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WIDTH CONSTRICTIONS IN OPEN CHANNELS

J. W. Hugh Barrett

Colorado State University

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ABSTRACT

WIDTH CONSTRICTIONS IN OPEN CHANNELS

The purpose of this study is to compare the existing methods of calculating the backwater due to, or discharge through, a constriction in an open channel, and to show how these methods are but particular expressions of a more general submerged flow equation.

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LIST OF SYMBOLS

Symbol	Definition
A_1	Area of flow including backwater at section I
A_n	Normal area of flow at bridge site before the bridge is in place
A_{n1}	Area of flow below normal water surface at section I
A_{n2}	Area of flow below normal water surface at section II
A_p	Flow area obstructed by piers
A_4	Area of flow at section IV at which normal water surface is reestablished
a	Area of flow in a subsection of a channel
B	Width of channel
b	Width of constriction
C	Chezy resistance coefficient; or free flow discharge coefficient
C_1	Submerged flow discharge coefficient
C_c	Coefficient of contraction
C_{cn}	Contraction coefficient for width of live stream flow at normal depth
C_{DA}	D'Aubuisson's pier coefficient
C_K	Kindsvater's discharge coefficient
C'_K	Kindsvater's discharge coefficient for standard condition
C_{NA}	Nagler's pier coefficient
C_{RE}	Rehbock's pier coefficient
E_1	Specific energy at section I after bridge placed
E_{1A}	Specific energy at section I before bridge placed, for abnormal stage-discharge condition
E_4	Specific energy at section IV

LIST OF SYMBOLS - (Continued)

Symbol	Definition
E_d	Specific energy at section downstream from constriction
E_f	Energy loss due to friction between section I and II
E_L	Energy loss $E_u - E_d$, $E_1 - E_n$ or $E_1 - E_{1A}$
E_n	Specific energy at normal depth
E_R	Ratio of E_d/E_u , E_n/E_1 or E_{1A}/E_1
E_{rt}	Transition energy ratio
e	Eccentricity of bridge centerline from channel centerline
$\mathbb{F}$	Froude number
$\mathbb{F}_1$	Froude number at section I, $V/(gy_1)^{1/2}$
$\mathbb{F}_n$	Froude number at normal depth, $V/(gy_n)^{1/2}$
g	Acceleration due to gravity (32.2 ft/sec^2)
H_L	Head loss $y_u - y_d$
j	Ratio of area obstructed by piers to gross area of bridge waterway
K^*	Total backwater coefficient, $K_b + \Delta K_p + \Delta K_e + \Delta K_s$
K_b	Backwater coefficient from base curve
ΔK_p	Incremental backwater coefficient for piers
ΔK_e	Incremental backwater coefficient for eccentricity
ΔK_s	Incremental backwater coefficient for skew
L^*	Distance from point of maximum backwater to water surface on upstream side of road embankment, measured parallel to centerline of stream
M	Bridge opening ratio, b/B
m	Contraction ratio (1-M)
n	Manning roughness coefficient
n_1	Exponent, in the free flow equation, and in numerator of submerged flow equation

LIST OF SYMBOLS - (Continued)

Symbol	Definition
n_2	Submergence exponent in the denominator of the submerged flow equation
Q	Total discharge, cfs
$Q_{E_L=1}$	Value of Q where $E_L=1$
$Q_{H_L=1}$	Value of Q where $H_L=1$
R	Hydraulic radius
S	Submergence, ratio of two flow depths, always less than 1
S_e	Slope of energy line
S_t	Transition submergence
V	Flow velocity
V_1	Average velocity at section I
V_4	Average velocity at section IV
V_{n2}	Average velocity in constriction for flow at normal depth, Q/A_{n2}
y	Flow depth
y_1	Flow depth at section I
y_4	Flow depth at section IV
y_1^*	Total backwater or rise above normal depth at section I
y_{1A}	Abnormal (non-uniform) stage at section I prior to placement of bridge constriction
y_c	Critical depth
y_d	Flow depth at a section downstream from constriction
y_n	Normal flow depth
y_u	Flow depth at a section upstream from constriction
α_1	Coefficient which corrects for nonuniformity of velocity distribution at section I

Symbol	Definition
δ_o	Pier shape factor
ϕ	Correlation coefficient between contraction and resistance backwater

Chapter 1

INTRODUCTION

Importance

The study of the hydraulics of bridge constrictions has generally been approached from one of two directions, either to determine the backwater caused by placing a bridge across a stream of known discharge, or to determine the discharge through a constriction having some measure of the backwater due to the constriction.

The importance of being able to determine the backwater due to a given bridge constriction is given by Bradley (4)*:

> "Structural designers are well aware of the economies which can be attained in the structural design of a bridge of a given overall length. The role of hydraulics in establishing what the length and vertical clearance of a bridge should be and even where it should be placed is less well understood. Confining the flood water unduly may cause excessive backwater with resultant damage to upstream land and improvements and overtopping of the roadway or may induce excessive scour endangering the bridge itself. Too long a bridge may cost far more in added capital investment than can be justified by the benefits obtained. Somewhere in between is the design which will be the most economical to the public over a long period of years. Finding that design is the ultimate goal of the bridge designer.
>
> "It is seldom economically feasible or necessary to bridge the entire width of a stream as it occurs at flood flow. Where conditions permit, approach embankments are extended out onto the flood plain to reduce costs, recognizing that, in so doing, the embankments will constrict the flow of the stream during flood stages. This is an acceptable practice. When carried to extremes, however, constriction of the flow can then result in damage to bridges, costly maintenance, backwater damage suits, or even contribute to the complete loss of the bridge or the approach embankments."

Alternatively, the applicability of using a bridge constriction as a flow measuring device to determine peak discharge is given by

---

* Numbers in parenthesis indicate references.

Kindsvater, Carter and Tracy (11):

> "Measurement of peak discharge directly by the usual current-meter method is often impossible; roads become impassable; structures from which current meter measurements could be made are washed out, knowledge of the flood rise may not be available sufficiently in advance to permit reaching the site near the time of peak; the flow of debris or ice may prevent the use of the current meter, or the rise and fall of the stream may be too rapid to allow a complete measurement even if an engineer is at the site with the necessary equipment. Consequently at times it is necessary to use indirect methods of determining peak discharge."

Purpose

Practical methods have been developed for obtaining the backwater that may be expected due to placing a bridge across a stream for a given design flood. The most commonly used method is that outlined in the Bureau of Public Roads (BPR) bulletin, "Hydraulics of Bridge Waterways" (4). This bulletin was compiled from research efforts by Liu, Bradley and Plate at Colorado State University (CSU).

Similarly, a practical method for computing peak discharge through a contraction, where the maximum backwater can be measured, is embodied in the U.S. Geological Survey (USGS) Circular 284 (11), "Computation of Peak Discharge at Contractions," based on the research work of Kindsvater, Carter and Tracy at the Georgia Institute of Technology.

Both these methods were based on model studies and have often shown large errors in application to prototype structures. Also, detailed investigation has not been undertaken to arrive at a satisfactory solution to the problem where abnormal stage-discharge conditions exist. In fact, Bradley (4) went so far as to say, "This is a case where it is more important to understand the problem than to attempt precise computations."

A different approach to analyzing the hydraulics of flow through bridge constriction was undertaken by Skogerboe, Austin and Chang (17) by applying their previously developed method of submerged (subcritical) flow analysis. Their study was primarily concerned with the abnormal stage-discharge condition.

A need therefore exists to compare the aforementioned methods of analysis and to determine the inter-relationships among the various methods, thereby disclosing any advantages or disadvantages of one technique in comparison with the other techniques. The purpose of this study is to evaluate these currently existing methods of predicting the effects of a bridge constriction on stream flow, as compared to the method of subcritical flow analysis.

Scope

The method of submerged flow analysis is extended to conditions of both uniform and non-uniform flow in the channel prior to a constriction being placed in it. The practicality of the method is improved by eliminating dependence on channel slope, and hence the necessity to carry out computations in terms of total energies. Instead, flow depths may be employed. In addition, the interrelationships between the method of submerged flow analysis and the previously developed methods of analysis are shown.

Chapter 2

LITERATURE REVIEW

The first rigorous investigation of flow through a contracted channel section was probably carried out by Boussinesq (3) in 1877. Followup work on Boussinesq's mathematical approach was carried out by Jaeger in 1948. Other early investigators, such as Nagler (14) in 1918, Lane (12) in 1920, Rehbock (16) in 1921, and Yarnell (22,23) in 1934 employed the empirical approach. Houk (6) in 1918 was probably the first to describe the contracted opening method, although he credited S. M. Woodward with the procedure. The above investigators were mainly concerned with the effect of a contraction caused by bridge piers and piles.

Kindsvater and Carter (10) in 1955, and Tracy and Carter (20) in 1955, were the first to obtain results of general value considering the effects of bridge abutments in a waterway. The use of dimensional analysis was of considerable value in modifying experimental procedure and data evaluation for developing generalized relationships. Liu, Bradley and Plate (13) extended the scope of the investigations substantially by utilizing a tilting flume. Further basic research was reported by H. R. Vallentine (21) using sharp-edged constriction plates. In 1962, Biery and Delleur (2) analyzed the case of single span arch bridge constrictions, while Davidian, Carrigan and Shen (5) extended the work of Kindsvater, Carter and Tracy to multiple opening constrictions.

The technique developed by Hyatt (7) for describing subcritical flow at open channel constrictions was used by Skogerboe, Austin and

Chang (17) to determine the backwater due to bridge constrictions under "abnormal stage-discharge" conditions.

Historical Development

The early investigations by Nagler, Lane and Yarnell were concerned with developing coefficients for the constriction discharge formulas proposed by D'Aubuisson and Weisbach. According to the D'Aubuisson equation (22) which results from the simultaneous solution of the continuity and energy equations, the velocity in the contraction zone (Fig. 1) is

$$V_2 = C_{DA}\sqrt{2g(E_1 - y_2)} = C_{DA}\sqrt{2g\left(\frac{V_1^2}{2g} + y_1 - y_2\right)} \quad \ldots\ldots \qquad (1)$$

or

$$y_1 - y_2 = \frac{Q^2}{2g}\left\{\frac{1}{C_{DA}b^2 y_2^2} - \frac{1}{B^2 y_1^2}\right\} \quad \ldots\ldots \qquad (2)$$

where

C_{DA} is D'Aubuisson's pier coefficient;

E_1 is the specific energy at Section I, in feet;

y_1 is the depth at Section I, in feet;

y_2 is the depth at Section II, in feet;

V_1 is the velocity at Section I, in ft/sec;

g is the gravitational acceleration, in ft/sec^2;

Q is the total discharge, in cfs;

b is the width of constriction, in feet; and

B is the width of channel in feet.

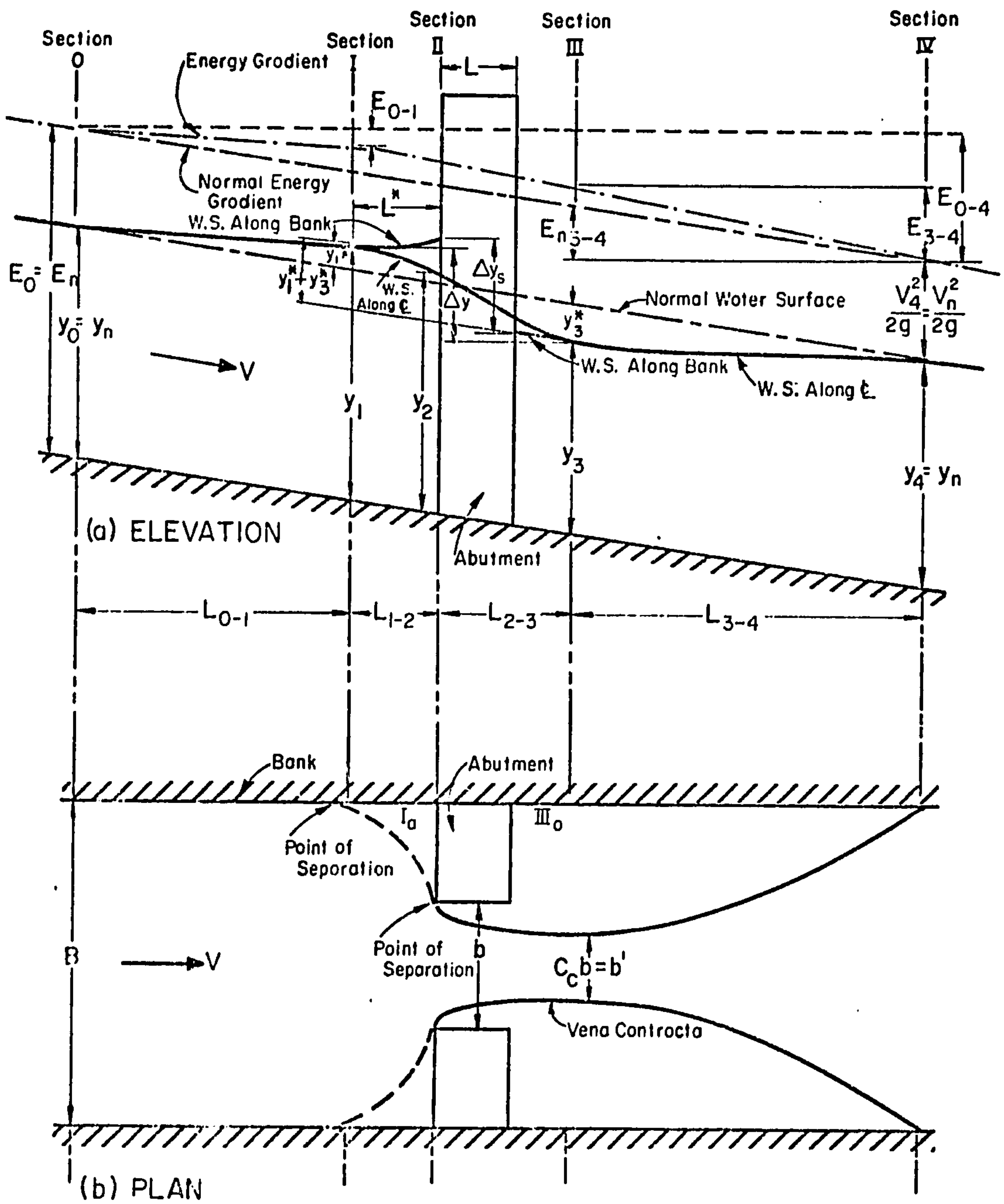

Figure 1. Definition sketch of simple vertical board constriction.

The true maximum backwater is

$$y_1^* = y_1 - y_n = y_1 - y_4 \quad \ldots\ldots\ldots \tag{3}$$

instead of $y_1 - y_2$ where

y_4 is the depth at Section IV, in feet; and

y_n is the normal flow depth, in feet.

For practical purposes, however, y_n can be substituted for y_2, which results in

$$C_{DA} = \frac{Q}{\sqrt{2gb^2 y_n^2 (y_1^* + \frac{V_1^2}{2g})}} \quad \ldots\ldots\ldots \tag{4}$$

or

$$y_1^* = \frac{1}{C_{DA} M^2} \frac{V_n^2}{2g} - \frac{V_1^2}{2g} \quad \ldots\ldots\ldots \tag{5}$$

where

M is the ratio, constriction width: channel width; and

V_n is the normal velocity, in ft/sec.

Nagler's equation was adapted specifically to the bridge pier problem, resulting in

$$Q = C_{NA} b\sqrt{2g}(y_n - \theta' \frac{V_n^2}{2g}) - \sqrt{y_1 - y_n + \beta' \frac{V_1^2}{2g}} \quad \ldots\ldots\ldots \tag{6}$$

where

C_{NA} is the Nagler pier coefficient;

θ' is a correction factor $= \frac{y_n - y_2}{V_n^2/2g}$ .... and (7)

β' is a function of the contraction ratio.

Nagler assumed that $\theta' = 0.3$.

The first known investigator in the United States to have studied simple width constrictions in the laboratory is E. W. Lane (12). His work, however, was concerned primarily with higher Froude numbers than usually involved in natural flow under bridges, and was limited to a few boundary forms. He correlated the discharges and difference in water surface elevation upstream and downstream from the constriction by introducing empirical discharge coefficients, but was unable to develop a definite unique correlation.

Rehbock (16) conducted extensive research to determine the backwater height caused by piers. He divided the channel flow passing through a constriction into three classes:

1. ordinary or "steady" flow, in which the water passes the obstruction with very slight or no turbulence;
2. intermediate flow, in which the water passing the obstruction displays a moderate degree of turbulence; and
3. "changed" flow, in which the water passing the obstruction becomes "completely" turbulent.

The three flow conditions are separated respectively by the two equations:

$$j = \frac{1}{0.97+21\frac{\mathbb{F}^2}{2}} - 0.13 \ \ldots\ldots. \qquad (8)$$

and

$$j = 0.05 + (0.9-2.5\ \frac{\mathbb{F}^2}{2})^2 \ \ldots\ldots. \qquad (9)$$

where $\mathbb{F}$ is the Froude number of the unobstructed flow, and j is the total width of the piers divided by the channel width. Rehbock assumed that the maximum backwater y_1^* is proportional to the velocity head of the unobstructed flow,

$$y_1^* = C_{RE} \frac{V_n^2}{2g} \quad \ldots\ldots\ldots \tag{10}$$

where C_{RE} is Rehbock's pier coefficient.

To compute C_{RE} for class 1 flow, he proposed the formula

$$C_{RE} = \{\delta_o - j(\delta_o - 1)\}\{0.4j + j^2 + 9j^4\}\{1 + \mathbb{F}^2\} \ldots\ldots\ldots \tag{11}$$

where δ_o is a pier shape factor.

D. L. Yarnell (23), in his research on flow past bridge piers, ignored the effect of bridge abutments and sought to verify the different backwater formulae developed to that time, such as those of D'Aubuisson, Weisbach, Nagler and Rehbock. He dismissed as theoretically unsound the Weisbach formula, which assumes the total flow through a constricted section is the sum of an orifice discharge and a weir discharge. Among other things, he concluded that as long as velocities are slow enough to keep within Rehbock's class 1 flow, any of the three other formulae will give results close enough for practical purposes, provided the proper coefficient is used. Yarnell also proposed a two-class flow classification system instead of Rehbock's system. The system used by Rehbock is strictly empirical, whereas Yarnell's system has physical meaning. In his classification, "Iowa Class B" is for the situation where critical depth occurs in the constriction, and "Iowa Class A" where flow throughout the constriction is subcritical. The backwater that occurs in conjunction with critical depth conditions is referred to as "contraction backwater," whereas the backwater that occurs when subcritical flow exists in the constriction is called "resistance backwater." Contraction backwater is not affected by downstream conditions. Resistance backwater is primarily a function of the energy losses occurring in the flow expansion downstream from the constriction.

The flow characteristics at bridge constrictions have been described by Kindsvater and Carter (10); Liu, Bradley and Plate (13); and by the Task Committee on Hydraulics of Bridges of ASCE (1). The following description has been extracted primarily from these sources, as summarized by Skogerboe, Austin and Chang (17).

The flow through a constriction such as a highway bridge crossing is usually of subcritical regime, and produces gradually varied channel flow far upstream and downstrean, with rapidly varying flow occurring at the constriction. The effect of the constriction on the water surface profile, both upstream and downstream, is conveniently measured with respect to the normal water surface profile, which is the water surface in the absence of the constriction under uniform flow conditions. Upstream from the constriction, an M1 backwater profile occurs, where the velocities, and consequently the rate of loss of flow energy, are less than for normal flow conditions. The backwater may extend for a considerable distance upstream, to a point where the constricted and the normal surface profiles practically coincide as shown at Section 0 in Fig. 1.

Near the constriction, the central body of water begins to be accelerated at Section I, while deceleration occurs along the outer boundaries. A separation zone (zone Ia) is formed in the corners upstream from the constriction. As the flow is accelerated at the constriction, the water surface profiles drops rapidly between sections II and III, with the jet stream contracting to a width somewhat less than the width of the opening. The spaces between the jet and the constriction boundaries (zone IIIa) are occupied by eddying water. Immediately downstream from the constriction, the jet stream begins to expand until uniform flow has been reestablished across the entire channel width at section IV, where the normal and constricted water surface profiles again coincide. Shear along the separation boundary in the reach between sections III and IV results in deceleration of the live stream, with average velocities and energy losses greater than for uniform flow due to the additional turbulent mixing resulting from the expansion process. Between sections 0 and IV, the total energy loss is the same as that for uniform flow.

The effect of the constriction is to cause a redistribution of the energy in the flow system over the reach between sections 0 and IV. At the constriction, the available energy is greater than the frictional resistance under uniform flow conditions by an amount required for the increased losses in the downstream reach. The increase in energy is a result of lower boundary drag loss (compared to uniform flow) upstream of the constriction. In the downstream reach, the increased energy losses, compared to frictional resistance for uniform flow, are due primarily to the increased turbulent mixing caused by diffusion of the jet as it expands from section III to section IV. These energy losses are a function of discharge, contraction ratio and constriction geometry. Therefore, these losses may be decreased by a decrease in discharge, a smaller contraction ratio, or by streamlining the abutment and constriction geometry to more nearly allow the jet to occupy the full width of the constricted opening. In general, the same statement is applicable to the backwater caused by the constriction.

Kindsvater and Carter (10) conducted a program of fundamental research on open channel constrictions at Georgia Institute of Technology using a horizontal steel flume 18 inches deep, 10 feet wide and with a usable uniform flow length of 21 feet. A combination of an energy equation and the continuity equation resulted in the discharge formula

$$Q = C_K b y_3 \sqrt{2g\{(y_1 - y_3) - E_f + \alpha_1 \frac{V_1^2}{2g}\}} \quad \ldots\ldots\ldots \tag{12}$$

where

- Q is the discharge in cfs;
- C_K is Kindsvater's discharge coefficient;
- b is the width of the contracted opening;
- y_1 is the flow depth at section I;
- y_3 is the flow depth at section III;
- g is gravitational acceleration
- V_1 is the average velocity at section I
- α_1 is a coefficient which corrects for velocity variation at section I; and
- E_f is the energy loss in feet due to friction between sections I and III.

Dimensional analysis shows the discharge coefficient to be a function of the following variables:

$$C_K = f(\mathbb{F}, m, y_3/b, L/b, e, \phi, \text{abutment type}) \quad \ldots\ldots\ldots \tag{13}$$

in which

$$\mathbb{F} = \frac{Q}{b y_3 \sqrt{g y_3}} \quad \ldots\ldots \tag{14}$$

which is a Froude number.

$m = 1 - b/B$ which is called the contraction ratio;

L is the length equivalent to the contracted opening in the flow direction;

e is the eccentricity of opening;

ϕ is the skew angle the axis of the abutments make to the direction of flow.

For an irregular, natural channel, the contraction ratio can be evaluated from

$$m = 1 - \frac{K_b}{K_B} \quad \ldots\ldots\ldots \qquad (15)$$

in which K_b is the conveyance of that part of the approach channel which occupies an area of width b, and K_B is the conveyance of the total section. Conveyance is defined in terms of the Manning's equation as

$$K = \frac{1.486}{n} AR^{2/3} \quad \ldots\ldots\ldots \qquad (16)$$

in which

A is the area;

R is the hydraulic radius; and

n is Manning's roughness factor.

As the ratio y_3/b was shown experimentally to be insignificant, Kindsvater and Carter ignored it and defined a standard condition such that

$\mathbb{F} = 0.5$

$e = 1$

$\phi = 0^{\circ}$

for a square-edged, vertical-faced abutment type. From the experimental data for this standard condition, a family of base curves showing the interrelationship between C_K, m and L/b was constructed (Fig. 2). The discharge coefficient for the standard condition is designated as C_K', and then adjusted for the variation of F , e , ϕ and abutment type from standard. The adjusted value of discharge coefficient is then substituted into Eq. 12 to compute the discharge. An example of the graphs used to adjust C_K' is given in Figs. 2 and 3 for a simple vertical board (Type I) constriction.

To apply this method for computing discharge, the stages of flow in the vicinity of the constriction must be obtained from field measurements in addition to such information as contraction ratio and abutment geometry. The process of computing discharge is the reverse to that for computing maximum backwater. In the latter case, the stages of flow in the vicinity of the constriction are unknown, but the flow rate is the design discharge for a certain flood frequency, and hence given. In Eq. 12, if Q and b are known and if C_K can be estimated, the remainder of the terms which represent the flow stages can be expressed as a function of the discharge and the discharge coefficient. Thus, a laboratory investigation intended to determine the discharge characteristics for an open channel constriction can be adopted to determine the maximum backwater as well, or vice versa (13).

By extending the investigation of Kindsvater and Carter (10), Tracy and Carter (20) developed a method for computing the maximum backwater. The maximum backwater, y_1^* measured a distance b upstream from the contracted inlet, can be divided by Δy, which is

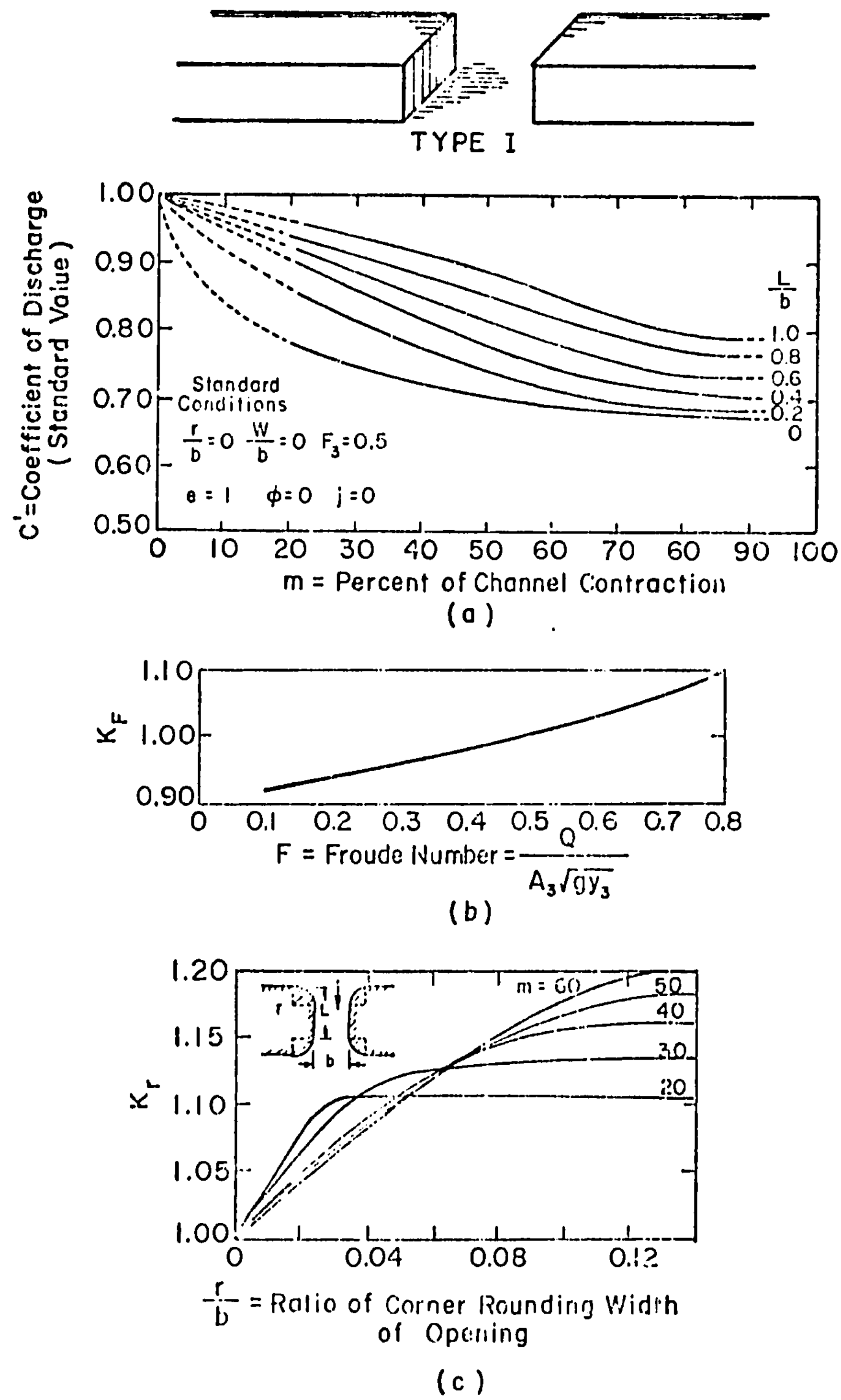

Figure 2. Discharge coefficient for constriction of Type I opening, vertical embankments and vertical abutments. (Taken from Kindsvater, Carter and Tracy, 1953).

