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ON THE INEVITABILITY OF FACTOR SUBSTITUTION

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16. Abstracts

This paper has examined the fact that the output and employment per unit of capital stock (investment) can be varied by any one of four quite different mechanisms which are systematically related and which all induce factor substitution in the same (neo-classical) direction with a change in relative factor prices. Verbal and graphic descriptions of these mechanisms ~~were~~ supplemented by derivation of elasticities of substitution and relationships between them. The necessary conditions for the assertion that "factor prices don't matter" are shown to be ~~terribly~~ stringent. ~~And~~ empirical elasticity estimates are shown typically to be (a) larger than the ex post instantaneous elasticity of optimal utilization theory and (b) smaller than the ex ante aggregate elasticity that is relevant to development planning.

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On the Inevitability of Factor Substitution *

There is an abundance of labor and a shortage of capital in poor countries. But along with these factor proportions, production techniques are frequently employed that are virtually identical to those in advanced countries where capital is abundant and labor is scarce. Two sorts of explanations for this paradox have been advanced:

(a) that the prices of capital and labor in poor countries are so similar to those in advanced countries — they so inaccurately reflect real scarcities — that it has been made profitable for firms to use advanced-country techniques even though those techniques are highly inappropriate and costly in output and employment, or

(b) that there is no choice in techniques of production so if products (especially industrial) are to be made at all, they must be made according to techniques rigidly determined by and embodied in advanced country capital equipment — the "shelf of technologies" is very precise, detailed and allows of no substantial modifications to make a technology more appropriate to poor countries and their factor endowments.

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* This paper was drafted while I was a visitor at Nuffield College and a guest of the Institute for Commonwealth Studies, Oxford. A.R. Khan, Henry Bruton, Earl McFarland, Steve Lewis and John Todd were especially helpful in spotting errors and making suggestions.

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Both explanations accept that factor endowments in poor countries are not reflected in the choice of technique — that their limited capital stock (expanded by painful domestic saving and risky foreign borrowing) should produce more output and more employment and that it would if the technologies chosen were less Western and more fitting to the circumstances of the country. They differ in their view of why. The first holds that factor prices are wrong so firms are rewarded for doing what is socially wrong and punished for doing what is right. The second holds that factor prices don't make any difference since, whatever they are, there is no choice open to the firm in the poor country, anyway.

The difference is terribly important to development policy.

The issue is whether it is possible technically to vary the proportions of men and machines to respond to changes in their relative prices. Whether, to put it negatively (which best suggests the effects of current development strategies), the wrong prices of capital and labor might help explain the tendency of poor countries to waste capital and leave abundant labor unemployed.

The empirical evidence is unanimous in showing that the use of capital and labor do respond to relative factor prices [1,2,9,13]. But that evidence appears not to be persuasive enough. So in Part I, we will examine the four mechanisms by which the output and employment associated with a given capital stock (or investment) can theoretically be increased or decreased by changing relative factor prices. In Part II, we will formalize the relationships between these sources of substitution, deriving the progression of elasticities of substitution that describes them. In Part III, we briefly consider what is measured by empirical elasticity estimates and what these do and don't

imply for possibilities of factor substitution in development planning. We will abstract throughout from technical change.

I.

Factor substitution can work through (a) the ex ante choice of which product to make[1], (b) the ex ante choice of a particular kind of plant with

1. "Ex ante" and "ex post" are used in the usual vocabulary of Putty-Clay growth analyses [10,11]. "Ex Post" describes substitution after a "plant" (capital stock) is installed, embodying a particular technology. "Ex Ante" in contrast, describes options before any (durable) committment has been made to a particular process. I was barely dissuaded from calling this paper "Putty-putty analysis" out of deference for Slim Gaillard.

which to make the product, (c) the ex post choice of the way to man that plant and (d) the ex post choice of the proportion of the time to operate that plant. These are not, of course, conceptually separate, sequential decisions. In optimal, static planning, they would ideally be determined simultaneously. But they can usefully be treated in sequence in the verbal discussion of this part, moving from the most aggregate to the most detailed level and forward in time from investment planning (ex ante) to operation (ex post).

A. Factor Substitution through the Choice of Products Ex Ante

If different products ("sectors") are typically made with processes that have different capital-labor relationships, the choice of what things an economy should make will be influenced by the prices of capital and labor. If capital is cheap relative to labor, those products and services that are made with relatively a lot of capital and little labor will look profitable — to a firm or to a planner. If capital is expensive, a different set of products and services will be most profitable. So the amount of employment

to be got from a given capital stock will be influenced by factor prices as they influence the composition of output. This kind of factor substitution would be eliminated only in the event that either there were no choice about what to produce domestically or that all products were made with techniques of the same capital and labor relationships.

