A DECISION - THEORETIC APPROACH TO UNCERTAINTY IN THE RETURN PERIOD OF MAXIMUM FLOW VOLUMES USING RAINFALL DATA

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ABSTRACT: The maximum seasonal runoff volume Q for an ungaged stream site is derived using (1) an event-based rainfall model for thunderstorms, and (2) a linear rainfall-runoff model. Major emphasis is placed on effect of uncertainty in parameters of rainfall inputs on the return period of maximum runoff volumes in a season. The event-based rainfall model, derived previously by the co-authors and others, has the following features: (1) the distribution of the number of events per season N is Poisson with mean m; (2) the distribution of point rainfall amount R per event is exponential with mean 1/u; (3) N and R are independent. More explicitly, we obtain a correct distribution function for the return period $T_Q(x)$ under the uncertainty in m and u, and demonstrate the necessity of following this approach for a decision-theoretic analysis of a water resource design problem. The approach enables us to design structures, relying only on rainfall data, on watersheds with ungaged streams by taking into account uncertainty of design site parameters. Also, we can tailor the design to a specific problem rather than use a pre-specified design flood, such as the magical 100-year flood.

ANALYSE DE DÉCISION DE L'INCERTITUDE DANS LES CALCULS DE PÉRIODE DE RECURRENCE D'ÉCOULEMENT A PARTIR DE DONNEES PLUVIOMÉTRIQUES

RÉSUMÉ: Le volume d'écoulement maximum est calculé à un site non instrumenté, en utilisant: (1) un modèle de pluie d'orage construit par événement; (2) un modèle pluie-débit linéaire. La manière dont l'incertitude sur les paramètres du modèle de pluie affecte la période de récurrence $T_Q(x)$ du volume d'écoulement maximum Q est analysée d'une manière quantitative. Le modèle de pluie d'orage a les caractéristiques suivantes: (1) le nombre d'événements par saison N suit une distribution de Poisson à moyenne m; (2) la quantité de pluie ponctuelle R par événement suit une distribution exponentielle de moyenne 1/u; (3) N et R sont des variables aléatoires indépendantes. Nous obtenons la fonction de distribution de $T_Q(x)$ tenant compte de l'incertitude sur m et u et montrons l'utilité de cette méthode pour une application correcte de la théorie de la décision à un problème de planification de ressources en eau. Nous pouvons ainsi de concevoir des ouvrages sur des bassins déversants sans données d'écoulement, à l'aide de données pluviométriques, tout en tenant compte de l'incertitude sur les paramètres. Par ailleurs, nous pouvons spécialiser la conception à chaque cas d'espèce au lieu d'utiliser une crue standard, telle la magique crue de période de retour centenaire.

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1.0 Introduction

Floods or stream discharges are properly described by their durations and volumes above a certain flow level and their instantaneous peak flows. Of these three properties, this paper is concerned with the uncertainty in the return period of maximum flow volumes which is a design parameter for flood protection and other structures. In particular, we consider the uncertainty due to inadequate data on small watersheds (up to 500 km^2).

It is well known that there is a good chance that a flow event Q with a large return period T_R may be exceeded at least once in an N-year design period. Typically, however, calculated risk diagrams (Gilman, 1964) do not consider the uncertainty in the return periods of rainfall and flow events. To a design engineer, the uncertainty of inadequate rainfall or flow data can result in either overinvestment (overdesign) or underinvestment (economic losses) in the design of flood retarding or retention structures or of water storage facilities (farm ponds or water supply reservoirs for small towns or industries). The Bayesian framework presented in this paper allows for an explicit consideration of hydrologic uncertainty as noted above and for a methodology to evaluate potential losses associated with that uncertainty.

Approaches taken to arrive at estimates of the return period of hydrologic flow properties include:

(a) Empirical fitting of probability density functions to historical data; in particular, the Soil Conservation Service (1965) fitted Pearson Type III distributions to flow volumes for various time periods in Arizona. This approach disregards any available information in precipitation records or any knowledge about the rainfall-runoff process.

