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## 9. ABSTRACT

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## ABSTRACT <br> DESIGN OF IRRIGATION DROP STRUCTURES

The object of the study described herein was to examine the performance of various drop structures and to attempt formulating a generalized design procedure. The approach channel and flow characteristics throughout the length of the drop structures were reviewed on the basis of open channel hydraulics. Many useful proposed stilling basins were also introduced and compared with each other in terms of both length and performance. Two types of drop structures were primarily investigated: first, a vertical drop structure with abrupt fall; and second, an inclined drop structure with a stilling basin.

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## TABLE OF CONTENTS

Page
LIST OF FIGURES ..... vii
Cnapter
I INTRODUCTION ..... 1
Statement of the Problem ..... 1
Purpose of the Study ..... 2
Scope of the Study ..... 3
Approach to the Study. ..... 5
II HYDRAULICS OF DROP STRUCTURES ..... 8
Review of Basic Concepts ..... 8
Hydraulics of Drop Inlets. ..... 13
Inclined Drops ..... 17
Metiod of solution to predict the flow depth along an incline. ..... 18
Effect of channel roughness, slope and length ..... 20
Vertical Drops ..... 22
Prediction of flow depth before a jump ..... 22
Nappe trajectory length. ..... 27
Hydraulic Jump ..... 35
Derivation of conjugate depth ..... 36
Length of hydraulic jump ..... 37
Types of jumps ..... 39
Stilling Basins. ..... 41
Straight drop spillway ..... 43
S.A.F. stilling basin. ..... 47
Dissipation bars ..... 49
USBR stilling basin III. ..... 53
USBR stilling basin IV . ..... 56
Appurtenances ..... 56
Tailwater. ..... 60
III DROP STRUCTURE DESIGN ..... 63
Vertical Drop Structures ..... 63
Vertical drop section. ..... 64
Stilling basin without appurtenances ..... 68
Stilling basin with appurtenances. ..... 73
Inclined Drop Structures ..... 77
Design of inclined section ..... 77
Stilling basin without appurtenances ..... 84
Stilling basin with appurtenances ..... 95

TABLE OF CONTENTS (Cont.)
Chapter Page
IV SUMMARY, CONCLUSIONS AND RECOMMENDATIONS ..... 101
Sunmary. ..... 101
Conclusions ..... 106
Recommendations ..... 107
BIBLIOGRAPHY ..... 109
APPENDIX ..... 117
Figure Page

1. Schematic drawing of a typical vertical drop ..... 4
2. Schematic of an inclined drop structure in an irrigation system ..... 4
3. Specific energy diagram ..... 10
4. Free overfall ..... 14
5. Constrictio.. (check structure) overfall ..... 14
6. End depth ratio for various channel slopes ..... 16
7. Inclined jet with standing pool on one side ..... 24
8. Condition of jet striking in inclined plane . ..... 24
9. Energy dissipation at the base of the free overfall ..... 28
10. Nappe trajectory of flow ..... 30
11. Graphical solution to nappe trajectory ..... 31
12. Definition sketch for free jet over a weir ..... 33
13. Types of hydraulic jump ..... 40
14. Straight drop spillway stilling basin ..... 45
15. SAF stilling basin ..... 48
16. Drop structure with dissipation bars ..... 51
17. Design c=iteria for scour hole ..... 52
18. Recommended geometry for USBR stilling basin III ..... 54
19. Length of hydraulic jump on a horizontal floor for USBR stilling basin III ..... 54
20. Height of baffle piers and end sill for USBR stilling basin III ..... 55
21. Geometry of the USBR stilling basin IV ..... 55

## LIST OF FIGURES (Continued)

Figure Page
22. Relation between Froude number and the unit discharge and the drop height in vertical drop structures ..... 66
23. Relation between Froude number and $Y_{1}$ in each unit discharge ..... 67
24. Relation between Froude number and sequent depth ratio for rectangular channel. ..... 69
25. Relation between the length ratio $L / Y_{2}$ and Froude number ..... 71
26. Stilling basin length for vertical drops ..... 75
27. Length of inclined section for $n=0.013$ and $\mathrm{F}_{1}=4.0$ ..... 80
28. Length of inclined section for $n=0.013$ and $F_{1}=4.5$ ..... 81
29. Length of inclined section for $n=0.013$ and $\mathrm{F}_{1}=5.0$ ..... 82
30. Length of inclined section for $n=0.013$ and $F_{1}=6.0$ ..... 83
31. Length of inclined section for $n=0.013$ and $F_{1}=7.0$ ..... 83
32. The length ratio of the inclined section for different Froude numbers and channel roughness ..... 85
33. Determination of flow regime ..... 86
34. Length of inclined section for $n=0.017$ and $F_{1}=4.0$ ..... 88
35. Length of inclined section for $n=0.017$ and $F_{1}=4.5$ ..... 89
36. Lencyth of inclined section for $n=0.017$ and $F_{1}=5.0$ ..... 89
37. Length of inclined section for $n=0.017$ and $F_{1}=6.0$ ..... 90

## LIST OF FIGURES (Continued)

Figure Page
38. Length of inclined section for $\mathrm{n}=0.017$ and $\mathrm{F}_{1}=7.0$ ..... 90
39. Length of inclined section for $n=0.021$ and $F_{1}=4.0$ ..... 91
40. Length of inclined section for $n=0.021$ and $\mathrm{F}_{1}=4.5$ ..... 91
41. Length of inclined section for $\mathrm{n}=0.021$ and $\mathrm{F}_{\underline{1}}=5.0$ ..... 92
42. Length of inclined section for $\mathrm{n}=0.021$ and $\mathrm{F}_{1}=6.0$ ..... 92
43. Length of inclined section for$\mathrm{n}=0.021$ and $\mathrm{F}_{1}=7.0$ • . . . . . . . . 92
44. Length of inclined section for
$n=0.025$ and $F_{1}=4.0$ ..... 93
45. Length of inclined section for $n=0.025$ and $F_{1}=4.5$ ..... 93
46. Length of inclined section for$\mathrm{n}=0.025$ and $\mathrm{F}_{1}=5.0$. . . . . . . . . 9447. Length of inclined section for$n=0-025$ and $F_{1}=6.0$. . .94
48. Length of inclined section for $n=0.025$ and $F_{1}=7.0$ ..... 94
49. Comparisons of stilling basin lengths in the inclined drop structures ..... 97

## CHAPTER

## $t$ INTRODUCTION

## Statement of the Problem


In the design of irrigation, drainage, and soil conservation systems, the problem of controlling flow veloE4: cities sufficiently to minimize erosion is continually confront 3 . Under many such conditions, drop structures have been successfully used to prevent excessive scour by dissipating a substantial fraction of the energy within the structure itself. Generally, the energy dissipation is accomplished with a hydraulic jump, a very effective and widely used energy dissipation phenomena. Although the drop itself provides a good means of dissipating energy, the effectiveness can often be improved by adding a stilling basin and their appropriate appurtenances.

The need for energy dissipation is underlined by the expense involved in most hydraulic structures, but most experimental studies have concerned themselves only with the very large conveyance works. In the few applications to smaller irrigation channels, most information describing the use of drop structures in specific uses and little general design information is available. Thus, a need exists for consolidating available experimental and design data into a generalized format for designing small drop $\underset{\sim}{6}$ st structures.

## Purpose of the Study

The purpose of this study is to develop generalized design information for small drop structures in irrigation channels. The drop structures must provide satisfactory flow conditions in the downstream channel, with minimum erosion potential, by including the stilling basin and its appurtenances.

To date, field observation of existing drop structure installations, including many commercial prefabricated structures, has indicated a general deficiency in providing adequate energy dissipation (Humphreys and Robinson, 1971). For example, a major problem encountered in designing small drop structures is that the Froude number of the flows at the bottom of the drop often fall in the range between 2.0 and 4.0 where hydraulic jumps are undular and unstable. Under these conditions, the erosive potential of the flows leaving the drop structure may be compounded by large surface waves.

An important objective of this study is to identify the limitations which should accompany the various design alternatives and evaluate the extension of the information to other conditions. This goal is to be achieved by following four primary work phases:

1. Identifying the general conditions where each drop geometry is best suited;
2. Predicting the flow characteristics at the bottom of the drop structure;
3. Determining the stilling basin geometry and appurtenances which most effectively dissipate the energy; and
4. Specify the rules covering design dimensions for various drop situations.

## Scope of the study

The use of a single structural geometry to meet the wide range of discharges and drop heights that may be encountered in irrigation, drainage, or soil conservation projects is impractical. Consequently, the scope of this study is limited to small drops being used in irrigation conveyance systems. Such drops are of two types: (1) vertical drop; and (2) inclined drop. In either case, the height of the drop does not generally exceed ten feet nor does the discharge exceed ten cubic feet per second per foot of width.

Vertical drops, as shown schematically in Fig. 1, provide an abrupt transistion from one elevation to another. The flow leaves the upstream channel as a freely falling nappe and then impinges on the stilling basin floor. Once the nappe strikes the floor of the stilling basin it begins to dissipate the energy of the drop through turbulence as indicated by the hydraulic jump.

The inclined drop structure is shown graphically in Fig. 2. Rather than transferring the flow between the upstream and downstream channels abruptly, as was the case for the vertical drop, the flow passes along a steep chute between upstream and downstream channels. At the


Fig. 1. Schematic drawing of a typical vertical drop structure in an irrigation system.


Fig. 2. Schematic of an inclined drop structure in an irrigation system.
end of the chute, the flow enters the stilling basin where much of its energy is dissipated by the hydraulic jump. The flows in both the vertical and inclined drop structures and subsequent stilling basin are affected by a number of parameters, such as discharge, drop height, bed roughness, side slope, and bottom width. In this study, only rectangular cross-sections are being considered to facilitate the analysis. In this manner, the author can develop the design procedure more fully for rectangular sections, thereby providing a better basis for later investigations by others regarding rectangular sections, as well as other geometric sections (e.g., trapezoidal, triangular, or circular).

## App,oach to the Study

The design procedures developed in this study summarize and apply the existing experimental information on drop structures, as well as the theory of open channel flow. The experimental data are used in conjunction with the characteristics of the flow to describe the geometries of the drop and stilling basins necessary to dissipate sufficient energy for a given discharge and . drop height.

The primary concern in the analysis of vertical drop structures is predicting the flow conditions in the drop structures. In order to accomplish this evaluation, the flow entering the stilling basin is analyzed for
given drop heights, discharges and channel geometries. The downstream flow conditions also would be evaluated from the viewpoint of the energy dissipation resulting from the hydraulic jump and the nappe trajectory of the freely falling jet. A knowledge of the flow conditions in the stilling basins is important in order to meet the requirements to the downstream channel. Various types of appurtenances shall be analyzed.

The variables needed in the analysis for the inclined drop are much more complex than that of the vertical drop. Therefore, it is more difficult to generalize the design procedure to provide simple rules and graphs for accomplishing the necessary design. Predicting the flow conditions in the upstream and downstream sections is the first step in formulating the design procedure for the inclined drop. Based on the flow conditions, the slope of the inclined drop is determined for a given drop condition, thereby producing a specific value of Froude number. Once this slope is determined, the specific features of stilling basin structures to satisfy downstream channel requirements may be determined.

Since the primary objective of this study is to develop general design rules for small drop structures, and specifically for the rectangular section for which the design procedure is well defined, no attempt will be made to verify the results by new experimental data.

Rather, efforts will be made to test various designs against existing recommendations and field observations.

## HYDRAULICS OF DROP STRUCTURES

The energy dissipation in drop structures is a subject which has received attention from many investigators in the past. Several factors are interrelated and serve to make the subject much more complicated than a superficial examination would indicate. To understand the phenomenon of the hydraulic jump and the energy dissipation in the drop structure, it is essential that basic concepts and hydraulic theories which are always associated with designs of drop structures are reviewed.

A drop structure usually includes an inlet section, a drop section in which the lowering is made, a stilling basin where the excess energies are dissipated, and an outlet section through which the water is discharged. In this chapter, the hydraulics of these important parts of a drop structure will be discussed.

## Review of Basic Concepts

The basic principle most often used in hydraulic analysis is the law of conservation of energy as expressed by the Bernoulli Equation,

$$
\begin{equation*}
E=z+\cos \theta+\alpha \frac{V^{2}}{2 g} \tag{1}
\end{equation*}
$$

in which:
$E$ is the total head at a section of the channel;
$z$ is the elevation above the datum plane;
$\theta$ is the slope angle of the channel bottom;
$y$ is the depth of the flow;
$V$ is the average velocity;
$\alpha$ is the volocity distribution coefficient; and $g$ is the acceleration due to gravity.

Specific energy in a channel section is defined as the energy per pound of water at any section of a channel measured with respect to the channel bottom. Thus, according to Eq. 1 with $z=0$, the specific energy becomes,
$E=y \cos \theta+\alpha \frac{V^{2}}{2 g}$
or for a channel of small slope and $\alpha=1$,

$$
\begin{equation*}
E=y+\frac{V^{2}}{2 g} \tag{3}
\end{equation*}
$$

which indicates that the specific energy is equal to the sum of the depth of flow and the velocity head. Eq. 3 can be plotted on the specific energy diagram which was introduced by Bakhmeteff (1912). It shows that for each value of E , two alternative depths of flow exist as shown in Fig. 3.

