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EASE OF FACTOR SUBSTITUTION IN AGRICULTURE

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16. Abstract: Variant approaches to estimating factor substitutability in agriculture produce [illegible] the same answer: the elasticity of capital-labor substitution in agriculture exceeds unity and is probably close to one and a half. This parameter size appears to be independent of the choice of time-period, inter-country versus intra-country [illegible] or time-series versus cross-section information, and the degree of aggregation [illegible] input, capital and labor, and the selection of functional forms for [illegible]. Three factor variable elasticity functions gave comparable results to two [illegible] multiple factor CES formulations.

[illegible] economy develops and the price of labor increases relative to commodities [illegible] substantial declines in demand for agricultural labor and high pay-offs [illegible] policies which assist rural to urban migration. Secondly, if factor [illegible] tions an undesirable degree of capital-intensity in nonagricultural sectors [illegible] capital for income distribution that similar distortions be avoided in [illegible] income declines in a sector which in many countries contains the poorest. [illegible] This trend is observable in Colombia and proposed [illegible] agricultural mechanization for that country are slow to relate to the policy [illegible].

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EASE OF FACTOR SUBSTITUTION IN AGRICULTURE

By

Wayne R. Thirsk

Fall, 1972

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Ease of Factor Substitution in Agriculture

Econometric estimation of a particular economic parameter has often left us with a disturbingly wide range of results from which to choose for empirical application. The width of the available range is perplexing because the conclusions of theoretical analysis are usually sensitive to the size of important parameters. Uncertainty of this kind hinders the contribution of economic theory to public policy debate. Moreover, the development of theory itself is retarded as contending schools of thought are created on the basis of differing assumptions about specific parameters.

Nowhere is this problem more evident than in the estimation of the elasticity of factor substitution. Issues involving the nature and significance of structural unemployment, the seriousness of unique factor supply constraints on economic growth, and the explanation of changed factor shares of total income have all relied on an inability to achieve agreement about the ease of factor substitution. In the nonagricultural sector(s) within the past ten years a proliferation of attempts to measure the elasticity of factor substitution has brought matters no nearer to resolution. In a comprehensive assessment of these efforts Nerlove concluded somewhat gloomily that "The major finding of this survey is the diversity of results: even slight variations in the period or concepts tend to produce drastically different estimates of the elasticity."[1] Nothing I am aware of has happened since 1967 to overturn that judgment.

In the agricultural sector, however, the news is more cheerful and a consensus

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[1] Marc Nerlove, "Recent Empirical Studies of the CES and Related Production Functions," The Theory of Empirical Analysis of Production, Studies in Income and Wealth, Volume 31, National Bureau of Economic Research, 1967.

on the ease of factor substitution appears to be emerging. The weight of available evidence presented here suggests the elasticity of substitution in agriculture exceeds unity and is closer to one and a half. The main goal of this paper is to describe the outcome of an attempt to estimate substitution elasticities by crop for Colombian agriculture using a rather narrow data base. A subsidiary ambition is to compare these results with those that have been estimated from data for other countries and which employ alternative methodologies.

1. Data Source for the Colombian Case

The information required to estimate factor substitution possibilities was extracted from detailed individual farm records collected by the Colombian Land Reform Agency INCORA.[2] These records or patrones are used to evaluate the production performance of different farm units and different crops in the various land reform districts.

It was possible to obtain information over a reasonably wide range of crops with this INCORA data. In addition to rice, cotton and corn, results were obtained for wheat and barley and an aggregate crop combination labelled SSS, representing pooled observations on sesame, soyabeans and sorghum. Wheat and barley were also combined because of the similarity in their production process and the relatively small numbers of observations for each separate crop.

The data consisted of a cross-section sample drawn from different geographic areas of the country in 1968. Regional variation in factor and product prices and individual farm differences in risk attitudes and immobile factors such as soil type and management were relied upon to identify production surfaces for each crop. Since factor and product prices can be taken as exogenous at the

[2] INCORA stands for the Instituto Colombiano para la Reforma Agraria.

level of the single farm unit, Ordinary Least Squares were used for estimation purposes along with Generalized Least Squares in those cases where simultaneous parameter estimation was attempted. Each record provided precise information on the quantity of single crop output produced per hectare in the season and the price received for this output. Information on intermediate input use permitted the construction of value-added series per hectare. Amounts of the various inputs per hectare along with their prices were also available. From this information it was possible to derive measures of the share of each factor in total factor cost. Labor input was measured in man-days and the price of labor was the wage rate paid. Land rents paid formed the variable for the price of land.

The only major data concern was the specification of the capital variable. The price of capital was simply the rental rate paid for the use of tractors, combines and spray planes. Because of limited degrees of freedom, it was necessary to develop some method of aggregation for the different components of farm machinery used. Ideally all three machinery rental prices should have been used independenly to explain deflated capital expenditures per hectare.[3] Instead, two different procedures were followed. One method involved deflating capital expenditures by the rental price of tractors to convert the capital spending aggregate to equivalent tractor-hours and then using the same tractor rental price as the price of capital. This procedure was less than satisfactory because it ignored variation in the other machinery rental prices but was used, nevertheless, because most of the mechanized observations employed only tractors. The second alternative considered capital expenditures deflated by an index of all the rental rates as the measure for the real quantity of farm machinery used, and took the same weighted

[3] Since capital is demanded for its physical services, it is a measure of real capital that should enter the production function.

average of the machinery rental prices as the price of capital with the weights determined by the share of each type of machinery in capital spending. The problem with this approach was that it measured an aggregate of different machine-hours and neglected substitution within the capital aggregate, such as the choice between the use of planes or tractors in applying insecticides.[4]

Two other shortcomings of the data may also be mentioned. Since the data are restricted to farm units normally occupying less than 20 hectares there is no guarantee the results would describe factor adjustments on much larger sized farms in Colombia, or elsewhere for that matter. Also, the maximum number of observations for any crop was 20 so there is considerable risk in generalizing these results to a larger universe. The entire effort at estimating substitution parameters for various crops was hampered by the small number of observations available for each crop.