(b) Use of phenomenological relations such as a linear transformation of rainfall volume to flow volume as a basis for obtaining probability density functions (pdf) of flow. The pdf of rainfall volume may be described empirically (with its consequent uncertainty) or from a process viewpoint wherein individual rainfall events are modeled as a stochastic process along the time axis (Duckstein et al. 1972).

(c) Use of detailed dynamical flow equations to relate pdf of rainfall properties to pdf of flow properties (Eagleson, 1972).

In this paper we use the second approach. Herein we build on previous work (Davis et al. 1972) where we evaluated the uncertainty in the return period of point rainfall amounts from summer thunderstorms. We define an event-based process in this case as a sequence of thunderstorms in time. The return period $T_R(k)$ of maximum point rainfall R (with k the rainfall amount or value of the random variable R) is derived by considering the following elements of the event-based process:

(a) The number N of events per season is Poisson distributed with mean m (of number of events per season):

$$f_N(j|m) = \frac{e^{-m}m^j}{j!} \tag{1}$$

(b) Rainfall events $R_1, R_2, \ldots,$ are independent identically distributed random variables.

(c) The amount R of point rainfall per event is exponentially distributed with parameter u (equal to reciprocal of mean amount rainfall per event):

$$f_R(k|u) = ue^{-uk} \tag{2}$$

(d) N and R are independent.

Then, the return period of k units of rain in a season, given the event-based parameters m and u, is

$$T_{\underset{\sim}{R}}(k|m,u) = [1-\exp(-me^{-uk})]^{-1} \tag{3}$$

Because m and u are uncertain due to small sample size, $T_{\underset{\sim}{R}}$ is uncertain.

To encode the uncertainty, the posterior distribution of m and u represents the likelihood of the values of m and u which produced the data. This posterior is given by the conjugate distributions for the exponential and Poisson distributions (de Groot, 1970, Chapt. 9). The distribution that is conjugate to both of these is the gamma:

$$g_{\underset{\sim}{X}}(x|a,b) = \frac{b^a x^{a-1} e^{-bx}}{\Gamma(a)} \tag{4}$$

For the Poisson distributution,

$x = m$, the parameter of the Poisson and estimated as $\hat{m}$.
$b = n$, the number of seasons.
$a = \hat{m}n$, the total number of rainfall events in n seasons.

For the exponential distribution,

$x = u$, the parameter of the exponential and estimated as $\hat{u}$.
$a = \hat{m}n$, the total number of rainfall events in n seasons.
$b = \hat{m}n/\hat{u}$, the total amount of rainfall for the mn events.

The resulting $F_{\underset{\sim}{X}}(x)$s in each case are posterior distributions and represent the likelihood that various values of m and u are the values describing the rainfall process that we are observing, after getting the data. These posterior distributions are used in a computer simulation to develop the posterior distribution of $T_{\underset{\sim}{R}}(k)$. The mean of this distribution is the expected return period $E[T_{\underset{\sim}{R}}(k)]$ for a k-inch rainfall. Computer results given by Davis, et al. (1972) indicate that the return period of point rainfall is subject to considerable uncertainty even with 20 years of data. The design and operational implications are obvious for flood control, dry farming with irrigation, and water supply. Next, we extend the procedure to uncertainty in return periods of seasonal flow volumes on small watersheds.

2.0 Extension to Seasonal Flow Volumes

If m is the total number of runoff producing rainfall events in a summer season, then the exact expected return period $T_Q(y)$ of the maximum seasonal runoff volume Q is, under our previous hypotheses,

$$T_{\underset{\sim}{Q}}(y|m,u) = [1-\exp(-m + m F_{\underset{\sim}{Q}}(y|u))]^{-1} \tag{5}$$

