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9. ABSTRACT

An analytical framework required for an integrated approach to the water-salinity management problem. In the section on the groundwater management model, such a framework is presented. In the section on decision rules, policy implications of this approach are examined, particularly as they relate to the difficulties associated with the use of economic incentives to bring about optimal water use patterns in a decentralized decision-making environment. It was found that relatively few adjustments are required in the groundwater management model for its applicability to a wider range of water management models, e.g., the conjunctive use of groundwater and surface water, interbasin management systems, and the like.

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Groundwater Management and Salinity Control

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A management model is presented that may be useful in analyzing decision rules for the conjunctive management of groundwater reserves for use in irrigation and salinity control. Alternative schemes for decentralized management via taxes and bribes are discussed. Taxes are described that bring about intradistrict efficiency in terms of water use and salinity control when downstream externalities are ignored. When externalities are considered, it is shown that a unique pattern of resource use requires a given institutional structure for the management of bribes.

Over recent years a great deal of attention has been given to the problems associated with the optimum control of groundwater reserves. Groundwater management has been examined analytically from a number of different viewpoints, e.g., the economic control of groundwater reserves [*Burt*, 1966], groundwater management under institutional restrictions [*Burt*, 1970], the conjunctive management of groundwater and surface water [*Burt*, 1964; *Cummings and Winkelman*, 1970], groundwater management and interbasin water transfers [*Kelso et al.*, 1973; *Martin and Young*, 1967, 1969], and groundwater management with saltwater intrusion [*Busch et al.*, 1966; *Cummings*, 1971].

The management of water stocks, ground or surface, in many regions of the world involves a further range of problems that have received relatively little attention thus far in the literature, viz., problems associated with increases in the concentration of salts in soils and water that result from continued irrigation. Such effects may also result in terms of the management of urban water supplies [*Wesner*, 1974]. The degradation of soils from the accumulation of salts is not a new phenomenon [*Evans*, 1974], nor is it a problem confined to small isolated regions; *Yaron* [1974] estimates that one fifth of the irrigated land in the United States and one third of the world's irrigated land are affected by salinity problems. Further, and of particular importance to economists, salt accumulations are amenable to management: soil salinity depends directly on water management policies, i.e., policies related to irrigation, leaching, and the structure of drainage facilities (a capital investment policy).

Several studies have recently appeared in the literature that bring into sharp focus the relevant management control issues associated with the conjunctive management of water and salinity. *Yaron et al.* [1972] provide estimates of response functions for crops, for which crop yields depend on the quantity and (saline) quality of applied water; particular stress is given in this work to the need for 'long-run' intertemporal analyses of salt accumulations. The stage for such long-run analyses is set in a companion work by *Bresler and Yaron* [1972] in which a static analysis of the optimum conjunctive management of water and salts is presented. In *Yaron* [1974] a long-run (intertemporal) analysis of the conjunctive management of surface water and salt accumulations is presented. *Howe and Orr* [1974] extend earlier works [e.g., *Hartman and Seastone*, 1970; *Gardner and Fullerton*, 1968] concerning the use of marketable water rights to include considerations of saline pollution. These later works by Yaron and Howe and Orr raise particularly interesting (and difficult) questions regarding the use of economic incentives for saline reduction. In the section on decision rules, attention is given to the complexities encountered in efforts to introduce economic incentives for the purpose of obtaining efficient resource use in a decentralized system.

A number of extensions to the works cited above are required for an integrated analysis of the water-salinity management problem. Specifically, the analytical frameworks suggested in the above ignore (1) the intertemporal problem of endogenously determined water scarcity and the ramifications of such scarcity for the opportunity cost of water used for leaching purposes and (2) optimal investment rates for capital and the role of capital in reducing salinity as well as in 'saving' scarce water supplies. Further, with reference to groundwater stocks the potential impact of irrigation on the quality of groundwater storage and possible externalities remain for consideration [*Konikow and Bredehoeft*, 1974].

The purpose of this paper is to suggest an analytical framework that will provide these extensions that appear to be required for an integrated approach to the water-salinity management problem. In the section on the groundwater management model, such a framework is presented. In the section on decision rules, policy implications of this approach are examined, particularly as they relate to the difficulties associated with the use of economic incentives for the purpose of bringing about optimal water use patterns in a decentralized decision-making environment. Concluding remarks are given in the final section.

Concern in this paper is limited to the management of groundwater stocks used for irrigation so that attention may be focused sharply on the interrelationships that exist between water and salinity management. The analysis may readily be extended to include surface water supplies and/or urban water uses along the lines suggested, for example, by *Burt* [1964] or *Cummings and Winkelman* [1970].

