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The Measurement of Technical Change Biases with Many Factors of Production

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THE MEASUREMENT OF TECHNICAL CHANGE BIASES WITH MANY FACTORS OF PRODUCTION

Technical change biases have generally been measured in two factor models, using value-added functions (David and van de Klundert, 1965, Sato, 1970, Lianos, 1971). A many factor generalization of these procedures has the following advantages: Primary factors can be disaggregated into different classes of labor and capital which would allow an investigation of how technical change affects each of the subclasses. Intermediate inputs can also be included so that questions such as natural resource biases or biases with respect to energy can be considered.[1] Also disaggregation is necessary when production processes are not separable between primary and intermediate inputs, which is a key assumption for fitting value-added functions.

The concept of Hicks' neutrality is used in this paper. But it is used in a slightly amended version which leads to a definition of biases in terms of factor shares.[2]

$$B_i = \frac{d\alpha_i^*}{dt}\frac{1}{\alpha_i} \tag{1}$$

where α_i is the share of factor i. The notation $d\alpha^*$ indicates that relative factor prices are held constant for this share change. Technical change is i-saving if $B_i < 0$, neutral if $B_i = 0$, and i-using if $B_i > 0$. This definition has the advantage that it leads to a single measure of bias for each factor in the n-factor case while Hicks' definition would lead to n-1 measures of bias for each factor.

Two models of measurements of biases are considered, both using the Translog cost function (Christensen, et. al., 1970).[3] Model A assumes variable rates of biases and is used to derive long-term series of biases. Model B assumes the biases to occur at constant rates and can be used with regression models. Since a cost function is used, very few constraints have to be imposed on the production process in both models (e.g., it does not have to be homogenous of degree one).

Both models are applied to U.S. agricultural data. For the period when data were available for both methods, the resulting estimates of bias were essentially the same, which gives support to the methodologies used.

I. The Translog Case

Model A

Every production function has a minimum cost function as its dual which relates factor prices to the cost of the output. The cost function contains all the information about the production process which the production function contains.

A minimum cost function with technical change in factor augmenting form and neutral economies or diseconomies of scale can be written

$$(2) \quad C = h(Y)\ \emptyset\left(\frac{W_1}{A_1}, \frac{W_2}{A_2}, \ldots, \frac{W_n}{A_n}\right)$$

where C is cost, Y is the output level, and the W's are the factor prices. The A's are augmentation parameters corresponding to the ones of the dual production function. A proportional change in A_i has the opposite effect on cost than a proportional change in the price of factor i.

Let $R_i = (W_i/A_i)$ the factor price of the augmented factor unit (A_iX_i). The Translog cost function can be written in logarithmic form as

$$(3) \qquad \ln C = \ln [h(Y)] + \ln \nu_o + \sum_i \nu_i \ln R_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln R_i \ln R_j$$

where h(Y) is a scale function of output and ν_o, ν_i, and γ_{ij} are the parameters of the cost function. The part before the double summation is a Cobb-Douglas function. If the cost function was Cobb-Douglas, then the production function also would be Cobb-Douglas (for proof, see Hanoch, 1970).[4] We, therefore, can think of the terms in the double summation as amendments to the Cobb-Douglas function which change the elasticities of substitution away from one (see equations (25) to (28) below). The function allows arbitrary and variable elasticities of substitution among factors.

The function can be considered a functional form in its own right or can be regarded as a logarithmic Taylor series expansion to the second term around input prices of one of an arbitrary twice differentiable cost function (Christensen, et. al., 1970). With the proper set of constraints on its parameters, it can, therefore, be used as an approximate to any one of the known costs and production functions. The following symmetry constraint holds for all Translog functions (equality of the cross derivatives).

$$(4) \qquad \gamma_{ij} = \gamma_{ji} \qquad \text{for all } i, j, i \neq j$$

The function must satisfy the following conditions:

i) Linear homogeneity in prices: When all factor prices double, the total cost has to double. It can be shown that this implies

(5) $$\sum_i \nu_i = 1; \qquad \sum_i \gamma_{ij} = 0; \qquad \sum_j \gamma_{ij} = 0 \qquad \text{for all } i,j.$$

ii) Monotonicity: The function must be an increasing function of the input prices, i.e.,

(6) $$\frac{\partial \ln C}{\partial \ln R_j} = \nu_i + \sum_j \gamma_{ij} \ln R_j \geq 0 \qquad i = 1, \ldots, n$$

iii) Concavity in input prices: This implies that the matrix

(7) $$\frac{\partial^2 C}{\partial R_i \partial R_j}$$

must be negative semi-definite within the range of input prices.[5]

To measure biases we need equations which explain factor shares in terms of factor prices.

