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A GENERAL CONSUMPTION TECHNOLOGY IN NEW DEMAND THEORY

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In Lancaster's "New Approach," utility is a function of the quantities of characteristics provided by goods, rather than of the quantities of goods themselves.[1] The characteristics available are determined by the consumption technology, which defines the characteristics frontier (the sets of characteristics attainable for any given expenditure) [4, 5]. This frontier is independent of individual preferences and has the same shape for all individuals facing the same set of prices. However, Lancaster assumes that no disposal processes exist that remove excess quantities of characteristics. This paper takes account of their existence.

I.

We assume one-to-one correspondence between goods and consumption activities, so that we can write the consumer-choice program in the form [5, p. 21]

$$\begin{aligned} &\text{Maximize } U(z) \\ &\text{subject to } px \leqslant k \\ &\text{with } z = Bx \\ &z, x \geqslant 0, \end{aligned} \tag{1}$$

where z is a vector of the quantities of characteristics provided; x, a vector of the quantities of goods consumed; B, a matrix linking characteristics and goods; p, a vector of the prices of goods; and k the maximum expenditure allowed.

The equation system $z = Bx$ defines the consumption technology. In Lancaster's words,

> for every point in his characteristics space, the consumer has a choice between different goods vectors. Given a price vector, this choice is a pure efficiency choice, so that for every characteristics vector the consumer will choose the most efficient combination of goods to achieve that col-

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1. An equivalent formulation was used earlier by Victor E. Smith [7,6]. In Smith's version goods had "attributes" instead of "characteristics." These attributes gave goods certain "goal-attainment" powers, determined by the technical relations between goods and their attributes, and summarized by a set of "goal-achievement functions." Utility was a function of the levels of attainment of one's several goals or objectives. Smith did not elaborate this reformulation of demand theory.

> lection of characteristics, and the efficiency criterion will be minimum cost. [4, p. 139]

The sets of characteristics attainable for a given k, applying this efficiency criterion, constitute the efficiency frontier or characteristics frontier. That is, the efficiency frontier for given p and k is the set of all z such that when we solve the problem

$$\text{Minimize } px, \text{ subject to } Bx = z \text{ and } x \geqslant 0,$$

the resulting value of px is equal to k.[2]

By using $Bx = z$ rather than $Bx \geqslant z$, Lancaster prevents the solution from containing costless slack variables, which he evidently feels have no counterpart in reality. However, disposal activities do exist, though not all are costless. Numerous goods require disposal activities for some of their characteristics. Consider the inedible parts of food or the packaging and wrapping of practically everything we buy.

We examine the consequences of the disposal phenomenon.

II.

Activities that remove or offset unwanted quantities of certain characteristics may either require the use of one's personal effort or energy or be purchased through the market. Costless disposal may be a limiting case of either the market or the personal process, but it is analytically convenient to treat it as a zero-price market process.

Examples of costless disposal often fall within the class of personal processes; i.e., the physiological functions that dispose of the excess quantities of some nutrients[3] or simple non-use (as of the full speed of a high-performance car in the hands of a law-abiding driver, or the full volume of high-fidelity sound equipment when the teenagers are out of the house). Other personal activities, like weeding the garden or carrying out the trash, involve a subjective cost.

Market disposal processes exist either in the form of direct services (labor to mow the lawn or peel the potatoes) or of goods that render disposal or offsetting services. Such goods are common. We buy a hi-fi set and "soundproof" the room for noise insulation; we buy a picture window for the view and put on draperies to reduce the glare; we eat rich food and buy an exercise machine for weight reduction.

A consumption technology frontier that gives no consideration to dis-

2. We owe this definition to R. L. Gustafson.

3. Such as vitamin C or protein. Within fairly wide limits excesses of vitamin C or protein (nitrogen) are excreted harmlessly in the urine.

posal processes leaves untouched much relevant consumption behavior. Whether some goods can be consumed at all may depend on the possibility (or cost) of disposing of unwanted characteristics. For instance, the absence of adequate disposal for automobile exhaust emissions has led to proposals that gasoline consumption be restricted in Los Angeles.

Formally, we restate the Lancaster programming model as follows:

$$
\begin{aligned}
&\text{Maximize } U(z) \\
&\text{with } z = w - y \\
&\text{subject to } Bx = w \\
&Cv = y \\
&p_x x + p_v v \leqslant k, \text{ and} \\
&x, v \geqslant 0,
\end{aligned}
\tag{2}
$$

where y is a vector of quantities of characteristics that negate unwanted excesses of the characteristics in z, w is a vector of the (gross) quantities of characteristics provided by the various consumption activities, x is a vector of consumption activities (each of which represents the consumption of one particular good), v is a vector of disposal activities, and C is a matrix linking the quantities of the characteristics disposed of to the levels of the disposal activities (or the quantities of the goods or services employed in those activities). (Where a good yields characteristics desired for their own sake as well as those that perform a disposal function, it is arbitrary whether we classify it in x or in v, as long as it appears in only one of these vectors.) We let p_x and p_v be vectors of the prices of the goods or services employed in x or v, as the case may be.[4]