The following notation is used in the sections below:

y_{it} volume of water allocated to area i for leaching purposes during the dormant season of period t, $i = 1, \cdots, I$, $t = 1, \cdots, T$;

w_{it} volume of water allocated to area i for irrigation (production and leaching) purposes during the growing seasons of period t, $i = 1, \cdots, I$, $t = 1, \cdots, T$;

v_{it} gross investment in capital stocks (for canal lining, drainage structures, pump equipment, etc.) in area i during t, $i = 1, \cdots, I$, $t = 1, \cdots, T$;

X_t stock of groundwater available at the beginning of period t, $t = 1, \cdots, T$;

S_{it} index of salt concentration in the root zone of area i soils at the beginning of period t, $i =$ [illegible] $= 1, \cdots, T$;

Z_t measure of the salt concentration in the stock of groundwater X_t at the beginning of period t, $t = 1, \cdots, T$;

K_{it} index of capital stocks in area i at the beginning of period t, $i = 1, \cdots, I$, $t = 1, \cdots, T$, that includes capital stocks for production, the condition of canals, and drainage facilities;

e_t groundwater recharge during period t, $t = 1, \cdots, T$;

$y_t = (y_{1t}, y_{2t}, \cdots, y_{It})$;

$w_t = (w_{1t}, w_{2t}, \cdots, w_{It})$;

$v_t = (v_{1t}, v_{2t}, \cdots, v_{It})$;

$S_t = (S_{1t}, S_{2t}, \cdots, S_{It})$;

$K_t = (K_{1t}, K_{2t}, \cdots, K_{It})$;

$t = 1, \cdots, T$.

Groundwater Management Model

Consider an irrigation district in an arid area for which groundwater supplies are the sole source of water. The district is divided into I zones or parcels, where the criteria for such divisions may include soil types, ownership, etc., but each parcel must be relatively homogeneous in terms of the salt content of its soil.

For each zone i, S_{it} is a measure of the level of salt concentration in the root zone of zone i soils at the beginning of period t. In this study, 'salt concentration' refers to a high value for the electrical conductivity of the saturation extract in soils and is assumed to be in units of some appropriate measure such as meq/l or mmho/cm at 25°C [*U.S. Department of Agriculture*, 1954]. A difference equation, describing the change in such concentration between the beginning and end of period t, is given by the following expression:

$$S_{it+1} = S_{it} - F_{it}(w_{it}, y_{it}, Z_t, K_{it}) + \eta_t \qquad (1a)$$

$$\forall\ i = 1, \cdots, I \qquad t = 1, \cdots, T$$

By (1*a*), zone i begins period t with salt concentrations S_{it}, which may increase during t as a result of natural or uncontrollable causes (chemical weathering etc.) as measured by η_t. The function F_{it} measures the change in salt concentrations that results under some conditions from water use and drainage control (capital stocks). Following *Israelsen and Hansen* [1967], 'irrigation must provide water for growth of crops and at the same time allow enough water to pass through the soil to leach out salts;' thus the use of water for irrigation purposes, our w_{it}, may also serve to leach out salts.

For generality we wish to include the possibility of water applications for leaching purposes during the dormant season. There are several justifications for such an inclusion. First, water tables and drainage conditions may be more favorable for leaching during the dormant season than during the growing season [*U.S. Department of Agriculture*, 1954, p. 39]; this situation is particularly true in areas that receive a major part of their rainfall during the dormant season [e.g., *Yaron and Olian*, 1973]. Pump capacity considerations may require off-season (dormant season) leaching, particularly when substantial leaching is required. Finally, leaching during the dormant season may be unavoidable in areas where water tables are high in relation to the root zone [*Israelsen and Hansen*, 1967; *U.S. Department of Agriculture*, 1954, p. 39]. This case would imply a modified functional form for (1*b*) introduced below. Some recent empirical findings regarding the effectiveness and timing of water used for leaching are found in *Van Schilfgaarde* [1974] and *Yaron* [1974].

Basically, leaching effects obtain only after water use brings soil moisture to field capacity. We define w^* and y^* as measures of water use levels during growing and dormant seasons that bring soils to field capacity during each intragrowing or intradormant season irrigation. It follows then that

$$\partial F_{it}/\partial w_{it} \le 0 \qquad 0 \le w_{it} \le w^* \qquad (1b)$$

$$\partial F/\partial w_{it} \ge 0 \qquad w_{it} > w^*$$

$$\partial F_{it}/\partial y_{it} \le 0 \qquad 0 \le y_{it} \le y^* \qquad (1c)$$

$$\partial F_{it}/\partial y_{it} \ge 0 \qquad y_{it} > y^* \qquad \forall\ i, t$$

This treatment of leaching has analytical appeal in terms of simplicity. Operational problems associated with this structure may generally be resolved by expanding the number of variables.

The quality of water applied, measured by Z_t, influences the impact of w_{it} and y_{it} on soil salinity; thus Z_t is included in F_{it}. It is assumed that $\partial F_{it}/\partial Z_t \le 0$ for all i and t.

The effectiveness of water use w_{it} and/or y_{it} in terms of leaching-out salts depends upon drainage conditions in each zone. Capital stocks for drainage facilities will therefore have a positive effect on reducing salt concentrations, and it is assumed that

$$\partial F_{it}/\partial K_{it} \ge 0 \qquad \forall\ t \qquad (1d)$$

We use K_{it} as an index of capital stocks in zone i at the beginning of period t, which is intended to include various types of capital, e.g., drainage facilities, the condition of canals, pumping equipment, etc. Thus K_{it} as used here is something of a composite measure of all capital items, and it is defined as such strictly for the purpose of simplifying the exposition. For many applications it will be desirable to introduce J terms K_{ijt}, $j = 1, \cdots, J$, to differentiate explicitly types of capital stocks. The transition equation for capital stocks is given by following difference equation

$$K_{it+1} = K_{it} - D_{it}(v_{it}, K_{it}) \qquad \forall\ i, t \qquad (2)$$

Equation (2) states that area i capital stocks at the end of period t equal initial stocks K_{it} minus net (of gross investment) depreciation. We assume that $\partial D_{it}/\partial v_{it} \le 0$ and $\partial D_{it}/\partial K_{it} \ge 0$. The use of K_{it} in D_{it} implies a 'capital decay' notion of gross depreciation. For some applications it may be useful to substitute $w_{it} + y_{it}$, total water use, for K_{it} in D_{it} for the purpose of tying gross depreciation to the use of capital stocks.