where $F_{\underset{\sim}{Q}}(y|u)$ is the distribution function of runoff per event $\underset{\sim}{Q}$ which we will write $F_{\underset{\sim}{Q}}(y)$ for simplicity. Our approach is to obtain $F_{\underset{\sim}{Q}}(y)$ from the distribution function $F_{\underset{\sim}{R}}(x)$ of rainfall $\underset{\sim}{R}$ per event, using the linear rainfall-runoff relationship

$$\underset{\sim}{Q} = \underset{\sim}{C}(\underset{\sim}{R}-A) \qquad 0 \leq \underset{\sim}{C} \leq 1 \tag{6}$$

where A are the initial abstractions depending on the watershed and $\underset{\sim}{C}$ is a coefficient depending on the rainfall characteristics for a given watershed, in particular, a time factor such as the maximum 15-minute intensity (Duckstein et al. 1972).

If we let

$$\underset{\sim}{P} = \underset{\sim}{R} - \underset{\sim}{A} \quad \text{for } \underset{\sim}{R} > A$$
$$= 0 \quad \text{for } \underset{\sim}{R} < A$$

then Equation (6) becomes $\underset{\sim}{Q} = \underset{\sim}{C}\underset{\sim}{P}$ or $y = cx$; the distribution function of $\underset{\sim}{P}$ is

$$F_{\underset{\sim}{P}}(x) = 1 - \exp(-u(x+A)) \text{ for } x > 0 \tag{7}$$

and that of $\underset{\sim}{Q}$ (Feller, 1967, Chapt. 2) is

$$F_{\underset{\sim}{Q}}(y) = \int_0^\infty F_{\underset{\sim}{P}}\left(\frac{y}{c}\right) f_{\underset{\sim}{C}}(c)\, dc \tag{8}$$

because $\underset{\sim}{C}$ is a random variable as noted in previous work by the coauthors (Duckstein et al. 1972). Since, physically, we cannot obtain more runoff than rainfall, then $0 \leq \underset{\sim}{C} \leq 1$, and a beta distribution for $\underset{\sim}{C}$ seems to be most appropriate:

$$f_{\underset{\sim}{C}}(c|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} c^{a-1}(1-c)^{b-1}, \quad 0 \leq c \leq 1 \tag{9}$$

The uncertainty on a,b will not be considered in the present study. To sum up, Equations (7), (8) and (9) may be combined to obtain

$$F_{\underset{\sim}{Q}}(y) = F_{\underset{\sim}{Q}}(y|u) = 1 - \exp\left[\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} e^{-uA} I(y|u)\right] \tag{10}$$

where

$$I(y|u) = \int_0^1 \exp\left(-\frac{y}{c} u\right) c^{a-1} (1-c)^{b-1}\, dc \tag{11}$$

Equations (10) and (11) are now substituted into Equation (5) to obtain an explicit expression of $T_{\underset{\sim}{Q}}(y|m,u)$. Because we have the sufficient statistics, $\hat{m}$ and $\hat{u}$, our knowledge of $\hat{m}$ and $\hat{u}$ can be expressed as a pdf (Tiao and Box, 1973). Hence, this encoded uncertainty results in a pdf on $T_{\underset{\sim}{Q}}(y|\hat{m},\hat{u})$.

3.0 Methodology

To obtain the pdf of the return period on hand, $T_{\underset{\sim}{Q}}(y|m,u,n)$ is a problem of transformation of random variables, where a closed form is beyond reach.

Thus, a simulation approach is used as follows: (a) consider a fixed yearly maximum flow volume $\underset{\sim}{Q} = y_o$, and (b) sample values m,u are drawn from the conjugate pdfs, $g_m(m|\hat{m},n)$ and $g_u(u|\hat{u},n)$, respectively, as noted in our discussion of Equation (4), (c) these sample values are substituted into $T_{\underset{\sim}{Q}}(y_o|m,u)$ to obtain one value of the return period $T_{\underset{\sim}{Q}}$, and (d) the process is repeated to obtain pdf of $T_{\underset{\sim}{Q}}$ for a fixed y (for example $y_o = y = 0.7$ inch in Table 1).

