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THE EFFECTIVE EXCHANGE RATE, EMPLOYMENT, AND GROWTH IN A FOREIGN EXCHANGE CONSTRAINED ECONOMY

Richard R. Nelson

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PREFACE

This Memorandum is part of a multifaceted study of Colombia being undertaken at The RAND Corporation under the sponsorship of the U.S. Agency for International Development, with supplementary funds provided by the Corporation. The RAND study of Colombia is designed to illuminate the problems and possibilities, promising strategies and desirable policies, for AID in its dealing with Colombia and for the Colombian government in its attempt to solve its economic problems. The study includes a set of linked substudies on various interactive phenomena: population growth and rural-urban migration, agricultural stagnation, the process and problems of industrialization, the balance of payments, unemployment and income distribution, education, and skill requirements. The substudies are being published separately. An overall report is being prepared to draw together and integrate the various parts.

The present study is concerned with the influence of the effective exchange rate on Colombia's ability to achieve its growth and employment objectives. The author is indebted to RAND colleagues Robert Aliber, Irving Fisher, Paul Johnson, John Koehler, Paul Schultz, and Robert Slighton for many useful comments and suggestions.

SUMMARY

In an earlier study of Colombia's manufacturing development the author presented an analysis of the way in which the balance of payments constraint limited the employment and growth possibilities open to Colombia. No explicit analysis was presented of how the effective exchange rate--the price of foreign exchange relative to domestic factor prices--influences the constraint and hence Colombia's ability to achieve faster growth and higher employment. The present study attempts such an analysis.

The study begins by examining certain characteristics of the conventional two-gap models that lead such people as, for example, J. Vanek to such pessimistic conclusions regarding Colombia's ability to increase its growth rate without increases in foreign assistance. These conclusions appear to make infeasible such programs as that proposed by Currie to deal with Colombia's massive unemployment problem. The key characteristics include the assumed lack of substitution possibilities between domestic resources and imports. This characteristic also lies behind another implication of the model--that the level of the effective exchange rate is not of particular policy concern.

A model is then developed that admits the possibility of substituting domestic inputs for both intermediate and capital goods imports. Within the framework of the model, Colombia's ability to deal with her growth and employment problems is reconsidered. The illustrative calculations suggest first that if the assumed substitution possibilities are not unrealistic, Colombia has far more room to maneuver than is suggested by studies such as that of Vanek. Second, a higher

effective exchange rate is the price of this room for maneuver. Third, many of Colombia's problems since the fall in coffee prices may reflect a target effective exchange rate that is too low given her employment and growth objectives, and given the shift to the left in the schedule of foreign exchange supply that the deterioration of world coffee markets implied.

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I. INTRODUCTION

In an earlier study I presented a brief analysis of the constraints on economic growth in Colombia.[1] Several considerations now appear to warrant the publication of a significantly extended and amended model--in particular a model that incorporates explicitly the possibility for factor substitution and examines the role of the exchange rate in determining relative factor intensities.

First, since the issuance of the earlier Memorandum, Vanek's study of foreign exchange requirements for economic development in Colombia has been published.[2] Vanek's stimulating and important analysis reaches discouraging conclusions regarding Colombia's ability to influence her growth rate through her own efforts alone; he argues that a substantial increase in the growth rate requires greater import capacity (from more foreign aid) and that increased "savings" yield little return. I believe that Vanek is essentially correct, if there is no change in the effective exchange rate. His analysis does not consider this possibility. Indeed, largely by omission but occasionally by commission he seems to imply that his conclusions are insensitive to the level of the effective exchange rate. In contrast, I am increasingly led to believe that the level of the exchange rate relative to domestic factor costs--that is, the "effective" exchange rate--may be extremely important in determining the ability of Colombia

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[1] R. R. Nelson, A Study of Industrialization in Colombia: Part I Analysis, The RAND Corporation, RM-5412-AID, September 1967.

[2] See J. Vanek, Estimating Foreign Resource Needs for Economic Development, McGraw-Hill Book Company, Inc., New York, 1967.

to increase its growth rate, with or without an increase in foreign assistance. One reason for this Memorandum is to provide theoretical backing for this belief.

Second, in thinking about Currie's proposals to increase employment in Colombia, I initially was led to a conclusion in the spirit of Vanek: just as domestic policy is very limited in its ability to increase the growth rate in the absence of an increase in foreign exchange availability, so is it sorely limited in its ability to increase employment.[1] Thus Currie's "Plan Colombia" appears infeasible with the given foreign exchange constraint. However, as my thinking evolved I became convinced that this conclusion, like Vanek's in regard to investment, assumes no change in the effective exchange rate. With a change in the effective exchange rate, domestic policy might well be more capable to increase employment. In light of the urgency of Colombia's unemployment problem, it seems to me that the theoretical backing for this conclusion should be presented.

Section II will examine the basic factors and assumptions that lie behind the impotency of domestic policy alone to increase investment and employment that are the implications of a "two-gap" model. There also will be a brief discussion of why the exchange rate, or exchange rate structure, should not be considered as independent of policy objectives. Section III will present the basic model, which incorporates factor substitution possibilities and factor prices. Sections IV and V reconsider the possibilities for expanding employment

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[1] L. Currie, Accelerating Development: The Necessity and the Means, McGraw-Hill Book Company, Inc., New York, 1967.

RAND RESEARCH
BRIEF

November 1968 RB-5680

RM-5680-AID, *The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy*, Richard R. Nelson, Rand Memorandum, November 1968, 65 pp.

PURPOSE: To analyze how the effective exchange rate--the price of foreign exchange relative to domestic factor prices--influences the balance of payments constraint and, hence, Colombia's ability to achieve faster growth and higher employment.

RELATED TO: Rand's multifaceted study of Colombia that is designed to illuminate the problems and possibilities, promising strategies, and desirable policies for AID in its dealings with Colombia and for the Colombian government in its attempt to solve its economic problems. An earlier analysis of the economic constraints on Colombia's growth is reported in RM-5412-AID, *A Study of Industrialization in Colombia: Part I Analysis*, September 1967.

THE METHOD: The first step in this analysis is the exposition of a "two-gap" model that incorporates the key features of existing "gap" models used to describe development problems. Fundamental to such models is the assumption of fixed factor-output ratios. Although it is misleading to say that either the savings or import constraint is not binding at any time in a "two-gap" model, there are sharp differences in the returns to more savings or more imports, in terms of an enhanced growth rate, depending on whether or not there is unutilized domestic capacity. If inputs and outputs are ordered such that import capacity is fully utilized while domestic output capacity is not, an increase in savings will have significantly less impact on growth than an increase in the supply of foreign exchange. A further characteristic of models employing fixed factor-output ratios is that employment and growth objectives are likely to be in conflict.

The second stage of analysis is the development of a "neoclassical" model of production designed to explore the possibilities of substituting domestic inputs for imports and labor for capital. To keep the model from becoming too complex, Cobb-Douglas relationships are employed throughout. With relative factor prices assumed flexible, continuous factor substitution is implied, and the sharp distinction between situations characterized as mainly import or savings constrained disappears. Instead, there are smooth diminishing returns in the long run to both increased savings and import capacity alone, and complementarity between them. Under the assumption that domestic productive capacity is not fully utilized, the key stimulus to factor substitution is the ratio of the exchange rate to the domestic wage rate. The analysis concludes with sample calculations of the effect of changes in the effective exchange rate and interest rate on employment and growth for various assumptions as to the parameters of the underlying production functions.

THE IMPLICATIONS: Based on the substitution possibilities provided by this model, the analysis suggests that domestic policy can do more to increase employment, can have greater impact on growth, and is more important as a complement to foreign aid in influencing the growth rate than the fixed coefficient two-gap model implies. However, achievement of these potentialities depends on the instruments of policy that a government can employ. In particular, their achievement requires, first, that the government have some ability to influence the consumer goods—investment level and mix; and, second, that it be able to influence the effective exchange rate and the equilibrium rate of return on capital. While an increase in foreign aid would make it "easier," particularly in making it possible to raise the real wage rate of those employed, it is possible for Colombia to meet her employment and growth objective with existing resources without any reduction in aggregate consumption. This will require recognition of the fact that the effective exchange rate target has been set too low, however--that the 1958 rate is a false parity.

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and investment and particularly examine how the exchange rate affects these possibilities. Section VI presents some numerical estimates, which must be taken with more than a grain of salt, but which may very roughly suggest the magnitudes involved.

The presentation will be much more formal and technical than in the other RAND Memoranda on Colombia. The analysis and implications will be presented much more informally in a forthcoming RAND Memorandum on Colombian policy to which this report may be considered an extended technical appendix.

II. THE BASIC TWO-GAP MODEL AND THE IMPLIED POLICY DILEMMA

Considering fine structure as well as basics, the variety of "gap" models that have been employed is great and proliferating. Vanek's model, applied to Colombia, is one of the richest and most interesting. The Chenery-Strout and Chenery-Eckstein models, which also have been applied to Colombia, are more aggregative and also differ in certain other ways from the Vanek model.[1] But underneath the apparent diversity, all the gap models have a roughly equivalent "two-gap" core, which can be developed in the following indirect but illuminating way.

Assume that an economy has available to it four basic activities: domestic production of investment goods, imports of investment goods, domestic production of consumer goods, and imports of consumer goods. A unit level of an activity can be chosen as "a dollar's worth." All activities require imports. The two production activities also require domestic inputs. The activity matrix is as follows:

		Activities			
		I_P	I_M	C_P	C_M
Output	I	1	1	0	0
	C	0	0	1	1
Input	M	a_1	1	a_2	1
	V	b_1	0	b_2	0

The column headings are the four activities, the subscripts, P and M, stand for production and imports respectively. The first two rows are

[1] H. Chenery and A. M. Strout, "Foreign Assistance and Economic Development," American Economic Review, September 1966; and H. Chenery and P. Eckstein, "Development Alternatives for Latin America," unpublished paper.

output of investment goods and consumer goods, I and C. The last two rows are use of imports, M, and domestic capacity V (for value added).[1] It is assumed that production of investment goods is more import and less domestic input intensive than production of consumer goods. Thus:

$$a_1 > a_2 > 0$$

$$b_2 > b_1 > 0$$

Assume a given capacity to import, and a given domestic production capacity. The constraints on the activities then are

$$M \geq a_1 I_P + a_2 C_P + I_M + C_M$$

$$V \geq b_1 I_P + b_2 C_P$$

For our purposes we thus are ignoring any specialization of domestic production capacity between investment and consumer goods.

These constraints limit the choice set for consumption and investment available to the economy to the frontier a - b - d in Fig. 1.A. If we ignored the two direct import activities the frontier would be a - b - c. But because import capacity not used up for intermediate goods can be directly converted, one to one, for consumer and capital goods, to the right of b the frontier is b - d, not b - c.[2]

---

[1] If V and M are measured in dollars, then $b_1 = 1 - a_1$ and $b_2 = 1 - a_2$.

[2] If V is measured in dollars, then the slope of the b - d facet can be shown to be -1. The slope of the a - b facet is $-\frac{a_2}{a_1}$.

