AGENCY FOR INTERNATIONAL DEVELOPMENT
WASHINGTON, D. C. 20523

BIBLIOGRAPHIC INPUT SHEET

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Batch 31

1. SUBJECT CLASSIFICATION

A. PRIMARY: Agriculture — AE70-0000-0000

B. SECONDARY: Distribution and marketing

2. TITLE AND SUBTITLE

The annual price determination mechanism

3. AUTHOR(S)

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4. DOCUMENT DATE	5. NUMBER OF PAGES	6. ARC NUMBER
1974	14p.	ARC

7. REFERENCE ORGANIZATION NAME AND ADDRESS

Mich. State

8. SUPPLEMENTARY NOTES (Sponsoring Organization, Publishers, Availability)

(In KASS working paper no. 74-2)

9. ABSTRACT

10. CONTROL NUMBER

PN-AAB-738

11. PRICE OF DOCUMENT

12. DESCRIPTORS

Price policy

13. PROJECT NUMBER

14. CONTRACT NUMBER

CSD-2975 Res.

15. TYPE OF DOCUMENT

590-1 (4-74)

PN-AAB-738 KASS working paper no. 2
CSD-2975 Res.

WP 74-2
Lloyd Teigen
June 19, 1974

The Annual Price Determination Mechanism

The mechanism described in this paper determines a set of prices which simultaneously equates commodity demand with the available supplies and satisfies a national budget constraint. This system is sufficiently general enough so that the prices of some of the commodities can be determined in the policy process. These policy determined price then act as exogenous constraints on the price determination process. A means of incorporating product reservation prices is presented which will guarantee positive prices for all commodities. Finally, a proof of convergence of the calculation process within two iterations is advanced.

The Model

We are given a system of demand equations which have constant price and income elasticities within any year. Instead of using a Cobb-Douglas type of equation system, the demand equation system is specified as a system of linear equations in which the stated elasticities are point elasticities at the previous set of prices and quantities. All income elasticities are multiplied by scalar (hereafter called the budget parameter) so that the aggregate

budget constraint can be simultaneously satisfied.

The model is specified algebraically with following notations:

E is the matrix of own and cross price elasticities of demand.

n is the column vector of income elasticities of demand.

c is the column vector of per capita consumption.

P is the column vector of consumer prices.

y is the scalar representing urban per capita income.

s is the scalar which is multiplied times the income elasticities in order to satisfy the budget constraint.

d(X) is a diagonal matrix whose diagonal elements are the same as those of vector X.

i is the subscript which denotes the number of the iteration in the current year's calculation process. The value of a variable with the subscript i=0 is the final value of that variable in the previous year.

Of the above items, E and n consist of system parameters, c and y are predetermined, and P and s are the endogenous variables to be determined in the price determination mechanism.

The linear demand equations are derived from the parameters and predetermined variables in the following manner:

The matrix of own and cross price derivatives is

$$AE_i = d(C_{i-1}) \quad E\left[d\ (P_{i-1})\right]^{-1}$$

The vector of income derivatives is

$$B_i = d\ (C_{i-1})\ n\ (1/y)$$

The vector of demand intercepts is

$$AO_i = C_{i-1} - S_{i-1}\ yB_i - AE_i\ P_{i-1}$$

The system of demand equations is represented by

$$C_i = AO_i + AE_i\ P_i + yB_iS_i$$

Market equilibrium occurs when per capita consumption is equal to the per capita supplies which are predetermined to be

$$C_i = (TDSUP_i + IMP_i - EXP_i - MKTLOSS_i)/POP$$

where TDSUP is the column vector of total domestic marketed surpluses, IMP is the column vector of imports, EXP is the column vector of exports and MKTLOSS is the column vector of marketing losses. POP is the urban population in the current year. All of these input data are determined outside this pricing mechanism.

The aggregate budget constraint states that per capita expenditure on all commodities (including saving) must exhaust per capita income. Since saving is not included as an element in the demand component, the per capita income is decreased by an exogenously determined saving rate.

In other words $y = C_i P_i$ where α is the average propensity to consume.

