COMPARISON OF BRIDGE BACKWATER RELATIONS

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INTRODUCTION

Practical methods have been developed for obtaining the backwater that may be expected due to placing a bridge across a stream for a given design flood. The most commonly used method is that outlined in the Bureau of Public Roads (BPR) bulletin, "Hydraulics of Bridge Waterways" (2,3). This bulletin was compiled from research efforts by Liu, Bradley and Plate (6) at Colorado State University (CSU).

A practical method for computing peak discharge through a contraction, where the maximum backwater can be measured, is embodied in the U.S. Geological Survey (USGS) Circular 284 (5), "Computation of Peak Discharge at Contractions," based on the research work of Kindsvater, Carter and Tracy at the Georgia Institute of Technology. Similar techniques have been more recently reported by Matthai (7), "Measurement of Peak Discharge at Width Contractions by Indirect Methods."

Both these methods were based on model studies and have often shown large errors in application to prototype structures. Also, detailed investigation has not been undertaken to arrive at a satisfactory solution to the

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problem where abnormal stage-discharge conditions exist. In fact, Bradley (2) went so far as to say, "This is a case where it is more important to understand the problem than to attempt precise computations."

A different approach to analyzing the hydraulics of flow through bridge constriction was undertaken by Skogerboe, Austin and Chang (8) by applying their previously developed method of submerged (subcritical) flow analysis. Their study was primarily concerned with the abnormal stage-discharge condition.

A need therefore exists to compare the aforementioned methods of analysis and to determine the inter-relationships among the various methods, thereby disclosing any advantages or disadvantages of one technique in comparison with the other techniques. The purpose of this paper is to evaluate these currently existing methods of predicting the effects of a bridge constriction on stream flow, as compared to the method of subcritical flow analysis.

FLOW CONDITIONS

An open channel will flow at normal depth if the discharge is constant, and the channel, within a sufficiently long reach, has a uniform cross section, a uniform surface roughness, and a uniform grade. If a constriction is installed in such a channel, the flow pattern will become that as shown in Fig. 1.

These stringent conditions would tend to suggest that uniform flow at a bridge site may be the exception rather than the rule. Non-uniform flow at a bridge site is due to downstream control, examples of which might include flood conditions at the confluence of two streams, downstream reservoir or spillway regulation, influence of tides, or changes in vegetative or moss conditions in flat gradient channels. A unique stage-discharge condition may no longer exist as it does for uniform flow. A definition sketch of this abnormal stage-discharge condition is shown in Fig. 2.

Regardless of the original flow condition, the flow through a constriction such as a highway bridge crossing is usually of subcritical regime, producing gradually varied channel flow far upstream and downstream, with rapidly varying flow occurring at the constriction. The effect of the constriction on the water surface profile, both upstream and downstream, is conveniently measured with respect to the normal water surface profile, which is the water surface in the absence of the constriction under the original flow conditions. Upstream from the constriction, an M1 backwater profile occurs, where the velocities, and consequently the rate of loss of flow energy, are less than for normal flow conditions. The backwater may extend for a considerable distance upstream, to a point where the constricted and the normal surface profiles practically coincide as shown at Section O in Fig. 1.

Near the constriction, the central body of water begins to be accelerated at Section I, while deceleration occurs along the outer boundaries. A separation zone (zone Ia) is formed in the corners upstream from the constriction. As the flow is accelerated at the constriction, the water surface profiles drop rapidly between sections II and III, with the jet stream contracting to a width somewhat less than the width of the opening. The spaces between the jet and the constriction boundaries (zone IIIa) are occupied by eddying water. Immediately downstream from the constriction, the jet stream begins to expand until the original flow condition has been reestablished across the entire channel width at section IV, where the normal and constricted water surface profiles again coincide. Shear along the separation boundary in the reach between sections III and IV results in deceleration of the live stream, with average velocities and energy losses greater than for the original flow condition due to the additional turbulent mixing resulting from the expansion process. Between sections O and IV, the total energy loss is the same as that for the unconstricted flow.

GENERAL SUBCRITICAL FLOW EQUATION

A theoretical subcritical flow discharge equation may be developed for the vertical board constriction shown in Fig. 1. The momentum equation may be written between sections 1 and 2 for the control volume in Fig. 3 to arrive at a general submerged flow equation for an open channel constriction. In the direction of flow, the momentum equation may be written as

$$F_1 - F_2 - F_c - F_f = Q\rho(\beta_2 V_2 - \beta_1 V_1) \quad \ldots\ldots \qquad (1)$$

in which F_1 and F_2 are the resultant forces due to the pressure distribution on the cross-section of flow at sections 1 and 2; F_c is the component of force in the direction of flow acting on the control volume of fluid due to the constriction; F_f is the friction or drag force acting on the surface of the control volume; Q is the theoretical discharge; ρ is the density of the fluid; β_1 and β_2 are momentum coefficients for the two flow sections; and V_1 and V_2 are the average velocities at sections 1 and 2. Assuming uniform velocity distribution and neglecting the friction force

$$F_1 - F_2 - F_c = Q\rho(V_2 - V_1) \quad \ldots\ldots \qquad (2)$$

(In prototype application, the assumption of uniform velocity distribution will not necessarily hold true. Also, in many cases, the friction force may not be neglected. However, a technique for incorporating these factors in the analysis will subsequently be shown in a later paper.)

Assuming hydrostatic pressure distribution

$$F_1 = \gamma B y_1^2/2 \quad \ldots\ldots \tag{3}$$

$$F_2 = \gamma b y_2^2/2 \quad \ldots\ldots \tag{4}$$

in which γ is the specific weight of the fluid; B is the width of the open channel; b is the width of the constriction; and y_1 and y_2 are the depths of flow at the two sections. The force acting on the control volume due to the constriction occurs at the upstream face of the constriction. Assuming the average depth of flow at the upstream face of the constriction is y_2,

$$F_c = \gamma (B-b) y_2^2/2 \quad \ldots\ldots \tag{5}$$

The momentum equation in the direction of flow can now be written as

$$\frac{\gamma B y_1^2}{2} - \frac{\gamma b y_2^2}{2} - \frac{\gamma (B-b) y_2^2}{2} = \frac{Q\gamma (V_2 - V_1)}{g} \quad \ldots\ldots \tag{6}$$

where g is the acceleration due to gravity.

Assuming steady flow, the continuity equation can now be employed.

$$Q = By_1V_1 = by_2V_2 \quad \ldots\ldots \tag{7}$$

Substituting the continuity equation into Eq. 6 and solving for the discharge

$$Q = \frac{\sqrt{g/2}\ B(y_1-y_2)^{1/2}}{\sqrt{\dfrac{(1-by_2/By_1)B}{by_2(y_1+y_2)}}} \quad \ldots\ldots \tag{8}$$

The opening ratio, b/B, may be represented by M and the submergence, y_2/y_1, by S. The denominator of the discharge equation can be made dimensionless by multiplying the numerator and denominator by y_1-y_2.

$$Q = \frac{\sqrt{g/2}\ B(y_1-y_2)^{3/2}}{\sqrt{\dfrac{(1-MS)(y_1-y_2)^2B}{by_2(y_1+y_2)}}} \quad \ldots\ldots \tag{9}$$

Therefore

$$Q = \frac{\sqrt{g/2}\ b(y_1-y_2)^{3/2}}{\sqrt{\dfrac{M(1-MS)(1-S)^2}{S(1+S)}}} \quad \ldots\ldots \tag{10}$$

For any particular channel constriction, b and M become constants and the discharge is a function of $(y_1-y_2)^{3/2}$ and S. If the submergence is held constant, the discharge becomes a function of $(y_1-y_2)^{3/2}$, alone. This suggests that a logarithmic plot of Q against y_1-y_2 would yield a family of straight lines with each line representing a constant value of submergence. The lines of constant submergence would each have a slope of 3/2.

OTHER WIDTH CONSTRICTION EQUATIONS

The most significant and recent work resulting in equations describing flow through constrictions in open channels has been embodied in the publications by Kindsvater, Carter, and Tracy (5) in 1953; Liu, Bradley, and Plate (6) in 1957; Bradley (2,3) in 1960 and 1970; and Biery and Delleur (1) in 1962. The discharge equations presented in each of these publications can be expressed in the form of a submerged (subcritical) flow equation. The development of such equations will be demonstrated for three of the publications (5,6,2).

Kindsvater, Carter and Tracy. By combining an energy equation and the continuity equation, Kindsvater and Carter (4) obtained the discharge formula

$$Q = C_K b y_2 \sqrt{2g\{(y_1-y_2) - E_f + \alpha_1 V_1^2/2g\}} \quad \ldots\ldots (11)$$

in which Q is the discharge in cfs; C_K is Kindsvater's discharge coefficient; y_1 is the flow depth at section I; y_2 is the flow depth at section II; V_1 is the average velocity at section I; E_f is the energy loss in feet due to friction between sections I and II; and

$$\alpha_1 = \frac{\Sigma(qv^2)}{QV_1^2} \quad \ldots\ldots \tag{12}$$

is a coefficient applied to the velocity head at section I to account for nonuniform velocity distribution, where q is the discharge in a subsection, and v is the average velocity in that subsection. By substituting $V_1 = \frac{Q}{By_1}$, and solving for Q,

$$Q = \frac{C_K b y_2 \sqrt{2g\{(y_1-y_2) - E_f\}}}{\sqrt{1 - \alpha_1 C_K^2 M^2 S^2}} \quad \ldots\ldots \tag{13}$$

where $M = b/B$ and $S = y_2/y_1$.

