A STOCHASTIC SNOW MODEL TO EVALUATE RESERVOIR OPERATION

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INTRODUCTION

A major portion of the annual flow of most streams in the western United States occurs during the spring and summer months as a result of melting snow that accumulates each winter on high mountain watersheds. Forecast of runoff from snowmelt can be classified into two general categories, water supply forecasting and rate-of-runoff forecasting (Gartska 1964).

Water supply forecasting is a complex technique that utilizes snow survey and other related information to predict seasonal streamflow, primarily, but also peak flow. These forecasts are made as much as several months in advance of the major runoff period while those concerned with floods from rapid snowmelt may involve only a few days prior notice.

The purpose of this paper is to present a methodology that will allow managers of water resources systems to exercise adaptive control procedures in regulating reservoir releases to minimize expected losses. Using an event-based precipitation model, a distribution function of water available for runoff is obtained. The amount of water available for runoff is the sum of the water equivalent of snow on the ground at the time the forecast is made, and the predicted amount of precipitation during the

forecast period. A melt coefficient, a random variable, is assumed to be a function of predicted temperature. The distribution function of snowmelt runoff is then taken to be the product of the two random variables involving the predicted precipitation and temperature. A one-period objective function is then defined to determine a reservoir release rate which would minimize expected losses.

PREDICTING SNOWMELT RUNOFF

Water supply forecasting

Seasonal water-yield forecasts have been developed to provide advance information to various water users on the amount of water that may be available for the remainder of the year. These forecasts are normally correlation analyses of varying complexity that use indices of precipitation and losses to estimate runoff, as direct measurements are almost impossible to obtain. Indices are assumed to provide adequate information for making these predictions in that snow accumulation data represent a combination of several storms which tends to reduce the local deviations that may occur during individual storms. In addition, seasonal accumulation tends to follow a relatively consistent pattern for large areas having comparable elevation and exposure (U. S. Soil Conservation Service, 1972).

Information on snowpack characteristics, water equivalent, density and depth, at a particular time and place is obtained from snow surveys. A major responsibility of the U. S. Soil Conservation Service is to conduct Federal-State cooperative snow surveys and to make water supply forecasts. To estimate streamflow, a Soil Conservation Service (SCS) forecaster will

examine at least 15 years of data that includes snow water equivalent, soil moisture, precipitation, base flow and temperature. These variables, assumed to be independent, are then inserted into a multiple linear regression analysis to obtain a predication equation for streamflow. An example of the general form of the prediction equation is as follows:

$$Y = b_o + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 \qquad (1)$$

where Y = runoff

X_1= base flow

X_2= fall precipitation

X_3= snow water equivalent

X_4= spring precipitation

b_i= regression coefficients

Past experience has indicated that snow water equivalent, used as an index of precipitation, has a very high correlation to runoff. Other precipitation indices such as winter and spring precipitation have a relative significance that is considered moderately high, while fall precipitation, antecedent streamflow and base flow have been found to be variables of lesser significance. Insofar as indices of losses are concerned, temperature has the greatest influence on the prediction of runoff accounting for as much as 25 percent of the variability. Soil moisture, wind, radiation and humidity have lesser influence.

Snowmelt floods

Forecasting the rate of runoff from snowmelt requires consideration of the sources of heat available for melting a specific snowpack and the

routing of the snowmelt through a particular watershed to a streamflow gaging site. The most commonly used methods of estimating snowmelt are the degree-day method and one that is based on a comprehensive analysis of all meteorological parameters that affect the transfer of heat to the snowpack. Comparisons of these methods to predict snowmelt rates have indicated that the simpler degree-day method using temperature as the only index of energy input may be used with little loss of accuracy (Anderson, 1968 and Pysklewyc et al., 1968). The degree-day equation that is generally used may be written as

$$M = K(T - T_o) \tag{2}$$

where M is snowmelt runoff

T is air temperature at median elevation of melting snowpack

T_o is a base temperature at which snowmelt is assumed to start (normally taken as 0°C)

K is a degree-day or melt factor

Values for base temperature and melt factor can be adjusted to conform to either mean daily or maximum daily temperature data. Melt factors are variable during the snowmelt season and can be specified daily or given as a function of accumulated runoff (U. S. Army Corps of Engineers, 1972).

