PB-223 944

EFFECT OF SETTLEMENT ON FLUME RATINGS

Tsu-yang Wu

Colorado State University

Prepared for:

Agency for International Development

August 1971

BIBLIOGRAPHIC DATA SHEET	1. Report No. 631.7-W959	3.	PB 223 944

4. Title and Subtitle

"Effect of Settlement on Flume Ratings"

5. Report Date: Aug 71

6.

7. Author(s): Tsu-yang Wu.

8. Performing Organization Rept. No.

9. Performing Organization Name and Address

Colorado State Univ.

10. Project/Task/Work Unit No. 931-11-120-115

11. Contract/Grant No. AID/csd-2460-[redacted]

12. Sponsoring Organization Name and Address

Department of State
Agency for International Development
Washington, D.C. 20523

13. Type of Report & Period Covered

14.

15. Supplementary Notes

16. Abstracts

Frequently, flow measuring flumes placed in unlined channels settle at the flume exit because of the scouring action of the high velocity flow leaving the flume. Usually, this settlement condition goes uncorrected in the field. A more satisfactory solution to the problem would be in evaluating the effect of this settlement upon the discharge rating of the flume.

Four cutthroat flumes were selected for hydraulic testing in the laboratory. These flumes had been previously rated with the flume floor horizontal. In this study, hydraulic data were collected under both free flow and submerged flow conditions for each flume at various degrees of flume floor slope.

The experimental results show that there is a very definite effect upon the discharge rating of a flume due to settlement. The amount of correction increases as the tilt angle increases. Also, the amount of correction increases as the flume length is increased.

17b. Identifiers/Open-Ended Terms

17c. COSATI Field/Group 631

18. Availability Statement

19. Security Class (This Report) UNCLASSIFIED

20. Security Class (This Page) UNCLASSIFIED

21. No. of Pages 100

22. Price $3.75

EFFECT OF SETTLEMENT ON FLUME RATINGS

Water Management Technical Report No. 12

by

Tsu-Yang Wu

Prepared under support of

United States Agency for International Development
Contract No. AID csd-2460
Improving Capacity of CUSUSWASH Universities
for Water Management for Agriculture

Colorado State University

Fort Collins, Colorado

August 1971

AET-71-72-TYW14

Reports published previously in this series are listed below. Copies can be obtained by contacting Mrs. Mary Fox, Engineering Research Center, Colorado State University, Fort Collins, Colorado 80521. The prices noted are effective as long as supplies last. After the supply of reports is exhausted, xerox copies can be provided at 10 cents per page.

No.	Title	Author	No. of Pages	Cost
1	Bibliography with Annotations on Water Diversion, Conveyance, and Application for Irrigation and Drainage, CER69-70KM3, Sept. '69.	K. Mahmood A.G. Mercer E.V. Richardson	165	$3.00
2	Organization of Water Management for Agricultural Production in West Pakistan (a Progress Report) ID70-71-1, May 1970	P.O. Foss J.A. Straayer R. Dildine A. Dwyer R. Schmidt	148	$3.00
3	Dye Dilution Method of Discharge Measurement, CER70-71WSL-EVR47, January 1971	W.S. Liang E.V. Richardson	36	$3.00
4	Water Management in West Pakistan, MISC-T-70-71RFS43 May, 1970	Robert Schmidt	167	$3.00
5	The Economics of Water Use, An Inquiry into the Economic Behavior of Farmers in West Pakistan, MISC-D-70-71DW44 March, 1971	Debebe Worku	176	$3.00
6	Pakistan Government and Administration: A Comprehensive Bibliography, ID70-71GNJ17, March, 1971	Garth N. Jones	114	$3.00
7	The Effect of Data Limitations on the Application of Systems Analysis to Water Resources Planning in Developing Countries, CED70-71LG35 May, 1971	Luis E. Garcia-Martinez	225	$3.00

No.	Title	Author	No. of Pages	Cost
8	The Problem of Under-Irrigation in West Pakistan: Research Studies and Needs, ID 70-71GNJ-RLA19	G. N. Jones R. L. Anderson	53	$3.00
9	Check-Drop-Energy Dissipator Structures in Irrigation Systems, AER 70-71, GVS-VTS-WRW3, May 1971	G. V. Skogerboe V. T. Somorey W. R. Walker	180	$3.00
10	Maximum Water Delivery in Irrigation	J. H. Duke, Jr.	213	$3.00
11	Flow in Sand-Bed Channels	K. Mahmood	292	$3.00

ABSTRACT

EFFECT OF SETTLEMENT ON FLUME RATINGS

Frequently, flow measuring flumes placed in unlined channels settle at the flume exit because of the scouring action of the high velocity flow leaving the flume. Usually, this settlement condition goes uncorrected in the field. A more satisfactory solution to the problem would be in evaluating the effect of this settlement upon the discharge rating of the flume.

Four cutthroat flumes were selected for hydraulic testing in the laboratory. These flumes had been previously rated with the flume floor horizontal. In this study, hydraulic data were collected under both free flow and submerged flow conditions for each flume at various degrees of flume floor slope.

The experimental results show that there is a very definite effect upon the discharge rating of a flume due to settlement. The amount of correction increases as the tilt angle increases. Also, the amount of correction increases as the flume length is increased.

Tsu-Uang Wu
Agricultural Engineering Department
Colorado State University
Fort Collins, Colorado 80521
August, 1971

ACKNOWLEDGMENTS

In completing this thesis, I owe a debt of gratitude to Mr. Gaylord V. Skogerboe, Associate Professor of Agricultural Engineering, who suggested this research and gave generously of his time and advice throughout the conduct of this study. His kindly guidance and encouragement are deeply appreciated.

I am also appreciative for the suggestions by my research committee: Mr. Donald L. Miles, Extension Irrigation Engineer, Agricultural Engineering Department, and Mr. Charles W. Thomas, Professor of Civil Engineering.

Thanks are also due to Mr. Wynn R. Walker, Ray S. Bennett, J. W. H. Barrett and Vason Bocckird for their help and painstaking efforts throughout the experimental program. Especially, the author is grateful to Mr. Walker for his assistance with the computer programming, which has made the analysis and results possible.

Special thanks go to my wife, Lichuan Lin Wu, whose constant encouragement and understanding helped in the completion of this research.

The effort reported herein was made possible through the support of the United States Agency for International Development under Contract No. AID/csd - 2460, "Improving Capacity of CUSUSWASH Universities for Water Management for Agriculture."

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

A:	Area of a section of the flow
A_1:	Area of flow at upstream gage section
A_2:	Area of flow at downstream gage section
B:	Width ratio b_2/b_1
b_1:	Width of the flume at upstream gage section
b_2:	Width of the flume at downstream gage section
b_0:	Width of the flume at throat section
C:	Coefficient for free flow discharge equation
C_1, C_2:	Coefficients in submerged flow discharge equation
E:	Energy head
E_1:	Energy head at upstream gage section
E_2:	Energy head at downstream gage section
F:	Force
Fr:	Froude Number
g:	Gravitational acceleration
H:	Upstream flow depth measured vertically to the bottom of the flume
h:	Total head on a section
ΔH:	Difference of depth measurement
h_f:	Friction loss
H_0:	Specific head

ℓ:	Distance between gage position to critical section, or two gage sections
L:	Length of cutthroad flume
L':	Length of the entrance converging section
M:	Momentum at a section
m:	Stage correction
n_1:	Exponent in free flow discharge equation
n_2:	Exponent in denominator of submerged flow discharge equation
P:	Pressure at a section
Q:	Discharge through the flume
q:	Discharge per linear foot of width
S:	Submergence
S_t:	Transition submergence
T:	Surface width for a flow section
t:	Time
V:	Volume of water in the control volume
W:	Width of a section
W_c:	Width of critical section
X:	Variable distance from a reference point
Y:	Depth of flow
Y_1:	Depth of flow at upstream gage point
Y_2:	Depth of flow at downstream gage point
Y_A:	Average hydraulic depth (A/T)

Y_c: Critical depth

$\overline{Y}$: Distance from free surface to centroid of the section area

Y_0: Depth at throat section

Z: Height from the datum to the bottom elevation of a section

α: Velocity distribution coefficient energy correction factor

β: Velocity distribution coefficient momentum correction factor

θ: Inclined or tilted angle

ρ: Fluid density

γ: Specific weight of fluid

CHAPTER I

INTRODUCTION

Flow measuring flumes

Water measuring devices have been very important in irrigation systems for many years. Recently, increased emphasis on water resource development to support rapidly increasing populations, with consequent expanding water demands by municipalities, industry, and agriculture, has rendered the optimum use of water resources extremely important. There is no doubt that water measuring devices will play an even greater role in the future allocation and distribution of water supplies. Water measuring devices are important for:

(1) successful business-like management;

(2) meeting legal obligations;

(3) water conservation; and

(4) insuring an equitable distribution of water.

There are many types of open channel water measuring devices available, among which the open channel flow measuring flumes are one of the most commonly used devices in irrigation systems. The characteristics of flow measuring flumes are:

(1) sturdy structures can be built which require little maintenance work;

(2) sufficient accuracy over a rather wide range of discharge is obtainable in most cases;

(3) they have no silting problems due to the increased velocities in the flume, thereby having low obstruction to transportation of suspended and bedload material through the flume; and

(4) they are suitable for permanent structures as well as for portable structures, depending upon the size.

A water measuring flume is an open channel structure containing a constricted section. The constriction can be constructed by raising the flume bottom or reducing the width between the flume walls. Although the discharge characteristics are the same for either type of constriction, a floor constriction is customarily regarded as a flow measuring weir and a wall constriction is designated as a flow measuring flume.

Problem

Although water measuring flumes have been intensively studied in the laboratory and are widely used in the field, there are some problems encountered in field installations. A very common problem is the difficulty of installing a flume so that the floor is horizontal both longitudinally (direction of flow) and transversely.

The flume may be tilted longitudinally or transversely. Tilting of the flume will affect the flow measurement. However, the longitudinal slope is not easily adjusted. Also, a positive slope flume floor sloping downward in the direction of flow has a more detrimental effect on the discharge measurement than a negative slope (12). Unfortunately, the positive slope condition may occur quite often in field installations of flumes in unlined open channels, because of the scouring action of the high velocity flow leaving the flume. The only satisfactory solutions, to date, for this problem include: raising the lower end of the flume so it is again level; placing a new level floor in the flume; or placing a liner in the existing flume and then grouting it into place. But usually, the problem is not corrected.

Purpose

Since many flumes installed in unlined channels experience settlement at the downstream end due to the scouring action of the high velocity jet leaving the flume, and because this settlement is usually not corrected in the field, a more satisfactory solution to the problem would be in evaluating the effect of this settlement (slope or tilt of flume floor) upon the discharge rating of the flume. The purpose of this study is to evaluate the effects of flume floor slope on the discharge rating as compared to the rating when the floor is horizontal. The type of flow measuring flume selected for study was the cutthroat flume because of its simple geometry.

Scope

A theoretical analysis was undertaken in an attempt to analyze the problem in a general manner. The intent of such an analysis is to predict the general format of the discharge rating as the slope of the flume floor is changed from horizontal to a positive slope.

Four cutthroat flumes were selected for hydraulic testing in the laboratory. These flumes had been previously rated with the flume floor horizontal. In this study, hydraulic data were collected under both free flow and submerged flow conditions for each flume at various degrees of flume floor slope. In all cases, the flume floor was sloped downward in the direction of flow (positive slope). The free flow and submerged flow ratings developed from the experimental data are compared with the ratings when the flume floor is horizontal.

CHAPTER II

DEVELOPMENT OF CUTTHROAT FLUME

Early Flume Studies

Studies on water measuring flumes have been carried out in many countries around the world. Other than studies conducted in the United States, which will be briefly described later, outstanding works (6, 17) have been reported in England by Jameson in 1925, Engel in 1933, and Linford in 1942; in India by Crump in 1922 and Inglis in 1928; in Italy by De Marchi in 1936, Contessini in 1936; Nebbia in 1936; and Citrini in 1939; by Khafagi in 1942 in Switzerland; and by Balloffet in 1955 in Argentina.

In the United States, experiments on flow measuring flumes, which were called Venturi flumes at the time, were reported by V. M. Cone (7) in 1917 at Colorado Agricultural Experiment Station, Colorado Agricultural and Mechanical College (currently designated Colorado State University). The flume had a converging entrance, a parallel-walled throat, and a diverging outlet. The walls were either vertical or inclined outward (trapezoidal), and the floor was horizontal.

Parshall (11) developed a Venturi flume which was later designated the Parshall flume. The Parshall flume has vertical walls, with an entrance section, throat section, and exit section. The floor of the entrance section is horizontal, whereas the floor in the throat section is sloped downward, and the exit section floor slopes upward in the direction of the flow. Parshall (11) developed this flume to cover a wide range of discharge conditions. Geometries have been developed for throat widths from 1-inch to 50-feet. Discharge ratings have been reported for both free flow and submerged flow conditions. Also, other investigators (18) have conducted additional studies on the Parshall flume.

A trapezoidal type of flow measuring flume, called the WSC flume, was reported by Chamberlain (5) in a publication of the Washington Agricultural Experiment Station, Washington State College. Trapezoidal flumes for open channel flow measurements were studied by A. R. Robinson and A. R. Chamberlain (15), and John A. Replogle (1969) in the United States. Also, laboratory studies on trapezoidal flumes have been reported by investigators in England (1) and Hungary (8).

Cutthroat Flume

Recently, investigators (19) have reported the development of a flow measuring flume which has a horizontal floor, with an entrance section and an exit section, but no throat length (Fig. 1). Hence, this flume has been called a cutthroat flume.

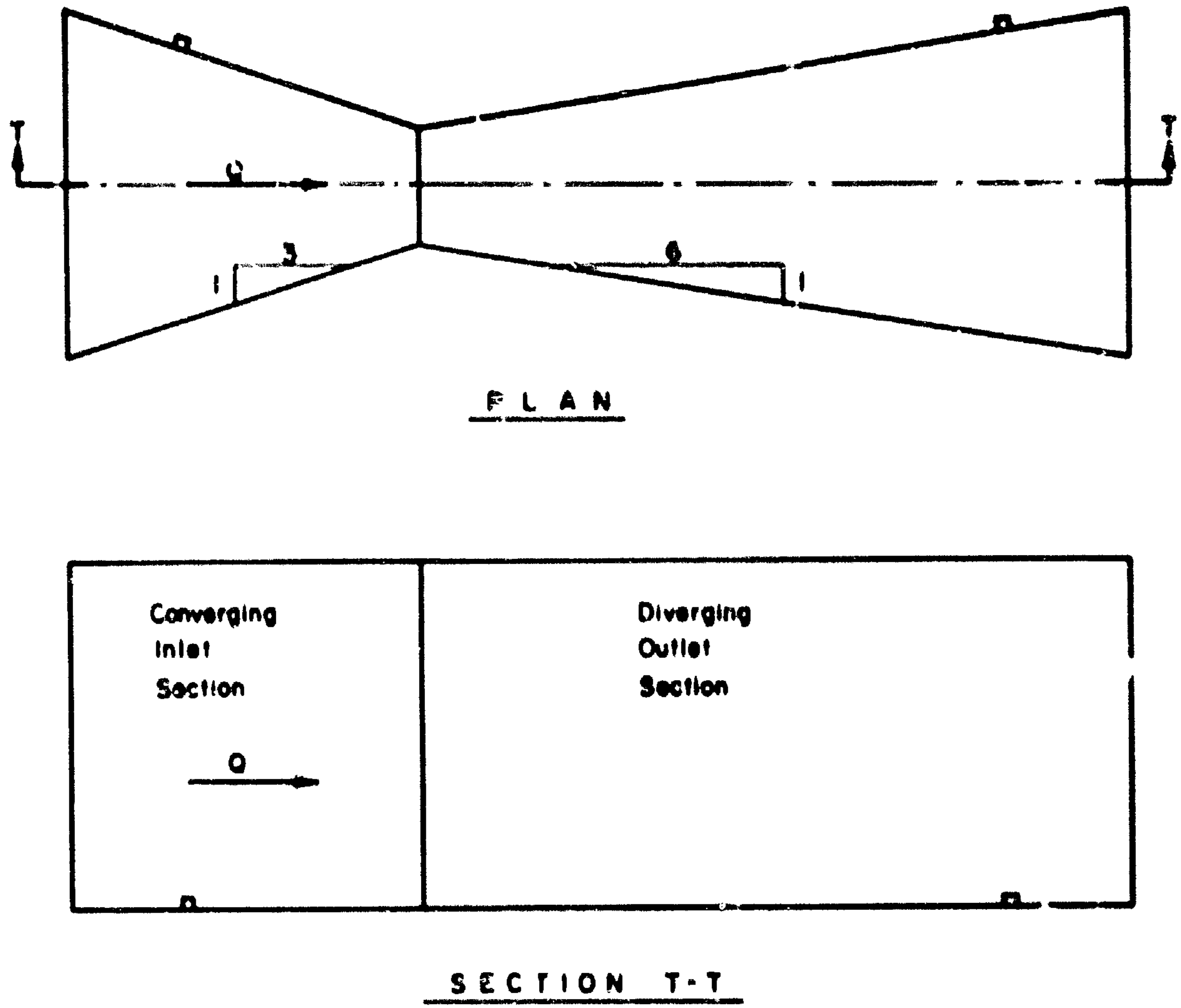


Figure 1. General sketch of cutthroat flume.

Ackers and Harrison (1) recommend a maximum convergence of 3:1 for a flume inlet section. The experimental work which led to the development of the cutthroat flume (19) indicated that such a convergence provided satisfactory hydraulic performance. Consequently, a 3:1 convergence (Fig. 1) is used for the entrance section.

Earlier studies by Hyatt (9) indicated that when the divergence of the flume exit section exceeded 6:1, flow separation would occur, and a major portion of the flow would adhere to one of the sidewalls. Although numerous divergences and lengths of exit section were tested during the development of the cutthroat flume (19), the 6:1 divergence (Fig. 1) proved most satisfactory as a balance between flow separation and fabrication costs.

Studies regarding the length of the throat section (19) showed that flow depths measured in the exit section of the flume resulted in more accurate submerged flow calibration curves than ratings employing flow depths measured in the throat section. The water surface profile changes rapidly in the throat section as compared with the exit section where the water surface profile is more nearly horizontal. Thus, there appeared to be no apparent advantage in having a throat section. Also, flow conditions in the exit section were improved by removing the throat section (19).

The most obvious advantage of a cutthroat flume is economy, since fabrication is facilitated by a flat-bottom (horizontal floor) and removal of the throat section. The initial investigations (19) were confined to a flume length, L, of 9 feet with throat widths, W, varying from 1-foot to 6-feet. For a constant flume length, various throat widths can be constructed by just moving the walls.

CHAPTER III

THEORETICAL DEDUCTION OF FLOW ANALYSIS

Basic Principles

Since the flow in a tilted water measuring flume is no more than open channel flow, the three basic principles of open channel flow will apply in the flow analysis. The three basic principles are conservation of mass (continuity equation), conservation of linear momentum (momentum equation), and conservation of energy (energy equation).

Continuity equation. The continuity equation is

$$\int_{cs} \rho \vec{v} \cdot d\vec{A} + \frac{\partial}{\partial t} \int_{cv} \rho \, dV = 0 \qquad (1)$$

where ρ is the mass density of the fluid, $\vec{v}$ is the velocity of the fluid, $d\vec{A}$ is the infinitesimal portion of a control surface, t is the time, and dV is an infinitesimal portion of the control volume. Under steady flow conditions, the mass density may be assumed as constant and the continuity equation can be simplified to

$$A_1 v_1 = A_2 v_2 \qquad (2)$$

where A_1 is the cross sectional area of flow in section 1, v_1 is the average velocity of flow in section 1, A_2 is the cross sectional area of flow in section 2, and v_2 is the average velocity of flow in section 2.

