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Further Exploration of the User Cost Problem in Economic Theory

Project Working Paper 75-1

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Preface

Contract AID/csd-2975 requires Michigan State University to perform a number of tasks and to render a number of services. One of these tasks involves the need to clarify or solve the user cost problem in economic theory as a prelude to improving, among other things, our ability to model investment and disinvestment in the agricultural sector. Net investment is growth. Net disinvestment is deterioration.

The following essay represents substantial progress from a position attained earlier by Francis S. Idachaba but does not yet fully solve the problem. Nonetheless the progress made herein fulfills Michigan State University's Contract requirement and provides a substantial basis for further work on this problem.

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Further Exploration of the User Cost Problem in Economic Theory

Simulation studies generally involve an analysis of changes in the level of durable assets. Often times these durables are inputs into a production process. For example, in the Korean Agricultural Sector Analysis and Recommended Development Strategies 1971-1985 [10], which involved the use of a simulation model to make projections about Korean agriculture, it was shown that following the recommendations of the Kass team would involve increasing beef production. This implies a need to increase the level of investment in the national beef herd. Similarly moving to a more mechanized agriculture implies that there would be decreases in the national draft animal herd and investment in machines to replace the animals.

Although changes in the levels of durable assets are of primary importance in most simulation studies, there have been few attempts to apply the maximizing principles of economic theory to the analysis of these changes. Thus there is a need to develop an abstract economic theory of investment and disinvestment as it relates to production in order to lend some economic content to changes in the levels of durable assets. The objective of this paper is to clarify the abstract maximization theory of economics as it applies to decisions relating to investments and disinvestments in durable assets. This will be presented in three sections. The first section will focus on the issue of investing in durable assets. The second section will focus on the issue of disinvestment. In this sec-

tion, disinvestment will be treated as more than just investment with a sign change. Finally in the third section, investment and disinvestment will be combined in an integrated theory of investment and disinvestment analytically linked through the production function.

I. Investment

In traditional production economics little attention has been focused on the problem of extracting services from durable inputs in the production process. Those studies which have acknowledged the "stock-flow" problem have assumed it away by fixing the rate of extraction [1]. Fixing the rate of extraction is theoretically unappealing as well as empirically unsupported. Casual empirical observation indicates that fixed durables are used at varying rates. For example on a farm, the tractor is seldom used the same number of hours each production period. Since the rate of utilization is variable in the real world, we should have a theory which would enable us to determine the optimal rate of utilization for durable inputs.

Following Idachaba [7] the following production function will be considered.

$$(1) \quad Y = f(X_1, X_2, X_3)$$

where

Y = output flow per unit of time

X_1 and X_2 are multiple use durables

X_3 = an input flow that is consumed in a single production period.

Furthermore X_1 and X_2 are used to produce services, but not entirely consumed in a single production period.

Since we are interested in determining the optimal rates of utilization for the durable inputs (X_1 and X_2), we should treat the rate of utilization as a variable, the level of which will be determined endogenously. In the production process, it is not the rate of utilization that we are interested in, but rather the flow of services generated by the utilization rate. In at least some instances increased rates of utilization are partial substitutes for marginal additions to the durable input. Also reduced rates of utilization may, under certain conditions, be substitutes for the disposal of durable inputs. It may also be possible to purchase flows of inputs as substitutes for stock purchases. Idachaba [7] provides a convincing argument for including the flow of services that results from changing the rate of utilization as <u>separate</u> inputs in the production function. Thus our production function becomes

$$(2) \quad Y = f\,(X_1, \theta_1, X_2, \theta_2, X_3)$$

where

θ_1 = flow of services resulting from changes in the rate of utilization of X_1

θ_2 = flow of services resulting from changes in the rate of utilization of X_2

X_i, $i = 1, 2, 3$ are the same as before.

It should be pointed out that

$$(3) \quad \theta_i = g_i\ (R_i)\ i = 1,2$$

where

R_i = the rate of utilization.

It is reasonable to assume that $\frac{dg_i}{dR_i} > 0$ and $\frac{dg_i^2}{dR_i^2} < 0$, where the latter may be expected to hold for levels of use close to the capacity level $\bar{R}_i$. R_i and $\bar{R}_i$ may both be functions of the age of X_i. In addition, it may be possible to increase R_i for X_i by complementary investments in X_{i+n} $n \neq 0$, i.e., it may be possible to increase the rate of use of a piece of machinery by investing in a larger tractor. This may also increase $\bar{R}_i$.

Changing the rate of utilization is not costless. We would expect increases in the rate of utilization to increase the physical wear and tear or depreciation of the durable. This can be expressed in the following relationship.

$$(4) \quad D_i = h_i\ (R_i)$$

where

D_i = the physical depreciation of an input as a function of R_i.

The determinants of the optimum rate of use will include the above relationship on the cost side. On the demand side the determinants will be influenced by the rate of output needed per production period to meet the firm's production schedule. This yields the following relationship for R_i

(5) $R_i = K_i\ (\theta_i,\ W^*)$

where

W^* is the optimal required output rate per period for the firm to meet its production schedule.

