PB-219 683

A GENERALIZED CROP-FERTILIZER PRODUCTION FUNCTION

James Garrett Ryan

North Carolina State University

Prepared for:

Agency for International Development

1972

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

PB 219 683

A GENERALIZED CROP-FERTILIZER PRODUCTION FUNCTION

James Garrett Ryan

AID/csd 2806

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
U S Department of Commerce
Springfield VA 22151

NORTH CAROLINA STATE UNIVERSITY
RALEIGH

211

BIBLIOGRAPHIC DATA SHEET	1. Report No. 631.88-R988	2.	3. Recipient's Accession No. PB-219 683
4. Title and Subtitle "A Generalized Crop-Fertilizer Production Function."			5. Report Date July 1972
			6.
7. Author(s) James G. Ryan			8. Performing Organization Rept. No.
9. Performing Organization Name and Address North Carolina State University, Department of Economics Raleigh, N.C. 27607			10. Project/Task/Work Unit No.
			11. Contract/Grant No. AID/csd-2806, Proj. 931-17-120-525
12. Sponsoring Organization Name and Address Department of State Agency for International Development Washington, D.C. 20523			13. Type of Report & Period Covered Thesis-6/70-7/72
			14.

15. Supplementary Notes

A thesis is submitted to the Graduate Faculty of N.C. State University

16. Abstracts

The problem examined in this study is the derivation of a generalized crop-fertilizer production function describing the response relation between measured soil characteristics, applied nutrients, weather and crop yield. It is hypothesized the function can be utilized to generate specific fertilizer recommendations to farmers based on soil analysis.

The data come from potato-fertilizer experiments conducted on 65 farm locations over a seven year period in the Sierra Region of Peru. A quadratic model was chosen as the most appropriate form to describe the response relationships in these data. Soil phosphorus and potassium were found to have statistically significant effects on response to applied nutrients, and thus on the optimum levels of application. These data did not consistently show that soil organic matter should directly affect nitrogen recommendations. On high organic matter soils it was found that risk due to weather variability falls as the levels of these three applied nutrients are increased up to about 120, 120 and 60 kilgrams per hectare respectively.

17. ~~[illegible]~~

17b. Identifiers/Open-Ended Terms

*Fertilizer research, *Crop yields, *Soil fertility

17c. COSATI Field/Group 631

18. Availability Statement	19. Security Class (This Report) UNCLASSIFIED	21. No. of Pages ~~209~~ 211
	20. Security Class (This Page) UNCLASSIFIED	22. Price $3.00/$0.95

FORM NTIS-35 (REV. 3-72)

USCOMM-DC 14952-P72

A GENERALIZED CROP-FERTILIZER PRODUCTION FUNCTION

by

JAMES GARRETT RYAN

A thesis submitted to the Graduate Faculty of
North Carolina State University at Raleigh
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

DEPARTMENT OF ECONOMICS

RALEIGH

1 9 7 2

APPROVED BY:

Chairman of Advisory Committee

ABSTRACT

RYAN, JAMES GARRETT. A Generalized Crop-Fertilizer Production Function. (Under the direction of ERNEST CALEB PASOUR JR.)

The problem examined in this study is the derivation of a generalized crop-fertilizer production function describing the response relation between measured soil characteristics, applied nutrients, weather and crop yield. It is hypothesized the function can be utilized to generate specific fertilizer recommendations to farmers based on soil analyses.

The data come from potato-fertilizer experiments conducted on 65 farm locations over a seven-year period in the Sierra region of Peru. A quadratic model was chosen as the most appropriate form to describe the response relationships in these data. Soil phosphorous and potassium were found to have statistically significant effects on response to applied nutrients, and thus on the optimum levels of application. These data did not consistently show that soil organic matter should directly affect nitrogen recommendations.

The applied nitrogen recommendations generated using the derived quadratic production function were 5 to 220 kilograms per hectare higher than the current recommendations of the National Center of Soil Analysis at La Molina, Peru. The size of the differential depended upon the levels of soil phosphorous, soil potassium and soil pH. Applied phosphorous recommendations derived here were up to 250 kilograms per hectare above those of the Soil Analysis Center. Applied phosphorous recommendations varied substantially depending on the soil pH level. At present, the Center does not use pH in their phosphorous recommendations.

It was found that on soils with less than about 3.7 percent organic matter, risk (measured by the variance of profits due to rainfall and temperature variations over time) increased as the levels of applied nitrogen, phosphorous and potassium were increased in a fixed ratio of 10:10:5. It may therefore be reasonable for highly risk averse potato farmers on these soils to apply no fertilizer.

On high organic matter soils it was found that risk due to weather variability falls as the levels of these three applied nutrients are increased up to about 120, 120 and 60 kilograms per hectare, respectively.

On these soils, aversion to risk (due to weather variations) would not appear to be a valid explanation for farmer applications below these levels.

A multivariate probability density function from a large sample of soils was derived with three soil nutrients and the pH as random variables. Given this density function, the expected value of using a blanket area recommendation from this study as opposed to current recommendations, amounted to S/. 8,600 per hectare annually (about $U. S. 200). The expected additional value of the soil test information was calculated for the same sample of soils as S/. 4,000 per hectare, or $U. S. 94. The current levels of fertilizer use are so low that an increase to the general level of the blanket recommendations is of far greater value than the additional benefits available from soil testing.

Widespread adoption of the fertilizer recommendations generated here would depress potato prices from about S/. 2,000 per metric ton to S/. 700 due to the resulting downward shift in the supply function. The net gain in economic welfare using partial equilibrium analysis was estimated as S/. 1,993 million per annum ($U. S. 46 million). This

excluded the cost of any research, extension or teaching to achieve the large increase in fertilizer use implied.

To Wendy, Stephen
Mum, Dad,
Ronda and Terry

BIOGRAPHY

The author was born March 11, 1941 in Sydney, Australia. In May 1962, he graduated as Bachelor of Science in Agriculture from The University of Sydney. From 1962 to the present, he has been employed as Research Economist with the New South Wales Department of Agriculture. In May 1969, he received the degree of Master of Science in Agriculture from The University of Sydney as an external student.

In 1968, he was granted study leave from the New South Wales Department of Agriculture to enter the Graduate School of North Carolina State University to pursue the Doctor of Philosophy degree.

The author married Miss Wendy Doran in 1964, and they have one child, Stephen, who is five years old.

ACKNOWLEDGMENTS

The author expresses his gratitude to Dr. R. K. Perrin for his direction and advice throughout the course of this study. Dr. J. A. Seagraves is also sincerely thanked for his assistance. Dr. E. C. Pasour Jr., Chairman of the author's Advisory Committee, was not able to participate directly in the project due to absence on study leave; however, his counsel during the course of his graduate program is much appreciated. Dr. D. M. Hoover and Dr. L. A. Nelson, members of the Advisory Committee, offered many helpful comments and criticisms. Dr. T. D. Wallace is also thanked for his advice.

Thanks are due to Dr. W. D. Toussaint for giving the author the opportunity to undertake this work and the Agency for International Development for providing generous financial support.

Dr. R. E. McCollum of the North Carolina State University Mission to Peru generously provided most of the original data for the study with the assistance of others in the Ministerio de Agricultura, Peru, including Dr. F. Queyodo and Dr. C. Valverde S. of the Estacion Experimental Agricola, La Molina; Ing. S. Villagarcia, Ing. F. Zapata, Ing. F. Ezeta, Ing. H. Villachica of the Universidad Agraria La Molina. S. A. Tuten and R. L. Sawyer, also of the Mission to Peru, and D. Oelsligle of the Peace Corps, rendered valuable assistance with data. All these persons are thanked.

Particular appreciation is due to Teddy Kovac who painstakingly carried out most of the hand computations and data coding and also drew the figures. Libby Amos performed the many computer programming routines required with the able assistance of Jim Goodnight, Sophia Bernat and Harriet McLaughlin.

The task of typing early drafts of the dissertation was competently fulfilled by Dianne Parker, Gloria Bowden, Ann Dees and Karen Smith. Ann Dees skillfully typed the final draft and Ruth Good undertook the final editing. The author is grateful to these ladies.

For granting study leave and providing some financial assistance in the first two years of his graduate program, the author thanks the New South Wales Department of Agriculture.

A special word of appreciation is expressed to the author's wife, Wendy, and son, Stephen, who spent many evenings and weekends at home on their own while he pursued his graduate program.

TABLE OF CONTENTS

Page

LIST OF TABLES viii

LIST OF APPENDIX TABLES x

LIST OF FIGURES xii

INTRODUCTION 1

PREVIOUS CROP-FERTILIZER RESPONSE STUDIES 4

Types of Response Models 5
Empirical Response Studies 11

Early Work 11
Incorporation of Soil and Site Characteristics 16
Methods Employed to Measure Weather 31

DEVELOPMENT OF RESPONSE EQUATION 37

The Data 37
The Model 39
Econometric Analysis 44

Generalized Least Squares Regression 44
Tests for Specification Bias 53
The Problem of Multicollinearity 67

GENERATING FERTILIZER RECOMMENDATIONS 82

Derivation of Economic Optima for Fertilizers 82

Sensitivity of Optima to Price Changes 86
Sensitivity of Optima to Soil and Weather Variables 89

Weather Uncertainty and Fertilizer Decisions 99

VALUE OF THE GENERALIZED FUNCTION AND SOIL TESTING 118

CONCLUSIONS 146

LIST OF REFERENCES 154

APPENDICES. 165

Appendix A. Developing the Appropriate Response Equation 166

Problems in Model Specification 167
Tests for Multicollinearity among the Data 174

TABLE OF CONTENTS (continued)

Page

Appendix B. Calculation of Economic Optima 180

Appendix C. Soil Test Probability Distribution, Computed Optima and Present Recommendations 187

LIST OF TABLES

Page

1. Expected signs of coefficients in the quadratic model 42

2. Generalized quadratic production function F_1 obtained from potato experiments in Peru, 1960-70 48

3. Analysis of variance structure for combined analysis of potato experiments, Peru, 1960-1970 50

4. Algebraic means of regression residuals ($\hat{\varepsilon}_{ij}$) by class intervals of applied phosphorous (P_A). 57

5. Values of applied variables to attain maximum physical and economic yields for quadratic models F_1, F_3, P_1 and P_2 and square root models F_2 and F_4 60

6. Generalized quadratic production function P_2 obtained from potato experiments in Peru, 1960-1970 62

7. Sensitivity of fertilizer optima to changes in price ratios 87

8. Optimum and recommended levels of applied nutrients for various soils with "average" weather 90

9. Changes in economic optima as levels of soil nutrients and weather change 97

10. Expected profits ($E(\Pi)$) and variance of profits (σ_Π^2) for an average soil at various rates of N_A applied to potatoes 107

11. Means of the soil variables from experimental data and the National Center of Soil Analysis (N.C.S.A.) at La Molina, Peru 121

12. Expected profits using computed optima and recommendations for N_A, P_A and K_A and the probability of their occurrence on soils with pH < 5.5 123

13. Expected profits using computed optima and recommendations for N_A, P_A and K_A and the probability of their occurrence on soils with $5.5 \leq pH \leq 6.5$ 126

14. Expected profits using computed optima and recommendations for N_A, P_A and K_A and the probability of their occurrence on soils with pH > 6.5 129

LIST OF TABLES (continued)

Page

15. Fertilizer input levels, potato yields, supply and demand estimates for a range of potato prices in Peru 138

LIST OF APPENDIX TABLES

Page

Appendix A

1. Means and standard deviations of variables from the experimental data for potatoes in Peru, 1960-1970 168

2. Analysis of variance on Peruvian potato data, 1960-1970 169

3. Generalized square root production functions F_2 and F_4 obtained from potato experiments in Peru, 1960-1970 170

4. Generalized quadratic equations F_3 and P_1 obtained from potato experiments in Peru, 1960-1970 172

5. Some simple correlation coefficients among independent variables 176

6. Generalized quadratic production functions M_1 and M_2 from randomly dividing data into halves for potato experiments in Peru, 1960-1970 177

Appendix B

1. Total costs of fertilizer in Peru, March 1971 181

2. Price ratios 182

3. Optimum levels of applied nutrients for various soils with "below average" weather 183

4. Optimum levels of applied nutrients for various soils with "above average" weather 184

5. Expected profits ($E(\Pi)$) and variance of profits (σ_Π^2) for a soil low in organic matter (N_s) at various rates of N_A, P_A and K_A applied to potatoes 185

6. Expected profits ($E(\Pi)$) and variance of profits (σ_Π^2) for soils high in organic matter (N_s) at various rates of N_A, P_A and K_A applied to potatoes 186

Appendix C

1. Means, computed optima and recommended levels of N_A, P_A and K_A for soils with pH < 5.5, average weather, and a range of values of N_s, P_s and K_s 188

LIST OF APPENDIX TABLES (continued)

Page

2. Means, computed optima and recommended levels of N_A, P_A and K_A for soils with $5.5 \leq pH \leq 6.5$, average weather, and a range of values of $\bar{N}_s$, P_s and K_s 190

3. Means, computed optima and recommended levels of N_A, P_A and K_A for soils with $pH > 6.5$, average weather, and a range of values of N_s, P_s and K_s 192

4. Estimated average fixed, average variable, average total and marginal costs of growing potatoes in the Sierra of Peru, at 1971 prices 194

LIST OF FIGURES

Page

1. Relationship between soil tests and relative yield for various levels of applied nutrients 27
2. Possibility of specification bias with the quadratic function 55
3. Ridge trace of coefficients of N_A, P_A, K_A, N_A^2, P_A^2, K_A^2 for P_2 71
4. Ridge trace of coefficients of N_s, P_s, K_s, pH, R, T for P_2 72
5. Ridge trace of coefficients of N_s^2, P_s^2, K_s^2, pH^2, R^2, T^2 for P_2 73
6. Ridge trace of coefficients of X_0, $N_s \cdot pH$, $N_s \cdot R$, $N_s \cdot T$, $R \cdot T$, SD_2 for P_2 74
7. Ridge trace of coefficients of $N_s \cdot N_A$, $P_s \cdot P_A$, $K_s \cdot K_A$, $pH \cdot P_A$, $N_A \cdot P_A$, $N_A \cdot K_A$ for P_2 75
8. Ridge trace of coefficients of $N_A \cdot R$, $N_A \cdot T$, $P_A \cdot K_A$, $N_A \cdot P_s$, $R \cdot P_A$ and error sum of squares for P_2 76
9. Frequency distribution of rainfall variable, Huancayo, 1921-1970 105
10. Frequency distribution of temperature variable, Huancayo, 1930-1970 106
11. Expected profits-variance map for an average soil 111
12. Expected profits-variance map for a soil low in organic matter 113
13. Expected profits-variance map for a soil high in organic matter 114
14. Possible effects of increased fertilizer use on potatoes in the Sierra of Peru, on production, prices and welfare 137
15. Cost Structure for an average soil with land fixed 141

CHAPTER I
INTRODUCTION

The problem to be examined in this study is the derivation of a generalized crop-fertilizer production function which adequately describes the response relation between measured soil characteristics, applied nutrients, weather and crop yield. The word "generalized" means that the function can be utilized under a wide range of soil and environmental conditions to replace site-specific functions for the potato growing areas of the Sierra of Peru.

It is hypothesized that the function can be utilized to generate specific fertilizer recommendations to farmers based on soil analyses. In the words of the National Academy of Science (1961, p. 49):

> One of the most challenging problems in statistical methodology relating to agronomic-economic research in response-surface studies then is that of developing theory and methods of combining results from a series of experiments to form a generalized production function. To be of practical usefulness, this function must be such that it may be employed over a comparatively large population of conditions by having at hand values for independent variables, such as soil-test results, to insert to account for local variations in conditions.

The empirical function derived in this study is subjected to rigorous statistical analysis to ensure it has not been adversely affected by specification bias and multicollinearity. The empirical estimation of crop-fertilizer response functions combining data over space and time has always proved to be a difficult task because of these two problems. The possibility of specification bias exists because the "laws of production" have not been adequately formulated into a multivariate functional form capable of depicting the exact mathematical relationships involved. Multicollinearity, on the other hand, is largely a data problem.

The data come from the potato-fertilizer experiments conducted in the Sierra region of Peru during the period 1960-70. The generalized function fitted to these data is used to measure the sensitivity of computed economic optima to changes in soil and weather characteristics and price relationships, which is one of the primary aims of the study. The current recommendations of the National Center of Soil Analysis at La Molina, Peru, are then compared with those generated from the fitted production function. Using a multivariate probability density function derived from a large sample of data from the La Molina laboratory (with soil organic matter, phosphorous, potassium and pH as random variables), the expected value of using the recommendations of this study compared with current recommendations is calculated. This measures the expected value of the generalized function.

Another primary aim is to compare the value of a general increase in fertilizer use with the value of soil testing. The value of soil testing is determined by measuring the expected value of the difference between the profit levels using recommendations of this study for the above sample of soils, compared to one blanket recommendation based on an average soil. The expected value is again calculated using the above sample multivariate probability density function. These calculations enable some policy statements to be made about the relative value of soil testing with a view to making farm-specific recommendations, versus a general program aimed at increasing the level of fertilizer use on all soils.

An analysis is made of the likely effects of increased fertilizer use on all farms on potato prices, production and economic welfare. If all potato farmers followed the recommendations derived in this study,

the price of potatoes would fall substantially due to the shift in the supply curve that would result and the inelasticity of demand for the product. Assuming all other resources are in completely elastic supply on potato farms, a new supply curve is derived which reflects the adoption of the recommendations of this study. The change in the equilibrium level of price and production is determined assuming a demand elasticity of -0.3. From these, an estimate is made of the change in economic welfare that would result from a program of research, education and extension aimed at increasing fertilizer use to the levels implied by the new supply curve.

The effect of weather risk on statements to farmers concerning "recommended" fertilizer applications is examined assuming rainfall and temperature as random variables. The hypothesis that the variance of profits is a monotonically increasing function of nitrogen fertilizer applications, as others such as Colyer and Kroth (1970) and Tollini and Seagraves (1970) found with corn, is tested using the potato data. The effect of interactions between soil and applied nutrients and the two weather variables on the expected profits-variance (E - V) map is considered. Previous work on E - V estimation has been restricted largely to single nutrient response with no soil factors entering the analysis.

CHAPTER II
PREVIOUS CROP-FERTILIZER RESPONSE STUDIES

Three major categories of problems have arisen in crop-fertilizer response research. The first concerns definition of the structure of the model. Included in this are the choice of the appropriate functional form to represent the response relationship and the way to incorporate soil and other site variables such as weather into the model.

The second problem deals with the statistical difficulties encountered in the empirical estimation of models. Within this category fall three related subclasses:

a. definition of statistical methods to sequentially "build up" equations so that the most important variables are included; multicollinearity among independent variables appears to cause most of the problems here;

b. the difficulty of fitting non-linear models, particularly when more than one independent variable is included, and the statistical properties of non-linear estimators;

c. determination of appropriate statistical tests when experimental data are combined over space and time.

The third class of problems relates to the use of fitted models in economic analyses. Included here are the questions of how to arrive at fertilizer recommendations for farmers, taking account of both profitability risk and how to evaluate the worth of empirical research compared to present farm practices.

The sizable literature on the first two problems will be specifically reviewed and discussed in this chapter. The economic aspects will be covered in Chapters IV and V.

Types of Response Models

Perhaps the most important and most common problem in response research has been the choice of an appropriate mathematical function to represent the relationship between fertilizer input and crop yield per unit of land. Ideally, one should hypothesize a functional relationship based on the "laws of production" as conceived by soil scientists and agronomists. Unfortunately, a concensus does not exist among these "lawyers" of production. As a result, the history of response research reveals that a plethora of functional forms has been utilized.

One of the first explanations of the relation between crop yields and plant nutrients was the "law of the minimum." According to Munson and Doll (1959, p. 134) de Saussere, a Swiss chemist, first alluded to this law in 1804 in his Reserches Chimiques sur la Végétation. It is more commonly known as Von Liebig's "law of the minimum," developed in 1855. The essence of this relationship is that it specifies a linear crop response to a given soil nutrient up to the yield ceiling, which occurs when other nutrients or factors become limiting. Thereafter yield (y) remains constant. It can be expressed as follows:

$$y = bX_1 \mid X_2, \ldots, X_n \quad (0 < X_1 < X_1^*)$$

$$y = A \mid X_2, \ldots, X_n \quad (X_1 \geq X_1^*) \tag{1}$$

where X_1^* is the nutrient level giving the maximum yield, A.

As Swanson (1963) points out, life in a Liebig world is uncomplicated. The response function can be estimated using ordinary least squares analysis and determination of the optimum level of nutrient is relatively simple as it involves corner solutions. Both Mitscherlich

and Lang, among others, refuted Liebig's "law," which implied that the various nutrients in the soil were not substitutes in production.

Mitscherlich (1909) developed the mathematical growth model which bears his name. It was a curvilinear form of the Liebig "law," with yield becoming asymptotic to an upper yield ceiling. For a single nutrient, it took the form:

$$y = A(1 - 10^{-cx}) \tag{2}$$

where y is absolute yield, x is the level of applied nutrient, A is the maximum physical yield and c is the response parameter.

Another version of the Mitscherlich function incorporated the level of soil nutrient, b:

$$y = A(1 - 10^{-c(x+b)}) \quad . \tag{3}$$

Unlike (2), equation (3) allowed yield to be positive at zero levels of x.

Baule (1918) modified Mitscherlich's function and showed that when one used the percentage yield concept instead of absolute yield, it becomes independent of the quantity of nutrient in the soil.[1] Baule also suggested that all nutrients should be in the response function, as shown in equation (4):

$$y = A(1 - 10^{-c_1x_1})(1 - 10^{-c_2x_2}) \ldots\ldots\ldots\ldots (1 - 10^{-c_nx_n}) \quad . \tag{4}$$

---

[1]The percentage (or relative) yield refers to the ratio of yield obtained with all nutrients but the one of interest available in "adequate" amounts, to yield with all nutrients, including the one of interest, at "adequate" levels. The nutrient of interest is set at an applied level of zero in order to obtain the yield for the numerator.

This specified that absolute and percentage yield was zero when an essential nutrient was absent in the soil and none was applied.

In 1923 Spillman developed his function which was similar to that of Mitscherlich except that R, the ratio of successive yield increments, was not assumed to be constant for a given nutrient, as in Mitscherlich's formulation. R was allowed to vary with soil and climatic conditions in the Spillman (1923) formulation. For multiple nutrients, the Spillman function was:

$$y = A(1 - R_1^{x_1})(1 - R_2^{x_2}) \ldots\ldots\ldots\ldots (1 - R_n^{x_n}). \qquad (5)$$

The R_i ($i = 1, 2, \ldots, n$) were the ratios by which marginal products of the applied nutrients X_i, ($i = 1, 2, \ldots, n$), declined as input levels were increased. Other features of this type of function were, as detailed by Heady (1956), and Heady and Dillon (1961, pp. 73-107)

a. the elasticity of response is less than one over all ranges of input levels; this was probably a reasonable assumption except on extremely deficient soils where a phase of increasing marginal returns to added nutrients may be expected;

b. the marginal product of each input was always positive as yield was asymptotic to A;

c. it does not allow isoclines to converge at the point of maximum yield, A; this implied that A can be attained with numerous input combinations;

d. the level of soil nutrients could be incorporated into the function by adding a soil nutrient variable to the powers of the R_i.

The main disadvantage of the Mitscherlich-Baule-Spillman models is that they require non-linear estimation procedures when the asymptote is not known. A number of studies have used these models such as Bray (1958) Yurtsever et al. (1965), Holford (1966), Eagle (1969), Ranganathan et al. (1969) and Cate (1971). Stevens (1951) and Edwards (1962) have outlined iterative procedures for handling general problems of non-linearity. Even with modern computers, the estimation problem becomes almost intractable when more than one independent variable is included.[2]

The "resistance" function was developed by Balmukand (1928) to relate soil and applied nutrients to crop yield. His function took the form:

$$y = \frac{a_1}{b_1+x_1} + \frac{a_2}{b_2+x_2} + \ldots\ldots + \frac{a_n}{b_n+x_n} + c \qquad (6)$$

where a_i ($i = 1, 2, \ldots, n$) are the parameters to be estimated, b_i ($i = 1, 2, \ldots, n$) are the levels of soil nutrients and x_i ($i = 1, 2, \ldots, n$) are amounts of applied nutrients.

This function allows:

a. yield to be asymptotic to 1/c;

b. linear isoclines which do not pass through the origin;

c. non-convergence of the isoclines at 1/c;

d. diminishing marginal products.

Heady and Dillon (1961, pp. 94-96) suggest that the "resistance" function is more appropriate for livestock feeding experiments. The function has not been widely used in response studies of any kind.

---

[2]For an excellent review of the history of the Liebig, Mitscherlich, Baule and Spillman functions together with an extensive bibliography, see National Academy of Science (1961). Heady (1963) also has a review of this early work.

The Cobb-Douglas function also has not been widely used in crop-fertilizer response studies. Its main limitation is the assumption that the marginal rate of substitution between nutrients is constant, as in the case of the Liebig function over certain ranges. The Cobb-Douglas is written:

$$y = b\, X_1^{c_1}\, X_2^{c_2} \ldots\ldots X_n^{c_n} \tag{7}$$

where b is a constant and the c_i $(i = 1, 2, \ldots, n)$ are the elasticities of production.

Other features of the Cobb-Douglas are:

a. the elasticities of production are constant;

b. it allows diminishing, increasing or constant marginal returns throughout but does not allow more than one of these phases;

c. it does not define a point of maximum yield and hence tends to overestimate the optimum level of inputs when the ratio of input price to output price is very small;

d. output is zero when one of the inputs is at zero level.

An extension of the Cobb-Douglas function is the Transcendental:

$$y = b\, X_1^{c_1}\, X_2^{c_2} \ldots\, X_n^{c_n}\, e^{d_1 x_1}\, e^{d_2 x_2} \ldots\, e^{d_n x_n}. \tag{8}$$

The main advantage of the Transcendental is that it allows phases of both increasing and diminishing marginal returns. It can be fitted with least squares regression procedures using log transformations. However, it is difficult to use for economic analyses.

By far the most used functional form has been the quadratic equation of the form:

$$y = a + b_1x_1 + b_2x_2 + \ldots + b_nx_n - b_{11}x_1^2 - b_{22}x_2^2$$
$$- \ldots - b_{nn}x_n^2 + b_{12}x_1x_2 + b_{13}x_1x_3 + \ldots + b_{1n}x_1x_n$$
$$+ b_{23}x_2x_3 + b_{24}x_2x_4 + \ldots + \ldots + b_{n-1,n}x_{n-1}x_n$$
$$+ b_{123} \ldots b_nx_1x_2 \ldots x_n. \qquad (9)$$

The properties of the quadratic which account for its popularity are:

a. it allows diminishing marginal returns for each input alone;

b. the interaction terms allow the function to represent either diminishing marginal returns for both factors or diminishing for one and increasing for the other;

c. the isoquants are allowed to intersect the input axes; hence, a zero level of one input does not imply a zero output;

d. total yield can reach a maximum, followed by negative marginal returns;

e. the isoclines are linear but because they are not forced through the origin of the input plane as in the Cobb-Douglas case, they allow changing input proportions along the expansion path;

f. first derivatives form linear equations which are easy to solve simultaneously.

A review of some of the empirical work using quadratic functions appears in a later section of this chapter.

A variation of the quadratic is where the x_i^2 terms are replaced by $x_i^{3/2}$ terms in equation (9). This generates non-linear isoclines which

do not pass through the origin of the input plane. Adding x_i^3 terms to equation (9) allows both increasing and diminishing marginal returns.

The square root function has also been used a great deal in response studies. Unlike the quadratic, it allows marginal products to decline at a diminishing rate rather than a constant rate. Its mathematical form is shown in equation (10):

$$\begin{aligned} y = a - b_1x_1 - b_2x_2 - \ldots - b_nx_n + b_{11}\sqrt{x_1} \\ + b_{22}\sqrt{x_2} + \ldots + b_{nn}\sqrt{x_n} + b_{12}\sqrt{x_1x_2} + b_{13}\sqrt{x_1x_3} \\ + \ldots b_{1n}\sqrt{x_1x_n} + b_{23}\sqrt{x_2x_3} + b_{24}\sqrt{x_2x_4} + \ldots \\ + \ldots b_{n-1,n}\sqrt{x_{n-1}x_n} + \ldots + b_{123} \ldots n\sqrt{x_1x_2 \ldots x_n}. \end{aligned} \tag{10}$$

It also has curved isoclines which pass through the origin, hence allowing input proportions to vary as output changes.

Empirical Response Studies

Very little empirical work was done on crop-fertilizer response prior to 1950, as noted by the National Academy of Science (1961, p. 6).

Early Work

Bray was a leader in the field of correlating the level of soil nutrients with the response of crop to applied nutrients. In his 1944 paper he fitted a Mitscherlich equation relating the level of exchangeable potassium in the soil to the percentage yield of corn. He only had one non-zero rate of potassium in the experiment and used this as justification for leaving the level of applied nutrient out of the equation. His aim was to estimate an equation that would enable prediction

of percentage yields for crops on a range of soils when only the soil test values were known. He chose percentage yield as his dependent variable as he found it gave a better fit to his data.

Bray's contention was that c, the coefficient of b, the level of the nutrient in the soil, was relatively constant for all soils in equation (11), which resembles the log form of equation (2). His aim was to utilize c to determine fertilizer requirements.

$$\text{Log } (A - y) = \text{Log } A - cb \tag{11}$$

where A = the maximum yield, and

y = absolute yield with non-zero level of nutrient applied.

However, Bray's (1944, pp. 318-320) analyses showed that other soil properties will affect c and that it will vary depending on the crop. No attempt was made to incorporate other soil properties into the function. Finally, Bray (1944, p. 320) admitted that his c applied only for soils in the mid-range for potassium.

In a later paper, Bray (1958) examined various rates of applied phosphorous on wheat and estimated the relationship between yield and the levels of soil and applied phosphorous. He again found that his estimated c values changed as the level of soil and applied phosphorous changed. However, soil type did not appear to affect them. Equations where percentage yields were included in place of (A - y) in equation (11) generally gave a better "fit" to the data.

Borden (1946) used Mitscherlich relationships to calculate the soil nutrient status using percentage yield as the predictor. As with most of the response research up to this time, the concern was to broadly delineate

soils on the basis of the relative magnitude of the yield response to "adequate" doses of fertilizer. Little or no interest was expressed in accurately defining just what was an "adequate" level of fertilizer.

The "renaissance" in the area of production function estimation came in the 1950s, and centered around the work of Heady and others at Iowa State University. Interdisciplinary research efforts between livestock researchers, agronomists, soil scientists, statisticians and economists seemed to blossom in this period.

The paper by Box and Wilson (1951) developed several concepts leading to more efficient estimates of production surfaces. They discussed multivariable quadratic response functions and the structure of the design matrix for most efficient estimation of the function. They pointed out that the likelihood of specification bias in the model depends upon the choice of experimental design. The $(X'X)^{-1}$ and what they refer to as the specification bias matrix provide an objective basis for comparing designs for both precision and bias. This was a considerable advance on the trial and error computational methods such as that of Mendum (1948). A later paper by Box (1954) emphasized the importance of considering all factors affecting yield, not simply one.

Johnson (1953), in comparing how a Cobb-Douglas quadratic and two variations of a Spillman function performed in describing corn-nitrogen response, found that the quadratic gave the best "fit." After this, empirical papers seemed to abound. Papers by Heady and Pesek (1954), Heady, Pesek and Brown (1955), Brown _et al_. (1956) set out generally to compare various functional forms such as the Cobb-Douglas, exponential, quadratic and square root. The quadratic or square root functions were generally chosen. Although it was not always clear what criteria were

used to choose between the fitted equations, it appeared to be largely based on the R^2 values.

All these papers included detailed economic analyses of the results. Isoquants, isoclines and least cost fertilizer combinations were calculated. The fact that isoclines turned out to be non-linear was frequently mentioned as further justification for using the square root function. However, this is a property of this function and cannot be used to justify the specification.

Brown and Oveson (1958) worked with wheat-nitrogen data available over a period of years in Oregon. They compared quadratic equations derived by combining "like" response years to estimate a general equation from the combined data. They measured the "cost of the wrong decision" as the margin over fertilizer cost of using the combined equation to estimate fertilizer optima versus the individual equations of "like" years. The average "cost of the wrong decision" from using a single rate each year was small--$1.58 per acre. Havlicek and Seagraves (1962) carried out a similar analysis for nitrogen on corn in North Carolina. Using a square root function, they found the cost of the wrong decision to be about $1.02 per acre. They also examined the value of price and weather information and compared the quadratic to the square root equation. They found that there was little to choose between the two equations.

In the late 1950s, two important books were published at Iowa. They contained many important papers by statisticians, agronomists, soil scientists, economists and others in the area of fertilizer response analysis. They were edited by Baum, Heady and Blackmore (1956) and Baum, Heady, Pesek and Hildreth (1957).

Heady, Pesek and Rao (1966) derived a quadratic production function for nitrogen and phosphorous on corn in Iowa. From this, they derived short- and long-run static supply functions for corn, and demand functions for the two nutrients. In deriving the optimum economic levels of the two nutrients, Heady, Pesek and Rao found the point estimates had rather wide statistical confidence intervals. Three methods were suggested to reduce these intervals:

a. increase the number of experiments;

b. add more variables to the equation;

c. improve the experimental designs.

Another example of a typical response study of the 1950s and 1960s where only applied nutrients were considered is that of Colyer and Kroth (1968). Using seven years of data on corn response to nitrogen and plant population at two locations, they derived quadratic equations for each of the seven years and also a combined equation. Both combined equations gave substantially reduced R^2 values, but most of the regression coefficients were significant. Soil tests were available on the sites but were not included in the analysis.

Colwell and Esdaile (1966) fitted square root response equations to 49 nitrogen and phosphorous field experiments on wheat in New South Wales, Australia. For each location, fertilizer optima were computed. This work was a forerunner to Colwell's later efforts at development of a generalized response equation involving incorporation of soil and other site characteristics.

The majority of the studies done up to the late 1950s were similar to those previously mentioned. Experiments were carried out at one or two sites over one or more years and involved primarily the fitting of

response curves for one or two applied nutrients and the calculation of economic optima. As soil testing techniques advanced, and the limitations of location specific functions for deriving general fertilizer recommendations came to be realized, more attention was given to the development of generalized production functions. As Munson and Doll (1959, pp. 164-165) noted:

> Few, (soil test-crop yield response studies) however, have considered economic levels of fertilization in conjunction with soil tests or the simultaneous effect of more than one soil-test nutrient in analyzing experimental results.

Incorporation of Soil and Site Characteristics

Apart from the earlier work of Bray (1944), Borden (1946) and others using relative yield concepts, one of the earliest attempts to incorporate soil test variables into empirical response equations was that of Fitts *et al.* (1953). They fitted linear functions relating the change in yield from applying 40 or 60 pounds of nitrogen to the rate of soil nitrification. Utilizing seven years data, they obtained significant results. Their results could have been improved had they incorporated plant stand, weather and rates of fertilizer nitrogen in a combined equation.

In a more sophisticated analysis using a Mitscherlich equation, Hanway and Dumenil (1955) derived a relationship between yield increase in corn and the rate of applied nitrogen and the level of soil nitrification, similar to equation (3). They derived economic optima for a range of soil nitrification rates but also failed to include weather variables, although they did examine a range of fertilizer rates.

Pesek (1956) discussed the use of *total* nutrient variables in a quadratic response function of the form:

$$y = c_1(k\alpha + N_F) + c_2(k\alpha + N_F)^2 + c_3\ (m\gamma + P_F)$$

$$+ c_4\ (m\gamma + P_F)^2 + c_5\ (k\alpha + N_F)(m\gamma + P_F) \qquad (12)$$

where y = yield increase;

α = soil test value of nitrogen;

γ = soil test value of phosphorous;

N_F, P_F = level of applied nitrogen and phosphorous, respectively.

In later work, Jensen and Pesek (1959a, 1959b) extended this model and performed statistical tests to determine whether such yield equations could be effectively generalized to include more than one fertility level. They fitted six separate quadratic equations to greenhouse yield responses to nitrogen and phosphorous applications for three fertility levels on each of two soil types. Then generalized equations similar to (12) were fitted to the data. The coefficients of the applied nutrient variables in the generalized equations were compared with those of the separate equations. They accepted the hypothesis that the coefficients were the same and hence that it is possible to develop a generalized equation to predict yields over a range of initial soil fertility levels. However, they cautioned that there was still a need to adequately quantify the contribution of soil fertility as a production factor in the generalized equation.

Anderson (1956) theoretically quantified the biases that can occur in yield predictions when the differences in initial soil fertility levels between experimental plots and farmers' fields are not recognized. Using this fact, Hurst and Mason (1957) estimated a single variable quadratic response function for total nitrogen which illustrated the

significance of these biases. Using Pesek's (1956) method, they generated an equation which also included initial soil fertility level.

Heady and Pesek (1957) suggested that the relevant functional form was the square root equation for soils which were deficient in nutrients as this allowed non-linear isoclines. Where soils are relatively rich in nutrients, the quadratic was appropriate. They did not specifically mention the inclusion of soil variables in the equations but appeared to suggest grouping soils on the basis of fertility and fitting separate functions. However, in a later paper, Heady (1957) did suggest that soil variables be included.

Stemberger (1957) utilized dummy variables for soil and weather effects on yield response to applied nutrients. He estimated a discrete response model giving only point estimates and a discrete model subject to diminishing returns along the lines of Hildreth (1956). The latter gave more efficient estimators than the discrete model and was of the form:

$$\text{Log } y_{ijk} = b_0 + s_i + w_j + b_1 N_{ijk} + b_2 N_{ijk}^2 + \varepsilon_{ijnt} \qquad (13)$$

where s_i = effect of the ith soil;

w_i = effect of the jth weather condition;

N_{ijk} = k^{th} level of applied nitrogen.

He found that there was little difference in optimum fertilizer rates as derived from the discrete model versus equation (13). Stemberger also compared the economic optima derived by Heady, Pesek and Brown (1955) and Johnson (1953) with that derived when the zero level of applied nutrients was deleted from the analysis. The optima derived for the quadratic, square root, quadratic-square root, Spillman and Cobb-Douglas functions

were all much closer together when the zero level was deleted than was the case when it was included. Stemberger explained this by saying that deletion of the check yields ensured diminishing returns throughout, which implied the slope coefficients were estimated more accurately.

C. H. Hildreth (1957) generated maximum likelihood estimates for λ in a model similar to Pesek (1956) shown in equation (12). Hildreth's model was:

$$y = f(x) + u \tag{14}$$

where $x = N + \lambda W$ = total of applied (N) and soil (λW) nutrient;

λ = factor of proportionality;

u = random error;

f = quadratic function.

He obtained different estimates of λ. The "best" values were calculated by fitting the model in equation (14) with various values of λ included and choosing the value of λ which gave the minimum error sum of squares. These chosen values were then used to generate ordinary least squares estimates of the other parameters.

Brown et al. (1962) used a similar model to Hildreth but utilized measurements of soil nutrient levels before and after fertilizer was added to obtain independent estimates of λ. They found that Hildreth's method of calculating λ gave similar numerical estimates to their method when a square root function was employed, but not when a hyperbolic was used.

In developing theoretical models for nitrogen release, Reuss and Geist (1970) used models similar to Hildreth, although no references

were made to his work. They pointed out that models with λ included are useful only if λ is constant from year to year. Geist et al. (1970) incorporated the models of Reuss and Geist (1970) on some barley-nitrogen experiments in Colorado. Using the coefficient of multiple determination (R^2) and the standard errors of the estimate ($s_{y \cdot x}$), they concluded that a model such as shown in equation (14) is "better" than models with organic nitrogen included as a separate variable or when it is not included at all.

There does not seem to be any particular advantage in attempting to measure a λ factor when the aim is to develop a generalized response function which will accurately estimate the importance of currently available soil nutrient measurements on yield response. In addition, the methods used to estimate λ all involve using the response data numerous times, with the attendant biases this introduces into estimates of the coefficients. Added biases result from the constrained least squares estimating procedure required when a particular λ is chosen.[3] None of the previously mentioned authors recognized this.

Using data from farm surveys, Rust and Odell (1957) employed a quadratic equation to relate corn yield to rates of N, P and K applied in the current and past years, an index of the kind and number of previous legume crops, and rainfall and temperature. The equation was fitted for seven soil types. The primary objective in the study was to determine the relative productivity of major Illinois soils. They found that weather accounted for most of the yield variation. Rust and Odell were

---

[3]For a discussion of this, see Goldberger (1964, pp. 256-258) and Toro-Vizcarrondo and Wallace (1968).

not specifically interested in fertilizer recommendations but rather standardizing yields for management levels which determine the amount of fertilizer applied. This was an attempt to classify soils empirically as opposed to the soil mapping techniques which concentrated primarily on visual and physical characteristics.

Kamprath and Miller (1958) were reasonably successful in predicting soybean yields on a sample of North Carolina farms using soil phosphorous as the only independent variable in a linear regression equation. However, the coefficient of multiple determination (R^2) was 0.28. No attempt was made to include applied phosphorous or the other soil measurements taken (pH, soil K, soil Ca) in a generalized function.

