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PLANNING SMALL WATER SUPPLIES IN DEVELOPING COUNTRIES

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PLANNING SMALL WATER SUPPLIES IN DEVELOPING COUNTRIES

BY
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16. Abstracts

This research included two principal objectives:

(1) develop a theoretical planning model for deciding water supply timing and scale in small communities of developing countries,

(2) initiate field studies to obtain data on the parameters of the model to make it operational

The work of model development had to focus on several communities instead of only one. Additionally, time in the model had to be made discrete because budgets are imposed at fixed points in time. Finally, the model had to include the considerations of Manne's model pertinent to developing countries: economies of scale, water supply benefits, increasing demand, the discount rate, etc.

While the first research objective is theoretical, the second is primarily applied. It was proposed to obtain at least preliminary information on water demand patterns in small communities, costs of water system construction, the economies of scale of water systems abroad, and by imputing, the benefits of publicly supplied water. All of the field data were obtained from Central America.

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PLANNING SMALL WATER SUPPLIES IN DEVELOPING COUNTRIES

by

Donald T. Lauria*

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Planning Small Water Supplies in Developing Countries

1. General Background

Public water supply systems are generally lacking throughout the world. In a study of 75 developing countries, Dieterich (1963) found that only one-third of the urban population and less than 10 percent of the total population was served with piped water into the home. An additional 26 percent of the urban dwellers obtained water from public outlets, but 40 percent of the urban and at least 70 percent of the total population had no access to piped water.

Dieterich estimated that within 15 years of his study, about 450 million urban dwellers in the countries he examined would need new, extended or improved systems. He estimated their cost at about 6.6 billion dollars. Bierstein (1968), however, thinks the estimate is too low; the cost is more likely to be 10 billion dollars.

The most serious problem facing developing countries regarding public water supplies is lack of funds. Other problems, however, also exist. Two of particular concern herein are the following: (1) once funds are earmarked in a particular country for water systems, decisions must be made on the towns to receive them, and (2) once towns have been selected, decisions must be made on the capacity of each system to be constructed.

The selection of towns to receive systems is basically a problem in investment timing. Assuming that all towns will eventually have supply facilities, the question is: should the particular town under consideration receive a system this year or not? In developing countries, this question is usually resolved by value judgement and political fiat.

Determination of water system capacity for each town is fundamentally a question of investment scale: how large should water systems be constructed? As in the economically advanced countries, water systems abroad are usually constructed with capacity to meet existing demands plus some excess. The amount of excess capacity is generally determined by applying design standards from the advanced countries. It is common, for example, to provide capacity beyond immediate needs for 20 or 25 years or even longer.

The deficiencies of current water supply planning practices abroad are readily apparent. Use of value judgement and political fiat can easily lead to misallocation of scarce funds, and unquestioned use of U.S. design standards in countries with significantly different economic conditions is clearly inappropriate. As a result, this research was undertaken to develop a more rigorous methodology for deciding water supply timing and scale in low income countries.

2. Related Work

The proposal for this research was based largely on the work of Alan Manne (1967) who developed mathematical models for investment in the chemical process industries. In the model most applicable to water supplies, demand is assumed to increase linearly with time as shown in Figure 1; the rate of demand increase is D units (million gallons per day, for example) per year. Initially (at time 0), the capacity of supply facilities is equal to the rate of demand. y years are allowed to elapse before making the first expansion. During this period, either the supply deficit can be made up by importing from another supplier or the deficit can be left unsatisfied; in either case, a cost is incurred. With importing. the cost is p dollars per unit imported, but if the demand goes unsatisfied, a social loss

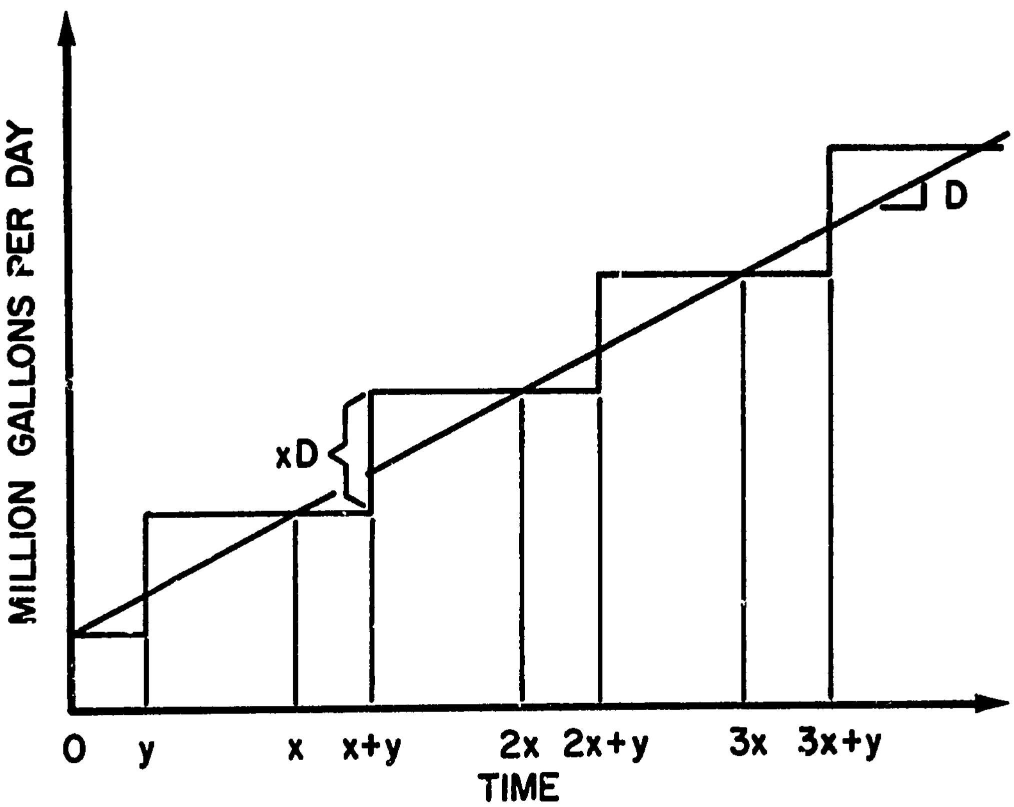

Figure I. Expansion Model With Deficits

results of p dollars per unit demanded but not supplied.

Assume at year y (the first time of construction) that the expansion has capacity xD (mgd) and costs C(xD) dollars. If the demand goes on forever and costs do not change with time, expansions of this same scale are required in years x + y, 2x + y, etc., and every construction period is preceded by y years of deficit, as shown. With this information, the following expression can be written for total present value cost over an infinite time horizon.

$$[\int_0^y e^{-rt}\ pDt\ \ dt\ +\ e^{-ry}\ C(xD)]/(1-e^{-rx}) \qquad (1)$$

The first term inside the bracket is the present value of importing or social costs during y years of deficit. The second term is the present value expansion cost. The entire bracketed expression is the total present value cost incurred each cycle of x years. The term outside the bracket is the present worth factor for an infinite number of cycles. In each term, r is the discount rate.

Both in the chemical and water supply industries, C(xD) is a function of the form

$$C(xD) = k(xD)^a, \qquad (2)$$

where k is a constant and "a", a proper fraction, is called the economy of scale factor. This function is shown in Figure 2. Note that the slope of line segment 0-C(z) is the average cost of a system of scale z. As scale increases, average cost decreases which is the condition that obtains when economies of scale exist.

Substituting (2) into (1) results in an expression with variables x and y and parameters p, D, r, k and a. y is the decision variable that denotes when to make the expansion (i.e., investment timing) and x is the decision variable associated with expansion scale. (x-y) is the number of years of excess capacity of each expansion which is generally called the

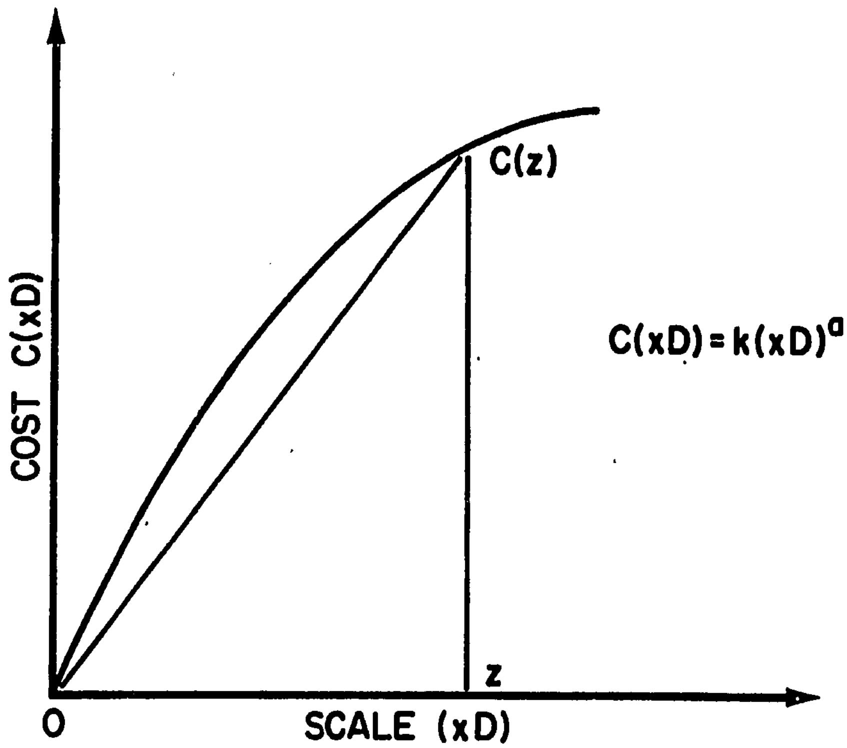

Figure 2. Power Cost Function

design period. To find the optimal values of the variables (i.e., the values that minimize total present value cost), the partial derivatives of (1) with respect to x and y can be set equal to zero and solved. The most important optimality condition that results is

$$y^* = \frac{r[k(xD)^a]}{pD} , \qquad (3A)$$

where the asterisk denotes an optimal value.

This completes Manne's model. It has valuable implications for water supply planning abroad. Most important, it presents a framework for making optimal decisions on expansion timing and scale taking account of demand, the discount rate, the social losses associated with unsatisfied demand, and the particular characteristics of water system cost functions. Besides providing this framework, (3A) shows that y^* approaches zero as p increases to infinity. But this implies that there should be no period of deficit when the social losses due to unsatisfied demand are very large. In the economically advanced countries, y^* is deliberately set to zero (i.e., supply deficits are disallowed) which implicitly assigns high value to p. In the developing countries, of course, the period of deficit is not usually zero.

Another important observation can be made from (3A) by rearranging it as follows

$$p = \frac{r[k(xD)^a]}{Dy^*} , \qquad (3B)$$

The numerator of the fraction is the product of the discount rate and the cost of expansion which is the annual opportunity cost of capital (a cost analagous to the annual interest charge). The denominator is the unsatisfied rate of demand at the time of expansion. In this form, the equation shows that a value is imputed to the social losses (or correspondingly, to the benefits of publicly supplied water) whenever a decision is made to invest in a water system. That is, by deciding to build at some point in time (that is assumed to be optimal) and to some particular scale (not necessarily optimal), a value is implicitly assigned to the benefits of public supply.

3. Research Objectives

With Manne's work as a guide, research was proposed which included two objectives:

(1) develop a theoretical planning model for deciding water supply timing and scale in small communities of developing countries.

(2) initiate field studies to obtain data on the parameters of the model to make it operational.

Although Manne's model provides valuable insights to water supply planning in developing countries, as it stands it is not suitable for use. In the first place, it is intended for expansion planning. This follows from the assumption that supply capacity and demand are initially equal. Clearly, water supply planning abroad is primarily for new systems that have initial outstanding demand, not for expansions.

More serious, Manne's model is for a single independent community. But towns abroad cannot be treated independently. Rather, planning is done regionally, performed by a central agency of the national government, and the basic problem is to allocate annual budgets among towns in need of systems. The budgets create economic interdependencies among systems (what is done in one town affects the others; for example, funds spent in A are automatically denied to B,C,D,...) which invalidates use of a single-system model like Manne's.

The work of model development, therefore, had to focus on several communities instead of only one. Additionally, time in the model had to be made discrete because budgets are imposed at fixed points in time. (Time in Manne's model is continuous.) Finally, the model had to include the considerations of Manne's model pertinent to developing countries:

economies of scale, water supply benefits, increasing demand, the discount rate, etc.

While the first research objective is theoretical, the second is primarily applied. It was proposed to obtain at least preliminary information on water demand patterns in small communities, costs of water system construction, the economies of scale of water systems abroad, and by imputing, the benefits of publicly supplied water. In connection with this last point, it was proposed to obtain field information on the parameters of the right hand side of (3B) for water supply investment decisions that had been made in the past. This includes the discount rate (r), construction cost $[k(xD)^a]$, and the unsatisfied rate of demand at the time of construction (Dy^*). Values of p were then to be calculated from (3B). All of the field data were to be obtained from Central America.

4. Regional Planning Model

The essential elements of the planning model developed in this research are presented in Appendix 1. The model, however, in the appendix is not for regional planning. Rather, it is basically a reformulation of Manne's model with discrete instead of continuous time and with the assumption relaxed that supply capacity and demand are initially in balance. With the conversion to discrete time, the model can easily be extended to accomodate multiple towns and budget constraints and thus be suitable for regional planning. The work required to extend the model is described in Appendix 1, and the actual model in extended form is included in sections 4 and 5 of Appendix 2. For the sake of completeness, the model is summarized herein.

Let time be divided into short (say, annual) periods denoted t. In certain of these periods (to be selected by the model user), let water supply systems be proposed for construction. These "construction opportunity periods" are denoted j. Let C_{ij} be the construction cost of the system proposed for town i in period j. C_{ij} is analagous to the cost function C(xD) of Manne's model. Instead of being curvilinear however, C_{ij} is a straight line approximation of C(xD) as shown in Figure 3. Clearly, it is accurate only for capacities between A and B.

C_{ij} is called a fixed charge function. In symbols,

$$C_{ij} = S_{ij}\ Z_{ij} + s_{ij}\ z_{ij}\ , \tag{4}$$

where S_{ij} is a fixed charge (or set-up cost) for the system proposed for town i in year j; s_{ij} is the cost per unit capacity (say, dollars per mgd), as shown. z_{ij} is a decision variable that denotes the scale (mgd) of the system to be constructed in town i in year j. Model solution will yield optimal values for the z's. Z_{ij} is also a decision variable but unlike z_{ij} which is continuous, Z_{ij} must be either 0 or 1. If a system is constructed at i in year j, then z_{ij} will be positive and Z_{ij} must equal 1 so that the fixed charge is incorporated in the cost function. If a system is not constructed, then $z_{ij} = Z_{ij} = 0$.

Construction cost incurred in period j can be discounted to present value (p.v.) by multiplying by a present worth factor, F_j. Total p.v. construction costs for water systems proposed for all towns in all periods is obtained by summing the discounted value of (4) over all i and j

$$\sum_i \sum_j F_j\ S_{ij}\ Z_{ij} + \sum_i \sum_j F_j\ s_{ij}\ z_{ij}\ . \tag{5}$$

As in Manne's model, let p be the social loss associated with each gallon of water demanded but not supplied. The units of p are dollars per gallon, and if the value of publicly supplied water varies among towns and from one period to another, we must use the symbol p_{it} to denote the particular value in town i in year t. Let y_{it} be the unsatisfied demand in town i in year t. y_{it} is a (continuous) decision variable to be evaluated from model solution; typical units are mgd. Let d_t be the duration of year t; in general,

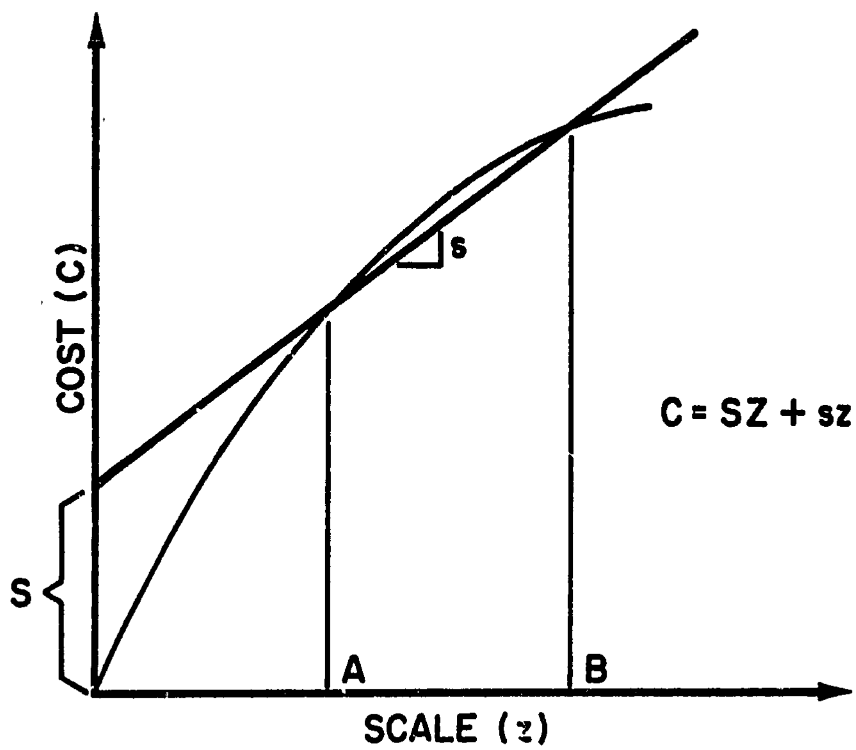

Figure 3. Fixed-Charge Cost Function

d_t is 365 days for all t. Then the amount of demand (in million gallons) left unsatisfied in town i in year t is $d_t\, y_{it}$. The social cost that results from this unsatisfied demand is $p_{it}\, d_t\, y_{it}$, and the total p.v. of social losses in all towns during all periods of the planning horizon is

$$\sum_i \sum_t F_t\, p_{it}\, d_t\, y_{it} \qquad (6)$$

where F_t is the present worth factor for year t. The total objective function to be minimized for this model is the sum of (5) and (6).

At this point, it is important to note that some of the decision variables (the z's and y's) are continuous whereas others (the Z's) are integers. Hence, this is a mixed integer programming (MIP) model which has nearly the identical format as a linear programming (LP) problem and indeed can be solved by repeated use of LP*.

To complete the model, several constraint sets are required. The Z's must be 0 or 1 and the z's and y's must be nonnegative.

$$Z_{ij} = 0 \text{ or } 1, \text{ all } i,j$$

$$z_{ij} \geq 0 \quad , \text{ all } i,j \qquad (7)$$

$$y_{it} \geq 0 \quad , \text{ all } i,t$$

As described above, Z must be 1 whenever project scale is positive to assure inclusion of the fixed charge. This is accomplished by the constraint

$$Z_{ij} \geq k_i\, z_{ij} \text{ , all } i, j, \qquad (8)$$

where k_i is a small number that is generally different for each town. Note

*LP and MIP models and their solutions are very well described in Hillier and Lieberman (1967) and McMillan (1970).

that when z_{ij} is positive, no matter how small, Z_{ij} will have to equal 1. z_{ij}, however, can never exceed $1/k_i$; k_i therefore, sets an upper bound on z_{ij} and in general should be assigned the reciprocal value of the upper limit of accuracy of the fixed charge function. In Figure 3 for example, k_i would be 1/B.

Perhaps the heart of the constraint set is the restriction placed on demand. For any town i in any year t, we require that

$$\begin{bmatrix}\text{initial}\\ \text{supply}\\ \text{capacity}\end{bmatrix} + \begin{bmatrix}\text{capacity}\\ \text{constructed}\\ \text{prior to t}\end{bmatrix} + \begin{bmatrix}\text{unsatisfied}\\ \text{demand}\\ \text{in t}\end{bmatrix} \geq \begin{bmatrix}\text{total}\\ \text{demand}\\ \text{in t}\end{bmatrix}$$

In symbols we have

$$Q_{io} + \sum_{j<t} z_{ij} + y_{it} \geq q_{it} \text{ , all i,t} \quad (9)$$

where Q_{io} is the existing supply capacity in town i at the start of the planning period and q_{it} is the water demand in town i in year t.

The final constraint requires that budgets not be exceeded. Let B_j be the funds available for construction in year j. If unused funds are forfeited at the end of the budget year, then we have

$$\sum_i S_{ij} Z_{ij} + \sum_i s_{ij} z_{ij} \leq B_j \text{ , all j.} \quad (10A)$$

However, if unused funds are allowed to carry over from one budget period to the next, we have

$$\sum_i \sum_{n=1}^{j} S_{in} Z_{in} + \sum_i \sum_{n=1}^{j} s_{in} z_{in} \leq \sum_{n=1}^{j} B_n \text{ , all j} \quad (10B)$$

While expressions (5) through (10) describe the basic model, there are a number of optional constraints that can be added to enhance realism.

Whenever a project is constructed, total supply capacity should at least equal existing demand. This can be accomplished by the following

$$Q_o + \sum_{n=1}^{j} z_{in} \geq Z_{ij} q_{ij} \text{ , all i,j.} \quad (11)$$

The method of cost accounting in the objective function discriminates against projects proposed near the end of the planning horizon. The entire cost of such projects lies within the horizon while useful life extends beyond it. To provide continuity into the future, a terminal constraint may be included. This provides for a minimum target level of excess capacity (or maximum level of undercapacity) in each town at the end of the planning horizon.

$$Q_{io} + \sum_j z_{ij} \geq q_{iT} + Q_{iT} \text{ , all } i \tag{12}$$

where q_{iT} is the demand in town i at the end of the horizon and Q_{iT} is (1) the minimum desired excess capacity in town i if > 0 or (2) the maximum allowable supply deficit if < 0.

We saw in Figure 3 that the fixed charge cost function is an accurate approximation of a power function like (2) only within certain limits. In many cases, difficulties might be encountered in selecting the values for A and B over which the fixed charge function applies. If A and B, for example, are selected so as to bracket the expected scale of initial construction, the resulting cost function would be inappropriate for capacity expansions with scale less than A.