Thus the firm wishes to maximize profit subject to the production function constraint. This is expressed in equation (6).

$$(6)\quad \text{Max. } \pi = P_y.Y - P_{x_1a}.X_1\ \{h_1\ [K_1\ (\theta_1,\ W^*)] + r + d_{x_1}\} - P_{x_2a}.X_2\ \{h_2\ [K_2\ (\theta_2,\ W^*)] + r + d_{x_2}\} - P_{x_3}.X_3$$

Before deriving the equilibrium conditions, it may prove useful to expand the first input cost term to see what costs are actually being considered.

$P_{x_1a}.X_1.\ \{h_1\ [K_1\ (\theta_1,\ W^*)] + r + d_{x_1}\}$ gives the per period cost associated with X_1. Expanding this by multiplying through by $P_{x_1a}.X_1$ yields.

$$(7)\quad P_{x_1a}.X_1\ h_1\ [K_1\ (\theta_1,\ W^*)] + P_{x_1a}.X_1\ r + P_{x_1a}.X_1\ d_{x_1}.$$

P_{x_1a} is the acquisition cost per unit of X_1, X_1 is quantity of X_1, $h_1\ [K_1(\theta_1,\ W^*)]$ from (4) and (5) is the depreciation rate, r is nominal interest rate, it represents the opportunity cost of funds committed to X_1, and d_{x_1} are costs per period associated with calendar time or age including obsolescence.

So the first term in (7) represents the total depreciation cost associated with X_1, the second term represents the total per period opportunity cost of funds committed to X_1 and the third term represents the total per period obsolescence cost associated with X_1. A similar expansion can be done for X_2 showing the three components that combine to form the total cost associated with X_2.

Equations (8) through (12) give the necessary conditions for profit maximization. Recall that the variables are X_1, X_2, X_3, θ_1, and θ_2.

$$(8) \quad \frac{\partial \pi}{\partial X_1} = P_y f_{x_1} - P_{x_1 a} \{h_1 [K_1 (\theta_1, W^*)] + r + d_{x_1}\} = 0$$

$$(9) \quad \frac{\partial \pi}{\partial X_2} = P_y f_{x_2} - P_{x_2 a} \{h_2 [K_2 (\theta_2, W^*)] + r + d_{x_2}\} = 0$$

$$(10) \quad \frac{\partial \pi}{\partial X_3} = P_y f_{x_3} - P_{x_3} = 0$$

$$(11) \quad \frac{\partial \pi}{\partial \theta_1} = P_y f_{\theta_1} - P_{x_1 a} . X_1 \left(\frac{\partial D_1}{\partial R_1} \frac{\partial R_1}{\partial \theta_1}\right) = 0$$

$$(12) \quad \frac{\partial \pi}{\partial \theta_2} = P_y f_{\theta_2} - P_{x_2 a} . X_2 \left(\frac{\partial D_2}{\partial R_2} \frac{\partial R_2}{\partial \theta_2}\right) = 0$$

where P_y = price of output, $P_{x_i a}$ is the acquisition price per unit of X_i ($i = 1,2,3$), $f_{x_i} = MPP_{x_i}$, and $f_{\theta_i} = MPP_{\theta_i}$. Thus we can specify the following:

$$(13) \quad MVP_{x_i}^{1/} = f_{x_i} . P_y \quad i = 1,2,3$$

$$(14) \quad MVP_{\theta_i} = f_{\theta_i} . P_y \quad i = 1,2$$

$$(15) \quad MfC_{x_i} = P_{x_i a} \{h_i [K_i (\theta_i, Y^*, W^*)] + r + d_{x_i}\} \quad i = 1,2$$

$$(16) \quad MfC_{x_3} = P_{x_3 a}$$

$$(17) \quad MfC_{\theta_1} = \left(\frac{\partial D_1}{\partial R_1} \cdot \frac{\partial R_1}{\partial \theta_1}\right) \left(1 + \frac{1}{\varepsilon_{\theta_1 a}}\right) P_{x_1 a} . X_1$$

$$(18) \quad MfC_{\theta_2} = \left(\frac{\partial D_2}{\partial R_2} \frac{\partial R_2}{\partial \theta_2}\right) \left(1 + \frac{1}{\varepsilon_{\theta_2 a}}\right) P_{x_2 a} . X_2$$

Two points should be noted. First there is a need to distinguish between acquisition and salvage prices. Normally transportation and positive transactions costs lead to acquisition prices being higher than salvage values [1].

The second thing that should be noted is that equations (17) and (18) give the marginal user cost, i.e., the cost incurred at the margin due to changes in the rate at which services are extracted from the fund of services. As expressed here marginal user cost is determined both internally and externally. Internally marginal user cost is determined by the

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1/ The MVP is used here since P_y is considered to be the marginal revenue as well as the average revenue. It is a simple matter to extend this analysis to handle imperfect product and factor markets. We restrict ourselves to the simpler case of perfect competition to better clarify the fundamental issues involved.

physical transformation of R_i into θ_i as well as by the change in physical depreciation per unit increase in the rate of utilization $(\frac{\partial D_i}{\partial R_i} \frac{\partial R_i}{\partial \theta_i})$. The marginal user cost is determined externally by what it would currently cost the firm to acquire a unit of X_i. As a result of this dual determination, the formula for the marginal user cost recognizes the fact that the factor price elasticities of supply for θ_i with respect to the acquisition price $(\varepsilon_{\theta_i a})$ cannot $= \infty$.