To obtain the efficiency choice for a given characteristics vector z^*, we

$$
\begin{aligned}
&\text{minimize } p_x x + p_v v \\
&\text{subject to } Bx - Cv = w - y = z^* \\
&x, v \geqslant 0
\end{aligned}
\tag{3}
$$

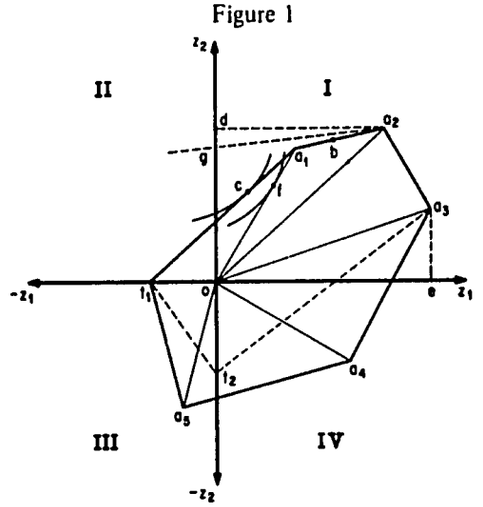
where z^* represents the set of net characteristics desired. The solution values of x and v, x^* and v^*, provide z^* at the least possible cost, $p_x x^* + p_v v^* = k^*$. Applying the scalar (k/k^*) to z^*, x^* and v^* determines the efficiency frontier point (characteristics set z) for budget k. Repeating the process for all z^* provides the whole frontier.

The frontier implied by (3) will not be the Lancaster frontier, although parts of the two frontiers coincide. Where a free disposal process exists, the ratio in which characteristics are consumed may differ from the ratio

4. We postpone the case in which the consumer's own time or energy is used.

in which they are purchased. In the Lancaster formulation it is sometimes necessary to incur a negative marginal utility from one characteristic in order to obtain the desired quantity of the other, or to reduce one's consumption of a desired characteristic in order to avoid consuming too large an excess of an undesired characteristic. Where free disposal processes exist these situations need not arise. The Lancaster frontier, based on the restraint $Bx = z^*$, appears in Figure 1 as the line $a_1a_2a_3$.[5] We deal here with only two characteristics, Z_1 and Z_2. The combinations of these characteristics indicated by points a_1, a_2 or a_3 may be attained by spending the whole budget (let us say one dollar) on either good X_1, X_2 or X_3.[6] Proportions between characteristics 1 and 2 other than those indicated at a_1, a_2 or a_3 may be had by dividing the expenditure between X_1 and X_2 (to reach positions along a_1a_2) or X_2 and X_3 (to reach positions along a_2a_3).[7] Selection among the points along $a_1a_2a_3$ is a subjective choice, determined by the characteristics of one's indifference map.[8]

Figure 1



5. Smith illustrated such an efficiency frontier for meeting certain nutrient requirements in [6, pp. 182-6], referring to it as the "set of efficient foods costing one dollar..." [p. 184]. Dorfman, Samuelson and Solow [3, p. 23] refer to this frontier as the "locus of efficient diets." On page 35 they use the phrase "efficiency frontier."

6. We assume that the act of buying is itself free of utility (although we know that for many of us this assumption is false)

7. Any point, b, on the efficiency frontier defines a set of characteristics levels, z^*, that will be provided exactly by x^*, where $Bx^* = z^*$, and the elements in x^* are chosen so that px is a minimum and that $px = k^* = \$1\ 00$.

8 We assume that the quantities of all other goods (and the total expenditure on them) are fixed. Thus the collection of characteristics provided by other goods is also fixed It may include specified amounts of Z_1 and Z_2. Our analysis concerns the cost and consequences of adding to (or subtracting from) whatever quantities of Z_1 and Z_2 may already be provided by these other goods.

Under Lancaster's assumptions, proportions lying outside the rays $0a_1$ or $0a_3$ cannot be attained. If dried beef, at a_1, is the least salty food available (here z_1 measures saltiness), it is assumed that there is no water available with which to leach out the salt. Furthermore, the Lancaster frontier may have segments with a positive slope (like a_1a_2). If a dollar's worth of dried beef does not provide enough protein (z_2), smoked herring at the price implied by $0a_2$ will provide more—but only if one takes still more salt with the protein. If the consumer does not want the extra salt, what are his options? Under the Lancaster assumption he must either limit himself to the protein provided at a_1 or choose a point such as b, which he will do if his indifference curve is tangent to a_1a_2 at b. This means, of course, consuming salt into the range where it has negative marginal utility.

If, however, there is in fact a costless disposal process for salt, our consumer need neither incur negative marginal utility from the consumption of salt nor limit his protein-salt ratio to those ratios found between a_1 and a_3. Moreover, X_1 disappears entirely from the characteristics frontier.

