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THE ELASTICITY OF SUBSTITUTION IN DEVELOPING COUNTRIES

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15. Supplementary Notes

16. Abstracts

In recent years a number of efforts have been made to measure substitutability in various sectors of developing countries. Such efforts are handicapped by lack of adequate data as well as by continuing conceptual and measurement difficulties. Enough results do seem available, however, to merit their systematic review at this time. It is the purpose of this report to make such a review as well as to present a series of new estimates. The most common (though not sole) measure of the substitutability of inputs is the elasticity of substitution. It is this measure to which attention is devoted in the present paper.

Part I consists of a quick review of the elasticity of substitution and the various approaches to its measurement. Part II is devoted to a consideration of a variety of estimates of the elasticity of substitution, and in Part III an effort is made to draw some conclusions from the estimates of Part II.

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The Elasticity of Substitution in Developing Countries

The extent of substitutability among inputs is of great importance with respect to a number of issues of concern to the development policymaker. Appropriate wage and employment policy depends on how one views the substitutability between capital and labor. So too does a satisfactory approach to a desired income distribution, to export promotion, to educational planning, to capital utilization, to saving rates required to reach growth targets. Also matters relating to the stability of the economy depend in large part upon how much substitution exists among inputs. In recent years a number of efforts have been made to measure substitutability in various sectors of developing countries. Such efforts are handicapped by lack of adequate data as well as by continuing conceptual and measurement difficulties. Enough results do seem available, however, to merit their systematic review at this time. It is the purpose of this report to make such a review as well as to present a series of new estimates. The most common (though not sole) measure of the substitutability of inputs is the elasticity of substitution. It is this measure to which attention is devoted in the present paper.

Part I consists of a quick review of the elasticity of substitution and the various approaches to its measurement. Part II is devoted to a consideration of a variety of estimates

of the elasticity of substitution, and in Part III an effort is made to draw some conclusions from the estimates of Part II.

I.

In the last decade a large number of estimates of the elasticity of substitution between capital and labor in manufacturing activities in the United States has been made.[1] The primary objective of these studies has been the determination of suitable production function. The specific question has been whether the evidence supported the assumption of an elasticity equal to unity and hence a Cobb-Douglas function, or whether the elasticity is other than unity (but constant) and thereby justifies use of a CES function. This of course is an important question, especially for certain issues, e.g., income distribution. As noted in the opening paragraph the more general question of the existence or non-existence of substitutability has implications for a wide range of policy matters. In these instances, interest is concentrated mainly on whether the elasticity exceeds zero and how high it can be assumed to be. In the present report attention is devoted primarily to these latter issues.

The elasticity of substitution relates the proportional change in the ratio of the wage rate to the cost of capital to the proportional change in the capital labor ratio. Write σ for the elasticity of substitution, K for capital, and L for

labor, w and r for the wage rate and cost of capital respectively, then

$$\sigma = \frac{d(K/L)/K/L}{d(w/r)/w/r}$$

$$\sigma = \frac{d \log (\frac{K}{L})}{d \log (\frac{w}{r})}$$

The most straightforward method of estimating σ would be from the regression equation

$$1) \quad \log (\frac{K}{L}) = \log a + b \log (\frac{w}{r})$$

In this equation b is σ, by definition of the latter. To estimate σ from Equation 1 requires data on capital, and for few countries are such data available.

Fortunately, one can arrive at estimates of σ without capital data, but with rather strong assumptions. If one begins with a CES production function, computes from that the marginal product of labor, sets the marginal product of labor equal to the wage rate, and converts to logs, there results (V is value added)

$$2) \quad \log (\frac{V}{L}) = \log a + b \log w + u$$

and b again is the elasticity of substitution.[3] For b in 2) to be an estimate of σ requires that all observations be generated by profit maximization in competitive labor and product markets, that the price of the product be the same in all markets, that all producers be operating on the same production function, and

that the quality of labor be about the same in all sectors. The CES production function also implies that σ is constant over all values of the capital labor ratio. If capital and rate of return data are available they may be used in corrollary eqation to 2). Thus

$$2a) \quad \log \left(\frac{V}{K}\right) = \log a + b \log r + u$$

and b is the elasticity of substitution if the same assumptions stated for 2) are met.

