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9. ABSTRACT

A description of a multiple-crop irrigation planning model developed to determine crop acreages, irrigation timing, and irrigation amounts on a representative farm in the irrigated area near Ft. Morgan, Colorado. The objectives of the study were to maximize the expected crop returns and minimize variances due to random precipitation. Recent experimental data from field trials on corn at Colorado State University were used. An evapotranspiration prediction model was developed to reconstruct soil moisture over the season for varying irrigation treatments. Regression analysis was used to determine parameters in production functions relating evapotranspiration in various time periods to yields. A dynamic programming model, also developed, determined optimal amounts of irrigation at prescheduled times for a single crop. The evapotranspiration prediction model and first the multiplicative and then the additive production functions were incorporated into this program. The model was employed to solve several types of case problems, including one involving unlimited water available at a predetermined price, and one involving the water's being limited during various time periods. The results of this latter program simulate the situation involving a crop competing with other crops for the limited water. Deterministic and nondeterministic cases were studied. In the first, all variables were assumed to take on their expected values, thus reducing the problem to a deterministic linear program of maximizing return. In the second, probabilistic rainfall was used as a supplement to known surface and groundwater supplies. A simulation program was formulated and run to evaluate the expected return and variance for various precipitation planning policies.

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Submitted by

Herbert G. Blank

In partial fulfillment of the requirements

for the Degree of Doctor of Philosophy

Colorado State University

Fort Collins, Colorado

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WE HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER OUR SUPERVISION BY Herbert G. Blank

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OPTIMAL IRRIGATION DECISIONS WITH LIMITED WATER

by

Herbert G. Blank

Abstract

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A dynamic programming model was developed which determined optimal amounts of irrigation at pre-scheduled times for a single crop. The adopted ET prediction model and first the multiplicative and then the additive production functions were incorporated into this program. Initially the problem is solved with unlimited water available at a pre-determined price. The model is solved again with water limited in various time periods. The results of this program simulate the situation when the crop competes with other crops for the limited water supply. Secondary data were used to determine timings and amounts of irrigation water for various production levels of the other crops considered in the study.

The multiple crop case incorporates the results of the single crop model. A deterministic and a non-deterministic case were studied. In the first case all variables were assumed to take on their expected values, thus reducing the problem to a deterministic linear program of maximizing return. In the second case, probabilistic rainfall, a supplement to known surface and groundwater supplies, was introduced. A simulation program, evaluating expected return and variance for various precipitation planning policies was formulated and run.

The multiple objective problem was reduced to a single objective of maximizing expected return since the change in variance for the policies tested was small. Finally, employing the optimal precipitation planning policy, the effect of restricting the known seasonal irrigation supply was evaluated. Expected return decreased and the variance of income increased as the water supply was increasingly restricted.

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Chapter I

INTRODUCTION

Irrigation is an agricultural practice followed by man for centuries, its purpose being to increase the productivity of land resources and to reduce the uncertainty associated with crop production. Though early man may not have thought in statistical terms, he did realize that yield could be increased and variation in quantity of yields could be reduced by providing water to crops in addition to that supplied by rainfall. This early attempt by man to control his environment for his own benefit has evolved over time to the present irrigation systems upon which the world depends for much of its food supply.

Although the land and water resources of the world are limited, the world's population continues to grow seemingly without limit. The result is that in order to feed this population, agriculture must become more productive. A goal of modern research is to provide information to farmers regarding increased productivity. One direction this research has taken is in trying to more efficiently use the land and water resources available to the farmer. This study attempts to continue this line of research.

The General Crop Planning Problem

A general research problem is to recommend to a farmer having limited resources, which crop or crops he should grow. This decision

depends not only on government restrictions and the farmer's limited resources of capital, land, labor, fertilizer and water, but on the farmer's regard for risk. This risk, measured by yearly variation in income, may be caused by such uncontrollable variables as crop prices or climatic events such as hail, frost, or flooding. It may also be caused by lack of precipitation combined with lack of irrigation supply or insufficient delivery capacity to meet the crop demands. The farmer's objective may vary from maximizing his expected return to minimizing the statistical variance of that return.

Solution of the general crop planning problem requires developing a model incorporating interaction among a number of disciplines, including agronomy, economics, statistical hydrology, etc. From the general problem various specific problems can be formulated, based on the particular characteristics of the application site and the purpose the model is expected to fill.

A Specific Crop Planning Problem

The specific problem dealt with in this study is how to recommend to a farmer which crops to grow given the following assumptions:

1. The farmer has a set of crops which can successfully be grown with his climatic conditions on his soil type.

2. The single variable input is water. Fertilizer application amounts are assumed known and constant for each crop, and salinity and other water quality variables are assumed constant throughout the season from all irrigation sources.

3. The seasonal water supply is limited and the rate of delivery within the season is limited.

4. Precipitation is a random variable contributing significantly to the water supplied to the crops.

5. Other variables are deterministic, taking on their expected values.

6. The farmer's objective is to maximize expected return, providing the variance of this return due to random precipitation remains at some "acceptable" level.

Other researchers have dealt with certain aspects of the foregoing problem (see Chapter II). The main contribution of this study is that it deals with the multiple time period, multiple crop case, and includes random precipitation in the analysis.

Due to availability of data and in an effort to complement a previous study (Young and Bredehoeft, 1972), the application site selected for the current study was a representative farm in the South Platte Valley near Ft. Morgan, Colorado. Under current water availability conditions, the current practice is to irrigate to field capacity at each irrigation. This study looks at a future time when water is limited and the price of water approaches its marginal value. Under these conditions. The farmer's decision, after the crops have been planted, is to optimally allocate his water resource among his mix of crops.

It may be desirable under water shortage conditions to irrigate one crop at a critical growth stage while shorting another crop. Another possibility is that it may be more profitable not to irrigate a crop since the increase in return (marginal return) is less than the cost of the water (marginal cost).

In order to make the water allocation decision quantitatively, the farmer must possess a production function for each crop enabling him to evaluate potential yields and returns under different water management alternatives. Included in the pre-season crop pattern optimization of this study is the optimal water allocation throughout the season based on random precipitation with other climate variables assuming their expected values.

Overview of the Optimization Model

The following, which may be classified as a "passive" programming approach, represents the overall optimization strategy. Figure 1.1 also summarizes the multiple crop optimization strategy. The next sections elaborate on some of the following points.

1. Determine optimal timings and amounts of irrigation for the single crop case as a function of water shortage in any period or combination of periods. This is accomplished by a dynamic program.

2. Solve a deterministic linear program for each discretized precipitation planning policy. The planned for precipitation amounts are subtracted from the crop water requirements. In this program, discretized crop production methods are considered.

3. Test the results of each precipitation planning policy by a simulation program which calculates the expected return and variance under each policy. In this program, linearized production functions are used.

4. Determine the optimal precipitation planning policy. The optimal crop acreages are determined by using the optimal precipitation planning policy in the deterministic linear program.

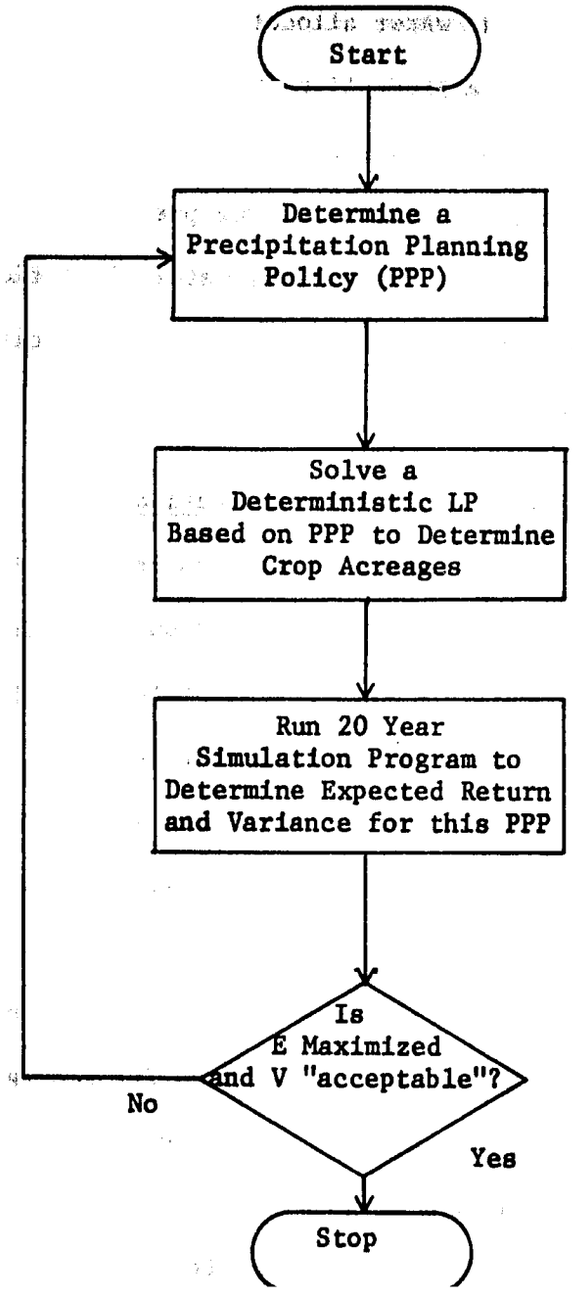


Figure 1.1

MULTIPLE CROP OPTIMIZATION PROCEDURE

Single Crop Optimization

In order to establish the multiple crop model, data from the single crop case are required. Various authors have developed models which determine optimal times and amounts of irrigation for a single crop when the seasonal supply is limited, but the farmer has complete flexibility regarding timing and amounts of irrigation (see Chapter II). For this study, a model is required which determines optimal irrigation timings and amounts when the capacity of the delivery system is limited and the single crop competes with other crops for this limited water supply. Thus, for a given water shortage in any time period and any possible combination of such shortages, the model must determine the optimal irrigation policy and the forecast yield and return. The optimal policy is thus determined as a function of the shortage in each of the time periods over the season.

The foregoing analysis is carried out for one crop, corn, for which recent agronomic data from field trials at Ft. Collins, Colorado, are available. For the other crops considered by the study, secondary data are used.

Multiple Crop Optimization

The analysis of the single crop case does not include random precipitation, but rather only specific values of possible shortages of water in each time period. In order to include precipitation in the multiple crop case, continuous functions relating water inputs to return are required. These functions are obtained from the results of the single crop optimization. In general, linear functions used to approximate non-linear relations are adequate only over a restricted

range. In this study, linear functions were obtained which appeared to be adequately representative of all the crops and shortages encountered in the Ft. Morgan situation. These functions are intended to be most accurate for a range from the mean precipitation amount (when adequate water is available to irrigate all crops fully), down to a seasonal precipitation shortage from the mean of 8 cm. This shortage occurs less than 10% of the time. For greater shortages, additional data points could be generated and other functions fitted.

The Precipitation Planning Policy

The approach of this study for handling random precipitation is to introduce a concept referred to as the "precipitation planning policy." A farmer adopting a 10% precipitation planning policy would plan for 10% of the mean precipitation in each time period and would allocate his fixed seasonal irrigation supply during the season according to the precipitation to date and a forecast of 10% of mean precipitation for the remainder of the season.

Under a 0% precipitation planning policy, rainfall would be neglected and, with irrigation water limited, a "conservative" mix of crops would be selected, one whose water requirements would always be met, but whose return would presumably be relatively low. Alternatively, under a 100% precipitation planning policy, a crop mix would be selected consisting of higher valued and higher water-using crops.

Growth Models

The optimization models described in the previous section all implicitly require as input a simulation model to predict return, or equivalently yield, from a crop under any given irrigation and

precipitation regime. A simple, accepted input-output relation between water inputs and yield would be useful; however, such a model does not exist. In this study an intermediate variable is introduced to link inputs to outputs.

Photosynthesis, the mechanism by which plants manufacture carbohydrates, may be viewed as the process which results in the final yield. Photosynthesis occurs when carbon dioxide, water and sunlight are all available to the plant. Carbon dioxide enters the plant through open stomata, while water vapor leaves through these stomata at a rate determined by atmospheric conditions and moisture availability. The water vapor that the plant loses through the open stomata makes up the major portion of the total water transpired by the plant. When the stomata close, the flow of both carbon dioxide into the plant and water vapor out of the plant are restricted. For further discussion see Appendix A.

Transpiration may thus be used as a proxy measure of plant growth. By maximizing transpiration for a given set of atmospheric conditions, photosynthesis is also maximized since when transpiration occurs the stomata are open and carbon dioxide is also entering the plant at the maximum rate for the given atmospheric conditions. Advancing one step further, it can be postulated that lack of transpiration, hence reduced carbon dioxide availability and reduced photosynthesis, would have greater effects on the final yield in some growth stages than in others.

Hanks (1974) used a growth index based on transpiration while in this study an index based on evapotranspiration ET is used. It is

assumed that the ratio of the ET index to the transpiration index would essentially be a constant, at least within the accuracy of the available data.

Two production functions are tested in this study relating ET in various time periods to yield. The functions are the additive relation

$$\frac{Y}{Y_{max}} = \sum_{i=1}^{n} A_i \left(\frac{ET_i}{ET_{max_i}}\right) \tag{1.1}$$

and the multiplicative relation

$$\frac{Y}{Y_{max}} = \prod_{i=1}^{n} \left(\frac{ET_i}{ET_{max_i}}\right)^{\alpha_i} \tag{1.2}$$

in which

i = growth stage index
Y = predicted yield
Y_{max} = maximum attainable yield
ET_i = ET in growth stage i
ET_{max_i} = maximum attainable ET in growth stage i
n = number of growth stages
A_i and α_i = parameters fitted to available data.

Both of the foregoing relations are incorporated into the single crop optimization model.

ET Prediction Models

In order to predict yield for any irrigation regime and with a growth model based on ET, a model which predicts ET is essential. In fact, any model which predicts crop water requirements must keep a soil moisture balance of the form

$$SM_{i+1} = SM_i + IRR_i + P_i - RO_i - D_i - ET \tag{1.3}$$

In which

i = time index
SM_i = depth of available soil moisture at time i
IRR_i = depth of irrigation at time i
P_i = effective precipitation at time i
RO_i = runoff at time i
D_i = drainage (deep percolation) at time i
ET_i = evapotranspiration at time i; all units in cm.

Runoff and deep percolation may be neglected if the sum of irrigation and precipitation amounts in a time period do not exceed the infiltration rate of the soil, and if they do not bring the soil moisture level above field capacity. Thus, updating soil moisture reduces to establishing an ET prediction model. For a discussion of ET prediction methods see Appendix A. For the requirements of this study, the Yaron ET prediction method, discussed in Chapter III, was considered adequate and was adopted. This method predicts ET based on soil moisture and stage of growth. Climate variables are included in the growth stage parameters with the assumption that at the given site the parameters are relatively insensitive to year to year climate variations.

Combining the ET prediction model with either the additive or the multiplicative production function results in a model to predict yield under any irrigation regime. An advantage of the yield prediction model developed in this study is that it can be directly integrated into an optimization model. Another advantage is that there are few parameters to fit: one parameter for each growth stage for each crop in the ET prediction model, and one for each growth stage for each crop in the production function. A disadvantage in determining these parameters, however, is that results from extensive field

trials are required. With experience from a number of trials on different soils and climates, an estimation procedures could be developed.

Organization of the Study

The next two chapters present the methodology and results of the study. Chapter II presents a literature review of irrigation models. Chapter III deals with a single crop, corn, for which recently available data are used. Parameters of the ET prediction model are determined and two production functions are fitted to available data. With this yield prediction model, then, an optimization program is presented which determines discretized production levels under conditions of water shortage in various growth stages. The results suggest that this procedure can readily be applied to any crop exhibiting differential impact on yield for the various growth stages.

In Chapter IV, data for other crops grown on the representative farm (alfalfa, sugar beets, and pinto beans) are presented and yield prediction models are determined for these crops. With discrete production levels for each crop, a linear program is formulated and solved to determine optimal crop acreages under conditions of known rainfall. Finally, the simulation program for evaluating expected return and variance under various precipitation planning policies is presented and the optimum precipitation planning policy is determined.

Chapter V presents a summary, conclusions and recommendation of the study. Appendices are included containing literature reviews from agronomy, economics, and systems engineering and flow charts of the computer programs developed in the study. Listings of the computer programs are available from the author or from the Civil Engineering Department of Colorado State University.

This chapter has presented an overview of the entire study. As in any study, some elements are borrowed and some are original. Among the borrowed features of this study are:

1. Adoption of the Yaron ET prediction model.

2. Use of an objective function with both multiplicative and additive terms in the objective function.

3. Use of linear programming and the basic principles of dynamic programming in the multiple and single crop optimization models, respectively.

4. Use of experimental data obtained by others for crops other than corn.

Among the original contributions of the current study are:

1. Analysis of newly available data for corn production.

2. Testing of several forms of production functions with available data.

3. Development of a dynamic program to establish optimum production methods under condition of interseasonal limitations on the irrigation supply.

4. Introduction of the concept of precipitation planning policy as a means of simplifying the decision structure and evaluating risk levels and the development of a simulation program to generate the Expectation-Variance curve.

Chapter II

REVIEW OF IRRIGATION MODELS

Introduction

Various models of irrigation systems have been proposed with varying purposes and varying degrees of sophistication. A distinction may be made between scheduling and planning models. Scheduling models attempt to aid the farmer during the season, determining optimal timings and quantities of irrigation. Scheduling models can be updated throughout the season, keeping track of some state variables related to plant growth and variables measuring resource availability. These models are generally daily models.

Planning models attempt to aid the farmer in choosing the acreage of each crop to be grown. The planning model must take into account resources known with certainty at the beginning of the season; this model must also deal in some way with inputs such as precipitation, solar radiation, and river flow which are known only probabilistically. Some form of scheduling model may be implicit in the planning model. However, since the planning model makes all decisions at the start of the season, whereas the scheduling model makes decisions as information becomes available during the season, the scheduling model incorporated in a planning model is generally rather gross.

This chapter reviews irrigation scheduling and planning models relevant to the current study. For each model the data base,

assumptions, calculations needed, methods of handling random variables, and multiple objectives and conclusions will be discussed. In a later section these models are discussed in view of their applicability to the current study.

Simulation Models

Jensen and Heerman

Jensen and Heerman (1970) described an irrigation scheduling program that has been used by the United States Department of Agriculture and the United States Bureau of Reclamation in advising farmers. The combination equation of Penman which has been described in Appendix A forms the basis of the program. Evapotranspiration ET is calculated on a daily basis from measured data and available soil moisture is updated by the program during the season.

At any time during the season, the next irrigation can be predicted using the formula:

$$N = \frac{D_o - D}{\overline{E}_t} \tag{2.1}$$

in which

D = current estimated depletion of soil moisutre (in.)
D_o = maximum allowable depletion for the present stage of growth (in.)
$\overline{E}_t$ = mean ET rate for the 3 previous days and 3 forecast days (in./day)
N = estimated number of days to next irrigation.

In another paper (Heerman and Jensen, 1970), the $\overline{E}_t$ value used was obtained from a graph showing $\overline{E}_t$ as a function of time, normally distributed about the peak ET day. From experiments at Akron, Colorado, better results were obtained by this method than with the previous method, which required a subjective forecast of $\overline{E}_t$

The next refinement in estimating an irrigation date was to add a term to N due to expected precipitation. The authors concluded that for the eastern Colorado region with relatively low precipitation, irrigation dates are not significantly affected by this refinement.

Kincaid and Heerman

Kincaid and Heerman (1974) describe a scheduling program for a programmable calculator. Again, the basis for the program is the Penman combination equation and associated crop coefficients and stress factors as described in Appendix A. As in the two previous papers, the authors assume an optimum soil moisture depletion level, stated as 50% of the total available moisture within the root zone. At an irrigation, the soil profile is returned to field capacity. The method of forecasting the irrigation date makes use of the normally distributed $\overline{E}_t$ function.

The scheduling programs described have a specific purpose: recommending next irrigation date based on maintaining the crop within previously determined optimal conditions. The assumption, basically, is that water is not scarce. The programs provide no information to the farmer who must choose, for example, between irrigating part of a crop fully or all of a crop lightly. The economic question of trading cost of water for increased return is not considered, or in other words, the cost of water is assumed to be far less than the marginal return due to a unit of water.

Hanks

Hanks (1974) tested a production function (described in Appendix B) for predicting grain yield from corn and sorghum. The author did

not, however, attempt to apply this model in a planning or scheduling sense. The model is limited by data in that it requires daily values of potential evapotranspiration and potential soil evaporation under the crop canopy.

In a later paper, Hill, Hanks, et al., (1974) described a program which predicts corn yield using the production function tested by Hanks. The program was used to predict the effect of supplemental irrigation on an otherwise rainfed site. The conclusion was that a supplemental irrigation system could be economically justified. The program as described in the paper was used as a simulation of an irrigation system, answering a question "what if" irrigation were available. The more general problem of optimal cropping pattern and optimal water allocation was not approached by the authors.

Yaron

Yaron, et al., (1973) developed a soil moisture simulation model as described in Chapter III. Using wheat data the authors fitted parameters to a Cobb-Douglas type function, an exponential function, and a Mitscherlich function (see Appendix B). The Mitscherlich function was adopted having the following independent variables:

1. Number of days during growth season with soil moisture above 12% (about 45% of available soil moisture),
2. A variable which measured the quality of the germination period, and
3. A year variable (4 years of data were used in the regression).

Upon obtaining a suitable yield prediction equation, 16 years of rainfall data were used to simulate the effect on yield of two approaches to irrigation scheduling. These were:

1. Irrigation on the basis of a predetermined time schedule, the quantities of water applied being equal to the moisture depletion in the root zone at the time of irrigation, and

2. Irrigating at the date on which the soil moisture is depleted to a predetermined critical level (Yaron, et al.,[10] 1973).

Taking into account costs of water, the authors concluded that the second policy is slightly better than the first according to three objectives: maximizing expected net return, minimizing variance, and maximizing income during years of low rainfall.

It should be noted that this is still a simulation approach, irrigation times and amounts were chosen according to two arbitrarily chosen rules and tested to determine net return. No attempt was made to analytically determine the optimal conditions, though the authors are apparently near the optimum according to all three objectives.

Stewart and Hagan

Stewart, et al., (1974b) describe 18 optimal methods of corn production with limited water supply. These methods are derived from data from field trials at Davis, California. Four irrigation days were specified during the season and irrigations were applied in one-inch increments up to field capacity. The irrigation days were scheduled to occur when 70% of the previous irrigation amount had been removed from the root zone. A pre-irrigation to field capacity occurred prior to planting. Yields were measured and profits due to irrigation were calculated, including a water cost and a labor cost of irrigation.

Similar to those models previously discussed, the model of Stewart, et al., did not involve optimization techniques. Also, a production function was not derived. The authors recommended that if a

fixed quantity of irrigation water per acre is known at the start of the season, the water should be applied according to the tables (see Tables 4 and 5, Stewart et al., 1974b) derived by the authors. This model is thus deterministic and assumes a single crop and an objective of maximizing return. The model could be adequate for the climatic conditions in the Central Valley of California, but is not readily transferable to other sites without repeating the full range of field trials.

Blank (1975) discusses how the results of Stewart, et al., can be applied to a crop planning model involving optimization. The model suggested is basically deterministic except that it may be extended to consider probabilistic water availabilities.

Single Crop Optimization Models

Hall

The models thus far discussed in this chapter have dealt with three aspects of the irrigation problem. Jensen and Heerman were concerned with scheduling and in particular, predicting date of next irrigation to obtain maximal yield. Yaron and Hanks were concerned with deriving production functions and then proceding to simulate crop yields under varying conditions. Stewart and Hagan's main contribution was in generating basic data relating water inputs to yields.

Hall, on the other hand, has worked with various people from the opposite end of the problem, starting with the optimization formulation and solution techniques, without concentrating on basic data.

Hall and Buras (1961) presented a problem of the optimal crop acreage for a known limited water supply. They dealt with a single

crop, for which return as a function of seasonal water input was known. The authors formulated a dynamic program to solve the problem and also developed a graphical solution technique. This model is limited in that it dealt with a single crop, was deterministic, and dealt only with the seasonal water input. Also, economies of scale were not discussed (i.e., as more land is brought into production, the total average cost decreases). The model did, however, consider the problem of limited water supply, concluding that, at least in the concave region of the production function (Stage 2) the policy should be to irrigate the selected acreage uniformly. The selected acreage, apparently, depends on the shape of the particular production function.

Hall and Butcher (1968) introduced additional complexity by considering the effect of time of water application on yields. Again the model dealt with a single crop and again the top down approach of assuming a production function was used. The form of the return function was

$$Z = P \prod_{i=1}^{n} a_i(d_i) \cdot Y_{max} - \sum_{i=1}^{n} c_i \cdot x_i \qquad (2.2)$$

in which

Z = return
P = price per unit of yield (\$/lb.)
Y_{max} = maximum yield (lbs.)
d_i = soil moisture deficit from field capacity at time i (in.)
$a_i(d_i)$ = dimensionless yield reduction coefficient for time period i
x_i = quantity of water applied during period i (acre-inch)
c_i = cost of water application during period i (\$/acre-inch).

After suggestions by Aron (1969) the model was presented in final form by Hall and Dracup (1970) as a dynamic program having three state variables which are

q = amount of water in storage (acre inch)

w = soil moisture level (in.)

and A = "state of the crop at any time as a result of the possible deficiencies before the time period" (Hall, 1969) (dimensionless).

The model may be classified as a single crop, deterministic, scheduling model. The model assumes a fixed supply of irrigation water to be applied to a known crop acreage. The results of the program are the optimal timings and amounts of irrigation water, determined on the basis of knowledge known at the beginning of the season. Precipitation and other random variables are apparently assumed to take on their mean values. The model is theoretical in that it is not based on actual data and is not applied to an actual site. In addition to the assumption regarding the multiplicative nature of the production function, the model assumes that daily evapotranspiration is a function only of the soil moisture level for that day, not of solar radiation, etc., though a more complicated relation could be adopted. Hall and Dracup (1970) discuss the problems of computation with a three-state variable dynamic program and suggest methods for speeding the program by restricting values of the state variables.

Minhas

Another single crop model was presented by Minhas, et al., (1974). The authors developed an evapotranspiration ET prediction model for wheat as a function of available soil moisture only. The function was of the form

$$f(x) = (1 - e^{-rx})/(1 - 2e^{-r\bar{x}} + e^{-rx}) \qquad (2.3)$$

in which

r = parameter fitted from data (1/in.)

x = available soil moisture ASM in root zone (in.)

$\bar{x}$ = ASM at field capacity FC (in.)

(x) = ratio of actual to potential ET for a plant when green cover is fully established.

The graph of the function is similar to Figure A.4. Actual ET is the product of $f(x)$; potential ET; and a crop weighting function, increasing from planting to full cover, constant until start of senescence, then decreasing to harvest. Parameters were fitted from wheat data from Delhi, India, and tested against results from alfalfa data of Mustonen and McGuinness (1968).

With an adequate ET prediction function, the authors used regression to fit parameters to the multiplicative function

$$Y = a[1 - (1 - x_1)^2]^{b_1} [1 - (1 - x_2)^2]^{b_2} [1 - (1 - x_n)^2]^{b_n} \tag{2.4}$$

in which

Y = yield

x_j = relative (i.e., fraction of maximum) ET in period j

$a, b_1, b_2, \ldots b_n$ are positive parameters fitted from data.

The data used were from 21 wheat experiments over 3 years. Dummy variables were introduced "to capture the effects of the differences in experimental designs, varieties used, amounts of fertilizers used, and the climatic factors (nonmoisture) between different years," (Minhas, et al., 1974). The resulting regressions generally had high values of R^2, but the parameters of interest tended to be nonsignificant.

The authors adopted a production function consisting of two time periods and formulated an optimization problem of maximizing yields subject to meeting a seasonal water availability constraint. The problem was solved via marginal analysis, equating marginal products of water in the two time periods.

This study is interesting in two respects. First, except for the regressions, the study was devised to avoid computer work. Integrals were evaluated analytically and the optimization problem was solved by traditional means. Secondly, the study attacked the entire problem from basic data to the optimization problem. The approach is limited in that it does not consider the effect of random variables, and assumes a single objective of maximizing yield from a previously planted field. Consistent with the results of Hall and Buras (1961), the optimal policy is to apply water uniformly throughout the field.

De Lucia

De Lucia (1969) devoted one chapter of his study to optimal irrigation timing of a single crop, applying the model to soybean production in Mississippi. The author formulated a three-state variable dynamic program with variables: quantity of water in surface storage, available soil moisture, and streamflow during the previous period. In addition, random variables of streamflow and rainfall are considered. Streamflow in a period is conditionally related to flow in the previous time period, thus necessitating the third state variable. Two sources of water are considered, surface and groundwater, at differing costs.

The production function is an additive function of each time period's contribution to growth. The objective is to maximize net return: price times yield less water costs. The growth function was adopted from Moore (1961), relating average soil moisture during a time period directly to growth. It is assumed that growth in all time periods contributes potentially equally to yield.

The model was solved using two probability levels of rainfall and four of river flow over 22 weeks. With a grid size of 16 acre-feet reservoir storage and 0.37" soil moisture levels, the computation time was 26 minutes. Obviously, extending the problem to more general cases such as the multi-crop problem would make computation time unreasonable.

The model, then, illustrates the complexity involved in the dynamic programming approach when random variables are introduced. Still, at this complex level, only a single crop and a simple additive return function which is only a function of available soil moisture is considered. By considering a return function similar to that of Hall, another state variable would be added. Considering deterministic stream flow would remove one state variable. These two modifications would thus keep the problem computationally feasible, but still the problem would be limited to a single crop with a single objective of maximizing expected return.

Dudley

Dudley, et al., (1971a) formulated a two-state variable dynamic program to determine optimal timing of irrigation for corn with a limited seasonal water supply. The state variables were available

soil moisture, ASM, and quantity of water in storage. The authors assumed an additive growth function with varying dollar values for growth in each time period. A "growth-no-growth" assumption was made, employing a concept similar to the stress-day concept of Flynn and Musgrave (1967) discussed in Appendix A. If ASM is high in relation to potential ET, ET occurs at a maximum rate and a growth day occurs, contributing to the dollar value of the crop. If ASM is low, ET occurs at a rate E_m, "the maximum rate which water moves into the plant from the soil mass," (Dudley, et al., 1971a) and a no-growth day is recorded, contributing nothing to the value of the crop.

A stochastic dynamic programming model was formulated to make use of 20 years of evaporation and precipitation data. The objective was to maximize expected return as a function of terminal soil moisture TSM, that is the ASM percentage at which an irrigation is to occur. Transition probability matrices of beginning soil moisture are generated for each TSM policy in each time period and for each level of water supply. Similar matrices are generated for beginning water supply and return.

This model is of interest because, although it does not consider multiple objectives, it does consider random precipitation and random free water evaporation, which is assumed to represent all other climatic variables. Employing the additive production function and the growth-no-growth concept limits the usefulness of the model, however. For corn, Hanks found the multiplicative function adequate to represent the possible large reduction in yield due to low soil moisture in critical periods. A final point which is brought up by the authors, is

that at each irrigation the soil profile is returned to field capacity. In an area of relatively high precipitation, this assumption does not seem reasonable since, should a rainfall occur after the irrigation, the water would be wasted in runoff and deep percolation.

The results of the stochastic dynamic program are employed in a second model described by Dudley, et al., (1971b). While the first model looked at optimal timing for a given acreage, the second looks at the optimal area to be planted to a single crop, adding an additional stochastic variable of reservoir inflow.

The problem solution technique is basically a simulation approach; an acreage is selected and expected return is calculated based on the 20 years of data and the optimal terminal soil moisture policies developed from the previous model. The process continues by varying the acreage and calculating return until an optimal return is achieved assuming return as a function of acreage is a unimodal function.

As the authors point out, the method assumes irrigating all acreage uniformly will achieve the optimal result. The method neglects the possibility of irrigating some of the crop while growing the remainder under dry land conditions or lightly irrigating it. Such a result may be chosen by a farmer having multiple objectives of meeting a minimum income and maximizing expected return. The profit maximizing result, as discussed previously, is to irrigate the entire acreage uniformly, providing operation is in Stage 2 of the production function.

Multiple Crop Optimization Models

Mobasheri

A first example of an effort to model a multiple crop situation is presented by Mobasheri (1968). The problem is formulated as a deterministic yearly linear program and is used as a subprogram of a reservoir sizing problem.

The model considers constraints on the total acreage available, the total amount of capital which is available for investment in the crops, the total amount of labor (in man days) available for the season, and the total amount of water available for irrigation. The model considers four crops with each crop possibly grown according to three different methods. Each crop can be grown with:

1. High capital investment and low labor usage,
2. Medium capital investment and medium labor, and
3. Low capital and high labor usage.

Each method has associated with it a certain potential yield per acre. The author considers levels of production in tons but to be consistent with later models equivalent equations are presented in acres.

Let x_i be acreage devoted to crop i. Consider different methods as different crops to avoid the use of double subscripts. The land constraint is

$$\sum_{i=1}^{n} x_i \leq X \tag{2.5}$$

in which X is the given total acreage. The capital constraint is

$$\sum_{i=1}^{n} c_i x_i \leq C \tag{2.6}$$

in which c_i is the capital required for one acre of crop i and C is the total seasonal capital available. The labor constraint is

$$\sum_{i=1}^{n} d_i x_i \leq D \tag{2.7}$$

in which d_i is the number of man-hours required for one acre of production of crop i and D is the total man-hours available in the season. The irrigation constraint is

$$\sum_{i=1}^{n} e_i x_i \leq E \tag{2.8}$$

in which e_i is the quantity of irrigation water required to produce one acre of crop i and E is the total water available in the season.

The simplex linear programming method is used to solve for the x_i which satisfy the foregoing constraints and which maximize the objective function given by

$$\max \sum_{i=1}^{n} r_i x_i \tag{2.9}$$

in which r_i is the known net return per acre of crop i. This model is probably too simplified to be applied to a specific situation. One problem is that the model is deterministic. That is, it assumes the amount of water available to the system is known at the start of the season with rainfall neglected. Secondly, the model is set up on a yearly basis. The model does not take into account seasonal variations in the supply of water or labor, or the crop demands for water or labor. According to the model, for example, sufficient yearly labor may be available on the average for a certain level of production while in actuality there may be shortages of labor at certain peak labor demand periods such as at harvest. Despite these drawbacks, the model is a good starting point for the discussion of multiple-crop models and it was useful in the particular case of determining

optimal reservoir storage, where a figure representing relative return from a variable yearly supply of water was needed.

Anderson and Maass

The simulation model developed by Anderson and Maass (1971, revised 1974) seems to present the next level of sophistication. The basis of the model is the rules that are established for irrigation to achieve maximum yields. Anderson and Maass neglected the role of soil moisture simulation models and turned, as did Mobasheri, to empirically established irrigation requirements for optimal yields with given soil conditions, climatic conditions and crop varieties. Thus, the authors eliminated the possibility of including climate related random variables.

The season is broken down into 14 two-week irrigation cycles and an empirically determined table is used to determine quantities and times of irrigation. The model assumes a 50% efficiency from the farmer's headgate to the root zone. Rainfall and climatic conditions are considered deterministic. If it is known with certainty that a rainfall will occur, for example, during the first two weeks of June then the irrigation requirements may be adjusted to take this into account.

In addition to the time and quantity table, there is a table which shows the loss in potential yield when a prescribed irrigation period is missed. There are further rules concerning losses which are incorporated into the simulation model such as crop loss if two successive irrigations are missed.

Other specific aspects of the model are that it considers varying types of irrigation distribution systems and it allows for reduction of acreage if water should become short during the season. The model does not include groundwater as a source of supply or possibilities of holdover surface storage.

The solution method has been termed a "steepest ascent" method by the authors, however, in a strictly mathematical sense it appears to be merely an ascent method, not necessarily steepest ascent. Interpreting the schematic algorithm, the crops are set at their minimum acreages and during each iteration the acreage of a crop is incremented, if possible, then a subroutine is called which finds the net benefits for this acreage pattern and tests to see if higher benefits would accrue if certain irrigations were missed. The crop pattern and benefits are stored and the algorithm cycles to increment acreages if possible thus repeating the process. The crop pattern and irrigation procedure yielding the highest return is selected to complete the procedure.

One problem with this method is that acreages are incremented in discrete amounts. The increments must be chosen sufficiently small in order to insure convergence, thus requiring much computer time.

A second problem is that the method may converge only to a local optimum. If the starting point is in some sense "near" a local optimum the method will converge to that point. No information is known about the overall nature of the net benefit function because of the complications introduced by considering missed irrigations. The problem could be eliminated if the solution method calculated all crop

pattern possibilities (a grid technique requiring an enormous number of calculations) or if a starting solution is chosen "near" the global optimum. With this particular formulation, it seems that an initial point could be determined sufficiently close to the suspected global optimum.

A third problem is that irrigations occur at the full amount or at zero amount. Employing a continuous production function would give more realistic results.

Despite the deterministic nature, the model does consider the importance of timing the water input. The model uses empirically determined optimal timings and amounts for a particular crop, soil type, and climatic condition, and hence is not readily transferable to another site.

Young and Bredehoeft

Young and Bredehoeft (1972) presented a multiple-crop planning model to determine a policy for conjunctive use of groundwater and surface water. Mobasheri, as did Anderson and Maass, considered several alternative methods of production for each crop. Young and Bredehoeft used the same idea, considering different amounts and timings of irrigation as different production methods. The optimal irrigation amount as developed by Anderson and Maass is one method, other methods correspond to skipping certain irrigations. Each method is associated with a certain net benefit per acre.

The model was simplified over Anderson's in that only four irrigation periods were considered. Groundwater was considered as an additional source of supply. A linear program was formulated similar to that of Mobasheri's except with the added dimension of time.

The irrigation planning problem was solved as a sub-program in a large simulation program. The authors did not consider the stochastic aspect of the problem due to the speed needed in computation. Similar to Anderson and Maass, the authors restricted themselves to a site specific model with a single objective of maximizing return and all or nothing irrigations.

Smith

Smith (1973) developed two irrigation planning models applied to expanding an irrigated area in what is now Bangladesh. The first model is a deterministic linear program of maximizing the net present value of the project considering capital investments in canals, drains and wells and returns due to a variable amount of land to be irrigated. Crop water requirements in various time periods and other data are known but are not presented in the paper. In an earlier study (Smith, 1970), reference is made to data obtained from "various unpublished memoranda of the Ganges-Brahmaputra Basin Study." Constraints include limits on pumping, rise in water table, labor, and acreages of certain high valued crops due to limited market demand. A minimum acreage of wheat is used as a parameter in the program to reflect a government objective of self-sufficiency in grain which conflicts with the profit maximizing objective.

A second model with basically the same constraints as the first, is formulated as a chance constrained program (see Appendix C), viewing rainfall and prices as random. The results presented in the paper, however, are just for the random water supply case. For a risk level of 30%, for example, the consumptive use coefficient is reduced by the

amount of effective rainfall that occurs with at least 70% probability, thus the constraints are violated 30% of the time. The result is a cropping pattern with return that occurs with 70% assurance. The question brought up in Appendix C of what happens when constraints are not met is answered by Smith. A 57 year simulation model was run with simulated rainfall employing the cropping patterns obtained from each of the risk levels (up to 30%). The results showed the effect on groundwater levels, returns, etc., when the constraints are violated. The expected return and variance of the returns would have been useful statistics generated from the simulation model, yielding an EV diagram (see Appendix C). These statistics were not discussed by the author.

It seems that for the simulation model to properly determine the effect of variable rainfall, a continuous production function would have to be incorporated. This point was mentioned in another report (Smith, 1970, p. E, 3). "This procedure assumes a linear relationship between crop yield and the water applied during any decision period." The data on which the relations are based was not discussed.

De Lucia

De Lucia (1969) formulated a multi-crop linear programming model which is solved at the beginning of the season. Other similar programs are solved as the season progresses, making use of the then current available knowledge of various random variables. This solution technique is referred to as a sequential linear programming model. The programs which are solved after the start of the season do not provide

additional information to the first program, which is a deterministic linear program, maximizing return based on the assumption that random variables will assume their expected values.

De Lucia does consider various methods of producing crops, similar to the method later used by Young and Bredehoeft. These production methods are developed for each soil type from the Moore type growth curves as discussed previously.

The initial linear program considers acreage constraints by soil type, minimum and maximum crop acreages, and crop water use constraints with coefficients removing expected effective rainfall and considering a known constant irrigation efficiency term. Surface and groundwater availabilities are known in various time periods. An overlapping growing season is handled by the first of the foregoing constraints.

Whereas the single crop model of de Lucia adequately handled stochastic variables, the second model does not consider them, at least from the point of view of the farmer at the start of the season. A single objective of maximizing return is adopted. The same criticism of the basic data applies to this model as to the single crop model: The Moore type additive relation between soil moisture and growth is not confirmed to adequately represent crop yields.

Hall

Hall and others in a report by the R. M. Parsons Co. (Parsons, 1970) applied Hall's work to a study of Indian irrigation. Data were obtained for two crops, wheat and jowar, (see Figures 2.1 and 2.2) and graphs were drawn for the coefficients $a_i(d_i)$ in the multiplicative

yield function, Equation 2.2. For these two crops a dynamic program was developed to determine optimal timings and amounts of irrigation. Fertilizer was also considered, under the assumption that for a given water application, yields were related to relative quantity of fertilizer applied or

$$Y = a_N(N)a_1(d_1)a_2(d_2) \ . \ . \ . \ a_n(d_n)Y_{max} \qquad (2.10)$$

in which $a_N(N)$ is given for maize by Figure 2.3. The program differed from that of Hall and Dracup (1970) in that the objective is to maximize yields and returns. Three state variables were considered: quantity of water in storage, soil moisture in the root zone, and available capital. The program allocates capital over the season between water and fertilizer. The results are optimal irrigation and fertilizer applications for a given level of available capital.

Various methods of production for the two crops are obtained from the dynamic program and these are used as input to a district wide linear program that considers, deterministically, optimal crop acreages. The objective is to maximize the net value of the output. The constraints considered by this program are water availability in various time periods, land use constraints, fertilizer availability, manpower availability, animal power availability, storage and transportation availability, and a constraint that limits the acreage of nonfood crops.

As can be seen from Figures 2.1 and 2.2, limited data were available for determining the production functions of wheat and jowar. In the Ferozepur district, for wheat and jowar the dynamic program determined four methods of irrigation, from fully irrigated to rainfed only.