Perhaps in the case of excess saltiness no truly costless disposal process exists,[9] but for some nutrients, such as vitamin C and protein, there is a range—often considerable—over which the body disposes of unneeded excesses in an essentially costless fashion.[10] If z_1 measures vitamin C and z_2 protein, and we recognize the existence of costless physiological disposal processes for both vitamin C and protein, the correct efficiency frontier is da_2a_3e, not $a_1a_2a_3$.[11] The positively sloped segment a_1a_2 disappears, and it becomes possible to enjoy Z_1 and Z_2 in any proportions one wishes. Furthermore, by spending his whole dollar on X_2 and using free disposal for any excess of Z_1 obtained, the consumer may have more protein than could have been obtained at any point along a_1a_2 (excluding a_2).

Where disposal is both free and automatic (involuntary), it can be argued that no range of negative marginal utilities exists—that the indifference curve is a straight line parallel to the axis beyond the point where the marginal utility of the characteristic becomes zero. If this is the case, the consumer for whom vitamin C is in excess will choose point a_2 if confronted with the Lancaster frontier, spending his whole dollar on X_2, just as he does when he chooses a point on the da_2 segment of our frontier.

9. Even soaking drief beef in water takes time and energy.

10. The prevalence of high blood pressure suggests that this is only true to a limited extent for excess salt.

11. Free disposal for characteristic 1 means that one may move leftward from any point on the diagram, as far as one wishes, with no extra cost.

There are two difficulties with this approach. (1) It confuses the objective and subjective sides of the problem by declaring that the form of the utility function, $U(z)$, depends upon whether or not free and involuntary disposal processes exist. (2) Free disposal becomes a special case, handled by a method that cannot be generalized to include other types of disposal processes. In today's world, where many formerly free disposal processes are rapidly becoming costly, the latter consideration is especially important. When outdoor burning of leaves and paper becomes proscribed by law, or my car is no longer permitted to leave exhaust fumes behind it as I travel, the cost of the disposal processes I must use has changed. The theory should be able to reflect this.[12]

That some disposal processes are becoming costly does not mean that the case of free disposal is unimportant. The phenomenon is more significant than we realize, for, being free, these processes do not come immediately to the attention of the economist. Consider the quality of durability. We complain when things wear out too soon, but we overlook the fact that again and again we purchase goods that provide more durability than we plan to use. We buy paint that will last for years, knowing that we are leaving our apartment in six months. The piano stands mute in our living room years after our children have ceased taking music lessons Our dresser drawers contain white shirts with narrow collars, now out of style, but with months of wear remaining. Time and again the goods available to us provide characteristics in fixed proportions that do not quite suit our wants. As Figure 1 shows, the optimal choice from a finite set of alternatives is often to buy an excess of durability (plotted on the z_1 axis in this case) in order to get the other characteristics that we want.

If a market cost is involved in using a disposal activity, we will not choose to reduce the quantity of an excess characteristic to the point where its marginal utility is zero. Assume that no free disposal processes exist, but that one dollar will dispose of t_1 units of the excess of Z_1, or t_2 units of Z_2.[13] Then the characteristics frontier in Figure 1 becomes $t_1a_1a_2a_3t_2t_1$.[14] (We shall consider points a_4 and a_5 later.) The consumer

12. There may also be a true change in my utility function with respect to exhaust fumes or air pollution from burning trash, the result of greater awareness of their potential for harm to myself and others. This is a factor distinct from, and in addition to, changes in the cost of disposal.

13 We consider only the case where the cost is a fixed amount per unit of the excess removed (lawn-mowing, for instance, paid at a fixed price per square rod mown)

14. The frontier is drawn to include t_1t_2, in the third quadrant, because negative quantities of the characteristics sometimes have real meaning. If Z_1 is heat and Z_2 is humidity, with the origin taken as the state of heat and humidity existing in the absence of any action, points along t_1t_2 may be the preferred points.

can obtain a higher ratio of Z_2 to Z_1 (along $t_1 a_1$) or of Z_1 to Z_2 (along $t_2 a_3$) than the Lancaster frontier would indicate. (Points outside the rays $0a_1$ and $0a_3$ are unattainable in the Lancaster formulation.) The consumer cannot, however, avoid the necessity of consuming an excess of one characteristic or the other whenever his optimal subjective choice lies outside the $a_2 a_3$ range of the frontier.[15] Outside that range the marginal utility of one of the characteristics consumed will necessarily be negative in the optimal position, if all disposal processes are costly. We see that such a choice has been made when the housewife remarks that she would really have preferred slightly less red in the upholstery of her new chairs.

As the unit cost of disposal diminishes, the situation approaches that for free disposal. Segments $t_1 a_1$ and $t_2 a_3$ rotate outwards, toward the limiting horizontal and vertical positions defined by our previous case—the costless processes When t_1, as it moves leftward, has passed the point where $t_1 a_1 a_2$ is a straight line, X_1 ceases to lie on the characteristics frontier. No consumer would then choose such a point as b, on $a_1 a_2$; his optimal selection will lie on the new frontier, ga_2.

