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121 Sewall Hall
WILLIAM MARSH RICE UNIVERSITY
Houston, Texas 77001

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THE LONG-RUN INCIDENCE OF GOVERNMENT SPENDING ON EDUCATION

by

Marian Krzyzaniak

and

Ibrahim Eris

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The Long-Run Incidence of Government Spending on Education*

1. Introduction

This paper attempts to attain two objectives. Its first and main concern is determining the combined long-run incidence of a selected tax (on profits and/or on factor incomes) the proceeds of which are spent on education. The increases in and dissemination of knowledge through expenditure on education are assumed to shift the aggregate production function outwards in the form of Harrod neutral technological progress.[1]

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[1]Students of economics of education work with the concept of human capital. It is usually introduced as one of two alternative formulations (see, for example, Jorgenson and Grilliches [5], and Conlisk [3].

In the first of them human capital enters the production function as an independent factor, say H. The aggregate production function may then be written: $Q = Q(K,H,L)$, where Q is the national product, K the homogenous capital and L the homogenous labor. If the labor is educated or becomes skilled, a part of its income is a return to unskilled labor and the rest a return to the human capital workers possess.

In the second approach the education level is combined with the labor to form an index L^* of effectiveness of labor. This index calls for computing two vectors. Vector A measures increased productivity due to various educational levels and vector N is made of numbers of labor units in each educational level. If we let L^* be the inner product or $L^*=A'N$, then the aggregate production function can be formulated as $Q = Q(K,L^*)$.

In a world of perfect divisibility, with only one level of education, the vectors will map into scalars A and L, where $L = \Sigma N_i$, and the aggregate production function can be written $Q = Q(K,AL)$ where A is also the Harrod neutral technological progress. In this paper we explain A as a function of past expenditures on education, which is also the position taken by Uzawa [16]. He studies the optimal level of employment in educational sector without explicitly considering how to implement such a policy in a capitalistic economy.

Second, it has been recognized in the past (see for example [15]), that fitting aggregate production functions for the U.S. economy to explain the growth of national product by the growth of factors alone leaves a large residual, which then is assigned to neutral technological progress. This assignment up to now was largely without a clear justification, a sort of deus ex machina in the Greek and Roman theater. The second objective is thus to substitute a logical explanation for this procedure.

One could try to account for the residual by Arrow's [1] "learning by doing." Inventors invent machines that are capital deepening and when put into operation are at first operated inefficiently until the labor using them learns to operate them. In the process labor reaps the benefit of higher wages. Basically such progress is of the Solow neutral type.

Sheshinsky [14] followed this lead using a model with the Cobb-Douglas production function (i.e., with elasticity of factor substitution $\sigma = 1$) and the Solow neutral technological progress. A moving equilibrium path resulted that was stable but inefficient. Inventors of machines causing "learning by doing" do not reap all the returns of their inventions, some spilling to the labor working with such machines. The supply of such machines under market conditions will, therefore, be inadequate. Sheshinsky proposed a fiscal scheme to improve efficiency, namely labor should be taxed and the tax proceeds transferred to inventors.

There is a flaw in Sheshinsky's argument. True, his models exhibit a stable moving equilibrium path in the presence of the Solow neutral technological progress. Uzawa [17], however, showed that for linear homogenous production functions, in general (i.e., for $\sigma \neq 1$), only the Harrod neutral technological progress rate is permitted. Thus Sheshinsky's case is a very

narrow one, being restricted to the Cobb-Douglas production function only.

Moreover, even in the Cobb-Douglas case we should not interpret the neutral technological progress rate to be other than Harrod neutral. This is because for $\sigma = 1$ we are unable to identify what type of neutrality we have. As for any deviation from $\sigma = 1$, no matter how small, only the Harrod neutrality is permitted; we should restrict even the case of $\sigma = 1$ by definition to this type of neutrality.

It is helpful to consider the various steps in our approach openly. The process starts with adoption of a fiscal policy. A tax scheme is selected first, then the tax proceeds are used to finance education of present and future workers. Existence of a better educated labor force (or the anticipation of it) induces inventors to invent machines requiring higher quality labor force because inventors of such machines find them more efficient. In the process inventors capture the marginal product of their inventiveness and workers receive wages in relation to their effort and to the education they acquired.

The central problem in our approach is that of transformation of government expenditure into education, which in turn results in Harrod neutral technological progress. Education is produced by teaching preadult population in daytime operations and the grown-up labor force in night schools. There is no need to build classrooms or provide any school equipment. Tax proceeds are spent to buy education time. Educators in turn are a part of adult population withdrawn from the labor force and paid the same wage[2] as the remainder of the labor force, called the productive labor force.

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[2]This is not so unrealistic as can be seen by comparing average wages of teachers and of skilled labor in the U.S.A.