One may introduce two alternative assumptions into the estimation of Expression 2, one involving lags and the other an assumption about first-order serial correlation in the residuals.[4] The simplest lag model would be the standard Koyck-type, so that

$$3) \quad \log \left(\frac{V}{L}\right)_t = \log a + \sigma (1 - \lambda) \log w_t + \lambda \log \left(\frac{V}{L}\right)_{t-1}$$

when a one year lag is employed. Additional lags might be introduced, but when this is done additional problems appear, especially those having to do with technical change.

On the other hand it may be assumed that Expression 2 holds without lags, but that when estimated there is first-order serial correlation of the residuals due to some (unknown) reason for mis-specification. If this serial correlation is of the form $u_t = pu_{t-1} + u_t$, and this expression is included in Expression 2), then the estimating equation becomes

$$4) \quad \log \left(\frac{V}{L}\right)_t = \log a + \sigma \log w_t + p \log \left(\frac{V}{L}\right)_{t-1} - \sigma p \log w_{t-1}$$

This model implies that the coefficient of the lagged wage rate (the only difference between Expressions 4 and 3) is negative and equal to the product of the coefficients of the other two explanatory variables. If the regression yields a coefficent of w_{t-1} that is in fact approximately equal to $-\sigma p_t$ and significant, then this would be strong evidence of the appropriateness of 4) as the estimating equation.

The elasticity of substitution in United States manufacturing has been computed from both cross section and times series data. The latter data yield uniformily lower estimates of σ. In general it would seem that cross section estimates are a bit more satisfactory, i.e., meet the necessary assumptions more completely. With time series data the chief difficulty is that of correcting for changing technology and for cyclical fluctuations around a trend. As emphasized earlier, the use of cross section data rests on assumptions that wage rates differ while product prices are the same from observation to observation, and technology is the same for all observations. These too are strong assumptions of course, but more easily dealt with than with those necessary for the use of time series. In the discussion to follow results from both time series and cross section data are reviewed.

II.

Three major efforts at estimating the elasticity of substitution in developing countries will be reviewed, those of Williamson for the Philippines,[5] of Katz for Argentina,[6] and of Daniels for eight developing countries.[7] Then some new estimates for Mexico are presented followed by another new set of estimates based on cross country regressions.

a. The eight countries included in Daniels's work are Argentina, Chile, El Salvador, Korea, Paraguay, Peru, Portugal, and Spain. Each country is represented, where possible, with observations for regions or states within the country. The number of observations vary from 13 to 54. Data were converted to United States dollars by means of a purchasing power parity measure. Data apply to various years between 1954 and 1961. The equation fitted for the ith industry is

$$\log \left(\frac{V}{L}\right)_{ij} = a + \sigma \log w_{ij} + u_{ij}$$

where $j = 1 \cdots m$, are the observations in the sample.

The manufacturing sectors of the eight countries are assumed to be similar enough that the assumption of a common production function is reasonable. This of course is a necessary assumption in all cross country analyses. The assumption that wage rate differences among the regions of a country are such that they produce observations along a production function is

a bit more suspect. If regions within a country are small and interregional mobility is great, then the observed differences in wage rates may represent simply random variations that do not affect factor combinations. It is quite probable (though Daniels does not include the data) that wage differences among countries is much greater than within countries. These former differences would then dominate the regression. A similar point holds with respect to product prices. Product prices are much more likely to be the same within the individual countries than among them. This seems likely to be the case even with a fairly satisfactory purchasing power parity measure.

The results of Daniels's calculations are shown in Column 2 of Table 1. The estimated value of the elasticities range from a low of .38 to a high of 1.80. Eight of the 17 are greater than unity. All are significantly different from zero on usual criteria. The R^2's (not adjusted for degrees of freedom) are generally such that one may conclude a fair degree of consistency between the model and the data. Evidently this set of estimates of σ support unambiguously the conclusion that there exists significant substitutability between capital and labor in all manufacturing sectors.

b. Jorge M. Katz has estimated σ for 15 Argentianian industries in 1946 and for 10 industries in 1954. He uses the same equation as Daniels, and his observations are drawn from the 22 Argentine provinces. As not all industries are found in

all provinces, the number of observations varied from 7 to 22. The range of differences in wage rates was 5:1, relatively modest compared to that obtained with cross country data. Presumably product prices are more or less the same among the provinces.

