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PROGRAMMING INVESTMENT IN THE STEEL INDUSTRY

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The Brookings Institution
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CHAPTER I

INTRODUCTION

This study presents a mixed integer programming model for use in the analysis of investment plans in the steel industry. The model is constructed by modifying a linear programming model so as to constrain a small subset of the variables to take on only the values of zero or one. There is one of these "integer" variables for each of a group of investment projects from among which it is desired to chose the "best" combination. The projects may be for the addition of capacity to various productive units within existing plants or for the construction of entirely new plants; they may consist of several alternative size productive units from which the model is to choose the "best" size and/or they may include projects for a number of different technologies for a particular productive unit from which the model is to choose the "best" technology.

The model is multi-period with the sub-model for each time period being very similar to a linear programming transportation problem. Thus, the regular variables (i.e. non-integer variables) which make up the largest part of the activities in the model are for production-transportation activities. In addition to the usual activities of shipments from plants to markets of final products, there are also activities for shipping intermediate products between plants, and activities for importing and for exporting each of the intermediate and final products.

The model is constrained to supply the requirements in every time period to each of the market areas for each product. Thus, the problem for the model is to select that combination of production-transportation activities in each time period and that combination of investments which minimize the present value of cost of meeting the product requirements of the market areas.

Through the inclusion of activities for the shipment of intermediate products between plants and activities for importing both intermediate and final products a mechanism is provided for studying the piecemeal addition of production capacity in a system of plants. For example, since there are economies of scale in the investment cost of most of the major productive units of a steel mill, i.e. the blast furnaces, steel furnaces, and rolling mills, it is considered desirable to install large units. One result of this phenomenon is that a substantial part of the capacity of the newly installed productive unit may be unused until the product requirements in the nearby market areas grow to the size of the unit. However, in a system of steel plants it is possible for the capacity structure of the plants to change over time in a way that makes the plants complementary to one another. In a region where two new steel mills are constructed the one mill might begin by installing the rolling units and the other the blast furnace and steel shop. Then the intermediate product could be supplied from the one facility to another for a time until both plants were built to full size.

Imports of intermediate and final products, just as shipments of intermediate products between plants, may enable the plants to reduce total investment cost over time and to insure better utilization of capacity. Imports may be used either to supply domestic requirements for final products for a time before the installation of a large productive unit to supply them and/or they may be used to supply intermediate products until the requirements for the final product rise to a sufficient level.

The mixed integer programming model presented here thus provides a tool for the analysis of investment plans within an existing system of plants.

1. Background of the Study

During recent years the use of mathematical programming models for economic analysis has been expanding rapidly. The continuing advance in the efficiency of electronic computers and of the algorithms written for them has made possible the solution of larger and larger models.

Models have been constructed for production units, plants, firms, sectors, and for entire economies. However, economists have shown greatest interest in recent years in the models at one end of this spectrum, the country-wide planning models.

Although the "Tableau Économique" of Quesnay, published in 1758, should be cited as the first step in the development of economy-wide

models, the modern-day development of input-output models by Leontief was the spark that lit the fires of recent-day creativity in this field.[1] The development of linear programming and, in particular, of the simplex algorithm by George Dantzig in 1947 cleared the way for the application of input-output models to country-wide planning. Since that date, input-output models have been constructed for many countries. One of the best of these is Bruno's model for Israel.[2] Another, the most recent and most sophisticated, is an eleven sector dynamic (i.e. multi-time period) programming model of the Indian economy by Eckaus and Lefeber.[3]

Concurrently with the development of input-output models for country-wide planning, there have been developments in the use of linear programming algorithms for solving transportation problems. Hitchcock originally formulated the transportation problem;[4] Koopmans clarified and amplified this formulation.[5] In recent years Lefeber[6] has applied

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[1] W. W. Leontief, "Quantitative Input and Output Relations in the Economic System of the United States," The Review of Economics and Statistics, 1936.

[2] Michael Bruno, Interdependence, Resource Use and Structural Change in Israel, Jerusalem, 1962.

[3] Though the final version of this model has not yet been published, the model is a generalization of the model described in R. S. Eckaus and L. Lefeber, "Capital Formation: A Theoretical and Empirical Analysis," Review of Economics and Statistics, May, 1962.

[4] F. L. Hitchcock, "Distribution of a Product from Several Sources to Numerous Localities," Journal of Mathematical Physics, Vol. 20, 1941.

[5] T. C. Koopmans, "Optimum Utilization of the Transportation System," Econometrica, Vol. 17, Supplement, 1949.

[6] L. Lefeber, Allocation in Space, North-Holland Publishing Company, Amsterdam, 1958.

this technique and the use of Lagrangian multipliers to the problems of the spatial location of productive units in an economic system. Vietorisz and Manne, working both together[1] and separately[2,3] have made advances on the problem of the spatial and time location of productive units in one sector of an economy, for sectors characterized by economies of scale and indivisibilities in investment.

Thus, while Manne and Vietorisz have made progress on the problem of when and where to construct new plants, no one has yet given close attention to investment problems where additions to capacity in the various productive units in each plant constitute as important a part of the investment in the industry as does the construction of new plants.

The steel industry is such an industry. Frequently, investments are designed to supplement the capacity of some productive units in the plant in order to make use of excess capacity in other parts of the plant. Also, even when new plants are constructed, the installation

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[1] Thomas Vietorisz and Alan Manne, "Chemical Processes, Plant Location, and Economies of Scale," in Manne, A. S. and H. M. Markowitz, eds., Studies in Process Analysis, Cowles Foundation Monograph No. 18, Wiley, New York, 1963, pp.136-158.

[2] Thomas Vietorisz, "Industrial Development Planning Models with Economies of Scale and Indivisibilities," mimeographed, IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y., presented at the Third European Congress of the Regional Science Assn., August 26-29, 1963 in Lund, Sweden.

[3] Alan S. Manne, "Plant Location Under Economies-of-Scale - Decentralization and Computation," Management Science, Vol. 11, No. 2, November, 1964, pp.213-35. Alan S. Manne, "Plant Size, Location, and Time Phasing," mimeographed, Center for International Studies, M.I.T., 1964.

of the various parts of the plant are usually phased over several years. This sort of "bottleneck investment" accounts for such a large part of the investment in the steel industry that it can not be safely ignored in analytical models for investment studies in this industry. Thus the models presented in this thesis are disaggregated to the extent of considering the capacity of each of the major production units within each plant and of considering requirements for four different final product groups.

Three types of models are included in this study; (1) an all integer model (a model in which all of the variables are constrained to take on positive integer values); (2) a linear programming model; and (3) a mixed integer programming model (a model in which some, but not all, of the variables are restricted to positive integer values).[1] These models use data for the cost of the production and transportation of flat steel products in all of Latin America (the all integer model), and especially in Brazil (the linear and the mixed integer programming models).[2]

---

[1] For those not familiar with mathematical programming, an effort has been made to explain each concept as it appears in the study. For example, in the section in the next chapter which gives a mathematical statement of the linear programming model several paragraphs following the equations are devoted to explaining the purpose and nature of each of the sets of equations. However, no comprehensive attempt is made in this study to explain mathematical programming. Any number of books on the subject are widely available. See, for example, Linear Programming, Method and Applications, (McGraw-Hill) by Saul I. Gass, or Linear Programming (Addison-Wesley) by G. Hadley.

[2] The reader who is not familiar with the terminology of steel technology is referred to Appendix A which provides a very general description of the production of flat steel products.

The all integer model (Appendix G) was developed first. It considers the case of a system of plants and markets with a single product, economies of scale in the investment cost, fixed demand at the market locations, a single time period, and no existing plants in the system. The problem is one of selecting that combination of plant locations which can serve the market requirements in the system at the minimum production and transportation costs. The model is presented in this thesis because of its usefulness in analyzing situations in which no plants now exist in the system and because of its contribution to computational experience with this type of model. Because this model is restricted to studying that class of problems in which there are no existing plants in the system, it could be called a "clean slate" model. Four different methods of solving this all integer variable problem were employed; a discussion of the relative merits of each method is given.

The linear programming model (Chapters II and III) is a single-time period model which is used for making an economic analysis of the present situation of the production and transportation of flat steel products in Brazil. A study of the response of the model to changes in the parameters was made. About twenty-five solutions to the problem were obtained to study the effects of changes in cost of inputs, changes in market requirements, and changes in transportation cost.

The mixed integer programming model (Chapters IV and V) is an extension of the linear programming model to cover three time periods

and to permit the introduction of investment variables into the system. These investment variables are restricted to the values of zero or one. The production-transportation activities remain free to take on any non-negative values. Thus, the model has some regular variables and some integer variables and is therefore called a mixed integer programming model.[1]

2. Assumptions

The assumptions made in this study fall into two general classes; (1) assumptions which are necessary because of the structure of the model, i.e. "model assumptions," and (2) assumptions which are necessary in order to construct a solveable-sized model of one subsector of the Brazilian economy, i.e. "institutional assumptions." These assumptions refer only to the linear and mixed integer programming models, as the assumptions of the "clean slate" model are discussed separately in that paper in Appendix G.

2.1 Model Assumptions. The model assumptions stem directly from the structure of the model. They are difficult to alter in the sense that the approaches presently known for modifying them are either

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[1]Chapters IV and V have been written as a self contained unit so that the reader who is interested only in the mixed integer programming model can skip directly from the end of this introductory chapter to the beginning of Chapter IV.

expensive in computational time or they may lead to unstable or otherwise unsatisfactory results.

A number of these assumptions are related to the price elasticity of demand and supply of the commodities of the model. For example, it is necessary to assume that the domestic demand for final steel products will be unaffected by changes in the prices of these products, that is that demand is perfectly inelastic and therefore is, in reality, a requirement. On the other hand the price elasticity of supply of raw material, manpower, and capital is assumed to be one, i.e. that the firms can obtain all the raw materials, manpower, and/or capital they need if they are willing to pay a certain fixed price. Thirdly, it is assumed that the supply of imports and the demand for exports are perfectly elastic.

A second group of the model assumptions have to do with linearity. It is assumed that the non-capital cost of production is a linear function of the amount produced, i.e. that the unit cost of production (other than capital cost) for producing one ton of steel products in a given plant is the same as the unit cost for producing 300 thousand tons of steel products. Capital costs are treated in a manner which permits the introduction of economies of scale and are, therefore, not subject to this assumption.

The third group of model assumptions concerns the independence of the cost of activities in the system. For example, it is assumed that the costs of production between plants and between productive units in the same plant are independent. Also, it is assumed that the

cost of one investment is not affected by whether or not another investment is accepted or rejected. However, this is not to say that the cost of operating the system is independent of the combination of investment projects that are accepted. Finally, it is assumed that the production cost in a given unit is not affected by whether or not a given investment is accepted or rejected. Thus, the choice of investment projects affects the levels of the production-transportation activities in the optimal solution, but not the cost of these activities.

Though the list of these assumptions is long, it will be shown in the body of this study that most of them are not too confining, so that it is possible to construct models which provide important insights into the flat products section of the Brazilian steel industry.

2.2 Instituional Assumptions. The construction of an optimizing model requires that certain abstractions be adopted. These assumptions are more easily modified than the model assumptions because they do not involve the development of new techniques, but rather the application of existing techniques to available data and/or the collection of additional data. However, the conditions which make these assumptions necessary are not only the availability of data, but also the cost of computer time.

For example, though there are between five and ten plants in Brazil that produce some flat steel products, the model described in this study includes only three plants; (1) COSIPA, Companhia Siderurgica Paulista, at Piacaguera near Santos; (2) CSN, Companhia Siderurgica

Nacional at Volta Redonda; and (3) USIMINAS, Usinas Siderurgicas de Minas Gerais S.A., near Ipatinga in the state of Minas Gerais. Figure 1 shows the location of these plants. Since these plants have better than three-fourths of the flat steel product capacity in the country, it was decided to limit the model to three plants. The model can be expanded to include any number of plants, provided the data are available and provided that the increase in the size of the model does not make it computationally unwieldy.

Secondly, the model assumes that the consumption of flat steel products is concentrated in Brazil's three largest cities: São Paulo, Rio de Janeiro, and Belo Horizonte. Steel men in Brazil estimate that about 90 per cent of the consumption of all steel products is centered in the area bounded by these three cities. The consumption of flat steel products is probably more concentrated in the three cities than the consumption of all steel products. The study assumes that 100 per cent of flat steel product consumption occurs in these cities. Like the previous institutional assumption, this one can be easily modified.

Thirdly, the study is confined to flat steel products. COSIPA and USIMINAS produce only flat steel products, each having a capacity in 1965 of roughly half a million tons of final products. CSN, however, produces both flat products and shapes. In 1965 CSN had a capacity of slightly less than a million and a half metric tons of ingot steel. More than three-fourths of that capacity is used for producing flat products. Some adjustments have been made in the model for the fact

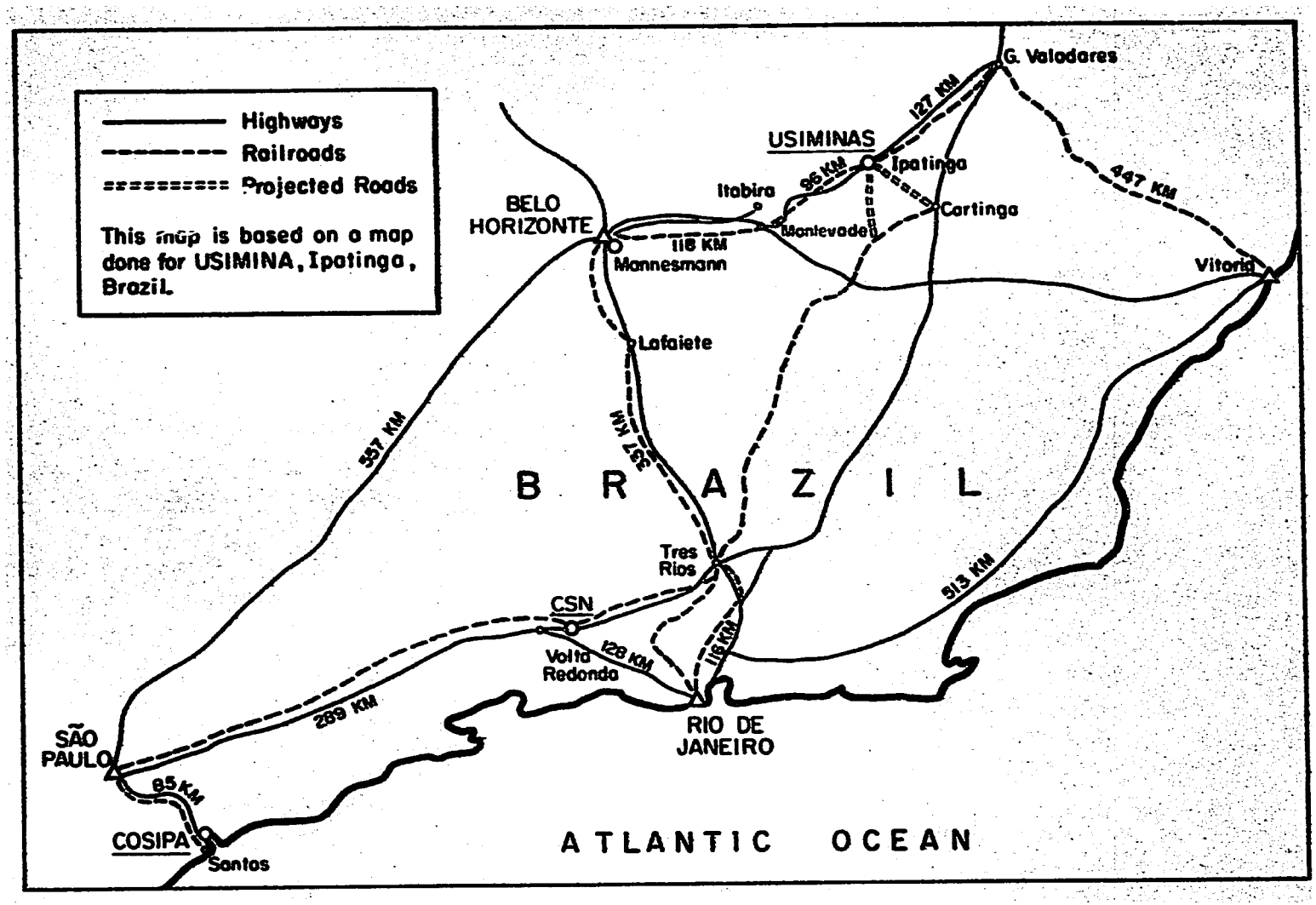

Figure I. Major Flat Product Steel Mills in Southern Brazil

that a part of CSN's steel making capacity is used to supply steel for shapes rather than for flat products. A more complete analysis could have been made if the study had considered both shapes and flat products. However, such an expansion of the study would have necessitated the inclusion of ten to fifteen products instead of the seven which are considered in the model. The model is perfectly general in the sense that any number of final and/or intermediate products may be included so long as their inclusion does not result in a computationally unwieldy system.

Another assumption of the study is that dividing the time span of the model into several $2\frac{1}{2}$ year periods provides a sufficiently disaggregated analysis. One year periods are the more usual divisions and could be used at the expense of longer computing time for each run of the model.

The study also assumes that there is no change in the cost of labor, or raw materials, or of capital over the time span of the model. If projections of the expected course of these prices over time were available, they could easily be included in the model.

It is assumed that the cost of capital is the same for all firms and that the cost is equal to the discount rate. The discount rate is used in the model to discount all cost to January 1, 1965. This assumption can also be modified easily.

Finally, the model assumes independence of investment decisions in the flat steel products sub-sector, from investment decisions in the

rest of the sub-sector and in other sectors. This assumption is more difficult to modify than those previously discussed because altering it would require the construction of a model that included a number of sub-sectors and/or sectors. If the same level of disaggregation as is used in the present model were used in such a multi-sub-sector model it would become large, i.e. computation time for a single solution of the model might rise from about one minute with an IBM 7094 to about ten minutes.

3. The Data

The data used in the model were obtained, in large part, by the author while visiting the three major steel mills in Brazil in January of 1965. In addition, parts of the data were obtained from the Banco Nacional de Desenvolvimento (BNDE) in Rio de Janeiro, from the Latin American Iron and Steel Institute (ILAFA), and from the Economic Commission for Latin America (ECLA) in Santiago.[1] Though the officials of these companies and institutions were generous in their assistance, the magnitude of the data collection effort to obtain the quantity and quality of information needed for a policy-making model exceeded

---

[1]Appreciation is due the officials of Companhia Siderurgica Paulista (COSIPA), Usinas Siderurgicas de Minas Gerais S.A. (USIMINAS), and Companhia Siderurgica Nacional (CSN) for providing data to the author and for permitting him to visit their plants. Likewise, the author is indebted to José Mariano Falcão of the BNDE, Fernando Aguirre Tupper of the ILAFA, and Bruno Leuschner, Armando P. Martijena, and Nuno Fidelino Figueiredo of the ECLA for providing data and assistance.

the time available to the author.[1] However, it was not the purpose of this study to develop a policy model. Rather it was to explore the usefulness of some types of mathematical programming models for investment planning.

---

[1]The highest quality data in the model are those for the specific consumption of materials in the production of steel products at the three plants. The data for the unit cost of these inputs and the data for capital costs are less reliable. The requirements estimates are based on the correlation between national steel consumption and the contribution to GNP of the manufacturing-construction sector and was the best projection available. The assumptions about the regional breakdown of the markets for particular types of products are weak. The transportation cost data and the estimates of the cost of imports and the profits to be made from exports are the weakest data in the model.

CHAPTER II

THE LINEAR PROGRAMMING MODEL

This section of the study presents a linear programming model of the production and transportation of flat steel products in Brazil in 1965. While this linear model offers a useful analytical tool for making an economic study of the current situation of a part of the Brazilian steel industry, it was not designed for this purpose. Rather, it was designed primarily to be the core of the non-linear mixed integer programming model which is used to study the problem of when and where to build new plants or to add to capacity in existing facilities when there is a growing demand for the products of the industry.

The development and use of the model consisted of five steps. First, the distances between the plants and markets were measured along existing roads, railroads, and inland waterways, and estimates of transportation cost over these routes were made. Then, a FORTRAN program was written to calculate the cost of using various routes and modes of transportation and to select the least-cost route in each case. This program, as all the programs for the Brazilian model, was written in a general form so that it could be expanded to perform the necessary calculations for any number of plants and markets.

Secondly, production cost estimates were made for pig iron and ingot steel cost at each of the three plants. Then, in a third step,

the linear programming matrix was constructed by combining the production and transportation cost estimates with data for capacities of the major production units in each plant, and market requirements for the products in each market area. FORTRAN programs were written to perform the required calculations and to set up and punch out the input deck for the linear programming (LP) routine. Each of these steps is explained in detail in succeeding sections of the paper.

In the fourth step, the components of the output of the linear programming routine were examined to determine their usefulness in analyzing the operation of the industry. After this, about twenty-five runs were made with the complete model, varying first one and then another of the parameters of the model to test their effects on the results.

While many programming studies of various aspects of the steel industry have been completed,[1] relatively few have focused on operating or investment problems in a system of steel plants. An outstanding exception is the recent careful and analytical study done for the location of a new steel mill for heavy shapes in Mexico.[2] This investigation used a complete enumeration of many possible combinations of

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[1]For a list of the published studies see S. L. Cook, "Applications in the Steel Industry," Ch. 9 in David B. Hertz and Roger T. Eddison, eds. Progress in Operations Research, Volume II, Wiley, New York, 1964.

[2]Comite para la programación de la industria siderurgica (Committee for the Planning of the Steel Industry), Programación del desarrollo de la industria de aceros comunes laminados (The Planning of the Development of the Rolled Steel Products Industry), Nacional Financiera, Mexico, D.F., November, 1964.

locations and technologies to find the least-cost combination of technology and location which would minimize the sum of production and transportation cost in the system.

Also, a linear programming study of the production and consumption of steel products in Chile is being done by the Compañia de Acero del Pacífico, S.A., (CAP). This study reportedly includes both chemical and capacity constraints on production in the various production units of the Huachipato plant of this firm, as well as a very detailed product breakdown for the Chilean market. Regrettably, this report has not been made publicly available.

A brief description of the objective function (1) and the sets of constraints (2), (3), (4) and (5) is in order. The objective function (1) states that the problem is to minimize the total cost of producing and transporting intermediate and final products, plus the cost of imports, minus the profits from exports. Term (a) of expression (1) represents the cost of producing and transporting ingot steel between plants as an intermediate good. Term (b) includes the cost of producing in all of the $i = 1,2,\ldots,n$ plants and shipping to all of the $j = 1,2,\ldots,m$ market areas each of the $k = 3,4,\ldots,p$ final products. Term (c) represents the cost of imports and term (d) the profits from exports (usually $d_i^k < 0$).

1. The Model

$$(1)\quad \text{Min } z = \underbrace{\sum_{i=1}^{n} \sum_{\substack{\ell=1 \\ \ell \neq i}}^{n} a_{i\ell} w_{i\ell}}_{(a)} + \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=3}^{p} b_{ij}^{k} x_{ij}^{k}}_{(b)} + \underbrace{\sum_{j=1}^{m} \sum_{k=3}^{p} c_{j}^{k} y_{j}^{k}}_{(c)} + \underbrace{\sum_{i=1}^{n} \sum_{k=3}^{p} d_{i}^{k} v_{i}^{k}}_{(d)}$$

Subject to:

$$(2)\quad \sum_{\substack{\ell=1 \\ \ell \neq i}}^{n} \alpha_{i}^{1} w_{i\ell} + \sum_{\substack{\ell=1 \\ \ell \neq i}}^{n} \beta_{i}^{1} w_{\ell i} + \sum_{k=3}^{p} \sum_{j=1}^{m} \tau_{i}^{k1} x_{ij}^{k} + \sum_{k=3}^{p} \tau_{i}^{k1} v_{i}^{k} \leq \min[\gamma_{i}^{0}, \gamma_{i}^{1}]$$

$$i = 1, \ldots, n$$

$$(3)\quad \sum_{k=3}^{p} \sum_{j=1}^{m} \tau_{i}^{ke} x_{ij}^{k} + \sum_{k=3}^{p} \tau_{i}^{ke} v_{i}^{k} \leq \gamma_{i}^{e}$$

$$e = 2, \ldots, z$$

$$i = 1, \ldots, n$$

$$(4)\quad \sum_{i=1}^{n} \lambda x_{ij}^{k} + \lambda y_{j}^{k} \geq \sigma_{j}^{k}$$

$$k = 3, \ldots, p$$

$$j = 1, \ldots, m$$

$$(5)\quad x_{ij}^{k} \geq 0$$

$$k = 3, \ldots, p$$

$$j = 1, \ldots, m$$

$$i = 1, \ldots, n$$

The set of constraints (2) requires that in each of the n plants the total amount of ingot steel shipped to other plants, minus the amount of ingot steel received from other plants ($\beta_j^1 < 0$), plus the amount used for producing domestically-used final products, plus the amount used for exported final products must be less than the ingot steel capacity of the plant. This capacity is defined as the lower of two capacities; (1) the ingot-steel-equivalent capacity of the blast furnace(s) (i.e. the pig iron capacity of the blast furnace(s) divided by the normal input of pig iron per ton of ingot steel produced), and (2) the ingot steel capacity of the steel shop.

The second set of constraints (3) requires that the amount of steel products passing through each of the production units e = 2,3,...,z in each of the plants must be less than the capacity of that unit. For example, the capacity of the hot strip finishing mill must be sufficient to provide all of the final product of hot sheet and strip desired as well as the intermediate product needed to produce the required quantities of both the cold sheet and strip and the tin plate. On the other hand, the capacity of the tin mill need be only large enough to produce the desired amount of tin plate.

Set (4) constrains the problem in such a way that for each product and each market the total amount produced domestically and shipped to the market areas plus the amount imported to the market areas from other countries will be greater than the requirement of each market area for that product.

In greater detail:

z = the total cost (other than capital cost) in US$[1] of producing and transporting the required final bill of goods for one year.

$w_{i\ell}$ = metric tons[2] of ingot steel shipped as an intermediate product from plant i to plant ℓ.

$a_{i\ell}$ = cost in US$/metric tons for each metric ton of ingot steel produced at plant i and delivered to plant ℓ, minus the cost of a metric ton of ingot steel from plant ℓ at plant ℓ, i.e. the extra cost to the system when plant ℓ uses ingot steel produced at plant i rather than in its own facilities.

x_{ij}^{k} = metric tons of product k produced at plant i and shipped to market area j.

$b_{ij}^{k} = u_i^k + t_{ij}^{*}$ = cost in US$/metric ton of product k produced at plant i.

$u_i^k = \sum_{h=1}^{g} f_i^j s_i^{kh}$ = cost in US$/metric ton of product k produced at plant i.

f_i^h = cost in US$ per unit of factor input h at plant i.

s_i^{kh} = specific input in units of the factor input h per metric ton of product k at plant i.

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[1] All calculations are made in US dollars rather than in cruzeiros because when the study was begun it was anticipated that it would be geographically expanded at a later date to include all of the countries in the Latin American Free Trade Area plus Venezuela. The free market exchange rate in existance at the time the data was created or published has been used to convert cruzeiros to dollars. It is assumed that the free market exchange rates approximate to equilibrium exchange rate.

[2] The term "tons" in this paper always means metric tons. In fact, exclusively metric system units are used throughout the paper.

t^*_{ij} = $\min_q (t^q_{ij})$ $q = 1,2,\ldots,r$ = minimum transportation cost in US$/m.ton for shipping products from plant i to market area j.

t^q_{ij} = transportation cost in US$/m.ton for shipping product from plant i to market area j over route q, where route q may consist of transportation by truck, train, barge, or ship or any combination of these modes.

y^k_j = imports in metric tons of product k to market area j.

c^k_j = delivered cost in US$/metric ton for imports of product k delivered to market area j.

v^k_i = exports in metric tons of product k from plant i.

d^k_i = cost in US$/metric ton for exports of product k from plant i. With $d^k_i < 0$ so long as it is profitable to export. Note that exports are treated here only as a means of reducing the total cost of domestic production.

$w_{i\ell}$ = shipment of ingot steel in metric tons from plant i to plant ℓ.

$w_{\ell i}$ = shipment of ingot steel in metric tons from plant ℓ to plant i.

α^1_i = capacity of production unit #1 in plant i required per metric ton of ingot steel produced. Normally $\alpha^1_i = 1$ or slightly greater than 1. The capacity of production unit #1 is defined as the lower of the two capacities, (1) the ingot-steel-equivalent capacity of the blast (or reduction) furnace(s), and (2) the ingot-steel capacity of the steel shop.

β_i^1 = capacity of production unit #1 in plant i required per metric ton of ingot steel produced. Normally $\beta_\ell^1 = -1$ since $w_{\ell i}$ is steel shipped <u>to</u> plant i from plant ℓ, therefore, the shipment must be added to the total ingot steel available at plant i, i.e. added to the right-hand side, but this is the same thing as subtracting it from the left-hand side of the constraint.

γ_i^{k1} = capacity of production unit #1 in plant i required per metric ton of product k produced at plant i. Note that the summation for k in this and the next term is from k = 3 to p. This results from the fact that products 1 and 2 are intermediate products (pig iron and ingot steel, respectively) rather than final products.

γ_i^{ke} = capacity of production unit e in plant i required per metric ton of product k produced in plant i. Note that these coefficients will be zero for all production units e, through which final product k is <u>not</u> required to pass, viz. the coefficient for the tin mill production unit with the final product hot sheet.

γ_i^o = ingot steel equivalent capacity of the blast furnace(s) or reduction furnace(s) at plant i in millions of metric tons per year, i.e. the pig iron capacity of the furnace divided by the specific input of pig iron normally used per metric ton of steel produced at plant i.

γ_i^1 = capacity in millions of metric tons of ingot steel per year for the steel furnaces (normally open hearths, electric furnaces, or LD converters) at plant i.

γ_i^e = capacity in millions of metric tons per year of production unit e at plant i.

$\lambda = 1$

σ_j^k = requirement in millions of metric tons per year for product k in market area j.

2. Transportation Cost

The transportation cost function used for calculating rail transport cost is a simple linear function

(6) $$T = a + bX + u$$

where: T = transportation cost over a given route in US$/metric ton

a and b = parameters

X = distance in kilometers over the route

u = error term.

The parameters in this function were estimated from data obtained from Annuario Estatistico Do Brasil. Annual observations for the five years 1959 to 1963 in cruzeiros per metric ton for shipments of steel bars and plates on Estrada de Ferro Central do Brasil were used.[1] The data included the freight cost for each year for distances of 100, 200, 300,...,1,000 kilometers.

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[1]The data for 1959-61 were taken from p.152 of Annuario Estatistico do Brasil for 1961, for 1962 from page 110 of the volume for 1962, and for 1963 from page 208 of the volume for 1963.

It was expected that a function of the form $T = aX^b$, $0 < b < 1$, i.e. a function that is linear in the logs of the two variables, would provide an adequate fit to the data. However, plotting the data revealed that the function (6) provided a closer fit. Regressions of the form (6) were run on the undeflated cruzeiro cost for the five years in order to obtain estimates for the parameters a and b for each of the five years. Then the parameters were deflated, using a price index for manufactured goods other than foods, and finally the parameters were converted from cruzeiros to US dollars by using the free exchange rate for the month in which the observations were made. The results are shown in Table 2.1.

Table 2.1

Parameters for the Railroad Freight Cost Function#

Year	Current Cruzeiros		Deflated Cruzeiros		US dollars		Exchange Rate	Price Index
	a	b	a	b	a	b		
1959	354.53	1.205	98.5	0.333	2.61	0.00896	136	360
1960	411.20	1.456	81.9	0.307	2.20	0.00778	187	473
1961	493.47	1.747	73.6	0.260	1.87	0.00662	264	671
1962	658.67	2.570	71.3	0.278	1.79	0.00700	367	924
1963*	955.00	3.896	54.9	0.224	1.54	0.00644	620	1740

*This last regression excluded the observations for 300 and 400 kilometers.
#The exchange rates used in this table are the free market exchange rates and were taken from International Financial Statistics, published by the International Monetary Fund. The price indices come from page 196 of Annuario Estatistico do Brasil for 1963.

The fits were so good (the minimum R square was greater than .991) that one supposes that the rate makers set the rates proportional to the length of haul with a fixed charge for handling and loading costs. If the statistics presented in Anmuario Estatistico do Brasil reflect the actual rates charged by Estrada de Ferro Central do Brasil, the freight rates failed to keep up with the inflation. This phenomena has been observed and commented upon in detail in an article by Baer, Kerstenetzky, and Simonsen.[1] From Table 2.1 it is apparent that the inflation resulted not only in a decrease in the deflated freight rates, but that it also caused a change in the structure of the rates, i.e. the cost of long distance shipping relative to short hauls decreased (note the decline of the parameter b both in deflated cruzeiros and in US dollars).

For most of the experimental runs of the model, the coefficients for 1962 were used so that the function was $T = 1.80 + 0.00700(X)$. Thus, for a shipment of a distance of 500 kilometers the railroad freight cost would have been estimated as US$5.30 per ton of steel bars or plates. A set of runs were made using different values of these parameters. This is discussed in Section 6 of Chapter III.

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[1] Werner Baer, Isaac Kerstenetzky, and Marios Henrique Simonsen, "Transportation and Inflation: A Study of Irrational Policy Making in Brazil," Economic Development and Cultural Change, Vol. XIII, No. 2, January 1965, p.188.

While Estrada de Ferro Central do Brasil is probably the railroad which carries the largest share of steel products in Brazil, a number of other railroad lines carry these products. However, as information was not available as to what part of the shipments is carried by each of these lines, the freight costs of Estrada de Ferro Central do Brasil have been used for all product shipments on railroads.

Since in some cases the rates charged by a railroad or a trucking concern may not reflect the real social cost of transportation over a given route, the FORTRAN program written for this study permits the introduction of toll cost on certain routes. For example, in the experimental runs made with this model a toll charge of US$0.50 per metric ton has been added to the freight cost for products transported (by both truck and train) between Santos and the rest of the country, and a charge of US$0.40 has been added to the cost for all products transported between Rio de Janeiro and the rest of the country. In each of these cases it is necessary to transport the products across the Great Escarpment (a mountainous barrier which separates the south central coast of Brazil from the interior). Similarly, toll charges could have been placed on routes where there is congestion.

As data for truck freight costs were not available, they were estimated from the railroad freight costs in the following manner. For distances less than 200 kilometers the truck freight costs were estimated to be 0.9 of the railroad freight costs; for distances

between 200 and 500 kilometers, 1.2 times the railroad freight cost; and for distances greater than 500 kilometers, 1.5 times the railroad freight cost.[1]

Barge and ship freight rates have not been estimated.[2] At present there is little or no shipping of steel products within Brazil by barges or ships; however, in the future there may be such shipments of final products from USIMINAS to Rio de Janeiro and to São Paulo. For the moment it has been assumed that all product from the USIMINAS plant will be shipped to these markets by either rail or truck transport.

The plant and market indices used in Table 2.2 are as follows: plant 1 = COSIPA, plant 2 = USIMINAS, plant 3 = CSN, market area 1 = São Paulo, market area 2 = Rio de Janeiro, and market area 3 = Belo Horizonte. This table gives distances between the markets and plants over feasible routes. The FORTRAN program written to calculate the minimum cost route from each plant to each market employs the cost function shown above, in conjunction with the toll charges, and the distance table to calculate the cost over each of the possible routes

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[1]The idea of using a step function came from Edgar M. Hoover's, The Location of Economic Activity, McGraw-Hill, New York, 1948, pp.19-20.

[2]I am grateful to Gonzalo Vargas for his assistance in studying ocean freight rates. I regret that the empirical part of the study never advanced to the many-country problem so that this information could be used.

and then to select the least-cost route. Table 2.3 shows the results based on the cost parameters estimated for 1962.

In addition to transportation cost for shipping final products, estimates have been made for the freight cost of shipping ingot steel between plants. These costs are based on the railroad freight cost for final products and thus probably slightly overestimate the rates actually charged for shipping these intermediate products.

Table 2.2

Distances Between Plants and Markets

Plant	Market	Route	Truck Km.	Train Km.	Barge (Nautical	Ship Miles)
1	1	1	85	0	0	0
1	1	2	0	85	0	0
1	2	1	5	0	0	220
1	2	2	0	582	0	0
1	2	3	5	0	220	0
1	3	1	642	0	0	0
1	3	2	0	837	0	0
2	1	1	771	0	0	0
2	1	2	0	846	0	0
2	1	3	85	451	480	0
2	2	1	667	0	0	0
2	2	2	0	640	0	0
2	2	3	0	451	260	0
2	3	1	214	0	0	0
2	3	2	0	214	0	0
3	1	1	353	0	0	0
3	1	2	0	353	0	0
3	2	1	146	0	0	0
3	2	2	0	252	0	0
3	3	1	473	0	0	0
3	3	2	0	473	0	0

Table 2.3

Transportation Costs

Plant	Market	Trans. Cost	Route
1	1	2.65	1
1	2	6.37	2
1	3	8.15	2
2	1	7.72	2
2	2	6.28	2
2	3	3.29	2
3	1	4.27	2
3	2	2.93	1
3	3	5.11	2

3. Production Cost

3.1 Form of the Function. Production costs are calculated as a linear function of the inputs and the costs of these inputs, using a function of the form

$$u_i^k = \sum_h f_i^h a_i^{kh} \qquad i = 1,\ldots,n \quad k = 1,\ldots,p \tag{7}$$

where u_i^k is the cost in US$/metric ton for producing a ton of product k in plant i, f_i^h is the cost in US$/unit for factor input h at plant i, and a_i^{kh} is the specific input in units of the factor input h per metric ton of product k at plant i.

A linear function has been used because the two elements of cost that would appear to be the most important non-linear components of cost, capital cost and labor cost, are either excluded from the function or else they account for a small share of total cost.

Capital cost is treated as sunk costs and therefore excluded. That is, for this first part of the study we assume that the interest charges on the loans secured to purchase the capital equipment must be paid, whether the equipment is used or not. Thus, the capital cost is unaffected by decisions as to when, where, and what to produce.

Labor costs, on the other hand, are assumed to be incurred in a linear relation to the scale of output of each plant. This may be a serious departure from reality. Data on labor costs in the production of steel products is so incomplete that it is impossible at present to judge the validity of this assumption.

The cost of production of a ton of pig iron and of ingot steel is shown in the following sections to be calculated as the sum of some ten to fifteen elements. This type of detailed breakdown of the elements of cost at each plant is possible because technical information on the specific consumption of blast furnaces and steel furnaces is relatively easy to come by. As it is more difficult to obtain this type of information for each of the many rolling mills in each steel plant, cost breakdowns for the products of each of these mills have not been made. Rather it has been assumed that the greatest part of the variation in the marginal cost of finished steel products, among steel plants in Brazil, can be attributed to differences in raw material cost and to the different technologies used at the various plants, e.g. the use of LD or open hearth furnaces.[1] Therefore,

[1] In the mixed integer programming model which follows in Chapters IV and V, this admittedly weak assumption was dropped and detailed cost calculations for the conversion cost at each stage of the process were made.

calculations of the costs of final products are based only on their ingot steel costs. The cost of each product is calculated as the ingot steel cost at the plant times the input coefficient for the tons of ingot steel required to produce a ton of the final product. Though there will be some variability in these coefficients from plant to plant; in this study they have been assumed equal for all plants. The coefficients used are 1.28 for steel plate, 1.24 for hot strip and sheet, 1.31 for cold strip and sheet, and 1.40 for tin plate. These coefficients were estimated from data for the plant at Volta Redonda in 1962.

3.2 Specific Inputs. This section and the next consist of descriptions of the specific inputs and unit cost data used for calculating pig iron and ingot steel cost at each of the plants. The abbreviations used in the tables of these two sections are explained in Table 2.4.

A set of specific inputs for the production of pig iron and ingot steel in the three major plants in Brazil is shown in Tables 2.5 and 2.6. This set, which was used for most of the runs of the model, is based on information obtained from the publications of the Instituto Latinoamericano del Fierro y el Acero, ILAFA (the

Latin American Iron and Steel Institute);[1] some studies of the Economic Commission for Latin America, ECLA;[2] annual reports of the steel companies; and data collected by the author while visiting eleven of the major steel mills in Latin America during the latter half of 1964 and the first month of 1965. Appendix B includes tables of the raw data accompanied by detailed comments on the sources.

Some comments on the data in Tables 2.5 and 2.6 are in order at this point. The vectors of input coefficients for the COSIPA plant near Santos are those anticipated by the engineers because neither the blast furnace nor the LD converters were yet in operation in January 1965. With the exception of the labor and maintenance inputs and the coke gas output the data come from engineering studies prepared by the firm and consultants for the firm. The labor and

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[1] See Repertorio de las Empresas Siderurgicas Latinoamericanas, 1962-63, published by ILAFA (Casilla 13810, Santiago, Chile) in 1962. A new edition of this report which gives names of officials of the companies, descriptions of equipment, capacities, and production is to be published in 1965. Also, see the monthly magazine of this organization, Revista Latinoamericana de Siderurgia, and a study done jointly by ILAFA and ECLA and sponsored by the Interamerican Development Bank, Economia Siderurgica Latinoamericana, in particular the "Monografias Nacionales" published by ILAFA in Santiago in 1963.

[2] ECLA, Economia Siderurgica Latinoamericana, op.cit. Also, United Nations, Interregional Symposium on the Application of Modern Technical Practices in the Iron and Steel Industry to Developing Countries, "The Iron and Steel Industry of Latin America, Plans and Perspectives, Steel Symp. 1963/Discussion Paper/ECLA 2, 30 October 1963.

Table 2.4

Abbreviations Used in Tables 2.5 and 2.6

ORE	Regular iron ore
OREF	Iron ore fines
SINTR	Sinter
PELLT	Pellets
RSCRAP	Plant scrap or return scrap
PIG	Molten pig iron
PIGIN	Pig iron in ingots
SPONG	Sponge iron
COALN	National Coal
COALM	Imported Coal
DOLO	Dolomite
LIMST	Limestone
QLIME	Quicklime
MANG	Manganese
ALUM	Aluminum
FLUOR	Fluorspar
FMANG	Ferromanganese
FSILC	Ferrosilicon
FUEL	Fuel Oil
NAGAS	Natural gas
OXY	Oxygen
ELEC	Electrical energy
ELTRG	Graphite electrodes
ELTRP	Paste electrodes
REFRA	Steel furnace refractories
BFGAS	Blast furnace gas
EFBAS	Electric furnace gas
CKGAS	Coke gas
DLAB	Direct labor
ILAB	Indirect labor
MAIN	Maintenance
TONS	Metric tons
1000M3	Thousand cubic meters
1000KH	Thousand kilowatt hours
MANHOR	Man hours
UNITS	Maintenance units (defined as US$ 1 of maintenance cost)

maintenance cost inputs come from the ECLA[1] study and the coke gas output statistic comes from a study on the Mexican steel industry mentioned earlier.[2] The blast furnace at the COSIPA plant is not initially equipped for fuel oil injection, thus the coke rate of 680 kilograms per ton. Once the furnace is modified to permit fuel oil injection one would expect this coefficient to decrease by about 100 kilograms per ton or more while the fuel oil coefficient will increase to between 50 and 100 kg per ton. It appears that there is an error in the COSIPA estimate of the specific input of water per ton of pig iron as this figure is much larger than for the other two plants with no apparent reason. The original COSIPA estimate was used for the linear programming model but modified before runs were begun with the mixed integer programming model.

The USIMINAS data reflect the actual operating conditions in the plant in the months prior to January 1965. At that time only one of the two blast furnaces was in operation and the plant was using a 100 per cent sinter charge. The data shown here assume that both blast furnaces are in operation. A vector of input coefficients for the one blast furnace operating with a 100 per cent sinter charge is shown in Appendix B.

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[1] ECLA, Economic Siderurgica Latinoamericana, op.cit., Ch. 7

[2] Comite para la Programacion de la Industria Siderurgica, op.cit., Volume II, Appendix VIII.

Table 2.5

Specific Inputs - Pig Iron
(Units per Ton of Product)

Input		COSIPA	USIMINAS	CSN
ORE	tons	.700	.470	.782
OREF	tons	0.000	0.000	0.000
SINTR	tons	.760	1.100	.794
PELLT	tons	0.000	0.000	0.000
SCRAP	tons	0.000	0.000	0.000
RSCRP	tons	0.000	0.000	0.000
PIG	tons	0.000	0.000	0.000
PIGIN	tons	0.000	0.000	0.000
SPONG	tons	0.000	0.000	0.000
COALN	tons	0.000	.371	.461
COALM	tons	0.000	.552	.589
COKE	tons	.680	0.000	0.000
DOLO	tons	0.000	.035	.035
LIMST	tons	.400	.132	.272
QLIME	tons	0.000	0.000	0.000
MANG	tons	.025	.040	.018
ALUM	tons	0.000	0.000	0.000
FLUOR	tons	0.000	0.000	0.000
FMANG	tons	0.000	0.000	0.000
FSILC	tons	0.000	0.000	0.000
FUEL	tons	0.000	0.000	.050
NAGAS	1000M3	0.000	0.000	0.000
OXY	1000M3	0.000	0.000	0.000
ELEC	1000KH	.007	.070	.070
ELTRG	tons	0.000	0.000	0.000
ELTRP	tons	0.000	0.000	0.000
WATER	1000M3	.650	.100	.015
STEAM	tons	.020	0.000	0.000
REFRA	tons	0.000	0.000	0.000
BFGAS	1000M3	-3.300	-2.624	-2.624
EFGAS	1000M3	0.000	0.000	0.000
CKGAS	1000M3	-.138	-.138	-.138
SLAG	tons	-.430	-.476	-.476
DLAB	MANHOR	.390	.390	.390
ILAB	MANHOR	.650	.650	.650
MAIN	UNITS	2.700	2.700	2.700

Table 2.6

Specific Inputs - Ingot Steel
(Units per Ton of Product)

Input		COSIPA	USIMINAS	CSN
ORE	tons	.040	.005	.111
OREF	tons	0.000	0.000	0.000
SINTR	tons	0.000	0.000	0.000
PELLT	tons	0.000	0.000	0.000
SCRAP	tons	.220	.187	.276
RSCRP	tons	0.000	.014	.039
PIG	tons	.880	.929	.628
PIGIN	tons	.016	0.000	.097
SPONG	tons	0.000	0.000	0.000
COALN	tons	0.000	0.000	0.000
COALM	tons	0.000	0.000	0.000
COKE	tons	0.000	0.000	0.000
DOLO	tons	0.000	.006	0.000
LIMST	tons	.080	.001	.042
QLIME	tons	.080	.059	0.000
MANG	tons	0.000	0.000	0.000
ALUM	tons	.004	0.000	0.000
FLUOR	tons	.008	.004	0.000
FMANG	tons	.007	0.000	0.000
FSILC	tons	.001	0.000	0.000
FUEL	tons	0.000	0.000	.118
NAGAS	1000M3	0.000	0.000	0.000
OXY	1000M3	.057	.050	.004
ELEC	1000KH	.022	.012	0.000
ELTRG	tons	0.000	0.000	0.000
ELTRP	tons	0.000	0.000	0.000
WATER	1000M3	.016	.029	.020
STEAM	tons	0.000	0.000	0.000
REFRA	tons	.009	.007	.043
BFGAS	1000M3	0.000	0.000	0.000
EFGAS	1000M3	0.000	0.000	0.000
CKGAS	1000M3	0.000	0.000	0.000
SLAG	tons	0.000	0.000	0.000
DLAB	MANHOR	1.980	1.980	.167
ILAB	MANHOR	0.000	0.000	.950
MAIN	UNITS	4.500	4.500	6.340

The coke input for the USIMINAS blast furnace has been divided into input coefficients for national and imported coal. The coke rate was 600 kg -- the low rate resulting from the high sinter charge even without fuel injection. Sinter has some fuel mixed with it; therefore the lower rates. The labor and maintenance input coefficients for all plants come from the ECLA study cited earlier and are based on operating experience of the Volta Redonda plant in 1962. Thus they provide a very rough approximation. Fortunately labor costs are not an important share of total cost in pig iron and ingot steel production. While technical information on raw materials inputs can be readily obtained, the equivalent information on labor input is often more closely held, so that representative data had to be used.

As in the case of COSIPA the coke gas output coefficient comes from the Mexican study mentioned above. The same is true for the blast furnace gas. The dolomite input coefficient may be classified as author's guess.

The CSN input vectors are based on information taken from flow charts for the operation of the plant in 1962. However, some modifications have been made. The estimated coke rate for the blast furnaces at Volta Redonda in 1962 was 764 kilograms per ton of pig iron. Since that date fuel oil injection equipment has been installed. Thus, it is assumed here that 50 kg of fuel oil is being injected per ton of pig iron produced, and that this injection has brought the coke rate

down to 686 kilograms per ton of pig iron. Table 2.5 shows the national and imported coal inputs required to produce 686 kilograms of coke. Modification of the other coefficients which may have occurred as a result of the injection has not been attempted here.

As with the other two plants the gas output coefficients come from the Mexican experience and the labor and maintenance input coefficients from the ECLA study. Since the ECLA statistics on labor input come from a survey conducted in 1962 at Volta Redonda, they offer a reasonable approximation to the present operating situation in the plant.

The specific input coefficients for ingot steel production shown in Table 2.6 come from the same sources as those described for the input coefficients for the production of pig iron. The exceptions to this rule are the labor and maintenance input coefficients. The statistics for CSN come from the ECLA study but those for COSIPA and USIMINAS do not. The 1.98 man hours per ton of ingot steel at USIMINAS is a number obtained by the author at the plant. The 4.50 units of maintenance is estimated to be somewhat less than the 6.34 for the equivalent input at Volta Redonda, since USIMINAS is a new plant with LD converters instead of open hearth furnaces. The direct labor and maintenance coefficients for COSIPA (also a new plant) are assumed equal to those for USIMINAS.

Except for exceptional cases such as the case of the sinter charge at USIMINAS these input coefficients do not change markedly with changes in the output level of the plant. At USIMINAS the sinter charge will decrease from the 100 per cent level now used to the level shown in Table 2.5 as soon as the second blast furnace is brought into operation.

The input coefficients may not be strongly affected by the level of output in a plant, but they do change significantly with modifications to the equipment. Because of the discrete nature of changes in technology, these changes require the replacement of a vector of specific inputs by a new vector. The facility with which these vectors can be changed, new cost calculations made, and the elements of the linear programming matrix revised, increases the usefulness of this type of model in studying the effects of technical changes. A series of tests with different technologies were conducted on this model. They are discussed in section 3 of Chapter III.

3.3 Unit Cost of Inputs. Table 2.7 gives the unit cost (delivered at the plant) of inputs used for most of the experiments conducted with the model. It has been assumed that the supply of these inputs is perfectly elastic at the prices shown.

The unit cost vector for COSIPA is based on estimates of the Company in 1964. The basic data were in cruzeiros of July 1964 and were therefore converted to dollars at the free exchange rate of

that date, 13,000 cruzerios per US dollars. The data for USIMINAS were for January 1963 and were in cruzeiros of that date. Therefore, they were converted to dollars at the free exchange rate of 475 cruzerios/U.S. dollar. The data for CSN were estimated by the author on the basis of the unit cost for inputs at USIMINAS and COSIPA and a knowledge of the source of the raw materials used at Volta Redonda. Labor and maintenance cost estimates were taken from the ECLA study mentioned above.

The cost of coke inputs at the different plants was calculated as a weighted average of the cost of national and imported coals delivered at the plant. The weights employed were the percentage of national and imported coals used in each of the plants. Both COSIPA and USIMINAS used 40 per cent national and 60 per cent imported coal while CSN used 44 per cent national and 56 per cent imported in 1962.[1]

Because of the tendency of most steel companies to prohibit the release of any cost information, it is difficult to obtain this kind of data. For planning models of this sort one needs not only observations on present cost of inputs (and better yet, some measure of the variability of these costs) but also some projections about how these costs may be expected to change over the time period included in the study.

The present state of data availability in Brazil limits one to present (or recent past) prices in the form of single observations

[1]This information was obtained by the author in interviews at the steel plants and company offices.

Table 2.7

Unit Cost of Inputs
(Dollars per Unit)

Input		COSIPA	USIMINAS	CSN
ORE	tons	5.69	2.84	3.50
OREF	tons	0.00	2.27	0.00
SINTR	tons	11.70	5.54	9.00
PELLT	tons	0.00	0.00	0.00
SCRAP	tons	23.10	32.20	24.00
RSCRP	tons	25.00	35.00	26.00
PIG	tons	34.45	30.48	38.15
PIGIN	tons	31.00	27.43	34.34
SPONG	tons	0.00	0.00	0.00
COALN	tons	25.60	29.90	27.00
COALM	tons	21.00	18.35	22.00
COKE	tons	29.85	30.65	31.85
DOLO	tons	0.00	0.00	0.00
LIMST	tons	4.61	5.04	5.00
QLIME	tons	6.00	6.00	6.00
MANG	tons	42.30	35.00	35.00
ALUM	tons	0.00	0.00	0.00
FLUOR	tons	20.00	20.00	20.00
FMANG	tons	280.00	280.00	280.00
FSILC	tons	344.00	344.00	344.00
FUEL	tons	27.60	25.00	28.00
NAGAS	1000M3	0.00	0.00	0.00
OXY	1000M3	40.00	40.00	40.00
ELEC	1000KH	16.70	14.00	16.00
ELTRG	tons	0.00	0.00	0.00
ELTRP	tons	0.00	0.00	0.00
WATER	1000M3	6.05	5.00	5.00
STEAM	tons	3.60	3.60	3.60
REFRA	tons	117.00	117.00	117.00
BFGAS	1000M3	2.18	1.50	1.50
EFGAS	1000M3	0.00	0.00	0.00
CKGAS	1000M3	12.60	7.00	7.00
SLAG	tons	0.00	0.00	0.00
DLAB	MANHOR	.42	.42	.42
ILAB	MANHOR	.50	.50	.50
MAIN	UNITS	1.00	1.00	1.00

without an indication of the variation of the prices. This situation is made more difficult by the inflation. While one can deflate data obtained for different time periods or can use the free market exchange rate, these measures assume that there is no change in the relative prices of inputs to iron and steel production.