At each time t and a given taxation scheme v, we partition the total labor force as follows: $L_t = L_{e,v,t} + L_{p,v,t}$, where $L_{e,v,t}$ stands for teachers (educators) and $L_{p,v,t}$ for the productive labor force. In turn, teachers and their pupils produce Harrod neutral endogenous technological progress coefficient $A_{v,t}$. As in Uzawa [16] we claim that the rate of this progress is a simple function of the ratio of educators to the total labor force $\varphi_{v,t} = L_{e,v,t}/L_t$. It is reasonable to assume that the rate of this progress would be an increasing function of $\varphi_{v,t}$. Hence we adopted the following transformation:

$$\dot{A}_{v,t}/A_{v,t} = \varphi^{\lambda}_{v,t} \text{ , where } 0 < \lambda < 1.^{3}$$

2. The Model

One would like to build growth models as general as possible. Unfortunately in such models, the less restricted is the production function the more difficult (if not outright impossible) it is to get an analytical solution for all the endogenous variables of the model. This need not be a problem if only some of such variables are sought, but in tax incidence analysis nearly all endogenous variables are of interest to the analyst. Analytical resolution usually becomes possible when we restrict ourselves to a very special case, that of Cobb-Douglas production function.[4] No wonder that in the early stages of economic analysis of some problems in growth theory it was customary to make this restrictive assumption. It is also comforting to find that many students of the U.S. economy find empirical evidence that the

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[3] This condition guarantees that $d^2\varphi^{\lambda}_v/d\varphi^2_v = \lambda(\lambda-1)\varphi_v^{(\lambda-2)} < 0$, i.e., there are decreasing returns to an increased role of teachers in the economy.

[4] One then deals with a linear homogenous production function with the elasticity of factor substitution, $\sigma = 1$.

elasticity of factor substitution for the U.S. economy is close to one.[5] This being an early stage in the analysis of the problem of incidence of government expenditure on education we do not feel that use of the Cobb-Douglas production function is damaging our effort unduly. Accordingly, we assume

$$Q_{v,t} = K^{\alpha}_{v,t}[A_{v,t}L_{p,v,t}]^{\beta}, \ \alpha > 0, \ \beta > 0, \ \alpha + \beta = 1. \tag{1}$$

$$L_t = L_o e^{ht}, \ h > 0, \tag{2}$$

$$L_t = L_{e,v,t} + L_{p,v,t}, \tag{3}$$

$$\dot{A}_{v,t} = (L_{e,v,t}/L_t)^{\lambda} A_{v,t} = \varphi^{\lambda}_{v,t} A_{v,t}, A_{v,o} = 1, \tag{4}$$

where

$Q_{v,t}$ = output at time t under tax scheme v, and

$K_{v,t}$ = capital at time t under tax scheme v.

With regard to taxes we assume an income tax with two, possibly different tax rates, the rate on profits, v_b, another tax on individual incomes, v_i. The vector $v = (v_b, v_i)$ defines the tax scheme.[6] The special cases i) $v = (v_b, 0)$, i.e. a tax on business income alone, ii) $v = (0, v_i)$ i.e. a tax on personal income alone, and iii) $v = (v_b, v_i)$ where $v_b = v_i$, i.e. a proportional tax on total income, has been mentioned in different contexts in the economics literature.

We also slightly generalize the distribution of business income. It is assumed that part of returns to capital are retained within business and

[5]Although individual point estimates vary from .35 to 1.1 given the error of such estimates they usually are compatible with hypothesis σ near 1. Franklin Fisher [4] shows that the aggregate production function of the Cobb-Douglas variety may not be a good description of the technical relationships in the U.S. economy, but insofar as the production function at the micro level yields constant factor shares, the aggregate factor shares will also be constant, and that, the Cobb-Douglas production function produces. Thus for tax incidence analysis the assumption of the Cobb-Douglas production function would be quite appropriate.

[6]From previous discussion it would be clear that we assume $v = (0,0)$

the rest distributed as dividends to the shareholders. Thus under the assumption of perfectly competitive markets we can write

$$T_{v,t} = v_b r_{v,t} K_{v,t} + v_i[(1-s_b) r_{v,t} K_{v,t} + w_{v,t} L_t], \tag{5}$$

and this revenue buys teacher time, i.e.

$$T_{v,t} = w_{v,t} L_{e,v,t}, \tag{6}$$

where $T_{v,t}$ = total tax revenue,

$r_{v,t}$ = marginal physical product of capital,

$w_{v,t}$ = marginal physical product of labor in commodity production sector, and also the real wage rate, and

s_b = the ratio of retained profits to total profits.

Equation (6), of course, is nothing but the requirement of a balanced budget.

As usual, investment in capital, $K_{v,t}$, is determined by available funds (i.e. by savings in the private sector). If we let s_i be the constant average saving rate by households out of income and $S_{v,t}$ the total savings, we obtain:

$$S_{v,t} = s_b(1-v_b) r_{v,t} K_{v,t} + s_i(1-v_i)(1-s_b) r_{v,t} K_{v,t} + s_i(1-v_i) w_{v,t} L_{v,t} \tag{7}$$

or

$$s_{v,t} = (S_{v,t}/Q_{v,t}) = \alpha s_b(1-v_b) + \alpha s_i(1-v_i)(1-s_b) + \beta s_i(1-v_i) + (s_i(1-v_i) w_{v,t} L_{e,v,t})/Q_{v,t}, \tag{7a}$$

and

$$K_{v,t} = s_{v,t} Q_{v,t}. \tag{8}$$

The first important thing to note about equations (1)-(8) is that they imply $\varphi_{v,t}$ and $s_{v,t}$ are constants over time under a given tax scheme even if

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for $t < 0$ and a constant $v = (v_b, v_i)$, $v_b \geq 0$, $v_i \geq 0$ for $t \geq 0$.

a moving equilibrium is not yet reached. This result is essentially due to assumptions of Cobb-Douglas production function and constancy of tax rates over time and would not be true out of equilibrium, for example, for CES production functions. Thus, the constancy of the rate of technological progress, in our model, is intimately related to the specification of the production function which points to the fact that ad hoc assumptions about such a constancy will in general fail to be true.