B. Factor Substitution through Choice of Techniques Ex Ante

After the decision has been made to produce a particular product, there exists, ex ante, an array of possible ways to produce it — configurations of capital stock, output and labor embodied in physical "plant". The "array" of course might consist of a single choice -- one kind of plant. But it might not. Which plant is deemed best, ex ante, will depend on relative factor prices — relatively low priced capital recommends the capital-intensive plant; low priced labor recommends the opposite.

Figures 1a through 1d are isoquant maps showing the four highly stylized possibilities ex ante. Figure 1a shows a nicely neoclassical array of many nicely neoclassical plants. Which of the many alternative plants, I_p, will be most profitable will be nicely sensitive to changing relative factor prices. The range of ex ante choice is described by the envelope curve, I_a, showing the fully range of ways to produce a given output, considering all different plant options. Figure 1b shows a Putty-Clay Growth sort of world in which each production technique has fixed coefficients, ex post, I_p, but there are enough of them that the range of ex ante choice, I_a, is not much different from the purely neoclassical case, 1a. The ex ante choice of plant is, once again, nicely sensitive to relative factor prices. Getting a bit ahead of the story, Figure 1c shows that there is only a single way to make the product — one

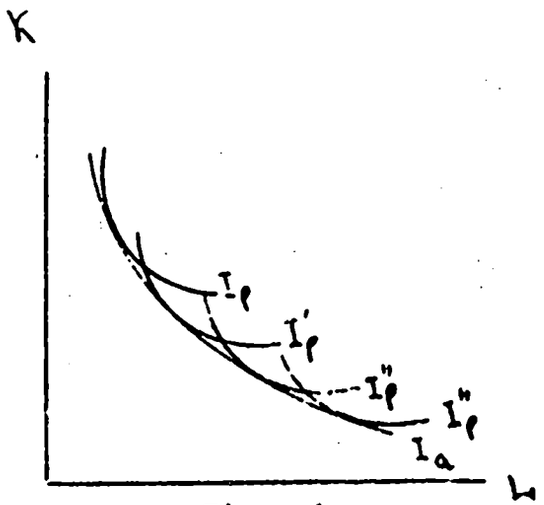

Figure 1a

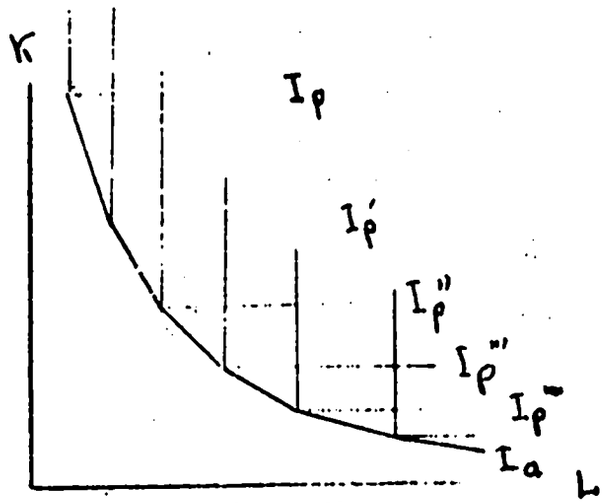

Figure 1b

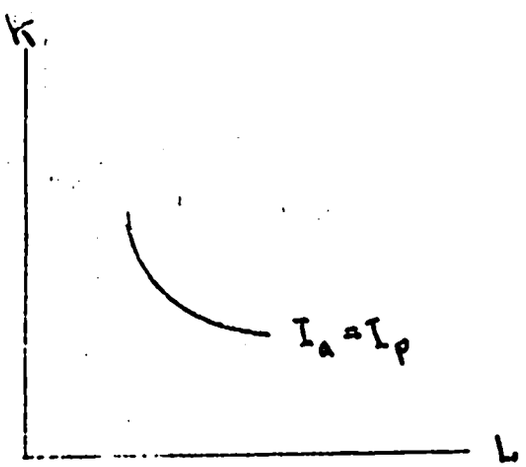

Figure 1c

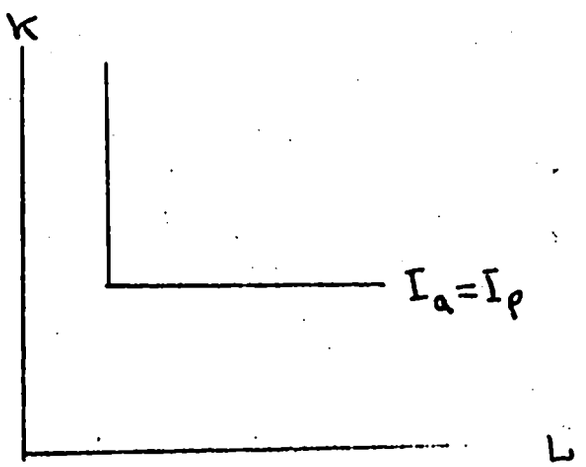

Figure 1d

plant that <u>can</u> produce it — so factor prices can't influence the choice of plants. But once the plant has been built, there'll be some choice, ex post, in how to man and run it. Finally, Figure 1d shows the most rigid case where there is only one way to make the product and, once the plant is built, there is only one way it can be operated (the meaning of this last statement is less clear than is usually assumed — a fact developed in D. below).

