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Unhanand, Komain and Kadir, Tariq N.

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9. ABSTRACT

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A Theory of the Combined Mole-Tile Drain System

KOMAIN UNHANAND AND TARIQ N. KADIR

Department of Agricultural and Irrigation Engineering, Utah State University, Logan, Utah 84322

A theory of water movement in the combined mole-tile drain system based on the transient state condition was developed. Two general equations were derived to describe the height of the water table at any location in the system at any elapsed time after the drainage process begins. One of the equations is applicable for the stage in which the water table is above the mole drains, and the other equation is for the stage in which the water table falls below the mole drains. The two general equations were simplified for the point located at midpoint between the tile drains and mole drains in the system. In the derivation, assumptions regarding the flow condition of groundwater and shape of the water table profile at certain boundaries were made. Field experiments were then conducted, and the test data were used in verifying the equation for the first stage. A reasonably good agreement between the theoretical analysis and field data was obtained for this type of research.

The drainage problems of heavy soils are more complicated than those of coarse-textured soils because the low infiltration rate necessitates an efficient surface drainage and the low hydraulic conductivity of the soils makes a close spacing of tile drains necessary. In clay soils the water movement is almost entirely confined to the cracks and fissures. The water movement is relatively fast through cracks and fissures but reduced to almost negligible flow when cracks and fissures are closed. Thus for tile drains in these soils to be effective for drainage they must be placed close together. The high initial cost of the drainage system resulting from the close spacing, plus the fact that such soils are often not suitable for growing high-income crops, makes the economic feasibility difficult to obtain.

Mole drains, constructed by pulling a bullet-shaped probe through the soil to form unlined channels, have been used to overcome the problem of the high initial cost of tile drains. But mole drains also are disadvantageous for being short-lived and requiring reconstruction every few years. Although the construction of mole drains is relatively simple, inexpensive, and requires little time, the repeated installation of their outlets could be time consuming and costly.

The problems of outlet construction may be eliminated by the use of the so-called 'combined mole-tile drain system.' In this system a set of tile drains is laid at a suitable depth, normally about 91.4 cm, and a set of mole drains is drawn about 30.5 cm above the tile drains in the direction perpendicular to the tile lines. The gravel envelope of the tile drain, being extended to approximately 30.5 cm from the ground surface, enables the mole drains that intersect the envelope to discharge the drainage water directly into the tile drains. With this type of construction the cost of the drainage system is reduced because the tile drain spacing could be made wider, and the mole drain outlet construction is eliminated.

The combined mole-tile drain system has been used in many countries in Europe [*Food and Agriculture Organization*, 1971] and Japan. To the author's knowledge, no information regarding its field performance is available. *Tomita* [1971] reported a theoretical analysis of the combined system in stratified soils based on the three-dimensional steady state equation (i.e., the Laplace equation). The authors presented the results in three-dimensional diagrams showing the equipotential lines and surface potential in the entire system for different arrangements of the hydraulic conductivity of the layers. The results were also presented in graphs from which the discharge in the collector tile drain may be computed if the tile drain spacing, the hydraulic conductivity of soil, and the mole drain spacing are given. In their study, mole drains having 7-cm diameter were drawn at the depth of 45 cm below ground surface, and the tile drains were laid at 75-cm depth.

Since no theoretical analysis of the water movement in the combined system based on the transient conditions could be found anywhere, and it appears that this system could be most practical for drainage of heavy soils, a theoretical analysis that may be used in the evaluation or design of the system is necessary.

The objectives of this paper are to present the following: a mathematical model for the combined mole-tile drainage system under a transient state condition, solutions for the above model for particular initial and boundary conditions, and the correlation of the solutions obtained with the data from a field experiment.

DESCRIPTION OF THE STUDY MODEL

A study model was chosen from an area bounded by a pair of mole drains and a pair of tile drains in a field in which a combined system was laid out indefinitely in all directions in order to eliminate the effect of the boundary conditions at the edge of the field (Figure 1). Since the theoretical analysis cannot be carried out unless certain conditions of water table and water movement in the study model are known and because no such information is available, a logical description of the water movement that will be used in the analysis is presented as follows.

At the beginning, all the drain outlets were closed, and the water table was a horizontal surface at a certain depth above the mole drains. As soon as the drain outlets were opened, the water table dropped quickly along the boundaries (i.e., the drains) and assumed the curved form shown in Figure 1. Curves *bac* and *dae* represent the water surface profiles at the sections midway between the mole drains and the tile drains, respectively. Point *a*, located at the intersection of the two curves, is the highest point on the water table in the system, since it is least affected by the drains.

The curves *bac* and *dae* divide the water table into four symmetrical regions 1–4. Because the flow is symmetrical in all regions, only one region, say region 1, will be used to describe the water movement. The direction of flow at any point on the curve *ba*, for example, at point 1 in Figure 2, is along the curve

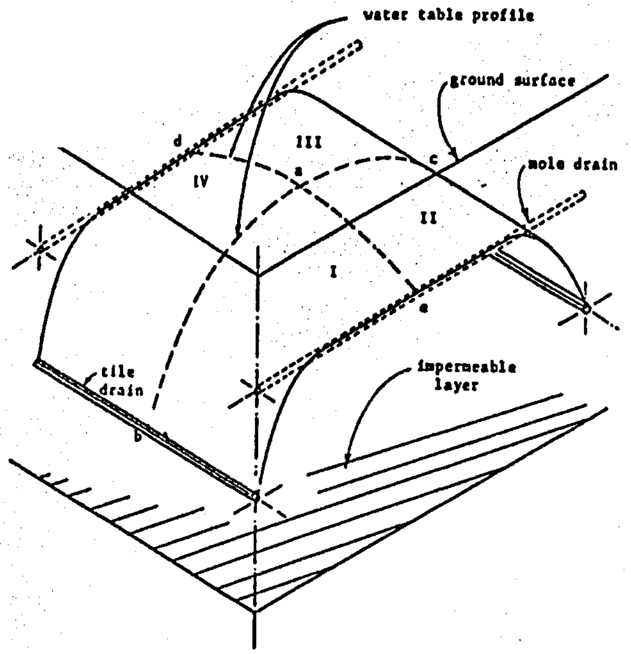

Fig. 1. Symmetric regions of the water table.

toward the tile drains because at any point on the curve *bac* the hydraulic gradient is zero in the direction parallel to the tile drain. Similarly, the flow at any point on the curve *ae* is along *ae* toward the mole drains. At any point on the surface within the region and not on the curves *ba* and *ae* or the boundaries the direction of flow will be affected by the hydraulic gradient components directed parallel to the tile drains and the mole drains (e.g., point 3 in Figure 2).