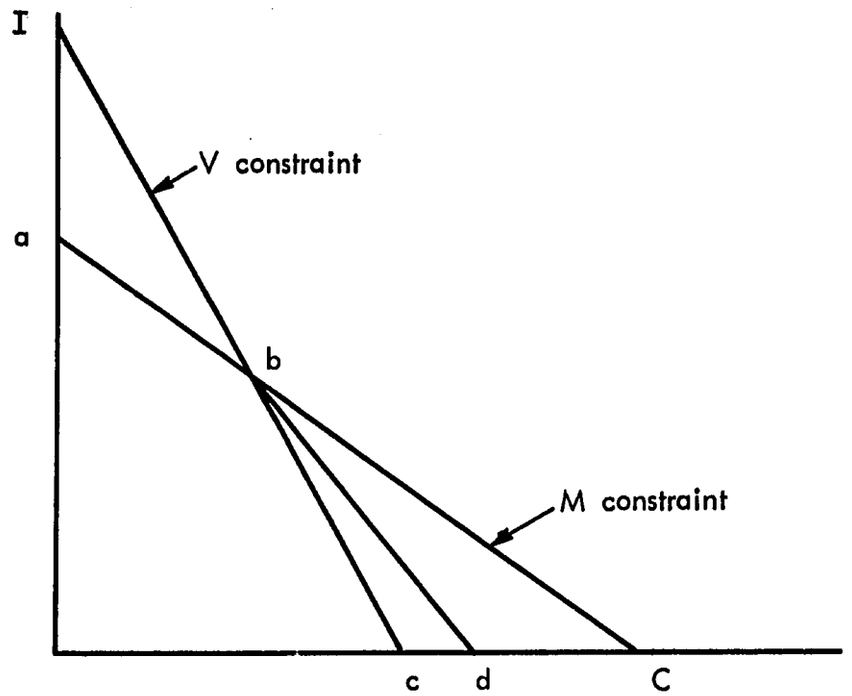
I
V constraint
a
b
M constraint
c
d
C

Fig. 1A

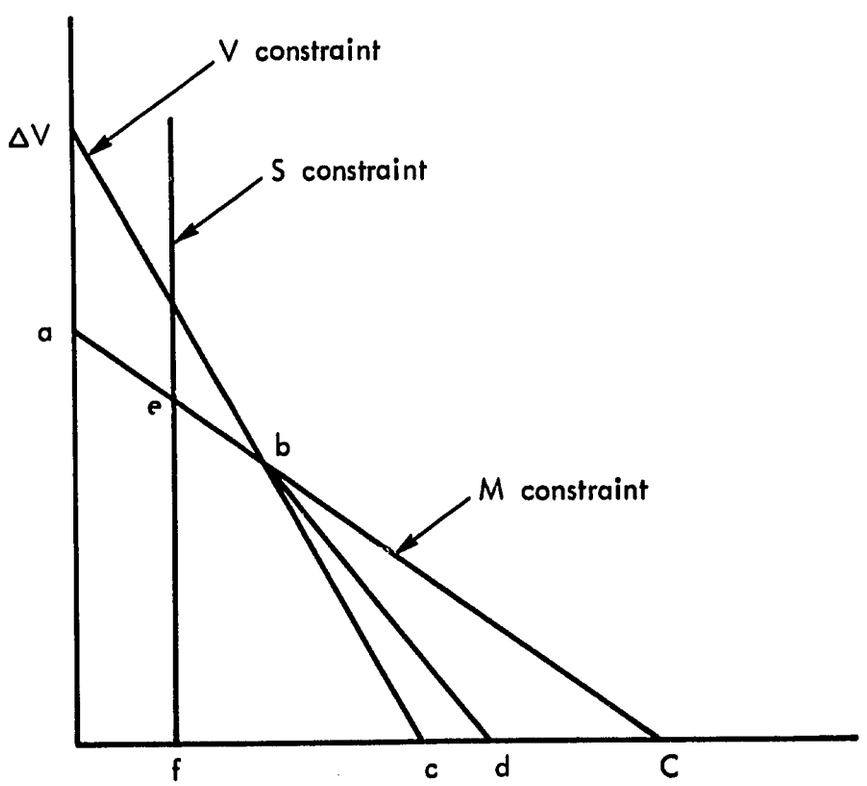
V constraint
ΔV
S constraint
a
e
b
M constraint
f
c
d
C

Fig. 1B

If the economy is operating in the b - d range, both domestic capacity and import capacity will be fully employed, imports will be employed in production of consumer goods and investment goods, and, in addition, some consumer and investment goods will be directly imported. Along a - b import capacity will be fully utilized, but there will be unutilized domestic capacity. The asymmetry between imports and domestic production capacity is fundamental to the two-gap model. Domestic inputs need to be complemented by imports to produce final product. Imports, in contrast, can directly provide final product (as well as complement domestic inputs).

The standard two-gap model assumes a constant capital to value added (or capital-output) ratio. Although labor may be required in production as well, it is assumed labor is in excess supply. With value added proportional to the capital stock, the increase in value added then will be proportional to the amount of investment. Or (with some changes in units) the earlier figure really depicts the growth, consumption, opportunities open to the economy at any time.[1]

In Fig. 1.B., the investment axis has been relabeled and calibrated, and a "savings" constraint introduced. In a later section I shall discuss this constraint in more detail, in particular its connection with constraints on fiscal and monetary policy. At the present, however, it can be best thought of as a "minimal consumption" constraint. Thus the economy is limited to areas to the right of the savings constraint, as well as below the two input constraints--that is, to the frontier e - b - d.

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[1]Specifically $\Delta V = B I$. Obviously depreciation is being ignored.

To bring this formulation in line with the more conventional formulation it is necessary to "get rid" of the V constraint.[1] This we can do by having the horizontal axis refer to $\frac{C}{V}$ and the vertical axis to the growth rate, $\frac{\Delta V}{V}$, which is proportional to $\frac{I}{V}$. The former V constraint now becomes a constant in the problem and over time (so long as b_1 and b_2 do not change). The two constraints in Fig. 2 that can be manipulated are the "savings" constraint and the foreign exchange constraint. We are back now with Vanek and Chenery. Much of their analysis, of course, is concerned with estimating the effects of shifting one or the other of these constraints.

Regarding the savings constraint, notice that the effect on the growth rate of shifting the savings constraint to the left (reducing minimal $\frac{C}{V}$) is significantly greater when the savings constraint is to the right of b than when (as depicted above) it is to the left of b. If the import coefficient for investment goods is two and a half times that for consumer goods (this may actually understate the difference), then the consumer good cost of a unit increase in investment (or a unit increase in the growth rate) is two and a half times as great to the left of b than to the right of b.[2] Vanek is arguing, in fact, that the current position is to the left of b; thus increased savings

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[1]By "get rid" I mean, of course, to eliminate it as a variable in the formulation of the problem. The domestic resource constraint on production always will exist (although it may not always be binding).

[2]See footnote 2, p. 5. We are assuming here that $a_1 = .5$, $a_2 = .2$. These are rough estimates of the direct and indirect dollar (or peso) import content per dollar (or peso) of investor and consumer goods respectively in Colombia.

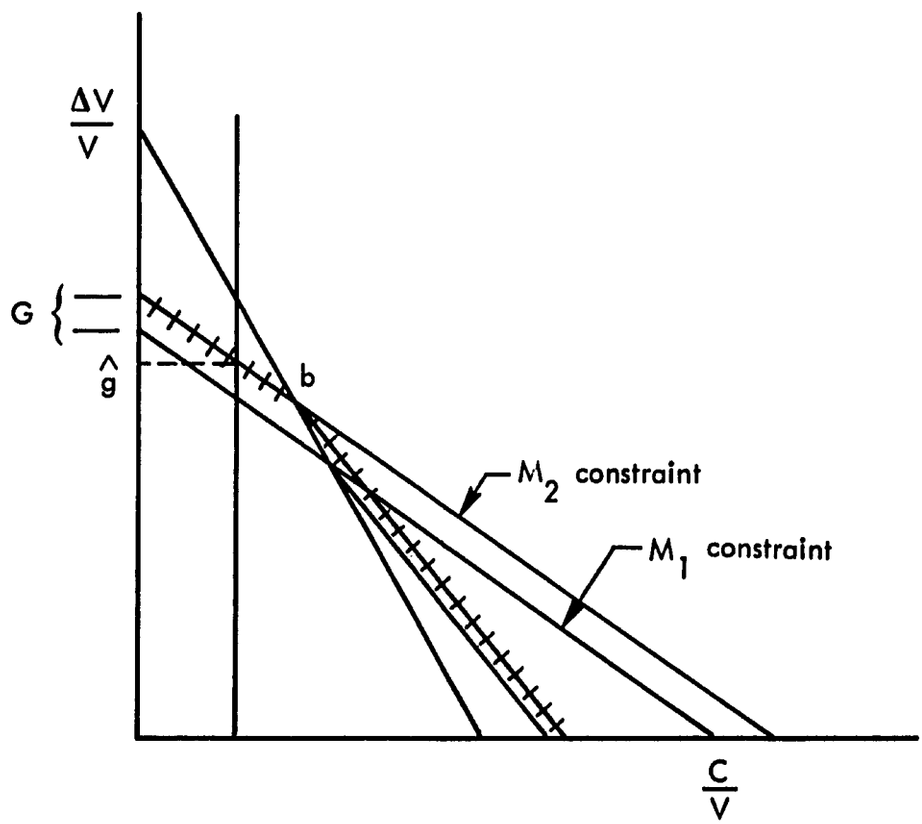

Fig. 2

(decreased consumption) will have limited effect on growth. And if this model does reflect reality, not only would it take a lot of reduced consumption to gain only a little added growth, there would also be rising unemployment. Although all of the imports released by cutting back consumption would be used for investment activity, the greater import intensity (lesser domestic input intensity) of investment means that the domestic resources so released would not be fully reemployed in investment. Thus employment and growth objectives are in conflict.

The relative returns to "more foreign assistance" (holding consumption constant) also vary depending on whether the economy is operating to the left or right of b. In Fig. 2 notice how much more the frontier is shifted upward to the left, as compared with to the right of b, as a result of an increase in import capacity from M_1 to M_2.

In the model, as posed above, an increase in savings, like an increase in import capacity, will always permit an increase in the growth rate.[1] Thus it is misleading to say that one or the other of these constraints is <u>not</u> binding at any time. Nevertheless there clearly are sharp differences in the returns to more savings and returns to more imports, in terms of an enhanced growth rate, depending on whether one is to the left or right of b. In a loose sense one can

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[1]In some of the earlier formulations imports used in the production of consumer goods were ignored. Thus a_2 was assumed to be zero--hence the curve of the import constraint to the left of b is flat. In this case reduced consumption does not permit any increase in investment, once consumer goods imports have been eliminated (that is, to the left of b).

say that to the right of b more savings is needed; to the left of b more import capacity is needed. This is the spirit of the two-gap distinction.

The foreign exchange "gap," which Vanek attempts to estimate, can be defined as the increase in import capacity needed to achieve a given growth target, given the savings constraint. If, in Fig. 2, the growth target is $\hat{g}$, the foreign exchange gap as a function of V is proportional to the distance between M_1 and M_2. That is, if in the absence of assistance the foreign exchange constraint is M_1, an increase in foreign exchange proportional to G on the vertical axis will be just sufficient to enable the growth objective to be met, given the existing savings rate.[1] (As a bonus the unemployment rate would be reduced as well.) Of course the "gap" would be smaller if the savings constraint could be pushed left. But given that the constraint already is to the left of b, it takes a lot of increased savings to gain only a slight reduction in the "gap."

I believe that this allegory has considerable validity as a description of the problems facing Colombia today, at least the short run problems. The system is plagued by both a low growth rate and high unemployment. So long as the import constraint is not relaxed, attempts

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[1]Specifically, the foreign exchange gap in dollars as a fraction of V is $\frac{a_1}{B}$ G where B is the value added to capital ratio, a_1 is the import intensity of investment, and $M_2 - M_1$ is the vertical distance between the M_2 and M_1 constraints. This can be shown as follows. The required increase in the growth rate, given the savings constraint, is G. This will require an increase in $\frac{I}{V}$ of $\frac{G}{B}$. The import requirements are $\frac{a_1}{B}$ G.

to resolve one problem through strictly internal policies are likely to compound the other. The other side of the coin is that additional aid could be extremely productive.

It is important to note, however, that a major cause of the difficulties is limited ability to substitute domestic resources for imports; in the model there are no substitution possibilities at all. For consideration of short run options this assumption probably is quite realistic, although even for the short run overly strong.