Take, as an illustration of all this, the production of ton-miles of freight movement per day. There might be a virtually infinite range of techniques to achieve a given output (Figures 1a and 1b), ranging from say, human porters through air freight. Or there might be only one way to move freight (1c, 1d) — excepting only trivial "alternatives" that don't matter because they would be more costly at <u>any</u> set of relative factor prices. Trucking, for instance, might be the only feasible technology to choose.

Then for any one technique, once it is embodied in extant capital -- whether chosen from the one or the many -- there may be alternative ways to operate (1a, 1c) or not (1b, 1d).

This is the sort of substitution that's most at issue in development policies because it describes the potential response to factor price policies. The extremes are the perfectly smooth neoclassical ex ante and ex post substitutability of 1a where everything is price sensitive in the smallest degree at all times and, on the other end, the "shelf of technologies", 1d, where for each product there exists a single technique (symbolized by "A Book of Technology") that describes in inflexible detail how a product is to be made wherever in the world its production takes place.

C. Factor Substitution by Choice of Techniques Ex Post

After the choice has been made among alternative possible plants (maybe a trivial choice among one alternative) and it is operating, there may still be variability in the way to run the plant -- in the number of men that can be used to operate the given capital stock, achieving different levels of output. If so, the number of men and machines per unit output will vary in response to changing relative factor prices. This is "instantaneous ex post" substitutability.

Because of the discussion in the next part, it is important to be clear about this sort of substitutability. It is what one would find if he walked into two physically identical plants existing in worlds of very different relative factor prices and watched them in operation. This gives a comparative static, "instantaneous" view of the process of production. If the two plants used different amounts of labor to produce different rates of output, these would be evidence of ex post instantaneous substitution. Take the truck

example again. With none of this sort of ex post instantaneous substitution, one would always and everywhere find (with a given age and sophistication of truck) the same number of drivers and helpers working on each truck at any moment. Never more nor less. Regardless of how cheap or expensive they might be, additional helpers technically could never increase the output of a truck (ton miles per day) by, for instance, decreasing the turnaround time to give a greater proportion of operating time on the road. On the other hand, with ex post instantaneous substitution, an additional driver's helper could technically increase the average rate of output. Whether he would be hired to do so would depend, of course, on his wage rate, but there would be some wage rate at which it would pay to add another helper.

Opportunity for this kind of substitutability is often thought to be quite restricted but the small amount of empirical evidence does not appear to support that consensus. A recent paper by Ranis, for instance, gives numerous examples from Japan, Korea and Taiwan in which a quite similar plant is operated with quite dissimilar numbers of workers and consequently quite different rates of output -- when relative factor prices make it profitable to do so [8].

D. Substitution by Changing Utilization — the Duration of Production and Idleness Ex Post

Even after the plant is constructed and even if there is no opportunity for instantaneous factor substitution, there is always opportunity for substitution by changing the daily (yearly) duration of production. Since a capital stock is typically idle much of the time and the proportion of time it is optimally idle depends on relative factor prices, it will be left idle more of the time if the price of capital is low relative to the price of labor [14]. The greater the amount of time the capital stock is idle, of course, the

less it will produce (the higher will be the capital-output ratio) and the fewer the man hours it will use (the higher will be the capital-labor ratio) — so with any given average work week per man, the less the employment it will generate.

In the truck example, this sort of substitution describes, simply, changes in the proportion of time a truck is operated. The truck may be used eight hours a day employing, say, one driver and one helper for a capital-labor ratio of one (truck) to two (men) and a capital-output ratio of one (truck) to 240 (ton miles per day).[1] But even if ex post instantaneous substitutability

1. In Part II I will more carefully describe this as a "capital(stock)-labor ratio" to distinguish it clearly as the ratio relevant to investment and growth — a quite different thing from the instantaneous capital-labor ratio that describes relative rates of factor service flows [14, 16].

is absent — so if a truck operates at all for instance it uses two men and produces at the rate of 30 ton miles per hour — doubling the number of hours of operation would halve the capital-labor ratio (now 1:4) as well as the capital-output ratio (now 1:480). In a 24 hour day, these crucial ratios can vary solely through changing utilization between one to two and one to six for the capital-labor ratio and between one to 240 and one to 720 for the capital-output ratio (considering only eight hour shifts).