Katz's estimates for 1946 are shown in Column 1 in Table 1. (His estimates from 1954 are generally different category by category, but all are positive and are more than double their standard error. Five of the 10 are greater than unity.) These estimates show a picture very similar to that of Daniels. In 1946 values of σ ranged from .45 to 2.02 and in 1954 from .47 to 1.73. Of the 25 σ's, there are three which are not significantly different from zero at the one per cent level. The R^2's also suggest consistency between data and model. There is considerable variation between the estimates for the two years, but the ranking of the sectors for which estimates are available in both years is very similar. Changes between the years did not seem to follow any pattern as to sectors or direction.

c. Jeffrey G. Williamson estimates σ in the context of a study of the demand for labor in the Philippines. He derives two functions which determine the demand for labor as follows:

$$\text{Log } L_i(t) = \epsilon a - \epsilon \log w_i(t) + \epsilon ct + \epsilon d \log Q_i(t) + (1 - \epsilon) \log L_i(t-1) + \epsilon u_i(t)$$

$$\text{Log } L_i(t) = \epsilon' \sigma' \log \left(\frac{r}{w}\right)_i(t) + \epsilon' a' + \epsilon' c' t + \epsilon' \log K_i(t) + (1 - \epsilon') \log L_i(t-1) + \epsilon' v_i(t)$$

where ϵ is a speed of adjustment coefficient
w_i is the wage rate of workers in Sector i deflated by the price of i
Q_i is value added in the ith sector
d is a measure of returns to scale
K is a measure of capital stock
r is a measure of the user cost of capital

The first equation provides an estimate of σ from the coefficient of the wage rate and the second from the (more theoretically appealing) ratio of the user cost of capital to the wage rate. Williamson is primarily interested in explaining the growth of employment in the Philippines. Both of his estimating equations are consistent with the CES and Cobb-Douglas production functions. They therefore represent methods of estimation similar to that used by the other investigators reported here. Coefficients are estimated from pooled cross section and time-series data for the period 1957-63. Again here it is not clear what the differences in wage rates really measure in the cross section data.

The two sets of estimates are shown in Table 2. For the first equation all but two estimates exceed unity. These two groups are more nearly capital goods producing activities. The same general picture emerges from the second set of estimates with the exception that Group II turns up with a relatively small σ. The fact that the σ's for Groups IV and V are very small supports the notion that the capital goods industries permit less substitution than do the consumer goods industries. This result however is not supported from other studies.

Under any circumstance the conclusion from these data is clear: substitutability between capital and labor in the manufacturing sector as a whole and in all individual sectors, except those in Groups IV and V, is quite marked. It may also be noted in passing that Williamson's estimates of ϵ, the speed of adjustment coefficient, is such that considerable lags are involved in the adaptation of employment to changing output and factor prices.

We now turn to two additional sets of estimates of the elasticity of substitution, one for Mexican manufacturing and the other derived from a cross section of developing countries.

d. Data for capital, employment, value added, and total wage bill in Mexican manufacturing are available on a state basis for the four digit classification of industries for 1960 and 1965, and for two digit in 1965. Employment and capital refer to that prevailing at the end of the year. The capital figure includes "intangibles" and working capital as well as reproducible tangible equipment. Wages and salaries include fringe benefits, and are obtained by dividing the annual wage bill by the number employed. Not all industries are located in all states, and calculations were limited to those that appeared in at least 20. The two key assumptions are: first, the average state of technology is the same from state to state. Thus the technology of the textile industry in the Federal District is the same as that in the Yucatan. This would appear to be a reasonably realistic

assumption. Second, wage rates vary from state to state due to the immobility of labor. As noted earlier, this second assumption is necessary in order that the observed differences in wage rates can be assumed to reflect differing factor supply situations. The assumption that product prices are the same in all states seems to be a realistic assumption.

The availability of capital data makes possible fitting the data to the definitional equation

$$\log \left(\frac{K}{L}\right) = \log a + \sigma \left(\log \frac{w}{r}\right)$$

where the r used is $\frac{VA - w\,L}{K}$. The defects of this latter measure are obvious. If it is assumed that the producing units are regularly equating wage rates with labor's marginal product, then VA - w L is determined residually, and would equal the equilibrium cost of capital services only if the residual equalled that equilibrium value. If, as is likely, producers are better able to maintain employment equilibrium than they are capital equilibrium, then this equation is less satisfactory than that in which the wage rate alone appears. In a really long run equilibrium of course no such problem appears.