Assuming $\alpha_1 \simeq 1.0$ and $E_f \simeq 0.0$,

$$Q = \frac{C_K\sqrt{2g}\ b y_2 (y_1-y_2)^{1/2}}{\sqrt{1 - C_K^2 M^2 S^2}} \quad \ldots\ldots \tag{14}$$

By multiplying both numerator and denominator by y_1-y_2 and dividing both by y_2,

$$Q = \frac{C_K\sqrt{2g}\ b(y_1-y_2)^{3/2}}{\sqrt{\dfrac{(1-C_KMS)(1+C_KMS)(y_1-y_2)^2}{y_2^2}}} \quad \ldots\ldots \tag{15}$$

Therefore,

$$Q = \frac{C_K\sqrt{2g}\ b(y_1 - y_2)^{3/2}}{\sqrt{\dfrac{(1 - C_K MS)\ (1 - S)^2}{S^2/(1+C_K MS)}}} \quad \ldots\ldots \qquad (16)$$

which is of similar form to Eq. 10, thereby indicating that for a given constriction geometry where b and M are constant, the discharge becomes a function of $(y_1 - y_2)^{3/2}$ alone for a given value of submergence.

By substituting values of y_1 at fixed values of submergence (thereby fixing y_2), values of discharge have been generated from Eq. 11 using typical values of C_K and α_1 and a range of bed slopes. These values have been plotted on logarithmic paper in Fig. 4 with discharge as the ordinate and $y_1 - y_2$ as the abscissa. As indicated by Eq. 16, values for a constant submergence describe a straight line on a logarithmic plot having a slope of 3/2. The corresponding submergence distribution is shown on Fig. 5, enabling the submerged flow equation to be obtained graphically.

The discharge equation may be obtained in terms of energies by substituting energy minus velocity head for flow depth and solving for discahrge.

$$Q = \frac{C_K\sqrt{2g}\ by_2(E_1 - E_2)^{1/2}}{\sqrt{1 + C_K^2(M^2S^2 - 1)}} \quad \ldots\ldots \qquad (17)$$

in which E_1 and E_2 are the specific energies at sections I and II, respectively. To eliminate the depth term, or to solve in terms of $(E_1 - E_2)^{3/2}$ is exceedingly difficult. As

this would tend to suggest, the data generated from Eq. 11, when plotted on logarithmic scales with discharge versus $E_1 - E_2$, yields a series of curved lines, one for each value of submergence.

Liu, Bradley, and Plate. From a plot of actual data for a vertical board model, together with dimensional analysis of the backwater phenomena, Liu, Bradley, and Plate (6) developed the empirical backwater equation

$$\left(\frac{y_1}{y_n}\right)^3 = 4.48\ F_n^2 \left[\frac{1}{M^2} - \frac{2}{3}\ (2.5 - M)\right] + 1 \quad \ldots\ldots \qquad (18)$$

in which y_n is the normal flow depth for the unconstricted flow; and F_n is the Froude number of the normal flow depth. By substituting

$$F_n = \frac{Q}{By_n\sqrt{gy_n}} \quad \ldots\ldots \qquad (19)$$

and solving for Q, an equation for discharge may be obtained

$$Q = B\sqrt{g}\sqrt{\frac{y_1^3 - y_n^3}{4.48\ \frac{1}{M^2} - \frac{2}{3}\ (2.5 - M)}} \quad \ldots\ldots \qquad (20)$$

Although the denominator is dimensionless, this equation is rather dissimilar to Eq. 10. However, for the same data, Izzard (4) suggested the equation

$$\frac{y_1}{y_n} = 1 + 0.45\left(\frac{F_n}{M}\right)^2 \quad \ldots\ldots\ldots \qquad (21)$$

Substituting for F_n and solving for Q, a discharge equation is obtained

$$Q = 1.49\sqrt{g}\ by_n(y_1-y_n)^{1/2} \quad \ldots\ldots \tag{22}$$

This may be alternatively expressed as

$$Q = \frac{1.49b\sqrt{g}\ (y_1-y_n)^{3/2}}{1/S - 1} \quad \ldots\ldots \tag{23}$$

which has a numerator of similar form to Eq. 10 and a dimensionless denominator. By substituting energy minus velocity head for flow depth in Eq. 22, the discharge equation can be described in terms of energies, although the term y_n persists.

$$Q = \frac{1.49\sqrt{g}\ by_n(E_1-E_n)^{1/2}}{\sqrt{1-1.11M^2(1-S^2)}} \quad \ldots\ldots \tag{24}$$

in which E_n is the specific energy at normal depth. Again, to obtain a concise expression for discharge purely in terms of energies, or in terms of $(E_1-E_n)^{3/2}$, is exceedingly difficult.

By solving Eq. 20 for given values of S and y_n, data has been generated and plotted according to the submerged flow analysis as shown in Figs. 6 and 8. However, despite the inability to express discharge in terms of $(E_1-E_n)^{3/2}$, the data plots perfectly, whether generated in terms of depths or energies with the following results:

$$\text{For } M = 1.245,\ Q = \frac{1.55(y_1 - y_n)^{3/2}}{-\log S} = \frac{1.59(E_1 - E_n)^{3/2}}{-\log E_R} \quad \ldots \quad (25)$$

$$\text{For } M = 0.497,\ Q = \frac{3.7(y_1 - y_n)^{3/2}}{-\log S} = \frac{3.8(E_1 - E_n)^{3/2}}{-\log E_R} \quad \ldots \quad (26)$$

$$\text{For } M = 0.733,\ Q = \frac{7.4(y_1 - y_n)^{3/2}}{-\log S} = \frac{7.4(E_1 - E_n)^{3/2}}{-\log E_R} \quad \ldots \quad (27)$$

in which E_R is the ratio E_n/E_1. The value of the submergence exponent, n_2, was found to be 1.0 for both depths and energies, as shown in Figs. 7 and 9.

Bureau of Public Roads. Working in conjunction with Colorado State University, the Bureau of Public Roads derived the expression for backwater

$$y_1^* = K^* \frac{V_{n2}^2}{2g} + \alpha_1 \left[\left(\frac{A_{n2}}{A_4}\right)^2 - \left(\frac{A_{n2}}{A_1}\right)^2 \right] \frac{V_{n2}^2}{2g} \quad \ldots\ldots \quad (28)$$

where y_1^* is the backwater, $y_1 - y_n$; K^* is the total backwater coefficient; A_{n2} is the cross-sectional flow area in the constriction at normal stage; A_1 is the area of flow including backwater at section I; A_4 is the area of flow at section IV at which normal water surface is reestablished; and $V_{n2} = Q/A_{n2}$. This expression is embodied in the manual by Bradley (2,3). Uniform flow is again assumed before placement of the constriction. Substituting $y_1^* = y_1 - y_n$ and expressing velocities in terms of discharges (continutiy equation) for a rectangular channel, Eq. 28 may be solved for discharge:

$$Q = \frac{\sqrt{2g}\ by_n(y_1 - y_n)^{1/2}}{\sqrt{K^* + \alpha_1 M^2(1-S^2)}} \quad \ldots \quad (29)$$

This may also be expressed as

$$Q = \frac{\sqrt{2g}\ b(y_1 - y_n)^{3/2}}{\sqrt{\frac{1-S^2}{S^2}\left[K^* + \alpha_1 M^2(1-S^2)\right]}} \quad \ldots\ldots \quad (30)$$

Again, the denominator is dimensionless.

The appropriate values of K^* were selected from the curves given in the BPR design manual (2,3), and Eq. 30 solved for given values of S and y_n. The difference between the two editions of the BPR design manual is the curves relating K^* and M. The results are plotted on Fig. 10, which was used in conjunction with Fig. 11 to obtain the submerged flow equation. The two sets of curves in Figs. 10 and 11 represent two different values of K^*. The values of energy for each flow depth were also obtained, for one of the values of K^*, which allowed Eq. 30 to be plotted in terms of energies as shown in Fig. 12. The energy ratio distribution is plotted on Fig. 13.

An expression for discharge in terms of energies may be obtained from Eq. 28 if energy minus velocity head is substituted for the backwater depth.

$$Q = \frac{\sqrt{2g}\ by_n(E_1 - E_n)^{1/2}}{\sqrt{K^* + M^2(\alpha_1 - 1)(1-S^2)}} \quad \ldots\ldots \quad (31)$$

Again, a solution purely in terms of energies has not been achieved.

Recognizing from Eq. 12 that $\alpha_1 = 1$ for a rectangular laboratory flume, the discharge equation becomes

$$Q = \frac{\sqrt{2g}\, by_n (E_1 - E_n)^{1/2}}{\sqrt{K^*}} \quad \ldots\ldots \qquad (32)$$

Hence, it may be seen that for a given constriction geometry located in a rectangular flume, using a constant value of y_n, that a plot of Q against $E_1 - E_n$ on logarithmic paper will yield a striaght line, with the slope being 0.5. This has been demonstrated in Fig. 12 where data was generated from Eq. 28 modified to the form of Eq. 29. Hence, by plotting $Q_{E_L = 1}$ against y_n (Fig. 14), the value of the coefficient $\sqrt{\frac{2g}{K^*}}\, b$ was confirmed, so that Eq. 32 may be expressed in the form

$$Q = C_1^* y_n (E_1 - E_n)^{1/2} \quad \ldots\ldots \qquad (33)$$

in which

$$C_1^* = \sqrt{\frac{2g}{K^*}}\, b \quad \ldots\ldots\ldots \qquad (34)$$

CONCLUSIONS

Flow conditions at a proposed bridge site on an open channel may be either uniform of non-uniform, depending on channel conditions. The effect of the constriction on the water surface profile, both upstream and downstream, is conveniently measured with respect to the normal water

surface profile, which is the water surface in the absence of the constriction under the original flow conditions. Irrespective of the original flow conditions existing at the bridge site, by using a combination of the momentum and continuity equations a general submerged flow equation may be obtained for the open channel constriction..

The equation used by Kindsvater, Carter and Tracy (5) was derived for non-uniform flow from a combination of an energy equation and the continuity equation. However, by making two simplifying assumptions and manipulating the remaining variables, this equation may be expressed in the form of a submerged flow equation.