2. Specification of the Variables and Parameters

Given the existence of cross-section data, the development of a long-run factor substitution model seemed appropriate for estimating purposes. A three-factor production model was taken as the most promising approach for examining the ease of substitution among different pairs of the factors: land, labor and farm machinery. An alternative procedure of analyzing a two-factor relationship

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[4]Comparisons with the same regression equation indicated that direct parameter estimates were not highly sensitive to the choice of one or the other specification. There seemed to be two reasons for this insensitivity. Few farms employed machinery other than tractors, and a relatively high tractor rental price was usually associated with relatively higher rental prices for other types of machinery. Thus, it was hardly ever the case that a high tractor rental was offset by a low combine rental as an influence on total capital spending; so that tractor rentals served as an adequate proxy for capital costs in those cases where machinery use comprised more than the use of tractors.

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involving only labor and farm machinery appeared unnecessarily restrictive since it would require assuming that land was either strictly complementary to the use of the other two factors or was equally substitutable for either. If either of these assumptions were inaccurate, an attempt to explain the farm-machinery/labor ratio by relative capital-labor prices would be unsuccessful whenever there was significant variation in the price of land to the price of either of the other two outputs. However, it is necessary to assume that all farms either operate on the same production function or, if not, that differences in efficiency are factor neutral and that each observation represents an equilibrium input position. Poor fits will occur to the extent these assumptions are unmet.

The variables and parameters employed in the regression analysis are listed below along with their definitions. The input levels for labor and machinery refer to the entire span of agricultural operations from initial land preparation to harvest.

A = land, measured in hectares

L = labor, measured in man-days per hectare

K = real capital input, measured as total machinery expenditures deflated by some combination of the rental prices of farm machinery

PA = land rental per hectare

PL = daily wage rate

PK = rental price paid for the use of farm machinery

Δj = share of input j in total costs, j = A, L, K

MC = marginal cost of production

P = output price

X = value-added measure of output

C = regression constant

ln = logarithmic operator

σ_{ij} = partial elasticity of substitution between factor I and factor j. $\sigma_{ij} = \sigma_{ji} \lessgtr 0$, and $\sigma_{ii} < 0$ are a priori restrictions on the values of the partial elasticities. The symmetry restriction $\sigma_{ij} = \sigma_{ji}$ was a maintained hypothesis in the estimating equations of the regression models below.

3. Specification of the Model and Estimating Equations

The basic model is described by three factor demand equations, a marginal cost equation and an equilibrium condition between price and marginal cost.

$$(1)\quad \ln A = C_1 + \Delta K\sigma_{ak} \ln PK + \Delta L\sigma_{a\ell} \ln PL + \Delta A\sigma_{aa} \ln PA + \ln X$$

$$(2)\quad \ln L = C_2 + \Delta K\sigma_{a\ell} \ln PK + \Delta L\sigma_{\ell\ell} \ln PL + \Delta A\sigma_{\ell a} \ln PA + \ln X$$

$$(3)\quad \ln K = C_3 + \Delta K\sigma_{kk} \ln PK + \Delta L\sigma_{\ell k} \ln PL + \Delta A\sigma_{ak} \ln PA + \ln X$$

$$(4)\quad \ln MC = C_4 + \Delta K \ln PK + \Delta L \ln PL + \Delta A \ln PA$$

$$(5)\quad P = MC$$

The particular form of these equations results from the assumption of long-run cost minimization with three variable factors of production. The model is cast in terms of derived demand functions rather than a production function because of the greater flexibility attached to the former. Known production function forms incorporating more than two factors impose the rigidity that either all partial elasticities are equal (the Cobb-Douglas and constant elasticity cases) or that their ratios or differences are equal.[5] Underlying the above factor demand equations is a constant returns to scale, variable elasticity of substitution, production function whose explicit form is unknown, and for our pur-

[5] An interesting and useful attempt to derive more flexible multi-factor production functions has been made by G. Hanoch, in "Cresh Production Functions," Harvard University, Harvard Institute of Economic Research, Discussion Paper Number 84, August 1969.

poses, and unimportant. It is unimportant because even if it were known, it would be convenient to work with the associated factor demand equations since these contain the partial elasticities as direct parameters. Estimating the production function, on the other hand, would likely require estimating a host of other production parameters in order to uncover the same partial elasticity. The availability of factor price and quantity data makes it possible to use factor demand relationships to infer something about the characteristics of the unobserved production function.

Although variable in theory, the partial elasticities are assumed to be constant over the observed range of the data for estimation purposes.[6] An additional constraint on the values of the partial elasticities is a homogeneity condition $\sum_{j}^{3} \Delta j \sigma_{ij} = 0$ which means that only three of the partial elasticities can be determined independently. Given estimates of σ_{ka}, $\sigma_{k\ell}$ and $\sigma_{\ell a}$ and information on factor shares, all of the own-substitution parameters, σ_{aa}, σ_{kk}, and $\sigma_{\ell\ell}$, follow from the homogeneity condition.

The relationships in the basic model were used to generate two different econometric specifications of the choice of factor proportions in mechanized agriculture. These different approaches can be considered as a family of regression models which differ according to the prior assumptions each imposes on

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[6]The partial elasticities have a conventional interpretation. If $\sigma_{ij} > 1$, then a one per cent increase in the factor price ratio Pi/Pj will provoke a larger than one per cent increase in the ratio of factor j to factor i, and the share of factor j in total costs or proceeds will rise. Also, if σ_{ij} is less than zero, factors i and j are complements in that an increase in the relative price of one of them will cause reduced use of the other. Factors i and j are substitutes if σ_{ij} is positive.