In a unit width of rectangular channel, Eq. 3
becomes,

$$
\begin{equation*}
E=\frac{q^{2}}{2 g y^{2}}+y \tag{4}
\end{equation*}
$$

in which $q$ is the discharge per unit width of the channel.
Differentiating Eq. 4 with respect to the depth and setting it equal to zero yields,

$$
\begin{equation*}
y_{\min }=\sqrt[3]{\frac{q^{2}}{g}} \tag{5}
\end{equation*}
$$

where $Y_{\text {min }}$ is the depth with minimum specific energy which is by definition, the critical depth of the flow, $y_{C}$. Hence,

$$
\begin{equation*}
Y_{C}=\sqrt[3]{\frac{g^{2}}{g}} \quad \text {. . . . . . . } \tag{6}
\end{equation*}
$$



Fig. 3. Specific onergy diagram. .

The concept of critical depth is also important in describing the action of gravitational and inertial forces acting on the flow. Generally, the ratio of the two forces is determined and is denoted the Froude Number, F, defined as,

$$
\begin{equation*}
F=\frac{V}{\sqrt{g y_{h}}} \tag{7}
\end{equation*}
$$

where $y_{h}$ is the hydraulic depth defined as the area of the flow cross-section divided by the surface width. In the case of a rectangular channel, the depth of the flow, y , is identical with the hydraulic depth, $\mathrm{y}_{\mathrm{h}}$. If Eq. 6 is rearranged to solve the critical velocity and substituted into Eq. 7, the value of the Froude number is shown to be 1. Thus, for subcritical or tranquil flow, $F<1$; at critical depth, $F=1$; and for supercritical or rapid flow, $F>1$.

Another fundamental principle which is involved in many hydraulic problems is a change in momentum, which is expressed by Newton's second law of motion. For the case of steady incompressible flow, the law may be stated in the following terms:

$$
\begin{equation*}
M=m \frac{V_{a}-V_{b}}{t} \tag{8}
\end{equation*}
$$

in which $M$ is momentum change in a time $t$, while $V_{a}$ and
$\mathrm{V}_{\mathrm{b}}$ are the velocity $a t$ sections a and b . In Eq. 8,
$-m$ is the mass of the volume of water flowing from section a to b. Since,

$$
\begin{equation*}
m_{l}=\frac{Q \gamma t}{g} \tag{9}
\end{equation*}
$$

Eq. 8 becomes,
$\quad \underset{m}{ }=\frac{\mathbf{Q \gamma}\left(\mathbf{V}_{\mathbf{a}}-\mathbf{V}_{\mathbf{b}}\right)}{\mathbf{g}}$
where $Q$ is the discharge; $\gamma$ is specific gravity and $g$ is the acceleration due to gravity. To date, a vast amount of research has been conducted to relate the preceeding principle directly to open channel flow, resulting in a great deal of success in formulating empirical estimates describing the flow. One of the most commonly used of the empirical formula for uniform channel flow is the Manning equation, which was published first in 1890.

$$
\begin{equation*}
V=\frac{1.486}{n} R^{2 / 3} S^{1 / 2} \tag{ll}
\end{equation*}
$$

, where $s_{e}$ is the slope of the energy line, $n$ is a roughness coefficient, and $R$ is the hydraulic radius given by,

$$
\begin{equation*}
R=\frac{A}{P} \tag{12}
\end{equation*}
$$

$A$ is the cross-sectional area of the $f l o w$, and $P$ is the dwetted perimeter. From Eq. 11 , the slope of the energy line (which is the same as the slope of the channel floor when uniform flow conditions exist' can be calculated as:

$$
\begin{equation*}
S_{e}=\frac{n^{2} V^{2}}{2.21 R^{4 / 3}} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \tag{13}
\end{equation*}
$$

The value of $n$ corresponding to the approximate boundary condition of the channel can be estimated from published sources such as Chow (1968).

## Hydraulics of Drop Inlets

The inlet section of a drop structure can consist of an approach channel having a geometry similar to irrigation canals or laterals, in which the water drops off the end of the channel. Numerous drop structure inlets in irrigation systems are used as check structures, thereby resulting in a number of possible conditions regarding the jet entering the stilling basin (Figs. 1 and 2). Usually, a turnout structure will be located immediately upstream from the check structure which is utilized to control the upstream water level and thus regulate the diversion through the upstream turnouts. The flow passing over the check is usually less than the design discharge for the channel, whereas the check structure might be removed when the full discharge capacity of the channel is to be conveyed downstream. The inlet to the drop must also be considered a control for the upstream channel to prevent channel scouring. The inlet should be symmetrical about the channel centerline and when possible located a sufficient distance downstream from horizontal bends in order to limit undesirable wave action due to unsymmetrical flow.

The flow condition at the inlet to the drop are shown in Figs. 4 and 5 in which the major variables are the flow depth at the end of the channel, called the brink depth, $Y_{b}$, or end depth, $Y_{e}$, and the roughriess coefficient, n .


Fig. 4. Free overfall.


Fig. 5. Constriction (check structure) overfall.

For the flow condition shown in Fig. 5, critical depth occurs in the vicinity of the constriction because both rectangular and trapezoidal check structures are commonly used in irrigation systems, the critical depth and its location for their condition can be computed with an accuracy sufficient for design purposes.

Rouse (1936) studied the discharge characteristics of the free overfall in a channel with a mild slope. It was found that the pressure distribution is no longer hydrostatic as the depth of flow decreased below the critical depth, $Y_{C}$, for uniform flow. From a series of experiments, Rouse concluded that the distance between the critical depth, $Y_{C}$, and the brink depth, $Y_{b}$, is equal to approximately four times the critical depth. Numerous measurements by Rouse give evidence that the ratio between the brink depth, $y_{b}$, and the critical depth, $y_{C}$, is 0.715, regardless of the ratio of discharge and the channel width.

Delleur, et al. (1956) showed that the ratio of the brink depth to the corresponding critical depth depends upon the relative slope of channel for a rectangular free overfall.

Diskins (1961) developed equations for the brink depth for exponential and trapezoidal channels with zero net pressure at the end section. Rajaratnam and Muralidhar (1964) improved Diskins' work and have developed brink depth relationships for various channel geometries such


Fig. 6. End depth ratio for various channel slopes.
as rectangular circular, trapezoidal, triangular and parabolic. They defined the flow depth at the end of the approach channel as the end depth, $y_{e}$. Rajaratnam and Muralidhar (1964) also found that the end depth ratio is 0.705 for the rectangular free overfall with an approach Channel of zero slope, which is slightly less than Rouse's value of 0.715 . The end depth ratio curves for the triangular, parabolic, rectangular and circular channels are shown in Fig. 6. The end depth ratio $Y_{e} / y_{c}$ is plotted against the slope ratio $s_{0} / s_{c}$, in which $s_{0}$ is the bed slope and $s_{c}$ is the bed slope corresponding to critical depth. A negative slope ratio means the bed slope has an adverse gradient. The end depth ratio curves for trapezoidal channels with different values of the shape parameter $z_{s} y_{c} / b$, in which $z_{s}$ is the side slope and $b$ is the bottom width, will lie between the curves for the triangular and rectangular channels.

## Inclined Drops

The effective design of an inclined drop can be possible when the variables such as design discharge, depth at the inlet, and channel shape, slope roughness, and length are properly treated by an adequate procedure. The usual case for inclined drops is for the slope of the section to be in the steep range so that the flow control point will be at the inlet. The most important hydraulic characteristic in designing the inclined
section is the flow depth at the bottom of the incline, which is also the flow depth preceeding the hydraulic jump in a properly designed stilling basin. Consequently, the major task in designing inclined drops is to determine what depth can be expected at the end of the incline. Method of solution to predict the flow depth along an incline. For the inclined drop section, there is no experimental data or analysis to evaluate the conditions at the end of the incline even with an extensive knowledge of the flow condition at the inlet section. Consequently, the design of such a structure depends upon computing the flow profile from one section to another. Although numerous techniques have been developed to examine flow profiles, a numerical method is probably most reasonable because of its applicability to computers. The method described herein was proposed by Prasad (1970) employing a numerical solution to the trial and error procedure. The differential equation of gradually varied flow is used in conjunction with Manning's formula for describing the energy slope. The gradually varied flow equation can be differentiated and expressed as,

$$
\begin{equation*}
\frac{d y}{d x}=\frac{S_{0}-S_{e}}{1-a \frac{Q^{2} T}{g A^{3}}} \tag{14}
\end{equation*}
$$

in which $y$ is the flow depth, $x$ is the distance along the channel bed, $s_{0}$ is the slope at the bed, $s_{e}$ is the energy gradient, $\alpha$ is the velocity head coefficient, $Q$ is the
discharge, $T$ is the top width of the channel section, $g$ is the acceleration due to gravity, and A is the channel cross-sectional area.

The term $\alpha\left(Q^{2} T / g A^{3}\right)$ in the gradually varied flow equation represents the kinetic flow factor and equals the Froude number squared. The coefficient $\alpha$ has been assumed to be constant from section to section, and in this study, a value of 1.00 has been used.

The evaluation of $s_{e}$ when using the Manning's formula can be expressed as,

$$
\begin{equation*}
S_{e}=\frac{n^{2} V^{2}}{2.21 R^{4 / 3}} \tag{15}
\end{equation*}
$$

where $n$ is the roughness coefficient, $V$ is the average flow velocity, and $R$ is the hydraulic radius. Eq. 15 can be substituted into Eq. 14 to compute the value of $d y / d x$.

The numerical solution reported by Prasad (1970) has been written for a digital computer and the program listing has been included in the Appendix. The procedure is based on the following equation. The depth at a section $i+1$ can be described as,

$$
y_{i+l}=y_{i}+\Delta y
$$

or

$$
\begin{equation*}
y_{i+1}=y_{i}+\left(\frac{d y}{d x}\right) \Delta x \tag{16}
\end{equation*}
$$

in which the subscript $i$ describes the ith station along the channel. The value of $\Delta y$ can be computed from the value obtained in Eq. Id multiplied by the incrementalw
value of distance along the channel, $\Delta x$. If the value of $\Delta x$ is very small, the value obtained in Eq. 14 can be assumed to vary linearly between stations in the solution. Assuming this condition, Eq. 16 can be rewritten to form the general basis for the solution.

$$
\begin{equation*}
Y_{i+1}=y_{1}+\left[\frac{\left(\frac{d y}{d x}\right)_{i+1}+\left(\frac{d y}{d x}\right)_{i}}{2}\right] \Delta x \quad . . . . \tag{17}
\end{equation*}
$$

The computational procedure, then is as follows:
(1) Compute $d y / d x$ from Eq. 14 using the initial value of $y$, or the previous value after the initial step solution,
(2) Assume $(d y / d x)_{i+1}=(d y / d x)_{i}$ as a first approximation,
(3) Evaluate the depth $y_{i+1}$ using the step
(4) Compute a new value of (dy/dx) ${ }_{i+1}$ from Fig. 14 based on the $y_{i+1}$ obtained in $i+1$ step (3); and
(5) Repeat steps (3) through (5) until the two estimates of (dy/dx) it come within the desired degree of accuracy, and then using the new value of depth, repeat this process until the last depth is obtained.

Following these five steps, a numerical solution of the equation of gradually varied flow can be obtained by digital computer in this study.

Effect of channel roughness, slope and length. Under the flow condition of Fig. 2, the variables which define the characteristic of flow are as follows:

Flow condition $=f\left(H, q, L, S, n, F_{1}\right)$
The channel roughness, $n$, can be varied to determine its effect upon flow conditions or, the optimum slopes for any value of roughness can be determined for the inclined
section by specifying the drop height and the Froude number, $F_{1}$. The Froude number, $F_{1}$ is computed from Eq. 7.

$$
\begin{equation*}
F_{1}=\frac{q}{g^{1 / 2} y_{1}^{3 / 2}} \tag{18}
\end{equation*}
$$

The length of the inclined section is expressed as,

$$
\begin{equation*}
\mathrm{L}=\frac{\mathrm{H}}{\mathrm{~S}} \tag{19}
\end{equation*}
$$

where $H$ is the inclined drop height and $S$ is the channel slope of the inclined section. The differential equation for the water surface profile and the hydraulic gradient slope is given by Eqs. 14 and 15 , respectively. From these relationships, the slope of the hydraulic gradient can be described as,

$$
\begin{equation*}
s_{e}=s_{0}-\frac{d y}{d x}\left(1-F_{1}^{2}\right) \tag{20}
\end{equation*}
$$

If the slope of the inclined section is determined for a given Froude number, $F_{1}$, by employing the optimization theory in order to compute an optimum slope, then the water profile can be determined by Prasad's (1970) method as described in the Appendix. The slope of hydraulic gradient, which includes the channel roughness, $n$ term, is calculated using Eq. 20. Consequently, it can be recognized how the channel roughness does affect the slope of an inclined section from these relationships. For uniform flow, however, since $d y / d x$ will be zero, the slope of the channel bed and the hydraulic gradient will be the same. In this case, the effect of the channel roughness, $n$, is directly proportional to the channel slope.