Liu, Bradley, and Plate (6) used empiricism and dimensional analysis to obtain a backwater equation for the case where uniform flow exists in the channel prior to placement of a constriction. The simplified equation suggested by Izzard for the same data may be readily manipulated into the form of a submerged flow equation.

The Bureau of Public Roads (2,3) obtained a backwater equation for the case of uniform flow in the unconstricted channel by considering energies at the section of maximum backwater and the section where uniform flow is reestablished downstream of the constriction. Again, this expression may be expressed as a submerged flow equation.

In all of the above cases, the original equations presented may be plotted on the coordinates used for submerged flow analysis, with head loss as the abscissa, dis-

discharge as the ordinate, and a series of parallel straight lines of slope 3/2 representing the different values of submergence.

The analytical expressions embodied in the current methods of measuring peak discharge through, or backwater due to, a bridge constriction may therefore be reduced to the form of a submerged flow equation. As these expressions were obtained for a particular flow condition (uniform or non-uniform), each represents only a particular case of the backwater phenomena. All could be encompassed by the more general submerged flow equation.

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APPENDIX II - NOTATION

The following symbols are used in this paper:

A_1 = area of flow including backwater at section I;

A_{n2} = area of flow below normal water surface at section II;

A_4 = area of flow at section IV at which normal water surface is reestablished;

B = width of channel;

b = width of constriction;

C_1 = submerged flow discharge coefficient;

C_K = Kindsvater's discharge coefficient;

E_1 = specific energy at section I after bridge placed;

E_2 = specific energy at section II;

E_f = energy loss due to friction between sections I and II;

E_L = energy loss E_u-E_d, E_1-E_n, or E_1-E_{1A};

E_n = specific energy at normal depth;

E_R = ratio of E_d/E_u, E_n/E_1, or E_{1A}/E_1;

F_1 = resultant force due to pressure distribution on the cross-section of flow at section 1;

F_2 = resultant force due to pressure distribution on the cross-section of flow at section 2;

F_c = component of force in the direction of flow acting on the control volume of fluid due to the constriction;

F_f = friction or drag force acting on the surface of the control volume;

F_n = Froude number at normal depth, $V/(gy_n)^{1/2}$;

g = Acceleration due to gravity (32.2 ft/sec^2);

H_L = head loss, $y_u - y_d$;

K^* = total backwater coefficient;

M = bridge opening ratio, b/B;

n_1 = exponent in numerator of submerged flow equation;

n_2 = submergence exponent in the denominator of the submerged flow equation;

Q = discharge;

$Q_{E_L=1}$ = value of Q where $E_L=1$ and $E_R=1$;

$Q_{E_L=1}$ = value of Q where $H_L=1$ and $S=1$;

q = unit discharge in a channel subsection;

s = submergence, ratio of downstream flow depth to upstream flow depth;

V_1 = average velocity at section I;

V_2 = average velocity at section II;

V_{n2} = average velocity in constriction for flow at normal depth, Q/A_{n2};

v = velocity in a channel subsection;

y_1 = flow depth at section I;

y_2 = flow depth at section II;

y_3 = flow depth at section III;

y_1^* = total backwater or rise above normal depth at section I;

y_n = normal flow depth;

α_1 = kinetic energy correction factor which corrects for nonuniformity of velocity distribution at section I;

β_1 = momentum coefficient at section I;

β_2 = momentum coefficient at section II;

γ = specific weight of the fluid; and

ρ = density of the fluid

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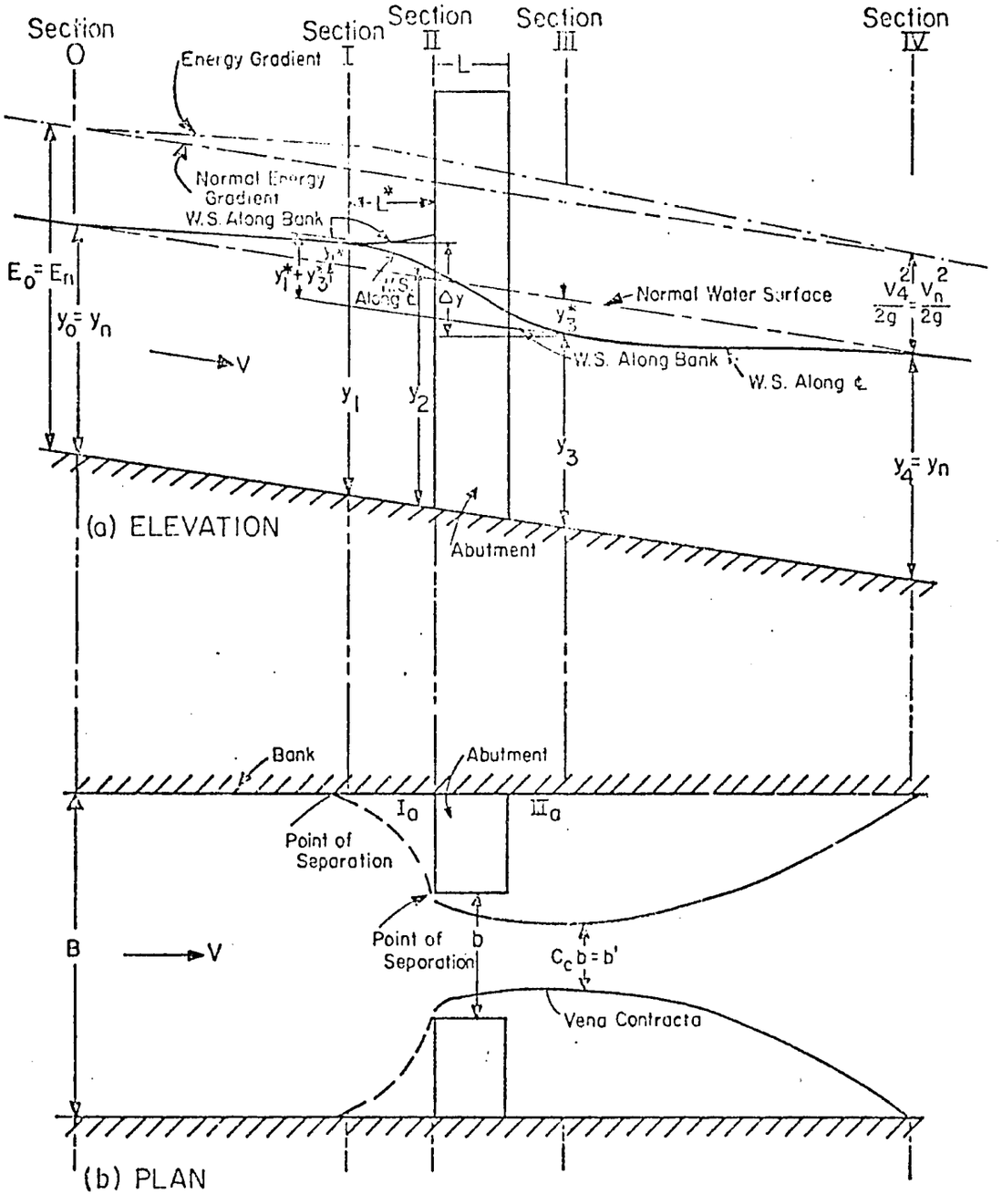