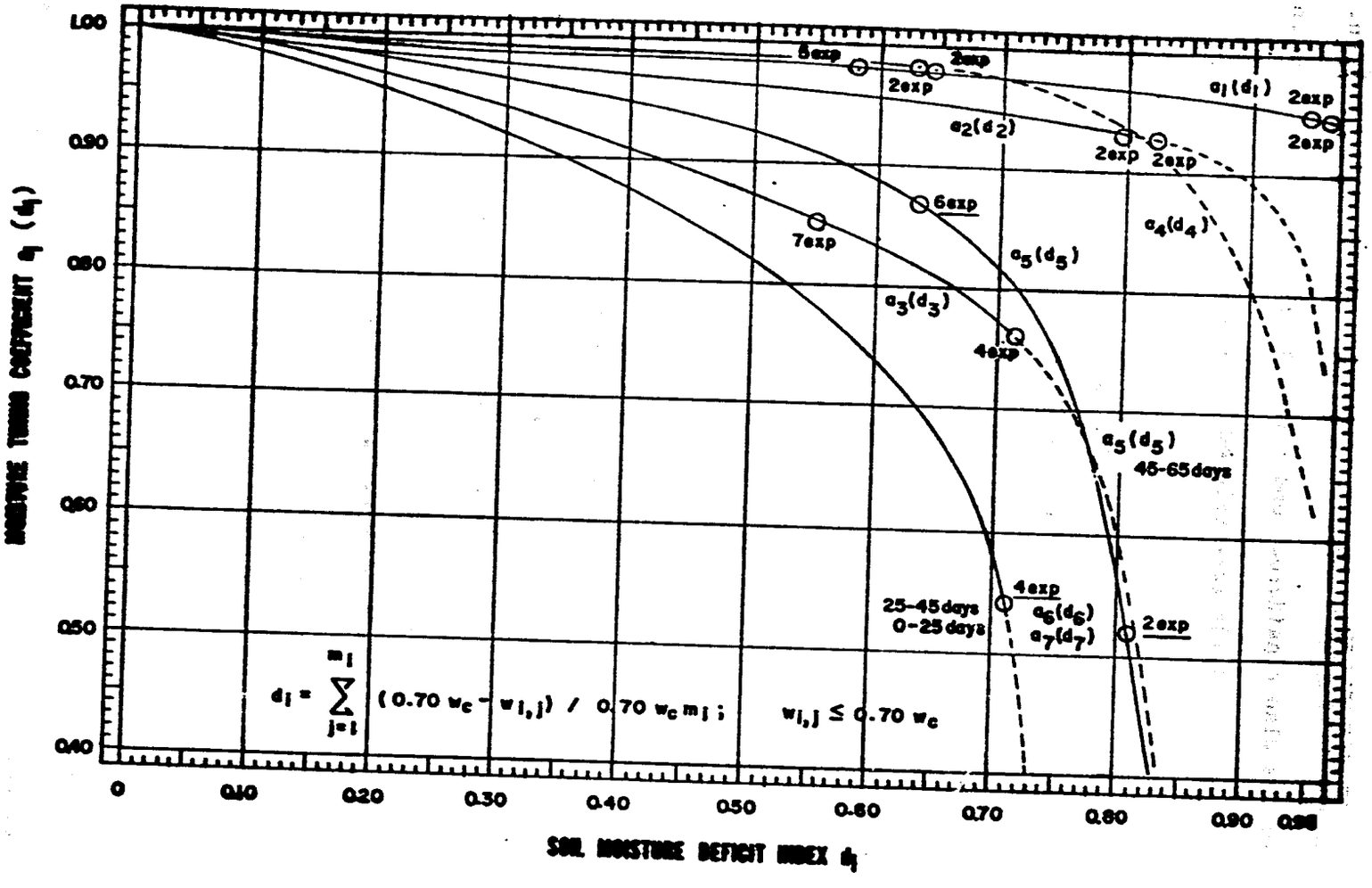


Figure 2.1

ESTIMATED SOIL MOISTURE DEFICIT FUNCTION, WHEAT (IMPROVED)
(From Parsons, 1970)

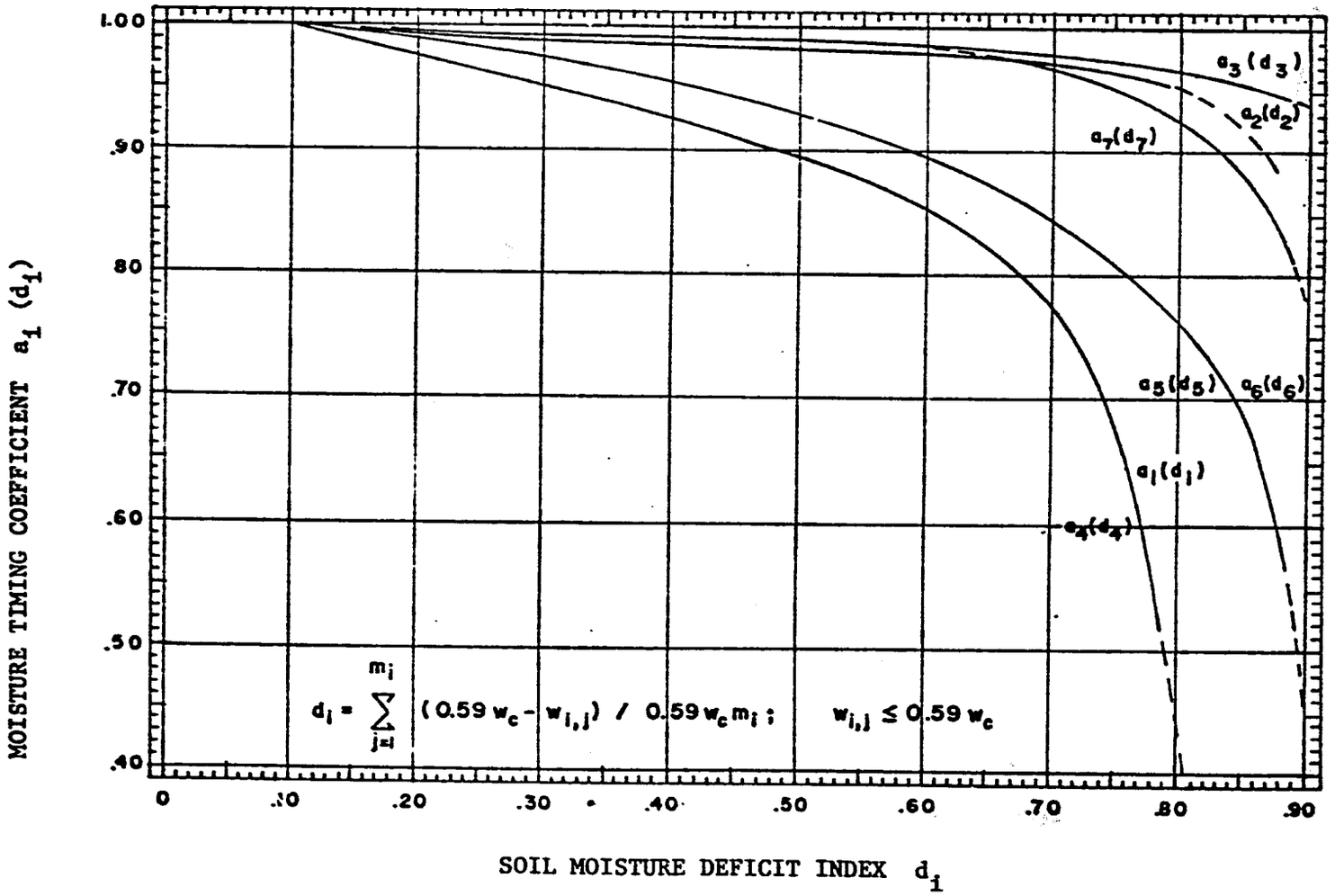
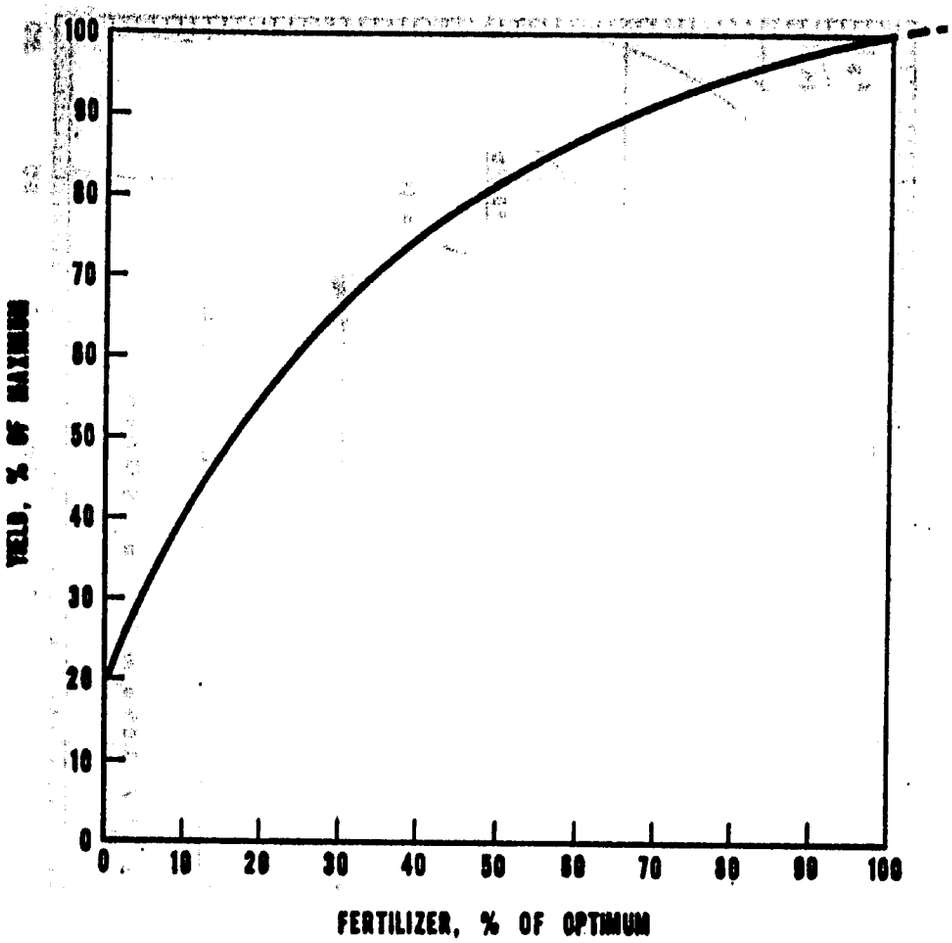


Figure 2.2

ESTIMATED SOIL MOISTURE DEFICIT FUNCTION, JOWAR (IMPROVED)
(From Parsons, 1970)



IMPROVED
IRRIGATED

N_{ij} = 120 kg/ha,N

Y_{ij} = 5000 kg/ha

Y_{ij}-N= 1000 kg/ha

Figure 2.3

RESPONSE OF MAIZE TO FERTILIZER IN ALLUVIAL SOILS, FEROZEPUR DISTRICT
(from Parsons, 1970)

For the other crops in the district (cotton, maize, rice, and gram), fewer methods of irrigation were considered. Water coefficients for these crops were determined from consumptive use and expected precipitation data. The consumptive use data for full irrigation (see Figure 2.4) were "established through research and careful field observations" under the direction of the American team leader, Wayne Criddle. The source of the consumptive use data for other than full irrigation methods was not discussed.

The three state variable dynasic program was written in ALGOL by the staff of the Parsons Company. To establish the program took 1,700 man hours and 20 hours of computer time. The final program run for a single crop in a single region took approximately 50 minutes. Obviously, repeating the program for each crop in each region of India, as was the initial intention of the study, would be a formidable task. Still at this level, the problem considered is deterministic.

Discussion of Multiple Crop Models

In this section various aspects of the multiple crop studies are compared and contrasted, in particular the following are discussed:

1. The problem statement
2. The basic data used
3. The growth model or production function adopted
4. Additive vs. multiplicative functions
5. Optimization and consideration of risk.

Problem Statements

Mobasheri (1968), Young and Bredehoeft (1972), Anderson and Maass (1971), Hall (Parsons, 1970), and de Lucia (1969) all consider basically the same problem: maximizing yearly yields or return from a fixed

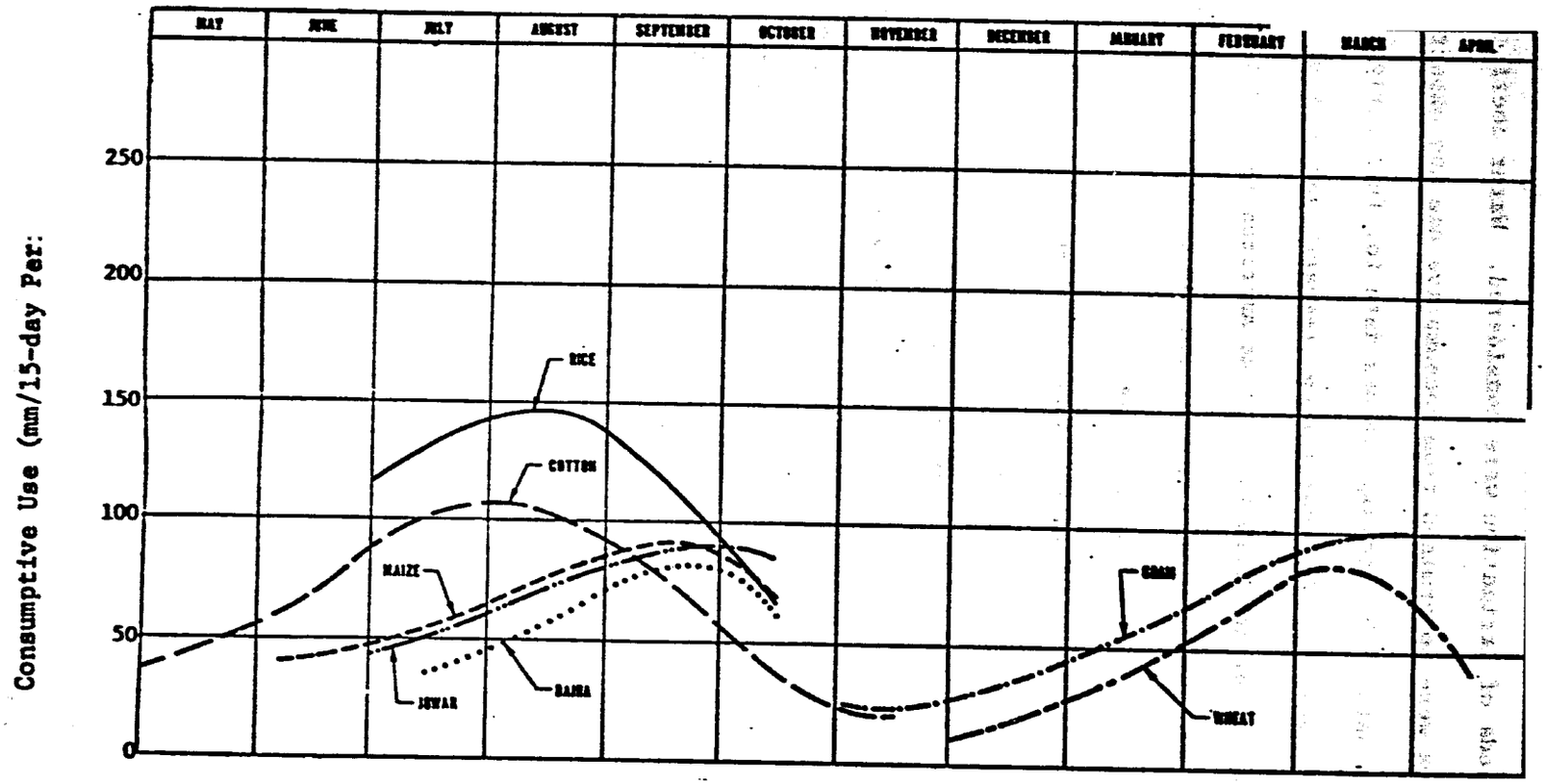


Figure 2.4

CONSUMPTIVE WATER REQUIREMENTS
FEROZEPUR DISTRICT
(From Parsons, 1970)

irrigated acreage, considering a given number of feasible crops. Smith (1973) is concerned with maximizing the net present worth of a planned expansion of a presently irrigated area, considering capital investments of the project and capacity dependent operation and maintenance costs, in addition to costs of water.

All of the previously mentioned authors consider linear constraints such as land constraints, water use constraints, etc. Smith (1973) and Hall (Parsons, 1970) consider crops grown in time periods extending throughout the entire year, but none of the studies consider more than one year and possible crop rotation requirements.

Basic Data

The data used by the authors range from being based on extensive field trials to being based on rather questionable assumptions. Stewart and Hagan (1973b) conducted field trials, growing corn under many different irrigation regimes.

Yaron, et al., (1973) and Minhas, et al., (1974) rely on data from a number of years to establish their respective production functions. A "year" term is often included in the regressions. When the year term accounts for much of the variation in observed yields, the model obviously has not been well constructed. A model of plant growth which includes soil moisture and climatic terms should not require a year term. Another alternative is to use data collected in a single year, thus eliminating complicating effects of climatic variability.

Smith (1973), de Lucia (1969), and Mobasheri (1968) devote little time to discussing the data on which their studies are based. Consumptive use figures for fully watered crops are available for many crops

in many locations. These data are adequate for a study such as Mobasheri's. In other studies, including Hall's and Anderson and Maass', it appears that data for yields under conditions of less than optimum water supply have been based, in some cases, on judgment resulting from limited observations. This is not meant to be a criticism of the studies, only a reflection on the lack of data and the lack of theory to predict crop yields.

Growth Models and Production Functions

Similar to the diversity in making use of basic data, diversity was noted in the growth models and production functions adopted by the various authors.

Stewart and Hagan (1973b) proposed a growth model linearly relating yields to seasonal ET. Yaron, et al., (1973) proposed a simple model for predicting ET while Jensen and Heerman (1970) and Hanks (1974) have more complicated models. Yaron used the stress day concept to develop a production function. Hanks related ET in various time periods to yields with a multiplicative function. Hall used a multiplicative production function with terms functions of soil moisture during the time periods. Updating soil moisture in Hall's model requires predicting ET. As in Yaron's formulation, Hall's ET (Hall and Butcher, 1970) is only a function of available soil moisture, ASM.

In the model of Minhas, et al., (1974), ET is a function of ASM, potential ET and a crop factor. Evapotranspiration is related to yields through a multiplicative production function. Dudley, et al., (1971a) predict actual ET from free water evaporation, a crop factor, and a soil factor. Yields are predicted based on the growth-no-growth concept which is based on daily ET values.

All of the previously mentioned authors rely on an ET estimation model. Some authors relate ET to yields while others, such as Hall, require estimates of ET in order to update soil moisture, which in turn is related to the yield coefficients in each time period. In any case, an ET estimation model is needed.

Additive versus Multiplicative Functions

Multiplicative production functions have been employed by Hanks, Yaron, Hall, and Minhas. Jensen (1968) proposed using the multiplicative relation for some crops, but the irrigation scheduling programs of Jensen assume only one method of production. Mobasheri, Anderson and Maass, Young and Bredehoeft do not employ continuous production functions.

Smith, in his simulation model, assumes a "linear relationship between crop yield and the water applied during any decision period" (Smith, 1970). De Lucia assumes an additive function, based on theory by Moore (1961) which does not appear to be justified for all crops (Hall and Dracup, 1970, p. 134; and Jensen, 1968). Dudley's growth-no-growth concept is an additive relation with each growth day contributing a dollar value to the crop.

The multiplicative relation says, for example, that if growth is only 70% of potential for a particular growth stage, then the maximum yield attainable by the crop is 70% of potential. According to the additive theory 70% of potential growth in a particular time period will only result in potential yields being reduced by 30% of that particular time period's potential contribution (see Figure 2.5).

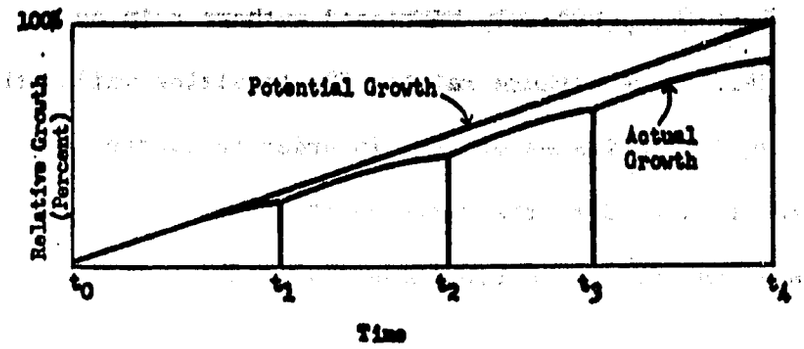


Figure 2.5
ACTUAL AND POTENTIAL GROWTH BY IRRIGATION CYCLES
(From Moore, 1961)

Again, the point is made that an adequate theory is not generally accepted and that currently available data are not sufficient to conclusively adopt any of the production functions described. As new data become available there is a need to test them with both approaches.

Optimization and Consideration of Risk

Simulation models have been formulated by Hanks (1974), Yaron, et al. (1973), and Anderson and Maass (1971). In the last two studies in some sense optimal results were obtained. Optimization problems formulated as dynamic programs were presented by Hall (Parsons, 1970), de Lucia (1969), and Dudley, et al. (1971a). Linear programs were formulated by Hall (Parsons, 1970), de Lucia (1969), Mobasheri (1968), Young and Bredehoeft (1972), and Smith (1973). Minhas, et al. (1974) formulated a mathematical programming problem and solved it using marginal analysis.

As was discussed previously, linear programming is well suited to handling the multiple cropping problem. Different methods of producing a single crop are easily incorporated into the model. Also,

problems of wider scope, such as that of Smith's which includes well and canal sizing, are also amenable to the linear programming format. In Mobasheri's and Young and bredehoeft's studies, the scope was decreased so that the linear program could be used as a sub-program in a larger problem.

All of the linear programming models employ constant water use coefficients except for that of Smith, which allows for random precipitation by including an effective precipitation term in his water use coefficients. De Lucia in his linear programming model assumes the expected value of precipitation will occur. The fixed water requirement models are limited in that they cannot consider random climatic variables besides precipitation. These models may also be criticized in that they do not get down to the actual physical process involved in plant growth. They assume for a given time period a fixed quantity of water is needed, whereas in actuality this quantity is a variable, dependent upon climatic variables such as temperature, humidity, wind speed, soil moisture status, previous condition of the plant, etc. However, the linear programming format appears to be the only method available for handling the multiple crop problem. Random water use coefficients could be handled, still retaining the basic linear programming format, by discrete stochastic programming methods (see Appendix C) if such methods were warranted.

Of the authors who have formulated multiple crop models with linear programs, only Smith (1973) has considered risk. Initially, he formulated a deterministic linear program, then he extended it to a chance constrained program with precipitation as the only source of risk. Finally, he ran a simulation program based on the results of

the previous program to determine the effects when the probabilistic constraints were violated. In the current study, further work is done employing the method of Smith. Moving to the discrete stochastic approach with the large programs that are generated does not seem justified with currently available data.

Summary

From the discussion of the previous sections, it is clear that a definitive multiple crop irrigation study has not been performed. Such a study will not be performed, of course, until an accepted theory for predicting yields is developed. Until that time, using the best available information, work can continue on improving and expanding existing studies. The remainder of this section presents research areas in which a new study should contribute.

A first area is basic data. Results of field trials for various crops and various water inputs and timings are needed to test theories of yield prediction. Soil moisture measurements throughout the season are needed. The accuracy and limitations of the data need to be discussed. Information regarding soil type and data on climatic variables need to be presented so that future studies may make use of the current data.

As discussed in a previous section, most researchers have recognized the need for ET prediction models and models which reconstruct the soil moisture curves over the season. Currently, for predicting yields at the start of the season, expected values of various climatic variables are used. A model such as Yaron's soil moisture simulation assumes constant climatic variables. Fitting parameters for the

local conditions to the basic data is required. One result of this exercise is that the reliability of the basic data (such as soil moisture measurements) becomes apparent.

With an accurate soil moisture simulation for the various field trials, production functions can be tested. These are not meant necessarily to explain the crop yields physically, but rather to allow further economic study of crop production. Efforts should be made, however, to define critical periods of growth and relate the growth index to yields in various time periods. The particular form of the function should not be a matter for argument but some "best fit" function should be adopted.

In general, single crop models have tended to be formulated as dynamic programs and multiple crop models as linear programs. With dynamic programming, each additional crop introduces at least one additional state variable, leading to dimensionality problems. Linear programming requires a linear production function or reformulation of a non-linear problem into linearized form. This has been done in various models by considering only discrete methods of production. For multiple crop studies, the flexibility of the linear programming technique is well established. Discretizing crop production methods does not limit the optimality of the result, if non-optimality is suspected, a finer grid can be placed on the production methods.

The models discussed have treated random variables by various methods, ranging from disregarding them to evaluating return for many combinations of possible values (in a dynamic programming sense) taken on by the random variables. In this study a technique similar to that

used by Smith is adopted where the optimum policy under a given precipitation level is found, then the effects of other historical precipitation amounts are evaluated in a simulation program.

Chapter III

THE SINGLE CROP MODEL

Introduction

In this chapter a model is established which determines levels of production and associated water use coefficients for a single crop, corn. The model is applied to a site in eastern Colorado and economic data regarding this site are presented. Field trial data from Colorado State University are used to determine parameters of the model. It is assumed that these parameters are representative of the application site.

The model that determines water use coefficients is actually broken down to three computer programs. These are

1. An ET prediction model
2. A program which determines the production function
3. A program for selecting optimal times and amounts of irrigation based on 1 and 2 (above) when water is limited in various time periods.

Hall and Butcher (1968) used a dynamic program to allocate a fixed seasonal supply of water to a single crop. This chapter analyzes a single crop which is competing for water with other crops. A model is developed which evaluates the returns for corn when grown under these intraseasonal water limiting conditions. The results are used as input to the next chapter, where the multiple crop problem is considered.

In this chapter, an additive production function and a multiplicative function are tested. Each is employed in a program to determine optimal irrigation amounts on fixed days, first with water not limiting. The additive function is also used in a program to determine optimal irrigation days and amounts. The additive function with fixed dates is used to determine the final optimal water use coefficients under deficient supply.

The ET Prediction Model

The Yaron ET Prediction Method

The soil moisture simulation of Yaron, et al., (1973) states that on a daily basis, ET is a linear function of available soil moisture, or

$$ET = b' \, ASM \tag{3.1}$$

in which b' are referred to in this study as "Yaron coefficients" having units of cm;

and ASM is available soil moisture percentage.

In this formulation when the soil profile is at field capacity, the value of ASM is one and hence b' may be interpreted as the ET rate which occurs at field capacity, or the daily average maximum ET rate.

Evapotranspiration may be viewed as the negative of the rate of change of soil moisture providing ET is the only means in which soil moisture decreases. This leads to the differential equation

$$ET = -\frac{dSM}{dt} = b \, SM \tag{3.2}$$

in which

SM is depth of available soil moisture (cm.).

b is related to b' in Equation 3.1 by the relation $b = b'/FC$ in which FC is the depth of available SM at Field Capacity, b is dimensionless.

Providing no irrigation or rainfall occurs, Equation 3.2 has the solution

$$SMf = SMIe^{-b_i \cdot t} \tag{3.3}$$

in which

t = time in days

SMI = initial soil moisture (cm.)

SMf = soil moisture after t days

b_i = Yaron soil moisture coefficient, constant in growth stage i (dimensionless)

If one irrigation occurs during the time period

$$SMf = (SMIe^{-b_i \cdot t} + IRR)e^{-b_i \cdot t} \tag{3.4}$$

in which

SMf = soil moisture after t + t' days

IRR = quantity of irrigation applied at day t

(Other terms as defined previously.)

Thus, within a growth stage, ET is given by

$$ET = SMI + IRR - SMF \tag{3.5}$$

in which

IRR = quantity of irrigation delivered to the root zone between initial and final times.

A modification of the basic Yaron approach is the concept that soil moisture contributes to ET by layers. Thus the contribution to total ET by the jth layer on a day in the ith time period is given by

$$ET_{ij} = b_{ij} ASM_{ij} \tag{3.6}$$

Total ET on a day in the ith time period ET_i is given as

$$ET_i = \sum_{j=1}^{n(i)} ET_{ij} \tag{3.7}$$

in which

n(i) is the number of layers which contribute to ET, i.e., the number of layers in the root zone at time period i.

For a comparison of the Yaron ET prediction method and other methods see Appendix B.

The Soil Moisture Simulation Model

A program was written to determine best fit coefficients to the ET prediction equation (Equation 3.6). A fit is desired which, for data from the given year, is able to predict ET under a wide range of soil moisture conditions, not just for the "well watered" case. Due to the limitations of the data, only three growth stages are considered, roughly corresponding to the early vegetative stage, pollination, and maturity.

The program is described in this section. A flow chart is included in Appendix D. The program was run with data from 1974 and was also run with more limited data from 1972. The results are compared in a later section. The comparison is desirable to test the sensitivity of the coefficients obtained.

Required inputs are number of soil layers, length of each growth stage, dates and amounts of irrigation, dates of soil moisture observations and soil moisture levels, initial soil moisture by layers, maximum available soil moisture by layers, initial values of the coefficients b_{ij}, and the allowable range of the coefficients.

The main program (SM SIM) reads in the data, determines which coefficient is to be optimized, and calls a subroutine ODS. This is a library subroutine which performs a Fibonacci search to minimize a unimodal function which is evaluated in RESTNT.

For the given current values of the coefficients, RESTNT performs a simulation for the entire year for each treatment. It proceeds on a daily basis, calculating ET, keeping track of irrigations and precipitation. On a soil moisture observation day, the absolute value of the difference between observed and calculated soil moisture is stored. The total of these absolute values for all treatments is calculated and passed back to ODS as the value of the function. Subroutine ODS continues to vary the value of the coefficient b_{ij} until the function is minimized. When this is accomplished, control is returned to SMSIM and the next coefficient is selected. The program terminates after determination of the last coefficient.

The program has been modified so that when the optimal coefficients are obtained, a microfilm output is produced, showing cumulative soil moisture by layers for each treatment (see Appendix B, Figures B.2, B.6).

A particular assumption of the program is that runoff is not considered. In the field trials, precipitation and irrigation amounts are measured at ground level. It is assumed that this entire depth of water enters the soil profile. A simple drainage model is considered. When field capacity for a particular level is reached, excess water is routed to the next deeper layer. A final point is that

ET is assumed to occur on irrigation days but not on rainfall days. Rerunning the program assuming ET on rainfall days did not improve fit to observed data.

Discussion of Program SM SIM

A question may be raised as to whether the optimization method used in the program does indeed yield a "best fit" to the observed data.

The criterion of minimizing the sums of the absolute values of the differences between calculated and observed values was employed by Yaron (1973) instead of minimizing the sum of the squares of the differences. The justification for using the former is that it does not give undue weight to outlying values. In this application there are obvious incorrect values (see Figure B.3) which the program should not attempt to fit. Thus this "best fit" criterion appears adequate for the current application.

Another point is that the program optimizes each coefficient with the other coefficients held constant. This method would be suitable if the time periods were independent, however, this is not the case. Determination of a coefficient in period 2 depends not only on the soil moisture observations during the time period, but on the soil moisture at the beginning of the time period, which depends on the coefficient in time period one.

An exact solution could be obtained by formulating a large mathematical programming problem which would have non-linear constraints and a quadratic objective function if the least squares

criterion were adopted. With no rainfall or irrigation the first several constraints would take the form

$$SM_1 = SMI \tag{3.8}$$

$$SM_2 - SM_1 - b_1 SM_1 \tag{3.9}$$

$$SM_3 = SM_2 - b_1 SM_2 \tag{3.10}$$

$$= SM_1 = b_1 SM_1 - b_1(SM_1 - b_1 SM_1)$$

$$= SM_1 - 2b_1 SM_1 + b_1^2 SM_1$$

The objective function is of the form

$$\min \sum_{j=1}^{n} (SM_{i(j)} - SMOB_j)^2 \tag{3.11}$$

in which

SMI = initial soil moisture

SM_i = soil moisture on day i

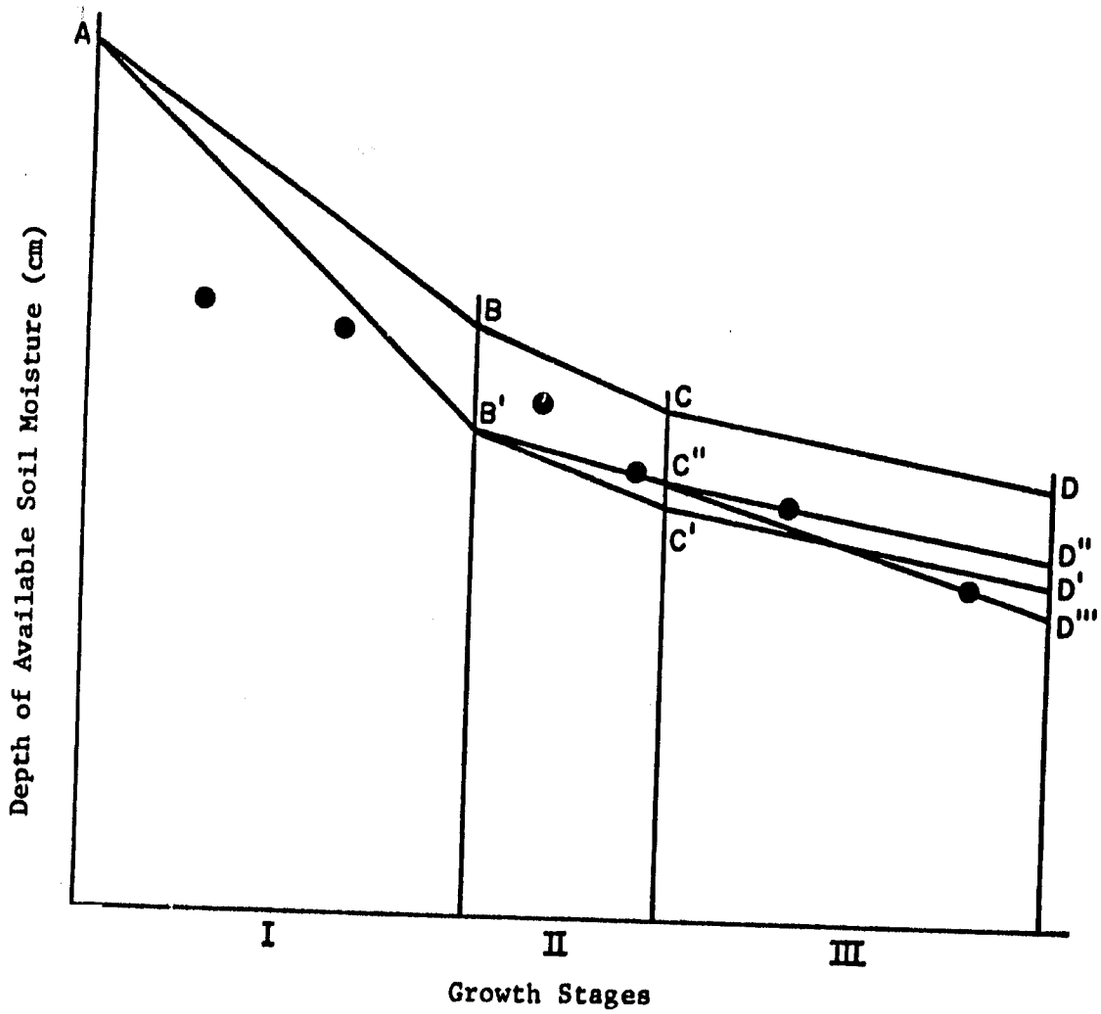
b_1 = Yaron coefficient in growth stage one.

$SMOB_j$ = j^{th} soil moisture observation

$SM_{i(j)}$ = calculated soil moisture on day i which corresponds to j^{th} SM observation

To avoid the complexities of the above programming approach, the assumption of independent time periods was made. A sufficiently close initial guess of the coefficients insures convergence to the best fit. This point may be demonstrated through Figure 3.1 Suppose the three initial guesses generate the three-line segments AB, BC, CD. By allowing b_1 to vary the new fit is AB', B'C', C'D'. Allowing b_2 to vary gives the fit AB', B'C'', C''D''. Allowing b_3 to vary gives the fit AB', B'C'', C''D'''.

For a test of convergence a second iteration may be performed, again allowing b_1 to vary with the new initial values of b_2 and b_3. With the computer runs, the second iteration was not performed since the initial results appeared adequate.



● Represents Soil Moisture Observations

—— Calculated Soil Moisture
Slopes of these Lines are Determined by the Parameter b_{ij}

Figure 3.1
GRAPHICAL ILLUSTRATION OF DETERMINATION OF PARAMETERS b_{ij}

The Results of Soil Moisture Simulation

The coefficients obtained from the program are shown in Table 3.1. These coefficients have units of cm. where SM is given as a fraction of field capacity FC. Thus, if the entire profile is at FC for a day in the first time period, the predicted ET is given by the sum of the coefficients, or 0.50 cm. These totals for the entire profile are used in the dynamic program in dimensionless form. Each coefficient is divided by the depth of available soil moisture at field capacity (33.2 cm.) to obtain the dimensionless value.

The results in Table 3.1 are misleading since they indicate that, early in the season, contributions to ET occur from all layers whereas later in the season, after roots would have penetrated the entire profile, contributions from deeper levels are reduced. This problem is explained in that available soil moisture at the beginning of the season was not measured. Selection of the initial SM level was arbitrarily set at field capacity and it appears this value was high. This may also be observed from Figures B.2 and B.5, where ET contributions from layer 7 occur early in the season. Adjusting the initial SM values downward would have decreased the coefficients obtained in the first time period.

Comparison with 1972 Results

The program was also run with 1972 corn data reported by Twyford (1973) from Colorado State University. These data were available for the entire soil profile (196 cm.) only. The results from the two years along with average measured pan evaporation for 1974 are presented in Table 3.2. It should be noted that potential evaporation

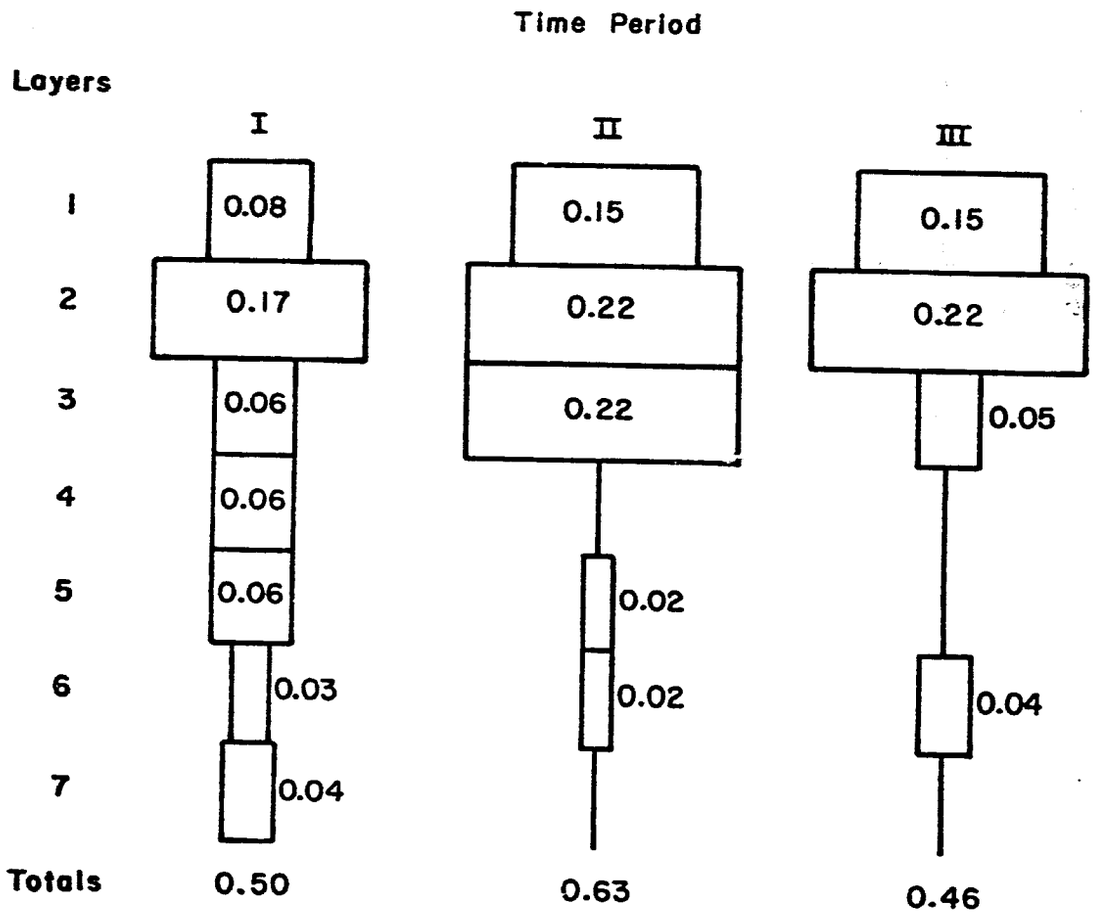


Table 3.1

CONTRIBUTION TO ET BY LAYERS IN TERMS OF ASM PERCENTAGE (Cm./Day)

Table 3.2

COMPARISON OF YARON COEFFICIENTS FROM 1972 AND 1974 COLORADO STATE UNIVERSITY CORN EXPERIMENTS

	Growth Stage		
	Early Vegetative	Pollination	Maturity
1972 Calculated Coefficients (cm./day)	0.41	1.25	0.38
1974 Calculated Coefficients (cm./day)	0.50	0.63	0.46
1974 Average Daily Pan Evaporation (cm./day)	0.63	0.66	0.57

should serve as an upper bound on the coefficients, since each coefficient represents the maximum predicted ET that can occur. The b_2 value for 1972 is obviously unreasonable since it excedes the estimate of potential evaporation provided by the pan measurement. This high value may have been obtained since only the A treatments were simulated, that is, those irrigated early in the silking period. Including the other treatments would have served to decrease the value of b_2 since lower ET rates occurred for these treatments.

The value for b_1 in 1974 is high as was mentioned previously because of high initial soil moisture.

It is difficult to draw conclusions from two years of data, but values of the coefficients are reasonable and the relative values are as would be expected. The maximum average ET rate during pollinatior

should be higher than the maximum average for the earlier period and it should be higher than for the later period which includes senescence.

Determination of Production Function Coefficients

Multiplicative Case

With the model to predict ET established, the second phase of this chapter is to relate ET to yields. The first function tested is the multiplicative relation

$$\frac{Y}{Y_{max}} = A \prod_{i=1}^{n} \left(\frac{ET_i}{ET_{max_i}} \right)^{\lambda_i} \tag{3.12}$$

in which Y is predicted yield

Y_{max} is maximum attainable yield

A is a constant to be determined

ET_i is measured ET in period i

ET_{max_i} is maximum ET in period i

λ_i is a parameter to be determined

n is number of growth stages.

A standard stepwise linear regression code was employed to fit coefficients in the equation using field trial data. Taking logarithms of both sides of the equation transformed it to a linear equation. The results of the regression runs are shown in Table 3.3. Values for ET_{max} were the maximum recorded ET during the time period and values for ET_i were calculated from field measurements of soil moisture and recorded precipitation (see Table 3.11).

In runs 1A and 1B, four time periods were considered. Thirteen treatments out of 48 showed non-negative ET in all those periods. In run 1A the constant A was held at one and in run 1B is was allowed to vary. Ideally a single treatment should exhibit maximum

Table 3.3

SUMMARY TABLE OF REGRESSION RUNS USING THE MULTIPLICATIVE RELATION

$$\frac{Y}{Y_{max}} = A\left(\frac{ET_1}{ET_{1max}}\right)^{\lambda 1}\left(\frac{ET_2}{ET_{2max}}\right)^{\lambda 2}\left(\frac{ET_3}{ET_{3max}}\right)^{\lambda 3}\left(\frac{ET_4}{ET_{4max}}\right)^{\lambda 4}$$

Run	I	II	III	IV	A	R^2
1A	.277	*	*	.217	-	.71
1B	.255	.315	*	*	.79	.87
2A	.174	.220	.331	-	-	.75
2B	.197	.232	.239	-	*	.82
3	*	*	.112	.330	0	.77
4	*	2.95	*	-	.84	.84
5	.533	-.129	.179	-	.96	.86

I - IV Time Periods
- Deleted or not defined
* Not significant at 5% level
A Intercept Value

Corn

Run 1A, 1B - 13 treatments, 4 time periods, CSU, 1974
Run 2A - 26 treatments, 3 time periods, CSU, 1974
Run 2B - 23 treatments, 3 time periods, CSU, 1974
Run 3 - water inputs only, 3 time periods, CSU, 1974
Run 4 - 9 treatments, 3 time periods, Davis, 1972

Grain Sorghum

Run 5 - 6 treatments, 3 time periods, Davis, 1973

ET rates in all time periods and should result in maximum yield. In this case, the constant A should be one. However, with the available data a better fit was obtained with the additional variable.

The ET data for time periods 3 and 4 appeared suspect with generally low values (including negative values) in period 3 and high values in period 4, leading to the hypothesis that soil moisture observations were incorrect on August 13. In the remainder of the runs, three growth stages were used as defined previously. In run 2A, 26 treatments with non-negative ET were tested and in run 2B, three treatments of high ET were removed resulting in a fit with R^2 of 82.

In run 3, water inputs instead of ET were related to yields resulting in an R^2 of 77. In run 4, corn data from Davis, California, was used. In run 5, the same equation was tested for grain sorghum resulting in a negative coefficient in the second time period, indicating that as ET increased in the time period, yield decreased. Whether this result has a physiological basis or the data are suspect is not known.

Additive Case

The additive relation

$$\frac{Y}{Y_{max}} = \sum_{i=1}^{n} A_i \left(\frac{ET_i}{ET_{max_i}} \right) + A_{n+1} \tag{3.13}$$

in which A_i are the coefficients to be determined, was also tested with results shown in Table 3.4. The values of ET_{max} were the

Table 3.4

SUMMARY OF REGRESSION RUN USING ADDITIVE RELATION

$$\frac{Y}{Y_{max}} = A_1\left(\frac{ET_1}{ET_{max1}}\right) + A_2\left(\frac{ET_2}{ET_{max2}}\right) + A_3\left(\frac{ET_3}{ET_{max3}}\right) + A_4$$

Run	A_1	A_2	A_3	A_4	R^2
6A	.236	.159	.573	-	.98
6A'	.2436	.1647	.5917	-	-
6B	.213	.113	.352	.246	.98
6C	.921	-	-	.275	.64
7	*	.664	*	-	.87

- Deleted or not defined

* Not significant at 5% level

Corn

6A - 3 time periods, 0 intercept, 23 treatments, CSU, 1974

6A' - values of 6A scaled to add to 1.00

6B - 3 time periods, non 0 intercept, 23 treatments, CSU, 1974

6C - yearly ET, non 0 intercept, 23 treatments, CSU, 1974

7 - 9 treatments, 3 time periods, 0 intercept, Davis, 1972

measured pan evaporation. In runs 6A and 6B the previously used 23 treatments were tested with zero and non-zero values of A_4. In run 6C total seasonal ET was related linearly to yields with non-zero intercept as was done by Stewart and Hagan (1973b). In run 7, the 1972 Davis data were tested using the additive relation of Equation 3.13.

It should be noted that the regression runs do not conclusively show a cause and effect relationship between ET and yields. They provide a means of obtaining the coefficients of empirical equations. Actually, ET is a proxy variable to measure plant growth, it is not meant to be a complete measure of the growth variable. The only physiological basis for using ET as discussed in Chapter I, is that ET occurs as does photosynthesis when the stomata are open in sunlight hours. It is hypothesized that closing the stomata, hence reducing ET, has different effects on yield dependent upon growth stage. The physical interpretation of the coefficients is that they indicate a relative contribution of ET in various growth stages to yield.

The analysis of Appendix B predicts that the coefficients obtained from the multiplicative run should be a first order approximation to those of the additive run when the treatments tested have relative ETs close to one. In this case, possibly due to the wide range of relative ET values tested, the results differ significantly.

With the existing theoretical basis for predicting crop yields under deficient water supplies, the two tested ET relationships appear adequate. One advantage of the ET relations is that if

atmospheric variables could be introduced into the ET prediction model, the yield prediction model would become more effective. A model relating inputs directly to yields completely neglects climatic influences.

The Combined Simulation Model

Combining the results of the soil moisture simulation with either the additive or multiplicative production function results in a model which can predict corn yield under any irrigation regime. Of course, this prediction is limited by the assumptions of the study, particularly that the Yaron coefficients are valid for the site and prediction year, that is, they are insensitive to climate variation, and that the production function adequately predicts yield in the forecast year at the application site.

The basic equation, Equation 3.1, states that ET is linearly related to available soil moisture percentage. A first approximation would assume that field capacity is a constant over the season and that roots may draw from any layer at any time. A better approximation is that the root zone is dynamic. That is, it expands, especially during the vegetative growth stage. In this study it is assumed that the entire soil profile is at field capacity at the start of the season. Thus, with the dynamic model, an increase in the quantity of water available occurs as the root zone enlarges.

A root zone growth model should be incorporated into the plant growth model. Simply stated, this model may be considered as viewing field capacity as a variable over the season. Initially, field capacity is determined by the average rooting depth during the first

time period. During the first time period, soil moisture is calculated through the Yaron equation. During the second time period, field capacity is increased, and soil moisture is increased by the difference of the present and previous field capacity values, or

$$SM_2 = SM_1 + (FC_2 - FC_1) \tag{3.14}$$

in which

SM_2 represents depth of available soil moisture at the start of period 2

SM_1 represents depth of available soil moisture at the end of time period 1

FC_2 represents the field capacity value in period 2

FC_1 represents the field capacity value in period 1.

Initially, the models of this chapter were formulated with the static root zone model. A single run was made, however, employing the dynamic root zone model. For this run, an average rooting depth of 3 feet was assumed during the first time period, and the full 7 foot depth for the remaining two periods. Under this assumption, the field capacity value during the first time period is calculated from Table 3.10 as 14.3 cm. The Yaron coefficients used were those of Table 3.1 disregarding those values in the first time period below the 3 foot depth. The results of this run are described in a later section.

Formulation of Optimization Models

Introduction

In the next section, the Yaron ET prediction model and first the multiplicative and later the additive production functions are incorporated into optimization models. Under the assumptions made,

the models determine quantities of irrigation water at various times in order to maximize return from the crop. First, a general problem with multiplicative production function is formulated. Next a more specific problem is presented. Thirdly, a problem is formulated and solved for irrigation on fixed dates using a multiplicative production function. Fourth, the problem is solved with the additive production function and fixed dates, and finally with the additive function and flexible dates.

General Problem

The general problem of maximizing yield for a single crop with irrigation quantity and timing as the variable means of production is stated in Equation 3.15. The formulation employs a multiplicative production function with the first term representing gross return. The second term represents variable water costs dependent on quantity of water applied. The third term represents labor costs dependent on number of irrigations. Other fixed and variable costs are constant and hence are not included in the optimization.