In augmented units Shephard's lemma, $\partial C/\partial W_i = X_i$, becomes

(8) $$\frac{\partial C}{\partial R_i} = \frac{\partial C}{\partial W_i}\frac{dW_i}{dR_i} = A_i X_i \ .$$

The first derivatives of the Translog function, with respect to the log of the factor prices, are equal to the shares:

(9) $$\frac{\partial \ln C}{\partial \ln R_i} = \frac{\partial C}{\partial R_i}\frac{R_i}{C} = \frac{(A_i X_i) R_i}{C} = \frac{A_i X_i (W_i/A_i)}{C} = \frac{W_i X_i}{C} = \alpha_i$$

Taking these derivatives, we have

$$\alpha_i = \nu_i + \sum_j \gamma_{ij} \ln R_j \qquad i = 1, \ldots, n. \tag{10}$$

Differentiating (10) totally

$$d\alpha_i = \sum_{j=1}^{n} \gamma_{ij} \; d\ln R_j \qquad i = 1, \ldots, n. \tag{11}$$

The proportional (log) change of a ratio is the difference of the proportional changes of its numerator and denominator. Then

$$d\alpha_i = \sum_{j=1}^{n} \gamma_{ij} \; (d\ln W_j - d\ln A_j) \qquad i = 1, \ldots, n. \tag{12}$$

Separating terms and using matrices

$$\begin{bmatrix} d\alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ d\alpha_n \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \gamma_{n1} & & \gamma_{nn} \end{bmatrix} \begin{bmatrix} d\ln W_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ d\ln W_n \end{bmatrix} - \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \gamma_{n1} & & \gamma_{nn} \end{bmatrix} \begin{bmatrix} d\ln A_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ d\ln A_n \end{bmatrix}$$

or

$$d\alpha = \gamma(d\ln W) - \gamma(d\ln A). \tag{13}$$

γ is not of full rank due to the homogeneity constraint. But calling an arbitrary factor the n'th factor

$$\gamma_{in} = - \sum_{i=1}^{n-1} \gamma_{ij}. \tag{14}$$

Using (14) to remove γ_{in} from (13), we have

$$d\alpha_i = \sum_{j=1}^{n-1} \gamma_{ij} \; dw_j - \sum_{j=1}^{n-1} \gamma_{ij} \; da_j, \tag{15}$$

where $dW_j = d\ln W_j - d\ln W_n = d\ln \left(\frac{W_j}{W_n}\right)$

and $da_j = d\ln A_j - d\ln A_n = d\ln \left(\frac{A_j}{A_n}\right)$.

Let Γ be the truncated (n-1) x (n-1) matrix of the γ_{ij} which is of full rank. Then

(16) $$d\alpha_{(n-1)x1} = \Gamma dw - \Gamma da,$$

which gives us the solution for the changes in the A ratios

(17) $$da = dw - \Gamma^{-1} d\alpha.$$

With the discrete time equivalent of (17), time-series of the augmentation series can be estimated, provided reliable estimates of the Γ^{-1} matrix are available. Going one step further, the share changes, which would have occurred in the absence of factor price changes, can be estimated directly. They are the share changes needed to estimate the biases according to equation (1). Call these changes $d\alpha^*$, which can be obtained from system (16) by setting $dw = 0$. Then

(18) $$d\alpha^* = - \Gamma da.$$

And substituting da from (17),

(19) $$d\alpha^* = d\alpha - \Gamma dw.$$

According to (19), we can immediately judge the nature of technical change for factors i=1, ..., n-1.[6] It has a nice simplicity to it. To find what the factor share changes would have been, had factor prices remained constant, simply subtract from the observed factor share changes that part which was caused by changing factor price ratios. The Γ matrix contains the substitution parameters which determine by how much the changes in factor price ratios alone could have altered the shares.

Before (19) can be used with time-series data, estimation of the coefficients of the matrix is necessary. This must be done with cross-section data where, ideally, all units are on exactly the same production function. We can then assume that all A_i's are equal to one for all cross-sectional units and rewrite equation (10):

$$\alpha_i = \nu_i + \sum_j \gamma_{ij} \ln W_j + \varepsilon_i \qquad i = 1, \ldots, n \tag{20}$$

and use this system of equations to estimate the γ_{ij} coefficients. Of course one will never find a cross-section where all units are on exactly the same production function. Ways to deal with this problem are discussed in Section II and footnote 9.

Model B

Model A assumes that the rate of biases is not constant over time. For shorter time periods it is, however, possible to assume that the biases are constant. If this is done, biased technical change at constant exogenous rates can be introduced in the Translog cost function in a

similar way to that which Christensen, et. al., (1970) introduced it into the corresponding production function:

$$(21) \qquad \ln C = \ln [h(Y)] + \ln \nu_o + \sum_i \nu_i \ln W_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln W_i \ln W_j + \nu_t \ln t + \omega_t (\ln t)^2 + \sum_i \omega_i \ln W_i \ln t$$

where t stands for time.

Upon differentiation the share equations become

$$(22) \qquad \frac{\partial \ln U}{\partial \ln W_i} = \alpha_i = \nu_i + \sum_j \gamma_{ij} \ln W_j + \omega_i \ln t \qquad i = 1,\ldots,n$$

which is estimation equation (20) with time entering as a variable. ω_i is the constant exogenous rate of the bias of factor i.

If (22) is used as a regression equation with a time-series or a combination of cross-section and time-series, the introduction of time in this way will ensure that biased technical change at constant rates will not bias the econometric estimates of the γ_{ij}'s. Furthermore, the coefficients $\hat{\omega}_i$ can be used to derive another set of price corrected shares series, say $d\alpha_i^{**}$, which can be used with equation (1) to estimate the biases for the particular period

$$d\alpha_i^{**} = \hat{\omega}_i \, d\ln t \qquad i = 1,\ldots,n.$$

Of course, this model cannot be used to extrapolate outside of the short regression period because then the assumption of a constant exogenous rate of bias is tenuous.