Fitts et al., in an unpublished paper, used 60 location-year experiments on corn in North Carolina to build up, using stepwise regression, a quadratic equation which included soil and applied nutrients and weather variables.[4] The criterion for choosing the "best" fitting equation was the size of the coefficient of multiple determination (R^2). Their final equation was:

$$y = b_0 + b_1\ OM + b_2 D + b_3 N_A + b_{22} D^2 + b_{33} N_A^{\ 2} + b_{23} D \cdot N_A + b_{24} D \cdot pH \qquad (15)$$

where OM = soil organic matter;

---

[4]See Fitts, J. W., D. D. Mason, D. Cooper, W. W. McPherson and J. Havlicek Jr., 1959. Determining yield response surfaces and economically optimum fertilizer rates for corn under various soil and climatic conditions in North Carolina. Paper presented at annual meeting of cooperators with TVA, Knoxville, Tennessee. Contract TV-13434A, Project Agreement No. N. C. 863.

D = drought index;

N_A = applied nitrogen;

pH = soil acidity-alkalinity.

Equation (15) generated an R^2 of 0.57. They compared this "generalized" function with individual location functions in a "cost of the wrong decision analysis," similar to that of Brown and Oveson (1958). The costs of the wrong decision were extremely low assuming the individual location functions were the true models.

Baird and Mason (1959), using similar data to that of Fitts et al., attempted to correlate soil phosphorous and potassium measurements with yield increases from the application of 75 pounds per acre of P_2O_5 and K_2O. Significant correlations were obtained on the Portsmouth soils, but not on the Norfolk soils. No attempt was made to develop a generalized response equation.

Again using the same data as Fitts et al., Nelson and McCracken (1962) developed a linear model to relate corn yield to soil properties. Stepwise regression methods were used to build up an equation. The three most important variables were the drought index, mineral index and soil nitrate production. Other important soil variables were depth of the A horizon, clay content of the B_2 horizon, amounts of acid extractable phosphorous and potassium, and exchangeable aluminum. Their R^2 values were generally higher than those of Fitts et al. and ranged from 0.66 to 0.94. This was due primarily to the different set of data used by Nelson and McCracken and their use of treatment means instead of replication yields. Seagraves attempted to combine all the soil, weather and applied variables from these North Carolina data into a generalized

quadratic model in an unpublished paper.[5] He concluded that it was more satisfactory to include interaction terms for soil nutrients, NO_3 and P_S, than λ values (see equation (14)). He also compared one general equation for Norfolk and Portsmouth soils compared with separate generalized equations for each series in terms of the fertilizer (soil test) recommendations implied by each. He found that one term for the log of P_S instead of two terms P_S and P_S^2 along with the interaction term with P_A allowed estimation of the optimum level of P_S to have in the soil. However, P_S was acting as a proxy (colinear with) for many other aspects of soil quality, so that this recommendation could not be taken seriously. He suggested a five-step procedure:

(1) Examine individual location regressions for applied nutrients so as to understand which nutrients had responses, group soil series; select functional forms and nutrients to be considered for that soil series;

(2) Examine simple r's for soil and weather variables;

(3) Construct regression analyses of soil and weather variables so as to understand multicollinearity problems;

(4) Partially study interactions among soil and weather variables and the applied nutrients;

(5) Construct general models with as few variables as possible, given what was learned from the partial analyses of the data.

Voss and Pesek (1967) suggested using simple correlation coefficients between independent variables and yield to decide what variables to

---

[5]See Seagraves, J. A. 1971. Agro-economic analysis of data from fertilizer trials: regression procedures. Proposed Econ. Res. Rep., Department of Economics, North Carolina State University, Raleigh.

include in the model when working with corn in Iowa. As with Seagraves' approach, this allows the data to help choose the model. However, since tests of hypotheses are invalidated by this procedure, it would appear to be preferable to select a model in conjunction with soil scientists and agronomists and adhere to it. Laird and Cady (1969) found an "agronomic" model to have better predictive power than a full model with all possible variables included. The latter may allow unimportant site variables to attain significant t-values on their estimated coefficients due only to high multicollinearity with other (agronomically important) variables which may in turn have non-significant coefficients. This particular problem is discussed in detail in Chapter III.

Reporting on fertilizer experiments in Sweden, Johnsson (1963) regarded an R^2 of about 0.45 as "disappointing" when soil and other site variables were included in a response equation. Walker and Long (1966), using stepwise regression procedures on soybean data from Tennessee, derived an R^2 of 0.50 with a quadratic equation containing a total of 17 soil and applied variables and their interactions. Russel (1968) obtained R^2 values of 0.50-0.70 on wheat in Australia using an equation with soil and applied nutrients and weather variables included. Byerlee and Anderson (1969) obtained an R^2 of 0.54 with a quadratic function on wheat in southern Australia. When working with large numbers of observations, one expects to have a lower R^2 than with smaller numbers. Also R^2 is not a good criterion of success when one is interested in the accuracy of the estimate of the parameters of the model, rather than on the ability of the model to predict.

Cate and Nelson (1965) and Cate (1971) worked with the relative yield concept to determine "critical levels" of soil nutrients below

which the probability of yield response to added fertilizer is high and above which it is low. Their model resembles those of von Liebig and Bray. In Cate's words (1971, p. 31):

> ...it is reasonable to expect a sharp break between adequate and non-adequate soil test values, corresponding to an abrupt cessation of further nutrient absorption at the point of maximum relative growth ....The critical level marks a sharp division between soils in which the nutrient is a strong limiting factor and those in which the supply of the nutrient is relatively adequate. (My emphasis.)

Cate then goes on to say (1971, p. 32):

> The decision of the plant to respond or not should be the main factor influencing the farmer's decision to spend or not. Soil fertility analyses provide the information to link the two.

The interpretation to be made of Cate's work is that he would recommend no added nutrients if the soil analyses were above the "critical level." If the analysis is below the "critical level" he would recommend the addition of nutrients. The latter appears to beg the question: how much should be applied? In addition, many soil scientists or agronomists disagree with the contention that crops respond in step function fashion.[6] Most soil laboratories recognize that the fitting of Mitscherlich type relative yield curves to soil analyses is generally the "best." Rouse (1968, p. 12) points out that such relative yield analysis "...does not indicate the addition needed to produce the yield potential on a soil that is deficient to some degree." He says that it is necessary to integrate fertilizer rate studies with soil analyses before recommendations can be made. Both Rouse (1968) and Ferrari (1959) plot separate relative

---

[6]For example, see Bray (1944, 1958), Holford (1966), Jordan et al. (1966), Colwell (1967b) and Rouse (1968).

yield curves for several rates of applied nutrients. Using these, recommendations are tailored for specific soil analyses. An example of these types of curves is shown in Figure 1.

In Cate's work, the amounts of applied nutrients used to generate his "step functions" are not held constant and are generally not specifically mentioned. It could be his functions are a hybrid of points like A, B, C, D, E, F, G and H in Figure 1.[7]

Fuller (1965), employing an exponential model, used information on previous yield outcomes to specify the optimum rate of nitrogen on corn. This was compared to applying a fixed rate of nitrogen each year. His model was:

$$Y_{tj} = \alpha_t + \beta_t \gamma^{N_{Ttj}} + b R_t \bar{Y}_j + \varepsilon_{tj} \tag{16}$$

and

$$N_{Ttj} = N_{Atj} + c_t + a_1 u_{t-1} + a_2[R_{t-1} u_{t-1} - E(R_{t-1} u_{t-1})] \tag{17}$$

$$c_t = N_{A,t-1} (c - de^{-kN_{A,t-1}}) \tag{18}$$

where Y_{tj} = yield per acre in year t of jth treatment;

N_{Ttj} = total nitrogen available in year t for treatment j;

N_{Atj} = applied nitrogen in year t on treatment j;

---

[7]It has since been discovered by Professor L. A. Nelson that some of the evidence Cate (1971) adduced in support of his "step function" hypothesis of crop response was incorrect. Cate cited lower R^2 values for the majority of his curvilinear relative yield curves compared to his "step function" R^2 values. On the basis of the refitting of one of the Mitscherlich functions, it appears Cate may have underestimated the R^2 values for these functions. The refitting of all of Cate's Mitscherlich functions, however, has yet to be completed to confirm this. Private communication, Professor L. A. Nelson, North Carolina State University, 1971.

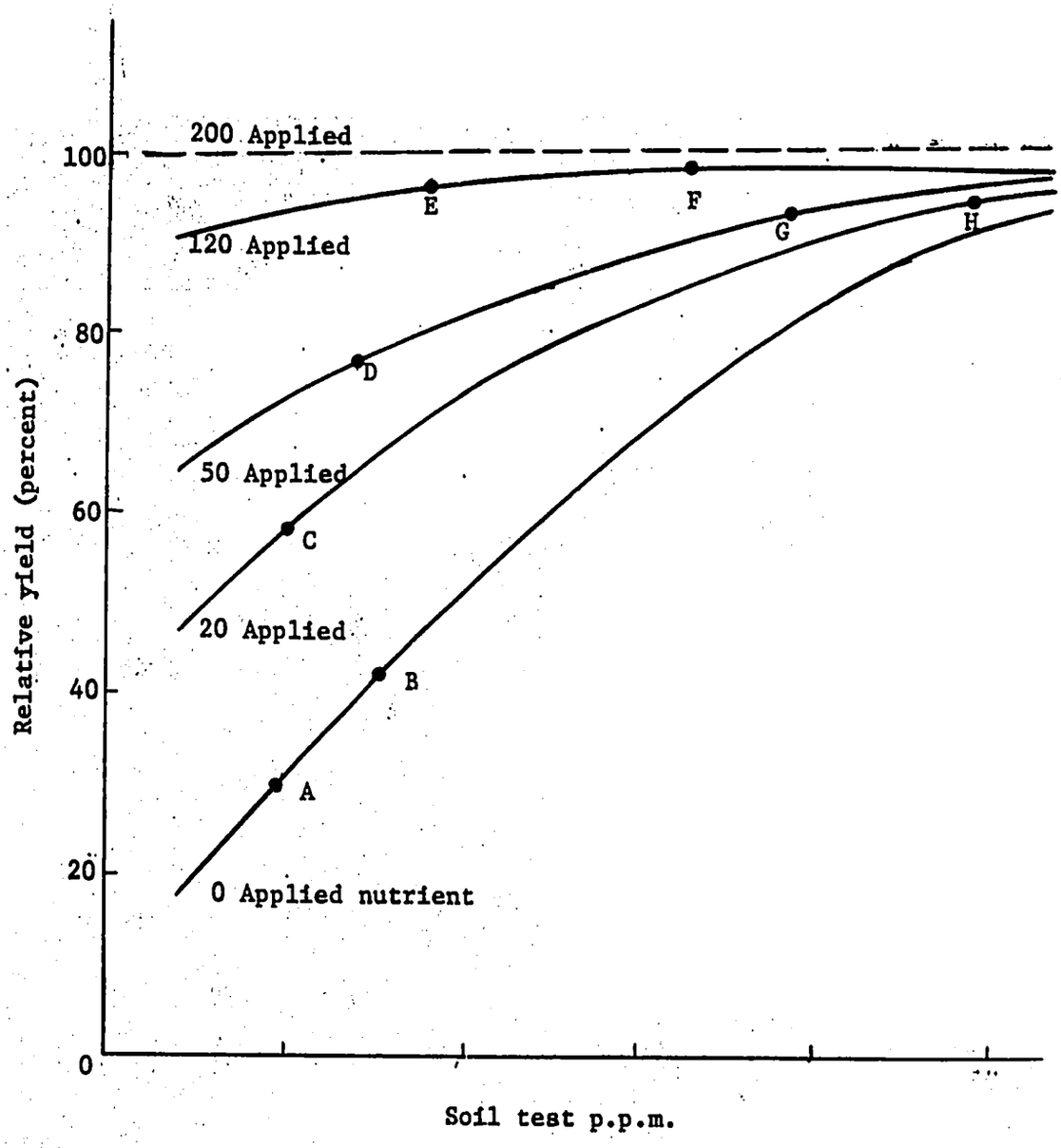

Figure 1. Relationship between soil tests and relative yield for various levels of applied nutrients

R_t = deviation from average rainfall;

$\bar{Y}_j$ = mean of treatment j;

c_t = carryover of available nitrogen from year t-1 to t;

u_t = deviation of Y_{tj} from expected value.

There appears to be little advantage in using the carryover analysis of Fuller compared to placing a variable which is equivalent to his c_t directly into an equation together with the amount of nutrient applied. Shrader et al. (1966) did this with an exponential model using corn-nitrogen data in Iowa. Their model was:

$$Y_{jk} = a + \gamma\, e^{\beta(N_j + N_k)} \tag{19}$$

where Y_{jk} = mean yield of kth nitrogen level of the jth soil;

N_j = available nitrogen of jth soil;

N_k = applied nitrogen.

The inclusion of weather variables might have improved their model as they found that the a and $\gamma\, e^{\beta N_j}$ components varied significantly from year to year.

Holford (1966) found that when response data were grouped on the basis of soil types, the R^2 values for separate square root equations involving soil and applied nutrients were much higher than for a combined equation with the same variables. However, he examined single nutrients only in each equation. It is likely that with more location variables included in his equation the "fit" would have improved with the combined equation. The equations with relative yield as the dependent variable gave a larger number of significant coefficients and had higher R^2 than when absolute total or incremental yields were used. Relative yield calculations standardize for many of the effects of location

variables such as harvest dates, soil physical conditions, crop varieties, climate and other nutrient deficiencies without actually using them. It is hence expected that such equations will have improved explanatory power. However, the deficiencies of the concept, as outlined earlier, remain.

Contrary to Hanway and Dumenil (1955), Jensen and Pesek (1959a, 1959b) and Reuss and Geist (1970), Colwell (1967a, 1967b, 1968) formulated a model which specified that crop yield response to added nutrients is functionally dependent on the level of soil nutrients and other relevant "site" variables. These other writers contended that the soil nutrient level shifted the origin of the response to applied nutrients. Colwell's single nutrient model was:

$$Y_{ij} = g_0 (T_i) + g_1 (T_i) X_{ij}^{1/2} + g_2 (T_i) X_{ij} \tag{20}$$

where Y_{ij} = yield at the ith site with the jth level of fertilizer;

T_i = soil test value for site i;

X_{ij} = the jth rate of fertilizer applied at the ith site;

g_i = functional relationship.

The magnitudes of the response coefficients (i.e., $g_0 (T_i)$, $g_1 (T_i)$, $g_2 (T_i)$) are a function of the soil test values, T_i, as shown in (20). For example, we may have as the exact form:

$$Y_{ij} = (a_0 + a_1 T_i^{1/2} + a_2 T_i) + (b_0 + b_1 T_i^{1/2} + b_2 T_i) X_{ij}^{1/2}$$

$$+ (c_0 + c_1 T_i^{1/2} + c_2 T_i) X_{ij}. \tag{21}$$

Colwell (1967a, p. 279) stated that if an ordinary least squares analysis was performed using a model like (21), the calculated residuals within

sites would be correlated and this would invalidate the significance tests. However, if all relevant site variables are included in the g functions, there is no reason to expect autocorrelation of residuals within sites. Furthermore, Laird and Cady (1969) have shown that if appropriate error mean squares are used when combining experiments, the statistical tests are not invalidated. If the blocks are random, then location (main) effects such as the coefficients of soil and weather variables, are tested against the pooled block within location error. Treatments (applied nutrient coefficients) and treatment x location interactions are tested against the combined experimental error if locations are considered fixed. If locations are random, treatments are tested against treatment x location interaction, as Cochran and Cox indicate (1950, pp. 394-396).

Colwell uses a combination of orthogonal polynomials and simultaneous regressions to attempt to overcome the problem of non-homogeneity of residuals between sites. He first fits an orthogonal response function to each site of the form:

$$Y_{ij} = c_{0i}\,\delta_{0j} + c_{1i}\,\delta_{1j} + \ldots\ldots + c_{\gamma-1,i}\,\delta_{\gamma-1,j} \qquad (22)$$

where γ = fertilizer treatment;

δ_{ij} = orthogonal polynomial coefficients;

c_{ki} = regression coefficients for polynomials of order $k = 0, 1, 2, \ldots, \gamma-1$.

The c_{ki} are then used as dependent variables in a series of k regressions with i observations in each, relating the k values of c at each site to location variables such as soil nutrient levels, pH and rainfall. Ordinary tests of significance are then made on these regressions and significant

coefficients are inserted back in (22) to estimate the generalized function in a form like (21).

Colwell does not make it clear how his orthogonal polynomials combined with simultaneous regressions actually overcomes the error heterogeneity problem. Colwell (1968) and Colwell and Esdaile (1968) have used the procedure to estimate generalized response functions for wheat and potatoes in Canada, and wheat in Australia, respectively.

Methods Employed to Measure Weather

The proper specification of the effects of climatic variables on yield response to added nutrients has come to be recognized as an important part of the development of "generalized" response functions. Invariably, studies that combined experiments over time and space showed that significant portions of yield variance can be explained by weather variability. Furthermore, the interaction between weather and other variables such as soil and applied nitrogen can materially affect the economic analysis. When weather is regarded as a random variable, the variance of profits resulting from increasing rates of application of nitrogen generally increases rapidly. This fact can affect the recommendations which might be made to farmers regarding "optimum" fertilizer rates. Knetsch (1959), Colyer (1969), Tollini (1969), Tollini and Seagraves (1970) and Colyer and Kroth (1970) have illustrated this. If farmers have any aversion to risk, then the amount of fertilizer they apply may be well below the point where marginal cost equals marginal revenue. This is elaborated upon in Chapter IV.

Several studies have used simple measures of rainfall and temperature as their weather variables, often with considerable success. One of

the earlier attempts was R. J. Hildreth's (1957) who used total growing season rainfall as one of the independent variables in a linear function to represent forage response. He then used rainfall records to calculate the probability that it will rain sufficiently to make it profitable to add fertilizer.

Rust and Odell (1957), Engelstad and Doll (1961) and Bondavalli et al. (1970) all used rainfall and temperature variables in studies involving quadratic, square root and Mitscherlich response functions. All found that these climatic variables explained a significant part of yield variability. Oury (1965) found that better predictions of aggregate corn yields in the United States were obtained with rainfall and temperature as independent variables than by calculation of complicated aridity indices. Doll (1967) found that rainfall explained 67 percent of aggregate yield deviations from the trend in the United States.

Drought indices have been used frequently in response studies. Using the Van Bavel criterion, Knetsch and Smallshaw (1958) demonstrated how to calculate the number of drought-days in a growing season. This involves consideration of daily rainfall, moisture loss, evaporation and soil water-holding capacity. Knetsch and Parks (1958) using the Knetsch and Smallshaw (1958) data for Tennessee fitted a quadratic yield equation to millet, involving nitrogen and drought-days as independent variables. Two experimental sites were involved and irrigation used to simulate a range of drought-days. In later papers, Knetsch (1959) and Parks and Knetsch (1959, 1960) set up drought indices which were calculated by regressing the mean yield of various irrigation treatments against the number of drought-days each treatment generated in each month. They included linear, quadratic and interaction terms among the periods. The

magnitudes of the coefficients derived from the regressions (excluding the constant term) were used as weights in defining the drought index for inclusion in the overall regression involving nitrogen rates and the index. Significant coefficients and very high R^2 values were obtained. They used the equations to measure the probable benefits from irrigation, given the probability distribution of the drought index in the absence of irrigation. Havlicek and Seagraves (1962) preferred the use of *á priori* weights on drought-days or moisture deficits to derive a drought index rather than the regression procedure of Knetsch (1959). As explained by Seagraves in an unpublished paper, the regression procedure measures the index with error and also uses the same yield data as in the ultimate response equation.[8] The R^2 values using the two types of indices were sometimes in favor of the *á priori* weights and sometimes in favor of the regression weights, although no statistical tests of differences were given. The *á priori* weights are also measured with some unknown error, although Seagraves does not mention this.[8] The evidence for *á priori* weights hence does not appear conclusive. It would have been interesting if they had also compared simple rainfall and temperature variables with the various indices.

Ewalt *et al.* (1961) compared simple rainfall and two methods of computing drought-days to explain corn yields in Missouri from 1905 to 1959. The first drought-day measure used Penman's method of calculating evapotranspiration in the soil water budget and the second used Thornwaite's.

---

[8]See Seagraves, J. A. 1961. Quantification of climatic factors for crop response functions. Paper presented at the TVA Symposium, Muscle Shoals, Alabama.

Penman's method generally gave slightly higher R^2 values when total drought-days and rainfall in the growing season were used. When weekly measures were employed, the equation with rainfall gave the highest R^2. When monthly or weekly measures were used, the drought-day variables exhibited multicollinearity whereas the rainfall variable did not.

Another popular technique for measuring weather effects has been Fisher's (1925) polynomial method. He developed the technique to explain the effects of weather on wheat yields at Rothamstead. Hendricks and Scholl (1943) used this idea to explain corn yields in Ohio, Indiana and Iowa. The essence of Fisher's model was that yield (y) was a function of a weather factor (or factors) X, at n different periods:

$$y = A_0 + A_1X_1 + A_2X_2 + \ldots + A_nX_n \tag{23}$$

He reasoned that the A_i could be expressed by a kth degree polynomial function involving time, t:

$$A_t = a_0 + a_1t + a_2t^2 + \ldots a_kt^k \tag{24}$$

and that if we use a quadratic (k=2), the following equations are obtained:

$$\begin{aligned} A_1 &= a_0 + a_1 + a_2 \\ A_2 &= a_0 + 2a_1 + 2^2a_2 \\ &\vdots \\ A_n &= a_0 + na_1 + n^2a_2 \end{aligned} \tag{25}$$

When these are substituted into (23), we obtain

$$y = A_0 + a_0(X_1 + X_2 + \ldots X_n) + a_1(X_1 + 2X_2 + \ldots nX_n)$$

$$+ a_2(X_1 + 2^2X_2 + \ldots + n^2X_n), \tag{26}$$

which can be written:

$$y = A_0 + a_0Z_0 + a_1Z_1 + a_2Z_2 \tag{27}$$

where $Z_0 = \sum_{i=1}^{n} X_i$, $Z_1 = \sum_{i=1}^{n} iX_i$, and $Z_2 = \sum_{i=1}^{n} i^2X_i$.

Equation (27) is estimated using regression procedures and the estimated coefficients are used in equation (25) to estimate A_i (i=1, 2, ..., n), the effects of weather on yield in each of the n periods. Instead of 104 separate variables, the effect of annual rainfall and temperature on yield can be represented by nine variables. Runge and Odell (1958, 1960) and Runge (1968) utilized Hendrick and Scholl's method to explain corn and soybean yields in Illinois.

Fitts _et al_. provided the above description of Fisher's orthogonal polynomial method.[9] Using the 60 location-year corn experiments reported by Baird and Fitts (1957), they compared Fisher's method with the drought index method outlined in Knetsch (1959) and Parks and Knetsch (1959, 1960). On the basis of R^2 values, the models with Fisher's index incorporated were "best." On the other hand, Colwell and Esdaile (1968) found that the Fisher variables using 52 weekly rainfall data for each of 49 sites were insignificant when included in a square root equation to explain wheat yields in Australia. Other variables in the equation were pH, soil phosphorous, soil potassium and clay content.

---

[9]See footnote 4.

Although by no means complete, the review of literature on the preceding pages indicates the type and extent of problems that have confronted those studying crop-fertilizer response.[10] In the pages that follow, specific attention is given to each of these problems in the development of a generalized production function for potatoes in the Sierra region of Peru.

---

[10] For a detailed bibliography on the subject, see Dillon (1968). Hutton (1955) provides a review of empirical work in southern United States prior to 1955 and Heady (1963) reviews United States work prior to 1963.

CHAPTER III
DEVELOPMENT OF RESPONSE EQUATION

The Data

The data chosen for this study were from the cooperative potato field experiments carried out in Peru by Professor R. E. McCollum of the North Carolina State University Mission to Peru, Dr. Carlos Valverde S. et al. of the Departmento de Suelos y Abonos, Estacion Experimental Agricola de La Molina, and Sven Villagarcia et al. of the Universidad Agraria.

The data from experiments conducted in 1960-64 were taken from the appendices of the publication by McCollum and Valverde S. (1968, pp. 48-55). Data from 30 experiments at 20 locations were used from this period. The locations included 1 in 1960-61, 11 in 1961-62, 6 in 1962-63, and 2 in 1963-64. These were all factorial experiments involving various levels of applied nitrogen (N_A), phosphorous (P_A) and postassium (K_A). On each of the 30 experiments measurements were taken of soil pH, percentage of organic matter in the soil (N_s), soil phosphorous (P_s) and soil potassium (K_s). The data from other experiments in McCollum and Valverde S. (1968) were not included as soil analyses were not available. Generally, each observation included in the analysis consisted of the mean of four replications in a randomized complete block design, including the soil analyses. The original data by replications were not available.

Unpublished data containing the results of 60 potato field experiments over the period 1967-1970 for 45 locations were obtained from Professor R. E. McCollum and his colleagues at the Estacion Experimental Agricola de La Molina in Lima, Peru. These trials were made up of central

composite and factorial response surface designs. Most were trials to determine response to N_A, P_A and K_A, but some examined the time of application and others compared various sources of nutrients.

The treatments from the time of application trials where P_A and K_A were applied at planting (siembra) and N_A was applied at planting and/or prior to the first cultivation (aporque) were used for the analysis.[11] All other treatments were not used. An analysis of variance was made on the 20 experiments which compared various sources of nitrogen, phosphorous and potassium on potato yield to determine if there were significant differences between sources. Twelve of the 20 experiments showed no significant differences among sources in potato yield response and these were included. The complete data from all the N_A, P_A and K_A rate experiments were included in the analysis also. The number of locations used for each year of the 1967-70 data were 12 in 1967-68, 17 in 1968-69 and 16 in 1969-70.

Other criteria for selection of experiments for inclusion in the analysis were that each had soil analyses made and that rainfall and temperature data were available. These data were necessary for development of the generalized production function. The 1967-70 data included yields and soil analyses for each block in the experiment, unlike the 1960-64 data, which, as previously mentioned, contained the mean analysis of four replications only.[12]

---

[11] This was done after consultation with Professor McCollum who stated that these data would be comparable with the N_A, P_A, K_A rate experiments.

[12] Electrical conductivity was also measured, but not considered relevant in the later analysis as salt problems were not evident in any of these experiments.

Overall, the experimental data consisted of 3,075 observations. Each observation involved eight variables--potato yield, level of soil organic matter, soil phosphorous, soil potassium, soil pH, applied nitrogen, applied phosphorous and applied potassium. Due to the experimental designs employed and the limited number of soil analyses per experiment, each of the 3,075 observations does not have unique values for each variable. For example, the 1960-64 experiments each have the same soil test values applying to all treatments used in the experiment. Appendix A Table 1 contains the means and standard deviations of all variables used in the analysis.[13]

Climatic data were not available for each of the 65 locations. The best that could be done was to use data from nine weather stations at Cajamarca, Huamachuco, Otuzco, Huancayo, Cuzco, Puno, Asangaro, Huaraz and Huancavelica. The data from these nine stations were applied to all experiments in their vicinity. A total of 22 different sets of "region by year" climatic data were utilized.

The Model

The aim is to develop a model which adequately represents the response of potatoes to soil nutrients, soil pH, applied nutrients and weather. In consultation with Professor R. E. McCollum, and using the concept of Colwell (1967a) that the response coefficients of applied nutrient variables are functionally dependent on soil and climatic variables, the following model was specified:

---

[13] Due to space limitations, the data are not presented here. A computer listing is, however, held by the author.

$$y = S + b_0 + b_1 N_s + b_2 P_s + b_3 K_s + b_4 pH + b_5 N_A + b_6 P_A$$
$$+ b_7 K_A + b_8 R + b_9 T + b_{11} N_s^2 + b_{22} P_s^2 + b_{33} K_s^2$$
$$+ b_{44} pH^2 + b_{55} N_A^2 + b_{66} P_A^2 + b_{77} K_A^2 + b_{88} R^2$$
$$+ b_{99} T^2 + b_{14} N_s \cdot pH + b_{15} N_s \cdot N_A + b_{18} N_s \cdot R$$
$$+ b_{19} N_s \cdot T + b_{26} P_s \cdot P_A + b_{25} P_s \cdot N_A + b_{37} K_s \cdot K_A$$
$$+ b_{46} pH \cdot P_A + b_{56} N_A \cdot P_A + b_{57} N_A \cdot K_A + b_{58} N_A \cdot R$$
$$+ b_{59} N_A \cdot T + b_{67} P_A \cdot K_A + b_{68} P_A \cdot R + b_{89} R \cdot T + \varepsilon \qquad (28)$$

where y = yield of potatoes in units of 100 kilograms per hectare,

S = a dummy variable for magnesium deficient locations,

b_0 = constant term,

N_s = soil organic matter percentage,

P_s = soil phosphorous in ppm of P,

K_s = soil potassium in ppm of K,

pH = soil pH,

N_A = applied nitrogen in kilograms per hectare,

P_A = applied P_2O_5 in kilograms per hectare,

K_A = applied K_2O in kilograms per hectare,

R = total rainfall in the six months October-March, in millimeters,

T = mean of minimum daily temperatures in January and February in degrees centigrade, and

ε = random error.

The quadratic form was chosen because it is simple to work with and represented the functional form most used in previous response studies.[14] It is also amenable to the usual tests of significance using ordinary least squares regression analysis and allows diminishing returns, which is expected to occur in these data. As the square-root function has also proved to be adequate in many studies, it is planned to fit an equation like (28) but with the squared terms deleted and replaced by variables raised to the 1/2 power and similarly with the interactions.[15] The Mitscherlich function was not considered because it has the severe disadvantage that when there is more than one variable, it does not allow variable input proportions. Furthermore, it becomes virtually impossible to estimate empirically due to the iterative methods which must be employed.

The expected signs of the coefficients in equation (28), determined in consultation with Professor McCollum and based on the work of McCollum and Valverde S. (1968) and Zapater (1970), are contained in Table 1. The uncertain sign on $pH \cdot P_A$ is due to the fact that at pH levels around five or less, the sign is expected to be negative. For pH levels in excess of seven, it is expected to be positive. It is uncertain between these ranges. Uncertain responses to K_A in the past led to the uncertainty of the $N_A \cdot K_A$ interaction. The sign of the $R \cdot T$ interaction depends on the levels of R and T, as in the case of $pH \cdot P_A$.

---

[14] In a paper by Heady (1963), of the 31 empirical studies he reviewed, 26, or 84 percent of them, either used the quadratic or stated that the quadratic or square-root functions fitted the data "best." Abraham and Rao (1966), Hutton (1955) and Hartley (1964) also preferred the quadratic and square-root functions.

[15] Mason (1957) illustrated the power of the square-root function with Anderson's (1956) data.

Table 1. Expected signs of coefficients in the quadratic model

Coefficient	Variable	Expected sign
	S	-
b_1	N_s	+
b_2	P_s	+
b_3	K_s	+
b_4	pH	+
b_5	N_A	+
b_6	P_A	+
b_7	K_A	+
b_8	R	+
b_9	T	+
b_{11}	N_s^2	-
b_{22}	P_s^2	-
b_{33}	K_s^2	-
b_{44}	pH^2	-
b_{55}	N_A^2	-
b_{66}	P_A^2	-
b_{77}	K_A^2	-
b_{88}	R^2	-
b_{99}	T^2	-

Table 1 (continued)

Coefficient	Variable	Expected sign
b_{14}	$N_s \cdot pH$	+
b_{15}	$N_s \cdot N_A$	-
b_{18}	$N_s \cdot R$	+
b_{19}	$N_s \cdot T$	+
b_{25}	$P_s \cdot N_A$	Uncertain
b_{26}	$P_s \cdot P_A$	-
b_{37}	$K_s \cdot K_A$	-
b_{46}	$pH \cdot P_A$	Uncertain
b_{56}	$N_A \cdot P_A$	+
b_{57}	$N_A \cdot K_A$	Uncertain
b_{58}	$N_A \cdot R$	+
b_{59}	$N_A \cdot T$	+
b_{67}	$P_A \cdot K_A$	+
b_{68}	$P_A \cdot R$	+
b_{89}	$R \cdot T$	Uncertain

For several reasons, simple rainfall and temperature variables were included in (28) instead of more complicated Fisher polynomials or drought indices. The first was that the drought indices require soil and daily rainfall measurements that were not available at each site. Second, Fisher's polynomials require a series of equally-spaced observations which were not available and furthermore they are of most use in reducing the number of weather variables to a manageable number. As we were only using two variables, the latter need did not arise. Third, rainfall and temperature variables are simple to work with. Fourth, probability density functions are relatively easy to determine for rainfall and temperature. Finally, Ewalt et al. (1961) found rainfall in any one week was less serially correlated with rainfall in any other week whereas drought days by weeks were highly correlated. When several weekly or monthly weather variables are used in a regression equation, simple rainfall and temperature may hence involve less multicollinearity problems.

Econometric Analysis

Generalized Least Squares Regression

The usual assumptions about the random error term ε in equation (28) are that:

$$E[\varepsilon] = 0 \tag{29}$$

$$E[\varepsilon\varepsilon'] = \sigma_\varepsilon^2 I \tag{30}$$

where σ_ε^2 = constant population variance parameter of the ε distribution.

The ordinary least squares estimator of the coefficients of (28) is obtained from the relation in (31):

$$\hat{b} = [X'X]^{-1} X'Y \tag{31}$$

where X is a 3,075 x 35 matrix,

Y is a 3,075 x 1 vector, and

$\hat{b}$ is a 35 x 1 vector.

The $\hat{b}$ estimators have the properties of unbiasedness and minimum variance if assumptions (29) and (30) hold.

The potato data for the 1960-64 period contained 420 observations of the mean of four replications, whereas the later 1967-70 data contained 2,655 observations from individual block data. Hence, assuming the population variance parameter σ_ε^2 to be the same in both, the properties of the random error were:

$$E \begin{bmatrix} \underset{m_1 \times 1}{\varepsilon_1} \\ \hline \underset{m_2 \times 1}{\varepsilon_2} \end{bmatrix} = \begin{bmatrix} \underset{m_1 \times 1}{0} \\ \hline \underset{m_2 \times 1}{0} \end{bmatrix} \tag{32}$$

where $m_1 = 420$, $m_2 = 2{,}655$, $n = 35$ and

$$E \left[\begin{bmatrix} \underset{m_1 \times 1}{\varepsilon_1} \\ \hline \underset{m_2 \times 1}{\varepsilon_2} \end{bmatrix} \begin{bmatrix} \underset{1 \times m_1}{\varepsilon_1} & \underset{1 \times m_2}{\varepsilon_2} \end{bmatrix} \right] = \begin{bmatrix} \underset{m_1 \times m_1}{\sigma^2 I} & \underset{m_1 \times m_2}{0} \\ \hline \underset{m_2 \times m_1}{0} & \underset{m_2 \times m_2}{4\sigma^2 I} \end{bmatrix}$$

$$= \sigma^2 \underset{(m_1 + m_2)\times(m_1 + m_2)}{\begin{bmatrix} I & 0 \\ \hline 0 & 4I \end{bmatrix}} \tag{33}$$

$$= \sigma^2 \Omega \, . \tag{34}$$

The Ω matrix is positive definite and to obtain linear, unbiased estimators of the b's in equation (31) with minimum variance, Aitken's Generalized Least Squares was used. The solution vector, as described in Goldberger (1964, pp. 231-238) is:

$$\tilde{b} = [X' \, \Omega^{-1} \, X]^{-1} \, [X' \, \Omega^{-1} \, Y] \, . \tag{35}$$

Now

$$\Omega^{-1} = \left[\begin{array}{c|c} I & 0 \\ \hline 0 & \frac{1}{4} I \end{array}\right]_{(m_1 + m_2) x (m_1 + m_2)}$$

$$= \left[\left[\begin{array}{c|c} I & 0 \\ \hline 0 & \frac{1}{2} I \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline 0 & \frac{1}{2} I \end{array}\right]\right]_{(m_1 + m_2) x (m_1 + m_2)}$$

$$= [M] \, [M] \, . \tag{36}$$

Hence (35) may also be written:

$$\tilde{b} = [X'M \, M \, X]^{-1} \, [X' \, M \, M \, Y]$$

or

$$\tilde{b} \, [X^{*\prime} \, X^*]^{-1} \, [X^{*\prime} \, Y^*] \tag{37}$$

where $X^{*\prime} = X'M$,

$X^* = M \, X$, and

$Y^* = M \, Y$.

The results of fitting the quadratic model F_1 (equation 28) using Aitken's Generalized Least Squares are shown in Table 2. The residual mean square from the combined regression analysis is not the appropriate error term to use in tests of significance on the coefficients of F_1. It is composed of the true residual error, σ_ε^2 plus any lack of fit error, σ_L^2, arising from failure to adequately specify the model. Furthermore, the residual errors within each location may be correlated, which invalidates the usual statistical tests. These problems are discussed in Cochran and Cox (1950, pp. 391-413).

Using Laird and Cady's (1969) method of assuming the blocks to be random and locations and treatments fixed, the analysis of variance structure for our data is as shown in Table 3. The main effect of locations (i.e., the variables involving only N_s, P_s, K_s, pH, R and T in the regression) is tested against the blocks in locations error, while treatments (N_A, P_A and K_A) and location by treatment interaction terms are tested against the pooled error. Appendix A Table 2 contains the results of the analysis of variance on the Peruvian potato data. The t-values in Table 2 were calculated using the formula:

$$t^* = \frac{t \cdot s_R}{s_A} \tag{38}$$

where t^* = adjusted t-value,

t = t-value computed from regression,

s_R = square root of residual error mean square from regression, and

s_A = square root of block in location or pooled error mean squares, as appropriate (from Appendix A Table 2).

Table 2. Generalized quadratic production function F_1 obtained from potato experiments in Peru, 1960-70[a]

Variable	$\tilde{b}$ coefficient[b]	t-value[c]
S	-198.1930***	-28.72
b_0	-1761.3330***	-19.18
N_s	8.8847***	12.87
P_s	-0.1955***	- 4.67
K_s	0.06462***	23.58
pH	14.3407***	14.63
N_A	-0.1476	- 1.14
P_A	1.01760***	9.75
K_A	0.8532***	22.69
R	0.1024***	10.27
T	26.3905***	15.12
N_s^2	-0.01432***	-10.87
P_s^2	0.001052***	9.66
K_s^2	-0.0000049***	-12.68
pH^2	-0.1147***	-15.33
N_A^2	-0.002089***	-17.23
P_A^2	-0.001903***	- 9.92
K_A^2	-0.004596***	-25.91
R^2	0.00000027	0.40
T^2	-0.1255***	-16.01
$N_s \cdot pH$	0.01794***	4.29
$N_s \cdot N_A$	-0.0002581	- 0.47
$N_s \cdot R$	-0.0005579***	-13.25

Table 2 (continued)

Variable	$\tilde{b}$ coefficient[b]	t-value[c]
$N_s \cdot T$	-0.07517***	-21.17
$P_s \cdot N_A$	-0.0004347***	- 2.43
$P_s \cdot P_A$	-0.002171***	-13.18
$K_s \cdot K_A$	-0.0001531***	-12.50
$pH \cdot P_A$	-0.01664***	-17.58
$N_A \cdot P_A$	0.002317***	12.84
$N_A \cdot K_A$	0.0008246***	4.56
$N_A \cdot R$	0.00005193***	4.62
$N_A \cdot T$	0.006960***	7.42
$P_A \cdot K_A$	0.001701***	8.54
$P_A \cdot R$	0.00006971***	7.18
$R \cdot T$	-0.001222***	-10.28
R^2	0.47	
C.V.	40.75 percent	

[a]Coefficients are associated with soil nitrogen (N_s) in percent organic matter x 10, soil phosphorous (P_s) in p.p.m. x 10, soil potassium (K_s) in p.p.m. x 10, (pH) x 10, applied nitrogen (N_A), applied P_2O_5 (P_A) and applied K_2O (K_A) in kilograms per hectare, rainfall (R) in millimeters x 10, mean minimum temperature (T) in degrees centigrade x 10.

[b]***, **, * indicate significance at the 0.01, 1 and 5 percent levels, respectively, using a two-tailed test.

[c]These are the adjusted t-values as explained in the text.

Table 3. Analysis of variance structure for combined analysis of potato experiments, Peru, 1960-1970[a]

Sources	Degrees of freedom	Expected mean square[b]
Locations[c]	$\ell-1$	$\sigma_\varepsilon^2 + t\,\sigma_B^2 + tb\,\frac{\Sigma(\gamma_j-\bar{\gamma})^2}{\ell-1}$
Treatments[c]	$t-1$	$\sigma_\varepsilon^2 + \ell b\,\frac{\Sigma(\tau_i-\bar{\tau})^2}{t-1}$
Blocks in locations[c]	$\ell(b-1)$	$\sigma_\varepsilon^2 + t\cdot\sigma_B^2$
Locations x treatments[c]	$(\ell-1)(t-1)$	$\sigma_\varepsilon^2 + b\cdot\sigma_{L.T}^2$
Pooled error	$\ell(b-1)(t-1)$	σ_ε^2

[a]This table is applicable to cases where there is a balanced design. In the case of the potato data, not all treatments appeared in every location, hence the analysis of variance structure is not exactly as shown in the table. See Appendix A Table 2 for the calculated analysis of variance.

