Slope - Discharge Ratings for Cutthroat Flumes

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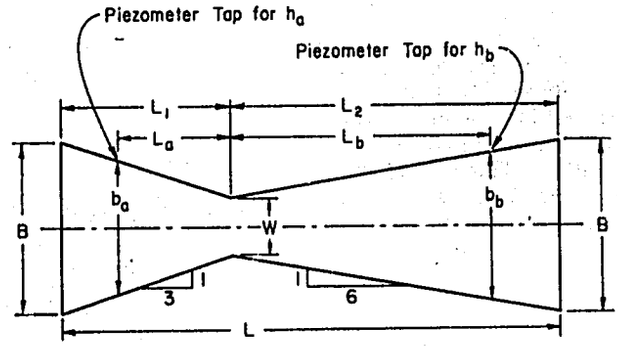
ALTHOUGH water measuring flumes have been intensively studied in the laboratory and are widely used in the field, there are some problems encountered in field installations. A common problem is the difficulty of installing a flume so that the floor is horizontal both longitudinally (direction of flow) and transversely. The flume may be tilted longitudinally or transversely. Tilting the flume will affect the flow measurement. However, the longitudinal slope is not easily adjusted. Also, a flume floor sloping downward in the direction of flow (positive slope) has a more detrimental effect on the discharge measurement than a negative slope. Unfortunately, the positive slope condition may occur quite often in field installations of flumes in unlined open channels, because of the scouring action of the high velocity flow leaving the flume. The only satisfactory solutions, to date, for this problem include: raising the lower end of the flume so it is again level; placing a new level floor in the flume; or placing a liner in the existing flume and then grouting it into place. But usually, the problem is not corrected.

Since many flumes installed in unlined channels experience settlement at the downstream end due to the scouring action of the high velocity jet leaving the flume, and because this settlement is usually not corrected in the field, a more satisfactory solution to the problem would be in evaluating the effect of this settlement (slope of flume floor) upon the discharge rating of the flume. The purpose of this study is to evaluate the effects of flume floor slope on the discharge rating as compared to the rating when the floor is horizontal. The type of flow measuring flume selected for study was the cutthroat flume because of its simple geometry.

Four Cutthroat flumes were selected for hydraulic testing in the laboratory. These flumes had been previously rated with the flume floor horizontal. In this study, hydraulic data were collected under both free flow and submerged flow conditions for each flume at various degrees of flume floor slope. In all cases, the flume floor was sloped downward in the direction of flow (positive slope). The free flow and submerged flow ratings developed from the experimental data are compared with the ratings when the flume floor is horizontal.



$B = W + \frac{2}{3} L_1 = W + \frac{1}{3} L_2$

Flume	W	L_1	L_2	L_a	L_b	L	B
3"x4.5'	3"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	1'-3"
* 6"x4.5'	6"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	1'-6"
*12"x4.5'	12"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	2'-0"
24"x4.5'	24"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	3'-0"
2"x3.0'	2"	1'-0"	2'-0"	0'-8"	1'-7⅝"	3.0'	0'-10"
4"x3.0'	4"	1'-0"	2'-0"	0'-8"	1'-7⅝"	3.0'	1'-0"
8"x3.0'	8"	1'-0"	2'-0"	0'-8"	1'-7⅝"	3.0'	1'-4"
16"x3.0'	16"	1'-0"	2'-0"	0'-8"	1'-7⅝"	3.0'	2'-0"
1"x1.5'	1"	0'-6"	1'-0"	0'-4"	0'-9⅞"	1.5'	0'-5"
* 2"x1.5'	2"	0'-6"	1'-0"	0'-4"	0'-9⅞"	1.5'	0'-6"
* 4"x1.5'	4"	0'-6"	1'-0"	0'-4"	0'-9⅞"	1.5'	0'-8"
8"x1.5'	8"	0'-6"	1'-0"	0'-4"	0'-9⅞"	1.5'	1'-0"

* Cutthroat flumes used in this study.