WINTER PRECIPITATION MODEL

Current procedures in water supply forecasting generally requires an estimate of future precipitation. Rather than use expected or mean values of anticipated precipitation, the proposed methodology requires the use of

a cumulative distribution function (cdf) of total precipitation expected during the forecast period. To accomplish this requires first, defining an event and second, obtaining the distribution (pdf) for the random number of events per unit time and the pdf for the random variable of the magnitude or depth of precipitation per event.

Defining an event

In previous papers, the authors have used more than one definition to describe the occurrence of a convective storm event (Fogel, et al., 1971; Duckstein et al., 1972). The events were essentially classified as to whether or not they produced runoff. While winter precipitation characteristics differ from summer events, it, too, can be defined several ways. The definition may be based on whether snow or rain fell, on threshold values for daily precipitation, and on when an event is deemed to be concluded. A major consideration in selecting a definition is that the required data be readily available.

For this analysis, a wet day was defined as one which has precipitation equal to or greater than 0.01 inch. Then, since a winter storm may last several days, the wet days were lumped together into storm groups. A storm group was defined as a sequence of consecutive wet days separated from other storm groups by one or more dry days. This definition is a slight modification of the one used by Kao et al. (1971).

Number of events per unit time

In the previous studies on thunderstorm rainfall, events were assumed to occur in an independent manner such that a Poisson variate can be used to represent the distribution of N, the number of events per

given time period. The occurrence of independent events follows a Poisson process if the interarrival times are exponentially distributed. Note that interarrival times are calculated from the beginning of one storm group to the beginning of another group (see Fig. 1).

In southwestern United States, winter storms occur relatively infrequently so that storm group durations are much shorter than the duration of dry spells. Therefore, as the pdf of interarrival times can be approximated by an exponential distribution, a Poisson distribution may be used to represent the pdf of number of storm group occurrences per season (Kao et al., 1971).

For areas where winter storms are more numerous, the interarrival time may no longer follow an exponential distribution. Thus, it is necessary to derive a more general equation to describe the pdf of number of occurrences per given time period. Gupta (1972) derived the required pdf which is of the form

$$f_N(j) = \sum_{i=0}^{\infty} \left(\frac{q}{p}\right)^n \left(1 - \frac{q}{p}\right)^i \binom{n+i-1}{i} \sum_{j=2n+i}^{\infty} e^{-m} \frac{m^j}{j!} - \sum_{i=0}^{\infty} \left(\frac{q}{p}\right)^{n+i} \left(1 - \frac{q}{p}\right)^i \binom{n+i}{i} \sum_{j=2n+i+2}^{\infty} e^{-m} \frac{m^j}{j!} \qquad (3)$$

where

m = mean number of events per time period

$p = 1-q$ = probability that an event has occurred

San Francisco data, assumed to be representative of winter storms in the western United States, was used to test the above relationship. Fig. 2 illustrates a good fit of the derived distribution to the historical record.

Amount of precipitation per event

Letting the amount of precipitation be expressed as an integral number of half-inch increments, the pdf of P, total precipitation per storm group, given that a storm group has occurred, may be expressed by a geometric distribution. A comparison of the fit of the postulated distribution to 69 years of San Francisco data is shown in Fig. 3. While the geometric distribution is acceptable, subsequent investigations have revealed that a J-shaped gamma distribution provides a better fit, primarily at the tail. This aspect is important when extreme events are considered.

Total seasonal precipitation

To determine the distribution of W, total precipitation per time period or season, it is assumed that the amounts of precipitation per storm group are mutually independent, identically distributed random variables and the number of storm groups per season is itself random. Then, it follo that

$$W = \sum_{1}^{N} P_i \tag{4}$$

With distributions assumed for P and N, it is possible to obtain the pdf of W by first deriving the generating function of W and then obtaining successive differentiations of this function. The procedure is detailed in Feller, chapter 12 (1957).