From the viewpoint that channel roughness determines the slope of the inclined section from the above approach, it may be also possible that the length of the inclined section can be controlled by the channel roughness of the inclined section for a given Froude number, $F_{1}$, and drop height, H.

## Vertical Drops

As was pointed out for the inclined drop, basic hydraulic information is also needed for the design of vertical drops. The flow depth in the stilling basin before a jump, $Y_{1}$, and the nappe trajectory length, $L_{d}$, are the most important parameters in determining each design dimension. There are a number of methods to calculate these parameters. A few of the available theoretical and experimental equations will be introduced in this section.
prediction of flow depth before a jump. In vertical drops, the aerated free falling nappe will occur beginning at the crest of the drop section. Rand (1955) indicated that for vertical drop structures, the flow depth before a jump $y_{1}$ and the conjugate depth, $y_{2}$ corresponding to $Y_{1}$ can be expressed as:

$$
\begin{align*}
& \frac{y_{1}}{H}=0.54\left(\frac{y_{c}}{H}\right)^{1.275} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot(21)  \tag{21}\\
& \frac{y_{2}}{y_{1}}=\frac{3.07}{\left(y_{c} / H\right)^{0.465}} \cdot \cdots \cdot \cdot \cdot \cdot(22) \tag{22}
\end{align*}
$$

These equations were developed from experimental data and therefore include energy loss.

A theoretical equation for determining the flow depth in the stilling basin before a jump was developed by White (1943). When a sheet of water strikes a flat surface, the flow pattern at the floor may be shown as in Fig. 7. From the momentum principle (Eq. 10), the change in the horizontal momentum is $\frac{\gamma}{g} v^{2} y_{a}(l-\cos \theta)$ in the upper sheet and $\frac{\gamma}{g} V^{2} y_{f}(1+\cos \theta)$ in the lower sheet. Equating these momentum changes gives:

$$
\begin{equation*}
\frac{y_{f}}{y_{a}}=\frac{1-\cos \theta}{1+\cos \theta} \tag{23}
\end{equation*}
$$

The flow to the left into the basin, $Q_{f}$, is determined in terms of $\theta$;

$$
\begin{equation*}
Q_{f}=\frac{Y_{f}}{Y_{a}} Q=\frac{1-\cos \theta}{1+\cos \theta} Q \tag{24}
\end{equation*}
$$

This is also the flow from the basin back into the jet. Assuming that this return flow has negligible momentum in the direction of the jet, the total momentum of the jet will not change in the mixing process. This determines $V_{m}$, the velocity after mixing. The momentum equation is:

$$
\begin{align*}
\frac{\gamma_{Q}}{Q} & =\frac{\gamma}{g}\left(Q_{f}\right) V_{m} \\
& =\frac{\gamma}{g}\left(Q+Q \frac{1-\cos \theta}{1+\cos \theta}\right) V_{m} \tag{25}
\end{align*}
$$

From Eq. (24)


Fig. 7. Inclined jet with standing pool on one side.

(a)

Fig. 8. Condition of jet striking inclined plane.

Since $V_{m}$ equals $V_{1}$ (the outflow velocity)

$$
\begin{equation*}
v_{1}=\frac{V}{2}(1+\cos \theta) \tag{26,a}
\end{equation*}
$$

and the corresponding depth of flow is

$$
\begin{equation*}
y_{1}=\stackrel{Q}{V_{1}} \tag{27}
\end{equation*}
$$

The cosine of the angle $\theta$ is the ratio of the horizontal velocity component, $\mathbf{V}_{\mathbf{x}}$, to the total velocity which is described as:

$$
\begin{equation*}
\cos \theta=\frac{\mathrm{V}_{\mathrm{x}}}{\mathrm{~V}} \tag{28}
\end{equation*}
$$

Since in the free overfall, $V=\sqrt{2 g \Delta h}$, where $\Delta h$ equals the sum of the specific head, $E$, and drop height, $H$. The velocity, V , can be expressed as:

$$
\begin{align*}
& v=\sqrt{2 g(H+E)}  \tag{29}\\
& v=\sqrt{2 g\left(H+\frac{3}{2} y_{c}\right)} \tag{30}
\end{align*}
$$

The horizontal velocity component, $\mathrm{V}_{\mathrm{x}}$, is found by equating the horizontal force at the critical section above the free overfall to the change in horizontal momentum between this point and any point in the free jet.

$$
\begin{equation*}
\frac{r_{2}}{y_{c}}={ }_{g}^{\gamma_{q}}\left(v_{x}-v_{c}\right) \tag{31}
\end{equation*}
$$

Because $q=V_{c} y_{c}$ and $V_{c}=\sqrt{g y_{c}^{\prime}}$ this gives,

$$
\begin{equation*}
\mathrm{v}_{\mathrm{x}}=\frac{3}{2} \mathrm{v}_{\mathrm{c}} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \tag{32}
\end{equation*}
$$

From Eq. $28, \cos \theta$ is substituted into Eq. 32 and Eq. $x$ 30 .

$$
\begin{equation*}
\operatorname{Hos} \theta=\frac{1.5 v_{c}}{\sqrt{2 g\left(H+\frac{3}{2} y_{c}\right)}}=\frac{1.06}{\sqrt{\frac{H}{Y_{c}}+\frac{3}{2}}} \tag{33}
\end{equation*}
$$

To determine the flowdepth just below the fall, use the i, continuity equation,

$$
y_{1}=\frac{q}{v_{1}} \quad \text { wet }
$$

and also, $\mathbf{y}_{\mathbf{C}}=\frac{\mathbf{q}}{\mathbf{v}_{\mathbf{C}}}$
Consequently,

$$
\frac{Y_{1}}{Y_{c}}=\frac{v_{c}}{v_{1}}=\frac{2 v_{c}}{v(1+\cos \theta)}
$$

Or man

$$
\begin{align*}
\frac{Y_{1}}{Y_{C}} & =\frac{2^{g y_{c}}}{\left.\sqrt{2 g\left(H+3 y_{c} / 2\right)\left[1+\frac{1.06}{H / y_{C}+3 / 2}\right.}\right]} \\
& =\frac{\sqrt{2}}{1.06 \sqrt{H / Y_{c}+3 / 2}} \ldots . \tag{34}
\end{align*}
$$

The total energy of the flow is found by specific energy equation as shown in Eq. 3. Then,

$$
\frac{E_{1}}{Y_{c}}=\frac{Y_{1}}{Y_{c}}+\frac{v_{1}^{2}}{2 g Y_{c}}
$$

0
is $\quad \frac{E_{1}}{Y_{C}}=\frac{\sqrt{2}}{1: 06 \sqrt{H / Y_{C}+3 / 2}}+\left(1.06+\sqrt{H / Y_{C}+3 / 2}\right)^{2} \cdot \cdots \quad$ (35)
Since $F_{1}=\left(y_{c} / y_{1}\right)^{3 / 2}$, the relative specific energy,
$E_{1} / Y_{C}$, retained by the flow leaving the section of impingement is'readily calculated from the general equation by inserting the values obtained from Eq. 4.

$$
\begin{equation*}
\frac{E_{1}}{y_{c}}=\frac{Y_{1}}{Y_{c}}\left[1+\frac{1}{2}\left(\frac{Y_{c}}{Y_{1}}\right)^{3}\right] \tag{36}
\end{equation*}
$$

This equation agrees quite well with experimental work by Moore (1943), as shown in Fig. 9, in which the values of $H / y_{c}$ versus $y_{c}$ for both experimental data and theoretical values are plotted. In Fig. 9, a horizontal line drawn between the initial head and experimental curve represents the head loss of the streail due to the existence of a basin upstream from the inpingement of the jet.

Nappe trajectory length. In a vertical drop, the length of basin consists two parts, the nappe trajectory length, $L_{d}$ and the hydraulic jump length, $L_{j}$ as shown in Fig. 1. The nappe trajectory length, $L_{d}$ can be defined as the distance from the crest to the place where the jet strikes the basin floor. In order to determine the nappe trajectory length, equations describing the nappe of a freely falling jet are used.

For the free overfall, the nappe trajectory length $L_{\mathrm{d}}$ can be determined using experimental data presented by Moore (1943), Bakhmeteff and Feodoroff (1943), and Rand (1955). They found that the flow geometry at a straight drop spillway can be described by functions of the drop number, which is defined as:

$$
\begin{equation*}
D=\frac{q^{2}}{g H^{3}} \tag{37}
\end{equation*}
$$

where $q$ is the discharge per unit width of crest of overfall, $g$ is the acceleration due to gravity, and $H$


Fig. 9. Energy dissipation at the base of the free overfall.
is the drop height. The functions which relate to the nappe trajectory length, $L_{d}$, are:

$$
\begin{equation*}
\frac{L_{\mathrm{d}}}{\mathrm{H}}=4.30 \mathrm{D}^{0.27} \tag{38}
\end{equation*}
$$

where the nappe trajectory length is the distance from the vertical wall to the position of the flow depth $Y_{1}$, while $y_{1}$ is the depth at the toe of the nappe or beginning of the hydraulic jump (Fig. 1).

One of the recent investigations by Donnelly and Blaisdell (1965) expresses the nappe trajectory length as an equation for nappe trajectory. The equation for $L_{d}$ is:

$$
\begin{equation*}
L_{d}=\frac{x_{f}+x_{s}}{2} \tag{39}
\end{equation*}
$$

in which $X_{f}$ is the horizontal distance from drop crest to the upper surface of the free-falling nappe at the elevation of the stilling basin as shown in Fig. 10. The equation for $X_{f}$ is:

$$
\begin{equation*}
\frac{X_{f}}{Y_{c}}=-0.406+\sqrt{3.195-4.386 \frac{H}{Y_{C}}} \tag{40}
\end{equation*}
$$

The equation for the upper surface of the submerged nappe trajectory above the tailwater level is the same as that for the free falling nappe. The point at which the upper nappe impinges on the tailwater is:

$$
\begin{equation*}
\frac{x_{t}}{Y_{c}}=-0.406+\sqrt{3.195-4.386 \frac{Y_{t}}{Y_{c}}} . \tag{41}
\end{equation*}
$$

in which $X_{t}$ is the horizontal distance from the drop crest to the point at which the surface of the upper nappe


Fig. 10. Nappe trajectory of flow.


Fig. 11. Graphical $^{\text {solution to nappe trajectory. }}$
plunges into the tailwater and $X_{t}$ is the vertical distance from the crest to the tailwater surface as shown in Fig. 10.

At a point where the upper surface of the submerged nappe trajectory strikes the stilling basin floor, the equation is:

$$
\begin{equation*}
\frac{x_{s}}{X_{c}}=\frac{0.691+0.228\left(X_{t} / Y_{c}\right)^{2}-\left(H / Y_{c}\right)}{0.185+0.456\left(x_{t} / y_{c}\right)} \tag{42}
\end{equation*}
$$

where $X_{s}$ is the horizontal distance from the drop crest to the point where the upper surface of the submerged nappe strikes the stilling basin floor. Therefore, from Eq. 39, the nappe trajectory length $L_{d}$ can be determined when the discharge and height of the vertical drop are given. This equation has been computed and plotted in Fig. 11. It should be noted that the origin of points $X$ and $y$ is at the crest of the drop.

Katsaitis (1966) has developed an equation to determine the nappe trajectory length from the weir condition of the vertical drop crest. The definition sketch of flow over a weir (Fig. 12) can be used to describe the development of an equation for determining $L_{d}$ which is the horizontal distance from the weir (or drop crest) to the centerline of the nappe trajectory where it strikes the stilling basin floor.

The reference point for the equation is the centeroid of the jet cross-section where the lower nappe has reached its highest elevation. The horizontal distance


Fig. 12. Definition sketch for free jet flow over a weir.
between the reference point and the weir crest is $H_{0}^{\prime} / 2$, where $H_{0}^{\prime}$ is the total head (specific energy) above the high point of the lower nappe.

The equations of motion can be written as:

$$
\begin{align*}
& x=\left(v_{0}\right)_{x} t \cdot  \tag{43}\\
& v_{z}=\left(v_{0}\right)_{z}+g t \cdot  \tag{44}\\
& z_{2}=\left(v_{0}\right)_{z} t+1 / 2 g t^{2}  \tag{45}\\
& v_{z}=\left(v_{0}\right)_{z}^{2}+2 g z \cdot \tag{46}
\end{align*}
$$

in which, $X$ is the horizontal distance from the reference point, $\left(V_{0}\right)_{x}$ is the initial horizontal velocity at the reference point, $\left(V_{0}\right)_{z}$ is the initial vertical velocity, $t$ is time, $V_{z}$ is the final vertical velocity, and $Z$ is the total height of fall measured from the reference point, where

$$
\begin{equation*}
\mathrm{Z}=\mathrm{H}+0.373 \mathrm{H}_{0} \tag{47}
\end{equation*}
$$

Therefore, the available head above the height $Z$ is $0.627 \mathrm{H}_{\mathrm{o}}$.