Difficulties of the type described above suggest that it is wise to use the results of present planning models with care -- but not to abstain from building the models. The data are improving and can be used with increasing confidence. More important, the models provide insight into the relative importance of different types of data. It may be that labor cost is such a small part of total cost in the steel industry and so stable over time that relatively rough measures may be used while coal costs are so volatile and vary so much from location to location that detailed study of them is very important. Furthermore, it may be that distances between plants are so great that even relatively large variations in cost would not affect competitive positions in key market areas. The same holds true for variations in rates of exchange. It may be that rates of exchange vary so much that economic studies across national boundaries are useless. This may be right, but this is something that one can not be sure about until it is determined how closely competitive plants in various countries will be for market areas in one or the other country, and until one knows something about the variation in exchange rates and can test the stability of investment study models within the limits of the variations that are known to take place.

The cost breakdowns given in Tables 2.8 and 2.9 provide a measure of the relative importance of the different inputs in determining the total cost of pig iron and ingot steel.

Though a systematic test of the model under changes in the prices of inputs has not been made, a number of runs with different cost parameters have been made to provide some indication of the properties of the model. The results of these tests are discussed in the next chapter.

4. The Matrix.

Once transportation and production costs are calculated, they are input to a third FORTRAN program which uses these values to calculate some of the objective function values. Other objective function values, constraint values, and matrix coefficients are read in by the program. The program then punches out, in the required format, the input deck for the linear programming problem.

A list of the abbreviations used in the linear programming matrix is given in Table 2.10 and the matrix itself is shown in Figure 2.1.[1] A brief introduction to the problem is followed by an explanation of each group of columns and rows in the matrix.

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[1]The page after Table 2.10 and prior to Figure 2.1 is a guide to the mnemonics which are most frequently used in this paper.

Table 2.8

Cost Breakdown
(Dollars per Ton of Pig Iron)

Input	COSIPA	USIMINAS	CSN
ORE	3.98	1.33	2.73
OREF	0.00	0.00	0.00
SINTR	8.89	6.09	7.14
PELLT	0.00	0.00	0.00
SCRAP	0.00	0.00	0.00
RSCRP	0.00	0.00	0.00
PIG	0.00	0.00	0.00
PIGIN	0.00	0.00	0.00
SPONG	0.00	0.00	0.00
COALN	0.00	11.09	12.44
COALM	0.00	10.12	12.95
COKE	20.30	0.00	0.00
DOLO	0.00	0.00	0.00
LIMST	1.84	.66	1.36
QLIME	0.00	0.00	0.00
MANG	1.05	1.40	.63
ALUM	0.00	0.00	0.00
FLUOR	0.00	0.00	0.00
FMANG	0.00	0.00	0.00
FSILC	0.00	0.00	0.00
FUEL	0.00	0.00	1.40
NAGAS	0.00	0.00	0.00
OXY	0.00	0.00	0.00
ELEC	.11	.98	1.12
ELTPG	0.00	0.00	0.00
ELTRP	0.00	0.00	0.00
WATER	3.93	.50	.07
STEAM	.07	0.00	0.00
REFRA	0.00	0.00	0.00
BFGAS	-7.19	-3.93	-3.93
EFGAS	0.00	0.00	0.00
CKGAS	-1.73	-.96	-.96
SLAG	0.00	0.00	0.00
DLAB	.16	.16	.16
ILAB	.32	.32	.32
MAIN	2.70	2.70	2.70
TOTAL	34.45	30.48	38.15

Table 2.9

Cost Breakdown
(Dollars per Ton of Ingot Steel

Input	COSIPA	USIMINAS	CSN
ORE	.22	.01	.38
OREF	0.00	0.00	0.00
SINTR	0.00	0.00	0.00
PELLT	0.00	0.00	0.00
SCRAP	5.08	6.02	6.62
RSCRP	0.00	.49	1.01
PIG	30.32	28.31	23.96
PIGIN	.49	0.00	3.33
SPONG	0.00	0.00	0.00
COALN	0.00	0.00	0.00
COALM	0.00	0.00	0.00
COKE	0.00	0.00	0.00
DOLO	0.00	0.00	0.00
LIMST	.36	0.00	.21
QLIME	.48	.35	0.00
MANG	0.00	0.00	0.00
ALUM	0.00	0.00	0.00
FLUOR	.16	.08	0.00
FMANG	1.96	0.00	0.00
FSILC	.34	0.00	0.00
FUEL	0.00	0.00	3.30
NAGAS	0.00	0.00	0.00
OXY	2.28	2.00	.16
ELEC	.36	.16	0.00
ELTRG	0.00	0.00	0.00
ELTRP	0.00	0.00	0.00
WATER	.09	.14	.10
STEAM	0.00	0.00	0.00
REFRA	1.05	.81	5.03
BFGAS	0.00	0.00	0.00
EFGAS	0.00	0.00	0.00
CKGAS	0.00	0.00	0.00
SLAG	0.00	0.00	0.00
DLAB	.83	.83	.07
ILAB	0.00	0.00	.47
MAIN	4.50	4.50	6.34
TOTAL	48.56	43.74	51.01

Table 2.10

Abbreviations Used in the Linear Programming Matrix

Abbreviation	Explanation
SA	COSIPA plant near Santos
US	USIMINAS plant near Ipatinga
VR	CSN plant near Volta Redonda
SP	market area around São Paulo
RJ	market area around Rio de Janeiro
BH	market area around Belo Horizonte
S	product index for ingot steel
P	product index for steel plate
H	product index for hot sheet and strip
C	product index for cold sheet and strip
T	product index for tin plate
S	steel furnaces
P	primary rolling mill
R	roughing mill
H	hot strip finishing mill
C	cold strip mill
T	tinning line

Guide to Mnemonics

Column Mnemonics

USSAS USIMINAS to Santos: ingot steel
VRSAS Volta Redonda to Santos: ingot steel

SASPP Santos to São Paulo: plate
SARJP Santos to Rio de Janeiro: plate
USRJP USIMINAS to Rio de Janeiro: plate
USBHP USIMINAS to Belo Horizonte: plate
VRSPP Volta Redonda to São Paulo: plate
VRRJP Volta R. to Rio de Janeiro: plate

SASPH Santos to São Paulo: hot sheet
USRJH USIMINAS to Rio de Janeiro: hot sheet
USBHH USIMINAS to Belo Horizonte: hot sheet
VRSPH Volta Redonda to Sao Paulo: hot sheet
VRRJH Volta R. to Rio de Janeiro: hot sheet

SASPC Santos to São Paulo: cold sheet
SARJC Santos to Rio de Janeiro: cold sheet
SABHC Santos to Belo Horizonte: cold sheet
USRJC USIMINAS to Rio: cold sheet
USBHC USIMINAS to Belo: cold sheet
VRSPC Volta Redonda to São Paulo: cold sheet
VRRJC Volta Redonda to Rio: cold sheet
VRBHC Volta Redonda to Belo: cold sheet

SASPT Santos to São Paulo: tin plate
USBHT USIMINAS to Belo: tin plate
VRSPT Volta Redonda to São Paulo: tin plate
VRRJT Volta Redonda to Rio: tin plate
VRBHT Volta Redonda to Belo: tin plate

IMSPP Imports to São Paulo: plate
IMRJP Imports to Rio de Janeiro: plate
IMRJH Imports to Rio de Janeiro: hot sheet
IMSPC Imports to São Paulo: cold sheet
IMRJC Imports to Rio de Janeiro: cold sheet
IMSPT Imports to São Paulo: tin plate
IMRJT Imports to Rio de Janeiro: tin plate

SAEXH Santos to Exports: hot sheet
VREXH Volta Redonda to Exports: hot sheet
SAEXC Santos to Exports: cold sheet
VREXC Volta Redonda to Exports: cold sheet
VREXT Volta Redonda to Exports: tin plate

Row Mnemonics

Capacity - Plants

SAS Santos: steel plant
SAP Santos: primary mill
SAR Santos: roughing mill
SAH Santos: hot strip mill
SAC Santos: cold strip mill
SAT Santos: tinning line

USS USIMINAS: steel plant
USP USIMINAS: primary mill
USR USIMINAS: roughing mill
USH USIMINAS: hot strip mill
USC USIMINAS: cold strip mill
UST USIMINAS: tinning line

VRS Volta Redonda: steel plant
VRP Volta Redonda: primary mill
VRR Volta Redonda: roughing mill
VRH Volta Redonda: hot strip mill
VRC Volta Redonda: cold " "
VRT Volta Redonda: tinning line

Requirements - Markets

SPP São Paulo: steel plate
SPH São Paulo: hot sheet
SPC São Paulo: cold sheet
SPT São Paulo: tin plate

RJP Rio de Janeiro: steel plate
RJH Rio de Janeiro: hot sheet
RJC Rio de Janeiro: cold sheet
RJT Rio de Janeiro: tin plate

BHP Belo Horizonte: steel plate
BHH Belo Horizonte: hot sheet
BHC Belo Horizonte: cold sheet
BHT Belo Horizonte: tin plate

		1 SAUSS	2 SAVRS	3 USSAS	4 USVRS	5 VRSAS	6 VRUSS
0	COST	18.54	6.63	8.90	2.90	11.52	17.43
1	SAS	1.00	1.00	-1.00		-1.00	
2	SAP						
3	SAR						
4	SAH						
5	SAC						
6	SAT						
7	USS	-1.00		1.00	1.00		-1.00
8	USP						
9	USR						
10	USH						
11	USC						
12	UST						
13	VRS		-1.00		-1.00	1.00	1.00
14	VRP						
15	VRR						
16	VRH						
17	VRC						
18	VRT						
19	SPP						
20	SPH						
21	SPC						
22	SPT						
23	RJP						
24	RJH						
25	RJC						
26	RJT						
27	BHP						
28	BHH						
29	BHC						
30	BHT						

Figure 2.1 - The LP Matrix

		7 SASPP	8 SARJP	9 SABHP	10 USSPP	11 USRJP	12 USBHP	13 VRSPP	14 VRRJP	15 VRBHP	16 SASPH	17 SARJH	18 SABHH	19 USSPH	20 USRJH	21 USBHH
0	COST	67.12	75.62	72.44	65.33		60.91	71.78	70.43	72.02	65.17	68.71	70.50	65.33	62.14	59.16
1	SAS	1.28	1.28	1.28							1.24	1.24	1.24			
2	SAP	1.28	1.28	1.28							1.24	1.24	1.24			
3	SAR	1.10	1.10	1.10							1.09	1.09	1.09			
4	SAH										1.05	1.05	1.05			
5	SAC															
6	SAT															
7	USS				1.28	1.28	1.28							1.24	1.24	1.24
8	USP				1.28	1.28	1.28							1.24	1.24	1.24
9	USR				1.10	1.10	1.10							1.09	1.09	1.09
10	USH													1.05	1.05	1.05
11	USC															
12	UST															
13	VRS							1.28	1.28	1.28						
14	VRP							1.28	1.28	1.28						
15	VRR							1.10	1.10	1.10						
16	VRH															
17	VRC															
18	VRT															
19	SPP	1.00			1.00			1.00								
20	SPH										1.00			1.00		
21	SPC															
22	SPT															
23	RJP		1.00			1.00			1.00							
24	RJH											1.00			1.00	
25	RJC															
26	RJT															
27	BHP			1.00			1.00			1.00						
28	BHH												1.00			1.00
29	BHC															
30	BHT															

Figure 2.1

		22 VRSPH	23 VRRJH	24 VRBHH	25 SASPC	26 SARJC	27 SABHC	28 USSPC	29 USRJC	30 USBHC	31 VRSPC	32 VRRJC	33 VRBHC
0	COST	69.14	68.39	69.[illegible]	[illegible]	72.[illegible]	73.9[illegible]	[illegible]	65.20	[illegible]	[illegible]	[illegible]	73.55
1	SAS				1.31	1.31	1.31						
2	SAP				1.31	1.31	1.31						
3	SAR				1.15	1.15	1.15						
4	SAH				1.12	1.12	1.12						
5	SAC				1.03	1.03	1.03						
6	SAT												
7	USS							1.31	1.31	1.31			
8	USP							1.31	1.31	1.31			
9	USR							1.15	1.15	1.15			
10	USH							1.12	1.12	1.12			
11	USC							1.03	1.03	1.03			
12	UST												
13	VRS	1.24	1.24	1.24							1.31	1.31	1.31
14	VRP	1.24	1.24	1.24							1.31	1.31	1.31
15	VRR	1.09	1.09	1.09							1.15	1.15	1.15
16	VRH	1.05	1.05	1.05							1.12	1.12	1.12
17	VRC										1.03	1.03	1.03
18	VRT												
19	SPP												
20	SPH	1.00											
21	SPC				1.00			1.00			1.00		
22	SPT												
23	RJP												
24	RJH		1.00										
25	RJC					1.00			1.00			1.00	
26	RJT												
27	BHP												
28	BHH			1.00									
29	BHC						1.00			1.00			1.00
30	BHT												

Figure 2.1

		34 SASPT	35 SARJT	36 SABHT	37 USSPT	38 USRJT	39 USBHT	40 VRSPT	41 VRRJT	42 VRBHT
0	COST	72.94	76.43	78.27	70.58	69.14	66.16	77.30	76.55	78.14
1	SAS	1.40	1.40	1.40						
2	SAP	1.40	1.40	1.40						
3	SAR	1.23	1.23	1.23						
4	SAH	1.20	1.20	1.20						
5	SAC	1.11	1.11	1.11						
6	SAT	1.05	1.05	1.05						
7	USS				1.40	1.40	1.40			
8	USP				1.40	1.40	1.40			
9	USR				1.23	1.23	1.23			
10	USH				1.20	1.20	1.20			
11	USC				1.11	1.11	1.11			
12	UST				1.05	1.05	1.05			
13	VRS							1.40	1.40	1.40
14	VRP							1.40	1.40	1.40
15	VRR							1.23	1.23	1.23
16	VRH							1.20	1.20	1.20
17	VRC							1.11	1.11	1.11
18	VRT							1.05	1.05	1.05
19	SPP									
20	SPH									
21	SPC									
22	SPT	1.00			1.00			1.00		
23	RJP									
24	RJH									
25	RJC									
26	RJT		1.00			1.00			1.00	
27	BHP									
28	BHH									
29	BHC									
30	BHT			1.00						1.00

Figure 2.1

		43	44	45	46	47	48	49	50	51	52	53	54
		IMSPP	IMSPH	IMSPC	IMSPT	IMRJP	IMRJH	IMRJC	IMRJT	IMBHP	IMBHH	IMBHC	IMBHT
		102.	102.	112.	122.	100.	100.	100.	120.	108.	108.	118.	128.
0	COS												
1	SAS												
2	SAP												
3	SAR												
4	SAH												
5	SAC												
6	SAT												
7	USS												
8	USP												
9	USR												
10	USH												
11	USC												
12	UST												
13	VRS												
14	VRP												
15	VRR												
16	VRH												
17	VRC												
18	VRT												
19	SPP	1.00											
20	SPH		1.00										
21	SPC			1.00									
22	SPT				1.00								
23	RJP					1.00							
24	RJH						1.00						
25	RJC							1.00					
26	RJT								1.00				
27	BHP									1.00			
28	BHH										1.00		
29	BHC											1.00	
30	BHT												1.00

Figure 2.1
Part 5

		55 SAEXP	56 USEXP	57 VREXP	58 SAEXH	59 USEXH	60 VREXH	61 SAEXC	62 USEXC	63 VREXC	64 SAEXT	65 USEXT	66 VREXT		
0	COST	-2.00	-2.00	-2.00	-3.00	-3.00	-3.00	-4.00	-4.00	-4.00	-5.00	-5.00	-5.00		
1	SAS	1.28			1.24			1.31			1.40			≤	$\min\left[\frac{1}{.880}(.72), 0.80\right]$
2	SAP	1.28			1.24			1.31			1.40			≤	1.80
3	SAR	1.10			1.09			1.15			1.23			≤	1.00
4	SAH				1.05			1.12			1.20			≤	1.50
5	SAC							1.03			1.11			≤	.35
6	SAT										1.05			≤	.00
7	USS		1.28			1.24			1.31			1.40		≤	$\min\left[\frac{1}{.929}(.70), 0.60\right]$
8	USP		1.28			1.24			1.31			1.40		≤	1.80
9	USR		1.10			1.09			1.15			1.23		≤	1.00
10	USH					1.05			1.12			1.20		≤	1.50
11	USC								1.03			1.11		≤	.15
12	UST											1.05		≤	.00
13	VRS			1.28			1.24			1.31			1.40	≤	$\min\left[\frac{1}{.629}(1.00)-.15, 1.62-.2\right.$
14	VRP			1.28			1.24			1.31			1.40	≤	1.40
15	VRR			1.10			1.09			1.15			1.23	≤	1.25
16	VRH						1.05			1.12			1.20	≤	1.50
17	VRC									1.03			1.11	≤	.80
18	VRT												1.05	≤	.28
19	SPP													≥	.02
20	SPH													≥	.20
21	SPC													≥	.30
22	SPT													≥	.15
23	RJP													≥	.16
24	RJH													≥	.12
25	RJC													≥	.18
26	RJT													≥	.06
27	BHP													≥	.02
28	BHH													≥	.08
29	BHC													≥	.14
30	BHT													≥	.04

Figure 2.1
Part 6

The matrix consists of an objective row, "COST"; thirty other rows and sixty-six columns. The columns fall into four groups. The first of these is the set of activities for producing ingot steel at one plant and shipping it to another plant. The second group is production-transportation activities such as SASPP, i.e. the production of steel plate at the Santos plant and its shipment to the market area of São Paulo. The third and fourth groups are, respectively, imports and exports.

The rows may be divided into three groups. The first group consists of a single row, which is the objective row, "COST." The second group of rows is the set of production constraint rows, which prevents the output of each of the production units from exceeding its capacity. The last group of rows requires that the total production and transportation of products to each market area be at least sufficient to supply the requirements for each good.

The problem presented in the matrix is akin to the classic machine-scheduling problem. The machine-scheduling problem is one of determining how much of each of a number of different products to manufacture, given that each product requires a certain time to process on each of a number of different machines. The capacity of each machine, in hours of use, and the profit to be gained from the production of each product are given. Similarly, in our problem the capacities of the blast furnace-steel shop, the primary mill, the roughing mill,

the finishing mill, and the cold strip mill, and the tinning line for each of the three plants are given. The profits for each product are not given, but rather the problem is constrained (by the third group of rows in the matrix) to produce or import at least the required quantity of steel plate, hot sheet and strip, cold sheet and strip, and tin plate for each of the market areas. And, in addition, it is required to do this at the minimum possible cost, given the cost of operating each of the activity vectors at unit level. That is, the problem is to find that combination of production-transportation activities, exporting activities, and importing activities that will minimize the total cost of flat steel products in Brazil, while meeting the requirements of each market area, and not exceeding the capacity constraints of the existing production units in the plants. An explanation of the elements of the matrix of Figure 2 will clarify the problem.

The first six columns of the matrix are activity vectors for the production of ingot steel in one plant and its shipment to another plant as an intermediate product. (Figure 2.1, Part 1.) For example, SAUSS (column #1) is the activity name for the production of ingot steel at Santos and its delivery to the USIMINAS plant at Ipatinga. The cost of this activity is US$18.54 per metric ton. This is equal to the cost of producing a ton of ingot steel at Santos, minus the cost of producing a ton of ingot steel at USIMINAS, plus the cost of

transporting the ton of ingot steel from Santos to USIMINAS. This is the net cost to the system of this shipment of intermediate product.

Proceeding down the first column one finds a 1.00 in the SAS row. This means that when activity SAUSS is operated at level x_1, that $(1.00)(x_1)$ units of the available steel capacity at Santos are used up. Thus, if $x_1 = .150$, then 150 thousand metric tons of ingot steel are produced at Santos and shipped to USIMINAS, thereby reducing the quantity of steel available for producing final products at Santos by 150 thousand tons. Page 6 of Figure 2 shows that the constraint for the SAS row is the minimum of (.72)/(.88) and (.80). This requires that the ingot steel used for shipment to other plants and for use in final products, whether consumed domestically or exported, must be less than the minimum of the steel-equivalent capacity of the blast furnace or furnaces and the capacity of the steel shop. The capacity of the blast furnace at Santos is 720 thousand tons, and the specific input of pig iron to ingot steel production is 880 kilograms of pig iron per ton of ingot steel. The capacity of the LD converters in the steel shop is 800 thousand tons per year.

Returning to column #1 of the matrix we encounter a value of minus one in the USS row. This means that each ton of ingot steel shipped from Santos to USIMINAS decreases by one unit the constraint on steel capacity at USIMINAS.

The remaining five columns of this group (columns 2-6) repeat the pattern of column one for each possible combination of shipping and receiving plant, with a plus one in the row for the steel shop of the shipping plant and a minus one in the row for the steel shop of the receiving plant.

Column seven begins the second group of activity vectors. This group includes columns 7 through 42 and contains an activity vector for the shipment to each of the three market areas from each of the three plants for each of the four final products, and thus includes (3)(3)(4) or 36 activity vectors.

The first vector SASPP (column 7) is for the production of steel plate at Santos and its shipment to São Paulo. Thus, US$67.12 is the cost per ton for the production and transportation of the product. The coefficient of 1.28 in the SAS rows means that for each ton of steel plate produced at Santos 1.28 tons of ingot steel are required. Similarly, the 1.28 in the second row, SAP, indicates that 1.28 tons of capacity in the primary mill at Santos is required for each ton of steel plate that is shipped. Likewise, before it is finished and ready for shipping the plate must pass through the roughing mill, but by the time the product reaches the roughing mill most of the scrap losses have already occurred. Therefore, only 1.10 tons of annual capacity at the roughing mill are required for each ton of plate.

Turning to page six of the matrix, we find that the capacity of the primary mill at Santos is 1.80 million tons per year and the capacity of the roughing mill is 1.00 million tons per year. It is interesting to observe the variation in the capacity of the five main production units at the new steel plant at Santos. The steel equivalent capacity of the blast furnace is slightly more than 800 thousand tons per year, the capacity of the steel shop is 800 thousand tons, the primary mill 1.8 million tons, the roughing mill 1.0 million tons, and the finishing mill 1.5 million tons. A part of this variation is because each of these units is required to process different amounts of product, even in a facility that is using all of them at capacity. However, a much more significant part of the variation is due to the technology of the equipment. The high capacity of the primary mill results from the fact that the economies of scale in the investment cost of this type of mill are strong up to capacities in this range. The same is true for the finishing mill, but not for the roughing mill where it is possible to install more stands as they are needed to increase the capacity. For the blast furnace and steel shop marginal investments can be made to increase substantially the present capacities. It is only mentioned here to point out why it is essential in a steel mill study to have a constraint row for each of the main production units and to think of investment as breaking first one bottleneck and then another, or as breaking two or three bottlenecks simultaneously.

In column 7 of the matrix there is a coefficient of 1.00 in row 19. This row is in the group of rows which requires that any solution to the problem be one for which the requirements of each market area for each product are satisfied. In this row, SPP, (the annual requirement for plate in São Paulo) there are coefficients of 1.00 in columns 10 and 13. These two activities are also for the shipment of steel plate to São Paulo, the first being for shipments from USIMINAS and the second for shipments from Volta Redonda. The only other entry in this row is in column 43; it is for the importation of plate to São Paulo. The right-hand side value for this constraint (on Part 6 of Figure 2.1) is .020. Thus, the annual requirement for steel plate in the São Paulo market area is 20 thousand tons, and this requirement must be fulfilled by shipments from some combination of the three plants and by imports.

Columns 7 through 9 are activity vectors for the production and shipment of steel plate from Santos to each of the three market areas, São Paulo, Rio de Janeiro, and Belo Horizonte. Columns 10-12 and 13-15 are for shipments of plate from USIMINAS and from Volta Redonda, respectively.

Column 16, SASPH, for the shipment from Santos to São Paulo of hot strip and sheets, differs only slightly from column 7. The scrap losses for hot sheet and strip are slightly less than for steel plate so the first three coefficients in the column (after the objective function value) are slightly less than the corresponding values in column 7. Also, there is the addition of a fourth coefficient.

This one being in row SAH, which is the capacity row for the hot strip finishing mill at Santos. Plate is not required to pass through the hot strip finishing mill, but hot strip and sheets are required to pass through this mill; therefore, the row SAH includes the coefficient shown.

Columns 16 through 24 are the activity vectors for the production and transportation of hot sheet and strip, and columns 25 through 33 and 34 through 42 are the equivalent vectors for cold sheet and strip and for tin plate.

This scheme portrays graphically the assumption of linearity of cost and independence of activities which is required of any linear programming problem. The cost associated with each activity is seen to be a linear function of the level of operation assigned to that activity and is not affected by changes in the activity level of any other vector. Thus, we are assuming that the quantity of tin plate produced by a plant does not affect the unit cost of the hot strip and sheet produced by the same plant.

The third group of columns (columns 43-54) is the activity vectors for the importation of steel products. There is one activity vector for each product for each market, or (4)(3)=12.

The cost of imports was set arbitrarily somewhat above domestic cost for the equivalent product on the assumption that the government would protect the industry. Tariffs collected by the government are assumed to be outside of the system and not used to minimize the total

cost of steel in the system. A base price of US$100 per ton for the importation of steel plate to Rio de Janeiro was set. Hot strip was assumed to be more expensive than plate, and cold strip and tin plate were assumed to be progressively more expensive than hot strip. Because the cost desired was the cost of the product delivered to the using firm in the market area, imports for the products to São Paulo were made US$2 per metric ton more than for the equivalent import to Rio de Janeiro. The similar differential between Belo Horizonte and Rio de Janeiro was assumed to be US$8 per ton.

While there is no constraint in this model on the total amount of imports, such a constraint could be provided either by putting upper bounds on the activity levels of the import activities or by adding another constraint row.

The last group of activity vectors (columns 55-66) include one vector for the export of each of the four products from each of the three plants. The coefficients within the matrix are the same as those for the production-for-domestic-consumption vectors. The objective function coefficients have been assigned negative values. Thus, exports are treated simply as a means of decreasing the total cost in the system. These values are the per unit profit for export activities and have greater absolute value for products which require more processing. For the same product the profit to be gained by each firm is assumed to be the same.

The availability of export profits induces the system to use each plant up to the capacity bottleneck of that plant, but since these profits are relatively small, the system first supplies the needs of the domestic economy. The greatest profits are on those products which require the most processing. Therefore, these are the preferred exports.

The less-than-or-equal-to elements of the right-hand side of the matrix (the constraint values in Part 6 of Figure 2.1) are the annual capacities of the production units in January of 1965. The three exceptions to this rule are the blast furnace and steel shop at Santos which were scheduled to go into operation in the summer of 1965, the hot and cold strip mills at USIMINAS which were also scheduled to be put into operation sometime in 1965, and the tinning line of Volta Redonda which has a capacity below the 280 thousand tons shown in the matrix, but which will reach that capacity within a year or so.

Though capacity constraints are given here as a single number, the capacity of a production unit in a steel mill is not an unambiguous concept. First, the capacity of output of a rolling mill depends upon the product mix. A primary mill which is used for producing a high percentage of slabs will have a higher capacity than the equivalent mill which is required to produce a high percentage of blooms. Secondly, the capacity of a blast furnace or a steel furnace is affected by changes in the inputs. A blast furnace with a high

percentage of sinter or pellet in the charge will produce more than the equivalent furnace charged with lump ore. Likewise, the percentage of scrap charged to an open hearth furnace affects the output of the furnace. Thirdly, the capacity constraints are not really absolute bottlenecks. For example, if a shortage of blast furnace capacity is the bottleneck in a plant the percentage of scrap in the steel furnace charge can be increased, or intermediate products of any desired degree of processing may be purchased. While the model permits the shipment of ingot steel between plants, it does not provide for the purchase of other forms of intermediate products such as pig iron, slabs, or coils of hot sheet. Fourthly, capacity information given by a company about its production units does not always agree with an economist's notion of what capacity is. Blast furnaces which were installed with a given rated capacity may, in fact, produce as much as twice their rated capacity through increases in their interior dimensions, fuel oil injection, higher blast temperatures and volumes, higher top pressures, more careful sizing of ores, the use of sinter and/or pellets in the charge, or the control of humidity in the furnace. Also, rolling mill capacities may be given as the capacity of the smallest unit in a line. Thus, a hot strip finishing mill of 2 million tons per year capacity which follows a semi-continuous rougher of 1 million tons per year may be classified as having a capacity of only one million tons. That is, the capacity stated is given according to the bottleneck. Fifthly,

capacity figures usually tell nothing at all about the cost of the modifications required to change the capacity of the unit. For example, a semi-continuous rougher installed so that additional stands can be easily added to increase the capacity is really something quite different from a continuous rougher of the same capacity which was installed in such a small working area that its capacity can not be increased.

The market requirements, shown as the constraint values for rows 19 thru 30, are based on an unpublished study of the Latin American Iron and Steel Institute, ILAFA,[1] and are indeed a very rough approximation of the reality -- especially so as a regional breakdown of the requirements for various products was not available.

The ILAFA study gives the apparent consumption of flat products in Brazil in 1962 as 860 thousand tons and shows a growth rate of around 100 thousand tons per year over the period of the previous few years. Assuming that steel consumption is growing at an increasing rate in Brazil, a figure of 1,200 million tons was decided upon for the apparent consumption of flat products in Brazil in 1965. It was assumed that 200 thousand tons of this would be steel plate, 400 thousand tons would be hot sheet and strip, and 600 thousand tons would be cold sheet and strip. Also, it was assumed that the country would consume 250 thousand tons of tin plate in 1965.

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[1] Instituto Latinoamericano del Fierro y el Acero, Series Historicas de Consumo Aparente, Produccion, Importacion y Exportacion de Productos Laminados en America Latina, Periodo 1951-1962, Santiago, Chile, January, 1964.

Because of the concentration of the ship-building industry in Rio de Janeiro, 80 per cent of the country's apparent consumption of steel plate was assigned to that region while the remaining 20 per cent was divided evenly between São Paulo and Belo Horizonte. Forty per cent of the hot sheet and strip requirement was assigned to Rio and forty per cent to São Paulo, with the remaining 20 per cent going to Belo Horizonte. The centralization of the automobile industry around São Paulo dictated the assignment of 50 per cent of the demand for cold sheet and strip to that market area, with Rio de Janeiro and Belo Horizonte receiving 37 and 23 per cent respectively. Sixty per cent of the tin plate requirement was assigned to São Paulo, 24 per cent to Rio and 16 per cent to Belo Horizonte.

5. The Algorithm

All computation with this model was done on the IBM 1620 computer of the Sloan School for Industrial Management of M.I.T.[1] This computer is equipped with two small disk drives permitting the use of the IBM 1620-1311 Linear Programming System.

The input deck for use with this programming system was punched out by the third of the three data organizing programs discussed above. Once the original data were read in and the problem solved (a process which required about ten minutes), the basis[2] of the solution

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[1]Appreciation is due the M.I.T. Industrial Management Computer Facility for use of the IBM 1620.

[2]"Basis" means the subset of activity vectors that comprise the basis for the optimal feasible solution to the problem.

was saved on the disk. Then to make additional test on the model some of the elements of the LP matrix were changed. The saved basis provided a starting point for the additional runs and permitted solution of each of them within an average time of about five minutes. Actually the solution time was probably shorter -- on the order of two minutes for each revision of the input date -- but as a substantial amount of output was desired for each solution, the output time required was two or three minutes.

For an industrial model a single solution to a linear programming problem is of limited usefulness. Even when high quality data are available for use in the model, uncertainty about the future means that any one of the parameters of the model may change enough to make an optimum solution no longer feasible or no longer optimal. The great value to be gained from using computer models for investment studies is that once the model is constructed, it provides an excellent laboratory. It is possible to hold all other parameters constant while varying one to determine its effect on the system. Thus, one can use the model to study the effects of changes in the cost of one of the inputs to steel production, or the effects of an investment in transportation which decreases freight cost in one leg of the system, or the impact of a new steel using industry being located in one market area rather than in another.

Models which are small enough that they require only a few minutes for solution on the available computer are thus much to be

preferred over models which require several hours of computation-time for solution on the available equipment and which thereby permit only two or three production runs in the course of a project.

The 1620-1311 LP System permits the user to call for a number of types of output of which four were used by the author. The names of these four types of output offer little in the way of descriptive value, but are useful in identifying the types of output. They are OUTPUT, CHECK, COST.R, and DO.D/J.

The OUTPUT output is shown in Table 2.4. The first section of the output gives the names and activity levels of the variables in the solution for the run. The type of variable is indicated by an F or W preceding the variable name and simply shows whether the variable is in the solution at an upper or lower bound or at some intermediate level. The W indicates a variable at its upper bound, the G a variable at its lower bound, and the F a variable at an intermediate level. Since no upper or lower bounds were placed on the activity vectors in this run all of the variables are shown to be at intermediate levels. Actually an implicit lower bound of zero was set as is done for all linear programming problems.

The variable names are the same as those used in the LP matrix. The activity level is the number of millions of metric tons shipped. For example 20 thousand tons of plate were shipped from Santos to the São Paulo market area and 160 tons of steel plate were shipped from USIMINAS to the Rio de Janeiro market area. One can see in this

Table 2.11

OUTPUT

VARBLS	TYPE	NAME	ACTIVITY LEVEL
		FSASPP	.020
		FUSRJP	.160
		FUSBHP	.020
		FSASPH	.160
		FUSRJH	.064
		FUSBHH	.080
		FVRRJH	.096
		FSASPC	.300
		FUSRJC	.006
		FUSBHC	.140
		FVRRJC	.154
		FSASPT	.000
		FUSBHT	.000
		FVRSPT	.150
		FVRRJT	.058
		FVRBHT	.040
		FIMRJT	.002
		FSAEXH	.122
		FVREXH	.215
		FSAEXC	.040
		FVREXC	.355

SLACKS	TYPE	NAME	ACTIVITY LEVEL	SIMPLEX MULT.
		FCOST	96,066	
		+WSAS		2.419
		+FSAPR	.980	
		+FSAR	.280	
		+FSAH	.824	
		+WSAC		.806
		+WSAT		41.455
		+WUSS		7.460
		+FUSPR	1.200	
		+FUSR	.477	
		+FUSH	1.185	
		+WUSC		.959
		+WUST		41.830
		+FVRS	.190	
		+WVRPR		2.419
		+FVRR	.020	
		+FVRH	.305	

cont'd.

Table 2.11 continued

SLACKS	TYPE	NAME	ACTIVITY LEVEL	SIMPLEX MULT.
		+WVRC		.806
		+WVRT		37.303
		-WSPP		70.217-
		-WSPH		68.170-
		-WSPC		72.570-
		-WSPT		120.750-
		-WRJP		73.438-
		-WRJH		71.390-
		-WRJC		75.960-
		-WRJT		120.000-
		-WBHP		70.458-
		-WBHH		68.410-
		-WBHC		72.980-
		-WBHT		121.590-

particular solution a part of the division of labor that exists between these three steel plants. USIMINAS has a relative specialization in steel plate, COSIPA at Santos in cold sheet and strip, and CSN at Volta Redonda in tin plate.

Only a single import activity appears in this solution, IMRJT, the importation of 2,000 tons of tin plate to Rio. On the other hand four export activities appear -- the export of hot and cold sheet and strip from both Volta Redonda and Santos. This corresponds roughly to the present situation of the flat products part of steel industry in Brazil. Once COSIPA and USIMINAS swing into full operation there will be something of an excess capacity in hot and cold rolled sheet and strip while tin plate capacity will continue to be pressed by present consumption rates.

All variables not among the twenty-one in Table 2.11 are at the zero activity level in this solution.

Activity levels and simplex multipliers are given for the slack variables in the bottom half of Table 2.11. These variables are called "slack" variables because in setting up an original tableau for solving a linear programming problem they are added to the matrix in such a way as to take up the slack in the inequalities and convert them to equalities. The reader will note that they are the constraints of the LP matrix described in the previous section.

The first slack variable is COST, the value of the objective function, and is given as US$96,066,000. This value is the total cost for one year of the production and transportation of flat steel

products in Brazil, plus the cost of imports, minus the profits from exports. It will be used repeatedly in later sections of the paper for comparing the total cost of various alternatives.

The plus and minus signs preceding the row name for the remaining rows in this table indicate whether the row is less-than-or-equal-to (+) constraint or a greater-than-or-equal-to (-) constraint. The capacity constraints are less-than-or-equal-to and the market requirement constraints are greater-than-or-equal-to.

As was mentioned above those variables in the solution at their upper limits are denoted by a W preceding the variable name. Those variables at intermediate levels are preceded by an F. Thus the W preceding SAS in the second row of the slacks shows that the steel shop (or blast furnace) at the COSIPA plant near Santos is being used to capacity in this solution.

The simplex multiplier (or shadow or dual price) shown in the third column of the table provides a measure of the economic value of increasing the capacity of the bottlenecks in the system. Thus <u>on the margin</u> each ton of additional capacity in the steel shop at the COSIPA plant will reduce the cost in the system by US$2.42 per ton. Glancing down the column of simplex multipliers we find the largest values for the tinning lines. For example, a tinning line constructed at Santos (given the assumptions of this model) would decrease the cost in the system <u>on the margin</u> by US$41.45 for each ton of capacity constructed.

For those production units which do not constitute bottlenecks in the system and have excess capacity there is no benefit to be gained from expansion, therefore the value of the simplex multiplier for these constraints is zero. The unused capacity for these production units is shown in the second column of the table. For example, 980 thousand tons of the installed capacity of 1.8 million tons of capacity of the primary mill at Santos would not be used.

The simplex multiplier has a slightly different interpretation for the market requirement constraints. It provides a measure of the cost to the system of an expansion in the requirement for a product in a market area. An expansion in the requirement of plate in the São Paulo market area (SPP) would cost the system US$70.22 per ton. An expansion in the requirement for tin plate in São Paulo (due to the fact that importation of tin plate is necessary in this solution) would cost the system US$121.75 per ton.

So cost in the system can be decreased either through additional investment to increase the capacity of the bottleneck production units or through the restriction of demand. The magnitude of the simplex multipliers gives an index of the effect that could be expected from each of the alternative measures.

The use of shadow prices in investment decisions must be done with care. An example will suffice to show this. The current shadow price for tin plate in each of the three markets is about US$120 per ton and the value of the simplex multiplier for each of the tinning

lines is in the neighborhood of US$40; however, if the capacity at any one of these mills was expanded by more than 2,000 tons (the amount of tin that is being imported) or if the market requirement for tin plate in the system was reduced by more than 2,000 tons, the shadow prices would fall to about US$80 per ton (near the domestic cost of production) and the simplex multipliers for the tinning lines would decrease substantially. Therefore, in using shadow prices one must be careful to investigate the quantity for which the shadow price is likely to hold. In general, for larger systems with smaller discontinuities the probability that a shadow price offers a good indicator for investment is increased. Even in the case cited above they offer a good indicator, but only on the margin.

The CHECK section of the output (see Table 2.12) is normally used as a check that the size of computational errors in the system has not exceeded some desired level. Since we are working with a relatively small problem the computation errors are not likely to reach serious proportions. However, this part of the output provides a useful table to see to what extent the capacity of each production unit is in use in the system. The column UPPER LIMIT gives the value of the capacity constraint on the unit and the column SOL. VALUE (solution value) gives the level of activity used in the solution. Thus we see that the steel shop at USIMINAS (USS) was used to capacity but only one-third of the 1.8 million tons of capacity of the primary mill (USPR) was needed.

Table 2.12

CHECK

ROW NAME	UPPER LIMIT	SOL. VALUE	LOWER LIMIT	ROW ERROR
COST		96.066	.000	.00000600
SAS	.820	.820		.00000006
SAPR	1.800	.820		.00000010
SAR	1.000	.720		.00000007
SAH	1.500	.676		.00000008
SAC	.350	.350		.00000002
SAT	.000	.000	1.000	.00000001-
USS	.600	.600		.00000001
USPR	1.800	.600		.00000000
USR	1.000	.523		.00000002-
USH	1.500	.315		.00000000
USC	.150	.150		.00000002
UST	.000	.000	1.000	.00000000
VRS	1.590	1.400		.00000010-
VRPR	1.400	1.400		.00000000
VRR	1.250	1.230		.00000006
VRH	1.500	1.195		.00000008
VRC	.800	.800		.00000000
VRT	.260	.260		.00000003
SPP		.020	.020	.00000000
SPH		.160	.160	.00000000
SPC		.300	.300	.00000001-
SPT		.150	.150	.00000000
RJP		.160	.160	.00000002
RJH		.160	.160	.00000000
RJC		.160	.160	.00000000
RJT		.060	.060	.00000000
BHP		.020	.020	.00000000
BHH		.080	.080	.00000000
BHC		.140	.140	.00000000
BHT		.040	.040	.00000000

*MAX ERROR = .00000600

The bottom part of the table shows that all of the market requirements were met as required.

Table 2.13 gives one of the most interesting sections of the output, COST.R. This table provides an indication of the stability of the solution of the model to changes in the parameters. The column CURRENT COST is the activity cost for the vector and the column HIGHEST COST is the cost to which that activity could rise before it would be replaced in the solution by another activity. In the current solution the market for plate in the São Paulo area is being served by the COSIPA plant near Santos at a cost to the plant (and to the system) of US$67.12 per ton. However, the plant could charge US$71.18 per ton for its plate delivered in São Paulo before its competitor at Volta Redonda (see column HI-VAR) could undersell it. From this table one could calculate the rents which may accrue to each plant. This type of information can be invaluable both to private competitors and to tax authorities who are eager to scoop up all rents in the system.

One begins to get some idea of the complexities of the readjustments in the system in response to a change in a single parameter by looking at the fifth row in this table, USRJH. If the cost for supplying hot strip and sheet from USIMINAS to Rio de Janeiro rose above US$66.65, then cost could be minimized in the system by having USIMINAS use its capacity for supplying plate to São Paulo. Some other plant, perhaps the one that lost the plate market in São Paulo, perhaps some other, will pick up the extra requirement for hot strip and sheet in Rio de Janeiro.

Table 2.13

COST.R

NAME	CURRENT COST	HIGHEST COST	HI-VAR	LO-VAR	LOWEST COST
SASPP	67.120	71.180	VRSPP	SPP	3.097-
USRJP	63.890	63.978	VRRJP	RJP	9.548-
USBHP	60.910	65.568	VRBHP	BHP	9.548-
SASPH	65.170	69.140	VRSPH	SPH	3.000-
USRJH	62.140	66.656	USSPP	VRRJP	62.054
USBHH	59.160	63.730	VRBHH	BHH	9.250-
VRRJH	68.390	68.476	VRRJP	USSPP	63.874
SASPC	68.570	72.710	VRSPC	SPC	4.000-
USRJC	65.200	66.188	USC	VRBHC	60.630
USBHC	62.220	66.790	VRBHC	BHC	10.760-
VRRJC	71.960	72.110	SARJC	USC	70.972
SASPT	72.940	77.230	SARJT		INFINITY-
USBHT	66.160	70.730	SARJT		INFINITY-
VRSPT	77.300	78.550	IMSPT	SARJT	73.010
VRRJT	76.550	80.840	SARJT	IMSPT	75.300
VRBHT	78.140	82.630	SABHT	USRJT	73.570
IMRJT	120.000	121.250	IMSPT	VREXT	81.550
SAEXH	3.000-	2.692-	SARJP	SAC	3.786-
VREXH	3.000-	1.938-	VREXP	SARJP	3.308-
SAEXC	4.000-	3.850-	SARJC	VRSPC	8.140-
VREXC	4.000-	3.169-	VRC	SARJC	4.150-

A quick glance down the two columns of current cost and highest cost reveals that the geographical spread of the three major steel plants for flat products in Brazil gives the market situation (of the model) a certain degree of stability. For eleven of the sixteen domestic market variables the separation between the current cost and the highest cost is greater than US$2 per ton. At the same time this form of stability indicates that the plants are in a position to reap substantial rents if provided a free enterprise situation in which to do so.

The last section of the output, DO.D/J (see Table 2.14), is similar to the previous section in that it shows how much the cost of an activity which is not in the solution must be reduced before that activity can enter the solution. From the discussion above we know that Volta Redonda could take the plate market in São Paulo away from COSIPA if it could reduce its cost for producing plate to US$67.12. Table 2.14 confirms that fact (see row VRSPP and column BASIS VALUE) and even carries out the subtraction to show that Volta Redonda would have to reduce its cost by US$4.06 per ton to secure the market. Likewise, the table reveals how low the cost of imports must go to compete in the domestic market and how high the profits from exporting must go in order to make it profitable to export a product which is not currently exported. While it would benefit the system to export steel plate from

Table 2.14

PART I DO.D/J

VBLS	TYPE	NAME	CURRENT COST	REDUCED COST	BASIS VALUE
		WSAUSS	18.540	13.500	5.040
		WSAVRS	6.630	9.049	2.419-
		WUSSAS	8.900	13.940	5.040-
		WUSVRS	2.900	10.360	7.460-
		WVRSAS	11.520	9.101	2.419
		WVRUSS	17.430	9.970	7.460
		WSARJP	70.660	.318	70.342
		WSABHP	72.440	5.078	67.362
		WUSSPP	65.330	4.662	60.668
		WVRSPP	71.180	4.060	67.120
		WVRRJP	70.430	.088	70.342
		WVRBHP	72.020	4.658	67.362
		WSARJH	68.710	.320	68.390
		WSABHH	70.500	5.090	65.410
		WUSSPH	63.580	4.660	58.920
		WVRSPH	69.140	3.970	65.170
		WVRBHH	69.980	4.570	65.410
		WSARJC	72.110	.150	71.960
		WSABHC	73.900	4.920	68.980
		WUSSPC	66.650	4.840	61.810
		WVRSPC	72.710	4.140	68.570
		WVRBHC	73.550	4.570	68.980
		WSARJT	76.480	4.290	72.190
		WSABHT	78.270	4.490	73.780
		WUSSPT	70.580	5.260	65.320
		WUSRJT	69.140	4.570	64.570
		WIMSPP	102.000	31.783	70.217
		WIMSPH	102.000	33.830	68.170
		WIMSPC	112.000	39.430	72.570
		WIMSPT	122.000	1.250	120.750
		WIMRJP	100.000	26.562	73.438
		WIMRJH	100.000	28.610	71.390
		WIMRJC	110.000	34.040	75.960
		WIMBHP	108.000	37.542	70.458
		WIMBHH	108.000	39.590	68.410
		WIMBHC	118.000	45.020	72.980
		WIMBHT	128.000	6.410	121.590
		WSAEXP	2.000-	1.097	3.097-
		WUSEXP	2.000-	7.548	9.548-
		WVREXP	2.000-	1.097	3.097-
		WUSEXH	3.000-	6.250	9.250-
		WUSEXC	4.000-	6.760	10.760-
		WSAEXT	5.000-	42.810	47.810-
		WUSEXT	5.000-	50.430	55.430-
		WVREXT	5.000-	38.450	43.450-

Table 2.14 cont'd.

PART II

ROWS TYPE	NAME	INCR B VALUE	DECR B VALUE
	+SAS	2.419	
	+SAPR		
	+SAR		
	+SAH		
	+SAC	.806	
RANGE	+SAT	41.455	
	+USS	7.460	
	+USPR		
	+USR		
	+USH		
	+USC	.959	
RANGE	+UST	41.830	
	+VRS		
	+VRPR	2.419	
	+VRR		
	+VRH		
	+VRC	.806	
	+VRT	37.303	
	-SPP		70.217
	-SPH		68.170
	-SPC		72.570
	-SPT		120.750
	-RJP		73.438
	-RJH		71.390
	-RJC		75.960
	-RJT		120.000
	-BHP		70.458
	-BHH		68.410
	-BHC		72.980
	-BHT		121.590

Santos if the profits to be gained from such enterprise were only US$1.09 greater, there would be no benefit to be had from exporting tin plate unless the international price increased by forty to fifty dollars per ton.

One parting comment about the first six rows of Part I of this table. The column REDUCED COST indicates the magnitude by which the cost of shipping ingot steel would have to be reduced before it would be worthwhile to engage in the shipment of intermediate product. In one of the test runs we decreased steel capacity at the COSIPA plant and hot and cold strip mill capacity at the USIMINAS plant to zero (the situation which existed in January of 1965) and just as might be expected we begin to get some shipment of intermediate product in the system. We will return to this later in a discussion of the tests that were conducted with the model.

Part II of Table 2.14 simply repeats a part of the information given in the OUTPUT section. It gives the dual prices.

CHAPTER III

TESTS WITH THE LINEAR PROGRAMMING MODEL

Some of the properties of the linear programming model and its usefulness as an analytical tool for studying a variety of policy problems are shown in this chapter. The six tests discussed here involve changes in one or a group of the parameters of the model while holding others constant to determine the effects of the changes. The tests were chosen both for their usefulness in investigating the properties of the model and for their similarity to economic policy problems of the Brazilian flat-product steel industry.

The first test "requirement shift," was a study of the effects on the system of locating a new automobile plant in Brazil. The model was run repeatedly with the new plant located in first one market area and then shifted to each of the other market areas in turn. Analyses were made of the changes in activity levels, shadow prices, total system cost, and profits accruing to the plants as this "chunk of product requirements" was moved around in the system.

Increases in the requirement levels in all markets simultaneously with constant production capacity was the subject of the second study. Market requirements were increased in five steps from their original levels to three and a half times their original levels.

In the third test changes in factor cost and changes in technology were examined. For example, a test was made of the effects on the system of the introduction of fuel oil injection in the blast furnace at the COSIPA plant.

The ability of the model to respond with shipments of intermediate products when part of the production capacity in the system was shut down was tested in the fourth test.

If all three plants in the system should simultaneously request import permits to purchase a new tinning line and the government decided that only one new tinning line was needed in the country at the time, which request should be granted? This is the subject of the fifth test.

Finally, a series of runs were made with the model to study the effects of changes in the transportation cost for final products.

Unless specific mention is made of a change in a parameter or group of parameters, their values will be the same as those shown in the tables of Sections 2, 3, and 4.

1. The Requirement Shift Test

Nearness to automobile plants, with their large requirements for hot and cold rolled sheet and strip as well as other steel products, is considered one of the prime determinants of the profitability of a steel plant location. To obtain more insight into the importance of this affect, we decided upon an automobile plant size

"chunk of product requirement" and moved it around in the system, then we doubled the size of the chunk and moved it around in the system again, solving the linear programming problem at each step.

Assuming that a small automobile uses about a half ton of flat steel products (three-fourths of this being cold rolled products and one-fourth hot rolled products) and assuming that a reasonable size automobile plant would produce 200 thousand units (production of automobiles and trucks in Brazil in 1964 was about one million units) an automobile plant size "chunk of product requirement" is determined to include 25 thousand tons of hot rolled products and 75 thousand tons of cold rolled products.

These quantities were added to the existing requirements for these products in first the São Paulo market area, then the Rio de Janeiro area, and finally the Belo Horizonte area. The amount was then doubled to 50 thousand tons of hot rolled and 150 thousand tons of cold rolled products and the tests repeated.

Table 3.1 gives the value of the objective function for each step as well as the average cost per ton of product.

The total cost of operating the steel system is seen to be relatively unaffected by the location of the automobile plant. São Paulo is the best location by a margin of US$170 thousand a year -- i.e. about US$1 per automobile. This small effect is not a particularly surprising result since there is a steel mill located near each of the market areas, and since there is not great variation between these plants in the cost of producing steel.

Table 3.1

Requirement Shift Test

Effect on Total System Cost and Average Cost per Ton of Product

New Auto Plant at	Original Solution	100 thou. tons			200 thou. tons		
		SP	RJ	BH	SP	RJ	BH
Total Cost (US$ million)	93.446	100.538	100.708	100.767	107.817	107.970	108.11
Average Cost (US$ per metric ton)	64.44	64.86	64.96	65.01	65.34	65.44	65.53

If on the other hand the automobile plant had been located in one of the southernmost states of Brazil or in the northwest part of the country the difference in the total system cost would have been more striking. Also, even within the area considered, the difference might have been more striking if all hot sheet and strip and all cold sheet and strip had not been treated as homogeneous commodities. In fact the width and quality of finish of hot and cold rolled products is a matter of paramount importance to the automobile industry. A more detailed product breakdown than the four products used in this study would be necessary to catch these distinctions in types of products.

The increase in the average cost per ton of product (see Table 3.1) resulted from the loss of some export profits (due to the increase in the total product requirement in the country).

Table 3.2 shows the effect of the requirement shift on the activity levels of the variables. The first column gives the activity levels of the variables in the original solution before the new automobile plant was added to the system. All other entries in the table give changes from the activity levels shown in the first column. The second, third and fourth columns show the effects of the addition of an auto plant of 200 thousand units (100 thousand tons of flat steel products) and the last three columns of the table show the effects of the addition of an auto plant of twice that size.

In the first step the new auto plant was located at São Paulo and the extra product requirement in the system was met by decreasing exports. The COSIPA plant at Santos decreased its exports of hot sheet and strip (SAEXH) by 25 thousand tons and its exports of cold sheet and strip (SAEXC) by 39 thousand tons, and shipped these products instead to the São Paulo market area (SASPH) and (SASPC), respectively.[1] With the added market requirements the capacity of the cold strip mill at COSIPA was not sufficient to permit all of the requirement for cold sheet and strip in the São Paulo market area to be satisfied by COSIPA alone. Therefore it was necessary for the CSN plant at

[1] The Guide to Mnemonics sheet contained in the second chapter will be useful in reading this and the following tables.

