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POLICY WORKSHOP PROPOSAL ON METHODS OF MATHEMATICAL PROGRAMMING WITH APPLICATIONS TO AGRICULTURAL POLICY ANALYSIS

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Agricultural Policy Analysis Project

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Objectives

1. To provide an understanding of the types of policy applications that can be made with programming models.
2. To provide training on how to integrate economic concepts into programming models.
3. To provide an acquaintance with some of the practical aspects of building an applied agricultural policy model.

Procedures

The workshop is organized into ten three-hour sessions. Each session, there is a break in the middle. Depending on the circumstances, there could be one or two sessions a day. For busy government staff members, for example, it might be best to have only one session per day, from eight to eleven in the morning or from three to six in the afternoon. Another option is to open the workshop with two days of two sessions each, and then drop to one session per day; in this way the workshop would occupy eight days in total.

In all of the sessions, emphasis is placed on numerical examples and tableaus, in an effort to show how the models are built in a practical sense.

Outline

Session No.	Contents
1	Introduction. Purposes and scope of the workshop. Methods of presentation of the material. Why optimizing models? Farm models and sector models. The sector as an economic unit. Normative vs. positive optimizing models. Sketch of some kinds of policy applications. The tableau. Tableau of a simple farm model. Algebra of the simple model.
2	Duality and the farm model. What are the Kuhn-Tucker conditions? Issues concerning the structure of the farm model: technologies, seasonality of input requirements. Monetary costs and opportunity costs.
3	Simple input supply functions. Marginal costs vs. average costs. Entering the model into the computer, part I.
4	Entering the model into the computer, part II. Interpreting the solution. Further issues in farm modelling.
5	Risk in the farm model. Empirical examples of risk modelling.
6	Introduction to the sector model. Models of multiple representative farms. Consumer demand functions. The market equilibrium. Computation of demand parameters. Shadow prices in the sector model. Geometry of the equilibrium solution.
7	Marketing and processing costs. Foreign trade activities. Subsidies. More policy applications.
8	Investment activities in the sector model. Applications to investment questions.
9	Entering a small sector model on the computer.
10	Review and discussion. Recommendations for improvement of the workshop and for future workshops.

Audience

The workshop is aimed at an audience with training to the master's level or beyond. It presupposes some elementary familiarity with what an LP model is, but it does not require any real working knowledge of these models. The workshop is suitable for counterparts working in planning and policy division of the ministries of Agriculture and other government agencies.

After completing the workshop, participants should be able to manage their own models, referring to references given in the workshop. In the latter part of the workshop, the participants would be putting small models onto micro computers.

References

The basic reference for the workshop is a manuscript by Peter B. R. Hazell and Roger D. Norton and soon to be published by Macmillan titled Mathematical Programming for Economic Analysis in Agriculture. Distributed with this proposal are Chapters 1 and 12 of the book plus the detailed Tables of Contents found at the beginning of each chapter.

Budget

For information on cost of the proposed policy workshop please contact:

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June 1985

MATHEMATICAL PROGRAMMING FOR ECONOMIC ANALYSIS IN AGRICULTURE

Peter B. R. Hazell and Roger D. Norton

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MATHEMATICAL PROGRAMMING FOR ECONOMIC ANALYSIS IN AGRICULTURE

Peter B. R. Hazell and Roger D. Norton

Chapter 1. INTRODUCTION

1.1 THE PURPOSE AND SCOPE OF THE BOOK

Mathematical programming has been used in agricultural economics for more than thirty years. It has become such a useful tool of analysis that its basic principles are taught in all colleges of agricultural economics, and its applications have been spreading geographically, particularly in the last decade. Programming models for agriculture have been used in a large number of developed and developing countries.

In the last ten to fifteen years there also have been a number of methodological advances in this field. The improvements have been in the direction of incorporating more economic theory, and observed institutional and economic reality, into the models. The more noteworthy advances have occurred in the areas of modelling consumer demand, market equilibrium in both product and factor markets, risk and risk aversion, and the role of instruments of economic policy. Also, the profession's ability to model decisions of the farming household has improved as well. The cumulative effect of these advances has been to provide a tool of analysis which is much more adaptable to different situations and potentially a more realistic portrayal of agricultural reality.

This book draws together many of the recent methodological contributions in the field, and it also provides a reasonably comprehensive review of the state of the art as regards agricultural programming models at the farm and the sector levels. Two principal aims have guided the writing of the book: to provide a textbook of the relevant theory and methodology, and also to provide detailed guidelines for practitioners regarding the process of building and implementing these models. To be useful, a model has to be well grounded in theory, but it also has to fulfill many practical requirements. It has to be appropriate to the problem at hand and to the available data, it requires an appropriate institutional framework, and the economics has to be expressed in the model in an appropriate and interpretable way.

Models provide the link between economic theory and data, on the one hand, and practical appreciations of problems and policy orientations, on the other. They are imperfect abstractions, but by virtue of their logical consistency frameworks they can provide the analyst and policy maker a valuable economic representation of the sector and a laboratory for testing ideas and policy proposals. However, experience has shown that the construction of a model requires not only a grasp of the relevant economics and an understanding of the issues to be addressed, but also a familiarity with sound techniques of building and applying models. Yet these techniques are rarely stressed in university curricula, and for agricultural programming models there are very few workshops that the practitioner can attend. Given the frequency with which agricultural

models are built, and the fact that they usually are fairly complex constructions, this gap is an important one. Hence a central focus of this book is on technique, both for building and for applying models. A subsidiary aim is to provide an appreciation of the range of possible applications that have and can be made with these models.

The book is intended for students, practitioners, and researchers. It is suitable for individual reading and for use in university courses. The theory and techniques presented are usable in widely differing situations, in both industrialized and developing countries. Existing texts give little attention to developing countries, so we have used a number of illustrations from case studies in those countries.

The presentation is generally aimed at the level of the master's student who has some familiarity with quantitative techniques. A few sections are more accessible to the Ph.D. student. The mathematical proofs in those sections may be omitted without loss of understanding of either the economics or the model-building procedures. The level of economic and mathematical preparation required includes microeconomic theory, introductory calculus and linear algebra, except for the above mentioned proofs which require multivariate calculus. Chapters 6 and 10, which deal with risk, also require some knowledge of basic statistics. For the student interested in a concise presentation of the relevant mathematical theory, a self-contained appendix is provided which develops the mathematics of linear programming with emphasis on economic interpretations. On

the other hand, the reader who is looking for an introduction to linear programming for farm models will find all the material in chapters 2 and 3 accessible without much mathematics. The same can be said for chapters 11 and 12 which concern practical techniques for building and using sector models.

The book is organized in three parts: I) the farm model, II) the sector model and III) applications for policy analysis. The first part (chapters 2 to 6) lays the microeconomic foundations for most of the rest of the book, with emphasis on modelling farm-level production functions and farmers' objectives. It also introduces the ways of organizing information in a linear programming framework, and discusses the principles and procedures for obtaining and interpreting a model's solution. Part II provides the model framework at the sector level, along with some additional theory for modelling market equilibrium and practical procedures for model construction and validation. Part III discusses the principles underlying useful applications and gives many examples of applications.

In algorithmic terms, the material is written for use in the context of either linear or nonlinear programming algorithms. Common examples of the latter are quadratic programming and mixed integer programming. However, for the treatment of risk, consumer demand and the objective function, emphasis is placed on linear programming, because it still is the most widely available algorithm and because it has some advantages in interpretability and it is amenable to extension to the general equilibrium case. As the material in this book shows, linear programming models are not necessarily linear in the economic behavior they represent.

1.2 THE AGRICULTURAL PROGRAMMING MODEL

Mathematical programming in agriculture had its origins in attempts to model the economics of agricultural production, including its spatial dimension. The mathematical programming format -- sometimes known as process analysis or activity analysis -- is a particularly suitable one for agriculture. Farmers, agronomists, and other agricultural specialists share a common way of thinking about agricultural inputs and outputs, in terms of the annual crop cycle and about input-output coefficients per acre or hectare or other unit of land. Yields are conceived of in tons or bushels per land unit, fertilizer applications in kilograms per hectare or like units, etc. In farm-level cost-of-production studies, input costs typically are disaggregated into labor, machinery services, draft animal services, fertilizer costs, other chemical costs, credit costs, etc., per land unit. For this way of visualizing agriculture production in numbers, it is but a short step to forming the column vectors of inputs and outputs that constitute the backbone of the programming model.

Similarly, agriculturalists often pose their problems in terms of inequality constraints, such as upper bounds on seasonal resource availability. And they are accustomed to the existence of slack resources in some seasons while the same resources are fully utilized in other seasons. This kind of thinking fits naturally into an analysis via programming models. For these reasons, experience has shown that it is feasible to review the production coefficients of a model directly with field experts, and to either verify or revise them accordingly. While the model has a mathematical expression,

much of its empirical content is accessible to experts in other professions.

Thus the programming model provides a rather natural framework for organizing quantitative information about the supply side of agriculture, whether at the farm level or the sector level. Indeed, one of the uses of a model is to help reconcile initially inconsistent data (as discussed in chapter 11).

Other uses of the model often involve different kinds of sensitivity analysis. At the farm level, the model can be useful in calculating the implications of different resource endowments, different market conditions, improved or new technologies, etc. This kind of information is generated by the model via variations in parameter values, with a new solution obtained for each set of parameter values.

At the sector level, parametric variations can be used to generate response functions which are implicit in the model's structure. Examples are factor substitution surfaces, supply response functions, response functions associated with particular policy instruments, and so forth. When used in this way, the model becomes a device for translating micro-level (farm-level) information into macro-level (sector-level) functions that are more familiar to many economists. At both the farm level and sector level, the model's solution also assign valuations to fixed resources, such as land and water supplies, whose price may not reflect their economic value.

It may be asked why the supply response functions are not taken from econometric studies. At the sector level, a set of estimated

supply and demand functions can indicate equilibrium levels of production and prices, toward which the sector would tend to move. Supply functions are estimated in agriculture, and of course they can be quite useful in understanding the sector's behavior. The main difficulties with relying only on econometrics are two-fold: data difficulties and changes in underlying economic structure. The data problem arises from the fact that, in many cases, large numbers of crops compete for the available fixed resources, and therefore cross-supply effects are important components of the supply functions. Normally there are not enough degrees of freedom in a time series data set to estimate both own and cross-supply elasticities. This is especially true in developing countries. Also, aggregate time series on production often are quite unreliable, again especially in developing countries. The programming model relies on cross-sectional farm budget data, which usually are more reliable, and other micro-level information to generate supply functions.

The question of changes in economic structure applies especially to technologies of production, market opportunities, and prices. Public policies can influence all of these, and in the policy options analyzed with the model, policy instruments may need to take on values lying outside the range of values observed historically. This possibility can make it unwise to base policy analyses on extrapolations from historically-estimated parameters. Also, programming models can be used to analyze the consequences of direct changes in economic structure, such as those which would arise from the introduction of new crop varieties, or from land reform which changes the size

distribution of farms. The consequences of these kinds of changes are difficult to capture in econometric models of supply, and yet failure to do so means that the estimated supply elasticities are not reliable if the structural changes are introduced.

In addition to these considerations, a programing model's supply functions provide information on associated responses of inputs, such as labor, agrochemicals, and the like. Seasonality of the responses also is taken into account. Thus the study of supply response by means of a programming model can answer many policy-oriented inquiries and not just provide supply elasticities. Other examples of policy applications can include evaluations of comparative advantage, assessment of the employment effects of different policies, generation of input demand functions, and joint evaluation of sets of investment projects. These, and other applications of sector models in several countries are discussed in chapters 12 and 13.

When the transition from farm model to sector model is made, an important shift occurs in the model's role, and that shift has not always been recognized by model builders. At the farm level, the programming model is explicitly a normative or prescriptive tool. The decision-maker, who may be the farmer himself, specifies his decision rule (profit maximization, cash-flow improvement, profit maximization subject to risk aversion, etc.), and the model helps simulate the consequences of that decision rule and the associated constraints on the farmer's choices. On the other hand, at the sector level, in a decentralized economy, there is no single decision maker. There basically are two levels of decision makers, the policy makers and the farmers, whose interests do not necessarily coincide. A model's

solution which is designed to, say, maximize agricultural export earnings will give an export-intensive cropping pattern but it will not indicate which, if any, policies exist that will induce farmers to adopt that cropping pattern. To deal adequately with maximization of a policy goal, the model would have to contain a specification of policy instruments and also a set of relationships which describe how producers will react to possible policy changes.

As discussed in chapter 7, it is difficult to specify mathematically and solve the two-level policy problem, and so in many cases it is more productive to develop a model which will help explain producers' reaction to external changes. While the policy model is normative, this second kind of model is descriptive or positive. Among other things, the descriptive model can be solved under different assumptions about policy parameters, and the corresponding solutions provide some information about the consequences of policy changes. For these reasons, this book places most of its emphasis on developing the descriptive model, although parts of chapters 7 and 12 discuss the policy model.

Since policy goals are not necessarily the farmers' goals, the descriptive model must be set up to maximize something other than a policy goal function. On the other hand, if, for example, all producers aimed to maximize profits, then a sector model which maximized aggregate profits would not be correct either, for it would give the monopolistic outcome. This is the well-known paradox of the competitive market. For the model a different kind of objective function is needed, one which drives its solution to a market-equilibrium outcome in the

presence of downward-sloping demand curves. These matters are taken up in the last part of chapter 7 and in chapters 8 and 9. Extensions to the case of simulating market equilibrium under risk are given in chapter 10, on the basis of the farm-level risk analysis given in chapter 6.

A common thread through all the chapters is the emphasis on ways of putting economics into agricultural models. Properly constructed, programming models can reflect a wide range of economic and institutional behavior, and they can be powerful tools of analysis. They provide an analytic and empirical link between economic theory and observed behavior. We hope this book helps stimulate others to continue with the challenging task of matching theory with the real world.

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Peter B. R. Hazell and Roger D. Norton

Chapter 12. METHODS OF POLICY ANALYSIS

12.1 POLICY INSTRUMENTS AND GOALS IN SECTOR MODELS

This chapter, and the one following, discuss ways in which programming models can be applied to questions of economic policy for agriculture. The emphasis is on sector-wide models, for it is at the sector level that most policy issues arise, but references also are made to the use of farm models and regional models where appropriate. Examples are given of applications made with existing models, and especially for Mexico, Turkey, Egypt, Peru, and Central America. To introduce those examples, some background information is provided on the key models used. The examples cover static and comparative static analysis, and they include issues of pricing policy, evaluation of investment projects, dynamic analysis, and, in chapter 13, analysis of public risk management policies.

It is not possible to make hard and fast rules for policy analysis; there is a bit of art to it. The main difficulty is in translating policy concerns, which may be rather broad, into specific analytic questions that can be addressed by the model. Sometimes, of course, that cannot be done, and even when it can the model analysis may deal with only a part of the policy concerns. Nevertheless, the model can provide relevant information for the decision process which could not be obtained otherwise, and this chapter suggests, with the aid of examples, some of the circumstances in which that is possible. We also discuss how to set-up the appropriate model experiments needed to obtain the relevant solutions.