II. Cross-Sectional Estimation of the Parameters of the Cost Functions

The cross-sectional estimation of the cost function used state data for the United States. A more detailed account of this estimation and the data used are given in Binswanger (forthcoming).

Four sets of cross-section data were obtained for 39 states or groups of states. The cross-sections were derived from census and other agricultural statistics for the years 1949, 1954, 1959, and 1964.

In general, Griliches' (1964) definitions of factors were used. He distinguishes the following five factors: land, labor, machinery, fertilizer, and all others. Intermediate inputs are included in this list and the function fitted corresponds to a gross-output function rather than a value-added function.[7]

The estimation equations are[8]

$$\alpha_{ikt} = \nu_i + \sum_j \gamma_{ij} \ln W_{jkt} + \omega_i \ln t + \sum_r \delta_{ir} d_r + \varepsilon_{ikt} \tag{24}$$

where $i,j = 1, \ldots, 4$ are the indices for the factors of production

$k = 1, \ldots, 39$ are the indices for the states

$t = 1, \ldots, 4$ are the indices for the time periods

δ_{ir} are share specific regional dummies for $r = 1, \ldots, 4$, and

$d_r = 1$ if $k \varepsilon r$
$\quad\;\; 0$ if $k \not\varepsilon r$.

The estimation equations are unaltered if neutral efficiency differences exist among states or time periods. Neutral differences would only alter the intercept ν_o of the cost function which drops out upon differentiation.

If any left out factor, such as education or research and extension, affects efficiency neutrally, leaving them out of the estimation equation will not bias the results.[9]

Nonneutral efficiency differences among the observational units will have the effect that the true α_i will differ for each observational unit; at equal factor prices, shares will not be equal.[10] If such differences occur among all units, the estimates of the coefficients of (24) will be biased. However, if such differences occur only among groups of states, the proper set of regional dummies will again lead to unbiased estimators. Regional dummies distinguishing 5 regions were, therefore, included in the regression equations. Nonneutral differences might arise due to educational differences, differences in research and extension, or differences in product mix.

If (24) is estimated with time-series data, a time trend in the estimation equation will solve the problem of estimation of biases over time, as explained in model B, provided the rates of biases stayed reasonably constant during the estimation period.

A detailed account of the error specification problem is given in Binswanger (forthcoming). Problems arise from the fact that (1) time-series and cross-section data are combined and (2) that, within each cross-section, the error terms of the shares equations are not independent.[11]

Both problems could not be handled simultaneously. For estimation purposes all cross-sections were combined and restricted generalized least squares (Theil, 1971) applied to the four shares equation, as if there was no problem of error interdependence over time. While this leads to

consistent estimates of the γ_{ij} coefficients, there is an efficiency loss and t-ratios of the estimates will be overstated to some extent.[12]

Prior to the estimation, several constraints were tested by applying the model to the 4 cross-sections individually using restricted generalized least squares. These tests, therefore, did have the desired asymptotic properties. The homogeneity constraint was not rejected in any of the 4 cross-sections (.05 significance level). The symmetry constraint was rejected only in the 1964 data set. However, the Cobb-Douglas constraint ($\gamma_{ij} = 0$ for all i,j) was rejected in all cross-sections.[13] Hence, homogeneity and symmetry were imposed in the estimation with all data pooled.

Pooling the cross-sections implies constancy of the γ_{ij} coefficients over time. This was tested as follows: a two-equation GLS model is fitted for each share with the 1949 data used for the first equation and the 1959 data for the second equation. The homogeneity constraint was imposed on the data.

The hypothesis is never rejected at the .01 level of significance, although it is rejected in 2 equations at the .05 level of significance. A test for constancy of coefficients can be interpreted as a test of the factor augmenting hypothesis, if there are no other specification errors. This test, therefore, implies a weak support for this hypothesis.

Table 1 reports the estimates of the pooled regressions. The t-ratios of the price coefficients appear to be low, despite the fact that they may be overstated to some extent. However, $\gamma_{ij} = 0$ implies that the corresponding partial elasticity of substitution is equal to one (see equations (25) and (26)). Therefore, we would expect, a priori, some of the γ_{ij} coefficients to be zero.

TABLE 1 - RESTRICTED ESTIMATES OF THE COEFFICIENTS OF THE TRANSLOG COST FUNCTION AND t-RATIOS[a,b]

Factor	Independent Variables										
	Land	Labor	Machinery	Fertilizer	Other[c]	Year	Intercept	MN	GF	SE	GS[d]
Land	.07747 (6.02)	-.03613 (3.25)	.00478 (.47)	.01066 (2.14)	-.05678	.00847 (1.47)	.2603 (9.96)	-.1021 (10.2)	-.0394 (4.1)	-.1073 (8.9)	-.0577 (4.7)
Labor		-.06367 (3.67)	-.00661 (.59)	-.02805 (4.97)	.13446	-.05482 (9.08)	.5218 (14.91)	.0194 (1.63)	-.0016 (.15)	.0169 (1.09)	.0246 (1.63)
Machinery	Symmetric		-.03485 (1.31)	-.00877 (.97)	.04545	.02498 (4.66)	.0926 (3.46)	-.0033 (.41)	.0369 (5.08)	-.0186 (1.86)	.0072 (.73)
Fertilizer				.00068 (.12)	.02548	.00178 (.63)	.0745 (5.6)	.0104 (2.5)	-.0041 (1.10)	.0370 (7.24)	-.0024 (.49)
Other					-.14861						

Source: USDA.