Along the boundary formed by the tile drain the water level is constant at the drain center line if one assumes that the drain is always half full of water. The condition along the mole drain is more complicated. It was found during a field experiment that during water table recession the discharge from the mole drain was initially very small and quickly diminished with time, whereas the general water table was still higher than the mole drain elevation. This characteristic seems to indicate that the water enters and flows in the mole channel, but most of it seeps out from the channel before reaching the tile drain. The water surface may leave the mole channel at some point *g* a distance x_0 from the tile drain, as shown in Figure 2. It should be noted here that the distance x_0 is not constant but increases with time during water table recession.

After the drainage has progressed for a certain period, a situation will be reached in which the water velocity component toward the mole drains is very small in comparison with the component toward the tile drain, and the flow toward tile drain predominates. In this situation the mole drain almost ceases to function, and the water surface along the mole drain begins to drop below the mole channel. The water table will gradually flatten out until the water velocity is completely in the direction toward the tile drain, a movement causing the flow to be two dimensional, the condition upon which the ordinary tile drain theories are based (Figure 3).

STAGES OF WATER MOVEMENT IN THE SYSTEM

The study model, shown in three dimensions in Figure 4, consists of tile drains spaced at S_t overlain orthogonally by mole drains spaced at S_m. The vertical distance between the mole drains and tile drains is d_2. The impermeable layer lies at a distance d_3 below the tile drains. The three Cartesian axes u, x, and y are also shown in the figure.

The stages of water movement in the combined system, which will be assumed in the theoretical analyses later, are described as follows.

Stage 1—water table above the mole drains. During this stage, both the mole drains and the tile drains function together in a combined fashion resulting in a two-dimensional flow pattern of which one component flows toward a mole drain and the other toward a tile drain.

At time $t = 0$ with all drain outlets closed, the water table is flat at a distance d_1 above the mole drains. Then it will be assumed that immediately after all drain outlets are opened simultaneously, the water table reorients itself into a curved surface as shown in Figure 1. Furthermore, the water table profile along any boundary will be assumed to be independent of time. Along the tile drains the water level is constant at the center line of the drain or $u = 0$. Along the mole drains the water table profile takes on a constant shape, which will be discussed later.

Stage 1 ends when the water table at every point in the study model is at or below the elevation of the mole drains (i.e., max $u = d_2$, see Figure 4).

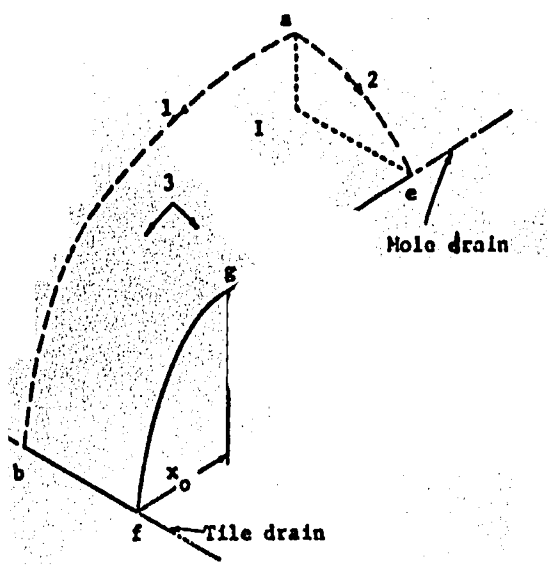

Fig. 2. Flow of groundwater, stage 1.

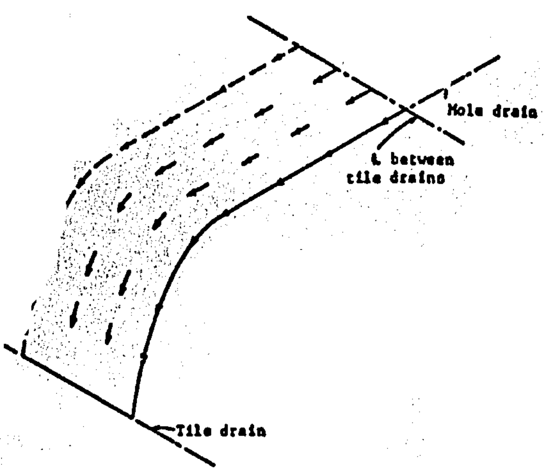

Fig. 3. Flow of groundwater, stage 2.