In chapter 7 it was pointed out that they policy problem is inherently a two-level planning problem in its mathematical structure. The two-level planning problem is difficult to solve, so most of this chapter deals with the more

practical one-level approach of simulating the sector's response to possible policy changes, i.e., using the positive model rather than a normative model. However, in section 12.9 we provide a feasible way of handling the two-level planning problem under some circumstances.

Given that the positive model is to be used, the objective function will typically be the sum of producer and consumer surplus. Policy goals will not appear explicitly in the objective function. They are variables in the model, or simple transforms of model variables which can be calculated ex post by hand or via a report generator. After a "base solution" is obtained, the essential procedure is to alter the model in a way that reflects a new policy, or new values of policy instruments, and then to solve the model again, recording the new values of the goal variables. By proceeding in this manner through a series of policy changes, a table can be built up that shows the relationship between policy options and their effects on different goals. (Such a table is discussed more fully in section 12.6 below.)

The agricultural sector frequently is expected to fulfill multiple, and conflicting, roles. Some common examples are creating more employment, earning foreign exchange, helping make (or keep) the country self-sufficient in food, increasing farm incomes, and keeping food prices low to consumers. Since these goals are not always mutually consistent, the analyst can be constructive by showing just which goals are advanced by particular policies, and which are not. It is not realistic to expect a policy maker to be able to quantify the policy preferences ("policy weights") among goals in advance, but by showing the multiple consequences of each policy with the aid of a model, the decision process can be facilitated.

Policy variables are represented in the model's structure by coefficients, in the matrix, the right-hand side, and/or the objective function. Thus a

policy variable is fixed in value for a given solution. One example would be the endowment of irrigated land, which is represented by the right-hand side coefficient in the constraint on the availability of irrigated land. More investment in irrigation would be reflected in an increase in that coefficient's value (or those coefficients' values, in the case of seasonal or monthly constraints).

Another example is the price of an input, such as irrigation water or fertilizer. A change in the pricing policy for irrigation would be reflected in a change in the coefficient c in the tableau in Table 11-5. At the macro-economic level, a change in exchange rate policy would be reflected in a change in the value of the coefficient e which appears in three places in the tableau in Table 11-8. Successful introduction of new technologies of production can be modelled by adding new column vectors of production coefficients with, for example, new values of the index t in the tableau in Table 7-1. A program that leads to reduction of marketing and storage costs can be represented by reducing the absolute value of the coefficients Δ_{jr} in the tableau of Table 7-3.

Many of the policy-related experiments with a programming model are carried out by making changes of this kind and solving the model again. Usually it is preferable to make only one change at a time, and then obtain a new solution before making further changes. This permits identification of the effects of each individual policy change. However, occasionally it is useful to make several changes before a new solution is obtained, when it is desired to simulate the effects of a package of policies. The joint effect of several policy changes is likely to be different than the sum of effects of individual changes, owing to interaction terms and to the inherent nonlinearity of the supply response

functions and of other aspects of the model's structure.

Clearly this procedure involves prior specification of policy changes which may be considered desirable and reasonable by at least some persons in policy making positions. Thus the model work requires close coordination with decision makers, or with someone representing them. Simulation of the consequences of pre-specified new policies may not seem as elegant as an attempt to derive an "optimal policy," but the latter is fraught with difficulties. First, there are the aforementioned computational difficulties associated with multi-level solutions. Second, notwithstanding the concept of Pareto optimality, opinions regarding what is optimal in an economic sense (as opposed to a mathematical sense) will differ from person to person. Some individuals may prefer a package of policies which generates more employment, others may prefer to emphasize foreign exchange earnings, and so forth. By simulating the multiple consequences of policy changes, the analyst can provide an objective basis on which these preferences can be debated.

In carrying out such procedures for policy analysis, it is important that goal variables <u>not</u> be entered into the model's objective function, and that their levels not be constrained. To include goal variables in the objective function would be to override its simulating (positive) role. The model's solution no longer would have a market-clearing character, and its interpretation would be unclear. The same comments apply to constraining the levels of goal variables. At the margin, constraints dominate the solution; only after they are met does the objective function have scope for improvement.

As discussed in chapter 7, when the model includes farm-level output and input choices the inclusion of goal variables in the objective function is equivalent to simulating the institutional framework of centralized planning, in which cropping decisions are made by government fiat. The same interpretation would apply to placing a constraint on the level of goal variables.

Therefore such procedures are inappropriate for analysis of market economies nd mixed economies. However, maximizing a goal variable directly can serve one analytic purpose, and that is to find the frontier, or maximum level of the goal which is conceivably attainable.

But even this frontier may not be very useful, because it bears little relationship to the points which are attainable under the market system and under the budgetary limitations on government policy. For example, suppose that in a country like Mexico cotton and sugar cane are the most labor-intensive crops. A solution which maximized employment would correspond to planting much of the country in those two crops (supposing it were agronomically feasible), to do so would cause domestic market prices for those goods to collapse and so farmers would suffer large financial losses. In such a hypothetical case, it would be out of the question to compensate the losses with subsidies, for lack of sufficient budgetary funds, and so the employment-maximizing cropping pattern could not even be approximated in reality. These considerations reinforce the arguments for using the positive model via a sequence of experiments involving changes in policy parameters, rather than using frontiers, to address policy issues.

12.2. SELECTED APPLIED MODELS[1]

This section discusses three sector-wide programming models which have been applied to questions of policy either by a government or by an international agency. The three were designed for the agricultural economies of Mexico, Egypt and Turkey. Each model has some special characteristic not found, in other models, and they provide unique insights into the power and scope of mathematical programming models for policy analysis.

12.2.1 The Mexican Model CHAC

CHAC[2] was the earliest positive programming model constructed for a developing country. As of this writing it has been used for more than ten years by the Mexican government to produce a number of policy documents for agriculture (examples are found in Secretaría de la Presidencia, 1973 and 1976; and L. M. Bassoco et al., 1982). It was developed as part of a collaborative research project carried out jointly by the World Bank and the Mexican government, and it has been documented in several publications over the years (principally

Goreux and Manne, 1973; Bassoco and Norton, 1975a and 1975b; and Norton and Solís, 1983).

The initial research project included economy-wide studies and studies of the energy and agriculture sectors (Goreux and Manne, 1973), and then the agricultural work was carried on independently. It always has had an institutional sponsor in Mexico, and it has been applied to specific questions arising from that sponsor's concerns. The first home for CHAC was the Central Bank of Mexico (Banco de México). Then it moved as key personnel involved in the model changed agencies. It has been in turn in the Ministry of the Presidency (Secretaría de la Presidencia); the office of Presidential Advisors, under the framework of the Mexican Food System (Sistema Alimentario Mexicano), and back to the Ministry of the Presidency, which now is called the Ministry of Programming and Budgeting (Secretaría de Programación y Presupuesto).

In structure, the model follows the general outlines presented in chapters 7 through 10, although the earliest versions did not contain the risk specifications. It was the first sector-wide programming model to use downward-sloping demand functions and to use efficient linearizations of those functions. The CHAC project sponsored the work on risk which led to the seminal paper by Hazell and Scandizzo (1974) and the material in chapter 10 of this volume.

The model contains supply and demand specifications for about 33 crops (that number has varied slightly from version to version), organized spatially in four major regions and twenty subregions. (This last number also varied over different versions.) The major regions were used mainly to define possibilities for labor migration, and the subregions were used for defining basic variations in available crops and technologies. For some of the subregions, further subdivisions were made on the basis of farm size and differences in irrigation facilities. The non-irrigated subregions were defined in terms of

altitude and annual rainfall, and therefore they are non-contiguous entities.

All together the model contains over 2,300 alternative production vectors, including over 200 for corn. Inputs are monthly in the case of land, labor, and water, and they are annual otherwise. The annual inputs are draft animal services, machinery services, chemical inputs, improved seeds, short-term credit, and a residual category of other inputs. On the supply side, family labor is distinguished from hired labor, and the former has a reservation wage which is lower than the market wage. The initial version of the model contained some 1,500 equations, and later versions about 2,200 equations.

The main data sources for the production coefficients were the regular cost-of-production studies of the agricultural banks and the national crop insurance company, supplemented by some special surveys.

Apart from formal questions of model design, the work with CHAC has been distinctive in two other respects. First, considerable emphasis was placed on flexibility in model structure, in order to facilitate subsequent changes in the model's structure. The reasoning was that, to be useful, the model would have to be solved a large number of times under many variations of structure and data. This meant a larger model in number of rows and columns, but the practical advantages were felt to outweigh the implied increase in computational cost. The other distinctive feature of the CHAC was the emphasis placed on design of meaningful policy experiments with a model that simulated market equilibrium, subject to specified government policy interventions. Many of the discussions in this chapter had their origin in that work.

12.2.2 The Egyptian Model HĀPY[3]

The Egyptian model was the outgrowth of another collaborative project, this time involving the World Bank and the Egyptian Ministry of Irrigation,

with the support of the United Nations Development Program. The motivation behind the model study, and other related studies, was the concern that the availability of water from the Nile could become a constraint on national growth by 1990. As agriculture consumes about 95 percent of the Nile waters, it is an obvious focus of this concern. The immediate goals of the analysis were to estimate agriculture's needs for water under different scenarios, to estimate the marginal productivity of water in agriculture, and to evaluate alternative investments in irrigation. Later, the analyses were expanded to include the effects of agricultural pricing policies. The initial version of the model is described in Kutcher (1980), and the investment version is discussed in Norton (1982).

HĀPY contains 25 crops and their production is specified for 15 canal command groupings, which are aggregations of the 50 canal commands used in operating the Nile irrigation system. Farm surveys by the national extension service provided the bulk of the data in the model. The initial version was based on 1978 data, and a revised version for investment analysis was developed on the basis of 1980 data. The model contains about one thousand equations.

Whereas CHAC contains downward-sloping demand functions for livestock d, but no explicit livestock activities, HĀPY contains livestock herds and their associated demands for feed. The herds, however, are exogenous in numbers of animals, so their purpose in the model is to account for feed requirements. Those requirements are expressed in nutrient terms.

In its initial applications, the model provided, among other things, results regarding the economic benefits of spreading water more thinly over all irrigated areas, in order to be able to irrigate the "new lands", and compared this with irrigating existing lands more intensively. In its second version it was used to jointly evaluate a large number of proposed irrigation investments.

Taken together, the proposed investments are large enough in scale that they would have non-marginal effects on sector production levels. Also, the increments in production would vary over crops, and hence relative prices would change as a consequence of the investments. For this reason, a joint evaluation of the investments using the model with endogenous prices yields different conclusions than does a traditional project-by-project evaluation under the assumption of fixed prices. This investment work with HĀPY contributed to the development of the material presented in section 12.8 below. As of 1985, the third round of analyses and reports with HĀPY was being developed in Cairo.

12.2.3 The Turkish Model TASM[4]

TASM was developed primarily to address issues of comparative advantage and pricing policy in Turkish agriculture, in a context of concern about the sector's future possibilities for expansion through increased export competitiveness. There have been two Turkish models, TASM I and TASM II. TASM I is similar to CHAC in the essentials of its structure but departs from both CHAC and HĀPY in including endogenous sizes of livestock herds. TASM II, which has a new data set, departs further in including marketing-processing activities (as per chapter 8), regional product demand functions, fertilizer response functions, and explicit alternative feed rations (as per chapter 11, section 11,6). The inclusion of regional supply, demand and processing activities necessitated the addition of interregional shipment activities for many products. TASM II contains five regions which are aggregations of Turkish provinces. There are about 1000 equations, with some 70 agricultural products, quarterly inputs for land and labor and annual inputs otherwise. The basic references are, for TASM I, Le-Si, Scandizzo, and Kasnakoglu (1982), and, for TASM II, Gencağa and Norton (1985).

Both TASMs were used in developing analyses for World Bank reports on Turkish agriculture, and as of this writing TASM II is operational in the Turkish Ministry of Agriculture. The models have been used to estimate growth prospects for the sector, to estimate regional differences in the degree to which arable land is a binding constraint on growth, to estimate input requirements, to explore the effects of changes in fertilizer pricing policy, and to measure Turkey's international comparative advantage by crop. The discussion in section 12.4 is based primarily on the work with TASM II, and to a lesser extent on the CHAC work.

12.2.4 Other Relevant Models

A number of other sector models have been built; five can be singled out as good illustrations of key points in the text. They are the Philippine model (MAAGAP) of Kunkel et al. (1978), the Tunisian model of Cappi and Condos (1976), the Brazilian model of Kutcher and Scandizzo (1981), the Central American model (MOCA) of Cappi et al. (1978), and the Malaysian model (TIGER) of Bell, Hazell, and Slade (1982, chapters 3 and 4). MAAGAP is especially good in the treatment of processing activities, the Tunisian model on livestock choices, the Brazilian model on crop consortiums, MOCA on inter-country trade linkages, and TIGER on mechanization and draft power choices in the context of fortnightly resource balances and constraints. Interesting programming models of investment choices at the local level are found in Husain and Inman (1977), Willis and Hanlon (1976), and Bassoco, Mutsaers, Norton, and Silos (1983). Several regional models for Mexico, by Kutcher, Pomareda and Simmons, Howell, Benito, and Candler, are found in Norton and Solis (1983).

12.3 AVERAGE AND MARGINAL COSTS OF PRODUCTION

Many prescriptions for economic policy are based on marginal equivalences. A well-known example is the recommendation to implement marginal-cost pricing in public enterprises. However, as noted in chapter 11, published agricultural data do not include marginal costs of production. At most average costs are reported, but even that is rare and such estimates usually are based on very few observations.

A programming model can be used to compute both average and marginal costs. The greater the disaggregation of farm types in the model, the more representative these costs will be. For a given crop, average costs can be computed on the basis of a model solution as follows:

$$C_a = \sum_i \sum_h \sum_r \sum_t \theta_{hrt} a_{ihrt} P_i / y_{hrt} \qquad (12.1)$$

where the indexes are i = input

h = farm type

r = region

t = technique of production

and where

θ_{hrt} = the share of acreage in the given crop, by indexes h,r, and t, in the validated base solution

a_{ihrt} = input-output coefficients (including land and other fixed resources)

P_i = input prices, including shadow prices for fixed resources

y_{hrt} = yield per hectare.

The input prices may be either exogenous or endogenous, or both, as the cost calculation is made after the model's solution is obtained.

Average costs calculated in this way differ somewhat in concept from those which are based on field surveys. The terms in equation (12.1) include the opportunity cost of fixed resources and, as appropriate, they may allow for seasonal variation in those opportunity costs. Thus from the viewpoint of economic theory estimates from equation (12.1) are preferable to those which are based on records of farmers' cash outlays. The latter usually exclude, or rely on arbitrary methods for accounting for costs associated with fixed resources.

Marginal costs may be derived from a model's solution in a number of ways. In a model structured so that price equals marginal cost, then it is necessary only to look for the price in the case of products that are sold on the domestic market. The simplest way to find the price is to look up the dual variable on the commodity balance. As shown in chapter 8, that dual variable is in fact the product price. If there are two commodity balances, one at the producer level and one at the consumer level (chapters 7 and 8), then the corresponding dual variables represent the farm gate and consumer prices, respectively.

