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MAXIMUM WATER DELIVERY IN IRRIGATION

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Fort Collins, Colorado

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by James Henry Duke, Jr.

COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO

AUGUST, 1971

WATER MANAGEMENT
TECHNICAL REPORT NO. 10

MAXIMUM WATER DELIVERY IN IRRIGATION

Water Management Technical Report No. 10

by

James Henry Duke, Jr.

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Optimum Utilization of Water Resources:
with Special Emphasis on Water Delivery
and Removal Systems and Relevant
Institutional Development

Engineering Research Center
Colorado State University
Fort Collins, Colorado
August 1971

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ABSTRACT

MAXIMUM WATER DELIVERY IN IRRIGATION

In order to increase the water delivery efficiency of an existing irrigation system, it is proposed that modern mathematical optimization techniques be applied to the management of irrigation water delivery. A deterministic mathematical model is developed to simulate the events that occur in delivering water, from a supply to the users, in an interconnecting, open channel irrigation system that contains reservoirs. The events considered are inflows, outflows, losses, return flows, demands and storage. The simulation model is developed so that it can be linked to nonlinear programming and optimal results obtained.

Because of the difficulty in obtaining some of the functions required for the nonlinear simulation model, linear approximations are made and a linearized simulation model is derived. This model is linked to linear programming so that optimal results can be obtained. The suggested objective function is the minimization of system losses and unrequired system outflows, a resource conservation objective.

Two example models were constructed, using the linearized simulation model, to illustrate model construction and solution. The results of these solutions show the model adequately fulfills the purpose for which it is intended. The model solutions also demonstrate the effects of including future time periods in an analysis and the effects of modifying a structure in a system.

Because of a lack of data, many of the parameters for the example models were estimated and no comparisons of the optimal strategies determined by the model and the strategies used in practice could be made. The model, however, shows which data are necessary to provide these comparisons and, further, those data that are necessary to apply the model for a particular system.

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Chapter I

INTRODUCTION

Water for irrigation is important to the agricultural production of many regions, including the Western United States. To supply the necessary water, irrigation systems have been constructed by private corporate, and governmental enterprises. Many of these systems, although built before the turn of the century, are still able to satisfy the needs for which they were designed. In the intervening years, however, irrigation demands have increased to such an extent that existing supplies of water are inadequate. This increase has made it necessary for new sources of supply to be sought.

There are two methods by which new supplies may be obtained: through the construction of new facilities, such as dams, for tapping undeveloped supplies, and through the reclamation of water now being wasted by existing facilities through evaporation, transpiration, and percolation losses. The first alternative is rapidly becoming unfeasible because of the lack of available supplies, the lack of suitable locations that possess the necessary engineering requirements for development, the desire to reserve sites for future development, or simply, increased public opposition to the destruction of the natural environment.

The alternative of reclaiming water is not independent of the alternative of constructing new facilities since the reclamation of wasted water will allow the preservation of both potential sites for structures and the natural environment. The importance of such an approach to water resources is exemplified in the statements by

Skinner in Pillsbury (1968), "...specified water saving measures should precede development of new supplies....," and Rockwell (1968), "A corollary to the search for more water is the more efficient utilization to existing supplies."

There are several approaches that can be used to reclaim water wasted from an irrigation system: existing facilities can be reconstructed to lessen the losses, well fields can be constructed to reclaim percolation losses, and system management techniques can be improved to reduce the losses inherent in providing the required services. In any case, the objective of these approaches is to lessen the volume of water which is made unavailable for use while still delivering the required volumes of water. To accomplish this objective, an increase in water delivery efficiency is required (U.S.B.R. (1963), Bishop (1961)). The approaches available to make irrigation water delivery more efficient are not independent, but must be examined simultaneously.

Two of the approaches, the reconstruction and consolidation of existing facilities and the construction of well fields, require justification that the measures are economically feasible, a source of money to pay for the measures taken, and the approval of the system users. In many irrigation systems which have been built and paid for, the users are hesitant to take such measures.

Purpose of the Present Study

The purposes of this study are: (1) to derive a method for the mathematical simulation of water delivery, from a source of supply to the consumer, in a surface irrigation network, and (2) to couple the derived simulation method with mathematical optimization techniques to provide a tool for water managers to use for increasing water delivery efficiency. By including efficiency as a criterion, it becomes necessary to account for losses and return flows in the simulation.

Scope of the Study

The derivation of a mathematical model for the simulation of irrigation water delivery for a particular system requires that mathematical statements be written not only for the structure which compose the system, but also for the institutional constraints that limit the operation of the system.

The model developed in this study is intended to be sufficiently general that it can be applied to any system which fulfills the assumptions made in the development. Thus, no institutional constraints are considered; however, the model shows the conditions that require the imposition of legal and administrative constraints.

In general, there are two classes of simulation models: the planning model, which is concerned with meeting long-term objectives, and the operational model, which is concerned with meeting short-term objectives. The difference between the two is the time horizon of the analysis.

The model presented here is an operational model, concerned with making an existing irrigation water delivery system as efficient as possible. A single irrigating season is considered as the time horizon. No restriction is placed on the division of the season into increments, only that the number of increments be finite.

An examination of the available literature indicates a majority of the published works describing applications of mathematical programming techniques to the analysis of water resource systems are concerned with the design and operation of new systems, or the operation of existing systems, with objectives of maximizing or minimizing some economic criteria. References to many of these studies may be found in the Bibliography.

The model developed here does not use economic criteria because the inclusion of such criteria implies an economic policy, a type of administrative constraint. Instead, the model is designed to incorporate physical criteria such as units of water lost, units of water delivered, and so forth. Such an approach, for resource conservation, is not in evidence in the literature.

Presentation of the Study

The study is presented in the following manner.

Chapter II is a brief review of the optimization techniques to be used in the study. The purposes of this chapter are to acquaint the reader, unfamiliar with optimization techniques, with both the terminology of operations research and the formulation of optimization problems. There is no discussion of solution methodology.

Chapter III is a detailed written description of the problem, the assumptions made in formulating the model, definitions of terms to be used in deriving the model, the model itself, and what the model does.

Chapter IV shows the derivation of the simulation model. No assumptions are made regarding the form of the functions that describe the losses and return flows in the model, so the model is termed nonlinear.

In Chapter V, assumptions are made to allow the expression of linear functions to account for the losses and return flows. These are substituted into the model of Chapter IV and a linearized model is obtained.

Chapter VI presents an application of the linearized model to a system that is representative of the type for which the model was developed. Estimates of many of the coefficients are required because few data are available. The latter half of the chapter discusses the results of the examples and the practical applications of the model.

Chapter VII is a summary of the study and the conclusions drawn from it.

Three appendices are included to assist in the understanding of the model:

Appendix A relates the modeling technique used in the study to the more standard "network" techniques found in the references on operations research.

Appendix B provides a series of four example formulations of the linearized model for a simple system. The examples consist of: a one time period model that excludes return flows; a one time period model that includes return flows; a two time period model that excludes return flows, and a two time period model that includes return flows.

Appendix C is a discussion of points which may either improve the applicability of the model or improve the usefulness of the model in practical applications. The points considered in this section are untested and must be regarded as suggestions for future research.

Chapter II

OPTIMIZATION TECHNIQUES

Mathematical techniques used to maximize or minimize mathematical functions are known as optimization techniques. There are several classes of optimization techniques that lead directly to optimal solutions, if certain conditions are satisfied. Three commonly used techniques are linear programming, nonlinear programming and dynamic programming. In this study only linear and nonlinear programming will be considered.

This chapter is devoted to defining common terminology of programming methods and the presentation of formats for the expression of problems so that linear and nonlinear programming techniques may be used. There is little discussion of solution methodology. Procedures for solving programming problems can be found in standard references, such as Hadley (1962,1964).

Concepts

For both nonlinear and linear programming, the problem formulation consists of writing a series of mathematical expressions describing relationships among variables which characterize the important features of the process under examination. The variables are called decision variables, and each of the expressions is called a constraint. The entire series of expressions is called a constraint set. The constraint set is a mathematical description of the process and the limitations on the decision variables of the process. Solutions are normally performed to obtain values for at least one of the decision variables.

In the formulation of a nonlinear or linear programming problem, there is no required relation between the number of decision variables and the number of constraints in a constraint set. However, there are normally more decision variables than constraints. Mathematically, this implies the existence of more than one set of values for the decision variables that will satisfy all of the constraints simultaneously. Each set of values is called a feasible solution to the problem. Most problems have several feasible solutions.

The optimal solution is the best feasible solution. To obtain the optimal solution to a problem, a measurement of the desirability of each feasible solution must be introduced. This is accomplished through the use of a special function, called the objective function. The objective function is used to rank the various feasible solutions for comparison. The feasible solution that yields the maximum or minimum value of the objective function, depending on the problem, is defined as the optimal solution.

It is possible that there will exist either a single feasible solution or no feasible solution to a problem. For those problems where there is only a unique feasible solution, it is the optimal solution. However, it is optimal only because there are no competing solutions.

If no feasible solution exists, there is no set of values for the decision variables that will satisfy all of the constraints simultaneously. In the methods for modeling irrigation systems, presented in this study, there are certain conditions which will result in the inability to obtain a feasible solution. The conditions and their implications are discussed in Chapter III.

Nonlinear Programming Problem Formulation

Any programming problem formulation consists of writing the constraints that describe the process and writing an objective function to mathematically express a desired policy as a maximization or minimization statement. For nonlinear programming, there is no restriction on the functional nature of the constraints or the objective function.

Using the notation of Hadley (1964), a set of m constraints that relate n decision variables is written as

$$g_i(x_1,\ldots,x_n)\{\leq, = ,\geq\}b_i, \quad i = 1,\ldots,m \ . \qquad (2\text{-}1)$$

The x_j, $j = 1\ldots,n$, are the decision variables and the b_i are constants. For a given constraint, only one of the signs $\{\leq, = ,\geq\}$, will be valid.

The objective function for nonlinear programming is written:

$$\text{(maximize or minimize)} \quad Z = f(x_1,\ldots,x_n). \qquad (2\text{-}2)$$

An objective function measures only the relative desirability of feasible solutions. Because constants have an equal effect on the value of an objective function for all feasible solutions, they are not included in the statement of an objective function.

Linear Programming Problem Formulation

Linear programming problem formulation is a special case of the nonlinear formulation described above. Computation techniques, however, may be quite different.

For linear programming, all constraints, as well as the objective function, must be linear. The constraint set is generally written (Hadley, 1964):

$$\sum_{j=1}^{n} a_{ij}\, x_j \{\leq, =, \geq\} b_i \quad i = 1,\ldots,m \, , \tag{2-3}$$

and the objective function is written:

$$\text{(maximize or minimize)} \quad Z = \sum_{j=1}^{n} c_j x_j \quad . \tag{2-4}$$

In practice, all inequality constraints of a linear programming problem are usually converted to equalities prior to solution. This is done by including an additional variable of the proper sign in each inequality. These variables are called <u>slack variables</u> if they convert less-than-or-equal-to inequalities to equalities, and <u>surplus variables</u> if they convert greater-than-or-equal-to inequalities to equalities. Slack and surplus variables are classified as decision variables and may, or may not, have physical significance.

With all constraints written as equalities, a linear programming constraint set may be written as a matrix equation,

$$A\bar{x} = \bar{b}, \tag{2-5}$$

in which the A matrix is composed of the a_{ij} coefficients, the $\bar{x}$ vector consists of unknown decision variables, including the slack and surplus variables, and $\bar{b}$ is a vector of constants. Should any of the b_i constants be linear combinations of the other elements in the $\bar{b}$ vector, such as

$$\sum_{j=1}^{n} a_{ij}x_j \{\leq, =, \geq\} \sum_{k=1}^{m} d_{ik}b_i \quad i = 1,\ldots,m \ , \tag{2-6}$$

equation (2-5) can be written

$$A\bar{x} = D\bar{b} \ , \tag{2-7}$$

in which D is a matrix relating the various elements of $\bar{b}$. A comparison of equations (2-5) and (2-6) shows that they are the same if D is a unit matrix. This observation will prove useful in analyzing the linearized model of this study.

Moreover, the elements of the matrix equation (2-6) and the objective function, equation (2-5), written as

$$\text{(maximize or minimize)} \quad Z = \bar{c}\,\bar{x} \tag{2-8}$$

can be partitioned into submatrices and subvectors. Equation (2-7), then becomes

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1T} \\ A_{21} & A_{22} & \cdots & A_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ A_{T1} & A_{T2} & \cdots & A_{TT} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_T \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1T} \\ D_{21} & D_{22} & \cdots & D_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ D_{T1} & D_{T2} & \cdots & D_{TT} \end{bmatrix} \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \vdots \\ \bar{b}_T \end{bmatrix} \tag{2-9}$$

and equation (2-8) becomes

$$(\text{maximize or minimize})\ Z = [\bar{c}_1\ \bar{c}_2\ \ldots\ \bar{c}_T] \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{x}_T \end{bmatrix} \qquad (2\text{-}10)$$

where T is the number of block submatrices in each row and column of the A and B matrices, and the number of subvectors in the $\bar{x}$ and $\bar{b}$ vectors.

In this study, partitioning proves useful in illustrating some of the features of the model developed. In addition, certain specialized linear programming algorithms, called decomposition algorithms, require a problem to have a particular partitioned form as a necessary condition for the use of the algorithm. Although not used in this study, decomposition algorithms could prove useful in solving models, of the type developed, for large systems.

Solution of Programming Problems

Programming techniques are used to find the best solution to any problem that has a number of alternative solutions. The constraint set defines the limits within which a process may operate. If formulated incorrectly, an incorrect solution results.

The objective function is the criteria by which the best solution is selected. It is the heart of an optimization problem.

Although the concept of an objective function is easy to grasp, the writing of an objective function for a particular problem may be difficult because it is a mathematical statement of a desired policy.

For complex problems, the writing of an objective function may require extensive study to ensure that the policy implied in the mathematical statement is equivalent to the desired policy.

There are many algorithms for solving programming problems, all of which are iterative. Descriptions of these algorithms can be found in references on mathematical programming, such as Hadley (1962,1964). The purpose of an algorithm is, of course, to obtain an optimal solution. For efficiency, developed algorithms ensure that once a feasible solution is found, all other feasible solutions examined by the algorithm will be closer to the optimal solution. This requirement reduces the number of iterations necessary to obtain the optimal solution because all feasible solutions are not examined.

Chapter III

THE PROBLEM AND THE MODEL

The Problem

An irrigation system has but one purpose: to transform the temporal and spatial distributions of a naturally available supply of water to the quantitative distributions required for crop production. An irrigation system has two components: physical structures, such as the conduits used to change the spatial distribution and the reservoirs used to change the temporal distribution; and a management policy, which governs the changes of distribution.

In the operation of an irrigation system, losses are incurred in the delivery of water from the supply to the consumer. These losses are due to evaporation, transpiration and infiltration from the structures composing the system. Thus, for a given system, the volume of water delivered is directly a function of the management policy governing the delivery.

The magnitude of delivery losses in 22 selected irrigation systems in the Western United States has been shown by Erie (1968). For the study period, 1949-1960, the average water diversion was 5.16 acre-feet per acre (ac-ft/ac), and the average farm delivery was 3.22 ac-ft/ac. These figures yield an average water delivery efficiency, the ratio of the average farm delivery to the average water diversion, of 62.4 percent.

Using the figures given by Erie (1968), an average delivery loss of 1.94 ac-ft/ac over the 828,000 ac represented by the project examined, it is found that 1,610,000 ac-ft of water was lost by these systems.* Were it possible to make these systems perfectly efficient (100%), the reclaimed water would be sufficient to irrigate an additional 500,000 ac at the present average farm delivery of 3.22 ac-ft/ac. Conversely, using the 1965 U.S. average municipal per capita water requirements of 157 gallons per capita per day (gpcpd),** increasing the irrigation water delivery efficiency to 100 percent would yield sufficient municipal water for 9,160,000 people, enough for a city with a population near that of Tokyo.

The Water Resources Council study (1968), based on data from 1965, found that of the 125,000,000 ac-ft of water delivered to irrigate 42,000,000 ac in the United States, 25,000,000 ac-ft were lost because of inefficient water delivery structures and practices, yielding a water delivery efficiency of 80 percent. This national water delivery efficiency (80%) is considerably better than that shown by Erie for the western systems.

If, however, it were possible to increase the national irrigation water delivery efficiency to 100 percent, the reclaimed water would irrigate an additional 8,400,000 ac at the 1965 delivery rate, or furnish water for an additional 142,200,000 people. Of course, it is not possible to obtain perfect efficiency in the delivery of irrigation water, but it is evident that an increase of only a few percent in the national water delivery efficiency would result in the conservation of sizeable quantities of water.

*Although not stated by Erie, it is presumed these are average annual losses.

**From the Water Resources Council (1968).

Reclamation of wasted water as an approach to water conservation is not new. It was advocated by Bishop, in 1961:

> "Along with the extensive use of water, irrigation is probably a major source of waste of the valuable water resource. This is due, in large measure, to the inefficiency of existing canals and distribution systems with their duplication and obsolescence."

And the U.S. Bureau of Reclamation, in 1963:

> "Conservation of the nation's water supplies, particularly in the western states, is becoming increasingly important as the demand for this vital quantity continues to increase and new sources of supply become increasingly scarce. The time is rapidly approaching when the only natural water supplies available will be the salvage of those now being lost through transpiration, evaporation, consumptive waste, and inefficient storage and transportation practices." (emphasis added)

In recent years, there has been a great deal of reseach devoted to methods of reclaiming water lost from irrigation systems. The methods developed are generally concerned with the modification of the structures in a system. The results of this research are best summarized by the Water Resources Council (1968):

> "Technical changes in irrigation development include changes in water storage, conveyance and application methods for the conservation of existing water supplies. Experimental use of evaporation-retarding films on reservoirs has been successful in reducing water losses. Control of phreatophytes makes additional water available for irrigation use in some areas. Seepage losses during conveyance have been reduced by lining irrigation canals with concrete and other impervious materials."

The use of any of these methods to improve an existing system requires that the improvements be proven economically feasible, that a source of money be available to pay for them, and that the system's users approve of them. Failure to meet any of these requirements results in failure to improve the system. In many small systems, already built and paid for, the users are hesitant to improve a system, even if the improvements are warranted and feasible.

An alternative to improving the structural efficiency of a system is better management of the water in the system. The approach, approximated in some systems, but never before described mathematically, consists of routing the water from the supply to the consumer in such a way that losses are minimized. Recent developments in mathematical optimization techniques and high-speed digital computer technology have made such a minimization procedure for improving the management of existing systems possible.

There are two steps in applying mathematical optimization techniques to irrigation water management. The first step is the derivation of a suitable mathematical model to simulate the process of irrigation water delivery, and the second step is the derivation of a mathematical statement that represents the desired operating policy in terms of the variables of the model.

The developments of this study are concerned with the first step, the derivation of a simulation model that is sufficiently general to be applied to any system that satisfies the conditions under which the model is developed. To keep the model broadly applicable, institutional constraints, such as legal, administrative and economic policies, are not included. The simulation model, as developed, can be used for analysis. The second step, a mathematical statement of the operating policy, is only necessary when the model is used in conjunction with programming techniques to determine an optimal water delivery strategy.

Model Description

The simulation model developed in this study is designed to be used as a tool for the management of water delivery from the supplies to the users, by a network of open-channel conveyance structures and reservoirs. To obtain an optimal strategy for any given time period, both the needs of the future and the influences of the past must be taken into account. Thus, the model is a multiple time period model.

It is assumed in the discussion to follow that future delivery strategies cannot influence the return flows of preceding time periods. The converse is not, however, assumed to be true. Delivery strategies for future time periods are assumed to be directly affected by return flows resulting from previously selected delivery strategies.

Model Structure

The simulation model, for each time period, consists of a series of mathematical statements that correctly describe an irrigation system: the conveyance of water from one point to another within the system, the storage of water within the system, the division or coalescence of flows at junctions within the system, and the losses and gains incurred in these actions.

Figure 1 is a diagram of the type of irrigation system considered in the study. It reveals the system is composed of three elements: reaches of open-channel conveyance structures, exemplified by the line A-L, any of which may be a natural stream, such as the line A-B; junctions of two or more conveyance structures, such as at points Y and P; and a number of storage structure (reservoirs), exemplified by the triangle L-M, which may be either "on-channel" or "off-channel." Throughout

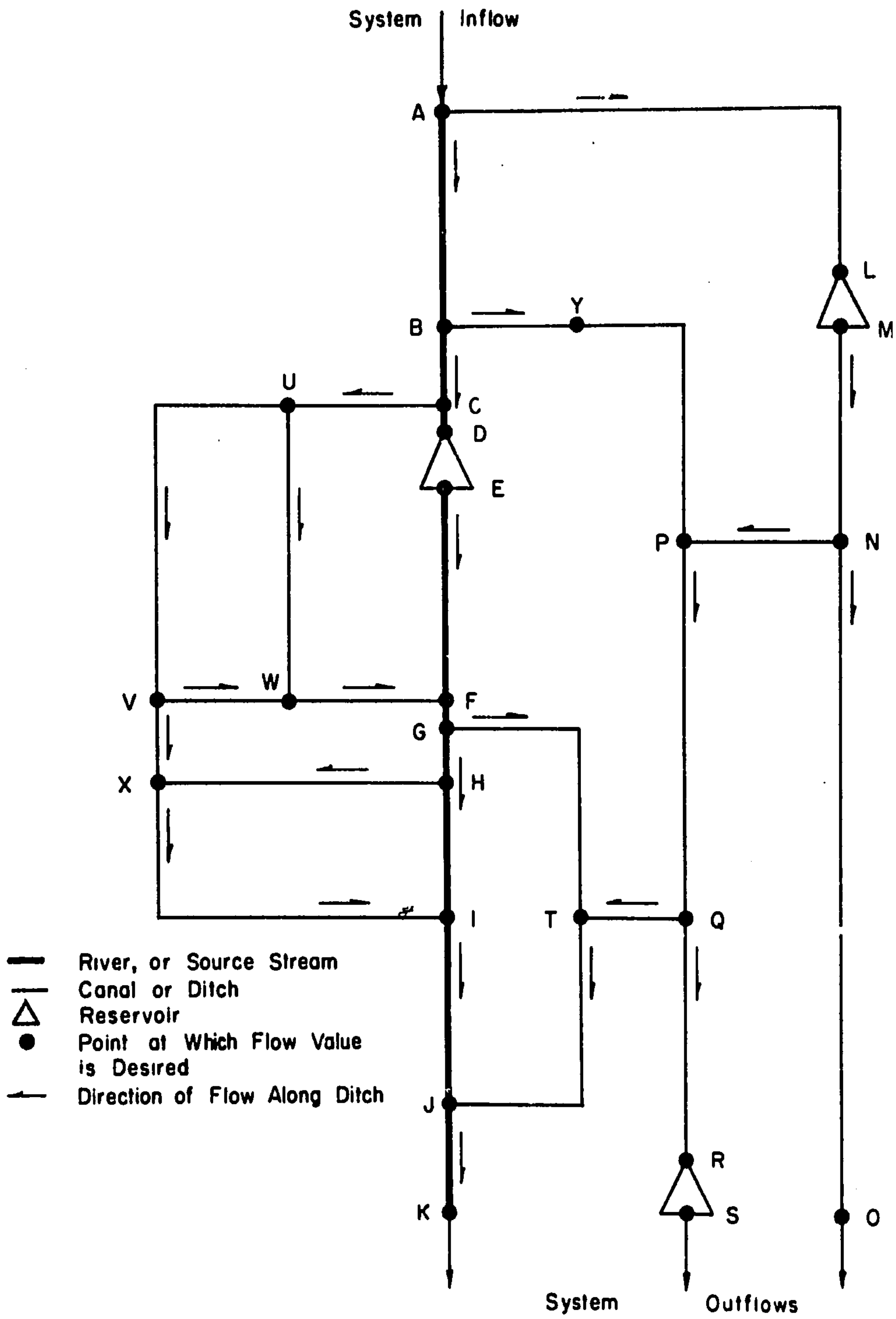


Figure 1. Typical System

the remaining text, these elements are referred to as ditch sectors, nodes and reservoirs. The relationship of each element to the others is shown in Figure 2.

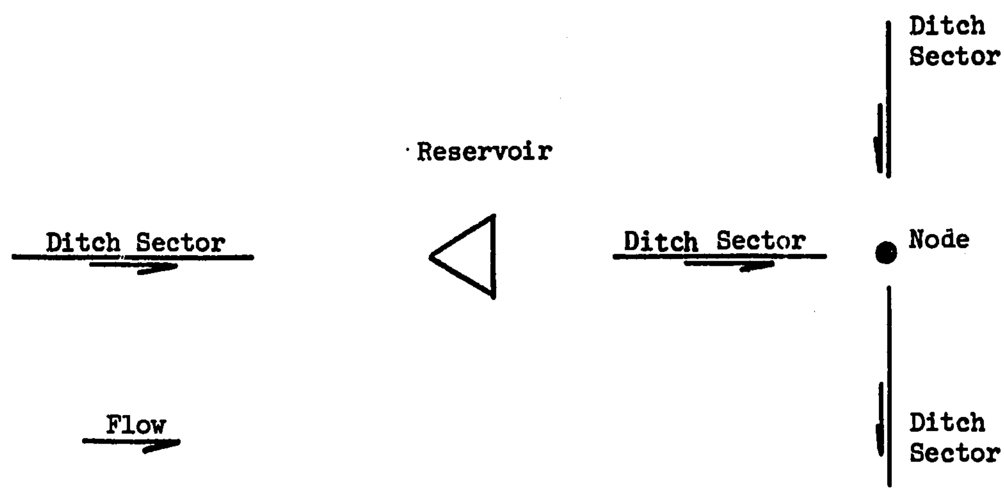


Figure 2. Relation of Elements

In developing a model to be used for operational purposes, the logical decision variables are the various ditch sector flows and reservoir contents. These decision variables must be measured in commensurable units. If the content of a reservoir is measured as the number of acre-feet in storage at the end of each time period and a ditch sector flow is measured as the volume of water released to the ditch sector in each time period (acre-feet per time period), then the requirement is satisfied. Ditch sector flows calculated in this manner can easily be converted to more conventional units of discharge such as cubic feet per second, or to such measures as headgate openings, if the proper conversion factors are used.

There are several events that can occur in each of the elements of a system: inflows, outflows, storages, losses, return flows and demands. For this study, these events are defined as follows.

An inflow to an element is the volume of flowing surface water that enters the element at the upstream end. For ditch sectors and reservoirs, inflows are considered to be the headgate releases into the structures, and for nodes, inflows are the excess flows leaving the downstream ends of the ditch sectors and reservoirs. All inflows are decision variables.

A system inflow is the volume of water, at a point, which is available for delivery and storage by the elements of the system in each time period. Point A, in Figure 1, is a system inflow point. All system inflows are assumed to occur at nodes and be known, or estimable, in quantitative time distribution.

The outflow from an element is defined to be the volume of flowing surface water that leaves the element at its downstream end. For ditch sectors, the outflow is generally the water remaining after losses, return flows, and demands have occurred, which is a release that cannot be directly controlled by an operator. For reservoirs, the outflow is usually a controlled release. For nodes, the outflows are the inflows to other elements of the system, the ditch sectors and reservoirs. All outflows are decision variables.

A system outflow is the volume of water that leaves a system at the downstream end of an element. Points K, O and S are the system outflow points of Figure 1. There are two types of system outflows, required and unrequired. Required system outflows are those flows which the system is required to pass downstream to other systems.

Unrequired system outflows are those flows released to downstream systems because the system of interest lacks sufficient storage capacity to retain them. Required system outflows are treated exactly like demands and are assumed to be known, or estimable, in both quantity and time distribution. Unrequired system outflows are unknown and assumed to be decision variables.

A demand is another event that can occur in each of the elements. It consists of a release of water from the system to a user in a single time period. Demands are not decision variables, but are considered to be volumes of water specified by the system users, for each time period, that must be delivered. This required delivery will create difficulties under some conditions; consequently, some discussion will be devoted to these difficulties later in this chapter.

Losses are those volumes of water removed from the system through uncontrollable evaporation, transpiration and infiltration. All ditch sectors and reservoirs in a system are assumed to have losses associated with their operation.

There are two specific losses that are useful enough in the model development to be specifically named, the seepage loss and the system loss. Seepage loss is that portion of a loss due to infiltration. System loss is the loss defined above, exclusive of those portions of the seepage loss that reappear in the system as return flows. The definition and use of system losses is superior to the use of losses, as defined above, in that it recognizes the importance of return flows to an irrigation system as water in temporary storage. An element that appears to experience extremely high losses may not, in reality, have a high system loss rate because a majority of its losses reappear as return flows.

Return flows are uncontrolled gains in ditch sector flows or reservoir storage volumes. Return flows are principally the result of a high groundwater table, a condition that can be caused by a great many factors. These causes and their effects on the simulation model will be discussed in the next chapter.

Storage is water reserved in certain locations within the system during periods of surplus system inflow to be released for use by the system during periods of deficient system inflow. Storage is considered to be controllable and occurs principally in reservoirs for a surface water irrigation network.

Model Construction

To use a mathematical model for duplicating the events that occur in an irrigation water delivery system, an accounting procedure must be used to keep track of the volumes of water available in the various portions of the system. Furthermore, a requirement must be specified that the water be delivered down the ditch sector from which it is to be taken. The method for simulation, developed in this study, contains these two features.

The continuity equation, which states that the change of mass in storage is equal to the difference between the mass entering an element and the mass leaving an element, is used to relate the quantities of water involved in the various events for each element of a system. The resulting relations are called mass balances.

Mass balances will be derived for the ditch sectors, reservoirs and nodes. However, there are certain assumptions, contained in the derivation of each mass balance, that must be examined.

A ditch sector mass balance is a mathematical abstraction of the events that occur in a ditch sector. These events are inflows, outflows, losses, return flows and demands. Changes in channel storage resulting from changes in flow between time periods are assumed to be negligible.

A reservoir mass balance is a mathematical abstraction of the events that occur in a reservoir. These events are inflows, outflows, losses, return flows, demands and storage.

A nodal mass balance is a mathematical abstraction of the events that occur at a node. These events are assumed to be only inflows and outflows. Losses, return flows, demands and storages are assumed not to occur. Physically, a node is the intersecting space of two or more ditches. This space is considered to be so small that storage, losses and return flows are negligible.

Demands are not considered to occur at the nodes for different reasons. If the demands for the various ditch sectors were satisfied at the nodes, there could exist alternate routes for their delivery. Because most demands are distributed along the ditch sectors, the modeling equations must require that the water be delivered along the proper ditch sector; this condition is not assured if the demands are included in the equations describing the relationships at the nodes.

Most man-made structures are limited in their capacity to transmit or store a quantity of water, and these restrictions must also be included in the simulation. The net result is a decrease in the number of feasible solutions, or strategies, for the delivery of the water. In the model developed, all structures are assumed to have both a minimum capacity restriction, at least zero, and a maximum capacity restriction. However, the expression of all of these restrictions may not be required.

In irrigation systems, all man-made ditches normally have maximum capacities that must be specified in the model. The minimum volume of water any element must transport or store is generally that volume of water that must be delivered by the element to the users. However, if a ditch sector is a natural stream, there is usually no maximum capacity restriction, but minimum capacity restrictions still exist.

Reservoirs are always subject to restrictions on maximum capacities which must be reflected in the model. Minimum capacity restrictions may be necessary in some cases, notably to maintain water levels for recreation or fish and wildlife conservation. In addition, under certain conditions the judicious use of minimum capacity restrictions can assist in the solution of the model.

Model Solution

As in a real system, there are innumerable feasible solutions to the simulation model. Given particular sets of solutions, the simulation model can be used for the analysis of certain problems, such as the legal problems described by Hartman and Seastone (1970) concerning the changing of points of diversion in Colorado.

The strength of the model, however, lies in its use with programming techniques for deriving an optimal strategy for the delivery of irrigation water to systems users. To derive an optimal strategy, an objective function must be introduced. Being a mathematical equation which represents a policy, it must be examined closely to ensure that it coincides with the desired policy.

In this study, only one objective function is used: to minimize the system losses and unrequired system outflows conjunctively. This function implies that a maximum volume of water be retained in the system. It is a conjunctive surface water - groundwater use policy, with emphasis on using surface transportation facilities to deliver the water. Other objective functions and policies can be defined, but for this study only one is used.

When the simulation model is used in conjunction with optimization techniques, the model becomes the constraint set. The individual mass balances and capacity restrictions are the constraints, and the ditch sector flows and reservoir storage values are the decision variables.

Multiple time period models are formulated as a series of a single time period models. However, complications are created by the return flows which reflect the influence of earlier delivery strategies on later time period delivery strategies. These complications will be further discussed in Chapters IV and V.

There are three conditions where no feasible solution to this optimization problem will exist. They are: (1) if the system supply is inadequate to meet the system demands; (2) if the demand for water from a single element is greater than the volume of water which that element can supply; and (3) if a system is simulated which has no outlet for unrequired system outflow, and the system supply is greater than the volume of water the system can store and use. Each of these problems exists in systems operations, and each has been solved in various ways.

The first is solved by legal restrictions that reduce the demand during the periods of deficient supply. The second is most generally solved by administrative restrictions which reduce the demand to that which the element can supply. The final problem can be solved by making the system inflow a decision variable to be calculated by the optimization procedure.

With the elucidation of these basic considerations, the model will now be derived and further discussed. The model, as first developed in Chapter IV, is nonlinear, with its application limited at the present time. In Chapter V, assumptions will be made to linearize the model, thus providing for more immediate usage.

Chapter IV

DEVELOPMENT OF THE MODEL

System Description

Any irrigation system can be described using distinct elements: ditch sectors, nodes and reservoirs. The level of description is the extent to which a system is described by these elements. This extent directly affects the accuracy of the results.

The most detailed description involves subdividing the system at each diversion point or headgate. A ditch sector is defined as the reach beginning just below one diversion point and extending to just beyond the next downstream diversion point. To preserve reality in the model, nodes are required to connect the various ditch sectors.

The advantage of such a detailed description is accuracy. The disadvantage is the immense size of the problem (numbers of equations and variables) generated for systems with large numbers of diversion points.

The least detailed description is an aggregation of all demands along a ditch into a single demand to be satisfied from the supply stream (see Figure 1), disallowing any interditch transfers. This minimizes the problem size and allows extremely large systems to be modeled. But this problem has only one solution at most, and is of no interest in this study.

Until efficient solution algorithms are developed that can handle a multitude of equations and variables, a level of description between the two listed above is suggested. This intermediate level of description consists of selecting the major ditches in the system and defining the node points as the junctions between them. Ditch sectors become those reaches of the major ditches between the node points. The individual demands, normally distributed along each of the ditch sectors, are aggregated to a single demand which is assumed to be delivered at the downstream end of each ditch sector, just upstream of the node. This is the least refined level of description that can be performed and still retain the alternate routing, or programming aspects of the model. It is this intermediate level of description that will be used in the models of a representative system in Chapter VI.

If, however, the exact ditch sector loss function is used in conjunction with the intermediate level of description, the losses from a ditch sector will be overestimated. This is because the demand water is assumed to travel through the entire sector, instead of being diverted all along it. This sacrifice in accuracy can be partially corrected either by adjusting the exact loss rate or by using additional nodes on long ditch sectors. Additional nodes result in an increased level of description.

Development of the Model

Developing the model consists of the derivation of equations to represent the ditch sectors, nodes and reservoirs and their concomitant capacity restrictions. To use the model with programming techniques, an objective function that expresses the operating policy must be derived.

In developing the model, it will be assumed that the system being simulated will be composed of M ditch sectors, P nodes, N reservoirs (of which U are modeled using the node definition and W are modeled using the ditch sector definition), and that an irrigation season consists of T time periods. At times, four indices will be required to properly express the space and time relations of the variables. It is imperative that the meaning of these indices be clear.

Two indices, i and j, will be used to express the spatial relations. The index i will always be used to denote an element of interest: a node, reservoir or ditch sector. The index j will always be used to represent those other elements which affect the element of interest, directly or indirectly. Both of these indices will be used as subscripts.

The remaining two indices, h and t, will be used to express the temporal relations. The index t will be always used to denote a time period of interest within the irrigating season. The index h will be used to represent preceding time periods which affect the time period of interest. Both of these indices will be used as superscripts.