The decision variables in the following formulation are quantities of irrigation water IRR_k applied to the root zone at various scheduled irrigation times. Evapotranspiration is a variable which depends on the state variable soil moisture SM_i and the decision variable IRR_k. The problem is formulated with ET represented on a daily basis by the following.

$$\underset{Irr_k}{\text{Max}}\ AY_{max} \prod_{j=1}^{jj} \left(\frac{ET^j}{ETMAX_j}\right)^{\lambda_j} - B \sum_{k=1}^{kk} Irr_k - C' \sum_{k=1}^{kk} DEL_k \qquad (3.15)$$

subject to

$$ET^j = \sum_{i=n(j)}^{n(j+1)} ET_i \qquad j=1, \ldots, jj$$

$$SM_1 = SMI$$

$$ET_i = b_i SM_i$$

$$SM_{i+1} = SM_i - ET_i + Irr_{k(i)}$$

and

$$0 \leq SM_{i+1} \leq SMI$$

in which

i = day number

j = index of growth stage

jj = maximum number of growth stages

k = index of irrigation days

kk = maximum number of irrigation days

Y_{max} = maximum yield

ET^j = actual ET during j^{th} growth stage, cm.

$ETMAX_j$ = maximum ET during j^{th} growth stage

IRR_k = quantity of water applied at the kth irrigation, cm. (decision variable)

DEL_k = 1 if $Irr_k \neq 0$, 0 otherwise

A, B, C = economic coefficients

λ_j = parameters of multiplicative production function

SMI = initial available soil moisture level, assumed equal to field capacity

SM_i = soil moisture on day i (state variable)

b_i = Yaron soil moisture coefficient on day i

ET_i = ET during day i

$k(i)$ = k if i = irrigation day T_k, 0 otherwise

T_k = vector of days when irrigation occurs

Irr_0 = 0

Formulation with Fixed Irrigation Dates

A practical management scheme for some soils and climates is that an irrigation can occur only at the change in growth stage and a maximum of one irrigation is considered within any growth stage. In this case, the constraints can be simplified over those in the previous formulation. Single constraints can be written for the entire growth period instead of the daily computations previously required. The formulation is:

$$\max_{\substack{Irr_{jj} \\ Irr_{j'}}} A\, Y_{max} \prod_{j=1}^{jj} \left(\frac{ET_j}{ET_{max_j}}\right)^{\lambda j} - B \sum_{j=1}^{jj} (Irr_j + Irr'_j) - C \sum_{j=1}^{jj} (DEL_j + DEL'_j) \qquad (3.16)$$

such that

$$SM_1 = SMI$$

$$SM_j' = (SM_j + IRR_j)e^{-b_j t_j'}$$

$$SM_{j+1} = (SM_j' + IRR_j')e^{-b_j t_j}$$

and

$$0 \le IRR_j \le FC - SM_j \qquad j=1, \ldots, jj$$

$$0 \le IRR_j' \le FC - SM_j' \qquad j=1, \ldots, jj$$

in which

A, B, C are economic coefficients

SMI = known initial soil moisture depth (cm.)

SM_j = soil moisture at the beginning of growth stage j, previous to an irrigation

SM_j' = soil moisture at an irrigation time within growth stage j

IRR_j = depth of irrigation at beginning of growth stage j

IRR_j' = depth of irrigation at a time within growth stage j

ET_j = ET during growth stage j

FC = depth of soil moisture at field capacity

ET_{max_j} = known maximum ET during growth stage j

b_j = known Yaron coefficient for growth stage j

t_j' = number of days in growth stage j previous to scheduled irrigation within growth stage j; zero if no irrigation is scheduled

t_j = number of days from scheduled irrigation within growth stage j to end of growth stage j

DEL_j = 1 if $IRR_j = 0$, 0 otherwise

DEL_j' = 1 if $IRR_{j'} = 0$, 0 otherwise

JJ = number of growth stages

Dynamic Programming Solution

Discussion of Solution Technique

In both of the previous formulations the objective function consists of additive and multiplicative terms similar to the dynamic programming formulation of Hall and Butcher (1968). The first formulation would require calculation of each possible value of ET^j for each possible irrigation time and amount and for each combination of irrigations during growth stage j. The second formulation is more amenable to the dynamic programming format, with many fewer combinations to be calculated.

A specific initial value sequential decision problem (Labadie, 1973) involves maximizing a separable objective function of the form

Problem A

$$\max_{\substack{x_{i+1},\, u_i \\ i=1,\, \ldots,\, n}} \sum_{i=1}^{n} f_i(x_i, u_i) + \phi(x_{n+1}) \tag{3.17}$$

subject to a Markovian or dynamic relation

$$x_{i+1} = g_i(x_i, u_i) \qquad i=1, \ldots, n$$

with x_1 given and

$x_i \varepsilon X_i$

$u_i \varepsilon U_i$ $\qquad\qquad\qquad\qquad i=1, \ldots, n$

The formulation may be written more generally with variables and functions as vectors. This formulation is discussed in terms of the irrigation problem in Appendix C, where x_i is the state variable representing soil moisture levels, u_i is the decision variable representing irrigation amounts, g_i represents the ET prediction model and f_i represents return as a function of soil moisture and irrigation amounts.

The solution of the foregoing problem rests on the separability of the objective function and the Markovian nature of the dynamic equation. By separating terms of the objective function at each stage i and forming a new function F_i, Problem A can be restated as an equivalent problem, Problem B, which can be solved efficiently. At each stage i, and for each value of the state variable x_i, an optimal u_i is found which maximizes $F_i(x_i)$ given by

<u>Problem B</u>

$$F_i(x_i) = \max_{\substack{u_i \,\varepsilon\, U_i \\ x_{i+1} = g_i(x_i,u_i) \,\varepsilon\, X_{i+1}}} [f_i(x_i,u_i) + F_{i+1}(x_{i+1})] \qquad i=1, \ldots, n \quad (3.18)$$

and

$$F_{n+1}(x_{n+1}) = 0 \text{ for all } x_{n+1}$$

in which x_1 is given and

x_i = state variable, $x_i \varepsilon X_i$

u_i = decision variable, $u_i \varepsilon U_i$

$F_i(x_i)$ = optimal return at time i for state variable x_i

The objective is to maximize $F_1(x_1)$. Maximizing $F_1(x_1)$ is equivalent to solving Problem A.

The nonseparable case as in Equation 3.16 may be represented by the following problem:

Problem C

$$\max_{\substack{x_{i+1},u_i \\ i=1,\ldots n}} \; pY_{max} \prod_{i=1}^{n} f_i(x_i,u_i) - \sum_{i=1}^{n} e_i(u_i) - \sum_{i=1}^{n} h_i(u_i) \qquad (3.19).$$

subject to

$$\begin{aligned} x_{i+1} &= g_i(x_i,u_i) \\ x_1 &= c \\ x_{i+1} &\in X_{i+1} \\ u_i &\in U_i \end{aligned} \qquad i=1, \ldots n$$

in which

Y_{max} = maximum yield

p = price per unit of yield, other terms as previously defined.

Here f_i are terms of the multiplicative production function and e_i and h_i represent irrigation costs. Actually e_i and h_i could be written as one function, but are written separately to represent water cost and labor cost of irrigation. The foregoing problem can be solved by an approach in which another state variable A_i is introduced. This variable represents the "state" of the crop at growth stage i, or mathematically, the allowable values of the multiplicative terms up to the present time. For all allowable values of the state variables, then, the optimal decision (irrigation amount) at the current decision stage is made as a function of the state variables.

By adding the state variable A_i and by employing the following general recursion relation, the solution (Labadie, 1973) may be obtained.

<u>Problem D</u>

$$R_i(x_i,A_i) = \max_{u_i} \; [-g_i(u_i) - h_i(u_i) + R_{i+1}(x_{i+1}, A_{i+1})] \quad (3.20)$$
$$i=1, \ldots, n$$

with
$$x_1 = c$$
$$A_1 = 1$$
$$x_{i+1} = \varepsilon X_{i+1}$$
$$A_{i+1} = A_i f(x_i,u_i) \varepsilon [0, 1]$$
and
$$R_{n+1}(x_{n+1},A_{n+1}) = A_{n+1}{}^{Y}\max^{P} .$$

The problem is solved when $R_1(x_1, A_1)$ is determined.

In the foregoing formulation, R_i represents the optimal return for the remainder of the season for given values of the state variables x_i, in this case representing soil moisture, and A_i, representing the state of the crop.

The following formulation, Problem D' is equivalent to Problem D; however, in the solution process, more information is stored at each stage i. The formulation is

<u>Problem D'</u>

$$\max R_1(x_1, A_1)$$

with

$$R_i(x_i, A_i) = \max_{u_i}[f(x_i, u_i)F_{i+1}(x_{i+1}, A_{i+1}) - (g_i(u_i) + G_{i+1}(x_{i+1}, A_{i+1})) - (h_i(u_i) + H_{i+1}(x_{i+1} A_{i+1}))] \quad (3.20')$$
$$i = 1, \ldots, n$$

with
$$x_1 = c$$
$$A_1 = 1$$

and state variable recursive relations

$$x_{i+1} = g_i(x_i, u_i)$$
$$A_{i+1} = f_i(x_i, u_i)A_i \qquad i=1, \ldots, n$$
$$x_i \in X_i$$
$$A_i \in [0, 1]$$

and return function recursive relations:

$$F_i(x_i, A_i) = f_i(x_i, u_i^*) \cdot F_{i+1}(x_{i+1}, A_{i+1})$$
$$G_i(x_i, A_i) = g_i(u_i^*) + G_{i+1}(x_{i+1}, A_{i+1})$$
$$H_i(x_i, A_i) = h_i(u_i^*) + H_{i+1}(x_{i+1}, A_{i+1})$$

with $F_{n+1} = 1$, $G_{n+1} = 0$, $H_{n+1} = 0$

u_i^* is the value that maximizes Equation 3.20'.

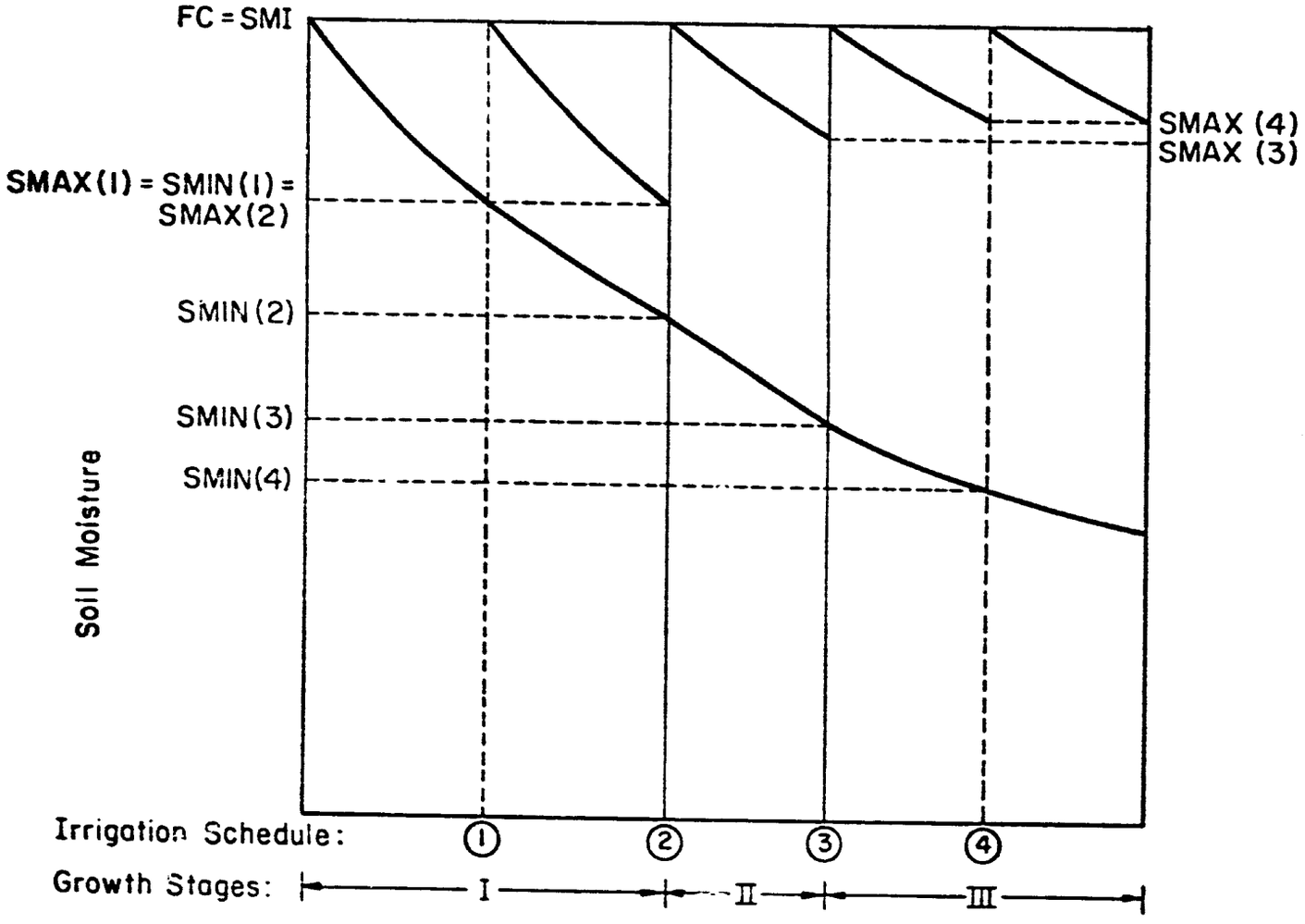
The additional stored information is F_i, the optimal multiplicative coefficient for the remainder of the season; G_i, the irrigation cost for the remainder of the season, and H_i, the irrigation labor cost for the remainder of the season. The specific formulation which is solved in this chapter is presented as Equation 3.21.

Multiplicative Case, Fixed Dates

Considering the three growth stages determined for the multiplicative production function and a maximum of four irrigations as in Figure 3.2, a two-state variable dynamic program was written. The state variables are soil moisture and state of the crop. Four stages are considered, corresponding to the three growth stages, with a fourth stage occurring at the last irrigation.

Within these assumptions, then, the purpose of the program, initially, is to determine amounts of irrigation water to be applied at scheduled times which maximize return for the crop. Two types of

Figure 3.2



GRAPHICAL ILLUSTRATION OF MAXIMUM AND MINIMUM SOIL MOISTURE LEVELS FOR THREE GROWTH STAGES AND FOUR IRRIGATIONS

costs are associated with the water, first a constant cost per unit volume and second a cost, which includes labor cost, per irrigation. The program is modified so that depths of irrigations are restricted in each time period. This could have been achieved by including additional state variables, but was done by making a number of computer runs, each time making a small modification.

The optimal returns which are calculated with the intraseasonal water restrictions are used in the multiple crop model to represent the situation when other crops compete for limited water during certain time periods. The problem, then, is a modification of the Hall and Butcher (1968) problem which optimally allocated a fixed seasonal supply of water.

Program OPT TIM starts at the fourth stage and works back in time. At the fourth stage, for each discretized soil moisture level and each discretized crop state, the program varies the decision variable, amount of irrigation to be applied, calculates the ET for the entire time period, then calculates the return function for the remainder of the season. The irrigation amount which maximizes this return function is then obtained.

The program stores as a function of the state variables, the optimal irrigation amount for the remainder of the season OAIRR, the optimal multiplicative ET coefficient OCF, and the optimal number of remaining irrigations ODEL. The program proceeds backward in time, at each stage calculating the optimal irrigation amount for each level of soil moisture and each value of the crop state variable A. Finally, the program picks out the optimal decisions that have been made. A flow chart of the program is included in Appendix D.

In this program, the values that are carried from one stage to the preceding one are OCF, OAIRR, and ODEL. With these values and with the values of the state variables and the current value of the decision variables, the return function for the remainder of the season can be evaluated. In general it is done in the following manner:

$$R_i(SM_i, A_i) = \max_{IRR_i} \left[a\left(\frac{ET_i}{ET_{max_i}}\right)^{\lambda_i} OCF_{i+1}(SM_{i+1}, A_{i+1}) - b(IRR_i + OAIRR_{i+1}(SM_{i+1}, A_{i+1})) - c(DEL + ODEL_{i+1}(SM_{i+1}, A_{i+1})) \right] \quad (3.21)$$

in which

R_i = return at decision stage i for remainder of season

a, b, c = economic coefficients

A_i = state variable at stage i measuring condition of the plant

SM_i = state variable measure soil moisture, $0 \leq SM_i \leq FC$

ET_i = ET in period i which is calculated as a function of SM_i and IRR_i

IRR_i = decision variable representing depth of irrigation applied at decision stage i, $0 \leq IRR_i \leq FC - SM_i$

OCF_{i+1} = optimal multiplicative coefficient for the remainder of the season

$OAIRR_{i+1}$ = optimal amount of irrigation for the remainder of the season

$ODEL_{i+1}$ = optimal number of irrigations for the remainder of the season

ET_{max_i} = maximum ET which can occur during period i.

Two recursive relations are required by this formulation. The first is used to update soil moisture and is given by

$$SM_{i+1} = (SM_i + IRR_i)e^{-b_i t} \quad (3.22)$$

The second is used to update the crop growth state variable. It is given by

$$A_{i+1} = A_i \left(\frac{ET_i}{ET_{max_i}} \right)^{\lambda_i} \tag{3.23}$$

It should be noted that a problem arises in distinguishing between growth stages and irrigation decision stages. At the fourth irrigation, the ET value for the entire third growth stage is required. Thus, although it is referred to as ET_4 it signifies the ET during the third growth stage. Similarly for the first irrigation, the ET value for the entire first growth stage is required. In order to obtain this value, the previous soil moisture level SM_{i-1} is calculated. For the fourth irrigation decision period it is calculated as

$$SM_3 = SM_4 \ / \ e^{-b_3 t_3} \tag{3.24}$$

in which b_3 = Yaron coefficient for the third growth stage
t_3 = number of days between third and fourth irrigation.

Final soil moisture SM_5 is calculated by the soil moisture recursive relation, Equation 3.22, and ET is obtained by

$$ET_4 = SM_3 + IRR_4 - SM_5 \tag{3.25}$$

In the third irrigation decision stage, again the ET value required is that for the entire growth stage. It is calculated by the following:

$$ET_3 = SM_3 + IRR_3 + OAIRR_4(SM_4, A_3) - (SM_4 + OAIRR_4(SM_4, A_3))e^{-b_3 t_4} \tag{3.26}$$

The implication of calculating ET for the entire growth stage is that $OCF_i(\quad)$ is unity in both the third and fourth irrigation

decision stages. The overlapping of irrigation decision stages requires the foregoing complications and limits applying the multiplicative production function to more general cases.

Additive Case, Fixed Dates

The program OPT TIM was modified to evaluate the additive ET production function instead of the multiplicative relation. This was accomplished with minor changes in the program. The same assumption of fixed irrigation dates was made. The values of the production function coefficients used were those obtained from Run 6A (see Table 2.5), proportionately scaled to add to one.

Additive Case, Flexible Dates

Another program, Program OPT DAY (see Appendix D) was written employing the additive production function. The objective of this program was to determine optimal times and amounts of irrigation for a single crop when irrigation dates are completely flexible. With this program, water was not limited, the only constraints were the variable cost of irrigation water and the labor cost per irrigation.

The problem is formulated as a one-state variable dynamic program. The formulation is essentially that of Equation 3.17 with x_i representing soil moisture and u_i representing the irrigation amount on day i, and $\phi = 0$. The daily return function f_i is calculated as

$$f_i(x_i, u_i) = a_i b_i(x_i + u_i) - Bu_i - C\Delta_i \qquad (3.27)$$

in which

x_i = available soil moisture, cm; $x_{i+1} = (x_i + u_i)(1-b_i)$

u_i = depth of irrigation applied to root zone (cm.)

B = variable cost of irrigation water

$\Delta_i = 0$ if $u_i = 0$, 1 otherwise

b_i = Yaron coefficient, $b_i(x_i + u_i) = ET_i$

a_i = dollar value of a unit of ET

The coefficients a_i are obtained from values of the additive production function (Run 6A, Table 2.5), number of days in the time period, maximum calculated ET during the time period (obtained from Yaron coefficients), and the expected price and maximal yields of the crop. The expression, in which i represents the index of the growth stage is

$$a_i = \frac{A_i \cdot Y_{max} \cdot P}{ETMAX_i} \tag{3.28}$$

in which

$$ETMAX_i = SMI \cdot b_i \cdot NDAY_i$$

and

a_i = dollar value of a unit of ET in growth stage i (\$/cm.)

A_i = percentage contribution of period i to yield (from Table 2.5)

Y_{max} = maximum yield (bu./acre)

P = price (\$/bu.)

$ETMAX_i$ = maximum calculated ET in period i (cm.)

SMI = SM at field capacity (cm.)

b_i = Yaron coefficient ($b_i \cdot$ SMI) represents max ET for any day in period i

$NDAYS_i$ = number of days in growth stage i.

The values of a_i obtained were

a_1 = \$1.43/cm.

a_2 = \$7.02/cm.

a_3 = \$4.84/cm.

Data for the Representative Farm

Economic Data

A detailed economic study of the Ft. Morgan, Colorado, irrigated area has been presented by Conklin (1974), based on data for 1973. The author distinguishes between three representative farm sizes and two types of operation: feed crop and cash crop. The cash crop farm of intermediate size (Size Category II, 280 acres) has been selected for the analysis of this study.

Tables 3.5-3.8 present variable and fixed cost data updated to 1974 (Conklin, 1975). These data are for cash crop production of corn, alfalfa, sugar beets, and pinto beans on Category II farms. Calculations of net returns to management are presented in Table 3.9 based on the 1974 prices and costs.

For the analysis of this chapter, water is seen as a variable input. In the dynamic programs described earlier in this chapter, a coefficient a was needed which represented gross return under conditions of maximum yield. The coefficient is obtained from Column 4 of Table 3.9.

The dynamic program determines gross return less irrigation quantity and irrigation labor costs. Still to be subtracted are other variable costs and fixed costs. The value of other variable costs and fixed costs is obtained by subtracting the sum of columns 7 and 8 from column 5 in Table 3.9. This calculation is required since column 5 includes water and labor charges which are already calculated by the dynamic program.

Table 3.5

COSTS AND RETURNS FOR SUGAR BEETS
ON 280 ACRE CASH CROP FARM, 1974
(From Conklin, 1975)

	Unit	Price or Cost/Unit	Quantity Per Acre	Value or Cost/Acre
1. Gross Return from Production				
Sugar Beets	TON	$30.00	19.0	$570.00
Total Receipts				$570.00
2. Variable Costs				
Fertilizer				
N	LB	0.22	120.0	26.40
P	LB	0.16	120.0	19.20
Seed	LB	5.50	2.0	11.00
Mildew Spray	ACRE	8.00	1.0	8.00
Herbicide	ACRE	9.00	1.0	9.00
Machinery Use				
Tractor Fuel and Lube				13.85
Tractor Repairs				5.30
Equipment Fuel and Lube				4.72
Equipment Repairs				12.85
Pump Operating Costs	AC-FT	3.00	1.5	4.50
Custom Work				
Fertilizer Spreader	ACRE	2.00	1.0	2.00
Labor - Machine Operation	HR	2.25	8.3	18.68
Irrigation	HR	2.25	3.5	7.88
Other - Thinning	ACRE	25.75	1.0	25.75
- Weeding	ACRE	14.00	1.0	14.00
Itemized Variable Costs				183.13
Miscellaneous Variable Costs				18.31
Interest on Operating Capital				12.09
Total Variable Costs				213.53
3. Income Above Variable Costs				356.47
4. Overhead Charge				19.17
5. Machinery and Equipment Ownership Fixed Costs (Deprec., Taxes, and Insurance)				
Tractor				11.04
Equipment				43.38
Total Machinery and Equipment Fixed Costs				54.42
6. Returns to Land, Risk, Management, and Capital				282.88
7. Land Charge				82.00
8. Interest on Machinery, Equipment and Overhead				31.56
9. Returns to Risk and Management				169.32

Table 3.6

COSTS AND RETURNS FOR PINTO BEANS ON 280 ACRE CASH CROP FARM, 1974 (From Conklin, 1975)

	Unit	Price or Cost/Unit	Quantity Per Acre	Value or Cost/Acre
1. Gross Return from Production				
Pinto Beans	CWT	$20.00	20.0	$400.00
Total Receipts				$400.00
2. Variable Costs				
Fertilizer				
Seed	CWT	40.00	0.6	24.00
Insecticide				
Herbicide	ACRE	7.00	1.0	7.00
Machinery Use				
Tractor Fuel and Lube				7.83
Tractor Repairs				2.92
Equipment Fuel and Lube				0.59
Equipment Repairs				3.08
Pump Operating Costs	AC-FT	3.00	0.67	2.00
Custom Work				
Fertilizer Spreader				
Apply & Incorporate Herbicide	ACRE	5.00	1.0	5.00
Combine	BU	0.50	33.3	16.66
Labor - Machine Operation	HR	2.25	3.8	8.55
Irrigation	HR	2.25	2.5	5.62
Other				
Itemized Variable Costs				83.25
Miscellaneous Variable Costs				8.32
Interest on Operating Capital				5.50
Total Variable Costs				97.07
3. Income Above Variable Costs				302.93
4. Overhead Charge				19.17
5. Machinery and Equipment Ownership Fixed Costs (Deprec., Taxes, and Insurance)				
Tractor				6.14
Equipment				12.24
Total Machinery and Equipment Fixed Costs				18.38
6. Returns to Land, Risk, Management, and Capital				265.38
7. Land Charge				82.00
8. Interest on Machinery, Equipment and Overhead				13.76
9. Returns to Risk and Management				169.62

Table 3.7

COSTS AND RETURNS FOR ALFALFA HAY
ON 280 ACRE CASH CROP FARM, 1974
(From Conklin, 1975)

	Unit	Price or Cost/Unit	Quantity Per Acre	Value or Cost/Acre
1. Gross Return from Production				
Alfalfa Hay (baled)	TON	$ 55.00	4.8	$264.00
Total Receipts				264.00
2. Variable Costs				
Phosphate (Every 3 yrs)	LB	0.16	30.0	4.80
Insecticide (Every 2 yrs)	ACRE	6.00	0.5	3.00
Machinery Use				
Tractor Fuel and Lube				
Tractor Repairs				
Equipment Fuel and Lube				
Equipment Repairs				
Pump Operating Costs	AC-FT	3.00	1.83	5.49
Custom Work				
Fertilizer Spreader(Every 3 yrs)	ACRE	2.00	0.33	0.67
Swath	ACRE	4.50	3.0	13.50
Bale and Swath	TON	9.00	4.8	43.20
Labor - Machine Operation				
Irrigation	HR	2.25	2.5	5.62
Other				
Itemized Variable Costs				76.28
Miscellaneous Variable Costs				7.63
Interest on Operating Capital				5.03
Total Variable Costs				88.94
3. Income Above Variable Costs				175.06
4. Overhead Charge				19.17
5. Machinery and Equipment Ownership Fixed Costs (Deprec., Taxes, and Insurance)				
Tractor				
Equipment				3.86
Total Machinery and Equipment Fixed Costs				3.86
6. Returns to Land, Risk, Management, and Capital				152.03
7. Land Charge				82.00
8. Interest on Machinery, Equipment and Overhead				4.36
9. Returns to Risk and Management				65.67

Table 3.8

COSTS AND RETURNS FOR CORN GRAIN
ON 280 ACRE CASH CROP FARM, 1974
(From Conklin, 1975)

	Unit	Price or Cost/Unit	Quantity Per Acre	Value or Cost/Acre
1. Gross Return from Production				
Corn	BU	$ 2.50	130.0	$325.00
Stalks for Grazing	ACRE	10.00	1.0	10.00
Total Receipts				$335.00
2. Variable Costs				
Fertilizer				
N(Dry)	LB	0.22	50.0	11.00
P(Dry)	LB	0.16	50.0	8.00
N(NH_3)	LB	0.17	100.0	17.00
Seed	BU	27.00	0.33	9.00
Insecticide	ACRE	3.00	1.0	3.00
Herbicide	ACRE	10.00	1.0	10.00
Machinery Use				
Tractor Fuel and Lube				10.41
Tractor Repairs				3.72
Equipment Fuel and Lube				
Equipment Repairs				2.77
Pump Operating Costs	AC-FT	3.00	1.0	3.00
Custom Work				
Fertilizer Spreader	ACRE	2.00	1.0	2.00
Labor - Machine Operation	HR	2.25	5.8	13.05
Irrigation	HR	2.25	2.5	5.62
Other				
Itemized Variable Costs				98.57
Miscellaneous Variable Costs				9.86
Interest on Operating Capital				6.51
Total Variable Costs				114.94
3. Income Above Variable Costs				220.06
4. Overhead Charge				19.17
5. Machinery and Equipment Ownership Fixed Costs (Deprec., Taxes, and Insurance)				
Tractor				9.18
Equipment				15.10
Total Machinery and Equipment Fixed Costs				24.28
6. Returns to Land, Risk, Management, and Capital				176.61
7. Land Charge				82.00
8. Interest on Machinery, Equipment and Overhead				17.72
9. Returns to Risk and Management				76.89

Table 3.9

CALCULATION OF NET RETURNS PER ACRE
FOR THE REPRESENTATIVE FARM

Crop	Unit	Prices $/Unit	Average Yields Unit/ Acre	Maximum Gross Return $	Variable and Fixed Costs	Net Returns to Management	Irrigation Costs	Irrigation Labor Costs	Net Returns Exclusive of Irr. Water and Labor Costs
Sugar Beets	ton	30	19.0	570	400.68	169.32	7.00	7.88	184.20
Pinto Beans	cwt.	20	20.0	400	230.38	169.62	4.50	5.62	179.74
Alfalfa	ton	55	4.8	264	198.33	65.67	7.99	5.62	79.28
Corn Grain	bu	2.50	140*	350	258.11	91.89	5.50	5.62	103.01

Column 7 is the sum of Pump Operating Costs from Tables 3.5-3.8 and $2.50/acre surface water charge.

Column 9 is the sum of columns 6, 7 and 8.

* Maximum yield per acre

The model also requires a coefficient b which represents cost per acre cm. of water delivered to the root zone. An average cost of irrigation water from groundwater and surface water was initially set at $2.50/acre foot. Assuming a 50% efficiency to the plant root zone results in an average cost of $5.00/acre foot for water at the root zone, converting to the needed units results in the b value used, $0.16/acre cm.

A third coefficient c is used by the programs to represent the labor cost per irrigation, assumed to be independent of quantity of water applied. That is, if any irrigation water is applied, a flat cost is incurred. The value used was $2.50, which was included under Miscellaneous Variable Costs in Tables 3.5-3.8.

On-farm water constraints and other economic data are discussed in Chapter IV.

Agronomic Data

The description of field trials for corn conducted at Colorado State University in 1974 is presented in Appendix A. It was necessary to define phenological growth stages according to the set measurement dates. Three stages were defined based on the hypothesis that the pre-silking period is critical for corn production. The stages are:

(1) Germination through Vegetative growth (from May 15 to July 16).

(2) Early Silking (July 16-July 23).

(3) Silking through Maturity (July 23-September 11).

Maturity to harvest (September 11, the date of last soil moisture observation, to mid-October) was neglected. Values of field capacity by layers are given in Table 3.10.

Table 3.10
AVAILABLE SOIL MOISTURE BY LAYERS
FOR CSU AGRONOMY FARM

Layer	Available Soil Moisture (cm. of water/ft. of soil)
1	3.74
2	5.30
3	5.27
4	4.88
5	6.00
6	3.76
7	4.21
TOTAL	33.20

For each of the 48 treatments, ET was calculated according to the mass balance equation, Equation 1.3, using the original neutron probe data, neglecting runoff and deep percolation. The assumptions of measuring ET by this method are discussed in Appendix A. The results of the ET calculations are presented in Table 3.11. A number of treatments have been deleted where ET calculations have resulted in negative values or values greater than pan evaporation during the same time period. Obviously, ET calculated by this method is crude, with errors introduced by unmeasured water inputs, instrument error, and/or user error.

Application of the Models to the Representative Farm

Water Unlimited Case

With water unlimited, the three dynamic programs were run using the data of the representative farm. With fixed irrigation days, the multiplicative and additive production functions gave the same

Table 3.11

CALCULATED ET BASED ON SOIL MOISTURE MEASUREMENTS IN THREE TIME PERIODS FROM FIELD TRIALS WITH CORN AT COLORADO STATE UNIVERSITY, 1974 (cm.)

Run	T+mt	July 1-July 16	July 17-July 23	July 24-Sept. 11	Yield (kg/ha)
1	I-1	1.52	1.99	6.83	4,544
2	I-12	2.20	5.13	10.54	4,723
3	I-3	3.14	5.87	10.00	7,547
4	I-10	8.93	3.22	13.70	7,322
5	I-4	7.60	3.01	13.93	8,995
6	I-5	4.29	7.65	18.83	8,820
7	I-6	13.60	2.78	22.79	8,958
8	I-7	15.26	5.27	20.89	8,569
9	II-11	3.22	5.13	11.76	5,147
10	II-3	8.73	1.35	12.51	6,827
11	II-10	9.76	2.29	9.96	6,210
12	II-4	4.04	10.22	7.98	8,059
13	II-9	16.27	2.41	9.88	7,674
14	II-5	9.44	13.05	6.91	8,059
15	II-8	14.57	8.30	4.85	8,415
16	III-2	1.38	2.71	10.17	3,563
17	III-10	7.88	2.39	14.37	6,661
18	III-4	9.89	1.80	15.48	6,370
19	III-8	11.00	3.31	18.83	8,748
20	III-6	8.05	18.60	4.22	9,083
21	IV-11	5.03	2.25	11.02	5,439
22	IV-5	2.47	8.35	4.01	3,515
23	IV-7	0.83	7.71	10.27	7,260
Pan Evaporation		10.22	4.64	28.58	

results: irrigate to field capacity at each irrigation. The OPT DAY program, with flexible irrigation dates, indicated nine irrigations, each time irrigating to field capacity. The results of these runs are summarized in Table 3.12.

It should be noted that these results are for quantities of water delivered to the root zone under conditions of zero rainfall. These values depend upon the assumptions of maximum yield of 140 bu./acre, price of $2.50/bu., water cost of $2.43/acre foot at the headgate with 50% efficiency to the root zone, and labor cost of $2.50/irrigation. The conversions to acre inches at the headgate are shown in Table 3.12 in parentheses.

Higher return under the flexible irrigation system would be expected; however, slightly lower return was obtained. This is probably explained by approximation occurring within the programs. The program OPT DAY calculates ET on a daily basis using Equation 3.1, whereas OPT TIM calculates ET over longer periods using the exponential form, Equation 3.3.

The program OPT DAY was rerun using a labor cost of $5 per irrigation. The number of irrigations is reduced to six as shown in Table 3.13. A rational farmer who has complete flexibility in setting the number of irrigations, who reports a labor cost of $2.50/irrigation, and who chooses to irrigate less than six times instead of the optimal nine, reveals a shadow price of labor greater than $5 per irrigation.

The additive ET--fixed date program was rerun to determine the marginal value of water (see Table 3.13). The cost of water was

Table 3.12

RESULTS OF PROGRAMS WITH FIXED
AND FLEXIBLE IRRIGATION SCHEDULES

Fixed Irrigation Schedule

Date	Depth of Water to Root Zone (cm.) Multiplicative	Additive	Quantity of Water at Headgate (acre in.)
June 15	12	12	(9.4)
July 17	14	14	(11.0)
July 24	4	4	(3.1)
Aug. 18	10	10	(7.9)
TOTAL	40	40	(30.4)
RETURN	$71.23	$71.16	

Flexible Irrigation Schedule

Date	Depth of Water to Root Zone (cm.)	Quantity of Water at Headgate (acre in.)
June 1	8	(6.3)
June 19	8	(6.3)
July 2	6	(4.7)
July 14	6	(4.7)
July 19	4	(3.1)
July 29	6	(4.7)
Aug. 6	4	(3.1)
Aug. 14	4	(3.1)
Aug. 23	4	(3.1)
	50 cm.	39.1
RETURN	$70.29	

Table 3.1:

OPTIMAL IRRIGATIONS FOR VARIOUS
WATER AND LABOR COSTS

Fixed Irrigation Schedule

Run	Water Cost $/acre ft. at Headgate	Water Cost $/acre cm. at Root Zone	Labor Cost $/Irr.	Optimal Irrigations (cm. at root zone)				Return $
1	2.43	0.16	2.50	12	14	4	10	(71.16)
2	9.72	0.64	2.50	12	14	4	10	(51.95)
3	14.58	0.96	2.50	10	14	4	10	(39.19)
4	20.00	1.28	2.50	0	20	4	10	(27.76)
5	2.43	0.16	5.00	12	14	4	10	(61.16)
6	2.43	0.16	7.50	12	14	0	12	(50.89)

Flexible Irrigation Schedule

Run	Water Cost $/acre ft. at Headgate	Water Cost $/acre cm. at Root Zone	Labor Cost $/Irr.	Irrigations Day	Amount	Return $
7	2.43	0.16	5.00	30	12	(61.66)
				61	14	
				76	8	
				84	4	
				92	4	
				103	6	
				TOTAL	48 cm.	

increased until no longer was the optimal policy to irrigate fully in each time period. This policy occurred for water costs between $0.64 and $0.96/acre cm. at the root zone which is equivalent to between $10 and $15/acre foot at the headgate.

The exact marginal value was not found, but assume it was $0.75/acre cm. At this cost of water, the added return due to the added 2 cm. of water equals the cost of the added water. This figure represents an approximation to the marginal value of water in the first time period, since this was the time period where the irrigation was not to field capacity. The marginal value of water in the other time periods is greater than $20/acre foot, as seen from Table 3.13.

The same program was rerun to test the sensitivity of the optimal policy to the labor cost per irrigation. With the existing crop price and water cost, if the labor cost increases to $7.50/irrigation, the optimal policy changes. Under this cost structure the third irrigation is skipped, with the remaining irrigations to field capacity.

The additive ET--fixed date irrigation program was selected for this and further analysis because it was thought to most adequately model the actual system. The additive relation gives a better fit to the field trial data and fixed irrigation dates are more representative of the application site, though more frequent irrigations are common practice.

All of the previous results have been for the static root zone model. The additive ET, fixed date program was modified to include a dynamic root zone model in order to test the sensitivity of the original model to changes in assumptions regarding the root zone.

The results are shown in Table 3.14. Significant differences from the original model were obtained. In further studies, the dynamic model should be included, particularly if more growth stages are included. With only three growth stages, the large increase in the root zone volume between periods one and 2, which results in addition of a large quantity of available soil moisture, is not a very realistic situation.

Table 3.14

COMPARISON OF RESULTS WITH DYNAMIC VERSUS STATIC ROOT ZONE MODEL

	Optimal Irrigation Requirements at root zone (Cm.)				Return
	1	2	3	4	
Dynamic	6	6	4	12	79.80
Static	12	14	4	10	71.16

Water Limited Case

The final computations for this chapter are the determination of optimal water use coefficients for corn under deficient water supply in specific time periods due to competition with other crops. In these runs, water availability was reduced in 2 cm. increments to a total of 8 cm. (6.3 acre inches at the headgate). In this range, every combination of water shortage in the various time periods was evaluated. In Chapter IV, the results are linearized and a good fit is required in the neighborhood of full irrigations. Thus the relatively small amounts of deficits are satisfactory for the analysis. For deficits greater than 8 cm., several additional calculations were made.

Table 3.15 gives the results of the water deficit runs. In run 2B, for example, the assumption is that water is short at the second irrigation so that the farmer can irrigate only to within 2 cm. of field capacity. In the other time periods the program selects the optimal quantities of water (with no limitation) to maximize return. The seasonal water use is 40 cm. since 2 additional cm. are used in period 3. The net return is $71.16 and the return net of all costs except for variable irrigation costs is $77.56. The last figure is needed for the optimization of Chapter III, where water costs are calculated by the linear program.

Another example is run 7 with 6 units short and the pattern 4/2 indicating 4 cm. short in period 2 and 2 cm. in period 3. Again, the interpretation is that the water supply is deficient so that the farmer lacks by 4 cm. of being able to refill the root zone to field capacity in period 2, and by 2 units of being able to fill it in period 3. Note that the total quantity applied in period 3 is 6 cm., whereas in the water unlimited case all that was required was 4 cm. to refill the root zone to field capacity. This method of generating water use coefficients, though generating some repetition and some results which appear unreasonable, insures every possible combination of water use coefficient.

Summary

This chapter has presented a thorough analysis of the single crop situation. Field trial data have been used to fit parameters in an ET estimation model and to fit parameters to two yield prediction models based on ET in three growth stages.

Table 3.15

IRRIGATION AMOUNTS AND EXPECTED RETURNS FOR DEFICIENT WATER SUPPLY

Run	Short	Irrigation (cm.) 1	2	3	4	Total	Net Return With 0. Irr. Cost
1	Full	12	14	4	10	40	77.23
2A	2	10	14	4	10	38	73.97
2B	2	12	12	6	10	40	74.78
2C	2	12	14	2	12	40	73.14
2D	2	12	14	4	8	38	71.44
3A	4	8	16	4	10	38	72.46
3B	4	12	10	8	10	38	71.56
3C	4	12	14	0	12	38	65.75
3D	4	12	14	4	6	36	65.65
4A	2/2	10	12	6	10	38	70.75
4B	2/2	10	14	2	12	38	69.89
4C	2/2	10	14	4	8	38	68.75
4D	2/2	12	12	4	10	38	69.80
4E	2/2	12	12	6	8	38	69.56
4F	2/2	12	14	2	8	36	66.79
5A	6	6	16	6	10	38	66.31
5B	6	12	8	8	10	38	68.33
5C	6	12	14	0	12	38	65.75
5D	6	12	14	4	4	34	59.86
6A	8	0	20	4	10	34	64.56
6B	8	12	6	10	10	38	65.12
6C	8	12	14	0	12	38	65.75
6D	8	12	14	4	2	32	54.07
7A	4/2	8	14	4	10	36	70.01
7B	4/2	8	16	2	12	38	68.37
7C	4/2	8	16	4	8	36	66.67
7D	4/2	10	10	8	10	38	67.52
7E	4/2	12	10	6	10	38	66.59
7F	4/2	12	10	8	8	38	66.33
7G	4/2	10	14	0	12	36	65.50
7H	4/2	12	12	0	14	38	64.11
7I	4/2	12	14	0	10	36	59.96
7J	4/2	10	14	4	6	34	62.40
7K	4/2	12	12	6	6	36	63.77
7L	4/2	12	14	2	8	36	61.56
8A	6/2	6	14	6	10	36	65.98
8B	6/2	6	16	4	10	36	65.12
8C	6/2	6	16	6	8	36	63.98
8D	6/2	10	8	10	10	38	64.30
8E	6/2	12	8	6	12	38	63.35
8F	6/2	12	8	8	8	36	62.54
8G	6/2	10	14	0	12	36	62.50
8H	6/2	12	12	0	14	38	62.38

Table 3.15 (Continued)

Run	Short	Irrigation (cm.) 1	2	3	4	Total	Net Return With 0 Irr. Cost
8I	6/2	12	14	0	10	36	59.96
8J	6/2	10	14	4	4	32	58.34
8K	6/2	12	12	6	4	34	57.98
8L	6/2	12	14	2	6	34	55.77
9A	4/4	8	12	8	10	38	67.35
9B	4/4	8	16	0	12	36	60.98
9C	4/4	8	16	4	6	34	60.88
9D	4/4	12	10	2	14	38	58.39
9E	4/4	12	10	8	6	36	60.54
9F	4/4	12	14	0	8	34	54.19
10A	2/2/2	12	12	4	8	36	64.01
10B	2/2/2	10	14	2	10	36	64.10
10C	2/2/2	10	12	6	8	36	64.96
10D	2/2/2	10	12	4	12	38	66.65
11	2/2/2/2	10	12	4	10	36	60.86
Other Runs							
12		10	14	2	2	28	39.23
13		10	10	0	0	20	4.94
14		0	14	0	0	14	-17.19
15		0	0	0	0	0	-97.79

The multiplicative and additive production functions were incorporated into dynamic programs to determine optimal water applications on four fixed irrigation dates. The results with the existing price of corn and cost of water and labor were the same with both production functions: to irrigate to field capacity at each irrigation.

A third program was written employing the additive production function without the restriction of fixed irrigation dates. With the same cost and price structure as previously used, nine irrigations were scheduled. When the price of labor was increased, the results were close to those of the previous program, thus indicating a shadow price of labor greater than the market price.

The additive production function program with fixed irrigation dates was adopted as most representative of the actual system being modeled. With this program the marginal value of water was determined to be between $10 and $15/acre foot early in the season, and greater than $20 an acre foot later in the season.

The models presented in this chapter employed a static root zone model. A dynamic root zone model was described and tested in one program. It was found that the results of the program are sensitive to the choice of root zone model and further research should be conducted.

Water use coefficients were determined under conditions of deficient supply. These coefficients provide alternative methods of production for corn under conditions of zero rainfall and with the constant climatic conditions as discussed in Chapter I. The introduction of rainfall, multiple crops, and multiple objectives will be analyzed in the next chapter.

Chapter IV

THE MULTIPLE CROP MODEL

Introduction

In this chapter the multiple crop problem is considered. The basic problem is to choose which crops to plant given an initial set of feasible crops and knowledge of returns with various quantities of water applied. In this chapter, precipitation is viewed as the only random variable. Other climate variables, crop prices, etc., are assumed known with certainty. Under these assumptions, the farmer is faced with a two objective problem: maximizing expected return from the crops and minimizing the variability of the return due to effects of random precipitation.

The general approach that has been adopted in this chapter is the passive programming approach as described in Appendix C. With this approach, a deterministic program is solved for each possible value of the random variables, in this case seasonal precipitation. For each policy determined by the deterministic program, a simulation model is run which calculates the expected return and variance under this policy. An EV curve is generated similar to Figure C.1. With knowledge of the farmer's utility curve or equivalently, with application of multiple objective techniques such as the surrogate worth trade off method, the optimal policy can be determined.

The interpretation of this policy, then, is that it is the quantity of precipitation which when included in a deterministic planning model, results in recommending crop acreages and irrigation timing which maximize the farmer's utility.

Two models are formulated and solved in this chapter. The first is a deterministic model which finds optimal crop acreages for given precipitation levels. This problem is formulated as a linear program similar to that of Young and Bredehoeft (1972).

The second is a simulation program which for each precipitation level, takes the optimal acreages and determines returns under the actual precipitation record for the site. At any given time period within any year, another linear program is solved which allocates water in the current time period based on actual rainfall to that date, forecast rainfall, and quantity of water remaining for the season. From this model, then, the expected return and variance for each policy is calculated.

Site Data

For the representative cash crop farm of the Ft. Morgan, Colorado, area, various assumptions have been made. Conklin (1974) states "typical" crop acreages of

Corn Grain	30 acres
Alfalfa	50 acres
Sugar Beets	50 acres
Pinto Beans	50 acres

Naturally, on any specific farm, various factors including availability of capital, machinery, and labor tend to dictate the crops grown.

Sugar beet production, for example, is limited to the capacity of nearby processing plants and before planting, the crop is contracted to the plant. For the entire survey region, Conklin reports, "Only 19 of the 89 farmers grow both beets and beans. Thirty-six farms raise only corn and hay. Machine costs are high enough to encourage farmers to specialize."

