PARAMETER OPTIMIZATION FOR SIMULATING SEMIARID WATERSHED HYDROLOGY

by

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TABLE OF CONTENTS

		Page
	LIST OF ILLUSTRATIONS	vi
	LIST OF TABLES	vii
	ABSTRACT	viii
1.	INTRODUCTION	1
2.	LITERATURE REVIEW	5
	Watershed Simulation Models	5
	Systems Models	6
	Component Models	9
	Model Calibration	12
3.	METHODS	18
	Experimental Watersheds	18
	Basis for Model Selection	22
	Model Development	23
	Watershed Geometry	25
	Component Relationships	28
	Parameter Selection	35
	Model Calibration	40
	Criteria Problem	40
	Optimization Strategy	42
4.	RESULTS AND DISCUSSION	46
	Calibration Results	46
	Validation Results	50
	Analysis of the Model	53
5.	CONCLUSIONS AND RECOMMENDATIONS	59
	APPENDIX: COMPUTER PROGRAM	61
	Listing of Programming Symbols	62
	Listing of Program	65
	LIST OF REFERENCES	72

LIST OF ILLUSTRATIONS

Figure		Page
1.	Topographic Map of Watershed HL-1	20
2.	Topographic Map of Watershed HL-2	21
3.	Finite Grid Representation of Watershed HL-2 . . .	24
4.	Rectangular Element Representation of Watershed HL-1	26
5.	Rectangular Element Representation of Watershed HL-2	27
6.	Method of Representing Watershed Geometry	29
7.	Objective Function Response Surface for Parameters P and Pore	37
8.	Objective Function Response Surface for Parameters P and A	39
9.	Objective Function Response during Optimization	52
10.	Simulated and Observed Hydrographs for Storm of 7/14/71	57

LIST OF TABLES

Table		Page
1.	Parameter Values of the Pattern Search	48
2.	Final Parameters of the Optimization Strategy . .	49
3.	Peak and Volume Comparisons for Final Calibration	51
4.	Peak and Volume Comparisons Using Final Parameter Values for Independent Data from Watershed HL-2	54
5.	Peak and Volume Comparisons Using Final Parameter Values for Independent Data from Watershed HL-1	55

ABSTRACT

The study was designed to explore the potential of numerical optimization techniques for obtaining unique optimum watershed model parameters. Successful parameter optimization would be valuable for correlations with watershed treatments or for regionalization for ungauged watersheds.

A digital watershed model capable of simulating surface runoff hydrographs from small desert watersheds was developed from previous modeling studies. Six parameters were selected for optimization. All other parameters were estimated from previous studies or were measured on the watershed.

The model was calibrated with data from a 14 acre watershed in the Tucson Basin. Independent data from the same watershed and data from a similar watershed were used to validate the calibrated parameter values. Calibration was carried out by an optimization strategy of response surface identification, followed by extensive surface climbing and concluding with a quadratic convergent fitting.

Optimization results discovered several fitting problems presented by the model but the final results for the calibrated events appeared to be good. In comparison,

the validation results were quite poor and indicated a large degree of data dependency in the optimization.

It appeared doubtful that the model calibration, even with extensive data, would be able to obtain parameter values capable of correlation with watershed treatments. With model improvements, the approach may provide parameters adequate for regionalization for ungauged watersheds.

CHAPTER 1

INTRODUCTION

With increased pressure from society for more effective land and water management, resource managers are faced with the difficult problem of evaluating their management designs before implementation. To accomplish this modeling is essential. A crucial component in the management of both land and water resources is the watershed system. Numerous watershed models exist. The variety is the result of varying objectives, the type of data available and the modeler's concept of the system.

The primary objectives of watershed models are generally one or some combination of the following:

1. to increase understanding of the system
2. to make predictions for gauged sites
3. to make predictions for ungauged sites
4. to evaluate effects of watershed management.

In terms of the management decision making process, a wide variety of predictions is useful. For flood frequency analysis, predictions of flood occurrences above a particular level may be needed. For reservoir operation, monthly flow volume simulations may be useful. For evaluating the

may be of interest. On the other hand, the treatment effects on flood peaks and timing of runoff may also be useful for watershed management evaluations.

Event based watershed models simulate specific runoff hydrographs from recorded rainfall hyetographs or from stochastic simulations of rainfall. Event models have the advantage that any predictions necessary for management decision making can be derived from event simulations. The disadvantage is the increased cost of analysis and the need for more refined data.

The steps in watershed simulation, regardless of the objectives, follow the same general format as explained by Dooge (1968):

First, it is necessary to choose a method of simulation. This involves selecting the type of model, for example electric analog, regression model or digital synthesis.

Second, it is necessary to develop the form of the model. This involves making hypotheses about watershed behavior.

Third, it is necessary to optimize the parameters according to some well designed criteria. Trial and error procedures are often used at this stage.

Finally, it is necessary to correlate the fitted parameters with watershed characteristics or watershed

condition in order to use the model for simulations on ungauged sites or for simulations of treatments.

Considerable work has been done on the first two steps. Some work has been done recently on parameter optimization of step three. However, there have been few attempts to correlate treatment effects or watershed characteristics with watershed model parameters. A major obstacle in doing so is obtaining unique optimum parameters in step three.

The purpose of the study was to determine the potential of numerical optimization for obtaining unique optimum parameters of an event simulation model of semiarid watershed hydrology.

To carry out the study, an event simulation model capable of simulating surface runoff hydrographs from small desert watersheds and computationally efficient enough for numerical optimization was developed. The model incorporates measurable watershed characteristics such as shape and topography and permits treatment simulations with proper changes in the parameters of component representations. A numerical optimization strategy was devised to calibrate the model and obtain the optimum parameters.

The specific objectives were: to analyze the effectiveness of the optimization strategy in calibrating

the model; to validate the optimized parameters with independent data on the same watershed; to evaluate the regional application of the estimated parameter values with independent data from a similar watershed; and to analyze the capabilities and limitations of the model for surface runoff simulation and optimization.

CHAPTER 2

LITERATURE REVIEW

A thorough but by no means exhaustive review of event based watershed simulation models and of model calibration methods is presented in this section. Model calibration is discussed only as applied to event based watershed simulation models. Additional references which were not discussed but which have relevance are the texts on the theory and application of optimization techniques by Wilde (1964) and by Beveridge and Schechter (1970) and the papers discussing the potential of numerical optimization in hydrology by Beard (1967) and by Green (1970).

Watershed Simulation Models

Within the category of event based watershed simulation models are large numbers of models. Several classifications are possible. A fairly satisfactory classification would be the distinction made by Amorocho and Hart (1964) between systems models and models based upon physical hydrology. The physically based models or component models are an attempt to incorporate the basic operation of each hydrologic component and their interaction into a complete

synthesis of the land phase of the hydrologic cycle. With systems models the complete process synthesis approach is discarded and the approach concentrates on applying a transformation function which produces outflow from a given input. Although it is often called a "black box" approach, systems investigations require at least general assumptions about the nature of the system. In systems modeling there is increasingly more reliance on knowledge of hydrologic phenomena for formulation of relationships and for rational choice of parameters. As a result a considerable amount of "grey area" exists between the two classifications. However, the distinction may be useful in clarifying the philosophy involved in model development and in analyzing its utility for watershed management simulations.

Systems Models

The simplest and most prolific systems models are the linear systems models. Like the well known unit hydrograph methods, the basic assumptions of the approach are superposition of response (linearity) and time invariance of response. The methods presuppose a synthesis of rainfall excess, or in systems terminology they require partial synthesis.

The time distribution of output for the linear system can be expressed in analytical form by the convolution integral:

$$Q(t) = \int_0^t h(t - \tau) \cdot i(\tau)d\tau$$

where h(t) is the instantaneous unit hydrograph, IUH, or the output from the system due to an instantaneous impulse of unit size, i(t) is the magnitude of the impulse or rainfall excess, and t and τ are fixed and variable time, respectively (O'Donnell 1966).

The problem of finding the IUH has been a topic of considerable research. The Nash model or cascade of equal linear reservoirs is often postulated as the general IUH model (Nash 1957). The impulse response of the IUH in the Nash model is the same as the gamma density function:

$$h(t) = \frac{1}{K\Gamma_n} e^{-t/K}(t/K)^{n-1}$$

where Γ_n is the gamma function which for integer n equals (n-1)!. The parameters n and K of the Nash model are estimated from streamflow data. Once these parameters are known, outflow can be determined by numerical integration of the convolution equation or, if n is an integer, by routing rainfall excess through n linear reservoirs with time constant K (Overton 1970).

Nash's lumped model can be contrasted with Dooge's time area method. Dooge (1959) postulates that output from

an element of a watershed incurs a translation delay and also passes through linear reservoirs. Assuming that all elements have the same delay and the same chain of storages to the outlet and that all storages have the same time constant K, a general impulse response function can be derived as in Nash's approach.

Researchers have also explored direct analysis methods which derive the IUH directly from the watershed data and thereby eliminate the need for a general model (O'Donnell 1966). However, these methods require watershed data and lack generality for ungauged sites.

The limitations inherent to all unit hydrograph or linear systems models are due to the assumptions of superposition and time invariance and to the difficulties in estimating parameters for ungauged sites. Researchers have had some success in correlating general IUH model parameters with watershed characteristics (Gray 1962, Nash 1959). This approach, although it may have some practical value, is criticized by Amorocho and Hart (1964) as a method which "combines the pitfalls of regression analysis with the weaknesses of the linearity assumptions."

In view of the shortcomings of the linearity assumptions, attempts have been made to analyze the watershed as a nonlinear system. Foremost among these is the work of Amorocho (1967). Amorocho theorizes that a functional

polynomial or its equivalent can be utilized to predict outflow from a watershed by a single operation and thus eliminate the need for partial synthesis, or it can be used for the representation of one or more of the components of a synthesis model. However, general nonlinear systems models are more difficult to formulate because there are an infinity of forms that nonlinear relations may take. Little progress is made unless the form of the relationship is specified beforehand.

In general, all the systems models are quite limited in evaluating the effects of treatment on streamflow response. Unless the treatment effects can be completely accounted for in the partial synthesis or incorporated into the model, the systems models will result in inadequate simulations.

Component Models

Considerable effort has been aimed at complete synthesis models since the inception of the Stanford Watershed Model (Crawford and Linsley 1962 and 1966). The Stanford Watershed Model can be distinguished from the systems models in that the modeling is shifted from an empirical representation of the system as a whole to a series of individual components with each one an empirical representation of a known hydrologic process. The

mathematical representations of hydrologic processes are expressed in general form applicable to all watersheds. To apply the model it becomes necessary to determine the parameter values for the specific watershed. A considerable amount of empiricism pervades in the Stanford Watershed Model's representation of the partial differential equations governing water transport. However, it has contributed immensely to the incorporation of hydrologic concepts in watershed simulations.

A more recent complete synthesis model is the USDAHL-70 model developed by Holtan and Lopez (1971). In the same spirit as the Stanford Watershed Model, a series of empiricisms provide the mathematical continuum of the physical processes. The model incorporates the variability in soils and other hydrologic factors through the use of land capability classes. Infiltration and rainfall excess are computed for each land capability class with empirical methods developed by the Agricultural Research Service. Utilizing hydrograph recession analysis, flow is routed through several discernible regimes of subsurface flow in addition to overland flow. Hydrologic capability classes are numbered in a down slope order so that computation can account for runoff cascades to successive capability classes.

More detailed descriptions of component processes have been available to modelers. Their use has been avoided

because of the considerable increase in computation. However, a model developed by Huggins and Monke (1966) at Purdue University attempts to incorporate some of these process representations in modeling the surface runoff response to rainfall on small agricultural watersheds. The model, hereafter referred to as the Purdue Model, gives a complete synthesis of surface runoff hydrographs. The model uses a finite grid technique for decomposing the watershed into surface units in an attempt to describe the detailed physical processes occurring within the watershed. The kinematic wave approximations of the equations of continuity and momentum were utilized to route overland flow from each element sequentially down slope.

More recently, Harley, Perkins and Eagleson (1970) developed a component model of surface runoff. The model, referred to as the MIT Catchment Model, was designed to simulate the essential features of the hydrograph while increasing computational efficiency relative to a finite grid method. Watershed geometry was represented by simple overland flow planes and rectangular stream segments. Both linearized and kinematic approximations to the equations of continuity and momentum were used in determining surface runoff. The linear routing method was used when the kinematic assumptions were violated.

The Purdue Model and the MIT Catchment Model are limited by the assumptions of the kinematic or linear routing methods, by the errors resulting from computational approximations of watershed configuration and by the errors in the finite differencing approximations used for computation (Kibler and Woolhiser 1970).