When the focus is on the long run, the assumption of no substitution possibilities clearly is absurd. Indeed, for at least the last 20 years the basic thrust of Colombia's industrial development has been import substitution, and policy has deliberately and effectively stimulated this process. Several of the two gap models do augment the basic core with a time trend in the coefficients.

Yet when focusing on the long run, and factor substitution is admitted--indeed is a major objective of policy--the question of incentives, costs, and factor prices, repressed in the conventional formulation, begs for attention. Colombia's problems are not new to her; indeed Colombia has been suffering from a shortage of imports, and a growing unemployment problem, since the coffee crisis of the mid 1950s, and with extreme severity for at least the last six years. In such a chronic situation the cost of imports relative to domestic factor prices ought to matter. Is it possible that Colombia has compounded her problems, and failed to consider an important path toward alleviating them, by having too high an effective exchange rate?

This is the basic question explored in the remainder of this

Memorandum. But before proceeding with the model, it seems important to digress briefly on the sensitivity of the exchange rate or exchange rate structure viewed as an instrument of policy, to policy objectives--that is on the concept of an "optimal" exchange rate structure.[1] (For simplicity here let me discuss the issue as if there were a single rate.) In recent years many people have suggested that the peso is overvalued, that the exchange rate is too low and ought to be increased. This obviously is a normative judgment. As such it must be related to policy objectives, and in the following sections I shall develop the argument that the objectives of achieving higher employment and more rapid growth argue for a higher exchange rate. But the key point that needs to be understood is that the "optimal" exchange rate cannot even be defined independently of specification of policy objectives (or a welfare function to be maximized). The "optimal" exchange rate varies with the objectives. If higher real wages for those employed, rather than greater employment, is the objective, or higher consumption rather than more investment, then the "optimal" exchange rate would be lower.

To most people the idea that the optimal exchange rate is a function of policy objectives seems reasonable enough. Yet there seems some resistance to an implication that one cannot define, much less measure, the equilibrium exchange rate independently of policy objectives. I will not engage, here, in any exegesis of the conditions under which the optimal exchange rate also will be the rate that equilibrates supply and demand for foreign exchange in a competitive

[1]Obviously the exchange rate structure involves not only a possible multiplicity of rates, but also tariffs, quotas, etc.

exchange market. Yet whether or not these conditions hold, it obviously is the case that the equilibrium exchange rate is not independent of a nation's objectives, or, more accurately, the policies employed to pursue them. If a country is willing to tolerate a high unemployment rate and a low growth rate, the equilibrium, as well as the "optimal" exchange rate, obviously is lower than if policies are pursued to reduce unemployment and increase investment. The demand for foreign exchange obviously will be lower in the first case than in the second and so will, and ought, its price.

Thus the equilibrium exchange rate certainly does reflect objectives and policies pursuing them, and it should. The following discussion is concerned explicitly with the "optimal" rate of exchange. But implicitly it also is an analysis of what will happen to the equilibrium rate when a government attempts to implement policies aimed at reducing unemployment and increasing growth.

III. A NEOCLASSICAL INPUT, OUTPUT, AND PRICE MODEL

It seems apparent that any model designed to explore the possibility of substituting domestic inputs for imports (and for exploring the possibility of increasing import capacity through exports) must incorporate explicitly the following characteristic of the Colombian situation. Imports are largely intermediate goods in consumer or producer goods industries, and direct purchases of capital goods, mostly machinery. Thus if one thinks of the economy as having two sectors--one producing consumer goods, the other concerned with building new plants and equipment--the key substitution possibilities are domestic resources--both capital and labor--for imports in both, not substitution of domestically produced consumer goods for imported final consumer goods. And the analysis of these substitution possibilities must recognize that one of the domestic inputs--capital--itself has an import content. Similarly in exploration of export opportunities one must recognize that imports go into exports, both through intermediate inputs and capital goods.

This makes for a rather complicated model. But it seems essential to build in this key characteristic explicitly and make simplifications elsewhere. One of the major simplifications that has been required is the use of Cobb-Douglas relationships throughout. The principal reason is that the structure of import demand described above leads to a webb of multiplicative relationships that are tractable under Cobb-Douglas assumptions, and next to impossible under other forms. There certainly are difficulties with using Cobb-Douglas throughout, in particular one would be tempted to assume a less than unitary elasticity

of substitution between domestic inputs and imports in investment.[1] However, this assumption does bring one partially offsetting bonus. It will be useful to experiment with some illustrative numbers and the parameters of a Cobb-Douglas model (like the parameters of a fixed coefficient) are easily estimated.

A second simplifying assumption is that the capital-labor ratio does not differ for investment and consumption. More explicitly, it will be assumed that there are three basic inputs--capital, labor, and imports, which are employed in both consumption and investment activity. However, the domestic input of capital and labor can be viewed as producing a domestic value added input, as follows:

$$V = A\, L^{\alpha}\, K^{1-\alpha} \tag{1}$$

where L and K stand for labor and capital respectively, and V is value added in real terms (the meaning of which will be apparent shortly). A is a productivity index, which may change over time. Consumption goods are produced from domestic inputs of capital and labor, and imports, as follows:

$$C = Z\, M_c^{\delta}\, V_c^{1-\delta} \tag{2}$$

where C and M are consumer goods and imports and V was defined above. Z is a constant that will not be important in the following analysis;

[1] It can be argued that a Cobb-Douglas production function provides a good first approximation to the substitution possibilities in a more general neoclassical model, provided that the (arc) elasticity of substitution is not very low or very high, and the changes in factor prices are not great. See R. R. Nelson, "The CES Production Function and Economic Growth Projections," Review of Economics and Statistics, August 1965.

all such unimportant constants will be denoted by Z without a subscript. New capital goods, I, are produced according to the following production function:

$$I = Z\, M_I^b\, V_I^{1-b} \qquad (3)$$

The Z in Eq. (3) will in general not be the same as in Eq. (2). Notice that both (2) and (3) can also be written in terms of the basic factors of production.[1] The simplifying assumption that permits (2) and (3) to be written in terms of M and V (as well as in M, L, and K) is not necessary, but very convenient.

Note that Eqs. (2) and (3), if taken literally, imply that all imports are intermediate goods. However, they can be interpreted as admitting the possibility of final, as well as intermediate, good imports, but constraining the mix of imported and domestic investment and consumption goods that can be sold (prices will be introduced later).

Also note that the model, if taken literally, assumes that domestic resources are perfectly fungible between investment and consumption. (This also is a characteristic of the basic two-gap model discussed in the preceding section.) This is not as bad as it seems. As will be discussed at some length later, the characteristics of the model imply that domestic resources, transferred from consumption to investment, or the other way, yield decreasing returns (yield increases

---

[1] Thus

$$C = Z\, M^{\delta}\, L^{\alpha(1-\delta)}\, K^{(1-\alpha)(1-\delta)}$$
$$I = Z\, M^{b}\, L^{\alpha(1-b)}\, K^{(1-\alpha)(1-b)}.$$

in output at the expense of rising marginal cutbacks in the other). Thus the model can be interpreted as implying a degree of specificity of domestic resources. Put another way, the specification of output as a Cobb-Douglas function of domestic inputs and imports (with different coefficients for consumption and investment) captures the specificity of domestic resources.

In order to lend analytic bite to the model it is necessary to make tractable assumptions about pricing and factor choice. Unfortunately I have not yet been able to develop an alternative to the competitive assumptions that permits reasonably easy crank turning. Therefore, it will be assumed that producers act as if labor is supplied elastically, that product markets are elastic, and given the existing capital stock hire labor up to the point where the value of the marginal product equals price. It also will be assumed that investors demand a certain rate of return on capital and that the capital stock is adjusted to achieve that rate of return; thus this is a long run equilibrium model.[1]

Let W be the going wage rate, i be the equilibrium rate of return on capital, P_I be the price of capital goods, and $r = P_I\, i$ the "rental" rate of a unit of capital. Then we have the following equilibrium relationships between inputs and outputs, and their prices.

$$\frac{L}{V} = \frac{\alpha P}{W} \tag{4}$$

$$\frac{K}{V} = \frac{(1 - \alpha) P}{r} \tag{5}$$

[1]This "equilibrium" rate of return will later be treated as a variable, as will the wage rate.

where P is the "price" of domestic value added. Note that (4) simply reflects the labor employment assumption, and (5) is the condition under which there will be a rate of return, i, on capital.[1]

The relationships can be viewed in three ways. First, they are the familiar "constant share" implications of a Cobb-Douglas competitive model. Second, at the same time they are estimating equations for α and $(1 - \alpha)$ which are, simply, the observed shares at any time. As discussed above, this simplicity of estimation is one of the motivations for the Cobb-Douglas formulation. Third, they are equations for the input coefficients (as a function of prices).

From the consumer goods production function:

$$\frac{M_C}{C} = \frac{\delta P_c}{E} \tag{6}$$

$$\frac{V_c}{C} = (1 - \delta) \frac{P_c}{P} \tag{7}$$

where P_c is the price of consumer goods, and E the exchange rate. And, from the investment goods production function:

$$\frac{M_I}{I} = \frac{b P_I}{E} \tag{8}$$

$$\frac{V_I}{I} = \frac{(1 - b) P_I}{P} \tag{9}$$

P_I, as above, is the price of a new capital good. In these formulations the value of inputs is proportional to the value of outputs;

[1]These equations of course are derived from solving for the conditions under which the value of the marginal product of labor equals the wage rate, and the value of the marginal product of capital equals capital's rental rate.

not as in the fixed coefficient model quantity proportional to quantity. And δ and b can be estimated from one observation.

Finally we can derive equations for the prices of various outputs as a function of the prices of various inputs. First we can write:[1]

$$P = \frac{Z}{A} W^{\alpha} r^{1-\alpha} \tag{10}$$

$$P_c = Z E^{\delta} P^{1-\delta} \tag{11}$$

$$P_I = Z E^{b} P^{1-b} . \tag{12}$$

It remains to get (10)-(12) in terms of the prices we will treat as basic--the rate of return on capital, the wage rate, and the exchange rate.

---

[1]The derivations are straightforward if tedious. For the first:

(1) $P = \frac{L}{V} W + \frac{K}{V} r$

(2) $\frac{V}{K} = A(\frac{L}{K})^{\alpha}$ so $\frac{K}{V} = \frac{(\frac{L}{K})^{-\alpha}}{A}$ from Eq. (1) in the text.

$$\frac{L}{V} = \frac{L}{K} \cdot \frac{K}{V} = \frac{(\frac{L}{K})^{1-\alpha}}{A}$$

(3) $\frac{L}{K} = (\frac{\alpha}{1 - \alpha}) \frac{r}{W}$ from Eqs. (4) and (5) in the text.

thus:

(4) $P = \frac{(\frac{\alpha}{1 - \alpha})^{1-\alpha} (\frac{r}{W})^{1-\alpha} W + (\frac{\alpha}{1 - \alpha})^{-\alpha} (\frac{r}{W})^{-\alpha} r}{A}$

(5) $P = \frac{\alpha [(\frac{W}{\alpha})^{\alpha} (\frac{r}{1 - \alpha})^{1-\alpha}] + (1-\alpha) [(\frac{W}{\alpha})^{\alpha} (\frac{r}{1 - \alpha})^{1-\alpha}]}{A}$

(6) $P = \frac{Z}{A} W^{\alpha} r^{1-\alpha}$

The other two equations are derived in the same way.

