, and readings from it have been used as a control for extraneous variables, which are presumed to affect both sections similarly. Curve #3 of Figure 9 represents the difference in amounts of creep shown by the two sections during the period from 1 January to 1 February 1965, whereas curves #1 and #2 show absolute values of creep for each of the two sections during the first thirty days of their history.

The results of this experiment indicate a 14.5% increase in the apparent flexural modulus of a glass fiber tape reinforced section as compared to the similar unreinforced section. This difference appears to be comparable for

Figure 8: Recovery of triangular test panel of figure 2 following removal of superimposed load.

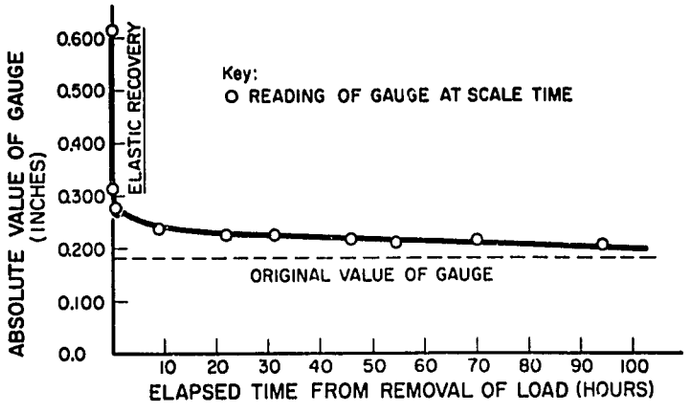


Figure 9: Comparison of structural performance of reinforced and unreinforced test beams (below).

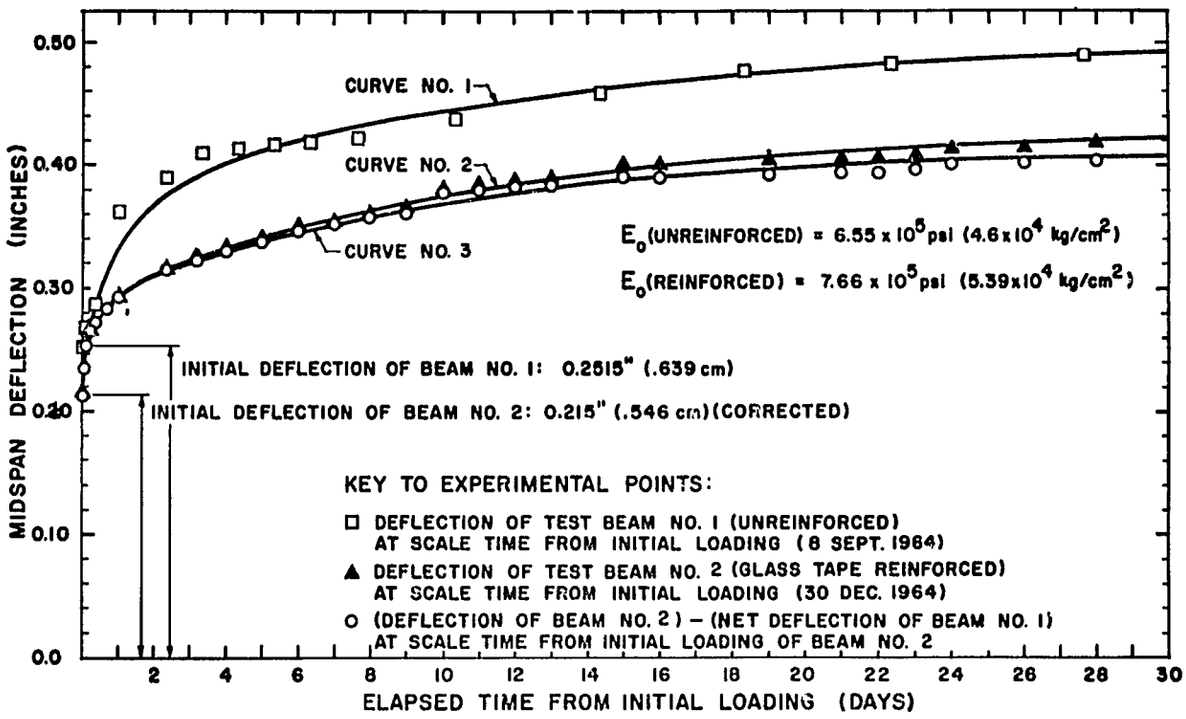


Figure 10a: Detail of floor joint of prototype structure.

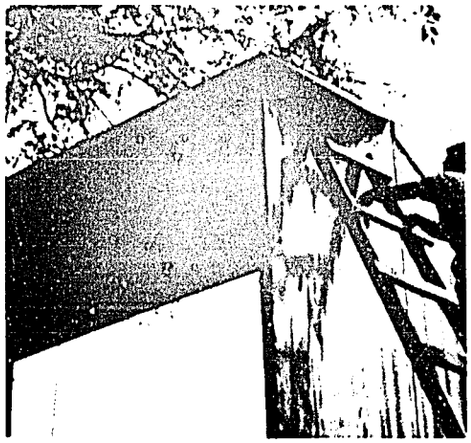


Figure 10b: Detail of roof joint of prototype structure.

instantaneous as well as delayed response, and the conclusion drawn is that reinforcement of structural sections with glass fiber tape appears to be an effective and economical means for gaining additional strength and stiffness.

c. Joint Design and Behavior. The structural system developed for the prototype structure has been conceived as a series of rigid, two-story bents, which do not require additional bracing or shear walls to resist horizontal loading. It was important, therefore, to achieve as much joint rigidity and as much continuity of the wall component as possible. Two questions have been explored: (1) Can a simple field joint be devised at the meeting of wall and roof which would afford a rigid connection at this point, without the need of accessory parts? (2) Can a similarly simple joint at the floor connection to the wall transmit loads only in shear to permit an unbroken wall component at this point?

Figure 10 illustrates the joints designed for these locations. Figure 10a illustrates the critical element in the shear joint between the floor member and the wall. One-half of a six-inch-wide glass fiber tape has been bonded to the wall component in such a way that the other half remains free to be bonded to the lower membrane of the floor. The floor component, whose ends are closed by a foam board or plywood diaphragm, is supported temporarily in place while the tapes are impregnated and cured. Finally, the space between the end of the floor and the wall is filled with polyester resin. The result is a joint which transmits all of the load either as tension in the glass tape or as shear between the paper and the foam.

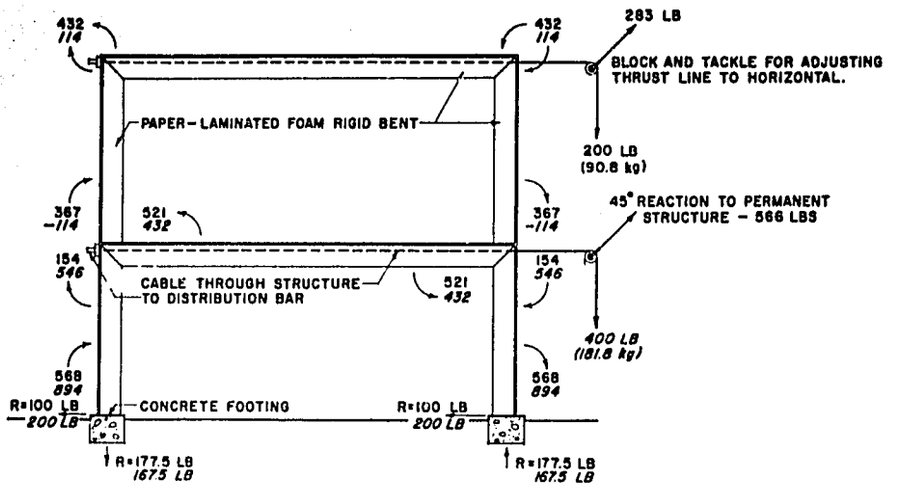
It has been found advisable to avoid details which depend only on the peeling resistance of the paper skin. A three-inch-wide analogy of the floor joint has been tested to failure, and found to have a theoretical ultimate capacity of 150 lb per inch of joint width (27.2 kg per cm). Critical delamination of the paper skin occurred at a value of 100 lb per inch (18 kg per cm), although there was no shear failure of the foam, even at ultimate load. Initial failure was observed in crushing of the foam at the edge of the lower membrane at a load of 50 lb per inch (9 kg/cm). While the detail is adequately strong for most purposes, and provides considerable evidence of failure prior to collapse, it cannot be recommended for reaction loads in excess of 32 lb per inch of joint width (6 kg per cm). In the prototype structure, this provides sufficient capacity for a total floor load of 45 lb per square foot (220 kg per m^2), a value felt to be adequate for this application.

The roof component shown in Figure 10b extends to the outside of the wall so that there is continuous bearing of the roof on the top of the wall. The top membrane of the roof component is extended approximately 18 inches (46 cm) beyond the wall face, folded down to cover the open end of the roof panel, and bonded to the exterior membrane of the wall with polyester resin. Initial tests of the triangular panel were performed with a similar end condition. The panel was subsequently retested with the ends simply supported, and comparison of data from the two tests indicates a joint rigidity equivalent to a minimum fixed end moment of 25%.

Further evidence of rigidity has been gained in the course of construction and testing of the prototype structure. Horizontal loads were imposed on the final bent of the structure prior to attachment to the sections already in place (Figure 11). Observed deflections were on the order of one quarter of the values predicted by taking into account the cantilevered resistance of the wall sections alone. Furthermore, measurements of the deflection of the wall section at various heights indicates a closer agreement with the ideal deflection curve for a rigid bent than with the curve for a pin connected, cantilevered bent. Figure 12 shows these relationships with a structural analogue of the actual bent. Of particular significance is the crossover point above the second floor line below which the measured deflections for the 200 lb load at the roof become less than those for the 400 lb load at the floor. The only analagous conditions of load which will produce this relationship in the ideal simple cantilever are those where the load at the roof line is from 1/6 to 1/3 the load at the floor.

In summary, the design of joints for structural systems of paper-laminated foam board can depend on the bonding and reinforcing properties of the impregnant polyester resin and glass fiber tapes to achieve any requisite degree of rigidity provided common sense and sound engineering principles are followed as with conventional materials.

d. Resin Impregnants and Paper. As has been suggested previously, the need for polyester impregnation of the paper skins of structural sections was established early in the investigation. The failure of sections due to buckling of the paper fibers was a critical problem to overcome before the full potential of the material could be realized. Polyester resin was found to be an adequate solution to this problem and was used in all of the early work.



NOTE: DEFLECTION MEASURED FROM BENTS ALREADY IN PLACE. INDUCED MOMENTS AT JOINTS, SHOWN THUS - ⌒100 - ARE IN FOOT-POUNDS AND ARE BASED ON JOINT RIGIDITY OF 100%. 200 AND 400 LB LOADS WERE IMPOSED SEPARATELY; ITALIC FIGURES RELATE TO 400 LB LOAD. NEGATIVE VALUE INDICATES THAT DIRECTION OF MOMENT IS OPPOSITE TO THAT SHOWN BY ARROW.

Figure 11: Diagram of loading and distribution of induced moments imposed on bent of prototype structure.

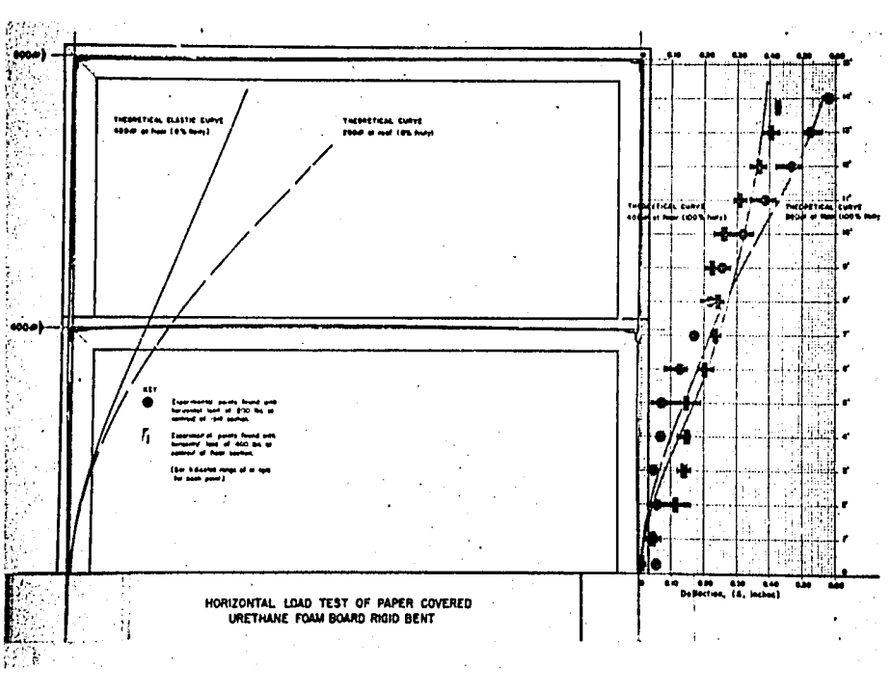


Figure 12: Comparison of lateral deflection of rigid bent of prototype structure with behavior of idealized rigid frame analogy.

It was felt, however, that other resins might be more effective than polyester, and also that different types of paper might have a significant effect on structural behavior. With the background already established, a series of experiments were conducted to provide some tentative answers to these questions.

The first step, as illustrated in Figure 13, was to test the triangular structure to failure in order to establish a foundation for future studies. The ends of the panel were redesigned and supported as shown in Figure 13a and 13b. On a span of 13.46 feet, the structure failed at a load equivalent to 70 lb per square foot (342 kg per square meter). Failure of the section was sudden and drastic, the result of a tension break at the point of maximum moment. With the loading as shown in Figure 13c, just prior to failure, deflection was recorded as 1.875 inches (average), and maximum tension at the extreme fiber was 5.2×10^3 psi (366 kg per cm^2). Figure 13d shows the tension side of the structure following failure. Most of the damage to the upper part of the section appeared to be the result of the loads falling, rather than of structural failure. There was no apparent failure of the upper skin.

Deflection of the panel was recorded at midspan for three increments of load, and intermittently as loading proceeded. Figure 14 is a tabulation of the deflections found at four points on the top skin. From this data the average value of the flexural modulus has been found to be 1.125×10^6 psi (7.91×10^4 kg per cm^2).

Following failure of the test panel, four samples were cut from the lower (tension) membrane. In addition, 36 other samples were prepared of a board with softer, shorter fibered pulp paper skins. These samples were divided into four groups of nine samples, six with the paper direction parallel to and three with the paper direction perpendicular to the long axis of the sample.

One of these groups was left unimpregnated, as a control. The other three groups were impregnated with various resins. The polyester resin used on the test structure was chosen for one group to enable comparison of the two types of paper. A humidity-curing urethane resin and a high-solids epoxy coating were selected for the other two groups. After impregnation and curing, the 36 new samples and the four from the test structure were trimmed to uniform size and prepared as shown in Figure 15.