[b]ℓ, t and b are the number of locations, treatments and blocks respectively; σ_ε^2, σ_B^2, and $\sigma_{L.T}^2$ are the variances associated with experimental error, blocks and locations x treatments, respectively; γ_j and $\bar{\gamma}$ are the location mean yields ($j = 1, 2, \ldots, \ell$) and the overall mean of all locations, respectively; τ_i and $\bar{\tau}$ are treatment mean yields and the overall treatment mean, respectively.

[c]The lack of fit error, σ_L^2, would be included in the locations, treatments, blocks in locations, locations x treatments expected mean squares.

A two-tailed t-test was used because, although we had a set of expected signs in Table 1, it was not felt these were held with sufficient conviction to justify the one-tailed test. Table 2 shows that all variables but N_A, R^2 and $N_s \cdot N_A$, which are not significant at 5 percent, have coefficients significant at the 0.01 percent level. The R^2 was 0.47 and the F-test was highly significant. The signs on P_s and N_A were negative while Table 1 shows they were expected to be positive. This is not a serious flaw in the case of N_A as the squared term is negative as expected, which means we have diminishing returns and a maximum point. With P_s^2 we have an unexpected positive sign which generates increasing returns and a minimum point. This implies the data range for P_s was such that yields did not begin to decline at high levels. The $N_s \cdot N_A$ term was nonsignificant although of the correct sign. This reinforces Professor McCollum's belief that organic matter may not be a good predicter of response to N_A in the high altitudes of the Sierra region. The highly significant negative coefficients on $N_s \cdot R$ and $N_s \cdot T$ were not expected. Upon further discussion with Professor McCollum after he was shown these results, he said they were not unreasonable. High organic matter soils are often poorly drained and tend to occur at higher elevations. High rainfall may not be advantageous in these instances. The negative sign on $N_s \cdot T$ is not as easy to justify, but in retrospect, McCollum thought the sign could be positive or negative. The coefficient on R^2 was unexpectedly positive although not significant.

Generalized least squares was also used to fit the square root function (F_2) to the data. The results are shown in Appendix A Table 3. All variables were significant at the 5 percent level except $\sqrt{N_A}$, P_s, R, $\sqrt{P_s \cdot N_A}$ and $\sqrt{N_A \cdot R}$. The R^2 of 0.47 was highly significant. The expectation

was that S would have a negative coefficient, all the single variable square root coefficients would be positive, all the linear terms negative, and all the square root interactions the same as shown in Table 1. On this basis, the square root function had correct signs on 25 of the 30 variables for which it was possible to denote an expected sign. As in F_1, the signs on $\sqrt{N_s \cdot R}$, $\sqrt{N_s \cdot T}$, R and $\sqrt{N_A}$ were not as expected. In addition, $\sqrt{N_s \cdot pH}$ had the wrong sign in F_2, although it was correct in F_1. Contrary to F_1, model F_2 had the correct signs on the $\sqrt{P_s}$ and P_s coefficients.

The quadratic equation F_1 and the square root F_2 were rerun with the rainfall and temperature variables R and T, and all their interactions deleted. In their place, 65 location dummies and seven year dummies were constructed. The purpose of this was to determine how the climatic variables fared in explaining yield variance over space and time.[16] The R^2 values increased from 0.47 to 0.84 for both equations when the dummy variables were included. However, F_3 and F_4, the dummy versions of F_1 and F_2 respectively, contained more soil test variables that were not significant at the 5 percent level than F_1 and F_2. In F_3 these soil variables were K_s, pH, K_s^2, $N_A \cdot P_s$ and $N_s \cdot pH$. In addition, three of the 65 location dummies were not significant as well as $N_A \cdot K_A$. Only two of the seven dummies for years were significant. In F_4 the soil variables $\sqrt{K_s}$, K_s, and P_s were not significant in addition to P_A and $\sqrt{N_A \cdot K_A}$ and two of the 65 location dummies. Only two of the seven dummies for years were significant in F_4. The signs on the coefficients

---

[16] Equation F_3 is contained in Appendix A Table 4 and F_4 is in Appendix A Table 3. Only the continuous variables are shown in these tables. The dummy variables are not included.

N_A, $N_s \cdot pH$ and $P_s \cdot N_A$ in F_3 were the opposite to those in F_1. In F_4 the signs on $\sqrt{P_s}$, $\sqrt{N_A}$, P_s, P_A and $\sqrt{N_s \cdot pH}$ were the opposite to those found in F_2.

The inclusion of location dummies caused a number of the coefficients for soil variables to become nonsignificant, especially for the quadratic (F_3). This indicates that the soil variables were explaining part of locational differences. In the presence of the location dummies, there apparently is not sufficient variation left for the soil variables to explain. The large increases in the coefficient of multiple determination when the location dummies were included suggests that the soil variables, rainfall and temperature, may not explain locational differences very well. Some other soil or environmental factors are involved. However, as the R and T variables and their interactions in F_1 and F_2 are significant, they do affect yield response. Also, large increases in the coefficient of multiple determination are expected when the number of variables is increased from 35 to 93. As long as the remaining variation between locations has the effect of raising or lowering the response function by a fixed amount (as the location dummies do), the inferences drawn in later economic analyses will not be affected.

Tests for Specification Bias

Cady and Laird (1969) and Anderson and Nelson in an unpublished paper[17] have examined the problem of fitting a model which may not be the "true" one. Cady and Laird suggested that various degree polynomials

---

[17] See Anderson, R. L. and L. A. Nelson. 1971. Some problems in the estimation of single nutrient response functions. Unpublished Mimeograph, Department of Statistics, North Carolina State University, Raleigh.

be fitted to the data and the one with the smallest lack of fit mean square chosen. Anderson and Nelson have observed that many response data seem to reach a yield plateau at relatively low nutrient levels and do not decline until fairly high nutrient levels. They postulated that the Mitscherlich-type function of the form shown in equation (3), Chapter II, which allows yield to be positive at zero levels of applied nutrient, would more adequately represent "true" response, at least over the relevant economic range (approximately AB in Figure 2). They suggested that if a quadratic model were fitted to data generating such a yield plateau, the estimated maximum yield and its accompanying input level would be biased upward, as would any economic optimum. This is illustrated in Figure 2. The quadratic would underestimate the slope of the response in the region AB and overestimate it in the region BC. Particularly for low value crops with a relatively high fertilizer/crop price ratio, this would overestimate "optimum" fertilizer requirements.[18]

It was decided to use both the quadratic and square root equations F_1 and F_2 to test the hypothesis that specification bias was present when the quadratic and square root equations were fitted when a Mitscherlich-type response was the "true" model. If it was in F_1, then one expects the calculated residuals ($\hat{\varepsilon}$) from F_1 ($\hat{\varepsilon} = Y - \hat{Y}$) to be negative at zero level of fertilizer, positive in the range of A'C', negative between C'E' and positive again thereafter. The square root function should be similar but have a somewhat steeper slope at low fertilizer levels.

---

[18] Note that the Mitscherlich may seriously overestimate maxima and optima when the price ratio is very low, as it is asymptotic with no downturn and with low price ratios the optima and maxima may occur to the right of AC.

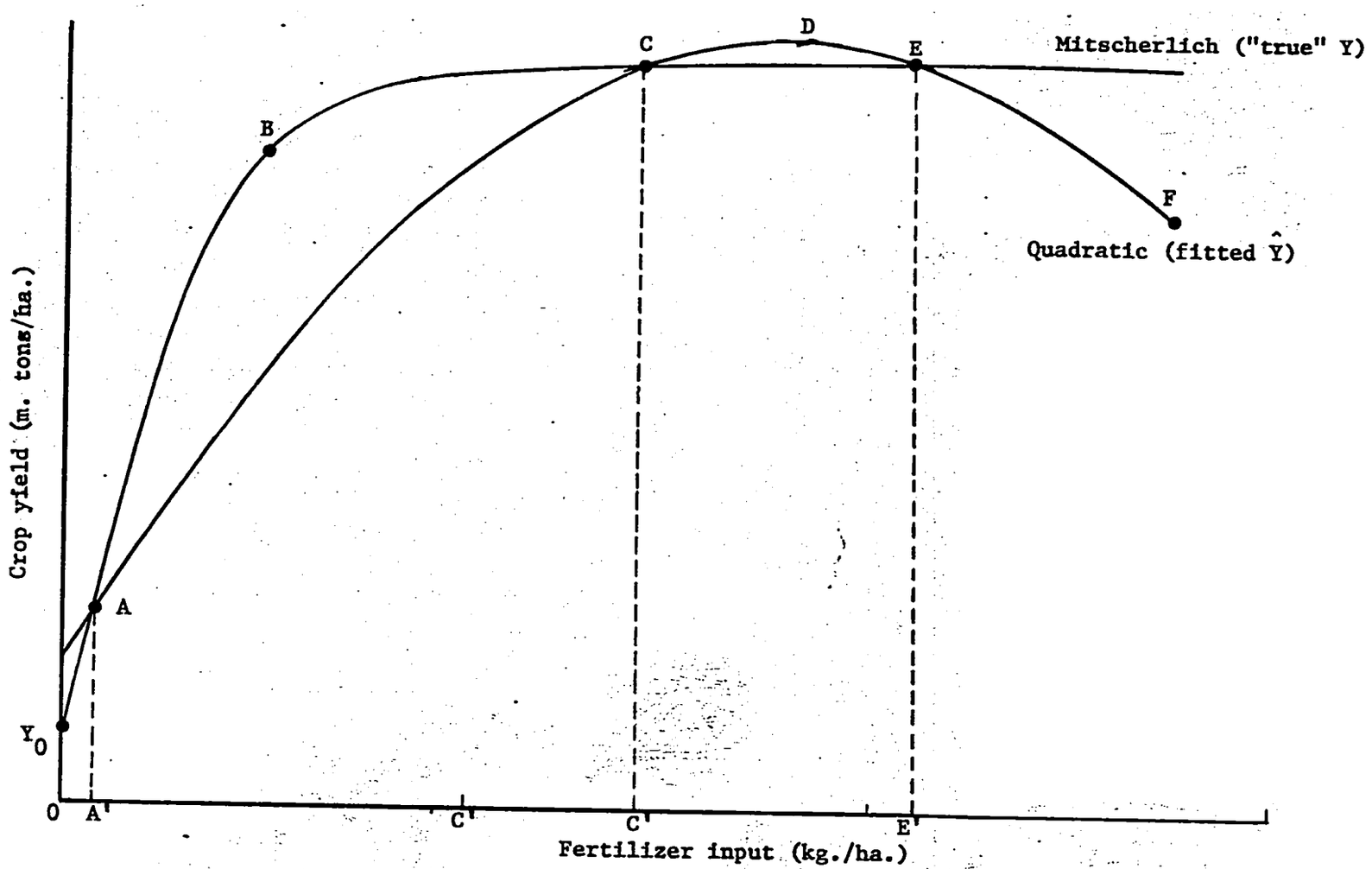

Figure 2. Possibility of specification bias with the quadratic function

To test for this specification bias, the calculated residuals from F_1 and F_2 were plotted against P_A, the level of applied phosphorous. High levels of phosphorous are not likely to damage yields but this variable was chosen simply because it contributed more to the explanation of yield than N_A or K_A. Four intervals were chosen for P_A: 0, 1-65, 66-180 and > 180 kilograms per hectare. Within each of these, the algebraic mean of the residuals was calculated, a t-test was made on each of the means to determine if they were significantly different from zero. Given the hypothesis that the Mitscherlich model is the "true" one over the range ABC, this indicates the probability that we have fitted a quadratic to a set of Mitscherlich-like data. To do this, it was assumed the residual mean square of each equation was the population variance ($\sigma^2_{\varepsilon_i}$) of the computed residuals. Then the 95 percent confidence limits were placed on the residual means using this. That is if:

$$\bar{\hat{\varepsilon}}_{ij} - 1.96 \frac{\sigma_{\varepsilon_i}}{\sqrt{n_j}} \leq 0 \leq \bar{\hat{\varepsilon}}_{ij} + 1.96 \frac{\sigma_{\varepsilon_i}}{\sqrt{n_j}} \tag{39}$$

for the jth class mean ($j = 1, 2, 3, 4$) in the ith equation ($\bar{\hat{\varepsilon}}_{ij}$), then it was not statistically significant from zero at the 5 percent level, where n_j is the number of observations in class j.

Table 4 contains the results of this analysis for equations F_1 to F_4 as well as for P_1 and P_2, which will be discussed presently. The quadratics F_1 and F_3 had significant positive residuals in the 1-65 kilogram class. The square root equation F_2 had a positive residual in the 1-65 class but it was not significant. F_4, the square root equation with the location dummies, had the smallest positive mean residual and

Table 4. Algebraic means of regression residuals ($\bar{\hat{\varepsilon}}_{ij}$) by class intervals of applied phosphorous (P_A)

Range of P_A	Number of observations[a] (n_j)	Equation number[b]					
		F_1	F_2	F_3	F_4	P_1	P_2
(kgs./ha.)							
0	822	-0.101	+0.892	-1.132	-0.123	-0.046	---
1 - 65	373	+12.092**	+3.100	+4.696*	+2.367	+11.083**	+9.589**
66 - 180	2900	-1.629	-0.250	-0.681	-0.585	-1.434[c]	-1.442
> 180	240	+1.233	-4.857	+4.806	+3.815	---	+2.520

[a]These are the number of observations included in the generalized least squares procedure used. The program called for all of the 420 observations for 1960-64 to be repeated four times. Hence, the total of 4335 here is 1260 more than the 3075 actual observations in the original data.

[b]**, * indicate significance at the 5 and 10 percent levels, respectively.

[c]These figures were derived using 2660 as the value of n_j in equation (39) as P_i had 240 observations deleted.

it was also not significant. No other class besides the 1-65 kilogram interval had significant residual means. The quadratics F_1 and F_3 hence appeared to have the worst specification bias on this criterion. However, it is not clear that only having a larger and more statistically significant residual mean in the 1-65 P_A range implies that the radius of curvature of the quadratic is a better or worse estimate of the "true" radius of curvature than the square root. The implication from Anderson and Nelson (see footnote 17) would be that significance of the residual means in the other three classes is also required before it could be positively stated that there definitely was specification bias of the type they hypothesized. This was not the case in equations F_1 and F_3.

To further test if the quadratic did have this specification bias the data were truncated the way Anderson and Nelson suggested.[19] All observations with P_A levels in excess of 187 kilograms per hectare were deleted. This was the level of P_A at which yield was a maximum.[20] This is expected to allow the quadratic to approximate the ABC portion of the Mitscherlich in Figure 2 more closely. Ideally, the data should be deleted beyond a point somewhere around C in Figure 2. However, due to the difficulty of accurately specifying such a point, D was used. The deletion should have the effect of shifting the point of maximum yield to the left

---

[19] See Anderson, R. L. and L. A. Nelson. 1971. Some problems in the estimation of single nutrient response functions. Unpublished Mimeograph, Department of Statistics, North Carolina State University, Raleigh, pp. 15,24.

[20] Equivalent to the point D in Figure 2. It was derived using the mean levels of variables entering the solution. The figure of 187 kg./ha. is much below the level of P_A for maximum yields of 379 kg./ha. when simultaneous solution for all three nutrients is performed.

and of increasing the radius of curvature of the function in the range A'C'. A total of 240 observations was deleted.

When the quadratic F_1 was rerun deleting all data which had $P_A \geq 187$, the result was equation P_1 in Appendix A Table 4. Table 4 in the text shows that P_1 is not significantly better using the residual mean criterion as far as this type of apparent specification bias is concerned than F_1. When the levels of the variables which gave maximum yields were calculated using the simultaneous methods for varying all three nutrients described in Chapter IV, Table 5 shows that P_1 gave much larger values than F_1. The expectation, given the hypothesis that the Mitscherlich was the "true" model, was that the level of P_A that gave maximum yields would be smaller for P_1 than for F_1. However, Table 5 shows the reverse to be the case. The radius of curvature of P_1 in the range A'C' was in fact less than in F_1, which was contrary to what was expected after the truncation. Hence the Anderson-Nelson (see footnote 19) truncation method of correcting this type of apparent specification bias existing in F_1 did not appear to be successful with this multinutrient equation. The results could be different were a further truncation made on the data. There is no criterion as to where or when to cease truncation. This makes it a subjective procedure, particularly when more than one nutrient is involved.

The 1-65 class in Table 4 is the only one exhibiting any significant non-zero residuals. However, the economic optima are likely to occur far to the right of this interval. The prior work of McCollum and Valverde S. (1968, p. 29) indicated optima of 272, 289 and 161 kg./ha. for N_A, P_A and K_A, respectively. Hence, even though there are large deviations at low

Table 5. Values of applied variables to attain maximum physical and economic yields for quadratic models, F_1, F_3, P_1 and P_2 and square root models F_2 and F_4[a]

Variables	Equations							
	F_1	F_2[b]	F_3	F_4[b]	P_1	P_2	M_1	M_2
	Level for maximum physical yields (kg./ha.):							
N_A	383	-	341	-	463	271	261	260
P_A	379	-	445	-	502	277	244	281
K_A	176	-	210	-	190	133	126	140
	Level for economic optimum (kg./ha.):							
N_A	346	-	304	-	414	251	246	239
P_A	336	-	376	-	441	250	221	253
K_A	164	-	185	-	173	126	120	131

[a]Significant and nonsignificant coefficients were included in the calculations.

[b]Equations F_2 and F_4 generated minimum physical yields and profits rather than maxima due to the existence of increasing returns at the mean levels of the soil and weather variables entering the simultaneous solution procedure outlined in Chapter IV.

levels of P_A in Table 4 for the equations F_1, F_3 and P_1, it is not clear that this is necessarily biasing the optima when the price ratio is so small and the optima so large. Since at the upper P_A levels there are no residuals significantly different from zero, the residual means test provides no basis for choosing between the models.

Another possible form of specification bias exists with F_1. This is that there is a phase of increasing returns at low levels of fertilizer such as the range Y_0B of Figure 2. This could be the cause of the relatively poor fit of F_1 to the data at low levels of P_A as shown in

Table 4. To test this hypothesis, equation F_1 was refitted but with all data having $P_A = 0$ deleted. This follows the technique of Stemberger (1957) and Anderson (1957). It was thought that this may reduce specification bias if there were a phase of increasing returns at low levels of P_A. As Table 4 shows, the equation that resulted (P_2) did have smaller residual means in the 1-65 and 66-180 P_A ranges. However, the 1-65 P_A mean was still significantly different from zero at the 5 percent level. Equation P_2 is shown in Table 6. The optima are shown in Table 5, and these are much lower than those obtained in either F_1 or P_1 and indicate it may be preferable on these grounds. The residual mean test gives no strong grounds for choosing between F_1 and P_2, particularly in the upper ranges of P_A where any large residuals are likely to be biasing the optima.

Comparing F_1 and P_2 in Tables 2 and 6, we see that they have similar sized coefficients on all but seven variables. The terms P_s, $P_s \cdot N_A$, $P_s \cdot P_A$, $P_A \cdot K_A$ and $P_A \cdot R$ in P_2 all have the same signs as their counterparts in F_1 but their absolute values are substantially different. This is no doubt due to the fact that the truncation involved primarily the variable P_A and this affected the coefficients of variables with P_s or P_A in them. In P_2, the variables N_A and $N_s \cdot N_A$ had nonsignificant positive coefficients but in equation F_1 the signs were negative and not significant. This again casts some doubts about the ability of soil organic matter percentages (N_s) to accurately predict the probable response to applied nitrogen (N_A). The variable R^2 had a significant positive coefficient in P_2 but in F_1 it was positive and not significant.

The radius of curvature of P_2 is larger than that of F_1 in the relevant economic range, and the residual mean test of Table 4 indicates it

Table 6. Generalized quadratic production function P_2 obtained from potato experiments in Peru, 1960-1970[a]

Variable	$\tilde{b}$ coefficient[b]	t-value[c]
S	-217.3824***	-28.50
b_0	-2084.2329***	-20.37
N_s	8.4717***	11.32
P_s	-0.3306***	-5.64
K_s	0.07329***	23.83
pH	13.6744***	12.24
N_A	0.1839	1.16
P_A	1.1604***	7.72
K_A	1.0122***	18.97
R	0.1052***	9.77
T	34.7394***	17.88
N_s^2	-0.01280***	-8.80
P_s^2	0.001359***	10.84
K_s^2	-0.00000594***	-14.03
pH^2	-0.1022***	-11.85
N_A^2	-0.002427***	-16.78
P_A^2	-0.001688***	-5.02
K_A^2	-0.004565***	-23.90
R^2	0.00000301***	4.04
T^2	-0.1666***	-19.00
$N_s \cdot pH$	0.02091***	4.38
$N_s \cdot N_A$	0.0002613	0.40
$N_s \cdot R$	-0.0005285***	-11.33

Table 6 (continued)

Variable	$\tilde{b}$ coefficient[b]	t-value[c]
$N_S \cdot T$	-0.07705***	-19.97
$P_S \cdot N_A$	-0.001124***	-5.25
$P_S \cdot P_A$	-0.0008689**	-3.67
$K_S \cdot K_A$	-0.0001659***	-10.90
$pH \cdot P_A$	-0.02289***	-15.29
$N_A \cdot P_A$	0.001684***	5.65
$N_A \cdot K_A$	0.0006271**	2.86
$N_A \cdot R$	0.00003912**	3.15
$N_A \cdot T$	0.006384***	6.07
$P_A \cdot K_A$	0.0008900**	2.92
$P_A \cdot R$	0.0001106***	9.25
$R \cdot T$	-0.001696***	-12.86
	$R^2 = 0.43$	
	C.V. = 39.23	

[a]See footnote a in Table 2.

[b]***, **, * indicate significance at the 0.01, 1 and 5 percent levels, respectively, using a two-tailed test.

[c]These are adjusted t's as explained in the text.

has slightly smaller residuals in this range. Thus, some of the apparent misspecification at low levels of P_A was perhaps caused by a phase of increasing returns. Removal of observations involving zero level of P_A improved the fit at low levels of P_A, but only marginally, and not so in the statistical sense. This marginal improvement in the fit with P_2 however, did have a marked effect on the economic and physical optima as shown in Table 5. The P_2 optima are about 100 kg./ha. below those in F_1 for N_A and P_A and about 40 kg./ha. below for K_A. Also, the P_2 optima are within the range of the experimental data, except for P_A, which only just exceeds the range, whereas in F_1, the optima for N_A and P_A far exceeded the data range.[21]

Table 5 shows how some other functional forms fared in generating physical and economic optimum fertilizer levels. The wide variation in Table 5 indicates that model specification can alter the optima considerably. The square root equations F_2 and F_4, although there was no indications from Table 4 that they had any Anderson-Nelson specification bias, exhibited increasing returns and thus provided no estimates of optimum application levels. This cautions against complete reliance on the residual mean test as a method of selecting among models with different specifications or data subdivisions.

The two quadratic equations F_1 and F_3 gave realistic optima as Table 5 illustrates. However, the optimum levels for N_A and P_A exceeded the data range used to derive the equation F_3 which has the location and year dummies generated lower physical and economic optima for N_A,

---

[21] The experimental maxima for N_A, P_A and K_A were 300, 240 and 320 kg./ha., respectively.

but unrealistically higher optima for P_A and K_A compared to F_1. The reason for this appears to be the deletion of the $N_A \cdot R$ and $N_A \cdot T$ terms from F_3, which were significantly positive in F_1. This had the effect of lowering the marginal product curve of N_A, thus lowering the optima. The higher optima for P_A and K_A are a result of flatter responses to these nutrients in F_3. P_1, as previously explained, gave unexpectedly higher optima than F_1 and cannot be considered a useful equation.

The final choice of equations seems to be between F_1 and P_2. As previously stated, the residual mean test did not indicate that P_2 was a clear improvement on F_1. Furthermore, as we saw with the square root models F_2 and F_4, specifications which pass the residual mean test may fail other tests such as computation of the economic and physical optima. It cannot be relied upon as an adequate screening device with multinutrient equations.

On the grounds that P_2 generated optima which appear more "realistic" to agronomists and soil scientists than F_1, it is chosen as the equation on which subsequent analyses will be made. There is no test known to the author which would enable a more objective choice to be made between F_1 and P_2. It is entirely possible that other model specifications and/or data truncations would be preferred to F_1 or P_2 which have not been considered here. Table 5 indicates that the computed optima are sensitive to the specification of the model and hence that the economic implications of alternative specifications may be different to those derived using P_2.

Hildreth (1956) has discussed the disadvantages of continuous models when specification bias is present. These biases do not disappear in large samples. He developed a discrete model with built-in restrictions such as diminishing marginal returns. In such form-free models, Hildreth

says the inefficiencies disappear as sample size increases. However, these models require iterative solutions which become difficult with more than one variable.

Another possible approach which could be employed to attempt to increase the radius of curvature of the quadratic would be to subdivide the data by level of application and fit two separate quadratics to the two halves of the data. The two equations could then be spliced together to generate a single model. A recent example of the use of this grafting procedure is the work of Fuller (1969). He specifies a point C in the data space of the independent variable where the production function assumes a different set of response parameters. Using the example of a single variable quadratic function, he demonstrates how to set up restrictions on the parameter space so that the curve and the derivative are continuous at the point C. The restrictions are linear in the parameters and reduce the number of independent parameters in the model. The parameters of the grafted model are estimated by using a constrained least squares regression model. The technique can be used to join several quadratics and could represent a curve with both increasing and decreasing returns. Fuller also illustrates the use of the procedure for functions in two or more variables. The choice of the point(s) where the functions are to be grafted is not specifically discussed by Fuller and is an important consideration in work of this nature. Quandt (1960), Robison (1964) and Hudson (1966) discuss the estimation of these "join" points. It would be of interest to attempt to fit two or more quadratics to these potato data and graft them in the way that Fuller illustrates.

The Problem of Multicollinearity

There was evidence that there was multicollinearity among some of the independent variables. Of the 578 possible simple correlations between the 34 independent variables, only 55 or 10 percent of them were not significant at the 5 percent level. The great majority of them were highly significant at the 0.01 percent level. Appendix A Table 5 contains some simple correlation coefficients for a number of variables.

Another indication of multicollinearity was the smallness of some of the eigenvalues (roots) of the original X'X matrix. If the eigenvalues are close to zero, the absolute size of the estimated coefficients is increased compared to their "true" values and so is their variance. The smallest eigenvalue was -32.3. This was small compared to the largest eigenvalue of 5.08 times 10^{18} and is close to zero in a relative sense only.

The fact that equation P_2 had a number of variables with unexpected signs on their coefficients, which were statistically significant, is also an indication that multicollinearity may be a problem.

Writers such as Engelstad and Doll (1961), Colwell and Esdaile (1966), Laird and Cady (1969), Colwell and Stackhouse (1970) and Seagraves (see footnote 25) have experienced problems of multicollinearity among the location variables. The procedures used to counter this problem have generally been either stepwise regression techniques, where variables are sequentially added according to the magnitude of their contribution to regression sum of squares or deletion of all nonsignificant variables. Recently, Hoerl and Kennard (1970a, 1970b) developed the ridge regression technique, which enables one to measure the effect that multicollinearity has on the estimated regression coefficients. If we consider the model:

$$Y = X\beta + \varepsilon \tag{40}$$

where Y is an m x 1 matrix, X is an m x n matrix of full rank, β is the n x 1 vector of population parameters and ε an m x 1 vector of experimental errors with the properties $E(\varepsilon) = 0$, $E(\varepsilon\varepsilon') = \sigma_\varepsilon^2 I_m$, the least squares estimate of β is

$$\hat{\beta} = (X'X)^{-1} X'Y. \tag{41}$$

If L^2 is the squared distance between $\hat{\beta}$ and β, Hoerl and Kennard (1970b) show that:

$$L^2 = (\hat{\beta} - \beta)' (\hat{\beta} - \beta)$$

and $$E(L^2) = \sigma_\varepsilon^2 \text{ Trace } (X'X)^{-1}$$

or $$E(\hat{\beta}'\hat{\beta}) = \beta'\beta + \sigma_\varepsilon^2 \text{ Trace } (X'X)^{-1}.$$

If the eigenvalues of (X'X) are:

$$\lambda_{max} = \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots\ldots \geq \lambda_n = \lambda_{min} > 0$$

then the average squared distance from $\hat{\beta}$ to β is given by:

$$E(L^2) = \sigma_\varepsilon^2 \sum_{i=1}^{n} \frac{1}{\lambda_i} > \sigma_\varepsilon^2/\lambda_n. \tag{42}$$

Hence, if the data results in an X'X matrix with one or more small eigenvalues, the average distance between $\hat{\beta}$ and β will be large. This is what occurs with multicollinearity in the data.

The ridge regression procedure tests the senstivity of the least squares solution to changes in the value of a small constant, Θ, added

to the diagonal elements of the X'X matrix. Adding Θ to the diagonal elements moves the smallest eigenvalue away from zero. The ridge solution is:

$$\hat{\beta}^* = [X'X + \Theta\, I]^{-1}\, X'Y \tag{43}$$

Ridge traces, showing the solution to equation (43) for 12 values of Θ are shown in Figures 3 through 8 using the truncated data of equation P_2. The twelve values of Θ were chosen in the range 2 to 67×10^{13}. For values of Θ in excess of this, the $\hat{\beta}^*$ coefficients all went to zero. The computation in (43) was accomplished by augmenting the original Generalized Least Squares X* matrix of equation (33) to derive Z where

$$Z_{(m+n)\times n} = \begin{bmatrix} X^*_{m \times n} \\ \hline \sqrt{\Theta}\, I_{n \times n} \end{bmatrix} \tag{44}$$

and $$W_{(m+n)\times 1} = \begin{bmatrix} Y^* \\ \hline 0 \end{bmatrix}$$

where m = original number of observations $(m_1 + m_2)$ and

n = number of parameters to be estimated,

and solving

$$\hat{\beta} = (Z'Z)^{-1}\, Z'\, W$$

or

$$\hat{\beta}^* = \left[\left[X^{*\prime} \;\vdots\; \sqrt{\Theta}\, I \right] \begin{bmatrix} X^* \\ \hline \sqrt{\Theta}\, I \end{bmatrix} \right]^{-1} \left[\left[X^{*\prime} \;\vdots\; \sqrt{\Theta}\, I \right] \begin{bmatrix} Y^* \\ \hline 0 \end{bmatrix} \right]$$

$$= \left[X^{*\prime}X + \Theta\, I \right]^{-1} \qquad X^{*\prime}\, Y^*$$

which is the same as equation (43).

According to Hoerl and Kennard (1970a, 1970b), if the $\hat{\beta}^*$ values exhibit great instability as Θ increases from zero, this is an indication that they have been affected by multicollinearity in the data. They suggest to choose as the estimates the vector of $\hat{\beta}^*$ values where they become relatively stable with respect to Θ while not sacrificing too much in increased error sum of squares.[22] The error sum of squares for the P_2 ridge analysis is shown in Figure 8.

Compared with the examples given by Hoerl and Kennard (1970a), the ridge trace in Figures 3 through 8 exhibit a great degree of instability. Many of the variables have their coefficients change sign; notably, N_A, T, R, R^2, $K_s{}^2$, R•T, N_s•pH and N_s•R. The simple correlation coefficients in Appendix A Table 5 show R and T to be highly correlated, also N_s and pH, N_s and T, K_s and P_A, K_s and R, and P_A and K_A. Explanations of the behavior of these coefficients lie largely in their correlations with other variables.

It appears that at a Θ value of approximately 17×10^5 (i.e., a $\log_{10} \Theta$ of about 6.3 in the figures) the coefficients reach some stability. The error sum of squares increases from some 19×10^6 to about 25×10^6, an increase of 32 percent at this value of Θ.

Recently, Wallace in an unpublished paper developed a statistical criterion for testing whether constrained estimators such as $\hat{\beta}^*$ are better or worse than ordinary least squares estimators $\hat{\beta}$.[23] At least one

---

[22] Hoerl and Kennard (1970b, pp. 58-59) show that the residual sum of squares for the solution to (43) is the ordinary least squares error plus $\Theta^2 \hat{\beta}^{*\prime} (X'X)^{-1} \hat{\beta}^*$. Hence the residual is a positive function of Θ.

[23] See Wallace, T. D. 1971. Weaker criteria and tests for linear restrictions in regression. Unpublished Mimeograph, North Carolina State University, Raleigh. Forthcoming in Econometrica.

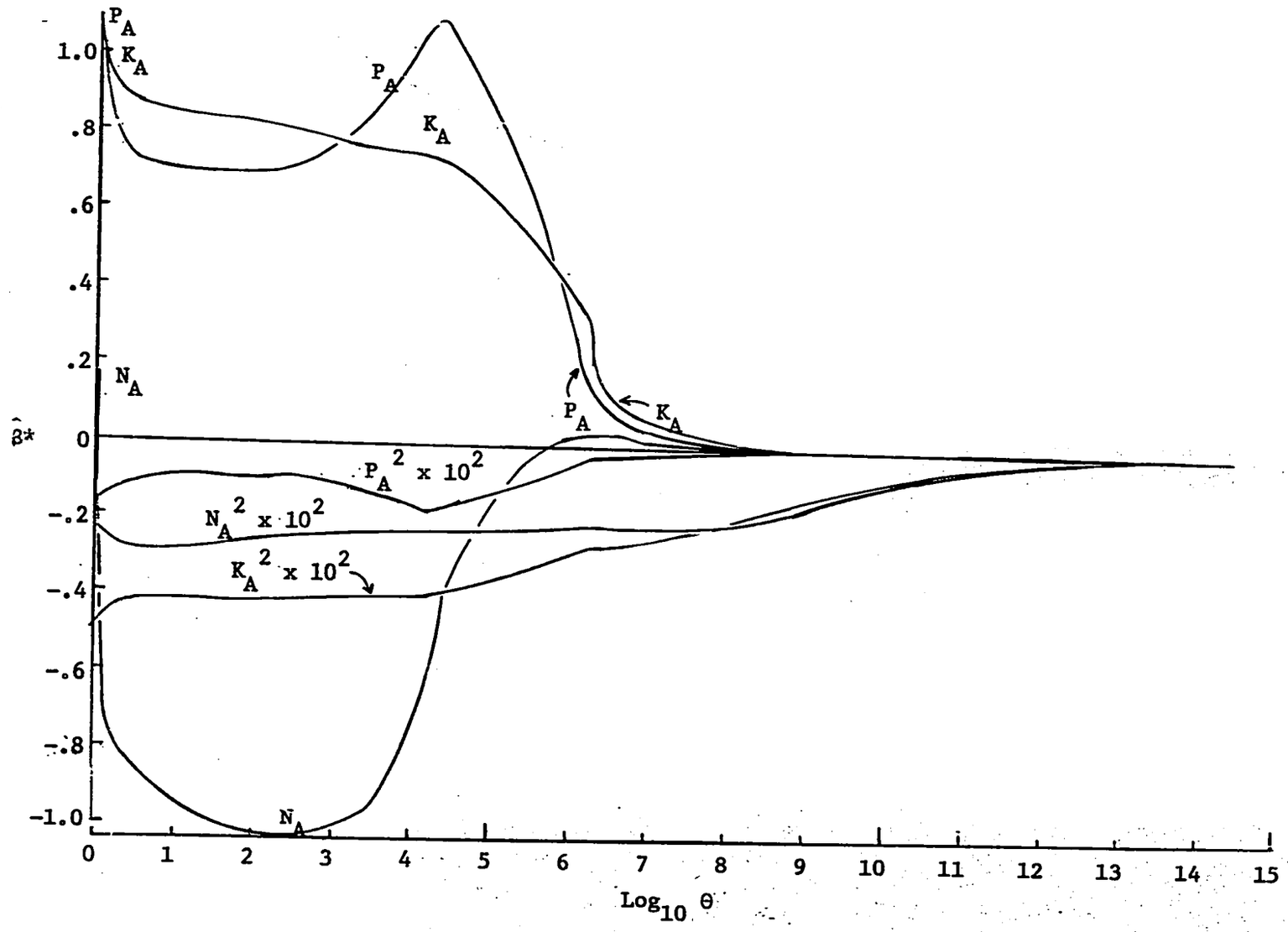

Figure 3. Ridge trace of coefficients of N_A, P_A, K_A, N_A^2, P_A^2, K_A^2 for P_2

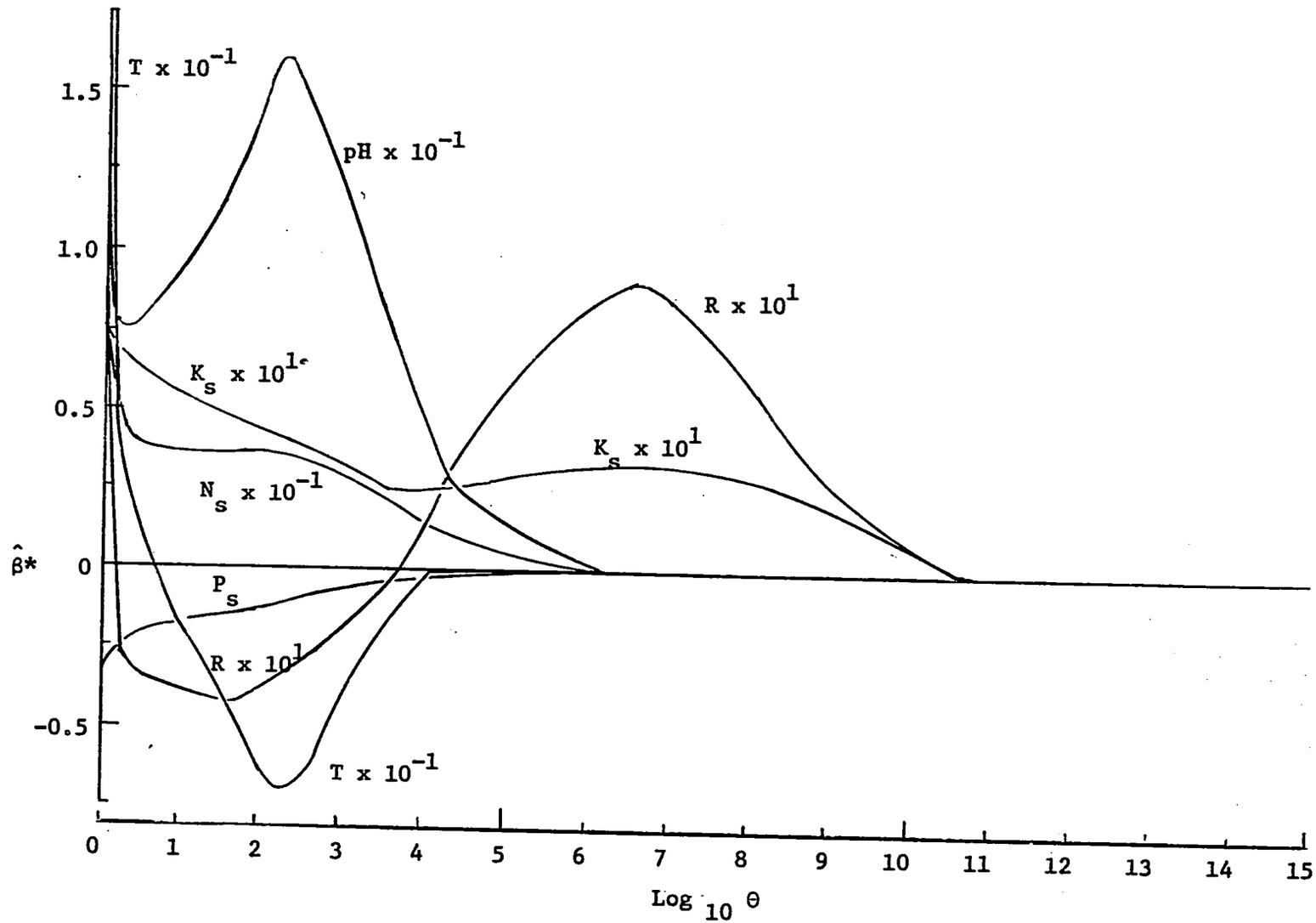

Figure 4. Ridge trace of coefficients of N_s, P_s, K_s, pH, R, T for P_2

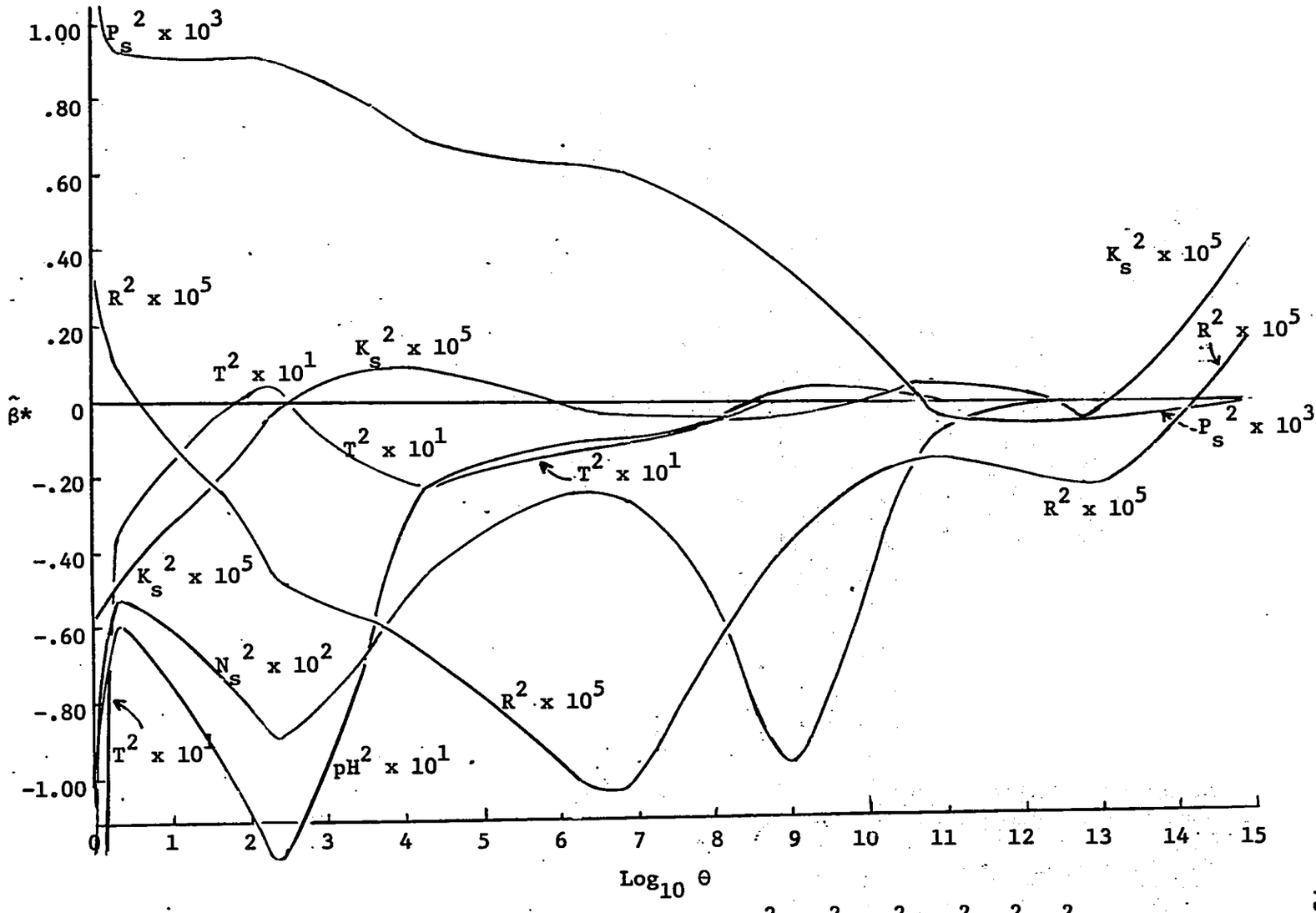

Figure 5. Ridge trace of coefficients of N_s^2, P_s^2, K_s^2, pH^2, R^2, T^2 for P_2