The proof is simple: from (3), (5) and (6) we have

$$(1-v_i)w_{v,t}L_{e,v,t} = v_b r_{v,t}K_{v,t} + v_i[(1-s_b)r_{v,t}K_{v,t} + w_{v,t}L_{p,v,t}].$$

Substituting this expression in (7a) we obtain:

$$s_v = s_{v,t} = \alpha s_b(1-s_i) + s_i - \alpha(s_b-s_i)v_b. \tag{7b}$$

On the other hand, dividing both sides of the same expression by $(w_{v,t}L_{p,v,t})$ and rearranging yields

$$\varphi_v = \varphi_{v,t} = [\alpha v_b + (1-\alpha s_b)v_i]/[\beta + \alpha(v_b + (1-s_b)v_i)]. \tag{9}$$

It is of interest to note that the rate of saving is not affected by non-business tax rate, that $s_v \gtreqless s_o$ depending on $s_b \gtreqless s_i$ and finally that $\partial s_v/\partial v_b \gtreqless 0$ depending on $s_b \gtreqless s_i$. On the other hand $\partial\varphi_v/\partial v_b > 0, \partial\varphi_v/\partial v_i > 0$ always. As we shall see, these facts have important bearing on incidence of government policies.

In Table (1) we summarize the after-tax value of variables which are of interest, and which remain constant, under different assumptions on tax rates.

With equations (7b) and (9) the model analytically becomes quite similar to the standard neoclassical one-sector growth model with constant rate of technological progress. One important difference remains though. The model here is subject to distinct regimes before and after $t = 0$. During $t < 0$

TABLE 1: Formulae of General and Special Tax Cases

Notation		$v = (v_b, v_i)$	$v = (v_b, 0)$	$v = (v, v)$	$v = (0, v_i)$
		Variant B	Variant B	Variant B	Variant B
1	s_o	$\alpha s_b + (1-\alpha s_b) s_i$	$\alpha s_b + (1-\alpha s_b) s_i$	$\alpha s_b + (1-\alpha s_b) s_i$	$\alpha s_b + (1-\alpha s_b) s_i$
2	s_v	$s_o - \alpha(s_b - s_i) v_b$	$s_o - \alpha(s_b - s_i) v_b$	$s_o - \alpha(s_b - s_i) v$	s_o
3	s_v / s_o	$1 - \frac{\alpha(s_b - s_i) v_b}{s_o}$	$1 - \frac{\alpha(s_b - s_i) v_b}{s_o}$	$1 - \frac{\alpha(s_b - s_i) v}{s_o}$	1
4	$F_{c,v}$	$\alpha[1 - v_b - (1-s_b) v_i]$	$\alpha(1 - v_b)$	$\alpha[1 - (1-s_b) v]$	$\alpha[1 - (1-s_b) v_i]$
5	$F_{p,v}$	$\beta(1 - v_i)$	β	$\beta(1 - v_i)$	$\beta(1 - v_i)$
6	$F_{e,v}$	$\alpha v_b + (1-\alpha s_b) v_i$	αv_b	$[1 + \alpha(1-s_b)] v$	$(1-\alpha s_b) v_i$
7	$F_{w,v}$	$\beta + \alpha v_b + \alpha(1-s_b) v_i$	$\beta + \alpha v_b$	$\beta + \alpha(2-s_b) v$	$\beta + \alpha(1-s_b) v_i$
8	$\varphi_v = \frac{F_{e,v}}{F_{w,v}}$	$\frac{\alpha v_b + (1-\alpha s_b) v_i}{\beta + \alpha[v_b + A(1-s_b) v_i]}$	$\frac{\alpha v_b}{\beta + \alpha v_b}$	$\frac{1 + \alpha(1-s_b) v}{\beta + \alpha(2-s_b) v}$	$\frac{(1-\alpha s_b) v_i}{\beta + \alpha(1-s_b) v_i}$

Notation: $F_{c,v}$ = relative share of capital (after tax),

$F_{p,v}$ = relative share of productive labor (after tax),

$F_{e,v}$ - relative share of educators (after tax),

$F_{w,v}$ = relative share of total labor (after tax).

there are no taxes, all labor is in the commodity production sector and there is no technological change due to education. At time t = 0 taxes are imposed.[7] Mathematically this leads to discontinuities in some of the solutions at t = 0, as can be seen in the following explicit solutions of the variables in system (1)-(9) which are obtained by techniques familiar to economists by now.