Model Calibration

Methods of watershed model calibration have largely depended on the type of model. For systems models numerous techniques have been explored. The method of moments has been used to derive the n and K parameters of the Nash model (Nash 1957). Other approaches using transform methods, orthogonal functions or matrix techniques have been employed to derive the ordinates of the Unit Hydrograph or IUH directly from the data. An excellent review of these methods is given by Jackson and Aron (1971). For nonlinear systems models, quasilinearization, a method based on the Taylor series expansion has potential for solving a large class of nonlinear models (Labadie and Dracup 1969). Other nonlinear models can only be solved by search techniques.

With the complete synthesis models such as the Stanford Watershed Model, estimation of unknown model parameters are often based on the hydrologist's knowledge of approximate parameter values. Parameter values are then adjusted until an appropriate fit between observed and

predicted hydrographs is obtained (Crawford and Linsley 1966). This approach, often called trial and error fitting, is aided by a thorough understanding of the hydrologic cycle and by familiarity with the model especially with the sensitivity of flows to parameter adjustment. Final parameter selection is largely subjective and several combinations of parameters may produce equally good results. Subjectivity in parameter estimation can cause havoc with attempts to relate parameter values with watershed characteristics or with watershed change (James 1970).

More objective procedures of parameter adjustment have been analyzed for synthesis models. Dawdy and O'Donnell (1965) first explored "hill climbing" optimization techniques to estimate parameters of a simplified component model. The techniques employed minimize the sum of squares of differences between synthesized streamflow and an observed record. In their study a unique set of parameter values was used to synthesize an observed record. Optimization techniques were then applied with a different set of parameter values. Thus the speed and accuracy of approaching the true parameter values could be compared for various optimization techniques. The Rosenbrock technique was found to be reasonably successful. It was also discovered that the greater the sensitivity of a model to a parameter, the closer and sooner the parameter was optimized.

Liou and James (1970) developed a self calibrating version of the Stanford Watershed Model. Criteria used for selection of parameters for calibration were that the parameters be difficult to measure and that the model be sensitive to parameter changes. Traditional optimization techniques were discarded in favor of a parameter adjustment technique which would approximate the trial and error adjustments based on hydrologic knowledge and on insight provided by sensitivity analysis. Adjustments continued until no further improvement in the sum of squares error of observed and predicted hydrographs could be achieved.

More recently Chapman (1971) reported the results of an optimization of a rainfall-runoff model for an arid zone watershed. The model simulates vertical water transport processes and makes no attempt to route the rainfall excess. Parameter values are restricted to ranges thought to have physical significance. Several error function criteria were analyzed. A strict sum of squares of differences between total observed storm runoff and total estimated storm runoff was found to be weighted heavily by large storms. In order to give more weight to threshold runoff storms, logarithmic, square root and cube root transformations were applied to observed and predicted values before computing the sum of squares error. Since final error function selection is a

value judgment, parameter sensitivity to error functions was considered undesirable. The square root transformation gave appropriate weights, but optimum values were only slightly affected by the other transformations. This result gives support to the use of the model.

Only one of the optimization techniques, the simplex technique, achieved convergence. Direct search strategies and the steepest descent technique were slow and often failed to reach the true optimum.

Perhaps the most comprehensive analysis of parameter optimization for watershed models has been by Ibbitt and O'Donnell (1971). The objectives of their paper were to show how the model builder can ease the fitting problems of a model and to show the designer of fitting techniques the range of problems that watershed models present.

The analysis showed that besides the standard problems confronting optimization techniques, local optima, saddle points and valleys, additional problems of plateaus and "pot holes" are characteristic of watershed models. Plateaus are usually the result of threshold parameters or parameters used in calculation potentials such as potential infiltration. "Pot holes" are multiple optima appearing as small perturbations in a confined region of the response surface. The causes are difficult to explain fully, but are

probably the result of any one or any combination of the following:

1. the model being an imperfect representation of the system

2. the effect of random data errors

3. serial correlation in the data used to compute the objective function.

Numerous suggestions were given for overcoming the problems presented by watershed models. Problems created by local optima and pot holes can be reduced by fitting the model to longer records. Random search can also be used to seek out the global optima. Solutions to the local optima problem will automatically solve the saddle point problem. Threshold parameters or potential calculations should be avoided when possible to prevent the plateau problem. However, fitting with data sets which bring all the parameters into operation can minimize the plateau effect. Parameter interaction is a major cause of valleys in the response surface. Some techniques are capable of handling valleys, but it is better to avoid their occurrence when possible. Altering model structure or careful parameter selection may reduce the parameter interaction problem. Using a wide range of data values, improving the quality of the data and employing certain objective functions can minimize the pot

hole effect. Perfections in the model will ultimately improve the fitting problems and the uniqueness of parameter values.

In summary, there are a wide variety of event based watershed simulation models. All models share the problem of determining unknown parameter values to apply a model to a specific watershed. Generally, the more physically based models have more parameters, require more data and have greater potential for regionalization than the more empirical models. To obtain unique parameter estimates for regionalization or for correlation with watershed condition, more objective methods of watershed model calibration need to be studied.

CHAPTER 3

METHODS

Data from Atterbury Experimental Watershed was used to carry out this study. Based on specific criteria, the most appropriate model was selected and modified to meet the needs of the study. A complete optimization strategy involving several methods was used in calibrating the model.

Experimental Watersheds

Atterbury Experimental Watershed, operated by The University of Arizona, is located about twelve miles east of Tucson, Arizona. The sparsely vegetated, low relief watershed is characteristic of desert watersheds in the Tucson Basin. Two small instrumented watersheds situated near the upstream end of Atterbury Experimental Watershed provided the data necessary for the study.

A single recording rain gauge is centrally located on each of the fourteen acre watersheds. Standard HL flumes, as described on page 26 in the U. S. Department of Agriculture's Agriculture Handbook No. 224 (1962), are used to measure runoff from the watersheds. Separate windup clocks drive the rainfall and runoff recording devices.

The watersheds, designated HL-1 and HL-2, were surveyed to obtain accurate areas and topographic representations as shown in Figures 1 and 2, respectively.

The vegetation on the watersheds is much denser along the channel than on the slopes. Vegetation consists mostly of creosote bush (Larrea tridentata) and desert mesquite (Prosopsis juliflora) with some cacti and scattered herbaceous vegetation.

The unconsolidated material forming the landscape is recently deposited (geologic time) alluvial material. The soils have thin loam and sandy loam surfaces and only moderate infiltration rates. The channels are relatively broad and flat and silt filled. Several clay lenses are scattered in the upper reaches of HL-1.

Only runoff producing events were selected for optimization so that the fitting procedure would "exercise" all of the parameters. Events preceded by dry spells were most desired for use in this study. When it was necessary to use events where antecedent precipitation occurred within a week before the event, soil moisture conditions were estimated with an exponential soil moisture retention model developed by Saxton and Lenz (1967) and calibrated with data from the Santa Rita test site of the IBP Desert Biome.

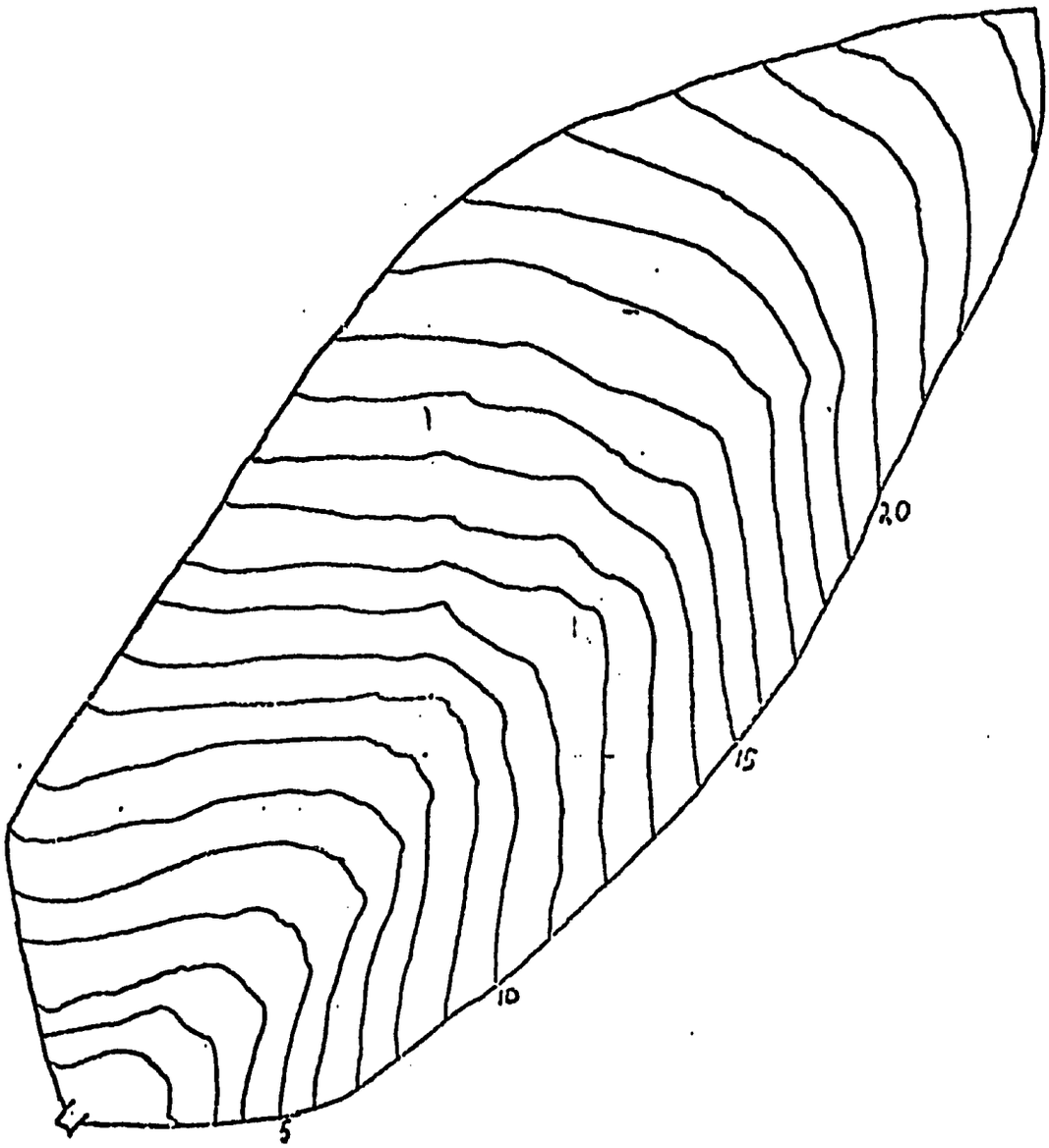


Figure 1. Topographic Map of Watershed HL-1.

