INTERACTIVE MULTI-OBJECTIVE DECISION MAKING UNDER UNCERTAINTY*

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ABSTRACT

The problems of sequential multiobjective problem solving under uncertainty are analyzed with the aid of an example on water quality management. The SEMOPS algorithm is constructed according to the Gestalt philosophy for subjectively viewing the entirety of a complex decision situation. The algorithm uses a nonlinear programming formulation to find goal values (six in the example: three dissolved oxygen levels in a stream, tax rates in two towns and profit level of a cannery) subsequent to a decision maker's specification of aspiration levels. The decision maker evaluates the current set of goal values and subjectively chooses adjustments in his aspiration levels for each goal. The system response to these levels is recomputed and evaluated. The algorithm terminates when the decision maker finds a satisfactum. The algorithm incorporates uncertainty by performing a sensitivity analysis on the final values of the decision variables. The sensitivity form of SEMOPS are simply an ad hoc concession to the substantial difficulties of incorporating Bayesian decision theory into the approach. These difficulties include the appropriateness of the axioms of utility theory for the decision problem outlined in this paper, the acquisition of prior distributions and utility functions for each goal, and the issue of a priori versus a posteriori weighting of each goal. This paper argues for a posteriori or subjective weighting because of the substantial measurement problems in eliciting prior probabilities, utility functions and weights.

1.0 Introduction

A method is proposed to combine uncertainty and multiple objective issues in decision making. We use the methodology to emphasize some of the problems in applying decision theory to natural resource problems.

It has been recognized in recent years that the standard one-period operations research or optimization models are inadequate for two reasons: (a) use of one-dimensional objective function; and (b) difficulties of inserting uncertainties in either the objective function (cost coefficients) or constraints (technological parameters as in resource availability). Recent research has sought to resolve this problem in various but separate ways (Cochrane and Zeleny, 1973; Wilcox, 1972). For example,

(a) Multidimensional decision problems have been modeled by using additivity axioms, dominance relations stemming from partial ordering of objectives (including standardized cost-effectiveness methodology (Kazanowski, 1968; Ko and Duckstein, 1972; Chaemsaithong, et al. 1973)), and interactive sequential decision-making algorithms (Monarchi, et al. 1973) that lead to complete ordering a posteriori.

(b) Stochastic elements have been introduced into mathematical programing (Bogardi, et al. 1973) but are far from describing the spectrum of uncertainties (in models, goals, data, forecasts). The issues in the latter have been reviewed by Kisiel and Duckstein (1972).

(c) Bayesian decision theory (BDT) has been used to imbed uncertainty into an objective function but there is difficulty in applying it to multiobjective situations (Davis, et al. 1972).

Specifically, we are considering the problems of introducing Bayesian decision theory into SEMOPS (a sequential multiple objective problem solving algorithm) whose potential usefulness has been demonstrated for a deterministic case study on investments in water quality control (Monarchi, et al. 1973). In the next section we outline the philosophy of decision making underlying SEMOPS. The remainder of the paper is organized as follows: review of the mechanics of SEMOPS and its amalgamation with Bayesian decision theory, incorporation of uncertainty into SEMOPS, an example using the Bow River Valley to demonstrate water quality control under uncertainty, and finally discussion of the problems in these methodologies, in particular as they pertain to natural resource problems.

1.1 Philosophy of decision making in SEMOPS

The research described herein has its origins in the Gestalt point of view (Köhler, 1947) which postulates that the perceptions of the individual are the result of the "context" or environment in which the stimuli are perceived. We can illustrate this with two examples:

1. A melody is more than just an enumeration of the notes to be played; it is our interpretation of the relationship of the notes to each other. In particular, our interpretation is time dependent based upon our present "mood."

2. A quotation taken out of context may no longer have the same meaning because words are related to the entire set of words surrounding them. Here the environment is the meaning or sense conveyed by the entire set of words.

The Gestalt view of perception may provide a realistic description of the way in which the DM defines and perceives both a decision problem and possible alternative solutions to that problem. In addition, the Gestalt orientation implies that the DM's judgment or evaluation of the worth of a solution is situation dependent; and, in fact, it is dependent upon the set of known alternatives because that information becomes part of the environment in which the decision must be made.

The preference structure (value system) of the individual determines the worth of the various alternatives to him. This preference structure is assumed to be completely implicit and perhaps known only imperfectly even to the decision maker (DM) himself. Past preferences are expressed in the individual's choice pattern over time. From this we can infer his ordering of the alternatives at the time the choices were made. An ordering is termed "complete" if there are no incomparable alternatives; otherwise it is a partial ordering. We assume in our development that there will be no incomparable alternatives for the multiple objective problem (MOP) among the set of feasible alternatives generated by SEMOPS. Our rationale for this assumption is that within the structure of a given decision problem, the alternatives represent variations of degree rather than of substance. To clarify this, we would say that

(profits of 6%, particulate atmospheric pollutant level of 400 mg/ℓ)

(profits of 10%, particulate atmospheric pollutant level of 1000 mg/ℓ)

represent two alternatives which differ in degree, whereas

(profits of 6%, particulate atmospheric pollutant level of 400 mg/ℓ)

versus

(stay home from work, go fishing)

represent two alternatives which differ in substance and both are not likely to be from the same decision problem. A variation in degree is a marginal comparison and a variation in substance is a structural comparison. We feel that alternatives which differ only in degree will be comparable and that those which differ in substance may or may not be comparable.

The DM's preference system is dynamic and changes as a result of environmental inputs, i.e., the perceived impingement of the real world upon the individual. At the same time, changes in the preference system alter the perceptual biases of the individual. These biases are responsible for the interpretation of the information from the environment so that the system feeds back upon itself. Although the structure is implicit and cannot be expressed functionally, the term "preference function" will be used to denote the psychological transformation of information from the environment into an assessment of the value of the alternatives to which that information relates. We note that this transformation also applies to the selective reception of the information itself. We assume that the preference function, like a mathematical function, is "single-valued" so that at any point in time the value of an alternative is uniquely determined. Equivalently, we can say that the individual cannot simultaneously have two evaluations of an alternative.