[a]Restrictions imposed: $\gamma_{ij} = \gamma_{ji}$ and $\sum_{j=1}^{n} \gamma_{ij} = 0$ for all i, j.

[b]Critical values with 578 degrees of freedom are t.05 = 1.96 and t.01 = 1.65. t-ratios may be overstated due to error interdependence over time.

[c]Implied estimates computed using the homogeneity constraint.

[d]MN, GF, SE, GS are dummies for Mixed Northern agriculture, Grain Farming states, Southeast, and Gulf States, respectively. The intercept stands for Western States and the coefficients of MN, GR, SE, GS are deviations from this intercept.

In the labor and the machinery equation the coefficients of time are significant. They imply that technical change has been labor-saving and machinery-using. Significant coefficients of regional dummies imply non-neutral regional efficiency differences.

The estimates of the γ_{ij} coefficients can be converted into point estimates of Allen partial elasticities of substitution (σ_{ij}) and of elasticities and cross elasticities of factor demand (η_{ij}) according to the following equations (for proof see Binswanger, forthcoming):

(25) $$\sigma_{ij} = \frac{\gamma_{ij}}{\alpha_i \alpha_j} + 1 \qquad \text{for all } i \neq j$$

(26) $$\sigma_{ii} = \frac{1}{\alpha_i^2} (\gamma_{ii} + \alpha_i^2 - \alpha_i) \qquad \text{for all } i$$

(27) $$\eta_{ij} = \frac{\gamma_{ij}}{\alpha_i} + \alpha_j \qquad \text{for all } i \neq j$$

(28) $$\eta_{ii} = \frac{\gamma_{ii}}{\alpha_i} + \alpha_i - 1 \qquad \text{for all } i.$$

The γ_{ij} coefficients have little intuitive meaning and, therefore, it is easier to evaluate them by what they imply for these elasticities. Table 2 shows the results using the unweighted average factor shares of the 39 states in the period 1949-1964.

The matrix of elasticities of substitution is negative semi-definite, which implies that the matrix of cross derivatives of the cost function is negative semi-definite, i.e., that the cost function is concave.[14]

TABLE 2 - ESTIMATES OF THE PARTIAL ELASTICITIES OF SUBSTITUTION AND OF FACTOR DEMAND WITH RESPECT TO OWN PRICE [a]

	Land	Labor	Machinery	Fertilizer	Other[b]
A) Elasticities of substitution					
Land	-2.225 (.57)	.204 (.24)	1.215 (.46)	2.987 (.93)	-.031
Labor		-3.028 (.19)	.851 (.25)	-1.622 (.53)	2.224
Machinery	Symmetric		-7.379 (1.23)	-.672 (1.72)	1.844
Fertilizer				-26.573 (4.61)	2.961
Other					-2.852
B) Elasticities of factor demand (η_{ii})	-.336 (.09)	-.911 (.06)	-1.089 (.18)	-.945 (.16)	-1.042

Source: USDA.

[a]Standard errors in parenthesis and computed as follows:

$$SE(\sigma_{ij}) = \frac{SE(\gamma_{ij})}{\alpha_i \alpha_j}; \qquad SE(\eta_{ii}) = \frac{SE(\gamma_{ii})}{\alpha_i}.$$

[b]Implied estimates computed using the homogeneity constraint.

All own demand elasticities have the correct sign. The demand for land appears very inelastic. The demand elasticities for machinery and other inputs are larger than 1, a fact to keep in mind, since it implies that a rise in the corresponding prices will, other things being equal, lead to a fall in the factor share.

III. The Empirical Measures of Biases in Efficiency Gains

This section presents the derived series of biases for the years 1912 to 1968 using Model A. It also presents the series of actual factor shares and factor prices. The data come from published USDA sources. The variables are constructed so that they correspond as closely as possible to the variables used in the cross section analysis. Total correspondence was, however, not achievable. For a detailed account of the data see Binswanger (1973).[15]

The basic estimation equation for the biases are equations (19) which when expanded reads

$$d\alpha_i^* = d\alpha_i - \sum_{j=1}^{n-1} \hat{\gamma}_{ij}\, d\ln w_j \qquad i = 1,\ldots, n-1 \tag{19a}$$

where the $d\alpha_i^*$ is the change in the share of factor i in the absence of ordinary factor substitution due to price changes; $d\alpha_i$ is the actual total change in share i, which includes the effect of the price changes; $d\ln w_j$ is the proportional change of the ratio of the price of factor i to the price of other inputs. For actual estimation purposes series of three-year moving averages of the shares and the factor prices were constructed. Then

discrete differences of these moving averages at four-year intervals were taken and used in the discrete change equivalent of (19). The $\hat{\gamma}_{ij}$ coefficients are taken from table 1. The resulting $d\alpha_i^*$ can be substituted into the discrete equivalent of equation (1) to compute rates of biases. Here, however, we compute series α_i^* which show how the shares would have developed after 1912 in the absence of factor price changes:

$$(29) \qquad \alpha_{it}^* = \alpha_{i,1912} + \sum_{\tau=0}^{t} \Delta \alpha_{it}^*$$