Partial elasticities are defined and described in R. G. D. Allen, Mathematical Analysis for Economists, St. Martin's Press, New York, pp. 503-508.

the data. In the first approach direct estimates of each substitution parameter σ_{ij} were obtained by substituting equation (4) into equations (1), (2) and (3) and using the homogeneity condition to yield the three sets of estimating equations below.

$$\text{(A1)} \quad \ln X/L = C + \sigma_{k\ell} \ln \frac{PL}{P} + \Delta A(\sigma_{a\ell} - \sigma_{k\ell}) \ln \frac{PL}{PA}$$

$$\ln X/L = C + \sigma_{k\ell} \ln \frac{PA}{P} + \Delta L(\sigma_{k\ell} - \sigma_{\ell\ell}) \ln \frac{PL}{PA}$$

$$\text{(A2)} \quad \ln X/A = C + \sigma_{a\ell} \ln \frac{PA}{P} + \Delta K(\sigma_{a\ell} - \sigma_{ka}) \ln \frac{PK}{PA}$$

$$\ln X/A = C + \sigma_{a\ell} \ln \frac{PK}{P} + \Delta A(\sigma_{aa} - \sigma_{a\ell}) \ln \frac{PK}{PA}$$

$$\text{(A3)} \quad \ln X/K = C + \sigma_{ka} \ln \frac{PK}{P} + \Delta L(\sigma_{ka} - \sigma_{k\ell}) \ln \frac{PL}{PK}$$

$$\ln X/K = C + \sigma_{ka} \ln \frac{PL}{P} + \Delta K(\sigma_{kk} - \sigma_{ka}) \ln \frac{PL}{PK}$$

Using set (A1) as an example, these equations can be interpreted in the following manner. Holding the factor price ratio PL/PA constant in the regression rules out substitution between the factor pair of land and labor. Thus an increase in the real price of labor PL/P implies an increase in the factor price ratio PL/PK and the subsequent substitution of capital for labor is picked up in the positive variation of average labor productivity X/L.

As long as factor shares Δj are considered to be parameters in the regression equation, the second equation in set (A1) must yield an identical estimate of the substitution parameter $\sigma_{k\ell}$, since it involves a linear combination of the same independent variables as in the first equation.[7] However, if factor shares are allowed

[7]This relationship is easy to discern. If B_j represents a regression coefficient, the first equation takes the regression form of $\ln X/L = C + B_1(\ln PL - \ln p) + B_2 (\ln PL - \ln PA)$. Adding and subtracting $B_1 \ln PA$ to this equation results in $\ln X/L = C + B_1 (\ln PA - \ln P) + (B_1 + B_2)(\ln PL - \ln PA)$ so that the regression form of the second equation $\ln X/L = C + B_3 \ln \frac{PA}{P} + B_4 \ln \frac{PL}{PA}$ implies the equalities $B_1 = B_3$ and $(B_1 + B_2) = B_4$ must hold between the two equations.

to vary, and the independent variables become share-weighted factor price ratios, the equalities above no longer hold and there is no guarantee that the alternative forms of any equation set will yield the same estimate of σ_{ij}. Specifications which regarded the factor shares as variable were thought to be more appropriate since only in the special Cobb-Douglas case would one expect constant factor shares to accompany varying factor prices. Also, by permitting factor shares to vary with each observation, it was possible to identify more than one substitution parameter in each equation. In set (A1), for example, σ_{aj} can be recovered from the regression coefficient for the variable $\Delta A \ln \frac{PL}{PA}$ given the point estimate of σ_{kj} from the regression coefficient for PL/P.

When a stochastic disturbance term is added to the different equation sets (A1), (A2) and (A3), the equations that have been described can be estimated by Ordinary Least Squares since each independent variable can be considered as exogenous. The usual assumptions about the disturbance term are made: that is, it is identically and independently distributed log-normal, and has an expectation of zero.

A second approach was developed as a safeguard against some of the difficulties associated with the first. A potential problem with the first regression set (A) is that it may be subject to measurement errors in value-added. Another shortcoming is the difficulty of imposing desirable cross-equation restrictions on the three sets of estimating equations. The second coefficient in any set may generate a value for σ_{ij} which differs from the value for σ_{ij} generated by the first coefficient in some other set. Moreover, two estimates of σ_{ii} are possible--one from the values for σ_{ij} and the other from the regression itself--because although homogeneity has been imposed, the estimates of σ_{ij} may not imply that it holds. Using equations (1) to (3) and imposing homogeneity to eliminate σ_{ii}

terms, the amounts of machinery and labor demanded per unit of land can be expressed in a way that permits the direct estimation of all σ_{ij} values. The two equations are:

$$\text{(B1)} \quad \ln \frac{K}{A} = C_o + \sigma_{ka}\{(1-\Delta L)\ln\frac{PA}{PK}\} + \sigma_{k\ell}\{\Delta L \ln \frac{PL}{PK}\} + \sigma_{\ell a}\{\Delta L \ln \frac{PA}{PL}\}$$

$$\text{(B2)} \quad \ln \frac{L}{A} = C_1 + \sigma_{ka}\{\Delta K \ln \frac{PA}{PK}\} + \sigma_{k\ell}\{-\Delta K \ln \frac{PL}{PK}\} + \sigma_{\ell a}\{(1-\Delta K) \ln \frac{PA}{PL}\}$$