Intuitively it is unlikely that the farmer will think that the supply curves for θ_i facing him are horizontal. His behavior in demanding a marginal unit of θ_i affects the price of θ_i.

The present value of total user costs for the two inputs over the time interval [0, T] is $\int_0^T [MfC_{\theta_1} + MfC_{\theta_2}] e^{-nt} dt$.

Equations (19) through (22) give the supplementary equilibrium equations.

(19) $Y^d = g_0(P_y)$

(20) $P_{x_i a} = g_i(X_i) \quad i = 1,2,$

(21) $B_i = g_{i+3}(\theta_i) \quad i = 1,2$

(22) $g_o(P_y) = f(X_1, X_2, X_3, \theta_1, \theta_2)$

where the B_i's are the acquisition costs per unit of θ_1 and θ_2, i.e., the marginal user costs.

When input prices are held constant, equations (8) through (22) give the implicit demand functions for the inputs. They also yield the optimal quantities X_1, X_2, and X_3 and the optimum flow of services θ_1 and θ_2. The demand functions can be written as

$$(23) \quad X_i^d = X_i^D \, (P_{x_i a}, P_{x_j a}, P_y, B_i, B_j)$$

$$(24) \quad \theta_i^d = \theta_i^D \, (B_i, B_j, P_y, P_{x_i a}, P_{x_j a})$$

It is important to realize that acquisition price is the relevant price to consider when demanding marginal units of flow inputs as well as stock inputs.

Since one of the interests of this paper is in the micro theory of investment in productive assets, it is necessary to make explicit the relationship between investment demand and the input demand function given by equation (23). Investment will be defined to be the positive time rate of change in the durable asset. In moving from input demand to investment demand, the linkages between the amount of services, input flow and real stocks of services must be recognized. Assume avail able services from stocks are functions of additions to these stocks in previous time period, i.e.,

$$(25) \quad X^s_{i_{tn}} = G \, (\int_{to}^{tn} M \, (t) \, dt)$$

where $X^s_{i_{tn}}$ is total amount of services available at time tn and M(t) is a functional form relating marginal additions to stock over the time interval [to, tn].

Investment demand is determined by the input flow and total services available as well as the required input demand in each period to meet anticipated output requirements. From equations (23), (24) and (25), the implied investment demand equation is obtained:

$$(26) \quad I_{tn} = I\,(P_{x_1a},\ P_{x_2a},\ B_1,\ B_2,\ r,\ \int_{to}^{tn} M(t)dt).$$

The user costs of inputs resulting from increasing the rates of utilization of X_1 and X_2 have been included to emphasize the substitution possibilities involved. The stock term captures the effect on the levels of user costs (B_1 and B_2) of operating at different distances from the capacity levels of the productive facilities.

It is plausible that $\frac{\partial I}{\partial (P_{x_1a}/P_y)} < 0$; $\frac{\partial I}{\partial (B_1/P_y)} < 0$ meaning that all else constant, if P_y rises relative to P_{x_1} or B_1 and the new levels are expected to be maintained, then investment can be expected to increase. Inferior inputs are ruled out. $\frac{\partial I}{\partial r} < 0$ conforms with the usual case.

When there is a relative fall in P_y or relative rise in P_{x_1a} which is expected to be maintained, disinvestment occurs, i.e., investment occurs at a lower rate. To determine the net effect of price changes in relative inputs alone on our stock demand for inputs alone, knowledge of the relevant elasticities of substitution among inputs and the price elasticities of demand for the product must be known.

The above is one approach along which a micro theory of investment could develop. Attention will now be focused on disinvestment.

II. Disinvestment

Much has been written relative to equipment replacement [2, 3, 6, 9, 11, 12], but no general theory of disinvestment exists. The latter is often treated as part of investment, sometimes in some form of opportunity cost [2, 3, 12]. One reason for the neglect of disinvestment in traditional invest ment literature [5, 8] is the argument that disinvestment is investment with a change in sign. Although equipment replace ment rules are plentiful, rules do not make a theory. As yet no integrated theory of production and disinvestment exists.

An asset generally has two prices: the acquisition pric and the salvage price [8]. We are generally interested in th behavior of producers, investors and disinvestors as they strive to maximize their net income streams.