FIG. 1 A series of Cutthroat flumes available for experimental design.

DEVELOPMENT OF CUTTHROAT FLUME

Earlier investigations (Skogerboe et al. 1967 and Skogerboe and Hyatt, 1967) reported the development of a flow measuring flume which has a horizontal floor, with an entrance section and an exit section, but no throat length (Fig. 1). Hence, this flume has been called a Cutthroat flume.

Ackers and Harrison (1963) recommend a maximum convergence of 3:1 for a flume inlet section, while earlier studies by Hyatt (1965) indicated that when the divergence of the flume exit section exceeded 6:1, flow separation would occur, and a major portion of the flow would adhere to one of the sidewalls.

Also, studies regarding the length of the throat section (Skogerboe et al, 1967 and Skogerboe and Hyatt, 1967) showed that flow depths measured in the exit section of the flume resulted in more accurate submerged flow calibration curves than ratings employing flow depths measured in the throat section.

The most obvious advantage of a Cutthroat flume is economy, since fabrication is facilitated by a flat-bottom (horizontal floor) and removal of the throat section. The initial investigations (Skogerboe et al, 1967 and Skogerboe and Hyatt, 1967) were confined to a flume length, L, of 9 ft with throat widths, W, varying from 1-ft to 6-ft. For a constant flume length, various throat widths can be constructed by just moving the walls. More recent investigations (Bennett, 1972 and Wu, 1971) have been concerned with flume lengths of 1.5 ft and 4.5 ft in order to

Article was submitted for publication on December 13, 1971; reviewed and approved for publication by the Soil and Water Division of ASAE on August 7, 1972.

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Acknowledgment: The effort reported herein was made possible through the support of the United States Agency for International Development under Contract No. AID/csd-2460, "Improving Capacity of CUSUSWASH Universities for Water Management for Agriculture."

This article is reprinted from the TRANSACTIONS of the ASAE (Vol. 16, No. 1, pp. 78, 79, 80 and 81, 1973)
Published by the American Society of Agricultural Engineers, St. Joseph, Michigan

extend the range of Cutthroat flume sizes.

Flow Conditions

The two most significant flow regimes, or flow conditions, under which the flume may operate are free flow and submerged flow. The distinguishing difference between the two is the occurrence of critical depth in the vicinity of the flume throat for the free flow condition. This critical flow control requires only the measurement of a flow depth at some location upstream (h_a) from the point of critical depth to obtain the free flow discharge.

When the downstream, or tailwater, depth is raised to the extent that the flow depths at every point through the structure becomes greater than critical depth, then the flume is operating under submerged (subcritical) flow conditions. With subcritical flow, an increase in tailwater depth will result in an increased upstream depth. A flume operating under submerged flow conditions requires that 2 flow depths be measured, 1 upstream (h_a) and 1 downstream (h_b) from the flume throat.

The definition given to submergence, S, is the ratio, often expressed as a percentage, of the downstream depth to the upstream depth, $S = h_b/h_a$. The value of submergence at which free flow changes to submerged flow, or vice versa, is referred to as the transition submergence, S_t. At this transition state, the discharge given by the free flow equation is exactly the same discharge as that given by the submerged flow equation.

Method of Flow Analysis

As stated earlier, the Cutthroat flume can be used to measure flow rates for either free flow or submerged flow conditions. The flow equation and the method of flow analysis is different for each type of flow (Skogerboe et al, 1967; Skogerboe and Hyatt, 1967 and 1968).