The stock of water in the aquifer underlying the irrigation district at the beginning of time t is measured by X_t. Periodic changes in water storage are assumed to be described by the following relation:

$$X_{t+1} = X_t + e_t - \sum_i [w_{it} + y_{it} - r_{it}(w_{it}, y_{it}, K_{it})] \qquad \forall\ t \qquad (3)$$

In (3), r_{it} measures the return flow to the aquifer from water use w_{it} and y_{it}. One would expect return flow to increase over some range with increases in water use and decrease with capital stocks (lined canals, drainage, etc.); thus we posit

$\partial r_{it}/\partial w_{it}$, $\partial r_{it}/\partial y_{it} \geq 0$ and $\partial r_{it}/\partial K_{it} \leq 0$. In areas where water tables are falling rapidly, as in areas in central Arizona, return flows may not 'catch up' with the falling water table, and a 'perched' water table is formed. In such cases, $\partial r_{it}/\partial w_{it}$, $\partial r_{it}/\partial y_{it}$ would be zero. Of course, this situation would require that well casings are sealed at 'perched' water table levels (depths).

The term $\sum_i [w_{it} + y_{it} - r_{it}(w_{it}, y_{it}, K_{it})]$ is thus a measure of the net reduction in groundwater stocks from pumping. Equation (3) then states that water stocks at the end of period t will equal initial stocks plus natural recharge e_t minus net water withdrawals.

In some instances, depending on the types of soils in the irrigation district and the geophysical properties of the aquifer, the quality (salt concentration) of water in the aquifer may be affected by return flows as was noted by *Konikow and Bredehoeft* [1974]. To allow for differing qualities of water in the aquifer, we define Z_t as a measure of the salt concentration of the aquifer at the beginning of period t and posit the following transition equation:

$$Z_{t+1} = Z_t - g(e_t) + R_t\{[r_{it}(w_{it}, y_{it}, K_{it})], S_t\} \qquad \forall\ t \tag{4}$$

In (4), $g(e)$ measures the impact of natural recharge on salt concentration in the aquifer, and we assume that $\partial g/\partial e_t \geq 0$. Parameter R_t measures the impact of return flows from water use in all zones i on the salt concentration of the aquifer. Thus

$$[r_{it}(w_{it}, y_{it}, K_{it})] \equiv [r_{1t}(w_{1t}, y_{1t}, K_{1t}), r_{2t}(w_{2t}, y_{2t}, K_{2t}), \cdots, r_{It}(w_{It}, y_{It}, K_{It})]$$

We assume that

$$\partial R_t/\partial w_{it} = \partial R_t/\partial r_{it}\ \partial r_{it}/\partial w_{it},\ \partial R_t/\partial y_{it} = \partial R_t/\partial r_{it}\ \partial r_{it}/\partial y_{it} \geq 0$$

Finally, for a given level of return flows r_{it} from zone i the impact of such return flows on the salinity of the aquifer stock will depend on the salinity of zone i soils S_{it}. Thus $\partial R_t/\partial S_{it}$ is assumed to be nonnegative for all i and t.

Total water use $w_{it} + y_{it}$ in any zone i during any period t is assumed to be bounded from above by a constraint reflecting capital stocks. Thus we assume that

$$w_{it} + y_{it} \leq G_{it}(K_{it}) \qquad \forall\ i, t \tag{5}$$

In (5), G_{it} measures periodic pumping capacity, which may be increased with larger capital stocks; i.e., $\partial G_{it}/\partial K_{it} \geq 0$. Of course, this very general expression, which is included to capture a major role of capital stocks, ignores many operational problems. Particularly, since G_{it} would most likely be in units like gallons per day, week, or irrigation period, most applications would require an additional time script, say τ, where τ is an irrigation period, and (5) would become $w_{it\tau} + y_{it\tau} \leq G_{it\tau}$, $\tau = 1, \cdots$.

Of course, for any period t we have the physical restrictions that total water use may not exceed water supplies:

$$\sum_{i=1}^{I} [w_{it} + y_{it} - r_{it}(w_{it}, y_{it}, K_{it})] \leq X_t + e_t \tag{6}$$

Within the system described by (1)–(6) above, we assume that the criterion for groundwater management is that of maximizing the present value of a T period stream of net benefits. Problems associated with the choice of T are discussed in the final section; problems associated with the choice of a 'period' t are discussed in the work by *Cummings and Winkelman* [1970]. Net benefits for any period t are assumed to be given by the following expression, where $\beta^t = (1 + k)^{-t}$ is the discount factor and k is the appropriate discount rate:

$$B^t(w_t, v_t, y_t, X_t, K_t, S_t)\beta^t = [b_t(w_t, y_t, v_t, X_t, K_t, S_t) - E_t(w_t, y_t)]\beta^t \qquad \forall\ t = 1, \cdots, T \tag{7a}$$