In addition the following two regressions were also fitted to the Mexican data:

$$\log \left(\frac{V}{L}\right) = \log a + \sigma \log w + u$$

$$\log \left(\frac{V}{K}\right) = \log a + \sigma \log r + u$$

These three estimates of σ for the two digit categories are shown in Columns 3, 4, and 5 of Table 1. Generally, though not in all cases, the estimates obtained from using the average value added of capital and an estimated rental rate yields the highest σ's and the highest $\bar{R}^2$.[8] More importantly however is the fact that the three equations yield estimates of σ not distressingly dissimilar, and rankings (with a couple of extreme exceptions) are virtually identical. All estimates are significant at the one per cent level. Again it is safe to conclude that these data support a conclusion that substantial substitutability exists between capital and labor in Mexican manufacturing activities. This conclusion is supported further by results, reported below, from the four digit categories.

e. In Columns 6 and 7 of Table 1 are shown estimates of σ obtained from cross country data. Data for value added, employment, and wage rates (wage bill over employment) were taken from the United Nations <u>The Growth of World Industry</u> for a variety of years. Value added and wage rates were converted to dollars by use of the exchange rates (see legend to Table 1 for details).

In Column 6 the estimates of σ are from Equation 4 of Part I, i.e.,

$$\log \left(\frac{V}{L}\right)_{1966} = \log a + \sigma \log w_{1966} + p \log \left(\frac{V}{L}\right)_{1965} - \sigma p \log w_{1965}$$

and in Column 7 from value added of labor on the wage rate. In the seriel correlation model, the sign of the coefficient of w_{65} was negative for all the categories except 35 reported in Table 1. Values for t varied from category to category, but were below 2.0 in only 4 equations. In 5 of the equations the coefficient of w_{1965} differed from σ p by more than .10, of which category 35 was the most flagrant violation. Given all this evidence, it seems that the seriel correlation model describes the data remarkably well. This is especially so, since (as Griliches points out [4]) it is difficult to account for a negative coefficient for w in any other way. Indeed, one would ordinarily expect that coefficient to be positive in the absence of the seriel correlation model.[9]

In Column 7 are shown values of σ estimated from the regression of the average value added of labor and the wage rate. These estimates show no surprises, and all meet the usual tests of significance.[10]

f. In Table 3 one last set of estimates of the elasticity of substitution are offered. These are again for Mexico, this time for the four digit categories of manufacturing in 1960 and 1965. These results are less satisfactory than those for the two digit categories. The three estimates for each year vary markedly among themselves, so that there is less grounds for confidence in the appropriateness of the estimating equation. The adjusted R^2's are also lower in general than are those for

the two digit categories, and this too suggests less consistency between model and data. Finally, there is considerable difference in a number of the sectors between the estimates of σ for the two years from the same equation. While a substantial change in σ in a given sector over five years is not impossible, a halving or a doubling of the estimated value does not add to one's confidence.

Despite all these qualifications the evidence cannot be thrown out. Most of the estimates of σ easily meet strong tests of significance. These results may not lead to a precise conclusion as to the value of the elasticity of substitution for the four digit categories of Mexican industries. We are, however, entitled to conclude that there does exist substitution between capital and labor in these categories.

The estimates of σ from the two digit categories generally exceeds the corresponding estimate from the four digit categories. Only in Sector 20 (Food) is the σ from the two digit less than any of the five four digit estimates, and this only in the regression of V/K on r. Only in three other instances (two for the V/K on r equation and one for the V/L on w equation) is the four digit estimate lower. These results support the suggestion of Solow[11] that the σ from two digit categories should exceed those from four digit because of the greater possibility of product substitution (induced by factor prices) in the former than in the latter. In a large number of instances the two digit value was much larger, suggesting that factor substitution via differing product mix is of considerable importance.

III.

What can be made of all this? One can easily question the quality of the data. So too can one find reasons to believe that the assumptions necessary for the regressions to produce the desired parameter estimate are not met. Still the exercise is far from empty. The data for the Philippines employed by Williamson (Table 2) and that for the United States used by Griliches (Column 8, Table 1) are open to less objection than those used in the other estimates. Yet the results obtained from the various sets of data look very similar. Values of R^2, of t, and of σ for Philippine and United States data behave very similar to those for the other countries.