A goal or an objective is defined as a state of affairs which is desirable and stipulate that the attainment or non-attainment of the goal must be measurable. For example, minimizing the particulate pollutant level in a stream is not a valid goal in our framework; but achieving a pollutant level less that 100 mg/ℓ is measurable and valid. It will also be stipulated that the number of goals which can be handled concurrently by an individual is less than 10. This figure is based on research reported by Johnsen (1968) and a paper by Geoffrion, et al. (1971) and probably is overstated for most situations of stress. The amount of information relevant to each goal is influenced by the total number of goals. Johnsen (1968) suggests a limit of 50 on the product of the number of bits or units of information per goal and the number of goals. In our MOP it is assumed that each goal must contain only one explicit bit of information. There is only one real number explicitly associated with attainment of each goal in the problem, and this defines a scalar MOP. The DM may, of course, be subjectively adding other bits of information from the environment as a whole but our algorithm is currently capable of dealing with only scalar MOPs.

The variables that can be controlled in order to attain goals are termed decision variables. A set of values for these variables is termed a policy vector, or simply a policy. These values comprise the domain of a set of functions which we term criterion functions (CFs). The CFs have as their range numeric values which can be used to determine goal attainment or non-attainment. The CFs are the familiar objective functions from classical optimization theory and there is one criterion function corresponding to each goal (this a result of a scalar MOP). The CFs play the role of predicting equations and allow us to forecast output for some given input.

For example, if production of at least 1000 widgets/day was a goal and if production were related to two decision variables, man-hours and raw materials, then a potential CF might be

$$\# \text{ units} = (\text{man-hours}) \times (\text{raw material}) \times (\text{dimensionality constant}).$$

Observe that we could simply count the number of units produced to determine if the goal had been met, but then we would have no predicting equation for alternate policy vectors. We note also that the choice of an appropriate CF is dependent upon the actual decision variables present in the problem.

The range of the CF is divided into "acceptable" and "unacceptable" regions by a concept termed "aspiration level," AL. The AL for a goal is simply the amount or degree of goal attainment that the individual actively seeks to attain. An AL is a fluid entity; it varies in a complex manner according to the past pattern of successes and failures that the individual has experienced in striving to attain the goal at each then current AL. It is also dependent upon the pattern of successes and failures with respect to the other objectives in the MOP. So we envision a highly interwoven psychological system in which aspirations are immersed in the individual's preference structure and in which the entire system changes as a result of experience. Uncertainty in state of nature and response of the system to a given input is imbedded in this process.

We distinguish a goal level (GL) from an AL by defining a GL as a requirement imposed externally on the decision maker DM which he may or may not aspire to attain. For example, Federal regulations may require a dissolved oxygen (DO) level in rivers of at least 5 mg/ℓ for water-based sports, but the administrators responsible for waste regulation may have a different aspiration.

Finally, we would like to distinguish between "optimal" and "satisfactory" choices. In the usual optimization sense, "optimal" means "best." However, there is no evidence that there is a psychological optimum for MOPs because of the dynamic nature of the preference structure. The only MOP we can conceive of having an optimum is one in which none of the goals compete for resources so that in effect the attainment of each goal is completely independent of the attainment of other goals. Additionally, a global optimum would imply knowledge of all possible alternatives and would require some assumptions concerning the convexity of the preference space and the preference function.

It is our contention that the DM "satisfices" in a particular decision situation rather than optimizes. "Satisfices" is used in the sense of Simon (1957) to denote the selection of any acceptable alternative, namely, a satisfactum. Consequently there may be many solutions to a MOP because there may be many satisfactory alternatives. In fact, these alternatives may represent incompatible but equally tenable preference structures for the individual (Shepard, 1964). We also note that the satisfactory alternatives need not be adjacent in the sense that the set of values of the decision variables which produce satisfactory results is continuous (although the decision variables themselves may be). We can infer then that the final determination of a satisfactory choice is dependent upon the particular set of alternatives considered -- there may be no unique satisfactum nor is there necessarily a unique search path to any satisfactum.

Our choice of an interactive technique implicitly postulates a serial process for multiple objective decision making. Koopmans (1964) states that "...almost all choices in real life are sequential, 'piecemeal,' choices between alternative ways of narrowing down the presently existing opportunity rather than 'once-and-for-all' choices between specific programs visualized in full detail." Simon's (1957) concept of "bounded rationale" explains that the constraints of memory and evaluative capacity limit our ability to consider numerous aspects of a situation. Shepard (1964) feels "...that the relative weights to be assigned to the component attributes (of multi-attributed alternatives) are not always determinate and may, in fact, depend on the adoption of one of several incompatible but equally tenable systems of subjective goals." He cites other research which indicates that people tend to view multiattributed alternatives as either "good" or "bad" rather than explicitly analyzing each part of the alternative. They are unable to combine numerous subjective weights at one time although their impression may be that they have done so.

It appears that a serial algorithm would not conflict with the "normal" decision making process. In fact, such a serial technique which involves the DM actively will help him generate a complete ranking of his goals through the information that is created in the form of feasible alternatives.

In summary, the key to multiobjective decision making seems to be both the amount of information and its timing. Too much causes confusion; too little results in indecision. We need information about our more important goals first. As the solution (to the decision problem) becomes more definite, we can assimilate additional information about the less important aspects of our alternatives.

We next summarize the algorithm, keeping in mind that we are, for the time being, dealing with deterministic decision making.

2.0 The SEMOPS Algorithm

SEMOPS, an interactive programming technique, dynamically invites the decision maker (DM) to evaluate the consequences of his earlier judgments on each objective. The procedure endeavors to draw on the strengths of the computer (to perform massive repetitive computations) and the strengths of the human mind (to evaluate, weigh and suggest new alternatives).

In the decision problem we have T goals and N continuous decision variables, $\underset{\sim}{x} = (x_1, \ldots, x_N)$. Allocation of each variable (oxygen resources, water storage, or lake level) is subject to L equality constraints, $\underset{\sim}{h}$; M inequality constraints, $\underset{\sim}{g}$; and N bounds on the decision variables, namely, a lower bound b_L and an upper bound b_U. This constraint set is written as vector sets:

$$\begin{aligned} \underset{\sim}{h} &= \underset{\sim}{H}(\underset{\sim}{x}) = 0 \\ \underset{\sim}{g} &= \underset{\sim}{G}(\underset{\sim}{x}) \geq 0 \\ 0 &\leq \underset{\sim}{b}_L \leq \underset{\sim}{x} \leq \underset{\sim}{b}_U \leq \infty \end{aligned} \qquad (1)$$

With each goal we identify a criterion function for predicting goal attainment with respect to the constraint set. In the set of T criterion functions (each of which is the classical objective function)

$$\underset{\sim}{z} = \underset{\sim}{Z}(\underset{\sim}{x}) \qquad (2)$$

the range of the $t^{\underline{th}}$ element of the vector $\underset{\sim}{z}$ is denoted $\Gamma(z_t)$. For the optimization routine associated with each criterion function and the interaction of the T goals, SEMOPS specifies that (a) $\underset{\sim}{x}$ is continuous and (b) $\underset{\sim}{h}$, $\underset{\sim}{g}$, and $\underset{\sim}{z}$ are all at least first order differentiable. Thus, both (1) and (2) may be non-linear.