If we let $k_{p,v,t} = K_{v,t}/L_{p,v,t}$ and assume the initial $k_{p,o,o}$ is given, then we obtain

$$k_{p,o,t} = e^{-ht}\left[k^{\beta}_{p,o,o} - (1-e^{h\beta t})(s_o/h)\right]^{1/\beta} \qquad \text{for } t < 0 \qquad (10a)$$

$$k_{p,v,t} = e^{-ht}\left[k^{\beta}_{p,o,o}/(1-\varphi_v)^{\beta} - (1-e^{(h+\varphi_v^{\lambda})\beta t})(s_v/(h+\varphi_v^{\beta})\right]^{1/\beta} \qquad \text{for } t \geq 0 \qquad (10b)$$

$$K_{o,t} = L_o e^{ht} k_{p,o,t} \qquad \text{for } t < 0 \qquad (11a)$$

$$K_{v,t} = (1-\varphi_v) L_o e^{ht} k_{p,v,t} \qquad \text{for } t \geq 0 \qquad (11b)$$

$$Q_{o,t} = L_o e^{ht} k^{\alpha}_{p,o,t} \qquad \text{for } t < 0 \qquad (12a)$$

$$Q_{v,t} = (1-\varphi_v) L_o e^{(h+\varphi_v^{\lambda}\beta)t} k^{\alpha}_{p,v,t} \qquad \text{for } t \geq 0 \qquad (12b)$$

$$w_{o,t} = \beta k^{\alpha}_{p,o,t} \qquad \text{for } t < 0 \qquad (13a)$$

$$w_{v,t} = \beta e^{\varphi_v^{\lambda}\beta t} k_{p,v,t} \qquad \text{for } t \geq 0 \qquad (13b)$$

$$r_{o,t} = \alpha k^{-\beta}_{p,o,t} \qquad \text{for } t < 0 \qquad (14a)$$

$$r_{v,t} = \alpha e^{\varphi_v^{\lambda}\beta t} k^{-\beta}_{p,v,t} \qquad \text{for } t \geq 0 \qquad (14b)$$

Now if $\varphi_v > 0$ then k and hence Q, w and r, have discontinuities at t = 0 since $k_{p,v,o} = k_{p,o,o}/(1-\varphi_v)$. But, as one would expect, K is a continuous

[7] For the sake of brevity we presented our model in the form given in (1)-(9). The reader should realize that actually two such systems, one referring to t<0, another to t ≥ 0, should be spelled out.

function of time. This has the effect that the usual definition of equilibrium of balanced growth is of little use in our model: the system, in general, cannot be in equilibrium both before and after the imposition of taxes. Assume the system was in equilibrium before taxes, i.e. assume $k^{\beta}_{p,o,o} = (s_o/h)$. Then in order to have the system in equilibrium during $t \geq 0$ we must also have $k^{\beta}_{p,o,o} = (s_v/h + \varphi^{\lambda}_v)(1-\varphi_v)^{\beta}$. But these two equalities are not consistent, for example, if $s_b > s_i$ (which is a reasonable assumption in the U.S. case).

3. Some General Remarks on Dynamic Incidence Analysis

All incidence analysis ultimately tries to answer the following question: In what way does the government taxation and expenditure policies affect the selected income or consumption variables of the model under consideration? The analyst determines how such variables would have evolved without government action and with it, and then tries to make some meaningful comparisons which might be of help in reaching a decision as to what policies to adopt.[8]

In order to be able to answer this question with as little subjectivity as possible at times certain simplifications have been made in the literature. Some of them have been misleading, as in the case of attempts to separate the effects of taxation from government expenditure. As long as one is willing to accept a budget constraint for the government (or the Walras law for the economy as a whole which is an identity), he has to acknowledge that only the <u>combined effects</u> of taxing and spending can be ascertained.

One often finds another simplification that is more difficult to accept. In general, government policy displaces the economy from its "equilibrium

[8] See, for example, Krzyzaniak [6], [7] [8], [9], [10].

path," if it was there to begin with. We know by now that in general neither stability of an equilibrium is guaranteed (especially in a world with more than two commodities), nor a speedy approach to it. It might take an extremely long time to get "close" to such an equilibrium.[9] In the past students of public finance evaluated incidence of a policy from the shift of variables in equilibria, i.e. they applied comparative statics, disregarding the dynamics of the problem. As long as the adjustment to the change in policy was speedy[10], or the direction of incidence in the short run did not differ[11] from that in the long run, the message of their approach was valid. But adjustments need not be speedy and if in addition the direction of incidence changes[12] between the short and the long run it may be misleading to disregard the dynamic change.[13]

There is also the question of definition of burdens. In the literature there does not seem to be agreement on this point.[14] Any government taxation and/or expenditure will in general affect the total output in the economy, incomes of individuals <u>and</u> their savings (or consumption). Thus the distributional effects of different taxation and expenditure schemes shall be different depending on whether one considers income burdens or

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[9] See, for example, [12], and [13].

[10] A low, near zero value of σ would guarantee speedy adjustment.

[11] This means that if a policy had a burden in the short run it had also a burden in the long run and vice versa.