The apparatus illustrated enabled rapid loading of the samples by pouring known weights of lead shot into the open tube and weighing the remainder at failure.

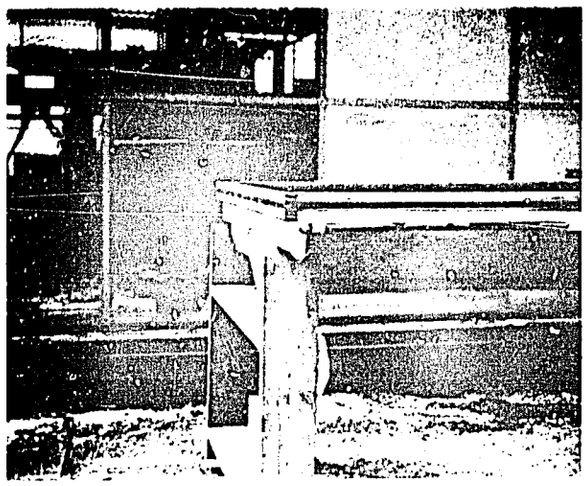


Figure 13a: Knife-edge and rocker support at one end of test panel.

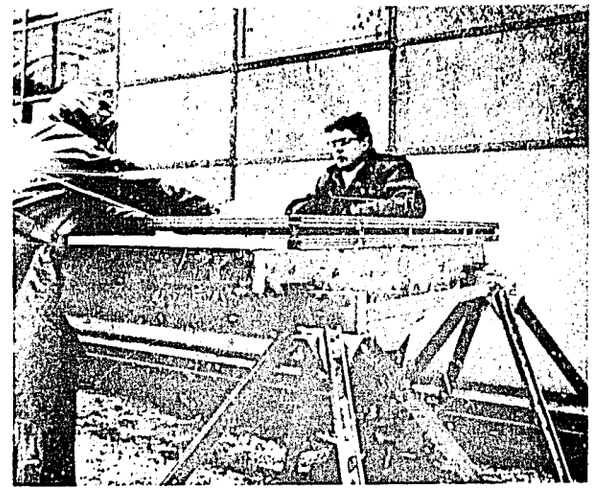


Figure 13b: Simple knife-edge support at opposite end of test panel.

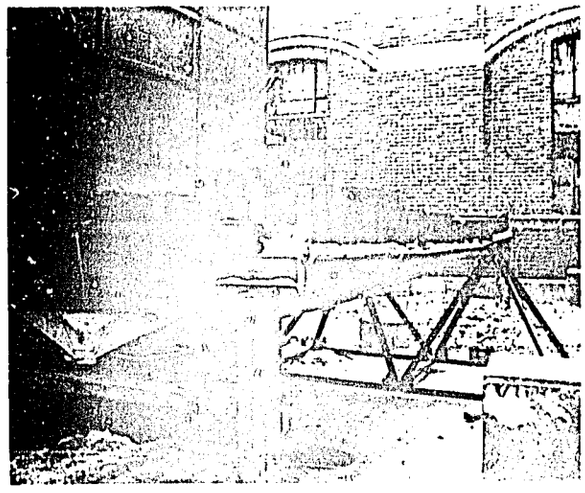


Figure 13c: Test panel under load of 4500 pounds (failure load).

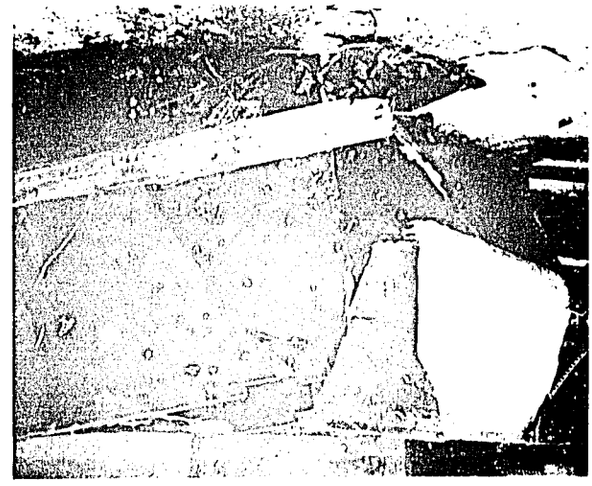


Figure 13d: Lower membrane of test panel following failure.

LOAD	NET MIDSPAN DEFLECTIONS			
	DIAL NO. 1	DIAL NO. 2	DIAL NO. 3	DIAL NO. 4
0 - 50 LBS./FT. (74.4 kg/m)	.175" (.445 cm)	.181" (.460 cm)	.186" (.473 cm)	.170" (.432 cm)
50 - 100 LBS./FT. (148.8 kg/m)	.195" (.495 cm)	.077" (.195 cm)	.086" (.215 cm)	.188" (.478 cm)
100-150 LBS./FT. (223.2 kg/m)	.190" (.483 cm)	.197" (.500 cm)	.198" (.503 cm)	.199" (.505 cm)
AVERAGE DEFLECTION	0.170" (.432 cm)			

Figure 14: Tabulation of midspan deflections recorded for fifty pound increments of load (test panel of figure 13).

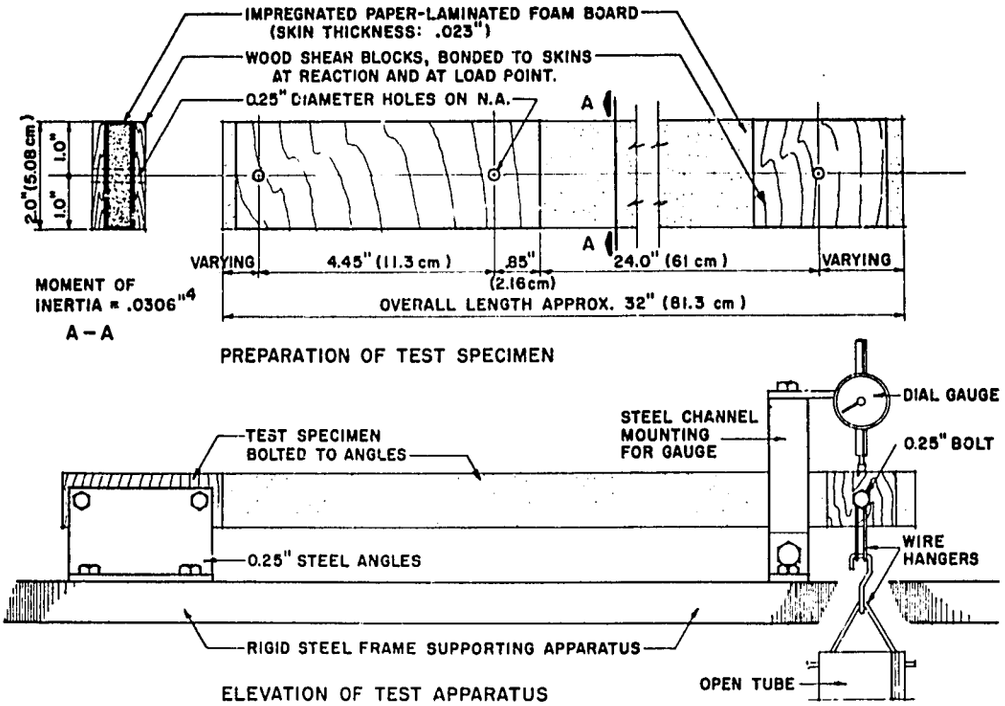


Figure 15: Preparation of impregnated board samples (top), and apparatus for testing samples to failure (bottom).

IMPREGNANT (TYPE AND AMOUNT)	TYPE OF PAPER AND FIBER DIRECTION RELATIVE TO LONG AXIS	1.10 LB(500 gr) DEFLECTION		LOAD AT FAILURE	
		AVERAGE	RANGE	AVERAGE	RANGE
NONE	PULP BOARD ∥	.286" (.726cm)	.260"-.300"	3.20 LB (1.455 kg)	2.80-3.47 LB
	⊥	——	——	2.39 LB (1.086 kg)	2.31-2.57 LB
POLYESTER (.045 LB/FT^2)	PULP BOARD ∥	.209" (.531 cm)	.190"-.220"	6.65 LB (3.020 kg)	5.03-7.47 LB
	⊥	——	——	5.00 LB (2.273 kg)	4.40-5.50 LB
	KRAFT FIBER ∥	.176" (.447 cm)	.155"-.187"	9.00 LB (4.085 kg)	7.86-9.69 LB
URETHANE (.081 LB/FT^2)	PULP BOARD ∥	.220" (.558 cm)	.190"-.270"	6.74 LB (3.057 kg)	6.03-7.02 LB
	⊥	——	——	5.62 LB (2.548 kg)	5.08-6.21 LB
EPOXY (.075 LB/FT^2)	PULP BOARD ∥	.222" (.564 cm)	.208"-.243"	5.75 LB (2.612 kg)	5.09-6.39 LB
	⊥	——	——	4.02 LB (1.827 kg)	3.79-4.15 LB

Figure 16a: Summary of results of testing impregnated samples.

IMPREGNANT	MAXIMUM FIBER STRESS AT FAILURE		FLEXURAL MODULUS, E ∥ TO PAPER DIRECTION
	PAPER DIRECTION ∥ TO LONG AXIS	PAPER DIRECTION ⊥ TO LONG AXIS	
UNIMPREGNATED	2600 psi (182 kg/cm^2)	1880 psi (132 kg/cm^2)	584,000 psi (41,000 kg/cm^2)
POLYESTER (PULP)	5270 psi (371 kg/cm^2)	3930 psi (276 kg/cm^2)	800,000 psi (56,200 kg/cm^2)
POLYESTER (KRAFT)	7190 psi (505 kg/cm^2)	——	940,000 psi (66,100 kg/cm^2)
URETHANE	5300 psi (373 kg/cm^2)	4420 psi (311 kg/cm^2)	759,000 psi (53,400 kg/cm^2)
EPOXY	4550 psi (320 kg/cm^2)	3160 psi (222 kg/cm^2)	750,000 psi (52,700 kg/cm^2)

Figure 16b: Effect of various impregnating resins on structural properties of paper-laminated polyurethane foam board.

Deflection of the samples was recorded at 1.1 lb (500 gr), which represented less than 10% of the deformation at failure load for all samples. Typical failure was in tension, close to the support, excepting for the unimpregnated samples, which exhibited "crippling," or separation of the paper layers on the compression side of the neutral axis.

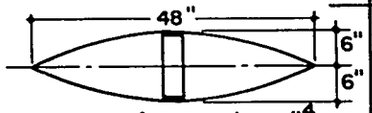
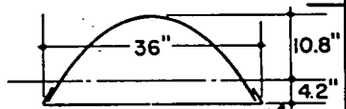
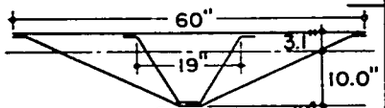
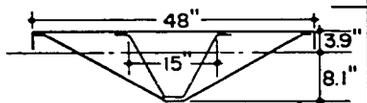
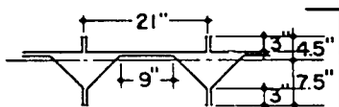
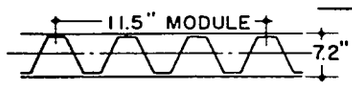
Figure 16a is a summary of the data collected from this experiment. From this data, the maximum fiber stress at failure for each of the groups of samples has been computed, as has the flexural modulus. This information appears in Figure 16b.

Inasmuch as the value of the glass fiber tape was not included in calculating the geometric properties of the test section, the values found for the modulus of polyester impregnated kraft fiber by the sample method and by the structural test appear to be in reasonable accord. As a method for comparing the properties of basically similar materials, it is felt to be adequate. However, comparison of the value found for ultimate strength of the polyester-kraft samples with the value found in the structural test indicates that this method cannot be relied upon as a test procedure to predict ultimate strength.

The relation between the effects of the various impregnants and the amounts used is of some economic importance; however, of greater significance for future work in this area is the effect of the change in the paper. Whereas the effect of the resin change is on the order of 15% difference in tensile strength and 10% difference in flexural modulus, the change from kraft fiber to pulp paper is found to reduce the tensile strength by nearly 27% and the modulus by nearly 15%. The conclusion drawn is that research is warranted to develop suitable structural papers with high fiber strength, high modulus of elasticity, and exceptional impregnating characteristics.

OTHER STRUCTURAL SYSTEMS

One of the objectives of the plastics research program has been to conceive a variety of structural alternatives within a given frame of reference rather than to propose an ideal structure. Selection of a particular system will depend on the conditions prevailing in each application. It has been considered the function of this project also to stimulate others in the development of structural systems utilizing the foam plastics. To this end, Figure 17 illustrates and summarizes the potentials of all of the structural units described thus far, and also presents two structural variations not covered in this paper. Each of these sections has

SECTION GEOMETRY	COVERING RATIO COVERAGE:AREA BOARD	FACTORY JOINTS PER FOOT OF WIDTH	FIELD JOINTS PER FOOT WIDTH	SPANNING* CAPABILITY	REMARKS
 SECTION A: $I = 90''^4$	1:2.1	0.5	0.25	—	CAPACITY LIMITED BY BUCKLING. POTENTIALLY ATTRACTIVE IF REINFORCED. SHIPS FLAT.
 SECTION B: $I = 232''^4$	1:2.5	0.66	0.33	—	FAILURE OF TEST BENT RESULT OF INADEQUATE IMPREGNATION. ULTIMATE CAPACITY UNKNOWN.
 SECTION C: $I = 155''^4$	1:2.7	1.00	0.20	12.4'	SECTION DESCRIBED IN TEST HAD TOP SKIN REINFORCED WITH FOUR PLY POLYESTER-PAPER. ACTUAL $I = 192''^4$
 SECTION D: $I = 112''^4$	1:3.0	1.25	0.25	13.5'	SECTION C AS MODIFIED FOR PROTOTYPE STRUCTURE. REQUIRES USABLE BOARD 54" WIDE.
 SECTION E: $I = 21''^4$/SECT.	1:2.9	1.14	2.28	9.2'	GOOD FOR NARROW BOARD MACHINES. SHIPS FLAT.
 SECTION F: $I = 16''^4$ PER FT. WIDE	DEPENDS ON ANGLE OF WEB. 1:4.0 AS DETAILED	DEPENDS ON ANGLE OF WEB. 2.39 AS DETAILED	DEPENDS ON WIDTH OF PANEL.	15.2'	WIDE BOARD MORE EFFICIENT, BUT NOT CRITICAL. CAPACITY MAY BE LIMITED BY COMPRESSION.

* SPANNING CAPABILITY IS PREDICATED ON A UNIFORMLY DISTRIBUTED LOAD OF 70 LB/FT^2 AND THE FOLLOWING ASSUMPTIONS:

1. AT THIS SPAN, TENSION FAILURE MAY BE EXPECTED WITH AN ULTIMATE STRESS NO GREATER THAN 5200 psi.
2. PAPER LAMINATED FOAM BOARD IS NOMINALLY 0.375" THICK, 2.5 LB/FT^3 DENSITY AND .023" KRAFT LINER SKINS.
3. ALL BOARD SURFACES ARE IMPREGNATED WITH AT LEAST .045 LB/FT^2 OF POLYESTER RESIN.
4. ALL LONGITUDINAL JOINTS ARE REINFORCED WITH GLASS FIBER TAPE.