To get interaction of goals we define goal levels (GL) as conditions imposed on the DM by external forces (natural or human, pre-existing air quality and building standards, property rights and other legal controls) and define aspiration levels (AL) as attainment levels personally sought by the DM. Frequently, the GLs and ALs are in conflict. Writing these two sets as $GL = (GL_1,\ldots,GL_T)$ and $AL = (AL_1,\ldots,AL_T)$, from the conventional widsom we recognize at least the following five types of constraints:

a) "at most" $z \leq AL$: AL may be maximum number of hikers desired by recreational interests or maximum river salinity concentration acceptable to irrigators (Colorado River water to Mexican farmers), or maximum nutrient concentration in lake influents (to control eutrophication),

b) "at least" $z \geq AL$: AL may be the river level desired by navigational interests,

c) "equals" $z = AL$: AL may be the firm power contracted to industry,

d) "within an interval" $AL_1 \leq z \leq AL_2$: bounds on lake levels and water temperature for recreation and/or fisheries,

e) "outside an interval" $z \leq AL_1$ and $z \geq AL_2$: less common; AL_1 and AL_2 may be the frequency limits within which a structure, such as a suspended bridge, may enter into resonance.

For each of the T goals, a "dimensionless indicator attainment," $d = DA(x)$ and $d > 0$, is defined by an appropriate transformation of any of the above constraint types. In each case, the inequality $d < 1$ implies that the goal is satisfied. The transformation tactic permits a "pooling" of the information on each goal as noted next. The "pooling" is achieved by a surrogate objective function, $s = \sum_{t \in T'} d_t$, defined on a subset T' of the set of T goals.

The function s is optimized at each iteration or cycle only to generate whatever information arises from the DM's prespecified GLs and ALs. The algorithm sequentially alternates between the DM's inputs and responses to the information generated at each cycle. Such information may be in terms of the effects of achieving or not achieving one goal on the aspirations of other goals. If such effects are intuitive or self-evident to the DM, then clearly the computer algorithm is not needed. But for most problems, the consequences of a DM's choice in a multivariate environment are non-intuitive and so the computer can be an invaluable aid.

The surrogate character of each cycle in SEMOPS recognizes our inability to define objectively the true preference function (of one goal versus others) of the DM. The value of d_t in s at each cycle reflects whether or not the $t^{\underline{th}}$ goal has been satisfied; note that the T' set is not the same in number and composition from cycle to cycle because of the DM's prerogatives in constraining certain goals at the next iteration and in inserting new information. Comparability of d_t values at each cycle is not possible because of the *nonlinear* transformations. The s is minimized only to help the DM reach a satisfactum or to help him decide that more information or research is necessary. Of course, the function s would be a "standard" objective function if the DM knew perfectly his preference or weighting function. The construction of DA and s has been completely arbitrary and is subject only to the directional property of d as discussed above. This property is used by the DM to decide on the set (T-T'), that is, those goals to be added to the constraint set because of their non-inclusion in s. The presence of a goal in s signifies that the DM is aware that he may fail to achieve the *desired* AL. But when a goal is entered as a constraint the DM *insists* that the AL be achieved. The distinction between *desiring* and *insisting* is central to SEMOPS.

Overall, SEMOPS is a three-step algorithm involving setup, iteration, and termination. Setup includes data acquisition, determination of state transition functions (on stream and lake flows, volumes, chemistry, biology), and transformation of the original problem into the DA format. Iteration is the truly interactive segment wherein one cycles between an optimization phase and an evaluation phase until a satisfactum is reached (if possible). The last is the termination step.

In the next section, we review the results of applying the algorithm to a deterministic formulation of a water quality control problem. This sets the stage for showing the problems of introducing uncertainty into the same example.

2.1 An Application of SEMOPS to a Deterministic Water Quality Control Problem

SEMOPS was illustrated by Monarchi (1972, Chapter 6) using a modified version of a realistic example developed by Dorfman and Jacoby (1969). The Bow River Valley, an artificial river basin, has a pollution problem stemming from three sources: (1) the Pierce-Hall Cannery, upstream from Bowville; (2) municipal effluent at Bowville; (3) municipal effluent at Plympton, downstream from Bowville. Between the cities lies Robin State Park;

the state boundary line is downstream from Plympton. Water quality is represented by the dissolved oxygen concentration DO, and effluent is characterized by its biochemical oxygen demand BOD. The Streeter-Phelps (1925) equation describes the dynamics of DO, given the BOD discharge and the stream characteristics. This ordinary differential equation is simply an accounting of the oxygen resources in a stream, independent of spatial variability and nonlinear effects.

The DM seeks to increase the collective utility of Bow Valley, by looking at six goals and defining six goal levels GL to evaluate the worth of any decision, namely: the DO levels at Bowville (AL_1 = 6 mg/ℓ), Robin State Park (AL_2 = 6 mg/ℓ), Plympton (AL_3 = 6 mg/ℓ), the percent return on investment at the Pierce-Hall Cannery ($AL_4 \geq 6.5\%$), the additional tax rate at Bowville ($AL_5 \leq 0.15\%$), and at Plympton ($AL_6 \leq 0.15\%$) that would be necessary to reach the desired DO level.