For the representative farm of this study, the above factors are assumed to work towards putting upper limits on the acreages of higher valued crops (beets and beans) and minimum acreages on the lowest valued crop, alfalfa.

Rainfall Data

Monthly precipitation data for the Ft. Morgan, Colorado, area during the irrigation season is presented in Table 4.1. The total average seasonal precipitation, based on 83 years of record, is 9.12".

In Table 4.2, precipitation amounts have been converted to cm. and adjusted to fit the time periods of the current study. Five time periods are considered. The third time period (July 16-25) corresponds to the critical silking period of corn. Entries in Table 4.2 have been obtained by multiplying entries in Table 4.1 by appropriate weights, assuming precipitation is distributed uniformly in any given month.

In Figure 4.1, a precipitation frequency analysis is performed based on the most recent 20 years of data. Seasonal precipitation values were placed in descending order, the ratio to the mean was calculated and plotted versus the plotting point.

$$T_r = \frac{n+1}{m} \qquad 4.1)$$

Table 4.1

IRRIGATION SEASON PRECIPITATION DATA
FOR FT. MORGAN, COLORADO
(Source: Climatological Data, Colorado,
U.S. Department of Commerce
Environmental Data Service)

Year	Month May	June	July	Aug.	Sept.	Seasonal Total (in.)
1955	3.65	2.60	1.97	1.36	2.06	11.64
1956	.59	2.78	6.70	1.36	.31	11.74
1957	4.50	1.97	2.02	1.79	.27	10.55
1958	2.17	4.04	1.97	.53	1.88	10.59
1959	4.46	.41	1.02	.55	.99	7.43
1960	2.23	.82	1.49	.21	1.11	5.86
1961	4.06	1.71	2.32	1.15	2.54	11.78
1962	3.79	3.00	3.56	.35	.42	11.47
1963	1.05	2.65	1.21	2.66	1.51	9.08
1964	1.63	2.54	.40	.76	.75	6.08
1965	1.26	3.33	5.36	1.15	2.00	13.10
1966	.52	2.67	.70	2.28	1.51	7.68
1967	6.53	3.94	1.74	1.10	.39	13.70
1968	2.00	1.56	.56	2.29	.18	6.59
1969	2.58	1.05	1.87	.07	.64	6.21
1970	.54	2.59	.54	2.13	.88	6.68
1971	.92	1.64	.40	.16	2.43	5.55
1972	.99	1.83	1.50	2.59	1.35	8.26
1973	2.21	1.22	1.93	.90	4.67	10.93
1974	.26	1.36	2.35	.34	.22	4.53
ong-term Average	2.56	2.13	1.90	1.45	1.08	9.12

Table 4.2

ADJUSTED PRECIPITATION DATA
FOR FT. MORGAN, COLORADO (cm)

Year	May 15-June 15	June 15-July 15	July 16-July 25	July 26-Aug. 19	Aug. 20-Sept. 30	Total
1	7.9	5.8	1.6	3.1	6.6	25.0
2	4.3	12.0	5.5	5.4	2.1	29.3
3	8.2	5.1	1.7	3.8	2.4	21.2
4	7.9	7.6	1.6	1.8	5.3	24.2
5	6.2	1.8	.8	1.4	3.1	13.3
6	3.9	2.9	1.2	1.1	3.0	12.1
7	7.3	5.1	1.9	2.9	7.6	24.8
8	8.6	8.3	2.9	2.3	1.4	23.5
9	4.7	4.9	1.0	4.7	6.5	21.8
10	5.3	3.7	0.3	1.4	2.7	13.4
11	5.8	11.0	4.4	4.4	612	3.18
12	4.1	4.3	0.6	3.9	6.1	19.0
13	13.3	7.2	1.4	2.6	2.1	26.6
14	4.5	2.7	0.5	3.8	2.7	14.2
15	4.6	3.7	1.5	1.0	1.7	12.5
16	4.0	4.0	0.4	3.6	4.3	16.3
17	3.3	2.6	0.3	0.4	6.3	12.9
19	4.4	4.0	1.6	2.3	12.7	25.0
20	2.1	4.7	1.9	1.7	0.9	11.3
Average	6.0	5.11	1.6	3.2	4.2	20.11

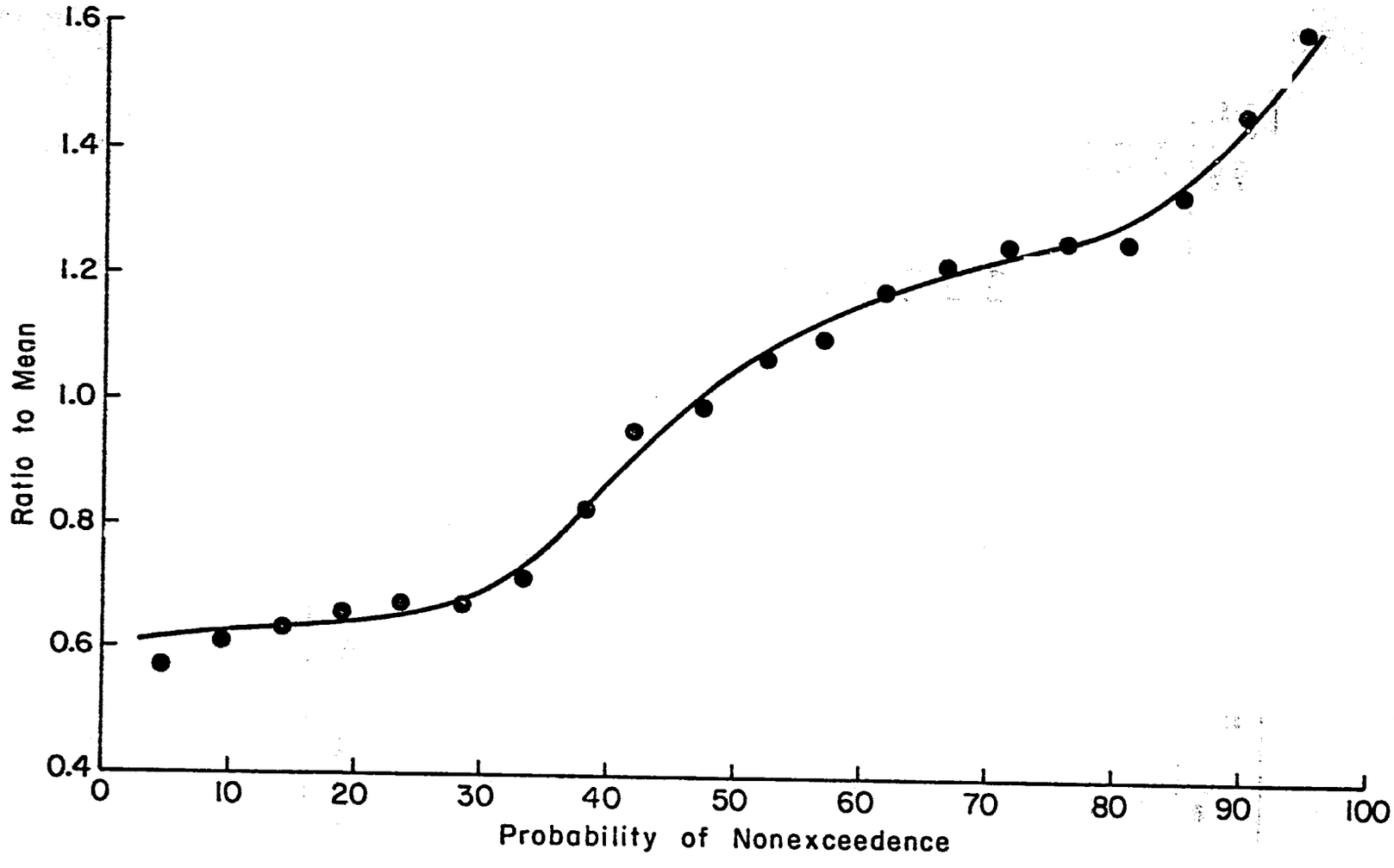


Figure 4.1

PRECIPITATION FREQUENCY ANALYSIS
FOR FT. MORGAN, COLORADO

in which

T_r = return period in years
m = rank of the observation
n = number of years of record.

Crop Water-Use Data

Crop water-use data for corn based on the field trials at Colorado State University is presented in Chapter III. The table of optimal returns with deficient supply (Table 3.15 is used as input to the deterministic linear program.

Data for irrigation of the remaining crops of the representative farm, particularly irrigating with deficient supply, is limited. Stegman, et al., (1965) present a summary of research data and suggest some values for ET rates and effects of stress for various crops. The results of the Stegman study will be employed in the current study, with the substitution of soybean data for the needed pinto bean data.

Table 4.3 presents the results of the Stegman study for the four crops of the representative farm. These values represent average daily ET under well-watered conditions. In Table 4.4 the ET rates have been adjusted by appropriate weights to conform to the time periods previously established.

Table 4.5 presents maximum daily ET rates. The data for corn is that from Table 3.1. Entries from Table 4.4 have been scaled upward by a factor 1.19 to convert from the "average well-watered conditions" to the maximum ET rate at Field Capacity for the application site. This factor was obtained by computing the ratio of the

ET corn results of Stegman with those of the current study during the peak ET time period and multiplying each entry by this ratio.

Table 4.3

AVERAGE DAILY ET BY
TIME PERIODS FOR FOUR CROPS
(From Stegman, et al., 1965)

Crop	ET (cm.) for the Given Days 1-30	31-60	61-90	91-120	121-150
Sugar Beets	.15	.30	.53	.53	.38
Beans	.15	.28	.53	.51	.25
Alfalfa	.30	.51	.48	.41	.33
Corn	.15	.28	.53	.51	.25

Table 4.4

AVERAGE DAILY ET ADJUSTED FOR
TIME PERIODS OF THE CURRENT STUDY (cm)

Crop	Dates: May 15-June 15	June 16-July 15	July 16-July 25	July 26-Aug. 19	Aug. 20-Sept. 30
	Days: 1-32	33-63	64-73	74-98	99-140
Sugar Beets	.16	.32	.53	.53	.45
Beans	.16	.30	.53	.52	.43
Alfalfa	.31	.51	.48	.45	.37
Corn	.16	.30	.53	.52	.43

Table 4.5

MAXIMUM DAILY ET FOR FOUR CROPS BY TIME PERIODS

Crop	Daily ET_{max} (cm.) for the Given Days 1-32	33-63	64-73	74-98	99-140
Sugar Beets	.19	.38	.63	.63	.53
Beans	.19	.36	.63	.62	.51
Alfalfa	.37	.61	.57	.53	.44
Corn	.50	.50	.63	.46	--

Table 4.6

MAXIMUM ET FOR FOUR FIXED IRRIGATIONS (CM.)

Crop	ET_{max} (cm.) for the Given Time Period Irrigation: 1 Day : 33	2 64	3 74	4 98
Sugar Beets	5.6	9.9	5.7	12.5
Beans	5.6	9.5	5.7	12.4
Alfalfa	10.0	14.4	5.2	10.9
Corn	12.7	12.4	5.7	9.7

Each entry of Table 4.5 is thus equivalent to the Yaron coefficients established for corn and employed in Equation 3.3. For each crop, values of maximum ET have been calculated according to Equation 3.3 based on four fixed irrigations and zero rainfall. These values are shown in Table 4.6. Note should be made that the values

obtained depend on the value used for depth of available moisture at Field Capacity. Lacking specific data for rooting depths of these crops, the value used in the calculation was that used for corn, 33.2 cm. This number is based on a rooting depth of 7' which is surely deeper than each of the other crops. The values in Table 4.6 thus represent an upper limit on ET rates.

Returns with Deficient Water Supply for Beets, Beans, and Alfalfa

For each of the crops other than corn, data were obtained from Stegman, et al., (1965) regarding the effect of "stress" on yield. Stress was not explicitly defined by the authors for some crops, but the assumption in this study is that each skipped irrigation induces a stress period.

For each crop, Tables 4.5 and 4.6 were used to determine ET and depth of water required to refill the root zone when one irrigation period is skipped. Again the Yaron relation, Equation 3.3, was used to determine these values. The first four columns of Tables 4.7-4.9 present these results. The first row represents maximum irrigation, the next rows show irrigation requirements when one irrigation is skipped. All values are in cm. at the root zone.

Column 5 in Tables 4.7-4.9 is the percentage of maximum yield obtained by each treatment. These values were obtained from the curves drawn by Stegman (1965), see Figures 4.2-4.4. For example, for pinto beans, using the soybean data, at day 64, the soil moisture is depleted by 13.5 cm. and a stress period is recorded. From Figure 4.2, the percentage of maximum yield obtainable is 64%. Similarly,

Table 4.7

CALCULATION OF RETURNS FOR SUGAR BEET PRODUCTION
UNDER DEFICIENT WATER SUPPLY

	1	2	3	4	5	6	7	8	9
Method of Production	Amount of Irrigation at Scheduled Times (cm) 1	2	3	4	% of Maximum Yield	Gross Return	Fixed and Variable Costs	Net Return with Zero Irrigation Costs	Net Return with Zero Water Cost
1	5.6	9.9	5.7	12.5	100	570.00	385.80	184.20	174.20
2	0.0	13.8	5.7	12.5	97	522.90	385.80	167.10	159.60
3	5.6	0.0	13.9	12.5	97	522.90	385.80	167.10	159.60
4	5.6	9.9	0.0	16.1	96	547.20	385.80	161.40	153.90
5	5.6	9.9	5.7	0.0	88	501.60	385.80	115.80	108.30

Table 4.8

CALCULATION OF RETURNS FOR PINTO BEAN PRODUCTION
UNDER DEFICIENT WATER SUPPLY

Method of Production	1	2	3	4	5	6	7	8	9
	Scheduled Irrigations				% of Maximum Yield	Gross Return $	Fixed and Variable Costs	Net Return with Zero Irrigation Costs	Net Return with Zero Water Cost
	1	2	3	4					
1	5.6	9.5	5.7	12.4	100	400	220.26	179.74	169.74
2	0.0	13.5	5.7	12.4	80	320	220.26	99.74	92.24
3	5.6	0.0	13.6	12.4	64	256	220.26	35.74	28.24
4	5.6	9.5	0.0	16.0	62	248	220.26	27.74	20.24
5	5.6	9.5	5.7	0.0	67	268	220.26	47.74	40.24

Table 4.9

CALCULATION OF RETURNS FOR ALFALFA PRODUCTION UNDER DEFICIENT WATER SUPPLY

Method of Production	1	2	3	4	5	6	7	8	9
	Scheduled Irrigations				% of Maximum Yield	Gross Return $	Fixed and Variable Costs	Net Return with Zero Irrigation Costs	Net Retur with Zero Water Cost
	1	2	3	4					
1	10.0	14.4	5.2	10.9	100	264.00	184.72	79.28	69.28
2	0.0	20.1	5.2	10.9	87	229.68	184.72	44.96	37.46
3	10.0	0.0	15.8	10.9	100	264.00	184.72	79.28	71.78
4	10.0	14.4	0.0	14.4	100	264.00	184.72	79.28	71.78
5	10.0	14.4	5.2	0.0	77	203.28	184.72	18.56	11.06

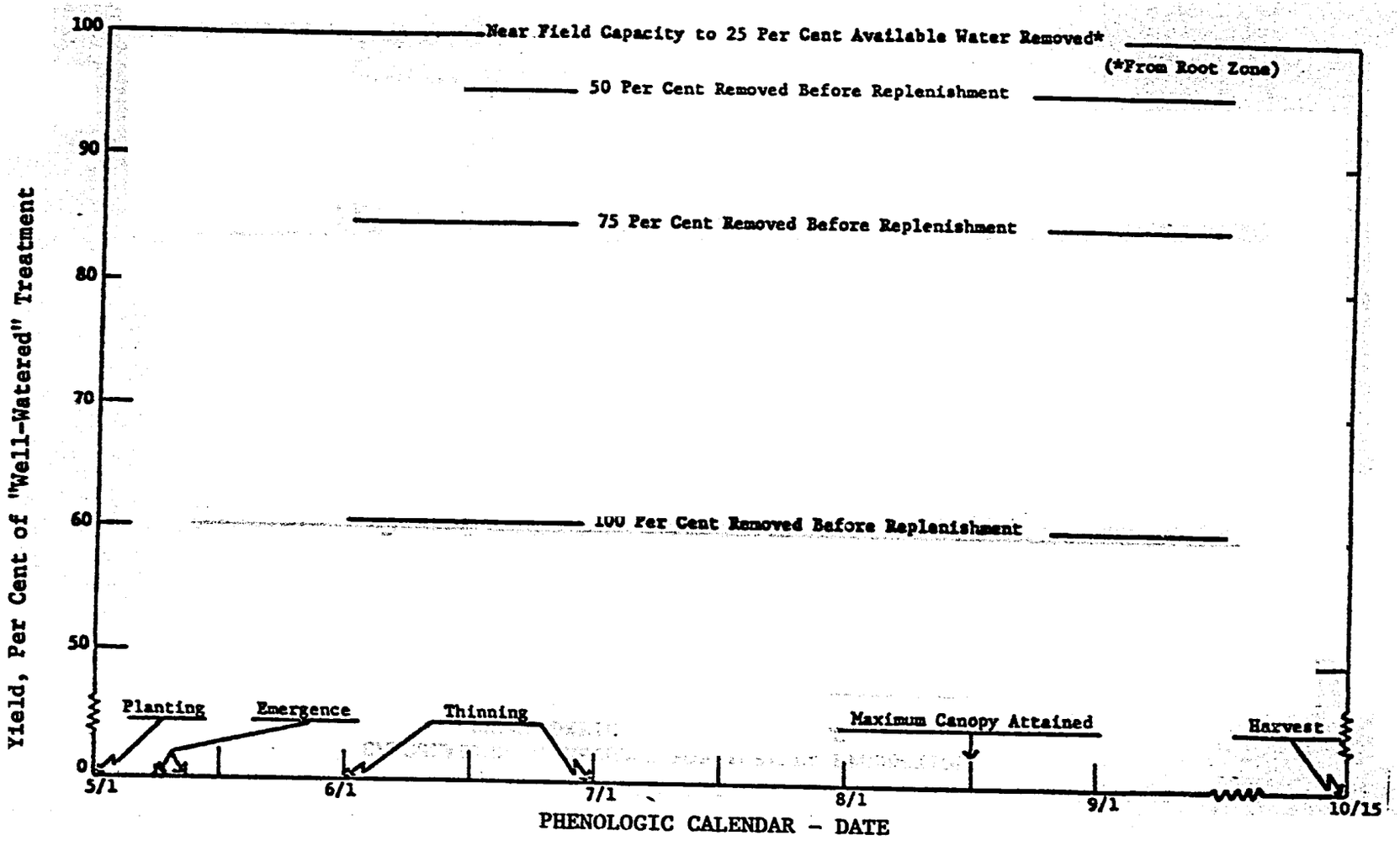


Figure 4.2

EFFECT OF SOIL WATER REGIME ON SUGAR BEET YIELD
(From Stegman, 1965)

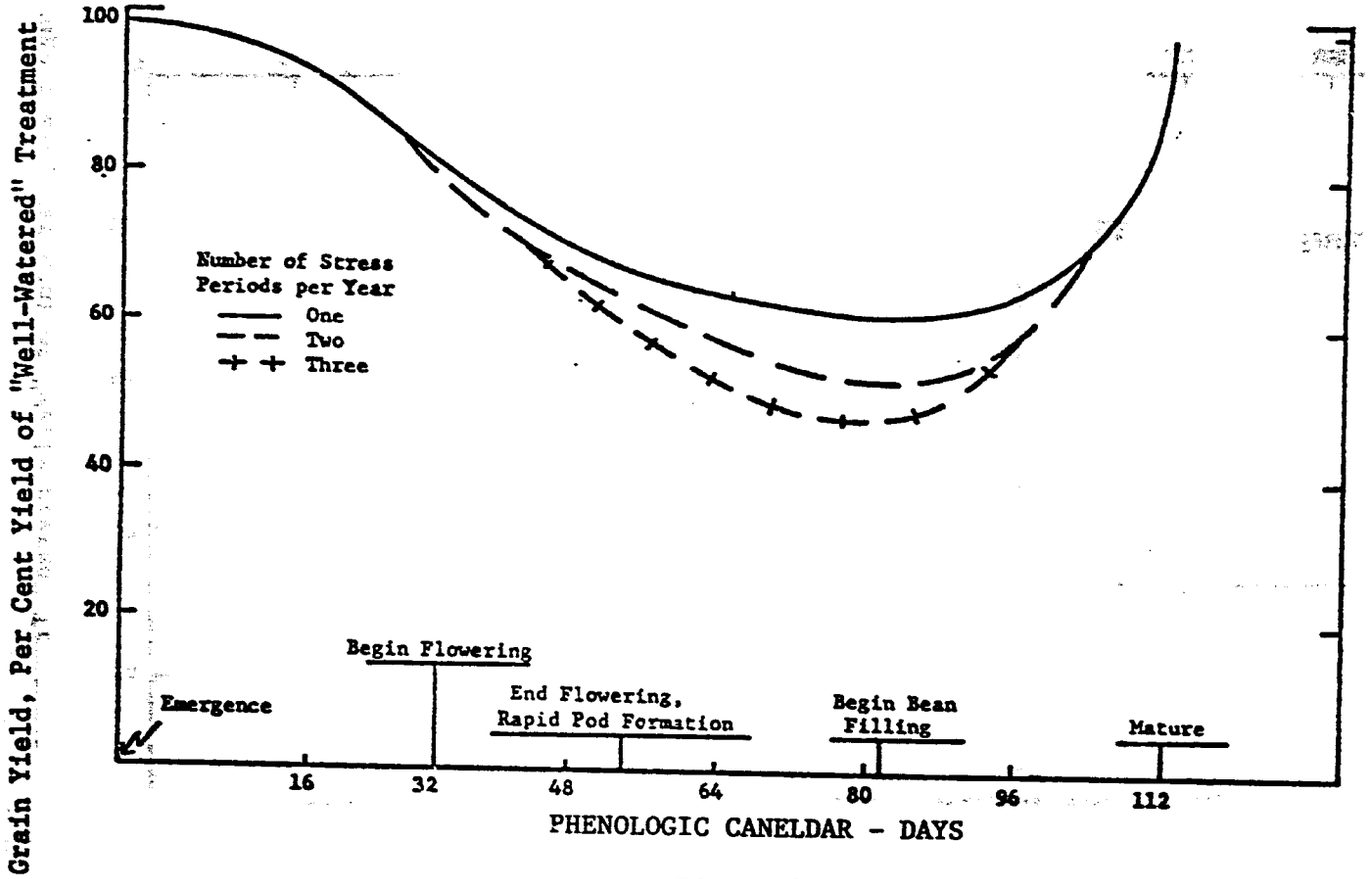


Figure 4.3

EFFECT OF MOISTURE STRESSES AT VARIOUS GROWTH STAGES ON GRAIN YIELD REDUCTION OF SOYBEANS (From Stegman, 1965)

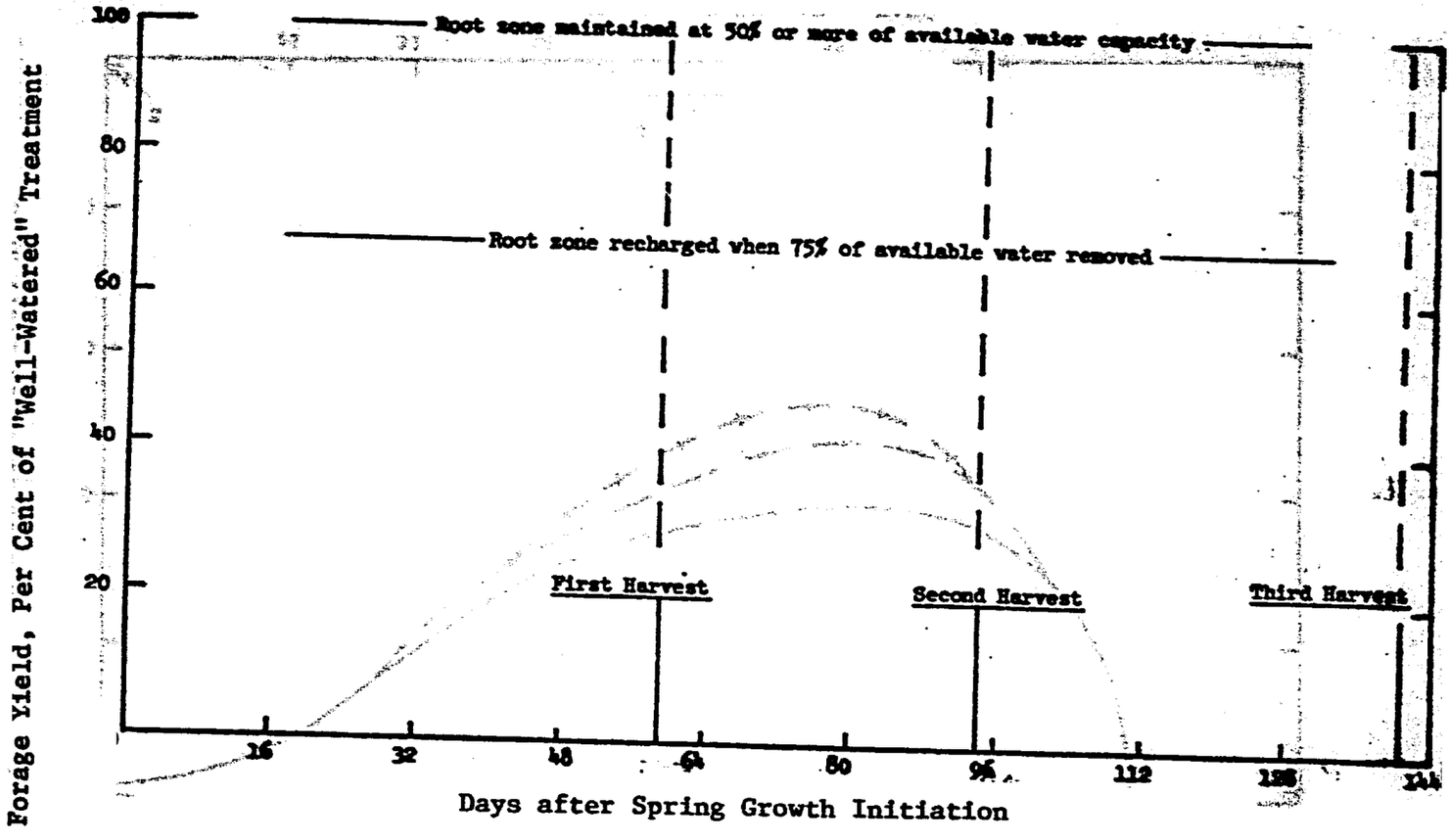


Figure 4.4

EFFECT OF EXTENT OF WATER DEPLETION OF ROOTING ZONE
BEFORE RE-CHARGE ON ALFALFA FORAGE YIELD
(From Stegman, 1965)

for sugar beets, skipping the first irrigation results in a depletion of 13.8 cm. or 42% of available soil moisture. Interpolation from Figure 4.1 results in a reduction in yield to 97% of maximum.

Column 6 of Tables 4.4-4.9 represents gross returns based on the maximum yields and prices given in Table 3.9 and the percentages of maximum yield in Column 5. Column 7 is the sum of fixed and variable costs assuming zero cost for irrigation water and labor. Column 8 is the net return for each method of production without irrigation labor or water charges. Column 9 is the net return including irrigation labor charge which is based on $2.50 per acre per irrigation. Column 9, then, is the net return with zero irrigation water charge. The values in Column 9 are input to the programs described later in this chapter.

Formulation of Linear Program to Determine Crop Acreages

The first model described in this chapter is a deterministic linear program for obtaining crop acreages which maximize return with a limited water supply. The program has been formulated considering one source of supply although at the application site, surface water and groundwater are available. The formulation is simplified, but the results are equivalent to the actual situation in which surface water supplies are used entirely and groundwater is used to meet the remaining crop requirements.

The problem is formulated as a linear program with objective of maximizing return. The first constraint is the acreage limitation. The second is the seasonal water availability constraint. The next four constraints limit water applied at each of the four scheduled

irrigations. The next three constraints are acreage constraints on individual crops. The last four constraints equate water consumed with water available. The mathematical formulation is

$$\max_{x_{ij}} \sum_{i=1}^{4} \sum_{j=1}^{j(i)} R_{ij} x_{ij} - C \sum_{k=1}^{4} S_k \tag{4.2}$$

such that

$$(1) \quad \sum_{i=1}^{4} \sum_{j=1}^{j(i)} x_{ij} \leq X \tag{4.3}$$

$$(2) \quad \sum_{i=1}^{4} \sum_{j=1}^{j(i)} \sum_{k=1}^{4} (a_{ijk} - r_k) x_{ij} \leq S \tag{4.4}$$

$$(3)\text{-}(6) \quad \sum_{i=1}^{4} \sum_{j=1}^{j(i)} (a_{ijk} - r_k) x_{ij} \leq SS_k \quad k=1, \ldots 4 \tag{4.5}$$

$$(7)\text{-}(8) \quad \sum_{j=1}^{j(i)} x_{ij} \leq X_i \quad i=2,3 \tag{4.6}$$

$$(9) \quad \sum_{j=1}^{j(4)} x_{4j} \geq X_4 \tag{4.7}$$

$$(10\text{-}(13) \quad \sum_{i=1}^{4} \sum_{j=1}^{j(i)} (a_{ijk} - r_k) x_{ij} = S_k \quad k=1, \ldots 4 \tag{4.8}$$

in which

i = crop index 1 represents corn, 2 represents beets, 3 represents beans, 4 represents alfalfa

j = method of production index

k = time period index

$j(i)$ = total number of method of production for crop i

R_{ij} = return per acre with zero water cost for crop i grown under method j (\$/acre)

x_{ij} = acreage of crop i grown by method j (acres)

C = cost of water delivered at the root zone (\$0.16/acre cm.)

S_k = water applied in period k (acre cm.)

X = maximum irrigated acreage (280 acres)

r_k = effective rainfall in period k

a_{ijk} = water requirement (acre cm.) of crop i grown by method j in time period k

S = maximum seasonal quantity of water to be delivered to the root zone (acre cm.)

SS_k = maximum quantity of water in time period k

X_2, X_3 = maximum acreages of beets and beans, respectively (50 acres)

X_4 = minimum acreage of alfalfa (50 acres)

The maximum and minimum acreages for the crops other than corn have been determined by the characteristics of the representative farm as discussed previously in this chapter. Note that providing sufficient water is available, the corn acreage will be set at its maximum value of 130 acres since it is a more highly valued crop than alfalfa. An interesting point of this program is that it determines how the production methods change as the precipitation varies.

Several other points should be made regarding parameters of the program. First, the program as written assumes rainfall is 100% effective. That is, all rainfall reaches the plant root zone. This assumption has been made by other authors (e.g., Stegman, et al., 1969), but is not entirely justifiable. As plants mature, particularly corn, interception reduces effective rainfall. A functional relationship between time and percent effective rainfall for each crop could easily be incorporated if these data were available.

A final point is the determination of total quantity of seasonal irrigation water. Under current conditions for the representative farm, groundwater pumping restrictions do not limit farmers' production. The purpose of this study is to look at a future time when pumping restrictions do limit production and to study the effects of

this policy on returns to farmers. Clearly, if the limit on pumping is set too high, the effect on returns is nil

In this study, the limit on available water S has been set so that during a year with average precipitation, the farmer has sufficient irrigation water to fully irrigate crops. In years with less than mean precipitation or when precipitation is low in critical periods, the farmer is affected.

The calculation of S is shown in Table 4.10. The value obtained is 6,288 acre cm. at the root zone, which converts to 410 acre feet, assuming 50% efficiency from headgate to root zone, or 1.46 acre feet of water per acre of irrigated land.

Table 4.10

CALCULATION OF TOTAL SEASONAL IRRIGATION WATER SUPPLY

Crop	Acreage	Irrigation Requirement Less Mean R.F. 1	2	3	4	Total Irrigation Required/Acre	Total Irrigation (Acre Cm. at Root Zone)
Beets	50	0.0	4.79	4.1	9.3	18.19	909.5
Beans	50	0.0	4.39	4.1	9.2	17.69	884.5
Alfalfa	50	4.0	0.00	14.2	7.7	25.9	1,295.0
Corn	130	6.7	7.29	4.1	6.5	24.59	3,196.7
TOTAL	280						6,287.5

Note: Skip irrigation 2 for alfalfa for maximum return.
Mean rainfall = 6.0 cm. in time period 1
5.11 cm. in time period 2
1.6 cm. in time period 3
3.2 cm. in time period 4

Results of the Deterministic Linear Program

The linear program formulated in the previous section was solved by the Simplex algorithm. The final form of the matrix is shown in Table 4.11. Scaling of some equations and variables was required in order to prevent roundoff error from affecting the solution.

The program was solved considering various precipitation levels, ranging from 0 cm. to the mean seasonal total of 20.11 cm. Note for example, for 50% of the mean seasonal precipitation, the values in each time period were assumed to be 50% of the mean.

The results of the computer runs are shown in Table 4.12 and graphically in Figure 4.5. Under most precipitation policies, 50 acres each of beets, beans, and alfalfa were planted and 130 acres of corn. Under the mean precipitation policy, 91 acres of corn were grown under method 1 and 39 acres under method 2A since some water shortage was encountered.

Under the 80% of the mean policy, 130 acres of corn were grown under method 7A. Since Table 3.15 was not complete, for policies below 70% of the mean precipitation level, other production methods were generated and were incorporated into the solution. Since these production methods are not necessarily optimal the program tends to slightly underestimate returns for precipitation policies less than 70%.

With the 40% policy, alfalfa production was increased to 101 acres while 79 acres of corn were grown under method 13. It is hypothesized that if Table 3.15 were complete, for each precipitation level, all corn acreage would be set at 130 acres and all acreage would be irrigated uniformly, as is the case for the 80% level. This

Table 4.11

LINEAR PROGRAMMING TABLEAU FOR CROP ACREAGE PROBLEM

	Corn Variables	Beet Variables	Bean Variables	Alfalfa Variables	Water Variables		
Objective Function:	Gross Returns				- Costs		
Constraints							
1. Acreage	1 1 . . .			. . . 1		≤	280
2. Seasonal Water Availability					1 1 1 1	≤	6288
3. Water Use in Each Time Period	Water Use Coeff. in Time Period 1—Forecast Effective Rf					≤	2500
4.	Water Use Coeff. in Time Period 2—Forecast Effective Rf					≤	2500
5.	Water Use Coeff. in Time Period 3—Forecast Effective Rf					≤	2500
6.	Water Use Coeff. in Time Period 4—Forecast Effective Rf					≤	2500
7. Maximum Beet Acreage		1 1				≤	50
8. Maximum Bean Acreage			1 1			≤	50
9. Minimum Alfalfa Acreage				1 1		≥	50
10. Mass Balance	Water Use Coeff. in Time Period 1—Forecast Effective Rf				-	=	0
11.	Water Use Coeff. in Time Period 2—Forecast Effective Rf				-1	=	0
12.	Water Use Coeff. in Time Period 3—Forecast Effective Rf				-1	=	0
13.	Water Use Coeff. in Time Period 4—Forecast Effective Rf				-1	=	0

Table 4.12

RETURN AS A FUNCTION OF PRECIPITATION LEVEL
AND ASSOCIATED CORN ACREAGES

Precipitation Level	Return	Corn Acreage	Method of Production (See Table 3.15)	
100%	29,692.00	91	1	
		39	2A	
80%	28,108.50	130	7A	
70%	26,666.62	8	7A	
		122	8J	
60%	24,260.43	104	12	
		26	8J	
40%	19,879.70	79	13	
		* 101	4	*
20%	16,054.27	127	14	
		3	15	(dry)
0%	13,215.79	95	14	
		35	15	(dry)
120%	29,932.45	130	1	

*Alfalfa acreage

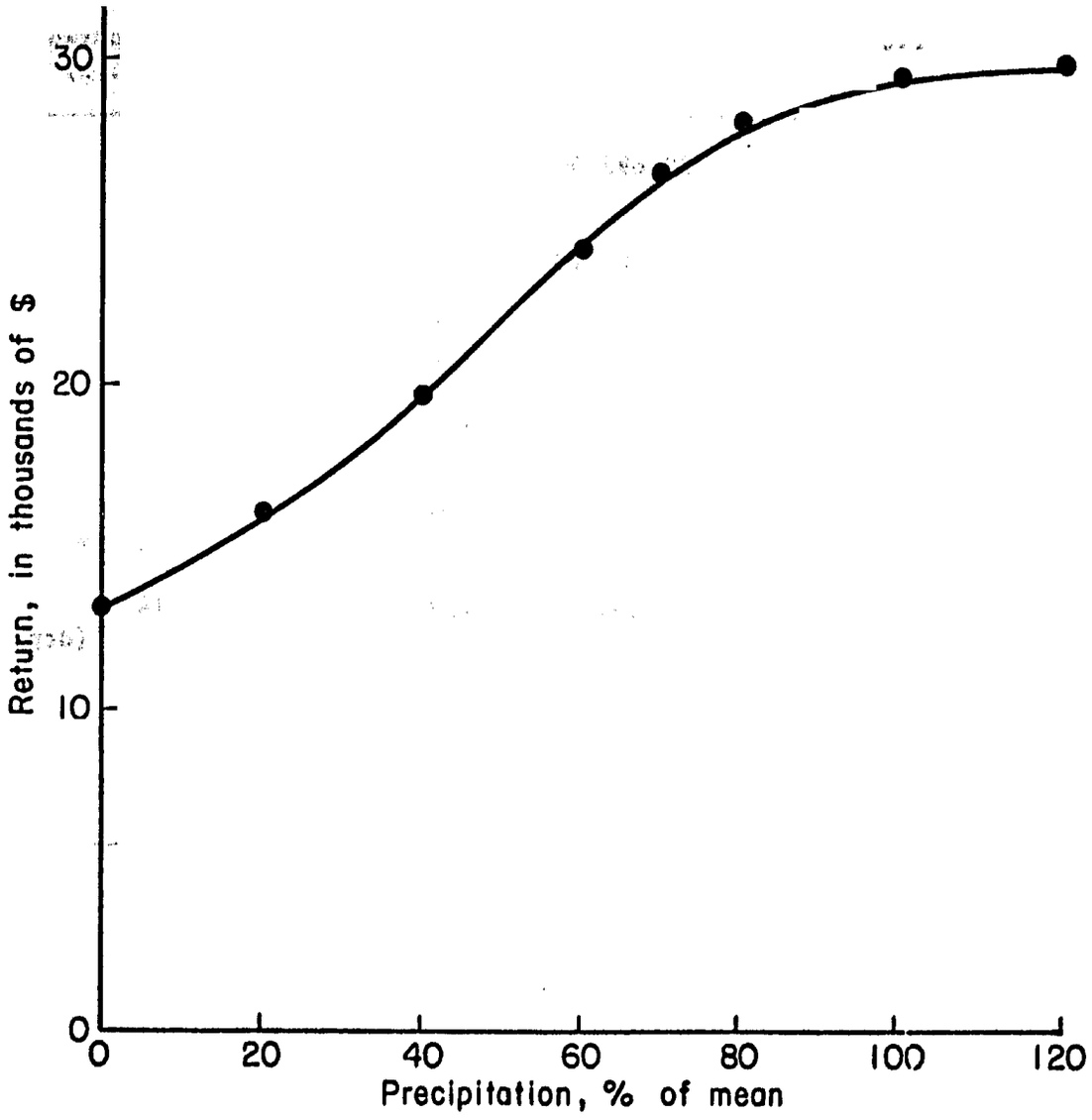


Figure 4.5

RESULTS OF DETERMINISTIC LINEAR PROGRAM SHOWING RETURN AS A FUNCTION OF SEASONAL PRECIPITATION WITH OTHER SOURCES OF WATER LIMITED

hypothesis is consistent with the results of Hall and Buras (1961) discussed in Chapter II. The discrete nature of the irrigation amounts generates policies such as that at the 100% level.

The linear programming results, it should be emphasized, are for the deterministic case, that is, when a farmer has complete knowledge of future precipitation amounts. The program determines return when a certain seasonal precipitation and distribution is forecast and that event occurs. The problem formulated in the next section looks at the non-deterministic case, that is, calculation of returns when an actual historical record is simulated.

The Multiple-Year Simulation

In the previous section the deterministic case was studied. For various levels of seasonal precipitation, returns were calculated based on foreknowledge of the seasonal precipitation and its distribution. In this section, a problem is formulated and solved which calculates the expected return and variance when random precipitation is included in the planning model.

The Precipitation Planning Policy

The general approach is to take a particular precipitation level, say 20% of the mean as the precipitation planning policy. With this policy, then, determine crop acreages, deterministically as was done in the previous section, and test the results of this policy with historical rainfall data.

A program is needed which simulates the farmer's decision of how much to irrigate and how much water to keep in reserve for the remainder of the season. In this program, acreages are set and the

irrigation decision is made at a particular time period based on the rainfall to date and the forecast rainfall for the remainder of the season.

In the previous section, discrete methods of production were considered. The program in this section requires continuous return functions. Linear return functions are developed for corn from Table 3.15 and for the other crops from Tables 4.6-4.8.

The program also requires a forecast of precipitation for the year. One approach would be to obtain historical forecasts of rainfall and assume that the farmer followed this forecast after initially planning his crop pattern according to a yearly forecast. Another approach is to assume that the farmer's decision is based on an overall philosophy, ranging from optimistic to pessimistic. The latter approach is adopted in this study. With the first approach the farmer could adopt the forecast, or depending on the accuracy of the forecast, and the farmer's view of risk, he could adopt some fraction of the forecast as his personal view of forecast rainfall. The latter approach is a more long-term view of risk. On a day-to-day basis the farmer has a better forecast than incorporated in this program, but the method adopted here reflects how expected income will vary and how the variance of that income will vary as the farmer assumes attitudes ranging from pessimistic (each year forecasting low rainfall) to optimistic (each year forecasting high rainfall).

Formulation of Problem

Since it is desired to run a 20-year simulation with four irrigation decision periods per year, the program formulation has been simplified for ease of computation. The return for each crop is

given as a linear function of water applied in each time period. A fifth time period has been added so that rainfall occurring after the last irrigation can still contribute to yield. The return for the jth crop is given as

$$R_j = \sum_{i=1}^{5} (a_{ij}Z_{ij} - cy_{kj})\hat{x}_j \tag{4.9}$$

in which

R_j = net return for the acreage of crop j

a_{ij} = dollar value of a volume of water applied to one acre of crop j in time period i

Z_{ij} = total volume of water applied (\$/acre cm./acre) to an acre of crop i in period j (acre cm.)

c = cost of irrigation water (set at \$0.16/acre cm. at the root zone, equivalent to \$5/acre ft. at the headgate

y_{kj} = volume of irrigation water applied to an acre of crop j in time period i (acre cm./acre)

$\hat{x}_j$ = acreage of crop j (acres), from the deterministic linear program.

The methods used for determining coefficients a_{ij} will be discussed in the next section.

The integral component of the simulation model is the model which allocates water at each time period k for a given year. This allocation is based on the quantity of water in storage at the kth time period, the depth of rainfall in the kth period, W_k', and the forecast rainfall for the remainder of the year $W_{k+1}, W_{k+2}, \ldots W_5$.

The problem is formulated considering 5 time periods and 4 crops. For the kth decision period, the model is formulated as:

$$\max_{Z_{ij}, y_{ij}} \sum_{i=k}^{5} \sum_{j=1}^{4} a_{ij}Z_{ij}\hat{x}_j - cy_{ij}\hat{x}_j \tag{4.10}$$

such that

(1) $Z_{kj} \leq W_k + y_{kj}$ (4.11)

(2) $Z_{ij} \leq W_i + y_{ij}$ $i=k+1, \ldots, 5$; $j=1, \ldots 4$ (4.12)

(3) $\sum_{i=k}^{5} \sum_{j=1}^{4} y_{ij}\hat{x}_j \leq S_k$ (4.13)

(4) $\sum_{j=1}^{4} y_{ij}\hat{x}_j \leq SS_i$ $i=k, \ldots 5$ (4.14)

(5) $Z_{ij} \leq FC - SM_{ij}$ $i=k, \ldots 5$; $j=1, \ldots 4$ (4.15)

(6) $y_{5j} \leq 0$ $j=1, \ldots 4$ (4.16)

in which symbols in the objective function have been defined previously and

W_k = actual rainfall during period k (cm.)

W_i = forecast rainfall in period i where $i > k$ (cm.)

S_k = amount of water in storage at period k (acre cm.)

SS_i = maximum water that can be delivered in time period i (acre cm.)

FC = depth of available soil moisture at field capacity (33.2 cm.)

SM_{ij} = depth of available soil moisture for crop j at time period i where $k \leq i \leq 5$ (cm.)

The first and second constraints state that depth of water applied (which contributes to return) is less than or equal to the sum of rainfall and irrigation. Usually this is an equality but z_{ij} may be constrained by (5). The third constraint is the seasonal quantity constraint on irrigation water. The fourth places an upper bound on water applied in any time period. Constraint (5) states that depth of water applied which contributes to return must not excede the deficit of the soil profile from field capacity. Constraint

(6) prevents irrigation at the end of the last time period, only rainfall contributes to return in this period.

Implicit in this linear program is a soil moisture simulation model. Again the model employed is that of Yaron. The set of constraints listed as constraint (5) represents the soil moisture simulation. At the kth time period, the value of SM_{kj} is known for all j, but the remaining values, SM_{ij} $i > k$, are variables, dependent on the z values, the quantity of water applied. For the jth crop at the k+1 time period, soil moisture is given by the Yaron model as

$$SM_{k+1,\ j} = (SM_{kj} + Z_{kj})e^{-b_{kj} \cdot t_k} \tag{4.17}$$

in which

b_{kj} = beta coefficient for the jth crop in the kth time period

t_k = length of time (days) of the kth time period.

Thus for time period k+1 and crop j, constraint (5) is given by

$$Z_{k+1,\ j} \leq FC - (SM_{kj} + Z_{kj})e^{-b_{kj} \cdot t_k} \tag{4.18}$$

or

$$C_{kj} \cdot Z_{kj} + Z_{k+1,\ j} \leq FC - c_{kj}SM_{kj} \tag{4.19}$$

in which

$$C_{kj} = e^{-b_{kj} \cdot t_k}$$

Similar expressions are derived for time period k+2, etc. Constraint (5) is thus represented by a lower triangular matrix with diagonal elements unity and other elements products of the constants c_{kj}. A schematic diagram of the entire matrix is shown in Table 4.13.