But before I go on any further about this, one very basic empirical question needs to be answered — whether there is, in fact, any <u>scope</u> for this sort of substitution-through-utilization in poor countries in the direction we're interested in — toward reducing the capital-labor and capital-output ratios; i.e., increasing utilization. Utilization must, in fact, be low enough in LDCs to be increased this way or our nice theoretical possibility simply isn't very interesting. The answer is in this sense encouraging. The present level

of manufacturing capital utilization in West Pakistan is 14% — capital is idle 86% of the time [15] — and evidence from Korea suggests utilization at 16% — capital idle 84% of the time [5] In U.S. manufacturing, capital is used about 25% of the time — it is idle over 75% [3]. These figures suggest that there is ample scope for factor substitution to work through increasing utilization.

Since this is a quite different sort of factor substitution, it isn't surprising that it can't be described in the usual sort of isoquant map like Figures 1 without modification. Consideration of utilization of the capital stock adds to the usual description of men and machines and output the additional dimension of the duration of operation. Figure 2 is intended to suggest this for one especially simple case. On the (k,l) plane is a simple instantaneous isoquant map for ex post production. As drawn, there is no ex post substitutability — at any moment of operation, machines and men must be combined in a fixed proportion in order to produce a given rate of output, like two men-one truck-half a ton mile per minute. Only one rate of output (isoquant) is shown.

Given the (say, hourly) rates of flows of inputs, k, l, and output, q, the total amount of input and output per day will clearly depend on the duration of production — what proportion of the twenty four hours production takes place. In Figure 2, the duration of operation is described by the value of f on the time axis and the extension of the instantaneous rate of output isoquant as a surface showing how long production lasts in the typical day. Operating periods of 8, 16 and 24 hours are indicated, producing 8q, 16q and 24q of output per day, respectively when q is the rate of output per hour. These different daily quantities of output are all made with the same capital stock. Moving out the time axis from duration of production f=0 to f=24 hours, constantly reduces the amount of capital stock needed to produce a unit of daily output — increased utilization increases capital productivity. It also increases the total man hours of labor employed (the area L = fl on