FIG. 1. - DEFINITION SKETCH OF SIMPLE VERTICAL BOARD CONSTRICTION

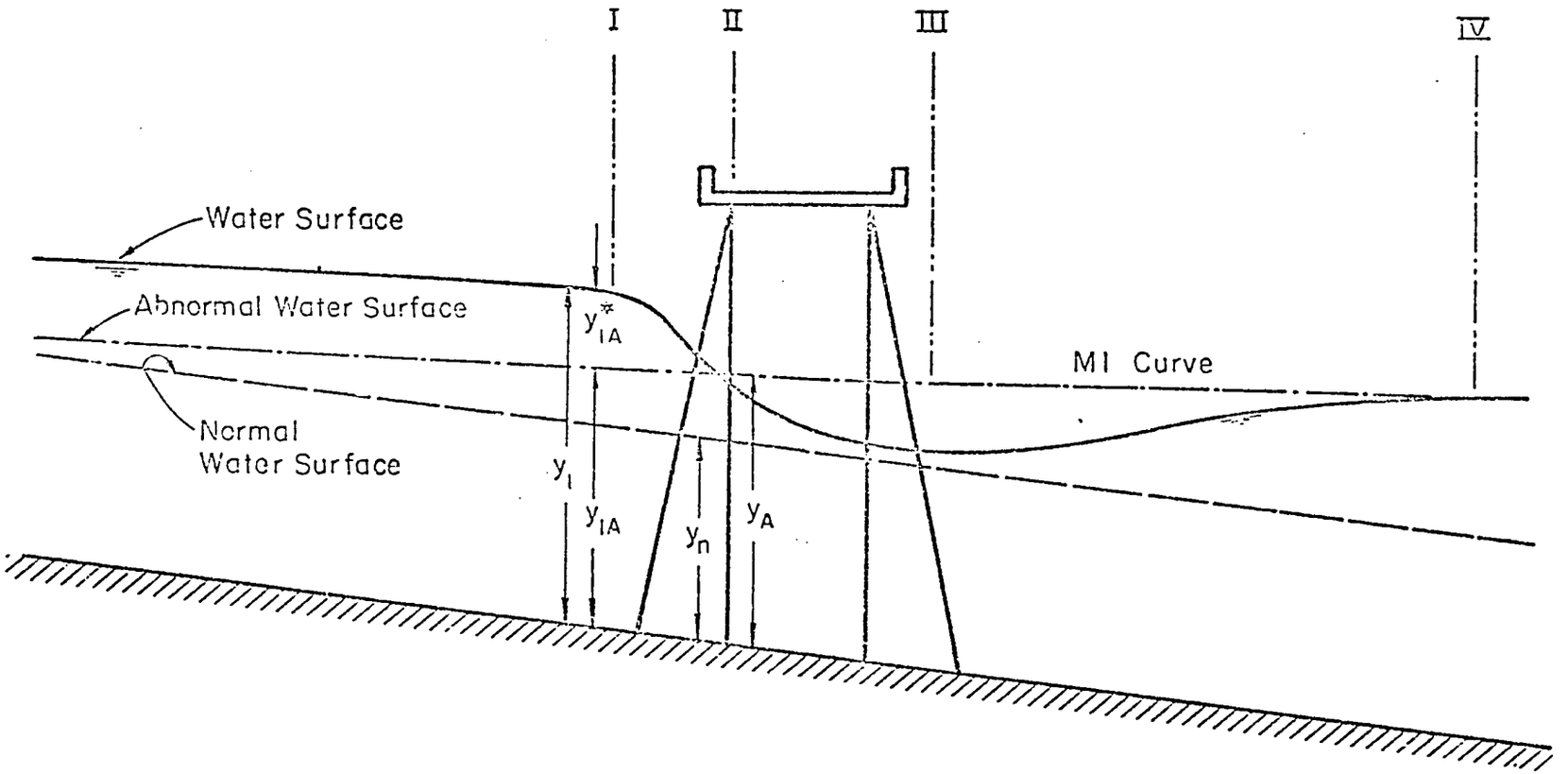

FIG. 2. - DEFINITION SKETCH OF ABNORMAL STAGE-DISCHARGE CONDITION

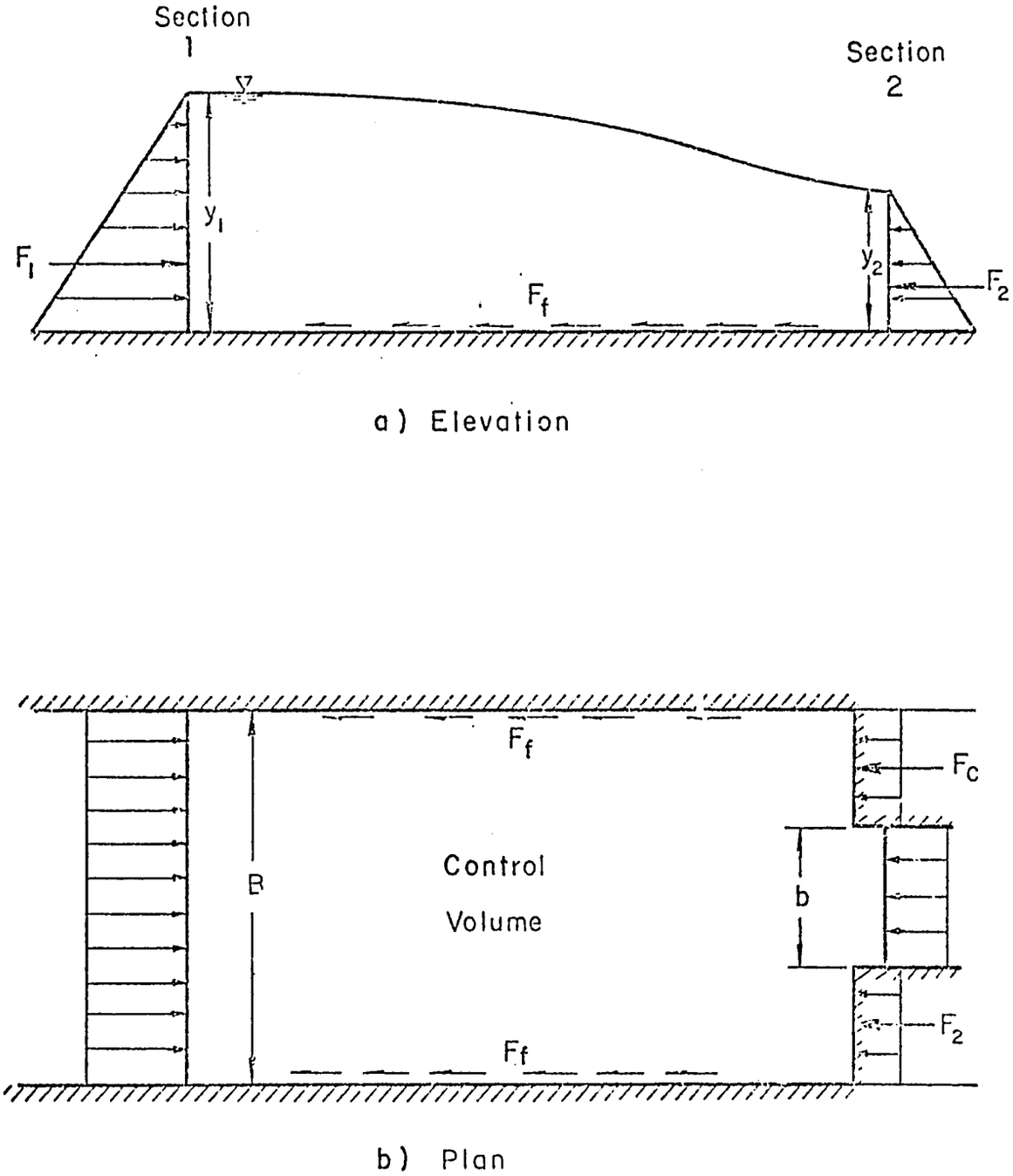

FIG. 3. - CONTROL VOLUME FOR A CONSTRICTION IN A RECTANGULAR CHANNEL

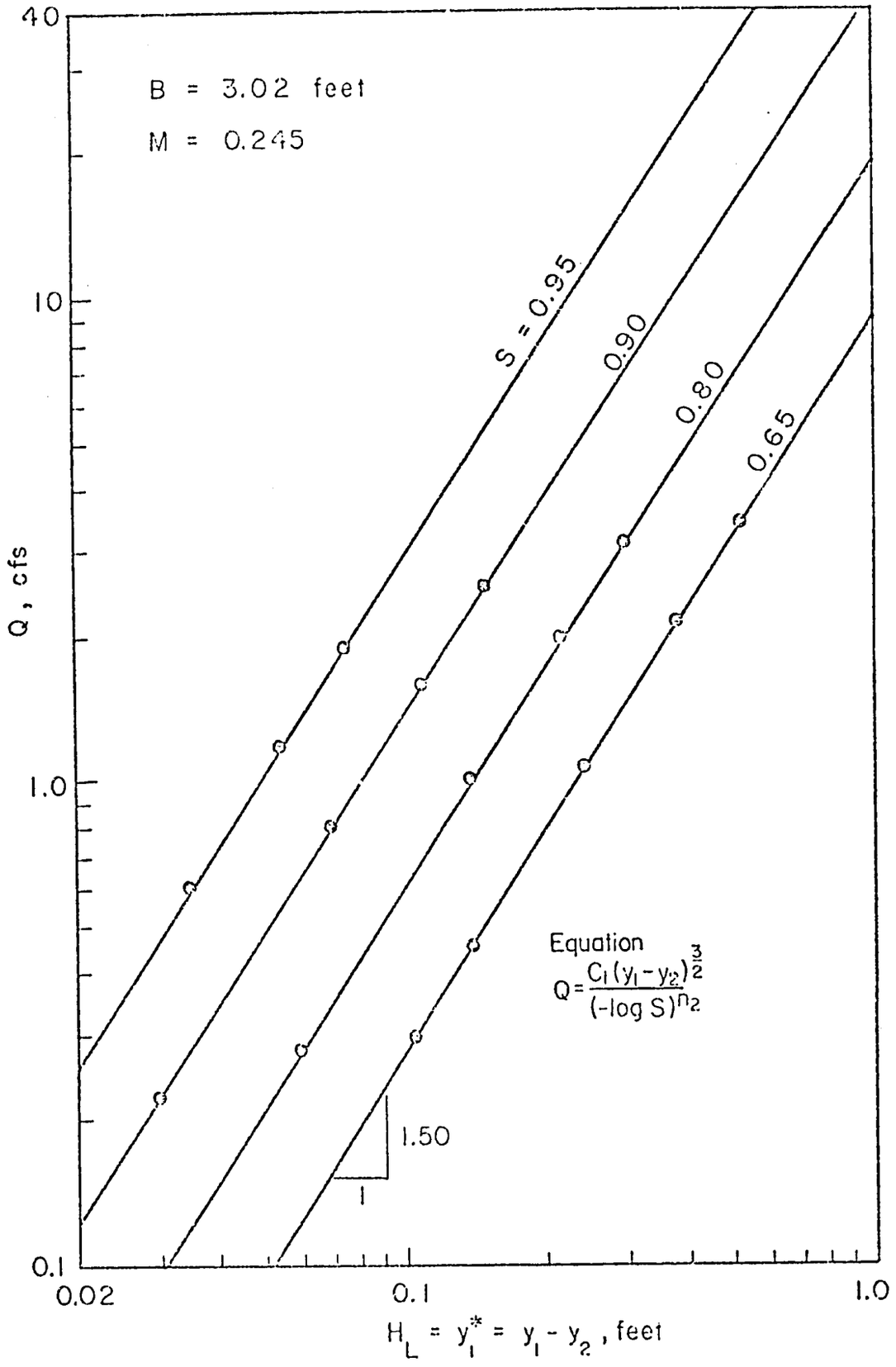