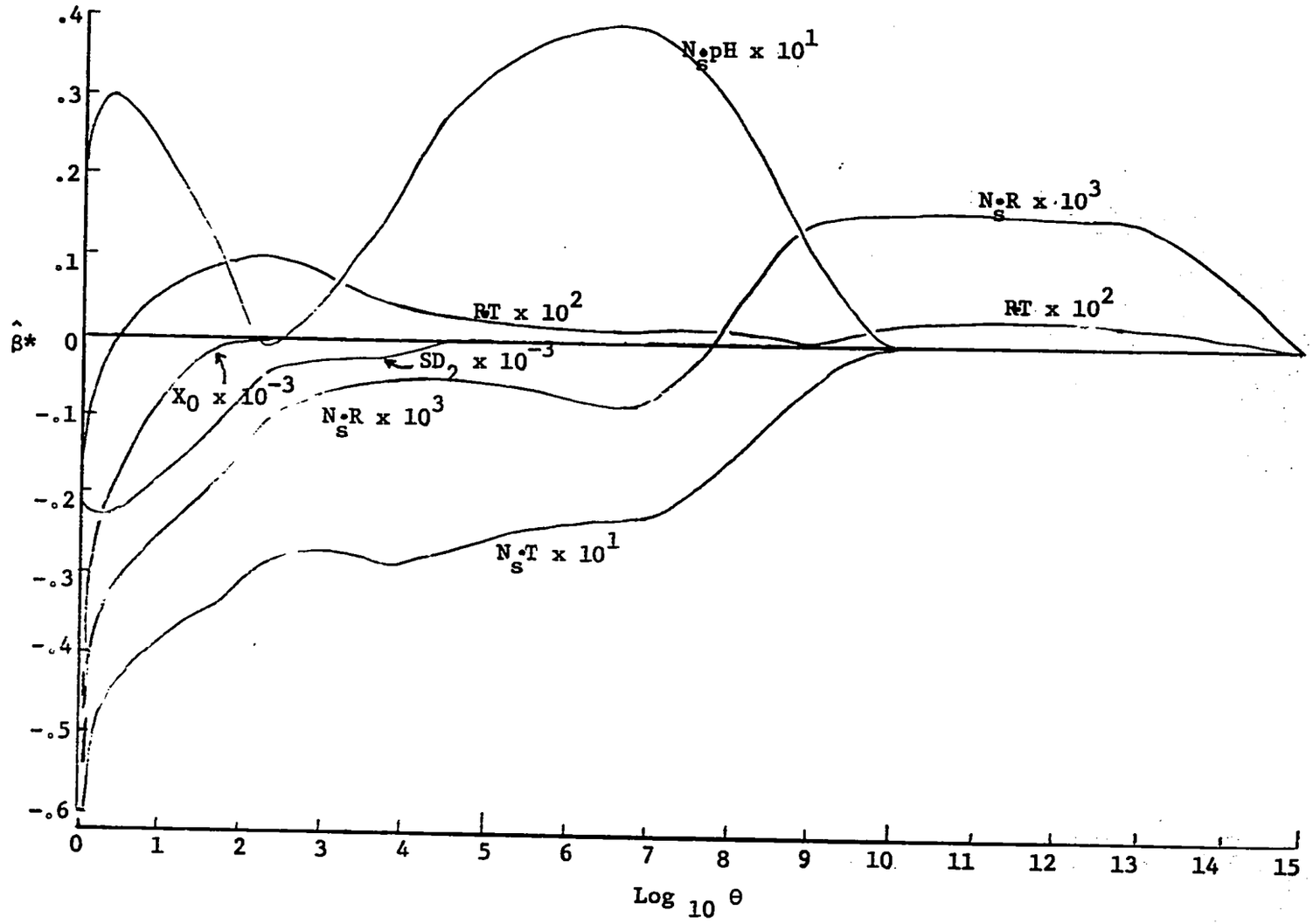

Figure 6. Ridge trace of coefficients of X_0, $N_s \cdot pH$, $N_s \cdot R$, $N_s \cdot T$, RT, SD_2 for P_2

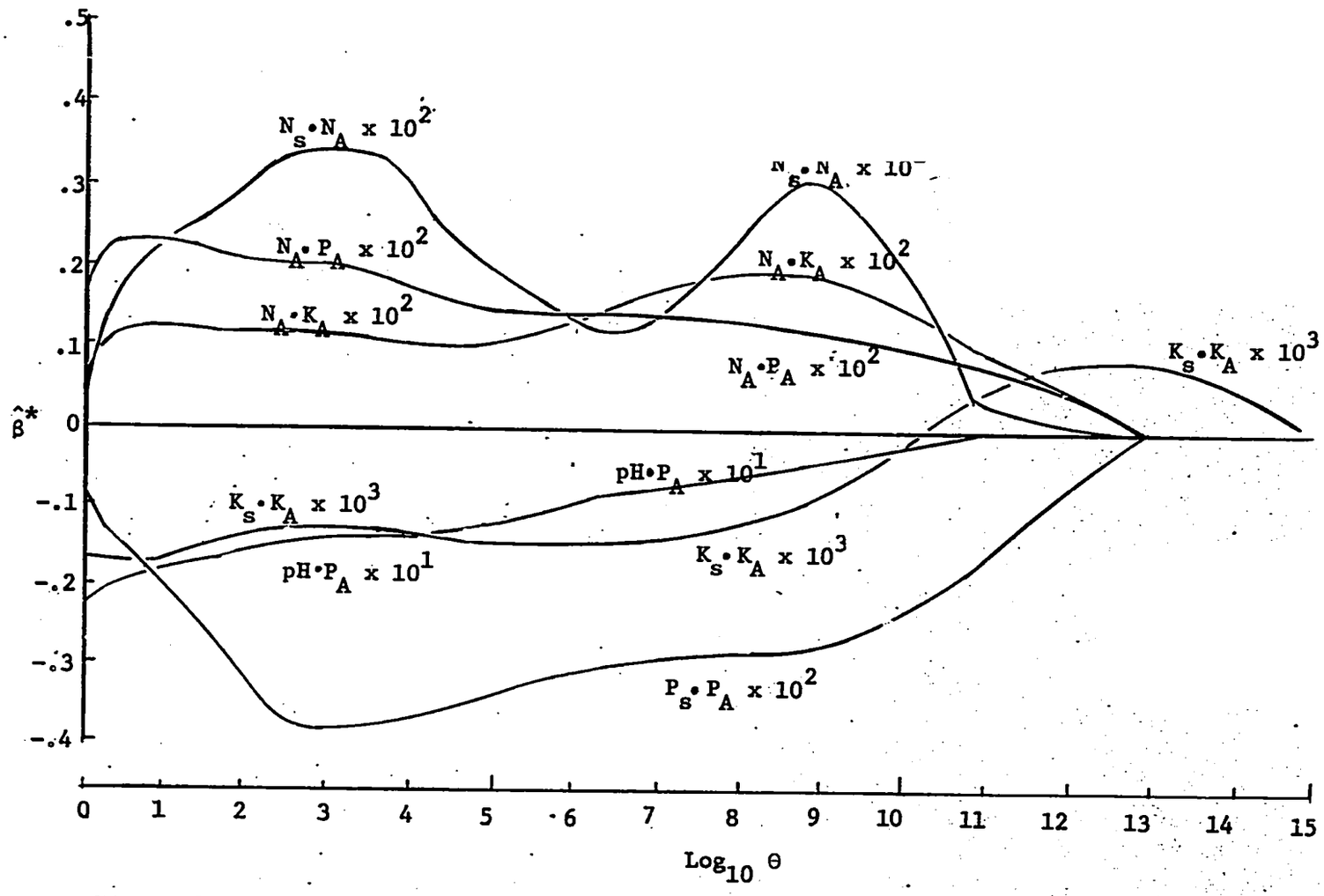

Figure 7. Ridge trace of coefficients of $N_s \cdot N_A$, $P_s \cdot P_A$, $K_s \cdot K_A$, $pH \cdot P_A$, $N_A \cdot P_A$, $N_A \cdot K_A$ for P_2

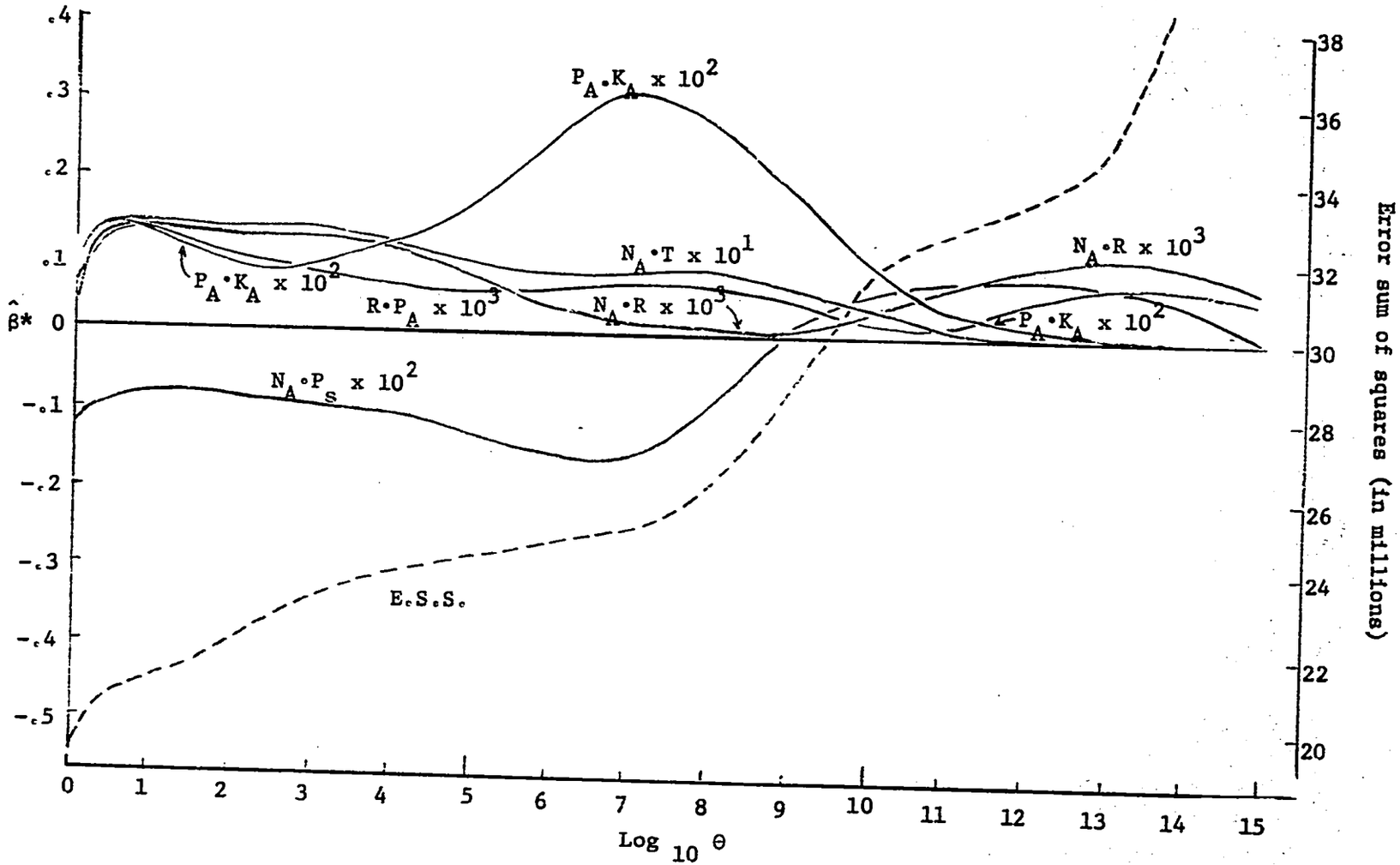

Figure 8. Ridge trace of coefficients of $N_A \cdot R$, $N_A \cdot T$, $P_A \cdot K_A$, $N_A \cdot P_s$, $R \cdot P_A$ and error sum of squares for P_2

of the elements of the constrained solution vector $\hat{\beta}^*$ is biased. However, all elements of $\hat{\beta}^*$ are estimated with smaller variances than the corresponding elements of the Generalized Least Squares solution vector $\hat{\beta}$. The Hoerl and Kennard (1970a, 1970b) ridge regression technique aims to minimize the average squared distance between $\hat{\beta}^*$ and β for each value of $\Theta > 0$. As they show (1970b, pp. 60-61), equation (42) can also be written:

$$E[L^2(\Theta)] = E[(\hat{\beta}^*-\beta)'(\hat{\beta}^*-\beta)] \tag{45}$$

$$= \sigma_\varepsilon^2 \sum_{i=1}^{n} X_i/(\lambda_i+\Theta)^2 + \Theta^2\beta'(X'X + \Theta I)^{-2}\beta \tag{46}$$

The first element of (46) is the sum of the variances of the elements of $\hat{\beta}^*$. The second element is the square of the bias introduced when $\hat{\beta}^*$ is used instead of $\hat{\beta}$. The sum of these is the mean square error of $\hat{\beta}^*$.

The Wallace[24] second weak mean square error test was carried out on the equation where Θ was approximately 17×10^5. The value of computed u was 21.8. The critical F value at the 5 percent level from the unpublished tables of Goodnight and Wallace was 2.70.[24] The null hypothesis that the selected ridge equation was better in weak mean squared error (i.e., a smaller average squared distance between $\hat{\beta}^*$ and β than between $\hat{\beta}$ and β) than the Generalized Least Squares equation was rejected. Hence, the estimates could not be improved using the ridge regression technique to correct for apparent multicollinearity.

To further test whether the existence of multicollinearity was leading to inexact estimates of the coefficients, the data were divided

---

[24] See Goodnight, J., and T. D. Wallace. 1971. Operational techniques and tables for making weak MSE tests for restrictions in regressions. Preliminary Unpublished Mimeograph, North Carolina State University, Raleigh. Forthcoming in Econometrica.

into two equal halves at random and model P_2 fitted to both halves. This was suggested by the unpublished work of Seagraves.[25] The results of fitting the two equations, called M_1 and M_2, are shown in Appendix A Table 6. A test of the equality of the two vectors of coefficients from M_1 and M_2 using the F test described by Johnston (1963, pp. 136-138) was performed. The calculated F was 0.95 and the critical F at the 25 percent level was 1.16. Choosing the 25 percent level gives a much more powerful test than lower levels of significance do. This test fails to reject the hypothesis that the vectors of coefficients in the two halves of the data are equal. Hence, this is convincing evidence that multicollinearity is not affecting the parameter estimates of P_2 significantly.

In equations F_1 and P_2, the coefficient estimates on N_A and $N_s \cdot N_A$ were not significant. The coefficient on $N_s \cdot N_A$ also had an unexpected sign in equation P_2. The only other coefficient that was not significant in the equations was that on R^2 in equation F_1. R^2 also had an unexpected sign in both F_1 and P_2. In the ridge analyses of Figures 3 through 8, the coefficients on these three variables were among the most unstable ones and their signs also changed. It is possible that these three variables are affected most by multicollinearity.

T-tests of the difference between all corresponding coefficients of the unconstrained models M_1 and M_2 were performed, as shown in Appendix A Table 6. The variables N_A and $N_s \cdot N_A$ had dissimilar coefficients, as illustrated by their very large t-values, as did $N_A \cdot R$ and $N_A \cdot T$. The

---

[25] See Seagraves, J. A. 1971. Agro-economic analysis of data from fertilizer trials: Regression procedures. Proposed Econ. Res. Rep., Department of Economics, North Carolina State University, Raleigh.

t-value on R^2 was not significant but it was greater than one. Hence, this again suggests it may be that the correlation of variables N_A, $N_s \cdot N_A$ and R^2 is causing disturbances in the parameter estimates for these and other variables such as $N_A \cdot R$ and $N_A \cdot T$.

It is notable that the other unstable variables in the ridge analyses, namely R, T, T^2 and R•T, did not show up in the t-tests on M_1 and M_2 as having dissimilar coefficients. These two methods of diagnosing multicollinearity problems hence do not give similar answers on the same data. This reduces the whole problem to a rather subjective evaluation of the evidence. It is also necessary to point out that the series of sequential t-tests performed on M_1 and M_2 in Appendix A Table 6 is not statistically valid as the t-tests are not independent. This, of course, is also true of all the usual t-tests in regression. The overall F-test on the two halves of the data failed to reject the hypothesis that the vector of coefficients in M_1 was identical to that in M_2. This is a statistically valid test.

To illustrate the economic significance of the equations fitted to the two halves of the data using P_2, the physical and economic optima for N_A, P_A and K_A were calculated for equations M_1 and M_2. These are shown in Table 5 of Chapter III. The optima were only 7 kg./ha. different for N_A in the two equations and about 30 kg./ha. apart for P_A and 11 for K_A. The economic significance of apparent multicollinearity is of little importance for N_A and K_A, but it may be for P_A. This is contrary to expectations as it was only the variables involving N_A which appeared different between M_1 and M_2.

To perform a more valid test of the significance of the difference between single coefficients in the two halves of the data, three more

regression models were fitted. The specific variables whose coefficients were chosen to be tested for multicollinearity effects were N_A, $N_s \cdot N_A$ and R^2. A constrained regression model was used whereby the coefficients on all variables but the one of interest were constrained to be equal in the two halves of the data. A single t-test was then performed on the difference between the coefficients of the variable of interest in the two halves. The procedure is described at the beginning of the section on multicollinearity in Appendix A.

The above t-values on the three coefficients were:

	t-value
N_A	1.35
$N_s \cdot N_A$	0.49
R^2	0.39

All of these t-values are not significant at the 5 percent level. This evidence that the coefficients of N_A, $N_s \cdot N_A$ and R^2 may not be affected by multicollinearity in the data is more valid than the previous t-tests on M_1 and M_2 in Appendix A Table 6.

The conclusion from all of this is that the evidence remains conflicting about the extent to which multicollinearity among the data may be affecting the parameter estimates in equation P_2, if at all. There may be "disturbances" in the parameters of P_2, but there are no unambiguous diagnostic methods of detecting these, as has been illustrated. Furthermore, even if the diagnosis were positive and definitive, there are no prescriptive devices available which allow objective measures of the degree of improvement in the resulting equation. The imposition of *á priori* constraints on the parameter space is one possible prescription

for multicollinearity, but as there were no independent parameter estimates available, this was not a possibility here.

For the above reasons it was decided to utilize equation P_2 as the best estimate of the potato-fertilizer production function in the Sierra region of Peru.

CHAPTER IV
GENERATING FERTILIZER RECOMMENDATIONS

The primary purpose of this study is to develop a basis for making sound fertilizer recommendations to potato farmers in the Sierra of Peru. The first step in this would appear to be to use the response equation P_2 to determine the economic optima for a range of soils. Optima here means the derivation of input levels using marginal analysis to generate the maximum net revenue or profit per hectare.

Derivation of Economic Optima for Fertilizers

To determine the optimum level of N_A, P_A and K_A to apply to potatoes, their marginal products must be set equal to their respective price ratios and a simultaneous solution obtained. From P_2 in Table 2 the marginal product equations were:

$$\frac{\partial y}{\partial N_A} = + 0.1839 - 0.004854\ N_A + 0.0002613\ N_s - 0.001124\ P_s + 0.001684\ P_A + 0.0006271\ K_A + 0.00003912\ R + 0.006384\ T \quad (47)$$

$$\frac{\partial y}{\partial P_A} = 1.1604 - 0.003376\ P_A - 0.0008689\ P_s - 0.02289\ pH + 0.001684\ N_A + 0.000890\ K_A + 0.0001106\ R \quad (48)$$

$$\frac{\partial y}{\partial K_A} = 1.0122 - 0.009130\ K_A - 0.0001659\ K_s + 0.0006271\ N_A + 0.000890\ P_A. \quad (49)$$

If we let:

$$\alpha = + 0.1839 + 0.0002613\ N_s - 0.001124\ P_s + 0.00003912\ R + 0.006384\ T \tag{50}$$

$$\beta = 1.1604 - 0.0008689\ P_s - 0.02289\ pH + 0.0001106\ R \tag{51}$$

$$\gamma = 1.0122 - 0.0001659\ K_s \tag{52}$$

and insert (50) into (47), (51) into (48) and (52) into (49), we obtain the following system of three equations in three unknowns. When solved, these equations give the optimum level of the three nutrients N_A, P_A and K_A for any given values of N_s, P_s, K_s, pH, R and T and the price ratios $\frac{P_{N_A}}{P_y}$, $\frac{P_{P_A}}{P_y}$, $\frac{P_{K_A}}{P_y}$:

$$- 0.004854\ N_A + 0.001684\ P_A + 0.0006271\ K_A = \frac{P_{N_A}}{P_y} - \alpha \tag{53}$$

$$0.001684\ N_A - 0.003376\ P_A + 0.000890\ K_A = \frac{P_{P_A}}{P_y} - \beta \tag{54}$$

$$0.0006271\ N_A + 0.00089\ P_A - 0.009130\ K_A = \frac{P_{K_A}}{P_y} - \gamma \tag{55}$$

In matrix form we have

$$A \cdot X = C$$

where A is the symmetric coefficient matrix

$$\begin{bmatrix} -0.004854 & 0.001684 & 0.0006271 \\ 0.001684 & -0.003376 & 0.000890 \\ 0.0006271 & 0.000890 & -0.009130 \end{bmatrix}$$

X is the solution vector

$$\begin{bmatrix} N_A \\ P_A \\ K_A \end{bmatrix}$$

and C is the matrix of constants:

$$\begin{bmatrix} \frac{P_{N_A}}{P_y} - \alpha \\ \frac{P_{P_A}}{P_y} - \beta \\ \frac{P_{K_A}}{P_y} - \gamma \end{bmatrix}.$$

The solution to this system of equations is given by:

$$X = A^{-1} C \tag{56}$$

where A^{-1} is the matrix:

$$\begin{bmatrix} -257.40 & -136.56 & -30.99 \\ -136.56 & -376.48 & -46.08 \\ -30.99 & -46.08 & -116.15 \end{bmatrix}.$$

The costs of fertilizer in Peru vary according to the distance from Lima. Appendix B Table 1 contains the range of prices which were being paid for fertilizer in March 1971. Appendix B Table 2 contains various conceivable price ratios that could be used to solve equation (56), using low, medium, and high fertilizer prices combined with low, medium and high potato prices.

The optimum levels of N_A, P_A, and K_A for an average year on an average soil were 251, 250 and 126 kg./ha. respectively, a ratio of 10:10:5.[26] This assumed a potato price of S/.200 per 100 kilos and average fertilizer prices of 9·1, 10·3 and 5·8 S/. kilo for N_A, P_A and K_A, respectively, as shown in Appendix B Table 1. In the 1961-62 period, N_A and P_A were priced much lower at approximately 7 S/. kilo for each of them. K_A has remained the same at about 6 S/. kilo according to the Food and Agriculture Organization (1963).

These optima for N_A and P_A are lower than those derived by McCollum and Valverde S. (1968, p. 29) of 272,289 and 161 kg./ha. for N_A, P_A and K_A, respectively. McCollum and Valverde S. used a lower potato price

---

[26]The average values of the weather and soil variables used for this calculation were obtained from the experimental data, namely R = 568 m.m., T = 6.83° C, P_s = 7.98 p.p.m., N_s = 3.73 percent, pH = 5.93 and K_s = 127 p.p.m. These were used to calculate α, β and γ in matrix C of equation (56). It should be noted that the coded values of R, T, P_s, N_s, pH and K_s were inserted in the above equations. The coded values are the ones shown in this footnote times 10.

of S/.150 per 100 kilos and prices of 12, 8·4 and 6 S/. kilo for N_A, P_A and K_A, respectively. Hence, since 1965-66, the price of N_A has fallen about 3 S/. kilo while the price of P_A has risen about 2 S/. kilo with K_A falling slightly. The limits of the McCollum and Valverde S. data were 160 kg./ha. for all three nutrients, whereas in the data used here it was 300, 240 and 320 kg./ha. for N_A, P_A and K_A, respectively. Part of the data used by McCollum and Valverde S. (1968) to derive their quadratic response equation was also used in the present analysis. Of the 3,075 original observations used here, 420 were from 1960-64, which were part of the data used by them.

The current (1971) recommendations of the National Center of Soil Analysis at La Molina, Peru are about 130, 40 and 105 kg./ha. for N_A, P_A and K_A respectively for an average soil. These are much below the optima derived here and the optima of McCollum and Valverde S. (1968). There is no evidence that at the levels of N_A derived in this study, potato quality would be affected sufficiently to affect the price received and hence the computed optima.[27]

Sensitivity of Optima to Price Changes

How sensitive are these optima to changes in the prices of potatoes and fertilizers? Table 7 contains the computed optima for nine combinations of three price ratios. The minimum and maximum value of each ratio has been combined with the minimum and maximum of the other two to generate the solutions in Table 7, together with the existing or average price ratios for all three nutrients.

---

[27] Private communication, Professor R. E. McCollum, North Carolina State University, Raleigh, North Carolina.

Table 7. Sensitivity of fertilizer optima to changes in price ratios[a]

Price ratio[b]			Optimum level of nutrient to apply (kg./ha.)		
$\frac{P_{N_A}}{P_y}$	$\frac{P_{P_A}}{P_y}$	$\frac{P_{K_A}}{P_y}$	N_A	P_A	K_A
Average	Average	Average	251	251	126
Min.	Min.	Min.	256	260	129
Min.	Min.	Max.	257	258	124
Min.	Max.	Min.	248	232	125
Min.	Max.	Max.	246	230	120
Max.	Min.	Min.	240	251	127
Max.	Min.	Max.	239	249	121
Max.	Max.	Min.	230	222	123
Max.	Max.	Max.	229	220	118

[a]Using the procedure outlined in Chapter IV of the text.

[b]Refers to the value of the relevant range of price ratios used in the analysis. See Appendix B Table 2 for the actual values of the ratios.

From Table 7, the optimum amount of N_A falls by about 16 kg./ha. when its price ratio goes from its minimum to its maximum value with the price ratios of P_A and K_A held constant. For example, with the P_A and K_A price ratios at their minimum values, the optimum level of N_A falls from 256 to 240 kg./ha. as its price ratio rises from its minimum to its maximum value. The N_A optimum is relatively insensitive to changes in the price ratios of P_A and K_A. It falls by a maximum of 11 kg./ha. when their price ratios move from their minima to their maxima, holding the N_A price ratio constant. It is possible that the prices of the three nutrients will move almost in unison. When they all change from their minima to their maxima, the optimum level of N_A falls by 27 kg./ha. from 256 to 229, or about 10 percent.

The optima for P_A are more sensitive to changes in the price ratios than those for N_A. Table 7 shows the optimum level of P_A falls by about 30 kg./ha. when its price ratio goes from its minimum to its maximum with the ratios for N_A and K_A held constant. The fall is only 11 kg./ha. when the price ratio of P_A is held constant and the ratios of N_A and K_A rise from their minima to their maxima. When the price ratios of all three nutrients increase from their minima to their maxima, the optimum level of P_A falls by about 40 kg./ha. from 260 to 220.

The optimum amount of K_A to use falls by only 5 kg./ha. as the price ratio of K_A rises from its minimum to its maximum value with the other price ratios held constant. When the ratios for N_A and P_A are held at their minimum values, Table 7 shows the optimum level of K_A to go from 129 to 124 kg./ha. as its price ratio rises from 0.018 to 0.064. The optimum K_A falls by 6 kg./ha. from 129 to 123 when the price ratio for K_A is held at its minimum value and the N_A and P_A ratios change from their minimum to maximum values.

When we consider the change in optimum levels of N_A, P_A and K_A that would occur should prices change from their present levels to the extremes shown in Table 7, the conclusion is that the optima are quite insensitive to price changes. At average prices and on an average soil the respective optima were 251, 250 and 126 kg./ha. An extreme change from the average price regime would change the N_A optimum by a maximum of 22 kg./ha., the P_A optimum by a maximum of 31, and K_A by a maximum of 8.

This analysis indicates that recommendations will not alter substantially when the prices of fertilizers and/or potatoes change within the bounds shown in Appendix B Tables 1 and 2. This conclusion was also made

by McCollum and Valverde S. (1968, pp. 28-29) using the 1960-64 potato data. Also, Table 7 shows the ratio of N_A: P_A: K_A to remain at approximately 10:10:5 for all price regimes, which compares with the 10:10:6 ratio derived by McCollum and Valverde S. (1968).

Sensitivity of Optima to Soil and Weather Variables

It is of prime importance to determine how fertilizer recommendations should change when the levels of N_s, P_s, K_s and pH in the soil change. In this way, some idea can be obtained of the necessity for soil testing before making fertilizer recommendations. If the optima do not alter substantially when the soil analyses change, this is an indication that general recommendations may be appropriate.

It is also important to know how recommendations would change when the weather turns out to be below or above average. Farmers in the Sierra of Peru cannot predict what the weather will be months in advance. It remains of concern, though, just how far from optimum their fertilizer decisions may be when they plan for an average year and a non-average one results.[28]

Table 8 shows the sensitivity of the optimum levels of N_A, P_A and K_A to various combinations of the soil test variables, for "average" weather. The three values of pH, P_s and K_s in Table 8 represent the mean, and the mean plus and minus one standard deviation using the experimental data. The interaction term $N_s \cdot N_A$ in P_2 was such that the optimum level of N_A was virtually independent of N_s. Hence, N_s was not

[28] Specific attention will be given to the risk aspects of fertilizer decisions in the next section.

Table 8. Optimum and recommended levels of applied nutrients for various soils with "average" weather

Solution number	Level of soil variables			Optimum and recommended levels of applied nutrients (kg./ha.)					Yield metric tons/ha.
	P_s p.p.m.	pH units	K_s p.p.m.	N_A[b] Opt.	P_A Opt.	P_A Rec.	K_A Opt.	K_A Rec.	
1	1.14	4.82	22	319	387	190	163	190	37.31
2	1.14	4.82	127	313	379	190	143	115	41.28
3	1.14	4.82	232	308	371	190	123	35	44.30
4	1.14	5.93	22	284	291	190	152	190	31.86
5	1.14	5.93	127	279	283	190	131	115	36.04
6	1.14	5.93	232	273	275	190	111	35	39.26
7	1.14	7.04	22	249	196	190	140	195	26.32
8	1.14	7.04	127	244	188	190	120	165	30.70
9	1.14	7.04	232	239	180	190	99	105	34.13
10	7.98	4.82	22	291	354	115	158	190	31.04
11	7.98	4.82	127	286	346	115	138	115	35.10
12	7.98	4.82	232	280	338	115	118	35	38.21
13	7.98	5.93	22	256	258	115	147	190	26.42
14	7.98	5.93	127	251	250	115	126	115	30.69
15	7.98	5.93	232	245	242	115	106	35	33.99
16	7.98	7.04	22	222	163	115	135	195	21.72
17	7.98	7.04	127	216	155	115	115	165	26.19
18	7.98	7.04	232	211	147	115	94	105	29.70
19	14.82	4.82	22	263	321	70	153	190	26.45
20	14.82	4.82	127	258	313	70	133	115	30.60
21	14.82	4.82	232	252	305	70	113	35	33.80
22	14.82	5.93	22	228	226	70	141	190	22.67
23	14.82	5.93	127	223	218	70	121	115	27.03
24	14.82	5.93	232	218	209	70	101	35	30.42
25	14.82	7.04	22	194	130	70	130	195	18.80
26	14.82	7.04	127	188	122	70	109	165	23.36
27	14.82	7.04	232	183	114	70	89	105	26.96

Table 8 (continued)

---

[a]Note that several of the optima exceed the range of the data for N_A and P_A of 300 and 240 kg./ha., respectively.

[b]As the optimum level of N_A to apply is virtually independent of N_S, the level of organic matter in the soil, the recommendation for N_A is not relevant here. As a guide, for soils in the range 2-4 percent organic matter, the recommendation was 120-160 kg./ha. For soils with < 2 percent organic matter, it was 160-180 kg./ha. and for those with > 4 percent, it was 100-140 kg./ha.

included in Table 8. All possible combinations of the above values of P_s, pH and K_s generated 27 optima in Table 8.

All the optima are sensitive to changes in the soil characteristics. For a soil low in P_s and K_s but very alkaline, like number 7 in Table 8, the optima are 249, 196 and 140 kg./ha. for N_A, P_A and K_A, respectively, generating a yield of 26.32 metric tons/hectare. On a rich, acid soil like 21, the corresponding optima are 252, 305 and 113 kg./ha., with a yield of 33.80 metric tons/hectare. The soil generating the highest yield is number 3 in Table 8. With high acidity, low P_s and high K_s, soil 3 generates a yield of 44.30 metric tons/hectare when 308, 371 and 123 kg./ha. of N_A, P_A and K_A are applied in an "average" year.

The Cate-Nelson analysis performed by McCollum and Valverde S. (1968, p. 27) on soil phosphorous and potatoes in the Sierra indicated a critical level of P_s of about 7 p.p.m. The implication from this was that little or no P_A was required if the soil P_s was above 7 p.p.m. Table 8 shows that even for soils with about 15 p.p.m. of P_s it is profitable to add in excess of 110 kg./ha. of P_A.

Of more significance is how the current recommendations of the National Center of Soil Analysis at La Molina, Peru compare to these computed optima. Table 8 contains such a comparison.[29] The recommendations for phosphorous are well below the computed optuma, particularly for soils with average and high P_s levels. For example, the P_A recommendation for soils with a P_s of about 15 p.p.m. is from 315 to 44 kg./ha.

---

[29] The recommendations were obtained by Mr. S. Tuten of the North Carolina State University Mission to Peru from the Soil Department and National Center of Soil Analysis, Agricultural Experiment Station, La Molina, Peru. They apply from April 1971.

below the computed optima. For soils low in P_s, such as 1 p.p.m., the recommendation is below the optima by 200 kg./ha at low pH but above it slightly at high pH levels. Generally, the recommendations are closer to the computed optima for P_A as the pH rises. Any build-up of P_s in the soil will be adequately taken account of by annual soil tests. Such tests will indicate available phosphorous and the optima will change accordingly.[30]

On soils with low K_s values of about 20 p.p.m., Table 8 shows the current recommendations to be too high by about 30-65 kg./ha. On average K_s soils of about 127 p.p.m., the recommendation is about 6-30 kg./ha of K_A too low on acid soils. On more alkaline soils with 127 p.p.m. of K_s, the recommendations are above the computed optima by 45-50 kg./ha. The recommendations on soils with high levels of K_s such as 232 p.p.m. are about 70-90 kg./ha. too low on the acid soils and about 5-15 kg./ha. too high on more alkaline soils.

It appears that no regard needs to be taken of the level of soil organic matter (N_s) in generating nitrogen recommendations if P_2 is accepted as the correct model.[31] Current nitrogen recommendations to farmers are far too low. On soils with less than 2 percent organic matter, the recommendations are between 5 and 160 kg./ha. too low, depending on the pH, N_s, P_s and K_s levels. On soils with 2 to 4 percent organic matter, they are between 20 and 200 kg./ha. too low, and on fertile soils

---

[30] It should be noted that the computed optima for P_A in Table 8 often exceed the range of the data used to derive the response equation P_2 of 240 kg./ha.

[31] The N_sN_A coefficient in P_2 was not significantly different from zero and its inclusion in P_2 did not alter N_A optima by more than a few kilograms when N_s was changed.

with greater than 4 percent organic matter they are 40 to 220 kg./ha. too low. N_A recommendations are closer to computed optima when soils are more alkaline and high in P_s and K_s.

Present recommendations for P_A do not vary as soil pH changes. Table 8 shows that as soils become more alkaline, the amount of P_A recommended should fall. When pH goes from about 4.8 to 5.9, the optimum level of P_A falls by about 100 kg./ha.[32] The P_A recommendation for a pH of around 7 should be another 100 kg./ha. below that again.

Recently, the K_A recommendations have been made to depend on the soil pH in addition to K_s. When pH exceeds 6.5, the K_A recommendations are increased. The analysis of Table 8 shows that the opposite should occur. As pH rises, the level of K_A should fall. However, an interaction term between K_A and pH was not included in equation P_2 to enable a definite conclusion to be made on this.[33]

Table 8 shows that the optimum levels of N_A, P_A and K_A vary significantly depending on their respective soil nutrients. For example, soil 1 had an optimum N_A of 319 kg./ha. whereas for soil 27, it was 183. The level of N_s was the same for both. Soil 1 had an optimum P_A of 387 kg./ha. whereas for soil 9 it was only 180, even though the level of P_s was similar in both. Soil 1 had an optimum K_A of 163 kg./ha. whereas on soil 25 with

---

[32]The level of pH which generates maximum physical yields varies substantially with the level of P_A (P_A and N_s are the only two variables entering the marginal product function of pH). For example, at an N_s level of 3.7 percent, the pH for maximum yields is about 6 when P_A is held at 100 kg./ha. When P_A is 250 kg./ha., the optimum pH is about 4.3.

[33]Professor McCollum did not believe there was a biological basis for a pH·K_A interaction. Furthermore, in their previous work, McCollum and Valverde S. (1968, pp. 25-29) only mentioned pH·P_A interactions, with no mention of pH·K_A.

the same K_s, it was 130. This illustrates the need for the Soil Testing Center to take account of the level of all soil nutrients in generating recommendations. The optimum level of each element depends not only on its level in the soil, but on the level of other relevant elements with which it has interactions. The procedures used in making recommendations should recognize the simultaneous nature of the solutions to equations such as (53), (54) and (55). Table 8 shows how much recommendations and yields can change when such a general approach is adopted.

For a soil laboratory to utilize the methods outlined in this study in making recommendations from the soil analyses would involve a series of about 15 calculations. First, they would calculate α, β and γ from equations (50), (51) and (52) and subtract them from their respective price ratios to form the C matrix. Then nine multiplications are required to compute the A^{-1} C solution. All this would take a clerk about 5 minutes per sample. Knowledge of the approximate average values of R and T based on the altitude from which each soil sample came would make recommendations based on this procedure even more reliable than use of regional average figures for R and T in equation (50). The quadratic form has a decided advantage over the square root form for these simultaneous solutions. The A^{-1} matrix need only be calculated once. It will not change when the values of the soil and weather variables alter, or when the price ratios change. For non-computerized laboratories, this is a valuable asset. The A^{-1} matrix must be recalculated for the square root equation when the price ratios change, although not when soil and weather variables do.