A similar procedure is then used to calculate the pdf of $(T_{\underset{\sim}{Q}})^{-1}$, which is the probability of exceedance of y_o. The design parameter of interest may be either $T_{\underset{\sim}{Q}}$ (for sizing a small dam) or $(T_{\underset{\sim}{Q}})^{-1}$ (estimating long-range replacement costs of structures.)

Finally, to be considered in a later study is the pdf of maximum seasonal flow Q that corresponds to a fixed return period. Such a pdf may be of interest for flood plain insurance purposes and can be calculated by the same simulation procedure as above.

4.0 Results

The results of the computer simulation are summarized in Tables 1 and 2 and Figures 1 and 2. In these we consider the variance of $\underset{\sim}{C}$, representative of conditions on the watershed, and the variance in our knowledge about rainfall parameters m and u.

Table 1 shows that u, the average rain per event, is much more important than m, the average number of storms per season, as judged by the variance of $T_{\underset{\sim}{Q}}$, (Var $\hat{T}_{\underset{\sim}{Q}}$), for different values of Var $\underset{\sim}{C}$. We also note the following

(a) As Var $\underset{\sim}{C}$ increases, $E[\hat{T}_{\underset{\sim}{Q}}]$ and Var $\hat{T}_{\underset{\sim}{Q}}$ decrease. Thus by not randomizing C the estimated return period of Q = 0.7 is much higher. By varying C the variable effects of rainfall intensity and watershed behavior on the return period are anticipated;

(b) Var $\hat{T}_{\underset{\sim}{Q}}$ increases dramatically when Var $\underset{\sim}{C}$ = 0 for joint uncertainty in m and u;

(c) The mean reciprocal return period (= exceedance probability = p) and Var $\hat{T}_{\underset{\sim}{Q}}^{-1}$ increase rapidly as Var $\underset{\sim}{C}$ increases. This result is shown because p is commonly used as the design parameter in hydrologic risk analysis.

These patterns hold for all values of runoff volume used in the sensitivity analysis (Q = 0.5, 0.7 and 0.9 inches of runoff) as shown in Table 2.

As expected, the Var $T_{\underset{\sim}{Q}}$ decreases with doubling of available data (10 to 20 years used in the simulation) as summarized in Table 2. The $E[\hat{T}_{\underset{\sim}{Q}}]$ is only slightly changed. A more general manifestation of the simulated process is evident in Figure 2 where the posterior pdf (of return periods for 0.7-inch runoff) based on 20 years of data has a much sharper modal value than the posterior pdf based on 10 years of data; note that mean $\hat{T}_{\underset{\sim}{Q}}$ is just to the right of the mode. While not shown, the posterior pdf's become more peaked as Var $\underset{\sim}{C}$ increases.

The effect of increasing runoff volume is to increase $E[\hat{T}_{\underset{\sim}{Q}}]$, Var $T_{\underset{\sim}{Q}}$ and coefficient of variation CV($\hat{T}_{\underset{\sim}{Q}}$) as shown in Table 2. The latter result about CV($\hat{T}_{\underset{\sim}{Q}}$) also implies that a Var $\hat{T}_{\underset{\sim}{Q}}$ increases more rapidly than $E[\hat{T}_{\underset{\sim}{Q}}]$. It is intriguing to note the dramatic effect that the introduction of Var $\underset{\sim}{C}$ has on the parameters.

The results in Table 2 for n = 10 years are shown in Figure 1, a plot on Gumbel extreme value paper. As previously noted, as Var $\underset{\sim}{C}$ increases the smaller $E[\hat{T}_{\underset{\sim}{Q}}]$. From the tabulated results we note that so-called confidence limits for each line would get wider as $\hat{T}_{\underset{\sim}{Q}}$ increases because Var $\hat{T}_{\underset{\sim}{Q}}$ increases with runoff volume. These confidence limits are narrower for n = 20 years of data as is evident from Table 2.

Of interest is the modest computer time (maximum of 25 seconds for 20 years of data) per simulation run on the CDC-6400. Given the number of uncertain parameters in this problem, it does not appear feasible to prepare charts and graphs for routine design use unless more exhaustive computer studies are performed.