The axis of the nappe may be considered perpendicular to a vertical section at a distance $0.5 H_{0}^{\prime}$ from the crest and the horizontal velocity of the filament in the nappe axis may be computed from the relation $v^{2} / 2 \mathrm{~g}$. In this case,

$$
\begin{equation*}
V_{x}=\left[\left(0.627 \mathrm{H}_{0}\right)(2 g)\right] \quad 1 / 2 \tag{48}
\end{equation*}
$$

At the reference point, the water particles have no vertical velocity. Consequently, the theoretical vertical velocity of the jet as it strikes the stilling basin floor can be obtained from Eq. 46 as;

$$
\begin{align*}
& V_{z}=(2 g Z)^{1 / 2} \cdot . . . . . . . . . . . . . \quad \text { (49) } \\
& v^{2}=v_{x}{ }^{2}+v_{z}{ }^{2}  \tag{50}\\
& t=(2 \mathrm{z} / \mathrm{g})^{1 / 2} \tag{51}
\end{align*}
$$

the horizontal distance traveled in this time, $t$ can be determined by combining Eq. 51 with Eq. 48.

$$
\begin{align*}
L_{\mathrm{d}} & =x+0.5 \mathrm{H}_{0}^{\prime} \\
& =\left(\mathrm{V}_{\mathrm{x}} t\right)+0.5 \mathrm{H}_{\mathrm{o}}^{\prime} \\
& =\left(0.627 \mathrm{H}_{\mathrm{o}} \mathrm{Z}\right)^{1 / 2}+0.5 \mathrm{H}_{0}^{\prime} \tag{52}
\end{align*}
$$

The above analysis is based on an aerated nappe. In practice, if the nappe is not fully aerated, the free jet will strike the stilling basin floor at a shorter distance from the overflow crest. Thus, the computation of $L_{d}$ using Eq. 52 would result in a conservative design.

## Hydraulic jump

The first investigation on the hydraulic jump conducted in the United States was during 1894 by Ferriday. Additional experiments regarding the hydraulic jump have been conducted by many investigators.

Bakhmeteff and Matzke (1936) presented the flow characteristics of the hydraulic jump together with a thorough discussion of the hydraulic jump in terms of dynamic similarity. By this study, they determined the general relationships that applied to hydraulic jumps of every size and nature. Also, Kinney (1935) conducted a series of experiments on the hydraulic jump.
(4) In 1954, a large number of experiments on the hydraulWhe ic jump were performed in the hydraulic laboratory of the U.S. Bureau of Reclamation (USBR). A total of 125 tests were run in six rectangular flumes varying in width from 1 to 5 ft . They verified the relationship between $y_{2} / Y_{1}$ and the Froude number, $F_{1}$, in order to show the applicability of the momentum formula to the hydraulic jump. In the experiments, the tailwater was varied until the hydraulic jump formed for a given Froude number. After completion of a series of these experiments, the U.S. Bureau of Reclamation (1964) published their extensive results on the performance of the hydraulic jump for energy dissipation in stilling basins. Derivation of conjugate depth. The flow depth after the hydraulic jump, $y_{2}$, can be determined by the sequent depth ratio, $Y_{2} / Y_{1}$ in which the subscripts refer to the sections after and before the jump, respectively. The most common technique used to describe the sequent depth ratio is through the use of the momentum principle of the flow.

From Eq. 10, the change in momentum $\Delta \mathrm{M}$ is $\Delta M=\frac{\gamma q\left(V_{1}-V_{2}\right)}{g} \cdot \cdots \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$

Since the change in hydrostatic pressure must equal the change in momentum according to Newton's second law of motion,
$\Delta p=\Delta M$
or

$$
\begin{equation*}
\frac{r q v_{1}}{g}+p_{1}=\frac{r q v_{2}}{g}+p_{2} \tag{54}
\end{equation*}
$$

For the rectangular channel, since $\Delta p=\frac{1}{2} \gamma\left(y_{2}^{2}-y_{1}^{2}\right)$, Eq. 54 becomes,

$$
\begin{equation*}
y_{2}^{2}-y_{1}^{2}=\frac{2 q}{g}\left(v_{1}-v_{2}\right) . \tag{55}
\end{equation*}
$$

Substituting $g / y$ for $V$, and solving for $y_{2}$ in terms of $Y_{1}$ and $V_{1}$,

$$
\begin{equation*}
Y_{2}=-\frac{Y_{1}}{2}+\sqrt{\frac{2 V_{1}^{2} Y_{1}}{g}+\frac{Y_{1}^{2}}{4}} . \tag{56}
\end{equation*}
$$

If $F_{1}^{2}=V_{1}^{2} / g y_{1}$ is substituted into Eq. 56 , the momentum formula becomes,
$\frac{Y_{2}}{y_{1}}=\frac{1}{2}\left(\sqrt{1+8 F_{1}^{2}}-1\right)$
Experiments were conducted in 1955 by the U.S. Bureau of Reclamation to verify Eq. 57. The agreement between theory and experimental values is excellent over the entire range, indicating that Eq. 57 is applicable even when the flow enters the jump at an appreciable angle to the horizontal. Silvester (1964) has probably presented the most complete analysis, along with a substantial literature review and generalization of experimental data. Jeppson (1965) provided a useful monograph for the solution to Eq. 57, as well as for the case involving triangular and trapezoidal channel shapes.

Length of hydraulic jump. The length of the hydraulic jump may be defined as the distance measured from the
front face of the jump to a point on the surface immediately downstream from the roller (Chow, 1959). This length cannot be determined easily by theory, but it has been investigated experimentally by many hydraulicians.

Bakhmet.eff and Matzke (1936) conducted an experiment to determine the longitudial element of the hydraulic jump. After examining the experimental results, they proposed that the length of hydraulic jump is a function of the Froude number and the height of jump. After Bakhmeteff and Matzka, numerous investigators (Aravin, 1935; Chertoussov, 1935; Ippen, 1950; Page, 1936; Posey, 1941; Woycieki, 1935; Wu 1949) proposed the formulas which determined the length of the hydraulic jump. Almost all the investigators described the length of jump in terms of the flow depth before the hydraulic jump and the height of hydraulic jump.

After examining the experimental results, the investigators proposed that the length of hydraulic jump is a function of the Froude number and the height of the hydraulic jump. The experimental data on the length of the hydraulic jump, $L_{j}$, can be plotted conveniently with the Froude number, $F_{1}$, against a dimensionless ratio, $L_{j} /\left(y_{2}-y_{1}\right), L_{j} / y_{1}$, or $L_{j} / y_{2}$.

From the experimental result of the USBR, the plot of $\mathrm{F}_{1}$ vs $\mathrm{L}_{\mathrm{j}} / \mathrm{yl}$ is probably the best, for the resulting curve can be best defined by the data. However, for practical purposes, the plot of $F_{1}$ vs $L_{j} / y_{2}$ is desirable
because the resulting curve shows a fairly flat portion for the range of Froude number that produces a well established hydraulic jump. This curve was developed primarily for the hydraulic jump occurring in rectangular channels. This experimental result may be applied throughout this study for determining the length of hydraulic jump in a horizontal stilling basin. The conjugate depth, $Y_{2}$, can be determined by Eq. 57, which is also well defined by the experiments.

Types of jumps. There are essentially four different forms of the hydraulic jump that may be encountered in the design of drop structures having a horizontal floor. It is important to note that the energy dissipation and internal characteristics of the hydraulic jump vary its performance considerably with each form. The following forms illustrated by Fig. 13 are taken from experiments conducted by the U.S. Bureau of Reclamation.

Each form has been classified in relation to the Froude number of the flow. When the Froude number is one, the water is flowing at critical depth; thus a hydraulic jump will not occur. For values of the Froude number between 1.0 and 1.7 , there is only a slight difference in the conjugate depth $Y_{1}$ and $Y_{2}$. As $F_{1}$ approaches 1.7, a series of small rollers develop on the water surface. When $F_{1}$ varies from 1.7 to 2.5 , the water surface is quite smooth, and the velocity throughout the crosssection is fairly uniform. The second form is that of the

D. Effective but Rough

[^1]transistion jump, occurring when $F_{1}$ varies from 2.4 to 4.5. This type of action is common in canal structures. Although some success in this range can be obtained with the use of wave suppressors (USBR Basin IV) it is very difficult to design a stilling basin for this form of hydraulic jump. When $F_{i}$ varies from 4.5 to 9.0 , well stabilized jumps are formed as depicted by Fig. 13. With values of $F_{1}$ exceeding 9.0 , the form of the hydraulic jump gradually changes and the effectiveness of the hydraulic jump is good, but the water surface is so rough it might need a specific type of stilling basin.

## Stilling Basins

The main function of stilling basins is to dissipate the excess energy of flowing water for downstream channel protection. Stilling basin designs are based primarily on laboratory investigations, model studies, and experience. However, the major hydraulic phenomenon which is associated with stilling basins is basically the hydraulic jump. Thus, the length of the hydraulic jump and the depth of water after the jump determined the basic dimensions of stilling basins. Reducing the stilling basin length can be accomplished using a number of proposed stilling basin appurtenances, each used for particular flow conditions.

There are many generalized stilling basin designs for energy dissipation. For example, Maxwell Stanley (1934) studied a simple stilling basin in which a pool was formed using a simple sill at the end of the basin; however, this
simplestilling basin is larger than more recently
developed stilling basins. Armin Schoklitsch (1937) has published results of experiments performed on an energy dissipator that was similar to Stanley's simple stilling basin, except that a silil or bucket was used at the basin entrance.

Jacob E. Warnock (1940) has described a rectangular stilling basin developed by the Bureau or Reclamation in which exploratory tests indicated that the basin is particularly efficient and that its size could be reduced considerably.

A Morris-Johnson drop structure was installed in the erodable sandy clay soil for the purpose of gully control a*ter at the U.S. Naval Auxiliary Air Station, Whiting Field, Milton, Florida. The performance was poor, showing excessive scour occurring just downstream of these structures. To improve this situation, Blaisdell and Donnelly (1950) conducted experiments and developed a drop spillway stilling basin. Its field operation was as successful as the laboratory tests had indicated. As a result of the favorable experience at Whiting Field, general design rules were developed for the straight drop spillway.

In many proposed 'designs of stilling basins, important elements to help the performance of drop structures are the straight drop spillway and dissipation bars for
20) vertical drops; and SAPF. stilling basin, USBR stilling basin III, and USBR stilling basin IV for inclined drops.

More recently, Humphreys and Robinson (1971)
investigated the performance of drop and check structures in the field, and arrived at the following conclusions:
(1). The commercial prefabricated structure did not generally provide adequate stilling basins for energy dissipation;
(2). The end sill caused turbulance that affected the downstream scour pattern and, in a small ditch, increased the total erosion volume;
(3). Structures having relative wide basins performed better than those with narrow basin;
(4). The coffer dam type structures gave fairly good hydraulic performance when used as a check with sufficient tailwater;
(5). With adequate tailwater depth, a trapezoidal stilling basin gave a good hydraulic performance; and
(6). For the relatively smail structures and water depth in the study, a nonaerated nappe contributes to good stilling within the structure.

They also concluded that laboratory studies are needed to investigate different methods of improving hydraulic design by verifying the effect of flared wingwalls, rounded corners, sills, and nonaerated nappes, etc.

Straight drop spillway. The straight drop spillway is used for erosion control in gullies, as a grade control structure in drainage ditches, as an irrigation drop and check structure, and as a spillway for earth dams.

Donneliy and Blaisdell (1965) reported a generalized straight drop spillway design after a series of

Fn experiments improving their previous work. . The tests were $^{\text {w }}$. performed in a channel having an approach section to the straight drop spillway that was 6 ft . wide, 10 ft . long, and 2 ft . deep. The resulting design specification is shown in Fig. 14. All the information presented in Fig. 14 was carefully checked and verified by experiments using downstream scour as the basis for evaluation.

The resulting design procedure is listed below.

1. The minimum length of the stilling basin $L_{b}$ is
given by:

$$
\begin{equation*}
I_{b}=L_{d}+X_{b}+X_{c}=I_{d}+2.55 y_{c} \cdot \cdots \tag{58}
\end{equation*}
$$

a. $L_{d}$ is the nappe trajectory length, which is the horizontal distance from the drop crest to the point where the average of the free and submerged upper nappe strikes the floor expressed by Eq. 39. Fig. 10, where $y_{c}$ is the critical depth of flow.
b. The distance from the point at which the surface of the upper nappe strikes the stilling basin floor to the upstream face of the floor block, $X_{b}$ is $X_{b}=0.8 y_{c}$.
c. The distance between the upstream face of the floor blocks and the end of the stilling basin, $x_{c}$ is $X_{c} \geq 1.75 y_{c}$.
2. The floor blocks are proportioned as follows:
a. The height of floor blocks is $0.8 Y_{C}$.