Table 4.13

LINEAR PROGRAMMING TABLEAU FOR PROGRAM IRR SIM

	Z Variables	Y Variables		Right Hand Sides
(1)	I	$-I$	$=$	W'_k
				W'_k
				W'_k
				W'_k
(2)				W_{k+1}
				W_{k+1}
				W_{k+1}
				W_{k+1}
				$\vdots$
(3)		$[x\ x\ \ldots\ x]$	$\leq$	S_k
		$[x]$		S_k
(4)		$[x]$		S_{k+1}
		$\ddots$		$\vdots$
		$[x]$		S_5
(5)	1			$FC\text{-}SM_{k,1}$
	1			$FC\text{-}SM_{k,2}$
	1			$FC\text{-}SM_{k,3}$
	1			$FC\text{-}SM_{k,4}$
	$C_{k,1}$			$FC\text{-}C_{k1}SM_{k,1}$
	$C_{k,2}$			$FC\text{-}C_{k2}SM_{k,2}$
	$C_{k,3}$			$FC\text{-}C_{k3}SM_{k,3}$
	$C_{k,4}$			$FC\text{-}C_{k4}SM_{k,4}$
	$C_{k,1}\cdot C_{k+1,1}$	$C_{k+1,1}$		$FC\text{-}C_{k1}\cdot C_{k+1,1}SM_{k,1}$
	$C_{k,2}\cdot C_{k+1,2}$			$FC\text{-}C_{k,2}\cdot C_{k+1,2}SM_{k,2}$
	$C_{k,3}\cdot C_{k+1,3}$			$FC\text{-}C_{k,3}\cdot C_{k+1,3}SM_{k,3}$
	$C_{k,4}\cdot C_{k+1,4}$			$FC\text{-}C_{k,4}\cdot C_{k+1,4}\cdot SM_{k,4}$
				$\vdots$
(6)		$[I]$	$\leq$	0
Objective Function	$a_{1j}\cdot\hat{x}_j$	$-c\cdot\hat{x}_0$		

Discussion of the Return Function Coefficients

The coefficients a_{ij} were determined by various methods because of the limitations of the available data. For corn, the data from Table 3.15 was used in a linear regression to determine the a_{ij} coefficients. Water applied in all time periods was significant at the 5% level and when the regression was constrained to a zero intercept, the resulting R^2 value was 96%. The results of this run are shown in Table 4.14. The largest residual was for zero water application. This was acceptable since the best fit is desired for water applications near the maximum since this occurs most frequently. A zero intercept was used to conform to the results with other crops. Results could be improved by loosening this restriction.

For alfalfa, deleting the first method of production in Table 4.9, four observations were available, generating four equations in four unknowns. When these equations were solved, however, one of the coefficients obtained was negative. For sugar beets and pinto beans, an exact solution was not possible. Applying regression to the five observations also resulted in some negative coefficient values. For these crops, water applied in each time period should contribute positively to return, hence another solution technique was attempted.

To obtain non-negative best fit coefficients for sugar beets and pinto beans, a linear program was formulated. In this formulation the constraints were less than inequalities containing the water use and return data of Tables 4.7 and 4.9 and the objective was to minimize the sum of the slack variables. The result of this formulation is that the coefficients obtained always under-predict actual return

because of the slack variables. An advantage of this formulation is that some observations can be fit exactly by placing a large penalty on the objective function coefficient of the corresponding slack variable. Hence for full irrigations of sugar beets, the generated coefficients provide an exact fit; whereas for less than full irriga- which occur infrequently if ever, the coefficients predict

Table 4.14

RETURN COEFFICIENTS FOR 4 CROPS
IN 5 TIME PERIODS

	Dollar Values of a Unit of ET by Time Periods				
Crop	1	2	3	4	5
Sugar Beets	4.90	5.13	5.28	4.70	1.20
Pinto Beans	4.75	4.98	5.12	4.56	1.20
Alfalfa	1.92	0.00	1.92	1.92	0.60
Corn	1.41	2.07	2.05	1.65	0.60

less than observed return.

For pinto beans, coefficients obtained in time periods two and three were zero, again an unacceptable answer. When these results were included in the program, the result was to skip the second and third irrigation, applying a large irrigation at the fourth time period. Because of the large coefficient here, higher than maximum returns were predicted. For the final computer runs, coefficients for pinto beans were obtained from those of sugar beets by reducing the beet coefficients by the percentage difference in return for beets

and beans. With these revised coefficients, pinto bean returns are accurately predicted for full irrigations. Actually this is satisfactory since in all cases beets and beans are irrigated fully

he coefficients for alfalfa were revised to accurately fit production method 3, under the assumption that irrigation in time period 2 was unnecessary and that each cm. of water applied in the other time periods contributed equally to final yield.

A final set of coefficients were needed for the dollar value of a unit of rainfall during the last time period. To obtain these coefficients, it was assumed that soil moisture was at field capacity after the last irrigation and a single rainfall event occurred midway through the time period. The increases in ET and in return were calculated for corn using Equation 3.28. The resulting value was used for corn and alfalfa since their returns are nearly equal. For sugar beets and pinto beans the value was doubled.

Description of Program IRR SIM

The 20-year irrigation simulation is programmed in Fortran IV as IRR SIM. A flow chart is included in Appendix D.

Initially the program reads the precipitation planning policy, the acreages determined by the deterministic linear program, the return function coefficients, the Yaron coefficients b_{ij}, the total seasonal available water S_o, and the maximum delivery in any time period SS_i.

The program initializes the year and time period and reads the actual rainfall for the first time period of the first year. With

this information, the first linear program is established in subroutine SET UP using the standard subroutine, SIMPLX. The solution is the quantity of water applied to each crop in that time period. Control returns to Subroutine SET UP which stores the z values (total quantity of water applied to each crop) which are used in calculating the soil moisture levels for the next time period and the Y values, the sum of which define the parameter USE, the quantity of irrigation water used in the time period.

The program next increments the time period, updating quantities of water in storage and soil moisture for each crop. Again the program reads the actual rainfall, sets up the linear program and solves it.

After the last time period, the actual return for the year is calculated according to the linear return functions of Equation 4.9. The program proceeds to the next year, repeating the entire process. After the calculations for the last year, the expected return and variance are calculated. The entire program is re-run for other precipitation planning policies and the expectation versus variance graph is generated.

As with the previous deterministic program, various assumptions are made. The basic assumptions of the Yaron soil moisture simulation are presented elsewhere. Other assumptions are that rainfall is 100% effective and that all rainfall during the time period is accumulated in the soil profile at the end of the time period. The exception is the last time period in which all rainfall during the time period is assumed to occur in the middle of the time period.

Results of the Multiple-Year Simulation

Two graphs of the results of program IRR SIM are shown in Figures 4.6 and 4.7. Figure 4.6 was generated with the original return function coefficients with some zero coefficients. Table 4.15 presents the results of the program with the revised coefficients of Table 4.14. Figure 4.7 shows these results in graphical form. The figures have been plotted with standard deviation versus expected return. For a normally distributed random variable, 68% of the observations lie within one standard deviation unit of the expected value. The results of this program are consistent with the previous statement. For each precipitation policy level, four of the 20 observations lie below one standard deviation unit.

The shape of the EV, or in this case expectation vs. standard deviation graph, in Figure 4.6 is consistent with that predicted by economic theory. At low precipitation planning levels, as the planning level is increased, variance rises and expected return rises. At high precipitation planning levels, the variance continues to increase but the expected return starts to decrease. Thus a point exists where expected return is maximized. The backward bending section of the curve is clearly an inferior region. For these points, points of equal expected return and less variance can be found.

In Figure 4.7 moving from 0% to 10%, the expected return increases and the variance decreases. As the planning percentage is increased, variance increases and expected return decreases. As with Figure 4.6, a point where expected income is maximized can be found.

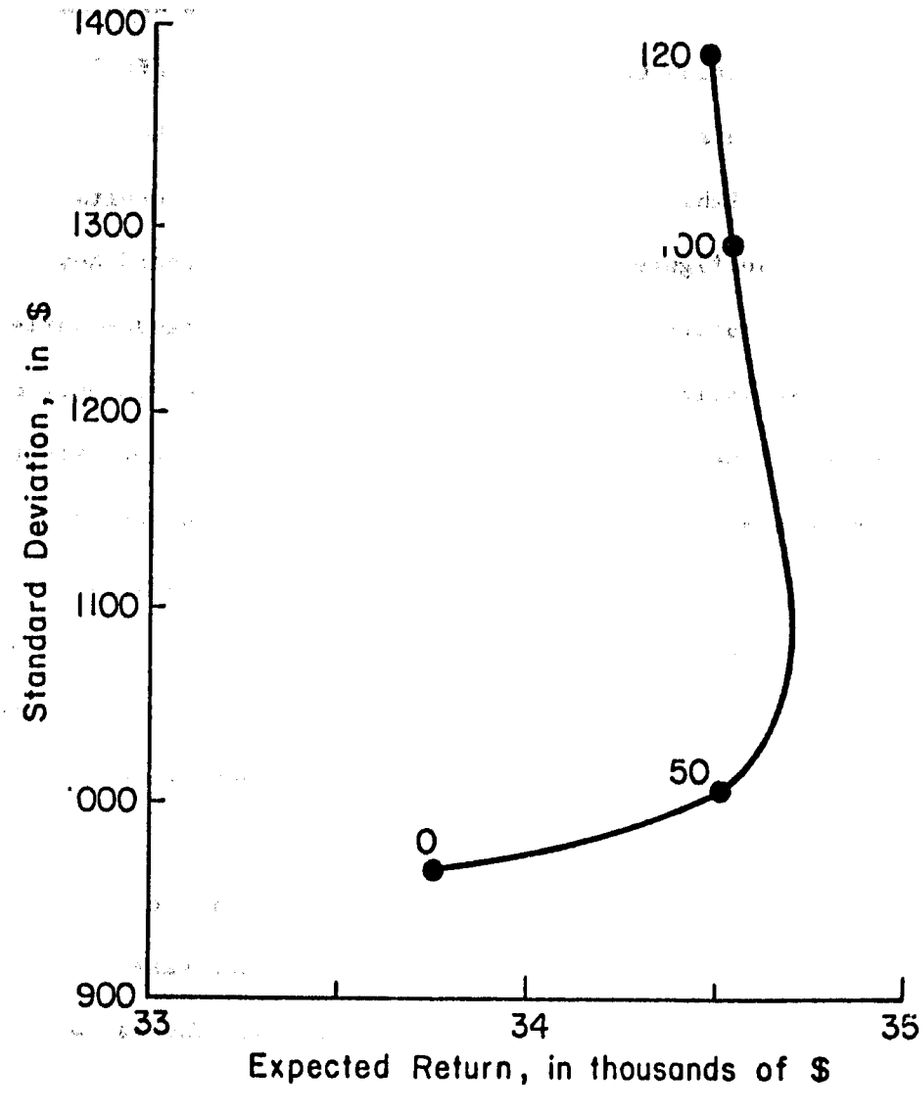


Figure 4.6

EV GRAPH WITH ORIGINAL
RETURN FUNCTION COEFFICIENTS

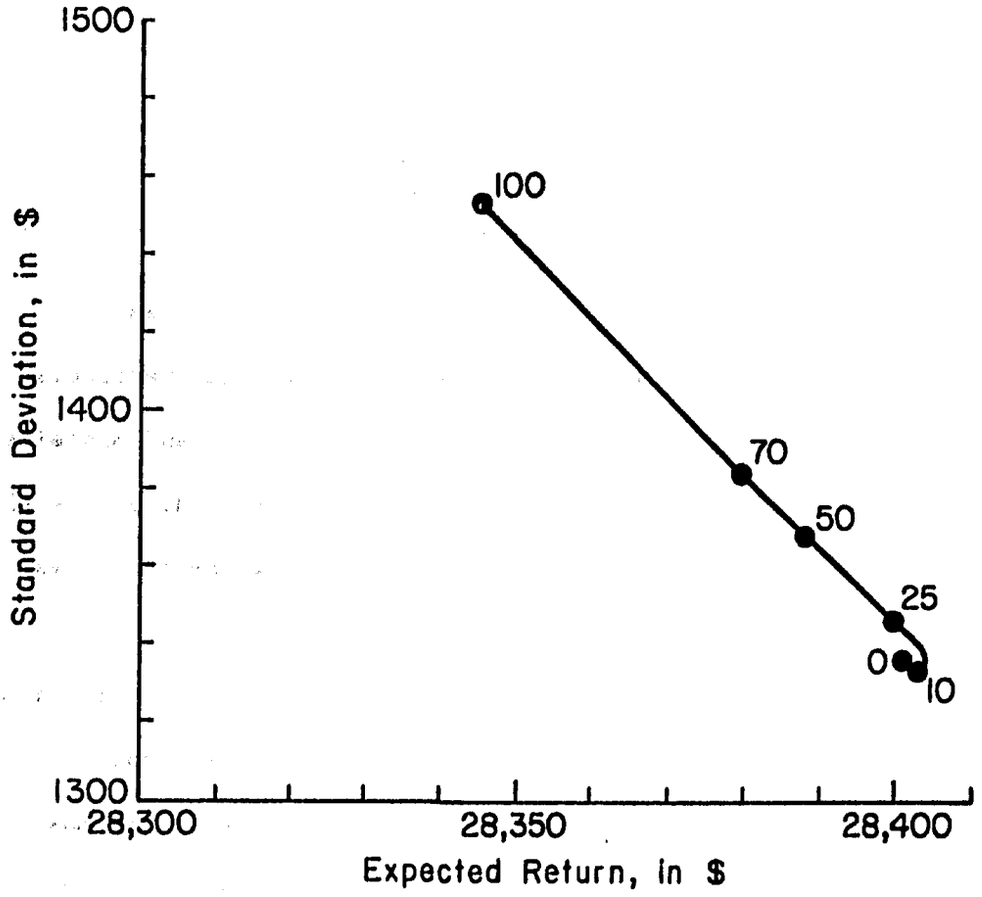


Figure 4.7

EV GRAPH WITH REVISED
RETURN FUNCTION COEFFICIENTS

Table 4.15

SUMMARY OF RESULTS OF PROGRAM IRR SIM
FOR VARIOUS PRECIPITATION PLANNING POLICIES

Precipitation Planning Policy	Expected Return ($)	Standard Deviation	Minimum Income in 20 Years	% of Mean
0	28,401	1,337	25,927	91.3
10	28,402	1,335	25,927	91.3
15	28,402	1,337	25,906	91.2
25	28,398	1,344	25,822	90.9
50	28,387	1,365	25,611	90.2
70	28,379	1,383	25,442	89.7
100	28,345	1,454	25,188	88.9

Discussion of Multiple Objective Problem

The multiple objective problem is to find the particular precipitation planning policy which maximizes return and minimizes the variance of that return. In general, in order to determine this policy, some knowledge must be obtained of how the farmer values a unit of return compared to a unit of variance.

In the specific case of Figure 4.7, the optimal precipitation planning policy is approximately 10%, since for the points calculated, the variance is minimized and expected return is maximized for this policy. Precisely why variance decreases between 0% and 10% is not clear. Though neither is it intuitively obvious that forecasting at 10% should result in greater variance than at 0%. In Figure 4.6, the multiple objective problem could be reduced to maximizing expected return since it is unlikely that a farmer could differentiate between a standard deviation of 3.8% of expected income (at 100%) and 2.9% of expected income (at 0%).

Returns Under Deficient Supply

A final question was posed, that is, once the optimal precipitation policy is determined, how is return affected as irrigation supply is more tightly restricted. Initially, the water supply was set at 6,288 acre cm. or 1.46 acre ft./acre. The simulation program was re-run using the optimal precipitation policy of 10% with water supply decreasing to 1 acre ft./acre. These results are shown in Table 4.16. The program results would not be accurate below the

Table 4.16

EFFECT ON EXPECTED RETURN AND VARIANCE OF LIMITED IRRIGATION WATER

Precipitation Policy (% of Mean)	Seasonal Supply (acre cm.)	Seasonal Supply (acre ft. / acre)	Expected Return	Standard Deviation	Minimum Income in 20 Years	% of Mean
10	6,288	1.46	28,402	1,335	25,927	91.3
10	5,278	1.23	27,237	1,838	24,135	88.6
10	4,267	1.00	25,694	2,223	22,219	86.5

1 acre ft./acre level since the return function coefficients were adjusted to give a best fit in the region of full irrigation. The coefficients could be readjusted to give a better fit providing data were available.

It may be concluded that when the seasonal water supply is restricted below that level required for maximum yields in an average year, return and variance are significantly affected. Figure 4.8 shows minimum observed income decreasing at a greater rate than expected income as the seasonal supply is decreased. In other words, as seasonal supply is reduced, the farmer becomes more dependent on precipitation, and thus the variance of his return increases.

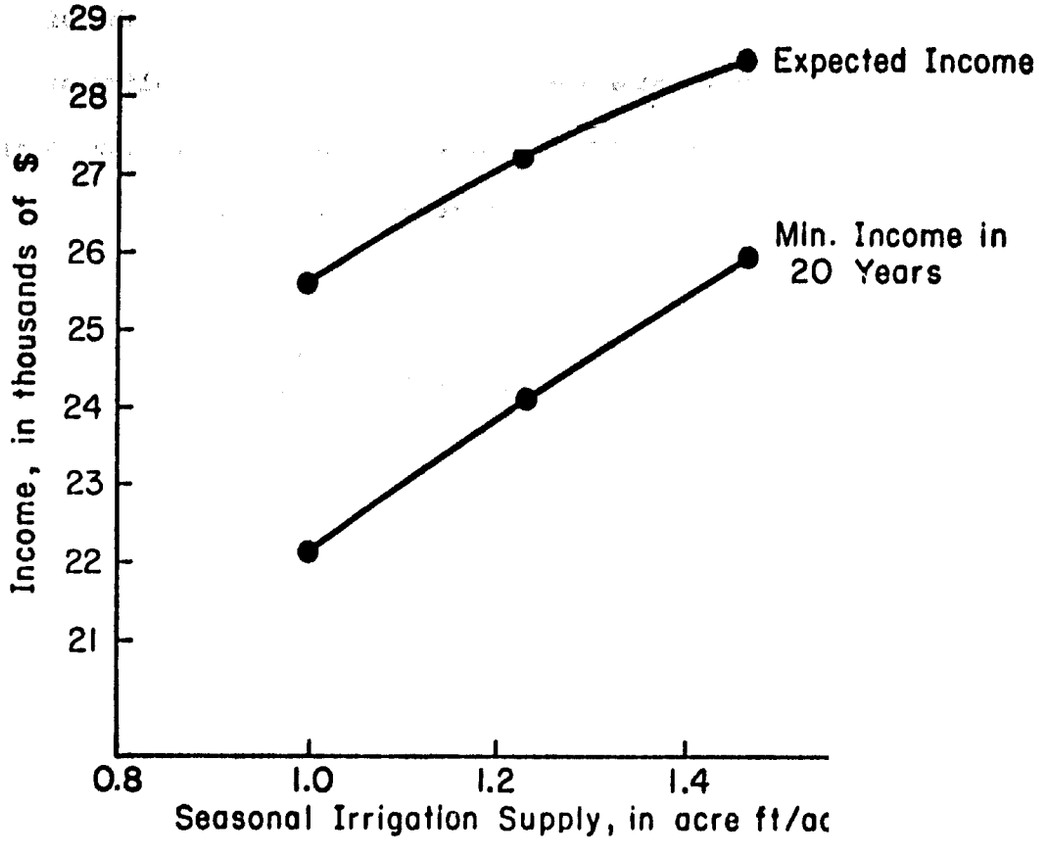


Figure 4.8

EXPECTED AND MINIMUM INCOME AS A FUNCTION OF SEASONAL SUPPLY FOR THE REPRESENTATIVE FARM

Summary

The main achievement in this chapter has been the determination of the optimal precipitation planning policy. This is the policy which when implemented results in maximizing expected return on a long-term basis. The variance of the various precipitation planning policies was calculated, but was found to not significantly influence the selection of the optimal policy.

The assumptions of the study should be kept in mind when viewing these results. The term "variance" refers to the variance of returns induced by random precipitation only. The "complete" problem of risk, with variance induced by other random climate variables, random water supply, random prices, etc., has not been studied. Neither has the problem of uncertainty been addressed.

Figures 4.7 and 4.8 show that the optimal precipitation policy is sensitive to the choice of the return function coefficients. More work could be done to improve the confidence in these coefficients.

It should be emphasized that the optimal policy found in this study is that precipitation level which maximizes return for this site over a long-term record with no other forecast information. On a month-to-month or year-to-year basis individual farmers have access to more accurate forecasts. However, planners of new irrigation projects and researchers studying other aspects of an irrigation system may have use for a percentage of seasonal precipitation which can justifiably be employed in their calculations and models. It would be expected that for arid areas, such as the Central Valley of California, the optimal precipitation policy would be 0%, while

in areas which rely heavily on precipitation, such as Iowa, the policy approaches 100%. Estimates of optimal precipitation planning policies could be determined for other geographic areas by employing rainfall records for these areas in the simulation program.

Chapter V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

This study has dealt with certain aspects of the multiple crop irrigation planning problem. The entire problem cuts across many disciplines and would require large amounts of data and many man hours of analysis. This study has taken currently available agronomic, economic and hydrologic data and formulated and solved several irrigation planning problems.

These problems may be generally stated as

1. How to decide which crops to plant given an initial feasible set of crops and a limited water supply.

2. How to include random precipitation in the analysis. The models developed have been applied to a representative farm in the irrigated area of the South Platte River Valley near Ft. Morgan, Colorado.

The key theoretical assumption of this study has been the relation proposed by Yaron, et al., (1973) that daily evapotranspiration ET is linearly related to available soil moisture ASM or equivalently that under conditions of no rainfall, soil moisture decays exponentially over time. Under this assumption, a computer program was written to determine these rate coefficients for field trial data for corn from Colorado State University.

The Yaron model allows prediction of ET for a crop under any irrigation regime. The next stage of the study has been to relate ET to yields. An additive and a multiplicative function of ET in three growth stages were tested and each was incorporated into a dynamic program to determine optimal water applications on four fixed irrigation dates under various water limiting conditions.

The next stage of the study, Chapter IV, incorporated the previous results into a multiple crop planning model. The optimal water applications for corn, along with limited data for other crops, were used in a linear program which determined crop acreages which maximize return under conditions of known rainfall.

The effect of random precipitation was studied in the next stage of the study. For this section, a continuous return function was developed. Regression analysis was employed to fit a linear return function to the corn results of Chapter III, obtaining a "best fit" in the region of full irrigation, since this region corresponds to actual practice. Linearized return functions were also obtained for the other crops of the study.

Also in this stage of the study, the concept of precipitation planning policy was introduced. A farmer adopting a 10% precipitation planning policy would plan for 10% of the mean precipitation in each time period and would allocate his fixed seasonal irrigation supply according to the actual rainfall to date and the forecast of 10% of mean precipitation for the remainder of the season.

The effect of various precipitation planning policies was tested using a 20-year simulation program. The expected return and variance

under each policy was calculated. The optimal precipitation planning policy is found, in general, by solving a multiple objective problem. However, in the particular case of this study the change in variance for the different policies was insignificant, thus reducing the problem to the single objective of maximizing expected return.

Conclusions

Marginal Value of Water

From the single crop model applied to the representative farm, the marginal value of water was obtained. The dynamic program was run, increasing the water cost until the optimal policy no longer was to irrigate to field capacity at each irrigation. Under the assumptions of the study, this calculation resulted in a marginal value of water between $10 and $15 per acre foot early in the season and greater than $20 per acre foot later in the season for the single crop.

Actually, the marginal value is a variable depending on seasonal quantity. It would be expected that the marginal value increases as total seasonal quantity of water decreases, and decreases as the total increases. From Figure 4.9, this marginal value calculation can be made. If the seasonal supply is initially limited to 1.46 acre feet/acre and then decreased to 1.23 acre feet/acre the expected income drops by $1,165 or $4.16/acre. Assuming linearity in this region of the curve, the marginal value of water is $18.09/acre foot. This figure reflects the expected value of a unit of water which could be applied any time during the season, while the previous figures reflect the value to a particular crop in particular time periods.

An objective of maximizing production with a secondary objective of minimizing water usage may be proposed. Minimizing water usage could be achieved through adopting the 10% precipitation policy. Under this policy, with the representative cropping pattern and the assumed 50% irrigation efficiency, the calculation of Table 5.1 results in a required seasonal supply of 2.3 acre feet/acre. This figure, it should be noted does not include any preplanting or other light irrigations during early growth of the crops. Under this production maximizing goal, expected return is approximately $30,000 while when seasonal water is limited to 1.46 acre feet/acre, the expected return is $28,402, leading to an approximate marginal value of water in this range of $7/acre foot.

The conclusion to be drawn from this section, providing the calculations are judged of sufficient accuracy, is that since the farmer of the representative farm is paying less than the marginal value of water, water cost should not be affecting production. The farmer should be irrigating fully, providing water supplies are not limited.

Policy Implications

Under the current operating structure, in which water is priced well below its marginal value, there may be a tendency for inefficient use, particularly over-irrigation. The State Engineer is concerned with meeting downstream water obligations and managing the groundwater aquifer. He should promote increased efficiency of water use while not placing undue burdens on the farmers of his area.

Some of the powers of the State Engineer are

1. Restrictions on digging new wells;

Table 5.1

CALCULATION OF SEASONAL WATER REQUIREMENT EMPLOYING A 10% PRECIPITATION PLANNING POLICY

Crop	Acreage	Water Requirement Less 10% P.P.P. (cm.)				Seasonal Depth of Water (cm. per acre)	Total Seasonal Water Requirement (acre cm.)
Beets	50	5.0	9.4	5.5	12.2	32.1	1,605
Beans	50	5.0	9.0	5.5	12.1	31.6	1,580
Alfalfa	50	9.4	0.0	15.6	10.6	35.6	1,780
Corn	130	12.1	11.9	5.5	9.4	38.9	5,057
TOTAL	280						10,022

$$10{,}022 \text{ acre cm.} \times \frac{\text{in.}}{2.54 \text{ cm.}} \times \frac{\text{ft.}}{12 \text{ in.}} \times \frac{1}{50\% \text{ efficiency}} = 657.6 \text{ acre feet}$$

$$\frac{657.6 \text{ acre feet}}{280 \text{ acres}} = 2.3 \text{ acre feet/acre}$$

2. Restrictions on pumping from existing wells;

3. Increasing the cost of pumping through some form of fee or tax;

4. Providing information to encourage improved water use efficiency.

Some possible measures under 4 (above) are promoting use of flow measuring devices and soil moisture measuring devices so the farmer has improved knowledge of his water use and needs. This service in some areas is provided as a package irrigation scheduling service.

Clearly, no one of the above measures may be adequate for meeting the downstream requirements particularly in years with low streamflow and low rainfall. Restrictions on digging new wells may not be effective in this area since the great majority of farmers currently operate wells. Providing information is a long-term proposal and farmer acceptance of scheduling services also may take years since the benefits of such activities may not be apparent each year.

Placing a tax on a well without coupling the tax to the quantity of water pumped may not serve to improve water use efficiency. Taxing quantity of water pumped requires metering of wells and some enforcement measures. Limiting pumping also requires enforcement though it may not require metering if pumping is allowed only on certain days or hours. Limiting is also more flexible than taxing since pumping hours or days can be changed through the season as downstream requirements dictate.

The limiting pumping alternative must be applied with sufficient knowledge of the outcome, however, Restricting pumping for only a short time during the critical silking period of corn, for example,

may seriously reduce farmers' returns. From Table 3.15, a shortage of 8 cm. at the root zone during the second time period (only 10 days) results in a reduction of return from the corn acreage of the representative farm of $911 from the maximum return, even though water use in the remainder of the season is not restricted.

Though an optimal groundwater management policy may be an impossibility in practice, a workable policy may be attained through combining the results of this study with the results of the Young and Bredehoeft study in which the spatial and time effects of pumping on downstream flows were obtained.

Recommendations

Production Functions

The conclusions of this study are dependent upon the accuracy of the assumptions and the accuracy of the theory that has been adopted. The major recommendation of this study is that further research be conducted on modeling the soil moisture versus ET relation and the plant growth versus yield relation. An ET prediction model must be included in order to predict irrigation requirements. Prediction of ET under less than the traditional "well watered" conditions is required. Predicting yield as a variable over the season is the second phase of the model. The current study has adopted a production function approach based on ET in various time periods. A theory is needed which can apply to a number of field crops and can be field tested.

Field Testing

The center sprinkler line with linearly decreasing application amounts appears as an acceptable method for testing the effects on

yield of varying soil moisture. Care should be taken in measuring soil moisture and, as has been done in the current year, soil moisture should be plotted immediately after measurement to check whether reasonable results are being obtained.

Although the center sprinkler line method varies amounts of irrigations, additional blocks of the crop are required to study the effect of different times of irrigation. Each additional variable that is included in the field trial, such as salinity, fertility, plant population, etc., multiplies the effort involved, as does repeating experiments for other crops. Thus the entire experiments are still costly.

For field trial data to be most useful, climatic measurements should be taken over the season. These should include pan evaporation data or data to evaluate the Penman equation.

Transfer of Data

Transfer of data on crops obtained from field trials to other geographical locations is a subject of further research. It is desired that a theory be developed so that the full range of experiments need not be repeated at each site. A complete analysis of the corn trials conducted at California, Arizona, Utah, and Colorado could provide insight to the data transfer problem.

In the current study, plant growth, all climate variables, and the root zone model are all included in the Yaron coefficients. These coefficients vary over the season to reflect the changing nature of the plant and the climate. One approach would be to develop these

coefficients for other crops and for other sites. Through experience, then, these coefficients could be applied to other locations.

An advantage of the Yaron model is that the coefficients for a given time period may be approximated by the maximum ET attainable by the plant during this time period. The value of these coefficients may be estimated by relating them to a percentage of pan evaporation, assuming the soil and climate are generally suited to the crop.

The data transfer problem could be approached through using the more theoretical energy balance approach. As with the Yaron method, this method does not include terms for differing soil types. Additionally, this method requires data which may not be available for the particular application site. Even with the required data, the energy balance method reduces to estimating "best fit" crop coefficients for the particular site. Thus, the data transfer problem, despite the method used, appears to reduce to a problem in obtaining experience in adjusting coefficients.

Extension to Include Additional Random Variables

Besides improving the physical modeling of crop growth, this study may be extended to include other random variables contributing to the variance of the farmer's return.

There are five types of random variables that can be included in a model which attempts to calculate expected return and the variance of that return. These may be classified as

1. Precipitation
2. Other climate variables

3. Price variables

4. Water supply variables, and

5. Variables reflecting catastrophic events such as hail, frost, flooding, etc.

The nature of the study determines which of these variables are included in the analysis.

When the study is restricted to determining an optimal policy, such as the optimal precipitation planning policy, it is sufficient to hold prices, for example, constant in the multiple year simulation. However, more accuracy would be obtained if other climate variables were included. That is, if the Yaron approach is used, in the multiple year simulation, the coefficients should be adjusted to reflect the actual climate conditions that occurred during the year. A method for adjusting these coefficients could be developed with sufficient ET data for a number of years.

Studies focusing on other random variables influencing the farmer's crop pattern decision could be carried out. An interesting study would be to look at price as the random variable. For each year of a multiple year simulation, price forecasts could be obtained ranging from "conservative" to "risky." Optimal cropping pattern decisions could be obtained for each discretized price level for each year and the multiple year simulation could be run, generating an EV curve. The solution of the two-objective problem would be the optimal price forecast level.

A further study could be carried out for the State Engineer, who may wish to determine the optimal seasonal water limitation based on streamflow forecasts.

To make the forementioned problems and the precipitation planning problem more accurately measure the actual variance experienced by the farmer, the effects of all the random variables should be included. However, if obtaining an optimal policy, such as the optimal precipitation policy, is the only objective of the study, random variables which do not influence the selection of that policy may be neglected.

The Precipitation Planning Policy in Dryland Agriculture

For the site used in this study, precipitation contributes relatively little to the water supply of the crop. Without irrigation, field crops of the type grown would surely be unprofitable at this site. The crop irrigation requirements shown in Table 4.10 would virtually never be met by precipitation alone. Higher optimal precipitation planning policies would have been obtained applying the analysis to a more humid non-irrigated site. In the dryland agriculture case, the precipitation planning policy which is adopted only affects the crop pattern decision, since there is no water allocation decision during the season.

A further study would be to determine from the farmer's view the dryland case and the alternative of installing irrigation equipment. In this case, measuring the variance of income due to precipitation and the decrease in variance due to the sure water supply is important. At some point, dependent upon precipitation amounts, crop and equipment prices, and the availability of capital, the farmer desires

to reduce the variance due to precipitation in exchange for an increasing investment, hence possibly reduced net return.

It is hoped, then, that from the current study, other research will be carried out in the general area of modeling the decisions encountered in irrigated agriculture. Areas of research arise in basic crop research and in applications from both the farmer's view and from the planning agency's view.

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Appendix A

AGRONOMIC BACKGROUND

Introduction

In order to develop an optimization model for irrigated agriculture, having a simple input-output model relating irrigation to yield would be convenient. Unfortunately, such a model does not exist. In this chapter, two approaches to predicting yield are discussed. The first approach determines water requirements to meet a potential yield. The second approach views yield as a variable which can be expressed by a continuous function of some growth measuring variables. Before discussing these plant growth models, basic agronomic concepts are introduced. Later in the chapter, empirical methods of testing the growth models are presented.

A complicating factor in the analysis is the timing of inputs. For example, for corn there are a number of growth stages commonly listed (see Table A.1). Water or more precisely the soil moisture level or even more precisely the plant water potential, has an influence on growth in each stage and on the final yield of the crop, but this relation is not precisely known.

Basic Concepts

Soil Moisture

In order to develop a simplified model based on available information, an understanding of various terms is needed. Soil moisture

Table A.1

STAGES OF GROWTH AND DEVELOPMENT OF GRAIN CROPS BASED ON EXTERNAL APPEARANCE (From Neghassi, 1974)

Stage	Description
Germination	The appearance of the radicle.
Tillering	The formation of tillers, i.e., branches produced from the base of the stem.
Jointing	The stage when two nodes can be seen, i.e., the beginning of shooting.
Shooting	The stage of elongation of internodes.
Booting	The end of the shooting stage and just prior to the emergence of the ears.
Heading) Earing)	The emergence of the ear from the tube formed by the leaf sheath.
Flowering	The opening of the flowers. In corn this stage is often divided into tasseling and silking being the time of appearance of the male and female flowers, respectively.
Grain formation	The period of grain development from fertilization until maturity. This period can be further subdivided into the following stages:
milk-ripe	grain contents have a milky consistency.
soft-dough	grain contents have a dough consistency.
waxy-ripe	grain contents have waxy appearance.
full-ripe	grain contents hard.
dead-ripe	grain ripe for cutting.

measurements are commonly expressed as percent of soil by weight Θ_w or by volume Θ_v. When the volume of soil is known, Θ_v can be converted to an equivalent measure of depth of water per unit depth of soil. Another common measure of soil moisture is soil moisture tension, commonly given in bars or atmospheres. According to Taylor and Ashcroft (1972, p. 11), tension is a measure of "the pull that must be placed on a unit area of water to prevent it from entering soil through a membrane that is permeable to water but not to soil or air." In order to convert soil moisture suction to volumetric soil moisture content for a particular soil, a desorption curve is used. This is a curve showing Θ_v versus tension during dewatering of the soil.

A fourth measure of soil moisture is available soil moisture (ASM). This is defined on a scale of 0 to 100 where zero corresponds to permanent wilting percentage (PWP) and one hundred corresponds to field capacity (FC). Wadleigh (1955) defines FC as "the moisture percentage of a soil, expressed on dry-weight basis, in the field two or three days after a thorough wetting." Thornthwaite and Mather (1955) state that at FC the soil contains no surplus of gravitational water and no deficit of capillary water.

Wadleigh defines PWP as "the moisture percentage of soil at which plants wilt and fail to recover turgidity." Turgidity or turgor pressure refers to the actual pressure in the plant cells (Vaadia, 1967). Cell walls are elastic and the cytoplasmic membranes are differentially permeable, allowing free passage of water but not of solutes. Barrs (1968) presents the equation for the water potential in a cell:

$$\psi = -\pi - \psi_m (\pm) P \qquad (A.1)$$

in which

ψ = total water potential (at 100% water content = 0).

π = osmotic potential (arising from the pressure of solutes)

P = turgor pressure ("hydrostatic pressure developed in the cell in response to the tensile strength of the cell wall")

ψ_m = matric potential ("arising from the imbibitional forces of colloids in the cell and forces of capillarity in the cell wall")

P is positive to indicate water in the cell is under compression.

According to Taylor and Ashcroft (1972) most of the water in the soil "accumulates as wedges at the contacts of soil particles." Water also adheres to soil particles; at FC the water film is about twelve molecular layers. Taylor and Ashcroft continue

> Living plants must get all of their water from these thin films and wedges. In so doing they must overcome forces of adhesion, cohesion, and osmosis that hold the water in the soil. . . . When there are about four or five molecular layers of water remaining, the attraction of the soil for water becomes so great that the plants can no longer serve their needs.

For most soils the soil moisture tension when the film is down to four or five molecular thicknesses, i.e., at PWP, is frequently estimated as about 15 bars (Wadleigh, 1955), though measured on a volumetric or weight basis, PWP can vary greatly depending largely on soil texture.

Transpiration

The major use of water by plants is through the process of transpiration. Jensen (1968) defines transpiration as "the loss of water in the form of vapor from (the cells of) plants. All aerial parts of plants may lose some water by transpiration, but most water is lost

through leaves in two stages: (1) evaporation of water from cell walls into intercellular spaces, and (2) diffusion through stomates into the atmosphere." Some water vapor also diffuses out through the epidermal cells of leaves and the cuticle. Evapotranspiration (ET) is the sum of water lost by transpiration and evaporation from the soil and plant surface.

One model views the plant as a "wet rag" transpiring water to the atmosphere at roughly a proportional rate as that from an open body of water. This view, according to de Wit (1955, p. 34), is correct providing the plant is "well supplied with water." Transpiration is caused by a source of energy (the sun) which provides the heat of vaporization of the water. Hence, ET rates are generally assumed to be less than evaporation from standard evaporation pans. Transpiration is also limited by soil water availability, however, and the "wet rag" model does not adequately predict ET in this case.

A second model views the entire plant as hydraulically connected to the soil. This model assumes the plant could grow normally providing soil moisture tension does not become too large. This is basically the view of Viehmeyer and Hendrickson as discussed by Gardner (1968). Growth is in some way inhibited when tension becomes too great, though again precise definition of "growth" has still not been made.

An important point to make is that the plant is not just a wet rag or cylinder filled with water, but is more complex. Vaadia and Waisel (1967) question the assumption that "all parts of the cell or tissue are at the same water potential. There are indications that considerable tensions can develop in some parts of corn plant leaves under conditions of normal transpiration, without any symptoms of leaf wilt or curl."

According to Kramer (in Norman, 1959, p. 53)

> A slight decrease in turgidity sometimes is accompanied by increased opening of stomata, but further reduction is nearly always accompanied by a decrease in stomatal aperture. . . . One effect of stomatal closure is to reduce transpiration, because by far the larger fraction of water loss occurs through the stomata. This reduction would be desirable in itself . . . but, unfortunately, they also reduce photosynthesis by reducing the supply of carbon dioxide.

The preceding statement is thus a justification for using transpiration as a proxy measure of plant growth. By maximizing transpiration, photosynthesis is also maximized since when transpiration occurs stomata are open and carbon dioxide is entering the plant. Advancing one step further one can postulate that lack of transpiration, hence photosynthesis would have greater effect in particular growth stages than in others. This reasoning leads to the production function discussed in Chapter III.

Another conclusion of the foregoing discussion is that transpiration is affected by two types of variables: those relating to soil moisture levels and those relating to climatic conditions. The relative importance of these two variables, especially with respect to timing, has not been determined. Relating seasonal ET directly to yields has been attempted by various authors (e.g., Stewart, et al., 1973a). However, for corn (see Figure A.1) the unexplained variance for this relation is large. It is evident that a more detailed understanding of the mechanisms of plant growth is needed.

Models of Plant Growth

Water Requirement Models

Perhaps the simplest and most widely used approach to planning water requirements is that proposed by Blaney and Criddle (Blaney,

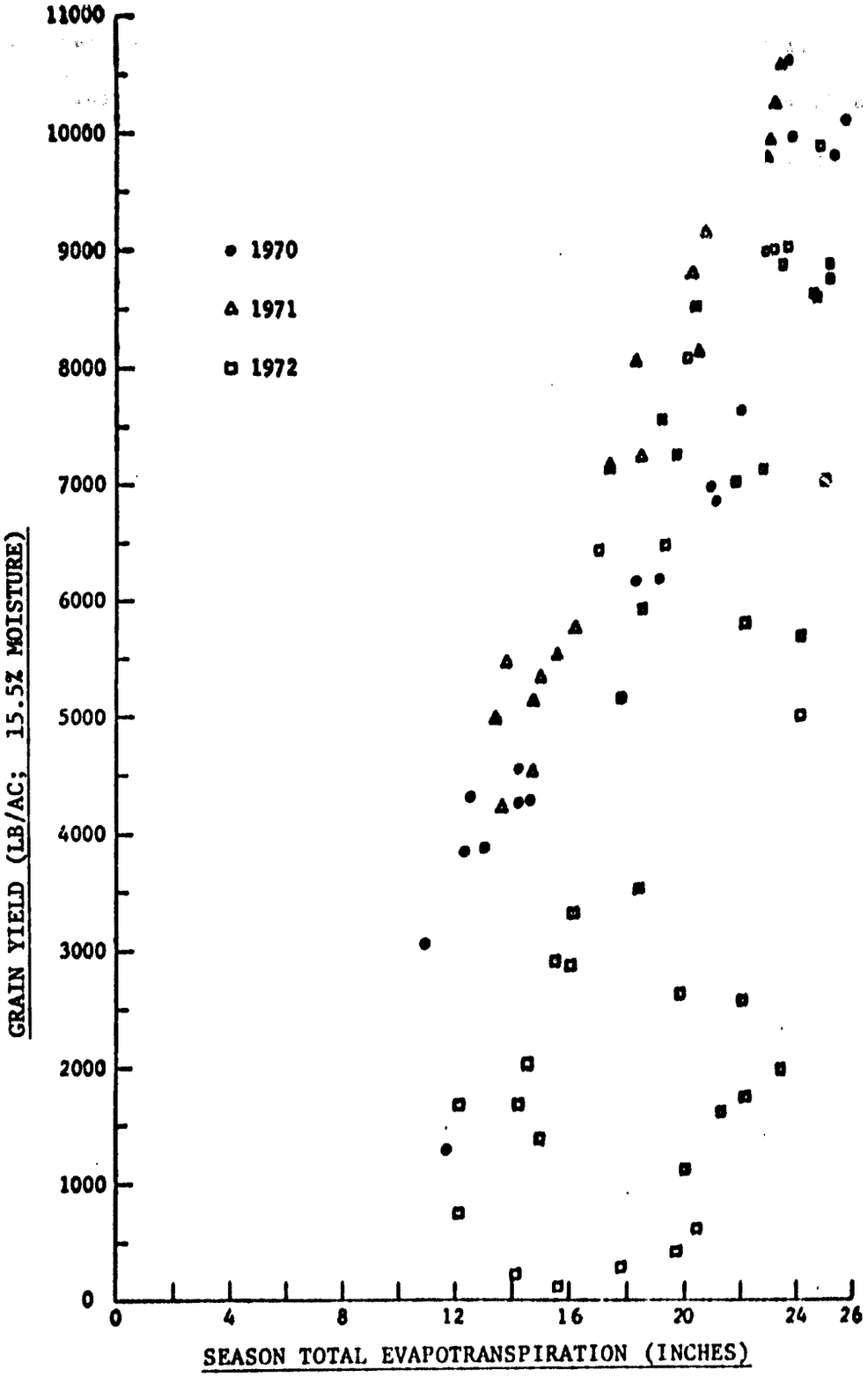

Figure A.1

Y VS. ET RELATIONS OF CORN FOUND IN THREE YEARS OF FIELD EXPERIMENTS AT DAVIS, CALIFORNIA, IN WHICH IRRIGATION DATES AND DEPTHS WERE VARIED, WITH OTHER GROWTH FACTORS NON-LIMITING (From Stewart, et al., 1973a)

1955). It is an empirical approach to estimating the water requirements (i.e., evapotranspiration) of a crop where water shortage for the crop is not considered. The formula is expressed as

$$U = \sum_{i=1}^{n} t_i p_i K \qquad (A.2)$$

in which

n = number of months in growing season

t_i = mean monthly temperature (degrees F.)

p_i = percentage of day time hours in the year during month i

K = a constant for a particular crop for a particular locality

U = consumptive use (ET) (in.).

An advantage of this method is that it has wide acceptance and the coefficients have been determined for various localities. The method has been successfully applied to proposed irrigation schemes to determine water requirements.

A disadvantage of the method is that it considers only one method of production, that is, by fully irrigating the crop. A major objection is that the method is site specific, requiring judgment of the engineer when applying the method to a new area. The lack of theoretical basis of the method is evident.

Hall Model

In contrast to the gross water requirement model of Blaney and Criddle, Warren A. Hall proposes a model that looks at the function of water in the plant. In this model, the individual growth stages of the plant are isolated by function--i.e., for corn these may be vegetative growth, which is necessary for continued growth; pollination, during

which number of grains are determined, and grain formation, where grain filling (volume and density) occurs. The yield may be expressed as

$$Y = \delta \eta \nu \rho \qquad (A.3)$$

in which

δ = some function of quality of vegetative growth, zero when death occurs, one when growth is maximum (dimensionless)
η = number of kernels
ν = volume of each kernal (average) (cm^3)
ρ = density of each kernel (average) ($kg./cm.^3$)

This model assumes the various growth periods are independent. Lacking the theoretical knowledge to determine the effect of soil moisture on each growth stage, Hall in a report to the Parsons Company (Parsons, 1968) has determined these effects empirically through field trials. This model was further discussed in Chapter II.

Indicator Approach

Theoretically, Hall's model could be used for planning cropping patterns or scheduling irrigation, whereas the Blaney Criddle method is used primarily in planning. Various methods have been proposed for scheduling irrigations, including those relying on some type of indicator. Soil moisture status is one type of indicator that will be discussed later. Included in the list of indicators presented by Haise and Hagan (1967) are plant color; plant, particularly leaf, movement; use of an indicator plant; and fruit, leaf, stem and trunk growth.

Hiler (1972) recommends the use of a "pressure bomb" to measure plant water potential. He has developed coefficients relating the plant water potential at various growth stages to yields. Limited data are presented to verify the method presented.

The general criticism of the Haise and Hagan indicators is that the plant may in some way be damaged before the indication is shown. A corn plant losing color indicates loss of turgor but does not predict when an irrigation should take place. A quantitative relation is needed between inputs and the indicator and between the indicator and yields. Lacking both of these, an optimization model is impossible. Eiler's method shows promise, but again, even if leaf water potential is highly correlated to yields, the theoretical connection is still needed between leaf water potential and quantity of water supplied in order to successfully model this system.