4.1 Comments on Results

In contrast to the classical empirical frequency approach in deriving $\hat{T}_{\underset{\sim}{Q}}$,

the event-based approach outlined here results in evaluation of uncertainty in $\hat{T}_Q$ from physically meaningful parameters like m and u. This is a much more efficient use of the available data on rainfall and runoff.

We have seen how the design would depend on the uncertainty in m and u and on the interaction between uncertainty in m and u and Var C. The end result, a posterior distribution on $\hat{T}_Q$, is of value to inference on hydrologic stochastic processes as discerned from limited data and of value to the next important step of invoking Bayesian decision theory for evaluating design decisions and for judging if better designs are possible by waiting for additional data.

It would be desirable to express the moments ($E[\hat{T}_Q]$ and Var $\hat{T}_Q$) of the posterior pdf in terms of m, u, Q and C, but this is intractable. The next best approach for thinking about our results in simpler terms is to consider the mean and variance of $\underset{\sim}{Q} = \underset{\sim}{C}\underset{\sim}{P}$:

$$E[\underset{\sim}{Q}] = E[\underset{\sim}{C}]\ C[\underset{\sim}{P}] \tag{12}$$

$$\text{Var } \underset{\sim}{Q} = E^2[\underset{\sim}{C}] \text{ Var } \underset{\sim}{P} + E^2[\underset{\sim}{P}] \text{ Var } \underset{\sim}{C} + (\text{Var } \underset{\sim}{C})\ (\text{Var } \underset{\sim}{P}) \tag{13}$$

as given by Benjamin and Cornell (1970, p. 169). When C is not random, these equations become $E[\underset{\sim}{Q}] = CE[\underset{\sim}{P}]$ and Var $\underset{\sim}{Q} = C^2$ Var $\underset{\sim}{P}$. The variance of Q (and thus its frequency of exceedance and its return period) is dramatically affected by randomization of C. It is common in hydrologic design to choose a "frequency factor" z (or standardized variate) in the relation $Q = E[\underset{\sim}{Q}] + z\ (\text{Var } \underset{\sim}{Q})^{1/2}$. To contrast properly this classical approach to finding a design flow Q with the method outlined in this paper would require a full-fledged decision theoretic analysis for a specific design problem. The evaluation would have to be repeated for each design use of the posterior pdf. Much work remains to be done in this direction.

4.2 Relationship of results to Bayesian decision theory

Let the loss function for the design of a flood protection structure, say a dike, be $L(h,\underset{\sim}{T})$ where h is the height of the dike and $\underset{\sim}{T}$ is a design return period such as T_Q or an exceedance probability $(T_Q)^{-1}$. The result of our investigation was to determine the posterior pdf $f_{\underset{\sim}{T}}(t)$ as given in Figure 2. Thus, we are now able to calculate Bayes risk, which corresponds to the optimum design h*

$$BR(h^*) = \min_h \int_0^{+\infty} L(h,t)\ f_{\underset{\sim}{T}}(t)dt \tag{14}$$

We can also calculate the worth of sample information to sharpen the estimate of T_Q (Davis et al. 1975) for each intended use of the data. Such studies are left for the sequel. It is very important to emphasize that the worth of data discerned by this methodology is based on the economic loss function associated with a particular design use of the data; the results are not in terms of the variance of the return estimate (return period in this case).

5.0 Conclusions

It is important to keep in mind when judging the results of the research reported here that we are dealing with maximum flow volumes generated by a sequence of thunderstorms during a season. Additional work is necessary to extend the

approach to other runoff-producing precipitation events (including snow) during the year. The use of the Gumbel distribution in this paper goes beyond its classical use for the instantaneous rainfall and flood maxima during the year.

We thus have found the following points in our theoretical and simulation analysis:

(a) The approach enables us to design structures, relying only on rainfall data, on watersheds with ungaged streams by taking into account uncertainty of the site parameters.