Fig. 14. Straight drop spillway stilling basin.
b. The width and spacing of floor blocks should be approximately $0.4 y_{C}$ but a variation of $0.15 \mathrm{Y}_{\mathrm{C}}$ from this limit is permissible.
c. The floor blocks should be square in plane and should occupy between $50 \%$ and $60 \%$ of
in the stilling basin width.
3. The height of end sill is $0.4 \mathrm{y}_{\mathrm{c}}$.
4. The side wall height above the tailwater level should be $0.85 \mathrm{y}_{\mathrm{C}}$.
5. The wing wall should be located at an angle of $45^{\circ}$ with the outlet centerline and should have a top slope of l:l.
6. The minimum height of the tailwater surface above the floor of the stilling basin, $y_{2}$, is $2.15 \mathrm{Y}_{\mathrm{C}}$.
7. The approach channel should have the following qualifications.
a. Be level with the crest of the drop.
b. Have the toe of the dike or the toe of the side slope intersect the approach channel floor at the end of the spillway notch, the approach channel at the headwall should have a bottom width equal to the spillway notch length.
c. Be protected by riprap or paving for a distance upstream from the headwall equal to two times the notch depth.
8. No special provision for aeration of the space beneath the nappe is required if the approach channel is shaped as recommended above.
S.A.F. stilling basin. Blaisdell (1948) developed the St. Anthony Falls (S.A.F.) stilling basin from the rectangular stilling basin reported by Warnock. This stilling basin should give adequate performance for a range of Froude numbers varying from 1.7 to l.7. Normally, the length of the basin is less than twice the sequent flow depth, $Y_{2}$. Thus, this particular structure is quite small and economical compared with Stanley's simple stilling basin. This is recommended for use on small structures such as small spillways, outlet works, and small canal structures.

1. The length of the stilling basin is determined by, $\mathrm{I}_{\mathrm{b}}=4.5 \mathrm{y}_{2} / \mathrm{F}_{1}{ }^{0.38}$.
2. The height of chute blocks and floor blocks is $y_{1}$ and the width and spacing are approximately $0.75 \mathrm{y}_{1}$.
3. The distance from the upstream end of the stilling basin to the floor blocks is $L_{b} / 3$.
4. No floor blocks should be placed closer to the sidewall than $3 y_{1} / 8$.
5. The floor blocks should be placed downstream from the openings between the chute blocks.
6. The floor blocks should occupy between 40-55\% of the stilling basin width.
7. The height of the end sill is given by $c=0.07 \mathrm{y}_{2}$ in which $y_{2}$ is the theoretical sequent depth corresponding to $y_{1}$.


Fig. 15. SAF stilling basin.
8. The tailwater above the stilling basin floor is given by;

$$
\begin{array}{ll}
y_{2}^{\prime}=\left(1.10-F_{1}^{2} / 120\right) y_{2} & \text { for } F_{1}=1.7-5.5 \\
y_{2}^{\prime}=0.85 y_{2} & \text { for } F_{1}=5.5-11 \\
y_{2}^{\prime}=\left(1.00-F_{1}^{2} / 8.0\right) y_{2} \text { for } F_{1}=11-17 . \tag{60}
\end{array}
$$

9. The height of side wall above the maximum tailwater depth is given by;
$z=y_{2} / 3$
10. Wingwalls should be equal in height to the stilling basin side walls. The top of the wingwall should huve a slope of l:l. It should be placed at an angle of $45^{\circ}$ to the outlet center line.
11. A cutoff wall of nominal depth should be used at the end of the stilling basin.

Dissipation bars. Katsaitis (1966) has developed a vertical drop structure having two rows of dissipation bars which are placed in the stilling basin. Dissipation bars proved very effective in a comparison with a model S.A.F. energy dissipator. A comparison test was made by replacing the S.A.F. structure with a dissipation bar structure of equal length. The result was that the dissipation bar structure passed 50 percent more than the design discharge without scour. The same discharge with the S.A.F. energy dissipator produced severe scour. The design recommendation is as follows:

1. Stilling basin length, $L_{b}$ is;
toot Aas $L_{b}=L_{1}+2 a_{1}+L_{2}+D$
in which $L_{1}$ is the distance from the crest to the first row of bars, which is expressed by 1.5 $L_{d}$; $D$ is $0.17 /\left(\mathrm{g} / \mathrm{V}^{2}\right)$; the width of bar, W , is $0.04 /\left(g / V^{2}\right)$; and $L_{2}$ is equal to $W$ as depicted in Fig. 16. $\mathrm{V}^{2}$ can be determined by Eq. 48, Eq. 49, and Eq. 50.
2. The height of bars, $h=2 Y_{c}$ would be sufficient to protect both bed and bank.
3. The height of bars should span the full depth of the flow when smooth flows are necessary for a downstream flow measuring device.
4. The end sill must be excluded because it destroys the streamlined flow produced by the bars.
5. The longitudinal shape of the scour bed downstream from the dissipation structure should be designed as shown in Fig. 17. Having found the scour depth, the value of radius R used to form the depression in the channel bed is found by, $\mathrm{R}=13 \mathrm{y}_{\mathrm{s}}$
6. The scour depth downstream in a channel may be computed by,
$y_{g} / H=2.728 \quad H_{0}\left[\left(g / V^{2}\right)\right]-0.2177 . .$.
in which $H$ is the drop height and $H_{0}$ is the specific energy above the crest.


Fig. 16. Drop structure with dissipation bars.


Fig. 17. pesign oriteria for scour hole.po . at giv
7. The tailwater depth may vary to any degree without affecting the performance of the bars.

USBR stilling basin III. The USBR stilling basin III, which is shown in Fig. 18, gives good performance for entrance Froude numbers, $F_{i}$, greater than 4.5. The length of this stilling basin is generally $2.75 \mathrm{Y}_{2}$, which is longer than the S.A.F. stilling basin, but only half the length of the free hydraulic jump. The design criteria developed by Peterka (1964) are listed below.

1. The stilling basin operates best at full sequent tailwater depth, $\mathrm{Y}_{2}$.
2. The length of basin can be obtained by consulting the curve sliown in Fig. 19.
3. Stilling basin III may be effective for values of Froude number as low as 4.0
4. The height, width, and spacing of chute blocks should equal the average depth of flow entering the basin, $Y_{1}$. The width of the blocks may be decreased, provided spacing is reduced a like amount. Should $y_{1}$ prove to be less than 8 inches, the block should be made 8 inches in height.
5. The size of baffle piers and end sills can be obtained by consulting the curves in Fig. 20.
6. It is recommended that a radius of reasonable length ( $R>4 y_{1}$ ) be used at the intersection of the chute and basin apron for slopes $45^{\circ}$ or greater.


Fig. 18. Recommended geometry for USBR stilling basin III.


Fig. 19. Length of hydraulic jump on a horizontal floor for USBR stilling basin III. . \%htate


Fig. 20. Height of baffle piers and end sill for USBR stilling basin III.


Fig. 21. Geometry of the USBR stilling basin IV. a Whe

USBR stilling basin IV. When Froude number is 2.5 to 4.5, an oscilliating jump will occur in the stilling basin, generating a wave that is difficult to dampen. The design criteria recommended by the USBR for this range of Froude number is as follows:

1. The length of the stilling basin is made equal to the length of the jump in a horizontal stilling basin without appurtenances, which can be determined from the curve in F'ig. 19.
2. The number of chute blocks shown in Fig. 21 is the minimum required to serve the purpose. For better hydraulic performance, it is desirable to construct the blocks narrower than indicated, preferably $0.75 \mathrm{y}_{1}$.
3. No baffle lers are needed in the basin. The addition of a small triangular sill placed at the ond of the apron for scour control is desirable. An end sill of the type used on Basin III is satisfactory.
4. Tailwater depth 5 to 10 percent greater than the conjugate depth is recommended for Basin IV.
5. The basin IV is applicable to rectangular crosssections only.

## Appurtenances

Appurtenances such as chute blocks, baffle piers and end sills are often installed to help improve the mo
performance of a stilling basin. In addition, they are helpful in stabilizing the flow, increasing the turbulence, distributing the velocities more evenly throughout the basin.

Schoklitsch (1938) described many of these appurtenances along with an energy dissipator he designed. He gave some descriptions of the effect of end sills in the design of his stilling basin previously mentioned, but made no attempt at evaluating the effect of floor blocks or baffle piers.

The USBR has made extensive studies on the appurtenances in connection with the hydraulic design of dams, and has evolved several different structural arrangements which apply to the specific conditions encountered.

Douma (1939) presented a rectangular stilling pool which he developed in the hydraulic laboratory of the USBR. This study gave the structural requirements necessary to produce satisfactory flow conditions at the discharge end of a stilling pool.

For determining the size of floor blocks, Warnock (1940) reported that the required height varied from 1/4 to $1 / 8$ of the downstream depth based on Douma's study.

Rusho (1948) presented a study which was more indicative of the type of work usually presented by the USBR. More complicateä studies of stilling basin appurtenances (Bwas performed by Blaisdell (1948), as previously mentioned in designing the SAF stilling basin. His
recomendations, insofar as they concern the size and spacing of the floor blocks, were that the best conditions of flow were obtained when:

1. The block height equaled the upstream flow depth;
2. The block width was three fourths of the block height; and
3. The aggregate block width was between 40 and 55 percent of the total stilling pool width.

A generalized approach to the problem of controlling the hydraulic jump is the study made by Forster and Skrinde (1949). On the basis of experimental data and theoretical analysis, they developed a diagram showing the relationships among, (1) Froude number of the approaching flow, (2) the ratio between the weir height and the approaching depth, $y_{1}$, and (3) the ratio between the distance from the toe of the jump to the weir crest and the sequent depth, $y_{2}$, upstream from the weir.

Weide (1951) made a special study of the use of floor blocks and their effect on the control of the jump. He suggested a block coefficient, $\mathrm{C}_{\mathrm{b}}$, dependent upon different forces exerted on the blocks, which can be determined by using the following equation:

$$
\begin{equation*}
c_{b}=\left(\frac{y_{2}}{y_{1}}\right)^{2}+\left(\frac{y_{2}}{y_{1}}\right)-2 F_{1}^{2} \tag{65}
\end{equation*}
$$

Also, Weide (1951) ' ndicated that when baffles were used, the conjugate depth can be found by:

$$
\begin{equation*}
y_{2}=\frac{y_{1}}{2}\left(\sqrt{1+4 \mathrm{Cb}+8 \mathrm{~F}_{1}^{2}}-1\right) \cdot \tag{66}
\end{equation*}
$$

: Harleman (1955) reported that appreciable depth reduction due co baffle piers was possible only if the toe of the hydraulic jump was within 20 block heights of the piers. It was also concluded that the depth reduction due to the piers be considered only as insurance against dangerous displacement of the hydraulic jump through. Changes in the Froude number or the tailwater depth.

More extensive studies were made by Bradley and Peterka (1957) with different shapes of baffle piers and sills to determine their efficiency as an appurtenance in stilling basins. From this investigation, it has been found that the best performance is given by baffle piers with a rectangular upstream face. For structural considerations, the downstream side may be made sloping. Pillai and Unny (1964) investigated wedge-shaped blocks as stilling basin appurtenances. As a result of model tests, blocks with an upstream angle of $120^{\circ}$ provide the best performance among the blocks tested and help to increase the energy dissipation in a shorter length. He also stated that the caviation damage would be reduced by the shape of the blocks.

Zimmerman and Maniak (1967) conducted a model test with eight types of end sills and baffle piers. They concluded that scour was reduced by more than $50 \%$ by the baffle piers. Furthermore, the baffle piers which contribute towards a stable position of the surface
roller within aatilling basin has considerable advanto tages. The baffle piers dimensions were independent of the drop height.

Rand (1965) has studied the effect of the vertical sill in an open channel. He described concisely the flow over a vertical sill by a set of five dimensionless parameters.

More recent investigations for appurtenances was made by Bhowmik (1971). Laboratory investigations were conducted for both the ordinary and forced hydraulic jumps on a horizontal floor in the Froude number range of 2.5 to 4.5. A set of appurtenances in a stilling basin which he called basin $L$ (including baffle piers, blocks and end sill ) was shown to perform satisfactorily. Based on this investigation, he provided design criteria of a stilling basin and appurtenance for the flow within the Froude number range of 2.5 to 4.5 .

## Tailwater

The formation of the hydraulic jump at the base of a stilling basin depends upon the relationship between the tailwater depth of the flow and the conjugate depth, $\mathrm{y}_{2}$, of the hydraulic jump. The conjugate depth of the hydraulic jump entirely depends upon the upstream condition. On the other hand, the tailwater depth results from downstream conditions. Therefore, in designing a stilling basin using the hydraulic jump as an energy dissipator, both the conditions upstream and downstream should be considered.

There may be three alternative patterns of tailwater depth that affect the hydraulic jump in a drop structure:

1. The case where the tailwater surface elevation is equal to the water surface elevation of the conjugate depth of the hydraulic jump, $y_{2}$;
2. The case where the tailwater surface elevation is less than the conjugate depth water surface elevation;
3. The case where the tailwater surface elevation is greater than the conjugate depth water surface elevation.

When the tailwater depth, $y_{t}$, is equal to the conjugate depth, $Y_{2}$, the hydraulic jump will occur in the stilling basin. For scour protection purposes, this is an ideal case. One big objection to this condition, however, is that a little difference between the actual and assumed values of the pertinent hydraulic coefficients may cause the jump to move downstream from its estimated position. Consequently, some means for controlling the position of the hydrauiic jump is always necessary.

When tailwater depth is less than the conjugate depth of the hydraulic jump, the hydraulic jump will move downstream to a point where Eq. 57 is again satisficd. This case must, if possible, be avoided in design, because the turbulence of the flow acting on an unprotected downstream channel will result in severe erosion. To prevent this condition, the designer should use a control structure in the channel bottom, which will increase the tailwater and thus ensure a hydraulic jump within the protected stilling basin. When the tailwater is nearly sufficient to cause
the jump to form, baffles or sills may be placed on the floor of the basin to increase the tailwatis depth.