A firm owning used inputs has two possible incomes he ca derive from them: income from selling these inputs on the used input market or income from retaining them in production Thus we can define profits from selling used inputs (π^*) as

$$(27) \quad \pi^* = P_{x_1 s} X_1^s + P_{x_2 s} X_2^s + \alpha_1 \theta_1^s + \alpha_2 \theta_2^s - P_y Y^*$$

where $P_{x_1 s}$ is the highest salvage value per unit of X_1 for this input outside the firm, Y^{**} is potential output if the i puts were retained in use in the production plan. Thus the

opportunity cost of inputs a producer plans to sell on the used input market is what they could have produced in the firm. X_i^s is the quantity of X_i being offered for sale and θ_1^s and θ_2^s are the amounts of input flows available for renting out for uses outside this firm. α_1 and α_2 represent the salvage price for θ_1^s and θ_2^s respectively. All other variables retain their former meanings. We maximize π^* subject to

$$(28) \quad X_i^s \leq X_i^o \qquad i = 1,2$$

$$(29) \quad \theta_i^s \leq \bar{\theta}_i \qquad i = 1,2$$

where X_i is the initial quantities on hand and $\bar{\theta}_i$ denotes physical capacity input flows and is a function of the rate of utilization, R_i. Forming the lagrangian of this we get

$$30) \quad L = P_{x_1 s} X_1^s + P_{x_2 s} X_2^s + \alpha_1 \theta_1^s + \alpha_2 \theta_2^s - P_y Y^{**}$$
$$+ r_1 [X_1^o - X_1^s] + r_2 [X_2^o - X_2^s] + r_3 [\bar{\theta}_1 - \theta_1^s]$$
$$+ r_4 [\bar{\theta}_2 - \theta_2^s]$$

From this we derive the necessary Kuhn-Tucker conditions for a saddle point:

$$(31) \quad \frac{\partial L}{\partial X_1^s} = P_{x_1 s} - P_y \frac{\partial Y^{**}}{\partial X_1^s} - r_1 \leq 0, \text{ if '<' holds,}$$
$$X_1^s = 0.$$

$$(32) \quad \frac{\partial L}{\partial Y^s} = P_{x_2 s} - P_y \frac{\partial Y^{**}}{\partial X_2^s} - r_2 \leq 0, \text{ if '<' holds,}$$
$$X_2^s = 0.$$

$$(33)\quad \frac{\partial L}{\partial r_1} = X_1^o - X_1^s \geq 0, \text{ if '>' holds, } r_1 = 0.$$

$$(34)\quad \frac{\partial L}{\partial r_2} = X_2^o - X_2^s \geq 0, \text{ if '>' holds, } r_2 = 0.$$

$$(35)\quad \frac{\partial L}{\partial \theta_1^s} = \alpha_1 - P_y \frac{\partial Y^{**}}{\partial \theta_1^s} - r_3 \leq 0, \text{ if '<' holds } \theta_1^s = 0.$$

$$(36)\quad \frac{\partial L}{\partial \theta_2^s} = \alpha_2 - P_y \frac{\partial Y^{**}}{\partial \theta_2^s} - r_4 \leq 0, \text{ if '<' holds } \theta_2^s = 0.$$

$$(37)\quad \frac{\partial L}{\partial r_3} = \bar{\theta}_1 - \theta_1^s \geq 0, \text{ if '>' holds } r_3 = 0.$$

$$(38)\quad \frac{\partial L}{\partial r_4} = \bar{\theta}_2 - \theta_2^s \geq 0, \text{ if '>' holds } r_4 = 0.$$

where $P_{x_i s}$ and α_i are unit salvage prices of X_i and θ_i. Equations (31) and (32) say that in equilibrium off firm salvage values of inputs are less than or equal to the within firm opportunity cost plus the imputed marginal cost of constraint (28). Equations (35) and (36) say the analagous things for θ_i. Equations (33) and (34) state that if the constraints are not binding then the imputed marginal costs of violating them should be set at zero. r_1 and r_2 reflect the prices per unit of X_i

equilibrium. We assume the second order conditions hold on the basis of a well behaved production function. When all inputs are variable, a well behaved utility function insures the second order conditions.

The above could be generalized to allow for less than perfect competition in the market for used inputs as well as in the product market, however, it is believed that such a generalization would obscure the more important issues being addressed here. By holding salvage prices constant for any given input, we can derive the firms supply function for used inputs in the used input market. Equations (31), (32), (35) and (36) yield the implicit supply functions of used inputs by the firm.

$$(39) \quad X_1^s = H_1 (P_{x_1 s}, P_{x_2 s}, r, P_y)$$

$$(40) \quad X_2^s = H_2 (P_{x_2 s}, P_{x_1 s}, r, P_y)$$

$$(41) \quad \theta_1^s = H_3 (\alpha_1, \alpha_2, r, P_y)$$

$$(42) \quad \theta_2^s = H_4 (\alpha_2, \alpha_1, r, P_y)$$

where X_1^s and θ_1^s are input quantities supplied and all other variables retain their earlier meaning. It is plausible to expect $\frac{\partial X_1^s}{\partial P_{x_1 s}} > 0$, $\frac{\partial X_1^s}{\partial (P_{x_1 s}/P_y)} > 0$, and $\frac{\partial \theta_1^s}{\partial (\alpha_1/P_y)} > 0$.

This approach enables us to identify the effects on net returns of the sale of an input from the portfolio of inputs.