Disposal processes become increasingly important (and costly) in the modern world. As population grows, what may once have been a disposal act of little cost or significance becomes costly, to society if not to the individual. The drain that eliminates excess water from my land creates floods farther downstream, necessitating expensive flood control projects. Increased volumes of sewage overload our rivers and require increasingly expensive sewage processing systems. Rapidly growing volumes of trash fill our landfills and drive us to consider building compacting presses or ski slopes from garbage hills.

If these increases in disposal costs are reflected in higher charges to consumers for the disposal services they employ, consumption patterns will be adjusted in favor of goods that produce smaller quantities of the disagreeable excesses, thus reducing the total magnitude of the problem of waste disposal for society. In Figure 1, as disposal costs rise for Z_1 and Z_2, t_1 and t_2 approach zero and the characteristics boundary for an expenditure of one dollar shrinks, approaching $0a_1 a_2 a_3 0$ as a limit. Given an indifference map for which point c is optimal when $t_1 a_1$ is the boundary, as the boundary becomes steeper, pivoting on a_1, the optimal consumption point tends to drift rightward, approaching such a point as f as a limit. The new consumption pattern creates less waste to be removed.

On the other hand, if disposal costs are born by the public and financed

15. As it may if his indifference curve is positively sloped In goods theory positive slopes are often ruled out by assuming that more of each good is always preferred. An analogous assumption in characteristics theory would exclude from study a large portion of the reality we hope to illuminate.

by general tax revenues, to the individual consumer disposal may appear free, or nearly so. The characteristics frontier will approximate the costless disposal case and the consumption pattern chosen will create larger quantities of wastes to be removed and greater burdens for public disposal systems.

We have discussed so far only pure disposal processes like Ot_1 and Ot_2. Each of these removes only one characteristic. The market offers also goods or services that yield compound disposal processes, represented by activity rays that enter quadrants II, III or IV in Figure 1. In quadrants II and IV the activity provides one characteristic and negates another (see point a_4); in quadrant III (point a_5) it provides only the two offsetting or negating characteristics. (Point a_5 might represent air conditioning, bought in order to dispose of heat and humidity—excess characteristics in the environment or in the total consumption process.)[16]

When consumption activities 4 and 5 are present along with the two costly pure disposal activities, the efficiency frontier between t_1 and t_3 becomes $t_1a_5a_4a_3$. If excesses of Z_2 are to be removed or offset, buying a combination of goods X_3 and X_4 is best in the a_4a_3 range of the frontier, but if the subjective choice lies in the a_5a_4 range, a combination of X_4 and X_5 will be more efficient. The pure disposal activity for Z_2 (shown by ray Ot_2) is no longer efficient; its price is too high. The disposal activity for Z_1, however, remains efficient and will be used, along with activity Oa_5, if the subjective choice lies along the t_1a_5 segment of the frontier.

III.

Personal disposal processes (the use of one's own time or energy) often involve a subjective cost, but they do not reduce the quantities of goods that can be purchased with a given money income. Instead, the choice is between the disutility of consuming a little more of the unwanted characteristic and the disutility of using time or energy to reduce the quantity of the characteristic that must be consumed. If some of the disposal process is employed, and the marginal utility of using time for disposal is negative, the excess characteristic will be consumed to the point where it too will have negative marginal utility.

When no subjective cost is involved, it is clear that the characteristics frontier depends solely upon objective factors. Subjective preferences determine what point on the frontier is chosen, but have no effect upon the determination of the frontier itself. But if one's own time or energy is

16. These offsetting characteristics must not be confused with Lancaster's "negative characteristics" [5, pp 94-96]. The latter are simply characteristics of which every consumer would prefer less to more. They perform no disposal function.

required, the cost of disposal is at least the utility foregone by diverting this time or energy from leisure-time activities that may be more pleasurable. In addition, the disposal activity may be disagreeable in itself—or agreeable, for that matter. (A man may not enjoy digging weeds from his lawn or spraying it with weed killer, but he may enjoy raking leaves on a crisp fall day—if there aren't too many of them.) Evaluation of the disutilities involved in personal-effort disposal is certainly a subjective matter.

If $0t_1$ in Figure 1 were a personal disposal process instead of a market process, we would have to replace the assumption that equal increments of market expenditure dispose of equal increments of the excess characteristic Z_1 by an assumption of increasing marginal disutility from the disposal activity. If we represent that marginal disutility by the money equivalent of the subjective sacrifice involved (the marginal rate of substitution between disposal time and money), we can use the same diagram, but t_1a_1, the segment of the efficiency frontier which involves the disposal process t_1, would no longer be linear. On the assumption that equal units of the disposal activity provide equal amounts of disposal, but that the marginal sacrifice increases with the level of the activity, t_1a_1 would be convex from above. Additional sacrifices of equal money value yield smaller and smaller quantities of disposal. The frontier would no longer be objective, for its shape would depend upon the subjective evaluation of the sacrifice involved. It would also vary from person to person, both because of the subjective component and because the physical effectiveness of equal amounts of disposal time varies among persons.