These series are presented in panel A of Table 3. Series of standardized values, i.e., $R_{it} = \alpha_{it}^*/\alpha_{i,\ 1912}$ are presented in Figure 1, which is in semilogarithmic scale. Hence, the slopes of the lines indicate biases according to equation (1), while the position of the line shows the cumulative bias since 1912.

How good are these series? From its assumptions the approach is not very restrictive. No separability is imposed on the production process, an assumption which is implicit whenever a production function is used with only capital and labor. This cost function approach also allows nonconstant returns to scale, but requires that the dual production function be homothetic. The model assumes simple cost minimization. This assumption is not violated even if the government intervenes in factor and goods markets. Government intervention would be recorded correctly either in quantities or price series and not bias the results. Only government control of both prices and quantities would violate the assumption.

TABLE 3 - PRICE CORRECTED SHARES (α_t^*), ACTUAL FACTOR SHARES, AND FACTOR PRICES USED IN COMPUTING (α_i^*)

	Year	Land	Labor	Machinery	Fertilizer	Other
A. Price Corrected Factor Shares α_i^*. Model A Estimates	1912	21.0	38.3	10.9	1.9	28.0
	1916	21.2	36.7	11.6	1.8	28.7
	1920	19.6	39.3	9.3	2.1	29.7
	1924	20.0	39.7	10.3	2.2	27.8
	1928	18.1	41.4	10.4	2.7	27.4
	1932	18.8	40.3	14.3	2.7	24.0
	1936	18.9	32.5	16.3	3.0	29.3
	1940	16.8	34.3	17.6	3.9	27.5
	1944	16.5	38.4	16.1	4.8	24.2
	1948	17.1	37.2	13.9	5.1	26.7
	1952	16.5	29.8	19.7	5.7	28.3
	1956	16.3	30.6	23.1	6.5	23.4
	1960	17.1	27.2	23.4	6.1	26.1
	1964	17.8	25.8	22.4	6.7	27.3
	1968	19.1	25.3	23.1	7.2	25.3
B. Actual Factor Shares α_i	1912	21.0	38.3	10.9	1.9	28.0
	1916	21.6	36.5	11.6	1.9	28.4
	1920	17.3	40.5	10.1	2.0	30.1
	1924	19.7	38.5	10.3	1.7	29.7
	1928	15.9	40.9	10.2	1.9	31.1
	1932	18.6	37.6	12.6	1.6	29.7
	1936	14.9	34.7	14.5	2.2	33.7
	1940	12.0	35.3	15.1	2.3	35.2
	1944	8.5	39.5	14.0	2.3	35.6
	1948	9.4	37.7	12.2	2.4	38.3
	1952	9.8	29.7	17.5	3.0	40.0
	1956	11.5	27.4	20.1	3.3	37.8
	1960	15.6	21.3	19.8	2.9	40.4
	1964	17.5	18.3	18.5	3.3	42.3
	1968	20.4	15.8	19.1	3.6	41.1
C. Factor Prices Relative to Agricultural Output Prices 1912=100	1912	100.0	100.0	100.0	100.0	100.0
	1916	113.3	106.8	110.0	105.7	103.8
	1920	79.0	104.3	81.3	85.7	105.0
	1924	119.0	134.5	111.7	93.1	106.6
	1928	104.8	154.1	128.5	90.0	118.9
	1932	160.8	194.7	231.5	128.6	101.5
	1936	69.4	113.4	189.2	99.6	110.9
	1940	87.3	179.0	288.8	103.4	160.1
	1944	65.2	217.2	244.2	63.0	211.7
	1948	73.0	247.8	226.6	50.4	222.8
	1952	91.3	274.3	301.1	53.6	214.6
	1956	145.8	407.9	423.7	65.9	229.6
	1960	254.1	502.7	550.3	63.0	241.5
	1964	338.1	610.0	651.2	63.2	270.9
	1968	481.0	766.9	735.8	58.2	280.4

Source: USDA.