The function b_t is a measure of net agricultural benefits from water use, which, as in many earlier studies, may be approximated by net agricultural income. Production relationships etc. are embedded in this general benefit function. Net agricultural benefits would generally be expected to increase with w_{it}, water applications during the growing season, in the sense of providing moisture in root zones required for plant growth. Inasmuch as w_{it} also serves as a leaching function, as was discussed above (equation (1*b*)), the level of w_{it} for leaching may be carried to levels $w_{it} > w^*$, the result being a loss of plant nutrients in areas where salt accumulations are a major problem [*Israelsen and Hansen*, 1967]. Consequently, net agricultural benefits could be affected adversely. Let $\hat{w}_i$ be the level of water use beyond which plant nutrients are lost. It is then assumed that

$$\begin{aligned} &\partial b_t/\partial w_{it} \geq 0 \qquad 0 \leq w_{it} \leq \hat{w}_i \\ &\partial b_t/\partial w_{it} \leq 0 \qquad w_{it} > \hat{w}_i \qquad \forall\ i, t \end{aligned} \tag{7b}$$

Water use during the dormant season y_{it} is included in b_t to allow for the possibility that off-season applications of water for leaching purposes may have a 'preirrigation' effect and thereby contribute positively to production.

$$\partial b_t/\partial y_{it} \geq 0 \qquad \forall\ i, t \tag{7c}$$

We assume that net benefits increase with X_t (reflecting, e.g., lower pumping costs) and K_{it} and decrease with respect to v_{it} (current investment costs) and S_{it} (reflecting downward shifts in crop production functions due to increased salinity):

$$\partial b_t/\partial X_t,\ \partial b_t/\partial K_{it} \geq 0 \qquad \partial b_t/\partial v_{it},\ \partial b_t/\partial S_{it} \leq 0 \qquad \forall\ i, t \tag{7d}$$

In some applications, drainage or tail water and/or soil directly or indirectly enters streams and results in a reduction of downstream benefits [*Howe and Orr*, 1974]. In such cases, externalities in terms of downstream costs associated with higher salt concentrations in streams may result from water use activities in our study area. To allow for this possibility, we include $E_t(w_t, y_t)$ in our expression of net benefits, where $E_t(w_t, y_t)$ is assumed to measure the benefits foregone by downstream users of streamflow as a result of water use in the study area. Parameter E_t is assumed to increase with w_t and y_t; i.e., $\partial E_t/\partial w_t$, $\partial E_t/\partial y_t \geq 0$.

The optimization problem of concern here is then that of maximizing

$$\sum_{t=1}^{T} [b_t(w_t, y_t, v_t, X_t, K_t, S_t) - E_t(w_t, y_t)]\beta^t \tag{8}$$

subject to the conditions set out in (1)–(6).

Consider the following Lagrangian expression

$$
\begin{aligned}
L = \sum_{t=1}^{T} \Big\{ & [b_t(w_t, y_t, v_t, X_t, K_t, S_t) - E_t(w_t, y_t)]\beta^t \\
& - \sum_{i=1}^{I} \lambda_{t+1}^i [S_{it+1} - S_{it} + F_{it} \\
& \cdot (w_{it}, y_{it}, Z_t, K_{it}) - \eta_t] \\
& - \sum_{i=1}^{I} \psi_{t+1}^i [K_{it+1} - K_{it} + D_{it}(v_{it}, K_{it})] \\
& - \Gamma_{t+1}\Big[X_{t+1} - X_t - e_t \\
& + \sum_{i=1}^{I} (w_{it} + y_{it} - r_{it}(w_{it}, y_{it}, K_{it}))\Big] \\
& - \Lambda_{t+1}[Z_{t+1} - Z_t + g_t(e_t) \\
& - R_t(r_{it}(w_{it}, y_{it}, K_{it}), S_{it})] \\
& - \sum_{i=1}^{I} \theta_{it}[w_{it} + y_{it} - G_{it}(K_{it})] \\
& - \Delta_t\Big[\sum_{i=1}^{I} (w_{it} + y_{it} \\
& - r_{it}(w_{it}, y_{it}, K_{it})) - X_t - e_t\Big]\Big\} \qquad (9)
\end{aligned}
$$

On the assumption that the functions in (9) are appropriately differentiable, necessary and sufficient conditions for a maximum in (9) include the following [*Hadley*, 1964; *Burt and Cummings*, 1970] when all variables are positive:

$$
\begin{aligned}
& (\partial b_t/\partial w_{it} - \partial E_t/\partial w_{it})\beta^t - \lambda_{t+1}^i\, \partial F_{it}/\partial w_{it} \\
& \quad - \Gamma_{t+1}(1 - \partial r_{it}/\partial w_{it}) + \Lambda_{t+1}\, \partial R_t/\partial w_{it} \\
& \quad - \Delta_t(1 - \partial r_{it}/\partial w_{it}) - \theta_{it} = 0 \qquad (10)
\end{aligned}
$$

$$\partial b_t/\partial v_{it}\beta^t - \psi_{t+1}^i\, \partial D_{it}/\partial v_{it} = 0 \qquad (11)$$