Recall that $r = P_I i$. Assuming foreign prices are fixed, using (10)-(12) one can derive:[1]

$$P = \frac{Z\, i^{\frac{(1-\alpha)}{\lambda}}\, W^{\frac{\alpha}{\lambda}}\, E^{\frac{b(1-\alpha)}{\lambda}}}{A^{\frac{1}{\lambda}}} \tag{13}$$

$$P_c = \frac{Z\, i^{\frac{(1-\alpha)(1-\delta)}{\lambda}}\, W^{\frac{\alpha(1-\delta)}{\lambda}}\, E^{\frac{(1-\alpha)b+\alpha\delta}{\lambda}}}{A^{\frac{1-\delta}{\lambda}}} \tag{14}$$

$$P_I = \frac{Z\, i^{\frac{(1-\alpha)(1-b)}{\lambda}}\, W^{\frac{\alpha(1-b)}{\lambda}}\, E^{\frac{b}{\lambda}}}{A^{\frac{1-b}{\lambda}}} \tag{15}$$

where $\lambda = \alpha + b - \alpha b$. With α and b both positive but less than one, so must be λ. Two other price equations will be useful. First the "rental" on capital:

$$r = \frac{Z\, i^{\frac{1}{\lambda}}\, W^{\frac{\alpha(1-b)}{\lambda}}\, E^{\frac{b}{\lambda}}}{A^{\frac{1-b}{\lambda}}} \tag{16}$$

---

[1]The first equation is derived as follows:

$$(1) \quad P = \frac{Z}{A} W^{\alpha} \left[i\, Z\, E^{b}\, P^{1-b} \right]^{1-\alpha}$$

where the term in brackets is the substitution for $r = P_I i$ from Eq. (12).

$$(2) \quad P^{(\alpha+b-b\alpha)} = \frac{Z}{A} W^{\alpha}\, i^{(1-\alpha)}\, E^{b(1-\alpha)}$$

$$(3) \quad P = \frac{Z\, i^{\frac{1-\alpha}{\lambda}}\, W^{\frac{\alpha}{\lambda}}\, E^{\frac{b(1-\alpha)}{\lambda}}}{A^{\frac{1}{\lambda}}}$$

Second, the real wage rate--that is the money wage rate divided by the price of consumer goods:

$$\frac{W}{P_c} = W^* = \frac{Z\, i^{\frac{-(1-\alpha)(1-\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{(1-\alpha)b+\alpha\delta}{\lambda}}}{A^{\frac{-(1-\delta)}{\lambda}}} \tag{17}$$

Notice how the structure of imports, described earlier and explicitly built into the model, is reflected in the price equations. Even the price of "domestic inputs" is not independent of the exchange rate; the reason, of course, is the effect of the exchange rate on the price (and rental) of capital as shown in Eqs. (15) and (16). The exchange rate influences the price of consumer goods through both the price of machinery and the price of intermediate goods imports. Thus the real wage (Eq. (17)) involves the exchange rate, as the diatribe relating to the effect on the cost of living of devaluation has stressed. And if, as we shall develop later, it is assumed that demand for Colombian exports are sensitive to the exchange rate, one cannot ignore that the cost of exports may be as well.

The model obviously is strictly neoclassical. It assumes competitive factor and product markets in the short run, and the equations relate to long run equilibrium conditions (in the sense that the capital stock is just such that the target rate of return, i, is achieved). This is a bothersome set of assumptions. I do not know the extent to which the implications derived in the following sections are sensitive to either the Cobb-Douglas specification, or the short run competitive equilibrium assumption, or the long run equilibrium

assumption. The specific quantitative results certainly are. I suspect, however, that most of the qualitative results would hold under much less restrictive, and much more realistic, assumptions. Certainly the implications above regarding prices are qualitatively what one would expect. And these would appear to be sufficient to generate the qualitative results on inputs and outputs derived in the following sections.

An important caveat is on the time frame of the model. The dual assumptions of a considerable (unitary) elasticity of substitution among the factors, and of short and long run equilibrium mean that the effects of changes in factor prices examined in the following sections cannot be assumed to occur overnight. The model is concerned with the long run implications of these changes. I admit I do not know how long the long run is. My conjecture is that something like a five year adjustment period is involved.

IV. THE CONSUMPTION-INVESTMENT CHOICE SET RECONSIDERED

It now is possible to reexamine the consumption-investment choice set within a model that admits input substitution. Recall the following specification of input coefficients in the production of investment and consumer goods in the presentation of the fixed coefficient two-gap model.

$$\frac{M_I}{I} = a_1 \qquad \frac{M_c}{C} = a_2 \qquad \frac{V_I}{I} = b_1 \qquad \frac{V_c}{C} = b_2 \quad .$$

Assuming the same capital labor ratios in both consumer and producer goods production:

$$\frac{K}{V} = c_1 \qquad \frac{L}{V} = c_2 \quad .$$

Ignoring the direct import activities the constraints of the two-gap model can be rewritten:[1]

$$M \geq a_1 \; I + a_2 \; C$$

$$K \geq c_1 \; b_1 \; I + c_1 \; b_2 \; C$$

$$L \geq c_2 \; b_1 \; I + c_2 \; b_2 \; C \; .$$

In terms of capital actually employed, K*;

$$L \geq \frac{c_2}{c_1} K^*$$

where, of course, $K \geq K^*$.

[1]The analysis will be focused on the region to the right of "b" in Fig. 3; hence there will not be any direct imports.

In our new formulation the constant input coefficients are turned into variables, by Eqs. (4)-(9). Equations (10)-(16), in turn, permit these input coefficients to be written in terms of the basic prices i, W, and E. The constraints then take the form:[1]

$$M \geq \frac{Z\, i^{\frac{(1-\alpha)(1-b)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-b)}{\lambda}} I}{A^{\frac{(1-b)}{\lambda}}} + \frac{Z\, i^{\frac{(1-\alpha)(1-\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-\delta)}{\lambda}} C}{A^{\frac{(1-\delta)}{\lambda}}} \tag{18}$$

$$K \geq (1-\alpha)(1-b)i^{-1}\, I + \frac{Z(1-\alpha)(1-\delta)\, i^{\frac{-(\alpha+\delta-\alpha\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(b-\delta)}{\lambda}} C}{A^{\frac{\delta-b}{\lambda}}} \tag{19}$$

$$L \geq \frac{Z\, i^{\frac{1}{\lambda}} \left(\frac{W}{E}\right)^{\frac{-b}{\lambda}} K^*}{A^{\frac{1-b}{\lambda}}} \tag{20}$$

The relationships have the derivatives one would expect. Import requirements of a given C, I bill are negatively related to the

---

[1]The first coefficient is derived as follows:

$$a_1 = \frac{M_I}{I} = \frac{b\, P_I}{E} \quad \text{from Eq. (8).}$$

$$\text{But } \frac{P_I}{E} = \frac{Z\, i^{\frac{(1-\alpha)(1-b)}{\lambda}}\, W^{\frac{\alpha(1-b)}{\lambda}}\, E^{\frac{b}{\lambda}-1}}{A^{\frac{1-b}{\lambda}}} \quad \text{from Eq. (15).}$$

Noting that $\lambda = \alpha + b - \alpha b$ and simplifying

$$a_1 = \frac{b\, Z\, i^{\frac{(1-\alpha)(1-b)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-b)}{\lambda}}}{A^{\frac{1-b}{\lambda}}} .$$

exchange rate, positively related to domestic factor prices W and i, reflecting the possibilities to substitute domestic inputs for imports. Capital requirements in the production of consumer goods are positively related to the wage rate, reflecting that a rise in labor costs induces substitution of capital for labor. They are negatively related to the exchange rate which influences the cost of a new machine, and to the expected rate of return on capital. Capital requirements per capital good produced are related only to the expected rate of return. At first glance this is a strange result, but it reflects simply the requirement that capital yield its "own" rate of return in the production of new capital (and the Cobb-Douglas assumptions). Labor employed per unit of capital depends negatively on W, and positively on the exchange rate (for the reason above) and on i.

For any given $(\frac{W}{E})$ and i there is a set of linear constraints on C and I, just as in Section II. A number of arbitrary constants, Z, in effect permit us to define units as we wish. Thus we can define both consumption and investment units as one peso or dollar's worth at some set of basic input prices, just as in Section II. Then, if we assume that capital goods are more import intensive and less domestic input intensive than consumer goods ($b > \delta$), the M constraint, and the K and L constraints, will have the relative slopes shown in Fig. 3. The special form of Eqs. (1)-(3), which together with cost minimization imply the same capital-labor ratio in both consumer and capital goods production, means that the labor and capital constraints have the same slope. The V constraint of Section II can be interpreted as the more binding of the K and L constraints in Fig. 3.

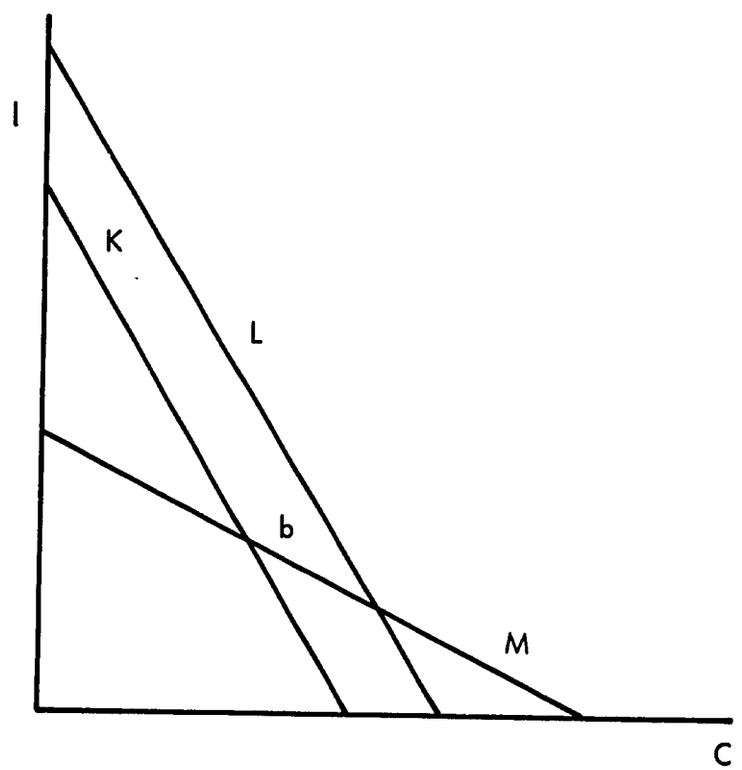

Fig. 3

With two products, three inputs, and a given set of factor prices, in general full employment of all inputs will be impossible. Thus in Fig. 3 there always will be unemployed labor. Further, unless the mix of output is right, one of the other inputs will be underemployed as well. At a high investment rate the import constraint will be binding and capital will be slack; at a high consumption rate the other way around. Of course to the right of b we cannot ignore direct imports of consumer and investor goods, but this range will not concern us in the following analysis. Thus we have, in effect, replicated the two-gap model.

But Fig. 3 assumes a given set of factor prices. We now are able to examine the effects of factor prices on the conclusions of the two-gap model. As factor prices change, the constraints will shift. A decrease in $(\frac{W}{E})$ or i will reduce the imports utilized for a given output and shift out the foreign exchange constraint.[1] Capital constraint can be shifted out by a decline in $(\frac{W}{E})$, or an increase in i. The labor constraint can be shifted out by a rise in $(\frac{W}{E})$ or a decline in i. There exists an i, $\frac{W}{E}$ combination such that, for any consumption-investment ratio, full employment of all three factors is possible and will be achieved. Further, if these factor prices obtain, the output of the economy, given its M, K, L endowments, is maximal along that C, I ray.

To see this, assume a given target ratio of $\frac{I}{C}$. Then, ignoring the labor constraint, one can plot alternative i, $\frac{W}{E}$ combinations that

[1] A change in $(\frac{W}{E})$ will not change capital requirements if all of output is investment.

will assure full utilization of the capital stock and full use of given foreign exchange. The K, M curve of Fig. 4 slopes down from left to right reflecting that an increase in either i or $\frac{W}{E}$ will induce a rise in $\frac{M}{K}$.[1] Therefore to maintain the same ratio, an increase in one must be compensated by a decline in the other. Similarly, ignoring the import constraint, a curve showing alternative i, $\frac{W}{E}$ pairs that will achieve full utilization of capital and full employment can be drawn. The K, L curve slopes up from left to right reflecting that a higher $\frac{W}{E}$ which will itself reduce $\frac{L}{K}$, requires a higher i, which itself would increase $\frac{L}{K}$, for the same capital-labor ratio to be cost minimizing. The two curves must cross.[2] Thus in the flexible coefficients model, full utilization of M, K and L is possible, for the right factor prices. In terms of Fig. 3, for some set of factor prices, the three constraints can all be made to go through the same point.