FIG. 4. - SUBMERGED FLOW (DEPTH) ANALYSIS OF EQUATION OF KINDSVATER, CARTER AND TRACY

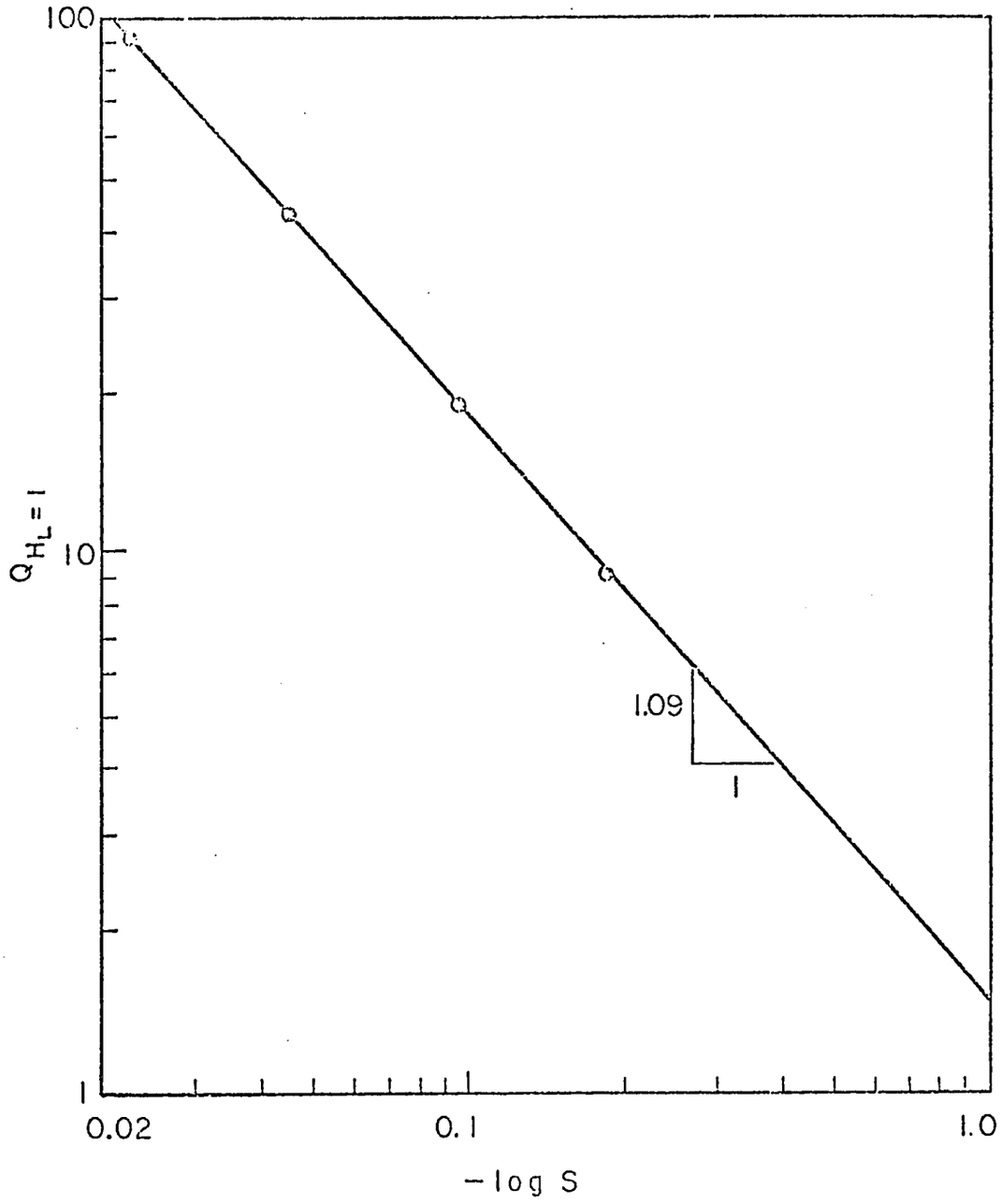

FIG. 5. - SUBMERGENCE DISTRIBUTION FOR EQUATION OF KINDSVATER, CARTER AND TRACY

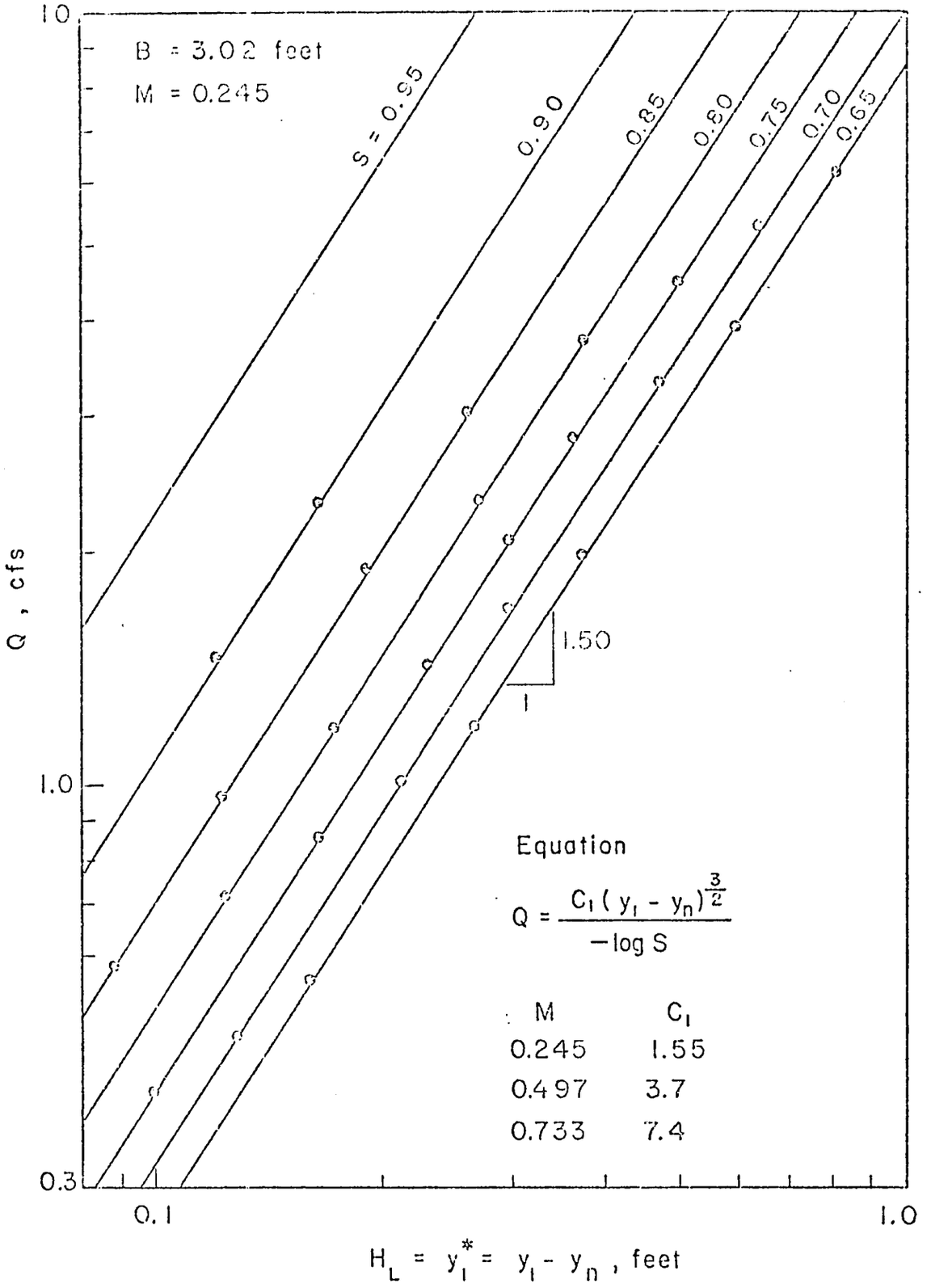

FIG. 6. - SUBMERGED FLOW (DEPTH) ANALYSIS OF EQUATION OF LIU, BRADLEY AND PLATE