(a) Base curve for coefficient of discharge

(b) Variation of discharge coefficient with Froude number

(c) Variation of discharge coefficient with entrance rounding

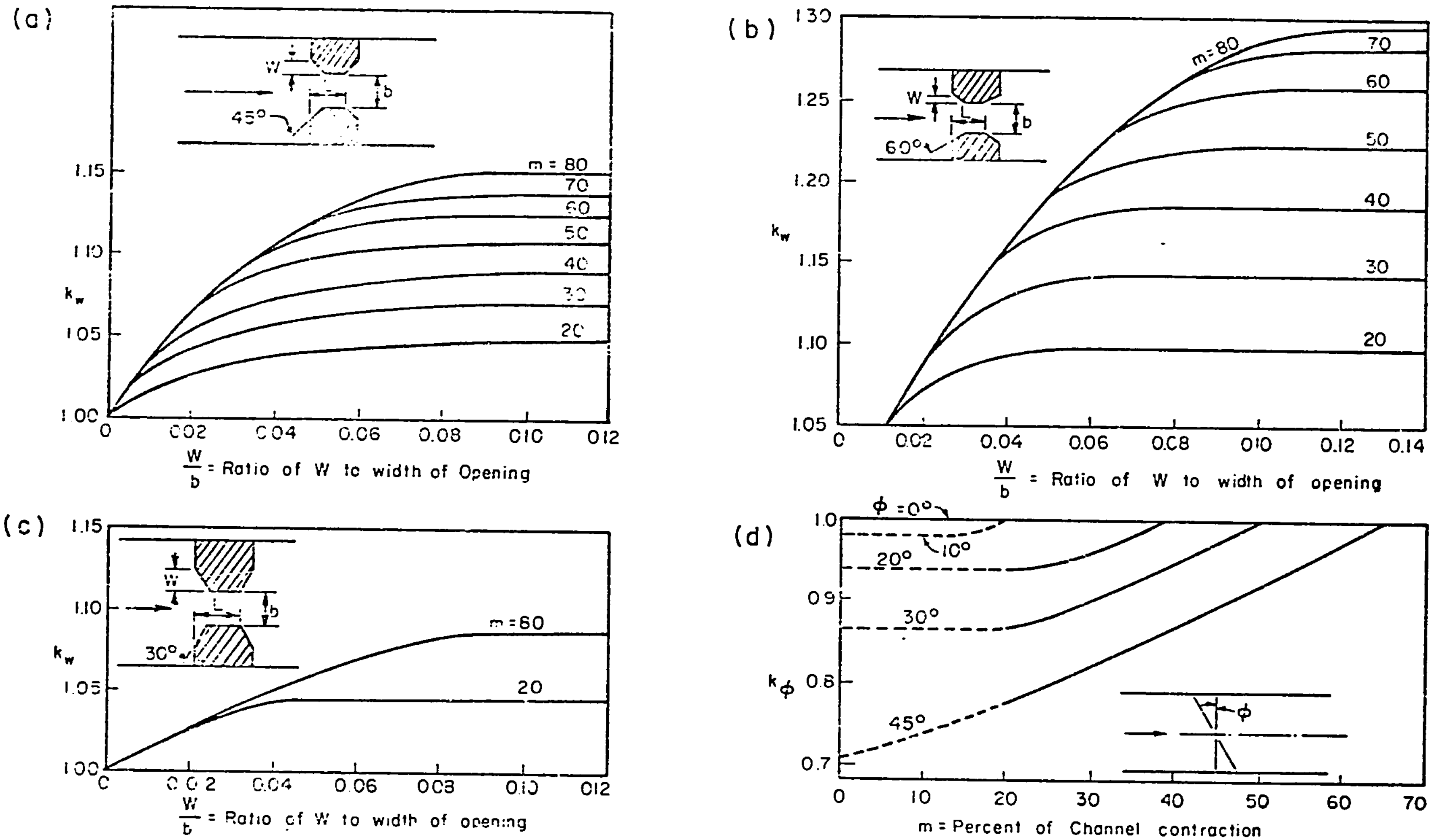

Figure 3. Adjustments to standard value of discharge coefficients. (Taken from Kindsvater, Carter and Tracy, 1953).

the difference in water surface elevation between section I and section III for the constricted channel, as shown in Fig. 1. The dimensionless ratio $y_1^*/\Delta y$ has been shown by Tracy and Carter's laboratory data to be a function primarily of the percentage of channel contraction. The influences of bed roughness and constriction geometry are secondary. Variable characteristics of the flow, such as the Froude number, flow depth and constriction length, are largely unimportant in their effect on this ratio. The variation of the backwater ratio $(y_1^*/\Delta y)_{base}$ with the contraction ratio, m, and the Manning's roughness factor, n, is shown in Fig. 4 in which $(y_1^*/\Delta y)_{base}$ is the ratio $y_1^*/\Delta y$ for a channel having a vertical faced constriction with square-edged abutments (simple vertical board model).

Letting

$$K_c = \frac{y_1^*/\Delta y}{(y_1^*/\Delta y)_{base}} \quad \ldots\ldots\ldots \tag{17}$$

in which $y_1^*/\Delta y$ is for any type of abutments, it was found that K_c varies with the contraction ratio as well as with the ratio of the existing discharge coefficient C_K to the discharge coefficient C_K' for the base condition, as shown in Fig. 4. The discharge coefficient C_K is Kindsvater's discharge coefficient discussed previously.

Tracy and Carter claimed that the quantity Δy can be computed from

$$\Delta y = \frac{V_3^2}{2gC_K^2} - \alpha_1 \frac{V_1^2}{2g} + E_f \quad \ldots\ldots\ldots \tag{18}$$

where $V_3 = \frac{Q}{by_3}$. In application, $y_1^*/\Delta y$ is selected from Fig. 4.

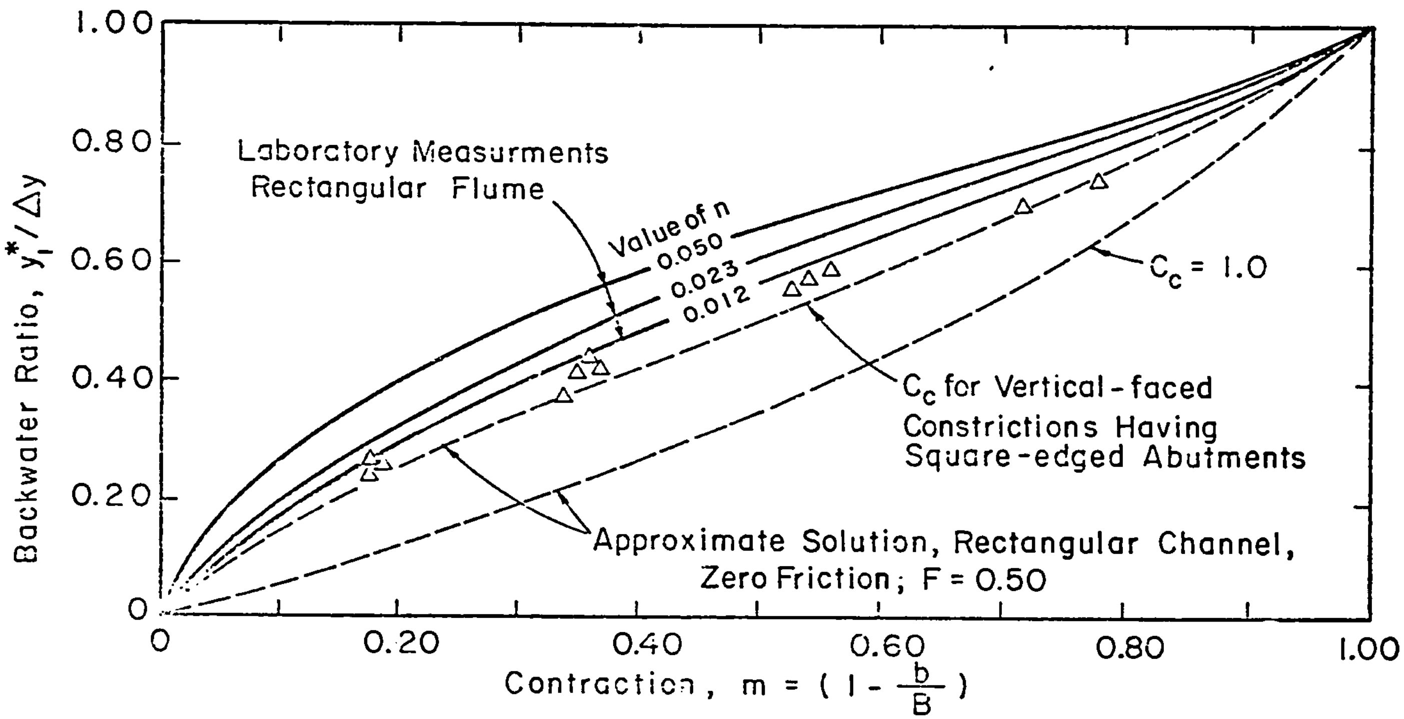

Figure 4. Variation of backwater ratio with contraction ratio and Manning's roughness coefficient. (Taken from Tracy and Carter, 1955).

The ratio $y_1^*/\Delta y$ is then adjusted for a constriction-geometry effect by the factor K_c obtained from Fig. 5. The adjusted ratio may be multiplied by Δy to yield the value of y_1^* .

The use of a horizontal flume by Tracy and Carter (20) presents some difficulty due to the inability to obtain uniform flow. Hence, there is difficulty in obtaining standards for conditions of unobstructed flow, which, according to Liu, Bradley and Plate, are generally essential for both theoretical and laboratory investigations.

Thus, Liu, Bradley and Plate, at Colorado State University, undertook hydraulic studies of model bridge constrictions in tilting flumes having widths of 4 feet and 7.9 feet. In most cases, the model was placed in the flume after uniform flow had been established, with a limited number of studies conducted for the abnormal stage-discharge condition. In addition to studying various geometries of bridge models, the roughness of the flume bed was varied in order to establish the effects of roughness upon backwater.

A combination of the continuity and energy equations was used to arrive at the general equation for the maximum backwater.

$$\left[\frac{y_1}{y_n}\right]^3 = \frac{3}{2}\,\mathbb{F}_n^2\left[\frac{9\phi}{4M^2}\right] -1 \quad \ldots\ldots\ldots \tag{19}$$

where $\mathbb{F}_n$ is the froude number at normal flow depth.

The coefficient, ϕ , corrects for:

1. nonuniform velocity distribution at sections I and II, as well as nonhydrostatic pressure distribution at Section II;
2. the deviation of the actual flow conditions from critical depth (free flow) conditions at the contraction inlet;

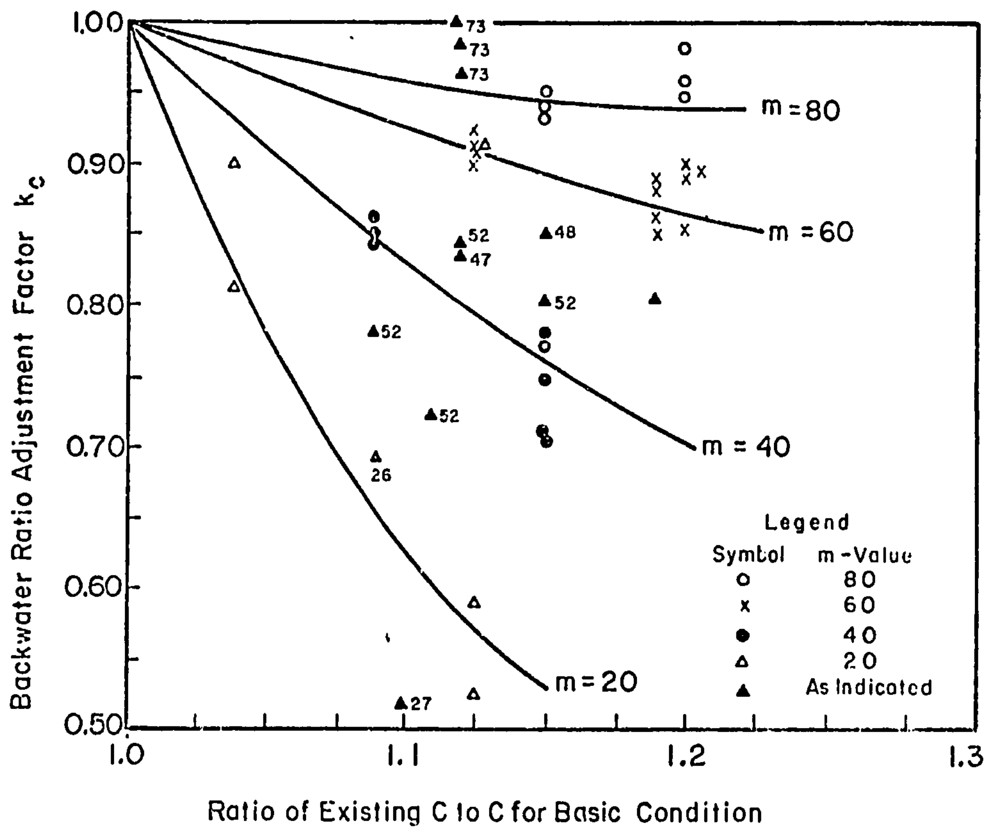

Figure 5. Variation of backwater ratio adjustment factor with discharge coefficient ratio. (Taken from Tracy and Carter, 1955).

3. certain approximations due to neglecting terms of higher order in the derivation of Eq. 19, which is only important when $M > 0.8$.

The variation of ϕ with the uniform flow Froude number $\mathbb{F}_n$, and the opening ratio, M , is shown in Fig. 6 for the vertical board (VB) model studied by Liu, Bradley and Plate (13). The coefficient, ϕ , approaches unity for all values of M as $\mathbb{F}_n$ approaches unity, whereas ϕ approaches infinity for all values of M as $\mathbb{F}_n$ approaches zero. From a plot of actual data for the vertical board model (Fig. 7), together with dimensional analysis of the backwater phenomena, an empirical backwater equation was developed.

$$\left[\frac{y_1}{y_n}\right]^3 = 4.48\mathbb{F}_n^2\left[\frac{1}{M^2} - \frac{2}{3}(2.5-M)\right] + 1 \quad \ldots\ldots\ldots \qquad (20)$$

By combining Eqs. 19 and 20, the relationship for ϕ can be obtained.

$$\phi = 1.33\left[1 - \frac{2}{3}M^2(2-M) - \frac{1}{3\mathbb{F}_n^2}\right] \quad \ldots\ldots\ldots \qquad (21)$$

Biery and Delleur (2) investigated the backwater caused by single span arch bridge constrictions. Arch bridges are unique in that the width of free water surface contracts as depth in the constriction increases. They compared the results of their hydraulic tests with the data collected at Colorado State University for the vertical board model. A comparison of backwater data for various bridge geometries is shown in Fig. 8. A generalized empirical equation for the backwater ratio can be written as

$$\frac{y_1}{y_n} = 1 + 0.47\left[\left(\frac{\mathbb{F}_n}{M'}\right)^{2/3}\right]^{3.39} \quad \ldots\ldots\ldots \qquad (22)$$

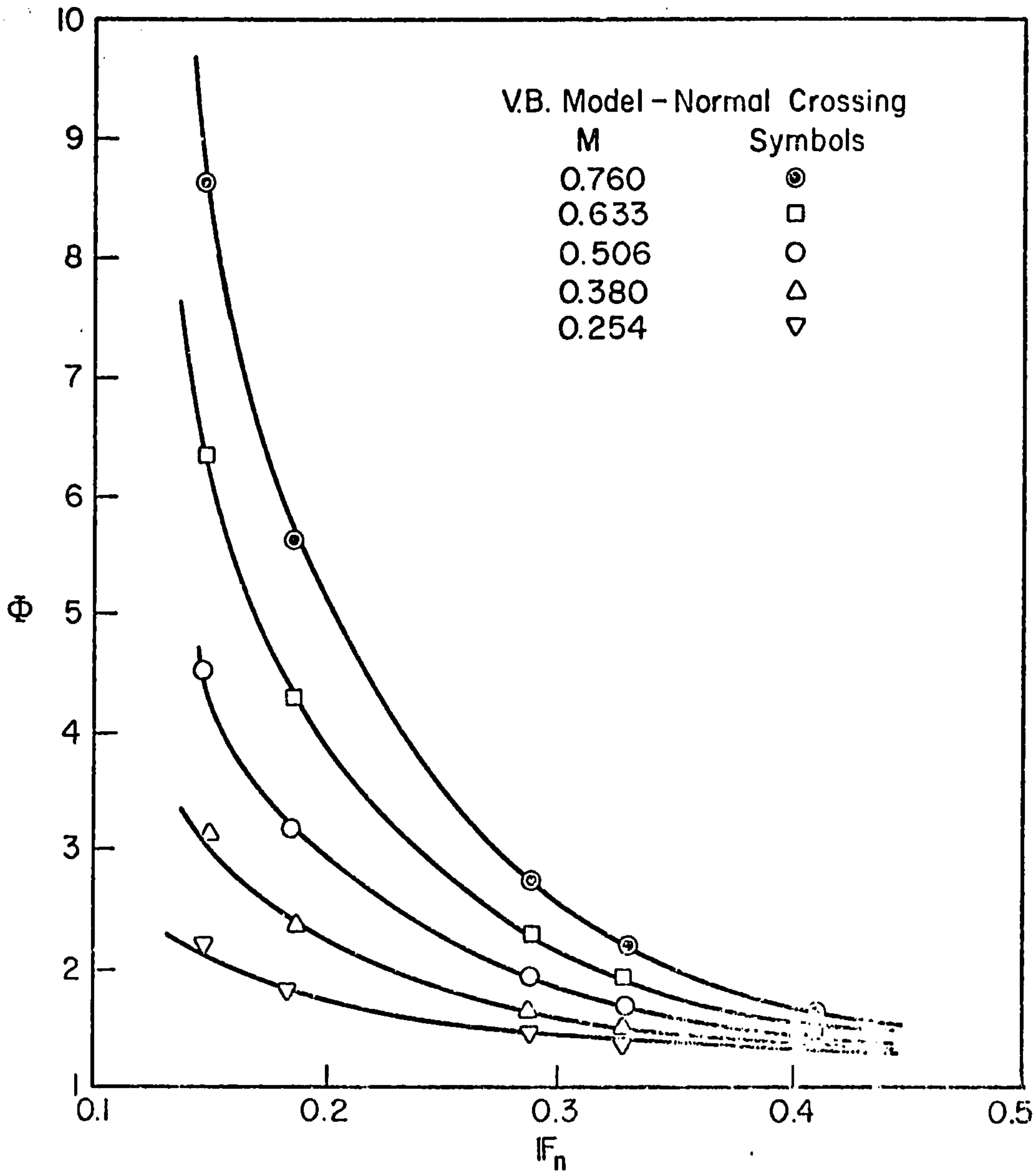

Figure 6. Variation of correction factor ϕ with Froude number $\mathbb{F}_n$ and opening ratio M for vertical board model. (Taken from Liu, Bradley and Plate, 1957).

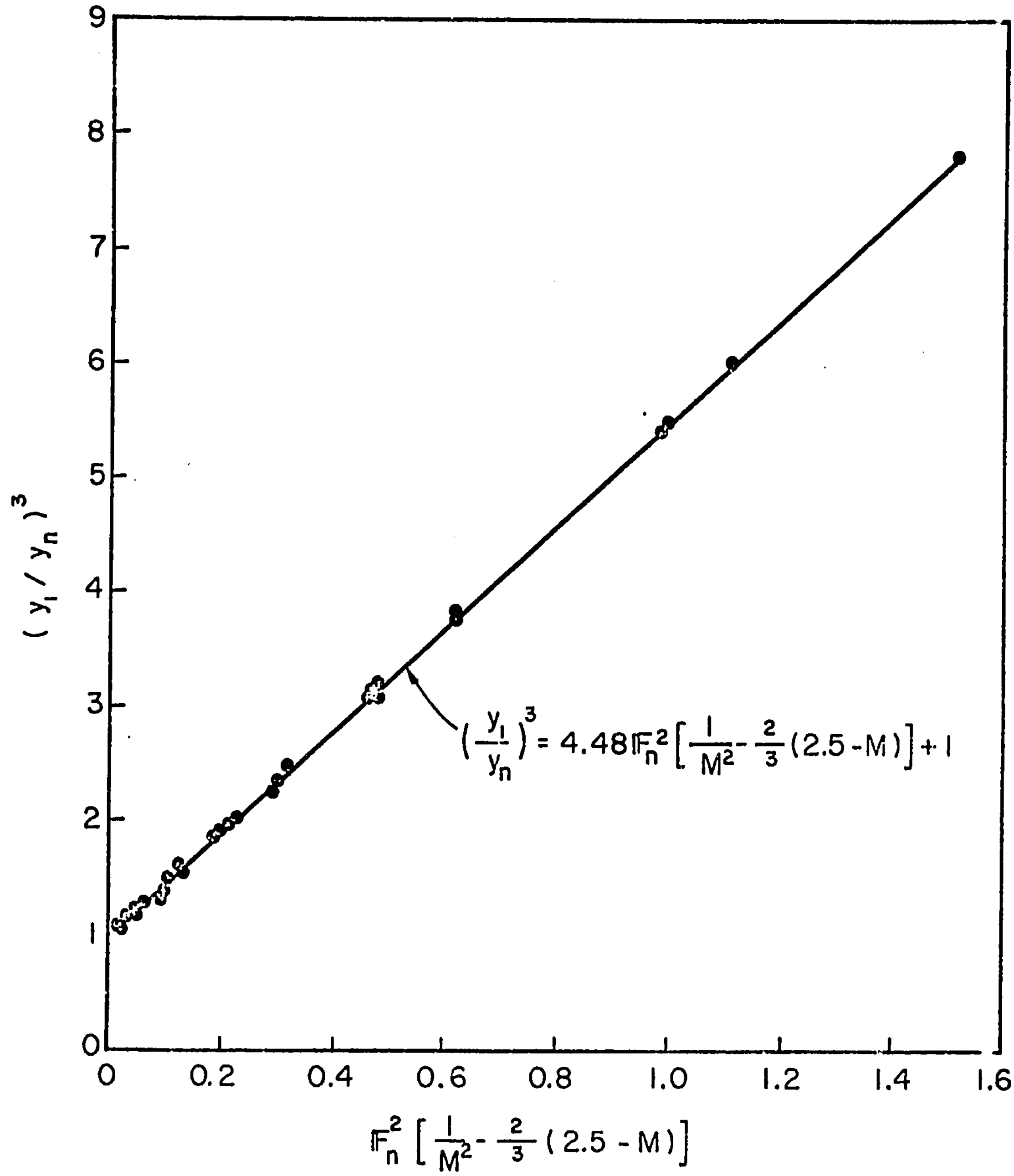

Figure 7. Empirical backwater equation compared to experimental data for vertical board model. (Taken from Liu, Bradley and Plate, 1957).

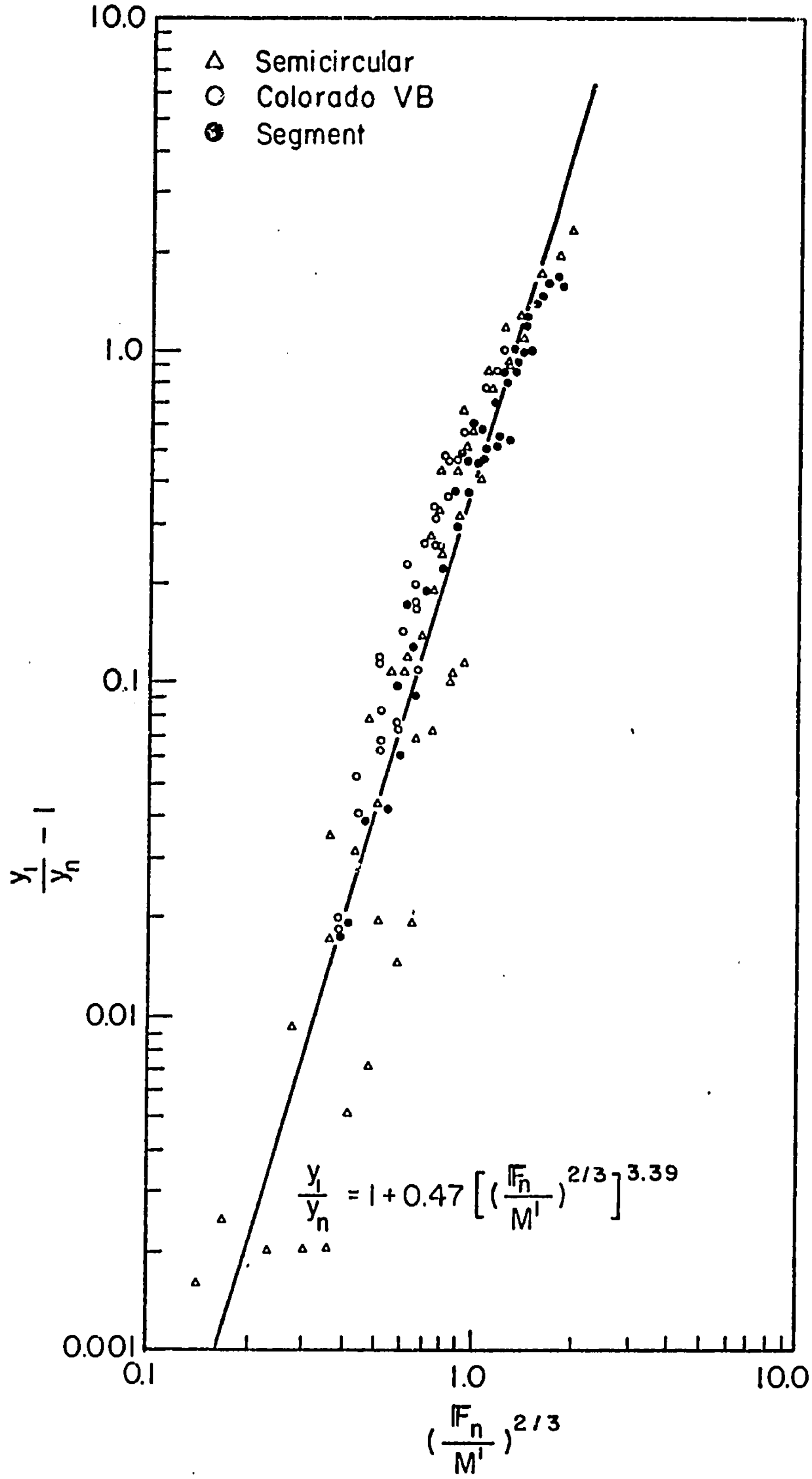

Figure 8. Generalized backwater ratio. (Taken from Biery and Delleur, 1962).

where M' is the channel opening ratio, which is b/B for rectangular constrictions, but is a function of flow depth for arch bridges (17).