<u>Momentum equation.</u> The momentum equation is

$$\Sigma \vec{F} = \frac{\partial}{\partial t}\{ \int_{cv} \rho \vec{v} dV \} + \int_{cs} \rho \vec{v} (\vec{v} \cdot d\vec{A}) \tag{3}$$

where $\vec{F}$ is a force acting on the control volume. When the flow is steady and ρ is assumed to be constant, the equation becomes

$$\Sigma \vec{F} = \rho (v_2 - v_1) Q \tag{4}$$

where Q is the discharge of the flow.

<u>Energy equation.</u> The energy equation is

$$E = \frac{\partial}{\partial t} \int_{cv} \{1/2\, \rho(\vec{v} \cdot \vec{v}) + P + \gamma Z \} dV + \int_{cs} 1/2\rho(\vec{v} \cdot \vec{v})$$

$$(\vec{v} \cdot d\vec{A}) + \int_{cs} P(\vec{v} \cdot dA) + \int_{cs} \gamma Z(\vec{v} \cdot d\vec{A}) \tag{5}$$

where E is the net external energy on the control volume, P is the pressure of the fluid, γ is the unit weight of the fluid, and z is the height from the datum. When the flow is steady and ρ is assumed to be constant, the equation becomes

$$\frac{v_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{v_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 \tag{6}$$

or

$$\frac{v_1^2}{2g} + h_1 + Z_1 = \frac{v_2^2}{2g} + h_2 + Z_2 \tag{7}$$

where h_1 is the pressure head at section 1, h_2 is the pressure head at section 2, and g is the gravitational acceleration.

Critical flow. The critical flow condition, which is the state of flow having the minimum energy required to pass a certain discharge, is very important in the study of flow measuring flumes. The condition for a flow to be critical flow is

$$F_r = 1$$

where F_r is the Froude number defined by

$$F_r = \frac{v}{\sqrt{\frac{gY_A \cos\theta}{\alpha}}} \quad (8)$$

where v is the average velocity, Y_A is the average depth (A/T, where T is the water surface width), θ is the inclined angle of the canal bed, and α is the velocity distribution coefficient energy correction factor. For a rectangular canal section, the relation between discharge and critical flow depth is derived to be

$$y_c = \left\{\frac{\alpha q^2}{g\cos\theta}\right\}^{1/3} \quad (9)$$

where y_c is the critical flow depth and q is the discharge per unit length of width.

Free Flow Discharge

The flow in an open channel water measuring device has to be divided into two flow conditions, namely free flow and submerged flow. The flow is free flow when it passes a critical section somewhere inside the flume. In this case, only one flow depth measurement, at some location supstream of the critical section, is necessary for determining the discharge. If the flow depth everywhere is larger than critical flow, then the flow is said to be submerged and two flow depth measurements are required for determination of the discharge, with the second flow depth being measured somewhere downstream from the minimum flow depth section (17).

In Fig. 2, for free flow to occur, there should be a control section existing some place near the throat of the flume. Assume section (b) to be the control section, which has a width, W_c, and critical flow depth, y_c, and section (a) to be the measuring gage point, which has a width, W, and a flow depth, y. Then,

$$F_r = \frac{v_c}{\sqrt{\frac{g y_c \cos\theta}{\alpha}}} = 1 \quad . \qquad (10)$$

Assuming $\alpha = 1$, then

$$y_c \cos\theta = \frac{v_c^2}{g} \qquad (11)$$

or

$$y_c \cos\theta = \frac{Q^2}{g W^2 y_c^2} \qquad (12)$$

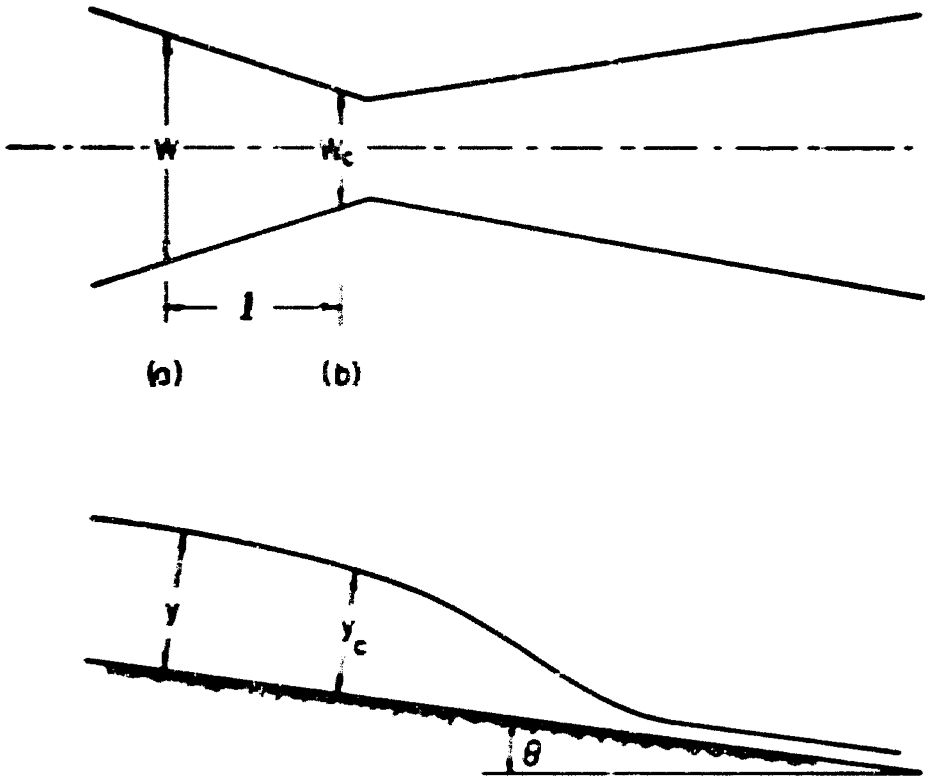


Figure 2. Free flow in the flume.

where Q is the discharge through the flume. Simplifying Eq. 12, or from Eq. 9,

$$y_c = \left(\frac{Q^2}{g W_c^2 \cos\theta}\right)^{1/3} \qquad (13)$$

or

$$y_c^2 = \frac{Q^{4/3}}{g^{2/3} W_c^{4/3} \cos^{2/3}\theta} \quad . \qquad (14)$$

Applying the energy equation to sections (a) and (b),

$$y_c \cos\theta + \frac{v_c^2}{2g} + h_f = y\cos\theta + \frac{v^2}{2g} + l \sin\theta \qquad (15)$$

where h_f is the loss of head between the sections and l is the distance between them.

Using Eq. 12 and Eq. 14, and rewriting v_c in terms of Q, W_c, and y_c, Eq. 15 can be changed to

$$\frac{3}{2}\,\frac{Q^{2/3}\cos^{2/3}\theta}{g^{1/3} W_c^{2/3}} = y\cos\theta + \frac{Q^2}{2gW^2y^2} + l\sin\theta - h_f \quad . \qquad (16)$$

The depth-discharge relations are given by Eq. 16. If h_f is negligible or known, the discharge, Q, can be determined by measuring only y.

For the practical use of Eq. 16, simplification is necessary. From dimensional analysis, it is possible to set

$$y = k\,\frac{Q^{2/3}}{g^{1/3} W^{2/3}} \qquad (17)$$

where k is a certain constant. Then,

$$Q^2 = \frac{gW^2y^3}{k^3} = \frac{gW^2y^3}{k_1} \qquad (18)$$

where k_1 is another constant which is equal to k^3. Putting Eq. 18 into Eq. 16 and simplifying,

$$\frac{3\cos^{2/3}\theta Q^{2/3}}{2g^{1/3}W_c^{2/3}} = (\cos\theta + \frac{1}{2k_1})y + l\sin\theta - h_f \quad . \qquad (19)$$

Since W_c, Q, and l are known values for this case, and if h_f is assumed to be negligible or known, Eq. 19 can be given in the form

$$C_0 Q^{2/3} = C_1 y + C_2 \qquad (20)$$

where

$$C_0 = \frac{3\cos^{2/3}\theta}{2g^{1/3}W_C^{2/3}} \qquad (21)$$

$$C_1 = \cos\theta + \frac{1}{2k_1} \qquad (22)$$

$$C_2 = l\sin\theta - h_f \qquad (23)$$

Then

$$Q = (\frac{C_1}{C_0}y + \frac{C_2}{C_0})^{3/2} \qquad (24)$$

or

$$Q = (C_1'y + C_2')^{3/2} \qquad (25)$$

where

$$C_1' = \frac{C_1}{C_0} \text{ and } C_2' = \frac{C_2}{C_0} \tag{26}$$

or

$$Q = C(y + m)^{3/2}$$

where

$$C = C_1'^{\,3/2} \text{ and } m = \frac{C_2'}{C_1'} \quad . \tag{28}$$

Equation 27 implies that the experimental form of the discharge formula should be in the format

$$Q = C(y + m)^n \tag{29}$$

where C, m, and n are constants for any particular flume geometry which must be determined by experiment.

It is to be noted that m is zero (assume $h_f = 0$) when the flume is set horizontally (since $\theta = 0$, $\sin\theta = 0$). So the discharge formula becomes $Q = C\,y^n$, which is consistent with the experimental formula obtained by many researchers. Also, it is interesting to note that Eq. 27 is the same form as the common discharge-stage rating curve relation for a stream gaging station.

In Eq. 27 for tilted flumes, allowing a certain permissible error over a fixed range of discharge, which is usually acceptable in practical use, the discharge formula could be given in the form

$$Q = Cy^n \tag{30}$$

where C and n have to be determined experimentally for each flume geometry.

The flow depth, y, has to be measured perpendicularly to the flume bottom according to the derivation of the theoretical formula. But this is not practical for field application except for the case $\theta = 0$ (horizontal floor). If it is possible to assume that the water surface is parallel to the flume floor in the vicinity where the flow depth is measured, then it is possible to set

$$y = H \cos \theta \tag{31}$$

where H is the depth measured vertically at the point. For Eq. 30 to be valid, a rather long gradually converging entrance of the flume is desirable.

An argument regarding the control section on the tilted flume should be given, since the discharge formula is primarily based upon the control section. In Fig. 3, x is the distance from the throat of the flume to a given section (x), θ is the tilted angle, k is ratio of the contraction of the walls, b is the width at the given section, y_b is the depth at the throat, and y_x is the depth at section (x). For simplicity, assuming hydro-static pressure distribution in the flume, and if the control occurs at section (x), then the following relations can be developed.

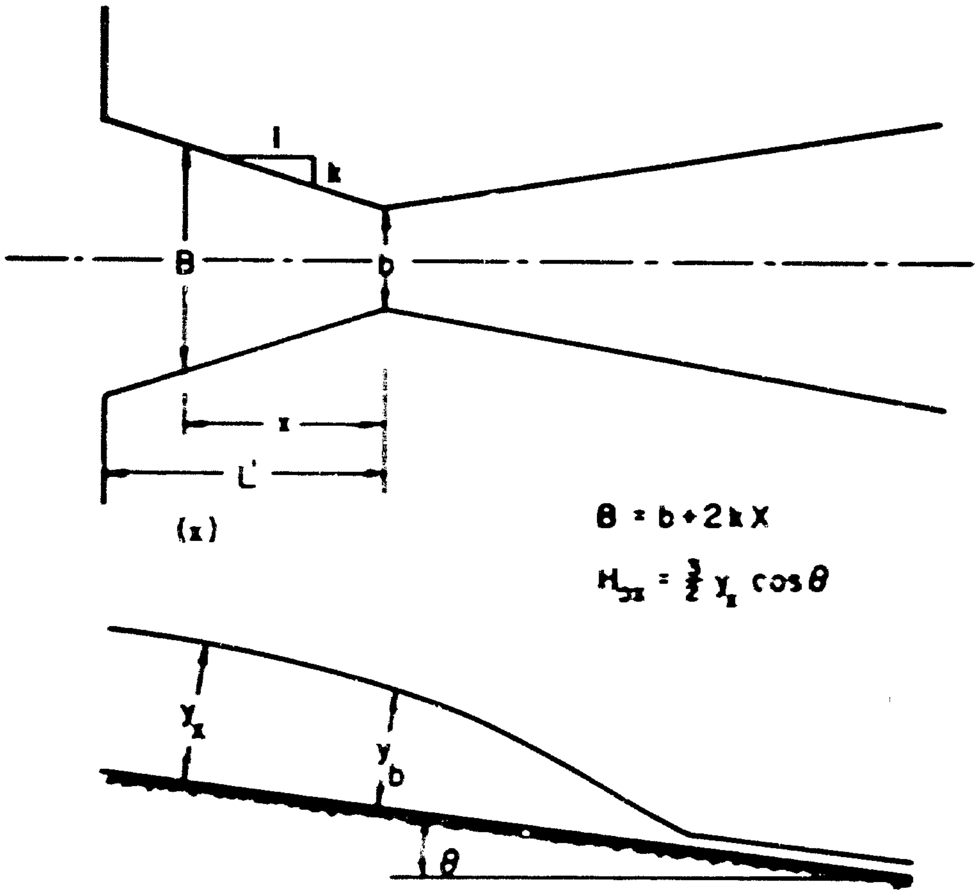


Figure 3. Control section for the flume.

$$H_{cx} = \frac{3Q^{2/3}\cos^{2/3}\theta}{2g^{1/3}W_c^{2/3}} = \frac{2Q^{2/3}\cos^{2/3}\theta}{g^{1/3}(b+2kX)^{2/3}} \tag{32}$$

where H_{cx} is the specific head (specific energy) at section (x).

Setting h_x to be the total head at section (x), with the datum elevation being the flume bottom elevation at the throat, then

$$h_x = X\sin\theta + \frac{3Q^{2/3}\cos^{2/3}\theta}{2g^{1/3}(b+2kX)^{2/3}} \quad . \tag{33}$$

The highest h_x that a given section in the flume has will be the possible minimum head for the flume to pass the discharge Q. Namely, the control section occurs at the section which gives the maximum h_x in Eq. 33. Since θ, Q, and g are regarded as fixed values or constants, Eq. 33 can be written as

$$h_x = a_1x + \frac{a_2}{(a_3 + a_4x)m} \tag{34}$$

where a_1, a_2, a_3, a_4, and m are positive constants.

Setting $\frac{dh_x}{dx} = 0$ and simplifying

$$(a_3 + a_4x)^{m+1} = \frac{a_2a_4m}{a_1} \tag{35}$$

and

$$\frac{d^2h_x}{dx^2} = \frac{m(m+1)a_2a_4^2}{(a_3+a_4x)^{m+2}} \quad . \tag{36}$$

Within the range $0 < x < L'$, where L' is the length of converging portion of the flume, Eq. 36 gives only positive values.

Hence, Eq. 35 gives only minimum h_x. From this argument, it is known that maximum h_x that is wanted has to be at one of the extremes of x, namely $x = 0$ or $x = L'$. Therefore, the control section occurs only at the throat of the flume or at the upstream edge of the flume.

There are three cases to determine the position of the control section as shown in Fig. 4.

Case 1. $\left\{\frac{dh_x}{dx}\right\}_{x>L'} = 0$, $h_{max} = h_{x(x=0)}$, control at neck

Case 2. $\left\{\frac{dh_x}{dx}\right\}_{x<0} = 0$, $h_{max} = h_{x(x=L)}$, control at upstream

Case 3. $\left\{\frac{dh_x}{dx}\right\}_{0<x<L'} = 0$, compare $h_{x(x=0)}$ and $h_{x(x=L')}$, the larger of which gives the control section.

By the above argument, the control section occurs either at the neck or the upstream edge of the flume and does not change location continuously. Consequently, it is clear that if the tilted angle is not large, the control section remains at the same position as that of the nontilted flume. In this case, Eq. 29 or its convenient form, Eq. 30, are valid and the coefficient and exponent in the formula will change continuously according to the change of tilted angle. If the tilted angle is sufficiently large, the discharge formula

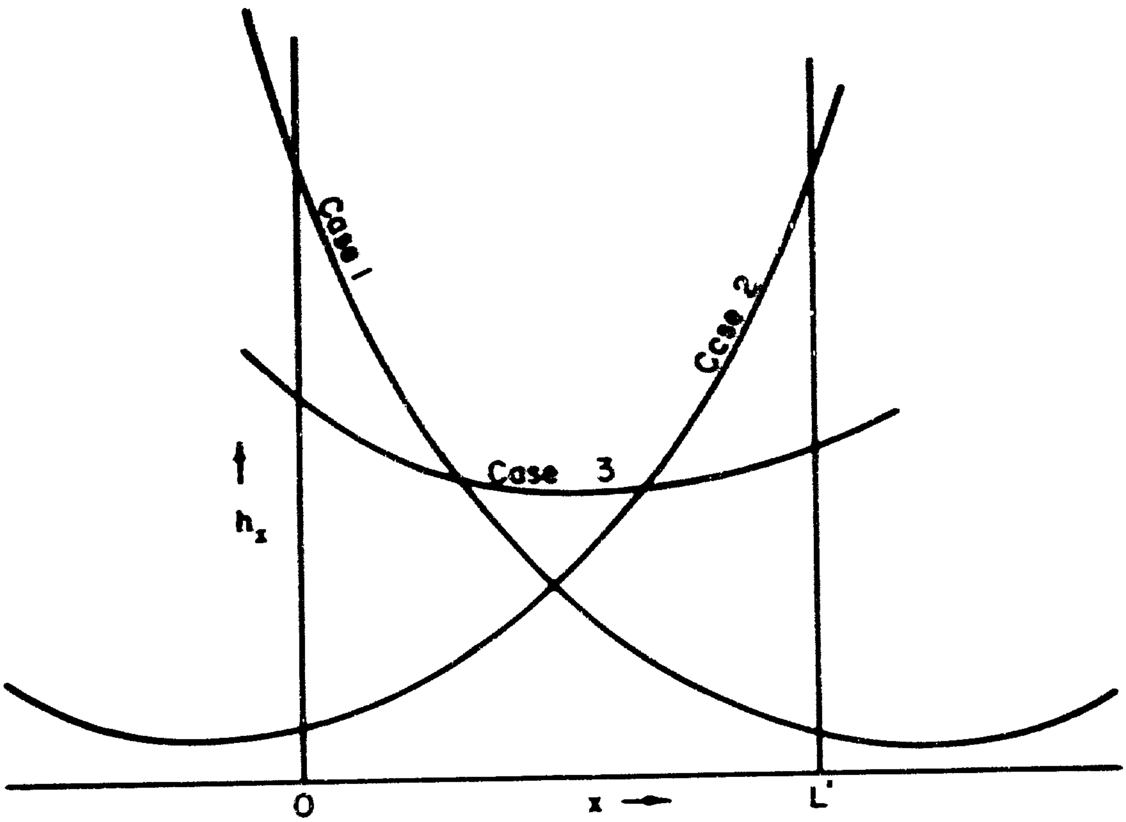


Figure 4. Cases for control section.

becomes independent of tilted angle but still takes the form of

$$Q = Cy^n \qquad . \qquad (30)$$

Submerged Flow Discharge

For submerged flow, two flow depth measurements are required. Theoretically, the energy equation can be used for deriving the discharge formula, but owing to the converging and diverging of the flow, there is some uncertainty as to the accuracy of using the energy equation in this problem. Usually, the momentum equation can be used in solving the problem when the energy equation is not sufficiently valid. Here, for completeness and for comparison, the problem will be dealt with in three forms: (1) by the energy equation; (2) by the momentum equation; and (3) by a combination of the energy and momentum equations.