Ditch Sector Mass Balance

Figure 3 schematically describes a typical ditch sector, i. In this ditch sector there are two inputs and three separate outputs. For a particular time period t, the inputs to a ditch sector are the volume of water entering the sector at the headgate, Q_i^t, and the return flow volume, R_i^t. In the same time period, the outputs are the losses, L_i^t, the demand, D_i^t, and the outflow, V_i^t. It is assumed that the return flows are not subject to losses in the sector, but occur in such a manner that they are available for diversion from the sector.

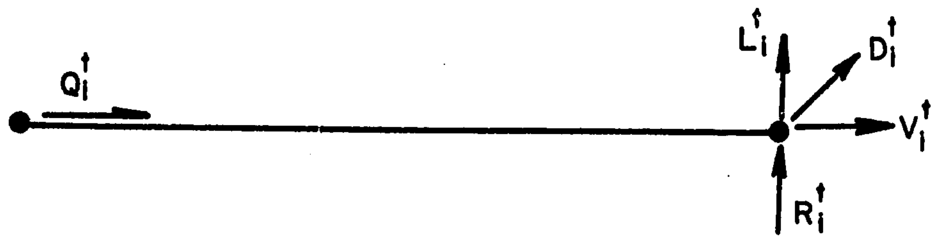


Figure 3. Schematic Representation of a Typical Ditch Sector, i .

From the continuity equation, written

$$\text{Input} = \text{Output} + \text{Storage}, \tag{4-1}$$

for a ditch sector,

$$Q_i^t + R_i^t = L_i^t + V_i^t + D_i^t \quad , \tag{4-2}$$

or

$$Q_i^t - L_i^t + R_i^t - V_i^t = D_i^t \; . \tag{4-3}$$

The subscript i denotes the ditch sector of interest, $i = 1,\ldots,m$ and the superscript t denotes the time period of interest, $t = 1,\ldots,T$. Equation (4-3) is an effective description of a ditch sector and is called the ditch sector mass balance.

For those ditch sector outflows V_i^t that leave the system, the required system outflows are assumed to be included in the demand D_i^t . Thus, the variable V_i^t becomes the unrequired system outflow.

Nodal Mass Balance

Figure 4 schematically describes a typical node. A node has one or more inflows, designated by the V's, and one or more outflows designated by the Q's. The reversal in the naming of the variables is needed because all inputs to the nodes are the outflows from ditch sectors and reservoirs and all outputs from the nodes are the inflows of the ditch sectors and reservoirs. Furthermore, the indices on the variables V and Q are derived from the indexing of the ditch sectors and reservoirs, and it is essentially impossible to make these indices consecutive. For this reason, let two sets, J_i and K_i be defined. The first set, J_i, is composed of the indices of those variables V which supply a node, i. The other, K_i, is composed of those variables Q that are supplied with water from a node, i. For example, the node illustrated in Figure 4 has the sets $J_i = \{1, 4, 6\}$ and $K_i = \{3\}$.

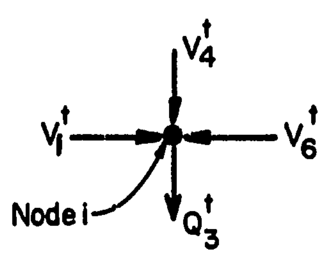


Figure 4. Schematic Representation of a Typical Node, i.

Using these definitions, the variables for a node i are related by the continuity equation as follows:

$$\sum_{j \epsilon J_i} V_j^t - \sum_{j \epsilon K_i} Q_j^t = 0 , \qquad (4\text{-}4)$$

for $i = 1,\ldots,P$ and $t = 1,\ldots,T$. The symbol ε is standard set notation for the phrase "is contained in the set." The simplicity of this expression is due to the assumptions that storage, demands, losses and return flows do not occur at a node.

For a node that is a system inflow point, the nodal mass balance becomes

$$\sum_{j \varepsilon K_i} Q_j^t = I_i^t \qquad \text{(4-4a)}$$

where the set J_i is empty and I_i^t is the volume of flow available as a system inflow at node i in time period t .

Reservoir Mass Balance

There are two methods for modeling a reservoir: as a node in which losses, return flows, demands and storage are considered, and as a ditch sector in which storage is considered. The node method will be examined first, because the ditch sector method is derived from the node method.

Figure 5 is a schematic representation of a typical reservoir for which the node definition should be used. In this figure there are four inputs to and four outputs from the reservoir, although these numbers will vary as with a node. Because of this, the sets J_i and K_i , used for expressing the nodal mass balance, must be used here as well.

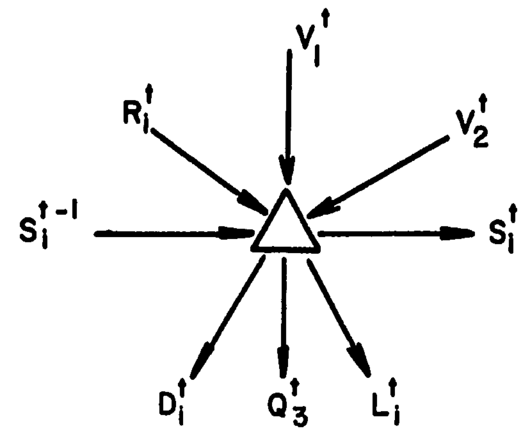


Figure 5. Schematic Representation of a Typical Reservoir, i , Node Definition.

Three of the inputs, V_1^t , V_2^t and R_i^t and three of the outputs, Q_3^t , D_i^t and L_i^t are previously defined in the derivations of the nodal and ditch sector mass balances. The additional variables S_i^{t-1} and S_i^t are the volumes of water in storage at the ends of time periods (t-1) and t , respectively. Justification for considering S_i^{t-1} an input and S_i^t an output requires that the problem be visualized in terms of the time domain. For any reservoir the volume of water in storage at the end of one time period automatically becomes a source of water for the next time period. Thus, S_i^{t-1} is an input for time period t . Similarly, S_i^t is an input to time period (t+1) . Because continuity must be preserved in time, as well as space, S_i^t is also an output from time period t .

From the continuity equation, after rearranging,

$$\sum_{j \varepsilon J_i} V_j^t + S_i^{t-1} - L_i^t + R_i^t - S_i^t - \sum_{j \varepsilon K_i} Q_j^t = D_i^t \;, \qquad (4\text{-}5)$$

for i = 1,...,U and t = 1,...,T , in which the V_j^t are the inflows to the reservoir from the ditch sectors, $j \varepsilon J_i$, the Q_i^t are the outflows from the reservoir to the ditch sectors, $k \varepsilon K_i$. Equation (4-5) is defined to be the reservoir mass balance using the node definition.

To derive the reservoir mass balance for the ditch sector definition from the node definition, it must be assumed that only a single ditch sector supplies a reservoir, that only a single ditch sector receives its supply from a reservoir, and that neither of these ditch sectors have losses or return flows. Figure 6 illustrates this relation.

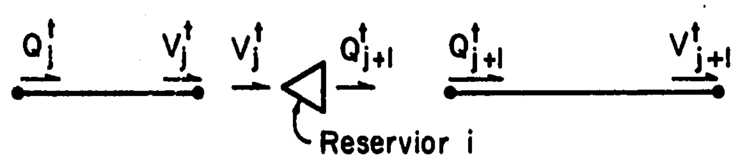


Figure 6. Relation Between the Two Reservoir Definitions.

The mass balances for the two ditch sectors shown in Figure 6 are

$$Q_j^t - V_j^t = D_j^t \tag{4-6}$$

and

$$Q_{j+1}^t - V_{j+1}^t = D_{j+1}^t . \tag{4-6a}$$

For the reservoir shown in Figure 6, the mass balance, using the node definition, is

$$V_j^t + S_i^{t-1} - L_i^t + R_i^t - S_i^t - Q_{j+1}^t = D_i^t , \tag{4-7}$$

in which $J_i = \{j\}$ and $K_i = \{j+1\}$. Substituting for Q^t_{j+1} and V^t_j in equation (4-7), from equations (4-6a) and (4-6) yields

$$Q^t_j + S^{t-1}_i - L^t_i + R^t_i - S^t_i - V^t_{j+1} = D^t_j + D^t_{j+1} + D^t_i \quad . \qquad (4\text{-}8)$$

Recognizing that the sum of the demands is an aggregated demand, relabeling it to be D^t_i and redefining the indices of Q^t_j and V^t_j to correspond with the index denoting the reservoir, yields

$$Q^t_i + S^{t-1}_i - L^t_i + R^t_i - S^t_i - V^t_i = D^t_i \ , \qquad (4\text{-}9)$$

for $i = 1,\ldots,W$ and $t = 1,\ldots,T$. This equation is defined to be the reservoir mass balance, using the ditch sector definition. Figure 7 is a schematic representation of a reservoir modeled according to the ditch sector definition.

The difference between a reservoir of the node definition and a reservoir of the ditch sector definition lies in the number of inflows to, and outflows from, the reservoir. The choice of the proper definition, for any particular reservoir, will help to minimize the problem size.

It is recommended that the ditch sector definition be used only if a reservoir has a single inflow and a single outflow, and that the node definition be used for all other cases. In Chapter VI, a representative system is modeled consistently using the ditch sector definition for all reservoirs. The fact that a larger problem size results is illustrated.

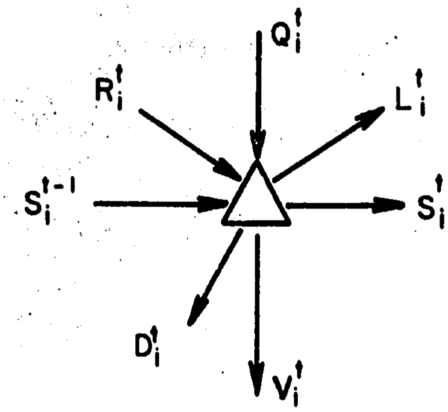


Figure 7. Schematic Representation of a Typical Reservoir, i , Ditch Sector Definition.

Capacity Restrictions

Capacity restrictions are expressed as inequalities and reflect the upper and lower limits of the values a decision variable can assume. For ditch sectors, the maximum and minimum capacity restrictions are expressed as

$$Q_i^t + R_i^t - L_i^t \leq Q_{i_{max}}^t \,, \qquad (4\text{-}10)$$

and

$$Q_i^t + R_i^t - L_i^t \geq Q_{i_{min}}^t \,, \qquad (4\text{-}11)$$

respectively, where $Q_{i_{max}}^t$ is the maximum capacity of the sector and $Q_{i_{min}}^t$ is the minimum capacity. Minimum capacity constraints for ditch sectors are seldom necessary unless to ensure that a variable remains greater than or equal to zero.

Reservoir maximum and minimum capacity restrictions are expressed:

$$S_i^t + R_i^t - L_i^t \leq S_{i_{max}}^t \,, \qquad (4\text{-}12)$$

and

$$S_i^t + R_i^t - L_i^t \geq S_{i_{min}}^t \tag{4-13}$$

Model Composition

Once the ditch sector mass balance, equation (4-3), the nodal mass balance, equation (4-4), the reservoir mass balances, equations (4-5) and (4-9), and the capacity constraints, equations (4-10), (4-11), (4-12) and (4-13), are defined it is possible to construct a simulation model for an irrigation system. The procedure consists of describing the system using ditch sectors, nodes and reservoirs and correctly writing the mass balances and capacity restrictions for each element.

With M ditch sectors, N reservoirs, and P nodes, the model will consist of M ditch sector mass balances, N reservoir mass balances, P nodal mass balances, $(M+N)$ maximum capacity restrictions and as few as zero, or as many as $(M+N)$, minimum capacity restrictions for each time period. For a single time period then, the total number of equations and inequalities in the model will lie between $(2M + 2N + P)$ and $(3M + 3N + P)$. For T time periods the total number of equations and inequalities in the model will lie between $T(2M + 2N + P)$ and $T(3M + 3N + P)$.

When used in conjunction with mathematical programming techniques, the model becomes the constraint set. Therefore, for T time periods, the programming model will be composed of between $T(2M + 2N + P)$ and $T(3M + 3N + P)$ constraints.

Furthermore, for each time period, the model contains two decision variables, Q_i^t and V_i^t, for each ditch sector mass balance, one decision variable, S_i^t , for each reservoir mass balance that uses the node definition, three decision variables, Q_i^t , V_i^t and S_i^t , for each reservoir mass balance that uses the ditch sector definition, one slack variable for each maximum capacity constraint and one surplus variable for each minimum capacity constraint. From these guidelines on the decision variables and constraints, problem size can be calculated while the system is being described in terms of its elements so that the model will be within the space limitations of the computer routine to be used for solution.

Losses and Return Flows

To calculate the loss and return flow volumes required for the ditch sector and reservoir mass balances, loss and return flow functions must be derived. The expression of the loss and return flow functions for a given element in a system requires consideration of the environment of the element, those factors that cannot be controlled by the manager of a system, and the past and present states of the element and all other elements of the system, those factors that can be controlled by the system manager. Furthermore, for the model to be completed, the loss and return flow functions must be expressed in terms of the decision variables Q_i^t , V_i^t and S_i^t .

The rates of evaporation, transpiration, and seepage control the rate of water loss from a ditch sector or reservoir. In turn, each of these rates is controlled by such environmental factors as wind speed and direction, air and water temperatures, radiation, vegetation density, soil types and permeability, etc. The detailed relation of these factors to the loss rates is not considered in this study, although including them would create a more accurate model. Instead, the effect of the past and present states of the system on the loss and return flow functions are considered.

In deriving the ditch sector and reservoir mass balances, it was assumed that a volume of water was lost from an element i in time period t. For evaporation the volume of water lost is the product of the evaporation rate per unit area, for the time period, and the water surface area exposed to the atmosphere. For transpiration the volume of water lost is the product of the transpiration rate per unit area, for the time period, and the area of vegetation. For seepage the volume of water lost is the product of the seepage loss rate per unit area, for the time period, and the area of the interface of the water and the structure containing it. Additionally, the seepage loss rate may change according to the depth of water in the structure.

If it is assumed that single valued functions can be defined to relate the various areas and depths to the volume of water a structure transmits or stores, then the losses can be related to the decision variables. Functionally this is denoted as

$$L_i^t = L(Q_i^t, V_i^t) \tag{4-14}$$

for ditch sectors and

$$L_i^t = L(S_i^{t-1}, S_i^t) \tag{4-15}$$

for reservoirs.

Return flows were defined earlier as being the result of a high groundwater table. A high groundwater table in the vicinity of the system is a function of such environmental factors as the areal extent and hydraulic properties of the aquifer, precipitation, vegetation above the aquifer, the extent and number of wells that remove water from the aquifer, etc. Again, the inclusion of each of these factors creates a more accurate model, but they are not of interest in this study.

Return flows that are of interest are those that have as their sources the seepage losses from the various ditch sectors and reservoirs. In addition, water that is released from the system to satisfy a demand but that is in excess of the crops requirement can return to the system. These excess waters are of interest because they are major contributors to the return flows.

Functionally, then, a return flow volume may be denoted for a time period t by

$$R_i^t = R_i^t(Q_j^h,\ V_j^h,\ S_k^h,\ D_\ell^h) \tag{4-16}$$

for $j = 1,\ldots,M$, $k = 1,\ldots,N$, $h = 1,\ldots,t$ and $\ell = 1,\ldots,M+N$. There are two exceptions to this statement: $i \neq j$ for $h = t$ for ditch sectors and $i \neq k$ for $h = t$ for reservoirs. These exceptions are because a return flow to an element in a given time period, due to a loss from the same element in the same time period, is simply a reduced loss. Equation (4-16) however, does allow a return flow to the same element from which a loss occurred if the return flow occurs in a later time period.

Final Mass Balance Expressions

By substituting equations (4-14) and (4-16) into equation (4-3) the final ditch sector mass balance can be defined as

$$Q_i^t - L(Q_i^t, V_i^t) + R_i^t(Q_j^h, V_j^h, S_k^h, D_\ell^h) - V_i^t = D_i^t \qquad (4\text{-}17)$$

for $i = 1,\ldots,M$, $j = 1,\ldots,M$, $k = 1,\ldots,N$, $t = 1,\ldots,T$, $h = 1,\ldots,t$, $\ell = 1,\ldots,M{+}N$ and $i \neq j$ for $h = t$. Substituting equations (4-15) and (4-16) into equations (4-5) and (4-9) yields the reservoir mass balance using the node definition

$$\sum_{g\varepsilon J_i} V_g^t + S_i^{t-1} - L(S_i^{t-1}, S_i^t) + R_i^t(Q_j^h, V_j^h, S_k^h, D_\ell^h)$$
$$- S_i^t - \sum_{g\varepsilon K_i} Q_g^t = D_i^t \qquad (4\text{-}18)$$

for $i = 1,\ldots,U$, and the reservoir mass balance using the ditch sector definition,

$$Q_i^t + S_i^{t-1} - L(S_i^{t-1}, S_i^t) + R_i^t(Q_j^h, V_j^h, S_k^h, D_\ell^h) - S_i^t - V_i^t = D_i^t$$

(4-19)

for $i = 1,\ldots,W$. In both equations, $j = 1,\ldots,M$, $k = 1,\ldots,N$, $t = 1,\ldots,T$, $h = 1,\ldots,t$, $\ell = 1,\ldots,M{+}N$ and $k \neq i$ for $h = t$. The nodal mass balance equation (4-4) is unchanged.

Objective Function

The objective function for this study is to minimize the system losses and unrequired system outflow conjunctively. At present, no mathematical statement of the function can be made because no approximations are assumed for the loss and return flow functions. However, in the next chapter, approximations are assumed for the loss and return flow functions and an objective function is written.

Chapter V

LINEARIZATION OF THE MODEL

The model, presented in Chapter IV, is a programming model in which the decision variables are the various ditch sector flows and the volumes of water in each of the reservoirs at the end of each time period. No assumptions were made regarding the relation of the decision variables to the loss and return flow functions.

In this chapter linear relations are assumed as approximations of the loss and return flow functions and these relations are substituted into the ditch sector and reservoir mass balances to obtain linear mass balance equations. Through this procedure linear programming can be used for optimization of the objective function; an advantage since linear programming routines are more commonly available than nonlinear programming routines.

In developing the linear approximations, it will be assumed that the ditch sectors and reservoirs are to be indexed consecutively with the indices $i = 1,\ldots,M$ denoting the ditch sectors and $i = M+1,\ldots,M+N$ denoting the reservoirs. This notation simplifies the expressions to be developed.

Linear Loss Functions

A linear loss function may be expressed as follows

$$L = \gamma Q \tag{5-1}$$

in which Q is a measure of the water transported, or stored, by an element and γ is a loss rate constant. To use this function, suitable expressions must be found for the γ and Q .

For ditch sectors, one equation for the calculation of γ that has been used in practice is

$$\gamma = u \sum_{i=1}^{p} (1 - u)^{i-1} \tag{5-2}$$

In the equation, u is defined to be a loss rate per unit length of ditch (such as per mile), or a unit loss rate, and p is the number of unit lengths in the ditch sector, such as the number of miles. Equation (5-2) is nonlinear with respect to distance. However, this presents no difficulty since γ is assumed to be a constant for any particular ditch sector, and linear programming only requires linearity with respect to the decision variables, represented by the Q of equation (5-1).

The measure of Q for ditch sectors is defined in this study to be a weighted average of the inflow to a ditch sector i in time period t, Q_i^t, and the outflow from the same ditch sector in the same time period, V_i^t, or

$$Q = a_i^t Q_i^t + b_i^t V_i^t , \tag{5-3}$$

for $i = 1,\ldots,M$, in which a_i^t and b_i^t are weighting factors defined such that

$$a_i^t + b_i^t = 1 . \tag{5-4}$$

Substituting equation (5-3) into (5-1) yields the linear loss function for a ditch sector:

$$L(Q_i^t, V_i^t) = a_i^t \gamma_{ii}^{tt} Q_i^t + b_i^t \gamma_{ii}^{tt} V_i^t \tag{5-5}$$

for $i = 1,\ldots,M$. The meaning of $L(Q_i^t, V_i^t)$ was defined previously as the volume of water lost from ditch sector i in time period t . The constant γ_{ii}^{tt} is a loss rate with respect to the volume of water transported by ditch sector i in time period t and is calculated using equation (5-2). The double subscript and superscript notation is used in this study to decrease the bulkiness of the expressions to be developed later. In standard mathematical notation, the meaning of γ_{ii}^{tt} is

$$\gamma_{ii}^{tt} = \delta_{ij}\delta_{ht}\gamma \qquad (5\text{-}5a)$$

where $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$, $\delta_{ht} = 1$ if $h = t$ and $\delta_{ht} \neq 0$ if $h \neq t$.

For reservoirs, the losses will be represented by equation (5-1) in the form

$$L = \gamma S + \beta \qquad (5\text{-}6)$$

in which γ is a constant, S is an appropriate value of storage, and β is the intercept on the L-axis where $S = 0$. Figure 8 illustrates this equation.

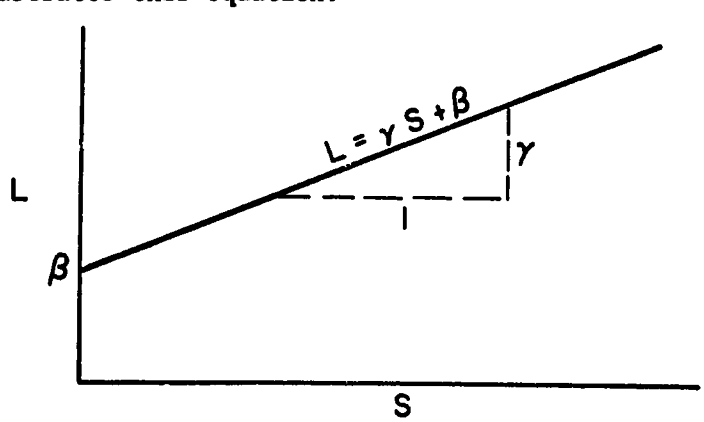


Figure 8. Relation of Reservoir Loss to Reservoir Volume.

Following the procedure used for deriving the ditch sector loss function, let S be determined as a weighted average of the initial and final volumes of water in storage in reservoir i in the time period t :

$$S = a_i^t S_i^{t-1} + b_i^t S_i^t \quad , \tag{5-7}$$

for $i = M+N,\ldots,M+N$, in which a_i^t and b_i^t are weighting factors such that $a_i^t + b_i^t = 1$. Substituting this function into equation (5-6) yields the reservoir loss function,

$$L(S_i^{t-1}, S_i^t) = a_i^t \gamma_{ii}^{tt} S_i^{t-1} + b_i^t \gamma_{ii}^{tt} S_i^t + \beta_i^t \tag{5-8}$$

for $i = M+N,\ldots,M+N$. This function is valid for both the node and ditch sector definitions of a reservoir.

The assumption of a linear loss function for a reservoir appears to be a gross approximation, but two points must be recalled. First, the losses are related to the volume of water in storage, rather than to the surface area or water depth as is usual. Second, any curvilinear function may be assumed to be composed of a series of linear segments, as exhibited in Figure 9. Through the judicious use of constraints and iterative linear programming, it is possible to insert a series of linear segments to solve a nonlinear problem. Using iterative linear programming allows closer approximations of reality, but it also increases the computational effort.

With these functions, equations (5-5) and (5-8), now defined, their substitution into equations (4-17), (4-18) and (4-19) only partially linearizes the ditch sector and reservoir mass balances. The linear return flow function, which completes linearization, remains to be defined.

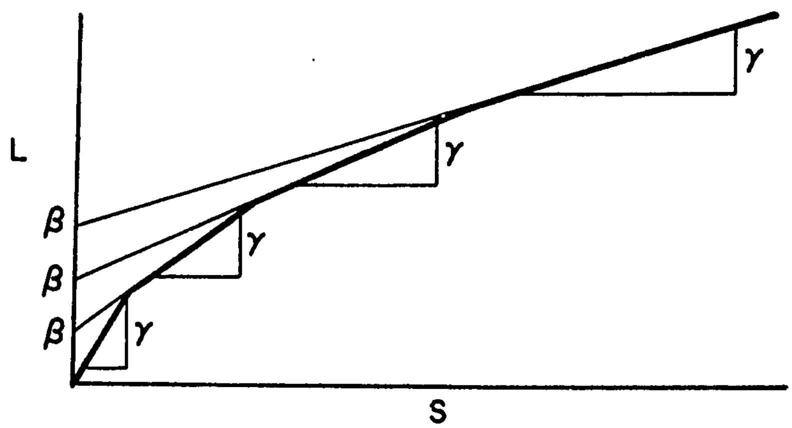


Figure 9. Linear Approximation of Nonlinear Function.

Linear Return Flow Functions

Return flows, as discussed in Chapter IV are a function of both environmental factors and system variables. In linearizing the return flow function, only those return flows created by the system operation will be considered. This is tantamount to assuming that the volume of water in underground storage, which can become available for return flows, is solely derived from the losses of the surface delivery system and the application of excess water to crops. Furthermore, return flows are not the result of a loss in a single time period, but are an accumulation which reflects the losses during all time periods preceeding, and including, the one of interest. This situation must be recognized in the equation derivation.

The total volume of return flow that results from the losses of any element, or from excess volumes of water released from an element to satisfy demands, can be, at most, equal to the original volumes lost or released. Generally the return flow volume is less than the original volumes lost or released because portions of those original volumes are consumed by evaporation, transpiration and deep percolation and are not available for return to the system. These considerations define a theoretical limit on the volume of return flow that a system can produce and this limit can be mathematically stated as

$$\sum_{h=1}^{t} \sum_{i=1}^{M+N} R_i^t(Q_j^h, V_j^h, S_j^h, D_j^h) \leq \sum_{h=1}^{t} \sum_{j=1}^{M+N} [L(Q_j^h, V_j^h) + L(S_j^{h-1}, S_j^h) + D_j^h] \ . \tag{5-9}$$

For clarity it is advantageous to separately derive the equations that describe the return flow due to losses from the delivery system and the return flows due to the application of excess water. After deriving these equations, the results will be combined to yield the linearized return flow functions.

First consider the return flows due to losses from the water delivery system. The water delivery system is composed of two elements that experience losses, ditch sectors and reservoirs. If it is assumed that some fraction, c_{ij}^{th} , of the original loss from any element j in time period h returns to a particular element i in time period t , then an incremental return flow relating the loss from one element and the return flow of another can be defined. The incremental return flow, $\Delta R_i^t(Q_j^h, V_j^h)$, to any element that results from ditch sector losses is

$$\Delta R_i^t(Q_j^h, V_j^h) = c_{ij}^{th} L(Q_j^h, V_j^h) \tag{5-10}$$

for $i = 1,\ldots,M+N$, $j = 1,\ldots,M$, $t = 1,\ldots,T$, $h = 1,\ldots,t$ and $i \neq j$ if $h = t$. The incremental return flow, $\Delta R_j^t(S_j^h)$, to any element that results from reservoir losses is

$$\Delta R_i^t(S_j^h) = c_{ij}^{th} L(S_j^{h-1}, S_j^h) \qquad (5\text{-}10a)$$

for $i = 1,\ldots M+N$, $j = M+1,\ldots,M+N$, $t = 1,\ldots,T$, $h = 1,\ldots,t$ and $i \neq j$ if $h = t$.

The total return flow to an element i in time period t , due to losses from all elements of the water delivery system, is the sum of the incremental return flows over all elements of the system that have experienced a loss in all time periods prior to, and including, the time period of interest. Mathematically this can be stated as

$$R_i^t(Q_j^h, V_j^h, S_j^h) = \sum_{h=1}^{t-1} \left[\sum_{j=1}^{M} \Delta R_i^t(Q_j^h, V_j^h) + \sum_{j=M+1}^{M+N} \Delta R_i^t(S_j^h) \right] + \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \Delta R_i^t(Q_j^t, V_j^t) + \sum_{\substack{j=M+1 \\ i \neq j}}^{M} \Delta R_i^t(S_j^t) \qquad (5\text{-}11)$$

for $i = 1,\ldots,M+N$ and $t = 1,\ldots,T$. The substitution of equations (5-10) and (5-10a), for the incremental return flows, into equation (5-11) yields

$$R_i^t(Q_j^h, V_j^h, S_j^h) = \sum_{h=1}^{t-1} \left[\sum_{j=1}^{M} c_{ij}^{th} L(Q_j^h, V_j^h) + \sum_{j=M+1}^{M+N} c_{ij}^{th} L(S_j^{h-1}, S_j^h) \right] + \sum_{\substack{j=1 \\ i \neq j}}^{M} c_{ij}^{tt} L(Q_j^t, V_j^t) + \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} c_{ij}^{tt} L(S_j^{t-1}, S_j^t) \qquad (5\text{-}12)$$

for $i = 1,\ldots,M+N$ and $t = 1,\ldots,T$.

Further substitution for the loss functions $L(Q_j^h, V_j^h)$, $L(S_j^{h-1}, S_j^h)$, $L(Q_j^t, V_j^t)$ and $L(S_j^{t-1}, S_j^t)$ from equations (5-5) and (5-8) yields two expressions, one for ditch sectors and one for reservoirs. Two expressions are necessary because of the exceptions that state that no element may have a return flow in any time period which is due to its own loss in the same time period. For ditch sectors,

$$R_i^t(Q_j^h, V_j^h S_j^h) = \sum_{h=1}^{t-1} \Bigg[\sum_{j=1}^{M} (a_j^h c_{ij}^{th} \gamma_{jj}^{hh} Q_j^h + b_j^h c_{ij}^{th} \gamma_{jj}^{hh} V_j^h)$$
$$+ \sum_{j=M+1}^{M+N} (a_j^h c_{ij}^{th} \gamma_{jj}^{hh} S_j^{h-1} + b_j^h c_{ij}^{th} \gamma_{jj}^{hh} S_j^h + c_{ij}^{th} \beta_j^h) \Bigg]$$
$$+ \sum_{\substack{j=1 \\ i \neq j}}^{M} (a_j^t c_{ij}^{tt} \gamma_{jj}^{tt} Q_j^t + b_j^t c_{ij}^{tt} \gamma_{jj}^{tt} V_j^t)$$
$$+ \sum_{j=M+1}^{M+N} (a_j^t c_{ij}^{tt} \gamma_{jj}^{tt} S_j^{t-1} + b_j^t c_{ij}^{tt} \gamma_{jj}^{tt} S_j^t + c_{ij}^{tt} \beta_j^t) \ , \tag{5-13}$$

for $i = 1,\ldots,M$, and $t = 1,\ldots,T$. For reservoirs

$$R_i^t(Q_j^h, V_j^h, S_j^h) = \sum_{h=1}^{t-1} \Bigg[\sum_{j=1}^{M} (a_j^h c_{ij}^{th} \gamma_{jj}^{hh} Q_j^h + b_j^h c_{ij}^{th} \gamma_{jj}^{hh} V_j^h)$$
$$+ \sum_{j=M+1}^{M+N} (a_j^h c_{ij}^{th} \gamma_{jj}^{hh} S_j^{h-1} + b_j^h c_{ij}^{th} \gamma_{jj}^{hh} S_j^h + c_{ij}^{th} \beta_j^h) \Bigg]$$
$$+ \sum_{j=1}^{M} (a_j^t c_{ij}^{tt} \gamma_{jj}^{tt} Q_j^t + b_j^t c_{ij}^{tt} \gamma_{jj}^{tt} V_j^t)$$
$$+ \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} (a_j^t c_{ij}^{tt} \gamma_{jj}^{tt} S_j^{t-1} + b_j^t c_{ij}^{tt} \gamma_{jj}^{tt} S_j^t + c_{ij}^{tt} \beta_j^t) \ , \tag{5-14}$$

for $i = M+1,\ldots,M+N$ and $t = 1,\ldots,T$. These equations relate return flows in a system to the losses due to water delivery in terms of the decision variables of the problem.

At this point, two definitions are made to reduce the bulkiness of equations (5-13) and (5-14). In both of the equations, the products of the constants $c_{ij}^{th}\gamma_{jj}^{hh}$ and $c_{ij}^{tt}\gamma_{jj}^{tt}$ are replaced by the constants γ_{ij}^{th} and γ_{ij}^{tt} , respectively.

That is,

$$\gamma_{ij}^{th} = c_{ij}^{th}\gamma_{jj}^{hh} \tag{5-15}$$

and

$$\gamma_{ij}^{tt} = c_{ij}^{tt}\gamma_{jj}^{tt} \; . \tag{5-16}$$

Inserting these definitions into equations (5-13) and (5-14) yields

$$\begin{aligned} R_i^t(Q_j^h,V_j^h,S_j^h) = & \sum_{h=1}^{t-1}\left[\sum_{j=1}^{M}(a_j^h\gamma_{ij}^{th}Q_j^h + b_j^h\gamma_{ij}^{th}V_j^h)\right. \\ & \left. + \sum_{j=M+1}^{M+N}(a_j^h\gamma_{ij}^{th}S_j^{h-1} + b_j^h\gamma_{ij}^{th}S_j^h + c_{ij}^{th}\beta_j^h)\right] \\ & + \sum_{\substack{j=1 \\ i\neq j}}^{M}(a_j^t\gamma_{ij}^{tt}Q_j^t + b_j^t\gamma_{ij}^{tt}V_j^t) \\ & + \sum_{j=M+1}^{M+N}(a_j^t\gamma_{ij}^{tt}S_j^{t-1} + b_j^t\gamma_{ij}^{tt}S_j^t + c_{ij}^{tt}\beta_j^t) \quad , \end{aligned} \tag{5-17}$$

for $i = 1,\ldots,M$, and

$$R_i^t(Q_j^h,V_j^h,S_j^h) = \sum_{h=1}^{t-1}\left[\sum_{j=1}^{M}(a_j^h\gamma_{ij}^{th}Q_j^h + b_j^h\gamma_{ij}^{th}V_j^h)\right.$$

$$\left. + \sum_{j=M+1}^{M+N}(a_j^h\gamma_{ij}^{th}S_j^{h-1} + b_j^h\gamma_{ij}^{th}S_j^h + c_{ij}^{th}\beta_j^h)\right]$$

$$+ \sum_{j=1}^{M}(a_j^t\gamma_{ij}^{tt}Q_j^t + b_j^t\gamma_{ij}^{tt}V_j^t)$$

$$+ \sum_{\substack{j=M+1\\ i\neq j}}^{M+N}(a_j^t\gamma_{ij}^{tt}S_j^{t-1} + b_j^t\gamma_{ij}^{tt}S_j^t + c_{ij}^{tt}\beta_j^t) \quad , \qquad (5\text{-}18)$$

for $i = M+1,\ldots,M+N$, and $T = 1,\ldots,t$ in both equations, respectively. Although equations (5-17) and (5-18) appear to be complicated linear functions, many of the γ_{ij}^{th} values are zero because the c_{ij}^{th} values are zero. A zero c_{ij}^{th} is primarily due to the lack of a hydraulic connection between two elements of a system. However, even if a connecting aquifer exists, it is doubtful that any two elements will receive return flows that originated from the other simultaneously, because the hydraulic gradient will be unfavorable. Therefore, of the "mirror image" return flow coefficients, such as γ_{12}^{th} and γ_{21}^{th} , only one will be non-zero. This reduces, by one-half, the number of return flow coefficients in any given problem. Equations (5-17) and (5-18) adequately describe the flows that return to the various elements of a system because of the losses occurring during water delivery.

Consider now the return flows to element i in time period t due to the application of excess water from element j to crops in previous time periods, $h = 1,\ldots,t$, $R_i^t(D_j^h)$. The maximum volume of excess applied water that may reappear in all portions of the system, from a single element j in time period h , E_j^h , is that volume not retained in the soil. Mathematically this is

$$E_j^h = (1 - e_j^h)D_j^h \, , \tag{5-19}$$

in which e_j^h is the irrigation application efficiency for water released from sector j in time period h. This efficiency is defined as the ratio of the volume of water necessary to increase the soil moisture to field capacity to the total volume of water applied.