The next important case is the pattern in which the tailwater depth, $Y_{t}$, is greater than, $Y_{2}$. In this case, the hydraulic jump will be forced upstream and may finally be drowned out at the stilling basin, becoming a submerged jump. This is the safest case in design because the position of the submerged jump can be most readily fixed and a high degree of energy dissipation results. When the vertical drop is used, high tailwater depths approaching the level of the approach channel bed will result in the jet moving along the water surface; thereby causing downstream bank erosion.

In practical design work, the designer should always be aware of the relationship between tailwater and conjugate depth of the hydraulic jump, and adjust the design to fit these conditions.

## CHAPTER III

## DROP STRUCTURE DESIGN

The design of both vertical and inclined drop structures involves three primary steps:
(1) Evaluation of the flow conditions at the inlet to the drop section;
(2) Prediction of the flow conditions at the base of the drop;
(3) Selection of a stilling basin and appurtenances which satisfactorily dissipate energy.

In the previous chapter, the theoretical and experimental procedures employed to accomplish each of these steps were reviewed. This chapter is presented to summarize the principles of the previous chapter into useful design procedures.

## Vertical Drop Structures

Vertical drops have been used in a variety of ways to control water. For example, vertical drops are often employed in irrigation, drainage, and erosion control structures to provide a safe transistion between two elevations in those systems. In addition, drops such as the measu-ing weir is a particular case of a vertical drop structure.

A vertical drop structure can be defined as a combination of a vertical drop and a stilling basin where the energy of flow is dissipated. The vertical drop
structure is well-suited for shall drops in irrigation channels, such as the typical six or eight feet elevation change in a large channel or the three and four feet drops in small ditches.

The normal approach to the design of vertical drops begins with the case where the nappe of the jet strikes the floor of the stilling basin and then forms a hydraulic jump at a downstream location as determined by the tailwater depth. Model tests on vertical drops, as reported in the literature, appear to deal with this case and they generally concentrate on reducing bed scour in the downstream channel. A great number of researchers indicate that for effective energy dissipation in vertical drop structures, a specific stilling basin should be employed along with needed appurtenances.

Vertical drop section. A typical vertical drop which is often used in irrigation channels was shown previously in Fig. 1, inäicating the various elements of the drop and stilling basin. As indicated by the drawing, generally given design parameters are the unit discharge, $q$, and the drop height, $H$.

For normal flow conditions of non-weir approach sections, a critical depth and brink depth for the rectangular cross-section can be computed using Eq. 6 and Fig. 6. Based on the brink depth, the flow depth before a jump can be determined by Eq. 34 as reported by White (1943). Using these relationships, the depth at the
base of the drop, $y_{1}$, was computed for a wide range of drop conditions. The Froude number at the base of the drop is expressed as a function of the drop height and the unit discharge. These relationships plot as straight lines on logarithmic paper, as shown in Fig. 22. In addition, a graphical relationship between $\mathrm{F}_{1}$ and $\mathrm{Y}_{1}$ can be established by Eq. 6 and Eq. 7 as shown in Fig. 23.

The two graphs showing the basic hydraulic characteristics of the flow are important in calculating the dimensions of the stilling basin. Using these graphs, the design can be modified when poor drop conditions produce low Froude numbers. These relations will be discussed later in the stilling basin section. The steps followed in determining the rroude number, $F_{1}$, and the flow depth before a jump, $y_{1}$, are:
(1) Based on the possible maximum discharge of the irrigation channel and the width of channel, the unit discharge, $q$ is computed;
(2) By a given drop height and a computed unit discharge, the Froude number before the hydraulic jump, $F_{1}$, is determined by using Fig. 22.
(3) If the Froude number does not fall within the range 4.0 to 9.0 , changes in sither drop height or the unit discharge should be taken.
(4) After selection of an adequate value of $F_{1}$ to safely transfer the flow through the given


Fig. 22. Relation between Froude number and the unit discharge and the drop height in vertical drop structures.


Fig. 23. Relation between Froude number and $y_{1}$ in each
unit discharge.
drop structure, the flow depth before the hydraulic jump, $Y_{1}$, is determined by consulting Fig. 23. The flow depth, $Y_{1}$, is used to determine the sequent depth $Y_{2}$, the length of stilling basin, and the dimensions of appurtenances.

Stilling basin design without appurtenances. The characteristics of the stilling basin without any appurtenances depends entirely on the hydraulic jump which is to occur within the stilling basin. Therefore, the important parameters relating to the hydraulic jump need clarification.

Once the flow depth, $Y_{1}$, and the Froude number, $F_{1}$, are determined for a particular drop condition, the computation of the sequent depth after a jump, $y_{2}$, becomes simple procedure. From the well known momentum equation applied to rectangular stilling basins, Eq. 57, the dimensionless depth ratio $y_{2} / y_{1}$, can be plotted with respect to the Froude number, $F_{1}$, as shown in Fig. 24. Thn line is virtually straight except for the left lower end. Examination of the figure shows that the correlation between the theory and the laboratory data is very good over the entire range, indicating that Eq. 57 is applicable when the flow enters the jump at an appreciable angle to the horizontal.

The entire length of the stilling basin consists of two main parts; the nappe trajectory length, $L_{d}$, and the hydraulic jump length, $L_{j}$.


4cFig. 24 . Relation between Froude number and sequent depth ratio for rectangular channel.

The nappe trajectory length can be determined by Eq. 39 or Eq. 52 as introduced in Chapter II. Using these equations, relations botween the dimensionless length ratio, $L_{d} / Y_{y}$. and the Froude number, $F_{1}$, can be developed as shown in rig. 25, For a given height $H$ and the discharge per unit wisth of the crest, $q$, both the sequent depth, $Y_{2}$, and the nappe trajectory length $L_{d}$, can be determined by Figs. 24 and 25.

Primarily in Fig. 25, the relation between tine Froude, number and nappe trajectory length is a curve having a mid, slope (except the left lower end), Itis recognized that, in this region, where the Froude number, $F_{i}$, is less thail 3.0 , the sequent Gepth, $y_{2}$, exceeds the drop height, H. Therefore, for this flow condition, the cifop crest may be submerged. Consequently, the turisulence of the flow may be converes for a considerable distance downstream.

A number of empirical relations are available to determine the length of the hydracilic jump, bui the usual practice is to use a length from 5 to 6 times the difference between the conjugate depths of the hydraulic jump $\left(y_{2}-y_{1}\right)$. Peterka (19154) has provided most of the currently available experimental data regarding hydraulic jump length. An analysis of the experimental data indicate that a good relationship between the length of. the jump and the height of the jump existed. Fig. 25 shows a plot of $I_{j} / Y_{2}$ versus Froude number, $F_{1}$. The top


Fig. 25. Relation between the length ratio $L / \underline{y}_{2}$ and Froude number.
curve is recommended by the USBR in determining the length of the hydraulic jump.

As previously mentioned in the vertical. drop section, in order to accomplish good energy dissipation in the stilling basin, as well as stabilization of the hydraulic jump, the Froude number should be within the range of 4.5 to 9.0. If possible, structures should be designed to insure that a hydraulic jump in this category will be formed. To control the Froude number, dimension changes in the vertical drop may be required.

Generally, the flow which produces small Froude numbers may occur in an irrigation channel because the drop height and unit discharge may be relatively small. For a given drop condition, to increase the Froude number, the following methods can be employed:
(1) Reduce the unit discharge by expanding the crest length;
(2) Increase the drop height by raising the crest elevation of lowering the stilling basin floor elevation; or
(3) A combination of (1) and (2).

For the flow condition having a Froude number, $\mathrm{F}_{1}>9.0$, the opposite changes can be made to decrease the Froude number.

The advantage of method (1) is that expanding the crest may result in erosion protection both upstream and downstream from the drop due to the decrease in
velocity. In the sume way, some advantages of raising the drop crest is reduced upstream velocities. In practice, raising of a rest is difficult to attain because of the backwater effect upstream of the channel.

In practical design, the economies would be checked between the cost of the expanded crest, or the cost of raising the crest, or the cost of lowering the stilling basin floor. The steps to determine the stilling basin length without appurtenances in the vertical drop structure is as follows:

1. Determine the conjugate depth, $Y_{2}$, from Fig. 24 for a given $F_{1}$ and $Y_{1}$.
2. Enter Fig. 25 which relates $L_{d} / Y_{2}$ and $F_{1}$ and determine $L_{d}$. In the same way, the hydraulic jump length $L_{j}$ can be determined by using Fig. 25.
3. The stilling basin length will be $L_{b}=L_{d}+L_{j}$ in which $L_{b}$ is total length of the stilling basin.
Stilling basin with appurtenances. For the purpose of decreasing the stilling basin length, many kinds of appurtenances have been proposed for different flow conditions. The important designs are the straight drop spill'way and dissipation bars reported by Blaisdell (1965) and Katsaitis (1966), respectively.

A comparison of the stilling basin length for the natural hydraulic jump (stilling basins without appurtenances), dissipation bars, and the straight drop spillway
is shown in Fig. 26 for a wide range of Froude numbers. The length of the stilling basin in this figure is the sum of nappe trajectory length, $L_{d}$, and the hydraulic jump length, $L_{j}$.

For the stilling basin length using dissipation bars, the slope of the curve is quite steep with the stilling basin length increasing rapidly as Froude number increases. In the region where Froude number is larger than 7.7, the length of stilling basin with dissipation bars becomes larger than the stilling basin length for the natural jump. This result is very surprising since the dissipation bars are expected to be highly effective in reducing the stilling basin length. In particular, the dissipation bars should result in a stilling basin length considerably less than the natural hydraulic jump. Thus, Fig. 26 indicates that the design procedure using dissipation bars should be modified.

An important dimension in determining the stilling basin length using dissipation bars is the distance "D" as shown in Fig. 15. By the recommended design procedure (Katsaitis, 1966), D is computed by using $0.17 \mathrm{~V}^{2} / \mathrm{g}$. The velocity of the nappe trajectory is affected a great deal by the drop height. Therefore, the value of $D$ may become very large by increasing the height of drop. As a result, the stilling basin length using dissipation bars exceeds the length of the natural jump at high Froude numbers (high velocities). Modifying the method


Fig. 26. Stilling basin length for vertical drops.
of calculating $D$ by employing the critical depth rather than the velocity of the nappe trajectory may produce a relation between stilling basin length and Froude number which is more plausible and similar to the relation for the straight drop spillway.

By comparison, the relation between stilling basin length and Froude number for the straight drop spillway nearly parallels the relation for the natural hydraulic jump. In fact, the straight drop spillway has a length slightly less than half the length of the natural hydraulic jump (including horizontal length of the nappe trajectory) for the entire range of Froude numbers.

The important differences in both stilling basins may be the resulting downstream bed scour for the same stilling basin length. Primarily, the design procedure for the dissipation bars is based upon model tests using qualitative evaluations of which is a commonly employed technique for cvaluating the performance of hydraulic structures. Since there is no available information regarding downstream conditions below the straight drop spillway, and dissipation bars structure, Fig. 26 provides the only readily available means for making a comparison.

Blaisdell (1965) recommended that the minimum tailwater depth be $2.15 y_{C}$ is all design recommendations are satisfied. On the other hand, for the dissipation bars structure, no tailwater depth was recommended, primarily because the bars should be effective at all tailwater depths.

An end sill is specified for the straight drop spillway, whereas Katsaitis (1966) specifically states that an end sill should not be used. A few recent investigators have indicated that the role of end sills for effective energy dissipation was questionable. They maintain that end sills produce more turbulent flow leaving the structure, thereby causing scour immediately downstream. An attempt to clarify the role cf end sills in the vertical drop structure should be undertaken in the laboratory.

## Inclined Drop Structures

According to a definition of the Bureau of Reclamation (1942), a chute which is 15 ft . or less in vertical height is called an inclined drop. Inclined drops have the advantage of low excavation costs and the structure can be installed to more readily conform with existing topography.

An attempt is made to prepare generalized designs for small inclined drop structures. The basic theory of the hydraulic jump is also used as with the vertical drop structures.

Design of inclined section. Factors involved in the design of an inclined drop are the height and length of the drop, discharge, channel geometries, and suriace roughness. The method used to predict the flow conditions at the base of the drop is a parametric application of the simple Fibonacci Search.