Izzard has suggested that Eq. 21 could be approximated by

$$\frac{y_1}{y_n} = 1 + 0.45\left(\frac{\mathbb{F}_n}{M'}\right)^2 \quad \ldots\ldots. \qquad (23)$$

and still fit the data closely. This equation can also be written as

$$y_1^* = \frac{0.45}{(M')^2} \frac{V_n^2}{g} = K \frac{V_n^2}{2g} \quad \ldots\ldots. \qquad (24)$$

which is of the same form as Bradley's equation, Eq. 25a (2).

Liu, Bradley and Plate (13) obtained a practical expression for backwater by applying the principle of conservation of energy between the point of maximum backwater upstream from the bridge, section I, and a point downstream from the bridge at which normal flow has been reestablished, section IV. The expression, incorporated into the design manual, "Hydraulics of Bridge Waterways" by Bradley (4) is

$$y_1^* = K^* \frac{V_{n2}^2}{2g} + \alpha_1 \left[\left(\frac{A_{n2}}{A_4}\right)^2 - \left(\frac{A_{n2}}{A_1}\right)^2\right] \frac{V_{n2}^2}{2g} \quad \ldots\ldots. \qquad (25)$$

where

K^* is the total backwater coefficient;

A_{n2} is the cross-sectional flow area in the constriction at normal stage; and

$V_{n2} = Q/A_{n2}$.

To compute backwater by Eq. 25, it is necessary to obtain the approximate value of y_1^* by using the first part of the expression:

$$y_1^* = K^* \frac{V_{n2}^2}{2g} \quad \ldots\ldots. \qquad (25a)$$

The value of A_1 in the second part of the expression, which depends on y_1^*, and which represents the difference in kinetic energy between sections IV and I, can then be determined:

$$\alpha_1 \left[\left(\frac{A_{n2}}{A_4}\right)^2 - \left(\frac{A_{n2}}{A_1}\right)^2 \right] \frac{V_{n2}^2}{2g} \quad \ldots\ldots\ldots \qquad (25b)$$

where

$\alpha_1 = \dfrac{\Sigma(qv^2)}{QV_1^2}$ = coefficient applied to velocity head at section I to account for non-uniform velocity distribution;

where (26)

q is the discharge in a subsection; and

v is the average velocity in that subsection.

A second approximation for y_1^* can then be computed from Eq. 25 and the procedure repeated until the backwater is evaluated.

The total backwater coefficient is the sum of a base coefficient, K_b, which is obtained from Fig. 9 for wingwall abutments; an incremental backwater coefficient for piers ΔK_p, obtained from Fig. 10; an incremental backwater coefficient for eccentricity, ΔK_e, obtained from Fig. 11; and an incremental backwater coefficient for skew, ΔK_s, which is taken from Fig. 12 for wingwall abutments. The expression for K^* becomes

$$K^* = K_b + \Delta K_p + \Delta K_e + \Delta K_s \quad \ldots\ldots\ldots \qquad (27)$$

A design procedure for determining the backwater at bridge constrictions when abnormal stage-discharge conditions exist in the main channel has been developed by Liu, Bradley and Plate (13) and incorporated in the design manual compiled by Bradley (4). A definition sketch of the abnormal stage-discharge condition is shown in Fig. 13.

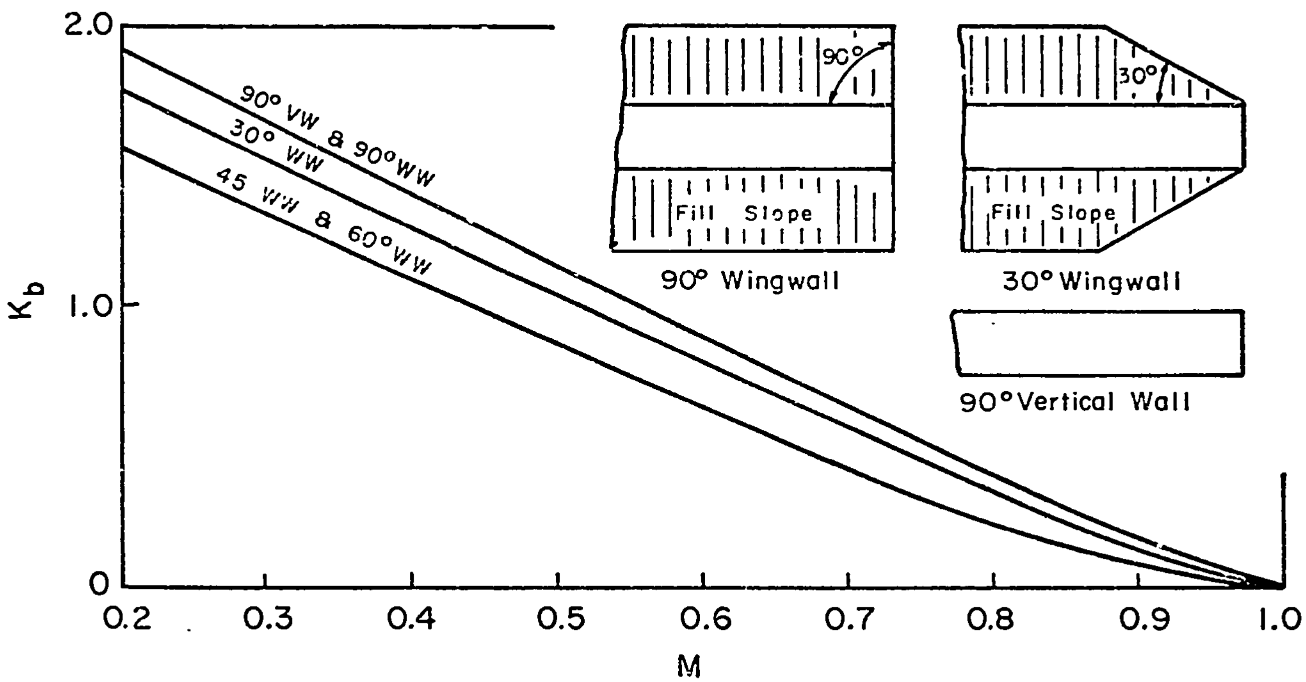

Figure 9. Base backwater coefficient curves for wingwall abutments. (Taken from Bradley, 1960).

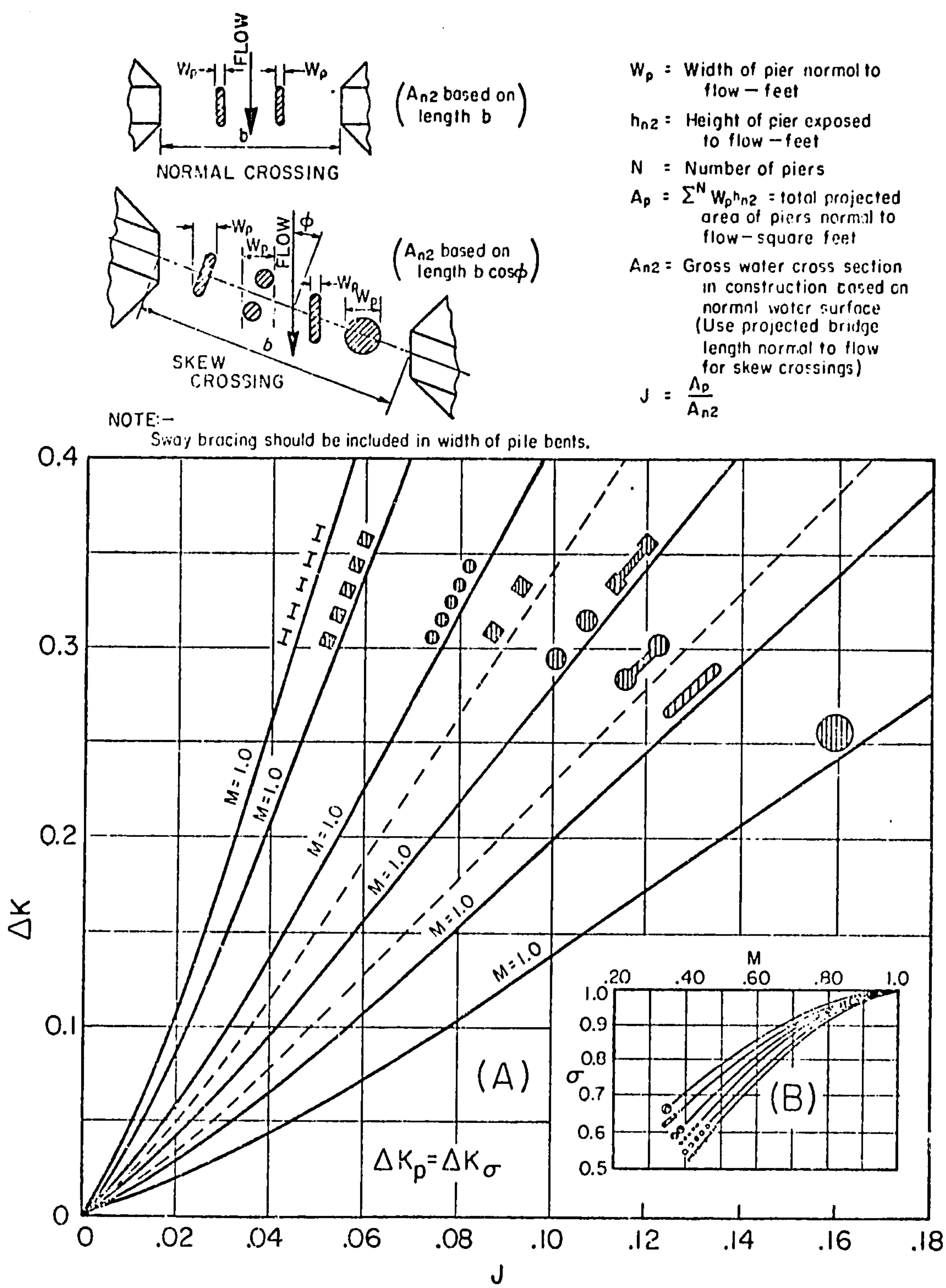

Figure 10. Incremental backwater coefficient for piers. (Taken from Bradley, 1960).

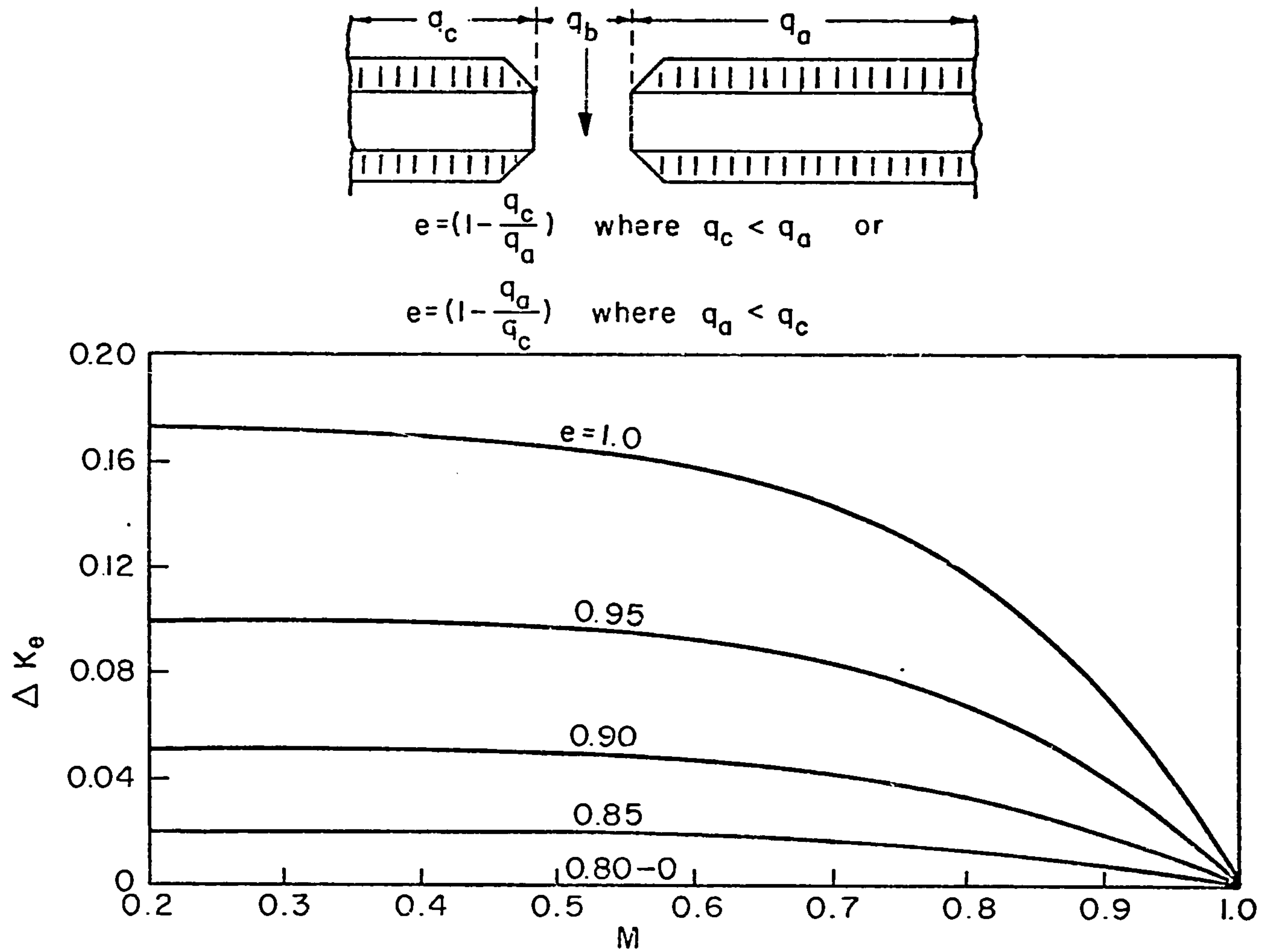

Figure 11. Incremental backwater coefficient for eccentricity. (Taken from Bradley, 1960).

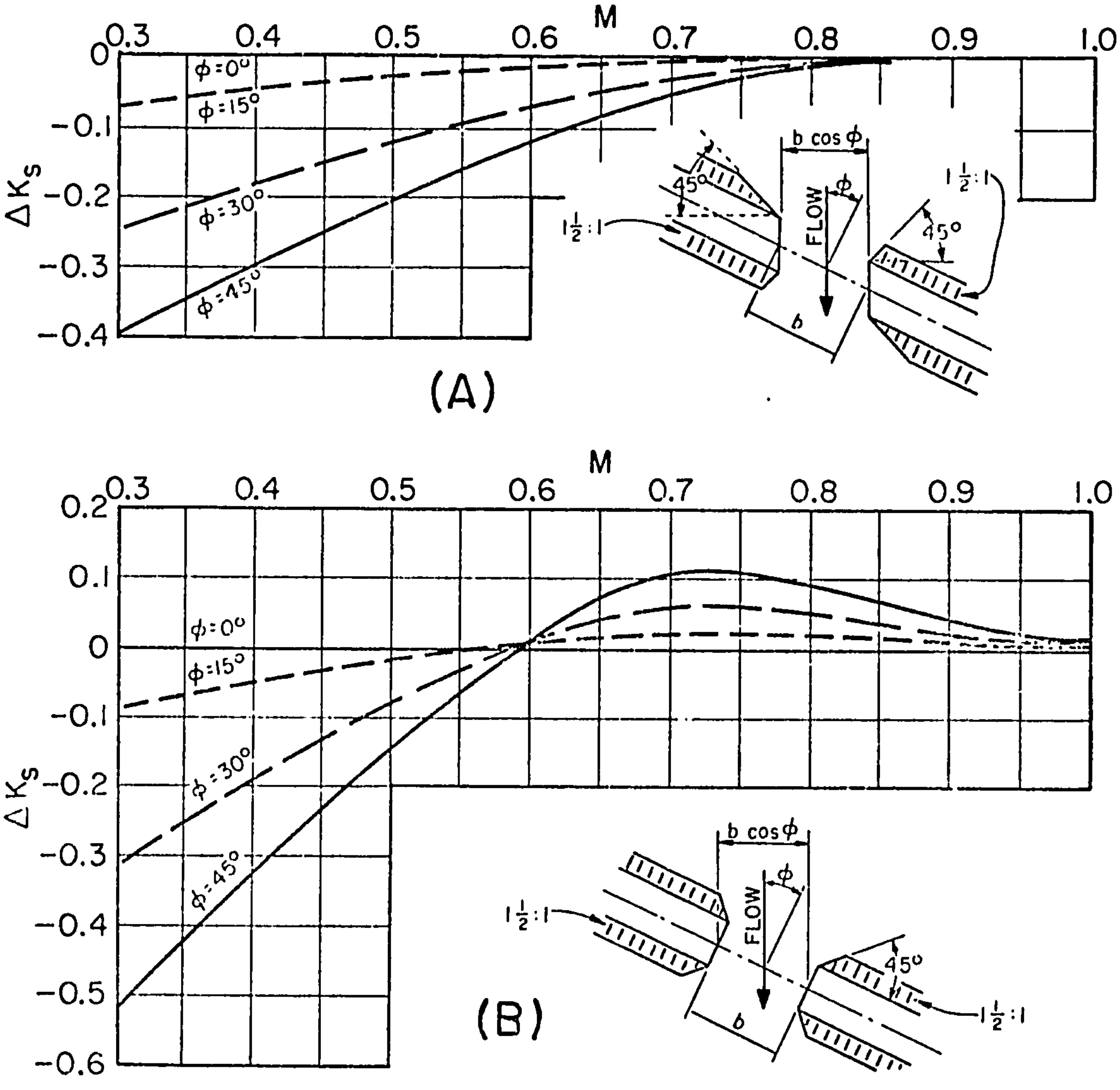

Figure 12. Incremental backwater coefficient for skew, wingwall abutments. (Taken from Bradley, 1960).

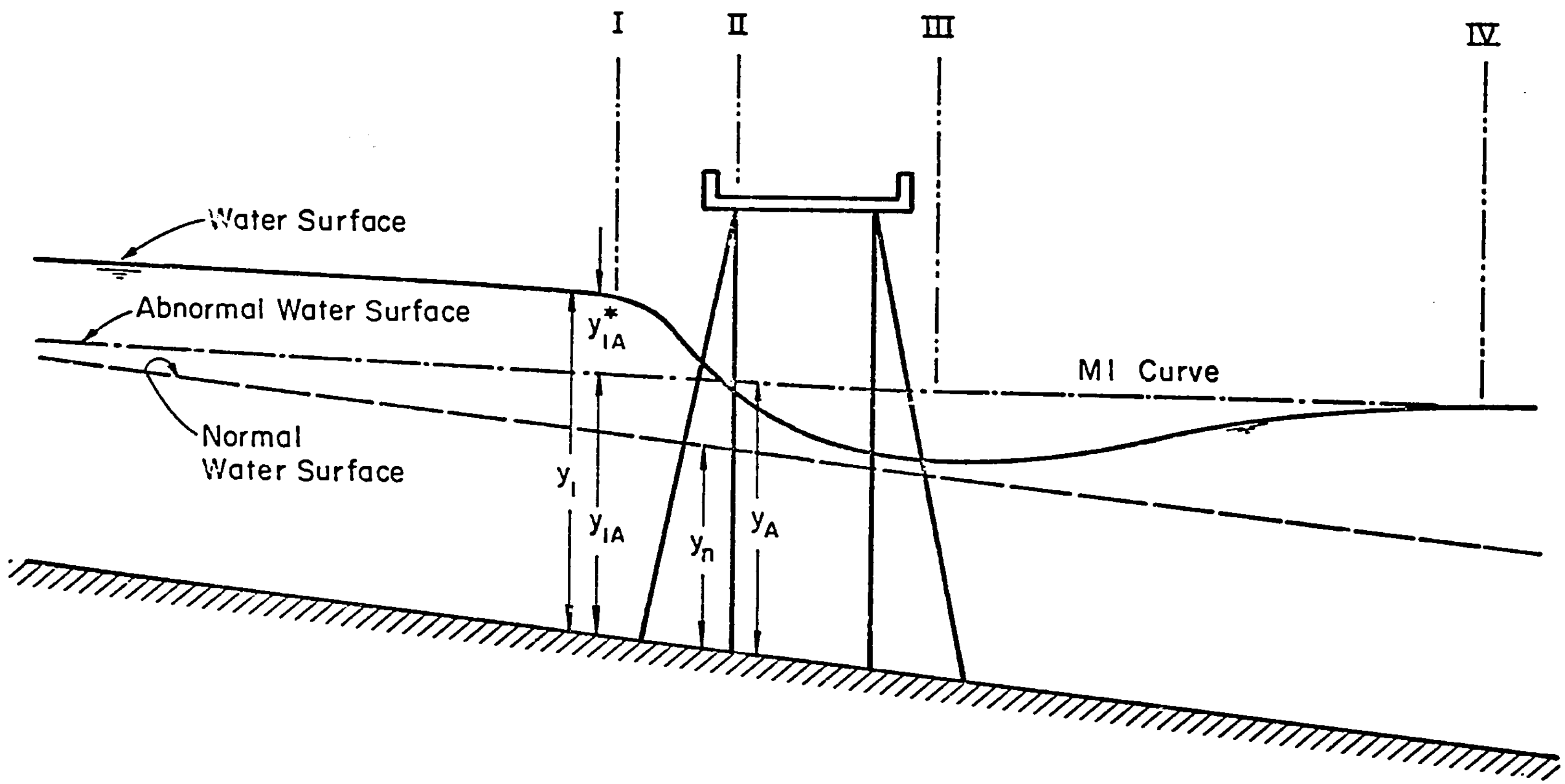

Figure 13. Definition sketch of abnormal stage-discharge condition.

This condition is caused by some form of downstream control, resulting in non-uniform flow at the bridge site. The abnormal stage used in the design procedure is the depth of flow, y_A, that would occur in the river channel at section II prior to construction of the bridge. The subscript A has been used to signify the abnormal condition.

The maximum backwater at section I under abnormal stage is computed from

$$y^*_{1A} = K^* \frac{V^2_{2A}}{2g} \quad \ldots\ldots\ldots \qquad (28)$$

where

$V_{2A} = \frac{Q}{A_{2A}}$ and A_{2A} is the cross-sectional flow area in the constriction for abnormal stage. To determine the total backwater coefficient, K^*, Eq. 27 is used in conjunction with Figs. 9, 10, 11 and 12. Because the solution for backwater under abnormal conditions is only a rough approximation, the terms involving the difference in kinetic energy between sections I and IV used in Eq. 25 have been omitted from Eq. 28.

Submerged Flow Analysis

By considering the change in momentum between an upstream section (section 1) and the section of minimum flow depth (section 2) in a flat-bottomed rectangular flow measuring flume, Skogerboe, Hyatt and Eggleston (19) derived a theoretical submerged flow discharge equation:

$$Q = \frac{(g/2)^{1/2} b(y_1 - y_2)^{3/2}}{\sqrt{\frac{(1-MS)(1-S)^2}{S(1+S)}}} \quad \ldots\ldots\ldots \qquad (29)$$

where

S is the submergence, defined as y_2/y_1 ;

b is the throat width;

y_1 is the depth at section I; and

y_2 is the depth at section II.

The free flow equation has been found by many previous researchers to have the form

$$Q = Cy_u^{n_1} \quad \ldots\ldots. \tag{30}$$

where

y_u is a flow depth upstream from the constriction;

C is the free flow coefficient; and

n_1 is an exponent dependent upon the constriction geometry.

The distinction between free flow and submerged flow is the occurrence of critical depth, usually in the constricted section. When free flow conditions exist, the flow is subcritical upstream from the constriction (depth of flow greater than critical depth), whereas in the constriction the flow is supercritical (depth of flow less than critical depth). With supercritical flow occurring in the constriction, a change in flow depth downstream from the constriction will not change the depth of flow upstream from the constriction.

Submerged (subcritical) flow conditions exist when the downstream, or tailwater, depth is raised to such a level that the flow depths of every point through the constriction become greater than critical depth. Under submerged flow conditions, a change in the tailwater depth also affects the upstream depth (17).

Having determined the form of the submerged flow discharge equation, (Eq. 29), Skogerboe, Hyatt and Eggleston (19) used dimensional analysis combined with a graphical and analytical approach to determine the approximate discharge equation for a flow measuring flume:

$$Q = \frac{C_1(y_u-y_d)^{n_1}}{\{-(\log S+C_2)\}^{n_2}} \quad \ldots\ldots \tag{31}$$

where

y_d is a flow depth downstream from the constriction;

S is the submergence $(= y_d/y_u)$;

C_1 and C_2 are coefficients; and

n_2 is the submergence exponent.

Usually C_2 is very small and can be taken as zero. The exponent n_2 varies between 1 and 3/2 for rectangular constrictions (n_2 approaches 1 for fully constricted channels and n_2 approaches 3/2 for channels having no constriction). The submerged flow equation can be plotted as a family of straight lines on logarithmic paper, with Q as the ordinate, y_u-y_d as the abscissa, and each straight line representing a particular value of submergence, S . A typical submerged flow plot is shown in Fig. 14.