During application, however, some of the maximum volume of excess released water, E_j^h, is lost to evaporation, transpiration and deep percolation. Therefore, the total volume of water that actually returns to the system, in all time periods concurrent with and following the time period of the release, is only a fraction, d_j^h, of the maximum available, $d_j^h E_j^h$ or $d_j^h(1 - e_j^h)D_j^h$. If it is further assumed that only a fraction, g_{ij}^{th}, of the actual volume that returns to the system appears in element i in time period t, then the incremental return flow to element i in time period t that results from an application of excess water from element j in time period h, can be defined as

$$\Delta R_i^t(D_j^h) = g_{ij}^{th} d_j^h (1 - e_j^h) D_j^h \, , \tag{5-20}$$

for $i = 1,\ldots,M+N$ and $j = 1,\ldots,M+N$. As with the return flows due to losses from the conveyance structures, certain terms in equation (5-20) can be assumed constant for a given system and can be represented by a single symbol. By defining

$$\alpha_{ij}^{th} = g_{ij}^{th} d_j^h (1 - e_j^h) \, , \tag{5-21}$$

equation (5-20) can be written as

$$\Delta R_i^t(D_j^h) = \alpha_{ij}^{th} D_j^h \ , \qquad (5\text{-}22)$$

for $i = 1,\ldots,M+N$, $j = 1,\ldots,M+N$, $h = 1,\ldots,t$ and $t = 1,\ldots,T$.

The total return flow, resulting from the application of excess water to the crops, to an element i in time period t is the sum of the incremental return flows that result from this application for all elements of the system in all time periods previous to, and concurrent with, the time period of interest. Mathematically,

$$R_i^t(D_j^h) = \sum_{h=1}^{t} \sum_{j=1}^{M+N} \Delta R_i^t(D_j^h) \ , \qquad (5\text{-}23)$$

for $i = 1,\ldots,M+N$ and $t = 1,\ldots,T$. In this equation there is assumed to be no restriction to prevent an element from receiving a return flow due to its own release of excess water in the same time period that the release occurs.

Substituting equation (5-22) into equation (5-23) yields the return flow function for the application of excess water to crops. It is

$$R_i^t(D_j^h) = \sum_{h=1}^{t} \sum_{j=1}^{M+N} \alpha_{ij}^{th} D_j^h \qquad (5\text{-}24)$$

for $i = 1,\ldots,M+N$ and $t = 1,\ldots,T$.

Combining equation (5-24) with equations (5-17) and (5-18) yields the linearized return flow functions for both losses from the system elements and the release of excess water from the system. For ditch sectors,

$$R_i^t(Q_j^h,V_j^h,S_j^h,D_j^h) = \sum_{h=1}^{t-1}\Bigg[\sum_{j=1}^{M}(a_j^h\gamma_{ij}^{th}Q_j^h + b_j^h\gamma_{ij}^{th}V_j^h)$$

$$+ \sum_{j=M+1}^{M+N}(a_j^h\gamma_{ij}^{th}S_j^{h-1} + b_j^h\gamma_{ij}^{th}S_j^h + c_{ij}^{th}\beta_j^h)\Bigg]$$

$$+ \sum_{\substack{j=1\\ i\neq j}}^{M}(a_j^t\gamma_{ij}^{tt}Q_j^t + b_j^t\gamma_{ij}^{tt}V_j^t)$$

$$+ \sum_{j=M+1}^{M+N}(a_j^t\gamma_{ij}^{tt}S_j^{t-1} + b_j^t\gamma_{ij}^{tt}S_j^t + c_{ij}^{tt}\beta_j^t)$$

$$+ \sum_{h=1}^{t}\sum_{j=1}^{M+N}\alpha_{ij}^{th}D_j^h \tag{5-25}$$

for $i = 1,\ldots,M$, and $t = 1,\ldots,T$. For reservoirs

$$R_i^t(Q_j^h,V_j^h,S_j^h,D_j^h) = \sum_{h=1}^{t-1}\Bigg[\sum_{j=1}^{M}(a_j^h\gamma_{ij}^{th}Q_j^h + b_j^h\gamma_{ij}^{th}V_j^h)$$

$$+ \sum_{j=M+1}^{M+N}(a_j^h\gamma_{ij}^{th}S_j^{h-1} + b_j^h\gamma_{ij}^{th}S_j^h + c_{ij}^{th}\beta_j^h)\Bigg]$$

$$+ \sum_{j=1}^{M}(a_j^t\gamma_{ij}^{tt}Q_j^t + b_j^t\gamma_{ij}^{tt}V_j^t)$$

$$+ \sum_{\substack{j=M+1\\ i\neq j}}^{M+N}(a_j^t\gamma_{ij}^{tt}S_j^{t-1} + b_j^t\gamma_{ij}^{tt}S_j^t + c_{ij}^{tt}\beta_j^t)$$

$$+ \sum_{h=1}^{t}\sum_{j=1}^{M+N}\alpha_{ij}^{th}D_j^h \ , \tag{5-26}$$

for $i = M+1,\ldots,M+N$ and $t = 1,\ldots,T$, which is sufficient for both the node and ditch sector definitions of a reservoir. Again, it should be emphasized that although equations (5-25) and (5-26) appear to be complicated, in modeling a real system, many of the coefficients will be zero.

Relation of Losses and Return Flows

Because of the difficulty in visualizing the assumed relationships between losses and return flows for the linearized model, graphical representations of a hypothesized loss from a ditch sector and the associated return flows are shown in Figure 10. A loss of 1.9 units of water from ditch sector i in time period 1 is shown in the graph on the right. Of 1.9 units lost, 0.3 units return to the sector, as shown in time periods 2 and 3. Other varying amounts return to the sectors i-2, i-1, i+1, i+2, and i+3 in the distributions shown. The total return flow to the system from sector i can be calculated and is found to be 1.18 units. The system loss, defined as the loss from an element less those volumes of water which reappear in the system as return flows, is 0.72 units of water.

The relations shown in Figure 10 can also be exhibited in a three-dimensional graph such as Figure 11. In this figure the horizontal plane is the element - time plane. The space below this plane shows the volume of water lost, and the space above shows the related return flows.

Figure 11, however, shows only the return flows for a loss from a single element in a single time period. To graphically describe the loss and return flow relationship for an entire system, similar figures would need to be constructed for each element that experiences a loss. The construction of these figures is neither necessary nor recommended.

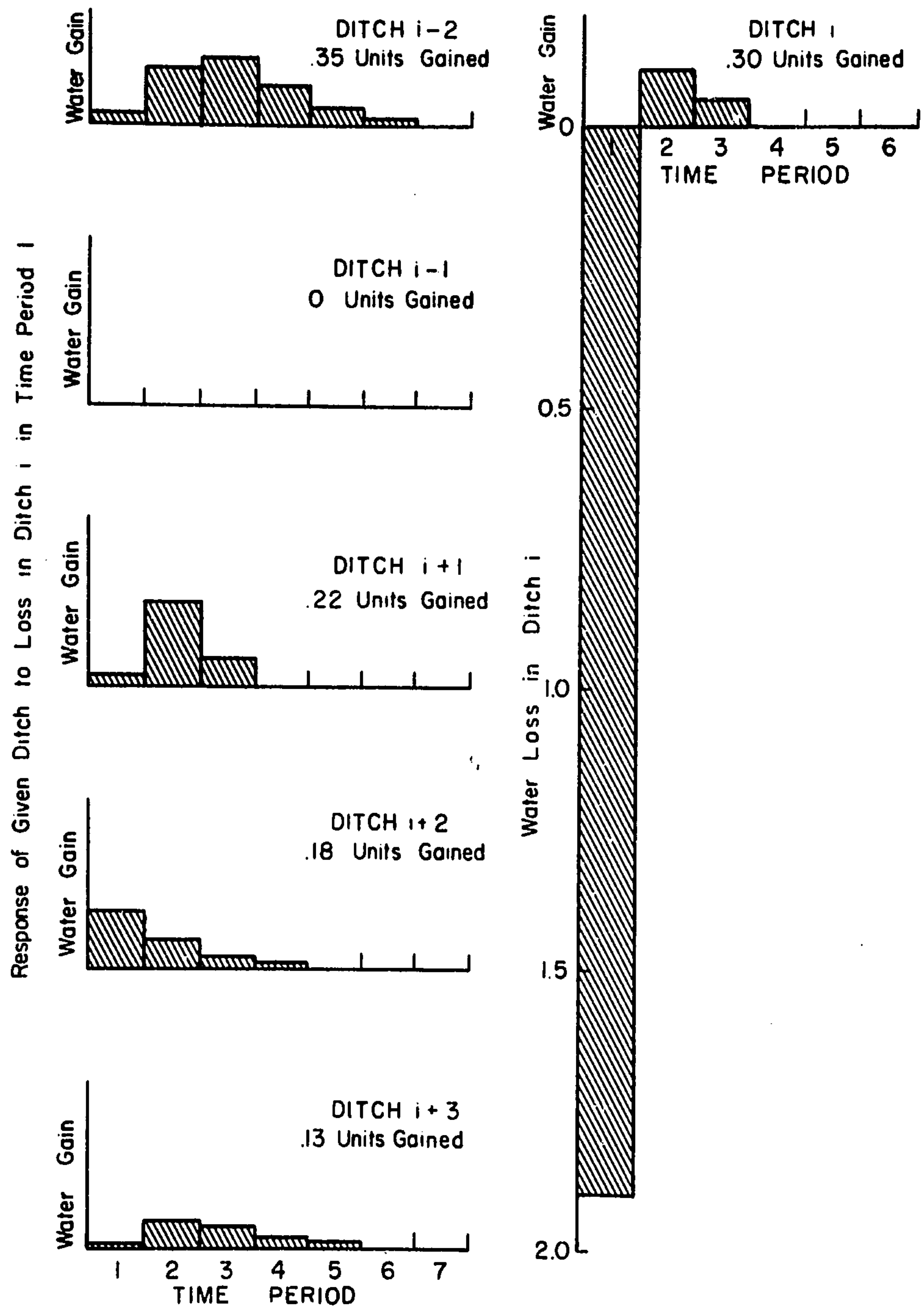


Figure 10. Two-Dimensional Representation of a Hypothesized Loss and Its Associated Return Flows

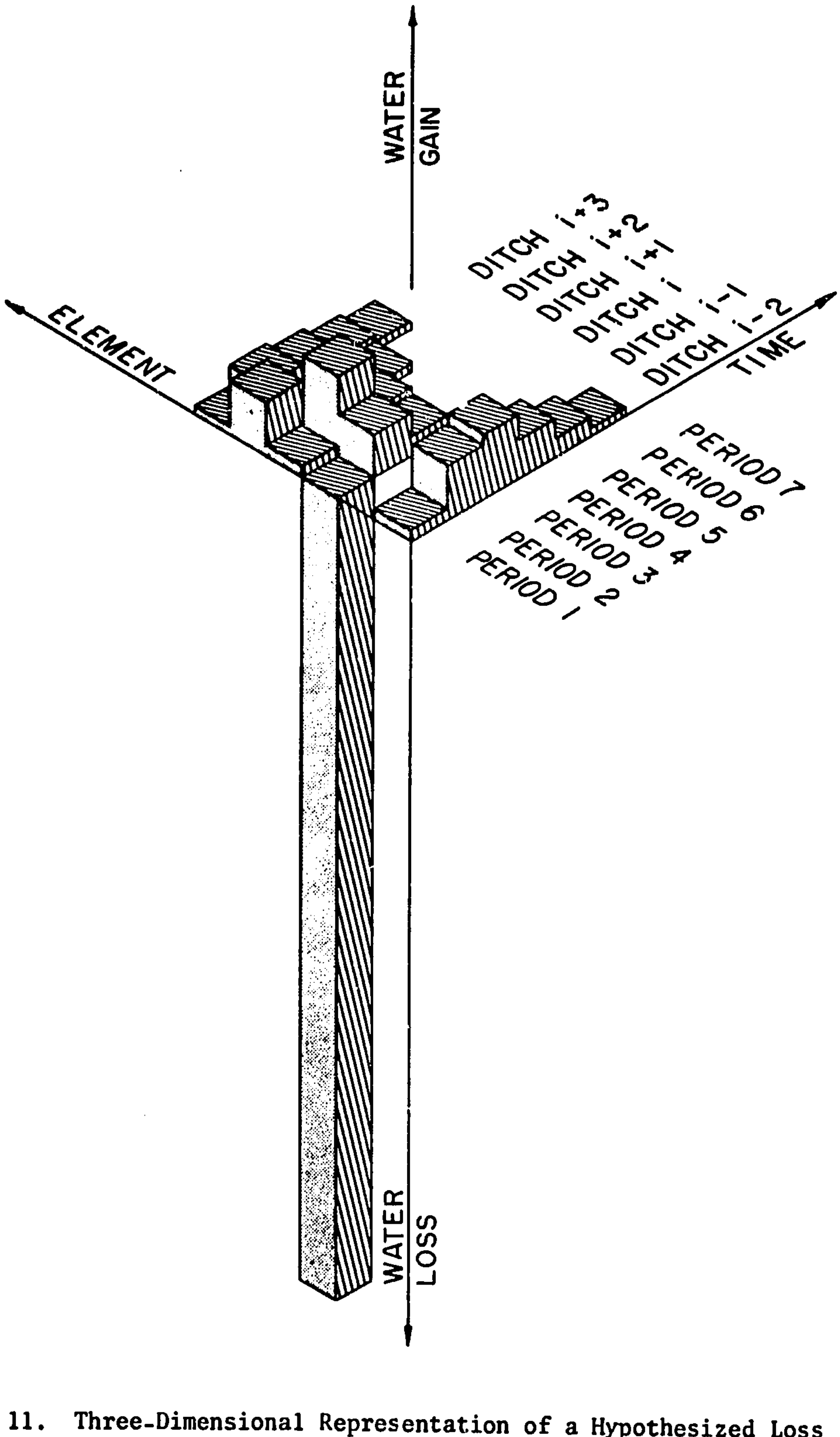


Figure 11. Three-Dimensional Representation of a Hypothesized Loss and Its Associated Return Flows.

They are presented here only to illustrate the relation between losses and return flows that are assumed in the linear model.

Linear Mass Balances

Linear mass balances are obtained by substituting the linear loss and return flow functions into the mass balance equations, derived in Chapter IV and repeated here for convenience. For ditch sectors,

$$Q_i^t - L(Q_i^t, V_i^t) + R_i^t(Q_j^h, V_j^h, S_j^h, D_j^h) - V_i^t = D_i^t \text{ ,} \tag{4-17}$$

for $i = 1,\ldots,M$; for reservoirs using the node definition,

$$\sum_{k\epsilon J_i} V_k^t + S_i^{t-1} - L(S_i^{t-1}, S_i^t) + R_i^t(Q_j^h, V_j^h, S_j^h, D_j^h) - S_i^t - \sum_{k\epsilon K_i} Q_k^t = D_i^t \text{ ,} \tag{4-18}$$

for $i = M+1,\ldots,M+U$; and for reservoirs using the ditch sector definition,

$$Q_i^t + S_i^{t-1} - L(S_i^{t-1}, S_i^t) + R_i^t(Q_j^h, V_j^h, S_j^h, D_j^h) - S_i^t - V_i^t = D_i^t \text{ ,} \tag{4-19}$$

for $i = M+U,\ldots,M+N$. In all equations $j = 1,\ldots,M+N$, $h = 1,\ldots,t$, and $t = 1,\ldots,T$, except that $i \neq j$ for $h = t$.

Substituting equations (5-5) and (5-25) for the loss and return flow functions, respectively, into equation (4-17) yields the linear ditch sector mass balance:

$$\sum_{h=1}^{t-1}\left[\sum_{j=1}^{M}(a_j^h\gamma_{ij}^{th}Q_j^h + b_j^h\gamma_{ij}^{th}V_j^h) + \sum_{j=M+1}^{M+N}(a_j^h\gamma_{ij}^{th}S_j^{h-1} + b_j^h\gamma_{ij}^{th}S_j^h)\right]$$

$$+ \sum_{\substack{j=1 \\ i \neq j}}^{M} (a_j^t \gamma_{ij}^{tt} Q_j^t + b_j^t \gamma_{ij}^{tt} V_j^t) + \sum_{j=M+1}^{M+N} (a_j^t \gamma_{ij}^{tt} S_j^{t-1} + b_j^t \gamma_{ij}^{tt} S_j^t)$$

$$+ (1 - a_i^t \gamma_{ii}^{tt}) Q_i^t - (1 + b_i^t \gamma_{ii}^{tt}) V_i^t = (1 - \alpha_{ii}^{tt}) D_i^t$$

$$- \sum_{h=1}^{t-1} \left[\sum_{j=M+1}^{M+N} c_{ij}^{th} \beta_j^h + \sum_{j=1}^{M+N} \alpha_{ij}^{th} D_j^h \right] - \sum_{j=M+1}^{M+N} c_{ij}^{tt} \beta_j^t - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \alpha_{ij}^{tt} D_j^t \qquad (5\text{-}27)$$

for $i = 1,\ldots,M$ and $t = 1,\ldots,T$. Substituting equations (5-8) and (5-26) into equation (4-18) yields the linear reservoir mass balance for the node definition:

$$\sum_{k \in J_i} V_k^t + \sum_{h=1}^{t-1} \left[\sum_{j=1}^{M} (a_j^h \gamma_{ij}^{th} Q_j^h + b_j^h \gamma_{ij}^{th} V_j^h + \sum_{j=M+1}^{M+N} (a_j^h \gamma_{ij}^{th} S_j^{h-1} + b_j^h \gamma_{ij}^{th} S_j^h) \right]$$

$$+ \sum_{j=1}^{M} (a_j^t \gamma_{ij}^{tt} Q_j^t + b_j^t \gamma_{ij}^{tt} V_j^t) + \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} (a_j^t \gamma_{ij}^{tt} S_j^{t-1} + b_j^t \gamma_{ij}^{tt} S_j^t)$$

$$+ (1 - a_i^t \gamma_{ii}^{tt}) S_i^{t-1} - (1 + b_i^t \gamma_{ii}^{tt}) S_i^t - \sum_{k \in K_i} Q_k^t = (1 - \alpha_{ii}^{tt}) D_i^t$$

$$+ \beta_i^t - \sum_{h=1}^{t-1} \left[\sum_{j=M+1}^{M+N} c_{ij}^{th} \beta_j^h + \sum_{j=1}^{M+N} \alpha_{ij}^{th} D_j^h \right] - \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} c_{ij}^{tt} \beta_j^t - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \alpha_{ij}^{tt} D_j^t \qquad (5\text{-}28)$$

for $i = M+1,\ldots,M+U$ and $t = 1,\ldots,T$. Further substitution of equations (5-8) and (5-26) into equation (4-19) yields the linear reservoir mass balance for the ditch sector definition:

$$Q_i^t + \sum_{h=1}^{t-1} \left[\sum_{j=1}^{M} (a_j^h \gamma_{ij}^{th} Q_j^h + b_j^h \gamma_{ij}^{th} V_j^h) + \sum_{j=M+1}^{M+N} (a_j^h \gamma_{ij}^{th} S_j^{h-1} + b_j^h \gamma_{ij}^{th} S_j^h) \right]$$

$$+ \sum_{j=1}^{M} (a_j^t \gamma_{ij}^{tt} Q_j^t + b_j^t \gamma_{ij}^{tt} V_j^t) + \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} (a_j^t \gamma_{ij}^{tt} S_j^{t-1} + b_j^t \gamma_{ij}^{tt} S_j^t)$$

$$+ (1 - a_i^t \gamma_{ii}^{tt}) S_i^{t-1} - (1 + b_i^t \gamma_{ii}^{tt}) S_i^t - V_i^t = (1 - \alpha_{ii}^{tt}) D_i^t + \beta_i^t$$

$$- \sum_{h=1}^{t-1} \left[\sum_{j=M+1}^{M+N} c_{ij}^{th} \beta_j^h + \sum_{j=1}^{M+N} \alpha_{ij}^{th} D_j^h \right] - \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} c_{ij}^{tt} \beta_j^t - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \alpha_{ij}^{tt} D_j^t$$

(5-29)

for $i = M+U,\ldots,M+N$ and $t = 1,\ldots,T$. The nodal mass balance remains the same as in equation (4-4),

$$\sum_{j \in J_i} V_j^t - \sum_{j \in K_i} Q_j^t = 0 \quad . \tag{4-4}$$

A close examination of the linear ditch sector and reservoir mass balances reveals three major parts: a mathematical description of the influence of the past system flows, contained in the $\sum_{h=1}^{t-1}$ term; a mathematical description of the present system flows, defined by all variables with the t superscripts; and a series of known terms that reduce to a single constant (those terms on the right of the equal sign). These observations are the basis for partitioning the linear programming matrix equation, equation (2-8).

Linear Programming Model

The linear programming model has the same features as the nonlinear model described in Chapter IV--the necessary linear mass balances and capacity restrictions to simulate the system for each time period.

For T time periods, there are T sets of these equations, and with the addition of the necessary slack and surplus variables to the capacity constraints, a matrix of the form of equation (2-6) can be written. Further, the matrix equation can be partitioned according to the time periods, just as equation (2-8) is partitioned. Such a matrix is shown in equation (5-30)

$$\begin{bmatrix} A_{11} & 0 & \cdots & 0 \\ A_{21} & A_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ A_{T1} & A_{T2} & \cdots & A_{TT} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_T \end{bmatrix} = \begin{bmatrix} D_{11} & 0 & \cdots & 0 \\ D_{21} & D_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ D_{T1} & D_{T2} & & D_{TT} \end{bmatrix} \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \vdots \\ \bar{b}_T \end{bmatrix} \tag{5-30}$$

Each of the submatrices in equation (5-30) has a particular physical meaning. Those on the principal diagonal of the A matrix represent the allocation of the supply in a particular time period to satisfy the demands in the same time period. For example, submatrix A_{11} represents the allocation of the supply available in time period 1 to satisfy the demands of time period 1, and A_{22} represents the allocation of the supply available in time period 2 to satisfy the demands of time period 2, etc. The off-diagonal submatrices of the A matrix represent the influence of an allocation of the supply available in one time period on the allocation of the supply available in another time period. For example, A_{21} , represents the influence of an allocation in time period 1 on the allocation in time period 2, and A_{T2} represents the influence of an allocation in time period 2 on the allocation in time period T .

The upper-triangular submatrices above the principle diagonal are the null or zero matrices because return flows resulting from future allocations cannot affect preceding strategies, as explained in Chapter III, in describing the model. Similar explanations may be made for the D_{th} submatrices.

The $\bar{x}_t$ and $\bar{b}_t$ are, respectively, the vectors of decision variables and constants, including the supplies, demands, and capacitie: for time period t , $t = 1,\ldots,T$. They correspond to the respective A_{th} and D_{th} submatrices, $h = 1,\ldots,t$, and $h \leq t$. Furthermore, when one has obtained the solution to a problem, the values of the $\bar{x}_t$ subvector will represent the optimal water delivery strategy in time period t .

The objective function, yet to be mathematically defined, can also be partitioned with justification. Such a partitioning is exhibited in equation (2-9) and repeated here:

$$(\text{max or min})\ z = [\bar{c}_1\ \bar{c}_2 \ldots \bar{c}_1] \begin{bmatrix} \bar{x}_1 \\ \bar{x}_1 \\ \cdot \\ \cdot \\ \cdot \\ \bar{x}_T \end{bmatrix} . \qquad (2\text{-}9)$$

The $\bar{x}_t$ of this equation correspond to the $\bar{x}_t$ of equation (5-30). The elements of the $\bar{c}_t$ are the coefficients of linear functions which involve the loss and return flow functions of the various elements.

<u>Objective Function</u>

With the definition of the linear approximations of the loss and return flow functions, a mathematical relation can be derived for the objective of minimizing system loss and unrequired system outflow conjunctively. This objective consists of two parts, the system loss and the unrequired system outflow. Equations will be defined for each of these parts, then combined to express the desired objective.

System losses have been defined as the losses due to evaporation, transpiration, and infiltration, exclusive of those seepage losses that reappear in the system as return flows. For a particular element i and time period t, the system loss is the loss as defined in Chapter III, from the element minus its incremental contribution to the return flows of the other elements of the system in the remaining $(T - t + 1)$ time periods. Mathematically this may be written as:

$$f_i^t = L(Q_i^t, V_i^t) - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \Delta R_j^t(Q_i^t, V_i^t) - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} \Delta R_j^h(Q_i^t, V_i^t) \qquad (5\text{-}31)$$

for ditch sectors, $i = 1, \ldots, M$, and $t = 1, \ldots, T$, and

$$f_i^t = L(S_i^{t-1}, S_i^t) - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \Delta R_j^t(S_i^{t-1}, S_i^t) - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} \Delta R_j^h(S_i^{t-1}, S_i^t) \qquad (5\text{-}32)$$

for reservoirs, $i = M+1, \ldots, M+N$ and $t = 1, \ldots, T$. In the equations f_i^t may be defined as the "cost" of using element i in time period t. The cost is measured in units of water.

There are $(M + N)$ f_i^t values for each time period, with each corresponding to a ditch sector or reservoir. For a ditch sector, $i = 1, \ldots, M$ and $t = 1, \ldots, T$, from equations (5-5), (5-10), (5-15), (5-16), and (5-31).

$$f_i^t = [\, a_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} a_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} a_i^t \gamma_{ji}^{ht}] Q_i^t + [b_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} b_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} b_i^t \gamma_{ji}^{ht}] V_i^t \,. \tag{5-33}$$

For a reservoir, $i = M + 1, \ldots, M+N$ and $t = 1, \ldots, T$, from equations (5-8), (5-10a), (5-15), (5-16), and (5-32), since the inflows, and outflows are assumed to experience no losses.

$$f_i^t = [\, a_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} a_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} a_i^t \gamma_{ji}^{ht}] S_i^{t-1} + [b_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} b_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} b_i^t \gamma_{ji}^{ht}] S_i^t \,. \tag{5-34}$$

The β terms and demands are not included because they are constants, as explained in Chapter II.

Unrequired system outflow involves the outflows from those elements at the extremeties of a system. Let two sets be defined, G_i and H_i, which have similar definitions to the sets J_i and K_i, previously used in defining the nodal mass balance. The set G_i will contain the indices of those variables V_j^t which represent outflows that leave the system, from ditch sectors and reservoirs modeled with the ditch sector definition, at system outflow point i. The set H_i will contain the indices of those variables Q_j^t which represent outflows that leave the system, at point i, from reservoirs modeled with the node definition. Unrequired system outflow can then be expressed as

$$\sum_{j\epsilon G_i} V_j^t + \sum_{j\epsilon H_i} Q_j^t \; . \tag{5-34a}$$

From these definitions, a general statement of the objective of minimizing system losses and unrequired system outflows can be written as

$$\text{Minimize } Z = \sum_{t=1}^{T} [\sum_{i=1}^{M+N} f_i^t + \sum_{j\epsilon G_i} V_j^t + \sum_{j\epsilon H_i} Q_j^t] \; . \tag{5-35}$$

Substituting equations (5-33) and (5-34) into (5-35) yields a usable mathematical statement of the objective:

$$\begin{aligned}
\text{Min } Z = & \sum_{t=1}^{T} \{ \sum_{i=1}^{M} [a_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i\neq j}}^{M+N} a_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} a_i^t \gamma_{ji}^{ht}) Q_i^t \\
& + (b_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i\neq j}}^{M+N} b_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} b_i^t \gamma_{ji}^{ht}) V_i^t] \\
& + \sum_{i=M+1}^{M+N} (b_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i\neq j}}^{M+N} b_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} b_i^t \gamma_{ji}^{ht}) S_i^t \\
& + \sum_{j\epsilon G_i} V_j^t + \sum_{j\epsilon H_i} Q_j^t \} \\
& + \sum_{t=2}^{T} \sum_{i=M+1}^{M+N} (a_i^t \gamma_{ii}^{tt} - \sum_{\substack{j=1 \\ i\neq j}}^{M+N} a_i^t \gamma_{ji}^{tt} - \sum_{h=t+1}^{T} \sum_{j=1}^{M+N} a_i^t \gamma_{ji}^{ht}) S_i^{t-1} \; .
\end{aligned} \tag{5-36}$$

With this objective function now defined, the linearized model derivation is complete.

Four examples of the structure of the A and D matrices an the $\bar{x}$ and $\bar{b}$ vectors are shown in Appendix B for a simple syste The equations are derived, both the constants and objective functi then put into the form of equation (5-30) for illustration. The n definition of a reservoir is used for modeling the reservoir shown

Relation of Derived Model to Network Model

The problem of irrigation water delivery is essentially a net problem, as described in Hadley (1962), and network terminology ca applied to the modeling of irrigation systems to yield similar res It is felt, however, that the model descriptions derived here have advantages over the use of network techniques; Appendix A discusse these advantages.

Chapter VI

MODEL OF A REPRESENTATIVE SYSTEM

As stated, the objective of this study is to develop a mathematical model for simulating the events that occur in an irrigation system and to use this simulation in conjunction with mathematical programming techniques to provide a tool for aiding in the management of irrigation water delivery. Such a simulation model is developed in Chapter IV. In Chapter V linear approximations are derived for the loss and return flow functions of the model and a linearized model is developed.

If the appropriate functions (or constants) are obtained, the nonlinear (or linear) programming model may be applied in two ways: for a preseason analysis to estimate the extent to which the demands on a supply will be satisfied during an irrigation season, and as an aid for decision making, which approximates strategies that minimize the losses resulting from the delivery and storage of water throughout an irrigating season. Both uses require quantitative estimates of future time and space distributions of supplies and demands. Further discussion of these uses of the model is included at the end of this chapter.

In this chapter the linearized model is used to simulate a representative system. A series of analyses are presented to show the change in routing strategy in a single time period according to the number of future time periods included in the analysis, and comparisons are made of optimal and nonoptimal strategies.

Prototype System

The system to be used for the examples has been built around the Cache La Poudre River in Northeastern Colorado. The Poudre River has its headwaters in the high mountains of Colorado and flows eastward toward the plains. It enters the plains at the downstream end of Poudre Canyon, near LaPorte, Colorado. There is a streamgage, the "mouth of the canyon gage," at this point. From the streamgage the river flows southeasterly through Fort Collins to Greeley, Colorado, where it becomes a tributary to the South Platte River.

The irrigation system consists of structures in both the mountains and the plains. The mountain structures are: small diversions for subsystems that are used primarily to irrigate hay meadows, reservoirs that are used to store water for later release to the plains subsystems, and transbasin diversions that have been constructed to provide additional water for the plains. The subsystems on the plains place the greatest demands on the river as a source of supply. In aggregate, there are 30 or so subsystems, each operated as a cooperative, and 50 or more reservoirs, mostly off-channel. The extent of the plains subsystem is shown in Figure 1?.

Most of the supply for the Poudre system is derived from the melting of winter snowpack in the mountains. The peak flow, occurring during spring thaw, is in May and June. The transbasin diversions contribute similarly to the supply.

The basin policy is to store water when there is surplus runoff and to use the stored water when the river flow is deficient. This policy is based only on hydrologic considerations. The detailed

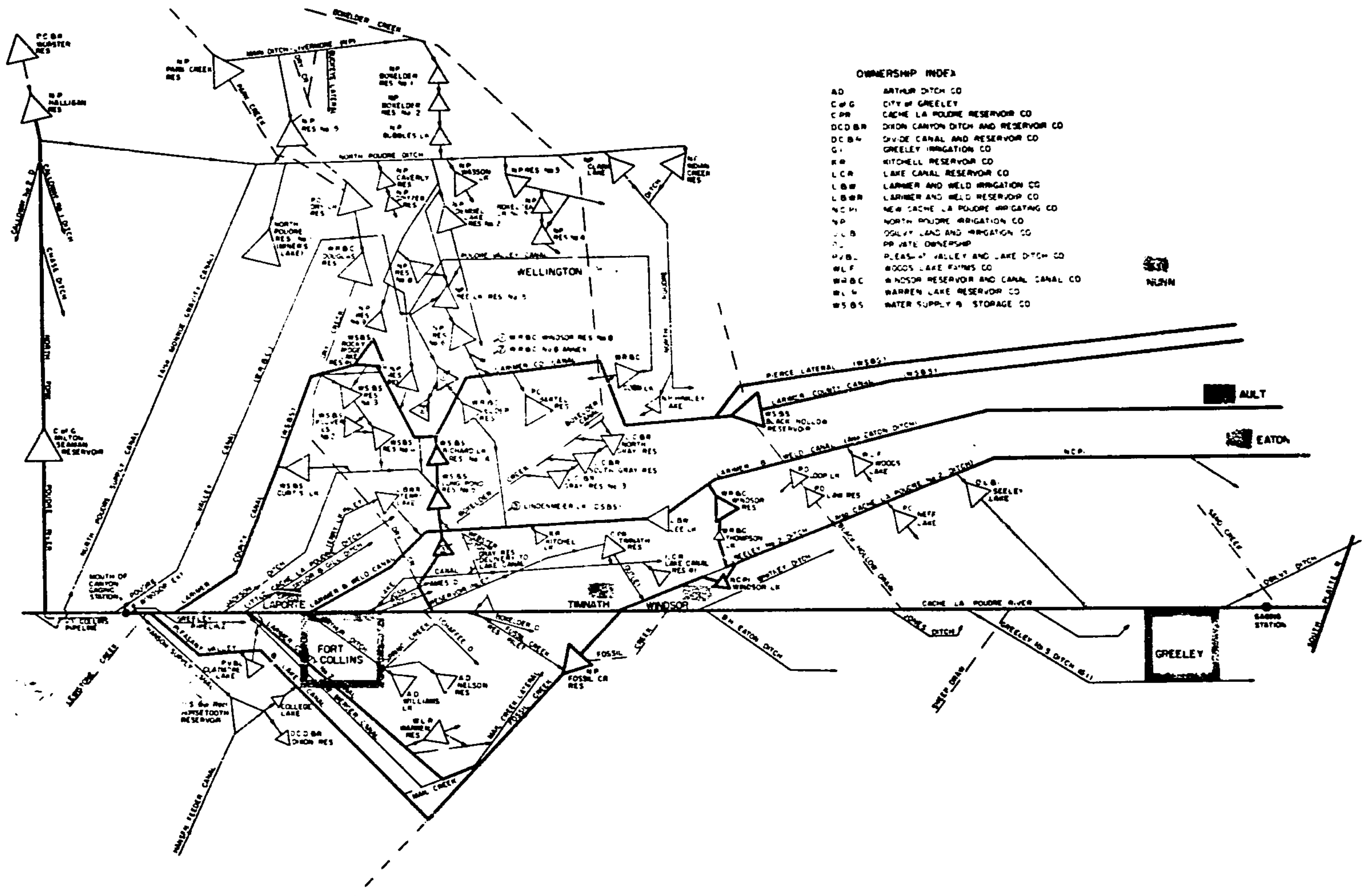


Figure 12. The Lower Cache La Poudre Ditch and Reservoir System (Courtesy of M.W. Bittinger and Associates).

procedures governing the delivery of demands from storage vary from subsystem to subsystem.

The state water law governing the diversions from the river and its tributaries is the Colorado Doctrine of Prior Appropriations and is administered by the State Engineer (Black, 1960; Danielson, 1958). For a number of years the river water has been insufficient to satisfy the demands on it, and cooperation among the various ditch companies in the system has affected an operating policy that allows more efficient delivery of the water than strict interpretation of the law would allow. The procedure involves the renting and trading of water. A discussion of this process is beyond the scope of this study, but can be found in such references as: Anderson (1963, 1961b, 1960); Biggs (1968); Davan, Anderson and Hartman (1962); Huzar, Seckler and Rohdy (1969); Hartman and Anderson (1963), and Hartman and Seastone (1970).

One transbasin diversion of special importance to the Poudre system is reflected by Horsetooth Reservoir, with a capacity of 151,800 ac-ft. It is part of the Colorado-Big Thompson Project, that was constructed to supply supplemental irrigation water to the Northeastern Colorado region. The water made available by this project is governed by an entirely different set of laws than those governing the river water, and will not be discussed further because it is excluded from the developed model. References relating to the project and its impact may be found in Dille (1958); Anderson and Hartman (1965); Davan, Anderson and Hartman (1962); Hartman and Anderson (1964); Hartman and Seastone (1970), and the annual reports of the Northern Colorado Water Conservancy District.

Model System

Only portions of the Poudre irrigation system are used for the model. They are indicated by the heavy black lines on Figure 12. Figure 13 is a schematic diagram, much like Figure 1, of the modeled system. It is composed of five ditches, eight reservoirs and the river. In modeling the system it is assumed that no legal system prevails. This implies that no ditch has priority over another to the river water.