A relevant question is why the current recommendations of the La Molina laboratory differ so markedly from those generated by P_2. The laboratory recently revised its recommendation procedures on the basis

of more research and experience. Whereas previously no account was taken of soil pH in generating recommendations, now La Molina uses pH in potassium recommendations. Also, potassium recommendations were increased recently. While the results of this study do not support these changes, it is an indication that La Molina is constantly reviewing its procedures. This study may be of some assistance in this process. At present they do not appear to have analytical procedures to address themselves to simultaneous consideration of all factors in crop response.

Appendix B Tables 3 and 4 contain the optimem levels of N_A, P_A and K_A to apply when weather is below and above average. When weather is dry and cold (below average), the optima and the resulting potato yields are all considerably reduced. When the weather is relatively wet and warm the optima are increased, especially for P_A, and yields are about 1 to 4 tons higher than when the weather is average. The calculated optima for P_A fall by about 80 kg./ha. when the weather goes from average to below average, and increase by 80 when it goes from average to above average.[34]

Table 9 summarizes the contents of Table 8 and Appendix B Tables 3 and 4. It shows how sensitive the calculated optima are to the soil characteristics and weather.[35] For example, with average weather, the

---

[34] However, as most of the P_A optima of Appendix B Table 4 exceed the 240 kg./ha. range of the experimental data, the quoted rise from average to above average weather in recommended P_A may not be valid.

[35] It should be noted that the figures in Table 9 are the average of nine values for P_A and K_A and 27 for N_A. These represent the optima for the element in question for a range of values of the other elements. Hence, Table 9 masks the changes in optima that occur for an element as the levels of the other element and the pH change.

Table 9. Changes in economic optima as levels of soil nutrients and weather change

Level of soil nutrient (p.p.m.)	Optimum level of applied nutrient (average of 9 computed optima over a range of values of P_s, K_s, pH)[a]		
	Rainfall and temperature[b]		
	Low	Average	High
N_s		Optimum N_A (kg./ha.)	
All levels	189	251	313
P_s		Optimum P_A (kg./ha.)	
1.14	205	283	362
7.98	172	250	329
14.82	139	218	296
K_s		Optimum K_A (kg./ha.)	
22	135	147	158
127	114	126	138
232	94	106	118

[a]The values from which these averages were calculated are shown in Table 7 and Appendix B Tables 3 and 4. The N_s optima are an average of 27 values.

[b]Low, average and high growing season rainfall and minimum temperatures were 432 m.m. and 5.13° C, 568 m.m. and 6.83° C, and 704 m.m. and 8.53° C, respectively. (These were all coded by a factor of 10 in the analysis.)

level of P_A to apply should be about 220 kg./ha. for a soil high in P_s and about 280 for a soil low in P_s. For K_A, the amount applied should rise from about 100 kg./ha. when soils have 230 p.p.m. of K_s to 150 when soils have only 22 p.p.m. of K_s.

The N_A optima do not change as the level of N_s changes. However, they do change substantially when the weather becomes warmer and wetter as Table 9 illustrates. The optimum rises from 190 kg./ha. to some 250 when weather changes from below average to average weather. It rises another 60 kg./ha. to 313 when the weather is above average.

The absolute changes in the optima for P_A and K_A as we go from high to low analysis soils are the same whether the weather is normal or otherwise. For P_A the optimum rises by 66 kg./ha. and for K_A it rises by 40 kg./ha. regardless of the weather. The optimum level of P_A changes with the weather due to the indirect effects of its interaction with N_A, which has weather interactions, and the $R \cdot P_A$ term in equation P_2. The optima for K_A change only as a result of interactions between it and N_A and P_A which have weather interaction terms.

For a given soil analysis, Table 9 shows that the optima change significantly with the weather for all three nutrients. With a soil analysis for P_s of about 8 p.p.m., the amount of P_A to apply falls from 330 kg./ha. for above average weather to 250 for average and 170 for below average weather. For a soil analysis of 127 p.p.m. for K_s, the corresponding optima are 138, 126 and 114 kg./ha. The decrements are the same when going from one weather situation to another regardless of the soil analysis. The decrease in optimum P_A is about 80 kg./ha. going from average to below average weather for all three soil test values in

Table 9. For N_A the drop is about 60 and for K_A only 12. This analysis shows that the value of improved weather predictions could hence be very high, particularly for N_A and P_A.[36]

In the next section an attempt will be made to consider the specific effects of weather uncertainty on profits and the associated variance of profits.

Weather Uncertainty and Fertilizer Decisions

Many writers have recognized that the calculation of economic optima along the lines of the previous section do not go far enough when the aim is to generate recommendations about fertilizer use. Four main sources of uncertainty have been alluded to in the literature which qualified statements about optimum fertilizer levels.

The first uncertainty involves the statistical confidence limits around the estimated parameters of the response equation, associated with the degree of experimental error, and how this affects the calculated optima. One of the early papers on this was that of Box and Hunter (1954) who demonstrated the theory and procedure involved in simultaneously placing confidence limits around maximum points on response surfaces. This paper was followed by Carter and Hartley (1958) who demonstrated how to place confidence limits around the marginal product curves of a Cobb-Douglas function. Doll _et al_. (1960) did the same thing for marginal product curves, isoquants and isoclines of a quadratic function. They noted that the confidence bands widened as the curves were further from

---

[36] In an excellent paper, Byerlee and Anderson (1969) used Bayesian statistics to measure the value of perfect versus imperfect information on rainfall for South Australian wheat. A similar approach could be used here although it is beyond the scope of the present study.

their mean points. This is a property of statistical confidence regions as they are a positive function of x_i, the deviation of independent variables from their means. Doll et al. (1960) suggest it may be reasonable to assume that yield and variance are positively correlated in the biological sense which statistical confidence bands of the above type do not reflect.

Many other papers have examined the statistical uncertainty around estimated parameters and the calculated optima. Among the most notable are the papers by Lu (1961), Fuller (1962, 1965), Havlicek and Seagraves (1962), Hoffnar (1963), Walker and Carmer (1967) and Anderson and Dillon (1968).

A second type of uncertainty relating to fertilizer recommendations is the degree of confidence in the functional form chosen for the empirical analysis. Johnson (1953), Havlicek and Seagraves (1962), Cady and Laird (1969), Anderson and Nelson (see footnote 17) and others have alluded to this problem. It was previously discussed in detail in Chapter III under the heading of specification bias.

The third uncertainty entering the process of generating fertilizer optima is that due to the vagaries of weather. Parvin (1954) was one of the first to point out the need to consider not just expected profits in "average" years, but also the variance due to weather over time. Empirical studies by Knetsch and Parks (1958), Knetsch (1959) and Smith and Parks (1967) on corn and millet in Tennessee considered the drought index in quadratic response equations as a random variable. Using historical probability distributions of this index, they calculated expected profits ($E(\Pi)$) for various levels of applied nitrogen, assuming all other

parameters and variables to be constants. They recommended the nitrogen rate which generated the highest $E(\Pi)$.

Colyer (1969) and Colyer and Kroth (1970) computed the $E(\Pi)$ and variance of profits due to weather ($\sum_{i=1}^{8} [\Pi_i - E(\Pi_i)]^2 = \sigma_\Pi^2$; where i is the number of years the experiments were conducted for various levels of nitrogen on corn in Missouri. This is termed the E-V map. They established that on non-irrigated plots, σ_Π^2 increased monotonically with $E(\Pi)$ and the level of nitrogen, up to the point of maximum yields. It then declined. This confirmed the earlier beliefs of Doll et al. (1960).

Tollini (1969) and Tollini and Seagraves (1970) did a similar analysis to Colyer and Kroth on corn in North Carolina. Unlike Colyer and Kroth, who used only eight years of experimental data to calculate their $E(\Pi)$ and σ_Π^2, without actually measuring weather, Tollini and Seagraves employed a cross-sectional moisture index in a response equation and applied historical probabilities to compute $E(\Pi)$ and σ_Π^2. They found that the variance of profit increases as the level of applied nitrogen increases. This may explain a significant amount of "under use" of fertilizer by farmers if they have a disutility of risk.[37]

The fourth uncertainty aspect in discussing fertilizer optima concerns that which exists at the farm level. It arises because the researcher really cannot know all the on-farm uncertainties facing the farm decision maker. Johnson (1957) suggested that unexplained residuals

---

[37]"Under use" is defined here as levels of fertilizer use where the marginal return is greater than the marginal cost of applying additional fertilizer. Other cuases could be a high proportion of tenant farmers bearing all fertilizer costs but receiving only a portion of the returns, capital rationing, pessimistic weather expectations, differences in management skills, different responses due to differences in the skills and high personal discount rates.

in experimental data may bear little or no resemblance to unexplained residuals in a farm situation. He says the size of experimental residuals and their correlation with unexplained residuals on farms is crucial as farmers form subjective estimates of the uncertainty of experimental results. In other words, he says that farmers are more interested in on-farm residuals than experimental residuals, which is obvious. Odell (1958, 1959) found that yields of crops on farms were roughly 80-90 percent of those achieved in experiments using similar treatments and weather. Davidson and Martin (1965) and Davidson et al. (1967) found that this ratio fell as experimental yields increased and also as the area of crop per farm rose. While these studies did not specifically say that the variability of farm yields is larger than experimental yields, they did seem to imply that.

Farmers respond to uncertainty in a variety of ways. Sometimes they formulate their own subjective probability estimates of possible outcomes of various farm activities, which may differ from those derived from historical probabilities of weather and the like. Their reaction to uncertainty may be to impose internal capital constraints on their activities. They may also react in a manner consistent with various game-theoretic models such as those of Wold, Hurwicz, Savage and Laplace. Examples of the use of game-theoretic models to "tailor" recommendations to farmers are the papers by Swanson (1957), Havlicek (1960), McQuigg and Doll (1961), Walker et al. (1964) and Swanson and Tyner (1965). All these models have implied assumptions about the shape of the E-V utility surface, some of which are quite unusual. Another possibility is to estimate an indifference curve in the E-V space as Officer and Halter (1968) did. There is some question as to whether the latter is consistent with

generally held concepts of utility theory, particularly the concept of non-measurability of utility.

In the context of potato farming in the Sierra of Peru, attempting to determine the E-V tradeoff would be a hazardous task which is beyond the scope of this study. Swanson's (1956) technique of incorporating fertilizer activities into a linear programming analysis, which has capital, rotation and minimum income constraints, requires intimate knowledge of the individual farm and farmer. This and game-theoretic models are hence not utilized in the subsequent analysis.

It is also not planned to consider the uncertainty generated by statistical considerations in this section. As noted in Chapter III, the data were used at least 20 times before equation P_2 was chosen as the appropriate empirical response function. The inherent biases that result from such sequential estimation procedures are well known and have been recently discussed by Ashar (1969). The usual statistical tests and confidence limits on parameter estimates and linear combinations of them (such as marginal product equations) are invalidated as a result. Furthermore, if the works of Brown and Oveson (1958), Knetsch (1959) and Havlicek and Seagraves (1962) are any guide, in practice the costs of the wrong decision on optimum fertilizer use as a result of inexact parameter estimates may be slight. For these reasons, attention will only be given here to the effect of weather uncertainty on fertilizer decisions. The uncertainty effects of nitrogen, phosphorous and weather are the only ones considered here as these were the only interactions assumed in the model.

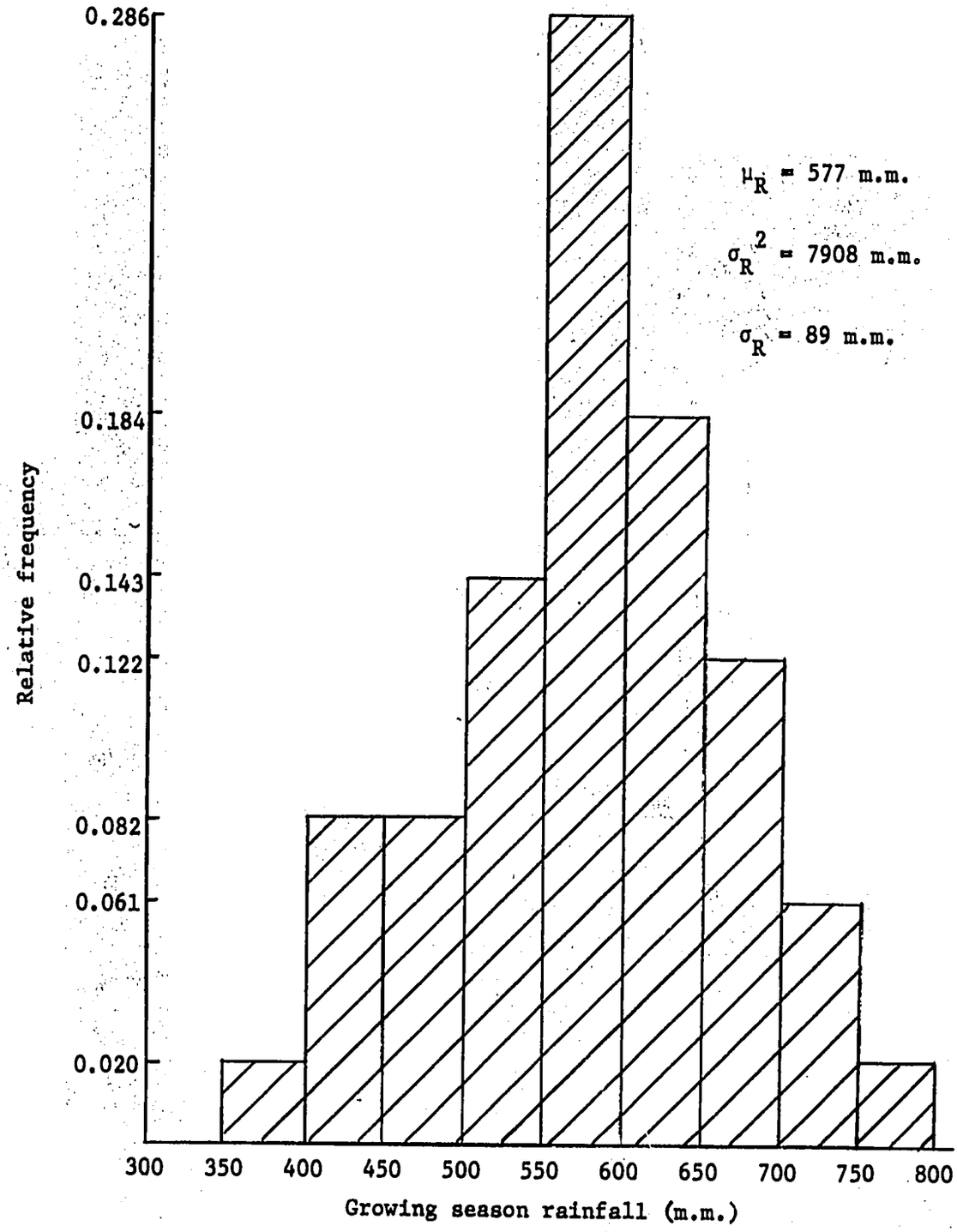

Figure 9. Frequency distribution of rainfall variable, Huancayo, 1921-1970

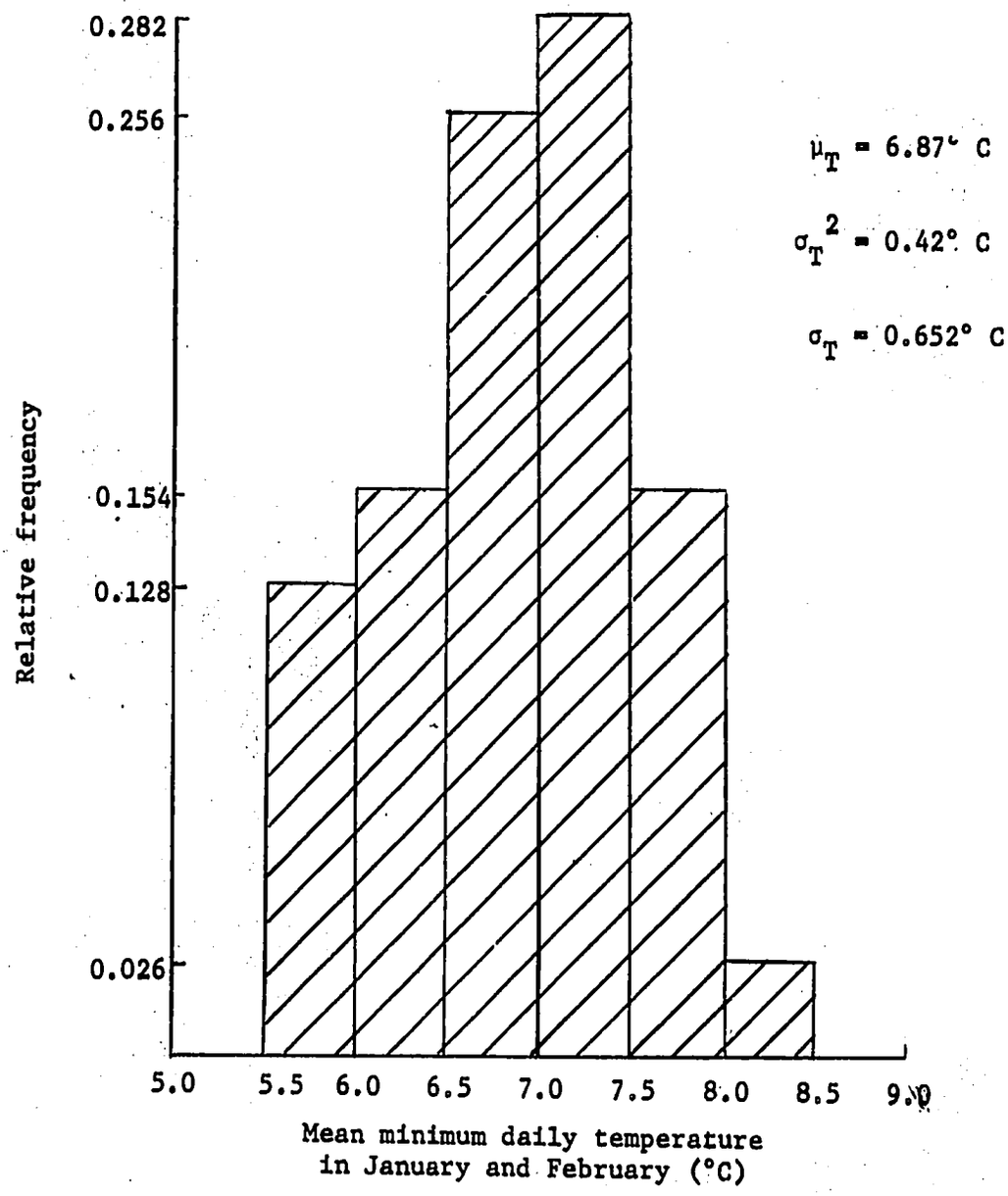

Figure 10. Frequency distribution of temperature variable, Huancayo, 1930-1970

Table 10. Expected profits ($E(\Pi)$) and variance of profits (σ_{Π}^2) for an average soil at various rates of N_A applied to potatoes[a]

N_A	P_A	K_A	$E(\Pi)$	σ_{Π}^2	λ[b]
(kg./ha.)			(S/.)	('000 S/.)	
0	0	0	14884	13,882	0.010503
20	20	10	20314	14,399	0.004307
40	40	20	25293	15,555	0.002524
60	60	30	29823	17,350	0.001675
80	80	40	33902	19,785	0.001181
100	100	50	37531	22,858	0.000732
140	140	70	43438	30,923	0.000387
180	180	90	47545	41,545	0.000175
220	220	110	49850	54,724	0.000032
260	260	130	50355	70,459	-0.000071
300	300	150	49060	88,751	-0.000149
340	340	170	45963	109,600	-0.000209
380	380	190	41066	133,006	-0.000258
420	420	210	34368	158,969	-0.000315
500	500	250	15570	218,564	-0.000378

[a]The levels of N_S, P_S, K_S and pH were set at 3.73 percent, 8 p.p.m., 127 p.p.m. and 5.93, respectively (all coded times 10). R and T were 568 m.m. and 6.83° C, respectively. These were the experimental data averages as shown in Appendix A Table 1. Average prices were used for fertilizers and potatoes as shown in Appendix B Tables 1 and 2.

[b]λ is the loss or increase in $E(\Pi)$ per unit change in σ_{Π}^2.

$$
\begin{aligned}
\sigma_{\Pi}^{2} = 40{,}000\ [&\tilde{b}_1^2\sigma_R^2 + \tilde{b}_2^2\sigma_T^2 + \tilde{b}_3^2\sigma_{T^2}^2 + \tilde{b}_4^2\sigma_{R^2}^2 + \tilde{b}_5^2N_s^2\sigma_R^2 + \tilde{b}_6^2N_s^2\sigma_T^2 \\
&+ \tilde{b}_7^2N_A^2\sigma_R^2 + \tilde{b}_8^2N_A^2\sigma_T^2 + \tilde{b}_9^2\sigma_{R\cdot T}^2 + \tilde{b}_{10}^2P_A^2\sigma_R^2 + 2(\tilde{b}_1\tilde{b}_2\sigma_{R\cdot T} + \tilde{b}_1\tilde{b}_3\sigma_{R\cdot T^2} \\
&+ \tilde{b}_1\tilde{b}_4\sigma_{R\cdot R^2} + \tilde{b}_1\tilde{b}_5N_s\sigma_{R\cdot R} + \tilde{b}_1\tilde{b}_6N_s\sigma_{R\cdot T} + \tilde{b}_1\tilde{b}_7N_A\sigma_{R\cdot R} + \tilde{b}_1\tilde{b}_8N_A\sigma_{R\cdot T} \\
&+ \tilde{b}_1\tilde{b}_9\sigma_{R\cdot RT} + \tilde{b}_1\tilde{b}_{10}P_A\sigma_{R\cdot R} + \tilde{b}_2\tilde{b}_3\sigma_{T\cdot T^2} + \tilde{b}_2\tilde{b}_4\sigma_{T\cdot R^2} + \tilde{b}_2\tilde{b}_5N_s\sigma_{T\cdot R} \\
&+ \tilde{b}_2\tilde{b}_6N_s\sigma_{T\cdot T} + \tilde{b}_2\tilde{b}_7N_A\sigma_{T\cdot R} + \tilde{b}_2\tilde{b}_8N_A\sigma_{T\cdot T} + \tilde{b}_2\tilde{b}_9\sigma_{T\cdot RT} + \tilde{b}_2\tilde{b}_{10}P_A\sigma_{T\cdot R} \\
&+ \tilde{b}_3\tilde{b}_4\sigma_{T^2\cdot R^2} + \tilde{b}_3\tilde{b}_5N_s\sigma_{T^2\cdot R} + \tilde{b}_3\tilde{b}_6N_s\sigma_{T^2\cdot T} + \tilde{b}_3\tilde{b}_7N_A\sigma_{T^2\cdot R} \\
&+ \tilde{b}_3\tilde{b}_8N_A\sigma_{T^2\cdot T} + \tilde{b}_3\tilde{b}_9\sigma_{T^2\cdot RT} + \tilde{b}_3\tilde{b}_{10}P_A\sigma_{T^2\cdot R} + \tilde{b}_4\tilde{b}_5N_s\sigma_{R^2\cdot R} \\
&+ \tilde{b}_4\tilde{b}_6N_s\sigma_{R^2\cdot T} + \tilde{b}_4\tilde{b}_7N_A\sigma_{R^2\cdot R} + \tilde{b}_4\tilde{b}_8N_A\sigma_{R^2\cdot T} + \tilde{b}_4\tilde{b}_9\sigma_{R^2\cdot RT} \\
&+ \tilde{b}_4\tilde{b}_{10}P_A\sigma_{T^2\cdot R} + \tilde{b}_5\tilde{b}_6N_s^2\sigma_{RT} + \tilde{b}_5\tilde{b}_7N_sN_A\sigma_{R\cdot R} + \tilde{b}_5\tilde{b}_8N_sN_A\sigma_{R\cdot T} \\
&+ \tilde{b}_5\tilde{b}_9N_s\sigma_{R\cdot RT} + \tilde{b}_5\tilde{b}_{10}P_AN_s\sigma_{R\cdot R} + \tilde{b}_6\tilde{b}_7N_sN_A\sigma_{T\cdot R} + \tilde{b}_6\tilde{b}_8N_sN_A\sigma_{T\cdot T} \\
&+ \tilde{b}_6\tilde{b}_9N_s\sigma_{T\cdot RT} + \tilde{b}_6\tilde{b}_{10}P_AN_s\sigma_{T\cdot R} + \tilde{b}_7\tilde{b}_8N_A^2\sigma_{R\cdot T} + \tilde{b}_7\tilde{b}_9N_A\sigma_{R\cdot RT} \\
&+ \tilde{b}_7\tilde{b}_{10}P_AN_A\sigma_{R\cdot R} + \tilde{b}_8\tilde{b}_9N_A\sigma_{T\cdot RT} + \tilde{b}_8\tilde{b}_{10}P_AN_A\sigma_{T\cdot R} + \tilde{b}_9\tilde{b}_{10}P_A\sigma_{RT\cdot R})].
\end{aligned}
\tag{59}
$$

The b_i are the estimated coefficients of terms with R or T in them in equation P_2, Table 6. They were:

$\tilde{b}_1 = 0.105172$ (R), $\tilde{b}_2 = -4.739410$ (T),

$\tilde{b}_3 = 0.166596$ (T^2), $\tilde{b}_4 = 0.00000301$ (R^2),

$\tilde{b}_5 = 0.0005285$ ($N_s \cdot R$), $\tilde{b}_6 = -0.0770512$ ($N_s \cdot T$),

$\tilde{b}_7 = 0.00003912$ ($N_A \cdot R$), $\tilde{b}_8 = 0.00638416$ ($N_A \cdot T$),

$\tilde{b}_9 = 0.0016959$ ($R \cdot T$), $\tilde{b}_{10} = 0.0001106$ ($P_A \cdot R$).

The σ^2 terms in (59) are the computed variances of the rainfall and temperature distributions from Huancayo and the σ terms are the respective covariance terms from the variables of the production function with the appropriate numerical values from the Huancayo data utilized. For example, $\sigma^2_{RT} = (\sigma_{RT})^2$ and $\sigma_{R \cdot T} = \sigma_{T \cdot R}$. After inserting the coded numerical values shown in Appendix A Table 1 into (59), it reduced down to a more manageable quadratic form

$$\sigma_{\Pi}^2 = 38{,}691{,}440 - 1{,}403{,}418\ N_s + 109{,}472\ N_A + 161{,}344\ P_A$$
$$+ 19{,}793\ N_s^2 + 123\ N_A^2 + 387\ P_A^2 - 3116\ N_s \cdot N_A$$
$$- 3880\ P_A \cdot N_s + 289\ N_A \cdot P_A. \qquad (60)$$

Table 10 shows the respective values of $E(\Pi)$ and σ_{Π}^2 from equations (58) and (60) for an average soil using average prices for potatoes and fertilizers. The profit equation for Table 10 with the assumed numerical values for the variables shown in footnote a of Table 10 inserted in (58) was

$$E(\Pi) = 14{,}884 + 143\ N_A + 62\ P_A + 154\ K_A - 0.4854 N_A^2$$
$$- 0.3376\ P_A^2 - 0.9130\ K_A^2 + 0.3368\ N_A \cdot P_A$$
$$+ 0.1254\ N_A \cdot K_A + 0.1780\ P_A \cdot K_A \qquad (61)$$

The variance equation for Table 10 was

$$\sigma_{\Pi}^2 = 13{,}881{,}752 - 6755\ N_A + 16620\ P_A + 123\ N_A^2 + 387\ P_A^2$$
$$+ 289\ N_A \cdot P_A. \qquad (62)$$

Table 10 shows that for an average soil, with average prices, the variance of profits begins to monotonically increase at an increasing rate as the level of fertilizer applied increases. The level of expected profits ($E(\Pi)$) at first increases until the optimum quantities of fertilizers are reached. This occurs when 260, 260 and 130 kg./ha. of N_A, P_A and K_A are applied, which are almost exactly the rates derived earlier for the average soil in solution 14, Table 8. The $E(\Pi)$ at these fertilizer levels is S/. 50,355. Figure 11 graphically illustrates the contents of Table 10.

The λ column in Table 10 measures the change in $E(\Pi)$ for each unit change in σ_Π^2. It represents the trade-off that must be made between increasing profits by adding more fertilizers and the attendant increase in the variance of those profits. If potato farmers on average soils were completely averse to the additional risk associated with the use of fertilizers, then they would not apply any N_A, P_A or K_A. In the Sierra of Peru these added risks associated with the use of more fertilizers as shown in Table 10 may help to explain why only about an average of 10 kg./ha. of nutrients is being applied, even though the marginal net return from increased fertilizers is substantial.

In making recommendations to potato farmers on average soils, risk considerations could conceivably drive the recommendation from about 260, 260, and 130 kg./ha. of N_A, P_A and K_A, respectively, down to zero. The current recommendation of the National Center of Soil Analysis at La Molina, Peru for this type of soil is 140, 115 and 115 kg./ha., respectively. The implied λ value of 0.0105 for a zero level of fertilizers is well above the average of 0.0002 obtained by Officer and Halter (1968,

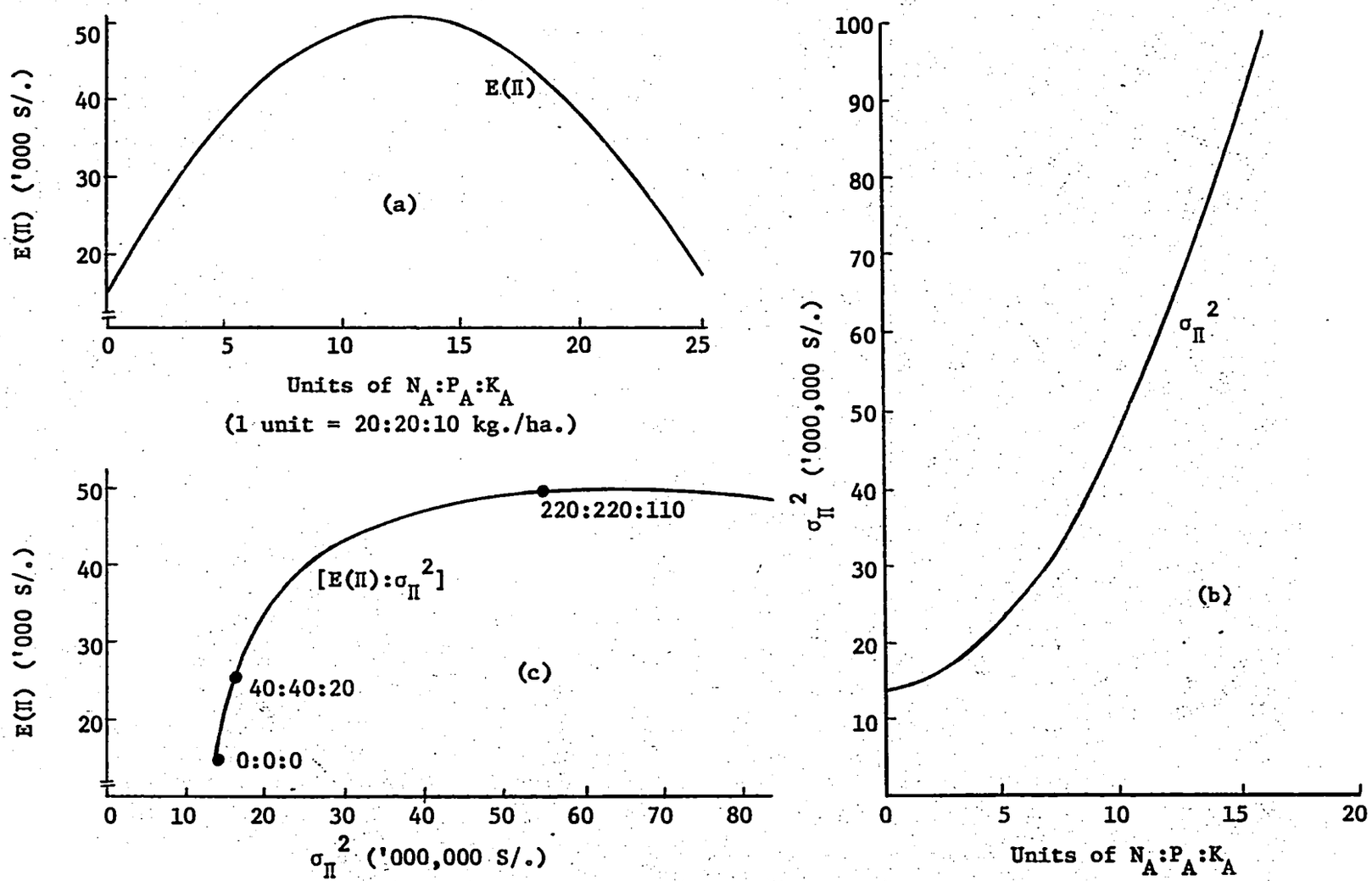

Figure 11. Expected profits-variance map for an average soil

p. 273) for five Australian farmers using utility analysis. It is, of course, not strictly valid to use the Officer-Halter average λ value as a measure of desired risk aversion levels in the Peruvian context, not only because they varied substantially from farmer to farmer in that study, but also they can be expected to vary between farmers of different countries. In addition, their sample was very small. However, if we use the λ value of 0.0002 they derived as the preferred position for Peruvian potato farmers, we arrive at a recommendation of about 160, 160 and 80 kg./ha. for N_A, P_A and K_A from Table 10.

Figures 12 and 13 have been drawn from Appendix B Tables 5 and 6 to illustrate the E-V trade-offs for soils with low and high levels of organic matter. For soils low in organic matter, Figure 12(b) shows that variance is a monotonically increasing function of the level of fertilizers. The maximum level of $E(\Pi)$ occurs at about 260, 260 and 130 kilos per hectare for N_A, P_A, and K_A, as was the case for the average soil.[38] There is a greater absolute variance in the less fertile soil at each level of fertilizers than in the average soil and also a greater increase in variance as fertilizer inputs increase. The recommendation one would make on a low organic matter soil is the same as for an average soil when farmers have absolute risk aversion; namely zero. If farmers on these soils have a lower degree of risk aversion similar to the λ value of 0.0002 found by Officer and Halter (1968, p. 273), the appropriate recommendation would be about 160, 160 and 80 kg./ha. of N_A, P_A, and K_A, respectively, the same as for an average soil.

---

[38] This is because the optimum level of N_A is independent of the level of N_S due to the coefficient on the $N_S \cdot N_A$ term in the fitted equation P_2.

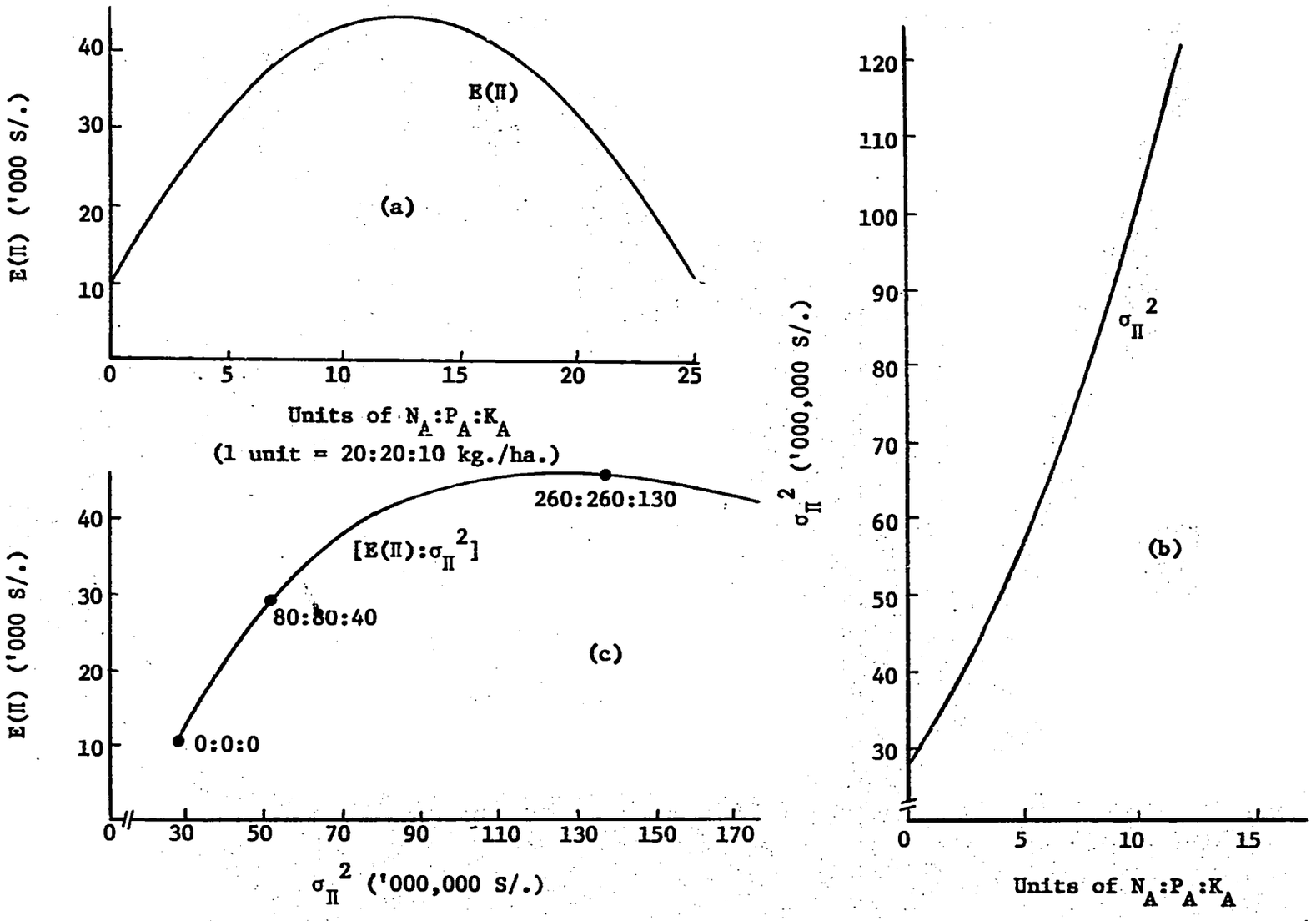

Figure 12. Expected profits-variance map for a soil low in organic matter

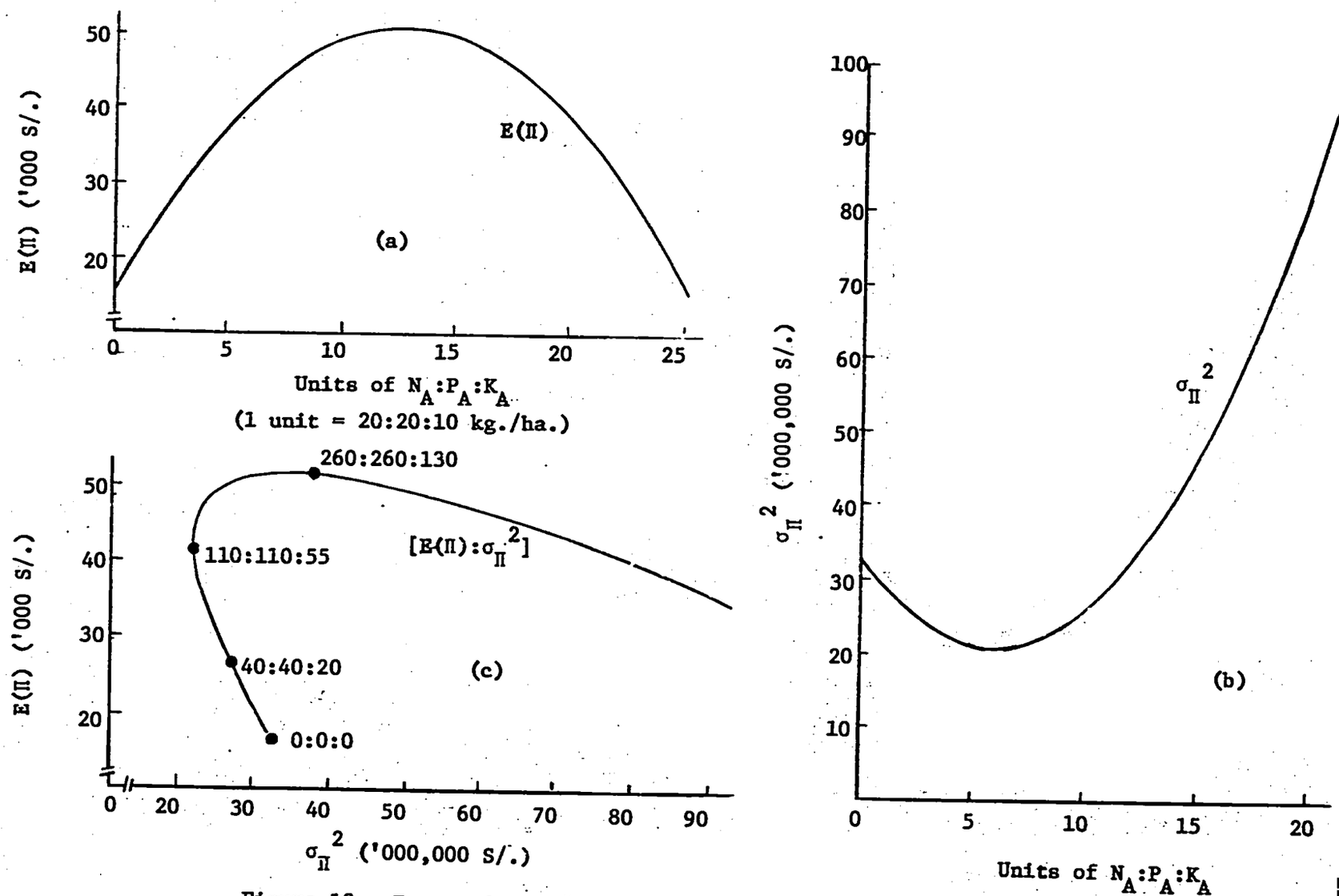

Figure 13. Expected profits-variance map for a soil high in organic matter

Figure 13(a) shows that the level of fertilizers generating the maximum $E(\Pi)$ for a high organic matter soil is about 260, 260 and 130 kg./ha. of N_A, P_A and K_A, the same as in both previous cases. However, there is a marked phase Figure 13(b) of decreasing variance as the level of fertilizers and $E(\Pi)$ rises. This occurs up to a level of about 120, 120 and 60 kg./ha. of N_A, P_A and K_A. For farmers who are completely averse to additional risk, one would recommend this latter level of fertilizers. This generates minimum variance and an expected profit from fertilizers of about S/. 36,000, which is S/. 15,000 below the maximum expected profit.