Output, for a given C, I ratio, will be maximal only at the intersection set of factor prices. If factor prices are in region 1 of Fig. 4, there will be excess demand for foreign exchange, together with unemployment of both labor and capital. In region 3, there will be excess demand for domestic labor and capital, and excess supply of

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[1]From Eqs. (6), (7), (4), (5), and (16), one can solve for the import capital ratio in consumption. From Eqs. (8), (9), (4), (5), and (16), one can solve for the ratio in investment. Both equations have the form

$$\frac{M}{K} = Z\, i^{\frac{1}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-b)}{\lambda}}$$

[2]The third curve in the figure, that of constant real wages, will be discussed later.

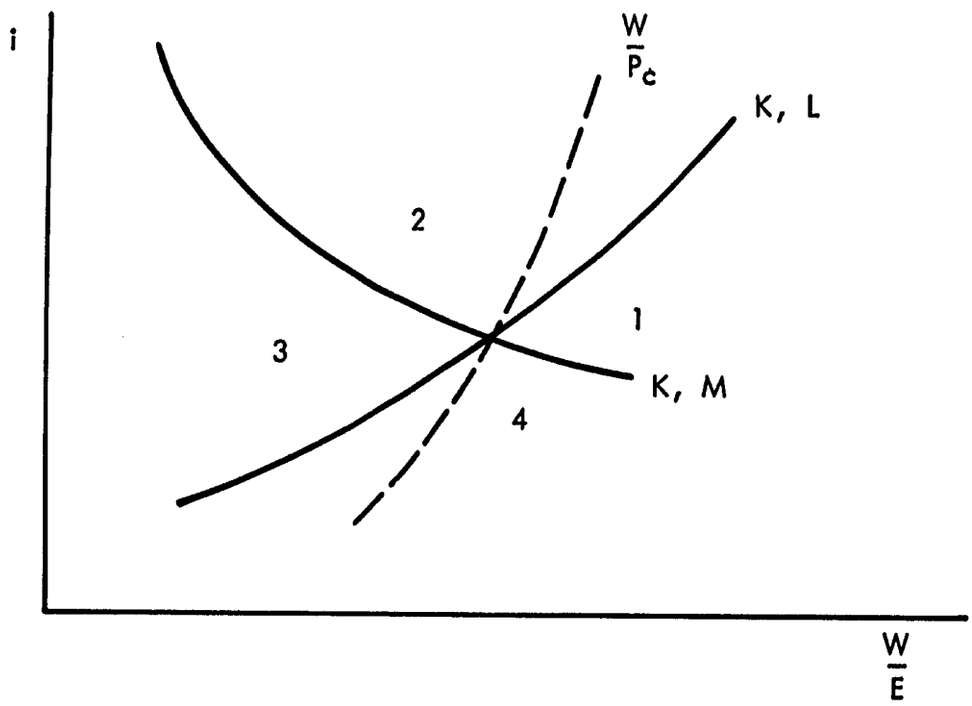

Fig. 4

imports. Regions 2 and 4 also will be marked by unemployment of at least one factor. Since our basic Cobb-Douglas model implies that the marginal productivity of all factors always is positive, it is clear that output cannot be maximal except at the equilibrium set of factor prices.

One important implication is that, with neoclassical substitution possibilities available, the activity analysis frontier cannot be efficient; there is available a neoclassical frontier that dominates it. In Fig. 5 the inner activity analysis frontier represents the case of unemployed labor, even at point b. It replicates Fig. 4. The analysis above demonstrates that for another set of factor prices output can be greater along the ray. The outer activity analysis frontier, through point c, is drawn for the set of factor prices that will generate full employment of labor as well as capital and imports for the consumption-investment ratio indicated by points b and c. But for other C, I ratios, these factor prices will not be efficient, for there will be unemployment of either domestic resources or imports. The neoclassical frontier (the dashed curve) is a full employment of all resources frontier; along the activity analysis frontier full employment of all resources occurs at most at one point.

In the following section we shall map out how factor prices must vary to achieve different C, I ratios along the frontier. Suffice it to flag here that the K, M curve of Fig. 4 has been drawn assuming a given C, I ratio. As that ratio changes, so does the output maximizing and full employment generating set of factor prices.

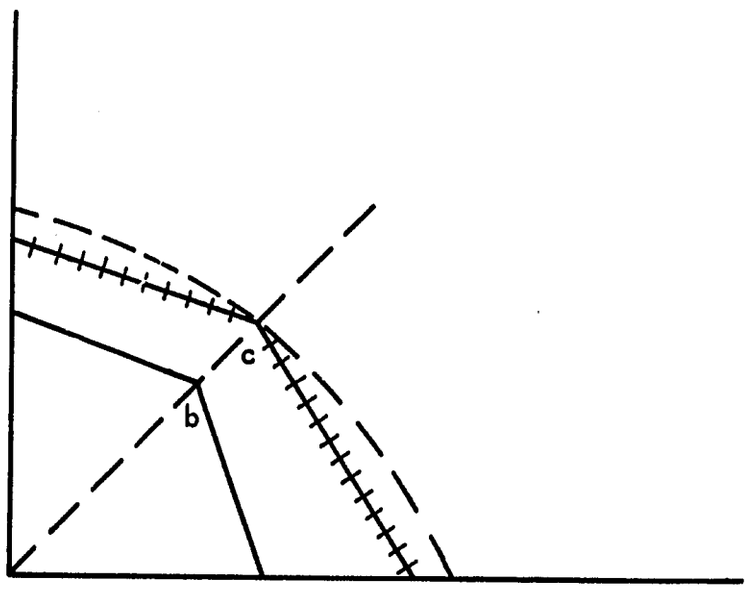

Fig. 5

Thus far we have ignored the possibility that export earnings, and hence import capacity, may also be sensitive to factor prices. Assume

$$M = B + Z\left(\frac{P_c}{E}\right)^{-\epsilon} \tag{21}$$

B can be interpreted as net foreign borrowing. The form of the second term on the right hand side of (21) assumes that only consumer goods are exported, and that foreign exchange proceeds of exports are positively related to the exchange rate, negatively related to their price in home currency.[1]

There is no reason, of course, why the exchange rate for exports has to be the same as the exchange rate for imports. There may be multiple nominal rates; in addition, the structure of tariffs and subsidies may cause these rates to diverge. If they are proportional to each other these discrepancies can be incorporated in the Z of Eq. (21). If they do not always stay in proportion the exchange rate in (21) must have a separate symbol. Although it can be argued that the two rates ought to move together (the assumptions under which they ought to be the <u>same</u> are more stringent) it sometimes is useful to consider them as separate and independent policy instruments, and in a later section I shall do so.

It also sometimes is useful to view the two rates as being in proportion to each other. If this is assumed, then it easily is seen that the introduction of an exchange rate sensitive export term to the import supply constraint simply makes the K, M curve steeper around any

[1] A necessary condition for a positive ϵ of course is that world demand is of greater than unitary elasticity.

observed point. For, solving for $\frac{P_c}{E}$ and substituting

$$M = B + g\left[\frac{i^{\frac{(1-\alpha)(1-\delta)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha(1-\delta)}{\lambda}}}{A^{\frac{1-\delta}{\lambda}}}\right]^{-\varepsilon} \tag{22}$$

Imports thus will be greater at a low $\frac{W}{E}$, and less at a high $\frac{W}{E}$, than were import capacity insensitive to the exchange rate.

With exports admitted, the C, I frontier of Fig. 5 is not, of course, the production possibility frontier. Rather, if it is to depict the consumption-investment opportunities available to the country as a whole for a given K, L endowment, and net foreign borrowing of B, the C of the frontier represents production of consumer goods minus exports. The corresponding I represents investment that is possible given domestic resources and imports (including those made possible by export proceeds, minus those resources and imports employed in the production of consumer goods for home use, C, and exports.[1]

Pulling together the results of this section, we have derived two important implications of admitting substitution possibilities to the basic two-gap model. First, the phenomenon of underutilization of domestic resources at high investment rates no longer is necessary. As we shall see in the following section (but which already should be intuitively obvious), a shift toward investment will require a rise in $\frac{E}{W}$ if employment of domestic inputs is to be maintained. But factor price adjustment can replace the higher unemployment adjustment of the

[1]Thus Eqs. (18)-(20) need to be rewritten with the supply of an input minus its use in exports on the left hand side.

"no substitution" two-gap models. Second, the C, I possibility frontier is both pushed outward and smoothed. The frontier is still concave; that is, the consumption costs of increasing investment rise as one moves from right to left along the frontier. But the sharp kink at b is eliminated.

Of course these conclusions require that factor prices vary as one moves along the frontier. But this would be so for any neo-classical model. Regardless of technological possibilities, no price changes, no substitution. And the qualitative results seem independent of the specific formulation. The admission of continuous substitution policies, or even the admission of a finite number of different activities for the production of C and I, can be expected to expand the possibility set, and smooth out its shape or at least eliminate the single sharp kink characteristic of the two-gap models. And relatedly, one would expect to find the "redundant input" phenomenon of the simple two-gap model converted into a smoother diminishing returns phenomenon where relatively full employment of all factors can be assured by suitable factor price adjustments. The qualitative implications regarding C, I possibilities thus far, then, do not seem to depend too much on the specific formulation of the model.

V. THE POLICY OPTIONS RECONSIDERED

The introduction of factor substitution possibilities suggests that domestic policy may be far less helpless, much more potent, in influencing employment and growth possibilities than the simple two-gap model of Section II suggests. In this section this will be discussed qualitatively; the following section will present some illustrative numbers.

With factor substitution possibilities it is possible to increase employment without reducing investment; indeed it is possible to increase employment and achieve an increase in investment and consumption at the same time. With a given capital stock an increase in employment (a higher labor-capital ratio) shifts the K, L curve of Fig. 4 to the left. For, as Eq. (20) shows, to achieve a higher $\frac{L}{K}$ requires a lower $\frac{W}{E}$ for any i. The simplest case to analyze geometrically is one of no change in the C, I ratio, hence the K, M curve does not shift. The new equilibrium, with greater employment, the same capital stock, and the same level of imports (or imports constrained by the export Eq. (22))[1] requires, then, a lower $\frac{W}{E}$ and a higher i.

The same conclusion can be seen another way. The initial conditions of unemployed labor, but fully employed capital and imports, are depicted by point b in Fig. 3. A lower $\frac{W}{E}$ will shift the M constraint outward (by reducing the import intensity of both consumption and investment), shift in the labor constraint (by inducing a rise in

[1]In either case the K, M curve of Fig. 4 does not shift.

the labor intensity of both activities), and pivot outwards the K constraint (by reducing capital intensity).[1] As shown in Fig. 6.A., output along the given C, I ray will unambiguously be increased. However a fall in $\frac{W}{E}$ alone will lead to a fall in the import-capital ratio (as shown in footnote 1, p. 29). Thus in the new situation the K and M constraints will intersect to the left of c and there will be slack imports. A rise in i will then be required to restore the old capital-import ratio (which is necessary if full employment of both is to be maintained). It is clear that some decrease in $\frac{W}{E}$, and some increase in i is capable of achieving the situation of Fig. 6.B. The K and L constraints have been made to coincide. They intersect the M constraint at d.