Free Flow

For free flow operation, a plot is made of flow rate, Q, against upstream depth, h_a, with Q as the ordinate and h_a as the abscissa. When these two variables are plotted on logarithmic paper, all of the points should fall on a straight line. The equation for this free flow rating can be written as:

$$Q = C\ h_a^{n_1} \quad \ldots\ldots\ldots\ldots \quad [1]$$

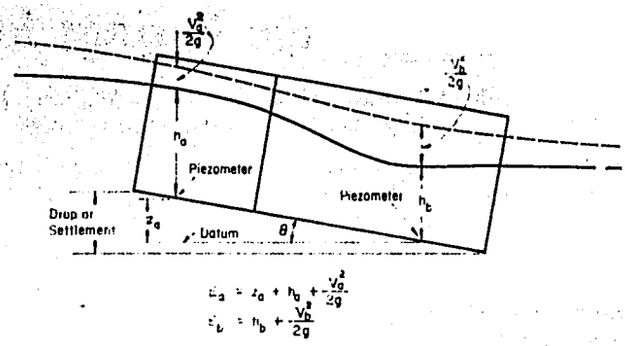

FIG. 2 Definition sketch for sloping flume.

where C is the free flow coefficient and n_1 is the free flow exponent.

Submerged Flow

Submerged flow calibration curves are determined for the Cutthroat flume by preparing 3-dimensional plots of the parameters describing submerged flow. The data is plotted on logarithmic paper with the discharge, Q, as the ordinate; difference in upstream and downstream depths of flow, $h_a - h_b$, as the abscissa; and the submergence, h_b/h_a, as the varying parameter (Skogerboe et al, 1967; Skogerboe and Hyatt, 1967 and 1968). Lines are then drawn connecting points of equal submergence. These are straight lines having a slope identical to the slope of the free flow rating curve (which is n_1) for the same geometry.

From the submerged flow plots, an equation has been developed (Skogerboe et al, 1967 and Skogerboe and Hyatt, 1967) which describes the flow rate through the Cutthroat flume. The equation is:

$$Q = \frac{C_1\ (h_a - h_b)^{n_1}}{(-\log S)^{n_2}} \quad \ldots\ldots\ldots \quad [2]$$

where C_1 is the submerged flow coefficient and n_2 is the submerged flow exponent. The value of C_1 and n_2 must be determined from a plot of the submerged flow data (Skogerboe et al, 1967; Skogerboe and Hyatt, 1967 and 1968).

Experimental Design

The purpose of this study is to evaluate the effect of settlement on flume ratings. The Cutthroat flume was selected because it would be a convenient type of flume for an initial study on this subject, due to the simplicity in geometry. There is a series of Cutthroat flumes, as described in Fig. 1, which had been constructed by personnel in the Agricultural Engineering Shop at Colorado State University for use in another study (Bennett, 1972). The sizes and dimensions of the flumes are tabulated in Fig. 1. The flume series consists of three flume lengths with four throat widths for each length, thereby making a total of 12 flumes. Two sizes (throat widths) from each of two flume lengths were selected for this study regarding the effect of settlement on flume discharge ratings, with the hope that a possibility would exist to extend the results for the whole series. The flumes selected for this study were the 6 in. by 4.5 ft, 12 in. by 4.5 ft, 4 in. by 1.5 ft, and 2 in. by 1.5 ft flume sizes, where the first number corresponds to the throat width and the last number to the flume length.

The primary purpose of the experimental program was to evaluate the change in discharge rating of a flow measuring flume under both free flow and submerged flow conditions, as the slope of the flume floor is increased. This portion of the study could be accomplished by selecting only one flume for hydraulic testing in the laboratory. The secondary purpose of the experimental program was to determine the effects of throat width upon the slope-discharge relations, as well as including flume length as a parameter by investigating two flume lengths (1.5 ft and 4.5 ft).

In this experimental study, the effect of sloping the flume floor was evaluated using three slopes in addition to horizontal. The slopes studied for each flume size are listed in Table 1, where Θ is the slope angle (slope of flume floor) in degrees as shown in Fig. 2.

Free Flow Results

For each flume and floor slope listed in Table 1, a free flow rating using equation [1] was developed. The results of this free flow analysis are given in Table 2. The most significant result concerns n_1, which is only a function of flume length for the Cutthroat flume geometry. The results show that n_1 is independent of throat width and the slope of the flume floor.