Table 3.2

Requirement Shift Test - Changes in Activity Levels
(Million metric tons)

Activity vector \ New Plant at	Original Solution	100 thou. ton increase			200 thou. ton increase		
		SP	RJ	BH	SP	RJ	BH
SASPP	.020						
USRJP	.160						
USBHP	.020						
SASPH	.200	+.025			+.050		
USRJH	.064			-.025			-.050
USBHH	.080			+.025			+.050
VRRJH	.056		+.025	+.025		+.050	+.050
SASPC	.300	+.039			+.039		
SABHC			[illegible]	+.039			+.039
USRJC	.005		[illegible]	-.005			-.005
USBHC	.140			+.005			+.005
VRSPC		+.036			+.111		
VRRJC	.155		+.075	+.005		+.150	+.005
VRBHC	—			+.032			+.107
SASPT	.001						
USBHT	.001						
VRSPT	.149						
VRRJT	.060						
VRBHT	.039						
SAEXH	.082	-.025			-.050		
VREXH	.094		-.025	-.025		-.050	-.050
SAEXC	.039	-.039		-.039	-.039		-.039
VREXC	.334	-.036	-.075	-.036	-.111	-.150	-.111
VREXT	.019						

Volta Redonda to begin supplying a part of the requirement for cold sheet and strip to São Paulo. This was accomplished by a reduction of the exports of that plant of cold sheet and strip (VREXC) and an equivalent shipment of that product to São Paulo (VRSPC). Only six changes in activity levels including one new activity (VRSPC) were necessary to restore a least-cost equilibrium.

More complex readjustments were required in the case where the new auto plant was located at Belo Horizonte. Eleven changes including the introduction of two new activities (SABHC) and (VRBHC) were necessary to restore the equilibrium.

The complexity of these changes shows the importance of flexibility in an economic system. A slight change in one part of the system may trigger changes that spread throughout the entire system before a new least-cost equilibrium is achieved. In a many-plant, many-market, many-product economy these changes are almost continuous and the required adjustments are of relatively small magnitude. But in a three-plant, three-market, four-product economy the adjustments are discrete and can be quite sharp.

An example of this sort of phenomenon may be seen in Table 3.3 which shows the effect of the "requirement shift" on shadow prices for final products. As in the previous table the first column gives the values for the original solution, and the other entries in the table show changes plus or minus from these values.

Table 3.3

Requirement Shift Test

Changes in Shadow Prices for Products

(U.S.$/metric ton)

Product \ New Plant at	Original Prices	100 thou. ton increase			200 thou. ton increase		
		SP	RJ	BH	SP	RJ	BH
SPP	67.92						
SPH	65.87						
SPC	70.27	+4.82		+0.15	+4.82		+0.15
SPT	80.68						
RJP	71.22						
RJH	69.19						
RJC	73.76						
RJT	79.35						
BHP	68.24						
BHH	66.21						
BHC	70.78			+5.15			+5.15
BHT	81.52						

In the case of the new 100 thousand ton (200 thousand unit) auto plant at São Paulo, capacity constraints at the nearby Santos mill of COSIPA required that a part of the additional cold sheet and strip be obtained from Volta Redonda. Table 3.2 shows this change. Since the Volta Redonda plant could not deliver cold sheet and strip to the São Paulo market area for as little as the Santos plant, the resulting shadow price for cold sheet and strip in São Paulo increased by US$4.82 per ton.

When the new auto plant was added to the system in the Rio de Janeiro market area there were no changes in the shadow prices, neither were there any new vectors in the basis. The original optimal basis

remained a feasible basis, and it was only necessary to change the activity levels of some of the vectors in the solution.[1] Such, however, was not the case when the auto plant was located at Belo Horizonte. The fourth column of Table 3.3 shows that the shadow price of cold sheet and strip in São Paulo increased by US$0.15 and the shadow price for the same product in the Belo Horizonte market area increased by US$5.15 per metric ton. Before the addition of the new capacity the market area at Belo Horizonte obtained all of its cold sheet and strip from the nearby USIMINAS plant (see Table 3.2). However, the market requirements added by the new auto plant made it necessary to purchase product from COSIPA (the second lowest cost supplier of cold sheet and strip to Belo Horizonte). But the capacity at COSIPA was not sufficient to supply all of the additional requirement so additional product had to be purchased from Volta Redonda.

The addition of an auto plant of 400 thousand units (200 thousand tons of final products) produced no more strain on the available capacity in the system than did the addition of an auto plant half that size. The shadow price increases were identical to those which occurred under the addition of the smaller plant to the

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[1] See D. Gale, The Theory of Linear Economic Models, McGraw Hill Book Company, 1960, page 304 for a proof that an optimal basis remains optimal under a change in the right-hand side (constraint coefficients) so long as the basis remains feasible. The same proof shows that the shadow prices also remain unchanged under these conditions.

system. This phenomenon occurred only because of the magnitudes involved. If a slightly larger auto plant had been added to the system, or if one of the productive units in one of the plants had had slightly less capacity than it had, a different combination of activities would have been needed to produce the least-cost solution.

Table 3.4 shows the "profits" accruing to the steel plants under different locations of the new automobile plant. The "profits" were calculated as follows. The cost of each activity in the solution was subtracted from the shadow price for the appropriate good and market to give a unit "profit."[1] For example, in the original solution the activity cost for the production and transportation of hot sheet and strip from USIMINAS to Belo Horizonte was US$66.21. Thus, the unit profit for this activity was US$8.67. To calculate the total profits accruing to each plant, the unit profits were multiplied by the activity levels of the solution and the profits for each plant were summed. Though a part of the profits should be attributed to transportation and a part to production, they have been treated here as though the steel plants owned the means of transportation.

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[1]These "profits" are not the same as those normally discussed, but are rather more like rents. Normal profits are calculated by subtracting total cost (including capital cost) from total revenues. In this definition of "profits" used here the cost (other than capital cost) of producing and transporting a good are subtracted from the shadow price in the market area.

Table 3.4

Requirement Shift Test

"Profits" Accruing to Plants
(US$ millions)

New Plant at / Plant & Product	Original Solution		São Paulo		Rio de Janeiro		Belo Horizonte	
	Sub-totals	totals	Sub-totals	totals	Sub-totals	totals	Sub-totals	totals
1. COSIPA		2.264		3.838		2.264		2.316
a. Plate	.062		.062		.062		.062	
b. Hot	.600		.675		.600		.600	
c. Cold	1.200		2.990		1.200		1.408	
d. Exports-Hot	.246		.171		.246		.246	
e. Exports-Cold	.156		-		.156		-	
2. USIMINAS		4.335		4.335		4.335		5.085
a. Plate	1.610		1.610		1.610		1.610	
b. Hot	1.250		1.250		1.250		1.250	
c. Cold	1.475		1.475		1.475		2.225	
3. CSN		3.241		3.241		3.241		3.245
a. Hot	.168		.168		.243		.243	
b. Cold	.620		.764		.920		.768	
c. Tin	.740		.740		.740		.740	
d. Exports-Hot	.282		.282		.207		.207	
e. Exports-Cold	1.336		1.192		1.036		1.192	
f. Exports-Tin	.095		.095		.095		.095	
4. TOTAL - ALL PLANTS		9.840		11.514		9.840		10.646

The first and second columns of Table 3.4 show a breakdown of the profits by product and by plant for the original solution before the new auto plant is added to the system. The remaining three pairs of columns give the same breakdown for the case in which a 200 thousand unit (100 thousand tons of steel products) automobile plant is located first at São Paulo, then at Rio de Janeiro, and finally at Belo Horizonte.

With the original conditions, the COSIPA plant at Santos would gain a profit of US$2.264 million, the largest part of its profit coming from the sale of cold sheet and strip. USIMINAS on the other hand would have a profit almost twice that of COSIPA, and Volta Redonda a profit about one and a half times that of COSIPA. Table 3.5 shows why this would occur.

The first four columns of Table 3.5 give "unit profits" for each product and the last four columns show the equivalent product flows. In the product flows section of the table the first row for each plant gives the total output of the plant in million metric tons and the following rows the amount of each product produced at each plant. The plants produced at full bottleneck capacity in every case but the product destination changed slightly from solution to solution. COSIPA produced 641 thousand tons of products, USIMINAS 469 thousand tons and CSN 806 thousand tons (flat products only).

Returning to the first column of Table 3.5 we find that the unit profits of USIMINAS were much higher than those for COSIPA and CSN. For example, in the original solution COSIPA earned US$4.00 on

Table 3.5

Requirement Shift Test

Unit "Profits" and Product Flows

Plant & Product \ New Plant at	Unit Profits (US$/metric ton)				Product Shipments (million metric tons)			
	Original	SP	RJ	BH	Original	SP	RJ	BH
1. COSIPA					.641	.641	.641	.641
a. Plate	3.10	3.10	3.10	3.10	.020	.020	.020	.020
b. Hot	3.00	3.00	3.00	3.00	.200	.225	.200	.200
c. Cold	4.00	8.82	4.00	4.15	.300	.339	.300	.339
d. Exports-hot	3.00	3.00	3.00	3.00	.082	.057	.082	.082
e. " cold	4.00	-	4.00	-	.039	-	.039	-
2. USIMINAS					.469	.469	.469	.469
a. Plate	8.95	8.95	8.95	8.95	.180	.180	.180	.180
b. Hot	8.67	8.67	8.67	8.67	.144	.144	.144	.144
c. Cold	10.18	10.18	10.18	15.33	.145	.145	.145	.145
3. CSN					.806	.806	.806	.807
a. Hot	3.00	3.00	3.00	3.00	.056	.056	.081	.081
b. Cold	4.00	4.00	4.00	4.00	.155	.191	.230	.192
c. Tin	5.00	5.00	5.00	5.00	.148	.148	.148	.148
d. Exports-Hot	3.00	3.00	3.00	3.00	.094	.094	.069	.069
e. " cold	4.00	4.00	4.00	4.00	.334	.298	.259	.298
f. Exports-Tin	5.00	5.00	5.00	5.00	.019	.019	.019	.019

each ton of cold sheet or strip while USIMINAS gained US$10.18 on each ton of the same product. This situation arose because of the low assembly cost of raw materials at USIMINAS and its proximity to the Belo Horizonte market, which was protected from the products of other plants by distance. On the coast COSIPA and CSN both had excess capacity and were forced to export; thereby bringing their unit profits down to the level of the profits they gained from exporting. (The model did not permit the plants to make greater profits on domestic sales than on exports.)

Another reason for the high profits of USIMINAS was that under the assumptions of the model as to input cost and specific consumption of raw materials as well as to transportation cost, etc., USIMINAS could deliver steel products to Rio de Janeiro for less than Volta Redonda. For example, USIMINAS could deliver hot sheet and strip to Rio de Janeiro for about US$6 per ton less than Volta Redonda. Thus USIMINAS could garner the high profit domestic markets and force CSN into the lower profit export markets.

In an earlier run of this same experiment, the capacity of the tin plate mill at Volta Redonda was assumed to be 260 thousand tons per year, or just slightly less than the domestic requirement for tin plate (including the scrap losses). The result was that it was necessary to import a few thousand tons of tin plate at a cost of US$120 per ton. The shadow prices for tin in all market areas jumped from around US$80 per ton to about the level of the import price, and the unit profits gained by Volta Redonda on tin plate increased to about US$40 per ton. Since CSN was the only plant in the system with a tinning line, the profits gained by that firm on the one product exceeded their profits on all other products together.

Proceeding with our analysis of the impact on profits of locating a new automobile plant in one market area or another, we return to Table 3.4. Column 4 of this table shows that locating the plant near São Paulo added substantially to the profits of COSIPA, increasing them from US$2.264 to US$3.838 million. This resulted in large part

from the increase in the shadow price of cold sheet and strip at São Paulo (see Table 3.3) by US$4.82 per ton. The reader may recall that this jump in the shadow price of cold sheet and strip at São Paulo occurred because the new auto plant at São Paulo caused the requirement for cold sheet and strip in that market to exceed the capacity of the cold strip mill in the nearby Santos plant of COSIPA.

Locating the auto plant at Rio de Janeiro rather than at São Paulo resulted in no changes of profit from the original solution. Though at first this seems to violate one's intuition about the matter, it follows from the fact that the original solution remains feasible under the change in the demand conditions. Readjustments could be made in the levels of activities already in the solution and it was not necessary to introduce any of the more expensive activities that had been excluded from the original optimal solution.

Building the new auto plant at Belo Horizonte resulted in a substantial increase in profits for USIMINAS and a slight change in profits for the other two plants. Of the three possible locations São Paulo offered the minimum total cost and the maximum profit to the system. However, the location giving the second greatest profit to the system (Belo Horizonte) is the third ranking solution by the cost minimizing standard.

We now turn our attention from a study of the effects of "requirement shift" to an analysis of the effects of increases in <u>total</u> product requirement.

2. The Requirement Increase Test

Holding capacity constant while increasing the product requirement in the entire system provides insight into where in the system bottlenecks may arise in the future. Since capital costs are not included in the model, the model can not be used for making exact decisions on the time and place to add to capacity; however, it can reveal future bottlenecks as indicators for investment.

Part I of Table 3.6 gives the requirements vector used in each run of the experiment. The vector ONE, which is used as the base for the increase shown in the remaining columns, is identical to ONCE with the exception that a slightly greater concentration of the national requirement for hot sheet and strip at São Paulo is assumed. The remaining vectors in Part I of the table were determined by increasing the vector ONE by factors of $1\frac{1}{2}$, 2, $2\frac{1}{2}$, and 3. Part II of the table shows the product breakdown for each step.

All parameters of the model were the same for this test as for the previous test, with the following exceptions. The capacity of the tinning line at Volta Redonda was assumed to be 260 thousand tons per year instead of 280 thousand tons per year. This assumption resulted in the tin imports discussed above. Also, a somewhat higher transportation cost was used (see Part III of Table 3.6) and the capacity of the blast furnace-steel shop and of the primary mill at Volta Redonda was assumed to be 200 thousand tons greater than in the previous runs.

Table 3.6

Requirements Increase Test

PART I.

Requirement Levels (million metric tons)

	OONCE	ONE	ONEH	TWO	TWOH	THREE
SPP	.020	.020	.030	.040	.050	.060
SPH	.160	.200	.300	.400	.500	.600
SPC	.300	.300	.450	.600	.750	.900
SPT	.150	.150	.225	.300	.375	.450
RJP	.160	.160	.240	.320	.400	.480
RJH	.160	.120	.180	.240	.300	.360
RJC	.160	.160	.240	.320	.400	.480
RJT	.060	.060	.090	.120	.150	.180
BHP	.020	.020	.030	.040	.050	.060
BHH	.080	.080	.120	.160	.200	.240
BHC	.140	.140	.210	.280	.350	.420
BHT	.040	.040	.060	.080	.100	.120

PART II.

Product Breakdown of Requirements (million metric tons)

	Plate	Hot	Cold	Tin		Total
ONE	.200 (14%)	.400 (28%)	.600 (41%)	.250 (17%)	=	1.450 (100%)
ONEH	.300	.600	.900	.375	=	2.175
TWO	.400	.800	1.200	.500	=	2.900
TWOH	.500	1.000	1.500	.625	=	3.625
THREE	.600	1.200	1.800	.750	=	4.350

PART III.

Transportation Cost (US$/metric ton)

Plant	Market	Cost
SA	SP	4.95
SA	RJ	8.49
SA	BH	10.27
US	SP	9.34
US	RJ	7.90
US	BH	4.91
VR	SP	5.89
VR	RJ	5.13
VR	BH	6.73

Thus in the first test, that part of the capacity in the blast furnace, steel shop, and primary mill at Volta Redonda which is used for producing materials for rolling into shapes instead of flats, was subtracted out of the total capacity available. All of these changes were held constant over the six runs made in this test.

Looking first at the effect on total cost in the system, one observes (see the last two rows of Table 3.7) that total cost increased and that average cost per ton of product also increased, but at a decreasing rate. This resulted from the decrease in exports (until they ceased altogether) and the steadily increasing share of imports in total national consumption.

The lower part of Table 3.7 shows that as the market requirements increased, first tin plate, then cold sheet and strip and steel plate, and finally hot sheet and strip were imported. This pattern resulted from the relative shortage of tin plate capacity and the relative surplus of hot strip mill capacity. Moving to the top of the table one can see the effect on the activity vector for the shipment of ingot steel from Volta Redonda to the COSIPA plant at Santos (VRSAS). At the original level of demand and at one-and-a-half times that level it was not efficient to make the shipment of the intermediate product. However, as the average cost of products in the system rose it becomes economic to ship the intermediate product, and thereby to make more efficient use of the excess rolling capacity at the COSIPA plant.

Table 3.7

Requirements Increase Test

Activity Levels of Variables (millions of tons)

Vector \ Run	ONCE	ONE	ONEH	TWO	TWOH	THREE
Objection Function (US$ million)	96.361	96.220	155.776	226.889	304.749	383.352
Average Cost per ton (U.S$/ton).	66.46	66.36	71.50	78.30	83.10	88.10
VRSAS	-	-	-	.190	.1190	.1190
SASPP	.020	.020	.002	.040	-	-
SARJP	.093	.054	-	.014	-	-
USRJP	.067	.106	.173	.125	-	-
USBHP	.020	.020	.030	.040	.050	.060
VRSPP	-	-	.028	-	.043	-
VRRJP	-	-	.067	.069	-	-
SASPH	.160	.200	.300	.400	.456	.456
USRJH	.160	.120	-	-	.078	.028
USBHH	.080	.080	.120	.160	.200	.240
VRSPH	-	-	-	-	.044	.144
VRRJH	-	-	.180	.240	.222	.166
SASPC	.300	.300	.340	.340	.340	.340
SARJC	.040	.040	-	-	-	-
USRJC	.006	.006	-	-	-	-
USBHC	.140	.140	.146	.146	.146	.146
VRSPC	-	-	.110	.260	.305	.235
VRRJC	.115	.115	.240	.115	-	-
VRBHC	-	-	.064	.134	.204	.274
VRSPT	.150	.150	.188	.168	.148	.128
VRRJT	.058	.058	-	-	-	-
VRBHT	.040	.040	.060	.080	.100	.120
IMSPP	-	-	-	-	.007	.060
IMRJP	-	-	-	.113	.400	.480
IMRJH	-	-	-	-	-	.166
IMSPC	-	-	-	-	.105	.325
IMRJC	-	-	-	.205	.400	.480
IMSPT	-	-	.037	.132	.227	.322
IMRJT	.002	.002	.090	.120	.150	.180
SAEXH	.026	.026	-	-	-	-
VREXH	.311	.311	.033	-	-	-
VREXC	.395	.395	.095	-	-	-

The amount shipped was in this case determined by the fact that the blast furnace and steel shop capacity at Volta Redonda was 190 thousand tons greater than the capacity of the primary mill.

The activity levels of the variables showed a tendency, as the requirements increased for each plant, to concentrate on serving the nearest market area, for interior markets to be served by domestic production, and for imports to go to markets near ports.

The change in flows in various parts of the system was not a simple expansion space of the increase in total requirements in the system. What appeared was a transportation planner's nightmare. Some flows increased, then leveled off and decreased, others started and stopped and started again, others remained static, and others rose steadily. Certainly had there been a price response of the transportation sector built into the model, the changes in the levels of the activities would have been less sharp. Also, rigidities in the system such as taste preferences for the products of one plant over those of another plant would have reduced the sharpness of the changes shown by Table 3.7. Finally, the magnitudes of the changes in product requirement used here were large. Changes of the magnitude shown in the table would be expected to require fifteen to twenty years.

For planning over shorter periods of time an experiment of this type could be most helpful to transportation planners. Also, the experiment could be easily modified to account for increases in capacity over time, with an LP run being made for each time period.

Table 3.7 also demonstrates how useful this kind of experiment could be for the economist concerned with the balance of payments. In countries where steel products represent a substantial portion of imports or exports, disaggregated projections on a product by product basis could be made with this type of model. For such planning it would be important to make a modification to the model by adding a constraint row for each imported raw material used in the production of steel products. Then the solution would provide estimation for the total amount of each of the raw material imports required.

Part I of Table 3.8 shows the increase in shadow prices as the national requirement increased. Discontinuities in changes of the shadow prices occurred between those steps where importation of a given product becomes necessary. For example there was a sharp jump in the shadow prices for steel plate, hot sheet and strip, and cold sheet and strip between steps ONEH and TWO, which corresponded to the initiation of imports of steel plate and cold sheet and strip in step TWO.

Part II of this table displays the effects on the shadow prices associated with each of the productive units in the system. As the tinning lines played a smaller and smaller part in restraining the system from reaching lower cost, their shadow prices fell. The steel shops, which acted as constraints on all final products, became increasingly the most important bottlenecks in the system and were assigned higher and higher shadow prices.

Table 3.8

Requirements Increase Test

Shadow Prices
(US$/ton)

PART I

Market & Product \ Run	ONCE	ONE	ONEH	TWO	TWOH	THREE	Import Prices
SPP	70.22	70.22	74.95	96.46	102.00	102.00	102
SPH	68.17	68.17	72.95	93.59	99.00	100.75	102
SPC	73.10	73.10	77.40	110.76	112.00	112.00	112
SPT	120.75	120.75	122.00	122.00	122.00	122.00	122
RJP	73.76	73.76	74.20	100.00	100.00	100.00	100
RJH	71.70	71.70	72.04	97.04	98.25	100.00	100
RJC	76.64	76.64	76.64	110.00	110.00	110.00	110
RJT	120.00	120.00	120.00	120.00	120.00	120.00	120
BHP	70.78	70.78	71.28	97.02	98.18	99.99	108
BHH	68.72	68.72	69.15	94.14	95.27	97.02	108
BHC	73.66	73.66	78.24	111.60	112.84	112.84	118
BHT	121.59	121.59	122.84	122.84	122.84	122.84	128

PART II

Production Unit \ Run	ONCE	ONE	ONEH	TWO	TWOH	THREE
SAS	2.42	2.42	6.12	22.92	27.28	28.70
SAPR						
SAR						
SAH						
SAC	1.32	1.32	.80	11.81	7.47	5.67
SAT	40.91	40.91	37.73	3.68	2.46	2.47
USS	7.71	7.71	8.05	28.21	29.12	30.53
USPR						
USR						
USH						
USC	1.30	1.30	5.31	12.06	12.11	10.31
UST	41.14	41.14	37.63	3.62	2.35	2.37
VRS				10.88	15.24	16.65
VRPR	2.42	2.42	2.42	11.70	8.31	8.31
VRR						
VRH						
VRC	.81	.81	.81	7.56	7.52	5.72
VRT	36.61	36.61	37.80	3.78	2.52	2.54

Both this test and the previous one involved changes in product requirements. The next test is one which employs a change in some of the cost parameters of the system.

3. Changes in Cost Parameters Test

The original calculation of the sinter charge at the Volta Redonda plant of CSN was .875 tons of sinter per ton of pig iron. This later proved to be in error and was corrected to .794 tons of sinter per ton of pig iron. This mistake provided us with the first test of the sensitivity of the results of the model to changes in the unit cost or changes in the specific consumption of inputs.

Part I of Table 3.9 shows the parameters for the various runs in this experiment and Part II gives the cost of ingot steel at each plant for each run. The parameters other than those shown are the same for this test as for the preceding test.

The CSN sinter charge for the first run was .875 and that for the second run was .794. All other parameters were held constant. The effect of the correction was to decrease the cost of ingot steel at CSN from US$52.20 to US$51.01 per ton, a sufficiently large reduction to permit Volta Redonda to compete effectively with USIMINAS for a share of the hot sheet and strip market in Rio de Janeiro, and with COSIPA for a larger share of the cold sheet and strip market in Rio (see columns 1 and 2 of Table 3.10). Exports of COSIPA were accordingly increased and those of Volta Redonda decreased.

Table 3.9

Changes in Cost Parameters Test

PART I - Values of Parameters

Parameter \ Run	1	2	3	4	5
1. CSN Sp.C of sinter	.875	.794	.794	.794	.794
2. CSN Ore cost (US$/ton)	3.50	3.50	4.50	3.50	3.50
3. COSIPA Sp.C of Coke	.680	.680	.680	.600	.680
4. COSIPA Sp.C. of Fuel oil	.000	.000	.000	.050	.000
5. USIMINAS Sp.C. of sinter	1.100	1.100	1.100	1.100	1.500

(for all these runs SPH = .200 and RJH = .120)

PART II - Cost of Ingot Steel (US$/metric ton)

Plant \ Run	1	2	3	4	5
1. COSIPA	48.56	48.56	48.56	47.66	48.56
2. USIMINAS	43.74	43.74	43.74	43.74	40.32
3. CSN	52.20	51.01	51.68	51.01	51.01

Run #2 with the corrected sinter charge then provided a basis for comparison for the other runs. In run #3 the cost of ore at the Volta Redonda plant was increased from US$3.50 per ton to US$4.50 per ton. This resulted in an increase of US$0.67 per ton in the cost of ingot steel at Volta Redonda. Comparing runs #2 and #3 in Table 3.10 one sees that this change was, however, sufficient to cause Volta Redonda to lose again just those gains it made in the correction of the sinter charge error. Thus a difference of US$0.67 is shown to produce a response in the system.

Run #4 examines the effects of the installation of fuel oil injection in the COSIPA blast furnace, assuming that this would result in a decrease of the coke rate from .680 to .600 tons of coke per ton of pig iron and an increase from .000 to .050 tons of fuel oil per ton of pig iron. This change enabled COSIPA to decrease its ingot steel cost by US$0.90 per ton, and was sufficient to permit this firm to gain 100 thousand tons of the Rio market as well as to increase its profits on all existing sales.

A comparison of the objective function values for runs #2 and #4 (see Table 3.10) shows that from the point of view of the system as a whole the decrease in annual cost resulting from this technical change was US$686 thousand per year. However, the "profit" gain to COSIPA from installing this technical change was less than the savings for the entire country. Table 3.11 shows the unit profits and product flows for this experiment, and Table 3.12 shows the profits accruing

Table 3.10

Changes in Cost Parameters Test

Effects on Activity Levels

Vector \ Run	1	2	3	4	5
Objective function	96.220	95.938	96.294	95.252	93.885
SASPP	.020	.020	.020	.020	.020
SARJP	.054		.054	.054	-
USRJP	.106	.160	.106	.106	.160
USBHP	.020	.020	.020	.020	.020
SASPH	.200	.200	.200	.200	.200
USRJH	.120	.064	.120	.120	.064
USBHH	.080	.080	.080	.080	.080
VRRJH	-	.056	-	-	.056
SASPC	.300	.300	.300	.300	.300
SARJC	.040		.040	.040	-
USRJC	.006	.006	.006	.006	.006
USBHC	.140	.140	.140	.140	.140
VRRJC	.115	.154	.115	.115	.154
SASPT	.000	.000	.000	.000	.000
USBHT	.000	.000	.000	.000	.000
VRSPT	.150	.150	.150	.150	.150
VRRJT	.058	.058	.058	.058	.058
VRBHT	.040	.040	.040	.040	.040
IMRJT	.002	.002	.002	.002	.002
SAEXH	.026	.082	.026	.026	.082
VREXH	.311	.255	.311	.311	.255
SAEXC	-	.040	-	-	.040
VREXC	.395	.355	.395	.395	.355

Table 3.11

Changes in Cost Parameters Test

"Unit Profits" and Product Flows

Plant & Product \ Run	Unit Profits (US$/metric ton)			Product Flows (million metric tons)		
	2	4	5	2	4	5
1. COSIPA - Santos						
a. Plate	3.10	3.10	3.10	.020	.074	.020
b. Hot	3.00	3.00	3.00	.200	.200	.200
c. Cold	4.00	5.03	4.00	.300	.340	.300
d. Exports - hot	3.00	3.00	3.00	.082	.026	.082
e. Exports - cold	4.00		4.00	.040	-	.040
2. USIMINAS						
a. Plate	9.55	8.71	13.93	.180	.126	.180
b. Hot	9.25	8.44	13.49	.144	.200	.144
c. Cold	10.76	10.76	15.24	.146	.146	.146
3. CSN - Volta Redonda						
a. Plate						
b. Hot	3.00	3.00	3.00	.056		.056
c. Cold	4.00	4.00	4.00	.154	.115	.154
d. Tinplate	43.45	43.45	43.45	.248	.248	.248
e. Exports - hot	3.00	3.00	3.00	.255	.311	.255
f. Exports - cold	4.00	4.00	4.00	.355	.395	.355

Table 3.12

Changes in Cost Parameters Text

"Profits" Accuring to Plants

(US$ millions)

	Run #2 Subtotal	Run #2 Total	Run #4 Subtotal	Run #4 Total	Run #5 Subtotal	Run #5 Total
1. COSIPA - Santos		2.268		2.617		2.268
a. Plate	.062		.229		.062	
b. Hot	.600		.600		.600	
c. Cold	1.200		1.710		1.200	
d. Exports - hot	.246		.078		.246	
e. Exports - cold	.160		-		.160	
2. USIMINAS		4.620		4.358		6.675
a. Plate	1.720		1.100		2.510	
b. Hot	1.330		1.688		1.940	
c. Cold	1.570		1.570		2.225	
3. CSN - Volta Redonda		13.719		13.723		13.719
a. Plate	-		-		[illegible]	
b. Hot	.168		-		.168	
c. Cold	.616		.460		.616	
d. Tinplate	10.750		10.750		10.750	
e. Exports - hot	.765		.933		.765	
f. Exports - cold	1.420		1.580		1.420	
4. Total Profit		20.607		20.698		22.662

to each plant. Comparing the profits for COSIPA for runs #2 and #4 we find that the profit gain for that firm was US$349 hundred thousand annually. Also, USIMINAS suffered a loss in profits of about US$262 thousand annually, as a result of a decrease in the shadow prices in markets in which it competed with COSIPA (see Table 3.11).

Thus, in this situation there is a sharp difference between the possible benefits to the country and to each of the plants from the technical change in one plant. The country at large would do well to subsidize COSIPA to make the investment. USIMINAS on the other hand would be wise to pay a fee to COSIPA to prevent the installation of the fuel oil injection.

A rather different situation arose when consideration was given to the effects of providing a large enough sinter plant to USIMINAS so that a 100 per cent sinter charge could be used there for both blast furnaces. It is seen by comparing run #5 with run #2 in Table 3.9 that this change permitted USIMINAS to produce ingot steel for US$3.40 less per metric ton than the cost when the plant used a specific consumption of sinter of 1.100. This result is largely attributable to the lower coke cost incurred when using a high-percentage sinter charge.

Table 3.10 shows that the annual saving to the system of the introduction of this change was US$2.035 million (the difference in the objective function values for runs #2 and #5).[1] Table 3.12 shows

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[1]These changes in costs are changes in operating cost only as there are no capital changes included in this linear programming model.

that the increased "profits" accruing to USIMINAS as a result of having sufficient sinter capacity to provide a 100 per cent sinter charge to its blast furnaces were US$2.055 million per year. Thus in this case the private benefit and the public benefit are shown to be almost the same. (A sinter plant large enough to provide a 50 per cent sinter charge for a one million ton per year plant involves an investment of roughly US$9 million.)

This analysis of the effects of some technical innovations producing changes in the cost structure of the system is very rough and rests on incomplete technical information. Also, this type of analysis assumes that the prices actually charged for the products are the same as their shadow prices.

4. Test of Shipments of Intermediate Product

As of the beginning of 1965 COSIPA had its rolling mills in operation but had not completed the construction of the LD converters and the blast furnace. USIMINAS, on the other hand, had completed the installation of its steel shop and its blast furnaces but did not have all of its rolling mills in operation. This situation gave rise to a shortage of ingot steel at COSIPA and a surplus at USIMINAS. The result was that COSIPA was buying ingot steel from USIMINAS as well as from CSN.

Most experimental runs made with this model assumed that the ingot steel production capacity at Santos and the full rolling capacity

at USIMINAS were in operation; however, in one run the steel shop-blast furnace capacity at Santos and the hot and cold strip mills at USIMINAS were assigned zero capacity. Also the blast furnace capacity at USIMINAS was reduced to 485 thousand tons per year to reflect the reality that only one of the USIMINAS blast furnaces was in operation. All other parameters of the model were the same as those for the first experiment described, i.e. lower transportation cost, higher tinning line capacity at Volta Redonda, and lower blast furnace-steel shop and primary mill capacity at Volta Redonda (this last item accounting for the fact that a part of the capacity of these two units at Volta Redonda is used to produce shapes rather than flats).

As was expected the model responded by a shipment of 229 thousand tons of ingot steel from USIMINAS to COSIPA and a shipment of 203 thousand tons of ingot steel from CSN to COSIPA. Table 3.13 shows the activity levels for the original run (same as the original run of the first experiment) and of the modified run. The results of the modified run should correspond roughly to the actual product flows in Brazil in 1964. Though some of the flows are known to correspond to reality, complete statistics on the flows are not yet available. For example, it is known that there were shipments of plate from USIMINAS to Rio de Janeiro, of cold sheets and strip from COSIPA to São Paulo, and of tin plate from Volta Redonda to São Paulo.

There were some exports of steel plate from USIMINAS in 1964. Though the results of the run do not show such shipments, the magnitude

Table 3.13

Intermediate Product Shipment Test

Activity Levels of Variables
(million metric tons)

Activity \ Run	Original	Modified
USSAS	-	.229
VRSAS	-	.203
SASPP	.020	-
USSPP	-	.020
USRJP	.160	.160
USBHP	.020	.020
SASPH	.200	.200
USRJH	.064	-
USBHH	.080	-
VRRJH	.056	.120
VRBHH	-	.080
SASPC	.300	.140
USRJC	.005	-
USBHC	.140	-
VRSPC	-	.160
VRRJC	.155	.160
VRBHC	-	.140
SASPT	.001	-
USBHT	.001	-
VRSPT	.149	.150
VRRJT	.060	.060
VRBHT	.039	.040
SAEXH	.082	-
VREXH	.094	-
SAEXC	.039	-
VREXC	.334	-
VREXT	.019	-

of the change that would result in this behavior is shown in Table 3.14. This table gives, in the column labeled "reduced cost," the required decrease in the cost of any activity for that activity to enter the basis, or in this case the required increase in the profitability of exports for these export activities to enter the basis. In the case of exports of steel plate from USIMINAS (USEXP) the required increase in profitability was only US$1.14, i.e. an increase in the profit per ton on the exports of steel plate from US$2.00 per ton to US$3.14 per ton. Likewise, the exports of hot and cold sheets and strip and of tin plate from Volta Redonda are shown to have very low "reduced cost."

The partial figures on USIMINAS production and exportation in 1964[1] point out some of the modifications which should be made in this and future models. First, pig iron production was substantially below the anticipation of the model. This may have resulted from the shipping bottleneck at the port of Vitoria which had not reached the expanded capacity required to insure a smooth flow of products. Or the difference may have resulted from the decreased level of economic activity in Brazil in 1965 which resulted from an effort to slow the inflation. Or the overestimate of the model may have resulted from failure to account for operational difficulties inherent in the start up of any new steel plant. Most likely the difference resulted from a combination of these and other factors not included in the model.

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[1] ILAFA, Revista Latinoamericana de Siderurgica, (monthly) Santiago, Chile, No. 59, March 1965, p.18.

Table 3.14

Intermediate Product Shipment Test

Cost Reduction Required for Activities Not in Bases to Enter Basis
(US$/metric ton)

Activity	Current Cost	Reduced Cost	Basis Value
SAEXP	-2.00	8.36	-10.36
USEXP	-2.00	1.14	-3.14
VREXP	-2.00	3.65	-5.65
SAEXH	-3.00	7.03	-10.03
USEXH	-3.00	13.29	-16.29
VREXH	-3.00	2.47	-5.47
SAEXC	-4.00	6.60	-10.60
USEXC	-4.00	13.35	-17.35
VREXC	-4.00	1.78	-5.78
SAEXT	-5.00	6.33	-11.33
USEXT	-5.00	13.57	-18.57
VREXT	-5.00	1.18	-6.18

Secondly, the USIMINAS data show substantial exports of pig iron, ingot steel, slabs, and steel plate in 1964 -- none of which were predicted by the model. This result shows the importance in further developments of the model of including the possibility of the exportation of intermediate products such as pig iron, ingot steel, and slabs as well as the exportation of final products.

5. Capacity Addition Test.

The cost of most capital equipment used in steel production is so great that many Latin American governments require government approval of all projects for any substantial investment requiring imported equipment. In other cases part of the financing of the investment comes from the government. In these instances there may at times be rival claims from several companies who want to expand their production facilities. Then the government may be faced with the problem of deciding between two or three plans. This experiment shows how a linear model may be used to provide a partial answer to this type of problem.

Here we consider the question of adding a new tinning line of 200 thousand tons per year capacity. A new tinning line is added to the capacity of each plant in turn to see where in the system it should be located to minimise total operating and transportation costs. (We assume that the capital cost of locating the new tin mill at each of the three steel mills would be the same, so that we need only consider the differences in operating and transportation costs.) All parameters in the model are the same as for the first test.

The value of the objective function for the original solution before the addition of the new tinning line was US$93,446 million. This amount was decreased by US$270 thousand by locating the new line at Santos, US$136 thousand by placing it at Volta Redonda, and US$50 thousand by installing it at USIMINAS. Thus, there is a significant annual savings gained by locating the new mill at the Santos plant. The proximity of the largest market in the country for tin plate, the São Paulo market area, was the key factor in determining the cost-minimising effects of installing the tinning line at the COSIPA plant. Another important factor was the availability of capacity in the cold strip mill that could be used for providing the raw material input for the tinning line. The COSIPA mill could give up a portion of its share of the cold sheet and strip market in São Paulo to the Volta Redonda plant at relatively little additional cost to the system, and then use the cold strip mill capacity to provide input for the tinning line. USIMINAS on the other hand could not so easily give up the capacity of its cold strip mill, since it was providing cold sheet and strip to the Belo Horizonte market area that no other plant in the system could provide for anything like as little cost to the system. Thus, the result of the runs was that a new tinning line of 200 thousand tons capacity would be immediately used to 80 per cent of capacity if installed at Santos but would only be used to 20 per cent of capacity if installed at USIMINAS.

Since the profitability of installing a new tinning line is apparently strongly affected by the availability of cold strip mill capacity, it would appear wise to make additional tests to consider the addition of a new tinning line together with an additional cold strip mill in each plant.

6. Transportation Cost Test

In Section 2 of this paper it was observed that there had been a secular decline in deflated railroad freight rates in Brazil over the period 1959 to 1963. In this test the parameters of the railroad transportation cost function were varied to see what might have been the effects on the steel industry of this decrease in railroad freight rates. It was assumed that truck freight rates decreased at the same rate as railroad freight rates.

In the railroad transportation cost function $T = a + bX + u$ we used values of 1.80 and 0.00700 for a and b respectively for most of the experiments with the model. Table 2.1 shows that the maximum value for the parameter a was 2.61 (1959) and the minimum value was 1.54 (1963). Similarly the maximum value for the parameter b was 0.00896 (1959) and 0.00644 (1963).

The values of the parameters a and b used in the experimental runs are shown in Table 3.15. Note that values used in the experiment have a range about twice that of the range observed between 1959 and 1963.

Table 3.15

Values of the Parameters "a" and "b" used in The Transportation Cost Test

Run	a	b
Original	1.80	0.00700
1	0.90	0.00350
2	1.35	0.00525
3	2.25	0.00875
4	2.70	0.01050

Table 3.16 shows that when the transportation cost was decreased from the original level there were some readjustments, but when the cost was increased there was no change in the activity levels. When the transportation cost decreased Volta Redonda lost part of its nearby market in Rio de Janeiro to the more distant plants of COSIPA and USIMINAS.

7. Limitations of the Model

The construction of an economic model may be compared to an optimizing problem where the objective is to approximate the reality with the strongest possible analytical tool and where this objective is constrained by (1) the rapidity of solution of the model, (2) the availability of data about the present, and (3) uncertainty of projections for the future.

7.1 Rapidity of Solution. If rapidity of solution were not a constraint one could simply write programs to consider all the possible

combinations of events and to select from them the least-cost combination. However, this type of exercise is prohibitively expensive, even for relatively small models such as the one presented here. With sixty-six possible activities to chose from (the number of columns in the LP matrix) and with constraints and a matrix such that the rank of the matrix is twenty-one (the number of activities in the solution), there are some $\binom{66}{21} = 9.05 \times 10^{16}$ possible extreme solutions. A linear programming algorithm is simply a way of beginning with one of these extreme solutions and considering only a small subset of them in arriving at an optimum solution.

So when a problem can be structured so as to closely approximate the non-linearities in the system with linear functions and at the same time to fit the other requirements of a linear programming problem, one can take advantage of this powerful computational method. Various types of non-linear programming models offer better approximations to the non-linear parts of the system but sacrifice computational efficiency. Thus, one is faced with a choice of using a non-linear model which gives a closer fit to a part of the reality but which limits very sharply the number of plants, markets, and products which can be included in the model, or of using a linear model with more plants, markets, and products, but with a poorer approximation to the non-linear parts of the system. For this part of the study a linear model has been used; for the next part a non-linear model will be used.

Table 3.16

Transportation Cost Test

Changes in Activity Levels of Variables
(million metric tons)

Activity	Original	1	2
SASPP	.020		
SARJP	-	+.054	+.054
USRJP	.160	-.054	-.054
USBHP	.020		
SASPH	.200		
USRJH	.064	+.056	+.056
USBHH	.080		
VRRTH	.056	-.056	-.056
SASPC	.300		
SARJC	-	+.040	+.040
USRJC	.006		
USBHC	.140		
VRRJC	.154	-.039	-.039
SASPT	-		
USBHT	-		
VRSPT	.150		
VRRJT	.060		
VRBHT	.040		
SAEXH	.082	-.056	-.056
VREXH	.094	+.056	+.056
SAEXC	.040	-.040	-.040
VREXC	.335	+.040	+.040
VREXT	.017		

What then is the extent of the cost of using a linear rather than a non-linear model for this part of this study? Linear programming models require that all cost be linear functions of their activity levels or use. As has been discussed earlier in this paper, this is not a restrictive assumption for raw material cost but is a violation of the reality of labor cost.

Labor cost is a non-linear function of the level of production in that a certain number of men are needed to operate and maintain the productive units whether they are used to capacity or not. Also, in most steel mills the firms are constrained by strong unions from laying off and adding to the labor force in the plant at will. However, the non-linearities in labor cost do not cause a significant bias in the results because the component of labor cost in total cost is small.

The linearity of the model also requires that the cost of each activity be independent of the level of activity of all other activities. This problem was mentioned earlier in the paper. The assumption would seem to be valid for independence in the cost of activities in different plants or in the cost of importing and exporting activities; however, it is less valid for independence among the activities within a given plant.

Even when using a linear programming model the rapidity of solution sharply constrains the number of products, markets, and plants which can be considered in a problem. Since it was desirable to have a model that could be solved in five to ten minutes on the IBM 1620 (a small computer), the problem was limited to four products, three market areas, and three plants. As was mentioned earlier the limitation to four

products prevents the model from catching in its analytical net many important distinctions. For example, the differentiation between heavy and light, narrow and wide steel plates is not considered. Likewise, there is no consideration in the model of the quality or width of hot and cold rolled sheets and strips. Also, the problem has been limited to a consideration of only flat steel products.

The limitation to three market areas prevents the model from considering the fact that a portion of the requirement for steel products is located in parts of the country distant from the southern triangle of São Paulo, Rio de Janeiro, and Belo Horizonte. However, in the case of flat steel products, which are used almost entirely as intermediate products for manufacturing other goods, the assumption of point concentration of markets in this area provides a close approximation to the reality.

Limiting the analysis to three plants excludes from consideration a number of other plants that produce flat products. However, aside from the firm Belgo-Mineria which has been producing about 100 thousand tons of flat products a year, none of the excluded plants is a large volume producer of flat steel products.

The desire for rapidity of solution of the problem also limits its ability to consider scheduling problems or fluctuations in requirements and output over the course of a year. The model permits no consideration of the fact that during a certain part of the year a consumer of steel products may be willing to pay a premium in order to have the

products available sooner. Likewise, transportation bottlenecks may prevent the smooth flow of product over the course of a year.

Though external economies and diseconomies have been excluded from implicit consideration in the model, they can be studied with this type of model. The requirements shift test described earlier is an example.

7.2 Availability of Data. The treatment of transportation cost in this paper provides a good example of the limitations placed on this type of study by the availability of data. The information which would ideally be used is the real social cost of transporting the products. These real social costs should reflect capacity constraints as well as other variables. That type of information not being available, one may resort to the rates actually charged to each steel company for shipment of each product over a wide variety of routes via three or four different means of transportation. That not being available, one may approximate these costs by using published data for railroad rates for steel products and estimating the functions described in an earlier section of the paper.

One of the most serious data constraints on the model is the lack of a detailed breakdown by type of product, market area, and year, of consumption of flat steel products. The availability of this information plus product flows in the system would enable one to test the model against the reality.

7.3 Uncertainty. Neither this model nor the mixed integer programming model discussed in the following chapters deal with the problem of uncertainty except in a very limited fashion. Considering the limited time available for preparing the study it was decided to concentrate first on the preparation of the mixed integer programming model under uncertainty and to postpone the added complication of uncertainty.

Uncertainty can be dealt with in this type of model by varying the parameters of the model within expected limits and running the model repeatedly with these changes. The tests described above for changes in transportation and other cost parameters and for changes in requirement levels are of this type. The facility with which the original elements of the matrix can be stored on disks or magnetic tape and then modified one at a time or in groups without the necessity of reloading the entire matrix for each run is a great aid in conducting this type of test.

A more systematic treatment of uncertainty may be made through the use of a modification of linear programming called "chance constrained programming."[1]

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[1] See Charnes, A. and W. W. Cooper, "Chance Constrained Programming," Mgmt. Science 6, pp.73-80, and an application of this technique to transportation investment planning by Pedro N. Taborga "Determination of an Optimal System of Transportation for Chile," mimeographed, Department of Civil Engineering, M.I.T., January 1965.

8. Improvements in the Model

As yet there has been little opportunity to check the results of the model against the actual product flows in the system. (This is not to say that the theoretical product flows should exactly correspond to the real flows, for example that all plant managers and consumers act in an optimizing fashion, but rather that the burden of proof is on the model to show its relevance to the situation.) However, the slight comparison that has been possible has indicated the importance of a number of modifications in the model.

First, the assumption of equal profits to all firms on exports of the same product must be altered to provide larger profits to those firms which produce at lower cost and which have good transportation access to foreign markets. This could be done by subtracting the estimated product cost (delivered to the nearest port) from the world price for the product.

Secondly, the model should be modified to include domestic trade in and export of other intermediate products. The present model permits domestic trade but not export of ingot steel. Activities for export of ingot steel, slabs, and pig iron as well as for domestic trade of slabs and pig iron should be included in the model. Likewise, consideration should be given to the inclusion of activities for domestic trade in blooms.

Thirdly, the detailed development of product cost which was accomplished in this model for pig iron and ingot steel should be extended

to cover the cost of slabs, blooms, plate, hot sheet and strip, cold sheet and strip, and tin plate.

9. Conclusions

The linear programming model of the production and transportation of flat steel products presented here has shown itself to hold great promise as an analytical tool for studying the operation of this sector of the Brazilian economy. Furthermore, it appears that a model which places capacity constraints on a number of the principal production units in each plant, disaggregates final product requirements into a number of different goods, permits shipment of intermediate products between plants, divides product requirement on a market area basis, includes transportation cost for final products to these market areas from the respective plants, and permits both exports and imports of the products, can provide a reasonably good approximation to an industry while remaining small enough to permit solution on a small computer in a short period of time.

The speed with which the model can be solved, coupled with the capability of the computer to store the input data on disks for modification of elements of the input without reloading the entire program, increases the usefulness of the model as an analytical tool. Repeated runs of the model can easily be made to test the effects of changes in the parameters and thereby to determine their economic effects.

CHAPTER IV

A MIXED INTEGER PROGRAMMING MODEL

This chapter presents a mixed integer programming model for studying the problem of when and where to add to capacity in an existing system of plants. This multi-time period model uses an expanded version of the linear programming model of the previous chapter as the submodel for each time period. An investment matrix is appended to the multi-time period model and additional rows are added to the bottom of the matrix to constrain the investment variables to take on only the values of zero or one. Thus, with the transportation-production activities of each time period free to take on any non-negative value and the investment activities constrained to 0-1 integer values, the model becomes a mixed integer programming model.

The expanded version of the linear programming model which is the sub-model for each time period permits the shipment between plants of the three intermediate products (pig iron, ingot steel, and slabs), rather than only the single intermediate product (ingot steel) of the previous model. Also, the enlarged version of the model permits the importation and exportation of each of the three intermediate products. As there is a substantial trade in intermediate products in the steel industry, this added capability of the model adds significantly to its realism.

In the previous version of the linear programming model, the blast furnace and steel shop were treated as a single productive unit. In the new version these two processes are treated as separate productive units.

The investment matrix consists of a separate activity for each of twenty-three investment opportunities. Four of the investment opportunities are for projects which could be completed by the beginning of the second time period, seven are for projects which could be completed for the third time period, and the remaining twelve are for projects which could be completed for the fourth time period. Thus, the problem of the model is one of choosing that combination of investment projects, of domestic production, of exports, and of imports which minimizes the present value of the cost of meeting the product requirements in each market in each year during the period July 1, 1967 to January 1, 1975 covered by the model.

The notation of the model was set up for a ten year span consisting of four time periods, each of two and a half years duration. The first time period of January 1, 1965 to July 1, 1967 was excluded from the model because investment decisions made now will not affect the structure of the industry for some years to come. Therefore, the model includes period 2 (July 1, 1967 to January 1, 1970); period 3 (January 1, 1970 to July 1, 1972); and period 4 (July 1, 1972 to January 1, 1975).

The form of this and the next chapter parallels that of the previous two chapters. In this chapter the model is written out in a

series of equations and inequalities; then the components of the data input to the model are discussed. In the next chapter the computational methods are explained, the results analyzed. The primary difference between this chapter and the chapter which described the linear programming problem is that here a discussion of the component parts of the matrix and the right-hand side of the problem is to be used as the vehicle for carrying the discussion of the data used in the model.

1. <u>The Model</u>

(1) <u>The Objective Function</u>.

$$\text{Min } Z = \sum_{t=2}^{4} \left(\underbrace{\sum_{i=1}^{n} \sum_{\substack{\ell=1 \\ \ell \neq i}}^{n} \sum_{k=1}^{3} a_{i\ell}^{kt} u_{i\ell}^{kt}}_{(a)} + \underbrace{\sum_{i=1}^{n} \sum_{k=1}^{3} b_{i}^{kt} v_{i}^{kt}}_{(b)} + \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=4}^{p} c_{ij}^{kt} w_{ij}^{kt}}_{(c)} + \underbrace{\sum_{j=1}^{m} \sum_{k=4}^{p} d_{j}^{kt} x_{j}^{kt}}_{(d)} - \underbrace{\sum_{i=1}^{n} \sum_{k=1}^{p} e_{i}^{kt} y_{i}^{kt}}_{(e)} \right) + \underbrace{\sum_{r=1}^{q} f_r z_r}_{(f)}$$

Subject to the following restrictions for each time period.

(2) <u>Capacity utilization constraint</u>.

$$\sum_{\substack{\ell=1 \\ \ell \neq i}}^{n} \sum_{k=1}^{3} \alpha_i^{ke} u_{i\ell}^{kt} \quad - \quad \sum_{\substack{\ell=1 \\ \ell \neq i}}^{n} \sum_{k=1}^{3} \alpha_i^{ke} u_{\ell i}^{kt}$$

(a) (b)

$$- \sum_{k=1}^{3} \alpha_i^{ke} v_i^{kt} \quad + \quad \sum_{j=1}^{m} \sum_{k=4}^{p} \alpha_i^{ke} w_{ij}^{kt}$$

(c) (d)

$$+ \sum_{k=1}^{p} \alpha_i^{ke} y_i^{kt} \quad - \sum_{r=1}^{q} \eta_{ir}^{et} z_r \leq \gamma_i^e$$

(e) (f)

for

$e = 1,\ldots,5$
$i = 1,\ldots,n$
$t = 2,\ldots,4$

(3) <u>Requirements constraints</u>.

$$\sum_{i=1}^{n} w_{ij}^{kt} + x_j^{kt} \geq \sigma_j^{kt}$$

for

$k = 1,\ldots,p$
$j = 1,\ldots,m$
$t = 2,\ldots,4$

(4) <u>Non-negativity constraints</u>.

$$u_{i\ell}^{kt} \geq 0 \qquad \begin{array}{l} k=1,\ldots,3 \\ \left.\begin{array}{l}\ell=1,\ldots,n \\ i=1,\ldots,n\end{array}\right\} i \neq \ell \\ t=2,\ldots,4 \end{array}$$

$$v_i^{kt} \geq 0 \qquad \begin{array}{l} k=1,\ldots,3 \\ i=1,\ldots,n \\ t=2,\ldots,4 \end{array}$$

$$w_{ij}^{kt} \geq 0 \qquad \begin{array}{l} k=4,\ldots,p \\ j=1,\ldots,m \\ i=1,\ldots,n \\ t=2,\ldots,4 \end{array}$$

$$x_j^{kt} \geq 0 \qquad \begin{array}{l} k=4,\ldots,p \\ j=1,\ldots,m \\ t=2,\ldots,4 \end{array}$$

$$y_i^{kt} \geq 0 \qquad \begin{array}{l} k=1,\ldots,p \\ i=1,\ldots,n \\ t=2,\ldots,4 \end{array}$$

(5) <u>Integer constraints</u>.

$$z_r = 0,1 \qquad r=1,\ldots,q$$

The objective function (1) states that the object of the problem is to minimize the present value of the cost of producing and transporting the required final bill of goods during the time periods covered by the model. The first summation in the equation sums the cost over the three time periods, t = 2,3,4. Term (a) of the equation represents the "extra" cost to the system of producing intermediate product k in time period t in plant i and shipping it to plant ℓ. The word "extra" refers to the fact that the activity cost, $a_{i\ell}^{kt}$, represents the difference between the cost of producing intermediate product k in plant i and shipping it to plant ℓ and the cost of producing it in plant ℓ. Term (b) includes the "extra" cost to the system of importing intermediate product k in time period t rather than producing it in plant i. "Extra" here means the difference between the cost of importing intermediate product k and the cost of producing it at plant i. In both term (a) and term (b) the k index runs from 1 to 3, while in terms (c) and (d) it runs from 4 to 7, and in term (e) it runs from 1 to 7. The products corresponding to each of these indices are as follows:

1 pig iron	intermediate products
2 ingot steel	
3 slabs	
4 plate	final products
5 hot sheet and strip	
6 cold sheet and strip	
7 tin plate	

Term (c) of the equation (1) represents the cost of producing in all of the $i = 1,2,\ldots,n$ plants and shipping to all of the $j = 1,2,\ldots,m$ market areas each of the $k = 4,5,\ldots,p$ final products. Term (d) includes the cost of importing each of the final products $k = 4,5,\ldots,p$ to each of the market areas $j = 1,2,\ldots,m$. The negative sign preceding term (e) shows that exports are treated as a way of decreasing the cost to the system of satisfying the domestic requirements. Finally, the term (f) represents the costs of investments $r = 1,2,\ldots,q$.