If only the mean and variance of W are required, they can be calculated directly from

$$E(W) = E(N)E(P)$$

$$\text{var}(W) = \text{var}(N)\,[E(P)]^2 + E(N)\,\text{var}(P) \tag{5}$$

DEVELOPMENT OF RUNOFF MODEL

A schematic diagram of the stochastic process of cumulative precipitation and runoff and snow on the ground is shown in Fig. 4. Precipitation is an event-based or intermittent process while runoff is continuous. In Fig. 5, a view is taken of the assumed process between two successive events. At the time of the $j^{\underline{th}}$ precipitation event, S_o represents the snow water equivalent on the ground, while B, a constant, is the loss incurred during the event such as from interception and evaporation. Thus, the snow on the ground following the $j^{\underline{th}}$ precipitation event is $S_o + P_j - B$. Between the two precipitation events, the duration of which is D_j, a random variable, the snowpack will have lost an additional amount of water. The total loss is lumped into the term A_j which is considered to be a random variable linearly related to the duration between events such that

$$A_j = kD_j + B \tag{6}$$

If the number of events can be described by a Poisson variate N, then D will follow an exponential distribution. The distribution function for $A, F_A(a)$, can readily be obtained from

$$F_A(a) = F_D\left[\frac{a - B}{k}\right] \qquad a > B \tag{7}$$

The next step is to define a new random variable X such that

$$X = P - A \qquad P > A \tag{8}$$

and

$$F_X(x) = \sum_a F_p(x + a)\, f_A(a) \tag{9}$$

where $f_A(a)$ is the probability distribution of A. The variable X can be interpreted to be the net precipitation per event.

Let Y be the sum of the random number of random variables $X_1, X_2, \cdots$

$$Y = \sum_{j}^{N} X_j \tag{10}$$

Since this relationship is now similar to equation 4 for determining the total seasonal precipitation, the same procedures for obtaining the pdf of Y and the mean and variance of Y can be used. In effect, the random variable Y can be assumed to represent the total net precipitation that will occur in a given time.

When a water supply forecast is made, the snow water equivalent is ascertained; and if an estimate of future precipitation is required, expected values are used. In the proposed procedure, a pdf of total precipitation in a forecast period is employed. Thus, the total amount of water available for runoff at the time a forecast is made is the sum of the current snow water equivalent, S_o, plus Y the amount of precipitation that is expected to occur in the forecast period, or until the end of the season. This defines a new random variable

$$Z = S_o + Y \tag{11}$$

which has a distribution function

$$F_Z(z) = F_Y(z - S_o) \tag{12}$$

The final step involves the assumption that not all of Z will show up as runoff. A further assumption is made that another random variable C, which has numerical values from 0 to 1, can be used to relate the

random variable Z to the runoff volume V in a manner that

$$V = CZ \tag{13}$$

Without any verification at this time, the belief is that C is related to a heat index such as degree-days and can be described by a beta distribution. The varible C, then, can be considered as being a runoff coefficient with a random component. The distribution function of runoff, then, becomes

$$F_V(v) = \sum_c F_Z \left(\frac{v}{c}\right) f_C(c) \tag{14}$$

In summary, then, the proposed model for predicting snowmelt runoff is of the form

$$V = C \left[S_o + \sum_j (P_j - A_j)\right] \tag{15}$$

in which C, P, and A are random variables.

APPLICATION TO RESERVOIR CONTROL

The Salt River Water Users' Association is involved in the delivery of water to more than one-half of the people in the state of Arizona as well as to a large irrigation project. Storage reservoirs on the Salt and Verde Rivers are available to meet these demands. During low runoff years, control of the storage facilities presents few problems. On the other hand, in above normal runoff years, such as the current one, managers are faced with regulating the release rate to minimize possible losses.

Low release rates cause little or no damage. A higher release rate

can result in some inconveniences as certain key roads become unavailable for traffic and also some flooding can occur in low lying areas. Still higher release rates can cause serious flooding below the dam. To operate such a reservoir, therefore, involves a trade off of the following three quantities:

1. The total water supply benefit B(V,R) which is a function of the total volume of water available V and the volume of water to be released R in the next time period (0,T)
2. The flood damage downstream from the dam C(R) caused by the planned release R
3. The flood damage at and downstream from the dam D(Q,R) caused by the flood peak Q; D is a decreasing function of R.