(b) Using this approach we can tailor the design to a specific problem rather than use a pre-specified design flood, such as the magical 100-year flood.

(c) Simulation is an appropriate method for evaluating uncertainty in estimates of physically-meaningful parameters arising in the event-based approach.

(d) Return period varies with record length, rainfall, and watershed events, etc. We have given an event-based approach to evaluate this variation.

(e) The sensitivity analysis demonstrates the dramatic importance of uncertainty in the average amount of rainfall per event and the importance of considering variability in the rainfall and watershed parameter called C in this paper.

(f) The results, if encoded in the posterior pdf of the return period T_Q, allow the user to exercise inference or to find sensitivity of the analysis to design decisions in the face of inadequate data. Bayesian decision theory is the framework suggested for undertaking the decision analysis.

The results have implications for design of a variety of hydraulic structures in both urban and rural watersheds, in temperate and arid climates, and in regions of the world confronted with inadequate hydrologic data. In the face of changing watershed conditions, as reviewed by Fogel, et al. (1972), the approach offered in this paper permits exercise of judgment on the effects of lack of knowledge and of nonstationary meteorologic and hydrologic parameters such as m, u and C. In our judgment, classical empirical frequency methods do not provide such a clear basis for evaluation. Extension to nonlinear watershed models are possible as noted by Duckstein, et al. (1972) and Fogel, et al. (1972).

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Table 1: Sensitivity analysis* on return period moments as function of uncertain parameters for rural watershed with only 10 years of data.

Uncertain parameters	Var C	Moments of return period $\hat{T}_Q$			Reciprocal return period	
		Mean $\hat{T}_{\underset{\sim}{Q}}$ (years)	Var $\hat{T}_{\underset{\sim}{Q}}$	CV($\hat{T}_{\underset{\sim}{Q}}$)**	mean	variance
m & u	0	41.82	538.	.555	.0314	.000299
	.0005	39.00	442.	.539	.0332	.000318
	.005	26.33	153.	.470	.0453	.000328
	.05	6.30	2.35	.243	.1685	.001809
only m	.0005	35.36	8.30	.081	--	--
	.005	23.41	3.89	.084	--	--
	.05	6.20	.19	.070	--	--
only u	.0005	37.61	383.	.521	--	--
	.005	24.14	110.	.435	--	--
	.05	6.52	1.96	.215	--	--

* Conditions for the analysis: A = 0.4 inches, mean C = 0.3 for beta distribution, Q = 0.7 inches on the average; rainfall is distributed on basis of an exponential distribution for amounts above 0.3 inches with an average of 14.0 storms/season and an average of 0.39 inches/event.

** Coefficient of variation of $\hat{T}_{\underset{\sim}{Q}}$.

Table 2: Sensitivity analysis on return period moments as a function of rainfall P, length of record n and variance of C; both m and u are uncertain; watershed is rural; conditions are as noted in Table 1.

Average runoff volume Q for season (inches)	n (years of data)	Var $\underset{\sim}{C}$	Moments of return period $\hat{T}_{\underset{\sim}{Q}}$		
			Mean $\hat{T}_{\underset{\sim}{Q}}$ (years)	Var $\hat{T}_{\underset{\sim}{Q}}$	CV($\hat{T}_{\underset{\sim}{Q}}$)
0.5	10	.001	11.70	58.07	.650
		.005	6.97	14.90	.553
		.05	2.94	.27	.177
0.5	20	.005	6.25	4.38	.335
0.7	10	0	41.82	538	.555
		.0005	39.00	441	.539
		.005	26.33	153	.470
		.05	6.30	2.35	.243
0.7	20	.0005	37.44	245	.418
		.005	24.36	68	.339
		.05	6.41	1.09	.163
0.9	10	0	271	48,085	.809
		.005	103	3,602	.582
		.05	14.29	21.25	.323
0.9	20	.005	95.95	1,721	.432

Figure 1: The effect of the variance of C on the return period of runoff volume.