The submerged flow equation (Eq. 31) has been found to be general for any form of width constriction in an open channel. Skogerboe, Austin and Chang (17) applied the subcritical flow analysis to evaluating the backwater due to bridge constrictions under abnormal stage-discharge conditions. Due to the loss in elevation between upstream and downstream sections in a sloping channel, it is useful to replace depths by energies in Eq. 31. If C_2 is set equal to zero, the submerged flow discharge equation becomes

$$Q = \frac{C_1(E_u-E_d)^{n_1}}{(-\log E_R)^{n_2}} \quad \ldots\ldots \tag{32}$$

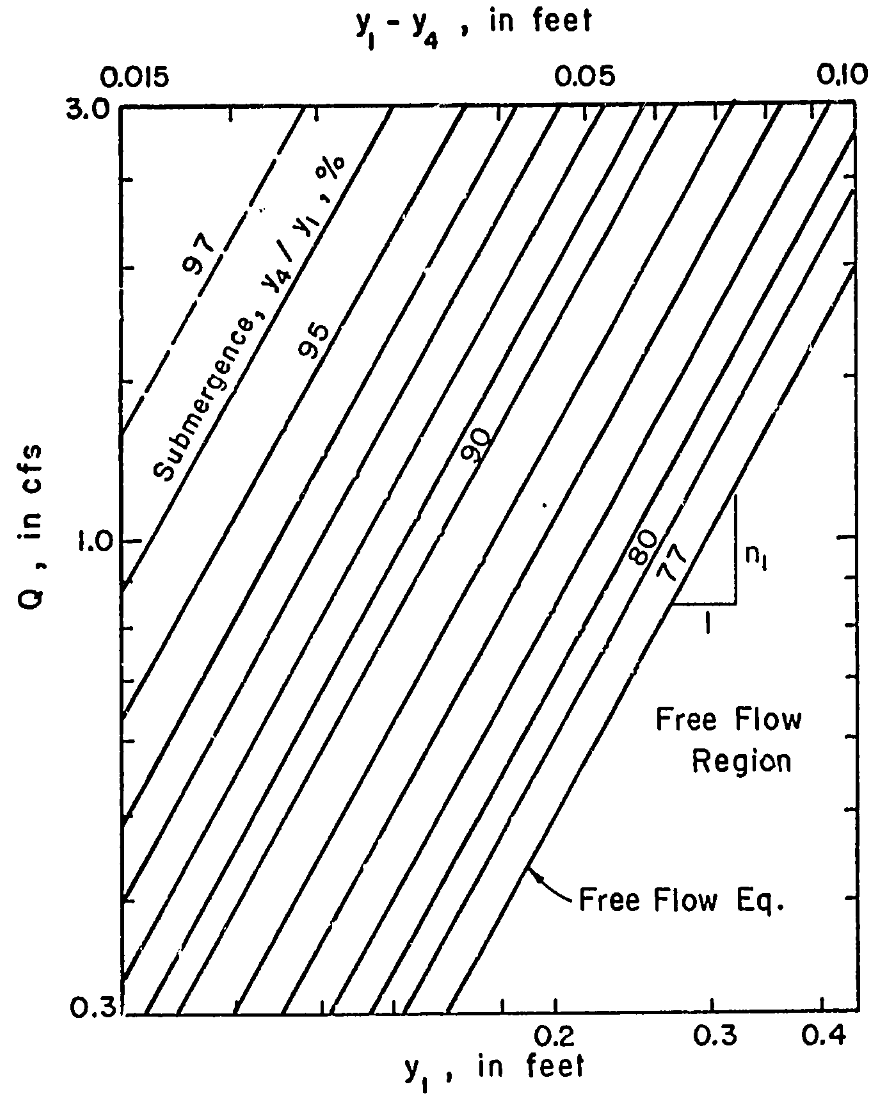

Figure 14. Typical example of submerged flow and free flow rating curves for a constriction. (Taken from Skogerboe and Hyatt, 1967).

where

E_u is the total energy at the upstream section above a datum;

E_d is the total energy at the downstream section with respect to the same datum; and

E_R is the energy ration, E_d/E_u .

The abscissa of a submerged flow plot now becomes E_u-E_d , which is the energy loss, E_L . A typical family of discharge-energy loss curves for a constriction is shown in Fig. 15.

To obtain the submerged flow discharge equation for a given constriction, the data obtained is plotted in the same form as shown in Fig. 15. The lines of constant energy ratio have a slope of n_1 . The discharge intercept at an energy loss of 1.0 for each line of constant energy ratio is then obtained and denoted as $Q_{E_L=1}$. It is then recognized that Eq. 32 reduces to

$$Q_{E_L=1} = \frac{C_1}{(-\log E_R)^{n_2}} \quad \ldots\ldots\ldots \tag{33}$$

By plotting $Q_{E_L=1}$ versus $-\log E_R$ on logarithmic paper, a linear relationship will result where C_1 is the value of $Q_{E_L=1}$ at $-\log E_R=1$, and n_2 is the slope of the straight line. Such a relationship is shown in Fig. 16.

The submerged flow discharge equation can be obtained in terms of depths in the same manner, where n_1 is the slope of the lines of constant submergence, as shown in Fig. 14. The coefficient C_1 and exponent n_2 are obtained by plotting the discharge where head loss equals one for each submergence $(Q_{E_L=1})$ against $-\log S$ on logarithmic paper.

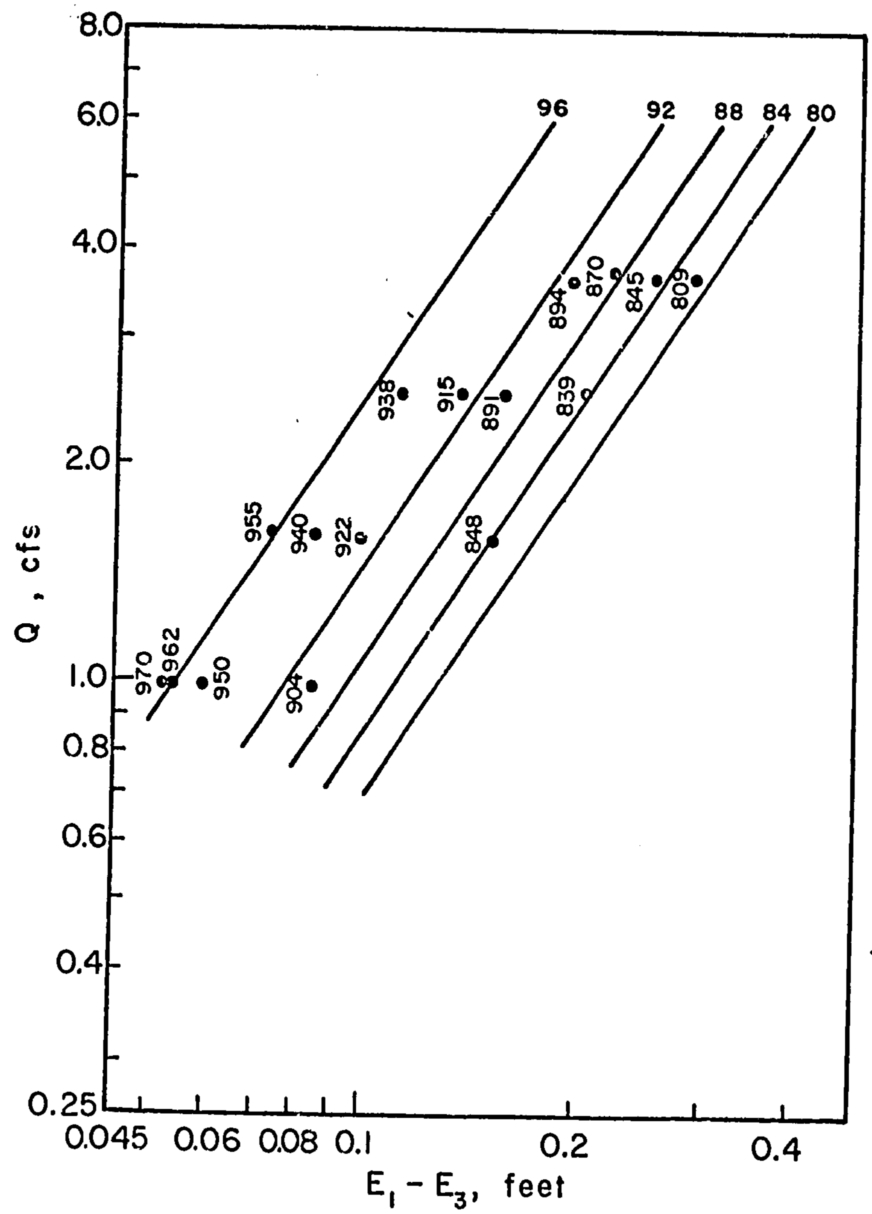

Figure 15. Typical discharge-energy loss curves for a constriction under subcritical flow conditions. (Taken from Skogerboe, Austin and Chang, 1970).

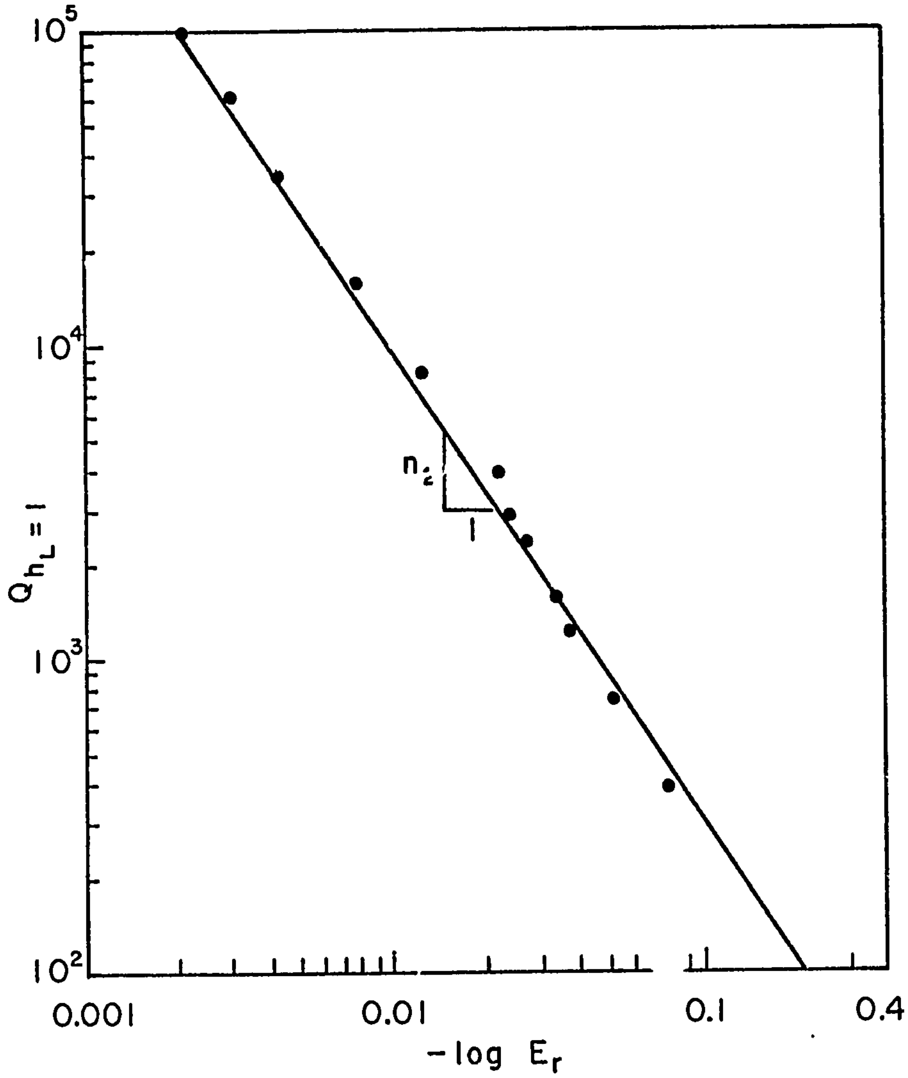

Figure 16. Energy ratio distribution for subcritical flow through a constriction.

Chapter 3

DEVELOPMENT OF SUBMERGED FLOW ANALYSIS

Momentum Theory

A theoretical submerged flow discharge equation may be developed for the vertical board constriction shown in Fig. 1. The momentum equation may be written between sections 1 and 2 for the control volume in Fig. 17 to arrive at a general submerged flow equation for an open channel constriction. In the direction of flow, the momentum equation may be written as

$$F_1 - F_2 - F_c - F_f = Q_t \rho (\beta_2 V_2 - \beta_1 V_1) \quad \ldots\ldots \quad (34)$$

where

F_1 and F_2 are the resultant forces due to the pressure distribution on the cross-section of flow at sections 1 and 2;

F_c is the component of force in the direction of flow acting on the control volume of fluid due to the constriction;

F_f is the friction or drag force acting on the surface of the control volume;

Q_t is the theoretical discharge;

ρ is the density of the fluid;

β_1 and β_2 are momentum coefficients for the two flow sections; and

V_1 and V_2 are the average velocities at sections 1 and 2.

Assuming uniform velocity distribution and neglecting the friction force

$$F_1 - F_2 - F_c = Q_t \rho (V_2 - V_1) \quad \ldots\ldots \quad (35)$$

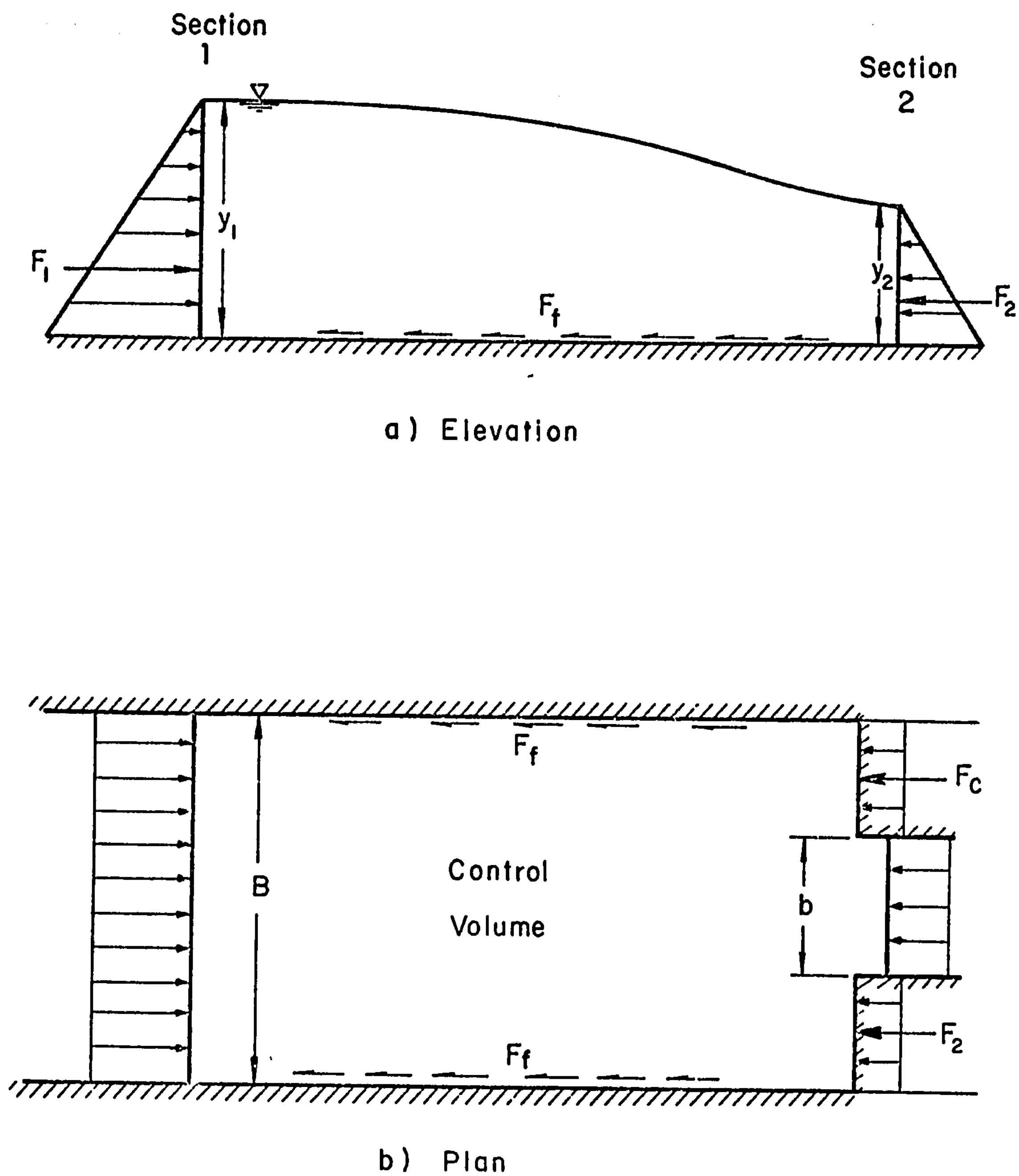

Figure 17. Control volume for a constriction in a rectangular channel.

(In prototype application, the assumption of uniform velocity distribution will not necessarily hold true. Also, in many cases, the friction force may not be neglected. However, these factors will subsequently be seen to be irrelevant).

Assuming hydrostatic pressure distribution

$$F_1 = \gamma B y_1^2/2 \quad \ldots\ldots. \tag{36}$$

$$F_2 = \gamma b y_2^2/2 \quad \ldots\ldots. \tag{37}$$

where

γ is the specific weight of the fluid;

B is the width of the open channel;

b is the width of the constriction; and

y_1 and y_2 are the depths of flow at the two sections.

The force acting on the control volume due to the constriction occurs at the upstream face of the constriction. Assuming the average depth of flow at the upstream face of the constriction is y_2,

$$F_c = \gamma(B-b)y_2^2/2 \quad \ldots\ldots. \tag{38}$$

The momentum equation in the direction of flow can now be written as

$$\frac{\gamma B y_1^2}{2} - \frac{\gamma b y_2^2}{2} - \frac{\gamma(B-b)y_2^2}{2} = \frac{Q_t\gamma(V_2-V_1)}{g} \quad \ldots\ldots. \tag{39}$$

where g is the acceleration due to gravity.

Assuming steady flow, the continuity equation can now be employed.

$$Q_t = By_1V_1 = by_2V_2 \quad \ldots\ldots. \tag{40}$$

Substituting the continuity equation into Eq. 39 and solving for the discharge

$$Q_t = \frac{\sqrt{g/2}\; B(y_1-y_2)^{1/2}}{\sqrt{\frac{(1-by_2/By_1)B}{by_2(y_1+y_2)}}} \quad \ldots\ldots\ldots \tag{41}$$

The opening ratio, b/B , may be represented by M and the submergence, y_2/y_1 by S . The denominator of the discharge equation can be made dimensionless by multiplying the numerator and denominator by y_1-y_2 .

$$Q_t = \frac{\sqrt{g/2}\; B(y_1-y_2)^{3/2}}{\sqrt{\frac{(1-MS)(y_1-y_2)^2 B}{by_2(y_1+y_2)}}} \quad \ldots\ldots\ldots \tag{42}$$

Therefore

$$Q_t = \frac{\sqrt{g/2}\; b(y_1-y_2)^{3/2}}{\sqrt{\frac{M(1-MS)(1-S)^2}{S(1+S)}}} \quad \ldots\ldots\ldots \tag{43}$$

For any particular channel constriction, b and M become constants and the discharge is a function of $(y_1-y_2)^{3/2}$ and S . If the submergence is held constant, the discharge becomes a function of $(y_1-y_2)^{3/2}$ alone. This suggests that a logarithmic plot of Q against y_1-y_2 would yield a family of straight lines with each line representing a constant value of submergence. The lines of constant submergence would each have a slope of $3/2$.

<u>Other Width Constriction Equations</u>

A complete review of the development of equations describing flow through constrictions in open channels has been presented in

Chapter 2. The most significant and recent work has been embodied in the publications:

(i) "Computation of Peak Discharge at Contractions," by C. E. Kindsvater, R. W. Carter and H. J. Tracy (11) in 1953;

(ii) "Backwater Effects of Piers and Abutments," by H. K. Liu, J. N. Bradley and E. J. Plate (13), in 1957; and

(iii) "Hydraulics of Bridge Waterways," by J. N. Bradley (4) in 1960.

The discharge equations presented in each of these publications can be expressed in the form of a submerged flow equation.

Kindsvater, Carter, and Tracy. By combining an energy equation and the continuity equation, Kindsvater and Carter (10) obtained the discharge formula

$$Q = C_K b y_2 \sqrt{2g\{(y_1 - y_2) - E_f + \alpha_1 \frac{V_1^2}{2g}\}} \quad \ldots\ldots. \qquad (44)$$

By substituting $V_1 = \dfrac{Q}{By_1}$, and solving for Q ,

$$Q = \frac{C_K b y_2 \sqrt{2g\{(y_1 - y_2) - E_f\}}}{\sqrt{1 - \alpha_1 C_K^2 M^2 S^2}} \quad \ldots\ldots. \qquad (45)$$

where $M = b/B$ and

$$S = y_2/y_1 \ .$$

Assuming $\alpha_1 \simeq 1.0$ and $E_f \simeq 0.0$

$$Q = \frac{C_K \sqrt{2g}\ b y_2 (y_1 - y_2)^{1/2}}{\sqrt{1 - C_K^2 M^2 S^2}} \quad \ldots\ldots. \qquad (46)$$

By multiplying both numerator and denominator by $y_1 - y_2$ and dividing both by y_2 ,

$$Q = \frac{C_K\sqrt{2g}\ b(y_1-y_2)^{3/2}}{\sqrt{\dfrac{(1-C_KMS)(1+C_KMS)(y_1-y_2)^2}{y_2^2}}} \quad \ldots\ldots\ldots \tag{47}$$

Therefore

$$Q = \frac{C_K\sqrt{2g}\ b(y_1-y_2)^{3/2}}{\sqrt{\dfrac{(1-C_KMS)(1-S)^2}{S^2/(1+C_KMS)}}} \quad \ldots\ldots\ldots \tag{48}$$

which is of similar form to Eq. 43, thereby indicating that for a given constriction geometry where b and M are constant, the discharge becomes a function of $(y_1-y_2)^{3/2}$ alone for a given value of submergence.

By substituting values of y_1 at fixed values of submergence (thereby fixing y_2), values of discharge have been generated from Eq. 44 using typical values of C_K and α_1 and a range of bed slopes. These values have been plotted on logarithmic paper in Fig. 18 with discharge as the ordinate and y_1-y_2 as the abscissa. As indicated by Eq. 48, values for a constant submergence describe a straight line on a logarithmic plot having a slope of $3/2$.

The discharge equation may be obtained in terms of energies by substituting energy minus velocity head for flow depth and solving for discharge.

$$Q = \frac{C_K\sqrt{2g}\ by_2(E_1-E_2)^{1/2}}{\sqrt{1 + C_K^2(M^2S^2-1)}} \quad \ldots\ldots\ldots \tag{49}$$

To eliminate the depth term, or to solve in terms of $(E_1-E_2)^{3/2}$ is exceedingly difficult, if not impossible. As this would tend to

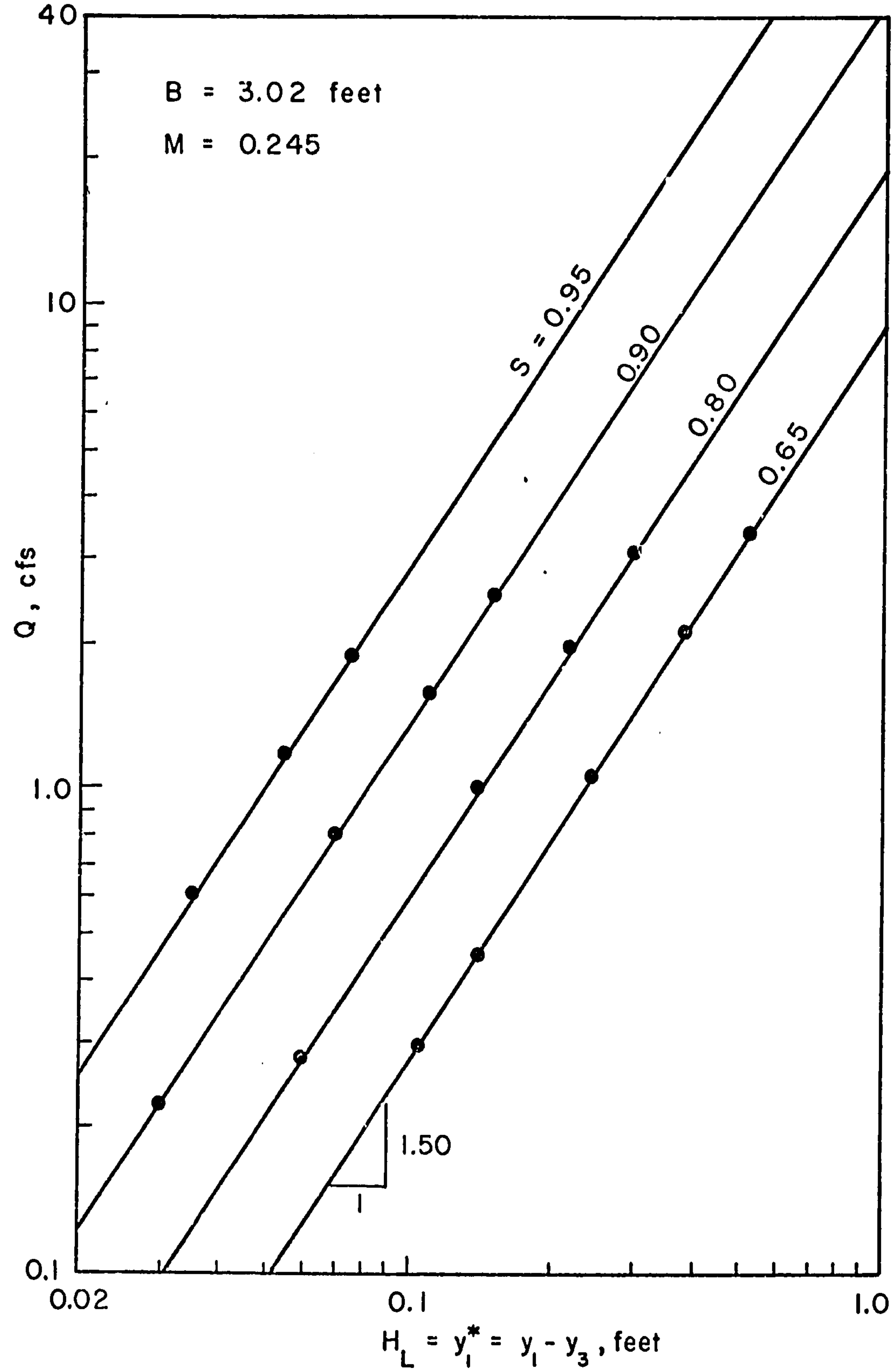

Figure 18. Submerged flow (depth) analysis of equation of Kindsvater, Carter and Tracy.