Up to now, only the runoff volume V and not the peak runoff rate Q has been considered. Where the peak rate is considered to be important, a linear volume to peak relationship may be assumed. In a regression analysis relating peak flow to volume of the Columbia River at The Dalles, the correlation coefficient was reported as 0.86 (U.S. Soil Conservation Service, 1972). Thus, it can be shown that the conditional cdf $G(q|v)$ is represented by a t - distribution. Then, the distribution function for Q is

$$F_Q(q) = \int_0^\infty G(q|v)\ f(v)\ dv \tag{16}$$

For the sake of simplicity, it is assumed that the release R takes place mostly at the beginning of (0,T) while floods come in the later part of that period.

From the trade offs mentioned above, a one-period objective function becomes

$$Z(Q,V,R) = -B(V,R) + C(R) + D(Q,R) \tag{17}$$

The goal is to determine R that minimizes the expected value of

$$\begin{aligned} \underset{R}{\text{Min}}\ z(R) = Z\,[z(Q,V,R)] = -\int_0^\infty B(v,R)f(v)dv \\ +C(R) + \int_0^\infty D(Q,R)f(q)dq \end{aligned} \tag{18}$$

Clearly, a multiperiod model could be formulated in a recursive manner (dynamic programming approach). The utility of such a model, however, would be greater for the definition of a fixed seasonal policy $(R_1, R_2, \cdots, R_N)$ rather than for the adaptive control being considered here.

As an example, let

$$\begin{aligned} B(V,R) &= b(V-R) && \text{for } V > R \\ &= 0 && \text{otherwise} \\ \\ C(R) &= 0 && \text{for } R < r_o \\ &= c(R-r_o) && \text{for } r_o < R \\ \\ D(Q,R) &= (Q-aR)^2 && \text{for } Q > aR \\ &= 0 && \text{otherwise} \end{aligned}$$

Equation 17 then becomes

$$\underset{R}{\text{Min}}\ z(R) = -b\int_R^\infty (v - R)f(v)dv + c(R-r_o) + \int_{aR}^\infty (q - aR)^2 f(q)dq \tag{19}$$

Equating the first derivative to zero results in

$$z^1(R) = +b\int_R^\infty f(v)dv - 2a\int_{aR}^\infty (q - aR)f(q)dq = 0 \tag{20}$$

or

$$b[1 - F(R)] - 2a \int_{aR}^{\infty} qf(q)dq + 2a^2 R[1 - F(aR)] = 0 \qquad (21)$$

This equation may be solved numerically for the optimum release R*.

DISCUSSION

The considerable variablity of snowfall in space and time has prompted an investigation into the role of stochastic snow models for water supply forecasting and flood prediction. Such an approach is motivated by the need to estimate risks associated with forecasts involving snowmelt runoff.

New procedures appear warranted in that estimates of snowmelt runoff do not appear to be significantly improved with the use of all the meteorological parameters that affect heat transfer over the method using only temperature as a heat index. Without questioning the inherent assumptions in a regression analysis, the utilization of such procedures often leaves a considerable amount of the variability unexplained. Thus, there appears to be a limit to the capability of current methods for predicting snowmelt runoff.

On the other hand, stochastic models do not have to make any apologies for poor correlation coefficients. Stochastic models start with the assumption that natural hydrologic systems are so complex that no exact laws have yet been discovered that can explain completely the natural phenomena. Procedures that recognize this uncertainty and utilize random variables that have some physical significance in their

relationships are generally preferred where there are managerial applications. The operation of water resources systems usually becomes critical when the occurrence of extreme events such as floods and drouths become imminent. Current procedures of forecasting snowmelt runoff generally use the mean or expected value approach, and hence, may run into difficulty in predicting the extreme event.

The proposed procedure tends to combine the relatively long term forecasting of water supplies with the short-term flood predictions. Distributions involving both precipitation and temperature are employed in an effort to characterize the uncertainty. Losses, from precipitation to streamflow,are assumed to be both additive and multiplicative. Verification of the characters of these relationships is currently in progress. In addition, there are other problems that require further study such as the rain on snow floods.

In conclusion, this paper presents a methodology for incorporating meteorologic uncertainty into a snowmelt runoff model that can be used for water supply forecasting and flood prediction. An example is presented to illustrate the utility of this information in selecting an optimal action that will reduce the risk or consequential effect of possible uncertain outcomes.