The capacity utilization constraints require that in each of the $e = 1,2,\ldots,s$ productive units in each of the plants in every time period that the capacity utilized to produce intermediate goods for other plants, final products for domestic consumption, and intermediate and final products for exports (terms (a), (d), and (e) respectively), must not exceed the capacity available. The negative terms in this set of constraints represent the means of easing these capacity bottlenecks through the receipt of intermediate products from other domestic plants, through imports of intermediate products, and/or through the construction of new capacity (terms (b), (c), and (f) respectively).

The set of constraints (3), requires that in each time period t, for each final product k and each market area j the total amount supplied domestically plus the amount imported will be greater than or equal to the requirement.

The non-negativity constraints (4) require that all variables in the problem be limited to values greater than or equal to zero.

One subset of the variables, the investment variables, z_r, are limited by the set of constraints (5) to take on only the values zero or one.

In greater detail:

Z = the present value in US$ million of producing and transporting the required final bill of goods during the time periods covered by the model.

$a_{i\ell}^{kt}$ = present value of cost (other than capital cost) in US$/metric ton for supplying over the $2\frac{1}{2}$ year length of time period t one metric ton per year of product k produced at plant i and delivered to plant ℓ, minus the cost (other than capital cost) of producing one metric ton per year of product k over the same length of time in plant ℓ, i.e. the extra cost to the system when plant ℓ uses intermediate product k produced at plant i rather than in its own facilities.

$u_{i\ell}^{kt}$ = million metric tons/year of product k shipped as an intermediate product from plant i to plant ℓ during time period t.

$u_{\ell i}^{kt}$ = million metric tons/year of product k shipped as an intermediate product from plant ℓ to plant i during time period t.

b_i^{kt} = present value of cost (other than capital cost) in US$/metric ton of importing over the $2\frac{1}{2}$ year length of time period t one metric ton per year of product k for use as an intermediate product at plant i, minus the cost (other than capital cost) of producing one metric ton per year of product k over the same length of time in plant i, i.e. the extra cost to the system of importing intermediate product k to plant i rather

v_i^{kt} = million metric tons/year of product k imported during time period t for use as an intermediate product at plant i.

c_i^{kt} = present value of cost (other than capital cost) in US$/metric ton for producing at plant i over the $2\frac{1}{2}$ year length of time period t one metric ton per year of product k and delivering it to market area j.

w_i^{kt} = million metric tons/year of product k produced at plant i and delivered to market area j during time period t.

d_j^{kt} = present value of cost (other than capital cost) in US$/metric ton for importing to market area j over the $2\frac{1}{2}$ year length of time period t one metric ton per year of product k.

x_j^{kt} = million metric tons/year of product k imported to market area j during time period t.

e_i^{kt} = present value in US$/metric ton of export profits gained by the system for the $2\frac{1}{2}$ years of time period t from exporting one metric ton per year of product k from plant i. Export profits per metric ton are defined as the f.o.b. prices for Brazilian exports of product k in 1964 less the sum of the production cost (other than capital cost) and the transportation cost to the Brazilian port (including loading cost).

y_i^{kt} = million metric tons/year of product k exported from plant i during time period t.

f_r = present value of cost of investment r, in US$ millions

z_r = an integer variable which is restricted to the values zero or one.

α_i^{ke} = metric tons of capacity required in production unit e of plant i for each metric ton of product k produced at plant i.

η_{ir}^{et} = million metric tons of capacity created in production unit e of plant i in time period t (and in all time period after t) by investment r.

γ_i^e = capacity in 1965 of production unit e in plant i, in millions of metric tons of product produced by the unit.

σ_j^{kt} = requirements of market area j for product k during time period t, in millions of metric tons per year.

2. The Matrix and the Right Hand Side

Figure 4.1 shows the structure of the mixed integer programming model. The upper case letters represent matrices, the underlined lower case letters represent vectors, and other lower case letters represent elements. The superscripts on the variables show the time period. No time superscripts are placed on the matrices because it is assumed that the capacity utilization aspect of the technology remains constant over time.

There are three large vectors and one large matrix in Figure 4.1. The row vector just above the matrix is the vector of activity cost. Its component vectors $\underline{a}^2$, $\underline{b}^2$, $\underline{c}^2$,... represent the correspondingly lettered elements in the objective functions. For example the vector $\underline{b}^2$ is composed of the elements b_i^{k2}. The same is true of the column vector of unknowns, that is the components of the vector $\underline{z}$ are z_r.

The right-hand side vector is shown to the right of the inequality sign. It consists of the $\bar{\underline{s}}$ vector which contains elements

for the capacity of each productive unit in each time period and elements for the requirement of each market area for each product in each time period. The $\bar{\underline{h}}$ vector is the lower part of the right-hand side and contains elements used to constrain the investment variable to integer values.

The principal components of the matrix are the three stair steps and the F and Q matrices. The F matrix is the investment matrix, and the Q matrix is the integer constraint matrix. Each stairstep consists of five matrices. The A matrix is for intermediate product shipments, the B matrix is for imports of intermediate products, the C matrix is for domestic production-transportation activities for final products, the D matrix is for imports of final products and the E matrix is for exports of both intermediate and final products.

The structure of this matrix is in many aspects similar to the multi-time period matrix used by Vietorisz in his fertilizer plant location study.[1]

2.1 The Elements of the Matrix and the Right Hand Side. Figure 4.2 shows the arrangement of the matrices A_i in the A matrix. The notation at the top of the matrix P_1,P_2, P_1P_3, etc. indicates the direction of product flow in that section of the matrix. For example, P_1P_2 is for shipments from plant 1 to plant 2. Thus, shipments from plant 1 to plant 2 use up capacity of the productive units in plant 1

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[1]Vietorisz, Thomas, "Industrial Development Planning Models...." op.cit., p.32.

$$\text{Min } Z = [\underline{a}^2\ \underline{b}^2\ \underline{c}^2\ \underline{d}^2\ \underline{e}^2 \mid \underline{a}^3\ \underline{b}^3\ \underline{c}^3\ \underline{d}^3\ \underline{e}^3 \mid \underline{a}^4\ \underline{b}^4\ \underline{c}^4\ \underline{d}^4\ \underline{e}^4 \mid \underline{f}] \left[\begin{array}{ccccc|ccccc|ccccc|c} A & B & C & D & E & & & 0 & & & & & 0 & & & \\ \hline & & 0 & & & A & B & C & D & E & & & 0 & & & F \\ \hline & & 0 & & & & & 0 & & & A & B & C & D & E & \\ \hline & & 0 & & & & & 0 & & & & & 0 & & & Q \end{array}\right] \left[\begin{array}{c} \underline{u}_2 \\ \underline{v}_2 \\ \underline{w}_2 \\ \underline{x}_2 \\ \underline{y}^2 \\ \hline \underline{u}_3^3 \\ \underline{v}_3 \\ \underline{w}_3 \\ \underline{x}_3 \\ \underline{y} \\ \hline \underline{u}_4^4 \\ \underline{v}_4 \\ \underline{w}_4 \\ \underline{x}_4 \\ \underline{y} \\ \hline \underline{z} \end{array}\right] \leq \left[\begin{array}{c} \bar{\underline{s}} \\ \hline \bar{\underline{h}} \end{array}\right].$$

Figure 4.1

The Multi-Period Matrix and the Right Hand Side

$$
\begin{array}{c}
\begin{array}{cccccc|ccc|ccccccccc|ccc|ccc}
P_1P_2 & P_1P_3 & P_2P_1 & P_2P_3 & P_3P_1 & P_3P_2 & IP_1 & IP_2 & IP_3 & P_1M_1 & P_1M_2 & P_1M_3 & P_2M_1 & P_2M_2 & P_2M_3 & P_3M_1 & P_3M_2 & P_3M_3 & IM_1 & IM_2 & IM_3 & EP_1 & EP_2 & EP_3
\end{array} \\
\left[\begin{array}{cccccc|ccc|ccccccccc|ccc|ccc}
A_1 & A_1 & -A_1 & 0 & -A_1 & 0 & -A_1 & & & C_1 & C_1 & C_1 & & & & & & & & & & A_1C_1 & & \\
-A_2 & 0 & A_2 & A_2 & 0 & -A_2 & & -A_2 & & & & & C_2 & C_2 & C_2 & & & & 0 & & & & A_2C_2 & \\
0 & -A_3 & 0 & -A_3 & A_3 & A_3 & & & -A_3 & & & & & & & C_3 & C_3 & C_3 & & & & & & A_3C_3 \\
 & & & 0 & & & & 0 & & & -I_{12} & & & -I_{12} & & & -I_{12} & & & -I_{12} & & & 0 &
\end{array}\right]
\end{array}
$$

Interplant Shipments of Intermediate Products

Imports of Intermediate

Plant to Market Shipments of Final Products

Imports of Final Products

Exports of both Intermediate and Final Products

Figure 4.2 The Single Period Sub-matrix

the elements of A_1 in this series of columns are positive), and they add to effective capacity in plant 2 (the elements of A_2 in this series of columns are negative). The elements of the matrices A_i are shown in Figure 4.3.

The large zero in the bottom half of the A matrix indicates that inter-plant shipments of intermediate products have no effect on the satisfaction of final product requirements. The elements σ_i^{kt} in the vector $\bar{\underline{r}}_t$ in the right-hand side, are the requirements for market area j for final product k in the tth time period. The $\bar{\underline{k}}$ vector is composed of elements, γ_i^e, which give the capacity of productive unit e in plant i at the beginning of the time covered by the model.

The B matrix represents imports of intermediate products. The matrices A_i in the B matrix are identical to those in the A matrix (except for sign) and consist entirely of negative elements because all imports of intermediate products serve to ease capacity constraints. The notation IP_1 indicates that the series of columns in that section of the matrix are for imports of intermediate products to plant 1.

The next section of the matrix, the C matrix, is the familiar linear programming production-transportation problem. Figure 4.3 shows that the elements of the matrices C_i are the metric tons of capacity required in productive unit e at plant i for each metric ton of final product k produced. The final products and productive units are:

$$A_i = \left[\alpha_i^{ke}\right]_{k=1,2,3} = \begin{bmatrix} \alpha_i^{11} & \alpha_i^{21} & \alpha_i^{31} \\ 0 & \alpha_i^{22} & \alpha_i^{32} \\ 0 & 0 & \alpha_i^{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

α_i^{ke} = metric tons of capacity required in production unit e of plant i for each metric ton of product k produced at plant i.

Intermediate Products

$$C_i = \left[\alpha_i^{ke}\right]_{k=4,5,6,7} = \begin{bmatrix} \alpha_i^{41} & \alpha_i^{51} & \alpha_i^{61} & \alpha_i^{71} \\ \alpha_i^{42} & \alpha_i^{52} & \alpha_i^{62} & \alpha_i^{72} \\ \alpha_i^{43} & \alpha_i^{53} & \alpha_i^{63} & \alpha_i^{73} \\ \alpha_i^{44} & \alpha_i^{54} & \alpha_i^{64} & \alpha_i^{74} \\ 0 & \alpha_i^{55} & \alpha_i^{65} & \alpha_i^{75} \\ 0 & 0 & \alpha_i^{66} & \alpha_i^{76} \\ 0 & 0 & 0 & \alpha_i^{77} \end{bmatrix}$$

Final Products

Figure 4.3

The Capacity Utilization Submatrices A and C.

Product Indices		Productive Unit Indices	
index	final product	index	productive unit
4	steel plate	1	blast furnace
5	hot sheet and strip	2	steel furnaces
6	cold sheet and strip	3	primary mill
7	tin plate	4	roughing mill
		5	hot strip mill
		6	cold strip mill
		7	tinning line

The arrangement of the matrices C_i in the C matrix is shown in Figure 4.2. The notation P_1M_1 indicates that the series of columns in this section of the matrix are for shipments of final products from plant 1 to market area 1. The subscripts on the identity matrices, I, indicate the number of rows and columns in these square matrices. Thus, the activities in the series of columns headed by P_1M_1 use up capacity in the productive units of plant 1 and at the same time satisfy the final product requirements of market area 1. The negative sign on the identity matrices and on the lower portion of the right-hand side vector, $\bar{\underline{r}}$, exist because all elements in these rows have been multiplied by minus one in order to reverse the inequality.

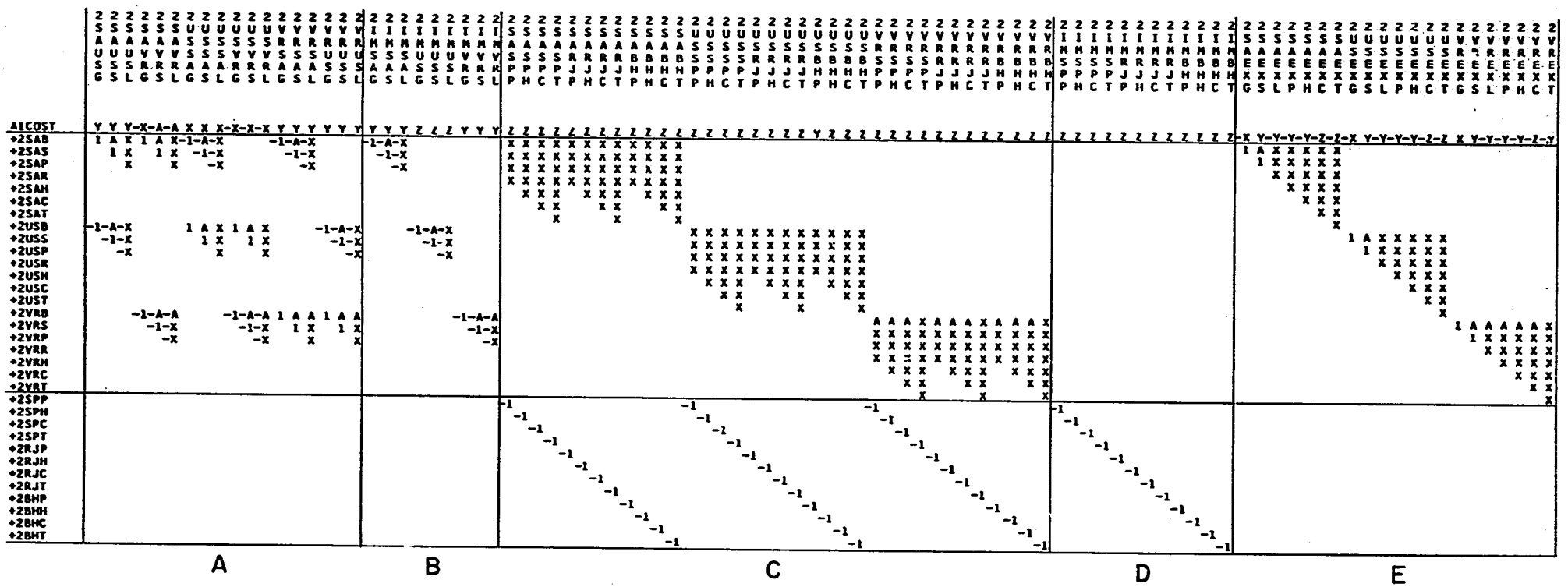
The D matrix represents the importation of final products. These activities affect only the satisfaction of market requirements.

The activities for exporting both intermediate and final products are included in the E matrix. The notation EP_i at the top of the matrix indicates that the activities in that section of the matrix are for the exportation of products from plant i. In the E matrix the sub-matrices A_i and C_i are placed side by side to create triangular matrices.

Figure 4.4 is a picture of a print-out of the matrix for the second time period. This print-out of the matrix was made with a program which is a part of the LP/11 system developed by Norman Driebeek of the Arthur D. Little Co. The guide to mnemonics contained in Chapter III is helpful for reading this picture. The legend gives the key to the magnitudes of the numbers shown in the picture.

Figure 4.1 shows that the investment matrix, F, is appended to the right-hand side of the multi-period model. The details of this matrix are shown in Parts I, II, and III of Figure 4.5. Part I of the figure contains the rows for the second time period as well as the corresponding right-hand side. Parts II and III of the figure show the similar sections for periods 3 and 4. In the matrix the parts of the figure would be arranged in a column with Part I on top.

The first seven rows of Part I of Figure 4.5 represent the production units of the COSIPA steel mill at Santos (SA) near São Paulo. The second seven rows are for the USIMINAS (US) plant in Minas Gerais state and the third group of seven rows are for the Volta Redonda (VR) plant of CSN located between Rio de Janeiro and São Paulo.



Legend

From		*To*
100.0	Z	1000.0
10.0	Y	100.0
1.0	X	10.0
0.1	A	1.0

Figure 4.4 A Picture of the Single Period Sub-matrix

		2ISAS	2USAS	2SAST	2VRTT	3SABB	3USSS	3ASSS	3UVSS	3SRSS	3ARST	3VRTT	4ISAB	4USSB	4VRBB	4SASS	4USSS	4VRSS	4SARR	4SACC	4UVSC	4VRCC	4SATC	4VRTT	RHS
1	2SAB																								.720
2	2SAS	-.80																							.800
3	2SAP																								1.800
4	2SAR																								1.000
5	2SAH																								1.500
6	2SAC																								.350
7	2SAT		-.18																						.000
8	2USB																								.900
9	2USS	-.80																							.600
10	2USP																								1.800
11	2USR																								1.000
12	2USH																								1.500
13	2USC																								.150
14	2UST																								.000
15	2VRB																								1.000
16	2VRS																								1.420
17	2VRP																								1.220
18	2VRR																								1.250
19	2VRH																								1.500
20	2VRC																								.800
21	2VRT			-.18																					.170
22	2SPP																								-.131
23	2SPH																								-.347
24	2SPC																								-.476
25	2SPT																								-.217
26	2RJP																								-.218
27	2RJH																								-.145
28	2RJC																								-.147
29	2RJT																								-.062
30	2BHP																								-.087
31	2BHH																								-.087
32	2BHC																								-.111
33	2BHT																								-.031

Figure 4.5 - PART I

The Investment Matrix "F", and the Right-Hand Side

Period 2

	2IS A S	2IU S S	2IS A T	2IV R T	3IS A B	3IU S B	3IS A S	3IU S S	3IV R S	3IS A T	3IV R T	4IS A B	4IU S B	4IV R B	4IS A S	4IU S S	4IV R S	4IS A K	4IS A C	4IU S C	4IV R C	4IS A T	4IV R T	RHS
3SAB				-.28																				.720
3SAS	-.80					-.80																		.800
3SAP																								.800
3SAR																								1.000
3SAH																								1.500
3SAC																								.350
3SAT		-.18							-.18															.000
3USB					-.36																			.900
3USS	-.80						-.80																	.600
3USP																								1.800
3USR																								1.000
3USH																								1.500
3USC																								.150
3UST																								.000
3VRB																								1.000
3VRS								-1.00																1.420
3VRP																								1.220
3VRR																								1.250
3VRH																								1.500
3VRC																								.800
3VRT			-.18							-.18														.170
3SPP																								-.164
3SPH																								-.433
3SPC																								-.598
3SPT																								-.274
3RJP																								-.274
3RJH																								-.181
3RJC																								-.184
3RJT																								-.078
3BHP																								-.109
3BHH																								-.108
3BHC																								-.138
3BHT																								-.039

Figure 4.5 - PART II

The Investment Matrix "F", and the Right-Hand Side
Period 3

	2ISA S	2IUS S	2ISA T	2IVR T	3ISA B	3IUS B	3ISA S	3IUS S	3IVS S	3IVR T	4ISA T	4IUS B	4IVR B	4ISA B	4IUS S	4IVR S	4ISA S	4IUS R	4IVR C	4ISA C	4IUS C	4IVR C	4ISA T	4IVR T	RHS
4SAB				-.28							-.80														.720
4SAS	-.80					-.80							-.80												.800
4SAP																									.800
4SAR																-1.00									1.000
4SAH																									1.500
4SAC																		-.35							.350
4SAT		-.18							-.18													-.18			.000
4USB					-.35							-.80													.900
4USS	-.80						-.80								-.80										.600
4USP																									1.800
4USR																									1.000
4USH																									1.500
4USC																				-.35					.150
4UST																									.000
4VRB												-.80													1.000
4VRS								-1.00								-1.00									1.420
4VRP																									1.220
4VRR																									1.250
4VRH																									1.500
4VRC																					-.35				.800
4VRT			-.18							-.18													-.18		.170
4SPP																									-.208
4SPH																									-.548
4SPC																									-.756
4SPT																									-.343
4RJP																									-.347
4RJH																									-.228
4RJC																									-.233
4RJT																									-.098
4BHP																									-.139
4BHH																									-.137
4BHC																									-.174
4BHT																									-.049

Figure 4.5 - PART III

The Investment Matrix "F", and the Right-Hand Side

The next four rows (22-25) are requirement rows for the market area of São Paulo (SP), and the eight final rows are the corresponding requirement rows for the market areas of Rio de Janeiro (RJ) and Belo Horizonte (BH). The map near the beginning of Chapter I shows the location of these plants and market areas.

The same mnemonics are used to identify the twenty-three investment opportunities. Thus, the variable name 2ISAS in the first column of the F matrix represents an investment in the steel furnaces at the plant at Santos. This project would be completed and ready to begin operation by the beginning of the second time period. Similarly, the variable 3IUSB represents an investment to be completed by the beginning of the third time period in the blast furnace at the USIMINAS plant.

The column name RHS is for the right-hand side. The entries in the first 21 rows in this column (which make up the vector $\bar{\underline{k}}$) are the capacity of each productive unit in 1965. The entries in the last twelve rows of the RHS column (which make up the $\bar{\underline{r}}_2$ vector) reflect the market requirements in the second-time period.

The entry of -.80 in the row 2SAS and the column 2ISAS indicates that the investment opportunity 2ISAS would add 800 thousand metric tons to the capacity of the steel shop in the Santos plant. In the same way, the project 2IVRT is for the installation of a tinning line of a capacity of 180 thousand metric tons of tin plate per year. The capacity of the existing electrolytic tin line at the Volta Redonda plant is 170 thousand metric tons per year, so the proposed investment would more than double the capacity of the tinning section of this steel mill.

Investments which would not be completed until the beginning of the third or the fourth time period have no effect on the capacity restrictions during the second time period. Therefore, the only entries in this top section of the F matrix are in columns for investment activities of the second time period. Part II of Figure 4.5 shows that projects completed at the beginning of both the second and the third time periods affect the capacity available in the third time period.

A comparison of the RHS column in Parts I, II and III of Figure 4.5 shows that the capacity vector $\bar{\underline{k}}$ remains constant over the time periods while the requirements vector $\bar{\underline{r}}_t$ varies with the time period (per the subscript). The capacity vector remains constant unless some of the original (i.e. 1965) productive units decrease in capacity due to aging and wearing of the equipment and/or are shut down and removed from the plant. Since two of the plants considered in this model have only been in operation for one or two years and the third plant is not likely to be forced to shut down any of its present production units during the next ten years, it is assumed that none of the major productive units in the existing plants will be removed or will decrease in capacity during the period covered by the model (1967 to 1975).

The requirements vector $\bar{\underline{r}}_t$ changes over time because requirements for steel products in the market areas are expected to increase during the time period covered by the model. Section 4 of this chapter contains a discussion of the derivation of these market area requirements.

The integer constraint sub-matrix Q, and the corresponding right-hand side vector $\bar{h}$, which fit heneath the F matrix and the top part of the right-hand side are shown in Figure 4.6.[1] Two additional rows are added to the matrix for each variable which is to be constrained to the values zero or one. The only entry in each of these rows is a one or minus one in the column of the variable which is to be constrained to zero or one (see Figure 4.6). The right-hand side values are then changed as follows to force the variable to take on (1) continuous values between and including zero and one, (2) exactly one, or (3) exactly zero:

	2ISAS		Alternative Right-Hand Side Values		
02ISAS	1	$\leq$	1	1	0
12ISAS	-1	$\leq$	0	-1	0
value of the variable			$0 \leq 2ISAS \leq 1$	2ISAS=1	2ISAS=0

The advantages of using this system for forcing the integer variables to take on the values zero or one is the facility with which right-hand sides can be changes in a series of linear programming runs with most linear programming codes.

---

[1]This method of constraining a subset of the variables in the model to integer values is outlined by Norman Driebeck (in a paper which is to be published in Management Science), "An Algorithm for the Solution of Mixed Integer Programming Problems," memo, Arthur D. Little, Inc., Cambridge, Mass. 1965.

	2ISAS	2IUSS	2ISAT	2IVRT	3ISAB	3IUSB	3ISAS	3IUSS	3IVRS	3ISAT	3IVRT	4ISAB	4IUSB	4IVRB	4ISAS	4IUSS	4IVRS	4ISAR	4ISAC	4IUSC	4IVRC	4ISAT	4IVRT	RHS
02ISAS	1																							1
12ISAS	-1																							0
02IUSS		1																						1
12IUSS		-1																						0
02ISAT			1																					1
12ISAT			-1																					0
02IVRT				1																				1
12IVRT				-1																				0
03ISAB					1																			1
13ISAB					-1																			0
03IUSB						1																		1
13IUSB						-1																		0
03ISAS							1																	1
13ISAS							-1																	0
03IUSS								1																1
13IUSS								-1																0
03IVRS									1															1
13IVRS									-1															0
03ISAT										1														1
13ISAT										-1														0
03IVRT											1													1
13IVRT											-1													0
04ISAB												1												1
14ISAB												-1												0
04IUSB													1											1
14IUSB													-1											0
04IVRB														1										1
14IVRB														-1										0
04ISAS															1									1
14ISAS															-1									0
04IUSS																1								1
14IUSS																-1								0
04IVRS																	1							1
14IVRS																	-1							0
04ISAR																		1						1
14ISAR																		-1						0
04ISAC																			1					1
14ISAC																			-1					0
04IUSC																				1				1
14IUSC																				-1				0
04IVRC																					1			1
14IVRC																					-1			0
o4ISAT																						1		1
14ISAT																						-1		0
o4IVRT																							1	1
14IVRT																							-1	0

Figure 4.6
The Integer Constraint Submatrix "Q", and Corresponding Right Hand Side.

2.2 The Coefficients in the Objective Function.

2.2.1 Discounting of the Cost for Production-Transportation Transportation Activities.

The coefficients in the objective function are the present values of the costs of operating each of the activities of the model at the unit level. These coefficients were calculated in three steps. First, the cost of producing and transporting a product, or exporting or importing it at a rate of one metric ton per year was calculated. Second, a discount factor was calculated for the production-transportation activities of each time period to discount all cost back to January 1, 1965 as follows:

R = discount rate per six month period

DISFA (K) = discount factor for the production-transporation cost incurred in the Kth time period

$$(6)\quad \text{DISFA (K)} = \frac{1}{(1+R)^{5(K-1)+1}} + \frac{1}{(1+R)^{5(K-1)+2}} + \ldots + \frac{1}{(1+R)^{5(K-1)+5}}$$

The reader may recall that each time period is made up of two and one-half years or 5 six months periods; therefore it is necessary to generate a discount factor for periods which are not annual. The function (6) adds up the five components of the discount factor of each time period. Thus the components for period 2 are the sixth through tenth six months periods.

It is assumed that all activities operate at a constant level within any single time period, i.e. the level of activities changes only

at the end of each two and a half year period. Also, it is assumed that the discount rate, R, does not change over time. This assumption could be easily modified if projections of the discount rate over time were made available.

Third, the activity cost per year is divided in half. This gives the cost of operating the activity at that level for six months instead of for one year. Then the discount factor is applied to give the present value of the cost of operating the activity over the two and a half years covered by the time period in which the activity takes place.

For example, the cost of producing a ton of steel plate at the Santos plant is calculated to be US$57.95 and the transportation cost from Santos to São Paulo is US$2.66. Therefore, the activity cost for SASPP (the production of one metric ton of steel plate at Santos and its shipment to São Paulo) is US$60.61. Then, the cost of supplying steel plate for six months at an annual rate of one million metric tons per year to the São Paulo market area from the Santos plant would be US$30.30 million. The present value of the cost of this activity, 2SASPP, in the second time period would then be (assuming a discount rate of 5 per cent per six months of 10 per cent per year):

$$C' = 30.3 \left(\frac{1}{(1+.05)^6} + \frac{1}{(1+.05)^7} + \ldots + \frac{1}{(1+.05)^{10}} \right)$$

$$C' = (30.3)\ (.746 + .711 + .677 + .645 + .614)$$

$$C' = (30.3)\ (3.393)$$

C' = US$102.80 million per million metric tons = the present value of the cost of the activity 2SASPP.

The method of discounting for the activity cost of the investment opportunities was somewhat different and will be discussed separately later.

The method of discounting described above was used for all production-transportation activities, including the trade in intermediate products, and the importation and exportation of all intermediate and final products. With an annual discount rate of 10 per cent, the discount factors for the three time periods were:

DISFA (2) = 3.393

DISFA (3) = 2.658

DISFA (4) = 2.083

2.2.2. Cost Calculation for the Production-Transportation Activities. For the linear programming model of the previous chapter detailed cost calculations were made for the cost of pig iron and ingot steel at each plant. Then, the costs of other products were approximated, depending upon the amount of ingot steel required to fabricate the final product, as a constant times the ingot steel cost. For the mixed integer programming model the detailed cost calculations made for pig iron and ingot steel cost at each plant have been extended to include a detailed production cost of slabs, steel plate, hot sheet and strip, cold sheet and strip, and tin plate.

A study was made at the Volta Redonda plant of CSN in 1962 to determine the scrap credit and the conversion cost at each step in the process of producing flat steel products.[1] The results of that study

[1]The scrap credit is the value of the scrap generated at each point in the production process. The conversion cost is the labor, energy, and material costs of processing the product at each production unit. Though a breakdown of the component parts of conversion cost was not available, it was noted that about US$17 of the conversion cost of US$24.93 for electrolytic tin was due to tin cost.

have been used in this paper as the basis for the cost calculations for rolled products for the mixed integer model. Since the 1962 study did not give a breakdown of conversion cost into its component parts and since data on conversion cost for final products in the other two plants were not made available, the conversion cost in all three plants was assumed to be the same. This assumption would appear to introduce some bias in favor of Volta Redonda because the two newer plants would be expected to be capable of producing with lower conversion cost.

Since scrap prices differ significantly at the three plants, separate estimates for the scrap credits at each stage in the process at each plant were made. This was accomplished by using the specific consumption data from Volta Redonda and multiplying the coefficient for the amount of scrap generated times the value of scrap at the plant concerned. Table 4.1 gives the results of these calculations, as well as the conversion cost at each step.

The costs of pig iron and ingot steel at each plant are calculated as described in Chapter II. The cost of the product leaving each of the thirteen productive units listed in Table 4.1 is calculated by multiplying the specific consumption times the cost per metric ton of the product leaving the previous productive unit in the line, subtracting scrap credit, and adding the conversion cost. The results of these calculations are shown in Table 4.2 and in greater detail in Appendix D.[1]

---

[1]The cost of pig iron and ingot steel at the three plants is slightly different in this table from the cost shown in the previous chapter. This is due to a change in the specific consumption of water in the production of pig iron at COSIPA from .650 to .065. Similarly, there is a change in the specific consumption of electrical energy in the production of pig iron at COSIPA from .007 to .070, and a change in the specific consumption of blast furnace gas at USIMINAS from -2.624 to -3.300 and at CSN from -2.624 to -3.715.

Table 4.1

Scrap Credit and Conversion Cost for Flat Steel Products

Process	Specific Consumption Product	Scrap Credit US$/metric ton COSIPA	USIMINAS	CSN	Conversion Cost US$/metric ton
1. Primary Mill	1.090	2.25	3.15	2.34	2.81
2. Scarfing	1.000	-	-	-	1.03
3. Reheating	1.000	-	-	-	2.56
4. Roughing Mill	1.028	.70	.98	.73	2.46
5. Hot Strip Mill	1.013	.32	.45	.34	3.62
6. Pickling Line	1.068	1.70	2.38	1.77	2.39
7. Cold Strip Mill	1.005	.12	.17	.13	5.79
8. Electrolytic Cleaning	1.017	.42	.59	.44	2.18
9. Annealing	1.000	-	-	-	2.13
10. Temper Mill	1.014	.35	.49	.36	2.83
11. Cleaning of Coils	1.071	1.77	2.48	1.85	.77
12. Electrolytic Tinning	.996	-	-	-	24.93
13. Inspection & Packaging	1.000	-	-	-	14.04

Note: The cost shown in this table are based on a study done of the Volta Redonda plant of Companhia Siderurgica Nacional in 1962. The free market exchange rate of 359 Cruzeiros/US$ which existed in May 1962 was used to convert Cruzeiro cost to dollar cost.

Table 4.2

Production Cost
(US$ per metric ton)

Plant	Pig Iron	Ingots	Slabs	Plate	Hot Strip	Cold Strip	Tin Plate
COSIPA (SA)	31.97	46.34	51.07	57.95	62.00	72.91	125.02
USIMINAS (US)	29.47	42.80	46.32	52.78	56.63	66.41	116.84
CSN (VR)	36.52	49.84	54.80	61.75	65.84	76.95	129.37

Since capital cost on all installed equipment is treated as sunk cost and therefore beyond the control of the decision makers, it is excluded from these estimates. The cost of working capital is also excluded but should not be.

A change in the method of calculating the transportation cost for shipping intermediate products between plants was made. In the previous model these costs were estimated from the transportation costs for final products over certain routes. Now they are calculated as follows:

$$\overline{T} = .7\ (a + bX)$$

where $\overline{T}$ is the transportation cost per metric ton over the distance X between plants and the parameters a and b are as estimated in Chapter II. The resulting transportation cost for shipping intermediate products between plants is as follows:

From	To	Cost (US$/metric ton)
COSIPA	USIMINAS	7.63
COSIPA	CSN	3.37
USIMINAS	CSN	5.51

The rates both to and from a given pair of plants are assumed to be the same; there are no back haul rates.

2.2.3 Objective Function Coefficients for Intermediate Products Shipments. The activity cost for all production-transportation activities delivering final products to market areas includes the cost of producing the pig iron, ingot steel, and slabs required to manufacture the final product. Therefore, when a plant receives shipments of one of the intermediate products from another plant, it is necessary to subtract the costs of producing the products in the receiving plant from the costs of producing the goods in the shipping plant and transporting them to the receiving plant. For example, the activity cost of SASPP includes the total cost of manufacturing a metric ton of steel plate at Santos and shipping it to São Paulo, i.e. including the cost of producing the pig iron ingot steel, slabs, etc. If Santos receives a part of its pig iron from USIMINAS there would be double accounting if the cost of producing that amount of pig iron at Santos were not subtracted out. Therefore, in calculating the activity cost for USSAG it is necessary to add together the cost of producing the pig iron at USIMINAS and the cost of shipping it from that plant to Santos and then to subtract from the sum the cost of producing a ton of pig iron at Santos. Thus, the costs for these activities, the coefficients $a_{i\ell}^{kt}$, is the extra cost to the system of producing the intermediate product k in plant i and shipping it to plant ℓ. Of course, this activity cost may be a negative number in cases where the difference in the cost of raw materials at the two plants is so large that the transportation cost between the plants is not sufficient to absorb the difference.

Even when the activity cost for an intermediate-product-shipment activity takes on a negative value, the capacity constraints and the necessity of filling the requirements of the market areas, plus the fact that all exports also have negative activity cost may cause the model not to choose the activity. Rather, these intermediate product shipment activities are used by the model when there is an imbalance in the capacity structure of two plants in their basic production units. Thus, if one plant has an excess of steel making capacity (relative to the capacity of its primary mill) and a second plant has a large primary mill, but insufficient steel making capacity -- then the activity will be chosen if it is more profitable to the system to do this than to export the excess ingot steel from the original plant and/or import ingot steel to the second plant.

Table 4.3 shows the activity cost for each of the intermediate product shipping activities in each of the time periods. The first three columns of the table give the present value of the cost of performing the activity at a level of one metric ton per year over the two and a half years of the time period. The fourth column gives a reference for comparison with the previous model, a number which is easier to compare to prices -- the undiscounted annual activity cost in US$/metric ton.

This table shows that under the assumptions of the model and <u>independent of capital cost</u> both COSIPA at Santos and USIMINAS can supply the range of intermediate products to Volta Redonda less expensively than they can be produced at Volta Redonda. However, as was noted above, this is not to say that they <u>should</u> supply intermediate products to Volta Redonda,

Table 4.3

Activity Cost for Intermediate Product Shipment Activities
(US$/metric ton)

Activity \ Time Period	Present Value of Cost 2nd	3rd	4th	Undiscounted Annual Activity Cost
SAUSG	17.18	13.46	10.55	10.12
SAUSS	18.94	14.84	11.63	11.19
SAUSL	21.01	16.46	12.90	12.41
SAVRG	-2.02	-1.58	-1.24	-1.19
SAVRS	-.22	-.18	-.14	-.13
SAVRL	-.61	-.47	-.37	-.36
USSAG	8.70	6.82	5.34	5.13
USSAS	6.94	5.44	4.26	4.10
USSAL	4.87	3.82	2.99	2.88
USVRG	-2.61	-2.05	-1.60	-1.54
USVRS	-2.58	-2.02	-1.59	-1.52
USVRL	-5.03	-3.94	-3.09	-2.97
VRSAG	13.44	10.53	8.25	7.94
VRSAS	11.65	9.12	7.15	6.88
VRSAL	12.03	9.42	7.38	7.11
VRUSG	21.32	16.70	13.09	12.58
VRUSS	21.29	16.68	13.07	12.57
VRUSL	23.73	18.60	14.57	14.00

because the total cost to the system may be less if the capacity in the basic production units, i.e. the blast furnace, steel shop, and primary mill, at COSIPA and USIMINAS is used to meet their own requirements for intermediate products.

2.2.4 Objective Function Coefficients for Imported Intermediate Products. The annual undiscounted cost for these activities, the coefficients b_i^{kt}, were calculated by (1) using Pittsburgh prices for pig iron, ingot steel, and slabs, (2) adding intercontinental transportation cost by ship, (3) adding transportation cost from the port to the plant, and (4) subtracting the cost of producing the intermediate product in the importing plant. Step (4) is necessary for the same reason that it is necessary in calculating the activity cost for the production and transportation of intermediate products between plants. This procedure yields an estimate of the added cost to the system of importing the intermediate product concerned rather than producing it domestically.

Pittsburgh prices for May 12, 1965, were obtained in US$ per ton and converted to US$/metric ton to give the following prices:[1]

pig iron	US$70.50/metric	ton	
ingots	88.30	"	"
slabs	92.50	"	"

The transportation cost used was US$15.00 per metric ton. This estimate was based on one used in the TECHINT study for the Propulsora plant in

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1 Steel. May 17, 1965, p.68.

Argentina for the transportation cost from the Benelux countries to Buenos Aires.[1]

The transportation cost from the port to the plant was assumed to be zero at Santos, since the plant is right on the water's edge; it was assumed to be US$3.00 per metric ton for USIMINAS and Volta Redonda.

2.2.5 Activity Costs for Production-Transportation Activities for Final Products. These activity costs, c_{ij}^{kt}, were calculated by adding the production costs at plant i (see Table 4.2) to the transportation costs for final product k from plant i to market area j. The transportation costs were estimated as discussed in Section 2 of Chapter II.

Table 4.4

Transportation Cost

Plant	Market Area	Cost (US$/metric ton)
COSIPA	São Paulo	2.66
COSIPA	Rio de Janeiro	6.37
COSIPA	Belo Horizonte	8.16
USIMINAS	São Paulo	7.72
USIMINAS	Rio de Janeiro	6.28
USIMINAS	Belo Horizonte	3.30
CSN	São Paulo	4.27
CSN	Rio de Janeiro	2.94
CSN	Belo Horizonte	5.11

[1]Propulsora Siderurgica,"Proyecto de un establecimiento siderurgica a ciclo integral," by TECHINT, Buenos Aires. Milano, October 1964.

For tables of distances between plants, and between plants and markets, as well as tables of specific inputs, costs of inputs, and cost breakdowns for pig iron and ingot steel in each of the three plants, see Appendix D.

2.2.6 Activity Costs of Importation of Final Products. These activity costs, d_i^{kt}, were calculated by taking (1) Pittsburgh prices of the products, (2) adding transportation cost to Brazilian ports, and (3) adding a sum for the cost of transportation from the port to the market area.

The Pittsburgh prices in US$/metric ton as of May 12, 1965 were as follows:[1]

Plates	US$122/metric ton
Hot rolled sheets	117
Cold rolled sheets	144
Tin Plate, elect.	201

The transportation cost was assumed to be US$15 per metric ton and the cost from the ports to the market areas was assumed to be zero for Rio de Janeiro, US$2 per metric ton for São Paulo, and US$8 per metric ton for Belo Horizonte.

2.2.7 Activity Cost of Exportation of Intermediate and Final Products. The profits, e_i^{kt}, from export activities were calculated by taking the difference between the cost of production plus trans-

[1]Steel, The Metalworking Weekly, May 17, 1965, p.68.

portation to the nearest port and the f.o.b. price for Brazilian exports in 1964. Table 4.5 shows the quantity, value, and the unit value of these exports.[1]

The transportation cost between the plants and ports exporting activities is assumed to be US$3/metric ton for the Volta Redonda and USIMINAS plants and US$1/metric ton for COSIPA at Santos.

Table 4.6 shows the resulting assumptions as to Brazilian f.o.b. prices in comparison with selected U.S. and Japanese prices. Since steel prices differ with different sizes, shapes, and qualities of products and the results shown in the table are also sensitive to the choice of exchange rate, the comparison is very rough. However, it would appear that the assumptions used in the model underestimate the price which the Brazilian steel companies could obtain for exports of pig iron and ingot steel. In fact, Table 4.7 shows that under the assumptions of the model as to export prices and costs of production, that in the short run the system, once the capital is installed, would

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[1]Since hot and cold sheets were included in the same category but divided between thick and thin sheets, an aggregate price was first obtained by weighting the unit price of the thick and thin sheets by the amount of each exported and dividing by the total amount exported. This yielded an aggregate export price for hot and cold sheets of US$128.82. Observing that the difference between the prices of hot and cold sheet in Pittsburgh was about US$27 per metric ton, the amount of US$13.50 per metric ton was subtracted from and added to US$128.82 to obtain prices of US$115.32 and US$142.32 for hot and cold sheet and strip respectively.

The amount of tin plate exported was so little and the price so far from US prices that it was decided to put the f.o.b. price for tin plate at US$10 under the Pittsburgh price, i.e. at US$191 per metric ton.

Table 4.5

Brazilian Exports of Selected Steel Products in 1964

Product	Quantity* (thousand metric ton)	Value* (thousand US$)	F.O.B.Unit Value (US$/metric ton)
Pig Iron	148.9	5,184.	34.80
Ingots	10.22	360.	35.30
Slabs,blooms,etc.	38.6	3,115.	80.60
Plates	12.4	1,344.	108.30
Hot & Cold Sheets between 3mm & 4.75 mm	7.9	1,043.	131.90
Hot & Cold Sheets less than 3mm	2.0	233.	116.50
Tin Plate	.012	1.3	109.20

*From Revista Latinoamericana de Siderurgia, published by the Instituto Latinoamericano del Fierro y el Acero, No. 62, June 1965, p.24. Santiago, Chile.

Table 4.6

Comparison of Brazilian f.o.b. Prices with Japanese and United States Prices
(US$/metric ton)

	Brazilian f.o.b. prices, 1964	Japanese 1965+	U.S.-Pittsburg 1965*	U.S.-Birmingham 1965+
Pig Iron	34.80		70.50	65.00
Ingots	35.30		88.30	
Slabs	80.60		92.50	
Plates	108.30	102.78	122.00	
Hot Strip	115.32		117.00	
Cold Strip	142.32	125.00	144.00	
Tin Plate	191.00		201.00	

+ From Steel, May 17, 1965 p.74.

* From Steel, May 17, 1965, p.68.

Table 4.7

Calculation of Undiscounted Annual Activity Cost for Export Activities

	F.O.B. Unit Value (US$/metric ton)	Production Cost+ plus transportation cost to port (US$/metric ton)	Undiscounted Annual Activity Cost (US$/metric ton)
SAEXG	34.80	32.97	-1.83
SAEXS	35.30	47.34	12.04
SAEXL	80.60	52.07	-28.53
SAEXP	108.30	58.95	-49.35
SAEXH	115.32	63.00	-52.32
SAEXC	142.32	73.91	-68.41
SAEXT	191.00	126.02	-64.98
USEXG	34.80	32.47	-2.33
USEXS	35.30	45.80	10.50
USEXL	80.60	49.32	-31.28
USEXP	108.30	55.78	-52.52
USEXH	115.32	59.63	-55.69
USEXC	142.32	69.41	-72.91
USEXT	191.00	119.84	-71.16
VREXG	34.80	39.52	4.72
VREXS	35.30	52.84	17.54
VREXL	80.60	57.80	-22.80
VREXP	108.30	64.75	-43.55
VREXH	115.32	68.84	-46.48
VREXC	142.32	79.95	-62.37
VREXT	191.00	132.37	-58.63

+"Production cost" here does not include capital cost.

receive very small gains from the exportation of pig iron and no gains at all from exports of ingot steel. On the other hand, Table 4.7 shows that in the short run the system could make large gains from exporting slabs, plates, hot strip, cold strip, and tin plate. However, in the long run, the system weighs both the gains from exporting and the necessity of meeting domestic requirements in deciding when and where to add additional capacity.

There is a certain level of export profits at which the model would decide to import the domestic requirements and to export domestic production and/or the model would decide to install all the new capacity possible in order to reap the large profits from exporting. However, as the results will show, and even though the author has little confidence in the data inputs to the model on import prices and export profits, the conclusions of the model roughly approximate a reasonable estimation of how this part of the Brazilian steel industry may be expected to perform during the next ten years.

The international trade model implicit in this model is that the government of Brazil is in a position to control all exports and imports of flat steel products, that the government wants to minimize the total use of Brazilian resources to satisfy the steel requirements of the country, and that export "profits" to the system are, in the short run, the difference between f.o.b. prices and the cost (with capital cost excluded) of producing the products and loading them on ships. With this sort of a model if there is excess capacity in any

of the productive units after the domestic requirements have been satisfied there is an incentive for the country to sell its excess abroad so long as the price to be obtained for it is above the cost (with capital cost excluded) of production and transportation cost to the port (including loading cost). Thus the model permits a practice which is called "dumping," i.e. one country selling its excess in another country at a price below the domestic price plus transportation charges between countries. Decisions on whether or not to install new capacity are made on the basis of minimizing total cost in the system over time. In the short run exports will be made whenever excess capacity is available and foreign prices are above marginal production cost plus transportation cost. In the long run, capacity will be installed for exporting activities provided the revenue to be gained from such enterprise is sufficient to cover the capital cost of the equipment, and the production and transportation cost.

2.2.8 Activity Costs of Investment Opportunities. The method used in calculating these costs is discussed in the following section of the paper on the treatment of investment decisions in the model.

3. Treatment of Investment in the Model.

If investment costs per ton of product were not a decreasing function of the size of the productive unit (between certain limits prescribed by the existing technology) but rather were a linear function of the size of the productive unit, then investment activities could

be treated like any other activity of the model. However, the technological reality of the steel industry is not only that investment costs are characterized by economies of scale between certain size limits, but also that many investment decisions are discrete in that the problem is to decide when and where to install a new productive unit of a size determined by other considerations.

For example, the present steel shops at the USIMINAS and COSIPA plants both consist of a pair of LD converters. Since it is necessary to reline these converters once every two or three weeks (an operation which takes several days to complete) and since it is desirable to maintain a steady flow of product through the plant, only one converter is maintained in operation at a time. Therefore, the effective capacity of the steel shops of these two plants is that of a mythical single converter which could be maintained in continuous operation without the necessity of relining the vessel.

When additional capacity is required in the steel shops of these plants a third converter of the same size as the first two will be installed. Then two converters will be maintained in operation at a time while the third converter is being relined. Thus, for less than half the cost of the original investment in the steel shop, the companies will be able to double the effective capacity of this part of their plants. So the question facing the investment planner for the Brazilian steel companies in the case of these particular units is not one of what size of converter to install but rather of when and where to install the new converter(s).

A similar situation arises in investments in some rolling mills. The roughing mill presently installed at the Santos plant of COSIPA is a reversing rougher. However, the plant was constructed so that at a later date additional roughing mills can be installed to convert the reversing rougher to a continuous rougher. Here once again the question facing the investment planner is not size but rather when to make the installation of the additional capacity.

A third example is presented by the blast furnaces of COSIPA and USIMINAS. Both blast furnaces were installed without fuel injection, so it would be anticipated that when additional capacity is desired in either of these furnaces that this modification will be made. The amount of oil injection will not be at issue as much as when to install the modification.

For other investments scale is a factor of major importance. For example when a new blast furnace is installed at Volta Redonda, the company officials will be able (within certain limits) to buy any size of blast furnace they desire.

So it seems useful for investment planning models to have the capability of considering both the question of when and where to make additions to capacity and the capability of considering the question of what size of unit to install. The investigation described in this paper has focused on the question of when and where to install units of given capacity. However, the method used here could be easily adapted to study the problem of investments under economies of scale.

This could be done by providing the model with projects for units of various sizes at the same plant in the same time period, and letting the model chose the desired size.

Also, it would be useful to explore the utilization of models in which investments in the near future, which are constrained by the technological situation of the existing productive units, would be assigned integer variables and projects for the more distant future would be assigned continuous variables, within certain bounds.

3.1 Selection of Investment Opportunities or Projects. The model most appropriate for considering the location of discrete additions to capacity is a model of the mixed integer type. This class of mathematical programming models is not difficult to formulate, but they are very difficult to solve when a large number of integer variables are used in the models. Even with recent advances in computation speeds and a new algorithm employing a very successful search technique, mixed integer problems with more than twenty or thirty integer variables are very expensive to solve.

This limitation of the number of integer variables makes it necessary to formulate the problem carefully before attempting to solve it. Thus, as a preliminary to specification of the model, projections of requirements for flat steel products in Brazil up to 1975 were compared to the capacity presently installed in the major productive units in each of the three plants. These rough calculations plus the experience from the linear programming model described in

the previous chapters gave an indication of what the bottlenecks in the system would be in 1967 or 1970 or 1972. Also, the expansion plans of the three companies were reviewed and estimates were made of the time required to complete various projects for additions to capacity.

As a result of this exercise some 23 "investment opportunities" were chosen to include in the model. Table 4.8 gives a listing of these projects, the number of metric tons they would add to present capacity, the cost of the project, the gestation time for the project, and the expected life of the equipment once it is installed.

The leading number in the mnemonic of the project name is the time period in which the project would go into operation. Thus, the project 2ISAS would be completed and in operation by the beginning of the second time period (July 1, 1967). The letter "I" in the project name indicates that this variable is for an investment activity. The second and third letters in the mnemonic in the project name indicate the plant at which the investment would be made, and the last letter indicates the productive unit in which the investment would be made. The symbols for the productive units are as follows:

B	blast furnace
S	steel shop
P	primary mill
R	roughing mill
H	hot strip mill
C	cold strip mill
T	tinning line

Table 4.8

Investment Opportunities

Project	Addition to Capacity (million metric tons)	Cost (US$ million)	Gestation Time (years)	Life of Equipment (years)
2ISAS	.80	7.90	2	18
2IUSS	.80	7.90	2	18
2ISAT	.18	8.00	2	20
2IVRT	.18	8.00	2	20
3ISAB	.28	1.80	1	20
3IUSB	.36	2.40	1	20
3ISAS	.80	7.90	2	18
3IUSS	.80	7.90	2	18
3IVRS	1.00	34.50	4	18
3ISAT	.18	8.00	2	20
3IVRT	.18	8.00	2	20
4ISAB	.80	28.00	4	20
4IUSB	.80	28.00	4	20
4IVRB	.80	28.00	4	20
4ISAS	.80	23.60	4	18
4IUSS	.80	23.60	4	18
4IVRS	1.00	34.50	4	18
4ISAR	1.00	25.00	3	20
4ISAC	.35	10.00	2	20
4IUSC	.35	10.00	2	20
4IVRC	.35	10.00	2	20
4ISAT	.18	8.00	2	20
4IVRT	.18	8.00	2	20

Relatively few projects are considered for the second time period because the present capacity will be sufficient (under the assumptions of the model) to serve most of the requirements and because the time between now (September 1965) and July 1967 is not sufficient to carry out a very large project.