Energy Balance Methods

The basis of energy balance methods is that evaporation from crops which are fully watered is a function of potential evaporation. According to Jensen and Heerman (1970), the most common equation for predicting potential evaporation is a combination equation containing radiation and aerodynamic terms presented by Penman.

$$E^* = \frac{\Delta}{\Delta+\gamma}(R_n - G) + \frac{\gamma}{\Delta+\gamma}(15.36)(1.0 + .01W)(e_s - e_d) \qquad \text{(A.4)}$$

in which

R_n = solar radiation term

G = soil heat flux term (empirically related to temperature)

$\frac{\Delta}{\Delta+\gamma}, \frac{\gamma}{\Delta+\gamma}$ = mean air temperature weighting terms

W = total wind run (miles)

e_d = estimated actual vapor pressure based on saturated vapor pressure at mean dew point

e_s = mean saturated vapor pressure.

For a complete definition of terms and complete discussion, see Jensen and Heerman (1970). Results of a program based on this equation are presented in Neghassi (1974) and are discussed in Appendix B.

Once an accurate prediction E^* is obtained and converted to units of inches of water per day, various crop factors are introduced to express actual evapotranspiration (E_t) as a function of E^*.

$$E_t = (K_{co}K_a + K_s)(.000673)E^* \tag{A.5}$$

in which

K_{co} = mean crop coefficient depending on stage of growth (ranging from .2 at planting to 1.0 at heading for corn, see Jensen, 1968)

K_a = function of available soil moisture

K_s = a coefficient to increase E_t on the first three days after a rainfall or irrigation.

The basic question raised by this equation is the precise form of the soil moisture function that determines K_a. Considering the crop on a particular day and neglecting the K_s term, the function that has been used is

$$E_t = \frac{C \cdot \ln(ASM + 1) \cdot E^*}{\ln(101)} \tag{A.6}$$

in which

C = constant term

ASM = available soil moisture

Hanks (1974) tested this relation using field data, but no theoretical justification for this formulation appears to have been presented. Further discussion is contained in the next section.

Soil Moisture Balance Methods

Whereas energy balance methods tend to relegate the effects of soil moisture to coefficient status, soil moisture balance methods tend to do the same for climatic effects.

The soil moisture balance method established by Thornthwaite and Mather (1955) used the equation

$$\Delta SM_i = P_i + IRR_i - RO_i - D_i - ET_i \qquad (A.7)$$

in which

i = time period
ΔSM = change in soil moisture
P_i = effective precipitation
IRR_i = irrigation
RO_i = runoff
D_i = drainage (i.e., deep percolation)
ET_i = evapotranspiration; all units are in cm.

In contrast to the argument of Viehmeyer and Hendrickson that ET remains at or near the maximum rate until a high soil moisture tension, Thornthwaite and Mather (1955) stated that "when one-half of the water is gone, the rate of ET falls to one-half of the potential rate. . . ."

A distinction may be made between potential ET and maximum ET. Potential ET, PET usually refers to a standard well watered reference crop such as alfalfa with 12 to 18 inches of top growth (Jensen and Heerman, 1970). PET has also been related to lake or pan evaporation (Mustonen and McGuinness, 1968). ET_{max} in this study refers to the maximum ET possible from the particular well watered crop for the given growth stage and given climatological conditions.

From Viehmeyer and Hendrickson's work and the work of Thornthwaite and Mather and others, various authors have attempted to relate

ET_{actual}/ET_{max} or ET_{actual}/PET to available soil moisture. Summaries of such relations are given by Mustonen and McGuinness (1968) and Minhas, et al., 1974), and are shown in Figure A.2.

Yaron, et al., (1973) adopted a linear relation which, when scaled to percentage ASM yields

$$ET_{ij} = b_{ij}(ASM_{ij}) \tag{A.8}$$

in which

j = index of month

i = index of soil layer

b_{ij} = empirically determined coefficient.

The coefficient is chosen to best fit available data and should theoretically sum (over the soil layers) to ET_{max} if the linear relation is assumed valid. The problem with this formulation is that all climatic effects are included in the coefficients b_{ij}.

Denmead and Shaw (1962) complicated the question of the shape of the ET versus ASM curve (see Figure A.3). They looked at transpiration only and regarded potential transpiration as equal to the transpiration rate at FC (T_{FC}). They introduced T_{FC}, which is affected by climate, as an additional variable in the diagram, and were thus able to explain nearly all of the scatter of the observed points.

In simplified form, then, the results of Denmead and Shaw indicate that on a day of low T_{FC} (i.e., a cool, humid, cloudy day), a plant could transpire near the maximum rate even at low ASM levels. Conversely, on a day of high T_{FC} (i.e., a hot, cloudless, windy day) the plant could transpire near the maximum rate only if ASM is near FC.

The curves given by Denmead and Shaw suggest the daily relation (see Figure A.4):

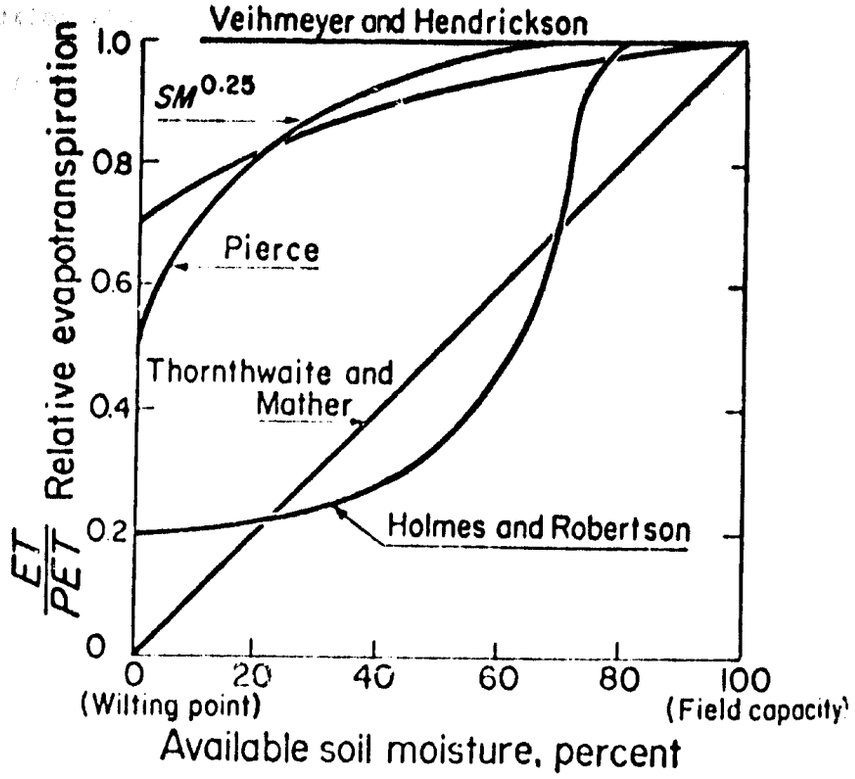

Figure A.2

RESULTS OF VARIOUS STUDIES WHICH SHOW THE DEPENDENCE OF RELATIVE EVAPOTRANSPIRATION ON SOIL MOISTURE
(From Mustonen and McGuinness, 1968)

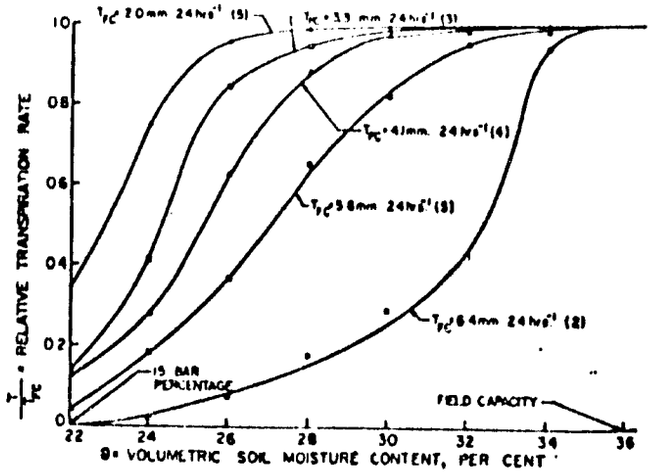

Figure A.3

RELATIVE TRANSPIRATION RATE AS A FUNCTION OF SOIL MOISTURE CONTENT FOR DIFFERENT POTENTIAL TRANSPIRATION CONDITIONS. THE CURVES REPRESENT DAYS ON WHICH THE TRANSPIRATION RATES AT FIELD CAPACITY HAD THE VALUE SHOWN IN THE BODY OF THE FIGURE. THE NUMBERS IN PARENTHESES REFER TO THE NUMBER OF DAYS OF OBSERVATION REPRESENTED BY EACH CURVE.
(From Denmead and Shaw, 1962)

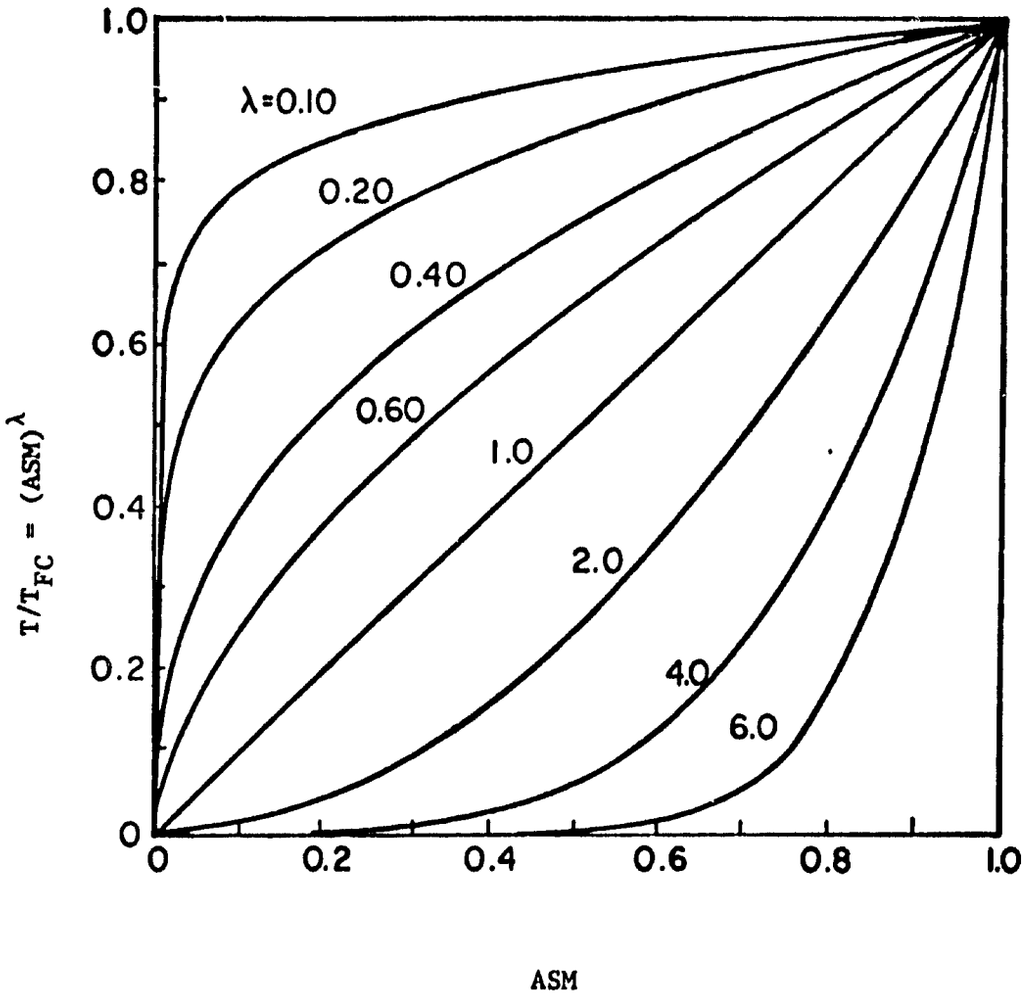

Figure A.4

RELATIVE TRANSPIRATION AS A FUNCTION OF
AVAILABLE SOIL MOISTURE WITH CLIMATE PARAMETER λ
(Adapted from Neghassi, 1974)

$$\frac{T}{T_{FC}} = (ASM)^{\lambda} \qquad (A.9)$$

in which

λ = some function of T such that when T_{FC} is low $\lambda<1$ and when T_{FC} is high $\lambda>1$.

Minhas, et al., (1974) describe a similar family of curves but do not relate the exponential parameter to climate.

Flynn and Musgrave (1967) employ SM in a different approach to those previously described. They use empirical data to fit a quadratic function between terminal soil moisture in period i and days of growth (d_i) in period i, in which d_i divided by total number of days in period i is an index of plant growth for the period. The d_i are assumed to be additive over the season. In addition to these assumptions the authors must assume some relation between irrigation inputs and terminal soil moisture. The authors do this by statistical curve fitting without looking at climatic conditions or the actual physical processes involved.

The problems of the approach of Flynn and Musgrave are obvious. By making such gross assumptions the authors generally obtain low values of the coefficient of multiple determination R^2 (a statistical measure of the percentage of explained variance). In addition, by neglecting climatic influences, the authors limit the predictive applicability of their work. The fitted equations assume a given level of solar radiation, etc.; hence are not relevant in the next year when these climatic conditions change.

In terms of a planning model the equations would be useful when planning to maximize expected value if the year of reference was in

some way "the mean" year and climatic conditions were in some way normal or at least not highly skewed about the reference year's climate.

The type of model developed by Flynn and Musgrave is termed "deterministic." That is, variables which are actually stochastic, i.e., known only with a probability distribution, are assumed constant. For planning purposes a deterministic model may be adequate, but caution is needed, especially when working with empirical data.

Agronomic Field Research

Lacking precise relations between input and yield for crops, and in order to separate climatic from soil moisture effects, rather large scale field trials (experiments) are necessary. In these trials the crop under study is grown in a number of plots under various irrigation regimes.

The Ideal Field Experiment

According to Hall and Dracup (1970), the ideal field experiment for corn, for example, would be to perform trials which stress the crop to varying degrees in each growth stage. Here, "stress" refers to lowering soil moisture to such an extent that reduced yields result. Suppose eight growth stages are considered and four levels of stress are desired. This would require 32 plots. Usually several replications (repetitions of the same experiment) are desired to assure accurate results. The plots thus far described assume stress in one period and full irrigation in the remaining periods. These trials would allow determination of curves such as Figure A.5 and hence the multiplicative terms of Equation A.3 as functions of SM.

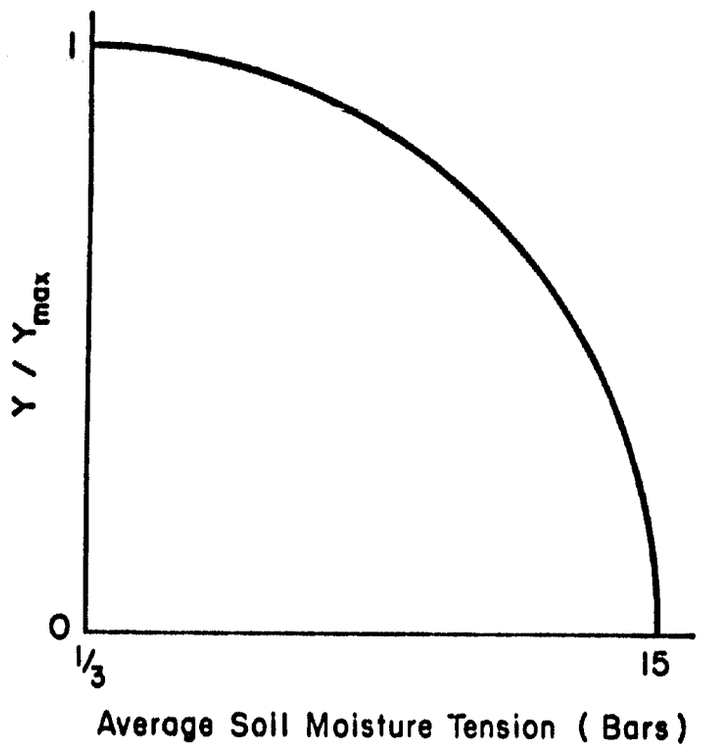

Figure A.5

RELATIVE YIELD AS A FUNCTION OF AVERAGE STRESS IN TIME PERIOD i GIVEN FULL IRRIGATION IN OTHER TIME PERIODS

The term "stress" is still not well defined at this point. It could refer to average SM (in tension units or units of ASM) over the time period or to terminal SM as used by Flynn and Musgrave.

In order to test fully the multiplicative relation of Equation A.3, plots which are stressed in various time periods are required. If every combination was desired the number would be 2^8 with one level of stress and 8 growth stages, $4 \cdot 2^8$ with four levels of stress and $3 \cdot 4 \cdot 2^8$ with three replications. It is clearly impractical to perform this many experiments.

In the absence of such exhaustive field trials a more realistic approach must be taken. This approach is to start with a certain hypothesis and test this with available data.

For this study, the hypothesis involves relating evapotranspiration to yields. The use of transpiration as a surrogate variable for measuring yields is not without theoretical basis as discussed in the Transpiration section. For the purpose of this study, ET is assumed to be a close enough measurement of transpiration. Other authors (e.g., Hanks, 1974), however, have developed empirical relations between transpiration and ET. The advantage of using ET is that it is readily calculated from field data.

In order to test the ET vs. yield hypothesis, relevant field experiments must be available. The requirements of the experiments are that (1) ET must be calculable, (2) there must be a sufficient number of plots to allow statistical analysis, (3) yields must be measured, and (4) other inputs such as plant populations, fertilizer levels, etc., must be held constant. Unfortunately, the number of experiments which fulfill the above criteria are few.

Lysimeter Experiments

Mustonen and McGuinness (1968) describe ET experiments on a deep rooted grass legume mixture using a field lysimeter. The lysimeter (see Figure A.6) weighs a large amount of soil which has been reconstructed to simulate the natural soil profile. The lysimeter is generally planted with the same crop as the surrounding area to minimize unusual aerodynamic effects. Runoff and percolate can be measured so that ET can be determined in a particular time period from the relation

$$ET_i = \Delta W_i + IRR_i + RF_i - RO_i - D_i \qquad (A.10)$$

in which

ΔW = change in weight converted to inches of water.
Other variables as previously defined.

The effect of dew is assumed negligible and is not measured. The lysimeter is the most accurate means of measuring ET providing the weighing mechanism is in good order. Mustonen and McGuinness describe the limitations of the lysimeter. The expense of the lysimeter limits its application in determining ET for the large number of field trials needed in determining the effect of ET on yields.

Field Measurement of ET

As an alternative to using lysimeters, ET can be determined *in situ* by measuring differences in soil moisture. The basic mass balance relation is

$$ET = \Delta SM_i + IRR_i + RF_i - RO_i - D_i \qquad (A.11)$$

in which all terms are in inches of water and ΔSM is the change in soil moisture in the root zone. Other terms are as previously defined. Providing rainfall and irrigation rates do not excede the soil infiltration rate and total quantities of water input do not bring soil

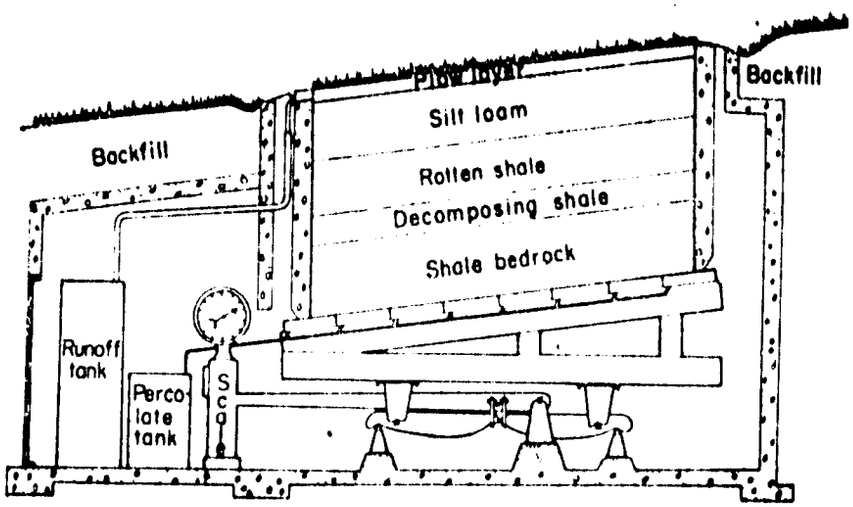

Figure A.6

SCHEMATIC DIAGRAM OF LYSIMETER
(From Mustonen and McGuinness, 1968)

moisture above FC, runoff and percolation can be assumed negligible. Another necessary assumption is that upward and lateral flow of water into the root zone are negligible.

Soil moisture can be measured in a number of ways (see Holmes, et al., 1968). Recently, the neutron probe (vanBavel, 1964) has become widely used. A source consisting of Ra-Be emits fast neutrons which lose energy as they collide with hydrogen atoms. The density of the resulting sphere of slow neutrons provides indirectly a measure of the water content of the soil.

In order to make a measurement the neutron probe is inserted into the soil profile through a small diameter access tube. Measurements are usually made every foot to the bottom of the root zone (approximately 7 feet for corn, but dependent upon soil profile characteristics), providing an average 6 inches above and below the probe. In the approximate top 6 inches of soil the instrument cannot accurately measure soil moisture.

Van Bavel (1962) states the "standard deviation from the (calibration) curve is about 0.5 percent of moisture by volume and presumably, this figure will be typical for a value estimated with aid of the calibration curve." Van Bavel recommends making a standard count with a known soil sample (a bucket of saturated sand) once every ten readings. A measurement problem may be encountered when the soil mass shrinks away from the access tube. In such cases, if measurements are made after a rain or irrigation, SM values may be higher than the actual values.

Description of Field Trials

Jensen and Sletten (1965) performed ET field experiments with wheat in Texas for three seasons. There were 6 irrigation treatments with 4 replications for each of 6 fertility levels. The ET values were calculated using Equation A.11 in which soil moisture measurements were found by taking core samples and determining water content by weighing before and after drying.

Three irrigation treatments were irrigated when SM tension reached prescribed levels, one treatment was pre-irrigated only, another was irrigated only once at jointing, and the final treatment was irrigated "according to precipitation distribution during the season."

Seasonal ET was related to yields as reproduced in Figure A.7. A trend was noted for a dry year similar to Figure A.4 indicating the relation

$$\frac{Y}{Y_{max}} = \left(\frac{ET}{ET_{max}}\right)^{\alpha} \qquad 0 \leq \alpha \leq 1 \qquad \text{(A.12)}$$

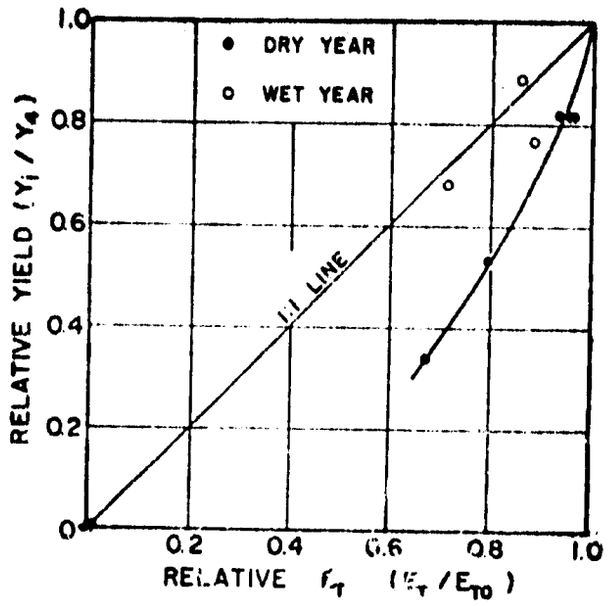

Figure A.7

RELATIVE YIELD DECREASED MORE RAPIDLY THAN RELATIVE SEASONAL EVAPOTRANSPIRATION IN A DRY YEAR (1956) WHEN IRRIGATIONS WERE DELAYED. IN A WET YEAR (1958) A DEFINITE TREND WAS NOT APPARENT.
(From Jensen and Sletten, 1965)

However, no relation was obtained for wet years. With only six treatments and some of those obviously over-irrigated, there were limited data points.

Stewart, et al., (1974a) describe field trials at Davis, California, in 1972 and 1973, with corn and grain sorghum. Stewart (1972) describes previous corn trials in 1969-71. The 1972 and 1973 experiments are most relevant since fertilizer application is held constant. Three growth periods are considered: vegetative, pollination and grain filling with eight irrigation regimes. Evapotranspiration was calculated weekly using Equation A.11 where neutron probes were used to determine soil moisture levels.

Danielson and Twyford (1974) reported on field trials on corn conducted in 1972 at Fort Collins, Colorado. Corn at three-plant population levels was irrigated under 11 irrigation regimes. Soil moisture measurements were made by neutron probes at least weekly during July and August and less frequently in June, September and October.

In the Fort Collins trials, the chief variable under study was the effect of irrigation during flowering. Twyford (1973) concluded (for the conditions at Fort Collins) that "a single irrigation, using a limited amount of water, early in the flowering period, proved greatly significant in increasing grain yield." In this trial, number of kernels was greater than in other trials receiving the same quantity of water but at different timings. He also concluded that a later irrigation (16 days after the first) increased kernel weight. These results support the theory of Hall summarized by Equation A.3.

In 1974, four universities (Colorado State University, Utah State University, Arizona State University and University of California, Davis) conducted field trials on corn. The data from Colorado State University was made available for this study and this experiment is described herein.

An early maturing field corn variety (Northrup King PX20) was planted in 30-inch rows in 4 blocks of uniformly deep Nunn clay loam on May 14, 1974. Each block consisted of 48 rows, approximately 100 feet in length.

The plant population level was uniform throughout. Fertilizer (102 lbs./acre of P_2O_5) was applied prior to planting. At planting, 120 lbs./acre of nitrogen fertilizer was applied.

A sprinkler line ran the length of each block, oriented on an east-west axis, the direction of prevailing winds. The sprinkler was designed to irrigate most heavily along the center of the block with application amount declining linearly so that outer rows were virtually unirrigated. Each block received a different irrigation schedule (see Table A.2).

Soil moisture measurements were made by neutron probes at access tubes located at 12 positions across each block. Readings were taken every foot to a depth of 7 feet. Soil moisture measurements were made at the beginning and end of the season and prior to each irrigation. Rain guages were located near each access tube so that Equation A.11 could be solved for ET at each tube, where the SM at the tube was assumed to be representative of the two rows on either side, toward and away from the sprinkler.

For each block the date when the first silks appeared was July 20. The milk to soft dough stage was approximately August 26. In each block 30 feet was harvested from all except the two outermost rows. Date of harvesting was mid-October. Yields were converted to kilograms of grain per hectare at 15.5% moisture.

This experiment was designed to obtain 48 trials, though assuming equal distribution of irrigation water on both sides of the center, there are 24 trials with 2 replications. The advantage of this experiment was that a large number of trials were generated at less expense than conventional methods. The disadvantage is that only four different timings were included.

Table A.2

IRRIGATION AND MEASUREMENT SCHEDULE FOR CORN TRIALS
AT CSU, 1974
(1 indicates irrigation, 0 indicates no irrigation)

		Irrigation Dates						
Measurement Sites	Block #	July 4	July 17	July 25	July 31	August 7	August 14	August 23
1-12	I	1	1	1	1	1	1	1
13-24	II	1	0	0	0	0	1	1
25-36	III	1	1	0	0	0	1	1
37-48	IV	0	1	0	0	0	0	0

Measurement Dates					
July 1	July 16	July 23		August 13	September 11
16 Days	7 Days		21 Days		29 Days

Summary

This chapter has presented information regarding the theory of predicting yield and background information regarding empirical research that has been carried out.

Two approaches to predicting yield have been presented. One is to determine water requirements which will meet a certain value of potential yield. A second approach is to view yield as a variable which may be expressed as a continuous function of some type of growth measuring variables which can be measured over the season. The Blaney-Criddle method, the indicator approach and the energy balance methods fall into the first category while the Hall model falls into the second category. If transpiration or ET is an acceptable growth measuring variable, a continuous function could be determined so that the energy balance methods or the soil moisture balance methods can be tied to predicting variable yield.

The second approach is that accepted by the systems analyst or economist who is concerned with determining the optimal allocation of a scarce resource, in this case, water. In the current study, this variable yield approach is adopted and incorporated into an optimization model.

In order to develop a continuous yield function, a growth measuring variable must be determined and tested. The variable selected for study is ET. The last section of this chapter has described two methods of measuring ET in field experiments and has described various field trials in which irrigation amounts and times have been varied to determine effects on yields. From these experiments the relation between inputs (time and amount of irrigation) and outputs

(yield) can be determined using ET as the proxy variable for plant growth.

Appendix B

THE PRODUCTION FUNCTION APPROACH

Introduction

The main concern of this study is to aid the farmer in determining his cropping pattern in order to maximize expected return. The amount of this return is determined, finally, by the quantity and quality of the crops that are harvested. The economic aspect of this study, then, is the determination of the relation between inputs, yields, and dollars returned.

As discussed in Appendix A, a simple relation between inputs and outputs does not exist. One problem is that for most crops, yield is not a variable that can be measured over the season. Some crops such as grasses and alfalfa are harvested for their forage value. For these crops, adding an additional day of growth may increase the yields linearly over a wide range of days providing adequate inputs are supplied.

In the more general case, in which crops are harvested for the value of their fruit, grain or root such as corn, wheat, sugar beets, etc., the effect of conditions during various physiological growth stages is important. These "conditions" combine, though not necessarily additively, to produce the final yields.

The "conditions" that affect the final yield may be broken down as controllable and uncontrollable variables. Soil moisture levels

and fertility levels are controllable through irrigation and fertilization while atmospheric factors are uncontrollable. Precipitation also contributes to the soil moisture level thus making soil moisture somewhat uncontrollable.

The uncontrollable (and even the controllable) factors are subject to uncertainty and risk. Rainfall, solar radiation, date of first frost, etc., are not known exactly, but are known with a probability distribution if that much data are known. Information regarding other factors affecting production and return such as availability of inputs and crop price may be totally lacking or unreliable.

These problems of risk and uncertainty together with the problems of measuring yield as a variable over the season complicate any attempt to construct a model of the system that will aid in decision making.

Production Functions

The Ideal Production Function

The ideal production function of economic theory (Dillon, 1968) relates one or more variable inputs, other inputs being held constant, to total product, in this case yield (see Figure B.1). It is assumed that this function consists of three stages. In stage one there are increasing returns to resource inputs, i.e., each additional unit of input adds more to output than the previous unit of input. In stage two, returns are positive but decreasing, and in stage three returns are negative. Stage two is regarded as the only stage of rational economic operation. For further discussion, see Heady and Dillon (1961).

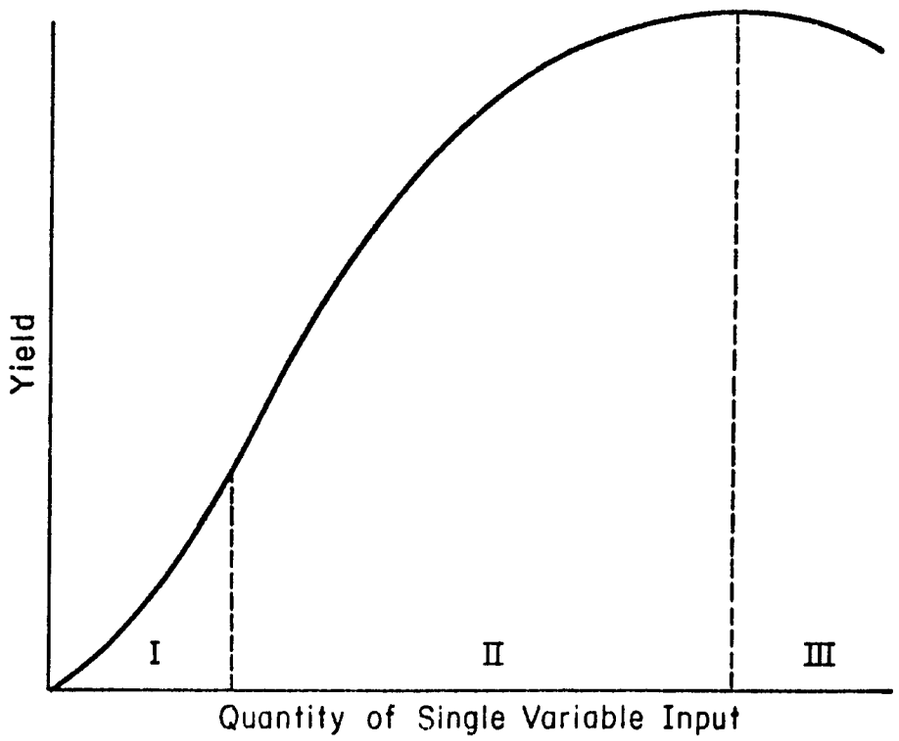

Figure B.1

STAGES OF PRODUCTION FOR AN IDEAL
PRODUCTION FUNCTION WITH ONE VARIABLE INPUT

In this study, water is viewed as the variable input. However, another variable, time, is also important. The time of application of the water input can shift the ideal production function to any of its three stages. Supplying water to corn early in the silking stage could result in increasing returns, while irrigating when the water table is high might result in negative returns.

Various mathematical forms of production functions have been proposed for crop production. In the next sections three functions which consider timing of inputs in various growth stages will be discussed. All of these are multiplicative functions as suggested for grain type crops by Jensen (1968). The limitations of multiplicative functions are discussed in a following section.

Mitscherlich Function

The Mitscherlich or Spillman equation is

$$Y = Y_{max} \prod_{i=1}^{I} (1 - B_i e^{-k_i x_i}) \qquad (B.1)$$

in which

Y = yield in suitable units
Y_{max} = maximum yield
i = index of the growth stage ($i = 1 \ldots I$)
x_i = dimensionless soil moisture index
B_i and k_i are parameters.

In this equation, the maximum yield is approached asymptotically where all x_i approach infinity and the minimum is where any of the x_i are zero and B_i is one or where all of the x_i are zero and none of the B_i are one. When all x_i are zero the yield is given by

$$Y = Y_{max} \prod_{i=1}^{I} (1 - B_i) \qquad (B.2)$$

Thus the B_i parameters are related to minimum yields.

For the one growth stage case the marginal product, MP, or rate of change of yield with respect to the single input x is given by

$$MP = \frac{dY}{dx} = - k(Y_{max} - Y) \qquad (B.3)$$

Thus the slope of the yield curve is proportional to the difference between the asymptotic yield and actual yield, and the function exhibits positive but decreasing returns for all values of x.

For the general multiple time period case the parameters k_i express the relative weights of the various time periods. The larger k_i, the larger the potential yield reduction for a given reduction in the soil moisture index.

An advantage of this function is that its parameters have some physical meaning as opposed to, for example, a polynomial formulation. Considering two time periods a polynomial would have the form

$$Y = a_0 + a_1x_1 + a_2x_2 + a_3x_1^2 + a_4x_2^2 + a_5x_1x_2 + \ldots \qquad (B.4)$$

in which the a_i are parameters to be determined from experimental data. This is a more general formulation, since any function may be approximated by its Taylor series, but the parameters have no physical meaning.

A disadvantage of the Mitscherlich function is that the soil moisture index must vary from 0 to ∞. A more natural index would vary from 0 at PWP to 1 at FC. Another disadvantage is that to fit parameters to actual data requires a non-linear least squares algorithm that restricts the k_i parameters to non-negative values. Neghassi (1974) encountered this problem when fitting the equation to data for wheat, obtaining negative parameters.

Cobb-Douglas Type Functions

A second function is the Cobb-Douglas type

$$Y = Y_{max} \prod_{i=1}^{I} (A_i)^{\lambda_i} \qquad (B.5)$$

in which

A_i is some index of growth, $0 \leq A_i \leq 1$

λ_i is a parameter, $0 \leq \lambda_i \leq 1$

The Cobb-Douglas type equation arises often in production economics. Similar to the Mitscherlich equation, the foregoing Cobb-Douglas formulation approximates only stage two of the ideal production function. The particular formulation of Equation B.5 differs from the usual in that the variables are not inputs, but rather are indices of growth which must in come way be related to inputs.

In the form of the Cobb-Douglas type equation of Equation B.5, the exponent is the parameter. This type is common in economic theory in the form where the base is quantity of input x_i and the exponent λ_i represents the output elasticity defined as

$$\lambda_i = \frac{\Delta Y/Y}{\Delta x_i/x_i} \qquad (B.6)$$

in which the parameter λ_i measures the percentage change in yield for a unit percentage change in inputs.

In the formulation of Equation B.5, however, the base is not an input but rather an index of growth. This index ranges from zero indicating crop loss to one indicating full growth during the time period. When all the A_i are one, the yield takes on its maximum value Y_{max}. Although the λ_i does not represent a conventional elasticity, it does express the relative weights of the index in each growth period.

Discussion of Multiplicative Functions

A general criticism of both the Cobb-Douglas types and the Mitscherlich functions is that all presuppose a multiplicative stress effect. That is, when a plant is stressed (i.e., a term on the right-hand side is less than 1) in periods i and j, the effect on yield is the product of the individual stresses.

Supposing that growth stages are well defined, as an example, let the stress effect in experiment I be 0.8 in growth stage one with remaining stages unstressed. The predicted yield is

$$Y_I = Y_{max}(.8)(1)(1) \ . \ . \ . \ (1) = .8Y_{max} \tag{B.7}$$

Let the stress effect in experiment II be 0.6 in stage three, while all other periods are unstressed. The predicted yield is

$$Y_{II} = Y_{max}(1)(1)(.6)(1)(1) \ . \ . \ . \ (1) = .6Y_{max}. \tag{B.8}$$

Next, consider a third experiment where the plant is stressed in stages one and three so that, as before, the terms are 0.8 and 0.6 respectively. According to the multiplicative relation, the yield must be

$$Y_{III} = Y_{max}(.8)(1)(.6)(1)(1) \ . \ . \ . \ (1) = .48Y_{max} \tag{B.9}$$

This multiplicative effect is present in all production functions discussed and limits their applicability. Experiments with some crops have revealed a conditioning effect (see for example Stewart, et al., 1974a), that is, once the crop is stressed initially and then stressed later in the season, the decreases in yields are not as great as if the crop is stressed only in the later time period. This conditioning effect cannot be shown with the multiplicative functions discussed here. The functions discussed will underestimate yields in cases of conditioning.

Another problem inherent in the multiplicative functions discussed in this chapter is that yield is not directly related to inputs. For the particular production function of Equation B.5, a model must be constructed that maintains a water balance. The model must keep track of irrigation amounts and timing, precipitation, and be able to predict ET from available soil moisture in the root zone and climatic variables.

Comparison of Additive and Multiplicative Functions

Suppose two functions, one multiplicative and one additive are proposed to measure relative yield using the variable $\underline{x}$ which is a measure of relative growth by growth stages i, with $x_i \leq 1$. Let the functions be given by

$$f(\underline{x}) = \prod_{i=1}^{n} (x_i)^{\lambda_i} \tag{B.10}$$

and

$$g(\underline{x}) = \sum_{i=1}^{n} \alpha_i (x_i) \tag{B.11}$$

in which λ_i and α_i are empirically determined parameters and n is the number of growth stages. It is known that $\lambda_i \geq 0$, $0 \leq \alpha_i \leq 1$ and that

$$\sum_{i=1}^{n} \alpha_i = 1. \tag{B.12}$$

It is desired to determine a correspondence between the two equations, at least in the neighborhood of maximum yields, i.e., where all x_i are "close" to one.

First, x^{λ} may be expanded as

$$x^{\lambda} = 1 + \ln(x) + \frac{(\lambda \ln(x))^2}{2!} + \tag{B.13}$$

Neglecting second order terms and higher results in

$$f(x) \approx \prod_{i=1}^{n} (1 + \lambda_i \ln(x_i)) \tag{B.14}$$

A further expansion is

$$\ln(1 + a) = a - 1/2a^2 + \frac{a^3}{3} \ldots \quad \text{for } -1 \leq a \leq 1 \tag{B.15}$$

Letting $x = 1 + a$ results in

$$\ln(x) = (x - 1) - 1/2(x - 1)^2 + \ldots \tag{B.16}$$

Again, with x_i close to one, the second order terms and higher may be neglected to give

$$f(\underline{x}) \approx \prod_{i=1}^{n} (1 + \lambda_i (x_i - 1)) \tag{B.17}$$

or, again neglecting higher order terms,

$$f(\underline{x}) \approx 1 + \sum_{i=1}^{n} \lambda_i (x_i - 1) + \ldots \tag{B.18}$$

which gives

$$f(\underline{x}) \approx 1 - \sum_{i=1}^{n} \lambda_i + \sum_{i=1}^{n} \lambda_i x_i . \tag{B.19}$$

If the λ_i are empirically shown to sum to one or if this result is shown by economic theory, the correspondence is achieved that

$$\lambda_i \approx \alpha_i . \tag{B.20}$$

Adoption of Growth Index and Production Function

Growth Indices

Hanks (1974) used a transpiration index; however, in this study an ET index is adopted. The general relationship is

$$Y = f(Y_{max}, \text{ growth index in various growth stages}). \tag{B.21}$$

The maximum yield is empirically determined for a particular site. The transpiration index of Hanks is the ratio of actual to potential

transpiration T_p. Potential transpiration is in turn an empirical function of free water evaporation, days since last wetting, and percentage of plant cover. Actual transpiration T is a function of T_p and available soil moisture ASM.

The ET index is the ratio of actual to maximum ET. In the Jensen formulation of Equation A.4, maximum ET would be potential evaporation multiplied by a crop coefficient, hence would be calculated as a function of growth stage, maximum and minimum daily temperatures, daily wind run, and daily solar radiation. Alternatively, maximum ET can be related to pan evaporation, a pan coefficient, and a crop coefficient (either daily or some other time period). In the Jensen approach, actual ET is related to ET_{max} and ASM through an empirical relation. The Yaron approach of Equation A.8, though also empirical, requires determination of only one coefficient in each time period.

The variable that has been chosen to measure growth over the season in this study is evapotranspiration ET. As discussed in Appendix A, transpiration is a superior measure since transpiration is more directly related to photosynthesis. Photosynthesis occurs when carbon dioxide is supplied during sunlight hours. Carbon dioxide enters through the open stomata, while water vapor leaves through the stomata at a rate determined by atmospheric conditions and the amount of moisture available to the plant. The water vapor that the plant loses through the open stomata makes up the major portion of the transpiration losses. Evapotranspiration includes water losses due

to evaporation, but when used as an index the ratio of ET/ET_{max} to T/T_{max} would be a constant at least within the accuracy of the data used.

Adoption of ET Prediction Method

Since the geographical region where this study is applied has a highly variable rainfall, it is logical that the study focus on precipitation and its contribution to available soil moisture and to the growth of the plant. Assumptions regarding other climatic variables are necessary, unfortunately, to allow computation.

As a result of the assumptions regarding climatic variables, the energy balance methods discussed in Appendix A have not been adopted in this study. Rather the simpler soil moisture balance method of Yaron has been used. In this method, climatic variables (except precipitation) are included in the coefficients b_{ij} in the equation

$$ET_{ij} = b_{ij}ASM_{ij}. \qquad (B.22)$$

Thus in fitting available data, it is assumed that climatic influences on the coefficients in the fitting year will be the same in the predicting year.

A computer program was written and tested using the 1972 and 1974 corn data from Colorado State University. The soil moisture data available from 1972 were for the entire profile (196 cm.), while the 1974 data were given for 1-ft. layers to a depth of 7 feet. Available soil moisture was set at 34.4 cm. or approximately 1.9 inches of available moisture per foot of soil.

Sample outputs are included in Figures B.2 - B.6. The flow chart of the program is included in Appendix D. A description of the program and results are included in Chapter III.

The advantage of the Yaron method is that no climatic data other than precipitation data are required. The program was modified, however, to accept other ET prediction methods. The logarithmic relation of Equation A.6 using evaporation pan data was first tested. Secondly, the following kinked linear relation was tested.

$$ET = \begin{cases} ET_{max} & \text{for } \%ASM \geq 50\% \\ \dfrac{ET_{max}}{0.5 \times 100} \%ASM & \text{for } \%ASM < 50\% \end{cases} \tag{B.23}$$

A final method, employing the combination equation (Equation A.4), was tested by Dr. Habte Neghassi, who adjusted the coefficients in the program for Fort Collins, Colorado.

The results showed that none of the models tested achieved significantly better fit than the Yaron model, at least in this special case where coefficients are desired to fit a number of treatments receiving quite varied timings and quantities of water. The obvious inaccuracy of the data may be ovserved in Figure B.2 where observed soil moisture levels (indicated by the numbers) on day 63 are greater than on day 48 with only a slight rainfall occurring between the two dates.