An alternative, but equivalent way to obtain the product price is via the primal solution. In a linear programming model, the prices on all segments of the demand curves are pre-specified, and so for a given product the price is the sum of segment prices times segment values in the solution. (No more than two segments will appear in the solution.) These two procedures for calculating price (marginal cost) will give the same figure, except for possibly minor discrepancies arising from the discontinuities involved in the segmentation of the demand curve. Any discrepancies can be reduced by increasing the number of demand segments in a given range of quantity (or price) variation. Thus,

a comparison of the primal and dual values of a product price can be a useful indicator of whether a finer demand segmentation is needed in a linear model.

Yet another approach, which is relevant for a linear or quadratic model, is to calculate the product price ex post by solving the inverse demand function on the basis of the quantity sold in the solution.

For products which are exported against a bound, the marginal cost of production can be found from the information provided in the shadow price on the export bound. The relation is

$$C_m = P_e - \pi_e \tag{12.2}$$

where P_e is the export price and π_e is the shadow price on the export bound. The latter represents the economic rent accruing to those who produce for export. Figure 12-1 shows π_e graphically; $\bar{e}$ in the figure is the bound on exports.

[Insert Figure 12-1]

If a product is both exported against a bound and sold on the domestic market, then the two marginal costs should be the same. The cost of obtaining a marginal unit for either market should be the same, unless access to the limited export market effectively is rationed. In that case, producers who enjoy access may have different production costs than those who sell on the domestic market. If in reality only certain producers export, the modeller may wish to specify a separate commodity balance for export, with the provision that only certain farm types have entries in that balance. Those farm types also would, of course, have access to the domestic market.

Table 12-1 shows the tableau for the case in which some farms (type 1) can sell only to the domestic market while others (type 2) can sell either to the domestic or the export market. Activities X_1 and X_2 represent production for the two farm types, and y_1, y_2 are the corresponding unit yields. The

variables M_2^e and M_2^d represent the export and domestic marketing activities for farms of type 2, and M_1 represents the (domestic) marketing activity for farms of type 1.

[Insert Table 12-1]

Average and marginal costs of production can and do differ significantly in most countries. That is, both more and less efficient producers co-exist in the sector. Many less efficient producers are not driven out for a variety of factors. For one thing, a producer may be relatively efficient in producing his principal crop but less efficient in producing his secondary crop. Yet the secondary crop's revenues may cover its variable costs, so he is willing to continue producing it, even though others do so at lower cost. Also, a higher income, or preferable life-style, may not be guaranteed in non-agricultural occupations, so poorer producers may prefer to stay in agriculture rather than endure the disruption of a change.

12.4 COMPARATIVE ADVANTAGE

It is often useful to develop measures of comparative advantage on the basis of both average and marginal costs (see, for example, Gencaga and Norton, 1985). A commonly used measure of comparative advantage is the domestic resource cost (DRC) of earning, or saving, a unit of foreign exchange. Algebraically, the DRC is written, for a single good,

$$DRC = \frac{C - eC^i}{eP^w - eC^i} \tag{12.3}$$

where C is the cost of production,

e is the exchange rate,

P^w is the border price of the good, and

C^i is the cost of imported inputs.

The cost C may be either average or marginal, and it is suggested that the DRC calculation be performed for each concept. The border price p^W should not be the international price but rather the actual c.i.f. import price or f.o.b. export price. For the cost of imported inputs, two approaches are possible. One is to include the costs of all inputs which are imported at least in part. The other is to include only the imported shares of those inputs, e.g., only 25% of fertilizer inputs if in fact only 25% of fertilizers are imported. The latter is probably preferable on grounds of accuracy, but the choice may depend upon the evolution of imported inputs over time and the prospects for new domestic production capacity in the input industry.

For the average cost case, the imported inputs can be identified readily from the model's input-output coefficients used in equation (12.1). For the marginal cost case, such identification is not quite as easy. There are two fairly simple ways of finding the marginal use of imported inputs, or any particular inputs. The first way is to identify the particular production vectors which are associated with the marginal points on the cost curve. This can be done by varying the product price (or rotating the demand curve as described in section 12-6 below) slightly, and seeing which production vectors come into the solution or drop out of it. Those are the marginal vectors on the cost curve. (The designation of which vectors are marginal will be conditional on the prices of other products, for they will influence the opportunity cost of fixed resources.) The input coefficients in those vectors will give the marginal requirements for particular inputs, per ton of output.

An alternative and simpler approach is to make use of accounting balances for use of inputs. When the price of the output is changed marginally, both the output level and the levels of input use will change. Those changes are, by definition, at the margin, and so the ratios of input changes to output changes are the marginal input-output coefficients.

Another commonly used measure of comparative advantage is the effective rate of protection, or EPC. For the average cost case, it can be expressed as

$$EPC = \frac{P^d - \left[\sum_i \sum_h \sum_r \sum_t \theta_{hrt} a_{ihrt} P_i^d / y_{hrt}\right]}{P^w - \left[\sum_i \sum_h \sum_r \sum_t \theta_{hrt} a_{ihrt} P_i^w / y_{hrt}\right]} \tag{12.4}$$

where the symbols are as defined for equation (12.1) above, P with no subscript is the product price, and the superscripts d and w represent domestic and border (world) prices respectively. For the marginal cost case, computation of the EPC requires identification of the marginal production vectors via solutions in the way described above. (A useful discussion of measures of comparative advantage for agriculture is found in Scandizzo and Bruce, 1980.)

Comparative advantage measures can be helpful in designing export marketing strategies and trade incentives. One of the first applications of a programming model to trade questions was made with the Mexican model CHAC. DRC calculations were made for 33 crops with the model, and the crops were ranked according to those calculations. Those crops which had a DRC lower than the exchange rate (in pesos/dollar) were deemed candidates for export promotion, especially those crops with very low DRCs. For those with higher DRCs, significant improvements in productivity would have to be achieved before exports or import substitution could be expected to take place. The first set of DRC calculations with CHAC was made on the basis of 1968 data; subsequently it turned out that the 1968-80 export and import growth rates by crop were closely correlated with those original DRC rankings. The rankings later were re-calculated on the basis of 1976 data and were found not to differ very much. Discussions

of these applications and others are found in Duloy and Norton (1973) and Bassoco and Norton (1983).

Interregional comparative advantage also may be explored with programming models. One indicator of a region's relative efficiency in production is provided by an experiment consisting of raising a product's price and recording the regional shares of the resulting incremental output. In cases in which regional production is tied to local processing capacity, running the model without processing activities (or without regional limits on processing capacity) may lead to a solution in which the regional cropping patterns differ significantly from the actual ones. This procedure was followed with CHAC; it was found that the model allocations for sugar cane indicated relatively more cane in the tropical regions and relatively less in the northwestern irrigated districts, as compared with the actual patterns, even though cane yields in the northwest were about twice as high as in the tropics. This was a clear indication that cane had a comparative advantage in the tropics. The reason was the high opportunity cost of irrigated land in the northwest, for the production of fruit, vegetables and other high-value crops. On the basis of this analysis, a decision was made by the Mexican government not to finance expansion of sugar milling capacity in the northwest (Bassoco and Norton, 1983, p. 151).

A quantification of interregional comparative advantage can be obtained by computing regional average costs of production on the basis of the model's coefficients and the vectors which appear in the solution. It is important that regionally fixed resources and their shadow prices be included in the computations. The relevant equation is the regional variant of (12.1), where the summation is not taken over the regional index r:

$$C_a^r = \sum_i \sum_h \sum_t \theta_{hrt} a_{ihrt} P_i / y_{hrt} \qquad (12.5)$$

Again, the values of C_a^r will be conditional upon the particular resource endowments and other elements of the solution. Changes in the model's parameters can be expected to lead to changes in the C_a^r.

12.5 INPUT USE AND INPUT PRICING

A major concern of agricultural policy makers is the future rate of input use in agriculture. One aspect of this concern is how the level of gainful agricultural employment may respond to agricultural growth and to relative price changes. The requirements for modern inputs such as agrochemicals, machinery, improved seeds, and credit may also change in response to sector growth and price changes.

By virtue of its detail on the input side, a sector programming model is a useful instrument for calculating the input requirements associated with different cropping patterns and policy strategies. In this regard, one of the most common applications of such models is the simple one of tabulating input use levels in the base solution and then in alternative solutions. Sometimes it is helpful to record the input requirements by region, farm size class, or crop group. These measures help identify the major sources of demand for modern inputs and give guidance for programs of supplying those inputs. Of particular interest is the projected growth of input demand. It can be expressed in terms of simple growth rates or in terms of elasticities with respect to output growth (For an example of the latter see the CHAC model, Bassoco and Norton, 1983, p. 142.)

Special interest resides in seasonal labor patterns, which often are highly peaked. The model generates seasonal agricultural employment, which is something that is not easily measured by direct surveys in developing countries. Figure 12-2 shows the seasonal employment patterns in Mexican

agriculture as they were calculated with the CHAC model. CHAC also was used to quantify the employment-creation effects of various trends and policy measures such as yield growth, irrigation investment, and a proposed tax on imports of agricultural machinery, in an effort to determine whether total employment creation in agriculture would match the net annual increase in the rural labor force, allowing for expected rates of rural-urban migration (Ministry of the Presidency, 1983, pp. 189-193).

[Insert Figure 12-2]

Input demands vary with prices. Chapter 7, section 7.5, discusses the use of a sector model to trace out relationships which show input demands as functions of input prices. TASM II was used to derive national demands for fertilizers, and to calculate the effects of higher fertilizer prices on agricultural production levels. Those effects were calculated from model solutions under different fertilizer prices (see Gencağa and Nortōn, 1985, pp. 6-7). It turned out that, for Turkish agriculture as a whole, raising the fertilizer price by reducing its subsidy did not have much effect on output levels owing to the shape of the fertilizer response function at prevailing levels of output. Other experts concurred in that finding; in general it is a useful rule to solicit the reactions of field experts or other specialists to the principal findings of a model-based study. If they do not agree, the model structure may (but not necessarily!) need revision. At minimum, a dialogue should develop regarding the reasons behind the model's results.

Another way of expressing input relationships is through isoquants, which link pairs of inputs. Isoquants are especially useful for inputs that are close substitutes, as in the case of labor and farm machinery. Once the isoquant is known, then direct calculations of the effects of changes in relative input prices on input use can be made. As with supply functions, isoquant-like

constructs are implicit in the model, and can be traced out numerically via appropriate solutions of the model.

As traditionally defined, an isoquant is a conditional concept: it shows input substitution possibilities when output is held constant. But in actuality, when relative input prices are altered output levels also change. Output will increase for some products and decrease for others, and the aggregate index of output will change. Similar effects will occur in a sector model. The input-substitution surface that can be traced out with the model is not therefore a true isoquant but rather a general locus of input substitution, or GLIS. For policy purposes, the GLIS is of more interest than the isoquant, for if input prices are changed there is no mechanism for holding output constant. The procedure for tracing out the GLIS is simply to calculate solutions under different pairs of values of input prices, e.g., under a sequence of values of the wage rate and the cost of agricultural machinery. In general, different levels of input prices will produce different solutions, even if the same ratio of input prices is maintained. This happens because of other inputs to the production process, whose relative prices to the price of the inputs under consideration are changed.

It is possible to use the model to trace out an approximate isoquant, and also an approximate iso-profits curve, by imposing a constraint on an index of output or profits. However, the presence of such a constraint will distort the shadow prices on the commodity balances so that they will no longer equal the corresponding primal prices, i.e. a competitive market solution is no longer obtained. The size of this error is an empirical matter, and there

may be cases in which it remains within acceptable bounds for purposes of generating the approximate isoquant.

The studies in Mexico provide examples of the GLIS and the approximate isoquant (Bassoco and Norton, 1983; Howell, 1983). Figures 12-3 and 12-4 show those examples. For both kinds of surfaces, the arc elasticities differ from point to point. Also, the first derivative is not continuous, and in some range of values the surfaces may not even be convex from below. These effects arise because of the discrete nature of the production functions and because movements along the surface affect first one product and then another.

[Insert Figures 12-3 and 12-4]

12.6 SUPPLY RESPONSE AND OUTPUT PRICING POLICIES

Supply response functions can be traced out numerically by varying the price of fixed price selling activities (as in chapter 7). When the model contains downward-sloping demand functions, a more convenient method is to rotate those functions in successive solutions. The rotation is accomplished by changing one coefficient, the right-hand side of the convex combination constraint. For example, three points on the supply function for a given crop can be found with right-hand side values of 1.0, 1.2, and 1.4 in the convexity constraint for that crop. Those values will generate the demand functions D_0, D_1, and D_2 in Figure 12-5. If the implicit supply response function is S, the corresponding supply-demand intersections will be points a, b, and c. If the supply response function is S', the intersections will be points a', b', and c'. For the case of function S, the modeller will observe the price-quantity pairs (P_0, Q_0), (P_1, Q_1) and (P_2, Q_2) in the successive solutions. These points

and others may be connected ex post to create the supply response function shown in the figure.

[Insert Figure 12-5]

The arc elasticity of supply may then be calculated. For the arc defined by points a and b, it is

$$\eta^{s}_{ab} = \frac{(Q_1 - Q_0)}{(P_1 - P_0)} \cdot \frac{(Q_0 + Q_1)}{(P_0 + P_1)} \tag{12.6}$$

A priori, it is impossible to know the magnitude of price and quantity responses associated with a given increase in the right-hand side of a convexity constraint. Responses depend on the shape of the unknown supply response function. Experience with programming models indicates that supply response elasticities may be quite variable as quantities increase, although they generally tend to diminish in value. Some early segments of the function may even be flat. The modeller should be prepared to experiment with a range of values of the convexity RHS before finding solutions that give significant increases in quantity along the supply response function.

Suppose it is desired to know the supply response functions for a particular region without changing the price in other regions. In this case, the marketing-processing activities may be used to set up the appropriate experiments. To move outward along a regional supply response function, the coefficient $-\Delta_{jr}$ found in Table 7-3 can be altered by the algebraic addition of a term Δ^* (shown on the vertical axis in Figure 12-6). This change corresponds to a regional price subsidy of amount $P_1 - P_0 < \Delta^*$, referring to Figure 12-6. In terms of the demand function facing the region's producers, it constitutes a parallel shift. The elasticity of supply response is then calculated ex post, as before.

[Insert Figure 12-6]

Some points of interpretation may be in order regarding the supply

response functions obtained from a sector model. First, following the distinction made in chapter 7, they are not conventional supply functions since prices of fixed resources and other outputs are not held constant. For purposes of policy analysis, the supply response concept is the more relevant, for in general other prices will not remain constant when one good's price is changed. That good will substitute for others in production and consumption, at least to some extent, and hence the market equilibrium prices of other goods will change.

Second, it may be asked whether the measured elasticity is a short-run or long-run concept. In one sense it is a long-run elasticity, because it refers to change in behavior between two equilibrium points after all adjustment processes have been completed. However, the static model does not allow for fixed capital formation, so in that respect the elasticity is a short-run concept. In measurements of short-run supply response based on historical time series, full responses to the price change often have not been worked out before another exogenous change intervenes to alter the response. To distinguish the elasticities derived from a programming model from other kinds of elasticities, they may be called "equilibrium short-run elasticities."

Because the programming model describes a situation of full adjustment, it is likely to give elasticities that are greater in value than corresponding econometric estimates. This need not always be the case, however. The CHAC model gave an elasticity of about 0.5 for maize, identical with an estimated elasticity based on historical time series data reported in Cartas and Cifuentes (1982). Also, at the farm level, Shumway and Chang (1977) found econometrically estimated elasticities and programming model elasticities to be comparable in value. The greater the differentiation of farm types in the programming model, the smaller the supply response elasticity is likely to be for modest price

increases. Also, inclusion of the risk-aversion specifications discussed in chapters 6 and 10 results in lower elasticities for the riskier crops (see chapter 13).