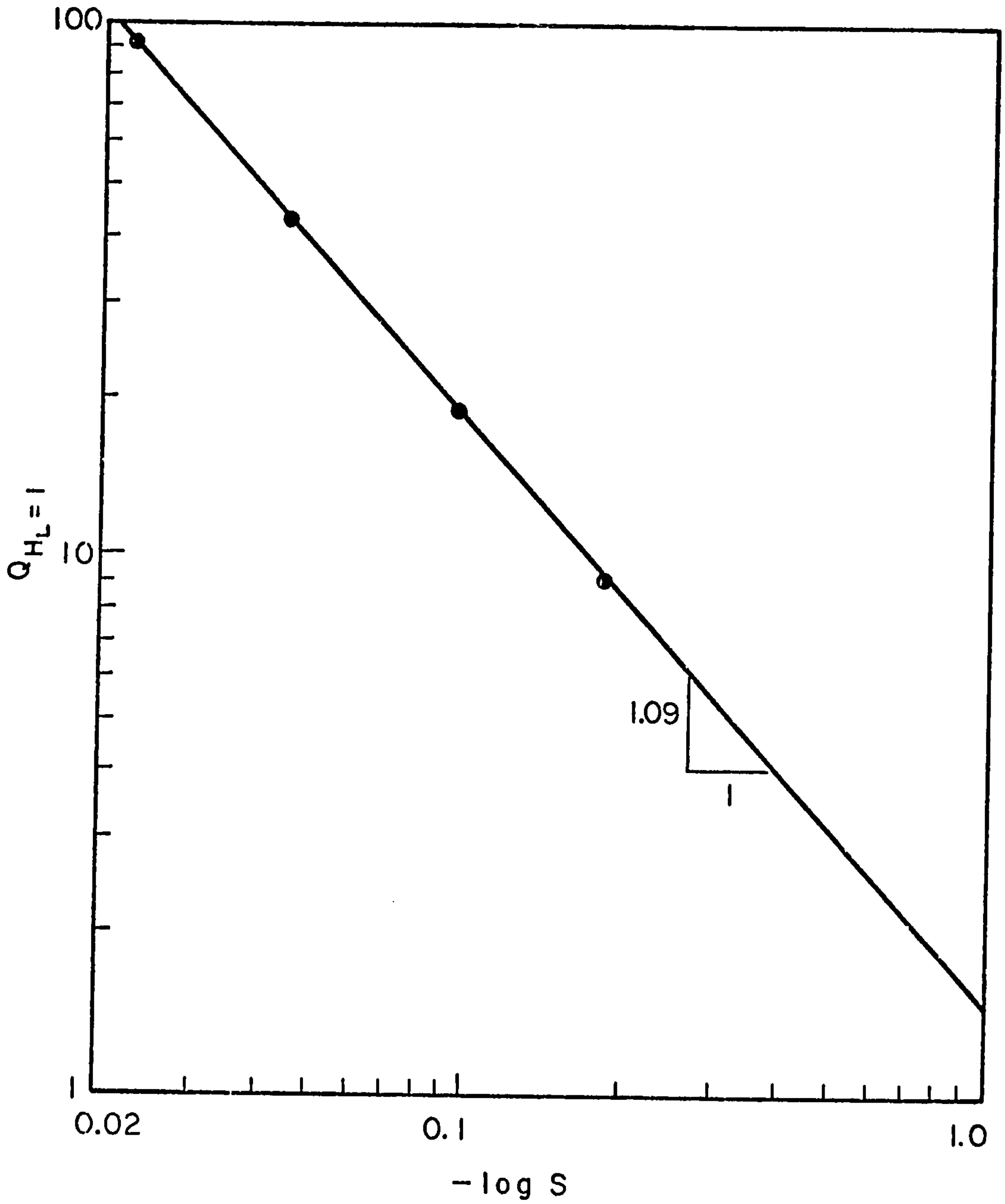

Figure 19. Submergence distribution for equation of Kindsvater, Carter and Tracy.

suggest, the data generated from Eq. 44, when plotted on logarithmic scales with discharge versus E_1-E_2, yields a series of curved lines, one for each value of submergence.

Liu, Bradley, and Plate. From a plot of actual data for a vertical board model, together with dimensional analysis of the backwater phenomena, Liu, Bradley, and Plate developed the empirical backwater equation

$$\left(\frac{y_1}{y_n}\right)^3 = 4.48\mathbb{F}_n^2\left[\frac{1}{M^2} - \frac{2}{3}(2.5-M)\right] + 1 \;\ldots\ldots\ldots \qquad (20)$$

By substituting

$$\mathbb{F}_n = \frac{Q}{By_n\sqrt{gy_n}} \qquad (50)$$

and solving for Q, an equation for discharge may be obtained

$$Q = B\sqrt{g}\sqrt{\frac{y_1^3-y_n^3}{4.48\left[\frac{1}{M^2} - \frac{2}{3}(2.5-M)\right]}} \;\ldots\ldots\ldots \qquad (51)$$

Although the denominator is dimensionless, this equation is rather dissimilar to Eq. 43. However, for the same data, Izzard suggested the equation

$$\frac{y_1}{y_n} = 1 + 0.45\left(\frac{\mathbb{F}_n}{M'}\right)^2 \;\ldots\ldots\ldots \qquad (23)$$

Substituting for $\mathbb{F}_n$ and solving for Q, a discharge equation is obtained

$$Q = 1.49\sqrt{g}\; by_n(y_1-y_n)^{1/2} \;\ldots\ldots\ldots \qquad (52)$$

This may be alternatively expressed as

$$Q = \frac{1.49b\sqrt{g}\ (y_1 - y_n)^{3/2}}{1/S - 1} \quad .... \tag{53}$$

which has a numerator of similar form to Eq. 43 and a dimensionless denominator. By substituting energy minus velocity head for flow depth in Eq. 52, the discharge equation can be described in terms of energies

$$Q = \frac{1.49\sqrt{g}\ by_n(E_1 - E_n)^{1/2}}{\sqrt{1 - 1.11M^2(1 - S^2)}} \quad .... \tag{54}$$

although the term y_n persists. Again to obtain a concise expression for discharge purely in terms of energies, or in terms of $(E_1 - E_n)^{3/2}$, is exceedingly difficult, if not impossible.

By solving Eq. 51 for given values of S and y_n, data has been generated and plotted according to the submerged flow analysis as shown in Figs. 20 and 22. However, despite the inability to express discharge in terms of $(E_1 - E_n)^{3/2}$, the data plots perfectly, whether generated in terms of depths or energies with the following results:

$$\text{For} \quad M = 0.245, \quad Q = \frac{1.55(y_1 - y_n)^{3/2}}{-\log S} = \frac{1.59(E_1 - E_n)^{3/2}}{-\log E_R} \quad .... \tag{55}$$

$$\text{For} \quad M = 0.497, \quad Q = \frac{3.7(y_1 - y_n)^{3/2}}{-\log S} = \frac{3.8(E_1 - E_n)^{3/2}}{-\log E_R} \quad .... \tag{56}$$

$$\text{For} \quad M = 0.733, \quad Q = \frac{7.4(y_1 - y_n)^{3/2}}{-\log S} = \frac{7.4(E_1 - E_n)^{3/2}}{-\log E_R} \quad .... \tag{57}$$

The value of the submergence exponent, n_2, was found to be 1.0 for both depths and energies, as shown in Figs. 21 and 23.

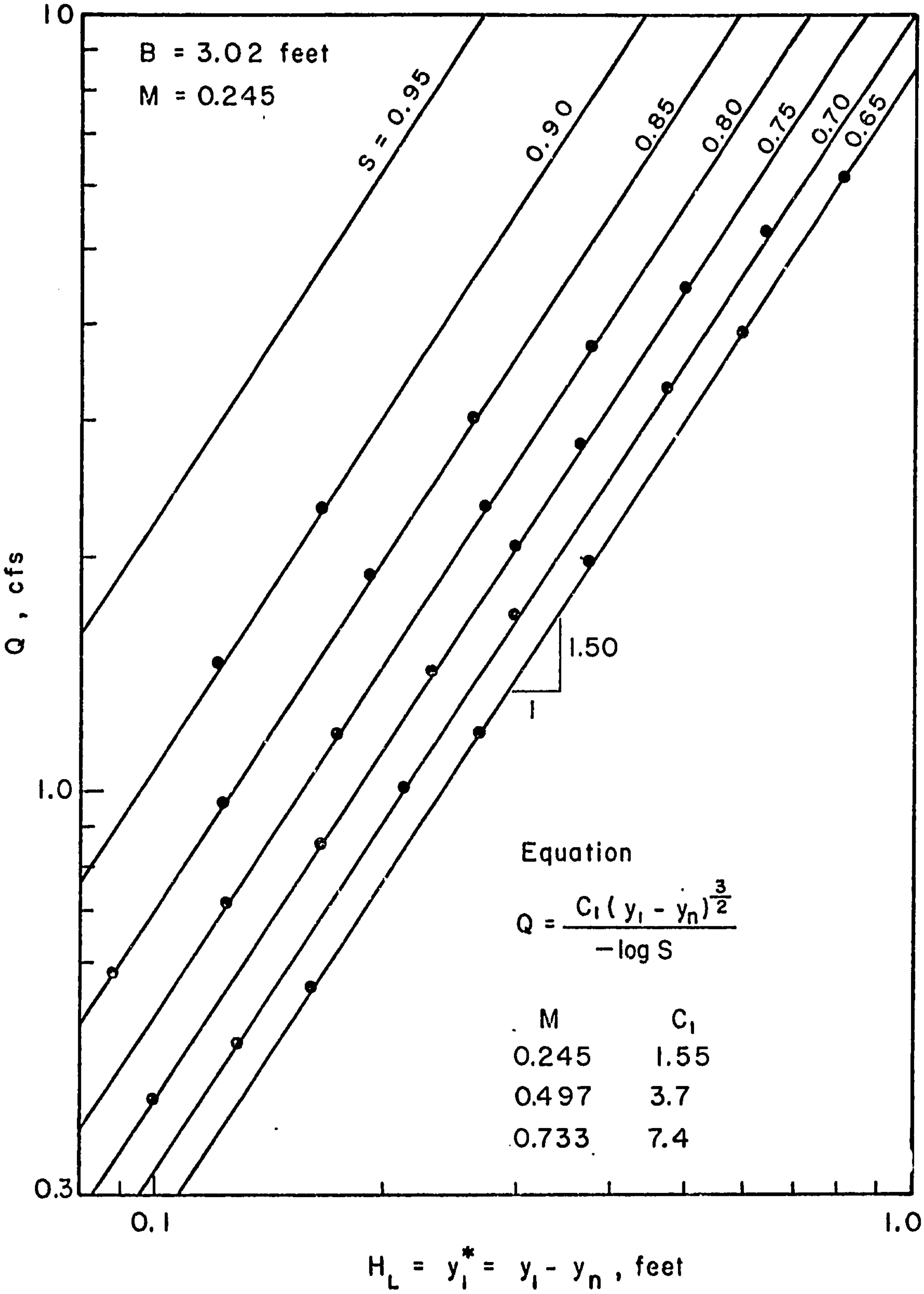

Figure 20. Submerged flow (depth) analysis of equation of Liu, Bradley and Plate.

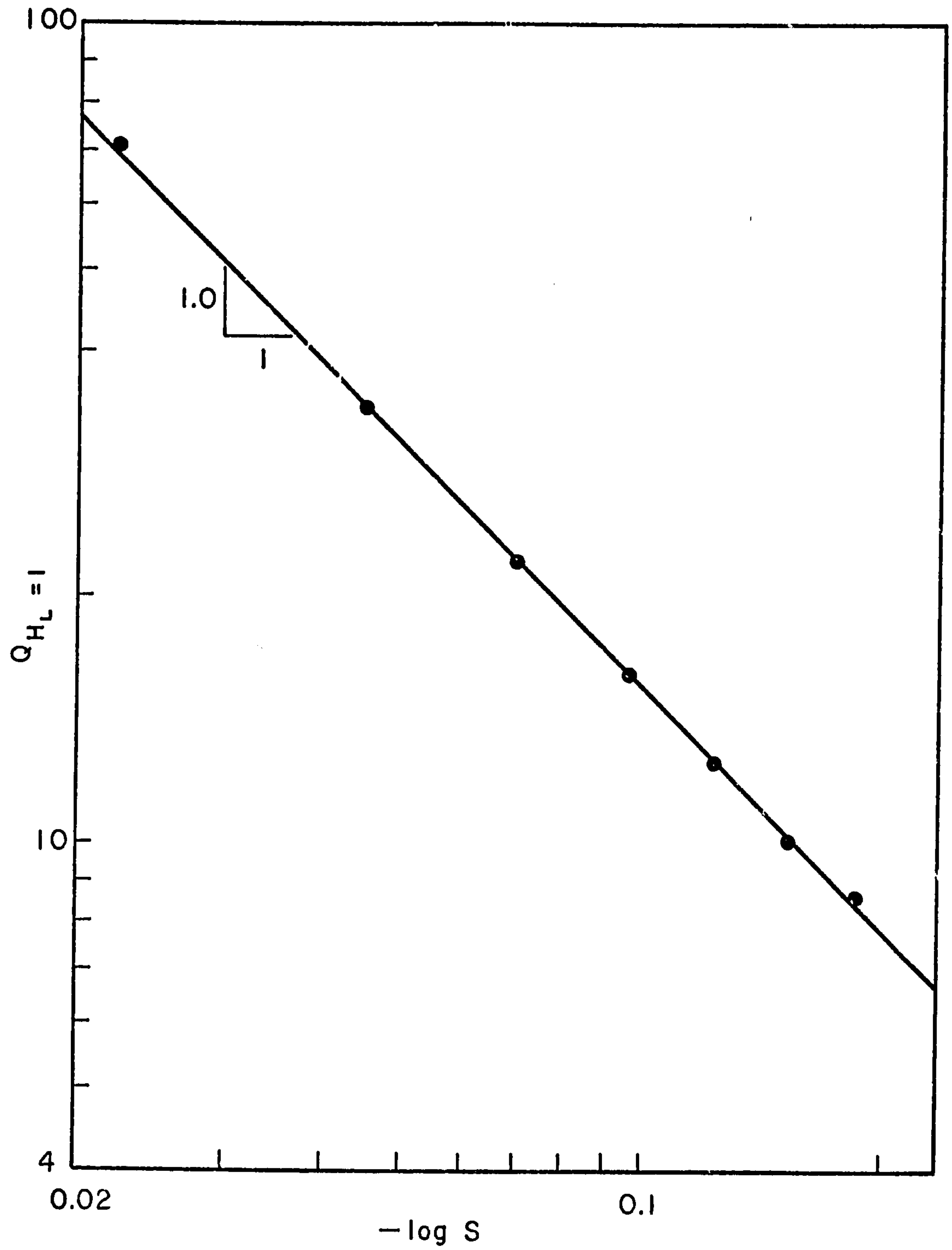

Figure 21. Submergence distribution for equation of Liu, Bradley and Plate.

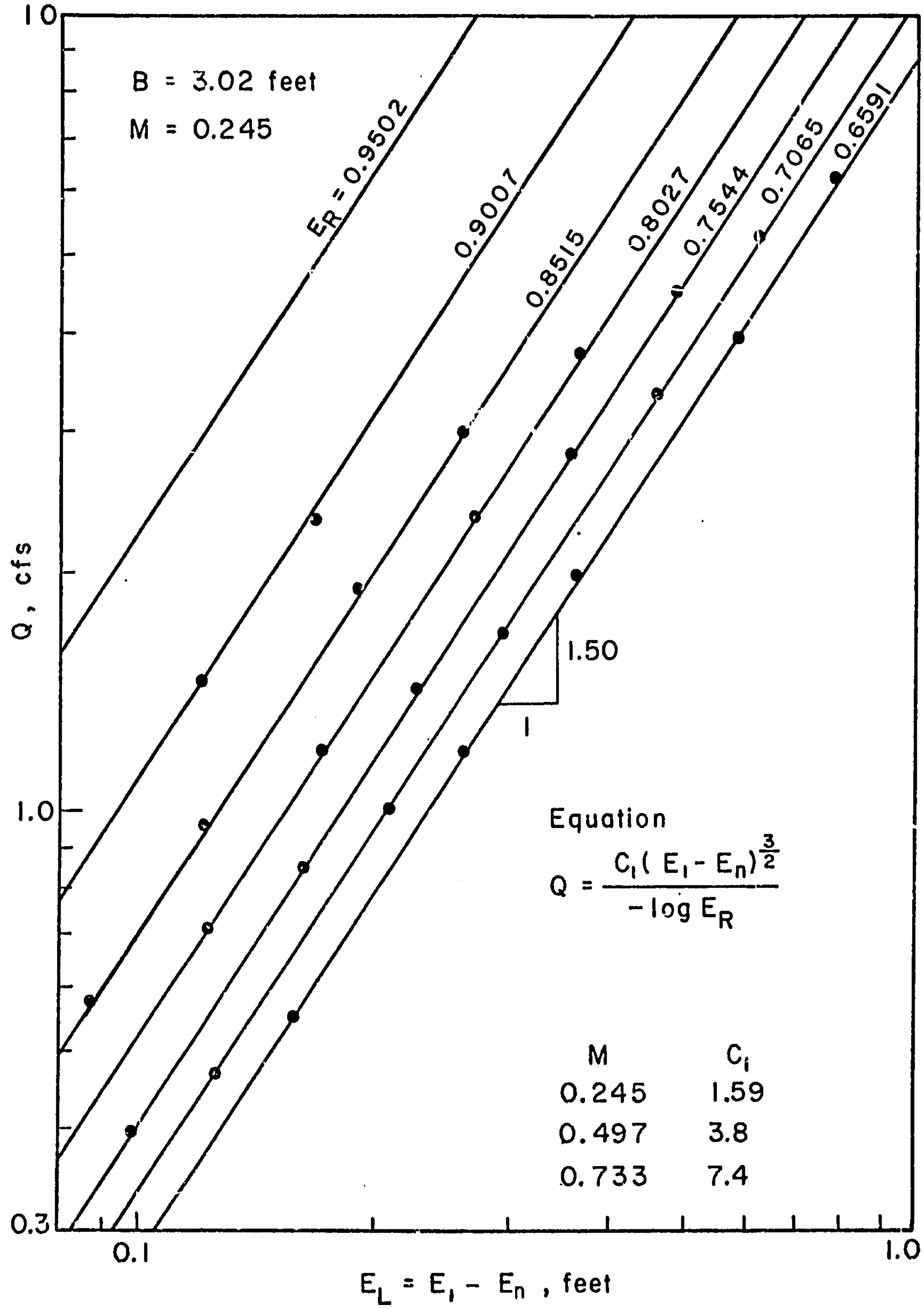

Figure 22. Submerged flow (energy) analysis of equation of Liu, Bradley and Plate.

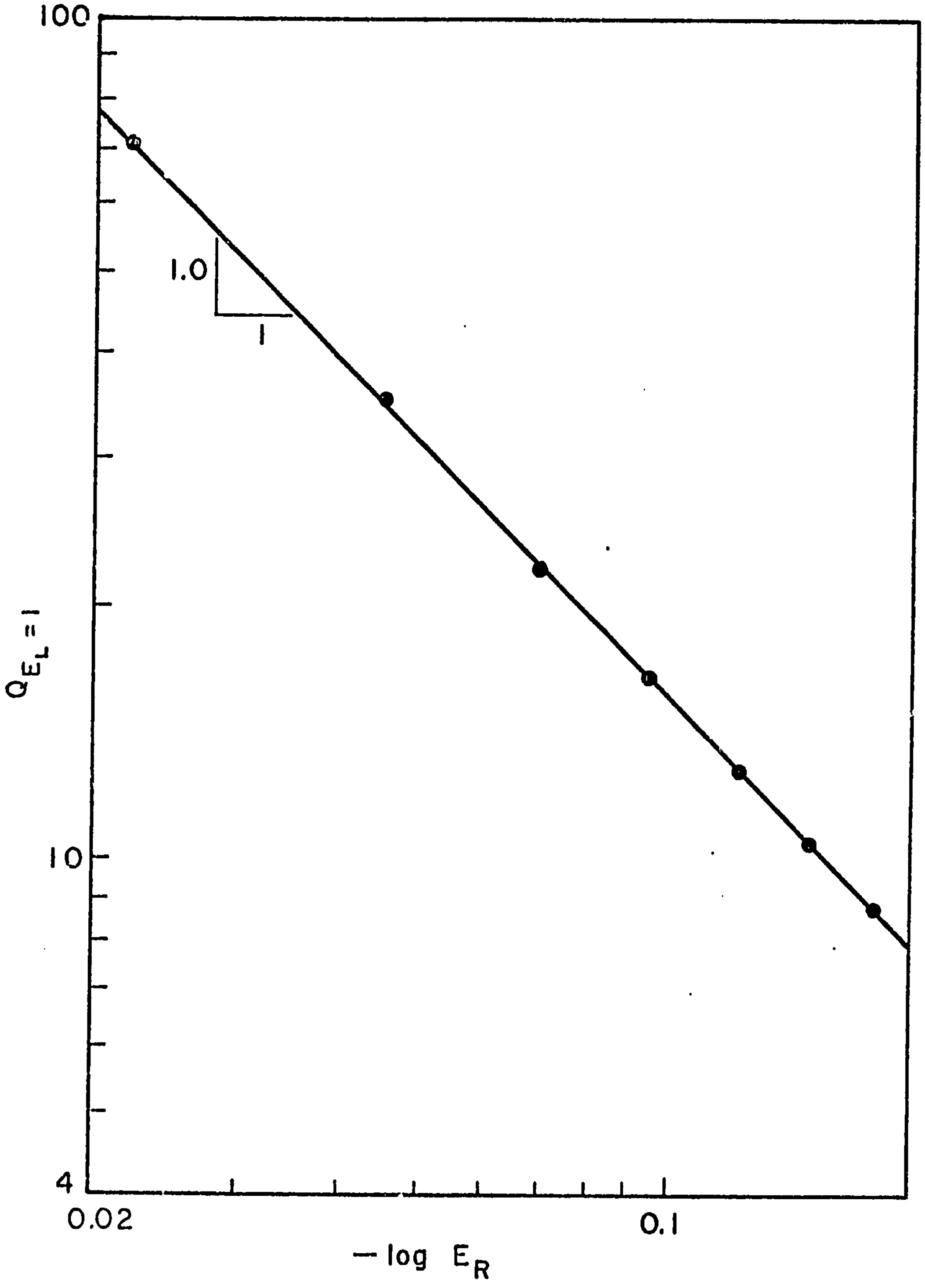

Figure 23. Energy ratio distribution for equation of Liu, Bradley and Plate.

Bureau of Public Roads. Working in conjunction with Colorado State University, the Bureau of Public Roads derived the expression for backwater

$$y_1^* = K^* \frac{V_{n2}^2}{2g} + \alpha_1\left[\left(\frac{A_{n2}}{A_4}\right)^2 - \left(\frac{A_{n2}}{A_1}\right)^2\right]\frac{V_{n2}^2}{2g} \quad \ldots\ldots. \tag{25}$$

which is embodied in the manual by Bradley (4). This analysis again assumes uniform flow before placement of the constriction. Substituting $y_1^* = y_1 - y_n$ and expressing velocities in terms of discharges (continuity equation) for a rectangular channel, Eq. 25 may be solved for discharge:

$$Q = \frac{\sqrt{2g}\ b y_n (y_1 - y_n)^{1/2}}{\sqrt{K^* + \alpha_1 M^2(1-S^2)}} \quad \ldots\ldots. \tag{58}$$

This may also be expressed as

$$Q = \frac{\sqrt{2g}\ b(y_1 - y_n)^{3/2}}{\sqrt{\dfrac{K^*}{y_n^2} + \dfrac{\alpha_1 M^2(1-S^2)(1-S)^2}{S^2}}} \quad \ldots\ldots. \tag{59}$$

However, in this case, the denominator is not dimensionless unless $K^* = f(y_n^2)$.

The appropriate values of K^* were selected from the curves given in the BPR design manual (4), and Eq. 58 solved for given values of S and y_n . The results were plotted on Fig. 24, with the submergence distribution plotting on a curve as shown in Fig. 25. The values of energy for each flow depth were also obtained, which allowed Eq. 58 to be plotted in terms of energies as shown in Fig. 26, with the energy ratio distribution falling on a straight line as shown in Fig. 27.

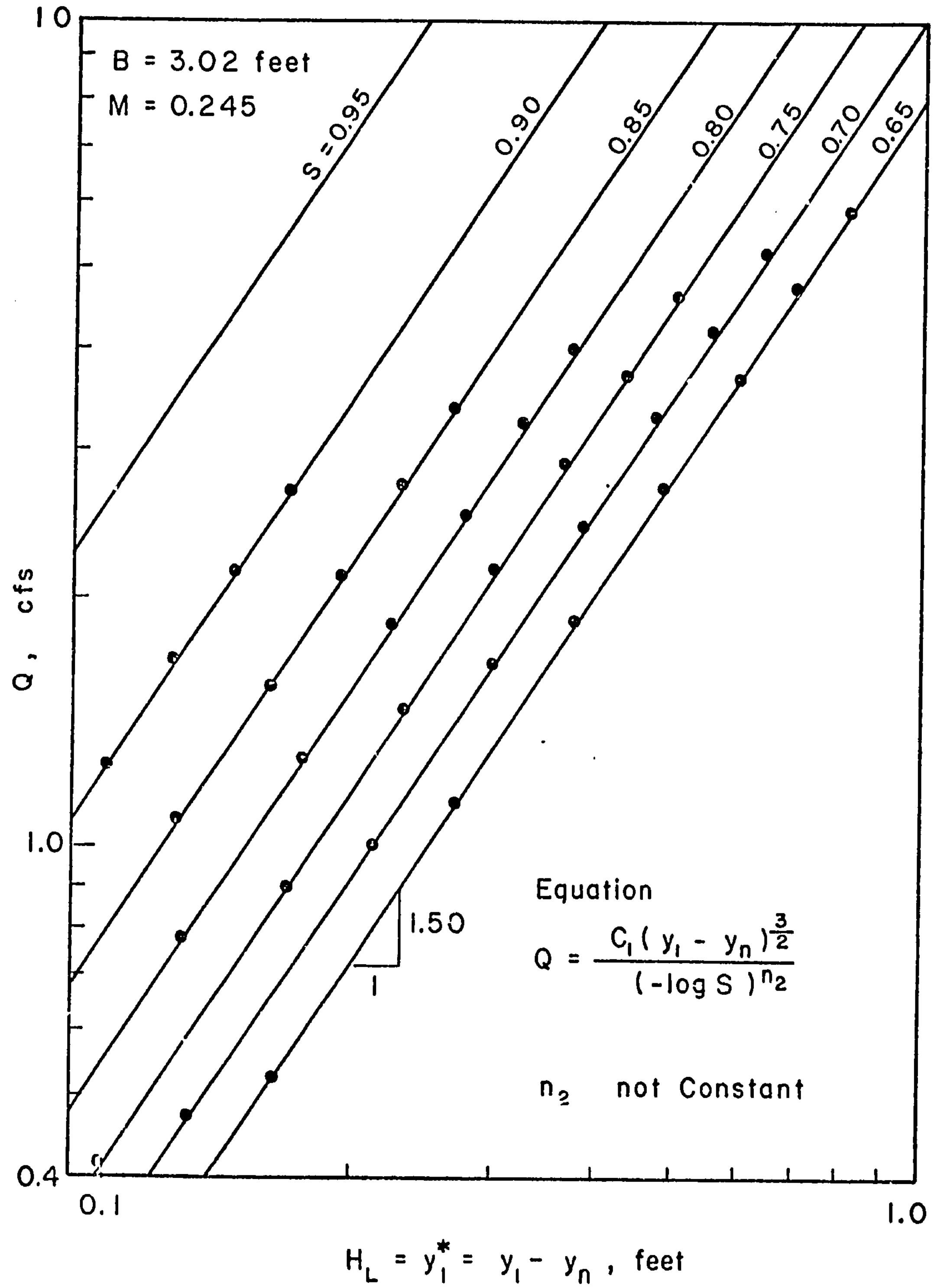

Figure 24. Submergence flow (depth) analysis of equation of Bureau of Public Roads.