Because both equations provide estimates of the same parameters, it was desirable to estimate both equations simultaneously under the imposed constraint that σ_{ij} have the same value in each equation. An adaptation of Generalized Least Squares (GLS) developed by Zellner was the estimating technique employed to satisfy these comditions.[8] This procedure is asymptotically more efficient than single-equation least squares because it takes account of zero restrictions on coefficients occurring in other equations. The first step in applying this procedure is to estimate each equation separately by Ordinary Least Squares and form a 2 x 2 variance-covar ance matrix based on the residuals of each equation. This matrix Σ is equal to $\begin{bmatrix} \sigma_{uu} & \sigma_{eu} \\ \sigma_{eu} & \sigma_{ee} \end{bmatrix}$ where σ_{uu}, σ_{ee} are the variances of the residuals from each equation and σ_{eu} is the covariance of these residuals. The inverse of Σ, Σ^{-1} is expressed as $\begin{bmatrix} \sigma'_{uu} & \sigma'_{eu} \\ \sigma'_{eu} & \sigma'_{ee} \end{bmatrix}$. If (Y_1, Y_2) is an n x 2 matrix of n observations of the dependent variables, $\ln \frac{K}{A}$ and $\ln \frac{L}{A}$, X_1 is an n x 4 matrix of n observations of the independent variables, unity, $(\Delta L - \Delta K) \ln \frac{PL}{PK}$, $(1 + \Delta L - \Delta K) \ln \frac{PA}{PL}$, and $(1 + \Delta K - \Delta L) \ln \frac{PA}{PK}$, and X_2 is an n x 1 vector of units, the constrained

[8] A Zellner, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of the American Statistical Association, June 1962, pp. 348-368.

GLS procedure can be expressed as:[9]

$$\begin{bmatrix} C_o \\ \sigma_{k\ell} \\ \sigma_{\ell a} \\ \sigma_{ka} \\ C_1 \end{bmatrix} = \begin{bmatrix} \sigma'_{uu}X'_1 X_1 & \sigma'_{eu}X'_1 X2 \\ & \\ & \\ & \\ \sigma'_{eu}X'_2 X_1 & \sigma'_{ee}X'_2 X_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sigma'_{eu}X'_1 Y_1 + \sigma'_{eu}X'_1 Y_2 \\ \\ \\ \\ \sigma'_{eu}X'_2 Y_1 + \sigma'_{ee}X'_2 Y_2 \end{bmatrix}$$

Elements of the diagonal of the inverse matrix above are estimated variances of the different parameter estimates.

In order to conserve on scarce degrees of freedom, the use of intermediate inputs was purposely overlooked. This omission could be a potential source of specification error biasing in either direction the measured from the true partial elasticity of substitution. This bias would not appear only if certain conditions were met. If, for example, any intermediate input, denoted by the subscript I, was added to the factor demand equations developed above, the labor to land ratio could be expressed as $\ln L/A = C + \Delta K(\sigma_{\ell k} - \sigma_{ka}) \ln \frac{PK}{PA} + \Delta L(\sigma_{\ell\ell} - \sigma_{a\ell}) \ln \frac{PL}{PA} + \Delta I(\sigma_{\ell i} - \sigma_{ai}) \ln \frac{PI}{PA}$. In a regression equation which omitted the last term, the expected value of the regression coefficient for the variable $\Delta K \ln \frac{PK}{PK}$ would be $(\sigma_{\ell k} \hat{-} \sigma_{ka}) + (\sigma_{\ell i} - \sigma_{ai}) B_I$ where $(\sigma_{\ell k} \hat{-} \sigma_{ka})$ is the regression coefficient obtained by including the intermediate input in the regression and B_I is the regression coefficient for the variable $\Delta K \ln \frac{PK}{PA}$ in the regression of $\Delta I \ln \frac{PI}{PA}$ on

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[9] In matrix notation this expression is $B = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y$ where $X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}$. The constraint of equal σ_{ij} values is achieved by adding the weighted factor price variables in each equation. Notice that the constant terms in each equation remain unconstrained.

$\Delta K \ln \frac{PK}{PA}$ and $\Delta L \ln \frac{PL}{PA}$. Therefore, no bias would appear if any one of three conditions could be established. If $\sigma_{\ell i} = \sigma_{ai}$ so that intermediate inputs were either strictly complementary with or equally substitutable for other factors, or if the covariance of $\Delta I \ln \frac{PI}{PA}$ and $\Delta K \ln \frac{PK}{PA}$ were zero, the omission of intermediate inputs would not bias the size of other parameter estimates.[10]

4. Regression Results

Table I presents the point estimates of the partial elasticities derived from regressions on the INCORA data. The complete regression results appear in Tables to in the Appendix. It is obvious that all of the crops analyzed did not meet with equal success. Rice and cotton were consistently the best performers in all cases with the trio of sesame, soya and sorghum not far behind. Corn Corn had a mixed record, performing well in some cases and badly in others. Wheat and barley were extremely erratic and not much confidence could be placed in any of the results for these crops. The combined crop regressions usually fared worse than most of the individual crops, tending to indicate the importance of different factor substitution relationships among different crops. Although the overall results were somewhat mixed, the fact that the most heavily mechanized crops produced plausible results was considered a small partial victory over the problem of small samples.

The partial elasticity of substitution between farm machinery and labor was invariably higher than the elasticities between other factor pairs in model A. Point estimates of $\sigma_{k\ell}$ for various crops gave significant values of 1.59 for rice

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[10] It is difficult to guess at the likely direction of any bias. If the covariance term is positive, and if intermediate inputs are a better substitute for land than for labor, the partial elasticities estimated from the per hectare regression forms may be downwardly biased from their "true" values.