DESIGN LOADS AND SPANS WILL MOST LIKELY BE GOVERNED BY DEFLECTION; REGARDLESS, AT THE SPANS INDICATED, ACTUAL LOADS SHOULD BE NOT MORE THAN 35 LB/FT^2 AND OF SHORT DURATION.

Figure 17: Evaluation of various structural systems utilizing paper-laminated polyurethane foam board as the principle structural material.

distinct advantages for use in various situations, and each has its long list of unresolved shortcomings, some of which are yet to be discovered. Only the first four have been tested to the extent already described. The performance of sections E and F has been predicted on the basis of data collected from the other sections. What their actual performance will prove to be is an open-ended question.

Section E is conceived as a system for roofing a space where walls and intermediate supports might be constructed of other materials. It is essentially a field fabricated system designed to be shipped economically from the factory. Section F, on the other hand, is conceived as a high capacity long span decking or wall panel which might be used over trusses or as the total structure of a building. Field fabrication of this section would be impractical, but its versatility lends itself to high volume factory production. All of the systems are based on a concept of a continuous, linear production system which includes the scoring, folding and bonding process of the material.

CONCLUSION

The prototype structure erected at The University of Michigan has in the course of its construction already served its principle function. It has demonstrated the feasibility of designing a structural system utilizing paper-laminated urethane foam board as a primary structural material. It has also provided overall direction to a testing program which has accompanied its construction. At this time, it is virtually completed, and is to be occupied as a temporary office space by members of the project staff in order to evaluate the system from the standpoints of weathering, maintenance, comfort and the like. Its cost has been proven low in comparison with conventional structures, even including, as an item of cost, the hand labor required where machines could have done the job better and more economically. Its potential as a low-cost system of construction is felt to be great. Ultimately, it is hoped that the building will give way to another experimental structure developed, as this one was, from the unique characteristics and properties of a new and challenging material.

Acknowledgments

This paper is a product of research on the "Structural Uses of Foam Plastics for Housing in Underdeveloped Areas," conducted at the Architectural Research Laboratory of The University of Michigan and sponsored by the Agency for International Development of the United States Department of State. This research is being conducted with my colleagues; Harold J. Borkin, Robert M. Darvas, James L. Haecker, C. Theodore Larson, Willard A. Oberdick, and Stephen C. A. Paraskevopoulos, project director.

We should like to express our sincere appreciation to Union Carbide Corporation for having contributed the materials for the two-story structure and the test bents, as well as for information and cooperation extended to the project. Also, we wish to acknowledge the contributions to this work of Atlas Chemical Industries, Raymond Chemical Company, and Armorlyte Corporation.

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2. Stephen C. A. Paraskevopoulos, "Architectural Research in the Use of Foam Plastics," Journal of Cellular Plastics, Vol. 1, No. 1, January 1965, pp 132-142.

3. J. M. Buist, "Advances in Urethane Rigid Foam Processes and Applications," Ibid, pp 101-120.

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APPLICATION OF MINIMUM STRUCTURE TO CELLULAR PLASTICS

Willard A. Oberdick, Professor of Architecture

Summary

The concept of structural membrane or minimum structure has been investigated as part of research intended to explore the parameters of utilizing cellular plastics for the construction of shell structures. Experimental work to date indicates that it is feasible to use these materials for primary structural applications provided that strict discipline is imposed in the selection of the structural shape.

APPLICATION OF MINIMUM STRUCTURE TO CELLULAR PLASTICS

Willard A. Oberdick, B. Arch., AIA, Professor of Architecture
(Architectural Research Laboratory, The University of Michigan, U.S.A.)

The topic is related to research on the structural use of cellular plastics conducted by the Architectural Research Laboratory of The University of Michigan and sponsored by the Agency for International Development (AID), United States Department of State. The objective is to explore the feasibility of using plastics and especially cellular plastics, for the resolution of housing problems in the underdeveloped areas of the world[1].

A preliminary feasibility study conducted by the project staff[2] resulted in the selection of various structural systems for detailed study and evaluation[3]. The intent is to investigate cellular plastics, either as primary structural materials or for their contribution in resolving structural problems in conjunction with other materials[4]. Two of these systems involve the creation of shell structures either by forming and rigidizing flexible membranes or by spraying foam on a form-giving device.

Because of the nature of cellular plastics, these structural systems depend on structural requirements necessitating the use of optimal structural shapes. To meet these requirements, the concept of the structural membrane or minimum structure has been explored. The nature of the problem, the related experimental work, and the principles of minimum structure are the subjects of this paper.

STRUCTURAL CRITERIA FOR CELLULAR PLASTICS

Reasonable cost in place, maintenance of desired mechanical and physical properties under adverse conditions, and limited time-related deformations, are essential requirements for any structural material. In terms of mechanical properties a major positive feature with regard to plastics is their relatively high strength-to-weight ratio. Problem areas, especially relating to cellular plastics, involve low service temperatures, low moduli of elasticity and high creep characteristics under sustained stress. However, the flexibility offered in formulating and producing plastics materials suggests the possibility of capitalizing on the positive aspects and of minimizing the effect of less desirable ones[5].

The mechanical properties of polyurethane foams are related to density, specific method of production, as well as to temperature and humidity conditions

during production. A change in density from 2 to 4 pounds per cubic foot will result in a 2.7 increase in the compressive strength and the modulus of elasticity[6]. In general, the same formulation produces a stronger material when poured rather than sprayed. Spraying usually results in a highly elongated cell structure in the direction of the rise. If the foam is required to perform in the other direction, as may be the case in shell structures, there could be a possible 50% decrease in strength. The quality of spray-produced foam greatly depends on environmental conditions and the field application of this system becomes increasingly difficult at higher densities. However, improvements in formulation and equipment may very well eliminate some of these shortcomings.

The rate of creep in cellular plastics is far in excess of that of other structural materials. A factor of 6.5 has been used in our laboratory to estimate the additional deformation over a 20-year period. Admittedly this is an extended projection, as no long-term experience is available to substantiate the figure or take into account any unforeseen changes in physical properties.

This estimate for creep, along with the low modulus of elasticity, represents a total 10% deformation for a stress of 13 psi for a 2.8 pcf rigid polyurethane foam. Such movements result in unacceptable deflections in bending or require such depth-to-span ratios as to make the use of cellular plastics uneconomical. The solution lies both in the selection of applications where large movements are tolerable and in limiting stress for a compatible unit deformation. For a 20 foot arch with a 4 foot rise a 2½% unit deformation would result in a 4 inch deflection at the crown. Such a deflection may well be tolerable in an arch but not acceptable in a flat bending structure. It is therefore logical that cellular plastics be used in such shapes where minimum stresses will result from sustained loads and where larger movements than normally associated with building structures can be accepted.

MINIMUM SURFACE

The goal of optimization in structure is expressed in the concept of a minimum surface. Soap films with surface of equal tension provide such surfaces over continuous edge systems. A wire frame bent in the form of a hyperbolic paraboloid saddle shell yields this geometric surface with a soap film. Prediction of the nature of a minimum surface is difficult in certain situations. The surface of revolution shown in Plate 1 is such that a point of instability is reached as the two wire frames are separated. At the conjugate or limiting stable point the film becomes discontinuous. The critical point has been predicted mathematically by Mary Sinclair[7]. The critical spacing of the two equal rings is 82% of their radius

and the radius of the interior film ring is 43%. Further, the angle between the tangents to the two catenary curves is 120 degrees.

The form shown in Plate 2 is similar to that in Plate 1 and would be identical if one of the supporting rings were the size of the film ring and the spacing were one-half of that in Plate 1. In the case of the single surface of Plate 2, if the two rings are gradually separated stability of the surface is maintained as long as the angle between the tangent to the curve and the perpendicular to the ring is greater than zero.

Comparison of the shape obtained by a soap film in Plate 2 and the shape obtained by stretching flexible foam in Plate 3 indicates that the latter is not a minimum surface. Partial slitting of the flexible foam in Plate 4 indicates a further effect of reducing or eliminating the hoop forces, which in the case of minimum surfaces are equal to the radial forces. The inequality of stress within the stretched membrane is further evidence of the variance of this form from that of a minimum surface since equality of forces is a characteristic of the liquid film.

A material such as rubber maintains nearly constant volume during extension, in which case Poisson's ratio approaches 0.5. For a rubber membrane restrained on two opposite sides and stretched in the perpendicular direction, the restraining lateral force would be 50% of the pulling force. Tests with flexible polyurethane foam have shown a Poisson's ratio of 0.64, which indicates a decrease in volume when in tension. Values approaching 1.0, a condition of equal stress, are impossible to obtain since this would mean a 100% decrease in volume. It can be concluded that elastic membranes cannot be used to precisely define a minimum surface.

MINIMUM STRUCTURE

Concepts of optimal structural form for two dimensional structural components have long been in common use in the case of the arch and suspension cable structures. The funicular or string polyon for the loaded cable and the pressure line of thrust of the arch are based on equilibrium of forces within limiting assumption of support and material quantity. The extension of these concepts to curved surfaces or three dimensional structures can be characterized as that of the structural membrane or minimum structure.

Frei Otto and Rudolf Trostel have developed concepts of the "Sail-Shells," stiffened pneumatic surfaces, and other tensile-stressed forms[8,9]. Application of

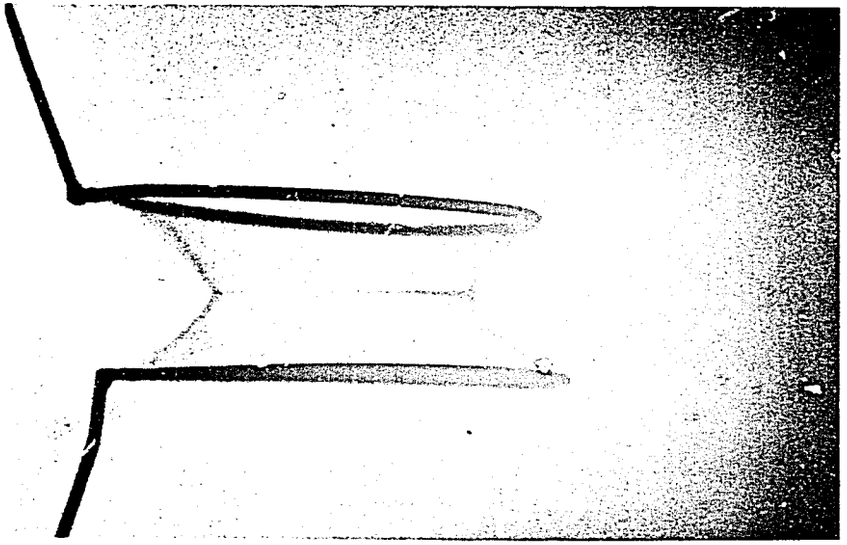


Plate 1 Soap film - Minimum surface of revolution with a transverse plane.

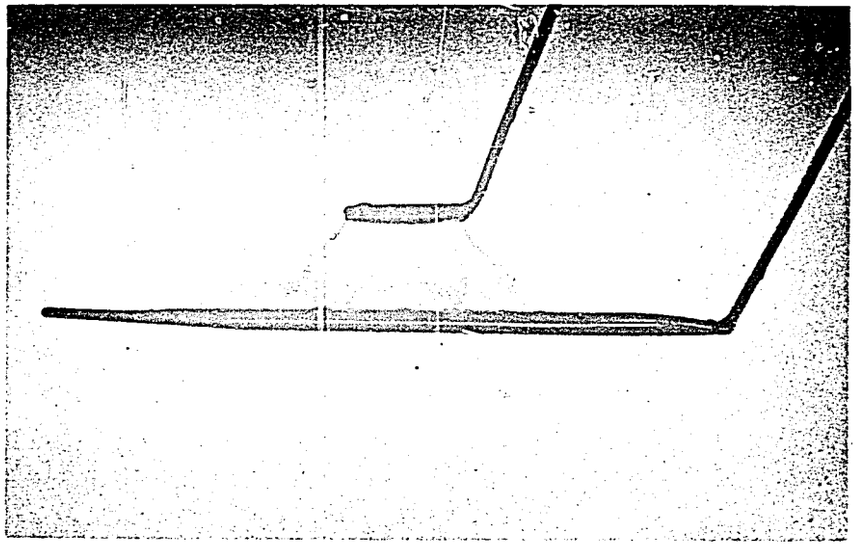


Plate 2 Soap film - Single minimum surface.

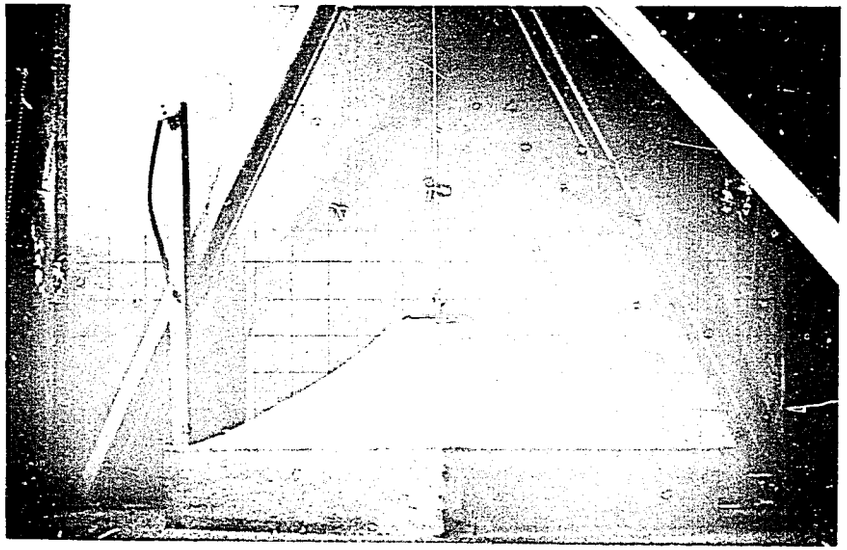


Plate 3 Elastic membrane of flexible polyurethane foam.