Consider the rectangular channel drop depicted eariier by Fig. 2. For a specified drop height, $H$, and Froude number, $F_{1}$, the slope of the inclined section can be determined to satisfy these conditions by employing the optimization approach. The steps in accomplishing the evaluation of slope are as follows:
(1) For a given Froude number, $\mathrm{F}_{1}$, and the unit discharge, $q$, the depth at the base of the incline represented by $y_{1}$ can be calculated from the following relationships:
$y_{1}^{*}=\frac{q^{2 / 3}}{g^{1 / 3} F_{1}^{2 / 3}}$..............
where $y_{1}^{*}$ is calculation value respect to a given $\mathrm{F}_{1}$ and g .
(2) Assuming that the maximum slope and minimum slope which can occur in the inclined drop are the slope of the Ogee Crest and critical slope, respectively. The value of slope for the specified conditions will be between these limits, $S_{\text {max }}$ and $S_{\text {min }}$.
(3) Generate Fibonacci numbers as follows:
$K_{0}=1$
$K_{1}=1$
$K_{n}=K_{n-1}+I_{n-2}$
in which K is the Fibonacci number
(4) Determine the search interval $\Delta$ from the Fibonacci number,

$$
\begin{equation*}
\Delta=\left(s_{\max }-s_{\min }\right) \frac{K_{n-2}}{K n} \tag{69}
\end{equation*}
$$

(5) Compute $y_{1}$ with respect to $S_{\max }$ and $S_{\min }$ by using the numerical solution of water profiles reported by Prasad (1970) and presented in the previous chapter.
(6) Compare $y_{1}$ and $y_{1}^{*}$ in steps 1 .
(7) Adjust the slopes by the step size $\Delta$, as follows and guess a new slope.
$S_{\text {min }}=S_{\text {min }}+\Delta$
$S_{\max }=S_{\max }-\Delta$
(8) Repeat step (5), (6), and (7) until $y_{1}$ is found to be the same as $y_{1}{ }^{*}$. At this point, the proper slope has been calculated.

By employing the optimization theory described herein for
a wide range of drop heights, unit discharges, base Froude numbers, the results can be graphically portrayed as shown in Figs. 27-31. From these figures, it is noticeable that the length of inclined section may be expressed as a function of the unit discharge, $q$, height of drop, $H$, channel roughness, $n$, and Froude number $F_{1}$. To test this hypothesis, the conditions in Fig. 2731 were repeated for Mannings $n$ values of $0.013,0.017$, 0.021 , and 0.024 and the results plotted again. These roughness represented the range generally associated


Fig. 27. Length of +wised section for $n=0.013$ and $F_{1}=$


Fig. 28. Length of inclined section for $n=0.013$ and $F_{1}=4.5$.


Fig. 29. Length of inclined section for $n=0.013$ and



Fig. 30. Length of inclined section for $n=0.013$ and $\mathrm{F}_{1}=6.0$.


Fig. 31. Length of inclined section for $n=0.013$ and $\mathrm{F}_{1}=7.0$.
with concrete channels which would be the most common construction material for inclined drops.

As a result of these additional analyses, an interesting result was discovered. If the incline lengths were sufficiently long, the length of the incline for various roughnesses could be expressed as unique functions of the roughness. For example, the data from figs. 27-31 calculated at an $n$ value of 0.013 were compared to the same data at different $n$ values. A length ratio is plotted against $n$ in Fig. 32., pointing out the results of this comparison.

If the flow approaches uniform depth, the length of the incline for specified conditions can be determined from Figs. 27-31. If such is not the case, the length must be determined for each situation. A useful aid in evaluating the regime for a particular design is presented in Fig. 33, indicating the dividing point between uniform and non-uniform flow down the incline.

Stilling basin without appurtenances. To determine the conjugate depth of the hydraulic jump, Fig. 24 which shows the relationship between the conjugate depth ratio, $y_{2} / y_{1}$, and the Froude number, $F_{1}$, is also valid for the inclined drop structures.

The length of hydraulic jump can ainu be determined by using the upper curve in Fig. 24, which is a plot of $I_{1} / Y_{2}$ versus $F_{1}$, for a certain range of flow conditions.


Fig. 32. The length ratio of the inclined section for different Froude numbers and channel roughnesses.


Fig. 33. Determination of flow region.


The important thing that should be considered in determinin; the length of stilling basin on the besis of the Froude number is that $F_{1}$ also affects the length of inclined section. If a large Froude number is desired, a shorter inclined section will result, but the length of stilling basin will be increased to compensate for the increased energy. Therefore, when an inclined drop structure is designed, the final decision should be based on an economical design taking into account the entire length of the inclined drop structure.

The steps for determining the length of inclined drop structure is as follows:

1. For a given drop height and unit discharge, assume a reasonable Froude number and the proper channel roughness;
2. Enter Fig. 33 and determine whether the flow is uniform flow or non-uniform.
3. Determine the length of the inclined section using Figs. 27-31 for uniform flow and Figs. 34-48 for non-uniform flow according to the channel roughness and Froude number.
4. By consulting Figs 23 and 24, determine the hydraulic jump length with respect to the assumed Froude number in step 1.
5. Compute the total length of inclined drop structure (horizontal length of incline plus length of hydraulic jump);


Fig. 34. Length of inclined secticn for $n=0.017$ and $F_{1}=4.0$.


Fig. 35. Length of inclined section for $\mathrm{n}=0.017$ and


Eig. 36. Length of inclined section for $n=0.017$ and $F_{1}=5.0$.


Fig. 37. Length of inclined section for $n=0.017$ and $F_{1}=$ 6.0 .


Fig. 38. Length of inalined section for $n=0.017$ and $F_{1}=$ 7.0 .


Fig. 39. Length of inclined section for $n=0.021$ and $F_{1}=$


Fig. 40. Length of inclined section for $n=0.021$ and $F_{1}=$ 4.5 .


Fig. 41. Length of inclined section for $n=0.021$ and $F_{1}=$


Fig. 42. Length of inclined sectionfor $n=0.021$ and $F_{1}=$


Fig. 43. Length of inclined section for $n=0.021$ and $F_{1}=7.0$.


Fig. 44. Length of inclined section for $n=0.025$ and $F_{1}=$


Fig. 45. Length of inclined section for $n=0.025$ and $F_{1}=$


Fig. 46. Length of inclined section for $n=0.025$ and $F_{1}=$


Fig. 47. Length of inclined section for $n=0.025$ and $F_{1}=$


Fig: 48. Length of inclined section for $n=0.025$ and $F_{1}=$
6. Adjust the value of $F_{1}$ to reduce the stilling basin length or inclined section length.
7. Repeat steps 1 through 6 until a satisfactory design is determined.

The control of the crest may be the same as mentioned before in the vertical drop, or a transistion inlet can be used to maintain a controlled head over the crest. The use of automatic gates or upstream checks, and flash boards are other means for controlling the upstream head.

Stilling basins with appurtenances. In practice, the stilling basin is seldom designed to confine the entire length of a free hydraulic jump, because such a basin would be too expensive. Consequently, appurtenances are installed in the stilling basin to control the jump. The main purpose of such control is to shorten the length of basin where the hydraulic jump takes place and thus reduce the size and cost of the stilling basin.

General functions of the appurtenances which are often used in inclined drop structures are as follows:

1. Chute blocks are placed at the entrance of the stilling basin to stabilize the hydraulic jump.
2. For baffle piers (floor blocks), the principle functions are to increase the turbulance in the stilling basin and help stabilize the formation of the hydraulic jump.
3. End sills are used primarily for scour control, (but this may be questionable).

The important stilling basin designs for the inclined drop structure which will be discussed herein are the SAF stilling basin, USBR stilling basin IV, and USBR stilling basin III. The appurtenances for the above stilling basins are listed in Table 1 which shows the dimensions of each appurtenance.

The USBR stilling basins and the SAF stilling basin are similar in appurtenances in that chute blocks, baffle piers and an end sill are used. The basin dimensions, however, are considerably different in some particulars. These differences seem to result from the differences in purpose of each structure. The SAF stilling basin tests were conducted in a movable bed channel. No actempt was made to accomplish entire energy dissipation within the stilling basin. On the other hand, the USBK insures that the energy dissipation will take place almost entirely within the stilling basin. This difference in experimental method and philosophy has resulted in the SAF stilling basin being shorter than the USBR stilling basin III. However, in the SAF stilling basin, the length of the stilling basin decreases as the Froude number increases. This is difficult to accept because i:lcreasing the Froude number will result in increased turbulence which will require additional stilling basin length to dissipate this excess energy in the region of Froude number 4.0 to 7.0. As shown in Fig. 49, the stilling basin length for the natural jump and che USBR stilling

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Fig. 49. Comparison of stilling basin lengths in inclined drop structures.

Table 1. Comparison of appurtenances in stilling basins.

basin III show an expected tendency of increased stilling basin length as the Froude is increased. Since the length of the SAF stilling basin decreased with increased Froude number, it is suggested that laboratory experiment is utilized to modify the design procedure for the SAF stilling basin.

The location of baffle piers is $0.8 y_{2}$ downstream from the chute blocks in the USBR stilling basin III. The length of stilling basin between Froude numbers of 4.0 and 9.0 are $2.07 \mathrm{y}_{2}$ and $2.70 \mathrm{y}_{2}$, respectively. Thus, the baffle piers are located from $1 / 2.59$ to $1 / 3.38$ times the basin length downstream from the chute blocks. The corresponding distance recommended for the SAF stilling basin $1 / 3$, which is very close as shown in Table l. A difference is that the baffle piers in basin III do not have to be staggered with the chute blocks. Staggering the blocks is recommended for best performance of the SAF stilling basin. Hallmark (1954) concluded that the staggering of floor blocks or baffle piers is highly effective for energy dissipation.

In the design of the USBR stilling basin IV, baffle piers are excluded. The Basin IV has been developed for flow conditions wherein the Froude number is between 2.5 and 4.5. In this region of $F_{1}$, it is very difficult to control the oscilliating wave. The purpose of appurtenances for Basin IV is to handle this wave action and to stabilize the hydraulic jump. In this study, the Froude
phe number can be taken from 7.0 to 4.0 in inclinea drop structures. Consequently, the USBR Basin IV is beyond the scope Novof this study, but it may be taken as a conservative design, or a proper design for flow conditions which does not permit the desired Froude number from 4.0 to 7.0 .

End sills are provided for the SAF stilling basin, the USBR Basin III and the USBR Basin IV. As previously mentioned in the vertical drop structure, end sills should be low, since high end sills may cause turbulent flow downstreani of the basin. In the past, the end sill was considered as indispensable appurtenance for controlling the scour immediately downstream from the stilling basin. The exact dimensions for end sills can be determined by laboratory experiments.

In conclusion, the USBR stilling basin III is most reasonable for inclined drop structures. However, other designs using the natural hydraulic jump and USBR stilling basin IV can also be considered for special conditions.

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS


#### Abstract

Summary The goal of this study has been to develop generalized design procedures for small irrigation drop structures by employing techniques which evaluate hydraulic characteristics at the inlet section, at the base of drops, and in the stilling basin. A number of analytical methods for various hydraulic elements were presented and developed in detail for designing drop structures. Current concepts for evaluating hydraulic characteristics of drops, along with procedures for the design of stilling basins including the behavior of the hydraulic jump, form the theoretical basis for the design procedures. A step-by-step approach has been proposed to allow for sufficient flexibility to adopt methods of attack to a wide variety of problems.

Recommended design procedures for drop structures are dealt with in Chapter III and are intended to give step-by-step explanations of the designs. To summarize these results, this chapter presents design procedures, thus lending a practical viewpoint to the analytical results.

Possible cases for example designs may be as follows: For vertical drop structures: a. Case I - The case where given drop conditions ( $H$ and $q$ produce an adequate base Froude number $\left(F_{1}=4.0-9.0\right)$.


b. Case II - The case where given drop conditions do not produce a proper Froude number for effective energy dissipation. (It is recommended for this case that the unit discharge be changed by modifying the channel bottom width, so that a proper Froude number, $F_{1}=4.0-9.0$, would be produced.

For inclined drop structure:
a. Case I - The case where a given drop height and unit discharge produce a uniform flow depending upon the channel roughness coefficient.
b. Case II - The case in which the flow remains entirely non-uniform along the entire length of the incline.

Vertical Drop Structures. Design procedures for vertical drop structures are listed below when the following design parameters are known: channel bottom width, b; and drop height, $H$; and design discharge, Q.

1. Compute the unit discharge, q.
2. Determine the Froude number at the base of the drop, $F_{1}$ (e.g., Fig. 22).
a. Case I - Froude number is within the range of 9.0 to $4: 0$ for the given conditions.
berox b. Case II-Froude number is less than 4.0 or larger than 9.0. (It is now
necessary to change the unit discharge or drop height so the Froude number will fall within the range 9.0 to 4.0 by consulting Fig. 22.
3. Compute the critical depth (Eq. 6) and determine the flow depth before the hydraulic jump, $y_{1}$ (Fig. 23).
4. Determine the value of $Y_{2} / Y_{1}$ (Fig. 24) and compute the conjugate depth, $\mathrm{Y}_{2}$.
5. Determine the stilling basin length (Fig. 26).
a. In the case of a natural jump.
b. In the case of the stilling basin for straight drop spillways.
6. If the straight drop stilling basin is adopted, recommendations for appurtenances are as follows; (Blaisdell (1964)):
a. The distance between the upstream face of the floor blocks and the end of stilling basin is $x_{c}=1.75 y_{c}$ (Fig. 14).
b. The height of floor blocks is $0.8 y_{c}$ and the width and spacing should be approximately 0.4 $y_{C}$, but a variation $0.15 y_{C}$ from this limit is permissible.
c. The floor block should be square in plane and should occupy between $50 \%$ and $60 \%$ of the stilling basin width.
d. The side wall height above the tailwater level should be $0.85 y_{C}$.
e. The wingwall should be located at an angle of $45^{\circ}$ with the outlet centerline and should have a top slope of l:1.
7. Both the conditions upstream and downstream from the drop have considerable influence on the adequacy of the design. Any uncertainties, particularly with respect to the tailwater elevation, must be provided for by lowering the stilling basin floor so that the tailwater level is equal to, or greater than, the conjugate depth water surface elevation, thereby insuring that the hydraulic jump occurs in the stilling basin.