To overcome this difficulty, the power function can be approximated with two (or more) fixed charge functions as shown in Figure 4. Here C^1_{ij} applies in the capacity range A-B and C^2_{ij} in the range B-C. The cost of the system proposed for town i in year j is

$$F_j S^1_{ij} Z^1_{ij} + F_j s^1_{ij} z^1_{ij} + F_j S^2_{ij} Z^2_{ij} + F_j s^2_{ij} z^2_{ij} \tag{13}$$

To assure that at most only one cost function is used, we impose the restriction

$$Z^1_{ij} + Z^2_{ij} \leq 1, \text{ all } i, j \tag{14}$$

While this formulation adequately provides an accurate cost function for initial construction and expansions, it introduces another integer and

A

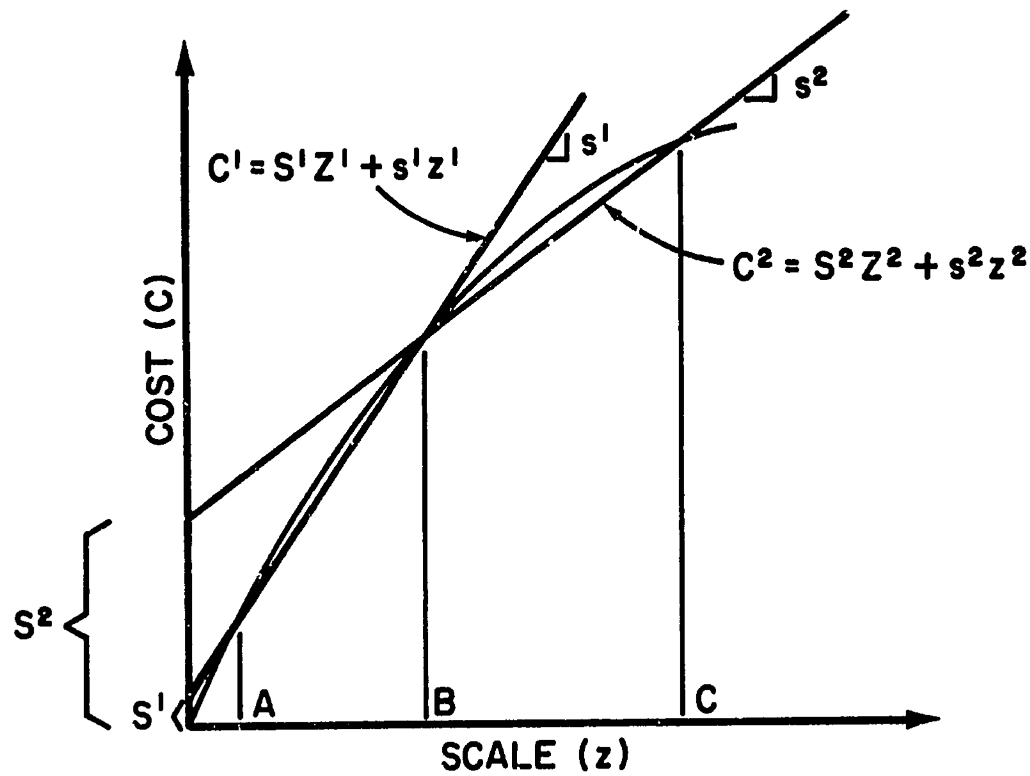

Figure 4. Multiple Fixed-Charge Cost Functions

continuous variable and another constraint for each construction opportunity period.

5. Parameter Values From Central America

The results of field studies conducted in Central America to determine (1) water system cost functions, (2) patterns of water demand, and (3) imputed values of water supply benefits (i.e., p values) are presented in sections 4, 5 and 6 of Appendix 3. For convenience, they are summarized herein.

Water System Costs

It has already been noted that water systems reflect economies of scale and have a cost function of the form shown in (2) and (15)

$$C(z) = k(z)^a , \tag{15}$$

where z is the scale variable with typical units, mgd. We have already noted that "a" is the economy of scale factor with values between 0 and 1. In Appendix 4, it is shown that "a" denotes the percentage change in cost per percent change in scale. In (15) by substituting $z = 1$, note that $C(1) = k$. Hence, k is seen to be the cost of a one mgd system.

(15) can be linearized by taking the logarithms of each side

$$\log C(z) = \log k + a \log (z) \tag{16}$$

$$Y = b + aX$$

In this form, the parameters of the function (a and b) can be readily determined by least squares analysis given values for Y and X.

Data were collected on 65 water systems in Central America for the least squares analysis. The systems were constructed between 1965 and 1969, are of the gravity type without filtration, include piped house services and public fountains, and were designed for towns with populations of 7500 or less.

The least squares analysis resulted in the following function

$$C(z) = 300{,}000\ (z)^{0.83} \qquad (17)$$

Hence, the cost of a one mgd system is seen to be $300,000 and the economy of scale factor is 0.83. It is important to note that larger economies of scale are associated with smaller values of "a". Hence, the economies reflected above are quite small. The data, however, from which (17) was developed did not adequately reflect planning and administration costs connected with project implementation. Had these costs been included, it is probable that "a" would be less than 0.83.

Water Demands

Data on water demand patterns were obtained from a study of 10 towns in Guatemala during the period 1967-71. The study was under the direction of Ing. Octavio Cordon of the Regional School of Sanitary Engineering at San Carlos University, Guatemala. The towns had populations ranging from 900 to 6200 with an average of 3100. The average age of the water systems is currently, 3.5 years.

The study revealed that on the average, 25 percent of the population was connected to the system by the end of the first year of operation. New connections were made at the approximate rate of 8 percent of the population per year. Those without connections generally rely for their water on public fountains and washing stations.

Average daily water use (from house meter records) ranged from 16 to 34 gallons per capita per day (60 to 130 liters per capita per day) with an average of 26 gpcd (100 lpcd). In general, per capita water use increased at the rate of 3 lpcd per year which implies a rate of about 3 percent per year (or perhaps something less).

Relationships of peak to average flow were determined from master meter data. The ratio of maximum daily to average daily demand (in English units) was found to be

$$1.09\ Q^{-.058}$$

where Q is average daily usage in mgd. If Q is average daily usage in liters per second, the ratio is 1.35 $Q^{-.058}$.

Comparison of master and house meter data provided a basis for determining unaccounted for and publicly used water. It was found that this was less than 2 percent of the total demand. This low value is due in part to minimal leakage associated with the newness of the systems. It also implies, however, that the amount of water demanded by public fountain users is extremely small, probably being not much different from that obtained from rivers and lakes prior to systems construction.

Imputed Water Supply Benefits

(3B) is an equation that can be used to impute the benefits of publicly supplied water in towns already served with systems. The assumptions under which (3B) can be used are described in Appendix 3. Imputed values (p's) were calculated for 65 towns in Central America. The method of calculation is as follows.

In "Town 1" of Appendix 3, the population at the time of water system construction was 453, and the cost of the system was \$9180. Assuming a discount rate of 10 percent, the annual opportunity cost of capital is \$918.00 (= .10 x 9180) per year. If the planners assumed that all inhabitants desired water at the rate of 30 gallons per capita per day, the unsupplied rate of demand immediately prior to project implementation was 13,500 (= 30 x 453) gallons per day or 4,930 thousand gallons per year.

The imputed value of p therefore is

$$p = \frac{918 \text{ dollars per year}}{4930 \text{ thousand gallons per year}}$$

or 18.6 cents per thousand gallons.

The p-values in Appendix 3 range from 8.9 to 76.0 cents per thousand gallons. The variance is 150.27 and the standard deviation is about 12.3. About 50 percent of the p-values are equal to or less than 20 cents per thousand and 90 percent are equal or less than 40 cents per thousand gallons.

Additional Work

In addition to model development and parameter estimation, extensions were made of Manne's models to facilitate their use and gain greater insights to water supply planning.

Expansion Model Without Deficits

The model of section 2 is for expansion planning when water has finite value. The optimality condition (3A) indicates that if p is infinite, the deficit period y is zero. This results in the expansion patterns shown in Figure 5 which is typical for the U.S. and other economically advanced countries.

With deficits eliminated, there is only a single decision variable, x, which denotes the period of excess capacity for each expansion. The total p.v. cost of an infinite number of expansions is

$$\frac{k(xD)^a}{1-e^{-rx}} \tag{18}$$

An expression for the optimal value of x can be obtained by setting the derivative of (18) with respect to x equal to zero and solving. The resulting equation is

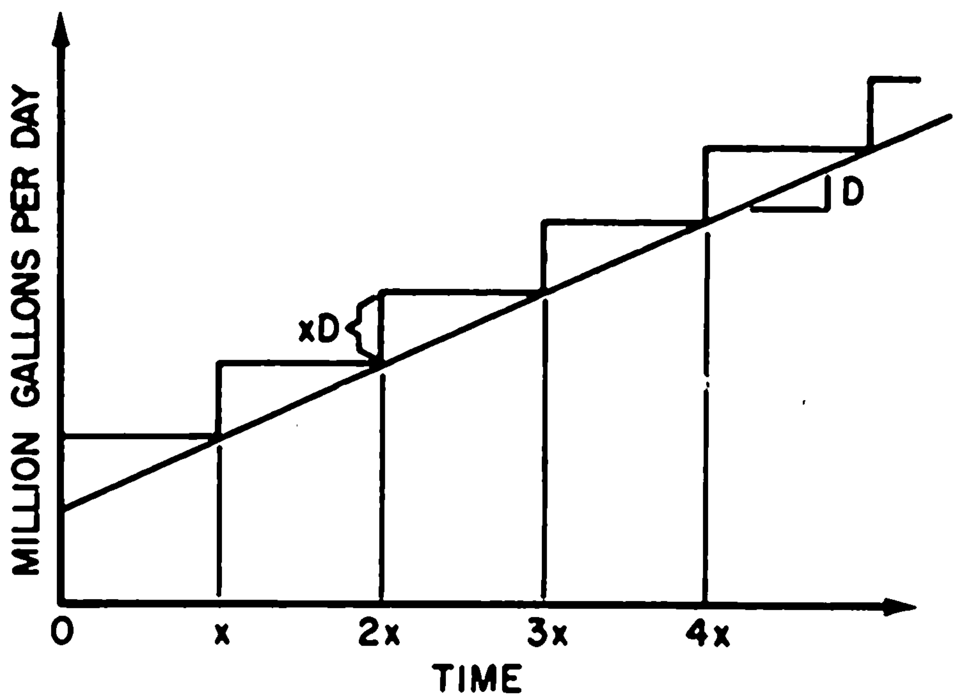

Figure 5. Expansion Model Without Deficits

$$a = \frac{rx^*}{e^{rx^*} - 1} \qquad (19)$$

(19) shows that the optimal period of excess capacity is a function of only two parameters, the economy of scale factor and the discount rate. Unfortunately, it is impossible to solve (19) for x^*; given values for "a" and r, x^* must be obtained by numerical methods (Newton's, for example) which are difficult to use. As a result, an approximating equation for (19) solved explicitly for x^* was developed as part of this research using statistical procedures.

$$x^* = \frac{2.6\ (1-a)^{1.12}}{r} \qquad (20)$$

The standard error of this equation for values of "a" between 0.60 and 0.85 and r between 0.05 and 0.20 is only 0.057 which implies an excellent fit.

To illustrate use of (20), assume a community with 20,000 present population anticipates future water demand increase at the rate of 12,000 gallons per day (gpd) per year. Assume further that the excess capacity of existing supply facilities is nearly exhausted so that within a year or so, an expansion will be needed. If the economy of scale factor for water supply facilities is 0.7 and the discount rate is 6 percent per year, the optimal period of excess capacity from (20) is 11.3 years:

$$x^* = 2.6\ (1 - .7)^{1.12}/.06 = 11.3,$$

and the optimal capacity of the expansion is 0.135 mgd (= $11.3 \times 12{,}000 \times 10^{-6}$).

Initial Construction

Manne's models cited in section 2 and above are for expansions only. In developing countries, however, the more common problem is design of initial facilities for which there is an outstanding demand. This situation is shown in Figure 6. Note that the initial project must meet existing demand D_o and have excess capacity for x_1 years at the end of which time a planning situation identical to that described above (i.e., with equally

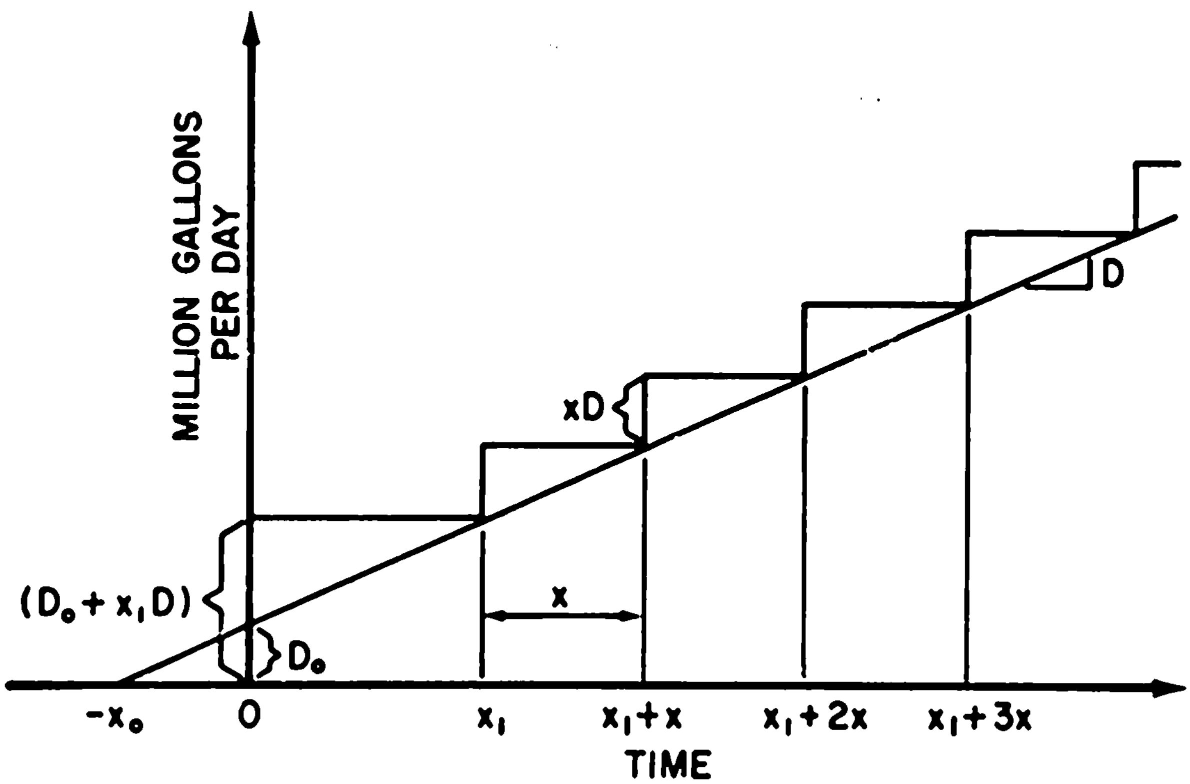

Figure 6. Model for Initial Construction and Expansion.

sized expansions) is encountered. The planning problem is to determine the optimal value of x_1, and the methodology for so doing has been developed by Thomas (1970).

The approach is similar to Manne's. An expression is written for total present value cost which includes initial construction plus the p.v. cost of an infinite number of future expansions discounted from year x_1 to time zero. The resulting objective function is

$$k(D_o + x_1 D)^a + \frac{e^{-rx_1} \; k(xD)^a}{1-e^{-rx}} \tag{21}$$

in which $x_o D$ may be substituted for D_o, where x_o is the elapsed period (in years) from the time of zero demand to the present as shown in Figure 6.

In this model, the two decision variables are x_1 and x. The optimal value of x, the excess capacity period of expansions, is found from the derivative of (21) with respect to x set equal to zero. As expected, the optimality expression is identical to that of Manne's model, (19), for which approximating equation (20) can be used. The optimal value of x_1, the excess capacity of initial construction, results from the derivative of (21) with respect to x_1 set equal to zero:

$$D \; a \; k(D_o + x_1^* D)^{a-1} = r \quad e^{-rx_1^*} \; \frac{k(xD)^a}{1-e^{-rx}} \tag{22}$$

As in the case of Manne's model, (22) cannot be solved explicitly for the decision variable, x_1^*. An approximating equation, however, has been developed as part of this research.

$$x_1^* = \frac{2.6 \; (1-a)^{1.12}}{r} + \frac{0.3 \; (1-a) \; x_o}{\sqrt{r}}^{0.85} \tag{23}$$

In (23), the parameters are the economy of scale factor (a), the discount rate (r), and the elapsed period to zero demand (x_0). Note that when x_0 is zero, there is no initial outstanding demand and the planning problem reduces to the case of expansions only; (23) correspondingly reduces to (20). Also note from (23) that the excess capacity of initial construction is always greater than that of expansions. It follows that it is erroneous to use the same design standards for new systems and capacity expansions. The fit of (23), unfortunately, is not as good as the previous approximation, but the equation is sufficiently accurate to be of practical value.

To illustrate use of (23), assume in the earlier example that per capita demand is 30 gallons per day; existing demand is therefore 600,000 gpd. Assuming no public water supply, the elapsed period to zero demand (x_0) is 50 years (= 600,000/12,000). From (23), the optimal period of excess capacity is 21.5 years:

$$x_1^* = 11.3 + \frac{0.3(1-.7) \quad 50^{0.85}}{\sqrt{.06}} = 11.3 + 10.2 = 21.5$$

Hence, optimal capacity of the initial project is 0.858 mqd (= 0.600 + 21.5 x 12,000 x 10^{-6}) assuming expansions are designed for 0.135 mqd.

A final observation on this model is pertinent. The assumed construction cost function is of the form $C(z) = kz^a$, where z is capacity in mgd. The average cost of a system of scale Z is k $Z^{(a-1)}$. From the derivative, the marginal cost of a system of scale Z is [a k $Z^{(a-1)}$]. It immediately follows that marginal cost is average cost times "a" with units dollars per mgd.*

Inspection of (22) shows that the left side is the product of D and the marginal cost of the initial project with units dollars per year. On the right side of (22), the fraction is the p.v. of an infinite number of expansions discounted to year x_1 (refer to 18). The exponential in front of

*Note that "a" must therefore be the ratio of marginal to average cost.

the fraction discounts this cost to present value at time zero. Hence, the right side is the product of r and p.v. expansion cost, also with units dollars per year. Hence, (22) can be rewritten in words

$$\begin{bmatrix}\text{annual}\\ \text{rate of}\\ \text{demand}\\ \text{increase}\end{bmatrix} \times \begin{bmatrix}\text{marginal}\\ \text{cost of}\\ \text{initial}\\ \text{system}\end{bmatrix} = \begin{bmatrix}\text{annual}\\ \text{discount}\\ \text{rate}\end{bmatrix} \times \begin{bmatrix}\text{p.v. cost}\\ \text{of all}\\ \text{future}\\ \text{expansions}\end{bmatrix} \qquad (24)$$

This condition must obtain for optimality. In this form, note that the number of future expansions need not be infinite nor is it necessary (for a local minimum) that the expansions be optimally sized.

Waiting Period Model

In both models of this section, the value of publicly supplied water is assumed to be infinite. Now let us assume that water in a community without existing supply facilities has finite value. This implies that construction of the initial system will be preceded by a waiting period as in the model of section 2. Once a system is constructed, however, a policy of disallowing deficits is imposed which implies an increase in the value of water to infinity. This situation is depicted in Figure 7.

An expression of total p.v. cost can be developed as before. This includes the social costs of deficit during the first y years, the p.v. cost of initial construction, and the present value cost of future expansions.

$$\int_{t=0}^{y} e^{-rt}\, p(D_0 + Dt)dt + e^{-ry}\ k(D_0 + yD + x_1 D)^a + e^{-r(y+x_1)}\ \frac{k(xD)^a}{1-e^{-rx}} . \qquad (25)$$

(25) includes three decision variables: y is the waiting period before constructing the first system, x_1 is the excess capacity period of this project, and x is the excess capacity period of future expansions. Optimal values of the

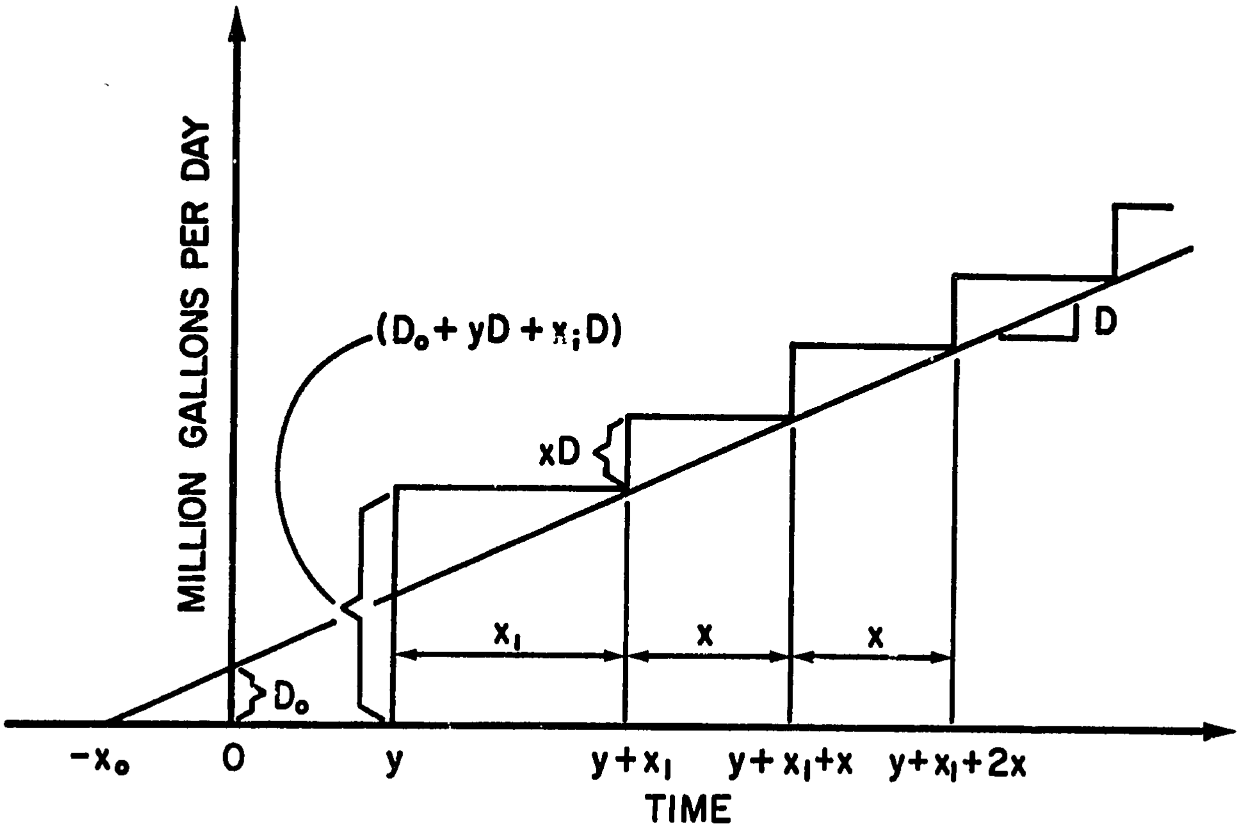

Figure 7. Deficit Model for Initial Construction and Expansions.

variables can be found by setting the appropriate partial derivatives of (25) equal to zero and solving.

The derivative with respect to x results in an expression identical to Manne's expansion model (19) which, as we have seen, can be approximated by (20).

The derivative with respect to x_1 results in an expression essentially identical to that obtained by Thomas in the initial construction model presented above (refer to 22). There is a slight difference in mathematical symbols, however. The term in parenthesis on the left hand side of (22) must be replaced by $(D_o + yD + x_1^*D)$ to account for increasing demand during the initial years of deficit. The word description of the optimality condition (24) remains unchanged.

The optimal waiting period is determined from the derivation of (25) with respect to y. The resulting expression is cumbersance

$$\underbrace{p(D_o + Dy^*)}_{I} + D \; a \; \underbrace{k(D_o + y^*D + x_1D)^{a-1}}_{II} = r \; \underbrace{k(D_o + y^*D + x_1D)^a}_{III} + r \; e^{-rx_1} \underbrace{\frac{k(xD)^a}{1-e^{-rx}}}_{IV} \qquad (26A)$$

Terms II and IV, however, can be eliminated from (26A) by substituting the optimality equation obtained from the derivative of (25) with respect to x_1. This assumes that the initial project is optimally timed and scaled. The resulting expression is

$$p(D_o + Dy^*) = r \; k(D_o + y^*D + x_1^*D)^a \qquad (26B)$$

But this is essentially identical to (3A) and (3B). It states that for the optimal planning of the initial project, the rate of social losses at the time of construction (the left hand side of 26B) must equal the annual opportunity cost of the initial project.