If $\frac{\partial X_i^s}{\partial P_{x_i s}} > 0$ $i \neq j$, then the inputs are complements. If

$\frac{\partial X_i^s}{\partial P_{x_i s}} < 0$, $i \neq j$, then the inputs are substitutes. The firm has to consider the substitutability and complementarity relationships between inputs in its decision to supply used inputs. One problem with the existing input replacement rules is that they typically look at the net return from selling a used input [2, 3, 12]. Our approach implies, for example, that if the salvage price of an asset goes up, it might pay to supply some of the complementary input on the used input market together with the first input. The implication of this is that the firm doesn't look at any asset in isolation. Rather it looks at the portfolio return of sub-categories of inputs according to whether they are complements or substitutes.

Suppose services lost [X^{SL}] from sales are a function of past reductions in stocks of inputs.

$$(43) \quad X_{tn}^{SL} = X^{SL} (-\Delta X_{t-1}, -\Delta X_{t-2}, \ldots -\Delta X_{t-n})$$

Given future output requirements and stock supply of used inputs on the used input market in previous periods, then from equations (39) through (43) the implied disinvestment supply function (DI) is:

$$(44) \quad DI = DI (P_{x_1 s}, P_{x_2 s}, P_y, r, \alpha_1, \alpha_2, dx)$$

where dx represents the costs associated with technological obsolescence and demand obsolescence. All other variables retain their earlier meaning. It is plausible to expect

$\frac{\partial DI}{\partial (P_{x_i}^{s}/P_y)} > 0$, meaning other things constant, a rise in product price relative to the salvage price of an input (and therefore a relative rise in the within-firm MVP of the input) that is expected to stay at its new level will result ultimately in a decrease in the firm's supply of disinvestment in the productive asset; conversely for a fall. It is also plausible that $\frac{\partial DI}{\partial \alpha_i} > 0$, which means that as the salvage value of θ_i increases, it may pay the firm to rent out the services of the inputs. It is also plausible to expect $\frac{\partial DI}{\partial r} > 0$.

In this section supply functions for used inputs and a disinvestment supply function have been derived. These were based on profit maximizing behavior. The recognition of the acquisition and salvage price differentials has been analytically important in the derivation of both an investment demand function and a disinvestment supply function.

III. Investment, Disinvestment and Production Theory

The theoretical considerations developed in the previous two sections form the basis for the extension of the Johnson-Edwards [5, 1] formulation that will be presented in this section.

For the inputs X_i, $i=1,2,\ldots n$, let $\infty \geq P_{x_i a} \geq P_{x_i s} \geq 0$, where $P_{x_i a}$ equals acquisition price of X_i and $P_{x_i s}$ equals the

salvage value of X_i. The optimal rate at which to extract services from the inputs will be endogenously determined. Thus we need not assume proportionality between stocks and the flow of services as was done in Edwards [1] and much of capital theory.

Assume an initial condition for a firm. Let G stand for the gain from reorganizing the firm, where reorganization refers to profit maximizing changes in the output and input vectors in response to relative prices of both products and inputs. In other words for relative prices constant and an initial equilibrium we have,

$$G \gtreqless 0 \text{ for } \Delta\theta_i \gtreqless 0, \quad i=1,\ldots n \text{ and}$$

$$\Delta X_i \gtreqless 0, \quad i=1,\ldots n.$$

Now

$$(45) \qquad G = \sum_{j=1}^{m} P_{y_j} (Y_j - Y_j^o) - \sum_{i=1}^{n} A_i \left(\sum_{j=1}^{m} X_{ij} - X_i^o \right)$$

where Y_j represents products and X_{ij} inputs; Y_j^o, $j=1,\ldots m$ stands for initial output of product j, and Y_j stands for the reorganized output of product j by the reorganized firm; X_i^o and X_{ij} similarly stand for the initial and reorganized input of X_i respectively. P_{y_j} and A_i are product and input prices respectively.

Three price equations can be defined for X_i: one of which states that if the reorganized quantity of X_i exceeds the initial quantity the relevant price for X_i is its acquisition

price; if the reorganized quantity of X_i is less than the initial quantity on hand, then the relevant price is its salvage price; if after reorganizing it does not pay to vary the quantity of X_i, then we say that X_i is fixed and the relevant price for X_i is its within firm opportunity cost or shadow price. The latter is the notion of an economic definition of asset fixity as opposed to a physical definition. This has been analyzed in much of Glenn Johnson's work [8]. These definitions are expressed in equations (46) through (48).

$$(46) \quad \text{if } \sum_{j=1}^{m} X_{ij} > X_i^o, \text{ then } A_i = P_{x_i a}, \quad i=1,\ldots,n$$

$$(47) \quad \text{if } \sum_{j=1}^{m} X_{ij} < X_i^o, \text{ then } A_i = P_{x_i s}, \quad i=1,\ldots,n$$

$$(48) \quad \text{if } \sum_{j=1}^{m} X_{ij} = X_i^o, \text{ then } P_{x_i s} < A_i < P_{x_i a}.$$