Inclined Drop Structures. Given the same conditions for designing inclined drop structures as for the vertical drop structures noted previously, the design procedure is as follows:

1. Assume a reasonable Froude number and the channel roughness coefficient (e.g., 0.013 for a first assumption).
2. Determine the flow regime with H and q (Fig. 33).
3. Determine the flow depth before the hydraulic jump, $y_{1}$, for the Froude number assumed in step 1 , (Fig. 23).
4. Determine the value of $y_{2} / Y_{1}$ (Fig. 24) and compute the conjugate depth, $y_{2}$.
5. Determine the inclined length.
(a. Case I (uniform flow) - Use Fig. 32 and Figs. 27-31.
b. Case II (non-uniform flow) - Use Figs. 34-48.
6. Determine the stilling basin length.
a. For the stilling basin without appurtenances. Use the curve in Fig. 49 for the natural hydraulic jump.
b. For USBR stilling basin III, use Fig. 49.
7. Repeat steps 1 through 6 for a range of possible Froude numbers and channel roughness coefficients in order to select the most economical design based on the stilling basin length.
8. Compare the total length of drop structure with tailwater depth, $y_{2}$ in each case.
9. Select the proper total inclined drop length to satisfy tailwater conditions and economical aspects such as earthwork costs, etc.
10. In the case using USBR stilling basin III, the appurtenances are as follows:
a. The height, width and spacing of chute blocks should equal $y_{1}$. The width of the blocks may be decreased, provided spacing is reduced a like amount. Should $y_{1}$ prove to be less than 8 inches, the blocks should be made 8 inches high (Fig. 18).
b. The dimensions of the baffle piers can be obtained by using Fig. 19 and Fig. 21. It is recommended that the baffle piers should be staggered with the chute blocks. A half space is recommended adjacent to the walls.
c. It is recommended that a radius of reasonable length ( $R>4 Y_{1}$ ) be used at the intersection of the chute and basin apron for slopes of $45^{\circ}$ or greater.
11. Check the tailwater surface elevation downstream and adjust the design to insure that the tailwater elevation is equal to, or greater than, the conjugate depth water surface elevation.

## Conclusions

Most information has been developed from proposed criteria and experimental results by other investigators regarding various components of drop structures. The major contribution of this thesis is the integration of the various criterion into a systematic step-by-step design procedure. For vertical drop structures, many useful design charts were presented which facilitate rapid design and the evaluation of alternative designs. For inclined drop structures, an analytical method was adopted which employs the Froude number of the base of the inclined drop as the criterion for evaluating the inclined section, thereby insuring a satisfactory hydraulic jump and consequent good performance in the stilling basin.

In an effort to evaluate several proposed stilling basins, comparisons were made for stilling basin length and appurtenances among various stilling basin designs cited in the literature. This comparison showed that the dissipation bar structure has a stilling basin length that exceeds the length of the natural hydraulic jump at high Froude numbers. However, a design modification has been proposed by the author which uses critical depth in determining stilling basin length. The length formula for the dissipation bars stilling basin becomes $L_{b}=1.5 L_{d}+$ $3.5 y_{c}$ which appears reliable, though it depends on how conservative the designer may be.

A design modification for the SAF stilling basin length was attempted using the Froude number as being directly proportional to the length ratio, $L / Y_{2}$. Since the height of baffle piers in the SAF stilling basin are less than those used in the USBR stilling basin III, it is obvious that the SAF stilling basin length should be longer than that of USBR stilling basin III, which is approximately $1.25 \mathrm{~F}^{0.38}$. Therefore, the proposed length of the SAF stilling basin has been modified to become $L_{B}=1.5 F_{1}{ }^{0.38}$.

## Recommendations

The present study can still be improved in many ways. Even though the proposed modified stilling basin length for the dissipation bars structure appears reasonable, laboratory work is needed to verify this hypothesis. Also, for the SAF stilling basin, the modified stilling basin length

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$$

should examine the basis of scour in the downstream basin using hydraulic laboratory models. Besides further investigating the relevancy of the modified stilling basin length, an experimental work should be undertaken to study the turbulence of the flows leaving the stilling basin, thereby providing a more effective evaluation of the hydraulic efficiency of various energy dissipators.

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## APPENDIX

COMPUTER PROGRAM

| Variables | Description |
| :--- | :--- |
| Al, A2 | Trial and error values related to solu- <br> tion of critical depth with a given <br> discharge |
| ALFA | The value of $\alpha$ in velocity head items |
| B | Channel bottom width |
| DEL | Incremental depth change to make Al = A2 |
| DX | Incremental chosen for computing profile |
| EL | Length of inclined drop |
| EN | Manning's roughness coefficient |
| Fl | Froude number at bottom of drop struc- |
| ture | Height of drop structure |
| HCD | Channel cross-section control |
| NOPER | Drop type control |
| PLSM | Computation control |
| Q | Discharge |
| Y | Hy |

INDROP
PROGRAM INDROP (INPUT, OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

## C

C this is A PROG. FOR THE INCLINED DROP
c
C EL=REACH LENGTH, $B=$ CHANNEL BOTTOM WIDTH, $Z=C H A N N E L ~ S I D E S ~ S L O P E ~$
C SO=CHANNEL SLOPE, EN=MANNING,S N, ALFA= VELOCITY HEAD COEFF., DX=REACH
C SUBDIVISION UNIT, PLSM=1 IF COMPUTING DOWNSTREAM AND -1 IF UPSTREAM,
C $\mathrm{Q}=\mathrm{DISCHRGE}, \mathrm{Y}(\mathrm{l})=$ STARTING DEPTH AND Y ARE THE DEPTH OF FLOW PROFILE.
C SE=ENERGE GRADIENT
C
DIMENSION F(22)
DO $600 \mathrm{M}=1,40$
READ (5,100) Q,B,Z,EN,ALFA,DX,PLSM,F1,H,SOMAX,SOMIN,NCD,NOPER
100 FORMAT (8F6.2,3F6.3,2I3)
IF (EOF (5) ) 800,7
7 IF (NCD.EQ.1)GO TO 700
$F 2=1.0$
CALL TRAPC ( $Q, B, Z, Y P, F 2$ )
$\mathrm{YC}=\mathrm{YP}$
GO TO 701
$700 \mathrm{YC}=(\mathrm{Q} * \mathrm{Q} /(\mathrm{B} * \mathrm{~B}) / 32.176) * * 0.3333$
701 CONTINUE
11 CONTINUE
IF (NCD.NE.1) GO TO 704
$\mathrm{YB}=0.8$ * YC
$\mathrm{R}=(\mathrm{B} * \mathrm{YB} /(2.0 * \mathrm{YB}+\mathrm{B})) * * 1.667$
$\mathrm{VB}=\mathrm{Q} / \mathrm{B} / \mathrm{YB}$
$\operatorname{SOMIN}=(\mathrm{VB} * E N / 1.486 / R) * * 2$
704 CONTINUE
EL=SQRT (H**2+(H/SOMIN) **2)
CALL DYDOT (YC,EL, Q,B,Z, SOMIN,ALFA, EN,DX, PLSM, YMAX)
EL=SQRT (H** $2+(\mathrm{H} / \mathrm{SOMAX}) * * 2)$
CALL DYDOT (YC,EL, Q,B,Z,SOMAX,ALFA,EN,DX,PLSM,YMIN)

```
PROGRAM
6
35
32
40

INDROP
IF (NCD.EQ.1) GO TO 702
Fl=F2
CALL TRAPC ( \(Q, B, Z, Y P, F 2\) )
\(\mathrm{Yl}=\mathrm{YP}\)
GO TO 703
\(702 \mathrm{Yl}=(\mathrm{Q} * \mathrm{Q} / \mathrm{B} / \mathrm{B} / \mathrm{Fl} * * 2 / 32.176) * * 0.3333\)
703 CONTINUE
IF (Yl.GT. YMAX) WRITE ( 6,101 ) Fl, YMAX,Yl IF (Yl.GT. YMAX) GO TO 600
101 FORMAT(1H ,*DOWNSTREAM DEPTH UNATTAINABLE AT THIS FROUDE NUMBER*, 13F10.3)
IF (Yl.LT. YMIN) WRITE \((6,101)\) Fl,YMIN,Yl
IF (Yl.LT. YMIN) GO TO 600
A=SOMIN
\(\mathrm{P}=\) SOMAX
DO1 \(\mathrm{I}=1,22\)
IF(I.GT.2) GO TO 2
\(F(I)=1.0\)
GO TO 1
\(2 \mathrm{~F}(\mathrm{I})=\mathrm{F}(\mathrm{I}-1)+\mathrm{F}(\mathrm{I}-2)\)
1 CONTINUE
111 CONTINUE
\(\mathrm{K}=\mathrm{J}-2\)
IF (K.EQ.2) GO TO 12
\(D E L=(P-A) * F(K) / F(J)\)
Z1=A+DEL
Z2=P-DEL
EL \(=\) SQRT ( \(\mathrm{H} * * 2+(\mathrm{H} / \mathrm{Zl})\) **2)
CALL DYDOT (YC,EL, \(\mathrm{Q}, \mathrm{B}, \mathrm{Z}, \mathrm{Zl}, \mathrm{ALFA}, E N, D \mathrm{X}, \mathrm{PLSM}, \mathrm{AZI})\)
EL=SQRT (H**2+(H/Z2)**2)
CALL DYDOT (YC, EL, \(\mathrm{Q}, \mathrm{B}, \mathrm{Z}, \mathrm{Z2}, \mathrm{ALFA}, E N, D \mathrm{X}, \mathrm{PLSM}, \mathrm{AZ} 2\) )
IF (AZl.GT.Y1) \(A=Z 1\)
```

PROGRAM INDROP
IF(AZ2.LT.Y1) P=Z2
J=J-1
GO TO lll
12 CONTINUE
SO=(A+P)/2.0
EL=SQRT(H**2+(H/SO)**2)
WRITE (6,500)H,EL,SO,Q,B,Z,Fl,EN
500 FORMAT(1H ,2F6.1,F8.4,4F6.1,F8.4)
6 0 0 ~ C O N T I N U E ~
800 STOP
END

```
12
30
\(\$\)


\section*{SUBROUTINE DYDOT}
```

                                    SUBROUTINE DYDOT(YC,EL,Q,B,Z,SO,ALFA,EN,DX,PLSM,YN)
                                    DIMENSION Y(100)
                                    YDOTF (B,S,EN,Z,AA,Q,YY)=(S- ((EN*Q)**2* (B+2.0*YY*SQRT (I.+Z*Z))**I.
    133333)/(2.21*((B+Z*YY)*YY)**3.33333))/(1.-((AA*Q*Q* (B+2.*Z*YY))/
2(32.17*((B+Z*YY)*YY)**3)))
Y(1)=0.8*YC
DO 300 I=l,14
AU=I
DEL=AU/105.0
DX=EL*DEL
YDOTI=YDOTF (B,SO,EN,Z,ALFA,Q,Y(I))*PLSM
YDOTJ=YDOTI
DO 203J=1.15
TEMP=YDOTJ
15
Y(I+l) =Y(I) +(YDOTI+YDOTJ)*DX*0.5
IF(Y(I+1)) 201,201,202
202 YDOTJ=YDOTF(B,SO,EN,Z,ALFA,Q,Y(I+I))*PLSM
IF (ABS (TEMP-YDOTJ)-.1E-06) 302, 302,203
203 CONTINUE
20. 201 WRITE (6,407) Y(I)
407 FORMAT(1H,4X,FlO.2)
302 CONTINUE
300 CONTINUE
YN=Y(15)
25
MPMURN
END

## SUBROUTINE TRAPC

## SUBROUTINE TRAPC ( $\mathrm{Q}, \mathrm{B}, \mathrm{Z}, \mathrm{YC}, \mathrm{F}$ )

IX=1. 0
$\mathrm{YD}=0.0$
101 continue
$\mathrm{A} 2=\mathrm{Q}^{*} \mathrm{Q}^{*}(\mathrm{~B}+2.0$ * Z *YD)
$\mathrm{Al}=\mathrm{F} *(32.176 *(\mathrm{YD} *(\mathrm{~B}+\mathrm{Z} * \mathrm{YD}))$ **3)
IF (Al-A2) 102,103,104
102 IF (IX.EQ.2) GO TO 105
IF (IX.EQ.3) GO TO 105
$\mathrm{YD}=\mathrm{YD}+0.1$
GO TO 101
104 IF (IX.EQ.3) GO TO 103
IX=2
YD=YD-0.01
GO TO 101
105
$\mathrm{IX}=3$
$\mathrm{YD}=\mathrm{YD}+0.001$
GO TO 101
103 CONTINUE
$\mathrm{YC}=\mathrm{YD}$
RETURN
END


[^0]:    *Reports are available from Mrs. Arlene Nelson, Engineering Research Center, Colorado State University, Fort Collins, Colorado 80523. Price: $\$ 3.00$ each until printing is exhausted; subsequent xerox copies obtainable at ten cents per page.

[^1]:    (1)

    Fig. 13. Types of hydraulic jump.