ACKNOWLEDGEMENT

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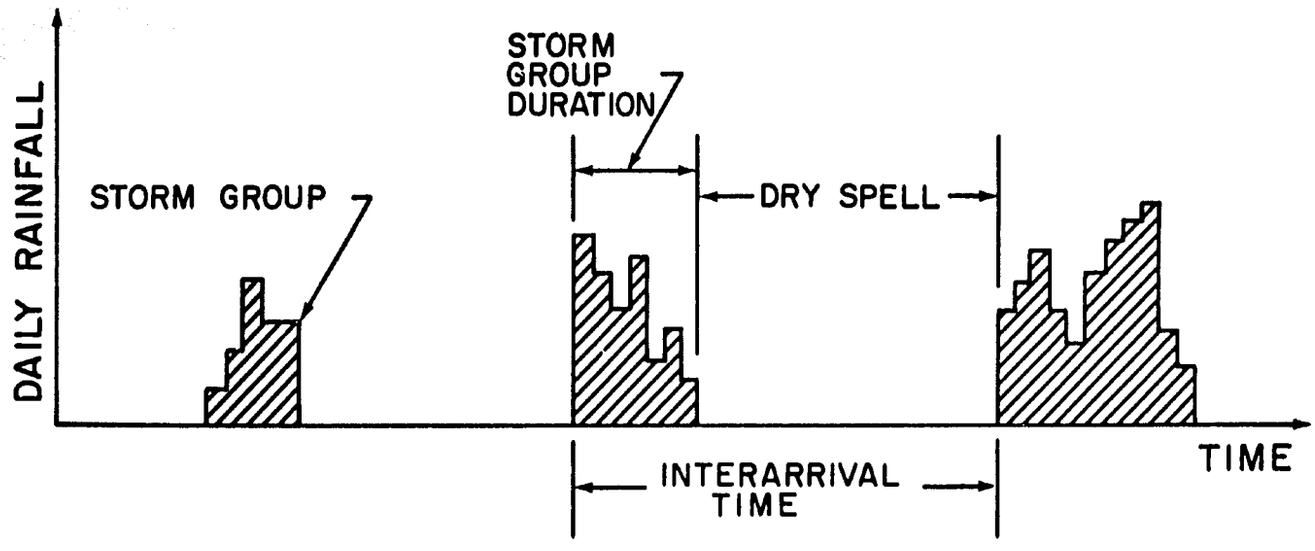


Fig. 1. Illustration of interarrival time of storm groups.