The control or action variable is the proportionate reduction of BOD $\underset{\sim}{x}$. Constraints are to maintain a certain level of DO at the state line and to bound $\underset{\sim}{x}$: $0.3 \leq \underset{\sim}{x} \leq 1$. Without entering into details of the rationale behind each cycle or iteration, the following results are obtained (Monarchi, 1972; Monarchi, et al. 1973):

<u>Initial Values</u>: $\underset{\sim}{x}_0 = 0.3$, and $\underset{\sim}{GL} = \underset{\sim}{AL}_0 = (6,6,6,6.5,.15,.15)$

<u>First Cycle</u>: $\underset{\sim}{AL}_1 = (5.96,4.07,5.77,6.42,.071,.159)$

If DM fixes Goal 6 at .150, then he obtains the vector

$$(6.14,5.35,6.17,5.50,.252,.150).$$

Second Cycle: DM fixes Goal 6 at .155; he thus obtains

(6.03,4.60,5.94,6.15,.125,.155).

Third Cycle: Goal 6 has been entered as a constraint with an AL equal to .155. DM, after an interactive phase, decides to change Goal 2 from 6 to 5 mg/ℓ. The resulting AL vector is

(6.10,5.00,6.07,5.76,.180,.155).

Fourth Cycle: In addition to the above constraints, Goal 4, which has already been reduced from 6.5 to 5.76, is further reduced to an AL of 5. The result is

(6.06,5.00,6.07,5.00,.187,.155).

Fifth Cycle: AL_5, the attainment level of goal 5, is raised to 0.190. The result is (6.0,5.0,6.0,0.190,0.155), which we assume to be a satisfactum for the DM. DM's policy decision $\underset{\sim}{x}$ is to impose waste reduction requirements of 88% on the cannery, 87% on Bowville and 82% on Plympton. We show the effects of uncertainty on these results.

But first we review the potential sources of uncertainty in the water quality control problem and then the limited role of Bayesian decision theory in this class of resource problems.

3.0 Uncertainties in the Multiple Objective Problem

The uncertainties in resource problems can be both technological and strategic (Kisiel and Duckstein, 1972). Strategic uncertainties include primarily our inability to forecast future social goals and socio-economic states of the world. Technological uncertainties include our ignorance about physical, chemical and biological processes in the environment. These include uncertainty about appropriate environmental models, about parameter estimates (given a true model), and about future functions of uncontrolled inputs into environmental models (given a true model and perfect parameter estimates). The relative importance of these two classes of uncertainties will vary with the problem.

Within the framework of the Bow River problem, the strategic uncertainties may exist in regional goals (future growth rates and concern with other ecologic parameters like air quality). Furthermore, the problem, as formulated, considers only a single attribute of water quality, namely, DO, but what of stream temperature, sediment, and so on? The DM's implied preferences may change with time or with better forecasts of future social and technologic conditions.

The SEMOPS algorithm assumes the adequacy of the Streeter-Phelps model of oxygen dynamics in a stream. Of course, more realistic models can be incorporated and the effects on the policy vector evaluated.

Given the correctness of the Streeter-Phelps model, our earlier use of SEMOPS assumed that streamflow, stream transfer coefficients, and stream reaeration and deaeration rates were constant. It is customary to determine waste treatment plant capacity $\underset{\sim}{x}$ by assuming a low flow that persists for seven days or more and that has an average return period of ten years. The low flow value is typically estimated from relatively short hydrologic records; this fact suggests the possible applicability of Bayesian decision theory (BDT) when a least cost design of waste treatment plants is the goal. In addition to uncertainty in the design streamflow, a major difficulty arises with estimates of stream reaeration coefficients; investments in plant design are rather sensitive to slight changes in such estimates (Yu, 1972).

Implementation of the Streeter-Phelps equation also depends on a forecast of future biochemical oxygen demands (BOD), exerted by wastes to be treated both in a waste treatment plant and in the river. That forecast in turn depends on forecasts of population and future per capita strength of the waste.

On the technological side there exists some possibility of formally analyzing the uncertainty in design flow, reaeration rate, and BOD load. The estimated DO in the stream is a function of efficiency of waste plant operation, flow rate, reaeration rate, future population growth, and the Streeter-Phelps model. Prior experience with each type of waste treatment plant can be used to encode the uncertainty in plant efficiency. Historical records of streamflow can be used as a basis for fitting a probability distribution function (pdf) like the lognormal or Gumbel's smallest value model to encode this uncertainty; uncertainty in the parameter of these distributions could be encoded in a prior pdf and BDT applied. Uncertainty in reaeration rates can only at present be encoded with the aid of field measurements and regression equations; BDT would be applied once the likelihood on the residuals was established (Metler et al. 1973). As for future population growth and appropriate model, their uncertainties are not as readily encoded, except perhaps in a subjective way as degrees of belief (say, through expert opinion).

With the above outline of uncertainties, the question naturally arises as to how best to incorporate these into SEMOPS. The alternatives include (a) to include singly each uncertainty in SEMOPS and then compare the policy vectors $\underset{\sim}{x}$ and (b) to introduce the uncertainty in $\underset{\sim}{x}$ directly by an additive error term with an assumed probability density function (pdf). The former approach assumes that decomposition of uncertainties is justified and that existing methodology can cope with each type of technological uncertainty; this is not so, however. The latter approach, while not computationally simple, skirts the explicitness of (a) by simply encoding all uncertainty as an uncertainty ε in the required degree of waste treatment x. Such uncertainty is propagated into the values of each of the goal values GL. In the next section we illustrate the use of SEMOPS with $\pm\varepsilon$ added to $\underset{\sim}{x}$.

4.0 Sensitivity Analysis in Bow River Valley Problem

In the previous section we suggested that there is no completely general way for incorporating Bayesian decision theory into our SEMOPS algorithm. The general nonlinearity of the goal functions also compounds the difficulties in such an effort. It should be emphasized that it is more natural to proceed with uncertainty analysis on specific components of $\underset{\sim}{GL}$ and then to choose $\underset{\sim}{x}$ on the basis of risk criteria arising in such analysis. As noted previously an ad hoc alternative is to add a random error term to the decision variables $\underset{\sim}{x}$

$\tilde{x}_i = x_i + \tilde{\varepsilon}_i$, $i = 1,2,3$ in the Bow River where the superwiggle indicates random variate. Then the $j^{\underline{th}}$ goal function becomes

$$\tilde{z}_j = f(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3), \quad j = 1, \ldots, 6 \text{ in the Bow River Valley}$$

If a pdf $\phi_1(\varepsilon_i)$, not necessarily symmetric between $\pm\varepsilon_i$, were assigned to ε_i, then, in principle, computer simulation would give a pdf $\phi_2(z_j)$ and a decision analysis might be pursued in terms of loss functions and risk propensity (aversion or proneness). In fact, the decision maker could use $\phi_2(z_j)$ at each cycle of SEMOPS to adjust his aspiration levels even though the computer program will still solve the problem deterministically. However, we note that computer solutions and analytical expressions for $\phi_2(z_j)$, or even the variance Var $\tilde{z}_j$, are hardly feasible except possibly through linearization of the nonlinear goal functions. Pertinent here is the fact that the expected value of the solution of a nonlinear function is not equal to the solution of a nonlinear function composed of expected values.