Using the intermediate level of description, suggested in Chapter IV, the example system is composed of seventeen ditch sectors, including those that represent the river, eight reservoirs, and sixteen nodes. Three time periods are considered in the analysis, corresponding to June, July and August in the prototype system.

For each time period, the model is composed of seventeen ditch sector mass balances, eight reservoir mass balances, and sixteen nodal mass balances. All reservoirs are modeled using the ditch sector definition, instead of the definition that would best fit each of the reservoirs. The disadvantage of consistently using a single definition for modeling reservoirs will be illustrated later in this chapter.

The model also contains nineteen maximum capacity and eight minimum capacity restrictions. All ditch sectors representing river flows are assumed to be unrestricted in maximum capacity. All ditch sectors, including those representing river flows, are assumed to be constrained in minimum flow only by the volume of water each must deliver. Each of the reservoirs is assumed to have both maximum and minimum capacity restrictions.

For each time period, then, the model consists of 68 equations ($M = 17$, $N = 8$, $P = 16$, with 19 maximum capacity and 8 minimum

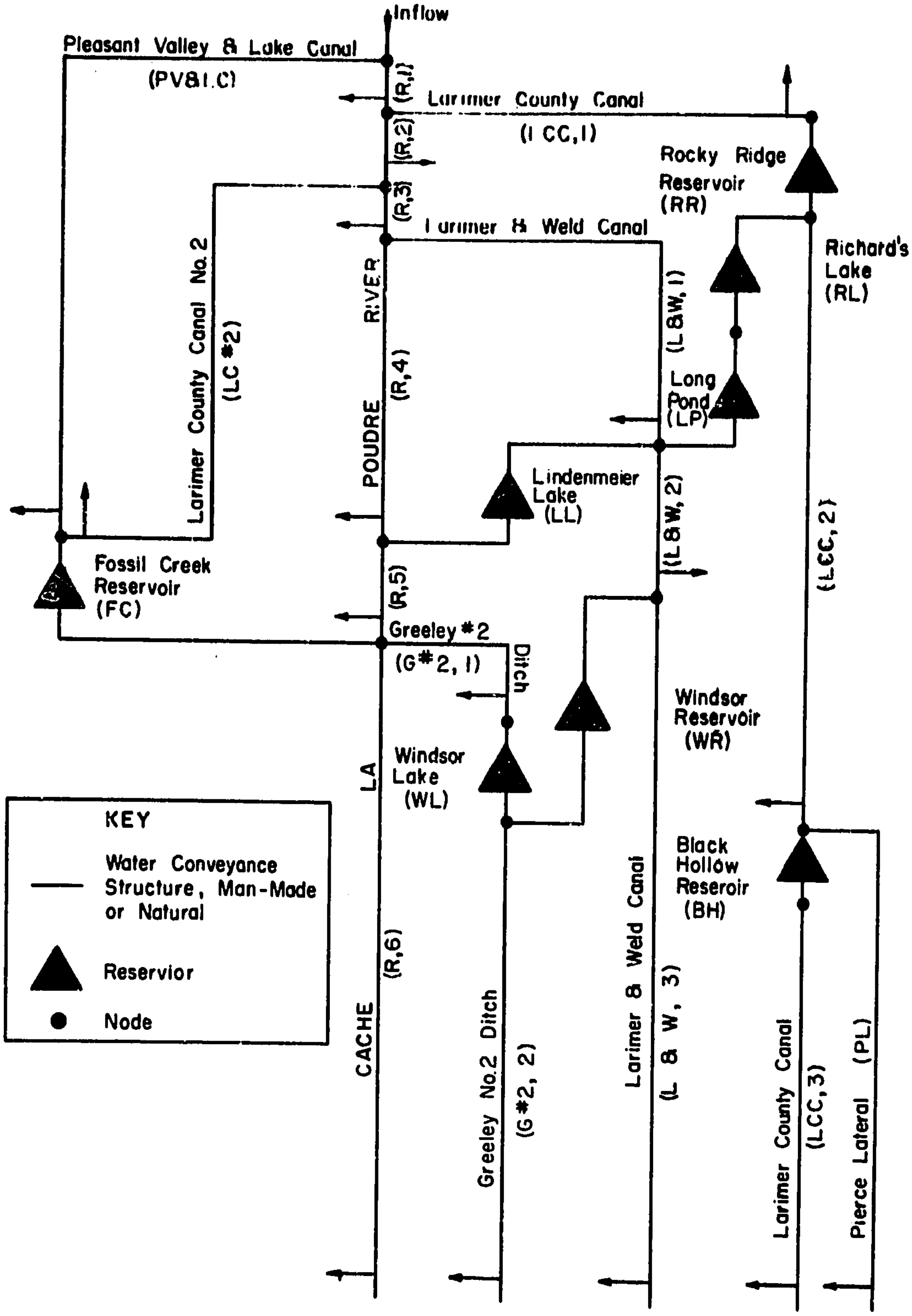


Figure 13. Schematic Diagram of Example System

capacity restrictions) and 85 decision variables (two for each ditch sector mass balance, three for each reservoir mass balance because the ditch sector definition is used, and one slack or surplus variable for each capacity restriction). For the three time period model, the numbers of equations and decision variables are three times those for a single time period model, or 204 equations and 255 decision variables.

Model Parameters

The parameters necessary for the model construction are the loss coefficients, the return flow coefficients, the maximum capacity values, and the minimum capacity values for each of the elements. To use the model, estimates must be available for the initial conditions and for the time and space distributions of the inflows, demands, and required system outflows.

Data for deriving the parameters and distributions necessary for the model were found to be virtually nonexistent. The only accessible data is from the State Engineer's Office; this data assembly is restricted to the diversions from natural systems. Detailed data on the actual deliveries made within the system are contained in the files of each of the ditch companies that operate in the system. These data vary both in their quality and assembly. Further difficulties are encountered if existing data are used because of implied operating policies that tend to bias a model based on them.

Because of these difficulties, the parameters used in the example solution are estimated. To make these estimates as close to reality as possible, discussions were held with representatives of the State Engineer's Office throughout the study. For some parameters guidelines were established, but for others only theoretical guidelines could be derived. The basis of the estimates will be discussed in the following sections as thoroughly as possible.

Inflows

The inflows used in the model were obtained by averaging the monthly flows, in acre-feet, for the 85 years of record at the mouth of the canyon gage (1883-1968). A slight adjustment was made to the inflow for the third time period so the supply would be adequate to satisfy the demands estimated for the season. The inflow values used were 56,100 ac-ft for the first time period, 24,500 ac-ft for the second, and 15,000 ac-ft for the third.

These flow values are not representative of the time distribution of the natural runoff of the stream because they include water released from high mountain reservoirs, not included in the model, and transbasin diversions. The timing and volume of the reservoir releases and transbasin diversions are controlled by the need for irrigation water in the plains, and, therefore, bias the data with an implied operating policy as discussed in the previous section. The effect of the bias is, however, considered to be negligible in the model presented here.

Demands

Because of the difficulties already cited, there was no real basis for estimating demands. Selected values, therefore, were made to exhibit both varying demand patterns on each of the ditch sectors and varying demand patterns over the system in each time period. No satisfaction of demands by diverting water directly from reservoirs to the fields was assumed.

The demand values used in the model are shown in Table I. Each sector is labeled by a shortened representation of the ditch name and sector number, and the key to the labeling is found on Figure 13. In addition, the indices to be used for the variables of the modeling equations are shown in the second column of Table I.

Two other studies are currently in progress which will alleviate the problems of demand estimation in the future. The results of these studies by Evans and Skogerboe (cited in the bibliography, but as yet, not published) should be available by the summer of 1971.

Required System Outflows

Although there are water rights on the South Platte River (downstream of its confluence with the Poudre River) that have a higher priority than some water rights on the Poudre, return flows are generally sufficient to satisfy them. Only occasionally is water required to be released from the Poudre system to satisfy them, generally in the early spring. Therefore, the required system outflow from the Poudre is essentially zero throughout an irrigating season and is assumed to be so in the model for all time periods.

Loss Coefficients

The loss coefficients for the ditch sectors were estimated using equation (5-2). The variable u was estimated from experience to range from .0025 per mile to .0100 per mile. Unit loss rates are difficult to estimate on a historical basis because of the influence of return flows.

Table I
Demands, Models I and II
(Acre Feet per Time Period)

Element	Index	Time Period 1	2	3
PV&LC	1	1000	500	500
LC#2	2	1000	1000	500
LCC,1	3	1000	2000	2000
LCC,2	4	2000	2000	1500
LCC,3	5	1000	1500	1500
PL	6	500	1000	1000
L&W,1	7	2000	3000	2000
L&W,2	8	1500	2000	2000
L&W,3	9	1500	2000	1500
G#2,1	10	2000	3000	2500
G#2,2	11	1000	1500	2000
R,1	12	2500	2500	2000
R,2	13	2000	1500	1500
R,3	14	1500	2000	1000
R,4	15	1500	1500	1500
R,5	16	1500	2000	1500
R,6	17	1500	1000	500
FC	18	0	0	0
RR	19	0	0	0
BH	20	0	0	0
RL	21	0	0	0
LP	22	0	0	0
LL	23	0	0	0
WL	24	0	0	0
WR	25	0	0	0

Two models of the system were analyzed, the difference between them was the loss rate used for the ditch sector LCC,1. In both models, the river was assumed to have the lowest unit loss rate and those ditch sectors farthest from the river were assumed to have the greatest unit loss rates. In Model I it was assumed the sector LCC,1 was lined to decrease its losses, thus giving it a lower unit loss rate than that of the river. In Model II the sector LCC,1 was assumed unlined, resulting in a higher unit loss rate than that of the river. Changing the unit loss rate from Model I to Model II illustrates the change in delivery strategy that results from system modification. The sector lengths, unit loss rates, and loss coefficients used in Models I and II are shown in Table II.

The reservoir loss functions required by the model are related to the volume of water that a reservoir contains. To derive the γ_{ii}^{tt} and β_i^t coefficients necessary for the model, the following procedure was used.

A unit loss rate was derived assuming the net pan evaporation was equal to the losses. The values used were 0.4 ac-ft/ac for the first time period, 0.6 ac-ft/ac for the second time period, and 0.5 ac-ft/ac for the third, based on the average measured pan evaporation and precipitation at Fort Collins. Seepage losses were assumed to be zero.

The volume of water lost from a reservoir was assumed to be the volume of water contained in the top layer of water with a thickness equal to the unit loss rate. Figure 14 illustrates this.

Table II

Ditch Sector Loss Coefficient Computations, Models I and II

Ditch Sector	Index (i)	Model I: Length (n)	Unit Loss Rate (u)	Loss Coefficient ($\gamma_{ii}^{\dagger\dagger}$)	Ditch Sector	Index (i)	Model II: Length (n)	Unit Loss Rate (u)	Loss Coefficient ($\gamma_{ii}^{\dagger\dagger}$)
PV&LC	1	24	.0100	.2143	PV&LC	1	24	.0100	.2143
LC#2	2	17	.0080	.1276	LC#2	2	17	.0080	.1276
LCC,1	3	15	.0010	.0149	LCC,1	3	15	.0040	.0583
LCC,2	4	23	.0060	.1293	LCC,2	4	23	.0060	.1293
LCC,3	5	26	.0090	.2095	LCC,3	5	26	.0090	.2095
PL	6	26	.0100	.2300	PL	6	26	.0100	.2300
L&W,1	7	3	.0050	.0149	L&W,1	7	3	.0050	.0149
L&W,2	8	13	.0060	.0753	L&W,2	8	13	.0060	.0753
L&W,3	9	46	.0080	.3089	L&W,3	9	46	.0080	.3089
G#2,1	10	5	.0060	.0296	G#2,1	10	5	.0060	.0296
G#2,2	11	40	.0080	.2748	G#2,2	11	40	.0080	.2748
R,1	12	1	.0030	.0030	R,1	12	1	.0030	.0030
R,2	13	3	.0030	.0090	R,2	13	3	.0030	.0090
R,3	14	3	.0020	.0060	R,3	14	3	.0020	.0060
R,4	15	5	.0020	.0100	R,4	15	5	.0020	.0100
R,5	16	9	.0030	.0267	R,5	16	9	.0030	.0267
R,6	17	34	.0030	.0971	R,6	17	34	.0030	.0971

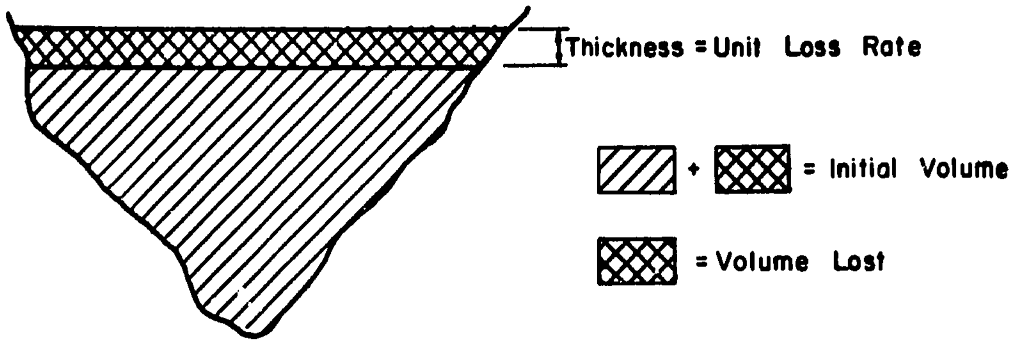


Figure 14. Volume of Water Lost from Reservoir.

Using the unit loss rates and available capacity tables for each of the reservoirs, the volume of water lost from a reservoir in a time period was calculated relative to the initial volume in the reservoir. The two volumes were then nondimensionalized by dividing by the maximum capacity of the reservoir, so the loss functions of the various reservoirs could be compared. A sample calculation is shown in Table III.

The next step involved the plotting of the nondimensionalized "volumes lost" against the nondimensionalized "initial volumes" on arithmetic graph paper and relating them with a straight line. Such a plot is shown on Figure 15. From the graph γ_{ii}^{tt} and $\beta_i^t/S_{i_{max}}^t$ were obtained, and multiplication of the latter by $S_{i_{max}}^t$ yielded the necessary β_i^t .

For the eight reservoirs used in the models, the relations were surprisingly linear. If in other cases, however, the graphs are found deviate significantly from a single straight line; a series of straight lines can be drawn and iterative linear programming used, as previously suggested. A severe deviation will always be found when the initial

Table III

Example Computations for Reservoir Loss Coefficients

Unit Loss Rate = 0.5 Ac-Ft/Ac

Stage (h)	Initial Volume (S_i^{t-1})	Final Volume S_i^t	$L_i^t = S_i^{t-1} - S_i^t$	$\frac{S_i^{t-1}}{S_{imax}^t}$	$\frac{L_i^t}{S_{imax}^t}$
1	29	14	15	.0255	.0132
2	62	45	17	.0545	.0150
3	97	79	18	.0853	.0158
4	135	115	20	.119	.0176
5	176	155	21	.155	.0185
6	219	197	22	.193	.0193
7	265	242	23	.233	.0202
8	312	287	25	.274	.0220
9	363	337	26	.319	.0229
10	416	388	28	.366	.0246
11	472	443	29	.415	.0255
12	531	501	30	.467	.0264
13	593	562	31	.521	.0273
14	658	626	32	.579	.0281
15	726	691	35	.639	.0308
16	800	762	38	.704	.0334
17	878	838	40	.772	.0352
18	960	919	41	.844	.0361
19	1047	1002	45	.921	.0396
20	1137	1092	45	1.000	.0396

S_{imax}^t = 1137 Ac. ft.

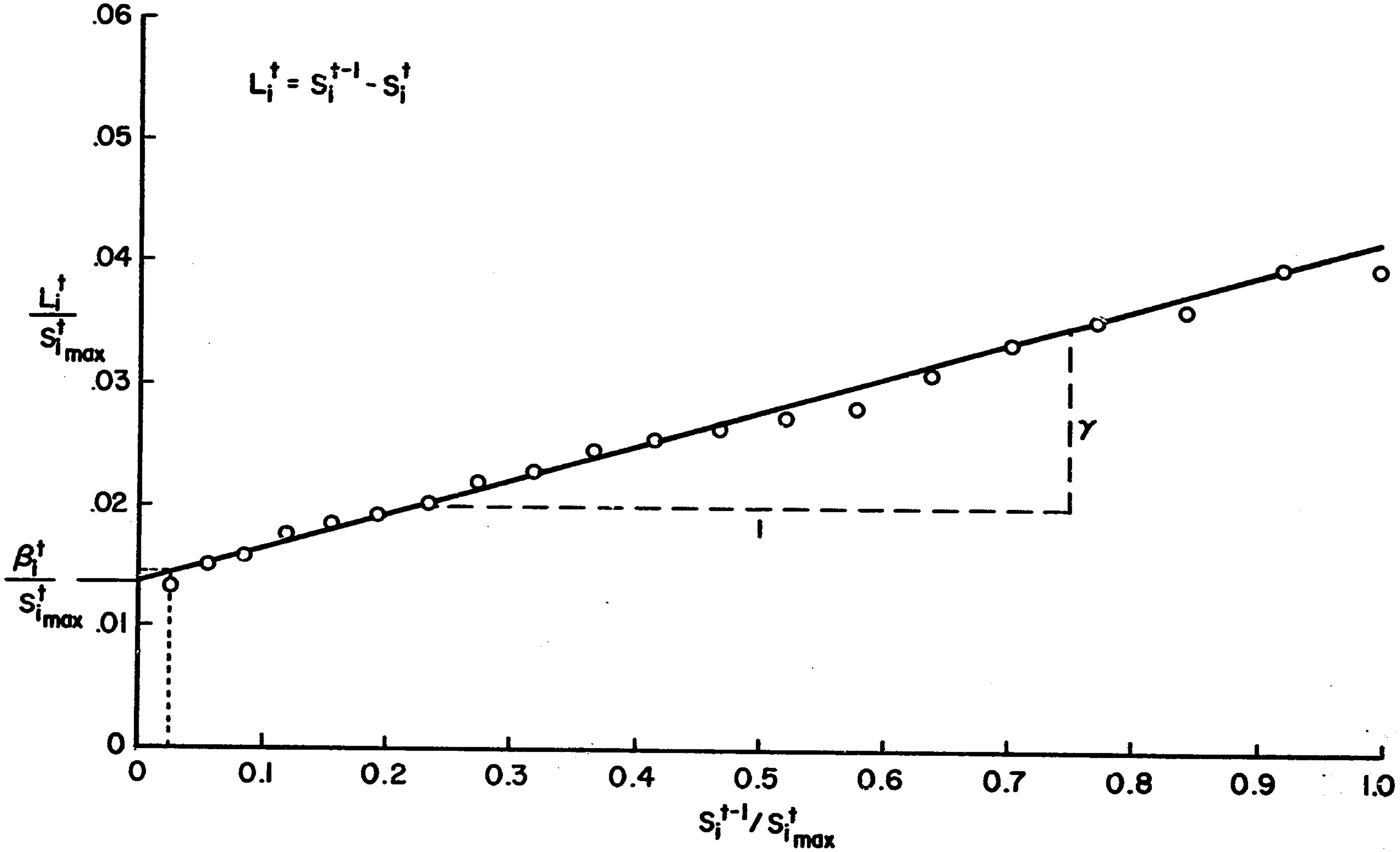


Figure 15. Graph for Reservoir Loss Computation

reservoir contents are near zero, but this deviation can be neglected by constraining the minimum capacity of the reservoir to be greater than the volume at which it occurs.

The reservoir loss coefficients, γ_{ii}^{tt} , and constant, β_i^t , for the sample models, were calculated for all reservoirs and time periods. The values obtained are tabulated in Table IV.

Table IV

Loss Coefficients and Constants for Reservoirs
Models I and II

		Time Period 1		Time Period 2		Time Period 3	
Reservoir	Index	γ_{ii}^{tt}	β_i^t	γ_{ii}^{tt}	β_i^t	γ_{ii}^{tt}	β_i^t
FC	18	.0299	52.80	.0443	78.00	.0372	66.00
RR	19	.0153	30.40	.0268	38.00	.0209	32.40
BH	20	.0257	51.20	.0397	76.80	.0328	64.80
RL	21	.0238	10.70	.0410	15.10	.0281	13.80
LD	22	.0299	22.80	.0302	37.20	.0233	33.20
LL	23	.0328	27.20	.0354	43.60	.0352	35.10
WR	24	.0240	54.00	.0325	104.40	.0277	77.40
WL	25	.0395	23.10	.0603	33.00	.0513	27.40

The weighting factors, a_i^t and b_i^t , were assumed to be 1.0 and 0.0, respectively, for all ditch sector mass balances in all time periods, and, 0.5 and 0.5, respectively, for all reservoir mass balances in all time periods. By choosing these weighting factors, the ditch sector losses are calculated entirely on the basis of the flows entering at the headgate, and the reservoir losses are calculated on the basis of the average volume of water in storage.

Return Flow Coefficients

As with the estimation of demands, there is a lack of data for the estimation of the spatial distribution of return flows. The magnitude of the difficulties posed by this problem is discussed in detail by Hartman and Seastone (1970). Therefore, in this study, coefficients for the spatial distribution of return flows were also estimated to illustrate features of the model, with equation (5-9) used as a restriction. Tables V, VI, and VII list the c_{ij}^{th} values used in Models I and II. All α_{ij}^{th} values were assumed to be zero, eliminating any return flow resulting from the application of excess water to crops.

For the ditch sectors of both Models I and II, the assumption of a loss function that is the same for all time periods creates equal return flow coefficients in corresponding time periods. For example, all $\gamma_{ij}^{t-1,t}$ values for the same i and j will be equal. For the reservoirs in the two models, the pattern is not evident because the loss functions change with the time periods. For a numerical example of this pattern, compare the coefficients of the ditch sector mass balance equations contained in equation sets I, II and III, Tables XII, XIII and XIV, developed later in this chapter.

Initial Conditions

Because channel storage is neglected in the ditch sectors, knowledge of the initial conditions is necessary for reservoirs only, and in an operating model these will be known. For the example of this chapter, not relative to any particular season, the initial

Table V

C_{ij}^{tt} Coefficients, Models I and II

Receiving Element \ Losing Element		PV&LC	LC#2	LCC,1	LCC,2	LCC,3	PL	L&W,1	L&W,2	L&W,3	G#2,1	G#2,2	R,1	R,2	R,3	R,4	R,5	R,6	FC	RR	BH	RL	LP	LL	WR	WL
	Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
PV&LC	1	X																								
LC#2	2	.05	X																							
LCC,1	3			X																						
LCC,2	4				X																					
LCC,3	5					X	.010																			
PL	6						X																			
L&W,1	7			.010				X												.010						
L&W,2	8				.005				X														.005			
L&W,3	9				.005	.003				X											.010					
G#2,1	10										X													.010		
G#2,2	11									.010		X													.010	
R,1	12	.030											X													
R,2	13	.010		.020										X												
R,3	14		.050	.010											X											
R,4	15		.010					.010								X										
R,5	16							.005	.010								X									
R,6	17										.002	.002						X	.010							
FC	18																		X							
RR	19																			X						
BH	20						.010														X					
RL	21			.010	.010																	X				
LP	22				.020																		X			
LL	23																							X		
WR	24								.010	.005															X	
WL	25																								.010	X

Table VI

$C_{ij}^{t-1,t}$ Coefficients, Models I and II

Receiving Element \ Losing Element	Index	PV&LC 1	LC#2 2	LCC,1 3	LCC,2 4	LCC,3 5	PL 6	L&W,1 7	L&W,2 8	L&W,3 9	G#2,1 10	G#2,2 11	R,1 12	R,2 13	R,3 14	R,4 15	R,5 16	R,6 17	FC 18	RR 19	BH 20	RL 21	LP 22	LL 23	WR 24	WL 25
PV&LC	1	.100																								
LC#2	2	.030	.070																							
LCC,1	3			.030																						
LCC,2	4				.040															.010						
LCC,3	5					.010	.030														.010					
PL	6						.100																			
L&W,1	7			.020				.020												.040		.030	.020			
L&W,2	8				.020				.050														.010			
L&W,3	9				.030	.030				.100											.040					
G#2,1	10								.030		.040													.020		
G#2,2	11									.030		.050													.030	
R,1	12	.010																								
R,2	13	.005		.040																						
R,3	14		.030	.030																						
R,4	15		.005					.030																.020		
R,5	16							.020	.040															.010		
R,6	17										.040	.040							.050							.030
FC	18	.030																								
RR	19																									
BH	20						.020																			
RL	21			.030	.050															.020						
LP	22			.010	.050															.010						
LL	23							.020	.010																	
WR	24								.020	.020																
WL	25																								.020	

Table VII

$C_{ij}^{t-2,t}$ **Coefficients, Models I and II**

Receiving Element \ Losing Element		PV&LC	LC#2	LCC,1	LCC,2	LCC,3	PL	L&W,1	L&W,2	L&W,3	G#2,1	G#2,2	R,1	R,2	R,3	R,4	R,5	R,6	FC	RR	BH	RL	LP	LL	WR	WL
	Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
PV&LC	1	.050																								
LC#2	2	.010	.050																							
LCC,1	3			.020																						
LCC,2	4				.020															.010						
LCC,3	5					.005	.020														.010					
PL	6						.050																			
L&W,1	7			.010				.010												.030		.010	.010			
L&W,2	8				.010				.040														.010			
L&W,3	9				.020	.010				.070											.020					
G#2,1	10								.010		.020													.010		
G#2,2	11									.010		.040													.020	
R,1	12	.005																								
R,2	13	.0025		.030																						
R,3	14		.010	.020																						
R,4	15							.020																.010		
R,5	16							.010	.030															.010		
R,6	17										.030	.030							.040							.020
FC	18	.010	.004																							
RR	19																									
BH	20						.010																			
RL	21			.020	.030															.010						
LP	22			.010	.040															.010						
LL	23							.010	.010																	
WR	24								.010	.015																
WL	25																								.010	

condition of each reservoir is assumed to be either 1/3 or 1/4 of the maximum capacity. The numerical values are shown in Table VIII.

Table VIII

Initial Reservoir Contents
Models I and II

Reservoir	Index (i)	S_i^o
FC	18	4000
RR	19	1000
BH	20	2000
RL	21	250
LP	22	1000
LL	23	250
WR	24	6000
WL	25	250

Maximum Capacity Values

For those ditch sectors that receive water directly from the Poudre River, maximum capacity values were available and used in the model. For the other ditch sectors, the maximum capacity values were unknown, but were assumed to be less than the capacity of the next upstream sector. For example, the sector LCC,2 was assumed to have a maximum capacity less than the known capacity of LCC,1. The maximum capacity of LCC,1 is known because its inflow comes directly from the river. Those ditch sectors representing the river were assumed to have no maximum capacities.

The variables representing the inflows to and outflows from the reservoirs, using the ditch sector definition, were assumed to have no maximum capacity restrictions. The maximum reservoir contents were given those values shown in available capacity tables, rounded to the nearest 1000 ac-ft. All maximum capacity values were assumed to be the same for both Models I and II for all time periods. They are listed in Table IX.

Table IX

Maximum Capacity Values
Models I and II

Element	Index	Maximum Capacities $Q^t_{i_{max}}$	Element	Index	Maximum Capacities $S^t_{i_{max}}$
PV&LC	1	6,500	FC	18	12,000
LC#2	2	11,000	RR	19	4,000
LCC,1	3	36,000	BH	20	8,000
LCC,2	4	24,000	RL	21	1,000
LCC,3	5	12,000	LP	22	4,000
PL	6	6,000	LL	23	1,000
L&W,1	7	50,000	WR	24	18,000
L&W,2	8	34,000	WL	25	1,000
L&W,3	9	17,000			
G#2,1	10	36,000			
G#2,2	11	24,000			

Minimum Capacity Values

The minimum capacity values for all ditch sectors, including th reservoir inflows and outflows, were assumed to be the required deli Thus, no explicit statement of the minimum capacity values were necessary.

The reservoir minimum capacity values, in reality zero, were assumed to be the volume at which the linear loss rate approximation (Figure 15) became inapplicable. For each reservoir the minimum capacity remained the same for all time periods, but among the reservoirs values varied from 2.5 percent to 11 percent of the maxim capacity. In Figure 15, for example, the minimum capacity value is $.025\ S^t_{i_{max}}$. Table X gives the minimum capacity values used for th reservoirs.

Linearized Simulation Models

The linearized simulation model consists of the linear mass balances and capacity restrictions. The equations of Model I are developed separately for each time period to clearly illustrate the construction of the model. Because only one column of the A matrix, that can be developed from the equations of Model I, is changed for Model II, no separate listing of the equations for Model II is inclu Instead, only the changed constants are listed.

The equations of the linearized model are those developed in Chapters IV and V: for the ditch sectors, equation (5-27); for the reservoirs, equation (5-29), using ditch sector definition; for the nodes, equation (4-4); for the ditch sector maximum capacities, equation (4-10) modified to

Table X

Minimum Capacity Values
Models I and II

Element	Index	Minimum Capacities $Q^t_{i_{min}}$	Element	Index	Minimum Capacities $S^t_{i_{min}}$
PV&LC	1	*	FC	18	1,320
LC#2	2	*	RR	19	100
LCC,1	3	*	BH	20	800
LCC,2	4	*	RL	21	25
LCC,3	5	*	LP	22	200
PL	6	*	LL	23	25
L&W,1	7	*	WR	24	450
L&W,2	8	*	WL	25	50
L&W,3	9	*			
G#2,1	10	*			
G#2,2	11	*			
R,1	12	*			
R,2	13	*			
R,3	14	*			
R,4	15	*			
R,5	16	*			
R,6	17	*			

* Minimum Capacity is that required to deliver the demand.

$$Q_i^t \le Q_{i_{max}}^t \; ; \tag{6-1}$$

for the reservoir maximum capacities, equation (4-12), modified to

$$S_i^t \le S_{i_{max}}^t \; ; \tag{6-2}$$

and for the reservoir minimum capacities, equation (4-13), modified tc

$$S_i^t \ge S_{i_{min}}^t \; . \tag{6-3}$$

These modifications presume that return flows and losses are small.

The tables used for the determination of the constants vary for each of the time periods and are stated in each section. Specific points of interest are discussed at the appropriate locations. A table relating the indices of variables used in the equations to the element abbreviations is given in Table XI.

Model I, Time Period 1

The data necessary to construct the ditch sector and reservoir mass balances are given in Tables I, II, IV, V and VIII; the data necessary for the maximum and minimum capacity restrictions are given in Tables IX and X. Equation set I, Table XII, results from the substitution of these data into the equations listed above. Before developing the model further, however, there are four points that should be discussed: the elimination of some nodal mass balances, with a consequent reduction in the number of decision variables; the nature of the nodal mass balances in the various time periods; the nature of the capacity constraints in the various time periods; and the nature of S_i^{t-1} in the first time period.

Table XI

Element Indexing
Models I and II

Element	Index	Element	Index
PV&LC	1	FC	18
LC#2	2	RR	19
LCC,1	3	BH	20
LCC,2	4	RL	21
LCC,3	5	LP	22
PL	6	LL	23
L&W,1	7	WR	24
L&W,2	8	WL	25
L&W,3	9		
G#2,1	10		
G#2,2	11		
R,1	12		
R,2	13		
R,3	14		
R,4	15		
R,5	16		
R,6	17		

Note: Ditch sector definition for reservoirs.

An examination of the equations for nodes 2, 4, 7, 14 and 16, in equation set I, shows they could be eliminated by substituting into the appropriate reservoir mass balance. To eliminate them would be the same as changing the reservoir mass balance to either a node definition or a mixed node-ditch sector definition. There is no reason why this should not be done when constructing a model for operational usage because it would reduce the problem size. For this example, however, the definition that best suits a reservoir was not used in order to show that a larger problem than necessary results from using only a single reservoir definition.

Because it is unlikely that a real system will be changed from one time period to the next, nodal mass balances will usually remain the same for all time periods. This condition is assumed in the model, so the nodal mass balances are the same for all three time periods, only the superscripts change (see equation sets II and III).

In some situations, it may be desirable to change the maximum or minimum capacity restrictions of one or more structures from one time period to the next. This flexibility is provided for in the model through specification of the maximum and minimum restrictions for each time period. The example discussed here, however, maintains the same capacity restrictions for all time periods, only the superscripts change.

In time period 1, the variable S_i^{t-1} is S_i^o, an initial condition. All initial conditions are presumed known and are treated as constants. Thus, for time period 1, a slightly different form of the linear reservoir mass balances is required. For the ditch sector definition,

$$Q_i^1 + \sum_{j=1}^{M} (a_j^1 \gamma_{ij}^{11} Q_j^1 + b_j^1 \gamma_{ij}^{11} V_j^1) + \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} b_j^1 \gamma_{ij}^{11} S_j^1 - (1 + b_i^1 \gamma_{ii}^{11}) S_i^1 - V_i^1$$

$$= (1 - \alpha_{ii}^{11}) D_i^1 + \beta_i^1 - \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} c_{ij}^{11} \beta_j^1 - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \alpha_{ij}^{11} D_j^1$$

$$- \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} a_j^1 \gamma_{ij}^{11} S_j^o - (1 - a_i^1 \gamma_{ii}^{11}) S_i^o , \qquad (6\text{-}4)$$

for $i = M+1,\ldots,M+U$, and for the node definition

$$\sum_{k \in J_i} V_k^1 + \sum_{j=1}^{M} (a_j^1 \gamma_{ij}^{11} Q_j^1 + b_j^1 \gamma_{ij}^{11} V_j^1) + \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} b_j^1 \gamma_{ij}^{11} S_j^1 - (1 + b_i^1 \gamma_{ii}^{11}) S_i^1$$

$$- \sum_{k \in K_i} Q_k^1 = (1 - \alpha_{ii}^{11}) D_i^1 + \beta_i^1 - \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} c_{ij}^{11} \beta_j^1 - \sum_{\substack{j=1 \\ i \neq j}}^{M+N} \alpha_{ij}^{11} D_j^1$$

$$- \sum_{\substack{j=M+1 \\ i \neq j}}^{M+N} a_j^1 \gamma_{ij}^{11} S_j^o - (1 - a_i^1 \gamma_{ii}^{11}) S_i^o , \qquad (6\text{-}5)$$

for $i = M+U,\ldots,M+N$.

Because the initial conditions are constant, they appear on the right of the equal sign. If the storage in a reservoir in time period 1 is greater than the demand for water directly from the reservoir in the same time period, the constants will reduce to a negative value. However, if the demands for water directly from a reservoir are greater than the initial reservoir contents, the constants will remain positive. This point is important because some solution algorithms require a positive constant on the right side of the equal sign. In the model developed here, all constants on the right side of the equal sign are negative for the reservoir mass balances of time period 1 because the demands have been assumed to be zero.

Finally, equation set I could be written in matrix form. The result would be the A_{11} and D_{11} submatrices and the $\bar{x}_1$ and $\bar{b}_1$ subvectors of equation (5-30). Specifically,

$$A_{11}\bar{x}_1 = D_{11}\bar{b}_1 \quad . \tag{6-6}$$

The elements of the submatrices and subvectors are not listed because of limitations of space.

Model I, Time Period 2

The data necessary to construct the ditch sector and reservoir mass balances and capacity constraints for time period 2 are given in Tables I, II, IV, V, VI, IX and X. The resulting equations are listed in equation set II, Table XIII.

Written in matrix form, the equation set becomes

$$A_{21}\bar{x}_1 + A_{22}\bar{x}_2 = D_{21}\bar{b}_1 + D_{22}\bar{b}_2 \quad , \tag{6-7}$$

in which $\bar{x}_1$ and $\bar{b}_1$ are the same as discussed in equation (6-6).

Model I, Time Period 3

The data necessary to construct the ditch sector and reservoir mass balances for time period 3 are listed in Tables I, II, IV, V, VI, VII, IX, and X. The resulting equations are listed in equation set III, Table XIV.