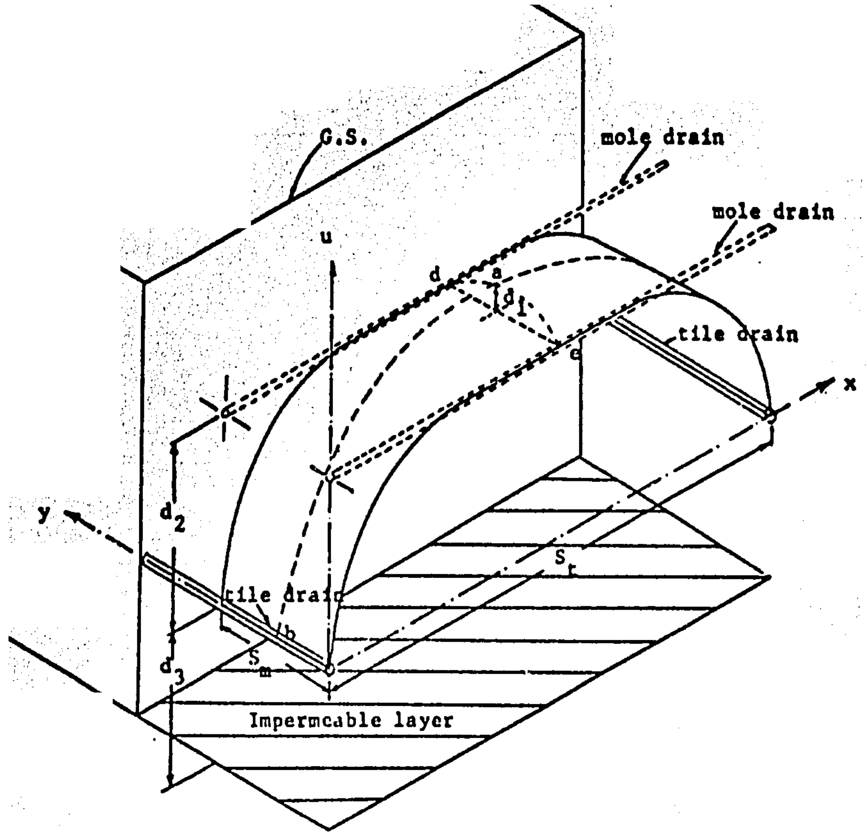

Fig. 4. Water table profile in a combined mole-tile drain system.

Stage 2—water table between the mole drains and the tile drains. During this stage, which starts with the condition of water table at the end of stage 1, only the tile drains are in action. At any point on the water table the hydraulic gradient is steepest in the direction toward the tile drains, a situation resulting in a two-dimensional flow pattern as occurs in an ordinary tile drain system (See Figure 3).

Although the shape of the water table at any vertical section parallel to the mole drains at the outset of this stage is assumed to be identical to the constant water table profile along the mole drains in stage 1 (Figure 3), the shape during stage 2 will be time dependent.

MATHEMATICAL MODEL

Assumptions. In setting up the mathematical model for the water movement in the combined mole-tile drain system under a transient state condition the following assumptions were used. (1) Soil is homogeneous and isotropic; (2) specific yield and hydraulic conductivity of the soil are constant; (3) Dupuit-Forchheimer assumptions are valid; (4) Darcy's law is applicable; (5) flow is under a transient state condition; (6) flow is completely gravitational; (7) land slope is small such that it has no effect on water movement; (8) height of the water table above the tile drains is very small in comparison with the vertical distance between the pipe drains and the impermeable layer; (9) tile drains are parallel, and mole drains are parallel and orthogonal to the tile drains; (10) spacing of mole drains is small in comparison with that of the tile drains such that $[(1/S_m^2) + (1/S_t^2)] = 1/S_m^2$; (11) first term of the infinite Fourier series is sufficient for convergence; and (12) horizontal water table is an initial condition. Mathematical models were set up for stage 1 and stage 2.

Stage 1—water table above the mole drains. The basic two-dimensional continuity equation governing the flow of water through the soil may be expressed as follows [*van Schilfgaarde et al.*, 1956]:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \alpha \frac{\partial u}{\partial t} \qquad (1)$$

in which u is the height of the water table above the tile drains at any time t and $\alpha = f/kd_3$, where f is the average specific yield of the soil (by volume), k is the average hydraulic conductivity of the soil, and d_3 is the vertical distance between the tile drains and the impermeable layer. It is assumed that d_3 is much larger than u.

The boundary conditions and the initial conditions are as follows:

Boundary conditions

$$u(x, 0, t) = f(x) \qquad u(0, y, t) = 0$$

$$u(x, S_m, t) = f(x) \qquad u(S_t, y, t) = 0$$

Initial conditions

$$u(x, y, 0) = d_1 + d_2 \qquad \text{horizontal water table}$$

where $f(x)$ represents the constant shape of the water table profile along the mole drain boundary.

Equation (1) is identical in form to the two-dimensional heat flow equation [*Carslaw and Jaeger*, 1959], and therefore its solution for the nonhomogeneous boundary conditions may be applied in solving (1) as follows.

The solution $u(x, y, t)$ of (1) may be expressed as the sum of two solutions $v(x, y)$ and $w(x, y, t)$:

$$u(x, y, t) = v(x, y) + w(x, y, t) \qquad (2)$$

where $v(x, y)$ is the steady state solution of the Laplace equation in rectangular regions,

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \quad (3)$$

with boundary conditions

$$v(x, 0) = f(x) \qquad v(0, y) = 0$$
$$v(x, S_m) = f(x) \qquad v(S_t, y) = 0$$

and $w(x, y, t)$ is the transient state solution of

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha \frac{\partial w}{\partial t} \quad (4)$$

with boundary conditions

$$w(x, 0, t) = 0 \qquad w(0, y, t) = 0$$
$$w(x, S_m, t) = 0 \qquad w(S_t, y, t) = 0$$

and an initial condition

$$w(x, y, 0) = (d_1 + d_2) - v(x, y)$$

where $v(x, y)$ is the solution obtained from (3).