In sum, the optimality conditions for this model are identical to those of the three separate models previously derived. In regard to the waiting period, initial construction should be delayed until social costs accrue at the same rate as the annual opportunity cost of the first project. Regarding the scale of the first project, it should be adjusted so that its marginal cost times the annual rate of demand increase equals the product of the discount rate and the present value cost of all future expansions. Finally, the optimal scale of expansion can be calculated from (19) or (20).

7. Dissemination of Research Results

Significant efforts have been made to disseminate the results of this research. Methods have included lectures, preparation of reports and technical papers, publication in professional journals, and personal communication.

Early findings of the study were presented in a lecture to about 40 faculty and students in the author's department at the University of North Carolina. Subsequent lectures were delivered to an additional 30 students at UNC. Recently, a lecture on some of the more basic concepts resulting from the study was delivered to a group of about 30 sanitary engineers in the Washington, D.C., area concerned with water supply planning abroad. Earlier this year, the content of Appendix 1 was presented to about 50 participants at a national meeting of the American Geophysical Union, and in June, 1972, the content of Appendix 4 was presented to about 50 conferees at the annual meeting of the American Water Works Association. In sum, about 200 individuals have been exposed to various aspects and findings of this work through lectures.

The reports and papers associated with this work (largely those included in the appendices) have been distributed to universities, institutions engaged in water supply planning abroad, firms of consulting engineers, and individuals. Approximately 20 copies of each of Appendices 1 and 4 were distributed within the author's department. In addition, copies of these papers were sent to another 10 to 15 professors in universities. Planning institutions that received copies include the World Health Organization, Pan American Health Organization, World Bank, Interamerican Development Bank, Technion (Israel Institute of Technology), and 2 or 3 more. Firms include Camp Dresser and McKee, Hazen and Sawyer, Hydroscience, Quirk Lawler and Matusky. Finally, copies of 2 or more of the appendices were sent to about 20 individuals. In all, about 120 reports and papers were distributed.

The papers in Appendices 1 and 4 have been accepted for publication in professional journals. The paper of Appendix 1 will be published in Water Resources Research and the paper of Appendix 4 will be published in the Journal of the American Water Works Association. At least one more paper will be prepared as a result of this research; it will be submitted to the Sanitary Engineering Division Journal, American Society of Civil Engineers.

Little information is available on the use being made of research results. The greatest interest has been by Technion, the World Bank, the Pan American Health Organization, and the Interamerican Development Bank. Technion and IDB have investigated programming algorithms via the writer for solution of the model of section 4, and it is hoped that serious efforts will be made by these organizations to apply the model. Other groups and individuals have expressed interest in the single-system models of section 6 and the method of imputing water supply benefits presented in section 5.

8. Needed Research

This research has identified the principal factors affecting the optimal planning of water supplies in developing countries: project capacity is largely dependent on economies of scale in construction; optimal timing is primarily a function of public water supply benefits; budgetary constraints create economic interdependencies among systems making it necessary to plan on regional bases. The research has also produced a mathematical model for improved investment decisions abroad, and preliminary steps have been taken to make the model operational. A great deal of work remains to be done, however, if this beginning is to be fruitful in producing more nearly optimal plans.

Although some experimentation has been done using ficticious data, the mathematical model needs to be applied to real planning problems. Computer studies should at first be undertaken for systems including only a few towns. Sensitivity analyses for these small systems should then be made by changing key parameters: for example, water supply benefits, future demands, construction cost functions, and budgetary constraints. Gradually, problem size should be increased by adding additional towns. This program of model application would (1) provide a basis for judging the realism of "optimal" results produced by the model, (2) identify the sensitivity of optimal plans to small changes (i.e., uncertainty) in model parameters, and (3) determine the cost and difficulty of computer solution for problems of increasing size.

Preliminary computer results indicate that solution costs increase rapidly with the number of integer variables in the model. In addition, the large amount of computer capacity required for execution of the algorithm used for solution severly limits the size of problems that can be run on existing machines.* As a result, an investigation should be made of computer programs that can solve the model more efficiently. If a better algorithm is not located, work should be done to modify existing programs or produce a new algorithm designed specifically for this model.

The model should be reformulated for simpler solution. By converting it to a linear programming (LP) model, for example, solution could be obtained on relatively small computers at low cost. While it is doubtful that the model itself can be changed to LP, it is probable that it can be alterred so that it can be solved by repeated use of LP. The model should also be simplified in order to obtain a first cut at an optimal plan. In its existing form, this might be done by using longer time intervals, a shorter planning horizon, and fewer construction opportunity periods. Similarly, the model can be refined and extended to make it more realistic.

Model improvement is only one aspect of needed research. In addition, field studies are required for evaluation of parameters. In particular, work should be done to accurately determine cost functions for water supply and distribution systems throughout the world. Such functions are required for systems employing different treatment methods (e.g., disinfection, chemical coagulation, slow sand filtration), for gravity and pumped systems, for systems with house services and/or public fountains, etc. Field data are also needed on demands in towns newly served with water supplies. Information should be obtained on the rate at which new connections are made, water demands for different user categories, unaccounted-for losses, demand variations, and the increasing rate of usage after connection .

Most difficult of the field work is that required for evaluation of water

*A branch and bound algorithm by R. Shareshian of IBM has been used to solve the model.

supply benefits. The method of imputing presented above is expedient but not entirely satisfactory. Determination of shadow health costs is an alternative method, and it might be possible to use differences in the market values of properties with and without water services as indicators of benefits. These and other suggested methods are described more fully in Appendix 4.

Finally, studies should be made of water planning institutions and practices throughout the world. Specific information is needed on how towns are selected to receive new supplies, how budget levels are set each year, engineering manpower limitation and its effect on design and planning work, and the need for regional water systems created by water scarcity problems.

Appendix 1

Water Supply Planning by Mixed Integer Programming

WATER SUPPLY PLANNING BY MIXED INTEGER PROGRAMMING

by

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A paper presented at the 53rd Annual Meeting of the American Geophysical Union in Washington, D. C. on April 19, 1972

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WATER SUPPLY PLANNING BY MIXED INTEGER PROGRAMMING

Abstract

Integer and mixed integer programming models for the expansion planning of water supply systems are presented. A cost minimization model is developed for the optimal sequencing of alternative expansions of given scale. The principal constraint set requires that projects be sequenced so as to continually satisfy water requirements. Also presented is a more general model for determining the optimal timing and scale of the next supply system expansion given that water requirements are temporarily allowed to go unsatisfied. This model, which is primarily intended for use in developing countries, (1) treats expansion timing and scale as decision variables, (2) seeks to maximize the present value of net benefits, (3) handles economies of expansion scale via fixed charge cost functions, and (4) accomodates arbitrarily varying water-requirement functions. Parametric study of the model reveals that with increasing future water requirements, expansion timing should be delayed as (1) fixed construction charges increase, or (2) the value of publicly supplied water decreases. Additionally, optimal excess capacity increases with decreasing cost per unit scale. Model results suggest a method for imputing water supply benefits.

INTRODUCTION

In a model for the optimal sequencing of water supply projects, Butcher et al. (1969) postulate a schedule of price-independent water demands that increase over a finite time horizon. A set of water supply projects of different scale and cost is given. The aggregate scale of the set of projects is equal to the maximum rate of demand at the end of the horizon. The planning problem is to determine the optimal sequencing of the projects so as to minimize total present value cost while satisfying demands

The sequencing problem incorporates several assumptions. Projects are independent and can be implemented in any order. The benefit of each unit of water demanded is constant. Since all water requirements must be satisfied, the total benefit of supplied water is fixed. Hence, benefits need not be explicitly considered since they are independent of the sequencing pattern. Minimization of total present value cost, therefore, is equivalent to maximization of present value net benefits.

In addition to the benefit of each unit of water being constant, its value is implicitly assumed to be infinite. To let even the smallest portion of demand go unsatisfied results in infinite cost. It is always less expensive, therefore, to construct facilities than incur a supply deficit.

The sequencing problem does not explicitly consider operating costs. Implicitly, however, they are proportional to the amount of water produced, and the cost per unit is identical for all projects. Total operating costs are therefore fixed because of the need to satisfy demands. Like benefits, operating costs can be ignored.

The problem is solved by dynamic programming. In a recent paper, Morin et al. (1971) show how a significant reduction in computational effort can be made still using dynamic programming. In the present paper, a model is develope for solution of the problem by 0,1 integer programming. This is followed by a similar 0,1 mixed integer programming model that focuses not on the optimal sequencing of projects but on the timing and scale of the first project to be implemented.

INTEGER PROGRAMMING MODEL

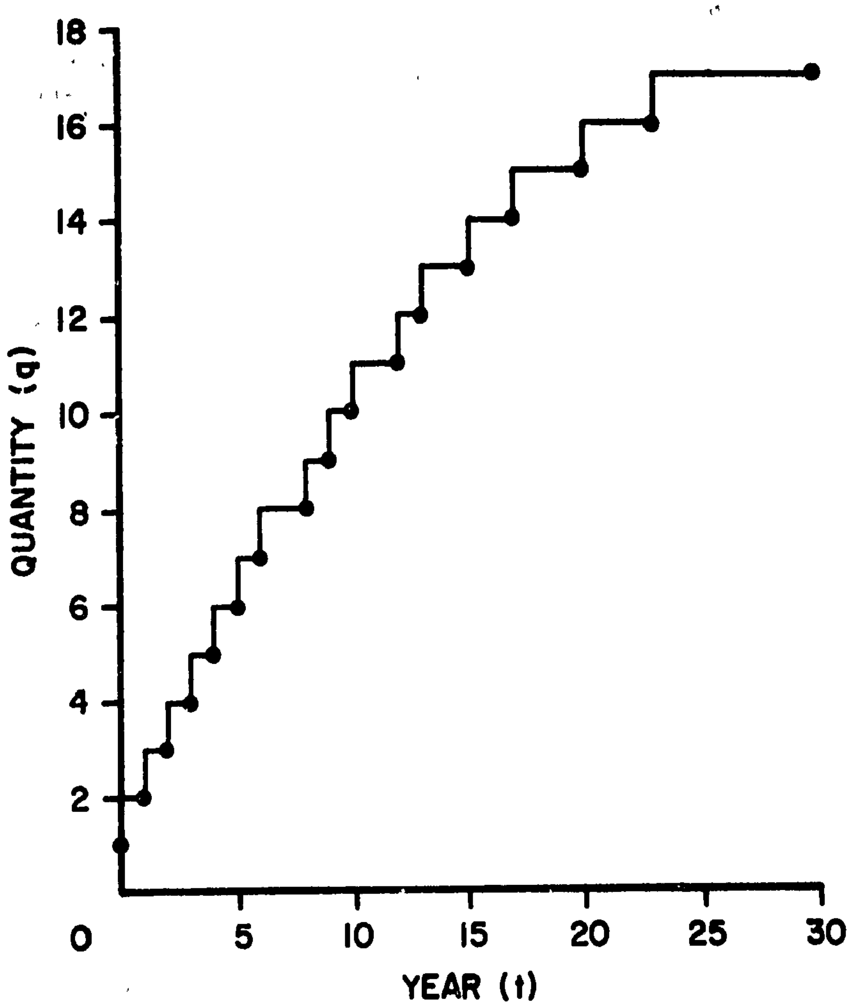
The water requirements schedule proposed by Butcher et al. in their numerical example is shown in Figure 1. For our purposes, the schedule appears as a step function. Water requirements change only at the start of a new year. q_t is the rate of demand in year t; convenient units are gallons per year (gpy).

Four alternative projects are considered. These are shown in Table 1. Q^s is the capacity (gpy) of project s and C^s is its cost.

Table 1
ALTERNATIVE PROJECTS

Project (s)	Capacity (Q^s)	Cost (C^s)
1	2	30
2	4	50
3	4	65
4	7	75

Both Morin (1971) and Erlenkotter (1967) have shown that it is never optimal to implement a new project while existing supply facilities have excess capacity. Rather, project implementation should be delayed until excess capacity is reduced to zero by increasing demand. For the numerical example, therefore, the possibility of construction need not be considered every year but only 11 times during the 30-year horizon. Such times are called construction opportunity periods.



Year (t)	1	2	3	4	5	6	8	9	10	12	13	15	17	20	23	30
Quantity (q)	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Figure I. Water Requirements Function

Table 2
TIMES OF ZERO-EXCESS CAPACITY

Number of Projects	Alternative Cumulative Capacity	Years of Zero-Excess Capacity
1	2, 4, 7	1, 3, 6
2	6, 8, 9, 11	5, 8, 9, 12
3	10, 13, 15	10, 15, 20
4	17	30

Clearly, one of the four projects must be constructed immediately if supply capacity is to exceed requirements. This first project will start production the end of year 0 and have capacity of either 2, 4, or 7. The excess capacity of this project will be exhausted in year 1, 3 or 6 at which time the second project must be implemented. The combined capacity of two projects will be 6, 8, 9 or 11. These are all the scale combinations without replacement of 2 out of the 4 projects. The corresponding times of zero-excess capacity are shown in Table 2. Proceeding in similar manner, we obtain the 11 construction opportunity periods (j) shown in Table 3; also shown are the associated demands (q_j) and present value (p.v.) factors (F_j).

Table 3
CONSTRUCTION OPPORTUNITY PERIODS

Year (t)*	0	1	3	5	6	8	9	10	12	15	20
Period (j)	1	2	3	4	5	6	7	8	9	10	11
Demand (q_j)	-	2	4	6	7	8	9	10	11	13	15
p.v. Factors (F_j)	1.000	.952	.864	.783	.746	.677	.645	.614	.557	.481	.377

*years of zero-excess capacity from Table 2

With 11 construction opportunity periods and 4 projects, there are 44 implementation alternatives. In general, the present value cost of project s proposed for construction in period j is $F_j C^s$. Let X_j^s be a 0,1 decision variable associated with this alternative. If X_j^s is 1, then project s should be implemented in period j; otherwise, the project should not be constructed. Total present value cost is therefore

$$\sum_s \sum_j F_j C^s X_j^s \quad , \tag{1}$$

which is the objective function to be minimized. For the numerical example herein, (1) is the sum of all elements in the following matrix.

$$\begin{array}{cc|cccccc} & & & & \text{Period } (j) & & & \\ & & 1 & 2 & & & & 11 \\ & 1 & 30X_1^1 & (.952)30X_2^1 & \cdot & \cdot & \cdot & (.377)30X_{11}^1 \\ \text{Project} & 2 & 50X_1^2 & (.952)50X_2^2 & \cdot & \cdot & \cdot & (.377)50X_{11}^2 \\ (s) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & 4 & 75X_1^4 & (.952)75X_2^4 & \cdot & \cdot & \cdot & (.377)75X_{11}^4 \end{array}$$

Four constraint sets complete the model. The binary decision variables must be restricted to integral values.

$$X_j^s = 0 \text{ or } 1, \text{ all } j, s \quad . \tag{2}$$

At most, one project can be built in each construction opportunity period.

$$\sum_s X_j^s \leq 1, \text{ all } j \tag{3}$$

Also, project s can be built at most only once.

$$\sum_j X_j^s \leq 1 \ , \text{ all } s \tag{4}$$

The capacity of the project implemented in period 1 must equal or exceed the demand through period 2, the next construction opportunity period. Similarly, the cumulative capacity of projects implemented in periods 1 and 2 must equal or exceed demand through period 3. In general, the total capacity of all projects implemented through any period j must equal or exceed requirements through period j+1. Hence we have

$$\sum_{j \leq j} \sum_s Q^s X_j^s \geq q_{j+1} \quad , \text{ all } j \quad . \tag{5}$$

For the problem herein, the first two constraints of this set are

$$2X_1^1 + 4X_1^2 + 4X_1^3 + 7X_1^4 \geq 2$$

$$2X_1^1 + 4X_1^2 + 4X_1^3 + 7X_1^4 + 2X_2^1 + 4X_2^2 + 4X_2^3 + 7X_2^4 \geq 4 \quad .$$

The remaining constraints are developed in similar manner. Note that for the last constraint (j = 11), the right hand side is q_{12} for which the bounds are $q_{11} < q_{12} \leq \sum_s Q^s$.

DISCUSSION

If J is the number of construction opportunity periods and S is the number of alternative projects, the total number of decision variables in the model is JS. In addition to integer restrictions on the variables, the total number of constraints is 2J + S.

For the numerical example, the number of decision variables is 44 and the number of constraints is 26. This problem was solved in less than 30 seconds on the IBM 360 computer with a branch and bound algorithm developed by Shareshian (1969). The algorithm uses the Land and Doig (1960) method.

Solution results are identical to those obtained by Butcher et al. The optimal sequence of projects is 4, 2, 3, 1 in years 0, 6, 12 and 20 with minimum present value cost of 159.81. Only slightly poorer is the sequence 4, 2, 1, 3 in years 0, 6, 12, 15 with present value cost of 160.28. This near optimal solution was identified in 32 iterations (i.e., branches) of the algorithm whereas the optimal solution required 3493 iterations. In general, integer programming formulations permit identification of such near optima that cannot be readily detected by dynamic programming. With large scale problems, this can be of significant importance.

Aside from the method of solution, the model itself can be modified to make it more useful. The scales of alternative projects in the model have been decided in advance. In practice however, project scale is seldom fixed. This is generally true whether the projects are separate supply systems or capacity expansions of the same system. Instead of assuming project scale is given, therefore, it might be better to treat it as a decision variable.

The solution of the sequencing problem results in an optimum construction schedule for all projects. There may be little value or interest, however, in an entire sequence. After all, the planner is only bound by his next decision. Furthermore, future changes may invalidate old results. It may be preferable, therefore, to concentrate only on the next project instead of a sequence.

In the United States, it is generally desired that water supply capacity equal or exceed water requirements. As shown by Manne (1967) and assumed in the sequencing model, this implies that the water from local facilities has infinite value. This however, is seldom true. Water can often be imported from a neighboring community at finite price, and where this is impossible as in the developing nations, the social losses due to unsatisfied demand are usually not infinite. If water value is made finite, however, supply deficits become permissible and it does not automatically follow that the first project should be implemented now. Hence, the question of optimal timing is raised.

In the next section, a model is presented that takes account of the above considerations. The purpose of the model is to determine the optimal timing and scale of the next water supply project given a water requirements function, finite water value, and project cost functions.

MIXED INTEGER PROGRAMMING MODEL

For simplicity, only a single project is under consideration. Construction opportunity periods (j) are preselected. The project, for example, might be proposed for construction immediately and every year of the next 5-year period or perhaps every other year of the next 8 or 10-year period. The exact schedule is left to the judgment of the planner. Identification of the optimal construction period results from model solution.

The water supply project is assumed to reflect economies of scale; average costs decrease with increasing scale. For this purpose, a fixed charge cost function as shown in Figure 2 is assumed. Consider the alternative of implementing the project in period j; S is its fixed charge (a set-up cost) and s is the cost per unit scale. If the project is constructed in j, its p.v. cost is

$$F_j \; (S \; Z_j + s \; z_j) \quad , \qquad (6)$$

where F_j is the p.v. factor for period j. Z_j is a 0,1 decision variable that indicates whether or not the project should be implemented in j; z_j, a continuous decision variable, indicates scale. The 0,1 variable must be restricted so that

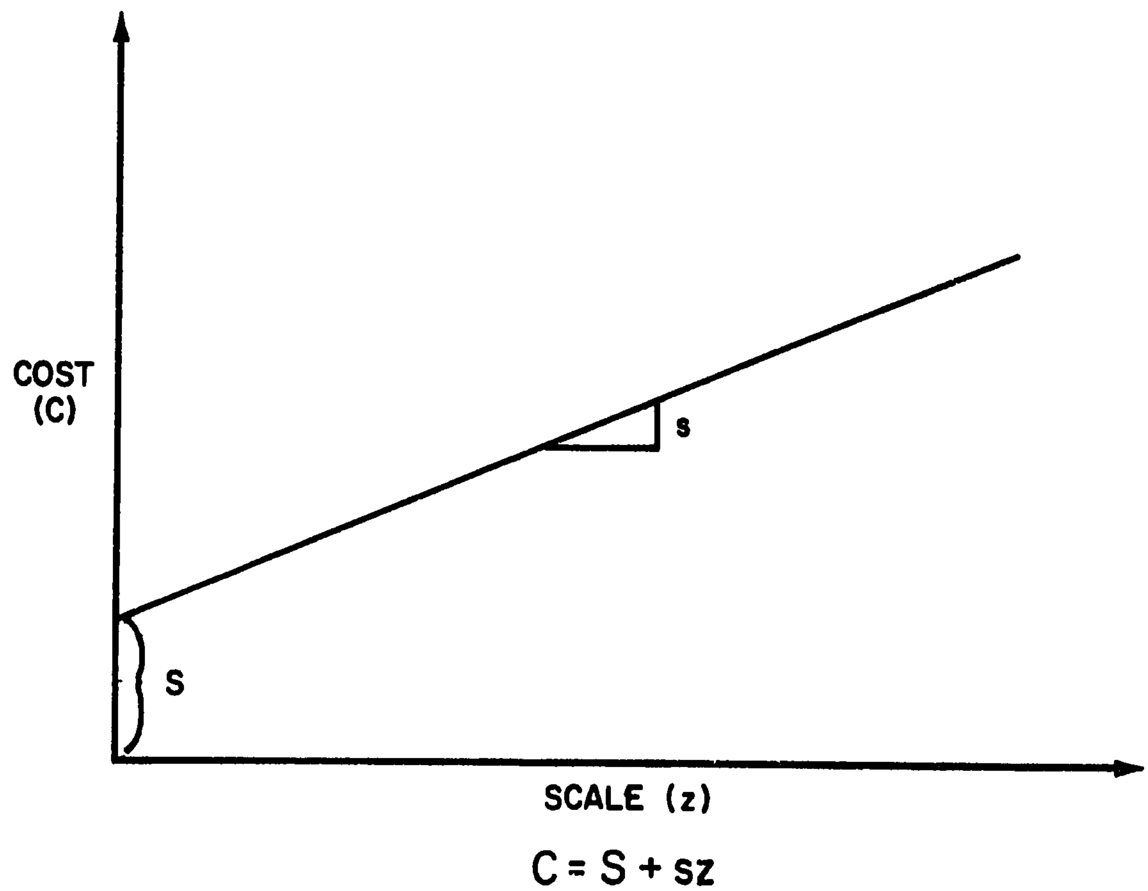


Figure 2. Fixed Charge Cost Function

if $z_j > 0$ (i.e., the project is constructed) $Z_j = 1$; otherwise $Z_j = z_j = 0$. Total p.v. implementation costs are obtained by summing (6) over all construction opportunity periods.

As in the previous model, the planning horizon is divided into time segments; 1-year periods are recommended. Specifically, d_t is the duration of t (years) and q_t is the corresponding rate of water demand (gpy). Each unit (gallon) of publicly supplied water is assumed to have a value of $\hat{p}$ dollars. If all water requirements in year t are satisfied by local community supply facilities, $F_t \hat{p} d_t q_t$ is the present value benefit for that year, a fixed constant which we shall call B_t. If, however, y_t (a decision variable) is the rate of demand left unsatisfied by the local system in t, then the present value water supply benefit that year is

$$B_t - F_t \hat{p} d_t y_t \tag{7}$$

Total p.v. benefits of public supply are obtained by summing (7) over all years of the horizon.