Table 1: Point Estimates of Substitution Elasticities

Regression Specification	Parameter	Rice	Cotton	Corn	SSS	Wheat and Barley
(A1) (not share weighted)	$\sigma_{k\ell}$	2.22	.90	1.48	.98	-.087
(A1) (share weighted)		1.59	.96	.90	.94	.03
(B) (GLS) I		1.43	1.88	1.44	1.30	1.07
(B) (GLS) II		1.44	1.87	1.44	1.30	1.08
(A2) (not share weighted)	$\sigma_{\ell a}$	1.18	.35	.28	.12	-.83
(A2) (share weighted) I		.36	.64	.55	.42	.02
(A2) (share weighted) II		.67	.63	.55	.20	.08
(A1) (share weighted)		.67	.61	.30	.89	-.55
(B) (GLS) I		.79	-.02	.78	.55	.44
(B) (GLS) II		.57	.54	.80	.44	.87
(A3) (not share weighted)	σ_{ka}	.25	.04	.63	-.04	-4.25
(A3) (share weighted) I		-1.09	-1.73	-.85	.26	-1.29
(A3) (share weighted) II		-.85	-1.72	-.80	.62	-1.19
(B) (GLS) I		-.34	-.13	-.65	.13	.04
(B) (GLS) II		-.29	-.50	-.69	.20	-.051

Notes: (a) Capital letters refer to the different specifications that were designated in the chapter text, and numbers refer to the specific equation within each specification. See pp. and .

(b) "Not share weighted" and "share weighted" refer to whether or not the factor price ratios were multiplied by factor cost shares prior to estimation.

(c) I and II refer to the different capital price and quantity concepts that were used on pages to .

(d) The complete regression results for each specification are shown in Tables to in the Appendix.

and about unity for cotton, corn and the combined sesame, soya and sorghum crops. Point estimates for the other elasticities were well under one with the elasticity between land and labor usually exceeding that between land and farm machinery. Most of the regression estimates produced a fairly strong and significant negative

elasticity for the machinery-land substitution parameter suggesting a complementary relationship existed for this factor pair, perhaps as a result of machinery indivisibilities. Thus, a higher machinery-labor ratio will tend to be accompanied by a higher ratio of land to labor. When point estimates of the difference between various partial elasticities were examined in model A, the above ranking of the elasticities in terms of relative size continued to hold up. The ease of substitution appeared to be highest between farm machinery and labor, next highest between land and labor, and least between land and farm machinery.

Generalized Least Squares estimates from model B, which imposed the constraint of equal substitution elasticities between the two equations, met with almost unqualified statistical success. These estimates are shown in Table . Except for cotton and wheat and barley, all of the estimated substitution parameters were significant to at least the 10 per cent level. Of the 30 estimated substitution elasticities, 26 were significantly different from zero and the gain in efficiency over single-equation estimates was to be expected.[11] For all of the crops the elasticity of substitution between farm machinery and labor exceeded one with a range of 1.07 to 1.88 and an unweighted average value of 1.43. Except for cotton, the substitution elasticity between land and labor was less than one and greater than zero with a range of .44 to .8. Between farm machinery and land, the substitution elasticity was significantly negative for rice, cotton and corn; barely positive for the SSS aggregate, and insignificant from zero in the case of wheat and barley. These results conform quite closely to those for model A and relative stability of the estimates between these two different models lends some support for their reasonableness.

In view of the small number of observations employed in these regressions, it would be reckless to generalize these findings. Because of small degrees of freedom,

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[11] It should be pointed out that there is an intractable "identification" problem here. A value for σ_{ij} insignificant from zero may indicate fixed proportions but is also consistent with either a bad specification or bad data.

not a great deal of confidence can be placed in the point estimates. They should be regarded as highly tentative pending additional evidence and confirmation. A noticeable deficiency of these estimates is that they assume cost minimization actually occurs so that the measured partial elasticities of substitution describe only the technological properties of various crops. However, if larger farmers, either through their fear or dislike of using labor, are not profit-maximizers, the substitution parameters one would measure from their behavior would necessarily be smaller than those estimated here, or would appear to lack reversibility over time. Accordingly, it is of some interest to examine the outcome of related research in other countries.

5. Results of Other Research into the Ease of Factor Substitution in Agriculture

The empirical results of the previous section supported the view of substantial sensitivity of the farm machinery-labor ratio to relative factor prices in Colombia. Only one other attempt to investigate the matter has been made and it tends to cast some doubt on the foregoing conclusion. John Sanders has used an aggregate stock adjustment model to estimate a price elasticity of demand for tractors of -.10 in the short run and -.36 in the long run. The price variable he used was the relative price of tractors to lagged crop output price and to lagged agricultural wage rates.[12]

There may be no clash between the earlier results and Sanders', however, if many of his observations are on the supply curve rather than on the demand curve for tractors. This could occur because the Colombian Agricultural Bank, the Caja Agraria, sets, by and large, both price of tractors and quantity supplied. In this kind of an excess demand situation, one cannot easily distinguish supply from demand

[12] John Sanders, "Government Policy and the Demand for Tractors in Colombia, 1950-1967," University of Minnesota, mimeo, 1971.

responses from observed data. If excess demand for tractors were a typical situation over this period, a tax up to the amount needed to put buyers on their demand curve would not reduce the amount demanded and would lend the impression of inelastic demand.

Outside of Colombia, empirical research into the case of agricultural factor substitution has been largely confined to the United States and Western Europe, including Britain. For the United States Lianos has recently sought to explain the decline in the agricultural labor's relative income share through the use of a neoclassical production framework.[13] Lianos employed time-series data over the period 1949-68 for a comprehensive definition of the agricultural sector encompassing both crops and livestock. Labor input consisted of man-hours of unpaid family members as well as paid workers. Capital was measured as the sum of expenditures on feed, livestock, seed, fertilizer, repairs and operations of capital items, depreciation and interest on farm mortgage debt. The value of this capital concept was deflated by an index of prices paid by farmers. An average cost of loans series for agriculture was taken as the price of capital while the price of labor was measured by a composite rate per hour.