If the characteristics frontier is no longer objective, an important operational advantage of the Lancaster theory has been lost. However, the apparent dependence of the frontier upon subjective evaluation is a consequence of the nature of the analysis, not of the problem. The present argument, which treats time as a cost to be subtracted from income, introduces the subjective element into the determination of the frontier, but if instead we regard the time or energy required for disposal of Z_1 as a third characteristic that itself enters the utility function, the separation of subjective and objective components is completely preserved.

Let utility be a function of z_1 and z_2, the quantities of Z_1 and Z_2 consumed, and of t_1, the time required to dispose of the Z_1 which is not consumed.[17] We assume that only two goods, X_2 and X_3, and two disposal processes, D_1 and V_1, are available. Each disposal process eliminates Z_1, but D_1 is a market process, which must be paid for, and V_1 requires the

17. Lancaster [4, pp. 98-9] has brief but illuminating comments on the interpretation of time or energy as a characteristic.

use of one's own time, T_1. In three dimensions the axes would measure t_1 and the levels of the two characteristics, z_1 and z_2. We use here a contour map of the three-dimensional efficiency frontier, obtained by taking sections through the z_2 axis parallel to the $t_1 z_1$ plane (Figure 2).

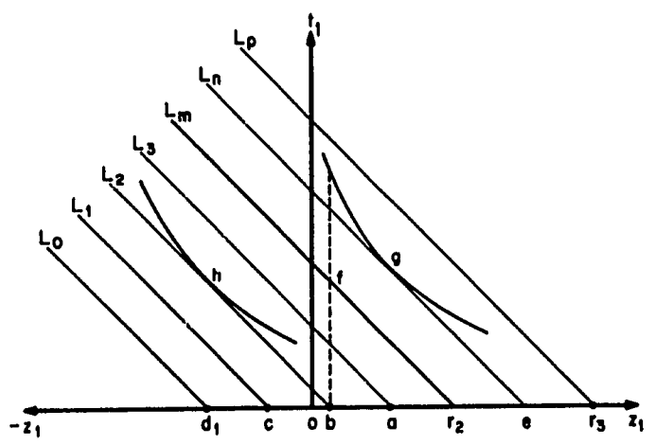
When $t_1 = 0$, the quantities of Z_1 available for consumption are shown along the $-z_1 z_1$ axis between d_1 and r_3. $0r_3$ is the maximum amount of Z_1 attainable (obtained by spending the whole dollar on X_3); $0d_1$ represents spending the whole dollar on D_1, which eliminates Z_1—the net consumption level is negative. The characteristics frontier for an expenditure of one dollar does not extend beyond d_1 and r_3.

$0r_2$ is the z_1 level obtained by spending the whole dollar on X_2. Dividing a dollar between X_2 and D_1 creates net quantities of Z_1 for consumption that are measured along the $-z_1 z_1$ axis between d_1 and r_2. At b, for instance, 50¢ has been spent on X_2 and 50¢ on disposal; at a, 75¢ on X_2 and 25¢ on disposal. Points between r_2 and r_3 represent the quantities of Z_1 that can be purchased by dividing a dollar between X_2 and X_3.

The amount of Z_2 to be consumed (z_2) varies with z_1. As z_1 falls from r_3 to r_2, z_2 rises. Expenditure is being shifted from X_3 to X_2, which raises the ratio of z_2 to z_1. At r_2, where the whole dollar is spent on X_2, z_2 is at its maximum. To the left of r_2, money is being spent on eliminating z_1, so less and less remains for the purchase of X_2; the level of z_2 diminishes. At d_1, where no X_2 is purchased, $z_2 = 0$.

When the personal disposal process, V_1, is used ($t_1 > 0$), there are more options. We can reduce z_1 without reducing z_2. We can spend the whole dollar on X_2, obtain the maximum amount of Z_2 that a dollar will buy, and exchange labor for the desired reduction in z_1. Starting from r_2, we

Figure 2



may reduce z_1 by increasing t_1, along line r_2L_m, with no change in the quantity of Z_2. ($L_m r_2$ is that contour of the efficiency frontier which represents the maximum quantity of z_2 attainable for a dollar.)

The personal disposal process is available at any level of z_1, so through every point on the diagram there runs a contour line parallel to $L_m r_2$. Contour L_3a represents the combinations of z_1 and t_1 that are consistent with the consumption of Z_2 at the level $0a$. We may adjust z_1 at will along this line without altering the consumption of Z_2, if we are willing to use the necessary amount of T_1.

The z_2 value of each contour declines as we move away from L_m in either direction, toward either L_o or L_p. The characteristics frontier for an expenditure of one dollar is bounded by L_0d_1, where $z_2=0$, by $L_p r_3$, where z_2 has a positive value, and by d_1r_3. (We rule out the possibility of negative purchases of goods, or the use of negative quantities of time.)

Subjective considerations determine what point will be chosen on this frontier, or on its boundaries. The solutions fall into two principal classes: (a) those involving both goods (between $L_m r_2$ and $L_p r_3$) and (b) those involving X_2 plus D_1 (between $L_m r_2$ and $L_o d_1$). The limiting cases involve only a single good or D_1 alone.