[12] This means the policy resulted in a burden in the short run, but in a gain in the long run or vice versa.

[13] Burmeister and Dobell [2, p. 57] on Sato's evidence conclude that "the possibly limited significance of the neoclassical model for purposes of social policy making became clear."

[14] For example, in his survey article P. Mieszkowski [11] mentions studies using burden defined both as a decline in income and as a decline in consumption.

consumption burdens.[15]

In differential incidence analysis a certain pattern of government expenditure (usually in terms of goods consumed by government) is taken to be given and then one tries to derive which of two different tax schemes that secure such a basket has a lesser burden or a larger gain. The ultimate objective is to determine a least "costly" way of reaching the objective.[16] The trouble is that there is no way of defining "cost" uniquely. The ordering of different tax schemes may vary depending on whether we define it in terms of income or consumption foregone (wealth lost).

A striking example of such a situation is supplied below in the context of expenditures on education. If "cost" is the change in incomes the least "cost" tax scheme dictates $v_b = 0$; if "cost" is the change in consumption the least "cost" calls for $v_i = 0$. Thus the choice of one or the other of these variables as the relevant burden or "cost" of government activity implicitly requires a value judgment.

4. Global Burden of Government Spending on Education

Let us define global income burden (gain), $B(v_b, v_i, t)$, and global consumption burden (gain), $C(v_b, v_i, t)$ as follows:

$$B(v_b, v_i, t) = Q_{v,t} - Q_{o,t} \quad \text{for all } t \geq 0$$

$$C(v_v, v_i, t) = (1-s_v)Q_{v,t} - (1-s_o)Q_{o,t} \text{ for all } t \geq 0,$$

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[15] Defining incidence as policy effects on real income (or wealth) of persons seems to be the older tradition. Krzyzaniak's past work was done along this track. Only with the advent of thinking with the help of growth models policy effects on consumption of persons were added as an alternative way of determining incidence. The distribution of consumption, however, may be viewed as a surrogate for distribution of wealth.

[16] Of course, desirability of least "cost" financing may also depend on its distributional impact on different individuals in the economy.

so that we talk of income gain at time t if $B(v_b,v_i,t) > 0$ and of income burden[17] at time t if $B(v_b,v_i,t) < 0$ and similarly for consumption.

To simplify our analysis we shall make two additional assumptions throughout the rest of the paper: (1) the economy was on a balanced growth path during $t < 0$ and thus would have remained so if no taxes were imposed; and (2) $s_b > s_i$. The following results crucially depend on these assumptions, but there is no difficulty in ascertaining how they should be modified if they are removed.

First we note some initial relationships and how they change with passage of time. It is easy to see that if $v_b > 0$ and $v_i > 0$, then $B(v_b,v_i,0) < 0$ and that there exists some $t_v > 0$ such that $B(v_b,v_i,t_v) = 0$ and for all $t > t_v, B(v_b,v_i,t) > 0$, that is, independent of the levels of taxes, in the "short run" there are income burdens and in the "long run" income gains. Furthermore, it can be shown that if $v = (v_b,v_i)$, $v' = (v_b',v_i')$ are such that $\varphi_v > \varphi_{v'}$ then $B(v_b',v_i',0) < B(v_b,v_i,0)$. There is no way to tell, however, whether $t_v \geq t_{v'}$. Similar statements hold for $C(v_b,v_i,t)$. It is of interest to note that in general $C(v_b,v_i,0) < B(v_b,v_i,0)$.

This shows that even if we neglect the distributional effects of expenditure on education and consider solely the global burdens there is a non-trivial problem of choice facing the policy makers. The choice will obviously depend upon how the government evaluates the short-run losses against long-run gains, and for what variable this evaluation shall be made. There is no way for the economist to make such a judgment purely on efficiency grounds.

Let us consider a familiar simple case. Assume that the government decision can be formulated either as

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[17] Krzyzaniak (see [6], [7], [8] and [9]) in the past defined burdens with a positive sign and realized transfers (see [10]) with a negative sign. The sign convention used here seems to be more natural.

$$\max \int_{t=0}^{\infty} e^{-\rho t} B(v_b, v_i, t)dt$$

subject to $0 \leq v_b \leq 1$ and $0 \leq v_i < 1$, or as

$$\max \int_{t=0}^{\infty} e^{-\rho t} C(v_b, v_i, t)dt$$

subject to $0 \leq v_b \leq 1$ and $0 \leq v_i < 1$,[18]

where $\rho > 0$. It is known that if ρ is too small a number (in our case if it is smaller than $1 + h$, 1 being the supremum of possible rates of technological change) for some $v = (v_b, v_i)$, the integral shall be infinite and thus the problem would have no solution.[19] It is intuitively clear, though, that for sufficiently large ρ there shall be a solution to either or both objectives. We give a discussion of solutions of these problems in the appendix. The expressions involved are algebraically too complex and not very enlightening. Therefore, here, we shall limit ourselves to some weaker results.