The discussion above smacks of the possibility of getting something for nothing. This is both true and untrue. It is true in the sense that the option is open to get both more consumption and more investment. The Keynesian situation obtains. It is untrue in that, as the real wage rate curve on Fig. 4 shows, an increase in employment (with balanced increase in C and I) will require a reduced real wage rate for people who are employed. The shifted left K, L curve means that the real wage rate at the new higher employment equilibrium is unambiguously lower. Thus there is a tension between higher employment and lower real wages for those employed. This is a classical tension, and a politically real one in such countries as Colombia.

Given a level of employment, the costs of shifting the mix of output toward more investment, less consumption, are clearly real.

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[1]The K and C constraints still must have the same slope--thus the L constraint must pivot as well.

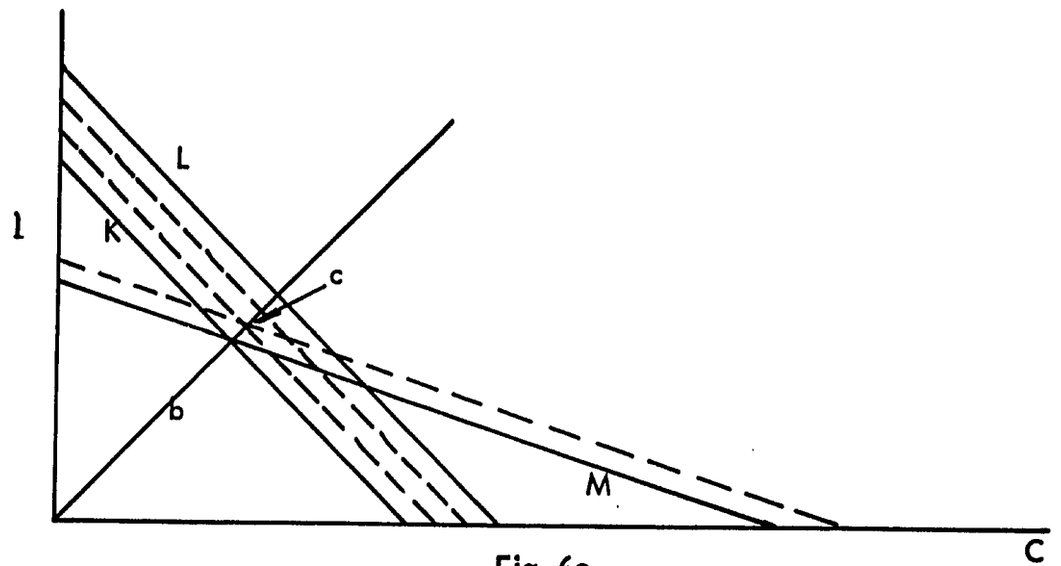

Fig. 6a

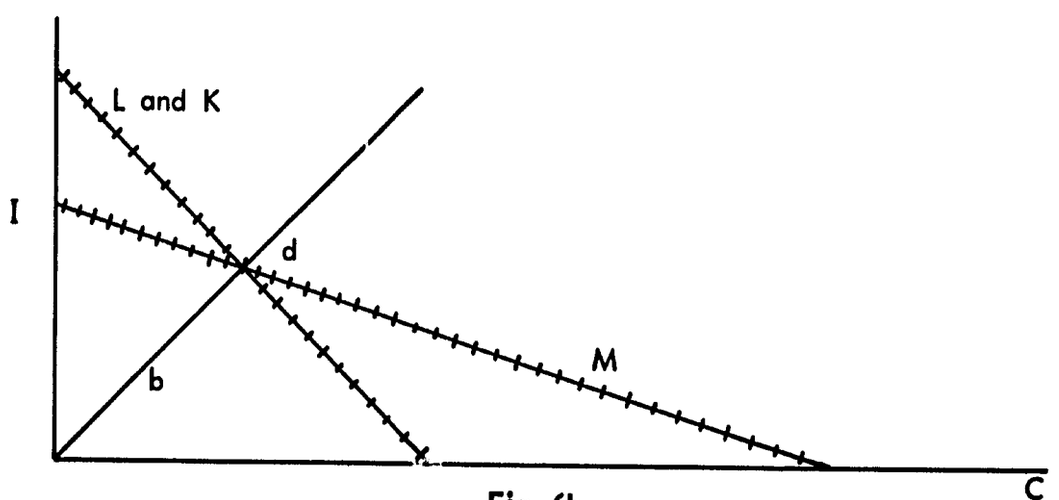

Fig. 6b

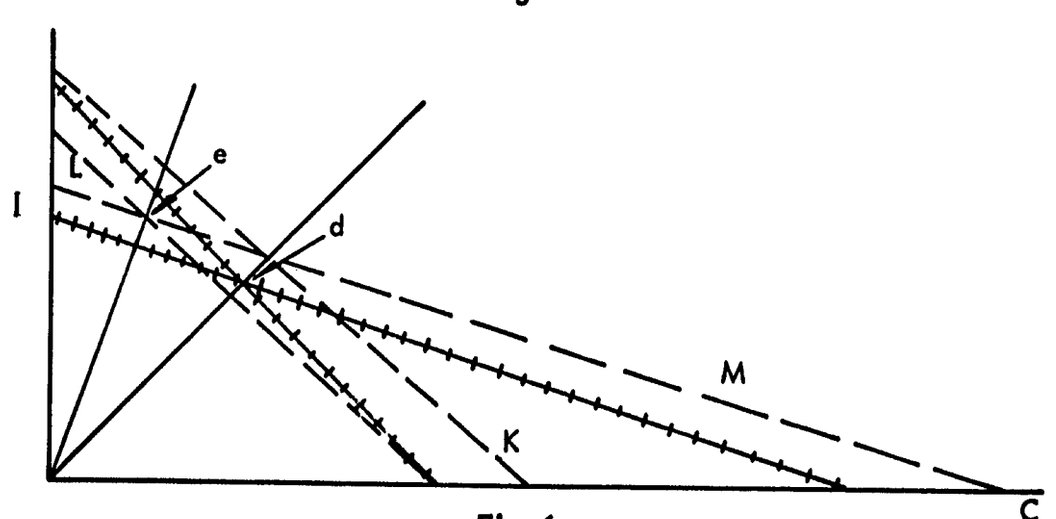

Fig. 6c

But in this model this shift is at least possible to accomplish without the added costs of increasing unemployment. In this model, as the fixed coefficient one, investment is more import intensive, less domestic resource intensive, than consumption. Thus an increase in investment relative to consumption will require a reduction in the import intensity of both. This will require a decline in the price of domestic inputs relative to imports. In terms of Fig. 4, an increase in the investment-consumption ratio shifts downward the K, M curve.[1] The K, L curve does not change reflecting the assumed equality of the capital labor ratio in consumption and investment. (Note the symmetry to the analysis in the preceding paragraph.) Full employment of all factors in the post shift situation thus requires a decline in $\frac{W}{E}$ and a decline in i.

As above, it also is possible to show this conclusion in terms of shifts in the constraints. Starting from the conditions of full employment of all inputs at point d in Figs. 6.B. and 6.C., a decrease in $\frac{W}{E}$ will shift out the import constraint, shift in the labor constraint, and pivot the capital constraint, just as before. Point e, with a higher ratio of investment to consumption, now is achievable with full employment of imports and labor. But with a fall in $\frac{W}{E}$ and no decline in i, the labor capital ratio will rise. With the labor constraint binding both before and after the shift, capital now must be redundant. Thus a decline in i, as well as of $\frac{W}{E}$, is required for full employment of all resources to be achieved at the higher ratio of investment to consumption.

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[1] This is because for a given $\frac{W}{E}$, a reduction in domestic input price relative to E means a lower i.

Putting both maneuvers together indicates the wide range of policy options available in a situation where initially there is unemployed labor. It is clear that a given increase in employment can be used totally to increase investment, or totally to increase consumption, or any of a wide variety of combinations in between--the proportional increase in C and I case is no magic. As shown in Fig. 7, the frontier of possible increases in C and I from a given increase in employment must be concave (just as the C, I frontier in general must be concave).[1] A greater than proportional increase in investment will require a greater decline in $\frac{W}{E}$, and a smaller rise in i, than the proportional increase case; a greater than proportional increase in C will require less of a decline in $\frac{W}{E}$, more of an increase in i. This makes good economic sense. The increase in employment relative to the capital stock calls for a decline in $\frac{W}{E}$ relative to i, but any point along this new K, L schedule will sustain the new higher labor-capital ratio. Since investment is more import intensive than consumption, a high investment-consumption ratio requires a higher exchange rate relative to W and i than does a high consumption-investment ratio.

Just as with the two-gap formulation, it is possible to translate the C, I choice set of the new classical model into a growth rate, consumption rate diagram. (See Fig. 8.) In contrast to the fixed coefficient model, where there is a fixed capital-value added ratio

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[1]This is the well known implication of linear homogeneous production functions, with one of the goods (investment) employing a different factor mix (a higher $\frac{M}{V}$) than the other. The formal proof will not be given here.

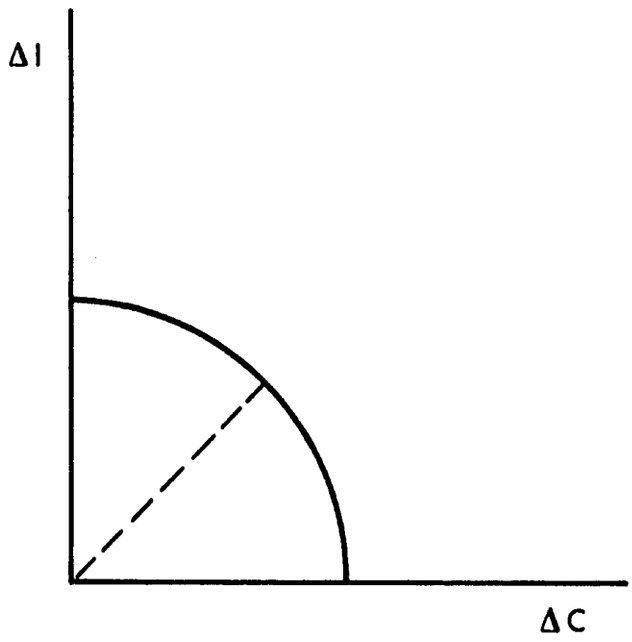

Fig.7

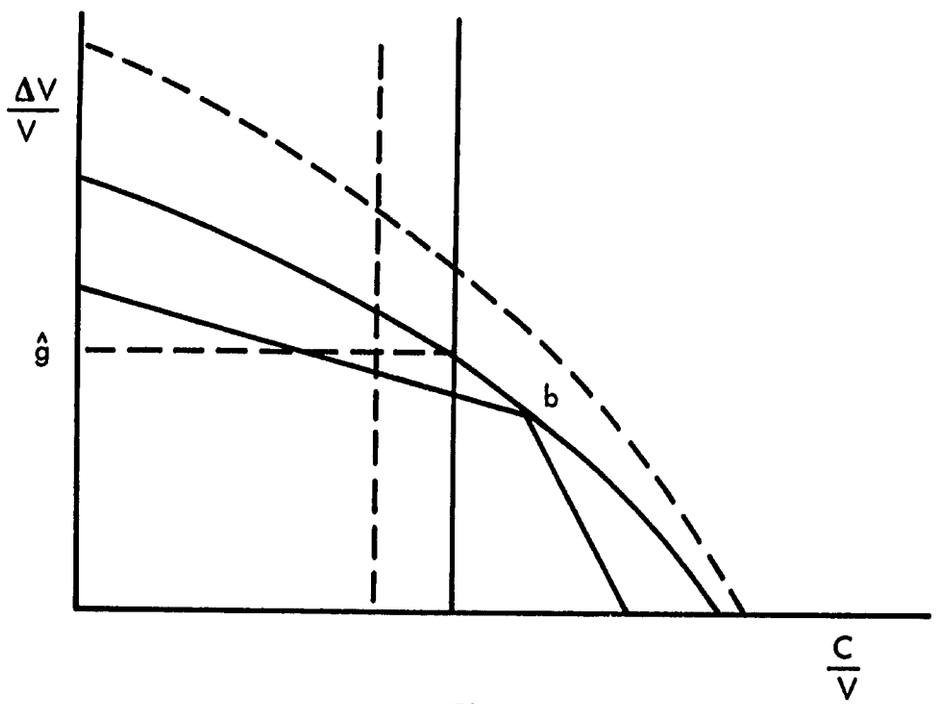

Fig. 8

and hence the growth rate is uniquely related to $\frac{I}{V}$, in the neoclassical model the growth rate also is related to the capital-labor ratio, the rate of growth of the labor force, and productivity growth, as will be discussed later. But taking these variables as given, one can derive Fig. 8 from Fig. 6.