After noting that labor's income share has fallen by about 50 per cent between 1949 and 1968, Lianos relies on the specification of an aggregate, two-factor, CES production function to explain this decline. By postulating that technical progress is of the factor augmenting type and can be represented by exponential trends, he derives two estimating equations capable of identifying both technical change and the elasticity of substitution σ:

1) $\ln K/L = C + \sigma \ln \left(\frac{PL}{PK}\right) + (\gamma_L - \gamma_K)(1 - \sigma) \ln t$

[13] Theodore D. Lianos, "The Relative Share of Labor in United States Agriculture," American Journal of Agricultural Economics, volume 53, No. 3, August 1971.

2) $\ln \Delta K = C + (1 - \sigma) \ln PL + \gamma_L (\sigma - 1) \ln t$

where previous notation applies except that γ_L, γ_K and t are new variables and refer to the rates of labor and capital augmentation and time respectively.

The first equation above fits the data well and generates a value for σ of 1.52 while the second equation, when it has been transformed and estimated by Autoregressive Least Squares, produces a similar value for σ of 1.60. The first, but not the second, equation also suggests that technical change has been predominantly labor rather than capital augmenting. These results are quite consistent with earlier efforts to estimate the demand elasticities for labor and tractors in U.S. agriculture. Elasticities of labor demand, $\frac{\partial \ln L}{\partial \ln PL}$, had been previously estimated by Bauer as -1.43 and by Wallace and Hoover as -1.48.[14] Griliches, on the other hand, had estimated the long-run price elasticity of demand for tractors, $\frac{\partial \ln T}{\partial \ln PT}$, at -1.5.[15] By inference then, the elasticity of substitution between tractors and labor, $\frac{\partial \ln (T/L)}{\partial \ln (PL/PT)}$, would also be close to one and a half, a value which would conform to Lianos' results if tractors were a reasonably stable proportion of his definition of total capital expenditure.

For Western Europe the issue of factor substitution in agriculture has been recently explored and summarized by Scott and Smyth.[16] They consider both cross-section data for 16 OECD countries and time-series information for Britain as a

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[14] L. Bauer, "The Effect of Technology on the Farm Labor Market," American Journal of Agricultural Economics, August 1969.

T. Wallace and D. Hoover, "The Income Effects of Innovation: The Case of Labor in Agriculture," Journal of Farm Economics, May 1966.

[15] Z. Griliches, "The Demand for a Durable Input: Farm Tractors in the United States, 1951-57," in A. C. Harberger, ed., The Demand for Durable Goods, University of Chicago Press, 1960, p. 181.

[16] H. G. Scott and D. J. Smyth, Demand for Farm Machinery - Western Europe, Royal Commission on Farm Machinery, Study No. 9, Queen's Printer, Ottawa, Canada, 1970.

means of illustrating the variability of the farm machinery - labor ratio T/L to the prices of these factors. T is measured either as tractor horsepower or alternatively as the number of tractors (excluding the garden variety). A stock concept of the number of people engaged in agriculture comprises the variable for labor input L. The price of labor PL is an index based on FAO data after conversion to a common U.S. dollar unit. A rough index of the cost of tractors PT was calculated based on tariffs and freight from Britain assuming British tractors were competitive in all of the western European countries.

The estimating equations used by Smyth and Scott rely on the existence of a multi-factor CES production function of the form $Y = A(\alpha_1 X_1^{-B} + \ldots + \alpha_n X_n^{-B})^{-1/B}$ in which the elasticity of substitution between any pair of factors is $\sigma = \frac{1}{1 + B}$. Assuming proportionality between factor prices and marginal products the ratio of T to L is given as $\frac{T}{L} = \left[\frac{\alpha T}{\alpha L} \frac{PT}{PL}\right]^{1/1 + B}$ or in logs as $\ln \frac{T}{L} = C + \sigma \ln \frac{PT}{PL}$. Fitted cross-sectionally to western Europe, this equation performed well and gave rise to a value for σ of about 2. Smyth and Scott note that the data display a positive correlation between farm size and the price of labor. They surmise that this correlation reflects the immobility of farm labor on small farms so that market wages are an inadequate indicator of opportunity costs for this group of farmers. In these circumstances, the $\frac{T}{L}$ ratio on a small farm, or in countries with many small farms, would be smaller than the model would predict on the basis of the relative tractor-labor price. When average farm size becomes an additional variable in the regression model the elasticity of substitution is reduced in size from 2 to 1.66 - 1.71.

Another possible source of bias that Smyth and Scott consider is the possibility of non-neutral technical progress. Specifically, if technical progress has been biased toward tractors, a portion of the higher tractor-labor ratio in richer countries would be attributable to their different technology rather than entirely to

their higher cost of labor. The impact of this effect would be an overestimate of the substitution elasticity. To test for the presence of such a possibility Smyth and Scott reformulate their model in rate-of-change terms: $\frac{d \ln T/L}{dt} = \frac{\sigma \, d \ln PT/PL}{dt} + \frac{a \, d \ln R}{dt}$ where R is average farm size. If the hypothesis of non-neutral technical change is incorrect, a regression in which the independent variables are percentage changes over time should appear with a zero constant term. In fact, when this regression is tried, they find a significantly positive constant term indicating tractor-using progress and a value for σ of 1.43.