Contour Interval = 1 ft.
1 in. = 192 ft.
Area = 13.8 acres

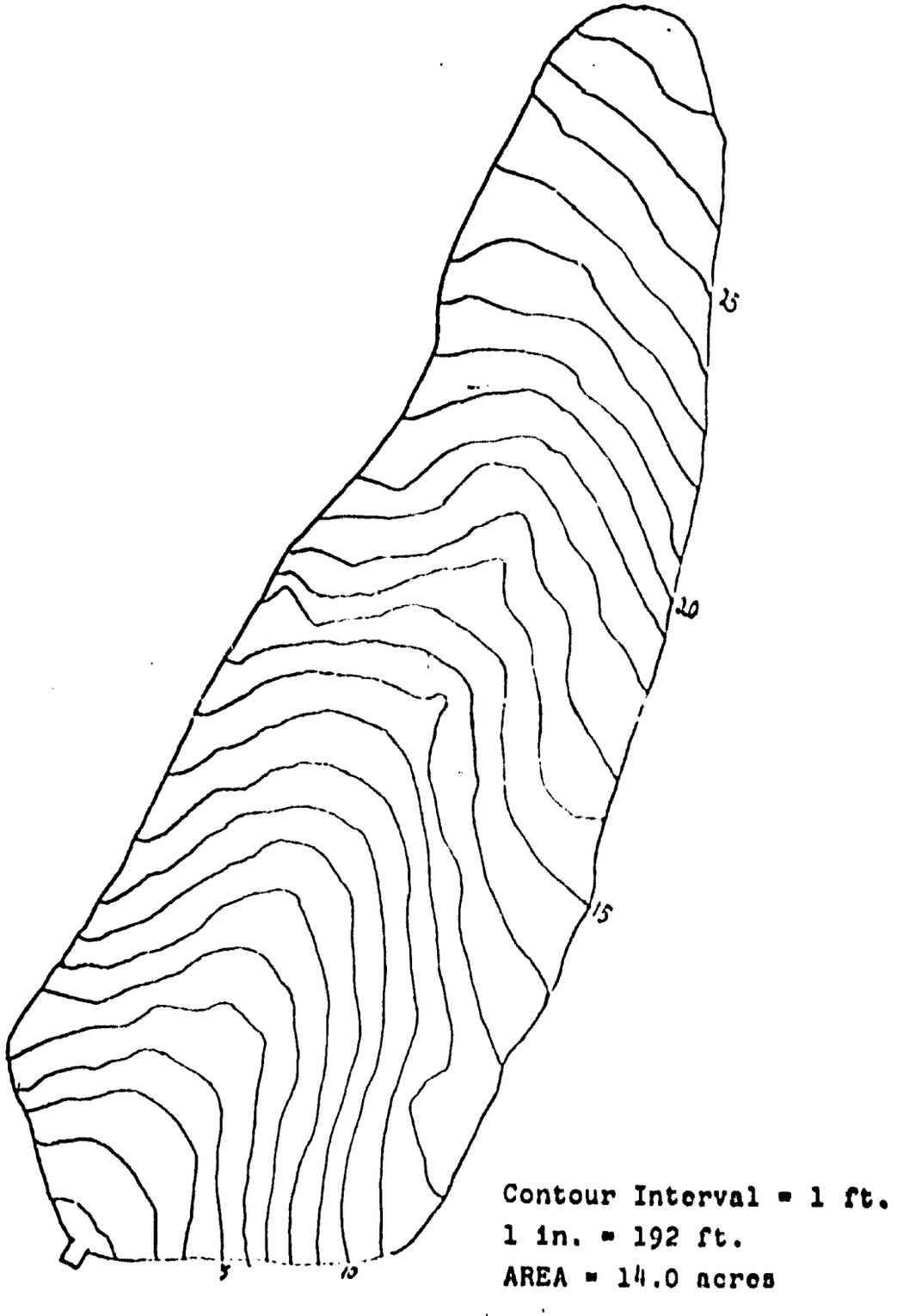


Figure 2. Topographic Map of Watershed HL-2.

Basis for Model Selection

Selection of an appropriate model was based on the following criteria:

1. The model should be applicable to small desert watersheds.

2. The model should be capable of representing watershed characteristics and watershed conditions.

3. The model should be computationally efficient to be optimized in a reasonable amount of computing time.

Of the watershed models currently available, the Purdue Model appeared to be the most applicable for this study. The model simulates overland flow from small watersheds making it ideally suited for small desert watersheds where overland flow dominates. The model inherently accounts for the effects of shape and general surface topography on runoff hydrographs. Spacial variations in soils and rainfall can also be accounted for in the model.

Since most of the complex processes and interactions are represented in the model, the parameters are closely related to specific watershed conditions. This feature makes attempts at correlating parameters with watershed conditions or regionalizing parameter values seem very promising. Success of such attempts would be very valuable for simulating treatment effects or for simulations on ungauged watersheds.

Figure 3 shows the finite grid representation of watershed HL-2 that was used in the Purdue Model. Slope and direction of flow are specified for each element. The portion of water flowing from each element to adjacent elements depends upon the direction of steepest slope. Outflow from each element is estimated as a function of storage on the element using the Chezy-Manning equation and the kinematic flow assumptions. Infiltration capacity is described as a function of soil moisture by the Holtan equation. Infiltration relations are allowed to vary among elements. For these watersheds infiltration stratifications for slope and channel elements appeared to be varranted.

Unfortunately the computing time required to optimize the Purdue Model as applied to HL-2 would be excessive. A six parameter optimization of the model would require more than ten hours of computing time on the CDC 6400. To overcome this disadvantage, an attempt was made to modify the model for use in this study while at the same time preserving the physical basis and component relationships of the model.

Model Development

The model developed for use in this study was based entirely on concepts and ideas presented in previous modeling studies. To increase computational efficiency, the

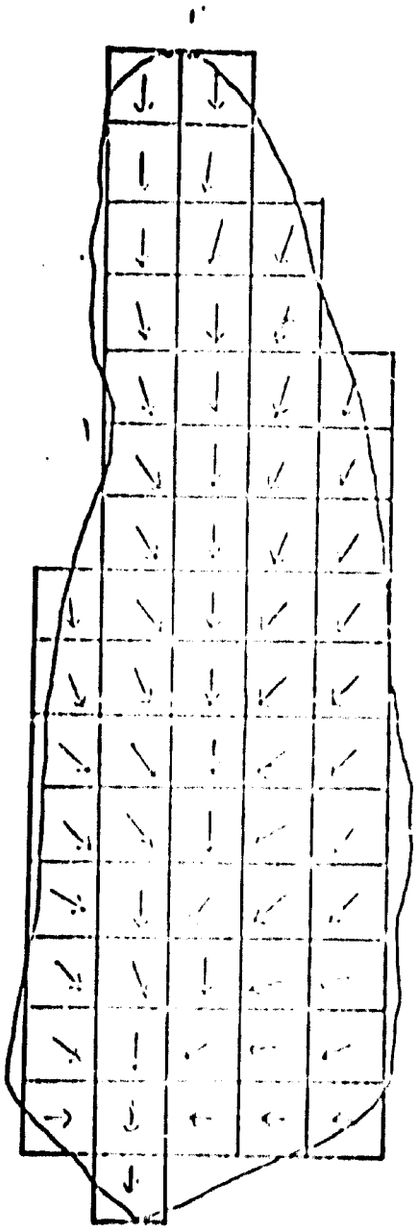


Figure 3. Finite Grid Representation of Watershed HL-2.

1 in. = 266 ft.

finite grid representation used by the Purdue Model was replaced by a system of rectangular slope and channel elements similar to the watershed geometry representation used in the MIT Catchment Model (Harley, Perkins and Eagleson 1970). Component process modeling with the exception of overland flow from slope elements is patterned after the Purdue model. The empirical overland flow method of the Stanford Watershed Model (Crawford and Linsley 1966) was used to route flow from the slope elements. The model is not proposed as a general model of surface runoff from desert watersheds. However, it may provide the basis for such a model.

Watershed Geometry

The criteria for choosing the arrangement and size of the elements is not firmly established. However, the elements should be as large as possible but sufficiently small so that slope, vegetation, length of overland flow, rainfall, lateral inflow and infiltration rates are relatively uniform within the boundaries of an element.

The basic element arrangements of watersheds HL-1 and HL-2 are shown in Figures 4 and 5, respectively. It should be noted that the overland flow elements are represented as being perpendicular to the stream segments. The method for choosing the length, DL, and the width, DX, of an element when flow is not perpendicular to the stream segment

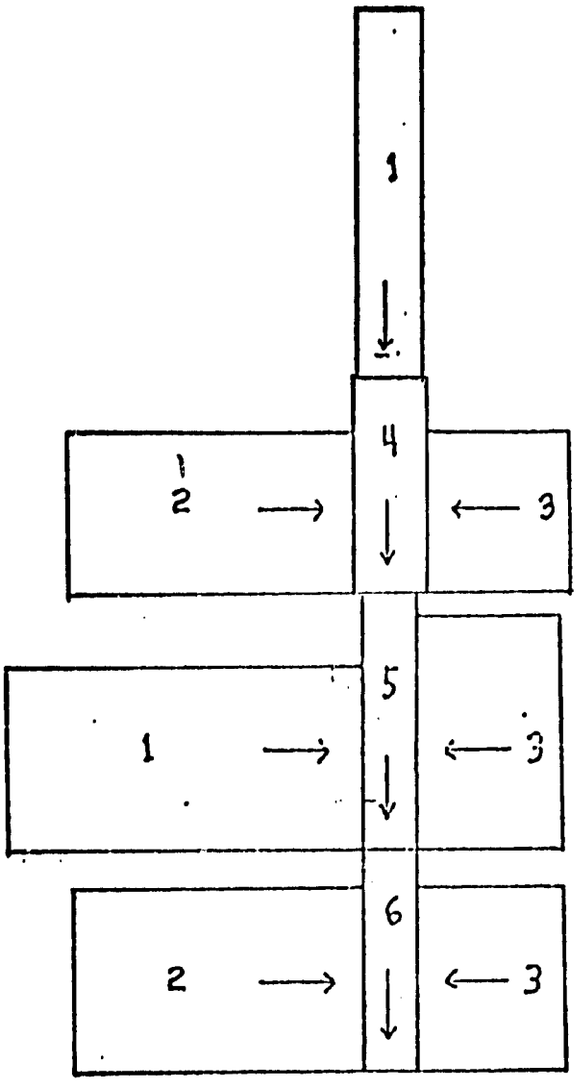


Figure 4. Rectangular element Representation of Watershed HL-1.

1 in. = 272 ft.

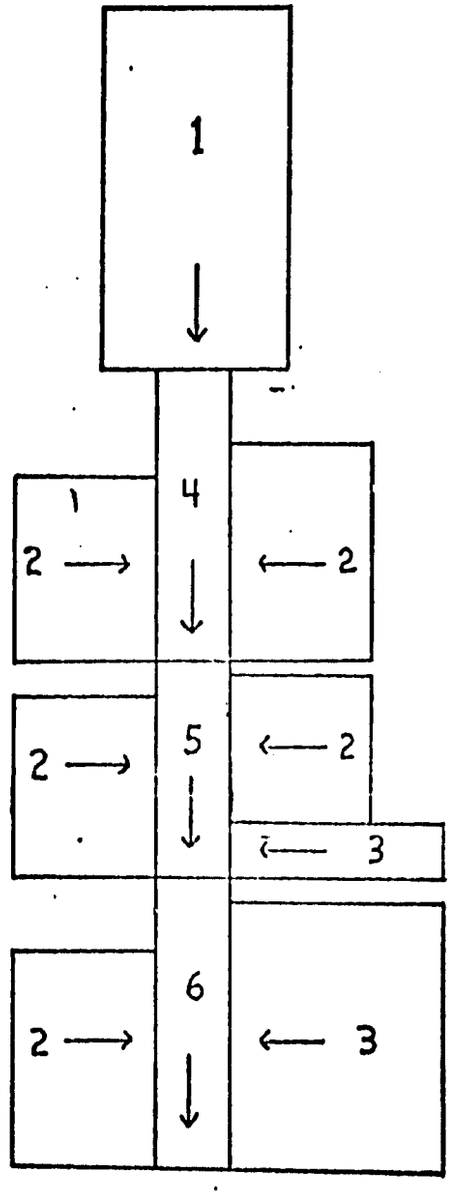


Figure 5. Rectangular Element Representation of Watershed HL-2.

1 in. = 272 ft.

is to determine the length of overland flow, DL, from the topographic map and then adjust the width, DX, so that the element area is equal to the prototype area and the flow is perpendicular to the stream segment as shown in Figure 6. It does not matter if the width of a slope element does not equal the length of a corresponding stream segment because all lateral inflows to a stream segment are lumped for determining outflow. Slope elements with the same length of flow, slope, soils and lateral inflow were combined for computational purposes. As a result, the watershed geometry representations of HL-1 and HL-2 can both be handled by three slope elements and three channel elements as numbered in Figures 4 and 5.

Component Relationships

The component processes characterized by the watershed model include interception, infiltration, depression storage and surface runoff. Interflow was not considered because it is not a common characteristic of small desert watersheds.