Adoption of a Production Function

In this study data were tested using the multiplicative Cobb-Douglas type equation

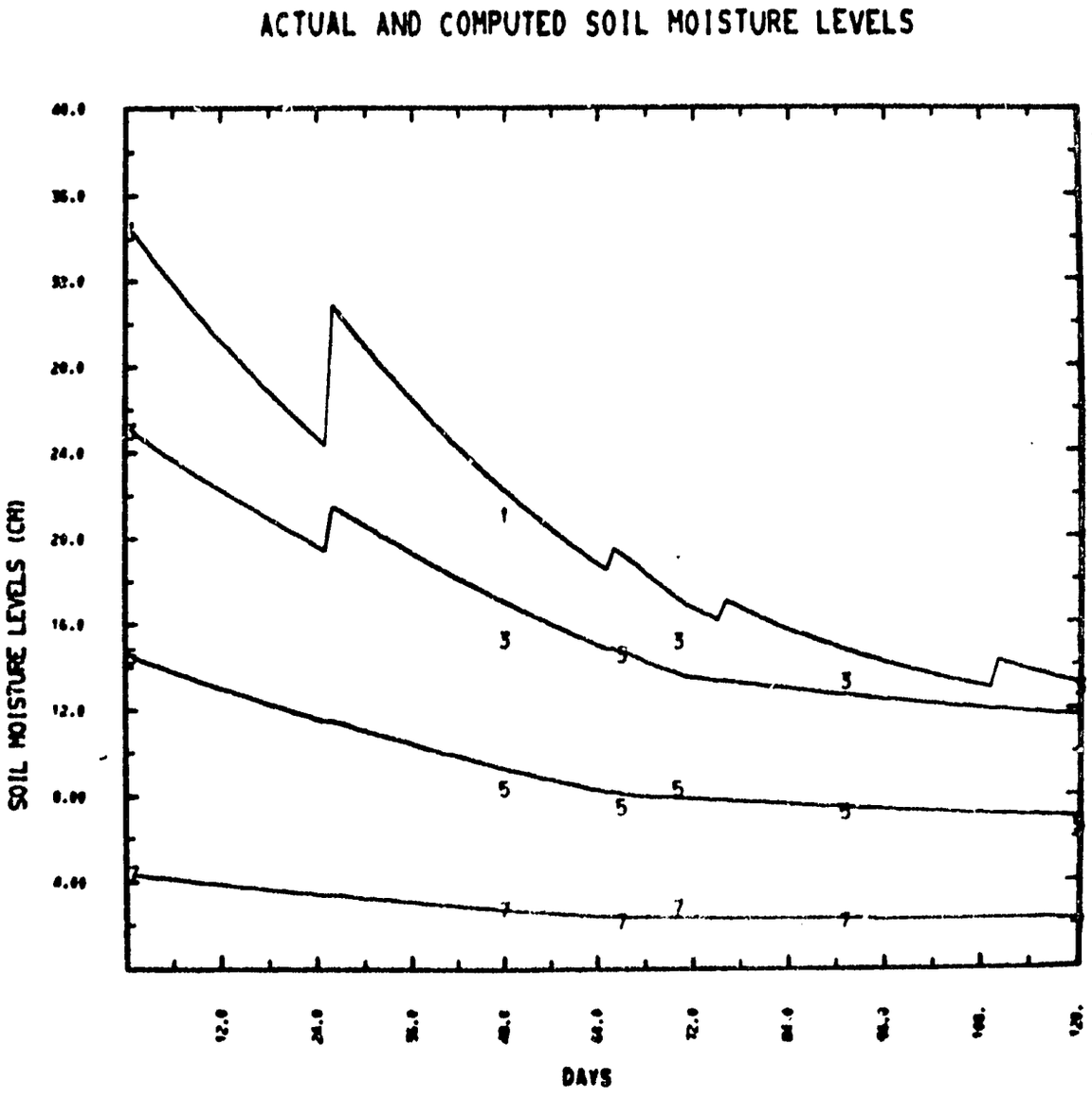

Figure B.2

SOIL MOISTURE SIMULATION FOR 1974 CSU
CORN DATA, TREATMENT I-1
(Numbers indicate actual soil moisture
measurements cumulative to the indicated depth)

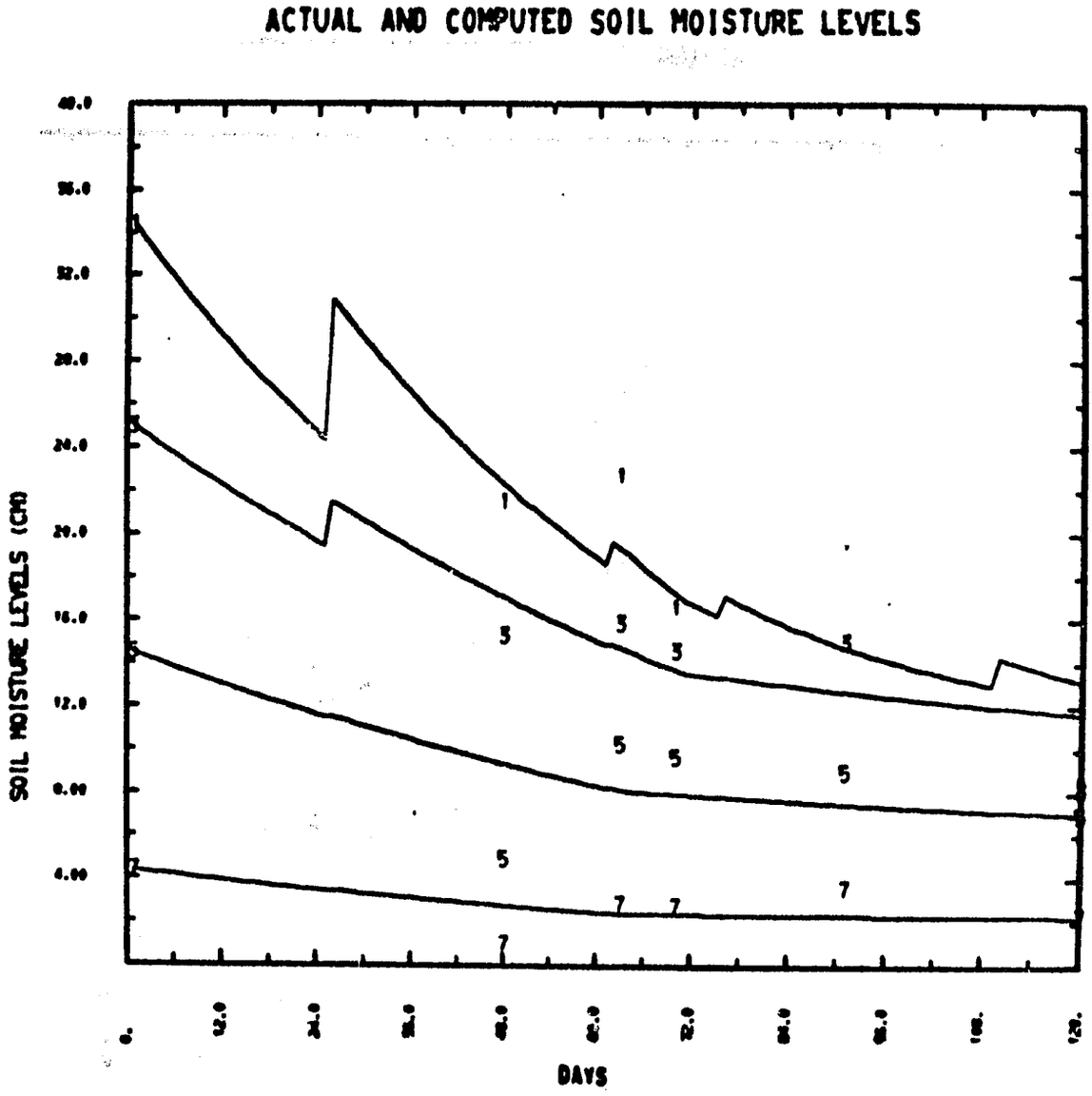

Figure B.3

SOIL MOISTURE SIMULATION FOR 1974 CSU
CORN DATA, TREATMENT I-12.
(Numbers indicate actual soil moisture
measurements, cumulative to the indicated depth)

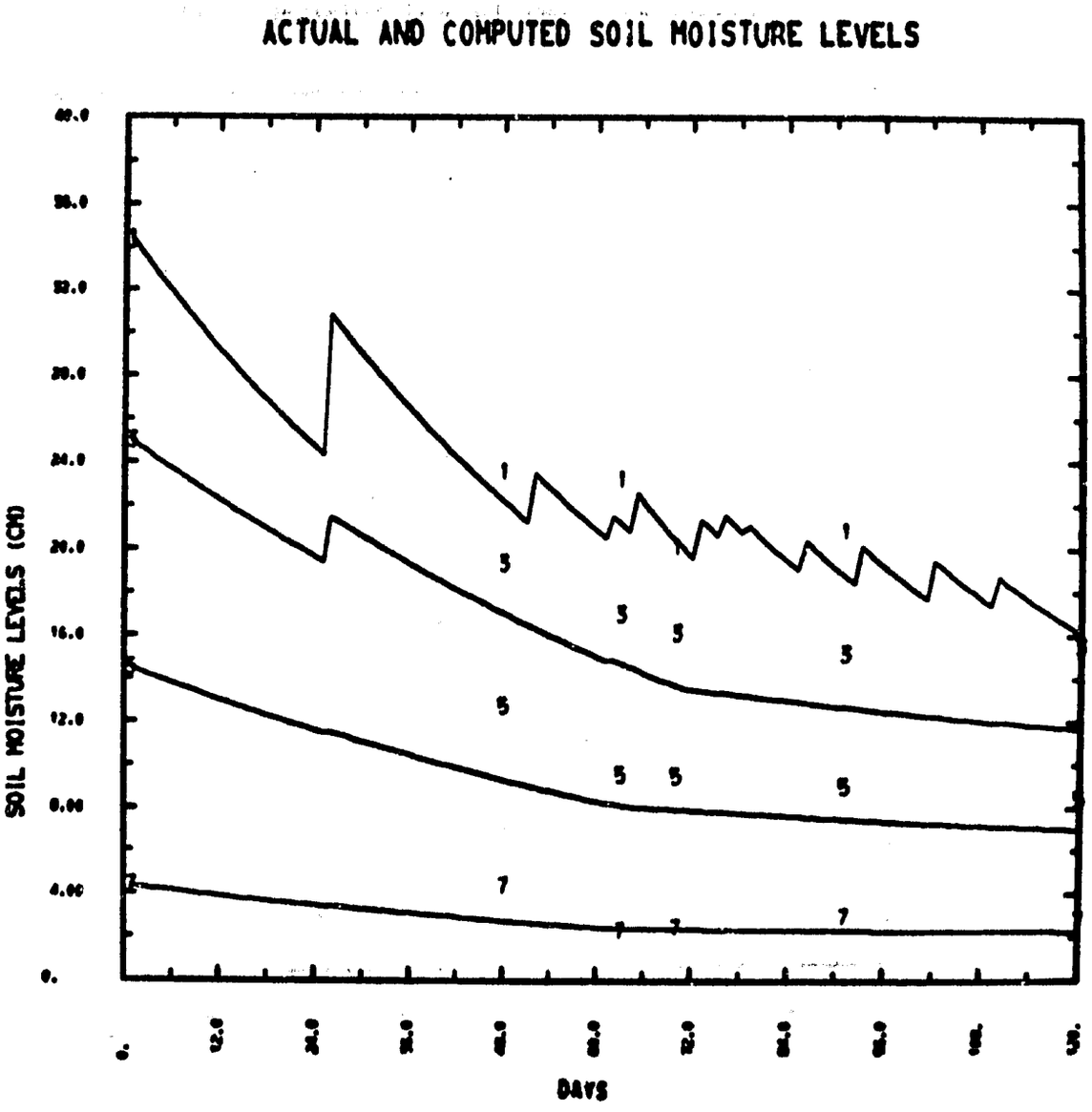

Figure B.4

SOIL MOISTURE SIMULATION FOR 1974 CSU
CORN DATA, TREATMENT I-3.
(Numbers indicate actual soil moisture
measurements, cumulative to the indicated depth)

ACTUAL AND COMPUTED SOIL MOISTURE LEVELS

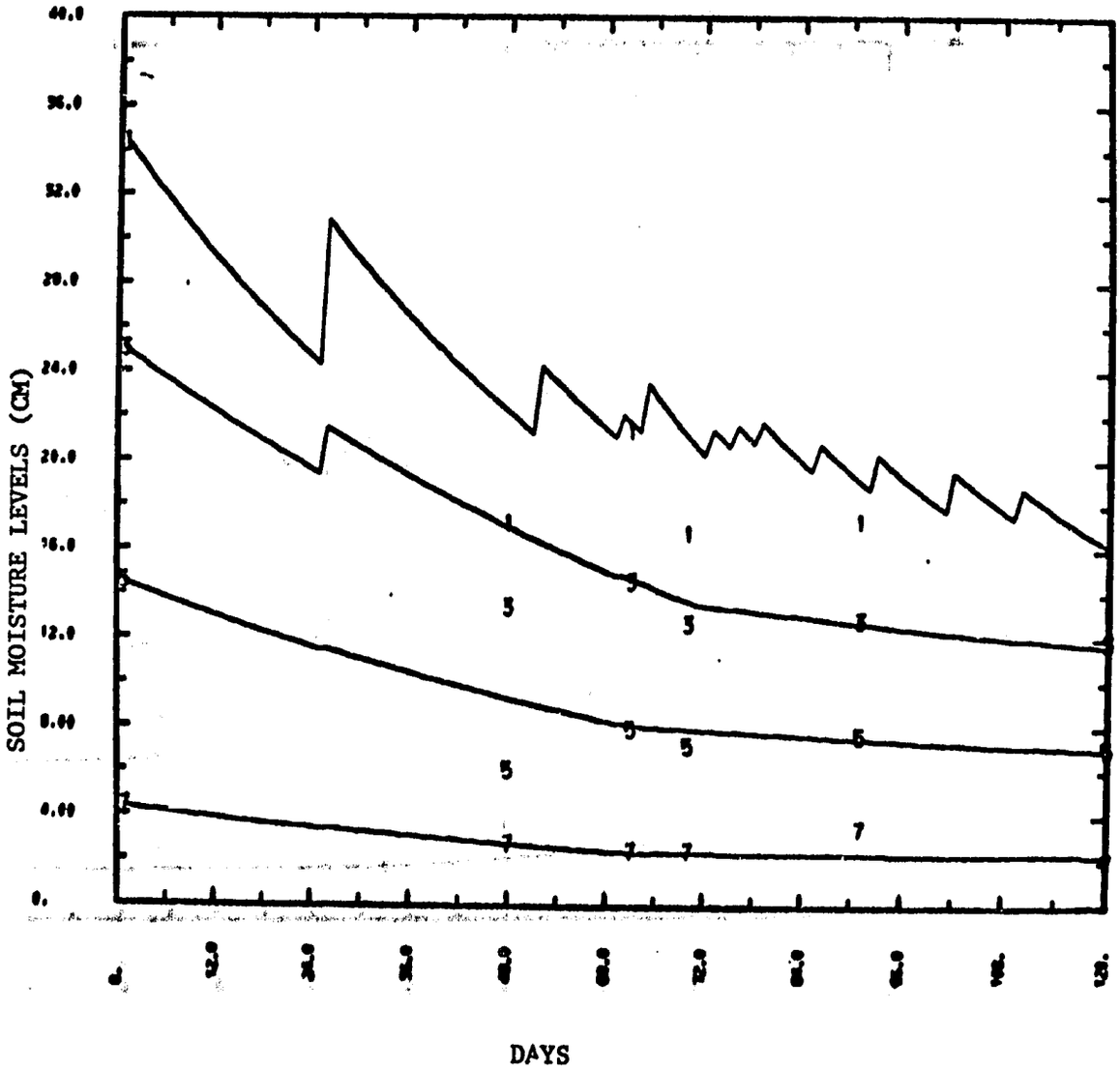

Figure B.5

SOIL MOISTURE SIMULATION FOR 1974 CSU
CORN DATA, TREATMENT I-10.
(Numbers indicate actual soil moisture
measurements, cumulative to the indicated depth)

ACTUAL AND COMPUTED SOIL MOISTURE LEVELS

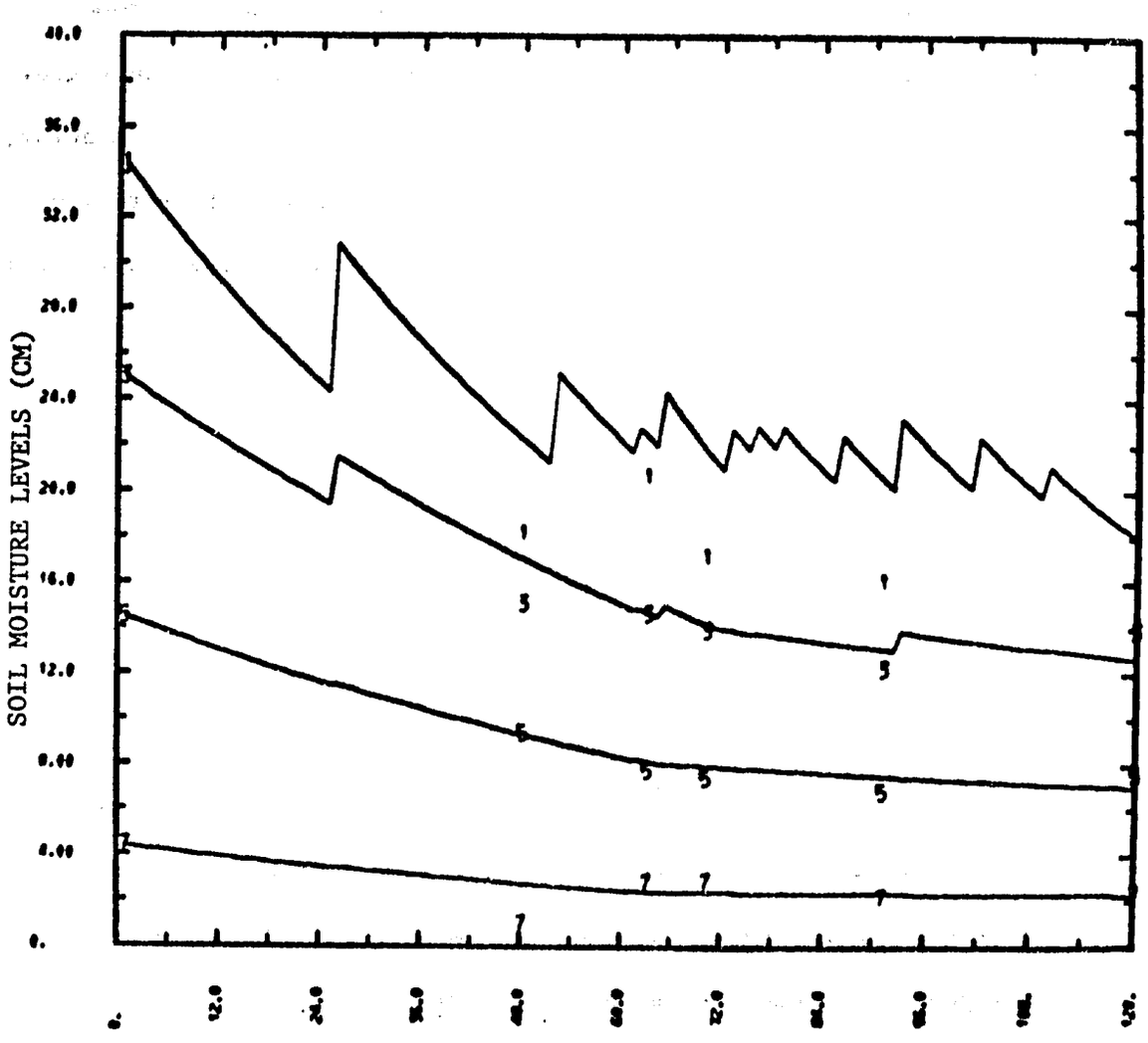

Figure B.6

SOIL MOISTURE SIMULATION FOR 1974 CSU
CORN DATA, TREATMENT I-4.
(Numbers indicate actual soil moisture
measurements, cumulative to the indicated depth)

$$Y = Y_{max} \prod_{i=1}^{I} (ET_i/ET_{max_i})^{\lambda_i} \tag{B.24}$$

and an additive equation

$$Y = Y_{max} \sum_{i=1}^{I} A_i (ET_i/ET_{max_i}). \tag{B.25}$$

Results are discussed in Chapter III.

Advantages of this formulation are that (1) There is some theoretical basis for using transpiration and hence ET as a growth index; (2) The parameters λ_i and A_i have some physical meaning, indicating the relative contribution of each growth stage to yield; and (3) In both equations, linear regression may be used to fit parameters to available data.

Discussion of Yield Prediction

The question remaining is how may the foregoing formulations which relate inputs to ET be applied in predicting future crop yields. One approach is to assume the future will be identical to the past. This assumption will result in a deterministic model.

Future daily values of pan evaporation or equivalently the variables that determine E^* in the combination equation (Equation A.4) must be known. If average values over some time period using historical data are employed, the results will be applicable providing the year is in some sense an average year.

Where a number of years of data are available, various more sophisticated methods are available to predict yield and assist in cropping pattern and irrigation decision making. However, the programming problem becomes more complex. These methods are discussed more fully in Appendix C.

A third approach is to assume that some variables are deterministic and some are stochastic. This study adopts this approach by assuming all climatic variables other than precipitation are deterministic. That is, temperature, wind, solar radiation, and pan evaporation values are assumed known with certainty while precipitation is the only random variable.

Assumptions

Within the assumption of deterministic climatic variables, then, an adequate model has been developed that predicts ET for each growth stage for corn over a wide variety of soil moisture conditions. For a particular timing and intensity of irrigation and rainfall during the season, the model will predict ET in each growth period.

The ET values are related to yield via production functions (Equation 3.24 and 3.25) that have been fitted from the 1974 CSU field data.

The foregoing procedure could be repeated for other crops; however, insufficient data are available. Another approach has been adopted for determining the Yaron coefficients for the other crops considered in this study.

Another key assumption is that conditions of the application site are assumed identical to those at Fort Collins, Colorado, which is roughly 70 miles to the west of Ft. Morgan. The soil series at the application site is classified as Nunn-Fort Collins, whereas the

soil at the agronomy farm is described as uniformly deep Nunn clay loam. This soil is generally heavier than the Nunn-Fort Collins soil. A brief comparison of the climates of the two sites is given in Table B.1. The general problem of transferability of data is discussed in Chapter V.

A third assumption is that quality of the corn grain is not affected by the management technique. That is, the same price per unit is paid whether the crop has been stressed severely or irrigated fully.

Summary

This chapter has concentrated on developing production functions for corn grown at the application site. Two production functions, one multiplicative and one additive, have been adopted for further study in Chapter III. Both require prediction of ET. An ET prediction model has been adopted in which ET is a linear function of available soil moisture. A program was written to fit coefficients for this model. Other ET prediction methods were tested and did not achieve significantly better fit to the available data.

With an adequate model to predict yields with given inputs, the question remains as to what are the optimal crops to be planted and the optimal irrigation decisions. Appendix C discusses optimization techniques having particular relevance to this study. In order to implement an optimization scheme for planning, other types of information, including economic information, regarding the particular site are necessary. The actual data used are presented as the model is developed in Chapters III and IV.

Table B.1

COMPARISON OF VARIOUS CLIMATIC VARIABLES
AT FT. MORGAN, COLORADO, AND FT. COLLINS, COLORADO

(Source: Climatological Data--Colorado,
U.S. Department of Commerce,
Environmental Data Service)

	Ft. Morgan, Colorado (83 years of record)	Ft. Collins, Colorado (94 years of record)
Average Annual Precipitation (through 1974)	13.2"	14.94"
Spring Precipitation (March, April, May, 1974)	2.77"	2.71"
1973	5.85"	5.12"
Long-term Average	4.37"	5.76"
Growing Season Precipitation (May-Sept., 1974)	4.53"	5.87"
1973	10.93"	6.41"
Long-term Average	9.12"	9.62"
Frost-free Days Mean (1969-1974)	155	147
Mean Monthly Temperature in Growing Season (through 1974)	(68 years of record)	(88 years of record)
May	58.0	55.6
June	67.5	64.3
July	74.0	70.8
August	71.9	68.9
September	62.2	60.0

Appendix C

METHODS OF SIMULATION AND OPTIMIZATION

Introduction

This chapter briefly discusses some quantitative methods used in predicting behavior of physical systems and planning optimal decisions relating to those systems. The methods discussed involve computer programs which in some sense "model" the physical system. The first half of the chapter discusses simulation and mathematical programming techniques applicable to deterministic models. The second half deals with methods of handling random variables.

This study deals with irrigation management. The simplest problem of irrigation management is to predict yields for a given cropping pattern and known water supply. A more general problem is to determine the cropping pattern and water allocation to obtain the maximum return. The most general problem is to determine decisions when some variables are considered random, that is, they are only known with a probability distribution. This problem will be referred to as the irrigation problem.

Simulation

Simulation is a method of predicting the physical response of a system for given inputs. For example, in this study a simulation model is desired which adequately predicts soil moisture levels in the soil profile over the growing season and hence evapotranspiration for given

water inputs. Since evapotranspiration can be related to yields, the model can be said to simulate plant growth.

Because of the complexity of physical systems, assumptions must be made which limit the simulation to only a rough approximation of the actual process. In this study, assumptions were made regarding climatic variables which would limit the validity of the model in any particular year.

The simulation model is basically a set of equations which attempt to reflect the physical process. These equations involve various parameters which can be manipulated to achieve some type of "best fit" to the observed data. Since these parameters are not known with great certainty, it is desirable that the model not be sensitive to changes in the parameters. That is, the modeler desires that a certain percentage change in a parameter would have less than that percentage change in the predicted outcome. It is also desirable that the number of parameters be small. A model that requires adjusting hundreds of parameters when applied to a different situation cannot be expected to be of widespread use.

Once a model is developed which contains a small number of insensitive parameters, it can be of use to planners and others interested in the "what if" question, i.e., what happens if a new policy is introduced. For example, a model that simulates yield can be used by a planner interested in limiting groundwater pumping. By limiting quantity and/or timing of irrigation water from pumping, the model can be rerun to predict the effect on yield. Simulation, then, can be used to test the effect of new policies.

A limitation of simulation models is that while varying one input, all other inputs are held constant. For example, for the yield model with limitations on groundwater pumping, the simulation model assumes that the farmer will allocate surface water in the same manner as before. Methods are available to avoid this problem while still using a simulation approach by estimating operating rules (see Anderson and Maass, 1971). Optimization methods allow a more realistic approach to the question of "what if" we apply a certain operating policy. When one input is varied, the model re-allocates all of the inputs to achieve optimal yields with available resources.

Optimization--The Mathematical Programming Problem

Whereas simulation models are useful for answering the "what if" question, they do not directly answer the question of "what is the best" operating policy. The general problems of water resource management, where planners are looking for the best set of operating rules, are optimization problems. For the problem of the study, the optimization problem is how to maximize returns from irrigated crops using available resources. The variables of this problem are acreages planted to various crops (x_i) and quantities of water applied to the crops in various time periods (q_{it}).

A general mathematical programming formulation of this problem (following Hall and Dracup, 1970) is

$$\max_{\underline{x},\underline{q}} R(x, q) \qquad \text{(C.1)}$$

subject to

$$\underline{g}(\underline{x}, \underline{q}, \underline{p}) \leq 0$$

in which R is the return function,

$\underline{x}$ is the vector of crop acreages

$\underline{q}$ is the vector of water applied to various crops in various time periods

$\underline{g}$ is a vector of constraint functions

$\underline{p}$ is a vector of parameters, including resource availability.

Equation C.1 is a very generalized statement of the problem, its purpose being to indicate the components of the formulation: decision variables, an objective function that is to maximized (it may also be written as an equivalent minimization) and constraints which serve to place limits upon functions of the decision variables. The constraints may be thought of as a simple simulation model.

Equation C.1 is written with a single objective of maximizing return. When several noncommensurate objectives exist, i.e., objectives not measured in comparable units, the problem is said to be multi-objective. The problem of maximizing return from an irrigated farm while minimizing the probability of attaining an income below a certain catastrophe level is a multi-objective problem. Approaches to dealing with the multi-objective problem will be discussed later in the chapter.

Analytical solutions to Equation C.1 have solutions only when the equations have some special form. However, often approximate solutions can be found.

Linear Programming

When the single objective function and all constraints are linear functions of the decision variables, the problem is called a linear program. The simplex and revised simplex algorithms are available as computer programs to solve this problem.

One characteristic of applying linear programming to a physical system, which is best described by non-linear relations, is that the problems tend to involve many variables very quickly. For example, in the irrigation problem, without deriving production functions, discrete production levels may be considered. If 5 crops are considered with 10 means of producing each crop, then already the problem contains 50 variables. Standard linear programming algorithms are not particularly efficient at solving large problems. Methods have been developed, in particular the Dantzig-Wolfe decomposition algorithm, which take advantage of the special structure of many large problems and allow efficient solution (see Lasdon, 1970).

An advantage of the linear programming formulation is that if a solution to the problem exists, the global minimum can be found. Another advantage is its flexibility, in that many problems which are not linear can be linearized over a region and solved using the algorithm. Finally, computer codes to solve linear programs are readily available.

An often mentioned disadvantage to linear programming is that the optimal solution always occurs at an extreme point.[1] For the irrigation problem formulated with the objective of maximizing return, the result may indicate planting as much as possible of the highest valued crop. Within the formulation of the problem, planting the highest valued crop is the solution. If this solution does not correspond to reality, then the fault is in the formulation, not the solution

---

[1]An extreme point is a feasible solution of the constraint equations which cannot be written as a convex combination of other feasible solutions.

technique. In reality, the farmer may have other objectives than maximizing return, such as minimizing risk.

Non-linear Programming

Non-linear problems fall into the categories of constrained and unconstrained problems. Minimizing (or maximizing) an unconstrained continuous, differentiable function can be done through methods of calculus. The Langrangian technique can be used in special cases with up to several constraints. Maximizing a function which is unimodal over a range, that is roughly, one which has only one peak, can be accomplished via search techniques such as the Fibonacci search (see Lasdon, 1970).

Constrained non-linear problems fall into categories of linear and non-linear constraints. With linear constraints the objective function can be linearized via various methods to transform the problem to a linear programming problem (see Geoffrion, 1969).

In the general case of non-linear objective function and non-linear constraints, questions of convexity arise. For a minimization problem where constraints form a convex set[2] and with convex objective function,[3] methods exist, e.g., gradient methods and linearization methods (see Himmelblau, 1972, and Geoffrion, 1969), which will converge to to the global minimum. Techniques applied to the more general case do not guarantee convergence to the global minimum.

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[2]A set in E^n is convex if for any two points in the set the line segment joining them lies in the set.

[3]A function is convex if the line segment drawn between any two points on the graph of the function never lies below the graph

Dynamic Programming

Whereas mathematical programming techniques are generally restricted to continuous, differentiable functions, dynamic programming, an example of an enumeration technique, can deal with more general functions.

The dynamic programming technique is adequately described in Bellman and Dreyfus (1962) and de Neufville and Stafford (1971). One general initial value formulation (Labadie, class notes) involves minimizing a separable objective function of the form

$$\min_{x_i, u_i} \sum_{i=1} f_i(x_i, u_i) + \phi(x_{n+1}) \qquad (C.2)$$

subject to a Markovian or dynamic relation

$$x_{i+1} = g_i(x_i, u_i)$$

with x_1 given and

$$x_i \varepsilon X_i$$

and $\quad u_i \varepsilon U_i \qquad i=1, \ldots n$

and $\quad h_i(x_i, m_i) \leq 0$

Thinking in terms of a single crop irrigation problem, i may be the index of the time period, x_i may represent soil moisture levels for the particular crop (a state variable) allowed to take on finite discrete values in set X_i and u_i may represent the irrigation decision variable, also allowed to take on finite discrete values in set U_i. The function h_i may represent limiting the soil moisture level to field capacity. In this formulation, yield would be an additive function of soil moisture and irrigation amounts (i.e., evapotranspiration) in each time period. Terms involving charges for water could also be

included. This formulation may be applied to a run of the river system where flows are known and/or to a system which includes storage.

An advantage of dynamic programming is that differentiability and convexity are not required. A disadvantage is that a separable objective function is required. Hall and Dracup (1970) present a method for handling a combination multiplicative and additive objective function. This method is discussed in detail in Chapter II.

A general limitation of the dynamic programming approach is the number of state variables. The example problem discussed has one state variable, soil moisture. Evapotranspiration could be thought of as a function of soil moisture, irrigation, and some variable measuring the previous condition of the plant. "Previous condition" would be carried in the program as another state variable. As additional state variables are introduced, computation time greatly increases. For the single crop irrigation management problem, only several state variables would be required. However, in multi-crop applications dimensionality problems may be encountered.

Conclusions Regarding Simulation and Mathematical Programming

Simulation and mathematical programming techniques are valuable tools for quantitative analysis of physical systems. Simulation can be used in some cases to answer the "what if" question. In more general cases optimization can be used to answer the "what if" question, and it can also be used to answer the "what is the best" question.

A limitation to the use of simulation and optimization techniques arises when modeling physical systems. Often the operation of the

physical system is not well known or sufficient data are not available. In the irrigation problem, for example, the data for predicting ET may not be available. Simplifying assumptions involving parameters which must be estimated allow the modeler to approximate the actual physical conditions.

A second limitation of these techniques, particularly optimization techniques, are the problems which arise with human interaction with the physical system. Here, questions of economics are present. The simplest assumption is that humans are motivated by profit maximization. Questions of the individual versus society and long and short term profit maximization serve to complicate the situation by introducing multiple objectives. With the techniques discussed, judgment must be used to model the situation.

A third limitation of the quantitative techniques previously discussed is that they do not inherently consider random variables. For the irrigation problem, a number of variables such as precipitation, crop prices, etc., may be viewed as occurring in some random manner. Methods of dealing with these random variables will be discussed in the next section.

Stochastic Inputs and Multiple Objectives

Introduction

In this section methods will be discussed which are extensions, in some sense, of the optimization and simulation techniques previously discussed. These "extensions" attempt to model physical systems and physical-human systems by considering an additional degree of complexity, complexity arising from the influence of random variables,

that is, variables which are known only with a probability distribution. In physical-human systems the complexity may arise from random variables or may be in the form of multiple objectives.

De Neufville and Stafford (1971) distinguish between situations involving certainty, risk and uncertainty. A certain situation exists where the outcome of a decision can be calculated in advance. For a decision made under risk, the probability distribution of the outcome is known. When decisions are made under uncertainty, the outcomes of the decisions are not known. This last case will not be considered in this study.

The relevant case to the irrigation problem is decisions made under risk. Viewing rainfall as the only random variable, its probability distribution is known and hence the probability distribution of the outcome of an irrigation decision can be calculated. Determining an optimal decision in this case is not a straightforward exercise. The next section introduces the concepts of expected value and variance and their roles as multiple objectives. It will be seen that when a random variable is introduced to a single objective problem, a multiple objective problem results. All of the methods discussed in the remainder of the chapter reduce the multi-objective problem to one of a single objective.

Expected Value and Variance

The expected value of the outcome of a decision variable x_j is expressed as $E(Z_j)$ where Z_j represents the return due to the jth decision. Introducing discrete random variable y_i having distribution $p(y_i)$, the return can be written as

$$Z_{ij} = f(x_j, y_i) \tag{C.3}$$

The expected return is given as

$$E(Z_j) = \sum_{i=1}^{n} p(Z_{ij}) Z_j \tag{C.4}$$

but

$$p(Z_{ij}) = p(y_i) \tag{C.5}$$

so

$$E(Z_j) = \sum_{i=1}^{n} p(y_i) f(x_j, y_i) \tag{C.6}$$

One obvious strategy is to choose the maximum expected value, i.e.

$$\max_j E(Z_j) \tag{C.7}$$

Maximizing the expected value, however, is not always an appropriate objective. Criticisms of this approach are that it neglects variance and it does not consider the gambler's ruin problem in which a gambler maximizing expected return virtually assures himself of ruin.

Variance is defined as the expected value of the sum of the squares of the deviation of an outcome from the mean outcome or

$$Var(Z_j) = \sum_{i=1}^{n} (Z_{ij} - E(Z_j))^2 p(Z_{ij}) \tag{C.8}$$

or

$$Var(Z) = \sum_{i=1}^{n} (f(x_j, y_i) - E(Z_j))^2 p(y_i) \tag{C.9}$$

It is possible that two decisions x_j and x_i result in near equal expected returns, however differing variances. The "appropriate" decision may be to choose the one with lower variance. This would certainly be the choice, for example, of the small farmer who would be forced out of business if his income in a particular year fell below some "catastrophic level" or did so over a series of years.

Hall and Dracup (1970) give the example of the gambler's ruin problem where the gambler knows with probability p where $0.5<p<1$

the outcome of a two result wager. In order to maximize his expected value over n trials, the gamblers "best" policy is to bet all his money at each trial. This procedure results in the highest expected value but the probability of attaining this result is p^n which approaches 0 as n approaches infinity. Hall points out "the essential deficiency of the analysis is the failure to simultaneously take account of the noncommensurate objective of avoiding risk."

When a random variable is introduced, it has been shown that complexity arises. When the probability distribution of the random variable is known, the expected value of a decision can be calculated. The choice of the optimal decision depends on the expected return and the variance. Determining a proper single objective function becomes a tradeoff between the relative weights of expected return and variance.

Monte Carlo Techniques

A simulation approach to handling situations of risk is the Monte Carlo technique.

For the irrigation problem, suppose 20 years of data could be generated from records or using random numbers with a known probability distribution. These data could include any number of random variables including precipitation, solar radiation, maximum and minimum temperatures, wind speed, etc.; whatever data the model uses to predict yields. The outcome of a particular decision can be evaluated, i.e., the return can be calculated for each of the years being simulated. Thus the basic Monte Carlo technique answers the "what if" question. Monte Carlo techniques can be combined with optimization

methods to determine optimal crop patterns, irrigation timings, and/or irrigation amounts providing a single suitable objective can be determined, otherwise multiple objective techniques must be introduced.

Chance Constrained Linear Programming

A general formulation of the chance constrained problem originally presented by Charnes and Cooper (1959) is given by Cocks (1968) and can be applied to the irrigation problem.

$$\max_{\underline{x}} \; E(\underline{c}'\underline{x}) + \phi Var(\underline{c}'\underline{x}) \qquad (C.10)$$

subject to $Pr[A\underline{x}<\underline{b}] \geq \underline{e}$

in which ϕ = risk aversion constant

x_j = decision variable (acreage of crop j)

A = matrix of technological constants (e.g., water use coefficients)

$\underline{c}'$ = linear return coefficients

$\underline{b}$ = vector of available resources

$\underline{e}$ = vector of probability levels.

In this formulation, with variable rainfall, the $\underline{b}$ vector includes random variables. An interpretation of a probability constraint is that the probability of water use exceeding supply occurs (1 - e)x100 percent of the time. That is, the constraint is violated with some set probability level. As e approaches 1, violation of the constraint is allowed less frequently and consequently the expected return is reduced.

Cocks (1968) criticizes this formulation, saying that "it says nothing about what to do or expect when constraints are violated." For the irrigation problem, when the constraints are violated the interpretation is that the farmer must allocate more water than he has

available. Since this situation is clearly impossible, the chance constrained approach seems more suitable for planning purposes than for real time management. What is desired is a program which does in fact determine the effect on yields when water supplies are less than optimal.

Discrete Stochastic Programming

Cocks (1968) describes a "discrete active programming model." In probabilistic programming, distinction is generally made between active and passive approaches. The passive approach involves solving a deterministic problem for each possible value of the random variable, then choosing the appropriate maximizing solution.

The active approach of Cocks, in the simplest case, solves for the maximum expected value by greatly increasing the number of variables and the number of constraints. A linear program with originally m constraints and n variables becomes a problem of $mn + kn + k$ variables and equal number of constraints where k is the number of discrete values taken by the random variable. The random variables considered in this formulation are the elements of the A matrix and the $\underline{c}'$ vector. If the A matrix is known deterministically, the formulation would be reduced to $mn + k$ variables. The author considers the multiobjective problem by including a variance term in the objective function.

The advantage of this "active" approach is that it solves the problem in one computer run. The author states that the method can be used to solve multistage allocation problems where more data becomes available as the season progresses.

The disadvantage of the method is the large programs that are generated. A reasonable 50 x 50 deterministic matrix becomes larger than 2500 x 2500 depending on the number of random variables and the number of discrete values they take.

Another disadvantage of this method is that in the formulation presented, the $\underline{b}$ vector is not allowed to vary. The $\underline{b}$ vector is generally regarded as resource availability, including resources such as water, labor, land, and precipitation. In a practical problem it is difficult to imagine a situation where the technological coefficients (elements of the A matrix) are random but the $\underline{b}$ vector is known with certainty.

The discrete stochastic approach of Cocks, then, cannot be regarded as a practical means of problem solution. A passive approach in which (1) the original matrix size is maintained and (2) any variable may be regarded as random, may be better suited to solving applied problems.

EV Frontier Method

Another method of dealing with random variables is the EV (expectation-variance) method arising from economic theory. An application of this method to agricultural production is presented by Lin, Dean and Moore (1974).

In this method an E versus V graph is generated. In the irrigation problem, for a particular cropping pattern decision, the expected return is plotted against the variance. Similar calculations are made for other cropping decisions and a curve is generated in which, in general, variance increases as expected income increases (see Figure C.1).

A farmer is assumed to possess a utility function, that is, he is willing to trade variance for income in some rational manner, with the objective of maximizing utility. Iso-utility curves are shown in Figure C.1 with utility increasing as one moves to the right from one curve to another. The maximization occurs at the unique point where the utility indifference curve is tangent to the EV curve.

This method or marginal analysis is attractive theoretically, but as in the previous case, computation of an actual problem is difficult. For each cropping pattern, each random variable must be discretized and for each event, the return, expected return and variance must be calculated. Generating the entire curve requires repeating the process many more times. Lin, et al., in their study measured means and variances "subjectively."

The authors tested various types of utility measures with data obtained from farmer interviews. The authors concluded that "none of the models (Bernoullian utility, lexicographic utility, and profit maximization) predicted actual behavior well, with a strong tendency for all models to predict more risky behavior than was in fact observed."

Lin, Dean, and Moore attempt to deal with the most general case, including all sources of risk and uncertainty. This current study concentrates on a specific case in which the only random variable is precipitation. Under this assumption and in the situation where the farmer has enough irrigation water to avoid the gambler's ruin problem, maximizing expected value may best represent the farmer's behavior.

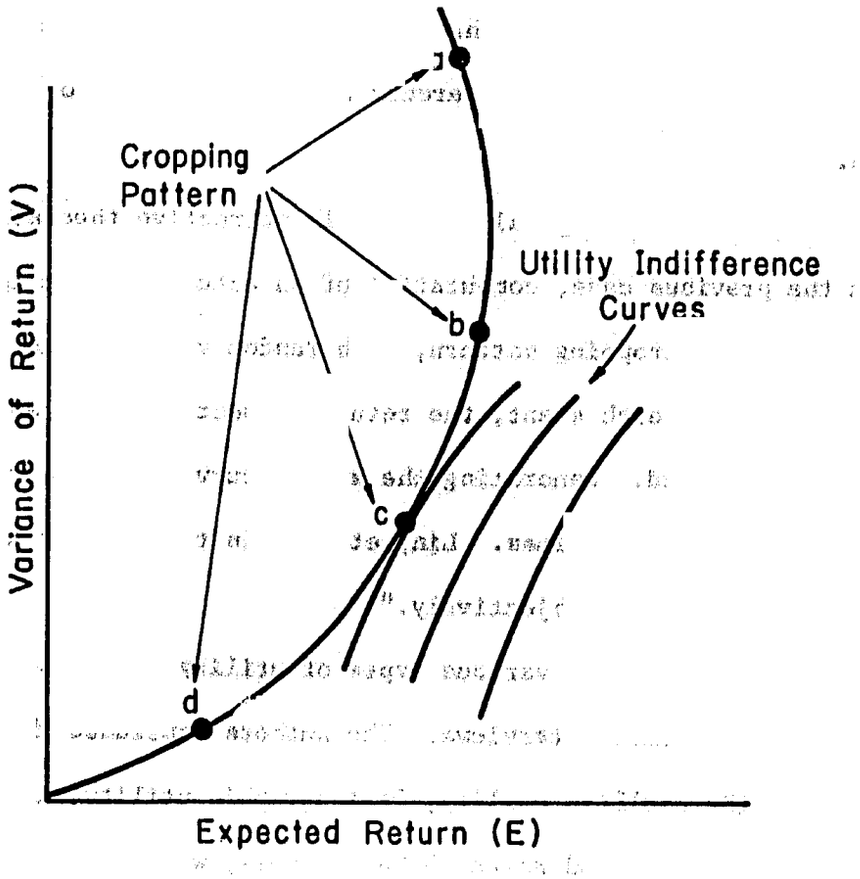


Figure C.1

UTILITY MAXIMIZATION BY EV FRONTIER METHOD
(After Lin, Dean and Moore, 1974)

The Surrogate Worth Trade Off Method

Whereas the EV frontier method deals with two noncommensurate objectives, the surrogate worth trade off method of Haimes and Hall (1974) deals with the general multiple-objective case.

The authors' method first reduces the problem to a noninferior region. With decision variable $\underline{x}$ and n objective functions, $f_j(x)$, $j=1, \ldots n$, which are to be minimized, a noninferior solution $\underline{x}^*$ is any solution such that "there does not exist another $\underline{x}$ such that

$$f_i(\underline{x}) \leq f_i(\underline{x}^*) \qquad i=1, \ldots n, \qquad (C.11)$$

strict inequality holding for at least one i." A mathematical program is constructed with one function to be minimized and the remaining functions as constraints with parameter ε_j. Noninferior points are generated by parameterizing ε_j and solving the resulting nonlinear programming problem.

The lagrange multiplier λ_{1j} is interpreted as the amount of the change in the first objective function by a unit change in the jth objective function, that is, the shadow proce of the jth objective function. The surrogate worth function W_{1j} is defined as a function of λ_{1j} and is found subjectively with the aid of a decision maker. This person is asked, essentially, to determine the number of units of the first objective that are equivalent to a unit change in the jth objective. Actually for two levels of λ_{1j}, the decision maker is asked to value the trade, on a scale of -10 to +10 with one unit of objective function j. An extrapolation is made to determine λ_{1j}^* i.e., the point where $W_{1j} = 0$.

The λ_{1j}^* is interpreted as the optimal number of units of the first objective function that are equivalent to a unit change in the jth objective function.

The essence of this approach is in determining λ_{1j}^*, since once this value is determined, the problem can be reduced to a single objective function problem. The jth objective function becomes commensurate with the first objective when multiplied by λ_{1j}^*.

Applied to the irrigation problem which has objectives of maximizing return and minimizing variance, the surrogate worth trade off method reduces to finding the shadow price of a unit of variance, that is, finding how much a farmer would be willing to increase his variance for a unit increase in expected return. In terms of utility theory as discussed previously, since λ_{1j}^* is constant, a linear utility function is assumed. It should also be noted that λ_{1j}^* is equivalent to ϕ in the formulation of Cocks.

The Focus Loss Method

Boussard and Petit (1967) propose a model of farmer behavior which assumes farmers "will maximize expected gain, provided that the possibility of ruin is so small that it can be neglected." Their method, in essence, reduces the multiple-objective problem to a single objective by adding additional constraints to the problem. The approach is "active" in that the entire problem is solved in one computer run.

A difficulty with the approach is that various parameters are introduced. These parameters are MINI, k and P_i. MINI represents the minimum permitted income, $1/k$ is the fraction of total loss permitted from any one crop, and P_i is the focal loss for crop i.

Focal loss is rather roughly defined as "the level of loss that a decision maker would be 'very surprised' to reach in any eventuality" (Boussard and Petit, 1967). In their study the authors determined focus loss parameters through consensus of extension workers, relating the focus loss to a "decennial catastrophe."

The study of Boussard and Petit took into account yield and price uncertainty; however, they did not consider the causes of yield variation such as climatic and precipitation variables. The current study attempts to look more closely at some causes of yield variation. The concept of maximizing expected return while avoiding the possibility of ruin has been adopted in the current study. The active programming approach, however, does not appear to be suited to this study. Rather a passive approach for determining expected value has been adopted.

Summary

This chapter has presented various techniques of solving deterministic and nondeterministic mathematical programming problems. The discussion has centered around applications to the irrigation decision problem and techniques were selected which relate directly to the problem.

Certainly this is not a complete presentation of all applicable techniques. Further discussion of the dynamic programming approach including methods of handling stochastic inputs could have been presented. Also applications of Bayesian decision making (see Halter and Dean, 1971), Markov chains, and decision tree analysis could have

been presented. The concentration has been, however, on methods which can handle the multiple time period and multiple crop nature of the irrigation problem.