The results do indicate that there is considerable substitutability between capital and labor. Only in isolated instances do the results indicate that σ may be zero. Even the stronger statement that the elasticity of substitution in developing countries is at least .50 is defensible. In many activities, it is evidently much greater.

The seriel correlation model (Columns 6 and most of 8 in Table 1) yields the most convincing estimates of σ. This is true in part because the model is appealing on a priori grounds. It is true in part because the model provides a test that can hardly be met unless the model is valid, and all the results shown in Column 6, Table 1 meet that test. Despite the questions surrounding the use of exchange rates to convert to a common

currency, the use of cross country data is more satisfactory than within country data as the variation in wage rates is so much larger in the former than in the latter case. Variations in technology from country to country in the sample used here are probably relatively minor. The major question has to do with the similarity of product prices from country to country, but even here differences do not seem great enough to defeat the model.[12] At the same time it should be noted that t values for σ from this model are generally smaller than those for the σ's obtained from the other equations. Also the estimate of σ from the regression of V/L on w seems somewhat more satisfactory than that from V/K on r, simply because measures of both r and K are less adequate than those for L and w.

The results also enable us to say something about where, in the economy, the greatest flexibility in choice of techniques is to be found. As shown in another paper,[13] employment growth is higher, for given rates of growth of productivity and wage rates, the higher is the σ. The estimates of Column 6, Table 1 show sectors 20, 22, 25, 26, 29, 30, 37, and 38 with σ in excess of unity. In these eight sectors, a rapid growth of productivity and output with wage rates constant would produce an even more rapid growth of employment. In the search for increasing labor intensity, these sectors stand out as prospective leaders.

From a comparison of the σ's for manufacturing activities in the United States and developing countries one important conclusion emerges. The figures in Column 8, Table 1 are higher

than those in the other Columns more often than they are lower, and sometimes markedly higher, e.g., categories 33 and 35. On the other hand, one could not conclude from Table 1 that the manufacturing sectors of the developing countries are less responsive to relative factor prices than are those sectors in the United States. Indeed, this evidence suggests that, by and large, the responsiveness is about the same in both.

The most general conclusion is the most important: Factor substitutability is alive and well in developing countries. Policies and models that assume otherwise mislead.

Table 1 ELASTICITIES OF SUBSTITUTION IN DEVELOPING COUNTRIES

			Mexico			Cross Country		U. S.
	1	2	3	4	5	6	7	8
	Argentina	Daniels	w/r	w	r			
20	1.35 (.25) .50	.75 (.08) .74	.69 (.06) .80	.80 (.07) .78	.80 (.03) .95	1.14 (.39) .96	.91 (.07) .89	.91* (.10) .69
21		1.35 (.23) .54	1.19 (.10) .84	1.22 (.07) .92	.86 (.07) .85	.81 (.32) .97	.98 (.06) .92	
22	1.76 (.30) .85	1.49 (.21) .82				1.17 (.47) .96	1.01 (.11) .80	1.09 (.54) .75
23	.98 (.19) .76	1.01 (.07) .84	.69 (.08) .69	.73 (.09) .67	.84 (.06) .88	.59 (.41) .98	.94 (.04) .96	.63 (.21) .90
24		.79 (.14) .36	.71 (.04) .89	.66 (.05) .86	.88 (.03) .95	.58 (.26) .99	.91 (.05) .93	1.17 (.20) .96
25	.93* (.18) .53	.86 (.08) .68	.72 (.07) .78	.74 (.07) .76	.85 (.05) .90	1.18 (.31) .98	.92 (.04) .97	1.04* (.05) .95
26		.80 (.16) .51	.74 (.06) .83	.78 (.06) .84	.87 (.05) .92	1.78 (.33) .98	.97 (.05) .95	1.67* (.30) .52
27	1.49 (.80) .42	1.34 (.16) .78	1.23 (.07) .94	1.33 (.06) .96	1.08 (.06) .95	.65 (.29) .98	.94 (.07) .90	.68 (.28) .86
28	.87 (.10) .72	.82 (.08) .71	.63 (.08) .67	.75 (.09) .70	.75 (.05) .88	.83 (.25) .98	.93 (.04) .95	.70 (.40) .67
29	.87 (.10) .76	.53 (.10) .40	.70 (.06) .84	.64 (.08) .67	.89 (.03) .97	1.06 (.25) .97	.94 (.04) .96	
30	.92 (.17) .90	1.31 (.13) .86	.69 (.08) .72	.77 (.09) .69	.79 (.04) .93	1.11 (.17) .98	.95 (.05) .95	.90 (.43) .85
31	.90 (.19) .56	1.09 (.11) .70	.92 (.12) .68	.97 (.12) .71	.91 (.06) .88	.91 (.15) .77	.66 (.15) .45	1.16 (.40) .85