TABLE 1. CUTTHROAT FLUME SIZES AND FLOOR SLOPES USED IN THE EXPERIMENTAL DESIGN

Flume size	Case 1 tan θ	Case 2 tan θ	Case 3 tan θ	Case 4 tan θ
2 in. x 1.5 ft	0	0.0278	0.0556	0.0833
4 in. x 1.5 ft	0	0.0278	0.0556	0.0972
6 in. x 4.5 ft	0	0.0185	0.0556	0.1078
12 in. x 4.5 ft	0	0.0185	0.0556	0.1064

TABLE 2. FREE FLOW RATINGS FOR EXPERIMENTAL CUTTHROAT FLUMES AT VARIOUS FLOOR SLOPES

Flume	Case	Floor slope	C	n_1
	1	0.0000	0.974	2.150
2 in. x 1.5 ft	2	0.0278	1.075	2.150
	3	0.0556	1.220	2.150
	4	0.0833	1.390	2.150
	1	0.0000	1.975	2.150
4 in. x 1.5 ft	2	0.0278	2.180	2.150
	3	0.0556	2.470	2.150
	4	0.0972	2.990	2.150
	1	0.0000	1.960	1.720
6 in. x 4.5 ft	2	0.0185	1.980	1.720
	3	0.0556	2.140	1.720
	4	0.1078	2.560	1.720
	1	0.0000	3.980	1.720
12 in. x 4.5 ft	2	0.0185	4.000	1.720
	3	0.0556	4.350	1.720
	4	0.1064	5.200	1.720

The argument may be presented that using values of n_2 different from 3/2 (n_1 is 1.72 and 2.15 in Table 2) indicates that the Froude Law is not completely valid. All of the free flow data for the Cutthroat flumes listed in Fig. 1 has been analyzed by Harrison (1971) using the equation

$$C_o = \frac{Q}{(g)^{1/2}\, W E_a^{3/2}} \quad \ldots\ldots \quad [3]$$

which fits the Froude Law. Harrison (1971) found the best logarithmic relationship fitting the data to be given by

$$C_o - 0.505 = 0.12(E_a/W)^{0.755} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [4]$$

where

$$C_o = \frac{Q}{g^{1/2}\,(W-0.02)\,(1 + 0.25W/L) \cdot [(E_a - 0.02L)\,(1 + 0.0002L/E_a)]^{3/2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [5]$$

The difficulty in using equations [4] and [5] is quite obvious. Thus, the authors prefer to use equation [1] in developing ratings for open channel constrictions.

The variation in free flow ratings with floor slope is taken into account with the free flow coefficient, C. Therefore, the development of relationships for the free flow coefficient involving flume length, throat width, and floor slope would be desirable.

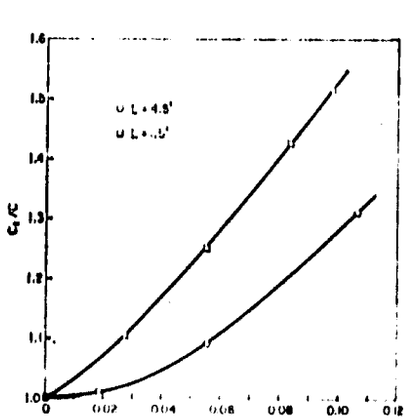

FIG. 3 Effect of tilt angle and flume length upon the free flow rating for a Cutthroat flume.

To denote the free flow coefficient for a Cutthroat flume with a sloping floor, the symbol C_t will be used, whereas C denotes a horizontal flume floor. If the ratio, C_t/C is plotted against floor slope, as shown in Fig. 3, the result is that a single unique curve exists for each flume length. This is an important discovery, because it reduces the amount of experimental work required in establishing each unique curve. Data do not have to be generated for each throat width. As would be expected, Fig. 3 shows that the degree of discharge correction, in comparison with a horizontal flume, increases with increasing slope. Also, the degree of correction decreases as the flume length is increased.