Operating costs are assumed to be proportional to the amount of water produced; $\bar{p}$ is the operating price (dollars per gallon). If all water requirements in t are satisfied from public supply, the p.v. cost is fixed; call it K_t. If, however, y_t is left unsatisfied, then the p.v. operating cost in t is

$$K_t - F_t \bar{p} d_t y_t \tag{8}$$

Combining (6), (7) and (8) after summing over appropriate time indices (j and t) results in an expression of total p.v. net benefits, the objective function to be maximized. The constant terms however can be ignored, and by defining $p = \hat{p} - \bar{p}$ and then multiplying the objective function by -1, we obtain an expression of total p.v. cost to be minimized

$$\sum_j F_j S Z_j + \sum_j F_j s z_j - \sum_t F_t p d_t y_t \tag{9}$$

For now, p can be considered the net cost per gallon of water demanded but not supplied by local facilities. In the case of importing, p is the net purchase price of water from an adjacent community. In the previous model, p has infinite value whereas here it is nonnegative.

Several constraint sets complete the model. Integer and nonnegative restrictions are as follows

$$
\begin{aligned}
Z_j &= 0 \text{ or } 1, \text{ all } j \\
z_j &\geq 0 \quad , \text{ all } j \\
y_t &\geq 0 \quad , \text{ all } t.
\end{aligned}
\tag{10}
$$

Z must be 1 whenever project scale is positive to assure inclusion of the fixed charge.

$$Z_j \geq k\, z_j, \text{ all } j, \tag{11}$$

where k is a constant that can be made arbitrarily small. When z_j is 0, Z_j will also be 0 because it adds nothing to production, only cost. However, as soon as z_j is positive, Z_j is 1. Clearly, z_j cannot exceed 1/k; k therefore sets an upper bound on project scale.

At most, only one project is to be constructed.

$$\sum_j Z_j \leq 1 \tag{12}$$

Demand constraints are similar to those of the previous model. For any year t,

initial supply capacity	+	capacity constructed prior to t	+	unsatisfied requirements in t	≥	total requirements in t

$$Q_o + \sum_{j<t} z_j + y_t \geq q_t, \text{ all } t, \tag{13}$$

where Q_0 denotes existing supply capacity at the start of the planning period.

Erlenkotter (1967) has shown that whenever a project is constructed, total supply capacity should at least equal existing demand. This can be accomplished by the following

$$Q_o + \sum_{j \leq J} z_j \geq Z_J q_J \text{ , all } J \tag{14}$$

q_J is the rate of demand in construction opportunity period J. If the project is constructed in J, Z_J is 1; otherwise it is 0. Hence, this constraint requires total supply capacity to (i) at least equal 0 if an expansion is not made, and (ii) at least equal existing demand if an expansion is made.

The method of cost accounting in the objective function discriminates against projects proposed near the end of the planning horizon. The entire cost of such projects lies within the horizon while useful life extends beyond it. To provide continuity into the future, a terminal constraint may be included. This provides for a minimum target level of excess capacity (or maximum level of undercapacity) at the end of the planning horizon.

$$Q_o + \sum_{j} z_j \geq q_T + Q_T \text{ ,} \tag{15}$$

where q_T is the demand at the end of the horizon and Q_T is (i) the minimum desired excess capacity if > 0 or (ii) the maximum allowable supply deficit if < 0.

NUMERICAL EXAMPLES

A numerical problem was programmed for the computer in which the first 20 years of Butcher's water requirements function was used. The assumed fixed charge cost function was

$$C(z) = 10 + 10 z$$

(Note that C(2), C(4) and C(7) are 30, 50 and 80 respectively which are close to the values used by Butcher et al. in their example.) Years 0, 1, 2, 3, 4, and 5 were selected as construction opportunity periods ($j = 1$ thru 6). Thus, if a project was to be constructed it would have to be done sometime in the next five years. The net value of water, p, was set equal to 2; the duration of all time periods, d_t, was 1 year; and the discount rate was 5 percent.

In general, the p.v. cost of the project proposed for construction in period j is F_j $(10\ Z_j + 10\ z_j)$. Total p.v. construction cost is therefore

$$10.000\ Z_1 + 10.000\ z_1 + 9.524\ Z_2 + \ .\ .\ .\ + 7.835\ z_6$$

The p.v. cost of unsatisfied demand in year t is $F_t\ 2\ y_t$. Total p.v. social costs are therefore

$$1.905\ y_1 + 1.814\ y_2 + 1.728\ y_3 + \ .\ .\ .\ + 0.754\ y_{20}\ .$$

The total objective function to be minimized is the sum of the above two cost expressions in which there are 6 integer and 26 continuous variables.

The maximum demand during the 20-year horizon is 15. Assuming the maximum scale project that would ever be built is, say, 20, the value of k for the fixed charge constraints is 0.05 (= 1/20). The resulting 6 constraints are

$$Z_j \geq .05\ z_j\ ,\ j = 1,\ 2,\ .\ .\ .\ .\ ,\ 6\ .$$

The redundancy constraint that allows construction only once is

$$Z_1 + Z_2 + \ .\ .\ ..\ + Z_6 \leq 1,$$

and the 20 demand constraints are :

$$
\begin{array}{ll}
z_1 & + y_1 \geq 2 \\
z_1 + z_2 & + y_2 \geq 3 \\
z_1 + z_2 + z_3 & + y_3 \geq 4 \\
z_1 + z_2 + z_3 + z_4 & + y_4 \geq 5 \\
z_1 + z_2 + z_3 + z_4 + z_5 & + y_5 \geq 6 \\
z_1 + z_2 + z_3 + z_4 + z_5 + z_6 & + y_6 \geq 7 \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \\
z_1 + z_2 + z_3 + z_4 + z_5 + z_6 & + y_{20} \geq 15
\end{array}
$$

The set of 6 minimum capacity constraints (14) assuring that existing demand at the time of construction is met can be rearranged as follows

$$
\begin{array}{ll}
Z_1 - z_1 & \leq 0 \\
2\,Z_2 - z_1 - z_2 & \leq 0 \\
3\,Z_3 - z_1 - z_2 - z_3 & \leq 0 \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \cdot \\
6\,Z_6 - z_1 - \cdot \quad \cdot \quad \cdot \; - z_6 & \leq 0
\end{array}
$$

Note that the coefficients of the Z's are the demands in years 0 thru 5 (construction opportunity periods 1 thru 6).

For this problem, a terminal constraint was not included. Thus the model required a total of 33 constraints in addition to integer and nonegative restrictions. Solution was obtained with Shareshian's algorithm. The optimal course of action is to build the project in year 2 with capacity of 11 in which case there will be excess capacity for 10 years. Total p.v. cost is 137.97.

The problem was reformulated with p values ranging from 1 to 10. The results are shown in Table 4. Solution time on the IBM 360 computer was less than 20 seconds per problem.

Table 4
EFFECT OF p ON OPTIMAL PLAN

p	Minimum p.v. Cost	Build in Year	Optimal Capacity	Excess* Capacity Period
1	107.26	Never	-	-
2	137.97	2	11	10
3	149.02	0	12	13
4	153.08	0	13	15
5	155.07	0	14	17
10	160.00	0	15	20

*The number of years of excess capacity following project implementation

DISCUSSION

(1) Table 4 shows the effect of p on the optimal timing and scale of construction. With large p, supply deficits should be eliminated by constructing early and providing sufficient excess capacity to satisfy demands. With decreasing p, deficits become permissible, both by delaying construction and providing less excess capacity. Table 4 shows that excess capacity periods (commonly called design periods) should be small where p is small, as in developing countries, and large where p is large, as in the economically advanced nations. U. S. design standards should not in general be used abroad.

(2) Examination of the model reveals that with increasing water requirements over time, project implementation should be delayed as fixed construction charges (S) increase. Additionally, the optimal amount of excess capacity increases as cost per unit scale (s) decreases. The exact effect of these cost parameters on timing and scale is largely affected by the demand function.

(3) The model can be readily modified to accomodate various planning situations. The parameters of the construction cost function, for example, can be made time

dependent to account for changing future costs. In this case S and s would be replaced by S_j and s_j. Similarly, p_t can replace p.

(4) Several projects instead of only one can be considered by adding another index to the parameters and variables of the construction cost function. Hence, z_{ij} would be the scale of project i proposed for implementation in period j. With multiple projects, changes result in both objective function and constraints, but these for the most part only include additional summations over i.

(5) Successive projects (or more realistically, successive expansions) instead of only the first can be handled if the cost of such expansions is independent of aggregate project scale. Assuming the same fixed charge cost function applies to every expansion, deleting (12) enables determination of the optimal construction pattern beyond the initial project.

(6) Minimum capacity constraints are included so that a project is never constructed with insufficient capacity to meet existing demands. If desired, $\hat{q}_j$ can be substituted for q_j in (14) so as to provide some arbitrary amount of excess capacity in the event of construction, where $\hat{q}_j > q_j$.

(7) Several questions surround the use of p in the model, the net value of locally supplied water. In the U. S., p is most realistically the net price of importing. Purchasing water from an adjacent community is an alternative to local supply. It is not, however, a substitute for distribution. Hence, if p is an import price, the model can only be applied to water supply, treatment and transmission facilities - not distribution.

(8) In developing countries, the model can realistically be applied to entire water systems (supply and distribution) in small communities. The alternative to local supply abroad is to go without public facilities. In this case, p is

a measure of the social losses due to unsatisfied demand. These are primarily amenity (quality-of-life) losses rather than economic development benefits foregone; for example, the lost value of labor due to water-related sickness and death, the value of time and energy spend in carrying water, etc.

The assumption in p that social losses are proportional to the quantities of water demanded but not supplied is reasonable for developing countries. In general only quantities to meet the basic necessities of life are demanded: water for drinking, cooking, hygiene. The value in use for any of these purposes is not much different. At the point where water is used for less essential purposes, however, its value will start to diminish and the proportionality assumption will not hold. For this reason, the model is less applicable to larger cities.

(9) Analysis of an expansion model by Erlenkotter (1967) that is similar to the one herein reveals that construction of a project should ideally be delayed until the social losses due to unsatisfied demand accrue at the same rate as the annual opportunity cost of capital invested in the project. Assuming y is the unsatisfied rate of demand at the time of construction (gpy), C is the implementation cost, r is the discount rate (per year), and p is the value of publicly supplied water (dollars per gallon), a mathematical statement of the optimality condition is $py = rC$, where the units on both sides are dollars per year. The data in Table 4 generally give the same result. Where $p = 2$, for example, $z^* = 11$, $C(z^*) = 120$, and $rC(z^*) = 6$; from the demand schedule, y in year 2 is 3 and $py = 6$.

Rearranging the optimality condition we obtain $p = rC/y$. In this form, it is possible to impute water supply benefit values either for decisions made in the past or ones currently under consideration. Assume for example that a new water system is being considered for a town of 10,000. The system will cost \$150,000 (C) for which the annual opportunity cost is \$7,500 if the discount rate (r) is 5 percent. Assuming the present demand (y) is 73 million gpy (20 gallons per day per capita), a decision to invest now would implicitly assign a net value (p) of about 10 cents per thousand gallons to publicly supplied water.

Inputed values for past decisions have only limited value for future planning purposes. They can, however, (i) indicate the general level of value placed on public water supply by previous investment, (ii) serve as guidelines for setting future p-values as a matter of planning policy, and (iii) provide a basis for comparison with water supply prices and other measurements of willingness to pay.

More important is the role of imputed p's for current investment alternatives. They can be compared with intuitive notions of public water supply value and thus be used as investment criteria. In the problem above, for example, if water is thought to be worth more than 10 cents (net) per thousand gallons, investment should have been made sometime in the past when rC/y was larger. Since this was not done, the project should be constructed now because by delaying, rC/y will decrease due to economies of scale.

SUMMARY

(1) As in the case of the first model, many dynamic programming problems can be reformulated for integer programming. An advantage of such formulation is that branch and bound techniques permit identification of near optimal solutions which can usually be obtained at significantly lower costs than global optima.

(2) In many cases, concern solely with the optimal sequencing of water supply projects is unrealistic. This assumes that project scales are decided in advance, locally supplied water has infinite value, and that it is important to know the subsequent construction pattern after the next project has been built. Instead of sequencing, it is often more important to know when to build the next project

and to what scale.

(3) The mixed integer programming model herein is specifically concerned with the timing and scale of public water facilities in a single community. The model can most realistically be used for the planning of water supply systems (without distribution) in the U. S. or complete rural water systems (including distribution) in developing countries.

(4) If the model is applied in the U. S., the price of importing water from neighboring communities less the local price of operation would most logically be used for the value of p in the objective function. When applied abroad where water systems are generally lacking, p reflects the social losses that result from having to go without publicly supplied water. Evaluation of p in this case is a stumbling block since entirely satisfactory methods have not yet been developed. Some alternative approaches for the determination of p include (i) imputing, as described herein, (ii) use of price data from towns served with water, assuming such data reflect willingness to pay (a more tenable assumption abroad than in the U. S.), (iii) questionnaires regarding willingness to pay, (iv) political fiat, (v) value judgement, (vi) differences in the market value of properties with and without public water service, and (vii) shadow health costs.

(5) While most of the assumptions of the mixed integer programming model fit within the community conditions of developing countries, the model cannot be applied abroad without modification. This is because water supply planning abroad is usually done by a central agency of the national government. Instead of considering each community independently as in the U. S., water systems for an entire group of towns must be planned simultaneously. This results from the requirement that the central planning agency alocate the national water supply sector budget among towns in need of systems. Annual budget constraints, therefore, create economic interdependencies among systems that are lacking in the U.S.

With centralized planning, the mixed integer programming (MIP) model would be merely a component of a larger budget allocation model. The MIP model would have to be repeated once for each community under consideration. This can be done simply by adding a location index to model variables and parameters. The objective function would then require additional summation over this index and the model would be completed by including a set of budgetary constraints.

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Appendix 2

Water Supply Investment Models for Developing Countries

WATER SUPPLY INVESTMENT MODELS FOR DEVELOPING COUNTRIES

1. General Problem

Water supply planning in developing countries is usually performed by an agency of the national government. The principal investment problem is allocation of budgets among communities in need of systems. With such centralized planning, it is not possible to consider each system separately. Rather, all systems must be planned simultaneously because when funds are invested in one project, they are automatically denied to the others.

The principal investment questions are when to construct water systems and how large to make them. Since it can be demonstrated that new water systems in developing countries should have excess capacity for 10 to 15 years, the most difficult problem is determination of investment timing.

2.1 Sequencing by Absolute Advantage

Suppose that water systems are to be constructed in several towns and that the scale of each is decided in advance. The investment problem is to determine investment sequencing among the alternative projects.

2.2 Definitions

C_{it} = present value cost of the system proposed for town i in year t.

x_{it} = binary decision variable (0 or 1) associated with C_{it}.

K_{it} = construction cost of the system proposed for town i in year t.

B_t = available budget for year t.

2.3 Objective Function

The objective function to be minimized is the total present value cost of investment; i.e.; the sum of all elements of the following matrix:

$$
\text{Town } (i) \quad
\begin{array}{c}
 \\ 1 \\ 2 \\ \cdot \\ I
\end{array}
\overset{\text{Time Period } (t)}{\begin{array}{cccc}
1 & 2 & \cdots & T \\
\end{array}}
\begin{bmatrix}
C_{11} & C_{12} & \cdots & C_{1T} \\
C_{21} & C_{22} & \cdots & C_{2T} \\
\cdot & \cdot & \cdots & \cdot \\
C_{I1} & C_{I2} & \cdots & C_{IT}
\end{bmatrix}
$$

Minimize $\sum_i \sum_t C_{it} x_{it}$ (2.1)

2.4 Constraints

Integer:

If a system should be constructed in town i in year t, then x_{it} is 1; otherwise it is 0. Hence

$$x_{it} = 0 \text{ or } 1, \text{ all } i,t \qquad (2.2)$$

Redundancy:

In each town, it is possible to construct a system once and only once. Hence

$$\sum_t x_{it} = 1, \text{ all } i \qquad (2.3\ a)$$

Alternatively, we may want to require that a system in each town need not be constructed, but if it is, it can only be constructed once. Hence

$$\sum_t x_{it} \leq 1, \quad \text{all } i \qquad (2.3\text{ b})$$

Budget:

The total cost of construction in any year cannot exceed the available funds. Hence

$$\sum_i K_{it}\, x_{it} \leq B_t, \quad \text{all } t \qquad (2.4\text{ a})$$

With this constraint, the unused budget from each year is forfeited. If unused funds are allowed to accumulate, we then require cumulative budgets through year $\hat{t}$ to equal or exceed cumulative construction costs. Hence

$$\sum_i \sum_{t \leq \hat{t}} K_{it}\, x_{it} \leq \sum_{t \leq \hat{t}} B_t, \quad \text{all } \hat{t} \qquad (2.4\text{ b})$$

Note that $\hat{t}$ assumes the same values as t.

2.5 Comments

The simplest form of this model includes (2.1), (2.2), and (2.3 a). The expected solution would prescribe construction of all systems in the final time period because in general, $C_{it} > C_{it+1}$.

If (2.3 a) is replaced by (2.3 b), the optimal solution would be to do nothing; i.e. $x^*_{it} = 0$ for all i and t. This would continue to be the solution if budget constraints (2.4 a) or (2.4 b) were added.

A more reasonable model would require all towns to have systems without exceeding budget constraints. Such a model would include (2.1), (2.2), (2.3 a) and (2.4 a). Care must be taken that the total budget equals or exceeds total construction cost. If it does not, the problem has no solution.

Probably the most realistic model would replace the budget constraint by a restriction requiring a minimum number of systems to be constructed each year. Defining

n_t = minimum number of required systems in year t,

we have

$$\sum_i x_{it} \geq n_t, \text{ all } t \qquad (2.5)$$

This model would therefore include (2.1), (2.2), (2.3 b), and (2.5).

3.1 Sequencing by the Efficiency of Investment

The above model only requires that total present value investment cost be minimized. It does not consider the "efficiency" of investment. If an implicit goal of investment is to allocate the budget so as to serve as many people as possible, then an appropriate planning criterion would be present value construction cost per capita. Assuming that this criterion is to be minimized, preference would generally be given to larger systems where greater economies of scale exist. The function of the model would then be to determine investment timing for each community so as to meet restrictions either on budgets or the minimum number of systems to be constructed each year.

3.2 Definitions

E_{it} = present value construction cost per capita of the system proposed for town i in year t. Other definitions as in section 2.2.

3.3 Objective Function

The objective function to be minimized is the total present value investment costs per capita. Hence

$$\text{Minimize} \quad \sum_i \sum_t E_{it} x_{it} \qquad (3.1)$$

3.4 Constraints

The constraints are identical to those of the previous model.

3.5 Comments

In general, the comments of section 2.5 apply herein. The simplest form of the model would still prescribe construction in the last period since E_{it} is generally greater than E_{it+1} due to both economies of scale and the discount rate.

The most realistic form of the model includes a constraint on the minimum number of systems to be built each year. It might also include an upper bound on construction cost. Care must be taken that these two constraints do not conflict

Minimize

$$\sum_i \sum_t E_{it} x_{it}$$

Subject to $\sum_t x_{it} \leq 1$, all i — Redundancy

$\sum_i x_{it} \geq n_t$, all t — Minimum No.

$\sum_i K_{it} x_{it} \leq B_t$, all t — Budget

$x_{it} = 0$ or 1, all i,t — Integer

4.1 Sequencing by Comparative Advantage

Although the previous model is an improvement on the first, it is still deficient because it does not consider comparative advantages among project. For this, it is necessary to include not only the present value cost of construction but also the economic losses that accrue by delaying implementation from one period to another. In other words, to decide optimal timing, it is necessary to examine the benefits foregone by not constructing now.

4.2 Definitions

Assume the planning horizon is divided into T periods each of 1-year duration; t is the time period index. Further assume that during T years of the horizon, there are J opportunities to construct a system of predetermined scale (in general, the scale will meet existing demands and provide excess capacity for 10 or 15 years). Then j is the index of the construction opportunity period.

Q_{ij} = water supply capacity of the project proposed for town i in period j (with such units as gallons per day)

C_{ij} = present value cost of the system proposed for town i in period j

x_{ij} = binary decision variable (0 or 1) associated with C_{ij}

p_{it} = present value net benefit of publicly supplied water in town i during period t (with such units as dollars per gallon)

K_{ij} = construction cost of the system proposed for town i in period j

d_t = duration of period t (in days)

y_{it} = continuous decision variable that denotes the rate of unsatisfied demand in town i during period t (with such units as gallons per day

q_{io} = existing demand in town i at start of the planning horizon

q_{it} = incremental increase in demand in town i during period t

Q_{io} = capacity of existing supply facilities in town i at start of the planning horizon

B_j = available budget in year j.

4.3 Objective Function

The total present value construction cost is

$$\sum_i \sum_j C_{ij} x_{ij}$$

To this must be added the cost of benefits foregone during periods when the demand for publicly supplied water is not satisfied. The rate of unsatisfied demand in town i during period t is $d_t y_{it}$, the present value cost of this unsatisfied demand is $p_{it} d_t y_{it}$, and total present value cost is

$$\sum_i \sum_t p_{it} d_t y_{it}$$

The total objective function to be minimized is therefore

$$\sum_i \sum_j C_{ij} x_{ij} + \sum_i \sum_t p_{it} d_t y_{it}. \qquad (4.1)$$

4.4 Constraints

Integer:

$$x_{ij} = 0 \text{ or } 1, \text{ all } i\ j \qquad (4.2)$$

Nonnegative:

$$y_{it} \geq 0, \text{ all } i, t \qquad (4.3)$$

Redundancy:

This model is concerned with deciding when to build a single system during the planning horizon. Hence we have

$$\sum_j x_{ij} \leq 1, \text{ all } i \qquad (4.4)$$

Demand:

For each year $\hat{t}$ of the planning horizon, the sum of existing capacity in town i plus expansion capacity through $\hat{t}$ and the rate of unsatisfied demand in $\hat{t}$ must equal or exceed cumulative demand through $\hat{t}$. Hence

$$Q_{io} + \sum_{j \leq \hat{t}} Q_{ij} x_{ij} + y_{i\hat{t}} \geq q_{io} + \sum_{t \leq \hat{t}} q_{it}, \text{ all } i, \hat{t} \qquad (4.5)$$

Budget:

Where unused budgets are forfeited we have

$$\sum_i K_{ij} x_{ij} \leq B_j, \text{ all } j \qquad (4.6\text{ a})$$

and where unused budgets accumulate

$$\sum_i \sum_{j \leq \hat{j}} K_{ij} x_{ij} \leq \sum_{j \leq \hat{j}} B_j, \text{ all } \hat{j} \qquad (4.6\text{ b})$$

4.5 Comments

It can be shown that the values for p are the gross benefits of publicly supplied water less the cost of producing water in local supply facilities.

The model is a problem in mixed integer programming. It can be solved by one of the branch and bound techniques.

The present value cost of construction for projects proposed near the end of the planning horizon must be adjusted. Otherwise, demand backlogging rather than construction will be preferred because the entire cost of such projects lies within the horizon while useful life extends beyond it.

5.1 A Model for Determining Optimal Timing and Scale

The four previous models are concerned only with optimal timing. It is implicitly assumed that the proposed scale of each alternative project is optimal. While it can be shown that water systems in general should have excess capacity for 10 or 15 years, the exact scale will depend on particular local conditions. Hence, scale should ideally be treated as a continuous variable. This is done in the model herein.