Written in matrix form, this equation set becomes

$$A_{31}\bar{x}_1 + A_{32}\bar{x}_2 + A_{33}\bar{x}_3 = D_{31}\bar{b}_1 + D_{32}\bar{b}_2 + D_{33}\bar{b}_3 \quad , \tag{6-8}$$

Table XII

Equation Set 1

Type	Element	Equation Number	
Ditch	PV&LC	1	$.7857Q_1^1 - V_1^1 = 1000$
Ditch	LC#2	2	$.0107Q_1^1 + .8724Q_2^1 - V_2^1 = 1000$
Ditch	LCC,1	3	$.9851Q_3^1 - V_3^1 = 1000$
Ditch	LCC,2	4	$.8707Q_4^1 - V_4^1 = 2000$
Ditch	LCC,3	5	$.7905Q_5^1 + .0023Q_6^1 - V_5^1 = 1000$
Ditch	PL	6	$.7700Q_6^1 - V_6^1 = 500$
Ditch	L&W,1	7	$.0001Q_3^1 + .9851Q_7^1 + .0001S_{19}^1 - V_7^1 = 1999.70$
Ditch	L&W,2	8	$.0006Q_4^1 + .9247Q_8^1 + .0001S_{22}^1 - V_8^1 = 1499.89$
Ditch	L&W,3	9	$.0006Q_4^1 + .0006Q_5^1 + .6911Q_9^1 + .0001S_{20}^1 - V_9^1 = 1499.49$
Ditch	G#2,1	10	$.9704Q_{10}^1 + .0002S_{23}^1 - V_{10}^1 = 1999.73$
Ditch	G#2,2	11	$.0031Q_9^1 + .7252Q_{11}^1 + .0001S_{24}^1 - V_{11}^1 = 999.46$
Ditch	R,1	12	$.0064Q_1^1 + .9970Q_{12}^1 - V_{12}^1 = 2500$
Ditch	R,2	13	$.0021Q_1^1 + .0003Q_3^1 + .9910Q_{13}^1 - V_{13}^1 = 2000$
Ditch	R,3	14	$.0011Q_1^1 + .0064Q_2^1 + .0001Q_3^1 + .9940Q_{14}^1 - V_{14}^1 = 1500$
Ditch	R,4	15	$.0013Q_2^1 + .0001Q_7^1 + .9900Q_{15}^1 - V_{15}^1 = 1500$
Ditch	R,5	16	$.0001Q_7^1 + .0008Q_8^1 + .9733Q_{16}^1 - V_{16}^1 = 1500$
Ditch	R,6	17	$.0006Q_{10}^1 + .0055Q_{11}^1 + .9029Q_{17}^1 + .0002S_{18}^1 - V_{17}^1 = 1499.47$
Reservoir	FC	18	$Q_{18}^1 - 1.0150S_{18}^1 - V_{18}^1 = -3887.20$
Reservoir	RR	19	$Q_{19}^1 - 1.0076S_{19}^1 - V_{19}^1 = -962.00$
Reservoir	BH	20	$.0023Q_6^1 + Q_{20}^1 - 1.0128S_{20}^1 - V_{20}^1 = 1923.20$
Reservoir	RL	21	$.0001Q_3^1 + .0013Q_4^1 + Q_{21}^1 - 1.0119S_{21}^1 - V_{21}^1 = -236.32$
Reservoir	LP	22	$.0026Q_4^1 + Q_{22}^1 - 1.0150S_{22}^1 - V_{22}^1 = -962.20$
Reservoir	LL	23	$Q_{23}^1 - 1.0164S_{23}^1 - V_{23}^1 = -218.70$
Reservoir	WR	24	$.008Q_8^1 + .0015Q_9^1 + Q_{24}^1 - 1.0120S_{24}^1 - V_{24}^1 = -5874.00$
Reservoir	WL	25	$Q_{25}^1 + .0001S_{24}^1 - 1.0198S_{25}^1 - V_{25}^1 = -222.49$
Node	1	26	$Q_1^1 + Q_{12}^1 = 56,100$
Node	2	27	$V_1^1 + V_2^1 - Q_{18}^1 = 0$
Node	3	28	$V_{12}^1 - Q_3^1 - Q_{13}^1 = 0$
Node	4	29	$V_3^1 - Q_{19}^1 = 0$
Node	5	30	$V_{19}^1 - Q_4^1 - Q_{21}^1 = 0$
Node	6	31	$V_4^1 - Q_6^1 - Q_{20}^1 = 0$
Node	7	32	$V_{20}^1 - Q_5^1 = 0$
Node	8	33	$V_{13}^1 - Q_2^1 - Q_{14}^1 = 0$
Node	9	34	$V_{14}^1 - Q_7^1 - Q_{15}^1 = 0$

Equation Set 1 (continued)

Type	Element	Equation Number	
Node	10	35	$V_7^1 + V_{22}^1 - Q_8^1 - Q_{23}^1 = 0$
Node	11	36	$V_8^1 - Q_9^1 - Q_{24}^1 = 0$
Node	12	37	$V_{15}^1 + V_{23}^1 - Q_{16}^1 = 0$
Node	13	38	$V_{16}^1 + V_{18}^1 - Q_{10}^1 - Q_{17}^1 = 0$
Node	14	39	$V_{10}^1 - Q_{25}^1 = 0$
Node	15	40	$V_{24}^1 + V_{25}^1 - Q_{11}^1 = 0$
Node	16	41	$V_{21}^1 - Q_{22}^1 = 0$
Maxcap*	PV&LC	42	$Q_1^1 \leq 6,500$
Maxcap	LC#2	43	$Q_2^1 \leq 11,000$
Maxcap	LCC,1	44	$Q_3^1 \leq 36,000$
Maxcap	LCC,2	45	$Q_4^1 \leq 24,000$
Maxcap	LCC,3	46	$Q_5^1 \leq 12,000$
Maxcap	PL	47	$Q_6^1 \leq 6,000$
Maxcap	L&W,1	48	$Q_7^1 \leq 50,000$
Maxcap	L&W,2	49	$Q_8^1 \leq 34,000$
Maxcap	L&W,3	50	$Q_9^1 \leq 17,000$
Maxcap	G#2,1	51	$Q_{10}^1 \leq 36,000$
Maxcap	G#2,2	52	$Q_{11}^1 \leq 24,000$
Maxcap	FC	53	$S_{18}^1 \leq 12,000$
Maxcap	RR	54	$S_{19}^1 \leq 4,000$
Maxcap	BH	55	$S_{20}^1 \leq 8,000$
Maxcap	RL	56	$S_{21}^1 \leq 1,000$
Maxcap	LP	57	$S_{22}^1 \leq 4,000$
Maxcap	LL	58	$S_{23}^1 \leq 1,000$
Maxcap	WR	59	$S_{24}^1 \leq 18,000$
Maxcap	WL	60	$S_{25}^1 \leq 1,000$
Mincap**	FC	61	$S_{18}^1 \geq 1,320$
Mincap	RR	62	$S_{19}^1 \geq 100$
Mincap	BH	63	$S_{20}^1 \geq 800$
Mincap	RL	64	$S_{21}^1 \geq 25$
Mincap	LP	65	$S_{22}^1 \geq 200$
Mincap	LL	66	$S_{23}^1 \geq 25$
Mincap	WR	67	$S_{24}^1 \geq 450$
Mincap	WL	68	$S_{25}^1 \geq 50$

*Maxcap = Maximum Capacity Restriction

**Mincap = Minimum Capacity Restriction

Table XIII

Equation Set II

Type	Element	Equation Number	
Ditch	PV&LC	1	$.0214Q_1^1 + .7857Q_1^2 - V_1^2 = 500$
Ditch	LC#2	2	$.0064Q_1^1 + .0089Q_2^1 + .0107Q_1^2 + .8724Q_2^2 - V_2^2 = 1000$
Ditch	LCC,1	3	$.0004Q_3^1 + .9851Q_3^2 - V_3^2 = 2000$
Ditch	LCC,2	4	$.0052Q_4^1 + .0002S_{19}^1 + .8707Q_4^2 - V_4^2 = 1999.70$
Ditch	LCC,3	5	$.0021Q_5^1 + .0069Q_6^1 + .0003\ S_{20}^1 + .7905Q_5^2 + .0023Q_6^2 - V_5^2 = 1499.49$
Ditch	PL	6	$.0230Q_6^1 + .7700Q_6^2 - V_6^2 = 1000$
Ditch	L&W,1	7	$.0003Q_3^1 + .0003Q_7^1 + .0008S_{19}^1 + .0007S_{21}^1 + .0006S_{22}^1 + .0001Q_3^2 + .9851Q_7^2$ $+ .0001S_{19}^2 - V_7^2 = 2997.62$
Ditch	L&W,2	8	$.0026Q_4^1 + .0038Q_8^1 + .0003S_{22}^1 + .0006Q_4^2 + .9247Q_8^2 + .0001S_{22}^2 - V_8^2 = 1999.88$
Ditch	L&W,3	9	$.0039Q_4^1 + .0063Q_5^1 + .0309Q_9^1 + .0010S_{20}^1 + .0006Q_4^2 + .0006Q_5^2 + .6911Q_9^2$ $+ .0002S_{20}^2 - V_9^2 = 1997.18$
Ditch	G#2,1	10	$.0023Q_8^1 + .0012Q_{10}^1 + .007S_{23}^1 + .9704Q_{10}^2 + .0002S_{23}^2 - V_{10}^2 = 2999.02$
Ditch	G#2,2	11	$.0093Q_9^1 + .0137Q_{11}^1 + .0007S_{24}^1 + .0031Q_9^2 + .7252Q_{11}^2 + .0002S_{24}^2 - V_{11}^2 = 1497.34$
Ditch	R,1	12	$.0021Q_1^1 + .0064Q_1^2 + .9970Q_{12}^2 - V_{12}^2 = 2500$
Ditch	R,2	13	$.0011Q_1^1 + .0006Q_3^1 + .0021Q_1^2 + .0003Q_3^2 + .9910Q_{13}^2 - V_{13}^2 = 1500$
Ditch	R,3	14	$.0005Q_1^1 + .0038Q_2^1 + .0004Q_3^1 + .0011Q_1^2 + .0064Q_2^2 + .0001Q_3^2 + .9940Q_{14}^2$ $- V_{14}^2 = 2000$
Ditch	R,4	15	$.0006Q_2^1 + .0004Q_7^1 + .0007S_{23}^1 + .0013Q_2^2 + .0001Q_7^2 + .9900Q_{15}^2 - V_{15}^2 = 1499.46$
Ditch	[illegible]	16	$.0003Q_7^1 + .0030Q_8^1 + .0003S_{23}^1 + .0001Q_7^2 + .0008Q_8^2 + .9733Q_{16}^2 - V_{16}^2 = 1999.73$
Ditch	R,6	17	$.0012Q_{10}^1 + .0110Q_{11}^1 + .0015S_{18}^1 + .0012S_{25}^1 + .0006Q_{10}^2 + .0055Q_{11}^2 + .9029Q_{17}^2$ $+ .0002S_{18}^2 - V_{17}^2 = 995.89$
Reservoir	FC	18	$.0064Q_1^1 + .9778S_{18}^1 + Q_{18}^2 - 1.0222S_{18}^2 - V_{18}^2 = 78.00$
Reservoir	RR	19	$.9866S_{19}^1 + Q_{19}^2 - 1.0134S_{19}^2 - V_{19}^2 = 38.00$
Reservoir	BH	20	$.0046Q_6^1 + .9802S_{20}^1 + .0023Q_6^2 + Q_{20}^2 - 1.0198S_{20}^2 - V_{20}^2 = 76.80$
Reservoir	RL	21	$.0004Q_3^1 + .0065Q_4^1 + .0003S_{19}^1 + .9795S_{21}^1 + .0001Q_3^2 + .0013Q_4^2 + Q_{21}^2 - 1.0205S_{21}^2$ $- V_{21}^2 = 14.49$
Reservoir	LP	22	$.0001Q_3^1 + .0065Q_4^1 + .0002S_{19}^1 + .9849S_{22}^1 + .0026Q_4^2 + Q_{22}^2 - 1.0151S_{22}^2 - V_{22}^2$ $= 36.90$
Reservoir	LL	23	$.0003Q_7^1 + .0008Q_8^1 + .9823S_{23}^1 + Q_{23}^2 - 1.0177S_{23}^2 - V_{23}^2 = 43.60$
Reservoir	WR	24	$.0015Q_8^1 + .0062Q_9^1 + .9838S_{24}^1 + .0008Q_8^2 + .0015Q_9^2 + Q_{24}^2 - 1.0162S_{24}^2 - V_{24}^2$ $= 104.40$
Reservoir	WL	25	$.0005S_{24}^1 + .9698S_{25}^1 + Q_{25}^2 + .0002S_{24}^2 - 1.0302S_{25}^2 - V_{25}^2 = 30.88$
Node	1	26	$Q_1^2 + Q_{12}^2 = 24.500$

Equation Set II (continued)

Type	Element	Equation Number	
Node	2	27	$V_1^2 + V_2^2 - Q_{18}^2 = 0$
Node	3	28	$V_{12}^2 - Q_3^2 - Q_{13}^2 = 0$
Node	4	29	$V_3^2 - Q_{19}^2 = 0$
Node	5	30	$V_{19}^2 - Q_4^2 - Q_{21}^2 = 0$
Node	6	31	$V_4^2 - Q_6^2 - Q_{20}^2 = 0$
Node	7	32	$V_{20}^2 - Q_5^2 = 0$
Node	8	33	$V_{13}^2 - Q_2^2 - Q_{14}^2 = 0$
Node	9	34	$V_{14}^2 - Q_7^2 - Q_{15}^2 = 0$
Node	10	35	$V_7^2 + V_{22}^2 - Q_8^2 - Q_{23}^2 = 0$
Node	11	36	$V_8^2 - Q_9^2 - Q_{24}^2 = 0$
Node	12	37	$V_{15}^2 + V_{23}^2 - Q_{16}^2 = 0$
Node	13	38	$V_{16}^2 + V_{18}^2 - Q_{10}^2 - Q_{17}^2 = 0$
Node	14	39	$V_{10}^2 - Q_{25}^2 = 0$
Node	15	40	$V_{24}^2 + V_{25}^2 - Q_{11}^2 = 0$
Node	16	41	$V_{21}^2 - Q_{22}^2 = 0$
Maxcap	PV&LC	42	$Q_1^2 \leq$ 6,500
Maxcap	LC#2	43	$Q_2^2 \leq$ 11,000
Maxcap	LCC,1	44	$Q_3^2 \leq$ 36,000
Maxcap	LCC,2	45	$Q_4^2 \leq$ 24,000
Maxcap	LCC,3	46	$Q_5^2 \leq$ 12,000
Maxcap	PL	47	$Q_6^2 \leq$ 6,000
Maxcap	L&W,1	48	$Q_7^2 \leq$ 50,000
Maxcap	L&W,2	49	$Q_8^2 \leq$ 34,000
Maxcap	L&W,3	50	$Q_9^2 \leq$ 17,000
Maxcap	G#2,1	51	$Q_{10}^2 \leq$ 36,000
Maxcap	G#2,2	52	$Q_{11}^2 \leq$ 24,000
Maxcap	FC	53	$S_{18}^2 \leq$ 12,000
Maxcap	RR	54	$S_{19}^2 \leq$ 4,000
Maxcap	BH	55	$S_{20}^2 \leq$ 8,000
Maxcap	RL	56	$S_{21}^2 \leq$ 1,000
Maxcap	LP	57	$S_{22}^2 \leq$ 4,000
Maxcap	LL	58	$S_{23}^2 \leq$ 1,000
Maxcap	WR	59	$S_{24}^2 \leq$ 18,000
Maxcap	WL	60	$S_{25}^2 \leq$ 1,000

Equation Set II (continued)

Type	Element	Equation Number	
Mincap	FC	61	$s_{18}^2 \geq 1,320$
Mincap	RR	62	$s_{19}^2 \geq 100$
Mincap	BH	63	$s_{20}^2 \geq 800$
Mincap	RL	64	$s_{21}^2 \geq 25$
Mincap	LP	65	$s_{22}^2 \geq 200$
Mincap	LL	66	$s_{23}^2 \geq 25$
Mincap	WR	67	$s_{24}^2 \geq 450$
Mincap	WL	68	$s_{25}^2 \geq 50$

Table XIV

Equation Set III

Type	Element	Equation Number	
Ditch	PV&LC	1	$.0107Q_1^1 + .0214Q_1^2 + .7857Q_1^3 - V_1^3 = 500$
Ditch	LC#2	2	$.0021Q_1^1 + .0064Q_2^1 + .0064Q_1^2 + .0089Q_2^2 + .0107Q_1^3 + .8724Q_2^3 - V_2^3 = 5000$
Ditch	LCC,1	3	$.0003Q_8^1 + .0004Q_3^2 + .9851Q_3^3 - V_3^3 = 2000$
Ditch	LCC,2	4	$.0026Q_4^1 + .0002S_{19}^1 + .0052Q_4^2 + .0003S_{19}^2 + .8707Q_4^3 - V_4^3 = 1499.32$
Ditch	LCC,3	5	$.0010Q_5^1 + .0046Q_6^1 + .0003S_{20}^1 + .0021Q_5^2 + .0069Q_6^2 + .0004S_{20}^2 + .7905Q_5^3 + .0023Q_6^3 - V_5^3 = 1498.72$
Ditch	PL	6	$.0115Q_6^1 + .0230Q_6^2 + .7700Q_6^3 - V_6^3 = 1000$
Ditch	L&W,1	7	$.0001Q_3^1 + .0001Q_7^1 + .0005S_{19}^1 + .0002S_{21}^1 + .0003S_{22}^1 + .0003Q_3^2 + .0003Q_7^2 + .0011S_{19}^2 + .0012S_{21}^2 + .0006S_{22}^2 + .0001Q_3^3 + .9851Q_7^3 + .0001S_{19}^3 - V_7^3 = 1995.72$
Ditch	L&W,2	8	$.0013Q_4^1 + .0030Q_8^1 + .0003S_{22}^1 + .0026Q_4^2 + .0038Q_8^2 + .0003S_{22}^2 + .0006Q_4^3 + .9247Q_8^3 + .0001S_{22}^3 - V_8^3 = 1999.23$
Ditch	L&W,3	9	$.0026Q_4^1 + .0021Q_5^1 + .0216Q_9^1 + .0005S_{20}^1 + .0039Q_4^2 + .0063Q_5^2 + .0309Q_9^2 + .0016S_{20}^2 + .0006Q_4^3 + .0006Q_5^2 + .6911Q_9^3 + .0002S_{20}^3 - V_9^3 = 1495.26$
Ditch	G#2,1	10	$.0008Q_8^1 + .0006Q_{10}^1 + .0003S_{23}^1 + .0023Q_8^2 + .0012Q_{10}^2 + .0007S_{23}^2 + .9704Q_{10}^3 + .0002S_{23}^3 - V_{10}^3 = 2498.51$
Ditch	G#2,2	11	$.0031Q_9^1 + .0110Q_{11}^1 + .0005S_{24}^1 + .0093Q_9^2 + .0137Q_{11}^2 + .0010S_{24}^2 + .0031Q_9^3 + .7252Q_{11}^3 + .0001S_{24}^3 - V_{11}^3 = 1995.02$
Ditch	R,1	12	$.0011Q_1^1 + .0021Q_1^2 + .0064Q_1^3 + .9970Q_{12}^3 - V_{12}^3 = 2000$
Ditch	R,2	13	$.0005Q_1^1 + .0004Q_3^1 + .0011Q_1^2 + .0006Q_3^2 + .0021Q_1^3 + .0003Q_3^3 + .9910Q_{13}^3 - V_{13}^3 = 1500$
Ditch	R,3	14	$.0013Q_2^1 + .0003Q_3^1 + .0005Q_1^2 + .0038Q_2^2 + .0004Q_3^2 + .0011Q_1^3 + .0064Q_2^3 + .0001Q_3^3 + .9940Q_{14}^3 - V_{14}^3 = 1000$
Ditch	R,4	15	$.0005Q_2^1 + .0003Q_7^1 + .0003Q_{23}^1 + .0006Q_2^2 + .0004Q_7^2 + .0007Q_{23}^2 + .0013Q_2^3 + .0001Q_7^3 + .9900Q_{15}^3 - V_{15}^3 = 1498.86$
Ditch	R,5	16	$.0001Q_7^1 + .0023Q_8^1 + .0003Q_{23}^1 + .0003Q_7^2 + .0030Q_8^2 + .0004Q_{23}^2 + .0001Q_7^3 + .0008Q_8^3 + .9733Q_{16}^3 - V_{16}^3 = 1499.29$
Ditch	R,6	17	$.0009Q_{10}^1 + .0082Q_{11}^1 + .0012S_{18}^1 + .0008S_{25}^1 + .0012Q_{10}^2 + .0110Q_{11}^2 + .0022S_{18}^2 + .0018S_{25}^2 + .0006Q_{10}^3 + .0055Q_{11}^3 + .9029Q_{17}^3 + .0002S_{18}^3 - V_{17}^3 = 491.88$
Reservoir	FC	18	$.0021Q_1^1 + .0064Q_1^2 + .9814S_{18}^2 + Q_{18}^3 - 1.0186S_{18}^3 - V_{18}^3 = 66.00$
Reservoir	RR	19	$.9896S_{19}^2 + Q_{19}^3 - 1.0104S_{19}^3 - V_{19}^3 = 32.40$
Reservoir	BH	20	$.0023Q_6^1 + .0046Q_6^2 + .9836S_{20}^2 + .0023Q_6^3 + Q_{20}^3 - 1.0164S_{20}^3 - V_{20}^3 = 64.80$
Reservoir	RL	21	$.0003Q_3^1 + .0039Q_4^1 + .0002S_{19}^1 + .0004Q_3^2 + .0065Q_4^2 + .0005S_{19}^2 + .9860S_{21}^2 + 0001Q_3^3 + .0013Q_4^3 + Q_{21}^3 - 1.0140S_{21}^3 - V_{21}^3 = 12.74$

Equation Set III (continued)

Type	Element	Equation Number	
Maxcap	FC	53	$S^3_{18} \leq 12{,}000$
Maxcap	RR	54	$S^3_{19} \leq 4{,}000$
Maxcap	BH	55	$S^3_{20} \leq 8{,}000$
Maxcap	RL	56	$S^3_{21} \leq 1{,}000$
Maxcap	LP	57	$S^3_{22} \leq 4{,}000$
Maxcap	LL	58	$S^3_{23} \leq 1{,}000$
Maxcap	WR	59	$S^3_{24} \leq 18{,}000$
Maxcap	WL	60	$S^3_{25} \leq 1{,}000$
Mincap	FC	61	$S^3_{18} \geq 1{,}320$
Mincap	RR	62	$S^3_{19} \geq 100$
Mincap	BH	63	$S^3_{20} \geq 800$
Mincap	RL	64	$S^3_{21} \geq 25$
Mincap	LP	65	$S^3_{22} \geq 200$
Mincap	LL	66	$S^3_{23} \geq 25$
Mincap	WR	67	$S^3_{24} \geq 450$
Mincap	WL	68	$S^3_{25} \geq 50$

Equation Set III (continued)

Type	Element	Equation Number	
Reservoir	LP	22	$.0001Q_3^1 + .0052Q_4^1 + .0002S_{19}^1 + .0001Q_3^2 + .0065Q_4^2 + .0003S_{19}^2 + .9884S_{22}^2 + .0026Q_4^3 + Q_{22}^3 - 1.0116S_{22}^3 - V_{22}^3 = 32.52$
Reservoir	LL	23	$.0001Q_7^1 + .0008Q_8^1 + .0003Q_7^2 + .0008Q_8^2 + .9824S_{23}^2 + Q_{23}^3 - 1.0176S_{23}^3 - V_{23}^3 = 35.10$
Reservoir	WR	24	$.0008Q_8^1 + .0046Q_9^1 + .0015Q_8^2 + .0062Q_9^2 + .9862S_{24}^2 + .0008Q_8^3 + .0015Q_9^3 + Q_{24}^3 - 1.0138S_{24}^3 - V_{24}^3 = 77.40$
Reservoir	WL	25	$.0002S_{24}^1 + .0006S_{24}^2 + .9744S_{25}^2 + Q_{25}^3 + .0001S_{24}^3 - 1.0256S_{25}^3 - V_{25}^3 = 24.00$
Node	1	26	$Q_1^3 + Q_{12}^3 = 15,000$
Node	2	27	$V_1^3 + V_2^3 - Q_{18}^3 = 0$
Node	3	28	$V_{12}^3 - Q_3^3 - Q_{13}^3 = 0$
Node	4	29	$V_3^3 - Q_{19}^3 = 0$
Node	5	30	$V_{19}^3 - Q_4^3 - Q_{21}^3 = 0$
Node	6	31	$V_4^3 - Q_6^3 - Q_{20}^3 = 0$
Node	7	32	$V_{20}^3 - Q_5^3 = 0$
Node	8	33	$V_{13}^3 - Q_2^3 - Q_{14}^3 = 0$
Node	9	34	$V_{14}^3 - Q_7^3 - Q_{15}^3 = 0$
Node	10	35	$V_7^3 + V_{22}^3 - Q_8^3 - Q_{23}^3 = 0$
Node	11	36	$V_8^3 - Q_9^3 - Q_{24}^3 = 0$
Node	12	37	$V_{15}^3 + V_{23}^3 - Q_{16}^3 = 0$
Node	13	38	$V_{16}^3 + V_{18}^3 - Q_{10}^3 - Q_{17}^3 = 0$
Node	14	39	$V_{10}^3 - Q_{25}^3 = 0$
Node	15	40	$V_{24}^3 + V_{25}^3 - Q_{11}^3 = 0$
Node	16	41	$V_{21}^3 - Q_{22}^3 = 0$
Maxcap	PV&LC	42	$Q_1^3 \leq 6,500$
Maxcap	LC#2	43	$Q_2^3 \leq 11,000$
Maxcap	LCC,1	44	$Q_3^3 \leq 3,600$
Maxcap	LCC,2	45	$Q_4^3 \leq 24,000$
Maxcap	LCC,3	46	$Q_5^3 \leq 12,000$
Maxcap	PL	47	$Q_6^3 \leq 6,000$
Maxcap	L&W,1	48	$Q_7^3 \leq 50,000$
Maxcap	L&W,2	49	$Q_8^3 \leq 34,000$
Maxcap	L&W,3	50	$Q_9^3 \leq 17,000$
Maxcap	G#2,1	51	$Q_{10}^3 \leq 36,000$
Maxcap	G#2,2	52	$Q_{11}^3 \leq 24,000$

in which $\overline{x}_1$ and $\overline{b}_1$ are the same as discussed in equation (6-6) and $\overline{x}_2$ and $\overline{b}_2$ are the same as discussed in equation (6-7).

Model II, All Time Periods

The equations for Model II are the same as those for Model I, except that the coefficients for the variable Q_3^t are changed. The data for calculating the new coefficients are listed in the various tables already discussed for Model I. The Model II values are listed in Table XV along with the Model I values they are to replace.

Linear Programming Models

Three linear programming problems are formulated using each model (the equations in Tables XII, XIII and XIV for Model I and the same equations, with the necessary changes, listed in Table XV, for Model II). The first problem to solve is to obtain the optimal delivery strategy in time period I, using knowledge of the events to occur in time period 1 only. The constraints for this case are represented by equation (6-6), formed from equation set I, and the optimal water delivery strategy is represented by the numerical values obtained for the vector $\overline{x}_1$.

The second problem to solve is to obtain the optimal delivery strategy in time period 1, based on knowledge of events to occur in time periods 1 and 2. The constraints for this case are formed from equation sets I and II and are represented by equations (6-6) and (6-7) in the form:

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \end{bmatrix} = \begin{bmatrix} D_{11} & 0 \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \overline{b}_1 \\ \overline{b}_2 \end{bmatrix} . \qquad (6\text{-}9)$$

Table XV

Changes in Equation Sets I, II and III, for Model I, to Obtain Model II

Type	Element	Equation Set	Equation Number	Replace Model I Parameter	With Model II Parameter
Ditch	LCC,1	I	3	$.9851Q_3^1$	$.9417Q_3^1$
Ditch	L&W,1		7	$.0001Q_3^1$	$.0006Q_3^1$
Ditch	R,2		13	$.0003Q_3^1$	$.0012Q_3^1$
Ditch	R,3		14	$.0001Q_3^1$	$.0006Q_3^1$
Reservoir	RL	I	21	$.0001Q_3^1$	$.0006Q_3^1$
Ditch	LCC,1	II	3	$.0004Q_3^1$	$.0017Q_3^1$
			3	$.9851Q_3^2$	$.9417Q_3^2$
Ditch	L&W,1		7	$.0003Q_3^1$	$.0012Q_3^1$
			7	$.0001Q_3^2$	$.0006Q_3^2$
Ditch	R,2		13	$.0006Q_3^1$	$.0023Q_3^1$
			13	$.0003Q_3^2$	$.0012Q_3^2$
Ditch	R,3		14	$.0004Q_3^1$	$.0017Q_3^1$
			14	$.0001Q_3^2$	$.0006Q_3^2$
Reservoir	RL		21	$.0004Q_3^1$	$.0017Q_3^1$
			21	$.0001Q_3^2$	$.0006Q_3^2$
Reservoir	LP	II	22	$.0001Q_3^1$	$.0006Q_3^1$
Ditch	LCC,1	III	3	$.0003Q_3^1$	$.0012Q_3^1$
			3	$.0004Q_3^2$	$.0017Q_3^2$
			3	$.9851Q_3^3$	$.9417Q_3^3$
Ditch	L&W,1		7	$.0001Q_3^1$	$.0006Q_3^1$
			7	$.0003Q_3^2$	$.0012Q_3^2$
			7	$.0001Q_3^3$	$.0006Q_3^3$
Ditch	R,2		13	$.0004Q_3^1$	$.0017Q_3^1$
			13	$.0006Q_3^2$	$.0023Q_3^2$
			13	$.0003Q_3^3$	$.0012Q_3^3$
Ditch	R,3		14	$.0003Q_3^1$	$.0012Q_3^1$
			14	$.0004Q_3^2$	$.0017Q_3^2$
			14	$.0001Q_3^3$	$.0006Q_3^3$
Reservoir	RL		21	$.0003Q_3^1$	$.0012Q_3^1$
			21	$.0004Q_3^2$	$.0017Q_3^2$
			21	$.0001Q_3^3$	$.0006Q_3^3$
Reservoir	LP	III	22	$.0001Q_3^1$	$.0006Q_3^1$
			22	$.0001Q_3^2$	$.0006Q_3^2$

The optimal solution that results from this problem is the optimal water delivery strategy for time period 1, contained in the subvector $\bar{x}_1$, and the optimal water delivery strategy for time period 2, contained in the subvector $\bar{x}_2$. However, only those values in $\bar{x}_1$ are of interest.

The final problem to solve is for an optimal routing strategy in time period 1, based on knowledge of events to occur in time periods 1, 2 and 3. The constraints are formed from equation sets I, II and III and are represented by equations (6-6), (6-7) and (6-8) in the following form:

$$\begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix} = \begin{bmatrix} D_{11} & 0 & 0 \\ D_{21} & D_{22} & 0 \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \\ \vec{b}_3 \end{bmatrix} \tag{6-10}$$

For this problem, the optimal solution yields the optimal water delivei strategy in time period 1, contained in the subvector $\bar{x}_1$; the optima: water delivery strategy in time period 2, contained in the subvector $\bar{x}_2$; and the optimal water delivery strategy in time period 3, contained in the subvector $\bar{x}_3$. Again, only the values contained in $\bar{x}_1$ are of interest.

Objective Functions

For each of the three problems for Model I and Model II, an objective function is required. In Chapter V, a mathematical statemen for minimizing system losses and system outflows, conjunctively, was made, (see equation (5-36)). The data necessary to construct the objective functions are contained in Tables II, IV, V, VI, and VII.

The objective functions for Model I are:

$$\text{MIN } Z = .1396Q_1^1 + .0984Q_2^1 + .0106Q_3^1 + .0839Q_4^1 + .1974Q_5^1 + .1725Q_6^1$$
$$+ .0128Q_7^1 + .0546Q_8^1 + .2286Q_9^1 + .0251Q_{10}^1 + .2254Q_{11}^1 + .0030Q_{12}^1$$
$$+ .0090Q_{13}^1 + .0060Q_{14}^1 + .0100Q_{15}^1 + .0267Q_{16}^1 + .0971Q_{17}^1 + .0343S_{18}^1$$
$$+ .0183S_{19}^1 + .0304S_{20}^1 + .0315S_{21}^1 + .0285S_{22}^1 + .0313S_{23}^1 + .0261S_{24}^1$$
$$+ .0480S_{25}^1 + V_5^1 + V_6^1 + V_9^1 + V_{11}^1 + V_{17}^1 \qquad (6\text{-}11)$$

for the first problem;

$$\text{MIN } Z = .1396Q_1^1 + .0984Q_2^1 + .0106Q_3^1 + .0839Q_4^1 + .1974Q_5^1 + .1725Q_6^1$$
$$+ .0128Q_7^1 + .0546Q_8^1 + .2286Q_9^1 + .0251Q_{10}^1 + .2254Q_{11}^1 + .0030Q_{12}^1$$
$$+ .0090Q_{13}^1 + .0060Q_{14}^1 + .0100Q_{15}^1 + .0267Q_{16}^1 + .0971Q_{17}^1 + .0343S_{18}^1$$
$$+ .0183S_{19}^1 + .0304S_{20}^1 + .0315S_{21}^1 + .0285S_{22}^1 + .0313S_{23}^1 + .0261S_{24}^1$$
$$+ .0480S_{25}^1 + V_5^1 + V_6^1 + V_9^1 + V_{11}^1 + V_{17}^1 + .1561Q_1^2 + .1066Q_2^2$$
$$+ .0121Q_3^2 + .0995Q_4^2 + .2005Q_5^2 + .1909Q_6^2 + .0134Q_7^2 + .0623Q_8^2$$
$$+ .2579Q_9^2 + .0266Q_{10}^2 + .2446Q_{11}^2 + .0030Q_{12}^2 + .0090Q_{13}^2 + .0060Q_{14}^2$$
$$+ .0100Q_{15}^2 + .0267Q_{16}^2 + .0971Q_{17}^2 + .0384S_{18}^2 + .0215S_{19}^2 + .0340S_{20}^2$$
$$+ .0333S_{21}^2 + .0257S_{22}^2 + .0333S_{23}^2 + .0280S_{24}^2 + .0540S_{25}^2 + V_5^2 + V_6^2$$
$$+ V_9^2 + V_{11}^2 + V_{17}^2 \qquad (6\text{-}12)$$

for the second problem, and

$$\begin{aligned} \text{MIN } Z = {} & .1396Q_1^1 + .0984Q_2^1 + .0106Q_3^1 + .0839Q_4^1 + .1974Q_5^1 + .1725Q_6^1 \\ & + .0128Q_7^1 + .0546Q_8^1 + .2286Q_9^1 + .0251Q_{10}^1 + .2254Q_{11}^1 + .0030Q_{12}^1 \\ & + .0090Q_{13}^1 + .0060Q_{14}^1 + .0100Q_{15}^1 + .0267Q_{16}^1 + .0971Q_{17}^1 + .0343S_{18}^1 \\ & + .0183S_{19}^1 + .0304S_{20}^1 + .0315S_{21}^1 + .0285S_{22}^1 + .0313S_{23}^1 + .0261S_{24}^1 \\ & + .0480S_{25}^1 + V_5^1 + V_6^1 + V_9^1 + V_{11}^1 + V_{17}^1 + .1561Q_1^2 + .1066Q_2^2 \\ & + .0121Q_3^2 + .0995Q_4^2 + .2005Q_5^2 + .1909Q_6^2 + .0134Q_7^2 + .0623Q_8^2 \\ & + .2579Q_9^2 + .0266Q_{10}^2 + .2446Q_{11}^2 + .0030Q_{12}^2 + .0090Q_{13}^2 + .0060Q_{14}^2 \\ & + .0100Q_{15}^2 + .0267Q_{16}^2 + .0971Q_{17}^2 + .0384S_{18}^2 + .0215S_{19}^2 + .0340S_{20}^2 \\ & + .0333S_{21}^2 + .0257S_{22}^2 + .0333S_{23}^2 + .0280S_{24}^2 + .0540S_{25}^2 + V_5^2 + V_6^2 + V_9^2 \\ & + V_{11}^2 + V_{17}^2 + .1940Q_1^3 + .1199Q_2^3 + .0143Q_3^3 + .1242Q_4^3 + .2089Q_5^3 \\ & + .2254Q_6^3 + .0147Q_7^3 + .0737Q_8^3 + .3043Q_9^3 + .0290Q_{10}^3 + .2693Q_{11}^3 \\ & + .0030Q_{12}^3 + .0090Q_{13}^3 + .0060Q_{14}^3 + .0100Q_{15}^3 + .0267Q_{16}^3 + .0971Q_{17}^3 \\ & + .0184S_{18}^3 + .0103S_{19}^3 + .0162S_{20}^3 + .0140S_{21}^3 + .0115S_{22}^3 + .0174S_{23}^3 \\ & + .0136S_{24}^3 + .0256S_{25}^3 + V_5^3 + V_6^3 + V_9^3 + V_{11}^3 + V_{17}^3 \quad , \qquad (6\text{-}13) \end{aligned}$$

for the third problem. The objective functions for Model II are the same as for Model I, except that the coefficient of the variable Q_3^1 is changed to .0396 in equation (6-11), the coefficients of the variables Q_3^1 and Q_3^2 are changed to .0396 and .0461, respectively, in equation (6-12), and the coefficients of the variables Q_3^1 , Q_3^2 , and Q_3^3 are changed to .0396, .0461 and .0553, respectively, in equation (6-13).