32	2.02 (.06) .30							
33	1.19 (.11) .85	1.11 (.07) .85	1.10 (.08) .86	1.09 (.08) .86	1.07 (.06) .91	.57 (.45) .97	.95 (.04) .96	2.37 (.47) .89
34		1.80 (.38) .58				.97 (.54) .82	.98 (.23) .52	1.20 (.28) .74
35	.87 (.22) .49	.97 (.18) .57	.93 (.04) .93	.92 (.04) .93	.97 (.04) .96	.02 (.12) .73	.36 (.13) .26	2.00 (.29) .87
36			.76 (.05) .90	.77 (.05) .88	.82 (.04) .94	.57 (.20) .99	.92 (.03) .98	1.53 (.53) .59
37	.45 (.16) .58	.38 (.16) .15	.73 (.04) .90	.66 (.05) .83	.87 (.03) .97	1.21 (.28) .99	.92 (.05) .94	.96* (.55) .11
38	.46 (.20) .30	.47 (.10) .47	.96 (.04) .94	.93 (.05) .92	.99 (.03) .97	1.06 (.33) .98	.90 (.01) .88	.75 (.43) .26

In each column the first figure in the set of three is the estimate of the elasticity of substitution and the second is the standard error of that estimate. In columns 1 and 2, the third figure is the coefficient of determination, and in the remaining columns it is the coefficient of determination corrected for degrees of freedom.

Column 1. Estimates for Argentina for 1946 from Jorge M. Katz, Production Functions, Foreign Investment and Growth, North-Holland, Amsterdam London 1969, p. 47. Estimated from $\log \frac{V}{L} = \log a + \sigma \log w + u$. Categories 25 and 26 were combined by Katz as were categories 36 and 37.

Column 2. Mark R. Daniels, "Differences in Efficiency among Industries in Developing Countries", The American Economic Review 59, March 1969, p. 165. Estimates based on data from eight countries. See text for details. Estimated from $\log \frac{V}{L} = \log a + \sigma \log w + w$.

Columns 3 - 5. Estimates of for 1965 for Mexican industry calculated from following equations:

Col. 3 $\log \frac{K}{L} = \log a + \sigma \log \frac{W}{\ } + u$

Col. 4 $\log \frac{V}{L} = \log a + \sigma \log w + u$

Col. 5 $\log \frac{V}{K} = \log a + \sigma \log w + u$

Data from which all calculations are made are taken from VIII Censo Industrial, Resumen General 1965, compiled by the Director General of Statistics of the Ministry of Industry and Commerce, Mexico City 1966.

Col. 6 - 7. Estimate of using data from individual countries converted to United States dollars by exchange rates. Data for value added, employment, and wage rates are taken from The Growth of World Industry 1967 and 1968 editions, United Nations, New York 1969 and 1970. Exchange rate used are from various issues of the international Fianacial Statistics published by the International Monetary Fund. The list of countries included in each regression is available on request. The same

set of countries could not be used in each equation. Equations used were

Col. 6 $\log \frac{V}{L}_{1966} = \log a + \sigma \log W_{1966} + p \log \frac{V}{L}_{1965} - \sigma p \log W_{1965}$

Col. 7 $\log \frac{V}{L}_{1966} = \log a + \sigma \log W_{1966} + u$

Equations were estimated for Sector 32 (Petroleum products) but yielded in most instances nonsense results.

Col. 8 Estimates of the elasticity of substitution for United States manufacturing activities from Griliches, in the source cited in footnote 4. Estimates are for 1958 with states as observations. Estimates from the seriel correlation model are shown unless they were inconsistent with other regressions in which case the estimate from labor productivity and the wage rate are shown. The latter are marked with an asterisk. The R^2's are not adjusted for degrees of freedom by Griliches.