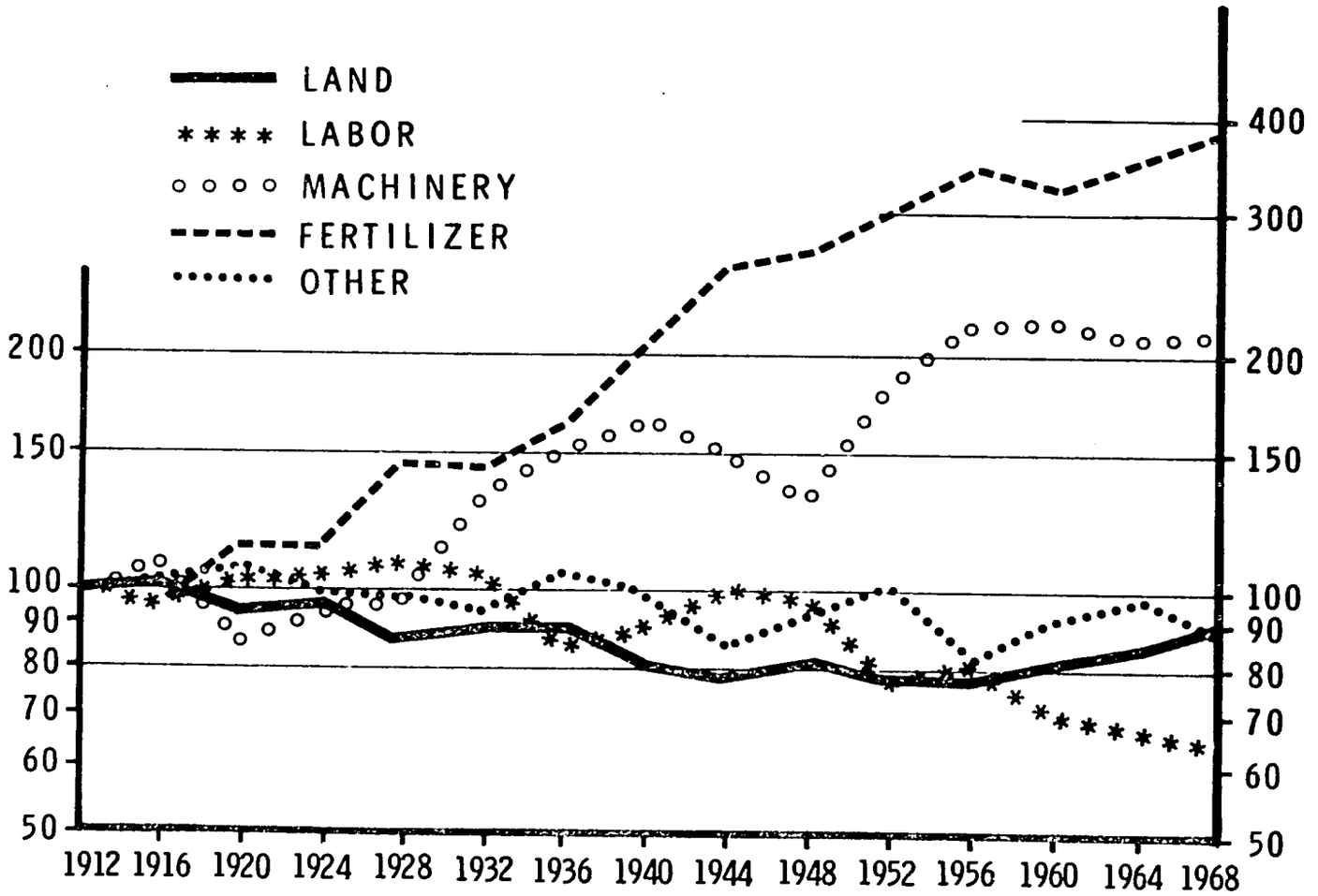

FIGURE 1. U. S. indices of biases in technical change : Model a estimates of α^*_{it} / α_i, 1912 1912 = 100

The key assumption of the approach is the constancy of the γ_{ij} parameters over time. This assumption was not supported as well as one might wish, but it cannot be avoided since data to estimate the γ_{ij} for the beginning of the period are not available. The quality of the α^* series depends completely on the quality of the $\hat{\gamma}_{ij}$ estimates. If they were really totally wrong chances would be that over the long periods involved, which include two World Wars and the depression, some strange result would be immediately apparent in the α^* series. Such a result might be if one of the α^* series became negative. Smaller errors in the $\hat{\gamma}_{ij}$ coefficients are, of course, not ruled out by such considerations. The errors could even be large enough to make inferences from small direction changes of the series impossible. Some conclusion on the quality of the α^* series and the underlying γ_{ij} estimates can be obtained by comparing the model A estimates presented above to the model B estimates which use the time coefficient of the regressions reported in Table 1. The price corrected share changes $\Delta\alpha^{**}$ for Model B are computed for the period 1948-1964 according to equation (23) under the assumption that the rate of the bias for each factor remained constant during that particular period or, alternatively, that $\hat{\omega}_i$ parameters measure an average rate of bias. Apart from the fact that the $\hat{\gamma}_{ij}$ and the $\hat{\omega}_i$ were estimated in the same equations, the Model A estimates $\Delta\alpha_i^*$ have nothing to do with Model B estimates $\Delta\alpha_i^{**}$ and, therefore, there is no reason, apart from chance, that they would be similar if either set of estimates were incorrect. Table 4 shows the comparison of the model B estimates with the model A estimates reported in the graphs.

TABLE 4 - COMPARISON OF MODEL A AND MODEL B ESTIMATES OF BIASES FOR THE PERIOD 1948-1964 FOR THE UNITED STATES

Factor	1948 Level of Shares	Estimated Share Change Due to Technical Change Alone: Model A 1948-1964 ($\Delta\alpha$*)	Estimated Share Change Due to Technical Change Alone: Model B 1948-1964 ($\Delta\alpha$**)
	- - - - - - - - - - - - - -percent- - - - - - - - - - - - - -		
Land	9.4	+2.3	+.7
Labor	37.7	-15.1	-11.4
Machinery	12.2	+6.9	+8.5
Fertilizer	2.8	+ .5	+1.6

Source: USDA.