If $t_1>0$ in the preferred position, the preference surface will be tangent to one of the contours, as at g or h. At any such tangency point the subjective marginal rate of substitution of Z_1 for T_1 equals the amount of T_1 that uses up one unit of Z_1. To the left of $L_m r_2$, MU_{z_1} (the marginal utility of Z_1) is negative.[18] (Money is being spent on the market disposal process, D_1.) Consequently it is wise to use T_1 in the disposal of Z_1 until the marginal utility of T_1 also becomes negative. At such a tangency point as h, the marginal disutility of consuming another unit of Z_1 equals the marginal disutility of using the additional T_1 required to dispose of that unit of Z_1 so that it will not have to be consumed.

To the right of $L_m r_2$, MU_{z_1} is positive. In this area combinations of X_2 and X_3 are being purchased, with implies that Z_2 is being given up in order to obtain more Z_1. (We assume that MU_{z_2} is positive at all levels of consumption.) Equilibrium along a contour to the right of $L_m r_2$ implies that MU_{t_1} is positive; we are willing to give up some of Z_1 (which has positive marginal utility), in order to gain the pleasures associated with using more of T_1. Indeed, we buy more of X_3 than is needed to provide

18. We include in MU_{z_1} the loss of utility that occurs when a unit of time is taken away from other leisure-time (non-market) uses plus any increment or decrement of utility specifically associated with activity V_1.

the Z_l we plan to consume, in order that there will be an excess of Z_l to be removed by the disposal process. The man who plays paddleball in order to keep his weight down is likely to eat a more fattening diet if he enjoys the game than if he plays solely from a sense of duty.

Along $L_m r_2$ itself, $MU_{z_l} \gtrless 0$. The characteristics surface is discontinuous as we cross that line. If equilibrium occurs along $L_m r_2$, however, it remains true that T_l will be used to the point at which the gain (loss) in utility from consuming less Z_l equals the loss (gain) in utility from using more T_l. If $MU_{t_l} = 0$ at equilibrium, the equilibrium point will lie along $L_m r_2$.[19]

There is also substitution between T_l and D_l and between T_l and Z_2: At b, 50¢ is being spent on X_2 and 50¢ on D_l; $t_l = 0$. Increasing t_l while holding z_l constant at $0b$ (moving vertically upward between b and f) means moving to higher z_2-contours, each of them representing a larger expenditure on X_2 and smaller expenditure on D_l. Anywhere to the left of $L_m r_2$ an increase in T_l reduces D_l and increases Z_2. If the most preferred point is in this area, it will be where the (negative) marginal utility of T_l just balances the (positive) utility of the extra Z_2 that can be had by diverting expenditure from the D_l replaced by T_l to the purchase of more X_2. This would be represented geometrically by a tangency point in the $z_2 t_l$ plane.

Above $L_m r_2$ an increase in T_l, for a given Z_l, is associated with a decrease in Z_2. To replace the Z_l used up as T_l increases, expenditure must shift from X_2 to X_3, reducing the amount of Z_2 obtained for a dollar. If the most preferred point lies above $L_m r_2$, the utility lost when Z_2 decreases is just balanced by the gain in utility associated with the increase in T_l.

Introducing the personal disposal activity leaves the characteristics frontier free of subjective elements, but it is no longer the same for all individuals unless one is willing to assume that the personal disposal process has the same technical coefficients for all. As this is equivalent

19. R. L. Gustafson has pointed out that once "personal effort" activities are introduced into the model there is no reason to use them only for disposal purposes Once we recognize that positive marginal utility is possible for a disposal activity, there is no clear line of distinction between a disposal activity and any other non-market activity The whole "do-it-yourself" phenomenon opens up for inclusion in the model For instance, $0t_l$ in Figure 2 could represent the use of time in an activity that is engaged in for its own sake, but requires T_l and Z_l as inputs No change would be needed in the analysis, but Z_l might no longer be found in excess amount. In the equilibrium position MU_{t_l} would be positive and MU_{z_l} might be

If the "do-it-yourself" activity produces Z_l instead of consuming it, and if T_l is the only input required, the same diagrammatic technique can be used, but the contour lines will have positive rather than negative slopes. Further extensions come readily to mind.

to assuming that each one of us would require the same amount of time or energy to accomplish a given task, the assumption must be rejected. The efficiency frontier involving personal disposal processes is itself personal. It is free of subjective elements, but not common to all individuals. If the personal disposal process is linear, however, the frontiers for one individual at different expenditure levels remain parallel, as in the original Lancaster case.

IV.

Finally, an algebraic illustration: Let d_l and v_l be the levels of the respective market and personal disposal activities, D_l and V_l. One unit of either D_l or V_l disposes of one unit of Z_l; D_l has a market price, p_l, while one unit of V_l uses up one unit of disposal time, T_l. The total leisure time available, t, is divided between t_l and t_0, all other uses of leisure time. We assume t to be given. Thus k, the income to be spent for the z_l and z_2 quantities of characteristics Z_l and Z_2, is fixed, as in our earlier examples.[20] X_2 and X_3 are the only goods available for purchase; their prices are p_2 and p_3, while x_2 and x_3 are the quantities purchased. The amount of Z_i provided by one unit of x_j is given by a_{ij}.