5.2 Definitions

As in the previous model, t is the time period index and j is the construction opportunity period index

F_j, F_t = present worth factor for periods j and t, respectively

S_{ij} = a fixed cost that is incurred if project i is implemented in period j

Z_{ij} = a binary (0,1) decision variable associated with S_{ij}

s_{ij} = construction cost per unit scale (e.g., $ per mgd) for project i with implementation in period j.

z_{ij} = continuous decision variable for the scale of project i proposed for implementation in period j

$p_{it}, d_t, y_{it}, q_{io}, q_{it}, Q_{io}, B_j$, as in previous model*

k_i = the reciprocal of the largest scaled project that could be constructed in town i

q_{iT} = minimum required excess capacity in town i at end of planning horizon

5.3 Objective Function

The present value implementation cost of the project proposed for town i in period j is

$$F_j\,(S_{ij}\,Z_{ij} + s_{ij}\,z_{ij})$$

The present value of the benefits foregone by not satisfying the demand for publicly supplied water in town i during period t is

$$F_t\,p_{it}\,d_t\,y_{it}$$

Summing these costs over all towns and construction opportunity periods results in total present value cost

$$\sum_i \sum_j F_j(S_{ij}\,Z_{ij} + s_{ij}\,z_{ij}) + \sum_i \sum_t F_t\,p_{it}\,d_t\,y_{it} \qquad (5.1)$$

5.4 Constraints

Integer:

$$Z_{ij} = 0 \text{ or } 1, \text{ all } i,j \qquad (5.2)$$

---

* except p_{it} is not a present value herein.

Fixed Charge:

Whenever an expansion is made, a fixed charge is incurred; that is, when $z > 0$, $Z = 1$. This can be done with the following

$$Z_{ij} \geq k_i z_{ij}, \text{ all } i,j. \quad (5.3)$$

By making k_i very small, Z_{ij} will have to equal 1 when z_{ij} is positive. Actually, k_i sets an upper bound on expansion scale and hence should be chosen accordingly.

Non-negative:

$$z_{ij}, y_{it} \geq 0, \text{ all } i,j,t \quad (5.4)$$

Redundancy:

As before, at most one system can be constructed in each town. For this reason, the planning horizon should not exceed 10 or 15 years. Hence

$$\sum_j Z_{ij} \leq 1, \text{ all } i \quad (5.5)$$

Demand:

As in the previous model, the sum of existing capacity in town i plus expansion capacity through $\hat{t}$ and the rate of unsatisfied demand in $\hat{t}$ must equal or exceed cumulative demand through $\hat{t}$. Hence

$$Q_{io} + \sum_{j \leq \hat{t}} z_{ij} + y_{i\hat{t}} \geq q_{io} + \sum_{t \leq \hat{t}} q_{it}, \text{ all } i,t \quad (5.6)$$

Excess Capacity:

It can be shown that whenever a system is constructed, total supply capacity should at least equal total demand. Hence

$$Q_{io} + \sum_{j \leq \hat{j}} z_{ij} \geq Z_{ij} (q_{io} + \sum_{t \leq \hat{j}} q_{it}), \text{ all } i, \hat{j} \quad (5.7)$$

Terminal:

It is reasonable to require that by the end of the planning horizon, each town should have a supply system that provides some degree of excess capacity over demand. Hence

$$Q_{io} + \sum_{j} z_{ij} \geq q_{io} + \sum_{t} q_{it} + Q_{iT}, \text{ all } i \quad (5.8)$$

If Q_{iT} is chosen to be < 0, a maximum level of unsatisfied demand is implied.

Budget:

If unused budgets are allowed to accumulate, we have

$$\sum_{i} \sum_{j < \hat{j}} [S_{ij} Z_{ij} + s_{ij} z_{ij}] \leq \sum_{t \leq \hat{j}} B_t, \text{ all } \hat{j} \quad (5.9\text{ a})$$

If unused budgets are forfeited, we have

$$\sum_{i} [S_{i\hat{j}} Z_{i\hat{j}} + s_{i\hat{j}} z_{i\hat{j}}] \leq B_{\hat{j}}, \text{ all } \hat{j} \quad (5.9\text{ b})$$

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Appendix 3

Interim Report on the Optimal Design of Small Water Supplies In Developing Countries

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Interim Report On The Optimal Design of Small Water Supplies In Developing Countries

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Interim Report On the Optimal Design of Small Water Supplies in Developing Countries

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1. Introduction

Community water supply engineering is significantly different in developing countries than in the United States. Most water systems abroad are planned by a central agency of the national government while in the U.S. planning is done by individual municipalities. The central agencies operate under budget restrictions that make it necessary to plan several systems simultaneously. In the U.S. however, water systems can usually be planned individually without having to consider the allocation of a budget among different communities. The economic conditions are much more stringent in the low income countries than in the U.S. Most water supply construction abroad is for new systems while in the U.S. it is for expansions. It is common in developing countries for at least part of the water demand of large segments of the population to be periodically unsatisfied, but in the U.S. the total demand is nearly always supplied. Water systems abroad primarily serve domestic needs, but U.S. systems additionally meet large commercial and lawn irrigation requirements.

Despite the differences, U.S. planning practices are widely used in developing countries. It is not uncommon, for example, to find water plants designed for more than twenty years and water mains for more than forty. As in the U.S., expansion policy usually assumes without question that the capacity of water supply facilities should always equal or exceed demand, although budget and other limitations often force unwanted supply deficits. Design values for the annual rate of demand increase are often based more on U.S. experience than that of developing countries, and this is sometimes true as well for per capita rates of water consumption.

Because of the differences in planning conditions, it is generally recognized that U.S. criteria will not produce optimal designs abroad.* U.S. practice is of course used because neither the technology nor the planning parameters have been developed that are specifically pertinent to the water supply situation in low income countries. The goal of this study therefore includes developing the theory and methodology for field evaluation of design criteria that will improve water supply planning abroad.

The primary concern of this study is with the scale and timing of investment in water supplies. These factors are the principal determinants of cost and are the most basic parameters in need of investigation. Thoretical insights regarding timing and scale can be gained from an examination

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* Indeed, the accuracy of conventional design criteria within the U.S. has been questioned in recent years.

of the mathematical models of Alan Manne*. Hence, the pertinent aspects of his work are summarized and discussed herein. To make Manne's models practicable for water supply systems, field evaluation is needed of his key planning parameters. Presentation of completed field work in this regard follows the section on theory. This includes work in Central America to evaluate water demands and analyze cost and other data records. The report concludes with suggestions for additional studies needed to complete the project.

2. Theory

Although Manne has developed several models for the timing and scale of investments, only two are discussed herein. One is called the "no-backlogs" model and the other is called the "time-phased imports" model.

No Backlogs Model

The no-backlogs model is for the planning of a single isolated project. This implies that when the model is used for water supplies in developing countries, selection

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* c.f. Manne, A.S., "Capacity Expansion and Probabilistic Growth", Econometrica, v 29, n 4, pp 632-649, Oct. 1961. Also, Investments for Capacity Expansion: Size, Location and Time Phasing, MIT Press, Cambridge, Mass., 1967

has already been made of the town that is to receive a new system or expansion.*

Initially, a supply system is assumed to exist, and the rate of demand (in our case, the demand for water) is exactly equal to production capacity. While this assumption is more appropriate for expansion of existing systems than construction of new ones, it is useful for model development and will ultimately be relaxed.

The demand for water is assumed to increase at the constant rate of D mgd per year. This increase continues forever (i.e., the time horizon is infinite). Demand is known with certainty and is not affected by the selling price of water.

The cost of a system expansion depends only on its size. As is common in the field of water supply, the cost of a system increases as its size increases but at a decreasing rate. The cost of a 6 mgd water system, for example, is less than twice the cost of a 3 mgd system.

The capacity of water supply facilities must always equal or exceed demand**. The capacity curve must therefore lie

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* This report does not consider the question of how to select the towns that are to receive water systems. Although this is a difficult planning problem abroad, its treatment is deferred to the final report.

** Although this policy is commonly followed in the U.S. and is often the desired policy abroad, it is not too realistic for developing countries and consequently is relaxed in the next model.

on or above the demand curve. This results in the step function shown in Figure 1. Because demand and supply are presently in balance (i.e., now, at time zero) now is the time for the first expansion, and the next will be required when demand has again grown equal to supply. With constantly increasing demand, an infinite time horizon, and unchanging costs and discount rate, the future is identical from each point where supply and demand are in balance. Hence, the expansion scale that is optimal at the present point of balance is optimal at every other such point.

Based on the above assumptions, the mathematical model for determining optimal expansion scale can be developed. If x is the design period in years and D is the rate of demand increase in mgd per year, then the scale of each expansion is xD mgd*. The expansion cost function is

$$C = k(xD)^a.$$

If the value of the exponent, a, were one, this function would be linear and cost would be directly proportional to scale. The cost, for example, of a 6 mgd expansion would then be twice that of a 3 mgd system. In the water supply field, the value of "a" is between zero and one. The

* The design period is the time between subsequent expansions. With this model, x is also (i) the period of excess capacity following an expansion, and (ii) the period between points where demand and supply are equal.

parameter k is the cost of a one mgd system which is immediately apparent by setting expansion scale, xD, equal to unity.

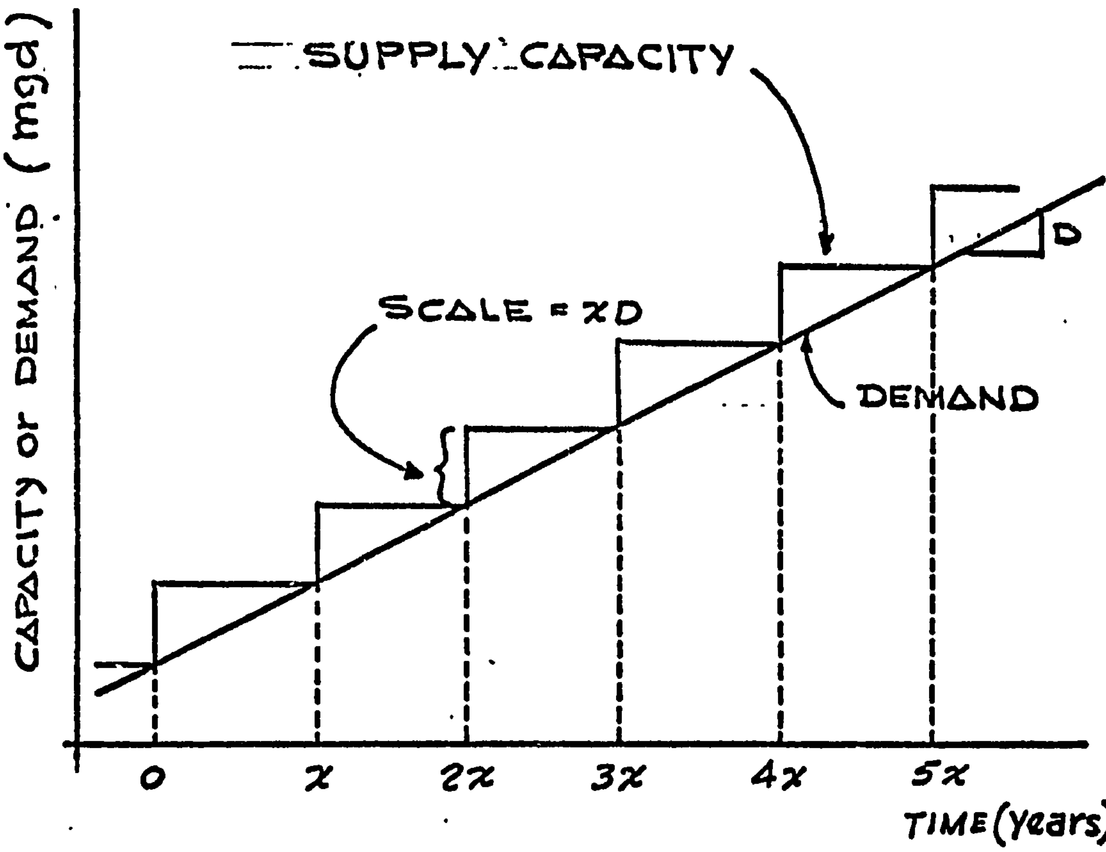
The planning problem is to find the optimal design period (x^*) that minimizes cost. The mathematical expression for total present value cost of all future expansions can be developed using the device of a recursion equation. Defining K as the present value of all future expansion costs from <u>any</u> point where supply and demand are in balance, we can write

$$K = k(xD)^a + e^{-rx} K.$$

This says that the present value of future costs at time zero is the sum of the first expansion cost (which is already a present value) and the discounted value of all future expansion costs from the next point where supply and demand are equal. Discounting is obtained using the factor e^{-rx} which is approximately equivalent to the more conventional discount operator $1/(1+r)^x$, where r is the rate of interest. Solving the equation for total present value cost results in the following

$$K = k(xD)^a/(1-e^{-rx}).$$

To find the design period that minimizes this cost, x^*, the derivative can be set equal to zero. The resulting



D = growth rate of demand (mgd per year).

x = design period (years)

xD = projet scale (mgd)

FIGURE 1 - CAPACITY & DEMAND VS TIME
(no - backlogs model)

optimality condition is

$$a = rx^*/(e^{rx^*}-1).$$

The above equation, which cannot be solved explicitly for optimal design period x^*, indicates that the best design period is a function of the exponent "a", which is called the economy of scale factor, and r which is the interest rate.* Numerical results from solution of this equation are presented in Figure 2. This graph shows that the design period should decrease as either "a" or r (or both) increase. This suggests that in developing countries, were no backlog policies to be adopted, water supply expansions should serve for relatively shorter periods of time than in the economically advanced ones because of higher discount rates. Also, where economies of scale are lacking, the optimal design period is zero indicating that excess capacity should not be built ahead of demand.

Assuming that the discount rate in developing countries lies in the range of 5 to 15 percent and the economy of scale factor is between .6 and .8 (for water treatment plants in the U.S., "a" is about .65), the following table shows the optimal design periods that would apply.

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* The numerical value of "a" indicates the economies of scale associated with water system construction. We have already seen that expansion costs are directly proportional to scale when "a" is one. In such a case, economies of scale are absent. With smaller values of "a", greater economies result. Doubling scale when "a" is 0.7 for example increases expansion cost by about 62 percent.

Table 1

Optimal Design Periods
(in years)

No-Backlogs Model

r/a	.60	.65	.70	.75	.80
.05	19.0	16.2	13.5	11.0	8.6
.10	9.5	8.1	6.8	5.5	4.3
.15	6.3	5.4	4.5	3.7	2.9

Water supply agencies abroad often use design periods of 20 or 25 years. Based on the above, it appears that such values are too high. This implies that scarce resources are being tied up unproductively too long. It might be asked, what are the economic consequences of such overdesign? Manne's present value cost function provides the answer. If the erroneous policy of overdesign is followed forever, the function

$$K = k(xD)^{a}/(1-e^{-rx})$$

indicates excessive costs. For example, where "a" is .65 and r is 10 percent, conditions that presumably apply to many water systems in developing countries, the optimal design period is 8.1 years with total p.v. costs, K, of 7.02*.

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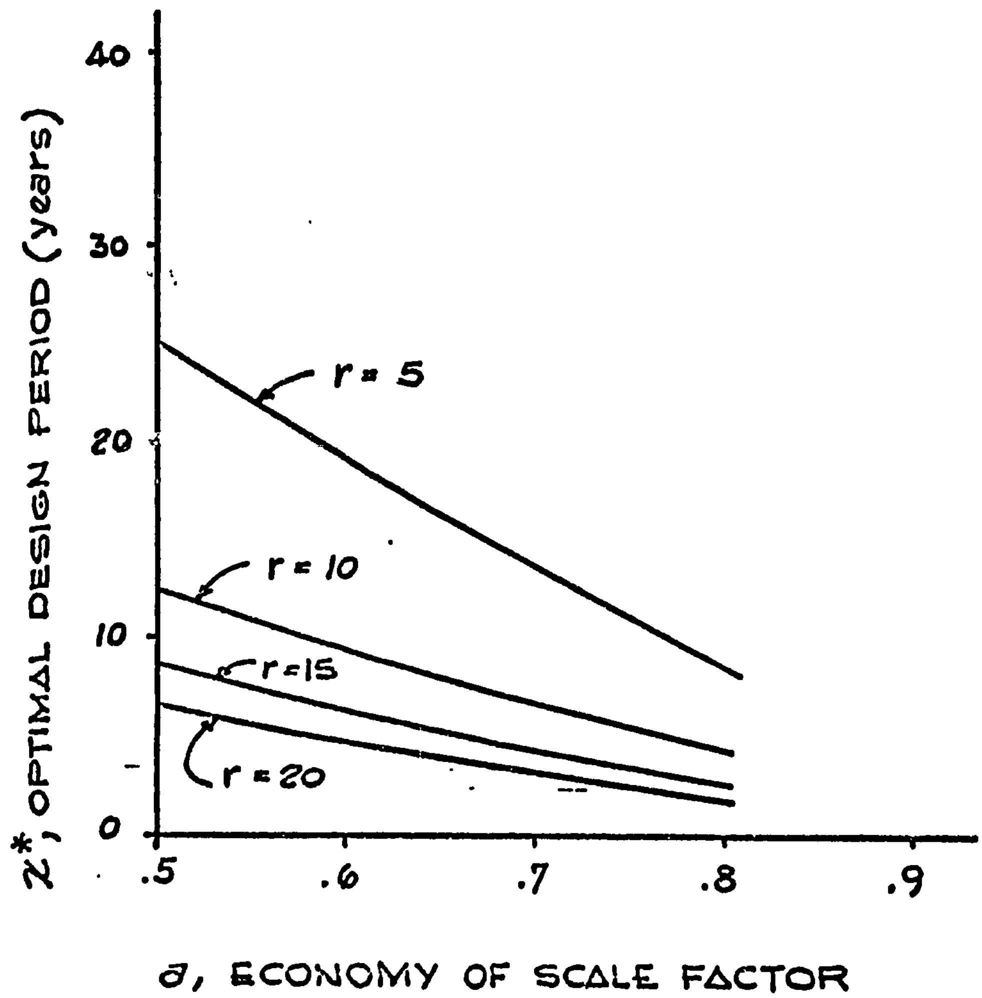
* To obtain K, k and D were set equal to unity which results in no loss of generality.



x^* = optimal design period (years)
r = discount rate (% per year)
a = economy of scale factor

FIGURE 2 - OPTIMAL DESIGN PERIODS
(no-backlogs model)

By erroneously using a 25-year design policy, K is 8.83 resulting in excess present value cost of 26 percent. More generally, the consequences of over or underdesign can be seen from Figure 3 which is a graph of the above cost function. The flatness of the curve suggests that slightly erroneous policies are not too serious, at least relatively speaking, although the actual dollar amounts may be substantial.

Time-Phased Imports Model

The basic assumptions of this model are nearly identical to those of the previous one. Only a single water supply project is considered, initially supply capacity and demand are equal, demand increases linearly, the time horizon is infinite, and the expansion cost function reflects economies of scale. The important difference with this model is that supply capacity need not always equal or exceed demand. Instead, demand is periodically allowed to rise above capacity which results in deficits as shown in Figure 4.

During periods when the water system is inadequate, excessive demands must be left unsatisfied because no other supply alternative exists. Under other circumstances, importing water from a neighboring community might be possible. This however cannot be done because the demand and supply functions of the model implicitly include distribution. A demand, for example, of three mgd not only is a requirement for a certain quantity of water but also implies that the flow must be distributed to users throughout the town. Similarly, system capacity of, say, four mgd implies the ability to produce and distribute this amount of water. Hence, excessive demand implies a deficiency both in production and distribution facilities that cannot be met by importing.

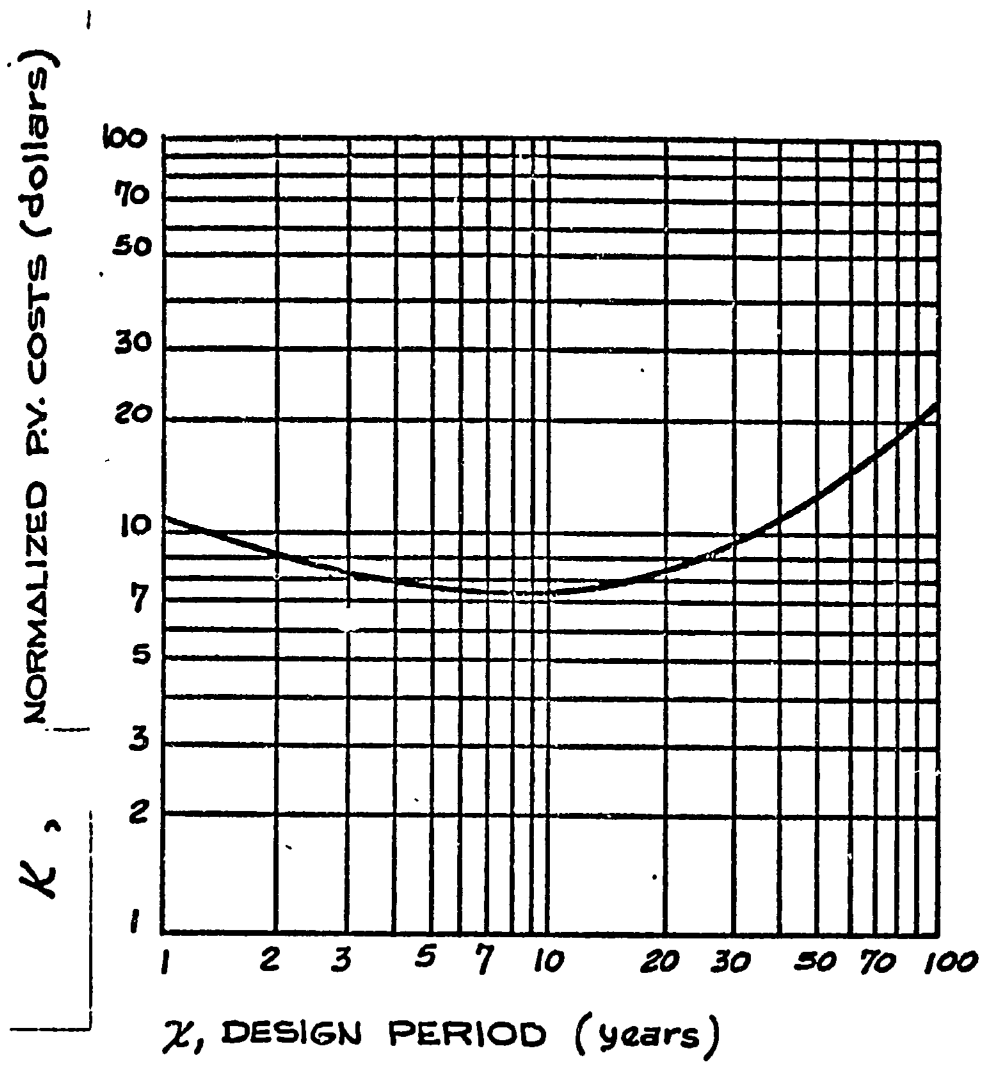
From Figure 4, three important observations can be made. The future from every point where capacity and demand are equal is identical. Hence, the policy that is optimal now at the first point of balance is optimal at every other such point. This accounts for the even timing between expansions and constant expansion scale.

Secondly, the design period (x) is the time between successive expansions. An expansion wipes out the y years of deficit that preceed construction and provides x-y years of excess capacity.

Finally, with this model there are two decision variables, expansion scale and timing. No longer is timing fixed by the requirement that capacity equal or exceed demand as in the other model. Rather, the optimal waiting period is now a matter for decision.