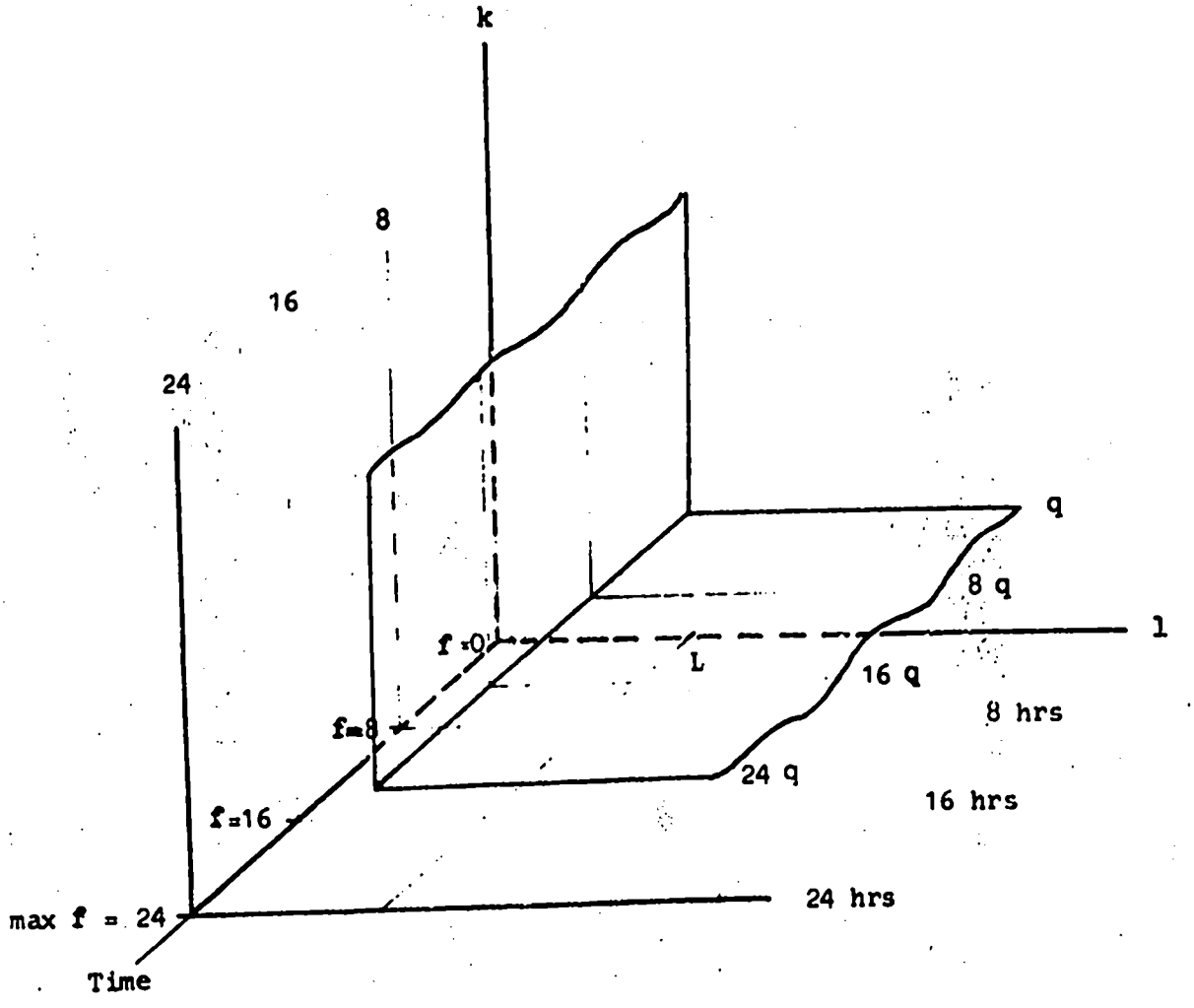

Figure 2

the (t,l) plane) with the given capital stock to produce the constant rate of output.

What would have to be shown, if it hadn't been done elsewhere in considerable detail [14, 16], is that (a) the daily duration of production can optimally be less than the engineering maximum (twenty-four hours less maintenance time) and (b) the optimal duration of production is sensitive to relative factor prices — an increase in the relative price of capital will increase the duration of production.

One quite relevant aspect of this sort of factor-substitution-through-utilization is that it operates better, more powerfully, the less is instantaneous factor substitution possible ex post. The maximum amount of substitution from changing duration of production and idleness will come when there is zero instantaneous elasticity of ex post substitution.

This is easy to explain. The periods when idleness of the capital stock is optimal are periods of relatively high costs of operation — nights, week-ends, winters, summers, etc., depending on the production process and its inputs. Relative factor prices affect the optimal duration of idleness by making the cost increases that occur during these periods relatively more or less important in total costs. If higher night time labor costs, say, were the sole explanation for optimally leaving plant idle, a rise in the price of capital would make any given increase in night time labor costs relatively less important — simply by making capital costs relatively larger. The incentive would be to economize on the now-higher-priced capital stock by using it more of the time even though this meant incurring somewhat higher labor costs by extending operations further into the night. As the capital price increased, therefore, so would the optimal duration of production [14,16].

If there were some instantaneous ex post substitutability, then if production were extended into periods of higher wage rates, part of the potential

increase in labor costs could be avoided by reducing the number of workers per machine — instantaneous substitution allows that. So the man hours of labor used with the capital stock would not increase proportionally to the increase in utilization — nor would output. If, in contrast, there were no instantaneous substitutability — a fixed number of men was required to operate each unit of capital stock — then the increase in man hours would be proportional to the increase in the duration of production. Zero instantaneous ex post elasticity of substitution, in effect, prevents firms from slithering out from under the higher night time wage rate by reducing labor input.[1]

In the trucking example, one last time, if there were a fixed crew of, say, two men per truck (driver and helper), doubling the duration of operation of the truck would double — no more, no less — both employment and output. But if it were possible, instead, to operate the truck either with two men (driver and helper) or with one (driver alone producing ton-miles at a lower rate), then a doubling of the period of production might only increase employment by half and output would be less than doubled. The truck would run during the day with a two man crew when labor was cheap, but at night with only a driver because labor costs were higher. At the night time wage rate, the helper wouldn't pay for himself even though it would still be profitable to operate the truck at night rather than let it sit idle.

So the more valid is the assertion of no ex post instantaneous substitution between factors, the greater will be the factor substitution induced through changes in the rate of utilization — in periods of operation and idleness.

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1. Another instantaneous flow elasticity -- the ex ante flow elasticity -- also influences optimal utilization in determining the relative importance in unit costs of the rhythmically varying input in the investment decision. It is not essential to this discussion th ough it figures prominently in the theory of idle capital [14].

II.

This can all be put more formally in elasticities — though diminishing returns to formality are reached rather quickly. It is helpful to start at "the other end" than the verbal description of Part I — to begin with ex post instantaneous elasticity and work up to ex ante, aggregate elasticity.

A. The Instantaneous Ex Post Elasticity of Substitution

The basic concept is an instantaneous production function that describes the production process at a duration-less moment of time [4, 6, 14] as a relationship between rates of flow — of output, q, and factor services, k and l,

$$q = \phi(k, l).$$

The elasticity of substitution for this is the instantaneous ex post elasticity,

$$(1) \qquad \sigma = \frac{d\left(\frac{k}{l}\right) \frac{w}{p}}{d\left(\frac{w}{p}\right) \frac{k}{l}}$$

where w is the wage rate (the price of labor service flow) and p is the cost of capital service flow. So σ describes how the number of people manning the capital stock ("plant") at any moment of operation changes in response to changes in wage rate or capital service price. As in Part I, above, when σ is zero, the plant must be manned by a crew of fixed size — more workers at any moment can't increase the rate of output at all; fewer will reduce it. A larger σ describes greater responsiveness in the number of workers tending the plant to changes in relative factor prices. All very standard stuff except, perhaps, in being carefully confined to instantaneous substitution.