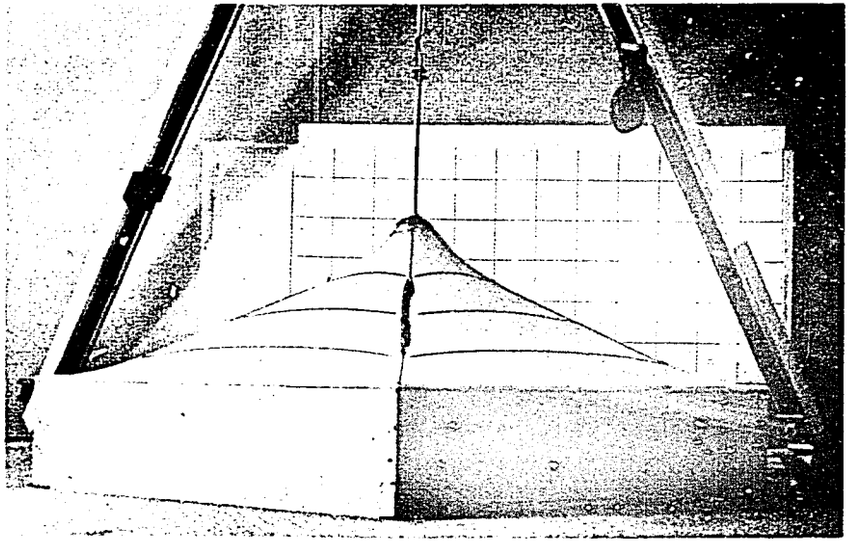


Plate 4 Elastic membrane with discontinuous hoop fcrces.

such concepts to concrete shells has been proposed by Kolbjorn Saether[10]. The authors of these studies conceive of structural form as being determined by the equilibrium of forces rather than geometry and, further, in certain applications have assumed that a reversal of the tensile-stressed structure results in a parallel reversal of stresses from tension to compression.

One might theorize that the concept of the structural membrane would be synonymous to that of the minimum surface. The hyperbolic paraboloid in Plate 5, figure a, is a minimum surface and in a case of uniform load in horizontal projection acting parallel to the reference axis, it is subject to uniform stresses over the entire surface. However, both tension and compression exist in the rigid surface and as such it is not easily determined on the basis of an equilibrium of forces in terms of a structural membrane.

Equilibrium can be determined by a series of cables supported by inclined edges, as shown in Plate 5, figure b. The cables for a uniform load in horizontal projection would be second degree parabolas. A membrane could be used if stretched in a direction perpendicular to the carrying direction, as indicated in Plate 5, figure c. This in effect is the type of prestressing or tie down that has been used in certain suspended roof structures to prevent flutter. The umbrella shell, shown in Plate 5, figure d, consisting of four warped parallelograms or hyperbolic paraboloids does meet the desired design criteria for cellular plastics; i.e., low uniform stresses. However, stiffening ribs sufficient to support the shear developed at the edge of the quarter panels would be required. This would require a change in concept from that of a monolithic umbrella to that of a combination of four components[11].

Structural membranes may be classified as either anticlastic or snyclastic depending on the curvature of the surfaces. The umbrella form similar to that of the liquid film in Plate 2 is an anticlastic surface. A soap bubble and certain pneumatic structures are synclastic surfaces.

Draped cables under self-weight or superimposed loads are subject to variations in stress along the cable. In a similar manner structural membranes will be subject to variations in surface or membrane forces. This would infer that thickness of the material will need to be varied over the surface to maintain the requirement of equality of stress. Further, this characteristic of a structural membrane points to the basic difference between the structural membrane and the minimum surface.

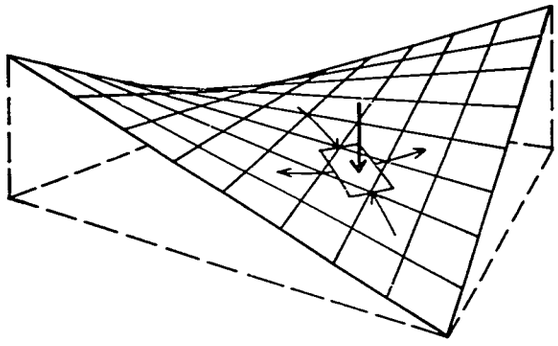


FIGURE a

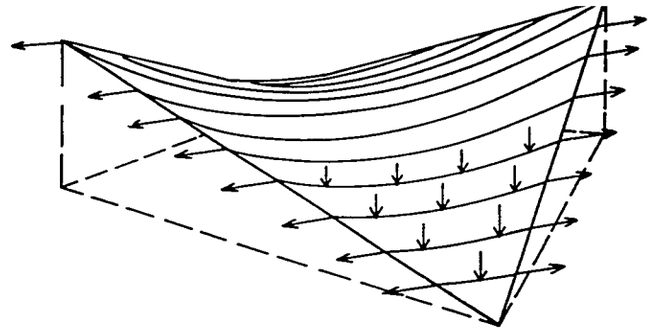


FIGURE b

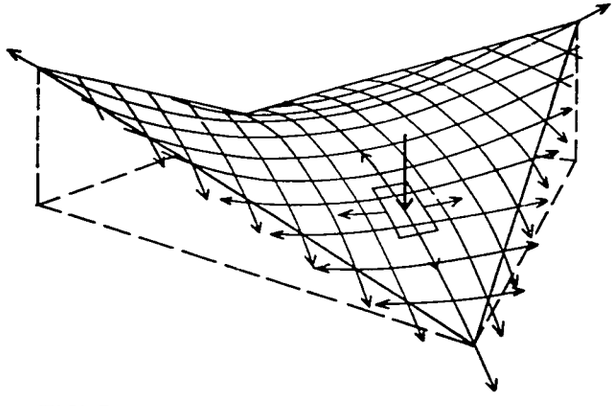


FIGURE c

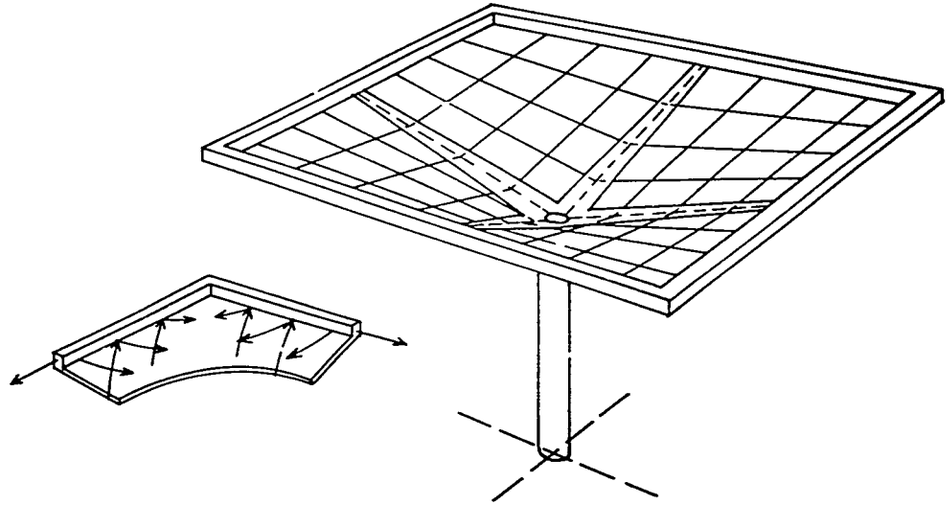


Plate 5 Comparison of hyperbolic paraboloid structures.

In certain cases of anticlastic structural membranes it would be possible under certain specific loads to obtain the same form as that of the minimum surface. This would have significance as to the optimum quantity of material used. However, it would have no significance as to the nature of the system of forces.

ANTICLASTIC STRUCTURAL MEMBRANES

An examination of the minimum surface of the liquid film in Plate 2 indicates a possibility for a structural form which could be obtained with plastic foam. The low modulus of elasticity and the high rate of creep for the foam plastics imposed strict discipline on the selection of a structural form. As a consequence, it was assumed that all forces and loads that would influence the creep of the foam should be taken into account in the equilibrium condition of the structural membrane.

The factors influencing the structural membrane were explored through structural models of elastic membranes. Flexible polyurethane foam was used for these experiments. Adhesives with similar stretch characteristics as the flexible foam permitted the fabrication of large uniform sheets from this material. The structural models were considered as isolated units and not as representing any projected prototypes. As a consequence loads were selected for a study of membrane behavior and do not necessarily correspond to projected service loads.

In the first study a 6 foot polyurethane flexible foam diaphragm was suspended with calibrated springs within an adjustable metal frame, as indicated in Plate 6. The center point was raised sufficiently to result in only horizontal forces in the edge supports. The turnbuckles provided adjustment so that the wood frame edge members were not in contact at the corners, indicating that all of the horizontal forces were taken by the springs. The membrane in Plate 6 is in equilibrium in reference to the weight of the wood edge strips and the weight of the foam.

In another test series the hexagonal edge members were made continuous so that the membrane and superimposed loads were maintained in equilibrium by the vertical spring in the center. Plate 7 indicates an equilibrium condition for an equivalent uniformly distributed surface load and an edge load. This is designated as type 1A on Plate 10. The relative change in shape for a condition of a predominant edge load is shown in Plate 8 and designated as type 1B on Plate 10.

The difference in the rise of the structural membranes is a function of the stress strain relationship of the membrane as well as that of the system of forces.

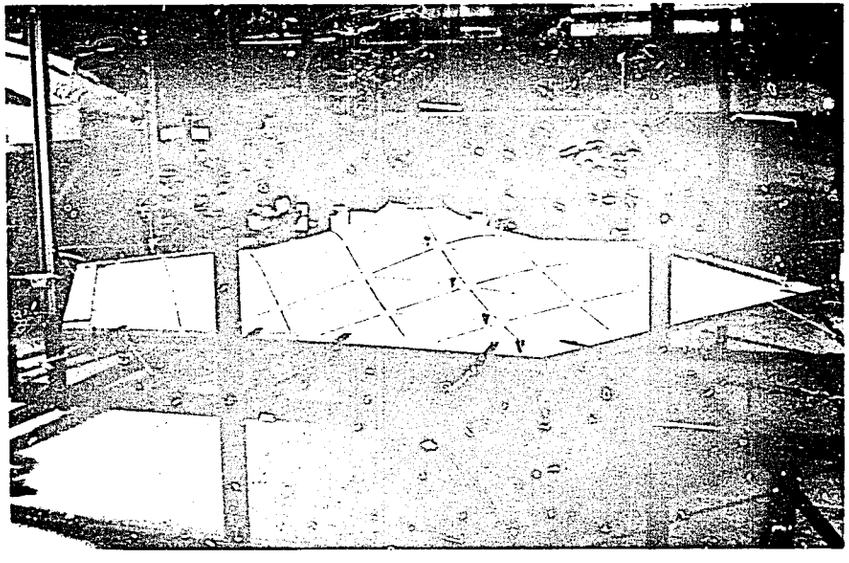


Plate 6 Anticlastic structural membrane in equilibrium with spring supports.

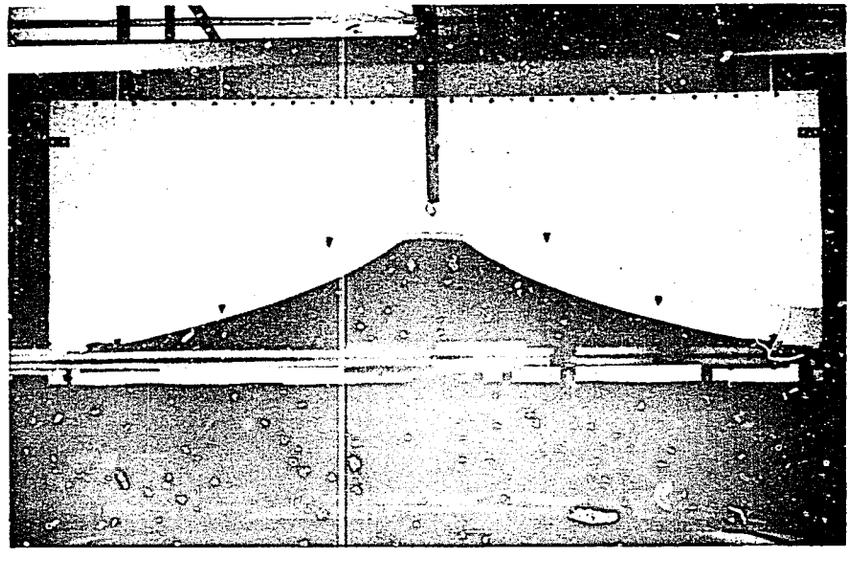


Plate 7 Structural membrane under surface load in equilibrium with a continuous hexagonal edge member.

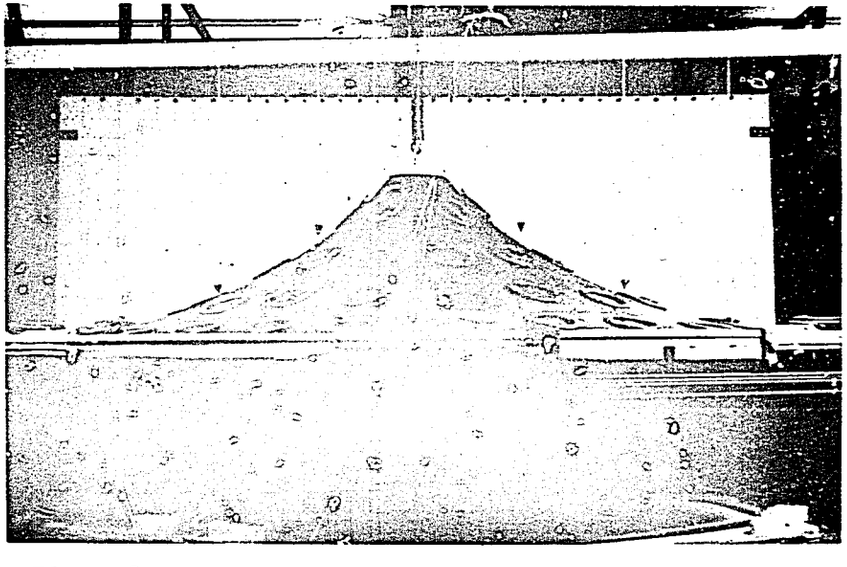


Plate 8 Structural membrane under edge load in equilibrium with a continuous hexagonal edge member.