For purposes of applied work, it is best to regard the programming and econometric elasticities as different constructs, each with its own uses. For purely forecasting questions about the supply effects of price changes on a single commodity, the econometric elasticities are usually preferable. The programming simulations of supply response are particularly useful in three areas of analysis:

1) Comparative simulations of supply response. For example, whether greater supply responses are likely to come from changes in output or input prices, or whether, say, farm income is likely to be increased more by a subsidy applied to the maize price, to the sorghum price, or to the wheat price.

2) Simulations of cross-supply effects and other derived effects from price changes. It can be helpful to project not only the supply response to own-price changes but also the associated crop substitution effects. An additional ton of maize may be gained at the cost of 0.30 tons of sorghum, 0.10 tons of soybeans, etc. Also, owing to the complexities of crop rotations and competition for resources on the supply side, some crops will be complements rather than substitutes, and the model's solutions will indicate the strength of these complementarity relationships. The model also provides simulations of the net effect ($\pm$) of output price changes on employment, foreign exchange earnings, and other aggregate goal variables.

3) Simulations of the structure of supply response. Sometimes it is of interest to know which regions and farm size classes respond most to a change in price incentives, and consequently, to calculate the net income effects for each type of farm. Another aspect of the structure of supply response is its disaggregation into net expansion of cultivated area, substitution for other crops, and yield increases. The programming model is well suited for investigating these kinds of questions.

These points are quite relevant to the formation of pricing policy, for policy usually does not seek to promote the production of a given crop for its own sake, but rather as a means to greater levels of farm income, increased incomes for certain classes of farmers, more agricultural foreign exchange earnings, etc. In this perspective, a model is most useful when comparing the effectiveness of different policy instruments in achieving given ends; for example, when the consequences of changes in prices of different outputs and inputs are simulated.

Tables 12-2 and 12-3 provide an example of comparative evaluations of different instruments of pricing policy taken from one of the CHAC studies carried out by the Mexican government in 1982. To construct the tables, it was necessary to work out the implications for the government budget of each price support instrument. To develop measures of the effectiveness of policies per unit of government expenditures, the model's simulated effects of the instruments were divided by the associated budgetary requirements. The instruments are price supports for different crops and subsidies on certain inputs. It can be seen that the instruments vary in their effects, but that the maize price support appears to be the most efficient instrument for generating employment and additional net farm income.

[Insert Tables 12-2 and 12-3]

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Table 12-4 provides an example of the analysis of supply response for Costa Rica, carried out with a submodel of the multi-country model MOCA (Cappi et al., 1978).[7] The demand function for corn was rotated rightward through increases in the right-hand side of its convex combination constraint. The results show the price and quantity effects for four basic crops. It can be seen that the substitution effects are strongest with respect to beans and are weakest with respect to sorghum. For the entire range of corn prices shown in Table 12-4, the own-price (arc) elasticity for corn is 0.589, and the cross-price elasticities are: rice-corn, -0.213; sorghum-corn, -0.022; and beans-corn, -0.411.

[Insert Table 12-4]

Table 12-5 shows the net effects of the corn price change on Costa Rica's foreign trade in corn. It can be seen that it would take an unrealistically large price increase to convert Costa Rica from an importer to an exporter of corn. The budgetary consequences of attempting to promote corn self-sufficiency for Costa Rica were also calculated, as shown in Table 12-6. For those calculations, it was assumed that the consumer price remain unchanged and that the higher producer price would be attained through subsidies to producers. The budgetary consequences of a policy of corn self-sufficiency are considerable.

[Insert Tables 12-5 and 12-6]

12.7 ANALYSIS OF STRUCTURAL CHANGE

Programming models are especially well suited for the analysis of structural change. In dealing with structural change, it should be borne in mind that the line between structural change and mere parameter changes is a blurred one. For example, if a new export market is opened up for a particular product, it may be possible to represent that change in the model simply by raising the RHS upper bound on exports. Is that a case of structural change or merely parametric change within the given structure? Another example could be the removal of a long-standing subsidy on an input, which is represented in the model by raising the exogenous price of the input. Again, this may be viewed as a structural change or a parametric variation.

For specifying the model experiments, however, the definition of structural change may not be so important. The basic point is that the model is a helpful tool for understanding the consequences which follow when a parameter, or set of parameters, takes on values not experienced historically.

For agriculture, two especially relevant examples concern technological change in production and land reform. Technological change often occurs through the introduction of a new plant variety, and its associated package of inputs. Such a change would be modelled by enlarging the model's set of column vectors to include one or more additional vectors that represent the new production possibilities. (Of course, care has to be taken to be sure that the new coefficients are realistic at the farm level, and not only based on data from experiment stations.)

Depending on the intensity of inputs, and their associated costs, the new technology might or might not be adopted in the model solution. Sensitivity tests could be conducted to determine the wage rate, or the prices of other inputs, at which the new technology enters or leaves the optimal basis. The model solutions would also show the effects of the new technology on

farm incomes, employment, output price levels, and other target variables. Procedures like these were utilized in the TIGER model for the ex post analysis of an investment project in Malaysia (Bell, Hazell and Slade, 1982).

When land reform experiments are conducted with a programming model, it is a simple matter to explore alternative assumptions about the productivity of inputs on the reformed land. The reallocation of resources which is attendant on land reform is modelled by changing the RHS values for land endowments for the different farm classes, while leaving the labor endowments unchanged. If it is assumed that the smallholders who are beneficiaries of the reform will apply their previous production practices to the reformed lands, then no change need be made in the production coefficients matrix of the model. If, on the other hand, agricultural extension, credit, etc., will be provided along with the reform, then it may be useful to conduct another experiment with higher yields, and possibly with greater intensities of modern inputs, in the production coefficients for the smallholders. Land reform experiments were conducted with the model for northeast Brazil (Kutcher and Scandizzo, 1981) and with a revised version of the MOCA model for Central America (SIECA, 1982).

12.8 EVALUATION OF ALTERNATIVE INVESTMENT PROJECTS

12.8.1 Programming Models and Project Evaluation

Although the benefits of investment activities extend over several years, investments often can be evaluated with a static model. The procedure is applicable not only when benefit streams are uniform over time, but also when they vary in the initial years after completion of the investment project.

What are the advantages of using a model, as opposed to the traditional procedures for evaluating projects? Principally they are three:

1) A model like the ones discussed in this book will simulate producers' changes in cropping and input use in response to the additional resources made available through the investment. This can help avoid the observed tendency in many traditional project evaluations to overestimate the extent to which the high value crops will be successfully promoted by the project, especially in the case of irrigation projects.
2) Because the model simulates producer responses to the project, it can provide information in addition to the economic efficiency of the project on its probable performance in contributing to such goals as employment creation and foreign exchange earnings.
3) In cases of programs of investments which are non-marginal in magnitude, relative prices are likely to be affected, and hence traditional, micro-level evaluation procedures will be biased by their assumption of constant prices. A programming model can give a joint evaluation of a package of investments, and rankings of the components of the package, in an environment in which prices will be determined in part by the level and composition of investment.

A programming model may be used in different ways to address issues of investment. One way is to use it to attempt to quantify only the benefits of the project(s). A second way is to use it to compute internal rates of return for the projects, using information on project costs as well as on factors that influence benefits. Both these procedures involve use of the positive (simulating) model. A third way is to use a two-stage planning procedure which also involves a normative model defined in terms of explicit goals of investment policy. These three procedures are discussed in turn.

12.8.2 Evaluating Project Benefits

Using a model to quantify only the benefit side of a project is most useful in the pre-feasibility stage of work, when a decision is being considered whether to make the expenditures necessary to conduct a formal feasibility study. That kind of study provides detailed estimates of the project's costs, but a decision on whether the study is needed can be made on the basis of a very rough project costing plus estimates of project benefits, and the model is a useful tool for supplying the latter.

The first step in indirect project evaluation is to specify the types of resources that will be expanded by the investment: irrigable land, irrigation water supplies, rainfed land, livestock herds, stocks of trees, etc. The next step is to specify some hypothetical increases in those resources as changes in right-hand side parameters of the project region in the model. By solving the model both with and without these changes, comparison of the solutions will show the changes in farm income, production, employment and other variables that would arise from the project.

The model also provides estimates of the shadow prices of the affected resources. These shadow prices measure the economic welfare benefits (change in producer and consumer surplus) deriving from unit increases in the resource endowments. As such, they constitute upper bounds on the unit costs of expanding the resources if the project's benefit-cost ratio is to remain at or above 1.0.

Table 12-7 shows the results of such a procedure carried out with a submodel of CHAC representing the Río Colorado Irrigation district in northwestern Mexico (Bassoco, Norton, and Silos, 1974 and 1983). It can be seen from the table that there is a general (although not universal) tendency for all categories of benefits to increase at a decreasing rate as irrigation water continues to be expanded. The table also shows that the cut-off point for project scale will vary with the types of benefits emphasized. For example, in this case, employment creation encounters sharply diminishing returns after the irrigation supplies are increased by 15%, whereas farm incomes continue to increase substantially even when irrigation is expanded beyond 20%.

[Insert Table 12-7]

A somewhat different example was provided by Bassoco, Norton and Silos (1974, 1983) using the El Bajio submodel of CHAC. From solutions of that model the shadow prices of rainfed and irrigation land were computed, and the difference between the two figures was interpreted as the upper limit, according to criteria of economic efficiency, on the cost of creating new irrigated land, if the benefit-cost ratio of the project is to remain above 1.0. An implicit assumption in this procedure was that cropping patterns in newly irrigated areas within El Bajío would be similar to those on existing irrigated areas in that region.

Using a model to evaluate a project is also helpful in determining how much the project's financial benefits are affected by changes in pricing policy, and in answering questions about how complementary investments would affect the project's viability. The same Río Colorado model was used to review the question of changes in yields and project benefits. At the time of the study, cotton

yields in the project area were declining owing to increases in soil salinity, so explorations were made of the effects of those declining yields. An additional solution was carried out under the assumption that cotton yields were 13% lower, a decline that appeared likely to occur over a period of four or five years on the basis of past trends. This reduction lowered the shadow price of irrigation water by 26%, i.e., it reduced the returns to investments, in terms of economic efficiency, by that amount.

Given the uncertainty attached to the outcome of agricultural research and extension (R & E), it is impossible to say what kinds of R & E efforts could avoid the projected decrease in yields. However, the following conclusion was drawn: if in the judgement of experts the allocation of less than 26% of the investment budget, say 10% to 15% to be safe, would be likely to halt the decline in yields, then such an allocation would be justified by the increase in returns to the total investment package in the district. A 10-15% allocation of the total investment budget would in fact have increased the prevailing R & E effort in that district several-fold, and specialists felt it would have been very likely to arrest the decline in cotton yields.

12.8.3 Explicit Project Choices in the Model[8]

Explicit project choices can be analyzed with a model when it incorporates both the benefits and costs of projects. An investment project is specified as a column vector of numbers that represent the costs and physical consequences of the project. The model translates these consequences into economic benefits via its simulation of producers' reactions to the increased resource endowments. But in this case the increase in the resource

endowments is endogenous, depending on which projects are in the optimal solution, and at what levels.

The programming model utilizes additional resources only as long as their marginal contribution to the objective function is at least as great as their marginal cost. In the optimal solution these two measures will be equalized. Given this characteristic, the model can be used as a tool for project appraisal. To show this, first the way of entering investment activities in the model is presented, and then the interpretation of the solution is discussed.

Suppose the following technical data are given for three irrigation investment projects in a certain region:

	I_1	I_2	I_3
Investment cost per hectare ($000)	5.52	6.10	7.49
Annual operating cost per hectare ($)	100	92	112
Maximal size of the project (000 has.)	34.4	15.7	28.3

Suppose also that the average annual operating costs per hectare for the existing irrigation facilities in the region are $80. Then the incremental operating costs for the three projects are $20, $12, and $32, respectively. It is these incremental costs which need to be entered as part of the investment activities, for the model already will be charged the base cost of $80 per hectare through the activities which currently supply irrigation water. The investment activities do not supply water directly but rather raise the upper bound on irrigation water or irrigable land. If the bound is expressed in terms of land, the mini-tableau for these investment activities is as shown in Table 12-8.

[Insert Table 12-8]

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In the tableau, the initial endowment of irrigable land is L_0, and the investment activities release that constraint. If all three projects were implemented fully, the amount of irrigable land available would be + 34.4 + 15.7 + 28.3. The activity ITOT, for total investment, registers the total cost of the selected projects, in thousand dollars. The real interest rate times that cost is charged to the objective function, as are the incremental operating costs. When an investment activity enters the optimal solution, it expands the resource endowment or makes other, specific physical improvements (such as permitting the use of more efficient technologies). Its cost is the exogeneous annualized cost of capital, or the real interest rate plus depreciation.

The model results refer to a representative year t in some period in the future. For that year, investments are undertaken up to the level at which the annual flow of net benefits equals the annualized investment cost. For the moment, the flows of benefits are assumed to be uniform over time; that assumption is relaxed later. The benefits are simply the increment in the objective function's value that occurs through use of the additional resource endowments provided by the investment. That increment is created via production, sales, and other activities, and hence it cannot be computed before knowing the model solution.

The nature of the interest rate in the model can be explored by assuming that the time stream of benefits associated with the project could be simulated by a sequence of identical solutions of the model - one for each year subseqequent to completion of the project - and by doing some ex post discounting. Each solution is subscripted t, and the benefits of time t are defined as

the increment in the objective function value, with respect to the value it would have taken in the absence of the investment project. The following demonstration shows that the interest rate in the model is equivalent to an internal rate of return.

In the optimal solution projects are chosen so that, for the last unit of investment

$$B_t = rI, \tag{12.7}$$

where B_t is the net benefit at time t associated with the marginal amount of investment in the solution, r is the real interest rate, and I is the marginal amount of investment in dollars.[9/]

For the total amount of investment, the relation is

$$B_t \geq rI^* \tag{12.8}$$

where $I^* = ITOT$ in Table 12-8.

The present value of the stream of benefits in the model associated with I^* is

$$B_p = \sum_{t=1}^{T} \frac{B_t}{(1+r)^t} \geq rI^* \sum_{t=1} \frac{1}{(1+r)^t} \tag{12.9}$$

and as $T \to \infty$, B_p approaches its limiting value:

$$\lim_{T \to \infty} B_p \geq rI^* \frac{1}{r} = I^* \tag{12.10}$$

In the limit, the present value of the benefits is greater than or qual to the (present value of) the investment, which is the defining characteristic of the internal rate of return, (IRR). Thus the model chooses the amount I, and its composition, so that every component project has an IRR at least as great as r. If no projects have an IRR as great as r, then there will be no investment in the optimal solution.

The normal procedure for working with an investment model of this sort is to vary r over different solutions and record the amounts of investment, by project, in the solution. This procedure will show the minimum IRR for different packages of investment and will trace out a marginal efficiency-of-investment (MEI) schedule.