Thus in a generalized matrix form, the budget constrained price determination problem is stated as

$$(1) \quad \begin{pmatrix} C_2 - AO(2) \\ C_2 - AO(2) \\ \alpha y \end{pmatrix} = \begin{pmatrix} AE(1,1) & AE(1,2) & yB(1) \\ AE(2,1) & AE(2,2) & yB(2) \\ C'_1 & C'_2 & 0 \end{pmatrix}_i \begin{pmatrix} P_1 \\ P_2 \\ s \end{pmatrix}_i$$

with the objective to determine P_1, P_2 and s at iteration i. P_1 and P_2 are two subsets of the price vector, but could be visualized equally as well if they were thought to be the only two prices to be determined.

Suppose now that the values of P_1 are determined by the policy process, rather than by market forces, and we have to determine P_2 and s. Then the problem requires the following system of equations to be solved.

$$(2) \quad \begin{pmatrix} C_2 - AO(2) - AE(2,1)\ P_1 \\ \alpha y - P'_1\ C_1 \end{pmatrix} = \begin{pmatrix} AE(2,2) & yB(2) \\ C'_2 & 0 \end{pmatrix} \begin{pmatrix} P_2 \\ s \end{pmatrix}$$

Unfortunately the vector of constants on the left hand side is not independent of the factors to be determind on the right because the per capita consumption (demand) is given by

$$C_1 = AO(1) + AE(1,1)\ P_1 + AE(1,2)\ P_2 + yB(1)\ s.$$

In order to make the vector of constants on the left hand

side of (2) entirely exogenous, the demand for C_1 has to be substituted back into the price determining equations.

$$(3)\quad \begin{pmatrix} C_2 - AO(2) - AE(2,1)\ P_1 \\ y - P'_1AO(1) - P'_1AE(1,1)\ P_1 \end{pmatrix} = \begin{pmatrix} AE(2,) & B(2)y \\ C'_2 + P'_1AE(1,2) & P'_1B(1)y \end{pmatrix} \begin{pmatrix} P_2 \\ s \end{pmatrix}$$

Thus the problem of price determination with policy determined prices which is conceptually posed in equation system (1) must be operationally solved by the system of equations (3).

There is an aspect of duality in this solution process. When the total quantity levels are externally determined, the prices internally adjust to these quantities. When prices are externally determined (ie. in the policy process), the quantities demanded must adjust to these prices. Either quantities or prices, but not both, must be specified in this model. While it is conceptually possible to specify the same number of independent restrictions on either quantity or price as there are demand equations independently of the current concern for maintaining the intra-commodity duality, the price determination mechanism currently does not have that capability. This means that only 32768 of the 155 Billion possible price-quantity policy configurations can be addressed in a 15 equation demand system. However, since many of the configurations are neither interesting nor feasible, the small fraction should not concern us.

Market Prices and Reservation Prices

For every commodity there is a price which is the lowest observed price at which output is forthcoming. This will be called the reservation price. The theoretical basis for these reservation prices is in the theory of cost functions. If the price of the product falls below the minimum point on the average variable cost curve, then the firm will not sell any more of its product. This is because the product is costing the firm more to produce than it will generate in sales.

The operation of the reservation price mechanism will now be described. The price determining mechanism is allowed to operate unconstrained for one iteration. Then a check is made to see if the market-determined consumer prices are higher than the reservation price levels. If all prices are above those levels, no adjustment is needed.

If just one price is below the reservation price level, the consumer price for that product is set equal to the reservation price and that commodity is removed from the price determining mechanism. Then a new series of prices is calculated which conforms to that reservation price and the aggregate budget constraint.

If more than one price is below the reservation price

level, a slightly different process is followed. Only one commodity is withdrawn from the market mechanism at any iteration. In this way the free-market mechansim is given the greatest opportunity to operate. The commodity for which the price difference is largest is the commodity which is removed from the price determining process.

This iteration process continues until all commodities have consumer prices higher than the reservation price level. The reservation price mechanism is designed so that even if a commodity is temporarily excluded from the price determining mechanism in one year, it will still return to the price determining mechanism in the following year.

In rather general terms, the effect of setting the market price equal to the reservation price is to increase all prices by means of the substitution effect and to lower the prices through the income effect for all commodities which have postive income elasticities.