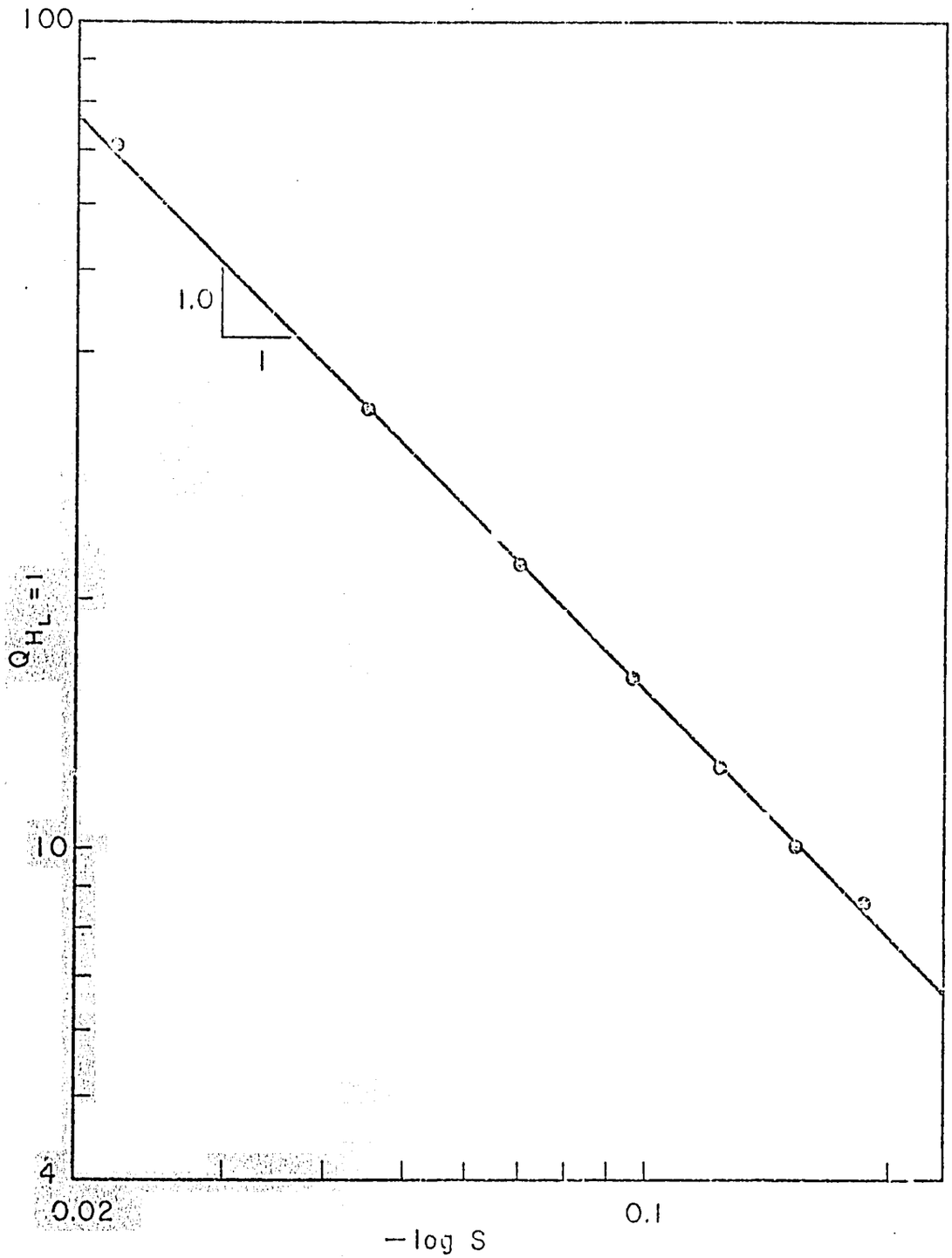

FIG. 7. - SUBMERGENCE DISTRIBUTION FOR EQUATION OF LIU, BRADLEY AND PLATE

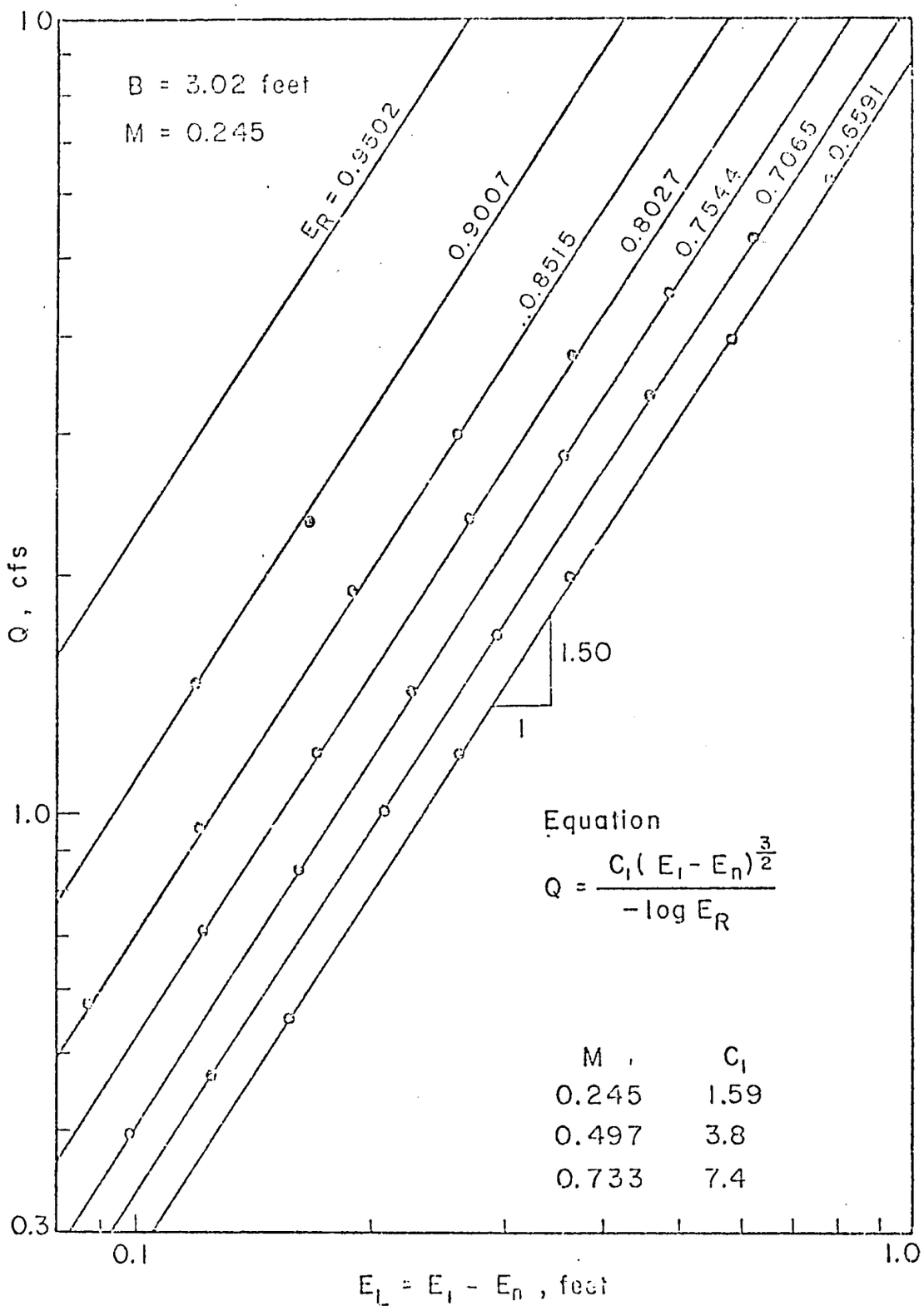

FIG. 8. - SUBMERGED FLOW (ENERGY) ANALYSIS OF EQUATION OF LIU, BRADLEY AND PLATE

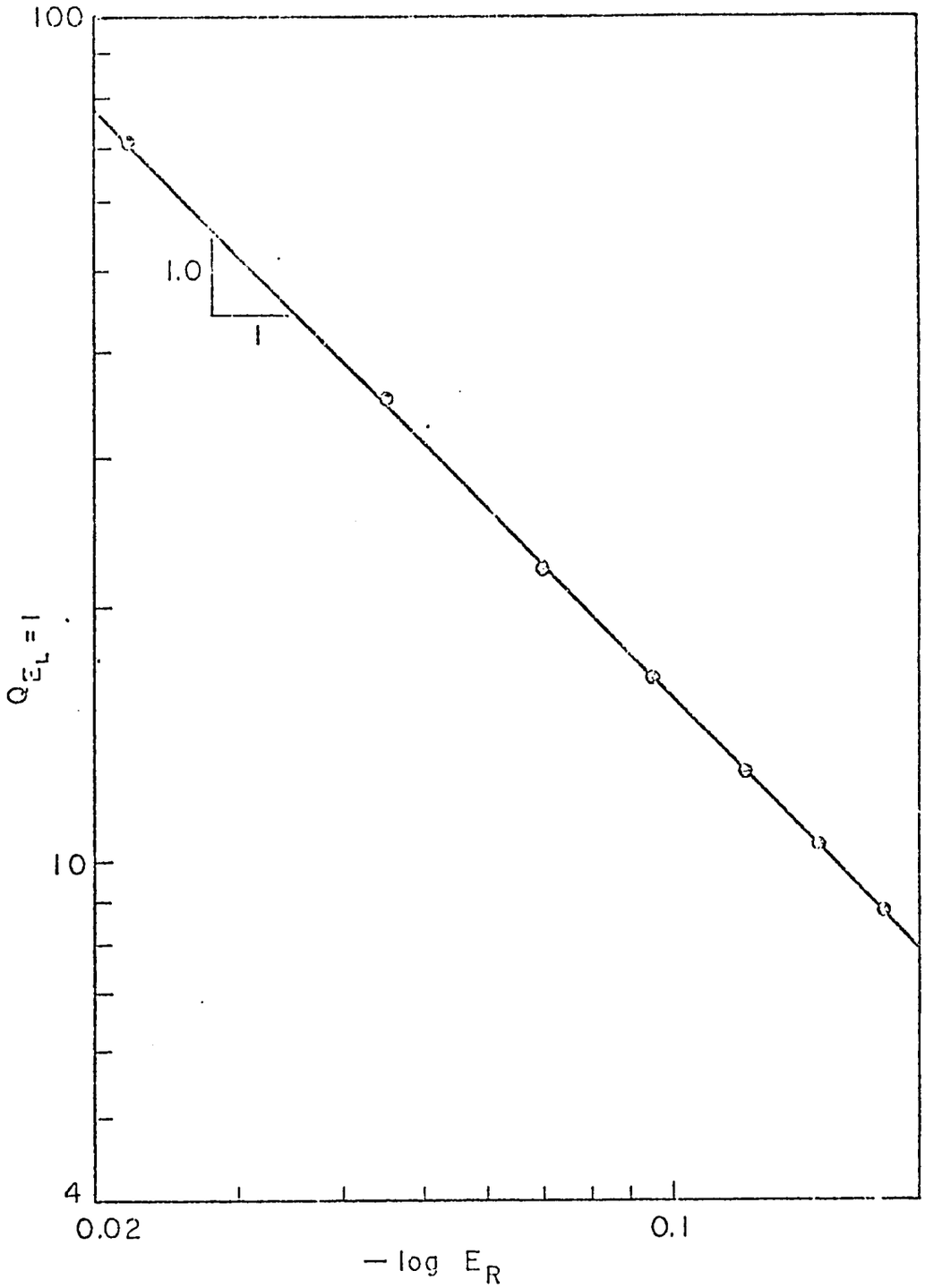

FIG. 9. - ENERGY RATIO DISTRIBUTION FOR EQUATION OF LIU, BRADLEY AND PLATE