TABLE 2

Estimates of the Elasticity of Substitution in Philippine Industry

Industry Group		Estimate of σ from text			
		Equation 1	R^2	Equation 2	R^2
I	20, 21, 22	1.82	.94	1.22	.94
II	23, 24, 29	1.01	.96	.35	.88
III	25, 26	1.14	.96	1.15	.94
IV	30, 31	.62	.97	.34	.89
V	34-38	neg	.82	neg	.75
VI	27, 28, 33, 39	1.24	.95	1.34	.94
All manufacturing		1.59	.94	.89	.90

Source: Williamson [5], pp. 56 and 58-59. For Equation 2, Williamson computes another set of regressions excluding capital gains from the w/r ratio. These equations yield lower estimates of σ than those shown here.

TABLE 3

Elasticity of Substitution in Mexican Manufacturing Activities

	1960			1965		
Category	w/r	w	r	w/r	w	r
2025	.52	.61	.80	.29	.33	.85
	(.05)	(.07)	(.04)	(.06)	(.06)	(.07)
	.76	.69	.93	.38	.46	.82
2052	.06	1.15	.57	.25	.49	1.09
	(.04)	(.35)	(.10)	(.07)	(.05)	(.05)
	.03	.25	.48	.30	.77	.95
2061	.53	.66	.84	.62	.67	.88
	(.05)	(.09)	(.03)	(.07)	(.05)	(.07)
	.80	.61	.97	.73	.85	.82
2093	.55	.59	.86	.59	.51	.96
	(.05)	(.07)	(.03)	(.08)	(.07)	(.04)
	.78	.69	.96	.61	.59	.95
2096	.51	.81	.84	.15	-.19	.98
	(.15)	(.18)	(.07)	(.02)	(.99)	(.005)
	.27	.40	.82	.64	-.03	.99
2141	.57	.66	.81	.94	1.06	.94
	(.09)	(.11)	(.04)	(.16)	(.10)	(.06)
	.56	.53	.92	.53	.80	.90
2411	.27	.39	.68	.59	.62	.80
	(.08)	(.14)	(.05)	(.09)	(.11)	(.06)
	.28	.22	.87	.63	.53	.88
2421	.16	.24	.69	.31	.33	.88
	(.05)	(.08)	(.07)	(.06)	(.05)	(.06)
	.21	.20	.78	.47	.57	.88
2431	.18	1.08	.28	.67	.62	.86
	(.09)	(.21)	(.12)	(.04)	(.05)	(.03)
	.07	.45	.13	.90	.84	.96
2441	.41	.62	.66	.53	.42	.82
	(.12)	(.14)	(.08)	(.06)	(.06)	(.04)
	.32	.43	.74	.74	.60	.92
2533	.30	.51	.80			
	(.08)	(.09)	(.07)			
	.33	.51	.81			

Category	w/r	w	r	w/r	w	r
2611	.59 (.07) .68	.74 (.09) .69	.70 (.05) .85	.75 (.06) .85	.78 (.06) .92	.86 (.05) .92
2617	.32 (.09) .45	.33 (.17) .16	.70 (.08) .84			
2813	.16 (.11) .03	1.00 (.17) .54	.27 (.10) .16	.50 (.07) .64	.58 (.08) .66	.66 (.07) .75
2921	.24 (.07) .26	.53 (.15) .34	.65 (.07) .77	.54 (.06) .74	.50 (.07) .67	.84 (.04) .93
3011	.35 (.05) .56	.18 (.25) -.02	.42 (.14) .20	.66 (.05) .85	.63 (.06) .77	.84 (.03) .96
3145	.55 (.09) .59	.80 (.15) .52	.79 (.04) .95	.59 (.06) .80	.55 (.10) .57	.84 (.04) .95
3312	.59 (.08) .69	.71 (.12) .56	.72 (.05) .90	.91 (.05) .90	.88 (.06) .88	.97 (.03) .97
3354	.46 (.09) .50	.70 (.24) .23	.84 (.05) .92	.84 (.10) .74	.80 (.11) .65	.93 (.06) .91
3355	.69 (.07) .74	.86 (.08) .78	.75 (.06) .85	.97 (.08) .82	1.00 (.06) .89	.93 (.07) .86
3511	.41 (.08) .48	.45 (.15) .24	.82 (.03) .95	.75 (.06) .85	.68 (.08) .75	.89 (.04) .95
3520	.48 (.06) .68	.64 (.09) .64	.54 (.04) .87	.75 (.04) .90	.74 (.05) .87	.86 (.03) .95
3617	.22 (.06) .26	.43 (.14) .23	.19 (.08) .14	.71 (.05) .86	.73 (.05) .88	.85 (.05) .90