B. The Ex Post Elasticity of Substitution between Capital Stock (Investment) and Man-hours of Labor

For the same "plant", consider the relationship between the capital stock (the plant) and the daily (or yearly) labor service flow measured in man-hours and the response of this relationship to changes in relative factor prices. The number of daily man hours that will be used with a given plant, ex post, is determined by the number of men manning the plant at any moment during operation (A. above) and how long the plant is operated — the rate of utilization. The optimal utilization — duration of production in a typical day (or year) — is a function of relative factor prices [14]. The ex post elasticity of substitution between capital stock and man-hours is

$$(2) \qquad \sigma = \frac{\sigma - E}{1 - E}$$

where σ is the instantaneous elasticity of substitution (1) and

$$E = \frac{du}{d\left(\frac{W}{P_k}\right)} \frac{\frac{W}{P_k}}{u}$$

is the elasticity of the duration of production with respect to relative factor prices.[1] E is always negative for $\sigma < 1$ and positive for $\sigma > 1$ [14, 5].

---

1. Daily (yearly) production is

$$Q = \int_0^u \phi(k,l)\, dt, \qquad u = f/24$$

where $0 \leq u \leq 1$ is the proportion of the day (year) the production process is in operation. The rate of capital service flow, k, will be constant over the period of operation $[0, u]$ for reasons explained in [14]. If the rate of labor service flow, l, is constant, too, then daily (yearly) factor service flows are $K = uk$ and $L = ul$ and daily (yearly) output $Q = uq$. The production function $Q = \phi(K,L)$ is the same as $q = \phi(k,l)$.

Since the size of the capital stock determines capital service flow in

operation, $k = \bar{K}$ when the plant is in operation (but $k = 0$ when the plant is idle, so capital flows cannot be inferred from the existence of capital stocks, as is common). k is the number of machine-instants of capital service flow; $\bar{K}$ is number of machines. For labor, daily (yearly) man-hours, L, and employment $\bar{L}$ are interchangeable if there is a fixed average workday, δ ($\delta \leq 24$ u hours) per worker, so $\delta\bar{L} = L$.

The price of the capital stock is the rental rate or "user cost" [14], P_k, incurred per day or year of owning the capital. $P_k = up$. The price of labor service flow relevant to optimal production is w per man hour where $w = W/u$ when

$$W = \int_0^u w(t)\, dt \qquad w'(t) > 0$$

is the daily wage cost of filling a position. Though w(t) could be the "instantaneous wage rate" relevant to (1) above, the restriction that l is constant over [0,u] while $w' > 0$ means that w is the relevant wage rate for the instantaneous elasticity, (1) and for the stock elasticity (2).

So the instantaneous ex post elasticity (1)

$$\sigma = \frac{d\left(\frac{k}{l}\right)\frac{W}{P_k}}{d\left(\frac{W}{P_k}\right)\frac{k}{l}}$$

and the ex post elasticity between capital stock and man-hours (2) is

$$\bar{\sigma} = \frac{d\left(\frac{K_s}{L}\right)\frac{w}{P_k}}{d\left(\frac{w}{P_k}\right)\frac{K_s}{L}} = \frac{d\left(\frac{k}{ul}\right)\frac{W}{P_k}}{d\left(\frac{W}{uP_k}\right)\frac{k}{l}}$$

Then,

$$\bar{\sigma} = \frac{d\left(\frac{1}{u}\cdot\frac{k}{l}\right)\frac{W}{P_k}}{d\left(\frac{1}{u}\cdot\frac{W}{P_k}\right)\frac{k}{l}}$$

$$= \frac{[d(k/l) - (k/l)\ du/u]\ W/P_k}{[d(W/P_k) - (W/P_k)\ du/u]\ k/l}$$

$$= \frac{\sigma - E}{1 - E} \quad .$$

So the elasticity of substitution between capital stock and man-hours or employment is always greater than the instantaneous elasticity of substitution

if $\sigma \neq 1$. Which makes sense. A change in relative factor prices will (a) induce a change in the number of men per machine (if $\sigma \neq 0$) and (b) induce a change in the optimal duration of operation (if $E \neq 0$). Both affect the number of man-hours used with the machine in the course of the day. Note that even if there is no instantaneous substitutability ($\sigma = 0$), there will nonetheless be a change in the number of man-hours (employment) per machine in response to changes in factor prices (so $\bar{\sigma} > 0$) when $E \neq 0$. The fact that the plant must, to use Solow's phrase, be operated by "a crew of fixed size" is not the same thing as zero ex post elasticity of factor substitution between capital stock (investment) and man-hours or employment. This appears to be an important fact for much of the theory of growth [10, et al].