Given the above situation we settled on an unsophisticated way of letting the DM introduce his own "priors" with respect to the upper and lower bounds of the uncertainties in the $\underset{\sim}{x}$ values. No pdf is imposed on values within those bounds. The resulting flow chart is given in Figure 1. After the results of each cycle of SEMOPS are printed out, the program asks the DM if he wants to explore the sensitivity of the results. At this time the DM can input a vector, call it $\underset{\sim}{Dx}$, and the program will compute the maximum and minimum values of each goal for all combinations of plus Dx and minus Dx around the "nominal" x values resulting from the solution of the deterministic problem. The idea is to have the

DM put in several $\underset{\sim}{Dx}$ corresponding to different "confidence" levels on the x_i in order to observe the variation in the six goal values z_j.

The results of such computer studies for two $\underset{\sim}{Dx}$ are shown in Table 1. Also shown therein are the results of the "certainty" case reviewed in Section 2.1 of this paper. The 1% and 3% figures are used for illustrative purposes; actual field conditions may not permit such sensitive design and subsequent operation of waste treatment plants.

In the sensitivity version of SEMOPS, DM has the option to vary the uncertainty bounds $\underset{\sim}{Dx}$ according to his sense of what is uncertain in the inputs to the algorithm. For a 3% increase in $\underset{\sim}{x}$, the required efficiency of waste treatment plant, we find slightly higher values of DO at each of the three river stations, a higher return on investment at the cannery, and higher tax rates at both Bowville and Plympton, all of this in relation to nominal values. Whereas, the reverse consequences are obtained for a 3% decrease in $\underset{\sim}{x}$. How is DM to respond to these results?

DM may give two interpretations to each ε_i. In the first, a deterministic design context, DM simply wants information on the effects of perturbations in $\underset{\sim}{x}$ on the goal levels z_j. Supposedly, no uncertainty enters his mind during evaluation of that information. In the second interpretation, the one intended in this paper, DM is aware of the many sources of uncertainty in the design problem as noted in Section 3.0. In contrast to the first interpretation, if DM is not sure why he chose a particular $\underset{\sim}{Dx}$, he now has a much more difficult task of evaluation. Should he

Table 1: Results of sensitivity analysis on last cycle of SEMOPS with uncertainty in required waste treatment plant efficiency. Nominal x_i, i = 1,2,3, values are respectively 0.8771, 0.8688 and 0.8182.

Case	Goal values in the principal problem					
	GL_1	GL_2	GL_3	GL_4	GL_5	GL_6
$\varepsilon_i = 0$ for $\tilde{x}_i = x_i + \varepsilon_i$	6.0	5.0	6.0	6.0	1.90	1.55
$\varepsilon_i = \pm 0.01$ Maximum Value:	6.084	5.108	6.099	6.114	2.057	1.659
Minimum Value:	6.039	4.922	6.043	5.871	1.760	1.450
$\varepsilon_i = \pm 0.03$ Maximum Value:	6.129	5.313	6.060	6.037	2.437	1.911
Minimum Value:	5.993	4.749	5.990	5.558	1.521	1.275
Dimensional Units	Dissolved oxygen, milligrams/liter	→	→	% return on investment	Dollar change in tax rate per $1000 assesed valuation	→

attribute ε_i to one source of uncertainty at a time, such as to operational efficiency of waste treatment plants or to stream reaeration coefficients, or should he attribute ε_i to all sources of uncertainty simultaneously? In the former case, DM is presumed to have prior knowledge about the most important source of uncertainty; this is a large assumption. In the latter case, DM must decide whether or not his decisions require a subjective weighting of the various sources of uncertainty. If only the total uncertainty is important to him, he has simplified the problem and is then faced with questions about costs of over- and under-investment, about probabilities to be associated with values of $\underset{\sim}{x}$ in the interval $(-Dx, +Dx)$, and about risk propensity (aversion or proneness).

In the first interpretation of ε_i, for the plus 3% case, how should DM trade-off the higher stream quality and percent return to the cannery against the higher tax rates in the two communities? and vice versa for the minus 3% case? In the design context the answer (or decision) depends on the utilities associated with each goal value (assuming perfect prediction of each value). If DM decides to go along with the plus 3% case, the community assigns greater utility to the additional stream quality and profits to the cannery than it does to the additional expenditure of real tax dollars. Decision theory under uncertainty should be applicable in these situations as an aid to the DM.

In the second interpretation the decision depends on DM's degree of belief in the possible occurrence of different values of the uncertainty and on his utility or loss function for the entire range of uncertainties in the interval (-Dx,+Dx). The nominal values of x_i are not true values for if they were true then the ε_i would lack meaning. Thus, a probability distribution $\phi_1(x_i) = \phi_1(\varepsilon_i)$ over the interval (-Dx, +Dx) would encode uncertainty in the true values x_i. However, because $\phi_1(x_i)$ could only be obtained subjectively and because its use to find $\phi_2(z_j)$ is fraught with difficulty as noted earlier, we are not able to consider explicitly the use of Bayes and minimax decision criteria for each of the six goal levels z_j.

But let us pretend that we could. Then we might be inclined to use the expected value viewpoint implied in the use of a minimum Bayes risk for the DO levels associated with GL_1, GL_2 and GL_3, but inclined to use the minimax rule for GL_4 (% profit to the cannery) and for GL_5 and GL_6 (change in tax rates at Bowville and Plympton). This subjective choice is suggested only to emphasize that different decision rules can be chosen for each z_j. To get at an evaluation of the set of z_j would require a multidimensional utility function. It is not at all clear at this point whether DM could specify such a function in a consistent way or whether DM may even agree with the idea. DM may simply prefer to rank the minimum Bayes risks for z_1, z_2, and z_3 and the minimax losses for z_4, z_5 and z_6; in the end with the aid of these and other criteria he makes a rather subjective choice of $\underset{\sim}{x}$.