Finally, when a stock adjustment version of the model is adapted to time-series data for the period 1948-65 in Britain, substantial substitution possibilities are also indicated. Using tractor horsepower, but not tractor stock, as the dependent variable in the regression model produces estimates of the elasticity of substitution between 1.1 and 1.54 depending on which tractor price variable (with or without quality adjustment) is employed. Smyth and Scott feel that the 1.1 value is probably too low since it is associated with the regression in which a constant-quality tractor price index is used and this procedure neglects the likelihood of concomitant growth in the quality of labor services. They suggest that σ is well above unity but not much above the value of 1.5.

6. Conclusions

Unlike the introductory quotation by Nerlove, the main finding of this exercise and partial survey is the uniformity of results. Widely variant approaches to the problem of estimating the degree of factor substitutability in agriculture produce essentially the same answer: the elasticity of capital-labor substitution in agriculture exceeds unity and is probably close to one and a half. It has been shown that this parameter size appears to be virtually independent of the choice of time-

period, the use of inter-country versus intra-country data or of time-series versus cross-section information, and, as well, the degree of aggregation of output, capital and labor and the selection of functional forms for the production function. Three factor variable elasticity functions gave comparable results to two factor and multiple factor CES formulations. It must be admitted, however, that these comparisons have not been carried out on a systematic basis. That is, the stability of σ has been observed for different specifications on different data sources. Ideally, insensitivity to aggregation, specification, and time period should be tested on the same data source. Until such a data source exists it is hard to believe that the clustering of values for σ about a common level is all happenstance. The chances are that it accurately reflects the wide range of alternative agricultural techniques available for adoption at any moment of time.

If this conclusion is accepted as being valid, it has strong implications for agricultural policy in developing countries. As an economy develops and the price of labor becomes expensive relative to commodities there are apt to be substantial declines in demand for agricultural labor and high payoffs to mobility policies which assist rural to urban migration. Secondly, if factor market distortions are responsible for an undesirable degree of capital-intensity in nonagricultural sectors, it is vital on income distribution grounds that similar distortions be avoided in the agricultural sector.[17] Otherwise the share of labor in agricultural income will decline in a sector which in many countries contains the poorest of the poor. This trend is observable in Colombia and proponents of agricultural mechanization for that country have been slow to relate it to the policy they advocate.

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[17] An attempt to assess the impact of such distortions in Colombia is found in W. Thirsk, "Income Distribution, Efficiency, and the Experience of Colombian Farm Mechanization," Discussion Paper 33 in this series.

APPENDIX

Table 1. Partial Substitution Elasticity Between Capital and Labor With and Without Factor Share Weights

(1) $\ln \frac{VA}{L}$	C	$\ln \frac{PL}{PQ}$	$\ln \frac{PL}{PA}$	#	R^2	D.W.
Rice	-1.48 (-1.19)	2.22* (2.33)	-.89* (-1.87)	13	.47	1.75
Cotton	.979 (2.45)	.90* (2.95)	-.155 (-1.14)	11	.56	2.02
Corn	-.70 (-.66)	1.48* (2.34)	-.54* (-1.82)	19	.27	1.84
SSS	1.12 (3.73)	.98* (5.27)	-.007 (-.04)	19	.67	1.04
Wheat and Barley	3.75 (2.58)	-.087 (-.103)	.84 (1.03)	14	.09	2.23

(2) $\ln \frac{VA}{L}$	C	$\ln \frac{PL}{PQ}$	$\Delta A \ln \frac{PL}{PA}$	#	R^2	D.W.
Rice	.039 (.041)	1.59** (1.62)	-.915* (-2.00)	13	.49	2.6
Cotton	1.026 (5.16)	.96* (4.46)	-.37* (-2.49)	11	.71	2.31
Corn	.47 (.65)	.90** (1.68)	-.601* (-3.05)	19	.40	1.46
SSS	1.086 (6.04)	.94* (5.23)	-.145 (-.80)	19	.68	1.19
Wheat and Barley	2.73 (2.96)	.03 (.03)	.584 (.69)	14	.05	1.83

Notes: (a) The form for both sets is:

$$\ln \frac{VA}{L} = C + \sigma_{k\ell} \ln \frac{PL}{PQ} + (\sigma_{a\ell} - \sigma_{k\ell}) \Delta A \ln \frac{PL}{PA} .$$

In the first set the share of land ΔA is assumed constant; in the second set it is entered as a variable. C is a constant.

(b) t-values are entered in parentheses under each regression coefficient; # refers to the number of observations; R^2 is the coefficient of determination, and D.W. is the Durban-Watson statistic.

(c) SSS refers to the aggregation of sesame, soyabeans and sorghum.

(d) For description of the price and quantity variables, see p. 5 in the text.

(e) A single asterisk (*) denotes significance at the 5 per cent level; a double asterisk (**) at the 10 per cent level.

Table 2: Partial Substitution Elasticity between Land and Labor With and Without Factor Share Weights

(1)	$\ln \frac{VA}{A}$	C	$\ln \frac{PA}{PQ}$	$\ln \frac{PK}{PA}$	#	R^2	D.W.
	Rice	1.38 (.51)	1.18 (.72)	.72 (.51)	13	.18	1.95
	Cotton	3.6 (4.5)	.35 (-.57)	-.94** (-1.51)	11	.42	2.10
	Corn	2.26 (5.56)	.284* (1.79)	-.286** (-3.15)	19	.60	2.32
	SSS	2.82 (15.9)	.12 (1.31)	-.255** (-1.39)	19	.41	1.28
	Wheat and Barley	4.25 (5.07)	-.83** (-1.66)	-1.097* (-1.95)	14	.26	2.02