Interception. Interception is modeled strictly in terms of interception storage. Evaporation losses are not considered. Interception storage refers to the volume of water held on the plant surfaces. It is usually satisfied during the early stages of the more intense storms. In the

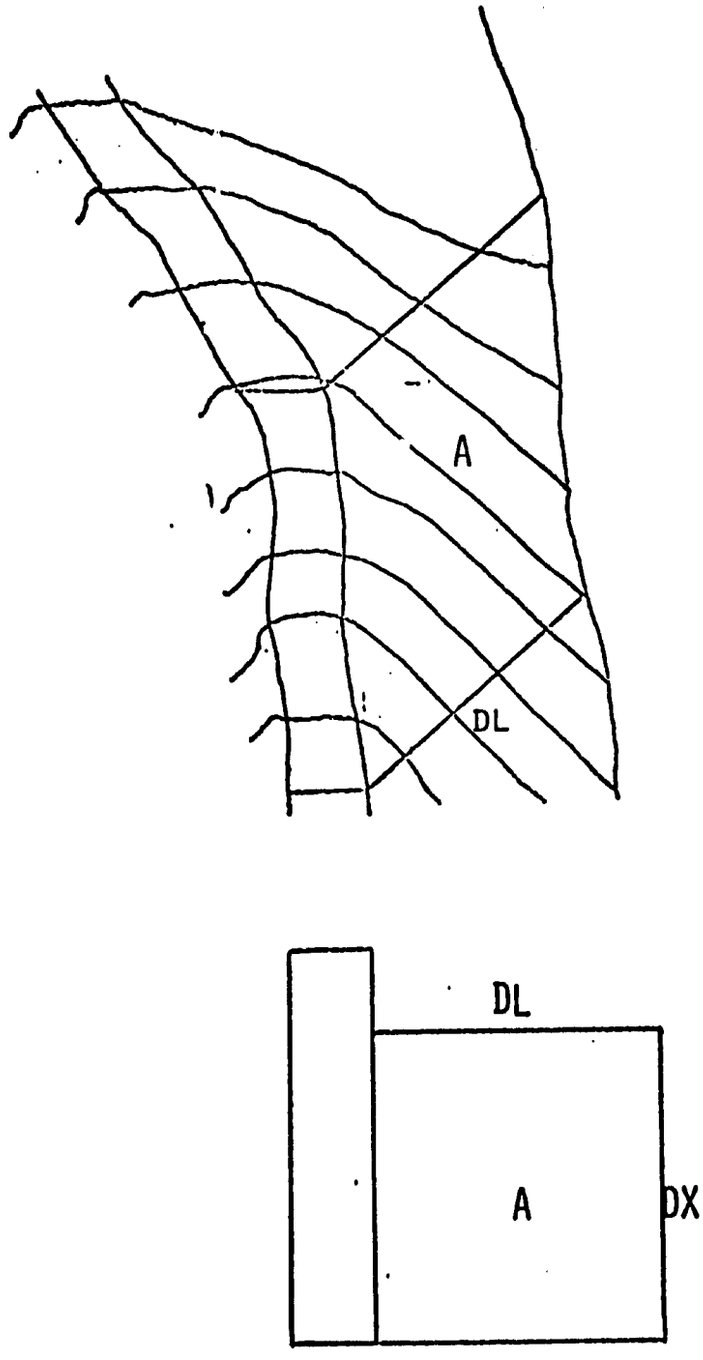


Figure 6. Method of Representing Watershed Geometry.

model, the rate of interception prior to satisfaction of interception storage is computed as the fraction of the total area covered by horizontal projected leaf surfaces times the rainfall rate.

Calculations require estimates of potential interception storage volume and the precent of total area covered by foliage. Up until potential interception is satisfied, intercepted rainfall rate is determined as follows:

$$Rate^* = Rate - Per \times Rate$$

where Rate* is the intercepted rainfall rate

Rate is the measured rainfall rate

Per is the percent of total area covered by foliage.

Reasonable estimates of Per and potential interception storage volume, Pit, are made. Errors due to oversimplification of the interception model would be more significant than small errors in parameter estimation. Fortunately, the magnitude of interception losses is relatively small on desert watersheds. Thus errors due to oversimplification of the interception model and to inaccuracies in parameter specification are not considered critical in the overall model.

Infiltration. Water that is not intercepted becomes available for infiltration upon reaching the ground surface. The infiltration relation used in this study is the form

of the Holtan equation used by Huggins and Monke in the Purdue Model. The equation, which estimates infiltration capacity as a function of soil moisture, has the form:

$$f = f_c + A[S - F)/Pore]^P$$

where f is the infiltration capacity, in/hr

f_c is the final or steady state infiltration rate, in/hr

S is the storage potential of the soil, in.

Pore is the total pore space, in.

F is the accumulated infiltration, in.

P is a parameter related to the rate of decrease in infiltration capacity with increases in soil moisture

A is the maximum potential increase in infiltration capacity above the steady state value, f_c.

An infiltration control depth can be visualized as a depth required for the hydraulic gradient to approach unity or as the depth to an impeding strata. This control depth is utilized in estimating the total pore space of the soil.

Depression Storage. Once the demands of interception and infiltration have decreased to a level below the rainfall rate, depression storage or at least a portion of it would normally be satisfied before water became available for surface runoff. This component is incorporated as part

of the functional relationship between the volume of water in storage on an element and the rate of outflow from the element. A threshold concept of depression storage or retention storage is used. Only surface detention storage, the portion of storage beyond this threshold, is available for surface runoff.

Surface Runoff. The time distribution of surface runoff from an element is determined by combining the storage-outflow relation with the following finite difference form of the continuity equation:

$$I_1 + I_2 - Q_1 + (2 \times S_1)/\Delta t = Q_2 + (2 \times S_2)/\Delta t$$

where I equals the inflow rate or rainfall excess

Q equals the outflow rate

S equals the volume of water in storage

Subscripts 1 and 2 refer to the values at the beginning and end, respectively, of time increment Δt.

Assuming zero initial conditions and a known storage-outflow relation, the equation can be solved numerically.

Determination of the storage-outflow relation vary for slope and channel elements. However, both assume a threshold value of surface depression storage and the existence of turbulent flow conditions.

The Stanford Watershed Model's overland flow techniques were used to determine surface runoff as a function of storage for slope elements. The Chezy-Manning equation was used to determine the relationship among surface detention storage at equilibrium (equilibrium occurs when, for a constant inflow, the rate of change of storage is zero), the supply rate to overland flow, Manning's n and the length and slope of the flow plane. The following relationship was derived:

$$D_e = 0.00818 \times i^{0.6} \times n^{0.6} \times L^{1.4}/S^{0.3}$$

where D_e is surface detention at equilibrium in ft^3/ft

i is the supply rate in in/hr

S is the slope in ft/ft

L is the length of overland flow in feet.

For surface runoff to occur, water must accumulate on the surface to some finite depth. The Chezy-Manning equation was again used to determine the rate of discharge as a function of surface detention depth as follows:

$$Q = 1.486/n \times y^{5/3} \times S^{1/2}$$

where y is the surface detention depth in ft

S is the slope in ft/ft

n is Manning's roughness coefficient

Q is discharge in ft^3/ft

To calculate the discharge from an element it is necessary to calculate the depth, y, at the lower edge of the flow plane. In the Stanford Watershed Model, y is empirically related to surface detention volume, D, by the following equation:

$$y = (D/L)[1.0 + 0.6(D/D_e)^3]$$

During recession flow when D_e is less than D, the ratio D/D_e is assumed to be one. The relation appears to be valid for uniform slopes with uniformly distributed inputs.

Outflow from adjacent slope elements and upslope channel elements are computed as inflow to channel elements. The rate of discharge from the channel elements is also based on the Chezy-Manning equation. The empirical overland flow methods of the Stanford Watershed Model could not be used because of the non-zero upslope boundary conditions. Instead, downslope surface detention depth, y, was assumed to be equal to the surface detention storage, D, divided by the area of an element as was done in the Purdue Model. The validity of this simplification decreases with longer elements. The approach was assumed to be valid for 300 to 400 ft. long channel elements.

Comparisons of the modified model with the Purdue Model with the same parameter values show close agreement for large events but greater differences for the smaller

events. However, the real test of the model is not how well it matches the Purdue Model but how well it fits the actual watershed and how well the optimized parameters adapt to regionalization or correlation with watershed treatments.

Parameter Selection

The primary criteria for selecting parameters to be optimized are that the parameters be difficult to measure and that the model be sensitive to changes in the parameters.

Sensitivity analysis of Purdue Model parameters performed by Huggins and Monke (1966) suggests that, of the non-measurable parameters, infiltration parameters, Manning's roughness coefficient and surface retention volume had the most influence on predicted hydrographs. Interception storage and other parameters had only minor influence. A sensitivity analysis of the modified model applied to HL-2 showed similar relationships.

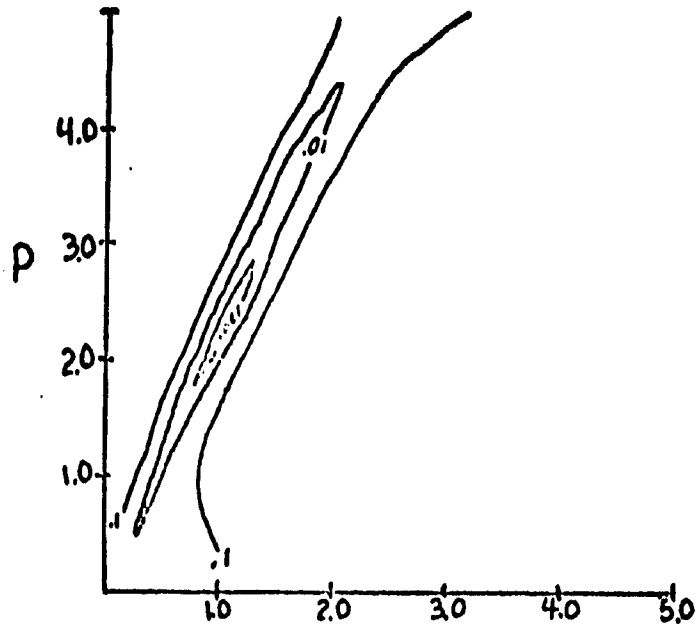
Another factor to be considered in parameter selection is parameter interaction. Parameter interaction may cause valleys in the response surface which slow the optimization process and widen the parameter range for possible pot hole occurrence. Ibbitt (1970) discovered that maximum potential infiltration rate was linearly related to the exponential decay constant in the Hortan infiltration equation. Analysis of various infiltration parameters in the

form of the Holtan equation used in this model revealed similar relationships.

The analysis, explained in detail in the next section on model calibration, employed a random grid technique to reveal the objection function response surface for variations in any two parameters. Three storms were used in obtaining objective function values for unique data (data generated from the storms by the model with a unique set of parameter values) and for actual data. Unique data response surface showed that Pore and P are linearly related as shown in Figure 7-a. Thus it is possible to arrive at various combinations of parameters along the valley which produce nearly optimal response.

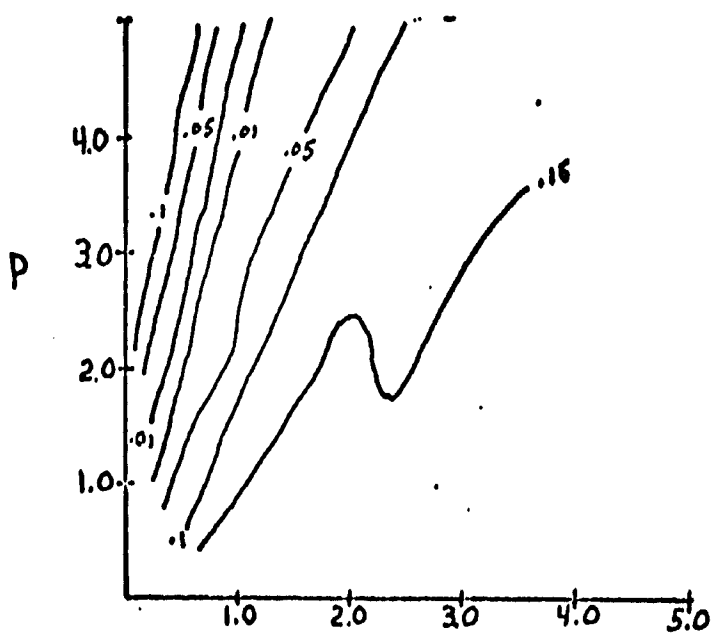
To speed up the optimization and reduce possible bias due to pot holes, it appeared desirable to fix one of the parameters as one would in trial and error calibration and optimize the other. Since it seemed possible to obtain reasonable estimates of Pore for the two soil stratifications, it was fixed and the parameter P was chosen as the one to be optimized.

The same two-parameter response surface with real data, shown in Figure 7-b, further substantiates the linear relationship between the two parameters. The indication of relatively small total pore space on the watershed agrees



a. Using Unique Data

Pore



b. Using Actual Data

Pore

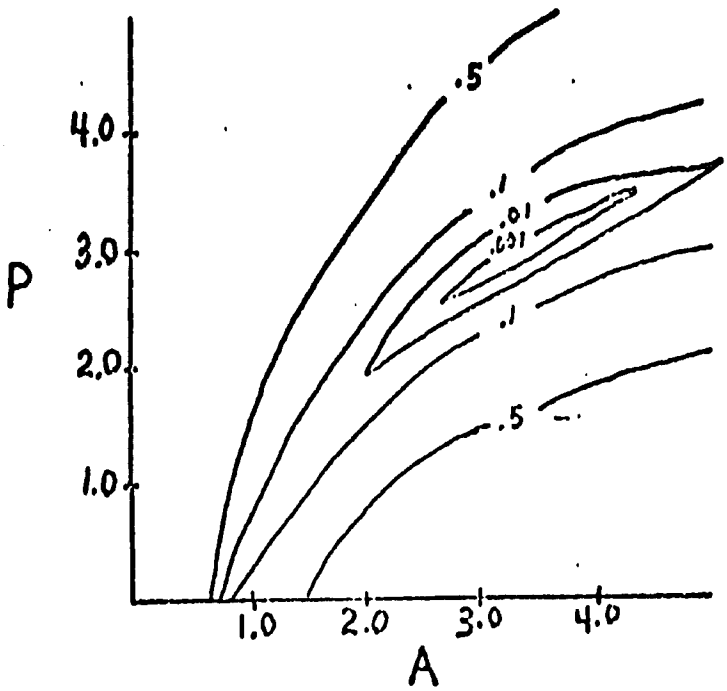
Figure 7. Objective Function Response Surface for Parameters P and Pore.

with intuitive estimates. However, there is evidence that total pore space is greater in the channels than on the slopes so two estimates were made.