C. The Elasticity of Substitution Ex Ante

Before building the plant we've been talking about — at the investment planning stage — there was the array of possible plant designs for producing this product. Each potential plant in that array would embody a different production function, hence a different ex post instantaneous elasticity of substitution, σ, (and E) and a different ex post elasticity of substitution on capital stock, $\bar{\sigma}$. Denote these for the ith potential plant, ex ante, by σ_i and $\bar{\sigma}_i$ and the corresponding elasticity of utilization, E_i. Of course, if there is, ex ante, only one possible plant to build to make that product, the "set of alternatives" is the trivial set of one plant -- which would make the analysis all very easy if rather unlikely.

But even this result is useful because if there is only a single ex ante alternative, then the ex ante elasticity of (stock) substitution, $\bar{\sigma}^*$, will be identical to the corresponding ex post elasticity, $\bar{\sigma}$. If there is more than one ex ante alternative plant, the ex ante substitution possibilities will generally be greater — additional alternatives will reduce the elasticity, ex ante, vis a vis a single alternative only under unusual circumstances and then only over part of the range of feasible capital-labor ratios. So we can express the ex ante elasticity of substitution as

$$(4) \quad \bar{\sigma}^* = \max \bar{\sigma}_i + \alpha(i)$$

where $\alpha(i)$ depends on the array of alternatives, their technical characteristics and their number. $\alpha(i)$ will usually be positive. Graphically, this is the envelope curve in Figures 1a or 1b that includes all ex post possibilities and the possibility of choice among them.

D. The Elasticity of Capital Stock Substitution Ex Ante for Many Products

Now consider ex ante an aggregate of products — a sector or an economy. It includes not only the elasticity of substitution for each single product, but it encorporates the way changes in relative factor prices can change the aggregate factor use by inducing changes in the mix of products made. This aggregate ex ante (stock) elasticity, $\bar{\sigma}^{**}$, will be a weighted average of the individual product ex ante (stock) elasticities plus a term expressing the factor price induced shift among industries. So, over j industries, the aggregate ex ante elasticity will be

$$(5) \quad \bar{\sigma}^{**} = \sum_j (\bar{K}_j/\bar{K})\, \bar{\sigma}^*_j + \sum_j \frac{(\bar{K}_j/L_j)}{K/L} \frac{d(L_j/L)}{d(W/P_k)} .$$

E. The Sources of Response to Relative Factor Price Changes

Combining these four expressions, we get an elasticity of substitution ex ante for a multi-product sector (or economy) that is a function of the instantaneous flow elasticity and the positive valued parameter $\alpha(i)$ — both weighted by the sectoral distribution of the capital stock — and the sensitivity of the sectoral distribution of labor to relative factor prices.

$$(6) \quad \bar{\sigma}^{**} = \sum_j \frac{\bar{K}_j}{\bar{K}} \left[\max. \frac{\sigma_i - E_i}{1 - E_i} + \alpha(i)\right] + \frac{\bar{K}_j/L_j}{K/L} \frac{d(L_j/L)}{d(W/P_k)}$$

This expression describes the necessary conditions for the assertion that "factor prices don't matter" since they can't influence factor use — the necessary conditions for ex ante elasticity of substitution to be zero. In order for this to be true — that there be no response in factor use to changes in factor prices — it is clearly necessary that simultaneously:

1. Factor use is not affected by product mix $d(L_j/L)/d(W/P_k) = 0$ either because there is only one product in the economy or because the composition of output is fixed and immutable so its use of factors is not variable by variations in product mix — there is effectively one composite product,

AND

2. Factor use is not affected by choice of techniques $[\alpha(i) = 0]$ because there is only a single kind of ex post plant, i, that can produce for each of the j industries that make up the sector or economy,

AND

3. Utilization is not affected by factor prices $[E = 0]$ because, ex post, all capital is utilized at its engineering maximum though that maximum is much greater than the optimal-desired levels of utilization suggested by

all empirical studies of capital utilization [5, 12, 15]. (Empirically, the maximum comparable level of capital utilization is that of the U.S. where capital is utilized about 25% of the engineering maximum [3]. In poor countries, the rate is much lower [5, 15].)

AND

4. The crew required to operate each piece of capital is everywhere inflexibly set [$\sigma = 0$] so increasing the number of workers will never increase the rate of production and decreasing the number will always idle capital.