Energy equation. The geometry of the flume is shown in Fig. 5. Then

$$y_1 \cos\theta + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g} + h_f - \ell_{1-2} \sin\theta \qquad . \qquad (37)$$

Using the continuity equation expressed in terms of the discharge and solving for Q, the following relationship is obtained.

$$Q = \sqrt{\frac{2g\{(y_1 - y_2)\cos\theta - h_f + \ell_{1-2}\sin\theta\}}{\dfrac{b_1^2 y_1^2 - b_2^2 y_1^2}{b_1^2 y_1^2 b_2^2 y_2^2}}} \qquad . \qquad (38)$$

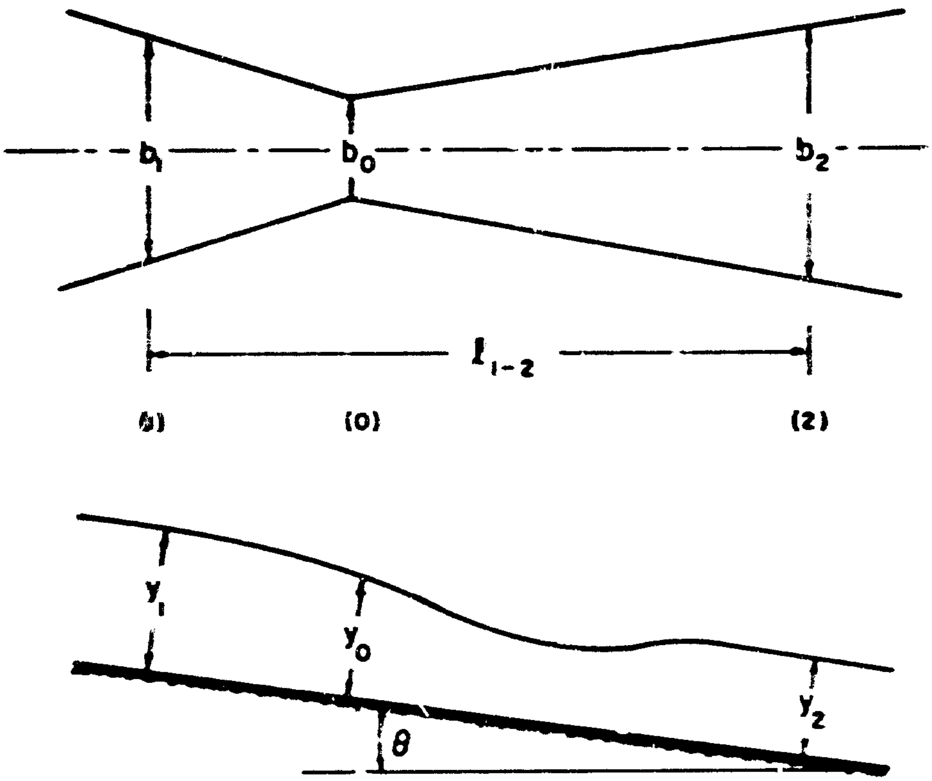


Figure 5. Submerged flow in a flow measuring flume.

Assume that $h_f + \ell_{1-2}\sin\theta$ is a function of $(y_1 - y_2)$. Namely, $h_f = c_1(y_1 - y_2)$, where c_1 is a constant which is dependent upon the flume geometry. Then,

$$Q = \sqrt{\frac{(2g\cos\theta - C_1)(y_1 - y_2)}{\frac{b_2^2 y_2^2 - b_1^2 y_1^2}{b_1^2 y_1^2 b_2^2 y_2^2}}} \quad . \tag{39}$$

Let

$$\frac{b_2}{b_1} = B, \ \frac{y_2}{y_1} = S \quad . \tag{40}$$

Multiplying both the numerator and the denominator by $(y_1 - y_2)$ and simplifying,

$$Q = \frac{(2g\cos\theta - C_1)^{1/2} b_2 (y_1 - y_2)^{3/2}}{\sqrt{\frac{(1 - S^2 B^2)(1 - S)^2}{S^2}}} \tag{41}$$

<u>Momentum equation.</u> In Fig. 5,

$$\gamma\bar{y}_1 A_1 \cos\theta - \gamma\bar{y}_2 A_2 \cos\theta - F_f + W\sin\theta + F_w + F_b$$
$$= \rho\beta_2 \frac{Q_2}{A_1} - \rho\beta_1 \frac{Q^2}{A_2} \tag{42}$$

where $\bar{y}_1$ and $\bar{y}_2$ are the distances from the free surface to the centroid of the flow areas. β_1 and β_2 are the velocity distribution coefficient momentum correction factors, F_f is the friction force, W is the weight of water within the control volume, F_w is the force from the walls, and F_b is the force from the bottom. Assume that

β_1 and β_2 = 1. F_f is negligible, and $W\sin\theta = F_b$ (forces on the bottom floor balance themselves). Assuming

$$F_w = \gamma\left(\frac{y_1+y_2}{2}\right)\cos\theta\left(\frac{y_1+y_2}{4}\right)(b_1 - b_2)\lambda$$

$$= \frac{\gamma(b_1 - b_2)}{8}(y_1 + y_2)^2\lambda \qquad (43)$$

where λ is a correction coefficient. Then, the momentum equation, Eq. 42, becomes

$$1/2\,\gamma b_1\cos\theta y_1^2 - 1/2\,\gamma b_2\cos\theta y_2^2 + \frac{\gamma(b_1 - b_2)}{8}(y_1 + y_2)^2$$

$$= Q\left(\frac{b_1y_1 - b_2y_2}{b_1y_1b_2y_2}\right) \qquad (44)$$

Since b_1, b_2, and $\cos\theta$ are fixed values, set

$$1/2b_1\cos\theta = k_1, \quad 1/2\,b_2\cos\theta = k_2$$

$$\frac{\lambda(b_1 - b_2)}{8} = k_3 \qquad (45)$$

Then

$$k_1y_1^2 - k_2y_2^2 + k_3(y_1 + y_2)^2 = \frac{Q^2}{g}\left(\frac{b_1y_1 - b_2y_2}{b_1y_1b_2y_2}\right). \qquad (46)$$

Looking at the algebra form of the left side of the above equation, it is possible to write the momentum relation in the following form.

$$Q^2 = \frac{g\{(m_1y_1 + m_2y_2)(m_3y_1 - m_4y_2)\}}{\dfrac{b_1y_1 - b_2y_2}{b_1y_1b_2y_2}} \qquad (47)$$

where m_1, m_2, m_3, and m_4 generally will be some fixed positive value for a fixed flume geometry. Upon multiplying the numerator and denominator by $(m_3 y_1 - m_4 y_2)^2$, setting $b_2/b_1 = B$, and $y_2/y_1 = S$, and simplifying,

$$Q = \frac{g^{1/2} b_2 (m_3 y_1 - m_4 y_2)^{3/2}}{\sqrt{\dfrac{(1-BS)(m_3 - m_4 S)^2}{S(m_1 + m_2 S)}}} \quad . \qquad (48)$$

Allowing some error in determining the discharge and setting, $m_3 = m_4$, Eq. 48 will be in the following form, which is the same as previously derived by others (19), when $\theta = 0$ and assuming $F_w = y_1$.

$$Q = \frac{(g/2)^{1/2} \, b_2 (y_1 - y_2)^{3/2}}{\sqrt{\dfrac{(1-BS)(1-S)^2}{S(1+S)}}} \quad . \qquad (49)$$

<u>Combination of energy and momentum equations.</u> In this method, the energy equation is used to describe the flow condition upstream from the throat and the momentum equation for describing flow conditions downstream from the throat. Using Fig. 5, neglecting h_f, and applying the energy equation between section (1) and the throat section, it is known that

$$Q^2 = \frac{2g\cos\theta(y_1 - y_0)}{\dfrac{b_1^2 \, y_1^2 - b_0^2 \, y_0^2}{b_0^2 \, b_1^2 \, y_1^2 \, y_0^2}} \quad . \qquad (50)$$

Applying the momentum equation between the throat and section (2), it is possible to write

$$Q^2 = \frac{g(k_1 y_0^2 + k_2 y_0 y_2 - k_3 y_2^2)}{\dfrac{b_0 y_0 - b_2 y_2}{b_2 y_2 b_0 y_0}} \quad . \tag{51}$$

Elimination of y_0 from Eq. 50 and Eq. 51 will give the discharge as a function of y_1 and y_2 as well as the other geometric variables such as b_1, b_2, and θ. But it is not an eacy task to obtain a discharge formula. A graphical solution to the problem is possible and will be described here. As shown in Fig. 6, y_1, y_0 and Q are plotted as parameters according to Eq. 50 in the upper portion, and y_2, y_0 and Q are plotted as parameters according to Eq. 51 in the lower portion of the graph. To determine the discharge from measurements of $y_1 = a$ and $y_2 = b$, two horizontal lines at $y_1 = a$ and $y_2 = b$ are to be drawn as shown in Fig. 5. A vertical line representing y_0 moves transversly. When this y_0 line intercepts $y_1 = a$ and $y_2 = b$ and reads the same Q in both parts of the graph, the Q so determined is the discharge required.

By the foregoing discussion, it is clear that the discharge formula for submerged flow is to be in the form

$$Q = \frac{C(y_1 - y_2)^{3/2}}{f(B, S)} \tag{52}$$

where C is a constant.

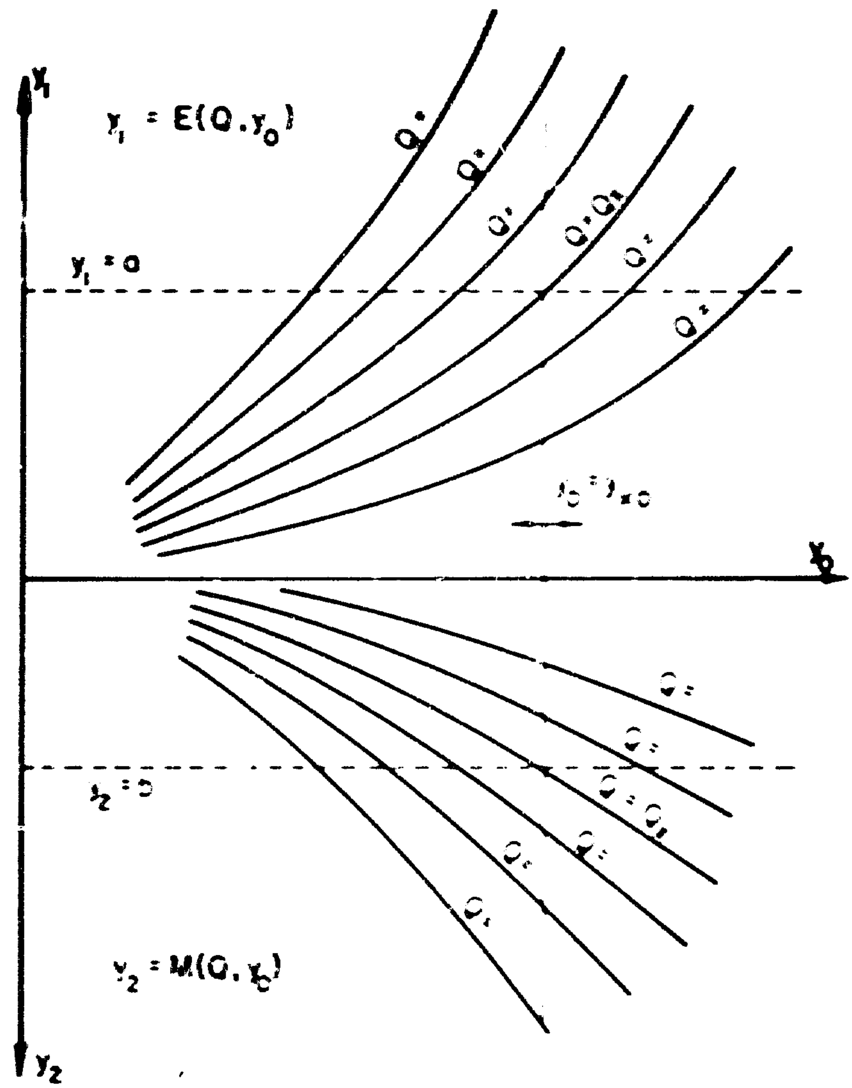


Figure 6. Schematic graph for determining the discharge through a flume by the energy-momentum equation.

The numerator is quite consistent in both the energy and momentum equations, while the denominator has a little different form depending upon which formula is used. Since the coefficients within each formula are usually determined experimentally, it is believed that either formula will be sufficient for evaluating the format of the discharge rating.

Equation 52 suggests that the experimental formula has to be in the form

$$Q = \frac{C(y_1 - y_2)^n}{f(B, S)} \quad . \tag{53}$$

By dimensional analysis, and introducing F_{rm}, S, and y/y_m as dimensionless pi-terms (F_{rm} is the Froude number at minimum flow depth, S is the submergence, and y_m is the minimum flow depth), Skogerboe, Hyatt, and Eggleston (17) were able to develop an experimental form of discharge formula for non-tilted flumes. Also, they were able to show that their formula is quite consistent with theoretical formulas. The experimental formula (17) has the form

$$Q = \frac{C_1(\Delta y)^{n_1}}{\{-(\text{Log}S + C_2)\}^{n_2}} \tag{54}$$

where C_1 and C_2 are constants, n_1 is the free flow exponent, and n_2 is the submerged flow exponent.

On considering the general form of the discharge formula which is given by Eq. 52, it is true that Eq. 54 will be valid when the flume is tilted. However, the constants in the discharge equation (Eq. 54) have to be determined experimentally for each flume geometry over a practical range of flume bottom slopes.

Corrections for Tilted Flumes

As mentioned earlier, methods for correcting the discharge rating of a non-tilted flume due to settlement at the flume exit is highly desirable. First of all, the free flow case will be discussed. Before calculating corrections, whether or not the control section remains at its original position must be evaluated by the argument for locating the critical section given previously in the section "Free flow discharge." The procedure is to calculate x by Eq. 35 in order to determine which of the three cases the problem belongs, noting that Q for the maximum and minimum discharges of the channel have to be used in Eq. 33, which is related to Eq. 35. If the analysis shows that the location of the control section remains unchanged, the corrections can be accomplished by the procedure described below.

In the derivation of Eq. 33, k_1 is independent of the tilted angle θ. Therefore, it is reasonable to assume k_1 remains the same for the case of a tilted flume. Setting $\theta = 0$ and neglecting h_f, k_1 can be calculated from Eq. 19 as

$$k_1 = \frac{g^{1/3}\ W_c^{2/3}\ y}{3Q^{2/3} - 2\ g^{1/3} W_c^{2/3}\ y} \tag{55}$$

The nontilted flume discharge formula is

$$Q = C_{non} y^{3/2} \tag{56}$$

where C_{non} is the free flow coefficient for the non-tilted (horizontal) flume. Combining Eqs. 55 and 56 gives

$$k_1 = \frac{g^{1/3} W_c^{2/3}}{3C_{non}^{2/3} - 2g^{1/3} W_c^{2/3}} \tag{57}$$

Using the above expression and the argument used in developing Eqs. 19, 20 and 27, the C and for the tilted flume discharge formula (Eq. 27) is calculated as

$$C = \frac{\left(2g^{1/3} W_c^{2/3} (\cos\theta - 1) + 3C_{non}^{2/3}\right)^{3/2}}{3^{3/2} \cos\theta} \tag{58}$$

$$m = \frac{2 l \sin\theta\, g^{1/3} W_c^{2/3}}{2g^{1/3} W_c^{2/3} (\cos\theta - 1) + 3C_{non}^{2/3}} \ . \tag{59}$$

Thus, the theoretical formula for the tilted flume can be obtained from the corresponding non-tilted flume based upon its geometry and the tilt angle. With regard to developing a general procedure for determining the corrections to be applied for tilting flumes, the following general formulas for C and m can be reasonably used.

$$C = \frac{\{2g^{1/2n} W_c^{1/n} (\cos\theta - 1) + 3C_{non}^{1/n}\}^n}{3^n \cos\theta} \qquad (60)$$

$$m = \frac{2l \sin\theta g^{1/2n} W_c^{1/n}}{2g^{1/2n} W_c^{1/n} (\cos\theta - 1) + 3C_{non}^{1/n}} \qquad (61)$$

At this point, it should be mentioned that Eqs. 60 and 61 are valid for the case of flumes having negative slope (upstream settlement). It is also very interesting to know that the above correction method can be extended for use in any type of flow measuring flume. Also, it is important to realize that the depth y is to be measured normal to the flume floor.

When the control section shifts to the flume entrance, the above corrections are no longer valid. Due to the complete change in location of the control section, new discharge formulas based upon the new control section location must be derived. This can be accomplished by using the energy equation.

Corrections for evaluating submerged flow in a tilted flume are not so straight-forward as the corrections for free flow conditions because of the assumptions made in deriving the discharge formula being much more complicated. The development of such submerged flow corrections is still possible, though.

Setting $\theta = 0$ in Eq. 38 and setting this equation equal to the already known nontilted discharge formula, the loss of head can be evaluated as a function of the known constant in the non-tilted discharge formula. Then, placing the value of h_f already described for the non-tilted case back into Eq. 38, the discharge for the tilted flume can now be calculated.

In using the momentum equation, the correction is to be determined by setting $\theta = 0$ and obtaining Eq. 48 for the non-tilted flume by following the procedure described in the derivation of Eq. 48. Then, the only unknown, λ, which is involved in determining k can be evaluated. Using this value of λ, along with the tilted angle, the corresponding values of m_1, m_2, m_3, and m_4, in Eq. 48 can be calculated. Thus, the discharge for the tilted flume can then be obtained. This same procedure for determining submerged flow corrections can be used for flow measuring flumes other than cutthroat flumes.

Transition Submergence

The formulas derived previously are termed free flow and submerged flow discharge formulas. The free flow formula is only valid in the free flow range where the tailwater does not raise the water level in the constriction above critical depth at the control section. Theoretically, the submerged flow formula is valid for all ranges of flow, free flow as well as submerged flow, since the energy or momentum theory applies everywhere in open channel flow. Even the experimental form of the submerged flow formula is logically valid over the entire range of submerged flow and free flow. However, the submerged flow formulas are provided only for the submerged flow case due to the complexity and lack of accuracy as compared with the free flow formula. Therefore, attention must be paid to the upstream-downstream water-depth relations when using the discharge formulas, free flow or submerged flow. It is also to be mentioned, by the way, that the experimental range for submerged flow formula is desirable as much as concentrated in the submerged range in order to get better accuracy within the applying range.

It is clear that there are two cases of flow in water measuring flumes, namely free flow and submerged flow.

The submergence, S, in a flow measuring flume is defined by

$$S = y_2/y_1 \tag{62}$$

where y_1 is a flow depth measured upstream from the flume throat and y_2 is a flow depth measured downstream from the flume throat.