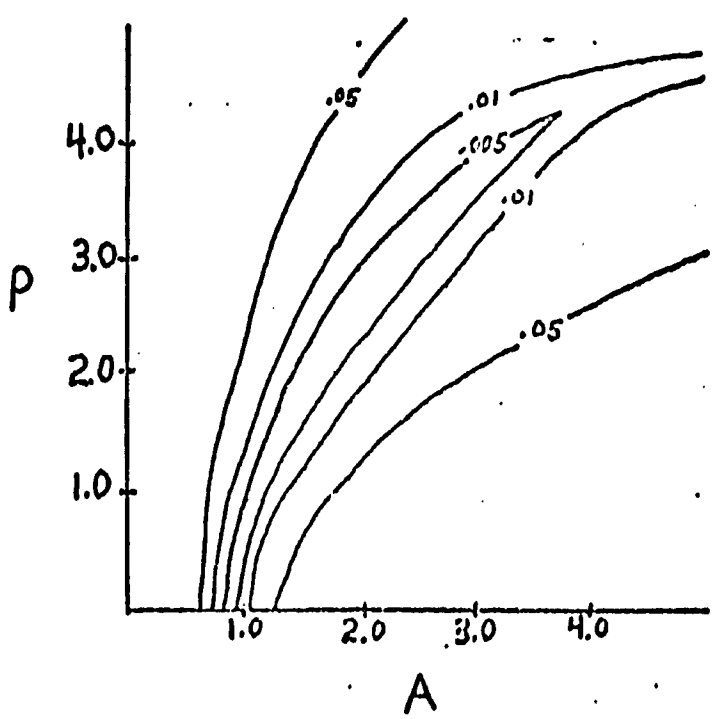
Comparisons of the P parameter with the A coefficient in the infiltration equation indicate a quadratic relation as shown in Figures 8-a and 8-b. Since it is possible to obtain realistic estimates of maximum potential infiltration, it appeared practical to fix the A parameter and optimize P.

The final selection of parameters for optimization was a retention storage parameter, Manning's roughness coefficient and, for each of the two soil stratifications, an infiltration decay parameter and a final infiltration rate.

Success at obtaining realistic parameter values would depend on the ability of the model to represent the real system, the closeness of the constants of fixed parameters to their actual values, the quality of the data and the success of the optimization technique at reaching a global optimum. Shortcomings on these points would result in biased estimates of parameter values and would limit the success in regionalizing parameters and in correlating parameters with watershed treatment.



a. Using Unique Data



b. Using Actual Data

Figure 8. Objective Function Response Surface for Parameters P and A.

Model Calibration

Model calibration was accomplished using a variety of optimization techniques available in an optimization package developed by Huelsman (1968). A complete optimization strategy beginning with response surface identification followed by extensive surface climbing and concluding with quadratic fitting was followed in calibrating the model.

Criteria Problem

Optimization methods require the selection of an error function or objective function to measure the goodness of fit of the model with an observed record. The choice of the objective function depends on the intended use for which the fitting is undertaken. Since this model is a hydrograph model, it seems appropriate to fit the entire hydrograph. The "F" criterion used by Ibbitt and O'Donnell (1971) does so by summing the squares of differences between corresponding ordinates of observed and predicted hydrographs. However, clock synchronization errors limit the usefullness of this criterion. For one of the events, records show that runoff started four minutes before rainfall began. This obvious synchronization error and any others could seriously bias the final parameter values. Additional bias may result from the F criterion if the distribution of residual errors is not normal.

To avoid bias due to synchronization errors a weighted peak and volume criterion could be selected. The criterion is compatible with most uses since peak and volume of flow events are of primary concern in water management. Maximum likelihood estimates would be extremely difficult to determine for a weighted peak-volume criteria. The "U3" criterion, a weighted peak-volume criterion, was formulated by Lichty, Dawdy and Bergman (1968) to produce parameter values without large bias. The criterion was given as:

$$U3 = 0.5\ \Sigma(\ln \text{simulated volume} - \ln \text{observed volume})^2 + \Sigma(\ln \text{simulated peak} - \ln \text{observed peak})^2 .$$

However, there is a chance of obtaining very small or even zero simulated peaks or volumes which would bias estimates when using the log transformation. Chapman (1971) discovered that the square root transformation produced similar contributions to the error function for various storm magnitudes as the logarithmic transformation. In addition, the square root transformation would not produce large bias at very small outputs.

Final selection of an objective function constitutes a value judgment. Comparisons of error distributions of peak and volume for various criteria and parameter values were analyzed to aid in the selection of an appropriately

weighted criterion. The following objective function was selected to give approximately equal weights to volume and peak and to allow greater contribution from the larger peaks while allowing smaller storms more contribution in the volume portion of the criterion:

$$OF = 1.5\Sigma\left|(\text{observed volume})^{\frac{1}{2}} - (\text{predicted volume})^{\frac{1}{2}}\right| + \Sigma[\text{observed peak} - \text{predicted peak}]^2 .$$

Optimization Strategy

The optimization strategy is based upon the hypotheses that response surface identification and previous hydrologic work can be used to obtain starting values near enough to the true optimum to allow hill climbing techniques to approach the global optimum and that the response surface exhibits quadratic behavior near the optimum. A random search technique is used to gain confidence in the above hypotheses or to allow adjustments to overcome possible violations.

The initial phase of the optimization strategy involved response surface examination. The response surface is the mapping of the objective function for a range of parameter values. Analysis in this phase was useful in selecting parameters for optimization, in identifying prominent features of the response surface and in choosing

starting values. A random grid technique was used to reveal the response surface for any two parameters. Figures 7-a, 7-b, 8-a and 8-b illustrate the response surface for pairs of infiltration parameters as delineated by contour lines of equal criterion values. Response surface identification can be time consuming. With the original eight parameters in the model, twenty-seven response surface comparisons would have been necessary to examine all combinations of pairs. To conserve computer time, the infiltration parameters for the two soil types were lumped for comparisons with other parameters and only three events were used in identifying the response surface. The events of 7/25/68, 8/07/69 and 8/13/69 were selected for this purpose.

Two separate response surface comparisons were made: one with unique data or data generated by a unique set of parameter values, and the other with actual data. The objective function for the unique data was a least squares comparison of hydrograph ordinates of unique hydrographs with predicted hydrographs. The objective function for actual data was the weighted peak-volume criteria. Thus a comparison of criterion values between the two response surfaces is not appropriate.

The second phase of optimization consisted of extensive surface movement toward the optimum. Six runoff

events from watershed HL-2 were selected for this phase of optimization. Comparisons of the fitting with independent data were made periodically to test the validity of the optimization. Good starting values help avoid local optima and can increase the speed of optimization. To overcome possible inadequacies in the chosen starting values, a random search technique capable of large steps was used initially to locate improvements in the objective function over that of the starting value. If all improvements were found within ± 30 percent of every starting value, a deterministic pattern search would proceed from the best point. Otherwise the search would proceed from the most successful point outside the range and from the initial starting values.

The pattern search continued until one unsuccessful iteration with reduced step size had been completed. In the case where more than one search was conducted, phase three began from the best point when the final parameter values were close and from both points when discrepancies still existed. Points with unrealistic parameter values were eliminated.

In phase three, the parameters were assumed to be near enough to a local or global optimum to be fit by a quadratic convergent method in just a few iterations. Six iterations of the Fletcher-Powell method were necessary for

quadratic convergence. To confirm the final parameter values, a random search technique with reduced step sizes was employed about the apparent optimum. The random search permits the optimization to jump out of small pot holes near the true optimum.

In the event that more than one optima was located and the parameter values were realistic, the point with the lowest criterion value was selected as the true optimum. In addition, all parameter sets were validated with independent data.

CHAPTER 4

RESULTS AND DISCUSSION

The results and analysis are presented for the model calibration, for validation of the optimized parameters in the model and for the model itself.

Calibration Results

Following the strategy outlined in the previous section, optimization began with response surface identification. These results were helpful in deciding to eliminate total pore space, Pore and the A coefficient as parameters to be optimized. Certain features of the response surface became evident such as a negative linear relation between the two steady state infiltration rates and a quadratic relation between the steady state infiltration rate and the coefficient. These interactions could slow the optimization and produce conditions which may result in biased final parameter values.

The analysis in phase one was used to obtain the starting values for the random search in phase two. In addition, Pore values were fixed at 1.0 in. for slope elements and 2.0 in. for the channel elements. The A

coefficients were estimated to be 3.7 in/hr for slope elements and 4.0 in/hr for the channel elements.

The random search discovered improved points outside the vacinity of the starting point. Therefore the deterministic pattern search proceeded from two points--the starting point and the most improved point. Table 1 shows the initial and final parameter values and the corresponding objective function values for both searches. The final parameter values seem to suggest the pot hole occurrence observed by Ibbitt (1970).

Phase three fitting was attempted from both of the final points determined by the pattern search. The Fletcher-Powell method reached the thirty-minute time limit before completing the six iterations required for quadratic convergence. Little improvement occurred. The failure of the Fletcher-Powell method was evidently due to the flat valleys encountered and the irregularity of the response surface. This would tend to discount the hypothesis of quadratic behavior near the optimum.

The random search with scaled step sizes to allow the search to explore valleys was started from the most improved point. Slight improvement occurred. The final parameters of the entire strategy are given in Table 2. Sensitivity studies about this point show high sensitivity to Manning's coefficient, moderate sensitivity to the steady

Table 1

Parameter Values of the Pattern Search

'arameters	Starting Point Search Initial Values	Starting Point Search Final Values	Improved Point Search Initial Values	Improved Point Search Final Values
'anning's 'oefficient	0.035	0.035	0.04	0.037
(slope)	5.0	16.475	9.35	18.335
c (slope)	0.3	0.938	0.449	0.926
(channel)	6.0	13.8	10.0	15.014
etention epth	0.01	0.004	0.0	0.001
c (channel)	0.4	0.31	0.7	0.42
bjective unction alue	0.914	0.290	0.532	0.285

Table 2

Final Parameters of the Optimization Strategy

Parameter	Value
Manning's Coefficient	0.0355
P (slope)	19.3963
f_c (slope)	0.9464
P (channel)	14.4620
Retention storage	0.0059
f_c (channel)	0.3240
Objective Function Value	0.2827

ate infiltration rate on the slope elements and low sen-
tivity to the other parameter values. Comparisons of the
served and predicted peaks and volumes for the final param-
er values are displayed in Table 3. The comparisons are
ose but the validation results are necessary to determine
e adequacy of the optimized parameter values.

The results of the calibration indicate a highly
regular response surface with extensive flat valleys. The
mbination of random and pattern searches was most success-
l in calibrating the model. However, the behavior of the
sponse surface and the occurrence of pot holes limits the
ccess of any technique for finding the unique optimum
rameter values.