$$
\begin{aligned}
& (\partial b_t/\partial y_{it} - \partial E_t/\partial y_{it})\beta^t - \lambda_{t+1}^i\, \partial F_{it}/\partial y_{it} \\
& - \Gamma_{t+1}(1 - \partial r_{it}/\partial y_{it}) - \theta_{it} - \Delta_t(1 - \partial r_{it}/\partial y_{it}) \\
& + \Lambda_{t+1}\, \partial R_{it}/\partial y_{it} = 0 \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
& \partial b_t/\partial K_{it}\beta^t - \lambda_{t+1}^i\, \partial F_{it}/\partial K_{it} \\
& \quad - \psi_t^i + \psi_{t+1}^i(1 - \partial D_{it}/\partial K_{it}) \\
& \quad + (\Delta_t + \Lambda_{t+1}\, \partial R_t/\partial r_{it} + \Gamma_{t+1})\, \partial r_{it}/\partial K_{it} \\
& \quad + \theta_{it}\, \partial G_{it}/\partial K_{it} = 0 \qquad (13)
\end{aligned}
$$

$$\forall\ i = 1, \cdots, I,\ t = 1, \cdots, T$$

$$\partial b_t/\partial X_t\beta^t + \Delta_t - \Gamma_t + \Gamma_{t+1} = 0 \qquad (14)$$

$$t = 1, \cdots, T$$

$$-\sum_i \lambda_{t+1}^i\, \partial F_{it}/\partial Z_t - \Lambda_t + \Lambda_{t+1} = 0, \qquad \forall\ t \qquad (15)$$

$$\partial b_t/\partial S_{it}\, \beta^t + \Lambda_{t+1}\, \partial R_t/\partial S_{it} - \lambda_t^i + \lambda_{t+1}^i = 0 \qquad (16)$$

$$\forall\ i, t$$

Given negativity in any of the above equations, the associated variable has a zero value [*Hadley*, 1964].

Prior to moving to an analysis of the policy ramifications for water management implied by the conditions (10)–(16) above, it is useful at this point to consider the economic measures given by the Lagrangian multipliers λ_{t+1}^i, ψ_{t+1}^i, Γ_{t+1}, Λ_{t+1}, Δ_{it}, and θ_{it} introduced into the Lagrangian expression (9).

For the inequality constraints (5) and (6) an economic interpretation of the associated multipliers θ_{it} and Δ_t is straightforward [*Smith*, 1961, appendix]. The multiplier θ_{it} measures the change in benefits that would result from an incremental relaxation of constraint (5). This implies a benefit change brought about by a change in G_{it} via an increment in K_{it}; thus θ_{it} may be interpreted as the marginal 'capacity' value of an increment in the capital stocks of i during t. The multiplier Δ_t is the marginal scarcity value of water to the area in period t and is nonzero only in those periods after which the groundwater stock has been exhausted.

The remaining multipliers are associated with equality constraints (equations (1)–(4), respectively) and require some manipulations in order to get at their economic interpretation. Consider λ_{t+1}^i. By equation (16) we may derive the following expression:

$$\lambda_t^i = \partial b_t/\partial S_{it}\, \beta^t + \Lambda_{t+1}\, \partial R_t/\partial S_{it} + \lambda_{t+1}^i \qquad (17)$$

$$\forall\ i, t$$

Since (17) holds for all t, it follows that

$$\lambda_{t+1}^i = \sum_{\tau=t+1}^{T} [\partial b_\tau/\partial S_{i\tau}\, \beta^\tau + \Lambda_{\tau+1}\, \partial R_\tau/\partial S_{i\tau}] \qquad \forall\ i \qquad (18)$$

The multiplier λ_{t+1}^i then measures at the margin the present value of the impact on benefits during all periods $t + 1, \cdots, T$, of an incremental change in soil salinity during period t and is therefore referred to here as marginal land salinity cost. We treat λ_{t+1}^i as nonpositive for all i and t given the assumption that $\partial b_t/\partial S_{it} \leq 0$ and the additional assumption, which has intuitive appeal for all but the most pathological cases, that $\Lambda_t \leq 0$. The term λ_{T+1} is omitted. It can be shown [*Burt and Cummings*, 1970, appendix] that λ_{T+1} measures the marginal terminal value (cost) of S_{iT}. Terminal values are discussed in the concluding section.

To simplify the exposition that follows in the section on decision rules the following notation is defined:

$$
\begin{aligned}
\hat{\lambda}_{t+1}^i &\equiv \sum_{\tau=t+1}^{T} \partial b_\tau/\partial S_{i\tau}\, \beta^\tau \\
\lambda_{t+1}^{*i} &\equiv \sum_{\tau=t+1}^{T} \Lambda_{\tau+1}\, \partial R_\tau/\partial S_{i\tau} \qquad (19) \\
\lambda_{t+1}^i &= \hat{\lambda}_{t+1}^i + \lambda_{t+1}^{*i}
\end{aligned}
$$

By using the method outlined above for the computation of λ_{t+1}^i the following expressions may be derived for the multipliers ψ_{t+1}^i, Γ_{t+1}, and Λ_{t+1} by using (13), (14), and (15), respectively:

$$
\begin{aligned}
\psi_{t+1}^i = \sum_{\tau=t+1}^{T} & [\partial b_\tau/\partial K_{i\tau}\, \beta^\tau \\
& - \lambda_{\tau+1}^i\, \partial F_{i\tau}/\partial K_{i\tau} + \theta_{i\tau}\, \partial G_{i\tau}/\partial K_{i\tau} \\
& + (\Delta_\tau + \Lambda_{\tau+1}\, \partial R_\tau/\partial r_{i\tau} + \Gamma_{\tau+1})\, \partial r_{i\tau}/\partial K_{i\tau}] \\
& \cdot \prod_{s=t+1}^{\tau-1} \cdot (1 - \partial D_{is}/\partial K_{is}) \qquad \forall\ i, t \qquad (20)
\end{aligned}
$$