Provided that the submerged flow formula and the free flow formula for a particular flume geometry are sufficiently accurate, the transition submergence can be computed by setting the free flow equation equal to the submerged flow equation, thereby obtaining a value of S, which is the transition submergence for the flume. If the exponents (n_1) are assumed to be the same in the free flow and the submerged flow equations, the transition submergence will be given by

$$C = f(S_t)\,(1 - S_t)^{n_1} \tag{63}$$

where C and n_1 are the coefficient and exponent in the free flow formula, S_t is the transition submergence, and $f(S_t)$ is a function of S_t depending upon the type of submerged flow formula. When the energy form of the submerged flow formula is used, $f(S_t)$ is

$$f(S_t) = \frac{C'}{\sqrt{\dfrac{(1-S^2B^2)(1 - S_t)^2}{S_t^2}}} \tag{64}$$

and when the dimensional analytical form (17) of the formula is used,

$$f(S_t) = \frac{C_1}{\{-(\mathrm{Log}\,S_t + C_2)\}^{n_2}} \tag{65}$$

Some discussion should be given regarding the application of Eq. 63 for obtaining the transition submergence. Both the free flow and submerged flow equations are not perfectly true hydraulically and they are no more than two independent experimental formulas as long as some theoretical unknowns are involved. Therefore, it cannot be expected that a perfect solution for the transition submergence can be obtained from Eq. 63. Letting

$$F = f(S_t)(1 - S_t)^{n_1} \tag{66}$$

Fig. 7 displays the characteristics of the solution to Eq. 63 where the solution of S_t is inside the case of

$$F = C \tag{67}$$

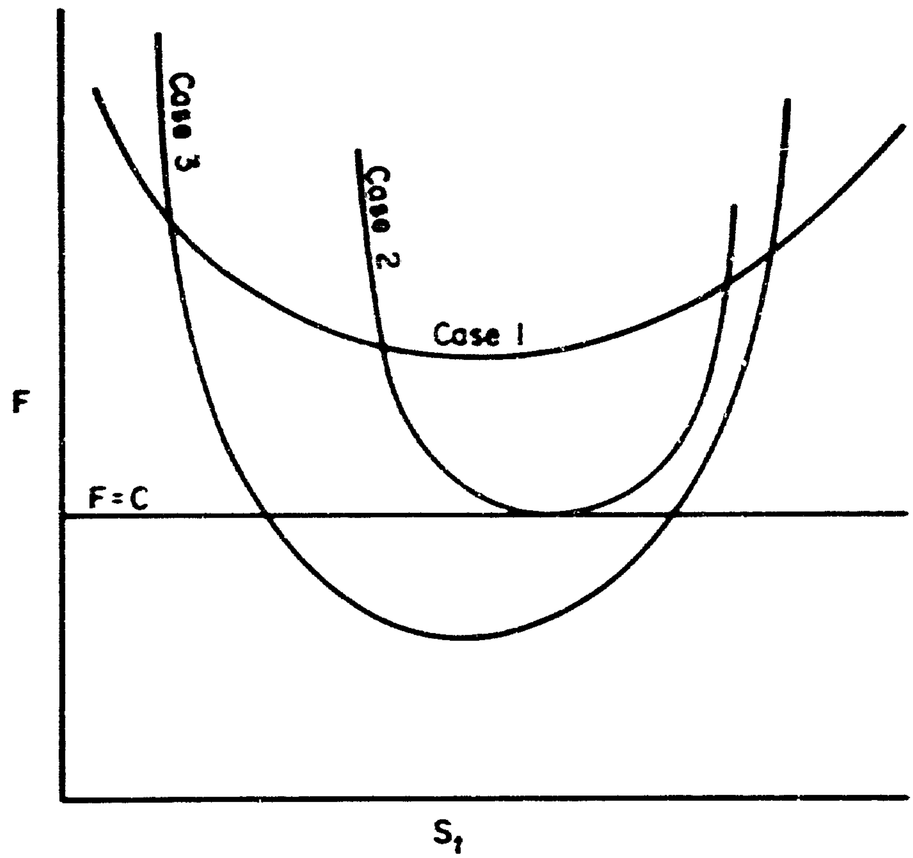


Figure 7. Functional relationship for the transition submergence.

Since F is a function independent of C, there are three cases for the solution of S_t. The solutions are: (1) no solutions as the case 1 in Fig. 7; (2) only one solution as shown by case 2; and (3) more than one solution (generally two) as shown by case 3. When there are two solutions, it is logical that one of the solutions is sufficiently close to the true transition submergence since the discharge should be the same for both formulas. If only one solution is obtained, it is assumed to be the solution for S_t. If more than one solution is obtained, judgment gained from available literature, experience, or experimental observations is required. However, the solutions give some help in determining the transition submergence. When there is no solution, there is no way of logically determining the transition submergence, but a rather good estimate in the lowest point in case 3 (Fig. 7) is the transition submergence.

CHAPTER IV

EXPERIMENTAL DEVELOPMENT OF FLOW ANALYSIS

Free Flow Discharge

Under free flow condition, the discharge, Q, through a flow measuring flume depends only upon the upstream depth of flow. Experiments on free flow discharge calibrations have been conducted by many earlier researchers. The results show that the free flow equation takes the form of

$$Q = CH^{n_1} \tag{68}$$

which is quite consistent with the theoretically developed format of the free flow equation. Many discharge equations, tables, or charts are available for many types of flow measuring flumes. For a cutthroat flume having a length of 9-feet, the free flow discharge equation (19) is

$$Q = Ch_a^{1.56} \tag{69}$$

where h_a is the depth of flow at the gage in the entrance section. The value of C for each size of flume (throat width) may be obtained from the following equation.

$$C = 3.5\, W^{1.025} \tag{70}$$

where W is the throat width in feet. The discharge is given in units of cubic feet per second (cfs). By combining Eqs. 69 and 70, the free flow discharge can be obtained for the rectangular cutthroat flume for throat widths from 1 to 6 feet and having a total length of 9-feet. Tables are also provided for practical use. Trapezoidal cutthroat flumes were also studied, with discharge equations and tables being provided (19).

To obtain the free flow discharge equation from experimental data is rather simple. The determination of the free flow coefficient, C, and the free flow exponent, n_1, can be obtained by curve fitting with the least squares method or by plotting Q against H (or h_a) on logarithmic paper, with the slope of the straight line being n_1, whereas C is the value of Q for H = 1.

Submerged Flow Discharge

Submerged flow exists in a flow measuring flume when a change in flow depth downstream from the flume causes a change in flow depth upstream of the flume throat for any particular constant discharge. Not much emphasis was placed on submerged flow in water measuring flumes by earlier investigators, but it is very important from a practical point of view.

To obtain a submerged flow discharge equation from experimental data is rather complicated as compared to developing the free flow discharge equation. In the first place, the form of the equation has to be determined. The form can be obtained in various ways: theoretically developed forms, dimensional analytical forms, or other statistical regression forms. Once the format of the formula is determined, the discharge equation can be obtained by curve fitting using the least squares method or any other convenient method. The theoretical forms were discussed in the previous chapter and other statistical regression methods are considered to be out of the scope of this study. The dimensional analytical method for determining the submerged flow discharge equation, which was developed by Skogerboe, Hyatt, and Eggleston (17), will be used in this study and is described below.

In Fig. 8, the variables involved in the determination of discharge for a particular flume can be written as

$$Q = f(g, y_1, y_2, S) \qquad (71)$$

where S is the submergence and y_1 and y_2 are the upstream and downstream flow depths, respectively. With five independent quantities and two dimensions, three pi-terms are necessary (9). The pi-terms are taken as follows.

$$\pi_1 = F_m = \frac{V_m}{\sqrt{g y_m}} = \frac{Q}{W g^{1/2} y_m^{3/2}} \qquad (72)$$

$$\pi_2 = S = y_2 / y_1 \qquad (73)$$

$$\pi_3 = \frac{\Delta y}{y_m} = \frac{(y_1 - y_2)}{y_m} \qquad (74)$$

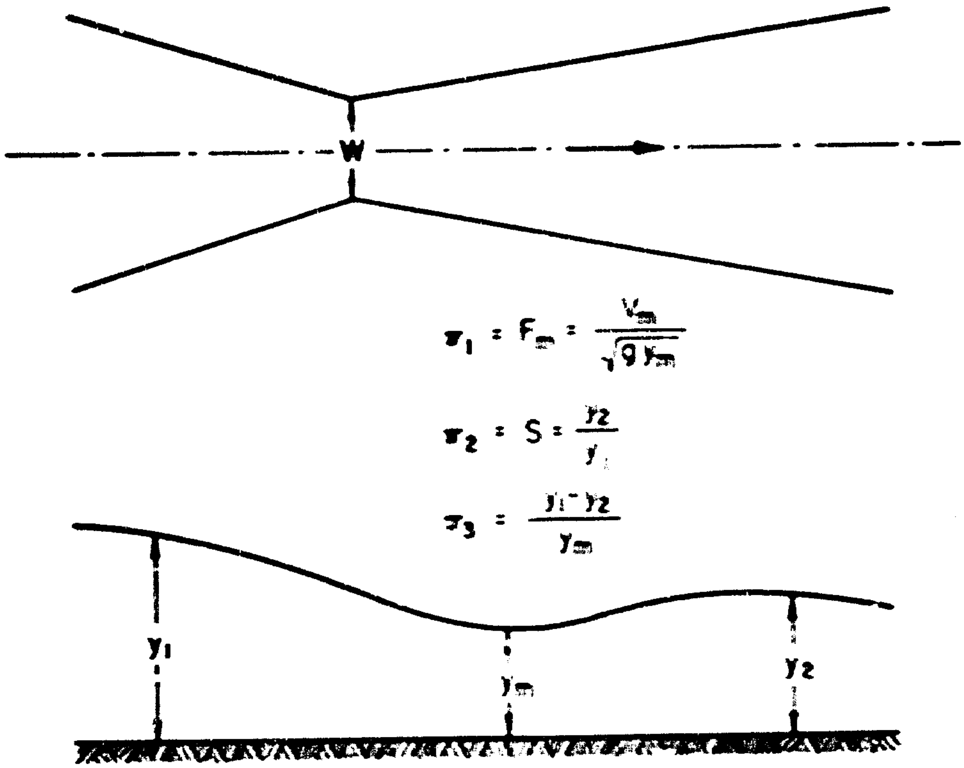


Figure 8. Definition sketch of submerged flow in a flume.

where W is the throat width of the flume, y_m is the minimum flow depth in the flume, F_m is the Froude number for a rectangular section corresponding to the flow depth y_m, and V_m is mean flow velocity at the cross-section having the minimum flow depth.

Using the submerged flow data collected by Hyatt (9), the logarithm of π_2 can be plotted against π_3 as shown in Fig. 9. The curve can be approximated by a straight line with some sacrifice in accuracy of the submerged flow calibration plot, thereby allowing the following relation to be obtained,

$$\text{Log}S = - k_1 \frac{y_1 - y_2}{y_m} - C_2 \tag{75}$$

where k_1, m_1, and C_2 are constants for any particular flume geometry.

A logarithmic plot of π_1 against π_3 will yield a straight-line relationship which can be combined with Eq. 75 to yield an approximate submerged flow discharge equation. In so doing, the discharge equation will contain $(y_1 - y_2)^{3/2}$ because F_m contains $y_m^{3/2}$.

A plot of y_m against $(y_1 - y_2)$ can be prepared on logarithmic paper with Q as the third variable (Fig. 10). An equation relating the three variables can be developed by plotting the value of y_m when $(y_1 - y_2)$ is equal to one for each line of constant discharge. These intercepts, designated C_m, can be plotted against Q as shown in Fig. 11. The empirical equation resulting from Fig. 10 has the form

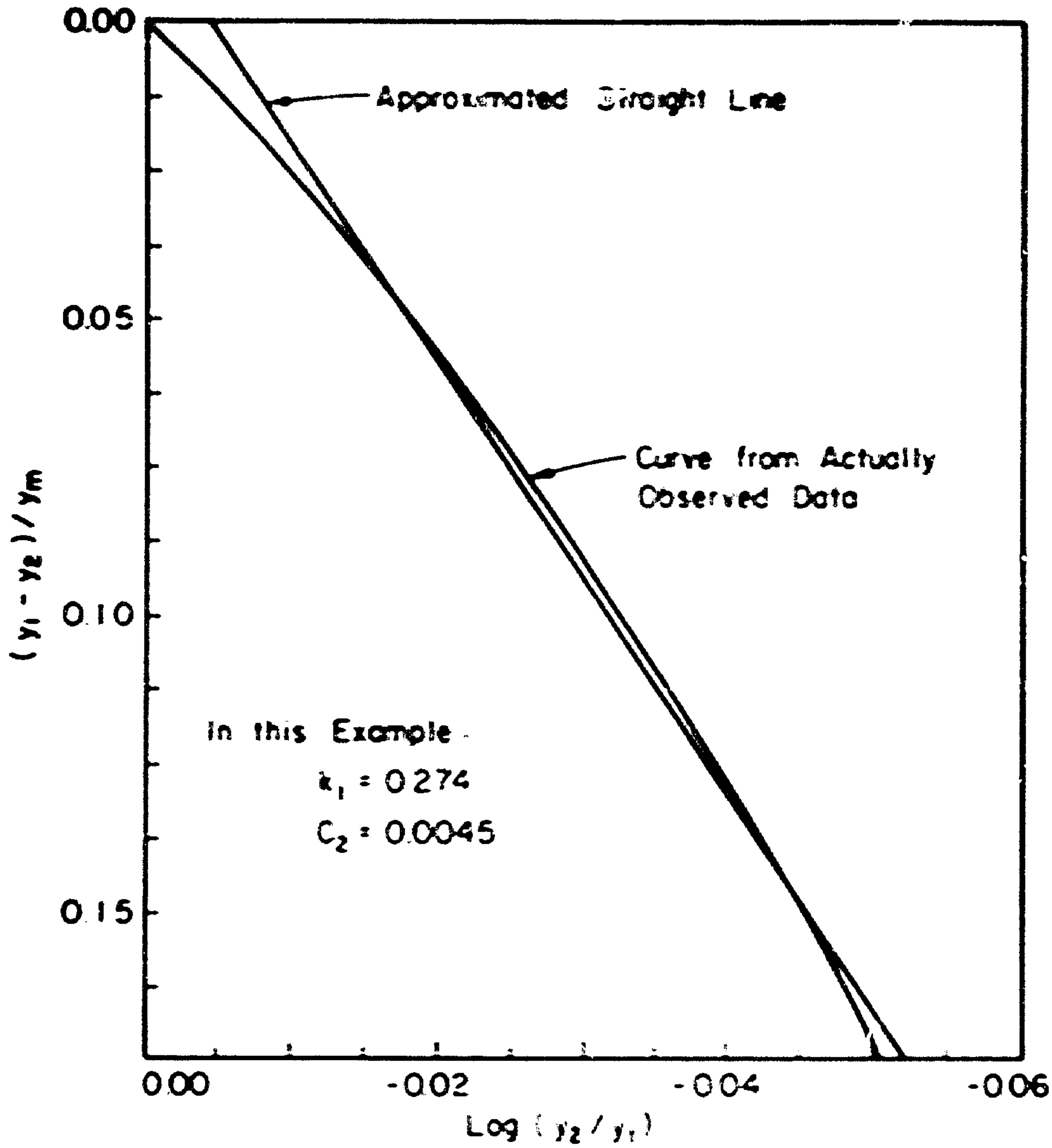


Figure 9. Typical relationship between τ_2 and τ_3.

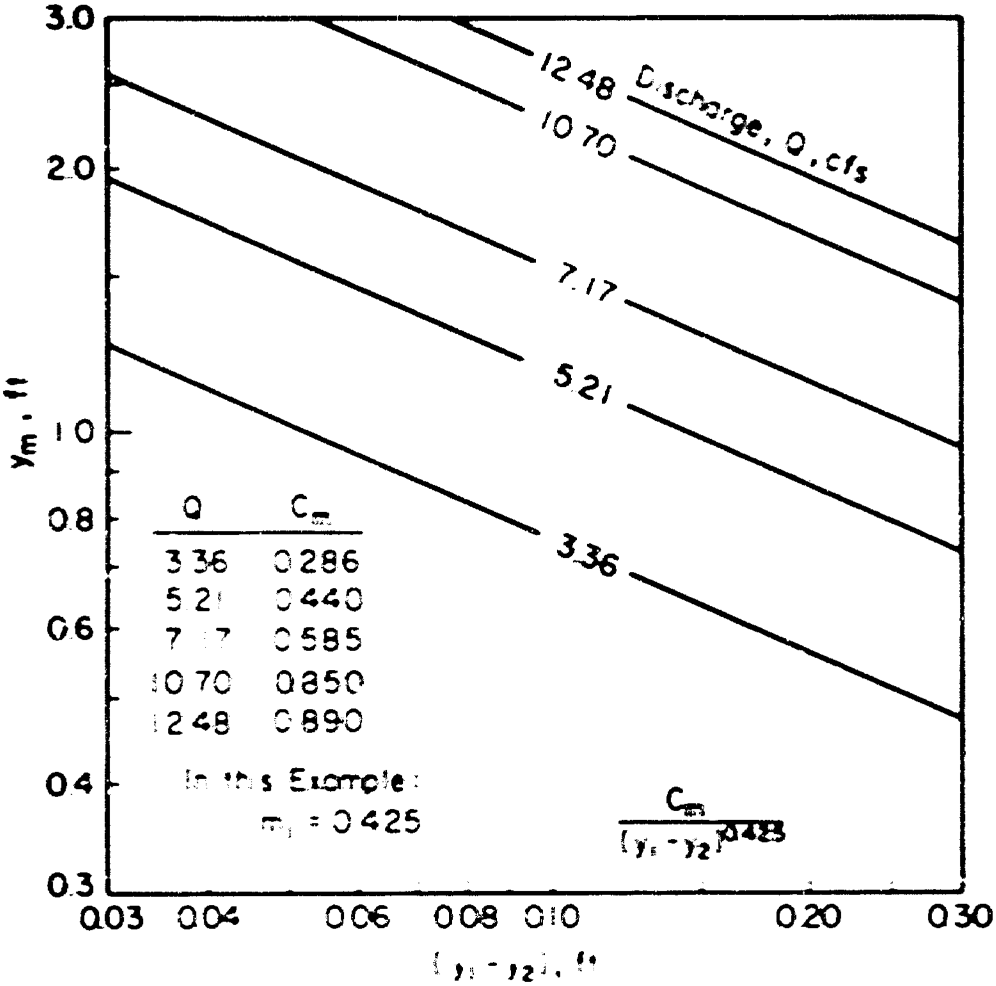


Figure 10. Plot of minimum flow depth, change in water surface elevation, and discharge.