In his previous work[20] Krzyzaniak has noted that taxation of business leads to high income burdens for the economy. We obtain an analagous result here also, i.e. <u>if $\hat{v} = (\hat{v}_b, \hat{v}_i)$ is the solution to maximization of gains in income then $\hat{v}_b = 0$</u>. (For proof, see the appendix). Thus, if any technological progress is desirable at all it should be paid by the individual taxes.

When one considers burdens or gains in terms of consumption the picture completely changes and the desirability of different tax rates becomes very

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[18] The reason for not allowing $v_i = 1$ is that in such a case $\varphi_v = 1$ and equation (10b) (and therefore the rest of the equations) do not make sense.

[19] One might argue in such a case that at least the desirability of certain tax schemes has been established.

[20] See Krzyzaniak [6], [7], [8] and [9].

much dependent on the discount rate and other parameters. In one case, one obtains a condition almost completely opposite of the one given above: *if* $[s_i/1-s_i(1-s_b)] \geq \alpha$ *and if* $\hat{v} = (\hat{v}_b, \hat{v}_i)$ *is the solution to the second problem above then either* $\hat{v}_i = 0$ *or* $\hat{v}_b = 1$ *or both*.[21] (For proof see the appendix).

In the light of these propositions let us consider the standard problem of differential incidence analysis, that is, obtaining a given government objective in the "least costly" way. It looks natural to us that in our problem the given objective of the government has to be to "buy" a certain amount of rate of technological progress. Thus the problem can again be formulated exactly as above except that one would put the additional constraint on each problem that $\varphi_v = \varphi^*$ where φ^* is the given rate of technological progress. The necessary conditions given above hold also in this case (see the appendix). But these conditions have completely different distributional implications. Apart from ethical judgments there is no reason why one should prefer either of these expressions as *the* cost or burden to an economy.

5. Distributional Effects of Government Spending on Education

The objectives considered above are concerned only with the aggregate performance of the economy. One might also consider the case where burdens of different subgroups within the economy might enter the objective function of the government separately. For example, it might be of interest to study separately the effects of different taxes on wages bill and to total profits.

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[21] We do not want to imply that the condition stated here is realistic. All we are trying to show is the possibility of such a result. We should also add that it seems like much weaker conditions on parameters might lead to the same result. (See the appendix).

Another interesting question would be to ascertain the popularity of government taxation for education, that is how does the government policy affect an individual or household? Is it possible that taxation for education despite its future benefits fails to pass, say, a plebiscite? To answer such questions would require, in addition to what is postulated in this paper, data on distribution of wealth among the households and their preference functions. Here we shall only try to determine the effects on wage and profit rates, total wages bill, and total profits. Then we shall make some suggestions as to what type of results could be obtained under different assumptions about distribution of wealth.

Table 2 below gives the definitions of income burdens when it is decomposed into factor incomes. In order to discuss the expressions involved there, let us first determine the effective wage and profit rates. Equations (13) and (14) give the pretax rates, not the after-tax rates. Let us define effective wage rate, $\hat{w}_{v,t}$, and effective interest rate, $\hat{r}_{v,t}$, as

$$\hat{w}_{v,t} = F_{w,v}Q_{v,t}/L_t \tag{15}$$

$$\hat{r}_{v,t} = F_{c,v}Q_{v,t}/K_{v,t}. \tag{16}$$

where $F_{w,v}$ and $F_{c,v}$ are given in Table 1.

From these and previous formulas by routine calculations one can show that

$$\hat{r}_{v,o} = [1-v_b-(1-s_b)v_i](1-\varphi_v)^{\beta} r_{o,o} \text{ and}$$

$$\lim_{t\to\infty} \hat{r}_{v,t} = [1-v_b-(1-s_b)v_i][(h+\varphi_v^{\lambda})s_o/hs_v]r_{o,o}.$$

Thus we see that if $v_b > 0$ or $v_i > 0$ then $\hat{r}_{v,o} < r_{o,o}$, that is the rate of profit declines at $t = 0$. Whether the equilibrium (limiting) value of $r_{v,t}$ is greater or less than what it would have been, $r_{o,o}$ will depend on what

TABLE 2: Definition of Policy Burdens
(taxing incomes and spending the proceeds on education)

No.	Economy	Real national product for private use	Real net (after tax) profits	Real net (after tax) wages	Real net (after tax)	
					wages of productive labor	wages of educators
1.	In absence of taxes	$Q_{o,t}$	$\pi_{o,t}$	$W_{o,t}$	$W_{o,t}$	--
2.	With tax	$Q_{v,t}$	$(\pi_{v,t}-T_{c,v,t})$	$(W_{v,t}+T_{c,v,t})$	$(W_{p,v,t}-T_{p,v,t})$	$(T_{c,v,t}+T_{p,v,t})$
3.	The difference (burdens or gains)	$(Q_{v,t}-Q_{o,t})$	$(\pi_{v,t}-\pi_{o,t})-T_{c,v,t}$	$(W_{v,t}-W_{o,t})+T_{e,v,t}$	$(W_{p,v,t}-W_{o,t})-T_{p,v,t}$	$(T_{c,v,t}+T_{p,v,t})$

Notation:

π = total profits
W = total wages
T_c = tax liability of capitalists
T_w = tax liability of labor
T_p = tax liability of productive labor
T_e = tax liability of educators

level of technological progress is chosen and how it is financed. It is possible that rate of profit will never reach its previous level. From this we cannot conclude though that the after tax <u>total profits</u> shall also be lower. In effect, total profits will evolve exactly like total output, since in the notation of table (2), $\pi_{v,t} - T_{c,v,t} = \hat{r}_{v,t} K_{v,t} = F_{c,v} Q_{v,t}$.