The four projects considered for the second time period are (1) the installation of a third LD converter of an annual capacity of 800 thousand metric tons, (2) the installation of a third LD converter of the same size at USIMINAS, (3) the addition of a tinning line of 180 thousand metric tons per year capacity to the rolling mill section of the COSIPA plant, and (4) the addition of a tinning line of the same size to the plant at Volta Redonda. So the problem of the model for this time period is to decide whether to install a third LD converter at COSIPA or at USIMINAS, at both, or at neither, the converters to be in operation by July 1, 1967. Also, the model must decide whether to install a new tinning line at Santos or at Volta Redonda, to install both, or to install neither.

Seven investment possibilities are considered for the third time period. Two of these are for investments in blast furnaces, three are for investments in steel shops, and two are for investments in tinning lines. The first project is for the installation of petroleum injection in the blast furnace at Santos. It is estimated that this would add 280 thousand metric tons per year to the present 800 thousand metric tons per year capacity of the furnace. The second investment is for the installation of fuel injection in the two blast furnaces at the USIMINAS plant. This investment, it is assumed, would add 300 thousand metric tons per year to the capacity of these furnaces.[1]

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[1]These estimates of the increase in capacity that could result from fuel injection are at best guesses and are based in a rough way on the experience at the blast furnace of the Chilean steel company, CAP.

The third and fourth investments in the third time period, 3ISAS and 3IUSS, are identical to the projects for addition of a third LD converter to the steel shops at Santos and USIMINAS in the second time period. Since it is the usual practice to add a third converter and then to add a fourth and fifth as a pair, the model should have been constrained to prevent it from investing in a third LD converter at Santos in the second time period and the doing the same thing again in the third time period.

The fifth investment in this time period, 3IVRS, is for the installation of a pair of LD converters at Volta Redonda. The present steel shop at Volta Redonda consists of 8 open hearth furnaces, but it has been assumed that when additional capacity is installed in the steel shop at this plant, it will be in a pair of LD converters.

The last two projects for this period, 3ISAT and 3IVRT, are identical to the two investments in tinning lines in the second time period. The constraint to prevent the installation of a tinning line at Santos or at Volta Redonda in both time periods is not necessary because the cost of an additional tinning line is relatively independent of the number of tinning lines already installed at a particular plant.

There are twelve projects which could be completed by the beginning of the fourth time period in July of 1972. The first three of these projects are for the installation of blast furnaces with annual capacities of 800 thousand metric tons at each of the three

plants. The next two projects, 4ISAS and 4IUSS, are for the installation of a pair of LD converters at Santos and/or at USIMINAS. It was assumed, and this time with good results, that the model would choose to install a third LD converter at each of the plants for either the second or third time period so that further increments to capacity in the steel shops of these plants for the fourth time period would require the addition of new pairs of LD converters.

The investment 4IVRS is identical to the investment for the previous time period in the steel shop at Volta Redonda.

The next project in the list, 4ISAR, is for the installation of an additional stand in the roughing mill at the COSIPA plant at Santos. This mill was referred to earlier in a discussion of the discrete nature of many investments in the steel industry in the short run. The investment would convert the mill from a reversing mill to a continuous mill, and is estimated that it would increase the capacity of the present unit from one million metric tons per year to two million metric tons per year.

Projects for the installation of new cold strip mills at Santos, USIMINAS, and/or Volta Redonda follow. Each of these mills would have a capacity of 350 thousand metric tons per year of cold sheet and strip. The last investments, 4ISAT and 4IVRT, provide the model with another opportunity to install tinning lines of 180 thousand metric tons per year capacity at Santos and/or Volta Redonda.

3.2 Treatment of Investment Cost in the Model.[11] Data on the cost of productive units used in the manufacture of flat steel products and a description of the use of this data for estimating the cost of the investment opportunities is given in Appendix E.

Once the cost of an investment project was determined it was necessary to discount these costs to obtain the present value of cost of the project. Since the outlays for the project are normally made over a period of several years while the project is being planned, equipment designed and constructed, buildings erected, and equipment installed and tested, the present value of cost calculations should be made by obtaining data on the stream of payments over time and discounting all of the payments to the present (January 1, 1965 in this model). However, as this sort of data is difficult to obtain, it is assumed that the payments are fairly evenly distributed about the mid-point of the "gestation period" for the project. As an approximation to this distribution, it will be assumed that all capital cost is concentrated at the mid-point of the gestation period, and this cost will then be discounted to January 1, 1965. If data were available to support them, more detailed expressions of construction cost profiles could be introduced into the model. Given the data at hand, this is a satisfactory approximation.

[1]Though he can not claim credit for any of the remaining errors in this section of the paper, I am greatly indebted to Henry Jacoby for his assistance in guiding me over the pitfalls of capital theory.

For example, Table 4.8 lists the gestation period for the project 2ISAS as two years. That is, it is estimated that two years would be required to plan, construct, and install a third LD converter at the Santos plant of COSIPA. Since the project is to go into operation at the beginning of the second time period on July 1, 1967, it is assumed that the US$7.9 million outlay for the project is all made on July 1, 1966 (the mid-point of the two year gestation period.)

Since the model has a fixed horizon, one further modification in the treatment of capital cost was necessary in order to avoid distorting edge effects. For projects in the latter time periods of the model there would be a bias against making the investment and in favor of importing products or increasing interplant shipments. In order to avoid these effects, capital costs have been converted to the equivalent uniform payment series, and the payments have been cut off at the end of the period covered by the model.[1] So, in effect, the system is only forced to pay for that portion of capital services which are utilized within the fixed time horizon of the model. Or, to express it another way, it is as though the system paid a rent for the use of the capital equipment during the life of the equipment.

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[1] I am indebted to Steve Marglin for the suggestion that capital cost be converted to the equivalent uniform payment series and the payment series cut off at the end of the time period covered by the model.

Let C be the arithmetic sum of the capital costs for the installation of a project, R be the discount rate per six-month period, NZ be the expected life of the equipment in number of six-month periods, and L be the size of the payments in the uniform series. Then

$$L = C \frac{R(1 + R)^{NZ}}{(1 + R)^{NZ} - 1} .$$

Furthermore, let C' be the present value of the cost of the project, M be the number of the six-month period (assuming January 1, 1965 to July 1, 1965 is the first six-month period -- January 1, 1965 being the date to which all costs are discounted) which is the mid-point of the gestation period of the project, and let NQ be the number of six-month periods in the time spanned by the model. Then

$$C' = \frac{L}{(1 + R)^{M}} + \frac{L}{(1 + R)^{M + 1}} + \ldots + \frac{L}{(1 + R)^{\min(M + NZ, NQ)}} .$$

Take for example the investment project 4IVRC. For this project C = 10.0, and NZ = 40. For this model NQ = 20. The gestation time of the project is two years; thus, the time from the mid-point of the gestation period to completion of the project is two six-months periods. Since the project is to be completed by July 1, 1972 (the 16th six-month period of the twenty six-months periods between the date to which all costs are discounted and the end of the time span covered by the model) the investment can be treated as though all payments were made at the beginning of the 14th six-month period. Actually, however, an error was made in writing the programs to perform these calculations,

with the result that all investments are treated as though they were made six months before the proper date.[1] Therefore, the project is treated as though all payments were made at the beginning of the 13th six-month period, and M = 13. R is assumed equal to five per cent (a discount rate of 10 per cent per year). Then

$$L = 10.0 \left[\frac{.05(1 + .05)^{40}}{(1 + .05)^{40} - 1}\right]$$

$$L = (10.0)\ (.0582)$$

$$L = .582$$

and

$$C' = .582 \left[\frac{1}{(1 + .05)^{13}} + \frac{1}{(1 + .05)^{14}} + \ldots + \frac{1}{(1 + .05)^{20}}\right]$$

$$C' = (.582)\ (3.598)$$

$$C' = \text{US\$}2.095\ .$$

Though empirically the difference is not large between the discounting method used and the method shown below, it would have been better to use[2]

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[1]This error resulted in an over estimation of capital cost by approximately 9, 11, and 17 per cent for projects for the second, third, and fourth periods respectively.

[2]I am indebted to Henry Jacoby for suggesting this treatment of the discounting.

$$C' = \underbrace{\sum_{t=M}^{H-1} \frac{(C)(R)}{(1+R)^t}}_{(a)} + \underbrace{\sum_{t=H}^{\min[H+NZ-1,NQ]} \left[\frac{R(1+R)^{NZ}}{(1+R)^{NZ}-1} \cdot \frac{1}{(1+R)^t} \right]}_{(b)}$$

where C' is the present value of cost of the project

C is the undiscounted cost of the project

R is the discount rate per six-month period

M is the number of the six-month period which is at the mid-point of gestation period of the project. If G is the gestation period of the project in number of six-months periods, then $M = H - \frac{G}{2}$.

H is the number of the six-month period in which the project is put into operation

NZ is the life of the equipment in number of six-month periods

NQ is the number of the six-month period which is the last period covered by the model.

In this equation the term (a) represents the discounted cost of using the resources between the mid-point of the gestation period and the beginning of the operation of the project, and term (b) represents the discounted cost of the uniform series of payments. The uniform series of payments is cut off at the end of the period spanned by the model or at the end of the life of the equipment, whichever occurs first.

4. The Right Hand Side of the Model, Capacities and Requirements

The vector of the right hand side of the model consists of a series of elements specifying the capacities of the productive units and the requirements of the market areas in each time period. This was shown in Figure 4.6 of this chapter. Also, the right-hand side vector includes in its bottom section elements designed to constrain the integer variables to zero-one values. (See Figure 4.7).

The vectors of capacities are constant in all time periods unless it is anticipated that some productive units will be forced to shut down during the period covered by the model. A discussion of the meaning of the capacities is contained in Chapter II.

The market requirements in each time period stem from projections for the consumption of rolled steel products. A number of studies have been made to forecast future consumption of steel products in Brazil. The results of three of these are shown in Table 4.9. The first of these was reported in a study mimeographed by ILAFA;[1] the second is an unpublished study done for the Brazilian Iron and Steel Institute;[2] and the third is an unpublished study done by Osao Kome of the Banco Nacional de Desenvolvimento (BNDE) of Brazil.[3]

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[1]Instituto Latinoamericano del Fierro y el Acero, "Estimacion del consumo probable de acero laminado para los años 1965, 1970 y 1975," Santiago, Chile, 1964.

[2]Instituto Brasileiro de Siderurgia, Viaduto Dona Paulina 80 - 19° and, São Paulo, Brasil, 1964.

[3]Banco Nacional de Desenvolvimento, Economic Department, Rua Sete de Setembre, 48, Rio de Janeiro, Brasil.

Table 4.9

Projections of Steel Consumption in Brazil
For Rolled Steel Products
(in million ingot metric tons)

	1965	1970	1975
ILAFA	4.374	7.173	11.723
OCHOA	4.863	7.857	12.750
BNDE	4.382	6.841	10.630

The BNDE study was chosen as the basis for the market requirements used in this study. The basic equation of the BNDE projections is

$$\log y = 1.41393 \log x - 1.72297$$

where y is the expected consumption of rolled steel products in millions of ingot metric tons and x is the amount of the gross national product which is generated in the construction and manufacturing industries.

The BNDE study assumes the following breakdown of demand between flat products and shapes:

	1965	1970	1975
shapes	53%	51%	50%
flats	47%	49%	50%

This breakdown was applied to the projection given in Table 4.9 and separate calculations were made for tin plate to obtain the following projection for final products (in millions of ingot metric tons):

	1965	1970	1975
plates, sheet, strip	1.709	2.805	4.465
tin plate	.351	.547	.850 .

These data are converted to projections in million of metric tons of final products by dividing the plates, sheets, and strips by 1.28 and the tin plate by 1.40. The results were then plotted on graph paper and smooth curves drawn between the points so as to obtain the values of the projections for $1967\frac{1}{2}$ and $1972\frac{1}{2}$. Table 4.10 gives the results.

Table 4.10

Projection of Consumption of Flat Steel Products in Brazil
(in millions of tons of final products)

	1965	$1967\frac{1}{2}$	1970	$1972\frac{1}{2}$	1975
plates, sheet, strip	1.332	1.750	2.190	2.770	3.490
tin plate	.250	.310	.391	.490	.607
total	1.582	2.060	2.581	3.260	4.097

Of the total amount of plates, sheets, and strips it was assumed that 25 per cent would be plate, 33 per cent would be hot sheet and strip and 42 per cent would be cold sheet and strip. Applying this percentage breakdown to Table 4.10 yielded Table 4.11, which follows on the next page.

Two further assumptions were needed to complete the calculation of the market requirements. The first of these was that 100 per cent

Table 4.11

Projection of Brazilian Consumption of Flat Steel Products
(millions of tons of final product)

	1965	1967½	1970	1972½	1975
Plates	.332	.437	.547	.693	.872
Hot Sheet and Strip	.440	.579	.722	.914	1.150
Cold Sheet and Strip	.560	.734	.920	1.162	1.468
Tin Plate	.250	.310	.392	.490	.607
Total	1.582	2.060	2.581	3.260	4.097

of the consumption of flat products in Brazil occurs in the "Southern Triangle" of Brazil bounded by São Paulo, Rio de Janeiro, and Belo Horizonte. This assumption doubtlessly overstates the concentration of flat product using industries in Brazil, but only slightly. Secondly, it is necessary to make assumptions about the distribution of consumption for flat steel products among the three market areas which comprise the "Southern Triangle." These are shown in Table 4.12 and are based on some guesses about the distribution which were made by Brazilian steel men and given to the author. The application of these assumptions to the results obtained above yielded the market requirements used in the model. Table 4.13 shows these requirements. The way in which these were incorporated into the model is shown in Figure 4.6.

Table 4.12

Regional Breakdown of Consumption of Flat Steel Products
(in per cent of total consumption in the Southern Triangle)

	São Paulo	Rio de Janeiro	Belo Horizonte
Plate	30	50	20
Hot Sheet and Strip	60	25	15
Cold Sheet and Strip	65	20	15
Tin Plate	70	20	10

Table 4.13

Projection of Product Requirements for Flat Steel Products
(millions of tons of final products)

Market and Product	Mnemonic	1965	$1967\frac{1}{2}$	1970	$1972\frac{1}{2}$	1975
1. São Paulo						
Plate	SPP	.100	.131	.164	.208	.262
Hot	SPH	.264	.347	.433	.548	.690
Cold	SPC	.364	.476	.598	.756	.954
Tin	SPT	.175	.217	.274	.343	.425
2. Rio de Janeiro						
Plate	RJP	.166	.218	.274	.347	.436
Hot	RJH	.110	.145	.181	.228	.288
Cold	RJC	.112	.147	.184	.233	.294
Tin	RJT	.050	.062	.078	.098	.121
3. Belo Horizonte						
Plate	BHP	.066	.087	.109	.139	.174
Hot	BHH	.066	.087	.108	.137	.172
Cold	BHC	.084	.111	.138	.174	.220
Tin	BHT	.025	.031	.039	.049	.061

As more detailed studies of Brazilian steel consumption become available, the assumptions made in this section can be modified or computor runs of the model can be made under a variety of assumptions about the growth of steel consumption in Brazil to determine how these assumptions affect the investment policy for the expansion of capacity in the industry.

CHAPTER V

SOLUTIONS TO THE MIXED INTEGER MODEL

The model was first solved without any constraints on the investment variables, i.e. the system was allowed to install 3.4 blast furnaces if it so desired. Then, the problem was solved again with the investment variables constrained to be less than one, but permitted to take on any values between zero and one. Next, a mixed integer solution to the model was obtained, forcing the investment variable to take on only the values zero or one. Then with the mixed integer solution as a starting point, some additional lattice point solutions to the problem were obtained through the use of the time-sharing system of Project MAC.[1]

1. Solution with No Upper-Bounds on the Investment Variables

In an investment problem where there were no indivisibilities and/or no economies of scale in the investment cost, a solution without upper bounds on the investment variables would be optimal. For the model of this paper the solution shown in Table 5.1 is not optimal, but it is interesting. The values of the investment variable provide an indication of how much capacity the system would like to have in each productive unit in each time period. However, it is a distorted

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[1]Appreciation is due to Project MAC at M.I.T. for permitting the use of the compatible time-sharing system to solve the model.

picture because the system will make repeated modification to the same blast furnace (the value for 3ISAB is 2.766), i.e. additional capacity can be obtained much less expensively through modification of existing blast furnaces than through addition of new blast furnaces to the system.

The value of the objective function for this run was US$725.91 million.

Table 5.1

Solution with No Upper-Bounds on the Investment Variables

Activity Levels of Investment Variables

	Second Time Period	Third Time Period	Fourth Time Period
ISAB	-	2.766	.000
IUSB	-	2.152	.000
IVRB	-	-	.000
ISAS	1.086	.000	.000
IUSS	1.500	.000	.000
IVRS	-	.000	.000
ISAR	-	-	.949
ISAC	-	-	1.186
IUSC	-	-	1.357
IVRC	-	-	.000
ISAT	.976	.000	.577
IVRT	.360	.000	.000

2. Solution with an Upper-Bound of Unity on the Investment Variables

Additional rows were added to the matrix to constrain the investment variable to values less than or equal to one. The new constraints resulted in an increase in the value of the objective function to US$759.45 million.

Table 5.2 shows the activity levels of the investment variables. The dashes in the table denote lack of a project for a particular productive unit in a given time period. Of the twenty-three projects, six are at the upper bound; eight are at the lower bound, and the remaining nine are at fractional levels.

Weingartner points out that the shadow prices on the rows used to constrain the investment variables to be less than or equal to one and the "reduced-cost" on the investment variables may be useful for ranking projects.[1]

For values at their upper bounds in the optimum solution to the continuous problem, the cost to the system of pushing these variables below the level of unity will be proportional to the shadow price on the row which constrains them to be less than or equal to zero. Likewise, for those values at the lower bound of zero in the continuous solution, the "reduced-cost" serves a function like the shadow price for the variables in the continuous solution at their upper bounds. That is, in a marginal

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[1] H. Martin Weingartner, Mathematical Programming and the Analysis of Capital Budgeting Problems, Prentice-Hall, Inc., Englewood Cliffs, N.J. 1963, p.55.

Table 5.2

Solution with an Upper-Bound of Unity on the Investment Variables

Activity Levels of Investment Variables

	Second Period	Third Period	Fourth Period
ISAB	-	1.000	.548
IUSB	-	1.000	.576
IVRB	-	-	.000
ISAS	.003	1.000	.000
IUSS	.500	1.000	.000
IVRS	-	.000	.000
ISAR	-	-	.949
ISAC	-	-	1.000
IUSC	-	-	1.000
IVRC	-	-	.543
ISAT	.864	.000	.000
IVRT	.000	.473	.578

sense, the change in the value of the objective function which will result from increasing one of these variables from the zero level will be proportional to the value of the reduced-cost for the variable.

3. Mixed Integer Solution to the Problem

A new algorithm written by Norman Driebeek was used for obtaining

the mixed integer solution to the model.[1] This algorithm utilizes and expands upon the results pointed out by Weingartner.

3.1 Driebeek's Algorithm. Driebeek's algorithm is a search routine for finding that lattice point which is the global optimum of the problem. It begins by calculating a continuous solution to the problem with the integer constrained to take on values less than or equal to one (and greater than or equal to zero -- by the non-negativity constraint). From this solution a set of penalties is calculated. For a cost minimizing problem these penalties represent the additional cost (over and above the value of the objective function for the continuous solution) to the system of forcing the integer variables to take on integer values.

Two classes of penalties are calculated, "true" and "pseudo" penalties. The true penalties are associated with those variables which are in the continuous solution at the upper and lower bound. For those variables in the continuous solution at the upper bound the shadow price on the row which constrains the variable to 1.0 represents the minimum amount that the functional value would have to increase if the variable were forced from one to zero. The change in the functional value will be greater than the magnitude of the associated shadow price if forcing the variable to zero results in some infeasibilities. Then the value of the objective function will be forced even higher in the effort to restore feasibility.

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[1]Driebeek, Norman, op.cit. The author is most grateful to N. Driebbek for making his algorithm available for use.

Table 5.3

Table of Penalties

Variable	Penalty	Type	Variable	Penalty	Type
02ISAS	-.00081678	Pseudo	04ISAB	-.16786252	Pseudo
12ISAS	-1.4566695	"	14ISAB	-.13010491	"
02IUSS	-1.5214000	"	04IUSB	-.16576417	"
12IUSS	-	True	14IUSB	-.15594265	"
02ISAT	7.6962725	Pseudo	04IVRB	-	True
12ISAT	.2482545	"	14IVRB	1.2800000	"
02IVRT	-	True	04ISAS	-	"
12IVRT	.00608250	"	14ISAS	3.1610000	"
03ISAB	11.355522	"	04IUSS	-	"
13ISAB	-	"	14IUSS	1.7000000	"
03IUSB	15.218798	"	04IVRS		"
13IUSB	-	"	14IVRS	9.8800000	"
03ISAS	.4090000	"	04ISAR	-13.289987	Pseudo
13ISAS	-	"	14ISAR	-.30777554	"
03IUSS	1.8700000	"	04ISAC	1.8950715	True
13IUSS	-	"	14ISAC	-	"
03IVRS	-	"	04IUSC	1.4542383	"
13IVRS	16.960000	"	14IUSC	-	"
03ISAT	-	"	04IVRC	-.34415418	Pseudo
13ISAT	.01880407	"	14IVRC	-.66487139	"
03IVRT	-.86180102	Pseudo	04ISAT	-	True
13IVRT	-.75323275	"	14ISAT	.01673345	"
			04IVRT	-.82462921	Pseudo
			14IVRT	-.7027302	"

When another lattice point is selected, dual iterations are performed to restore feasibility. If the value of the objective function goes above the value of the functional obtained with the previous integer solution, the trial is cut off and another lattice point is selected. The algorithm proceeds in this fashion until all of the eligible lattice points have been tested.

The success of this sort of algorithm depends heavily on the method of choosing the lattice point for the original integer solution and the method of the search. Driebeek's method of choosing the first lattice point has a great deal of intuitive appeal. Though he does not reveal the search routine used, its performance has been impressive on a number of problems.

3.2 Computational Experience. Two versions of the model presented in this paper were solved with Driebeek's algorithm. The first of these contained eleven integer variables and the second twenty-three integer variables. In neither case did the first lattice point chosen turn out to be the optimum. In the first case, the computer run was cut short after only a few trials and subsequent calculations showed that the optimum had not been found at the time of cut off. The solution which it is now assumed is the optimum for this problem would have been tested on the sixteenth trial. The problem had used about four minutes of computer time on an IBM 7090 at the time it was cut off (it would likely have required several more minutes to reach the solution which is now regarded as the optimum). In addition to the eleven integer variables, the problem contained 122 rows, 433 other columns, and slightly less than 1800 matrix entries.

In the second version of the model which was solved with the algorithm, the number of integer variables was increased to twenty-three; the number of rows was increased to 146; the number of other columns remained at 433, and the number of entries was 1800. The value of the objective function for the continuous solution was US$759.45 million.

The lowest functional value found by the algorithm was US$776.34 million. Later, Driebeek reported to the author that there had been a bug in the program on this run -- a bug which resulted in poor performance of the algorithm.

After this solution was obtained from the algorithm, additional calculations were made by the author which showed that there was a feasible solution at the objective function value of US$769.49. The author reported the lattice point which yielded this solution to Driebeek. Driebeek then checked the output of the algorithm run and reported that if the bug had not been present the lattice point attempted on about the 95th trial would have yielded this solution. While it is very difficult to estimate the computation time that would have been required by the algorithm to arrive at this solution, a half an hour on an IBM 7094 appears to be a reasonable guess.

3.3 <u>The Results</u>. Table 5.4 provides a comparison of the activity levels for the investment variables for the continuous and the best integer solution. The dashes, as before, are used to denote the lack of an investment project for a particular productive unit in a given time period.

Table 5.4

Activity Levels for the Continuous and Mixed Integer Solutions to the Twenty-Three Integer Variable Problem

Investment Activities

	Second Time Period		Third Time Period		Fourth Time Period	
	Continuous	Integer	Continuous	Integer	Continuous	Integer
ISAB	-	-	1.000	1	.548	1
IUSB	-	-	1.000	1	.576	1
IVRB	-	-	-	-	.000	0
ISAS	.003	0	1.000	1	.000	0
IUSS	.500	1	1.000	1	.000	0
IVRS	-	-	.000	0	.000	0
ISAR	-	-	-	-	.949	1
ISAC	-	-	-	-	1.000	1
IUSC	-	-	-	-	1.000	1
IVRC	-	-	-	-	.543	1
ISAT	.864	1	.000	0	.000	0
IVRT	.000	0	.473	0	.578	1
Objective Function	759.45	769.49				

In this particular case, the best integer solution found to date was obtained by rounding off the values of the variables obtained in the continuous solution.

For the second time period the model chose to install the third LD converter at USIMINAS and not to install the one at Santos. Then for the third time period it chose to install another third LD converter at USIMINAS and a third LD converter at Santos. Since a third converter can not be installed at USIMINAS for both the second and third time periods, additional runs were made later to remove this inconsistency from the results. As was suggested earlier, the inconsistency could have been avoided by inserting a constraint in the model to prevent its setting both 2IUSS and 3IUSS equal to one.

In addition to the new converter at USIMINAS, the model decided to invest in a tinning line at Santos -- both the converter and the tinning line to be installed by July 1, 1967.

For the third period the model decided to install the modifications to the blast furnaces at both Santos and at USIMINAS. As mentioned before, it decided to install a third LD converter at both USIMINAS and at Santos, but it decided not to install the two new converters at Volta Redonda. This probably occurred because the capacity could be obtained for much less cost by installing third converters at the newer plants. It might also have been due to the fact that the model was not provided the opportunity to invest in a project to increase the blast furnace capacity at Volta Redonda for either the second or third time periods.

Also, for the third period, Table 5.4 shows that the model rejected a project for the installation of additional tinning lines at both the Volta Redonda and the Santos plants. The model then chose in the

fourth period to install another tinning line at Volta Redonda. The fact that the model chose to install a tinning line at Santos for the second time period and at Volta Redonda for the fourth time period seems logical because the greatest part of the requirement for tin products is in São Paulo very near to Santos, and Santos has a cold strip mill but no tinning line at present. Once Santos had been chosen as the site for a new tinning line for the second time period, it did not have enough excess capacity in its cold strip mill to be able to supply yet another tinning line, but Volta Redonda had some extra capacity in its cold strip mills.

For the fourth time period, the model chose to construct new blast furnaces at Santos and USIMINAS and rejected the same project at the same cost at Volta Redonda. Also, it rejected projects for the construction of new pairs of LD converters at all three plants, probably because sufficient steel making capacity was installed in the second and third time periods. Given the opportunity to install new cold strip mills of identical capacity and for the same cost at the three plants, the model accepted all three projects. Finally, as was mentioned above, the model decided to install an additional tinning line at Volta Redonda for the fourth time period. These results offer a good opportunity to make an analysis of why the model chose the options it did. In this case, the model systematically rejected investment projects in the blast furnace and steel shop at Brazil's oldest integrated steel mill, Volta Redonda, in favor of additions to capacity in these primary units at the two newer

plants at Santos and USIMINAS. There are at least three possible explanations for this propensity of the model, two of which are fairly obvious and a third which is not so obvious.

The most obvious explanation is that, under the assumptions of the model, the cost of production for steel products is greater at Volta Redonda than at either Santos or at USIMINAS. The difference is so large that even with the proximity of Volta Redonda to the large market area of Rio de Janeiro, USIMINAS can produce and deliver steel products to Rio for less than Volta Redonda. Thus, it may be that the model wanted to install new capacity at those plants which, over the long run, would be able to supply the market areas at less cost to the country.

Secondly, the investment projects in the primary units at Volta Redonda may have been rejected in favor of those at Santos and USIMINAS because of the present structure of capacity in the three plants. As the oldest plant of the three, Volta Redonda is presently using almost all of the capacity in its primary productive units to the fullest extent. Thus, if more capacity is installed in the steel shop, additional capacity must at the same time be installed in pig iron production, and in a new primary mill. The plants at Santos and USIMINAS, on the other hand, have excess capacity in a number of units and the capability of making relatively inexpensive modification to existing primary units to add substantially to their capacity. Thus, increments to national capacity can be made at least cost to the country in the short run through additions to capacity in these plants.

The phrase "in the short run" in the previous paragraph is the key to the third and less obvious explanation for the model's behavior. The model is instructed to minimize the total cost of the production and transportation cost of meeting the steel requirements of the country during the period covered by the model. Therefore, the cost of meeting the steel needs of Brazil in the period 1975 to 1985 is of no interest to the model. Thus, if the model can meet the needs of the system in the short run by expanding piecemeal the existing plants rather than constructing the basis for even greater expansions in the distant future, it would be expected to follow the first strategy. For this reason, there is a great deal to be said for building a model to cover a period of time somewhat longer than the period for which investment decisions must be made.

Table 5.5 gives the activity levels for the production transportation activities in the three time periods. The model appears to have taken good advantage of the capability of shipping intermediate products between plants. The products shipped and the sending and receiving plants change from one time period to another as new production units are installed, thereby shifting the capacity bottleneck in the plant from one productive unit to another.

There is one interesting import activity of a type that one might expect to see more of if import costs were somewhat lower than those used in the model. In the third time period the model chose to import some tin plate to Rio de Janeiro (see the activity IMRJT near the bottom

Table 5.5

Activity Levels for the Continuous and Mixed Integer Solutions to the Twenty-Three Integer Variable Problems

Production Transportation Activities

	Second Time Period		Third Time Period		Fourth Time Period	
	Continuous	Integer	Continuous	Integer	Continuous	Integer
SAVRG		.002				
SAVRL			.084	.084		
USSAS						.200
USSAL					.336	.337
USVRG	.063	.062			.047	
USVRL			.097	.097	.177	.176
VRSAS	.220	.220	.135	.135	.198	
IMSAL	.015	.017				.045
SASPP	.131	.131	.126	.126	.208	.208
SASPH	.347	.347	.433	.433	.548	.548
SASPC	.021		.180	.155	.396	.342
SASPT	.148	.167	.148	.171	.148	.157
SARJP					.213	.259
SARJH					.100	.100
USRJP	.074	.074				
USRJH	.145	.145	.166	.167	.128	.128
USRJC			.008	.008		
USBHP	.087	.087	.109	.109	.139	.139
USBHH	.087	.087	.108	.108	.137	.137
USBHC	.111	.111	.138	.138	.174	.174
VRSPP			.038	.038		
VRSPC	.455	.476	.418	.442	.360	.413
VRSPT	.069	.050	.126	.102	.195	.186
VRRJP	.144	.144	.274	.274	.134	.088
VRRJH			.015	.014		
VRRJC	.147	.147	.097	.159	.233	.233
VRRJT	.062	.062	.078	.020	.098	.098
VRBHT	.031	.031	.039	.039	.049	.049
IMRJC			.079	.017		
IMRJT				.058		
SAEXG						.363
SAEXC	.159	.159			.124	.167
USEXG						.200
USEXL			.072	.071		
USEXH	.174	.176	.379	.379		
USEXC	.035	.035			.311	.311

of Table 5.5) rather than to install a new tinning line at Volta Redonda. The reader may recall from the earlier discussion that the model was offered the option of installing tinning lines at Santos and at Volta Redonda in each of the time periods. However, it chose to install only two units -- one at Santos in the second time period, and one at Volta Redonda in the fourth time period. Therefore the conclusion of the model in this case was that it was less expensive to postpone the installation of the added tinning line at Volta Redonda from the third to the fourth time period and to import a small amount of tin plate to Rio de Janeiro in the third time period.

The activity levels for exports approximate expected developments for the decade to come. For example, all of the exporting is done by Santos and USIMINAS, the two plants which are in the best locations for exporting. It does not appear that under the present assumptions of the model there is sufficient incentive for Brazil to build up a large export industry in steel products. If the model had not rejected a number of the investment opportunities available to it, and if it did not have the incentive to export all products except ingot steel (recall that ingot steel exports had positive rather than a negative activity cost), the failure to build up capacity for exporting would be less convincing. A study of the effects on the model of increases and/or decreases in the profitability of exporting should prove to be interesting.

Among the domestic production-transportation activities, Santos originally served only the market area around São Paulo, but in the fourth

time period it began to take a larger share of the Rio de Janeiro market as well. USIMINAS devoted itself to Belo Horizonte and to exports with some shipments to Rio; Volta Redonda shipped to both São Paulo and Rio, but lost a part of both of these markets in the fourth time period.

3.4 The Use of Time Sharing to Improve on the Algorithm Results. When a search routine is used to solve a problem and when the problem is too large to make an exhaustive search, one feels uncertain about the result no matter how impressive the performance of the algorithm has been on similar problems with known solutions. Also, with this type of mixed integer problem in which there are a relatively small number of integer variables, the model builder often has an intuitive feel for those combinations of integers which are most likely to form the optimum combination. When this intuition is aided by an algorithm to guide the model builder to a lattice point, which if it is not the optimum is very close, the result may be much better than using simply the algorithm or simply intuition.

The capabilities of a time sharing system lend themselves very well to this kind of algorithm-intuition problem. With the results from the use of Driebeek's algorithm in hand, a program was written to permit the console operator on the time sharing system to modify the right-hand side of the problem so as to force the solution to a chosen lattice point.[1] Appendix F contains listings of (i) this program (INTRHS), (2)

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[1]This program as well as the input smoothing and output smoothing programs prepared for the mixed integer programming model were written by David Kelleher to whom the author is indebted for excellent programming work.

the input smoothing program (TRANSM and subroutines), which performed all the transportation and production cost calculations, carried out the discounting, and punched out the input deck for the linear programming routine, and (3) the output smoothing program (OUTPUT), which read the output from the linear programming routine and organized it into output formats which were more easily read and analyzed.

The program INTRHS and the linear programming routine, MFOR, with which it is used, have the capability of using multiple right-hand sides, i.e. of solving a linear programming problem, altering some or all of the elements in the right-hand side and running the program again. With this model the linear programming routine required about 90 seconds with an IBM 7094 to reach the first solution but only ten to twenty seconds to obtain each additional solution. Thus the model builder can begin with the lattice points and the other results of the algorithm solution and try in rapid succession a number of lattice points that he feels offer a better solution than that obtained by the algorithm.

An economic planning office, be it in a private firm or in a national government, is often faced with the task of presenting alternatives and the associated cost of each alternative. The time sharing system offers the capability of obtaining results from a number of combinations with the opportunity to look at the results almost immediately, and to choose on the basis of those results other combinations of investments which might be interesting to examine.

With the results from the use of Driebeek's algorithm in hand to study, the author was able in one case to find a better lattice point than the algorithm solution. This occurred in the results from the eleven integer variable problem. (The eleven integer variable problem was identical to the twenty-three integer variable problem except that none of the investment projects for the fourth period was included in the model.) An analysis of the results showed that the model chose to invest in additional tinning lines for the Santos plant for both the second and third time periods, but used less tin capacity at Santos in the third and fourth time periods than was used in the second time period. A run was made with the new lattice point, which excluded the investment in the tinning line at Santos in the third period, and the result was a decrease in the functional value from the algorithm value of US$806.82 million to the new value of US$803.72 million. This is by no means to demean the value of using the algorithm, however. Without the algorithm solution in hand to begin with, it would have been virtually impossible to choose this particular lattice point from the 2048 possibilities. Since the continuous solution functional value for this problem is US$799.93 million, the chances of finding a better solution are relatively small.

The time sharing system was used on the results of the twenty-three integer variable problem with two different purposes in mind. First, an attempt was made to check the results of the algorithm and to see if a better solution could be found, and second, the time sharing system was used to remove an inconsistency from the results.

Had the algorithm not had a bug in it, it is unlikely that the use of time sharing system would have resulted in an improvement on the algorithm solution. However, because of the bug, a lower solution was obtained. That solution was the round-off solution, i.e. the lattice point for the solution was obtained by rounding off the values of the investment variable obtained in the continuous solution. The functional value for this round-off solution was US$769.49 million. The functional value for the continuous solution was US$759.45.

The time sharing system was also used to remove an inconsistency from the results. To prevent the model from installing a third LD converter at USIMINAS in both the second and third time periods a solution was run which permitted the installation of a third LD converter at USIMINAS in the second time period, of a third LD converter at Santos in the third time period, and of an additional pair of LD converters at USIMINAS in the fourth time period. This solution yielded a functional value of US$773.06 million.

4. A Sensitivity Test on the Discount Rate

Since it is very difficult to estimate exactly what discount rate should be used for this type of study, a sensitivity test was made by using discount rates of five, ten, and fifteen per cent per year. Though the value of the objective function changed very sharply (from US$1026.42 million for the five per cent discount rate to US$759.45 million for the ten per cent rate, and US$572.39 million for the fifteen per cent rate), the investment projects chosen by the model did not

change very much. The first three columns of Table 5.6 show the resulting values of the investment variables (in these solutions the investment variables were given an upper bound of one, but permitted to take any value between and including zero and one). The most noticeable change is that with a five per cent per year discount rate the model chose to install a tinning line at Volta Redonda for the second time period and to reject the equivalent project at the same cost at Santos. With a ten and fifteen per cent discount rate this decision was reversed. The explanation for this is that even though the two projects cost the system the same, the added steel capacity required to make the projects useful required much more capital cost at Volta Redonda than at Santos.

The fact that the investment decisions of the model are relatively insensitive to changes in the discount rate stems partially from the treatment of capital cost. One normally expects that with an increasing discount rate, projects in the later time periods will become relatively less expensive in present value terms and the model would shift to choose more projects in the later periods. That does not occur in the model because the system is only charged for capital services within the time horizon of the model. In calculating the amount which must be paid in each time period by the uniform payments series formulas, the payments per period are an increasing function of the discount rate within the relevant range. For example, with an equipment life of 20 years, NZ = 40 six-months period, with R equal to the discount rate per six-month period, and Q equal to the uniform series payments, where

$$Q = \frac{R(1+R)^{NZ}}{(1+R)^{NZ}-1}$$

Table 5.6

Effect on Investment Decisions of Changes in the Discount Rate

Variable	5%	10%	15%	No Discount Cost	Present Value of Cost of Investments 5%	10%	15%
2ISAS	.082	.003	-	7.90	4.58	5.06	5.37
2IUSS	.500	.500	.500	7.90	4.58	5.06	5.37
2ISAT	-	.864	.864	8.00	4.35	4.94	5.34
2IVRT	.864	-	-	8.00	4.35	4.94	5.34
3ISAB	1.000	1.000	1.000	1.80	.60	.63	.62
3IUSB	1.000	1.000	1.000	2.40	.80	.84	.83
3ISAS	1.000	1.000	.975	7.90	3.10	3.19	3.13
3IUSS	1.000	1.000	1.000	7.90	3.10	3.19	3.13
3IVRS	-	-	-	34.50	16.03	16.96	17.18
3ISAT	-	-	-	8.00	2.94	3.11	3.11
3IVRT	.473	.473	.473	8.00	2.94	3.11	3.11
4ISAB	.619	.548	.523	28.00	7.63	7.74	7.40
4IUSB	.517	.575	.597	28.00	7.63	7.74	7.40
4IVRB	-	-	-	28.00	7.63	7.74	7.40
4ISAS	-	-	-	23.60	6.85	6.76	6.37
4IUSS	-	-	-	23.60	6.85	6.76	6.37
4IVRS	-	-	-	34.50	10.01	9.88	9.31
4ISAR	.949	.949	.949	25.00	6.05	6.05	5.71
4ISAC	1.000	1.000	1.000	10.00	2.12	2.10	1.95
4IUSC	1.000	1.000	1.000	10.00	2.12	2.10	1.95
4IVRC	.543	.543	.179	10.00	2.12	2.10	1.95
4ISAT	-	-	-	8.00	1.70	1.68	1.56
4IVRT	.578	.578	.578	8.00	1.70	1.68	1.56
Objective	1026.42	759.45	572.39				

Then Q = .040, 058, and .079 when R = .025, .050, and .075, respectively.

Therefore, with the present value of cost of an investment activity equal to the undiscounted cost of the activity times the discount factor (which is a decreasing function of the discount rate) and times the uniform series payment factor, Q, (which is an increasing function of the discount rate) there is no clear-cut effect on the present value of costs of investment activities which results from an increase in the discount rate.

Another possible explanation for the relative insensitivity of the model to changes in the discount rate is that there is no great difference between the capital costs and the operating cost among the alternative projects.

5. Conclusions

The primary conclusion of this study is that computer speeds and algorithm efficiencies -- for both linear programming and mixed integer programming -- have advanced to the point that it is possible to construct multi-period, multi-product, sectorial models for studying discrete additions to capacity through investments in existing plants and/or the addition of new plants to the system. Furthermore, computer speeds and algorithm efficiencies are sufficient so that the models may include shipments of intermediate products between plants, exports and imports of intermediate products, and exports and imports of final products. This flexibility of the models makes them powerful tools for studying questions of investments under economies of scale, discrete investments, import substitution, installation of domestic capacity for exporting

intermediate and final products, and choice of technology.

A second conclusion of this study is that the advent of time sharing systems for large computers adds another valuable tool to the available methods for studying investment problems. The combination of algorithm-intuition which these systems offer is shown to be a very useful device in helping to solve mixed integer programming problems. Also, time sharing systems are shown to provide the possibility of obtaining cost estimates on a great variety of investment strategies within a few minutes. Thus, it becomes easy for an analyst to make the calculations for a wide variety of choices which can be submitted to the decision makers (be they corporate managers or politicians) for decision.

Thirdly, programming models are ideally suited for studying the operation of an economic system to decide which elements in the system are of greatest importance in determining the conclusions of the model, and thereby for pointing the way to the parts of the system that deserve more detailed investigation. For example, the magnitude of economies of scale in investment cost is a problem that appears to merit much more careful attention than it has yet been given by economists. It will be a relatively simple matter to run a model of the sort discussed in this paper under a variety of assumptions about the magnitude of economies of scale in investment cost in the steel industry. If the results of the model are very sensitive to changes in these parameters, then more empirical research on economies of scale

will be in order; however, if the effects are small, then the efforts of investigators are more wisely devoted to other problems.

Finally, this type of model is shown to be most useful for studying the effects on an economic system of changes in transportation cost, shifts in demand, and changes in the cost of inputs.

APPENDIX A

A BRIEF DESCRIPTION OF THE PRODUCTION OF FLAT STEEL PRODUCTS

As our purpose here is only to provide the reader with the necessary glossary for understanding the terms used in this paper, the person who is interested in making a detailed study of steel technology is referred to a publication of the United States Steel Corporation, The Making, Shaping, and Treating of Steel (originally written by Camp and Francis).

The production of steel products may be divided into four basic processes; (1) the mining and preparation of raw materials, (2) the reduction of iron ore to pig iron, (3) the refining of pig iron to steel, and (4) the rolling of the steel ingots to shape them into final products.

1. The Mining and Preparation of Raw Materials.

Iron ore is mined in open pit mines and is usually crushed and sized near the mines. The ores are transported to the steel mill where the lump ores may be charged directly to the blast furnace and the ore fines (fine particles of ore) are mixed with coal fines and agglomerated to produce a high quality and uniformly-sized input for the blast furnace. This mixture of ore fines and coal fines, called sinter, is widely used in Latin America. Also, a few pelletizing plants are being constructed to concentrate the ores into uniformly-sized pellets. Pelletizing has not been rapidly adopted in the region because Latin American ores are generally of such high iron content that pelletizing has not been deemed economic.

In many countries in the region national coals contain such a high percentage of volatile material that it is necessary to mix them with imported coals in order to have a cokable mixture. The coals are mixed before they are heated in coke ovens to produce coke and coke gas. A number of chemical by-products are extracted from the coke gas and the remaining gas is used for heating purposes elsewhere in the plant.

2. The Reduction of Iron Ore to Pig Iron.

Coke, iron ore (and/or sinter and/or pellets), and limestone constitute the principal materials charged to the reduction furnace, which is usually a blast furnace, though at least two major plants in Latin America have electric reduction furnaces.

The ore and coke is charged at the top of the blast furnace and a blast of heated air is forced in through opening in the sides of the furnace near the bottom and up through the charge. Often petroleum or natural gas is injected through these openings along with the blast. The ores are reduced (i.e. oxygen is removed from the iron oxide ores to reduce them to purer forms of iron) in the furnace and molten pig iron is tapped from the bottom of the furnace. Also, molten slag is tapped from the bottom of the furnace and may be sold for use in cement manufacturing or for other industrial purposes.

The blast is converted to blast furnace gas, a relatively low specific heat gas, as it is forced up through the furnace. The gas leaves the furnace at the top, is cleaned, and is then passed through a series

of heat exchangers which are used to heat the incoming blast. Only about one-third of the blast furnace gas is required to heat the incoming blast. Therefore, the remainder is used elsewhere in the plant or sold to other plants for use as a fuel.

3. The Refining of Pig Iron to Steel.

The molten pig iron is transported in special railroad cars to the steel furnaces (sometimes called the steel shop). There it is poured into huge ladles and transported to the furnaces. The majority of the integrated steel plants in Latin America have open hearth steel furnaces. These furnaces are shaped like great baths. They are charged with molten pig iron, scrap, ferroalloys, iron ore, and limestone. Then hot air or flames are blown across the bath to maintain it at very high temperatures. Fuel oil is often used as a fuel though a mixture of coke and/or blast furnace gases or other types of fuels may be used.

The average time for a heat, i.e. the time from one charge to the next, averages from eight to eleven hours. Many plants have reduced this time by a matter of hours by injecting oxygen through lances in the roof of the furnace.

There is a marked tendency for new plants to install LD (Linz-Donawitz) converters rather than open hearth furnaces. An LD converter is something like the more familiar Bessemer converter except that oxygen rather than air is injected into the furnace and the injection is through a lance from the top rather than through openings in the bottom of the

converter. The LD converters, because of very short heat-time, have substantially smaller capital cost per unit of output than the open hearths. The open hearths on the other hand are said to be able to operate with higher percentages of scrap content in the charge than the LD converters.

A few steel mills in the region use electric steel furnaces. These furnaces are charged from the top with molten pig iron, scrap, etc. Carbon electrodes protrude through the roof of the furnace down into the charge and electric current is arcked between these electrodes to heat the charge.

The hot metal is discharged from the steel furnace into a ladle, and the ladle is transported by an overhead crane to an area where its contents are poured into ingot molds. The molten metal is allowed to cool sufficiently so that when the mold is stripped off it leaves a solid ingot of steel. The ingot is usually of rectangular cross-section and about eight to ten feet high.

4. The Shaping of the Steel Ingots to Make Final Products.

Within the last few years a few steel plants in Latin America have installed continuous casting plants. In these plants the molten metal is poured from the ladle into a container which feeds a continuous flow of liquid steel into a vertical column. The steel hardens as it descends and is cut off at the bottom of the mold. Though first used only for light shapes, this method is now being adopted for the production of large slabs as well.

In the more conventional mills the ingots are transported to soaking pits where they are heated until they attain uniform temperatures throughout. The ingot is removed from the soaking pit by overhead cranes and taken to the primary mill, where it is flattened and elongated by being rolled back and forth through the mill as the rollers are brought closer together on each pass. The process forms the ingot into either blooms or slabs. Blooms are square in cross-section and are used for producing shapes (angles, rods, beams, pipe, rails, etc.), while slabs are rectangular in cross-section and are used for producing flat products (sheets, strip, plate, coils, tin plate, etc.). A primary mill which is used principally for producing blooms is called a blooming mill, and a mill used mainly for shaping slabs is called a slabbing mill.

Since a continuous casting unit replaces the ingot molds, soaking pits, and primary mill, it offers large savings in capital cost over conventional methods. Also, because of reduced scrap losses, it reduces operating cost.

The slabs produced either by the conventional method or by tontinuous casting are allowed to cool, and then are cleaned, checked, and stacked in the slab yard to await further rolling. There are several reasons for permitting the product to cool at this point. First, it is important to check carefully the slabs for flaws that may have resulted from faults in the ingots. Second, the outside edges of the slabs may leave slight marks on the rollers so it is desirable, when possible, to schedule the rolling in such a fashion as to roll from wide to narrow, i.e. to roll the widest products first and then move down to the narrower products.

Once the slab is selected for rolling it is shoved into a reheating furnace where it remains until it is uniformly heated. It is then passed through a small mill called a scale breaker before entering the roughing mill (or plate mill). This mill may be either reversible, semi-continuous, or continuous. A reversible rougher passes the product back and forth through the mill as it rolls it thinner with each pass. This mill may be either a two-high or a three-high. The two-high mill has two rollers which are reversed each time the product is reversed. The three-high mill does not require reversing the three rollers. Rather the tables on each side of the mill lower or raise the product to feed it through the mill first in one direction, then in the other.

A semi-continuous roughing mill usually consists of a reversing rougher followed by a single-pass mill. In some of the newer mills either a reversing or a semi-continuous rougher has been installed with space provided for the addition of more mills to the line. Thus, a semi-continuous mill may be easily converted to a continuous mill in which the product moves continuously through a number of mills (or stands). As a rough estimate one might say that a plant with a capacity of less than half a million metric tons per year would be equipped with a reversing rougher, a one million ton plant with a semi-continuous rougher and plants greater than one-and-a-half million tons of capacity would have a continuous rougher.

That part of the product which is to be sold as plate leaves the main line at this point and is passed through a plate mill where very

little reduction in thickness takes place but where the product is smoothed before being cut, checked, and prepared for shipment.

That part of the product which continues down the main line from the roughing mill passes to the finishing mill, which may be either reversible or continuous. A reversible finishing mill is often called a Steckle mill, after its inventor. A Steckle mill is a four-high mill (two smaller working rolls through which the product passes, backed up by two larger rollers) with a coiler on each side of the mill. The product is passed back and forth through the mill until it is reduced enough to be rolled up on one of the coilers. Then it is reversed, being unrolled from the one coiler, reduced as it passes through the mill, and then rolled up on the coiler on the other side. Because of their relative slowness Steckle mills are ordinarily used in plants of less than half a million tons per year capacity.

A continuous finishing mill (sometimes called a hot strip mill) consists of four, five, or six stands placed about twenty feet apart. Each of these stands is a four-high mill. The product shoots through these stands at a speed of about 25 miles per hour, being slightly reduced in thickness at each stand. The product leaving the mill is called hot sheet or hot strip. After leaving the hot strip mill it is either cut and packaged or it is rolled up into coils. The sheet may be as much as six feet or more in width, though four feet is a more common width. The largest users of this product are automobile and appliance factories.

The product which is destined for further processing is transported in coils to the pickling line, a sulphuric acid solution bath which cleans and deoxidizes the product. Most pickling lines are continuous, i.e. the end of one coil is welded to the beginning of the next. Then at the other end of the bath the product is cut again and re-rolled in coils.

The coils then advance to the cold reduction mill (often called a cold strip mill). Cold reduction mills normally consist of three to five stands (each stand being a four-high mill) placed about six feet apart. As with the continuous hot strip mill, the product is reduced slightly in thickness as it passes through each mill.

Since in the cold reduction mill the product is rolled at room temperature, it is hardened and becomes somewhat brittle. Therefore, the coils are taken from the cold reduction mill to the group of annealing furnaces, where they are soaked in heat for a number of hours in order to provide the desired degree of ductility. The coils are removed and are given a finishing pass in a temper mill -- a single stand mill. The cold sheets and strip may then be sold as coils; they may be cut and packaged for sale or they may be sent on to the tinning line or to the galvanizing line.

The galvanizing line prepares the product to be sold as galvanized sheets, mostly for construction purposes. The tinning line may be either batch or continuous. If it is a batch line the product is cut into small sheets before it is dipped in molten tin to be coated. The

continuous lines are referred to as electrolytic tinning lines as the tin plating is accomplished by an electrolytic process. Since a large percentage of tin products are used for making food containers, very careful inspection of the tin plate for pinholes or other defects is necessary. Electronic devices to do this checking have not yet been perfected; therefore, it must be done by hand, thereby increasing sharply the labor intensiveness of tin products.

APPENDIX B

RAW DATA FOR SPECIFIC INPUTS AND UNIT COST OF INPUTS

1. Specific Inputs.

1.1 Blast Furnaces. Table B.1 gives the raw data for specific inputs. The information for COSIPA was supplied by the firm and is theoretical, as the blast furnace at the plant was not yet in operation in February of 1965 when the data was received. This vector of specific inputs does not include fuel oil injection since this equipment was not originally installed on the blast furnace.

It appears that there may be an error in the estimate of the specific consumption of water. The estimate is 650 cubic meters per ton of pig iron as compared to the normal estimate of 100 cubic meters or less. Therefore it appears that this is a decimal point error in these estimates and that the figure should be 65 cubic meters per ton of pig iron. Their estimate of electrical energy consumption may also be off by a factor of ten; however, this is less certain. The estimate of COSIPA is 7 kilowatt hours per ton of pig iron. Comparing this to an estimate of 70 kilowatt hours per ton of pig iron for the new USIMINAS blast furnace and an estimate of 35 kilowatt hours per ton of pig iron made in an unpublished Mexican study, leads one to believe that this statistic may be off by a decimal point.