Validation Results

To validate the parameter values obtained during
timization, the model was run with independent data from
e same watershed and with data from a different watershed
ing parameter values from every other successful iteration.
e results as displayed in Figure 9 show the progression
the calibration and the response of the corresponding
lidations during the optimization. Some improvement in
e validation samples appear to occur during the initial
ages of optimization. However, the results diverge or
cillate as the fitting progresses. It is evident that

Table 3

Peak and Volume Comparisons for Final Calibration

	PEAK (in/hr)		VOLUME (in.)	
Date	Observed	Predicted	Observed	Predicted
8/18/67	1.24	1.200	0.483	0.403
8/07/69	0.33	0.495	0.108	0.168
8/13/69	0.1665	0.1918	0.0537	0.054
7/08/70	0.2763	0.2305	0.0942	0.082
8/14/70	0.0035	0.0101	0.0023	0.002
7/14/71	1.4	1.4435	0.6862	0.684

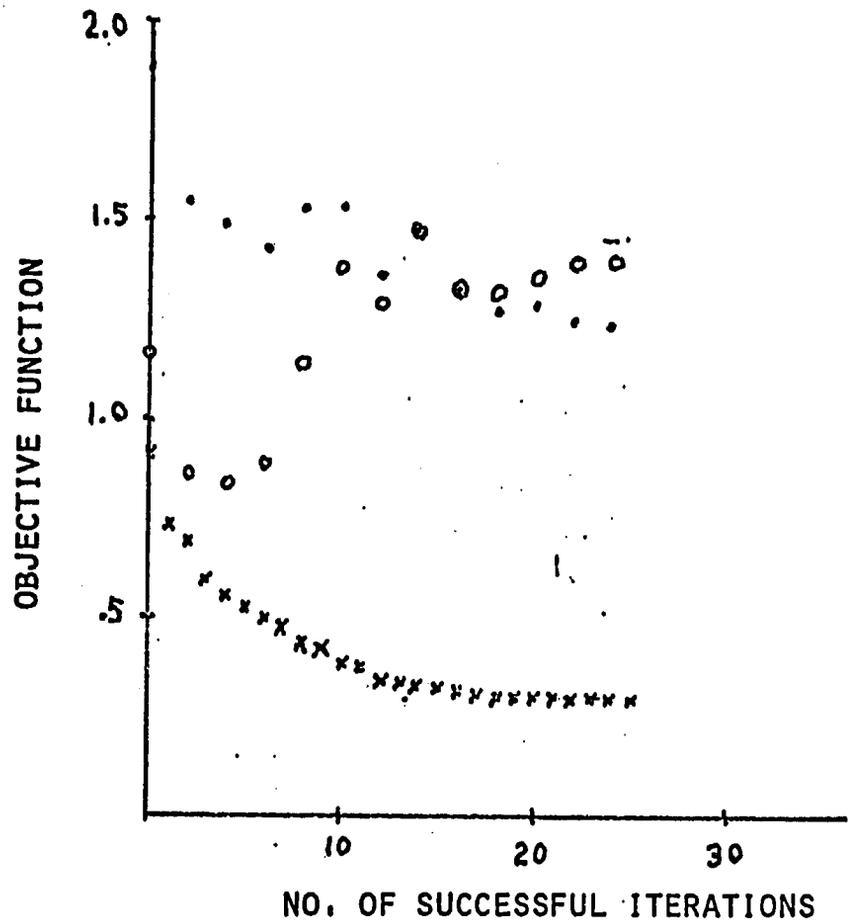


Figure 9. Objective Function Response during Optimization.

X Calibration Sample Watershed HL-2
O Validation Sample Watershed HL-2
• Validation Sample Watershed HL-1

more than six events would be required for adequate calibration of the model. The prospects of obtaining unique parameters even with a much larger calibration sample are remote.

Comparisons of the observed and predicted peaks and volumes of the validation samples using the final parameter values are given in Tables 4 and 5. Variations appear for all events. The major portion of the error cannot be attributed to one event. However, runoff from the long moderate intensity storm of 9/03/70 in Table 4 was always underpredicted with the parameters used in Figure 9. The duration and intensity of this storm makes it particularly sensitive to the steady state infiltration rate. This would indicate that the steady state infiltration rates may have been too high.

Analysis of the Model

Analysis of the final parameter values indicate that surface retention storage is very small and could possibly be eliminated from the model. However, if management activities are to be considered then retention storage should not be eliminated because of the possible effect treatments might have on this component. The final values of Manning's coefficient and the steady state infiltration rate of the channel elements were about the same as might

Table 4

Peak and Volume Comparisons Using Final Parameter Values for Independent Data from Watershed HL-2

	PEAK (in/hr)		VOLUME (in.)	
Date	Observed	Predicted	Observed	Predicted
7/17/67	0.788	0.3909	0.446	0.182
7/28/68	0.036	0.0607	0.0175	0.019
7/21/70	1.02	0.7564	0.427	0.33
7/31/70	0.1909	0.0612	0.0628	0.021
8/01/70	0.3503	0.0568	0.1178	0.026
9/12/70	0.037	0.0038	0.0134	0.001

Table 5

Peak and Volume Comparisons Using Final Parameter Values for Independent Data from Watershed HL-1

	PEAK (in/hr)		VOLUME (in.)	
Date	Observed	Predicted	Observed	Predicted
7/08/68	0.0608	0.0181	0.0148	0.003
8/05/69	0.004	0.0018	0.001	0.000
8/07/69	0.519	0.5413	0.192	0.171
7/08/70	0.182	0.074	0.0707	0.027
7/31/70	0.532	0.1927	0.1607	0.064
9/03/70	0.1426	0.001	0.147	0.000

be expected. However, the P coefficients and the steady state infiltration rate on the slope elements were much higher than expected. It is worthwhile to note that where the validation sample from watershed HL-2 showed the best results in Figure 9, the parameter values could be considered as appropriate intuitive estimates. Perhaps intuitive estimation aided by hydrograph analysis would be more effective than optimization for this type of component model. In any event, the computation time needed for optimization would make the intuitive estimation worthy of consideration.

The computational requirement is an important aspect of the model for optimization. Although the model is relatively efficient requiring one or two seconds for evaluating each event, the optimization time is very demanding. The deterministic and random searches of phase two required over one hour of computing time on the CDC 6400. The Fletcher-Powell method was not completed and it still required over one hour of computing time. The final random search ran for about thirty minutes. The obvious need for more data for effective calibration would make optimization of the model very expensive.

The simulated and observed hydrographs for the large high intensity storm of 7/14/71 are shown in Figure 10. The peak and volume comparisons are excellent and the hydrograph

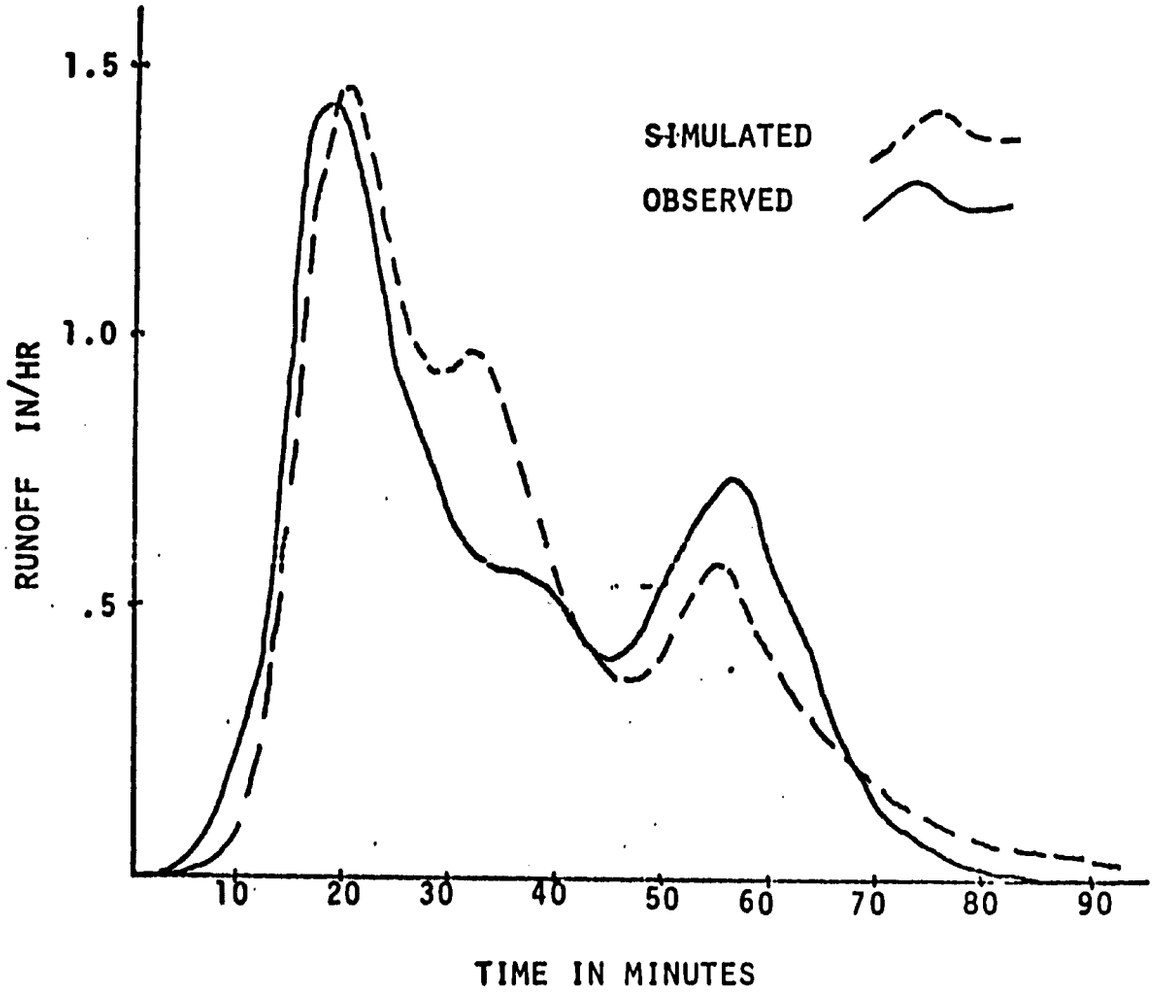


Figure 10. Simulated and Observed Hydrographs for Storm of 7/14/71.

imulation is representative. The fit for all of the large igh intensity storms was good even for those in the valida-ion sample. Simulation of these storms would be less ensitive to the infiltration component while the routing spects would be more critical. Although the channel outing method was suspect, it would appear that the infil-ration component of the model requires the most improvement specially for optimization purposes.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

The ultimate aim of this study was to explore the potential of numerical optimization for obtaining unique optimum parameter values of a watershed model. The occurrence of pot holes and the variability in parameter values due to pot hole occurrence along extensive valleys makes estimation of true optimum parameter values extremely difficult. It also indicates that model modification is needed if the model is to be calibrated effectively. Modifications should reduce parameter correlations and eliminate threshold values or infiltration potential concepts.

The data dependency of the optimized parameters further complicates the problem. It seems likely that model modification could eliminate elongated valleys and pot holes and even improve accuracy. This would greatly reduce the data dependency effect. With sufficient data, it may be possible to optimize for regional values. However, a model's lack of equivalence to the physical system may prevent an optimization from distinguishing treatment effects on the optimized parameter values from variations caused by data.

Because of the high cost of optimizing the more complex physically based models and because of the problem posed by the response surface, it is believed that multivariable optimization of these models is not economical at this time. Research in developing physically based models where all but one or two parameters must be measured or specified might prove worthwhile especially where simulation of treatments is of concern.

It may also be possible to develop more empirical models with smoother response surfaces which may be optimized effectively for regional application. The capability of these models to represent the system would affect the data dependency of the parameter optimization. An adequate method of handling infiltration would be of primary concern. The thesis by Ibbitt (1970) and the Stanford Watershed Model (Crawford and Linsley 1966) would provide a good starting point for such a study.

APPENDIX

COMPUTER PROGRAM

The program, written in Fortran IV, was developed from the Purdue Model program of Huggins and Monke (1966). All data input is accomplished in the main program, MODEL. The essential features of the model are developed in Subroutine ANLYZ. ANLYZ can either be called directly from the main program or be called by any techniques in the optimization package by Huelsman (1968). For the latter, the optimization technique would be called from the main program. Newton's method of finding the zero of a function was utilized to solve the continuity equation for each element.

Functions RAIN and FILT were taken directly from the Purdue Model to determine interception and infiltration, respectively. The purpose of subroutine REVISE is to set initial conditions and convert functions into convenient form for computation. Subroutine DRY, which was also extracted from the Purdue Model, was used to reduce the computational effort when overland flow is not occurring on the watershed. The other subroutines (RDOPT, PROPT, OPT2, OPT3, OPT4 and OPT9) are explained in the paper by Huelsman (1968).

Listing of Programming Symbols

Programming symbols necessary to follow the program are defined in the following list.