When the benefit streams are non-uniform, a conversion factor may be applied in the model to account for the non-uniformities. The most likely case is that of a project start-up period, when it may take several years

before the benefits attain their level at full development. For that case, the conversion factor is of the form

$$k = \frac{\bar{B}}{B} \tag{12.11}$$

where $\bar{B}$ is the average benefit over the life of the project, and B is the annual benefit at full development. Now since the benefits are endogenous in the model and derive from diverse activities, the factor k cannot be applied directly to the benefits. But it can be applied in an equivalent way to the investment costs, which are exogenous parameters in the model.

Let us re-write the fundamental equality (12.7)

$$kB = rI \tag{12.12}$$

valently,

$$B = r\left(\frac{I}{k}\right) = r\hat{I} \tag{12.13}$$

so that I/k may be entered into the model instead of the original investment cost I. That is, once k is known, the figures 5.52, 6.10, and 7.49 in Table 12-8 would be divided by k before being entered into the tableau. (If k differed by project, each figure would be divided by its own k.)

The starting point for the calculation of k for a given project is to decompose the benefit stream into its components for the start-up period and for the period of full development:

$$\bar{B} \sum_{t=1}^{p} \frac{1}{(1+r)^t} = \sum_{t=1}^{g} B_t \frac{1}{(1+r)^t} + B \sum_{t=g+1}^{p} \frac{1}{(1+r)^t} \tag{12.14}$$

where p is the number of years in the project's lifetime and g is the number of years in the start-up period.

To substitute (12.11) into (12.14), it is easier to normalize the benefits so that B = 1, yielding

$$k = \frac{\sum_{t=1}^{g} B_t^f \frac{1}{(1+r)^t} + \sum_{t=g+1}^{p} \frac{1}{(1+r)^t}}{\sum_{t=1}^{p} \frac{1}{(1+r)^t}} \qquad (12.15)$$

where B_t^f represents the year t benefits expressed as a fraction of the full-development benefits B. The parameter B_t^f usually is known by project engineers, along with g and p, and thus (12.15) can be calculated for a given level of r.

The application of (12.15) is illustrated below for a project with a 40-year lifetime, a seven-year start-up period, and with benefits beginning at 50% of their full-development level. The calculation of k is made twice, once for r = .04 and once for r = .06. The time stream of benefits is assumed to be as follows:

year:	1	2	3	4	5	6	7	8	... 40
B_t^f :	.5	.5714	.6428	.7142	.7856	.8570	.9284	1.0	... 1.0

Then at r = .04,

$$k = \frac{4.2195 + 13.7907}{19.7928} = 0.9099$$

and at r = .06,

$$k = \frac{3.8943 + 9.4639}{15.0463} = 0.8878$$

For the case of r = .04, in Table 12-8 the entries 5.52, 6.10, and 7.49 all are divided by 0.9099.

When investment activities are entered explicitly into the model, the IRR resulting from the analysis is a financial rate of return, not an economic ate of return. Inputs such as labor and foreign exchange (imported inputs)

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are not "shadow priced" in the primal, but rather are costed at prevailing market prices, or true opportunity costs in the case of family labor. This is in keeping with the descriptive nature of the model. If it is felt that, say, the exchange rate is out of equilibrium, then in order for the analysis to yield an economic rate of return the exchange rate would have to be shadow priced at the estimated equilibrium rate, and the prices of imports and exports would have to be revised accordingly.

The programming model could be used to simulate what would happen if the exchange rate were moved to an equilibrium level. Similarly, it could be used to simulate different wage levels. But it must be borne in mind that these kinds of solutions are in fact simulations of hypothetical policy changes.

The Egyptian model HĀPY was modified in the ways described in this section to include investment activities like those shown in Table 12-8. Some 69 investment activities were included for possible projects involving expansion of the land irrigable with the Nile waters (via installation of canals, drains, and other infrastructure). In the aggregate, these investment projects were sufficient to increase total production significantly and to alter relative prices. Hence the ranking of projects which resulted from the model, in terms of their IRRs, was different than the ranking which had been obtained with traditional project-by-project evaluations.

Another example of a model with endogeneous investment activities is found in the farm-level study of Husain and Inman (1977). The area under study was a proposed cooperatively-managed farm of some 1600 hectares in a World Bank project area in the Middle East. The model contains six alternative crops, alternative planting dates for each crop, inputs specified in 24 bi-monthly

periods, and 28 alternative types of agricultural machinery which could be purchased and utilized. Product prices were fixed since the study was carried out at the farm level. The specification of machinery purchase options was as in chapter 11, but it was more complex owing to the complementarities between some pieces of machinery. The model, called SARF, also includes crop-specific irrigation response functions.

The main purpose of the model was to analyze the effects of different interest rates on the composition of machinery purchases. But it was also used as a normative model to explore the consequences of the farm's adoption of different objective functions. The alternative objectives explored were a) maximization of profits, and b) maximization of wages plus profits. These two alternatives were designed to express the trade-offs between incomes of shareholders in the cooperatives and non-shareholders. The latter supplied labor but received no share of the profits. The model quantified the extent to which low interest rates, and hence large purchases of machinery, benefitted shareholders at the expense of non-shareholders, via displacement of labor. In terms of project design, the SARF model indicated appropriate combinations of machinery purchases, and associated cropping patterns, under different interest rates and wage rates.

In their conclusions, Husain and Inman pointed out that models like SARF can be used for farm budget analysis, cropping pattern planning, the planning of machinery stock acquisition and usage, and planning the timing and amounts of irrigation releases. They made some interesting general observations:

"The underlying premise of this model is that project design could be improved and project appraisal facilitated if analysts could explore a larger set of alternatives than they are presently able to do (by manual

procedures). This premise has two parts: (i) the specification of technical and behavioral relationships, which is the art and science of the analyst and (ii) given a specified set of relationships, the evaluation of the effect of alternative policy instruments, which is essentially a question of technology (existence of algorithms like linear programming and easy access to computers). The first part of The premise is the more difficult; it could be aided by theory as well as feedback from selective empirical adventures but it requires sensitive judgments from analysts. The second part is simpler ... At this stage, however, [lack of modelling skills] is the binding constraint on improving project designs/appraisals" (Husain and Inman, 1977, p. 36).

12.8.4 A Normative Investment Model

The project rankings calculated in the manner of the HĀPY and Mexican regional models reflect only one policy objective: to maximize the efficiency of the investments in generating financial returns to farmers, as expressed in the IRR criterion. Farm-level studies can explore different objective functions, but at the sector-level incorporation of other policy objectives in the analysis has to be done in a somewhat roundabout way. Different combinations of investment projects can be defined a priori and then each combination can be put in the model and its corresponding solution obtained. By comparing the solutions, it then is possible to compare the different combinations in terms of their effects on farm income, employment, foreign exchange, and other goal variables. The final decision could be made on the basis of these comparisons, if appropriate.

A more direct procedure would involve exploration of the Euclidean space of policy goals and investment projects. Such an exploration could be conducted with a model whose detached coefficients tableau looked like the tableau in

Table 12-9. The tableau shows each project's effects on the goals via the coefficients e_1 through x_n, and the goal variables E through X represent the effect on each goal of the total investment package. Each project is limited in the extent to which it may be implemented, and the total package is bounded by the amount of available investment financing, FL. The objective function weights z_1 through z_5 are measures of the hypothetical policy preferences with respect to the goals, and one application of the model would consist of varying those weights over different solutions to see the consequences for the investment program. The question is how to generate the model in the tableau.

[Insert Table 12-9]

If a descriptive programming model is available, then a two-stage procedure can be devised. In the first stage, the descriptive model is solved n times, once with each project included in the model as per Table 12-8 above. This procedure yields n simulations of the effects of the projects, though there will be fewer than n if some of the solutions result in the project not being undertaken. Assume that m of the solutions contain an investment at a positive level. The information from those m solutions is then arranged in the form of the column vectors in the left part of the matrix in Table 12-9, i.e., the column vectors I_j. The right-hand side information is added, plus the identity matrices, and the normative model is ready for use except for the z_j coefficients, which are discussed later.

Thus far, the main drawback to this procedure is the implicit assumption of fixed prices, if the investment program is large relative to the sector's existing resource endowment. To relax this assumption, the first stage solutions can be conducted somewhat differently. Instead of putting one investment activity in the model for each solution, combinations of investment activities can be

used. Each alternative combination should fully utilize the available financing. The combinations should represent a sufficient expansion of the resource base that product prices are affected. The creation of the tableau in Table 12-9 then proceeds as before, except that now each I_j represents a combination of projects. The right-hand side values B_j of the bounds on projects can be changed to be 1.0 in each case, i.e., each combination can be funded fully or at any lower value.

A procedure similar to this was followed in the case of an investment programming model for Peru (Norton, Santaniello, and Echevarría, 1983). That study also addressed issues related to funding an aggregate, multi-year investment program before all the individual projects of the program were known. A decision was made to identify representative projects, past or present, for each line of investment, and to conduct the analysis of choices in terms of the amount of funding for each line. Information on the representative projects was used as a basis for calculating the benefits of each line of investment, and so with this direct approach it was possible to skip the first stage of solving the descriptive model and go directly to the normative model shown in Table 12-9. In this case, all of the principal agricultural products were traded goods, so the working assumption was made that the investment program would not be likely to affect product prices.

The selected investments were to be made in the first period and the benefits would accrue over a horizon as long as forty years. Some summation and discounting equations were added to create variables representing the discounted forty-year stream of benefits. Five categories of benefits were included: sector income, income in the poorer regions, production of basic foods, employment creation, and foreign exchange earnings.

In highly simplified form, the equations of the model (called TICLIO) were as follows:

$$\text{Max } \hat{Z} = \sum_i w_i Z_i \tag{12.16}$$

Subject to

$$Z_i = \sum_j \sum_t \left[\frac{z_{ijt}}{(1+d)^t}\right] \sum_p I_{pj} \tag{12.17}$$

$$\sum_p \sum_j c_{pj} I_{pj} \leq F \tag{12.18}$$

$$\sum_{j \varepsilon J_m} \sum_p I_{pj} \leq R_m \tag{12.19}$$

where the variables are

$\hat{Z}$ policy objective function

Z_i policy goal variables

I_{pj} investment activities by project p and line j

and the parameters are

z_{ijt} benefit in terms of goal i arising from investment activity j at time t

c_{pj} investment cost coefficients

F aggregate financial limit on the investment program

R_m other bounds on components of the investment program

In basic structure the model was simple, but a number of interesting details were incorporated via equations (12.19). One of the equations (12.19) was an administrative capacity limit for irrigation investments; i.e., an expression of the fact that public agencies have limits on their capacity to implement projects. Another set of equations in (12.19) referred to physical limits

in each region on the volume of projects; i.e., the number of hectares susceptible to irrigation, reforestation and so forth. Yet another set constituted "representativeness" constraints on the projects which exemplified each line of investment. For each line of investment there were several illustrative projects, and if a project was typical of, say, only 25% of the investments in its line, then the representativeness constraints prevented that project from representing in the solution more than 25% (or a somewhat higher share) of the investment in that line.

The model was applied in a variety of ways. First, it was found that the proportionate increase in all goal variables was higher for a relaxation of the administrative capacity constraint than for a relaxation of the financing constraint. In other words, the model indicated the program would have benefited more from improvements in the staff's implementation capacity than from more project funding, and in fact after the program was approved implementation capacity turned out to be very much the limiting factor.

A set of solutions was designed to explore the consequences for composition of the investments of different policy goals. Table 12-10 shows the solutions for each case in which one $w_i = 1.0$ and the others are equal to zero. Through a series of additional solutions and discussions with officials of the government and the funding agency (The Interamerican Development Bank), it was determined that the preferred program had policy weights (w_i) of 0.75 for sector income and 0.25 for employment creation. This set of weights was then adopted for officially measuring the progress of the program as individual projects were implemented.

[Insert Table 12-10]

Another set of experiments was addressed to tracing out the trade-offs,

or frontiers, among pairs of policy goals. Figure 12-7 shows the frontier for sector income vs. output of basic foods, and the associated composition of the investment program for each node on the frontier. The composition is expressed in terms of shares of each of the four major lines of investment: irrigation, reforestation, research and extension, and marketing. In general, these frontiers revealed that the trade-offs (slopes) were quite extreme as the axes were approached. The implication of this finding that the preferred program was not likely to lie along an axis, that is, it would not correspond to giving one policy goal a weight of 1.0 and the others zero weights.

[Insert Figure 12-7]

Finally, the study pointed out that historical data can provide bounds on the relative values of the policy weights, as starting points in a dialogue about their values. For two policy goals, let their increments over the past, say, ten years be $\hat{w}_1$ and $\hat{w}_2$. Suppose that policy makers are unhappy with past trends in that they want to place more emphasis on increasing goal 2 (perhaps employment). Then for the weights in the policy model it can be inferred that

$$\frac{w_2}{w_1} > \frac{\hat{w}_2}{\hat{w}_1} \tag{12.20}$$

12.9. OTHER MULTI-LEVEL PROCEDURES

Multi-level procedures involve more than one model, and the models are linked in the sense that the solution to one model provides inputs into another model. Although in principle these procedures can involve many models, almost all practical experiences are limited to two models. Multi-level procedures are required under the following kinds of circumstances:

(a) One large model that includes all the relevant considerations in a single set of equations may be too large to solve in a practicable way.

therefore it has to be broken into component models which are solved individually in an iterative fashion, with solutions from each model providing information for revising coefficients in another model. The outcome of such an iterative process (eventually) provides a joint solution of the set of models which is equivalent to a solution of the original large model. Dantzig and Wolfe (1963) devised the first procedure like this, which was called decomposition.

(b) The objective functions that are appropriate to different levels of decision models may differ, and therefore no single model can encompass them. This is the multi-level programming problem discussed in chapter 7.

(c) There may be an informational problem in the sense that one model cannot be specified completely without information which is obtained from solutions of another model.

Decomposition procedures are not used much in practice, because today's computing power usually is sufficient to solve even very large models, and direct solution is more rapid than solutions via decomposition. However, the literature on decomposition techniques has provided added insights into the structure of optimization models and also has helped to further understand the nature of actual multi-level decision processes in an institutional sense. Some of the seminal works in this area are by Malinvaud (1969), Kornai (1967, 1969), and Heal (1973).

The information problem, which also has institutional analogues, was explored in Kornai (1969), Duloy and Norton (1973b), and Manne (1973b). For the sake of illustration, let us suppose that the two levels are the economy-wide (macro) level and the sector level. They sometimes are called, the upper and lower levels, respectively, the center and the periphery, or the inner and outer level problems.

To model the choices in the entire economy, it is necessary to know the production functions -- or activity analysis vectors -- at the level of each sector. In many circumstances it is not possible to obtain that information without effectively constructing a sector model. But once a sector model is constructed, it may not be efficient to combine it with a macro model -- because of model size and differences in objective functions, and also because the sector model contains a lot of information that is irrelevant to macro considerations. A macro model builder does not want to be bothered with monthly land and labor constraints in each of many different localities, or with the consequences for crop rotations of planting corn in March or May.

In these circumstances, a workable two-level procedure involves solving the sector model several times under variations in macro parameters, e.g., wage rates, interest rates, and the exchange rate. Each solution is then summarized in aggregate form as a vector of numbers representing total sector production, employment, exports and imports, use of capital, use of chemical inputs, energy, and other industrial inputs, and so forth. A set of vectors like this describes the sector's feasible space from a macro viewpoint -- i.e. the alternative vectors or vectors that are achievable under reasonable variations in macro parameters.