The data for USIMINAS were collected by the author while visiting the plant near Ipatinga in January of 1965. Two vectors of specific

Table B.1

SPECIFIC INPUTS - BLAST FURNACES
(Units of input per metric ton of pig iron)

Input	Units	COSIPA	USIMINAS with 70% sinter	USIMINAS with 100% sinter	CSN	CSN	CSN
Iron Ore(regular)	Metric tons	.700	.470		40%	.782	
Iron Ore(fines)	"						
Sinter	"	.760	1.100	1.500	60%	.794	.692
Pellets	"						
Scrap(regular	"						
Return scrap	"						
Pig Iron(liquid)	"						
Sponge Iron	"						
National Coal	"				40%	.513	
Imported Coal	"				60%	.655	
Coke	"	.680	.600	.560	.631	.764	.787
Dolomite	"					.035	
Limestone	"	.400	.132	.100		.272	
Quick Lime	"						
Manganese	"	.025	.040	.030		.018	
Aluminum	"						
Fluorspar	"						
Ferromanganese	"						
Ferrosilicon	"						
Fuel Oil	"				16.6*		
Natural Gas	1000 m^3						
Oxygen	"						
Electrical Energy	1000 kwh	.007		.070			
Graphite Electrodes	Metric tons						
Electrodes de Pasta	"						
Water	1000m^3	.650		.100			
Steam	Metric tons	.020		.003			
Steel Shop refractories	"						
Blast furnace gas	1000 m^3	-3.300				-3.715	
Electric furnace gas	"						
Coke Gas	"						
Slag	Metric tons	-0.430				-0.476	
Direct Labor	Man hours						
Indirect labor	US$						
Maintenance	"						
Blast Temperature	°C	980	800		800		
Blast Volume	1000m^3/min						
Top Pressure							
Production capacity of unit	million metric tons/year						

*in liters

inputs are shown, one for a 70 per cent sinter charge and one for a 100 per cent sinter charge. The blast furnace had been operated with sinter charges of both percentages so this data reflect the actual operating experience of the plant. Neither of these input vectors includes petroleum injection. Engineers in the plant estimated that the coke rate could be lowered to less than 500 kilograms per ton of pig iron once fuel oil injection was used.

The three columns of inputs shown for the blast furnaces of CSN at Volta Redonda are for observations from three different sources. The left-hand column is information collected by the author while visiting the plant in January of 1965. However, this information is not of the same technical quality as that obtained at the USIMINAS plant. Higher quality information is shown in the middle column. These data were calculated from a flow sheet showing total input and output flows at the Volta Redonda plant in 1962. Since that date fuel oil injection has been installed in the blast furnaces at this plant so there have probably been significant changes in the specific consumption of a number of the inputs. The data in the third column were taken from an article published in the May 1964 issue of the Revista of the Latin American Iron and Steel Institute (page 22).

1.2 Steel Furnaces. Table B.2 gives the specific inputs for the production of steel in the LD furnaces of COSIPA and USIMINAS and in the open hearth furnaces of CSN at Volta Redonda.

Table B.2

SPECIFIC INPUTS - STEEL FURNACES
(Units of input per metric ton of ingot steel)

		COSIPA LD Furnace	USIMINAS LD Furnace	USIMINAS LD Furnace	CSN Open Hearth Furnace	CSN Open Hearth Furnace
	Metric tons					
Iron Ore(regular)	"	.04	.020	.005	.106	.111
Scrap(regular	"	.22	.169	.187	.358	.276
Return scrap	"		.015	.014		.030
Pig Iron(liquid)	"	.88	.959	.929	.698	.628
Pig Iron(in ingots)	"	.016				.097
Dolomite	"			.006		
Limestone	"	.08		.001	.042	.042
Quick Lime	"	.08	.054	.059		
Aluminum	"	.045				
Fluorspar	"	.008		.004		
Ferromanganese	"	.007				
Ferrosilicon	"	.0006				
Fuel Oil	"					
Natural Gas	$1000m^3$					
Oxygen	"	.057	.055	.050		
Electrical Energy	1000kwh	.022	.019	.017		
Water	$1000m^3$	.016		.029		
Steel Shop Refractories	Metric tons	.008		.007	.035	.043
Coke Gas	$1000m^3$			.020		
Direct labor	Man hrs.			1.98		
Indirect labor	US$					
Maintenance	"					
Capacity of furnace	Metric tons					
Number of furnaces						

The data for COSIPA in the first column of the table are theoretical as the LD furnaces of the plant were not yet in operation at the time the company prepared the data in January of 1965. Though the data are theoretical; they compare closely with the operating experience of USIMINAS in the LD furnaces in operation at that plant.

Two vectors of inputs are shown for USIMINAS. The left-hand column of data was given to the author by officials at the company headquarters in Belo Horizonte. The right-hand column of information was obtained at the plant near Ipatinga in January of 1965. It is supposed that the data supplied by the company headquarters represent the experience of an earlier period in the performance of the furnaces.

The left-hand column of observations for CSN are from data collected by the author at the Volta Redonda plant in January of 1965. The right-hand column of figures were calculated from the flow sheet for the plant for 1962. The data calculated from the flow sheet provide a more detailed breakdown of the metallic charge to the open hearth furnaces than do the data collected at the plant. Neither vector shows oxygen injection to the open hearth furnaces. In January of 1965 three of the eight open hearth furnaces were equipped with oxygen injection equipment. The oxygen plant had sufficient capacity to provide injection to only one of the three furnaces at a time. One would suppose that the capacity of the oxygen plant will be expanded to provide sufficient capacity for injection of oxygen to all eight furnaces.

2. Unit Cost of Inputs

Columns of unit cost information for the inputs to both pig iron and ingot steel production are shown in Table B.3. No information is shown for CSN because the only data available on unit cost for the inputs of the Volta Redonda plant of that firm are contained in an as yet unreleased study of the Economic Commission for Latin America. The estimates used in this study for specific input cost at Volta Redonda are based on the ECLA study and on the information collected by the author on input cost at the COSIPA and USIMINAS plants.

The data for the COSIPA plant were provided by officials of that company and were given in cruzeiros of July 1964. In this table the data have been converted to cost in US dollars per unit using the free market exchange rate of 1300 cruzeiros/US dollar which prevailed in July 1964.

The data for USIMINAS were also provided by officials of that firm. The information provided was in cruzeiros of January 1963. Consequently these data were converted to US dollars at the free market exchange rate of 475 cruzeiros per US dollar which held in January of 1963.

Since price indices on each of the types of inputs were not available, no attempt was made to deflate the data in cruzeiros to a common base before converting it to dollars.

Table B.3

UNIT COST OF INPUTS

		COSIPA July 1964 US$/unit	USIMINAS Jan. 1963 US$/unit
Iron Ore(regular)	Metric tons	5.69	2.84
Iron Ore(fines)	"		2.27
Sinter	"	11.70	5.54
Pellets	"		
Scrap(regular)	"	23.10	32.20
Return scrap	"		
Pig Iron(liquid)	"		
Pig Iron(in ingots)	"		
Sponge Iron	"		
National Coal	"	25.60	29.90
Imported Coal	"	21.00	18.35
Coke	"	33.00	35.10
Dolomite	"		
Limestone	"	4.61	5.04
Quick Lime	"		
Manganese	"	42.30	
Aluminum	"		
Fluorspar	"		
Ferromanganese	"		
Ferrosilicon	"		
Fuel Oil	"	27.60	
Natural Gas	$100m^3$		
Oxygen	"		40.00
Electrical Energy	1000kwh	16.70	14.00
Graphite Electrodes	Metric tons		
Paste Electrodes	"		
Water	$1000m^3$	6.05	0.17
Steam	Metric tons	3.60	
Steel Shop Refractories	"	117.00	
Blast furnace gas	$1000m^3$	2.18	
Electric furnace gas	"		
Coke Gas	"	12.60	
Slag	Metric tons		
Direct labor	Man hrs.		
Indirect labor	US$		
Maintenance	"		
Exchange Rate Cruzeiros per US$		1300.	475.

APPENDIX C

EQUIVALENCE TABLE FOR MATHEMATICAL NOTATION AND FORTRAN VARIABLE NAMES

INDICES

Mathematical Index	Index	FORTRAN Index	Explanation
i	1	SA	COSIPA plant near Santos
i	2	US	USIMINAS plant near Ipatinga
i	3	VR	CSN plant at Volta Redonda
j	1	SP	market area around São Paulo
j	2	RJ	market area around Rio de Janeiro
j	3	BH	market area around Belo Horizonte
k	1		product index for pig iron
k	2	S	product index for ingot steel
k	3	P	product index for steel plate
k	4	H	product index for hot sheet and strip
k	5	C	product index for cold sheet and strip
k	6	T	product index for tin plate
e	0		reduction furnaces (usually blast)
e	1	S	steel furnaces
e	2	PR	primary rolling mill
e	3	R	roughing mill
e	4	H	hot strip finishing mill
e	5	C	cold strip mill
e	6	T	tin mill
n		NP	number of plants
m		NM	number of market areas
p			number of products
z		NPROC	number of processes
g		NI	number of inputs
r_{ij}		NUM(I,J)	number of alternative routes from plant I to market J
		NC	number of columns in LP matrix
		NRP	positive rows in LP matrix, i.e. the number of rows for which $\leq$ holds

VARIABLES

Mathematical Variable Name	FORTRAN Variable Name	Explanation
$w_{i\ell}$	$e_1e_2e_3e_4S$	where e_1e_2 are identification initials for the steel plant i and e_3e_4 are identifica- initials for the steel plant l, viz. SAVRS, shipment from Santos to Volta Redonda of ingot steel.
x_{ij}^k	$e_1e_2e_3e_4e_5$	where e_1e_2 are identification initials for the steel plant i and e_3e_4 are identification initials for the market area j and e_5 is for product k, viz. USBHP, shipment from USIMINAS to Belo Horizonte of steel plate.
$a_{i\ell}$	COLCOS, or BAT(l,J)	ingot steel shipment activity cost
b_{ij}^k	COSL(I,J,K) COLCOS,BAT(l,J)	activity cost for producing product k at plant i and delivering it to market area j.
u_i^k	COST(I,J)	production cost for product k at plant i.
f_i^h	C(I,K)	unit cost of factor input h at plant i.
s_i^{kh}	S(I,M,K)	specific input of factor input h for product k at plant i.
t_{ij}^*	BTRACO(I,J)	minimum transportation cost from plant i to market area j.
t_{ij}^q	TRACO(I,J,L)	transportation cost from plant i to market area j over route q.
	PTRACO(I,J)	transportation cost for ingot steel from plant i to plant j.
y_j^k	$IMe_1e_2e_3$	imports of product k to market area j, where e_1e_2 are identification initials for the market area and e_3 is the identification initial for the product, viz. IMSPT, the imports of tin to São Paulo.
c_j^k	BAT(I,J)	activity cost for importing product k to market area j.

Mathematical Variable Name	FORTRAN Variable Name	Explanation
v_i^k	$e_1e_2EXe_3$	exports of product k from plant i, where e_1e_2 are identification initials for the plant and e_3 is the identification initial for the product, viz. VREXH, the export of hot strip and sheet from Volta Redonda.
d_i^k	BAT(1,J)	profit for exporting product k from plant i.
α_i^1	UNITY	the number one.
β_i^1		the number minus one.
τ_i^{ke}	COEF COFN BAT(I,J)	capacity of production unit e in plant i required per metric ton of product k produced in plant i.
γ_i^e	RS1	capacity of production unit e at plant i, top group of elements in the right-hand side of the linear programming matrix.
σ_j^k	RS1	requirement for product k in market area j, bottom part of the vector for the right-hand side of the linear programming matrix.
	ROW(I)	row names for the linear program (LP) matrix BAT.
	COL(I)	column names for the LP matrix BAT
	FOBJ	row name for the objective row, COST
	M1,M2,M3	indices used to control amount of output from programs
	SROW(I)	names of factor inputs
	UNITS(I)	names of units for factor inputs
	SUBCO(I,J,K)	component of cost for plant I, process M, and factor input K.
	PERNC(I)	percentage of national coal used in coke production of plant I.
	PERIC(I)	percentage of imported coal used in coke production at plant I.
	DTe_1(I,J,K)	distance in kilometers on means of transport e_1 from plant I to market area J along route K, where e_1 = K for trucks, R for railroads, B for barges, and S for ships.

Mathematical Variable Name	FORTRAN Variable Name	Explanation
	$CTe_1(I,J,K)$	cost in US$/metric ton for transporting steel products from plant I to market J along route K for that part of the cost arising from the use of means of transportation e_1, where e_1 = K for trucks, R for railroads, B for barges, and S for ships.
	AAA	element of transportation cost function
	BBB	element of transportation cost function
	$TOLLe_1$	a toll charge in US$/metric ton charge for using certain legs of the transportation e_1.
	TRANSM	name of transportation cost program
	PCOSTM	name of production cost program
	LINERM	name of program for setting up the LP matrix and punching out an input deck for the LP routine.

```
ZZJØB                                   3126020161563 02-614

ZZFØR
*FANDK1204
*LDISKTRANSM
C       TRANSPØRTATIØN CØST RØUTINE
        DIMENSIØN NUM(3,3), DTK(3,3,3), DTP(3,3,3)
        DIMENSIØN DTB(3,3,3), DTS(3,3,3)
        DIMENSIØN CTR(3,3,3),CTK(3,3,3),CTB(3,3,3),CTS(3,3,3)
        DIMENSIØN TPACØ(3,3,3),PTPACØ(3,3)
        DIMENSIØN CØST( 3, 6)
        CØMMØN M1,PTPACØ,M2,CØST,NP,NM,M3
        READ 13,M1,M2,M3
   13   FØRMAT (3I4)
        READ 2, NP,NM
    2   FØRMAT (2I4)
        READ 4,((NUM(I,J),J=1,NM),I=1,NP)
    4   FØRMAT(3I4)
        READ 17,AAA,BBB
   17   FØRMAT (2F10.5)
        PUNCH 96
   96   FØRMAT (8X,32HPLANT  MARKET  TRANS CØST  RØUTE)
        READ 5, TØLL1,TØLL2, TØLL3, TØLL4
    5   FØRMAT (4F8.2)
        DØ 30 I=1,NP
        DØ 30 J=1,NM
        NUMD = NUM(I,J)
        DØ 30 L=1,NUMD
        READ 6, DTK(I,J,L),DTP(I,J,L),DTB(I,J,L),DTS(I,J,L)
    6   FØRMAT (4F8.0)
        CTR(I,J,L)= AAA+BBB*DTP(I,J,L)
        IF (DTR(I,J,L)) 510,510,512
  510   CTR(I,J,L)=0.
  512   CTKS=AAA+BBB*DTK(I,J,L)
        IF(DTK(I,J,L)) 514,514,516
  514   CTKS =0.
  516   DTKS=DTK(I,J,L)
        IF (DTKS - 200.)  1, 1,8
    8   CTK(I,J,L) = 1.2*CTKS
        GØ TØ 14
    1   CTK(I,J,L) = 0.9*CTKS
        GØ TØ 18
```

```
      PUNCH 93, I,J,L,DTK(I,J,L),DTR(I,J,L),DTB(I,J,L),DTS(I,J,L)
   93 FØRMAT (I3,2I8,3X,4F8.0)
   80 CØNTINUE
   34 CØNTINUE
      CALL LINK (PCØSTM)
  253 CØNTINUE
      END
```

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ZZZZ

ZZJØB                                   3126001615630 2-614

ZZ      KENDRICK PCØSTM

ZZFØR

*FANDK1204
*LDISKPCØSTM
C       PRØDUCTIØN CØST ROUTINE
        DIMENSIØN S( 3,2,40),C( 3,40),CØST( 3,6)
        DIMENSIØN PERNC( 3),PERIC( 3)
        DIMENSIØN SUBCØ( 3,2,40)
        DIMENSIØN BTRACØ( 3, 3)
        DIMENSIØN SRØW(36)
        DIMENSIØN UNITS (36)
        CØMMØN ''1,BTRACØ,''2,CØST,NP,N'',''3
        READ 10,  NP,NPRØC,NI
   10   FØRMAT (3I4)
        READ 11, (PERNC(I),PERIC(I),I=1,NP)
   11   FØRMAT(2F8.4)
        READ 15,(SRØW(I),I=1.NI)
   15   FØRMAT (10(A6,1X))
        READ 15, (UNITS(I),I=1,NI)
        DØ 12 I=1,NP
        DØ 12 K=1,NI
        READ 16,(S(I,M,K),M=1,NPRØC)
   16   FØRMAT (2F8.4)
   12   CØNTINUE
        READ 19,((C(I,K),I=1,NP),K=1,NI)
   19   FØRMAT (3F8.4)
        DØ 38 I=1,NP
        EQPNC=1.392*C(I,10)
        PCØIN=PERNC(I)*EQPNC+PERIC(I)*C(I,11)
        C(I,12)=PCØIN+ 3.
        DØ 38 M=1,NPRØC
        SUM=0.
        DØ 36 K=1,NI
        SUBCØ(I,M,K)=S(I,M,K)*C(I,K)
   36   SUM=SUM+SUBCØ(I,'',K)
        CØST(I,M)=SUM
```

```
      PUNCH 81
   81 FØRMAT (14X,44HCØST BREAKDØWN - DØLLARS PER TØN ØF PIG IRØN)
      PUNCH 64
      PUNCH 69,(SRØW(K),(SUBCØ(I,1,K),I=1,NP),K=1,NI)
   69 FØRMAT (1X,A6,9X,F8.2,5X,F8.2,6X,F8.2)
      PUNCH 77,(CØST(I,1),I=1,NP)
   77 FØRMAT (1X,5HTØTAL,10X,F8.2,5X,F8.2,6X,F8.2)
      PUNCH 110
      PUNCH 83
   83 FØRMAT (12X,47HCØST BREAKDØWN - DØLLARS PER TØN ØF INGØT STEEL)
      PUNCH 64
      PUNCH 69,(SRØW(K),(SUBCØ(I,2,K),I=1,NP),K=1,NI)
      PUNCH 77,(CØST(I,2),I=1,NP)
   99 PUNCH 110
      PUNCH 100
  100 FØRMAT (28X,15HPRØDUCTIØN CØST)
      PUNCH 102
  102 FØRMAT (27X,17H(DØLLARS PER TØN))
      PUNCH 72
   72 FØRMAT (1X,5HPLANT,10X,53HPIG       INGØT      PLATF    HØT STRIP CØL
     1D STRIP    TIN)
      DØ 75 I=1,NP
      PUNCH 74,  I,(CØST(I,M),M=1,NPROC)
   74 FØRMAT (I3,7X,2F10.2)
   75 CØNTINUE
  255 CØNTINUE
      PUNCH 110
      CALL LINK (LINERM)
      END
```

```
      DØ 133 K=1,5
      IF (K-1) 132,133,132
  132 CØSL(I,J,K)=BTRACØ(I,J)+CØFN(K)*CØST(I,2)
  133 CØNTINUE
      K=1
      DØ 142 I=1,NP
      DØ 142 J=1,NP
      IF (I-J) 140,142,140
  140 CØLCØS(K)=CØST(I,2)-CØST(J,2)+PTRACØ(I,J)
      K=K+1
  142 CØNTINUE
      KW =NP*(NP-1)+1
      DØ 134 K=2,5
      DØ 134 I=1,NP
      DØ 134 J=1,NM
      CØLCØS(KW)=CØSL(I,J,K)
      KW=KW+1
  134 CØNTINUE
      NRP=NRP+1
      IF (M3) 169,147,169
  147 PUNCH 148
  148 FØRMAT(18HINPUT.CØ2000YFAMAN)
      PUNCH 150
  150 FØRMAT (6HRØW.ID)
      PUNCH 152,FØRJ
  152 FØRMAT (12X,A6)
      PUNCH 154,(RØW(I),I=1,NRP)
  154 FØRMAT (11X,1H+,A6)
      PUNCH 156,(RØW(I),I=NRP,NP)
  156 FØRMAT (11X,1H-,A6)
      PUNCH 158
  158 FØRMAT (6HCØL.ID)
      PUNCH 160,(CØL(I),I=1,NC)
  160 FØRMAT (12X,A6)
      PUNCH 162
  162 FØRMAT (6HMATRIX)
  169 NZ=NP+1
      DØ 157 I=1,NZ
      DØ 157 J=1,NC
  157 BAT(I,J)=0.
      K=1
      DØ 151 J=1,NP
```

```
      DØ 151 I=1,NP
      IF (I-J) 153,155,153
  153 MZ=1+6*(I-1)+1
      BAT(MZ,K)=-1.
      K=K+1
  155 CØNTINUE
  151 CØNTINUE
      DØ 166 J=1,NC
  166 BAT(1,J)=CØLCØS(J)
      M=1
      UNITY=1.
      ND=NP-2
      DØ 172 I=1,NPP,6
      N=M+ND
      DØ 170 J=M,N
      II=I+1
      BAT(II,J)=UNITY
  170 CØNTINUE
      M=M+NP-1
  172 CØNTINUE
      LN=3
      M=NP*(NP-1)+1
      KM=1
      DØ 186 L=1,4
      NK=1
      DØ 184 K=1,NP
      LM=1
      NMM=NK+LN-1
      DØ 182 I=NK,NMM
      N=M+NM-1
      DØ 180 J=M,N
      II=I+1
      BAT(II,J)=CØEF(KM,LM)
  180 CØNTINUE
      LM=LM+1
  182 CØNTINUE
      M=M+NM
      NK=NK+6
  184 CØNTINUE
      KM=KM+1
      LN=LN+1
  186 CØNTINUE
```

```
      LZT=0
      M=NP*(NP-1)+1
      NRR=NRP+1
      DØ 192 I=NRP,NP
      N=M+NP*NM-1
      DØ 190 J=M,N,NM
      II=I+1
      BAT(II,J)=UNITY
      NAB=M
  190 CØNTINUE
      M=NAB
      M=M+NP*NM
      IF(M-42) 192,192,193
  193 M=NP*(NP-1)+1
      LZT=LZT+1
      M=M+LZT
  192 CØNTINUE
C     MATRIX CØEFFICIENTS FØR IMPØRT SECTIØN
      NT=NP*(NP-1)+NP*NM*4+1
      NL=NT+NM*4-1
      READ 196,(BAT(1,J),J=NT,NL)
  196 FØRMAT (10F6.2)
      DØ 194 I=NRP,NP
      DØ 194 J=NT,NL
      IW=J-(NT-NRP)
      IF (I-IW) 197,195,197
  195 II=I+1
      BAT(II,J)=1.
  197 CØNTINUE
  194 CØNTINUE
C     MATRIX CØEFFICIENTS FØR EXPØRT SECTIØN
      M=NP*(NP-1)+NP*NM*4+NM*4+1
      LS=M+NP*4-1
      READ 196,(BAT(1,J),J=M,LS)
      LN=3
      KM=1
      DØ 226 L=1,4
      NK=1
      DØ 224 K=1,NP
      LM=1
      NMM=NK+LN-1
      DØ 222 I=NK,NMM
```

```
      II=I+1
      BAT(II,M)=COEF(KM,LM)
      LM=LM+1
  222 CONTINUE
      M=M+1
      NK=NK+6
  224 CONTINUE
      KM=KM+1
      LN=LN+1
  226 CONTINUE
  161 DO 163 J=1,NC
      PUNCH 164, COL(J),FOBJ,BAT(1,J)
      IF(M3) 163,167,163
  167 DO 168 I=2,N7
      II=I-1
      IF (BAT(I,J)) 165,168,165
  165 PUNCH 164, COL(J),POW(II),BAT(I,J)
  168 CONTINUE
  163 CONTINUE
  164 FORMAT (6X,2A6,2X,F10.2)
      READ 198,(PS1(I),I=1,NR)
  198 FORMAT (10F6.3)
      IF (M3) 220,199,220
  199 PUNCH 200
  200 FORMAT (11HFIRST.BONCE)
      PUNCH 202,(POW(I),PS1(I),I=1,NR)
  202 FORMAT (12X,A6,2X,F10.3)
      PUNCH 206
  206 FORMAT (6HENDATA)
      PUNCH 208
  208 FORMAT (11HMIN     ONCE)
      PUNCH 210
  210 FORMAT (6HOUTPUT)
      PUNCH 212
  212 FORMAT (6HCHECK.)
      PUNCH 214
  214 FORMAT (6HCOST.P)
      PUNCH 216
  216 FORMAT (6HDO.D/J)
      PUNCH 218
  218 FORMAT(6HENDJOB)
  220 CALL EXIT
```

ZZXFOSTRANSM

```
1   1   1
3   3
2   3   2
3   3   2
2   2   2
1.80        .007
  .50       .50     .40     .40
  85
            85
  5                         220
            582
  5                 220
  642
            837
  771
            846
  85        451     480
  667
            640
            451     260
  214
            214
  353
            353
  146
            252
  473
            473
3   2   36
 40         60
 40         60
 44         56
ØRE     ØREF    SINTR   PELLT   SCRAP   RSCRP   PIG     PIGIN   SPØNG   CØALN
CØALM   CØKF    DØLØ    LIMST   QLIME   MANG    ALUM    FLUØR   FMANG   FSILC
FUEL    NAGAS   ØXY     ELFC    ELTRG   FLTRP   WATER   STEAM   REFRA   BFGAS
EFGAS   CKGAS   SLAG    DLAB    ILAB    MAIN
TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS
TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS    TØNS
TØNS    1000M3  1000M3  1000KH  TØNS    TØNS    1000M3  TØNS    TØNS    1000M3
1000M3  1000M3  TØNS    MANHØR  MANHØR  UNITS
```

700	~~04~~
760	
	22
	88
	016
680	
400	08
	08
025	
	004
	008
	007
	001
	057
007	022
650	016
020	
	009
-3300	
-138	
-430	
39	198
65	
270	450
470	005
1100	
	187
	014

	929
371	
552	
035	006
132	001
	059
040	
	004
	050
070	012
100	029
	007
-2624	
-138	
-476	
39	198
65	
270	450
782	111
794	
	276
	039
	628
	097
461	
589	

035		
272	042	
018		
050	118	
	004	
070		
015	020	
	043	
-2624		
-138		
-476		
39	167	
65	95	
270	634	
569	284	350
	227	
1170	554	900
2310	3220	2400
2500	3500	2600
2560	2990	2700
2100	1835	2200
3330	3510	
461	504	500
600	600	600
4230	3500	3500
2000	2000	2000

28000	28000	28000
34400	34400	34400
2760	2500	2800
4000	4000	4000
1670	1400	1600
605	500	500
360	360	360
11700	11700	11700
218	150	150
1260	700	700
42	42	42
50	50	50
100	100	100

30 18 66

SAS	SAPP	SAP	SAH	SAC	SAT	USS	USPP	USP	USH
USC	UST	VRS	VRPP	VRP	VRH	VRC	VRT	SPP	SPH
SPC	SPT	RJP	RJH	RJC	RJT	BHP	BHH	BHC	BHT
SAUSS	SAVRS	USSAS	USVRS	VRSAS	VRUSS	SASPP	SARJP	SABHP	USSPP
USRJP	USBHP	VRSPP	VRRJP	VRBHP	SASPH	SARJH	SABHH	USSPH	USRJH
USBHH	VRSPH	VRRJH	VRBHH	SASPC	SARJC	SABHC	USSPC	USRJC	USBHC
VRSPC	VRRJC	VRBHC	SASPT	SARJT	SABHT	USSPT	USRJT	USBHT	VRSPT
VRRJT	VRBHT	IMSPP	IMSPH	IMSPC	IMSPT	IMRJP	IMRJH	IMRJC	IMRJT
IMBHP	IMBHH	IMBHC	IMBHT	SAFXP	USFXP	VRFXP	SAFXH	USFXH	VRFXH
SAFXC	USFXC	VRFXC	SAFXT	USFXT	VRFXT				

COST

128	128	110							
124	124	109	105						
131	131	115	112	103					
140	140	123	120	111	105				
102	102	112	122	100	100	110	120	108	108
118	128								
-2	-2	-2	-3	-3	-3	-4	-4	-4	-5
-5	-5								
820	180	100	150	35	001	60	180	100	150
15	001	142	120	125	150	80	28	02	20
30	15	16	12	16	06	02	08	14	04

APPENDIX D

TRANSPORTATION AND COST DATA FOR THE MIXED INTEGER PROGRAMMING MODEL

T0295 3017 TAPE.6 LONG FOR T0295 3017

	PLANT	MARKET		TRANS COST	ROUTE
SA	1	1	SP	2.66	1
	1	2	RJ	6.37	2
	1	3	BH	8.16	2
US	2	1		7.72	2
	2	2		6.28	2
	2	3		3.30	2
VR	3	1		4.27	2
	3	2		2.94	1
	3	3		5.11	2

TRANS COST	FROM PLANT	TO PLANT
7.63	1	2
3.37	1	3
7.63	2	1
5.51	2	3
3.37	3	1
5.51	3	2

DISTANCES BETWEEN PLANTS AND MARKETS

PLANT	MARKET	ROUTE	TRUCK KM	TRAIN KM	BARGE (NAUTICAL MILES)	SHIP
1	1	1	85	-00	-00	
1	1	2	-00	85	-00	
1	2	1	5	-00	-00	
1	2	2	-00	582	-00	
1	2	3	5	-00	220	
1	3	1	642	-00	-00	
1	3	2	-00	837	-00	
2	1	1	771	-00	-00	
2	1	2	-00	846	-00	
2	1	3	85	451	480	
2	2	1	667	-00	-00	
2	2	2	-00	640	-00	
2	2	3	-00	451	260	
2	3	1	214	-00	-00	
2	3	2	-00	214	-00	
3	1	1	353	-00	-00	
3	1	2	-00	353	-00	
3	2	1	146	-00	-00	
3	2	2	-00	252	-00	
3	3	1	473	-00	-00	
3	3	2	-00	473	-00	

DISTANCES BETWEEN PLANTS

FROM PLANT		TO PLANT	DISTANCE (KM)
SA	1	2	1300.00
	1	3	430.00
US	2	1	1300.00
	2	3	868.00
VR	3	1	430.00
	3	2	868.00

PRODUCTION COST
(DOLLARS PER TON)

PLANT	PIG IRON	INGOTS	SLABS	PLATE	HOT STRIP	COLD STRIP	TIN PL
1	31.97	46.34	51.07	57.95	62.00	72.91	125.
2	29.47	42.80	46.32	52.78	56.63	66.41	116.
3	36.52	49.84	54.80	61.75	65.84	76.95	129.

SPECIFIC INPUTS - PIG IRON

INPUT		COSIPA	USIMINAS	CSN
ORE	TONS	.700	.470	.782
OREF	TONS	-0.	-0.	-0.
SINTR	TONS	.760	1.100	.794
PELLT	TONS	-0.	-0.	-0.
SCRAP	TONS	-0.	-0.	-0.
RSCRP	TONS	-0.	-0.	-0.
PIG	TONS	-0.	-0.	-0.
PIGIN	TONS	-0.	-0.	-0.
SPONG	TONS	-0.	-0.	-0.
COALN	TONS	-0.	.371	.461
COALM	TONS	-0.	.552	.589
COKE	TONS	.680	-0.	-0.
DOLO	TONS	-0.	.035	.035
LIMST	TONS	.400	.132	.272
QLIME	TONS	-0.	-0.	-0.
MANG	TONS	.025	.040	.018
ALUM	TONS	-0.	-0.	-0.
FLUOR	TONS	-0.	-0.	-0.
FMANG	TONS	-0.	-0.	-0.
FSILC	TONS	-0.	-0.	-0.
FUEL	TONS	-0.	-0.	.050
NAGAS	1000M3	-0.	-0.	-0.
OXY	1000M3	-0.	-0.	-0.
ELEC	100CKH	.070	.070	.070
ELTRG	TONS	-0.	-0.	-0.
ELTRP	TONS	-0.	-0.	-0.
WATER	1000M3	.065	.100	.015
STEAM	TONS	.020	-0.	-0.
REFRA	TONS	-0.	-0.	-0.
BFGAS	1000M3	-3.300	-3.300	-3.715
EFGAS	1000M3	-0.	-0.	-0.
CKGAS	1000M3	-.138	-.138	-.138
SLAG	TONS	-.430	-.476	-.476
DLAB	MANHOR	.390	.390	.390
ILAB	MANHOR	.650	.650	.650
MAIN	UNITS	2.700	2.700	2.700

SPECIFIC INPUTS - INGOT STEEL

INPUT		COSIPA	USIMINAS	CSN
ORE	TONS	.040	.005	.111
NOREF	TONS	-0.	-0.	-0.
SINTR	TONS	-0.	-0.	-0.
PELLT	TONS	-0.	-0.	-0.
SCRAP	TONS	.220	.187	.276
RSCRP	TONS	-0.	.014	.039
PIG	TONS	.880	.929	.628
PIGIN	TONS	.016	-0.	.097
SPONG	TONS	-0.	-0.	-0.
COALN	TONS	-0.	-0.	-0.
COALM	TONS	-0.	-0.	-0.
COKE	TONS	-0.	-0.	-0.
DOLO	TONS	-0.	.006	-0.
LIMST	TONS	.080	.001	.042
QLIME	TONS	.080	.059	-0.
MANG	TONS	-0.	-0.	-0.
ALUM	TONS	.004	-0.	-0.
FLUOR	TONS	.008	.004	-0.
FMANG	TONS	.007	-0.	-0.
FSILC	TONS	.001	-0.	-0.
FUEL	TONS	-0.	-0.	.118
NAGAS	1000M3	-0.	-0.	-0.
OXY	1000M3	.057	.050	.004
ELEC	1000KH	.022	.012	-0.
ELTRG	TONS	-0.	-0.	-0.
ELTRP	TONS	-0.	-0.	-0.
WATER	1000M3	.016	.029	.020
STEAM	TONS	-0.	-0.	-0.
REFRA	TONS	.009	.007	.043
BFGAS	1000M3	-0.	-0.	-0.
EFGAS	1000M3	-0.	-0.	-0.
CKGAS	1000M3	-0.	-0.	-0.
SLAG	TONS	-0.	-0.	-0.
DLAB	MANHOR	1.980	1.980	.167
ILAB	MANHOR	-0.	-0.	.950
MAIN	UNITS	4.500	4.500	6.340

INPUT		COSTPA	USTMINAS	CSN
ORE	TONS	5.69	2.84	3.50
OREF	TONS	-0.	2.27	-0.
SINTR	TONS	11.70	5.54	9.00
PELLT	TONS	-0.	-0.	-0.
SCRAP	TONS	23.10	32.20	24.00
RSCRP	TONS	25.00	35.00	26.00
PIG	TONS	31.97	29.47	36.52
PIGIN	TONS	28.77	26.52	32.87
SPONG	TONS	-0.	-0.	-0.
COALN	TONS	25.60	29.90	27.00
COALM	TONS	21.00	18.35	22.00
COKE	TONS	29.85	30.66	31.86
DOLO	TONS	-0.	-0.	-0.
LIMST	TONS	4.61	5.04	5.00
QLIME	TONS	6.00	6.00	6.00
MANG	TONS	42.30	35.00	35.00
ALUM	TONS	-0.	-0.	-0.
FLUOR	TONS	20.00	20.00	20.00
FMANG	TONS	280.00	280.00	280.00
FSILC	TONS	344.00	344.00	344.00
FUEL	TONS	27.60	25.00	28.00
NAGAS	1000M3	-0.	-0.	-0.
OXY	1000M3	40.00	40.00	40.00
ELEC	1000KH	16.70	14.00	16.00
ELTRG	TONS	-0.	-0.	-0.
ELTRF	TONS	-0.	-0.	-0.
WATER	1000M3	6.05	5.00	5.00
STEAM	TONS	3.60	3.60	3.60
REFRA	TONS	117.00	117.00	117.00
BFGAS	1000M3	2.18	1.50	1.50
EFGAS	1000M3	-0.	-0.	-0.
CKGAS	1000M3	12.60	7.00	7.00
SLAG	TONS	-0.	-0.	-0.
DLAB	MANHOR	.42	.42	.42
ILAB	MANHOR	.50	.50	.50
MAIN	UNITS	1.00	1.00	1.00

COST BREAKDOWN - DOLLARS PER TON OF PIG IRON

INPUT	COSIPA	USIMINAS	CSN
ORE	3.98	1.33	2.74
OREF	0.	-0.	C.
SINTR	8.89	6.09	7.15
PELLT	0.	0.	C.
SCRAP	-0.	-0.	-0.
RSCRP	-0.	-0.	-C.
PIG	0.	0.	C.
PIGIN	0.	0.	C.
SPONG	0.	0.	0.
COALN	-0.	11.09	12.45
COALM	-0.	10.13	12.96
COKE	20.30	-0.	-0.
DOLO	0.	-0.	-0.
LIMST	1.84	.67	1.36
QLIME	-0.	-0.	-0.
MANG	1.06	1.40	.63
ALUM	0.	0.	0.
FLUOR	-0.	-0.	-C.
FMANG	-0.	-0.	-C.
FSILC	-0.	-0.	-0.
FUEL	-0.	-0.	1.40
NAGAS	0.	0.	0.
OXY	-0.	-0.	-0.
ELEC	1.17	.98	1.12
ELTRG	0.	0.	0.
ELTRP	0.	0.	0.
WATER	.39	.50	.07
STEAM	.07	-0.	-0.
REFRA	-0.	-0.	-0.
BFGAS	-7.19	-4.95	-5.57
EFGAS	0.	0.	0.
CKGAS	-1.74	-.97	-.97
SLAG	0.	0.	0.
DLAB	.16	.16	.16
ILAB	.32	.32	.32
MAIN	2.70	2.70	2.70
TOTAL	31.97	29.47	36.52

INPUT	COSIPA	USIMINAS	CSN
ORE	.23	.01	.39
OREF	0.	-0.	0.
SINTR	-0.	-0.	-0.
PELLT	0.	0.	0.
SCRAP	5.08	6.02	6.62
RSCRP	-0.	.49	1.01
PIG	28.13	27.38	22.94
PIGIN	.46	-0.	3.19
SPONG	0.	0.	0.
COALN	-0.	-0.	-0.
COALM	-0.	-0.	-0.
COKE	-0.	-0.	-0.
DOLO	0.	-0.	0.
LIMST	.37	.01	.21
CLIME	.48	.35	-0.
MANG	-0.	-0.	-0.
ALUM	-0.	0.	0.
FLUOR	.16	.08	-0.
FMANG	1.96	-0.	-0.
FSILC	.34	-0.	-0.
FUEL	-0.	-0.	3.30
NAGAS	0.	0.	0.
OXY	2.28	2.00	.16
ELEC	.37	.17	-0.
ELTRG	0.	0.	0.
ELTRP	0.	0.	0.
WATER	.10	.14	.10
STEAM	-0.	-0.	-0.
REFRA	1.05	.82	5.03
BFGAS	-0.	-0.	-0.
EFGAS	0.	0.	0.
CKGAS	-0.	-0.	-0.
SLAG	0.	0.	0.
DLAB	.83	.83	.07
TLAB	-0.	-0.	.47
MAIN	4.50	4.50	6.34
TOTAL	46.34	42.80	49.84

APPENDIX E

EVIDENCE ON EQUIPMENT COST

Steel companies and equipment manufacturing concerns so zealously guard information on equipment cost that the information which is publicly available is precious little. However, there are a number of places to search for the evidence.[1]

The Instituto Latinoamericano del Fierro y el Acero has recently made a survey of the major companies in Latin America to obtain information on equipment cost. Data from this survey are helpful for estimating costs of coke plants, blast furnaces, and oxygen converters, but are too aggregative to be of any help in estimating the costs of rolling mills and other equipment.

The Wall Street Journal also provides some clues about equipment cost in the steel industry. However, the stories in this newspaper usually carry incomplete technical descriptions of the equipment and usually explain that the cost of the equipment was not announced by the company but that "industry sources" estimated that the amount was so much.

Also, some of the technical publications of the industry occasionally provide some cost information. The Revista Siderurgicas of ILAFA and Blast Furnace and Steel Plant are two of the most helpful.

Studies of the United Nations, particularly of the Economic Commission for Europe and the Economic Commission for Latin America,

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[1]For reference citations, see Table E.1.

also provide some information. However, the results of these studies are usually to give an equation of the form

$$C = aX^b$$

where C is the cost of the equipment, a is a constant, X is the annual capacity of the unit, and b is a constant such that $0 \leq b \leq 1$. Sources from which the data to estimate the function were obtained are vague and the data are not usually published.

Finally, there are feasibility studies. These studies are always confidential, are difficult to obtain, and can not be quoted. Also, the data they contain are expectations about what equipment costs will be rather than statements about what the costs were.

Table E.1 contains the evidence which the author was able to obtain. Some of the companies are referred to with letters of the alphabet in order not to violate the confidence of the institutions which supplied the data. The investment cost given in the second column of the data should be regarded as the cost of purchasing and installing the unit, unless otherwise noted.

The estimates of capital costs for the projects included in the model were made on the basis of the information contained in Table E.1 and on some additional information which will be discussed in the section to follow.

The estimates of the cost of modifying the blast furnaces at COSIPA and USIMINAS to increase their productivity were based on

Table E.1

EVIDENCE ON EQUIPMENT COST FOR THE MAKING OF STEEL AND THE ROLLING OF FLAT PRODUCTS

PART I.

Unit Plant	Location	Capacity (million metric tons/year)	Investment (US$ million)	Data Source
1. COKE PLANT				
(a) A		.487	15.7	ILAFA
(b) B		.250	17.2	ILAFA
(c) COSIPA	Santos, Brazil		5.9*	Revista #45 p. 15
(d) C - (43 ovens, 15 tons each)		.220	13.9	ILAFA
(e) D			11.8	ILAFA
(f) Jones & Laughlin - (59 ovens)		.360	6.0	WSJ - 12/1/58 19;2
(g) Bethlehem Steel Co. - (76 ovens)			7.5	WSJ - 12/15/59 2;4
(h) E - (74 ovens)		.450	18.6	AID
2. SINTER PLANT				
(a) Pittsburg Steel, Monessen, Pa.		.500	3.5	WSJ - 3/6/62
(b) COSIPA	Santos, Brazil		4.4*	
(c) The experience of German plants		"small" plants - US$13-14/metric ton of capacity		UN Steel Symp.1963
		"large" plants - US$11.5-12.5/metric ton of capacity		Tech Paper/A.7

*equipment cost only - f.o.b.

Table E.1 continued

PART II.

Unit Plant	Location	Capacity (million metric tons/year)	Investment (US$ million)	Data Source
3. BLAST FURNACES				
(a) A		.515	23.7	ILAFA
(b) C		.220	12.4	ILAFA
(c) F		.450	12.0	ILAFA
(d) D	(2 blast furnaces)	.464	9.4	ILAFA
(e) E		.440	13.2	AID
(g) G		1.220	43.7	
(h) Propulsora, La Plata, Arg.(P)		.635	20.0	Project Study

PART III.

4. OXYGEN CONVERTERS				
(a) B	-2 LD's, 45 tons each 2 mixers,800 tons each	.600	16.9	ILAFA
(b) F	2 LD's	.500	20.0	ILAFA
(c) E	2 LD's,80 tons each	.470	15.3	AID
(d) G	2 LD's,170 tons each	1.400	51.4	
(e) G	first two vessels add a third vessel add a fourth & fifth	1.400	51.4 29.6+ 13.0 7.5+ 39.5 23.1+	
(f) COSIPA	2 LD's	.800	7.6+	Revista, #45, p.15
(g) US Steel	Gary,Indiana 3 LD's		30.0	WSJ - 9/24/63 32;2
(h) National Steel, Weirton, Pa.	2 B.D.F.	1.5/2.0	30.0	WSJ - 8/12/60 20;2
(i) Republic Steel-2 B.D.F's,	220 tons each		40.0	WSJ - 3/25/64

+ Equipment cost only, purchase price

Table E.1 continued

PART IV.

Unit Plant	Location	Capacity (million metric tons/year)	Investment (US$ million)	Data Source
5. OXYGEN PLANTS				
(a) COSIPA - for 2 LD converters with a capacity of 800 thousand metric ton/year			2.6+	Revista, #45 p.15
(b) Wheeling Steel Company		.150	6.0	WSJ 6/18/59 16;3
6. CONTINUOUS CASTING				
(a) F		slabs .200 bars .200	4.0 3.0	
(b) National Steel, Weirton, Pa. (4 stand low overhead) (slabs up to 9" thick & 40" wide)		slabs 1.500	10.0	WSJ 3/25/64 NYT 3/18/64
7. PRIMARY MILL				
(a) G	46"x90" slabbing mill	4.000	57.0	
(b) COSIPA	Santos, Brazil 44" wide	1.800	5.7*	Revista, #45, p.15
8. PLATE MILL				
(a) G	140" wide plate mill		82.0	
(b) COSIPA, 110" wide plate mill			7.8*	Revista, #45, p.15

+ Equipment cost only, f.o.b.

* Mechanical equipment only - f.o.b.

Table E.1 continued

PART V.

Unit Plant	Location	Capacity (million metric tons/year)	Investment (US$ million)	Data Source
9. HOT STRIP MILLS				
(a) Pittsburg Steel Co. 56" wide		1.7	45.0	WSJ-5/15/59 7;1
(b) Republic Steel Corp.			55.0	WSJ-6/23/60 6;4
(c) Jones & Laughlin-Cleveland Plant			60.0	WSJ-11/18/61 8;3
(d) G 80" wide		3.5	92.0	
(e) COSIPA 66" wide		1.5	4.6*	Revista,#45 p.15
(f) Armco - Middletown		3.6	125.0	BF&SP,Jan.65 Vol.53,#1,p.81
10 COLD STRIP MILLS				
(a) US Steel Corp. - 6 stand			20.0	WSJ 12/17/63 17;3
(b) Inland - 54"-3 stand-5,000 ft/min. coils up to 48,000 lbs.			5.0+	WSJ 5/6/63 10;5
(c) Kaiser - 3 stands for light tin plate			15.0	WSJ 3/25/64 8;
(d) COSIPA	Santos, Brazil	.350	1.9*	Revista,#45, p.15
(e) Propulsora, La Plata, Argentina (may include shearing line and/or pickling line)		.300	20.0	Project Study

* - Mechanical equipment only, f.o.b.

\+ - Mechanical and electrical equipment only.

Table E.1 continued

PART VI.

Unit Plant	Location	Capacity (million metric tons/year)	Investment (US$ million)	Data Source
11. TINNING LINES				
(a) F		.100	5.0	
(b) Kaiser Steel Corp. - thin tin plate 3 stand			15.0	WSJ,6/13/63 3;4
(c) Republic Steel Corp., Warren 54" - 2 stand			10.0 5.0+	WSJ,5/6/63 10;5
(d) National, Weirton, 2 stand			15.0	WSJ 3/24/65 8;
12. GALVANIZING LINE				
(a) F		.100	3.0	
(b) Bethlehem, California 54" wide 400 to 500 ft/min		.280	20.0	BF&SP,Jan 1965 Vol.53,No.1,p.59

+ Mechanical and electrical equipment only

Key to references:

1. ILAFA = unpublished survey by the Instituto Latinoamericano del Fierro y el Acero, "Inversiones Siderurgicas Latinoamericanas," Santiago, Chile, 1965.
2. Revista = Revista Siderurgicas, monthly magazine published by ILAFA
3. WSJ = The Wall Street Journal
4. UN Steel Symp. = United Nations, "Interregional Symposium on the Application of Modern Technical Practices in the Iron and Steel Industry to Developing Countries," 1963.
5. AID = The Agency for International Development, Washington, D.C.
6. NYT = The New York Times
7. BF&SP = Blast Furnaces and Steel Plant.

estimates of the cost of modifying a large blast furnace in another Latin American country. Data from that estimate is shown in Table E.2. Though the blast furnace at COSIPA has just been installed and probably already contains some of the features in the list of modifications in Table E.2, it is assumed that all of the modifications could be added to the COSIPA blast furnace at a cost of US$1.8 million with an increase in the capacity of the furnace from .720 to 1.000 million metric tons per year.

Since there are two blast furnaces at USIMINAS and space for one additional stove (to heat the blast air), it is assumed that the modification cost would be double for increasing the top pressure, adding humidity control, and the injection of petroleum, but that all other components of the cost would be as shown in Table E.2. Thus it is assumed that the output of the two blast furnaces could be increased from a rated capacity of .900 to 1.260 million metric tons per year with an investment cost of US$2.4 million.

To estimate the cost of the investments for building new blast furnaces Figure E.1 was constructed from the data in Table E.1. The letter on the points in the plot are for the corresponding plant name in the table. In addition there are two points labeled "Muller." These two points were calculated from the function

$$C = 40\ (x)^{.60}$$

where C is the capital cost for the investment in millions of US dollars and x is the capacity of the blast furnace in millions of metric tons

K+E 10 X 10 TO THE INCH 358-5D
KEUFFEL & ESSER CO. MADE IN U.S.A.

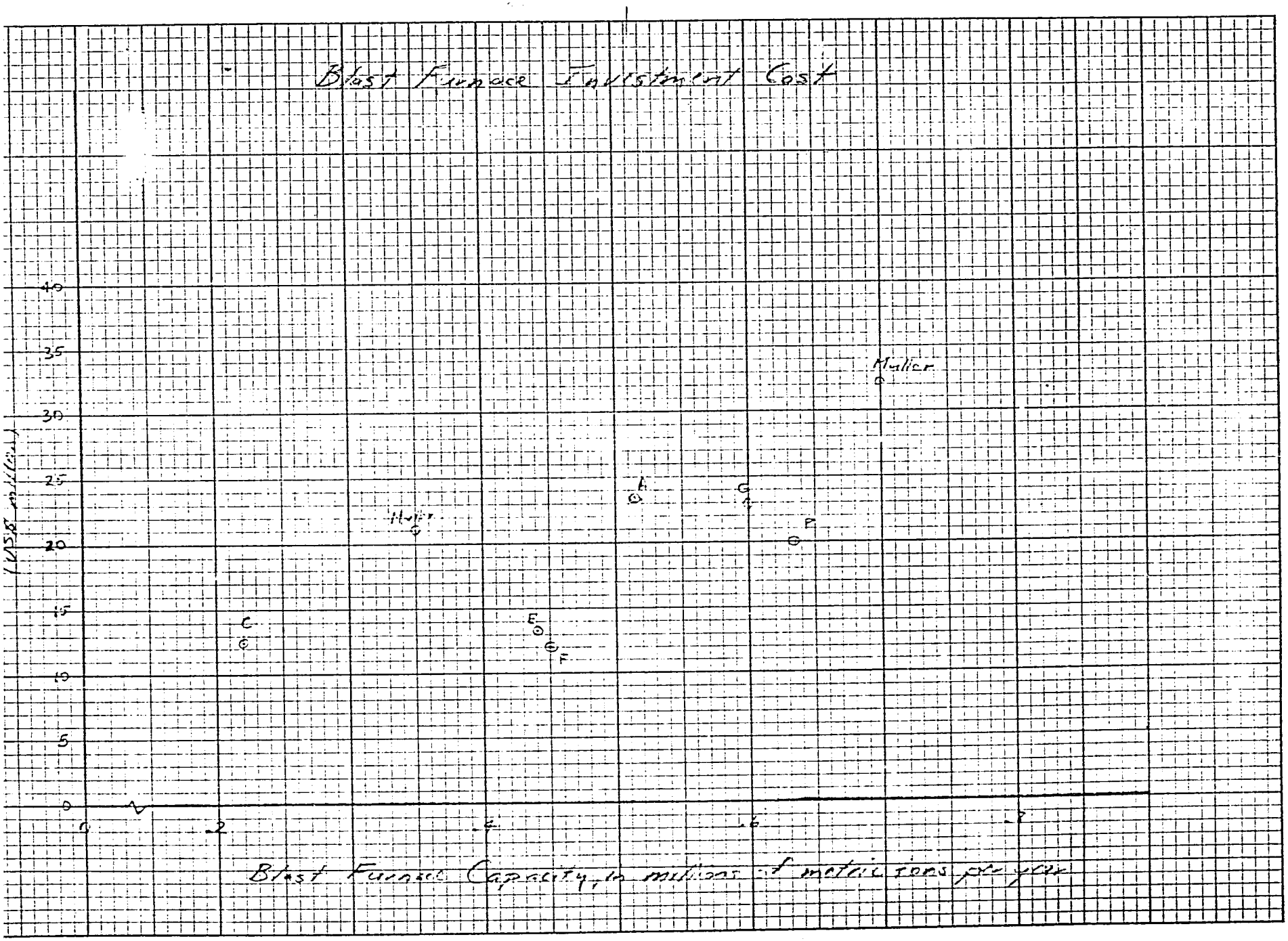

Table E.2

COST OF MODIFICATIONS TO A BLAST FURNACE

TOTAL COST --DOMESTIC AND FOREIGN IN THOUSANDS OF US DOLLARS

Modification	Cost
1. Increase Top Pressure	402.0
2. Humitidy Content	8.5
3. Addition of a Fourth Stove (25' diam., 110' high)	891.0
4. Injection of Petroleum	200.0
5. Distribution of Electrical Energy	9.0
6. Parts	45.0
7. Engineering	50.0
8. Contingencies	160.0
Total	1,765.5

of pig iron per year. This function was taken from a study by Gunther Muller[1] based on an earlier study by the Economic Commission for Europe.[2]

In recent years a series of modifications in blast furnace design and operation have resulted in significant increases in the productivity of existing blast furnaces, and, one would suppose, decreases in the cost per ton of capacity in new blast furnaces. For this reason greater

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[1]Muller, Gunther H., "Aspectos Economicos del Proceso HyL," ILAFA, Cuarto Congresso Latinamericana de Siderurgia, Mexico City, D.F. 1964.

[2]United Nations, Economic Commission for Europe, A Comparison of Steel Making Processes, New York, 1962.

weight should be given to the points E and P in Figure E.1, as they represent projects recently completed or not yet begun while the other points in the graph represent blast furnaces of slightly greater vintage than F and P. With greatest weight given to the four points, F,P,G, and E and making a very rough projection, a capital cost of US$28 million is chosen as the capital cost for the installation of a blast furnace of 800 thousand metric tons per year capacity. Though there would be some difference in the cost of erecting a blast furnace of this size at the three different plants and there would be some differences in the cost of capital to the different companies, those differences are assumed away in the model.

A plot of investment cost versus capacity for a pair of LD converters is shown in Figure E.2. As with the previous graph, the letters and names refer to plants listed in Table E.1. Muller's equation for converters is

$$C = 19.5\ (x)^{.71}$$

The equation for the points marked ECLA is not given in the study.[1]

From Figure E.2 it appears that a fair guess for the capital cost of a pair of LD converters with a capacity of 800 thousand metric tons per year would be US$21 million. Since it is also often necessary to add to the infrastructure of the plant when new productive units are installed

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[1]United Nations, Economic Commission for Latin America, Economia siderurgica latinoamericana, informe regional, Santiago, Chile

K+E 10 X 10 TO THE INCH 358-5D
KEUFFEL & ESSER CO. MADE IN U.S.A.

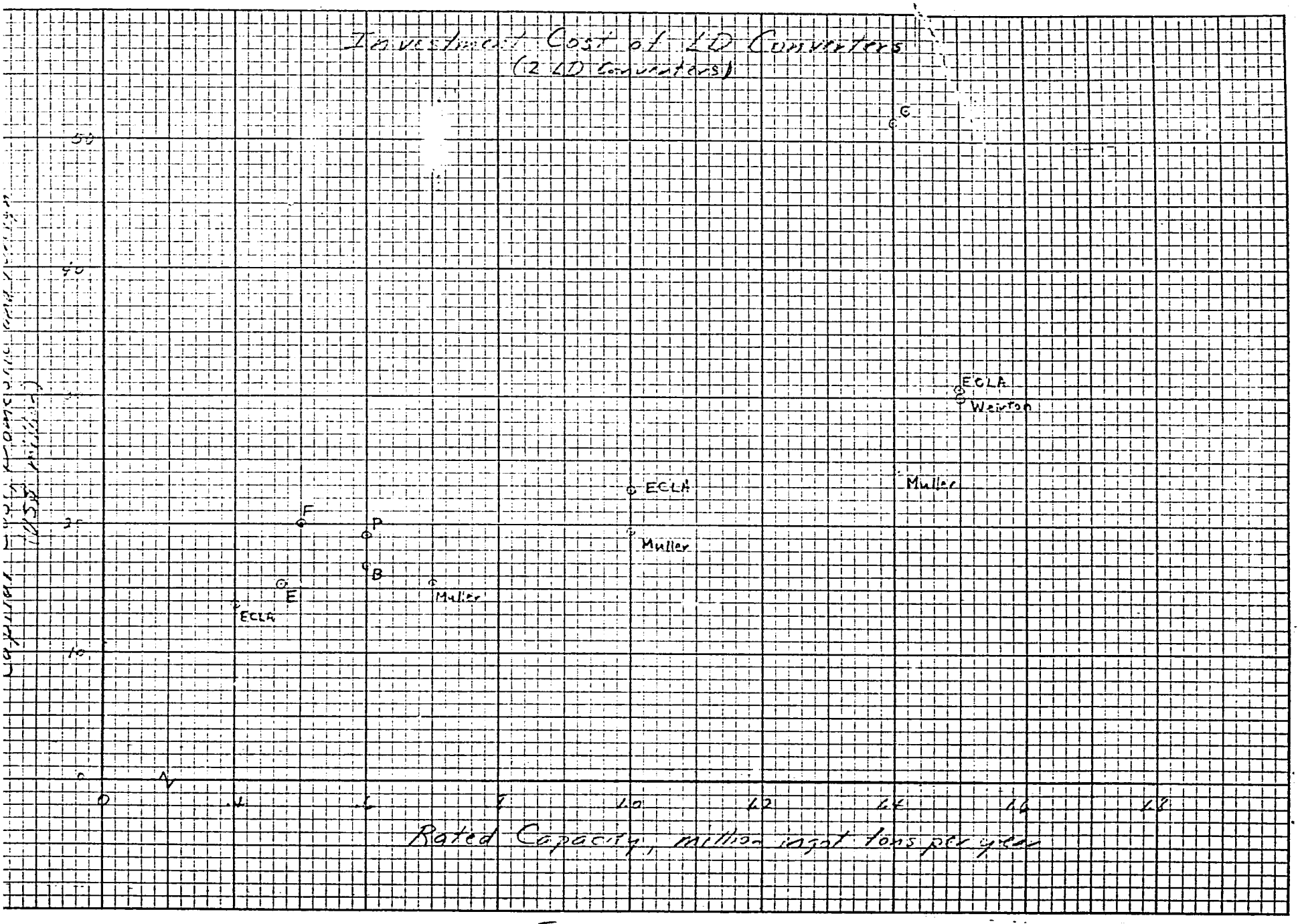

Figure E.2

and since infrastructure investment in a new plant may account for almost half of the total investment, but will certainly be less in the usual addition to plant capacity, it is assumed that the total investment cost associated with the installation of a pair of LD converters of 800 thousand metric tons is 1.5 (21) = US$31.5 million.

Table E.1 shows that in the case of plant G where plans were made for the installation of two LD converters, then a third, and then two more, that the capital cost of the third LD was roughly one-fourth of the cost of the original pair and that the cost of the final pair was about three-fourths of that of the original pair. Applying these proportions to the US$31.5 million for a pair of LD converters yields a cost of US$7.9 million for a third converter and of US$23.6 for a fourth and fifth.

The scant data obtained on the cost of plate mills makes it difficult to estimate the cost of adding a stand to the existing reversing rougher at COSIPA. The cost of the 140" plate mill at plant G is shown by Table E.1 to have been estimated at US$82 million, and according to a report in Revista Siderurgicas (see Table E.1) the cost of only the mechanical equipment of the 110" plate mill already installed at COSIPA was US$7.8 million. So, taking a wild guess it is assumed that the total cost of the equipment and the modifications required to increase the capacity of the roughing mill at COSIPA by one million metric tons from its present annual capacity of one million metric tons will be US$25 million.

APPENDIX F

COMPUTER PROGRAMS AND DATA INPUT FOR THE MULTI-PERIOD MODEL