(2)	$\ln \frac{VA}{A}$	C	$\ln \frac{PA}{PQ}$	$\Delta K \ln \frac{PK}{PA}$	#	R^2	D.W.
	Rice I	2.68 (2.48)	.364 (.77)	.017 (.035)	13	.16	2.76
	II	2.12 (2.87)	.67* (2.06)	.882** (1.40)	13	.30	2.93
	Cotton I	2.48 (7.58)	.64* (5.08)	.83** (1.72)	11	.79	1.97
	II	2.49 (7.80)	.63* (5.04)	.676* (1.84)	11	.80	1.98
	Corn I	2.02 (5.55)	.55* (3.93)	.61** (1.61)	17	.53	2.22
	II	2.00 (5.29)	.55* (3.72)	.51** (1.30)	17	.50	2.21
	SSS I	2.67 (16.05)	.42* (4.47)	.87* (3.03)	19	.56	1.23
	II	2.88 (15.34)	.20* (2.20)	.006 (.015)	19	.31	.72
	Wheat and Barley I	3.58 (6.09)	.02 (.10)	.71* (2.01)	13	.30	1.92
	II	3.50 (6.00)	.08 (.32)	.951* (2.03)	13	.31	1.82

Notes: (a) The format is the same as for Table 1.

(b) The regression form is:

$$\ln \frac{VA}{A} = C + \sigma_{a\ell} \ln \frac{PA}{PQ} + (\sigma_{a\ell} - \sigma_{ak}) \Delta K \ln \frac{PK}{PA} .$$

(c) A single asterisk (*) denotes significance at the 5 per cent level; a double asterisk (**) significance at the 10 per cent level.

Table 3: Partial Substitution Elasticity between Capital and Land With and Without Factor Share Weights

(1)	$\ln \frac{VA}{K}$	C	$\ln \frac{PK}{PQ}$	$\ln \frac{PL}{PK}$	#	R^2	D.W.
	Rice	-1.81 (-1.12)	.25* (2.09)	-1.71* (-3.28)	13	.73	2.75
	Cotton	2.40 (1.08)	.04 (.023)	-.512 (-.89)	11	.17	1.81
	Corn	.001 (.004)	.63* (1.76)	-2.77* (-3.46)	19	.82	.83
	SSS	2.18 (10.25)	-.04 (-.26)	-.48** (-1.43)	19	.13	1.24
	Wheat and Barley	10.54 (4.86)	-4.25* (-2.97)	-4.57* (-2.21)	13	.59	1.57

(2)	$\ln \frac{VA}{K}$		C	$\ln \frac{PL}{PQ}$	$\Delta K \ln \frac{PL}{PK}$	#	R^2	D.W.
	Rice	I	2.05 (3.38)	-1.09** (-1.74)	-.641** (1.57)	13	.48	2.41
		II	3.43 (4.56)	-.85 (-1.08)	-.878** (-1.73)	13	.24	2.36
	Cotton	I	4.52 (12.28)	-1.73* (-4.65)	3.98* (4.37)	11	.75	3.26
		II	4.32 (5.2)	-1.72* (-2.06)	3.56** (1.60)	11	.35	1.95
	Corn	I	3.62 (8.89)	-.85* (-2.34)	.87 (.97)	19	.37	1.49
		II	3.58 (8.97)	-.80* (-2.21)	1.12 (1.24)	19	.41	1.80
	SSS	I	.563 (4.58)	.26** (1.43)	.96* (2.91)	19	.40	1.38
		II	1.96 (12.12)	.62* (2.57)	.34 (.79)	19	.40	1.66
	Wheat and Barley	I	2.44 (5.92)	-1.286* (-3.05)	2.43* (5.11)	13	.78	2.00
		II	3.85 (8.51)	-1.19* (-2.59)	1.03* (1.99)	13	.51	1.85

Notes: (a) The format is the same as for Table 1.

(b) The regression form of set (1) is:

$$\ln \frac{VA}{K} = C + \sigma_{ka} \ln \frac{PK}{PQ} + (\sigma_{ka} - \sigma_{kl}) \Delta L \ln \frac{PL}{PK} ;$$

for set (2) it is:

$$\ln \frac{VA}{K} = C + \sigma_{ka} \ln \frac{PL}{PQ} + (\sigma_{kk} - \sigma_{ka}) \Delta K \ln \frac{PL}{PK} .$$

(c) For set (2) two regressions were run: one using the tractor rental for PK (I), and the other using a weighted rental price for all machinery (II) (see pages 2-4 in the text).

(d) A single asterisk (*) denotes significance at the 5 per cent level; a double asterisk (**) at the 10 per cent level.

Table 4: Constrained Simultaneous Estimation of Partial Substitution Elasticities

Crop		Parameters $\sigma_{k\ell}$	$\sigma_{a\ell}$	σ_{ka}
Rice	I	1.43* (28.6)	.79* (4.16)	-.34* (-2.13)
	II	1.44* (27.1)	.57* (5.85)	-.29* (-2.11)
Cotton	I	1.88* (29.3)	-.02 (-.0013)	-.13 (-.60)
	II	1.87* (29.2)	.54* (2.29)	-.50* (-3.29)
Corn	I	1.44* (30.6)	.786* (2.55)	-.655* (-2.17)
	II	1.44* (36.0)	.80* (3.40)	-.687* (-3.37)
SSS	I	1.30* (11.8)	.55* (5.34)	.133** (1.33)
	II	1.30* (40.6)	.44* (3.33)	.20** (1.63)
Wheat and Barley	I	1.07* (12.74)	.44* (3.60)	.04 (.028)
	II	1.08* (12.0)	.487* (1.85)	-.051 (-.038)

Notes: (a) The estimation method is constrained Generalized Least Squares (see pages 10-11 in the text).

(b) (I) refers to the use of tractor rental prices for PK and (II) to a weighted average of all rental prices.

(c) t-values are in parentheses below each coefficient.

(d) A single asterisk (*) denotes significance at the 5 per cent level; a double asterisk (**) at the 10 per cent level.

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