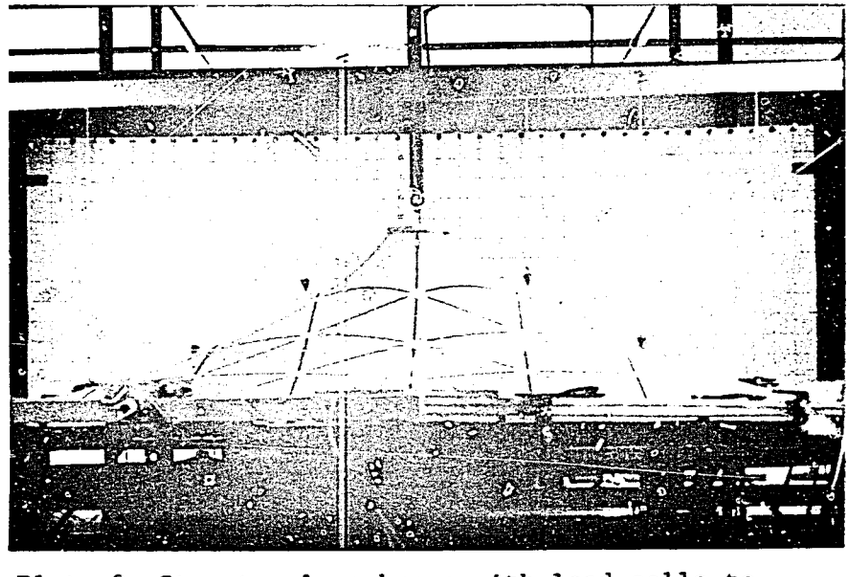


Plate 9 Structural membrane with load cells to measure bending in the hexagonal edge member

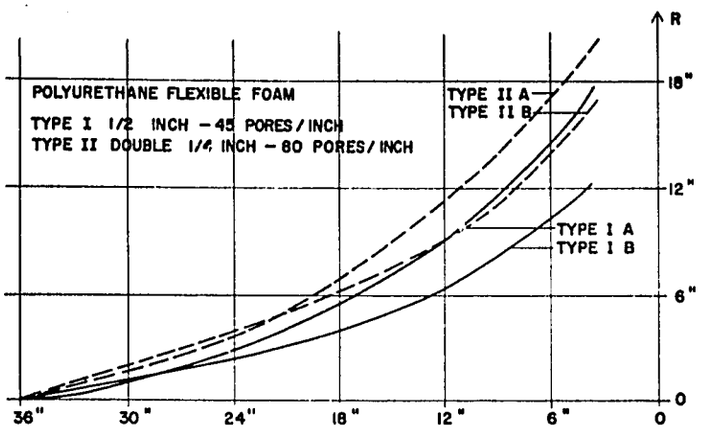
This is illustrated by comparing Plates 8 and 9 both of which are supporting the same edge loads. The membrane shown in Plate 8 consists of a 45 pores, ½ inch thick flexible polyurethane foam sheet, designated as type I on Plate 10, and the membrane in Plate 9 consists of two 80 pores, ¼ inch thick sheets, designated as type II in Plate 10. Type II exhibited a higher Poisson's ratio and a higher rate of creep than did type I. The plumb bob elevations in Plate 9 are those of the type II membrane subjected to surface load in addition to edge loads. The creep resulting from the higher stresses accounts in part for the particular form. The plumb bob elevations shown on Plates 7 and 8 are the same as those shown on Plate 9.

Surface strain was determined by measuring the relative extension of a grid mark on the surface on the membrane. The force strain relationships were determined by a series of tests one of which is illustrated in Plate 13. Curves for two of the materials are noted as figure a in Plate 11. The term "unit force" is used rather than "stress" since the numerical values are in pounds per inch width for the actual thickness. Poisson's ratio is determined by comparing unit elongation and unit lateral contraction. The values are noted parenthetically in Plate 11, figure a, and are shown to vary with the nature of the skin and the magnitude of the unit force.

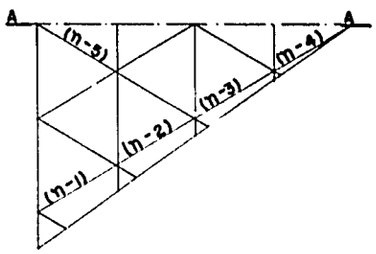
The specific values of radial and hoop unit forces tabulated on Plate 10 are determined from the graphs designated as figures b and c on Plate 11. As an example, the unit measured radial strain 0.340 at n-4 on membrane 1A results in a radial force of 3.5 and a hoop force of 1.9 pounds per inch. The identifying line in figure b illustrates this condition. The specific curves in figures b and c were determined from the formulae noted on Plate 11 by assuming that the net hoop strain was essentially zero.

On one of the structural models, moments were determined experimentally at the midpoint of one of the edges and at one of the corners. Load cells were secured to the top and bottom as shown on Plate 9. Moments were recorded at both locations for both surface and edge loads. The maximum moment, 46 inch pounds, was recorded at the midpoint of the side for the surface load. The existence of this moment can be explained by the slight difference between the slopes of the surface at the corner and at the midpoint of the edge.

A semi-graphical approach of successive corrections was used to corroborate the test results. This approach is illustrated on Plate 12 and refers to the



PROFILE AT SECTION A–A

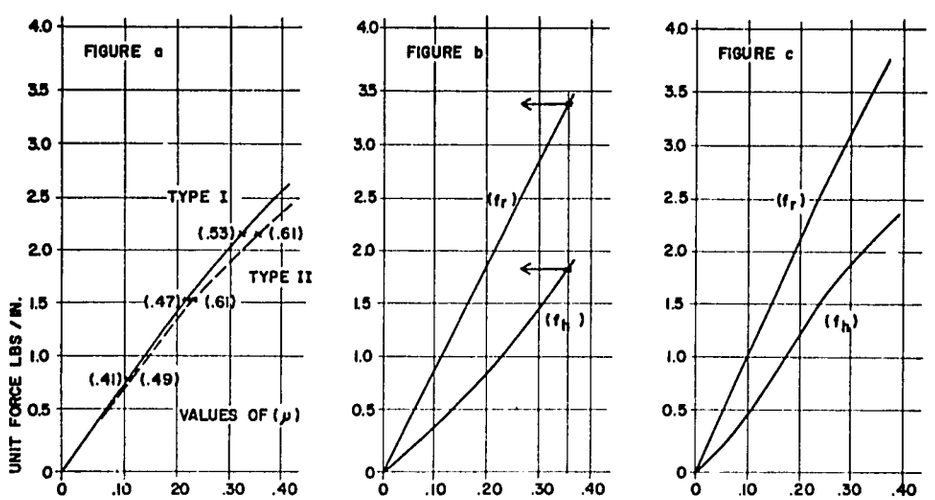


REFERENCE PLAN

TYPE	I A			I B			II A			II B		
SELF WEIGHT	2.45 LBS			2.45 LBS			1.91 LBS			1.91 LBS		
EDGE WEIGHT	24.84 LBS			24.84 LBS			24.84 LBS			24.84 LBS		
SURFACE LOAD	56.88 LBS			—			56.88 LBS			—		
MEASURED R	84.17 LBS			25.4 LBS			82.2 LBS			25.4 LBS		
	ϵ_r	f_r	f_h	ϵ_r	f_r	f_h	ϵ_r	f_r	f_h	ϵ_r	f_r	f_h
POSITION n – 1	.014	.15	.05	.005	–	–	.005	–	–	–	–	–
n – 2	.063	.58	.20	.019	.17	.05	.062	.58	.30	.029	.35	.14
n – 3	.115	1.0	.40	.057	.45	.18	.086	.87	.40	.052	.50	.25
n – 4	.340	3.5	1.9	.135	1.2	.47	.420	4.05	2.55	.176	1.85	1.05
n – 5	.034	.35	.10	.01	.06	.03	.017	.20	.07	.001	–	–

NOTE ϵ_h – HOOP STRAIN ASSUMED TO BE ZERO
ϵ_r – RADIAL STRAIN – MEASURED INCHES PER INCH
f_r – RADIAL UNIT FORCE POUNDS PER INCH WIDTH
f_h – HOOP UNIT FORCE POUNDS PER INCH WIDTH
f_r AND f_h ARE DETERMINED FROM THE GRAPHS ON PLATE II

Plate 10 Test results of anticlastic structural membrane models.



STRAIN (ϵ) – INCHES/INCH
TEST SPECIMEN

RADIAL STRAIN (ϵ_r) – INCHES/INCH
TYPE I – MEMBRANE
1/2 INCH 45 PORES/ INCH

RADIAL STRAIN (ϵ_r) – INCHES/ INCH
TYPE II – MEMBRANE
DOUBLE 1/4 INCH 80 PORES/ INCH

NOTATIONS
f_r = RADIAL FORCE (LBS/INCH WIDTH)
f_h = HOOP FORCE (LBS/INCH WIDTH)
ϵ_r = RADIAL STRAIN (INCHES/INCH)
ϵ_h = HOOP STRAIN (INCH/INCH)
E = MODULUS OF ELASTICITY (LBS/INCH WIDTH)
μ = POISSON'S RATIO (ϵ_x/ϵ_y)

FORCE STRAIN IN TWO PERPENDICULAR DIRECTION

$$f_r = \frac{(\epsilon_r + \mu\epsilon_h)E}{1-\mu^2} \quad \text{if } \epsilon_h = 0; \quad f_r = \frac{\epsilon_r E}{1-\mu^2}$$

$$f_h = \frac{(\epsilon_h + \mu\epsilon_r)E}{1-\mu^2} \quad \text{if } \epsilon_h = 0; \quad f_h = \frac{\mu\epsilon_r E}{1-\mu^2}$$

Plate 11 Force strain properties of flexible polyurethane foams.

structural membranes of Plates 7 and 8. The method consists of essentially three steps:

1. The determination of loads and the corresponding line of thrust for an independent radial section. This neglects the effect of hoop forces.
2. The determination of the corresponding unit hoop force using the radial unit surface forces measured from the ray polygon (not shown) and force strain relationships shown in Plate 11. The horizontal radial projected force is determined on the basis of circumferential pressure.
3. The drawing of a new line of thrust with the additional horizontal forces. The specific latter values noted on Plate 1_2 are for the second correction.

Steps 2 and 3 are repeated until the unit radial surface force assumed in Step 2 corresponds closely to the value in Step 3.

The curves determined by this method are compared to the profiles obtained from the structural membranes shown on Plate 7 and 8. The profile of the soap film shown on Plate 12 was obtained by a projection of a photograph of the actual liquid film of Plate 2 to the scale shown. The variation is consistent with the earlier comments on the elastic membrane which is essentially the condition of the skin weight and the edge load. However, the curve is much closer to that of the structural membrane obtained with surface load. A projection of the same film profile distorted proportionately for the vertical scale appeared to correspond closely to the curves shown. The semi-graphical method developed to corroborate the test results is proposed as an approximate method of determining the profile of an anticlastic structural membrane in cellular plastic for any given conditions of load dimensions and mechanical properties.

From the nature of the structural models it may be concluded that the elastic membrane can be used as a form-giving device in a larger scale prototype. However, the structural form of the rigid material will only be a structural membrane within the limits of compatibility of Poisson's ratios of the forming membrane and the rigid foam. The semi-graphical analysis could be used to define the form and formulate a set of parameters for the elastic membrane to achieve the minimum structure in cellular plastics

Examples.

As noted in the introduction, one of the structural systems selected for the study of its potentials involves the forming and rigidizing of flexible membranes. Flexible polyurethane foam was selected for this purpose. The method involves a

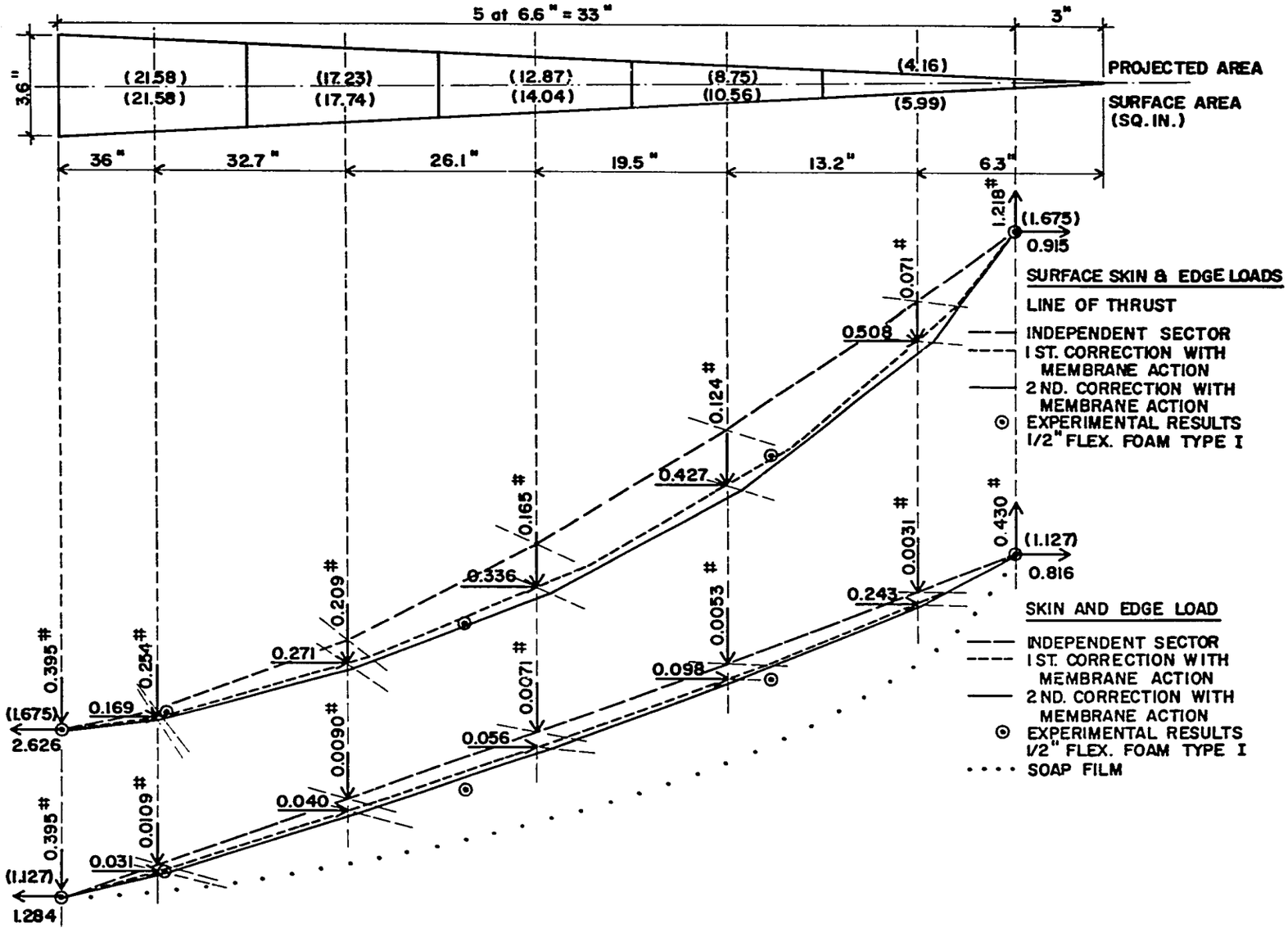


Plate 12 Semi-graphical analyses of the structural membrane models, Reference Plates 7, 8 and 10.

flexible foam sheet which is impregnated with a resin or another material, stretched or otherwise formed to the desired shape, and allowed to rigidize. This concept can take two directions: (1) the rigidized component becomes a primary homogenious structural member requiring only secondary applications, such as exterior surface treatment; and (2) the rigidized foam becomes a form-giving device for double-curved surfaces and can serve as the core material for sandwich-type construction.

Because of time limitations and other requirements related to our AID-sponsored research it was decided to follow the second direction for the erection of a prototype structure. For this purpose a reticulated flexible polyurethane foam was chosen. Through the reticulation process, the membranes of the foam cells are removed, resulting into a linear cell structure. Reticulation reduces the surface to be impregnated and as a result requires less impregnating resin.