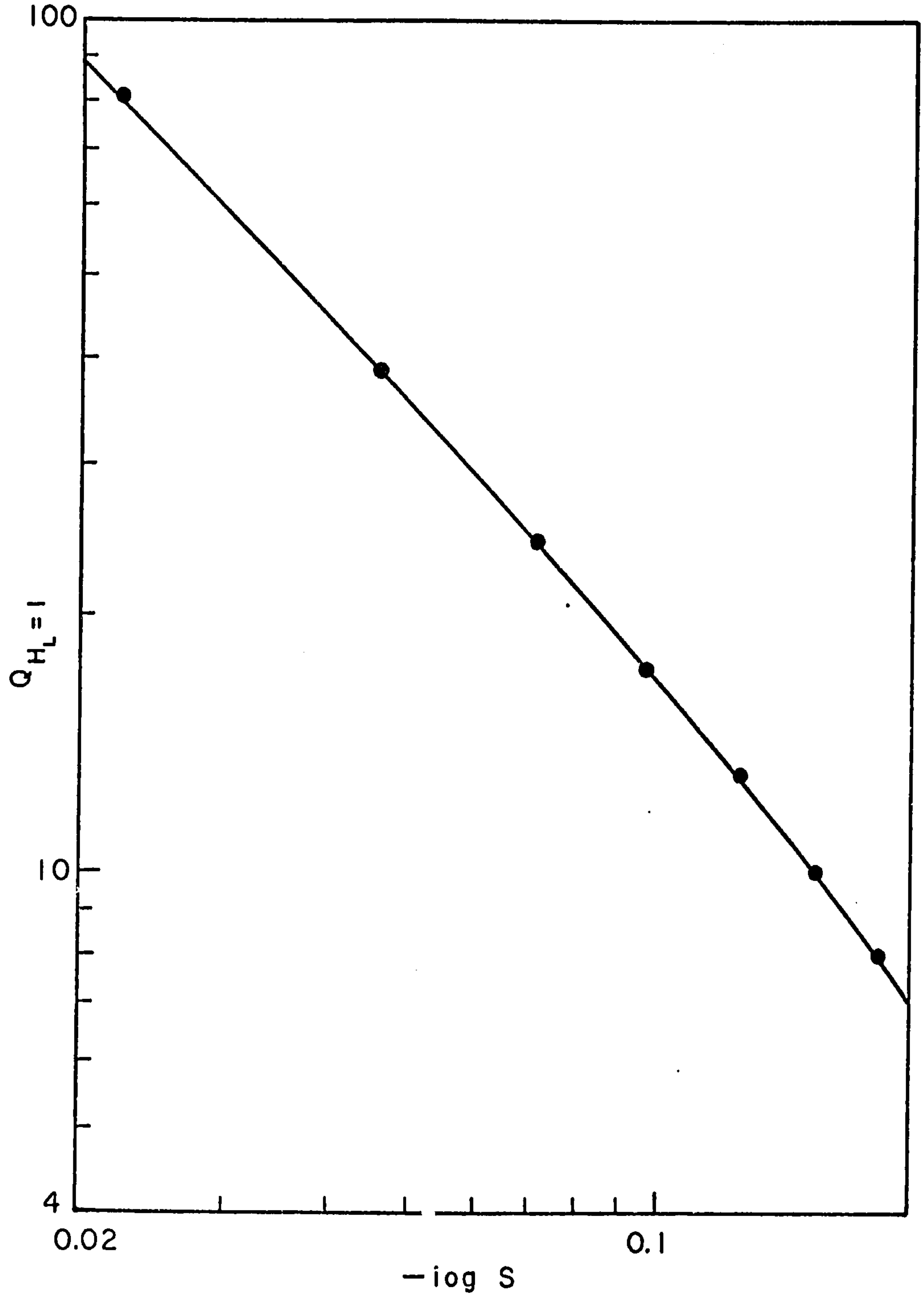

Figure 25. Submergence distribution for equation of Bureau of Public Roads.

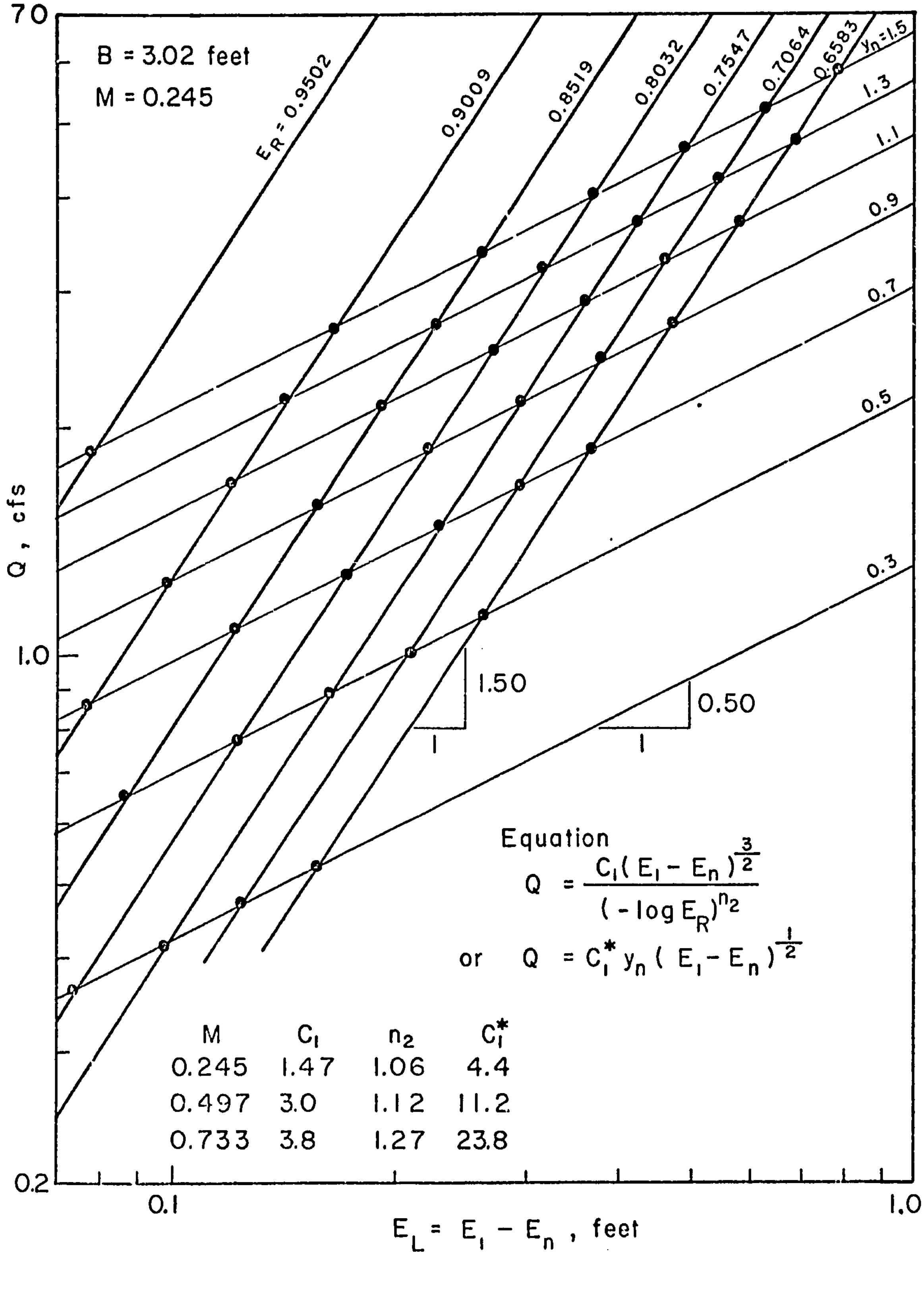

Figure 26. Submerged flow (energy) analysis of equation of Bureau of Public Roads.

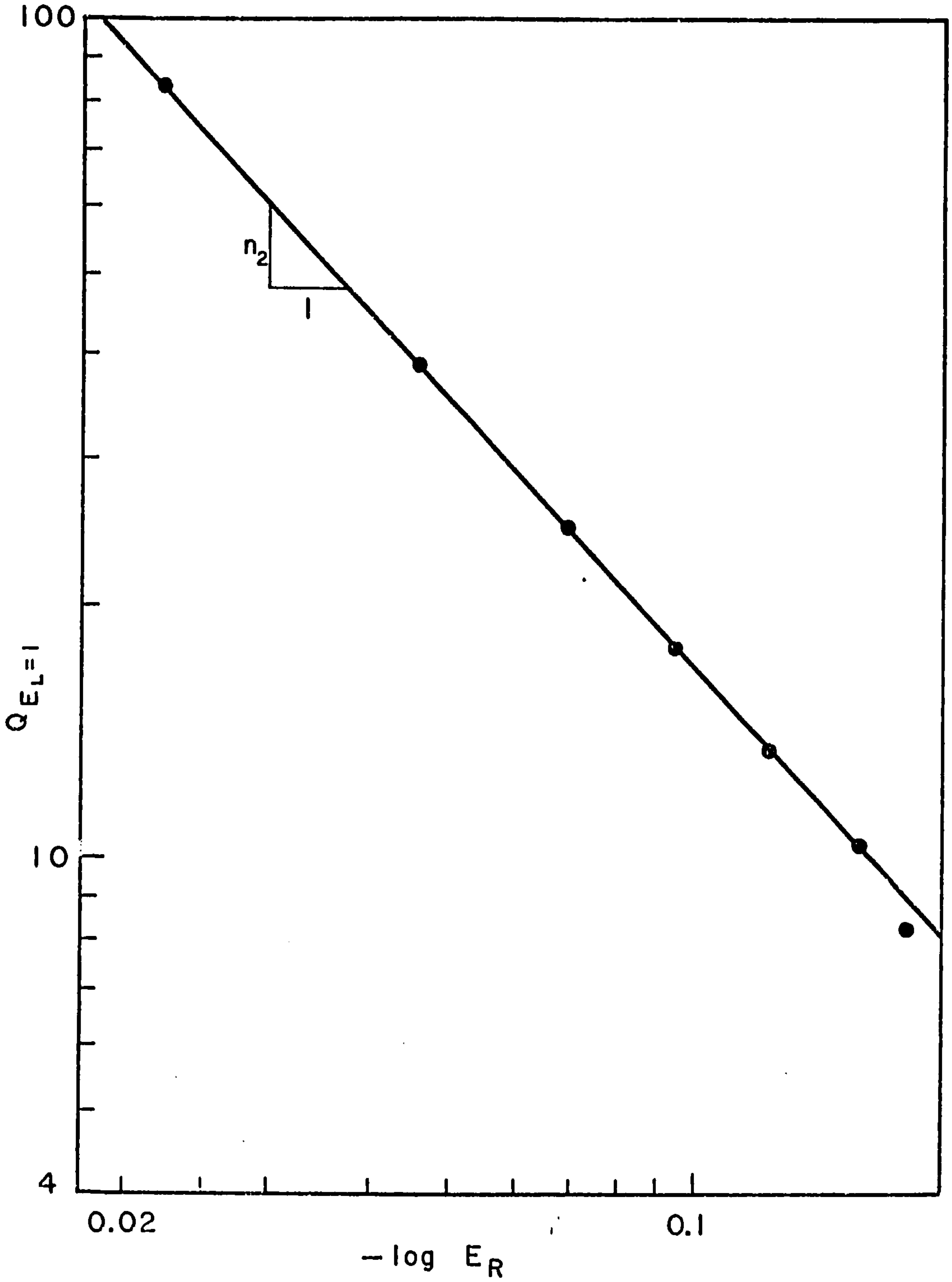

Figure 27. Energy ratio distribution for equation of Bureau of Public Roads.

An expression for discharge in terms of energies may be obtained from Eq. 25 if energy minus velocity head is substituted for the back-water depth.

$$Q = \frac{\sqrt{2g}\ by_n(E_1-E_n)^{1/2}}{\sqrt{K^* + M^2(\alpha_1-1)(1-S^2)}} \quad \ldots\ldots\ldots \tag{60}$$

Again, a solution purely in terms of energies has not been achieved.

Recognizing from Eq. 26 that $\alpha_1 = 1$ for a rectangular laboratory flume, the discharge equation becomes

$$Q = \frac{\sqrt{2g}\ by_n(E_1-E_n)^{1/2}}{\sqrt{K^*}} \quad \ldots\ldots\ldots \tag{61}$$

Hence, it may be seen that for a given constriction geometry located in a rectangular flume, using a constant value of y_n, that a plot of Q against E_1-E_n on logarithmic paper will yield a straight line, with the slope being 0.5. This has been demonstrated in Fig. 26 where data was generated from Eq. 25, modified to the form of Eq. 58. Hence, by plotting $Q_{E_L} = 1$ against y_n (Fig. 28) the value of the coefficient $\sqrt{\frac{2g}{K^*}}\,b$ was confirmed, so that Eq. 61 may be expressed in the form

$$Q = C_1^* y_n(E_1-E_n)^{1/2} \quad \ldots\ldots\ldots \tag{62}$$

where

$$C_1^* = \sqrt{\frac{2g}{K^*}}\,b \quad \ldots\ldots\ldots \tag{63}$$

As evidenced by the above analysis of discharge equations developed at Georgia Institute of Technology, Colorado State University

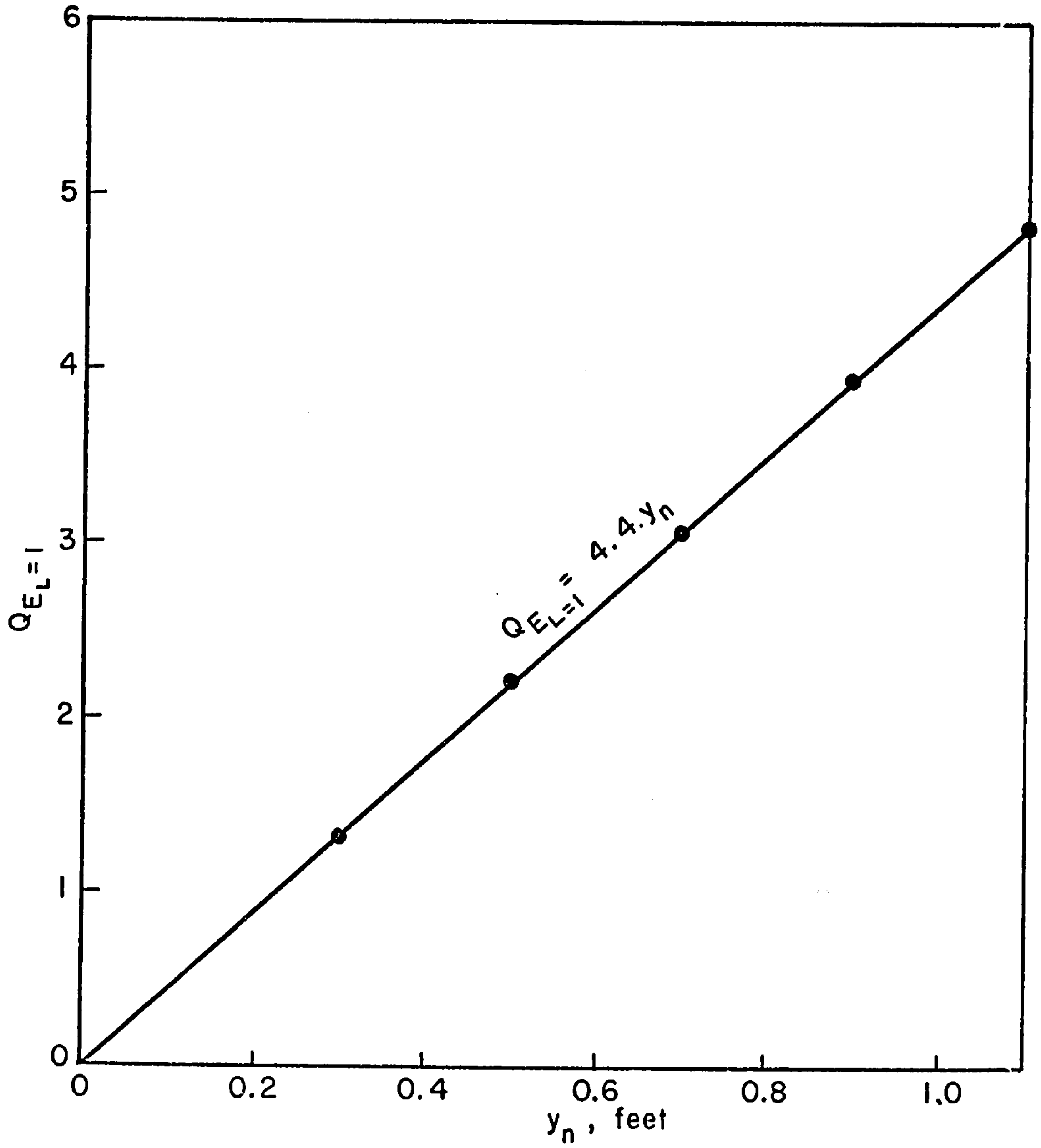

Figure 28. Normal depth distribution for equation of Bureau of Public Roads expressed in terms of specific energies.

and Bureau of Public Roads, the analytical expressions embodied in the current methods of measuring peak discharge through, or backwater due to, a bridge constriction may be reduced to the form of a submerged flow equation. Hence, it may be said that these expressions represent only particular cases of the backwater phenomena, and all could be encompassed by the more general submerged flow equations.

Relationship Between Submergence and Froude Number

The theoretical discharge through a rectangular constriction may be expressed as

$$Q_t = \frac{\sqrt{g/2}\ b(y_1-y_2)^{3/2}}{\sqrt{\dfrac{M(1-MS)(1-S)^2}{S(1+S)}}} \quad \ldots\ldots\ldots \tag{43}$$

Since

$$(y_1-y_2)^{3/2} = y_1^{3/2}(1-S)^{3/2} \quad \ldots\ldots \tag{64}$$

Eq. 43 may be expressed as

$$Q = \frac{\dfrac{\sqrt{g}}{\sqrt{2}}\ by_1^{3/2}(1-S)^{1/2}}{\sqrt{\dfrac{M(1-MS)}{S(1+S)}}} \quad \ldots\ldots\ldots \tag{65}$$

If S is specified for a given constriction

$$\frac{\dfrac{1}{\sqrt{2}}(1-S)^{1/2}}{\sqrt{\dfrac{M(1-MS)}{S(1+S)}}} = \xi \ \ldots\ldots\ldots \tag{66}$$

where ξ is a constant. Therefore

$$Q = \xi\sqrt{g}\ by_1^{3/2} \quad \ldots\ldots\ldots \tag{67}$$

or

$$\xi = \frac{Q}{by_1\sqrt{gy_1}} \quad \ldots\ldots\ldots \qquad (68)$$

which will be recognized as a Froude number. Hence, at a specified submergence, the Froude number is constant for a given constriction.

The problem remains to determine the exact form of the relationship between submergence and Froude number. The study undertaken by Colorado State University (13) produced the expression

$$\left(\frac{y_1}{y_n}\right)^3 = 4.48\mathbb{F}_n^2\left[\frac{1}{M^2} - \frac{2}{3}(2.5-M)\right] + 1 \quad \ldots\ldots\ldots \qquad (20)$$

for a vertical board constriction, where it will be recognized that

$$\left(\frac{y_1}{y_n}\right)^3 = \left(\frac{1}{S}\right)^3 \quad \ldots\ldots\ldots \qquad (69)$$

The relationship between submergence and Froude number suggests that data generated from a submerged flow equation for a particular constriction should plot equally well on the format used by Liu, Bradley and Plate (see Fig. 7). Hence, equations obtained by Skogerboe, Austin and Chang (17) for a 3.02 foot wide flume with constriction ratios of 0.245, 0.497 and 0.733 were used to generate values of the parameters employed by Liu, Bradley and Plate. A plot of this generated data is shown in Fig. 29. Since the equations used by Skogerboe, Austin and Chang were obtained from a best fit of actual data, and hence subject to error, the minor scatter in the points would appear insignificant. The value of 3.78 obtained for the coefficient (compared to 4.48 obtained by Liu, Bradley and Plate) may be explained by the presence of scale effects. For the larger channel, the viscous

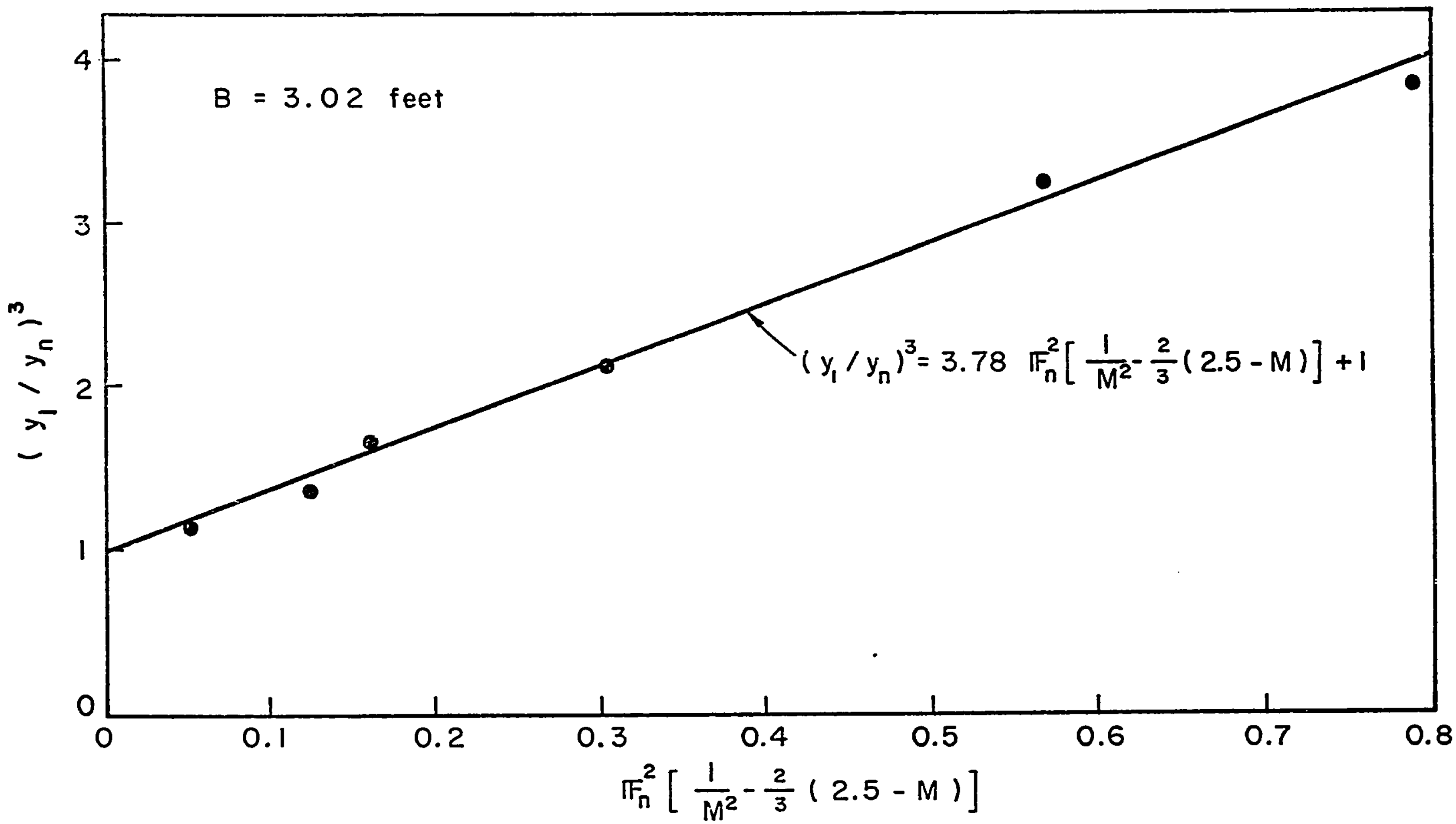

Figure 29. Comparison of equations by Skogerboe, Austin and Chang with parameters used by Liu, Bradley and Plate.

drag apparently becomes significant, tending to increase the magnitude of the backwater for a given Froude number.

An expression similar to Eq. 20 may be obtained by considering energies at the section of maximum backwater and at a section in the vicinity of the constriction where normal depth has to occur (see Fig. 1). For a channel in which uniform flow occurs before placing the constriction

$$y_1 + \frac{V_1^2}{2g} = y_1 + \frac{V_2^2}{2g} \quad \ldots\ldots\ldots \qquad (70)$$

Since normal depth must reoccur in the vicinity of the constriction, section II may be redefined for this case as the section where this occurs. Therefore, since $y_2 = y_n$

$$y_1 + \frac{V_1^2}{2g} = y_n + \frac{V_2^2}{2g} \quad \ldots\ldots\ldots \qquad (70)$$

From continuity, for a rectangular channel,

$$V_o B y_o = V_n B y_n = V_1 B y_1 = V_2 C_{cn} b y_n \quad \ldots\ldots\ldots \qquad (71)$$

Therefore,

$$V_2 = \frac{V_n B}{C_{cn} b} \quad \ldots\ldots\ldots \qquad (72)$$

where C_{cn} is a contraction coefficient giving the width of the actual live streamflow at normal depth in the vicinity of the contraction. Also, from Eq. 71

$$V_1 = V_n \frac{y_n}{y_1} \quad \ldots\ldots\ldots \qquad (73)$$

Substituting into Eq. 70

$$y_1 + \frac{V_n^2}{2g}\left(\frac{y_n}{y_1}\right)^2 = y_n + \frac{V_n^2}{2gC_{cn}^2 M^2} \quad \ldots\ldots\ldots \qquad (74)$$

where $M = b/B$

Multiplying both sides of Eq. 74 by y_1^2/y_n^3 and collecting terms gives

$$\left(\frac{1}{S}\right)^3 = \left(\frac{1}{S}\right)^2 \left(\frac{\mathbb{F}_n^2}{2C_{cn}^2 M^2} + 1\right) - \frac{\mathbb{F}_n^2}{2} \quad \ldots\ldots\ldots \quad (75)$$

or

$$\frac{1}{S} = \frac{\mathbb{F}_n^2}{2}\left(\frac{1}{C_{cn}^2 M^2} - S^2\right) + 1 \quad \ldots\ldots\ldots \quad (76)$$

both of which may be compared to Eq. 20.

By expressing the Froude number in terms of discharge, and collecting terms, a discharge equation may be obtained from Eq. 76.

$$Q = \frac{\sqrt{2g}\ by_n(y_1-y_n)^{1/2}}{\sqrt{\frac{1}{C_{cn}^2} - M^2S^2}} \quad \ldots\ldots\ldots \quad (77)$$

Again, this may be expressed in the submerged flow form of Eq. 43 by multiplying numerator and denominator by y_1-y_n, and dividing by y_n.

$$Q = \frac{\sqrt{2g}\ b(y_1-y_n)^{3/2}}{\sqrt{\frac{(1-C_{cn}^2M^2S^2)(1-S)^2}{C_{cn}^2S^2}}} \quad \ldots\ldots\ldots \quad (78)$$

Alternatively, Eq. 77 may be expressed in terms of energies as

$$Q = \frac{\sqrt{2g}\ by_n(E_1-E_n)^{1/2}}{\sqrt{\frac{1}{C_{cn}^2} - M^2}} \quad \ldots\ldots\ldots \quad (79)$$

Although the derivation of Eq. 77 was based on a rectangular channel, the principle may be applied to any channel or constriction shape by substituting for B or b the width of a rectangular section of the same depth and having the same cross-sectional area as the actual channel or constriction section.