```
C       MAIN PRØGRAM FØR INPUT SMØØTIHING
C       TRANSPØRTATIØN CØST RØUTINE                                     00010
        DIMENSIØN NUM(3,3), DTK(3,3,3), DTR(3,3,3)                      00020
        DIMENSIØN DTB(3,3,3), DTS(3,3,3)                                00030
        DIMENSIØN CTR(3,3,3),CTK(3,3,3),CTB(3,3,3),CTS(3,3,3)           00040
        DIMENSIØN TRACØ(3,3,3), BTRACØ(3,3)                             00050
        DIMENSIØN CØST (3,7) , PTRACØ (3,3)                             00060
        DIMENSIØN C (3,40) , BAT(34,100)      ,DBPR(3,3)                00070
        CØMMØN M1,BTRACØ,M2,CØST,NP,NM,M3   ,PTRACØ,C     , BAT   ,M4   00080
        CØMMØN IBASE,ITAPE,ITAB,ITAK                                    00090
        IBASE=0                                                         00100
        ITAPE=IBASF+2                                                   00110
        ITAB =IBASE+6                                                   00120
        ITAK=IBASE+5                                                    00130
        PRINT 501                                                       00140
    501 FØRMAT(8HM1-4,4I2/                                              00145
       124HM1 (TRANS) 0=ØUT,1=NØNE./                                    00155
       225HM2 (CØST) 0=FULL,1=SHØRT./                                   00165
       348HM3 (MATRIX) 0=CØMPLETE W/F,-1=W/F',1=ICØST ØNLY./            00175
       431HM4 (LP) 0=MFØR,1=LP/90,-1=BØTH.)                             00185
        READ 502,M1,M2,M3,M4                                            00195
    502 FØRMAT(4I2)                                                     00205
        READ INPUT TAPE ITAPE, 2, NP,NM                                 00215
      2 FØRMAT (2I4)                                                    00225
        READ INPUT TAPE ITAPE, 4,((NUM(I,J),J=1,NM),I=1,NP)             00235
      4 FØRMAT(3I4)                                                     00245
        READ INPUT TAPE ITAPE, 17,AAA,BBB                               00255
     17 FØRMAT (2F10.5)                                                 00265
        READ INPUT TAPE ITAPE, 117 , ((DBPR (I,J) ,J=1,NP ),I=1,NP)     00275
117     FØRMAT (9F6.0)                                                  00285
        WRITE ØUTPUT TAPE ITAB, 96                                      00295
     96 FØRMAT (8X,32HPLANT  MARKET  TRANS CØST  RØUTE)                 00305
        READ INPUT TAPE ITAPE, 5, TØLL1,TØLL2, TØLL3, TØLL4             00315
      5 FØRMAT (4F8.2)                                                  00325
C SETS NUMBER ØF ALTERNATIVE RØUTES TØ BE CØNSIDERED FØR                00335
C EACH LEG ØF SYSTEM                                                    00345
```

```
      DØ 30 I=1,NP
      DØ 30 J=1,NM
      NUMD = NUM(I,J)
      DØ 30 L=1,NUMD
      READ INPUT TAPE ITAPE, 6, DTK(I,J,L),DTR(I,J,L),DTB(I,J,L),DTS(I,J
     1,L)
C CALCULATES TRANSPØRTATIØN CØST FØR RAILRØADS
    6 FØRMAT (4F8.0)
      CTR(I,J,L)= AAA+BBB*DTR(I,J,L)
      IF (DTR(I,J,L)) 510,510,512
C CALCULATES TRANSP CØST FØR TRUCKS
  510 CTR(I,J,L)=0.
  512 CTKS=AAA+BBB*DTK(I,J,L)
      IF(DTK(I,J,L)) 514,514,516
  514 CTKS =0.
  516 DTKS=DTK(I,J,L)
      IF (DTKS - 200.)  1, 1,8
    8 CTK(I,J,L) = 1.2*CTKS
      GØ TØ 14
    1 CTK(I,J,L) = 0.9*CTKS
      GØ TØ 18
   14 IF (DTKS - 500.) 18,18,3
    3 CTK(I,J,L) = 1.5*CTKS
   18 CØNTINUE
C ADDS TØLL CØST TØ TRUCK TRANSP CØST
      IF(CTK(I,J,L)) 40,40,42
   42 IF (I-1) 46,44,46
   44 CTK(I,J,L) = TØLL1+CTK(I,J,L)
      GØ TØ 40
   46 IF (I-3) 40,48,40
   48 IF (J-2) 40, 7,40
    7 CTK(I,J,L) = TØLL3+CTK(I,J,L)
C ADDS TØLL CØST TØ RAIL TRANSP CØST
   40 IF(CTR(I,J,L)) 27,27,9
    9 IF (I-1) 22,20,22
   20 CTR(I,J,L) = TØLL2+CTR(I,J,L)
```

```
      GØ TØ 27
   22 IF(I-3) 27,23,27
   23 IF(J-2)27,25,27
   25 CTR(I,J,L) = TØLL4+CTR(I,J,L)
C SETS BARGE AND SHIP TRANSP CØST ARBITRARILY HIGH
   27 CTB(I,J,L) = 9.*DTB(I,J,L)
      CTS(I,J,L) = 9.*DTS(I,J,L)
C CALCULATES TRANSP CØST FØR EACH ØF THE ALTERNATIVE RØUTES
C AND CHØØSES LØWEST CØST RØUTE
      TRACØ(I,J,L) = CTR(I,J,L)+CTR(I,J,L)+CTB(I,J,L)+CTS(I,J,L)
      IF (L-1) 28,28,21
   28 TRMIN = 99999.
   21 IF (TRACØ(I,J,L) - TRMIN) 24,24,26
   24 TRMIN = TRACØ(I,J,L)
      LS=L
      BTRACØ(I,J) = TRMIN
   26 CØNTINUE
      IF (L - NUM(I,J)) 30,29,29
C ØUTPUTS TRANSPØRTATIØN CØST ØVER LØWEST CØST RØUTE
   29 WRITE ØUTPUT TAPE ITAB, 31, I,J,BTRACØ(I,J),LS
   31 FØRMAT (6X,2I6,4X,F8.2,4X,I4)
   30 CØNTINUE
C     TRANSPØRTATIØN CØST BETWEEN PLANTS
      WRITE ØUTPUT TAPE ITAB, 115
115   FØRMAT(10X,10HTRANS CØST,3X,10HFRØM PLANT,3X,8HTØ PLANT)
      DØ 102 I=1,NP
      DØ 102 J=1,NP
      PTRACØ(I,J)=0.
      IF(I-J) 101,102,101
101   PTRACØ(I,J)= .7 * (AAA+BBB*DBPR(I,J))
      WRITE ØUTPUT TAPE ITAB , 116 , PTRACØ(I,J) ,I,J
116   FØRMAT (12X,F8.2,  7X,I6,5X,I6)
102   CØNTINUE
      IF (M1) 87,87,34
C ØUTPUTS DISTANCE DATA
   87 WRITE ØUTPUT TAPE ITAB, 90
```

```
   90 FØRMAT ( 9X,36HDISTANCES BETWEEN PLANTS AND MARKETS)
      WRITE ØUTPUT TAPE ITAB,  2
   32 FØRMAT (54HPLANT    MARKE     RØUTE     TRUCK    TRAIN    BARGE    SHIP)
      WRITE ØUTPUT TAPE ITAB,  2
   92 FØRMAT (28X,28HKM       KM  (NAUTICAL MILES))
      DØ 80 I=1,NP
      DØ 80 J=1,NM
      NUMD = NUM(I,J)
      DØ 80 L=1,NUMD
      WRITE ØUTPUT TAPE ITAB, 93, I,J,L,DTK(I,J,L),DTR(I,J,L),DTB(I,J,L)
   93 FØRMAT (I3,2I8,3X,4F8.0)
   80 CØNTINUE
      WRITE ØUTPUT TAPE ITAB,119
  119 FØRMAT (9X,24HDISTANCES BETWEEN PLANTS)
      WRITE ØUTPUT TAPE ITAB,120
  120 FØRMAT(9X,10HFRØM PLANT,3X,8HTØ PLANT,3X,13HDISTANCE (KM))
      DØ 121 I=1,NP
      DØ 121 J=1,NP
      IF(I-J)122,121,122
  122 WRITE ØUTPUT TAPE ITAB,123,I,J,DBPR(I,J)
  123 FØRMAT(13X,I6,7X,I6,10X,F13.2)
  121 CØNTINUE
   34 CØNTINUE
      CALL PCØSTM
      CALL LINERM
  253 CØNTINUE
      CALL EXIT
      END
```

```
      SUBRØUTINE PCØSTM
C     PRØDUCTIØN CØST RØUTINE
      DIMENSIØN S( 3,2,40),C( 3,40),CØST( 3,7)
      DIMENSIØN PERNC( 3),PERIC( 3)
      DIMENSIØN SUBCØ( 3,2,40)
      DIMENSIØN BTRACØ( 3, 3)
      DIMENSIØN SRØW(36)
      DIMENSIØN UNITS (36), BAT(34,100)
      DIMENSIØN PTRACØ (3,3)
      CØMMØN MI, BTRACØ, M2,CØST,NP,NM,M3,PTRACØ,C,BAT,M4
      CØMMØN IBASE,ITAPE,ITAB,ITAK
      READ INPUT TAPE ITAPE, 10,     NPRØC,NI
   10 FØRMAT (3I4)
      READ INPUT TAPE ITAPE, 11, (PERNC(I),PERIC(I),I=1,NP)
   11 FØRMAT(F6.2,2X,F6.2)
      READ INPUT TAPE ITAPE, 15,(SRØW(I),I=1,NI)
   15 FØRMAT (10(A6,1X))
      READ INPUT TAPE ITAPE, 15, (UNITS(I),I=1,NI)
      DØ 12 I=1,NP
      DØ 12 K=1,NI
      READ INPUT TAPE ITAPE, 16,(S(I,M,K),M=1,NPRØC)
   16 FØRMAT (F7.3,1X,F7.3)
   12 CØNTINUE
      READ INPUT TAPE ITAPE, 19,((C(I,K),I=1,NP),K=1,NI)
   19 FØRMAT (F6.2,2X,F6.2,2X,F6.2)
C     CALCULATIØN ØF CØST ØF INPUTS
      DØ 38 I=1,NP
C     ** CØKE INPUTS IN PARTICULAR
      EQPNC=1.392*C(I,10)
      PCØIN=PERNC(I)*EQPNC+PERIC(I)*C(I,11)
      C(I,12)=PCØIN+ 3.
C     ** ALL ØTHER INPUT CØST
      DØ 38 M=1,NPRØC
      SUM=0.
      DØ 36 K=1,NI
      SUBCØ(I,M,K)=S(I,M,K)*C(I,K)
```

```
   36 SUM=SUM+SUBCØ(I,M,K)
      CØST(I,M)=SUM
      IF (M-1) 38,39,38
C     ** MØLTEN AND INGØT PIG IRØN CØST
   39 C(I,7)=CØST(I,M)
      C(I,8)=.9*CØST(I,M)
   38 CØNTINUE
      CALL NEWPC
      IF (M2) 51,51,99
C     ØUTPUTS SPECIFIC INPUT DATA
   51 WRITE ØUTPUT TAPE ITAB, 110
  110 FØRMAT (1H1)
      WRITE ØUTPUT TAPE ITAB, 89
   89 FØRMAT (22X,26HSPECIFIC INPUTS - PIG IRØN)
      WRITE ØUTPUT TAPE ITAB, 52
      WRITE ØUTPUT TAPE ITAB, 64
      WRITE ØUTPUT TAPE ITAB, 66,(SRØW(K),UNITS(K),(S(I,1,K),I=1,NP),K=1
     1,NI)
   66 FØRMAT (1X,A6,2X,A6,1X,F8.3,5X,F8.3,6X,F8.3)
      WRITE ØUTPUT TAPE ITAB, 110
      WRITE ØUTPUT TAPE ITAB, 91
   91 FØRMAT (20X,29HSPECIFIC INPUTS - INGØT STEEL)
      WRITE ØUTPUT TAPE ITAB, 52
      WRITE ØUTPUT TAPE ITAB, 64
      WRITE ØUTPUT TAPE ITAB, 66,(SRØW(K),UNITS(K),(S(I,2,K),I=1,NP),K=1
     1,NI)
      WRITE ØUTPUT TAPE ITAB, 110
C     ØUTPUTS CØST ØF INPUTS DATA
      WRITE ØUTPUT TAPE ITAB, 50
   60 FØRMAT (25X,19HUNIT CØST ØF INPUTS)
      WRITE ØUTPUT TAPE ITAB, 62
   62 FØRMAT (27X,16HDØLLARS PER UNIT)
      WRITE ØUTPUT TAPE ITAB, 54
   64 FØRMAT (1X,5HINPUT,13X,34HCØSIPA      USIMINAS            CSN   )
      WRITE ØUTPUT TAPE ITAB, 68,(SRØW(K),UNITS(K),(C(I,K),I=1,NP),K=1,N
     1I)
```

```
   68 FØRMAT (1X,A6,2X,A6,1X,F8.2,5X,F8.2,6X,F8.2)                    00730
      WRITE ØUTPUT TAPE ITAB, 110                                     00740
C     ØUTPUTS CØST BREAKDØWN                                          00750
      WRITE ØUTPUT TAPE ITAB, 81                                      00760
   81 FØRMAT (14X,44HCØST BREAKDØWN - DØLLARS PER TØN ØF PIG IRØN)    00770
      WRITE ØUTPUT TAPE ITAB, 64                                      00780
      WRITE ØUTPUT TAPE ITAB, 69,(SRØW(K),(SUBCØ(I,1,K),I=1,NP),K=1,NI) 00790
   69 FØRMAT (1X,A6,9X,F8.2,5X,F8.2,6X,F8.2)                          00800
      WRITE ØUTPUT TAPE ITAB, 77,(CØST(I,1),I=1,NP)                   00810
   77 FØRMAT (1X,5HTØTAL,10X,F8.2,5X,F8.2,6X,F8.2)                    00820
      WRITE ØUTPUT TAPE ITAB, 110                                     00830
      WRITE ØUTPUT TAPE ITAB, 83                                      00840
   83 FØRMAT (12X,47HCØST BREAKDØWN - DØLLARS PER TØN ØF INGØT STEEL) 00850
      WRITE ØUTPUT TAPE ITAB, 64                                      00860
      WRITE ØUTPUT TAPE ITAB, 69,(SRØW(K),(SUBCØ(I,2,K),I=1,NP),K=1,NI) 00870
      WRITE ØUTPUT TAPE ITAB, 77,(CØST(I,2),I=1,NP)                   00880
   99 WRITE ØUTPUT TAPE ITAB, 110                                     00890
  255 CØNTINUE                                                        00900
      RETURN                                                          00910
      END                                                             00920
```

```
      SUBRØUTINE NEWPC
C     CALCULATIØN ØF SLAB,PLATE,HØT,CØLD,AND TIN CØST
      DIMENSIØN C(3,40), SCRCR (3,15)
      DIMENSIØN CØST (3,7),PRCØS(3,15)
      DIMENSIØN SPCØN (15),CØNCØ(15),BAT(34,100)
      DIMENSIØN BTRACØ(3,3),PTRACØ(3,3)
      CØMMØN M1,BTRACØ,M2,CØST,NP,NM,M3,PTRACØ,C,BAT,M4
      CØMMØN IBASE,ITAPE,ITAB,ITAK
      READ INPUT TAPE ITAPE,602,(SPCØN(K),CØNCØ(K),K=3,15)
  602 FØRMAT (2F7.3)
      DØ 600 I=1,NP
      DØ 610 K=3,15
      IF (SPCØN(K)-1.) 605,605,608
  608 SCRCR (I,K)=C(I,6)*(SPCØN(K)-1.)
      GØ TØ 610
  605 SCRCR(I,K)=0.
  610 CØNTINUE
      PRCØS(I,3)=SPCØN(3)*CØST(I,2)-SCRCR(I,3)+CØNCØ(3)
      DØ 604 K=4,15
  604 PRCØS (I,K)=SPCØN(K)*PRCØS(I,K-1)-SCRCR(I,K)+CØNCØ(K)
      CØST (I,3)=PRCØS(I,3)
      CØST (I,4)=PRCØS(I,6)
      CØST (I,5)=PRCØS(I,7)
      CØST (I,6)=PRCØS(I,9)
      CØST (I,7)=PRCØS(I,15)
  600 CØNTINUE
      WRITE ØUTPUT TAPE ITAB,611
  611 FØRMAT (28X,15HPRØDUCTIØN CØST)
      WRITE ØUTPUT TAPE ITAB,613
  613 FØRMAT (27X,17H(DØLLARS PER TØN))
      WRITE ØUTPUT TAPE ITAB, 615
  615 FØRMAT (80H   PLANT    PIG IRØN    INGØTS      SLABS      PLATE  HØT S
     ITRIP CØLD STRIP TIN PLATE)
      WRITE ØUTPUT TAPE ITAB, 617, (I,(CØST(I,K),K=1,7),I=1,NP)
  617 FØRMAT (I6,3X,7F10.2)
      RETURN
```

END 00370

```
      SUBROUTINE LINERM
C     PREPARATION OF DATA FOR  INFAR PROGRAM
      DIMENSION RS1(99)
      DIMENSION COFF (3,7,7)
      DIMENSION ROW (33),COL(96)
      DIMENSION BTRACO (3,3)
      DIMENSION COST (3,7)
      DIMENSION PTRACO (3,3)
      DIMENSION BAT(34,100),CAT(1,300)
      DIMENSION C (3,40), EXTR(3), EMPR(7)
      DIMENSION NI(7),NLIFE(100),BA(140)
      DIMENSION ICOL(100),F(100,50),NGES(30,7),ADCAP(100)
      DIMENSION NIT(100),NIP(100),NID(100)
      COMMON M1, BTRACO,M2,COST,NP,MM,M3,PTRACO,C,BAT,M4
      COMMON IBASE,ITAPE,ITAP,ITAK
      READ INPUT TAPE ITAPE, 106,NP,NPP,NC,NT
  106 FORMAT (4I4)
      NPP=(NT-1)*NP
      NCC=(NT-1)*NC
      READ INPUT TAPE ITAPE, 109,(COL(I),I=1,NC)
  109 FORMAT (8(1X,A5,1X))
      READ INPUT TAPE ITAPE, 107, (ROW(I),I=1,NP)
  107 FORMAT (10(1X,A3,1X))
      READ INPUT TAPE ITAPE,111, FOBJ
  111 FORMAT (A6)
      NZ=NP+1
      DO 157 I=1,NZ
      DO 157 J=1,NC
  157 BAT (I,J)=0.
C     I=PLANT INDEX, M=PRODUCTION UNIT INDEX, K=PRODUCT INDEX
      DO 800 I=1,NP
      DO 800 M=1,7
      READ INPUT TAPE ITAPE , 130, (COFF (I,M,K),K=1,7)
  800 CONTINUE
      KW=3*NP*(NP-1)+3*NP+1
C     CALCULATES DELIVERED COST OF FINAL PRODUCTS
```

```
      DØ 133 I=1,NP
      DØ 133 J=1,NM
      DØ 133 K=4,7
      BAT (I,KW)=PTRACØ(I,J)+CØST (I,K)
      KW=KW+1
  133 CØNTINUE
C     CALCULATES CØST TØ THE SYSTEM ØF INTERMEDIATE PRØDUCT SHIPMENTS
      K=1
      DØ 146 I=1,NP
      DØ 144 J=1,NP
      IF (I-J) 140,144,140
  140 DØ 142 KJ=1,3
      BAT(I,K)=CØST(I,KJ)-CØST(J,KJ)+PTRACØ(I,J)
      K=K+1
  142 CØNTINUE
  144 CØNTINUE
  146 CØNTINUE
C     READS IN IMPØRT CØST ØF INTERMEDIATE PRØDUCTS
      LG=NP*(NP-1)*3+1
      LH=NP*3-1+LG
      READ INPUT TAPE ITAPE, 130,(BAT(1,J),J=LG,LH)
  130 FØRMAT (9F8.2)
      J=NP*(NP-1)*3+1
      DØ 145 I=1,NP
      DØ 147 K=1,3
      BAT(1,J)=BAT(1,J)-CØST(I,K)
      J=J+1
  147 CØNTINUE
  145 CØNTINUE
C     READS IN IMPØRT CØST ØF FINAL PRØDUCTS
      LR=LH+NP*NM*4+1
      LT=LR+NM*4-1
      READ INPUT TAPE ITAPE,130,(BAT(1,J),J=LR,LT)
C CALCULATIØN ØF EXPØRT PRØFITS
      READ INPUT TAPE ITAPE,130,(EXTR(I),I=1,NP)
      READ INPUT TAPE ITAPE,130,(EXPR(K),K=1,7)
```

```
      LQ=NP*(NP-1)*3+NP*3+NP*NM*4+NM*4+1
      DØ 135 I=1,NP
      DØ 137 K=1,7
      BAT(I,LQ)=CØST(I,K)+FXTR(I)-FMPR(K
137   LQ=LQ+1
135   CØNTINUE
C SETS UP MATRIX CØEFFICIENTS IN INTERMEDIATE PRØDUCT SECTIØN
C ** THE POSITIVE CØEFFICIENTS
      MP=1
      DØ 159 I=1,NP
      MY=(I-1)*7+2
      DØ 151 J=1,NP
      IF (I-J)158,151,158
158   DØ 155 K=1,3
      MZ=MY
      DØ 153 M=1,3
      BAT(MZ,MP)=CØEF(I,M,K)
      MZ=MZ+1
153   CØNTINUE
      MP=MP+1
155   CØNTINUE
151   CØNTINUE
159   CØNTINUE
C ** THE NEGATIVE CØEFFICIENTS
      KW=1
      DØ 170 J=1,NP
      DØ 171 I=1,NP
      MQ=7*(I-1)+2
      IF (I-J)173,171,173
173   DØ 176 K=1,3
      MN=MQ
      DØ 178 M=1,3
      BAT(MN,KW)=-CØEF(I,M,K)
      MN=MN+1
178   CØNTINUE
      KW=KW+1
```

```
176   CØNTINUE
171   CØNTINUE
170   CØNTINUE
C     MATRIX CØEFFICIENTS FØR IMPØRTS ØF INTERMEDIATE PRØDUCT
      KR=NP*(NP-1)*3+1
      DØ 160 I=1,NP
      MY=(I-1)*7+2
      DØ 162 K=1,3
      KM=MY
      DØ 164 M=1,3
      BAT(KM,KR)=-CØEF(I,M,K)
      KM=KM+1
  164 CØNTINUE
      KR=KR+1
  162 CØNTINUE
  160 CØNTINUE
C SETS UP MATRIX CØEFFICIENTS FØR FINAL PRØDUCTIØN SECTIØN
C ** CAPACITY CØEFFICIENTS
      KR=NP*(NP-1)*3+NP*3+1
      DØ 182 I=1,NP
      MY=(I-1)*7+2
      DØ 182 L=1,NM
      LN=4
      DØ 182 K=4,7
      KM=MY
      DØ 180 M=1,LN
      BAT(KM,KR)=CØEF(I,M,K)
      KM=KM+1
180   CØNTINUE
      LN=LN+1
      KR=KR+1
182   CØNTINUE
C **REQUIREMENT CØEFFICIENTS
      KR=NP*(NP-1)*3+NP*3+1
      MX=7*NP+2
      MF=MX+4*NM-1
```

```
      DØ 190 I=1,NP
      DØ 190 J=MX,ME
      BAT(J,KR)=-1.
      KR=KR+1
190   CØNTINUE
C MATRIX CØEFFICIENTS FØR IMPØRTS ØF FINAL PRØDUCTS
      DØ 196 J=MX,ME
      BAT(J,KR)=-1.
      KR=KR+1
196   CØNTINUE
C MATRIX CØEFFICIENTS FØR ALL EXPØRTS
      DØ 226 I=1,NP
      LN=1
      MY=(I-1)*7+2
      DØ 224 K=1,7
      KM=MY
      DØ 222 M=1,LN
      BAT(KM,KR)=CØEF(I,M,K)
      KM=KM+1
222   CØNTINUE
      KR=KR+1
      LN=LN+1
224   CØNTINUE
226   CØNTINUE
C     READS IN DATA FØR INVESTMENT ACTIVITIES
      READ INPUT TAPE ITAPE, 230, R, NT
  230 FØRMAT (F6.2,I4)
      READ INPUT TAPE ITAPE, 232, NIV, (NI(K),K=2,NT)
  232 FØRMAT (15I4)
      KW=1
      DØ 238 K=2,NT
      NX=NI(K)
      DØ 238 L=1,NX
      READ INPUT TAPE ITAPE, 240,ICØL(KW),F(I,KW),NGES(L,K),ADCAP(KW),NI
     IT(KW),NIP(KW),NIU(KW),NLIFE(KW)
      KW=KW+1
```

```
  238 CØNTINUE
 240  FØRMAT(1X,A5,F6.2,I6,F6.2,4I6)
C     MATRIX CØEFFICIENTS FØR INVESTMENT ACTIVITIES
      DØ 108 KW=1,NIV
      NQ=NIT(KW)
      DØ 108 L=NQ,NT
      NIR=NIU(KW)+(NIP(KW)-1)*7+(L-2)*NR+1
      F(NIR,KW)=-ADCAP(KW)
  108 CØNTINUE
C READS IN RIGHT HAND SIDE VALUES
      READ INPUT TAPE ITAPE,228,(RS1(I),I=1,NRR)
228   FØRMAT(11F6.3)
      CALL DISFAC (R,NT,NI,F,NGES,CAT,NC,NLIFE)
C     READS IN BASIS
      READ INPUT TAPE ITAPE, 244, NEB
  244 FØRMAT (I4)
      READ INPUT TAPE ITAPE, 248, (BA(I),I=1,NEB)
  248 FØRMAT (6X,11A6)
      CALL LPØUT (CØL,RØW,RS1,FØBJ,F,NIV,NR,NC,NT,NRR,NCC,CAT,ICØL,BA,NE
     1B,NRP)
  220 RETURN
      END
```

```
      SUBRØUTINE DISFAC (R,NT,NI,F,NGES,CAT,NC,NLIFE)
C     CALCULATIØN ØF AND APPLICATIØN ØF DISCØUNT FACTØRS
      DIMENSIØN BTRACØ(3,3),PTRACØ(3,3),CØST(3,7),C(3,40)
      DIMENSIØN BAT (34, 100)
      DIMENSIØN NI(7),NGES(30,7)
      DIMENSIØN DISFA(7),DF(30,7)
      DIMENSIØN F(100,50),NLIFE(100)
      DIMENSIØN CAT(1,300)
      CØMMØN M1,BTRACØ,M2,CØST,NP,NM,M3,PTRACØ,C,BAT,M4
      CØMMØN IBASE,ITAPE,ITAB,ITAK
      NQ=NT*5
C     DISCØUNT FACTØR FØR PRØDUCTIØN AND TRANSPØRTATIØN CØST
      KW=1
      DØ 710 K=2,NT
      LT=(K-1)*5+1
      LE=LT+4
      CATCH=0.
      DØ 714 J=LT,LE
  714 CATCH=CATCH+1./(1.+R)**J
      DISFA(K)=CATCH
C     DISCØUNT FACTØRS FØR INVESTMENT
      NX=NI(K)
      DØ 720 I=1,NX
      M=(K-1)*5-NGES(I,K)
      NZ=NLIFE(KW)
      NW=1
      SUM=0.
      DØ 719 J=M,NQ
      SUM=SUM+(1./(1.+R)**J)
      IF (NW-NZ)713,717,717
  713 NW=NW+1
  719 CØNTINUE
  717 DF(I,K)=SUM
      KW=KW+1
  720 CØNTINUE
  710 CØNTINUE
```

```
      WRITE ØUTPUT TAPE ITAB,716,(DISFA(K),K=2,NT)
  716 FØRMAT (1X,5HDISFA, 7F8.4)
      DØ 718 K=2,NT
      NX=NI(K)
  718 WRITE ØUTPUT TAPE ITAB,722,(DF(I,K),I=1,NX)
  722 FØRMAT (1X,2HDF,10F8.3)
C     DISCØUNT ØF ØBJECTIVE FUNCTIØN VALUES
      DØ 730 K=2,NT
      DØ 734 J=1,NC
      ML=J+NC*(K-2)
  734 CAT(1,ML)=DISFA(K)*(BAT(1,J)/2.)
  730 CØNTINUE
      LM=1
      DØ 740 K=2,NT
      NX=NI(K)
      DØ 744 I=1,NX
      NZ=NLIFE(LM)
      F(I,LM)=DF(I,K)*F(I,LM)*(R*(1.+R)**NZ)/((1.+R)**NZ-1.)
  744 LM=LM+1
  740 CØNTINUE
      RETURN
      END
```

UNIFORM-COURIER 6-1105 STOCK FORM

```
      SUBRØUTINE LPØUT (CØL,RØW,RS1,FØBJ,F,NIV,NR,NC,NT,NRR,NCC,CAT,ICØL
     1,BA,NEB,NRP)
      DIMENSIØN BTRACØ(3,3),CØST(3,7),PTRACØ(3,3),C(3,40),BAT(34,100)
      DIMENSIØN CØL(96),RØW(33),RS1(99)
      DIMENSIØN ICØL(100),F(100,50)
      DIMENSIØN CAT(1,300),ICØL(100)
      DIMENSIØN BA(140) ,FPC(100)
      CØMMØN M1,BTRACØ,M2,CØST,NP,NM,M3,PTRACØ,C,BAT,M4
      CØMMØN IBASE,ITAPE,ITAB,ITAK
      UNITY=1.
      UNITM=-1.
      IØUT=0
      IF(M4)4,4,2
C     ØUTPUT ØF LP INPUT DECK FØR MFØR
    4 IF(M3)610,601,602
  610 WRITE ØUTPUT TAPE ITAK,10
   10 FØRMAT(5HBEGIN/6HEXTRA3/11HRØW    ICØST/3HRHS)
      KR=1
      DØ 15 N=2,NT
      DØ 15 I=1,NR
      WRITE ØUTPUT TAPE ITAK,14,N,RØW(I),RS1(KR)
      KR=KR+1
   15 CØNTINUE
   14 FØRMAT(12X,I1,A5,2X,F10.3)
      GØ TØ 602
  601 WRITE ØUTPUT TAPE ITAK,40
   40 FØRMAT(5HBEGIN/6HEXTRA3/11HRØW    ICØST/4HTAPE/2H07)
      WRITE ØUTPUT TAPE 7,902
  902 FØRMAT(3HRHS)
      KR=1
      DØ 901 N=2,NT
      DØ 901 I=1,NR
      WRITE ØUTPUT TAPE 7,14,N,RØW(I),RS1(KR)
      KR=KR+1
  901 CØNTINUE
      WRITE ØUTPUT TAPE 7 , 903
```

```
903   FØRMAT(3HEØF)
      WRITE ØUTPUT TAPE ITAK,904
904   FØRMAT(6HMATRIX)
      GØ TØ 363
C     INSERT F' RHS RESTRAINTS
620   CØNTINUE
602   WRITE ØUTPUT TAPE ITAK,18
18    FØRMAT(3HEND/6HMATRIX)
      GØ TØ 363
24    IF(M3)32,32,28
32    WRITE ØUTPUT TAPE ITAK,384,((N,RØW(I),N,RØW(I),UNITY,I=1,NR),
     1 N=2,NT)
      IF(M3)603,604,604
604   WRITE ØUTPUT TAPE ITAK , 42 , (ICØL(I),ICØL(I),UNITY,
     1 ICØL(I),ICØL(I),UNITY, I=1,KNIV)
42    FØRMAT(6X,1H0,A5,1H0,A5,F7.2/6X,1H1,A5,1H1,A5,F7.2)
      GØ TØ 630
C     INSERT F' DUMMY SLACKS
603   CØNTINUE
630   PRINT 632
632   FØRMAT(15HBASIS TAPE (I2))
      READ 633,MK5
633   FØRMAT(I2)
      IF(MK5)639,37,639
639   IF(MK5-2)636,631,636
636   WRITE ØUTPUT TAPE ITAK,637,MK5
637   FØRMAT(3HEND/4HTAPE/I2/6HINVERT/5HCRASH/2HGØ)
      GØ TØ 640
631   WRITE ØUTPUT TAPE ITAK, 44
44    FØRMAT (3HEND/5HBASIS)
      WRITE ØUTPUT TAPE ITAK, 48,(BA(I),I=1,NEB)
48    FØRMAT (6X,11A6)
      WRITE ØUTPUT TAPE ITAK, 52
      GØ TØ 640
52    FØRMAT (3HEND/6HARTRØW/3HEND/6HINVERT/5HCRASH/2HGØ)
37    WRITE ØUTPUT TAPE ITAK,36
```

```
   36 FØRMAT(3HEND/5HSØLVE)                                          00730
  640 WRITE ØUTPUT TAPE ITAK,913                                     00740
  913 FØRMAT(4HTAPE/2H09)                                            00750
      END FILE ITAK                                                  00760
      PRINT 550,ITAK                                                 00770
  550 FØRMAT(8HEND FILE,I3)                                          00780
      IF (M4) 2,420,420                                              00790
C     ØUTPUT ØF LP INPUT DECK FØR LP90                               00800
    2 ITAK=ITAK+10                                                   00810
      IØUT=1                                                         00820
      WRITE ØUTPUT TAPE ITAK, 350                                    00830
  350 FØRMAT(6HRØW ID)                                               00840
      WRITE ØUTPUT TAPE ITAK,352,FØBJ                                00850
  352 FØRMAT(12X,A6)                                                 00860
      DØ 354 N=2,NT                                                  00870
      WRITE ØUTPUT TAPE ITAK,356,(N,RØW(I),I=1,NR)                   00880
  356 FØRMAT(11X,1H+,I1,A5)                                          00890
  354 CØNTINUE                                                       00900
      WRITE ØUTPUT TAPE ITAK, 353,(ICØL(I),I=1,NIV)                  00910
  353 FØRMAT (11X,1H+,A5,1HR)                                        00920
      WRITE ØUTPUT TAPE ITAK, 355                                    00930
  355 FØRMAT (29HINTEGER VARIABLES USING RHS 1)                      00940
      WRITE ØUTPUT TAPE ITAK, 357,(ICØL(I),I=1,NIV)                  00950
  357 FØRMAT (7X,A5,2H,1)                                            00960
      WRITE ØUTPUT TAPE ITAK, 362                                    00970
  362 FØRMAT(6HMATRIX)                                               00980
  363 NZ=NR+1                                                        00990
      KR=1                                                           01000
      DØ 364 N=2,NT                                                  01010
      DØ 366 J=1,NC                                                  01020
      IF(CAT(1,KR))369,367,367                                       01030
  367 WRITE ØUTPUT TAPE ITAK, 368,N,CØL(J),FØBJ,CAT(1,KR)            01040
      GØ TØ 371                                                      01050
  369 IF(IØUT)510,367,510                                            01060
  510 X=-CAT(1,KR)                                                   01070
      WRITE ØUTPUT TAPE ITAK, 373,N,CØL(J),FØBJ,X                    01080
```

```
373 FØRMAT (6X,I1,A5,A6,1H-,F6.2)
371 KR=KR+1
368 FØRMAT(6X,I1,/ ,A6,F7.2)
    IF(M3)372,372, 66
372 DØ 378 I=2,NZ
    II=I-1
    IF(BAT(I,J))375,378,376
376 WRITE ØUTPUT TAPE ITAK, 384,N,CØL(J),N,RØW(II),BAT(I,J)
    GØ TØ 378
384 FØRMAT (6X,I1,A5,I1,A5,F7.2)
375 IF(IØUT)512,376,512
512 X=-BAT(I,J)
    WRITE ØUTPUT TAPE ITAK, 385, N,CØL(J),N,RØW(II),X
385 FØRMAT (6X,I1,A5,I1,A5,1H-,F6.2)
378 CØNTINUE
366 CØNTINUE
364 CØNTINUE
    NTFC=NRP*(NT-1)
C   READ F' ACTIVITY CØSTS
    READ INPUT TAPE ITAPE,715,(FPC(I),I=1,NTFC)
715 FØRMAT(10F7.2)
    IF(M3)701,702,702
701 KC3=1
    DØ 800 K1=2,NT
    DØ 800 K2=1,NRP
C   WRITE F' ACTIVITY CØSTS
    WRITE ØUTPUT TAPE ITAK,720,K1,RØW(K2),FØBJ,FPC(KC3)
720 FØRMAT(7X,I1,1HI,A3,A6,1X,F6.2)
    KC3=KC3+1
C   INSERT F' MATRIX ENTRIES
    DØ 800 K3=K1,NT
800 WRITE ØUTPUT TAPE ITAK,801,K1,RØW(K2),K3,RØW(K2),UNITM
801 FØRMAT(7X,I1,1HI,A3,I1,A3,3X,F5.2)
    GØ TØ 721
702 PRINT 820
820 FØRMAT(25HNUMBER ØF INVESTMENTS(I2))
```

```
      READ 633,KNIV
      DØ 392 KW=1,KNIV
      IF(IØUT)950,951,950
950   WRITE ØUTPUT TAPE ITAK, 394, ICØL(KW),FØBJ,F(1,KW)
 394  FØRMAT (7X,A5,A6,1X,F6.2)
      GØ TØ 952
951   WRITE ØUTPUT TAPE ITAK,953,ICØL(KW),FØBJ,F(1,KW)
953   FØRMAT(6X,A5,1X,A6,1X,F6.2)
952   IF (M3) 397,397,392
 397  KX=2
      DØ 399 N=2,NT
      DØ 399 L=1,NR
      IF(F(KX,KW))413,416,414
414   IF(IØUT)960,961,960
960   WRITE ØUTPUT TAPE ITAK,422,ICØL(KW),N,RØW(L),F(KX,KW)
      GØ TØ 416
961   WRITE ØUTPUT TAPE ITAK,962,ICØL(KW),N,RØW(L),F(KX,KW)
962   FØRMAT(6X,A5,1X,I1,A5,1X,F6.2)
      GØ TØ 416
 422  FØRMAT (7X,A5,I1,A5,1X,F6.2)
 413  IF(IØUT)514,961,514
 514  X=-F(KX,KW)
      WRITE ØUTPUT TAPE ITAK, 423, ICØL(KW),N,RØW(L),X
 423  FØRMAT (7X,A5,I1,A5,1H-,F6.2)
 416  CØNTINUE
      KX=KX+1
 399  CØNTINUE
      WRITE ØUTPUT TAPE ITAK , 391 , ICØL(KW),ICØL(KW),UNITY,
     1 ICØL(KW),ICØL(KW),UNITM
 391  FØRMAT(6X,A5,1X,1H0,A5,F7.2/6X,A5,1X,1H1,A5,F7.2)
 392  CØNTINUE
 721  IF (IØUT) 395,99,395
  99  IF (M4) 24,24,395
 395  IF (M3) 398,398,396
 398  WRITE ØUTPUT TAPE ITAK,400
 400  FØRMAT (7HFIRST B)
```

```
      KM=1                                                              01810
      DØ 406 N=2,NT                                                     01820
      DØ 406 I=1,NR                                                     01830
      IF(RS1(KM))407,405,405                                            01840
  405 WRITE ØUTPUT TAPE ITAK, 408,N,RØW(I),RS1(KM)                      01850
      GØ TØ 409                                                         01860
  407 X=-RS1(KM)                                                        01870
      WRITE ØUTPUT TAPE ITAK,411,N,RØW(I),X                             01880
  411 FØRMAT(12X,I1,A5,1H-,F6.3)                                        01890
  408 FØRMAT (12X,I1,A5,F7.3)                                           01900
  409 KM=KM+1                                                           01910
  406 CØNTINUE                                                          01920
      WRITE ØUTPUT TAPE ITAK, 425,(ICØL(I),UNITY,I=1,NIV)               01930
  425 FØRMAT (12X,A5,1HR,F6.2)                                          01940
      WRITE ØUTPUT TAPE ITAK,412                                        01950
  412 FØRMAT(3HEØF)                                                     01960
      END FILE ITAK                                                     01970
      PRINT 550, ITAK                                                   01980
   28 CØNTINUE                                                          01990
  396 CØNTINUE                                                          02000
  420 RETURN                                                            02010
      END                                                               02020
```

6-1185 STOCK FORM UNIFORM-COURIER

```
3   3                                                                   00020
2   3   2                                                               00030
3   3   2                                                               00040
2   2   2                                                               00050
1.80        .007                                                        00060
  0  1300   430  1300     0   868   430   868     0                    00070
  .50       .50   .40     .40                                           00080
   85                                                                   00090
             85                                                         00100
    5                     220                                           00110
            582                                                         00120
    5             220                                                   00130
  642                                                                   00140
            837                                                         00150
  771                                                                   00160
            846                                                         00170
   85       451   480                                                  00180
  667                                                                   00190
            640                                                         00200
            451   260                                                   00210
  214                                                                   00220
            214                                                         00230
  353                                                                   00240
            353                                                         00250
  146                                                                   00260
            252                                                         00270
  473                                                                   00280
            473                                                         00290
2  36                                                                   00300
 40       60                                                            00310
 40       60                                                            00320
 44       56                                                            00330
ØRE     ØREF    SINTR   PELLT   SCRAP   RSCRP   PIG     PIGIN   SPØNG   CØALN   00340
CØALM   CØKE    DØLØ    LIMST   QLIME   MANG    ALUM    FLUØR   FMANG   FSILC   00350
FUEL    NAGAS   ØXY     ELEC    ELTRG   ELTRP   WATER   STEAM   REFRA   BFGAS   00360
EFGAS   CKGAS   SLAG    DLAB    ILAB    MAIN                                    00370
```

6-1105 STOCK FORM UNIFORM-COURIER

TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS		00380
TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS	TØNS		00390
TØNS	1000M3	1000M3	1000KH	TØNS	TØNS	1000M3	TØNS	TØNS	1000M3		00400
1000M3	1000M3	TØNS	MANHØR	MANHØR	UNITS						00410
700		040									00420
											00430
760											00440
											00450
		220									00460
											00470
		880									00480
		016									00490
											00500
											00510
											00520
680											00530
											00540
400		080									00550
		080									00560
025											00570
		004									00580
		008									00590
		007									00600
		001									00610
											00620
											00630
		057									00640
070		022									00650
											00660
											00670
065		016									00680
020											00690
		009									00700
-3300											00710
											00720
-138											00730

UNIFORM-COURIER STOCK FORM 6-1185

-430		00740
390	1980	00750
650		00760
2700	4500	00770
470	005	00780
		00790
1100		00800
		00810
	187	00820
	014	00830
	929	00840
		00850
		00860
371		00870
552		00880
		00890
035	006	00900
132	001	00910
	059	00920
040		00930
		00940
	004	00950
		00960
		00970
		00980
		00990
	050	01000
070	012	01010
		01020
		01030
100	029	01040
		01050
	007	01060
-3300		01070
		01080
-138		01090

6-1195 STOCK FORM UNIFORM-COURIER

-476		01100
390	1980	01110
650		01120
2700	4500	01130
782	111	01140
		01150
794		01160
		01170
	276	01180
	039	01190
	628	01200
	097	01210
		01220
461		01230
589		01240
		01250
035		01260
272	042	01270
		01280
018		01290
		01300
		01310
		01320
		01330
050	118	01340
		01350
	004	01360
070		01370
		01380
		01390
015	020	01400
		01410
	043	01420
-3715		01430
		01440
-138		01450

UNIFORM-COURIER STOCK FORM 6-1105

-476			01460
390	167		01470
650	950		01480
2700	6340		01490
569	284	350	01500
	227		01510
1170	554	900	01520
			01530
2310	3220	2400	01540
2500	3500	2600	01550
			01560
			01570
			01580
2560	2990	2700	01590
2100	1835	2200	01600
3330	3510		01610
			01620
461	504	500	01630
600	600	600	01640
4230	3500	3500	01650
			01660
2000	2000	2000	01670
28000	28000	28000	01680
34400	34400	34400	01690
2760	2500	2800	01700
			01710
4000	4000	4000	01720
1670	1400	1600	01730
			01740
			01750
605	500	500	01760
360	360	360	01770
11700	11700	11700	01780
218	150	150	01790
			01800
1260	700	700	01810

```
  42      42      42
  50      50      50
 100     100     100
1.090   2.811
1.000   1.028
1.000   2.557
1.028   2.460
1.013   3.621
1.068   2.390
1.005   5.791
1.017   2.181
1.000   2.134
1.014   2.833
1.071    .772
 .996  24.928
1.000  14.039
33  21  96   4
SAUSG  SAUSS  SAUSL  SAVRG  SAVRS  SAVRL  USSAG  USSAS
USSAL  USVRG  USVRS  USVRL  VRSAG  VRSAS  VRSAL  VRUSG
VRUSS  VRUSL  IMSAG  IMSAS  IMSAL  IMUSG  IMUSS  IMUSL
IMVRG  IMVRS  IMVRL  SASPP  SASPH  SASPC  SASPT  SARJP
SARJH  SARJC  SARJT  SABHP  SABHH  SABHC  SABHT  USSPP
USSPH  USSPC  USSPT  USRJP  USRJH  USRJC  USRJT  USBHP
USBHH  USBHC  USBHT  VRSPP  VRSPH  VRSPC  VRSPT  VRRJP
VRRJH  VRRJC  VRRJT  VRBHP  VRBHH  VRBHC  VRBHT  IMSPP
IMSPH  IMSPC  IMSPT  IMRJP  IMRJH  IMRJC  IMRJT  IMBHP
IMBHH  IMBHC  IMBHT  SAEXG  SAEXS  SAEXL  SAEXP  SAEXH
SAEXC  SAEXT  USEXG  USEXS  USEXL  USEXP  USEXH  USEXC
USEXT  VREXG  VREXS  VREXL  VREXP  VREXH  VREXC  VREXT
SAR  SAS  SAP  SAR  SAH  SAC  SAT  USR  USS  USP
USR  USH  USC  UST  VRB  VRS  VRP  VRR  VRH  VRC
VRT  SPP  SPH  SPC  SPT  RJP  RJH  RJC  RJT  BHP
BHH  BHC  BHT
1CØST
   1.00     .90    1.15    1.15    1.11    1.18    1.26
```

```
                 1.00     1.28     1.28     1.24     1.31     1.40                        02180
                          1.28     1.28     1.24     1.31     1.40                        02190
                                   1.10     1.09     1.15     1.23                        02200
                                            1.05     1.12     1.20                        02210
                                                     1.03     1.11                        02220
                                                              1.05                        02230
        1.00      .93     1.19     1.19     1.15     1.22     1.30                        02240
                 1.00     1.28     1.28     1.24     1.31     1.40                        02250
                          1.28     1.28     1.24     1.31     1.40                        02260
                                   1.10     1.09     1.15     1.23                        02270
                                            1.05     1.12     1.20                        02280
                                                     1.03     1.11                        02290
                                                              1.05                        02300
        1.00      .75      .96      .96      .93      .98     1.05                        02310
                 1.00     1.28     1.28     1.24     1.31     1.40                        02320
                          1.28     1.28     1.24     1.31     1.40                        02330
                                   1.10     1.09     1.15     1.23                        02340
                                            1.05     1.12     1.20                        02350
                                                     1.03     1.11                        02360
                                                              1.05                        02370
       85.50   103.30   108.00    88.50   106.30   111.00    88.50   106.30   111.00     02380
      139.50   134.50   161.50   238.50   137.50   132.50   159.50   236.50   145.50     02390
      140.50   167.50   244.50                                                            02400
        1.00     3.00     3.00                                                            02410
       34.80    35.30    80.60   108.30   115.32   142.32   191.00                        02420
      .05     4                                                                           02430
      23    4    7   12                                                                   02440
    2ISAS   7.90      2     .80      2      1      2     36                               02450
    2IUSS   7.90      2     .80      2      2      2     36                               02460
    2ISAT   8.00      2     .18      2      1      7     40                               02470
    2IVRT   8.00      2     .18      2      3      7     40                               02480
    3ISAB   1.80      1     .28      3      1      1     40                               02490
    3IUSB   2.40      1     .36      3      2      1     40                               02500
    3ISAS   7.90      2     .80      3      1      2     36                               02510
    3IUSS   7.90      2     .80      3      2      2     36                               02520
    3IVRS  34.50      4    1.00      3      3      2     36                               02530
```