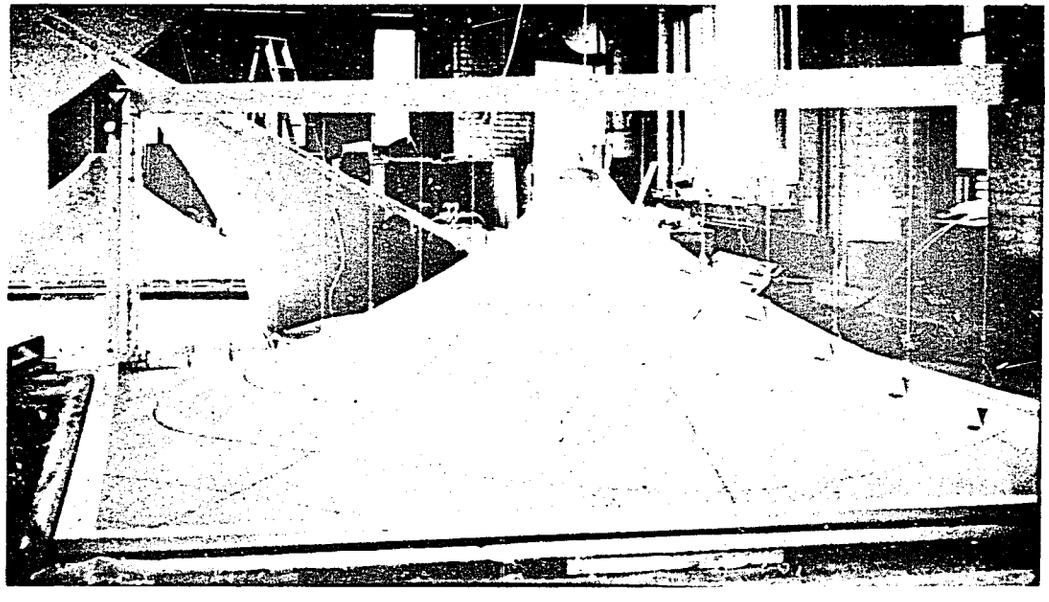
The elastic membrane was used in forming the 12 x 12 foot umbrella-shaped components shown in Plate 14. A one-inch thick reticulated flexible foam was impregnated with a moisture-curing urethane resin, attached to a wooden frame and stretched into shape on a template by lifting the membrane at the center. After the urethane resin had cured, chopped glass fibers and polyester resin were sprayed on the one surface to the desired thickness. After this surface had cured the shell was removed from the template and the skin applied to the other side in a similar manner. Completed shells are shown in the background of Plate 14.

Although the form requirements are not as critical for a sandwich-type shell, the concept of minimum structure was considered in the formation of these shells. Through this method of forming the weight of the impregnating resin, the restraining force at the column location and the tie down restraint at the edges all parallel the system of forces essential for the equilibrium of the structural membrane.

In order to explore the feasibility of using the flexible foam only as a form-giving device, without relying on it for shear transfer between skins, one of the shells was constructed without impregnating the foam. However, small spots in the vicinity of the corners were impregnated with a spray gun to stiffen the relatively flat areas. Both types are presently undergoing comparative testing to ascertain the relative performance of each.

The umbrella-shaped components are designed to be mounted on top of tubular columns and linked to each other. Arrangements were made for a first such structure to be erected at the grounds of Rice University, as shown on Plate 15 to parallel our testing program. A similar structure is scheduled to be erected

Plate 14 Measuring profiles of a 12 x 12 foot structural membrane.



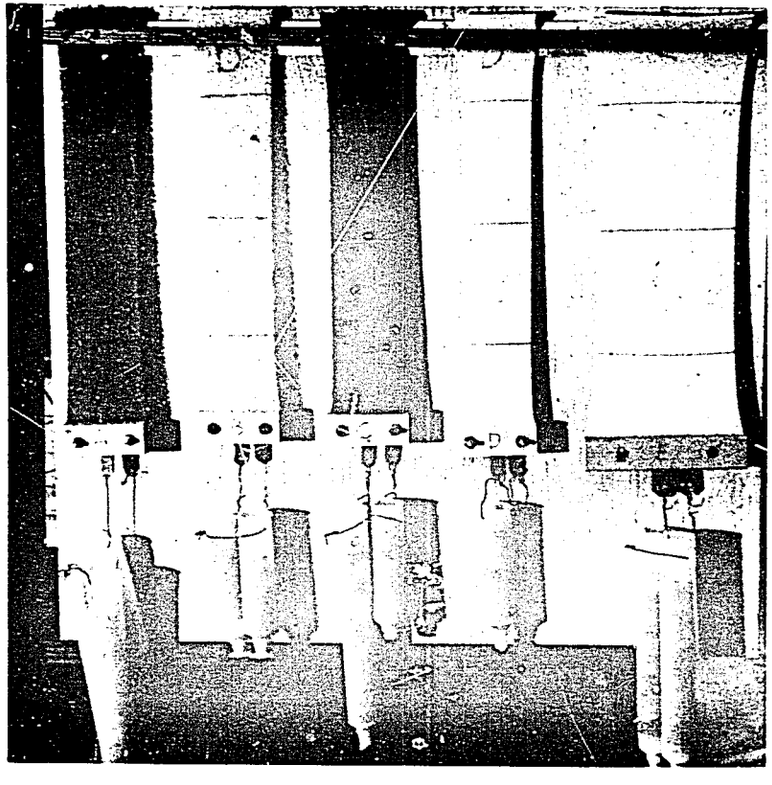


Plate 13 Force strain tests of flexible polyurethane foam.

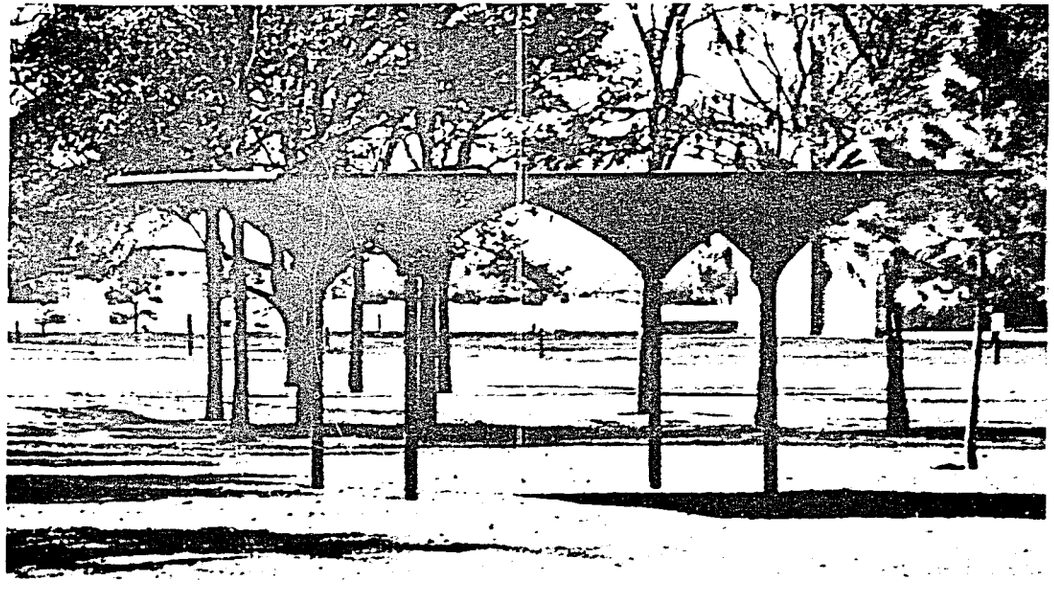
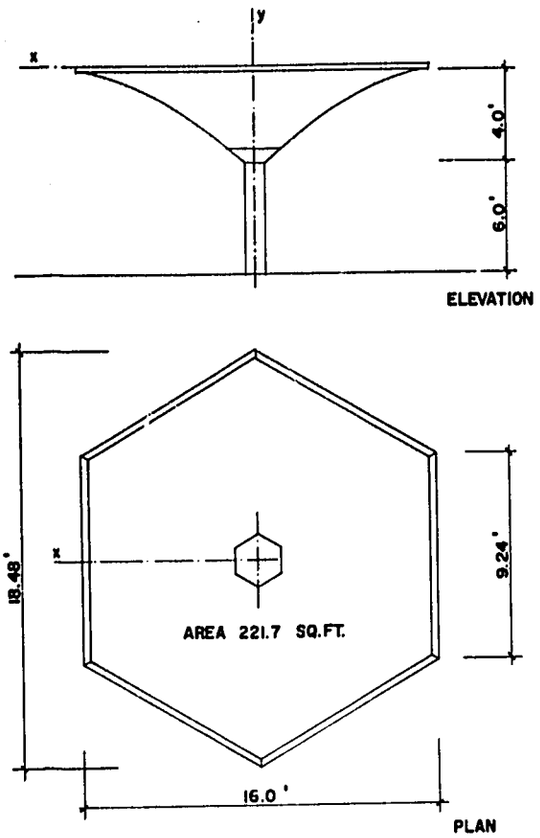


Plate 15 Four inverted umbrella components erected on tubular columns to form a 24 x 24 foot structure.



MATERIALS

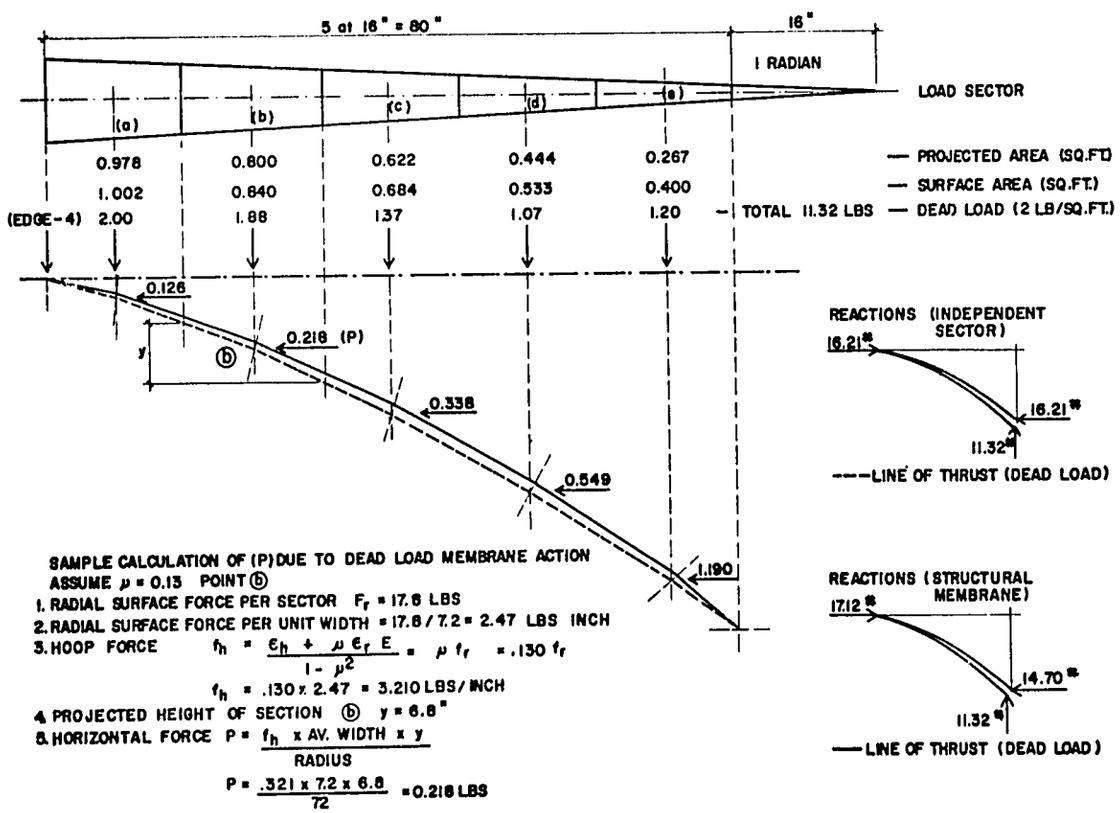
EDGE WOOD DOUGLAS FIR
- D = 30 P.C.F
- f_c = 1200 P.S.I.
- f_b = 1500 P.S.I.
- E = 1.76 x 10^6
- LAMINATED 5 - 2" x 4"

SHELL SPRAYED POLYURETHANE RIGID FOAM
- D = 2.8 P.C.F.
- f_c = 15 P.S.I.
- f_b = 22 P.S.I.
- E = 1000 P.S.I.
- μ = 0.13
- SF = 2.0
- CREEP = 6.5 x ELASTIC STRAIN
- t = 7" AT x = 16"
- t = 5" AT x = 32"
- t = 3 1/2" AT x = 96"

SURFACE DEFINITION - VALUES IN INCHES

x	0	16	32	48	64	80	96
y	-48	-40	-29.2	-19.2	-11.2	-4.6	0

Plate 16 Assumptions and design conclusions for a cellular plastics inverted umbrella shell.



SAMPLE CALCULATION OF (P) DUE TO DEAD LOAD MEMBRANE ACTION
ASSUME μ = 0.13 POINT (b)

1. RADIAL SURFACE FORCE PER SECTOR F_r = 17.6 LBS
2. RADIAL SURFACE FORCE PER UNIT WIDTH = 17.6 / 7.2 = 2.47 LBS INCH
3. HOOP FORCE $f_h = \frac{\epsilon_h + \mu \epsilon_r E}{1 - \mu^2} = \mu f_r = .130 f_r$

   f_h = .130 x 2.47 = 3.210 LBS/INCH
4. PROJECTED HEIGHT OF SECTION (b) y = 6.8"
5. HORIZONTAL FORCE $P = \frac{f_h \times \text{AV. WIDTH} \times y}{\text{RADIUS}}$

   $P = \frac{.321 \times 7.2 \times 6.8}{72}$ = 0.218 LBS

Plate 17 Determination of shell profile for a cellular plastic inverted umbrella shell.

outside our laboratory after completion of the present testing procedure in order to obtain performance data under different climatic conditions.

The example of a proposed 16-foot hexagonal umbrella made of sprayed polyurethane foam is included as an illustration of applying the principle of minimum structure. The assumptions and design conclusions are shown on Plate 16 while the semi-graphical determination of the form is included on Plate 17.

Initially, the approximate dimensions were determined on the basis of an independent radial sector subjected to full dead and live load as well as anticipated membrane action. The specific Poisson's ratio of the foam was taken into account. Since this differs from that used in the structural membrane, the resulting form will differ accordingly. The design was checked by relating a surface of thrust for live load to the actual surface in a manner similar to that for an arch to determine the magnitude of bending stresses. Expected horizontal loads on the edge member were also considered in the design. It should be noted that the concept of structural membrane assumes adequate edge restraint. This results in tension and bending in the hexagonal wood tie edge member and has to be taken into account in establishing the size of this member. The nature of the structural membrane is such that a shell can be inverted from the position shown without any change in the membrane state of stress.

SYNCLASTIC STRUCTURAL MEMBRANE

An inflated balloon is a structural membrane subjected to uniform air pressure. The absence of bending stresses and the existence of uniform surface stresses offer an optimum condition for cellular plastics.