Furthermore, according to *Powers* [1972], $v(x, y)$ is the sum of two solutions

$$v(x, y) = v_1(x, y) + v_2(x, y) \quad (5)$$

where $v_1(x, y)$ is the solution of

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} = 0 \quad (6)$$

with boundary conditions

$$v_1(x, 0) = f(x) \qquad v_1(0, y) = 0$$
$$v_1(x, S_m) = 0 \qquad v_1(S_t, y) = 0$$

and $v_2(x, y)$ is the solution of

$$\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2} = 0 \quad (7)$$

with boundary conditions

$$v_2(x, 0) = 0 \qquad v_2(0, y) = 0$$
$$v_2(x, S_m) = f(x) \qquad v_2(S_t, y) = 0$$

Solving (6) for $v_1(x, y)$ results in the following infinite Fourier series [*Kreider et al.*, 1966]:

$$v_1(x, y) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{S_t} \sinh \frac{n\pi}{S_t}(S_m - y) \quad (8)$$

where

$$A_n = \frac{2}{S_t \sinh (n\pi S_m/S_t)} \int_0^{S_t} f(x) \sin \frac{n\pi x}{S_t}\, dx \quad (9)$$

Similarly, by solving (7) for $v_2(x, y)$,

$$v_2(x, y) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{S_t} \sinh \frac{n\pi y}{S_t} \quad (10)$$

where

$$B_n = \frac{2}{S_t \sinh (n\pi S_m/S_t)} \int_0^{S_t} f(x) \sin \frac{n\pi x}{S_t}\, dx \quad (11)$$

By substituting $v_1(x, y)$ from (8) and $v_2(x, y)$ from (10) in (5) and noting that $A_n = B_n$,

$$v(x, y) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{S_t} \cdot \left[\sinh \frac{n\pi}{S_t}(S_m - y) + \sinh \frac{n\pi y}{S_t}\right] \quad (12)$$

where A_n is expressed in (9).

By using the trigonometrical identity

$$\sinh a + \sinh b = 2 \sinh \frac{(a + b)}{2} \cdot \cosh \frac{(a - b)}{2}$$

equation (12) may be written as

$$v(x, y) = 2 \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{S_t} \cdot \sinh \frac{n\pi S_m}{2S_t} \cosh \frac{n\pi}{2S_t}(S_m - 2y) \quad (13)$$

By solving (4) for $w(x, y, t)$ according to the work of *Kreider et al.* [1966],

$$w(x, y, t) = \sum_{\substack{m=1\\n=1}}^{\infty} A_{mn} \sin \frac{n\pi x}{S_t} \sin \frac{m\pi y}{S_m} \cdot \exp\left[-\frac{\pi^2}{\alpha}\left(\frac{m^2}{S_m^2} + \frac{n^2}{S_t^2}\right)t\right] \quad (14)$$

where

$$A_{mn} = \frac{4}{S_m S_t} \int_0^{S_m} \int_0^{S_t} g(x, y) \sin \frac{n\pi x}{S_t} \sin \frac{m\pi y}{S_m}\, dx\, dy \quad (15)$$

$$g(x, y) = [d_1 + d_2] - v(x, y) \quad (16)$$

Equation (13) contains the infinite Fourier series, and (14) involves the integration of an infinite series. Including more than one term of the infinite series generally involves extensive mathematical calculations that are not suitable for practical application. An analysis conducted to determine the effect of truncating of the infinite series of (13) indicates that an overestimation of about 30% of the value of $v(x, y)$ may exist. For the sake of simplicity of mathematical calculations, however, all other terms in the series except the first will be neglected, and (13) may be reduced to

$$v(x, y) \doteq \left(\frac{4}{S_t}\right)(\psi)(A) \sin \frac{\pi x}{S_t} \cosh \frac{\pi}{2S_t}(S_m - 2y) \quad (17)$$

where

$$A = \int_0^{S_t} f(x) \sin \frac{\pi x}{S_t}\, dx \quad (18)$$

$$\psi = (\sinh \xi/2)/\sinh \xi \quad (19)$$

$$\xi = \pi S_m/S_t \quad (20)$$

If only the first term of the infinite series is used, (14) reduces to

$$w(x, y, t) = \left(\frac{4}{S_m S_t}\right)(B) \sin \frac{\pi x}{S_t} \sin \frac{\pi y}{S_m} \cdot \exp\left[-\frac{\pi^2}{\alpha}\left(\frac{1}{S_m^2} + \frac{1}{S_t^2}\right)t\right] \quad (21)$$

where

$$B = \int_0^{S_m} \int_0^{S_t} g(x, y) \sin \frac{\pi x}{S_t} \sin \frac{\pi y}{S_m} \, dx \, dy \qquad (22)$$

By substituting $v(x, y)$ from (17) and $w(x, y, t)$ from (21) in (2), $u(x, y, t)$ during stage 1 may be expressed as

$$u(x, y, t) = \left(\frac{4}{S_t}\right)(\psi)(A) \sin \frac{\pi x}{S_t} \cosh \frac{\pi}{2S_t}(S_m - 2y) + \left(\frac{4}{S_m S_t}\right)(B) \sin \frac{\pi x}{S_t} \sin \frac{\pi y}{S_m} \exp\left[-\frac{\pi^2}{\alpha}\left(\frac{1}{S_m^2} + \frac{1}{S_t^2}\right)t\right] \qquad (23)$$

where ψ, A, B, S_m, S_t, α, and t are as defined previously.

Stage 2—water table between the mole drains and the tile drains. During this stage, mole drains no longer take part in the drainage, and the water movement can be described by the equation of continuity for ordinary tile drains:

$$\frac{\partial^2 u}{\partial x^2} = \alpha \frac{\partial u}{\partial t} \qquad (24)$$

with boundary conditions

$$u(0, t) = 0 \qquad u(S_t, t) = 0$$

and an initial condition

$$u(x, 0) = f(x)$$

where t is the time measured from the beginning of stage 2, and $f(x)$ represents the water table profile at $t = 0$ (see Figure 5).