Both series estimate biases with the same sign and of about the same magnitude. Because of the differences in the underlying assumption, they cannot be expected to agree perfectly. The consistency of the estimates provides support for both methodologies of measuring biases.

IV. Conclusions

There has been a very strong fertilizer-using bias at a fairly constant rate throughout the period. In the absence of the marked price reduction of fertilizer, the fertilizer share would have risen from less than 2 percent to 7.2 percent. The actual rise was much less, to 3.6 percent. This result is consistent with Griliches' 1958 findings.

There also has been a strong machinery-using bias, despite the marked rise in machinery prices. In the absence of any price changes the machinery share would have more than doubled. The rate of bias was, however, not constant and between 1940 and 1948 there was even a machinery-saving bias.

It is also interesting to note that the strong rise in the machinery prices[16] did not lead to a rise in the actual machinery share. This is because the own price elasticity of machinery implied in the γ_{ij} parameter is -1.09. A rise in the price, therefore, tends to decrease the share.

Until 1944 technical change was labor neutral, but after that a labor-saving bias occurred at a rapid rate. The finding of a labor-saving bias is consistent with Liano's (1971) finding of a labor-saving bias in a two-factor CES framework. The labor-saving bias explains, however, only a little more than one-half of the decrease in the actual labor

share between 1944 and 1968. A large part of this drop must be explained as a simple price effect. Note that the elasticity of the labor demand with respect to its own price is less than one.

There was a slight land-saving bias between 1912 and 1950 and a small land-using bias thereafter. The actual land share has, however, first decreased and then increased much more than the price-corrected share. The behavior of the actual land share is almost entirely due to price effects and not to biased technical change. It changes very dramatically because the demand elasticity for land is very small.

Biases have been an important source of share changes of fertilizer, machinery,and labor, but not of land and other inputs where price effects dominated the share changes. For labor both the price effects and the bias tended to reduce the share, whereas for fertilizer the bias and the price effects worked in opposite directions. Thus diversity of the way in which technical change and factor prices explain shares is quite remarkable.

A much more detailed picture about the interaction of prices and biases in determining factor-shares emerges from considering 5 factors rather than only 2. The series of the biases also show that it would be inappropriate to assume constant rates of biases over long periods of time. Such a procedure could never detect the important change from labor-neutral technical change to labor-saving technical change which took place right after World War II.

FOOTNOTES

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[1]The author is presently engaged in such a study with Australian data.

[2]Hicks' definition is as follows: Technical change is said to be neutral, labor-saving, or labor-using depending on whether, at a constant capital-labor ratio, the marginal rate of substitution stays constant, increases, or decreases. Mathematically this can be expressed as follows:

$$\text{(1a)} \qquad \frac{d}{dt} MRS = \frac{d}{dt} \frac{f_K}{f_L} = - \frac{d}{dt} \frac{dL}{dK} \gtreqless 0$$

where f_K and f_L stand for the marginal products and the capital labor ratio is held constant. Neutrality is, therefore, a homothetic inwards shift of the unit isoquant. If at a constant factor ratio the marginal rate of substitution (or the ratio of the capital price to the labor price) is rising, then the labor share is declining. This leads immediately to definition (1).

To estimate biases it is, however, not possible to simply look at historical factor share changes. The observed share changes have come about through biased technical change and through ordinary factor substitution in response to changes in the prices of the factors. The basic problem is, therefore, to sort out to what extent the share changes have been due to biased technical change and to what extent to price changes. This can only be done, in a graphic sense, if the curvature of the isoquant is known. The substitution parameters of the production process have to be estimated before any biases can be measured.

[3]A general model for measuring biases in the many factor case for arbitrary twice differentiable production functions can be found in Binswanger (1973).

[4]The parameters of the function are estimated later in the paper and the Cobb-Douglas restriction ($\gamma_{ij} = 0$ for all i,j) is tested and rejected.

[5]This condition can be translated into the condition that the matrix (σ_{ij}) of partial elasticities of substitution be negative semi-definite. (Binswanger, forthcoming). For the values of σ_{ij} found in the cross-sectional estimation this was the case.

[6]Since $\sum_{i=1}^{n} d\alpha_i^* = 0$ the solution for the n'th factor is simply $d\alpha_n^* = - \sum_{i=1}^{n} d\alpha_i^*$.

[7]Most of the cross-section data come from published USDA sources. Expenditures on factors usually are actual expenditures and, where applicable, imputed expenditures for wages of family members, interest charges, depreciation, and taxes. Quantity data are derived as price weighted indexes of physical units (land and fertilizer), or the sum of individually deflated expenditures (all other), or a combination of these methods (machinery and labor). The quantity data were already computed in Fishelson (1969), who used Griliches' (1964) data with slight changes. Expenditure and quantity data are consistent with each other. The price data were obtained by dividing the expenditure data by the quantities.

[8]Due to the homogeneity constraint (5) only n-1 share equations are linearily independent and can be estimated simultaneously.