A large scale structural model of a suspended structural membrane is shown in Plate 18. In contrast to the model tests for the anticlastic structural membranes, this model was to serve as a direct basis for defining the profiles of a prototype structure.

The membrane skin is ¼ inch flexible urethane foam, similar to that used in the anticlastic umbrella surface shown on Plate 9. The skin has been marked with a grid on a flat surface. Plywood disks have been bolted to the corners to provide supports, while no other stiffening edge members are being used. The projection of the surface beyond the lines of support was used to insure a surface coverage within these lines. The supporting turnbuckles were adjusted to obtain an acceptable sag for the total form. Details of the loading device to assimilate uniform loading are shown in Plate 20. The unit elongation of the surface was small and, as such,

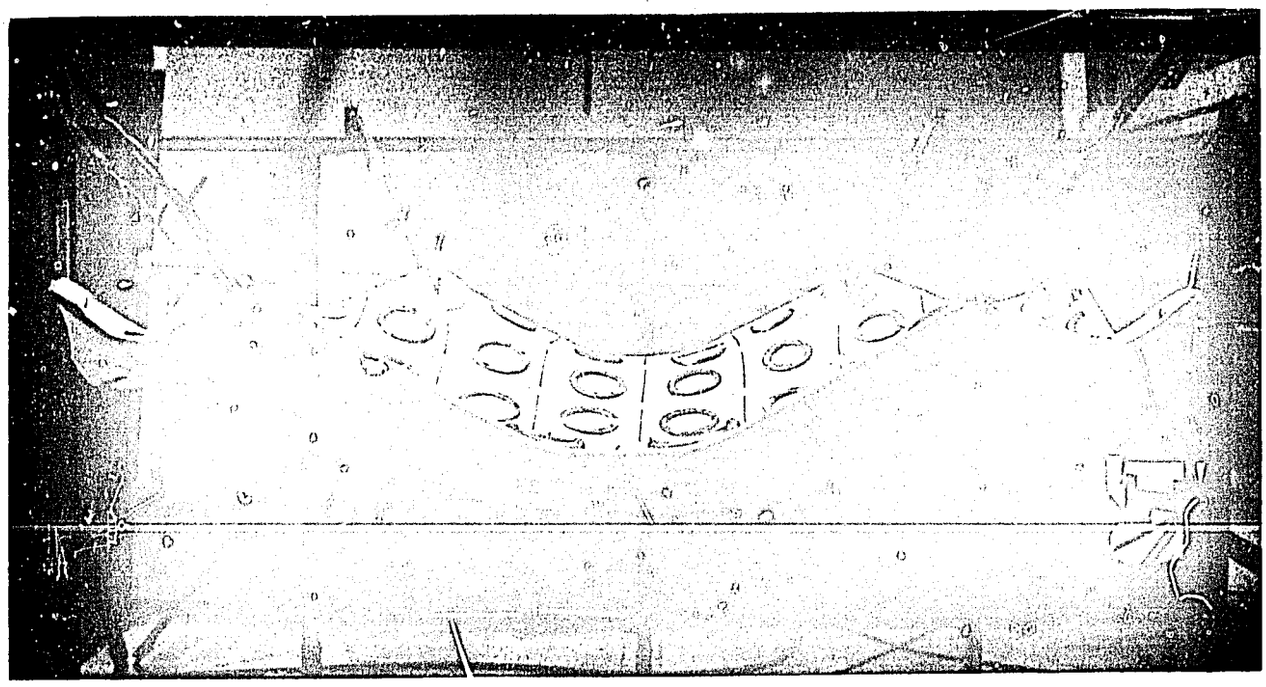


Plate 18 Synclastic structural membrane subject to surface load.

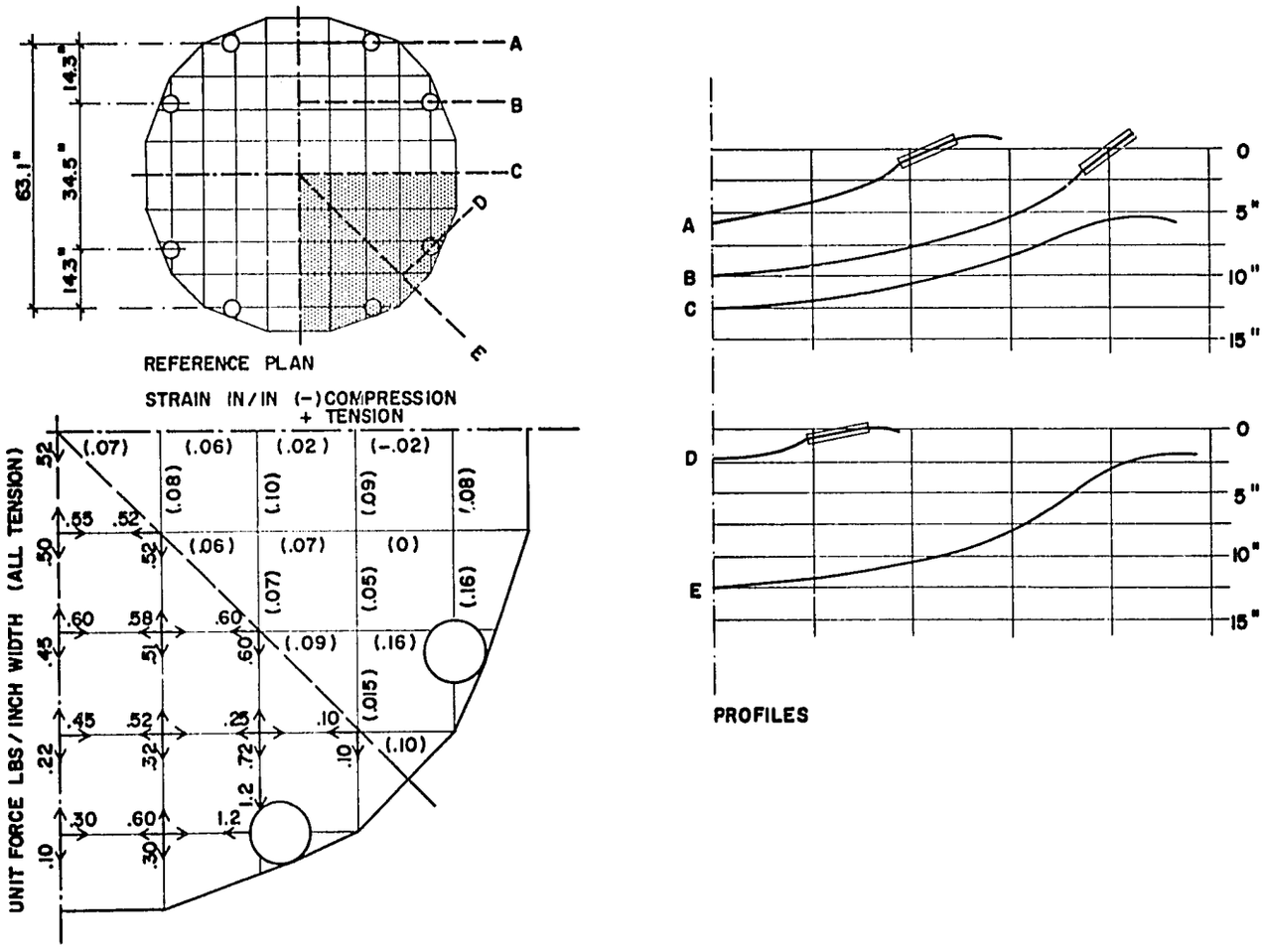


Plate 19 Test results of synclastic structural membrane.

did not require a variation of loading from that defined by the original surface grid. Loads at the edge are subdivided to provide a closer picture of the profile at this point which results in a reverse curve.

The required magnitude of the load is dependent on the desired sag and the physical characteristics of the membrane. This simple relationship is possible since only surface loads are considered. Results of this test are shown in summary on Plate 19. Measured strains in both directions are used to determine the corresponding unit forces. Values of the modulus of elasticity and Poisson's ratio are obtained from figure a on Plate 11 except that all forces for the type II foam have been reduced by 50% to account for the single ¼ inch skin. Profiles included on Plate 19 are intended to serve as a basis for proposed shells in cellular plastics.

A surface of thrust is shown by the structural membrane for a half loaded shell in Plate 21. The supports were held in the rigid position as defined by the symmetrically loaded structural membrane. Comparison of the two membranes gives a graphic presentation of the importance of using the predominantly critical load as a determinant of form.

Examples.

Economy of structure is not only dependent on the efficiency of structural form in relation to materials but also on the efficiency of the production and erection methods used. In reinforced concrete shell construction the high cost of the formwork for a spherical dome negates in part the advantages in terms of cost resulting from an efficient structural surface. The same point would be magnified if formwork were to be constructed to assimilate a synclastic structural membrane. The importance of this point is relative and will of course vary in accordance with the labor-material cost ratio in each country. The development of "airform" for the construction of a concrete dome-shaped house by E. Noyes and W. Neff, and the development of the "lift shape" made of reinforcing steel by J. Marsh and B. Evans[12] are examples of attempts to obtain more economic construction methods for shell structures.

In the spring of 1964 our project staff erected in collaboration with Wyandotte Chemicals Corporation a foam plastics shell structure by spraying polyurethane foam on a folding armature prepared for this purpose in our laboratory.

The wood lattice grid was preassembled in four parts and transported to the site in a collapsed condition. The four parts were joined on a concrete floor

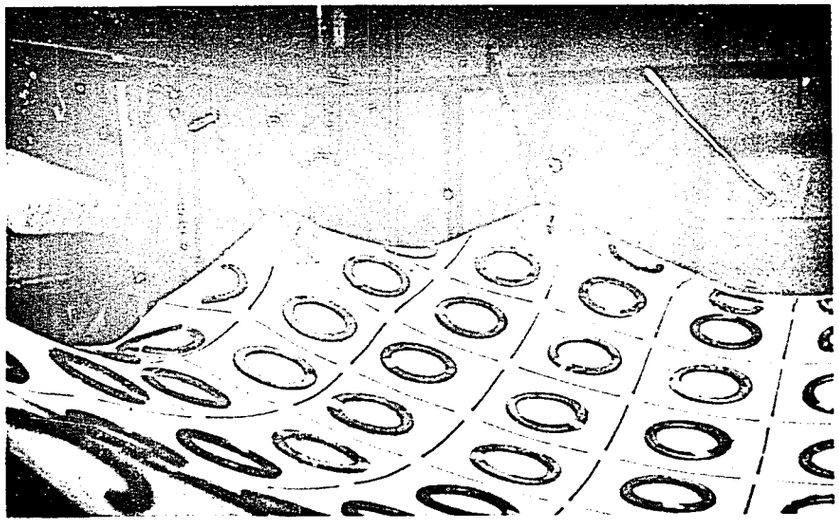


Plate 20 Loading detail for structural membrane.

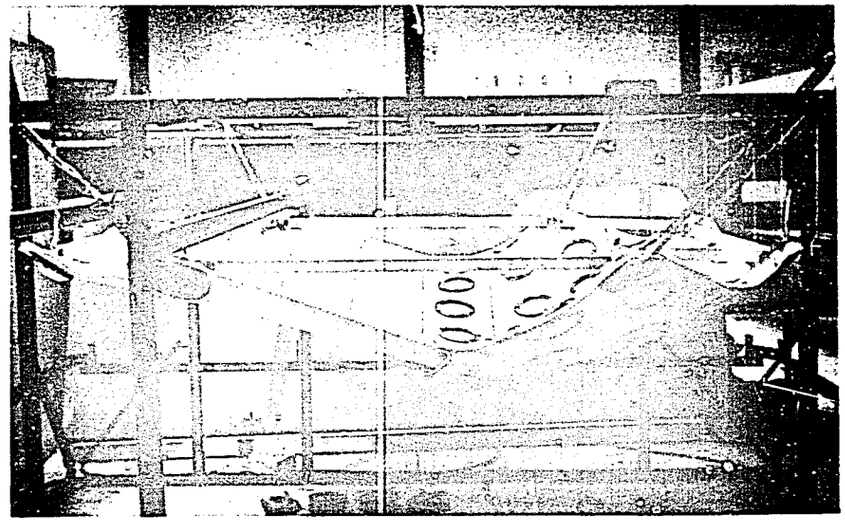


Plate 21 Surface of thrust for one-half loaded shell.

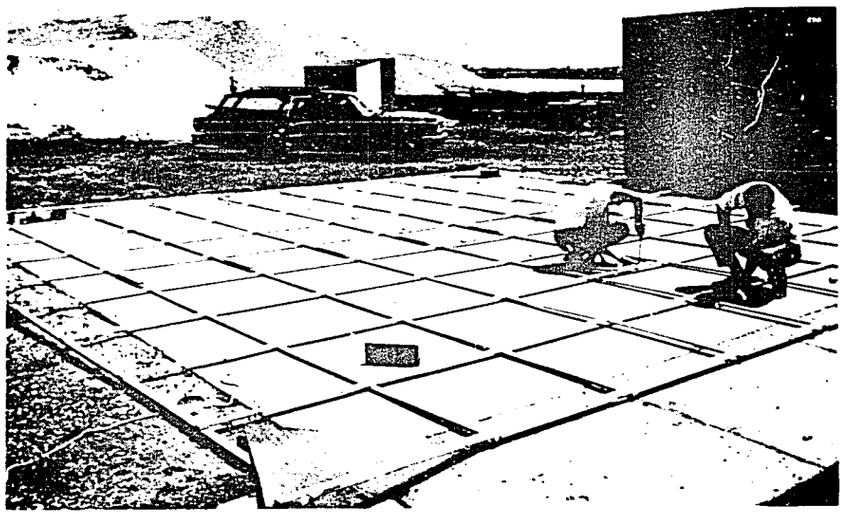


Plate 22 Wood armature, 27 x 27 foot before

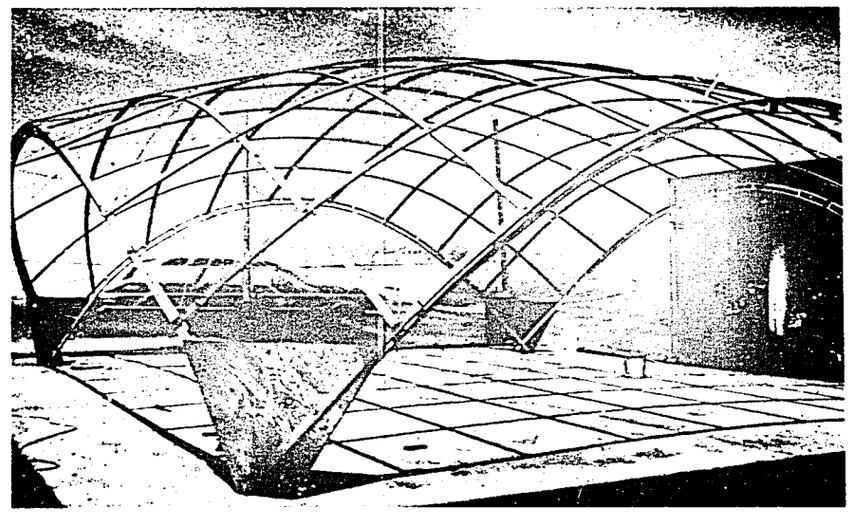


Plate 23 Erected wood armature, 21 x 21 foot base.

slab and assembled into a 27 x 27 foot grid as shown in Plate 22. The bolted joints were left loose to permit rotation of the wood. The armature was then lifted into place and attached to four corner anchor plates, forming a 21-foot square shell. The shape of the armature was adjusted to the desired rise; the joints were bolted tight and intermediate wires were introduced to rigidize the lattice frame. Stiffening edge members were added to the completed armature, as illustrated in Plate 23. The armature was then covered with a stapled-on nylon reinforced paper-skin, stretched on the upper side of the wood lattice. Finally, a 2.5 pcf density polyurethane foam was sprayed from inside the armature with an air gun to an average thickness of 4 inches, as shown in Plate 24.