In the KASS model the total annual supply is externally determined, ie it is price inelastic. When the reservation price mechanism is incorporated, this is equivalent to describing the intrayear supply curve for the products in the KASS model

as in Figure 1. Below the reservation price level RP, no product will be forthcoming. At the price level RP, an intermediate level of product determined by the location of the demand curve will be sold. At price levels above RP, the total annual supply will be sold.

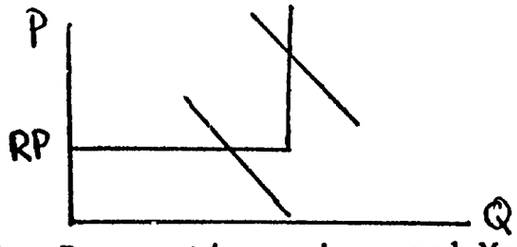

Figure 1. Reservation prices and Market Prices.

For the commodity groups in the KASS model there were two sources of data that were utilized in setting these reservation prices. Ideally, one should use production cost figures for all commodities when they are available. Unfortunately, production cost data is available at this time for only rice, barley, and wheat. For the remaining commodities, the reservation price is specified as 60% of the world export price level.

Because barley and wheat are grown outside of the primary growing season, the primary alternative for the farmer's time and resources seems to be non-agricultural pursuits. Because of this, all production costs for barley and wheat should be considered to be variable. These costs should include both the cash and non-cash costs to the farmer. Because rice is grown during the main growing season, a stronger case could be made for assessing only those variable costs of production in setting the reservation

prices for rice. However, for purposes of consistency, the total production cost of rice was also used as the reservation price. The base year for which this production cost data was taken is 1971. Table 1 presents the current data for the reservation prices.

To summarize, the reservation price system is a mechanism derived from the economic theory of costs which specifies a price level below which no product will be forthcoming. Thus, it puts an effective lower bound on each of the market prices which are calculated within the price determining model. Therefore, there need be no further concern about the possibility of negative prices being generated by the price determining mechanism.

Table 1: Reservation Price Assumptions

	1971 Assumed Export Rice 1/ W/kg	1971 Reservation Price 2/ W/kg	1971 Production Cost Less Selfservice3/ W/kg	1971 Purchase Disbursement 3/ W/kg
Rice	100	66.825	27.288	22.838
Barley	33.6	57.285	22.695	14.805
Wheat	40	47.948	15.869	9.556
Other Grains	44	26.4		
Fruits	100	60.0		
Pulses	103.2	61.92		
Vegetables	90	54.0		
Potatoes	100	60.0		
Beef	680	408.0		
Milk	78	46.8		
Pork	365	213.6		
Poultry	484	290.4		
Egg	266	159.6		
Fish	280	168.0		
Agricultural Residual	0	0.0		

1/ Source: Korean Agricultural Sector Study

2/ For Rice, Barley, and Wheat, total production cost in 1971 per kilogram was used

3/ Source: Yearbook of Agriculture and Forestry Statistics, 1973

History of Model Development

In the history of the annual price determination mechansim within the KASS model there have been at least four stages of development that have transpired.

Initially, the annual price adjustment mechanism was formulated to consist of sequential solution of the budget constraint to first determine the budget parameter (s) conditional upon initial price levels. Then, conditioned upon this estimated budget parameter and initial price levels, the excess demand for each commodity was calculated. Then the price levels were adjusted proportional to the ratio of the percentage excess demand to the own price elasticity of demand.

The next stage of development retained the sequential (flip-flop) solution of the budget constraint and demand equations to determine the budget parameter (s) and the prices, respectively. The improvement over the first form of the model was to use a matrix inversion routine to determine the current iteration prices (conditional upon the budget parameter).

The third level of development recognized the simultaneous dependence of prices on the budget parameter and vice versa. The method of solution of these equations was also matrix inversion for the system of demand equations augmented by the budget constraint. Because the per capita consumption of the commodities with policy determined prices was treated as independent of the

market determined prices, equation system (2) was solved in the price determining process. For selected levels of imports and exports this method converged to price levels that were considered reasonable.

The final level of development represents the transition from equation system (2) to equation system(3). In this mechanism the dependence of the consumption of commodities with policy determined prices on the market determined prices for the other commodities is explicitly recognized. The mechanism to fit in reservation prices was developed at this stage.