This description of the feasible space then is inserted into the macro model as a set of column vectors of the input-output type. Clearly, this is possible only if the "macro" model is of the activity analysis type, as in Manne (1973a). This procedure eliminates the need to iterate with the sector model as in a decomposition approach, but note that multiple solutions of the sector model are required in order to specify the macro model.

In most cases of policy planning in market economies, macro and sectoral policy deliberations are not closely enough linked to benefit from such

procedures. But at the sector level itself there is a two level problem, as discussed in Chapter 7 and briefly above in connection with the Peruvian investment model. The two levels are 1) the decentralized level of producer and consumer decisions, which is the topic of most of this book, and 2) the policy choice level, at which decisions are made on policy instruments.

In general, this two-level procedure can be modelled in much the same way as the multi-level informational problem. The descriptive sector model can be solved several times under different values of the policy instruments, and then each of those solutions can be summarized in the form of a vector of target variables (employment, export earnings, etc.) and instruments (price support levels, input prices, investment amounts by type, etc.) and instruments (price support levels, input prices, investment amounts by type, etc.). The policy model then looks somewhat like the tableau in Table 12-9. The budgetary consequence of each instrument can also be evaluated from the sector model's solutions, and entered into a budget constraint row in the policy model.

Proceeding in this way permits formal exploration of the policy options under different weights in a policy objective function. It also provides a tool for defining the "policy feasible space", - i.e. the set of values of the target variables which could be attained under acceptable variations in the value of policy instruments. The policy feasible space is necessarily convex, as shown in the illustration in Figure 12-8, which is taken from Ballenger (1984).

[Insert Figure 12-8]

Tracing out the policy feasible space also reveals the policy feasible frontier. Based on the few studies conducted along these lines so far, it is likely that the actual policy configuration will be found to line inside the frontier, i.e., with existing budgetary limitations, a new combination of policies can

be found that will increase at least one target variable without decreasing any others (Candler and Norton, 1977; Ballenger, 1984; Ballenger and Norton, 1985). Furthermore, each of the policies in such a combination would have been deemed acceptable beforehand, or else they would not have been included in the analysis to begin with.

The policy feasible space shown in Figure 12-8 was constructed from a policy model which in turn was derived from a truncated version of the CHAC model for Mexico. Each of the circled points on the solid frontier was obtained as a solution to the policy model under different weights in the policy objective function. The circled points below and to the left of the solid frontier correspond to individual solutions of the truncated CHAC model. The base point corresponds to the CHAC solution with no subsidies. That part of the policy feasible space which lies to the right of the vertical dashed line is the complementary subset of the space; it is the area in which either target variable may be increased without decreasing the other target variable. In this case, it can be seen that most of the points on the frontier imply trade-offs between the two policy goals. Conceptually, the main drawback of this procedure is that there is no guarantee that the true frontier (of the policy feasible space) actually has been found. However, in practice, with enough exploration, via solutions of the policy model under different objective function weights, the analyst can be reasonably confident that the frontier has in fact been found.

This kind of policy analysis is generalizable to many different types of government intervention in the agricultural sector. It would appear to be

among the more practicable of the various multi-level methods that have been proposed and tried.

12.10 COMPARATIVE STATICS AND DYNAMICS

12.10.1 Comparative Statics

As the previous sections in this chapter demonstrate, much of the applied analysis with programming models is carried out via simulation of alternative static equilibria. The simulations represent hypothetical outcomes toward which the sector would tend if only one parameter, or selected parameters, were changed. But there is no time period associated with them.

Comparative statics solutions can be conducted in a time dated fashion. The appropriate parameters of the model are adjusted to their presumed values for some year in the future, and then the model is solved. In interpreting the solution, the implicit assumption is that a new equilibrium is established by the designated year, that is, that markets will have adjusted. Experience in projecting sector growth rates with programming models suggests that this is not an unreasonable assumption.

Projections of growth prospects are in fact the most common kind of comparative statics analysis. Usually, several alternative projections are made in order to explore the possible effects of external factors, such as growth in export markets, and of domestic policies, such as investment in irrigation and other kinds of land improvement. As in the other analyses, much of the interest lies in examining the different effects of alternative policies.

A comparative statics analysis requires at least two solutions: a base solution and a projection solution, the latter referring to a particular year in the future. To set up the projection solution, the following kinds of

changes need to be made in the model's coefficients:

a) rotate the demand functions

b) change export and import bounds and prices, as appropriate

c) change the resource endowments in the right-hand side vector

d) change input prices and supply functions, as appropriate

e) change the technology coefficients for production and processing, particularly the crop and livestock yields in production.

It should be noted that projections are carried out in constant prices, so changes in the prices of exports, imports, and production inputs should reflect expected changes in real prices, i.e., prices deflated by a general price index. Clearly, these prices are an example in which considerable uncertainty may be associated with the projections of parameter values, and so it may be desirable to perform sensitivity analysis in the form of alternative projections.

Demand functions are rotated via changes in the right-hand side values of the convex combination constraints. Suppose that aggregate real income is expected to increase by 30% over the projection period, and that the income elasticity of demand for a product is 0.5. In that case, the domestic demand function needs to be rotated rightward by 15%. This rotation is accomplished by increasing the right-hand side of the convex combination constraint from a value of 1.0 to a value of 1.15.

Resource endowment changes can represent important allocations of public funds, as in the case of investment programs for irrigation and expanding the amount of non-irrigated cultivable land. Some attention has to be paid to endowments of agricultural labor, which are increasing in some countries and decreasing in others.

Part of the changes in technology over time can be captured by endogenous shifts along the production function in the static model. However, typically the entire production function will be shifting outward over time, and this effect can be best represented simply by increasing unit yields. The rate of increase selected normally is the historical rate, unless there are specific reasons for projecting a different rate. As with other parameters in the model, the technological parameters can be used to conduct interesting sensitivity tests. The consequences for incomes, employment, etc. can be traced out for different rates of yield growth.

12.10.2 Dynamics

The procedures for comparative statics analysis are the same at the farm level and the sector level, but in the case of dynamic analysis the procedures may differ. At the farm level, the dynamic linkages are of three kinds: 1) the farmer's income this year will affect his savings and therefore may affect his possibilities for investments; 2) investments affect the capacity for production in later years; and 3) there exist biological intertemporal linkages, such as the yield curve over time in tree crops and other perennials, and reproduction and growth rates for livestock. Also, a farmer may reasonably be expected to have a subjective discount rate, and his objective function will include, among other variables, some transformation of his discounted income stream. Hence it often is important to make farm models multi-period, as discussed in chapter 5, section 5.3.

In a sector model, not all these dynamic linkages are present. Savings are not endogenous to the model. They could be, but many investments in the sector require public funding, so there is no point in specifying a savings-

investment relationship in the model. Also, the objective function does not admit of discounted income streams, since it is a market-simulating function. Hence, in procedural terms, a multi-period analysis with a sector model usually is carried out as a sequence of comparative statics solutions.

In the event that some investments are endogenous, in the manner discussed in section 12.8.3 then ex post linkages can be made among the time sequence of static solutions. The endogenous level of investment in the period t model could be used to calculate the right-hand side parameters that represent resource endowments in the model for period t+1.

Other linkages of this type can be envisaged. For example, if a rural-urban migration function were available (outside the model), then the endogenous agricultural income in the solution for period t could be applied, via a side calculation, to determining the endowments of rural labor in the right-hand side of the model for period t+1. The extent to which these kinds of linkages are desirable or practicable depends on the situation being modelled. A special form of intertemporal linkage has been developed, called recursive programming, and it is reviewed below.

12.10.3 Recursive Programming

In its most general form, a recursive programming model can be defined as "a sequence of optimization problems in which one or more parameters or coefficients in any problem in the sequence are functionally dependent on the optimal variables of preceding members of the sequence" (Day 1963b). A simple example would be a sequence of comparative static solutions in each of which the resource constraints were updated on the basis of the investment decisions of the optimal solution for the previous period.

A more specific form of recursive programming was developed by Henderson (1959) and Day (1961, 1963b) in an attempt to incorporate behavioral dynamics into programming models. Even when a model is well specified in terms of incorporating farmers' price expectations, and their behavior towards risk, consumption, leisure, and investment decisions, it is often the case that time sequences of comparative static solutions only track history poorly in simulating actual adjustments from year to year in cropping patterns and resource use. Discrepancies may arise from incorrect model specification and from data errors. But they may also arise if key factors affecting farmers' decisions over time are not incorporated in the model. The latter may include lags in the dissemination of knowledge about new technologies or market conditions, changes in farmers' perceptions about the profitability and riskiness of new technologies, or simply farmers' reluctance to change from established cropping and technology practices.

These kinds of considerations are inherently difficult to measure and incorporate in a model in an explicit way, particularly in a dynamic setting in which markets and technologies, and hence perceived information,change frequently. Henderson (1959) and Day (1963b) argued that such information difficulties lead farmers to react cautiously in adjusting their cropping patterns from one period to the next, and they proposed modelling such cautious behavior through a set of flexibility constraints. These constraints are imposed on individual cropping activities, and take the form

$$(1-\beta_{j,\,min})\, X_{jt-1} \leq X_{jt} \leq (1+\beta_{j,\,max})\, X_{jt-1}. \qquad (12.21)$$

Here X_{jt} denotes the level of the jth cropping activity in the model solution for period t, and this level is constrained to lie within a range determined in part by the activity's level in the model solution for the

previous period. The coefficients $\beta_{j,\ min}$ and $\beta_{j,\ max}$ are, respectively, the minimum and maximum proportions by which farmers are assumed to be willing to deviate from the cropping level of the previous period.

Recursive programming models have to be initialized to some base period or year. For example, in period t=1, the values of $X_{jt-1} = X_{jo}$, all j, would need to be initialized. Thereafter, sequences of solutions are obtained, each solution of which relates to a specific period or year. Given a fixed set of model coefficients, a sequence of solutions will typically converge to an equilibrium solution in which none of the flexibility constraints will be binding. If the model is then perturbed by changing a coefficient, such as a commodity price, then a new sequence of solutions will follow. The flexibility constraints may initially restrict the speed with which the optimal cropping pattern can adjust to the price change, but eventually the sequence of solutions should onverge to a new equilibrium solution in which the flexibility constraints are no longer binding.

Since the flexibility constraints play a key role in controlling a model's solutions when in a disequilibrium state, then the estimated values of $\beta_{j,\ min}$ and $\beta_{j,\ max}$ are crucial to the model's ability to track or predict actual adjustment paths. Unfortunately, there are no really satisfactory ways of estimating the β coefficients.

In practice, the β coefficients are assumed to be constant over time, and they may simply be based on the largest upward and downward adjustments observed in the area of a crop over some historical period. Another approach is to sort pairs of adjacent observations from time series data on X_j into two groups - those in which $X_{jt} > X_{jt-1}$, and those in which $X_{jt} < X_{jt-1}$. (Note that one can form as many pairs as the original number of observations less one, since, for

example, the series X_{j1}, X_{j2}, and X_{j3} leads to the pairs X_{j1}, X_{j2} and X_{j2}, X_{j3}). Within each group a regression equation of the form $X_{jt} = \beta_j X_{jt-1}$ can then be run to provide estimates of β_j. The estimated β_j for the group satisfying $X_{jt} < X_{jt-1}$ is the estimate of $\beta_{j,\ max}$, and the estimated β_j for the remaining group is the estimate of $\beta_{j,\ min}$. Estimated values of the β coefficients are usually adjusted during model validation tests to enable the model to best track some historical period.

Day and Singh (1977) provided a particularly interesting application of recursive linear programming in which they attempt to track the impact of the green revolution on the farm economy of the Indian Punjab.

FOOTNOTES

1. Portions of this section are adapted from Kutcher and Norton (1982).
2. CHAC is named after the Mayan rain god.
3. HĀPY is an early Egyptian god of the harvest.
4. TASM is the acronym for Turkish Agricultural Sector Model.
5. Consumer and producer surplus are not accepted as true welfare measures, whereas the compensating and equivalent variations are, but Willig (1976) has shown that in many cases the surpluses can be very close approximations to the variations.
6. This model is presented in Ballenger (1984) and Ballenger and Norton (1985).
7. These calculations were made by Carlos Pomareda.
8. This section is adapted in part from Bassoco, Mutsaers, Norton and Silos (1983).
9. For simplicity, we ignore depreciation of the project's capital assets.

MATHEMATICAL PROGRAMMING FOR ECONOMIC ANALYSIS IN AGRICULTURE

Peter B. R. Hazell and Roger D. Norton

Chapter 12. METHODS OF POLICY ANALYSIS

Table 12-1. Mini-Tableau for Export Allocation over Farm Types

	X_1	X_2	M_2^e	M_2^d	M_1	
Production balance, farm type 1	$-y_1$				+1	≤ 0
Production balance, farm type 2		$-y_2$	+1	+1		≤ 0
Export balance			-1			≤ 0
Consumption balance				-1	-1	≤ 0

Table 12-2. Mexico: Changes in Sector Objectives per 10 Million Pesos Spent on Alternative Price Support Investments

Objective \ Instrument	Maize	Beans	Sorghum	Wheat	Rice	Barley	Soybeans	Sesame	Safflower
Producer profits	1.024	.123	.569	.048	.229	.233	.396	.378	.589
Sector income	.820	.156	.525	.054	.343	.224	.516	.420	.685
Irrigated area	.325	-.088	.290	.006	.074	.075	.247	.140	.411
Northwest	.084	-.098	.113	.006	.049	.037	.022	.070	.068
North	.044	.010	.059	.006	.025	0	.022	0	.205
Center	.193	-.010	.196	-.012	0	0	.202	0	.103
South	.004	0	0	0	0	0	0	0	0
Rainfed areas	.495	.234	.236	.048	.270	.112	.280	.280	.274
Northwest	.008	.010	.010	.006	.025	.037	.011	.070	.034
North	.048	.020	.039	0	0	0	0	0	.103
Center	.294	.068	.020	.012	.025	.037	-.045	.280	-.034
South	.143	.137	.162	.030	.196	.075	.291	0	.103
Employment	.172	.082	-.148	.011	.160	.032	.252	.110	.291

Source: Calculated from solutions of the model CHAC and reported in Sistema Alimentario Mexicano (1982).

Note: Units are in 10 millions of pesos, except for employment, which is expressed in thousands of man-yea
Sector income includes the income of hired labor.

T-' le 12-3. . Mexico: Changes in Sector Objectives per 10 Million Pesos Spent on Alternative Input Sub..dies

Objective \ Instrument	Subsidies at a Rate of 30% of Input Prices			
	Agro-Chemicals	Seeds	Surface Irrigation Water	Machinery
Producer	.550	.325	1.18	1.47
Sector income	.450	.316	1.16	.574
Irrigated areas	.243	.162	1.26	.115
Northwest	- .033	.044	.696	.013
North	.037	.007	.435	.040
Center	.224	.103	.130	.135
South	.007	.007	.043	- .072
Rainfed areas	.202	.154	- .01	.457
Northwest	0	.007	.043	.013
North	0	- .029	0	.031
Center	- .018	.096	- .174	.233
South	.210	.074	0	.175
Employment	.118	.022	- .043	- .269
Foreign exchange savings	.163	0	0	.761
Maize production	.875	- .007	.043	n.a.