where

$$\prod_{s=t+1}^{t} (1 - \partial D_s/\partial K_s) \equiv 1$$

and

$$\prod_{s=t+1}^{t+1} (1 - \partial D_s/\partial K_s) \equiv (1 - \partial D_{t+1}/\partial K_{t+1}$$

$$\Gamma_{t+1} = \sum_{\tau=t+1}^{T} \partial b_\tau/\partial X_\tau \beta^\tau \geqq 0 \qquad \forall\ t \qquad (21)$$

$$\Lambda_{t+1} = -\sum_{\tau=t+1}^{T} \left(\sum_{i=1}^{I} \lambda_{\tau+1}{}^{i}\ \partial F_{i\tau}/\partial Z_\tau \right) \leqq 0 \qquad \forall\ t \quad (22)$$

Consider the earnings of an increment in investment during t. In each future period the increment (net of depreciation) earns (1) marginal direct benefits from a more efficient production process $\partial b_\tau/\partial K_\tau$; (2) marginal benefits from less salinity in soils that result from larger capital stocks (the marginal cost of soil salinity $\lambda_\tau{}^{i}$ times the decline in soil salinity attributable to an increment in capital $\partial F/\partial K$); (3) the marginal 'capacity' value of capital $\theta_{i\tau}$ times the change in capacity associated with an increment in K, $\partial G/\partial K$; and (4) given the effect of an increment in K on return flows $\partial r/\partial K$, the marginal values, or costs, associated with a $\partial r/\partial K$ unit return flow (which includes the marginal scarcity value of water Δ_t, the marginal costs of salinity in the aquifer Λ_{t+1}, and the marginal value of groundwater in stock Γ_{t+1}). This of course is the measure given by $\psi_{t+1}{}^{i}$ in (20), hence our interpretation of $\psi_{t+1}{}^{i}$ as the marginal value of capital in zone i during t.

The value Γ_{t+1} is seen to measure the marginal present value of an increment to groundwater stocks in all periods $t+1, \cdots, T$ by (21) and will be referred to as the marginal value of water in storage during t. Similarly, the nonpositive term Λ_{t+1} is seen to measure the marginal cost of salinity in the aquifer.

With the above development of scarcity values and costs, attention may now be turned to an analysis of optimal decision rules for groundwater management.

Decision Rules for the Conjunctive Management of Groundwater and Salinity

Decision rules for water used during irrigation and dormant seasons, as well as for periodic investment, may be deduced from (10)–(12), which are repeated here after rearranging terms:

$$(\partial b_t/\partial w_{it} - \partial E_t/\partial w_{it})\beta^t - \lambda_{t+1}{}^{i}\ \partial F_{it}/\partial w_{it} = (\Gamma_{t+1} + \Delta_t)(1 - \partial r_{it}/\partial w_{it}) - \Lambda_{t+1}\ \partial R_t/\partial w_{it} + \theta_{it} \qquad (23)$$

$$(\partial b_t/\partial y_{it} - \partial E_t/\partial y_{it})\beta^t - \lambda_{t+1}{}^{i}\ \partial F_{it}/\partial y_{it} = (\Gamma_{t+1} + \Delta t)(1 - \partial r_{it}/\partial y_{it}) - \Lambda_{t+1}\ \partial R_t/\partial y_{it} + \theta_{it} \qquad (24)$$

$$\partial b_t/\partial v_{it}\beta^t = \psi_{t+1}{}^{i}\ \partial D_{it}/\partial v_{it} \qquad \forall\ i, t \qquad (25)$$

Consider first optimal water applications during the irrigation or growing season, where the optimal value of w_{it} is such that $w_{it} > w_i^*$ ($\rightarrow \partial F_{it}/\partial w_{it} \geq 0$) and $w_{it} < \hat{w}_t$ ($\rightarrow \partial b_t/\partial w_{it} \geq 0$). Given the raison d'être for y_{it}, $y_{it} > y^*$ is assumed throughout this section. The left-hand side of (23) then measures the present value of marginal benefits associated with an increment in water use during the growing season of period t and includes marginal direct benefits (net agricultural incomes), net of external costs, and ($\lambda_{t+1}{}^{i} \leq 0$), the value of such water use in terms of reducing soil salinity for all future periods weighted by the impact of marginal water use on soil salinity ($\partial F/\partial w$) in period t. The right-hand side of (23) measures the opportunity cost of a net (of return flows) increment in use and includes the scarcity value of water (the marginal stock value of water Γ_{t+1} and the marginal cost of exhausting the aquifer stock Δ_t) plus the cost of return flows $\partial R/\partial w$ in terms of groundwater salinity $\Lambda_{t+1} \leq 0$ and the marginal capacity value of capital stocks θ_{it}.

Under these conditions, water use during the growing season is carried to the point where the present value of marginal net benefits equals the present value of the opportunity cost of an increment in water use.

If the opportunity cost of water is quite low in t, it may be optimal to continue applying water beyond $\hat{w}$, at which point marginal crop (agricultural, $\partial b/\partial w$) benefits become negative. This situation would require that the marginal value of water use for leaching is sufficiently large to offset negative marginal crop benefits in equating benefits with opportunity costs from water use.

On the other hand, high scarcity values for water will in some instances imply periodic rates of water use that are less than w^*. In such cases, marginal 'current' benefits weight heavily in relation to the present value of future salinity costs.