Equation (24) may be solved by a method similar to that used by R. E. Glover [*Dumm*, 1954, 1964]. From the work of *Kreider et al.* [1966],

$$u(x, t) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{S_t} \exp\left(-\frac{n^2\pi^2}{\alpha S_t^2} t\right) \qquad (25)$$

where

$$A_n = \frac{2}{S_t} \int_0^{S_t} f(x) \sin \frac{n\pi x}{S_t} \, dx \qquad (26)$$

If using only the first term of the infinite Fourier series is sufficient, (25) becomes

$$u(x, t) = C \sin \frac{\pi x}{S_t} e^{-} \qquad (27)$$

where

$$C = \frac{2}{S_t} \int_0^{S_t} f(x) \sin \frac{\pi x}{S_t} \, dx \qquad (28)$$

$$\phi = \pi^2/\alpha S_t^2 \qquad (29)$$

Shape of water table profile along the mole drain $f(x)$. The function $f(x)$ represents the shape of water table profile along the mole drains during stage 1 and at the beginning of stage 2. Since no information of any kind is available, $f(x)$ will have to be assumed.

During stage 1, $f(x)$ may be expressed mathematically as follows:

$$f(x) = f_1(x) \qquad 0 \le x \le x_0$$
$$f(x) = d_2 \qquad x_0 \le x \le (S_t - x_0) \qquad (30)$$
$$f(x) = f_2(x) \qquad (S_t - x_0) \le x \le S_t$$

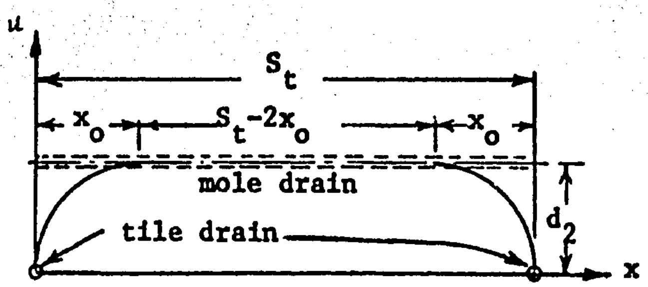

Fig. 5. Water table profile along the mole drains at time $t = 0$, stage 2.

where $f_1(x)$ and $f_2(x)$ describe the curves og and $o'g'$ in Figure 4, respectively. Six cases of $f(x)$ were investigated: case 1, zero-degree polynomial; case 2, first-degree polynomial; case 3, second-degree polynomial; case 4, third-degree polynomial; case 5, fourth-degree polynomial; and case 6, sine wave equation.

Polynomial equations. The general form of the nth degree polynomial equation may be expressed as

$$u = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n \qquad (31)$$

The constants $C_0, C_1, C_2, \cdots, C_n$ may be evaluated from the following boundary conditions:

$$u(0) = 0 \qquad u(x_0) = d_2$$
$$u'(x_0) = u''(x_0) = \cdots = u^{(n)}(x_0) = 0$$
$$u(S_t) = 0 \qquad u(S_t - x_0) = d_2$$
$$u'(S_t - x_0) = u''(S_t - x_0) = \cdots = u^{(n)}(S_t - x_0) = 0$$

The derivatives are equated to zero at $x = x_0$ and $x = S_t - x_0$ to obtain smooth curves at those points. Applying the above boundary conditions to solve for the constants in (30) 31 and simplifying the general equations for $f(x)$ may be expressed as follows:

$$f_1(x) = d_2 - (-1)^n \frac{d_2}{x_0^n}(x - x_0)^n \qquad 0 \le x \le x_0$$
$$f(x) = d_2 \qquad x_0 \le x \le S_t - x_0 \qquad (32)$$
$$f_2(x) = d_2 - \frac{d_2}{x_0^n}(x - S_t + x_0)^n \qquad S_t - x_0 \le x \le S_t$$

where n is the degree of polynomial.

Sine wave equation. The profile of water table along the mole drain $f(x)$ using a sine wave equation is shown by the curve $ogg'o'$ in Figure 6. Actually, only the portion og and $g'o'$ are parts of a sine curve. If the general equation for the sine curve has a period of S_t,

$$u(x) = \delta \sin(\pi x/S_t) \qquad (33)$$

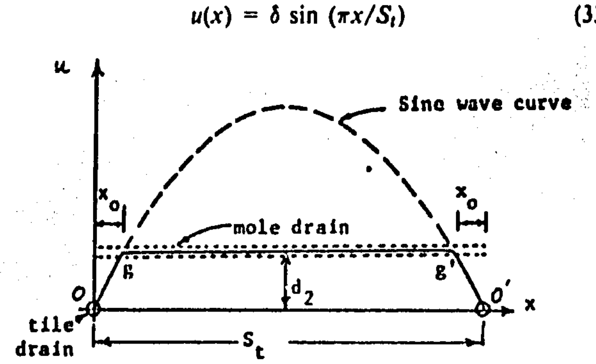

Fig. 6. Water table profile along the mole drain as a sine curve.

and by substituting the boundary condition $u(x_0) = d_2$ in (33),

$$\delta = d_2/\sin(\pi x_0/S_t)$$

The $f(x)$ may then be described as follows:

$$f(x) = \delta \sin \frac{\pi x}{S_t} \qquad 0 \leq x \leq x_0$$

$$f(x) = d_2 \qquad x_0 \leq x \leq S_t - x_0 \qquad (34)$$

$$f(x) = \delta \sin \frac{\pi x}{S_t} \qquad S_t - x_0 \leq x \leq S_t$$

where δ, d_2, S_t, and x_0 are as previously defined.

EVALUATION OF EQUATIONS (23) AND (27)

By choosing $f(x)$ for any particular case from (32) or (34) and substituting in (18), A is obtained. With the known A, $v(x, y)$ is determined from (17), and consequently, $g(x, y)$ from (16) by using the value of $v(x, y)$ just obtained. Then by substituting $g(x, y)$ in (22), B may be determined. The general solution for $u(x, y, t)$ for stage 1 may be found from (23) by using A and B obtained from the procedure described above.

The general solution for $u(x, t)$ during stage 2 may be obtained from (27) by substituting the same $f(x)$ chosen in stage 1 in (28) and by using ϕ obtained from (29).