Submerged Flow Results

A definition sketch for a sloping flow measuring flume is shown in Fig. 2. The datum for hydraulic computations is taken as the elevation of the flume floor at the cross-section where the downstream flow depth, h_b, is measured. Under such conditions, it is necessary to describe the discharge ratings using energy rather than flow depths in order to use the submerged flow equation. For example, one of the problems is that under conditions of high submergence, the water surface elevation at the h_b piezometer may be higher than the water surface elevation measured at the h_a piezometer, thereby resulting in $h_a - h_b$ being negative. Thus, the energy at the piezometer taps in the entrance and exit sections of the Cutthroat flume is used, where

$$E_a = z_a + h_a + V_a^2/2g \quad \ldots \quad [6]$$

$$E_b = h_b + V_b^2/2g \quad \ldots\ldots\ldots \quad [7]$$

$$E_r = E_b/E_a \quad \ldots\ldots\ldots\ldots \quad [8]$$

The free flow equation is then rewritten as

$$Q = CE_a^{n_1} \quad \ldots\ldots\ldots\ldots \quad [9]$$

The submerged flow equation becomes

$$Q = \frac{C_1 H_L^{n_1}}{(-\log E_r)^{n_2}} \quad \ldots\ldots\ldots \quad [10]$$

where H_L is the head loss defined by

$$H_L = E_a - E_b \quad \ldots\ldots\ldots \quad [11]$$

For each flume and floor slope listed in Table 1, a submerged flow rating using equation [10] was developed. The results of this submerged flow analysis are given in Table 3. As shown previously for the free flow ratings, n_1 is a function of flume length, only, for the Cutthroat flume geometry. A comparison of Tables 2 and 3 shows that n_1 has a different value for the energy analysis as compared with using flow depths. In fact, the value of n_1 for each flume length based upon energy was developed from the free flow ratings in Table 2 by converting flow depths to energy, plotting discharge against $E_a = h_a + V_a^2/2g$, and then obtaining new values of C and n_1.

The submerged flow energy analysis did show that the submerged flow exponent, n_2, is not a function of throat width or floor slope, but is a function of flume length, only, for the Cutthroat flume geometry. This important result allows the variation in submerged flow ratings to be reflected entirely in the submerged flow coefficient, C_1, thereby allowing a similar presentation of results as depicted for the free flow ratings with various degrees of floor slope.

The submerged flow coefficient for a Cutthroat flume with a sloping floor will be denoted by the symbol C_{1t}. If the ratio, C_{1t}/C_1 is plotted against floor slope (Fig. 4), the result is a single unique curve for each flume length, which corresponds with the results of the free flow ratings (Fig. 3). Therefore, if curves could be developed for a range of flume lengths, the curve for

TABLE 3. SUBMERGED FLOW ENERGY RATINGS FOR EXPERIMENTAL CUTTHROAT FLUMES AT VARIOUS FLOOR SLOPES

Flume	Case	Floor slope	C_1	n_1	n_2
	1	0.0000	0.4719	1.922	1.563
2 in. x 1.5 ft	2	0.0278	0.4513	1.922	1.563
	3	0.0556	0.4428	1.922	1.563
	4	0.0833	0.4393	1.922	1.563
	1	0.0000	0.8417	1.922	1.563
4 in. x 1.5 ft	2	0.0278	0.8055	1.922	1.563
	3	0.0556	0.7904	1.922	1.563
	4	0.0972	0.7828	1.922	1.563
	1	0.0000	0.9754	1.688	1.389
6 in. x 4.5 ft	2	0.0185	0.7540	1.688	1.389
	3	0.0556	0.6499	1.688	1.389
	4	0.1078	0.5750	1.688	1.389
	1	0.0000	1.8588	1.688	1.389
12 in. x 4.5 ft	2	0.0185	1.4369	1.688	1.389
	3	0.0556	1.2417	1.688	1.389
	4	0.1064	1.0781	1.688	1.389

intermediate flume lengths could be developed by interpolation.