To treat explicitly the endogenous determination of the optimum rate of utilization we will extend equation (45) to the following:

$$(49) \quad G = \sum_{j=1}^{m} P_{y_j} (Y_j - Y_j^o) - \sum_{i=1}^{n} A_i \left(\sum_{j=1}^{m} X_{ij} - X_i^o \right) - \sum_{i=1}^{n} W_i \left(\sum_{j=1}^{m} \theta_{ij} - \theta_i^o \right)$$

For θ_i we can define three price equations analagous to the ones for X_i.

$$(50) \quad \text{if } \sum_{j=1}^{m} \theta_{ij} > \theta_i^o, \text{ then } W_i = B_i$$

$$(51) \quad \text{if} \sum_{j=1}^{m} \theta_{ij} < \theta_i^o, \text{ then } W_i = \alpha_i$$

$$(52) \quad \text{if} \sum_{j=1}^{m} \theta_{ij} = \theta_i^o, \text{ then } \alpha_i < W_i < B_i$$

where W_i is the price of a unit of θ_i from equation (49).

Equation (50) indicates that when more than θ_i is being used we price θ_i at its acquisition price, B_i. From equation (51), when less than θ_i^o is being used the relevant unit price is α_i, its salvage value. When it doesn't pay to vary θ_i the unit price is W_i, the within firm opportunity cost. B_i is the user cost of θ_i: $\frac{\partial D_i}{\partial R_i} \frac{\partial R_i}{\partial \theta_i} P_{x_i a} \cdot X_i$ is our earlier derivation of user cost. Marginal user cost is $\frac{\partial D_i}{\partial R_i} \frac{\partial R_i}{\partial \theta_i} P_{x_i a} \cdot X_i$ $[1 + 1/\varepsilon_{\theta_i a}]$. This is what it costs the firm to acquire a unit of θ_i at the margin by buying X_i. When X_i is not varied, marginal user cost is $\frac{\partial D_i}{\partial R_i} \frac{\partial R_i}{\partial \theta_i} \cdot A_i \cdot X_i$ $[1 + 1/\varepsilon_{\theta_i a}]$, where $P_{x_i s} \leq A_i \leq P_{x_i a}$. (Note that B_i would equal marginal user cost if the supply curve for θ_i faced by the firm were horizontal, in general this is not the case.) B_i and marginal user cost are determined both internally and externally. $P_{x_i a}$ is not the historical price, but the current price per unit of X_i. α_i is determined by the off firm opportunity cost of θ_i (using θ_i outside the firm without the present owner losing his stock ownership of X_i, e.g., renting θ_i).

After reorganizing from an initial equilibrium in response to relative price changes, the firm would have bought some inputs, sold some inputs, hired some input services, or rented out some input services. Let W_i = amount of X_i purchased and V_i = amount of X_i sold. Similarly let θ_i^A = amount of θ_i "purchased" and θ_i^S = amount of θ_i sold (rented). The following restrictions can then be imposed.

$$(53) \quad \sum_{j=1}^{m} X_{ij} \leq X_i^o + W_i - V_i$$

$$(54) \quad X_i^o - V_i \geq 0$$

$$(55) \quad \sum_{j=1}^{m} \theta_{ij} \leq \theta_i^o + \theta_i^A - \theta_i^S$$

$$(56) \quad \theta_i^S \leq \bar{\theta}_i$$

where (53) indicates that the reorganized vector of inputs cannot exceed the initial quantity plus new purchases less the quantity sold; (54) says that the firm cannot sell more than it has; (55) is analogous to (53) while (56) says that the firm cannot rent out more than the capacity level for θ_i, $\bar{\theta}_i$. If firms operate at capacity levels, then θ_i loses some of its importance. The fact that firms do not always operate at the capacity levels of their inputs reflects the cost associated with increasing the rates of utilization. In general, whether it pays to operate at the capacity level is determined by both the associated user cost and demand conditions. Thus we want to maximize

$$(57)\quad G^* = \sum_{j=1}^{m} P_{y_j} (Y_j - Y_j^o) + \sum_{i=1}^{n} S_i V_i$$

$$+ \sum_{i=1}^{n} \alpha_i \theta_i^s - \sum_{i=1}^{n} A_i W_i - \sum_{i=1}^{n} B_i \theta_i^A$$

where $S_i = P_{x_i s}$ and all other variables retain their earlier definitions. Equation (57) is maximized subject to the constraints specified in (53) through (56) by forming the following lagrangian function.

$$(58)\quad L = \sum_{j=1}^{m} P_{y_j} (Y_j - Y_j^o) + \sum_{i=1}^{n} S_i V_i$$

$$+ \sum_{i=1}^{n} \alpha_i \theta_i^s - \sum_{i=1}^{n} A_i W_i - \sum_{i=1}^{n} B_i \theta_i^A$$

$$+ \sum_{i=1}^{n} \lambda_i (X_i^o - V_i + W_i - \sum_{j=1}^{m} X_{ij})$$

$$+ \sum_{i=1}^{n} \rho_i (\bar{\theta}_i - \theta_i^s + \theta_i^A - \sum_{j=1}^{m} \theta_{ij})$$

$$+ \sum_{i=1}^{n} \mu_i (X_i^o - V_i) + \sum_{i=1}^{n} \delta_i (\bar{\theta}_i - \theta_i^s).$$