In order that there be no factor response to changing factor prices, it is necessary that all of these conditions hold simultaneously.[1]

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1. Equation (6) reveals other permutations that could yield a zero aggregate elasticity:

a. If $\sum_j (\bar{K}_j/\bar{K})\, \bar{\sigma}^*_j = \sum_j \frac{\bar{K}_j/L_j}{\bar{K}/L} \frac{d(L_j/L)}{d(W/P_k)}$, the average aggregate response of each industry's factor use would just exactly offset an opposite shift of product between industries, the change in factor prices inducing a net shift into the industries using the larger amounts of the now-more-expensive factor.

b. If the product-mix term of (6) were zero and $\bar{\sigma} = -\alpha(i)$, every industry would have to have at least one feasible ex post technique with fixed stock coefficients (maximum utilization and all) and that ex post technique would be the optional choice for every industry in the tangle of factor prices being examined.

c. If the instantaneous elasticity and utilization elasticity were of the same value and sign, $\sigma = E$. There is no empirical evidence to show that this is not possible. But then there is no empirical evidence at all on either σ or E. When the concept of σ has been identified or described, both the (one piece of) empirical literature [8] and the speculative-theoretical literature [10] deal entirely with values of σ near zero. And this analysis suggests that σ is much less than estimated elasticity. It seems unlikely that σ is greater than one.

III.

Empirical estimates of elasticities of substitution have been made for both advanced and underdeveloped countries with generally similar results: that estimated values range, sector by sector, from around .5 to the order of 1.6 with virtually all estimates significantly different from zero. Cross section data generally yield higher elasticity estimates than do time series. See [1] for an excellent summary of the literature.

Empirical estimates are based on aggregates. Two useful statements can be made about these estimated values on the basis of the preceding analysis.

First, empirical elasticity estimates will be larger than the underlying instantaneous, ex post elasticities. So observed, aggregate elasticities can be much greater than one and still be consistent with a [illegible] elasticity less than one. This, importantly, preserves the relevance of the theory of optimally idle capital [14] in face of high empirical estimates of elasticity of substitution greater than one.

Second, empirical elasticity estimates will generally underestimate the possibilities for substitution open with new investment -- the elasticity relevant to development planning. Empirical estimates of elasticity are an amalgum of ex ante and ex post elasticities. Consider a time series estimate first (and abstract entirely from technical change). In the simplest scenario, there will be (1) a set of plants in existence at t_o then (2) a (one-shot) change in relative factor prices at t_o, (3) after which additional plants will be built and then (4) at t_1 elasticity is

estimated. The elasticity of substitution between the two dates will measure (a) a response ex post by the plants existing at both t_o and t_1 and (b) a response ex ante as new, post-t_o plants are chosen in light of the new factor prices. Ex ante elasticity $\bar{\sigma}^*$ is greater than ex post elasticity $\bar{\sigma}$ and the weight that the larger, ex ante element will get in an empirical estimate will depend on the proportion of the t_1 capital stock that was built <u>after</u> the price change at t_o. With a one shot price change, the estimated elasticity of substitution would be

$$(7) \quad \bar{\sigma}^{**} = \Sigma \left[\frac{\bar{K}_{pj}}{\bar{K}} \bar{\sigma}_j + \frac{\bar{K}_{aj}}{\bar{K}} \bar{\sigma}^*_j \right] + \Sigma \frac{K_i/L_i}{K/L} \frac{d(L_i/L)}{d(W/P_k)}$$

where the amount of capital in industry j built prior to the price change is $\bar{K}_{pj}$, and after the price change is $\bar{K}_{aj}$. So ceteris paribus, the greater the gross rate of investment the greater will be the estimated elasticity of substitution. The more recent (or faster) the relative price change, the smaller will be the estimated elasticity. All with the same actual elasticity, $\bar{\sigma}^{**}$. And all this abstracts from technical change.

So empirical estimates of elasticity of substitution will <u>understate</u> the "pure" ex ante elasticity of substitution that's relevant to forecasting price responsiveness in development <u>planning</u>. The observed elasticity of substitution will underestimate the substitution that can be expected with new investment because empirical estimates include influence of the lower ex post response of plants already in existence at the time of the price change. Only if a time series measured response so long after a (one-shot) factor price change that all capital had been replaced since the price change ($\bar{K}_{pj} = 0$) -- only then would empirical time series estimates reflect the full opportunity for factor substitution.

Cross section elasticities should better reflect the "pure" ex ante or Planner's elasticity since the variation in factor prices will be more completely reflected in the capital stock. Indeed, if each observation were taken from an environment in which the factor prices had prevailed long enough that in each case the entire capital stock had adjusted to them, the cross section elasticity estimates would yield an accurate picture of ex ante substitution possibilities. This may well explain the fact that higher elasticity estimates are found in cross section than in time-series data [1].

IV.

This paper has examined the fact that the output and employment per unit of capital stock (investment) can be varied by any one of four quite different mechanisms which are systematically related and which all induce factor substitution in the same (neoclassical) direction with a change in relative factor prices. Verbal and graphic descriptions of these mechanisms were supplemented by derivation of elasticities of substitution and relationships between them. The necessary conditions for the assertion that "factor prices don't matter" are shown to be terribly stringent. And empirical elasticity estimates are shown typically to be (a) larger than the ex post instantaneous elasticity of optimal utilization theory and (b) smaller than the ex ante aggregate elasticity that's relevant to development planning.

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