The utility function is

$$U = f(z_l, z_2, t_0, t_l) \tag{4}$$

given the quantities of the characteristics provided by the fixed amounts of goods other than X_2 and X_3 that are being consumed. However, t_0 and t_l are not independent, for $t = t_0 + t_l$, and t is constant. Thus we may write

$$U = g(z_l, z_2, t_l) = f(z_l, z_2, t_0, t_l) \tag{5}$$

For given levels of z_l and z_2,

$$dU = (\partial g/\partial t_l)dt_l = (\partial f/\partial t_0)dt_0 + (\partial f/\partial t_l)dt_l$$

But $dt_l = -dt_0$, so

$$dU/dt_l = \partial g/\partial t_l = (\partial f/\partial t_l) - (\partial f/\partial t_0)$$

The total marginal utility of time used for disposal ($MU_{t_l} = dU/dt_l$) equals the marginal utility or disutility ($\partial f/\partial t_l \gtreqless 0$) specific to T_l minus the marginal utility ($\partial f/\partial t_0 > 0$) of the non-market activities being given up.

20. Our concern is with the allocation of a fixed income among consumption goods, not with the allocation of time between market and non-market uses. Becker [2; 1, pp. 45-47, 162-71] has dealt with the latter problem.

The complete model follows.

Maximize $U = g(z_1, z_2, t_1)$, where

$$
(6)\quad
\begin{array}{ll}
\text{(a)} & p_1 d_1 + p_2 x_2 + p_3 x_3 = k \\
\text{(b)} & -d_1 + a_{12} x_2 + a_{13} x_3 - v_1 = z_1 \\
\text{(c)} & a_{22} x_2 + a_{23} x_3 = z_2 \\
\text{(d)} & v_1 = t_1
\end{array}
$$

We assume $\partial g/\partial z_2 > 0$, and $\partial^2 g/\partial z_1^2$, $\partial^2 g/\partial z_2^2$, $\partial^2 g/\partial t_1^2 < 0$.

The subjective elements of the problem are entirely contained in function g, which provides an ordinal measure of utility. The restraints, (a) through (d), involve only objective facts.

Forming the Lagrangian leads to the following first order equilibrium conditions, each of which defines the optimal level for a single variable.

$$
(7)\quad
\begin{array}{lll}
 & & \textit{Variable} \\
\text{(a)} & a_{12}(\partial g/\partial z_1) + a_{22}(\partial g/\partial z_2) = \lambda p_2 & x_2 \\
\text{(b)} & a_{13}(\partial g/\partial z_1) + a_{23}(\partial g/\partial z_2) = \lambda p_3 & x_3 \\
\text{(c)} & -(\partial g/\partial z_1) = \lambda p_1 & d_1 \\
\text{(d)} & +(\partial g/\partial z_1) = +(\partial g/\partial t_1) & v_1
\end{array}
$$

In addition, there are the four equations in (6), (a)-(d), which require that any solution point lie on the characteristics frontier. Equations (7) determine where on that frontier the solution lies. As there are four restraints in (7), (a)–(d), and only three levels of characteristics to be chosen, we need have no more than three of these holding simultaneously for any given point of equilibrium. Furthermore, we impose the condition that d_1, v_1, x_2 and $x_3 \geqslant 0$. Therefore, if one of these equations can only be satisfied by a negative value of d_1, v_1, x_2 or x_3, it is replaced by $d_1 = 0$, $v_1 = 0$, $x_2 = 0$, or $x_3 = 0$, as the case may be.

Equations (7), (a) and (b), require that the marginal utility of each good consumed at a positive level shall equal its price expressed in utility units. (The factor λ defines the worth of a dollar in utility units.) By (7c), $d_1 > 0$ only when $\partial g/\partial z_1 = MU_{z_1} < 0$. If both d_1 and $v_1 > 0$, (7c) and (7d) both hold, and $MU_{t_1} = -\lambda p_1$. If (7c) does not hold, $d_1 = 0$, but (7d) may still hold with $v_1 > 0$. If so, MU_{t_1} and MU_{z_1} may be > 0. We have already seen

that

$$MU_{t_1} = (\partial f/\partial t_1) - (\partial f/\partial t_0)$$

While we may assume that $\partial f/\partial t_0$, the marginal utility of alternative leisure-time activities, is always positive, we have $\partial f/\partial t_1 \gtrless 0$ for the specific marginal utility of disposal activity V_1, so $MU_{t_1} \gtrless 0$.