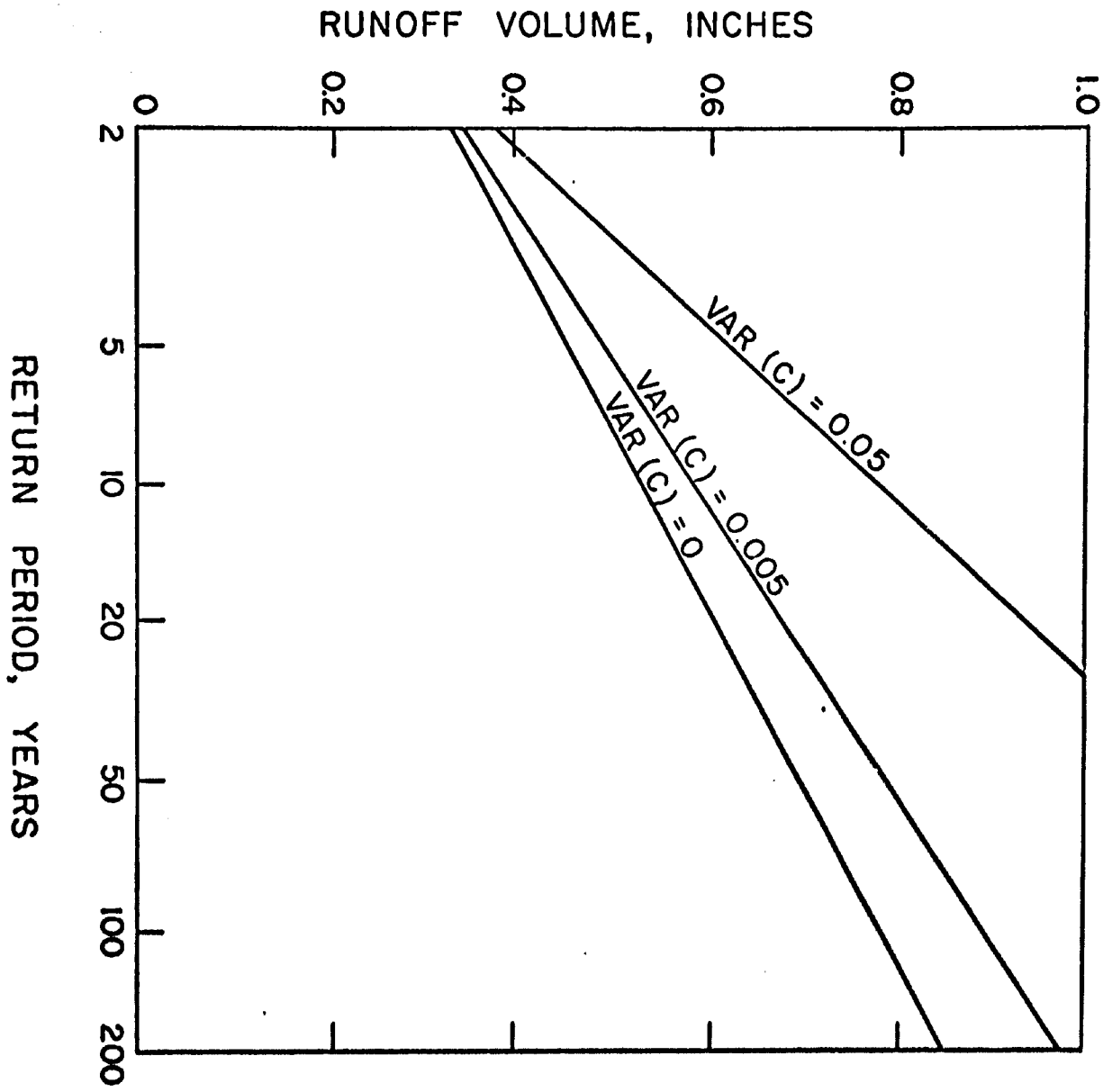
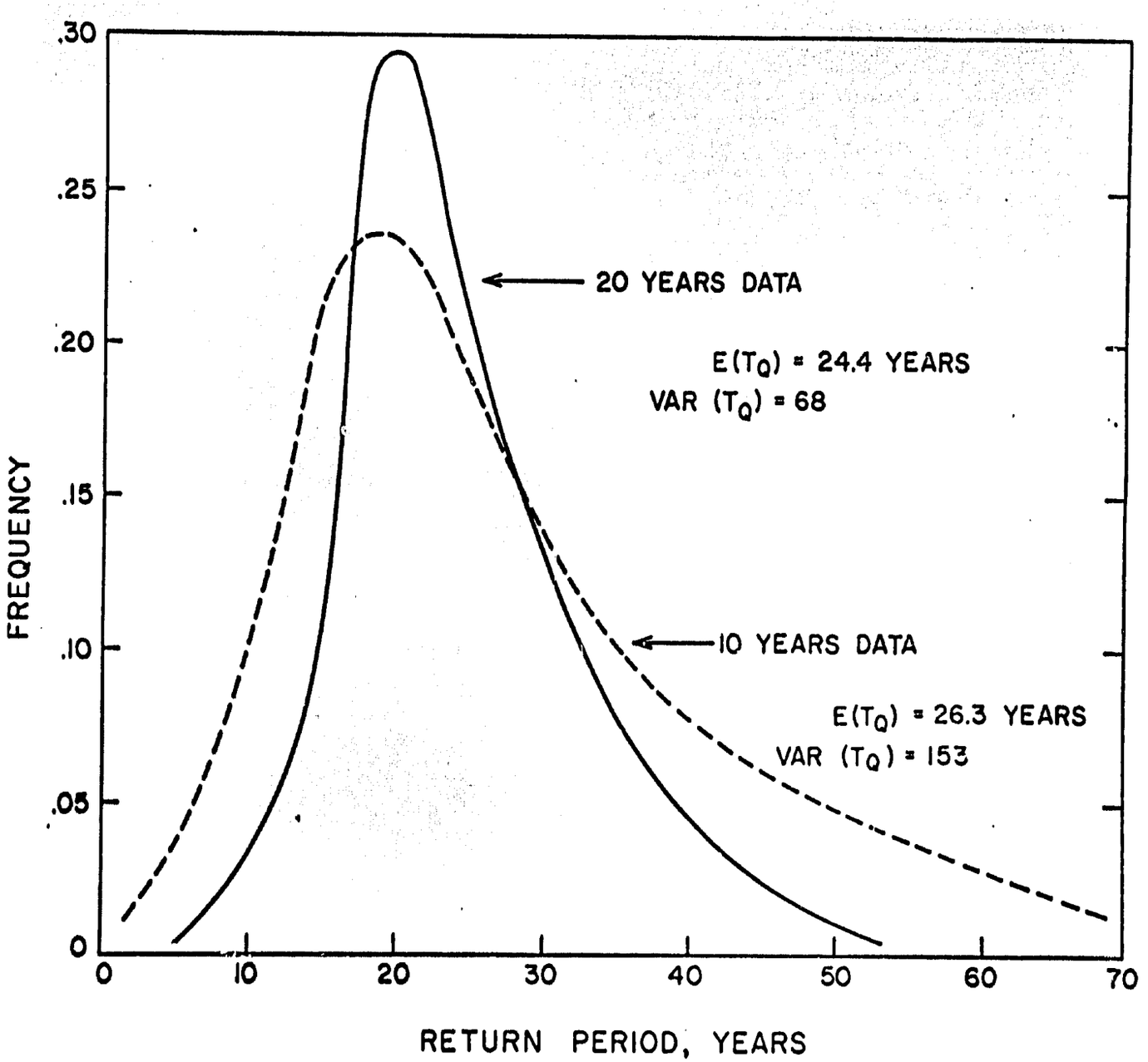
Figure 2: Posterior probability density function of return periods for 0.7-inch runoff as function of record length.