The hypothesized reality of our problem formulation simply requires DM to choose in some way between the maximum and minimum values of GL_j in each sensitivity analysis. Some appreciation of subjective probabilities and utilities for each GL_j might allow DM to be more incisive in his evaluation. While the positive utilities for the maximum DO values may be high, their probabilities may be quite small; then the chance of over-investment would be large. On the other hand, to choose the set of minimum values as a basis for decision, DM must tradeoff the negative utility of lower DO values (if true) and lower profit to the cannery against the savings in costs of overdesign, and the reduced taxes in Bowville and Plympton; probabilities of each GL_j will strongly influence the judgment. Thus, in the sensitivity approach to SEMOPS, DM must subjectively combine the utility and probability for each GL_j and in turn decide subjectively over the set of six GL_j on the final set $\underset{\sim}{x}$ of decision variables. It is not clear that an informed DM could really do this; for example, he may subtly allow his subjective probabilities to be influenced by his utilities for each GL_j. At this juncture, we are confronted with the reality of irrationality of both decision making and utility theory. The axioms of utility theory are not necessarily compatible with the actual decision making.

In the next section we discuss the overall problem of using SEMOPS under uncertainty, including some preliminary thoughts on a complete decision system versus a practical decision system, the place of SEMOPS and BDT in the broader cost-effectiveness framework, the problem of a priori specification of preference functions or weights and their use in a normative or predictive sense, and the compatibility of SEMOPS under uncertainty with the axioms of utility theory.

5.0 Discussion

In a complete decision system, it would be ideally desirable to have an interactive program wherein the uncertainty associated with each goal could be attacked completely by Bayesian decision theory. Implementation would be a computer programming problem involving computation of expected opportunity loss, expected expected opportunity losses, expected value of sample information, etc. However, this does not imply that such a system would be best because the axioms of BDT may not hold across the set of goals. Practically speaking, the complete decision system would require prior distributions, likelihood functions on the data, and loss functions. These requirements impose severe measurement problems to acquire such information. We conclude then that a complete decision system is not achievable with the current state of decision theory and measurement techniques.

In a practical decision system, it may be possible to have a multi-objective Bayesian decision theoretic framework that is, however, not interactive and that is imbedded in a larger cost-effectiveness framework. The goals would be satisfied separately. Our experience with applying BDT to water resource design still suggests computational and conceptual difficulty in implementing it for the single objective case (Davis, et al. 1972; Duckstein et al. 1973) although the problems are gradually yielding. The value of BDT in small sample cases can be sharpened by contrasting it with non-Bayesian approaches (say, the factor of safety viewpoint) as has been done by Duckstein et al. (1973) in their evaluation of flood levee design. The results of BDT analysis for each goal in a non-interactive mode may be either in terms of the Bayes risk or minimax criterion for each goal. Such criteria are only a part of the vector of criteria used to evaluate alternative systems.

To undertake this evaluation we prefer the standardized cost-effectiveness (CE) procedure proposed by Kazanowski (1968, 1972) and adapted in our work (Chaemsaithong, et al. 1973; Ko and Duckstein, 1972):

a) Identify multiple goals to be achieved.

b) Give the requirements or specifications for meeting the goals.

c) Identify the criteria for evaluating the alternative systems to be used in trying to achieve goals.

d) Specify distinctly different alternative systems.

e) Choose a fixed cost or fixed effectiveness approach.

f) Evaluate the merits of each alternative system in terms of the criteria given in (c). BDT analysis enters this step where feasible.

g) Prepare an array of systems versus criteria. This includes ranking of the criteria, clustering of criteria to which DM is indifferent, and identifying criteria that DM finds incomparable.

h) Analyze the array by ranking, identifying dominated alternatives or criteria, or using lexicographic analysis (MacCrimmon, 1972; Roy, 1972). At this stage, no decision is made by the analyst; SEMOPS may be invoked for each of the alternative systems.

i) Perform sensitivity analysis on the goals, requirements, criteria, revision of alternative systems and choice of approach. This step may be undertaken with the aid of the decision-making group in order to facilitate implementation by them at a later date.

j) Document the rationale of the entire procedure.

Clearly, SEMOPS and BDT are only a part of the above broad structure called the CE methodology. SEMOPS in steps (g) and (h), through its interactive mode, helps to order criteria (in CE) or goal levels (in SEMOPS). BDT analysis, if feasible, would generate values of Bayes risk and expected opportunity loss for each system and for each goal. These criteria take their place along with other criteria like minimax, environmental quality, aesthetic, etc.

Our experience with CE suggests that it is extensively appreciated, notwithstanding its unstructured and non-formal orientation. In fact, the latter may be assets in broader and amorphous problems. We sense that SEMOPS under uncertainty may be appreciated by DM provided more justification is generated for its use. Usefulness of the combination of SEMOPS and BDT will have to be demonstrated because of data requirements, possible complexity of implementation, and axiomatic difficulties in a multiobjective environment.

The multidimensional preference function of DM <u>implicitly</u> arises in CE at step (h) and, of course, in SEMOPS. We cannot define a priori such functions but we can observe past behavior. But does this provide a valid basis for inference about future behavior? All weighting schemes based on past behavior and current interviews or questionnaires may be fine, but they need to be validated. Can such schemes stand up to the strong test of forecasting the evolution of individual, group and social value structures? This question does not seem answerable at present in the affirmative. We note from the literature that Keeney (1969) assumes in his normative multidimensional utility theory that preference functions are known beforehand, Dyer (1971) in his goal programming under uncertainty assumes prior weighting of goals, Drobny et al. (1971) choose to collapse the array (steps (g) and (h)) of the cost-effectiveness procedure into a single effectiveness index by assigning weights, and Major (1969) assigns weights to each of a set of goals for water resource planning. Assignment of weights may be meaningless unless an individual or a group has a <u>commitment</u> to assume responsibility for them. Even then, the weights or preferences may change because humans do change their attitudes and because we presumably learn from the consequences of past weights or choices. We note that SEMOPS allows for evolution of the value structure of the group or individual DM.