After the completion of the spraying operation, some deflection was observed at the midpoint of the edge member. This increased over a period of six weeks to a point where the edges of the shell sagged from an initial average height of 84 inches at the center, to 70 inches. The appearance of the shell at the end of six weeks took the shape shown in Plate 25. A wire model of the same armature (Plate 26) was studied in considerable detail. Surface loads on the model caused a similar behavior to that of the prototype. An empirical design solution, developed on the basis of model tests, was applied to the shell with the conclusion that bending stresses in the triangular support sections were approximately 16 psi. This was excessive and would eventually have resulted in failure due to creep.

Based on the same model study, the structure was pulled back to its original shape by applying loads at the four corners by means of springs as shown in Plate 27. Relaxation occurred in the springs indicating a gradual pull back to the original form. Stiffening edge members forming a wood and wire truss were installed at the edge line as shown in Plate 28. In effect, the stiffened edge members now resist the tendency of the supports to buckle, while the foam shell works above the arch level.

With these modifications, the form can be repeated without major problems. However, the problem of creep encountered during this experiment did suggest a more precise study of the form of the shell. This was sought through an application of minimum structure. Model studies with synclastic structural membranes were used as a guide to determine optimum form. In this case, the objective was the design of a shell resting on eight supports, representing a modification of the first prototype study. A 16 foot armature, (Plate 29) was constructed similarly to that described previously, and manipulated to reflect the shape of the structural membrane in Plate 18. Proportions of rise-to-span were maintained at the edges and at the

Plate 24 Spraying polyurethane foam on paper-covered wood armature.

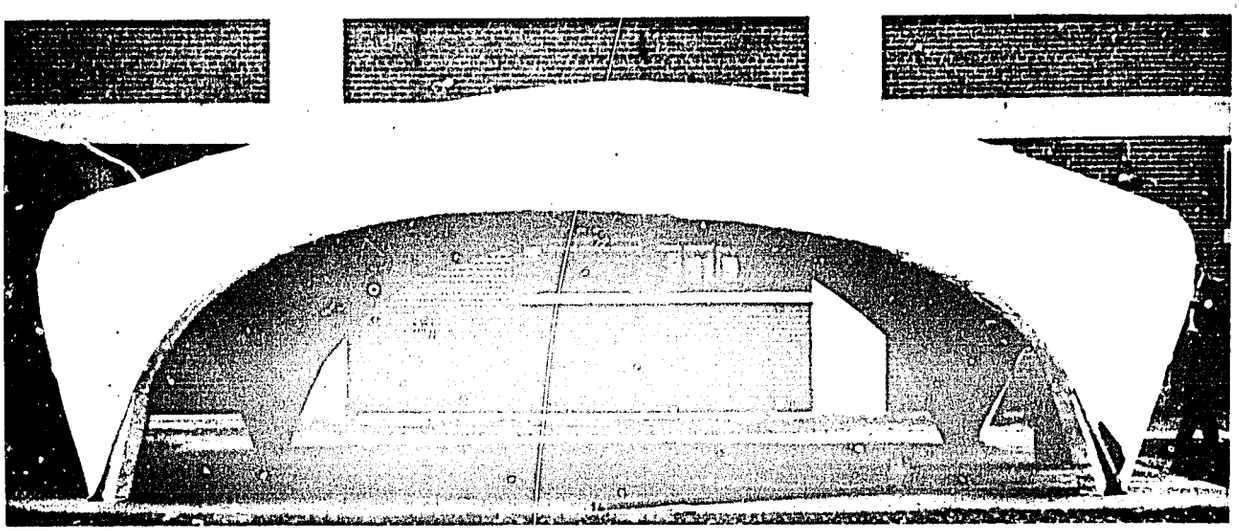


Plate 25 Deflected position of shell after 6 weeks of creep.

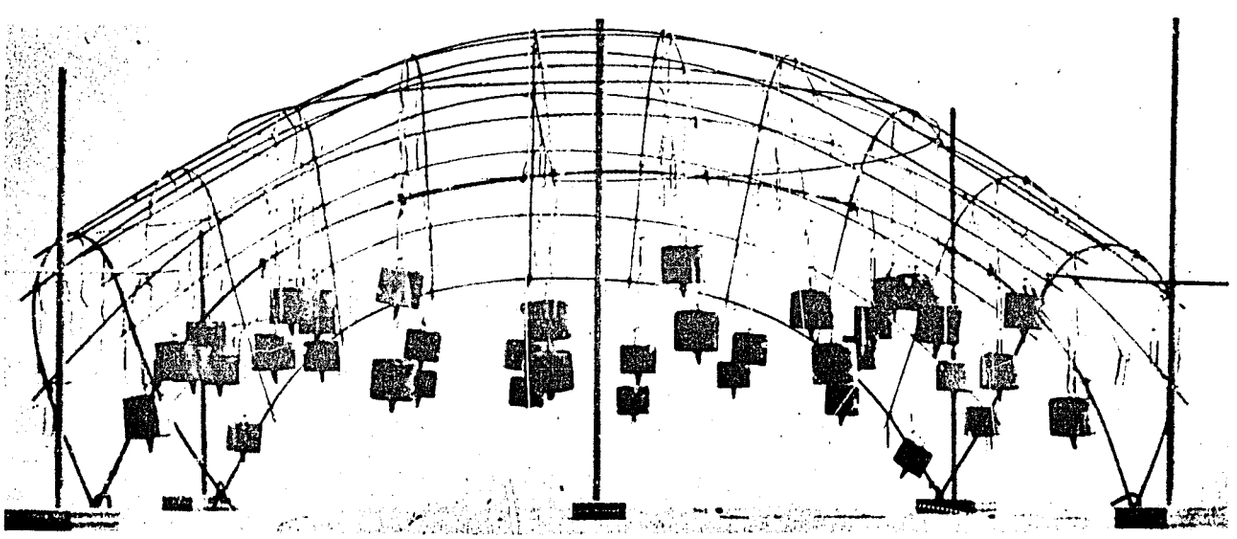


Plate 26 Loaded wire model of prototype wood armature.

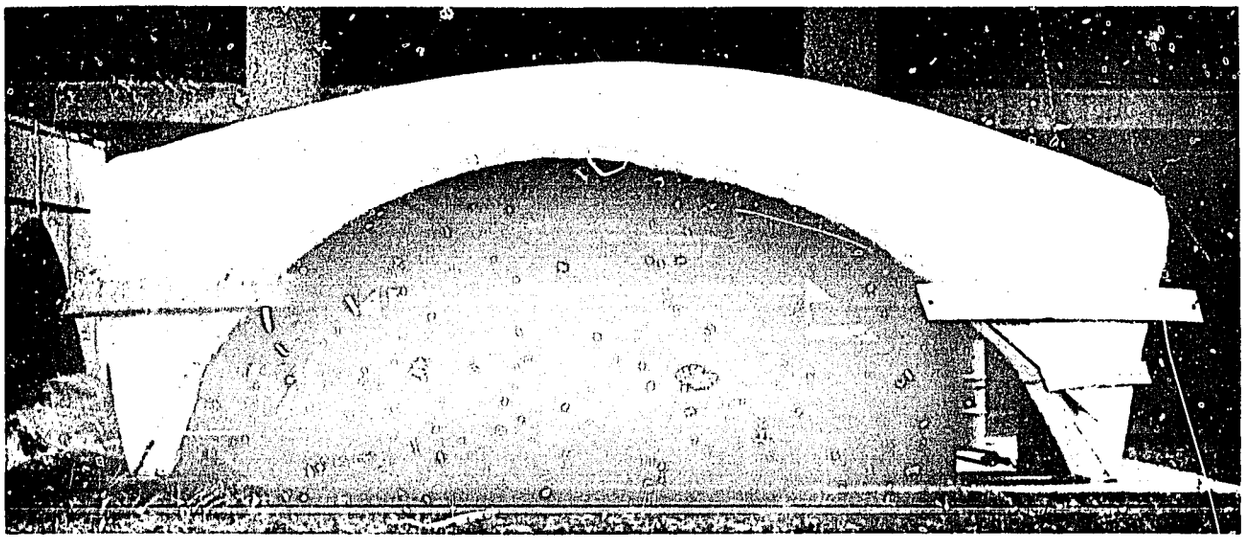


Plate 27 Pulling back of shell to original shape.

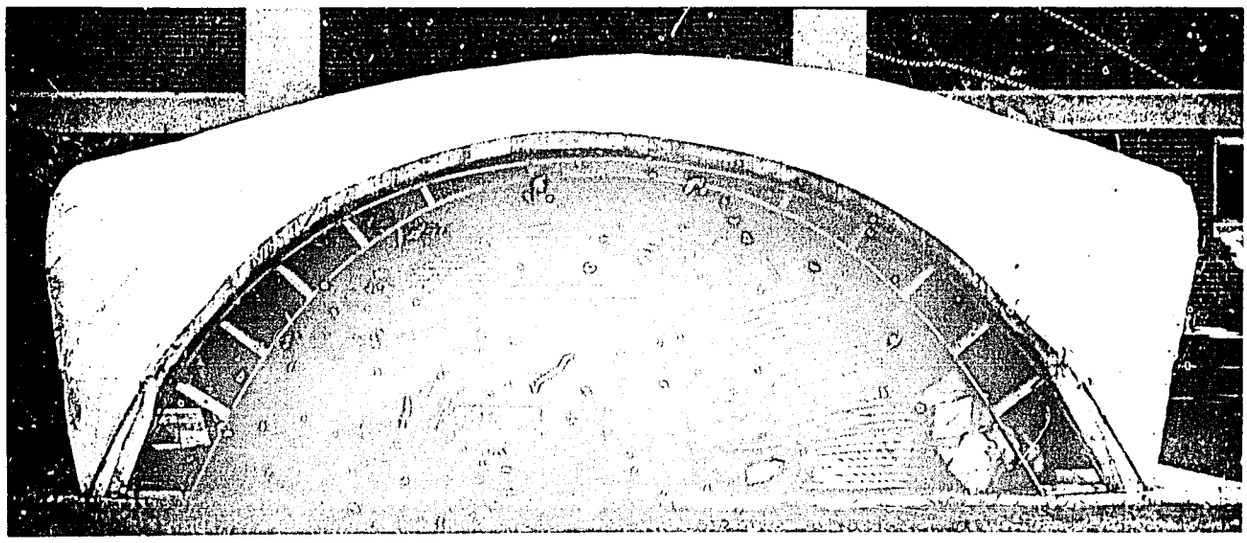


Plate 28 Shell with stiffened edge trusses.

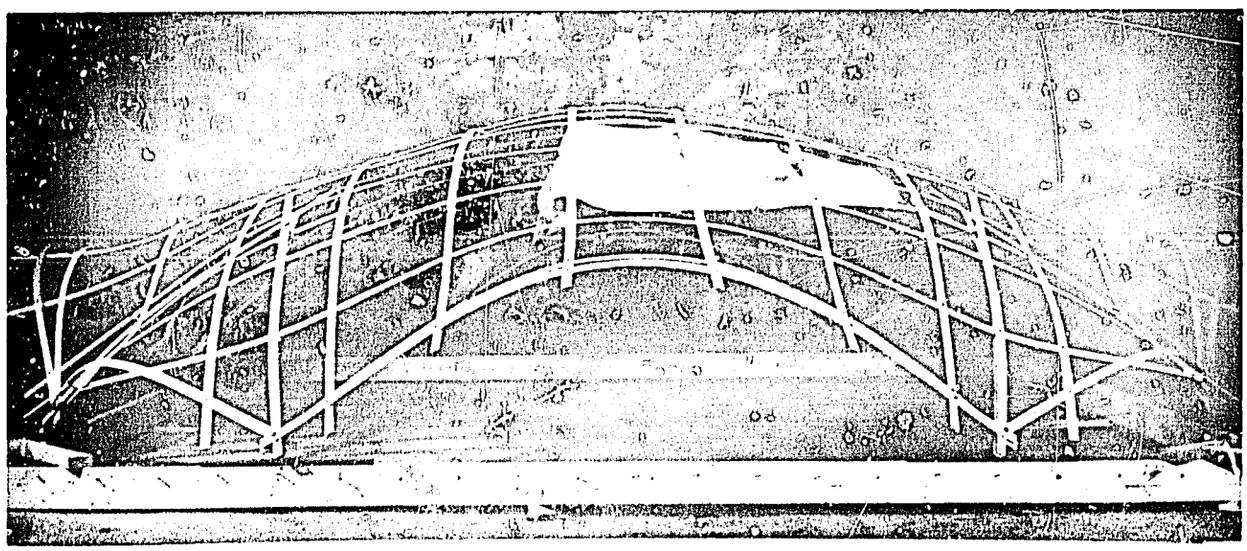


Plate 29 16 x 16 foot wood armature in the form of a structural membrane. Reference Plate 18.

diagonals. The particular curvature at the edge was obtained by substituting a slightly longer member than that of the grid.

The 16 foot armature is to serve as the form-giving device for two experimental spray applications with polyurethane foam. In the first instance, the armature is to be sprayed from the top and separated from the foam shell. Then, the armature is to be sprayed from underneath as in the case of the first prototype structure, with the wood armature remaining as an integral part of the shell. The objective of this experiment is to study the effect of form as well as the effect of the armature within the shell.

CONCLUSIONS

On the basis of the comparison of the concepts of minimum surface and minimum structure, it can be concluded that for an anticlastic shell the minimum surface indirectly indicates an optimum form. Whether such surfaces will be structurally more efficient than the synclastic structural membranes has not been determined. More basic work is required to explore these relationships.

In relation to application of minimum structures in cellular plastics, two methods have been outlined for translating concepts of equilibrium into minimum structures with cellular plastics. In both cases the structural membrane has been assumed to be essential for the realization of primary structural applications. Conclusions from our work to date indicate that it is feasible to use cellular plastics as primary structural components or systems. Whether these in effect will represent optimum building structures will depend not only on structural performance, but also on the efficiency of production methods and construction techniques, as well as on the usability of optimum structural shapes in specific architectural applications. Before these systems will be accepted by the building industry, more research is needed in the chemistry, production and behavior of these materials to insure predictable long-term performance.

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