The necessary Kuhn-Tucker conditions are given by the following equations.

$$(59)\quad \frac{\partial L}{\partial X_{ij}} = P_{y_j} \frac{\partial Y_j}{\partial X_{ij}} - \lambda_i \leq 0, \ \frac{\partial L}{\partial X_{ij}} \cdot X_{ij} = 0, \ X_{ij} \geq 0$$

$$(60)\quad \frac{\partial L}{\partial W_i} = \lambda_i - A_i \leq 0, \ \frac{\partial L}{\partial W_i} \cdot W_i = 0, \ W_i \geq 0$$

$$(61)\quad \frac{\partial L}{\partial V_i} = S_i - \lambda_i - \mu_i \leq 0, \ \frac{\partial L}{\partial V_i} V_i = 0, \ V_i \geq 0$$

$$(62)\quad \frac{\partial L}{\partial \lambda_i} = X_i^o - V_i + W_i - \sum_{j=1} X_{ij} \geq 0, \quad \frac{\partial L}{\partial \lambda_i} \cdot \lambda_i = 0$$

$$\lambda_i \geq 0$$

$$(63)\quad \frac{\partial L}{\partial \mu_i} = X_i^o - V_i \geq 0 \quad \frac{\partial L}{\partial \mu_i} \cdot \mu_i = 0, \quad \mu_i \geq 0$$

$$(64)\quad \frac{\partial L}{\partial \theta_{ij}} = P_{y_j} \frac{\partial Y_j}{\partial \theta_{ij}} - \rho_i \leq 0, \quad \frac{\partial L}{\partial \theta_{ij}} \cdot \theta_{ij} = 0,$$

$$\theta_{ij} \geq 0$$

$$(65)\quad \frac{\partial L}{\partial \theta_i^A} = \rho_i - B_i \leq 0, \quad \frac{\partial L}{\partial \theta_i^A} \cdot \theta_i^A = 0, \quad \theta_i^A \geq 0$$

$$(66)\quad \frac{\partial L}{\partial \theta_i^S} = \alpha_i - \rho_i - \delta_i \leq 0, \quad \frac{\partial L}{\partial \theta_i^S} \cdot \theta_i^S = 0,$$

$$\theta_i^S \geq 0$$

$$(67)\quad \frac{\partial L}{\partial \delta_i} = \bar{\theta}_i - \theta_i^S \geq 0, \quad \frac{\partial L}{\partial \delta_i} \cdot \delta_i = 0, \quad \delta_i \geq 0$$

$$(68)\quad \frac{\partial L}{\partial \rho_i} = \theta_i^o - \theta_i^S + \theta_i^A - \sum_{j=1}^{m} \theta_{ij} \geq 0,$$

$$\frac{\partial L}{\partial \rho_i} \cdot \rho_i = 0, \quad \rho_i \geq 0.$$

Supplementary equations determining input and product prices analagous to equations (19) through (22) may be added.

Equation (59) states that the optimal reorganization of a firm in response to price changes involves having the marginal value productivity of each factor in producing a given product less than or equal to its within firm opportunity cost (λ_i). (60) states that in equilibrium reorganization,

the optimum purchases of X_i involve an acquisition price equal to the within firm opportunity cost of X_i. (61) states that when the salvage value of X_i (S_i) is less than $\lambda_i + \mu_i$, the firm ceases to sell the input. Equation (62) is an inequality rather than an equality as in Johnson and Edward's formulation. This is because λ_i cannot be allowed to be negative in any use unless two conditions hold: (1) the marginal cost of disposing of a unit of the input exceeds the marginal opportunity cost and (2) X_i has a by-product whose value is not captured by its marginal productivity and the value of which exceeds the opportunity cost in alternative uses. The condition that V_i and W_i cannot be simultaneously positive (i.e., the firm cannot buy and sell at the same time) can be relaxed. A used asset of a given vintage in the hands of a producer that he plans to sell at the margin does not have the same MVP as a different asset of the same vintage that the farmer plans to acquire at the margin. Therefore he can buy and sell inputs at the same time because the units are not perfect substitutes in production.

Equation (64) states that the optimally reorganized firm will have the MVP of services coming from increases in the rates of utilization less than or equal to their within firm opportunity cost (ρ_i). Equation (65) says that if the opportunity cost of θ_i is less than its acquisition price B_i, the firm reduces its "purchases" of θ_i until $\rho_i = B_i$ or until $\theta_i^A = 0$, whichever comes first. Equations (66) states that when the

salvage value of θ_1 (α_1) is less than $\rho_1 + \delta_1$ the firm ceases to rent the services. Equation (67) states that the firm cannot rent more than the capacity level. Equation (68) is analagous to equation (62).

This analysis could be further extended by allowing for less than perfect competition in the product and factor markets. The second order conditions are assumed to hold on the basis of a well behaved production function.