In Fig. 8, if the target growth rate exceeds $\hat{g}$, there will be a foreign exchange "gap." But if we are to the right of the savings constraint, say at point b, increases in savings can do more to close the gap than is implied by the fixed coefficient model. Further, if we are on the savings constraint, but there is unemployed labor, by moving to the neoclassical frontier the gap can be reduced.

Notice that with the smoothing out of the frontier--the disappearance of the kink--there also disappears the split of the diagram into two regimes--one with high growth returns from a shift left of the savings constraint and low returns to increased import capacity, the other with high returns to greater import capacity and low returns to a higher savings rate. Instead there are smooth diminishing returns to both increased savings and increased aid alone, and complementarity between them. The diminishing returns to increased savings (reduced consumption) simply reflects the diminishing returns to domestic inputs, holding imports constant. Diminishing returns to aid reflects the same phenomenon the other way.

That aid yields greater returns in terms of growth at a high savings rate than at a low one is retained in this model, reflecting the greater import intensity of investment. But an increase in aid increases the marginal productivity, in terms of growth, of increased domestic savings.

Thus substitution possibilities mean that domestic policy can do more to increase employment, can have greater impact alone on growth, and is more important as a complement to foreign aid in influencing the growth rate than the fixed coefficient two-gap model implies. However, achievement of these potentialities depends on the instruments of policy that a government can employ. In particular, their achievement requires first that the government have some ability to influence the C, I level and mix; and second, that it be able to influence the effective exchange rate and the equilibrium rate of return on capital.

Implicit in the conventional two-gap formulation are some assumptions about the power of government to employ fiscal-monetary policy, and of the limits on that power. The savings or minimum consumption constraint really has to be interpreted as a limit on government taxing power resulting either from problems with the collection system, or from a political or welfare judgment, or both. Presumably the government has some power to permit consumption to exceed the minimal level by reducing taxes, or providing public (non-investment) services. Similarly, it is implied that the government has the power, through relaxing or tightening credit, to vary the level of investment to fill the "room" left by savings.

In principle, conventional fiscal and monetary policy instruments plus ability to set the exchange rate and the long run interest rate will do the trick. I will not discuss here either the exceptions to this, or the problems involved in really being able to control the exchange rate and the long-term interest rate.

In conclusion to this section it seems worthwhile to return to the discussion at the end of Section II on the equilibrium or optimum exchange rate. Equation (22) (reproduced for convenience) defines the supply of foreign exchange.

$$M = B + g\,Z\,[i^{\frac{(1-\alpha)(1-\delta)}{\lambda}} (\frac{W}{E})^{\frac{\alpha(1-\delta)}{\lambda}}]^{-\epsilon}\,. \qquad (22)$$

The supply will vary positively with the exchange rate divided by the money wage rate, negatively with the equilibrium rate of return on capital (which enters the cost of consumer goods for export). If we assume a given employment of capital and labor, i is an increasing function of $\frac{W}{E}$, and we can then draw a supply curve of foreign exchange as a function of $\frac{E}{W}$ (with i varying so as to maintain the capital-labor ratio).[1]

Equation (18) (reproduced below) defines the demand for foreign exchange.

$$M = Z\,i^{\frac{(1-\alpha)(1-b)}{\lambda}} (\frac{W}{E})^{\frac{\alpha(1-b)}{\lambda}}\,I + Z\,i^{\frac{(1-\alpha)(1-\delta)}{\lambda}} (\frac{W}{E})^{\frac{\alpha(1-\delta)}{\lambda}}\,C. \qquad (18)$$

For a given production of I and C, demand will be negatively related to the exchange rate divided by the wage rate, positively related to the equilibrium rate of return on capital, for the reasons discussed earlier. If we assume that i and $\frac{W}{E}$ are on the K, L curve of Fig. 4, import demand is a function of C and I, given K, L. This is shown in Fig. 9.

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[1] From Eq. (20).

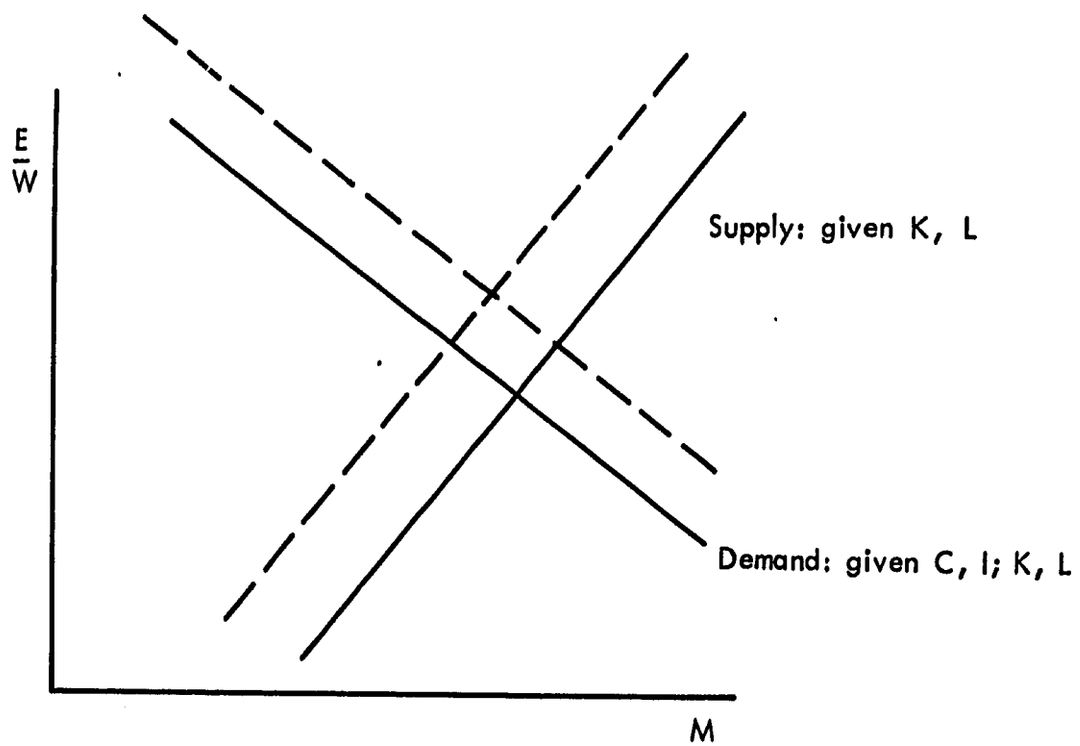

Fig. 9

We now can follow the policy maneuvers examined earlier in this section in terms of the demand for and supply of foreign exchange. An increase in employment, with an associated rise in output of consumer or investor goods, obviously shifts the demand curve of foreign exchange to the right. The increase in i relative to $\frac{W}{E}$ required to achieve the higher labor capital ratio shifts the supply curve to the left. Thus the conclusion, described earlier in terms of Fig. 5, that an increase in employment requires an increase in the equilibrium effective exchange rate is transparently obvious in terms of analysis of demand for and supply of foreign exchange.

The increase in I relative to C, holding K and L constant, also shifts the demand curve to the right.[1] The supply curve as defined above does not shift. Here, again, the earlier result that an increase in I relative to C requires an increase in $\frac{E}{W}$, described in terms of Fig. 4, is transparent when one considers what happens to the demand for and supply of foreign exchange.

The supply of and demand for foreign exchange diagram also permits us to interpret Colombia's recent economic malaise within the framework of the model. The falloff of coffee prices in the mid 1950s can be interpreted as a shift to the left in the supply of foreign exchange schedule facing Colombia (Fig. 9), or a shift down in the K, M curve (Fig. 4). For full employment equilibrium to be maintained this required either an allocation of resources <u>away</u> from investment and toward consumption (which would conserve on foreign exchange) or an

[1]This is not immediately obvious from the equations because of the constants Z. However, other equations imply that the import intensity of investment exceeds that of consumption.

increase in $\frac{E}{W}$ which would both increase foreign exchange earnings and tend to push down the import intensity of both investment and consumption.

The 1960 development plan calls, if anything, for an increase in the investment rate. The implication that this required an effective exchange rate above the level of the period prior to the coffee price decline appears not to have been recognized. The result is that the system has operated in region 4 of Fig. 4--excess demand for foreign exchange, excess supply of domestic inputs. The investment rate has in fact fallen, but not sufficiently to avoid the development of significant domestic slack.[1] From time to time devaluation has been undertaken. But almost always the target has been (implicitly) to restore the effective exchange rate of the mid 1950s, not to achieve a higher rate. As a result, even after effective devaluations there still have been the "two-gap" symptoms, and a belief has developed that they are a necessary fact of life. The possibility that the effective exchange rate target has been set too low for the achievement of Colombia's growth and employment objectives--that the 1950 rate is a false parity--does not appear to have received as much consideration as it should.

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[1]The decline of course has not been deliberate.

VI. SOME CALCULATIONS

The preceding section presented qualitative conclusions. But one of the advantages of the Cobb-Douglas formulation is that the parameters are easily estimated. This section examines some of the rough quantitative magnitudes involved, using estimates of the parameters for Colombia.

Earlier it was pointed out that the exchange rate for exports and the import exchange rate can be considered as separate policy instruments.[1] This section makes this separability assumption and focuses on the "import" rate. We first shall explore the possibilities for expanding employment assuming a given K and M, and second, of increasing investment at the expense of consumption for a given K, M, and L. The possible effects of devaluation on exports have been much discussed but the effects upon domestic factor substitution little examined, and it is useful to focus on the latter and not contaminate the analysis with the former. Also to simplify the equations we shall hold A (total factor productivity) constant and absorb it in Z. And for simplicity we shall assume all changes in $\frac{W}{E}$ are the result of changes in E; thus changes in E are changes in $\frac{W}{E}$.

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[1]Thus, analytically we can separate between, first, a change in the export rate alone, and, second, a change in the import rate alone together with import cost subsidy to exporters to keep their factor costs from changing. The former can be viewed as an action that changes the M versus K, L endowments available for production of investment goods and consumer goods for use at home. That is, an increase in the export rate makes more M available, and less K and L available (because more has gone into export production) for home use. The latter can be viewed as a change in the import rate for fixed endowments of M, K and L available for C and I.