Central to the problem of decisions under uncertainty and to the problem of weighting are the axioms of utility theory:

a) Complete ordering of alternatives and criteria is possible.

b) Individuals or groups are transitive in their choices.

c) There exists a continuity and ordering of probabilities between two choices, that is, a convex combination of probabilities can be found between two choices.

d) There exists consistency in the choice of alternatives in relation to other irrelevant alternatives, that is, choice between two alternatives is independent of the surrounding environment. Problems of ecologic management cut to the heart of deficiencies in this axiom.

It is clear that we cannot have Bayes risk and interactive Bayes procedures without the axioms. In fact, we may need further axioms that might make the problem even more unrealistic. In SEMOPS it is conceivable that DM encounters incomparable choices, thus violating the first axiom (Roy, 1972), and even the remaining three axioms. Concerning the second axiom, Slovic et al. (1973) discuss the individual who is a money pump because of his intransitive choices. Concerning the fourth axiom, it is quite common to encounter external factors that suggest additional alternatives; independence is difficult to justify and is violated purposely in the Gestalt philosophy embodied in our SEMOPS algorithm. It should be evident that efforts to evaluate, in the sense of the behavioral sciences, the above four axioms require measurement techniques (in general, not adequate to meet our needs) to obtain the utility and probabilistic structures of individuals and groups.

In Section 4.0 we reviewed the possible incorporation of BDT into SEMOPS and suggested the need to separate utility functions and pdfs. As Ferrell (1972) notes, people do not necessarily give accurate prior pdfs because they tend not to believe the evidence before them. Slovic et al. (1973) suggest something similar in terms of human fallibility in probabilizing past observations of extreme natural hazards like floods and droughts. They note that humans do well with hydrologic events that have magnitudes in the vicinity of the mean value. Conrath (1973) notes that DMs are more comfortable with point estimates than with probability distributions. Thus, to implement BDT and its combination with SEMOPS is also an educational problem, not just a computational and axiomatic one. There are two problems. First, how best to teach people to believe in the evidence, in particular, probabilistic evidence before them? Second, how best to teach people to distinguish between utility and probability? An overall question arises: how best to develop human intuition about probability concepts and more generally the uncertainty issues existing in natural and socio-economic processes?

In summary, in this paper we have sought to develop the problems of incorporating uncertainty analysis, in particular Bayesian decision theory, into an algorithm on multi-objective problem solving. Our algorithm at present copes with a scalar MOP, that is, only one water quality parameter (DO) in the Bow River example. Our analysis requires quantification of the goal levels and requires that the goal function for each goal be at least first-order differentiable; this may not always be the case. It is hoped that our evaluation of the merits of SEMOPS and BDT serves to sharpen the judgment as to the best direction for the next increments of improvement in multiobjective problem solving under uncertainty.

6.0 Conclusions

From our analysis of the problems of implementing SEMOPS under uncertainty, we suggest the following conclusions on the premise that the algorithm begins to capture important realities of the actual decision process:

a) Complete integration of one-period SEMOPS and Bayesian decision theory is impossible because the axioms of utility theory are not completely compatible with the Gestalt viewpoint inherent in SEMOPS.

b) While complete incorporation of uncertainty into SEMOPS is not presently possible, the addition of a sensitivity analysis to SEMOPS does provide more guidance to the DM than that obtained without a sensitivity analysis.

c) SEMOPS requires no prior weighting of alternatives or of goal levels as do other schemes proposed for multi-objective problem solving under uncertainty. All weighting schemes to solve the multi-attribute problem must be validated as to their predictive abilities.

d) To resolve the problem of implementing and improving decision theories, more work is necessary on the elicitation of probability distributions from people, and in educating people to believe in the evidence before them and to distinguish between utilities and probabilities.

e) There is need to consider extension of SEMOPS under uncertainty to the multi-period design, planning and operation problems arising in natural resources.

f) We may need a system model (in the sense of Wymore (1972)) to study interaction between goals under uncertainty.

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Figure 1

Logic Flow of User Interaction with New SEMOPS Program
All input is from teletype by user decision maker (DM)