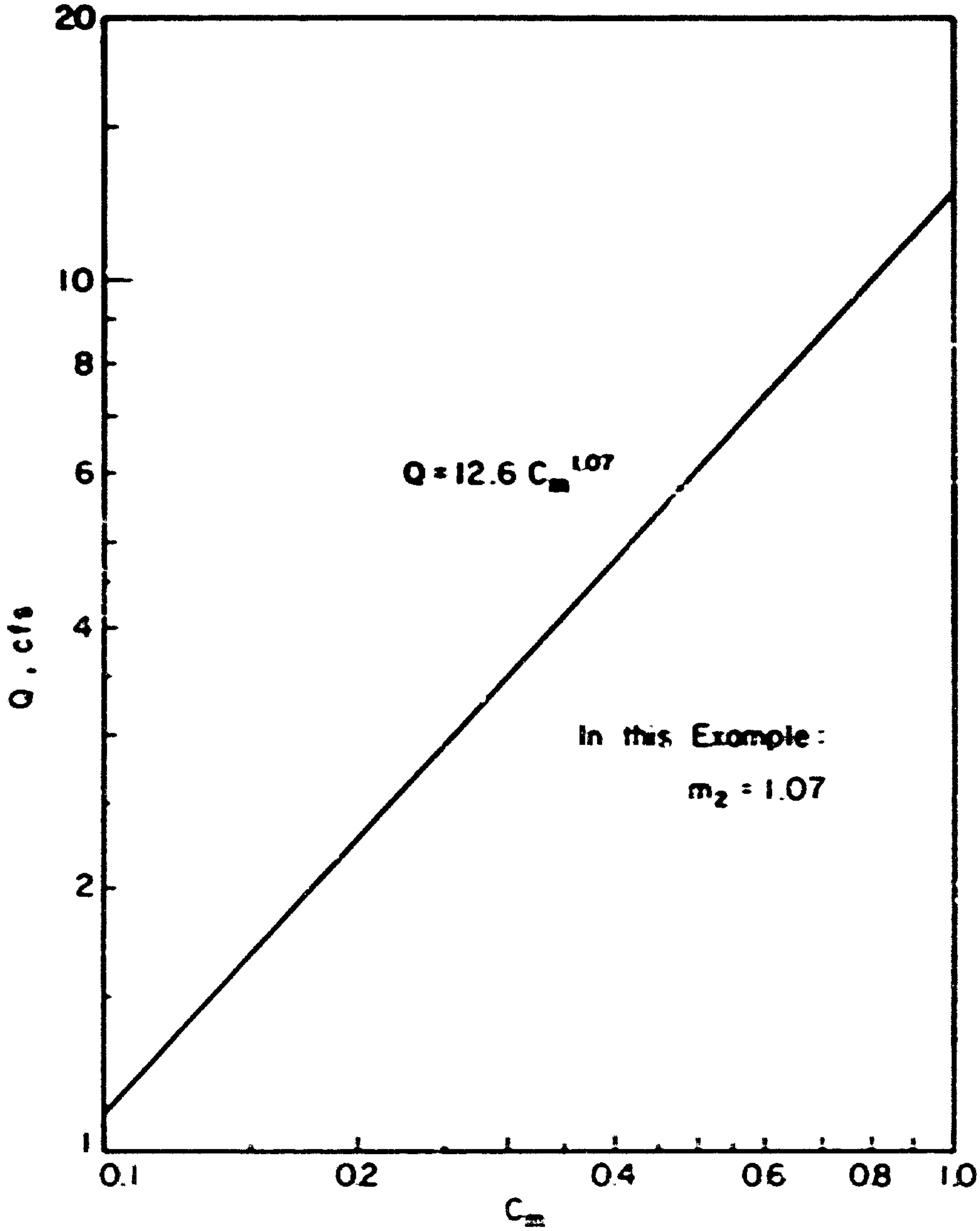


Figure 11. Development of relationship between minimum flow depth, change in water surface elevation, and discharge.

$$y_m = \frac{C_m}{(y_1 - y_2)^{m_1}} \tag{76}$$

where m_1 is a certain constant. The empirical equation resulting from Fig. 7 (11) has the form

$$Q = k_2\, C_m^{\;m_2} \tag{77}$$

where k_2 and m_2 are constants for the particular geometry under study. Combining Eqs. 76 and 77

$$Q = k_2 y_m^{\;m_2} (y_1 - y_2)^{m_3} \tag{78}$$

where

$$m_3 = m_1 m_2$$

An approximate submerged flow discharge equation can be obtained by combining Eqs. 75 and 78

$$Q = \frac{C_1 (y_1 - y_2)^{n_1}}{\{-(\mathrm{Log}S - C_2)\}^{n_2}} \tag{79}$$

where

$$n_2 = m_2 \tag{80}$$

$$C_1 = k_2 k_1^{\;m_2} \tag{81}$$

$$n_1 = m_2 + m_3 \tag{82}$$

Equation 79 is the general form of the submerged flow discharge equation derived from dimensional analysis combined with a graphical and analytical approach.

The transition submergence can be obtained by setting Eq. 79 equal to Eq. 68. Thus, the following expression will give the transition submergence, which is the same result as described at the end of the previous chapter.

$$\frac{C_1}{C} = \frac{(-\text{Log}S_t - C_2)^{n_2}}{(1 - S_t)^{n_1}} \quad . \qquad (83)$$

In order to solve for S_t from Eq. 83, a trial and error procedure must be used.

The general form of the submerged flow discharge equation, Eq. 79, can be represented in graphical form as shown in Fig. 12. Also, the graph (Fig. 12) can portray the free flow discharge equation as well.

Now, if C_2 is set equal to zero in Eq. 79, the submerged flow discharge equation becomes

$$Q = \frac{C_1(y_1 - y_2)^{n_1}}{(-\log S)^{n_2}} \quad . \qquad (84)$$

In calibrating any particular flow measuring flume, the value of n_1 is obtained from a plot of the free flow data. Thus, in Eq. 84, the value of C_1 and n_2 must be determined from a plot of the submerged flow data. This can be accomplished by determining the discharge

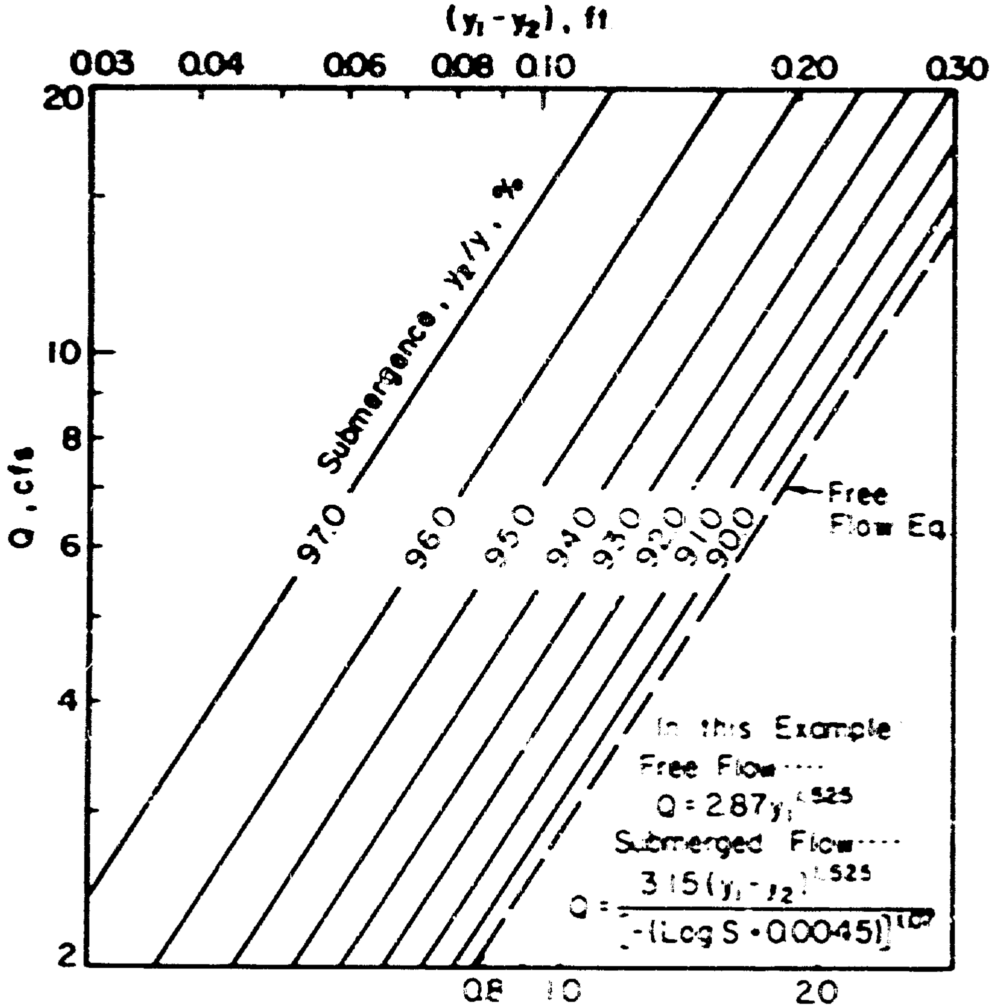


Figure 12. Free flow and submerged flow calibration for a flume.

intercept at $y_1 - y_2 = 1.0$ by the symbol $Q_{\Delta y=1}$ and recognizing that $(y_1 - y_2)^{n_1}$ is equal to one when $y_1 - y_2 = 1.0$, Eq. 84 can be reduced to

$$Q_{\Delta y=1} = \frac{C_1}{(-\log S)^{n_2}} \tag{85}$$

By plotting $Q_{\Delta y=1}$ against -log S on logarithmic paper, a linear relationship will result where C_1 is the value of $Q_{\Delta y=1}$ at -log S = 1 and n_2 is the slope of the straight line. A typical relationship is shown in Fig. 13.

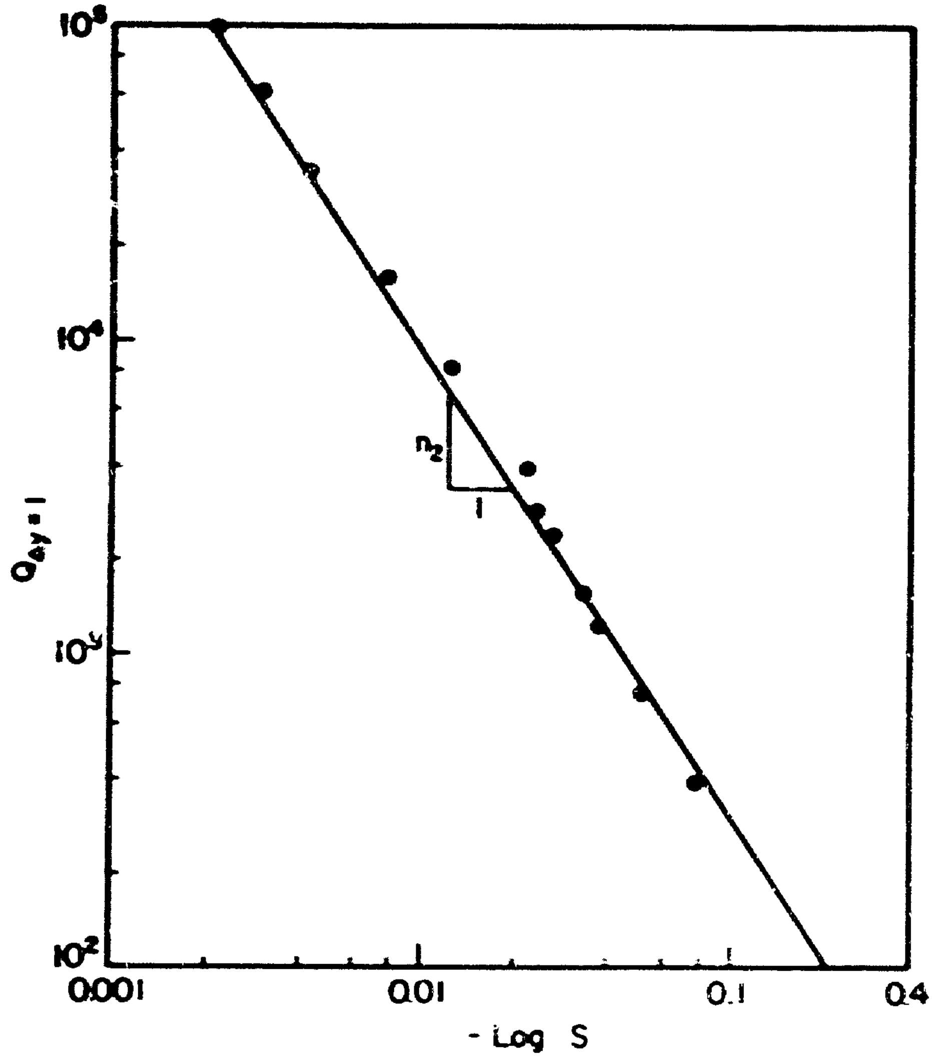


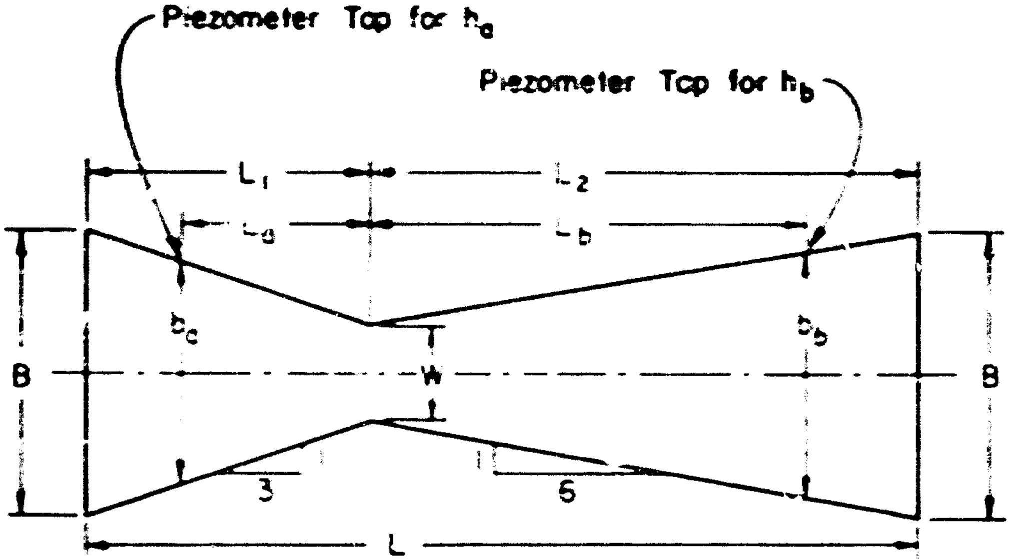
Figure 13. Typical relationship for distribution of submergence.

CHAPTER V

EXPERIMENTAL DESIGN

The purpose of this study is to evaluate the effect of settlement on flume ratings. The cutthroat flume was selected because it would be a convenient type of flume for an initial study on this subject, due to the simplicity in geometry. There is a series of cutthroat flumes, as described in Figure 14, which had been constructed by personnel in the Agricultural Engineering Shop at Colorado State University for use in another study. The sizes and dimensions of the flumes are tabulated in Fig. 14. The flume series consists of three flume lengths with four throat widths for each length, thereby making a total of 12 flumes. Two sizes (throat widths) from each of two flume lengths were selected for this study on the effect of settlement on flume discharge ratings, with the hope that there is a possibility to extend the result for whole series. The flumes selected for this study were the 6" x 4.5', 12" x 4.5', 4" x 1.5', and 2" x 1.5' flume sizes, where the first number corresponds to the throat width and the last number to the flume length.

The primary purpose of the experimental program was to evaluate the change in discharge rating of a flow measuring flume under both free flow and submerged flow conditions, as the slope of



$$B = W + \frac{2}{3} L_1 = W + \frac{1}{3} L_2$$

Flume	W	L_1	L_2	L_a	L_b	L	B
3"x4.5'	3"	1'-6"	3'-0"	1'-0"	2'-5$\frac{1}{2}$"	4.5'	1'-3"
6"x4.5'	6"	1'-6"	3'-0"	1'-0"	2'-5$\frac{1}{2}$"	4.5'	1'-6"
12"x4.5'	12"	1'-6"	3'-0"	1'-0"	2'-5$\frac{1}{2}$"	4.5'	2'-0"
24"x4.5'	24"	1'-6"	3'-0"	1'-0"	2'-5$\frac{1}{2}$"	4.5'	3'-0"
2"x3.0'	2"	1'-0"	2'-0"	0'-8"	1'-7$\frac{3}{8}$"	3.0'	0'-10"
4"x3.0'	4"	1'-0"	2'-0"	0'-8"	1'-7$\frac{3}{8}$"	3.0'	1'-0"
8"x3.0'	8"	1'-0"	2'-0"	0'-8"	1'-7$\frac{3}{8}$"	3.0'	1'-4"
16"x3.0'	16"	1'-0"	2'-0"	0'-8"	1'-7$\frac{3}{8}$"	3.0'	2'-0"
1"x1.5'	1"	0'-6"	1'-0"	0'-4"	0'-9$\frac{7}{8}$"	1.5'	0'-5"
2"x1.5'	2"	0'-6"	1'-0"	0'-4"	0'-9$\frac{7}{8}$"	1.5'	0'-6"
4"x1.5'	4"	0'-6"	1'-0"	0'-4"	0'-9$\frac{7}{8}$"	1.5'	0'-8"
8"x1.5'	8"	0'-6"	1'-0"	0'-4"	0'-9$\frac{7}{8}$"	1.5'	1'-0"

Figure 14. A series of cutthroat flumes available for experimental design.

the flume floor is increased. This portion of the study could be accomplished by testing only one flume in the laboratory. The secondary purpose of the experimental program was to determine scale effects. For example, the 2" x 1.5' flume is a one-third scale model of the 6" x 4.5' flume and the 4" x 1.5' flume is a one-third scale model of the 12" x 4.5' flume.

In this experimental study, the effect of tilting the flume floor was evaluated using three tilt angles in addition to collecting information when the flume floor was horizontal. The tilt angles studied for each flume size are listed in Table 1, where θ is the tilt angle (slope of flume floor) in degrees.

Table 1. Cutthroat flume sizes and floor slopes (tilt angle) used in the experimental design.

Flume Size	Case 1 tan θ	Case 2 tan θ	Case 3 tan θ	Case 4 tan θ
6" x 4.5'	0	0.0185	0.0556	0.1078
12" x 4.5'	0	0.0185	0.0556	0.1064
4" x 1.5'	0	0.0278	0.0556	0.0972
2" x 1.5'	0	0.0278	0.0556	0.0833

CHAPTER VI

EXPERIMENTAL FACILITY

This hydraulic experimental study utilized the 4-feet wide by 60-feet long recirculating flume located in the Engineering Research Center at Colorado State University.

The layout of the experimental recirculating flume system is shown in Fig. 15. The cutthroat flume to be tested was set inside the recirculating laboratory flume. Water was pumped into the flume system by a centrifugal pump through a discharge pipe into a headbox located at the entrance to the laboratory flume. Water was obtained from the large sump under the laboratory floor and pumped (using a small pump) into the sump of the pump-discharge pipe-flume system. After a suitable volume of water was obtained for the recirculating system, the valve gate connecting the large sump to the small sump could be closed. The flow in the system could be adjusted with the valve in the discharge pipe.

At the end of the 4-foot laboratory flume, there was a tank which received the falling water from the flume exit. The water was discharged from the tank through an orifice which was used as a flow measuring device. The dimensions of this orifice are shown in Fig. 15. The width of the orifice is controlled by a sliding gate

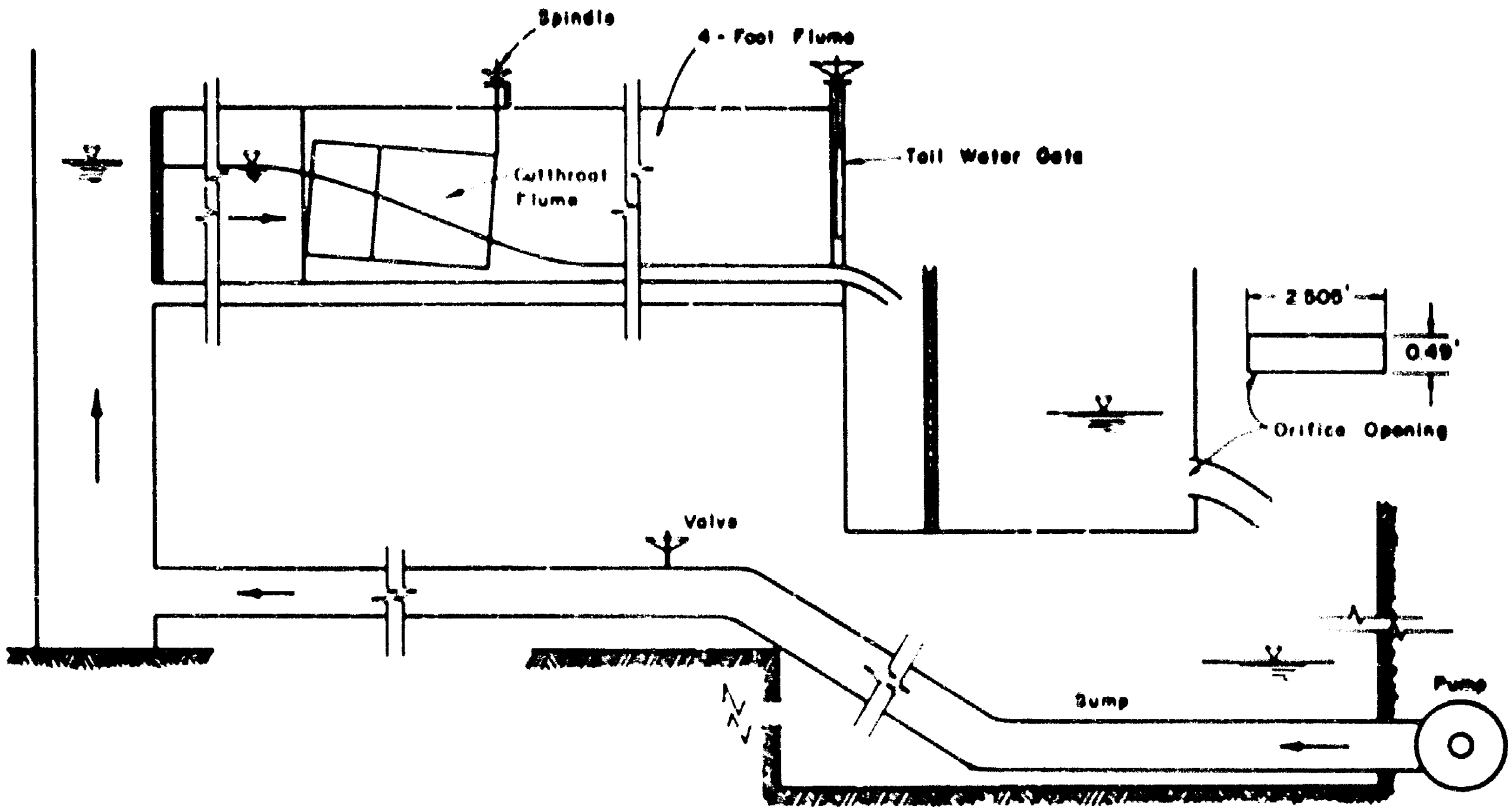


Figure 15. Experimental flume facility.

that moves wideways, which can be set for openings of 0.5 feet, 1.0 feet, 1.5 feet, and full opening. Although increments of 0.5 feet were commonly used for the orifice width, it is possible to set any opening between zero and full opening, providing the width is accurately measured. The depth of water above the orifice was measured with a hook gage in a stilling well set beside the tank, which was connected by means of a flexible tube to a piezometer tap located in the wall of the tank. The water discharging from the orifice dropped into the sump and then passed through the centrifugal pump, again.