There are two major difficulties in utilizing the results shown in Fig. 4. First of all, using energy in the rating requires that flow depths measured in the field be converted to energy before using the submerged flow equation (equation [10]) to determine discharge. Correspondingly, beginning with energy, the flow depth can only be computed using a trial-and-error solution, which is very cumbersome. Secondly, an initial glance at Fig. 4 leaves the impression that the discharge decreases as the floor slope is increased, which is not the case. Unfortunately, using energy requires a knowledge of energy-depth relations with changing submergence (or energy ratio) in order to interpret the variation of C_{it}.

Conclusions

Frequently, flow measuring flumes placed in unlined channels settle at the flume exit due to scouring action. Usually, the settlement is not corrected. Therefore, the discharge will be affected and corrections to the discharge coefficient varies and discharge increases as the flume floor slope increases.

The most significant findings resulting from this study are that both the free flow and submerged flow exponents have a unique value for each flume length. These exponents are not affected by throat width or flume floor slope.

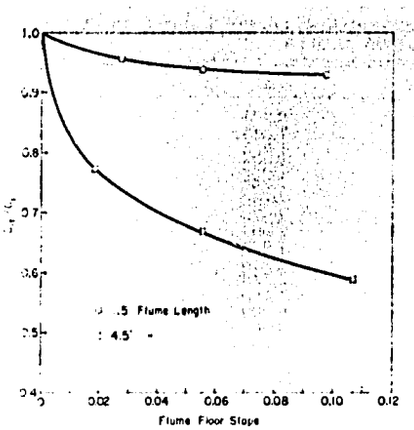

FIG. 4 Effect of floor slope and flume length upon the submerged flow energy rating for a Cutthroat flume.

The above findings have allowed the effect of flume floor slope to be analyzed in terms of the free flow coefficient and submerged flow coefficient. These coefficients are uniquely related to the flume floor slope for each flume length, but are independent of throat width. Thus, generalized slope-discharge relations could be developed for Cutthroat flumes by extending these studies to longer flume lengths.

References

1 Ackers, P., and A. J. M. Harrison. 1963. Critical-depth flumes for flow measurement in open channels. Hydraulic Research Paper No. 5, Hydraulics Research Station, Department of Scientific and Industrial Research, Wallingford, Berkshire, England, April.

2 Bennett, R. S. 1972. Cutthroat flume discharge relations. Unpublished M.S. Thesis, Agricultural Engineering Department, Colorado State University, Fort Collins, Colorado, March.

3 Harrison, A. J. M. 1971. Personal communication, June.

4 Hyatt, M. L. 1965. Design, calibration, and evaluation of a trapezoidal measuring flume by model study. Unpublished M.S. Thesis, Civil Engineering Department, Utah State University, Logan, Utah, March.

5 Skogerboe, G. V., M. L. Hyatt and K. O. Eggleston, 1967. Design and calibration of submerged open channel flow measurement structures: Part 1—Submerged flow. Report WG 31-2, Utah Water Research Laboratory, Utah State University, Logan, Utah, February.

6 Skogerboe, G. V., M. L. Hyatt, R. K. Anderson and K. O. Eggleston. 1967. Design and calibration of submerged open channel flow measurement structures: Part 3—Cutthroat flumes. Report WG 31-4, Utah Water Research Laboratory, Utah State University, Logan, Utah, April.

7 Skogerboe, G. V., M. L. Hyatt, 1967. Rectangular Cutthroat flow measuring flumes. Proceedings of ASCE, Journal of the Irrigation and Drainage Division, 93 (IR4): Proc. Paper, 5628, pp. 1-13. December.

8 Skogerboe, G. V., and M. L. Hyatt, 1968. Submerged flow in Parshall flumes. Transactions of the ASAE 11(1):142-145.

9 Wu, Tsu-Yang. 1971. Effect of settlement on flume ratings. Unpublished M.S. Thesis, Agricultural Engineering Department, Colorado State University, Fort Collins, Colorado, August.