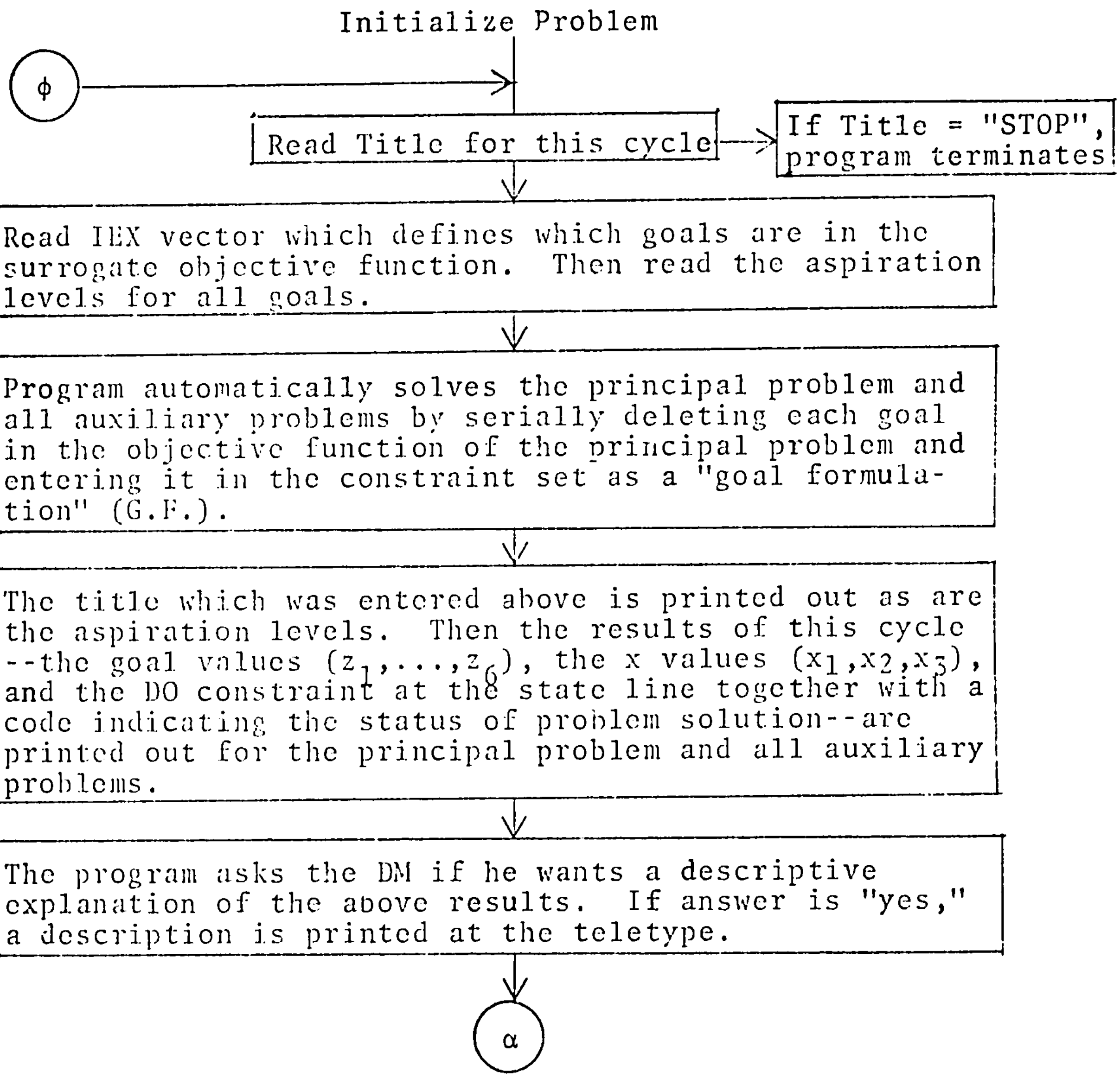

Figure 1 (continued)

α

Program asks the DM if he wants to explore the sensitivity of the results by "wiggling" the x values for each problem by some amount (supplied by the DM). The purpose of this part is to provide a "poor man's version" of a stochastic problem. The DM can supply "reasonable" amounts of variation for each x value which might correspond to subjective (Bayesian) estimates of variance. When the resulting variations are observed by the DM, he may wish to concentrate on the goals, by entering them as constraints, in a different order than he might otherwise have done from the "deterministic results" printed previously. This, presumably, would result from his desire to reduce the uncertainty on the goals which are most important to him. Even though all of the x's (x_1, x_2, x_3) may be varied by the same amount, the variations in the goal values will differ in magnitude because the goal functions are nonlinear.

If answer to question (1^{st} sentence above) is no, program goes on to next section.

A

β

Figure 1 (continued)

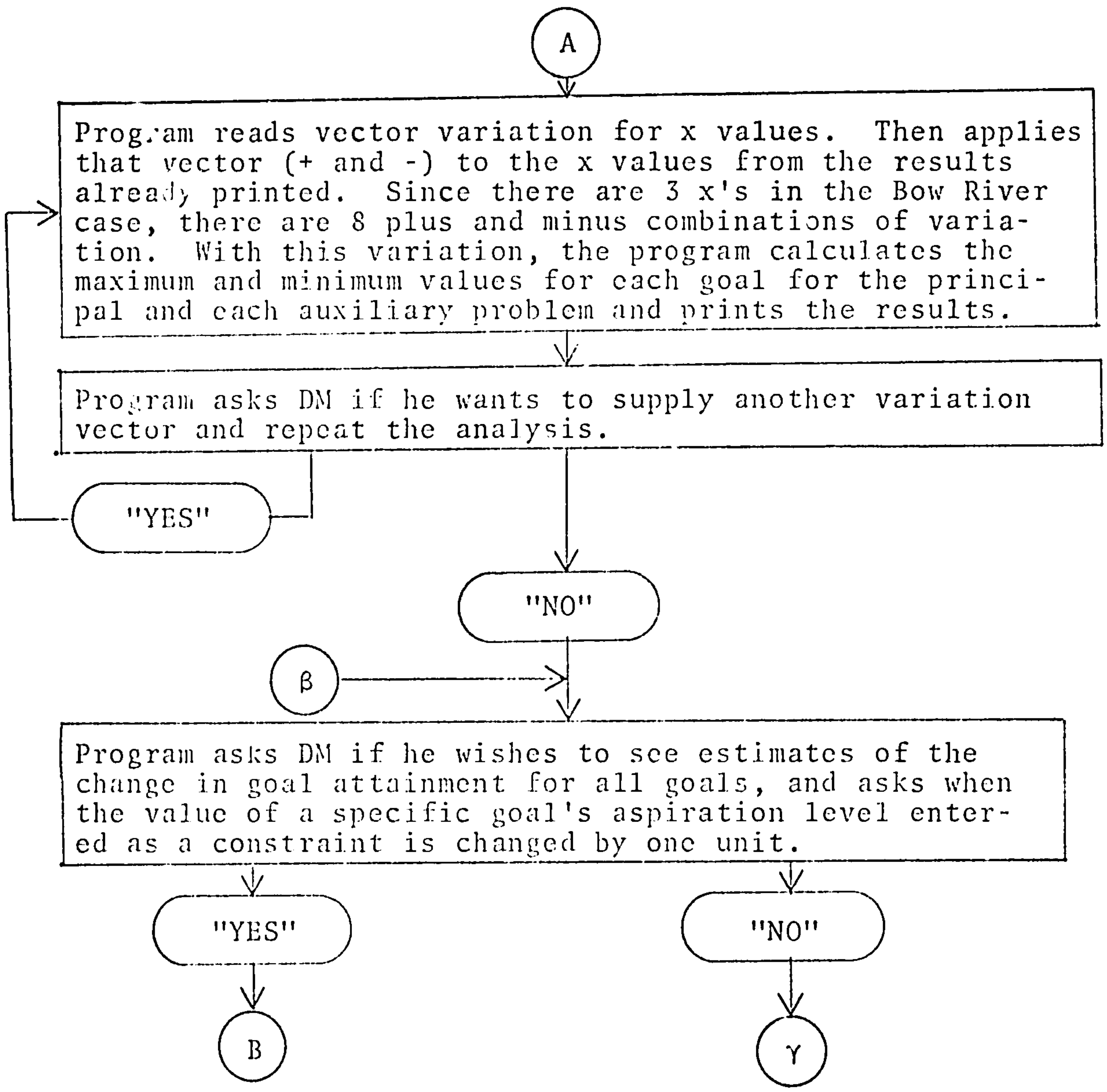

Figure 1 (continued)