Not all of equations (7) can be satisfied simultaneously if we require x_1, x_2, d_1, and $v_1 \geqslant 0$. Assume, as we have in our geometric illustrations, that G_2 provides less z_1 and more z_2 per dollar than does G_3 ($a_{12}/p_2 < a_{13}/p_3$, $a_{22}/p_2 > a_{23}/p_3$). Then (7a), (7b), and (7c) cannot hold simultaneously. By (7c), $\partial g/\partial z_1 < 0$. If $x_2 > 0$, (7a) holds and

$$[(a_{12}/p_2)(\partial g/\partial z_1)] + [(a_{22}/p_2)(\partial g/\partial z_2)] = \lambda$$

But the comparable expression derived from (7b),

$$[(a_{13}/p_3)(\partial g/\partial z_1)] + [(a_{23}/p_3)(\partial g/\partial z_2)] = \lambda$$

is not true if (7c) and (7a) hold, for both terms within the parentheses are algebraically smaller for X_3 than for X_2. If the solution contains both x_2 and d_1 at positive levels, $x_3 = 0$.

By a similar argument, if $MU_{x_1} = 0$, $x_3 = 0$ and $d_1 = 0$, but x_2 and v_1 may be at positive levels.

When both x_2 and x_3 are in the solution at positive levels, the usual equilibrium condition holds:

$$\begin{aligned} MRS_{x_2x_3} &= MU_{x_2}/MU_{x_3} \\ &= [a_{12}(\partial g/\partial z_1) + a_{22}(\partial g/\partial z_2)]/[a_{13}(\partial g/\partial z_1) + a_{23}(\partial g/\partial z_2)] = p_2/p_3 \end{aligned}$$

Solving (7a) and (7b) simultaneously gives

$$\begin{aligned} MRS_{z_1z_2} = MU_{z_1}/MU_{z_2} &= (a_{22}p_3 - a_{23}p_2)/(a_{12}p_3 - a_{13}p_2) \\ &= [(a_{22}/p_2) - (a_{23}/p_3)]/[(a_{12}/p_2) - (a_{13}/p_3)] \end{aligned}$$

The ratio of MU_{z_1} to MU_{z_2} is the inverse of the ratio of the difference between the quantities of Z_1 that each of the two goods provides for a dollar to the difference between the quantities of Z_2/dollar that the two goods provide. This is, of course, the slope of the efficiency frontier in the z_1z_2 plane.

If V_1 is also in the solution, (7d) gives $MRS_{t_1z_1} = MU_{t_1}/MU_{z_1} = 1$. Also

$MRS_{t_1 z_2} = MRS_{z_1 z_2}$. From (7), (a), (b) and (d) together,

$$MU_{t_1} = MU_{z_1} = \lambda(a_{22}p_3 - a_{23}p_2)/(a_{13}a_{22} - a_{12}a_{23})$$

In equilibrium MU_{t_1}/λ, the marginal rate of substitution of time for income (the money equivalent of the marginal utility of personal disposal time), is determined by the prices and technical qualities of X_2 and X_3.

When x_2 and d_1 exceed zero, $x_3 = 0$, $MU_{z_2} = \lambda\,[(p_2 + a_{12}p_1)/a_{22}]$, $MU_{z_1} = -\lambda p_1$, and $MRS_{z_1 z_2} = -[p_1 a_{22}/(p_2 + a_{12}p_1)]$.

If v_1 is also positive,

$$MU_{t_1} = MU_{z_1} = -MU_{d_1} = -\lambda p_1,\ MRS_{t_1 d_1} = -MRS_{t_1 z_1} = -1$$

and $$MRS_{t_1 z_2} = -[p_1 a_{22}/(p_2 + a_{12}p_1)]$$

We note that the marginal utility of a characteristic and of personal disposal time may be either positive or negative (or zero) in an optimal position, and that the *MRS* between characteristics and between time and Z_2 depends upon whether the market disposal process is in use. If the personal disposal process did not exist, it would be impossible to adjust z_1 without adjusting z_2, and vice versa.

V.

A neglected aspect of the problem of consumer choice is the fact that the commodities available to us on the market frequently do not possess the attributes we desire in exactly the right proportions. Standard economic theory cannot deal with the problem because the characteristics (attributes) of commodities are not represented explicitly in the model. The Lancaster model has the capacity, but must be extended to include disposal activities before the matter can be handled satisfactorily.

Optimal choice among commodities must take account of the possibility (and cost) of disposal for those unwanted characteristics that every household obtains along with its regular weekly purchases. Costly disposal affects both our choices among goods and our consumption patterns for characteristics. Goods that would not appear on the efficiency frontier at all when disposal costs are low may become important objects of consumption as disposal costs rise. The rising costs of disposal that manifest themselves in so many ways as our land becomes more crowded promise to force us to consume more of many unwanted characteristics (smog, noise and traffic jams, among others), at higher and higher marginal disutility levels for the characteristics involved.

The extended Lancaster method used in this paper provides an objective efficiency frontier that takes account of disposal processes. Upon this general frontier the private choice among characteristics, as represented by indifference curves or surfaces, can now be superimposed to determine individual consumer equilibrium. To the extent that the costs of disposal are objectively determined and the same for all persons the frontier is invariant among individuals, as Lancaster has claimed. When personal disposal processes are employed, however, this invariance disappears, because such processes cannot be expected to be the same for all. This means that separating objective from subjective factors will be difficult in empirical studies where personal disposal processes are important.

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