The situation for wage rates is quite different. We see that

$$\hat{w}_{v,o} = \left\{[\beta + \alpha v_b + \alpha(1-s_b)v_i]/\beta\right\}(1-\varphi_v)^{\beta} w_{o,o}$$

$$= [\beta + \alpha v_b + \alpha(1-s_b)v_i]^{\alpha} [\beta(1-v_i)]^{\beta} w_{o,o}/\beta$$

$$\lim_{t\to\infty} \hat{w}_{v,t} = \lim_{t\to\infty} [F_{w,v}(1-\varphi)L_o e^{(h+\varphi_v^{\lambda}\beta)t} k_{p,v,t}^{\alpha}] = \infty$$

As is clear from the first equation, it is possible that the wage rate (and hence the total wages bill) will <u>increase</u> at $t = 0$. For example if $v_b > 0$ and $v_i = 0$ (or sufficiently small relative to v_b) then $\hat{w}_{v,0} > w_{0,0}$.

From the analysis of the previous section it follows that if education is to be provided some segment of the population has to sacrifice in the short run, but the result about wages above shows that not everybody needs to forego income or consumption. In order to determine precisely what segment of the economy will pay for the education under different tax schemes or how popular such a measure would be depends upon the distribution of wealth among individual labor units. If capital is distributed evenly among the population, then it is clear that everybody will have to carry part of the burden in the short run. On the other hand, if capital is very unevenly distributed then education financed by taxes on capital may turn out to be a very popular policy, since for a large part of the population profits income would be a small percentage of the total income and, therefore, it is possible

that gains in wages will cancel out the losses in profit income.[22]

Even if the distribution of wealth is uneven, expenditure in education may still turn out to be a popular measure, depending upon the "length of short run" and the age composition and rates of time preference in the economy. Given the parameters of our system, one can calculate the time at which benefits of education will be felt for each income component. If the population is a "young" one, one would expect parts of it would benefit from education within their lifetime. Of course, we should also consider the possibility that the benefits to upcoming generations will affect the utility of the present generation favorably.

Once more we should stress that also at the individual level one should consider not only effects of taxes on incomes but also on consumptions. The latter is the one which we in general assume the consumer considers in his decisions.

All this points to a question of practical political significance: is there a tax-subsidy system which a government can use under which some supply of education is desirable on the part of the majority of the population? If so, what would the most popular expenditure pattern be?

We should hasten to add that our stress in this section with the views of the present generation is not meant to imply that education should not be supplied if it is not desirable from the point of view of the present generation. Though one should admit that in an electoral system it might be extremely difficult to do otherwise.

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[22] Another reasonable conjecture at this point would be that the income distribution will become more even over time as benefits of the education accrue to labor.

Appendix

Let $$f(v_b, v_i) = \int_{t=0}^{\infty} e^{-\rho t} B(v_b, v_i, t)\,dt$$

and $$g(v_b, v_i) = \int_{t=0}^{\infty} e^{-\rho t} C(v_b, v_i, t)\,dt$$

From calculus it is known that if $\hat{v}_b, \hat{v}_i$ maximize f then one of the following holds at $v_b = \hat{v}_b, v_i = \hat{v}_i$

i) $\partial f/\partial v_b \leq 0, \quad \partial f/\partial v_i \leq 0, \quad \hat{v}_b \partial f/\partial v_b = \hat{v}_i \partial f/\partial v_i = 0$

ii) $\partial f/\partial v_b > 0, \quad \partial f/\partial v_i \leq 0, \quad \hat{v}_b = 1, \quad \hat{v}_i \partial f/\partial v_i = 0.$

Similarly for g. The case $\hat{v}_i = 1$ is excluded by the constraints. It is also true that

$$\partial f/\partial v_b = \int_{t=0}^{\infty} e^{-\rho t} \partial B/\partial v_b\,dt$$

and $$\partial f/\partial v_i = \int_{t=0}^{\infty} e^{-\rho t} \partial B/\partial v_i\,dt.$$

Similar expressions hold for g.