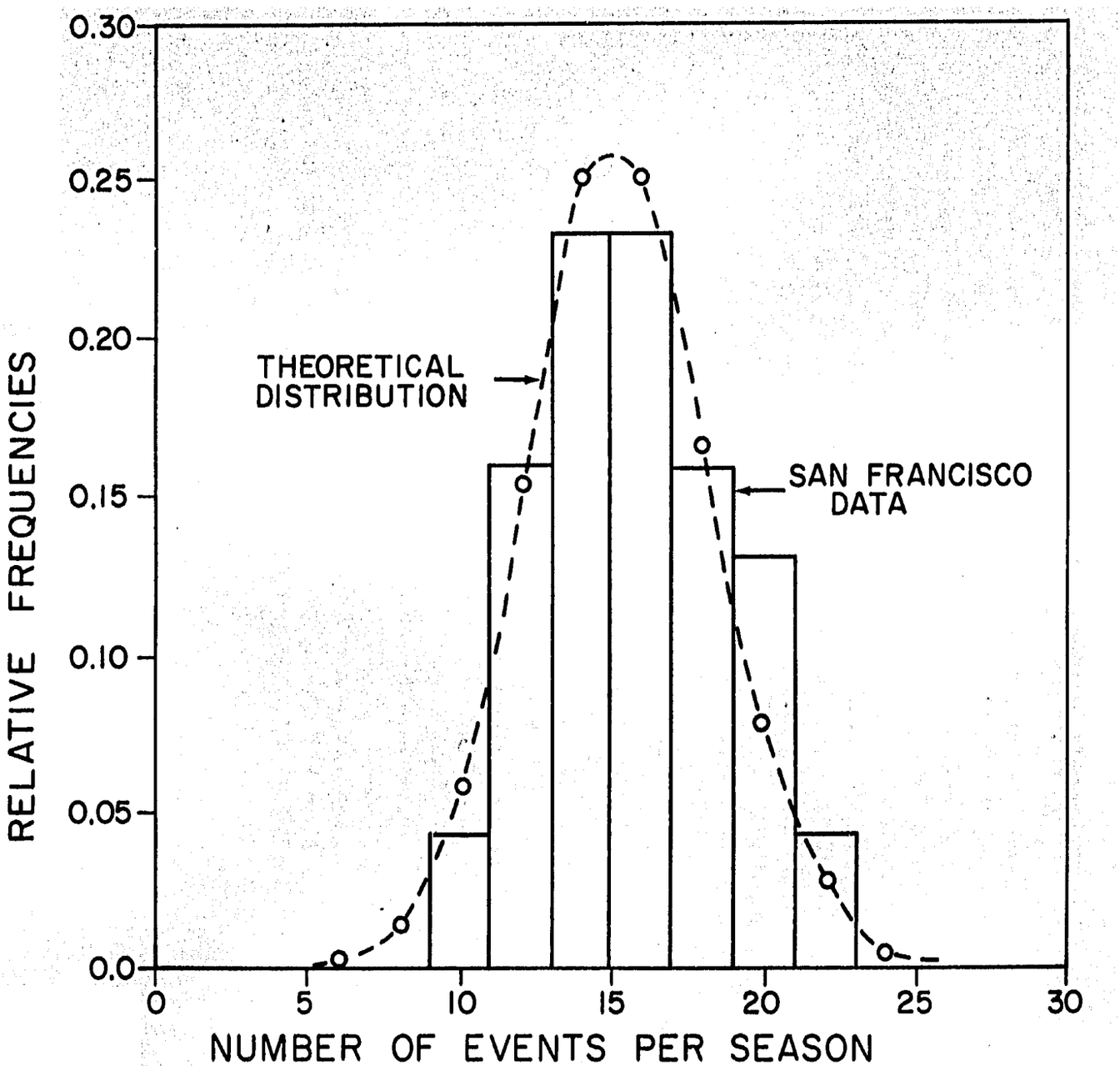


Fig. 2. Distribution of number of storm groups per season.