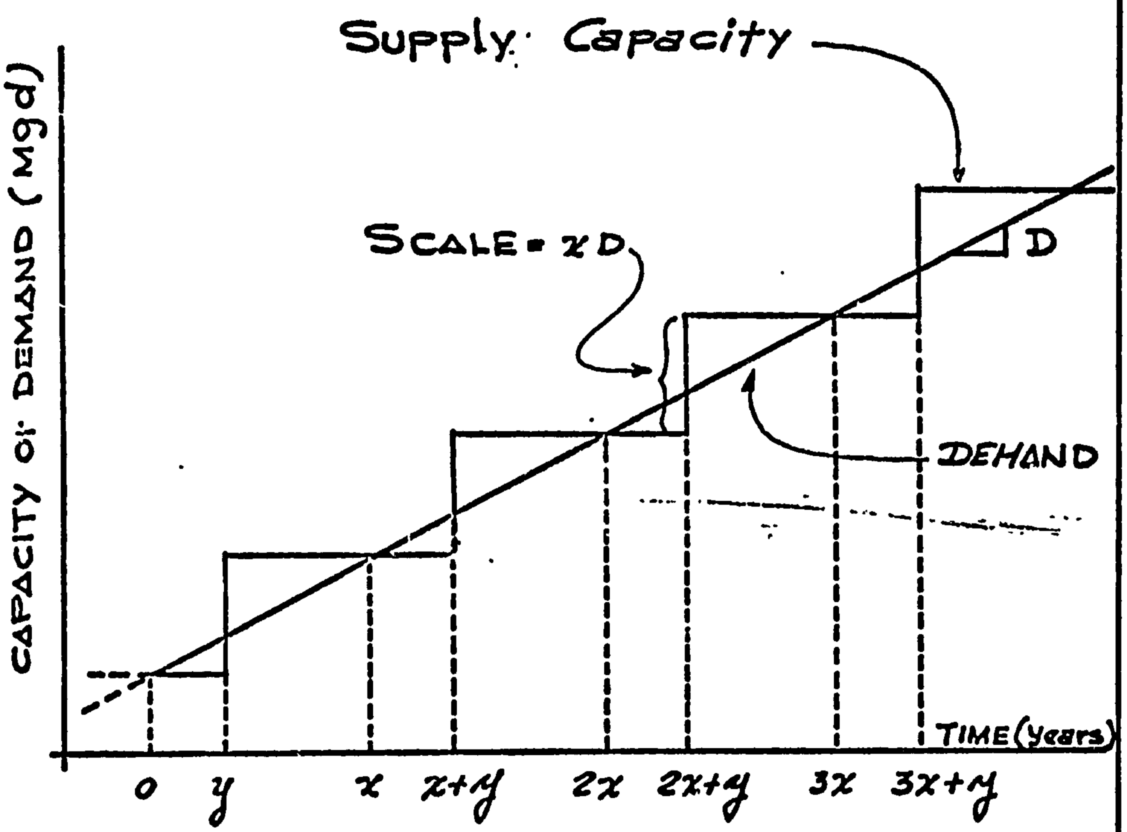
As with the no-backlogs model, the planning objective is to minimize total present value cost. This includes expansion costs, operation and maintenance costs, and social costs that result when part of the demand is not satisfied. These latter are called backlogging costs.

In the no-backlogs model, the output of the water system



r = 10 = discount rate (% per year)
a = .65 = economy of scale factor
X^* = 8.07 = optimal design period (years)
K = 7.02 = minimum p.v. cost (dollars)

FIGURE 3 - P.V. COSTS VS DESIGN PERIOD (no-backlogs model)



D = growth rate of demand (mgd per year)

x = design period (years)

y = import-backlog period (years)

FIGURE 4- CAPACITY & DEMAND VS TIME

(time-phased imports model)

is equal to the total demand which is a fixed constant. Operation and maintenance costs are assumed to be proportional to system output which means that they are independent of project scale and can be ignored.* In the imports model however, system output is not equal to the total demand. The fact that demand is fixed no longer reduces operating and maintenance costs to a constant. Hence, they must be explicitly considered in model formulation.

The total present value cost function of this model can be developed by referring to Figure 4. As with the no-backlogs model, a recursion equation is used. The general approach is as follows:

(1) The present value cost of operation is calculated as if the total demand is supplied by the system. The result is a constant as in the case of the no-backlogs model, but its value is too large because it erroneously includes the operating cost of periodically supplying water beyond system capacity.

(2) In order to correct this error, the present value of excess operating costs must be substracted for the periods of deficit.

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* It is assumed that operating costs equal the price of operation multiplied by the amount of water produced and then discounted to time zero. But the amount of water produced in the no-backlogs case is equal to total demand which is a constant, and the price is constant. Hence the product of these terms is also a constant that is independent of design period. Its derivative in the expression for total present value cost is therefore zero, and it follows that operating costs can be ignored.

(3) Now however, the present value social or backlogging costs from not meeting total demand during the same periods of deficit must be added.

(4) Finally, present value expansion costs must be included.

In the strategy of model development, the erroneously high operating costs is a constant that does not affect the optimality conditions and hence can be ignored. The downward adjustment in operating costs during periods of capacity deficit can be combined with the social costs of the same periods. Finally, expansion costs are included and the entire expression put in the framework of a recursion equation.

For developing the recursion equation, G is defined as the present value of future variable costs from any point where supply capacity and demand are equal. G is a function of expansion costs, backlogging costs for periods of capacity deficit, and operating cost "credits" to compensate for the error in the assumptions that total demand is met by the system.

$$G = \begin{bmatrix} \text{present value} \\ \text{backlogging} \\ \text{costs during} \\ \text{period 0 to y} \\ \text{(I)} \end{bmatrix} - \begin{bmatrix} \text{present value} \\ \text{excess operating} \\ \text{costs during} \\ \text{period 0 to y} \\ \text{(II)} \end{bmatrix} + \begin{bmatrix} \text{present value} \\ \text{expansion} \\ \text{cost at} \\ \text{time y} \\ \text{(III)} \end{bmatrix} + \begin{bmatrix} \text{present value} \\ \text{cost of G} \\ \text{at time x} \\ \text{(IV)} \end{bmatrix}$$

If p_1 is the social cost per gallon of water demanded but not supplies during periods of deficit and p_2 is the operating cost per gallon of water, the combination of these prices for items I and II in the above expression is the net "backlogging" price p, where p is the difference between p_1 and p_2. Typical units of p are dollars per gallon, and its value is always positive, for should the social cost of not meeting demand (p_1) be less than the operating cost (p_2), it would be more economical to let demands go unsatisfied and never construct a water system.

The backlogging price p is a measure of the net social losses due to unsatisfied demand. It represents the net benefits foregone by not having a public water supply system. Such social losses primarily include amenity benefits not achieved, although for larger water systems, foregone economic development benefits would also be included. Typical lossses for small systems include the value of labor due to sickness and death and the value of time and energy spent in carrying water, losses that would have been avoided had a public water system been in existence.

The mathematical expression for the above recursion equation can be written as follows

$$G = \underbrace{\int_0^y e^{-rt}\, pDt\, dt}_{(I\ \&\ II)} + \underbrace{e^{-ry}\, k(xD)^a}_{(III)} + \underbrace{e^{-rx}\, G}_{(IV)}$$

After integrating and solving for G, the optimal waiting period prior to expansion (y^*) and the optimal design period (x^*) can be found by setting the appropriate partial derivatives equal to zero. The resulting optimality conditions are

$$y^* = r\,k(xD)^a/pD$$
$$a = [x^*\,(e^{rx^*} - 1)] \,/\, [y^*/(e^{ry^*} - 1)]\,.$$

The first expression above has valuable implications for water supply planning in developing countries. As the backlogging price, p, increases, the deficit period, y^*, approaches zero. But when y^* is zero, the situation is identical to the no-backlogs model. Hence, the restriction disallowing deficits is equivalent to assigning an infinite value to the backlogging price.

Aside from indicating the conditions for the single best system expansion, the first equation above indicates an optimal planning policy. This means that given a value for design period x, optimal or not, total present value costs will be minimized by delaying construction y^* years from the time capacity and demand are equal. If the design period is the optimal value (i.e., x^*), the absolute minimum will be obtained, but if x is non optimal, a relative minimum that is the lowest possible value for the given x will result.

Knowledge of the optimal waiting period is of little practical value for the planning of new water systems because

y^* is measured from the point of equilibrium between capacity and demand which does not exist. However, the optimality condition can be rearranged to other forms that are more useful, the following for example

$$y^* = [kD^a / pD] rx^a .$$

The term in brackets is called the "penalty factor". As we have already seen, k is a measure of construction cost and p is a measure of the social costs due to leaving part of the demand unsatisfied. For convenience, let us assume that the value of D is unity.

The penalty factor describes a cost ratio between meeting and not meeting water demands. High values of the factor imply that construction is relatively more expensive than backlogging and vice versa. In places like the U.S. where the benefits assigned to publicly supplied water are very high, p is large and the penalty factor approaches zero. Conversely, in developing countries p generally has low value and the factor is large.

Figure 5 shows the optimal levels of the waiting period (y^*) and design period (x^*) as functions of the penalty factor for arbitrarily selected "a" and r of 0.65 and 10 percent. The most important observation from this graph is that (x^*-y^*) is a maximum when the penalty factor is zero and it decreases as the penalty factor increases. But ($x^* - y^*$) is the optimal period of excess capacity following an expansion.

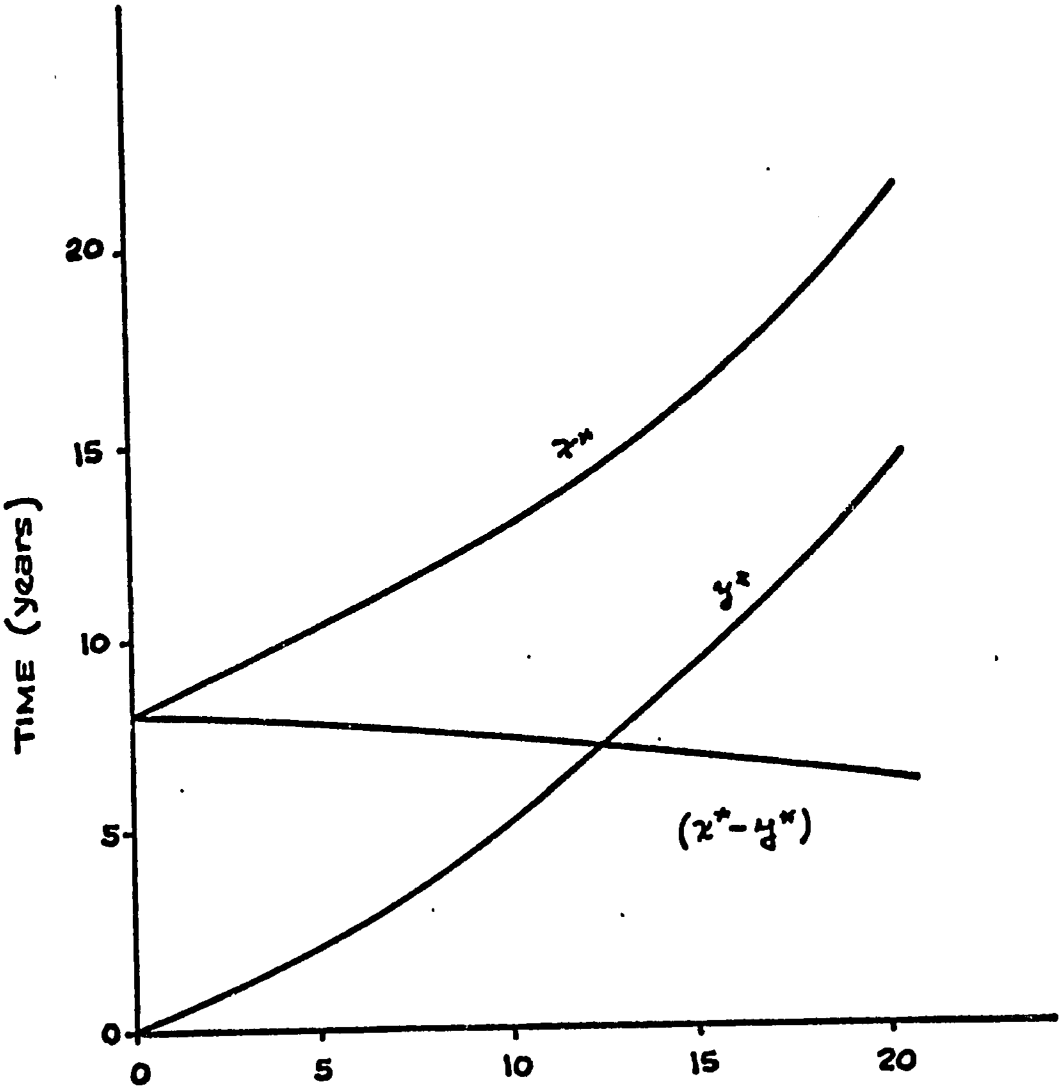
Hence we can conclude that whether there are supply deficits or not, the optimal period of excess capacity for a water system should never exceed the design period values obtained for the no-backlogs model. Once the decision on when to construct a system is made, its maximum scale can be determined in the absence of information on backlogging prices.

An example will illustrate. Assume town population is 10,000 and the rate at which water is needed is 20 gallons per day per capita. Assuming no existing water system, the present rate of unsatisfied demand is .20 mgd. Suppose the growth rate of demand is .005 mgd per year and "a" and r are .7 and 10 percent. From Table 1, the maximum period of excess capacity is 6.8 years which results in maximum excess capacity of .034 mgd (6.8 x.005). Hence, the scale of the expansion to be made now should not exceed .234 mgd.

In this example, the decision is made *a priori* that a water system is to be constructed now. The possibility exists however that now is not the best time for expansion; perhaps it would be more economical to delay construction. Although the optimality equation above cannot indicate the best time for implementation unless numerical values are available for all the parameters, it can assist in deciding whether to build now or not even if an exact value is not available for p.

Consider the equation in the following form

$$p = r[k(xD)^a] / Dy^* .$$



k/p = PENALTY FACTOR

a = .65 = economy of scale factor

r = 10 = discount rate (% per year)

x^* = optimal design period (years)

y^* = optimal backlog period (years)

k/p = penalty factor

FIGURE 5- OPTIMAL DESIGN & BACKLOG PERIOD VS PENALTY FACTOR (time-phased imports model)

The term in brackets is the cost of expansion and the numerator is the annual interest on investment in dollars per year. The denominator is the unserved rate of demand at the time of expansion. By substituting numerical values for the right hand side, p can be estimated. This is the value of p that would be implicitly assigned by undertaking the proposed construction. The calculated p is called the imputed value, and by comparing it with what the planners think is the true value of publicly supplied water, the decision can be made to build now or wait. If the imputed p is too large, building now would assign greater benefits than what the water is really worth, and hence the project should be delayed, but if it is too small, implementation should proceed.

The previous example can be used for illustration. The planners know that if a system is to be constructed, its scale should not exceed .234 mgd. Suppose they decide to use this scale. The questions is, should the system be built now or not?

Suppose the estimated cost of a .234 mgd system is $145,000*. Also assume that the discount rate is 10 percent per year. Then the interest on construction is $14,500 per year. If construction is made now, the unserved rate of

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* The assumed construction cost function from which this value derives is $C=400{,}000\ (xD)^{.7}$, where xD is scale in mgd and C is cost in dollars.

in the economy of scale factor, "a". With numerical values for "a" and the discount rate (r), it is possible to calculate optimal design period, x^*, for the no-backlogs model. Although x^* is of little value from the standpoint of applying the no-backlogs model, it is of considerable interest as an upper limit on the period of excess capacity.

(2) Optimal design period is of little planning value by itself. Our interest is in the optimal scale of construction which depends equally on the design period and the rate of demand. Specifically, information is needed on the growth rate of water demand in communities newly served with supply systems. Hence, development of statistical prediction models using such data from developing countries should be made.

(3) With accurate information on both design period and water demand, it should be possible to make fairly good estimates of optimal water system scale. The flatness of the total present value cost function suggests that serious errors should not result even if the scales deviate somewhat from optimality. The problem of construction timing however still exists. Although the scale might be correct, investment at the wrong time might lead to serious misallocation of scarce funds. Solution to the timing problem depends on numerical data for the net benefits of publicly supplied water. Although it appears that accurate estimates of public water supply benefits will be long in coming, work on this problem can be started using the method of imputing to analyze cost and other data from developing countries.

(4) Neither the comments above nor the models of this section have taken proper account of the type of centralized planning in the presence of budgetary constraints that is employed in developing countries. Hence, additional theoretical work is needed to develop planning models more suitable to the conditions abroad, and these should then be applied to specific water supply planning problems.

In connection with each of the above points, some work has been started. With exception of item 4 which will be discussed in the final report, some of the results that have been obtained are presented in the following sections herein.

3. Construction Cost Function

The upper limit on the excess capacity that should be built into a water system depends largely on the economy of scale factor, "a". This factor can be estimated by statistically fitting the cost function

$$C = k(xD)^a$$

to water system construction data. With values for cost (C) and design scale (xD), the parameters k and "a" can be determined by least squares analysis.

As a substitute for design scale, the statistical analysis can be made using data for design population. In this case, it is necessary to replace xD by the product of per capita water demand (q) and design population (w). Assuming per capita demand is the same for all systems, the cost function to be fitted to the data is

$$C = k(qw)^a = (kq^a)\ w^a = k_1 w^a \ .$$

For convenience, C is cost in thousands of dollars and w is the expected number of inhabitants to be served by the end of the design period. k_1 therefore is the cost (in thousands of dollars) of serving a town with a design population of one.

In order to use least squares analysis for estimating the economy of scale factor, it is necessary to take the log transform of the above function. This results in a linear equation

$$Y = b_0 + b_1 X,$$

where Y and X are the logarithms of system cost and design population, respectively, b_0 is the log of k_1, and b_1 is the economy of scale factor. The corresponding equations for parameter evaluation are

$$b_0 = (\Sigma Y)/n - b_1(\Sigma X)/n$$
$$b_1 = [\Sigma XY - (\Sigma X)(\Sigma Y)/n]/[\Sigma X^2 - (\Sigma X)^2/n],$$

where n is the number of data points in the sample.

The appendix includes cost data for 65 water systems constructed in Central America from 1965 through 1969. All are of the gravity type and are new rather than extensions of existing water works. From the least squares analysis, b_0 was found to be -3.07 and b_1 is 0.83. The equivalent exponential form of the cost function is

$$C = .046\ w^{.83}\ .$$

If per capita water consumption is 30 gallons per day, the design population for a one mgd system is about 33,300. The corresponding cost of this scale system calculated by the above equation is approximately $260,000. Hence, the cost equation in its original form is

$$C = 260\ (xD)^{.83}\ ,$$

where design scale (xD) is in mgd. If per capita consumption is 25 gallons per day instead of 30, the coefficient in the equation is 300 instead of 260.

The large "a" value obtained from this analysis implies that economies of scale associated with water supply systems are rather small. Referring to Figure 2, the period of excess capacity (x*) for a discount rate of 10 percent should apparently be less than 5 years. This is considerably shorter than the 20 or 25 year periods currently in use, but before recommending that design periods in developing countries be drastically reduced, it is necessary to examine the basis on which the economy of scale factor was estimated.

The most obvious weakness of the statistical analysis is that it is based on a relatively small data sample. The conditions in 65 towns hardly constitute a firm basis for changing design policy. What is needed, therefore, are additional data from newly constructed water systems that will improve the confidence that can be placed in the economy of scale value. At this point we can only observe that water supply systems in Central America seem to be overdesigned. Although the results reported herein and those given earlier* for some systems in Guatemala imply that a design period of 10 years would be preferable to one of 20 or 25 years, it is still too soon to strongly recommend a definite design value.

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* Lauria, D.T., Report on Water Demand Study, Community Water Supply Branch, U.S. AID, Washington, D.C., 1969

Another problem herein is with the data used for the statistical analysis. To large extent, the cost data are accounting costs that primarily cover construction. But the cost function implicitly includes planning, engineering, legal and administrative costs in addition to those of construction, and unless these are reflected in the data, somewhat erroneous results will be obtained. Actually, the economies of scale associated with planning and engineering are probably far greater than those of construction. Because some planning and other costs are not included in the data of the appendix, the value of the economy of scale factor is probably too high. Correspondingly, the indicated period of excess capacity is probably too short. It is doubtful, however, that even if all costs were taken into account, the period would exceed 10 years.

4. Water Demand

The optimal sizing of water supply systems is not only a function of the design period. Additionally, it depends on the future expected rate of water usage. Although many assumptions have been made about rates of water demand and various values are currently in use for design purposes, little has been done in developing countries to actually measure consumption. Consequently, in 1967 a study was started in Guatemala to obtain demand data in communities newly served with supply systems.*

Preliminary water demand studies have been completed in ten towns, and additional studies are in progress. The ten towns have populations ranging from 900 to 6200 with an average of 3100. The oldest water system in these towns has been in existence about four years and the newest about one year; the average age is 2.4 years.

The town with the smallest percentage of connections serves only one person in eight with piped water into the home. The town with the largest percentage serves about eight in ten, and the mean number connected is 44 percent. On the average, about 25 percent of the population is connected by the end of the first year and new connections are made at the approximate rate of 8 percent of the population per year. Those without house connections generally rely for their water on public fountains and washing stations distributed throughout the town.

From an analysis of house meter records, it was found that average consumption ranged between 60 and 130 liters

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* For a preliminary report on this study, see: Cordon, Octavio, "Demandas de Agua: Progreso de la Investigacion", paper presented at the 13th Meeting of the Interamerican Society of Sanitary Engineers (AIDIS), Caracas, Venezuela August, 1970. (Most of the data of this section are from this report.)

per capita per day (lpcd). The average was about 100 lpcd which is equivalent to approximately 26 gallons per capita per day (gpcd). As water users became accustomed to having a system, average demand increased, rapidly at first and then more slowly. At one extreme, the average rate of increase for the period of record was 5.36 lpcd per year; but overall, the ten towns showed demand growth of about 3 lpcd per year. Based on 100 lpcd average consumption, this rate is something less than 3 percent per year. Had the systems been somewhat older, it is expected that the average increase would have been less. Consequently, 3 percent may be an upper limit on growth.

The system of water rights employed in Guatemala entitles households to 30 cubic meter per month without having to pay excess usage charges (about 260 gallons per day). With an average of 5.7 persons per house connection, this amounts to about 45 gpcd. The actual average usage of only 26 gpcd during the initial years of system operation is far below this limit. Additionally, it was found that 90 percent of monthly consumption during the early years is equal or less than 30 cubic meters, and 50 percent of the consumption is less than 15 cubic meters.

In addition to the analysis of individual meter records, data were collected on total community water demands. Nutating disc meters, either alone or in parallel, were installed in the supply main of each of the ten systems under investigation

The master meter for each town was kept in operation for one month during which time the flow was recorded (by hand) every 15 minutes.

Based on the master meter data, mathematical models have been developed for relationships between average and extreme flows. The resulting function that is probably most important for design purposes is $R = 1.35/\hat{Q}^{.058}$ where $\hat{Q}$ is the average daily demand in liters per second and R is the ratio of maximum daily to average daily demand. The equivalent function in English units is $R = 1.09/Q^{.058}$, where Q is average daily usage in mgd. As expected, the value of R decreases as system scale increases.

The master and house meter data together provide a measure of unaccaounted for and publicly used water. It was found that this is less than 2 percent of the total demand. Such a low value is due in part to the fact that the studied systems were new and leakage minimal. However, the value also implies that for those users dependent on public fountains and washing stations, the amount of water demanded is extremely small, probably being not much different from that obtained from natural sources prior to system construction.