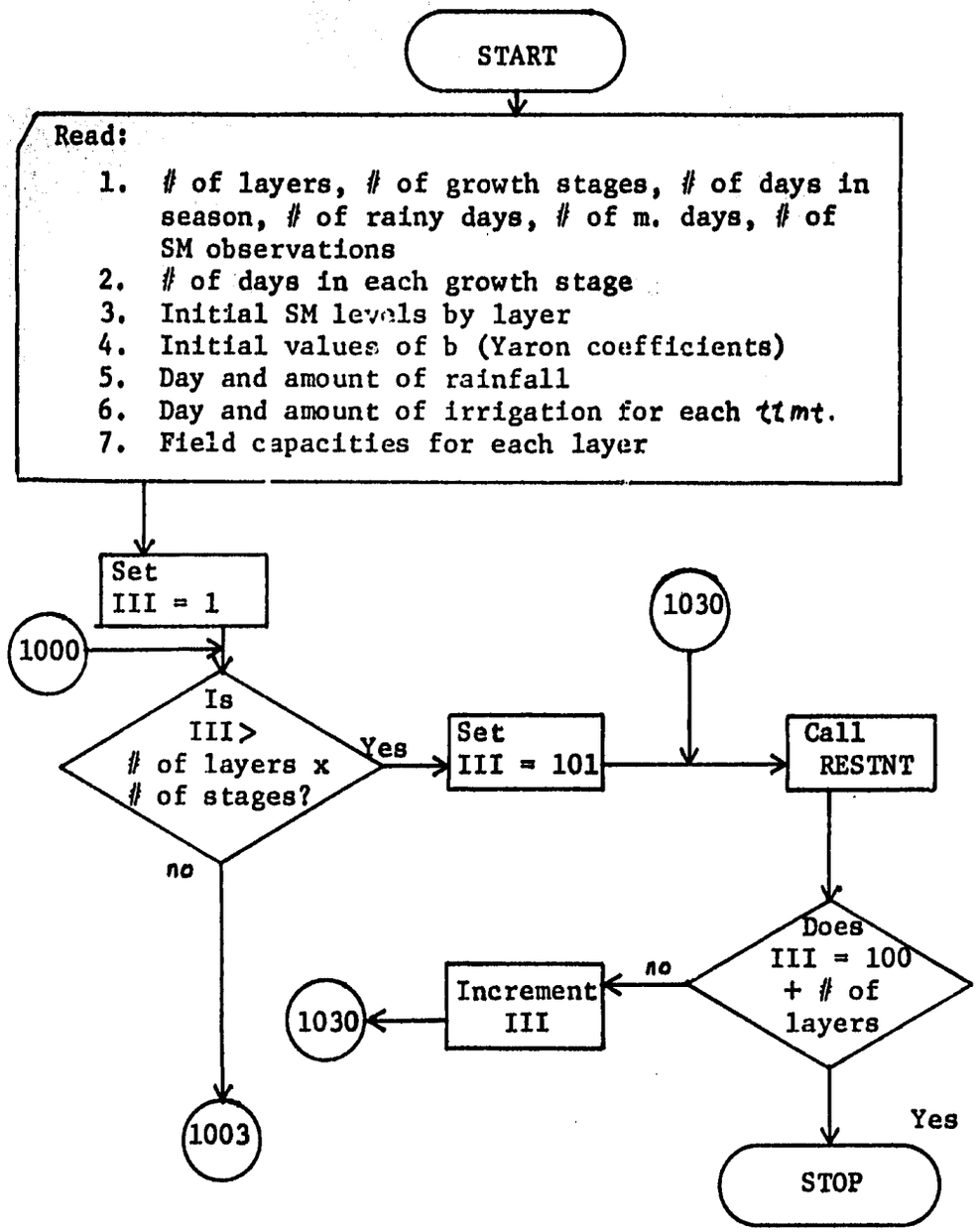
Regarding the introduction of random variables, it is hoped that some insight has been gained regarding the complexity introduced by considering the nondeterministic case. In this study, by considering precipitation as a random variable, a certain amount of complexity arises. By additionally considering random variables such as solar radiation and prices, complexity would increase greatly.

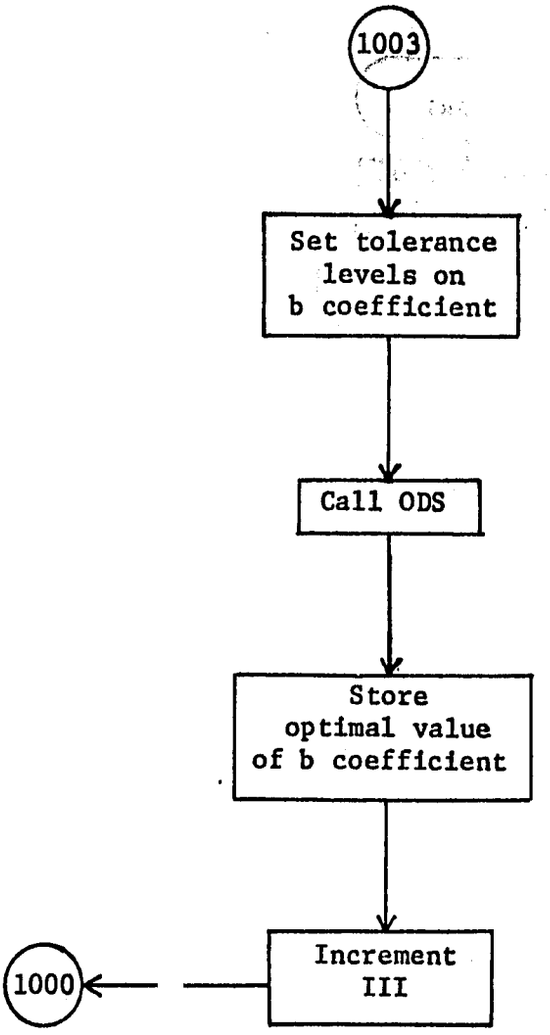
The multiple-objective problem is not great for the irrigation problem. Clearly, when risk is considered there are two objectives: maximizing expected return and minimizing variance. The most direct means of dealing with these two objectives is to form a single objective function consisting of an income term added to a constant coefficient times the variance term. The more general case in which the coefficient is a variable may be handled by the surrogate worth trade off method or by the EV frontier method.

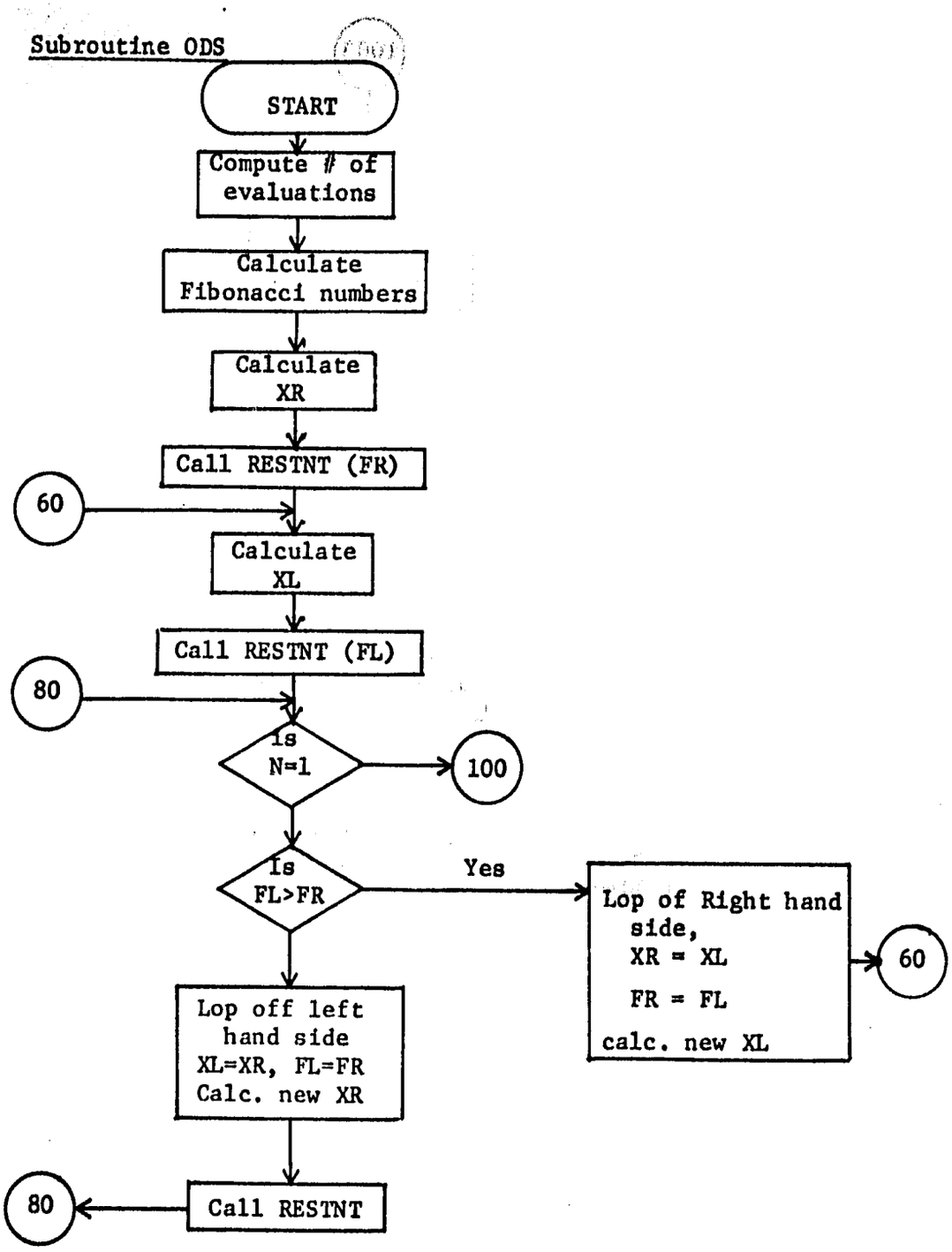
Appendix D

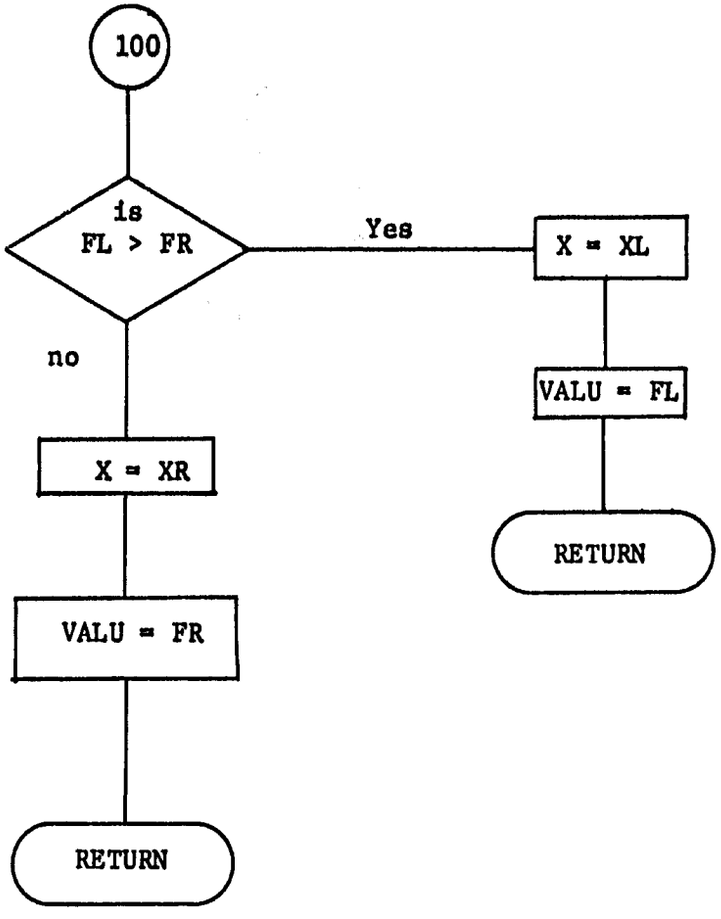
COMPUTER PROGRAM FLOW CHARTS

Program SM SIM

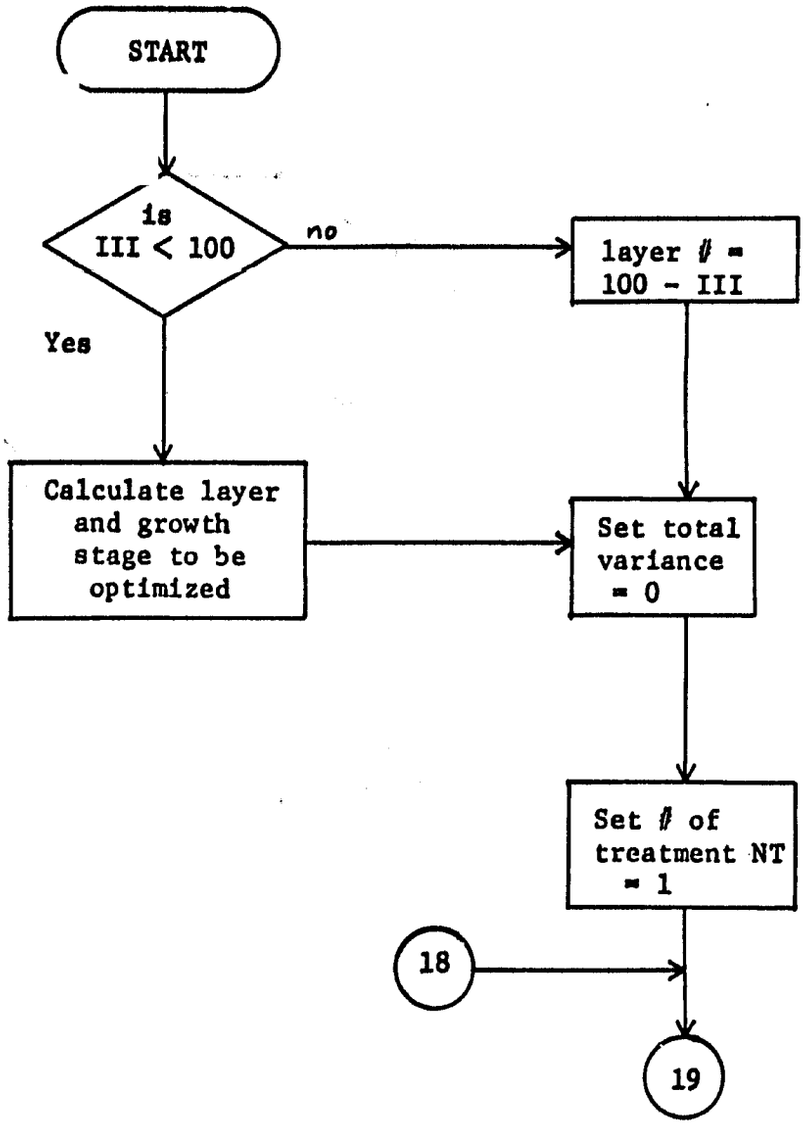


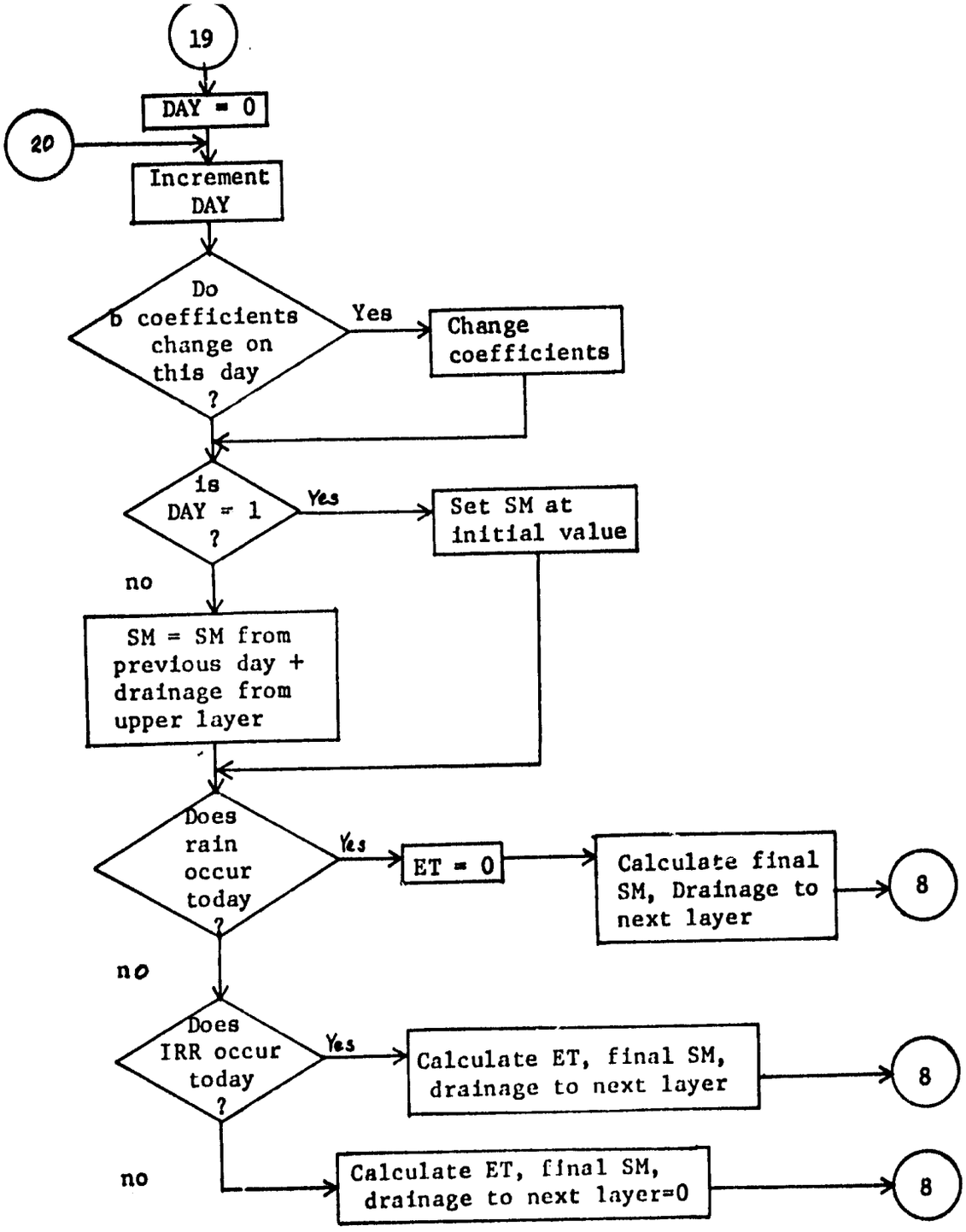

1003
Set tolerance levels on b coefficient
Call ODS
Store optimal value of b coefficient
Increment III
1000

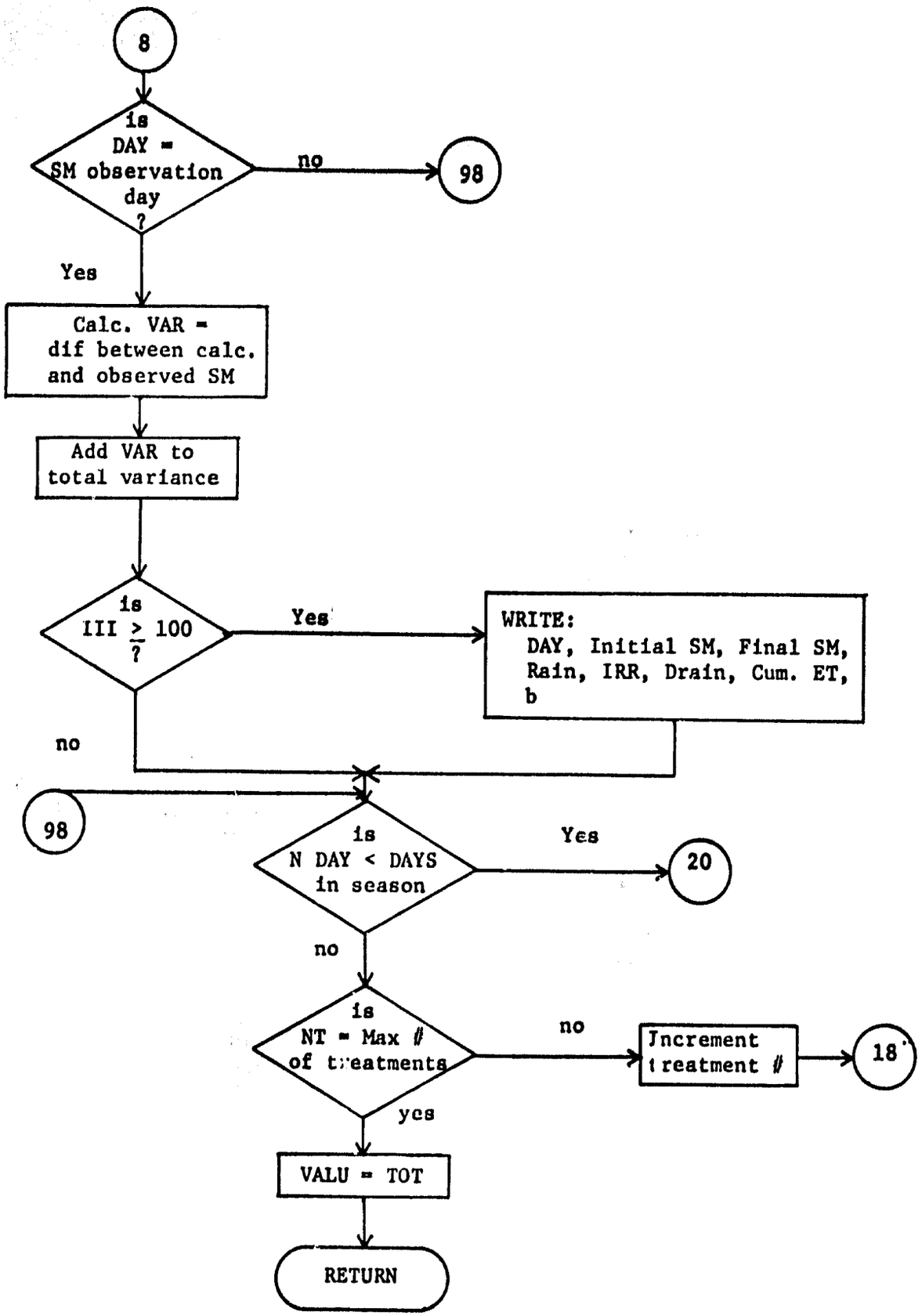

Subroutine ODS
START
Compute # of evaluations
Calculate Fibonacci numbers
Calculate XR
Call RESTNT (FR)
60
Calculate XL
Call RESTNT (FL)
80
Is N=1
100
Is FL>FR
Yes
Lop of Right hand side,
XR = XL
FR = FL
calc. new XL
60
Lop off left hand side
XL=XR, FL=FR
Calc. new XR
Call RESTNT
80


100
is
FL > FR
Yes
X = XL
VALU = FL
RETURN
no
X = XR
VALU = FR
RETURN

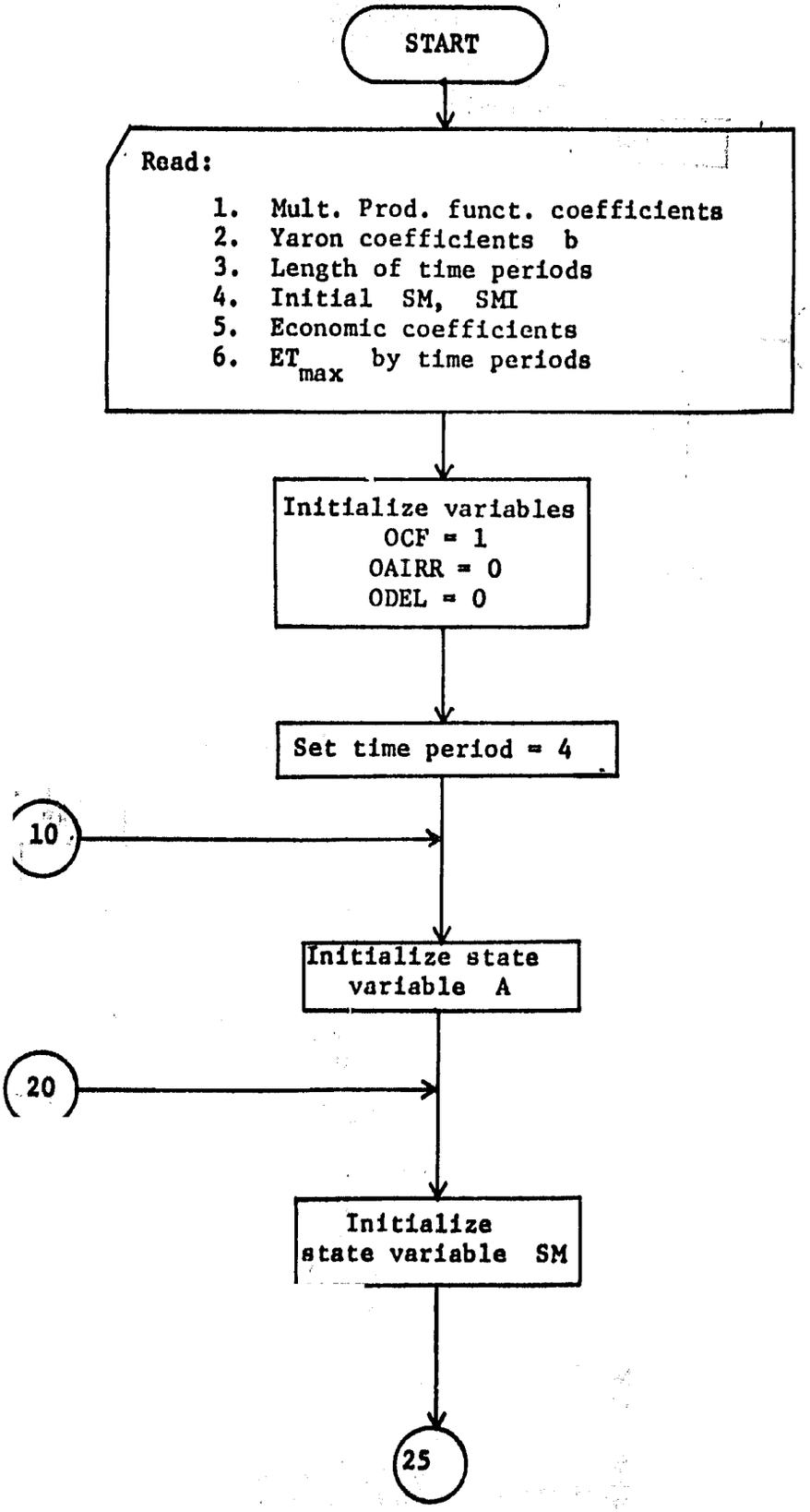
Subroutine RESTNT

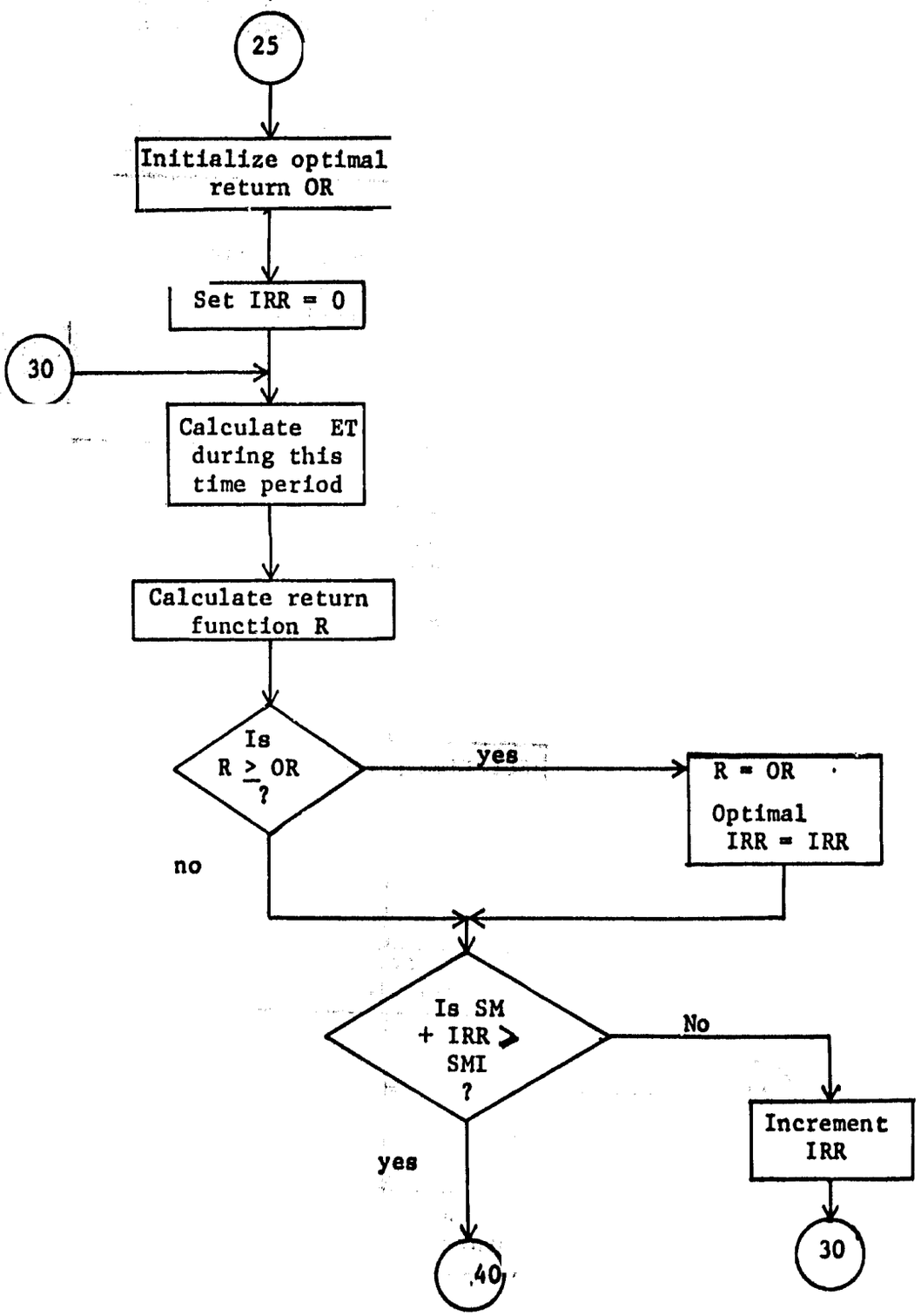


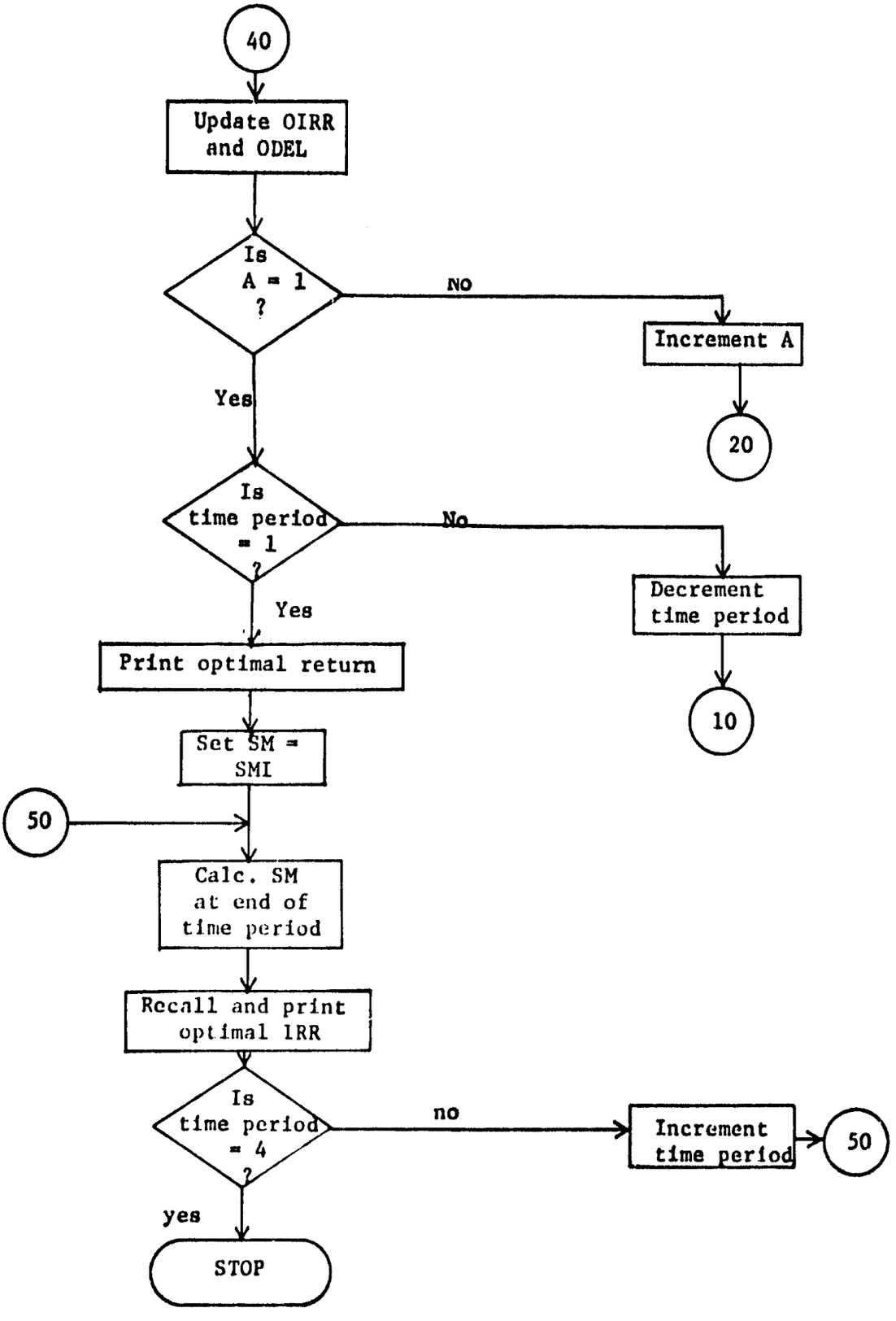

19
DAY = 0
20
Increment DAY
Do b coefficients change on this day ?
Yes
Change coefficients
is DAY = 1 ?
Yes
Set SM at initial value
no
SM = SM from previous day + drainage from upper layer
Does rain occur today ?
Yes
ET = 0
Calculate final SM, Drainage to next layer
8
no
Does IRR occur today ?
Yes
Calculate ET, final SM, drainage to next layer
8
no
Calculate ET, final SM, drainage to next layer=0
8


8
is DAY = SM observation day ?
no
98
Yes
Calc. VAR = dif between calc. and observed SM
Add VAR to total variance
is III ≥ 100 ?
Yes
WRITE:
DAY, Initial SM, Final SM, Rain, IRR, Drain, Cum. ET, b
no
98
is N DAY < DAYS in season
Yes
20
no
is NT = Max # of treatments
no
Increment treatment #
18
yes
VALU = TOT
RETURN

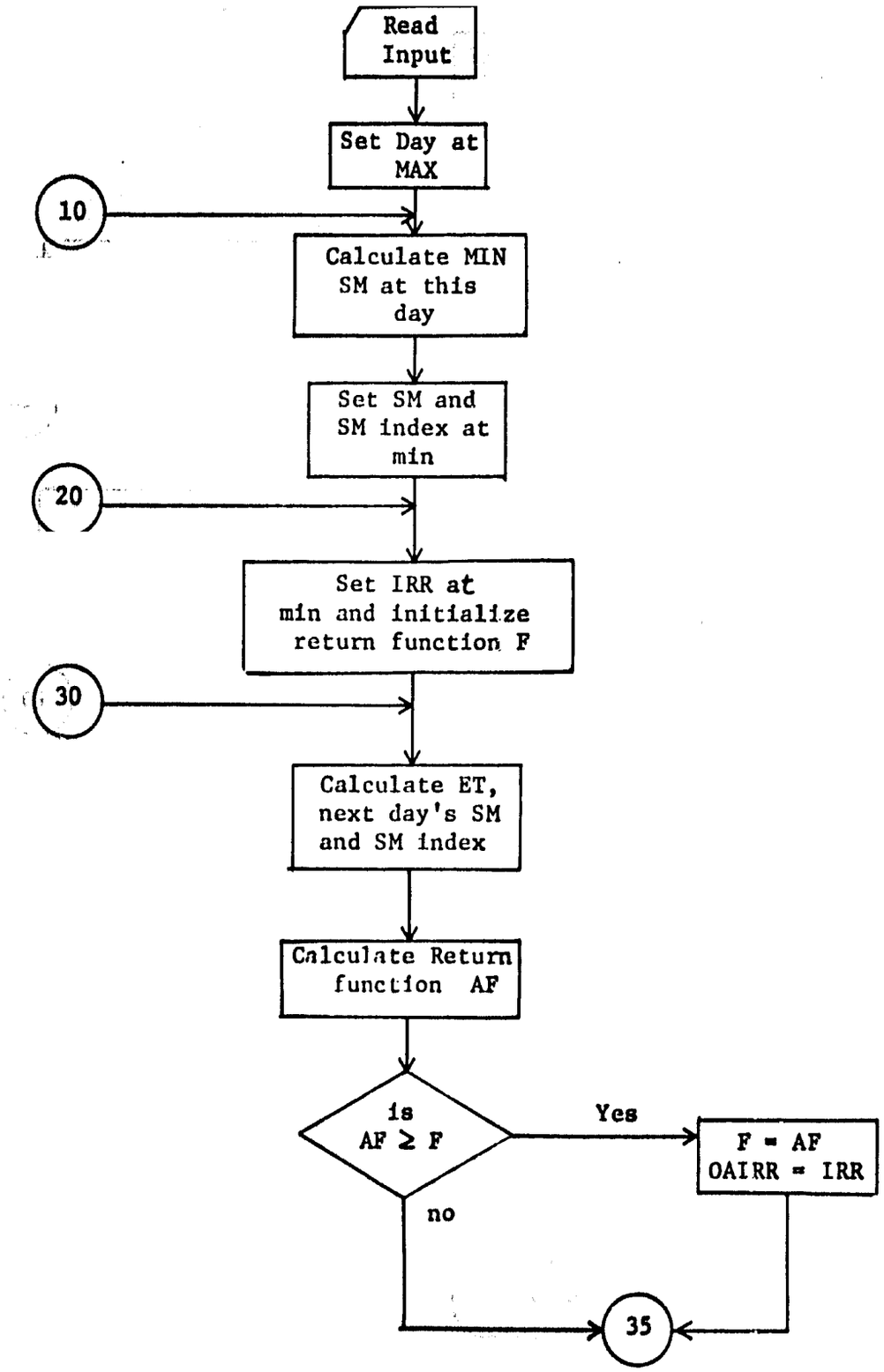
Program OPT TIM

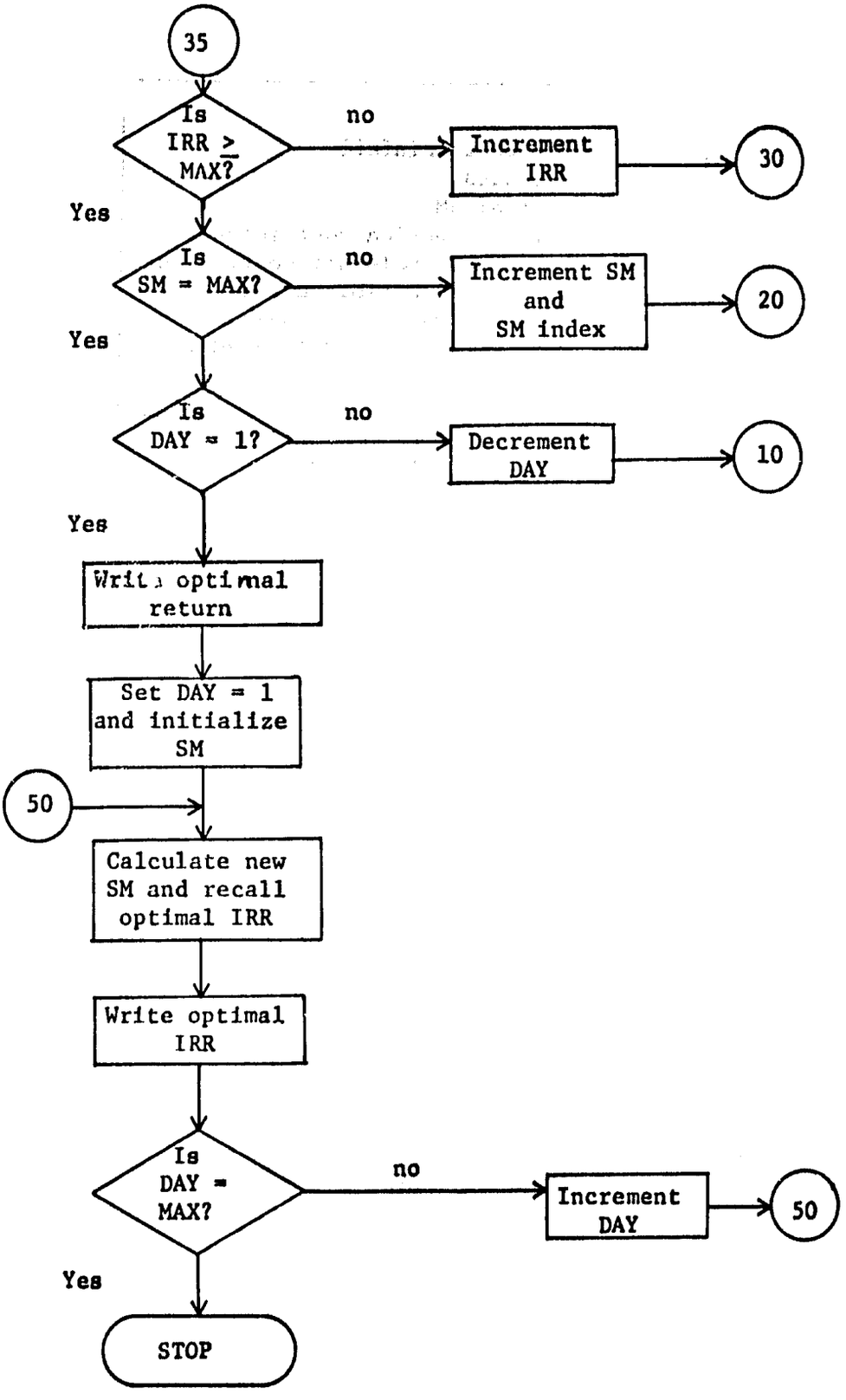
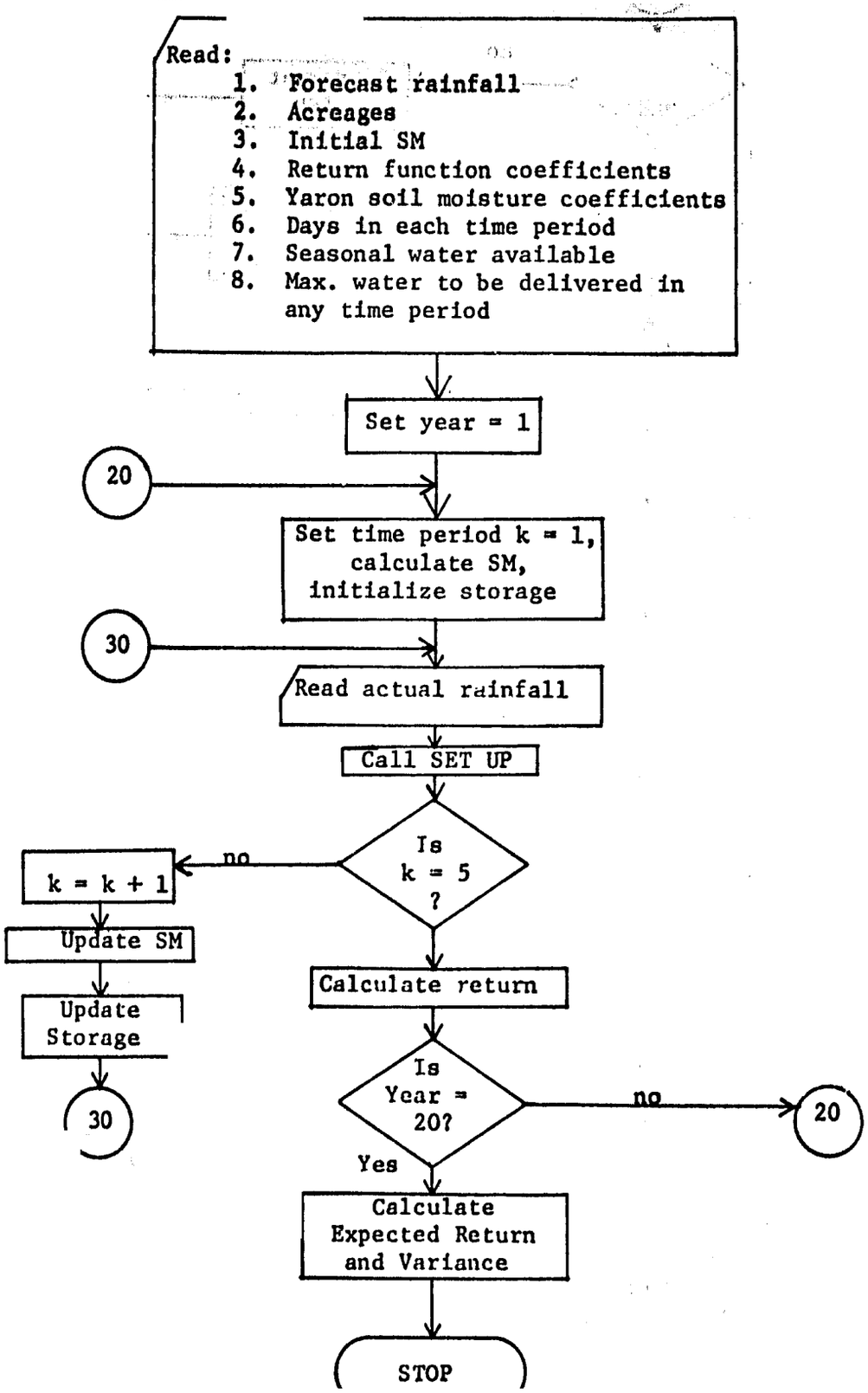



25
Initialize optimal return OR
Set IRR = 0
30
Calculate ET during this time period
Calculate return function R
Is R ≥ OR ?
yes
R = OR
Optimal IRR = IRR
no
Is SM + IRR > SMI ?
No
Increment IRR
30
yes
40


40
Update OIRR and ODEL
Is A = 1 ?
NO
Increment A
20
Yes
Is time period = 1 ?
No
Decrement time period
10
Yes
Print optimal return
Set SM = SMI
50
Calc. SM at end of time period
Recall and print optimal IRR
Is time period = 4 ?
no
Increment time period
50
yes
STOP

Program OPT DAY




35
Is IRR ≥ MAX?
no
Increment IRR
30
Yes
Is SM = MAX?
no
Increment SM and SM index
20
Yes
Is DAY = 1?
no
Decrement DAY
10
Yes
Write optimal return
Set DAY = 1 and initialize SM
50
Calculate new SM and recall optimal IRR
Write optimal IRR
Is DAY = MAX?
no
Increment DAY
50
Yes
STOP

Program IRR SIM