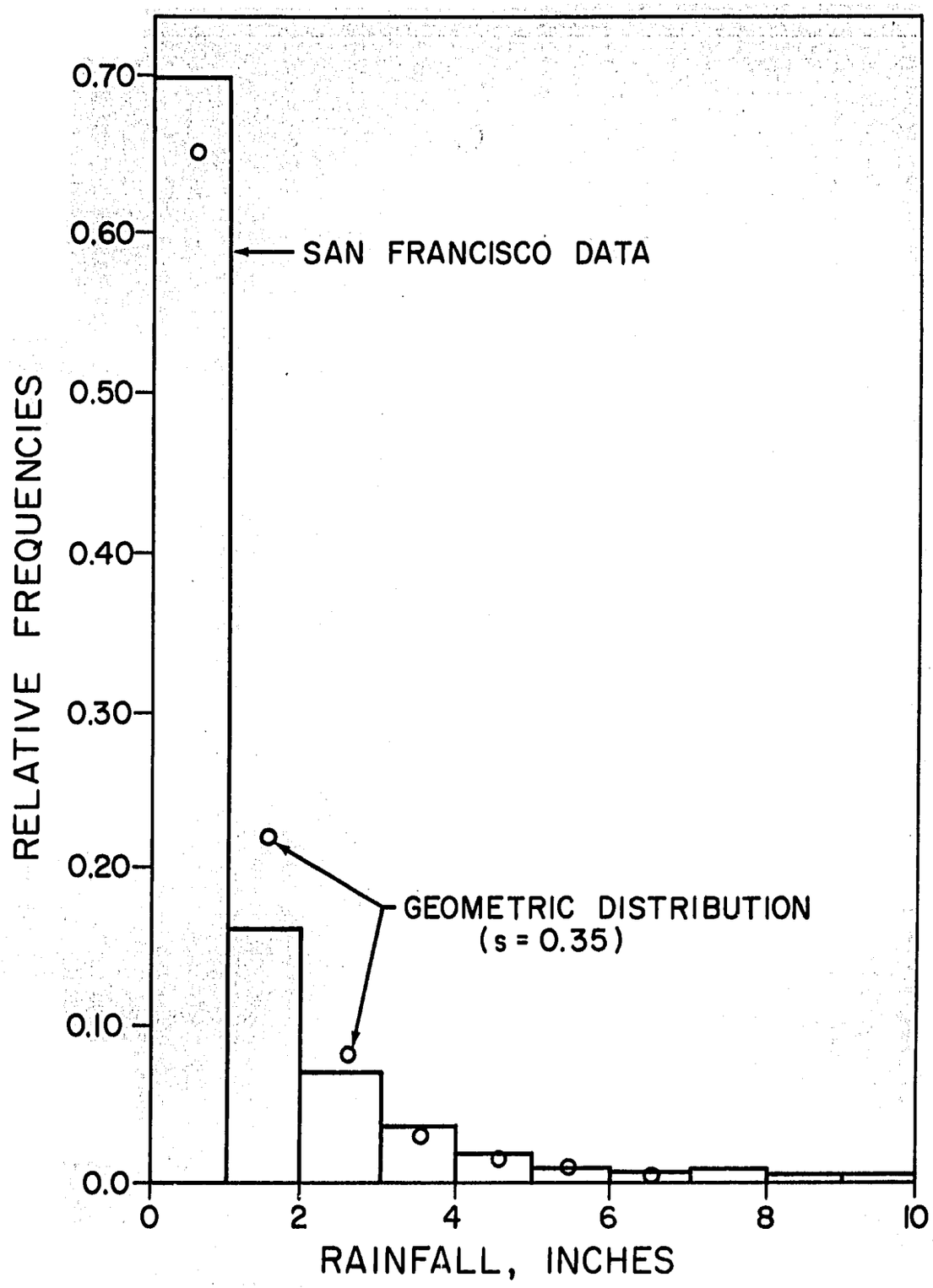


Fig. 3. Distribution of precipitation depth per storm group.