Let $$A(v_b, v_i, t) = k^{\beta}_{\Gamma\ ,o}/(1-\varphi_v)^{\beta} - (e^{-(h+\varphi_v^{\lambda})\beta t} - 1)[s_v/(h+\varphi_v^{\lambda})].$$

Then taking the derivative and rearranging one obtains:

$$\partial B/\partial v_b = A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda})t} \Big\{ \partial\varphi_v/\partial v_b [\alpha(k^{\beta}_{p,o,o}/(1-\varphi_v)^{\beta})(1/1-\varphi_v) e^{-(h+\varphi_v^{\lambda})\beta t}$$
$$- A + (1-\varphi_v)\lambda\varphi^{\lambda-1}(\beta t A + \alpha t\, s_v/(h+\varphi_v^{\lambda})$$
$$+ (e^{-(h+\varphi_v^{\lambda})\beta t} - 1)\alpha s_v/\beta(h+\varphi_v^{\lambda})^2)] - \partial s_v/\partial v_b (1-\varphi_v)(e^{-(h+\varphi_v^{\lambda})\beta t} - 1)[\alpha/\beta(h+\varphi_v^{\lambda})]$$

So we can write

$$\partial B/\partial v_b = A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda})t} [U(v_b, v_i, t)\partial\varphi_v/\partial v_b - Y(v_b, v_i, t)\partial s_v/\partial v_b]$$

Now it is easy to show that

$$\partial B/\partial v_i = A^{(2\alpha-1)\beta} L_o e^{(h+\varphi_v^{\lambda})t} U \partial\varphi_v/\partial v_i .$$

Furthermore, let us note that

$$A(v_b, v_i, t) > 0 \quad \text{and} \quad Y(v_b, v_i, t) < 0 \text{ for all } t \geq 0.$$

Assume $\hat{v}_b, \hat{v}_i$ maximizes $f(v_b, v_i)$, where $0 \leq \hat{v}_b \leq 1$ and $0 \leq \hat{v}_i < 1$. Then by the necessary conditions stated above

$$\partial f/\partial v_i = \partial\varphi\ \partial v_i \int_{t=0}^{\infty} A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda}-\rho)t} U dt \leq 0$$

where all the expressions are evaluated at $v = \hat{v} = (\hat{v}_b, \hat{v}_i)$.

But then

$$\partial f/\partial v_b = [(\partial f/\partial v_i)(\partial\varphi_v/\partial v_b)/(\partial\varphi_v/\partial v_i)] - (\partial s_v/\partial v_b) \int_{t=0}^{\infty} A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda}-\rho)t} Y dt < 0$$

since

$\partial s_v/\partial v_b < 0$, $\partial\varphi_v/\partial v_b > 0$ and $\partial\varphi_v/\partial v_i > 0$. Thus $\hat{v} = 0$ should hold, which proves the first assertion in the text.

Using the same notation as above one can easily show that

$$\partial C/\partial v_b = A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda})t} \left\{ (1-s_v)(\partial\varphi_v/\partial v_b)U - (\partial s_v/\partial v_b)[(1-s_v) + (1-\varphi_v)A] \right\}$$

$$\partial C/\partial v_i = A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda})t} (1-s_v)(\partial\varphi_v/\partial v_i) U.$$

Now if $\hat{v}_b, \hat{v}_i$ maximizes $g(v_b, v_i)$, then $\partial g/\partial v_i \leq 0$, evaluated at the maximum point. But this time we have

$$\partial g/\partial v_b = [(\partial f/\partial v_i)(\partial\varphi_i/\partial v_b)/(\partial\varphi_v/\partial v_i)] - (\partial s_v/\partial v_b) \int_{t=0}^{\infty} A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda}-\rho)t} [(1-s_v)Y + (1-\varphi_v)A] dt.$$

Unfortunately, the sign of this last expression is not easy to determine. To prove the special case mentioned in the text let us note that

$$(1-s_v)Y + (1-\varphi_v)A = (1-\varphi_v)\left\{(k^{\beta}_{p,o,o}/(1-\varphi_v)^{\beta})\ e^{-(h+\varphi_v^{\lambda})\beta t}\right.$$

$$- [e^{-(h+\varphi_v^{\lambda})\beta t} -1][(s_v-\alpha)/\beta(h+\varphi_v^{\lambda})]$$

$$=(1-\varphi_v)\left\{[(k^{\beta}_{p,o,o}/(1-\varphi_v)^{\beta})-(s_v-\alpha)/\beta(h+\varphi_v^{\lambda})]e^{-(h+\varphi_v^{\lambda})\beta t}\right.$$

$$\left. + (s_v-\alpha)/\beta(h+\varphi_v^{\lambda})\right\}$$

This expression is non-negative for all t if $s_v \geq \alpha$. The condition given in the text guarantees this result. So if $\partial g/\partial v_i = 0$ then $\partial g/\partial v_b > 0$ which implies $v_b = 0$. If $\partial g/\partial v_i < 0$ then $v_i = 0$, hence the result stated in the text is true. As mentioned before, and in view of the above formulae, it should be pointed out that the condition given in the text is too strong for the desired results. The result depends upon the positivity of the integral:

$$\int_{t=0}^{\infty} A^{(2\alpha-1)/\beta} L_o e^{(h+\varphi_v^{\lambda}-\rho)t}[(1-s_v)Y + (1-\varphi_v)A]dt$$

at $\hat{v}_b, \hat{v}_i$, but not on the positivity of the function within the integral which the given condition ensures.

In the case of differential incidence φ_v is set to be a constant as an additional constraint. Then the reader can see that if $\partial\varphi_v/\partial v_b$ and $\partial\varphi_v/\partial v_i$ are identically set equal to zero in the formulae for the derivatives of f and g, one finds that the same results apply to this case also.

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