A(I)	= Infiltration coefficient for the I-th element.
ADIR	= Average surface retention depth in inches.
AREA(JK)	= Area of watershed JK in acres.
AR(I,JK)	= Area in sq. ft. of element I on watershed JK.
ASM(I,J,JK)	= Antecendent soil moisture on element I, for storm J and on watershed JK.
B(I)	= coefficient for runoff function for the I-th element.
CA(I)	= Maximum potential increase in infiltration capacity above the steady state value--in/hr. Used in computing the A coefficient.
COEF(I)	= Coefficient used in determining depth at equilibrium on element I.
CONV(JK)	= Constant to convert outflow from watershed JK from ft^3/sec to in/hr.
CU(I)	= Constant to convert rates on element I from in/hr to ft^3/sec.
CU1(I)	= Constant for converting from inches per element area to cubic feet.
DIR	= Twice the surface retention volume divided by DT in ft^3/sec.
DT	= Time increment used in the analysis in sec.
DT1	= Time increment used in a previous simulation in sec.
DTM	= Time increment used in the analysis in min.
DX(I,JK)	= Width of element I on watershed JK in feet.

DY(I,JK)	= Length of element I on watershed JK in feet.
F1(I,JK)	= Portion of outflow from element 1 on watershed JK flowing into element I.
F2(I,JK)	= Portion of outflow from element 2 on watershed JK flowing into element I.
F3(I,JK)	= Portion of outflow from element 3 on watershed JK flowing into element I.
FC(I)	= Steady-state infiltration capacity rate of I-th element in ft^3/sec.
FLIN	= Net rate at which water is flowing into an element in ft^3/sec.
FLINS(I)	= The sum of the inflow, outflow and the rate of change of storage for the I-th element at the end of each iteration in ft^3/sec.
GWC(I)	= Gravitational water capacity of the I-th element in ft^3.
IDATE(KK,JK)	= Date of storm KK on watershed JK.
ISTART	= Number controlling printout of output. = 1 gives no printout. = 2 gives all printout including hydrograph. = 3 gives peak and volume estimates.
NEXP	= Drainage exponent used in infiltration calculations.
NSET	= Number of data sets.
NOBS(JK)	= Number of observations for watershed JK.
OBPK(KK,JK)	= Observed peak for storm KK on watershed JK in in/hr.
OBVOL(KK,JK)	= Observed volume for storm KK on watershed JK in inches.
P(I)	= Infiltration coefficient for the I-th element.
PER	= Ground surface covered by foliage in percent of total area.

PIT	= Potential interception storage volume in inches.
PIV(I)	= Moisture deficiency (relative to saturation) for the I-th element in ft^3.
PR	= Word controlling printout of input data.
PREC	= Total amount of rainfall of current storm in inches.
PRPK(KK,JK)	= Predicted peak for storm KK on watershed JK in inches per hour.
PRVOL(KK,JK)	= Predicted volume for storm KK on watershed JK in inches.
Q1	= Watershed outflow rate in/hr.
Q2	= Rate of discharge from an element in ft^3/sec.
QI(I)	= Discharge rate from I-th slope element of I-1 channel element in ft^3/sec.
Q(I)	= Discharge rate from the I-th element in ft^3/sec.
RATE	= Rate at which water is being supplied to an element by rainfall in ft^3/sec.
RC(I,KK,JK)	= The rainfall intensity corresponding to the I-th time interval for storm KK on watershed JK--in/hr.
RIT	= Rate of interception in ft^3/sec.
RN	= Manning's roughness coefficient.
SITE(JK)	= Name of watershed JK.
S(I)	= Twice the element storage volume divided by the time increment in ft^3/sec.
SS(I)	= The change in value of S(I) during an iteration.
SSTOR	= Twice the volume of water in storage divided by the time increment in ft^3/sec.

SUPP. = Rate at which water is being supplied to satisfy infiltration capacity in ft^3/sec.

T = Time in min.

TC(I,KK,JK) = The time at which a change in rainfall intensity occurred for storm KK on watershed JK in min.

VOL = The total volume of runoff predicted by simulation in inches.