The cutthroat flume to be tested was placed inside the 4-foot flume with the upstream edge serving as the hinge for tilting the flume. A flexible rubber sheet was used between the headwall of the flume and the channel iron cutoff attached to the walls and floor of the laboratory flume, thereby allowing the cutthroat flume to be tilted without leakage of water occurring. The downstream edge of the cutthroat flume was supported down from an overhead frame attached to the 4-foot laboratory flume. A spindle was attached to the frame which allowed the downstream edge of the cutthroat flume to be moved upward or downward by turning the spindle, thereby allowing various angles of tilt to be studied. The maximum drop of the flume exit was approximately 6 inches.

Two hook gages were used in the stilling wells, which were connected by flexible tubes to the piezometer taps (h_a and h_b) located in the flume wall. The capacity of the recirculating flume system was approximately 5 to 6 cfs with a cutthroat flume installed. The range of discharges utilized in the experimental program ranged from less than 1 cfs to the capacity of the system.

A tailwater gate was provided at the end of the 4-foot flume for adjusting the tailwater depth in order to induce submerged flow conditions.

In running the hydraulic tests, the cutthroat flume being tested was set horizontally and a constant discharge was run and readings for the entrance stilling well (h_a) were read. Then, the flume was set at a particular predetermined tilt angle and the water surface elevation in both stilling wells was read, along with the stilling well associated with the free orifice flow measuring device located in the downstream tank. Then, the flume was set at the predetermined second tilt angle (tilt angle 2) and the stilling wells were again read. Next, tilt angle 3 was set and the three hook gages were again read. Then, the tailwater gate was lowered in order to increase the flow depth upstream and the stilling wells were read. Next, tilt angle 2 was set and readings taken, and then tilt angle 3 was set with hook gage readings being collected. Again, the tailwater gate is lowered and the above procedure repeated. After data has been collected for high degrees of submergence (greater than 95 percent), a new discharge is set and the procedure repeated.

CHAPTER VII

RESULTS

Application of Theory

The theoretical analyses reported earlier in this study were used to predict the effect of settlement on the discharge ratings of the cutthroat flumes under study. The theoretical analysis is capable of predicting the effect of settlement on the free flow discharge ratings. By just modifying the coefficients in the theoretical analysis, the predicted free flow ratings can be easily made to conform with the experimental results.

Unfortunately, the theoretical submerged flow analysis only gives a very rough indication of the effect of settlement on the submerged flow discharge ratings. In order for the theoretical analysis to conform with the experimental results, changes in the general format of the theoretical equations are required. Thus, the difficulties are more involved that just simply adjusting some coefficients. To achieve conformity between the theoretical submerged flow equations and the experimental results reported below, considerable effort would be required. Consequently, attempts to use the theoretical equations for predicting discharge ratings when the flume floor has a downward slope in the direction of flow were not pursued any further.

Experimental Results

A definition sketch for a tilting flow measuring flume is shown in Fig. 16. The datum for hydraulic computations is taken as the elevation of the flume floor at the cross-section where the downstream flow depth, h_b, is measured. Under tilted conditions, it is necessary to describe the discharge ratings using energy rather than flow depths in order to use the submerged flow equation (Eq. 84). Thus, the energy at the piezometer taps in the entrance and exit sections of the cutthroat flume is defined by

$$E_a = z_a + h_a + v_a^2 / 2g \tag{86}$$

$$E_b = h_b + v_b^2 / 2g \tag{87}$$

$$E_r = E_b / E_a \quad . \tag{88}$$

The free flow equation is then rewritten as

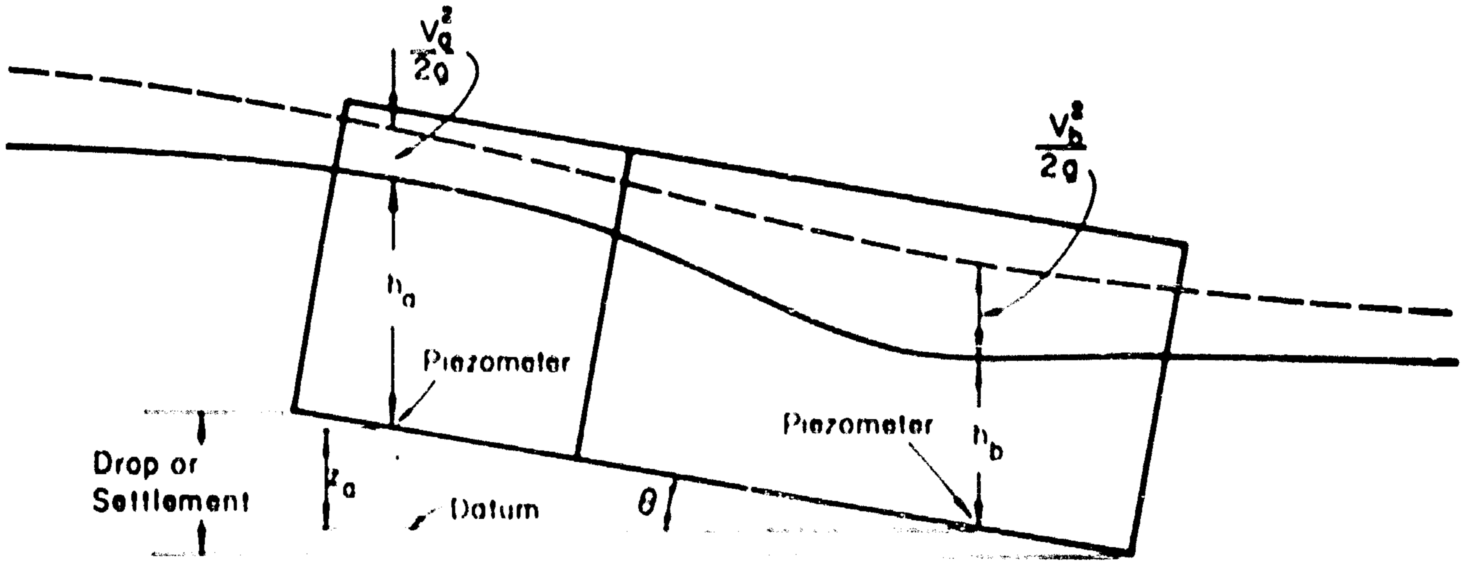
$$Q = CE_a^{n_1} \quad . \tag{89}$$

The submerged flow equation becomes

$$Q = \frac{C_1 H_L^{n_1}}{(-\log E_r)^{n_2}} \tag{90}$$

where H_L is the head loss defined by

$$H_L = E_a - E_b \quad . \tag{91}$$



$$E_a = z_a + h_a + \frac{V_a^2}{2g}$$

$$E_b = h_b + \frac{V_b^2}{2g}$$

Figure 16. Definition sketch for tilting flume.

A summary of the free flow and submerged flow ratings for the four cutthroat flumes tested in the laboratory under various angles of tilt are listed in Table 2. The coefficients and exponents listed in this table were developed by using Eqs. 89 and 90. The values of n_1 and n_2 developed from an analysis of hydraulic data for each horizontal flume case were used for the values of n_1 and n_2 under conditions of tilt. This assumption is based upon results from the theoretical analysis for free flow conditions. Using Eqs. 89 and 90, but allowing n_1 and n_2 to vary in accordance with the data, resulted in some variation in n_1 and n_2.

The variation of the free flow coefficient, C, and the submerged flow coefficient, C_1, listed in Table 2, is plotted in Figs. 17, 18, 19, and 20. The variation in C and C_1 for the flumes having a length of 1.5 feet is shown in Figs. 17 and 18, while the variation in these coefficients for the 4.5-foot flumes is shown in Figs. 19 and 20. A comparison of these graphs shows that the discharge ratings for various angles of tilt are affected considerably more by the longer flumes. This would be expected since the longer flumes allow a greater degree of flow acceleration, thereby more significantly decreasing the flow depths in the flume.

The variation of discharge when the flume is horizontal, Q_L, and the discharge when the flume is tilted, Q_T, is shown in Figs. 21, 22, 34, and 24 as a function of the flume floor slope (lower

Table 2. Summary of free flow and submerged flow ratings for experimental cutthroat flumes using the energy analysis.

Flume	Case	Tilt Angle*	C	n_1	n_2	C_1
2" x 1.5'	1	0.0000	0.7859	1.922	1.563	0.4716
	2	0.0278	0.7361	"	"	0.4415
	3	0.0556	0.7206	"	"	0.4261
	4	0.0833	0.6945	"	"	0.4120
4" x 1.5'	1	0.0000	1.3923	1.922	1.563	0.8417
	2	0.0278	1.3803	"	"	0.8232
	3	0.0556	1.3316	"	"	0.8147
	4	0.0972	1.2594	"	"	0.8132
6" x 4.5'	1	0.0000	1.8224	1.688	1.389	0.9754
	2	0.1085	1.3615	"	"	0.7435
	3	0.0556	1.0000	"	"	0.6453
	4	0.1078	0.7800	"	"	0.5556
12" x 4.5'	1	0.0000	3.4738	1.688	1.389	1.8588
	2	0.0185	3.0641	"	"	1.4516
	3	0.0556	2.5450	"	"	1.2504
	4	0.1064	2.1099	"	"	1.0899

*Tilt angle is slope of flume floor in ft./ft.

NOTE: Energy datum at down-stream gage point.

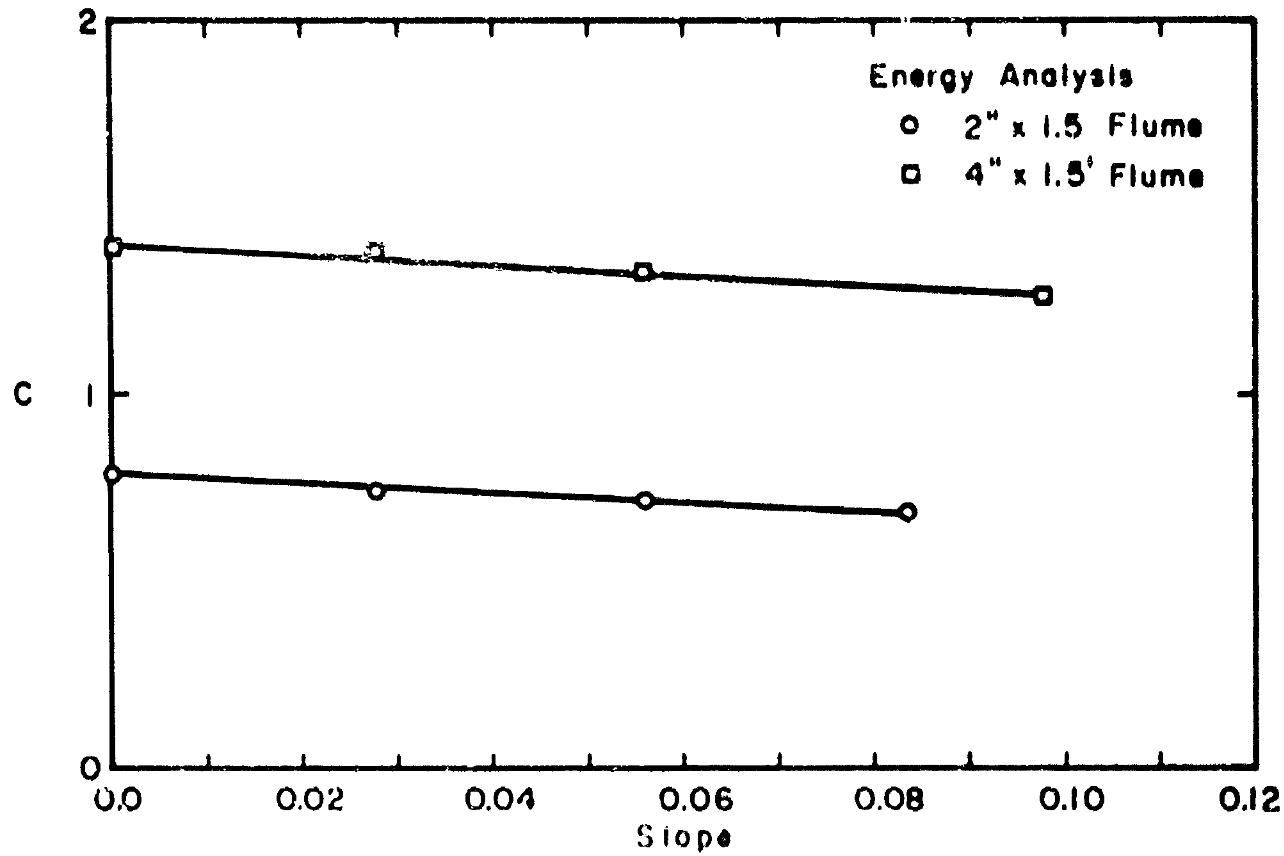


Figure 17. Variation of free flow coefficients with slope of flume floor for 2" x 1.5' and 4" x 1.5' flumes.

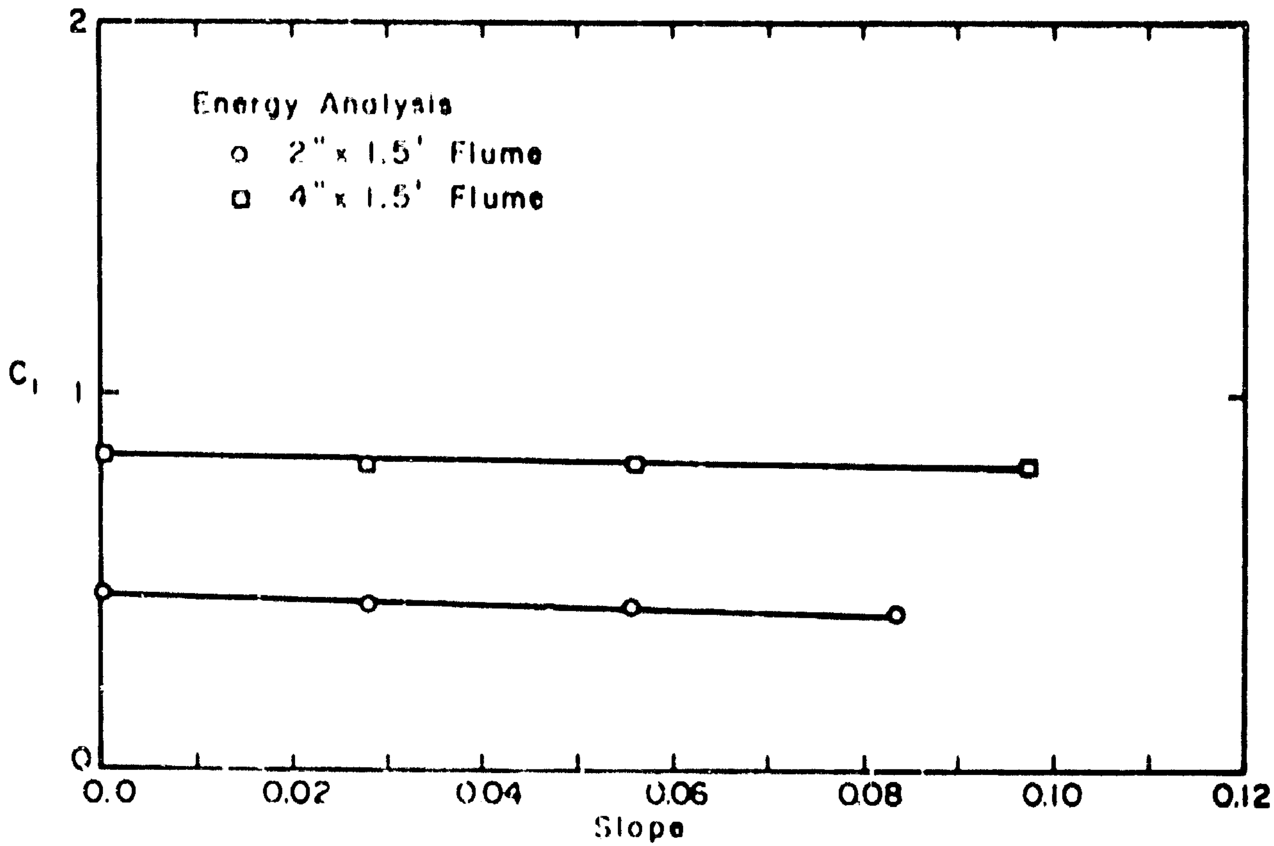


Figure 18. Variation of submerged flow coefficient with slope of flume floor for 2" x 1.5' and 4" x 1.5' flumes.