Optimal use rates for water during the dormant, or nongrowing, season are given by (24), interpretations for which are similar to those for w_{it}; viz., water is applied during t in each area i until marginal net benefits equal the opportunity cost for water. By solving (23) and (24) for $\Gamma_{t+1} + \Delta_t$ (the measures for the scarcity value of water in the district) it may be seen that the marginal value of water used in the irrigation season, net of soil salinity, groundwater salinity, and capacity costs, must equal the marginal net value of water used during the dormant season.

By (25), investment for each area i in all t is carried to the point where the marginal investment costs equal the marginal value of capital (equation (20)).

Suppose that irrigation takes place in our hypothetical district under conditions where no incentives exist for the conservation of water nor for concern regarding external salinity effects; assume also that capital stocks are fixed. We allow, however, for concern on the part of each irrigator in terms of salt accumulation on land under his control. Within the context of the model given in (9), each irrigator i (i.e., each 'zone' is associated with an individual irrigator) will choose a periodic level of water use such that the following conditions obtain:

$$\partial b_t/\partial w_{it}\ \beta^t - \hat{\lambda}_{t+1}{}^{i}\ \partial F_{it}/\partial w_{it} = \theta_{it} \qquad (26)$$

$$\partial b_t/\partial y_{it}\ \beta^t - \hat{\lambda}_{t+1}{}^{i}\ \partial F_{it}/\partial y_{it} = \theta_{it} \qquad (27)$$

This formulation implies that b is additive over individual farmers; i.e., $b_t = \Sigma_i b_{it}$, in which case the individual's marginal income (benefits) $\partial b_t/\partial w_{it}$ may be expressed as $\partial b_{it}/\partial w_{it}$.

By (26) and (27) in the absence of incentives for water conservation, water use in both the growing and the dormant seasons is carried to the point where either net benefits are zero ($\theta_{it} = 0$ for $w_{it} + y_{it} < G_{it}$) or pumping capacity is exhausted. Our hypothetical farmer, motivated by the desire to maximize income, ignores all external costs, i.e., the impacts of his actions in terms of diminishing the groundwater stock Γ_{t+1}, the quality of the groundwater Λ_{t+1} and $\lambda_{t+1}{}^{*t}$, and any external costs imposed on downstream users of water $\partial E_t/\partial w_{it}$, $\partial E_t/\partial y_{it}$. This solution is analogous to the classical 'commonality' solution wherein the common resource is exploited by individual users until marginal value products associated with resource use are zero [*Millman*, 1956; *Cummings*, 1969].

Thus water use in the irrigation district is characterized by I users independently choosing periodic pumping rates that satisfy conditions (26)–(27). For users with relatively large pumping capacities the buildup in soil salinity during production seasons may be offset by leaching at levels of water use where $w_{it} + y_{it} < G_{it}(K_{it})$. Other users will use water at maximum rates $G_{it}(K_{it})$, and conditions (26)–(27) obtain. One could conceivably observe rates of water use during some period t resulting in I different rates of earnings at the margin.

In such circumstances, policy questions relate to alternative means by which rates of goundwater use and accumulations of salt may be made more efficient. One policy alternative is the use of taxes or charges [e.g., *Kamien et al.*, 1966]. An optimum tax structure, i.e., those taxes that bring about the rates of water use given by (23) and (24) under decentralized management, may be deduced from conditions (23) and (24). Consider a simple case where return flows are zero and where externalities $\partial E/\partial w$, $\partial E/\partial y$ are the same for all zones i and do not vary between growing and dormant seasons (e.g., $\partial E/\partial w = \partial E/\partial y$). The optimum tax P_t^1 is given by

$$P_t^1 = \partial E/\partial w_t\, \beta^t + \Gamma_{t+1} + \Delta_t \qquad \forall\ t \qquad (28)$$

In the system described by operating decisions analogous to those in (26)–(27) the tax P_t essentially a 'Pigouvian' tax [*Baumol*, 1972; *Buchanan*, 1969; *Mishan*, 1971], will bring about optimality in terms of resource allocation [*Baumol*, 1972, p. 312]. We are not prepared at this time to address ourselves to the question concerning the disposition of such taxes once collected, Baumol's recommended 'lump sum payment' notwithstanding [*Baumol*, 1972, p. 312].

With return flows introduced a single tax will not serve given the heterogeneity of farms in terms of soil salinity and the potential differential impacts of such salinity on groundwater quality. If rates of return flow are the same for all zones and do not vary between growing and dormant seasons (i.e., $\partial r/\partial w = \partial r/\partial y$ are the same for all i), taxes for each zone i are given by the following expression:

$$P_{it}^2 = \partial E_t/\partial w_t\, \beta^t + (\Gamma_{t+1} + \Delta_t)(1 - \partial r_t/\partial w_{it}) - \Lambda_{t+1}\, \partial R_t/\partial w_{it} + \lambda_{t+1}{}^{*t}\, \partial F_{it}/\partial w_{it} \qquad (29)$$

$$i = 1, \cdots, I \qquad \forall\ t$$

Of course, in most cases, return flows will vary over zones and seasons, in which case a tax structure to bring about efficiency could be most difficult to compute and administer [*Kneese*, 1964, pp. 54–67, 85–98; *Whipple*, 1966].