The above is an integrated theory of production, investment and disinvestment. For a theory of investment, equations (59) and (60) yield the implicit demand equations for inputs from which the investment demand function can be derived as was done in section I of this paper. In a similar manner, equations (64) and (65) yield implicit demand equations for θ_1 where these conditions determine the possibility for substitution between working inputs more intensively and investing in new marginal units.

For a theory of disinvestment equations (61) and (63) define the implicit supply equations for used inputs to be sold on the used input market. From these we can derive the implied disinvestment supply equation, in a manner analagous to what was done in section II of this paper [equations (39) through (44)]. In the general case the disinvestment supply function would be a function of the salvage price of the input, user costs, the salvage value of input flows available for renting, and costs associated with calendar time.

$$(69) \quad DI_i = DI\,(S_i, \lambda_i, S_j, \lambda_j, \alpha_i, \rho_i, \alpha_j, d_{x_i}(t))$$

where

DI_i is the disinvestment supply of asset i,

S_i is salvage price of input X_i

λ_i is opportunity cost of X_i in the firm

S_j is some weighted index of opportunity costs of other inputs

λ_j is some weighted index of opportunity costs of other inputs

α_i is salvage price of input flows

ρ_i is the opportunity cost of Θ_i

α_j is a weighted index of opportunity costs of other Θ_i's

$d_{x_i}(t)$ are costs associated with time.

It is plausible to expect $\frac{\partial DI_i}{\partial (S_i/\lambda_i)} > 0$ meaning that a rise in salvage price of X_i relative to its within firm opportunity cost with other things constant results in an increase in the firm's disinvestment supply. It is also plausible to expect $\frac{\partial DI_i}{\partial \alpha_i} < 0$ and $\frac{\partial DI_i}{\partial \rho_i} < 0$; $\frac{\partial DI_i}{\partial \alpha_j}$ and $\frac{\partial DI_i}{\partial \rho_j}$ have ambiguous signs.

Equations (66) and (67) implicitly define the supply equations for Θ_i where these conditions emphasize the substitution possibility between decumulating the sources of income streams over time and decumulating them through loss of stock ownership at a point in time.

IV. Summary

his paper has attempted to present an integrated theory of production, investment and disinvestment. The theory is based on the assumption that producers attempt to maximize profits. The analysis in this paper is consistent with previous work done by Glenn Johnson [8] in the area of investment and disinvestment. It extends his analysis by treating the rate of utilization explicitly in the production process. In his analysis when $P_{x_i}s < MVP_{x_i} < P_{x_i}A$ no dynamic adjustment toward any long run equilibrium level of stocks of inputs or assets is involved. Neither investment nor disinvestment occurs. The analysis presented here implies that for this to result in no changes in the level of production, it must be the case that it does not pay to vary the rates of utilization of the firm's inputs for any relative price changes. Thus this presentation broadens the usual definition of asset fixity. The concept of two prices for an asset was shown to be analytically applicable to θ_i, the input flows coming mainly from changes in the rate of utilization.

The concept of the rate of use and the associated user cost as well as the integrated production, investment and disinvestment theory may find application in a wide variety of areas. For example, this analysis could be applied to determine the optimal rate of use of land resources. It could also be applied to the determination of the optimal size and the optimal

rate of use of animal herds. Another area of possible application is rural-urban migration theory. The Harris-Todaro model [5] might form the appropriate point of departure for such an application.

There are several issues that were not addressed in this paper. The issue of lagged adjustments in both investment demand and disinvestment supply was not addressed. Any adjustment mechanism would have to recognize the fact that producers make mistakes, that they have imperfect knowledge and that information is not costless. Since our objective here was to clarify economic theory as it applies to production, investment and disinvestment at the firm level, aggregation problems were not touched on.

In developing and clarifying the theory little attention has been focused on decisions rules, primarily because rules do not make a theory. It was deemed more important to clarify the economic theory from which rules may be derived.

Before the theory presented here can be applied to simulation studies, attention will have to be focused on the issues not addressed in this paper. It is the author's belief that further research is needed to more accurately specify the production process. The production function specified in equation (1) masks the interaction that occurs when the rate of utilization of durables is allowed to vary. A better formulation of the production process might be the following.

$$(70) \quad Y = f(\theta_1, \theta_2, X_3)$$

$$\theta_1 = f_1(X_1, \theta_2, X_3, R_1)$$

$$\theta_2 = f_2(\theta_1, X_2, X_3, R_2)$$

where all variables retain their former meanings. This formulation expresses output as a function of the flow of services from the inputs. These flows are determined by the stock level of the input, the input flows of other inputs, and the rate of use of the input in question. Research is needed to determine the appropriate technique for solving this system of equations. Three possible alternatives for solution are (1) linear programming, (2) some form of non-linear programming or (3) numerical techniques.

Another area that was not treated by this paper is the derivation of appropriate investment and disinvestment decision rules. Deriving decision rules will probably encompass the issues of lagged adjustments, imperfect knowledge and discounting.

This paper has attempted to lay the foundation upon which further work in the areas of investment and disinvestment as they relate to simulation studies can be built.

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