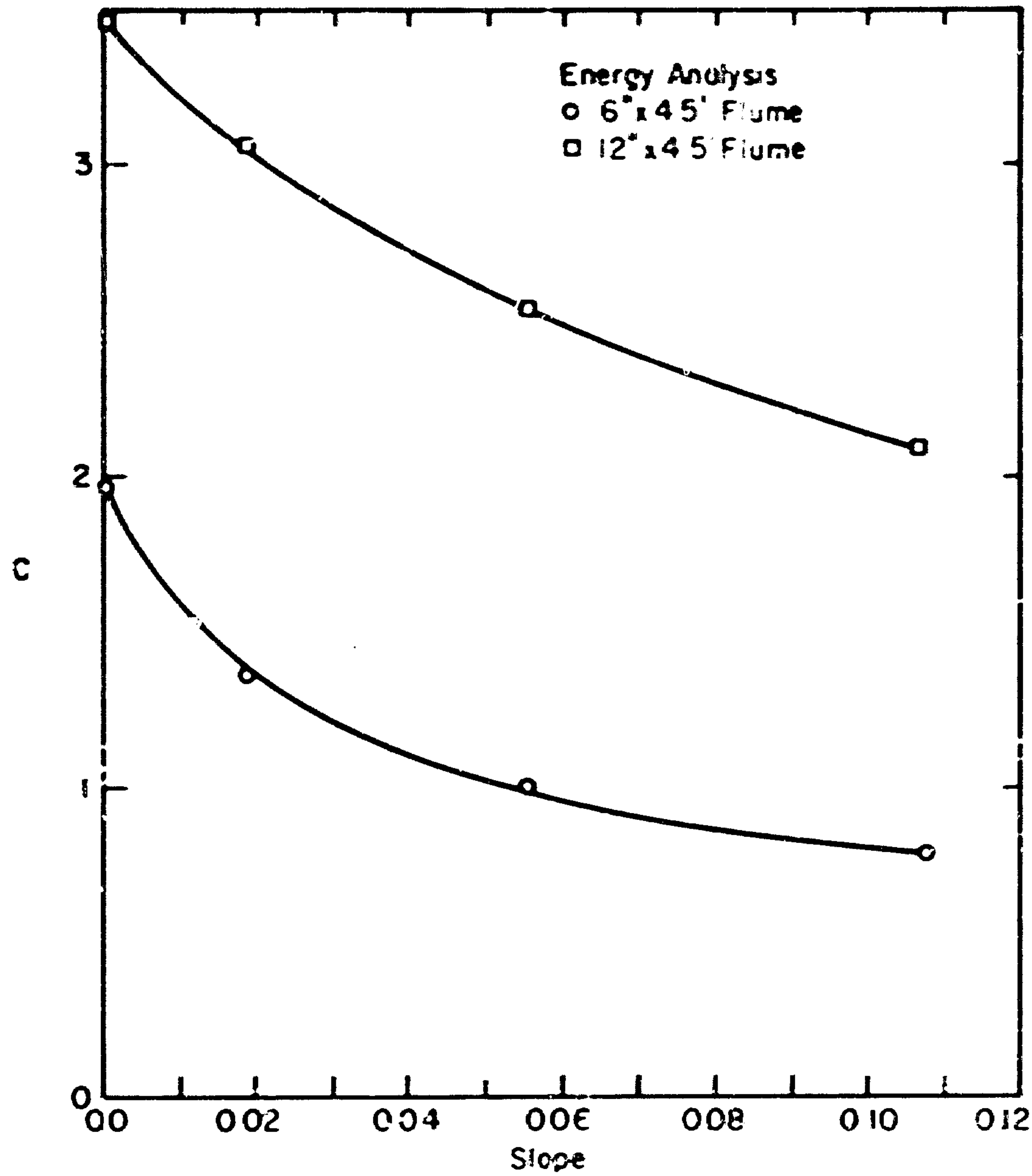


Figure 19. Variation of free flow coefficient with slope of flume floor for 6" x 4.5' and 12" x 4.5' flumes.

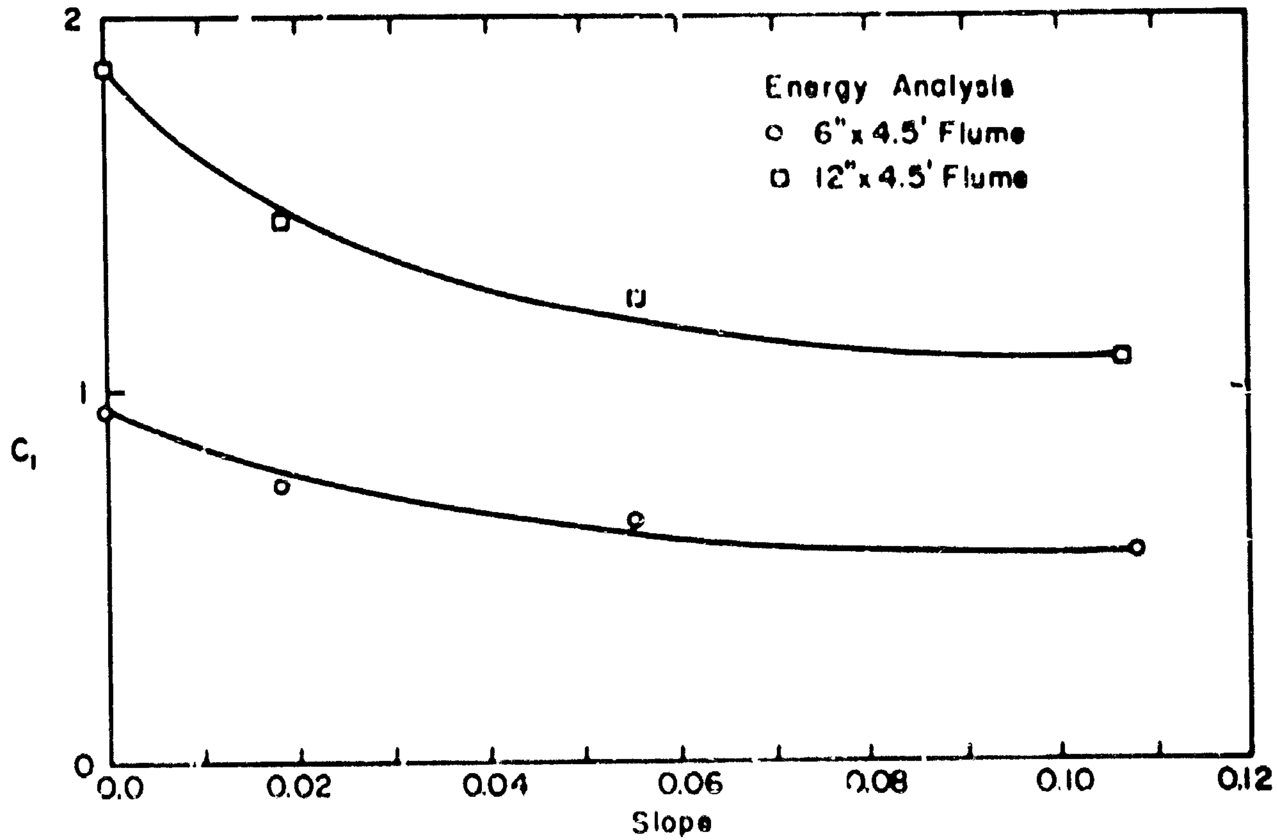


Figure 20. Variation of submerged flow coefficient with slope of flume floor for 6" x 4.5' and 12" x 4.5' flumes.

abscissa) or settlement (upper abscissa). Again, these graphs were prepared based upon the results listed in Table 2. Referring again to the definition sketch for a tilted flume (Fig. 16), the energy analysis uses E_a, which includes the potential energy due to slope, z_a. Thus, Figs. 21-24 do provide a check on the consistency of the data but do not provide an absolute comparison on the effect of slope upon discharge. The discharge ratio is affected much more in the 4.5-foot flumes as compared with the 1.5-foot flumes.

In order to obtain an absolute measure on the effect of slope upon discharge, the results using the energy analysis must be converted to flow depths. This transfer requires a trial-and-error procedure. Using the free flow equation (Eq. 68) involving flow depth, h_a, the variation of the free flow coefficient with flume floor slope is shown in Figs. 25 and 26. The free flow coefficient for the 2" x 1.5' is affected only slightly by slope, with a 10% slope affecting the discharge by only 5%. For the 4" x 1.5' flume, a 10% slope increases the discharge by 27%. In comparison, the 4.5-foot long flumes with a 10% slope show a 79% increase in discharge.

An analysis was made to determine the accuracy in using a small measuring flume as a model to predict the discharge rating for a larger, geometrically similar flume. The model: prototype relationship was established for both free and submerged flow according to the principles of dimensional analysis for a Froude model.

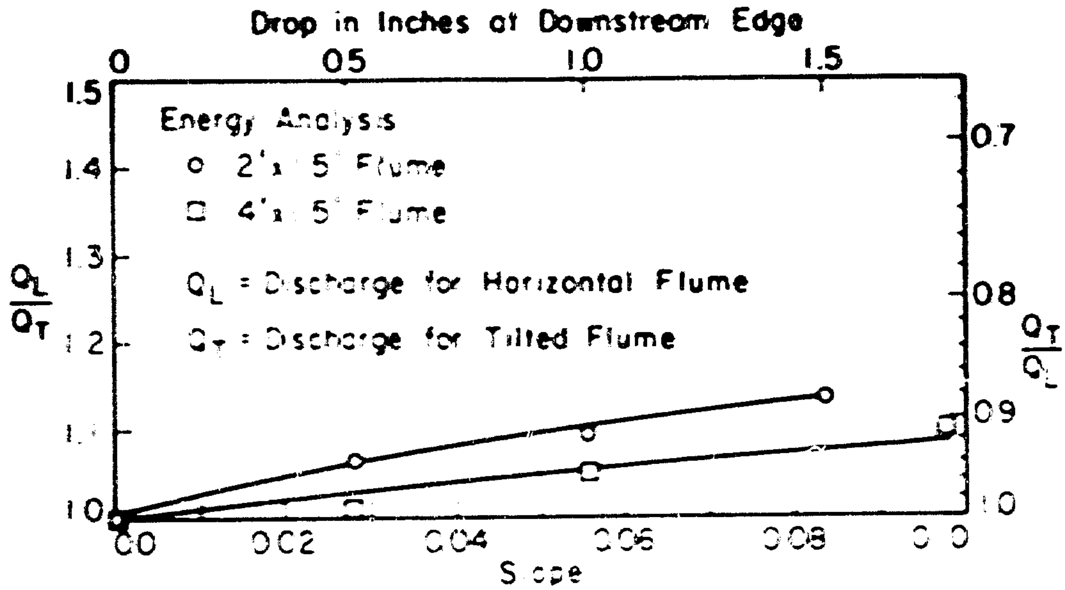


Figure 21. Comparison of free flow discharge ratios under horizontal and tilted conditions for 2' x 1.5' and 4' x 2.5' flumes.

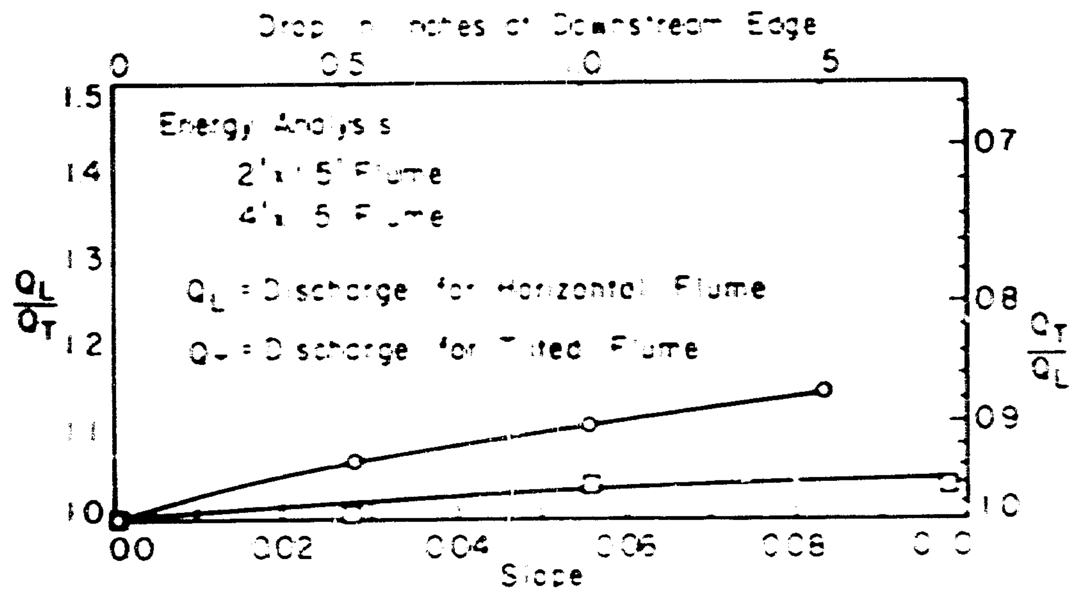


Figure 22. Comparison of submerged flow discharge ratios under horizontal and tilted conditions for 2' x 1.5' and 4' x 1.5' flumes.

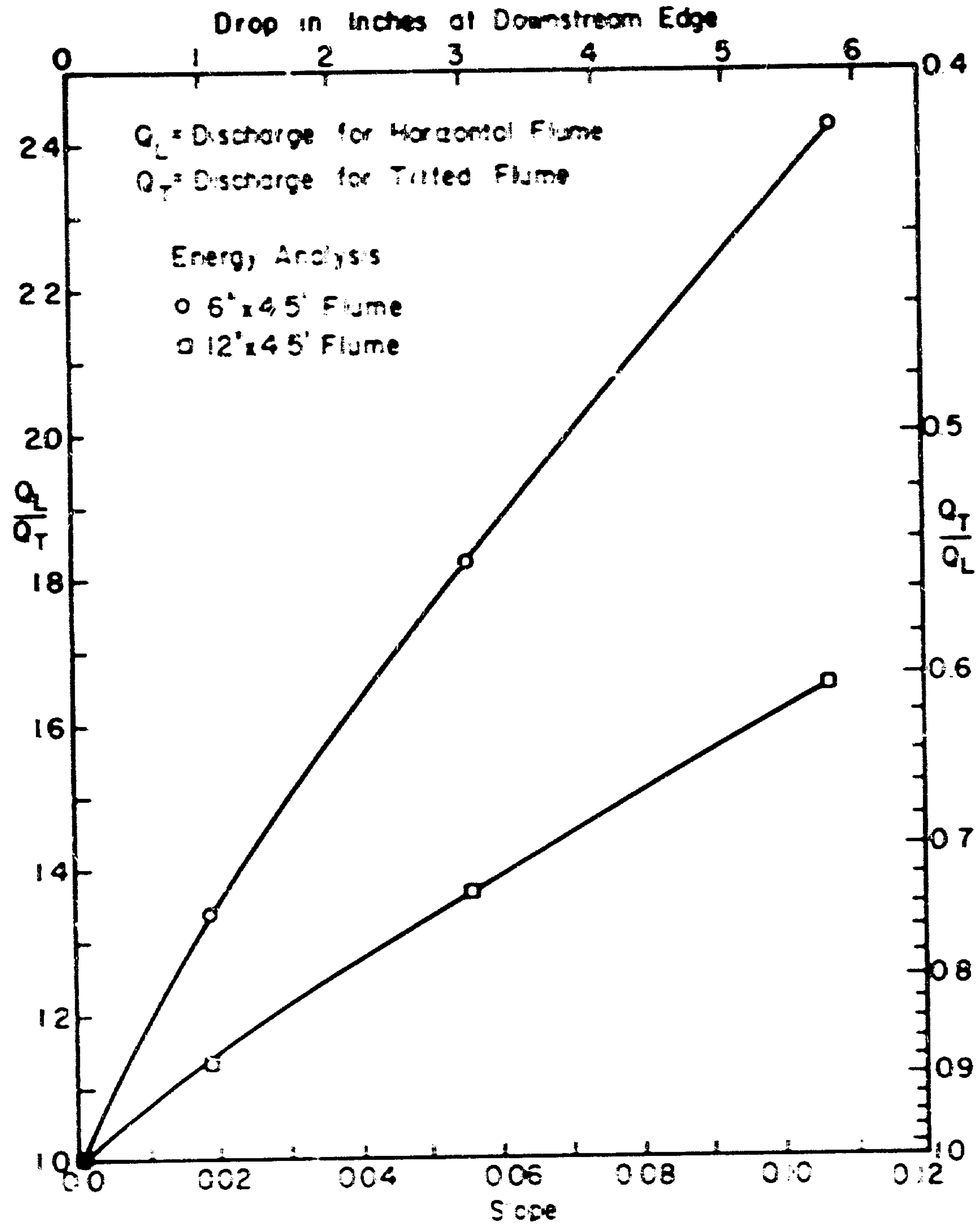


Figure 23. Comparison of free flow discharge ratios under horizontal and tilted conditions for 6' x 4.5' and 12' x 4.5' flumes.

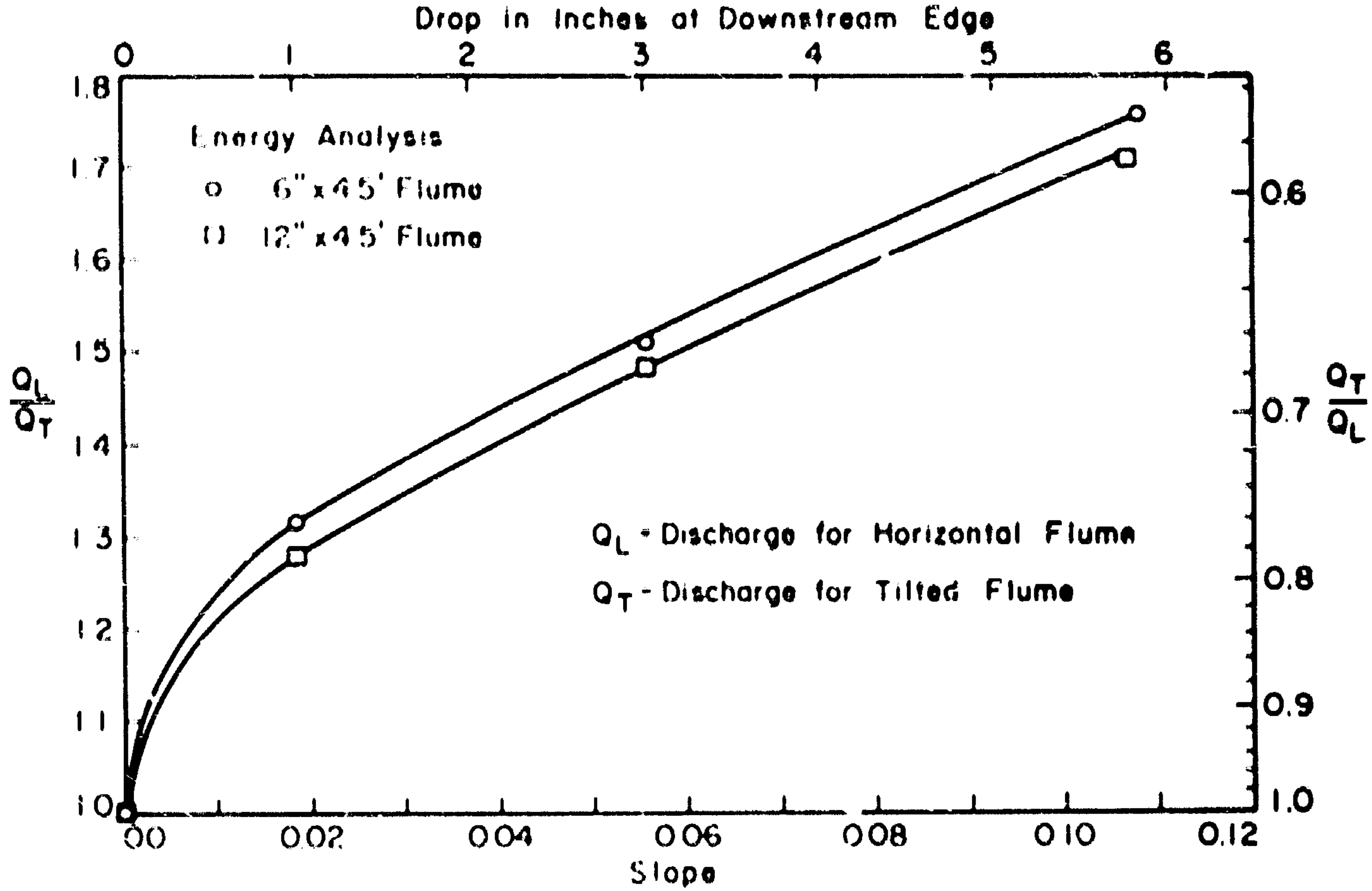


Figure 24. Comparison of submerged flow discharge ratios under horizontal and tilted conditions for 6" x 4.5' and 12" x 4.5' flumes.