Due to the fact that in practice the spacing of the mole drains is very much smaller than the spacing of the tile drains (i.e., $2m \leq S_m \leq 5m$ and $30m \leq S_t \leq 150m$), in this mathematical analysis it was assumed that $[(1/S_m^2) + (1/S_t^2)] = 1/S_m^2$.

Since the water table condition at midpoint between the drains is most interesting in drainage engineering, general solutions of (23) and (27) that express the height of water table at any location in the system at any time will not be presented here. Instead, only the solutions for the height of water table at midpoint will be given.

SOLUTIONS EVALUATED AT MIDPOINT

The solutions of (23) and (27) evaluated at midpoint may be written in general forms as follows.

General form of solution for stage 1. The solution for stage 1 may be expressed as

$$u_m = K_1 \cdot e^{-\zeta t} + K_2 \qquad (35)$$

in which $u_m = u(S_t/2, S_m/2, t)$ or height of water table at midpoint,

$$K_1 = \frac{16(d_1 + d_2)}{\pi^2} - \frac{4d_2}{\pi}(\chi) \qquad (36)$$

$$K_2 = (2d_2)(\psi)(\chi)$$

$$\zeta = \frac{\pi^2}{\alpha S_m^2}$$

$$\psi = \frac{\sinh \xi/2}{\sinh \xi} \qquad (37)$$

$$\xi = \frac{\pi S_m}{S_t}$$

and the values of χ varied depending on the particular case considered (i.e., $f(x)$ chosen) and are listed as follows: for case 1, $\chi = 4/\pi$; for case 2, $4/\pi \cdot \sin\beta/\beta$; for case 3, $8/\pi \cdot (1 - \cos\beta)/\beta^2$; for case 4, $24/\pi \cdot 1/\beta^2 \cdot [1 - (\sin\beta)/\beta]$; for case 5, $48/\pi \cdot 1/\beta^2 \cdot \{1 - [2(1 - \cos\beta)/\beta^2]\}$; and for case 6, $2/\pi \cdot [(\beta/\sin\beta) + \cos\beta]$ (note that $\beta = \pi x_0/S_t$).

Equation (35) may be rearranged as

$$S_m = \{\pi^2 t/\alpha \ln [K_1/(u_m - K_2)]\}^{1/2} \qquad (38)$$

in which u_m is the height of water table above the tile drains at midpoint of the system. All other terms are as previously defined.

General form of solution for stage 2. Similarly to stage 1 the general form of solution for stage 2 may be expressed as

$$u\left(\frac{S_t}{2}, \frac{S_m}{2}, t\right) = \chi d_2 e^{-\phi t} \qquad (39)$$

in which $\phi = \pi^2/\alpha S_t^2$.

By rearranging (39) and substituting u_m for $u(S_t/2, S_m/2, t)$,

$$S_t = [\pi^2 t/\alpha \ln (\chi d_2/u_m)]^{1/2} \qquad (40)$$

Equations (38) and (40) may be used in the design of the combined mole-tile drain system.

FIELD EXPERIMENT

A field experiment was carried out on the Utah State University drainage farm, Logan, Utah, during the summer of 1972.

Four perforated plastic drains, 10 cm in diameter, were laid in a trench at a depth of 84 cm and a spacing of 36.58 m. The plastic pipe was surrounded with a graded gravel envelope to the depth of 30.5 cm below the ground surface. The top soil was then used to fill the trench to the ground surface. Ten mole drains were drawn orthogonally to the tile drains at the depth of 53 cm below the soil surface. The typical arrangement of the tile and mole drains (see Figure 7) clearly shows that the water can flow directly from the mole channel through the gravel envelope into the tile drains and no outlet construction for the mole drains was required. The mole drains, about 7.6 cm in diameter, were drawn at a spacing of 1.83 m.

Observation wells were constructed at midpoints and other locations in the experiment plot. Each well was made by drilling a hole about 10 cm in diameter and 1.22 m deep. The side of the hole was roughened by a metal brush before a 2.5-cm-diameter perforated PVC pipe about 1.52 m long was placed in the center of the hole on a 5-cm gravel bedding, and the space outside the pipe was then filled with graded gravel.

The experiment was conducted by building up the groundwater table in the plot with a sprinkler system while all the tile drain outlets were closed. When the general water table elevation in the area was just below the ground surface and flat, the sprinkler system was shut off, and the water table was allowed to adjust itself for about 24 hours. Then the water surface elevations at observation wells were measured, and the drainage process was started by opening the tile drain outlets simultaneously. Measurements of the water surface elevation in each observation well were repeated at various elapsed times from the beginning of the drainage process.

Other data necessary for the theoretical analysis such as the hydraulic conductivity of the soil were also collected. The hydraulic conductivity was determined by using the single auger hole method. Several holes were tested to obtain the average value of the hydraulic conductivity. The test was made in three stages at the depths of 0.91, 1.83, and 2.74 m in each hole to locate the impermeable layer. The hydraulic conductivity was computed by the formula developed by *Maasland and Maskew* [1957] for finding the hydraulic conductivity of

TABLE 1. Height of Water Table Above Tile Drains at Midpoint Measured at Various Elapsed Times

Height of Water Table u_m, cm	Elapsed Time t, d
Test Run 1, September 15-18, 1972	
65.2	0.000
63.1	0.071
59.4	0.169
54.6	0.342
54.3	0.597
53.6	0.875
45.1	0.960
44.8	1.899
40.2	2.407
39.0	2.888
Test Run 2, September 21-26, 1972	
89.3	0.00
89.9	0.094
88.7	0.226
88.1	0.388
87.2	0.640
86.6	1.059
83.8	1.194
79.6	1.381
75.0	1.963
57.9	2.407
51.2	3.211
47.2	3.374
44.8	4.067
41.2	4.391
40.2	5.076
39.3	5.366

the saturated soil below the water table. Table 1 shows the data of the height of water table at midpoint measured at various elapsed times. Here are the data on the physical character of the combined system and the soil: S_m = 1.83 m, S_t = 36.58 m, d_1 = 34.1 cm in test run 1 and 58.2 cm in test run 2,