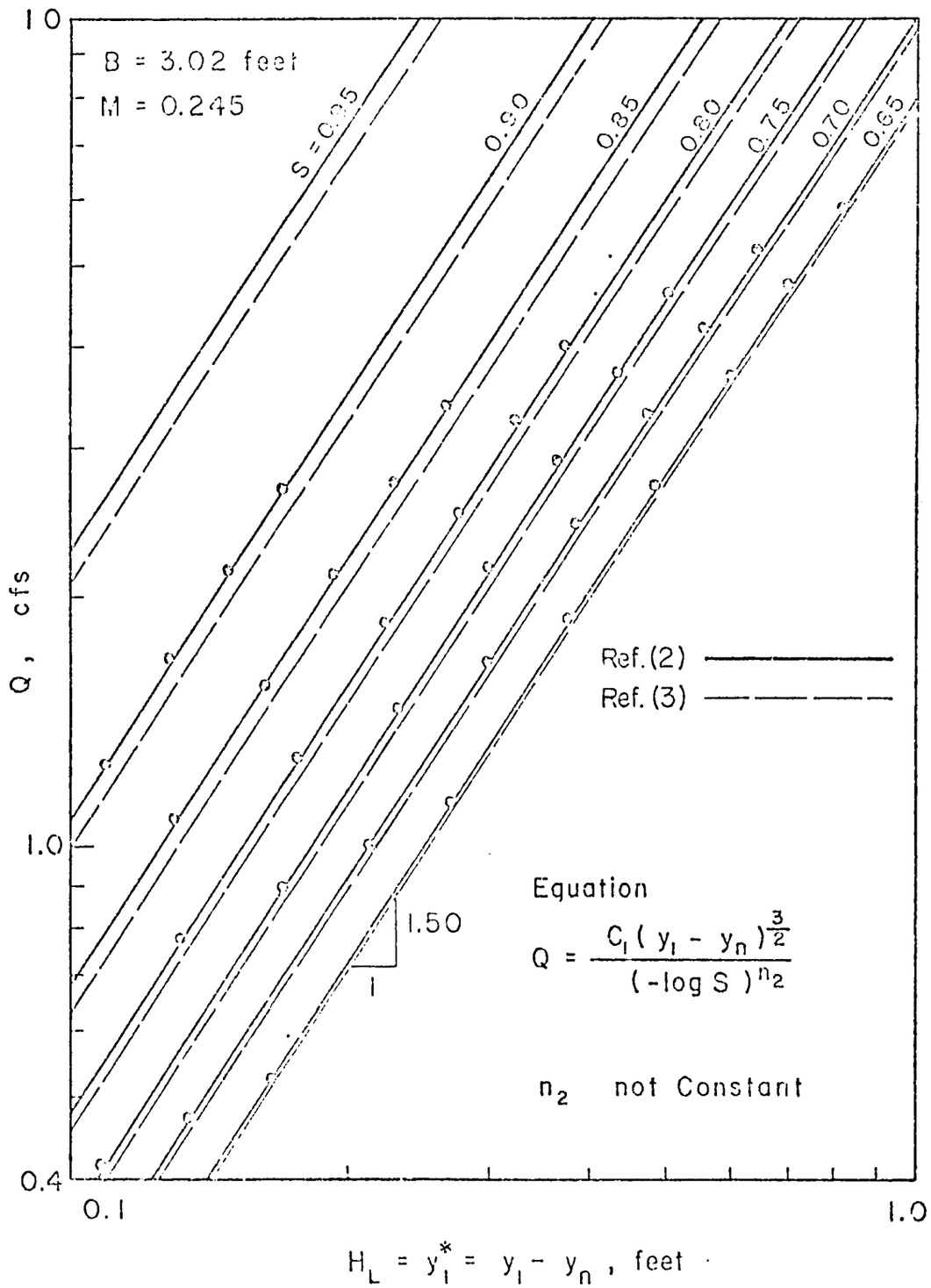

FIG. 10. - SUBMERGENCE FLOW (DEPTH) ANALYSIS OF EQUATION OF BUREAU OF PUBLIC ROADS

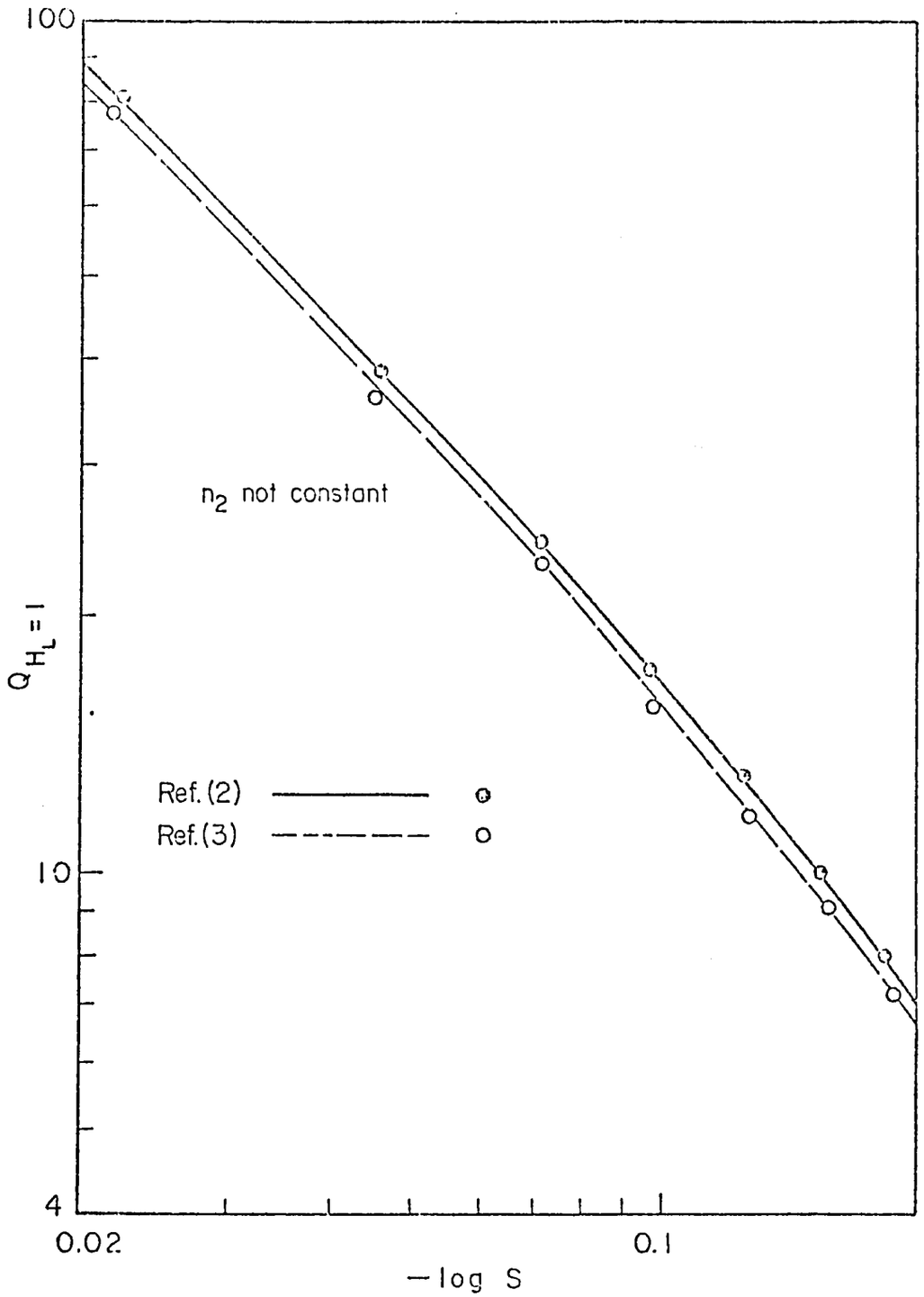

FIG. 11. - SUBMERGENCE DISTRIBUTION FOR EQUATION OF BUREAU OF PUBLIC ROADS

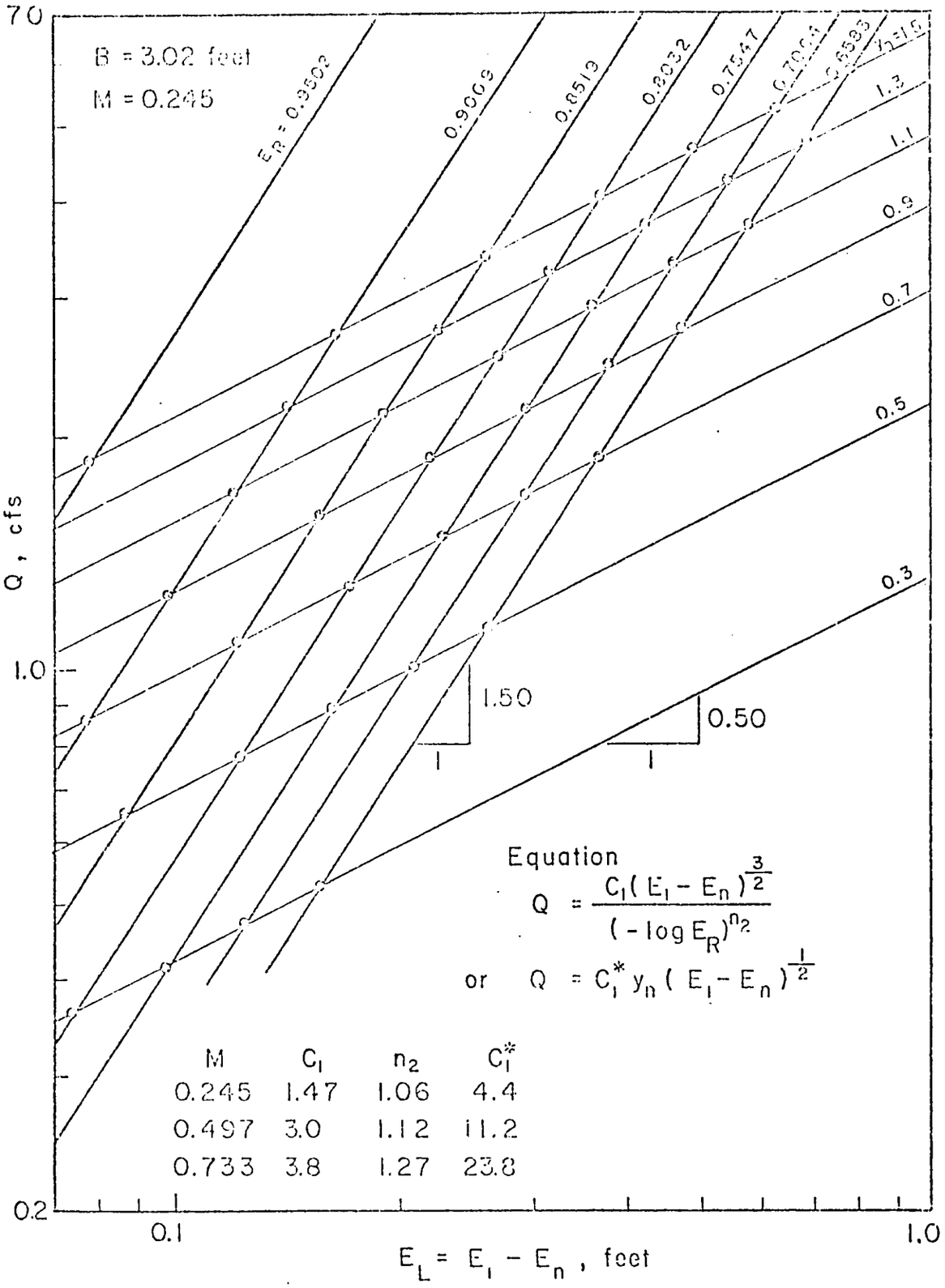