Chapter 4

COMPARISON WITH PREVIOUS METHODS OF ANALYSIS

Position of Measurement and Flow Conditions

Previous analyses to obtain an expression for the discharge through, or backwater due to, an open channel contraction have inevitably considered an upstream flow section and another section further downstream. Also, an initial flow condition (uniform or nonuniform) has invariably been assumed for the unconstricted channel. A combination of either the energy and continuity equations, or momentum and continuity equations, has been written between the two sections, and a discharge or backwater equation obtained, whichever was required.

Kindsvater and Carter (10) and Tracy and Carter (20) defined an upstream section, Section 1, as the section at which acceleration of the flow approaching the constriction begins, with this section being one opening width, b , upstream from the beginning of the constriction. Thus, the difference in water levels between the normal and the backwater profiles at section 1 is the backwater measure adopted. The downstream section considered in the analysis, section 2, is located at the point of minimum width of the contracted live stream. To provide a practical measuring point, the depth at section 3, which is arbitrarily defined as being located in the relatively quiet zones of eddying fluid at the downstream side of the constriction, is substituted for section 2. This substitution is based on observations of model studies carried out in the horizontal flume at the Georgia Institute of Technology. These results would appear verified by the model studies carried out in the tilting flume at Utah State University for cases where subcritical flow occurs at every section. For supercritical flow at the section of minimum flow depth, a unique stage-discharge condition exists.

The work of Kindsvater, Carter and Tracy was based on data collected in a level flume, which meant that non-uniform flow conditions existed for all runs. However, according to Liu, Bradley and Plate (13), uniform flow is a necessary standard condition from which to work:

> "The difficulty in using the data from a level channel is the lack of standards representing the unobstructed flow conditions, because in a certain channel the velocity, the depth, and the energy gradient of the unobstructed flow vary from section to section for a given discharge (which means that the flow is non-uniform). Such standards are in general very essential for both theoretical and laboratory investigation."

Therefore, Liu, Bradley and Plate carried out model studies in a tilting flume at Colorado State University, generally setting uniform flow in the flume before placing a constriction. An energy equation was written between section I, where maximum backwater occurs, and section IV, where uniform flow is again reestablished downstream of the constriction. An equation developed from this analysis was used in the design manual prepared by Bradley (4).

A limited number of tests were conducted for the case of an abnormal stage-discharge condition occurring in the channel before the constriction was placed. However, no attempt was made to analyze this condition, and Bradley suggested only an intuitive procedure for handling the problem.

Skogerboe, Austin and Chang (17) recognized that uniform flow at a bridge site may be the exception rather than the rule. Non-uniform flow at a bridge site is due to downstream control, examples of which might include flood conditions at the confluence of two streams, downstream reservoir or spillway regulation, influence of tides, or changes in vegetative or moss conditions in flat gradient channels. A unique stage-discharge condition may no longer exist as it does for uniform flow. Analysis for this condition was carried out between section I, where maximum backwater occurs, and section IV, where the abnormal stage for the unconstricted flow is again reestablished. This approach was based on the experience of Skogerboe and others with flow measuring devices, where the necessity exists to measure depths at both an upstream and downstream section for subcritical flow throughout the length of the constriction.

In application, all of the above methods have serious shortcomings. The calculation procedure outlined by Kindsvater, Carter and Tracy is tedious, and has shown serious error in application to prototype structures, probably due to scale effects. The method adopted by the Bureau of Public Roads is limited to uniform flow in the channel at the bridge site, with an approximate hypothesized equation offered for abnormal stage-discharge conditions. This method is also tedious. The method suggested by Skogerboe, Austin and Chang necessitates estimating where the abnormal stage for the unconstricted flow will be reestablished downstream of the constriction. For non-uniform flow, the equation cannot be solved in terms of backwater unless the flow profile may be estimated by some other means. Also, the value of the exponent $n_1 = 3/2$ is at best dubious, considering the scatter exhibited by the data. In addition, friction loss was ignored between the two flow sections.

A more suitable method would appear to be an expression of the submerged flow equation (Eqs. 29, 30) in terms of depths or specific energies at section I (section of maximum backwater) before and after the constriction is placed. The expression in terms of depths is more practical, since it allows a direct solution for discharge. Hence, Eq. 29 for uniform flow in the unconstricted channel becomes

$$Q = \frac{C_1 (y_1 - y_n)^{n_1}}{(-\log S)^{n_2}} \quad \ldots\ldots\ldots \tag{80a}$$

where y_n is the normal depth. For the abnormal stage-discharge condition

$$Q = \frac{C_1 (y_1 - y_{1A})^{n_1}}{(-\log S)^{n_2}} \quad \ldots\ldots\ldots \tag{80b}$$

where y_{1A} is the depth at section I before the constriction is placed. The subscript A denotes abnormal stage. Significantly, the form of the equation is identical, regardless of the initial flow regime. Steady flow is required, but it is not necessary to be tranquil in the constriction.

The approach eliminates the necessity to account for channel slope. In the earlier analyses by Skogerboe and others, the effects of bed slope were accounted for by expressing the submerged flow equation in terms of total energies above an arbitrary datum, rather than depths. Expressed in this form, the submerged flow equation for a given constriction retained the same values of C_1, n_1 and n_2 regardless of channel slope. However, this placed a severe limitation on the practical application of the equation, as it could not be solved for discharge directly.

Perhaps even more significantly, for the limited data available, the submerged flow equation has invariably been found to give a value of n_1 exactly equal to 3/2, irrespective of whether the submerged flow equation is expressed in terms of depths or energies, or for uniform or non-uniform flow. This precise figure, although analytically derived, had generally eluded the earlier submerged flow researchers. This followed from their studies on flow measuring devices where the influence of the downstream point of measurement on the exponent n_1 varied with the geometry of the device. The deviation of n_1 from 3/2 may be explained by the failure to account for the friction loss between the two measuring points. By measuring at the upstream section only, this loss is eliminated.

The only data available describing flow conditions before and after placement of a constriction has been reported by Colorado State University (13). Data from this report has been plotted for both depths (Figs. 30 and 31) and energies (Figs. 32 and 33) for the abnormal stage-discharge condition, while Figs. 34 and 35 present data for uniform flow in the unconstricted channel.

For the abnormal stage-discharge conditions, only one discharge had been run for a given constriction. Therefore, the value of $n_1 = 3/2$ was arbitrarily chosen. However, the consistency with which the plotted points fall on a straight line in the plot of $Q_{H_L = 1}$ versus $-\log S$ and $Q_{E_L = 1}$ versus $-\log E_R$ verifies this choice. Again, as with the theoretically generated data, C_1, n_1 and n_2 are the same constants using either depths or energies in the discharge equation for a given constriction.

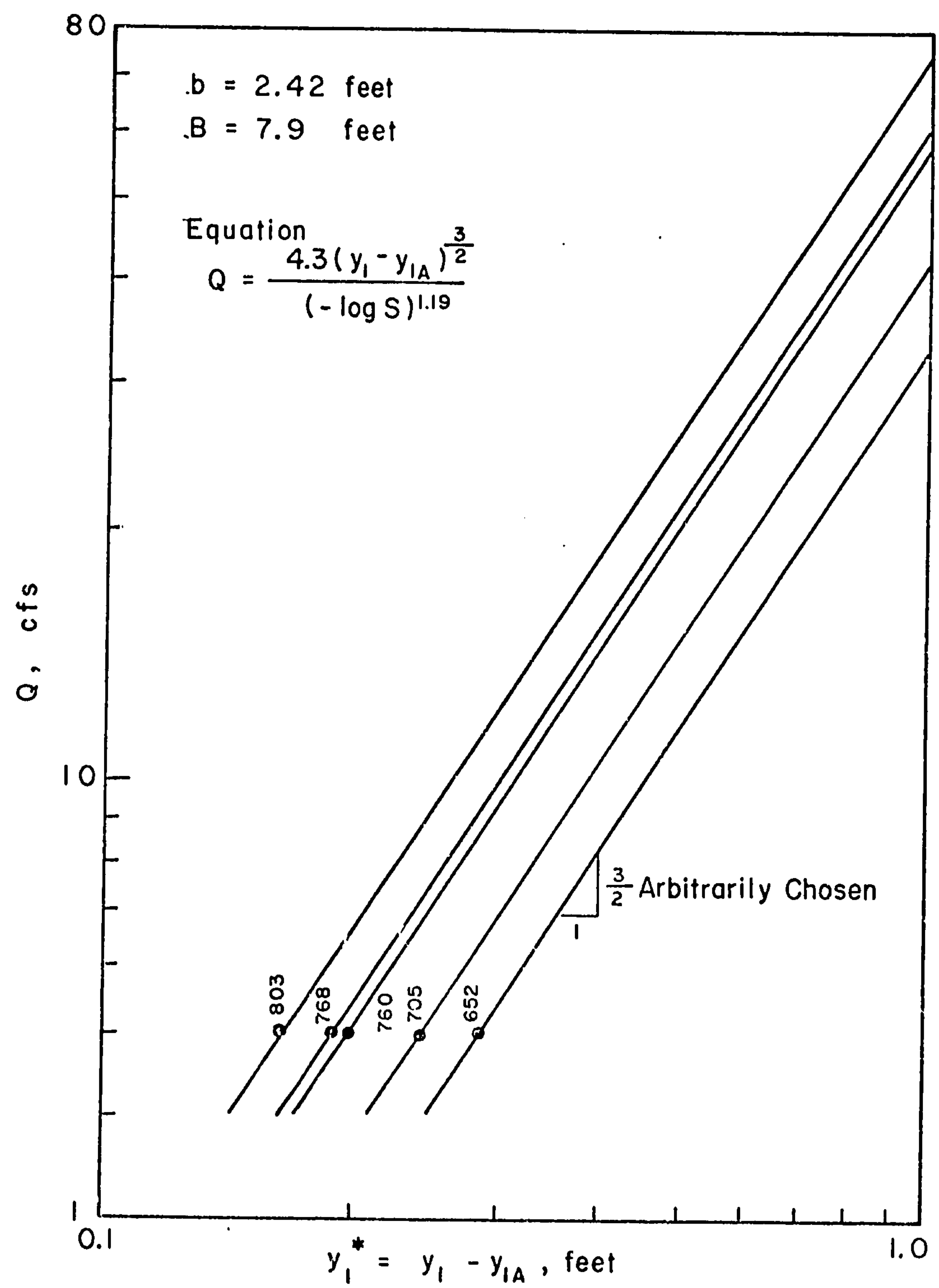

Figure 30. Submerged flow (depth) plot of abnormal stage-discharge data from Colorado State University report. (From Table 2, 45°WW abutments).

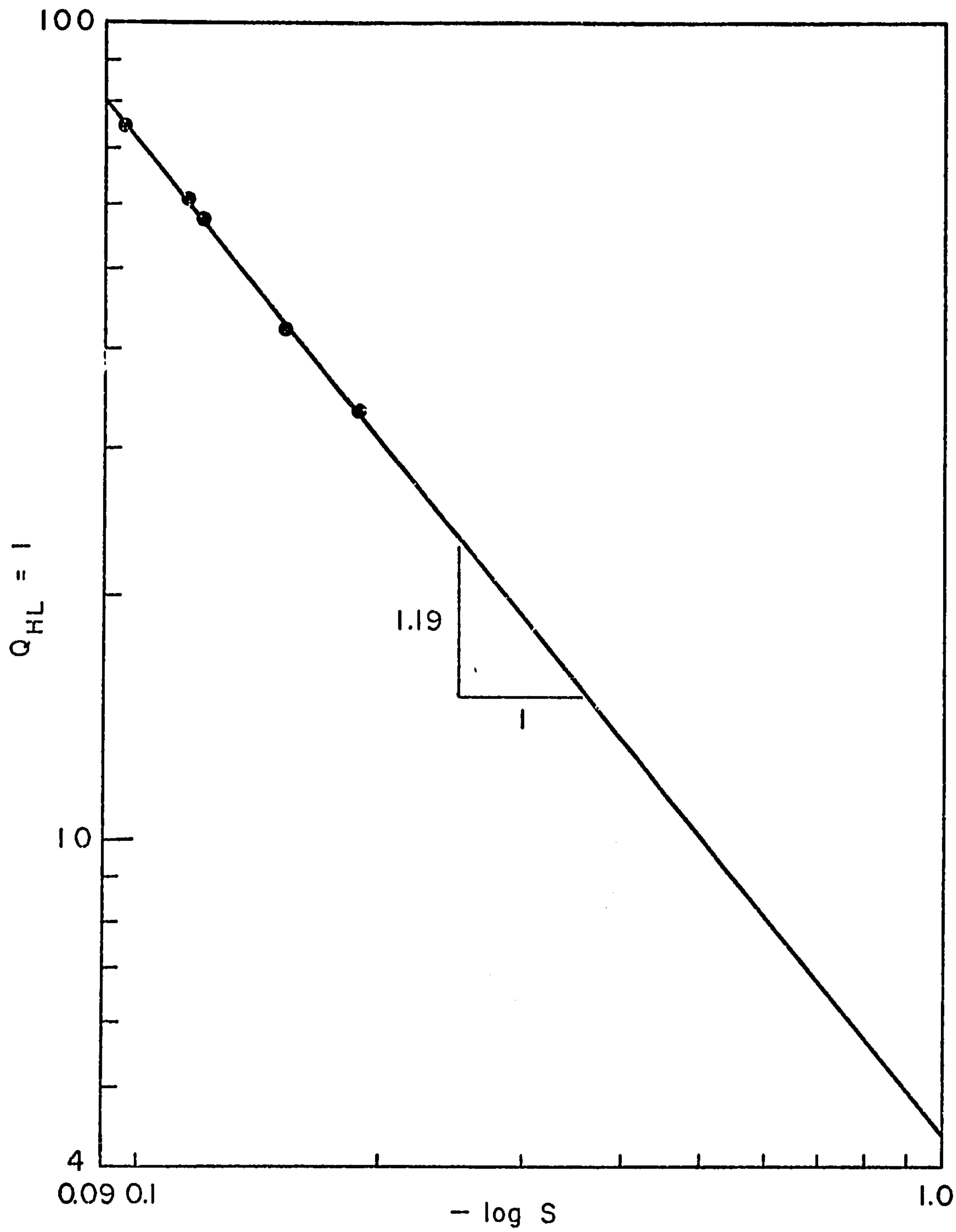

Figure 31. Submergence distribution of abnormal stage-discharge data.

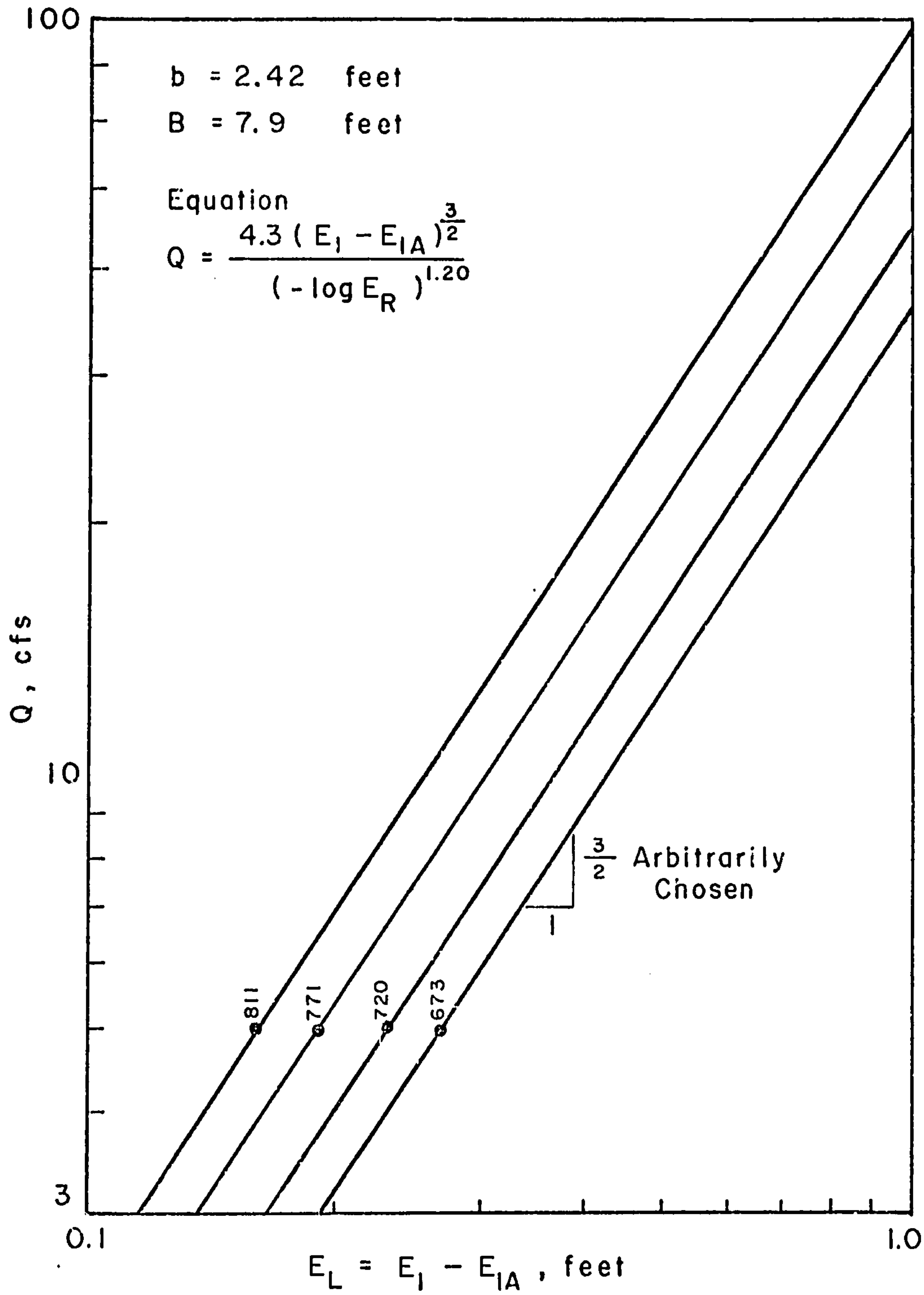

Figure 32. Submerged flow (energy) plot of abnormal stage-discharge data from Colorado State University report. (From Table 2, 45°WW abutments).

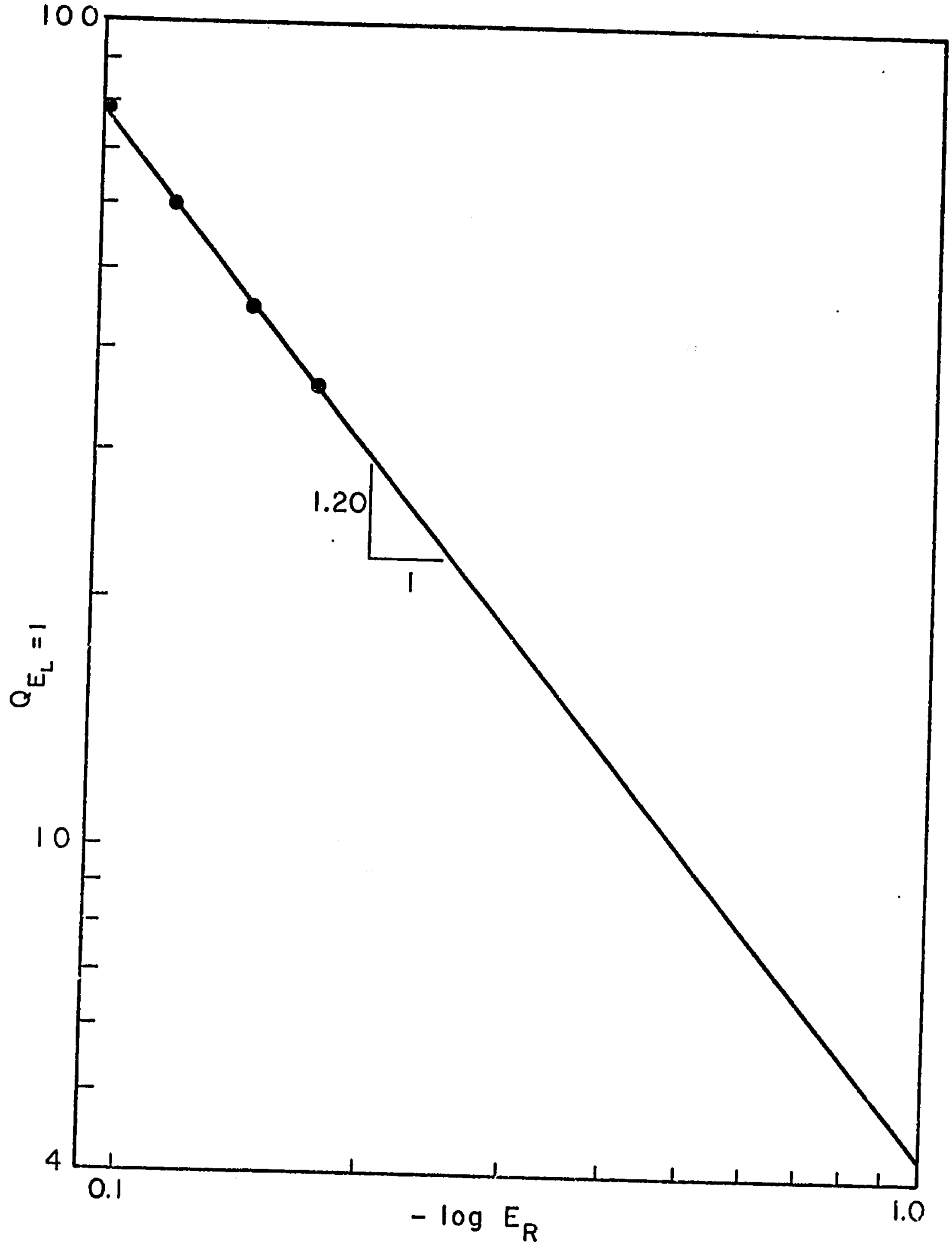

Figure 33. Energy ratio distribution of abnormal stage-discharge data.

A greater range of data was available for conditions of uniform flow in the channel prior to placement of the constriction. Again, the value of $n_1 = 3/2$ was arbitrarily chosen and found to be an excellent fit to the data. However, as shown in Fig. 36, the plots of $Q_{H_L=1}$ against $-\log S$ and $Q_{E_L=1}$ against $-\log E_R$ fall on a curve, indicating that n_2 is not a constant. An attempt to linearize this curve would necessitate a significant increase in the value of n_1. However, as careful observation of Figs. 34 and 35 shows, this is impossible if the lines of constant submergence (or energy ratio) are to fit the data correctly. Hence, it must be concluded that the correct value of n_1 is precisely 3/2. Although n_2 is not constant, Fig. 36 shows that a common curve fits both the submerged distribution (using flow depths) and energy ratio distribution (using energies), which means C_1, n_1 and n_2 are, for all practical purposes, identical for both depths and energies. A quantitative explanation of this result has not been deduced. However, qualitatively, it is felt that this may be due to the small differences in velocity head when the position of measurement is at section I rather than at an upstream and downstream measuring point.

An analysis to determine the variables affecting n_2 has not been conducted. For practical purposes, a straight line of best fit through the points on the submergence distribution plot will give results of sufficient accuracy.

Some of the abnormal stage-discharge data from the Colorado State University report (13) has also been reduced by Froude model laws and superimposed on the submerged flow plots obtained by Skogerboe, Austin and Chang (17). The data used was for a 7.9 foot wide flume, whereas

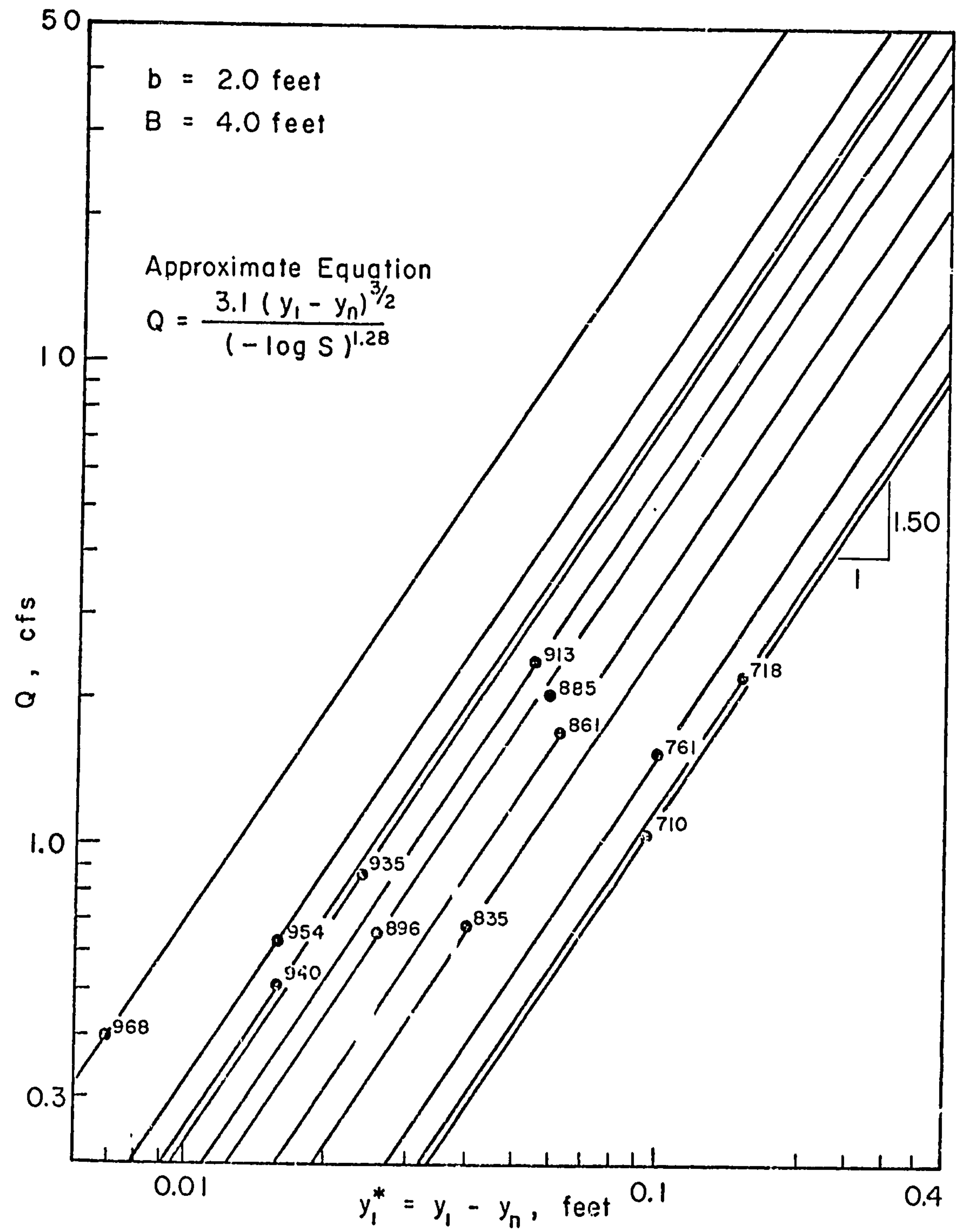

Figure 34. Submerged flow (depth) plot of uniform flow data from Colorado State University report. (From Table 1, 45°WW abutments).