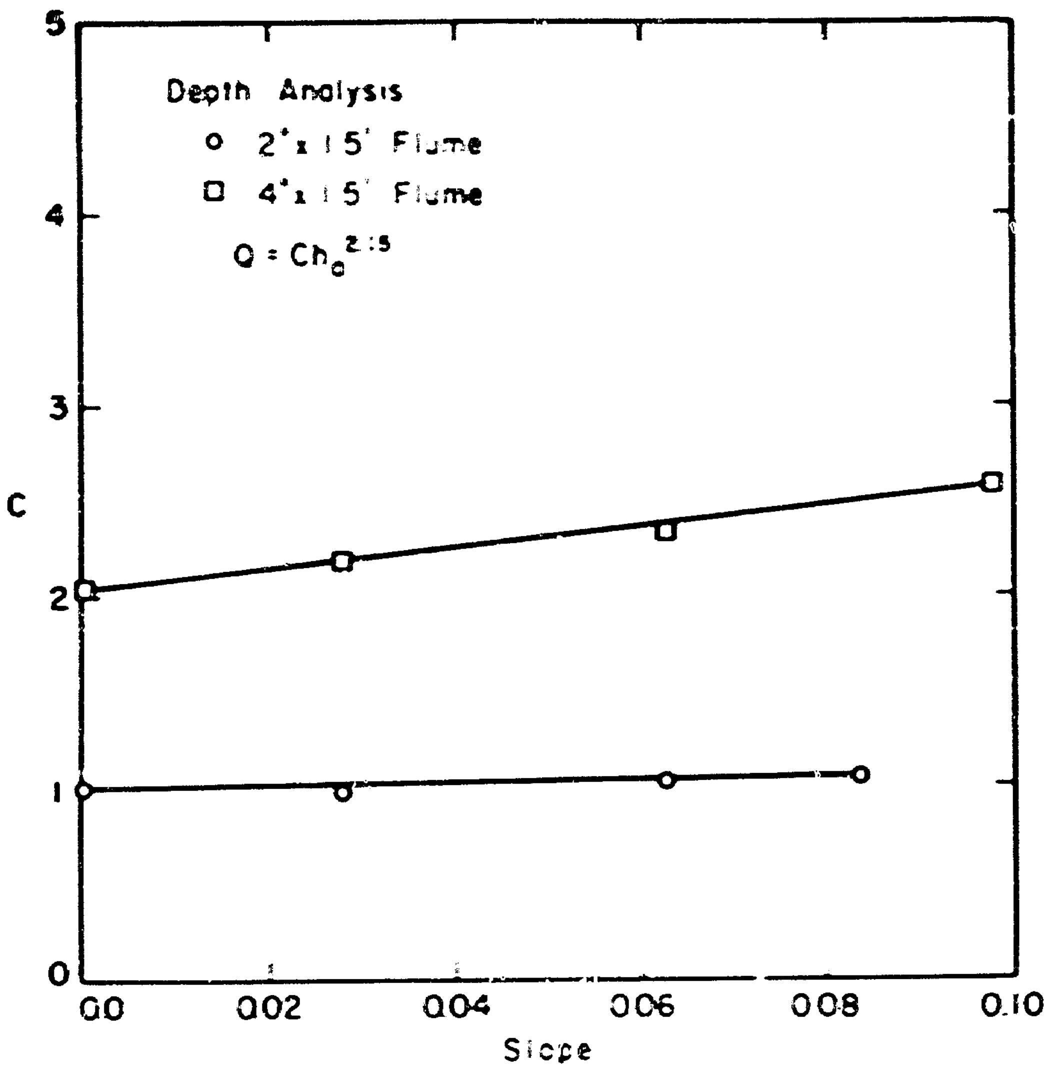


Figure 25. Variation of free flow coefficients with slope of flume floor for 2' x 1.5' and 4' x 1.5' flumes using flow depth (h_a) analysis.

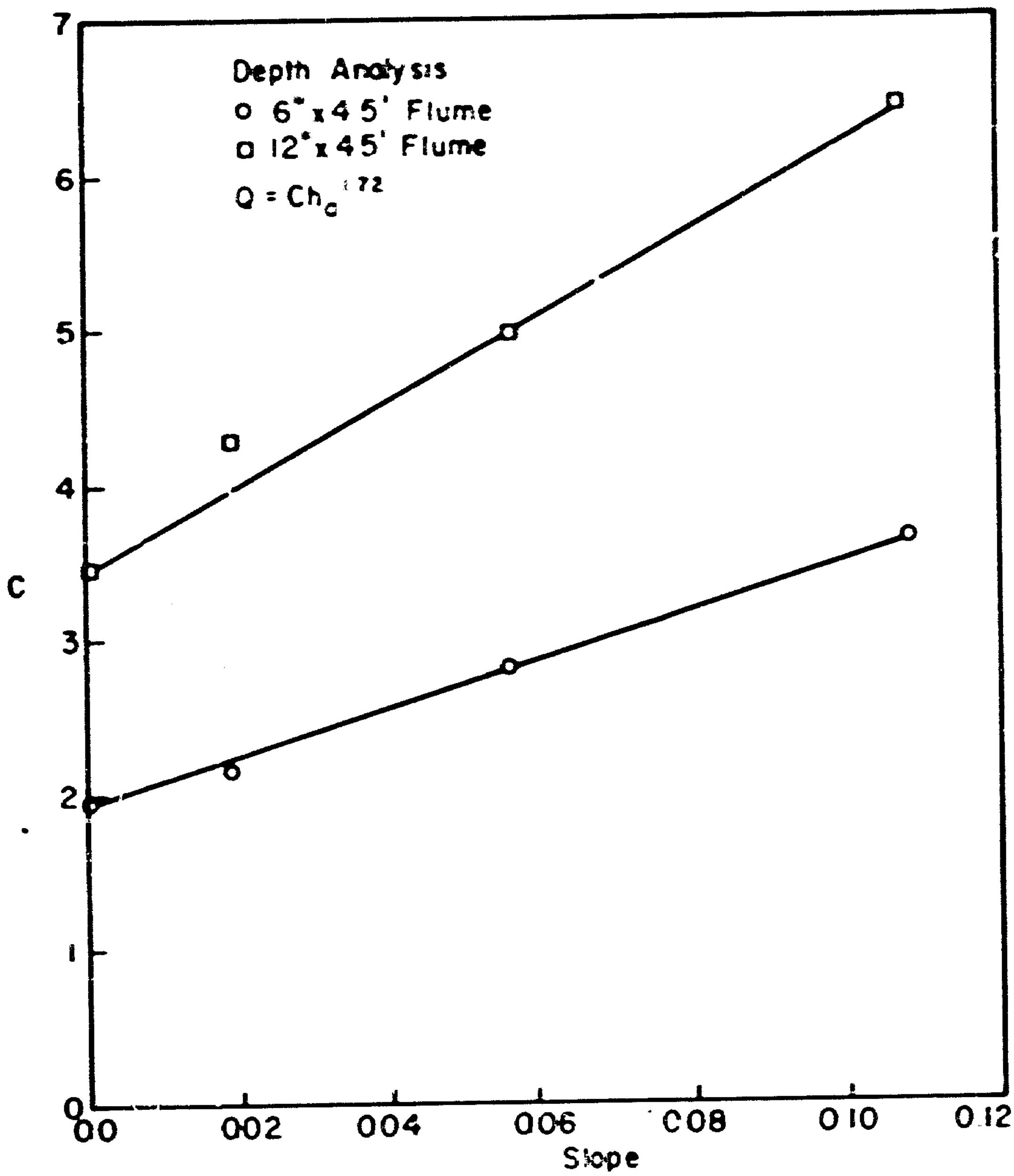


Figure 26. Variation of free flow coefficient with slope of flume floor for 6" x 4.5' and 12" x 4.5' flume using flow depth (h_a) analysis.

A comparison of the free flow discharge of the 6" x 4.5' prototype flume was predicted from the 2" x 1.5' model flume, along with the actual calibrated discharge of the prototype flume, is shown in Table 3. The comparison is shown for cases where the flume is both level and tilted.

A similar comparison for free flow through the 12" x 4.5' prototype flume as predicted from the 4" x 1.5' model flume, along with the actual calibration, is shown in Table 4. Tables 5 and 6 show the comparisons carried out for the submerged flow.

The error between the calibrated discharge and the predicted discharge for the prototype flume has been taken as positive where the predicted discharge is larger. Although for the free flow case the point of minimum error does not occur at any consistent upstream energy, the error becomes increasingly positive with depth for both level and tilted flumes. The error also becomes increasingly positive with depth for the submerged flow case. However, the error decreases with increased submergence.

The scale effect study indicates the degree of error that might be expected when a model is used to predict the actual prototype calibration plots. The error obtained indicates that a direct calibration of a flume should be made in order to obtain accurate results. If a model calibration is required, a field check of the prototype structure should also be undertaken to correct the rating.

Table 3. Comparison of free flow calibration for the 6" x 4.5' (prototype) flume with the 2" x 1.5' (1/3 model) flume.

E_a (ft)	Horizontal flume			Flume slope=0.0556		
	Prototype discharge cfs	Predicted discharge cfs	Error %	Prototype discharge cfs	Predicted discharge cfs	Error %
0.5	0.57	0.39	-31	0.31	0.36	16
1.0	1.82	1.48	-19	1.00	1.36	36
1.5	3.61	3.23	-11	1.98	2.96	50
2.0	5.87	5.62	- 4	3.22	5.15	60
2.5	8.56	8.63	1	4.70	7.91	68
3.0	11.6	12.2	5	6.39	11.23	76

Table 4. Comparison of free flow calibrations for the 12" x 4.5' (prototype) flume with the 4" x 1.5' (1/3 model) flume.

E_a (ft)	Horizontal flume			Flume slope=0.0556		
	Prototype discharge cfs	Predicted discharge cfs	Error %	Prototype discharge cfs	Predicted discharge cfs	Error %
0.5	1.08	0.69	-36	0.79	0.66	-16
1.0	3.47	2.63	-24	2.54	2.51	- 1
1.5	6.89	5.73	-17	5.05	5.48	9
2.0	11.2	9.96	-11	8.20	9.52	16
2.5	16.3	15.3	- 6	11.9	14.6	22
3.0	22.2	21.7	- 2	16.3	20.7	28

Table 5. Comparison of submerged flow calibration for the 6" x 4.5' (prototype) flume with the 2" x 1.5' (1/3 model) flume.

		Flume level			Flume slope=0.0556		
E_a (ft)	S %	Prototype discharge cfs	Predicted discharge cfs	Error %	Prototype discharge cfs	Predicted discharge cfs	Error %
0.5	80	0.51	0.41	-20	0.34	0.37	9
	85	0.49	0.39	-21	0.32	0.35	8
	90	0.45	0.35	-23	0.30	0.32	6
	95	0.38	0.28	-25	0.25	0.26	2
1.0	80	1.65	1.55	- 6	1.09	1.40	28
	85	1.58	1.46	- 7	1.04	1.32	27
	90	1.45	1.32	- 9	0.96	1.19	24
	95	1.22	1.07	-12	0.81	0.97	20
1.5	80	3.27	3.38	3.3	2.16	3.05	41
	85	3.13	3.19	2.1	2.07	2.88	39
	90	2.88	2.88	0.1	1.90	2.60	37
	95	2.43	2.34	-3.5	1.61	2.12	32
2.0	80	5.31	5.88	11	3.52	5.31	51
	85	5.08	5.55	9	3.36	5.01	49
	90	4.68	5.01	7	3.09	4.52	46
	95	3.95	4.07	3	2.61	3.68	41
2.5	80	7.74	9.02	16	5.12	8.15	60
	85	7.40	8.51	15	4.90	7.69	57
	90	6.82	7.69	13	4.51	6.95	54
	95	5.75	6.25	9	3.80	5.65	48
3.0	80	10.5	12.8	22	6.97	11.6	66
	85	10.1	12.1	20	6.61	10.9	64
	90	9.27	10.9	18	6.13	9.86	61
	95	7.82	8.87	13	5.17	8.02	55

Table 6. Comparison of submerged flow calibration for the 4" x 1.5' (prototype) flume with the 12" x 4.5' (1/3 model) flume.

		Flume level			Flume slope=0.0556		
E_a (ft)	S %	Prototype discharge cfs	Predicted discharge cfs	Error %	Prototype discharge cfs	Predicted discharge cfs	Error %
0.5	80	0.97	0.73	-25	0.65	0.70	7.6
	85	0.93	0.68	-26	0.62	0.66	6.3
	90	0.85	0.62	-27	0.57	0.60	4.3
	95	0.72	0.50	-30	0.48	0.49	0.5
1.0	80	3.14	2.76	-11	2.11	2.67	26
	85	3.00	2.61	-13	2.02	2.52	25
	90	2.76	2.35	-14	1.86	2.28	22
	95	2.33	1.91	-17	1.56	1.85	18
1.5	80	6.23	6.02	- 3	4.19	5.83	39
	85	5.95	5.69	- 4	4.00	5.51	37
	90	5.48	5.14	- 6	3.68	4.97	34
	95	4.62	4.17	- 9	3.11	4.04	30
2.0	80	10.1	10.4	3.5	6.82	10.1	48
	85	9.67	9.89	2.2	6.51	9.57	47
	90	8.92	8.93	0.2	5.99	8.65	44
	95	7.51	7.26	-3.3	5.05	7.03	39
2.5	80	14.7	16.0	9	9.92	15.5	56
	85	14.1	15.1	7	9.48	14.7	55
	90	12.9	13.7	5	8.73	13.2	52
	95	10.9	11.1	2	7.37	10.7	46
3.0	80	20.0	22.8	13	13.5	22.1	63
	85	19.1	21.5	12	12.9	20.8	61
	90	17.6	19.4	10	11.8	18.8	58
	95	14.9	15.8	6	10.0	15.3	52

CHAPTER VIII

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

A very common problem encountered with water measuring flumes located in unlined channels is the settlement of the downstream edge of the flume. Under this situation, corrections for the discharge rating are required. Four cutthroat flumes, 2" x 1.5', 4" x 1.5', 6" x 4.5' and 12" x 4.5' were selected for studying the effects of flume settlement on both free flow and submerged flow ratings. Theoretical analysis for both free flow and submerged flow of the flume was developed with the flume floor being horizontal or at some angle of tilt. Experiments were conducted in the laboratory. The technique developed from dimensional analysis for developing discharge ratings was used in the analysis of the experimental results. The free flow and submerged flow ratings for the four cutthroat flumes at various angles of tilt are reported.

Conclusions

The discharge corrections for a tilted flume can be made theoretically from the discharge equations of the nontilted flumes. Theory can be used to predict the effect of settlement on free flow ratings easily, but satisfactory results cannot be obtained for predicting the submerged flow discharge corrections due to the complexity of the theoretical equations. For the free flow case, for practical use, only one stage correction, m, seems to be required.

The experimental results show that there is a very definite effect due to settlement. The discharge coefficient varies considerably as the tilted slope of the flume is increased. The discharge increases as the tilted angle, or flume floor slope, increases. As the flume length, or size, is increased, the effect of settlement upon the discharge rating becomes more significant.

Using the results of the experiments for the four cutthroat flumes, a study on predicting discharge by model similarity was attempted. The 1/3 scale Froude models resulted in large errors in predicting the prototype discharge, whether the flume was horizontal or if settlement existed. The prediction error was large for both free flow and submerged flow conditions.

Recommendations

Frequently, flow measuring flumes placed in unlined channels result in settlement at the flume exit due to scouring action. Usually, the settlement is not corrected. Therefore, the discharge will be affected and corrections to the discharge rating are necessary. In addition to the study conducted, calibrations for other cutthroat flumes are needed in order that general discharge correction curves could be prepared for field use.

Parshall flumes are widely used for flow measurement in irrigation systems. Therefore, studies on the effect of settlement upon discharge ratings for Parshall flumes are also necessary.

REFERENCES

1. Ackers, P., and A. J. M. Harrison, 1963. Critical-depth flumes for flow measurements in open channels. Hydraulic Research Paper No. 5. Hydraulics Research Station, Department of Scientific and Industrial Research, Wallingford, Berkshire, England. April.

2. American Society of Agronomy. 1967. Irrigation of agricultural lands. No. 11 in the Series AGRONOMY.

3. Austin, Lloyd H., Gaylord V. Skogerboe, and Ray S. Bennett, 1970. Subcritical flow at open channel structures: open channel expansions. Report PRWG 17-1, Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah.

4. Bondurant, James A., Allan S. Humpherys, and A. R. Robinson, 1969. Cast-in-place concrete trapezoidal measuring flumes. ARS 41-155. Agricultural Research Service, United States Department of Agriculture.

5. Chamberlain, Adrian R, 1952. Measuring water in small channels with WSC flume. Washington Agricultural Experiment Stations, Institute of Agricultural Sciences, State College of Washington.

6. Chow, Ven Te, 1959. Open-channel hydraulics. McGRAW-HILL Book Co.

7. Cone, V. M., 1917. The Venturi flume. Journal of Agricultural Research, 9 (4): 115-123. April 23.

8. Haszpra, O., 1962. A tortlapu atmenet. Hidrologiai Kozlony, 2 sz., p. 153-157. Hungary.

9. Hyatt, M. L., 1965. Design, calibration, and evaluation of a trapezoidal measuring flume by model study. MS Thesis, Utah State University, Logan, Utah. March.

10. Linsley, Ray K., Jr., Max A. Kohler, and Joseph L. Paulhus. 1958. Hydrology for engineers. McGRAW-HILL Book Co.

11. Parshall, R. L., 1926. The improved Venturi flume. Transactions, ASCE, 89:841-851.

12. Replogle, John A., 1968. Flow measurement with critical-depth flumes. Contribution from the Soil and Water Conservation Research Division, Agricultural Research Service, U.S. Department of Agriculture. Submitted for 7th Congress I.C.I.D., Mexico City, 1969, Question 24: "Hydraulic structures on small channels." Preliminary presentation before U.S. Committee, Phoenix, Arizona, March 28.

13. Robinson, A. R., 1966. Water measurement in small irrigation channels using trapezoidal flumes. Transactions of the ASAE, Vol. 9, No. 3, pp 382-385 and p 388.

14. Robinson, A. R., 1968. Trapezoidal flumes for measuring flow in irrigation channels. ARS 41-140. Agricultural Research Service, United States Department of Agriculture.

15. Robinson, A. R., and A. R. Chamberlain, 1960. Trapezoidal flumes for open channel flow measurement. Trans. ASAE 3 (2): 120-128.

16. Shames, Irving H., 1962. Mechanics of fluid. McGRAW-HILL Book Co.

17. Skogerboe, Gaylord V., Leon M. Hyatt, and Keith O. Eggleston, 1967. Design and calibration of submerged open channel flow measurement structures. Part 1 - Submerged flow. Report WG 31-2. Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah.

18. Skogerboe, Gaylord V., Leon M. Hyatt, Joe D. England, and Raymond J. Johnson, 1967. Design and calibration of submerged open channel flow measurement structures. Part 2 - Parshall flumes. Report WG 31-3. Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah.

19. Skogerboe, Gaylord V., Leon M. Hyatt, Ross K. Anderson, and Keith O. Eggleston, 1967. Design and calibration of submerged open channel flow measurement structures. Part 3 - Cutthroat flumes. Report WG 31-4. Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah.

20. Skogerboe, Gaylord V., Leon M. Hyatt, and Lloyd H. Austin, 1967. Design and calibration of submerged open channel flow measurement structures. Part 4 - Weirs. Report WG - 57. Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah.

21. Skogerboe, Gaylord V., Lloyd H. Austin, and Kuan-Tao Chang, 1970. Subcritical flow at open channel structures: bridge constrictions. Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah.

22. Thomas, Charles W., 1957. Structures and methods for measuring irrigation water. Proceedings Third Congress, International Commission on Irrigation and Drainage, San Francisco, 1957. International Commission on Irrigation and Drainage, New Delhi, India, 1957.

23. Thomas, Charles W., 1960. World practices in water measurement at turnouts. Proceedings ASCE, Paper No. 2350, Journal of Irrigation and Drainage Division, American Society of Civil Engineers.

24. U. S. Bureau of Reclamation, 1967. Water measurement manual. A Water Resources Technical Publication.