[9]That equation (24) measures the γ_{ij} parameters of the functional form (3) can be seen as follows: If there is only one cross-section, and all cross-sectional units have the same A_i parameters (no efficiency differences) factors can be rescaled to make all A_i equal to one. Hence $W_{jk} = R_{jk}$. If there are neutral efficiency differences among the cross-sectional units, all the A_i will differ by the same proportion among the 2 units. This proportion can be absorbed in a separate intercept term for each unit. These intercepts drop out upon differentiation so that we can again set all $A_i = 1$. If more than one cross-section is used, (24) still gives the correct estimates for (3), provided that for all cross-sectional units each A_i changes at the constant rate ω_i during the period under investigation.

[10]Nonneutral efficiency differences between 2 states imply that at equal factor prices, the states will use factors in differing proportions. Factor shares will, therefore, differ even at equal factor prices.

[11]Problem (1). For each share equation, data from 4 cross-sections are combined. This poses the familiar problem of combining cross-section and time-series data. Despite the 5-year interval between the cross-sections, this problem is still important: the correlation coefficients of the OLS residuals of the share equations between the cross-sections of 1949 and 1965 ranged between 0.62 and 0.87. If first order auto-correlation was the true error specification over time, this would imply first order auto-correlation coefficients larger than 0.9.

Problem (2). Within each of the 4 cross-sections, the error terms of the n-1 estimation equations are not independent, since for each state the same variables which might affect the shares in addition to the prices were left out of the model. If restrictions across equations ($\gamma_{ij} = \gamma_{ji}$) are imposed, OLS estimators are no longer efficient, despite the fact that all equations contain the same explanatory variables on the right-hand side (Theil 1971). Therefore, the seemingly unrelated regression problem applies.

[12]The Computer Program used was Triangle Universities Computing Center: Two and Three Stage Least Squares (TTLS).

[13]All 3 tests were performed against a completely unconstraint model.

[14]The elasticities of substitution also show that the production process is not separable between primary and intermediate inputs. Separability would imply, inter alia, that the partial elasticities of substitution of fertilizer with the primary factors land, labor, and machinery be equal (Berndt and Christensen, forthcoming). This is obviously not the case.

[15]No quality adjustments were made for any of the factors because we are really not interested in earnings and biases of some adjusted efficiency unit of labor or land, but in the earnings of natural units of factors. Any quality adjustment before measuring the biases is, therefore, inappropriate.

[16]The machinery price is the price of machinery services, which includes interest charges, depreciation, fuel, and repair costs. Depreciation rates and repair costs have strongly increased over time.

REFERENCES

E. R. Berndt and L. R. Christensen, "The Internal Structure of Functional Relationships: Separability, Substitution, and Aggregation," Rev. Econ. Studies, forthcoming.

H. P. Binswanger, "The Measurement of Biased Efficiency Gains in U.S. and Japanese Agriculture to Test the Induced Innovation Hypothesis," unpublished doctoral dissertation, North Carolina State University 1973, University Microfilms, Ann Arbor, Michigan.

_____, "A Cost Function Approach to the Measurement of Factor Demand Elasticity and of Elasticities of Substitution," Amer. J. Agric. Econ., forthcoming, 1974.

L. R. Christensen, D. W. Jorgenson, and J. L. Lau, "Conjugate Duality and the Transcendental Logarithmic Production Function," unpublished paper presented at the Second World Congress of the Econometric Society, Cambridge, England, September 1970.

_____, _____, and _____, "Transcendental Logarithmic Production Frontiers," Rev. Econ. Statist., Feb. 1973, 55, 28-45.

P. A. David and Th. van de Klundert, "Biased Efficiency Growth and Capital Labor Substitution in the U.S., 1899-1960," Amer. Econ. Rev., June 1965, 55, 357-394.

W. E. Diewert, "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function," J. Polit. Econ., May-June 1971, 79, 481-505.

G. Fishelson, "Returns to Human and Research Capital, United States Agriculture, 1949-1964," unpublished doctoral dissertation, North Carolina State University 1968, University Microfilms, Ann Arbor, Michigan.

Z. Griliches, "The Demand for Fertilizer: An Economic Interpretation of a Technical Change," J. Farm Econ., Aug. 1958, 40, 591-606.

_____, "Research Expenditure, Education, and the Aggregate Agricultural Production Function," Amer. Econ. Rev., Dec. 1964, 54, 961-974.

G. Hanoch, Generation of New Production Functions through Duality, Harvard Institute of Economic Research Discussion Paper 118, Cambridge, Mass. 1970.

T. P. Lianos, "The Relative Share of Labor in the United States Agriculture," Amer. J. Agric. Econ., Aug. 1971, 53, 411-422.

R. Sato, "The Estimation of Biased Technical Progress," Int. Econ. Rev., June 1970, 11, 179-207.

R. M. Solow, "Some Recent Developments in the Theory of Production," in The Theory and Empirical Analysis of Production, NBER, Studies in Income and Wealth, M. Brown, ed., Vol. 31, 1967.

H. Theil, Principles of Econometrics, New York, 1971.

Triangle Universities Computing Center, Program Manual for Two and Three Stage Least Squares, Research Triangle Park, North Carolina 1972.