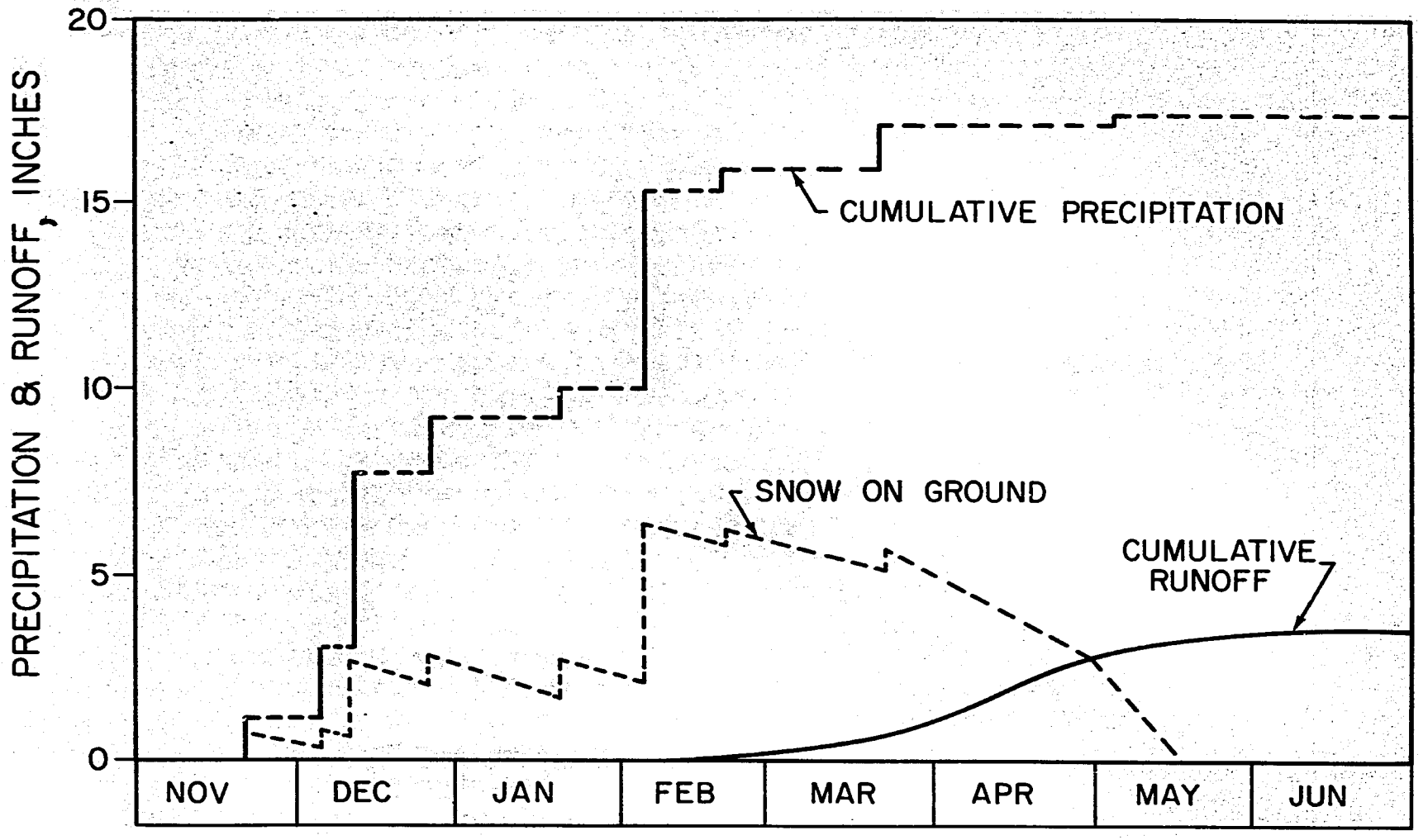
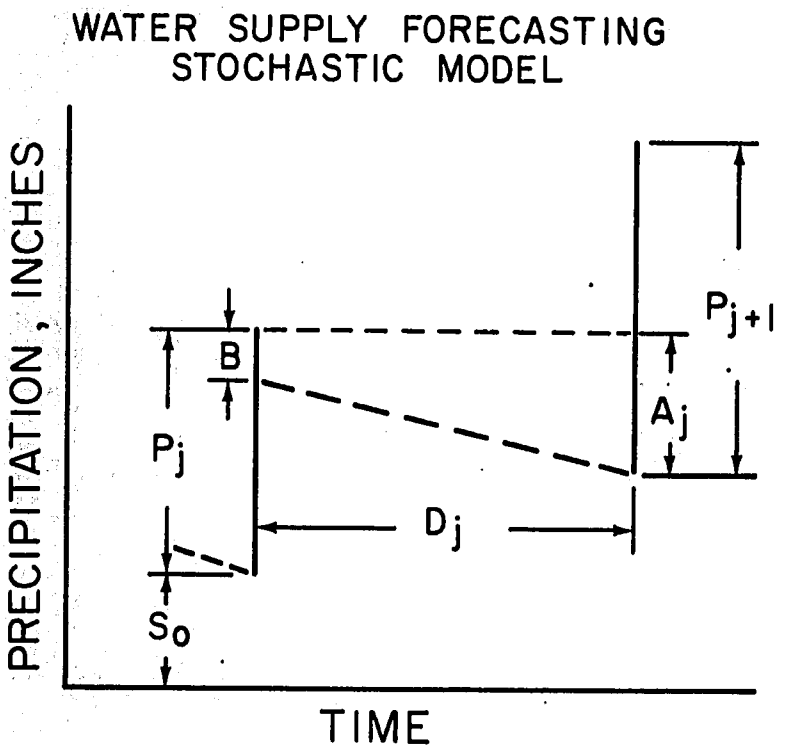


Fig. 4. Schematic representation of cumulative precipitation, snow on the ground and runoff.



$$Q = C\left[S_o + \sum_j (P_j - A_j)\right] \quad \text{in which}$$

C, P and A are random variables

Fig. 5. Defining basic elements of stochastic snow model.