```
3ISAT  8.00    2   .18    3    1    7   40                                  02540
3IVRT  8.00    2   .18    3    3    7   40                                  02550
4ISAB 28.00    4   .80    4    1    1   40                                  02560
4IUSB 28.00    4   .80    4    2    1   40                                  02570
4IVRB 28.00    4   .80    4    3    1   40                                  02580
4ISAS 23.60    4   .80    4    1    2   36                                  02590
4IUSS 23.60    4   .80    4    2    2   36                                  02600
4IVRS 34.50    4  1.00    4    3    2   36                                  02610
4ISAR 25.00    3  1.00    4    1    4   40                                  02620
4ISAC 10.00    2   .35    4    1    6   40                                  02630
4IUSC 10.00    2   .35    4    2    6   40                                  02640
4IVRC 10.00    2   .35    4    3    6   40                                  02650
4ISAT  8.00    2   .18    4    1    7   40                                  02660
4IVRT  8.00    2   .18    4    3    7   40                                  02670
 .720  .800 1.800 1.000 1.500  .350  .000  .900  .600 1.800 1.000            02680
1.500  .150  .000 1.000 1.420 1.200 1.250 1.500  .800  .170 -.131            02690
-.347 -.476 -.217 -.218 -.145 -.147 -.062 -.087 -.087 -.111 -.031            02700
 .720  .800 1.800 1.000 1.500  .350  .000  .900  .600 1.800 1.000            02710
1.500  .150  .000 1.000 1.420 1.200 1.250 1.500  .800  .170 -.164            02720
-.433 -.598 -.274 -.274 -.181 -.184 -.078 -.109 -.108 -.138 -.039            02730
 .720  .800 1.800 1.000 1.500  .350  .000  .900  .600 1.800 1.000            02740
1.500  .150  .000 1.000 1.420 1.200 1.250 1.500  .800  .170 -.208            02750
-.548 -.756 -.343 -.347 -.228 -.233 -.098 -.139 -.137 -.174 -.049            02760
99                                                                           02770
     2USSPH 2ISAS2SASPP2SAP  2SAH  2SAEXC2SASPT2USVRG2USS  2USP  2USR        02780
     2USH  2USEXC2USBHT2USBHP2VRSAS2VRRJP2USRJP2VRH  2VRSPC2VRSPT2SAR        02790
     2SASPH3IMRJC 2ISAT2VRP  2USRJH2VRRJC2VRRJT2USEXH2USBHH2USBHC2VRBHT      02800
     4VRRJC3SASPP3VRSAS3SAVRL3SAH  2VRT  3SASPT4VRSAS3USEXL 3IUSS3USBHP      02810
     3USH  3VRRJT3USBHT 3IUSB3VRS  3VRSPP 3ISAB3VRH  3VRSPC3VRSPT3VRRJH      02820
     3SASPH3SASPC3USRJC3VRRJP3USRJH3VRRJC4SASPC3USEXH3USBHH3USBHC3VRBHT      02830
     4VRS  4SAEXC4SASPP 4ISAR2SAS  4SASPT 4ISAT 4ISAC4VRRJH3USSAS4SARJP      02840
     4USH  4USBHC4USBHT3SAEXL4USVRG4USSAL4USVRL4VRH  4VRSPC4VRSPT3SAVRG      02850
     4SASPH3USEXG4VRRJT4VRRJP4USEXC4SARJH 2IVRT4USBHP4USBHH 4IUSC4VRBHT      02860
  1.    2.    3.    4.    5.    6.    7.    8.    9.   10.                  02870
 11.   12.   13.   14.   15.   16.   17.   18.   19.   20.                  02880
 21.   22.   23.   24.   25.   26.   27.   28.   29.   30.                  02890
```

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	02900
41.	42.	43.	44.	45.	46.	47.	48.	49.	50.	02910
51.	52.	53.	54.	55.	56.	57.	58.	59.	60.	02920
61.	62.	63.								02930

```
C       PRØGRAM FØR ALTERING RHS VALUES ØN INTEGER CØNSTRAINT RØWS
        IEØF=23192079408                                          00010
        DIMENSIØN NAME (100), VAL1(100),VAL2(100)                 00020
        DIMENSIØN CØPY(100,5)                                     00030
        NIV=-1                                                    00040
        DØ 600 J=1,100                                            00050
  600   READ INPUT TAPE 7,601,(CØPY(J,I),I=1,5)                   00060
  601   FØRMAT(5A6)                                               00070
  401   READ INPUT TAPE 7,403,IF,NAME(NIV+2),VAL1(NIV+2)          00080
  403   FØRMAT(A6,7X,A5,F12.1)                                    00090
        NIV=NIV+1                                                 00100
        IF(IF-IEØF)500,420,500                                    00110
  500   READ INPUT TAPE 7,405,VAL2(NIV+1)                         00120
  405   FØRMAT(18X,F12.1)                                         00130
        GØ TØ 401                                                 00140
  420   PRINT 5,NIV                                               00150
    5   FØRMAT(37HNUMBER ØF INVESTMENTS ØN INPUT TAPE =,I3)       00160
        REWIND 7                                                  00170
        PRINT 701                                                 00180
  701   FØRMAT(33HENTER 7 FØR MAIN RUN,9 FØR ALTERB)              00190
        READ 11,ITAP                                              00200
        IF(ITAP-7)99,702,99                                       00210
  702   CALL DELETE (12H.TAPE.      7)                            00220
   99   PRINT 9                                                   00230
    9   FØRMAT(42H1 TØ ADD, 2 TØ ALTER, 3 TØ LØØK, 4 TØ DRØP)     00240
        READ 11,J                                                 00250
   11   FØRMAT(I1)                                                00260
        IF(J-2)10,20,20                                           00270
   10   PRINT 13                                                  00280
   13   FØRMAT(4HNAME)                                            00290
        NIV=NIV+1                                                 00300
        READ 15,NAME(NIV)                                         00310
        NI=NIV                                                    00320
        GØ TØ 102                                                 00330
   15   FØRMAT(A5)                                                00340
   36   PRINT 17                                                  00350
```

```
   17 FØRMAT(31HVALUE (0,1, ØR 2 FØR CØNTINØUS))
      READ11,ISW
      IF(ISW-1)100,101,102
  100 VAL1(NI)=0.
      VAL2(NI)=0.
      GØ TØ 99
  101 VAL1(NI)=1.
      VAL2(NI)=-1.
      GØ TØ 99
  102 VAL1(NI)=1.
      VAL2(NI)=0.
      GØ TØ 99
   20 PRINT 7
    7 FØRMAT(16HENTER INVESTMENT)
      READ 15,IX
      DØ 24 K=1,NIV
      IF(NAME(K)-IX)24,26,24
   24 CØNTINUE
      L1=1
      L2=NIV
      GØ TØ 320
   26 IF(J-3)366,38,250
  366 NI=K
      GØ TØ 36
  250 NIV=NIV-1
      DØ 251 N=K,NIV
      NAME(N)=NAME(N+1)
      VAL1(N)=VAL1(N+1)
  251 VAL2(N)=VAL2(N+1)
      GØ TØ 99
   38 L1=K
      L2=K
  320 PRINT 17
      IF(NIV)999,99,350
  350 DØ 340 KØUNT=L1,L2
      IF(VAL1(KØUNT)-10.*VAL2(KØUNT)-1.)321,322,323
```

```
 321 ISW=0
     GØ TØ 324
322  ISW=2
     GØ TØ 324
 323 ISW=1
324  PRINT 325,NAME(KØUNT),ISW
325  FØRMAT(A5,2H =,I2)
340  CØNTINUE
     IF(J-3)99,204,99
204  IF(L2-NIV)99,201,999
201  PRINT 200
200  FØRMAT(35H0 TØ CØNTINUE,1 TØ QUIT,2 TØ STACK0)
     READ 11, ISW
     IF(ISW)999,99,425
 425 PRINT 428,NIV
428  FØRMAT(38HNUMBER ØF INVESTMENTS ØN ØUTPUT TAPE =,I3)
     IF(ITAP-9)703,704,703
704  WRITE ØUTPUT TAPE ITAP,705
705  FØRMAT(6HALTERB)
     GØ TØ 855
703  DØ 602 J=1,100
602  WRITE ØUTPUT TAPE ITAP,601,(CØPY(J,I),I=1,5)
855  IF(NIV)999,426,800
 800 WRITE ØUTPUT TAPE ITAP,801,(NAME(I),VAL1(I),NAME(I),VAL2(I),
    1 I=1,NIV)
801  FØRMAT(12X,1H0,A5,F12.1/12X,1H1,A5,F12.1)
426  IF(ITAP-9)710,711,710
710  WRITE ØUTPUT TAPE ITAP,802,IEØF
     END FILE ITAP
     GØ TØ 888
711  WRITE ØUTPUT TAPE ITAP,715
715  FØRMAT(3HEND/6HINVERT/5HCRASH/2HGØ)
 802 FØRMAT(A6)
888  ITAP=9
     IF(ISW-1)999,999,99
 999 CALL EXIT
```

END

01080

```
C     ØUTPUT SMØØTHING PRØGRAM
      DIMENSIØN NAME(150),VL(150,3),NA(12),A(12),VLX(12,3)
      EQUIVALENCE (ANAME,INAME),(NA,A)
      ANAME=6H  NAME
      A(1)=4HISAS
      A(2)=4HIUSS
      A(3)=4HISAT
      A(4)=4HIVRT
      A(5)=4HISAP
      A(6)=4HIUSB
      A(7)=4HIVRS
      A(8)=4HIVRB
      A(9)=4HISAR
      A(10)=4HISAC
      A(11)=4HIUSC
      A(12)=4HIVRC
   22 I=0
      IØ=0
      ISW=1
      JJ=2
    2 READ INPUT TAPE 6 , 1 , ITEST
    1 FØRMAT(1X,A6)
      IF(ITEST-INAME)2,4,2
    4 READ INPUT TAPE 6 , 1 , ITEST
      PRINT 103,I
   10 READ INPUT TAPE 6 , 3 , J , NAM , VAL
    3 FØRMAT(1X,I1,A5,F18.6)
      IF(J-1) 8,8,6
    6 IF(J-JJ)34,32,30
   34 IØ=I
      GØ TØ (36,30),ISW
   36 ISW=2
      JJ=2
      DØ 40 K=1,12
      DØ 40 KK=1,3
   40 VLX(K,KK)=-1.
```

```
   46 DØ 42 K=1,12
      IF(NA(K)-NAM)42,44,42
   42 CØNTINUE
      PRINT 103,I
      ISW=1
      GØ TØ 8
   44 VLX(K,J-1)=VAL
      JJ=J
      READ INPUT TAPE 6,3,J,NAM,VAL
      IF(J-JJ)30,46,46
30    I=IØ
      JJ=J
 32    I=I+1
      J=J-1
      NAME (I)=NAM
      VL(I,J)=VAL
      GØ TØ 10
 8    WRITE ØUTPUT TAPE 1,11
 11   FØRMAT(3H1   ,8HVARIABLE,5X,6HSECØND,9X,5HTHIRD,10X,6HFØURTH//)
      DØ 50 K=1,12
50    WRITE ØUTPUT TAPE 1, 13,NA(K),(VLX(K,KK),KK=1,3)
      DØ 20 K=1,I
      IF(VL(K,1)+VL(K,2)+VL(K,3))18,20,18
 18   WRITE ØUTPUT TAPE 1 , 13, NAME(K),(VL(K,KK),KK=1,3)
13    FØRMAT(3X,A5,3(3X,F12.6))
 20   CØNTINUE
      PRINT 103,I
 103  FØRMAT(2HI=,I3)
      GØ TØ  (999,22),ISW
999   CALL EXIT
      END
```

6-1185 STOCK FORM UNIFORM-COURIER

APPENDIX G

A PLANT LOCATION MODEL WITH ECONOMIES OF SCALE - SOME COMPUTATIONAL EXPERIENCE

Introduction

In industries characterized by strong economies of scale, decisions as to the location of new plants or the expansion of existing facilities involve consideration of the swap off between the lower production cost and the higher transportation cost which result from concentrating production in fewer locations. The model outlined in this paper uses data from the Latin American iron and steel industry as a case study to find the most efficient combination and location of plants to serve fixed market requirements for a homogenous good in a given year in the future. A later part of this project will include the development of a multi-period model.

The Model

As with the linear programming transportation problem the model used here assumes fixed requirements, r_j, for a homogenous good at a number of market locations, j. The amount of the good shipped from each production point, i, to each market is denoted, x_{ij}. The cost of shipping the final goods is assumed to be a linear function of the amount shipped over a given route and is written, t_{ij}. However, unlike the linear programming transportation problem this model assumes that the total amount available to be shipped from any given plants is not fixed but rather unrestricted. The total cost function at each plant i, is characterized by economies of scale and is approximated by a function of the form

$$c_i = a_i y_i + b_i x_i$$

where

c_i = total cost of steel production at location i, millions of dollars per year

a_i = fixed charge portion of total cost, millions of dollars per year

y_i = an integer variable restricted to the values 0 and 1 such that

$$y_i = 0 \text{ when } x_i = 0$$

$$y_i = 1 \text{ when } x_i > 0$$

b_i = linear portion of total cost, dollars per ton

$x_i = \sum_j x_{ij}$ = production at location i, tons per year.

This type of model was applied by Vietorisz and Manne (1963) to the chemical industry in Latin America.

The problem is solved by minimizing the total transportation and production cost subject to the constraints that all $x_{ij} \geq 0$ and that the requirements of each market are fulfilled, i.e.

$$\min Z = \sum_{ij} x_{ij} t_{ij} + \sum_i a_i y_i + \sum_i b_i x_i$$

subject to

$$x_{ij} \geqq 0 \text{ for all } i,j$$

$$\sum_i x_{ij} \geqq r_j \text{ for all } j.$$

The scale of production at any one plant location is unconstrained, i.e. it is assumed that in the long run considered here that a plant of any size can be constructed at any location. This assumption, when coupled with those of linear transportation cost and decreasing unit cost of production produces the result that no market is ever served by more

than one plant. This result will hold so long as each market is free to buy from the plant of its choice. If at any point in time a market found itself buying from two different plants at the same price, the market would benefit by concentrating all of its purchasing from one or the other plant. This would enable the plant chosen to produce at a lower unit cost and enable the market to fulfill its requirements at a smaller total cost.

The Data.

The data presently available on the production and transportation cost of flat steel products in the Latin American Free Trade Area is neither of the quantity or quality to justify their use in a twelve plant, twelve market model of the industry. Therefore, heroic assumptions were made where necessary in order to construct a model of this size. Because of the small quantity of data on which this model was based only the most limited policy conclusions can be drawn. In particular, it would be unwise to draw conclusions from this data as to the relative efficiency of various locations for steel mills. It is anticipated that more data of better quality will be available in a later phase of this project. For now it is deemed desirable to obtain computational experience with a model of this size.

1. Transportation Cost. Twelve plant sites and twelve market areas were chosen and estimates were made of the cost of transportation between the plants and the markets. The twelve production sites chosen

are locations where flat steel products are already produced or locations at which it appears likely that a steel mill might be constructed at some future date. There are already steel works of one sort or another at ten of the locations. The other two locations, Buenos Aires and Veracruz were chosen to determine the viability of seacoast locations near major market areas. The twelve locations and the number and symbol assigned to them for use in the remainder of the paper are:

1. San Nicolas, Argentina (SN)
2. Buenos Aires, Argentina(BA)
3. Volta Redonda, Brazil (VR)
4. São Paulo, Brazil (SP)
5. Belo Horizonte, Brazil (BH)
6. Paz del Rio, Colombia (PR)
7. Huachipato, Chile (HA)
8. Monclova, Mexico (ML)
9. Monterrey, Mexico (MT)
10. Veracruz, Mexico (VC)
11. Chimbote, Peru (CH)
12. Orinoco, Venezuela (OR)

The twelve market locations are major industrial centers where flat steel products are used. Since the bulk of flat products are intermediate products used in either the automobile, appliance, or container industry the assumption of point consumption is a close representation of reality. The markets chosen, their number and alphabetic abbreviation are listed below.

1. Buenos Aires, Argentina (BA)
2. Montevideo, Uruguay (MV)
3. Rio de Janeiro, Brazil (RJ)
4. São Paulo, Brazil (SP)
5. Belo Horizonte, Brazil (BH)
6. Bogota, Colombia (BG)
7. Santiago, Chile (SG)
8. Mexico City, Mexico (MC)
9. Monterrey, Mexico (MT)
10. Lima, Peru (LI)
11. Caracas, Venezuela (CA)
12. Orinoco, Venezuela (OR)

Transportation costs were calculated by measuring the distance over which the finished products must be transported by rail, inland water, or maritime transport and then multiplying by the appropriate rate for each type of transportation. Also trans-shipment costs of $1.20

per ton (one loading and one unloading) and Panama Canal charges of $1.25 per ton were assigned where appropriate.[1] Since the trans-shipment cost often amounts to 20 per cent or more of the total transportation cost the number of times which goods must be trans-shipped is an important factor in determining the most efficient production locations. Table G.1 is the transportation cost matrix obtained from these calculations.

2. Production cost. Cost data for use in this model were based on information taken from a recent UN publication (1963) on the iron and steel industry of Latin America. In this study the Economic Commission for Latin America (ECLA) made a breakdown of cost for pig iron, steel, and flat rolling production cost for hypothetical plants of 100, 200, 400, 500, 800, 1,000, and 1,500 thousand tons per year. Within these categories cost elements are further subdivided into salaries and wages, capital charges, raw materials, fuel, etc. In addition the report includes cost data for alternative methods of steel production, i.e. open hearth, electric furnace, and LD process; however, in this paper only the data for the open hearth method was used.

The ECLA data are used to separate production cost into three components, capital charges, labor cost, and other cost. For each of these three elements of cost, curves were plotted and linear approximations were made by cutting the curve at plant sizes of 200 and 1,500 thousand tons per year. This gave a reasonably close fit in every case. (See Figure G.1.)

---

[1]The cost for freight rates, trans-shipment cost and Panama Canal charges were obtained from the unpublished paper of Ana Maria Martirena-Mantel, "Integration and Economies of Scale: the Steel Industry." Yale University, 1963. The freight rates which Mrs. Martirena-Mantel took from an ECLA study are

railway transport	0.00848	dollars	per	ton-kilometer	
inland water tran	0.00399	"	"	"	"
maritime transport	0.00234	"	"	"	"

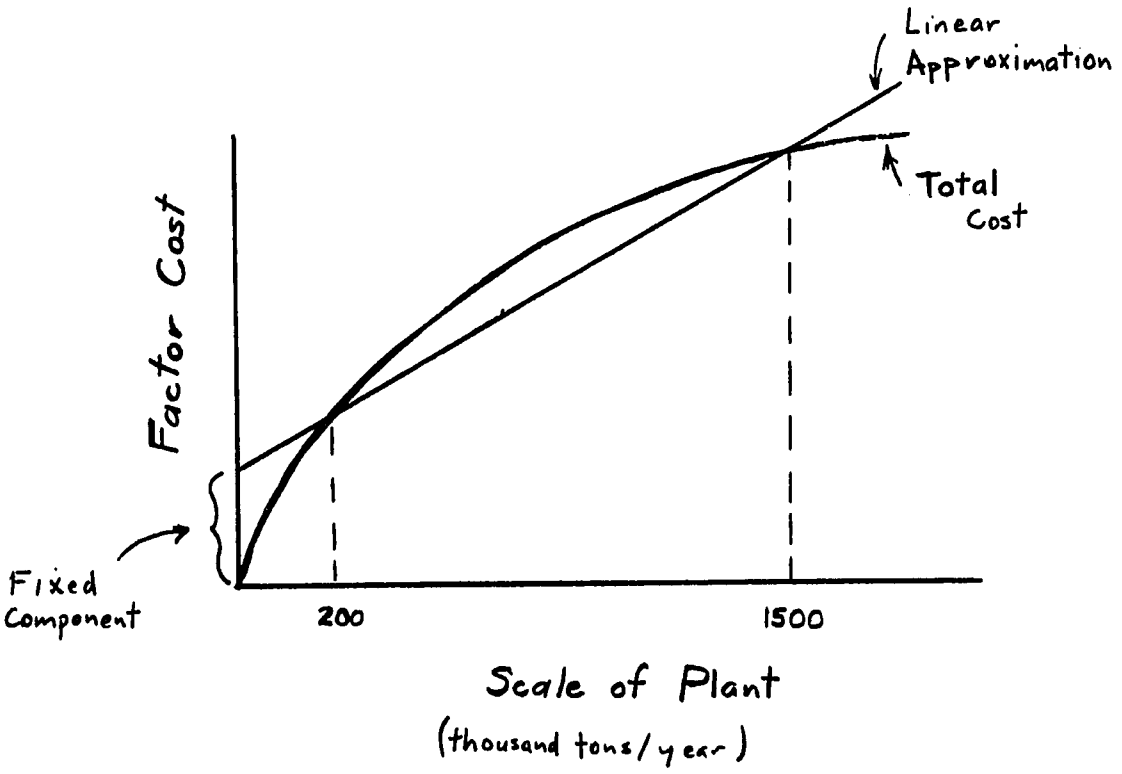
Linear
Approximation
Total
Cost
Factor Cost
Fixed
Component
200
1500
Scale of Plant
(thousand tons/year)

Figure G.1

Table G-1

TRANSPORTATION COST

(Dollars per ton)

Market	Plant											
	SN	BA	VR	SP	BH	PR	HA	ML	MT	VC	CH	OR
BA	1.68	.00	4.93	5.52	7.44	24.50	7.40	23.20	21.50	16.80	11.45	12.98
MV	2.16	1.68	4.70	5.30	7.25	24.30	7.20	23.00	21.30	16.60	11.25	12.78
RJ	4.68	4.20	2.05	4.18	4.60	20.70	10.00	19.40	17.70	14.50	14.00	9.10
SP	6.00	5.52	3.58	.00	6.70	21.20	11.30	19.90	18.20	15.00	15.30	9.60
BH	7.92	7.44	3.60	6.70	.00	24.45	13.20	21.60	19.90	16.70	17.20	11.30
BG	23.30	22.82	21.50	20.55	23.60	2.90	18.75	16.70	15.00	12.30	11.50	12.00
SG	11.45	10.57	13.27	13.00	15.80	20.32	4.36	10.87	19.17	16.67	7.81	14.62
MC	21.42	20.04	19.20	19.70	21.75	17.30	16.90	.70	8.00	4.15	11.65	11.40
MT	22.00	21.50	19.70	20.20	22.25	17.80	17.40	1.70	0.00	7.25	12.15	11.90
LI	12.35	11.85	14.55	14.30	17.10	16.65	6.00	13.10	11.40	9.15	2.77	9.25
CA	13.46	12.98	11.20	11.70	13.75	12.80	12.95	14.54	12.84	7.40	7.65	3.90
OR	13.46	12.98	11.20	11.70	13.75	12.80	12.95	14.54	12.84	7.40	7.65	.00

A closer approximation to the cost curves can be obtained by fitting curves, but this increases the complexity of the computation problem considerably since marginal costs are no longer constant. If a smooth curve were used rather than the linear approximation it would be necessary to calculate the unit cost of production for all plants each time that the plants are surveyed to see which can supply the various markets at the minimum cost. When better cost data is available some regressions will be fitted and a program written and tested to determine the sensitivity of the results to closer curve fitting and the additional computation time required to obtain this improved approximation to reality.

Capital cost and labor cost were assumed to be the same for all production locations. (This assumption will be modified later when more complete data is available.) Costs other than capital or labor (referred to hereafter as "other cost") were corrected for the cost of assembling raw materials at the different production locations under consideration.

Curves of the type shown in Figure G.1 were then plotted for the capital cost and labor cost elements and the following parameters were obtained:

	fixed component million $ per year	Variable component $ per ton per year
capital cost	5.0	12.9
labor cost	2.2	1.8

The cost data in the UN steel symposium paper mentioned earlier were based on an assembly cost of raw materials of $28.30 per ton of pig iron. This is the cost of raw material assembly at the Huachipato plant in Chile. The assembly cost at other locations were given in the UN paper as follows:

San Nicolas, Argentina	$41.82
Monclova, Mexico	23.78
Chimbote, Peru	37.43
Orinoco, Venezuela	22.44

Using these assembly costs as guidepost, estimates for other locations were made as follows:

Buenos Aires, Argentina	$39.00
Volta Redonda, Brazil	28.30
São Paulo, Brazil	30.00
Belo Horizonte, Brazil	27.00
Paz del Rio, Colombia	28.30
Monterrey, Mexico	25.00
Veracruz, Mexico	38.30

Then calculations were made for "other cost" for each of the seven plant sizes in each of the twelve locations and cost curves were plotted and the linear approximations made, as was done for capital and labor cost.

Table G.2

"OTHER COST"

Location	fixed component million $ per year	variable component $ per ton per year
San Nicolas	10	90
Buenos Aires	10	87
Volta Redonda	9	76
São Paulo	9	79
Belo Horizonte	9	76
Paz del Rio	9	76
Huachipato	9	76
Monclova	9	71
Monterrey	10	73
Veracruz	9	83
Chimbote	9	75
Orinoco	9	70

Since the fixed component of cost differed from $9 million per year in only three cases, was relatively small in comparison with the variable component of cost, and the margin or error in the method of estimation was at least $1 million per year, it was decided to use a fixed cost component of $9 million per year for all plant locations. Also, using the same fixed cost component for all plants substantially reduces the calculation time in parts of the problem. Thus the fixed component of cost is

the same for all twelve plants and is equal to the sum of the fixed component of capital, labor, and other cost. The a_i in the equation $c_i = a_i y_i + b_i x_i$ is then $a_i = 5.0 + 2.2 + 9 = \$16.2$ million per year with a_i the same for all i. The parameter b_i is equal to the sum of the variable components of capital and labor cost ($12.9 and $1.8 per ton per year) plus the variable component of other cost for each plant.

The variable-component-of-cost parameter, b_i, and the transportation cost matrix, t_{ij}, are added to give a linear cost matrix, d_{ij}, Table G.3. This matrix gives the linear cost component at which a ton of flat steel products can be manufactured at plant i and transported to market j.

3. Market requirements. The market requirements, r_j, were obtained from demand projections made by ECLA in the UN study (1963) for the consumption of flat products in 1970. The projections are for countries rather than for cities, but due to the intermediate product nature of flat products, the consumption is so highly centralized that it is assumed in this study that all the demand for the country is concentrated in the major cities. Thus for countries in which there are several major industrial centers the projection for the country's consumption was rather arbitrarily divided among these centers; in the case of Brazil; 40 per cent, 40 per cent, 20 per cent between Rio de Janeiro, São Paulo, and Belo Horizonte; of Mexico, 75 per cent, 25 per cent between Mexico City and Monterrey; and of Venezuela, 87 per cent, 13 per cent between Caracas and Orinoco. (See Table G.4.)

Table G.3

LINEAR COST MATRIX
(Cents per ton)

	PLANT											
	SN	BA	VR	SP	BH	PR	HA	ML	MT	VC	CH	OR
BA	10638	10170	9563	9922	9814	11520	9810	10890	10920	11450	10115	9768
MV	10686	10338	9540	9900	9795	11500	9790	10870	10900	11430	10095	9748
RJ	10938	10590	9275	9788	9530	11140	10070	10510	10540	11220	10370	9380
SP	11070	10722	9428	9370	9740	11190	10200	10560	10590	11270	10500	9430
BH	11262	10914	9430	10040	9070	11515	10390	10730	10760	11440	10690	9600
BG	12800	12452	11220	11425	11430	9360	10945	10240	10270	11000	10120	9670
SG	11575	11227	10397	10670	10650	11102	9506	10657	10687	11437	9751	9932
MC	12612	12264	10990	11340	11245	10800	10760	9540	9570	10185	10135	9610
MT	12670	12320	11040	11390	11295	10850	10810	8740	8770	10495	10185	9660
LI	11705	11355	10525	10800	10780	10735	9670	9880	9910	10685	9247	9395
CA	11816	11468	10190	10540	10445	10350	10365	10024	10054	10510	9735	8860
OR	11816	11468	10190	10540	10445	10350	10365	10024	10054	10510	9735	8470

Note that the Table is in cents per ton. Thus the linear cost in dollars of manufacturing a ton of flat products at San Nicholas (SN) and sending it to Buenos Aires (BA) is $106.38.

Table G.4

MARKET REQUIREMENTS - 1970

(million of tons of flat products per year)

Buenos Aires	1.65
Montevideo	.14
Rio de Janeiro	1.25
São Paulo	1.25
Belo Horizonte	.66
Bogota	.57
Santiago	.47
Mexico City	1.20
Monterrey	.49
Lima	.28
Caracas	1.20
Orinoco	.18

The Computational Experience

Since economies of scale introduce non-linearities into the problem, the standard linear programming techniques can not be used to find the most efficient combination of plants. Therefore integer variables, y_i, which are restricted to the values 0 and 1 are used to make a linear approximation to the cost functions, and computational methods to permit integer values are required. Three different types of algorithms for integer programs have been or will be applied to the model. Complete enumeration and steepest ascent one point move algorithms (SAOPMA) have already been written and applied. An integer programming algorithm developed by Gomory and applied by Vietorisz to plant location problems has been used to a limited extent.

1. Complete Enumeration. In problems involving a small number of 0-1 integer variables it is possible to make a complete enumeration of the possibilities; however, the number of enumerations required

doubles each time a new integer variable is added so the computation time for anything more than the smallest of problems is very large. Vietorisz and Manne (1963) made a complete enumeration for a plant location problem involving five plants and two processes, which involved ten integer variables and required $2^{10} = 1024$ iterations. Martirena-Mantel (1963) has done five and seven plant, single process models for the Latin American iron and steel industry.

A Fortran program was written to accomplish the complete enumeration. It proceeds in several steps. The first step is assigning zeros and ones to the twelve integer variables. A set of twelve nested DO loops are used to generate the $2^{12} = 4096$ combinations of zeros and ones. Those plants to which zero variables are assigned are omitted from the problem and those to which ones are assigned remain in the problem. Each market is served by that plant of those remaining in the problem which can serve the market at the least linear cost. The requirement for the market is then multiplied by the linear cost component. This is done for each market and the total linear cost is summed and added to the product of the fixed cost per plant times the number of plants in the problem. In this procedure a plant which is assigned a one, and is therefore in the problem for the iteration, may not have low enough cost to serve any of the markets; i.e. there may be a lower bidder for each market. When this occurs the solution will be dominated by another solution, in particular it will be dominated by that solution which includes all the plants which are assigned to the problem but which do not serve any

of the markets. The program written for the complete enumeration prints out only those solutions which are not dominated. In the complete enumeration runs discussed later only about one tenth (roughly 400) of the solutions were printed out.

The program was first run on the IBM 1620 and required a computation time of 6 seconds per iteration. The time to complete the 4096 iterations would have been about $6\frac{1}{2}$ hours on the IBM 1620 or about 8 minutes on the IBM 7094. A program of this length is so expensive as to discourage one from using it to run a series of calculations based on changes in the cost parameters of the model. Also expansion of the program to handle more variables would not seem wise.

Professor J. C. Emery of the School of Industrial Management at M.I.T. became interested in the program and rewrote it, improving its efficiency by a factor of ten. The complete enumeration program was then run on the IBM 1620 in forty-five minutes, roughly one-half second per iteration. Then the same program with a change in the parameters of the model was run on the IBM 7094.[1] It required a little more than one minute for the entire program and less than half a minute to complete the iterations once the program had been compiled. With this more efficient program the model could be expanded to 16 integer variables and still run in roughly eight minutes on the IBM 7094, or with fifteen integer variables the time required would be small enough to permit the change of variables desired. All this attention to the details of run time is needed because the size of model to be used in a later phase of this study depends critically on the time required to complete the calculations.

[1]Thanks are due to the M.I.T. Industrial Management Computer Facility for the use of the IBM 1620 and to the M.I.T. Computation Center for the use

Allowance for different fixed components of cost at various plant locations would add slightly to the time required for running the program but this addition is not expected to increase the run time significantly.

Since the other algorithms used for solving problems involving integer variables either often find local optima (SAOPMA) or show poor convergence characteristics (integer programming algorithms) the complete enumeration technique provides the only sure way of finding the optimum. Actually it has been proven that the integer programming algorithms now in use will converge within a finite period of time, but for large problems there is not yet sufficient experience with these algorithms to estimate the length of time required.

There is an additional benefit to be obtained from using the complete enumeration method. One finds not only the most efficient combination of plants but also a ranking of all combinations by the value of the minimand (the total cost required to meet the market demands when a given subgroup of the twelve plants are in operation). In a world where many considerations other than economic efficiency are involved in plant location and size decisions, it is valuable to know the ranking by relative efficiency of various combinations of plant locations and to be able to estimate the difference in efficiency between various combination of plant locations.

For the original data input given in Tables G.1, G.2, G.3 and G.4 the most efficient combination of plants is that of a single plant located in Venezuela. (See Table G.5). This result is based on the very low assembly

Table G.5

OUTPUT OF COMPLETE ENUMERATION PROGRAM, ORDERED BY VALUE OF MINIMAND

ORIGINAL DATA

Minimand	Plants in Solution												Plant Which Serves Market J											
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
901.82	0	0	0	0	0	0	0	0	0	0	0	1	12	12	12	12	12	12	12	12	12	12	12	12
911.89	0	0	1	0	0	0	0	0	0	0	0	1	3	3	3	3	3	12	12	12	12	12	12	12
912.68	0	0	0	0	0	0	0	1	0	0	0	1	12	12	12	12	12	12	12	8	8	12	12	12
913.18	0	0	0	0	0	0	0	0	1	0	0	1	12	12	12	12	12	12	12	9	9	12	12	12
914.53	0	0	0	0	1	0	0	0	0	0	0	1	12	12	12	12	5	12	12	12	12	12	12	12
916.02	0	0	0	0	0	0	1	0	0	0	0	1	12	12	12	12	12	12	7	12	12	12	12	12
916.26	0	0	0	0	0	1	0	0	0	0	0	1	12	12	12	12	12	2	12	12	12	12	12	12
916.76	0	0	0	0	0	0	0	0	0	0	1	1	12	12	12	12	12	12	11	12	12	11	12	12
917.27	0	0	0	1	0	0	0	0	0	0	0	1	12	12	12	4	12	12	12	12	12	12	12	12
922.74	0	0	1	0	0	0	0	1	0	0	0	1	3	3	3	3	3	12	12	8	8	12	12	12
923.25	0	0	1	0	0	0	0	0	1	0	0	1	3	3	3	3	3	12	12	9	9	12	12	12
925.38	0	0	0	0	1	0	0	1	0	0	0	1	12	12	12	12	5	12	12	8	8	12	12	12
925.72	0	0	1	0	1	0	0	0	0	0	0	1	3	3	3	3	5	12	12	12	12	12	12	12
925.89	0	0	0	0	1	0	0	0	1	0	0	1	12	12	12	12	5	12	12	9	9	12	12	12
926.09	0	0	1	0	0	0	1	0	0	0	0	1	3	3	3	3	3	12	7	12	12	12	12	12
926.32	0	0	1	0	0	1	0	0	0	0	0	1	3	3	3	3	3	6	12	12	12	12	12	12
926.83	0	0	1	0	0	0	0	0	0	0	1	1	3	3	3	3	3	12	11	12	12	11	12	12
926.87	0	0	0	0	0	0	1	1	0	0	0	1	12	12	12	12	12	12	7	8	8	12	12	12
927.11	0	0	0	0	0	1	0	1	0	0	0	1	12	12	12	12	12	6	12	8	8	12	12	12
927.37	0	0	1	1	0	0	0	0	0	0	0	1	3	3	3	4	3	12	12	12	12	12	12	12
927.38	0	0	0	0	0	0	1	0	1	0	0	1	12	12	12	12	12	12	7	9	9	12	12	12
927.61	0	0	0	0	0	0	0	1	0	0	1	1	12	12	12	12	12	12	11	8	8	11	12	12
927.62	0	0	0	0	0	1	0	0	1	0	0	1	12	12	12	12	12	6	12	9	9	12	12	12
928.12	0	0	0	0	0	0	0	0	1	0	1	1	12	12	12	12	12	12	11	9	9	11	12	12
928.13	0	0	0	1	0	0	0	1	0	0	0	1	12	12	12	4	12	12	12	8	8	12	12	12
928.63	0	0	0	1	0	0	0	0	1	0	0	1	12	12	12	4	12	12	12	9	9	12	12	12
928.72	0	0	0	0	1	0	1	0	0	0	0	1	12	12	12	12	6	12	7	12	12	12	12	12
928.96	0	0	0	0	1	1	0	0	0	0	0	1	12	12	12	12	5	6	12	12	12	12	12	12
929.46	0	0	0	0	1	0	0	0	0	0	1	1	12	12	12	12	5	12	11	12	12	11	12	12
929.98	0	0	0	1	1	0	0	0	0	0	0	1	12	12	12	4	5	12	12	12	12	12	12	12
930.10	0	0	1	0	0	0	0	1	0	0	0	0	3	3	3	3	3	8	3	8	6	8	8	8
930.46	0	0	0	0	0	1	1	0	0	0	0	1	12	12	12	12	12	6	7	12	12	12	12	12
931.19	0	0	0	0	0	1	0	0	0	0	1	1	12	12	12	12	12	6	11	12	12	11	12	12
931.28	0	0	1	0	0	0	0	0	1	0	0	0	3	3	3	3	3	9	3	9	9	9	9	9
931.47	0	0	0	1	0	0	1	0	0	0	0	1	12	12	12	4	12	12	7	12	12	12	12	12
931.71	0	0	0	1	0	1	0	0	0	0	0	1	12	12	12	4	12	6	12	12	12	12	12	12
931.81	0	0	0	0	0	0	1	0	0	0	1	1	12	12	12	12	12	12	7	12	12	11	12	12
932.21	0	0	0	1	0	0	0	0	0	0	1	1	12	12	12	4	12	12	11	12	12	11	12	12
934.84	0	0	1	0	0	0	0	0	0	0	1	0	3	3	3	3	3	11	11	11	11	11	11	11
936.57	0	0	1	0	1	0	0	1	0	0	0	1	3	3	3	3	5	12	12	8	8	12	12	12
936.82	0	0	1	0	0	0	0	1	0	0	1	0	3	3	3	3	3	11	11	8	8	11	11	11
936.94	0	0	1	0	0	0	1	1	0	0	0	1	3	3	3	3	3	12	7	8	8	12	12	12
937.07	0	0	1	0	1	0	0	0	1	0	0	1	3	3	3	3	5	12	12	9	9	12	12	12
937.18	0	0	1	0	0	1	0	1	0	0	0	1	3	3	3	3	3	6	12	8	8	12	12	12
937.33	0	0	1	0	0	0	0	0	1	0	1	0	3	3	3	3	3	11	11	9	9	11	11	11
937.45	0	0	1	0	0	0	1	0	1	0	0	1	3	3	3	3	3	12	7	9	9	12	12	12
937.68	0	0	1	0	0	0	0	1	0	0	1	1	3	3	3	3	3	12	11	8	8	11	12	12

cost of raw materials estimated by ECLA for a plant located at Orinoco (often called the Matanzas plant) $22.44 per ton of pig iron at Orinoco as compared to $28.29 per ton of pig iron at Huachipato. Secondly, it is assumed in this set of calculations that the Orinoco plant is located on a river of sufficient depth that the final products from the plant can be loaded directly onto ocean going vessels, thus avoiding the trans-shipment cost involved in first transporting the products by rail or inland water routes and then trans-shipping them to an ocean liner.

Since a single plant at Orinoco would have to produce 9.34 million tons per year to meet the requirements of all the markets in the region while the largest steel mills in existence are somewhat smaller than this size, there is no data available which would confirm or negate the assumptions used here about the cost of production in a plant of that size. However, since the cost data used in this study reflect cost in a plant up to a size of only 1.5 million tons per year the empirical basis in this model for making policy decisions on plants is very weak. Yet in all plants over 1.5 million tons per year the linear approximation causes an overestimation of cost of production if the process continues to show economies of scale. The overestimation of production cost would cause a bias in favor of less concentration of production.

The second ranked solution (at only one per cent greater total cost) includes two plants -- one at Volta Redonda and one at Orinoco. The plant at Volta Redonda serves the five markets in Brazil, Uruguay, and

Argentina -- requiring a plant of 4.95 million tons. The Orinoco plant (4.39 million tons) would serve Mexico and the north and west coast of South America.

At an increment in total cost of less than one per cent more, and the tenth ranked solution, there are three plants in the solution, Volta Redonda, Orinoco, and Monclova. The Mexican plant would serve the two Mexican markets at Mexico City and Monterrey (totaling 1.69 million tons per year), the Volta Redonda plant would serve the markets of Brazil, Uruguay, and Argentina, and the Orinoco plant would be reduced to 2.70 million tons per year and would supply the north and west coast of South America. Even with the quality of the data input for these calculations it seems likely that the flat steel product requirements of the region could be met with considerably increased efficiency if production of these types of steel products were concentrated in relatively few locations. Changes in technology may certainly alter this conclusion in the future, but it is not clear that such changes are likely to occur in the near future. Continuous casting has been mentioned as one technology that might make small steel plants more efficient vis-a-vis their larger brothers. However, even with continuous casting the economies of scale in hot and cold rolling of flat products appear to weigh heavily in favor of large plants.

Perhaps at a later point in the study it will be possible to obtain cost data for continuous casting operations and thus test the reasoned judgment advance in the previous paragraph.

To test the sensitivity of the results to changes in the parameters a ten per cent increase in the variable component of production cost at Orinoco was made. This raised the linear production cost component for that plant from $70 to $77 per ton. Production cost at Orinoco were then about average for the region.

With this change in data the single-plant-at-Orinoco solution drops from its first-ranked position to that of 190th and the value of the minimand at that point is 4 per cent above that of the new global minimum.

The most efficient solution with the new parameters is a two plant solution a 5.42 million ton per year plant at Volta Redonda serving the east coast plus Santiago and a 3.92 million ton per year plant at Monclova serving Mexico, Venezuela, Colombia, and Peru. (See Table G.6.)

With the change in parameters the highest ranked three-plant solution ranks 4th in the total listing (less than 1 per cent above the minimand of the optimum). It adds a plant at Chimbote (Peru) to that of the two plant optimum. The Chimbote plant (2.70 million tons per year, then serves Chile, Peru, Colombia, and Venezuela, while the Monclova plant serves only Mexico and the Volta Redonda plant only the east coast of South America.

The isolated nature of the Bogota market up in the mountains with the Paz del Rio plant nearby is shown by the fact that a fourth plant can be added to the three-plant solution above with that plant serving only Bogota and the value of the minimand is less than 2 per cent above that of the optimum.

Table G.6

OUTPUT OF COMPLETE ENUMERATION PROGRAM, ORDERED BY VALUE OF MINIMAND

CHANGED DATA

Minimand	Plants in Solution												Plant Which Serves Market J											
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
930.11	0	0	1	0	0	0	0	1	0	0	0	0	3	3	3	3	3	8	3	8	8	8	8	8
931.28	0	0	1	0	0	0	0	0	1	0	0	0	3	3	3	3	3	9	3	9	9	9	9	9
934.85	0	0	1	0	0	0	0	0	0	0	1	0	3	3	3	3	3	11	11	11	11	11	11	11
936.83	0	0	1	0	0	0	0	1	0	0	1	0	3	3	3	3	3	11	11	8	8	11	11	11
937.33	0	0	1	0	0	0	0	0	1	0	1	0	3	3	3	3	3	11	11	9	9	11	11	11
939.20	-	-	1	0	0	0	0	1	0	0	0	1	3	3	3	3	3	8	3	8	8	8	12	12
939.96	0	0	1	0	0	0	0	0	1	0	0	1	3	3	3	3	3	9	3	9	9	9	12	12
940.51	0	0	0	0	1	0	0	1	0	0	0	0	5	5	5	5	5	8	5	8	8	8	8	8
941.29	0	0	1	0	0	1	0	1	0	0	0	0	3	3	3	3	3	6	3	8	8	8	8	8
941.52	0	0	1	0	0	0	0	0	0	0	0	1	3	3	3	3	3	12	3	12	12	12	12	12
941.53	0	0	1	0	0	0	1	1	0	0	0	0	3	3	3	3	3	8	7	8	8	7	8	8
941.68	0	0	0	0	1	0	0	0	1	0	0	0	5	5	5	5	5	9	5	9	9	9	9	9
942.30	0	0	1	0	0	1	0	0	1	0	0	0	3	3	3	3	3	6	3	9	9	9	9	9
942.62	0	0	1	0	0	0	1	0	1	0	0	0	3	3	3	3	3	9	7	9	9	7	9	9
943.93	0	0	1	0	1	0	0	1	0	0	0	0	3	3	3	3	5	8	3	8	8	8	8	8
944.06	0	0	0	0	1	0	0	0	0	0	1	0	5	5	5	5	5	11	11	11	11	11	11	11
945.11	0	0	1	0	1	0	0	0	1	0	0	0	3	3	3	3	5	9	3	9	9	9	9	9
945.58	0	0	1	1	0	0	0	1	0	0	0	0	3	3	3	4	3	8	3	8	8	8	8	8
946.04	0	0	0	0	1	0	0	1	0	0	1	0	5	5	5	5	5	11	11	8	8	11	11	11
946.54	0	0	0	0	1	0	0	0	1	0	1	0	5	5	5	5	5	11	11	9	9	11	11	11
946.71	0	0	1	0	0	1	0	0	0	0	1	0	3	3	3	3	3	6	11	11	11	11	11	11
946.76	0	0	1	1	0	0	0	0	1	0	0	0	3	3	3	4	3	9	3	9	9	9	9	9
947.47	0	0	0	1	0	0	0	1	0	0	0	0	4	4	4	4	4	8	8	8	8	8	8	8
947.93	0	0	1	0	0	0	0	0	0	0	1	1	3	3	3	3	3	11	11	11	11	11	12	12
948.67	0	0	1	0	1	0	0	0	0	0	1	0	3	3	3	3	5	11	11	11	11	11	11	11
948.69	0	0	1	0	0	1	0	1	0	0	1	0	3	3	3	3	3	6	11	8	8	11	11	11
948.71	0	0	0	1	0	0	0	0	1	0	0	0	4	4	4	4	4	9	4	9	9	9	9	9
949.20	0	0	1	0	0	1	0	0	1	0	1	0	3	3	3	3	3	6	11	9	9	11	11	11
949.52	0	0	0	0	1	0	0	1	0	0	0	1	5	5	5	5	5	8	12	8	8	8	12	12
949.89	0	0	1	0	0	0	1	0	0	0	1	0	3	3	3	3	3	11	7	11	11	11	11	11
949.91	0	0	1	0	0	0	0	1	0	0	1	1	3	3	3	3	3	11	11	8	8	11	12	12
950.28	0	0	0	0	1	0	0	0	1	0	0	1	5	5	5	5	5	9	12	9	9	9	12	12
950.32	0	0	1	1	0	0	0	0	0	0	1	0	3	3	3	4	3	11	11	11	11	11	11	11
950.38	0	0	1	0	0	1	0	1	0	0	0	1	3	3	3	3	3	6	3	8	8	8	12	12
950.42	0	0	1	0	0	0	0	0	1	0	1	1	3	3	3	3	3	11	11	9	9	11	12	12
950.63	0	0	1	0	0	0	1	1	0	0	0	1	3	3	3	3	3	8	7	8	8	7	12	12
950.65	0	0	1	0	1	0	0	1	0	0	1	0	3	3	3	3	5	11	11	8	8	11	11	11
950.67	0	0	0	0	1	0	1	1	0	0	0	0	7	7	5	5	5	8	7	8	8	7	8	8
950.98	0	0	1	0	0	1	0	0	1	0	0	1	3	3	3	3	3	6	3	9	9	9	12	12
950.99	0	0	0	1	0	0	0	0	0	0	1	0	4	4	4	4	4	11	11	11	11	11	11	11
951.16	0	0	1	0	1	0	0	0	1	0	1	0	3	3	3	3	5	11	11	9	9	11	11	11
951.30	0	0	1	0	0	0	1	0	1	0	0	1	3	3	3	3	3	9	7	9	9	7	12	12
951.69	0	0	0	0	1	1	0	1	0	0	0	0	5	5	5	5	5	6	5	8	8	8	8	8
951.76	0	0	0	0	1	0	1	0	1	0	0	0	7	7	5	5	5	9	7	9	9	7	9	9
951.84	0	0	0	0	1	0	0	0	0	0	0	1	5	5	5	5	5	12	12	12	12	12	12	12
951.67	0	0	1	0	0	0	1	1	0	0	1	0	3	3	3	3	3	11	7	8	8	11	11	11
951.96	0	0	1	0	0	1	0	0	0	0	0	1	3	3	3	3	3	6	3	12	12	12	12	12
952.08	0	0	0	1	1	0	0	1	0	0	0	0	5	5	5	4	5	8	5	8	8	8	8	8
952.26	0	0	1	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3	3	3	3	3	3	3
952.30	0	0	1	1	0	0	0	1	0	0	1	0	3	3	3	4	3	11	11	8	8	11	11	11

2. SAOPMA. The "steepest ascent one-point move algorithm" introduced by Reiter and Sherman (1963) and applied to a plant location problem by Manne (1963) was tested on this model. The algorithm uses an arbitrarily chosen initial combination of plants, i.e. one of the 4096 possible choices. Then a one-point move is made in each of twelve directions by changing each of the twelve integer variables from a zero to a one or from a one to a zero, depending upon its original value, making the calculation for the new combination of plants, resetting the integer at its original value, and then moving to the next of the twelve variables and repeating the operation. For example in a four-plant case with an arbitrarily chosen original combination 0 1 0 0, the first step would involve calculating the value of the minimand for the combinations 1 1 0 0, 0 0 0 0, 0 1 1 0, and 0 1 0 1. Then the one of the four combinations which provided the lowest cost would be chosen for the new setting and the stepping process would be repeated. If none of the steps from the original value produced a lower value of the minimand then that value would be assumed to be the optimum. As was mentioned earlier this type of algorithm may often find local optima.

The efficiency of a SAOPMA-type algorithm in locating the global optimum was tested by writing a Fortran program which was used to make six test runs on the IBM 1620. In the first three runs the original data was used and in the second three the changed data (reflecting a 10 per cent increase in the linear cost component of the Orinoco plant) were used.

In each of the first three runs a different batch of ten randomly selected original settings of plant combinations (with twelve 0-1 variables in each setting) were used as starting points for the algorithm. The same three batches were used for the second three runs with the changed parameters.

The global optimum was reached with at least one of the ten original settings in all six runs, but the probability of the global optimum being found with any given original setting was strongly affected by the change in the parameters of the model. With the original data the global optimum was found with each of the ten original settings for all three runs, but with the changed data the global optimum was found with only 4 out of 10, 5 out of 10, and 2 out of 10 original settings for the three runs.

Though the performance of the algorithm was relatively poor with the changed data, the magnitude of the difference between the local optimum and the global optimum was never large. The mean value of the difference was five-tenths of one per cent of the value of the global minimand and the largest difference was one point five per cent of the global minimand. In the 19 out of 30 cases in which a local rather than the global optimum was found, the second rank solution (from the complete enumeration) was found in 9 cases, the third ranked in 4 cases, the ninth ranked in 5 cases, and the sixteenth ranked in 1 case.

Thus, if the SAOPMA could be run on the computer in much less time than the complete enumeration its use could be recommended for

large problems; however, with the experience obtained here, using ten different original settings in each run, the computation time on the IBM 1620 was about thirteen minutes per run, of which five minutes was spent in compiling the Fortran deck. This compares with forty-five minutes for the complete enumeration run of which three or four minutes were used to compile the program; the result being that in our experience with large programs the SAOPMA reduces the computation time by a factor of five but may fail to find the global optimum.

3. Integer Programming. An integer programming algorithm developed by Gomory and applied by Vietorisz to plant location problems was used on the data developed for this model. The FORTRAN program used was written for the IBM 7094 by H. Serenson and Vietorisz.[1] It was applied to two ten-plant problems and to two twenty-plant problems. The ten-plant problems were identical to the two twelve-plant problems used in the complete enumeration runs except for the exclusion of the plants at Buenos Aires and Veracruz. For the twenty-plant cases cost estimates for an additional eight plants were made.

In both the ten and the twenty-plant problems, runs were made for the original cost data and for the increased-cost-at-Orinoco data. Convergence was obtained in both runs with the ten-plant problem, the first run in one integer program (IP) iteration and the second in

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[1]Thanks are due to both Thomas Vietorisz and to Herb Serenson for assistance in making the integer programming calculations and to the Thomas J. Watson research labs of IBM for the use of their IBM 7094.

173 IP iterations. The second run took an estimated 1.7 minutes on the IBM 7094. For the twenty-plant problem convergence was obtained immediately with the original data but had not been reached after more than 300 IP iterations once the changed data input was used. The ease with which convergence was obtained with the original data in both the ten and twenty-plant problems can probably be attributed to the fact that the global optimum in each case is a single plant location while for the changed data the global optimum includes two-plant locations. Experience with the IP plant location algorithm is as yet too limited to determine definitely whether or not the number of IP iterations required for convergence is a function of the number of plants in the global optimum.

David Kendrick
Cambridge, Massachusetts
June 29, 1964.

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