A second alternative that has been suggested in the literature is the establishment of marketable water rights. *Howe and Orr* [1974] are concerned with the establishment of a system of marketable water rights along a river basin in which to avoid the complications introduced by variations in state water laws for river basins that involve many states (the Colorado River), a central agency is established that stands ready to buy or sell water rights at a stated price. In terms of intradistrict efficiency in water use and salt accumulations we will argue that a marketable water (stock) rights program may allow efficiency with decentralized decision making. However, the introduction of downstream externalities without the central control posited in the basic model of this paper prohibits the generation of unique water use solutions in the absence of a given institutional administrative structure.

Ignore for the moment the downstream externalities $E(w, y)$, and as before assume fixed capital stocks and that the initial groundwater stock X_1 is allocated among all users $i = 1, \cdots, I$. Each user i has marketable rights to his stock x_{it}; he may increase his stock by buying stock rights from other users or decrease his stock by selling his stock rights. An interior solution $\theta_{it} = 0$ for profit-maximizing individuals now becomes, in contrast with (26),

$$(\partial b_t/\partial w_{it}\, \beta^t - \hat{\lambda}_{t+1}{}^t\, \partial F_{it}/\partial w_{it})(1 - \partial r_t/\partial w_{it})^{-1} = (\Gamma_{it+1} + \Delta_{it}) \qquad (30)$$

where Γ_{it+1} and Δ_{it} are scarcity values that relate to the particular stock of water of i. A term analogous to Λ_{t+1} does not appear inasmuch as even with rights to a particular 'stock' the quality of the groundwater is still commonly determined.

Suppose now that an agency stands ready to buy and/or sell water rights at the price $\bar{P}_t$ (which is the measure of P_t^2 in (29), adjusted for return flows, that results from the optimization model with fixed capital stocks and $E(w, y)$ excluded). It is easily shown that users equate the right-hand side of (30) with $\bar{P}_t$ by selling or buying stocks (stock rights). If the scarcity value of i stocks, as measured by the right-hand side of (30), is greater (less) than $\bar{P}_t$, stock rights are purchased (sold). Equilibrium in the system results in the time path of water use as given in (23) and (24) (excluding of course externalities $E(w, y)$).

When externalities (downstream pollution) are reintroduced, the effectiveness of a system of marketable water rights in terms of establishing efficiency is difficult to evaluate. Given an initial 'commonality' state wherein farms use decision rules given by (26) and (27), the establishment of water rights does not introduce a decentralized mechanism that gives immediate impetus to the evolution of optimal resource allocation when downstream parties stand ready to offer bribes. If w_1 and w_2 are optimum use rates for groundwater that result from the initial 'commonality' state (decision rules (26) and (27)) and the 'efficient' state (decision rules (23) and (24)), respectively, the assumed convexity of $E(w, y)$ implies that the marginal bribe $\partial E/\partial w$, as well as the total bribe [*Whipple*, 1966], is larger for w_2 than for w_1. If bribes can be affected by the individual polluters and if downstream damages are relatively high, the potential exists for polluters to trade off some efficiency in terms of resource allocation for bribery gains [*Davis and Whinston*, 1962].

The tax (charge) versus bribery issue is argued extensively elsewhere in the literature [e.g., *Kneese*, 1964; *Kneese and Bower*, 1968; *Kamien et al.*, 1966; *Bramhall and Mills*, 1966; *Freeman*, 1967] and lies beyond the scope of this paper. It is hoped, however, that the forgoing observations serve to give our analysis some perspective vis-à-vis alternatives for bringing about optimum control of groundwater and salinity with decentralized decision making.

CONCLUDING REMARKS

Relatively few adjustments are required in the model presented in the section on groundwater management for its applicability to a wider range of water management models, e.g., the conjunctive use of groundwater and surface water, interbasin management systems, etc. The raison d'être of this work has been simply to suggest a method by which the salinity control problem may be integrated with the general problem of water resource management in agriculture.

Reference is made in the text to the problem of choosing T, the relevant time horizon for managing the groundwater supplies. As the problem is stated in the section on groundwater management, the choice of T may be crucial inasmuch as the 'scarcity' of the water resource is affected by the choice of T for relatively small values of T; i.e., if T is small, it may be impossible to extract the entire groundwater stock under any set of conditions in this given T period horizon, in which case the resource is 'free,' not scarce, and the rule for water use becomes trivial: use water until its net marginal product is zero. Whereas this problem is discussed in detail elsewhere [*Burt and Cummings*, 1970], two options for selecting T that have operational appeal may be mentioned here. First, one may simply choose T arbitrarily; if in the optimal solution the aquifer is not exhausted, T is then increased until either the stock becomes exhausted or (depending on the discount rate) the variations in benefits, associated with changes in T, are 'acceptably small.' For the choice of an infinite time horizon see the appendix of the work by *Burt and Cummings* [1970].

Second, problems associated with T may be avoided in most part by the use of a 'terminal value function' that measures the value (theoretically, from $T + 1$ to infinity) of the terminal stock of groundwater (i.e., the value of X_{T+1}). This is discussed more extensively in the works by *Burt and Cummings* [1970] and *Burt* [1964].

Of course, the relationship between T and water stocks discussed above is also relevant to the other state variables in the system S_t, Z_t, and K_t. If land has any productive uses in agriculture after groundwater stocks are depleted, terminal value (cost) relations may be required in order to reflect the impact of decisions made over the T period horizon on the productivity of resources for all periods $t > T$.

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