Examining equations (6-11), (6-12) and (6-13), it is evident tha equation (6-11) could be written in matrix form as

$$\text{MIN } Z = \bar{c}_1\bar{x}_1 \; ; \qquad \text{(6-14}$$

equation (6-12) could be written as

$$\text{MIN } Z = [\bar{c}_1 \; \bar{c}_2] \begin{matrix} \bar{x}_1 \\ \bar{x}_2 \end{matrix} \qquad \text{(6-15)}$$

and, equation (6-13) could be written as

$$\text{MIN } Z = [\bar{c}_1 \; \bar{c}_2 \; \bar{c}_3] \begin{matrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{matrix} \; . \qquad \text{(6-16)}$$

In each of the equations, the contents of the $\bar{c}_1$ subvector represent the "costs" of using any particular element in time period 1, the contents of the $\bar{c}_2$ subvector represent the "costs" of using any particular element in time period 2, and the contents of the $\bar{c}_3$ subvector represent the "costs" of using any particular element in time period 3. The $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$ are the vectors of decision variables for each time period discussed previously with respect to equations (6-6), (6-9) and (6-10).

A close examination of equations (6-11), (6-12) and (6-13) reveals that they are composed of exactly the same terms for corresponding time periods. The implication of this is that even though the purpose of the first problem formulation of either Model I or Model II is to find an optimal water delivery strategy in time period 1, based only on knowledge of events to occur in the time period, the return flows for time periods 2 and 3 are considered in the objective function. Likewise, even though the purpose of the second problem formulation is to find an optimal water delivery strategy in time period 1, based on knowledge of events to occur in time periods 1 and 2, the return flows in time period 3 are considered in the objective function.

There are two reasons for using these objective functions. The physical reason is that once a delivery strategy is selected and used the return flows will occur regardless of whether the system is operated in the future. Just because these return flows do not occur within the time period of analysis is no justification for discounting them as a system loss. Further, for the types of analysis discussed in the following sections, the use of this type of objective function provides results that are more easily compared.

Solution of Linear Programming Models

The three linear programming problem formulations for each of the two models were solved using the Control Data Mathematical Programming System 3 (CDM3) routine, adapted to the CDC 6400 computer in the University Computer Center at Colorado State University. The routine, as adapted, can solve a problem of 300 rows by 600 columns with no more than 2,400 non-zero coefficients in the A matrix. For the three time period models, characterized by equations (6-10) and (6-16), the largest solved, containing 204 rows, 255 columns and 712 non-zero coefficients, the solution time was approximately two minutes.

Results from Linear Programming Models

The optimal water delivery strategy for each problem of Models I and II are shown in Tables XVI and XVII, respectively. Results from these tables are combined with the diagram of the system, Figure 13, and presented in Figures 16 and 17. These figures give a visual representation of how the water should be delivered to satisfy the various demands so that the system losses and unrequired system outflows are minimized.

There are three major points related to the results of the two models that should be discussed. The first is the difference in strategies between Models I and II themselves; the second is the variation in optimal stragegies for time period 1 resulting from the inclusion of additional time periods in both models; and the third is that in both models there is no system outflow.

Comparing Figures 16 and 17 it can be seen that the corresponding water delivery strategies are different for some elements. It should be recalled that the difference between the two models is the unit loss rate assumed for the ditch sector LCC,1. In Model I, LCC,1 is assumed to be lined and in Model II, unlined.

From Figures 16 and 17, it can be seen that the major difference in strategies centers about ditch sectors LCC,1 and L&W,1. In Model I, the sector LCC,1 has a lower loss rate than even the river, and can supply six reservoirs (RR, BH, RL, LP, LL and WR) and seven ditch sectors (LCC,1; LCC,2; LCC,3; PL; L&W,2; L&W,3; and G#2,2). To minimize the system losses in Model I it is advantageous then to divert as much water as possible into LCC,1. This is evidenced in Figure 16, and is done regardless of the number of time periods in the analysis.

Table XVI

Results from Example Model I for Optimal Water Delivery Strategies in Time Period 1

		Number of Time Periods in Analysis		
Element	Index	1	2	3
PV&LC, Inflow	1	1273	1273	1273
LC#2, Inflow	2	1131	2117	1131
LCC,1, Inflow	3	36000	36000	36000
LCC,2, Inflow	4	5532	5570	6936
LCC,3, Inflow	5	1263	1263	1263
LCC,3, Outflow	5	0	0	0
PL, Inflow	6	649	649	649
PL, Outflow	6	0	0	0
L&W,1, Inflow	7	2126	2164	3121
L&W,2, Inflow	8	17286	17286	16868
L&W,3, Inflow	9	2163	2163	2162
L&W,3, Outflow	9	0	0	0
G#2,1, Inflow	10	4288	3313	3329
G#2,2, Inflow	11	1366	1366	1367
G#2,2, Outflow	11	0	0	0
R,6, Outflow	17	0	0	0
FC, Inflow	18	0	860	0
FC, Contents	18	3830	4677	3830
FC, Outflow	18	0	0	0
RR, Inflow	19	34464	34464	34464
RR, Contents	19	8000	8000	8000
RR, Outflow	19	27365	27365	27365
BH, Inflow	20	2167	2200	3390
BH, Contents	20	2793	2826	4000
BH, Outflow	20	1263	1263	1263
RL, Inflow	21	21833	21795	20429
RL, Contents	21	1000	1000	1000
RL, Outflow	21	21068	21030	19666
LP, Inflow	22	21068	21030	19666
LP, Contents	22	4000	4000	4000
LP, Outflow	22	17985	17947	16586
LL, Inflow	23	798	798	798
LL, Contents	23	1000	1000	1000
LL, Outflow	23	0	0	0
WR, Inflow	24	12325	12325	11940
WR, Contents	24	18000	18000	17620
WR, Outflow	24	0	0	0
WL, Inflow	25	2162	1216	1231
WL, Contents	25	1000	72	87
WL, Outflow	25	1366	1366	1367

Table XVII

Results from Example Model II, for Optimal Water Delivery Strategies in Time Period 1

		Number of Time Periods in Analysis		
Element	Index	1	2	3
PV&LC, Inflow	1	1273	1273	1273
LC#2, Inflow	2	2493	1131	1131
LCC,1, Inflow	3	16123	18617	20063
LCC,2, Inflow	4	3214	5570	6936
LCC,3, Inflow	5	1263	1263	1263
LCC,3, Outflow	5	0	0	0
PL, Inflow	6	649	649	649
PL, Outflow	6	0	0	0
L&W,1, Inflow	7	20381	20283	18842
L&W,2, Inflow	8	17290	17195	15777
L&W,3, Inflow	9	2166	2163	2162
L&W,3, Outflow	9	0	0	0
G#2,1, Inflow	10	4288	3314	3331
G#2,2, Inflow	11	1366	1366	1367
G#2,2, Outflow	11	0	0	0
R,6, Outflow	17	0	0	0
FC, Inflow	18	1189	0	0
FC, Contents	18	5001	3830	3830
FC, Outflow	18	0	0	0
RR, Inflow	19	14183	16531	17893
RR, Contents	19	8000	8000	8000
RR, Outflow	19	7084	9433	10794
BH, Inflow	20	149	2200	3390
BH, Contents	20	800	2825	4000
BH, Outflow	20	1263	1263	1263
RL, Inflow	21	3871	3863	3858
RL, Contents	21	1000	1000	1000
RL, Outflow	21	3089	3083	3080
LP, Inflow	22	3089	3083	3080
LP, Contents	22	4000	4000	4000
LP, Outflow	22	0	0	0
LL, Inflow	23	798	798	798
LL, Contents	23	1000	1000	1000
LL, Outflow	23	0	0	0
WR, Inflow	24	12325	12241	10931
WR, Contents	24	18000	17917	16622
WR, Outflow	24	0	0	0
WL, Inflow	25	2162	1216	1233
WL, Contents	25	1000	73	88
WL, Outflow	25	1366	1366	1367

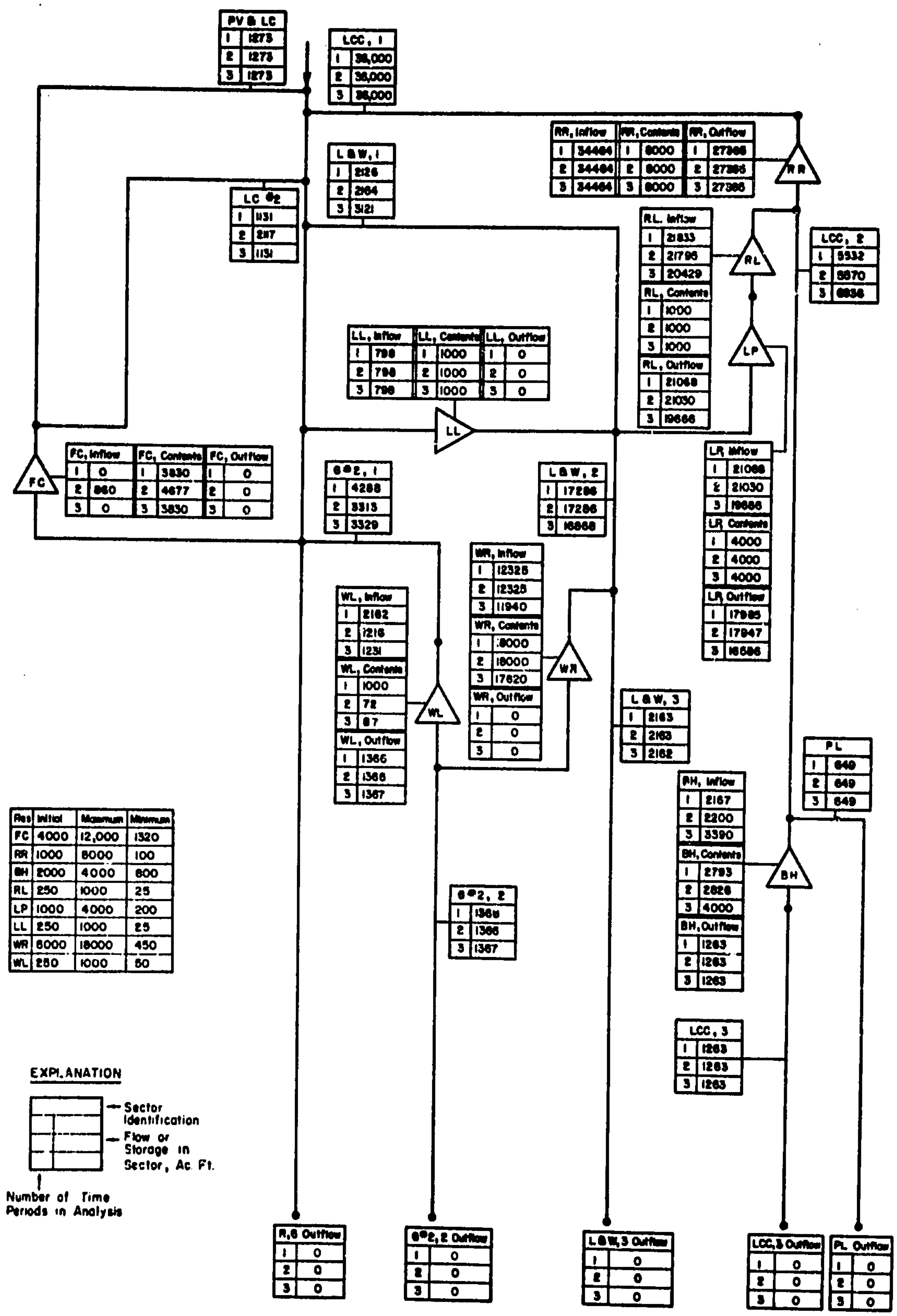


Figure 16. Optimal Water Delivery Strategies, Model I

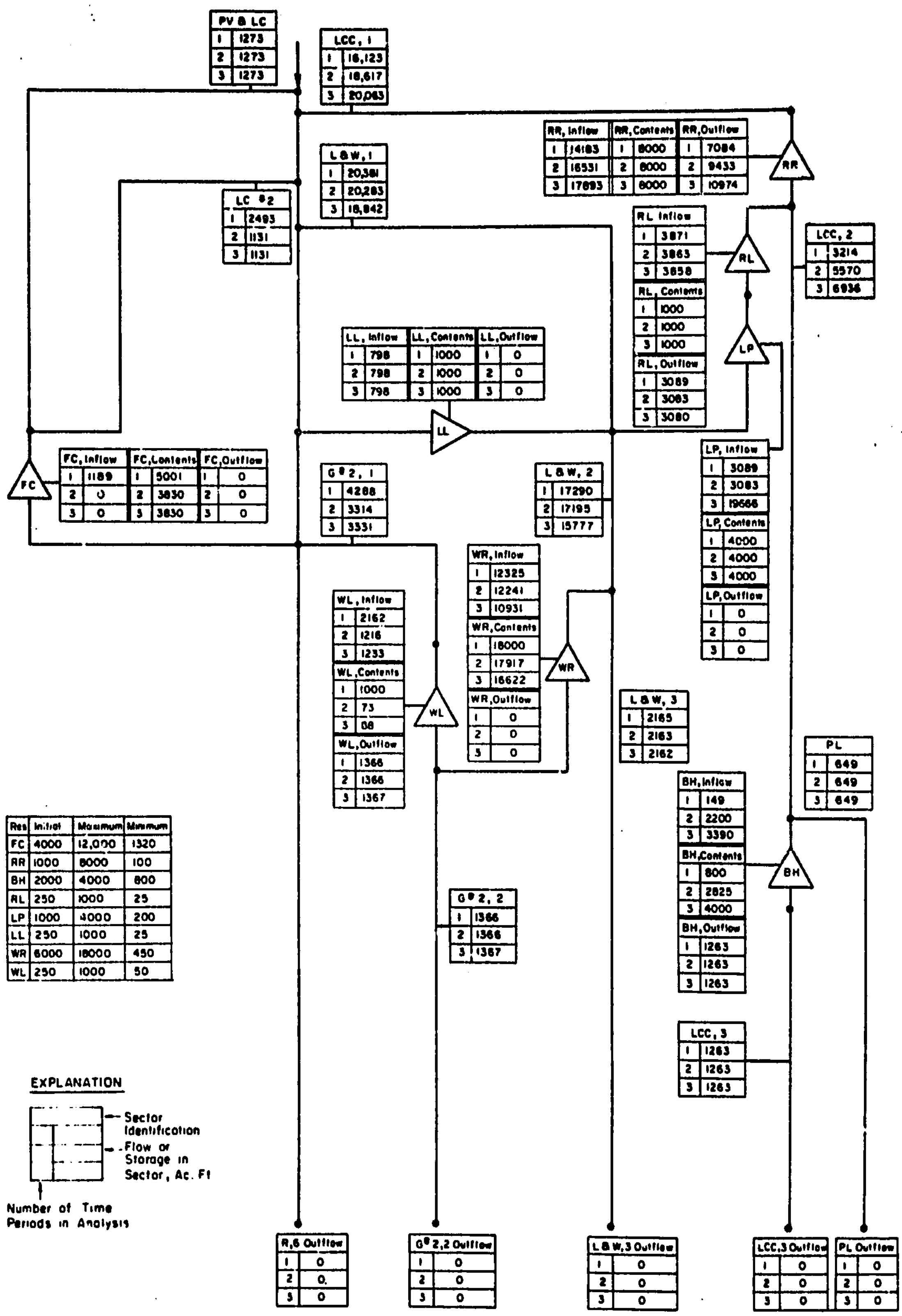


Figure 17. Optimal Water Delivery Strategies, Model II

In Model II, however, LCC,1 has a higher loss rate than both the river and the sector L&W,1 (.0583 as opposed to .0149 and .0149 respectively). The model solutions show a better strategy is to divert water into ditch sector L&W,1 for storage in the reservoirs LL and WR or delivery by ditch sectors L&W,2 and L&W,3.

Those ditch sectors with no change in delivery strategy are those with only one ditch sector that can deliver the demand. These results from the models are both reasonable and to be expected.

A comparison of the optimal strategies in time period 1 according to the number of time periods in the analysis, using either Figure 16 or 17, indicates that for some ditch sectors and reservoirs the optimal strategy is a function of the number of time periods in the analysis. This is also expected. Bellman's principle of optimality states (Hadley, 1964, p. 362):

> "...we cannot have an optimal value of the objective function for k stages unless for any x_k selected for stage k, the value of the remaining $k-1$ stages is optimal given the x_k for stage k."

The reason for the variation in optimal strategies in physical terms, is that an optimal strategy in one time period may not store water in a location where it is available to satisfy a future demand. To obtain an optimal strategy throughout the period of analysis, there must be a balance of losses in delivering the water to a reservoir. in storing the water in a reservoir for one or more time periods, and in delivering the water to satisfy a demand. The derivation of optimal strategies for each time period independently of the others, while yielding the absolute minimum system losses and unrequired system outflows, does not guarantee all demands can be met in the future because in minimizing system losses, surplus water in early time periods may be stored in reservoirs that cannot supply areas where a water deficiency exists in later time periods.

As stated, solving for the optimal water delivery strategy for a single time period, using only knowledge of events to occur in that time period, will yield the absolute minimum value of the objective function. When more than one time period is used in an analysis, the value of the objective function for the time period of interest is required to increase if the strategy is changed to satisfy the additional conditions imposed by those additional time periods. The first column of numbers in Table XVIII exhibits the values of the objective functions resulting from the optimal water delivery strategies for time period 1 of Models I and II, respectively, according to the number of time periods in the analysis. As expected, there is some additional system loss incurred in time period 1 to ensure that the demands in time periods 2 and 3 can be satisfied. Again, these results are reasonable and to be expected.

Finally, in both Figures 16 and 17, it can be seen that the outflows from the ditch sectors R,6; G#2,2; L&W,3; LCC,3 and PL are all zero regardless of the number of time periods in the analysis, even though the system inflow is 31,100 ac-ft greater than the demand in time period 1. This occurs because the system has adequate storage to retain all of the excess water and because the objective function specifies that the unrequired system outflow is to be minimized.

Table XVIII

Values of the Objective Functions at the Optimal Solutions, Nonoptimal Solutions and Comparisons of the Two for Models I and II

	Time Periods in Analysis	Value of Objective Function for Optimal Solution	Value of Objective Function for Nonoptimal Solution	Ratio: Nonoptimal / Optimal
	1	5,278	38,642	7.32
Model I	2	5,296	22,450	4.24
	3	5,307	17,104	3.22
	1	5,977	38,417	6.43
Model II	2	6,052	26,548	4.39
	3	6,072	6,422	1.06

To provide a comparison of an alternate, but nonoptimal solution to the problem with the optimal solutions exhibited in Tables XVI and XVII and Figures 16 and 17, the first feasible, but nonoptimal, solution examined by the computer was printed. Although not necessarily the worst of all possible feasible solutions to the problem, the first feasible solution is the worst solution examined by the computer. These results are listed in Table XIX, for Model I, and in Table XX for Model II, and shown in Figures 18 and 19. Also contained in Table XVIII is a comparison of the values of the objective function for time period 1 for both the optimal solutions and the nonoptimal solutions. It can be seen that considerable water can be saved if proper management is used in system operations.

In progressing from feasible, but nonoptimal, solutions such as those shown in Tables XIX and XX, to the optimal solutions, such as those shown in Tables XVI and XVII, decisions are made by the computer not only to store water in reservoirs, but to store it in the reservoirs with the least system losses.

Practical Applications

It has already been stated in Chapter III that the power of the simulation model lies in its use with programming techniques to derive optimal water delivery strategies that satisfy given demands. Toward this end, the model can provide a preseason analysis for determining how much of the demand on a system can be satisfied during the irrigating season, based on forecasts of the supply available to the system. The object of such an analysis is to find the maximum amount of the demand that can be satisfied during the season, not to determine the delivery strategies that should be used during the season.

This consists of looking for the last increment of demand that yields a feasible solution. If all demands cannot be satisfied, a legal policy is required to specify those demands that are to be satisfied and those demands that are not.

With this assistance, a farmer will have better estimates of the volume and time distribution of his water supply. However, it should be expected that more than one preseason analysis will be required as the irrigating season approaches and forecasts of the supplies and demands become more accurate.

A logical extension of the preseason analysis, and the use for which the model was derived, is the application of the model as a tool

Table XIX

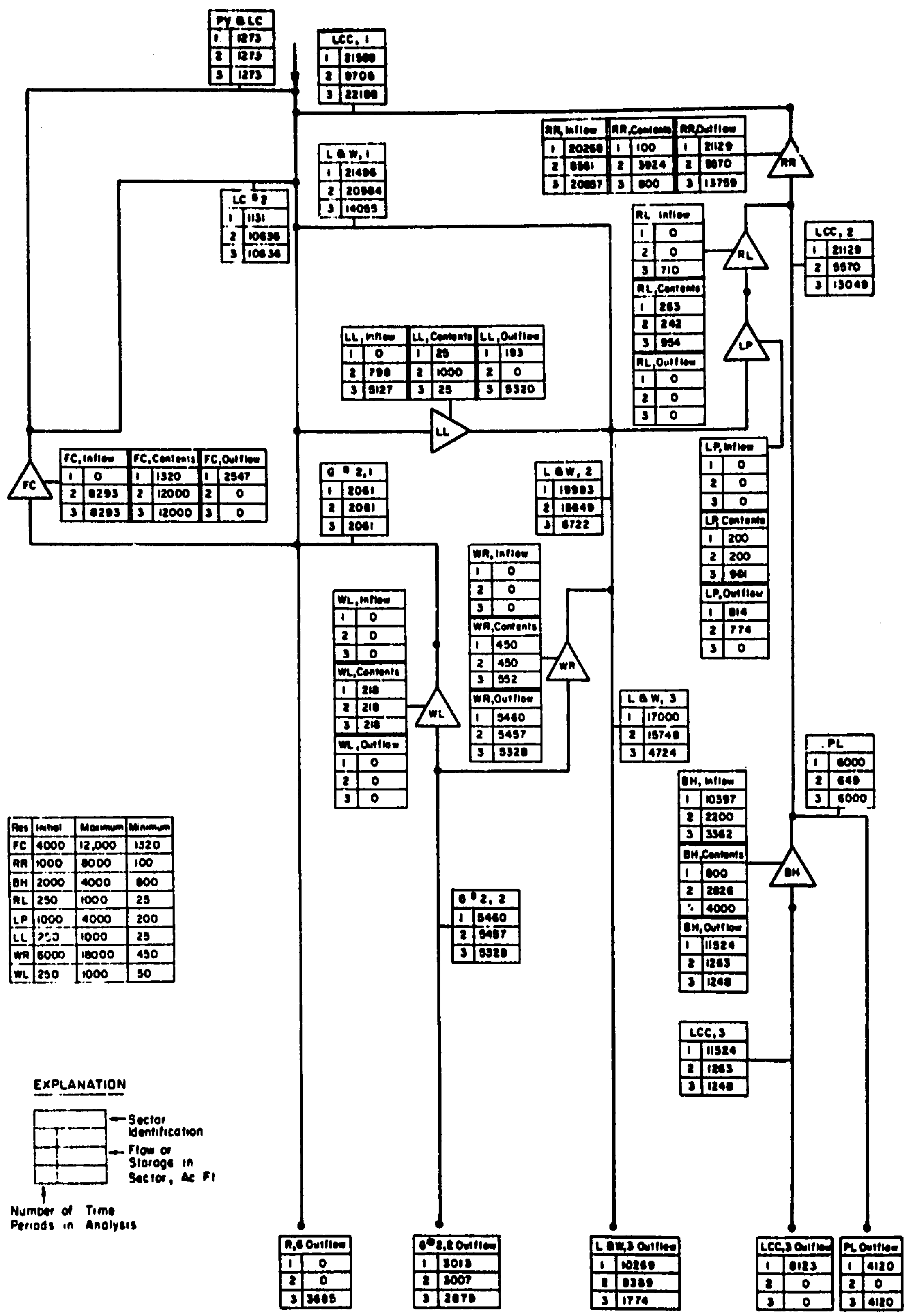
Results from Example Model I, for Nonoptimal Water Delivery Strategies in Time Period 1

Element	Index	Number of Time Periods in Analysis 1	2	3
PV&LC, Inflow	1	1273	1273	1273
LC#2, Inflow	2	1131	10636	10636
LCC,1, Inflow	3	21589	9706	22188
LCC,2, Inflow	4	21129	5570	13049
LCC,3, Inflow	5	11524	1263	1248
LCC,3, Outflow	5	8123	0	0
PL, Inflow	6	6000	649	6000
PL, Outflow	6	4120	0	4120
L&W,1, Inflow	7	21496	20984	14055
L&W,2, Inflow	8	19993	18649	6722
L&W,3, Inflow	9	17000	15748	4724
L&W,3, Outflow	9	10269	9389	1774
G#2,1, Inflow	10	2061	2061	2061
G#2,2, Inflow	11	5460	3007	5328
G#2,2, Outflow	11	3013	5457	2879
R,6, Outflow	17	0	0	3685
FC, Inflow	18	0	8293	8293
FC, Contents	18	1320	12000	12000
FC, Outflow	18	2547	0	0
RR, Inflow	19	20268	8561	20857
RR, Contents	19	100	3924	8000
RR, Outflow	19	21129	5570	13759
BH, Inflow	20	10397	2200	3362
BH, Contents	20	800	2826	4000
BH, Outflow	20	11524	1263	1248
RL, Inflow	21	0	0	710
RL, Contents	21	263	242	954
RL, Outflow	21	0	0	0
LP, Inflow	22	0	0	0
LP, Contents	22	200	200	981
LP, Outflow	22	814	774	0
LL, Inflow	23	0	798	5127
LL, Contents	23	25	1000	25
LL, Outflow	23	193	0	5320
WR, Inflow	24	0	0	0
WR, Contents	24	450	450	552
WR, Outflow	24	5460	5457	5328
WL, Inflow	25	0	0	0
WL, Contents	25	218	218	218
WL, Outflow	25	0	0	0

Table XX

Results from Example Model II, for Nonoptimal Water Delivery Strategies in Time Period 1

Element	Index	Number of Time Periods in Analysis 1	2	3
PV&LC, Inflow	1	1273	1273	1273
LC#2, Inflow	2	1131	10636	4960
LCC,1, Inflow	3	21628	9161	15965
LCC,2, Inflow	4	20228	8488	6936
LCC,3, Inflow	5	10740	1256	1263
LCC,3, Outflow	5	7503	0	0
PL, Inflow	6	6000	3239	649
PL, Outflow	6	4120	1994	0
L&W,1, Inflow	7	21488	21535	25805
L&W,2, Inflow	8	19993	20001	18289
L&W,3, Inflow	9	17000	17000	2162
L&W,3, Outflow	9	10268	10255	0
G#2,1, Inflow	10	2061	2061	2061
G#2,2, Inflow	11	5460	5460	1367
G#2,2, Outflow	11	3013	3013	0
R,6, Outflow	17	0	0	1649
FC, Inflow	18	0	8293	3340
FC, Contents	18	1320	12000	7121
FC, Outflow	18	2547	0	0
RR, Inflow	19	19367	7627	14035
RR, Contents	19	100	100	8000
RR, Outflow	19	20228	8488	6936
BH, Inflow	20	9613	2152	3390
BH, Contents	20	800	2791	4000
BH, Outflow	20	10740	1256	1263
RL, Inflow	21	0	0	0
RL, Contents	21	247	239	233
RL, Outflow	21	0	0	0
LP, Inflow	22	0	0	0
LP, Contents	22	200	200	966
LP, Outflow	22	812	781	0
LL, Inflow	23	0	0	5142
LL, Contents	23	25	215	25
LL, Outflow	23	103	0	5335
WR, Inflow	24	0	0	13254
WR, Contents	24	450	450	17569
WR, Outflow	24	5460	5460	1367
WL, Inflow	25	0	0	0
WL, Contents	25	218	218	220
WL, Outflow	25	0	0	0



Res	Initial	Maximum	Minimum
FC	4000	12,000	1320
RR	1000	8000	100
BH	2000	4000	800
RL	250	1000	25
LP	1000	4000	200
LL	250	1000	25
WR	6000	18000	450
WL	250	1000	50

Figure 18. Nonoptimal Water Delivery Strategies, Model I

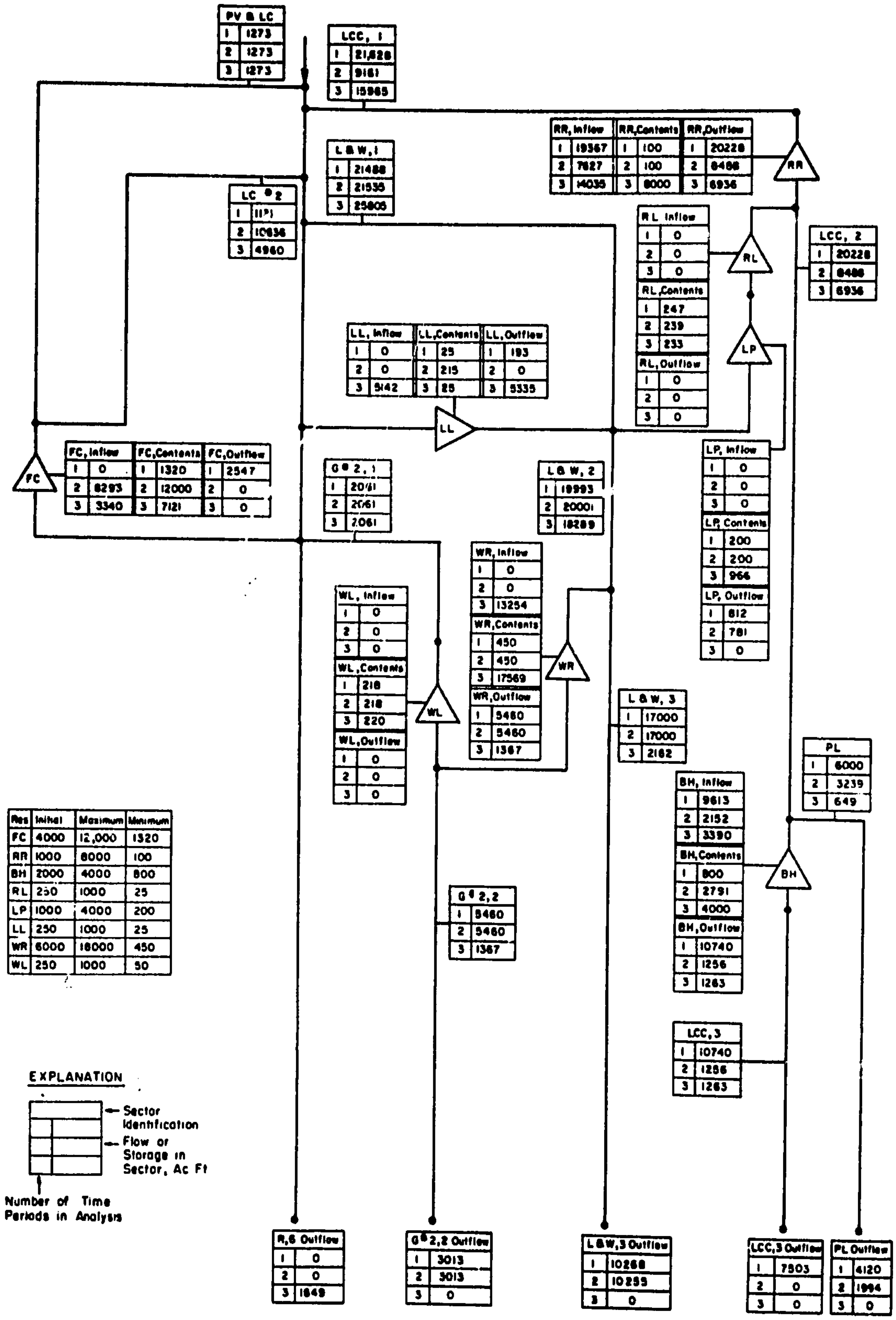


Figure 19. Nonoptimal Water Delivery Strategies, Model II

for finding optimal water delivery strategies during an irrigation season. This use is illustrated in the example Models I and II.

The procedure consists of solving for the optimal delivery strategy for each time period of the season, considering the distributions of the inflows and the demands in future time periods. An accou ting procedure is required for all time periods following the first so that return flows are properly considered and the problem size is reduced as the remaining irrigation season becomes shorter. For the linearized model, this consists of programming the following matrix equation of the form of equation (5-30):

$$\begin{bmatrix} A_{t,t} & 0 & \cdots & 0 \\ A_{t+1,t} & A_{t+1,t+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{T,t} & A_{T,t+1} & \cdots & A_{T,T} \end{bmatrix} \begin{bmatrix} \bar{x}_t \\ \bar{x}_{t+1} \\ \vdots \\ \bar{x}_T \end{bmatrix} = \begin{bmatrix} D_{t,1} & \cdots & D_{t,t} & 0 & \cdots & 0 \\ D_{t+1,1} & \cdots & D_{t+1,t} & D_{t+1,t+1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ D_{T,1} & \cdots & D_{T,t} & D_{T,t+1} & \cdots & D_{T,T} \end{bmatrix} \begin{bmatrix} \bar{b}_1 \\ \vdots \\ \bar{b}_t \\ \bar{b}_{t+1} \\ \vdots \\ \bar{b}_T \end{bmatrix} \begin{bmatrix} A_{t,1} & A_{t,2} & \cdots & A_{t,t-1} \\ A_{t+1,1} & A_{t+1,2} & \cdots & A_{t+1,t-1} \\ \vdots & \vdots & \ddots & \vdots \\ A_{T,1} & A_{T,2} & \cdots & A_{T,t-1} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_T \end{bmatrix} \tag{6-17}$$

for the derivation of a water delivery strategy in the t^{th} time period.

There is no restriction that the time periods used in the model must be of equal length, but care must be exercised in expressing loss and return flow functions if the irrigation season is divided into time periods of unequal length. If the ditch sectors have high capacities, however, the assumption that channel storage is negligible may not be valid when very short time periods are assumed.

Chapter VII

SUMMARY AND CONCLUSIONS

Persons using surface irrigation systems are major consumers of water in the United States, and the demand for irrigation water is continually increasing. To meet present and projected demands, new sources of supply must be sought. An alternative to the construction of new facilities such as reservoirs, for tapping undeveloped supplies, is the reclamation of water wasted through evaporation, transpiration and percolation from the structures that compose a surface irrigation water delivery system.

There are two methods that can be used to reclaim wasted water: improvement of the structures that compose a system, and improvement of the irrigation water delivery management. The improvement of the water delivery management is particularly attractive if a system is composed of several reservoirs and interconnecting ditches that allow alternate routes for the delivery of water to satisfy some of the demands on a system.

In this study, a nonlinear model has been developed to simulate the events that occur in an irrigation system. These events are inflows, outflows, losses, return flows, demands and storage. It was shown that any interconnecting irrigation system can be described by three elements: reaches of ditches, called ditch sectors; junctions of ditches, called nodes; and reservoirs. The events ditch sectors were assumed to experience were inflows, outflows, losses, return flows and demands. The events reservoirs were assumed to experience were inflows, outflows, losses, return flows, demands and storage.