The findings reported herein and the results of the cost analysis of the previous section provide a basis for calculating the scale of new water supply systems abroad. Presumably, the scale so determined will be more nearly optimal than if conventional design criteria were used. An example will illustrate.

Suppose that water is to be provided for a town with present population of 5000. Assuming population increases at 2 percent per year and the design period is 10 years, the population for which the system should be designed is 6100. For those connecting to the system now, an average demand of about 26 gpcd is expected. This demand will grow to 32 gpcd by the end of the design period if the annual rate of increase is 2 percent.* By the end of the design period, the entire town should be served in the home if connections are made at the same rate as that found in the study. Of course, at that time some of the connections will be new while others will have been in existence from the beginning. Assuming 25 percent of the population connects initially and the remaining connections are evenly distributed over the 10-year design period, the resulting average per capita demand is about 30 gpcd, and the corresponding total demand is .183 mgd. If at the end of the design period the public and unaccounted for usage is, say, 5 percent, the average demand will be about .195 mgd or 32 gpcd (121 lpcd).

Having estimated the average future demand, a problem to

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* Study results imply an upper limit on demand growth of 3 percent annually. The 2 percent value has been arbitrarily selected.

be decided is whether the system should be designed for this flow or a higher rate to meet peak demands. Assuming the system is to supply maximum daily requirements, its capacity must be 1.20 times the average daily rate (or .234 mgd) based on the equation for R reported above. The corresponding per capita demand is about 38 gpcd or 145 lpcd. Using the project implementation cost function of the previous section, a system of this scale would cost about 16 percent more than if the facilities were only designed for average daily demand.*

While the study thus far has provided general information on the demand phonemenon, more specific data are still needed. Mathematical functions should be developed to relate household usage to measurable environmental factors. In particular, the wide variation in average per capita usage (from 60 to 130 lpcd) needs to be explained. Public and unaccounted for demand must be more carefully measured and broken into components; e.g. public fountains, washing stations, leakage, etc. Data are needed on how the rate of demand changes with time (particularly with the age of the system), and more information should be collected on the rates at which new users are connected to the system. Perhaps most important, an analysis is needed to determine the economic

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* If initial population were 500 instead of 5000, R would be 1.37, the maximum daily demand would be 44 gpcd (166 lpcd), and system cost would be 30 percent higher by designing for maximum daily instead of average daily demand.

consequences of designing for various peak rates of flow because at present, there is little economic justification for basing scale on maximum daily demand.

5. Imputed Water Supply Benefits

Whenever a decision is made to construct a new water system or extension, a certain value is implicitly assigned to publicly supplied water in the community. In essence, the planners say that the value of water is such that construction is less expensive than allowing demands to go unsatisfied any longer. In most cases, the planners do not know the numerical value they attach to public supply, but it can be estimated (or "imputed") using the mathematical model presented in section 2.

Let us assume that when a decision was made in the past to construct a water system, the planners felt it was correctly sized and timed. That is, both the design scale of the system and the size of the town to be served were thought to be optimal. With this assumption, the implicitly assigned value of water can be imputed by the equation

$$p = r \, [\, k(xD)^a \,] \, / \, Dy \, .$$

As previously defined, p is the difference between the gross value of water and the price of production in local supply facilities. The numerator is the annual rate of interest on project cost (dollars per year), and the denominator is the rate of unsupplied demand at the time of construction (thousands of gallons per year). In the remainder of this section, the denominator, Dy, is replaced by the product of town population at construction time (w) and the per capita demand for water (q). For towns not previously served by water systems, wq is equivalent to Dy.

Two important observations need to be made about the above equation. The first is that when relatively long design periods are used for sizing supply facilities, p-values are relatively large. This is because the design period affects project cost upon which p is dependent. Hence, the implicitly assigned value of water may be significantly different for two alternative projects that would serve the same town but with different excess capacities.

The second observation is that, for a given policy regarding design period, p-values in small towns will generally be greater than those in larger communities. This is because the denominator of the equation for p increases directly as town population increases, but the numerator increases at a decreasing rate due to economies of scale. Hence, communities of different size will have different p's even if the period of excess capacity is identical.

In calculating p, estimating data rather than data on actual conditions should be used. This is particularly necessary in the case of project costs. Estimates are made prior to investment decisions and form a basis for action, while accounting costs follow decisions and reveal little about how they were reached. Engineering estimates, however, are often quickly set aside after project implementation and are difficult to obtain. Consequently, accounting cost data are used herein with the assumption that there was close agreement between estimated and actual costs.

The pertinent data from the appendix for imputing the value of publicly supplied water include population at construction time and project cost. For town 1, these are 453 inhabitants and 9180 dollars, respectively. Assuming a discount rate of 10 percent, the annual interest on project cost is 918 dollars per year. If the planners assumed that all inhabitants desired water at the rate of 30 gpcd, the unsuplied rate of demand immediately prior to project implementation was 13.5 thousand gallons per day (453 x 30) or 4930 thousand gallons per year. Hence,

$$p = 918 / 4930 = \$.186/M \text{ gallons} = 18.6\ ¢/M$$

The values for the other towns can be similarly imputed.

The p-values in the appendix range from 8.9 to 76.0 cents per thousand gallons with an average of 24.4. The variance of the p's is 150.27 and the standard deviation is about 12.3. As noted above, the variation is due in part to differences in community size at the time of project implementation and to differences in the design period of excess capacity. For this set of values, community size ranges from 210 to 3912 with and average of 950. Additionally, the ratio of design to existing population ranges from 1.35 to 2.62 which, with an annual growth rate of 2 percent implies that excess capacity is provided from 15 to nearly 50 years. On the average, the population ratio is 1.76 and the corresponding design period is 28 years.

In the absence of budget constraints, it is possible to use the imputed values of publicly supplied water to make decisions regarding the timing of new projects. The rationale is as follows:

1. The true value of water in small communities is assumed to be constant.
2. The imputed p's constitute a sample of measurements on the true value.
3. Investment should not be made in any system where the implicitly assigned value is greater than what the water is truly worth.
4. Hence, each potentital system must be examined to see whether investment now would overassign value.
5. If this would result, implementation should be delayed.
6. However, if the assigned value is not too large, implementation should proceed.

Let us now examine the rationale in more detail.

The assumption that publicly supplied water has the same value is based on the fact that supply systems in small towns essentially satisfy only the basic necessities of life. The value of a gallon of water for drinking is probably not much different than the value of a gallon for personal hygiene. Where water is uded for less essential purposes as is often the case in larger towns and cities, the assumption no longer holds.

It follows from this assumption that the imputed p's are measurements of the true value of water. The 65 values in the appendix constitute a sample from the population of all possible measurements from a specific geographical region. From the sample, statistical inferences can be made about the population, particularly its mean.

To build a water supply system too soon implies building it when town population is too low. As already seen, the implicitly assigned value of water is relatively high when the size of the town to be served is small. Delaying implementation until the population increases causes the assigned value of p to decrease. Consequently, by comparing the value of water that would be assigned by constructing now with the true value of water inferred from the imputed p's. the decision can be made to proceed with implementation or delay. If the implicitly assigned value exceeds the true value, implementation should be delayed until an increase in population reduces it to the acceptable limit. However, if it falls short of the true value, it would have been better had the investment been made earlier.

An example will illustrate. Suppose a town of 100 is being considered for a water system. Assuming growth at the rate of 2 percent per year and excess capacity for 30 years, the estimated project implementation cost (using the equation of section 3) is $3920. The resulting value that would be implicitly assigned by constructing now is 35.9 cents per thousand gallons, assuming an interest rate of 10 percent and per capita usage of 30 gallons per day.

To determine whether this price exceeds the true value of water, a null hypothesis is made that the mean of the population of p's equals or exceeds 35.9. An alternative hypothesis is that the mean is less than 35.9. If the data sample leads us to accept the null hypothesis, then the water system should be constructed, but if it is rejected, we should delay. For testing the hypothesis, a significance level of 5 percent is used.*

Assuming the null hypothesis is true, the statistic $(\bar{p} - p_o)\sqrt{N}/s$ has the standard normal distribution**, where $\bar{p}$ is the sample mean, p_o is the hypothesized value, N is sample size, and s is the standard deviation. The hypothesis should be rejected if the value of the statistic is less than -1.645. In this case, its value is -7.55 $(=[24.4 - 35.9]\sqrt{65}/12.3)$. Hence we reject and consequently decide to delay construction. By a similar calculation, it can be shown that the implicitly assigned value cannot exceed 26.9 cents per thousand for implementation to be currently acceptable.

In this example, the excess capacity period is 30 years which is too long for a discount rate of 10 percent and an

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* Hence, if we reject, the probability of being in error is equal or less than .05.

** N is sufficiently large to use the normal rather than t distribution economy of scale factor or .83. A more nearly optimal period is 10 years. Suppose therefore that the planning office changes its design policy to 10 years. The town of 100 would then have an implicitly assigned water value of 23.9 cents per thousand, and the proposed system would be acceptable. Hence, reduction of overdesign policies has the effect of permitting construction in towns of smaller size.

6. Additional Work

The outstanding work required to complete this project includes the following. Additional theoretical models are needed that more closely reflect the water supply planning conditions of developing countries. Also, studies should be continued to obtain and analyze field data for implementation of planning models. Finally, efforts should be made to apply the models in actual planning situations. The work of these items is described in more detail in the remainder of this section.

Theoretical Models

1. The planning models of Alan Manne which form a basis for much of the work herein assume that supply and demand are initially equal. This assumption is erroneous for new water supply systems. Consequently, the models, particularly the one dealing with imports, should be expanded to consider an initial supply deficit.*

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* Muhich has already made such a modification for the no-backlogs model, but nonlinear programming is required for solution. c.f. Muhich, A.J., "Capacity Expansion of Water Treatment Facilities", unpublished Ph.D. Thesis, Harvard University, Cambridge, Mass., 1966.

2. On page 14 herein, the equation $y^* = r\ k(xD)^a/pD$ is explained to represent not only the timing for the single best system expansion, but also an optimal planning policy. That is, given a decision on expansion scale (xD), this equation indicates the time of contruction (y*) that minimizes total present value cost. Similarly, a mathematical model should be developed to indicate the policy for optimal expansion scale given a decision on the time of construction. Indeed, in water supply planning, it is more common to encounter situations where the decision has been made to build now and the question is, to what scale?

3. While Manne's models are valuable for planning an isolated system in the absence of binding budgetary constraints, they are not completely applicable to the centralized water supply planning practices of developing countries. Consequently, a simple model should be developed to illustrate the principles of budget allocation among few systems. It would probably be best for the model to be developed for solution by calculus.

4. For more practial purposes, a larger programming model is needed that can be used by central planning offices for deciding the location, timing, and scale of water supply investments. Lauria has developed two such models, one for solution by linear programming and the other using mixed integer programming, but additional work is needed to improve their efficiency of solution and manageability.*

Field Data

5. Manne's models show that the economy of scale factor of the expansion cost function plays a major role in determining the optimal design period. Additional data are needed to confirm and expand the results already obtained. Specifically, "a" values are needed for different system components including supply, treatment and distribution facilities. In addition, data from several countries should be analyzed to identify regional differences in economies of scale. It also seems desirable to analyze

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* c.f. Lauria, D.T., "The Location, Timing and Scale of Water Supply Investments in Developing Countries", unpublished Ph.D. thesis, University of North Carolina, Chapel Hill, N.C., 1970

engineering cost data to determine optimal planning periods.

6. One of the most difficult studies to be continued is the investigation of water demands. Unlike the work of the previous item, major efforts are needed to collect as wells as analyze data on water usage. As outlined in section 4, the work to be performed includes developing statistical models for demand prediction that relate consumption to measurable community characteristics. Public and unaccounted for usage needs to be more carefully measured and explained. Information is required on rates at which new connections are made. Much more data are needed on demand variation, and studies should be made to determine the economic consequences of designing for various extreme values.

7. The work of Manne and others* shows that a knowledge of the value of water is indispensable for proper budget allocation among alternative projects. The method of imputing described herein is expedient but not completely satisfactory for determining the benefits associated with public supply systems. Consequently, while additional data from previous investments should be collected and analyzed to impute p-values, more rigorous benefit studies should also be started.

Application

8. The goal of this project is to develop the theory and improve the practice of water supply planning in developing countries. As the above items of work progress, attempts should be made to apply these principles and models to specific design situations. Two approaches are possible. The planning models can be used with past data to compare the decisions that would have been made with what was actually done. Alternatively, the models can be used with current data to determine future courses of action. In either case, practical application is needed in order to demonstrate the value of the approach described herein.

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* See, for example, Marglin, S.A., Approaches to Dynamic Investment Planning, North-Holland Pub. Co., Amsterdam, 1963

APPENDIX

Water Supply System Data

Town No.	Cost ($)	Present Population	Design Population	Population Ratio Design/Present	Imputed Price (p)
1	9,180	453	720	1.59	18.5 ¢/M gal
2	17,520	430	800	1.86	37.2
4	13,780	675	1150	1.70	18.7
5	33,890	904	1500	1.66	34.3
6	9,970	350	500	1.43	26.0
7	16,970	815	1110	1.36	19.0
8	12,160	400	800	2.00	27.8
9	24,020	633	950	1.50	34.7
11	11,800	540	1000	1.85	20.0
12	19,750	779	1087	1.40	23.2
13	16,500	726	1000	1.38	20.8
14	19,430	1159	2300	1.98	15.4
15	45,280	1905	3000	1.57	21.7
16	46,250	1920	3840	2.00	22.0
18	59,710	3175	4300	1.35	17.2
19	70,570	1230	2215	1.80	52.4
22	94,220	2645	5300	2.00	32.5
23	21,250	613	1226	2.00	31.7
24	34,170	1129	1850	1.64	27.7
26	15,100	491	800	1.63	28.1
28	42,190	919	1800	1.96	42.0
29	9,260	576	864	1.50	14.7
30	13,700	618	1000	1.62	20.3
31	11,160	950	1292	1.36	10.7
32	11,330	400	600	1.50	25.9
33	4,070	400	600	1.50	9.3
35	17,460	800	1600	2.00	19.9
36	4,270	300	408	1.36	13.0
37	17,290	1200	1800	1.50	13.1
38	12,060	403	800	2.00	27.3
39	14,240	950	1700	1.79	13.7
41	9,850	613	1200	1.96	14.7
43	12,320	459	712	1.55	24.5
45	35,060	822	2147	2.62	39.0
51	31,840	1563	3000	1.92	18.6
52	21,500	1076	1800	1.68	18.3
53	72,120	3875	7500	1.93	17.0
54	23,620	880	1200	1.36	24.5
55	45,660	550	1100	2.00	76.0
56	25,450	672	1150	1.71	34.6
57	2,560	250	500	2.00	9.3
58	2,560	550	1100	2.00	17.2

Town No.	Cost ($)	Present Population	Design Population	Population Ratio Design/Present	Imputed Price (p)
59	10,800	536	938	1.75	18.4
60	8,770	210	420	2.00	38.1
62	15,550	350	700	2.00	40.6
63	14,770	817	1400	1.71	16.5
64	12,840	831	1500	1.80	14.1
66	32,490	666	1300	1.95	44.7
67	15,330	1499	2500	1.67	9.3
68	43,810	1532	3100	2.02	26.1
69	27,250	817	1500	1.84	30.4
70	16,140	400	800	2.00	36.9
71	17,890	761	1400	1.84	21.4
72	25,170	1196	2000	1.67	19.2
73	27,020	800	1600	2.00	30.9
74	25,920	1201	2300	1.91	19.7
75	17,490	840	1300	1.55	19.1
76	21,750	1584	3168	2.00	12.5
77	12,960	783	1580	2.02	15.2
78	8,070	770	1200	1.56	9.6
79	16,760	275	600	2.18	55.7
80	17,840	765	1150	1.50	21.3
81	11,400	446	802	..80	23.4
82	20,240	923	1800	..95	20.0
83	38,050	3912	6200	..58	8.9

Appendix 4

Water Supply Planning in Developing Countries: a general statement

WATER SUPPLY PLANNING IN DEVELOPING COUNTRIES
a general statement

by

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A paper presented at the Annual Conference of the American Water Works Association in Chicago, Illinois, on June 8, 1972

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EXCESS CAPACITY

The business world is often alarmed when production facilities are not used to full capacity. There has been recent concern in Europe, for example, with idle capacity in the textile industry due to imports of synthetic fibers from Asia. Disastrous economic consequences have periodically resulted from unused capacity in hotels, train and transportation facilities, agriculture, and manufacturing.

Alarm over under utilization of production facilities might seem strange to environmental engineers. After all, water supply and sewerage systems are deliberately provided with excess capacity. In the U. S., it is common to design sanitary facilities with sufficient capacity to meet demands for the next 15, 20 or even 50 years. Such design times are actually periods of excess capacity and represent the expected number of years between construction and the time when demand will have grown equal to system scale thus requiring expansion. Rejecting the assumption that water is inherently different, it is important in light of the business world experience to ask why excess capacity is provided in sanitary facilities, particularly water supply systems.

The primary reason is well known to most: when faced with increasing demands over time, excess capacity is provided because of economies of scale. Figure 1 shows a typical cost curve for a water system that reflects such economies. The equation of the curve is $C = kz^a$, where z is project scale in mgd (million gallons per day) and C is cost. The concavity of the function is due to "a", the economy of scale factor, whose value is between 0 and 1. When "a" is 1, costs vary linearly with scale and economies are absent; large economies on the other hand are associated with small values of "a".

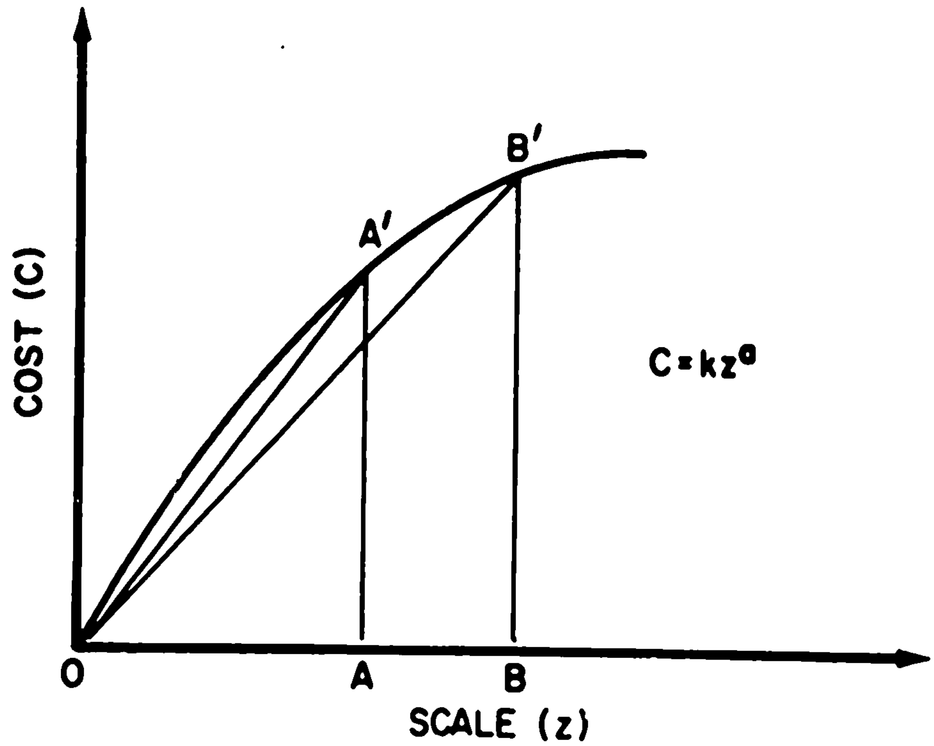

Figure 1. Power Cost Function

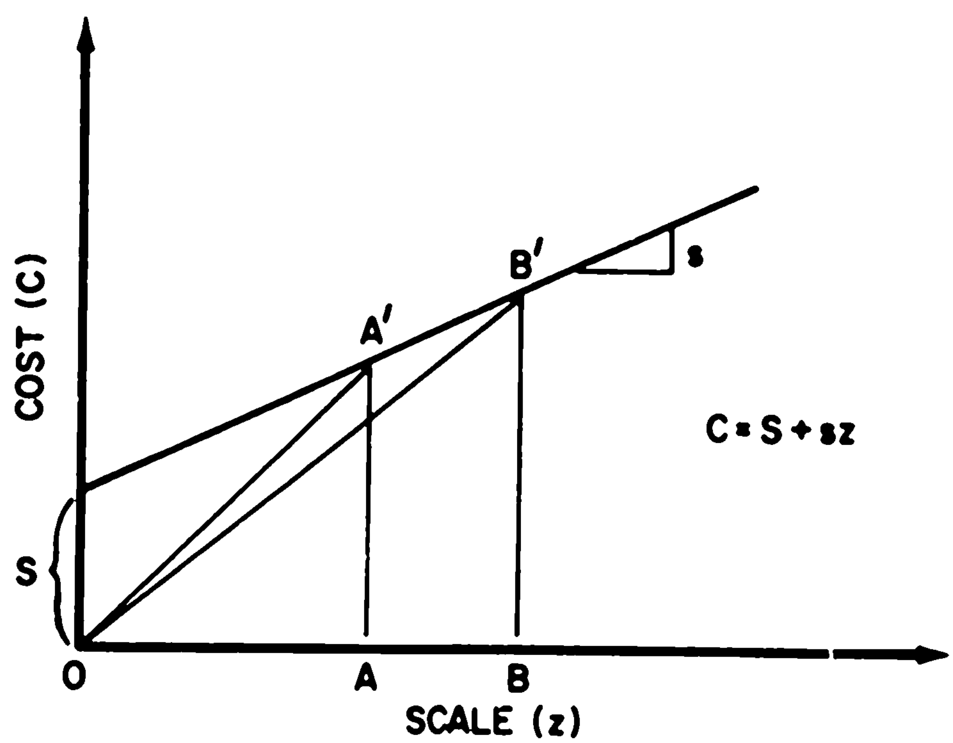

Figure 2. Fixed-Charge Cost Function

The average cost of a project of scale A in Figure 1 is the slope of line segment OA'; similarly, the average cost of B is the slope of OB'. In general, with economies of scale, average costs decrease as scale increases.

When z in the cost function is 1 mgd, C = k. Hence, k is the cost of a 1-mgd system. The significance of "a" is not so apparent. By taking the derivative of the function with respect to z, however, we find that $a = (dC/C)/(dz/z)$. In this form, the economist will recognize that "a" is a measure of elasticity; specifically, the percentage change in cost per percent change in scale.

Another cost function that reflects economies of scale is shown in Figure 2. In this fixed charge function, S is a set-up cost and s is the cost per unit scale, a marginal cost. Again we note that average costs decrease with increasing scale.

Economies of scale in water projects is a phenomenon largely associated with project initiation. Consulting engineers know of the work connected with land acquisition, state approvals, setting up for design, holding public information meetings, referenda, bond issues, having the contractor move onto the site, etc. Once these tasks are done, it makes relatively little cost difference whether the project has, for example, 5 mgd capacity or 6. To take full advantage of the large costs associated with project start-up, it becomes economical to provide capacity beyond that needed for immediate demands.