First, then, consider the first maneuver discussed in the preceding section--increasing employment with balanced expansion of C and I, assuming a given K and M. From Eq. (2) we can derive the change in employment for a given capital stock as a function of changes in E and i as follows:

$$d \log L = \frac{b}{\lambda} d \log E + \frac{1}{\lambda} d \log i \tag{23}$$

Of course, given K:

$$d \log V = \alpha \, d \log L \tag{24}$$

Instead of assuming, as in the previous section, that C and I are expanded proportionately, for mathematical computation it is more convenient to assume that labor (and value added) are expanded proportionately in the production of C and I. Thus

$$d \log V - d \log V_I = d \log V_C \tag{25}$$

Total demand for imports can be written:

$$M = [(\frac{M}{V})_C \frac{V_C}{V} + (\frac{M}{V})_I \frac{V_I}{V}] \, V \tag{26}$$

Using Eqs. (6)-(9) and (13)[1]

$$\left(\frac{M}{V}\right)_C = Z \left(\frac{\delta}{1 - \delta}\right) i^{\frac{(1-\alpha)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha}{\lambda}} \tag{27}$$

$$\left(\frac{M}{V}\right)_I = Z \left(\frac{b}{1 - b}\right) i^{\frac{(1-\alpha)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha}{\lambda}} .$$

Thus

$$M = Z\, i^{\frac{(1-\alpha)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha}{\lambda}} \left[\frac{b}{1 - b} \left(\frac{V_I}{V}\right) + \frac{\delta}{1 - \delta} \left(\frac{V_C}{V}\right)\right] V . \tag{28}$$

Since the supply of imports has not changed, M cannot change. With the terms in brackets constant this implies

$$0 = \frac{(1 - \alpha)}{\lambda} d \log i - \frac{\alpha}{\lambda} d \log E + d \log V \tag{29}$$

Substituting for d log V using Eqs. (23) and (24) and simplifying:

$$d \log i = \alpha (1 - b)\, d \log E \tag{30}$$

Thus i must move in the same direction as E, as was demonstrated geometrically in the preceding section.

Substituting back in (25) and (24), we can obtain simpler expressions for the increases in value added and employment.

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[1]From (6) and (7)

$$\left(\frac{M}{V}\right)_C = \frac{\delta}{(1 - \delta)} \left(\frac{P}{E}\right)$$

From (14)

$$\frac{P}{E} = Z\, i^{\frac{(1-\alpha)}{\lambda}} \left(\frac{W}{E}\right)^{\frac{\alpha}{\lambda}}$$

The same steps suffice to derive (27).

$$d \log V = \alpha \, d \log E \tag{31}$$

$$d \log L = d \log E \, . \tag{32}$$

The policy maneuver in question, then, permits employment to expand in exactly the same percentage as the devaluation. To the extent the model is even roughly correspondent with reality, the effect of devaluation in permitting an increase in employment with no increase in demand for foreign exchange is not to be sneezed at.[1]

The effect on real investment can be derived as follows: Investment can be written:

$$I = Z \, M_I^b \, V_I^{1-b} = Z \, (\tfrac{M}{V})_I^b \, V_I \, . \tag{33}$$

Using the earlier equation for $(\frac{M}{V})_I$:

$$I = Z \, [i^{\frac{(1-\alpha)}{\lambda}} \, (\tfrac{W}{E})^{\frac{\alpha}{\lambda}}]^b \, V_I \tag{34}$$

Using Eqs. (25) and (30):

$$d \log I = \alpha \, (1 - b) \, d \log E. \tag{35}$$

Similarly it can be shown that

$$d \log C = \alpha \, (1 - \delta) \, d \log E. \tag{36}$$

These results should have been intuitively obvious. The same percentage decrease in $(\frac{M}{V})$ in both C and I occurs and the same percentage increase in V (and L). Since total M is constant, imports used in each activity must be also. The change in output in each then is strictly

[1]Note again that there is a relatively long term adjustment period before potential employment increase can be realized.

the result of the (same percentage) increase in L, which equals the percentage devaluation. The coefficients in (35) and (36) are, simply, the elasticities of output with respect to labor.

The key parameters are α, b, and δ. Labor's share in net value added in Colombian industry is about .4, so assume $\alpha = .4$. Rough calculations suggest that direct plus indirect imports account for about half of the value of investment, and 20 percent of the value of consumption. Let us assume, therefore, that $b = .5$ and $\delta = .2$.

Slighton's study of urban unemployment in Colombia suggests an unemployment rate of well over 10 percent.[1] In view of the likelihood that an increase in urban employment would induce additional migration to the cities, an increase in employment of 15 percent certainly seems a reasonable objective. The model (Eq. (32)) suggests that a devaluation of about 15 percent might do the trick. To prevent excess demand for capital from developing, Eq. (30) indicates that such a devaluation ought to be accompanied by a 3 percent increase in the long run interest rate. Under the balanced expansion assumption, the increased employment would generate a 3 percent increase in output of investment goods, and a (slightly more than) 5 percent increase in production of consumer goods.

Let us turn now to the C, I tradeoff maneuver. In the import demand Eq. (28) V is constant as well as M since L is given as well as K. But the expression in brackets--which is a function of the allocation of domestic resources between investment and consumption--is not

[1]Robert L. Slighton, Urban Unemployment in Colombia: Measurement, Characteristics, and Policy Problems, The RAND Corporation, RM-5393-AID, January 1968.

constant. Indeed, a shift in resources is the heart of the maneuver. Let the terms in brackets be denoted by $Q\left(\frac{V_I}{V}\right)$.[1] Then instead of (29):

$$0 = \left[\frac{1-\alpha}{\lambda} d \log i - \frac{\alpha}{\lambda} d \log E\right] + \frac{d \log Q}{d \log \frac{V_I}{V}} d \log \left(\frac{V_I}{V}\right) \qquad (37)$$

The first term above can be recognized as the proportional change in $\frac{M}{V}$ in both I and C. This can be seen from Eq. (27). Equation (37), then, is the condition under which change in import demand resulting from a change in import intensity in C and I is just offset by change in demand resulting from a shift in resources between C and I.

We are constrained to have a constant capital-labor ratio. Equation (20) shows that this requires

$$i = \left(\frac{W}{E}\right)^b \qquad (38)$$

One immediate consequence of this constraint is that the first bracketed term in Eq. (37), which is already identified as equal to the proportional change in $\frac{M}{V}$ in both I and C, collapses to $-\, d \log E$.[2] One might have intuited this if one had noted that the relationship between i and E in Eq. (38) means that a decline in i exactly offsets the effect of an increase in E on the price of domestic value added. (See Eq. (14).)[3] Thus Eq. (38) amounts to a constant price of domestic

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[1] Note that $\frac{V_C}{V} = 1 - \frac{V_I}{V}$.

[2] $-\left(\frac{1-\alpha}{\lambda}\right) b\, d \log E - \frac{\alpha}{\lambda} d \log E = -\frac{[\alpha + b - b\alpha]}{\lambda} d \log E.$

But $\lambda = \alpha + b - b\alpha$.

[3] Remember W is constant.

value added constraint. With Cobb-Douglas production functions throughout, hence a unitary elasticity of substitution, it follows that the percentage change in the ratio of imports to value added must exactly equal the percentage change in the price of imports, holding constant the price of value added.

We now can derive an explicit relationship between the change in the exchange rate and the change permitted in the fraction of (domestic) resources allocated to investment. With the bracketed term collapsing to -d log E, (37) can be written:

$$d \log \frac{V_I}{V} = \frac{1}{\dfrac{d \log Q}{d \log \frac{V_I}{V}}} \, d \log E \tag{39}$$

But

$$\frac{d \log [Q]}{d \log \frac{V_I}{V}} = \frac{\left[\frac{b}{1-b} - \frac{\delta}{1-\delta}\right] \frac{V_I}{V}}{\left(\frac{b}{1-b}\right) \frac{V_I}{V} + \left(\frac{\delta}{1-\delta}\right) \left(1 - \frac{V_I}{V}\right)} \tag{40}$$

For convenience denoting $\dfrac{d \log Q}{d \log \frac{V_I}{V}}$ as $\frac{I}{J}$

$$d \log \frac{V_I}{V} = J \, d \log E. \tag{41}$$

The change in quantity of investment can be derived as follows. Rewriting Eq. (33) for convenience:

$$I = Z \left(\frac{M}{V}\right)_I^b V_I \, . \tag{33}$$

It was earlier shown that $d \log \left(\frac{M}{V}\right)_I = -d \log E$. With V constant $d \log V_I = d \log \frac{V_I}{V}$ which equals $J \, d \log E$. Thus:

$$d \log I = [-b + J] \, d \log E \tag{42}$$

Through analogous steps it can be shown that:

$$d \log C = [- \delta - J \frac{V_I}{V_C}] \, d \log E \tag{43}$$

Since $J > 1 > b$, it is clear that I will increase with E, and C contract, as shown geometrically earlier.

We thus have arrived at explicit expressions for the C, I possibility frontier, and further, can associate movements along the frontier with changes in E (and changes in i). It will be illuminating to make one particular calculation. Assume as before that $b = .5$ and $\delta = .2$. Let the existing position on the frontier be defined by $\frac{V_I}{V} = .15$ which seems about right for Colombia. This implies that $J = 3.4$. Then it can be calculated, using the equations above, that a roughly 7 percent devaluation (together with a 3.5 percent decrease in i) will permit a roughly 20 percent increase in I, at the cost of a 5 percent reduction in C. These calculations assume fixed M, K, and L.

Having made these "frontier" calculations we now can go back and estimate how much investment could be increased as a result of a 15 percent <u>increase</u> in employment for a given K and M, if consumption were held constant instead of expanding by 5 percent as in earlier calculations. This maneuver amounts to the resultant of first employing the proportional expansion maneuver analyzed earlier and then the "along the frontier" maneuver of the paragraph above. The result is that investment could be increased by 3 + 20 = 23 percent.[1] The magnitude of the required devaluation is 15 + 7 = 22 percent.

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[1]Although this seems at first glance like a very large number, some secondary calculations suggest it is quite reasonable. Thus

In this model, an increase in investment of roughly the above magnitude is required to get the growth rate of Colombian industry up from its present level to the estimate of the 1960 development plan.[1] Differentiating the value added Eq. (1), we obtain the familiar result:[2]

$$\frac{\dot{V}}{V} = \frac{\dot{A}}{A} + \alpha \frac{\dot{L}}{L} + (1 - \alpha) \frac{\dot{K}}{K} \tag{44}$$

If we constrain the capital-labor ratio by Eqs. (4) and (5):

$$\frac{\dot{V}}{V} = \frac{\dot{A}}{A} + \frac{\dot{K}}{K} + \alpha \left(\frac{\dot{r}}{r} - \frac{\dot{W}}{W}\right) \tag{45}$$

$$\frac{\dot{L}}{L} = \frac{\dot{K}}{K} + \left(\frac{\dot{r}}{r} - \frac{\dot{W}}{W}\right) \tag{46}$$

Let us assume $\frac{\dot{A}}{A}$, $\frac{\dot{W}}{W}$ and $\frac{\dot{r}}{r}$ are constant and are not affected by the devaluation and increase in investment and employment.[3] The growth of output and employment then, will be uniquely related to the rate of

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assume that, contrary to the analysis above, all of the 15 percent increase in employment, the 6 percent increase in value added, went into investment with V_C, M_C, and M_I all held constant. Then C would stay constant as assumed. The increase in V_I would be 6 percent/ 15 percent = 40 percent. With (1 - b) = .5, even this nonoptimal use of the increased resources now employed would yield a 20 percent increase in I. The 23 percent figure represents an "optimal" allocation.

Of course the reason why a small percentage reduction in C can permit a large percentage increase in I is, simply, that C is significantly larger than I.

[1]Government of Colombia, Plan General Desarollo Economico y Social, Cali, Colombia, 1962.

[2]This equation, of course, does not include the effect of the initial increase in employment.

[3]Again, we ignore here the shift in factor prices as a direct result of the devaluation.

growth of the capital stock. And with a given initial K, a given percentage increase in I = K yields an exactly equal percentage increase in $\frac{\dot{K}}{K}$.

The present value of $\frac{\dot{V}}{V}$ is about .065 and $\frac{\dot{K}}{K}$ about .050. The result of a 23 percent increase in I would be to bring the growth rate of V up to the estimate of the plan (just above 8.0 percent). Assuming as above no change in rates of change in r and W, the rate of employment growth would be lifted from its present rate of about .030 a year to over .040 a year. All of these changes of course are over and above those achieved by the initial adjustment to policy. Although it certainly would be "easier" to accomplish these objectives with an increase in foreign aid (in particular consumption levels and the real wage rate could be higher), in this model at least Colombia has the theoretical capability to do it on her own; and, given present unemployment levels, to do it with no reduction in aggregate consumption.