FIG. 12. - SUBMERGED FLOW (ENERGY) ANALYSIS OF EQUATION OF BUREAU OF PUBLIC ROADS

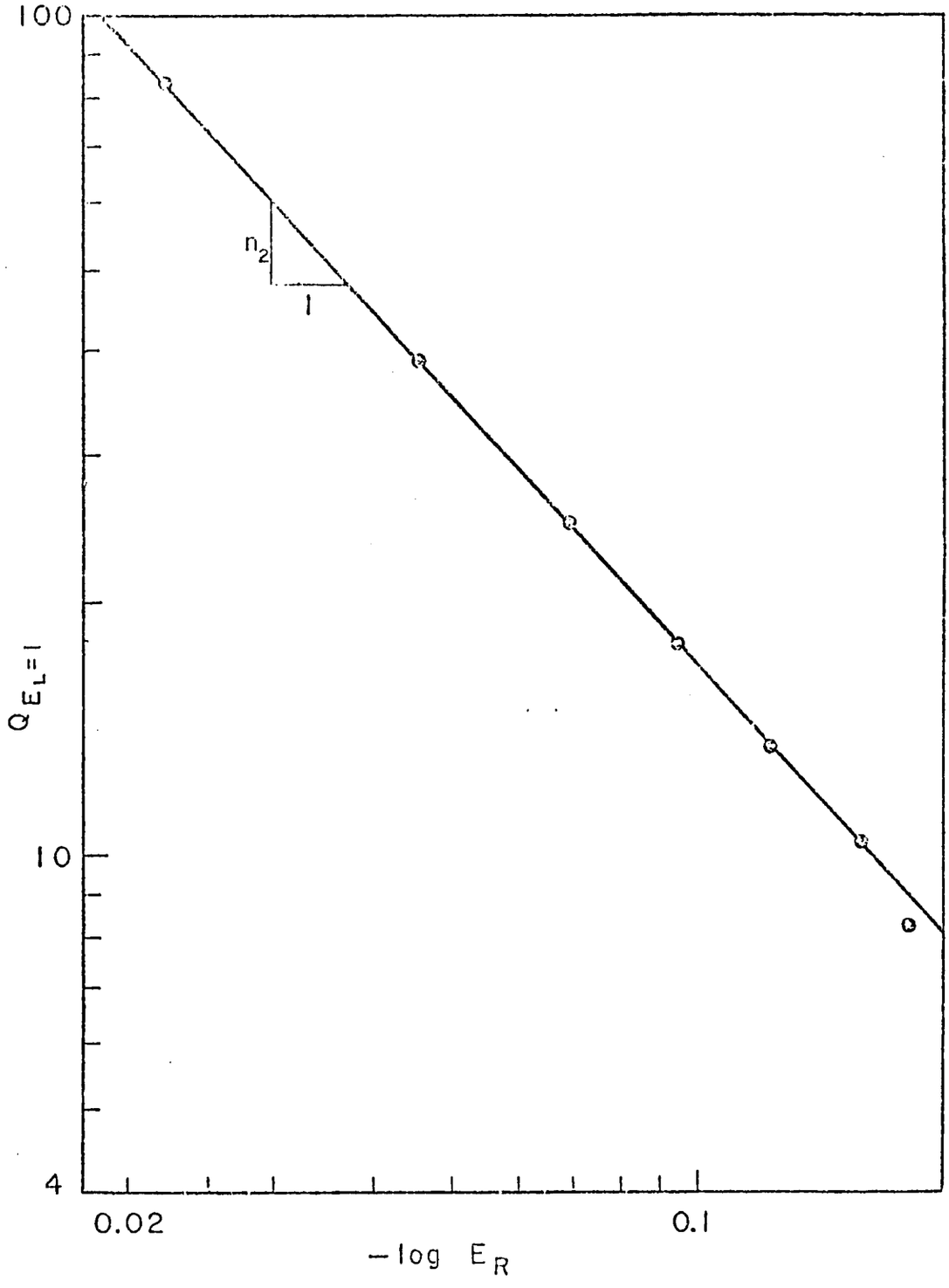

FIG. 13.- ENERGY RATIO DISTRIBUTION FOR EQUATION OF BUREAU OF PUBLIC ROADS

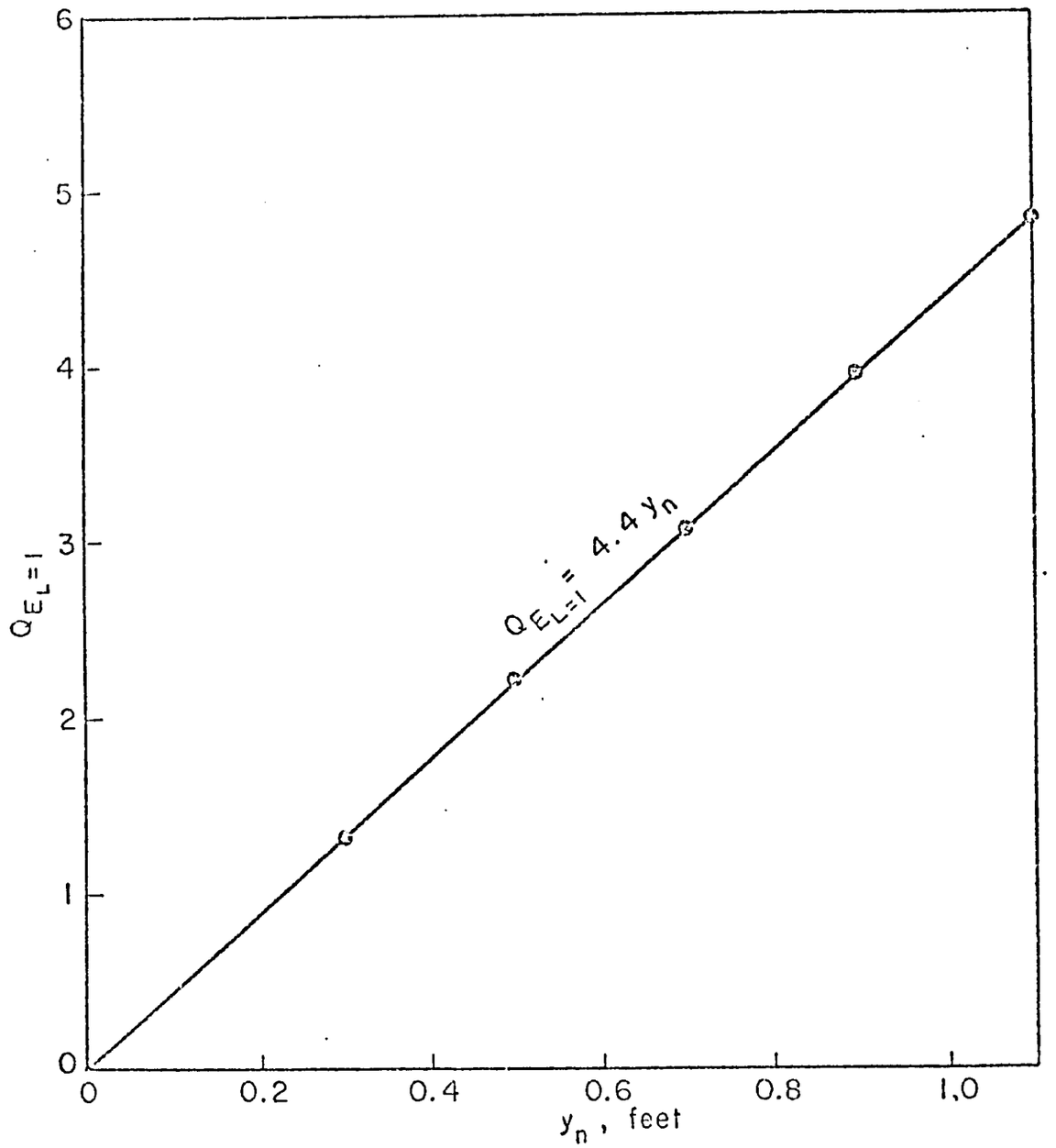

FIG. 14.- NORMAL DEPTH DISTRIBUTION FOR EQUATION OF BUREAU OF PUBLIC ROADS EXPRESSED IN TERMS OF SPECIFIC ENERGIES