Source: Calculated from solutions of the model CHAC and reported in Sistema Alimentario Mexicano (1982).

Note: Units are in 10 millions of pesos, except for employment, which is expressed in thousands of man-years, and maize production, which is in thousand tons.

12.4 Supply and Response and Cross-Price Effects Arising from Changes in in the Price of Corn, Costa Rica, 1976 a/

Corn		Rice		Sorghum		Beans	
Price	Quantity	Price	Quantity	Price	Quantity	Price	Quantity
0.080	62.69	0.074	107.90	0.051	14.51	0.026	11.67
0.093	75.82	0.085	107.00	0.058	14.50	0.0236	8.62
0.133	95.83	0.119	104.31	0.079	14.42	0.355	6.89
0.258	115.79	0.225	86.91	0.153	14.19	0.811	6.88

Note: Prices are in Central American dollars per kilo and quantities are in thousand tons. The prices are producer prices.

a/ The corn price was changed exogenously by rotating the domestic demand curve. All other values in the table are determined endogenously to the model.

Source: Calculated from solutions of the model MOCA.

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Table 12-5. Foreign Trade Effects of Different Levels of the Corn Price, for Costa Rica

Producer Price	Total Production	Net Production	Imports(-) Exports(+)	Domestic Consumption	Degree of National Self-Sufficiency
($CA/kg)		(thousand tons)			
.080	62.69	59.55	-41.65	101.2	58.8
.093	75.12	72.03	-29.17	101.2	71.2
.133	95.83	91.04	-10.16	101.2	90.0
.258	115.79	110.00	+ 8.8	101.2	108.7

Source: Calculated from solutions to the model MOCA.

Table 12-6.

Effects of the Corn Price on Selected Macroeconomic Variables, Costa Rica

Producer Price	Producers' Gross Margin	Cost of Imports	Total Cost of Supply	Average Cost of Supply	Consumer Expenditures	Gross Export Earnings	Implicit Subsidy(-)or Tax(+) on Consumers	Implicit Subsidy(-) on Exports	Fiscal Income(+) or Outlays(-)	Tariff Income	Net Effect on the Balance of Payments
.080	5.02	3.33	14.84	0.147	16.19	-	+ 1.35	-	1.35	1.35	-3.35
.093	7.00	2.33	15.86	0.157	16.19	-	+ 0.33	-	0.33	0.33	-2.33
.133	12.79	0.81	20.19	0.199	16.19	-	4.00	-	-4.00	0.18	-0.81
.258	29.91	-	37.11	0.337	16.19	0.71	-17.95	-2.26	-20.21	-	+0.71

Note: The price is in $CA/kg., and all other figures are in $CA million.

Source: Calculated from solutions to the model MOCA.

Table 12-7. Effects of Making More Irrigation Water Available in the Rio Colorado District

Item	Base-solution value	Percentage increase in water				
		10	15	20	25	30
Farmers' incomes (millions of pesos)	305.0	335.4	341.4	362.5	374.2	385.8
		(+10.0)	(+11.9)	(+18.9)	(+22.7)	(+26.5)
District income (millions of pesos)	388.0	425.4	434.7	458.5	471.6	484.7
		(+9.6)	(+12.0)	(+18.2)	(+21.5)	(+24.9)
Employment (thousands of man-years)	31.60	34.14	35.59	36.21	36.55	36.89
		(+8.0)	(+12.6)	(+14.6)	(+15.7)	(+16.7)
Shadow price of gravity-fed water (pesos per 10,000 cubic meters)	1,077	1,052	937	937	737	737
		(-2.3)	(-13.0)	(-13.0)	(-31.6)	(-31.6)

Note: Figures in parentheses denote percentage change over base solution value.

Source: The Book of CHAC, p. 473.

Table 12-8. Mini-Tableau for Irrigation Investment Activities

	Irrigation Activities					
Row	I1	I2	I3	ITOT		RHS
January irrigable land	-1.0	-1.0	-1.0		$\leq$	L_o
⋮	⋮	⋮	⋮		⋮	⋮
December irrigable land	-1.0	-1.0	-1.0		$\leq$	L_o
Investment cost	5.52	6.10	7.49	-1	$\leq$	0
Limit, I1	1.0				$\leq$	34.4
Limit, I2		1.0			$\leq$	15.7
Limit, I3			1.0		$\leq$	28.3
Objective function	-0.020	-0.012	-0.032	-r	Maximize	

Table 12-9. Tableau for a Normative Investment Model

	Irrigation Activities						Goal Variables						
	I1	I2	I3	. . .	In	ITOT	E	F	Y	YS	X		RHS
Employment(E)	e_1	e_2	e_3	. . .	e_n		-1					=	0
Foreign Exchange(F)	f_1	f_2	f_3	. . .	f_n			-1				=	0
Farm Income(Y)	y_1	y_2	y_3	. . .	y_n				-1			=	0
Income on Small Farms(YS)	ys_1	ys_2	ys_3	. . .	ys_n					-1		=	0
Food Production(X)	x_1	x_2	x_3	. . .	x_n						-1	=	0
Bound, I1	1											$\leq$	B_1
Bound, I2		1										$\leq$	B_2
Bound, I3			1									$\leq$	B_3
.				.									.
.					.								.
.						.							.
Bound, In					1							$\leq$	B_n
Total Investment	c_1	c_2	c_3			-1						=	0
Financial Limit						1						$\leq$	FL
Objective Function							z_1	z_2	z_3	z_4	z_5		Maximize

Table 12-10. Policy Trade-offs as Measured with the Investment Model for Peru

	Solution Number				
	1	2	3	4	5
Objective function	Sector income	Income in poor regions	Employment	Foreign exchange	Basic Food production
Increments in:					
Sector income	651.90	619.60	534.05	604.07	586.67
Income in poor regions	540.29	619.60	519.87	474.75	534.61
Employment	2656.15	2881.55	3044.16	2529.31	2678.11
Foreign exchange	486.72	399.02	298.10	586.70	455.65
Basic food production	449.49	453.21	249.38	383.47	518.37
Irrigation investment:	42.74	30.79	12.72	50.99*	50.99*
(coast)	(22.29)	(-)	(-)	(33.31)	(9.62)
(mountains)	(20.46)	(21.51)	(12.72)	(17.68)	(20.65)
(jungle)	(-)	(9.28)	(-)	(-)	(20.72)
Forestry investment:	26.12*	26.12*	26.12*	17.19	-
(coast)	(-)	(-)	(4.35)	(-)	(-)
(mountains)	(17.08)	(17.08)	(12.73)	(8.14)	(-)
(jungle)	(9.05*)	(9.05*)	(9.05*)	(9.05*)	(-)
R & E investment	29.41*	26.85	29.41*	29.41*	29.41*
(coast)	(2.56*)	(-)	(2.56*)	(2.56*)	(2.56*)
(mountains)	(13.68*)	(13.68*)	(13.68*)	(13.68*)	(13.68*)
(jungle)	(13.18*)	(13.18*)	(13.18*)	(13.18*)	(13.18*)
Marketing investment	-	17.00*	17.00*	-	17.00*

Notes: a) An asterisk denotes that the line of investment is at its financial upper limit.
b) R & E refers to research and extension.

Source: Norton, Santaniello, and Echevarría (1983, p. 167).

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Chapter 12. METHODS OF POLICY ANALYSIS

FIGURE CAPTIONS

12-1. Measuring the Marginal Cost of Production for an Exported Good

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Figure 12-1. Measuring the Marginal Cost of Production for an Exported Good

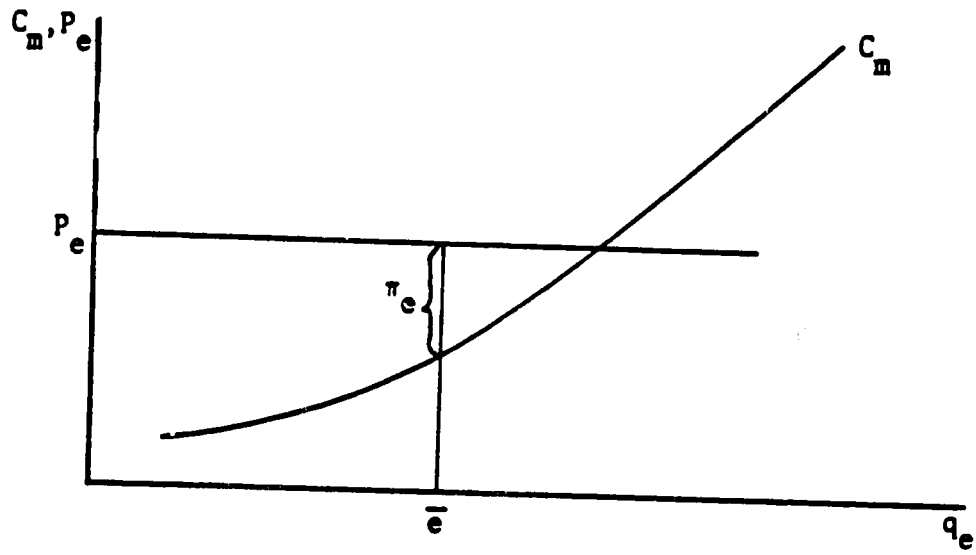

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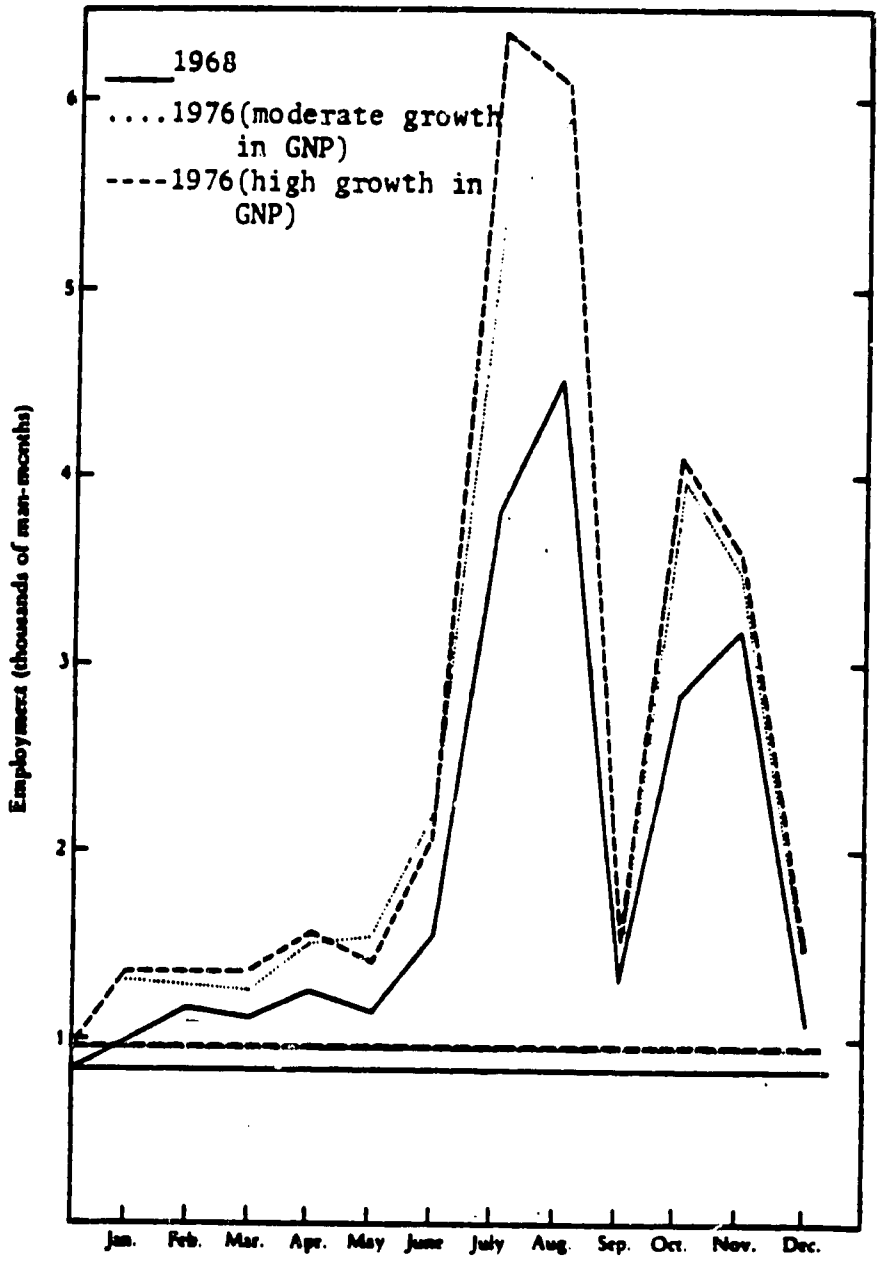
Figure 12.2. Seasonal Employment Projections in CHAC



Source: The Book of CHAC, p. 140. These solutions were made in 1973.

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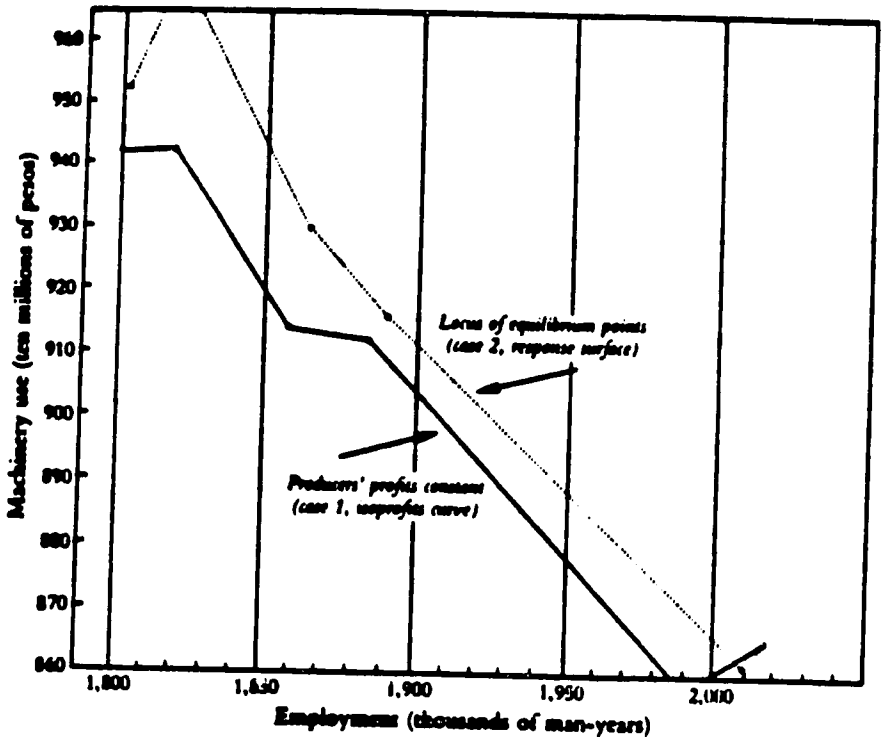
Figure 12-3. Sector Isoquants for Capital and Labor in CHAC



Source: The Book of CHAC, p. 132.