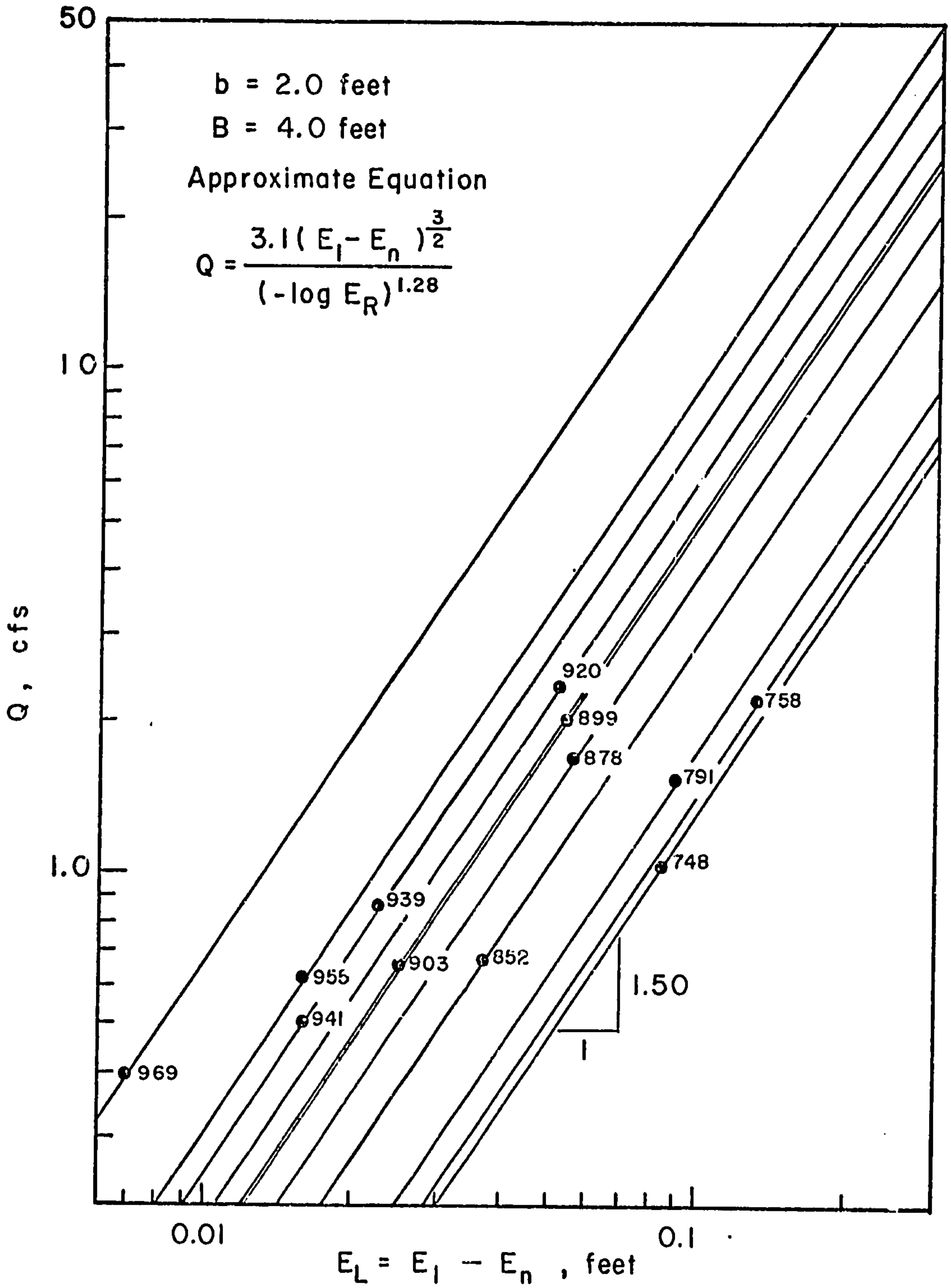

Figure 35. Submerged flow (energy) plot of uniform flow data from Colorado State University report. (From Table 1, 45°WW abutments).

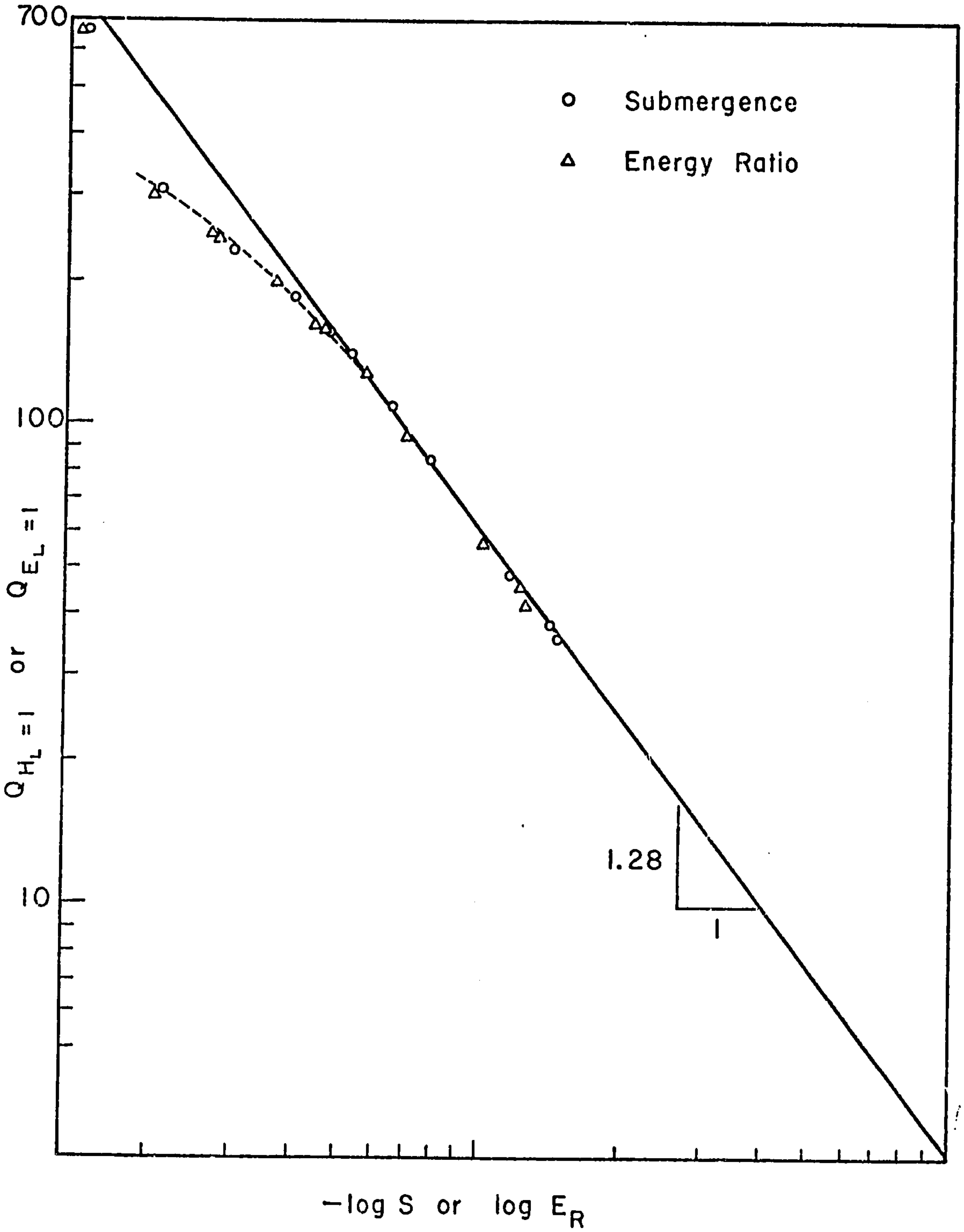

Figure 36. Submergence and energy ratio distribution of uniform flow data.

the flume used at Utah State University was 3.02 feet wide. The correlation obtained between these two sets of data is shown in Fig. 37. The CSU data fits the USU model rating equally as well as the original data. This would suggest that the Froude laws hold in going from model to prototype, providing the model is sufficiently large to overcome scale effects.

Constancy of Discharge Coefficients

Currently used methods of computing peak discharge through, or backwater due to bridge constrictions, rely on discharge coefficients initially obtained from model studies. Using independent model data, a check has been made on the constancy of these coefficients for given constrictions under a range of flow conditions.

Data obtained from the report of Skogerboe, Austin and Chang (17) was substituted into the equation of Kindsvater, Carter and Tracy (Eq. 12) and solved for Kindsvater's discharge coefficient, C_K. Section 1 as defined by Tracy and Carter (20), the position adopted for the measure of backwater, was found to disagree markedly with the position of maximum backwater observed in the flume at Utah State University, including the level flume case. Hence, elevations at sections I and II used by Skogerboe, Austin and Chang were substituted into Eq. 12. The mean values of C_K obtained for the standard conditions specified by Kindsvater, Carter and Tracy (11) are listed in Table 1. As data were available for only three different contraction ratios, each with a different length:width ratio, a plot as shown in Fig. 2a could not be obtained. However, the results do have the same trend as the curves of Fig. 2a, considering the different points of

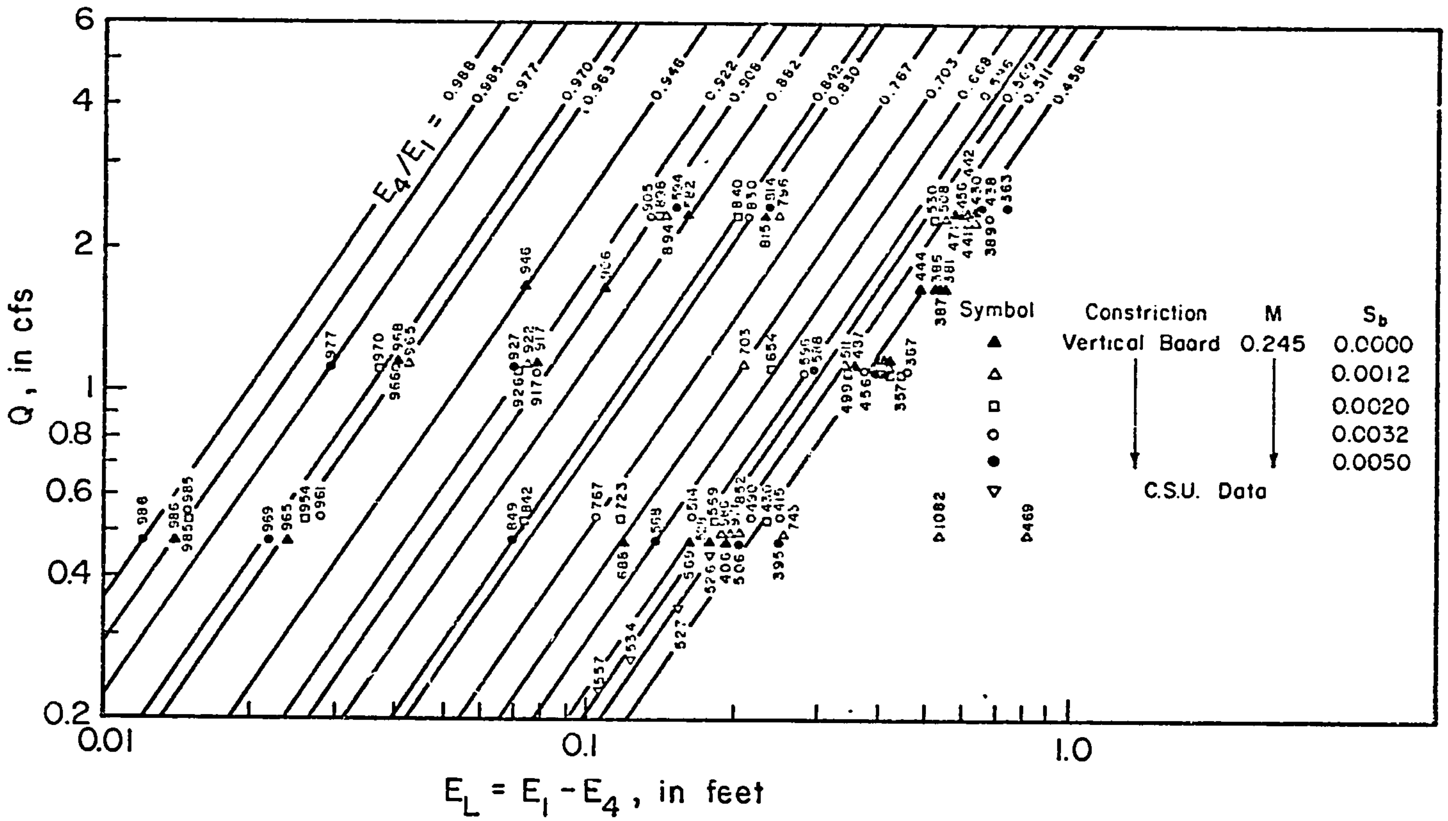

Figure 37. Abnormal stage-discharge data from Colorado State University report, reduced by Froude model laws and plotted with Utah State University data.

measurement adopted. By using the values of C_K where the standard conditions (Fig. 2a) were met, the data was able to be plotted in the form of Fig. 2b, showing the correction factor for the Froude number variation (see Fig. 38). Considering the scatter in the plotted points, it is felt this curve (Fig. 38) is not sufficiently different to that obtained by Kindsvater, Carter and Tracy to refute their analysis.

Table 1

Value of Kindsvater's Contraction Coefficient, From Utah State University Data

Contraction Ratio, m	$\frac{L}{b}$	C_K
0.7555	1.35	0.815
0.503	0.666	0.841
0.267	0.452	0.890

In a similar manner, the Utah State University data was substituted into the equation developed by the Bureau of Public Roads (Eq. 24) and solved for the total backwater coefficient, K^*. Any data obviously incorrect, together with data where supercritical flow occurred in the constriction, was eliminated. The value of K^* obtained, however, for a given constriction was far from constant, and differed markedly (Table 2) from the values obtained from the Bureau of Public Roads publication (Fig. 9). For a given channel slope and a given discharge, the value of K^* was observed to always increase as the depth at section I was increased. Hence, to compute the standard deviation of this data would be meaningless. The application of Eq. 25 to abnormal stage-discharge conditions is

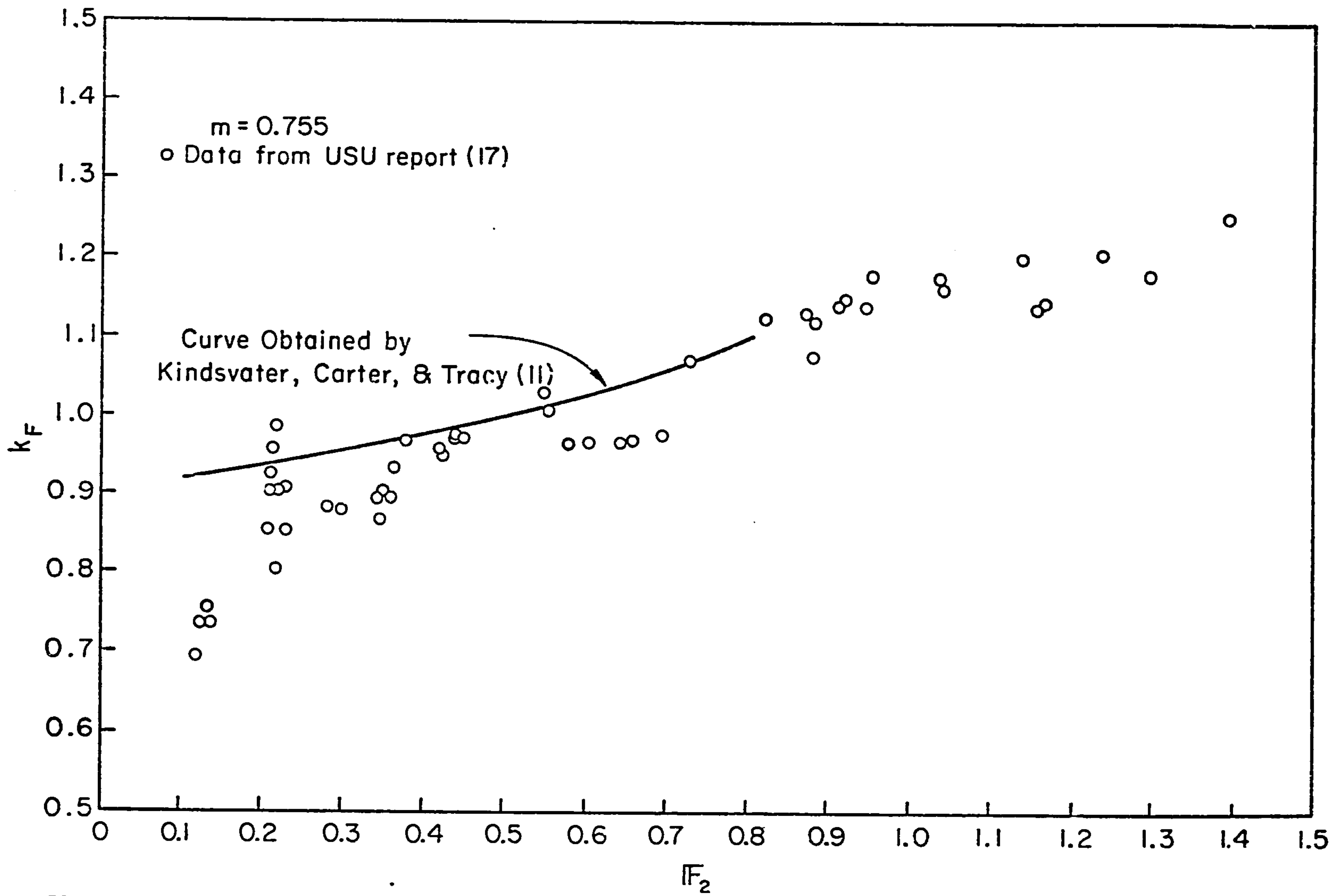

Figure 38. Correction factor for non-standard Froude number in Eq. 12. (Kindsvater, Carter and Tracy).

therefore obviously invalid. Data was not available to substitute into the proposed abnormal stage-discharge equation (Eq. 28).

Table 2

Value of Bureau of Public Roads Total Backwater Coefficient, From Utah State University Data

Contraction Ratio, m	Mean Value of K^*	BPR Value of K^*
0.755	1.33	1.8
0.503	2.26	1.15
0.267	2.23	0.55

Comparison of Different Methods

By extracting data from the Colorado State University report, it was possible to compare the accuracy of predicting discharge using the submerged flow equation with that obtained using either the Geological Survey equation developed by Kindsvater, Carter and Tracy, or the Bureau of Public Roads equation, developed by Bradley. Comparisons were made for both uniform flow (Table 3) and the abnormal stage-discharge condition (Table 4).

In using the Bureau of Public Roads equation, the precise values of the parameters of the equation were available. However, in using the equation of the Geological Survey, the value of the downstream depth parameter y_3 had not been measured. Instead, a downstream stagnation depth was used, this being the depth measured at the point of stagnation where the downstream face of the constriction meets the flume wall. Using this stagnation depth may introduce a slight error, tending to underpredict the actual discharge, since this depth is greater than y_3 , thereby giving a lower head loss.

Table 3

Comparison of Discharge Computed by Geological Survey Method, Bureau of Public Roads Method and Submerged Flow Equation - Uniform Flow

Depths, ft.			Channel	Discharges, cfs				Errors, %		
y_1	y_3	y_n	Slope	G.S.284	B.P.R.	Subflow	Actual	G.S.284	B.P.R.	Subflow
0.446	0.340	0.384	0.0012	0.988	1.132	1.583	1.72	42.6	34.2	8.0
0.250	0.204	0.224	0.0012	0.274	0.396	0.639	0.66	58.7	40.0	3.2
0.516	0.394	0.457	0.0012	1.331	1.250	1.920	2.05	35.2	39.0	6.3
0.327	0.199	0.232	0.0036	0.463	0.979	1.038	1.06	56.4	7.7	2.1*
0.532	0.295	0.382	0.0024	1.228	2.016	2.155	2.25	45.4	10.4	4.2*
0.242		0.202	0.0024		0.498	0.645	0.68		26.8	5.2
0.418	0.252	0.318	0.0024	0.786	1.333	1.499	1.57	49.9	15.1	4.5*
0.266		0.250	0.0008		0.292	0.641	0.51		42.7	-25.6
0.370	0.320	0.346	0.0008	0.597	0.508	1.065	0.87	31.2	41.6	-22.4
0.218		0.211	0.0004		0.129	0.422	0.40		67.8	-5.4
0.348		0.332	0.0004		0.352	0.912	0.63		44.1	-44.8
0.629		0.574	0.0004		1.404	2.483	2.40		41.5	-3.4

* Supercritical flow in constriction

Table 4

Comparison of Discharge Computed by Bureau of Public Roads Method and Submerged Flow Equation - Abnormal Stage-Discharge Condition

Depths, ft.		Discharges, cfs			Errors, %	
y_1	y_{1A}	B.P.R.	Subflow	Actual	B.P.R.	Subflow
0.809	0.527	4.71	4.79	5.00	5.8	4.2
0.817	0.576	4.76	4.78	5.00	4.8	4.4
0.821	0.624	4.66	4.74	5.00	6.8	5.2
0.838	0.673	4.60	4.74	5.00	8.0	5.2

Little data was available in the Colorado State University report for any particular constriction geometry with the abnormal stage-discharge flow condition. The available data represented Froude numbers in the constriction which were too high to allow solution by the Geological Survey method (11). The comparison in Table 4, therefore, is only between the Bureau of Public Roads equation and the submerged flow equation for the abnormal stage-discharge condition.

In all cases, the discharge predicted by the submerged flow analysis is closer to the actual discharge measured than the discharge predicted by either the BPR method or the USGS method. For uniform flow (Table 3), the relative superiority of the submerged flow equation for the given sample may be clearly seen. The three points where the submerged flow analysis gives significant error in prediction may be readily identified on Fig. 36, where the plotted points curve away from the straight line of approximate fit.

For the available abnormal stage-discharge condition data (Table 4), the discharges predicted by both the BPR method (4) and the submerged flow analysis do not differ significantly from each other or from the measured discharge. However, in view of the poor results obtained using the BPR method for uniform flow, it is conjectured that a greater range of data would reveal greater errors in the BPR method than in the method of submerged flow analysis.

Chapter 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

The dual intentions of this study have been to compare and analyze the interrelationship between the most significant existing methods for evaluating the effects of width constrictions on open channel flow, and to eliminate from the method of submerged flow analysis the constraints which have previously limited its utility in application to this problem.

Analyses devoted specifically to the bridge constriction problem date back to the nineteenth century. A review of methods currently in use has been presented, together with the method of submerged flow analysis which has been developed from recent studies on flow measuring flumes.

By writing a momentum equation between the section of maximum backwater and the contracted section of flow through a constriction, a theoretical submerged flow equation has been obtained.

$$Q_t = \frac{\sqrt{g/2}\; b(y_1-y_2)^{3/2}}{\sqrt{\frac{M(1-MS)(1-S)^2}{S(1+S)}}} \quad \ldots\ldots\ldots \qquad (43)$$

For a given constriction geometry and a constant submergence, the discharge is a function of $(y_1-y_2)^{3/2}$. Hence, the relationship for flow through a given constriction may be plotted on logarithmic paper with y_1-y_2 and Q as coordinates, yielding a series of lines of constant submergence, each having a slope of 3/2 .

The three works most commonly referred to for the analysis of the width constriction problem are:

(i) "Computation of Peak Discharge at Contractions," by C. E. Kindsvater, R. W. Carter and H. J. Tracy;

(ii) "Backwater Effects of Piers and Abutments," by H. K. Liu, J. N. Bradley and E. J. Plate; and

(iii) "Hydraulics of Bridge Waterways," by J. N. Bradley.

The equations presented in each of these publications can be expressed in the form of a submerged flow equation with a head loss term having an exponent of 3/2 . The equations may be plotted on submerged flow coordinates in terms of flow depths.

For a given submergence, the Froude number at a given section is constant. A relationship between submergence and Froude number had previously been developed by Liu, Bradley and Plate for the case of uniform flow in the unconstricted channel. By recognizing that normal depth will reoccur in the vicinity of the constriction, an alternative expression may be obtained which can be more readily expressed in the form of a submerged flow equation than the semi-empirical equation of Liu, Bradley and Plate.

The earlier analytical expressions for describing the effects of width constrictions may therefore be more generally expressed in the form of a submerged flow equation. By considering the depths at section I before and after placement of the constriction, this equation has the advantage over previous analytical methods of being independent of flow condition (uniform or non-uniform), flow regime (subcritical or supercritical) at the constriction and channel slope. This also ensures that the exponent n_1 is exactly equal to 3/2 , removing its dependence on the positions of depth measurement.

The only data available describing flow conditions before and after the placement of a width constriction are listed in the Colorado State University report by Liu, Bradley and Plate. Appropriate portions of this data were plotted on submerged flow coordinates, and the corresponding submerged flow equations obtained. The data was then substituted into the submerged flow equations, together with existing equations for discharge through a constriction, in order to compare the accuracy of discharge predictions. In all cases, the submerged flow equation made a closer prediction of the discharge actually measured.

Data taken at Utah State University was substituted into the equation of Kindsvater and Carter, together with the equation used by the Bureau of Public Roads, and solved for the respective coefficients. The results tended to verify the analysis of Kindsvater and Carter.

Conclusions

The methods outlined in this study are only a first step in evolving an accurate method for obtaining either the backwater due to, or peak discharge through, a bridge constriction in a natural stream. The specific results presented are only for constriction in rectangular channels, but the method has been shown to be applicable to any channel section and any type of constriction.

Of particular virtue is the method's applicability to any flow condition or regime and its independence of channel slope. The coefficient C_1 , and one exponent, n_2 , remain to be evaluated.

Earlier research indicates that these parameters are probably dependent upon the constriction geometry, as well as possibly being interrelated.

Prototype studies may indicate that the procedure necessary to obtain the coefficient and exponent of the submerged flow equation will be as tedious as that for obtaining coefficients in the existing methods. However, it may be anticipated that this will not be the case, as dependence on channel roughness and energy coefficients has been eliminated by considering only the upstream flow section, while dependence on the Froude number has been shown to be accounted for by the submergence parameter.

Recommendations

Given the wide range of applicability of the submerged flow method of analysis, it would appear justified to conclude that further research should be conducted to determine the exact relationships between the remaining parameters, such as wingwall shape, eccentricity, skew, etc., together with the submerged coefficient, C_1 , and submerged flow exponent, n_2 . The basis of these relationships may be established in an experimental flume by setting a particular flow condition and discharge before placing the constriction. However, the laboratory results should be checked, and where necessary modified through extensive prototype tests, before venturing to suggest that it should replace existing methods of peak discharge and backwater computation.

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