The events nodes were assumed to experience were inflows and outflows only. Using these elements to describe a system allows any system to be modeled to any desired degree of refinement.

After the derivation of the nonlinear simulation model was completed, the model was linked to nonlinear programming so that optimal results could be obtained. The objective chosen for the optimization was a resource conservation objective, to minimize system loss and unrequired system outflow.

The mathematical functions that describe the losses and return flows of a system in a nonlinear model are both highly particular to the system and difficult to obtain. Therefore, linear approximations of the loss and return flow functions were derived and the substitution of these approximations into the equations of the nonlinear simulation model yielded a linearized simulation model. This linearized simulation model was then linked to linear programming so that optimal results could be obtained. The objective of minimizing the system loss and unrequired system outflow, expressed for the nonlinear model, was also used in the linear programming model. The ability to use linear programming is advantageous because linear programming routines for computers are more readily available than nonlinear programming computer routines.

Using the linearized simulation model, two example models were constructed for a representative system and used to obtain optimal water delivery strategies under various conditions. The available data were not adequate to define the model parameters, so a majority of the parameters were estimated.

The results obtained from the linear models indicate an optimal water delivery strategy, for a particular time period, is highly dependent on the number of future time periods included in the analysis and that the modification of even a single structure in a system such as lining a ditch to reduce seepage, can markedly affect optimal strategies. These results, concerning the influences of future time periods and system modifications, are reasonable and show the model performs its purpose well. Because of the lack of adequate data, no comparisons of the optimal strategies determined by the model and the strategies used in practice could be made.

The model developed in the study, in either its nonlinear form or its linear form, is designed to be calibrated for a particular system before the model is used. Therefore, it is imperative that data are available to calibrate the model. If data are not available, the model cannot be used with any degree of reliability. If a data collection program is anticipated, the model itself indicates those data that are important. They are the inflows to and outflows from the various ditch sectors and reservoirs composing the system. The exact procedure for obtaining the loss and return flow functions is not clear at this time, but some suggestions are presented in Appendix C.

The simulation model, developed in this study, has a number of very practical applications. It can be used as a tool to analyze a system for a great number of problems, such as the best locations for system improvement. However, the most important use of the simulation model is in conjunction with mathematical optimization techniques to minimize losses in delivering water to system users. Any interval of time may be used in the model, but time periods of at least one week are suggested.

By using forecasts of system inflows and demands throughout an irrigation season, the model provides a tool for increasing the efficiency of a water delivery system through an entire season, under the specification that certain demands must be met. The model includes the effects of return flows, and recognizes the importance of these flows as water in temporary storage. The efficiency obtained in any solution to the model, using forecasts of events to come, is directly related to the accuracy of the forecasts and continual updating is necessary to obtain a maximum efficiency. The structure of the model under conditions where updating is required is shown in the study.

As developed in the study, the model simulates one facet of irrigation water management, that of minimum loss water delivery. There are several improvements, or extensions, that are immediately evident, such as the inclusion of legal constraints. Some of these improvements are briefly discussed in Appendix C. Each improvement or extension will increase the detail to which a system can be modeled, with a resulting increase in the complexity of the model. But the increased detail and complexity do not necessarily imply increased reliability of the results. Therefore, any model results should not be taken as absolute, but should be tempered with engineering judgement. The model is simply a tool for the irrigation system manager.

In the context of managing a total water resource for irrigation, the model developed here is a first but necessary step. With further developments toward including the institutional arrangements that affect water management, such as the legal criteria, the end result will be a technique for the integrated management and operation of irrigation systems for water resource conservation.

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APPENDIX A

RELATION OF DERIVED MODEL TO NETWORK FLOW MODELS

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In this appendix it is shown that the model derived in the study is equivalent to network models (Hadley, 1962, Ch. 10), and that the derived model is the more desirable. The derived model accounts for all sources and sinks of water in all elements, the ditch sectors, the reservoirs, and the nodes. Network flow models account for all sources and sinks only at the nodes, with minimum capacity constraints explicitly stated to ensure demands are delivered down the proper legs of the network.

Consider the following network of ditch sectors:

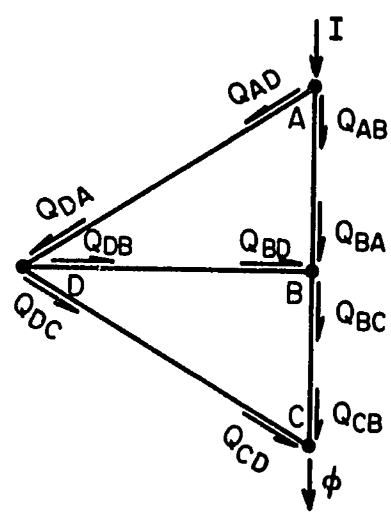

Figure A-1. Network of Ditch Sectors

in which the ditch sectors, AB, AD, DB, BC and DC, each experience losses, L_{AB}, L_{AD}, L_{DB}, L_{BC} and L_{DC}; return flows, R_{AB}, R_{AD}, R_{DB} R_{BC}, and R_{DC}; and demands, D_{AB}, D_{AD}, D_{DB}, D_{BC}, and D_{DC}, respectively. The symbols I and ϕ denote the system inflow and outflow.

Applying the continuity equation to each ditch sector and each node, the following definitions are obtained - for the ditch sectors,

Along AB, $Q_{AB} - L_{AB} + R_{AB} - D_{AB} = Q_{BA}$ (A-1)

Along AD, $Q_{AD} - L_{AD} + R_{AD} - D_{AD} = Q_{DA}$ (A-2)

Along DB, $Q_{DB} - L_{DB} + R_{DB} - D_{DB} = Q_{BD}$ (A-3)

Along BC, $Q_{BC} - L_{BC} + R_{BC} - D_{BC} = Q_{CB}$ (A-4)

Along DC, $Q_{DC} - L_{DC} + R_{DC} - D_{DC} = Q_{CD}$ (A-5)

and for the nodes

At A, $I - Q_{AD} - Q_{AB} = 0$ (A-6)

At B, $Q_{BA} + Q_{BD} - Q_{BC} = 0$ (A-7)

At C, $Q_{CB} + Q_{CD} - \phi = 0$ (A-8)

At D, $Q_{DA} - Q_{DB} - Q_{DC} = 0$ (A-9)

where

Q_{AB} is the outflow from node A directed toward node B

Q_{AD} is the outflow from node A directed toward node D

Q_{DB} is the outflow from node D directed toward node B

Q_{BC} is the outflow from node B directed toward node C

Q_{DC} is the outflow from node D directed toward node C

Q_{BA} is the inflow to node B from node A

Q_{DA} is the inflow to node D from node A

Q_{BD} is the inflow to node B from node D

Q_{CB} is the inflow to node C from node B

and Q_{CD} is the inflow to node C from node D.

To account for all sources and sinks of water at the nodes, as in a network model, the ditch sector equations, (A-1) through (A-5), are substituted into the node equations, (A-6) through (A-9). The best decision variables for the simulation of an irrigation water delivery system are those that represent the headgate settings on the various ditch sectors, Q_{AB}, Q_{AD}, Q_{DB}, Q_{BC} and Q_{DC}. Therefore, in substituting the ditch sector equations into the node equations, the variables Q_{BA}, Q_{DA}, Q_{BD}, Q_{CB} and Q_{CD} are eliminated. These substitutions yield

At A, $$I - Q_{AD} - Q_{AB} = 0 \tag{A-10}$$

At B, $$Q_{AB} - L_{AB} + R_{AB} - D_{AB} + Q_{DB} - L_{DB} + R_{DB} - D_{DB} - Q_{BC} = 0 \tag{A-11}$$

At C, $$Q_{BC} - L_{BC} + R_{BC} - D_{BC} + Q_{DC} - L_{DC} + R_{DC} - D_{DC} - \phi = 0 \tag{A-12}$$

At D, $$Q_{AD} - L_{AD} + R_{AD} - D_{AD} - Q_{DB} - Q_{DC} = 0 \tag{A-13}$$

and rearranging yields

At A, $$Q_{AD} + Q_{AB} = I \tag{A-14}$$

At B, $$Q_{AB} - L_{AB} + R_{AB} + Q_{DB} - L_{DB} + R_{DB} - Q_{BC} = D_{AB} + D_{DB} \tag{A-15}$$

At C, $$Q_{BC} - L_{BC} + R_{BC} + Q_{DC} - L_{DC} + R_{DC} - \phi = D_{BC} + D_{DC} \tag{A-16}$$

At D, $$Q_{AD} - L_{AD} + R_{AD} - Q_{DB} - Q_{DC} = D_{AD} \; . \tag{A-17}$$

These equations, (A-14) through (A-17), preserve continuity for the water delivered by the system, but in the substitution process the requirement that demand volumes must be delivered down the proper ditch sector has been lost. In essence, this model places the demands at the nodes.

To satisfy the requirement that the demands must be delivered down the proper ditch sector, explicit minimum capacity restrictions must be stated:

Along AB, $$Q_{AB} - L_{AB} + R_{AB} \geq D_{AB} \tag{A-18}$$

Along AD, $$Q_{AD} - L_{AD} + R_{AD} \geq D_{AD} \tag{A-19}$$

Along DB, $Q_{DB} - L_{DB} + R_{DB} \geq D_{DB}$ (A-20)

Along BD, $Q_{BC} - L_{BC} + R_{BC} \geq D_{BC}$ (A-21)

Along DC, $Q_{DC} - L_{DC} + R_{DC} \geq D_{DC}$. (A-22)

Equations (A-14) through (A-22) are a proper formulation of the problem as a network model. Equations (A-14) through (A-19) account for the water flowing through the system, so that continuity is maintained, and equations (A-18) through (A-22) require that demands be delivered down the proper ditch sectors. This model consists of nine equations and eleven decision variables, after surplus variables are added to convert the minimum capacity restrictions to equalities.

Adding the surplus variables, V , to equations (A-18) through (A-22), the following equations are obtained,

Along AB, $Q_{AB} - L_{AB} + R_{AB} - V_{AB} = D_{AB}$ (A-23)

Along AD, $Q_{AD} - L_{AD} + R_{AD} - V_{AD} = D_{AD}$ (A-24)

Along DB, $Q_{DB} - L_{DB} + R_{DB} - V_{DB} = D_{DB}$ (A-25)

Along BC, $Q_{BC} - L_{BC} + R_{BC} - V_{BC} = D_{BC}$ (A-26)

Along DC, $Q_{DC} - L_{DC} + R_{DC} - V_{DC} = D_{DC}$. (A-27)

The substitution of these equations into equations (A-14) through (A-17) yields

At A, $Q_{AD} + Q_{AB} = I$ (A-28)

At B, $V_{AB} + V_{DB} - Q_{BC} = 0$ (A-29)

At C, $V_{BC} + V_{DC} - \phi = 0$ (A-30)

At D, $V_{AD} - Q_{DB} - Q_{DC} = 0$. (A-31)

The equations, (A-23) through (A-31), contain both the requirement that the demands must be delivered down the proper ditch sector and account for all water in the system. They result in exactly the model derived in the study. Further, recognition that

$$V_{AB} = Q_{BA} \quad \text{(A-32)}$$

$$V_{AD} = Q_{DA} \quad \text{(A-33)}$$

$$V_{DB} = Q_{BD} \quad \text{(A-34)}$$

$$V_{BC} = Q_{CB} \quad \text{(A-35)}$$

$$V_{DC} = Q_{CD} \quad \text{(A-36)}$$

reveals equations (A-23) through (A-31) are exactly the same as the definitions, equations (A-1) through (A-9), on which the network flow model, equations (A-14) through (A-22), are based.

Further examination of equations (A-23) through (A-31) reveals the model to be composed of nine equations and eleven decision variables, exactly the same problem size as the network flow model. Thus, no reduction in problem size is gained by using the network flow model.

There are additional advantages of using the derived model rather than a network model: (1) the requirement that a demand must be carried down the proper ditch sector is implicit and cannot be inadvertently neglected; (2) two flow values in each ditch sector are available for use in the expression of loss and return flow functions, the volume of water entering the sector and the volume of water leaving the sector; (3) the relations in the model are equivalent to the definitions on which a standard network formulation is based, and the algebra necessary to obtain a network formulation is eliminated as one possible source of error in problem formulation; and, (4) all ditch sectors are modeled with equations of the same form, allowing for easier problem formulation.

APPENDIX B

EXAMPLE LINEARIZED MODELS FOR A SIMPLE IRRIGATION SYSTEM

APPENDIX B

EXAMPLE LINEARIZED MODELS FOR A SIMPLE IRRIGATION SYSTEM

Applying the linearized model to a simple system illustrates and clarifies the developments in Chapter V because the results of an application can be seen without the aid of the model. Such an application is illustrated in the following text.

A simple irrigation system is illustrated in Figure B-1.

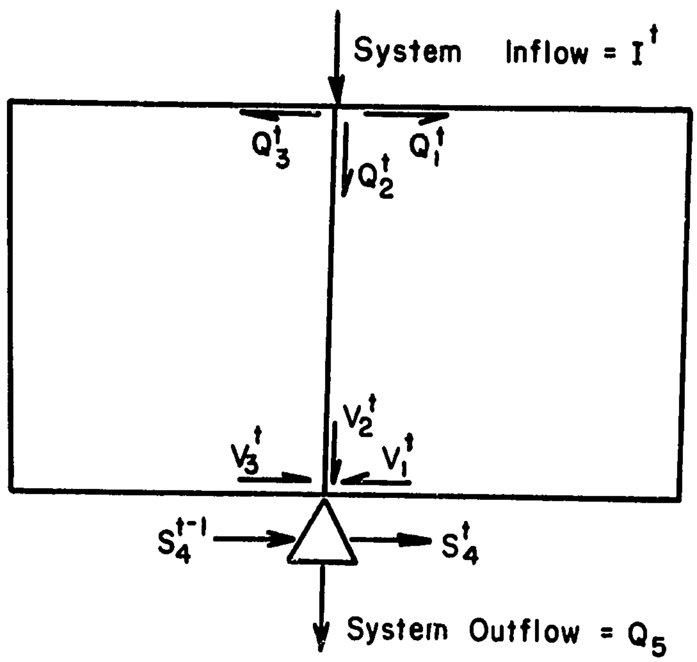

Figure B-1 Simple System

Four linear models are developed for this system: a single time period model with return flows excluded, a single time period model with return flows included, a two time period model with return flows excluded, and a two time period model with return flows included. General notation is used for all constants in the problem. The A and D matrices in their partitioned forms and the $\bar{x}$ and $\bar{b}$ vectors also in their partitioned form are presented.

The first step in simulating the simple system is to describe it by its elements. From Figure B-1, we can see there are three ditch sectors, one reservoir, and one node. Thus, $M = 3$, $N = 1$ and $P = 1$. To simulate the system, three ditch sector mass balances, one reservoir mass balance, and one nodal mass balance must be written for each time period. In all four linear models the node definition is used for writing the reservoir mass balance, because there are three sources of inflow, and the minimum capacity restrictions of all elements are assumed to be the volume of water necessary to deliver the demand, thereby eliminating the need to explicitly state them.

Single Time Period with Return Flows Excluded

From equation (5-27) the ditch sector mass balances are:

$$(1 - a_1^1\gamma_{11}^{11})\, Q_1^1 - (1 + b_1^1\gamma_{11}^{11})V_1^1 = D_1^1 \tag{B-1}$$

$$(1 - a_2^1\gamma_{22}^{11})Q_2^1 - (1 + b_2^1\gamma_{22}^{11})V_2^1 = D_2^1 \tag{B-2}$$

$$(1 - a_3^1\gamma_{33}^{11})Q_3^1 - (1 + b_3^1\gamma_{33}^{11})V_3^1 = D_3^1 \quad . \tag{B-3}$$

The superscript is both t and T because only one time period is examined.

The nodal mass balance is written according to equation (4-4a) because it is a point of system inflow. The set K_1 is $K_1 = \{1,2,3\}$, because all ditch sectors receive water from the node. The nodal mass balance is written

$$Q_1^1 + Q_2^1 + Q_3^1 = I_1^1 \tag{B-4}$$

in which I_1^1 is the system inflow to the node.

For the reservoir the set J_4 is $J_4 = \{1,2,3\}$ because all ditch sectors empty into the reservoir. The set K_4 is $K_4 = \{5\}$ because of the outflow from the system, Q_5^1. Using equation (5-28), the reservoir mass balance can be written

$$V_1^1 + V_2^1 + V_3^1 - (1 + b_4^1\gamma_{44}^{11})S_4^1 - Q_5^1 = D_4^1 + \beta_4^1$$

$$- (1 - a_4^1\gamma_{44}^{11})S_4^0 \qquad \text{(B-5)}$$

in which Q_5^1 is the unrequired system outflow, and S_4^0 is the initial reservoir storage, presumed known.

The equations used to define the capacity restrictions are the modified equations (6-1) and (6-2), defined in Chapter VI. Denoting the maximum capacities of the three ditch sectors and the reservoir by $Q_{1_{max}}^1$, $Q_{2_{max}}^1$, $Q_{3_{max}}^1$, and $S_{4_{max}}^1$, respectively, the maximum capacity restrictions are,

$$Q_1^1 \leq Q_{1_{max}}^1 \qquad \text{(B-6)}$$

$$Q_2^1 \leq Q_{2_{max}}^1 \qquad \text{(B-7)}$$

$$Q_3^1 \leq Q_{3_{max}}^1 \qquad \text{(B-8)}$$

$$S_4^1 \leq S_{4_{max}}^1 \quad . \qquad \text{(B-9)}$$

The unrequired system outflow, Q_5^1, is assumed to be unrestricted.

Converting the maximum capacity inequalities into equalities requires the addition of a slack variable, X_i^t, to each inequality. Rewriting, we obtain

$$Q_1^1 + X_1^1 = Q_{1_{max}}^1 \qquad \text{(B-10)}$$

$$Q_2^1 + X_2^1 = Q_{2_{max}}^1 \tag{B-11}$$

$$Q_3^1 + X_3^1 = Q_{3_{max}}^1 \tag{B-12}$$

$$S_4^1 + X_4^1 = S_{4_{max}}^1 \quad . \tag{B-13}$$

Equations (B-1), (B-2), (B-3), (B-4), (B-5), (B-10), (B-11), (B-12), and (B-13) can be written as a matrix equation of the form of equation (2-5):

$$A\bar{x} = \bar{b} \quad . \tag{2-5}$$

The elements of the A matrix and the $\bar{x}$ and $\bar{b}$ vectors are listed in Exhibits I and II. Since there are twelve decision variables and only nine rows in the A matrix, there exists more than one solution to the problem. In fact, there are 220 possible optimal solutions; the best of these must be sělected according to the objective function.

The same objective function is used here as was used for the example models in the study, Chapter VI. Mathematically the objective function is

$$\text{MIN } Z = a_1^1\gamma_{11}^{11}Q_1^1 + a_2^1\gamma_{22}^{11}Q_2^1 + a_3^1\gamma_{33}^{11}Q_3^1 + b_4^1\gamma_{44}^{11}S_4^1 + b_1^1\gamma_{11}^{11}V_1^1$$
$$+ b_2^1\gamma_{22}^{11}V_2^1 + b_3^1\gamma_{33}^{11}V_3^1 + Q_5^1 \tag{B-14}$$

in which the sets J_4 and H_4 for the unrequired system outflows, are empty and $H_4 = \{5\}$, respectively. This completes the linearized model construction for the simplest model, the model for a single time period with return flows excluded.

$$
\begin{bmatrix}
(1-a_1^1\gamma_{11}^{11}) & 0 & 0 & 0 & 0 & -(1+b_1^1\gamma_{11}^{11}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (1-a_2^1\gamma_{22}^{11}) & 0 & 0 & 0 & 0 & -(1+b_2^1\gamma_{22}^{11}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1-a_3^1\gamma_{33}^{11}) & 0 & 0 & 0 & 0 & -(1+b_3^1\gamma_{33}^{11}) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1+b_4^1\gamma_{44}^{11}) & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT I: A Matrix, Single Time Period Example, Excluding Return Flows.

$$\bar{x} = \begin{bmatrix} Q_1^1 \\ Q_2^1 \\ Q_3^1 \\ S_4^1 \\ Q_5^1 \\ V_1^1 \\ V_2^1 \\ V_3^1 \\ X_1^1 \\ X_2^1 \\ X_3^1 \\ X_4^1 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} D_1^1 \\ D_2^1 \\ D_3^1 \\ D_4^1 - (1-a_4^1\gamma_{44}^{11})S_4^o + \beta_4^1 \\ I_1^1 \\ Q_{1_{max}}^1 \\ Q_{2_{max}}^1 \\ Q_{3_{max}}^1 \\ S_{4_{max}}^1 \end{bmatrix}$$

EXHIBIT II: $\bar{x}$ and $\bar{b}$ Vectors, Single Time Period Example, Excluding Return Flows.

Single Time Period with Return Flows Included

The model for a single time period that includes return flows is similar to the previous model. The nodal mass balances and the capacity constraints remain the same, but the ditch sector and reservoir mass balances change; the changes include more of the terms in equations (5-27) and (5-28).

For the ditch sectors the mass balances become:

$$(1-a_1^1\gamma_{11}^{11})Q_1^1 + a_2^1\gamma_{12}^{11}Q_2^1 + a_3^1\gamma_{13}^{11}Q_3^1 + b_4^1\gamma_{14}^{11}S_4^1 - (1+b_1^1\gamma_{11}^{11})V_1^1 + b_2^1\gamma_{12}^{11}V_2^1$$

$$+ b_3^1\gamma_{13}^{11}V_3^1 = (1-\alpha_{11}^{11})D_1^1 - \alpha_{12}^{11}D_2^1 - \alpha_{13}^{11}D_3^1 - c_{14}^{11}\beta_4^1 - a_4^1\gamma_{14}^{11}S_4^0 \quad , \qquad \text{(B-15)}$$

for ditch sector 1;

$$a_1^1\gamma_{21}^{11}Q_1^1 + (1-a_2^1\gamma_{22}^{11})Q_2^1 + a_3^1\gamma_{23}^{11}Q_3^1 + b_4^1\gamma_{24}^{11}S_4^1 + b_1^1\gamma_{21}^{11}V_1^1 - (1+b_2^1\gamma_{22}^{11})V_2^1$$

$$+b_3^1\gamma_{23}^{11}V_3^1 = - \alpha_{21}^{11}D_1^1 + (1-\alpha_{22}^{11})D_2^1 - \alpha_{23}^{11}D_3^1 - c_{24}^{11}\beta_4^1 - a_4^1\gamma_{24}^{11}S_4^0 \qquad \text{(B-16)}$$

for ditch sector 2; and

$$a_1^1\gamma_{31}^{11}Q_1^1 + a_2^1\gamma_{32}^{11}Q_2^1 + (1-a_3^1\gamma_{33}^{11})Q_3^1 + b_4^1\gamma_{34}^{11}Q_4^1 + b_1^1\gamma_{31}^{11}V_1^1 + b_2^1\gamma_{32}^{11}V_2^1$$

$$- (1+b_3^1\gamma_{33}^{11})V_3^1 = - \alpha_{31}^{11}D_1^1 - \alpha_{32}^{11}D_2^1 + (1-\alpha_{33}^{11})D_3^1 - c_{34}^{11}\beta_4^1 - a_4^1\gamma_{34}^{11}S_4^0 \qquad \text{(B-17)}$$

for ditch sector 3. The reservoir mass balance equation becomes

$$a_1^1\gamma_{41}^{11}Q_1^1 + a_2^1\gamma_{42}^{11}Q_2^1 + a_3^1\gamma_{43}^{11}Q_3^1 - (1+b_4^1\gamma_{44}^{11})S_4^1 - Q_5^1 + (1+b_1^1\gamma_{41}^{11})V_1^1$$

$$+ (1+b_2^1\gamma_{42}^{11})V_2^1 + (1 + b_3^1\gamma_{43}^{11})V_3^1 = - \alpha_{41}^{11}D_1^1 - \alpha_{42}^{11}D_2^1 - \alpha_{43}^{11}D_3^1 + (1-\alpha_{44}^{11})D_4^1$$

$$- (1 - a_4^1\gamma_{44}^{11})S_4^0 + \beta_4^1 \ . \qquad \text{(B-18)}$$

The matrix equation describing these constraints is defined by equation (2-6):

$$A\bar{x} = D\bar{b} \quad . \tag{2-6}$$

The elements of the A and D matrices and the $\bar{x}$ and $\bar{b}$ vectors are listed in Exhibits III, IV and V.

The objective function for minimizing system losses and unrequired system outflows conjunctively is

$$\begin{aligned} \text{MIN } Z = {} & (a_1^1\gamma_{11}^{11} - a_1^1\gamma_{21}^{11} - a_1^1\gamma_{31}^{11} - a_1^1\gamma_{41}^{11})Q_1^1 + (a_2^1\gamma_{22}^{11} - a_2^1\gamma_{12}^{11} - a_2^1\gamma_{32}^{11} \\ & - a_2^1\gamma_{42}^{11})Q_2^1 + (a_3^1\gamma_{33}^{11} - a_3^1\gamma_{13}^{11} - a_3^1\gamma_{23}^{11} - a_3^1\gamma_{43}^{11})Q_3^1 \\ & + (b_4^1\gamma_{44}^{11} - b_4^1\gamma_{14}^{11} - b_4^1\gamma_{24}^{11} - b_4^1\gamma_{34}^{11})S_4^1 + (b_1^1\gamma_{11}^{11} - b_1^1\gamma_{21}^{11} - b_1^1\gamma_{31}^{11} - b_1^1\gamma_{41}^{11})V_1^1 \\ & + (b_2^1\gamma_{22}^{11} - b_2^1\gamma_{12}^{11} - b_2^1\gamma_{32}^{11} - b_2^1\gamma_{42}^{11})V_2^1 + (b_3^1\gamma_{33}^{11} - b_3^1\gamma_{13}^{11} - b_3^1\gamma_{23}^{11} \\ & - b_3^1\gamma_{43}^{11})V_3^1 + Q_5^1 \quad . \end{aligned} \tag{B-19}$$

This completes the model construction for both the single time period models.

Two Time Periods with Return Flows Excluded

For the example of two time periods with return flows excluded, the constraint matrix equation (2-5) is partitioned into

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix} \tag{B-20}$$

$$
\begin{bmatrix}
(1-a_1^1\gamma_{11}^{11}) & a_2^1\gamma_{12}^{11} & a_3^1\gamma_{13}^{11} & b_4^1\gamma_{14}^{11} & 0 & -(1+b_1^1\gamma_{11}^{11}) & b_2^1\gamma_{12}^{11} & b_3^1\gamma_{13}^{11} & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{21}^{11} & (1-a_2^1\gamma_{22}^{11}) & a_3^1\gamma_{23}^{11} & b_4^1\gamma_{24}^{11} & 0 & b_1^1\gamma_{21}^{11} & -(1+b_2^1\gamma_{22}^{11}) & b_3^1\gamma_{23}^{11} & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{31}^{11} & a_2^1\gamma_{32}^{11} & (1-a_3^1\gamma_{33}^{11}) & b_4^1\gamma_{34}^{11} & 0 & b_1^1\gamma_{31}^{11} & b_2^1\gamma_{32}^{11} & -(1+b_3^1\gamma_{33}^{11}) & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{41}^{11} & a_2^1\gamma_{42}^{11} & a_3^1\gamma_{43}^{11} & -(1+b_4^1\gamma_{44}^{11}) & -1 & (1+b_1^1\gamma_{41}^{11}) & (1+b_2^1\gamma_{42}^{11}) & (1+b_3^1\gamma_{43}^{11}) & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT III: A Matrix, Single Time Period Example, Including Return Flows.

$$
\begin{bmatrix}
(1-\alpha^{11}_{11}) & -\alpha^{11}_{12} & -\alpha^{11}_{13} & -\alpha^{11}_{14} & -c^{11}_{14} & -a^{1}_{4}\gamma^{11}_{14} & 0 & 0 & 0 & 0 & 0 \\
-\alpha^{11}_{21} & (1-\alpha^{11}_{22}) & -\alpha^{11}_{23} & -\alpha^{11}_{24} & -c^{11}_{24} & -a^{1}_{4}\gamma^{11}_{24} & 0 & 0 & 0 & 0 & 0 \\
-\alpha^{11}_{31} & -\alpha^{11}_{32} & (1-\alpha^{11}_{33}) & -\alpha^{11}_{34} & -c^{11}_{34} & -a^{1}_{4}\gamma^{11}_{34} & 0 & 0 & 0 & 0 & 0 \\
-\alpha^{11}_{41} & -\alpha^{11}_{42} & -\alpha^{11}_{43} & (1-\alpha^{11}_{44}) & +1 & -(1-a^{1}_{4}\gamma^{11}_{44}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT IV: D Matrix, Single Time Period Example, Including Return Flows.

$$\bar{x} = \begin{bmatrix} Q_1^1 \\ Q_2^1 \\ Q_3^1 \\ S_4^1 \\ Q_5^1 \\ V_1^1 \\ V_2^1 \\ V_3^1 \\ X_1^1 \\ X_2^1 \\ X_3^1 \\ X_4^1 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} D_1^1 \\ D_2^1 \\ D_3^1 \\ D_4^1 \\ \beta_4^1 \\ S_4^0 \\ I_1^1 \\ Q_{1_{max}}^1 \\ Q_{2_{max}}^1 \\ Q_{3_{max}}^1 \\ S_{4_{max}}^1 \end{bmatrix}$$

EXHIBIT V: $\bar{x}$ and $\bar{b}$ Vectors, Single Time Period Example, Including Return Flows.

where A_{12} is null. This yields

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix} \qquad \text{(B-21)}$$

The equations comprising the first row of the partitioned matrix equation (B-21), $A_{11}\,\bar{x}_1 = \bar{b}_1$, are exactly the same as those derived for the single time period model with return flows neglected, so they will not be repeated here; they are, however, included in the listing of the matrix elements, Exhibits VI and VII.

Writing the mass balance equations and capacity restrictions for the second time period as a matrix equation yields the second row of equation (B-21), $A_{21}\bar{x}_1 + A_{22}\bar{x}_2 = \bar{b}_2$. The partitioning of this equation is based on the superscripts of the variables in the mass balance equations. Because return flows are not included, the A_{21} submatrix contains only one non-zero element. This results because neglecting return flows leaves only the volume of water in the reservoir at the end of time period 1 that can affect time period 2.

The ditch sector mass balances for the second time period of this problem are: for ditch sector 1,

$$(1-a_1^2\gamma_{11}^{22})Q_1^2 - (1 + b_1^2\gamma_{11}^{22})V_1^2 = D_1^2 \quad ; \qquad \text{(B-22)}$$

for ditch sector 2,

$$(1-a_2^2\gamma_{22}^{22})Q_2^2 - (1 + b_2^2\gamma_{22}^{22})V_2^2 = D_2^2 \quad ; \qquad \text{(B-23)}$$

and, for ditch sector 3,

$$(1-a_3^2\gamma_{33}^{22})Q_3^2 - (1 + b_3^2\gamma_{33}^{22})V_3^2 = D_3^2 \quad . \tag{B-24}$$

The reservoir mass balance is

$$(1-a_4^2\gamma_{44}^{22})S_4^1 - (1 + b_4^2\gamma_{44}^{22})S_4^2 - Q_5^2 + V_1^2 + V_2^2 + V_3^2 = D_4^2 + \beta_4^2 \quad , \tag{B-25}$$

and the nodal mass balance is

$$Q_1^2 + Q_2^2 + Q_3^2 = I_1^2 \quad . \tag{B-26}$$

With the addition of the slack variables, the capacity constraints are

$$Q_1^2 + X_1^2 = Q_{1_{max}}^2 \tag{B-27}$$

$$Q_2^2 + X_2^2 = Q_{2_{max}}^2 \tag{B-28}$$

$$Q_3^2 + X_3^2 = Q_{3_{max}}^2 \tag{B-29}$$

and

$$S_4^2 + X_4^2 = S_{4_{max}}^2 \quad . \tag{B-30}$$

These equations form the second row of the partitioned constraint matrix equation (B-21). The elements of the A_{11}, A_{21}, and A_{22} submatrices and the $\bar{x}_1$, $\bar{x}_2$, $\bar{b}_1$, and $\bar{b}_2$ subvectors are listed in Exhibits VI, VII, VIII, IX and X.

The objective function for minimizing system losses and unrequired system outflows for this case is

$$
\begin{bmatrix}
(1-a_1^1\gamma_{11}^{11}) & 0 & 0 & 0 & 0 & -(1+b_1^1\gamma_{11}^{11}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (1-a_2^1\gamma_{22}^{11}) & 0 & 0 & 0 & 0 & -(1+b_2^1\gamma_{22}^{11}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1-a_3^1\gamma_{33}^{11}) & 0 & 0 & 0 & 0 & -(1+b_3^1\gamma_{33}^{11}) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1+b_4^1\gamma_{44}^{11}) & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT VI: A_{11} Submatrix, Two Time Period Example, Excluding Return Flows.

$$\bar{x}_1 = \begin{bmatrix} Q_1^1 \\ Q_2^1 \\ Q_3^1 \\ S_4^1 \\ Q_5^1 \\ V_1^1 \\ V_2^1 \\ V_3^1 \\ X_1^1 \\ X_2^1 \\ X_3^1 \\ X_4^1 \end{bmatrix} \qquad \bar{b}_1 = \begin{bmatrix} D_1^1 \\ D_2^1 \\ D_3^1 \\ D_4^1 - (1-a_4^1\gamma_{44}^{11})S_4^0 + \beta_4^1 \\ I_1^1 \\ Q_{1_{max}}^1 \\ Q_{2_{max}}^1 \\ Q_{3_{max}}^1 \\ Q_{4_{max}}^1 \end{bmatrix}$$

EXHIBIT VII: $\bar{x}_1$ and $\bar{b}_1$ Subvectors, Two Time Period Example, Excluding Return Flows.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-a_4^2\gamma_{44}^{22}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

EXHIBIT VIII: A_{21} Submatrix, Two Time Period Example, Excluding Return Flows.

$$
\begin{bmatrix}
(1-a_1^2\gamma_{11}^{22}) & 0 & 0 & 0 & 0 & -(1+b_1^2\gamma_{11}^{22}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (1-a_2^2\gamma_{22}^{22}) & 0 & 0 & 0 & 0 & -(1+b_2^2\gamma_{22}^{22}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1-a_3^2\gamma_{33}^{22}) & 0 & 0 & 0 & 0 & -(1+b_3^2\gamma_{33}^{22}) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1+b_4^2\gamma_{44}^{22}) & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT IX: A_{22} Submatrix, Two Time Period Example, Excluding Return Flows.

$$\bar{x}_2 = \begin{bmatrix} Q_1^2 \\ Q_2^2 \\ Q_3^2 \\ S_4^2 \\ Q_5^2 \\ V_1^2 \\ V_2^2 \\ V_3^2 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix} \qquad \bar{b}_2 = \begin{bmatrix} D_1^2 \\ D_2^2 \\ D_3^2 \\ D_4^2 + \beta_4^2 \\ I_1^2 \\ Q^2_{1_{max}} \\ Q^2_{2_{max}} \\ Q^2_{3_{max}} \\ Q^2_{4_{max}} \end{bmatrix}$$

EXHIBIT X: $\bar{x}_2$ and $\bar{b}_2$ Subvectors, Two Time Period Example, Excluding Return Flows.

$$\text{MIN } Z = a_1^1\gamma_{11}^{11}Q_1^1 + a_2^1\gamma_{22}^{11}Q_2^1 + a_3^1\gamma_{33}^{11}Q_3^1 + (b_4^1\gamma_{44}^{11} + a_4^2\gamma_{44}^{22})S_4^1$$

$$+ b_1^1\gamma_{11}^{11}V_1^1 + b_2^1\gamma_{22}^{11}V_2^1 + b_3^1\gamma_{33}^{11}V_3^1 + a_1^2\gamma_{11}^{22}Q_1^2 + a_2^2\gamma_{22}^{22}Q_2^2$$

$$+ a_3^2\gamma_{33}^{22}Q_3^2 + b_4^2\gamma_{44}^{22}S_4^2 + b_1^2\gamma_{11}^{22}V_1^2 + b_2^2\gamma_{22}^{22}V_2^2 + b_3^2\gamma_{33}^{22}V_3^2$$

$$+ Q_5^1 + Q_5^2 \quad , \tag{B-31}$$

thus completing the construction of the two time period model with return flow is excluded.