TABLE 2. Results of Data Analysis by Using Equation (38) (Zero-Degree Polynomial Case)

u_m, cm	t, d	S_m, m	S_c, m	S_c/S_m
	Test Run 1, September 15-18, 1972			
65.2	0			
63.1	0.071	2.02	0.98	0.500
59.4	0.169	2.86	1.45	0.507
54.6	0.342	3.61	1.89	0.522
54.3	0.597	4.78	2.73	0.574
53.6	0.875	5.75	3.50	0.613
45.1	0.960	4.64	2.49	0.539
44.8	1.899	6.64	4.15	0.625
40.2	2.407	5.89	3.39	0.582
39.0	2.888	5.47	3.03	0.559
	Test Run 2, September 21-26, 1972			
89.3	0			
89.0	0.094	2.67	1.54	0.575
88.7	0.226	4.15	2.59	0.628
88.1	0.388	5.42	3.61	0.670
87.2	0.640	6.93	4.93	0.712
86.6	1.059	8.98	6.79	0.757
83.8	1.194	9.15	6.93	0.757
79.5	1.381	9.24	6.96	0.754
75.0	1.963	10.41	8.01	0.770
57.9	2.407	8.84	6.33	0.716
51.2	3.211	9.14	6.52	0.713
47.2	3.374	8.63	5.96	0.693
44.8	4.067	8.96	6.25	0.698
41.2	4.391	8.19	5.48	0.669
40.2	5.076	8.54	5.78	0.678
39.3	5.366	8.30	5.55	0.669

d_2 = 31.1 cm, d_3 = 98.5 cm, and hydraulic conductivity k = 22.6 cm/d.

When k = 22.6 cm/d, f was found to be 0.045 from a graph showing the relationship of the specific yield and hydraulic conductivity such as given by *Luthin* [1973].

RESULTS AND DISCUSSION

The theory was correlated to the field experiment by investigating the validity of (38), which is applicable for stage 1. A similar correlation could have been made by using (39), should the field data for stage 2 have been available.

The S_m was computed from (38) by using values of u_m and t selected to cover the entire range of the field data. The fixed point iteration technique [*Stark*, 1970] was used to solve (38). The result of the computation for S_m is shown in the third column of Table 2.

Since (38) was derived with an assumption that the depth of the impermeable layer was very large in comparison with the height of water above the drains, but in the experimental field the depth to the impermeable layer was only slightly greater than twice the height of water table above the drains, therefore the following correction method based on a paper by *Moody* [1966] was applied.

Hooghoudt's equation neglecting convergence toward drains may be expressed as

$$S_m^2 = (½)hC + Cd \qquad (41)$$

where $C = 8kh/q$, d is the depth of the impermeable layer below the drain, k is the hydraulic conductivity of the soil, h is the height of water table above the drains at midpoint, S_m is the mole drain spacing, and q is the discharge rate in the drain.

If $\hat{d}$ is the equivalent depth of the impermeable layer, (41) may be written as

$$S_c^2 = (½)hC + C\hat{d} \qquad (42)$$

where S_c is the corrected spacing of drains.

The relationship between d and $\hat{d}$ was given by *Moody* [1966] as

$$\frac{\hat{d}}{d} = \left\{1 + \frac{d}{S_c}\left[\frac{8}{\pi}\ln\frac{d}{a} - \alpha\right]\right\}^{-1} \qquad 0 \le \frac{d}{S_c} \le 0.3 \qquad (43)$$

where

$$\alpha = 3.55 - 1.6\frac{d}{S_c} + 2\left(\frac{d}{S_c}\right)^2 \qquad (44)$$

and a is the drain radius. For $d/S_c > 0.3$,

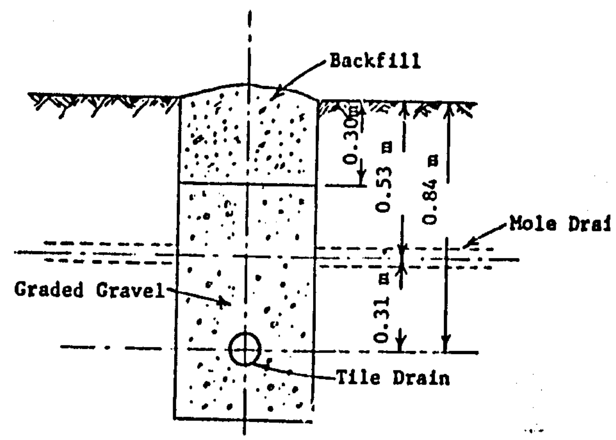

Fig. 7. Typical arrangement of the mole drains and tile drains used in the field experiment.

$$\frac{\hat{d}}{S_c} = \left\{\frac{8}{\pi}\left[\ln\left(\frac{S_c}{a}\right) - 1.15\right]\right\}^{-1} \quad (45)$$

By dividing (42) by (41) and rearranging,

$$S_c = S_m \left[(\tfrac{1}{2}h + \hat{d})/(\tfrac{1}{2}h + d)\right]^{1/2} \quad (46)$$

Equations (43), (45), and (46) are used to compute the corrected spacing S_c for the case in which the depth of the impermeable layer is not sufficiently large. For these experiments the corrected spacings S_c computed from the above equations are shown in the fourth column in Table 2. The correction ratios S_c/S_m are also shown in the same table.

In general, the corrected spacings of the mole drains S_c in Table 2 are much larger than the spacing of 1.83 m actually used in the experiment. Test run 1 yielded the average corrected spacing of 2.62 m, which is closer to the actual spacing than that of test run 2, which yielded the average corrected spacing of 5.55 m. The poor agreement of the corrected spacings and the actual spacing in test run 2 could be attributed to the fact that ponding existed in some areas within the experimental plot, including the midpoint, at the beginning of the test.