Summary

The annual price determination mechanism is a model which calculates the set of prices that simultaneously equates market demand with the available supplies while satisfying a national budget constraint. When some prices in the set are policy determined, the corresponding demand equations are eliminated from the simultaneous solution by substituting them into the budget constraint.

A method of incorporating the concept of prices below which no product will be forthcoming was presented. This mechanism operates to guarantee that all market prices will be at least as large as the reservation prices.

A short historical development of the evolution of the various price setting mechanisms in the KASS model is given. This related

the current model to those which preceded it.

Finally, the algebraic solution of the model is given in the appendix. By substituting their definitions for the parameters in the demand equations, we were able to prove that only a single calculation is required for each set of exogenous quantities and prices, regardless of any of the possible configurations of parameter values.

Further Desirable Work

The theoretical refinement of the KASS urban demand component, as such seems to have reached a vicinity of its economic limit. The only major theoretical addition which could be suggested is a mechanism to allow the specification of conditions on any n prices or quantities in the n equation system. A three commodity version of such a mechanism which does not incorporate the overall budget constraint already exists for use in the food grain management program.

The refinement of the empirical estimates of the demand elasticities is a subject of higher priority affecting the urban demand component.

Another more general theoretical addition to the KASS structure would be the development of a means to simultaneously compute urban and rural demands rather than the current sequential mechanism.

Appendix

In order to solve the system given by equation (3) we must invert the matrix of coefficients on the right hand side. This inverse is

$$\left(\begin{array}{c|c} AE(2,2)^{-1}\ (I+yB(2)\ Z^{-1}(C_2+P'_1AE(1,2))AE(2,2)^{-1} & -AE(2,2)^{-1}B(2)yZ^{-1} \\ -Z^{-1}(C'_2+P'_1AE(1,2))\ AE(2,2)^{-1} & Z^{-1} \end{array}\right)$$

Where $Z=(P'_1B(1) - (C'_2+P'_1AE(1,2))\ AE(2,2)^{-1}B(2)\ y$

The vector of constants for that system consists of the following two elements:

$$C_2-AO(2)-AE(2,1)P_1 = AE(2,2)P_{2,i-1}+S_{i-1}\ yB(2)+(C_{2-1}-C_{2,i-1}) -AE(2,1)\ (P_{1,i}-P_{1,i-1}) \text{ and}$$

$$\alpha y-P'_1AO(1)-P'_1AE(1,1)P_1 = \alpha y-P'_{1i}C_{1,i-1}+S_{i-1}\ yP'_{1\ i}B(1) + P'_{1,i}\ AE(1,2)P_{2,i-1} -P'_{1,i}\ AE(1,1)\ (P_{1i}-P_{1,i-1})$$

When analyzing this system two important points must be noted regarding behavior beyond the second iteration. Because the set of prices P_1 is policy determined and not subject to change within the year, $P_{1i} = P_{1,i-1}$ for $i = 2$. Secondly, because no intrayear supply response is assumed for any of the commodities, and in particular for the commodities with market determined prices, $C_{2i}=C_{2,i-1}$, also for $i=2$.

The solution to the system of equations for all iterations within the year except the first one is as follows:

$$P_{2i}=P_{2i-1} - yAE(2,2)^{-1}\ B(2)\ Z^{-1}\quad (\alpha y-P'_{1i}C_{1,i-1}-C'_{2\ i}P_{2,i-1})$$
$$+\ S_{i-1}\ y\ AE(2,2)^{-1}\ B(2)\quad (I-Z^{-1}Z)\ \text{and}$$
$$S_i=S_{i-1}+Z^{-1}\quad (\alpha y-P'_{1i}C_{1i-1}-C'_{2\ i}\ P_{2i-1})$$

The portions of the solution which are multiples of the $(C_{2i}-C_{2i-1})$ and $(P_{1i}-P_{1i-1})$ differences have been omitted since these differences are zero for iterations other than the first. Substituting $P_{1,i-1}$ for P_{1i} and C_{2i-1} for C_{2i} we see that the budget constraint for the previous iteration causes the other terms to drop out.

Thus, the remarkable conclusion is that (regardless of the values of elasticities in the system) the solution process will converge after the second iteration. In fact, the second calculation is only necessary to verify that convergence indeed had occured.