The reasons for the decline in variance are the negative signs on the N_s N_A and P_A N_s interaction terms in equation (60). At high levels of N_s these terms dominate the positive effects of increased levels of N_A and P_A on variance. The cause of the negative N_s N_A and P_A N_s terms in equation (60) is the negative signs on the N_s R and N_s T coefficients of the fitted equation P_2. These were opposite to the a priori expected signs, but after the equation was fitted Professor McCollum thought the negative N_s R term was logical and the sign of the N_s T term was really uncertain anyway.

This analysis leads to the apparently anomalous conclusion that when risk is to be averted the amount of fertilizers to apply should increase as the level of N_s in the soil rises. If farmers on high N_s soils have less than absolute risk aversion, like the λ value of 0.0002 of Officer and Halter (1968, p. 273), Figure 13(c) and Table 6 in Appendix B show the recommendation would be about 200, 200 and 100 kg./ha. of N_A, P_A and K_A compared to the figure of 120, 120 and 60 with complete aversion to risk.

The E-V patterns found on soils with organic matter levels greater than about 3.5 percent are all similar to those shown in Figure 13. That is, the variance of profits with respect to fertilizers has a U shape. This is contrary to the relationship found in most studies of this kind. Colyer (1969), Colyer and Kroth (1970), Tollini (1969) and Tollini and Seagraves (1970), all established that σ_{Π}^{2} increased monotonically with $E(\Pi)$ as the level of nitrogen on unirrigated corn was increased. Colyer and Kroth found σ_{Π}^{2} began to decrease after maximum yields were attained. Tollini and Seagraves found it continued to increase.

However, Colyer and Kroth (1970, pp. 489-490) obtained similar results to ours for irrigated corn. As plant population was increased, they found σ_{Π}^{2} at first fell and then rose to give a U shape, particularly at high levels of applied nitrogen. Of course, the irrigation reduced the variance due to rainfall, which is included in the present analysis, so their results are not strictly comparable to ours. Also the interaction of plant population and moisture is more complicated than that between nitrogen and moisture due to the added variability of weed population with the former.

Internal capital rationing is another possible way in which farmers' behavior is altered by risk. They may apply fertilizer according to a rule such as:

$$\frac{\partial Y}{\partial N_A} \quad \frac{P_Y}{P_{N_A}} = \delta > (1+r) \tag{63}$$

and δ is the internally imposed constraint. If we had knowledge of the appropriate δ for each farmer, specific recommendations could be made. However, we do not have this information.

One of the main conclusions of this analysis is that considerations of weather risk when making fertilizer recommendations should be less important on soils which are high in organic matter. On soils with an N_s level in excess of about 3.5 percent, the added risk in using the recommendations which generate maximum profits (see Table 8) would be minimal, given the U-shaped variance map. The implications of this is that on highly fertile soils, aversion of risk due to weather is not a valid reason for the "under use" of fertilizer by farmers.[39] Where climatic variables have negative interactions with soil and applied nutrients, as was the case here, variance of profits can be reduced by using more rather than less fertilizer on certain soils. Other reasons must be sought to explain farmer behavior in these instances.[40]

On the other hand, this analysis shows that for average and below average soils there is support for the widely held view that risk aversion could be one of the main causes of "under use" of fertilizer. Variance of profits was a monotonically increasing function of N_A, P_A and K_A and $E(\Pi)$ on these soils. To the extent that the previously mentioned studies by Colyer (1969), Colyer and Kroth (1970), Tollini (1969), and Tollini and Seagraves (1970) were applicable to average soils, the present analysis for potatoes in the Peruvian Sierra supports their conclusions for these soils.

---

[39] See footnote 37.

[40] See footnote 37 for some other possible explanations. To the extent that high organic matter soils are often at high elevations or are poorly drained, the E-V equations do not adequately reflect the risks of growing potatoes under such unfavorable circumstances. The overall risk of crop failure could make high levels of purchased inputs unlikely in such places.

CHAPTER V
VALUE OF THE GENERALIZED FUNCTION AND SOIL TESTING

On the presumption that the hypothesized model in equation (28) of Chapter III is in fact the "true" one, an evaluation of the benefits that would accrue to farmers and the Peruvian economy from using P_2, the empirically-derived (truncated) form of (28), can be made. This is an important consideration in view of the extensive research which is proceeding on potatoes in Peru and the fact that it is the principal crop in that country.[41]

A small number of writers have attempted to calculate the "costs of the wrong decision" after deriving response functions. Havlicek and Seagraves (1962) computed these "costs" assuming that 37 location-year functions for corn in North Carolina were "correct." The optimum level of nitrogen was calculated for each location using a generalized function including soil phosphorous and a moisture deficiency index. This was then used in the respective location-year functions to calculate net revenue. This net revenue was compared to that generated by using the level of nitrogen that was optimum employing each of the 37 separate functions. They found the average "cost" of using the generalized function to be quite low, at $1.02 per acre. They recognized that this "cost" would become the advantage of a generalized function if the hypothesis was that the generalized function was the "true" model.

---

[41] In 1967 the value of potato production in Peru was S/. 3.443 billion, which was in excess of S/. 1.3 billion more than the next most important crop, alfalfa. Potatoes were second to corn in area cultivated at 272,000 hectares, more than 95 percent of which was grown in the Sierra region where all the data used in this study were derived. Source: Ministerio de Agricultura (1970, p. 32).

Colwell (1968) performed a similar analysis to Havlicek and Seagraves using generalized response functions for wheat and potatoes in Ontario, Canada. However, Colwell only chose an arbitrary "average" recommendation to compare with that generated by inserting the relevant soil test values into his generalized functions. He found that the gross gains from the use of the soil test fertilizer recommendations were negligible when in the vicinity of the "average" recommendation, and only became appreciable when the divergence was large. He emphasized the need for separation of the benefits derived from soil testing from those due to the application of fertilizer. For example, in a region that is uniformly very deficient in a particular nutrient, the benefits from fertilizers will be high. However, since the district is uniform, the benefits from soil test recommendations will be almost identical with that from a proper estimate of the "average" fertilizer requirement. Here, soil testing may not be economically justified.

The aim in this chapter is first to generate two sets of recommendations--one using the generalized function P_2 in Table 2, and the other using the procedures of the National Center of Soil Analysis at La Molina in Peru. Then, using a multivariable probability distribution of soil analyses derived from a sample of records from the Center, calculate the expected benefits per hectare from using the P_2 recommendations compared to existing recommendations. The difference measures the expected value of the generalized function. This is more ambitious than any of the previous studies of this kind as it uses actual recommendations rather than simulated ones; it involves recommendations for three nutrients rather than one; and to compute expected benefits, a four-variable probability distribution is employed.

Second, a general recommendation is to be derived for an "average" soil, and the expected profits from using this regardless of soil test analyses are to be compared with the expected profits from using P_2 to generate recommendations based on soil tests. The difference measures the value of carrying out soil tests.

A total of 144 classes was established representing the range of soil test values to be expected for the four variables pH, N_s, P_s and K_s. The relevant intervals were:

pH: < 5.5, 5.5 - 6.5, > 6.5,

N_s (percent o.m.): < 2.0, 2.1 - 4.0, > 4.0,

P_s (p.p.m.): < 4 , 5 - 8, 9 - 15, > 15,

K_s (p.p.m.): < 60 , 60 - 120, 121 - 230, > 230.

these were all cross classified using a random sample of 1,541 soil test results from La Molina for potato soils in the Sierra.[42] The mean value of each of the four soil variables for each of the 144 cells was calculated, together with the relative frequency of occurrence. The means are shown in Table 11, which also contains those of the soil variables from the experimental data. T-tests of the significance of the differences between the experimental and the La Molina data were made. The computed t-values are shown in Table 11. All the means were significantly different at the 0.01 percent level. The sample of data from the National Center of Soil Analysis at La Molina had significantly larger means for all four soil variables. This indicates that the

---

[42] These data were kindly extracted by Mr. S. A. Tuten and his staff at the North Carolina State University Mission to Peru, Lima.

Table 11. Means of the soil variables from experimental data and the National Center of Soil Analysis (N.C.S.A.) at La Molina, Peru

Variable	Units	Experimental data	N.C.S.A. data	t-value[a]
N_s	percent o.m.	3.73	4.26	6.57***[b]
P_s	p.p.m. P	7.98	10.78	10.09***
K_s	p.p.m. K	127	158	9.69***
pH	standard units	5.93	6.65	20.99***
Number of observations		4,335	1,541	

[a]The t-values were calculated using the formula

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1 - \bar{X}_2}}$$

and using a pooled estimate of s^2 in calculating the denominator.

[b]*** represents a significant difference between the means at the 0.01 percent level.

potato farmers who utilize the soil testing services in Peru generally have better quality and more alkaline soils than those on which the experiments were conducted. The joint probability distributions of the two samples of data were also quite dissimilar.

Using the class means from La Molina, the optimum levels of N_A, P_A and K_A to apply were calculated using the procedure outlined in Chapter IV, equations (47) through (56). Then, the appropriate recommendations using the procedures of the National Center of Soil Analysis at La Molina were derived for the same soil nutrient means. These are shown in Appendix C Tables 1 through 3.

Particularly for N_A and P_A, the recommendations generally fall well below the computed optima, as was the case in Chapter IV, Table 8 and Appendix B Tables 3 and 4. For K_A, the optima are closer to the recommendations and often exceed them as K_s falls. The recommendations generally are further below the optima for N_A the higher the level of N_s. The same occurs with P_A as P_s rises. Hence, the Soil Analysis Center is more conservative on fertile soils than on infertile ones. This is an elaboration of the analysis of Chapter IV.

The expected profit function (using equation P_2) as shown in equation (58) of Chapter IV was calculated for each cell of Appendix C Tables 1 through 3, first using the computed optima for N_A, P_A and K_A, and then using their recommended levels.[43] Rainfall and temperature were set at the Huancayo average values of 577 m.m. and 6.87° C, respectively, together with the means of R.T, T^2 and R^2. The results of these profit calculations are shown in columns (2) and (3) of Tables 12 through 14.

The value of the generalized function P_2 in making fertilizer recommendations on the basis of soil tests is shown in column (5) of these tables. It is calculated by subtracting column (3) from column (2). It can be seen that the benefits of using the generalized function could be quite large. On some soils it could be as much as S/. 24,000 per hectare. Generally, the soils high in N_s and P_s, where recommendations tend to be far below the computed optima, derive the largest benefits from the use of the generalized function P_2.

---

[43] It should be noted that a number of the computed optima for P_A in Appendix C Tables 1 and 2 exceeded the range of the experimental data of 240 kg./ha.

Table 12. Expected profits using computed optima and recommendations for N_A, P_A and K_A and the probability of their occurrence on soils with pH < 5.5[a]

Soil analysis[b] (1)	Expected profits with N_A, P_A and K_A at: (S/.ha.) Optimum levels (2)	Recommended levels (3)	One general recommendation (4)	Value of generalized function (S/.ha.)[c] (5)	Value of soil testing[d] (6)	Probability of soil analysis (7)
1	63014	44738	49129	18276	13885	.001298
2	68254	53956	56728	14298	11526	.000649
3	66461	57183	59261	9278	7200	.001947
4	-	-	-	-	-	0
5	56454	32728	44597	23726	11857	.000649
6	-	-	-	-	-	0
7	65731	55975	58145	9756	7586	.001298
8	-	-	-	-	-	0
9	-	-	-	-	-	0
10	-	-	-	-	-	0
11	-	-	-	-	-	0
12	-	-	-	-	-	0
13	44765	27221	41549	17544	3216	.000649
14	41267	28088	39958	13179	1309	.000649
15	47800	39494	46510	8306	1290	.000649
16	94655	83241	83292	11414	11363	.000649
17	64597	46166	52852	18431	11745	.004543
18	68814	51520	56786	17294	12028	.006489
19	71078	58936	63268	12142	7810	.012980
20	78050	64748	70156	13302	7894	.001947
21	59581	41727	49543	17854	10038	.006489
22	61545	47363	54505	14182	7040	.002596
23	65677	53058	58339	12619	7338	.003894
24	75729	61810	69538	13919	6191	.001298

Table 12 (continued)

Soil analysis (1)	Expected profits with N_A, P_A and K_A at: (S/.ha.)			Value of generalized function[c] (S/.ha.) (5)	Value of soil testing[d] (6)	Probability of soil analysis (7)
	Optimum levels (2)	Recommended levels (3)	One general recommendation (4)			
25	-	-	-	-	-	0
26	57481	36445	47848	21036	9633	.001947
27	63445	50134	57621	13310	5823	.001947
28	63673	45312	56679	18361	6994	.000649
29	41788	23470	39559	18318	2229	.000649
30	-	-	-	-	-	0
31	48388	35709	47194	12679	1194	.000649
32	54317	43127	52588	11190	1729	.000649
33	64501	46187	54773	18315	9729	.007788
34	68498	50142	58505	18356	9993	.026609
35	74242	56541	63114	17701	11128	.026609
36	83551	66780	74362	16771	9189	.011033
37	59426	36238	47207	23188	12219	.003245
38	60846	43344	52621	17502	8225	.009086
39	66704	51136	58293	15568	8411	.012980
40	76565	59637	68579	16928	7986	.007139
41	52728	28416	43361	24312	9366	.001947
42	55593	33176	46288	22417	9305	.005192
43	57836	39731	51095	18105	6741	.004543
44	70158	52677	63509	17481	6649	.003894
45	45914	31070	42887	14844	3027	.003894
46	51336	41477	48500	9859	2836	.020119
47	52043	43656	50193	8387	1850	.013629
48	68810	60735	64344	8075	4466	.004543

Table 12 (continued)

---

[a]Expected profits were calculated using the respective values of soil characteristics shown in Appendix C Table 1 and the mean value of R, T, R^2, T^2 and R.T at Huancayo. Blank entries occur when column (7) is zero.

[b]See Appendix C Table 1 for the details of the anlyses corresponding to the numbers.

[c]This is column (2) minus column (3).

[d]This is column (2) minus column (4).

Table 13. Expected profits using computed optima and recommendations for N_A, P_A and K_A and the probability of their occurrence on soils with $5.5 \leq pH \leq 6.5$[a]

Soil analysis[b] (1)	Expected profits with N_A, P_A and K_A at: (S/.ha.) Optimum levels (2)	Recommended levels (3)	One general recommendation (4)	Value of generalized function (S/.ha.)[c] (5)	Value of soil testing[d] (6)	Probability of soil analysis (7)
49	52391	44472	49206	7919	3185	.003245
50	52790	46057	50166	6733	2624	.001298
51	64425	59255	61360	5170	3065	.005192
52	66535	58285	64635	8250	1900	.000649
53	45167	39138	43755	6029	1412	.001298
54	50466	44543	48412	5923	2054	.003245
55	54055	51236	53298	2819	759	.004543
56	65409	60080	62579	5329	2830	.000649
57	-	-	-	-	-	0
58	44753	37201	43651	7552	1101	.007139
59	46618	44788	46580	1830	38	.001298
60	60160	57003	56958	3157	3202	.001298
61	36640	22395	35650	14245	990	.001947
62	40572	28442	40008	12130	564	.002596
63	44829	37476	43121	7353	1708	.000649
64	-	-	-	-	-	0
65	54128	43718	50794	10410	3334	.005192
66	57460	48903	54565	8557	2895	.014278
67	65627	59716	63006	5910	2621	.009735
68	69841	60555	66644	9287	3198	.004543
69	44025	36657	42939	7368	1086	.001298
70	49164	41801	47877	7363	1287	.002596
71	56907	51546	55535	5361	1372	.009735
72	66579	58079	64741	8500	1838	.007139

Table 13 (continued)

Soil analysis[b] (1)	Expected profits with N_A, P_A and K_A at: (S/.ha.) Optimum levels (2)	Recommended levels (3)	One general recommendation (4)	Value of generalized function (S/.ha.)[c] (5)	Value of soil testing[d] (6)	Probability of soil analysis (7)
73	45338	35421	43792	9917	1546	.001947
74	45717	38743	45217	6974	500	.007139
75	51763	47721	51483	4042	280	.007788
76	63956	57507	61970	6449	1986	.005841
77	38404	25621	37582	12783	822	.000649
78	39819	30696	39293	9123	526	.003245
79	47388	41901	46427	5487	959	.005841
80	59228	52365	55785	6863	3443	.001947
81	58504	46227	54977	12277	3527	.004543
82	61649	49607	57782	12042	3867	.012980
83	67052	57349	63668	9703	3384	.012980
84	75759	61353	72381	14406	3378	.005841
85	51356	39220	48751	12136	2605	.001298
86	56140	46176	53666	9964	2474	.005192
87	60913	52691	59193	8222	1720	.012980
88	67084	56550	64990	10534	2094	.004543
89	-	-	-	-	-	0
90	51541	40427	50004	11114	1537	.003245
91	53079	46156	52384	6923	695	.011033
92	63105	53888	61687	9217	1418	.007788
93	38681	28551	38234	10130	447	.000649
94	54648	49310	50069	5337	4579	.002596
95	47882	42028	47161	5854	720	.013629
96	73693	71098	64168	2595	9525	.014278

Table 13 (continued)

---

[a]Expected profits were calculated using the respective values of soil characteristics shown in Appendix C Table 2 and the mean value of R, T, R^2, T^2 and R.T at Huancayo. Blank entries occur when column (7) is zero.

[b]See Appendix C Table 2 for the details of the analyses corresponding to the numbers.

[c]This is column (2) minus column (3).

[d]This is column (2) minus column (4).

Table 14. Expected profits using computed optima and recommendations for N_A, P_A and K_A and the probability of their occurrence on soils with pH > 6.5[a]

Soil analysis[b] (1)	Expected profits with N_A, P_A and K_A at: (S/.ha.) Optimum levels (2)	Recommended levels (3)	One general recommendation (4)	Value of generalized function[c] (S/.ha.) (5)	Value of soil testing[d] (6)	Probability of soil analysis (7)
97	32963	24979	30874	7984	2089	.011682
98	35489	26864	33135	8625	2354	.015576
99	43091	38400	41255	4691	1836	.010384
100	50088	44110	47408	5978	2680	.003245
101	40289	35115	39858	5174	431	.005841
102	35291	29553	33001	5738	2290	.005192
103	43751	39621	43341	4130	410	.010384
104	57073	52567	55480	4505	1593	.001947
105	27874	22297	25879	5577	1995	.001298
106	35474	31879	34221	3595	1253	.006489
107	37696	36968	35414	728	2282	.007788
108	38488	32951	27780	5537	10708	.001947
109	26804	21418	21162	5386	5642	.001298
110	31987	23640	31407	8347	580	.000649
111	39204	37783	36774	1421	2430	.000649
112	52386	48271	36928	4115	15458	.004543
113	37519	27831	35746	9688	1773	.033099
114	43199	35365	42278	7834	921	.046728
115	49183	43703	48271	5480	911	.024662
116	57757	50272	55802	7485	1955	.008437
117	35900	28503	34525	7397	1375	.017523
118	38429	32419	36878	6010	1551	.029854
119	41879	36878	39261	5001	2617	.021417
120	54163	48421	52520	5742	1643	.009086

Table 14 (continued)

Soil analysis[b] (1)	Expected profits with N_A, P_A and K_A at: (S/.ha.) Optimum levels (2)	Recommended levels (3)	One general recommendation (4)	Value of generalized function[c] (S/.ha.) (5)	Value of soil testing[d] (6)	Probability of soil analysis (7)
121	32956	28023	31434	4933	1522	.009086
122	36135	31104	34007	5031	2128	.020119
123	43499	41854	41992	1645	1507	.014278
124	46997	42496	41157	4501	5840	.007788
125	32443	25609	29810	6834	2633	.002596
126	35837	31033	31758	4804	4079	.016225
127	45707	44495	36144	1212	9563	.009735
128	59839	57209	41358	2630	18481	.014278
129	45048	34961	44373	10087	675	.015576
130	49444	38818	49000	10626	444	.023364
131	54681	47886	54323	6795	358	.017523
132	62980	55399	62078	7581	902	.009086
133	43270	34817	42777	8453	493	.003894
134	44440	35854	43483	8586	957	.018172
135	48294	39127	46721	9167	1573	.013629
136	63411	54502	60823	8909	2588	.005841
137	37628	31082	36770	6546	858	.002596
138	40938	34867	39847	6071	1091	.007139
139	46409	43781	44936	2628	1473	.006489
140	56711	52005	51788	4705	4923	.004543
141	46073	41282	32936	4791	13137	.000649
142	38555	33633	34718	4922	3837	.009735
143	46766	45938	37695	828	9071	.012980
144	73230	70163	55148	3067	18082	.020119
					Total	1.000105

Table 14 (continued)

---

[a]Expected profits were calculated using the respective values of soil characteristics shown in Appendix C Table 3 and the mean value of R, T, R^2, T^2 and R.T at Huancayo.

[b]See Appendix C Table 3 for the details of the analyses corresponding to the numbers.

[c]This is column (2) minus column (3).

[d]This is column (2) minus column (4).

To compute the expected value of the generalized function per hectare of potatoes ($E(P_2)$) in the Sierra region of Peru, which is the loss from using the La Molina recommendations, equation (64) was used:

$$E(P_2) = \sum_{i=1}^{144} [E(\Pi_{0i}) - E(\Pi_{Ri})] \cdot P_i \qquad (64)$$

where i = the number of soil classifications used in the sample;

$E(\Pi_{0i})$ = expected profits using the optimum levels of N_A, P_A and K_A from column (2), Tables 12 through 14;

$E(\Pi_{Ri})$ = expected profits using the recommended levels of N_A, P_A and K_A from column (3), Tables 12 through 14;

P_i = probability of the soil analysis from column (7) of Tables 12 through 14.

As previously mentioned, the probabilities (P_i) for the sample of 1541 soil test analyses from the National Center of Soil Analysis do not resemble the distribution of soil analyses in the experimental data used to derive P_2, and the sample means are also significantly different. The experimental data were not a random sample of sites as the experiments were set up to cover a wide range of soil types rather than to randomly sample the population. Also, the choice of soils for the experiments depended on the cooperation of the farmers in the area, which again affected the randomness of the sample. However, farmers who send soils into the La Molina Laboratory also may not be a random selection so that neither sample may be completely random. With these qualifications, the results of the estimation of equation (64) should give an idea of the magnitude of the benefits to be derived from using equation P_2 for generating fertilizer recommendations for potatoes in the Sierra of Peru. The added assumption of constant prices is also made in this analysis,

which is unlikely in view of the large increase in production which would occur if fertilizer use were increased to the levels implied by the analysis.

The expected value of the generalized function from equation (64) using Tables 12 through 14 was S/. 8,593 per hectare of potatoes. In 1967, 226,735 hectares of unirrigated potatoes were grown in the Sierra region of Peru.[44] The average yield was 5.845 m.t./ha. with a gross value of S/. 11,910 per hectare. Of course, potato farmers, in the aggregate, would not be using the recommended nutrient levels of the Soil Analysis Center. Not even all of those who utilize the services of the Center would follow the recommendations precisely.[45]

The expected value of the current recommendations can be estimated by the following:

$$E(R) = \sum_{i=1}^{144} E(\Pi_{Ri}) \cdot P_i \qquad (65)$$

where E(R) = expected value of current procedures for generating recommendations and the other parts of (65) are as explained earlier for equation (64).

The E(R) was S/. 43,724 per hectare. This would not represent the <u>net</u> profit from the potato enterprise if current recommendations were used. Other costs of production of potatoes would also have to be deducted,

---

[44] Ministerio de Agricultura (1970, p. 254).

[45] The average fertilizer application rate is only six to ten kg./ha. of plant nutrients in the Sierra according to a private communication from S. A. Tuten of the North Carolina State University Mission to Peru, Lima. On the coast of Peru, an average of 90 kg./ha. of plant nutrients are used, although it is all irrigated with an average yield in excess of 13 m.t./ha.

making the figure less than S/. 30,000 per hectare. By utilizing the generalized function P_2 instead of current recommendations, the net profits of farmers in the Sierra of Peru could be increased annually by S/. 8,593 per hectare, or by more than 30 percent, which is the ratio of $E(P_2)$ to the estimated net profits from using current recommendations of S/. 30,000, (assuming constant prices). The benefits of using the optima from equation P_2 compared to the present farm practices of 6 to 10 kilos of nutrients per hectare could be about four times the value of the generalized function, as yields could be quadrupled from their present levels at a small increase in fertilizer cost compared to the added returns. However, if yields were quadrupled, potato prices would be substantially reduced, particularly if the area sown did not change. Furthermore, it is extremely unlikely that farmers in the Sierra of Peru would increase fertilizer use to the levels implied by that figure due to capital constraints, risk, sharefarming agreements and the like.

To measure the value of soil testing (E(S)) in generating recommendations compared to the use of one general recommendation based on an average soil of 219, 177 and 111 kg./ha. of N_A, P_A and K_A, respectively, the following was estimated:

$$E(S) = \sum_{i=1}^{144} [E(\Pi_{Oi}) - E(\Pi_{Gi})] \cdot P_i \qquad (66)$$

where $E(\Pi_{Gi})$ = expected profits using one general recommendation on all i soils from column (4) of Tables 12 through 14, and the rest of (66) are as explained for equation (64).[46]

---

[46] The general recommendation was based on the average soil from the La Molina data. The characteristics of this soil are shown in column (4) of Table 11.

The expected value of soil testing for potatoes was calculated to be S/. 4,053 per hectare, per annum over the Sierra region of Peru. The value of soil testing, as expected, was greatest on soils far removed from the average, such as soil numbers 144, 141 and 128. The range was from S/. 18,481 per hectare on soil number 128 to only S/. 38 per hectare on soil 59.

The figure of S/. 4,053 is only 47 percent of the value of the generalized function calculated earlier. This value of soil testing is quite low compared to the prospective annual increase in profits from increasing fertilizer use from the present levels of about 10 kilos of nutrients per hectare up to the above blanket recommendation levels (about S/. 35,000 per hectare).

It would appear from this that the highest priority ought to be given to increasing fertilizer use on all potato farms rather than on doing individual soil tests and generating farm-specific recommendations. Of course, as Williams and Couston (1962) point out, soil tests can often make farmers more confident about increasing fertilizer use, even though they may not follow recommendations precisely. Soil tests hence may be of prime value in a general extension program aimed at increasing fertilizer use. It seems clear though that it is not necessary at this stage to exhort farmers to have soil tests for the sole purpose of "tailoring" recommendations to each farm. Increasing general fertilizer use on all farms would appear to have the greatest rewards at the present time.

The Food and Agriculture Organization (1966) recommend the planning and design of simple experiments with as few treatments and replications

as possible in cultivators' fields to enable maximum cooperation and coverage of farmers. This may be another useful way of increasing general fertilizer use as an alternative or complement to a soil testing program.

As previously mentioned, if potato farmers in the Sierra of Peru followed the recommendations in this study, it is likely that production would be substantially increased. It was decided to attempt to estimate the effect of adoption of the blanket recommendation for an average soil on production, prices and economic welfare.

To do this, it was assumed the existing supply curve is perfectly elastic at the current farm price level of S/. 2,000 per metric ton. This assumes the supply curves for the land, labor and capital inputs required for potato production are also perfectly elastic. The existing supply curve for potatoes is shown as S_0S_0 in Figure 14.

To estimate the new supply curve S_1S_1 in Figure 14, the cost structure for growing 1 hectare of potatoes under an advanced but non-mechanized system, over a range of yields and fertilizer inputs was calculated.

First, the optimum levels of N_A, P_A and K_A were calculated for a range of potato prices assuming the fertilizer prices remained constant at the average values in Appendix B Table 1. These are shown in Table 15 along with the predicted yields associated with them. Fertilizer was assumed to be the only variable input in the analysis, as the labor and materials costs of land preparation, seeding, pesticides and harvesting were considered as fixed costs per hectare and independent of yield. The non-labor component of these costs was estimated to be

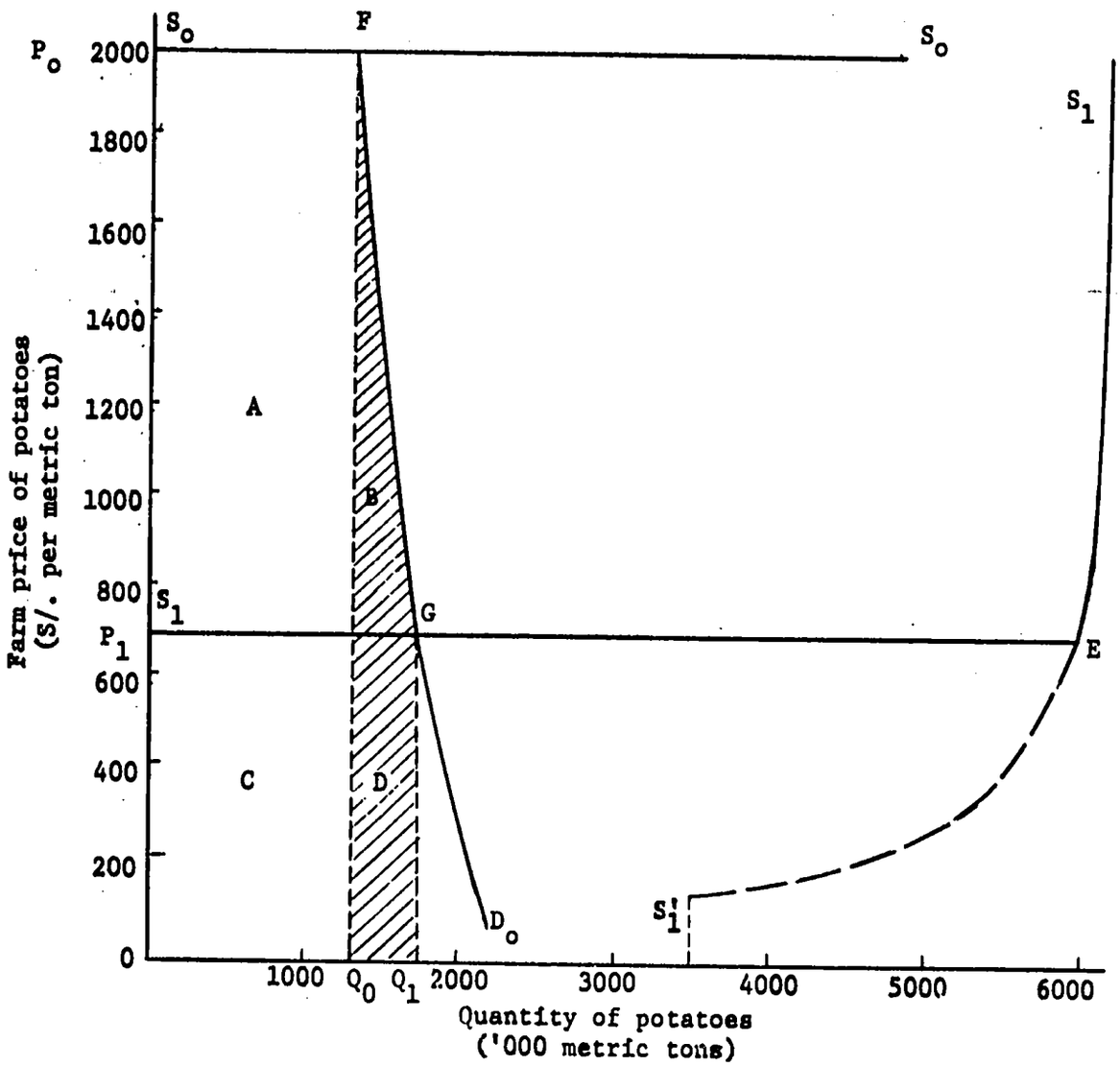

Figure 14. Possible effects of increased fertilizer use on potatoes in the Sierra of Peru, on production, prices and welfare

Table 15. Fertilizer input levels, potato yields, supply and demand estimates for a range of potato prices in Peru[a]

Farm prices of potatoes[b]	Optimum level of fertilizers			Yield of potatoes	Quantity supplied	Quantity demanded
	N_A	P_A	K_A			
(S/. per metric ton)	(kg./ha.)			(metric tons/ha.)	(1000 metric tons)	
2,000	219	177	111	28.10	6,371	1,325
1,500	212	168	109	27.98	6,344	1,432
1,000	199	150	104	27.70	6,281	1,586
800	189	137	101	27.43	6,219	1,670
600	173	115	95	26.84	6,086	1,765
400	140	70	83	25.10	5,691	1,885
200	42	65	47	19.62	4,449	2,037
100	0[c]	0[c]	0[c]	13.10	2,970	2,129

[a]This assumes an average soil with the characteristics shown in column (4), Table 11.

[b]The market price of potatoes was assumed to be S/. 175 per metric ton higher than each of the farm prices to allow for marketing costs. The market prices were used to construct the demand curve and farm prices were used in the supply analysis.

4,561 soles per hectare at 1965 prices by Coffey in an unpublished paper.[47] To allow for price inflation since 1965, Coffey's figure for materials costs was inflated by a factor of 0.71. This represented the 71 percent increase in the consumer price index since that time.[48] Present day labor costs were assumed to be 4 soles per manhour.[49] The combined labor and materials fixed cost per hectare at 1971 prices came to 9,102 soles.

The annual cost of land was estimated by calculating the per hectare costs of a traditional, non-mechanized system of growing potatoes (at 1971 prices), as outlined in Coffey's unpublished work[50] and subtracting this from the gross returns per hectare on an average crop of 5.845 metric tons. The gross return per hectare at a farm price of 2,000 soles per metric ton was 11,690 soles. The costs of potato production per hectare for this traditional system were estimated at 5,570 soles at 1971 prices giving a residual of 6,120 soles, which was attributed to the annual cost of land in potato growing. This residual can be correctly

---

[47]See the unpublished paper by J. D. Coffey, 1965, "Estimated Costs, Yield, and Resource Requirements for Crops and Livestock for the Cuzco Sierra Region of Peru." Faculty of Social Sciences, Universidad Agraria, Lima, Peru, p. 47.

[48]For the period 1965-1969 the source was Banco Continental (1970, p. 6) and for 1970 it was Banco Central de Reserva del Peru (1970, p. 5). For 1971 an estimated increase of 5 percent in the index was used, based on a private communication with Professor J. A. Seagraves, North Carolina State University, Raleigh, North Carolina.

[49]Source: Private communication, Professor J. A. Seagraves, North Carolina State University, Raleigh, North Carolina.

[50]See the unpublished paper by J. D. Coffey, 1965, "Estimated Costs, Yield, and Resource Requirements for Crops and Livestock for the Cuzco Sierra Region of Peru." Faculty of Social Sciences, Universidad Agraria, Lima, Peru, p. 41.

assumed to be the opportunity cost of land, as in equilibrium the marginal value product of land in all uses must be equal. Hence, there is no element of rents to potato land included in the calculated annual cost of land of 6,120 soles per hectare, even though it was estimated as a residual.

When this figure of 6,120 soles is added to the previously derived fixed cost of growing one hectare of potatoes under an advanced technology of 9,102 soles, we obtain an estimate of 15,222 soles as the total fixed cost of growing one hectare of potatoes. Again, this does not include the cost of fertilizers, which is considered the only variable cost in this analysis.[51]

Appendix C Table 4 contains estimates of the average variable, average fixed, average total and marginal costs of growing potatoes per metric ton at various yields per hectare for an average soil using the above cost figures. Figure 15 has been drawn from this table and shows that below a price of approximately 700 soles per metric ton, average total costs per metric ton (A.T.C.) would not be covered. The supply curve under an advanced management system with about 400 kilograms of fertilizers applied per hectare and yields of 27 metric tons is assumed to be perfectly elastic at this farm price of about 700 soles per metric ton. This, of course, assumes perfectly elastic supplies of land, labor and other productive inputs, at the above per unit prices of 6,120 soles, 4 soles and 9,102 soles, respectively, and perfectly elastic supplies of fertilizers.

---

[51] The labor costs involved in marketing are specified as a fixed cost per hectare by Coffey rather than varying with yield. The cost of jute bags is not included in costs as the farm price has been adjusted to take account of this.

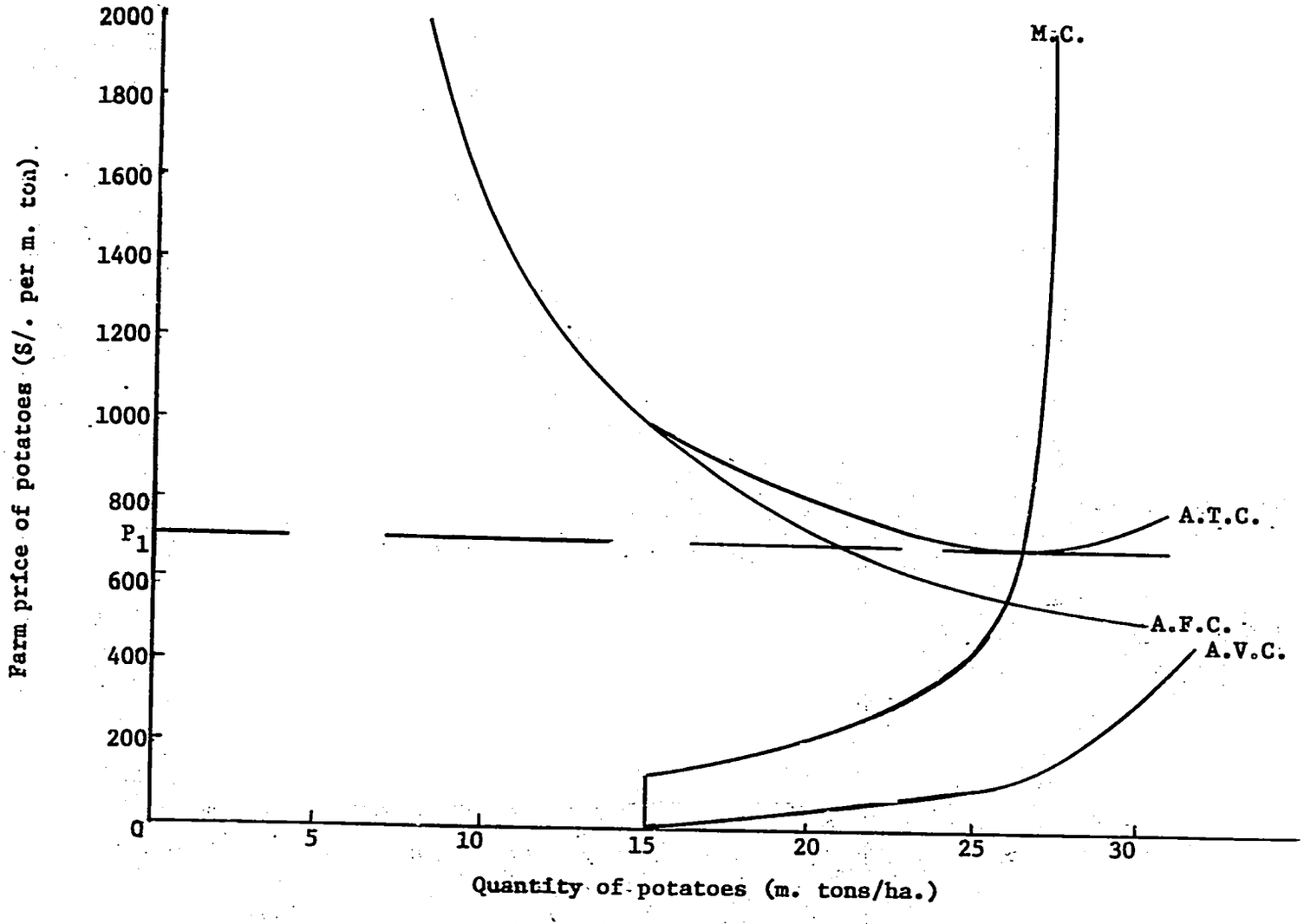

Figure 15. Cost structure for an average soil with land fixed

The curve $S_1'S_1$ has been drawn in Figure 14 to represent the production which would be forthcoming if the area devoted to potatoes remained at the present figure of 226,735 hectares, while the yields per hectare varied with the price of potatoes, as shown in columns (5) and (6) of Table 15. It indicates that if the supply of land becomes perfectly inelastic at 226,735 hectares (the present area of potatoes), the supply curve of potatoes will not be perfectly elastic above 6 million metric tons. However, given our estimate of demand, represented by D_0D_0 to be explained presently, this portion of the supply curve is not relevant.