Manne (1967) and others have developed mathematical models for determining the optimal design period (more accurately called the excess capacity period) for projects involving economies of scale. In their simplest model, one that is particularly appropriate for water supply and treatment systems in the U.S., a linearly increasing demand function as shown in Figure 3 is assumed. The rate of demand increase is D mgd (or mgy - million gallons per year) per year. At time 0, demand and capacity of existing facilities are equal. As is common in U. S. water practice, the capacity of supply facilities is required to equal or exceed demand; hence an expansion must immediately be made. Assuming it will have excess

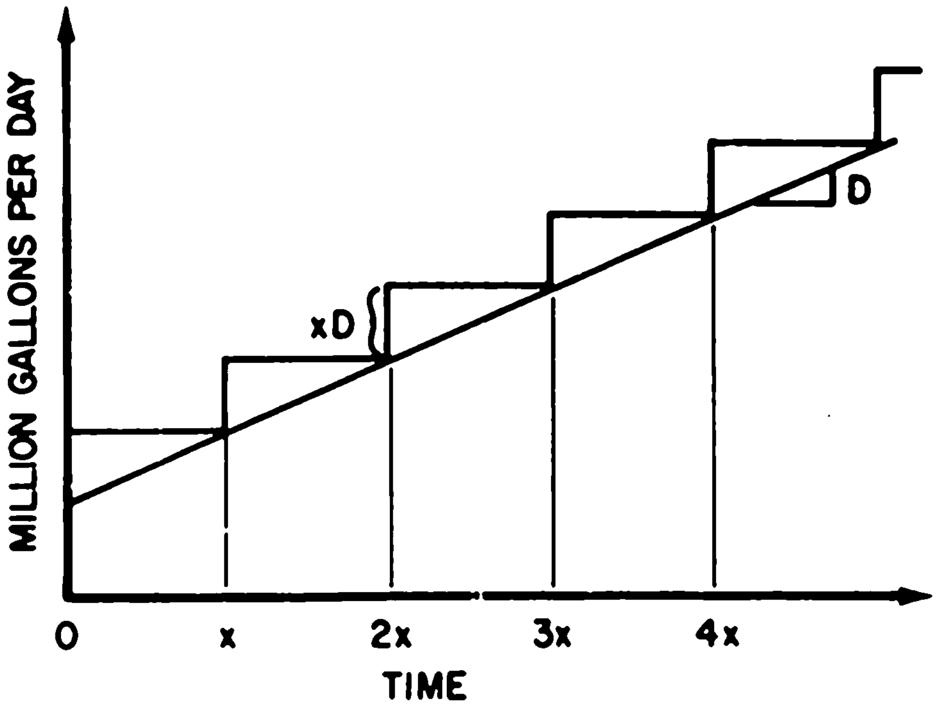

Figure 3. Expansion Model Without Deficits

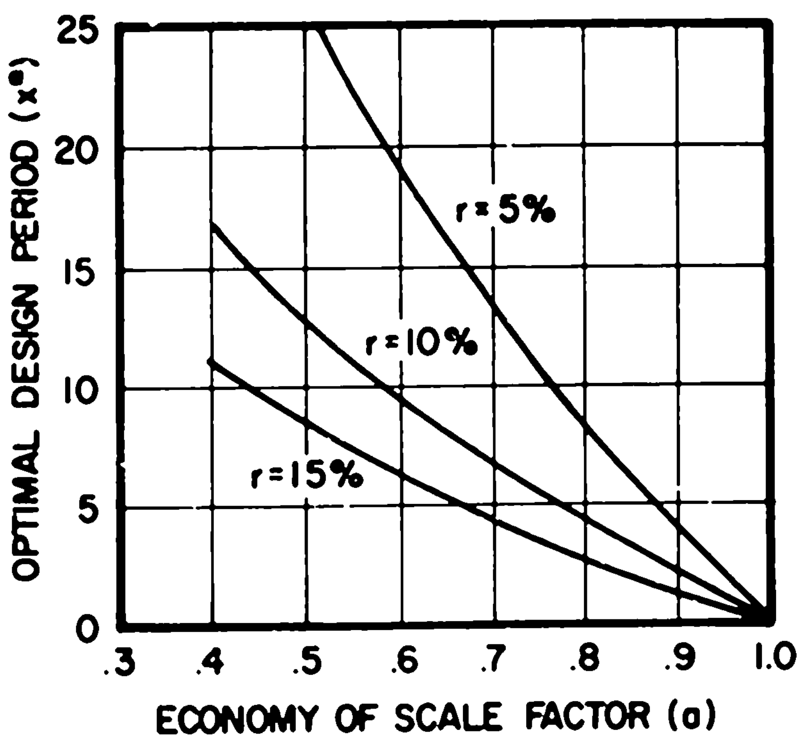

Figure 4. Optimal Design Periods

capacity for x years, its capacity is xD mgd and its cost is $k(xD)^a$ dollars. By always designing for the same excess capacity period, expansions of the same scale are required every x years. Summing the discounted costs of all expansions results in an expression of total present value cost. The optimal design period, x*, that minimized this expression is found by setting the derivative with respect to x equal to zero and solving. The resulting optimality condition for an infinite number of expansions is $a = rx^*/(e^{rx^*}-1)$, where r is the annual discount rate. A cross plot of this equation showing that x* is a decreasing function of "a" and r is in Figure 4.

TIMING

It was notedabove that in U. S. water practice, the capacity of supply facilities is required to equal or exceed demand. It is pertinent to ask why. Specifically, why are deficits in supply capacity disallowed? The answer is that if demands are not met by local facilities, the consequences are assumed to be terribly unpleasant. This however is not necessarily true. A case in point is the Chapel Hill, North Carolina,experience during the summer drought of 1968. The town's reservoir nearly ran dry, water use restrictions were imposed, and finally a connection was made to the City of Durham system for importing water. Once the connection was made, nearly normal living conditions resumed. Had water importing been planned, however, as the town approached zero excess capacity in its reservoir, much anguish could have been avoided.

If importing is a viable alternative to local supply, capacity need not always equal or exceed demand. Manne (1967) and Erlenkotter (1967) have analyzed this situation and a sketch of their model is shown in Figure 5. Assuming supply and demand are initially in balance (i.e., supply facilities have zero excess capacity), water can be imported at price p dollars per gallon for the next y years at which time an expansion is made. At the time of expansion, the rate of

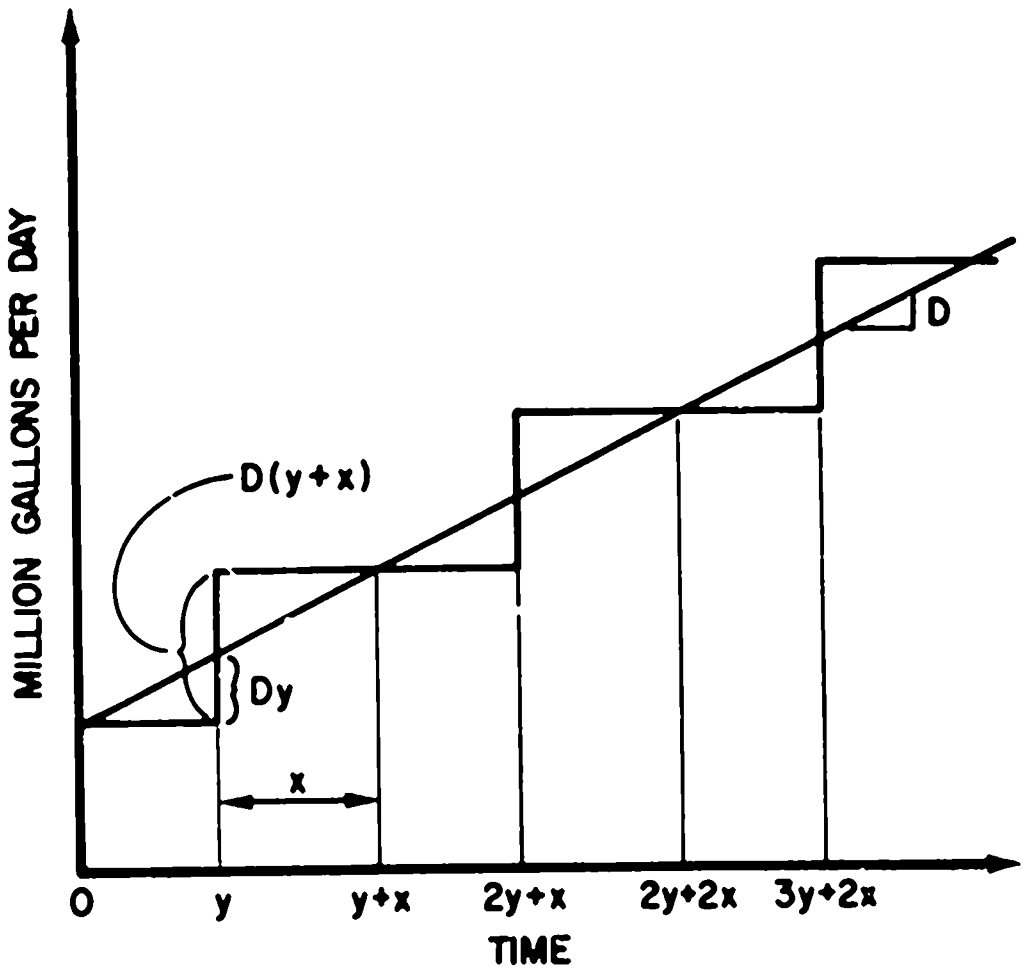

Figure 5. Expansion Model With Deficits

demand satisfied by imported water is Dy. The expansion has capacity $D(y + x)$ mgd, it provides excess capacity for x years and costs $k[D(y + x)]^a$ dollars. By summing discounted import and expansion costs over an infinite time horizon, an expression of total present value cost is obtained in terms of y and x. Setting the appropriate partial derivatives equal to 0 results in expressions for y^*, the optimal waiting period during which there is importing, and x^*, the optimal excess capacity period.

One of the optimality conditions from this analysis is that $pDy = rC$, where C is the cost of each expansion whether it is optimally scaled or not and the other parameters are as previously defined. The units on both sides of this expression are dollars per year. The left hand side is the rate of import charges at the time of expansion and the right hand side is the annual opportunity cost of capital which is similar but not the same as an annual interest charge. In this form, the optimality condition indicates that construction should be delayed until import charges accrue at the same rate as capital opportunity costs.

The optimality expression can be rearranged as follows, $y = rC/pD$. Of importance is the fact that y equals 0 when p equals infinity. But when y is 0, we have a situation identical to that shown in Figure 3 where supply capacity is required to equal or exceed demand. Hence, under the assumptions of this model, a policy that disallows supply deficits implies that the price of importing water from a neighboring community is infinite, a very unpleasant alternative indeed. If however water can be imported at finite price, one would conclude (under the assumptions of the model) that deficits in the supply from local facilities are permissible. In this case it does not automatically follow that expansions should be made when excess capacity is reduced to zero. Hence, the question of optimal timing is raised.

REGIONALIZATION

The majority of water supplies in the U.S. are planned and constructed without much consideration of neighboring systems. Although this situation is changing it is still largely true that if a town feels the need of expanding its water system, it retains an engineer and proceeds with design, financing and construction quite independently. Suppose, however, that two communities share the same source of supply that has inadequate capacity to meet the future needs of both. In this case, neither town can independently proceed with planning. Water taken by one user is denied to the other. The scarce resource, therefore, creates physical interdependencies between the systems and an allocation problem results. We can conclude that towns are able to plan and construct water systems without regard for their neighbors only so long as interdependencies created by scarce resources are absent. The increasing scarcity of water and particularly water quality in the U.S. has much to do with the current trend toward regionalization.

WATER SUPPLY PLANNING IN DEVELOPING COUNTRIES

The discussion so far has focused on water supply planning in the U.S. Now let us put these concepts within the perspective of developing countries. We intuitively know that water supplies abroad reflect economies of scale. Unfortunately, little work has been done to measure such economies. A study by Lauria (1969) of newly constructed gravity supply and distribution systems in Guatemala for small communities revealed an economy of scale factor of 0.77, and a more recent study of systems in Honduras (Lauria, 1971) showed "a" to be 0.85. In neither case did the data properly reflect planning, legal and administration costs which undoubtedly would reduce these values of "a". The main point, however, is that economies of scale exist abroad which means that excess capacity should be provided.

The simplicity of Manne's model shown in Figure 3 tempts one to want to use it for determining optimal design periods in developing countries. The underlying assumptions however are not applicable. Supply capacity, for example, does not equal demand at the start of the planning horizon. Although this is true in the U.S. where the principal planning problem is expansion of existing facilities, it does not hold in developing countries where most water supply planning is for new systems. While Manne's model does not apply abroad, it correctly shows that the optimal excess capacity period is a function of the discount rate which of course is different in developing countries than in the U.S. Based on this fact alone, we can conclude that U.S. design period standards should not be used abroad.

The question of construction timing has been shown to be intimately connected with the value of water. In the cited example, importing is assumed to be the alternative to local supply, and p is the import price. In developing countries, however, importing water from a neighboring community is not an alternative to local supply. What is more, importing is never an alternative to both supply and distribution. The most common alternative to public supply in developing countries is for the bulk of demand to go unsatisfied. In this case, the benefits of publicly supplied water are foregone, and p becomes a measure of the social losses associated with unserved demand.

Under these conditions, p is an opportunity cost rather than a purchase price. Only if publicly supplied water has infinite value will it be economical to require local supply capacity to equal or exceed demand. This however in developing countries is unlikely. Hence, assuming the costs associated with unsatisfied demand are finite, policies disallowing deficits should not be imposed. It does not follow, therefore, that public systems should be immediately built in all towns where they are lacking nor that expansions should be made when excess capacity is reduced to zero.

Water supply systems in the U.S. can be planned independently only where resource allocation problems do not exist. In most developing countries, physical interdependencies of the type caused by water scarcity are absent. On the other hand, economic interdependencies are common which make it necessary to plan water supply systems on a regional basis. In developing countries, water supply planning is performed by a central agency of the national government. The agency is equipped with annual budgets to be allocated among towns in need of systems. Funds invested in one system are automatically denied to the others. Hence, for optimal decision making, groups of systems must be planned simultaneously.

MIP MODELS

The above discussion identifies the basic water supply planning problems of developing countries. National water sector budgets must be allocated among towns needing systems. This involves decisions on when and how much to invest in individual communities. These are problems of construction timing and scale.

In many instances, planning policies can be adopted regarding the amount of excess capacity to be provided in new water supplies. Such policies might not be optimal, but they are expedient and in some cases near optimal. All new systems, for example, might have excess capacity for 10 or 15 years. If this is decided in advance, then the budget allocation problem reduces solely to a question of timing: when should community supplies be built?

It is important to note that, were it not for interdependencies created by budgets, each community could be treated individually as in the U. S., and a model like Manne's could be used to decide optimal timing and scale. The need to consider budgetary trade offs among systems, however, invalidates this approach.

In an attempt to solve the budget allocation problem, regional planning models for use abroad have been developed (Lauria, 1970). The most basic model assumes that project scales have been decided in advance. For model use, a set of communities needing water systems is selected. A finite planning horizon is divided into 1-year periods in each of which water demands for the separate towns are assumed to remain constant. Changes in demands, however, can take place at the beginning of each new year.

In each community, systems are proposed for construction the first year. With scales already decided (based on current and expected future demands), the cost of each proposed project is a known constant. A 0,1 integer variable is associated with each system to denote whether it should be constructed the first year or not. Projects of fixed scale are similarly proposed for other years of the planning horizon (not necessarily every year) for each town, and 0,1 variables are used to indicated whether or not they should be implemented.

If demands are not satisfied by local facilities, they are assumed to go unserved. A continuous variable is therefore included for each town each year to denote the amount of unsatisfied demand. Associated with each such variable is the price of social losses.

The objective function to be minimized is the sum of present value construction and social costs. One constraint set requires the sum of local supply capacity plus unsatisfied demand to equal or exceed demand in each town each year of the horizon. Another constraint set requires annual construction costs to fit within annual budgets, and a final constraint specifies terminal conditions at the end of the planning horizon.

The model that treates scale as variable instead of constant is nearly identical. Fixed charge cost functions are assumed. In this case, the cost of each proposed project has two components, a fixed charge that reflects set-up costs and a variable charge that depends on scale. A 0,1 integer variable is associated with each fixed charge, and the variable charge includes a continuous decision variable for project scale. The objective function is the sum of present value construction and social costs as before. In addition to former constraints, restrictions are included to assure that projects whenever constructed will have sufficient capacity to at least meet existing demands. Additionally, each 0,1 variable is restricted to the value 1 when a project is constructed and 0 otherwise. This in effect makes these variables indicators of optimal construction timing.

A model similar to that described above has been recently presented (Lauria, 1972). This model and the earlier one in which project scales are fixed can be solved by mixed integer programming (MIP). Solution has been obtained using a branch and bound algorithm developed by Shareshian (1969). Computer requirements are extensive and costs are large when a large number of communities is considered. Consequently, the second model in which both timing and scale are variable has been reformulated for separable programming.

WATER SUPPLY BENEFITS

Much has been written about the need for data on water supply benefits in developing countries, but little has been said about how such data would be used for planning. The MIP models, on the other hand, indicate the role of benefit information in planning, but they do not resolve the problem of obtaining such data.

Very little work has been done to estimate the benefits of public water supplies abroad. One of the classic benefit studies was made a decade ago in Puerto Rico (Pyatt and Rogers, 1962). Using concepts from Weisbrod (1961) and Dublin (1930), the researchers sought to express benefits in terms of additional worker income resulting from reduced morbidity and mortality following construction of public water systems. Although much interest in this approach has persisted, the basic problem remains: the physical (i.e., health) effects of public supplies abroad have not been adequately described. Until this is done (which implies long term prospective community studies), estimation of benefits via shadow health costs will not be practicable.

It is generally recognized that the benefits of a good or service can be measured by what one is willing to pay for them. With incomes and the type of imperfect water market that exist in the U. S., few would argue that what has to be paid for water represents willingness to pay. Price, therefore, is not an accurate indicator of water supply benefits in this country. In developing nations, however, it seems that the price of water in rural communities might more accurately reflect ability or willingness to pay. Hence, as a first cut at estimating water supply benefits abroad, there might be value in examining existing price structures. An alternative is to conduct surveys by questionnaire to inquire what people are willing to pay.

Water supplies in developing countries affect more than public health. A property served by a community system should be worth more than one without such service. Buyers are presumably willing to pay for the convenience of water at the front door or in the house. Warford (1972) has proposed that the real estate market abroad be examined to determine the effect public water supply has on property values. This work would require prospective community studies of the type needed to evaluate health effects.

Instead of attempting to measure water supply benefits for use in planning, they can be set by judgement and political fiat. This permits planners to decide the economic implications of their decisions and it allows orderly progress toward a future planning goal that disallows water supply deficits. As shown by Manne's model, smaller deficits are associated with higher water values. By setting annual benefits that increase to some large target value during a period of, say, 20 years, provision would be made for implementing a policy of no-deficits by the end of that period.

Although this proposal does not seem reasonable for larger cities, it might have merit for rural towns. To implement the proposal, it would be desirable to use the current level of benefits as a basis for setting higher values in the future. This can be achieved by imputing water supply values for recently made investment decisions.

An example will illustrate. The optimality condition obtained by Manne and Erlenkotter was reported above to be pDy = rC which can be solved for p as follows, p = rC/Dy. Suppose that a newly constructed water system in a town of 10,000 was thought to be properly sized and timed. The system cost $150,000 (C) which implies an annual opportunity cost (rC) of $15,000 if the discount rate (r) is 10 percent. Assuming the unsatisfied rate of demand (Dy) at the time of construction was 73 million gallons per year (20 gallons per day per capita), the decision to invest implicitly assigned a value (p) of about 20 cents per thousand gallons to publicly supplied water. Similar analyses of other new systems can reveal the current benefit level. It is important to note, however, that instead of using Manne's model that applies only to local planning, the imputing should be done using a regional model of the MIP type.

A variation of the above is to impute water supply benefits for current decisions under consideration and compare the results with intuitive notions of water value. As before, this can make planners aware of the economic implications of their plans. They can then decide to implement or not based on judgement.

For simplicity, the Manne model will again be used although strictly speaking it does not apply. If in the above example investment in the town of 10,000 had not just been made but rather was under consideration, the imputing calculation would show that construction now would assign a value to water supply of 20 cents per thousand gallons. If the planners feel that water in the town is worth less than this, construction should be delayed because by waiting, the ratio rC/Dy decreases for a given design period policy due to economies of scale.

A final comment on water supply benefits is pertinent. The MIP models and the discussion herein have focused on budget allocation. The budget was assumed to be known, but clearly, this is one of the most difficult decision problems in developing countries. With data on the benefits of water supply, however, the problem of budget setting becomes manageable, at least in theory.

NEEDED RESEARCH

The MIP models described herein represent a first attempt at developing a water supply planning model for developing countries. The models need to be made more realistic and extended for specific planning situations abroad. They should be altered to handle among other things price elasticity of demand, uncertainty, and cost functions unknown to the central planning agency. In addition, they should be reformulated if possible for simpler solution. Although work has been started in these areas using such techniques as chance constrained programming, duality analysis and separable programming, much still remains to be done.

More experience is needed with model application. Several problems involving few communities have been solved, but larger problems with perhaps 100 or more towns should be programmed.

Work is needed in developing countries to evaluate the benefits of public water supply. Most basic is the need to determine the physical effects of water works abroad. Once these have been identified, evaluation of economic effects should be quite straightforward. The primary physical effects involve health and economic development.

Other studies should also be made to estimate benefits. These include imputing using data from historical investment decisions, examination of property values, review of water price structures, and surveys to determine willingness to pay.

5. The structure of national water supply planning agencies should be investigated. Specifically, patterns of decision making, the role of political fiat, allocation of planning personnel, design policies and standards, and methods for setting national water supply sector budgets should be examined.
6. The costs of water supply abroad should be analyzed, particularly with regard to identifying economies of scale. Special attention needs to be given to planning, administration and hidden costs.
7. Most of the discussion herein applies to the planning of small water systems in developing countries. Planning in larger cities, however, poses special problems and these need to be examined. As a start, the planning context should be investigated.
8. Of particular importance is the need for accurate demand forecasting abroad. In the U.S., historical water use records are available for planning purposes, but in developing countries, such data are lacking. The problem abroad is to predict future demands in communities that have never had public supply systems, and for this work, innovative methodologies are needed.

CONCLUSIONS

1. The objective herein has been to provide an overview of the most fundamental water supply planning problems in developing countries. The discussion has been simplistic and many of the statements need to be treated with caution. Some important considerations ignored herein include (i) price elasticity of demand, (ii) the separate components of water systems, (iii) water quality, (iv) operating costs, (v) financing, (vi) uncertainty, (vii) planning objectives other than econimic efficiency, (viii) health versus economic development benefits, (ix) water collection from vendors and surface supplies, and (x) selection of communities to be considered for planning.

2. Although the real situation is far more complex, it seems safe to conclude that (i) the amount of excess capacity that should be provided in water systems is largely a function of economies of scale and the discount rate, (ii) construction timing primarily depends on the tension between implementation costs and the value of publicly supplied water, and (iii) regional planning is required abroad because of the need to allocate national water sector budgets.
3. Planning for an objective of economic efficiency requires data on water supply benefits. Three basic approaches can be used to obtain such information. (i) Market studies can be made for measuring benefits. Pertinent markets include labor, real estate, and the water market itself. (ii) Benefits can be set by value judgement and political fiat. (iii) Benefits can be imputed.
4. Much research, theoretical and applied, is needed to improve water supply planning in developing countries. New and better regional planning models should be developed. Studies should be conducted abroad to obtain information on water supply benefits. Current planning practices and agencies should be examined. Basic data should be collected on water supply costs, national water budgets, and community water demands.

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Appendix 5

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