Two Time Periods with Return Flows Included

The linearized model for two time periods with return flows included is the most complicated of the four models presented in this Appendix. The matrix equation from equation (5-30) defining the constraints is

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} D_{11} & 0 \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix} . \tag{B-32}$$

The A_{11} and D_{11} submatrices and the $\bar{x}_1$ and $\bar{b}_1$ subvectors are exactly the same as the A and D matrices and $\bar{x}$ and $\bar{b}$ vectors defined previously for the single time period model with return flows included, so the equations will not be repeated; however, the elements are listed in Exhibits XI, XII and XIII.

The A_{21}, A_{22}, D_{21} and D_{22} submatrices and the $\bar{x}_2$ and $\bar{b}_2$ subvectors, in the second row of the partitioned matrix equation (B-32),

$$A_{21}\,\overline{x}_1 + A_{22}\,\overline{x}_2 = D_{21}\,\overline{b}_1 + D_{22}\,\overline{b}_2 \;, \qquad \text{(B-33)}$$

are obtained by writing the ditch sector mass balances, the reservoir mass balance, the nodal mass balance and the maximum capacity constraints for the model.

The ditch sector mass balances are:

$$\begin{aligned}
& a_1^1\gamma_{11}^{21}Q_1^1 + a_2^1\gamma_{12}^{21}Q_2^1 + a_3^1\gamma_{13}^{21}Q_3^1 + (b_4^1\gamma_{14}^{21} + a_4^2\gamma_{14}^{22})S_4^1 + b_1^1\gamma_{11}^{21}V_1^1 \\
&\quad + b_2^1\gamma_{12}^{21}V_2^1 + b_3^1\gamma_{13}^{21}V_3^1 + (1 - a_1^2\gamma_{11}^{22})Q_1^2 + a_2^2\gamma_{12}^{22}Q_2^2 + a_3^2\gamma_{13}^{22}Q_3^2 \\
&\quad + b_4^2\gamma_{14}^{22}S_4^2 - (1 + b_1^2\gamma_{11}^{22})V_1^2 + b_2^2\gamma_{12}^{22}V_2^2 + b_3^2\gamma_{13}^{22}V_3^2 = -\alpha_{11}^{21}D_1^1 - \alpha_{12}^{21}D_2^1 \\
&\quad - \alpha_{13}^{21}D_3^1 - \alpha_{14}^{21}D_4^1 - c_{14}^{21}\beta_4^1 - a_4^1\gamma_{14}^{21}S_4^0 + (1 - \alpha_{11}^{22})D_1^2 - \alpha_{12}^{22}D_2^2 \\
&\quad - \alpha_{13}^{22}D_3^2 - \alpha_{14}^{22}D_4^2 - c_{14}^{22}\beta_4^2 \qquad \text{(B-34)}
\end{aligned}$$

for ditch sector 1;

$$\begin{aligned}
& a_1^1\gamma_{21}^{21}Q_1^1 + a_2^1\gamma_{22}^{21}Q_2^1 + a_3^1\gamma_{23}^{21}Q_3^1 + (b_4^1\gamma_{24}^{21} + a_4^2\gamma_{24}^{22})S_4^1 + b_1^1\gamma_{21}^{21}V_1^1 \\
&\quad + b_2^1\gamma_{22}^{21}V_2^1 + b_3^1\gamma_{23}^{21}V_3^1 + a_1^2\gamma_{21}^{22}Q_1^2 + (1 - a_2^2\gamma_{22}^{22})Q_2^2 + a_3^2\gamma_{23}^{22}Q_3^2 \\
&\quad + b_4^2\gamma_{24}^{22}S_4^2 + b_1^2\gamma_{21}^{22}V_1^2 - (1 + b_2^1\gamma_{22}^{22})V_2^2 + b_3^2\gamma_{23}^{22}V_3^2 = -\alpha_{21}^{21}D_1^1 \\
&\quad - \alpha_{22}^{21}D_2^1 - \alpha_{23}^{21}D_3^1 - \alpha_{24}^{21}D_4^1 - c_{24}^{21}\beta_4^1 - a_4^1\gamma_{24}^{21}S_4^0 - \alpha_{21}^{22}D_1^2 \\
&\quad + (1 - \alpha_{22}^{22})D_2^2 - \alpha_{23}^{22}D_3^2 - \alpha_{24}^{22}D_4^2 - c_{24}^{22}\beta_4^2 \qquad \text{(B-35)}
\end{aligned}$$

for ditch sector 2; and,

$$
\begin{aligned}
& a_1^1\gamma_{31}^{21}Q_1^1 + a_2^1\gamma_{32}^{21}Q_2^1 + a_3^1\gamma_{33}^{21}Q_3^1 + (b_4^1\gamma_{34}^{21} + a_4^2\gamma_{34}^{22})S_4^1 + b_1^1\gamma_{31}^{21}V_1^1 + b_2^1\gamma_{32}^{21}V_2^1 \\
&\quad + b_3^1\gamma_{33}^{21}V_3^1 + a_1^2\gamma_{31}^{22}Q_1^2 + a_2^2\gamma_{32}^{22}Q_2^2 + (1 - a_3^2\gamma_{33}^{22})Q_3^2 + b_4^2\gamma_{34}^{22}S_4^2 \\
&\quad + b_1^2\gamma_{31}^{22}V_1^2 + b_2^2\gamma_{32}^{22}V_2^2 - (1 + b_3^2\gamma_{33}^{22})V_3^2 = - \alpha_{31}^{21}D_1^1 - \alpha_{32}^{21}D_2^1 - \alpha_{33}^{21}D_3^1 \\
&\quad - \alpha_{34}^{21}D_4^1 - c_{34}^{21}\beta_4^1 - a_4^1\gamma_{34}^{21}S_4^0 - \alpha_{31}^{22}D_1^2 - \alpha_{32}^{22}D_2^2 + (1 - \alpha_{33}^{22})D_3^2 \\
&\quad - \alpha_{34}^{22}D_4^2 - c_{34}^{22}\beta_4^2 \qquad\qquad \text{(B-36)}
\end{aligned}
$$

for ditch sector 3. The reservoir mass balance is

$$
\begin{aligned}
& a_1^1\gamma_{41}^{21}Q_1^1 + a_2^1\gamma_{42}^{21}Q_2^1 + a_3^1\gamma_{43}^{21}Q_3^1 + (1 + b_4^1\gamma_{44}^{21} - a_4^2\gamma_{44}^{22})S_4^1 + b_1^1\gamma_{41}^{21}V_1^1 \\
&\quad + b_2^1\gamma_{42}^{21}V_2^1 + b_3^1\gamma_{43}^{21}V_3^1 + a_1^2\gamma_{41}^{22}Q_1^2 + a_2^2\gamma_{42}^{22}Q_2^2 + a_3^2\gamma_{43}^{22}Q_3^2 \\
&\quad - (1 + b_4^2\gamma_{44}^{22})S_4^2 - Q_5^2 + (1 + b_1^2\gamma_{41}^{22})V_1^2 + (1 + b_2^2\gamma_{42}^{22})V_2^2 \\
&\quad + (1 + b_3^2\gamma_{43}^{22})V_3^2 = - \alpha_{41}^{21}D_1^1 - \alpha_{42}^{21}D_2^1 - \alpha_{43}^{21}D_3^1 - \alpha_{44}^{21}D_4^1 - c_{44}^{21}\beta_4^1 \\
&\quad - a_4^1\gamma_{44}^{21}S_4^0 - \alpha_{41}^{22}D_1^2 - \alpha_{42}^{22}D_2^2 - \alpha_{43}^{22}D_3^2 + (1 - \alpha_{44}^{22})D_4^2 + \beta_4^2 . \qquad \text{(B-37)}
\end{aligned}
$$

The nodal mass balance and capacity constraints for this case are the same as those for the two time period model where return flows are excluded, equations (B-26), (B-27), (B-28), (B-29) and (B-30). They will not be repeated. The elements in the submatrices and subvectors comprising the two time period model with return flows included, A_{11}, A_{21}, A_{22}, D_{11}, D_{21}, D_{22}, $\overline{x}_1$, $\overline{x}_2$, $\overline{b}_1$, and $\overline{b}_2$, are shown in Exhibits XI, XII, XIII, XIV, XV, XVI, XVII, and XVIII. The A_{21} submatrix for this case, Exhibit XIV, has considerably more elements than the A_{21}

$$
\begin{bmatrix}
(1-a_1^1\gamma_{11}^{11}) & a_2^1\gamma_{12}^{11} & a_3^1\gamma_{13}^{11} & b_4^1\gamma_{14}^{11} & 0 & -(1+b_1^1\gamma_{11}^{11}) & b_2^1\gamma_{12}^{11} & b_3^1\gamma_{13}^{11} & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{21}^{11} & (1-a_2^1\gamma_{22}^{11}) & a_3^1\gamma_{23}^{11} & b_4^1\gamma_{24}^{11} & 0 & b_1^1\gamma_{21}^{11} & -(1+b_2^1\gamma_{22}^{11}) & b_3^1\gamma_{23}^{11} & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{31}^{11} & a_2^1\gamma_{32}^{11} & (1-a_3^1\gamma_{33}^{11}) & b_4^1\gamma_{34}^{11} & 0 & b_1^1\gamma_{31}^{11} & b_2^1\gamma_{32}^{11} & -(1+b_3^1\gamma_{33}^{11}) & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{41}^{11} & a_2^1\gamma_{42}^{11} & a_3^1\gamma_{43}^{11} & -(1+b_4^1\gamma_{44}^{11}) & -1 & (1+b_1^1\gamma_{41}^{11}) & (1+b_2^1\gamma_{42}^{11}) & (1+b_3^1\gamma_{43}^{11}) & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT XI: A_{11} Submatrix, Two Time Period Example, Including Return Flows.

$$
\begin{bmatrix}
(1-\alpha_{11}^{11}) & -\alpha_{12}^{11} & -\alpha_{13}^{11} & -\alpha_{14}^{11} & -c_{14}^{11} & -a_4^1\gamma_{14}^{11} & 0 & 0 & 0 & 0 & 0 \\
-\alpha_{21}^{11} & (1-\alpha_{22}^{11}) & -\alpha_{23}^{11} & -\alpha_{24}^{11} & -c_{24}^{11} & -a_4^1\gamma_{24}^{11} & 0 & 0 & 0 & 0 & 0 \\
-\alpha_{31}^{11} & -\alpha_{32}^{11} & (1-\alpha_{33}^{11}) & -\alpha_{34}^{11} & -c_{34}^{11} & -a_4^1\gamma_{34}^{11} & 0 & 0 & 0 & 0 & 0 \\
-\alpha_{41}^{11} & -\alpha_{42}^{11} & -\alpha_{43}^{11} & (1-\alpha_{44}^{11}) & +1 & -(1-a_4^1\gamma_{44}^{11}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT XII: D_{11} Submatrix, Two Time Period Example, Including Return Flows.

$$\bar{x}_1 = \begin{bmatrix} Q_1^1 \\ Q_2^1 \\ Q_3^1 \\ S_4^1 \\ Q_5^1 \\ V_1^1 \\ V_2^1 \\ V_3^1 \\ X_1^1 \\ X_2^1 \\ X_3^1 \\ X_4^1 \end{bmatrix} \qquad \bar{b}_1 = \begin{bmatrix} D_1^1 \\ D_2^1 \\ D_3^1 \\ D_4^1 \\ \beta_4^1 \\ S_4^0 \\ I_1^1 \\ Q_{1_{max}}^1 \\ Q_{2_{max}}^1 \\ Q_{3_{max}}^1 \\ S_{4_{max}}^1 \end{bmatrix}$$

EXHIBIT XIII: $\bar{x}_1$ and $\bar{b}_1$ Subvectors, Two Time Period Example, Including Return Flows.

$$
\begin{bmatrix}
a_1^1\gamma_{11}^{21} & a_2^1\gamma_{12}^{21} & a_3^1\gamma_{13}^{21} & (b_4^1\gamma_{14}^{21}+a_4^2\gamma_{14}^{22}) & 0 & b_1^1\gamma_{11}^{21} & b_2^1\gamma_{12}^{21} & b_3^1\gamma_{13}^{21} & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{21}^{21} & a_2^1\gamma_{22}^{21} & a_3^1\gamma_{23}^{21} & (b_4^1\gamma_{24}^{21}+a_4^2\gamma_{24}^{22}) & 0 & b_1^1\gamma_{21}^{21} & b_2^1\gamma_{22}^{21} & b_3^1\gamma_{23}^{21} & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{31}^{21} & a_2^1\gamma_{32}^{21} & a_3^1\gamma_{33}^{21} & (b_4^1\gamma_{34}^{21}+a_4^2\gamma_{34}^{22}) & 0 & b_1^1\gamma_{31}^{21} & b_2^1\gamma_{32}^{21} & b_3^1\gamma_{33}^{21} & 0 & 0 & 0 & 0 \\
a_1^1\gamma_{41}^{21} & a_2^1\gamma_{42}^{21} & a_3^1\gamma_{43}^{21} & (1+b_4^1\gamma_{44}^{21}-a_4^2\gamma_{44}^{22}) & 0 & b_1^1\gamma_{41}^{21} & b_2^1\gamma_{42}^{21} & b_3^1\gamma_{43}^{21} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

EXHIBIT XIV: A_{21} Submatrix, Two Time Period Example, Including Return Flows.

$$
\begin{bmatrix}
(1-a_1^2\gamma_{11}^{22}) & a_2^2\gamma_{12}^{22} & a_3^2\gamma_{13}^{22} & b_4^2\gamma_{14}^{22} & 0 & -(1+b_1^2\gamma_{11}^{22}) & b_2^2\gamma_{12}^{22} & b_3^2\gamma_{13}^{22} & 0 & 0 & 0 & 0 \\
a_1^2\gamma_{21}^{22} & (1-a_2^2\gamma_{22}^{22}) & a_3^2\gamma_{23}^{22} & b_4^2\gamma_{24}^{22} & 0 & b_1^2\gamma_{21}^{22} & -(1+b_2^2\gamma_{22}^{22}) & b_3^2\gamma_{23}^{22} & 0 & 0 & 0 & 0 \\
a_1^2\gamma_{31}^{22} & a_2^2\gamma_{32}^{22} & (1-a_3^2\gamma_{33}^{22}) & b_4^2\gamma_{34}^{22} & 0 & b_1^2\gamma_{31}^{22} & b_2^2\gamma_{32}^{22} & -(1+b_3^2\gamma_{33}^{22}) & 0 & 0 & 0 & 0 \\
a_1^2\gamma_{41}^{22} & a_2^2\gamma_{42}^{22} & a_3^2\gamma_{43}^{22} & -(1+b_4^2\gamma_{44}^{22}) & -1 & (1+b_1^2\gamma_{41}^{22}) & (1+b_2^2\gamma_{42}^{22}) & (1+b_3^2\gamma_{43}^{22}) & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT XV: A_{22} Submatrix, Two Time Period Example, Including Return Flows.

$$
\begin{bmatrix}
-\alpha^{21}_{11} & -\alpha^{21}_{12} & -\alpha^{21}_{13} & -\alpha^{21}_{14} & -c^{21}_{14} & -a^{1}_{4}\gamma^{21}_{14} & 0 & 0 & 0 & 0 & 0 \\
-\alpha^{21}_{21} & -\alpha^{21}_{22} & -\alpha^{21}_{23} & -\alpha^{21}_{24} & -c^{21}_{24} & -a^{1}_{4}\gamma^{21}_{24} & 0 & 0 & 0 & 0 & 0 \\
-\alpha^{21}_{31} & -\alpha^{21}_{32} & -\alpha^{21}_{33} & -\alpha^{21}_{34} & -c^{21}_{34} & -a^{1}_{4}\gamma^{21}_{34} & 0 & 0 & 0 & 0 & 0 \\
-\alpha^{21}_{41} & -\alpha^{21}_{42} & -\alpha^{21}_{43} & -\alpha^{21}_{44} & -c^{21}_{44} & -a^{1}_{4}\gamma^{21}_{44} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

EXHIBIT XVI: D_{21} Submatrix, Two Time Period Example, Including Return Flows.

$$
\begin{bmatrix}
(1-\alpha_{11}^{22}) & -\alpha_{12}^{22} & -\alpha_{13}^{22} & -\alpha_{14}^{22} & -c_{14}^{22} & 0 & 0 & 0 & 0 & 0 \\
-\alpha_{21}^{22} & (1-\alpha_{22}^{22}) & -\alpha_{23}^{22} & -\alpha_{24}^{22} & -c_{24}^{22} & 0 & 0 & 0 & 0 & 0 \\
-\alpha_{31}^{22} & -\alpha_{32}^{22} & (1-\alpha_{33}^{22}) & -\alpha_{34}^{22} & -c_{34}^{22} & 0 & 0 & 0 & 0 & 0 \\
-\alpha_{41}^{22} & -\alpha_{42}^{22} & -\alpha_{43}^{22} & (1-\alpha_{44}^{22}) & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

EXHIBIT XVII: D_{22} Submatrix , Two Time Period Example, Including Return Flows.

$$\bar{x}_2 = \begin{bmatrix} Q_1^2 \\ Q_2^2 \\ Q_3^2 \\ S_4^2 \\ Q_5^2 \\ V_1^2 \\ V_2^2 \\ V_3^2 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix} \qquad \bar{b}_2 = \begin{bmatrix} D_1^2 \\ D_2^2 \\ D_3^2 \\ D_4^2 \\ \beta_4^2 \\ I_1^2 \\ Q^2_{1_{max}} \\ Q^2_{2_{max}} \\ Q^2_{3_{max}} \\ S^2_{4_{max}} \end{bmatrix}$$

EXHIBIT XVIII: $\bar{x}_2$ and $\bar{b}_2$ Subvectors, Two Time Period Example, Including Return Flows.

submatrix for the two time period model with the return flows excluded, Exhibit VIII. This results from the influence of earlier strategies (the time period 1 strategy) on later strategies (the time period 2 strategy) because of the inclusion of return flows.

The objective function for minimizing system losses and unrequired system outflows for this case is

$$
\begin{aligned}
\text{MIN } Z = {} & (a_1^1\gamma_{11}^{11} - a_1^1\gamma_{21}^{11} - a_1^1\gamma_{31}^{11} - a_1^1\gamma_{41}^{11} - a_1^1\gamma_{11}^{21} - a_1^1\gamma_{21}^{21} - a_1^1\gamma_{31}^{21} - a_1^1\gamma_{41}^{21})Q_1^1 \\
& + (a_2^1\gamma_{22}^{11} - a_2^1\gamma_{12}^{11} - a_2^1\gamma_{32}^{11} - a_2^1\gamma_{42}^{11} - a_2^1\gamma_{12}^{21} - a_2^1\gamma_{22}^{21} - a_2^1\gamma_{32}^{21} \\
& - a_2^1\gamma_{42}^{21})Q_2^1 + (a_3^1\gamma_{33}^{11} - a_3^1\gamma_{13}^{11} - a_3^1\gamma_{23}^{11} - a_3^1\gamma_{43}^{11} - a_3^1\gamma_{13}^{21} - a_3^1\gamma_{23}^{21} \\
& - a_3^1\gamma_{33}^{21} - a_3^1\gamma_{43}^{21})Q_3^1 + (b_4^1\gamma_{44}^{11} + a_4^2\gamma_{44}^{22} - b_4^1\gamma_{14}^{11} - b_4^1\gamma_{24}^{11} \\
& - b_4^1\gamma_{34}^{11} - b_4^1\gamma_{14}^{21} - a_4^2\gamma_{14}^{22} - b_4^1\gamma_{24}^{21} - a_4^2\gamma_{24}^{22} - b_4^1\gamma_{34}^{21} - a_4^2\gamma_{34}^{22} \\
& - b_4^1\gamma_{44}^{21})S_4^1 + (b_1^1\gamma_{11}^{11} - b_1^1\gamma_{21}^{11} - b_1^1\gamma_{31}^{11} - b_1^1\gamma_{41}^{11} - b_1^1\gamma_{11}^{21} - b_1^1\gamma_{21}^{21} \\
& - b_1^1\gamma_{31}^{21} - b_1^1\gamma_{41}^{21})V_1^1 + (b_2^1\gamma_{22}^{11} - b_2^1\gamma_{12}^{11} - b_2^1\gamma_{32}^{11} \\
& - b_2^1\gamma_{42}^{11} - b_2^1\gamma_{12}^{21} - b_2^1\gamma_{22}^{21} - b_2^1\gamma_{32}^{21} - b_2^1\gamma_{42}^{21})V_2^1 + (b_3^1\gamma_{33}^{11} - b_3^1\gamma_{13}^{11} \\
& - b_3^1\gamma_{23}^{11} - b_3^1\gamma_{43}^{11} - b_3^1\gamma_{13}^{21} - b_3^1\gamma_{23}^{21} - b_3^1\gamma_{33}^{21} - b_3^1\gamma_{43}^{21})V_3^1 + (a_1^2\gamma_{11}^{22} - a_1^2\gamma_{21}^{22} \\
& - a_1^2\gamma_{31}^{22} - a_1^2\gamma_{41}^{22})Q_1^2 + (a_2^2\gamma_{22}^{22} - a_2^2\gamma_{12}^{22} - a_2^2\gamma_{32}^{22} - a_2^2\gamma_{42}^{22})Q_2^2 + (a_3^2\gamma_{33}^{22} \\
& - a_3^2\gamma_{13}^{22} - a_3^2\gamma_{23}^{22} - a_3^2\gamma_{43}^{22})Q_3^2 + (b_4^2\gamma_{44}^{22} - b_4^2\gamma_{14}^{22} - b_4^2\gamma_{24}^{22} - b_4^2\gamma_{34}^{22})S_4^2
\end{aligned}
$$

$$+ (b_1^2\gamma_{11}^{22} - b_1^2\gamma_{21}^{22} - b_1^2\gamma_{31}^{22} - b_1^2\gamma_{41}^{22})V_1^2 + (b_2^2\gamma_{22}^{22} - b_2^2\gamma_{12}^{22} - b_2^2\gamma_{32}^{22}$$

$$- b_2^2\gamma_{42}^{22})V_2^2 + (b_3^2\gamma_{33}^{22} - b_3^2\gamma_{13}^{22} - b_3^2\gamma_{23}^{22} - b_3^2\gamma_{43}^{22})V_3^2 + Q_5^1 + Q_5^2 \quad . \qquad \text{(B-3}$$

APPENDIX C

IMPROVEMENT OF THE MODEL

APPENDIX C

IMPROVEMENT OF THE MODEL

The model presented in the study is a basic model; considerable improvement could be made to increase its flexibility as a tool for the management of irrigation water delivery. This appendix is devoted to indicating the areas that need improvement.

It is the writer's opinion that the most profitable future work (in order of descending importance) should deal with: the derivation of procedures to include legal and administrative restrictions in the model; methods for deriving from field data either the coefficients of the linear model or the nonlinear loss and return flow functions; methods for making the model a conjunctive use model, including application to both surface and underground sources of water; solution algorithms that are more efficient, and methods for using the model to find the most profitable (water saving) locations for improving a system through sensitivity analyses. Each of these suggestions is examined in detail in the following sections.

Legal and Administrative Constraints

Because it is the function of a legal system to intervene only when the supply available to a system is less than the volume of water needed to satisfy the demands of the users, the assumption made in the example models, that no legal system was applicable, is equivalent to assuming the supply was always adequate to satisfy the demands. Because the losses are at the minimum, however, solutions from the model represent the maximum volume of water, with the available system inflow, that can be delivered to satisfy the demands. Any other routing strategy would not result in the satisfaction of as much demand.

When a supply is insufficient to meet the demands, even though the model is applied with a minimum loss objective, there is no feasible solution to the problem. To obtain a feasible solution, the demands must be reduced and most operating systems have a legal procedure to do this in an equitable manner.

If no feasible solution can be derived for a problem using the model in this study, a legal procedure can be manually applied to the model to reduce the demands. However, this can be a tedious and time-consuming process.

A more efficient procedure would be to simulate the legal procedure with a computer program and to use the simulated procedure in connection with the optimizing water delivery model as illustrated in Figure C-1. The solution methodology which results is iterative:

(1) Attempt to solve for an optimal strategy with the initial set of demands. If an optimal solution results, the legal procedure simulation was not necessary. If no optimal solution results, go to step 2.

(2) Call the legal procedure simulation to reduce the demands and attempt to solve for an optimal strategy with the new set of demands. If an optimal solution results, the problem is completed. If no optimal solution results, repeat this step until an optimal solution does result.

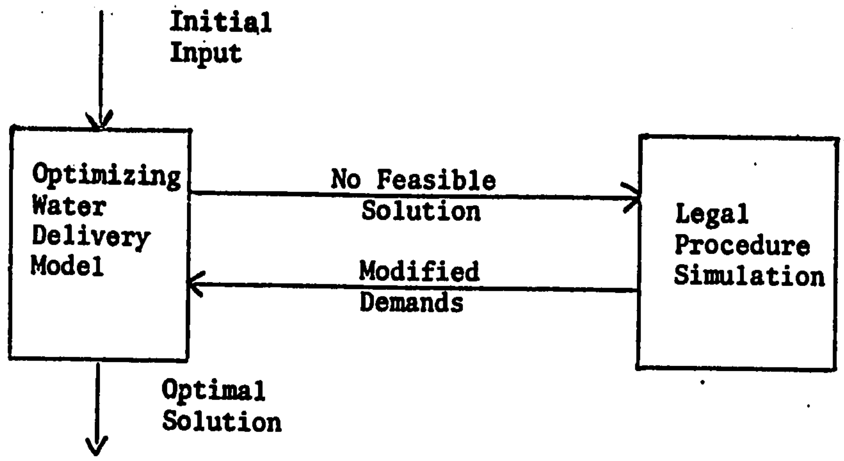

Figure C-1. Relation of System Model to Legal Model.

In the United States there are two broad legal doctrines of water rights: appropriations and riparian (Trelease, Bloomenthal and Geraud, 1965). Both of these doctrines treat water as property on a "here and now" basis. The application of a multiple time period irrigation water delivery model, such as the one developed in this study, could present difficulties in the interpretation of either of these doctrines: in the riparian doctrine because it does not consider the storage of water and in the appropriations doctrine because of the timing inherent in the reduction of the demands.

For example, if the demand for water in a system that is governed by the appropriations doctrine must be reduced, which demands are to go unsatisfied, those demands which are estimated to occur in the last time periods of analysis, or those which are estimated to occur in earlier time periods? The answers to these types of questions, of course, rest with lawyers, and the necessary modifications of existing statutes rest with the administrators and legislators of the various states.

Loss and Return Flow Function Derivations

In the development of the linearized water delivery simulation model, Chapter V, linear approximations were derived for the loss and return flow functions. In the example models of Chapter VI, estimated coefficients were used. For actual application of the linearized model as an aid in optimum water delivery strategy determination, a procedure for field data collection and reduction must be derived so that the coefficients are as real as possible.

If it is initially assumed that each element of a system is independent of all other elements in the system, that is, there are no return flows, then knowledge of the inflows, outflows, demands and storages of each element will yield estimates of the loss coefficients, γ_{ii}^{tt}. However, some variation in these loss coefficients should be expected. Comparison of these variations in the loss coefficients and the inflows, outflows, demands and storages of the other elements of the system could provide a beginning for obtaining the return flow coefficients. To perform these types of analyses, the data collection techniques must be very accurate.

Once these steps have been taken, the influence of other factors such as the environmental variables described in Chapter IV can be included, and eventually the nonlinear loss and return flow functions described in Chapter IV can be developed. The idea is to make the simulation model as accurate as possible, even to the point of using short-term forecasts of the environmental variables in the loss and return flow functions.

Conjunctive Use Model

If it is assumed that groundwater table levels are a function of the losses and return flows from a surface water delivery and irrigation system, various wells in the system, and inflows from, or outflows to, other groundwater storage basins, then it is conceptually possible to define "well sector" mass balances to account for the removal of water from underground storage to satisfy surface demands. The cost of this increased flexibility is increased complexity in the model. Changes in the groundwater table location in the vertical direction, will undoubtedly influence the loss and return flow functions in the surface water delivery system, and these influences would have to be mathematically described in a conjunctive use model.

The use of mass balances creates a "well-oriented" groundwater model as opposed to the "grid-oriented" models found in the literature (Bittinger, et.al., 1967). This could allow larger models to be solved. For groups of closely spaced wells, the effects could be combined to represent a single well.

Including wells in the model would make it applicable to the types of systems most commonly used. The Poudre system, used for the example models, is estimated to have 1500 wells that are used for irrigation. The objective function for a model that includes groundwater withdrawals will reduce to the minimization of evaporation losses, transpiration losses, and basin outflow from both surface and subsurface sources.

Solution Algorithms

The extent of a system can be described using elements is directly a function of the solution algorithm for a given amount of computer storage capacity. It is probable that use of the nonlinear model will require the preparation of a computer code for the solution algorithm. If so, the algorithm should be the most efficient possible, in terms of information storage, to allow for the greatest possible level of description.

Linear programming routines, for solution of the linearized model, are more available than nonlinear programming routines. If a routine is to be chosen that is already coded for a computer, it should allow a maximum amount of information to be stored.

If a linear programming routine is to be developed, specialized algorithms, such as decomposition algorithms, should be examined closely. An even more detailed study could develop the use of nonlinear programming routines to solve large linear programming problems. Such routines could result in greater efficiency in saving computer storage and time (Hayman, personal communication, 1969).

System Improvement Using the Simulation Model

After a programming model has been constructed for a system, certain analyses, called sensitivity analyses, can be made for various purposes (Orchard-Hayes, 1968; Au and Stelson, 1969). For the derived model, sensitivity analyses can be used to determine the effect a change in one or more of the loss or return flow coefficients can have on the value of the objective function.

A particular technique that appears to hold some promise, towards sensitivity analysis of this type, is an input-output analysis, described by Chenery and Clark (1959) and Miernyk (1965). A cursory examination indicates it could be of great value in designating those ditch sectors or reservoirs that could be most profitably reconstructed to save water.

APPENDIX D

NOTATION

APPENDIX D

NOTATION

Symbol	Interpretation
A	Matrix of coefficients for decision variables, linear programming description.
A_{ij}	Submatrix of partitioned A .
D	Matrix of coefficients for b constants, linear programming description.
D_{ij}	Submatrix of partitioned D .
D_i^t	Volume of water released from element i in time period t to satisfy demand for water.
E_j^h	Volume of water released as a demand, from element j in time period h , that is in excess of the volume required to saturate the soil.
G_i	A set for element i , the contents of which are the indices, j , of those flows, V_j^t , which are system outflows.
H_i	A set for element i , the contents of which are the indices, j , of those flows, Q_j^t , which are system outflows.
I_i^t	System inflow at node i in time period t .
J_i	A set for node i , the contents of which are the indices, j , of those flows, V_j^t , which supply a node.
K_i	A set for node i , the contents of which are the indices, j , of those flows, Q_j^t , which receive their supply from a node.
L	Measure of the volume of water lost.
$L(Q_i^t, V_i^t)$	Volume of water lost from a ditch sector i in time period t as a function of the ditch sector decision variables.
$L(S_i^{t-1}, S_i^t)$	Volume of water lost from a reservoir i in time period t as a function of the reservoir decision variables.

NOTATION - (Continued)

Symbol	Interpretation
L_i^t	Volume of water lost from an element i in time period t .
M	Number of ditch sectors.
N	Number of reservoirs.
P	Number of nodes.
Q	Measure of the volume of water transported or stored by an element.
Q_i^t	Inflow to an element i in time period t .
$Q_{i_{max}}^t$	Maximum capacity of element i in time period t .
$Q_{i_{min}}^t$	Minimum capacity of element i in time period t .
R_i^t	Total volume of return flow to element i in time period t .
$R_i^t(Q_j^h,V_j^h,S_k^h,D_\ell^h)$	Volume of return flow to element i in time period t as a function of the decision variables and demands of the model.
$R_i^t(Q_j^h,V_j^h,S_j^h)$	Volume of return flow to element i in time period t resulting from water delivery system losses.
$R_i^t(D_j^h)$	Volume of return flow to element i in time period t resulting from the release of excess water from element j in time period h .
$\Delta R_i^t(Q_i^h,V_j^h,S_j^h)$	Incremental volume of return flow to element i in time period t due to conveyance or storage loss from element j in time period h .
$\Delta R_i^t(D_j^h)$	Incremental volume of return flow to element i in time period t due to the release of excess water from element j in time period h .
S	Measure of water stored in a reservoir.
S_i^t	Volume of water stored in reservoir i at the end of time period t .

NOTATION - (Continued)

Symbol	Interpretation
$S^t_{i_{max}}$	Maximum volume of water that can be stored in reservoir i at the end of time period t .
$S^t_{i_{min}}$	Minimum volume of water to be maintained in reservoir i at the end of time period t .
S^o_i	Initial volume of water stored in reservoir i .
T	Number of time periods in the model.
U	Number of reservoirs modeled with the node definition.
V^t_i	Outflow from element i in time period t .
W	Number of reservoirs modeled with the ditch sector definition.
X^t_i	Slack or surplus variable.
Z	Objective function values (to be maximized or minimized).
a^t_i	Weighting factor for: Q^t_i for ditch sectors, S^{t-1}_i for reservoirs.
a_{ij}	Coefficient in the A matrix, linear programming description.
b_i	Constant in $\bar{b}$, linear programming description.
b^t_i	Weighting factor for: V^t_i for ditch sectors, S^t_i for reservoirs.
$\bar{b}$	Vector of constants.
$\bar{b}_i$	Subvector of $\bar{b}$ created by partitioning of the D matrix.
c_j	Constant in objective function, linear programming description.
c^{th}_{ij}	Fraction of water lost from element j in time period h that returns to element i in time period t .
$\bar{c}$	Vector of constants in objective function, linear programming description.

NOTATION - (Continued)

Symbol	Interpretation
$\bar{c}_i$	Subvector of $\bar{c}$ created by partitioning of A and D matrices.
d_j^h	Fraction of excess water applied to crops, released from element j in time period h , that returns to all elements in the system in the remaining (T-t+1) time periods.
d_{ik}	Element of D matrix, linear programming description.
e_j^h	Irrigation application efficiency for demand waters released from element j in time period h .
f(-)	Notation used for nonlinear objective function, nonlinear programming description.
f_i^t	System loss from element i in time period t .
g	Index for elements.
$g_i(-)$	Notation used for nonlinear constraints, nonlinear programming description.
g_{ij}^{th}	Fraction of excess water released from element j in time period h that returns to element i in time period t .
h	Index for time periods.
i	Index for elements.
j	Index for elements.
k	Index for elements.
ℓ	Index for elements.
m	Total number of constraints, nonlinear and linear programming descriptions.
n	Total number of decision variables, nonlinear and linear programming descriptions.
p	Number of unit lengths in a ditch sector.
t	Index for time periods, normally used for the time period of interest.
"	Unit loss rate for ditch sector loss rate computation.

NOTATION - (Continued)

Symbol	Interpretation
x_j	Decision variables, nonlinear and linear programming description.
x_k	Decision variables, quote from Hadley on Bellman's principle of optimality.
$\bar{x}$	Vector of decision variables.
$\bar{x}_i$	Subvector of $\bar{x}$, created by partitioning the A matrix.
y	Reservoir water level, used in reservoir loss computation.
α_{ij}^{th}	Fraction of demand, D_j^h, that returns to element i in time period t .
β	Intercept on loss axis, where storage is zero, reservoir loss computation.
β_i^t	Intercept on loss axis, where storage is zero, for loss function of reservoir i in time period t .
γ	Loss coefficient.
γ_{ii}^{tt}	Loss coefficient for element i in time period t .
γ_{ij}^{th}	Return flow coefficient for return flows to element i in time period t due to losses from element j in time period h .
ε	Used for phrase "is contained in the set."
δ_{ij}	Kronecker delta.
δ_{ht}	Kronecker delta.