Listing of Program

```
      PROGRAM MODEL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION TITLE(10),NUM(20)
      COMMON /OPT/X(20),XL(20),XU(20),KX(20),XP(20),H(20),R(20),W(20),G(
     120),PARAM(10,7),NOPT(10,10),N,NH,YERR,ITER,ERMIN,ITMAX,ALFA(8)
      COMMON /DAT/NEXP,SUPP,DT,PAR(6),TC(60,8,3),RC(60,8,3),PER,PINT,NDT
     1,PORC(6),CA(6),FCAP(6),ASM(2,6,3),AR(6,3),SLOP(6,3),AREA(3),CU(6,3
     2),CUI(6,3),IDATE(8,3),ISTART,KRAIN(8,3),COEF(6)   ,SITE(3),DX(6,3)
     3,DY(6,3),CONV(3),F1(6,3),F2(6,3),F3(6,3)
      COMMON /ERROR/OBPK(8,3),OBVOL(8,3),PRPK(8,3),PRVOL(8,3),JK,NOBS(3)
      DATA  NOBS(1),NOBS(2),NOBS(3)/6,6,6/
      DATA PRI /4HPRIN/
      READ(5,20)TITLE
      WRITE(6,21)TITLE
      READ(5,22)NSET
      DO 80 L=1,NSET
      N=NOBS(L)
      DO 30 I=1,N
      READ (5,66) IDATE(I,L),PR,SITE(L),(ASM(LI,I,L),LI=1,2)
      READ(5,69) OBVOL(I,L),OBPK(I,L)
      K = 1
    1 READ (5,58)J,TC(K,I,L),RC(K,I,L)
      K = K + 1
      IF (J) 2,1,2
    2 IF (K.GT. 60) GO TO 85
      K=K-1
      KRAIN(I,L)=K
      IF(PRI.EQ.PR)WRITE(6,53)IDATE(I,L),SITE(L),(TC(J,I,L),RC(J,I,L),J=
     11,K)
   30 CONTINUE
      READ(5,49)(DX(I,L),I=1,6)
      READ(5,49)(AR(I,L),I=1,6)
      AREA(L)=(AR(1,L)+AR(2,L)+AR(3,L)+AR(4,L)+AR(5,L)+AR(6,L))/43560.
      READ(5,49)(SLOP(I,L),I=1,6)
      DO 40 I=1,6
```

```
      READ(5,45)F1(I,L),F2(I,L),F3(I,L)
      CU(I,L)=AR(I,L)/43560.
      CU1(I,L)=AR(I,L)/12.
      DY(I,L)=AR(I,L)/DX(I,L)
   40 CONTINUE
      CONV(L)=CU(1,L)+CU(2,L)+CU(3,L)+CU(4,L)+CU(5,L)+CU(6,L)
   80 CONTINUE
      READ(5,59)PER,PINT,DT,NDT,NEXP,PORE(1),PORE(4),CA(1),CA(4),FCAP(1)
     1,FCAP(4)
      PORE(2)=PORE(1) $ PORE(3)=PORE(1) $ PORE(5)=PORE(4) $ PORE(6)=PORE
     1(4) $ CA(2)=CA(1) $ CA(3)=CA(1) $ CA(5)=CA(4) $ CA(6)=CA(4) $ FCAP
     2(2)=FCAP(1) $ FCAP(3)=FCAP(1) $ FCAP(5)=FCAP(4) $ FCAP(6)=FCAP(4)
      NEXP=3
      IF (NDT.GT.201) GO TO 70
      N=0
      CALL RDOPT
      JK=1
      ISTART=3
      CALL OPT3
      ISTART=2
      CALL PROPT
      GO TO 95
   20 FORMAT(10A8)
   21 FORMAT(///,5X,10A8)
   22 FORMAT(I5)
   45 FORMAT(3(5X,F5.3))
   49 FORMAT (6(2X,F8.4))
   53 FORMAT(1H1,30X,19HRAINFALL HYDROGRAPH/32X,9HSTORM OF A8,5X,9HWATER
     1SHED A6 / 1H0 20X  11HTIME - MIN. 7X 23HRAINFALL RATE - IN./HR. /
     2(20X F9.1, 18X,F6.2))
   56 FORMAT (52H1DATA INPUT EXCEEDS RANGE OF DIMENSION SPECIFICATION )
   58 FORMAT (I2,F8.0,F10.0)
   59 FORMAT(42X,F6.2,29X,F3.0,/17X,F5.0,38X,I3,/20X,I2,36X,F6.0,10X,F6.
     10,/,26X,F6.0,10X,F6.0,/,26X,F6.0,10X,F6.0)
   66 FORMAT (10X  A8, 6X A4, 6X,A6,10X,F5.4,5X,F5.4)
   69 FORMAT(10X,2F10.2)
   70 WRITE(6,68)
   68 FORMAT (46H1RAINFALL DATA EXCEEDS DIMENSION SPECIFICATION )
      STOP
   85 WRITE (6,68)
   95 STOP
      END


      SUBROUTINE ANLYZ
      DIMENSION O(6),OI(7),S(6),SS(6),A(6),PIV(6),P(6),FC(6),GWC(6),B(6)
     1,DIR(6),FLINS(6),T(1000),Q1(1000),PIT(6)
      COMMON /OPT/X(20),XL(20),XU(20),KX(20),XP(20),H(20),R(20),W(20),G(
     120),PARAM(10,7),NOPT(10,10),N,NH,YERR,ITER,ERMIN,ITMAX,ALFA(8)
      COMMON /DAT/NEXP,SUPP,DT,PAR(6),TC(60,8,3),RC(60,8,3),PER,PINT,NDT
     1,PORE(6),CA(6),FCAP(6),ASM(2,6,3),AR(6,3),SLOP(6,3),AREA(3),CU(6,3
     2),CU1(6,3),IDATE(8,3),ISTART,KRAIN(8,3),COEF(6)   ,SITE(3),DX(6,3)
     3,DY(6,3),CONV(3),F1(6,3),F2(6,3),F3(6,3)
      COMMON /EROR/OBPK(8,3),OBVOL(8,3),PRPK(8,3),PRVOL(8,3),JK,NOBS(3)
      ND1=200
      KPR=2
      LST=NOBS(JK)
```

```
      DO 39 I=1,6
   39 PAR(I)=X(I)
      DO 41 I=1,3
      B(I)=1.486/PAR(1)*DX(I,JK)*SQRT(SLOP(I,JK))
      COEF(I)=.000818*(PAR(1)/CU(I,JK))**.6*(DY(I,JK)**1.6)/SLOP(I,JK)**
     1.3
      P(I)=PAR(2)
      FC(I)=CU(I)*PAR(3)
   41 CONTINUE
      DO 42 I=4,6
      B(I)=1.486/PAR(1)*DX(I,JK)/(2./DT*DX(I,JK)*DY(I,JK))**1.6667*SQRT(
     1SLOP(I,JK))
      P(I)=PAR(4)
      FC(I)=CU(I)*PAR(6)
   42 CONTINUE
      DO 99 KK=1,LST
      IF(JK.EQ.3.AND.KK.GT.4) NDT=999
      Q1(1)=0.
      CALL REVISE(B,DIR,KPR,QI,A,PIV,P,FC,GWC,FLINS,Q,S,SS,PIT,KK,LST)
      PK=0.00000001
      ITR = 1
      VOL=0.
      PREC=0.
      DTM = DT/60.
      T(1) = TC(1,KK,JK)
      IF (ISTART.EQ.2) WRITE(6,9)IDATE(KK,JK),(PAR(I),I=1,6)
    9 FORMAT (1H0 30X 19H RUNOFF  HYDROGRAPH / 32X 9HSTORM OF  AR / 1H0
     1 ,10X,*PARAMETER SET      PAR(1) =*,F7.4,5X,*PAR(2) =*,F7.4,5X,*PAR
     2(3) =*,F7.4,/,28X,*PAR(4) =*,F7.4,5X,*PAR(5) =*,F7.4,5X,*PAR(6) =*
     3,F7.4)
      IF(ISTART.EQ.2)WRITE(6,10)T(1),Q1(1)
   10 FORMAT (1H0 31X 17HRUNOFF HYDROGRAPH / 1H0 20X 11HTIME - MIN. 9X 1
     19HDISCHARGE - IN./HR. / 20X F9.1, 18X F6.3)
      DO 60 L = 2,NDT
      T(L) = T(L-1)
      DO 40 J=1,KPR
      T(L) = T(L) + DTM
      IF (T(L) - TC(ITR,KK,JK)) 13,11,11
   11 IF (ITR - KRAIN(KK,JK)) 12,67,67
   12 ITR = ITR + 1
      PREC=PREC+RC(ITR,KK,JK)*(TC(ITR,KK,JK)-TC(ITR-1,KK,JK))/60.
   13 DO 50 M=1,3
      SSTOR = S(M) + SS(M)
      RATE=CU(M,JK)*RC(ITR,KK,JK)
      FLIN=RAIN(RATE,PIT(M),PER)
      SUPP = .5*SSTOR + FLIN
      FLIN = FLIN - FILT(A(M),PIV(M),P(M),FC(M),GWC(M))
      IF(FLIN)15,15,16
   15 DE=.0000000001
      GO TO 17
   16 DE=COEF(M)*FLIN**.6
   17 FHS = FLINS(M) +FLIN
      IF (FHS.GT.DIR(M)) GO TO 22
   18 S(M) = FHS
      SS(M) = 0.
      FLINS(M) = FLIN + FHS
      IF (Q(M).EQ.0.) GO TO 50
   19 D = -Q(M)
      Q(M) = 0.
      GO TO 48
```

```
22 EX= FHS*1.E-4
   IF (SSTOR.GT.DIR(M)) GO TO 28
24 SSTOR = FHS
   GO TO 28
26 SSTOR = SSTOR - FS/(1. + 1.66667*Q2/D)
28 D = SSTOR - DIR(M)
   D=D/2.*DT/DX(M,JK)
   IF(D.GT.DE) DE=D
   Y = D/DY(M,JK)*(1.0+.6*(D/DE)**3)
   Q2=B(M)*Y**1.66667
   FS = Q2 + SSTOR - FHS
   IF (FS.LT.-EX.OR.FS.GT.EX) GO TO 26
   D=Q2-Q(M)
   Q(M) = Q2
   SS(M) = SSTOR - S(M)
   S(M) =SSTOR
   FLINS(M) = FLIN + SSTOR - Q2
48 QI(M)=QI(M)+D
50 CONTINUE
   QI(4)=0.0
   DO 21 M=4,6
   SSTOR=S(M)+SS(M)
   RATE=CU(M,JK)*RC(ITR,KK,JK)
   RR=RAIN(RATE,PIT(M),PER)
   FLIN=QI(M)+RR+QI(1)*F1(M,JK)+QI(2)*F2(M,JK)+QI(3)*F3(M,JK)
   SUPP = .5*SSTOR + FLIN
   FLIN = FLIN - FILT(A(M),PIV(M),P(M),FC(M),GWC(M))
   FHS = FLINS(M) +FLIN
   IF (FHS.GT.DIR(M)) GO TO 23
   S(M) = FHS
   SS(M) = 0.
   FLINS(M) = FLIN + FHS
   IF (Q(M).EQ.0.) GO TO 21
   D = -Q(M)
   Q(M) = 0.
   GO TO 20
23 EX= FHS*1.E-4
   IF (SSTOR.GT.DIR(M)) GO TO 25
   SSTOR = FHS
   GO TO 25
27 SSTOR = SSTOR - FS/(1. + 1.66667*Q2/D)
25 D = SSTOR - DIR(M)
   Q2=B(M)*D**1.66667
   FS = Q2 + SSTOR - FHS
   IF (FS.LT.-EX.OR.FS.GT.EX) GO TO 27
   D=Q2-Q(M)
   Q(M) = Q2
   SS(M) = SSTOR - S(M)
   S(M) =SSTOR
   FLINS(M) = FLIN + SSTOR - Q2
20 QI(M+1)=QI(M+1)+D
21 CONTINUE
40 CONTINUE
   Q1(L)=QI(7)/CONV(JK)
   VOL = VOL + Q1(L)
   IF (Q1(L).LT..0005.AND.ITR.GT.2) CALL DRY  ( PIV,S,Q,A,P,FC,GWC,QI
  1,ITR,FLINS,PREC,PIT,T(L),JK,KK)
   IF(Q1(L)-PK)59,31,31
31 PK=Q1(L)
   TP=T(L)-T(1)
```

```
59 IF(ISTART.EQ.2)WRITE(6,65) T(L),Q1(L)
65 FORMAT (20X F9.1, 18X F7.4)
60 CONTINUE
   WRITE(6,79)
79 FORMAT(5X,*OUTFLOW EXCEEDS DIMENSION SPECIFICATION*)
67 IF(Q1(L-1).GT..001) GO TO 13
   VOL = (VOL - .5*Q1(L-1))*DT*FLOAT(KPR)/3600.
   MAXIT=ITMAX-1
   IF(ISTART.GE.2) WRITE(6,70)IDATE(KK,JK),(PAR(I),I=1,6),PREC,VOL,PK
70 FORMAT(1H0,32X,9HSTORM OF  A8 /  5X,*PARAMETER SET   PAR(1)=*,F7.4
  1,5X,*PAR(2)=*,F7.4,5X,*PAR(3)=*,F7.4,5X,*PAR(4)=*,F7.4,5X,*PAR(5)=
  2*,F7.4,5X,*PAR(6)=*,F7.4,/,5X,*RUNOFF VOLUME FROM *,F5.2,21H INCHE
  1S OF RAINFALL = F6.3, 4H IN.   ,4X,*PEAK FLOW =*,F7.4,* IN./HR.*)
   PRVOL(KK,JK)=VOL
   PRPK(KK,JK)=PK
99 CONTINUE
   ISTART=3
   RETURN
   END


   FUNCTION FILT(A,PIV,P,FC,GWC)
   COMMON /DAT/ NEXP,SUPP,DT
   IF (PIV) 45,50,5
 5 FILT = A*PIV**P + FC
   IF (FILT.GT.SUPP) FILT = SUPP
   IF (PIV.LT.GWC) GO TO 25
   PIV = PIV - FILT*DT
   RETURN
25 PIV = PIV - (FILT - FC*(1. - PIV/GWC)**NEXP)*DT
   RETURN
45 IF(ISTART.EQ.2) WRITE(6,46) PIV
46 FORMAT (52H WATER CONTENT OF SOIL EXCEEDS TOTAL POROSITY, PIV = 1P
  1 E15.7)
   PIV = 0.
50 IF (FC.GT.SUPP) GO TO 65
   FILT = FC
   RETURN
65 FILT = SUPP
   PIV = (FC - FILT)*DT
   IF (PIV.GT.GWC) PIV = GWC
   RETURN
   END


   FUNCTION RAIN(RATE,PIT,PER)
C  DETERMINATION OF NET RAINFALL RATE ** CONSTANT INTERCEPTION RATE
   IF (PIT) 40,50,10
10 RIT = PER*RATE
   IF (RIT - PIT) 15,15,20
15 RAIN = RATE - RIT
   PIT = PIT - RIT
   RETURN
20 RAIN = RATE - PIT
   PIT = 0.
   RETURN
```

```
40 IF(ISTART.EQ.2) WRITE(6,41) PIT
41 FORMAT (60H INTERCEPTION VOLUME EXCEEDS MAXIMUM POTENTIAL VOLUME,
  1PIT = 1PE15.7)
   PIT = 0.
50 RAIN = RATE
   RETURN
   END


   SUBROUTINE REVISE(B,DIR,KPR,QI,A,PIV,P,FC,GWC,FLINS,Q,S,SS,PIT,KK,
  1LST)
   DIMENSION Q(6),QI(7),S(6),SS(6),A(6),PIV(6),P(6),FC(6),GWC(6),B(6)
  1,DIR(6),FLINS(6), PIT(6)
   COMMON /OPT/X(20),XL(20),XU(20),KX(20),XP(20),H(20),R(20),W(20),G(
  120),PARAM(10,7),NOPT(10,10),N,NH,YERR,ITER,ERMIN,ITMAX,ALFA(8)
   COMMON /DAT/NEXP,SUPP,DT,PAR(6),TC(60,8,3),RC(60,8,3),PER,PINT,NDT
  1,PORE(6),CA(6),FCAP(6),ASM(2,6,3),AR(6,3),SLOP(6,3),AREA(3),CU(6,3
  2),CU1(6,3),IDATE(8,3),ISTART,KRAIN(8,3),COEF(6)   ,SITE(3),DX(6,3)
  3,DY(6,3),CONV(3)
   COMMON /EROR/OBPK(8,3),OBVOL(8,3),PRPK(8,3),PRVOL(8,3),JK,NOBS(3)
   LI=1
   DO 41 I=1,6
   IF(I.GT.3)LI=2
   DT1 = DT
   DT = 0.
   Q(I)=0.0
   S(I) = 0.
   SS(I) = 0.
   QI(I) = 0.
   TPOR = CU1(I,JK)*PORE(I)
   PIV(I) = (1.-ASM(LI,KK,JK))*TPOR
   A(I) = CU(I,JK)*CA(I)/TPOR**P(I)
   GWC(I)=(1.-FCAP(I))*TPOR
   PIT(I)=PINT*CU1(I,JK)/DT1
   SUPP=RC(2,KK,JK)*(1.-PER)*CU(I,JK)
   DIR(I)=PAR(5)/6.*AR(I,JK)/DT1
   FLINS(I) = SUPP - FILT(A(I), PIV(I),P(I),FC(I),GWC(I))
41 DT = DT1
   QI(7)=0.0
   IF(ISTART.NE.2) GO TO 94
   WRITE(6,50)IDATE(KK,JK)
50 FORMAT (1H1 13X 52HMATHEMATICAL SIMULATION OF SMALL WATERSHED HYDR
  1OLOGY/18X 35HPREDICTION OF RUNOFF FROM STORM OF   A8///1X )
94 RETURN
   END


   SUBROUTINE DRY  ( PIV,S,Q,A,P,FC,GWC,QI,ITR,FLINS,PREC,PIT,T,JK,KK
  1)
   DIMENSION Q(6),QI(7),S(6),SS(6),A(6),PIV(6),P(6),FC(6),GWC(6),PIT(
  16),FLINS(6)
   COMMON /DAT/NEXP,SUPP,DT,PAR(6),TC(60,8,3),RC(60,8,3),PER,PINT,NDT
  1,PORE(6),CA(6),FCAP(6),ASM(2,6,3),AR(6,3),SLOP(6,3),AREA(3),CU(6,3
  2),CU1(6,3),IDATE(8,3),ISTART,KRAIN(8,3),COEF(6)   ,SITE(3),DX(6,3)
   NRAIN=KRAIN(KK,JK)
   DO 10 I = 1,6
   DQ = .0005*CU(I,JK)
   IF (Q(I).GT.DQ) GO TO 80
```

```
10 CONTINUE
   DTS = DT
   L = 1
11 GO TO (14,13), L
13 ITR = ITR + 1
   PREC=PREC+RC(ITR,KK,JK)*(TC(ITR,KK,JK)-TC(ITR-1,KK,JK))/60.
   L = 1
14 T = T + 5.
   DT = 300.
   IF (T.LT.TC(ITR,KK,JK))GO TO 16
   DT=(TC(ITR,KK,JK)-T+5.)*60.
   T = TC(ITR,KK,JK)
   L = 2
16 RATIO = DTS/DT/2.
   DO 35 I = 1,6
   RATE=CU(I,JK)*RC(ITR,KK,JK)
   R=RAIN(RATE,PIT(I),PER)
   FLINS(I) = PIV(I)
   QI(I) = Q(I)
   IF (S(I).LT.0.) S(I) = 0.
   SUPP = S(I)*RATIO + R
   Q(I) = FILT(A(I),PIV(I),P(I),FC(I),GWC(I)) - R
   S(I) = S(I) - Q(I)/RATIO
   IF (Q(I).LT.0.) GO TO 38
35 CONTINUE
   I = 0
   IF(T-TC(NRAIN,KK,JK)) 11,40,40
38 T = T - DT/60.
40 DO 60 J = 1,6
   IF (J.GT.I) GO TO 50
   PIV(J) = FLINS(J)
   S(J) = S(J) + Q(J)/RATIO
   FLINS(J) = S(J) - QI(J)
   GO TO 55
50 FLINS(J) = S(J) - Q(J)
55 Q(J) = 0.
60 QI(J) = 0.
   QI(7)=0.
   DT = DTS
   IF(ISTART.EQ.2) WRITE(6,70)
70 FORMAT (25X 28HSIMULATION BY SUBROUTINE DRY )
80 RETURN
   END


   SUBROUTINE ERR
   COMMON /OPT/X(20),XL(20),XU(20),KX(20),XP(20),H(20),R(20),W(20),G(
  120),PARAM(10,7),NOPT(10,10),N,NH,YERR,ITER,ERMIN,ITMAX,ALFA(8)
   COMMON /EROR/OBPK(8,3),OBVOL(8,3),PRPK(8,3),PRVOL(8,3),JK,NOBS(3)
   LST=NOBS(JK)
   YERR=0.0
   DO 70 I=1,LST
   FN1=OBPK(I,JK)-PRPK(I,JK)
   FN2=SQRT(OBVOL(I,JK))-SQRT(PRVOL(I,JK))
   IF(ITER.EQ.ITMAX) WRITE(6,35)FN1,FN2
35 FORMAT(*      RESIDUAL ERROR  PEAK=*,F8.4,*  VOLUME=*,F8.4)
   YERR=YERR+FN1**2+1.5*ABS(FN2)
70 CONTINUE
   RETURN
   END
```

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