To estimate the demand curve for potatoes in Peru, use was made of the concept of the arc elasticity of demand. The formula used was:

$$\eta = \frac{Q_0 - Q_n}{Q_0 + Q_n} \cdot \frac{P_0 + P_n}{P_0 - P_n} \tag{67}$$

where Q_0 = current demand in 1967 of 1,325,000 metric tons;

Q_n = demand at the new price of P_n;

P_0 = present retail potato price of S/. 2,175 per metric ton;

P_n = potato price for which the new quantity demanded Q_n is to be calculated;

η = price elasticity of demand for potatoes in Peru of -0.3.[52]

The various values of P_n as shown in Table 15, each with a figure of S/. 175 added to cover cash marketing costs,[53] were inserted into equation (67) and the quantity demanded, Q_n, was calculated. This is shown in column (7) of that table.

---

[52] This estimate was taken from a recent study by Torres (1970, pp. 41-42).

[53] These were the costs of jute bags which were assumed to last about two seasons.

The demand and supply curves are drawn in Figure 14 which shows the effects of an autonomous shift in the supply curve due to adoption of the blanket recommendations of this study on prices, production and welfare. The equilibrium point would be shifted from F in Figure 14 to G. The reduction in farm price would be S/. 1,310, or from S/. 2,000 per metric ton to S/. 690. The increase in total utility for consumers of potatoes would be the shaded areas B plus D. This assumes the marginal utility of money is the same for all consumers. The added resource costs associated with this gain in utility, excluding the costs of any research, extension or education required to shift the supply curve from S_0S_0 to S_1S_1 are areas (C plus D) minus (A plus C). If we assume FG is linear, an approximation of these areas can be made as follows:

$$\text{Gain in utility} = G = B + D. \tag{68}$$

$$\text{Added resources} = R = (C + D) - (A + C)$$

$$\text{or } R = D - A. \tag{69}$$

Net gain in economic welfare = W = G - R, and from equations (68) and (69) this is

$$W = (B + D) - (D - A)$$

$$= B + A$$

or as an approximation,

$$W \cong (P_0 - P_1)\, Q_0 + \frac{1}{2}\,(P_0 - P_1)(Q_1 - Q_0)$$

$$\cong 1{,}993 \text{ million soles}$$

where P_0 = 2,000 soles per metric ton;

P_1 = 690 soles per metric ton;

Q_1 = 1,718,000 metric tons;

Q_0 = 1,325,000 metric tons.

The annual net welfare gain of approximately S/. 1,993 million is equivalent to about 46 million U. S. dollars. This gives a general idea of the possible net benefits to Peruvians generally from a program aimed at boosting fertilizer usage on all soils without recourse to a soil test. Of course, there would be a reduction in total revenue to potato farmers in the Sierra as a result of such a program due to the inelastic demand for potatoes in Peru. Those on marginal land with lower opportunity costs than was assumed in the above analysis would be most adversely affected by such a price decline. There would tend to be a redistribution of income away from owners of marginal potato land that has few other alternative productive uses.

If there were such a dramatic increase in fertilizer use in the Sierra and there were not a completely elastic supply of land and labor at the existing estimated prices of 6,120 and 4 soles per unit, respectively, then their prices would be bid down by the large substitution of fertilizer inputs for land and labor. To the extent that this occurs, the S_1E portion of the new supply curve in Figure 14 would be lowered. The net welfare gains, represented by the areas A plus B in Figure 14, would be increased. However, this gain in overall welfare would be at the expense of a redistribution of income away from potato farmers due to reduced factor rewards. Hence, there is an inverse relationship between the size of the gain in welfare and the amount of redistribution away from potato farmers in this situation.

If the supply of fertilizers is not perfectly elastic at today's prices, as was assumed in the analysis, the new supply curve S_1S_1 would shift up to the left and the welfare gains would be reduced. Empirical estimation of fertilizer supply elasticities in Peru would be an important topic for future research to enable a more accurate assessment of the welfare implications of expanded fertilizer use to be made.

With these qualifications, it may be a worthwhile policy to undertake a program of research, extension and education of potato farmers in the Sierra aimed at boosting fertilizer usage. For an annual return in the vicinity of $U.S. 46 million, expenditure of significant amounts of resources could be justified to achieve the desired increase. Fertilizer use would have to increase about 40 fold from the present level of around 10 kilograms to in excess of 400 kilograms per hectare to derive the above benefits.

The above analysis only gives an estimate of the possible economic benefits that may accrue from a program aimed at substantially increasing fertilizer use on potatoes in the Sierra. It would be an important topic for future research to investigate in detail the present supply function for potatoes and determine the likely changes in land use that may occur if potato prices fall to the levels implied by the previous analysis. Also, the use of a point estimate of the demand elasticity to draw the demand curve when employing the concept of arc elasticity introduces some biases in the above analysis. Only after a more refined analysis is made can definite statements be made about the impact of expanded fertilizer use on economic welfare.

CHAPTER VI
CONCLUSIONS

One of the conclusions of this study was that it is possible to develop a generalized crop-fertilizer production function involving many soil, weather and applied nutrient variables, compared to the hypothesis that no relationship exists between these variables. The quadratic model was found to be better than the square root function on the experimental potato data from Peru which were used in this study. The quadratic generated much more realistic figures for nitrogen, phosphorous and potassium physical and economic optima than the square-root models.

There was some evidence, using a regression residual mean criterion, that the quadratic entailed some specification bias in the rational economic region. One suggested truncation procedure to correct for this, involving deletion of data with large amounts of one applied nutrient, was not successful. A second truncation where all data with zero level of one applied nutrient were deleted resulted in marginal improvement in the specification bias. However, the computed physical and economic optima for this model were within the range of the data and appeared more realistic to soil scientists familiar with the region. The computed optima for the non-truncated quadratic model were mostly outside the data range for nitrogen and phosphorous. On these grounds, the quadratic model, with zero levels of applied phosphorous deleted, was chosen as the best representation of the generalized potato-fertilizer production function in the Peruvian Sierra. Many other types of truncations could have been attempted to improve the fit of the quadratic model as measured by the algebraic size of the regression residuals over various intervals of

the production surface. The analysis could have utilized nitrogen, potassium or the total amount of nutrients as the variable on which to base the truncation instead of phosphorous. Previously the data truncation procedure had only been employed with single nutrient production functions. Obviously, where multi-nutrient responses are involved, the problems of data truncation are much more complex. There is no clear relationship between an improvement in the residual mean test and an improved model specification for economic analysis.

There was some indication from a ridge regression analysis, the simple correlation coefficients, and incorrect signs on some coefficients, that multicollinearity may have been affecting some parameter estimates. Tests of the difference between coefficients for two (randomly selected) halves of the data indicated otherwise, however. An F-test supported the hypothesis that the two sets of coefficients were equal. T-tests on the difference between single coefficients for the two halves, with all other coefficients constrained to be equal for both halves, also supported the hypothesis of equal coefficients.

One suggested method of improving the estimates when apparent multicollinearity exists, namely constrained least squares using the constraints suggested by the ridge analysis, was no improvement on the quadratic model with zero applied phosphorous data deleted, using the weak mean square error test. From all this, it appears that ridge regression analysis was not successful in identifying coefficients sensitive to multicollinearity in the data, nor in choosing a set of constraints on the parameter space to improve the estimation of the model.

The quadratic model with the zero levels of applied phosphorous data deleted was used in the economic analyses. It was found that soil phosphorous, potassium and pH affect fertilizer recommendations considerably. These data and analyses did not consistently show that soil organic matter directly affects nitrogen recommendations. In some other models examined, soil organic matter did affect response to added nitrogen, while in others, including the chosen model, it did not. All computed optima were relatively insensitive to realistic changes in the various price ratios.

Nitrogen recommendations generated in this study for soils with less than 4 percent organic matter are 5 to 200 kg./ha. higher than current recommendations, depending on the level of soil phosphorous, potassium and the pH level. On high organic matter soils they are 40 to 220 kg./ha. higher, due to interactions between soil organic matter, pH and other soil nutrients.

Phosphorous recommendations from this study are 30 to 250 kg./ha. above the current recommendations on soils with average to high levels of soil phosphorous. The computed recommendations are further above the current recommendations the higher the soil phosphorous level. On high pH soils the two recommendations are closer. On acid, low phosphorous soils the computed optima are 85 to 200 kg./ha. above current recommendations but up to 10 kg./ha. below them on alkaline soils low in phosphorous.

On soils low in potassium the computed recommendations in this study are 30 to 65 kg./ha. below current recommendations. On soils medium and high in potassium with a pH less than 6, they are 5 to 90 kg./ha. above them. On soils with a pH greater than 6 and with medium to high

potassium levels, the computed recommendations are below the current ones by 15 to 55 kg./ha.

The Soil Testing Center at La Molina, Peru, should take into account the level of all nutrients in making recommendations, not just the one of interest. The procedures used should recognize the simultaneous nature of the economic solution for several nutrients, as shown in Chapter IV. Such methods could be utilized in making recommendations based on soil tests. It would not require the use of a computer, and each recommendation would take about five minutes to calculate after the soil tests are made.

On average soils with about 3.7 percent organic matter, the analysis showed that risk (measured by the variance of profits due to rainfall and temperature variations over time) increased as the levels of applied nitrogen, phosphorous and potassium were increased in a fixed ratio of 10:10:5. It may be reasonable for highly risk averse potato farmers on these soils not to apply any fertilizer. This compares with the figure of 251, 250 and 126 kg./ha. of applied nitrogen, phosphorous and potassium, respectively, which generates maximum expected profits on these soils, and the current recommendation of about 140, 115 and 115 kg./ha. respectively.

For high organic matter soils, the analysis shows that risk due to weather variability falls as the level of applied nitrogen, phosphorous and potassium is increased up to about 120, 120 and 60 kg./ha., respectively. This indicates there may be no necessity for modifying recommendations generated for soils in this range because of risk effects. Even completely risk averse farmers would be advised to put on at least these amounts.

Hence, on low and average organic matter soils there is support for the widely held view that risk aversion could be one of the main causes of "under use" of fertilizer. On highly fertile soils on the other hand, aversion of risk due to weather variations does not appear to be a valid reason for any "under use" of fertilizer by farmers. Other reasons must be sought to explain farmer behavior in these instances. Some possible explanations are a high proportion of tenant farmers bearing all fertilizer costs but receiving only a portion of the returns, capital rationing, pessimistic weather expectations, differences in management skills, different responses due to the differences in these skills and high personal discount rates.

One of the primary implications of the study was that the use of fertilizer recommendations as generated by the study, instead of current soil test recommendations, could increase annual profits on the average by 8,600 soles per hectare, assuming no price changes resulting from increased production. This is about $U. S. 200 per hectare and it is one measure of the value of the generalized function.

The use of the generalized function for generating recommendations for each farm based on a soil test compared to one blanket recommendation with no soil testing could result in increased annual profits of 4,000 soles per hectare, which is $U. S. 94 per hectare, again assuming no price changes. This represents the value of soil testing. The marginal net benefit of raising fertilizer use from current levels to the blanket recommendation of this study for an "average" soil is estimated to be some 35,000 soles per hectare, or $U. S. 815 per hectare annually, which is about eight times the value of soil testing. However, these figures

must be regarded as upper limits because potato prices would be substantially reduced if all farmers in the Sierra of Peru increased fertilizer use to the levels implied in this analysis. This would involve quadrupling production if the area in production remained constant. Indeed, it is unlikely that the average fertilizer use would be increased to these levels due to capital constraints, risk aversion and the like on the part of farmers.

The above estimates indicate that the value of soil testing is small compared to the value of a general increase in fertilizer use on all potato farms. Soil testing may not be relevant on most farms until fertilizer use is increased considerably. This is not to say that a soil testing program would not have a valuable role to play in achieving this general increase in fertilizer use. It merely implies that the returns from generating farm-specific recommendations are small compared to those to be derived from a general increase in fertilizer use.

An analysis of the likely impact of the widespread adoption of fertilizer recommendations generated in this study showed that the price of potatoes could fall from the present level of S/. 2,000 per metric ton to about S/. 690. This assumed that all farmers applied the optimum amounts of nitrogen, phosphorous and potassium fertilizers, which are rather heroic assumptions. The net gain in economic welfare from such a shift in the supply curve of potatoes could be some S./ 1993 million or $U. S. 46 million annually, excluding the resource cost of the extension, research and education program required to shift the supply curve. This assumes a completely elastic supply of all inputs at their current prices. There would be a reduction in total revenue to potato farmers in the Sierra of Peru upon such a price reduction due to the

inelastic demand for potatoes in that country. Those on marginal farms with few alternatives to potatoes and hence close to zero opportunity costs for their land and labor, would be most adversely affected by this price decline. There is a strong possibility of a significant redistribution of income away from the poorer farmers if fertilizer use were substantially increased. Consumers of potatoes would be the main beneficiaries of such a program. The more inelastic the supply curves of land and labor are, the greater will be the size of this welfare gain, due to lowered unitary costs of the remaining land and labor resources in potato production after the substitution of fertilizer for them.[54] However, at the same time, the more inelastic these input supply curves, the larger the redistributive effects on potato farmers would be due to reduced land and labor returns. Hence, there is an inverse relationship here between possible welfare gains and redistributive effects of a program to boost fertilizer use.

More detailed research is required to specify accurately the supply function for potatoes in the Sierra. Only after a more refined analysis can definite statements be made about the impact of expanded fertilizer use on economic welfare and income distribution.

In this study, no attempt was made to compare the generalized production function with individual functions for each site. It is possible that much explanation is lost with a generalized function compared to site-specific functions. However, the advantage of a generalized function is that it can be utilized for a wide range of soils. Site-specific

---

[54]This would be offset to some extent by a possible increase in the per unit cost of fertilizer.

functions have a much more limited sphere of applicability. There is a need for research on the optimum size of geographic areas or soil type groupings in which to employ a single generalized production function.

Another fruitful research topic for future attention would be to utilize the technique of grafting quadratic functions fitted to different parts of the potato response data, as described by Fuller (1969). This may be a more efficient way of estimating the "true" production function, especially if the "true" function exhibits a Mitscherlich shape at low levels of applied nutrients.

LIST OF REFERENCES

Abraham, T. P. and V. Y. Rao. 1966. An investigation on functional models for fertilizer response surfaces. Jour. Indian Soc. Agric. Stat. 28:45-61.

Anderson, J. R. and J. L. Dillon. 1968. Economic considerations in response research. Amer. Jour. Agric. Econ. 50(1):130-142.

Anderson, R. L. 1956. A comparison of discrete and continuous models in agricultural production analysis, pp. 39-61. In E. L. Baum (ed.), Economic Analysis of Fertilizer Use Data. Iowa State College Press, Ames.

Anderson, R. L. 1957. Some statistical problems in the analysis of fertilizer response data, pp. 187-206. In E. L. Baum (ed.), Fertilizer Innovations and Resource Use. Iowa State College Press, Ames.

Ashar, V. G. 1969. On the use of preliminary tests in regression analysis. Unpublished Ph.D. thesis, Department of Economics, North Carolina State University, Raleigh. University Microfilms, Ann Arbor, Michigan.

Baird, B. L. and J. W. Fitts. 1957. An agronomic procedure involving the use of a central composite design for determining fertilizer response surface, pp. 135-143. In E. L. Baum (ed.), Fertilizer Innovations and Resource Use. Iowa State College Press, Ames.

Baird, B. L. and D. D. Mason. 1959. Multivariable equations describing fertility--corn yield response surfaces and their agronomic and economic interpretation. Agron. Journal 51:152-156.

Balmukand, B. H. 1928. Studies in crop variation. V. The relation between yield and soil nutrients. Jour. Agric. Sci. 18:602-627.

Banco Central de Reserva del Peru. 1970. Boletin del Banco Central de Reserva del Peru (Bulletin of the Central Reserve Bank of Peru). 40:467.

Banco Continental. 1970. Peru in Figures. The Chase Manhattan Group, Lima, Peru.

Baule, B. 1918. Zu Mitscherlich's Gesetz der physiologischen Beziehungen (About Mitscherlich's law of physiological relationships). Landw. Jahrb. 51:363-385.

Baum, E. L., E. O. Heady and J. Blackmore. 1956. Methodological Procedures in the Economic Analysis of Fertilizer Use Data. Iowa State College Press, Ames.

LIST OF REFERENCES (continued)

Baum, E. L., E. Q. Heady, J. T. Pesek and C. G. Hildreth. 1957. Fertilizer Innovations and Resource Use. Iowa State College Press, Ames.

Bondavalli, B., D. Colyer and E. M. Kroth. 1970. Effects of weather, nitrogen and population on corn yield response. Agron. Jour. 62:669-672.

Borden, R. J. 1946. Modified Mitscherlich method for soil cultures. Soil Science 62(1):51-60.

Box, G. E. P. 1954. The exploration and exploitation of response surfaces. Biometrics 10(1):16-60.

Box, G. E. P. and J. S. Hunter. 1954. A confidence region for the solution of a set of simultaneous equations with an application to experimental design. Biometrika 41:190-199.

Box, G. E. P. and K. B. Wilson. 1951. On the experimental attainment of optimum conditions. Jour. Royal Stat. Soc. B13(1):1-38.

Bray, R. H. 1944. Soil-plant relations: The quantitative relation of exchangeable potassium to crop yields and to crop response to potash additions. Soil Sci. 58:305-324.

Bray, R. H. 1958. The correlation of a phosphorous soil test with the response of wheat through a modified Mitscherlich equation. Soil Sci. Soc. Amer. Proc. 22:314-317.

Brown, W. G., E. O. Heady, J. T. Pesek and J. A. Stritzel. 1956. Production surfaces, isoquants, isoclines and economic optima in fertilizer use. Iowa Agric. Exp. Sta. Bull. 441, Ames.

Brown, W. G., T. L. Jackson and R. G. Petersen. 1962. A method for incorporating soil test measurements into fertilizer response functions. Agron. Jour. 54:152-154.

Brown W. G. and M. M. Oveson. 1958. Production functions from data over a series of years. Jour. Farm Econ. 40:451-457.

Byerlee, D. R. and J. R. Anderson. 1969. Value of predictors of uncontrolled factors in response functions. Aust. Jour. Agric. Econ. 13(2):118-127.

Cady, F. B. and R. J. Laird. 1969. Bias error in yield function as influenced by treatment design and postulated model. Soil Sci. Soc. Amer. Proc. 33:282-286.

LIST OF REFERENCES (continued)

Carter, H. O. and H. O. Hartley. 1958. A variance formula for marginal productivity estimates using the Cobb-Douglas function. Econometrica 26:306-313.

Cate, R. B. Jr. 1971. Improving the interpretation of soil fertility correlation data--a comparison of continuous and discontinuous models, using a variety of data sets. Unpublished Ph.D. thesis, Department of Soil Science, North Carolina State University, Raleigh. University Microfilms, Ann Arbor, Michigan.

Cate, R. B. Jr. and L. A. Nelson. 1965. A rapid method for correlation of soil test analyses with plant response data. International Soil Testing Series Tech. Bull. No. 1, North Carolina State University, Raleigh.

Cochran, W. G. and G. M. Cox. 1950. Experimental Designs. John Wiley and Sons, Inc., New York.

Colwell, J. D. 1967a. Calibration and assessment of soil tests for estimating fertilizer requirements. I. Statistical models and tests of significance. Aust. Jour. Soil Res. 5:275-293.

Colwell, J. D. 1967b. The calibration of soil tests. Jour. Aust. Inst. Agric. Sci. 33(4):321-330.

Colwell, J. D. 1968. Calibration and assessment of soil tests for estimating fertilizer requirements. II. Fertilizer requirements and an evaluation of soil testing. Aust. Jour. Soil Res. 6:93-103.

Colwell, J. D. and R. J. Esdaile. 1966. The application of production function analysis for the estimation of fertilizer requirements of wheat in northern New South Wales. Aust. Jour. Exp. Agric. Anim. Husb. 6:418-424.

Colwell, J. D. and R. J. Esdaile. 1968. The calibration, interpretation, and evaluation of tests for the phosphorous fertilizer requirements of wheat in northern New South Wales. Aust. Jour. Soil Res. 6:105-120.

Colwell, J. D. and K. M. Stackhouse. 1970. Some problems in the estimation of simultaneous fertilizer requirements of crops from response surfaces. Aust. Jour. Exp. Agric. Anim. Husb. 10:183-195.

Colyer, D. 1969. Fertilization strategy under uncertainty. Canadian Jour. Agric. Econ. 17(3):144-149.

Colyer, D. and E. M. Kroth. 1968. Corn yield response and economic optima for nitrogen treatments and plant population over a seven-year period. Agron. Jour. 60(5):524-529.

LIST OF REFERENCES (continued)

Colyer, D. and E. M. Kroth. 1970. Expected yields and returns for corn due to nitrogen and plant population. Agron. Jour. 62(4):487-490.

Davidson, B. R. and B. R. Martin. 1965. The relationship between yields on farms and in experiments. Aust. Jour. Agric. Econ. 9(2):129-140.

Davidson, B. R., B. R. Martin and R. G. Mauldon. 1967. The application of experimental research to farm production. Jour. Farm Econ. 49(4):900-907.

Dillon, J. L. 1968. The Analysis of Response in Crop and Livestock Production. Permagon Press, London, England.

Doll, J. P. 1967. An analytical technique for estimating weather indexes from meteorological measurements. Jour. Farm Econ. 49(1)Part 1:79-88.

Doll, J. P., E. H. Jebe and R. D. Munson. 1960. Computation of variance estimates for marginal physical products and marginal rates of substitution. Jour. Farm Econ. 42(3):596-607.

Eagle, D. J. 1969. Determination of the nitrogen requirements of crops by analysis. (11) Soil nitrogen release values and nitrogen response in crops. "Nitrogen and Soil Organic Matter." Tech. Bull. Minist. Agric. Fish. Fd. 15:78-88.

Edwards, C. 1962. Non-linear programming and non-linear regression procedures. Jour. Farm Econ. 44:100-114.

Engelstad, O. P. and E. C. Doll. 1961. Corn yield response to applied phosphorous as affected by rainfall and temperature variables. Agron. Jour. 53:389-392.

Ewalt, R. L., J. P. Doll and W. L. Decker. 1961. Correlation of drouth indices with corn yields. University of Missouri, College of Agric. Exp. Sta. Res. Bull. 788, Columbia.

Ferrari, T. J. 1959. Factors affecting growth and methods of expressing growth response data, pp. 65-75. In Economic Optimum Fertilizer Use. Doc. Food and Agric. Project No. 393. OECD of European Productivity Agency, Paris, France.

Fisher, R. A. 1925. The influence of rainfall on the yield of wheat at Rothamstead. Royal Soc. Phil. Trans. Series B 213:89-142.

Fitts, J.W., W. V. Bartholomew and H. Heidel. 1953. Correlation between nitrifiable nitrogen and yield response of corn to nitrogen fertilization on Iowa soils. Soil Sci. Soc. Amer. Proc. 17:119-122.

LIST OF REFERENCES (continued)

Food and Agriculture Organization. 1963. Production Yearbook. Vol. 17. United Nations, Rome, Italy.

Food and Agriculture Organization. 1966. Statistics of crop response to fertilizers. United Nations, Rome, Italy.

Fuller, W. A. 1962. Estimating the reliability of quantities derived from empirical production functions. Jour. Farm Econ. 44:82-99.

Fuller, W. A. 1965. Stochastic fertilizer production functions for continuous corn. Jour. Farm Econ. 47(1):105-119.

Fuller, W. A. 1969. Grafted polynomials as approximating functions. Aust. Jour. Agric. Econ. 13:35-46.

Geist, J. M., J. O. Reuss and D. D. Johnson. 1970. Prediction of nitrogen fertilizer requirements of field crops. II. Application of theoretical models to malting barley. Agron. Jour. 62(3):385-389.

Goldberger, A. S. 1964. Econometric Theory. John Wiley and Sons, Inc., New York.

Graybill, F. A. 1961. An Introduction to Linear Statistical Models. McGraw-Hill Book Company, Inc., New York.

Hanway, J. and L. C. Dumenil. 1955. Predicting nitrogen fertilizer needs of Iowa soils: III. Use of nitrate production together with other information as a basis for making nitrogen fertilizer recommendations for corn in Iowa. Soil Sci. Soc. Amer. Proc. 19:77-80.

Hartley, H. O. 1964. Experimental designs for estimating the characteristics of response functions. OECD. Doc. Agric. Food 65:163-176.

Havlicek, J. Jr. 1960. Choice of optimum rates of nitrogen fertilization for corn on Norfolk-like soil in the coastal plain of North Carolina. Unpublished Ph.D. thesis, Department of Agricultural Economics, North Carolina State College, Raleigh. University Microfilms, Ann Arbor, Michigan.

Havlicek, J. Jr. and J. A. Seagraves. 1962. The "cost of the wrong decision" as a guide in production research. Jour. Farm Econ. 44:157-168.

Heady, E. O. 1956. Methodological problems in fertilizer use, pp. 3-21. In E. L. Baum (ed.), Economic Analysis of Fertilizer Use Data. Iowa State College Press, Ames.

LIST OF REFERENCES (continued)

Heady, E. O. 1957. Organization activities and criteria in obtaining and fitting technical production functions. Jour. Farm Econ. 39(2):360-369.

Heady, E. O. 1963. Interdisciplinary research: The estimation and application of technical input-output coefficients from production function studies in North America. OECD. Doc. Agric. Food 65:45-107.

Heady, E. O. and J. L. Dillon. 1961. Agricultural Production Functions. Iowa State University Press, Ames.

Heady, E. O. and J. T. Pesek. 1954. A fertilizer production surface with specification of economic optima for corn grown on calcareous Ida silt loam. Jour. Farm Econ. 36(3):466-482.

Heady, E. O. and J. T. Pesek. 1957. Some methodological considerations in the Iowa-TVA research project on economics of fertilizer use, pp. 144-167. In E. L. Baum (ed.), Fertilizer Innovations and Resource Use. Iowa State College Press, Ames.

Heady, E. O., J. T. Pesek and W. G. Brown. 1955. Crop response surfaces and economic optima in fertilizer use. Iowa State College Agric. Exp. Sta., Res. Bull. 424, Ames.

Heady, E. O., J. T. Pesek and V. Y. Rao. 1966. Fertilizer production functions from experimental data with associated supply and demand relationships. Iowa State Univ. Agric. Home Econ. Exp. Sta. Res. Bull. 543, Ames.

Hendricks, W. A. and J. C. Scholl. 1943. Techniques in measuring joint relationships: The joint effects of temperature and precipitation on corn yields. North Carolina Agric. Exp. Sta. Tech. Bull. 74, Raleigh.

Hildreth, C. G. 1956. Discrete models with qualitative restrictions, pp. 62-75. In E. L. Baum (ed.), Economic Analysis of Fertilizer Use Data. Iowa State College Press, Ames.

Hildreth, C. H. 1957. Possible models for agronomic-economic research, pp. 176-186. In E. L. Baum (ed.), Fertilizer Innovations and Resource Use. Iowa State College Press, Ames.

Hildreth, R. J. 1957. Influence of rainfall on fertilizer profits. Jour. Farm Econ. 39(2):522-524.

Hoerl, A. E. and R. W. Kennard. 1970a. Ridge regression: Applications to nonorthogonal problems. Technometrics 12(1):69-82.

LIST OF REFERENCES (continued)

Hoerl, A. E. and R. W. Kennard. 1970b. Ridge regression: Biased estimation for nonorthogonal problems. Technometrics 12(1):55-67.

Hoffnar, B. R. 1963. Some practical statistical notions for production function studies. Jour. Farm Econ. 45:1226-1231.

Holford, I. C. R. 1966. Phosphate and potash requirements of sugar cane in relation to soil chemical analysis and soil type. Aust. Jour. Exp. Agric. Anim. Husb. 6:409-417.

Hudson, D. J. 1966. Fitting segmented curves whose join points have to be estimated. Jour. Amer. Stat. Assoc. 61:1097-1129.

Hurst, D. C. and D. D. Mason. 1957. Some statistical aspects of the TVA-North Carolina cooperative project on determination of yield response surfaces for corn, pp. 207-216. In E. L. Baum (ed.), Fertilizer Innovations and Resource Use, Iowa State College Press, Ames.

Hutton, R. F. 1955. An appraisal of research on economics of fertilizer use. Report No. T 55-1, TVA, Division of Agric. Relations, Agric. Econ. Branch, Muscle Shoals, Alabama.

Jensen, D. and J. Pesek. 1959a. Generalization of yield equations in two or more variables: I. Theoretical considerations. Agron. Jour. 51:255-259.

Jensen, D. and J. Pesek. 1959b. Generalization of yield equations in two or more variables: II. Application to yield data. Agron. Jour. 51:259-263.

Johnson, G. L. 1957. Discussion: Economic implications of agricultural experiments. Jour. Farm Econ. 39(2):390-397.

Johnson, P. R. 1953. Alternative functions for analyzing a fertilizer-yield relationship. Jour. of Farm Econ. 35(4):519-529.

Johnsson, A. B. 1963. Input/output relationship in fertilizers and crops. OECD. Doc. Agric. Food 65.

Johnston, J. 1963. Econometric Methods. McGraw-Hill Book Company, Inc., New York.

Jordan, C. W., C. E. Evans and R. D. Rouse. 1966. Coastal Bermudagrass response to applications of P and K as related to P and K levels in the soil. Soil Sci. Soc. Amer. Proc. 30:477-480.

Kamprath, E. J. and E. V. Miller. 1958. Soybean yields as a function of soil phosphorous level. Soil Sci. Soc. Amer. Proc. 22:317-319.

LIST OF REFERENCES (continued)

Knetsch, J. L. 1959. Moisture uncertainties and fertility response studies. Jour. Farm Econ. 41:70-76.

Knetsch, J. L. and W. L. Parks. 1958. Interpreting results of irrigation experiments--A progress report. Report T 59-1 AE, Tennessee Valley Authority, Knoxville.

Knetsch, J. L. and J. Smallshaw. 1958. The occurrence of drought in the Tennessee Valley. Report T 58-2 AE, Tennessee Valley Authority, Knoxville.

Laird, R. J. and F. B. Cady. 1969. Combined analysis of yield data from fertilizer experiments. Agron. Jour. 61(6):829-834.

Lu, J. L. 1961. On variance estimate of marginal productivity. Jour. Farm Econ. 43(2):316-317.

McCollum, R. E. and I. Carlos Valverde S. 1968. The fertilization of potatoes in Peru. I. A summary and interpretation of data from field experiments completed from 1959 through 1964 in the Sierra. North Carolina Agric. Exp. Sta. Tech. Bull. No. 185, Raleigh.

McQuigg, J. D. and J. P. Doll. 1961. Economic analysis involving random weather inputs. Jour. Farm Econ. 43(4)Part I:909-920.

Mason, D. D. 1957. Statistical problems of joint research. Jour. Farm Econ. 39(2):370-381.

Mendum, S. W. 1948. A device for analyzing yields: Short cut to yield-curve problems. Jour. Farm Econ. 30(2):357-364.

Ministerio de Agricultura. 1970. Estadistica Agraria Peru 1967. (Agricultural Statistics Peru 1967). Volume 1. Oficina de Estadistica, Lima, Peru.

Mitscherlich, E. A. 1909. Das Gesetz des Minimums und das Gesetz des abnehmenden Bodenertrages (The law of the minimum and the law of diminishing returns of land). Landw. Jahrb. 38:537-552.

Munson, R. D. and J. P. Doll. 1959. Economics of fertilizer use in crop production. Adv. in Agron. 11:133-169.

National Academy of Science. 1961. Status and methods of research in economic and agronomic aspects of fertilizer response and use. Nat. Res. Council, Nat. Academ. Sci. Pub. 918, Washington, D. C.

Nelson, L. A. and R. J. McCracken. 1962. Properties of Norfolk and Portsmouth soils: Statistical summarization and influence on corn yields. Soil Sci. Soc. Amer. Proc. 26:497-502.

LIST OF REFERENCES (continued)

Odell, R. T. 1958. Soil survey interpretation—yield prediction. Soil Sci. Soc. Amer. Proc. 22(2):157-160.

Odell, R. T. 1959. Effects of weather on corn and soybean yields. Illinois Research, University of Illinois Agric. Exp. Sta., Urbana.

Officer, R. R. and A. A. Halter. 1968. Utility analysis in a practical setting. Amer. Jour. Agric. Econ. 50(2):257-277.

Oury, B. 1965. Allowing for weather in crop production model building. Jour. Farm Econ. 47(2):270-283.

Parks, W. L. and J. L. Knetsch. 1959. Corn yields as influenced by nitrogen level and drouth intensity. Agron. Jour. 51:363-364.

Parks, W. L. and J. L. Knetsch. 1960. Utilizing drought-days in evaluating irrigation and fertility response studies. Soil Sci. Soc. Amer. Proc. 24(4):289-293.

Parvin, D. W. 1954. The choice and use of functions in farm management studies, pp. 19-20. In Proceedings of Association of Southern Agricultural Workers, 51st Annual Convention, Agricultural Economics and Rural Sociology Section, Dallas, Texas.

Pesek, J. T. 1956. Agronomic problems in securing fertilizer response data desirable for economic analysis, pp. 101-112. In E. L. Baum (ed.), Economic Analysis of Fertilizer Use Data. Iowa State College Press, Ames.

Quandt, R. E. 1960. Tests of the hypothesis that a linear regression system obeys two separate regimes. Jour. Amer. Stat. Assoc. 55:324-330.

Ranganathan, V., R. Soundarajan and C. S. Balasundaram. 1969. Studies on the applicability of Mitscherlich-Bray equation for correlating crop responses. Fertilité 33:31-42.

Reuss, J. O. and J. M. Geist. 1970. Prediction of the nitrogen requirements of field crops. Part I. Theoretical models for nitrogen release. Agron. Jour. 62(3):381-384.

Robison, D. E. 1964. Estimates for the points of intersection of two polynomial regressions. Jour. Amer. Stat. Assoc. 59:214-224.

Rouse, R. D. 1968. Soil test theory and calibration for cotton, corn, soybeans and coastal Bermudagrass. Auburn University Agric. Exp. Sta. Bull. 375, Auburn, Alabama.

Runge, E. C. A. 1968. Effects of rainfall and temperature interactions during the growing season on corn yield. Agron. Jour. 60:503-507.

LIST OF REFERENCES (continued)

Runge, E. C. A. and R. T. Odell. 1958. The relation between precipitation, temperature and the yield of corn on the Agronomy South Farm, Urbana, Illinois. Agron. Jour. 50:448-454.

Runge, E. C. A. and R. T. Odell. 1960. The relation between precipitation, temperature and the yield of soybeans on the Agronomy South Farm, Urbana, Illinois. Agron. Jour. 52:245-247.

Russell, J. S. 1968. Nitrogen fertilizer and wheat in a semi-arid environment. 4. Empricial yield response models and economic factors. Aust. Jour. Exp. Agric. Anim. Husb. 8:736-747.

Rust, R. H. and R. T. Odell. 1957. Methods used in evaluating the productivity of some Illinois soils. Soil Sci. Soc. Amer. Proc. 21: 171-175.

Shrader, W. D., W. A. Fuller and F. B. Cady. 1966. Estimation of a common nitrogen response function for corn (Zea mays) in different crop rotations. Agron. Jour. 58(4):397-401.

Smith, W. G. and W. L. Parks. 1967. A method incorporating probability into fertilizer recommendations. Jour. Farm Econ. 49(5):1511-1515.

Spillman, W. J. 1923. Application of the law of diminishing returns to some fertilizer and feed data. Jour. Farm Econ. 5:36-52.

Stemberger, A. P. 1957. Economic implications of using alternative production functions for expressing corn-nitrogen production relationships. North Carolina Agric. Exp. Sta. Tech. Bull, No. 126, Raleigh, North Carolina.

Stevens, W. L. 1951. Asymptotic regression. Biometrics 7:247-268.

Swanson, E. R. 1956. Selecting fertilizer programs by activity analysis, pp. 171-187. In E. L. Baum (ed.), Economic Analysis of Fertilizer Use Data. Iowa State College Press, Ames.

Swanson, E. R. 1957. Problems of applying experimental results to commercial practice. Jour. Farm Econ. 39(2):382-389.

Swanson, E. R. 1963. Static theory of the firm and three laws of plant growth. Soil Sci. 95:338-343.

Swanson, E. R. and E. H. Tyner. 1965. Influence of moisture regime on optimum nitrogen level and plant population for corn: A game theoretic analysis. Agron. Jour. 57:361-364.

LIST OF REFERENCES (continued)

Tollini, H. 1969. Actual and optimal use of fertilizer. Unpublished Ph.D. thesis, Department of Economics, North Carolina State University, Raleigh. University Microfilms, Ann Arbor, Michigan.

Tollini, H. and J. A. Seagraves. 1970. Actual and optimal use of fertilizer. Econ. Res. Rep. No. 14, Department of Economics, North Carolina State University, Raleigh.

Toro-Vizcarrondo, C. and T. D. Wallace. 1968. A test of the mean square error criterion for restrictions in linear regression. Jour. Amer. Stat. Assoc. 63:558-572.

Torres, J. A. 1970. Analysis of the effects of alternative policies in the food sector of the Peruvian economy. Unpublished M. S. thesis, Department of Economics, North Carolina State University, Raleigh.

Voss, R. and J. Pesek. 1967. Yields of corn grain as affected by fertilizer rates and environmental factors. Agron. Jour. 60:567-572.

Walker, O. L., E. O. Heady and J. T. Pesek. 1964. Application of game theoretic models to agricultural decision making. Agron. Jour. 56:170-173.

Walker, W. M. and S. G. Carmer. 1967. Determination of input levels for a selected probability of response on a curvilinear regression function. Agron. Jour. 59(2):161-162.

Walker, W. M. and O. H. Long. 1966. Effect of selected soil fertility parameters on soybean yields. Agron. Jour. 58:403-405.

Williams, M. S. and J. W. Couston. 1962. Crop production levels and fertilizer use. FAO, United Nations, Rome, Italy.

Yurtsever, N., M. Atesalp and S. W. Melsted. 1965. A tentative correlation for the Olsen bicarbonate phosphorous soil test with wheat responses under Turkish soil conditions. Soil Sci. 100:163-167.

Zapater, J. M. 1970. Nitrogen supplying ability of soils from the Sierra of Peru. Unpublished M. S. thesis, Department of Soil Science, North Carolina State University, Raleigh.

APPENDICES

Appendix A

Developing the Appropriate Response Equation

Problems in Model Specification

Appendix A Tables 1 through 4 are concerned with the specification of the correct model.

Appendix A Table 1. Means and standard deviations of variables from the experimental data for potatoes in Peru, 1960-1970

Variable	Units	Mean	Standard deviation
y	metric tons/ha.	17.5	9.81
N_s	percent o.m.	3.73	2.87
P_s	p.p.m. P	7.98	6.84
K_s	p.p.m. K	127	105
pH	standard units	5.93	1.11
N_A	kg./ha.	127	62
P_A	kg./ha.	103	66
K_A	kg./ha.	96	60
R	m.m.	568	136
T	° C	6.83	1.7
Number of observations		4,335	

Appendix A Table 2. Analysis of variance on Peruvian potato data, 1960-1970

Source	d.f.	Sum of squares	Mean square
Loc.-treat. combinations[a] (*i.e.*, loc. + treat. + (loc. x treat.))	391	34,740,106	88,849
Blocks in locations	194	458,695	2,364
Pooled error	3,724	6,460,099	1,734
Corr. total	4,309[b]	41,658,902	9,668

[a]As every treatment did not appear at each location, the analysis involved an unbalanced design. It was not possible to separate out location, treatment and location x treatment mean squares.

[b]The 420 observations which represented the means of four replications were repeated four times to ensure a correct analysis of variance. This accounts for the apparent discrepancy between the number of observations in the regression (3,075) and the number here (4,309). (Twenty-six cards were also inadvertently omitted from the analysis of variance.)