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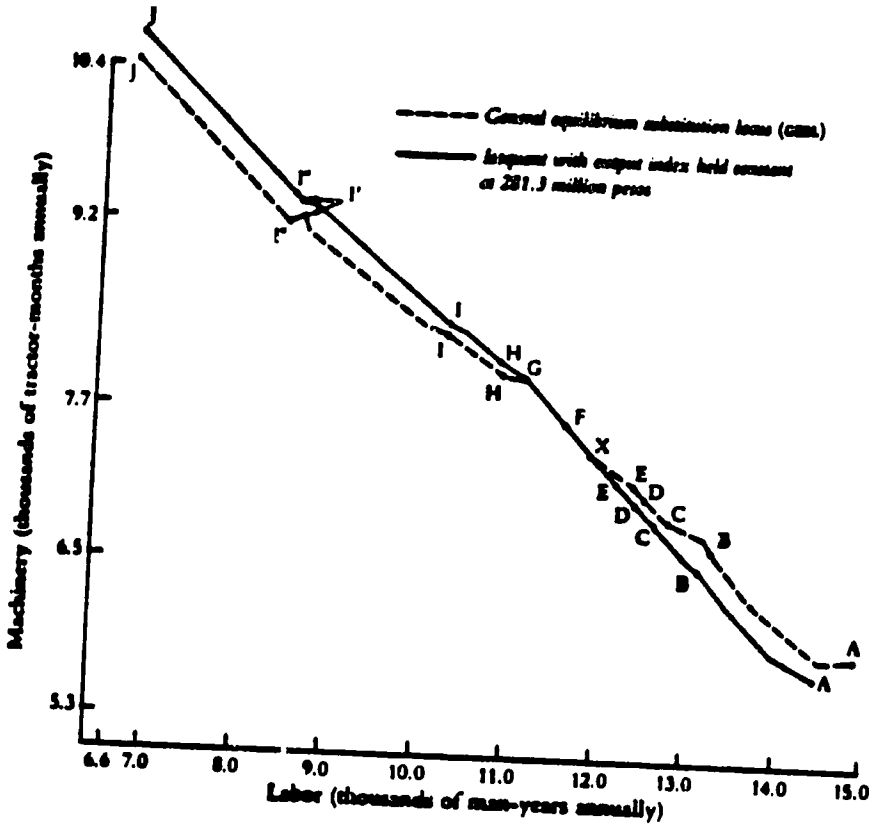
Figure 12-4. Machinery-Labor Substitution on All Farms as Induced by Variations in the Wage Rate



Source: The Book of CHAC, p. 397.

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Figure 12-5. Tracing Out a Supply Response Function

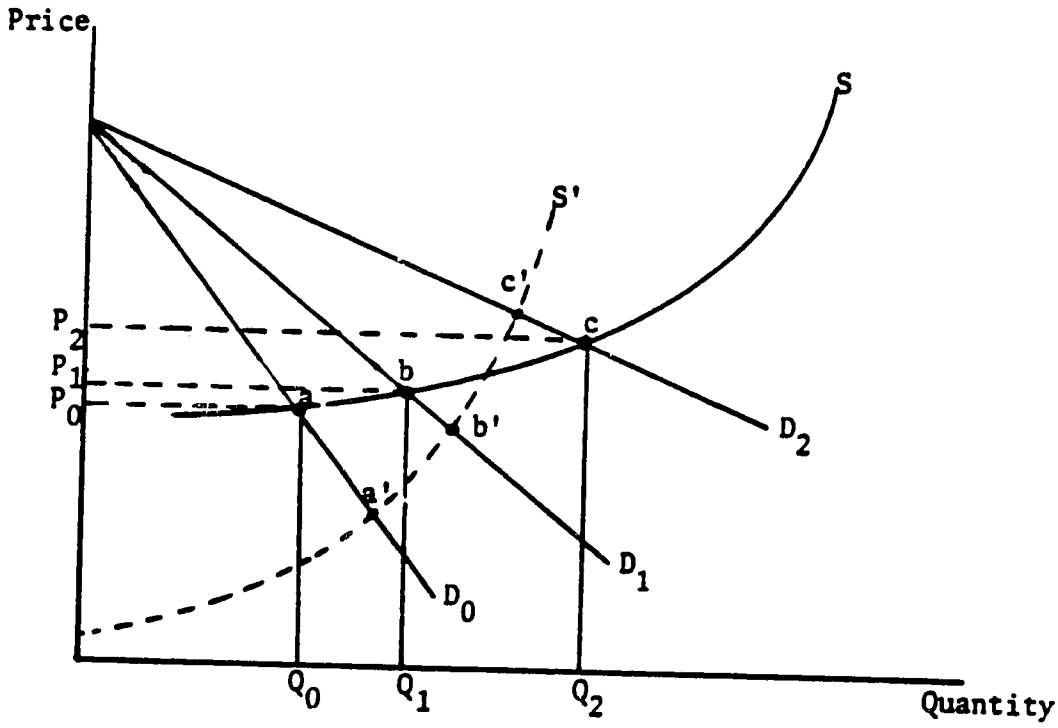

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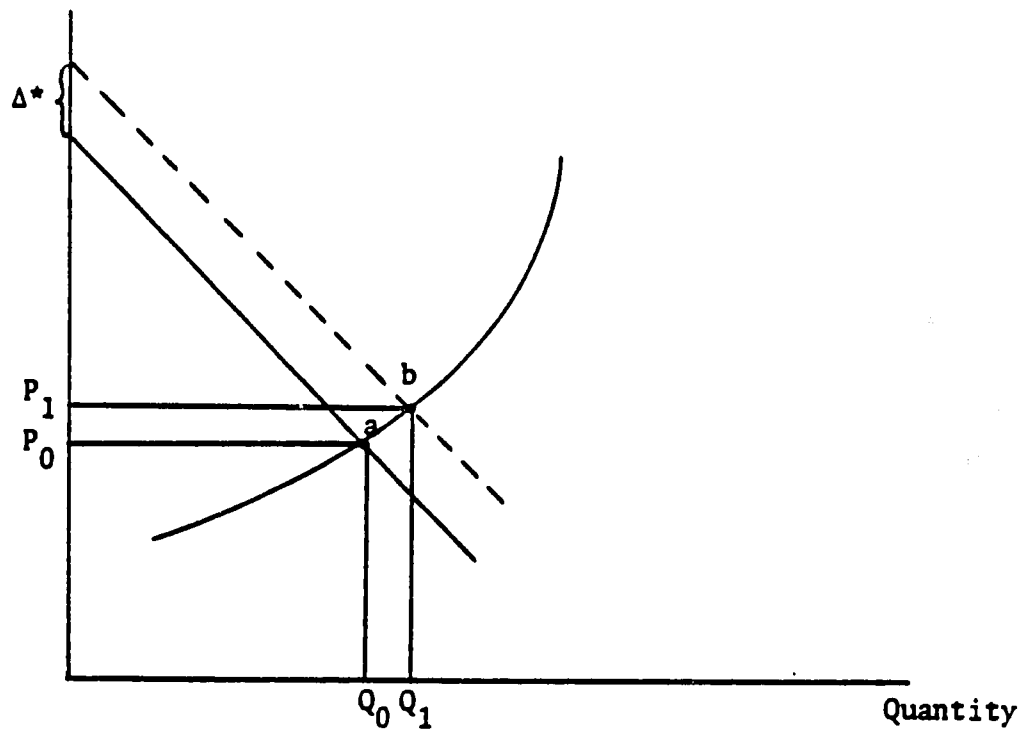
Figure 12-6. The Procedure for Finding a Regional Supply Function



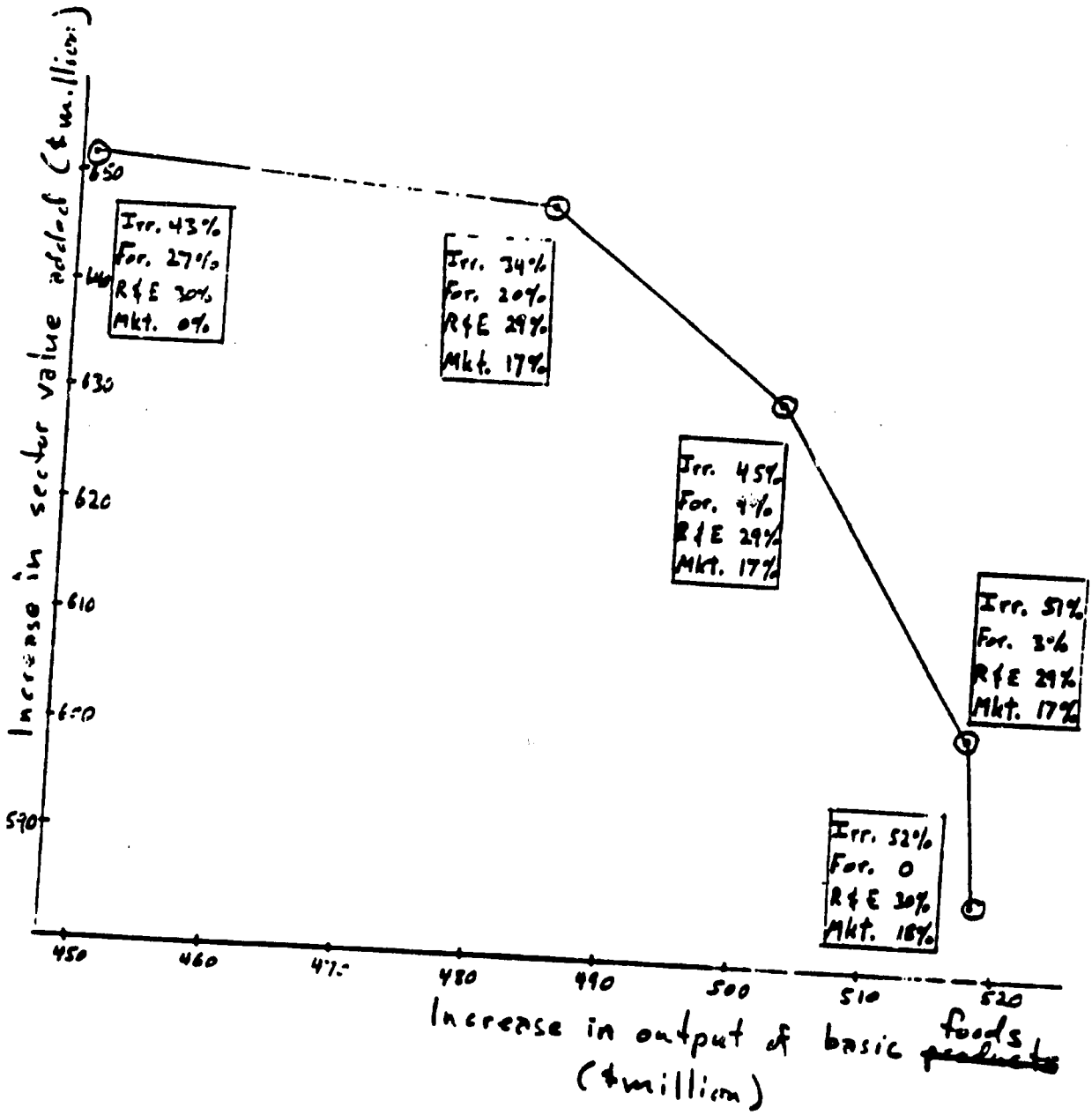


Figure 12-7. Production Choices between value added and output of basic foods

Source: Adapted from Norton, Santaniello, and Echeverría (1983, p. 170)

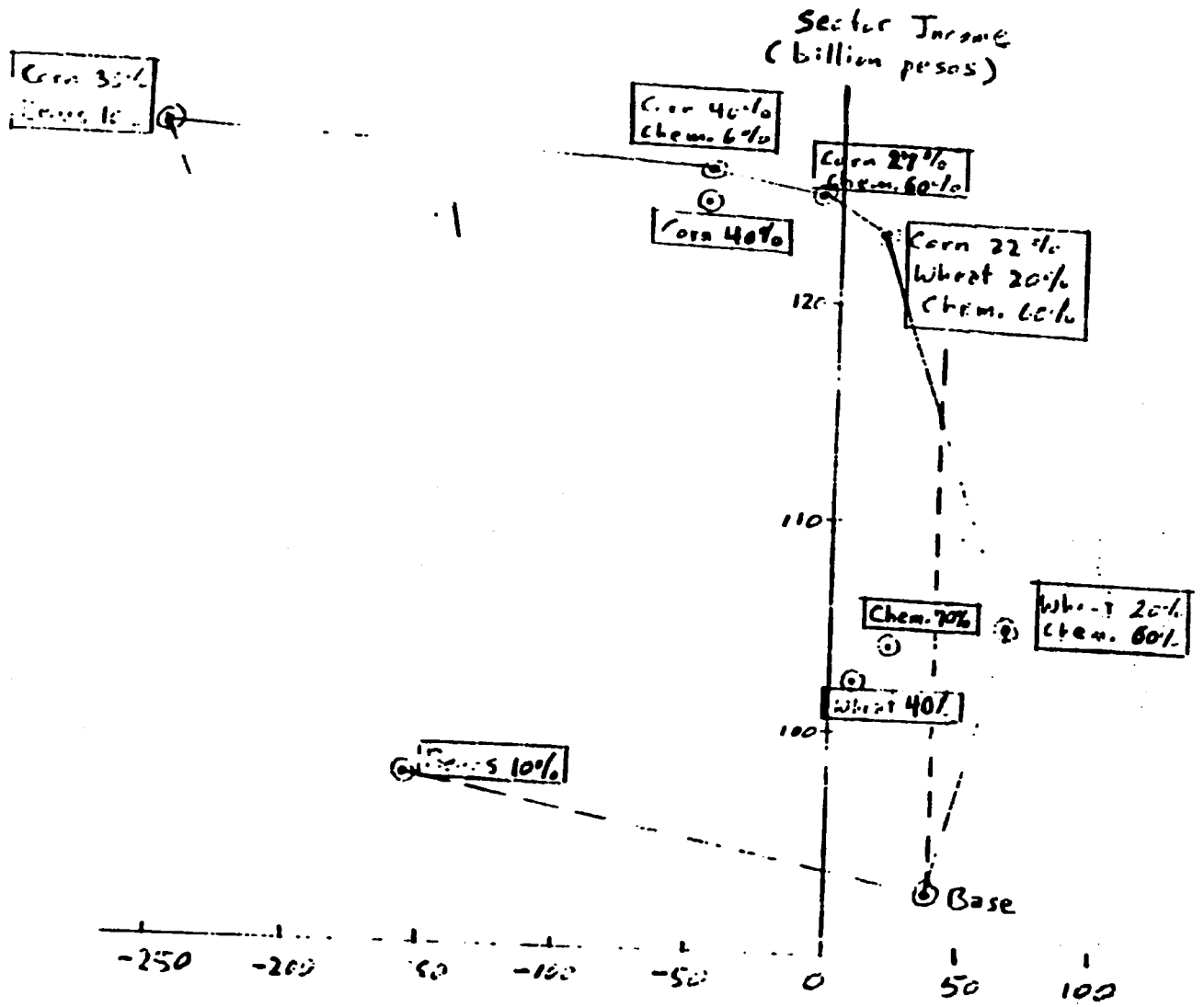


Figure 12-8. A P.L. Feasible Space

Note: The percent figures are subsidy rates by product

Source: Ballenger (1984, p. 135)

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