Category	w/r	w	r	w/r	w	r
3724	.28	.39	.61	.44	.41	.81
	(.05)	(.11)	(.04)	(.04)	(.04)	(.05)
	.52	.25	.87	.78	.79	.90
3726	.30	.11	.63	.47	.30	.85
	(.07)	(.17)	(.04)	(.07)	(.06)	(.04)
	.40	-.02	.90	.63	.46	.93
3841	.18	-.21	-.27	.62	.64	.97
	(.06)	(.47)	(.18)	(.08)	(.06)	(.06)
	.22	-.03	.04	.63	.77	.89
3853	.12	.51	.59	.44	.24	.95
	(.07)	(.18)	(.05)	(.07)	(.06)	(.02)
	.06	.21	.84	.58	.33	.99

Note: For each year the column headed w/r refers to results from the regression of K/L on w/r, that headed w to results from the regression V/L on w, and that headed r to results from the regression V/K on r.

References

1. An excellent review and evaluation of these many estimates is Marc Nerlove, "Recent Empirical Studies of the CES and Related Production Functions" in The Theory and Empirical Analysis of Production, Studies in Income and Wealth, Volume 31, Murray Brown, editor, Columbia University Press for the National Bureau of Economic Research, New York, 1967.

2. If one allows for the elasticity of substitution to change as the capital labor ratio changes, more complex measurement problems emerge. See T.C. Liu and G.H. Hildebrand, Manufacturing Production Functions in the United States, Cornell University Press, Ithaca, New York, 1965.

3. In the original derivation of this, K.J. Arrow, H.B. Chenery, B.S. Minhas, and R.M. Solow, "Capital-Labor Substitution and Economic Efficiency," The Review of Economics and Statistics, 1961, 3 (August 1961), pp. 225-250 begin with Expression 2, found it empirically valid, and then derived the production function which implied that relationship.

4. For a full discussion see Zvi Griliches, "Production Functions in Manufacturing: Some Preliminary Results," in the same volume as the Nerlove paper cited in the first reference. The next two paragraphs in the text rely heavily on Griliches's arguments.

5. Jeffrey G. Williamson, "Capital Accumulation, Labor Saving, and Labor Absorption Once More," Quarterly Journal of Economics, February 1971, 85(1), pp. 40-65.

6. Jorge M. Katz, Production Functions, Foreign Investment and Growth, North-Holland Publishing Company, Amsterdam and London, 1969.

7. Mark R. Daniels, "Differences in Efficiency Among Industries in Developing Countries," American Economic Review, March 1969, 59(1), pp. 159-171.

8. Phoebus J. Dhrymes, "Some Extensions and Tests for the CES Class of Production Functions," The Review of Economics and Statistics, November 1965, 47(4), pp. 357-366 suggests reasons why this result might be expected.

9. Equation 3 of Part I was also fit to these data, but the results were generally nonsense.

10. Estimates of σ for other years from the cross country data were also prepared. Estimates of σ from the seriel correlation model (1963/64) were quite different from those shown in the table in Categories 20, 21, 33, 35-38. Using labor productivity-wage rate equation, values of σ for the years 1963-66, only categories 31, 34, 35 showed change of more than a point or two.

11. Robert M. Solow, "Capital, Labor, and Income in

Manufacturing," in The Behavior of Income Shares, Princeton, New Jersey for the National Bureau of Economic Research, 1964, pp. 101-128.

12. Daniel M. Schydlowsky and Moises Syrquin, "The Estimation of CES Production Functions and Neutral Efficiency Levels Using Effective Rates of Protection as Price Deflators," Discussion Paper Number 212, Harvard Institute of Economic Research, Cambridge, Massachusetts, November 1971 computed regressions V/L on w directly and then sought to use effective rates of protection to make prices uniform. They found little difference between the two estimates of the elasticity of substitution.

13. Henry J. Bruton, "Employment Productivity and Import Substitution," Research Memorandum 44, March 1972.