Appendix A Table 3. Generalized square root production functions F_2 and F_4 obtained from potato experiments in Peru, 1960-1970[a]

Variable	Equation F_2		Equation F_4	
	$\tilde{b}$ coefficient[b]	t-value[c]	$\tilde{b}$ coefficient[b]	t-value[c]
S	- 192.9097***	-27.78	---[d]	---[d]
b_0	-5232.6547***	-22.26	-1485.6661**	-2.96
$\sqrt{N_s}$	138.0343***	9.28	99.0824**	3.97
$\sqrt{P_s}$	2.6255*	2.08	- 4.9225*	-2.45
$\sqrt{K_s}$	7.3063***	18.60	1.5461	1.66
$\sqrt{pH}$	577.9596***	18.67	305.7868*	2.49
$\sqrt{N_A}$	- 0.3996	- 0.12	7.5563***	8.19
$\sqrt{P_A}$	27.9784***	9.88	16.8862***	8.76
$\sqrt{K_A}$	12.3574***	19.99	6.6288***	9.60
$\sqrt{R}$	15.6957***	6.56	---[d]	---[d]
$\sqrt{T}$	408.2980***	16.34	---[d]	---[d]
N_s	- 1.4129***	- 6.34	- 1.2036*	-1.99
P_s	- 0.07462	- 1.33	0.1327	1.55
K_s	- 0.04165***	-10.17	- 0.004606	-0.55
pH	- 38.2886***	-20.40	- 18.2306*	-2.42
N_A	- 0.1750***	- 5.03	- 0.1319**	-3.57
P_A	- 0.09837**	- 2.41	0.02637	0.58
K_A	- 0.7321***	-17.98	- 0.3321***	-6.56
R	0.00600	0.49	---[d]	---[d]
T	- 11.9581***	-19.72	---[d]	---[d]
$\sqrt{N_s \cdot pH}$	7.3206***	9.15	- 8.5876**	-3.18

Appendix A Table 3 (continued)

Variable	Equation F_2		Equation F_4	
	$\tilde{b}$ coefficient[b]	t-value[c]	$\tilde{b}$ coefficient[b]	t-value[c]
$\sqrt{N_S \cdot N_A}$	- 0.2328**	- 2.36	- 0.6161***	- 6.70
$\sqrt{N_S \cdot R}$	- 0.5118***	- 6.14	---[d]	---[d]
$\sqrt{N_S \cdot T}$	-15.8833***	-23.54	---[d]	---[d]
$\sqrt{P_S \cdot N_A}$	0.05982	1.19	0.1771**	3.41
$\sqrt{P_S \cdot P_A}$	- 0.6405***	-14.78	- 0.3152***	- 6.71
$\sqrt{K_S \cdot K_A}$	- 0.1639***	-11.04	- 0.08387***	- 5.22
$\sqrt{pH \cdot P_A}$	- 3.5174***	-17.09	- 1.9580***	- 8.47
$\sqrt{N_A \cdot P_A}$	0.4022***	11.38	0.2398***	6.76
$\sqrt{N_A \cdot K_A}$	0.1612***	4.64	0.05175	1.48
$\sqrt{N_A \cdot R}$	0.01737	0.64	---[d]	---[d]
$\sqrt{N_A \cdot T}$	0.5992**	3.91	---[d]	---[d]
$\sqrt{P_A \cdot K_A}$	0.3007***	8.84	0.1777***	5.08
$\sqrt{P_A \cdot R}$	0.05101*	2.24	---[d]	---[d]
$\sqrt{R \cdot T}$	- 1.4346***	- 7.88	---[d]	---[d]
$R^2 = 0.47$			$R^2 = 0.84$	
C.V. = 40.88 percent			C. V. = 22.65 percent	

[a]See footnote a in Table 2 of the text.

[b]***, **, * indicate significance at the 0.01, 1 and 5 percent levels, respectively, using a two-tailed test.

[c]These are the adjusted t's as explained in the text.

[d]Variables R and T and their interactions were not included in F_4. Coefficients of the dummy variables for F_4 are not included in the table.

Appendix A Table 4. Generalized quadratic equations F_3 and P_1 obtained from potato experiments in Peru, 1960-1970[a]

Variable	Equation F_3		Equation P_1	
	$\tilde{b}$ coefficient[b]	t-value[c]	$\tilde{b}$ coefficient[b]	t-value[c]
S	---[d]	---[d]	- 196.2254***	-27.80
b_0	-305.5729*	- 2.48	-1784.0217***	-18.81
N_s	3.9450**	3.96	8.4492***	11.64
P_s	- 0.3909***	- 5.55	- 0.1646**	- 3.84
K_s	0.0103	1.91	0.06399***	22.16
pH	6.8144	1.73	15.5504***	15.46
N_A	0.7444***	17.24	- 0.1440	- 1.09
P_A	0.8549***	10.74	0.7329***	6.40
K_A	0.4156***	10.27	0.8882***	23.14
R	---[d]	---[d]	0.09476***	9.27
T	---[d]	---[d]	27.01080***	15.07
N_s^2	- 0.009977**	- 3.47	- 0.01418***	-10.37
P_s^2	0.0008651***	5.15	0.0009530***	8.56
K_s^2	- 0.0000002	- 0.30	- 0.00000498***	-12.02
pH^2	- 0.06338*	- 2.04	- 0.1256***	-16.35
N_A^2	- 0.001758***	-13.72	- 0.002085***	-17.10
P_A^2	- 0.0008868***	- 4.41	- 0.001596***	- 6.89
K_A^2	- 0.001912***	- 9.08	- 0.004717***	-26.04
R^2	---[d]	---[d]	0.00000092	1.32
T^2	---[d]	---[d]	- 0.1296***	-16.10
$N_s \cdot pH$	- 0.02505	- 1.84	0.02197***	5.04
$N_s \cdot N_A$	- 0.003321***	- 6.57	- 0.0003667	- 0.65

Appendix A Table 4 (continued)

Variable	Equation F_3		Equation P_1	
	$\tilde{b}$ coefficient[b]	t-value[c]	$\tilde{b}$ coefficient[b]	t-value[c]
$N_s \cdot R$	---[d]	---[d]	- 0.0005342***	-12.25
$N_s \cdot T$	---[d]	---[d]	- 0.07330***	-19.66
$P_s \cdot N_A$	0.0003099	1.66	- 0.0004860**	- 2.68
$P_s \cdot P_A$	- 0.0009690***	- 5.40	- 0.002140***	-12.04
$K_s \cdot K_A$	- 0.0000618***	- 4.35	- 0.0001500***	-11.78
$pH \cdot P_A$	- 0.01020***	- 8.98	- 0.01515***	-15.02
$N_A \cdot P_A$	0.001199***	6.63	0.002410***	13.14
$N_A \cdot K_A$	0.0001105	0.60	0.0008024***	4.40
$N_A \cdot R$	---[d]	---[d]	0.00005403***	4.69
$N_A \cdot T$	---[d]	---[d]	0.006723***	7.04
$P_A \cdot K_A$	0.0009726***	4.47	0.001432***	6.91
$P_A \cdot R$	---[d]	---[d]	0.00009692***	9.07
$R \cdot T$	---[d]	---[d]	- 0.001252***	-10.30
$R^2 = 0.84$			$R^2 = 0.48$	
C. V. = 22.87 percent			C. V. = 41.13 percent	

[a]See footnote a in Table 2 of the text.

[b]***, **, * indicate significance at the 0.01, 1 and 5 percent levels, respectively, using a two-tailed test.

[c]These are adjusted t's as explained in the text.

[d]These variables were not included in Equation F_3. The coefficients for the 65 location and seven year dummy variables are not shown here in order to save space.

Tests for Multicollinearity among the Data

The following discussion and Appendix A Tables 5 and 6 are concerned with diagnoses and prescriptions for the problem of multicollinearity.

The following constrained regression models were fitted to the subdivided data:

$$\begin{bmatrix} Y' \\ \hline Y'' \end{bmatrix} = \left[\begin{array}{c|c|c} X_0' \; X_1' \; X_2' \; . \; . \; . \; X_{k-1}' & X_k' & 0 \\ \hline X_0'' X_1'' X_2'' . \; . \; . \; X_{k-1}'' & 0 & X_k'' \end{array}\right] \left[\begin{array}{c} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{k-1} \\ \hline \beta_k' \\ \hline \beta_k'' \end{array}\right] \tag{1}$$

This generated two regressions:

$$Y' = X_0'\beta_0 + X_1'\beta_1 + \ldots\ldots + X_{k-1}'\beta_{k-1} + X_k'\beta_k' \tag{2}$$

$$Y'' = X_0''\beta_0 + X_1''\beta_1 + \ldots\ldots X_{k-1}''\beta_{k-1} + X_k''\beta_k'' \tag{3}$$

where Y', X_0', X_1', X_2', ... X_k' = the first half of the data,

Y'', X_0'', X_1'', X_2'', ... X_k'' = the second half of the data, and

X_k = the variable of interest (e.g., N_A, $N_s \cdot N_A$ and R^2).

The variables X_0, X_1, X_2, ... X_{k-1}, whose coefficients are not being tested, are constrained to be equal. The coefficients on the variable of interest, X_k' and X_k'', are not. To test the hypothesis that β_k' and β_k'' are the same, the t-test in Graybill (1961, p. 133) was used. The hypothesis tested was

$$r'\hat{\beta} = r_0 = 0$$

where

$$r'\hat{\beta} = [0 \; . \; . \; . \; 0 \vdots 1 \vdots -1] \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \cdot \\ \cdot \\ \hat{\beta}_{k-1} \\ \hline \hat{\beta}'_k \\ \hline \hat{\beta}''_k \end{bmatrix} = \beta'_k - \beta''_k \quad . \tag{4}$$

The t-test statistic was:

$$t = \frac{r' \; \hat{\beta}}{\sqrt{(r'(X'X)^{-1})r\hat{\sigma}^2}} \quad . \tag{5}$$

Appendix A Table 5. Some simple correlation coefficients among independent variables.

	N_s	P_s	K_s	pH	N_A	P_A	K_A	R	T
y	0.12	-0.08	0.26	0.04	0.32	0.31	0.20	-0.17	-0.03[a]
N_s	1	-0.22	0.22	-0.34	0.07	0.24	0.13	-0.08	-0.35
P_s		1	0.17	0.02[a]	0.03[a]	-0.07	0.03	-0.12	0.03[a]
K_s			1	0.06	0.17	0.33	0.24	-0.28	-0.07
pH				1	0.04	-0.06	-0.01[a]	-0.04	0.22
N_A					1	0.20	0.19	-0.13	-0.06
P_A						1	0.33	-0.20	-0.11
K_A							1	-0.11	-0.11
R								1	-0.42
T									1

[a]These were not significant at the 5 percent level. All other coefficients were. These coefficients are based on 4335 observations on each pair of variables, many of which are repeated observations. Hence, the significance levels are biased downward.

Appendix A Table 6. Generalized quadratic production functions M_1 and M_2 from randomly dividing data into halves for potato experiments in Peru, 1960-1970[a]

Variable	Equation M_1		Equation M_2		t-value of difference in coefficients
	$\tilde{b}$ coefficient[b]	t-value[c]	$\tilde{b}$ coefficient[b]	t-value[c]	
S	- 207.6948***	-19.21	- 223.8510***	-20.67	0.69
b_0	-2345.4784***	-15.88	-1913.7096***	-13.29	1.36
N_s	9.5176***	8.98	7.6630***	7.21	0.80
P_s	- 0.4083***	- 4.86	- 0.3065**	- 3.68	0.57
K_s	0.06868***	15.84	0.07648***	17.25	0.82
pH	14.4360***	9.14	13.2454***	8.29	0.35
N_A	1.3868***	5.82	- 0.7571**	- 3.42	3.68**
P_A	1.2479***	5.93	1.1936***	5.53	0.10
K_A	1.0733***	14.71	0.8617***	10.74	1.11
R	0.1424***	9.08	0.08299***	5.53	1.78
T	35.9808***	12.97	34.1503***	12.40	0.31
N_s^2	- 0.01369***	- 6.71	- 0.01263***	- 6.02	0.21
P_s^2	0.001478***	8.41	0.001316***	7.25	0.42
K_s^2	- 0.00000583***	- 9.91	- 0.00000594***	- 9.63	0.84

Appendix A Table 6 (continued)

Variable	Equation M_1		Equation M_2		t-value of difference in coefficients
	$\tilde{b}$ coefficient[b]	t-value[c]	$\tilde{b}$ coefficient[b]	t-value[c]	
pH^2	- 0.1038***	8.47	- 0.1008***	- 8.21	0.11
N_A^2	- 0.002397***	-11.76	- 0.002684***	-12.67	0.54
P_A^2	- 0.001712**	- 3.61	- 0.001842**	- 3.82	0.11
K_A^2	- 0.004826***	-18.05	- 0.004219***	-15.28	0.88
R^2	0.00000112	1.06	0.00000411**	3.90	1.30
T^2	- 0.1663***	-13.30	- 0.1685***	-13.51	0.08
$N_s \cdot pH$	0.01913**	2.86	0.02056**	2.99	0.09
$N_s \cdot N_A$	- 0.002875**	- 3.20	0.003427***	6.58	2.66**
$N_s \cdot R$	- 0.0005715***	- 8.53	- 0.0004848***	- 7.38	0.60
$N_s \cdot T$	- 0.08039***	-14.67	- 0.07448***	-13.66	0.50
$P_s \cdot N_A$	- 0.001429***	- 4.66	- 0.0007142*	- 2.29	0.91
$P_s \cdot P_A$	- 0.0002928	- 0.87	- 0.001353***	- 4.01	1.24
$K_s \cdot K_A$	- 0.0001542***	- 8.53	- 0.0001763***	- 8.30	0.44
$pH \cdot P_A$	- 0.02611***	-12.30	- 0.02144***	-10.06	0.86
$N_A \cdot P_A$	0.001193**	2.87	0.002056***	4.63	0.79

Appendix A Table 6 (continued)

Variable	Equation M_1		Equation M_2		t-value of difference in coefficients
	$\tilde{b}$ coefficient[b]	t-value[c]	$\tilde{b}$ coefficient[b]	t-value[c]	
$N_A \cdot K_A$	0.00004234	0.14	0.001196**	3.60	1.42
$N_A \cdot R$	- 0.00005445**	- 2.89	0.0001086***	6.43	3.60**
$N_A \cdot T$	0.001434	0.96	0.01054***	6.97	2.38*
$P_A \cdot K_A$	0.001349**	3.14	0.0008241	1.84	0.47
$P_A \cdot R$	0.0001194***	7.16	0.0001004***	5.83	0.44
$R \cdot T$	- 0.001793***	- 9.53	- 0.001652***	- 8.81	0.35
	$R^2 = 0.45$		$R^2 = 0.43$		
	C.V. = 39.16 percent		C.V. = 39.31 percent		

[a]See footnote a in Table 2 of the text.

[b]***, **, * indicate significance at the 0.01, 1 and 5 percent levels, respectively, using a two-tailed test.

[c]These are the adjusted t's as explained in the text.

Appendix B

Calculation of Economic Optima

Appendix B Table 1. Total costs of fertilizer in Peru, March 1971[a]

Element	Cost per kilogram (S/.)			
		Materials plus transport		
	Materials (F.O.B. Lima)	Lowest (F_L)	Average (F_M)	Highest (F_H)
N_A	8.358	8.758	9.118	9.758
P_A	9.500	9.900	10.260	10.900
K_A	5.000	5.400	5.760	6.400

[a]Source: Ing° Ramiro Fernandez O., Jefe del Dpto. de Fertilizantes.

Appendix B Table 2. Price ratios[a]

Prices used[b]	$\frac{P_{N_A}}{P_y}$	$\frac{P_{P_A}}{P_y}$	$\frac{P_{K_A}}{P_y}$
F_L/P_L	0.08758	0.09900	0.05400
F_L/P_M	0.04379	0.04950	0.02700
F_L/P_H	0.02786	0.03300	0.01800
F_M/P_L	0.09118	0.10260	0.05760
F_M/P_M[c]	0.04559	0.05130	0.02880
F_M/P_H	0.03039	0.03420	0.01920
F_H/P_L	0.09758	0.10900	0.06400
F_H/P_M	0.04879	0.05450	0.03200
F_H/P_H	0.03253	0.03633	0.02133
Range	0.027860-0.097580	0.033000-0.109000	0.018000-0.064000

[a]This is based on the price ratio required to solve the optimum input levels from the fitted equations; namely price of element in S/. per kilogram divided by price of potatoes in S/. per 100 kilograms, as these were the units used in the equation. Source: see footnote a, Appendix B Table 1.

[b]See code in Appendix B Table 1 for fertilizer (F) prices. P_L, P_M and P_H refer to the following potato prices--S/. 100, S/. 200 and S/. 300 per 100 kilos, respectively.

[c]Most likely price ratios.

Appendix B Table 3. Optimum levels of applied nutrients for various soils with "below average" weather[a]

Solution number	Level of soil variables			Optimum levels of applied nutrients			Yield metric tons/ha.
	P_s p.p.m.	pH units	K_s p.p.m.	N_A kg./ha.	P_A kg./ha.	K_A kg./ha.	
28	1.14	4.82	22	257	308	151	18.89
29	1.14	4.82	127	251	300	131	23.08
30	1.14	4.82	232	246	292	111	26.30
31	1.14	5.93	22	222	213	140	15.45
32	1.14	5.93	127	217	205	119	19.83
33	1.14	5.93	232	211	197	99	23.26
34	1.14	7.04	22	187	117	128	11.91
35	1.14	7.04	127	182	109	108	16.50
36	1.14	7.04	232	176	101	88	20.13
37	7.98	4.82	22	229	275	146	13.57
38	7.98	4.82	127	223	267	126	17.84
39	7.98	4.82	232	218	259	106	21.16
40	7.98	5.93	22	194	180	135	10.96
41	7.98	5.93	127	189	172	114	15.43
42	7.98	5.93	232	183	164	94	18.95
43	7.98	7.04	22	159	84	123	8.25
44	7.98	7.04	127	154	76	103	12.93
45	7.98	7.04	232	149	68	82	16.65
46	14.82	4.82	22	201	242	141	9.93
47	14.82	4.82	127	195	234	121	14.29
48	14.82	4.82	232	190	226	101	17.69
49	14.82	5.93	22	166	147	129	8.15
50	14.82	5.93	127	161	139	109	12.71
51	14.82	5.93	232	155	131	89	16.32
52	14.82	7.04	22	131	51	118	6.28
53	14.82	7.04	127	126	43	97	11.05
54	14.82	7.04	232	121	35	77	14.86

[a]"Below average" weather refers to rainfall of 432 m.m. and a mean minimum temperature of 5.13° C. The optima were derived by the procedures described in Chapter IV.

Appendix B Table 4. Optimum levels of applied nutrients for various soils with "above average" weather[a]

Solution number	Level of soil variables			Optimum levels of applied nutrients			Yield metric tons/ha.
	P_s p.p.m.	pH units	K_s p.p.m.	N_A kg./ha.	P_A kg./ha.	K_A kg./ha.	
55	1.14	4.82	22	381	466	175	41.43
56	1.14	4.82	127	376	458	155	45.20
57	1.14	4.82	232	371	450	135	48.00
58	1.14	5.93	22	347	370	164	33.98
59	1.14	5.93	127	341	362	143	37.95
60	1.14	5.93	232	336	354	123	40.96
61	1.14	7.04	22	312	275	152	26.44
62	1.14	7.04	127	307	267	132	30.61
63	1.14	7.04	232	301	258	111	33.82
64	7.98	4.82	22	353	433	170	34.21
65	7.98	4.82	127	348	425	150	38.07
66	7.98	4.82	232	343	417	130	40.96
67	7.98	5.93	22	319	337	158	27.59
68	7.98	5.93	127	313	329	138	31.65
69	7.98	5.93	232	308	321	118	34.75
70	7.98	7.04	22	284	242	147	20.89
71	7.98	7.04	127	279	234	127	25.15
72	7.98	7.04	232	273	226	106	28.45
73	14.82	4.82	22	326	400	165	28.67
74	14.82	4.82	127	320	392	145	32.62
75	14.82	4.82	232	315	384	125	35.60
76	14.82	5.93	22	291	304	153	22.89
77	14.82	5.93	127	285	296	133	27.04
78	14.82	5.93	232	280	288	113	30.23
79	14.82	7.04	22	256	209	142	17.02
80	14.82	7.04	127	251	201	121	21.37
81	14.82	7.04	232	245	193	101	24.76

[a]"Above average" weather refers to rainfall of 704 m.m. and a mean minimum temperature of 8.55° C. The optima were derived by the procedures described in Chapter IV.

Appendix B Table 5. Expected profits ($E(\Pi)$) and variance of profits (σ_{Π}^2) for a soil low in organic matter (N_s) at various rates of N_A, P_A and K_A applied to potatoes[a]

$N_A:P_A:K_A$	$E(\Pi)$	σ_{Π}^2	λ[b]
(kg./ha.)	(S/.)	('000 S/.)	
0 0 0	9,961	28,086	0.001191
20 20 10	15,361	32,619	0.000957
40 40 20	20,311	37,790	0.000774
60 60 30	24,810	43,601	0.000628
80 80 40	28,859	50,052	0.000508
100 100 50	32,458	57,141	0.000363
140 140 70	38,305	73,237	0.000217
180 180 90	42,352	91,891	0.000106
220 220 110	44,598	113,101	0.000019
260 260 130	45,043	136,867	-0.000052
300 300 150	43,687	163,191	-0.000109
340 340 170	40,530	192,071	-0.000158
380 380 190	35,573	223,509	-0.000200
420 420 210	28,815	257,503	-0.000250
500 500 250	9,897	333,161	-0.000310

[a]The levels of all variables were the same as described in footnote a of Table 9 of the text except N_s was 0.86 percent here.

[b]See footnote b in Table 9.

Appendix B Table 6. Expected profits ($E(\Pi)$) and variance of profits (σ_{Π}^2) for soils high in organic matter (N_s) at various rates of N_A, P_A and K_A applied to potatoes[a]

N_A:P_A:K_A	$E(\Pi)$	σ_{Π}^2	λ[b]
(kg./ha.)	(S/.)	('000 S/.)	
0 0 0	15,590	32,284	-0.001560
20 20 10	21,049	28,785	-0.001752
40 40 20	26,059	25,926	-0.002053
60 60 30	30,618	23,705	-0.002599
80 80 40	34,727	22,124	-0.003884
100 100 50	38,386	21,182	0.175529
140 140 70	44,354	21,216	0.001608
180 180 90	48,520	23,806	0.000460
220 220 110	50,886	28,953	0.000073
260 260 130	51,451	36,657	-0.000120
300 300 150	50,215	46,918	-0.000237
340 340 170	47,179	59,736	-0.000315
380 380 190	42,341	75,110	-0.000370
420 420 210	35,703	93,041	-0.000429
500 500 250	17,025	136,574	-0.000487

[a]The levels of all variables were the same as described in footnote a of Table 9 of the text except N_s which was 6.6 percent here.

[b]See footnote b in Table 9.

Appendix C

Soil Test Probability Distribution, Computed Optima and Present Recommendations

Appendix C Table 1. Means, computed optima and recommended levels of N_A, P_A and K_A for soils with pH < 5.5, average weather, and a range of values of N_s, P_s and K_s[a]

Cell number	Class intervals (p.p.m.)			Number of observations	Class means (p.p.m.)				Amount of fertilizer to apply (kg./ha.)					
									N_A		P_A		K_A	
	N_s	P_s	K_s		pH	N_s	P_s	K_s	Opt.	Rec.	Opt.	Rec.	Opt.	Rec.
1	≤2.0	≤4	≤60	2	4.5	1.0	4	55	317	170	402	175	159	180
2	≤2.0	≤4	61-120	1	4.8	1.5	2	98	313	165	383	190	149	160
3	≤2.0	≤4	121-230	3	5.2	1.8	3	142	295	165	340	180	135	110
4	≤2.0	≤4	>230	0	-	-	-	-	-	-	-	-	-	-
5	≤2.0	5-8	≤60	1	4.5	2.0	8	40	302	160	384	115	159	185
6	≤2.0	5-8	61-120	0	-	-	-	-	-	-	-	-	-	-
7	≤2.0	5-8	121-230	2	5.0	1.3	5	170	291	170	345	170	130	105
8	≤2.0	5-8	>230	0	-	-	-	-	-	-	-	-	-	-
9	≤2.0	9-15	≤60	0	-	-	-	-	-	-	-	-	-	-
10	≤2.0	9-15	61-120	0	-	-	-	-	-	-	-	-	-	-
11	≤2.0	9-15	121-230	0	-	-	-	-	-	-	-	-	-	-
12	≤2.0	9-15	>230	0	-	-	-	-	-	-	-	-	-	-
13	≤2.0	>15	≤60	1	4.5	2.0	40	60	170	160	229	20	131	175
14	≤2.0	>15	61-120	1	5.3	1.5	30	100	183	165	205	20	122	160
15	≤2.0	>15	121-230	1	5.4	2.0	32	170	169	160	181	20	106	105
16	≤2.0	>15	>230	1	5.1	1.7	60	320	56	165	61	20	59	0
17	2.1-4.0	≤4	≤60	7	4.8	3.3	3	48	313	135	382	185	158	180
18	2.1-4.0	≤4	61-120	10	4.7	3.0	3	94	314	140	387	185	150	160
19	2.1-4.0	≤4	121-230	20	5.1	3.3	3	171	297	135	347	185	131	105
20	2.1-4.0	≤4	>230	3	5.1	2.5	2	286	295	150	343	190	109	25
21	2.1-4.0	5-8	≤60	10	4.8	3.1	6	53	301	140	367	160	155	180
22	2.1-4.0	5-8	61-120	4	5.1	3.6	6	102	289	135	338	160	142	160
23	2.1-4.0	5-8	121-230	6	5.0	3.4	6	152	289	135	343	160	133	110
24	2.1-4.0	5-8	>230	2	5.2	3.6	5	350	277	135	315	170	94	0
25	2.1-4.0	9-15	≤60	0	-	-	-	-	-	-	-	-	-	-
26	2.1-4.0	9-15	61-120	3	4.5	3.2	11	93	288	140	366	110	146	160
27	2.1-4.0	9-15	121-230	3	5.0	2.7	9	197	274	150	325	110	122	100

Appendix C Table 1 (continued)

Cell number	Class intervals (p.p.m.)			Number of observations	Class means (p.p.m.)				Amount of fertilizer to apply (kg./ha.)					
									N_A		P_A		K_A	
	N_s	P_s	K_s		pH	N_s	P_s	K_s	Opt.	Rec.	Opt.	Rec.	Opt.	Rec.
28	2.1-4.0	9-15	>230	1	4.6	3.1	13	240	269	140	337	100	115	30
29	2.1-4.0	>15	<60	1	5.4	3.3	18	50	233	135	258	30	140	180
30	2.1-4.0	>15	61-120	0	-	-	-	-	-	-	-	-	-	-
31	2.1-4.0	>15	121-230	1	5.4	2.3	21	170	214	160	234	20	114	105
32	2.1-4.0	>15	>230	1	5.1	3.2	33	234	172	135	198	20	96	25
33	>4.0	<4	<60	12	5.1	5.7	2	48	309	120	362	190	156	180
34	>4.0	<4	61-120	41	5.0	6.5	2	95	311	115	367	190	148	160
35	>4.0	<4	121-230	41	4.8	6.6	2	159	314	115	380	190	137	110
36	>4.0	<4	>230	17	5.0	5.9	2	393	295	120	344	190	90	0
37	>4.0	5-8	<60	5	4.6	6.8	6	41	310	115	387	160	159	185
38	>4.0	5-8	61-120	14	4.9	6.9	7	104	293	115	351	160	143	160
39	>4.0	5-8	121-230	20	4.8	6.3	7	173	292	115	354	160	131	105
40	>4.0	5-8	>230	11	4.9	6.0	6	363	283	120	336	160	94	0
41	>4.0	9-15	<60	3	4.6	7.2	11	58	289	110	362	110	152	180
42	>4.0	9-15	61-120	8	4.5	6.1	12	89	286	120	363	105	147	160
43	>4.0	9-15	121-230	7	4.8	8.5	11	170	278	100	336	110	129	105
44	>4.0	9-15	>230	6	4.8	5.2	11	352	266	125	321	110	93	0
45	>4.0	>15	<60	6	4.6	6.6	42	52	162	115	213	20	130	180
46	>4.0	>15	61-120	31	4.9	7.3	44	94	143	110	174	20	117	160
47	>4.0	>15	121-230	21	5.0	6.8	37	172	164	115	193	20	107	105
48	>4.0	>15	>230	7	4.9	7.1	46	318	123	110	148	20	73	0

[a]Blanks indicate there were no soil samples in the relevant ranges from the La Molina laboratory.

Appendix C Table 2. Means, computed optima and recommended levels of N_A, P_A and K_A for soils with $5.5 \leq pH \leq 6.5$, average weather, and a range of values of N_s, P_s and K_s[a]

Cell number	Class intervals (p.p.m.)			Number of observations	Class means (p.p.m.)				Amount of fertilizer to apply (kg./ha.)					
									N_A		P_A		K_A	
	N_s	P_s	K_s		pH	N_s	P_s	K_s	Opt.	Rec.	Opt.	Rec.	Opt.	Rec.
49	≤2.0	≤4	<60	5	6.1	1.9	2	42	276	165	275	185	146	185
50	≤2.0	≤4	61-120	2	6.0	1.5	4	83	269	165	271	180	137	170
51	≤2.0	≤4	121-230	8	5.9	1.7	2	182	275	165	281	185	121	105
52	≤2.0	≤4	>230	1	6.1	2.0	4	300	255	160	246	175	95	10
53	≤2.0	5-8	≤60	2	6.3	1.8	6	50	253	165	238	160	139	180
54	≤2.0	5-8	61-120	5	6.0	1.5	6	90	260	165	260	160	135	160
55	≤2.0	5-8	121-230	7	6.3	1.7	6	166	247	165	229	160	117	105
56	≤2.0	5-8	>230	1	6.1	1.7	8	444	231	165	215	115	64	0
57	≤2.0	9-15	≤60	0	-	-	-	-	-	-	-	-	-	-
58	≤2.0	9-15	61-120	11	6.0	1.7	11	88	240	165	237	110	131	170
59	≤2.0	9-15	121-230	2	6.5	1.2	11	176	219	170	187	110	109	105
60	≤2.0	9-15	>230	2	6.2	1.4	13	480	205	170	180	105	52	0
61	≤2.0	>15	≤60	3	5.9	1.7	17	43	221	165	220	30	137	185
62	≤2.0	>15	61-120	4	5.8	1.7	22	108	200	165	200	20	121	160
63	≤2.0	>15	121-230	1	5.5	1.5	35	125	156	165	162	20	111	115
64	≤2.0	>15	>230	0	-	-	-	-	-	-	-	-	-	-
65	2.1-4.0	≤4	≤60	8	6.0	3.2	3	46	276	135	279	185	145	180
66	2.1-4.0	≤4	61-120	22	6.0	3.0	3	89	273	140	276	185	137	160
67	2.1-4.0	≤4	121-230	15	6.0	2.7	2	187	272	150	273	190	119	105
68	2.1-4.0	≤4	>230	7	5.8	2.9	3	256	271	150	280	180	107	30
69	2.1-4.0	5-8	≤60	2	6.5	2.8	6	35	248	150	222	160	140	190
70	2.1-4.0	5-8	61-120	4	6.2	2.9	7	79	251	140	240	160	134	170
71	2.1-4.0	5-8	121-230	15	6.1	2.9	6	153	255	140	248	160	122	110
72	2.1-4.0	5-8	>230	11	6.0	3.4	6	298	251	135	245	160	95	20
73	2.1-4.0	9-15	≤60	3	6.0	3.1	10	55	247	140	245	110	138	180
74	2.1-4.0	9-15	61-120	11	6.2	3.3	12	96	230	135	215	105	127	160
75	2.1-4.0	9-15	121-230	12	6.2	3.1	12	181	226	140	208	105	111	105

Appendix C Table 2 (continued)

Cell number	Class intervals (p.p.m.)			Number of observations	Class means (p.p.m.)				Amount of fertilizer to apply (kg./ha.)					
									N_A		P_A		K_A	
	N_s	P_s	K_s		pH	N_s	P_s	K_s	Opt.	Rec.	Opt.	Rec.	Opt.	Rec.
76	2.1-4.0	9-15	>230	9	6.1	3.3	11	401	222	135	205	110	70	0
77	2.1-4.0	>15	<60	1	6.0	2.3	16	50	222	160	216	35	135	180
78	2.1-4.0	>15	61-120	5	6.1	2.8	23	95	188	150	170	20	120	160
79	2.1-4.0	>15	121-230	9	5.9	3.0	28	178	170	140	157	20	102	105
80	2.1-4.0	>15	>230	3	5.8	2.8	33	358	143	150	128	20	65	0
81	>4.0	≤4	<60	7	6.1	5.5	1	49	282	125	280	190	146	180
82	>4.0	≤4	61-120	20	5.8	6.1	3	101	281	120	293	185	137	160
83	>4.0	≤4	121-230	20	5.8	5.9	3	177	277	120	287	185	123	105
84	>4.0	≤4	>230	9	5.9	5.6	2	330	270	125	271	115	93	0
85	>4.0	5-8	<60	2	5.9	6.5	7	58	264	115	269	160	141	180
86	>4.0	5-8	61-120	8	5.9	5.0	6	98	265	125	270	160	134	160
87	>4.0	5-8	121-230	20	6.0	6.7	6	176	259	115	256	160	118	105
88	>4.0	5-8	>230	7	5.9	5.8	6	268	257	120	257	160	102	25
89	>4.0	9-15	<60	0	-	-	-	-	-	-	-	-	-	-
90	>4.0	9-15	61-120	5	5.9	6.3	10	104	249	115	251	110	130	160
91	>4.0	9-15	121-230	17	6.0	6.2	12	159	235	115	228	105	117	110
92	>4.0	9-15	>230	12	6.0	6.1	12	345	226	115	214	105	81	0
93	>4.0	>15	<60	1	6.4	6.7	19	60	200	115	168	25	127	175
94	>4.0	>15	61-120	4	5.8	6.8	44	103	114	115	96	20	106	160
95	>4.0	>15	121-230	21	5.9	6.7	27	170	177	115	164	20	105	105
96	>4.0	>15	>230	22	6.1	7.0	44	403	89	115	47	20	45	0

[a]Blanks indicate there were no soil samples in the relevant ranges from the La Molina laboratory.

Appendix C Table 3. Means, computed optima and recommended levels of N_A, P_A and K_A for soils with pH > 6.5, average weather, and a range of values of N_s, P_s and K_s[a]

Cell number	Class intervals (p.p.m.)			Number of observations	Class means (p.p.m.)				Amount of fertilizer to apply (kg./ha.)					
									N_A		P_A		K_A	
	N_s	P_s	K_s		pH	N_s	P_s	K_s	Opt.	Rec.	Opt.	Rec.	Opt.	Rec.
97	≤2.0	≤4	<60	18	8.1	1.6	2	47	213	165	102	190	124	185
98	≤2.0	≤4	61-120	24	8.2	1.7	2	82	208	165	91	190	116	180
99	≤2.0	≤4	121-230	16	8.0	1.7	3	167	206	165	97	185	101	115
100	≤2.0	≤4	>230	5	7.9	1.3	4	303	198	170	90	175	75	30
101	≤2.0	5-8	<60	9	6.9	1.8	6	53	234	165	186	160	132	185
102	≤2.0	5-8	61-120	8	8.0	1.8	6	104	197	165	87	160	111	170
103	≤2.0	5-8	121-230	16	7.3	1.7	7	152	212	165	139	160	108	160
104	≤2.0	5-8	>230	3	7.3	1.7	5	348	210	165	134	170	72	20
105	≤2.0	9-15	<60	2	7.4	1.4	15	50	182	165	100	40	121	185
106	≤2.0	9-15	61-120	10	7.5	1.7	10	101	196	165	111	110	114	170
107	≤2.0	9-15	121-230	12	7.7	1.5	11	163	183	165	84	110	99	115
108	≤2.0	9-15	>230	3	8.7	1.8	13	280	145	165	0	100	67	35
109	≤2.0	>15	<60	2	7.5	1.8	25	40	138	165	44	20	114	185
110	≤2.0	>15	61-120	1	6.7	1.0	17	65	194	170	149	30	124	180
111	≤2.0	>15	121-230	1	7.3	2.0	18	170	167	160	85	30	97	115
112	≤2.0	>15	>230	7	7.9	1.6	30	404	102	165	0	20	41	0
113	2.1-4.0	≤4	<60	51	8.0	3.1	2	45	217	140	111	185	125	185
114	2.1-4.0	≤4	61-120	72	7.7	3.1	3	92	220	140	129	180	119	170
115	2.1-4.0	≤4	121-230	38	7.7	3.1	3	168	216	140	123	180	104	115
116	2.1-4.0	≤4	>230	13	7.7	3.0	3	316	209	140	112	180	75	30
117	2.1-4.0	5-8	<60	27	7.7	3.0	6	50	210	140	118	160	125	185
118	2.1-4.0	5-8	61-120	46	7.8	2.8	6	92	205	150	106	160	115	170
119	2.1-4.0	5-8	121-230	33	8.0	2.9	7	163	191	140	78	160	99	115
120	2.1-4.0	5-8	>230	14	7.5	3.2	7	289	200	135	112	160	80	35
121	2.1-4.0	9-15	<60	14	7.5	2.8	11	52	196	150	111	110	122	185
122	2.1-4.0	9-15	61-120	31	7.7	3.2	11	94	187	135	90	110	112	170
123	2.1-4.0	9-15	121-230	22	7.5	3.2	11	169	190	140	102	110	100	115

Appendix C Table 3 (continued)

Cell number	Class intervals (p.p.m.)			Number of observations	Class means (p.p.m.)				Amount of fertilizer to apply (kg./ha.)					
									N_A		P_A		K_A	
	N_s	P_s	K_s		pH	N_s	P_s	K_s	Opt.	Rec.	Opt.	Rec.	Opt.	Rec.
124	2.1-4.0	9-15	>230	12	8.2	2.9	10	298	165	140	36	110	68	30
125	2.1-4.0	>15	<60	4	6.9	2.6	25	53	157	155	95	20	118	185
126	2.1-4.0	>15	61-120	25	7.3	3.2	24	98	147	135	62	20	106	170
127	2.1-4.0	>15	121-230	15	7.5	3.1	32	176	104	140	0	20	83	115
128	2.1-4.0	>15	>230	22	7.9	2.9	33	459	95	140	0	20	31	0
129	>4.0	<4	<60	24	7.4	5.6	3	48	233	125	159	185	131	185
130	>4.0	<4	61-120	36	7.4	6.2	3	95	231	115	156	185	122	170
131	>4.0	<4	121-230	27	7.4	5.2	3	170	227	125	149	185	107	115
132	>4.0	<4	>230	14	7.2	5.3	4	307	222	125	151	175	82	30
133	>4.0	5-8	<60	6	7.1	5.3	6	46	230	125	170	160	132	185
134	>4.0	5-8	61-120	28	7.6	5.8	6	96	213	120	124	160	117	170
135	>4.0	5-8	121-230	21	7.8	6.0	6	160	203	120	102	160	103	160
136	>4.0	5-8	>230	9	7.4	6.3	6	394	204	115	118	160	62	0
137	>4.0	9-15	<60	4	7.3	4.2	10	54	207	130	133	110	125	185
138	>4.0	9-15	61-120	11	7.4	5.8	11	91	199	120	117	110	116	170
139	>4.0	9-15	121-230	10	7.5	5.5	11	169	191	125	103	110	100	115
140	>4.0	9-15	>230	7	7.6	5.2	13	394	168	125	67	105	54	0
141	>4.0	>15	<60	1	7.5	9.1	40	60	91	100	0	20	103	175
142	>4.0	>15	61-120	15	7.2	5.3	25	94	147	125	67	20	107	170
143	>4.0	>15	121-230	20	7.6	5.6	30	172	111	125	2	20	84	115
144	>4.0	>15	>230	31	7.4	6.1	42	395	77	115	0	20	41	0

[a]Blanks indicate there were no soil samples in the relevant ranges from the La Molina laboratory.

Appendix C Table 4. Estimated average fixed, average variable, average total and marginal costs of growing potatoes in the Sierra of Peru, at 1971 prices[a]

Yield (1)	Total variable costs[b] (2)	Average variable costs (3)	Average fixed costs[c] (4)	Average total costs (5)	Marginal costs (6)
(m.t./ha.)	(S/. per ha.)		(S/. per metric ton)		
13.10	0	0	1,162	1,162	
19.62	1,321	67	776	843	203
25.11	2,473	98	606	704	210
26.84	3,305	123	567	690	481
27.43	3,711	135	555	690	688
27.70	3,953	143	550	693	896
27.98	4,285	153	544	697	1186
28.10	4,452	158	542	700	1670

[a]Costs are calculated for one hectare. They are based on the unpublished data in J. D. Coffey, 1965, "Estimated Costs, Yield, and Resource Requirements for Crops and Livestock for the Cuzco Sierra Region of Peru." Faculty of Social Sciences, Universidad Agraria, Lima, Peru, p. 47.

[b]It is assumed fertilizer is the only item in variable costs. The levels of fertilizers to attain the yields in column (1) are shown in Table 14. The per unit costs of fertilizers used are shown in Appendix B Table 1, column (4).

[c]These are based on a cost per hectare of S/. 15,222 taken from Coffey's unpublished 1965 paper cited in footnote a above, after allowing for price inflation since 1965.