It is interesting to note that within a certain range of the hydraulic conductivity k, such as in this study, the error in the determination of k does not have a significant effect in the drain spacing determination because the mole drain spacing is proportional to $(k/f)^{1/2}$ as indicated in (38), and from the graph showing the relationship between the specific yield and hydraulic conductivity such as given by *Luthin* [1973] the value of $(k/f)^{1/2}$ is almost constant for k between 0.1 and 0.6 in./d (0.25 and 1.52 cm/d).

The probable reasons for not obtaining a better agreement between the theoretical and actual experimental spacings will be discussed as follows.

Equation (38) used in the correlation study has been derived from (23) and virtually consists of two terms: $w(x, y, t)$ and $v(x, y)$. The term $v(x, y)$ finally becomes K_2 in (38) and contains an infinite series of which all terms but the first were neglected in the derivation of the equation. Further investigation indicates that by including only the first term of the series the value of $v(x, y)$ could be overestimated by about 27% for S_m and S_t within the practical limits.

The term $w(x, y, t)$ consists of an infinite series of which each term contains an integral of $v(x, y)$. Here again only the first term of the series is maintained in the derivation of (38). The loss of accuracy in dropping other terms at this stage has not been investigated.

The overestimation of about 27% in the value of $v(x, y)$ and the unknown loss of accuracy in deriving the term $w(x, y, t)$ may be two of the causes affecting the overall accuracy of (38).

The term $f(x)$, which describes the profile of the water table along the mole drain during stage 1, has been assumed to be independent of time. Logically, the shape of water table profile should be dependent on x and time t. In other words, $f(x)$ should be replaced by $f(x, t)$. Consequently, K_1 and K_2 will no longer be constants but probably be functions of time t.

To remove any doubt of the possibility of whether the six different assumed shapes of $f(x)$ could affect the accuracy of the derived equations, an investigation was made to study the consequence resulting from the assumption made for $f(x)$. Since $f(x)$ affected only the χ term in each case, the characteristic of χ for all six cases has been analyzed. It is found mathematically that all χ converge to $4/\pi$ when x_0/S_t approaches zero. The graphical representation in Figure 8 also shows that when x_0/S_t is 0.05, all values of χ converge to about $4/\pi$. Since under actual conditions x_0/S_t is expected to be smaller than 0.05, it may be concluded that any assumed shape of the water table profile $f(x)$ should yield approximately the same results.

CONCLUSIONS

The study on the correlation between the developed theory and the field experiment by comparing the computed spacings of mole drains with the actual spacing indicates that further improvement of the equations to obtain a better accuracy should be attempted. The improvement could be made by including more than the first terms of the infinite series in the derivation and by replacing $f(x)$ with $f(x, t)$. Only the first term of the infinite series was used in this analysis for the sake of simplicity.

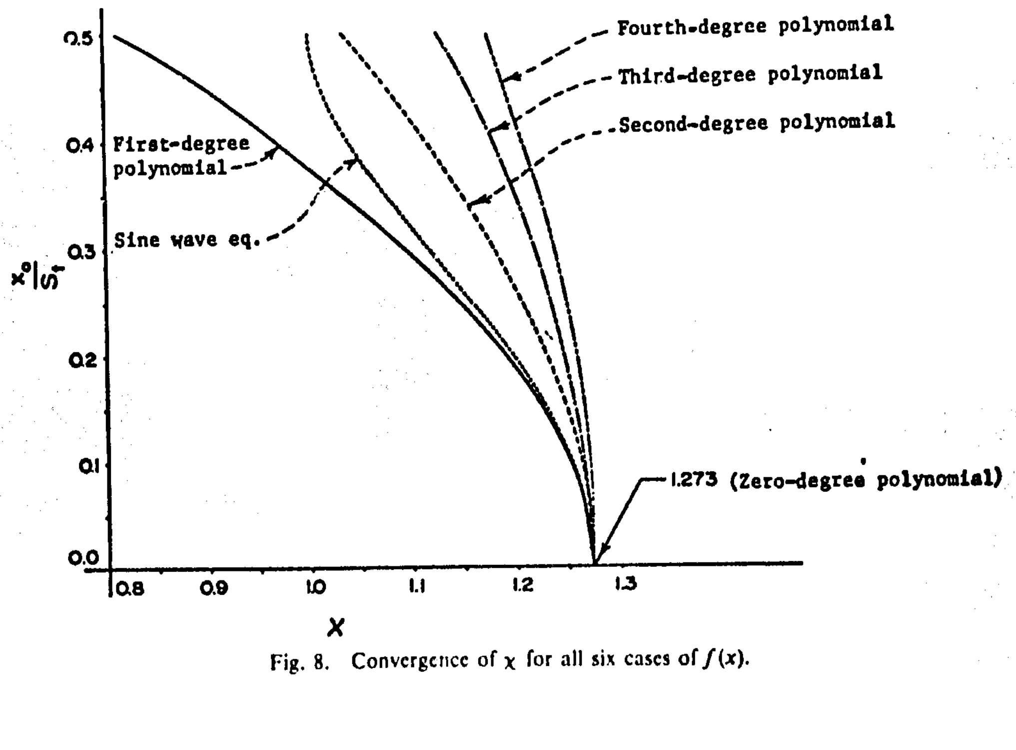

Fig. 8. Convergence of χ for all six cases of $f(x)$.

If one realizes that field experiments in drainage frequently encounter some uncontrollable or nonuniform conditions such as the variation of soil properties in the plot, the average computed spacing of 2.64 m should be regarded as being in reasonably good agreement with the actual spacing of 1.83 m. The equation developed should be very useful for the design of the combined mole-tile drain system considering the fact that until present, no other design equations have been developed.

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